# Entanglement, holography, and the quantum phases of matter

Yale University, September 10, 2012

Subir Sachdev

PHYSICS
HARVARD

Lecture at the 100th anniversary Solvay conference,
*Theory of the Quantum World*
arXiv:1203.4565

Liza Huijse

Max Metlitski

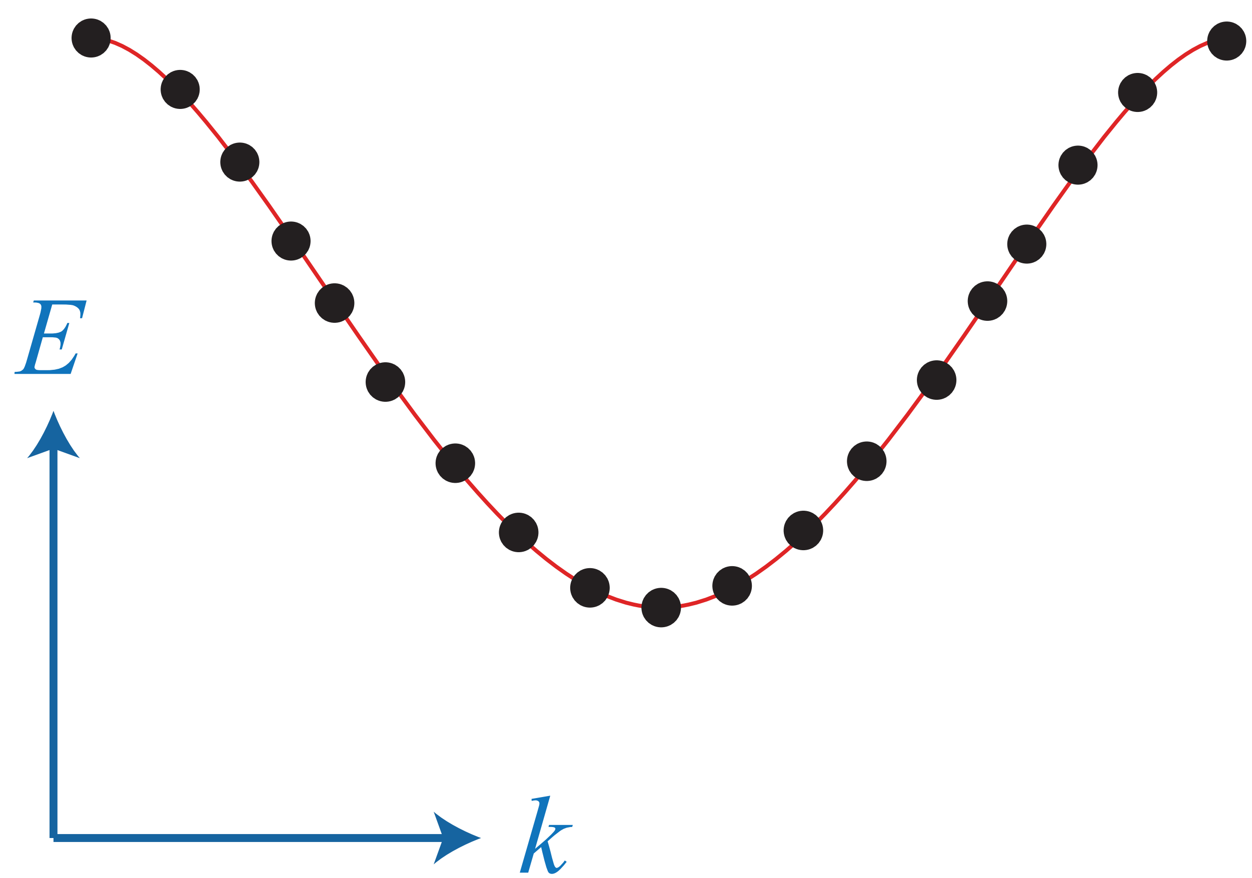
Brian Swingle

# Sommerfeld-Bloch theory of metals, insulators, and superconductors:

## many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states

# Band insulators




An even number of electrons per unit cell

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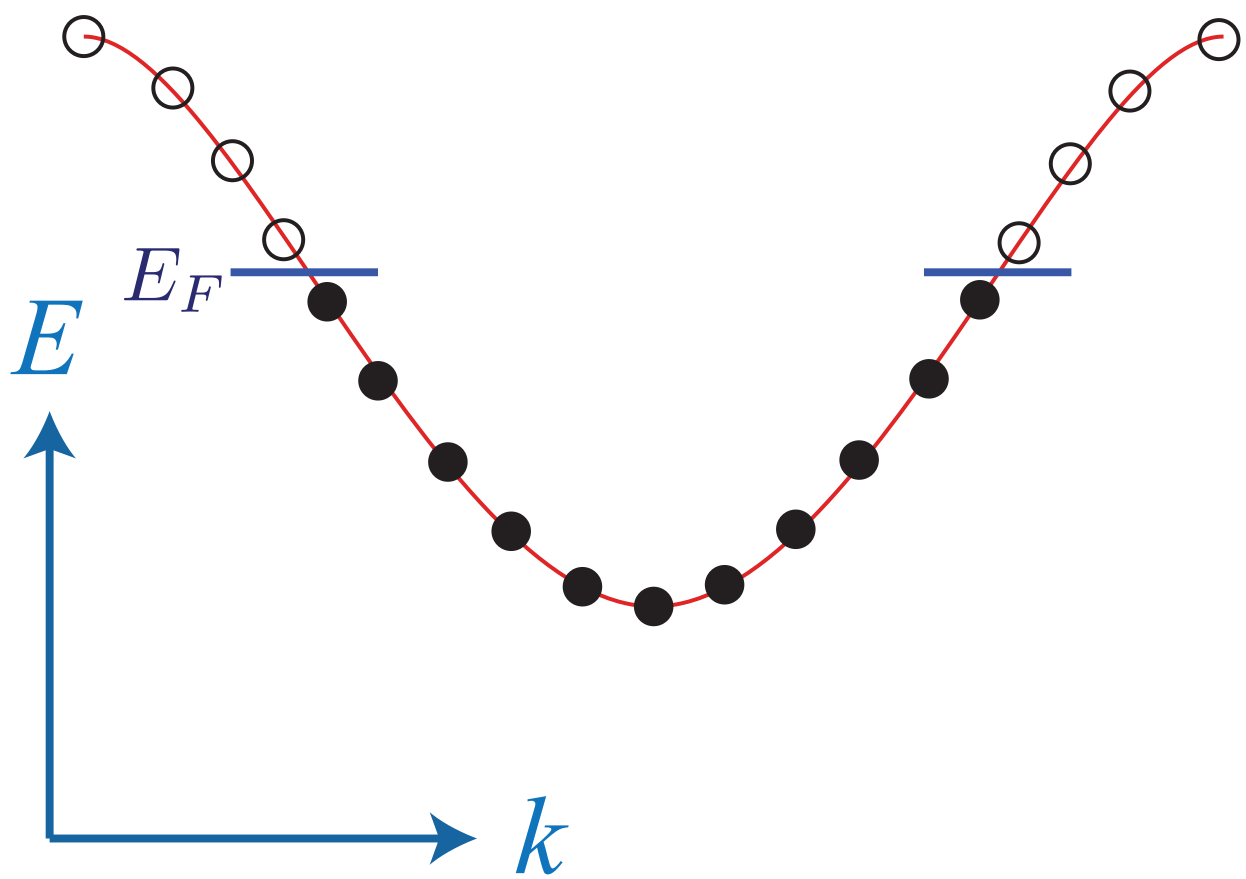
many-electron quantum states are adiabatically connected to independent electron states

## Metals




An odd number of electrons per unit cell

Sommerfeld-Bloch theory of
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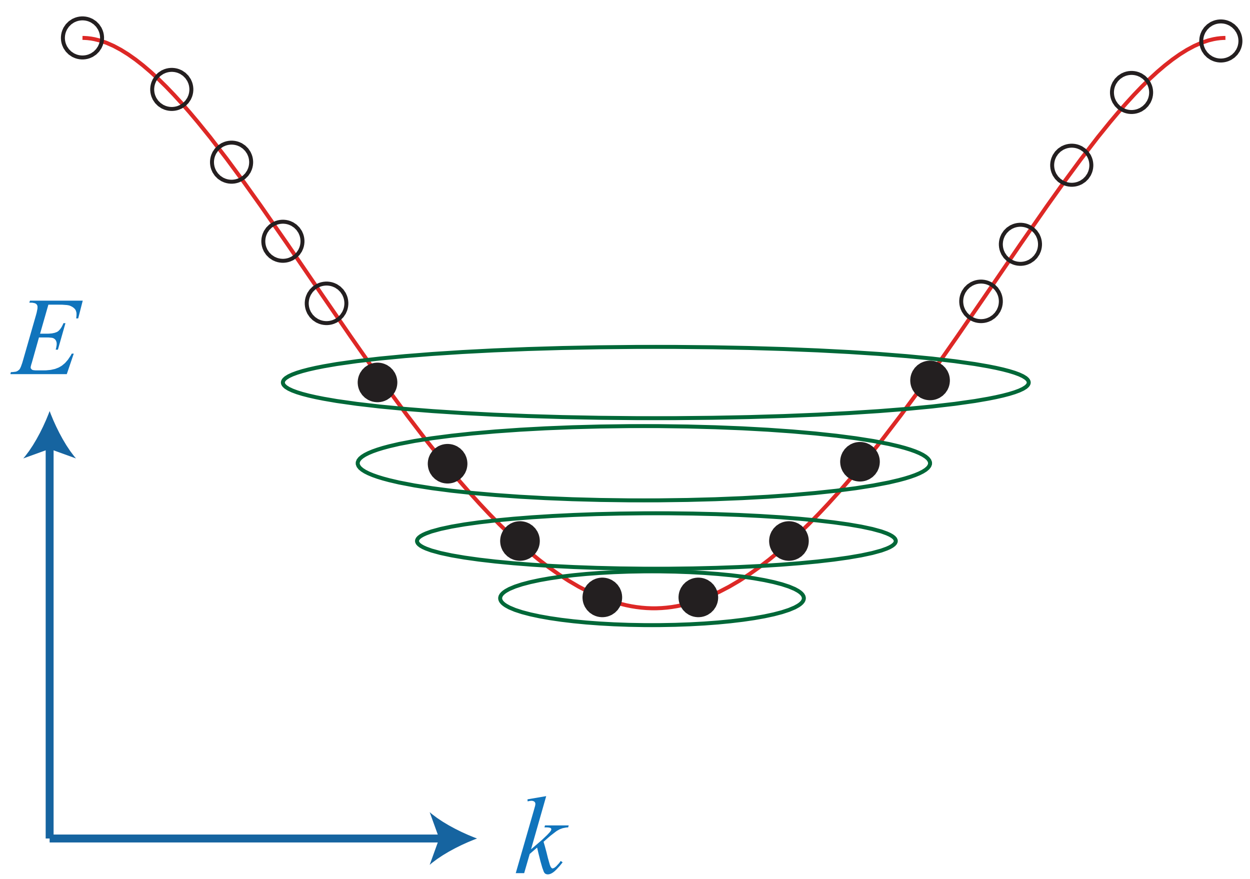
## Superconductors



## Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

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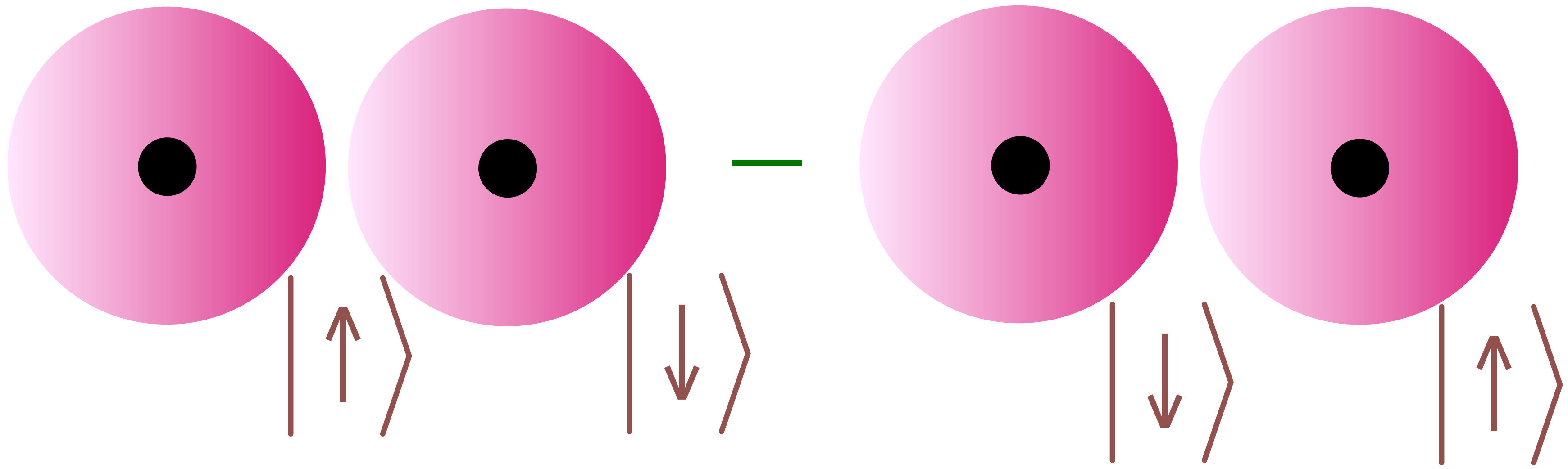
Not adiabatically connected
to independent electron states:

*many-particle*

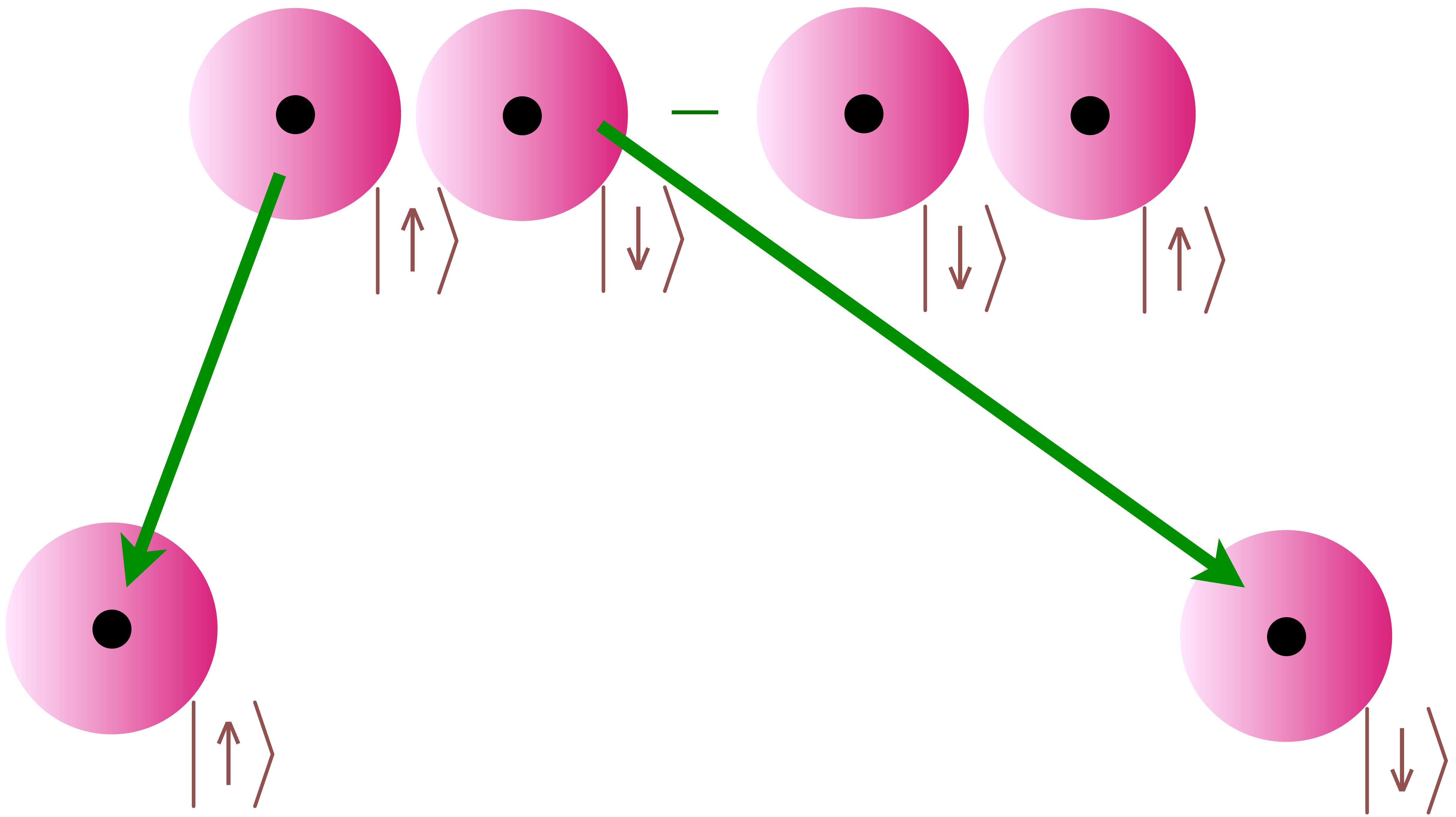
*quantum entanglement*

# Quantum Entanglement: quantum superposition with more than one particle

Hydrogen molecule:



# Quantum Entanglement: quantum superposition with more than one particle



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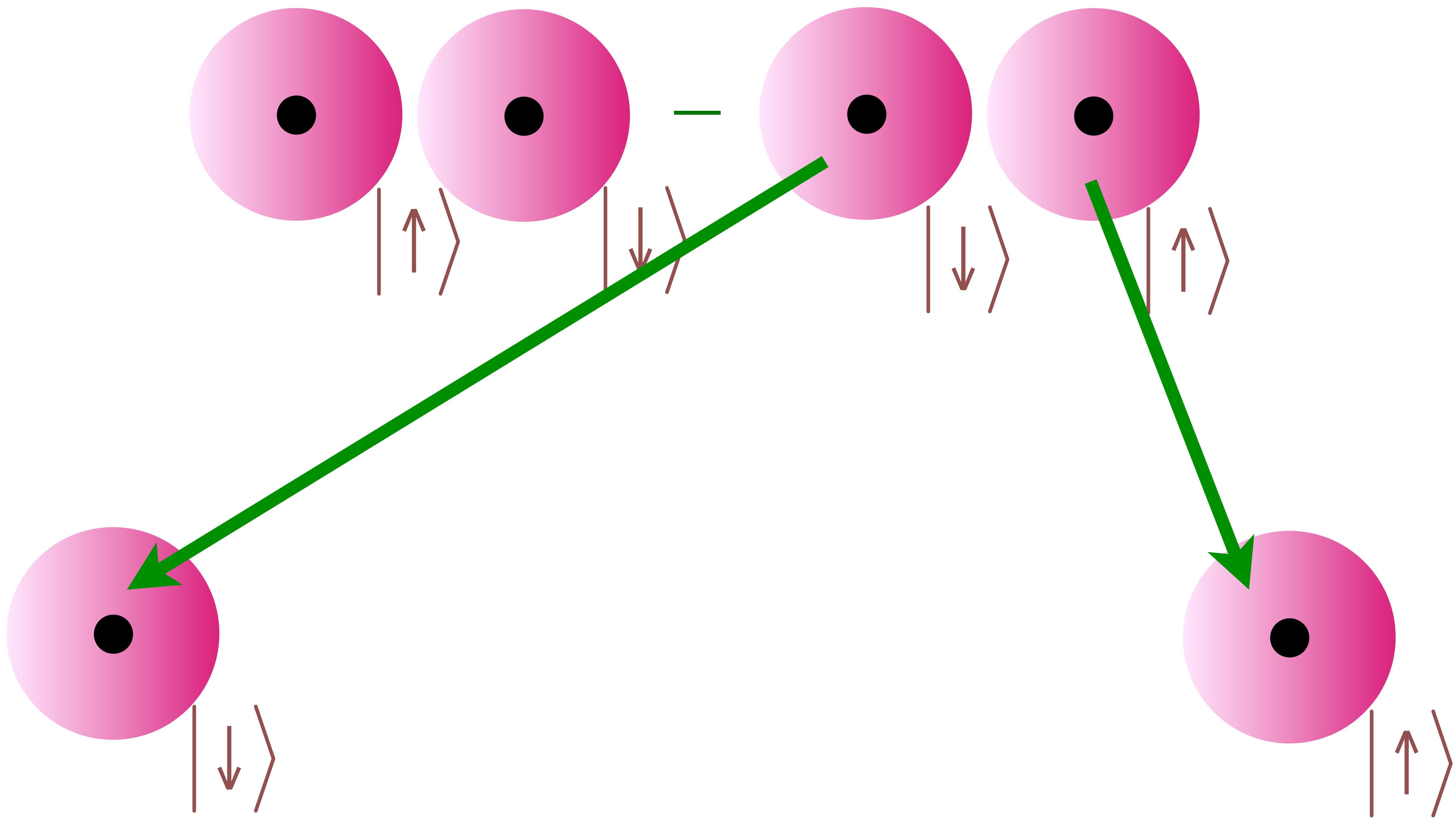

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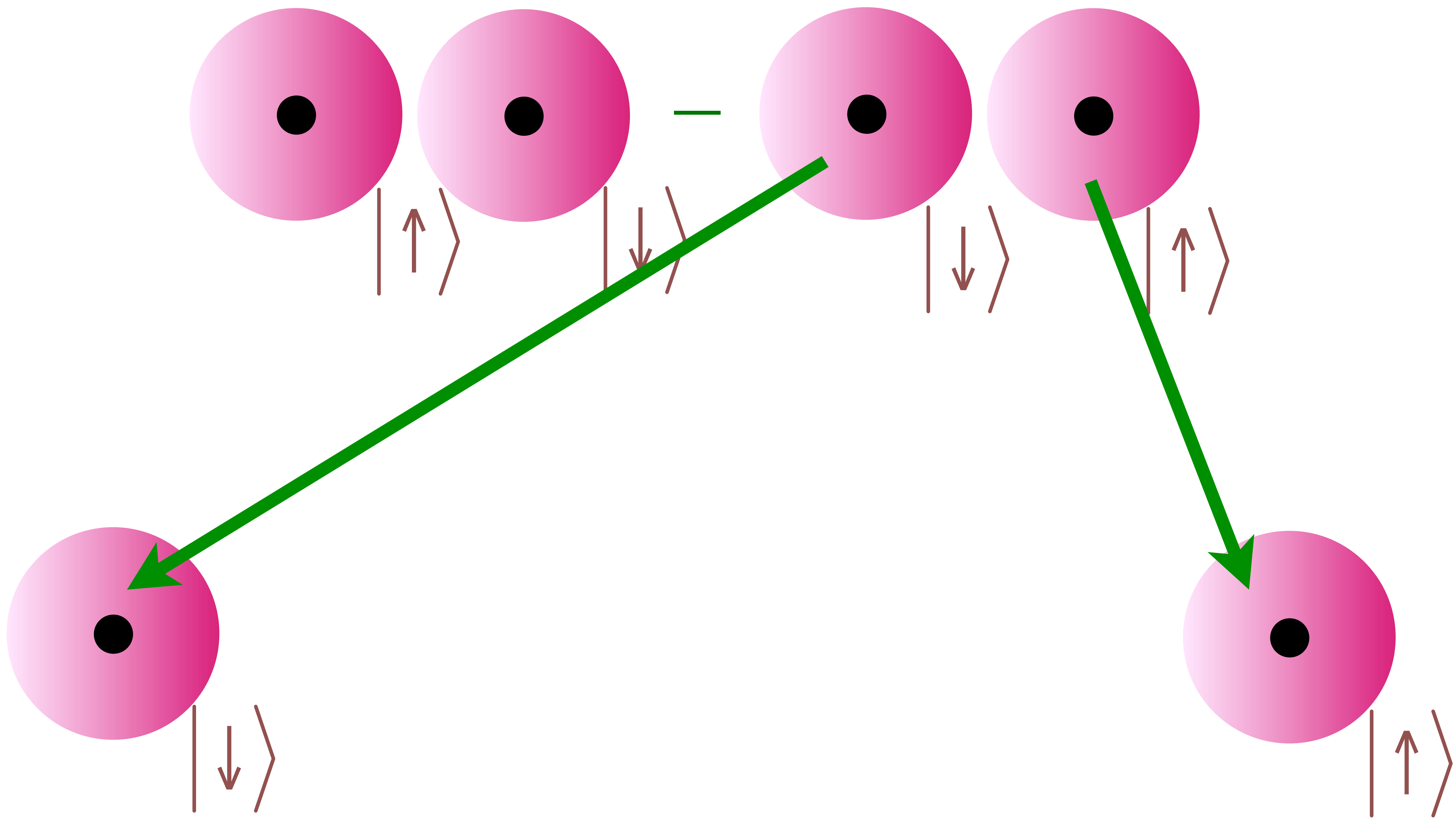
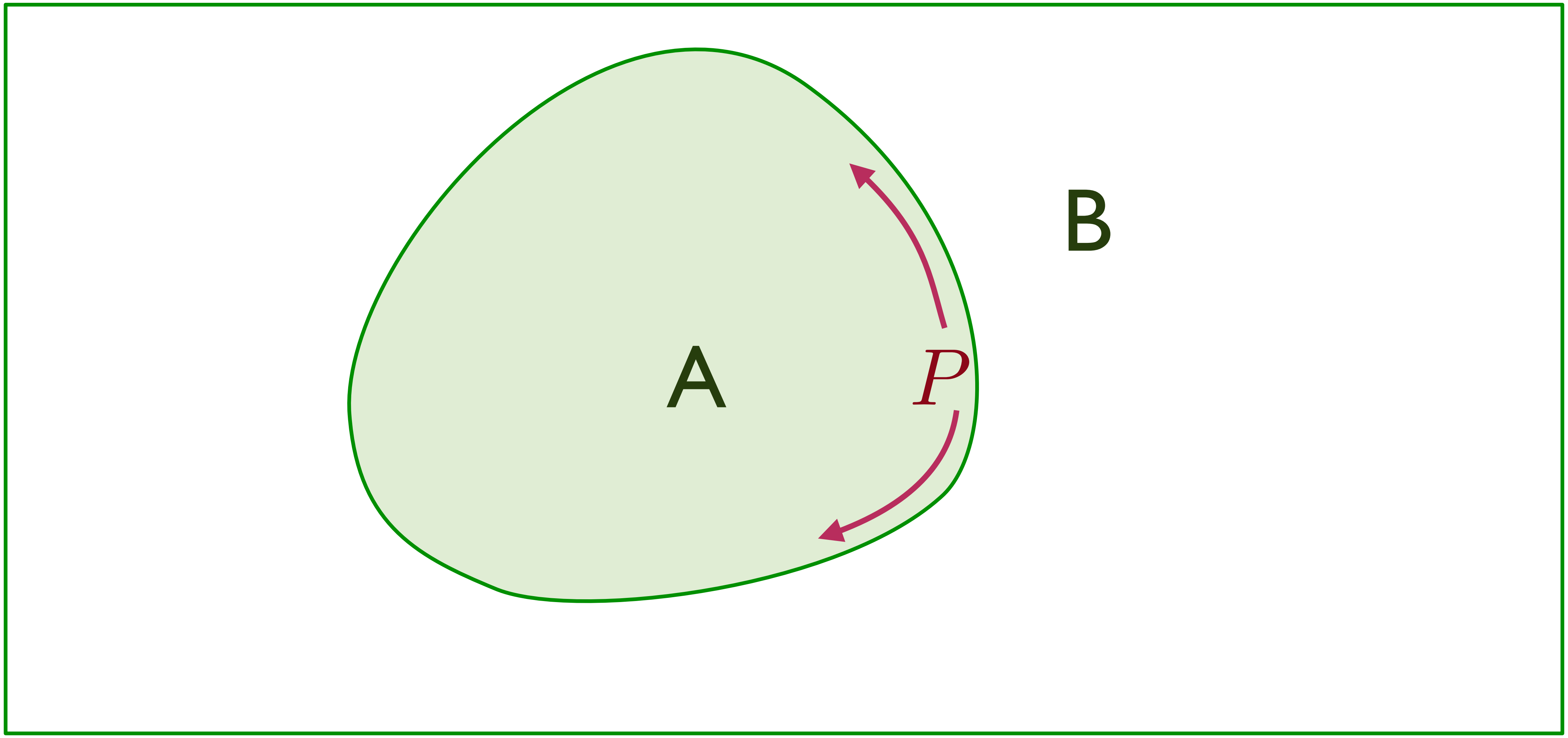



Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart

# Entanglement entropy




$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

**Entanglement entropy** $S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$

# “Complex entangled” states of quantum matter, *not* adiabatically connected to independent particle states

## Gapped quantum matter

*Spin liquids, quantum Hall states*

## Conformal quantum matter

*Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene*

## Compressible quantum matter

*Strange metals in high temperature superconductors, Bose metals*

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**topological field theory**

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**?**

# “Complex entangled” states of quantum matter in *d* spatial dimensions

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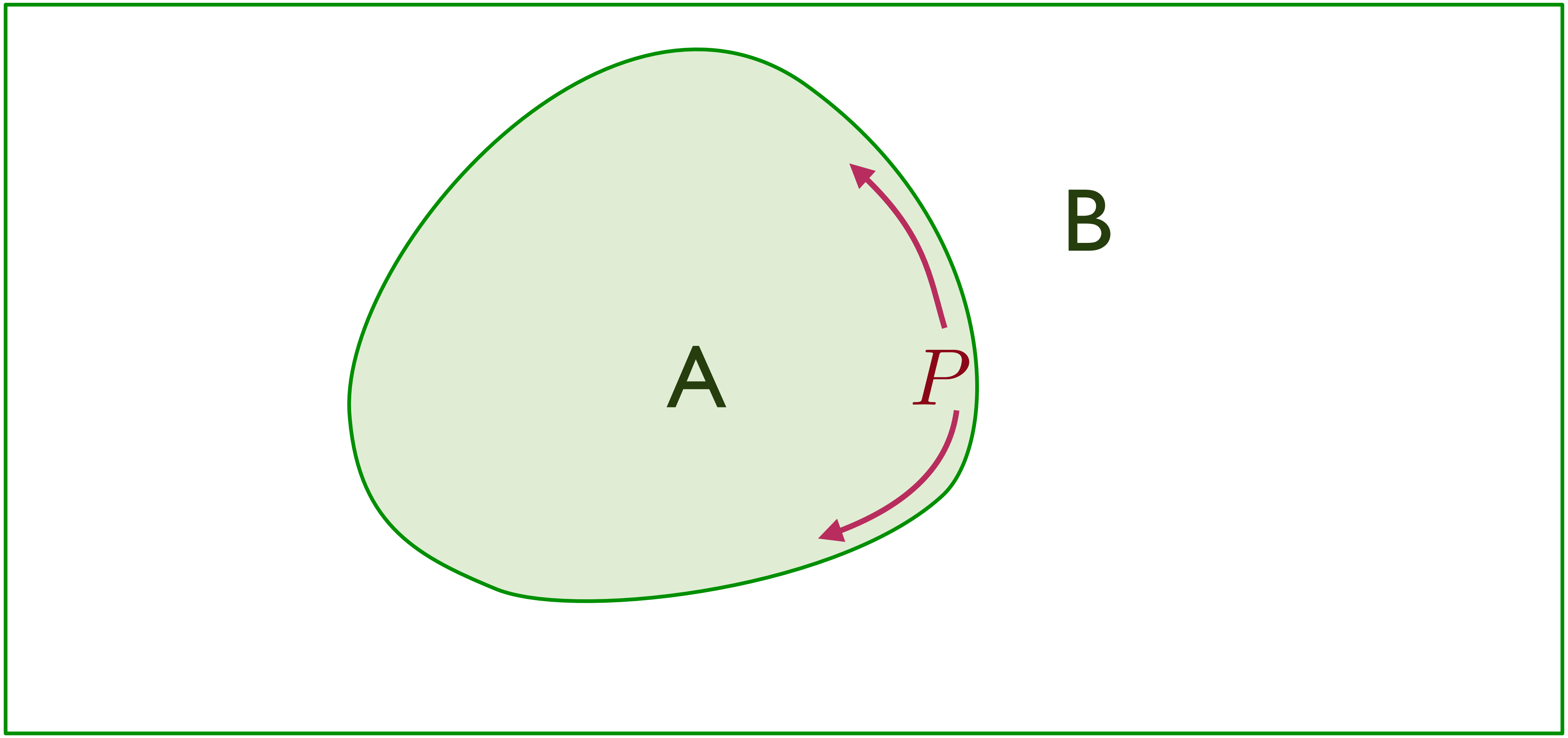
# Entanglement entropy of a band insulator

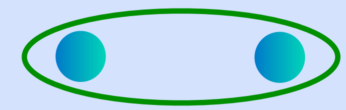



$$S_E = aP - b\exp(-cP)$$

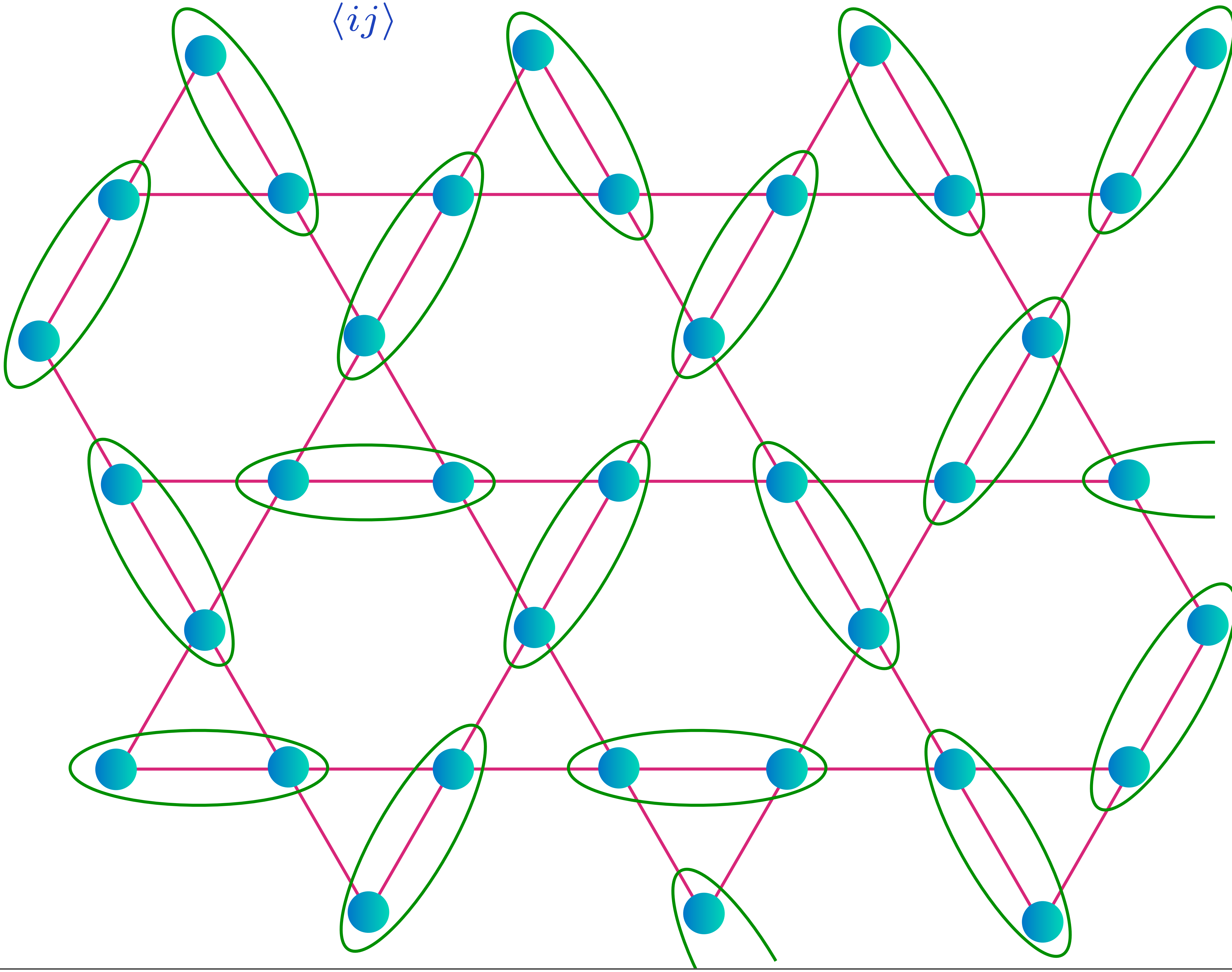
where $P$ is the surface area (perimeter) of the boundary between A and B.

# Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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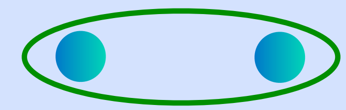
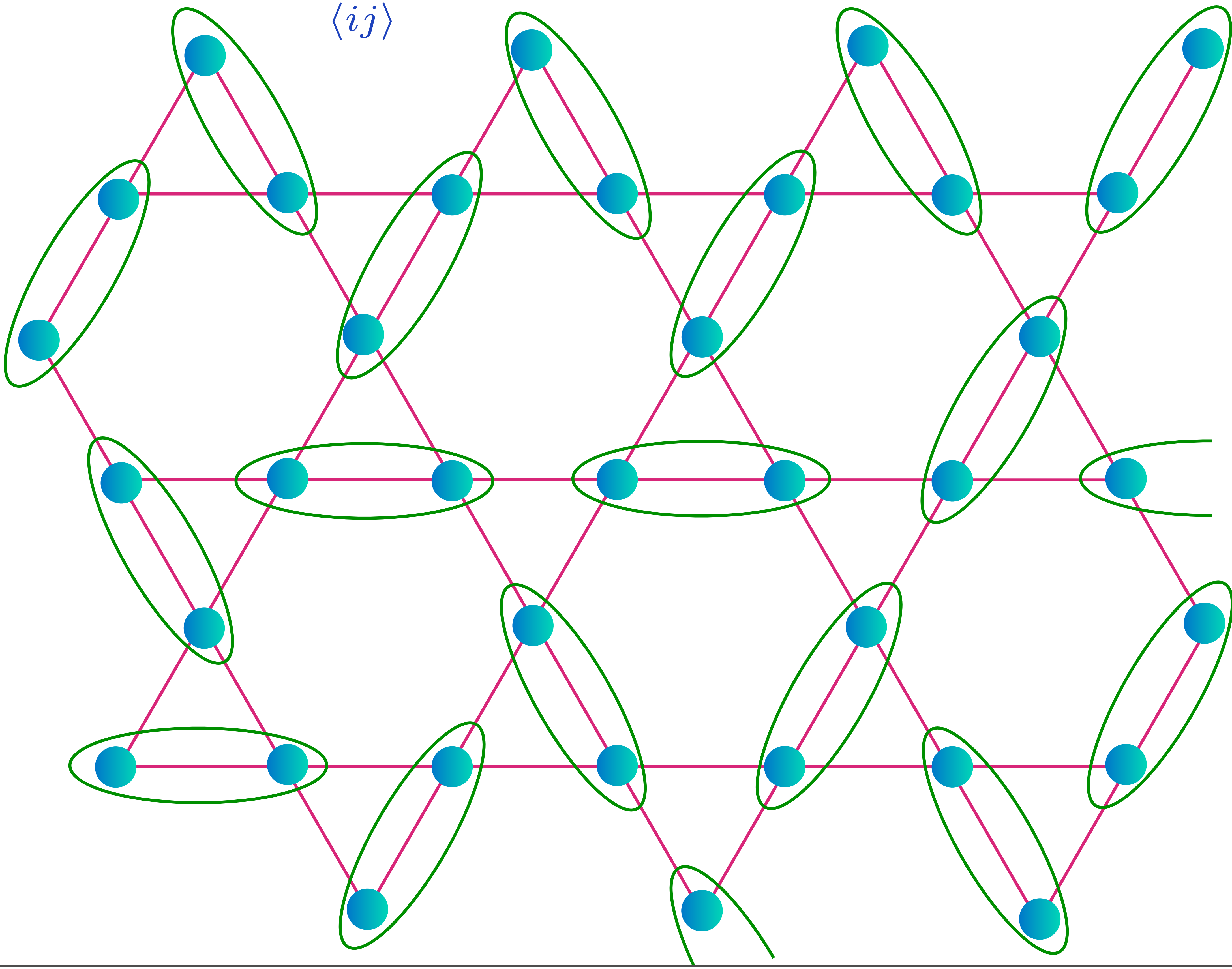
 $= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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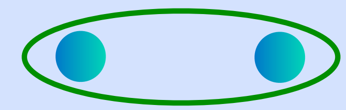
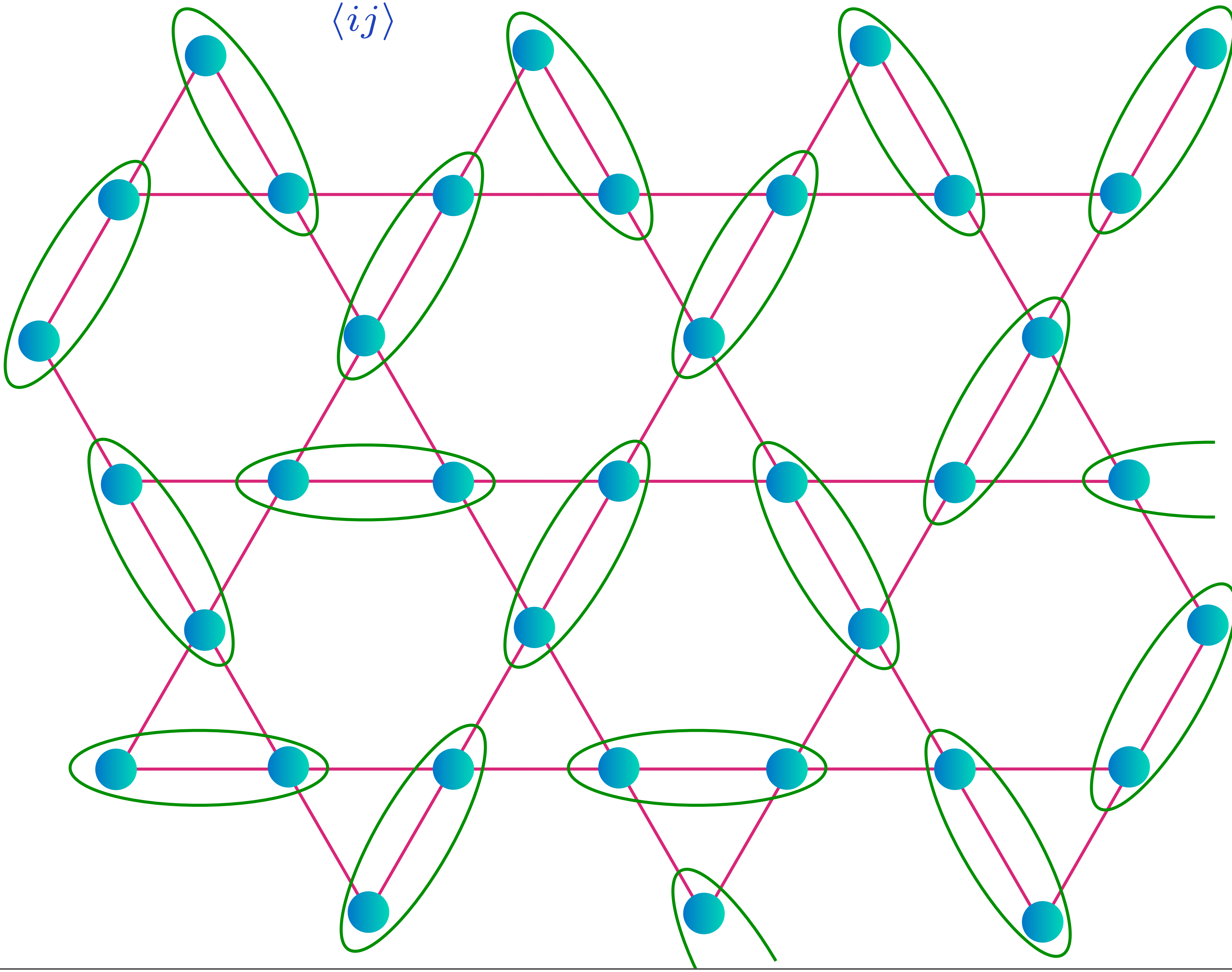
 $= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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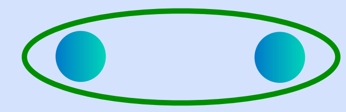

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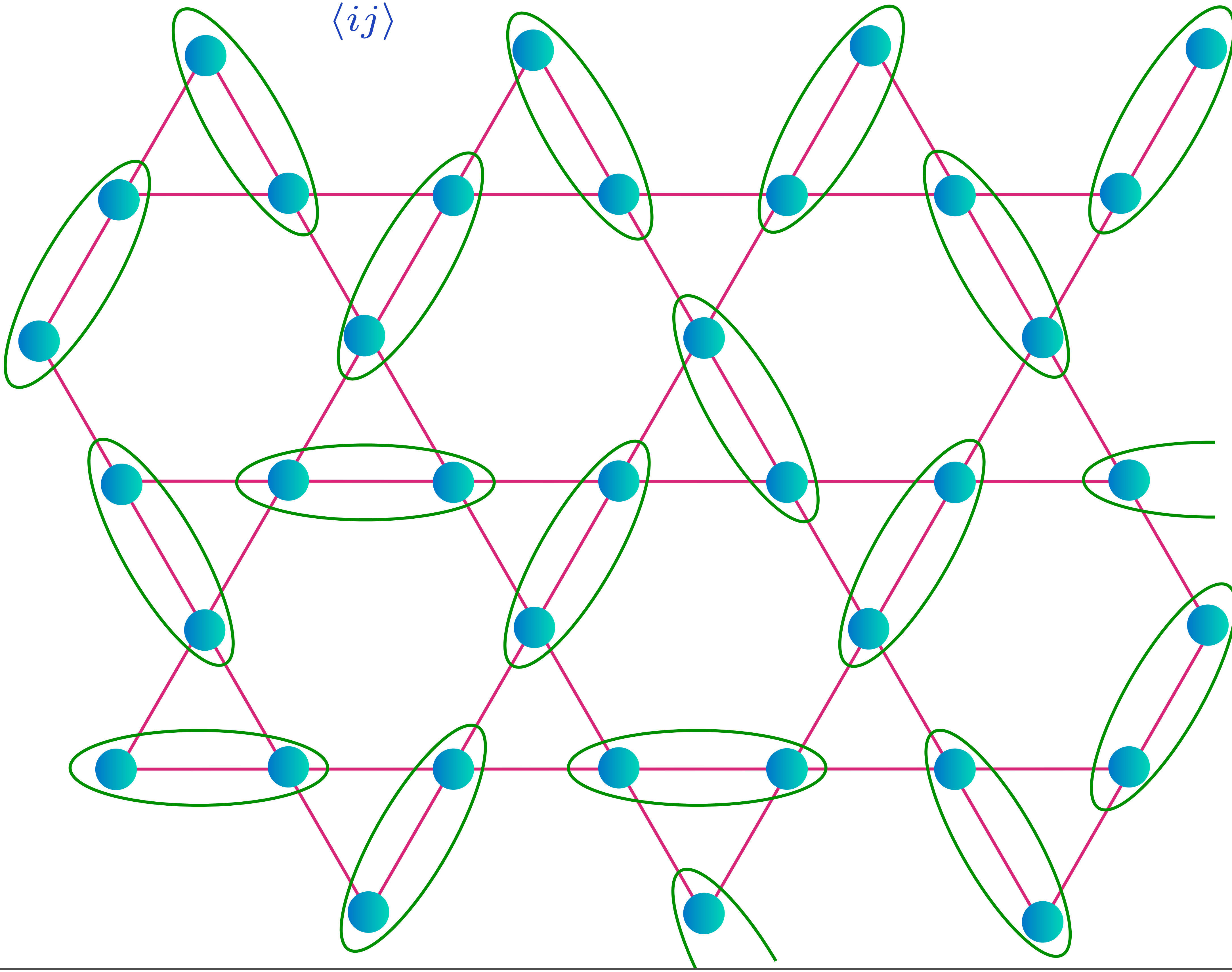

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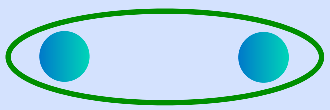

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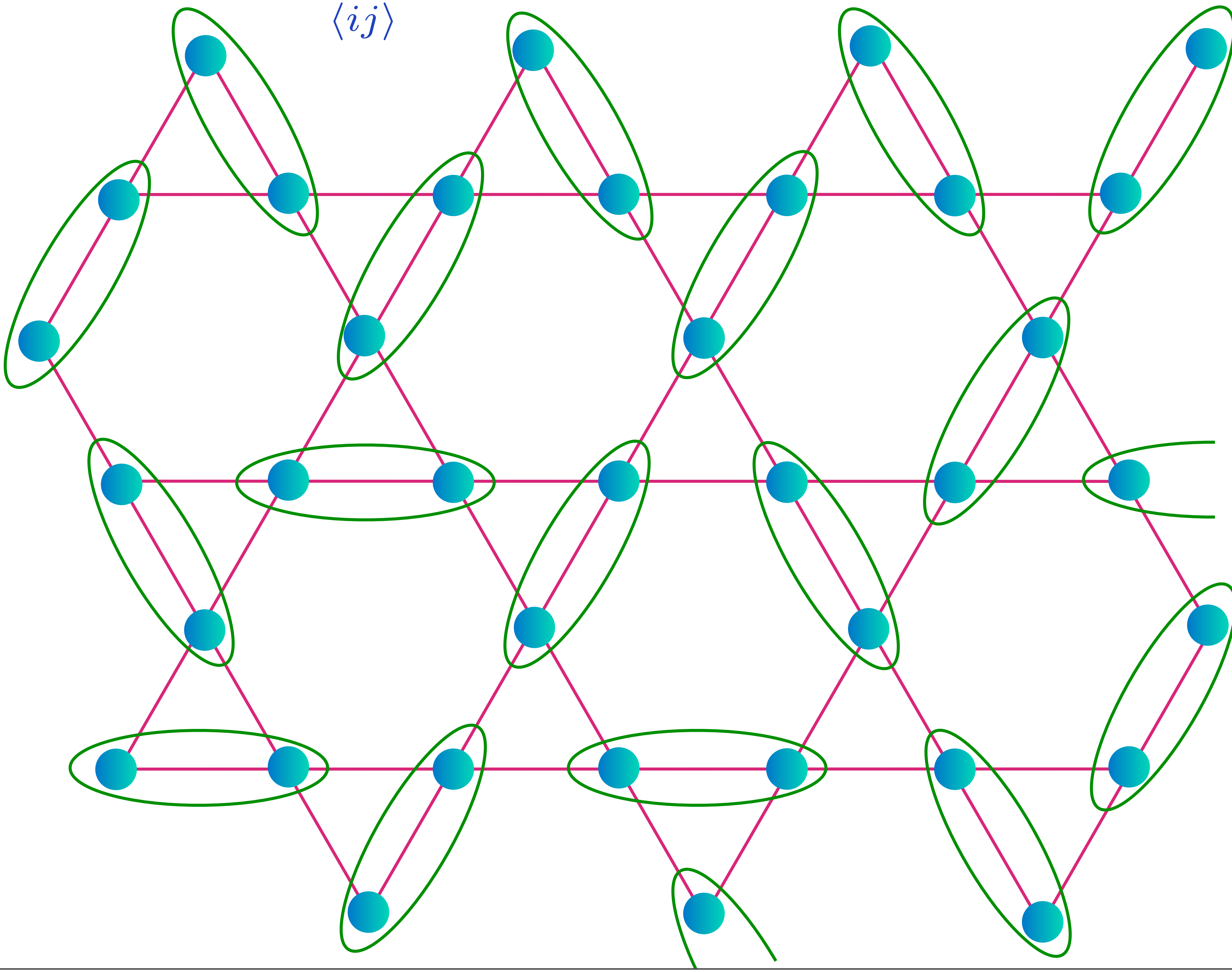

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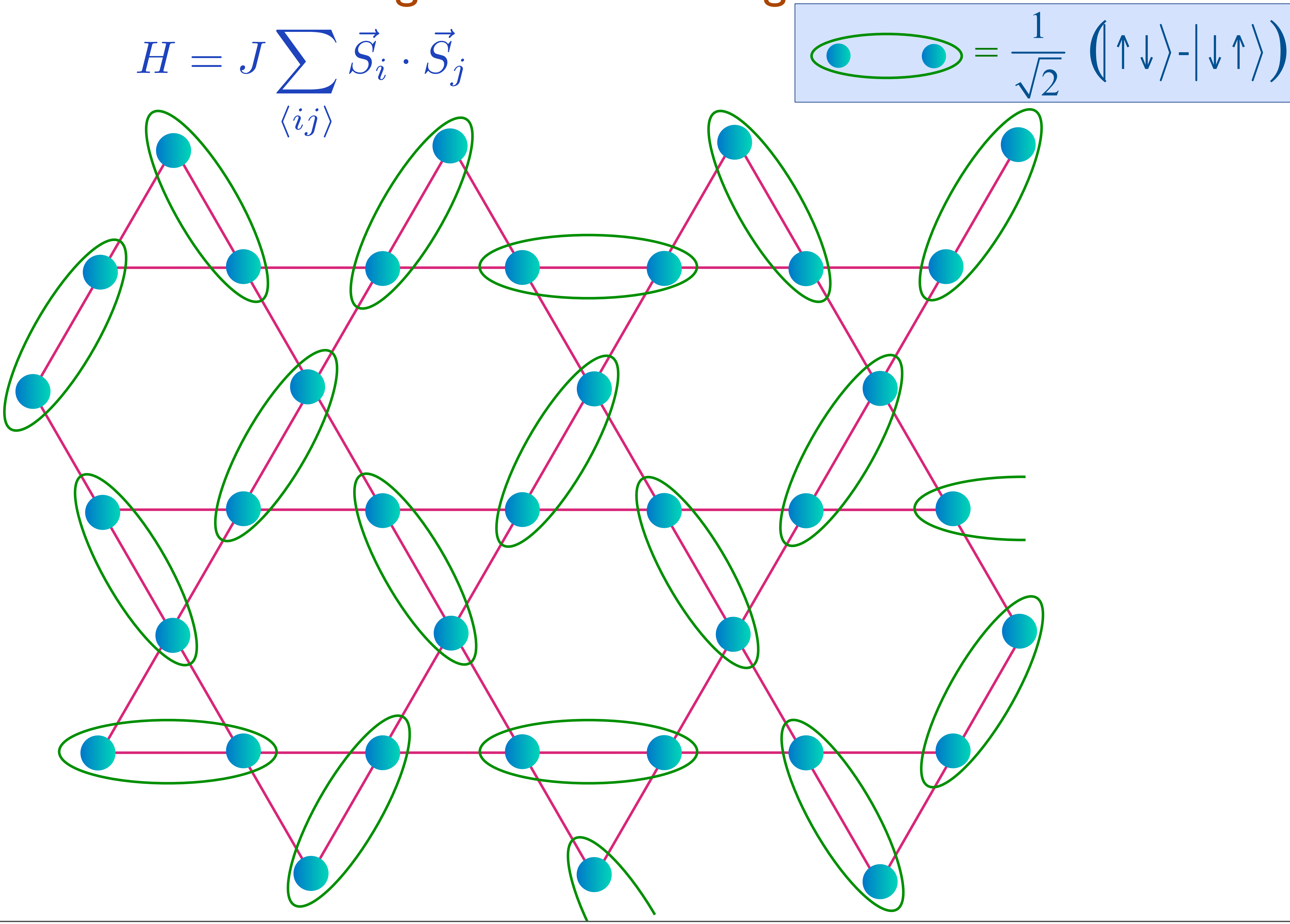

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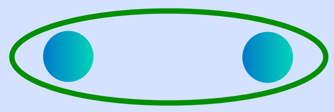
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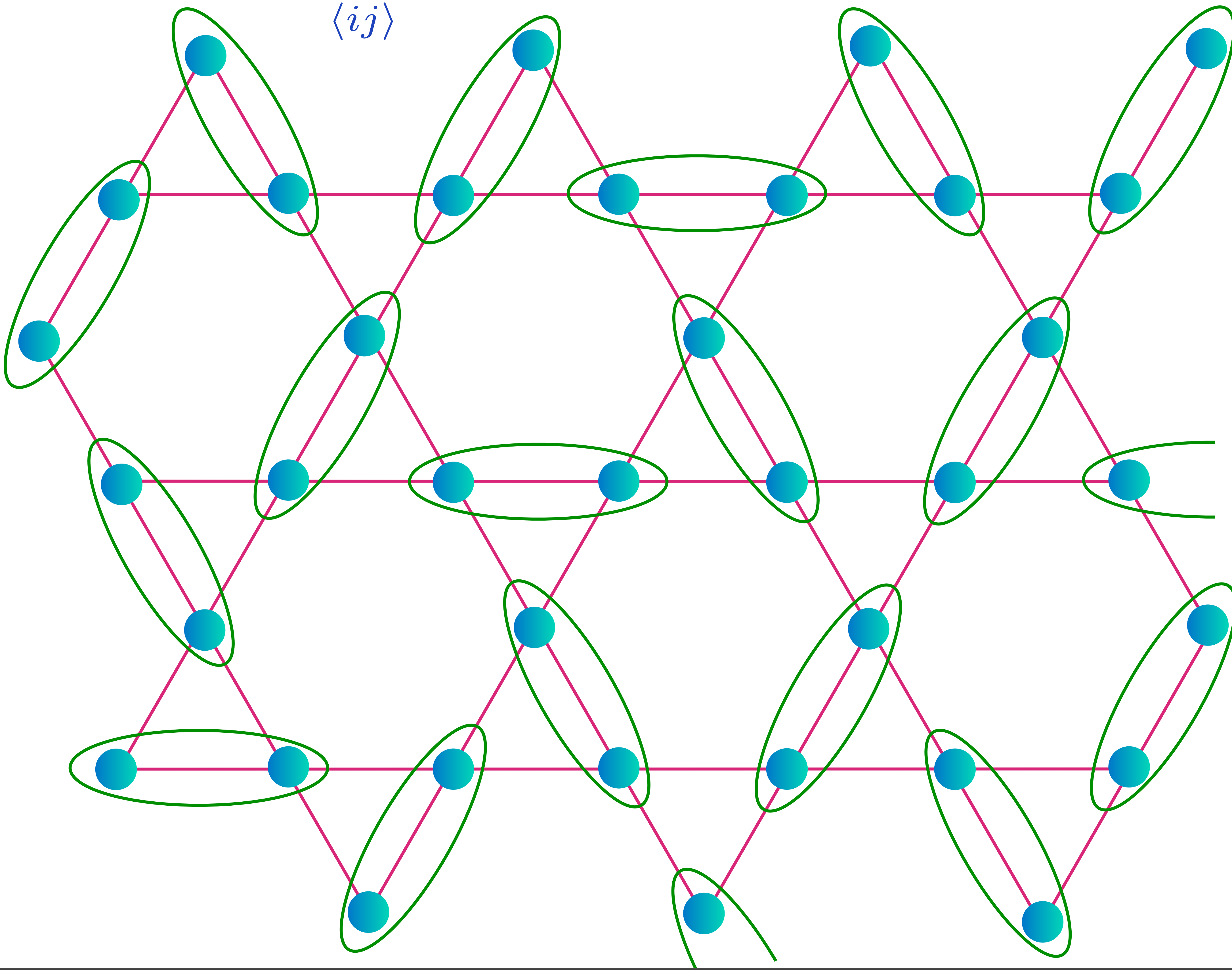
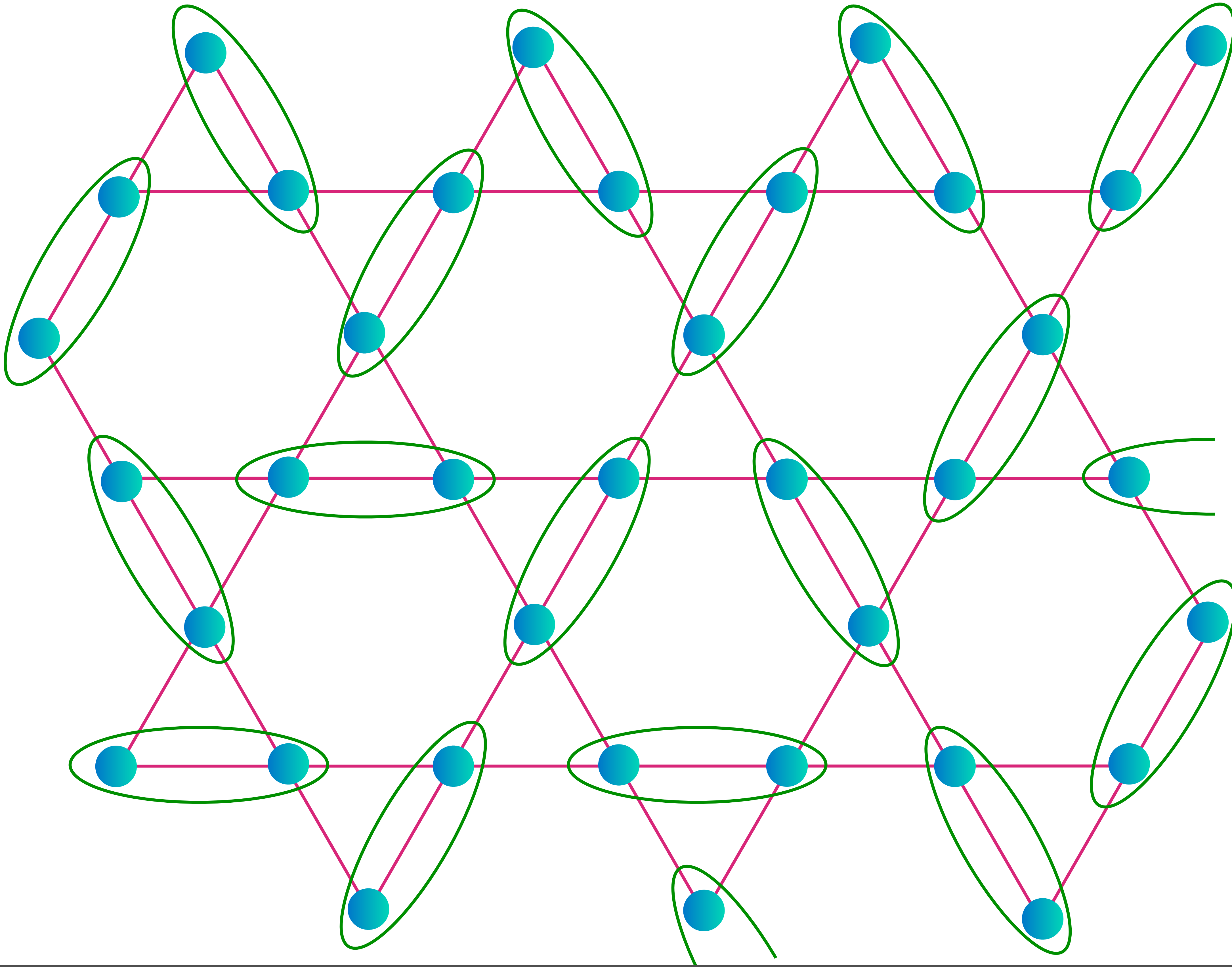


# Mott insulator: Kagome antiferromagnet

Alternative view

Pick a reference configuration



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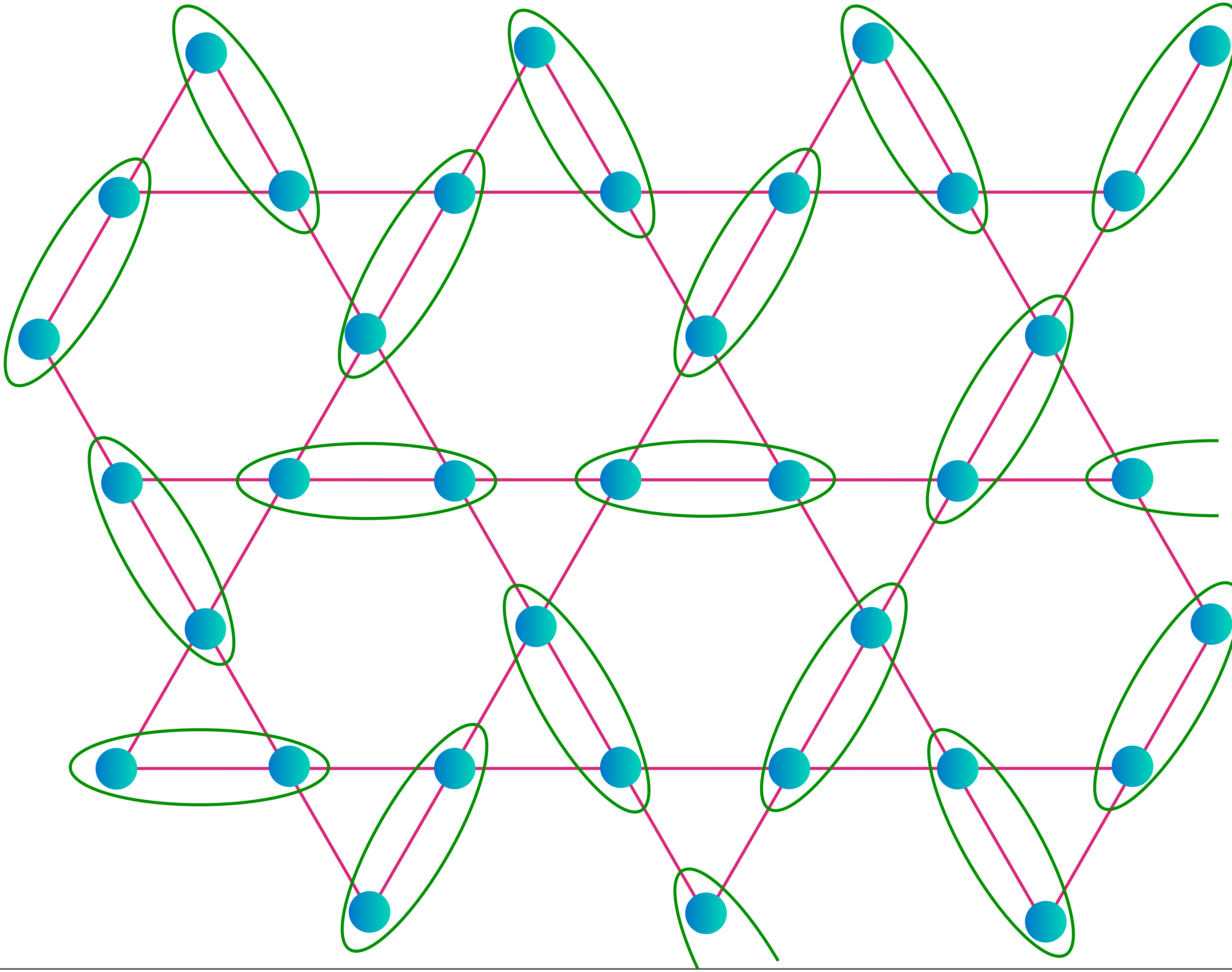
Alternative view

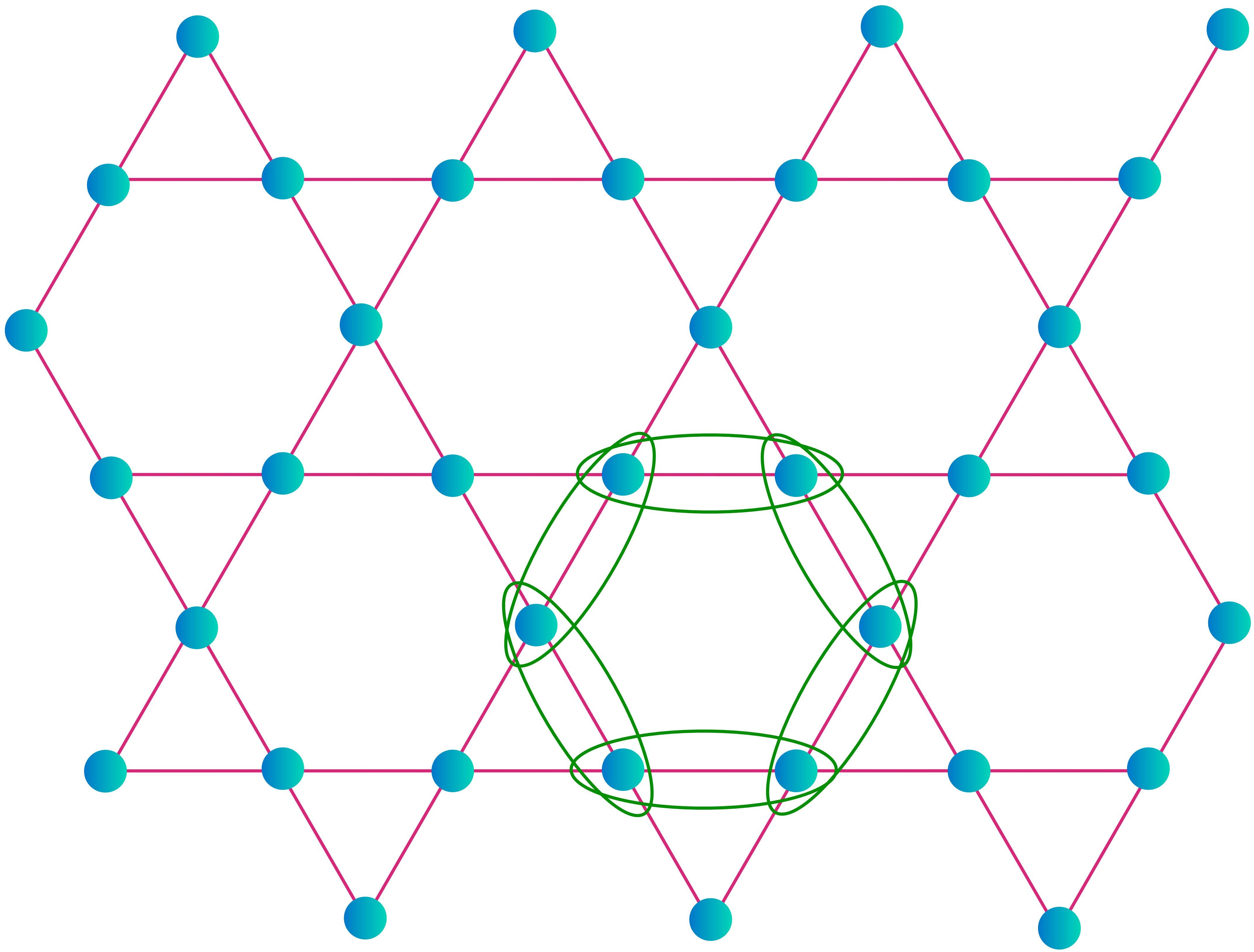
A nearby configuration



# Mott insulator: Kagome antiferromagnet

Alternative view

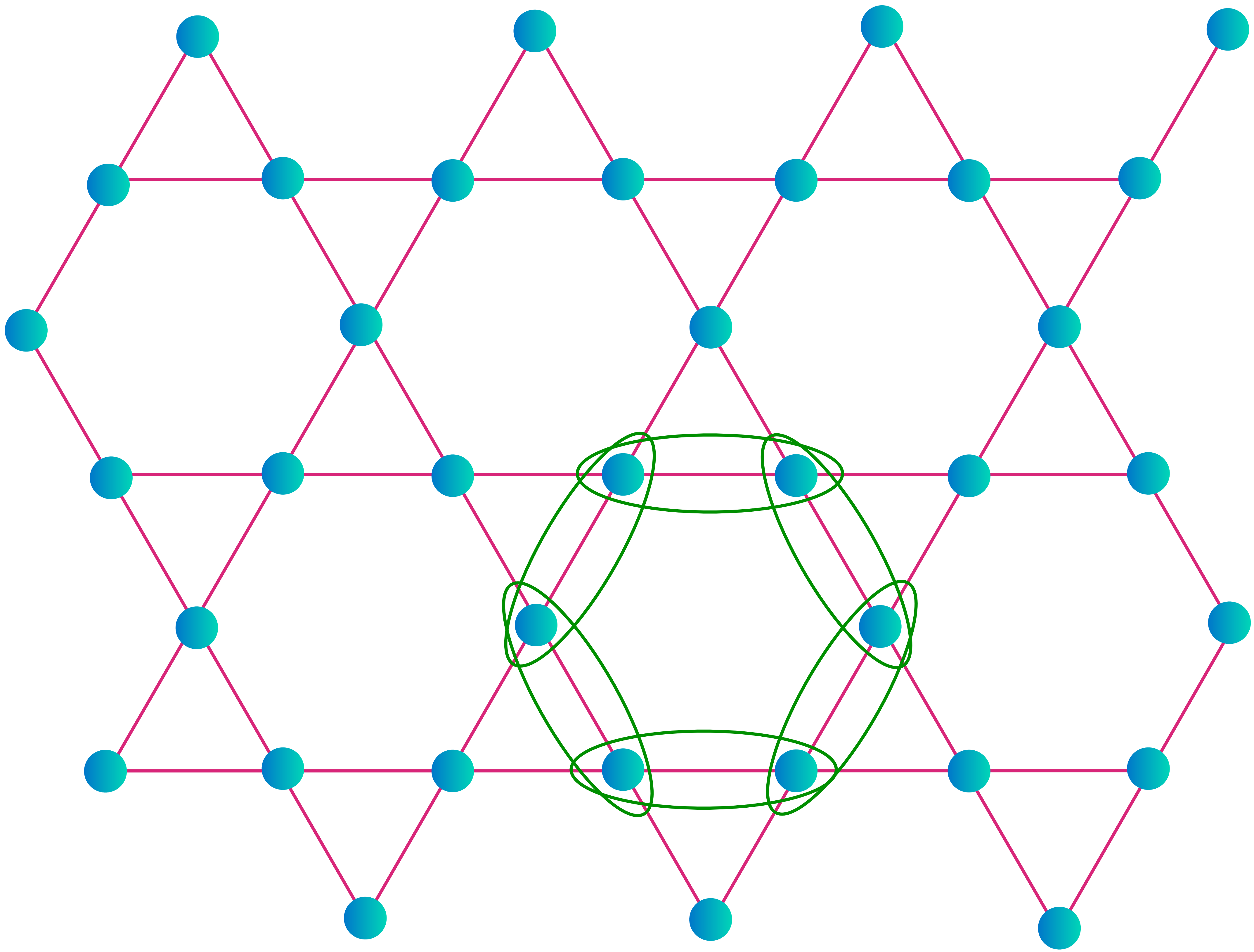
Difference: a closed loop



# Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops



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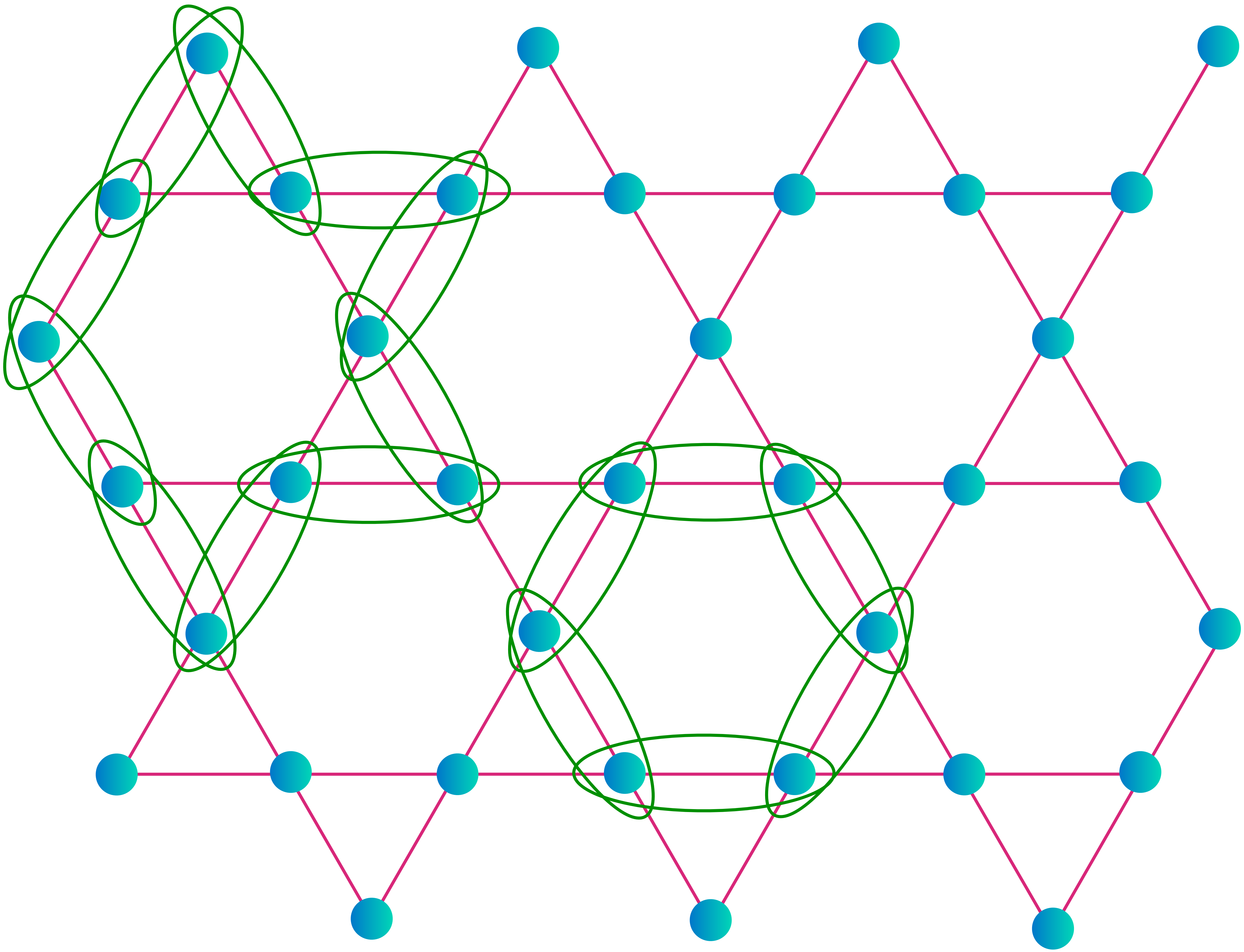

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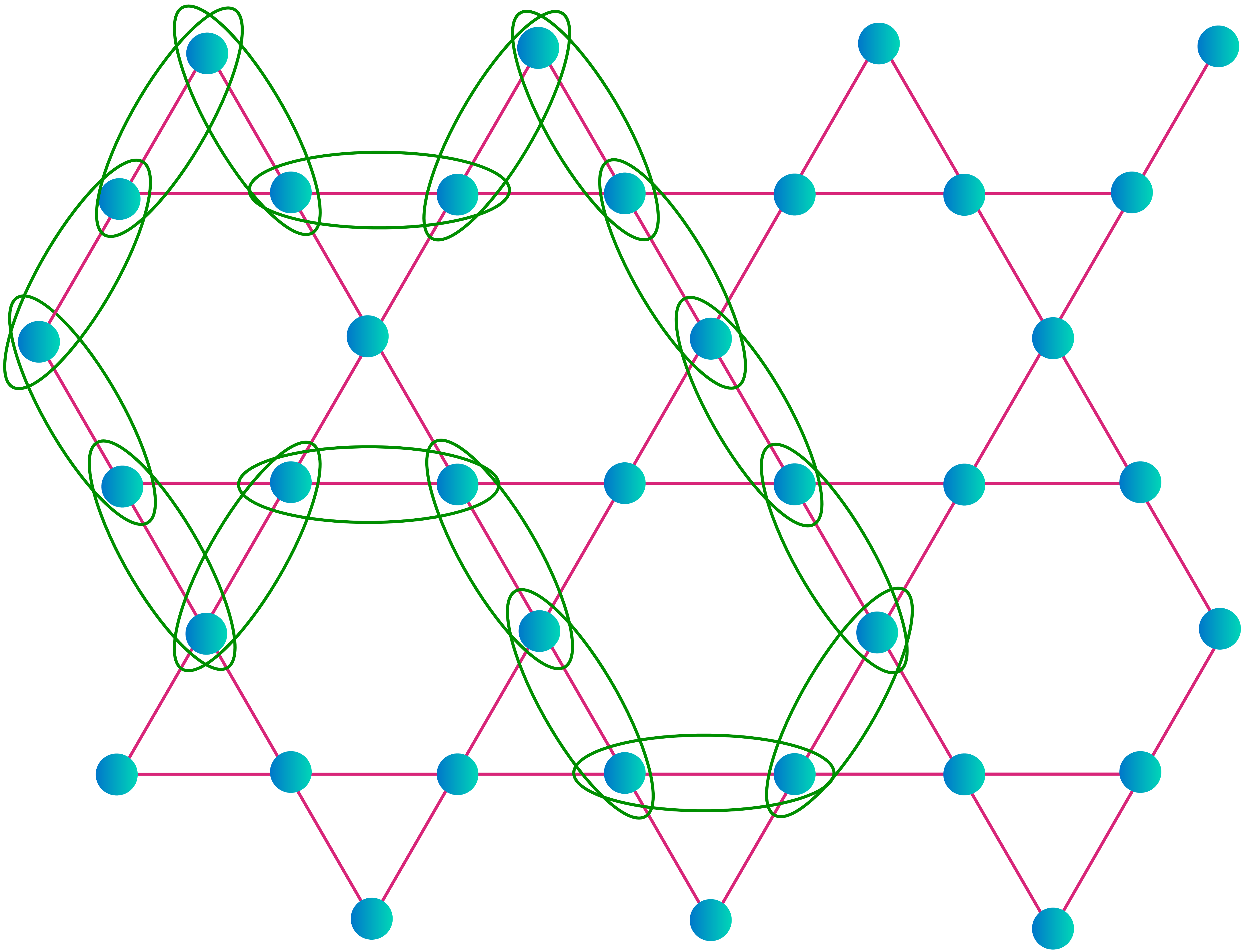

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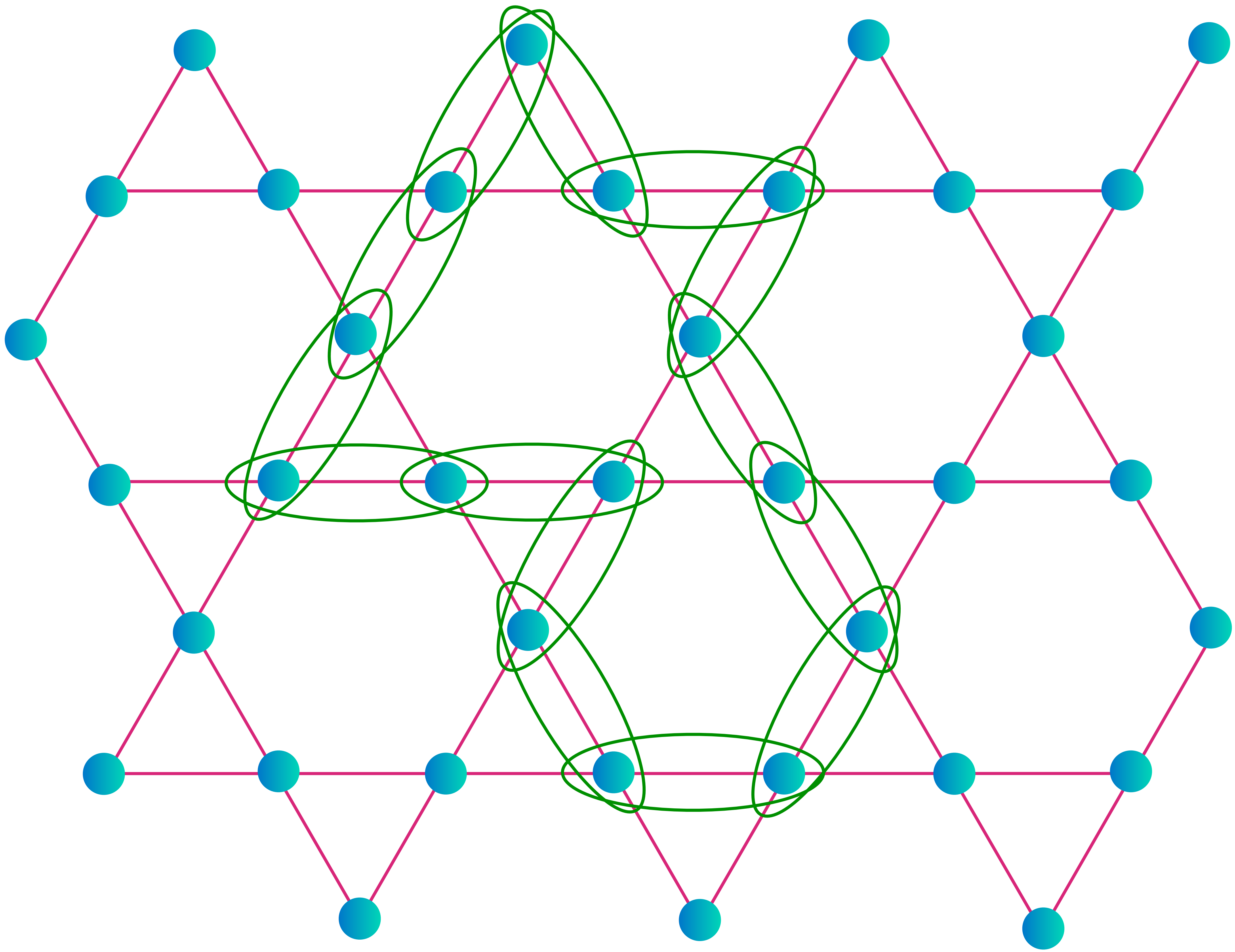
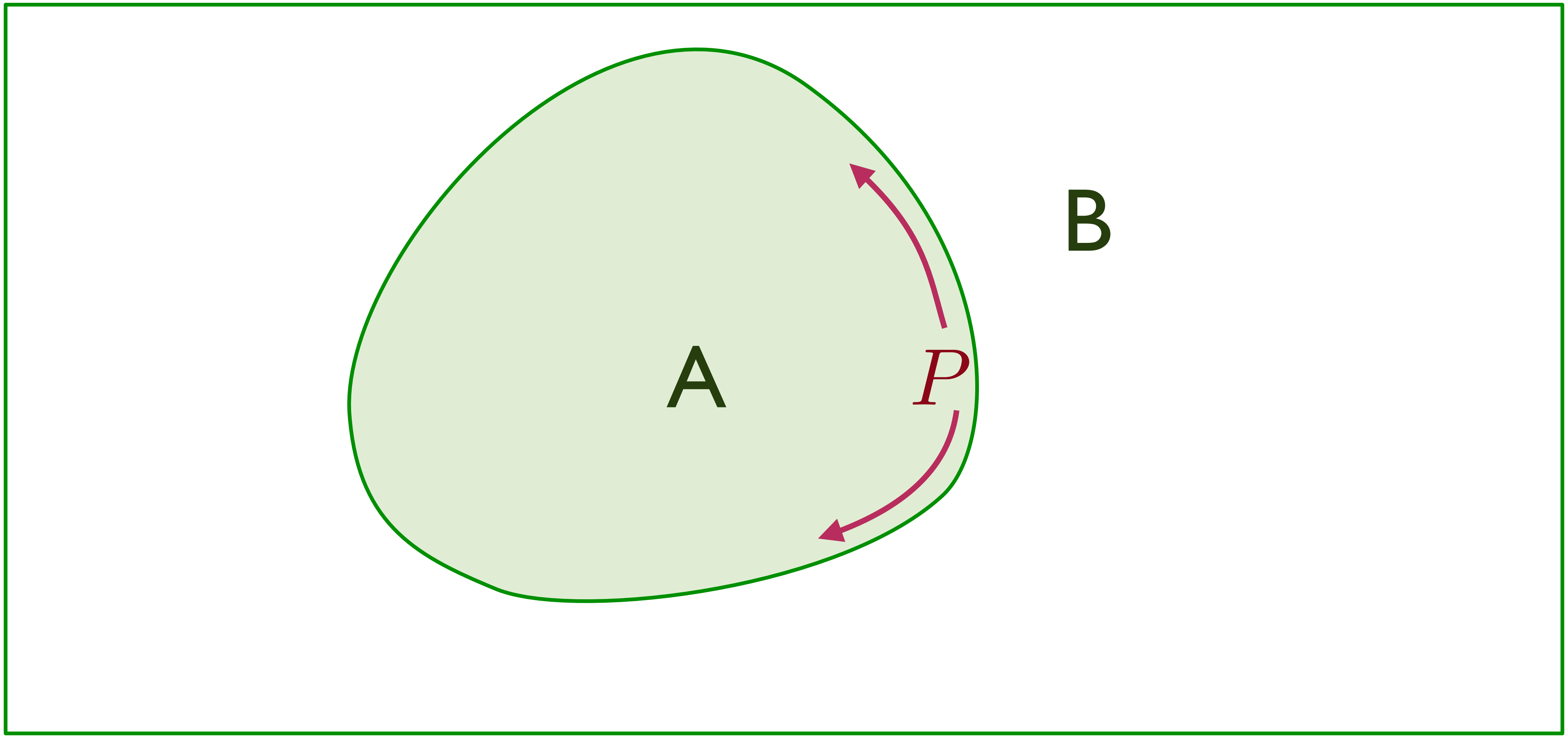


# Entanglement in the $Z_2$ spin liquid




The sum over closed loops is characteristic of the $Z_2$ spin liquid, introduced in
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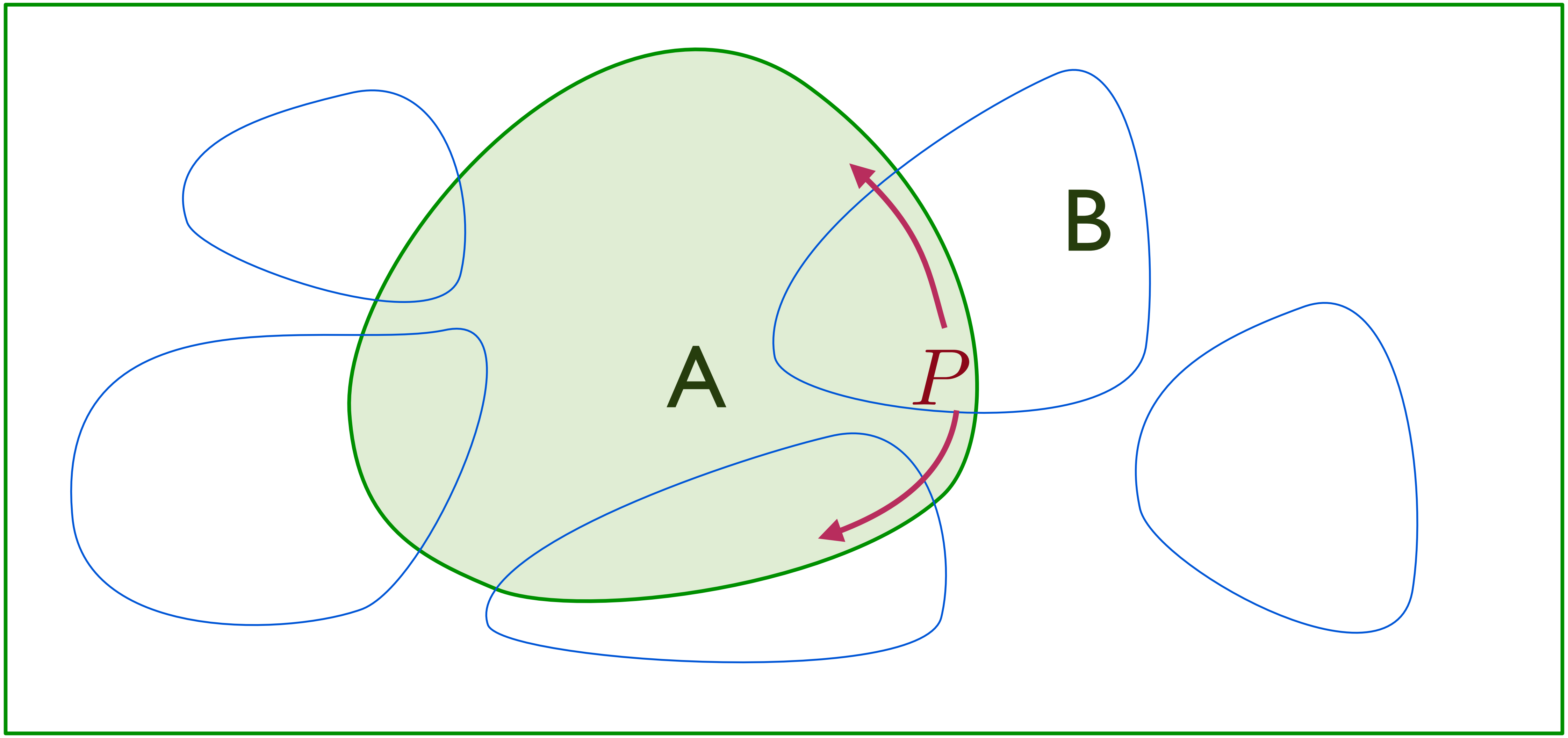


Sum over closed loops: only an even number of links cross the boundary between A and B

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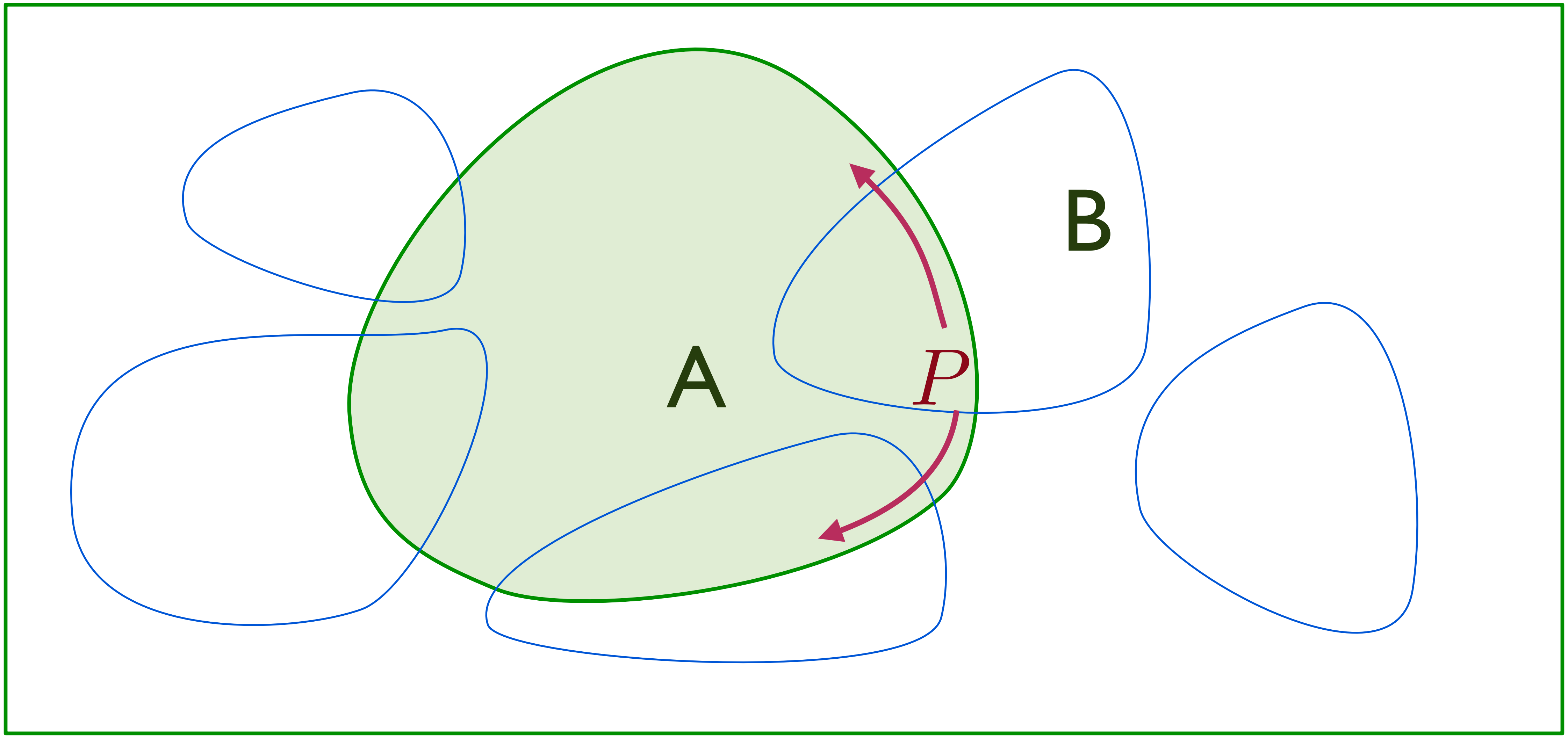


$$S_E = aP - \ln(2)$$

where $P$ is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006); Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

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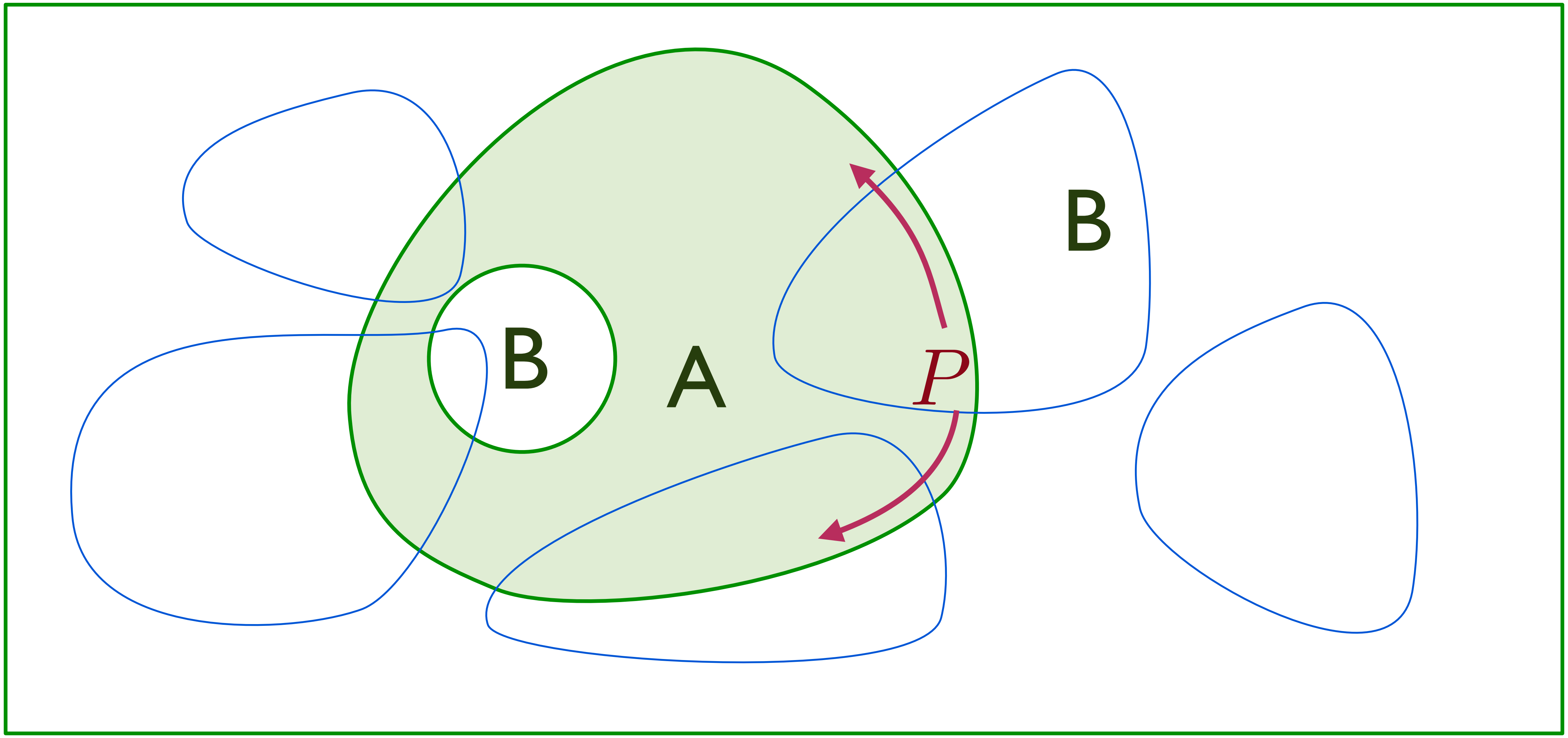


$$S_E = aP - \ln(4)$$

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M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
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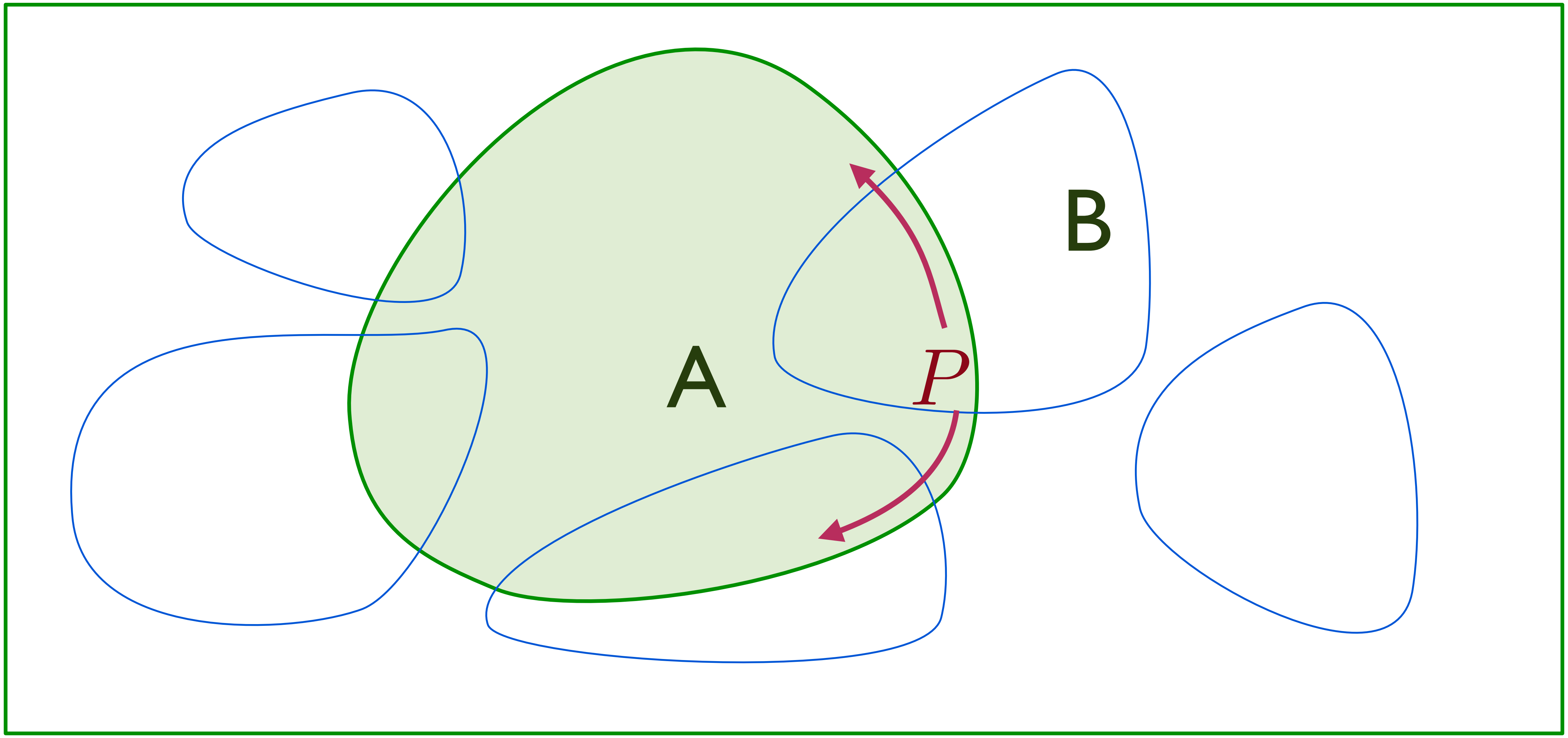


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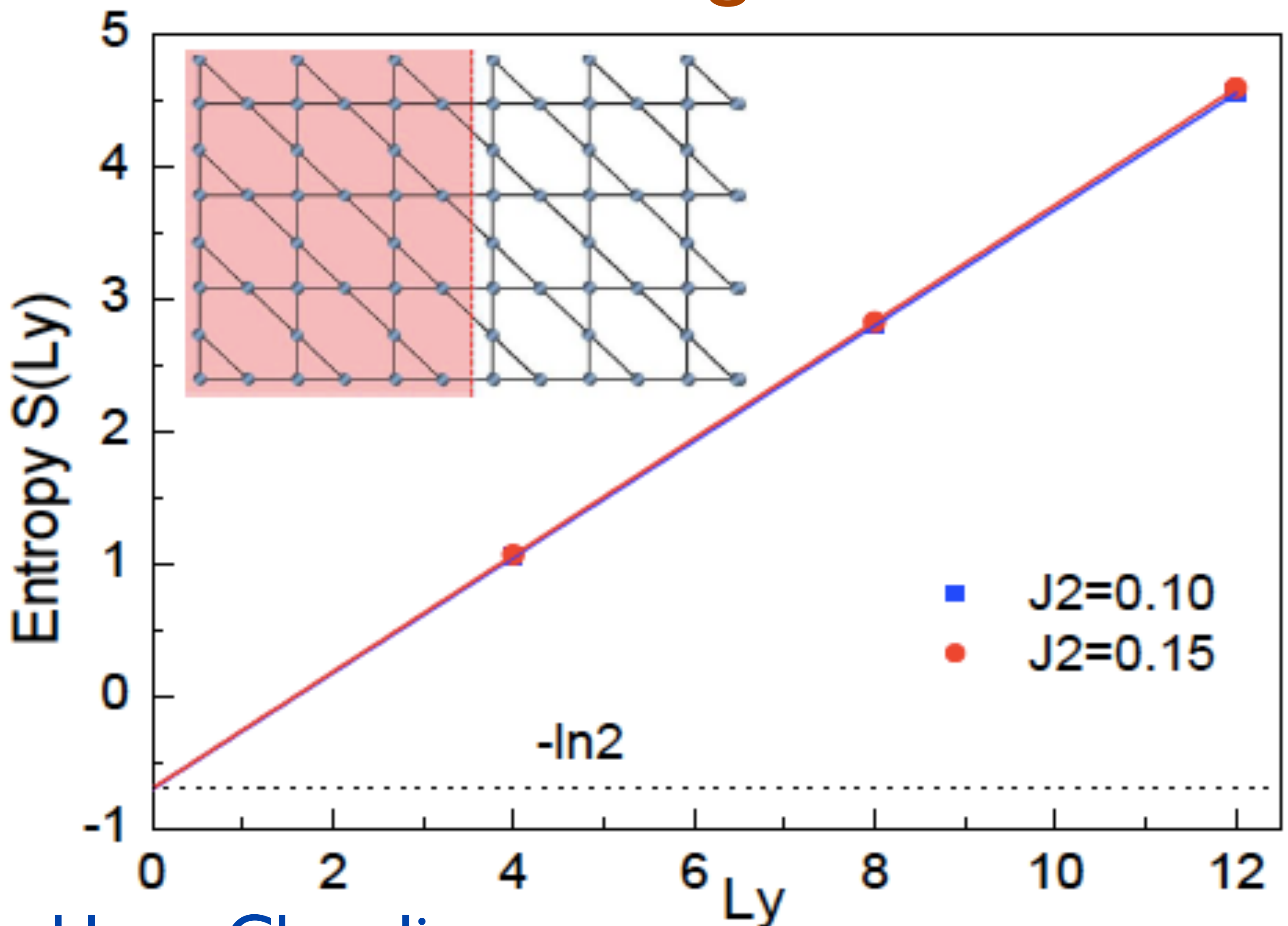
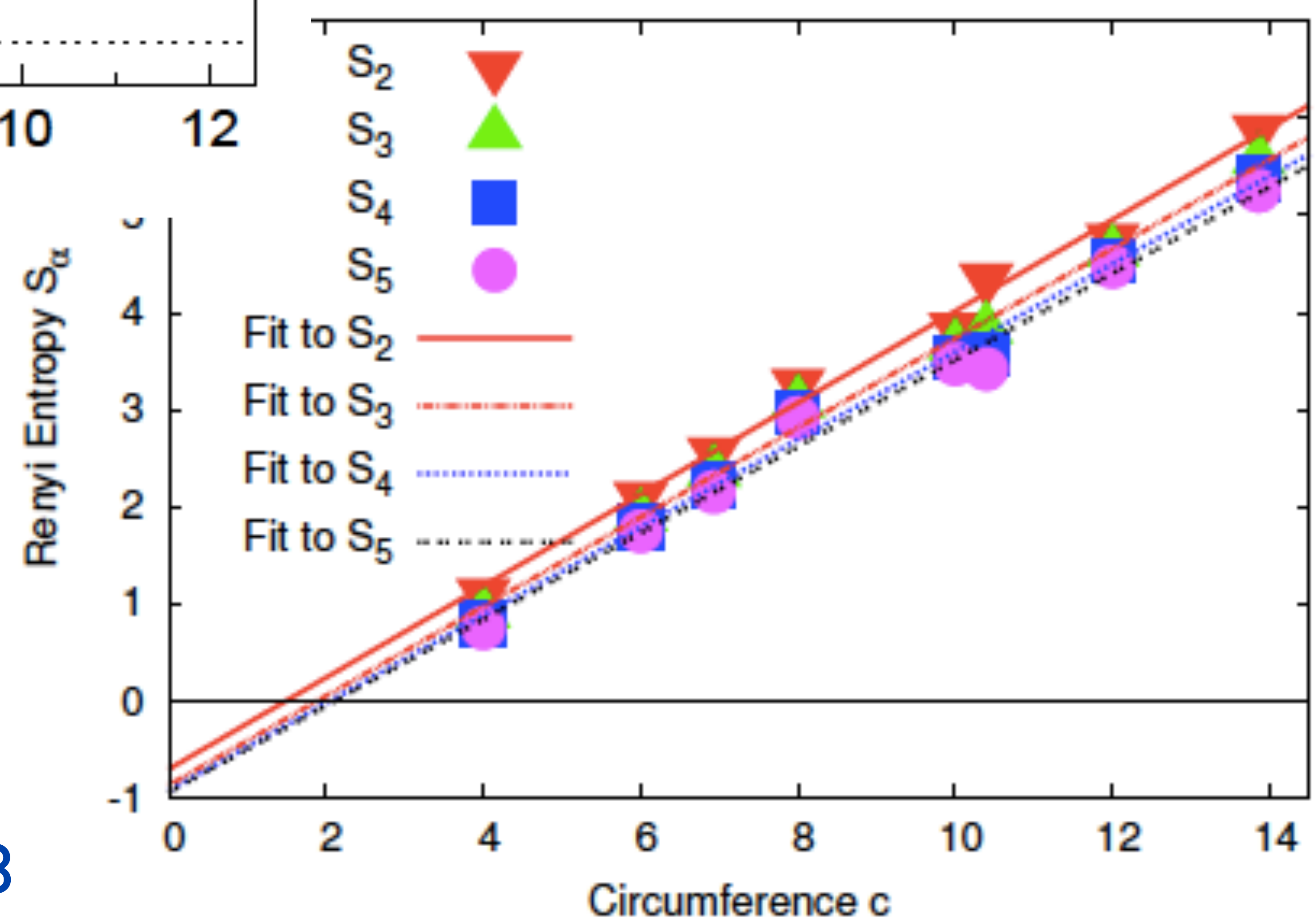



Strong numerical evidence for a $Z_2$ spin liquid

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289




S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

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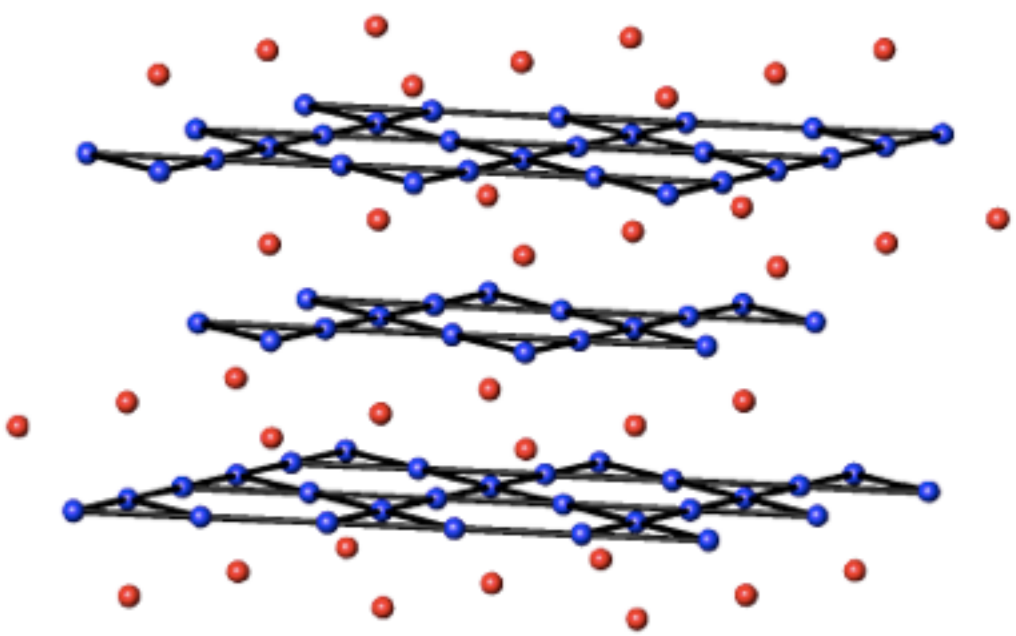
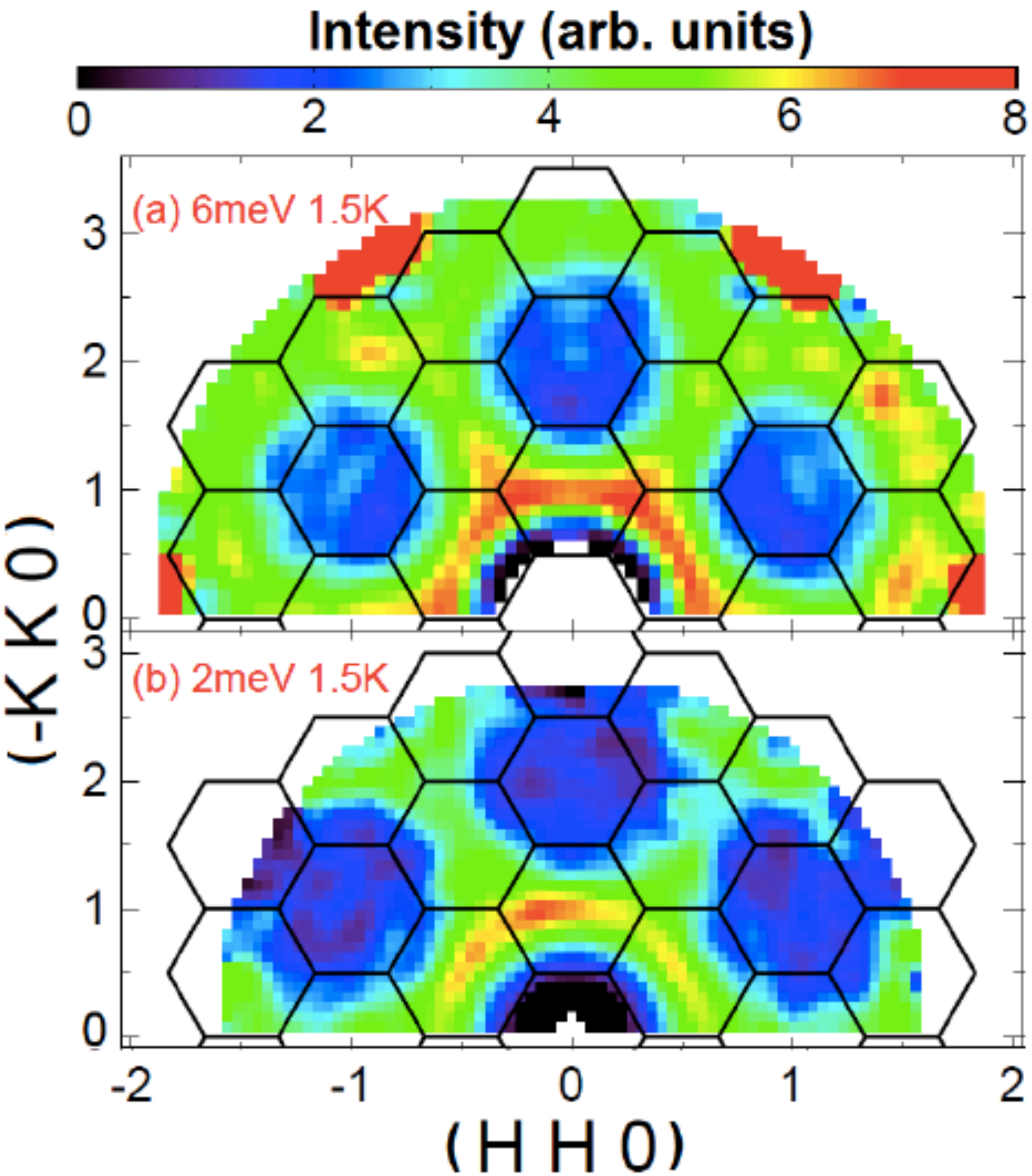
Evidence for spinons
Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





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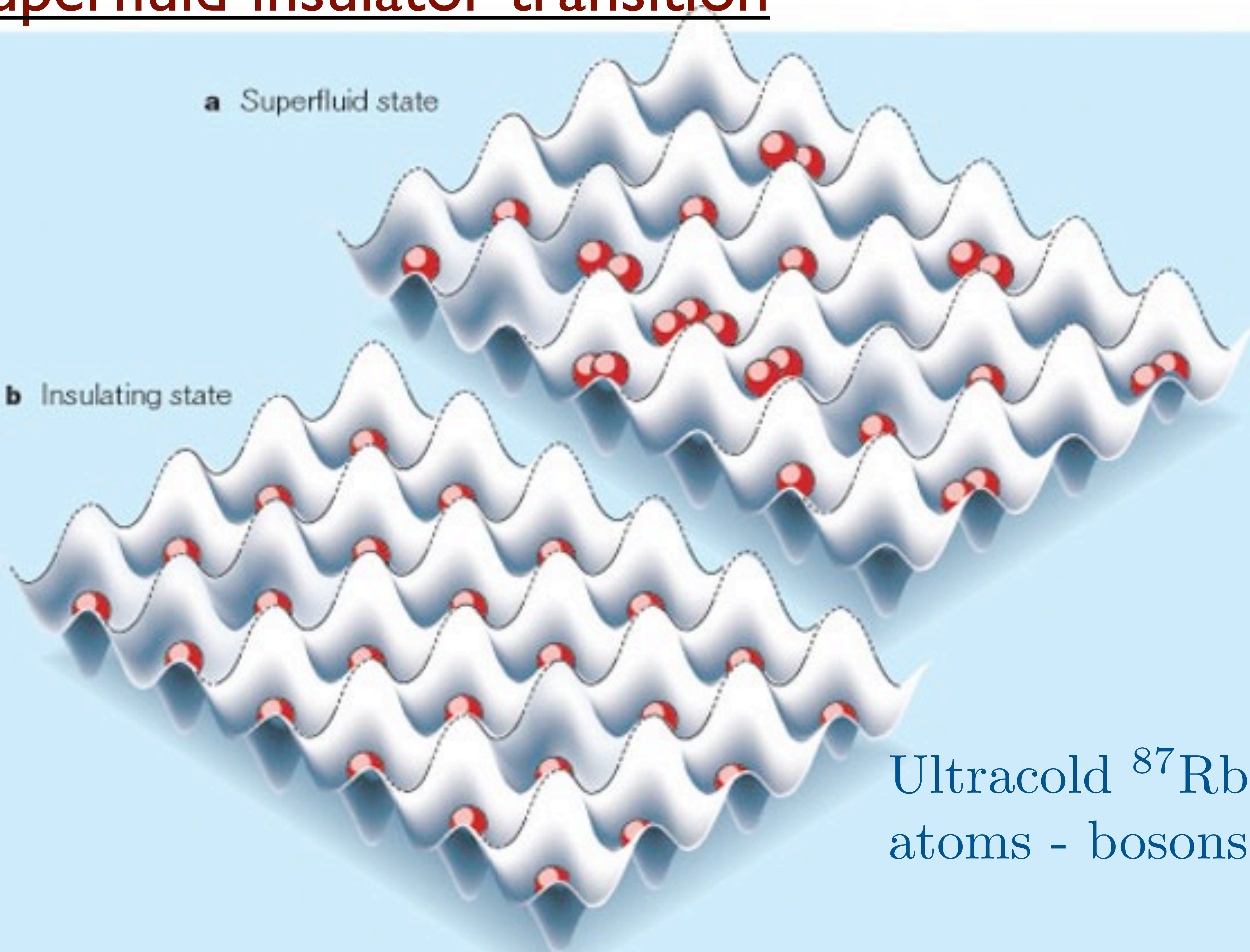
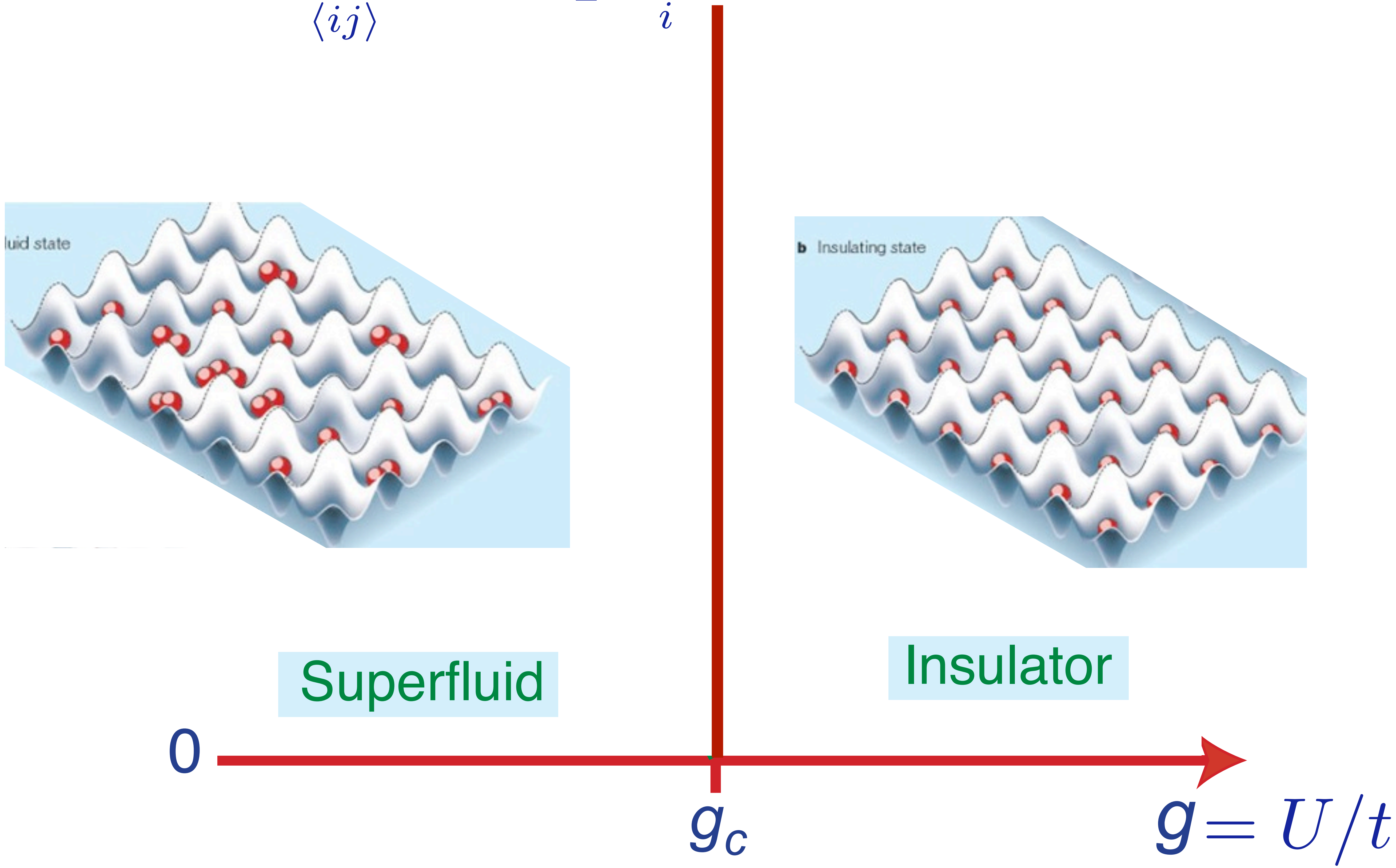
# Superfluid-insulator transition




Ultracold $^{87}$Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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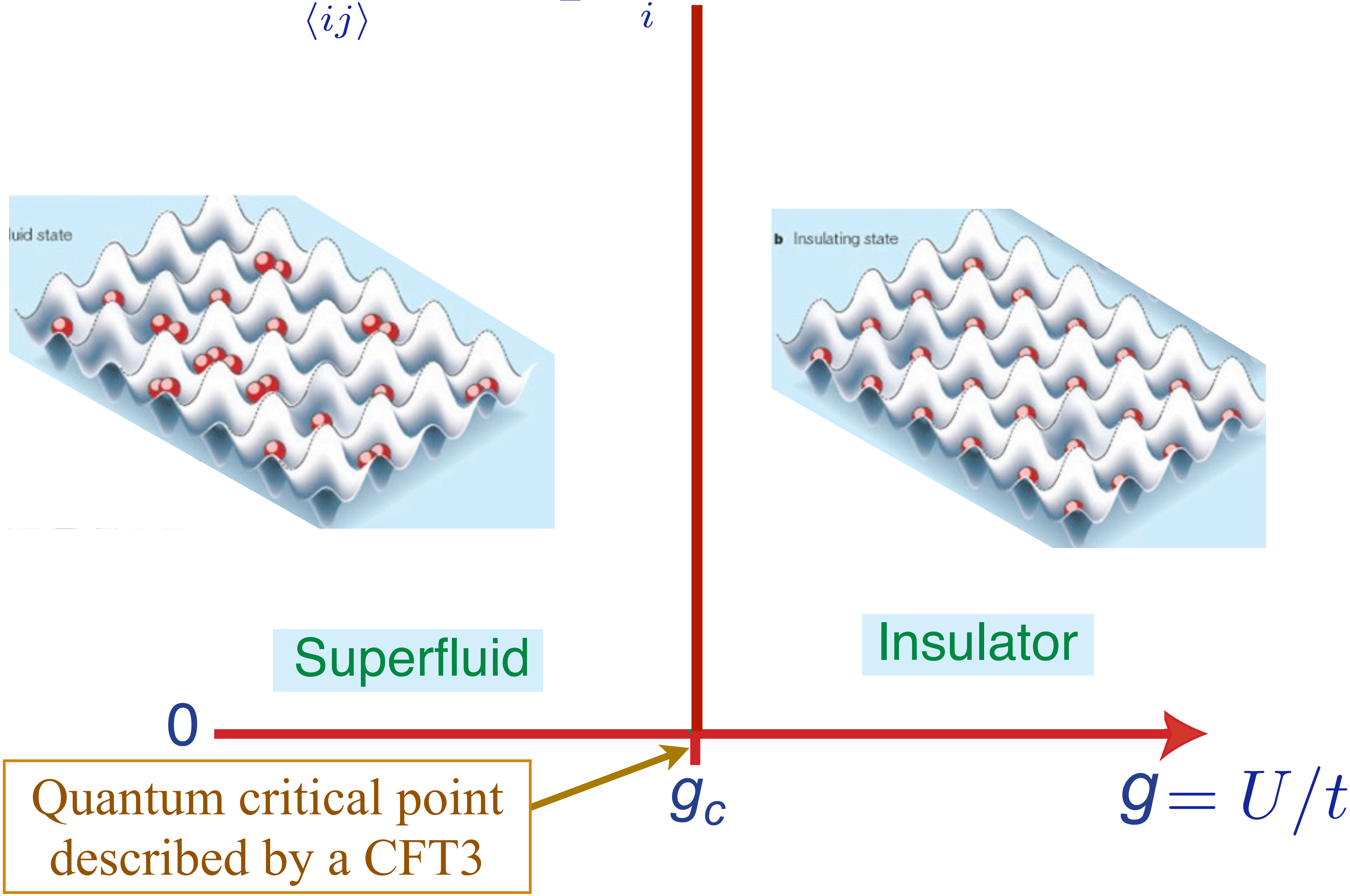
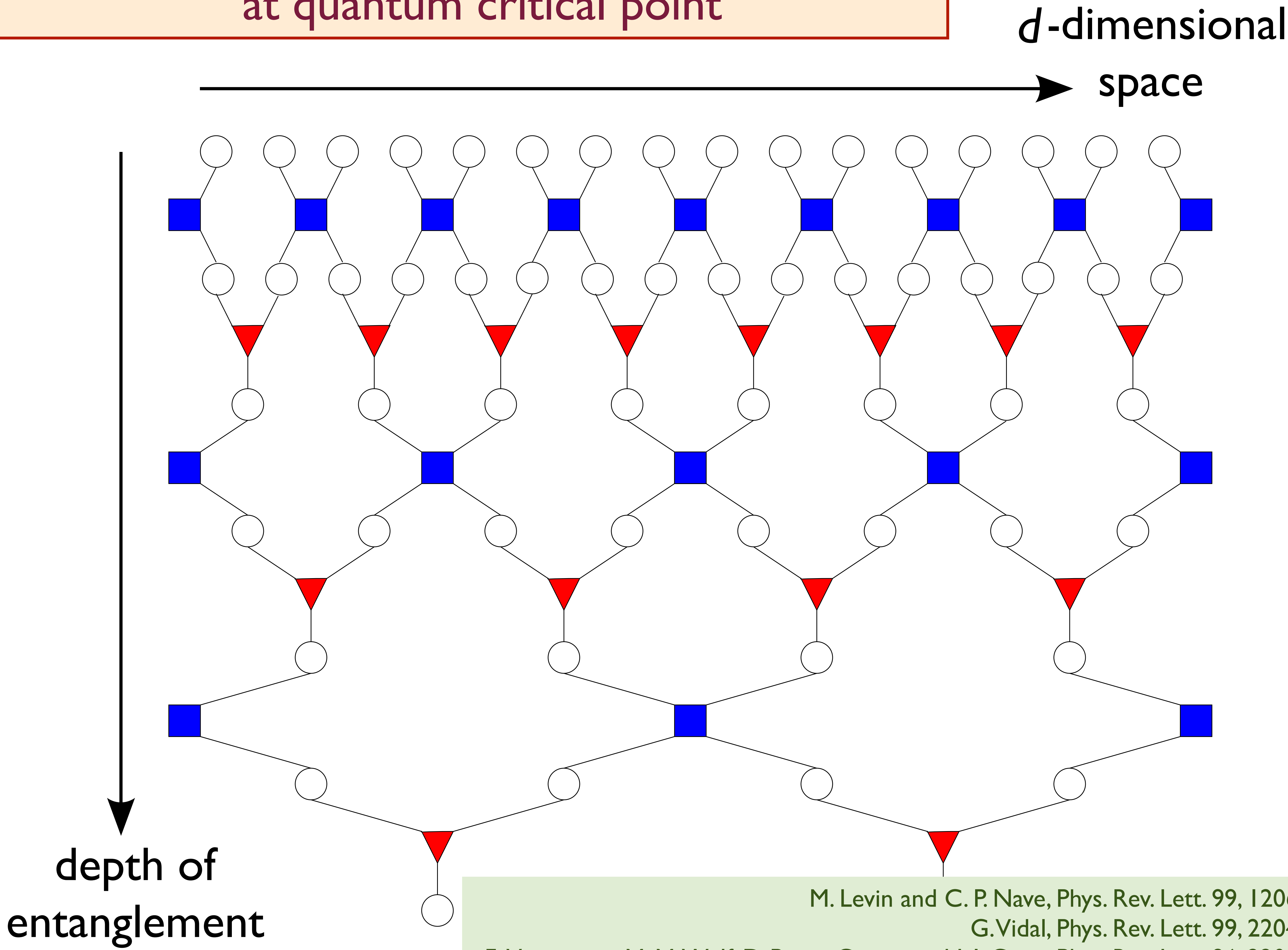


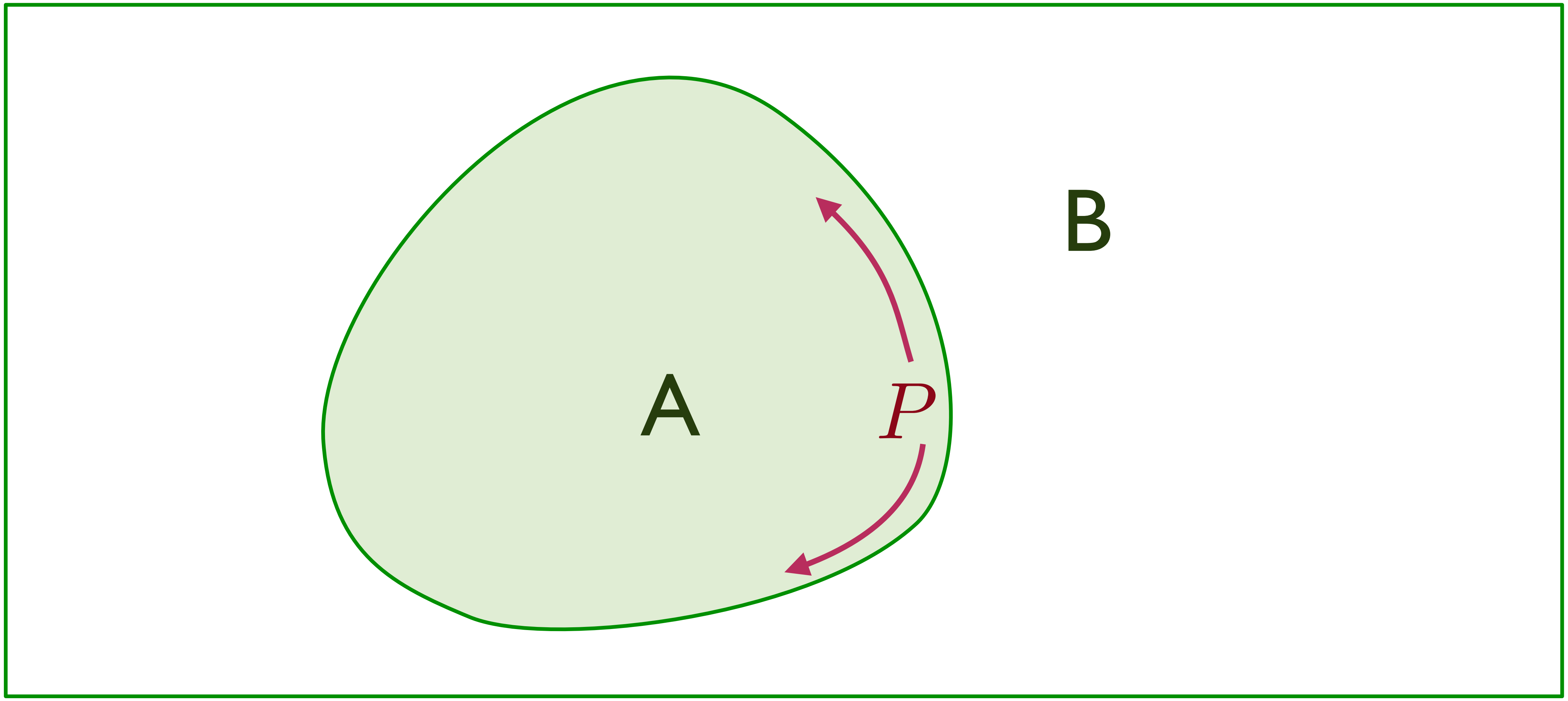
# Tensor network representation of entanglement at quantum critical point




M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

## Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where $\gamma$ is a shape-dependent universal number associated with the CFT3.




M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

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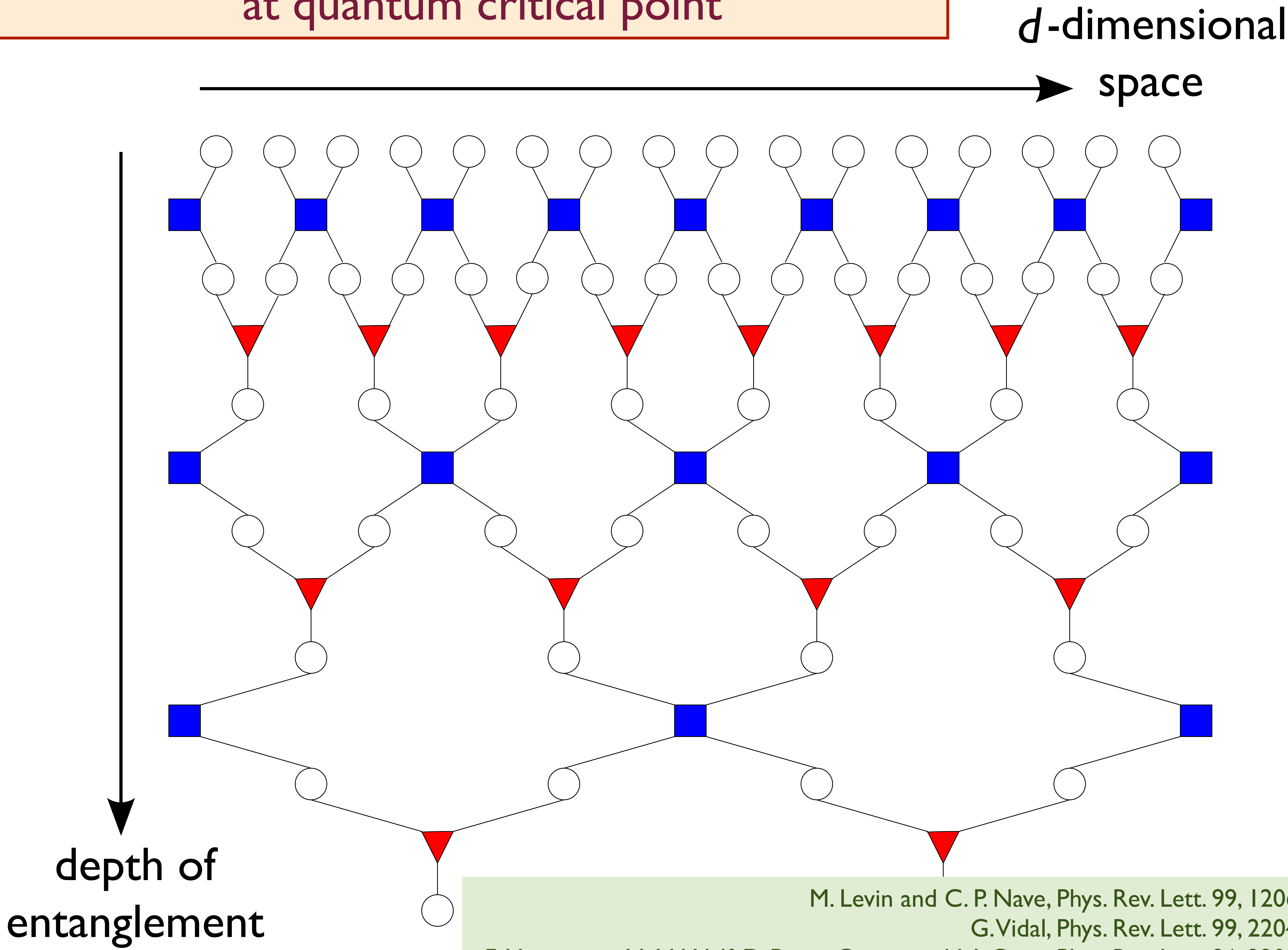


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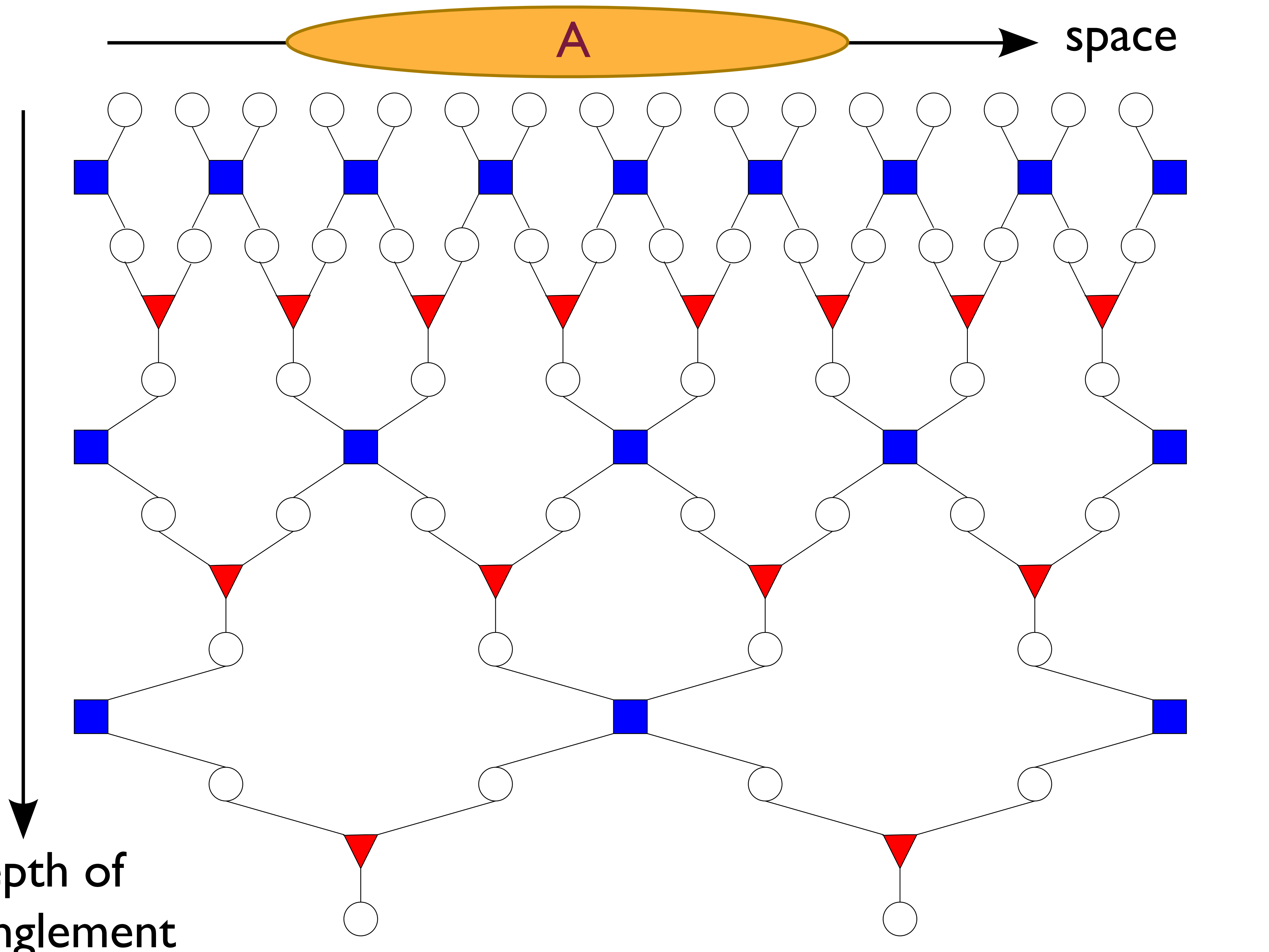


Brian Swingle, arXiv:0905.1317

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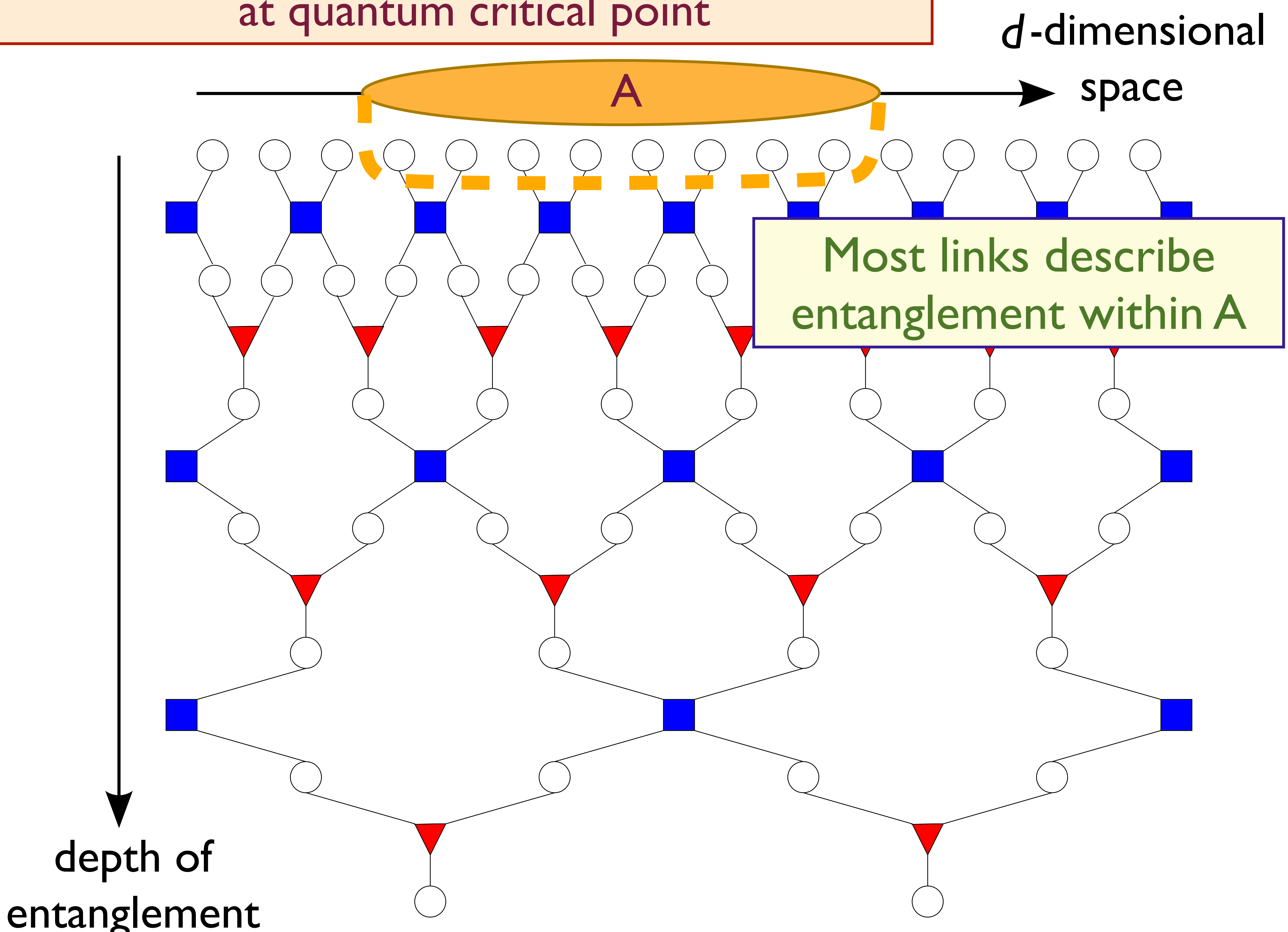
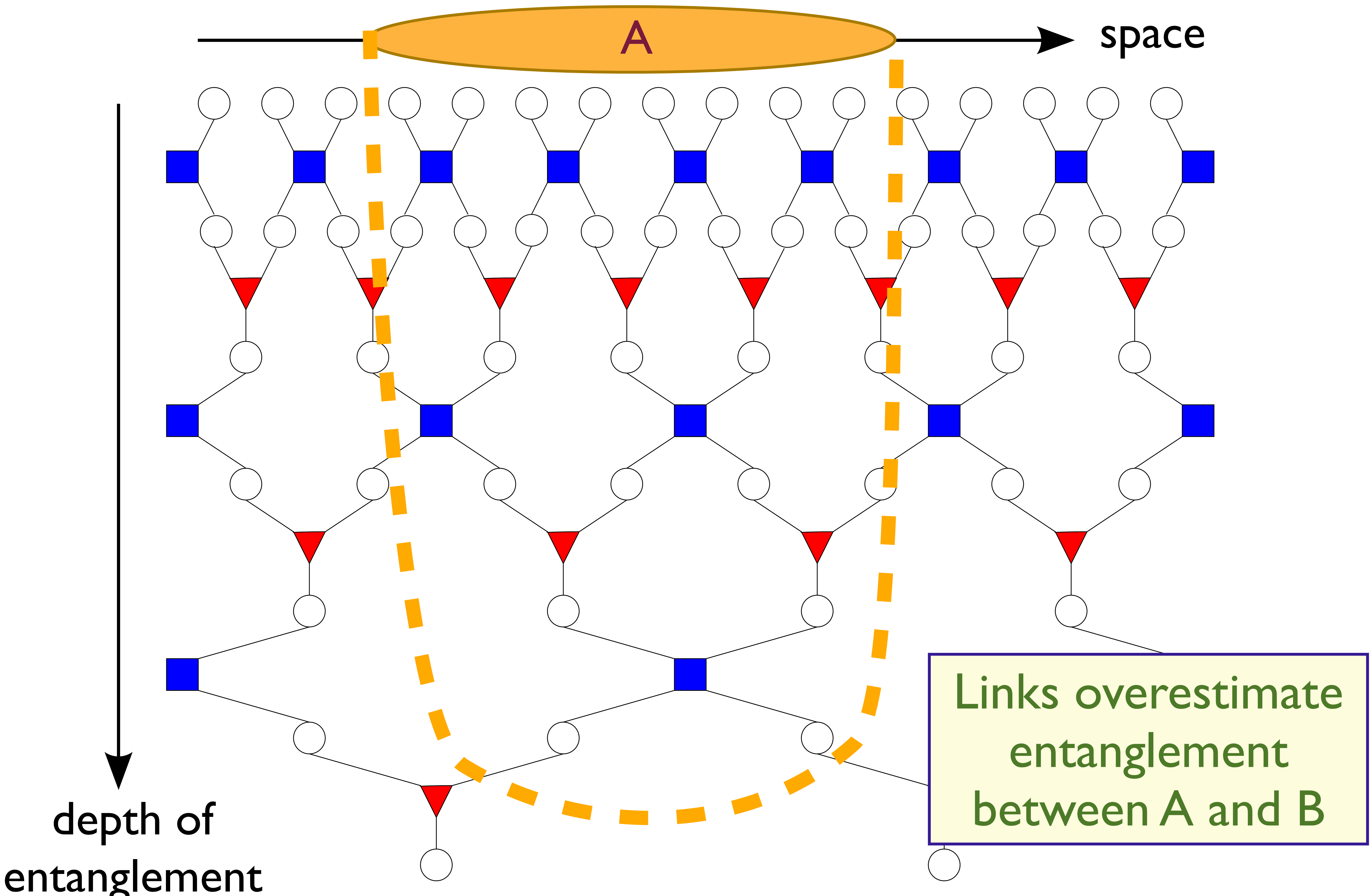



Brian Swingle, arXiv:0905.1317

# Tensor network representation of entanglement at quantum critical point




Brian Swingle, arXiv:0905.1317

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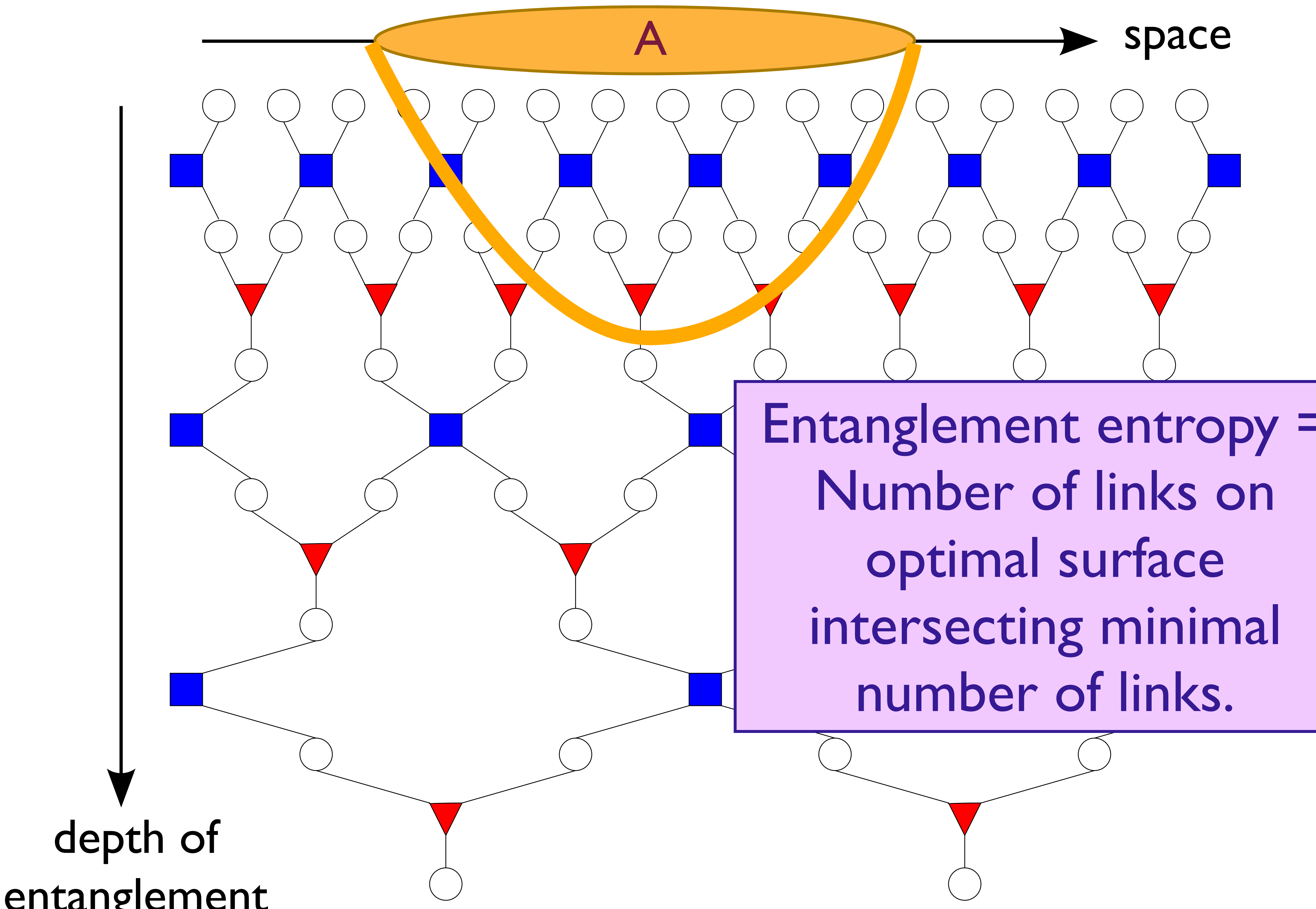
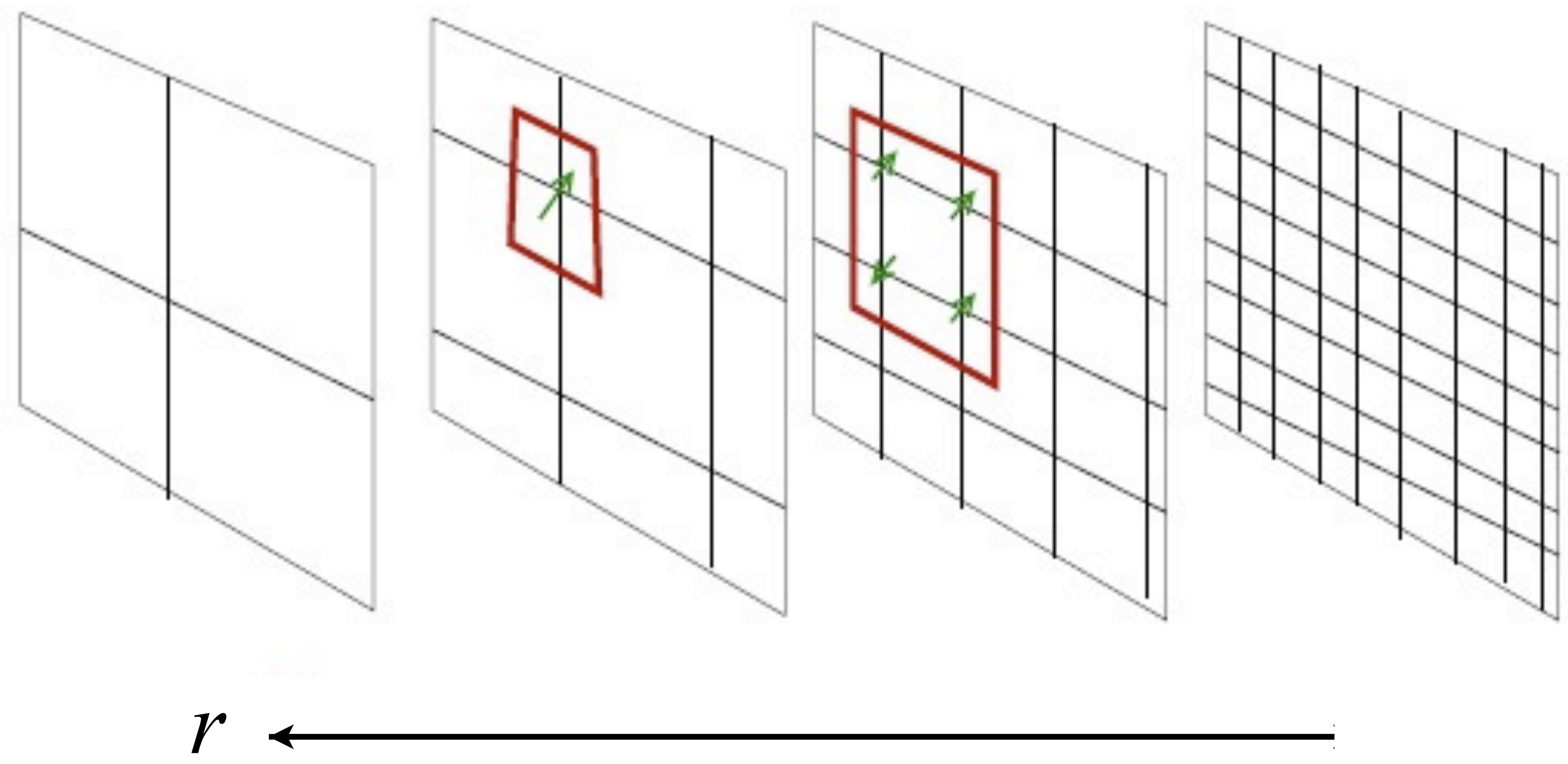



Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

# Holography




**Key idea:** $\Rightarrow$ Implement $r$ as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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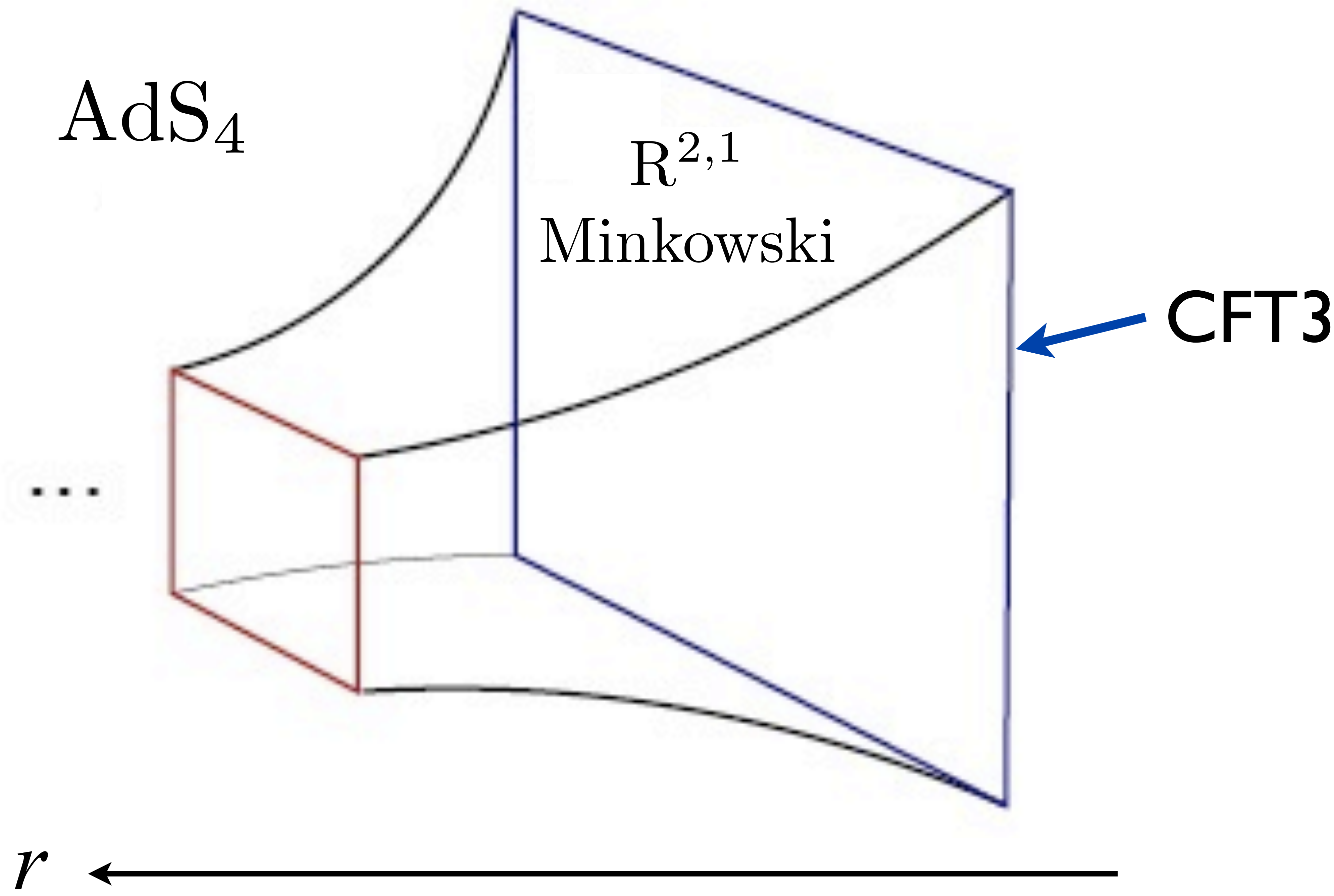
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in $r$ has been used to the prefactor of $dx_i^2$ equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space $\mathrm{AdS}_{d+2}$.

# AdS/CFT correspondence



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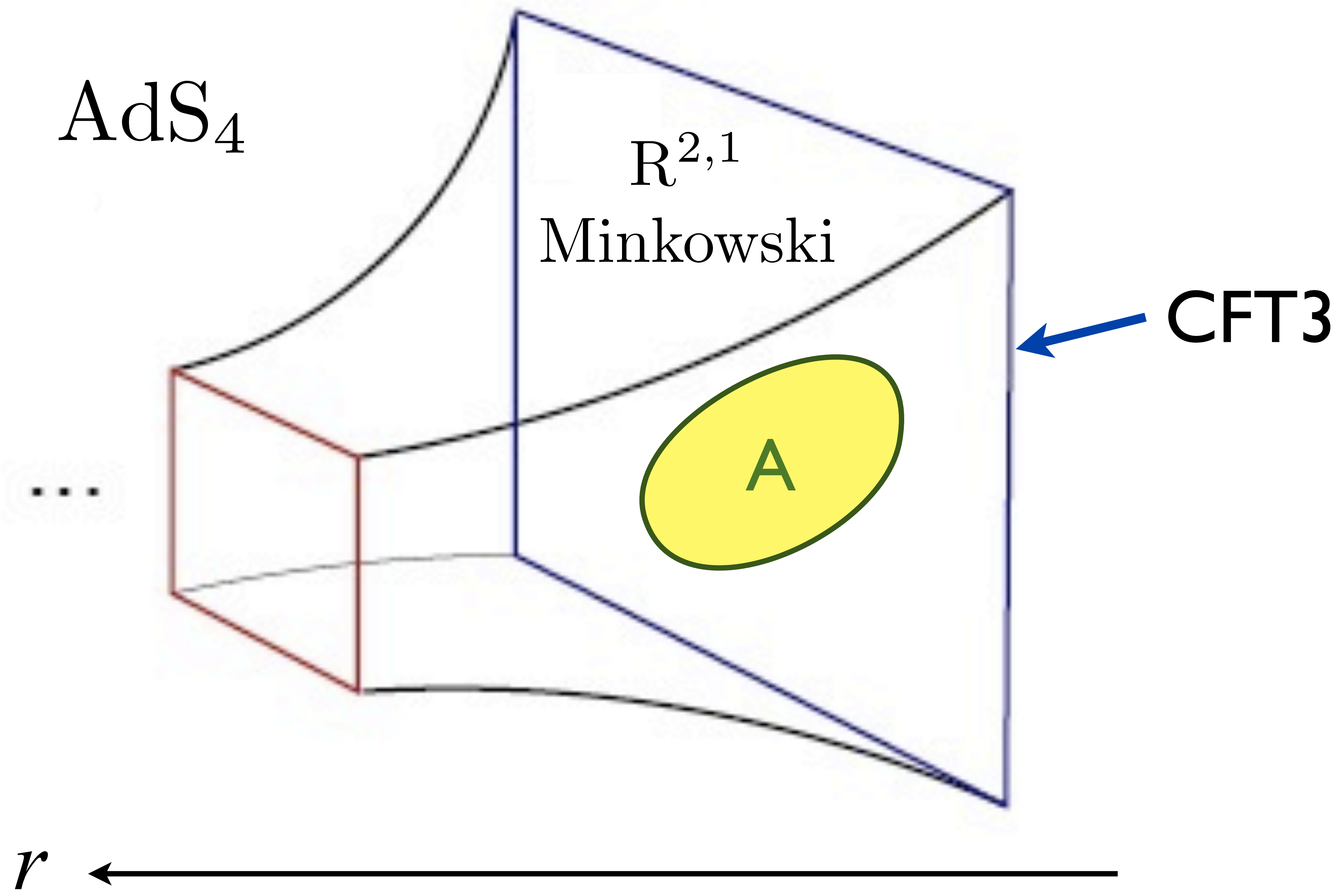

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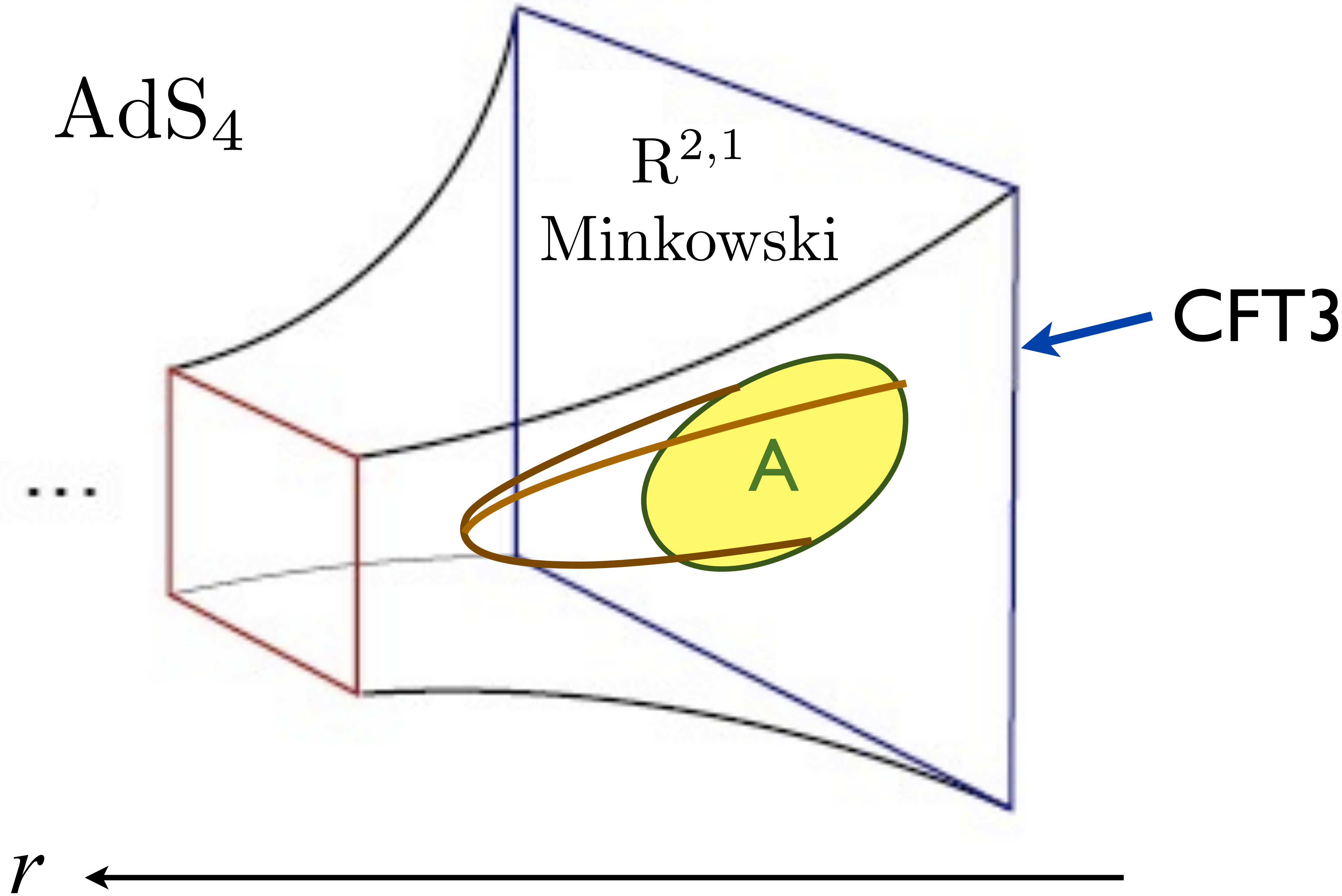
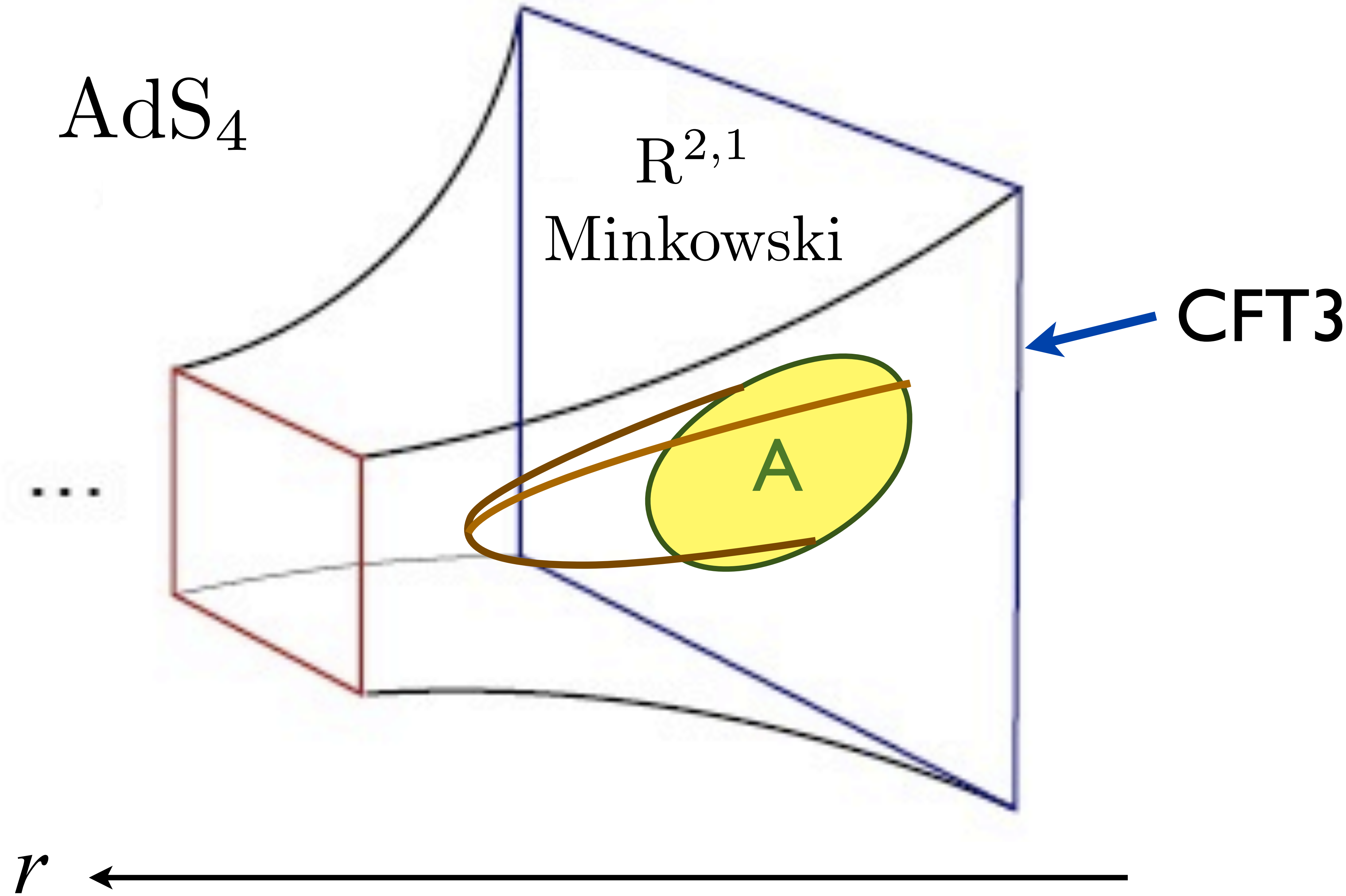



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

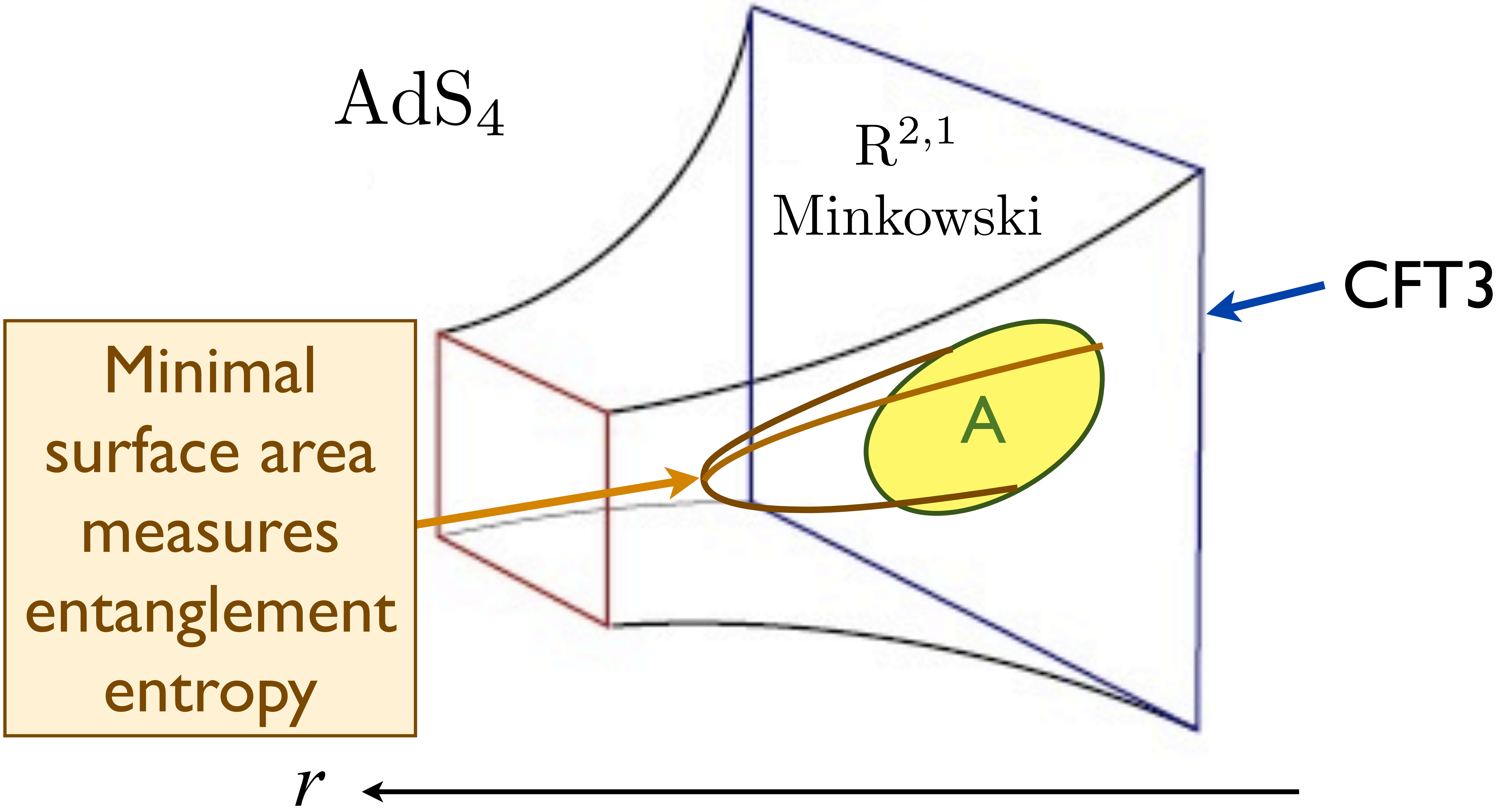
# AdS/CFT correspondence




The entropy of this region is bounded by its surface area
(Bekenstein-Hawking-'t Hooft-Susskind)

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

# AdS/CFT correspondence

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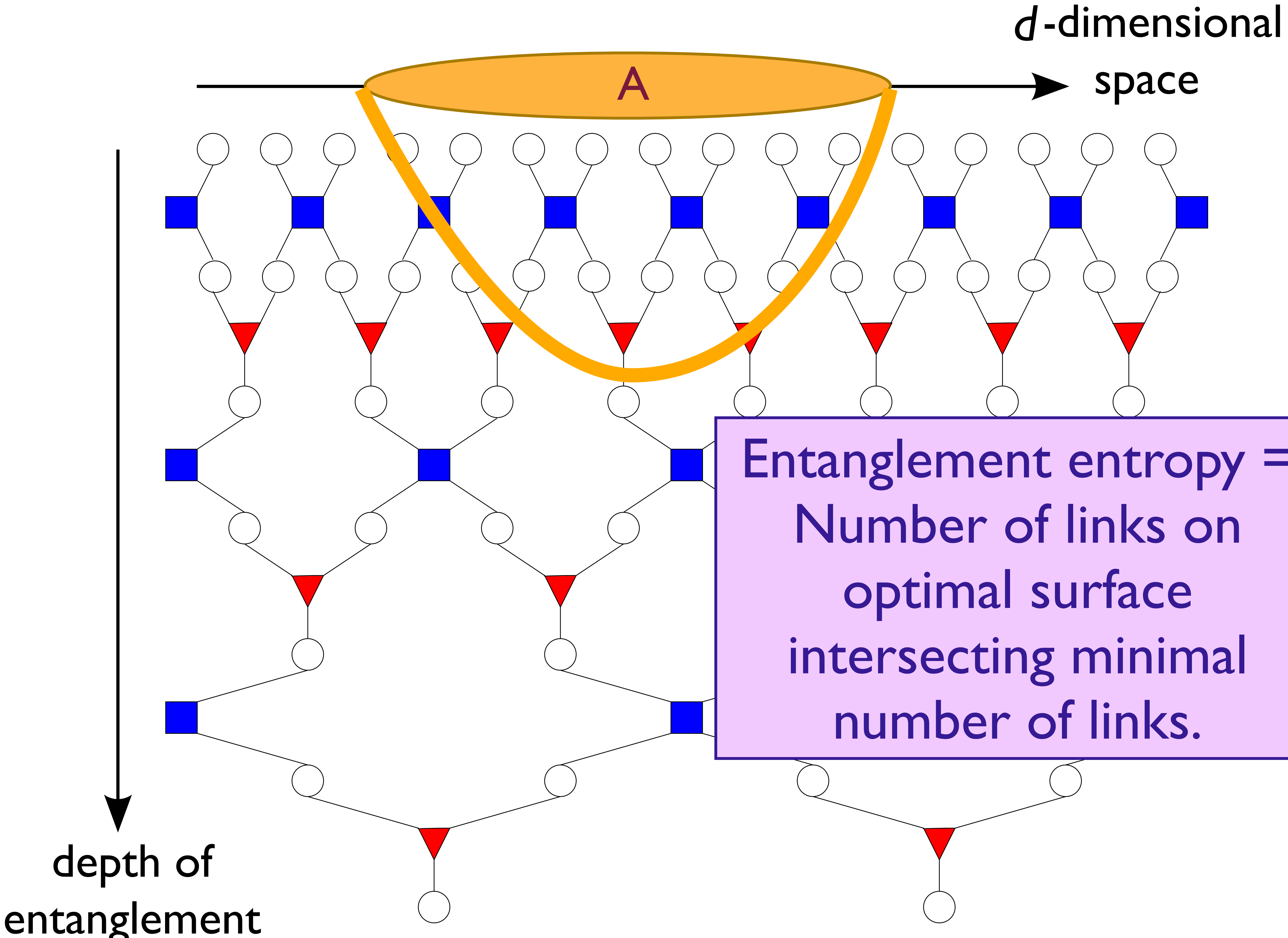
# Entanglement entropy




Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

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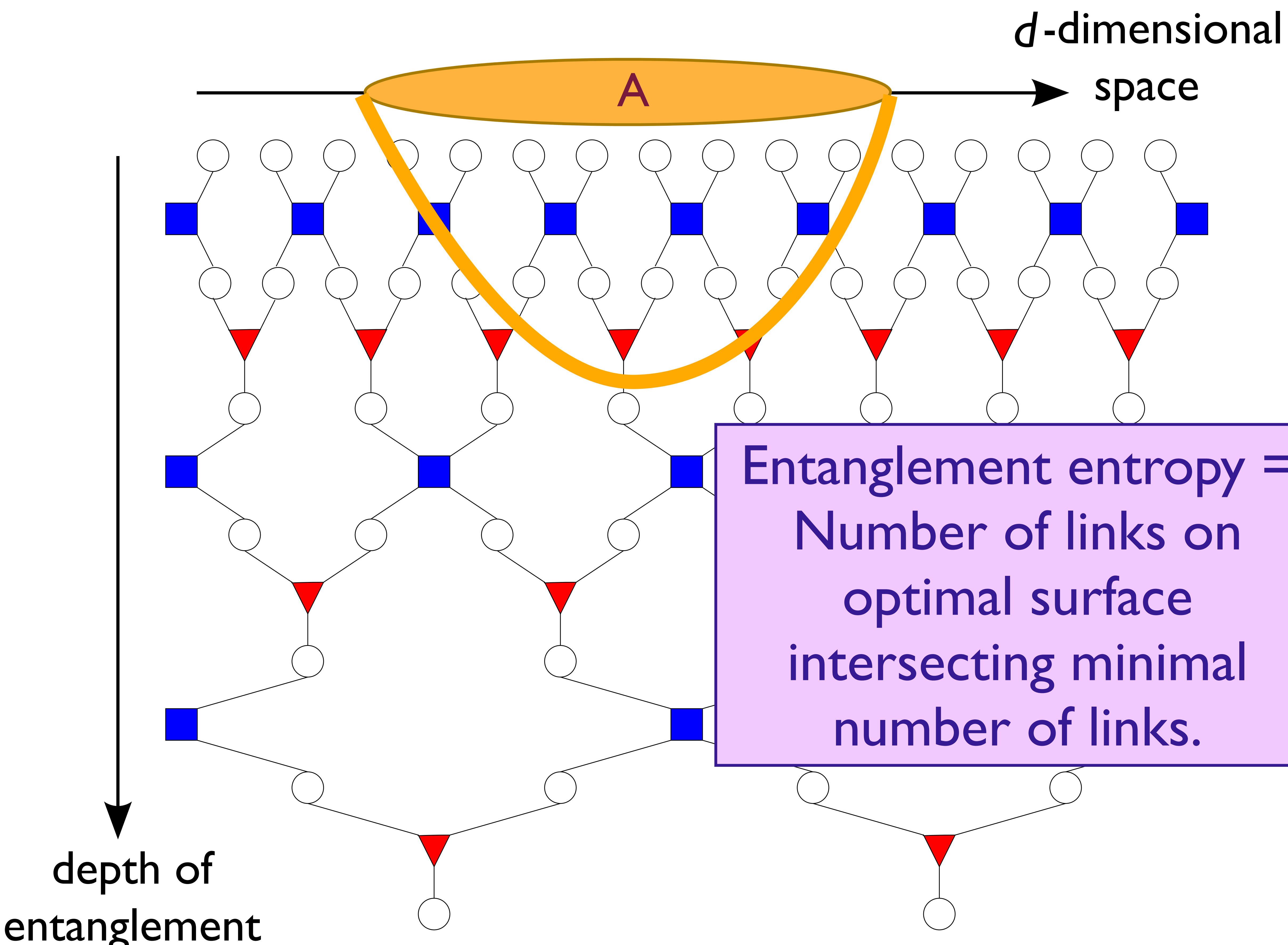


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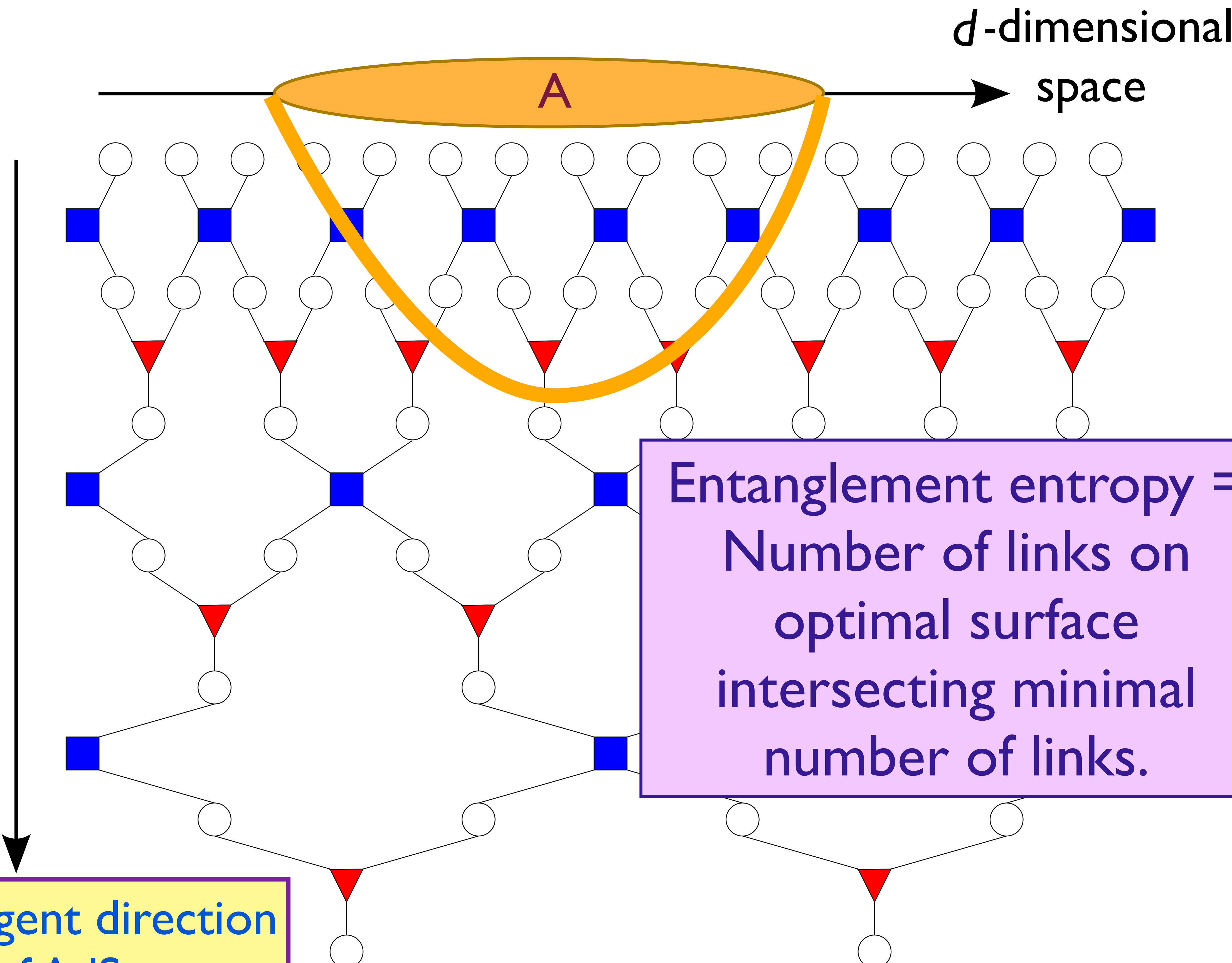


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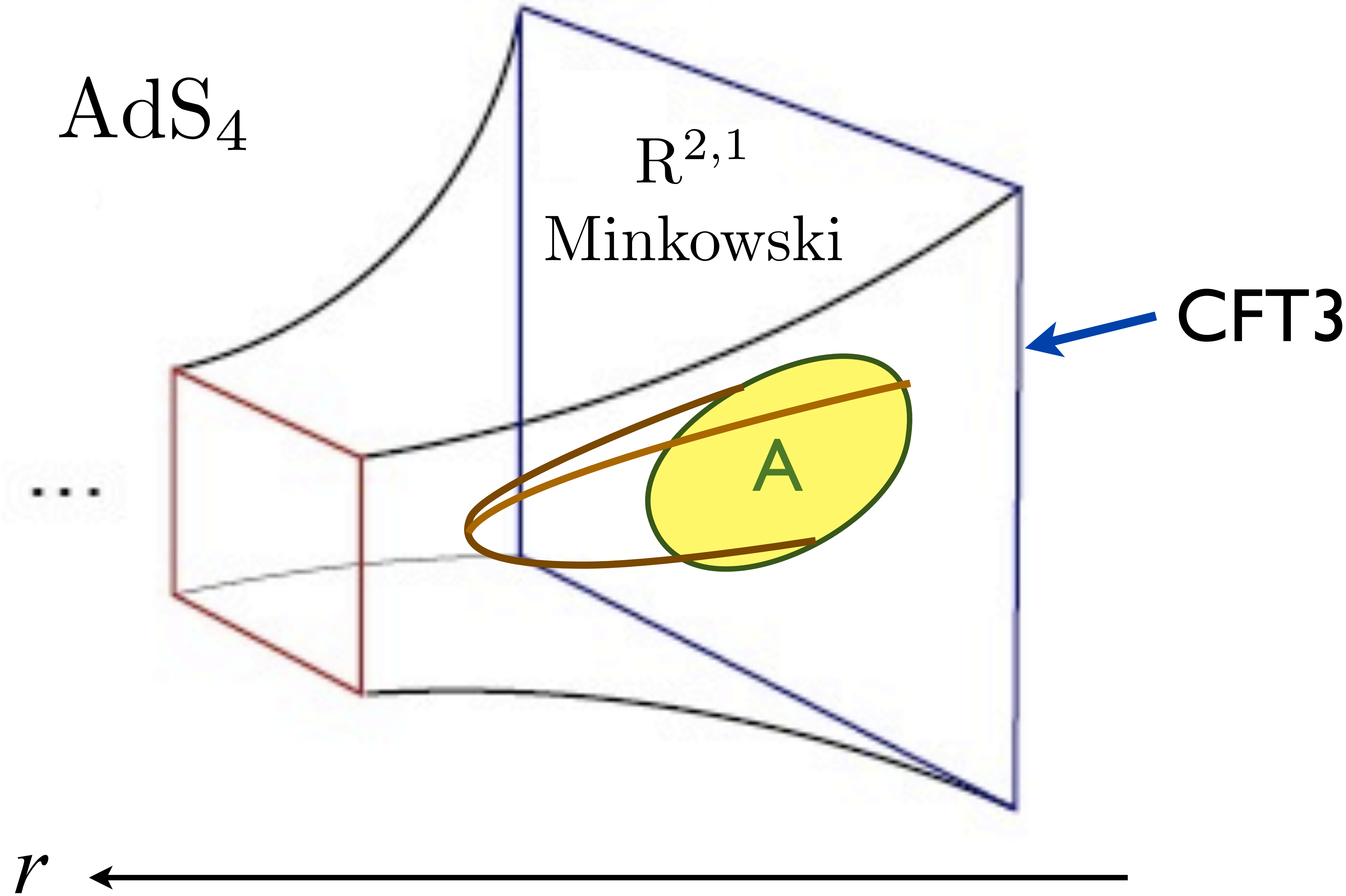
Emergent direction of $AdS_{d+2}$

Brian Swingle, arXiv:0905.1317

# AdS/CFT correspondence




- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

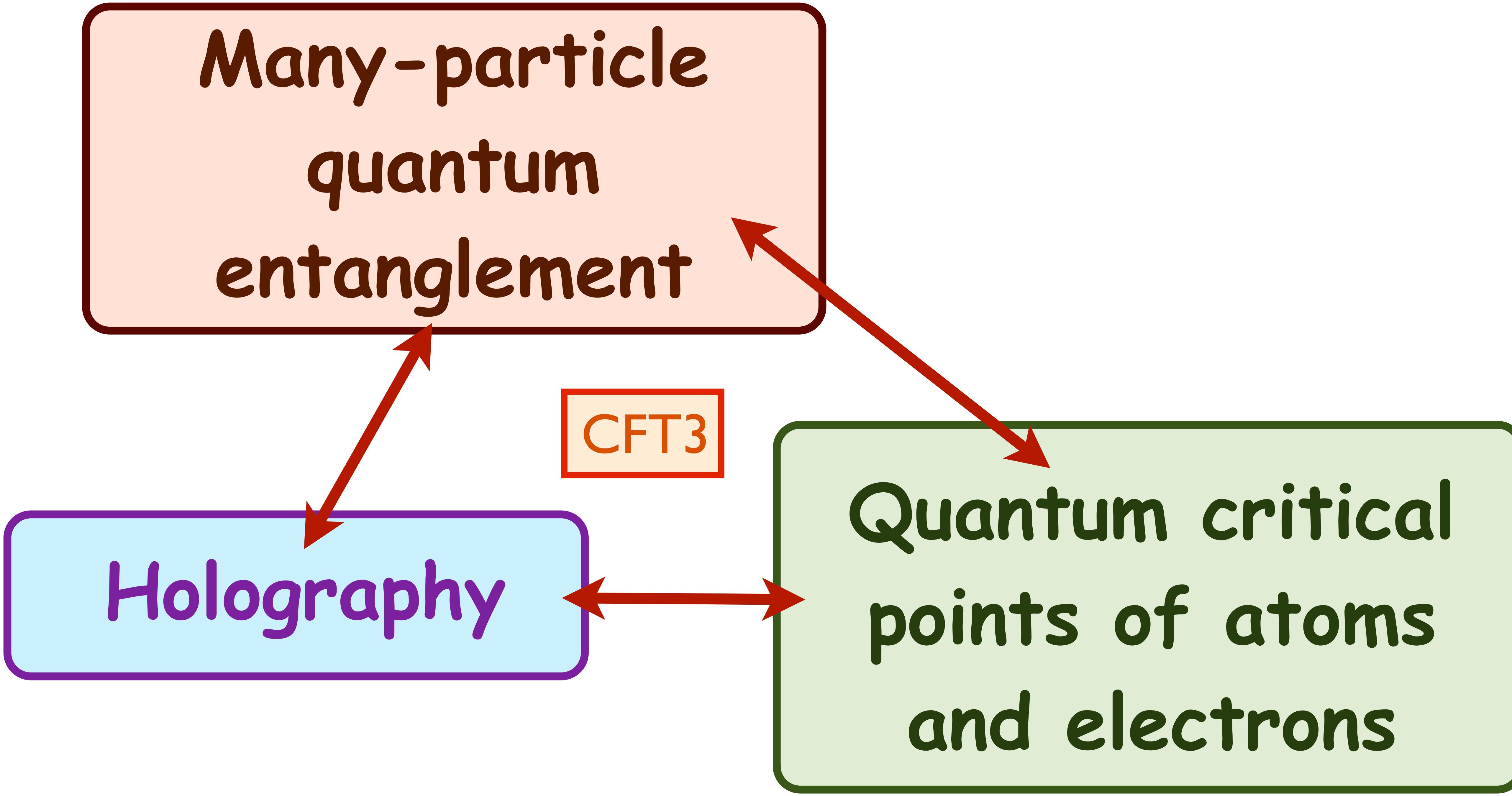
where $\gamma$ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Many-particle quantum entanglement

Holography

Quantum critical points of atoms and electrons


Many-particle quantum entanglement
CFT3
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Many-particle quantum entanglement

Holography and string theory

Quantum critical points of atoms and electrons

Black holes

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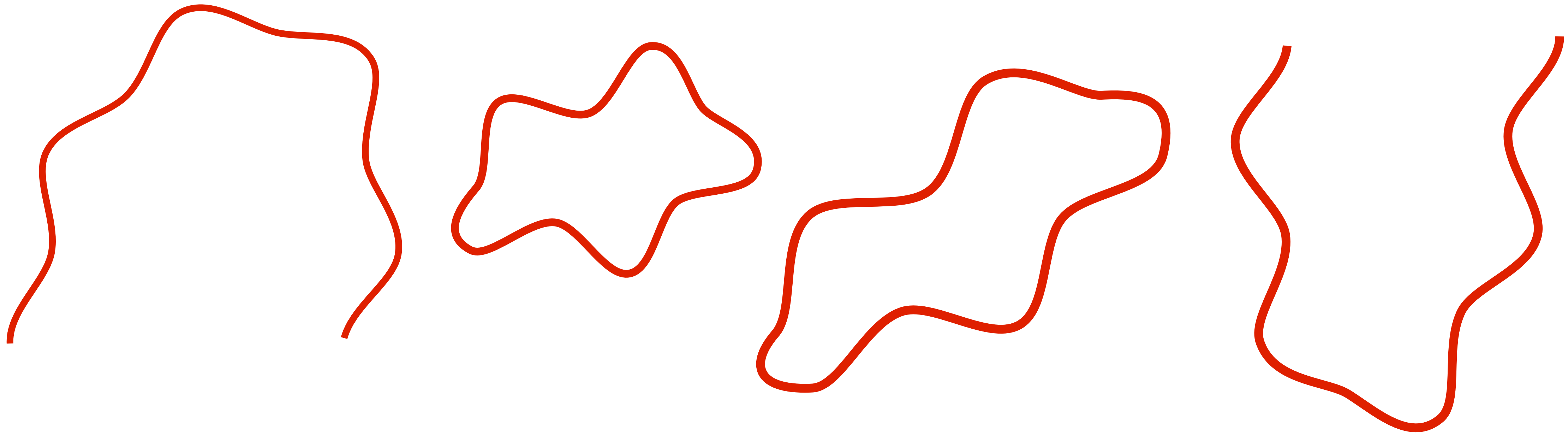
Holography and string theory

Quantum critical points of atoms and electrons

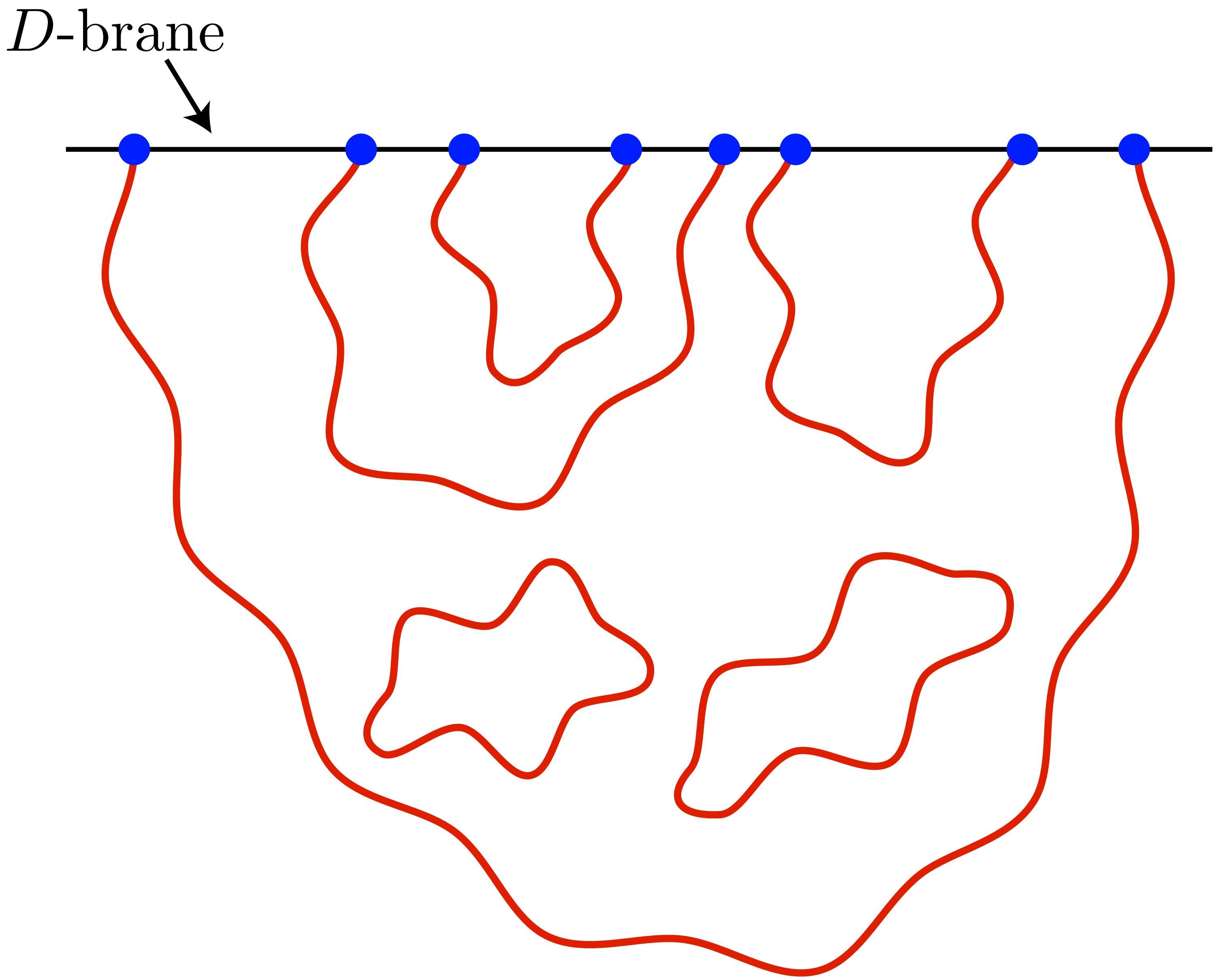
Black holes

# String theory

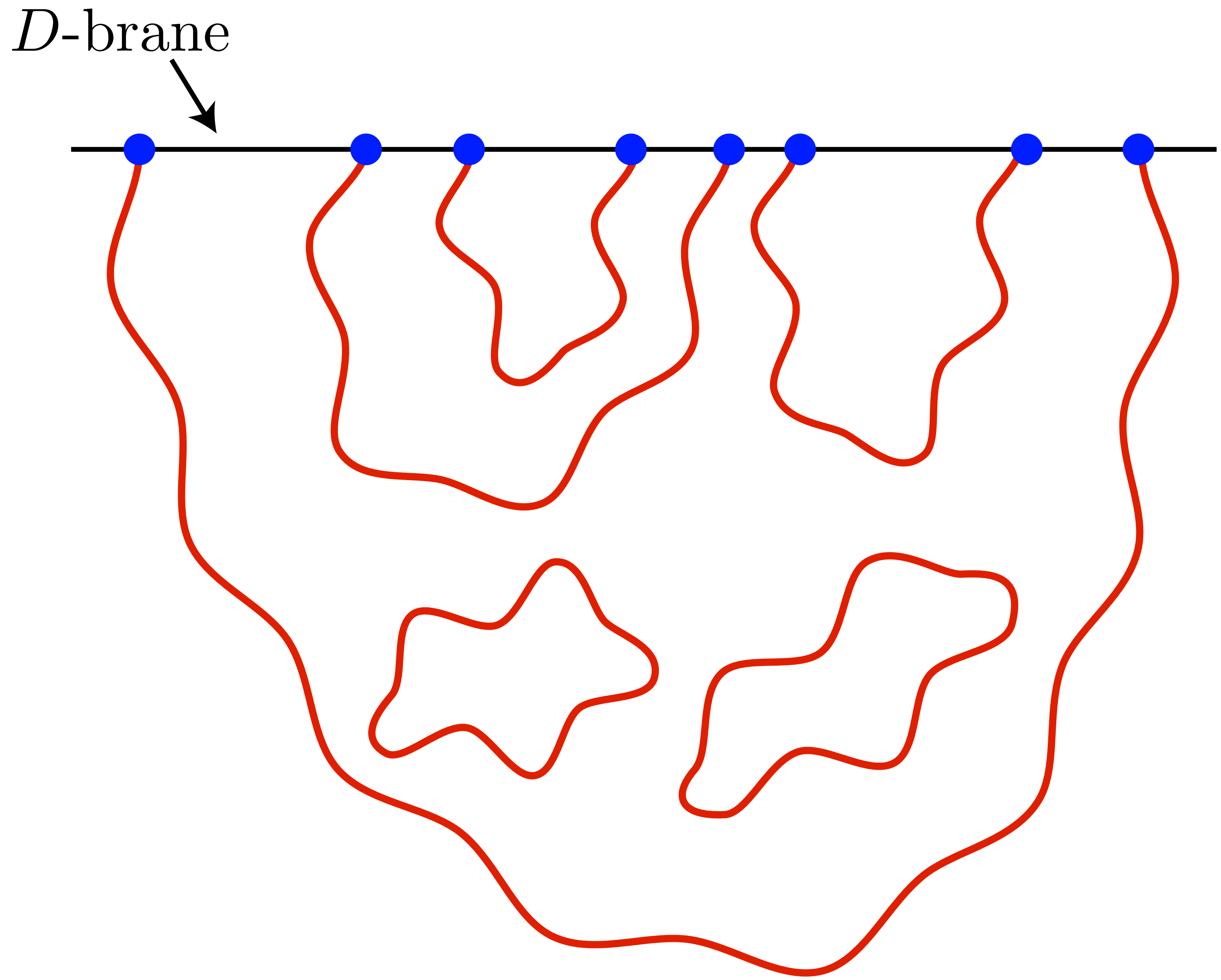


- Allows unification of the standard model of particle physics with gravity.

- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .




- A $D$-brane is a $d$-dimensional surface on which strings can end.

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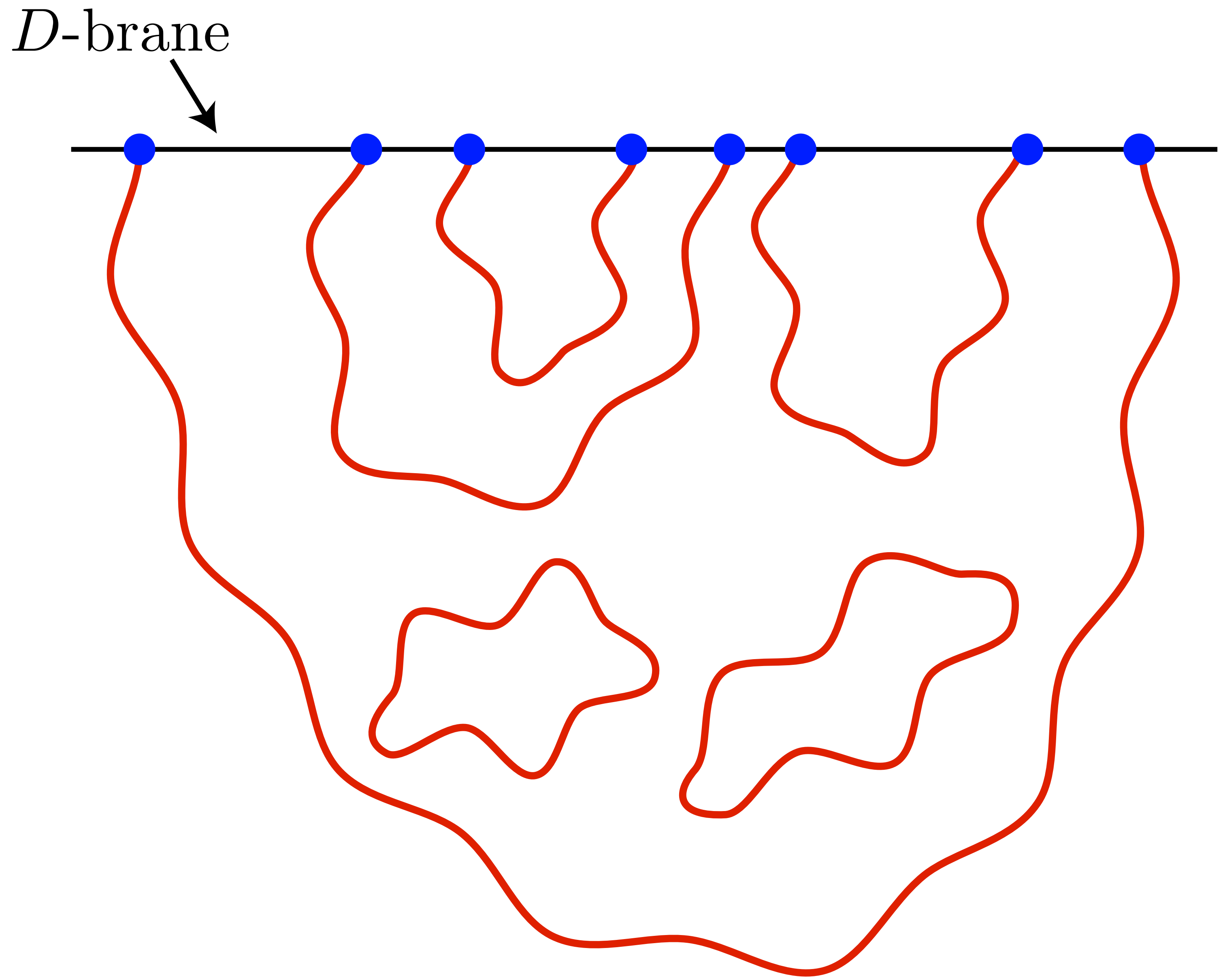


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- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

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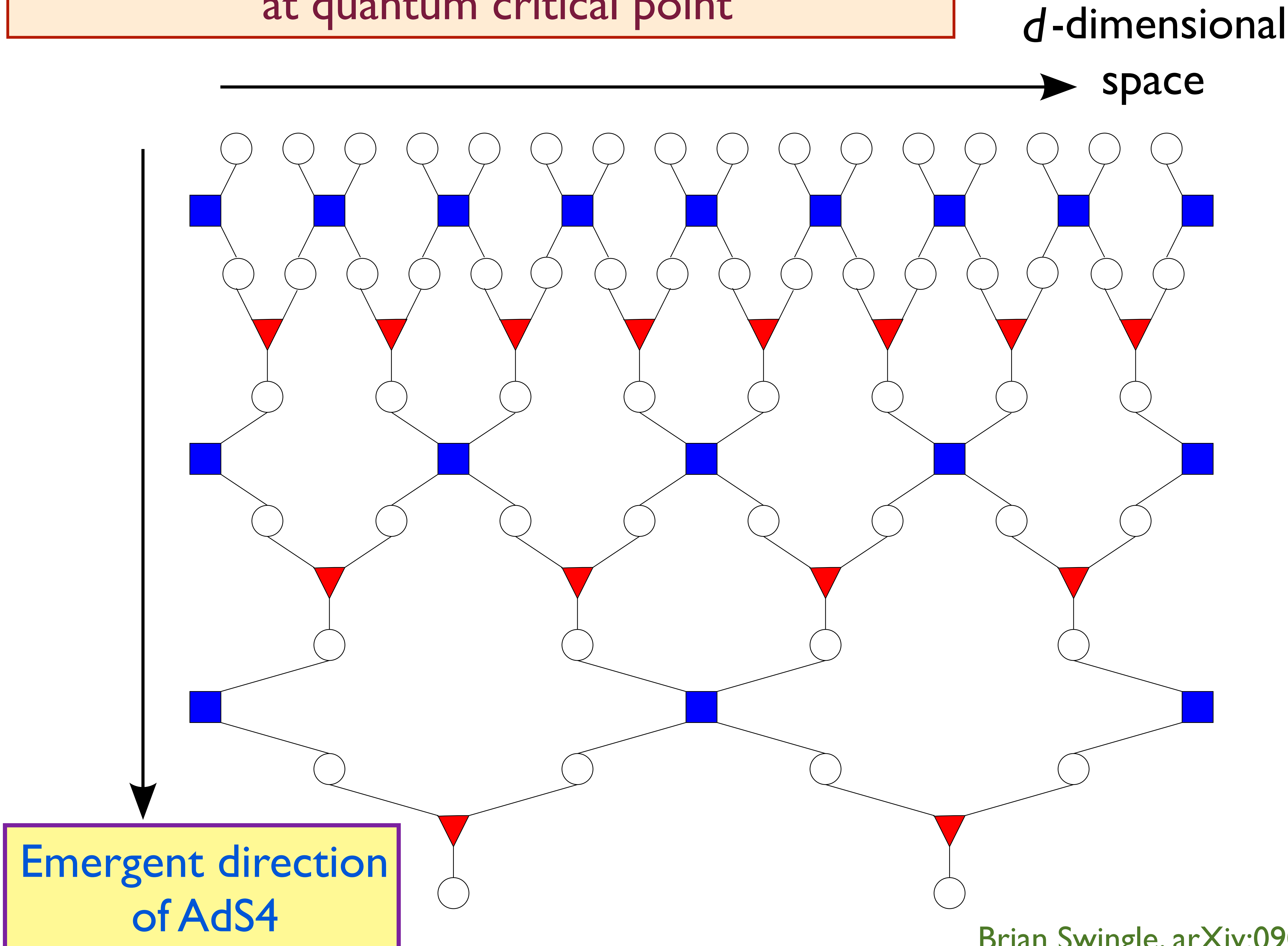


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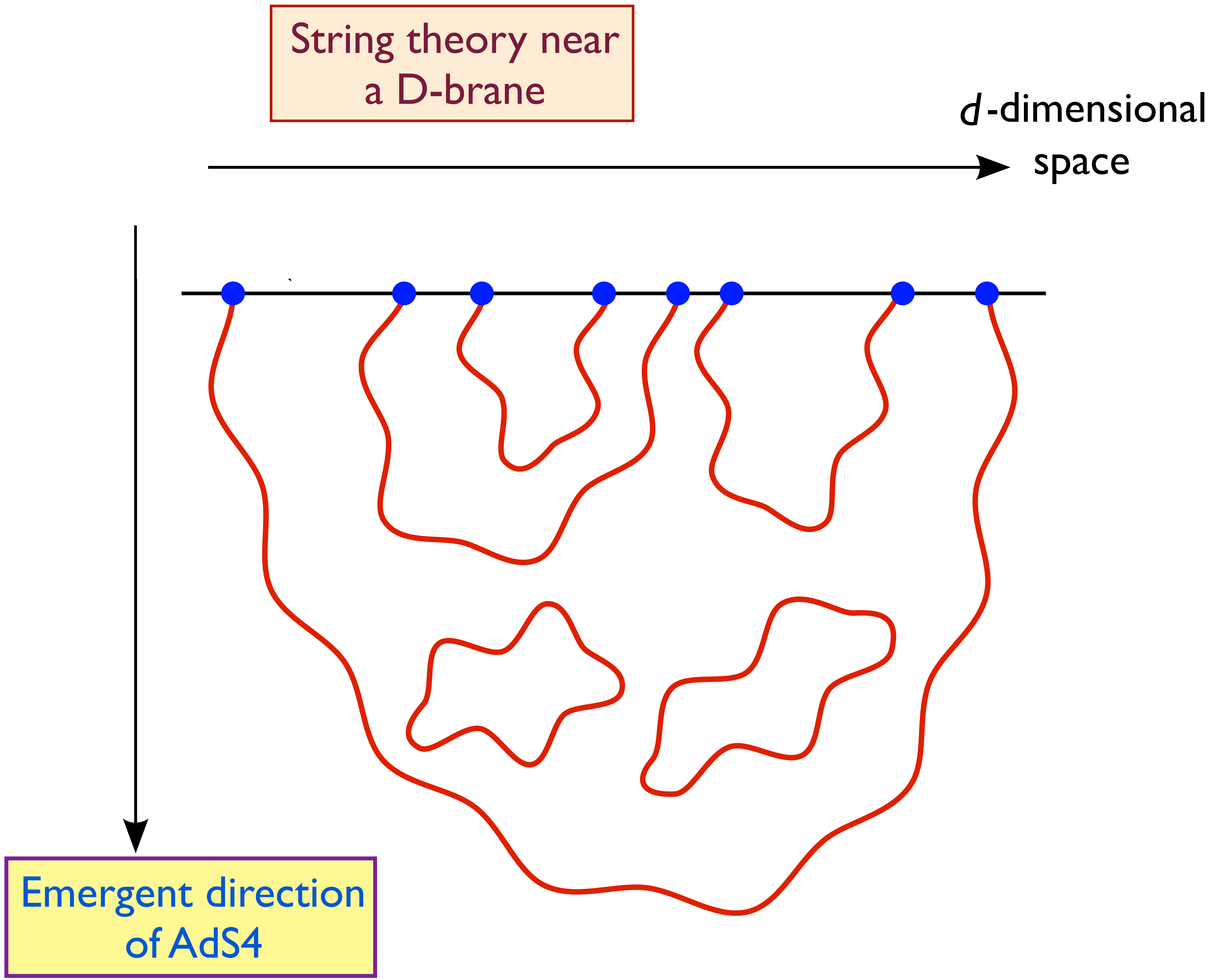
String theory near a D-brane
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Many-particle quantum entanglement

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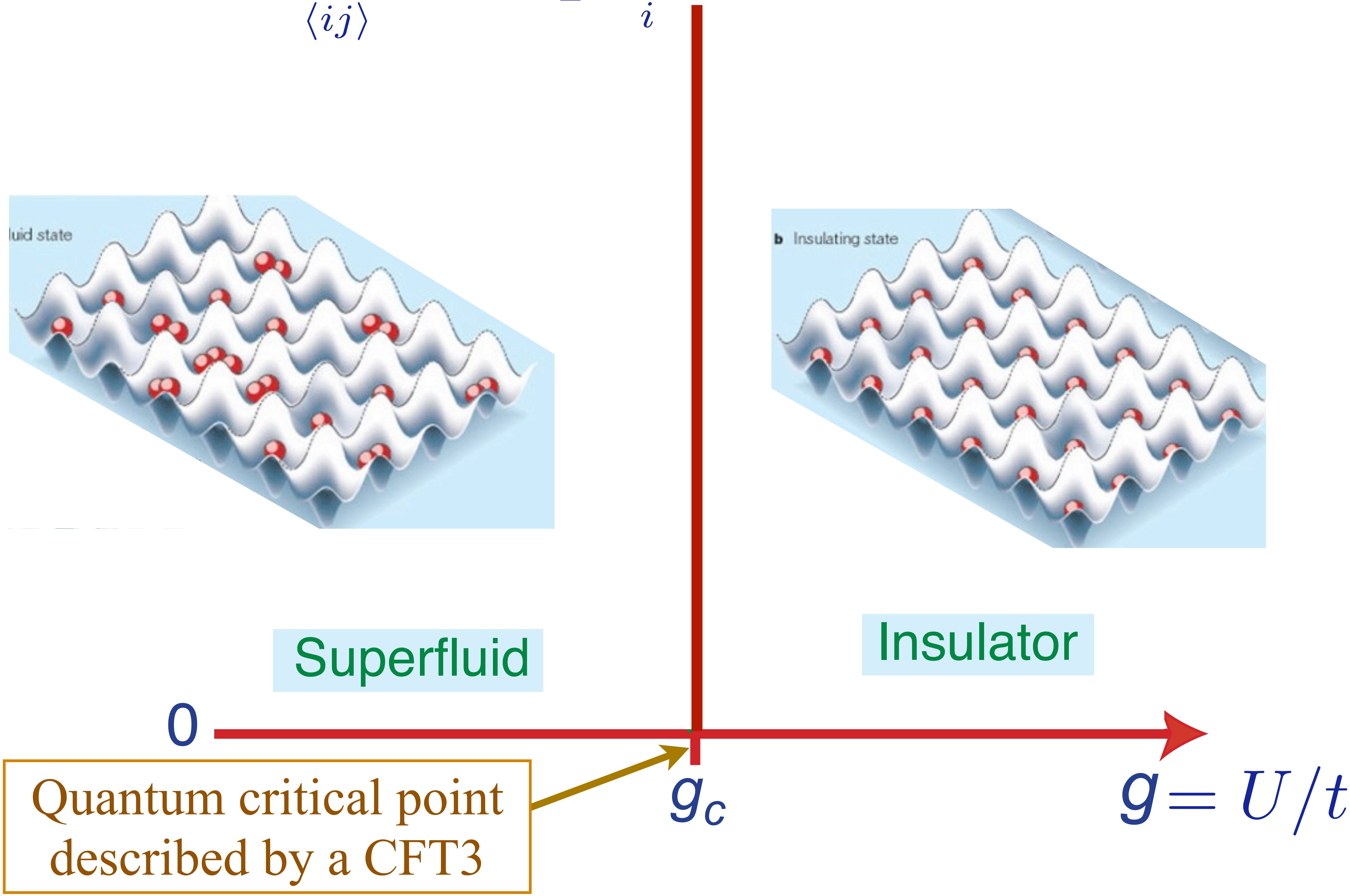
$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) \quad ; \quad n_i \equiv b_i^\dagger b_i$$



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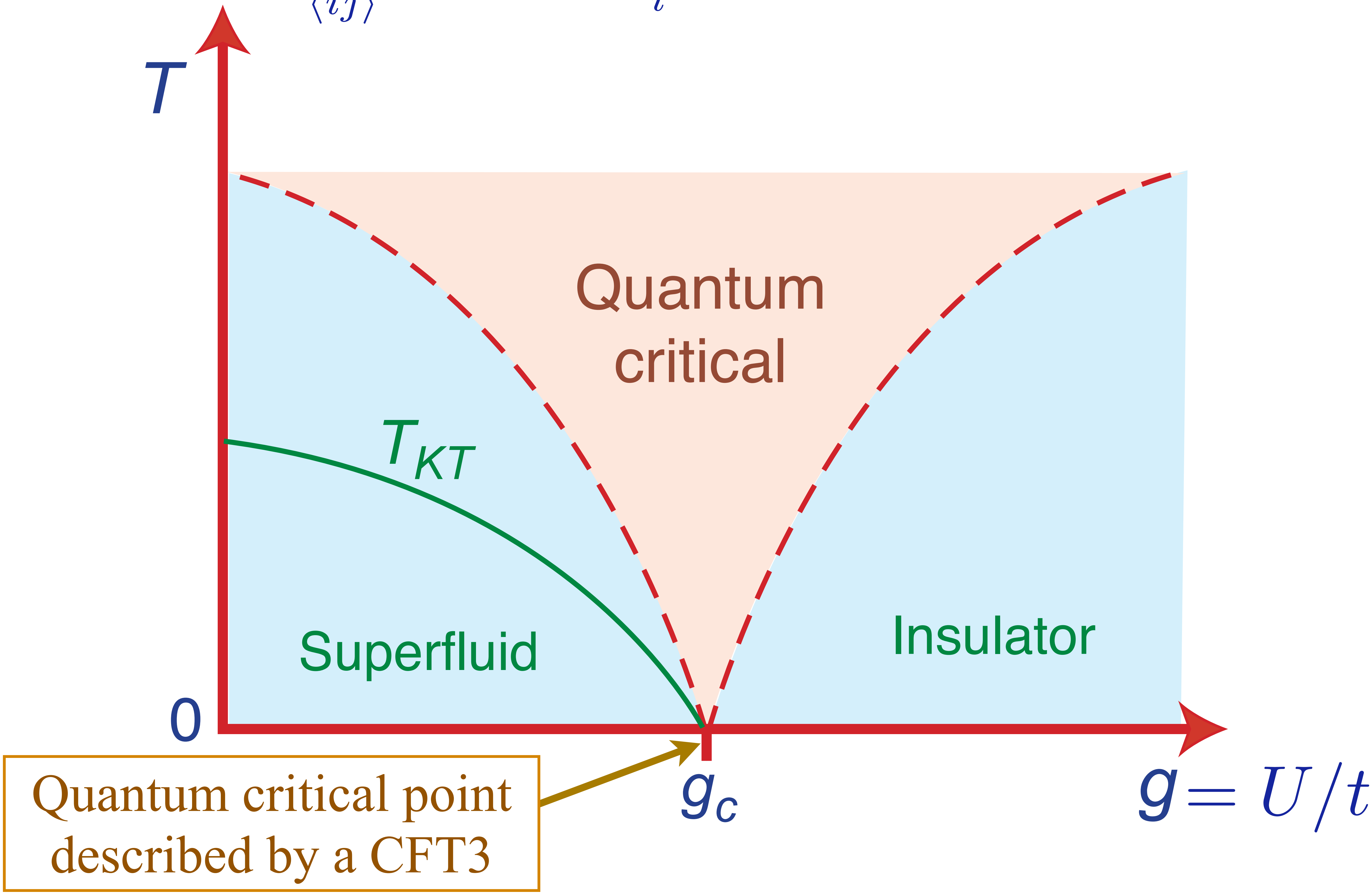

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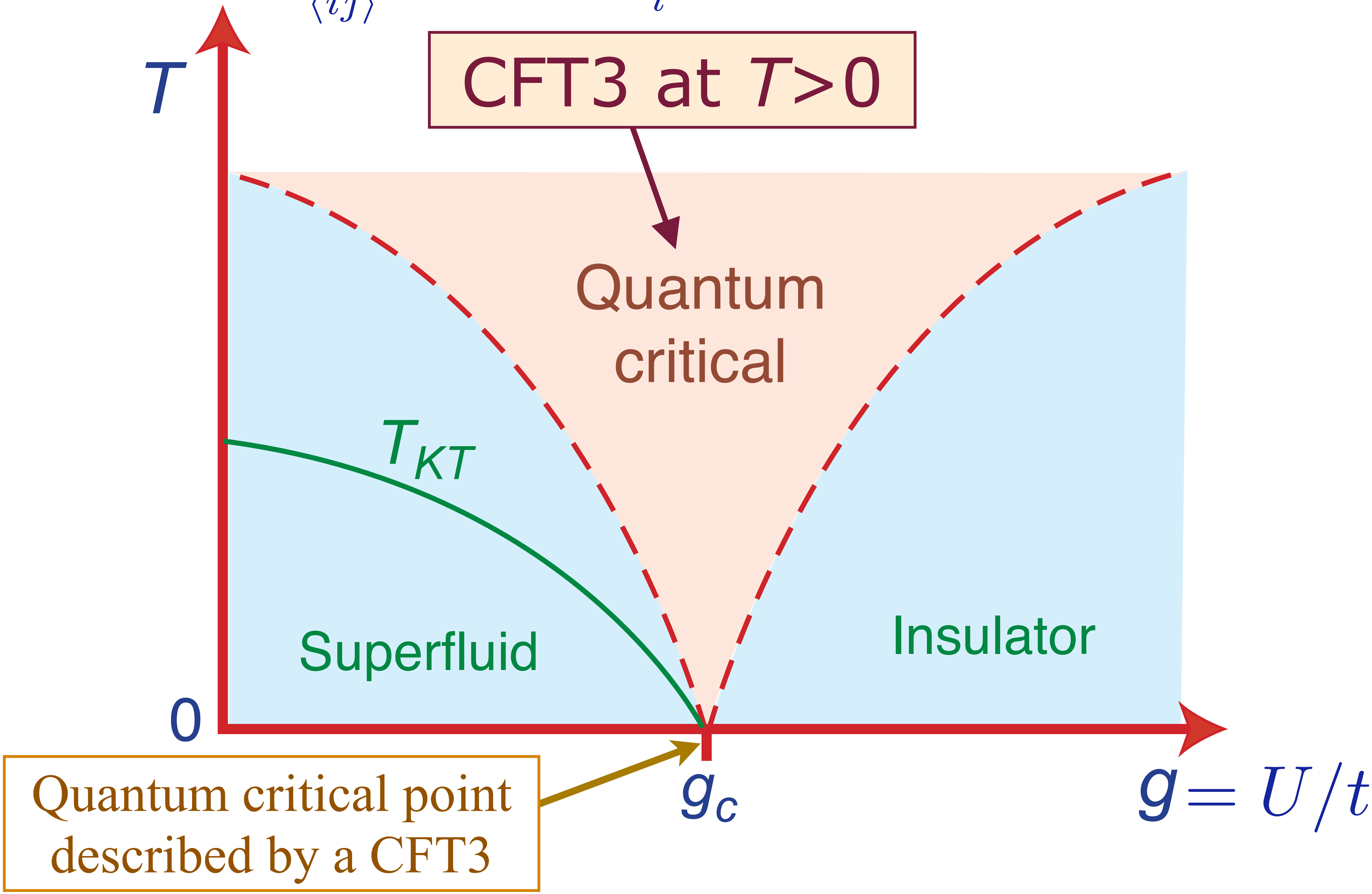

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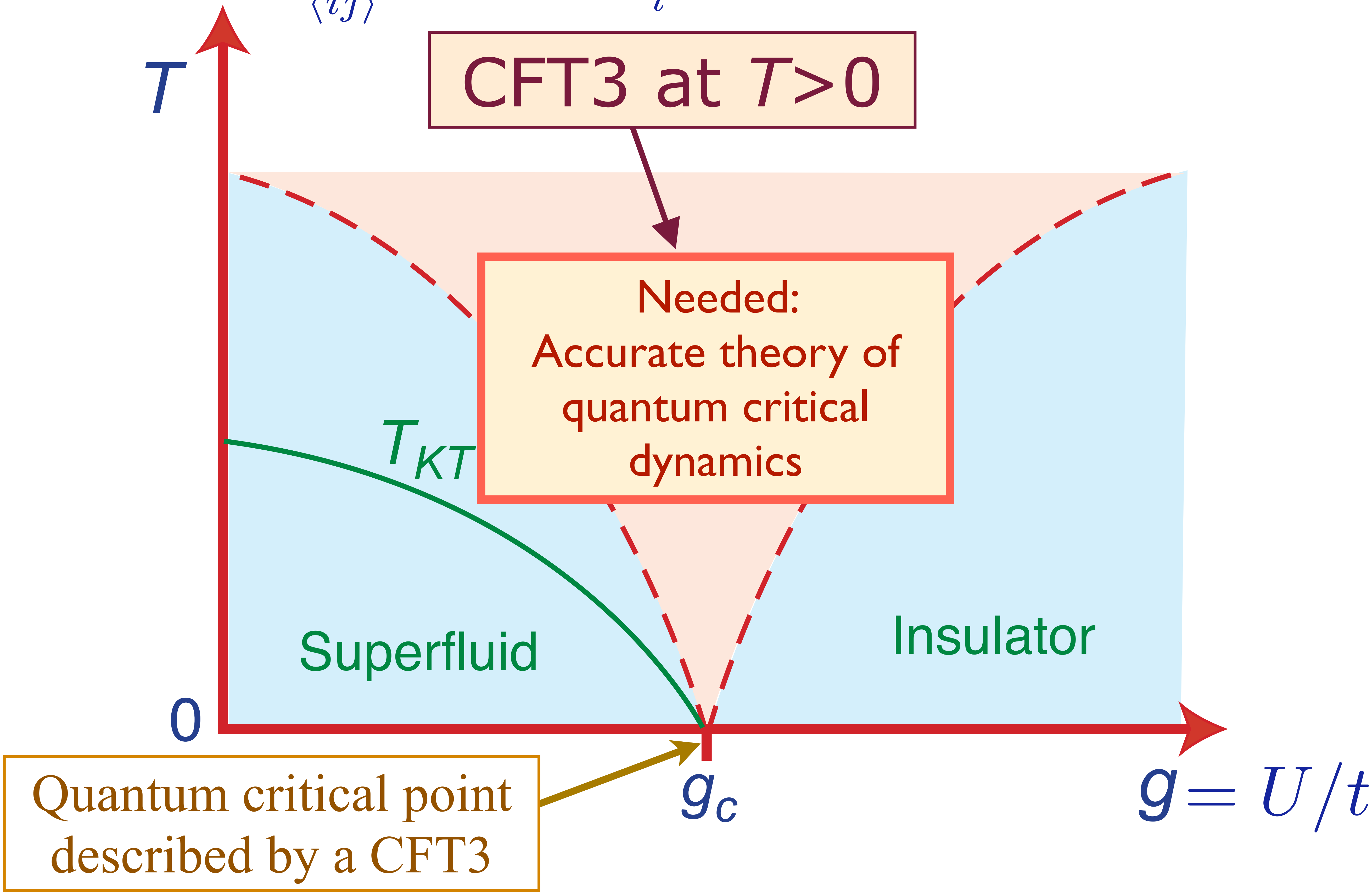
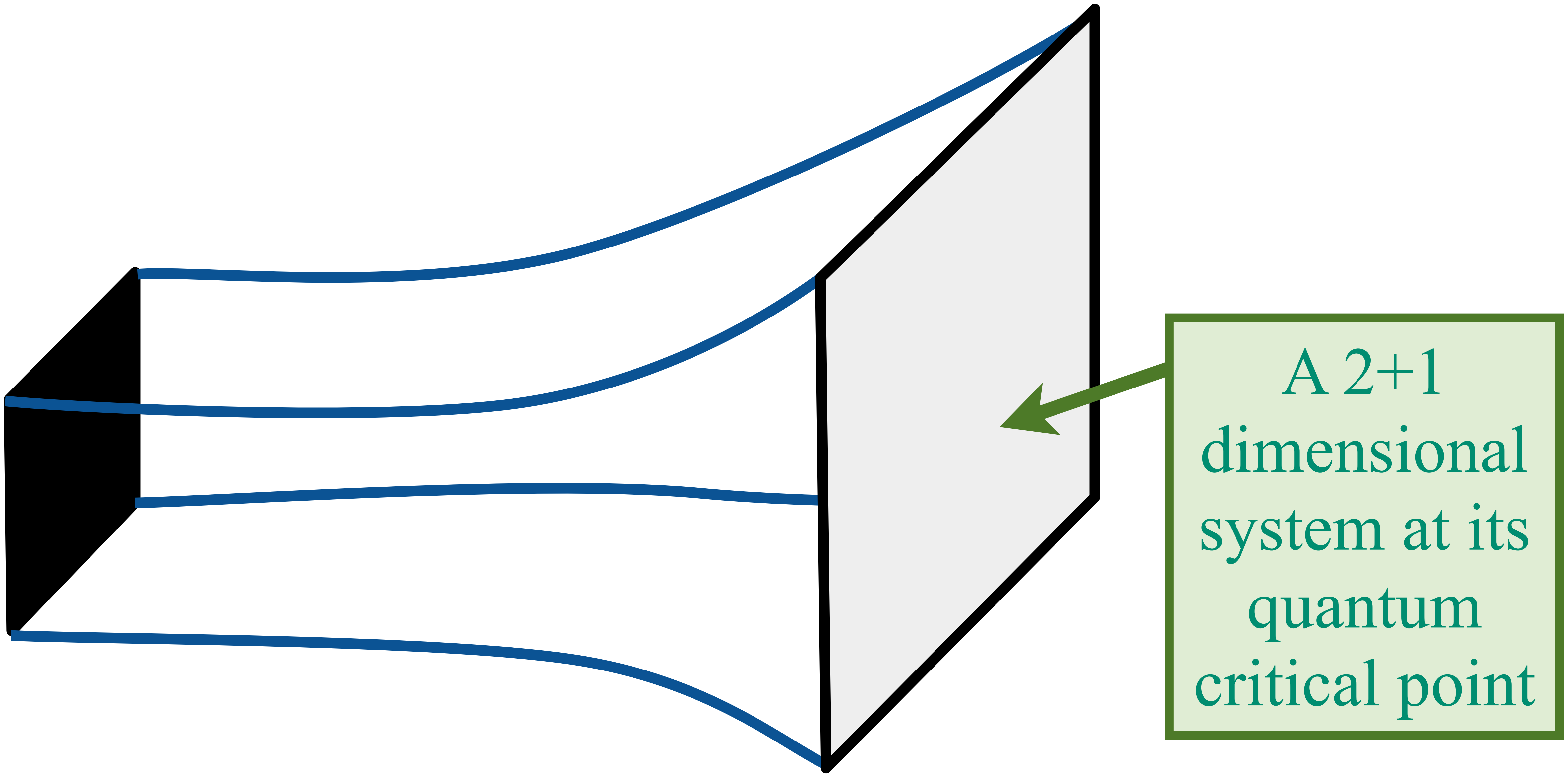


# String theory at non-zero temperatures



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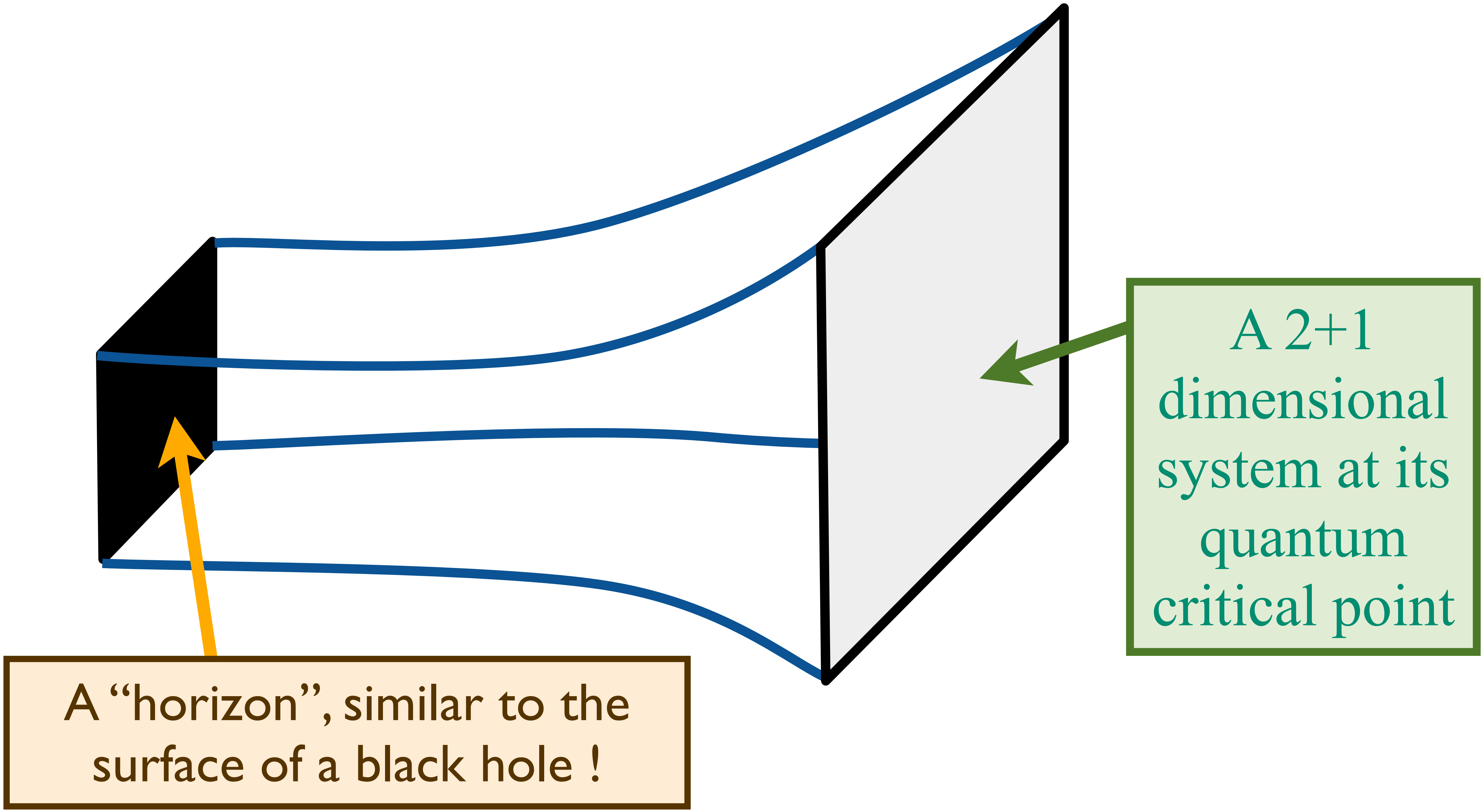
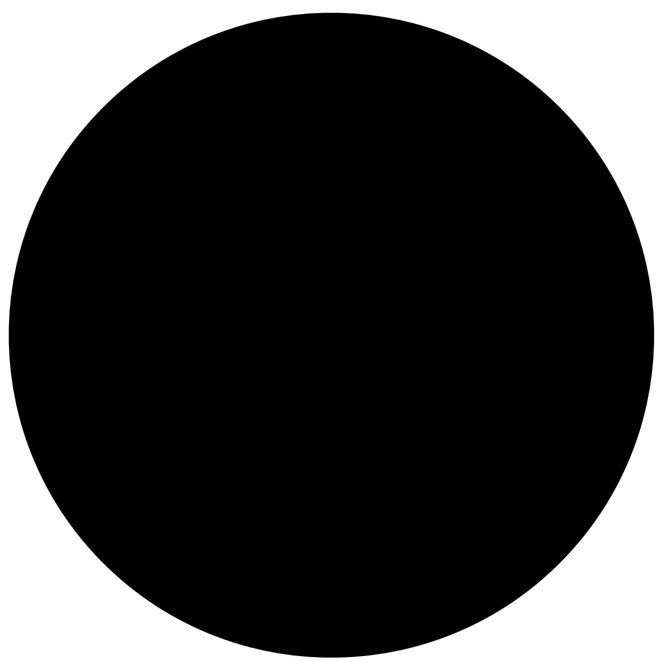


# Black Holes

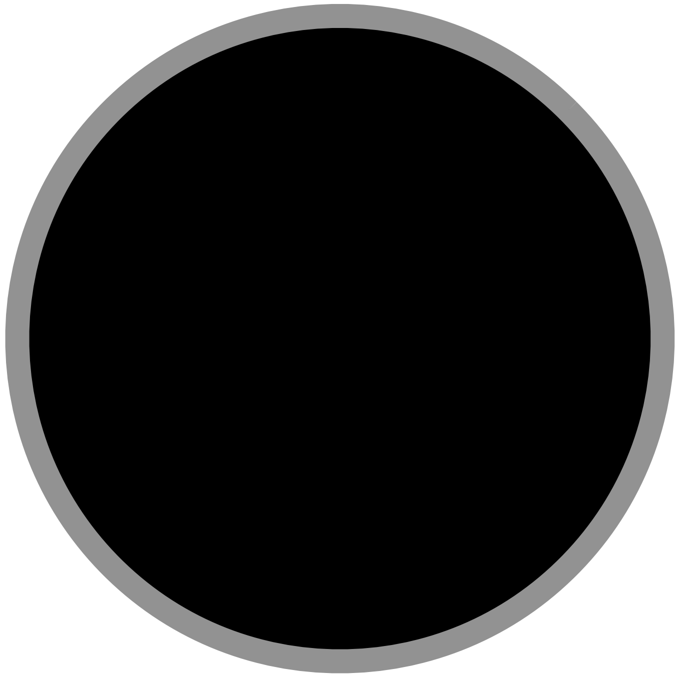
Objects so massive that light is gravitationally bound to them.



# Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

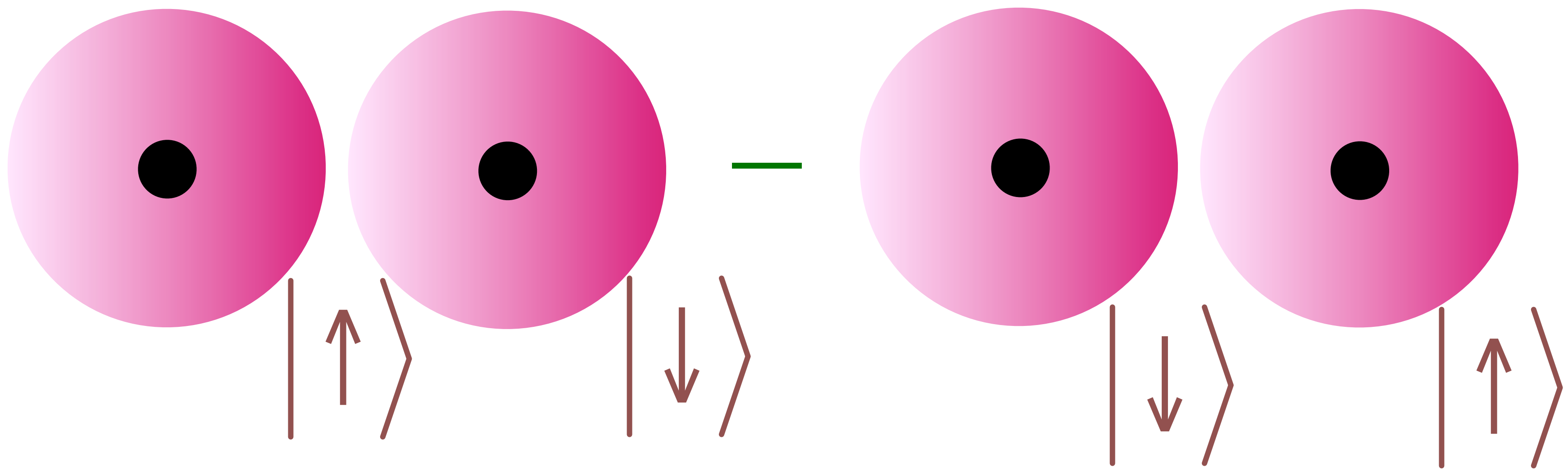


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

# Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

# Quantum Entanglement across a black hole horizon



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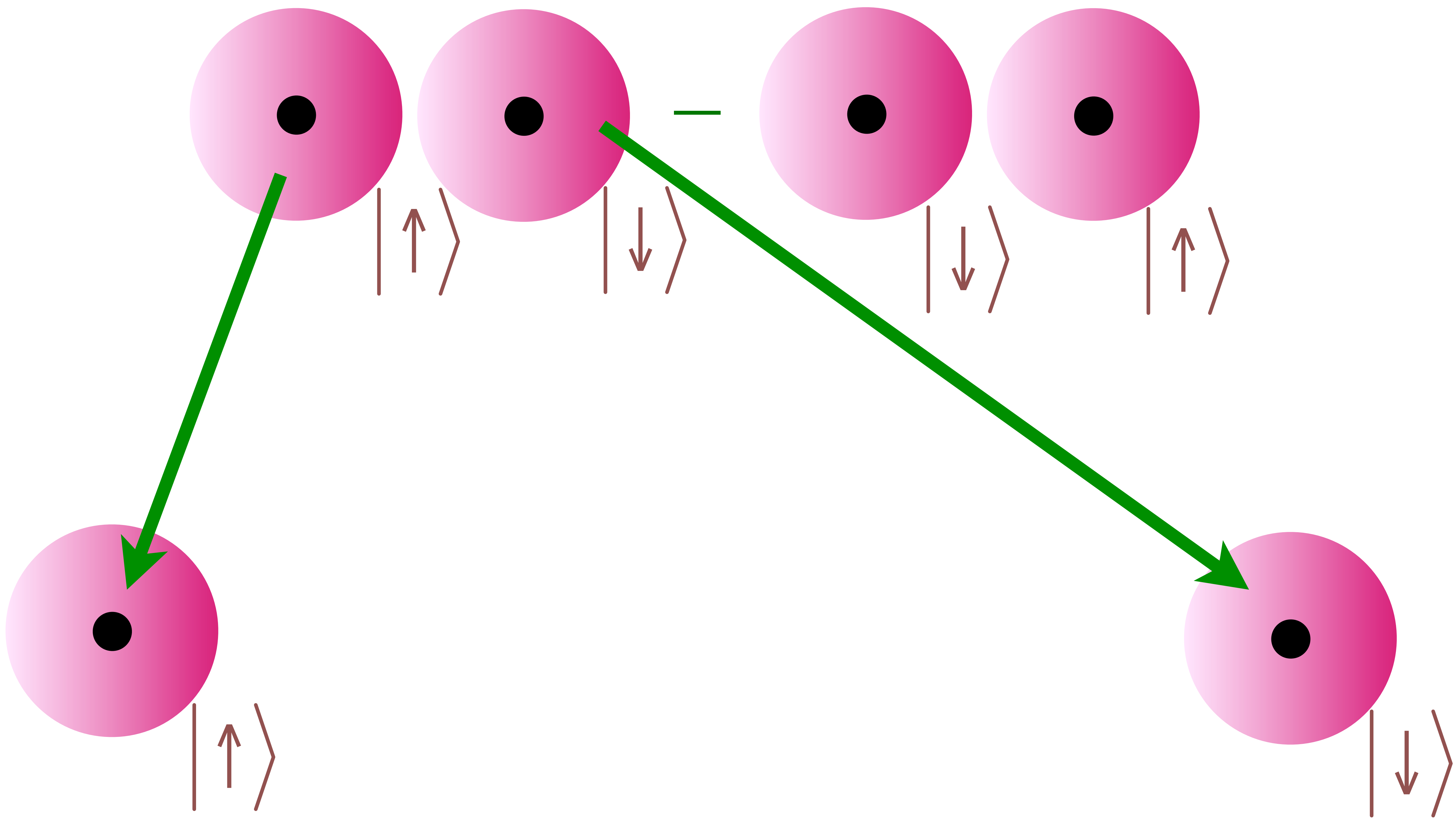

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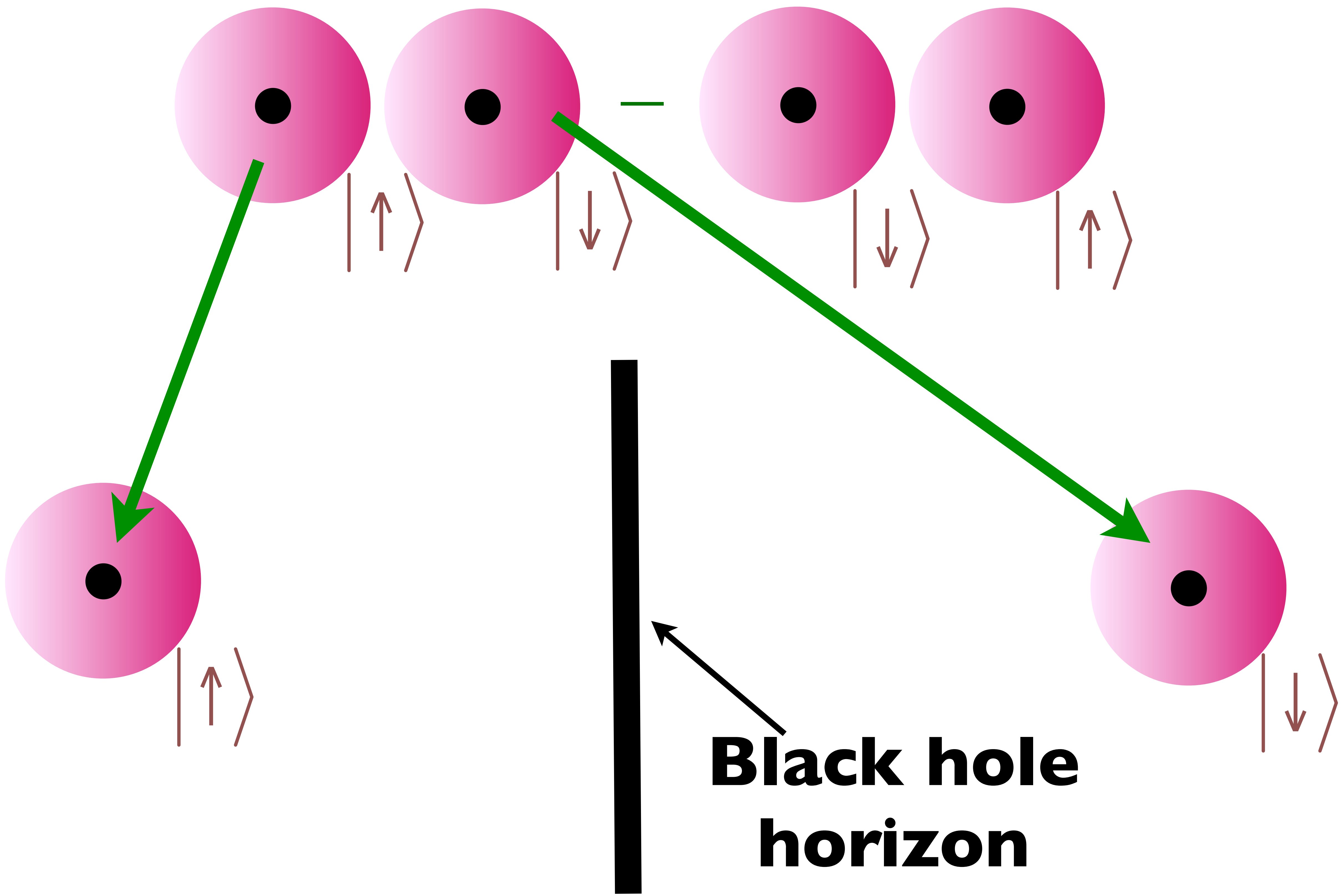

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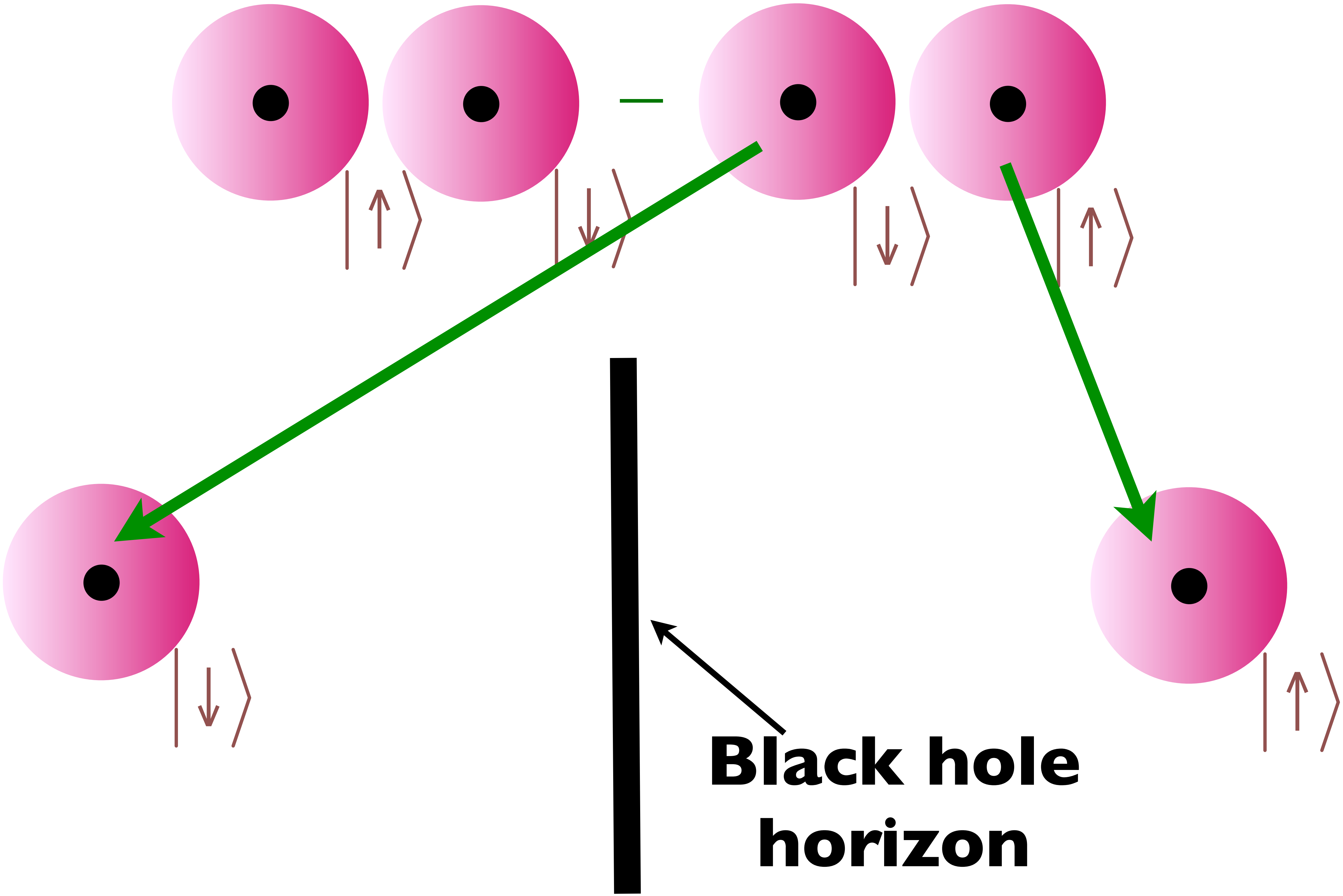
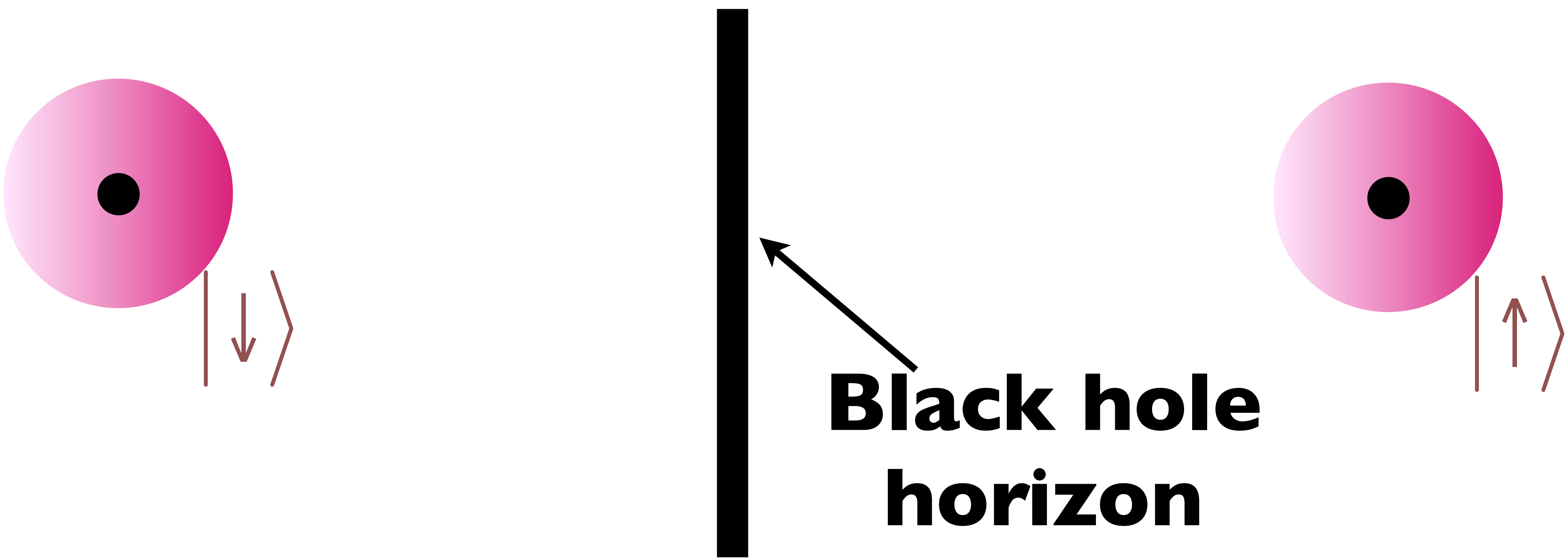


# Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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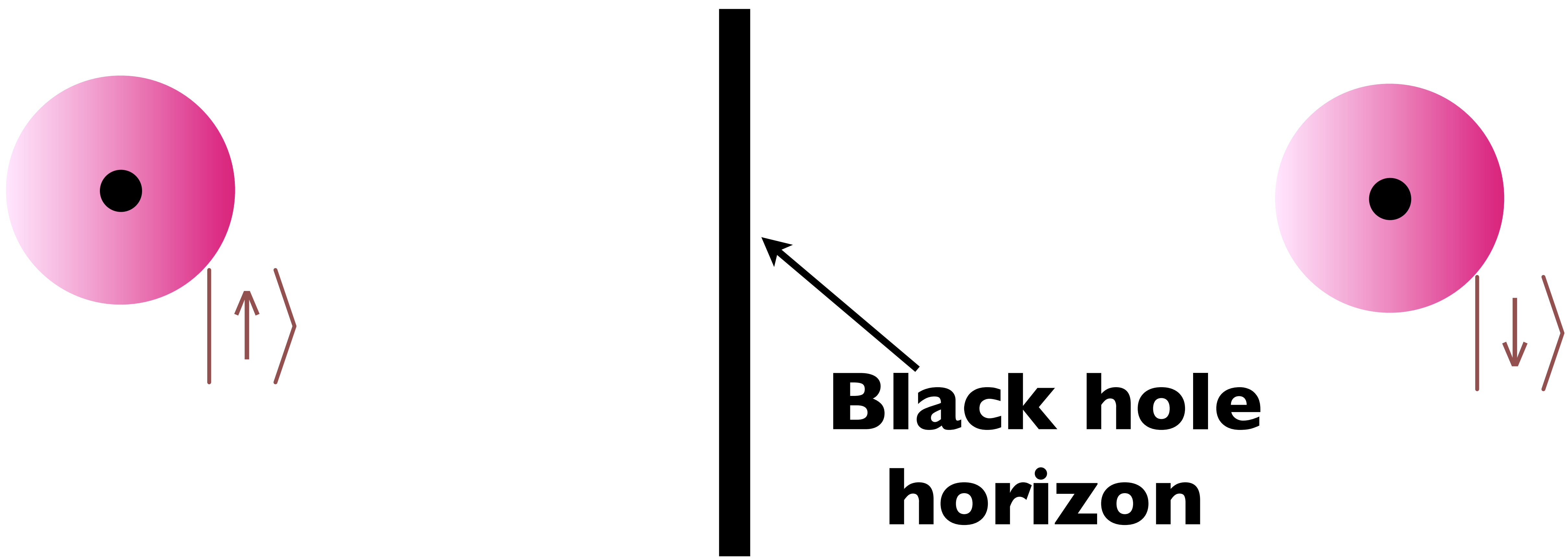
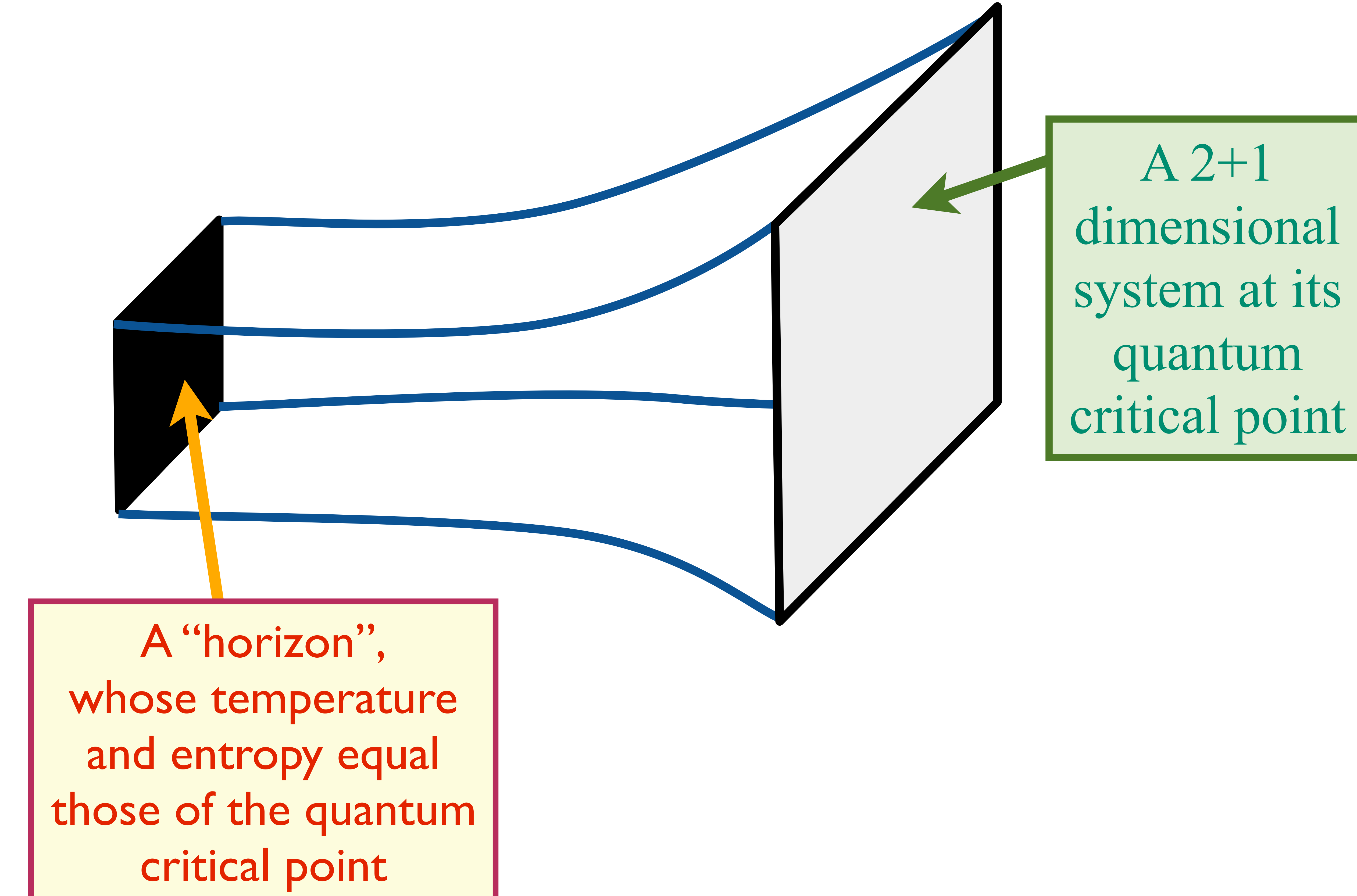


# Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

# String theory at non-zero temperatures



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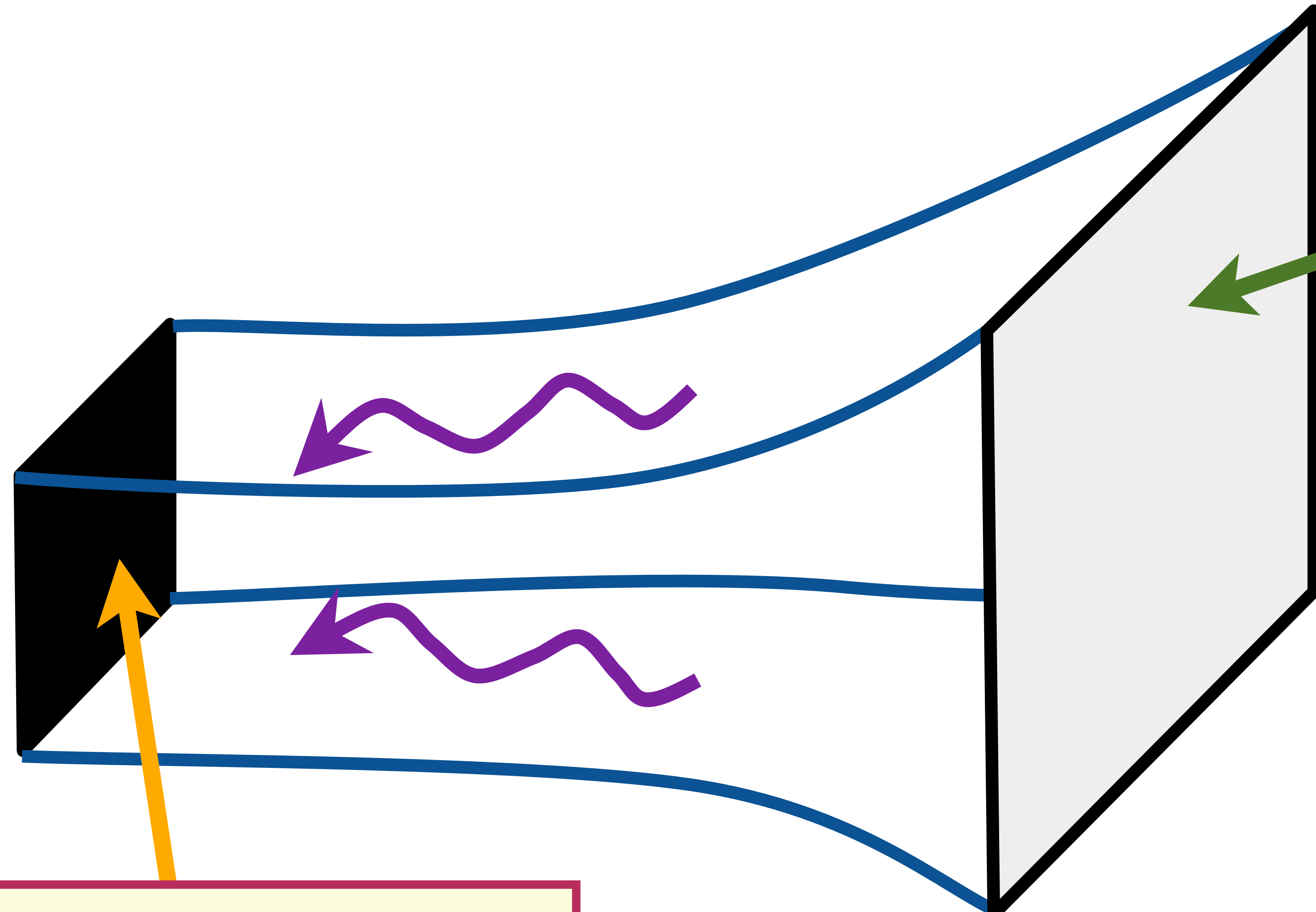
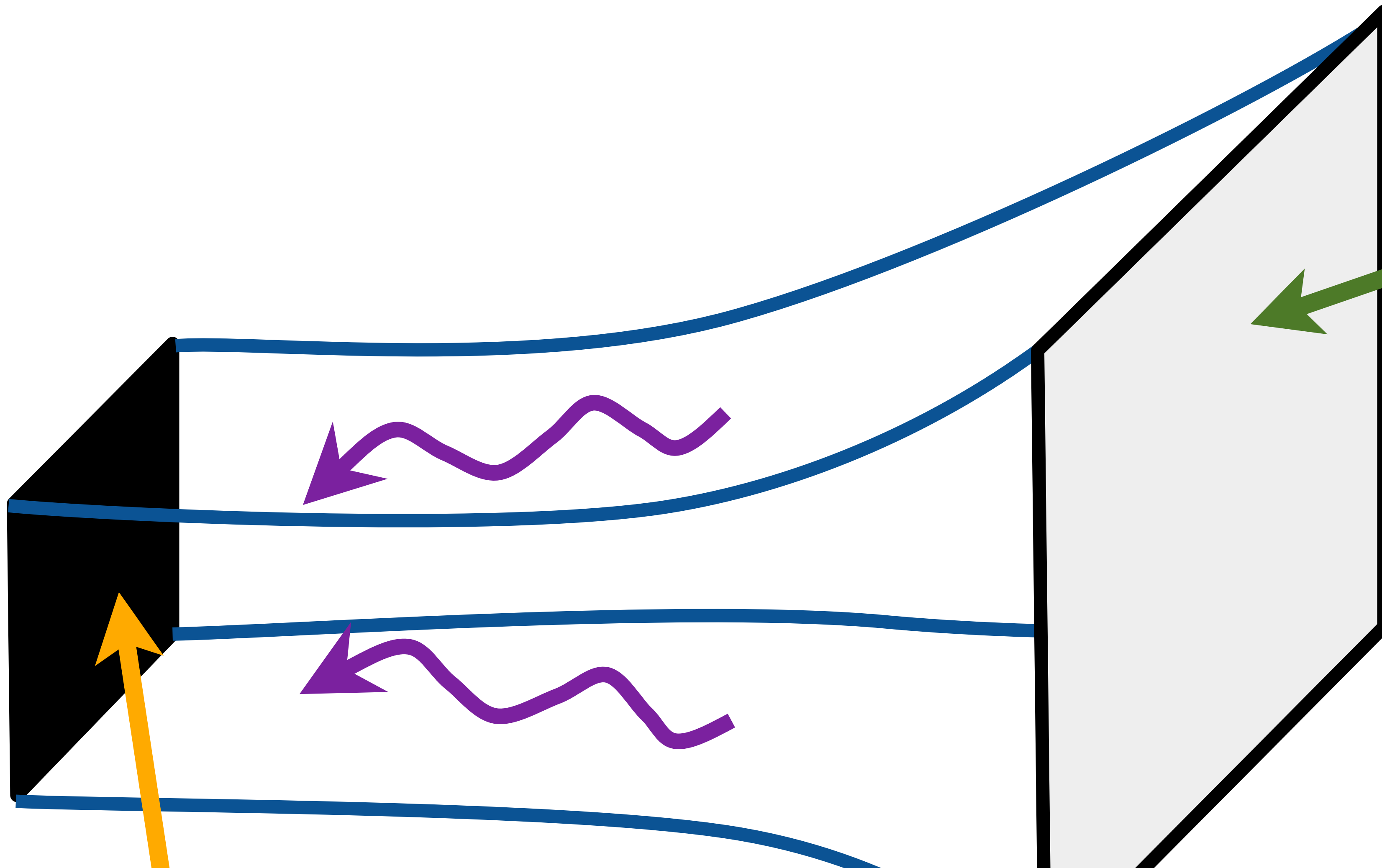


A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

**Friction of quantum criticality = waves falling into black brane**

# String theory at non-zero temperatures

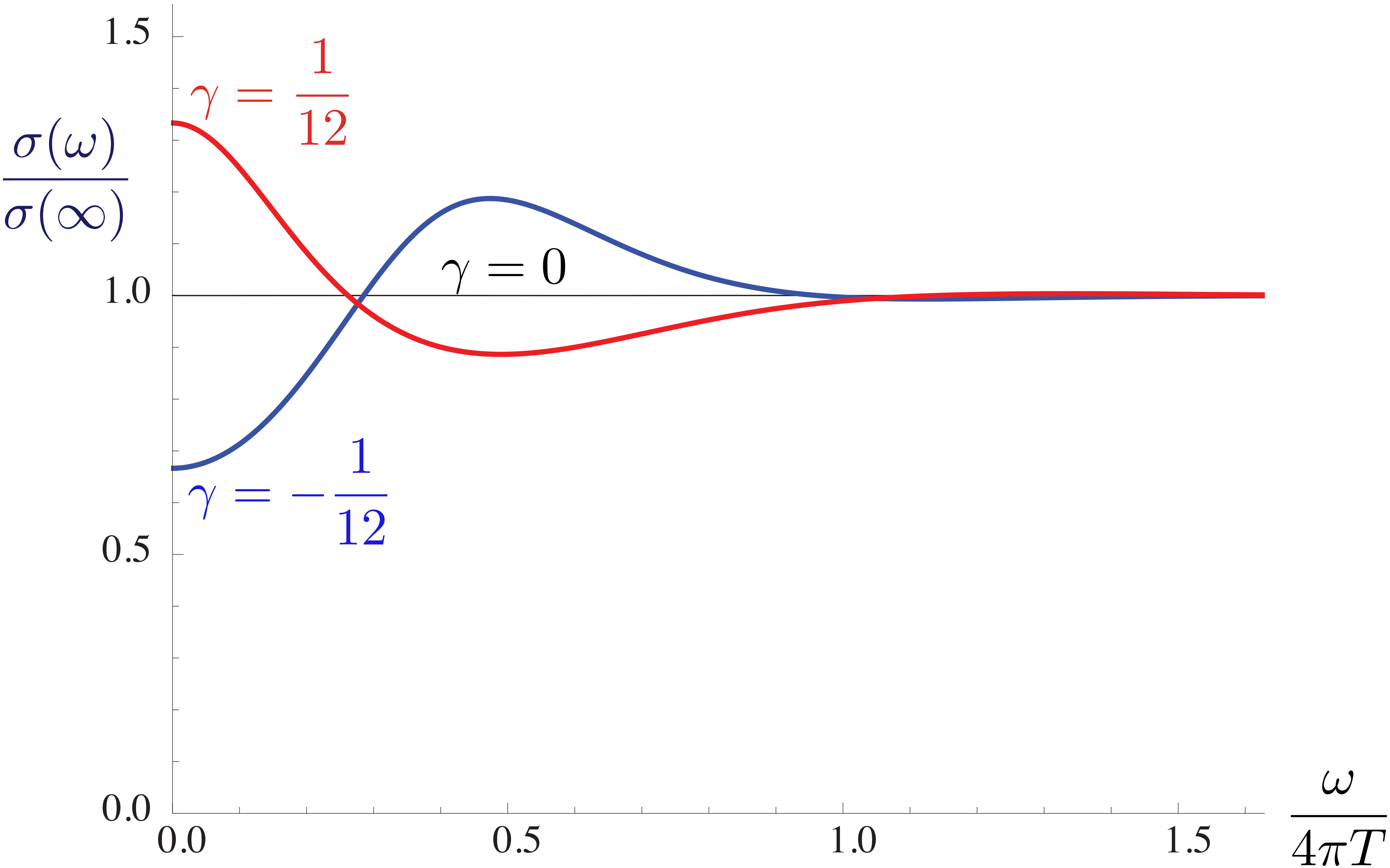


A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

# AdS$_4$ theory of charge transport in a CFT3




R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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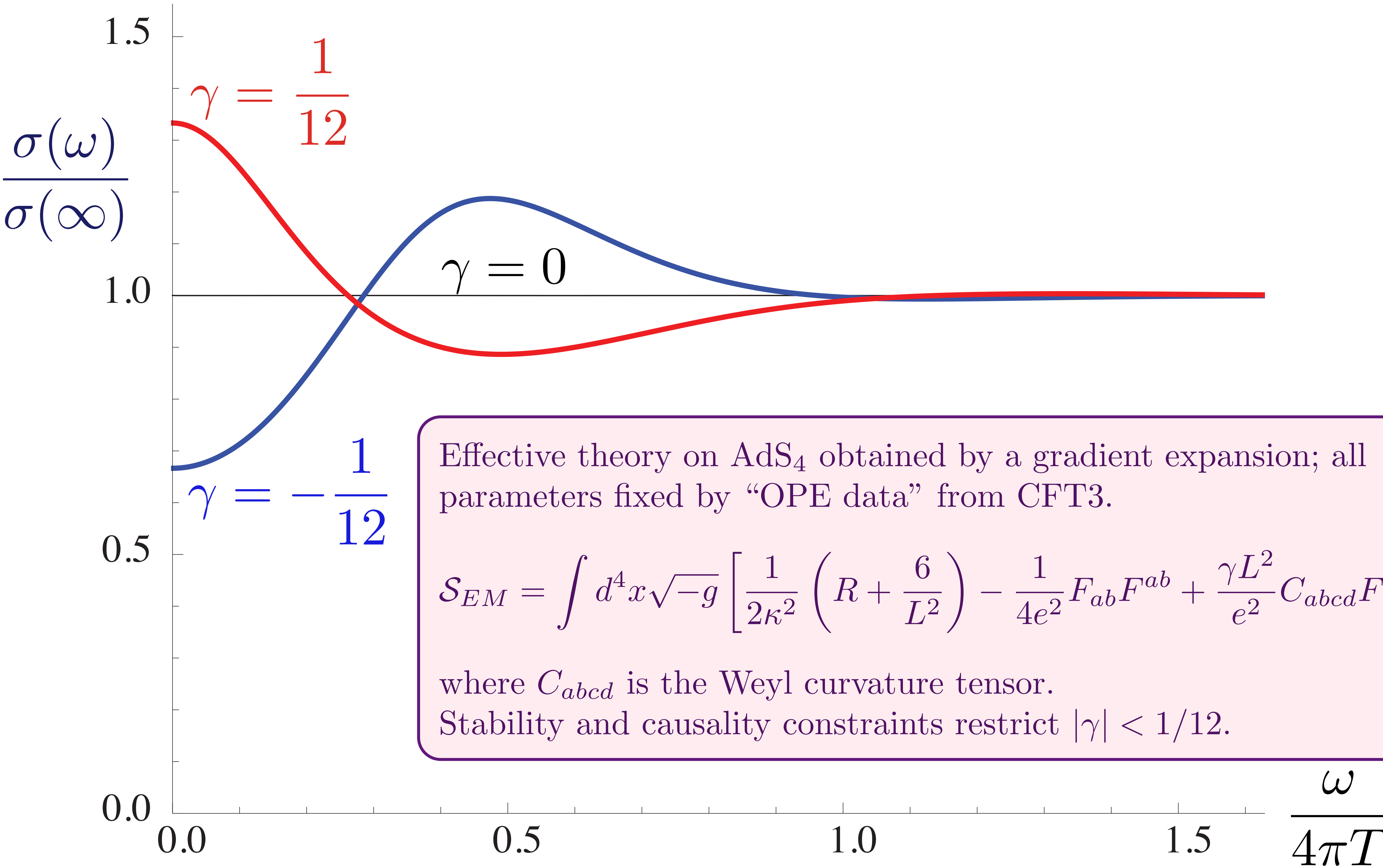


Effective theory on AdS$_4$ obtained by a gradient expansion; all parameters fixed by "OPE data" from CFT3.

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right]$$

where $C_{abcd}$ is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

# "Complex entangled" states of quantum matter in *d* spatial dimensions

## Gapped quantum matter

*Spin liquids, quantum Hall states*

## Conformal quantum matter

*Quantum critical points in antiferromagnets, superconductors, and ultracold atoms; graphene*

## Compressible quantum matter

*Strange metals in high temperature superconductors, Bose metals*

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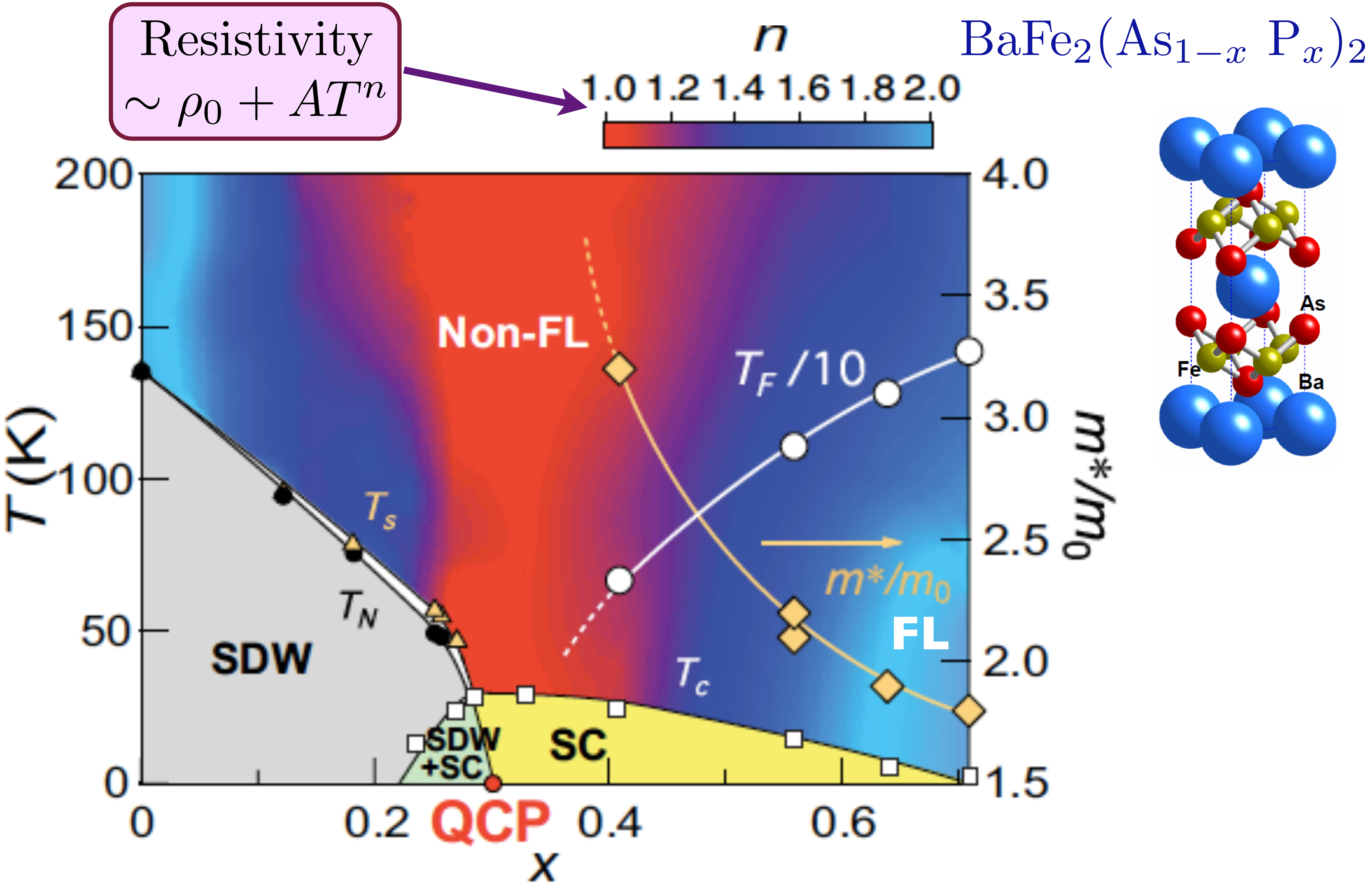
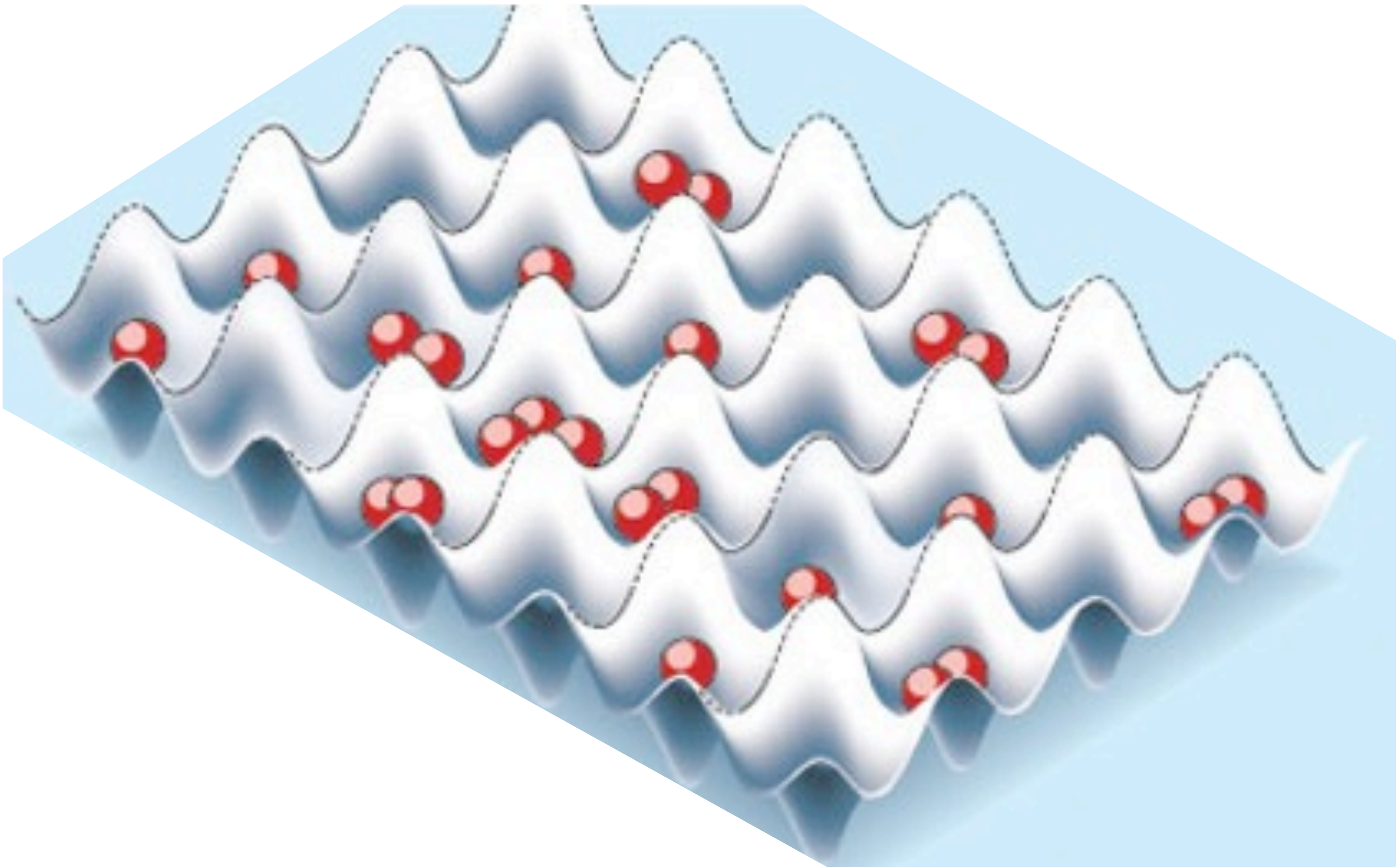



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M. A. Tanatar, H. Kitano, N. Salovich, R. W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).

# Bosons with correlated hopping

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i (n_i - 1) + w \sum_{ijk\ell \in \Box} b_i^\dagger b_k^\dagger b_j b_\ell$$



A *Bose metal*: a compressible phase of bosons which breaks no symmetries.

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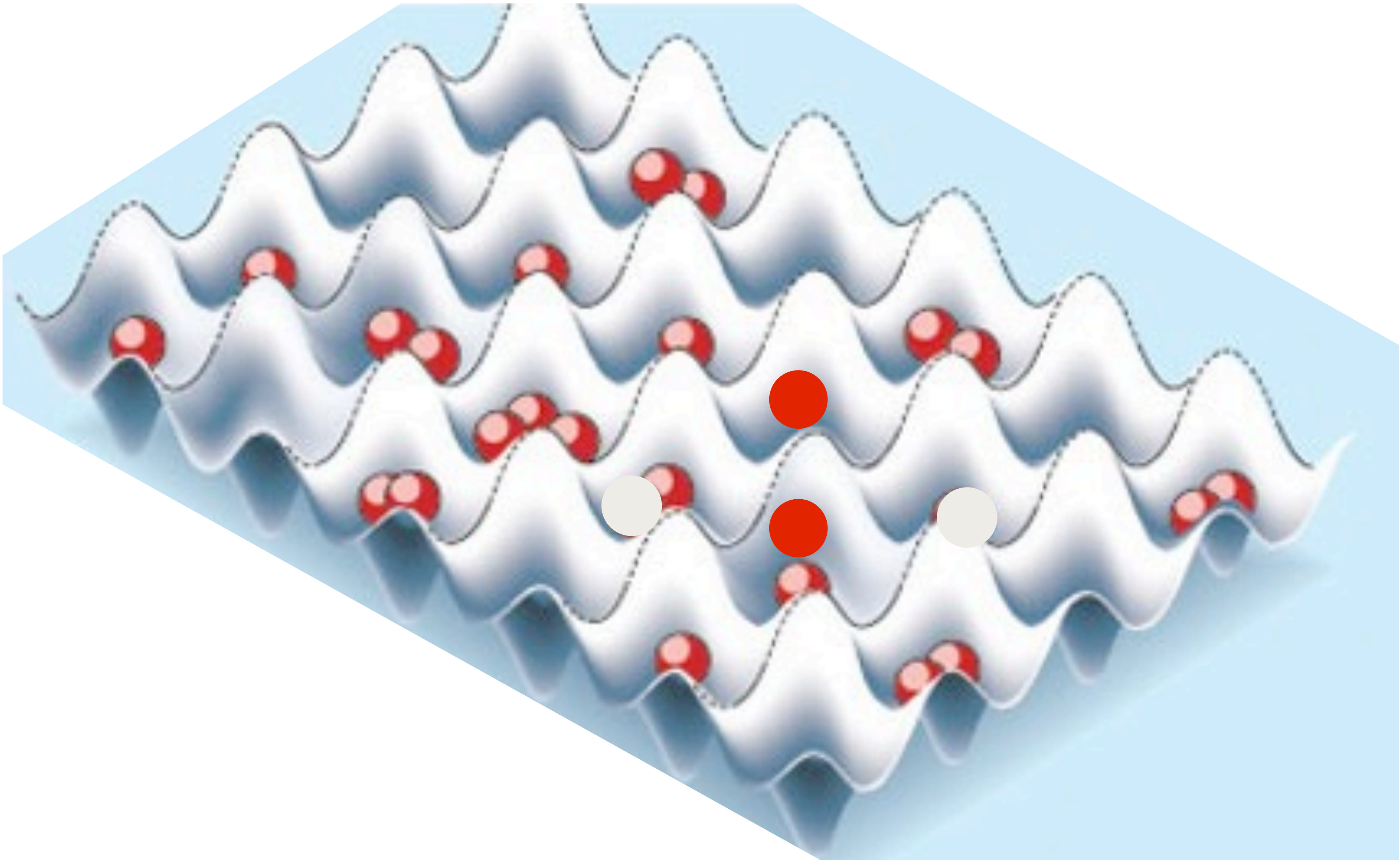

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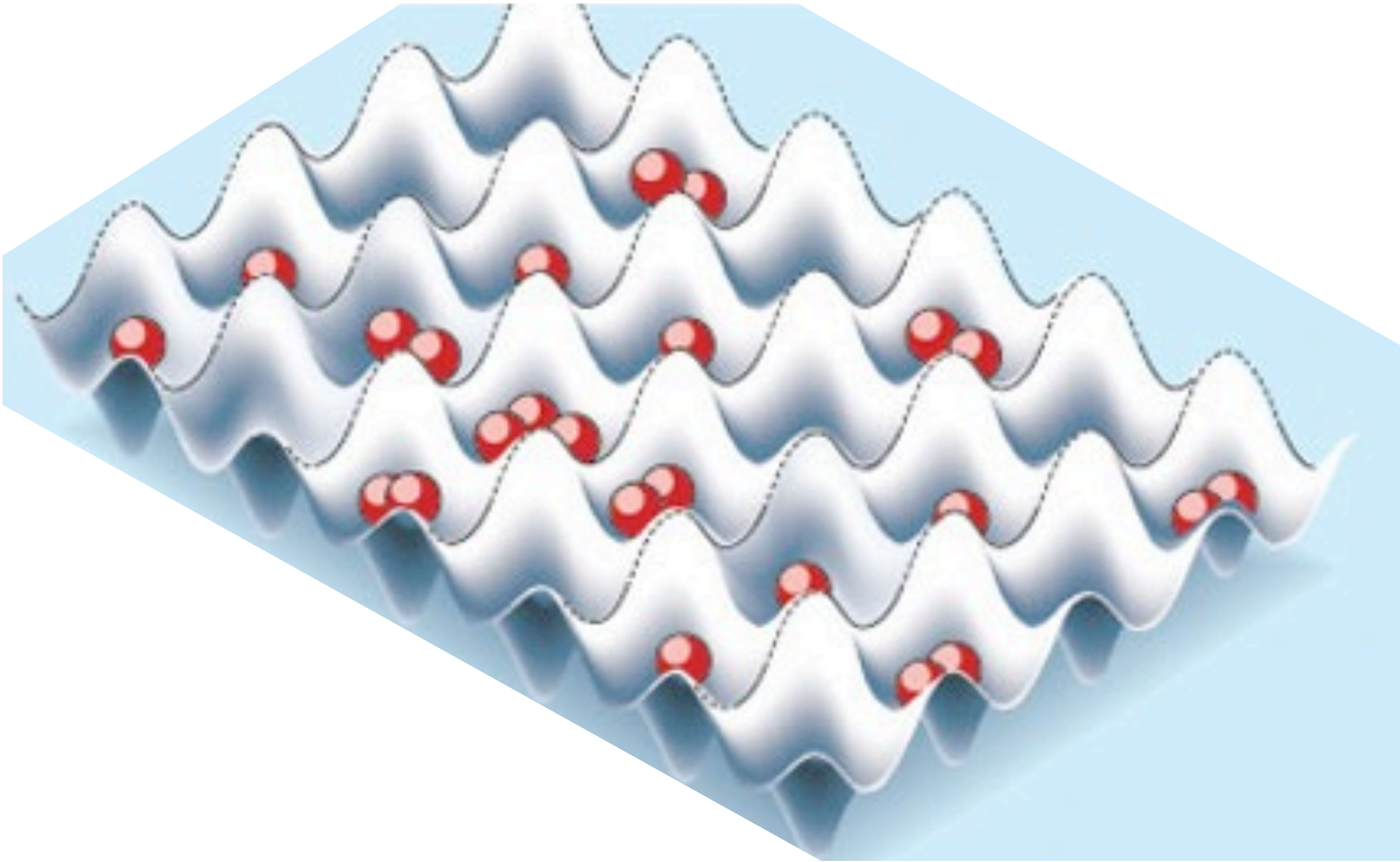
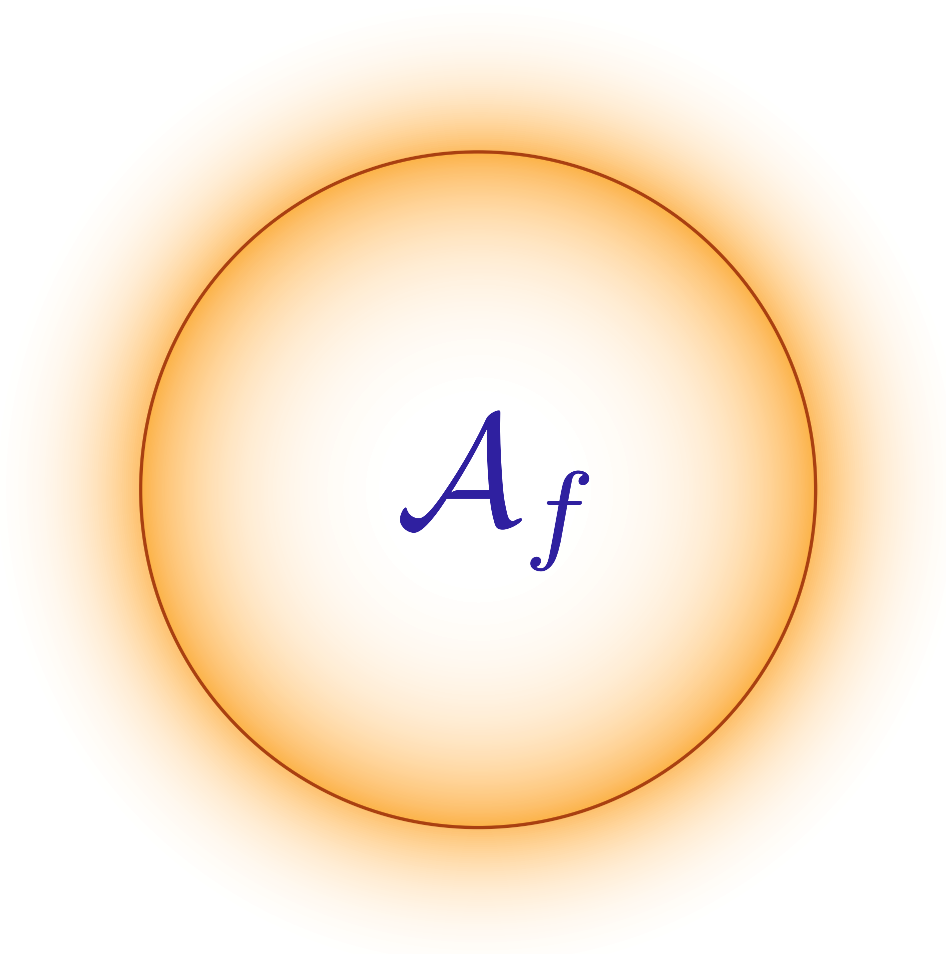


A *Bose metal*: a compressible phase of bosons which breaks no symmetries.

- *Bose metal*: the boson, $b$, fractionalizes into (say) 2 fermions, $f_1$ and $f_2$ ("*quarks*"), each of which forms a Fermi surface. Both fermions necessarily couple to an emergent gauge field, and so the Fermi surfaces are "*hidden*".



$$\mathcal{Q} = b^\dagger b$$

$$\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)
S. Sachdev, to appear

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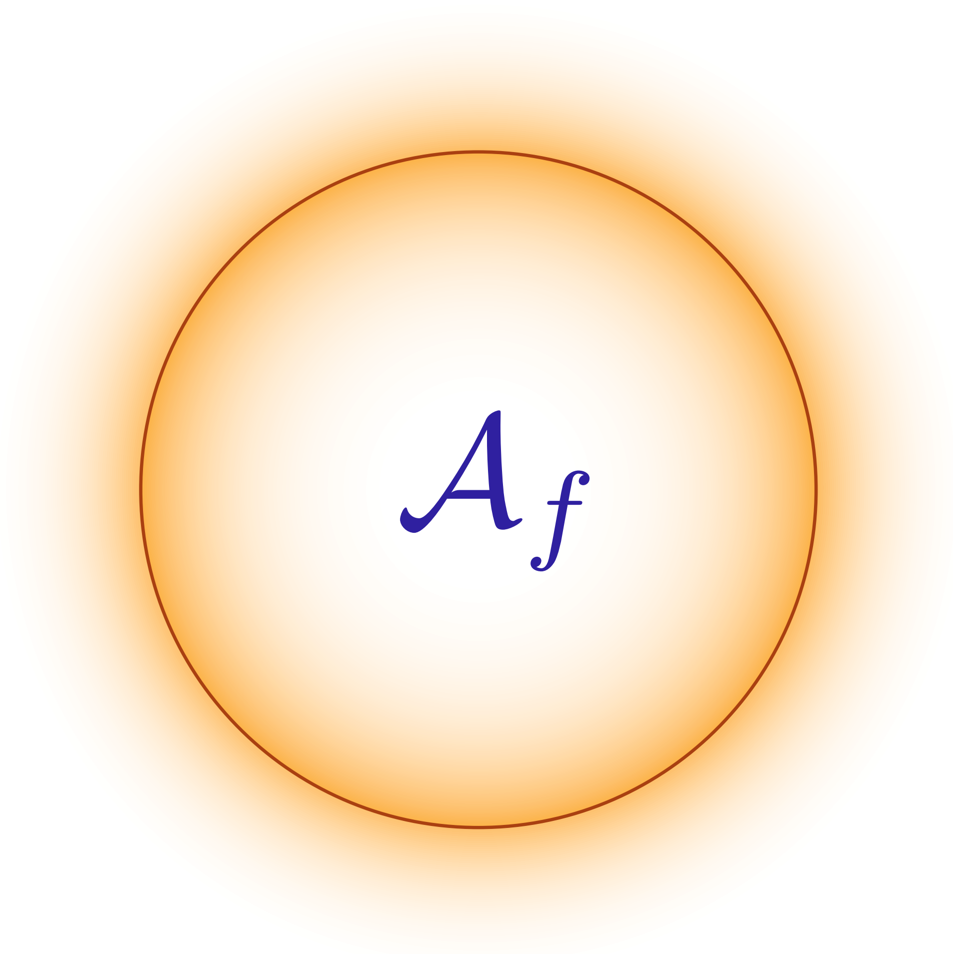


$$b \to f_1 f_2$$

Gauge invariance:

$$f_1(x) \to f_1(x) e^{i\theta(x)},$$
$$f_2(x) \to f_2(x) e^{-i\theta(x)}$$

O. I. Motrunich and M. P. A. Fisher, *Physical Review* B **75**, 235116 (2007)
L. Huijse and S. Sachdev, *Physical Review* D **84**, 026001 (2011)
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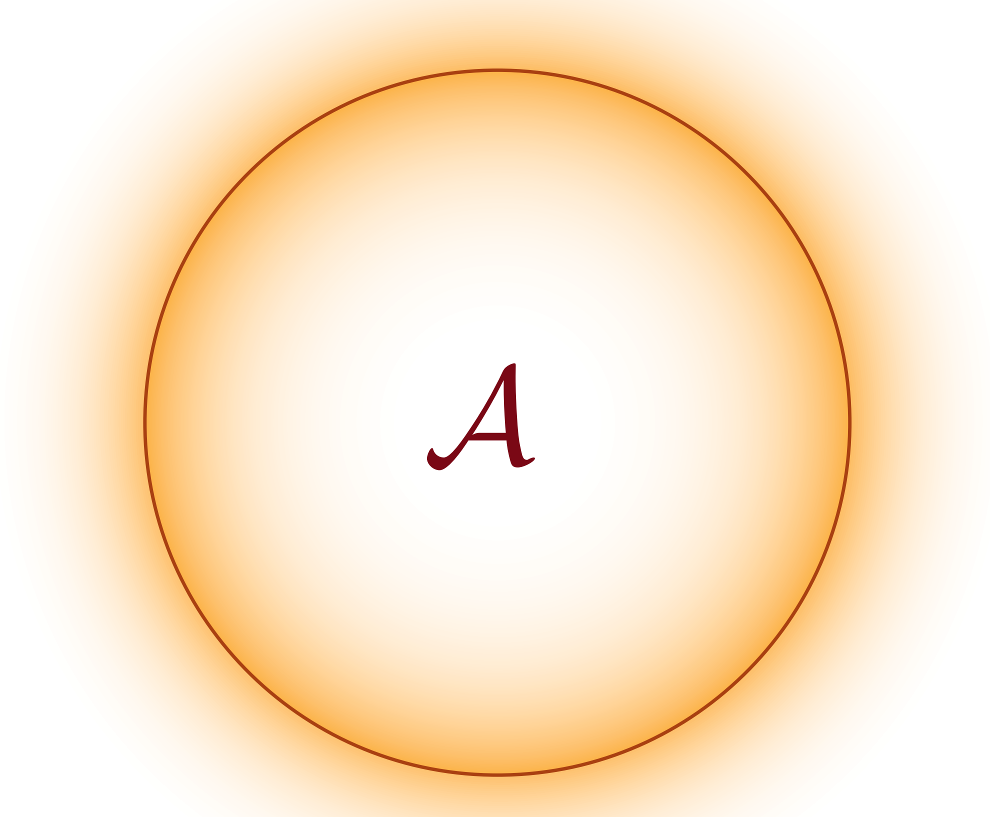
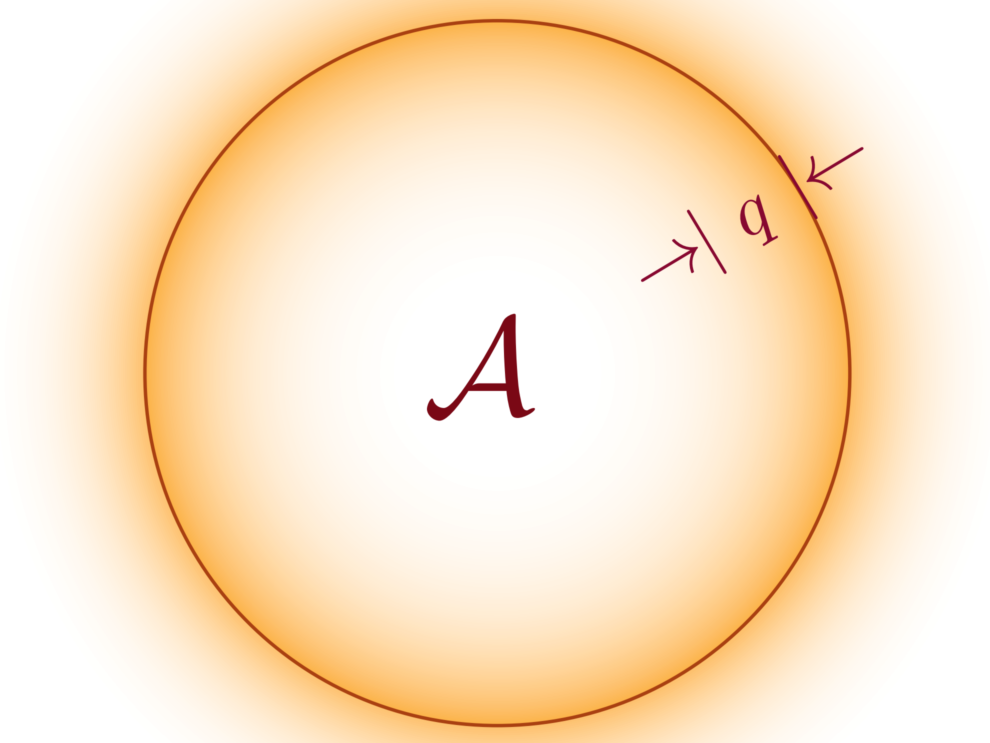
# Bose metals




- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

# Bose metals

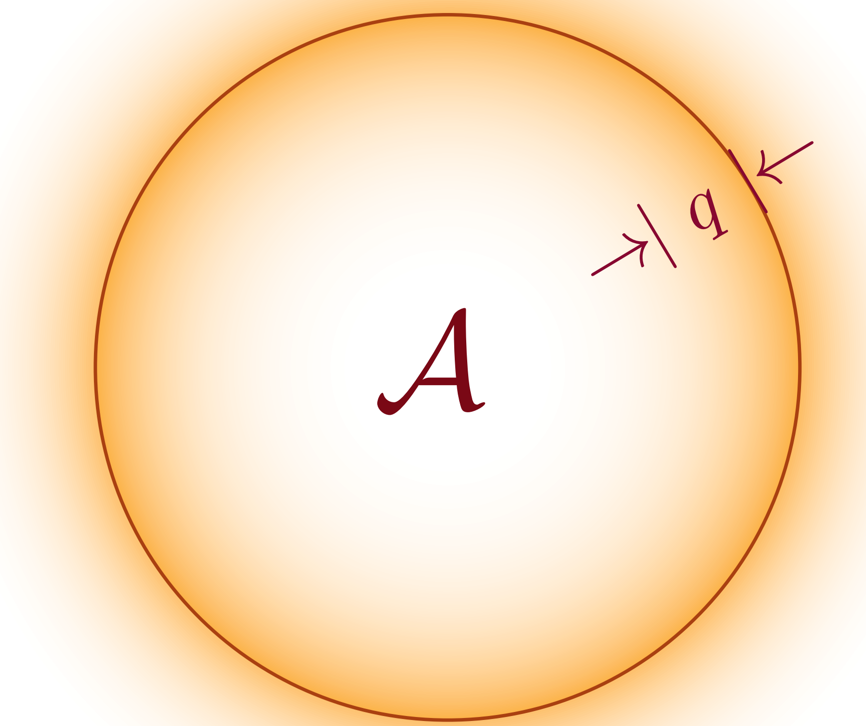



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$; three-loop computation shows $z = 3/2$.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

# Bose metals

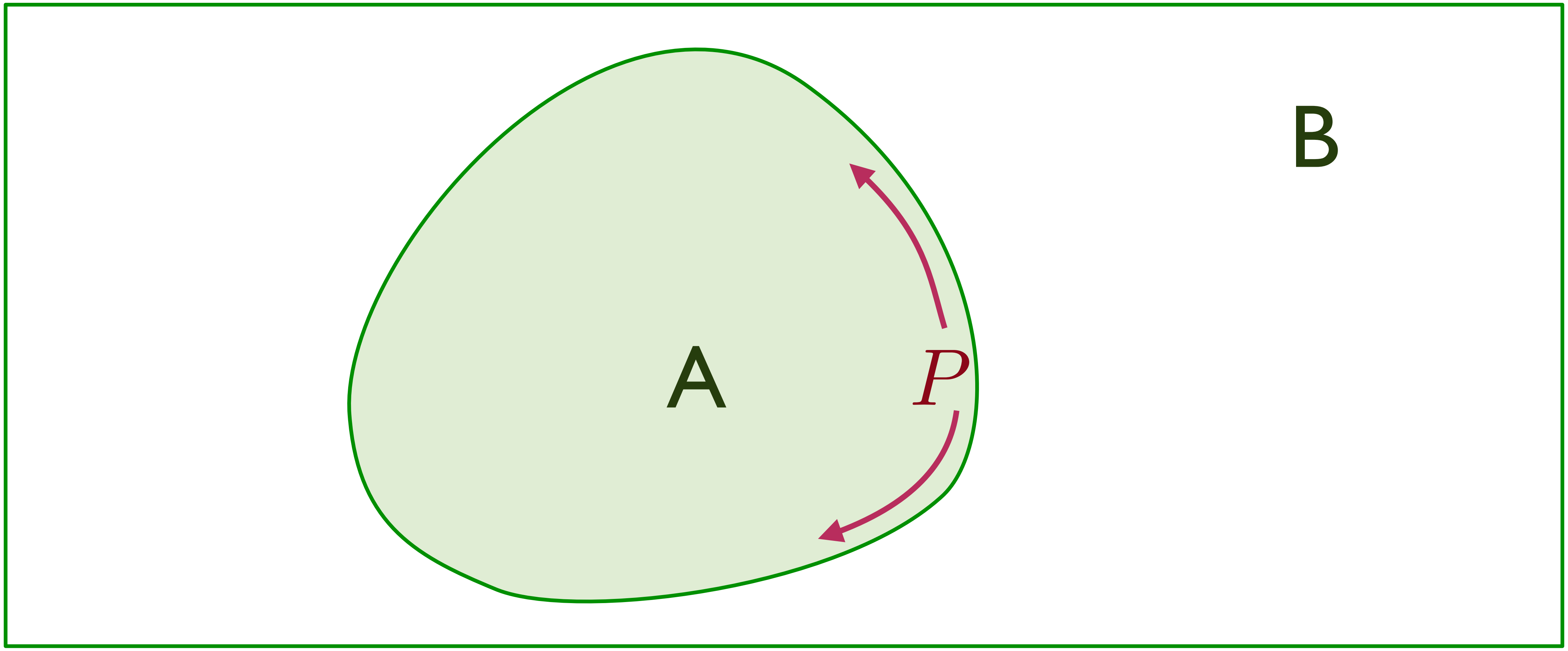



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|^z$; three-loop computation shows $z = 3/2$.

- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

# Entanglement entropy of Fermi surfaces



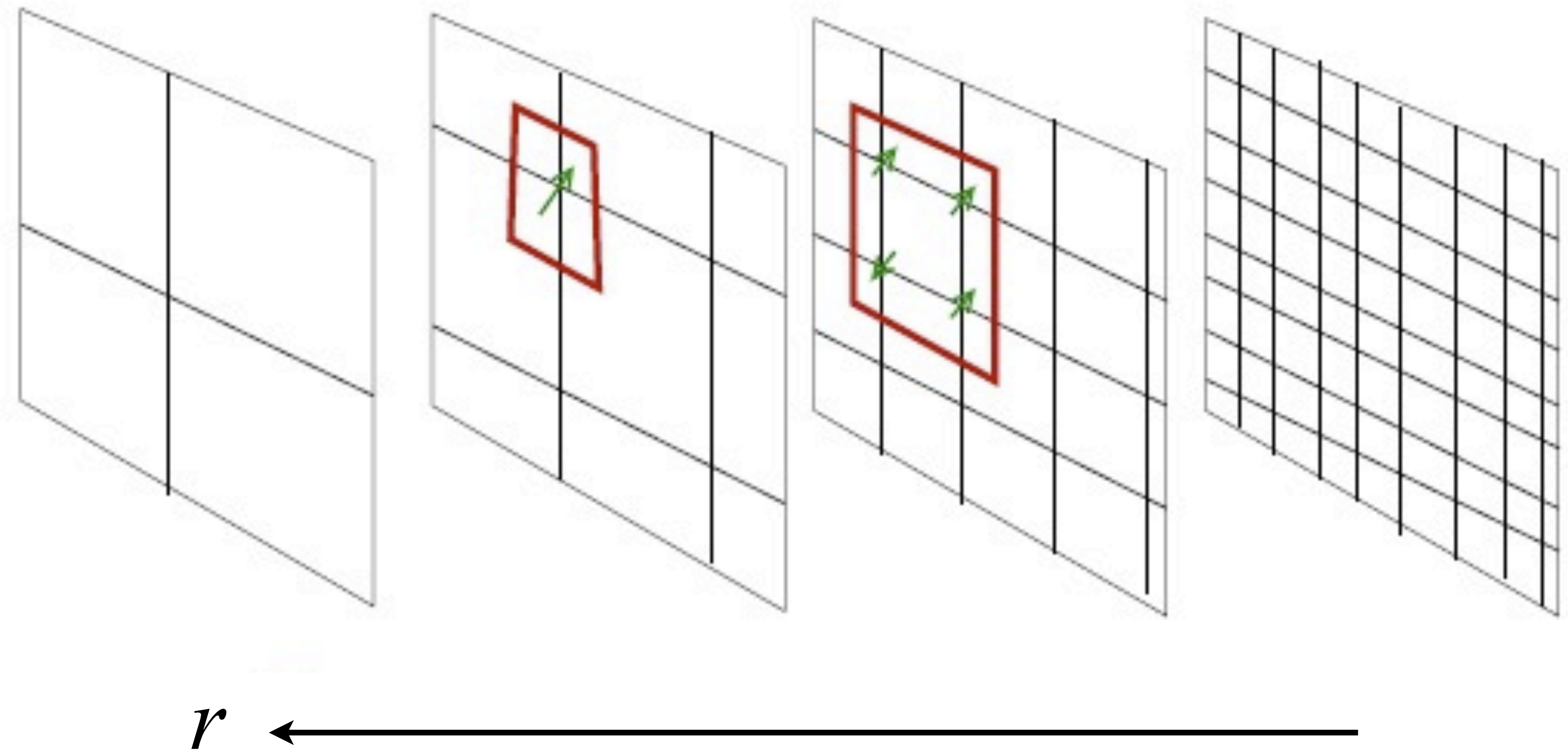

Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum $k_F$,
where $P$ is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

# Holography



Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies $z$ as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

$\theta$ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$\theta$ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

We have used reparametrization invariance in $r$ to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

At $T > 0$, there is a *horizon*, and computation of its Bekenstein-Hawking entropy shows

$$S \sim T^{(d-\theta)/z}.$$

So $\theta$ is indeed the violation of hyperscaling exponent as claimed. For a compressible quantum state we should therefore *choose* $\theta = d - 1$.
No additional choices will be made, and all subsequent results are consequences of the assumption of the existence of a holographic dual.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

# Holography of Bose metals

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

The null energy condition (stability condition for gravity) yields a new inequality

$$z \geq 1 + \frac{\theta}{d}$$

In $d = 2$, this implies $z \geq 3/2$. So the lower bound is precisely the value obtained from the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

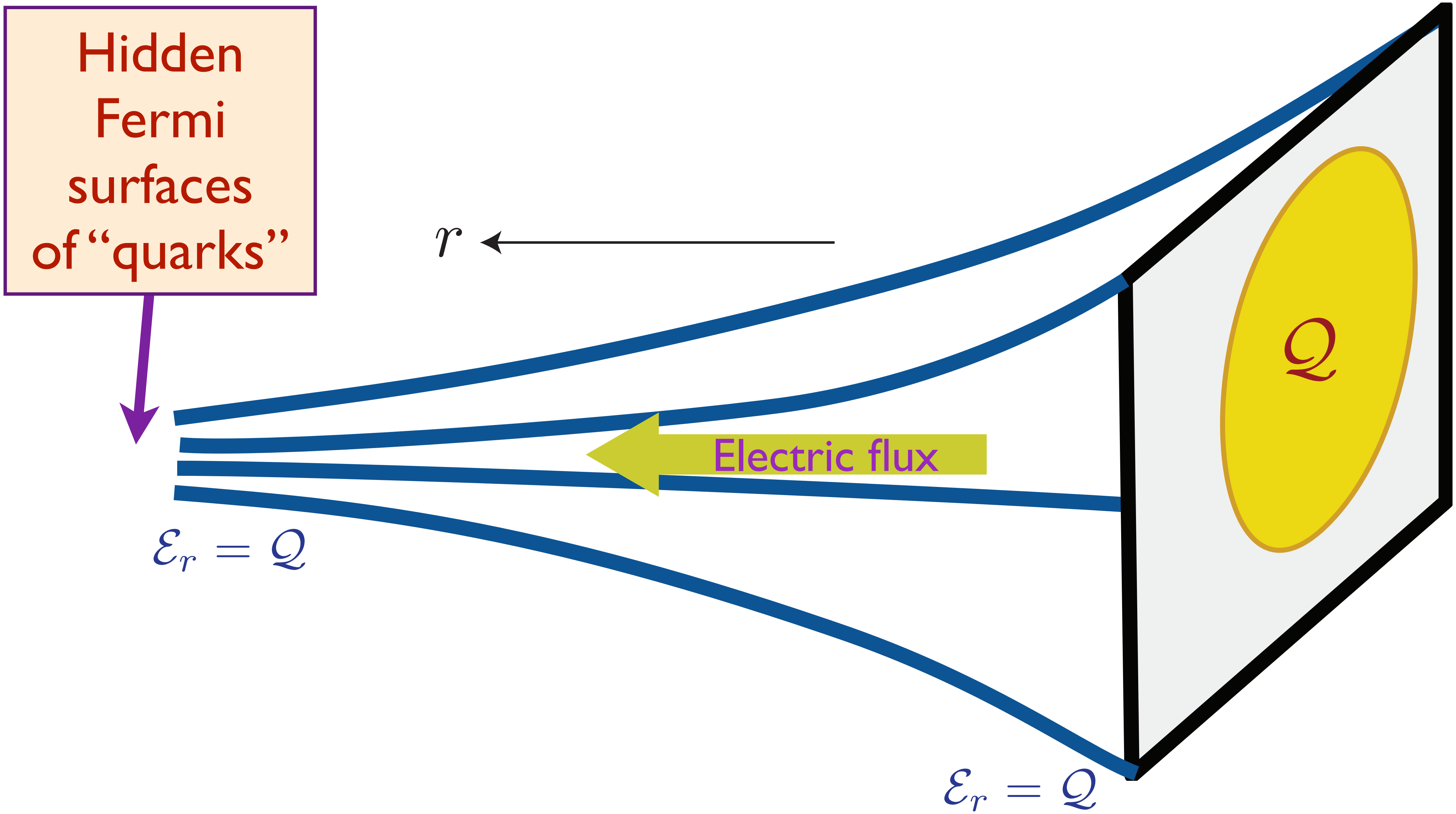
Application of the Ryu-Takayanagi minimal area formula to a dual Einstein-Maxwell-dilaton theory yields

$$S_E \sim \mathcal{Q}^{(d-1)/d} P \ln P$$

with a co-efficient *independent* of UV details and of the shape of the entangling region. These properties are just as expected for a circular Fermi surface with $\mathcal{Q} \sim k_F^d$.

N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

# Holographic theory of a Bose metal




Fully fractionalized state has all the electric flux exiting to the horizon at $r = \infty$

# Holography, fractionalization, and hidden Fermi surfaces

- Electric flux exiting the horizon corresponds to fractionalized component of the conserved density $\mathcal{Q}$, which is proposed to be associated with "hidden" Fermi surfaces of gauge-charged particles.

- Gauss Law and the "attractor" mechanism in the bulk
  $\Leftrightarrow$ Luttinger theorem on the boundary theory.

# Conclusions

Realizations of many-particle entanglement:
$Z_2$ spin liquids and
conformal quantum critical points

# Conclusions

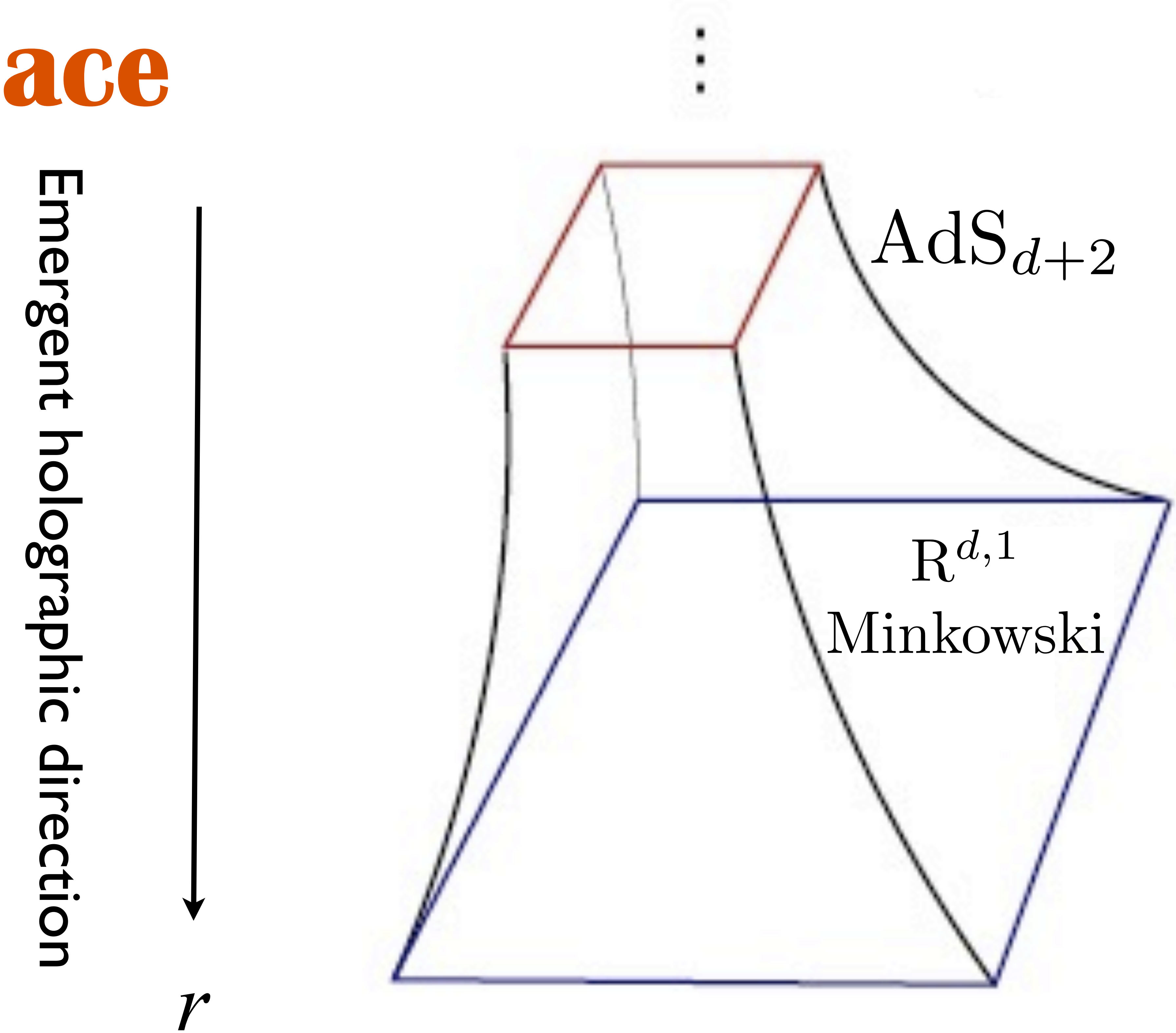
More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

# Conclusions

**String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.**

Much recent progress offers hope of a holographic description of "strange metals"

# anti-de Sitter space



# anti-de Sitter space