Quantum mechanics without quasiparticles:

applications to ultracold atoms and the high temperature superconductors

ETH Zurich
December 18, 2013

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Talk online: sachdev.physics.harvard.edu

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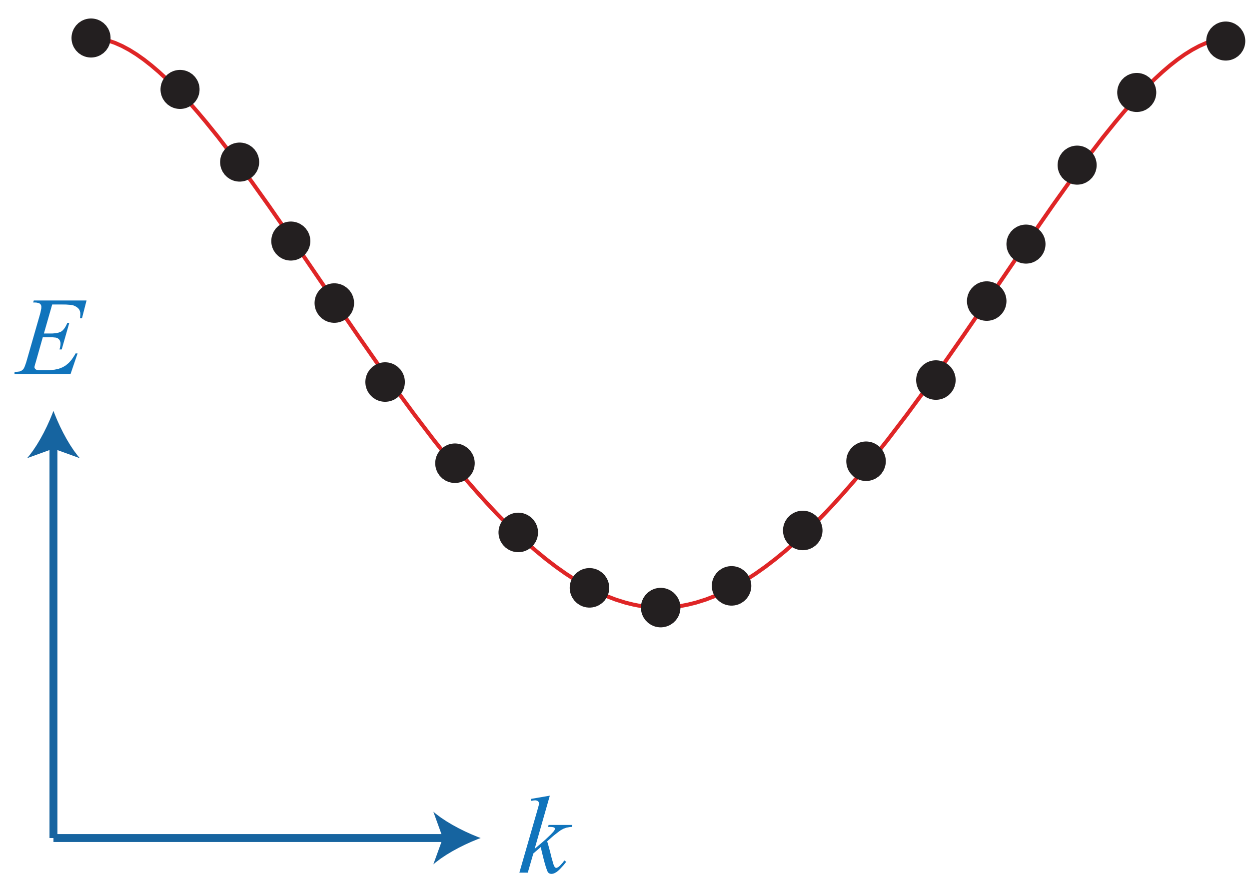
Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

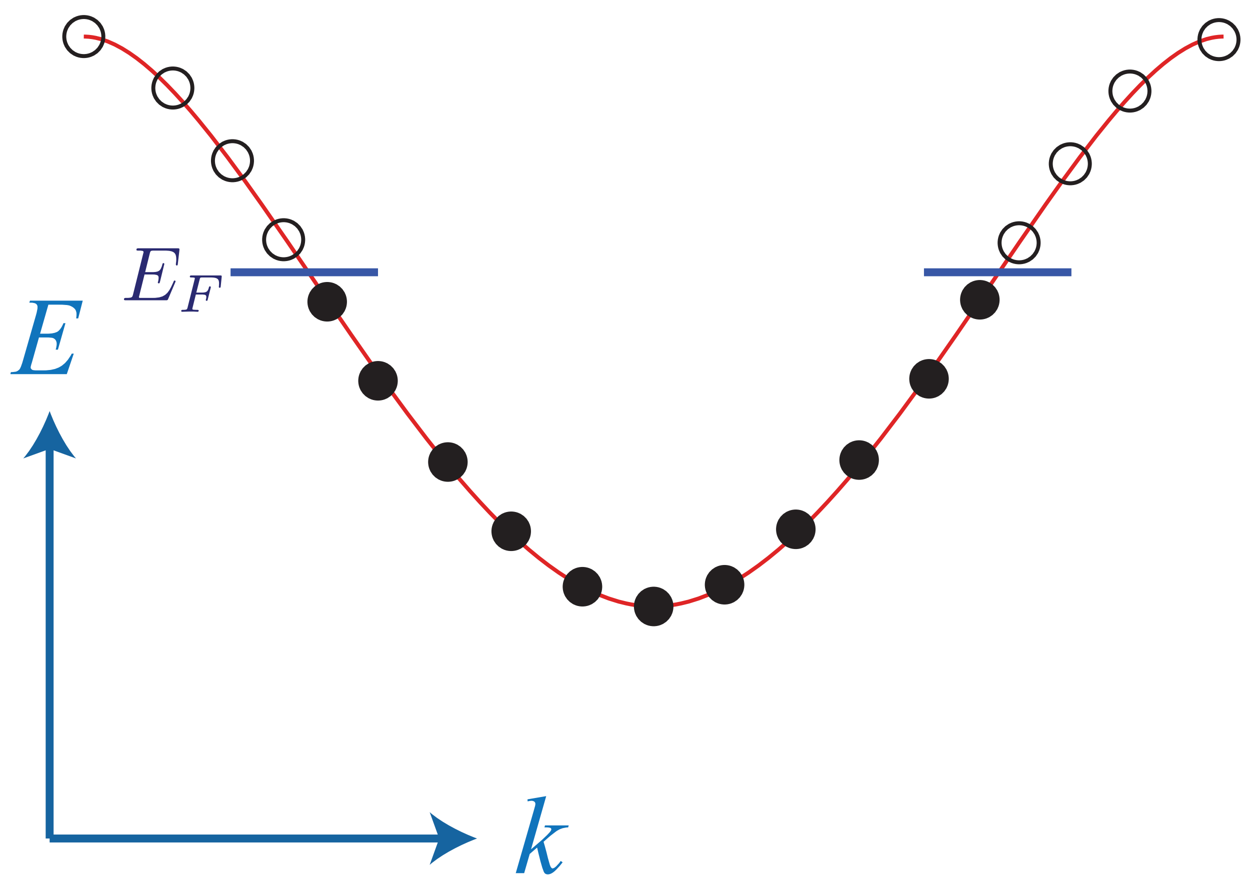
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

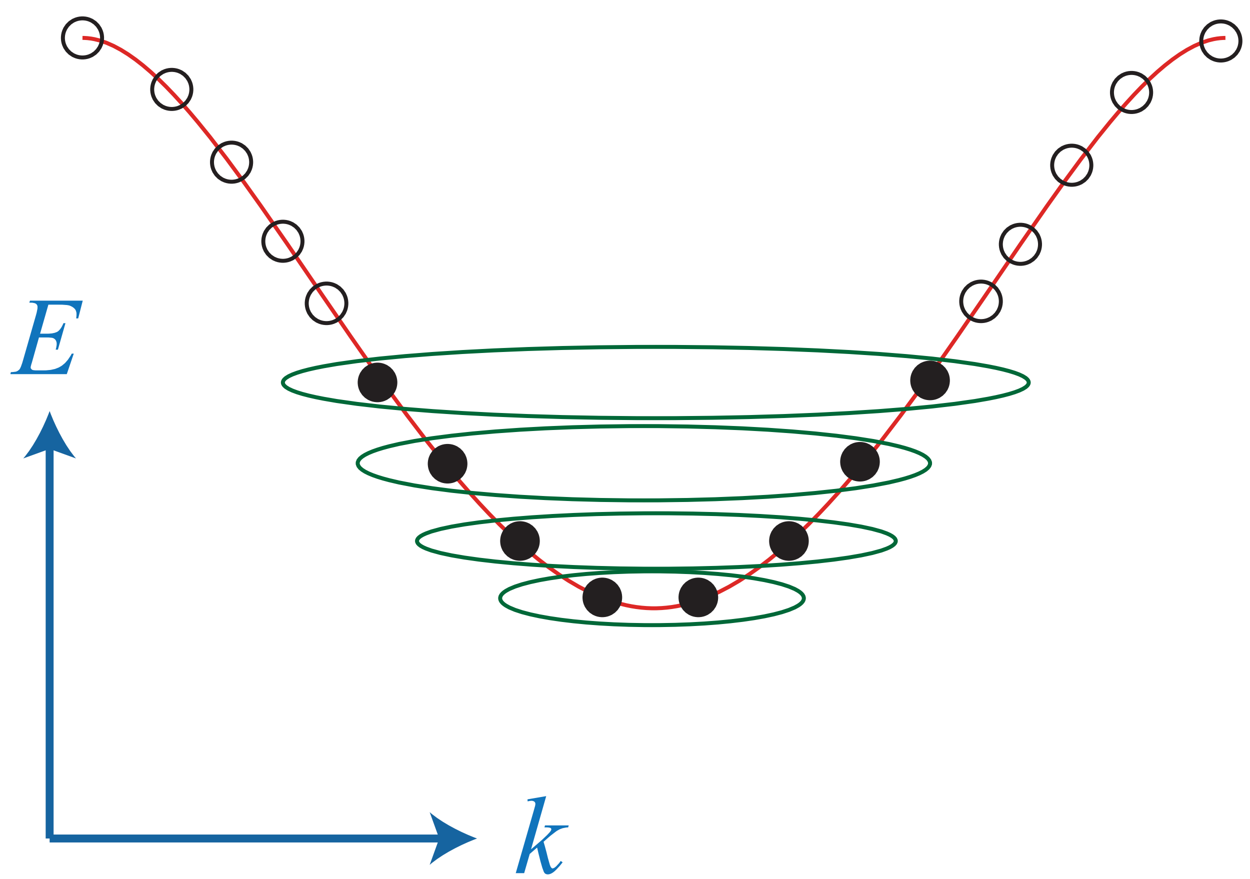
Metals



Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

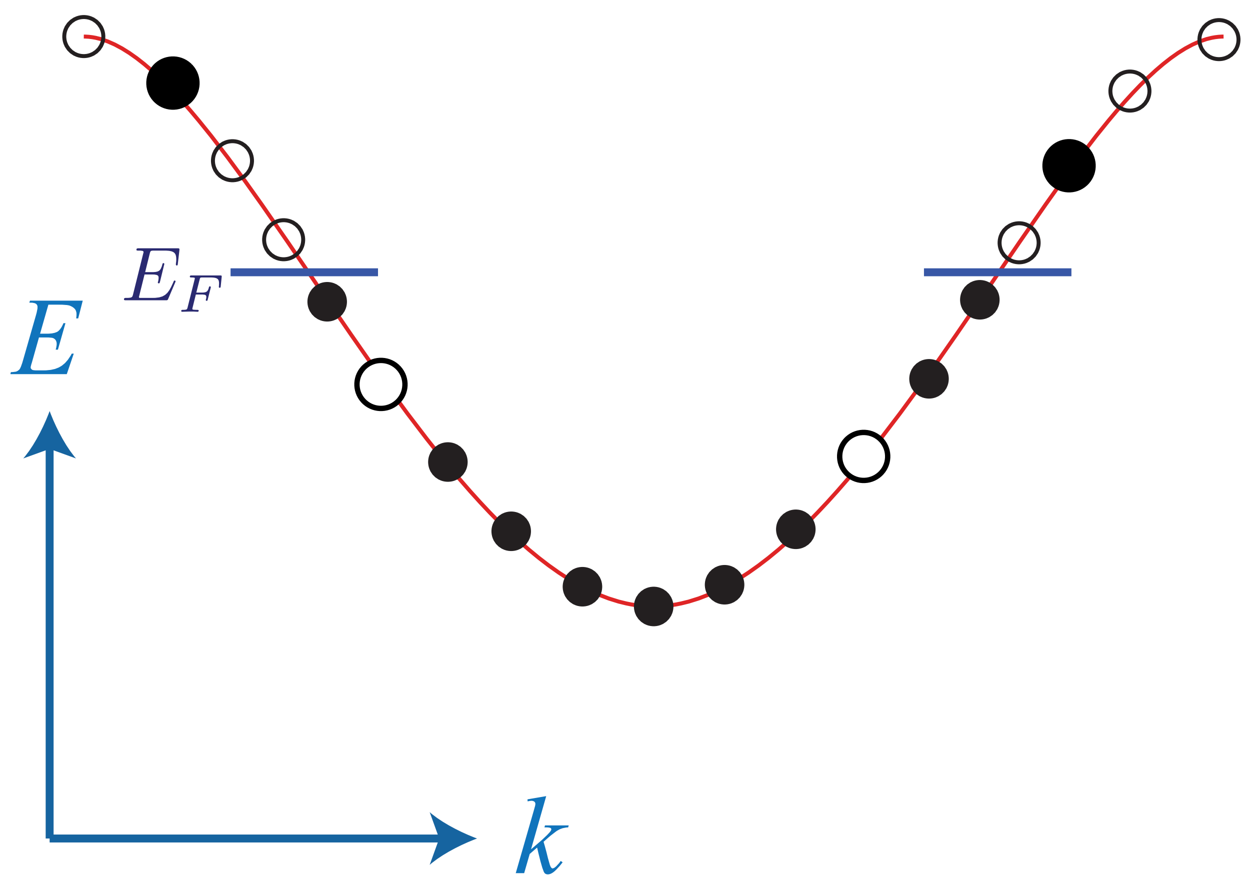
Superconductors



Boltzmann-Landau theory of dynamics of metals:

Long-lived ***quasiparticles*** (and ***quasiholes***) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement,

Modern phases of quantum matter

Not adiabatically connected to independent electron states:

many-particle quantum entanglement,

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle

quantum entanglement,

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

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Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

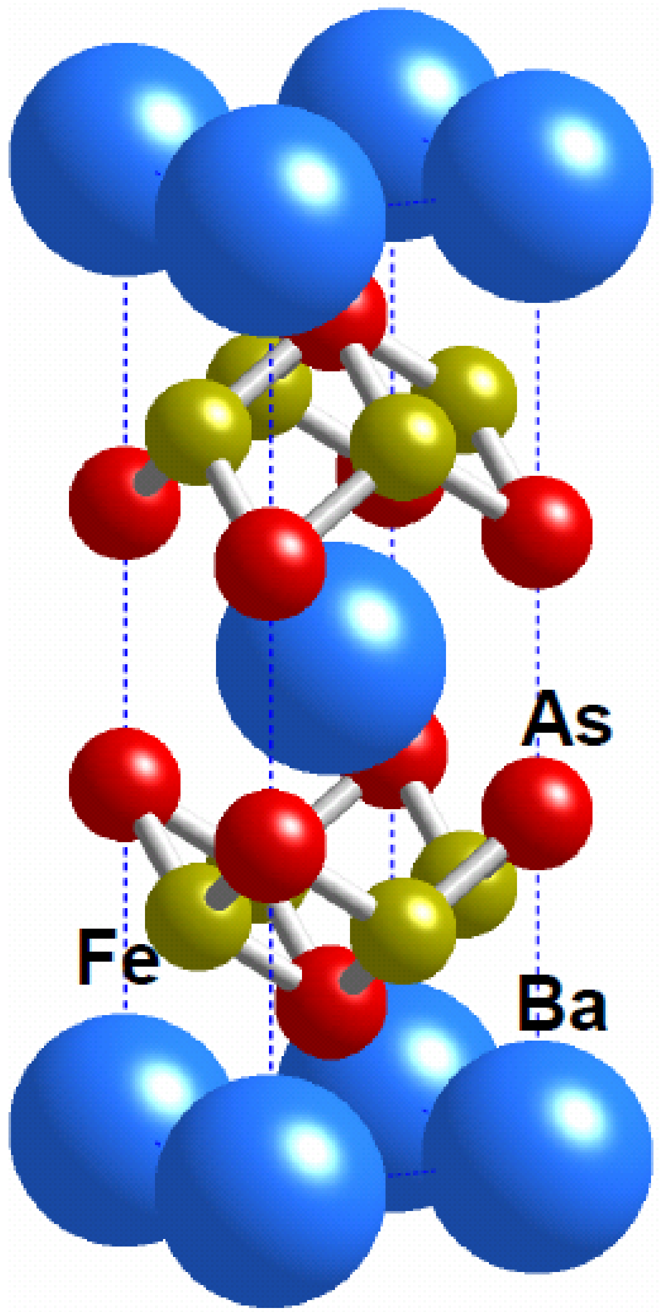
many-particle

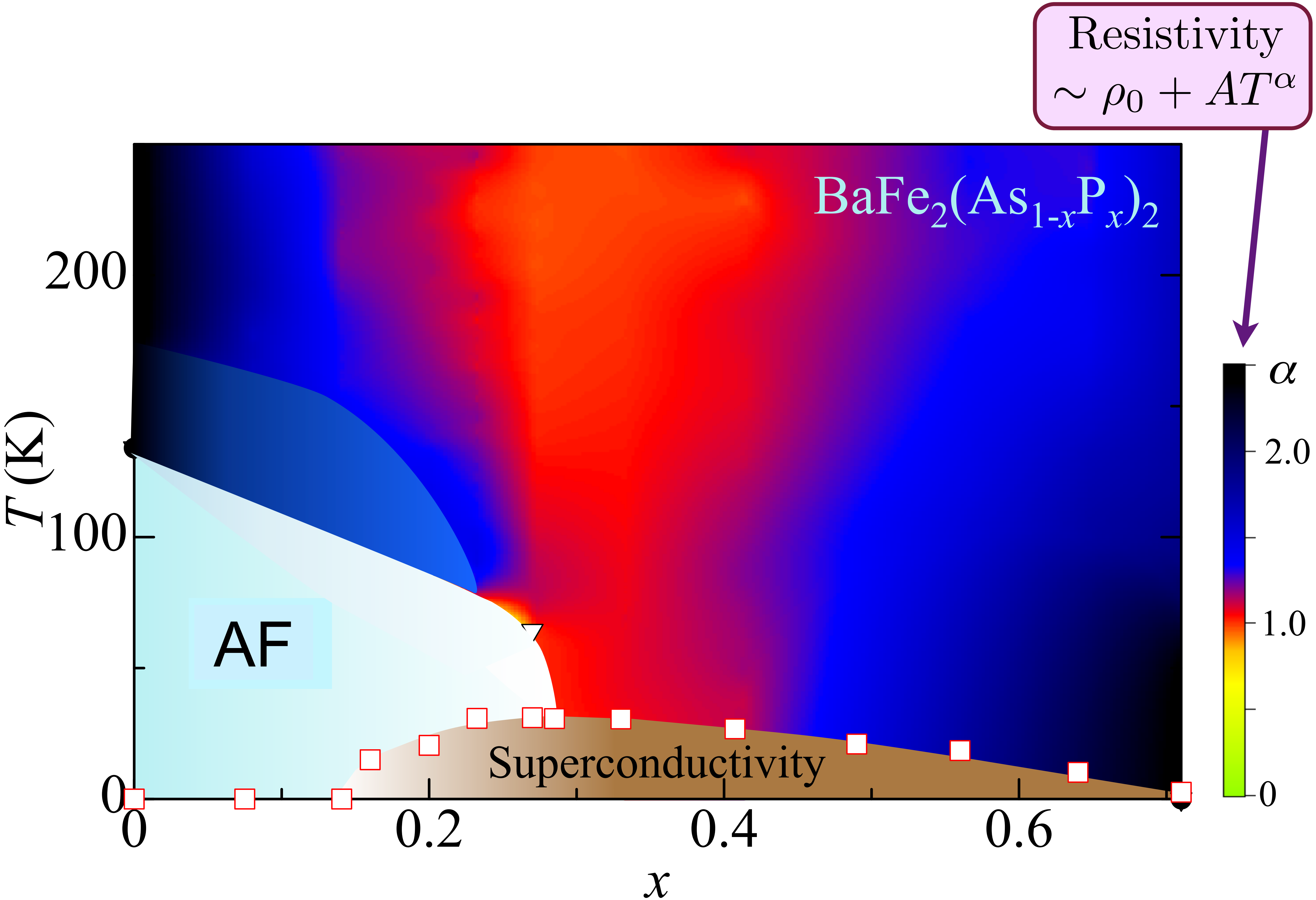
quantum entanglement,

and no quasiparticles

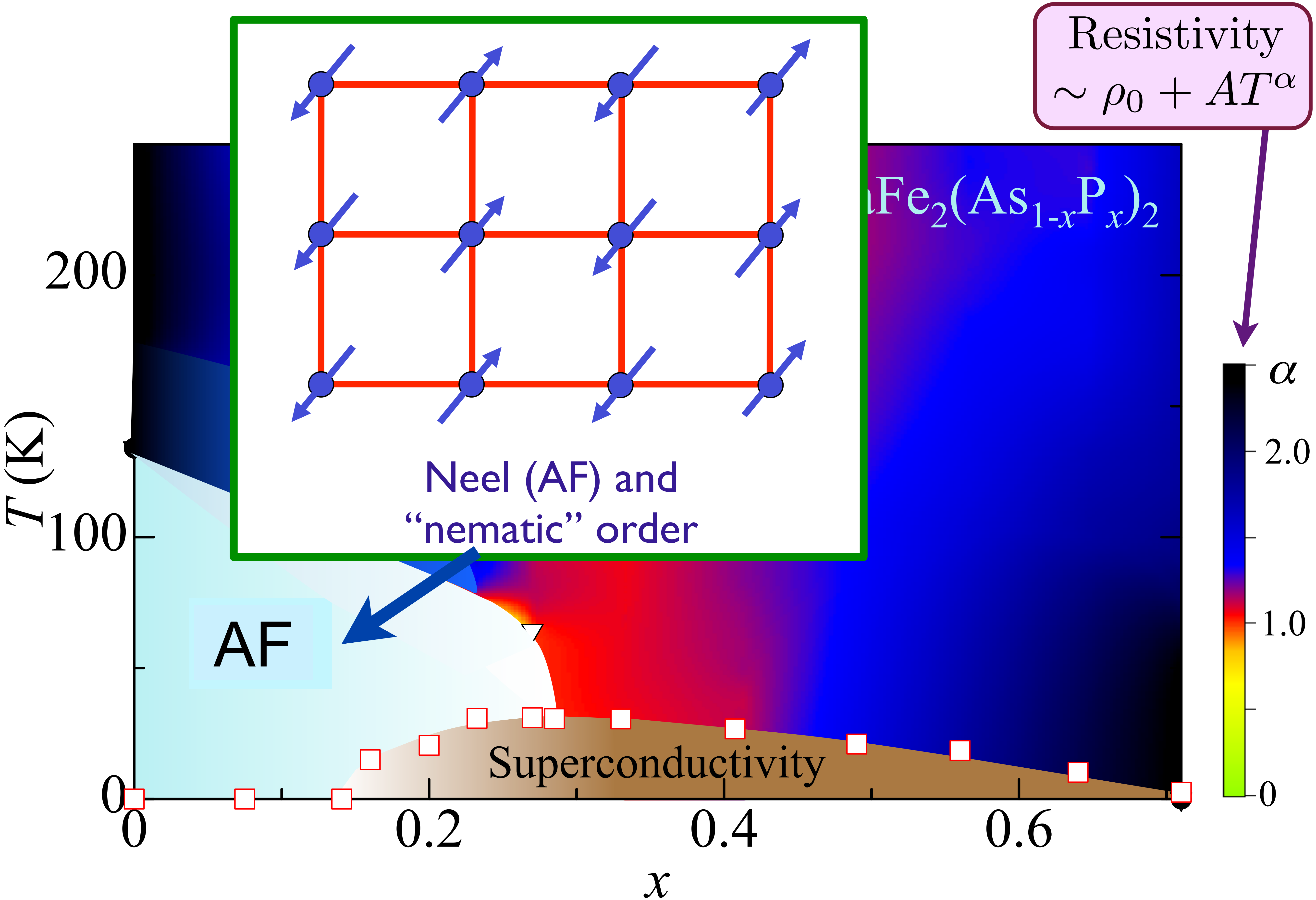
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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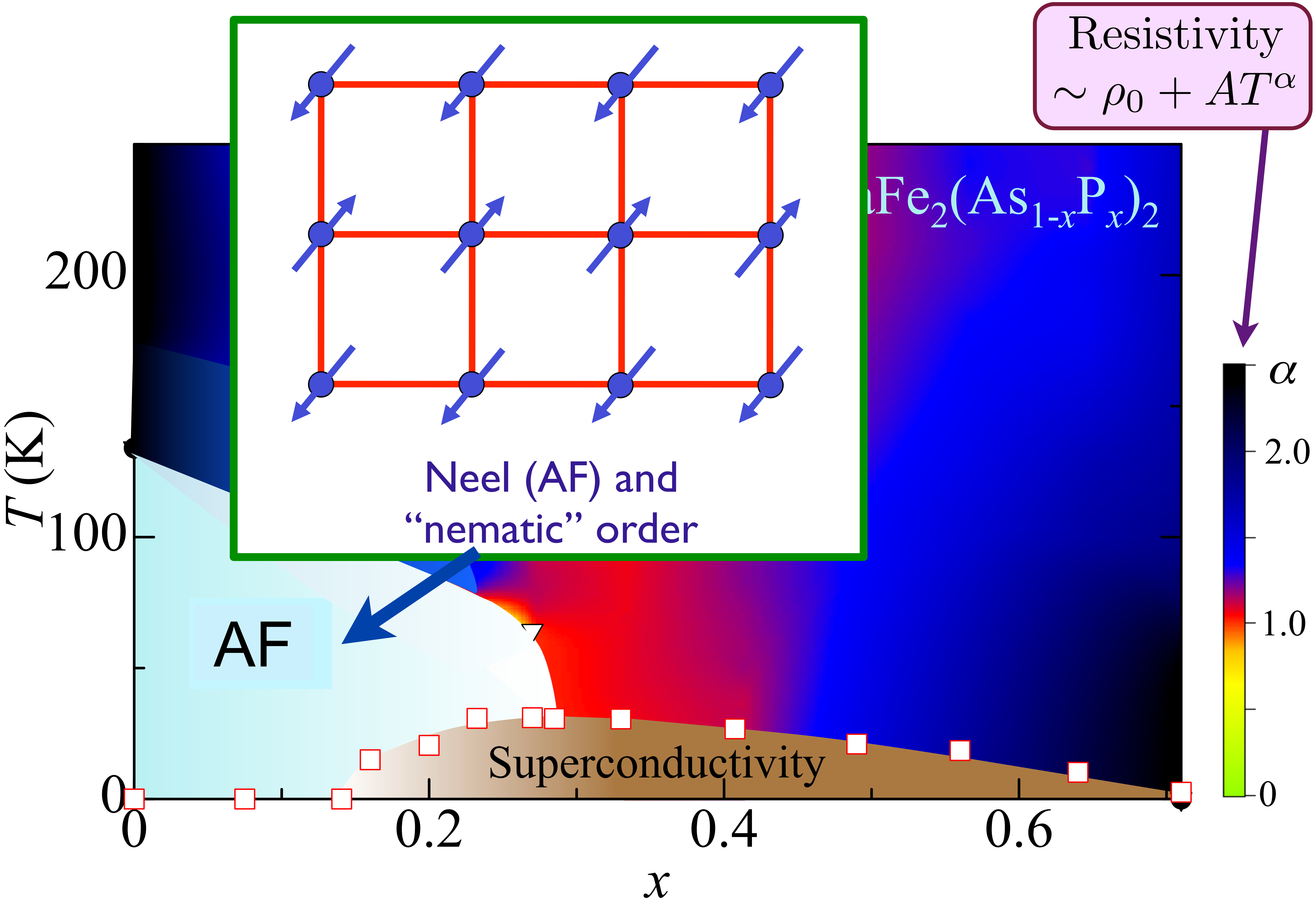

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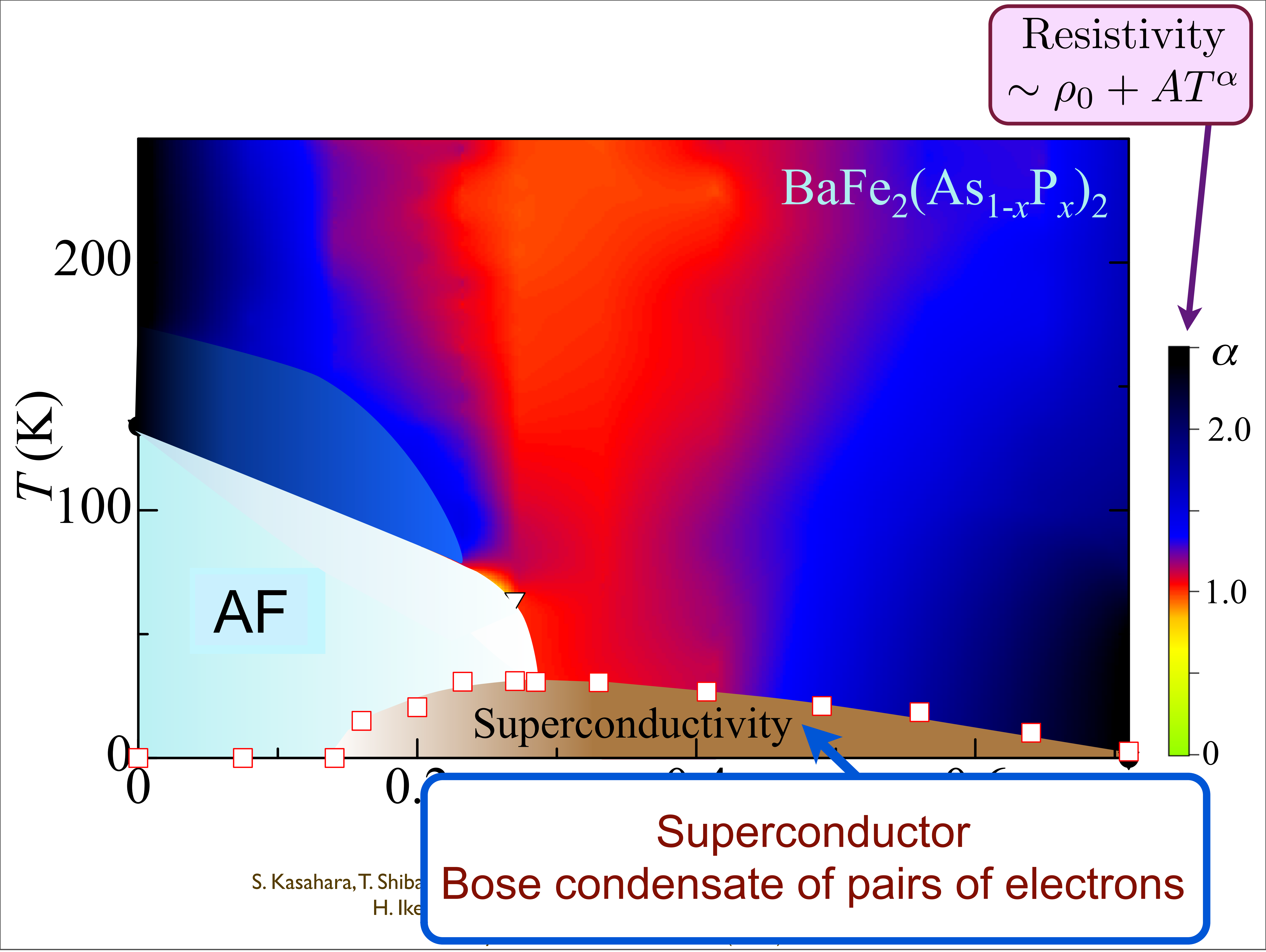
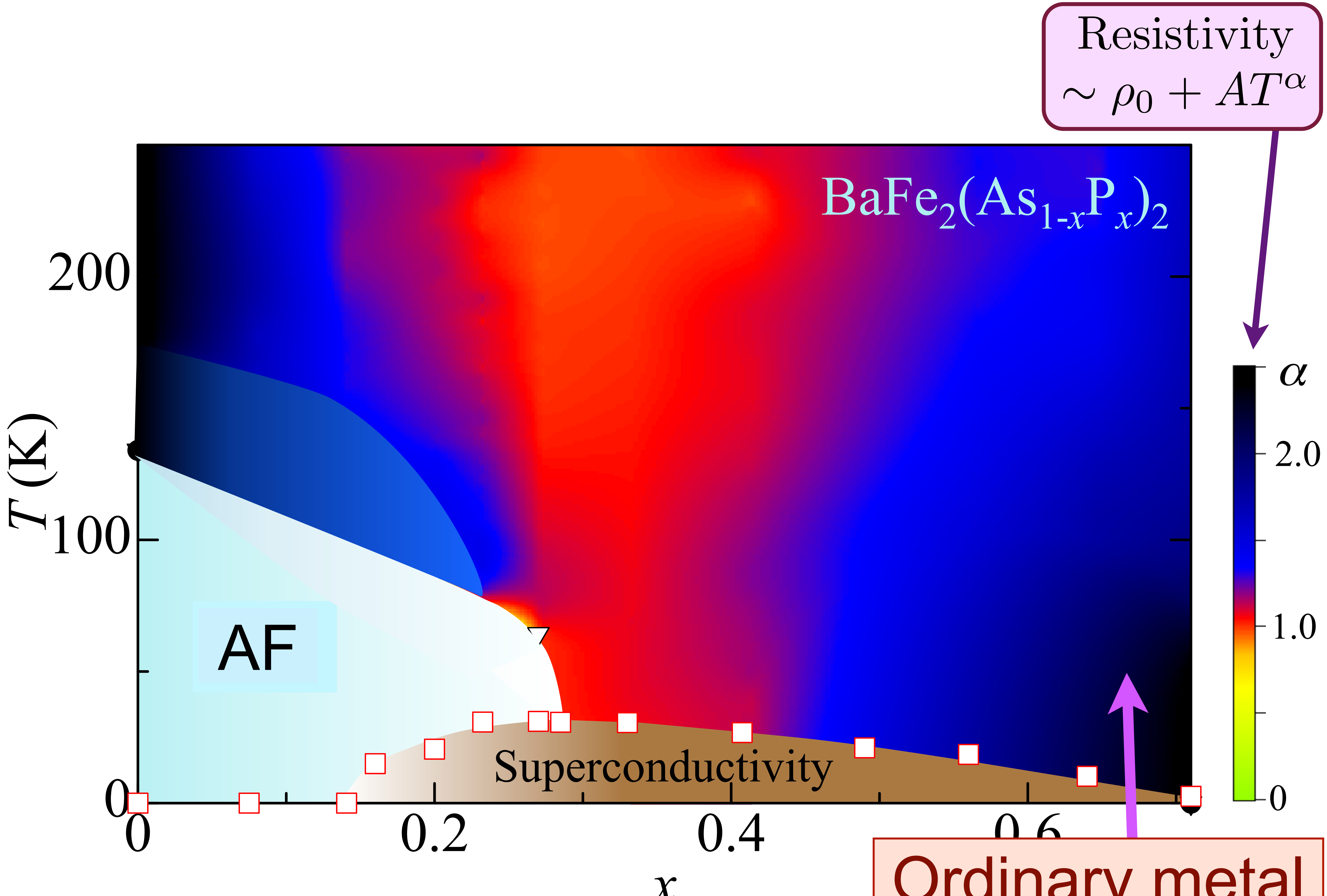

Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
T (K)
200
100
0
0
0.2
AF
Superconductivity
α
2.0
1.0
0
Superconductor
Bose condensate of pairs of electrons
S. Kasahara, T. Shiba
H. Ike



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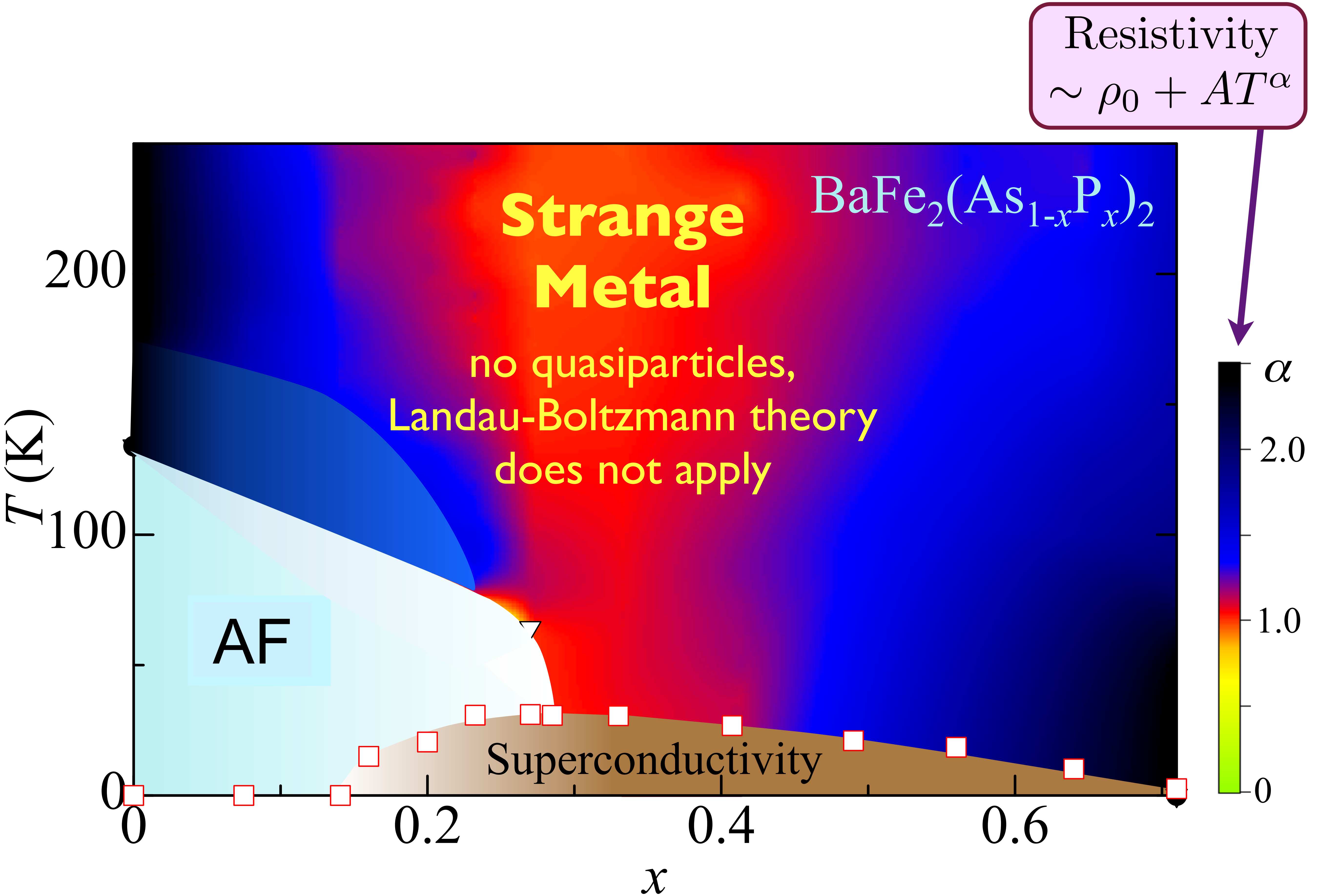

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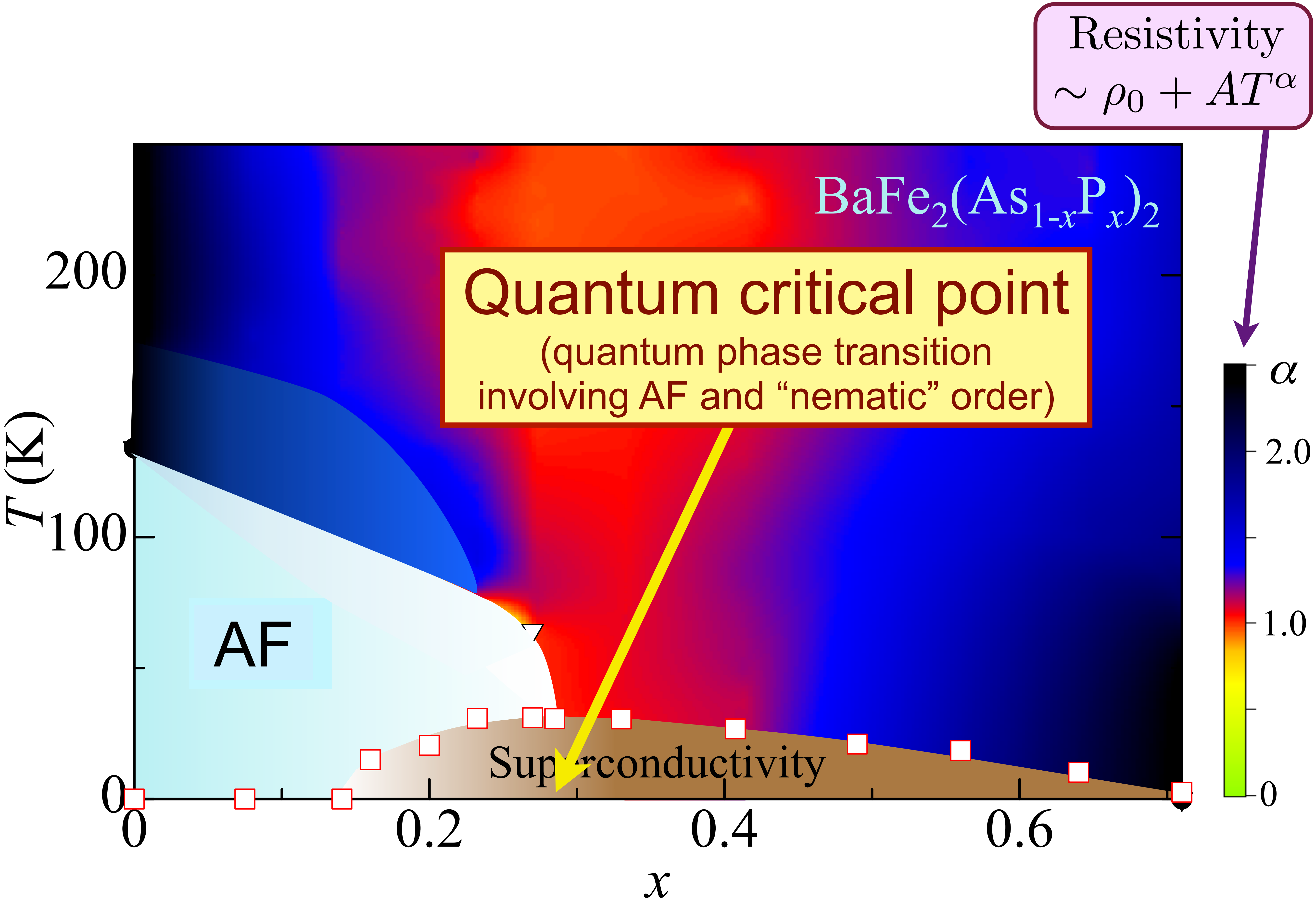

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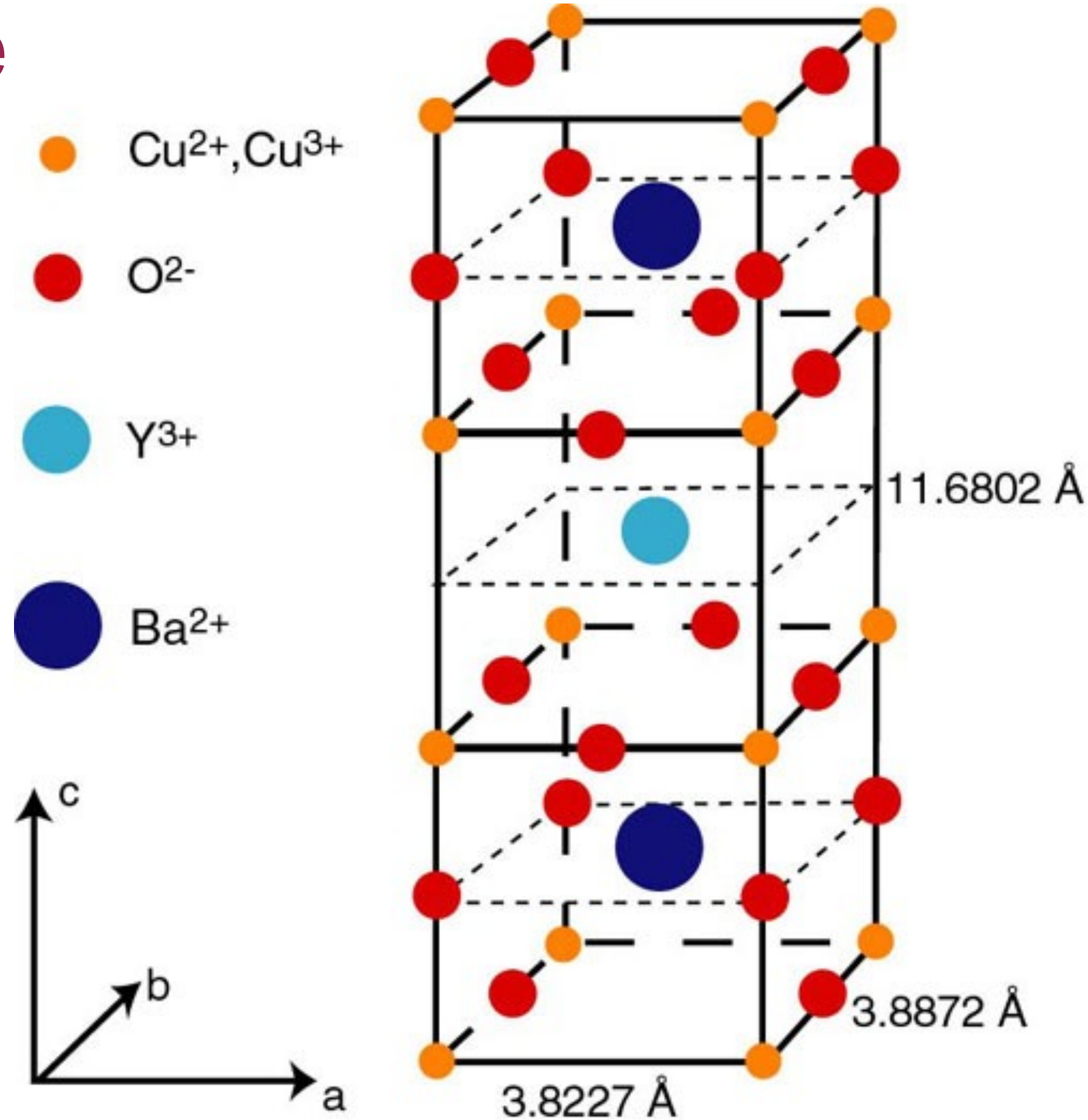
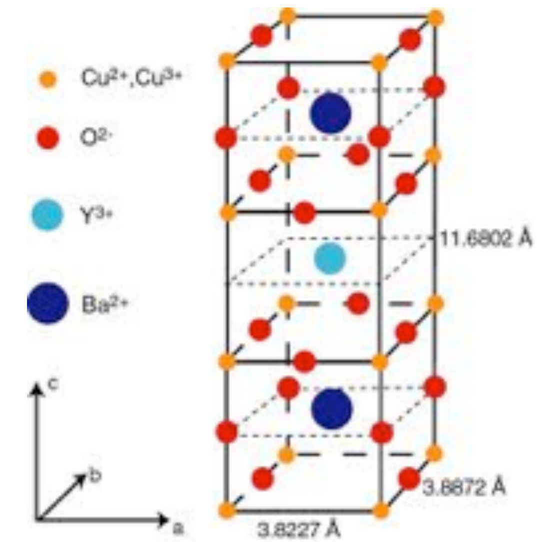
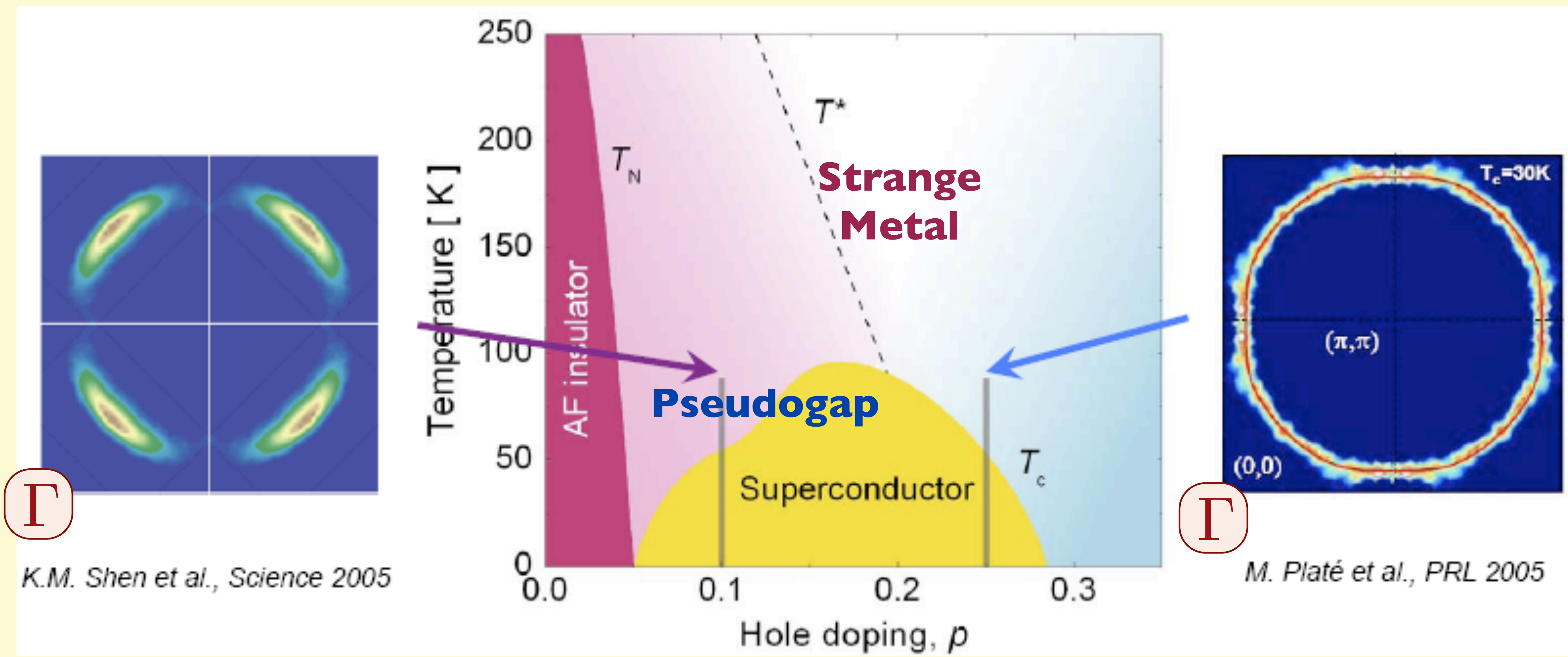
High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$





Smaller hole
Fermi-pockets

Large hole
Fermi surface

Outline

1. The simplest models without quasiparticles

A. Magnetic insulators in two dimensions

B. Ultracold atoms in optical lattices

C. Conformal field theories in

2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

High temperature superconductivity

and competing orders

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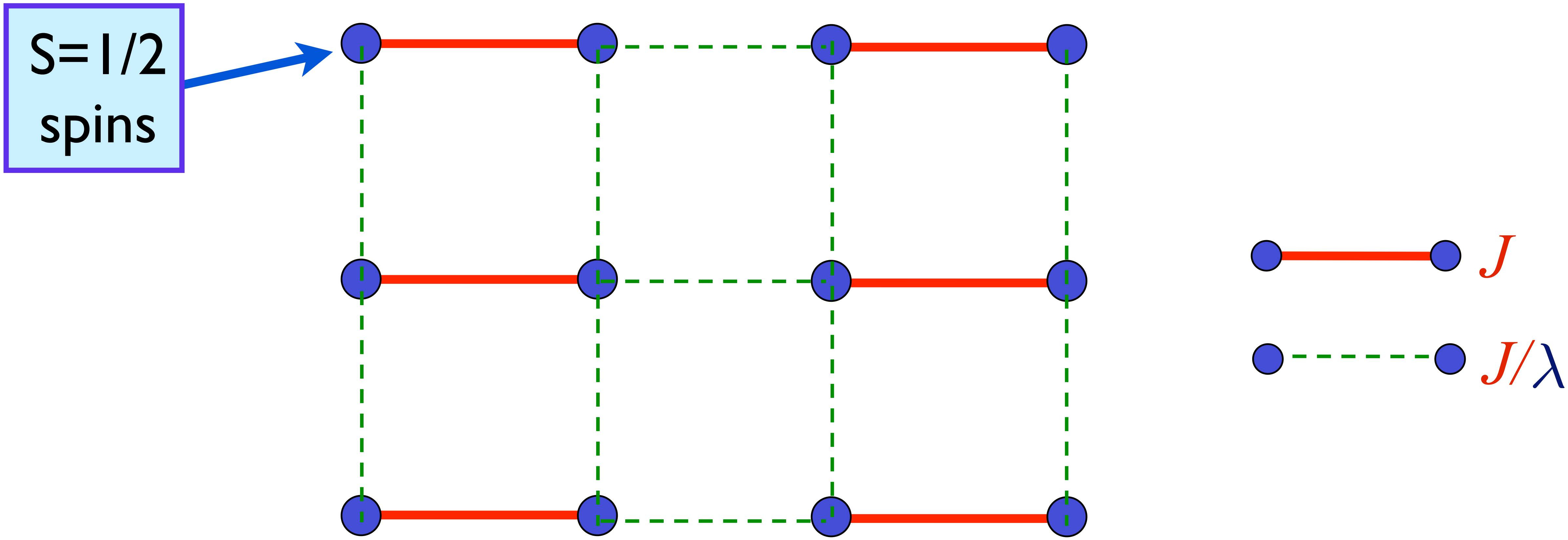
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Square lattice antiferromagnet

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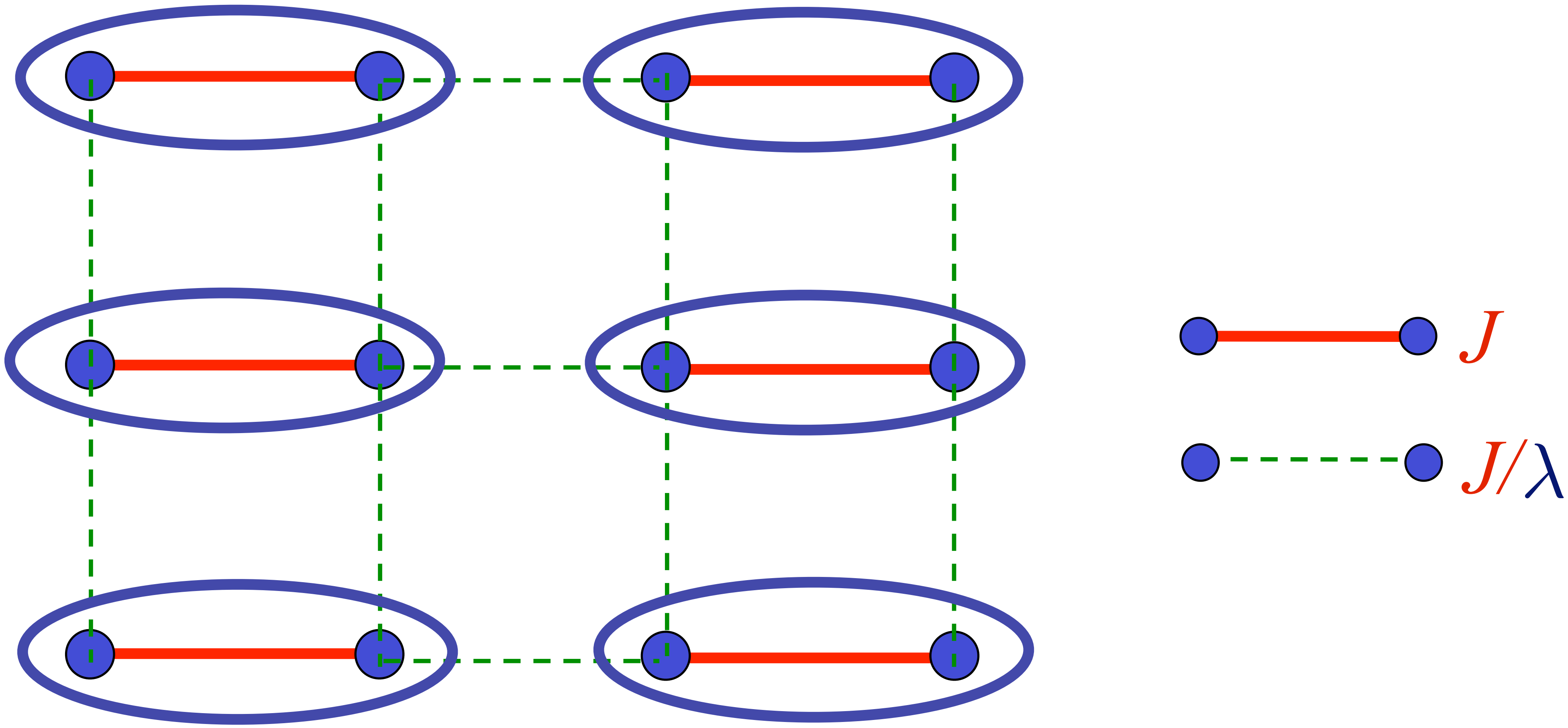
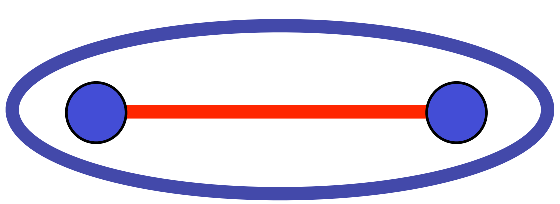


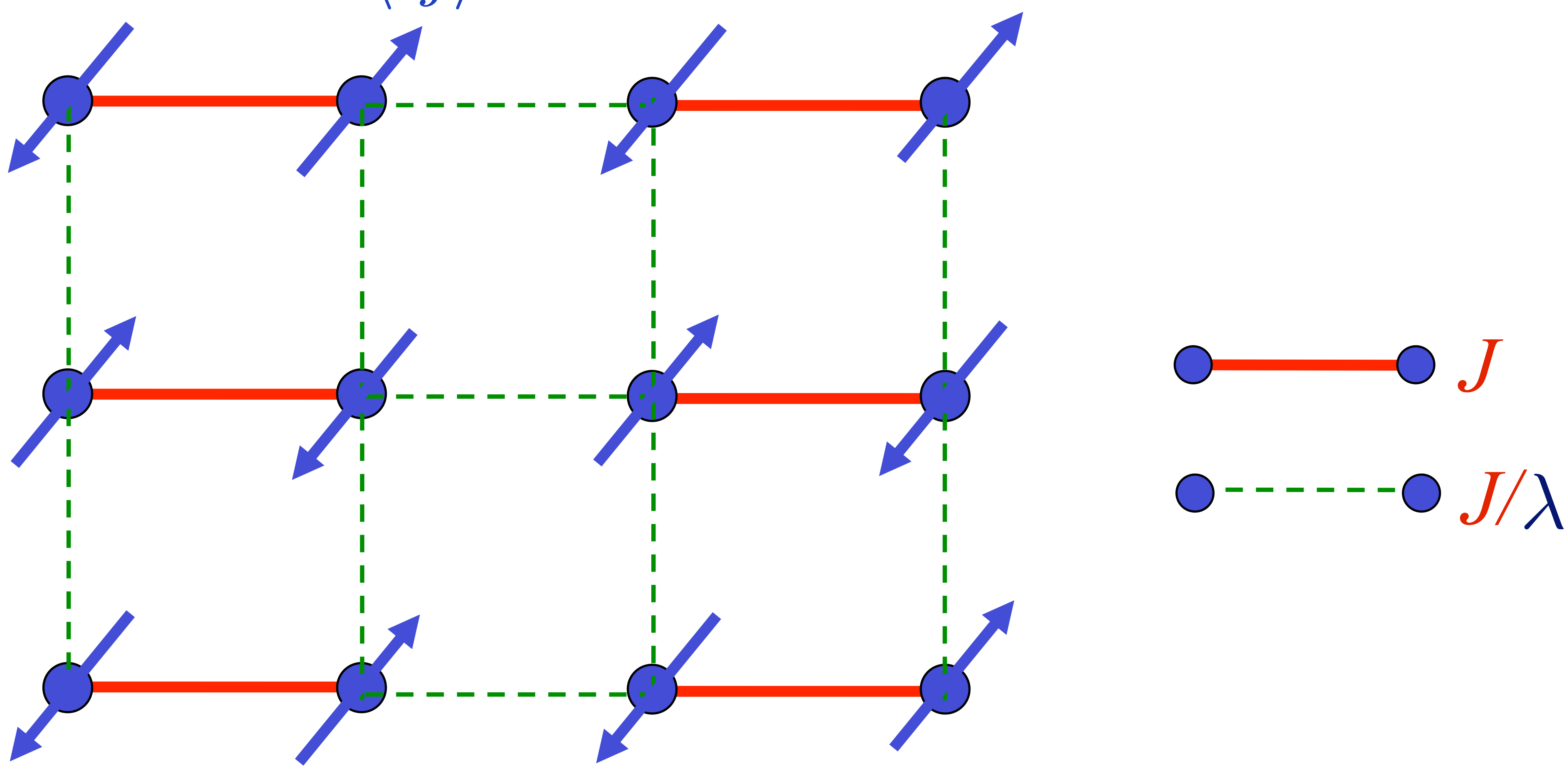


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

At large λ ground state is a “quantum paramagnet” with spins locked in valence bond singlets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

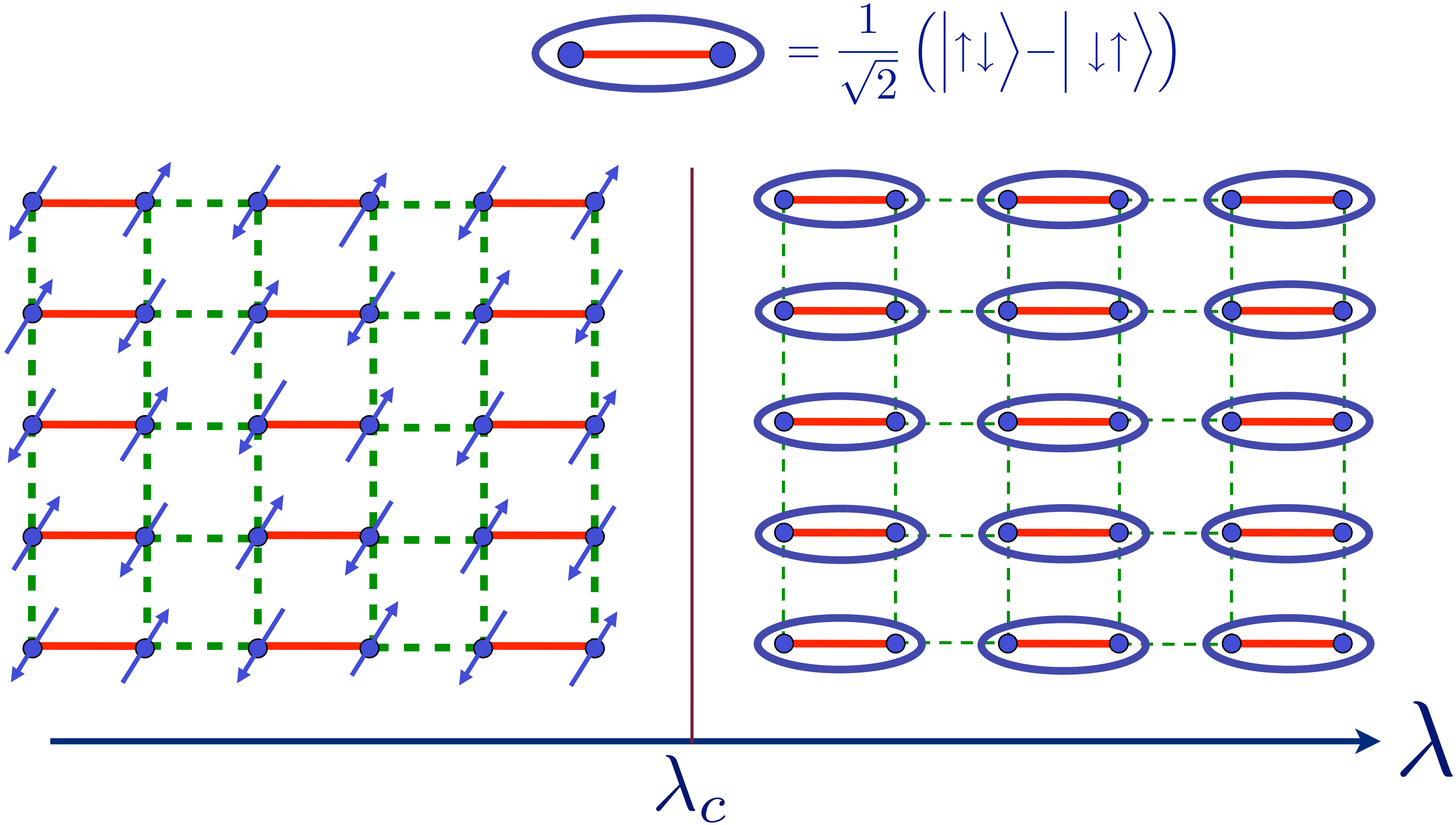


For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

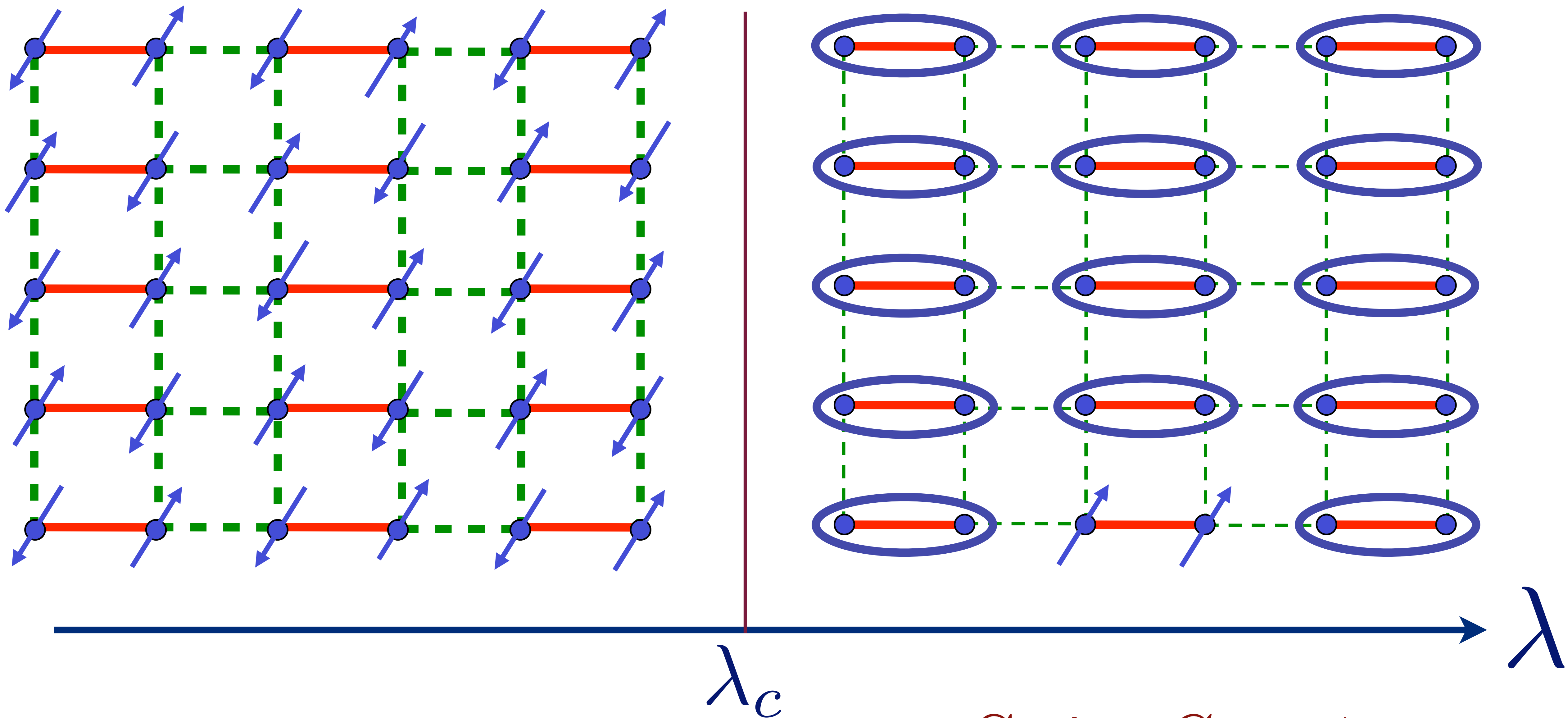
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ

Quasiparticles in the paramagnetic phase



Spin $S = 1$
"triplon"

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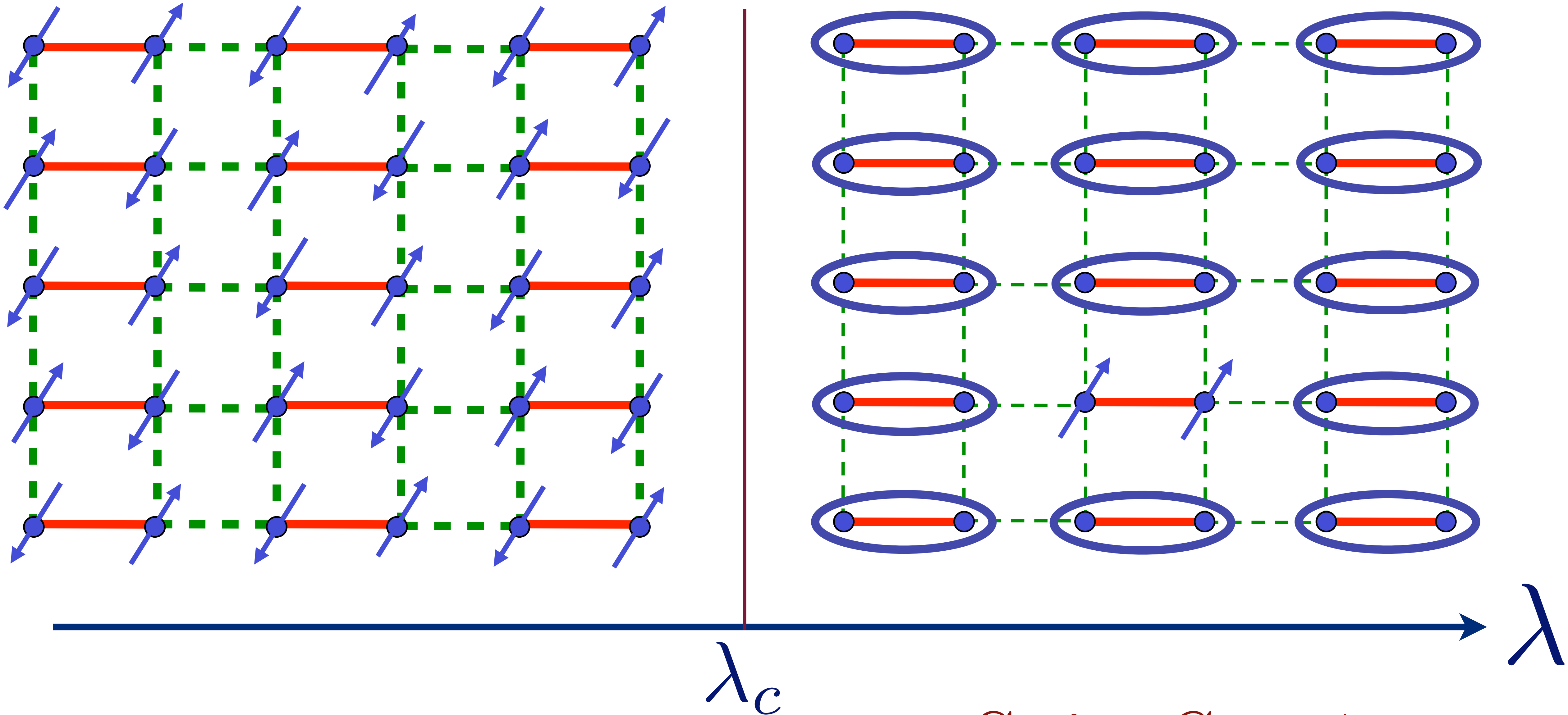

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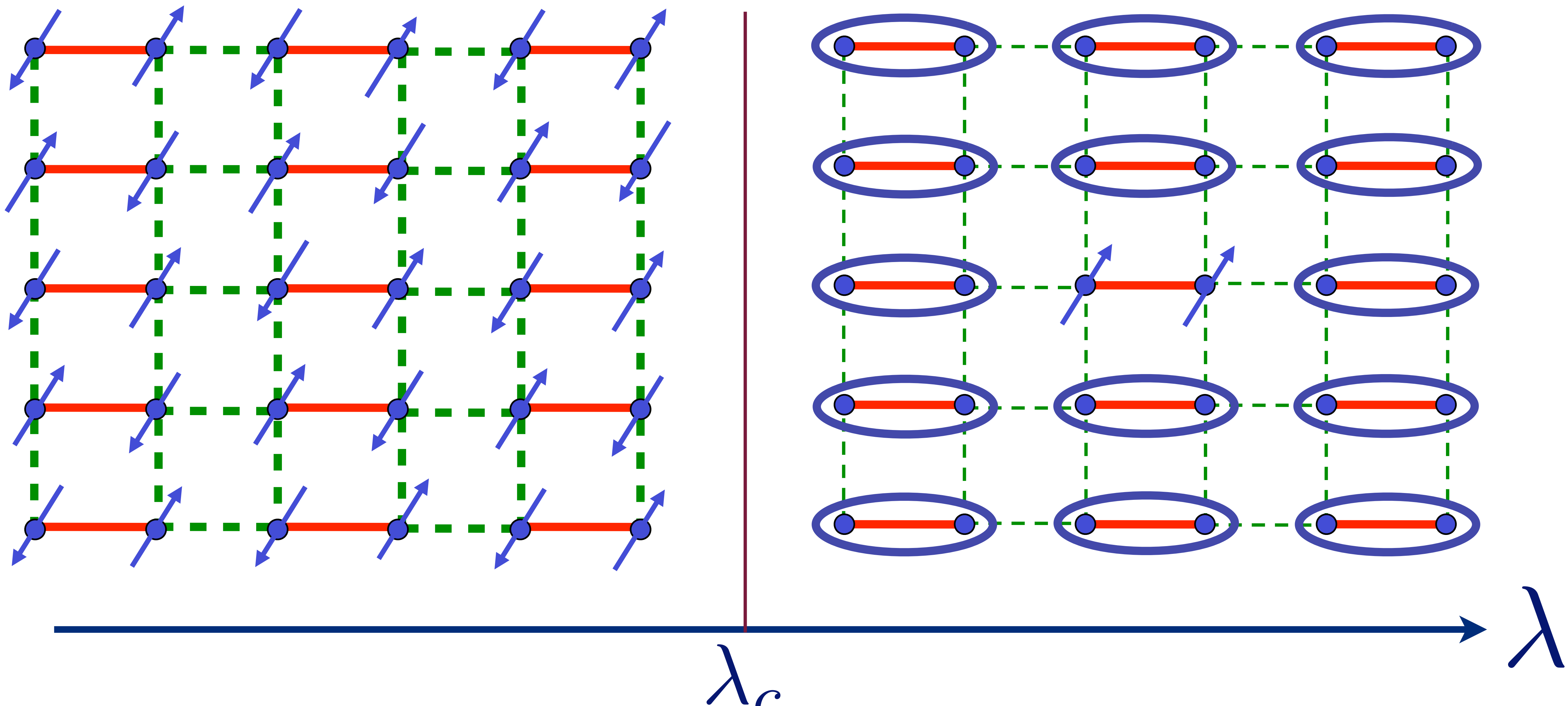

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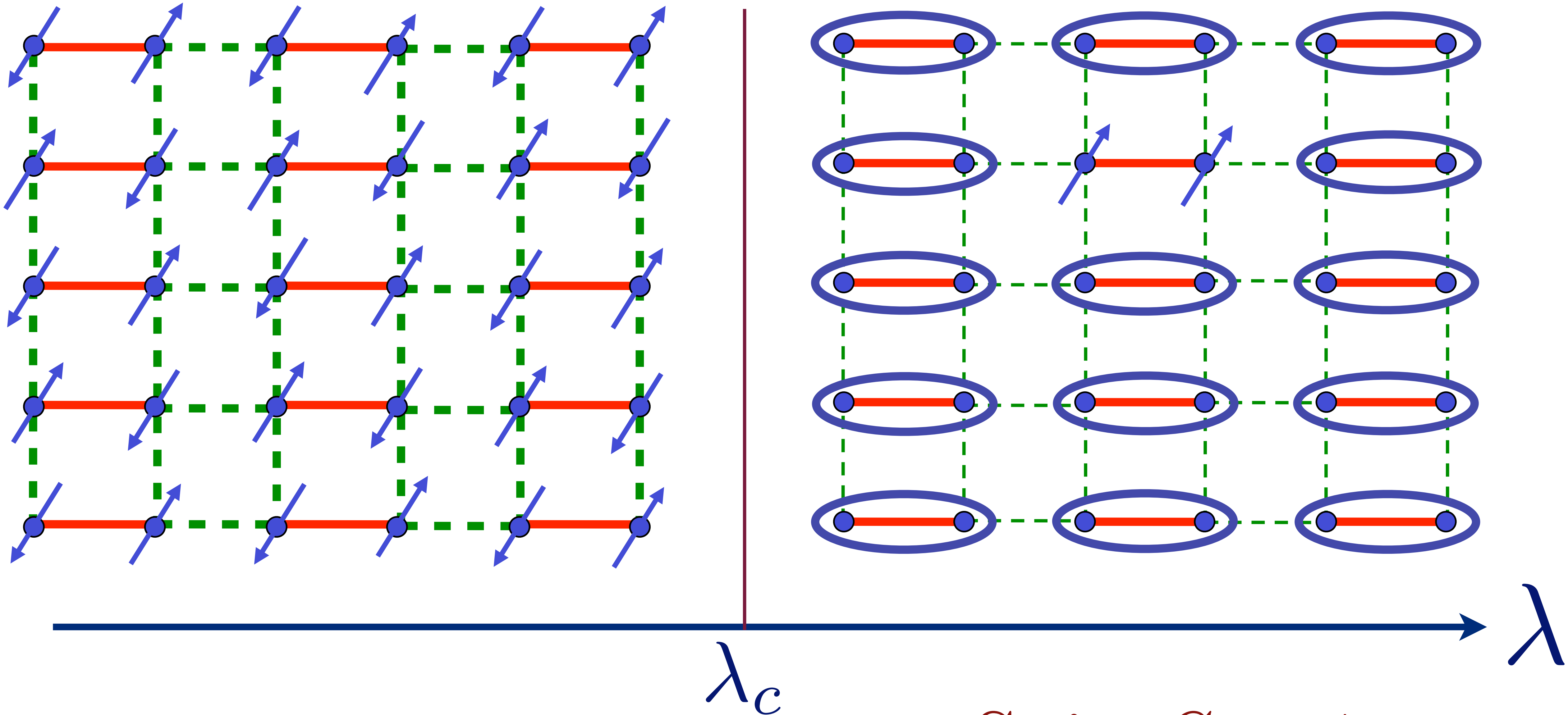

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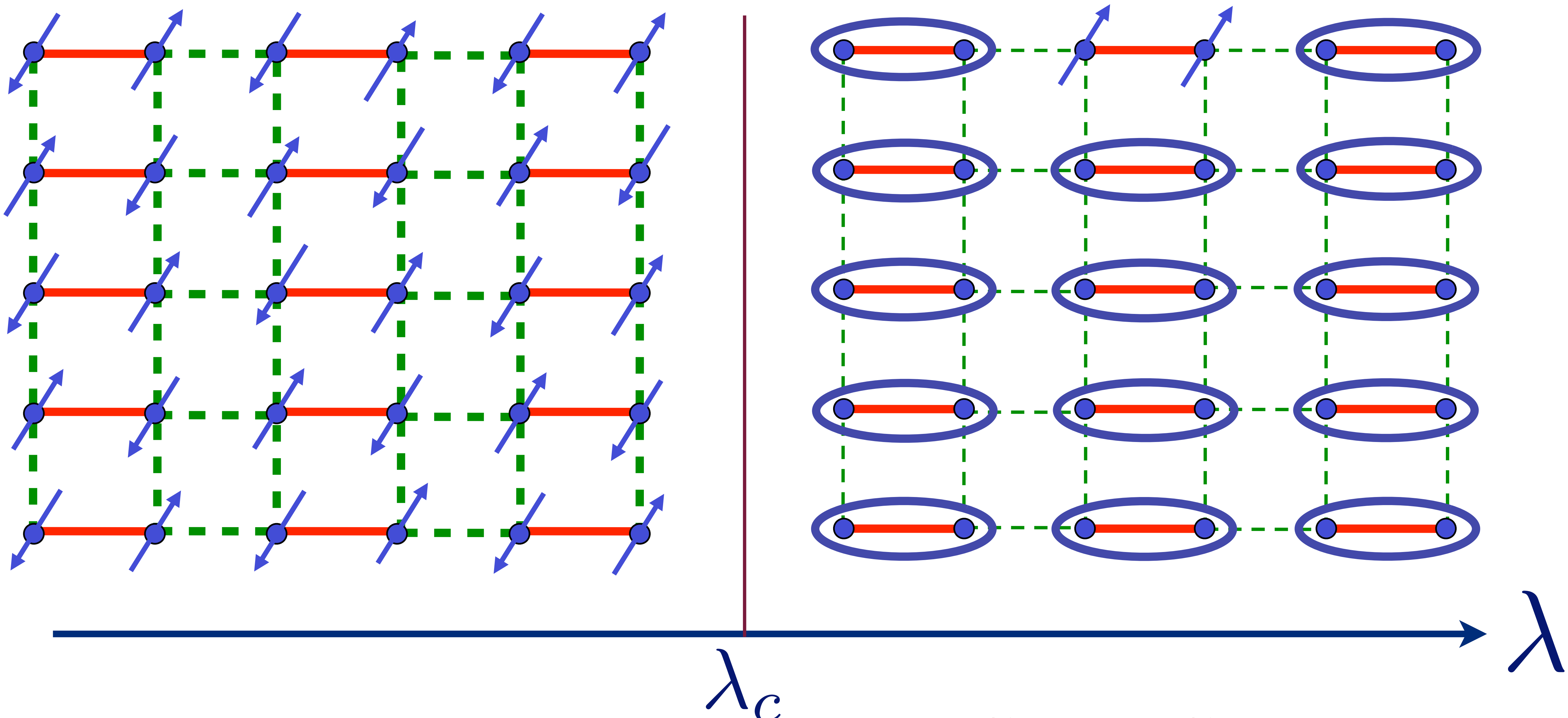

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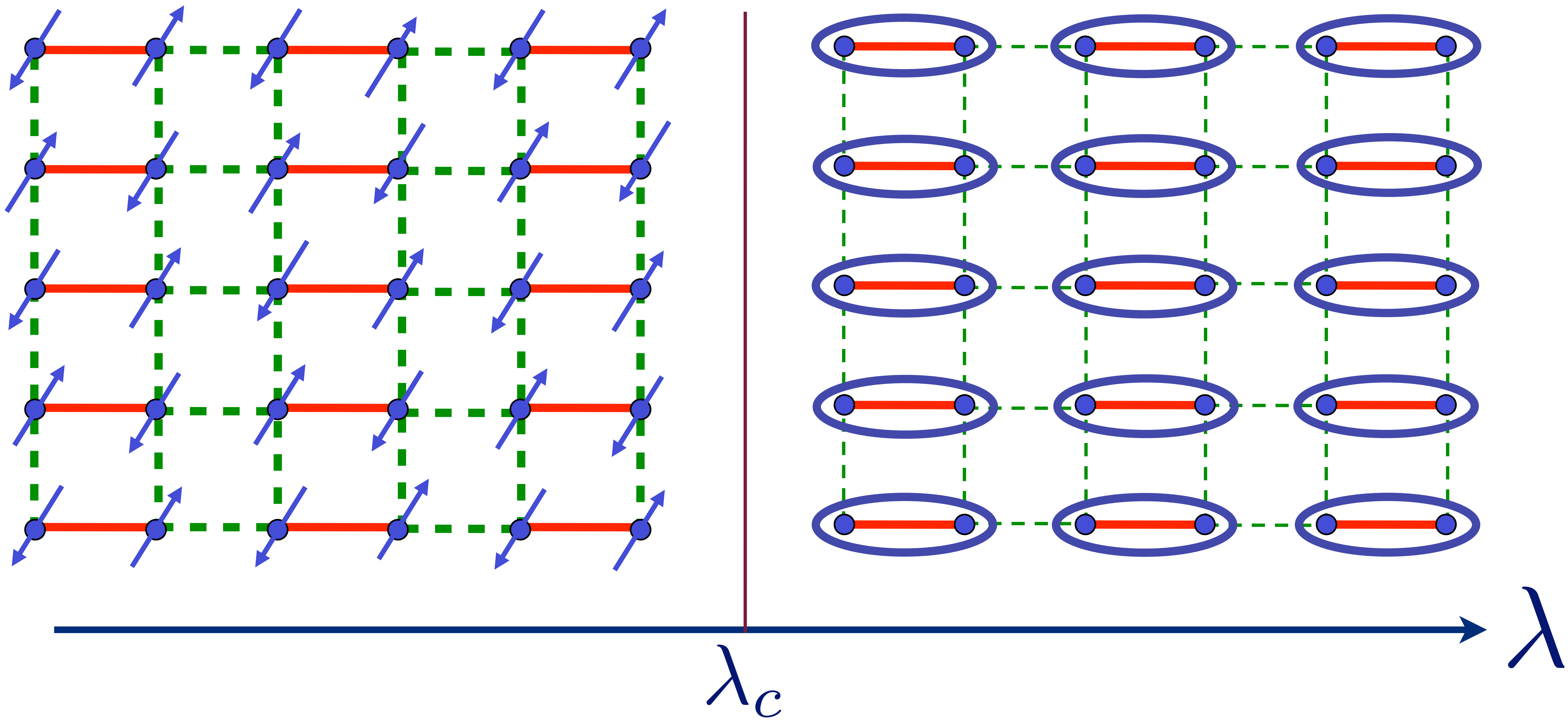
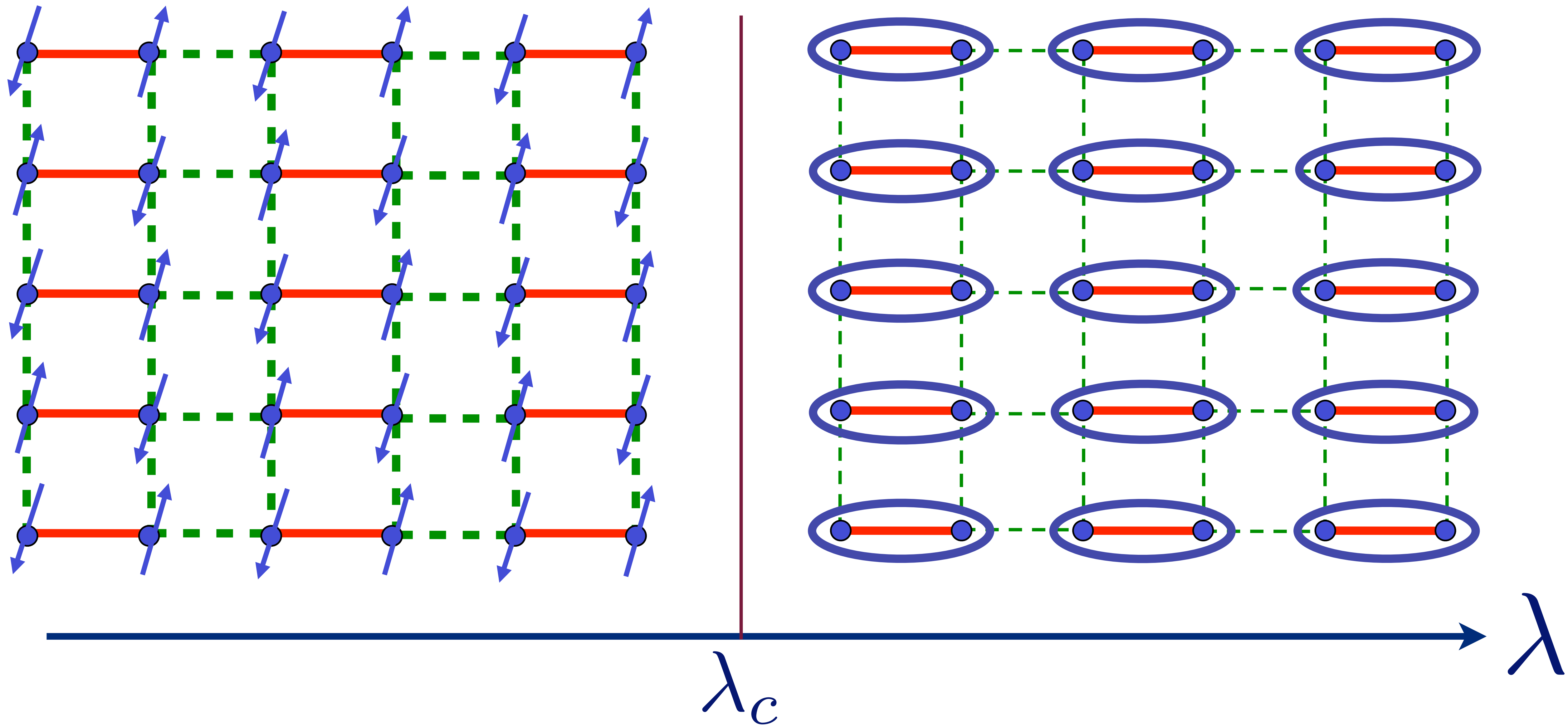
Quasiparticles in the Néel phase



Spin waves

Quasiparticles in the Néel phase



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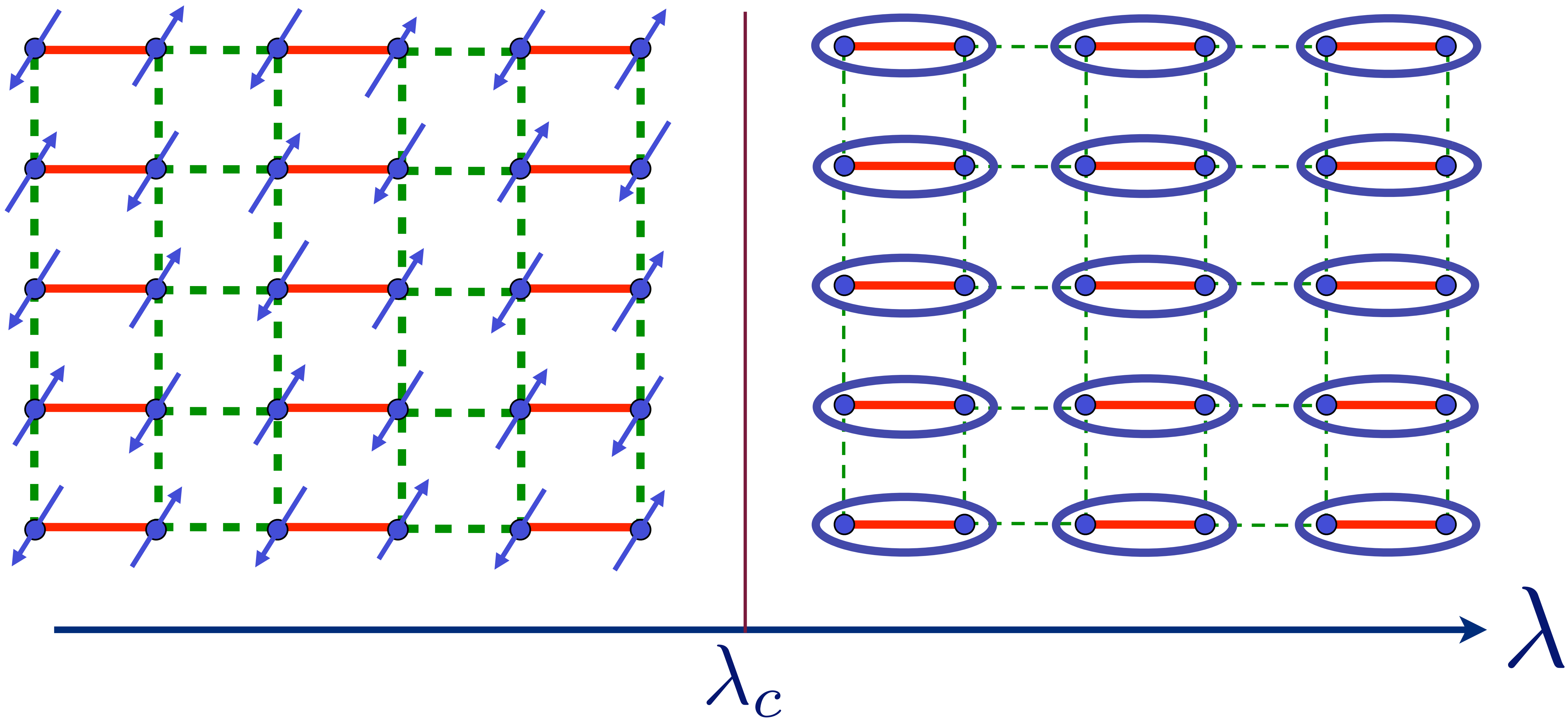
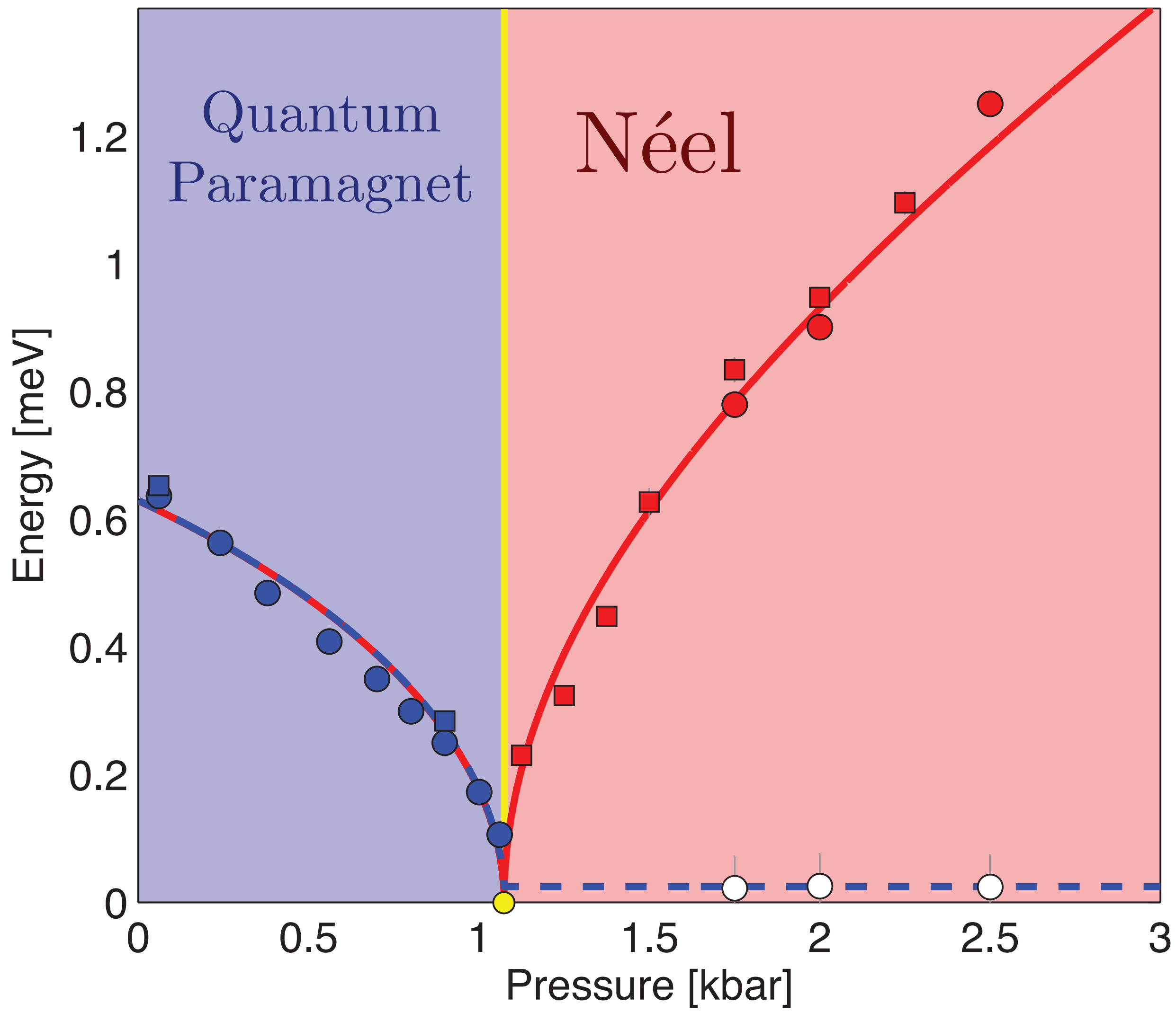


Spin waves

Excitations of $TlCuCl_3$ with varying pressure (in d=3)



Related observations in PHCC (also in d=3) by M. Thede, A. Mannig, M. Månsson, D. Hüvonen, R. Khasanov, E. Morenzoni, and A. Zheludev, arXiv:1310.7807

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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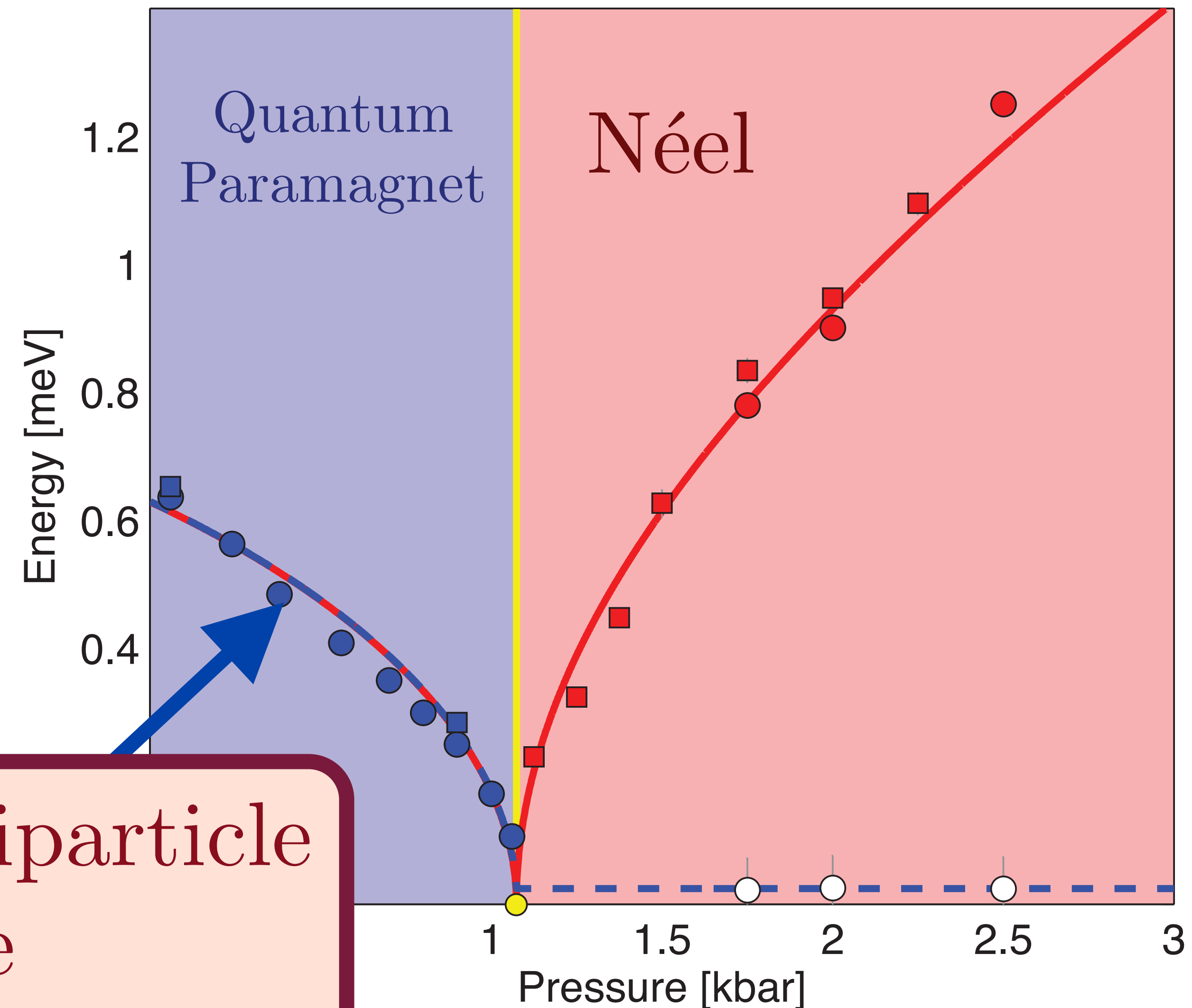

Triplon quasiparticle of the quantum paramagnet

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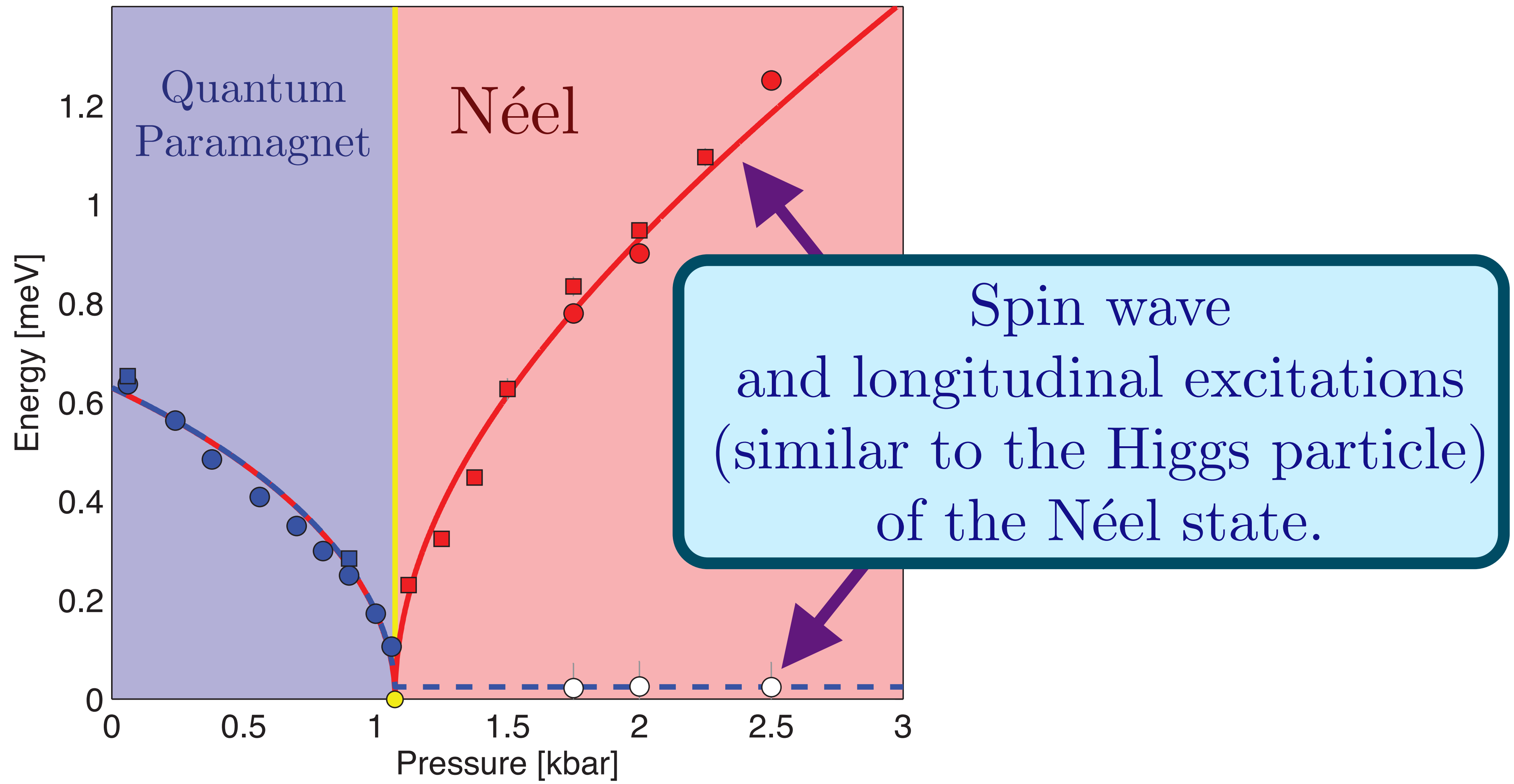
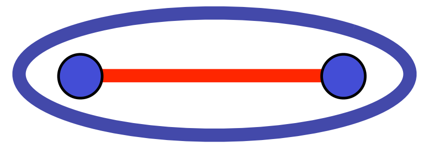


"Higgs" particle appears at theoretically predicted energy

S. Sachdev, arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)



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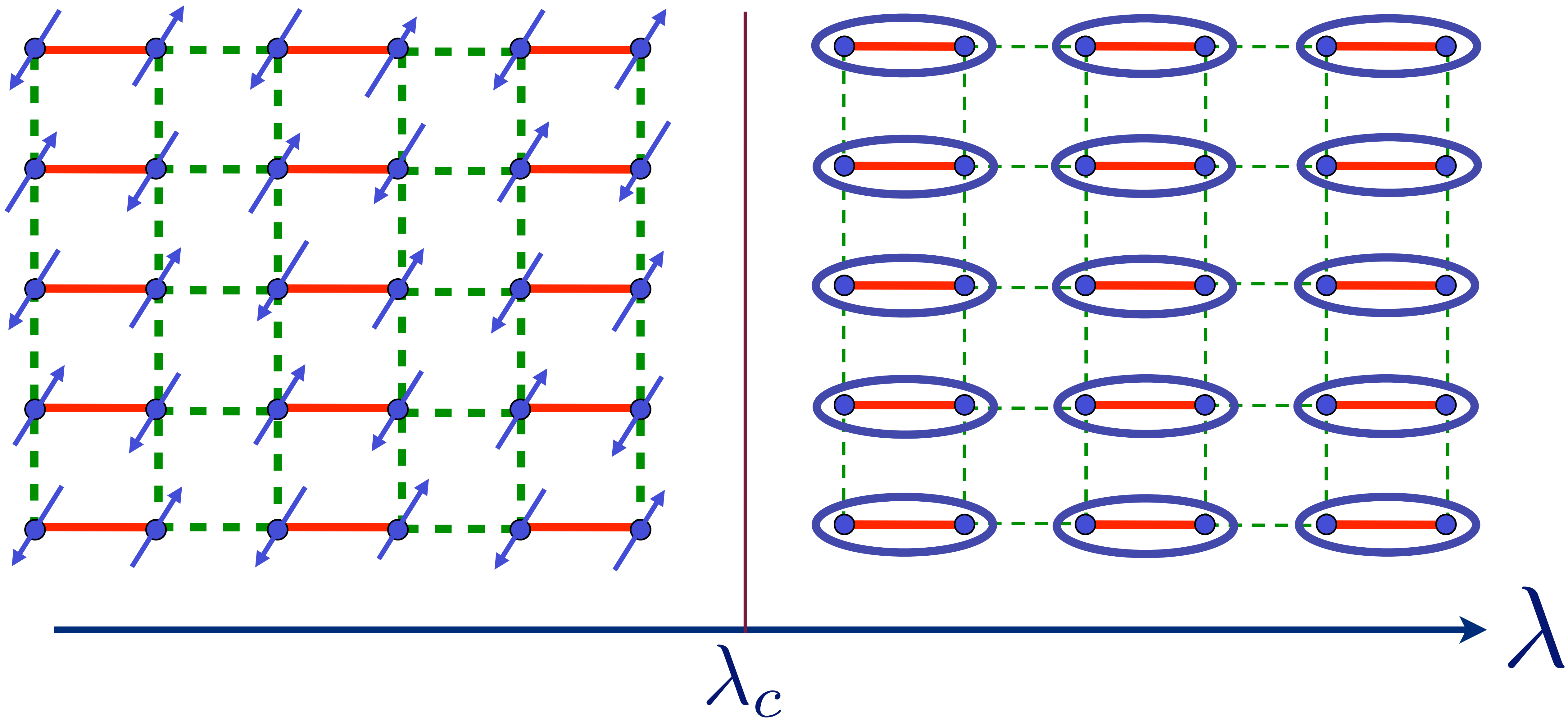

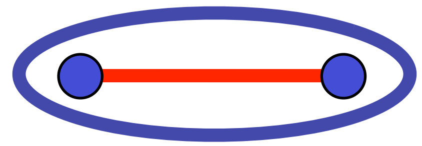

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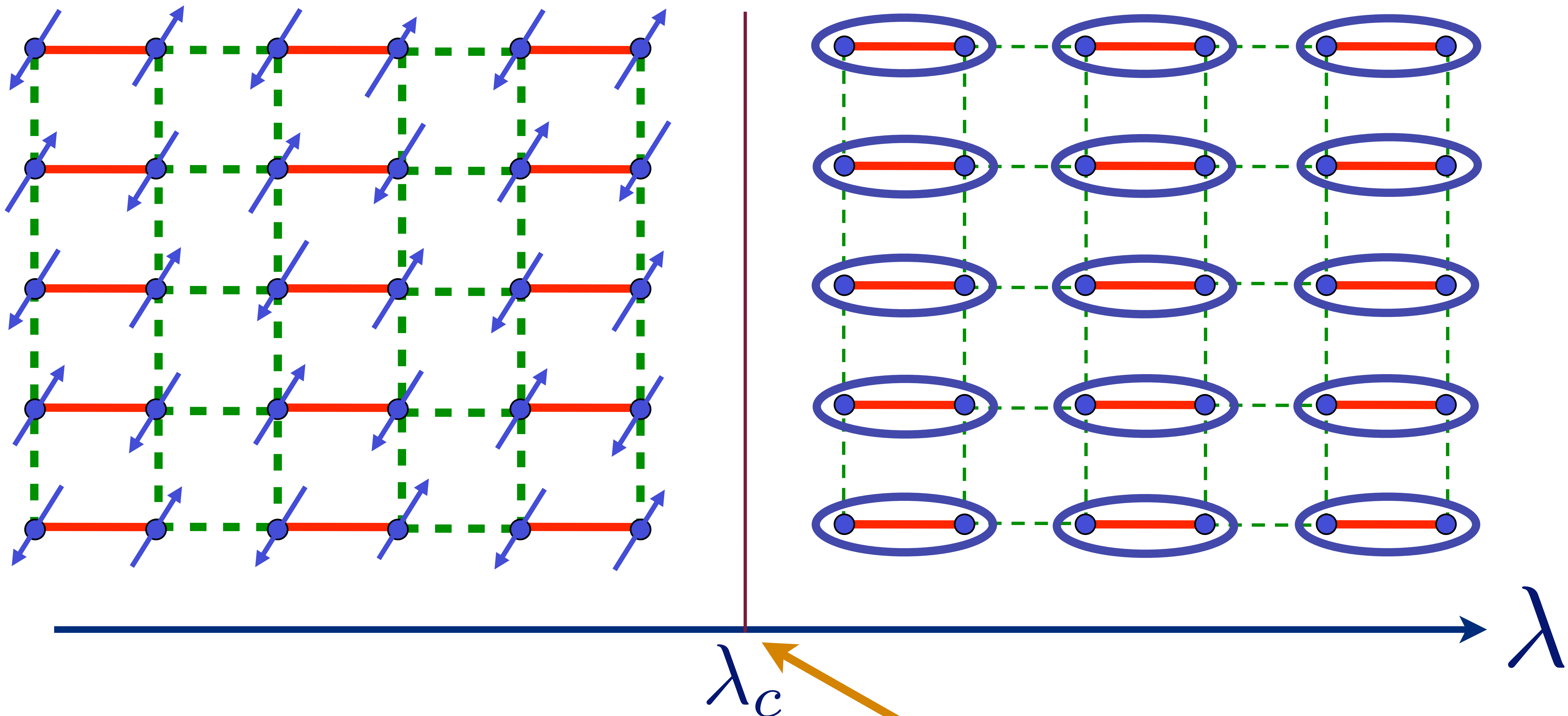
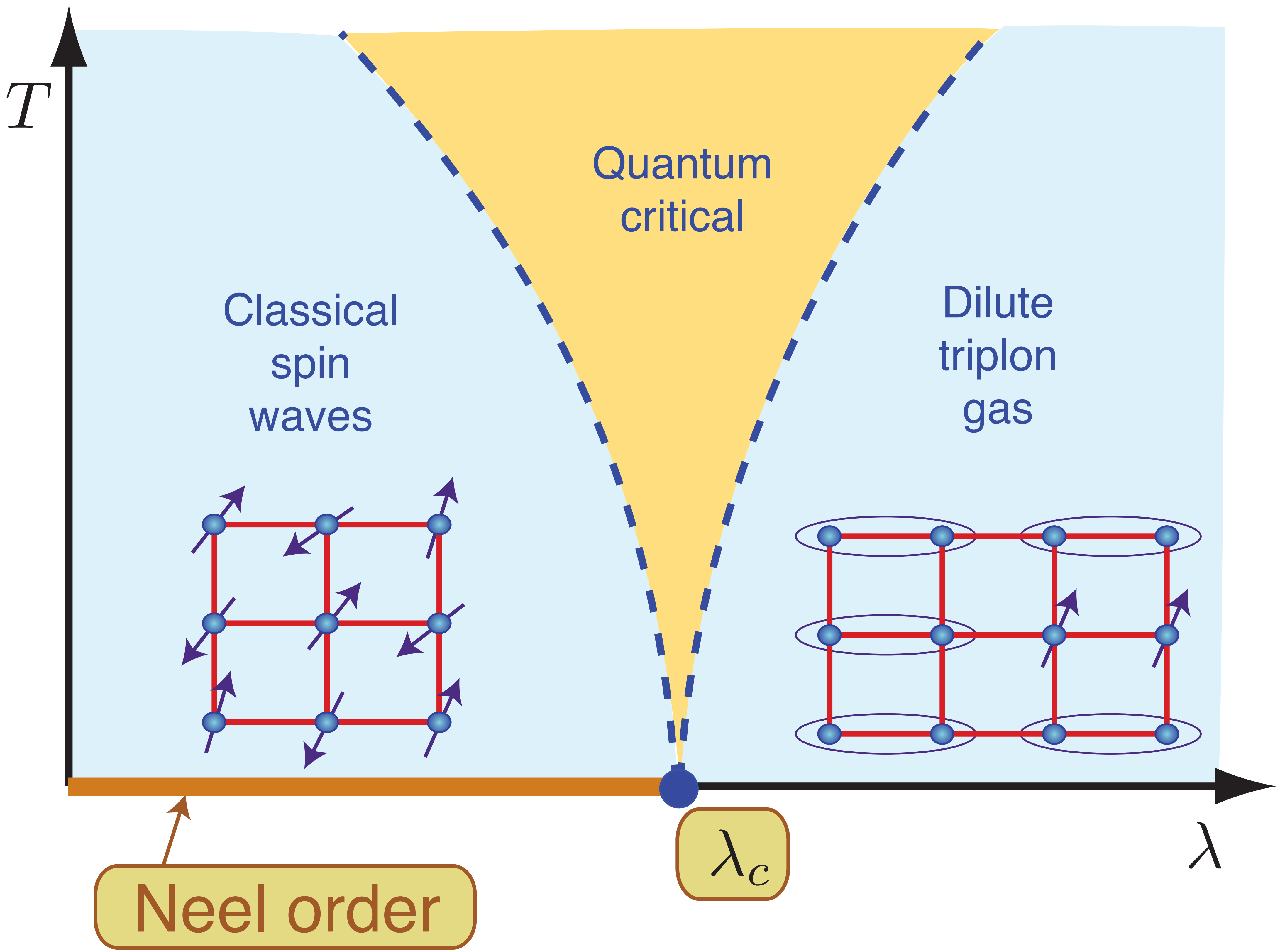
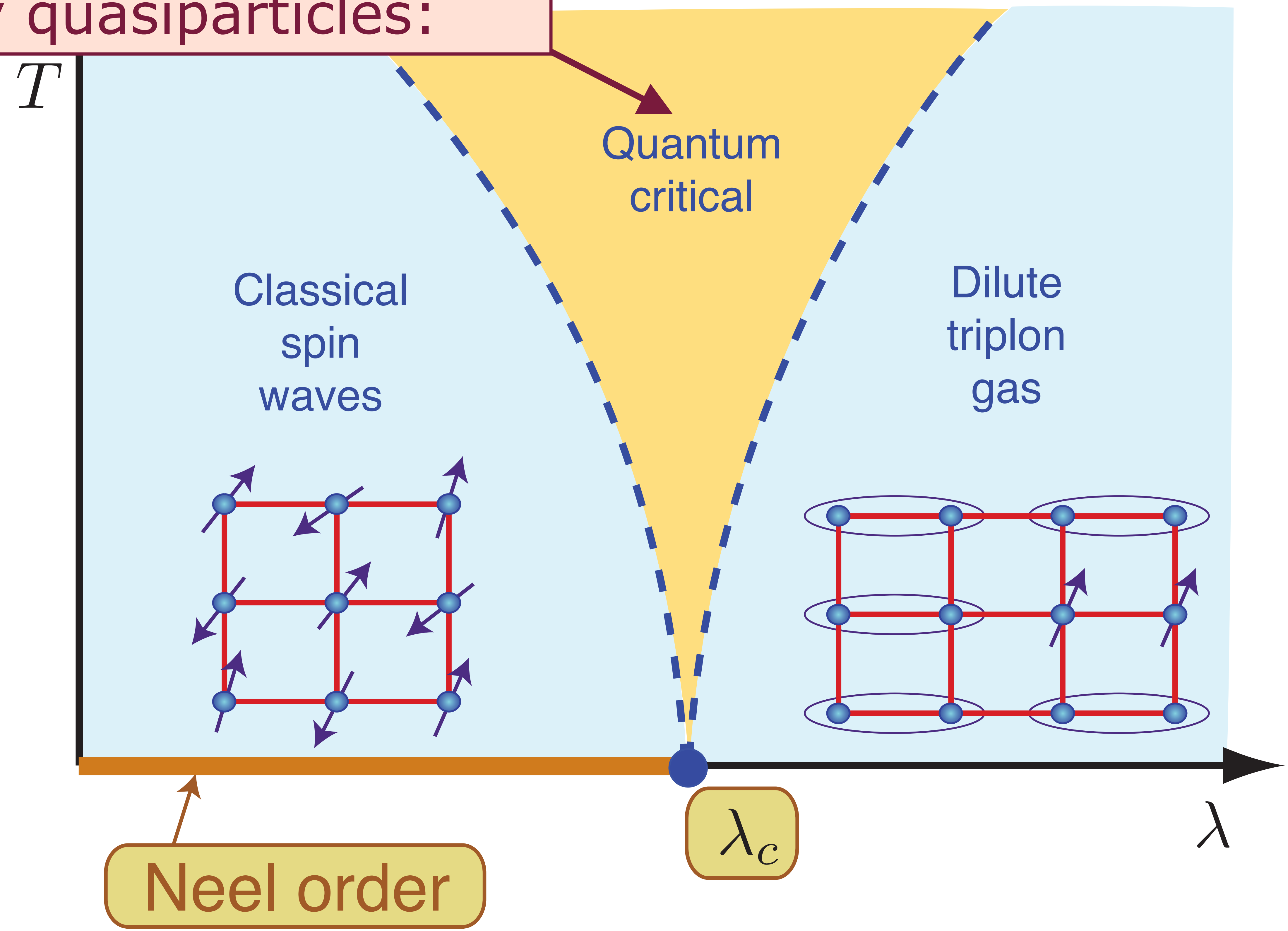


Quantum critical point:
A new state of matter
with long-range quantum entanglement
<u>and</u> no quasiparticles

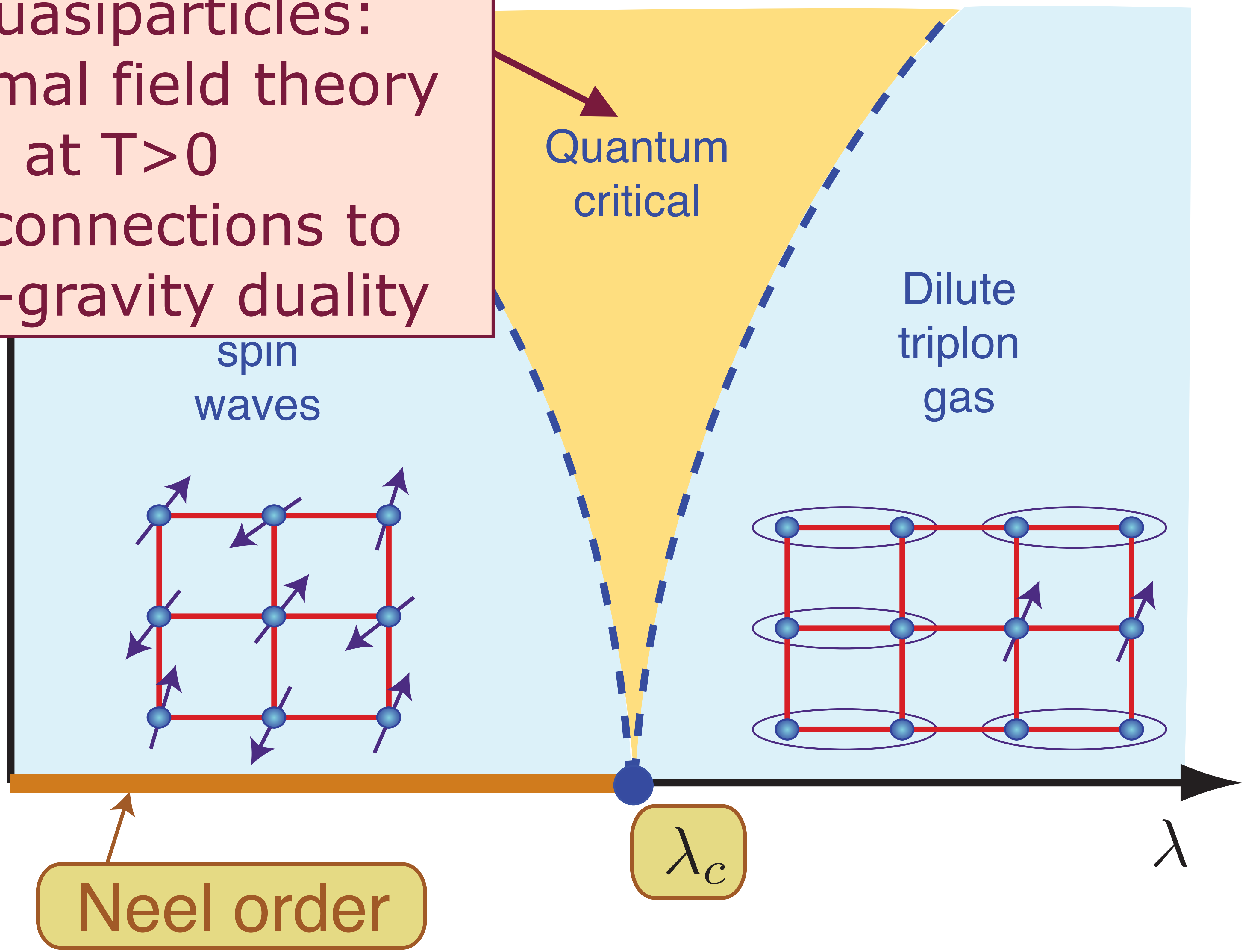

T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order

Dynamics not described by quasiparticles:



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Dynamics not described by quasiparticles: conformal field theory at T>0 with connections to gauge-gravity duality



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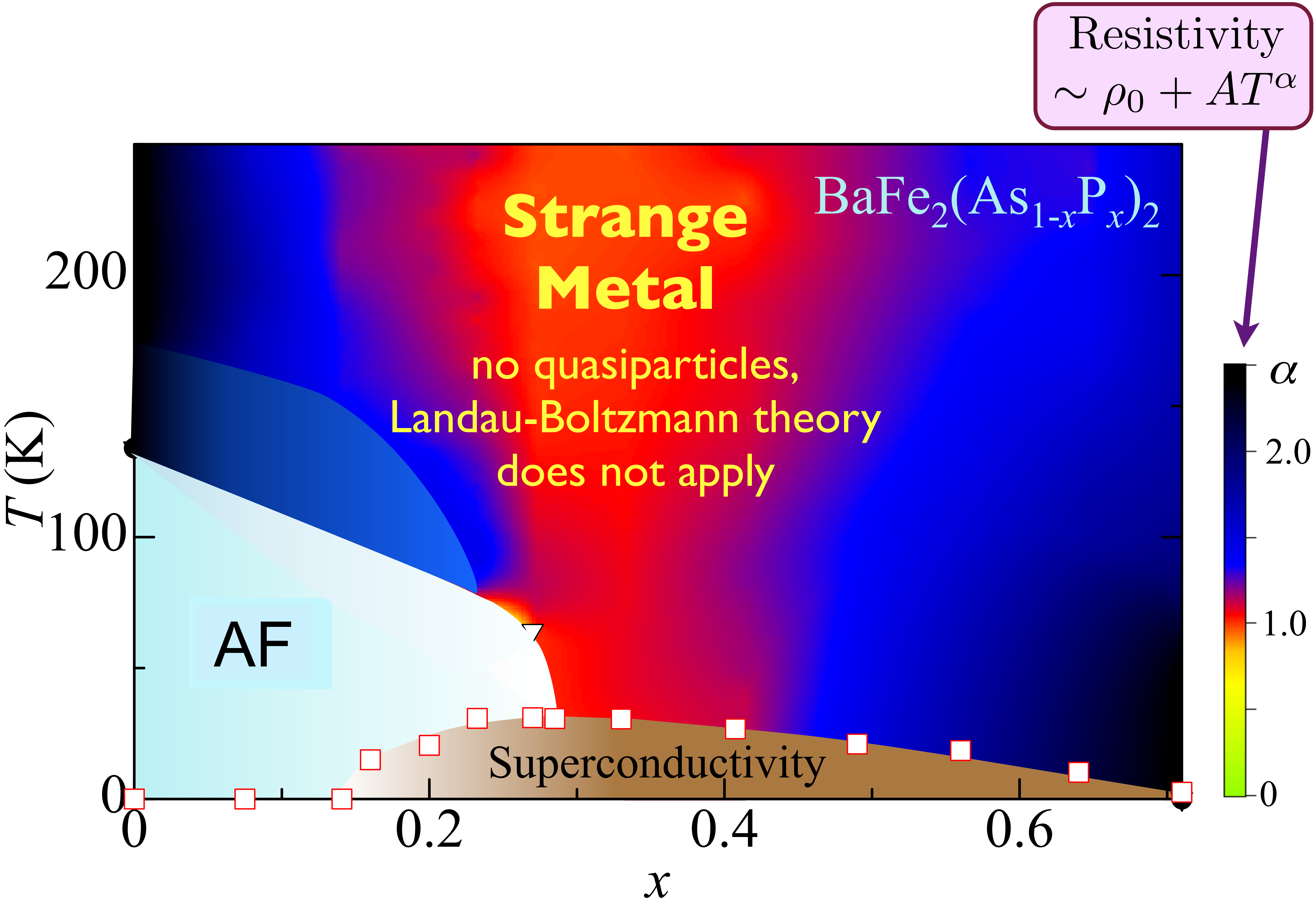

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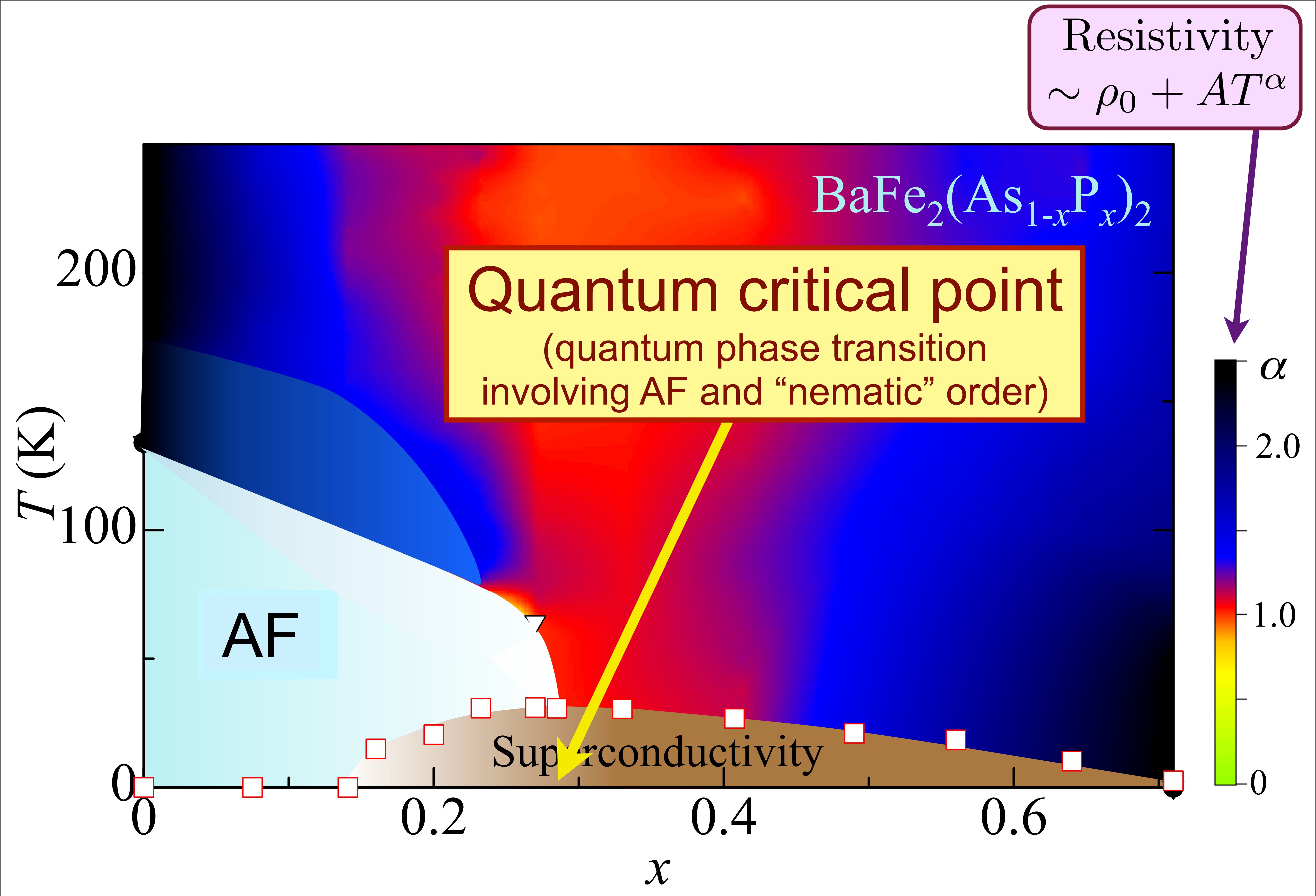

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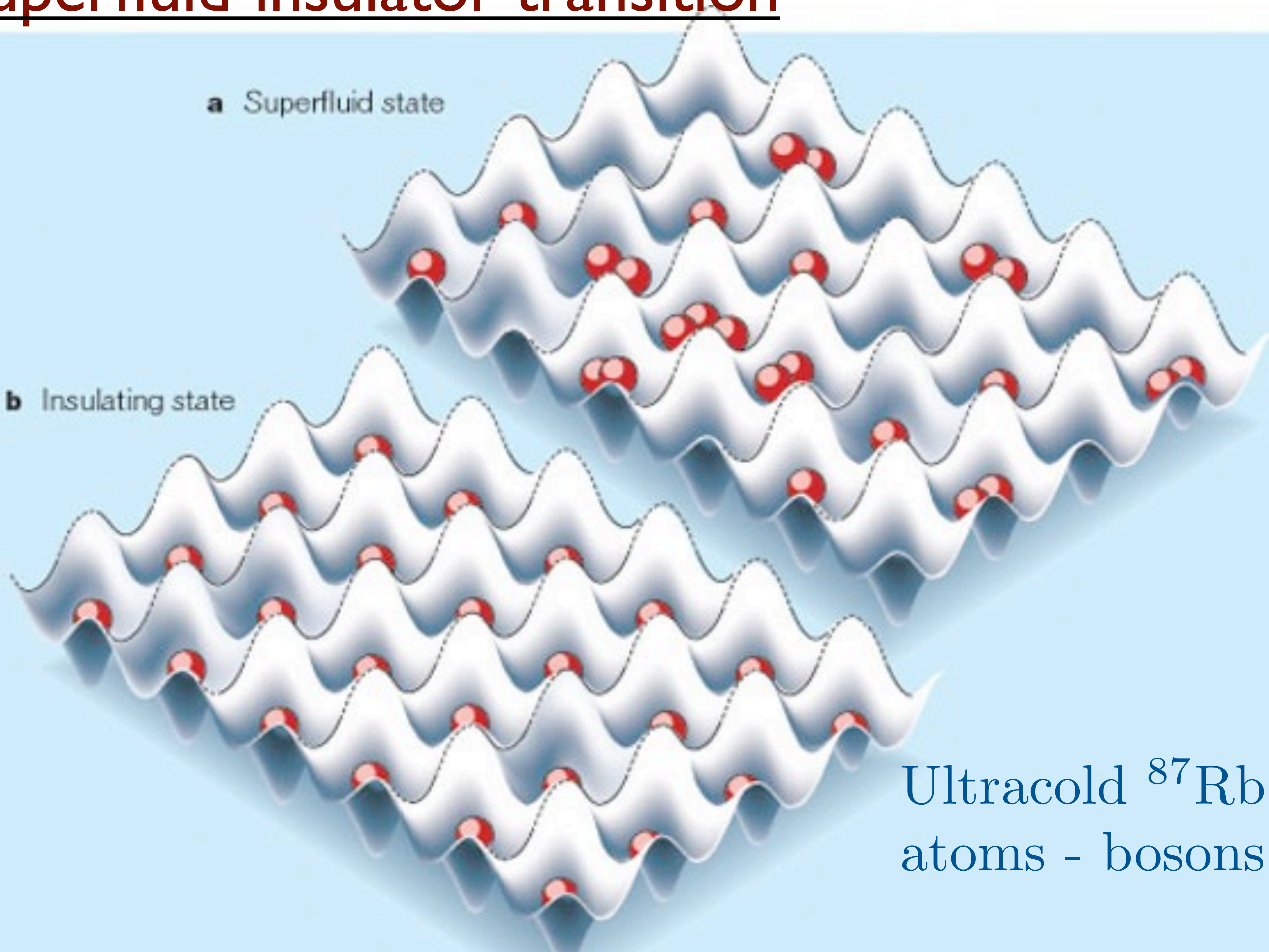
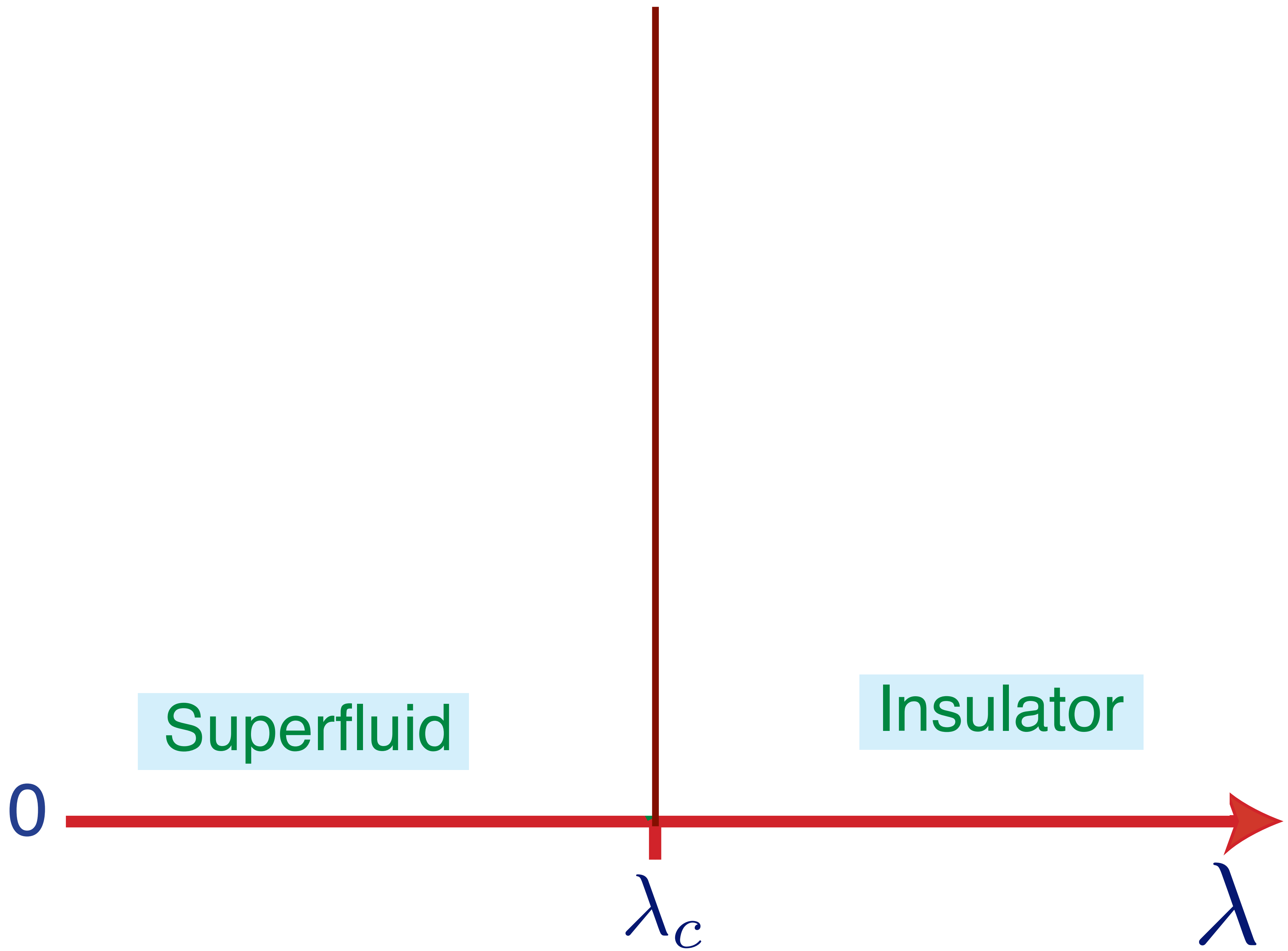
Superfluid-insulator transition

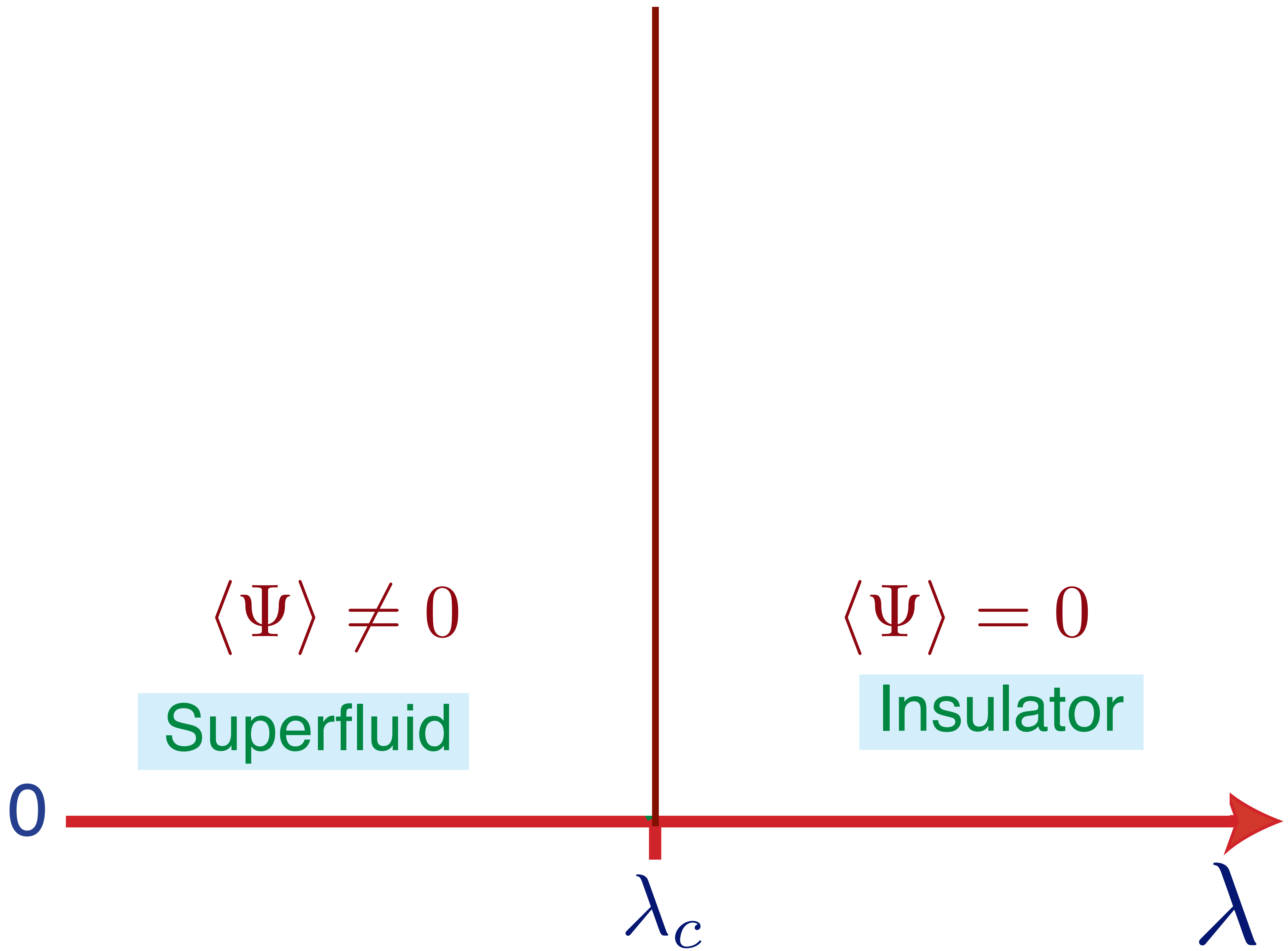


Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

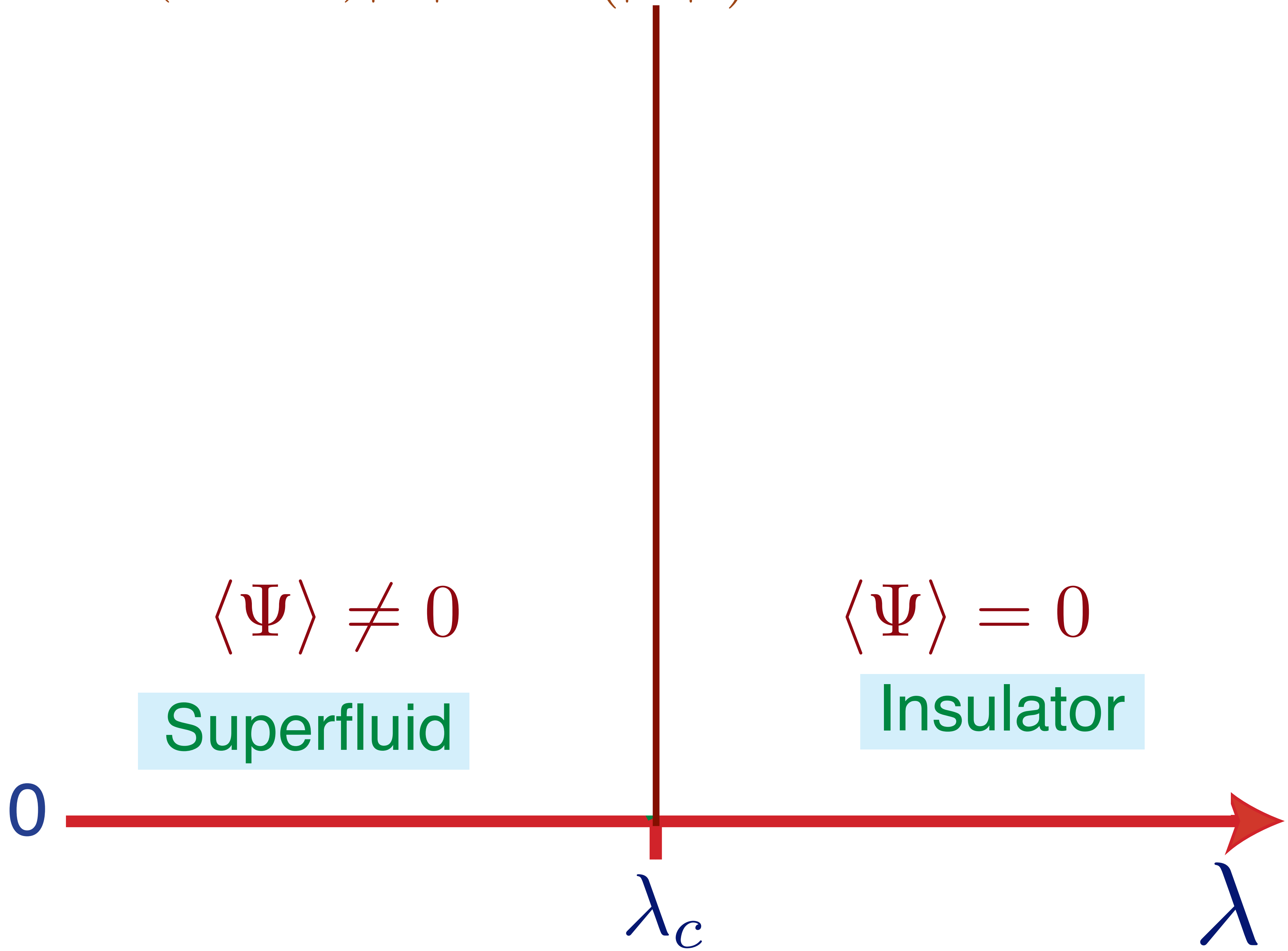

Superfluid
Insulator
0
λ_c
λ

$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



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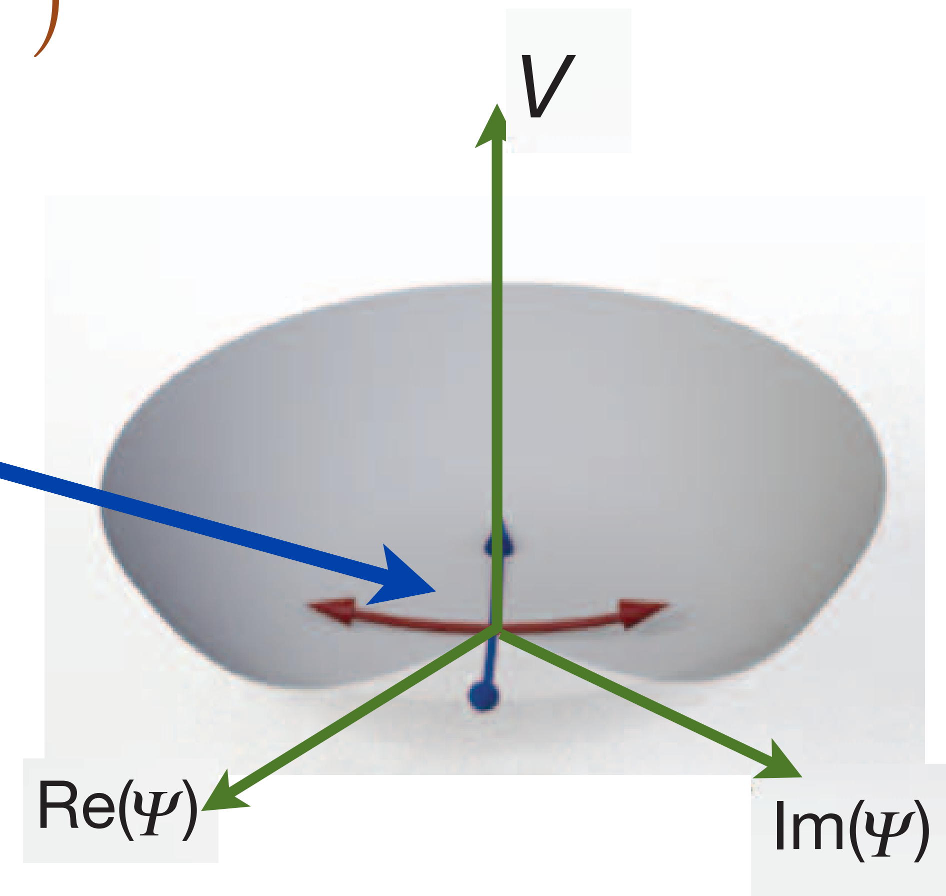
Quanta of oscillations of Ψ: quasiparticles of insulator



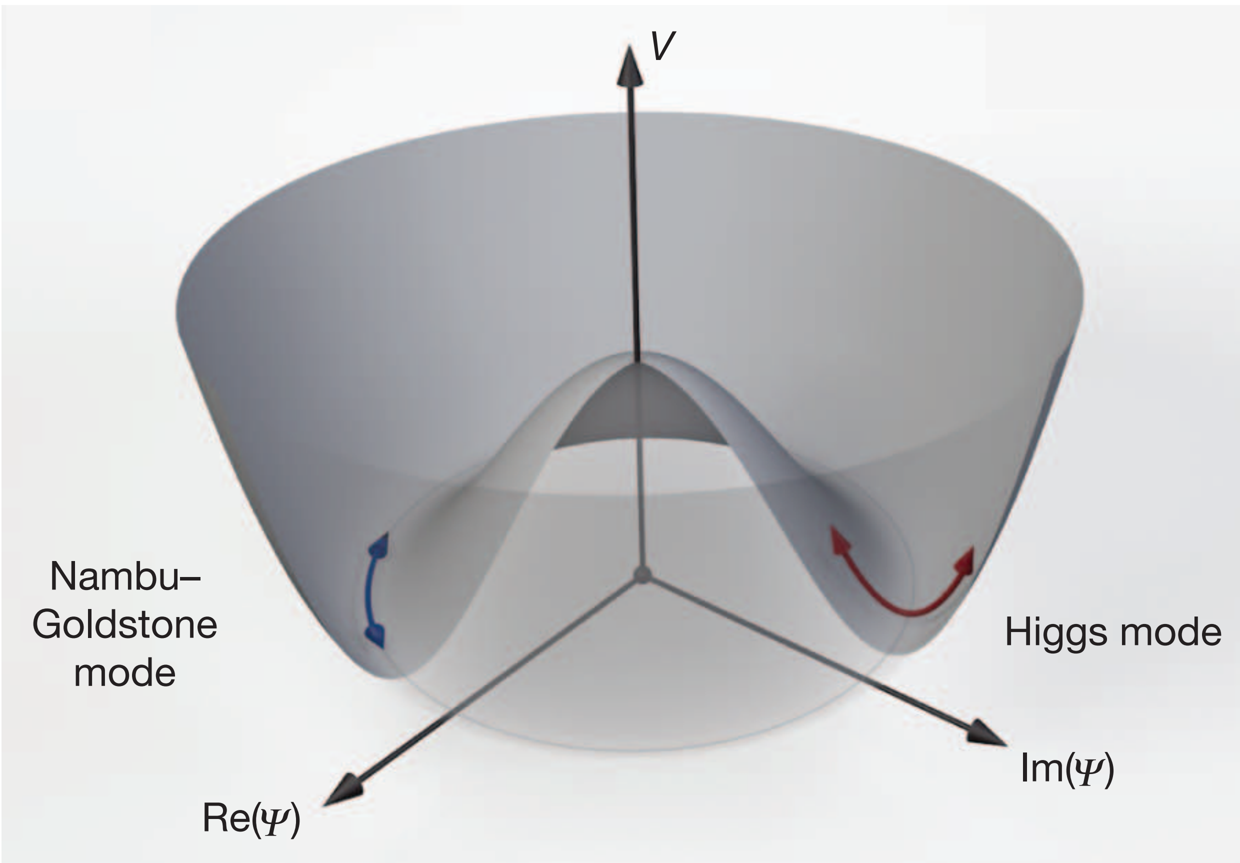
$\langle \Psi \rangle \neq 0$ — Superfluid

$\langle \Psi \rangle = 0$ — Insulator

0 λ_c λ

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

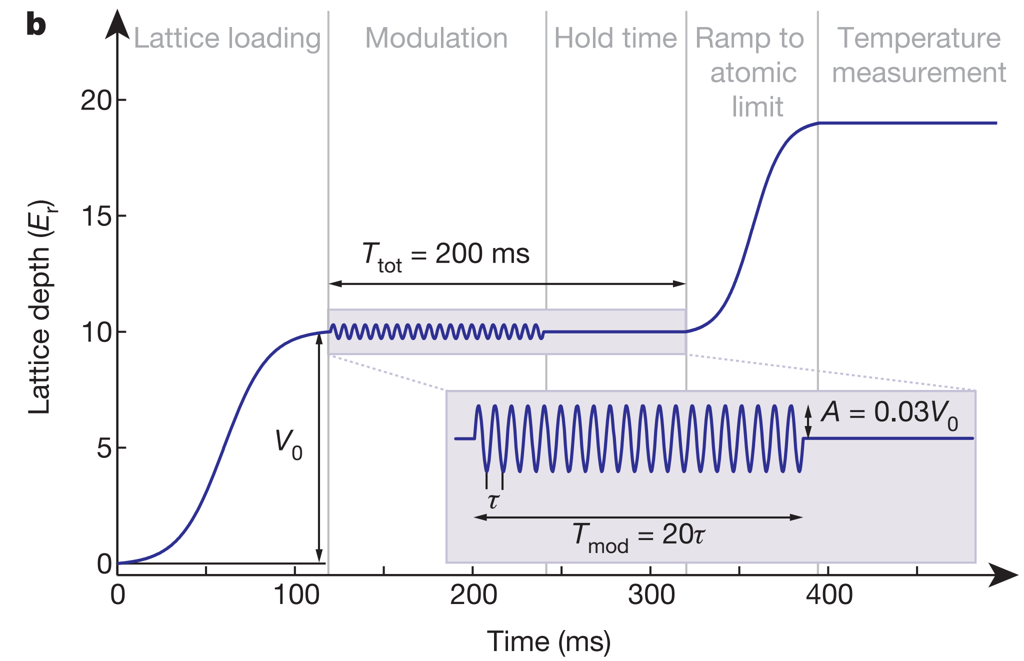
0

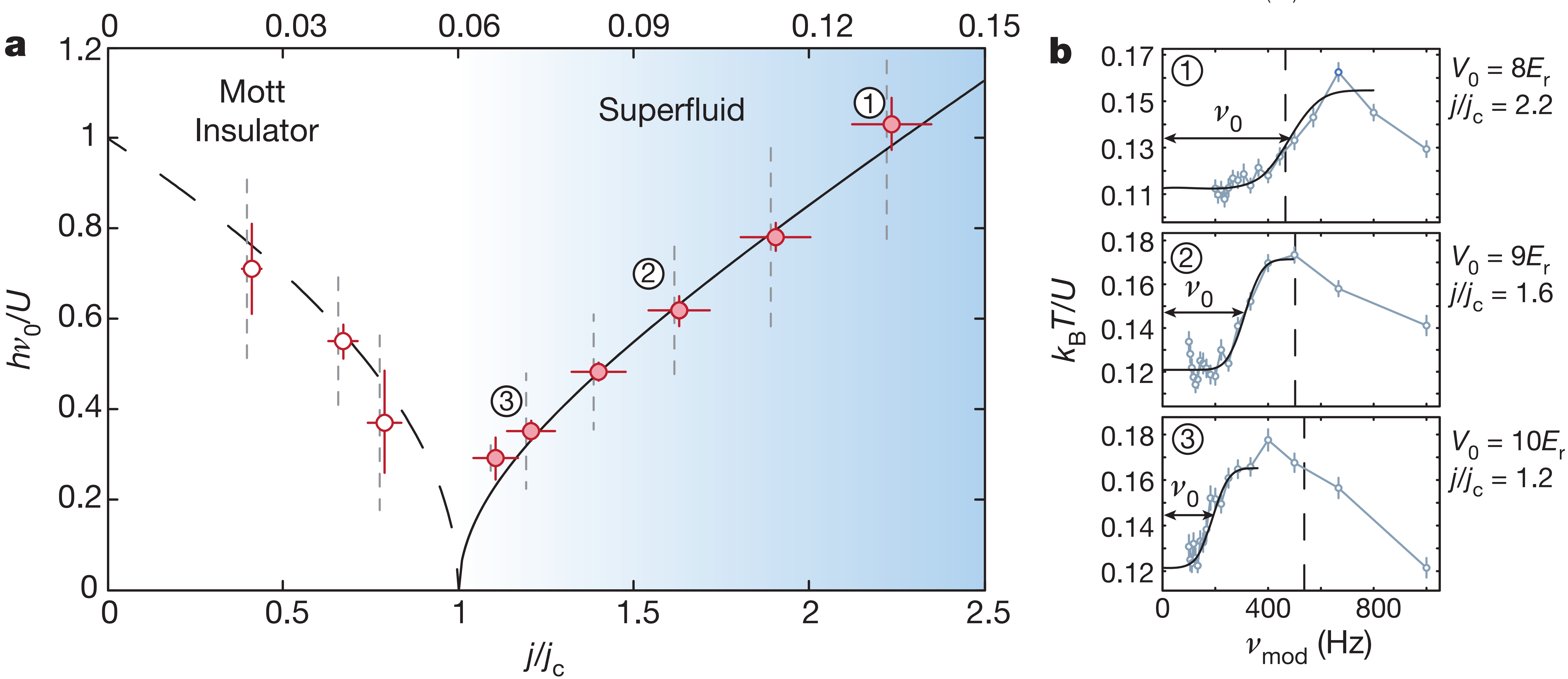
λ_c

λ

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

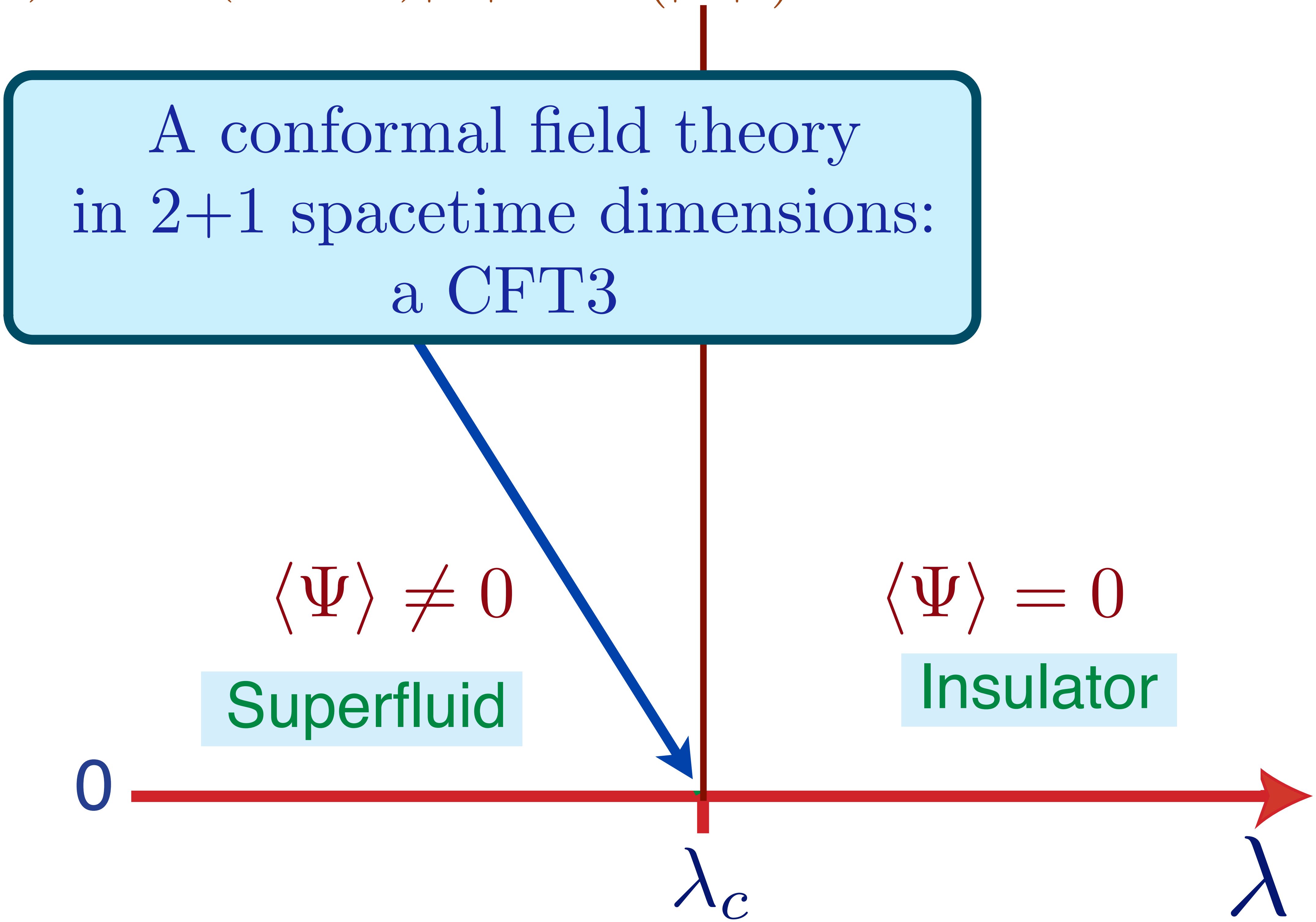




Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

$$\mathcal{S} = \int d^2 r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



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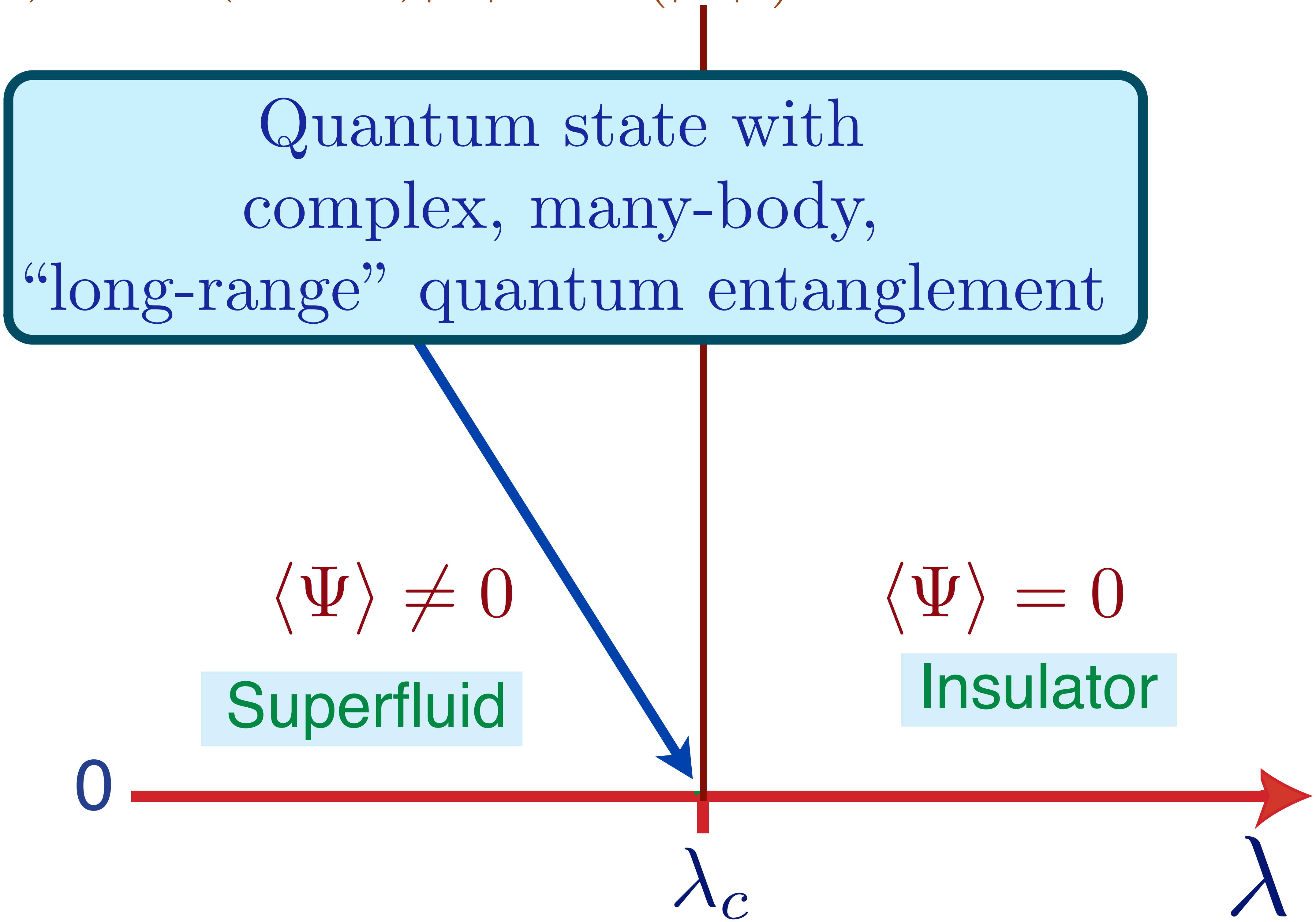

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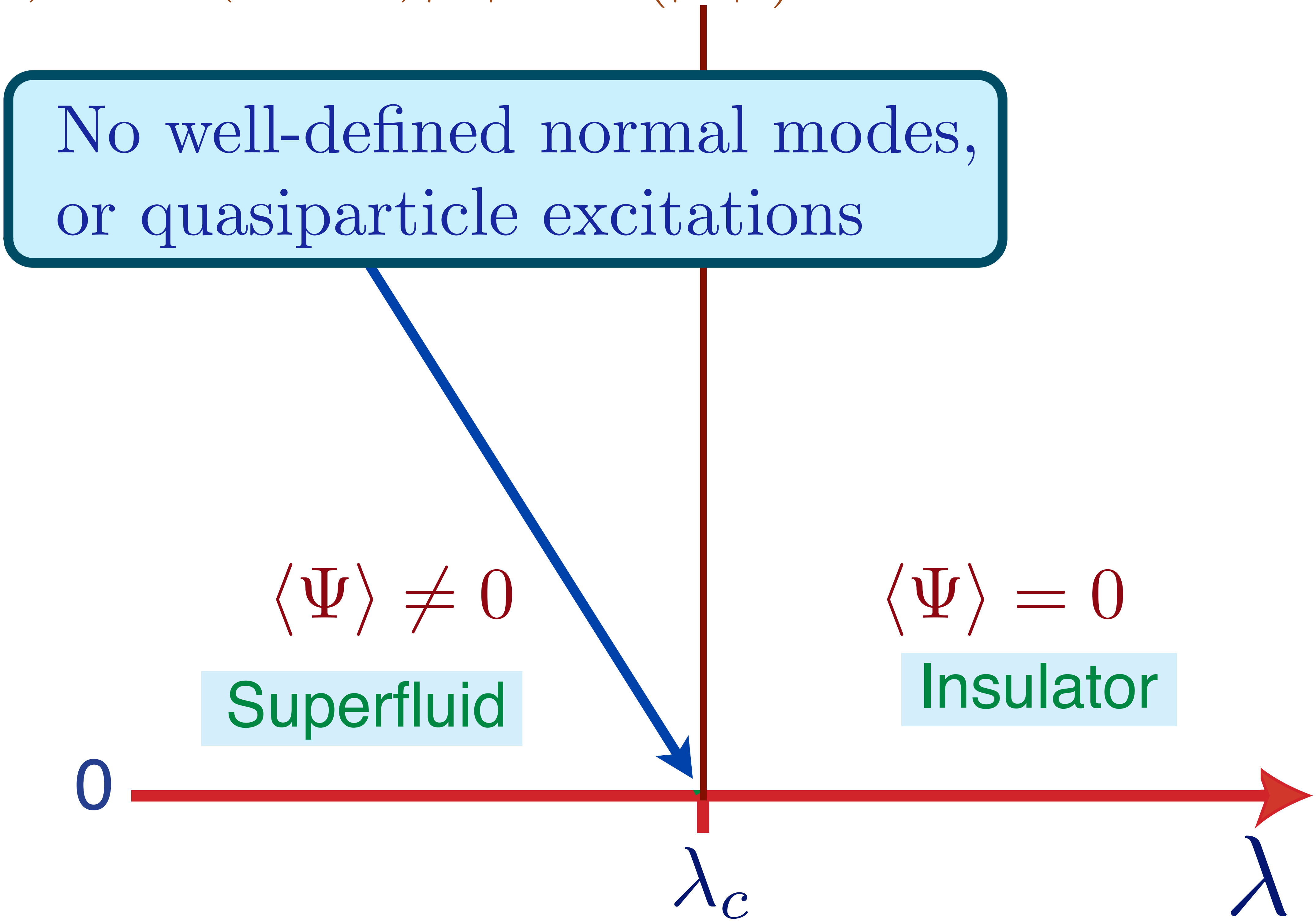
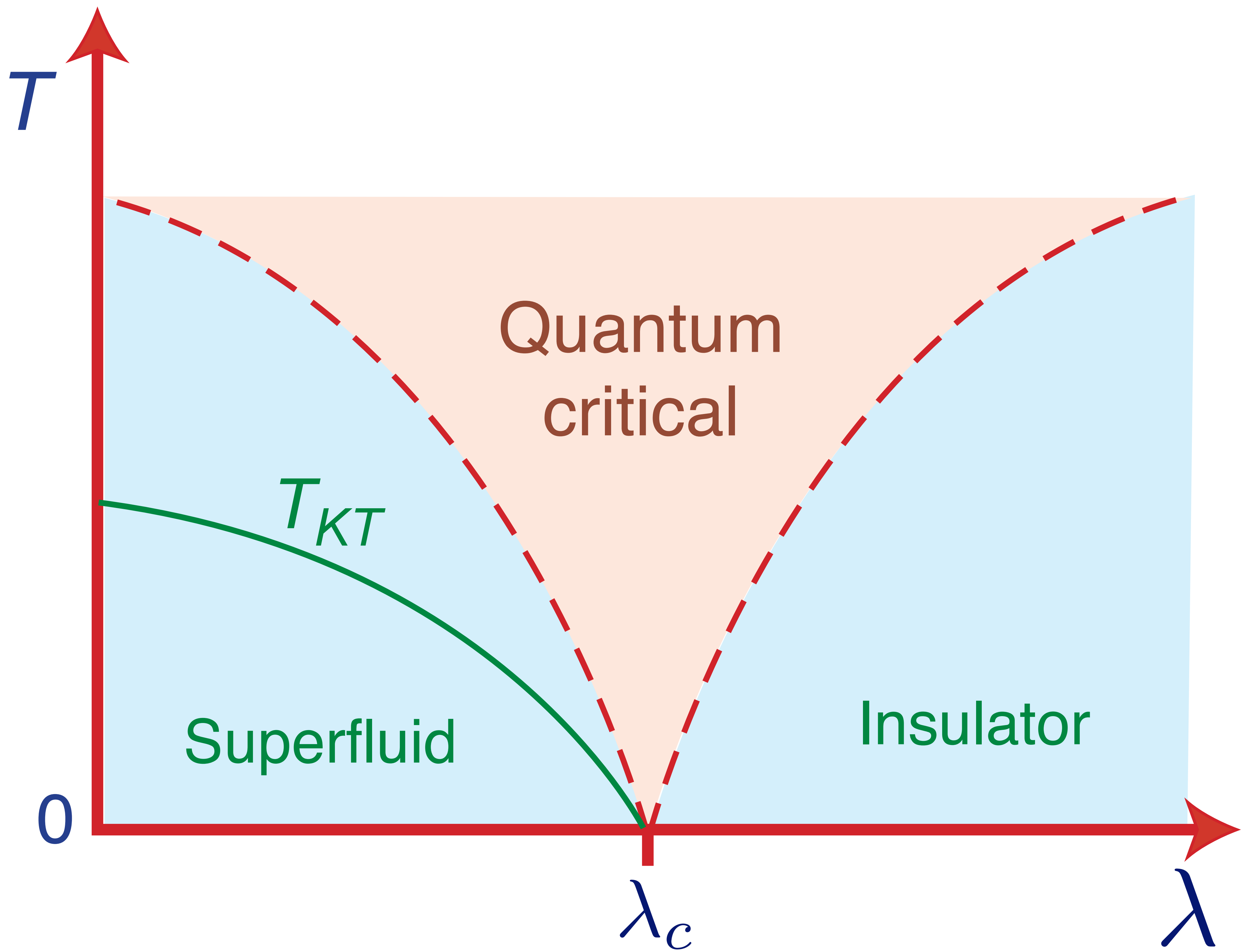
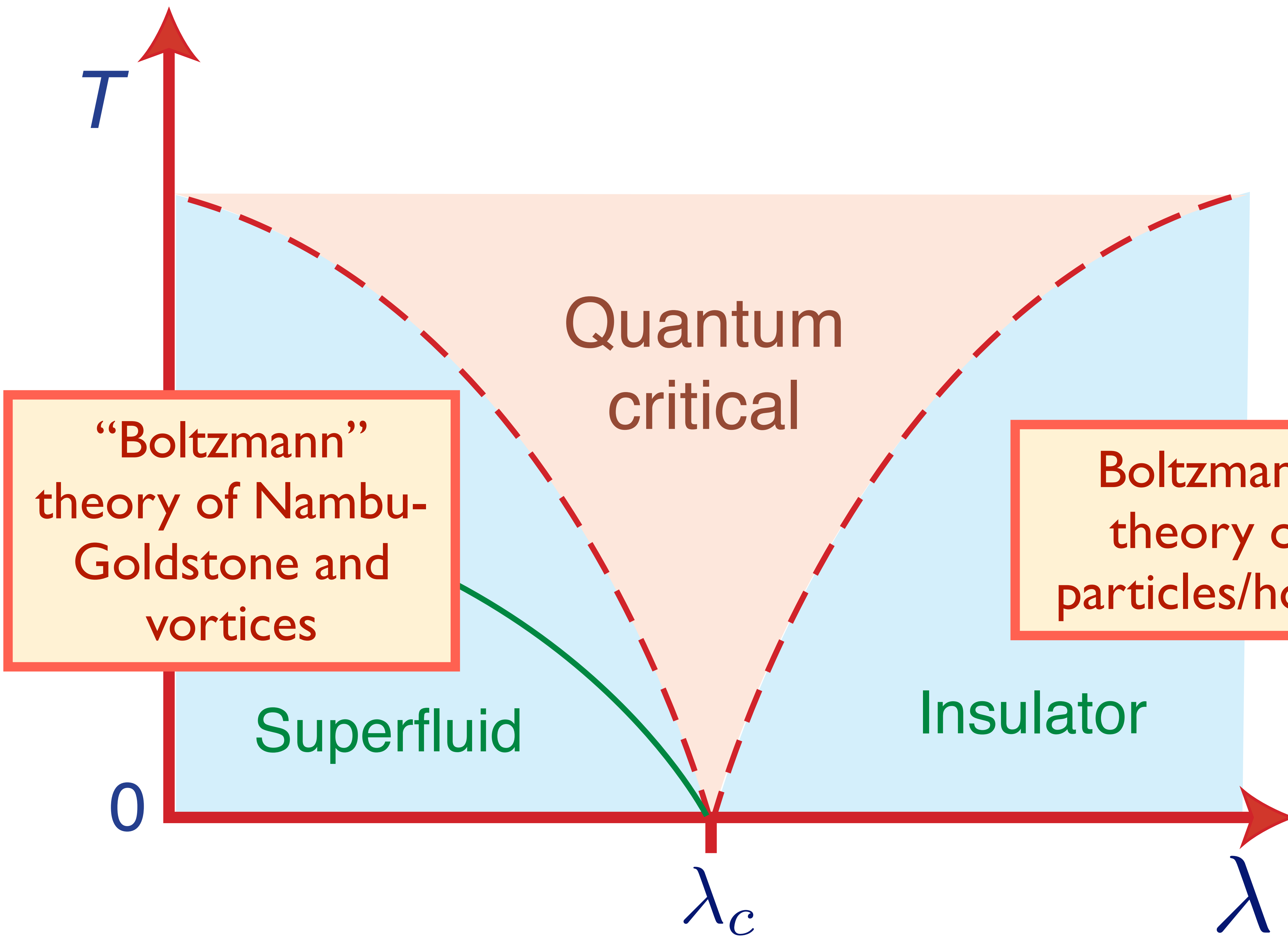
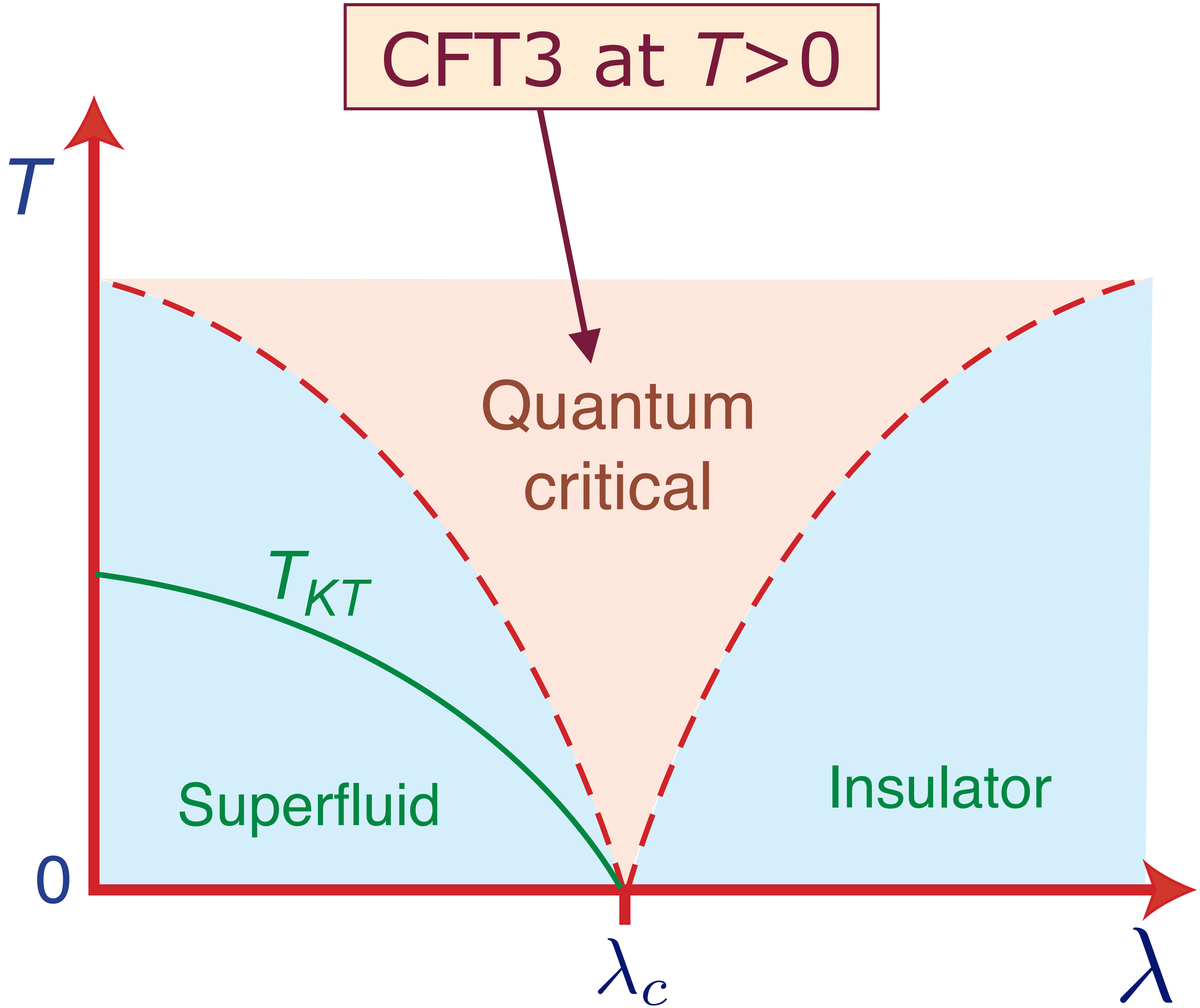
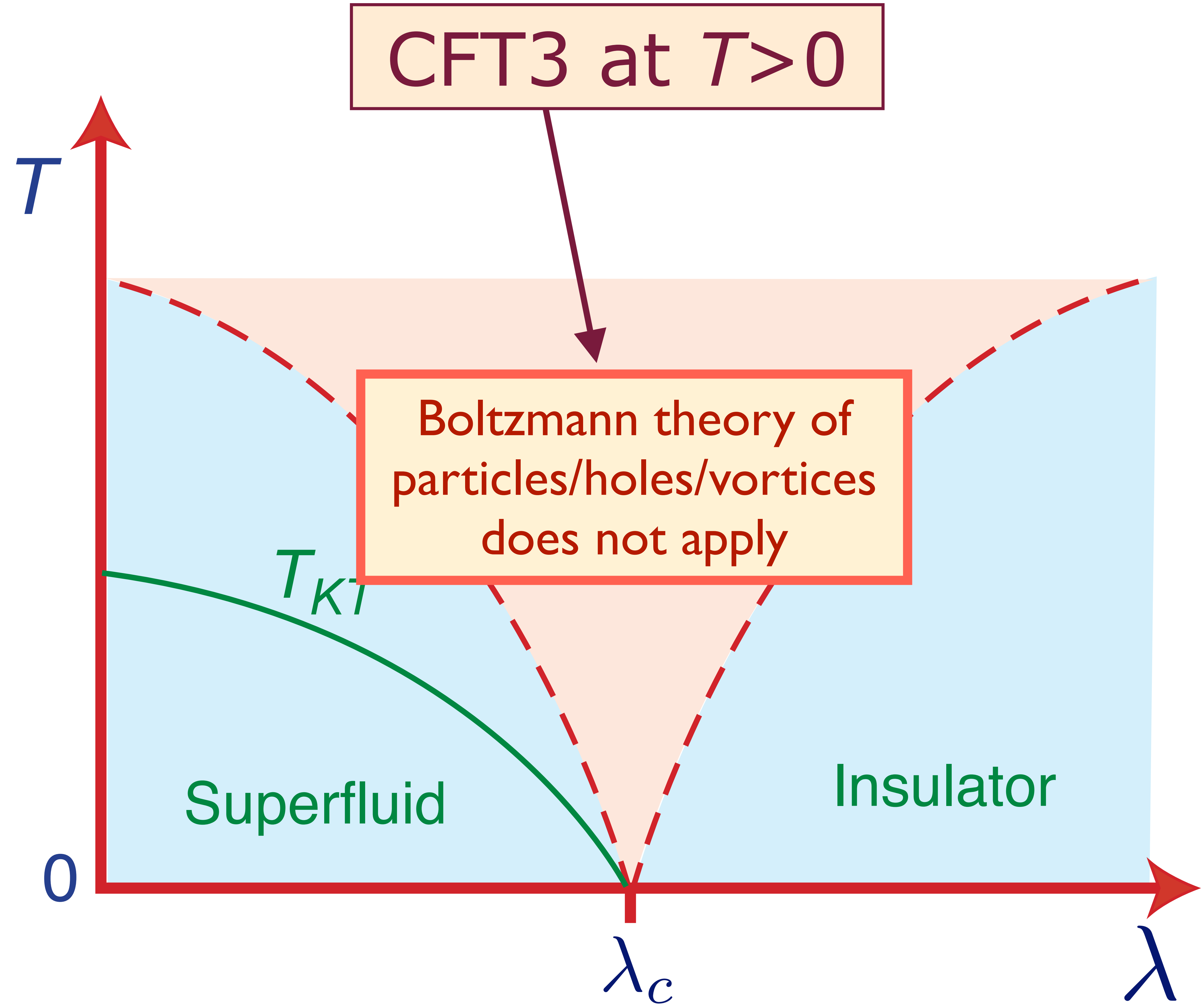


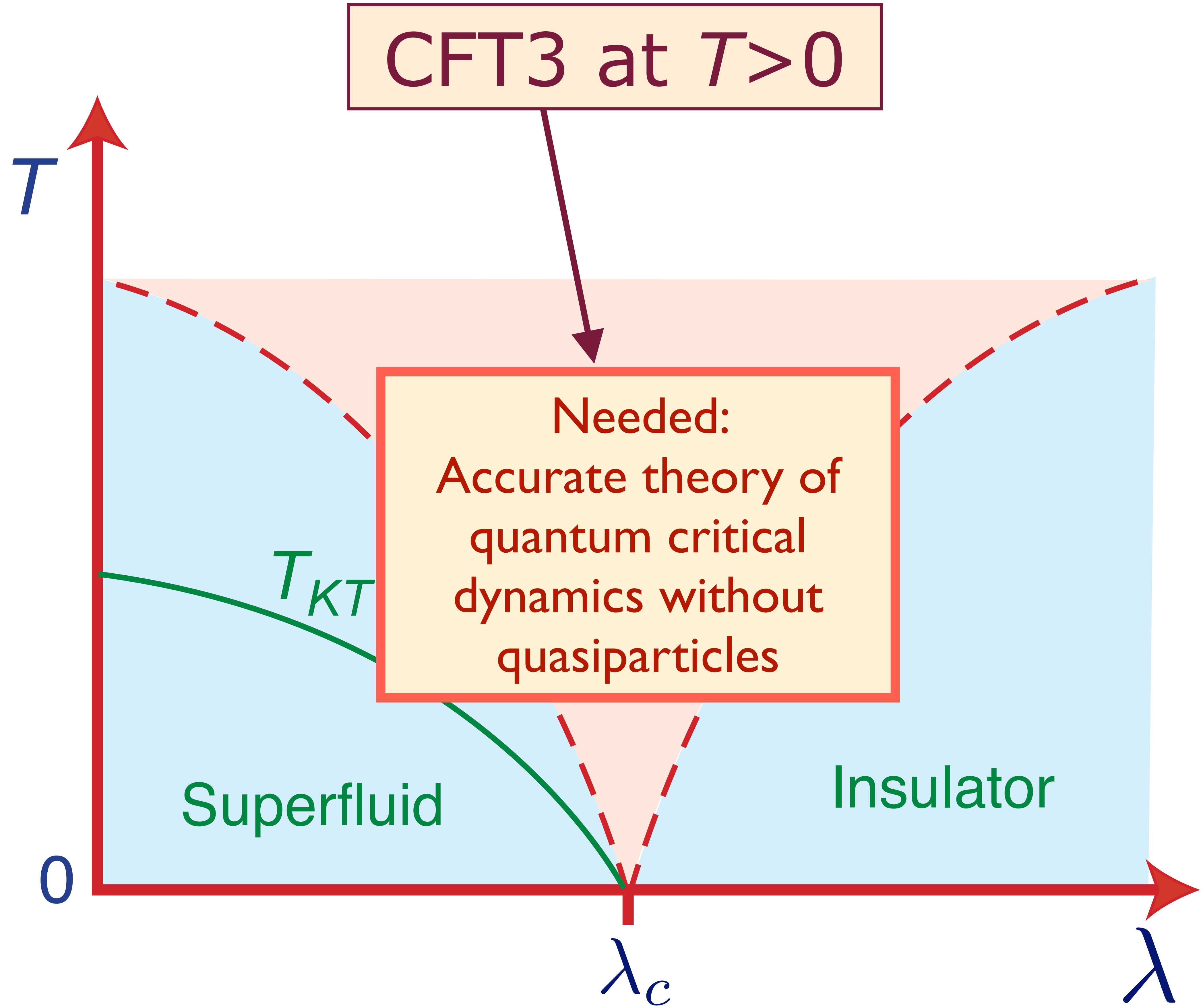


T
Quantum
critical
T_KT
Superfluid
Insulator
0
λ_c
λ


T
0
Quantum
critical
"Boltzmann"
theory of Nambu-
Goldstone and
vortices
Boltzmann
theory of
particles/holes
Superfluid
Insulator
λ_c
λ


CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at $T>0$
Boltzmann theory of particles/holes/vortices does not apply
T
T_{KT}
Superfluid
Insulator
0
λ_c
λ


CFT3 at $T>0$
T
Needed:
Accurate theory of
quantum critical
dynamics without
quasiparticles
T_{KT}
Superfluid
Insulator
0
λ_c
λ

Quantum critical dynamics

Quantum *"nearly perfect fluid"*
with shortest possible *local* equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C} \frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant.

Response functions are characterized by poles in LHP with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in the holographic theory.

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical dynamics

Transport co-oefficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

$$\sigma(\omega) = \frac{Q^2}{h} \times \left[\text{Universal function of } \frac{\hbar\omega}{k_B T}\right]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Outline

1. The simplest models without quasiparticles

A. Magnetic insulators in two dimensions

B. Ultracold atoms in optical lattices

C. Conformal field theories in

2+1 dimensions and

the AdS/CFT correspondence

2. Metals without quasiparticles

High temperature superconductivity

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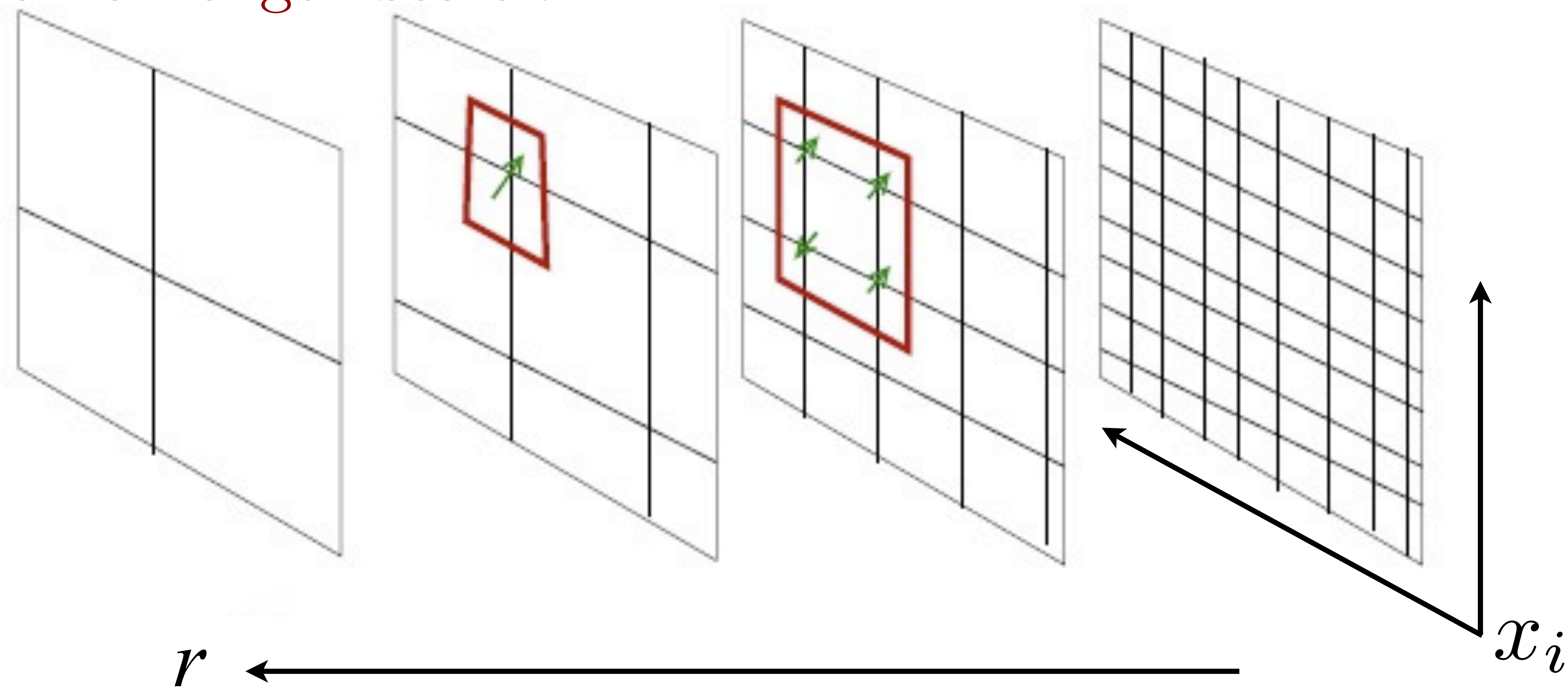
Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale r



J. McGreevy, arXiv0909.0518

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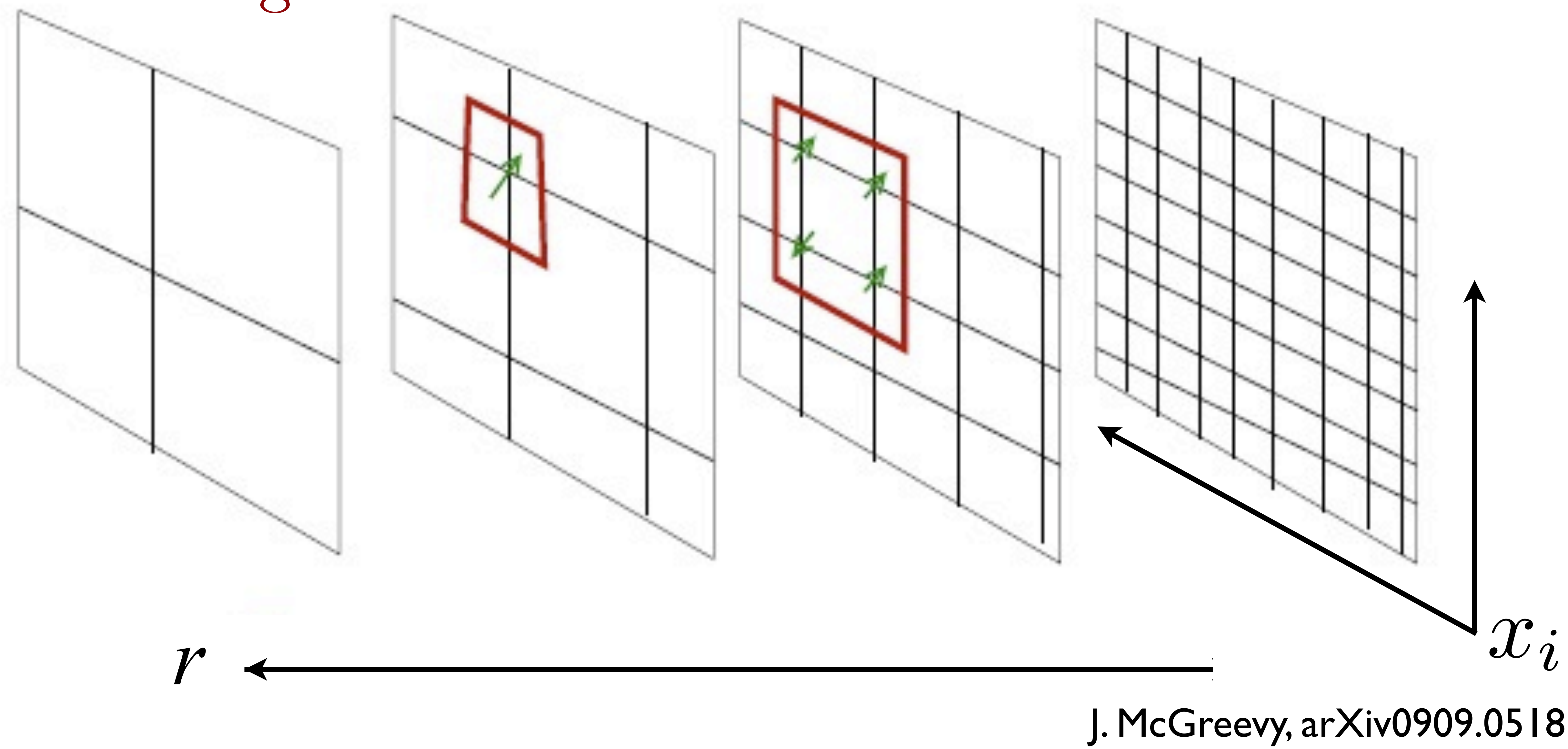
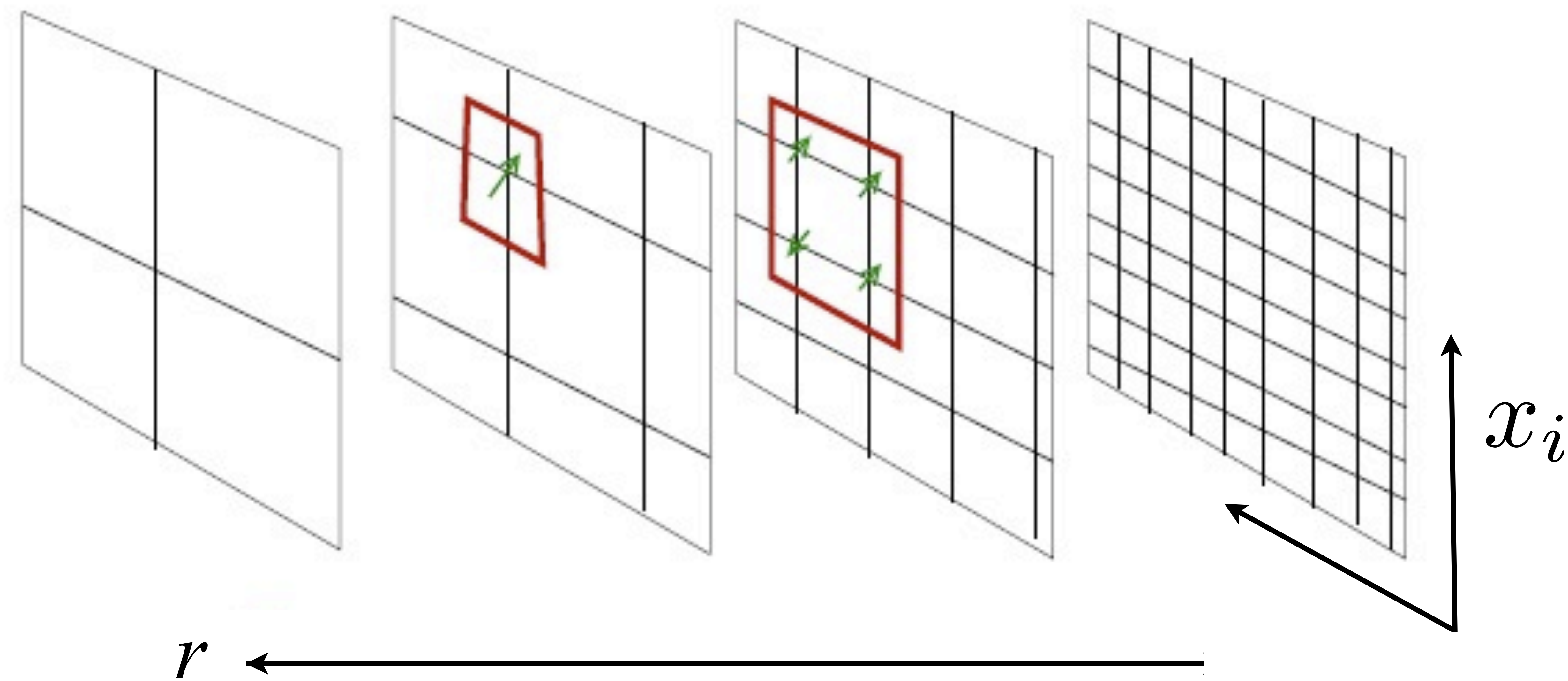


Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

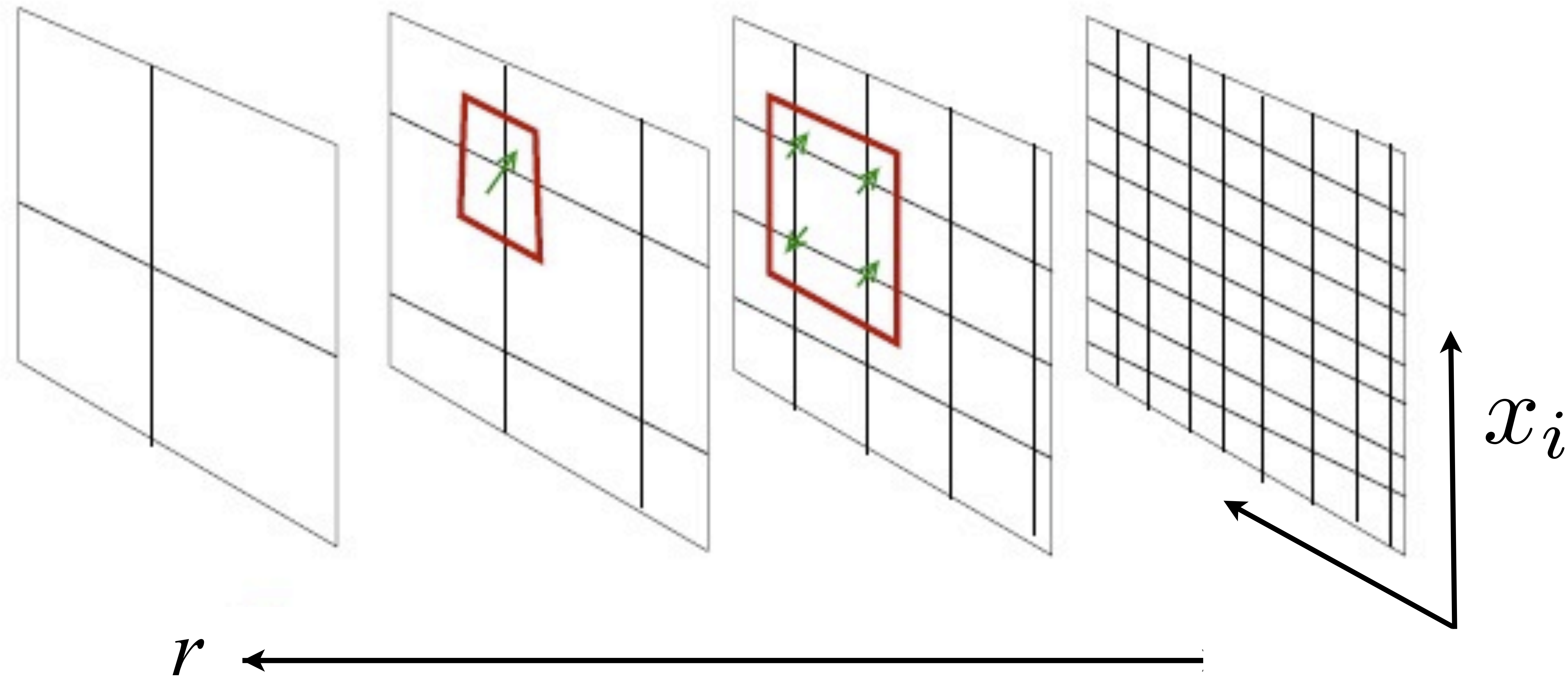
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \dots d$)

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography

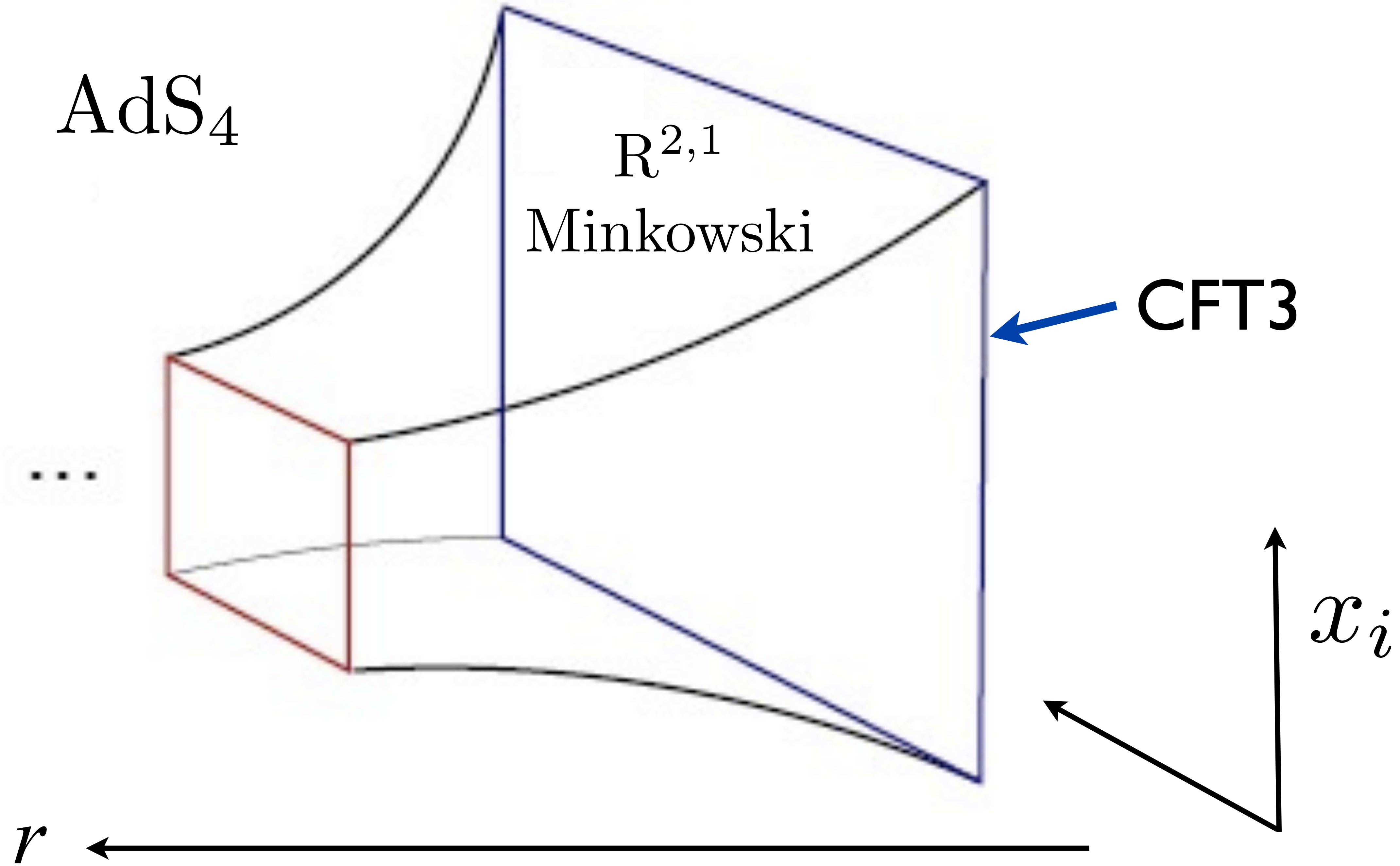


This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

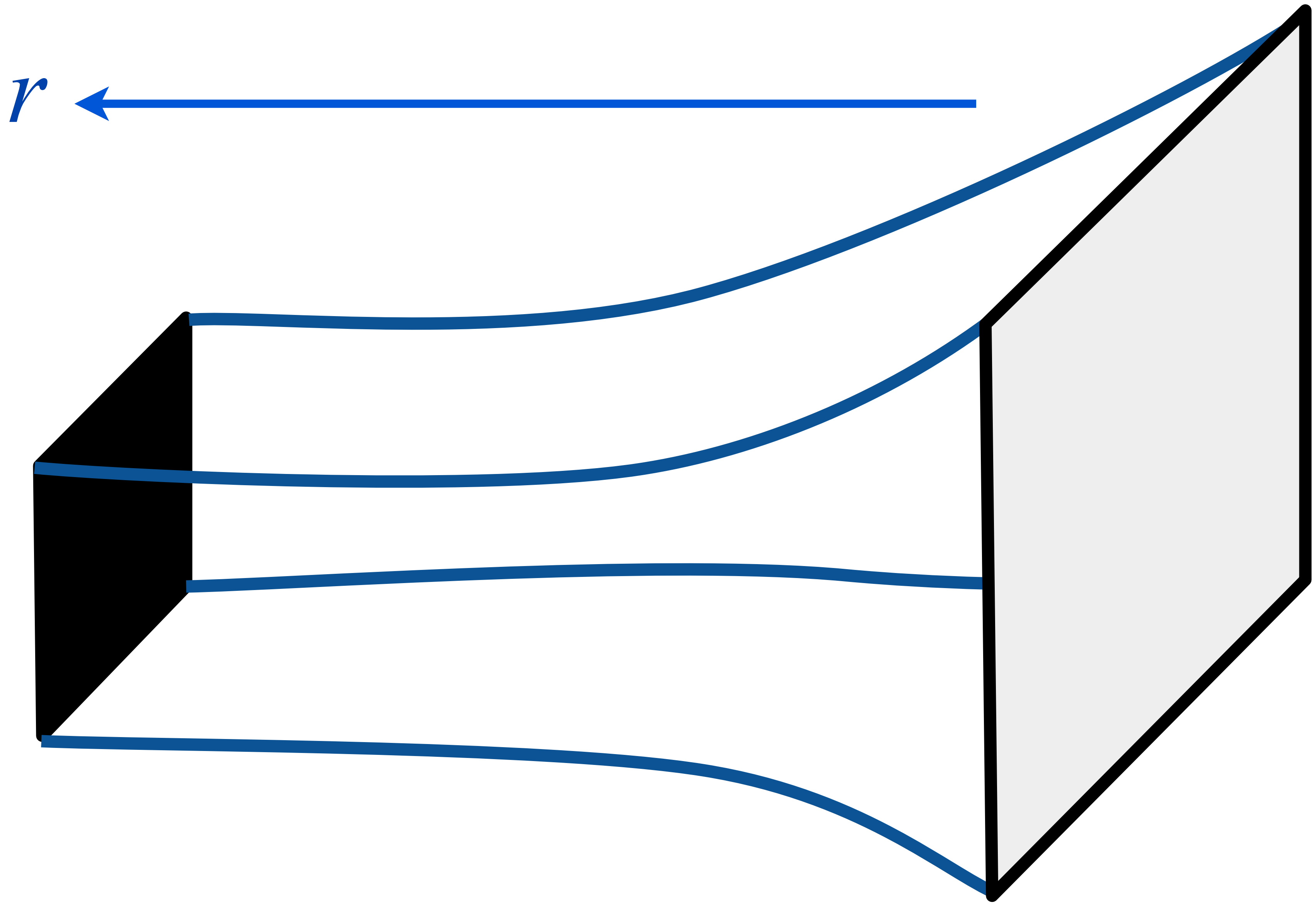
AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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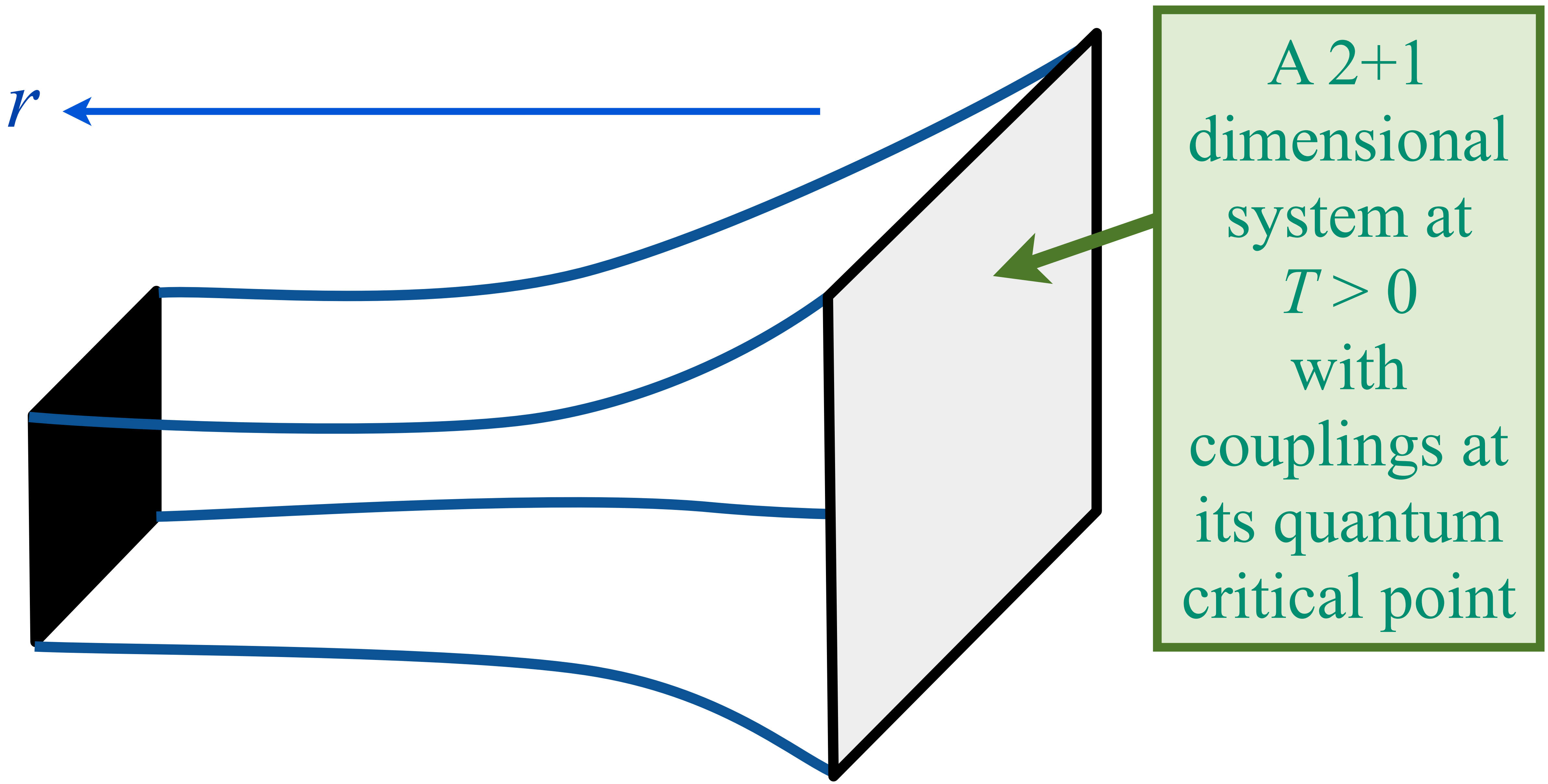

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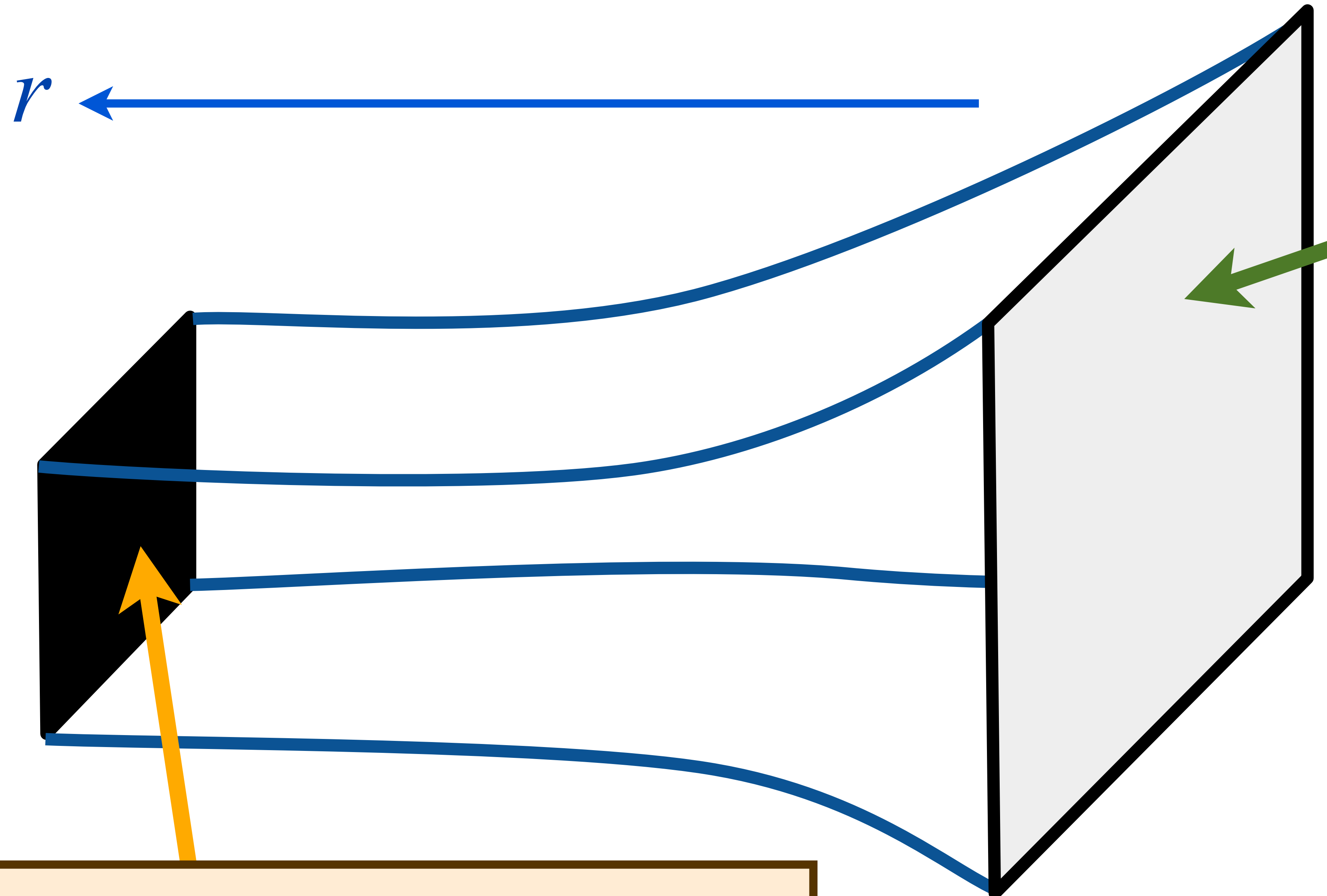

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

A "horizon", similar to the surface of a black hole !

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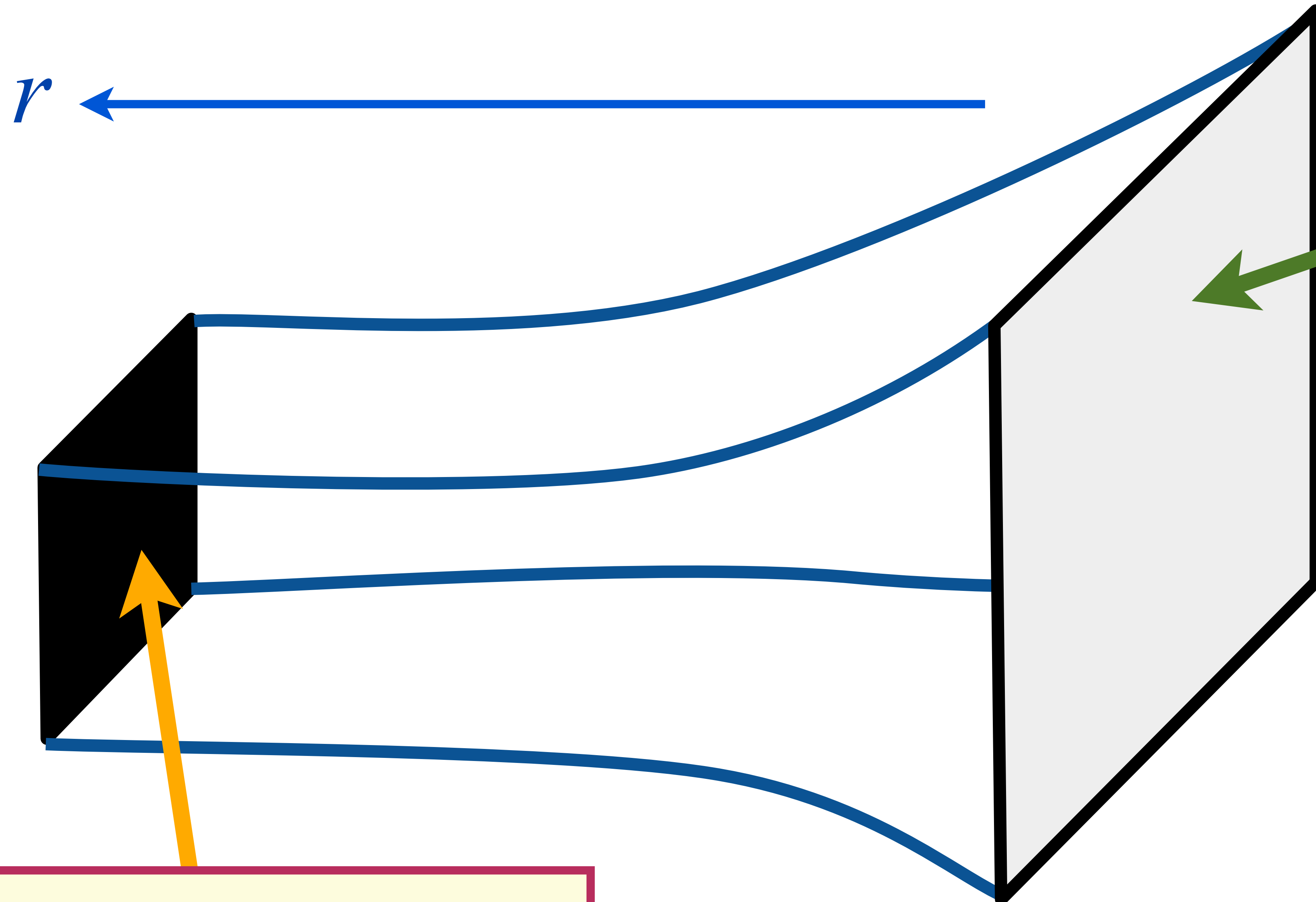

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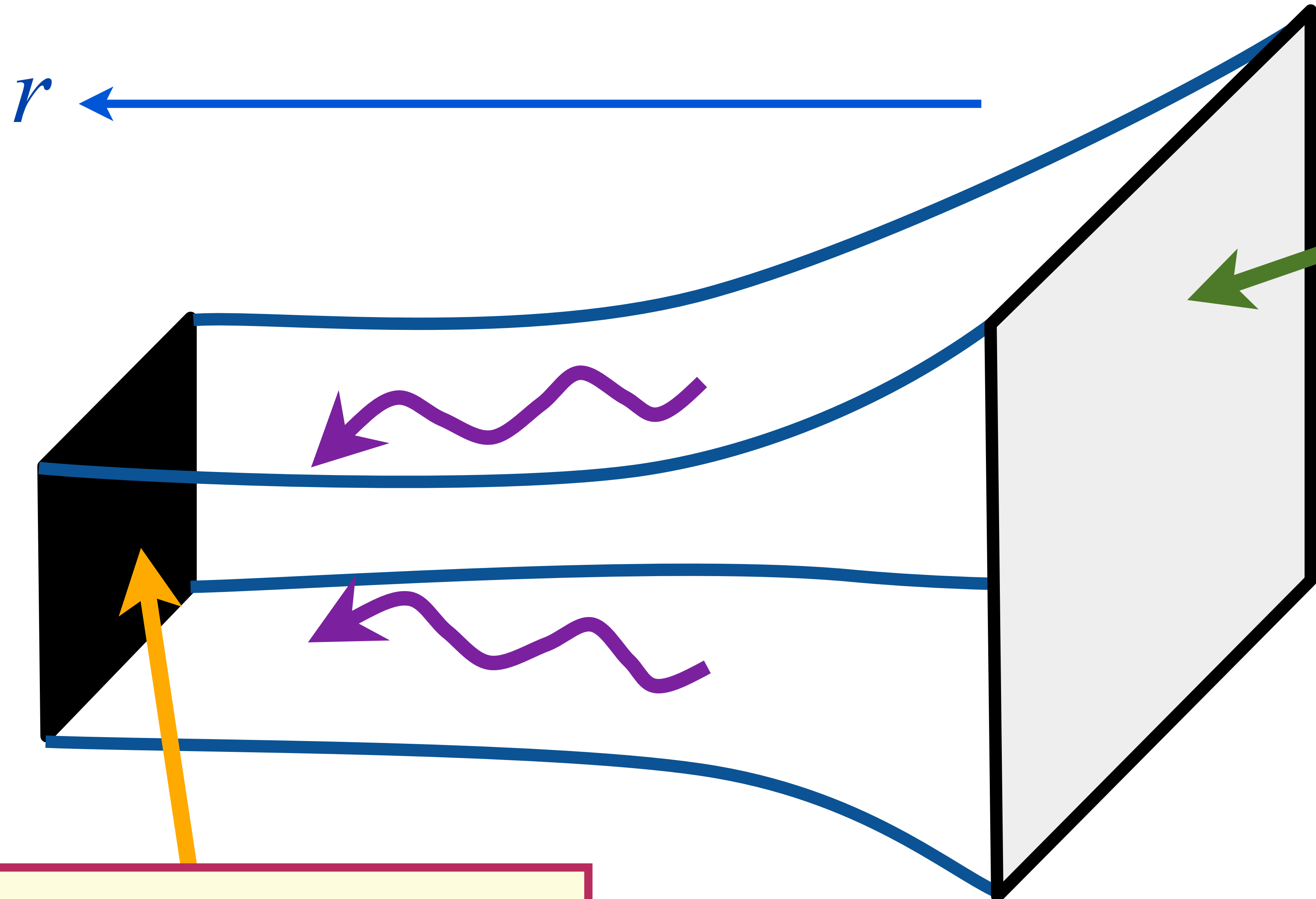
The temperature and entropy of the horizon equal those of the quantum critical point

Gauge-gravity duality at non-zero temperatures

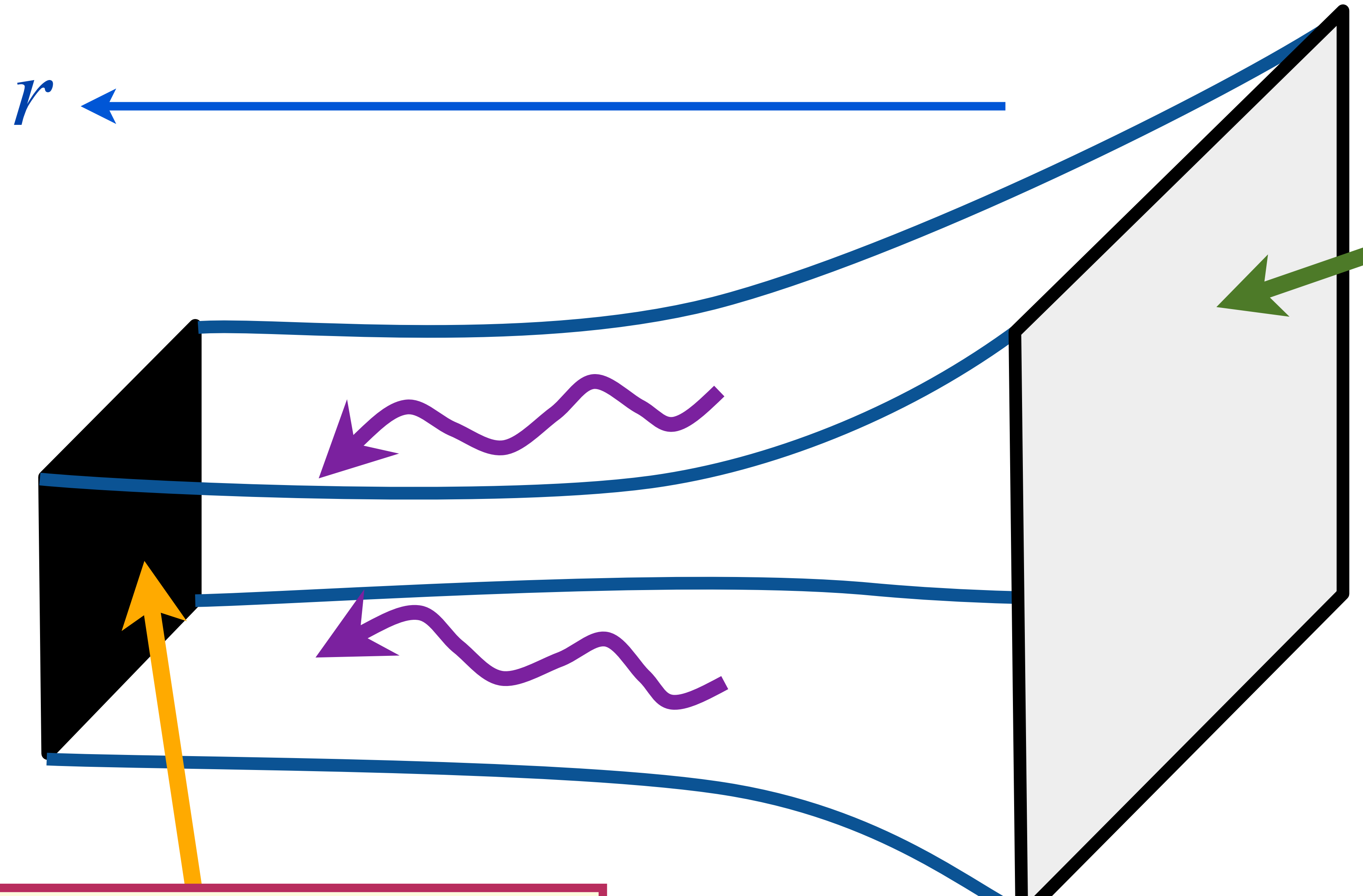


A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Quasi-normal modes of quantum criticality = waves falling into black hole

Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Characteristic damping time of quasi-normal modes: $(k_B/\hbar)\times$ Hawking temperature

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

AdS$_4$ theory of quantum criticality

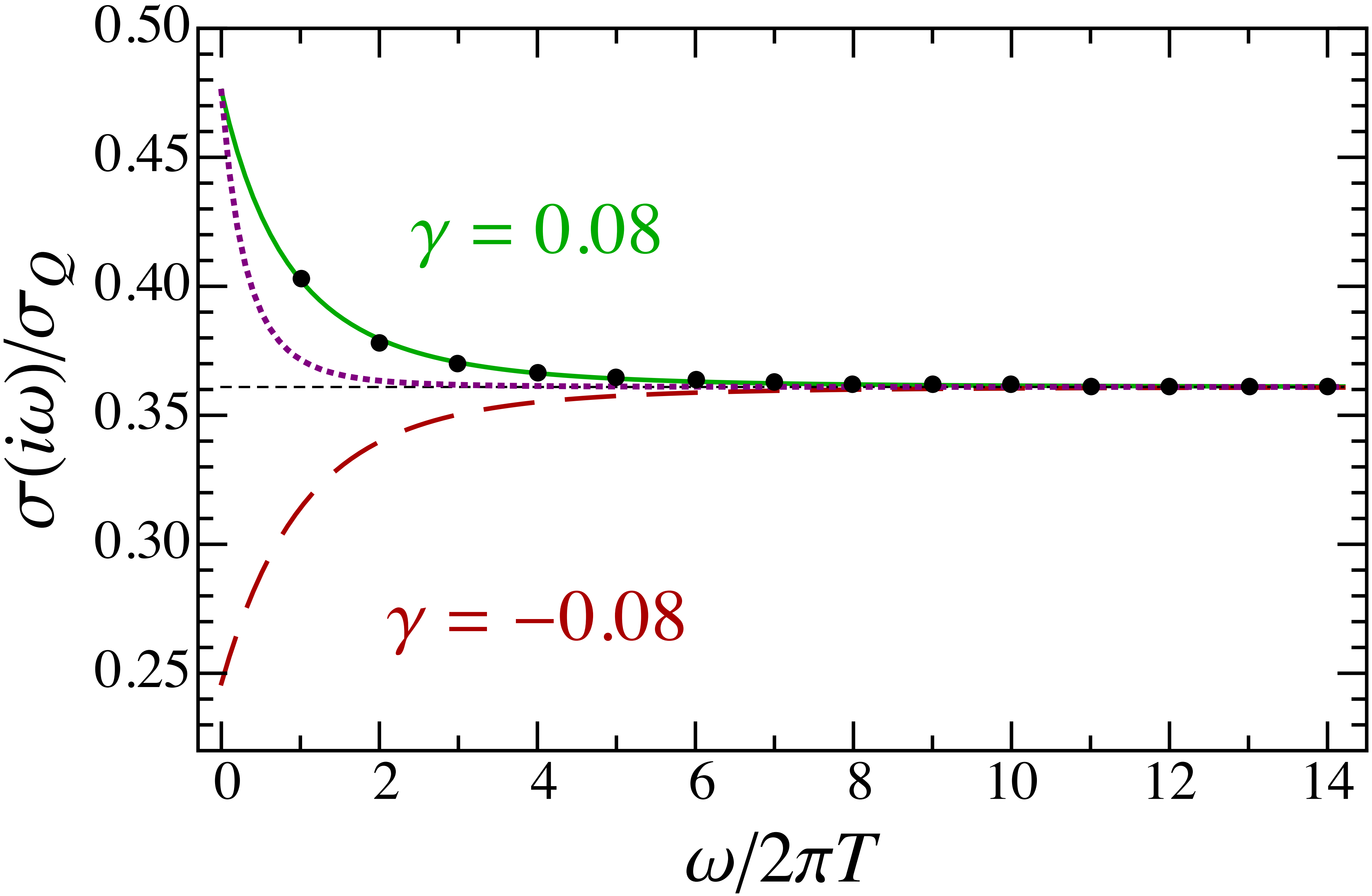
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS$_4$ theory of quantum criticality

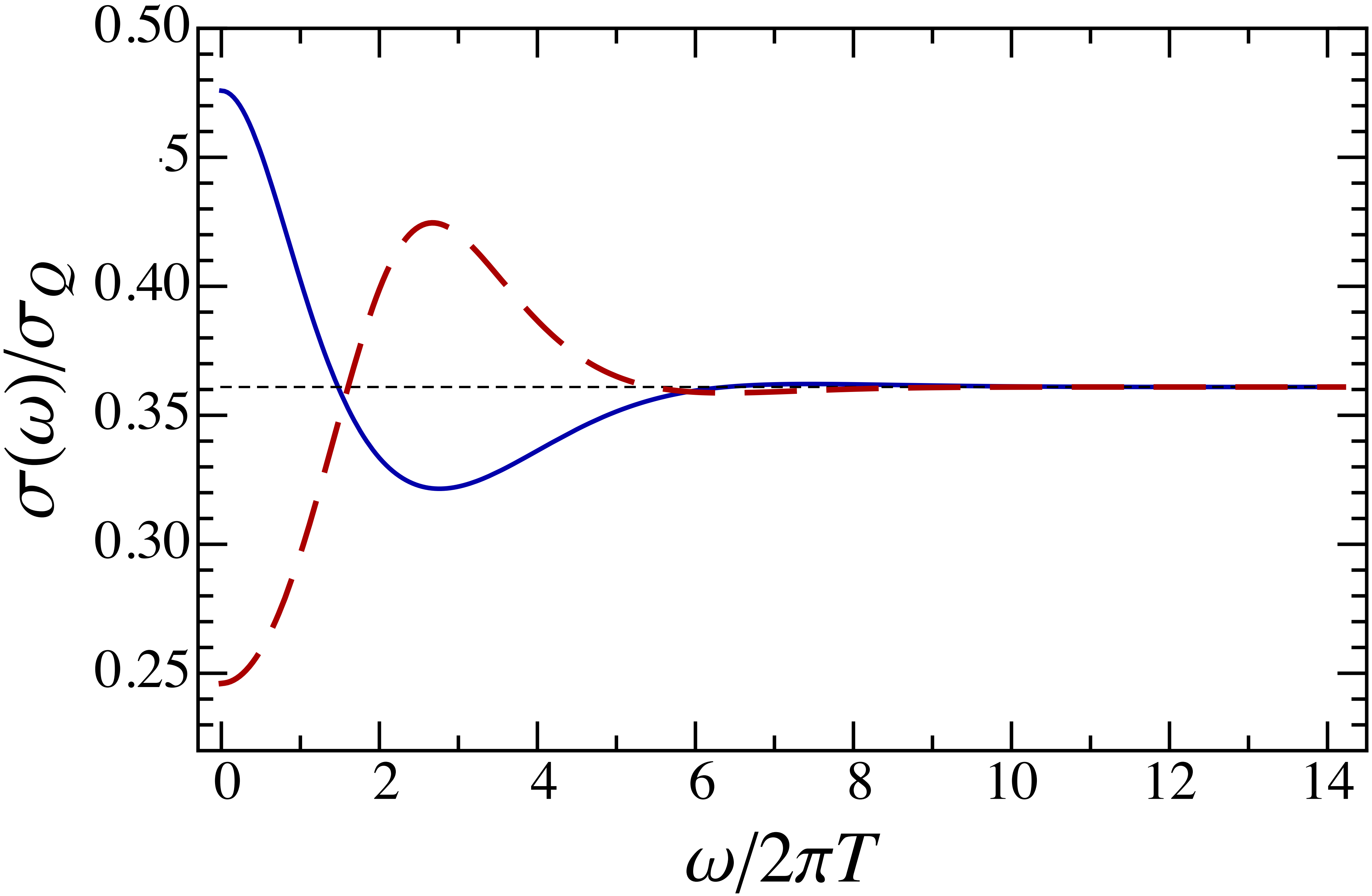


Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective T and taking $\sigma_Q = 1/g_M^2$.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

AdS_4 theory of quantum criticality



Predictions of holographic theory,
after analytic continuation to real frequencies

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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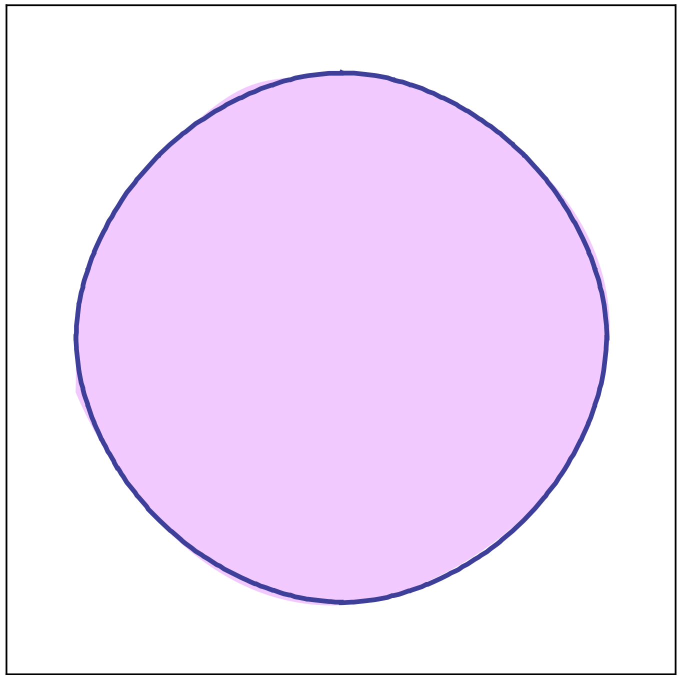
Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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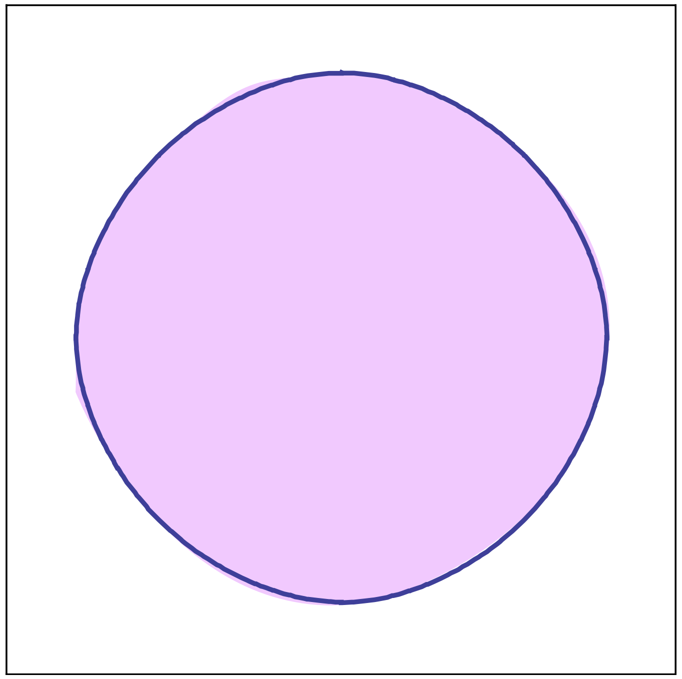
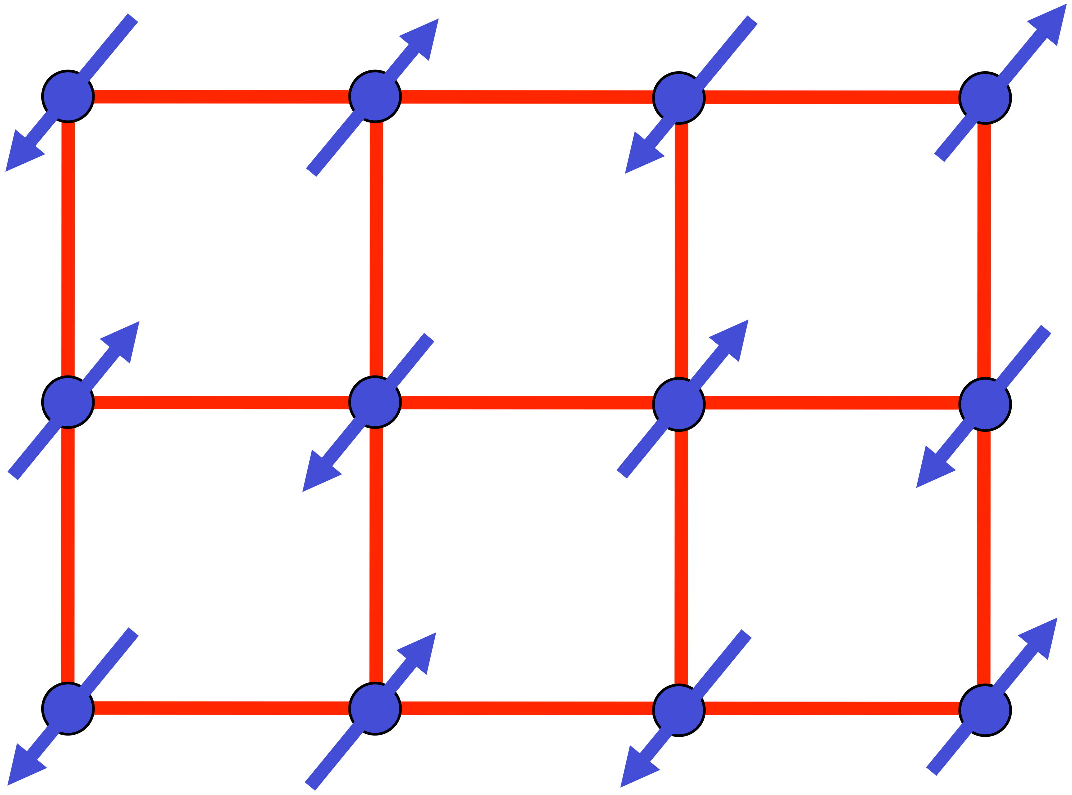


+

The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.



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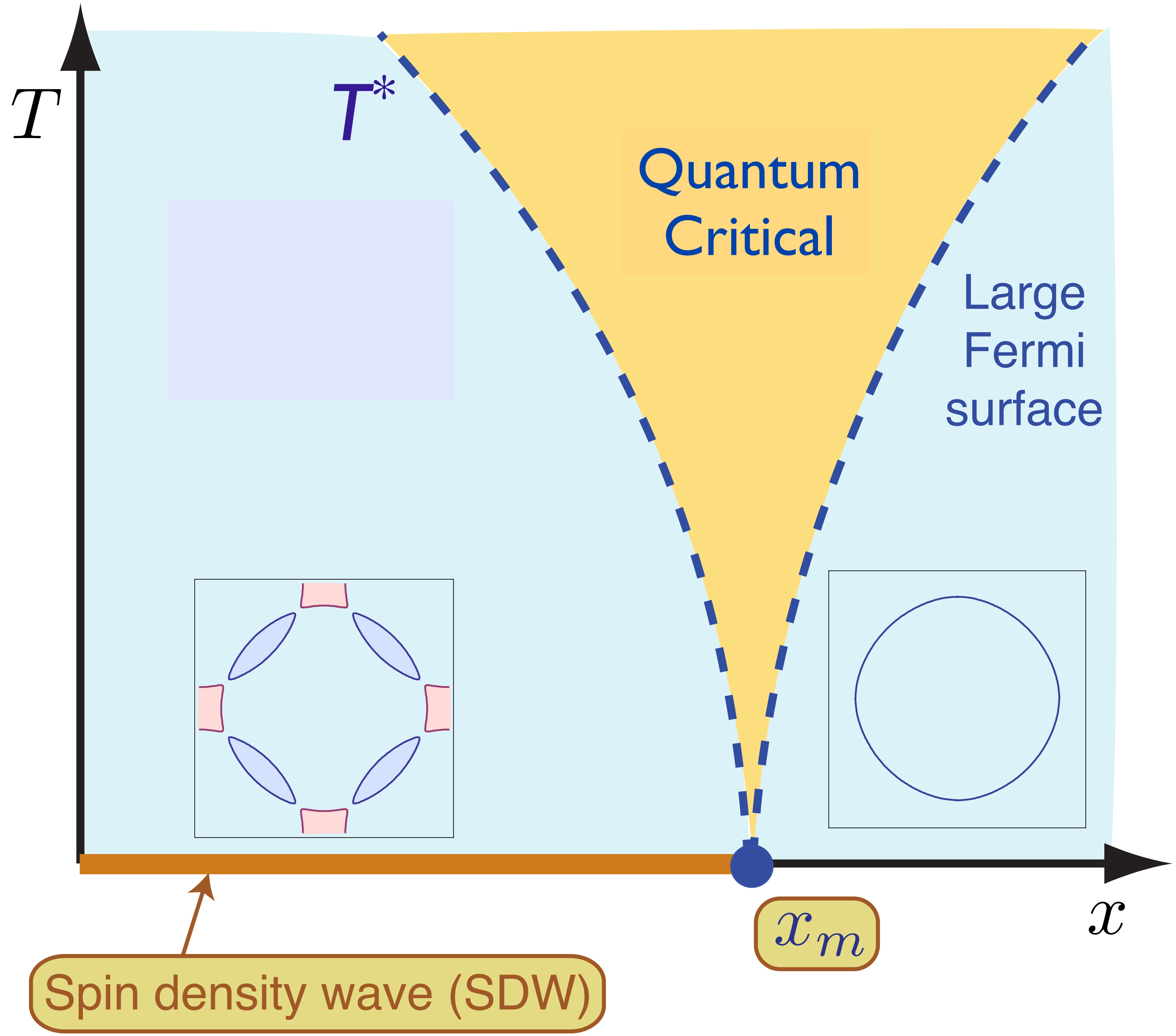
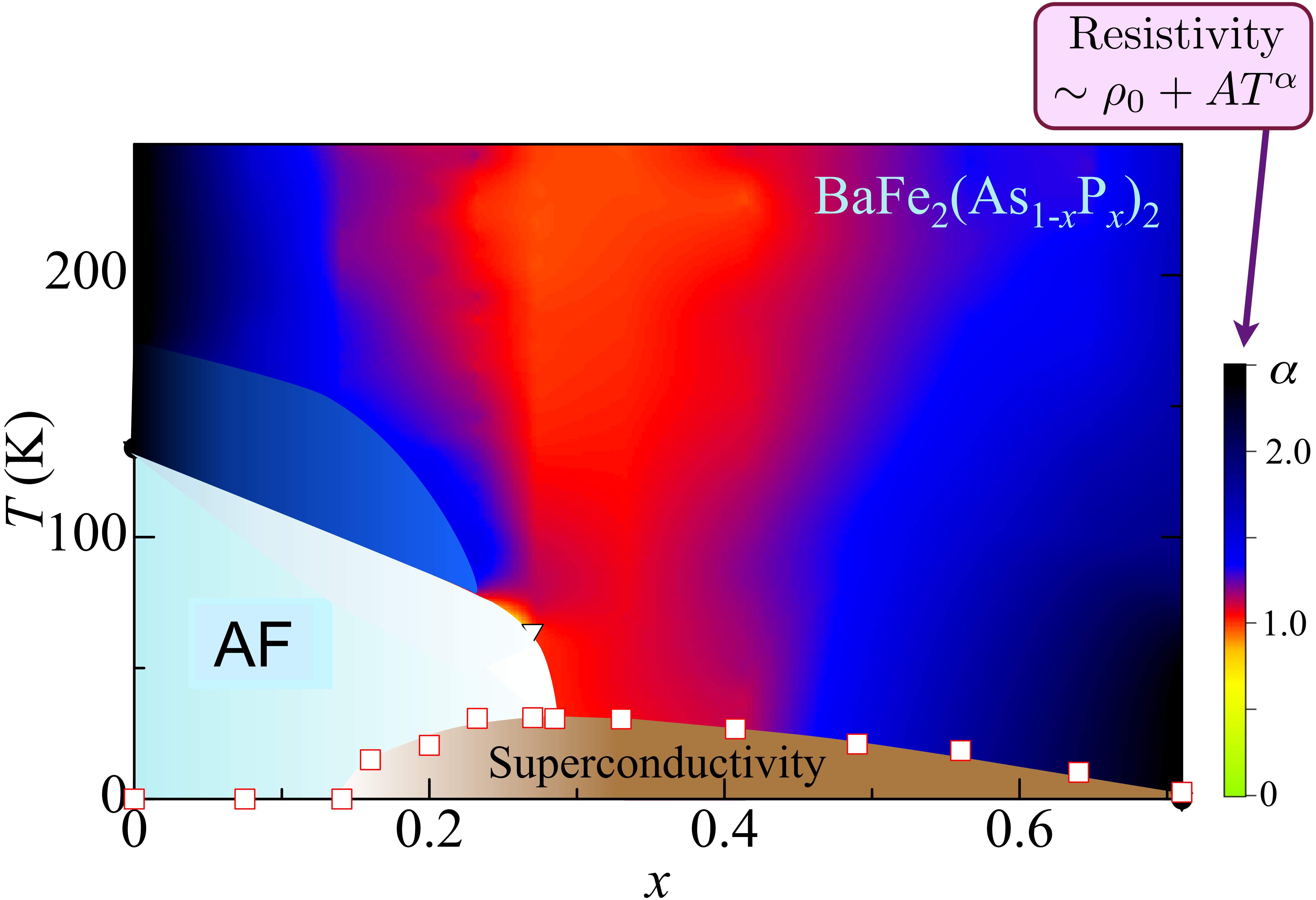


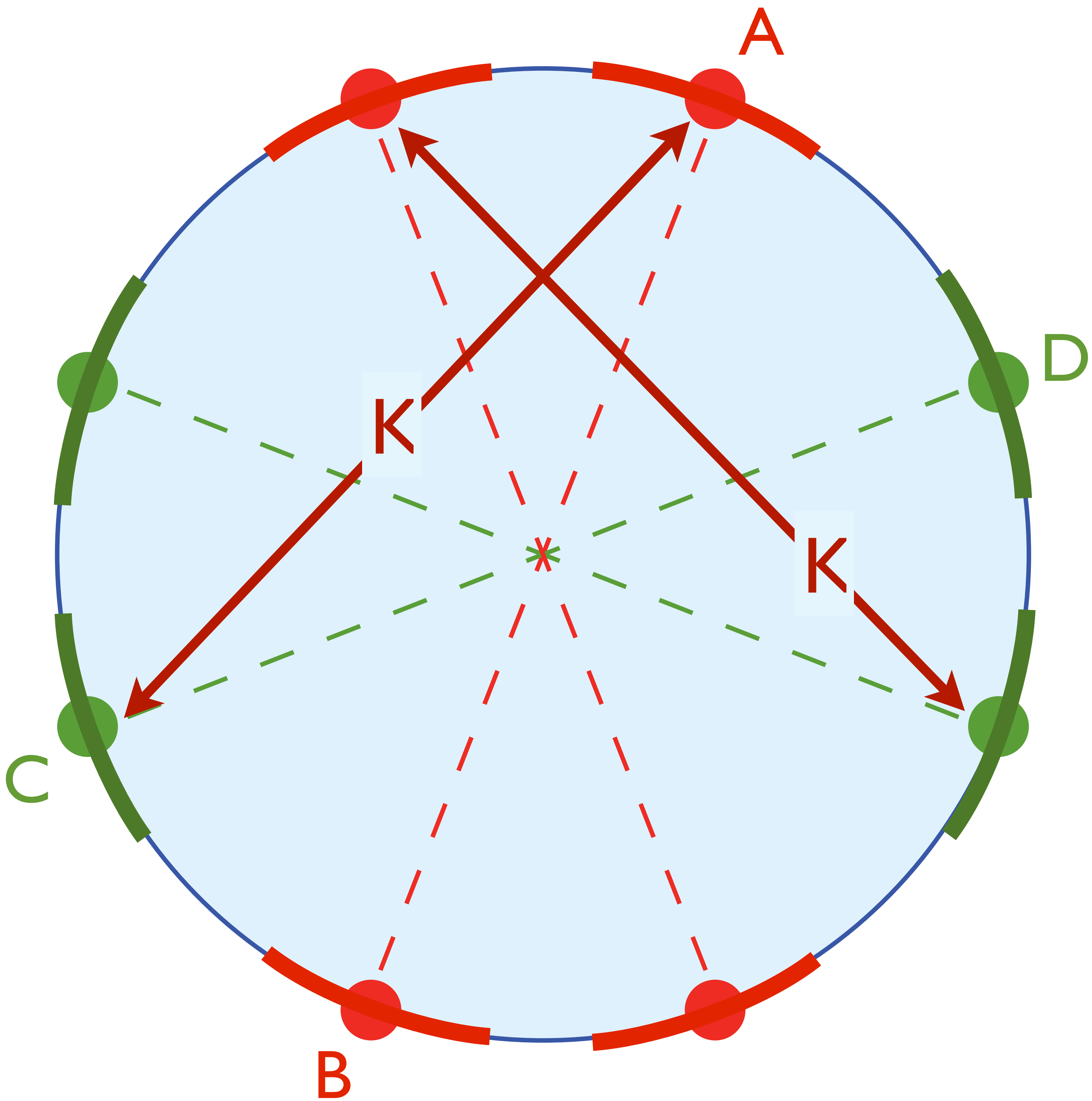
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Origin of superconductivity

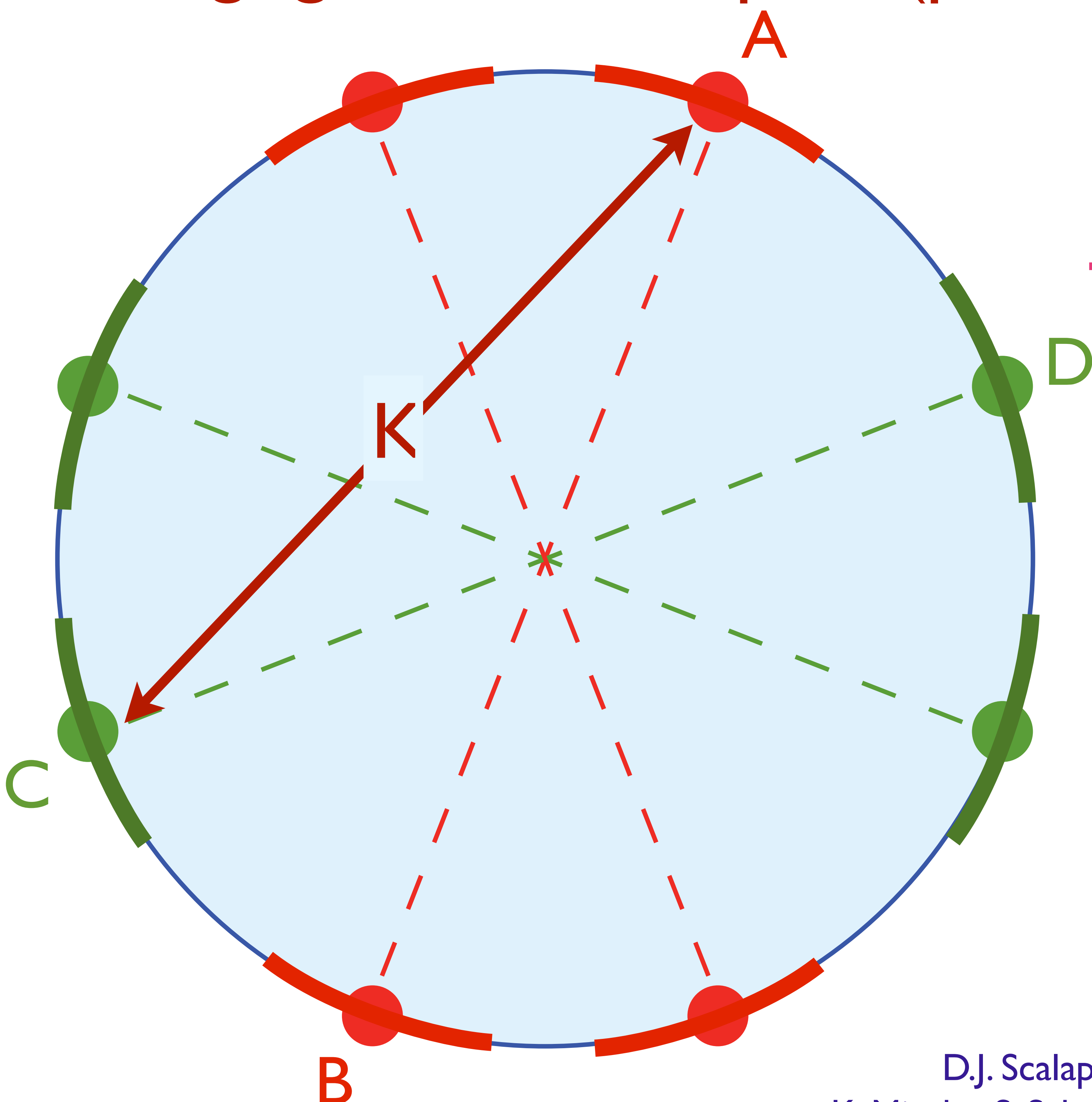
Focus on points on the Fermi surface separated by **K**

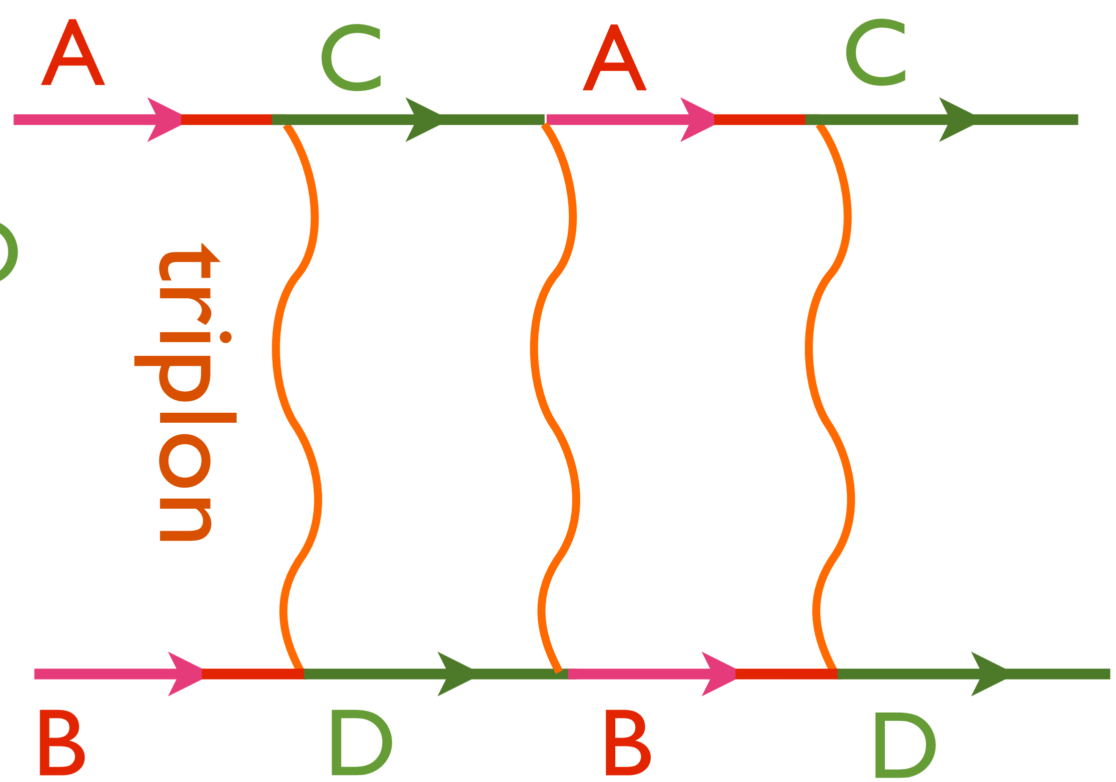


Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

Origin of superconductivity

Pairing "glue" from triplon (paramagnon) exchange





V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
Ar. Abanov, A.V. Chubukov, and A.M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

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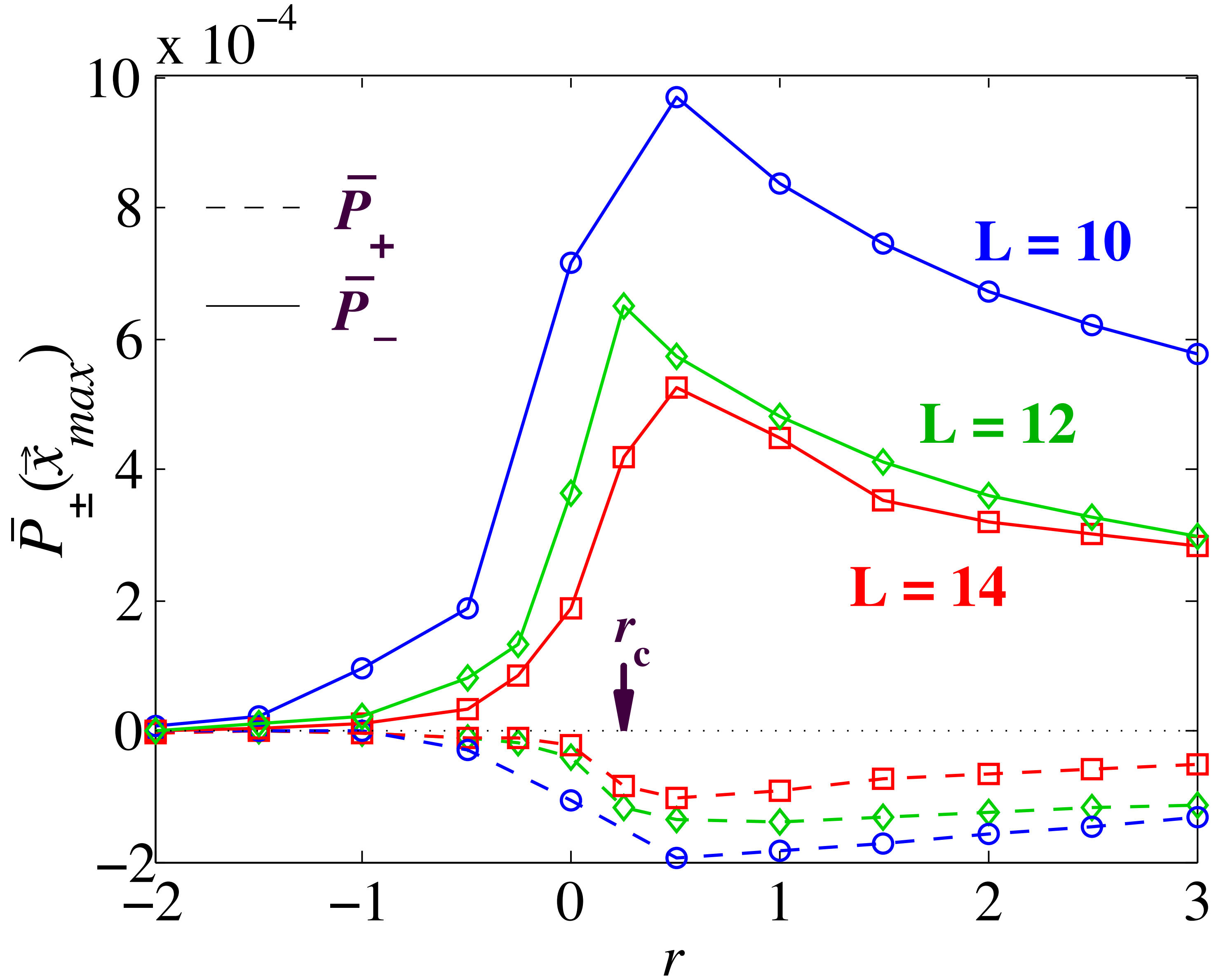
- An instability to charge-density-wave/bond order can become nearly degenerate with super-conductivity if the Fermi-surface is not too curved.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

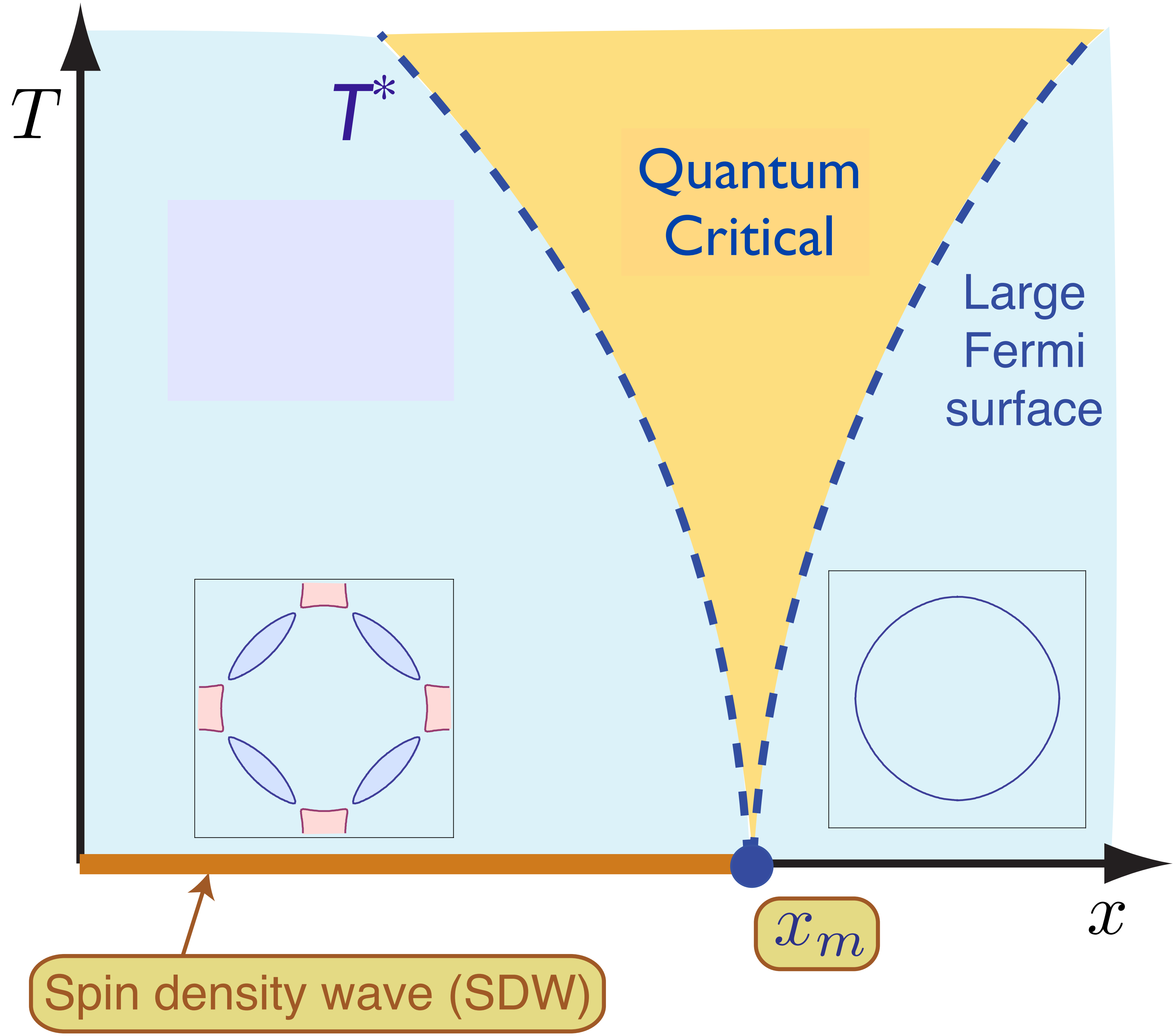
d-wave superconducting survives in the strong-coupling region across the quantum critical point



s/d pairing amplitudes P_+/P_- as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism



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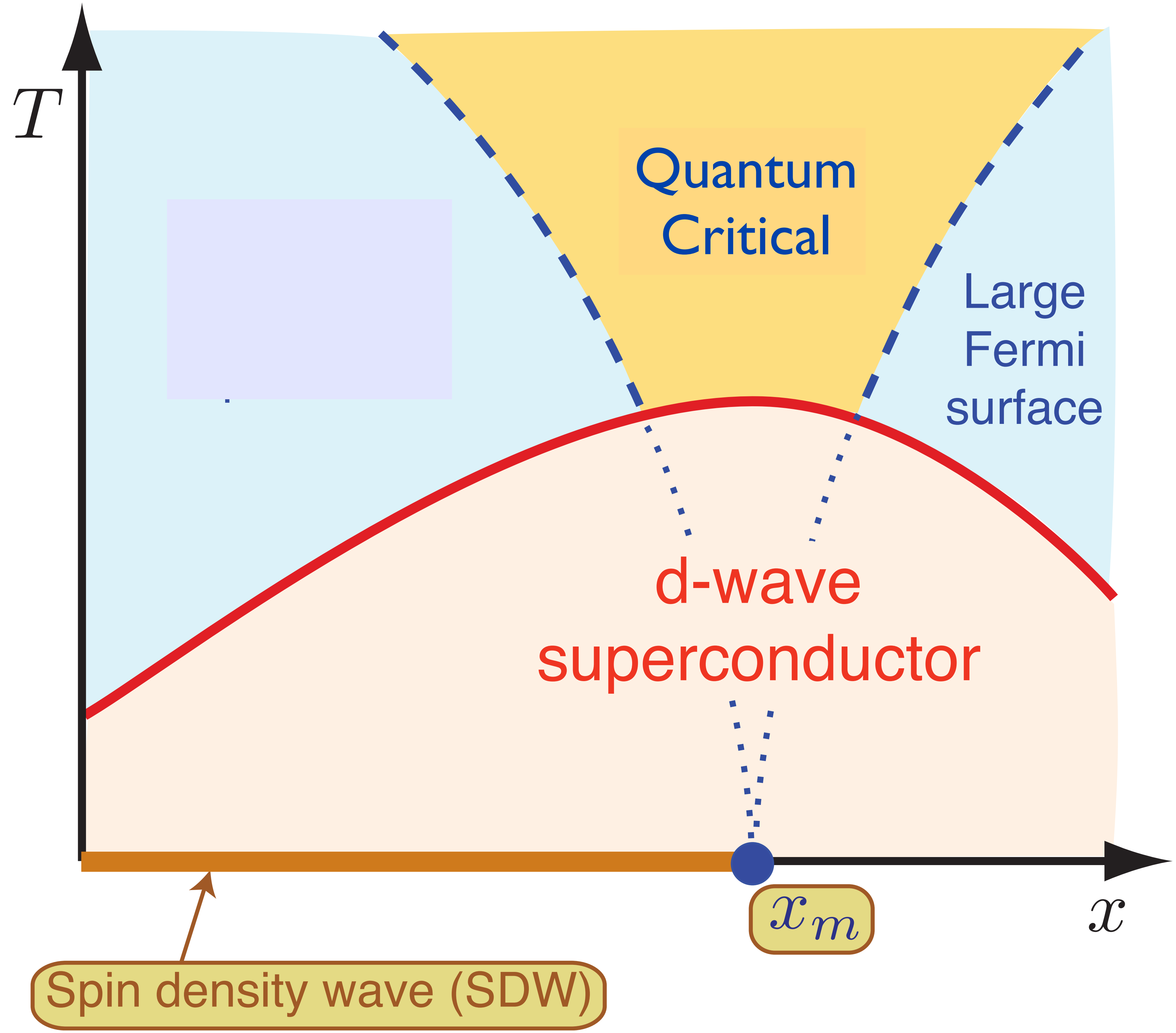
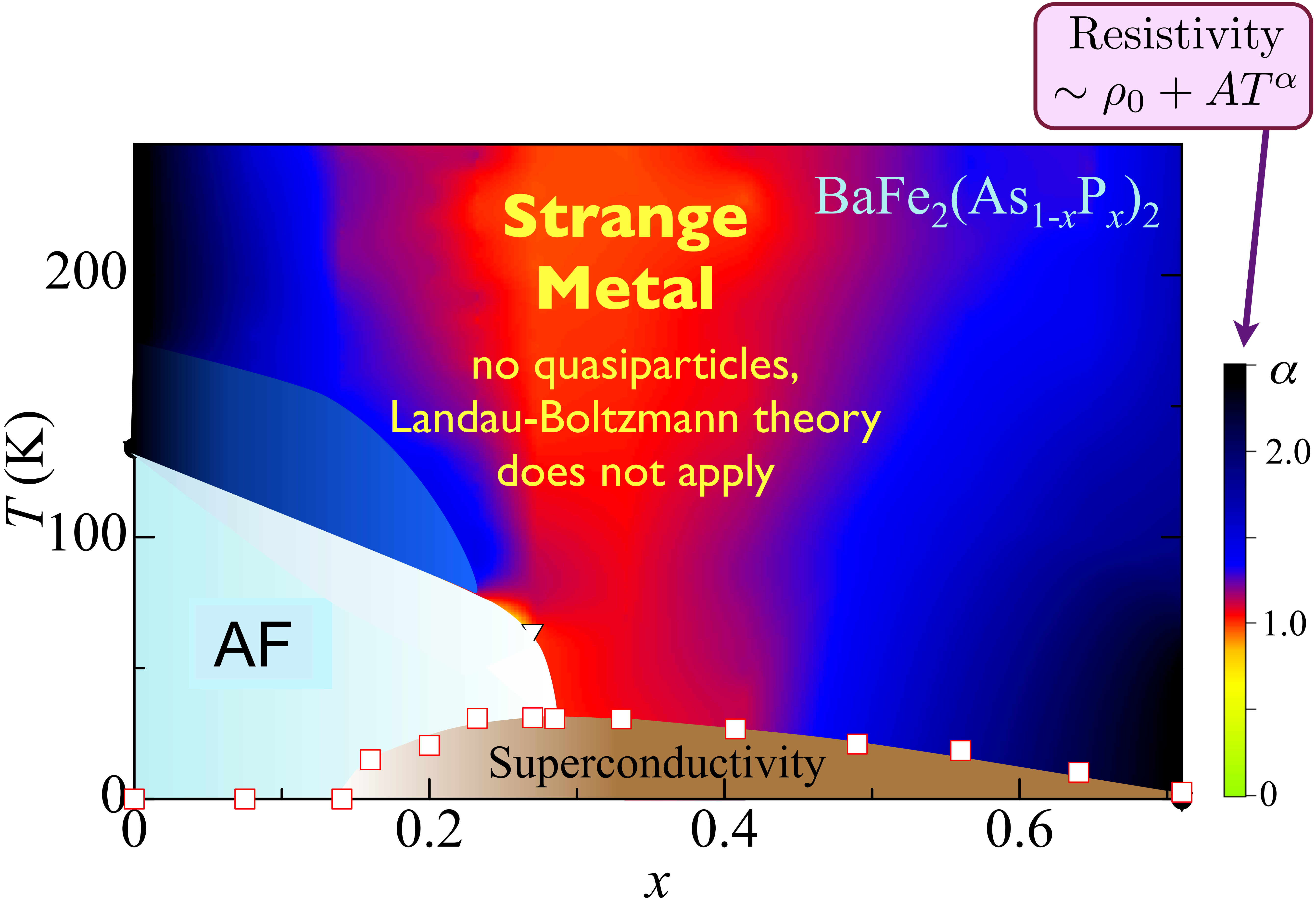
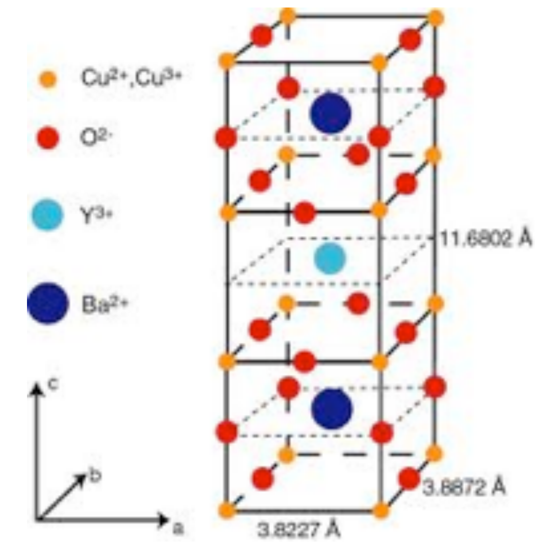
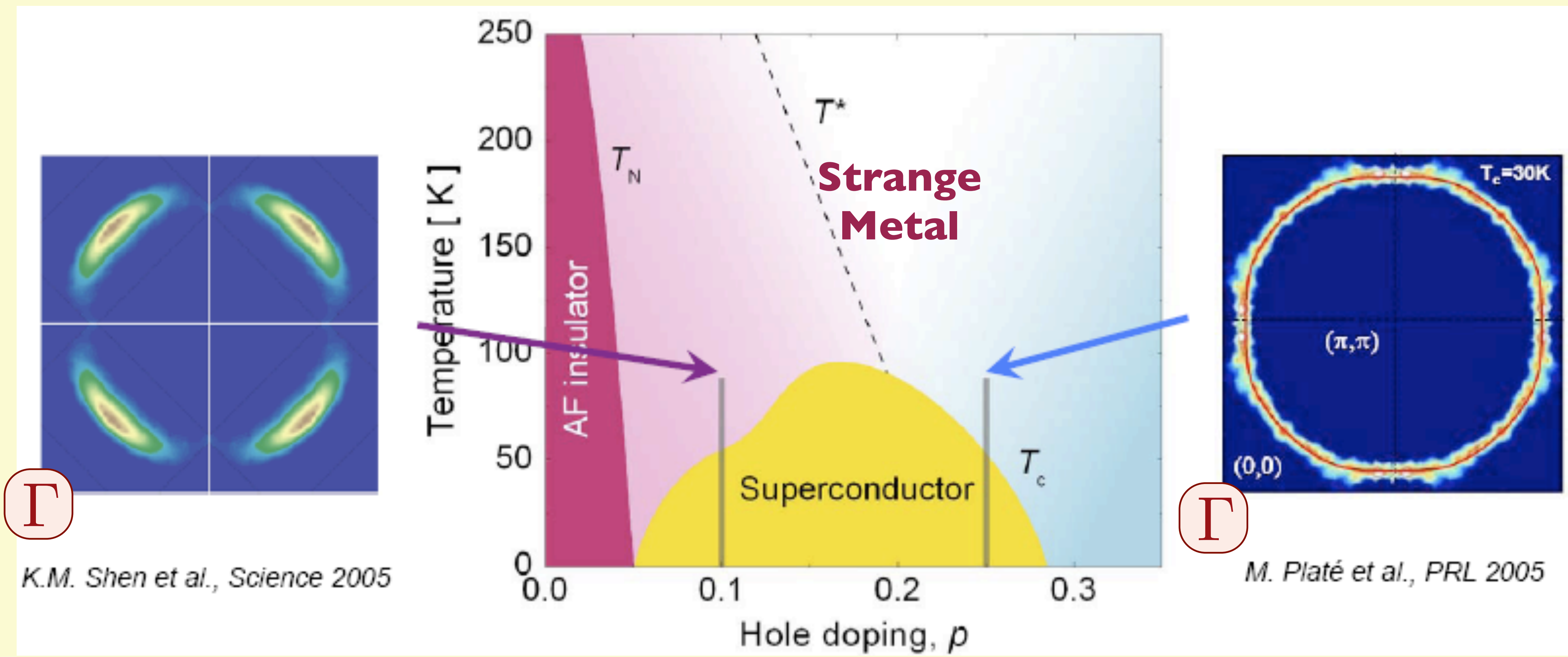


QCP for the onset of SDW order is actually within a superconductor



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)





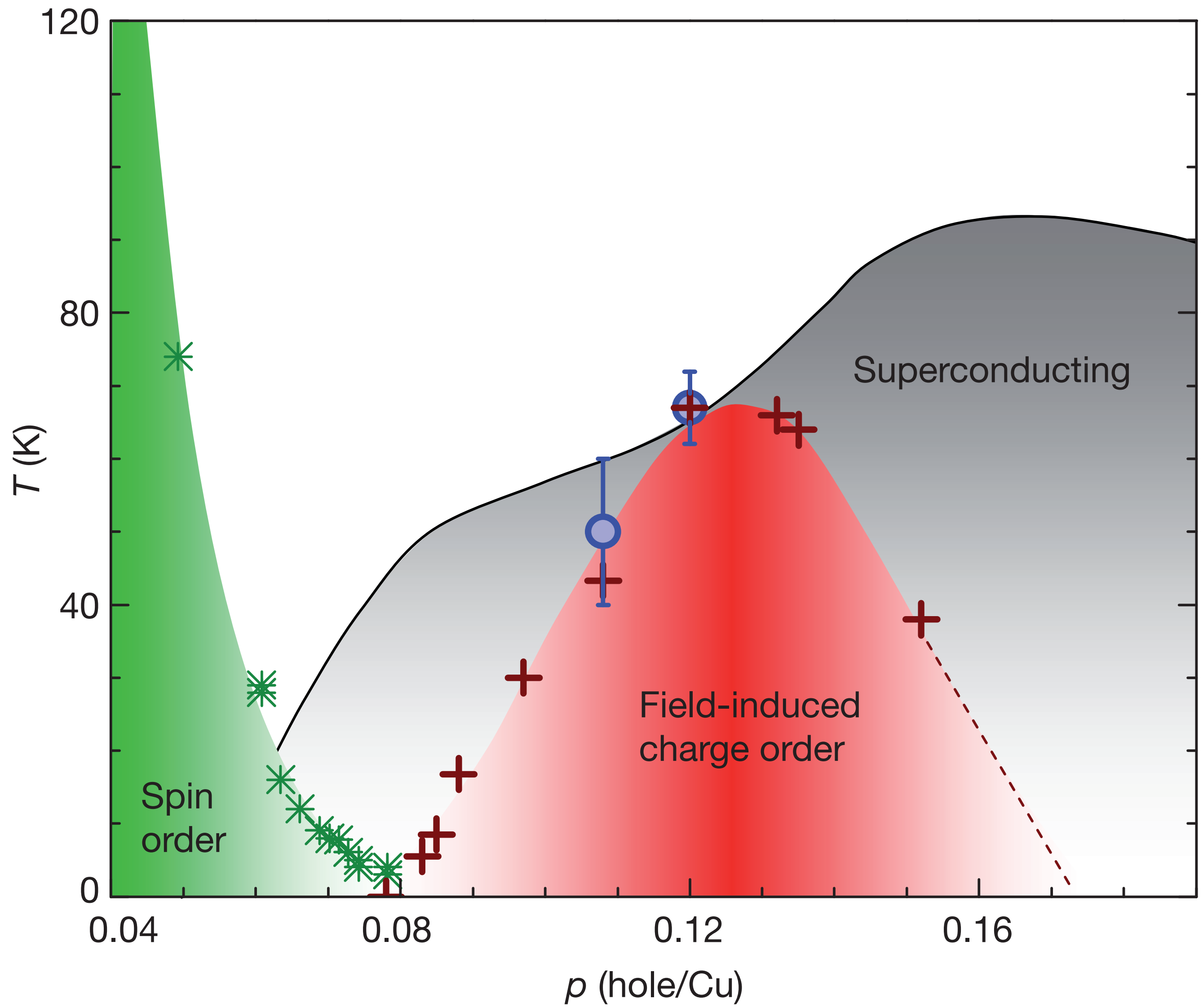
Smaller hole Fermi-pockets

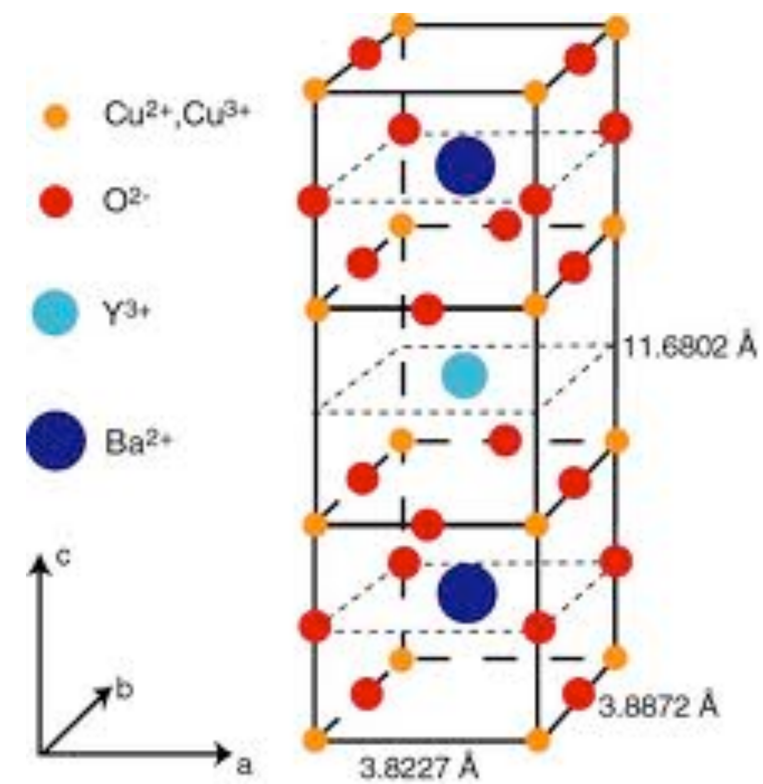
Large hole Fermi surface

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

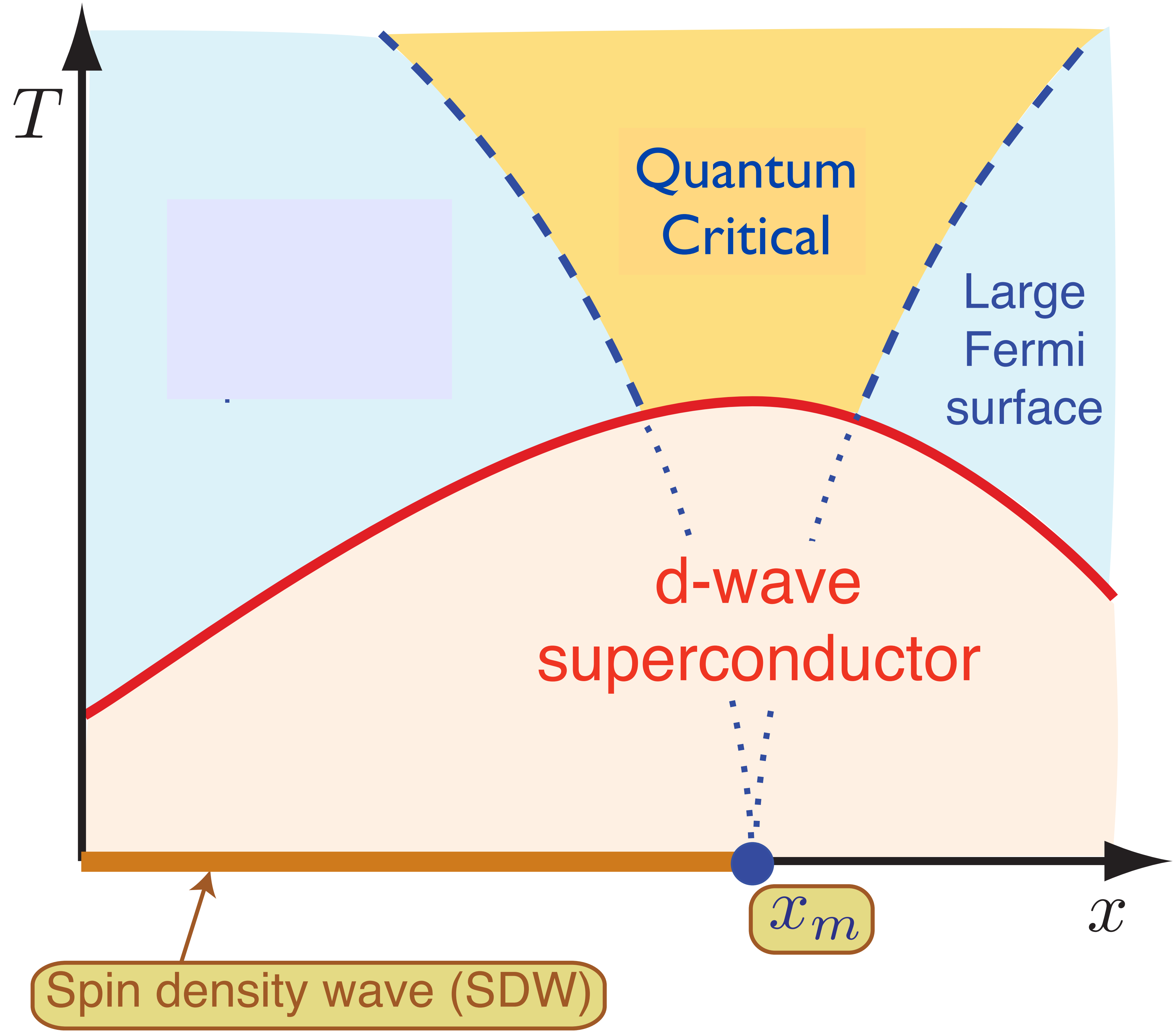
Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



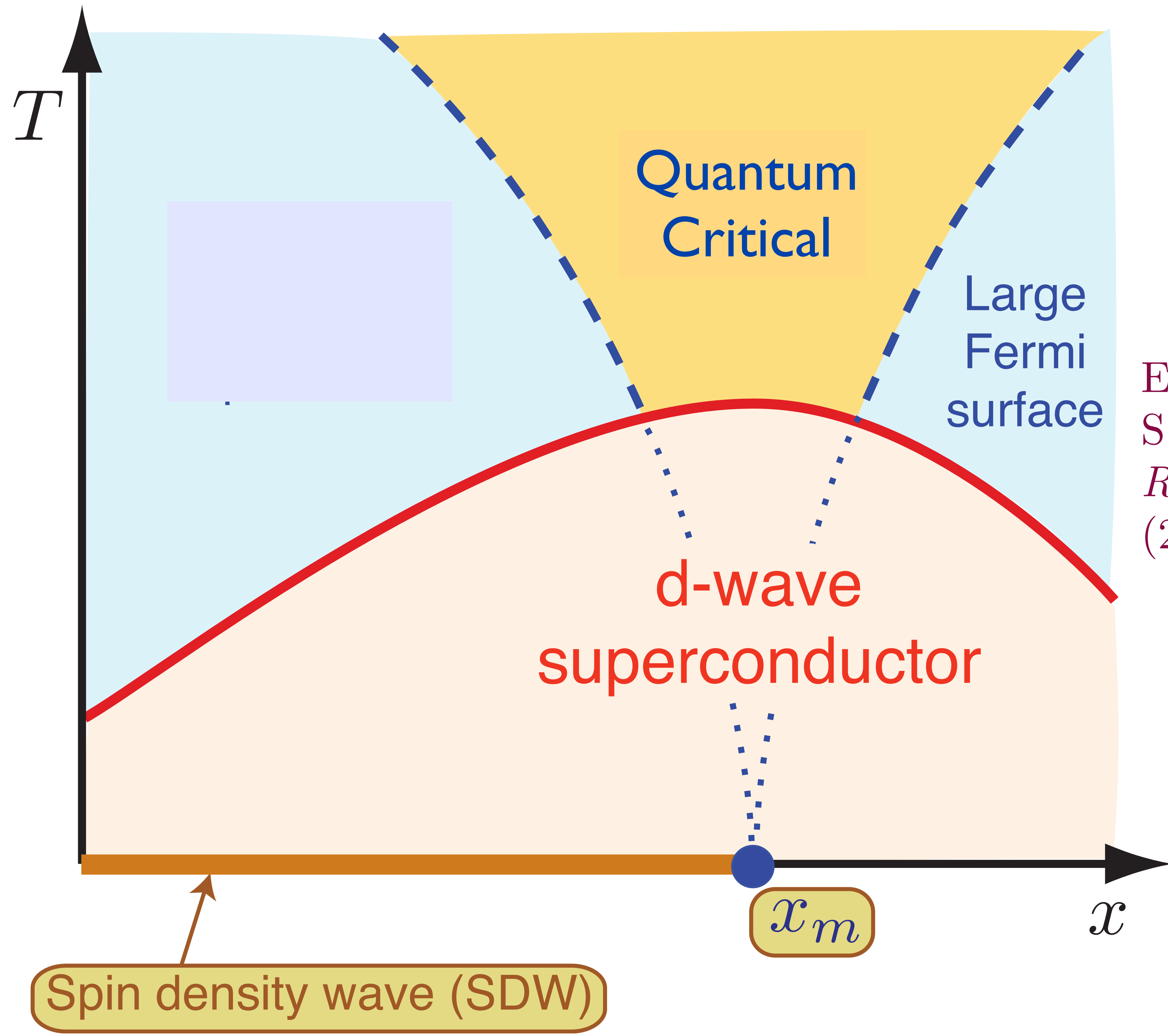


Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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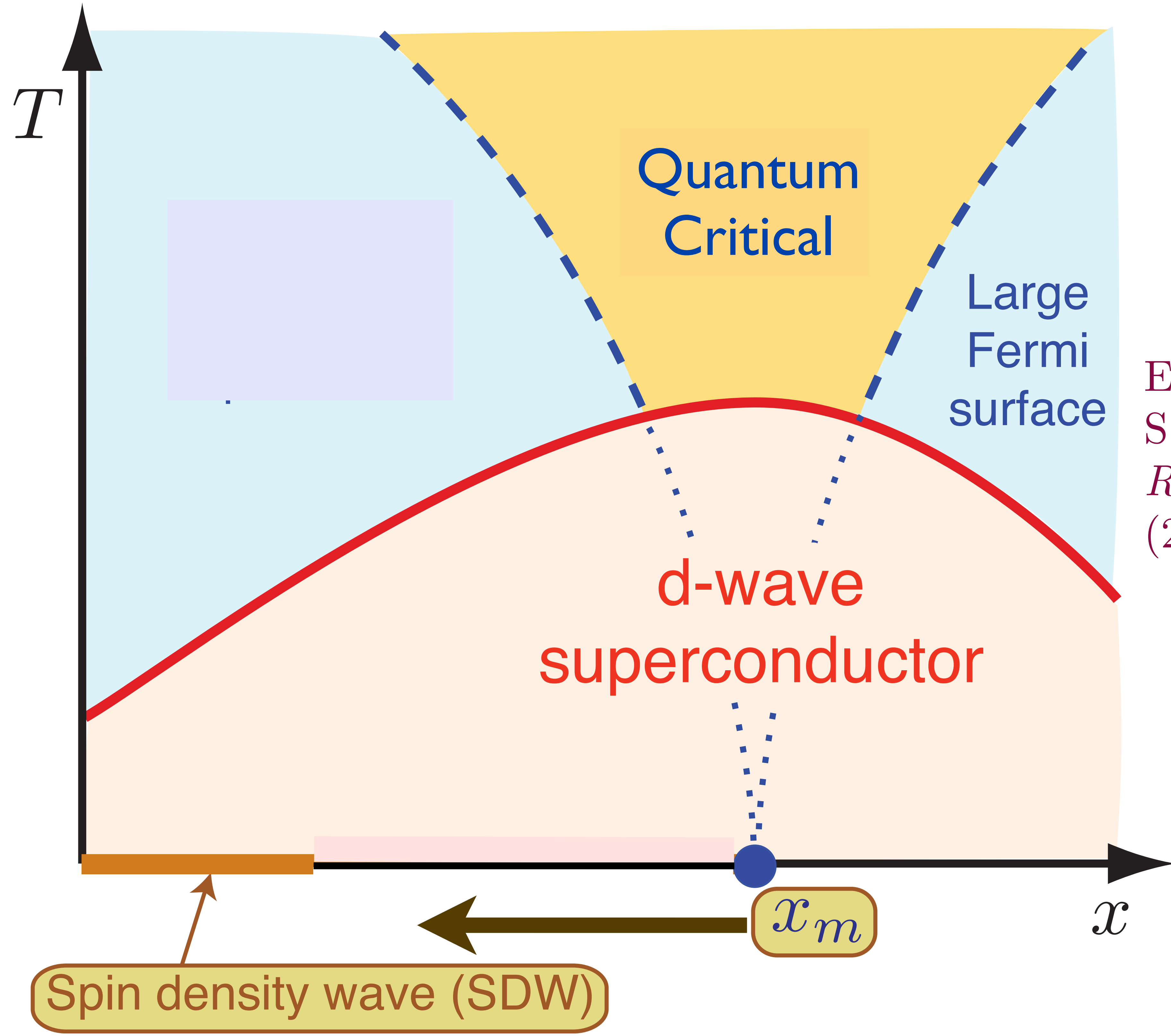

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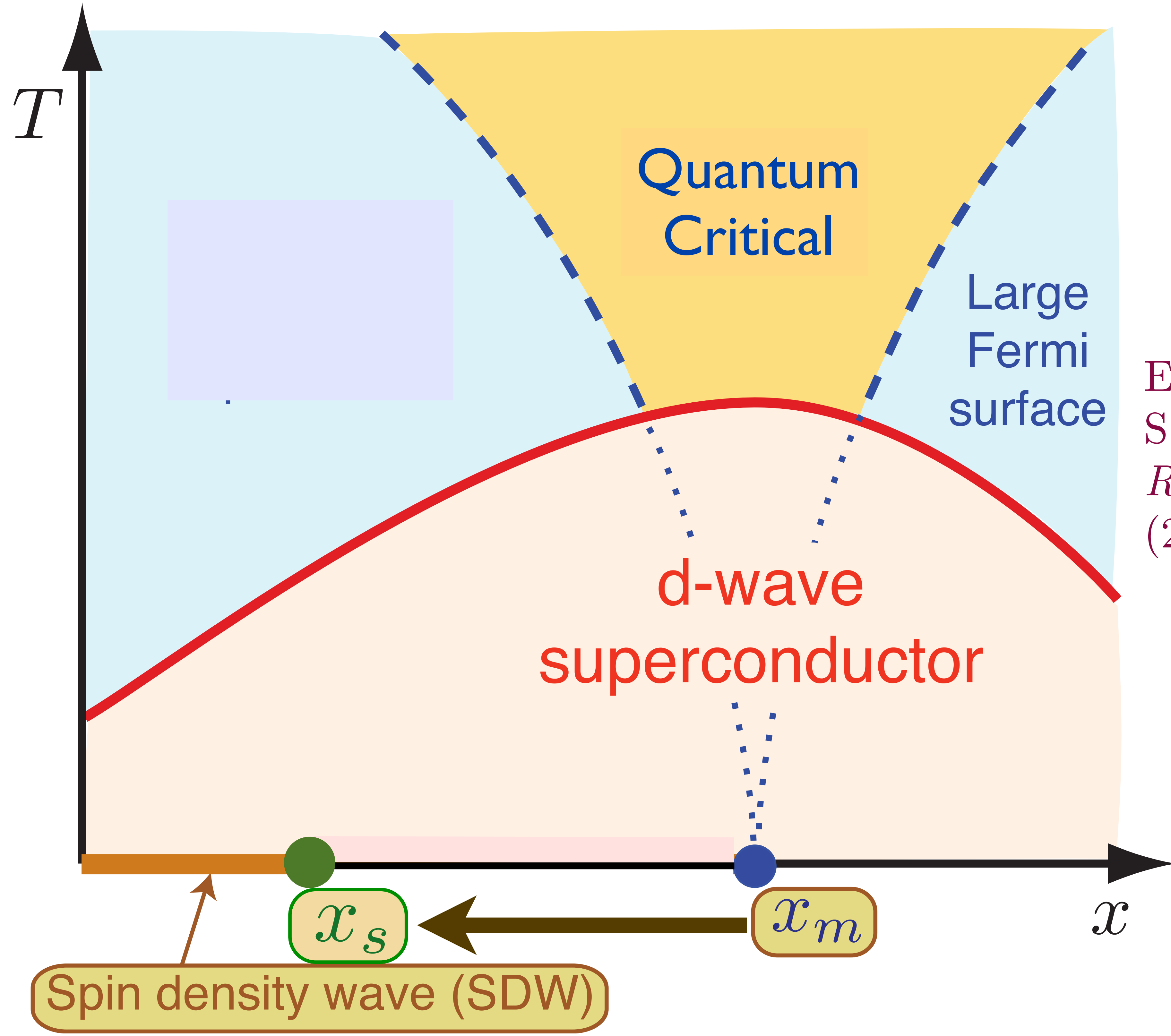

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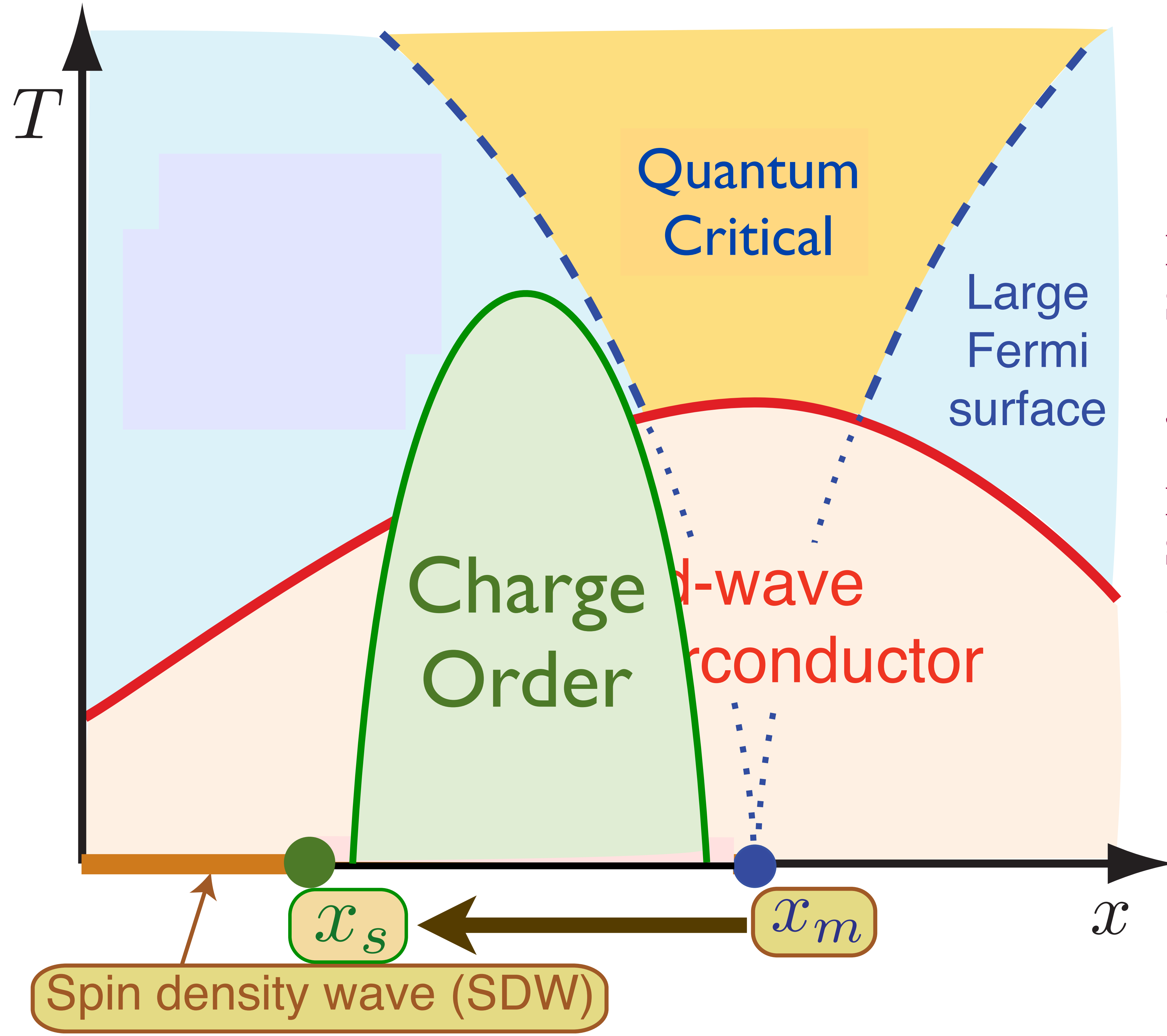


M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

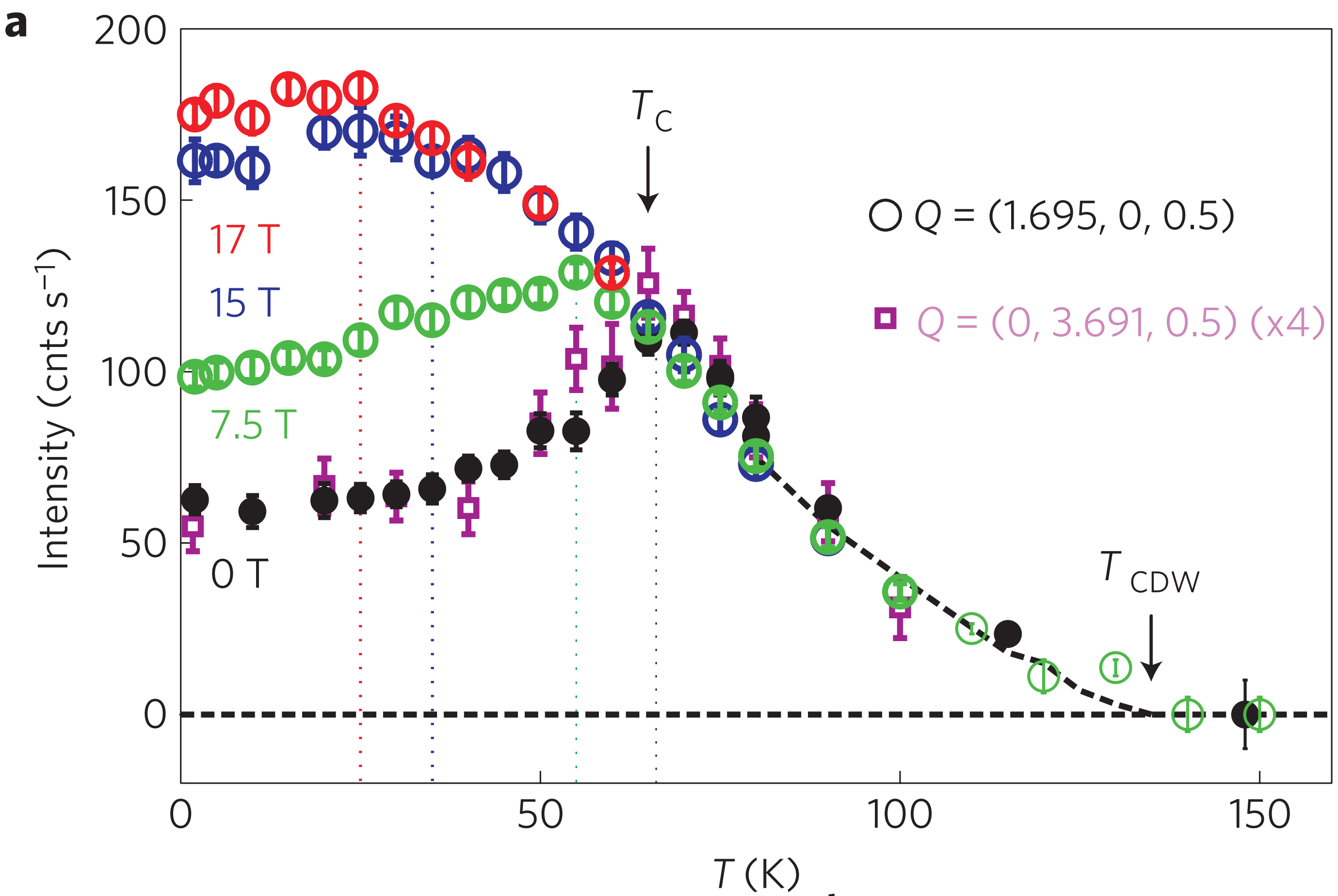
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

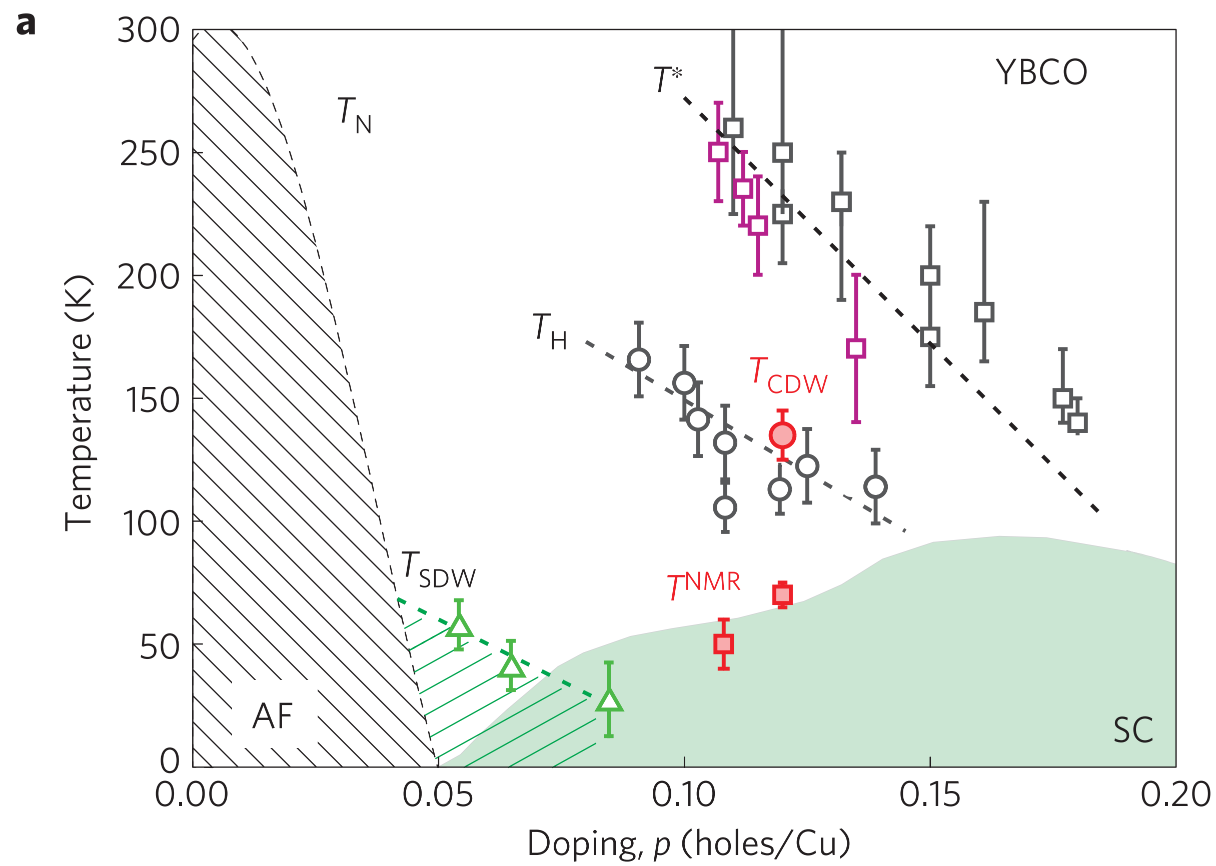
The metal has an instability to *both* d-wave superconductivity and a d-wave charge density wave (bond order).

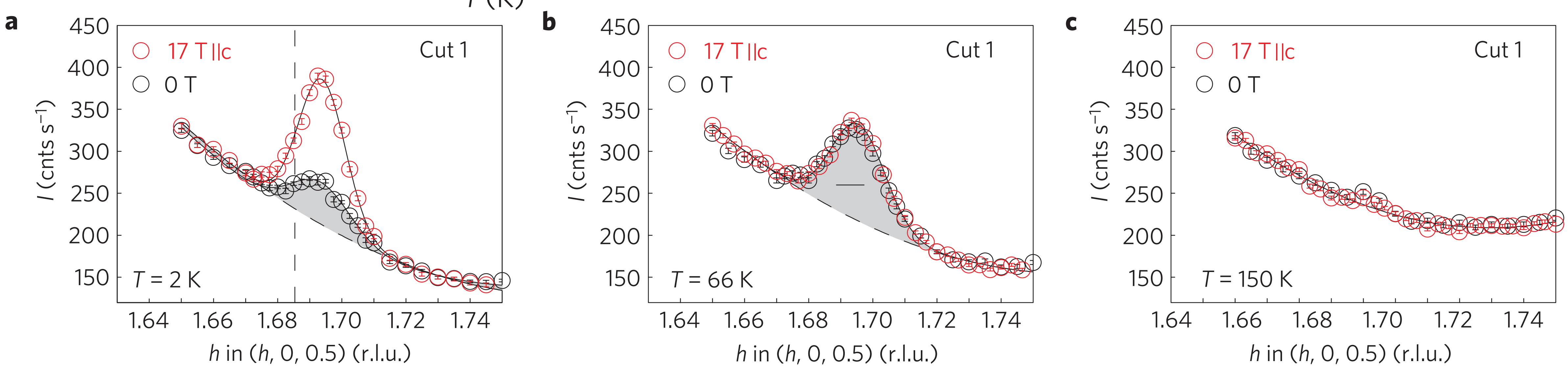
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]







Wednesday, December 18, 13

6-component order parameter for the cuprate superconductors

Superconducting order $\Psi(\mathbf{r})$:

$$\left\langle c_{i\alpha}^\dagger c_{j\beta}^\dagger \right\rangle = \varepsilon_{\alpha\beta} \left[\sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\left\langle c_{i\alpha}^\dagger c_{j\beta} \right\rangle = \delta_{\alpha\beta} \left[\sum_{\mathbf{k}} P_{\mathbf{Q}_x}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}_x\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x\left((\mathbf{r}_i + \mathbf{r}_j)/2 \right)$$
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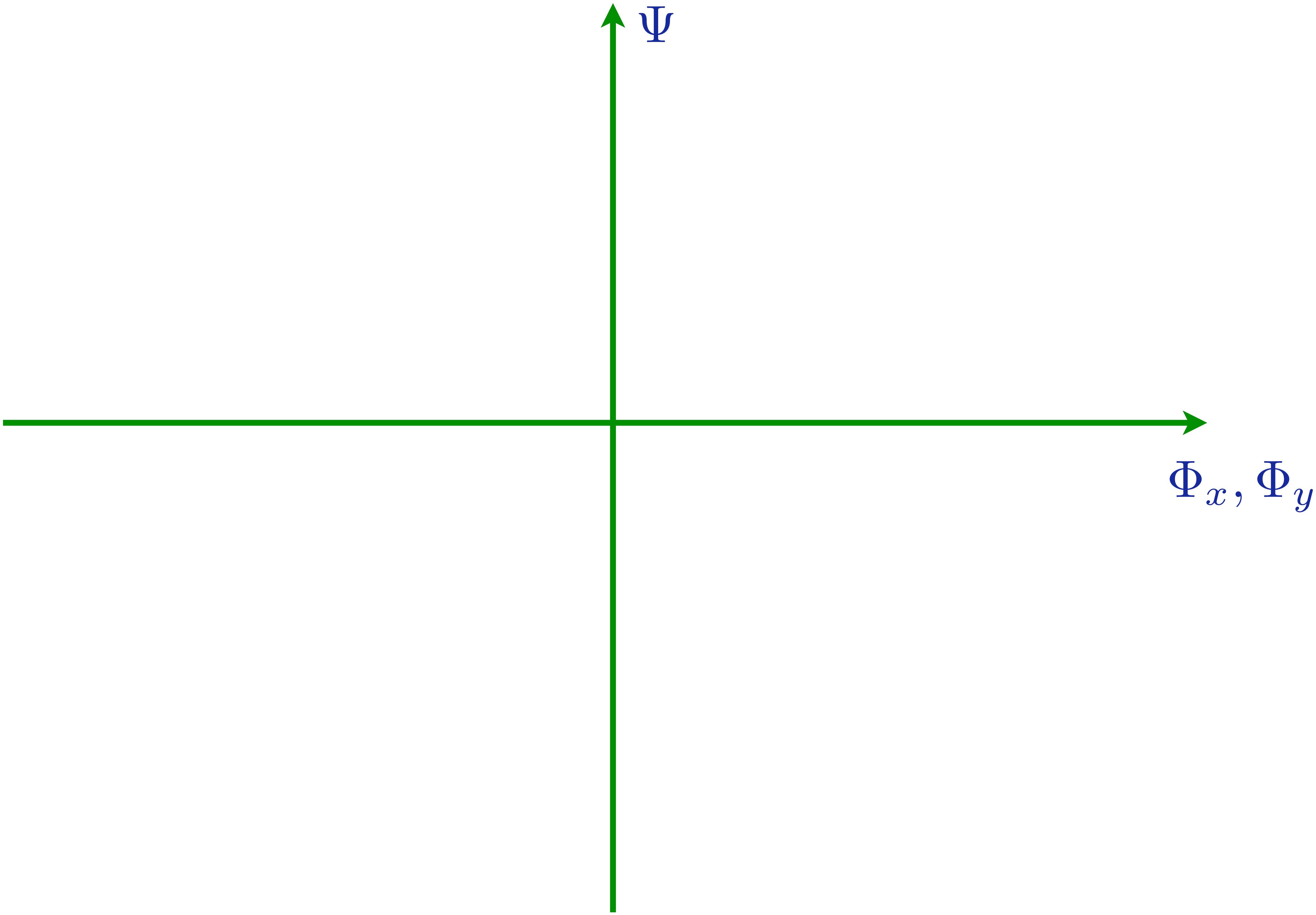
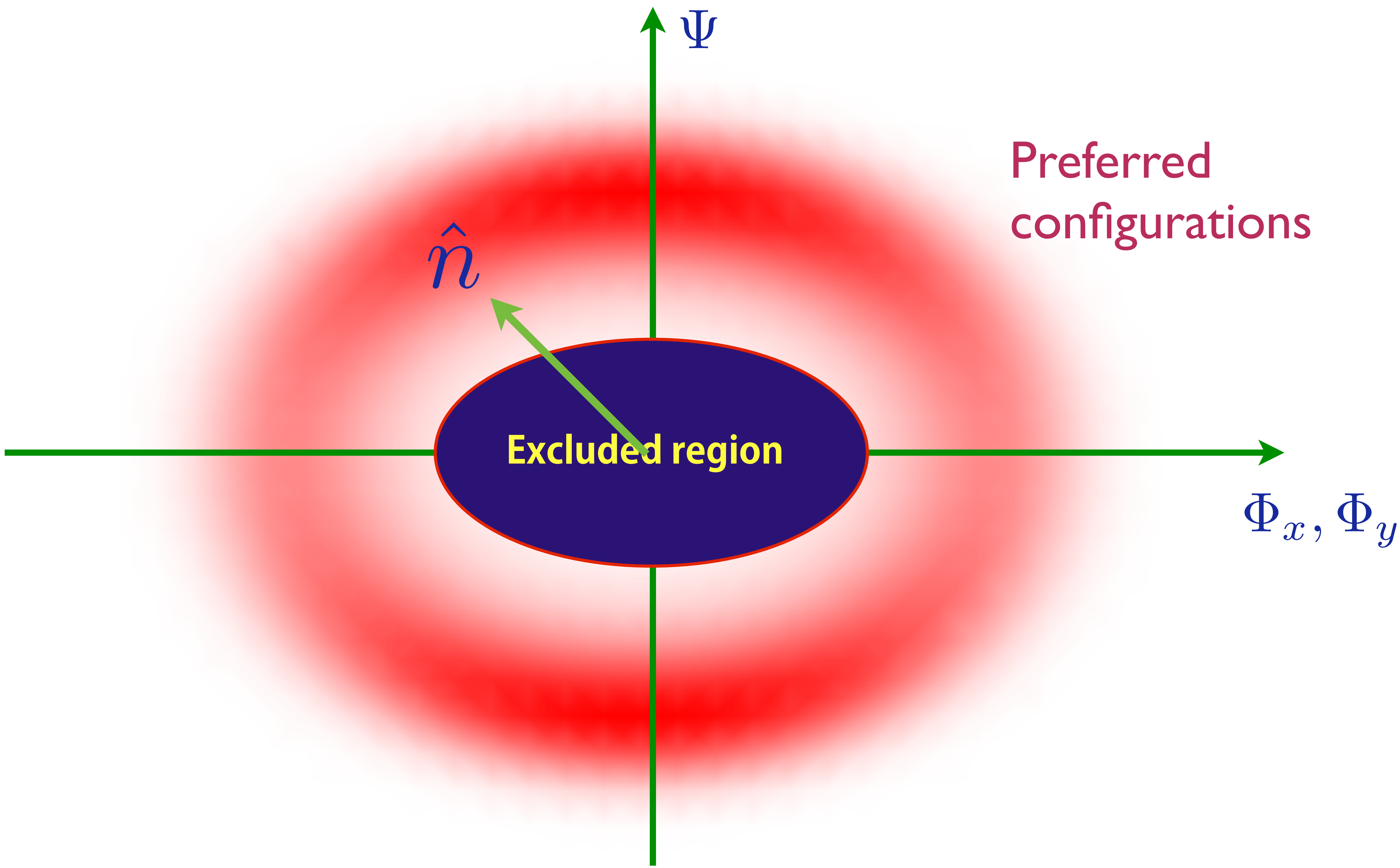
6-component order parameter



6-component order parameter
O(6)-sigma model for thermal fluctuations



Support from theory of antiferromagnetic quantum criticality

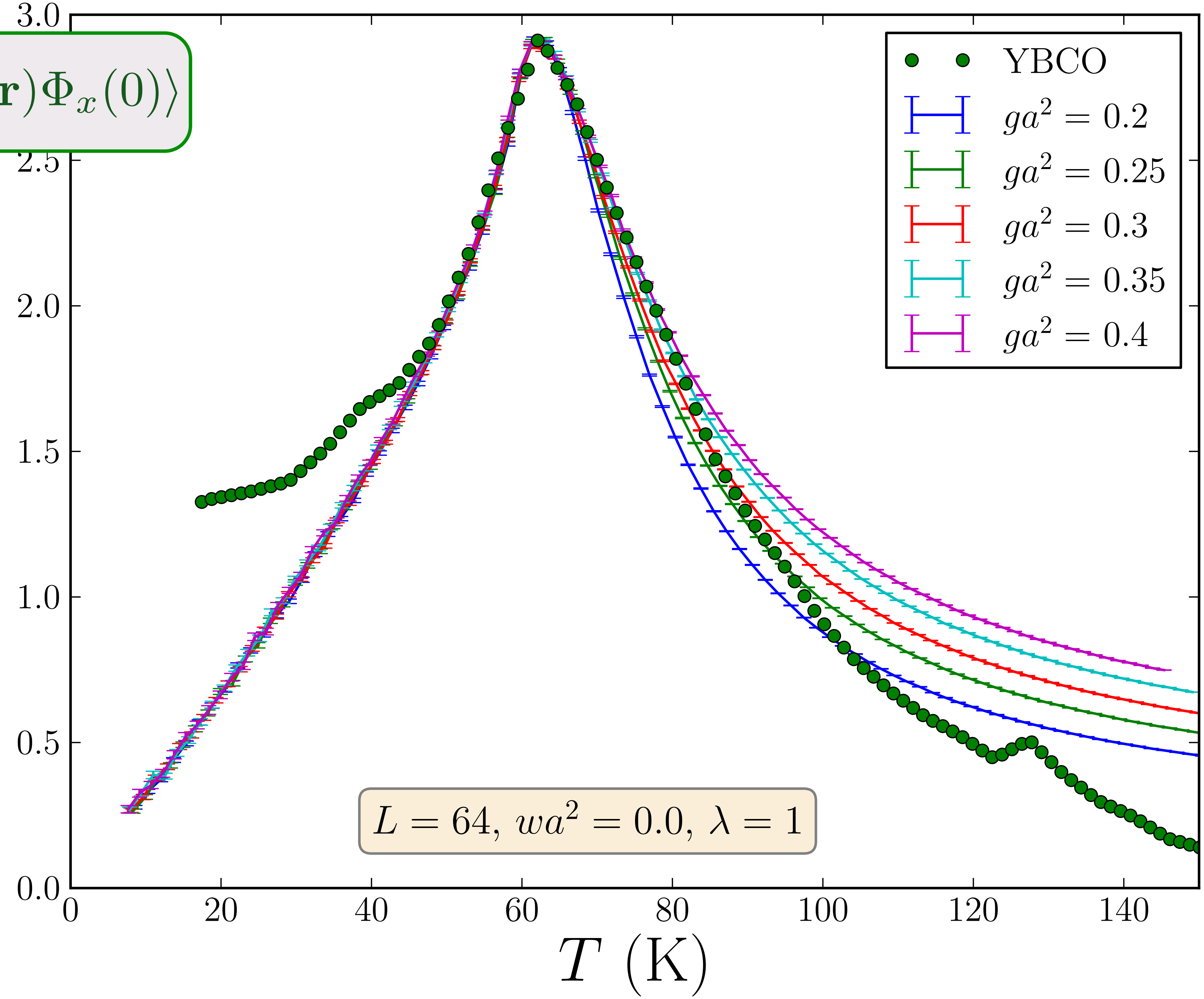
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

Comparison of Monte Carlo of O(6) model with expts

$$S_{\Phi_x} = \int d^2 r \, \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order structure factor S_{Φ_x}



For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160$K.
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

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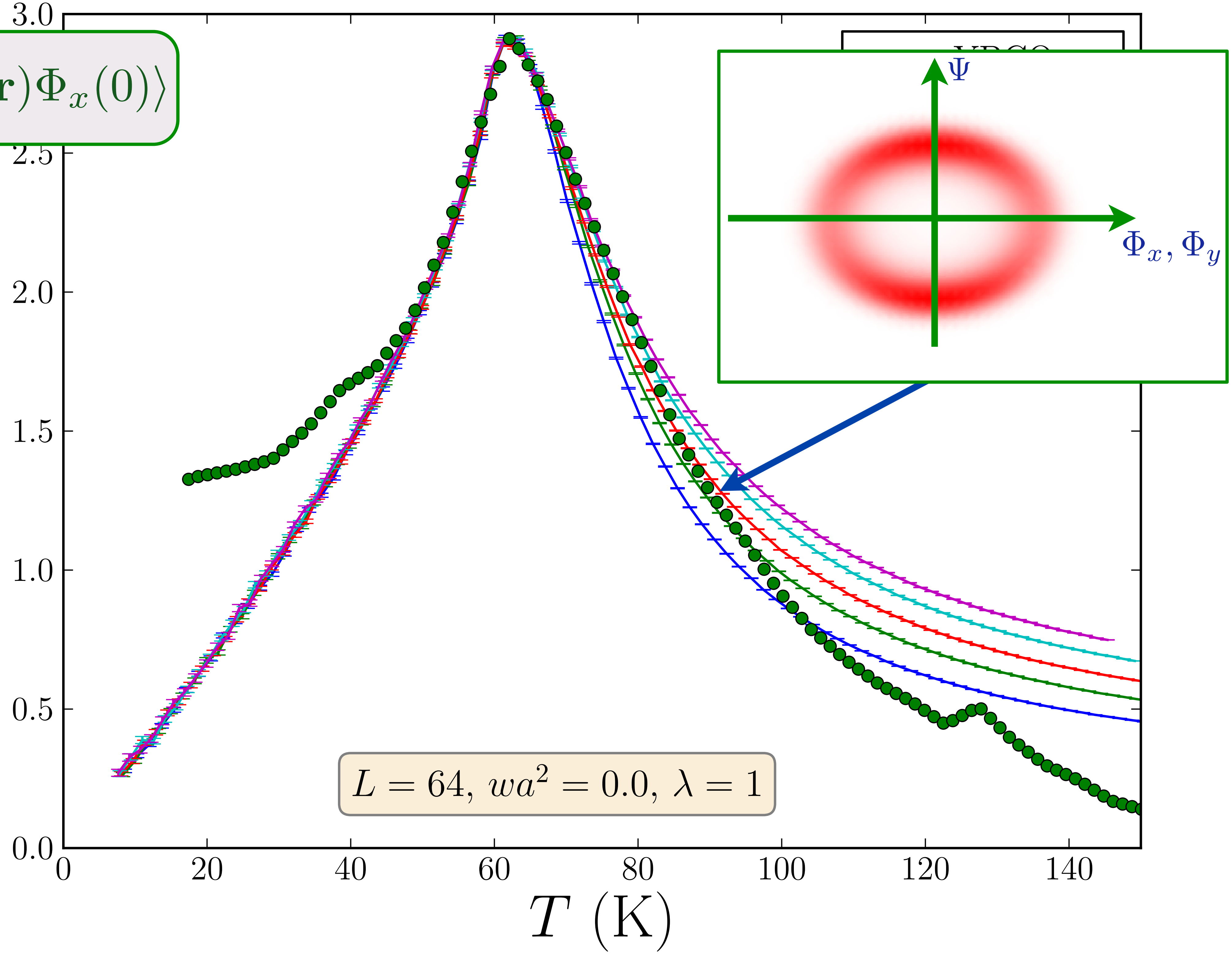

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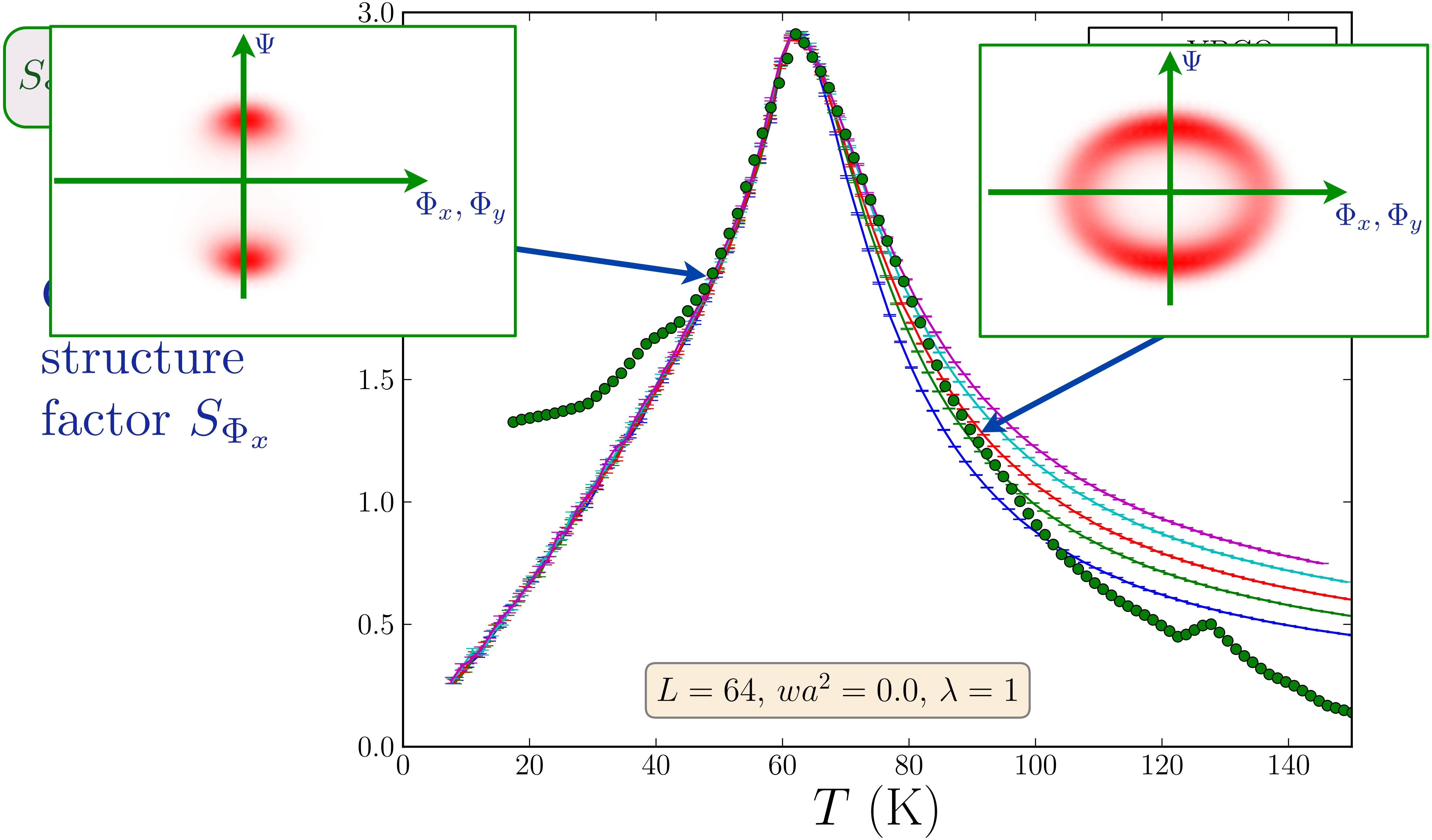

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