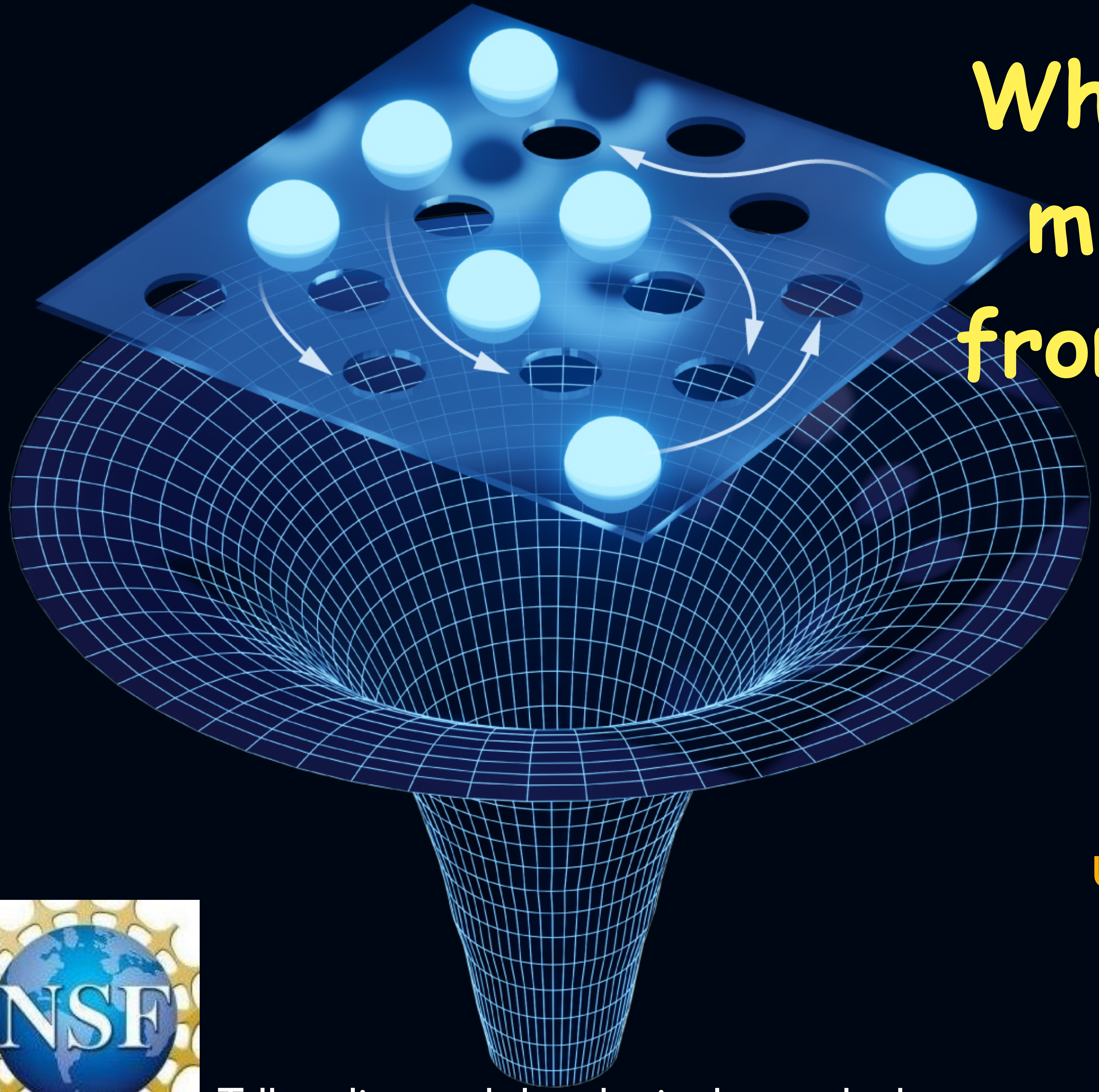

When nature entangles millions of particles: from quantum materials to black holes

Review articles:
arXiv:2304.13744, 2305.01001

University of Wurzburg
July 3, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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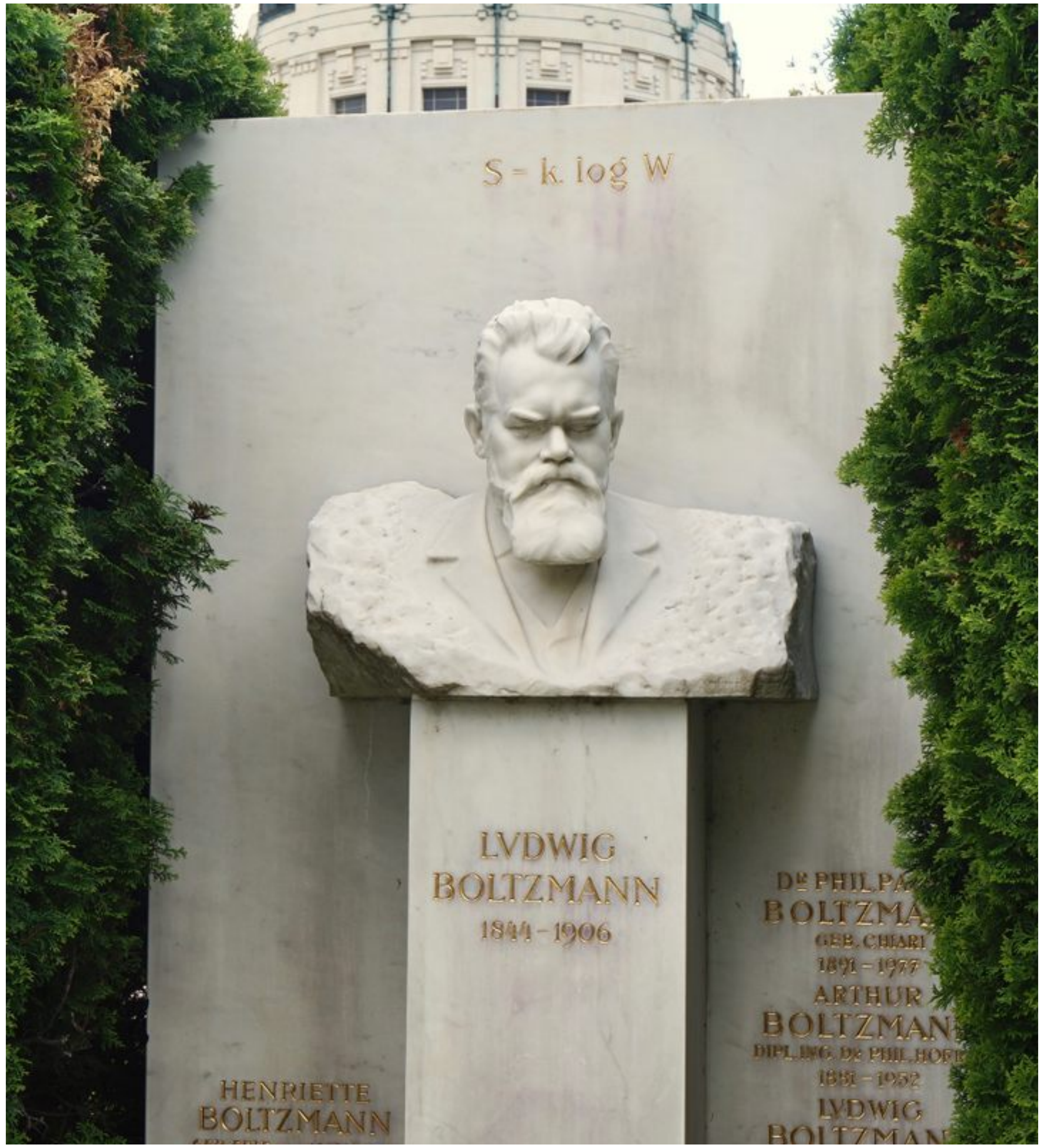
HARVARD

Foundations by Boltzmann



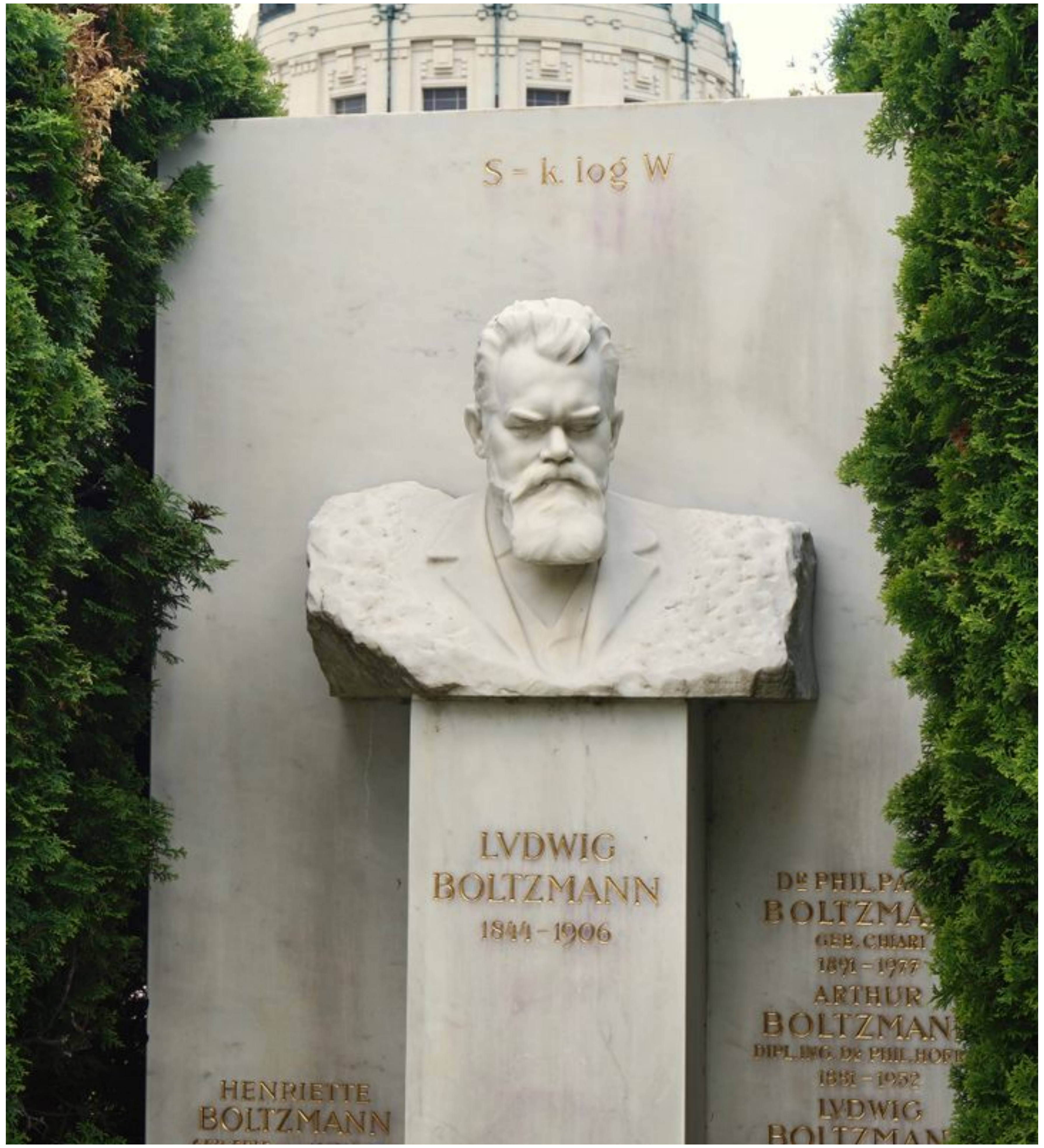
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

Boltzmann equation (1872)
Dilute classical gas



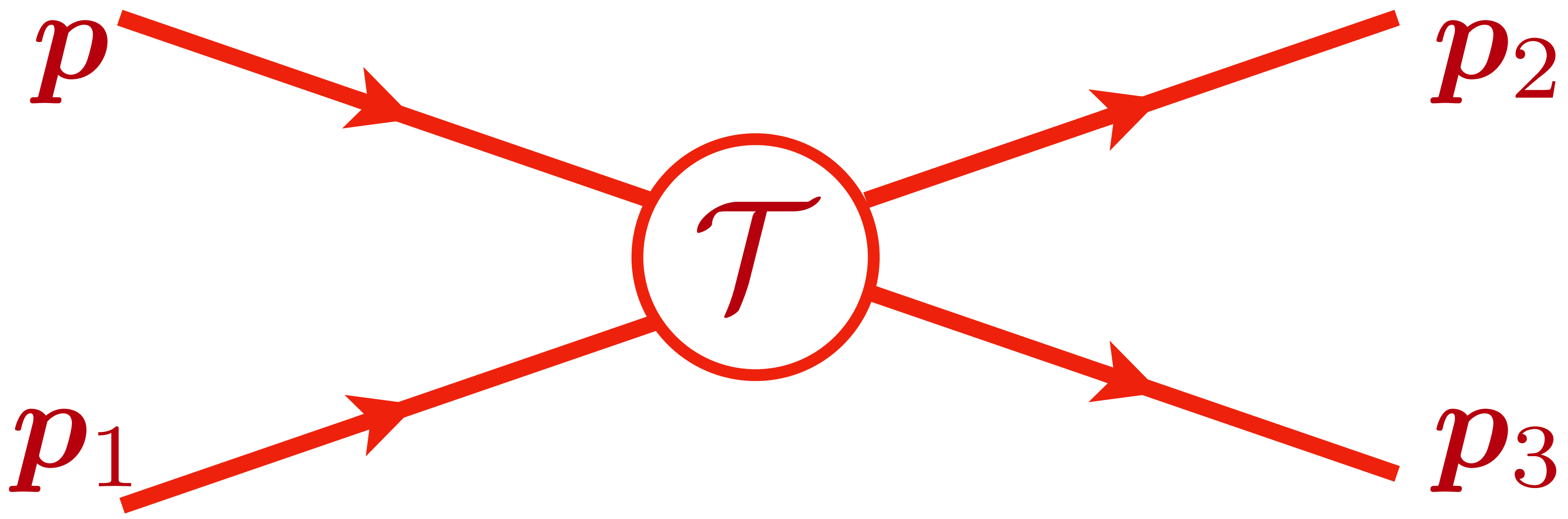
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Molecular chaos: successive collisions are statistically independent

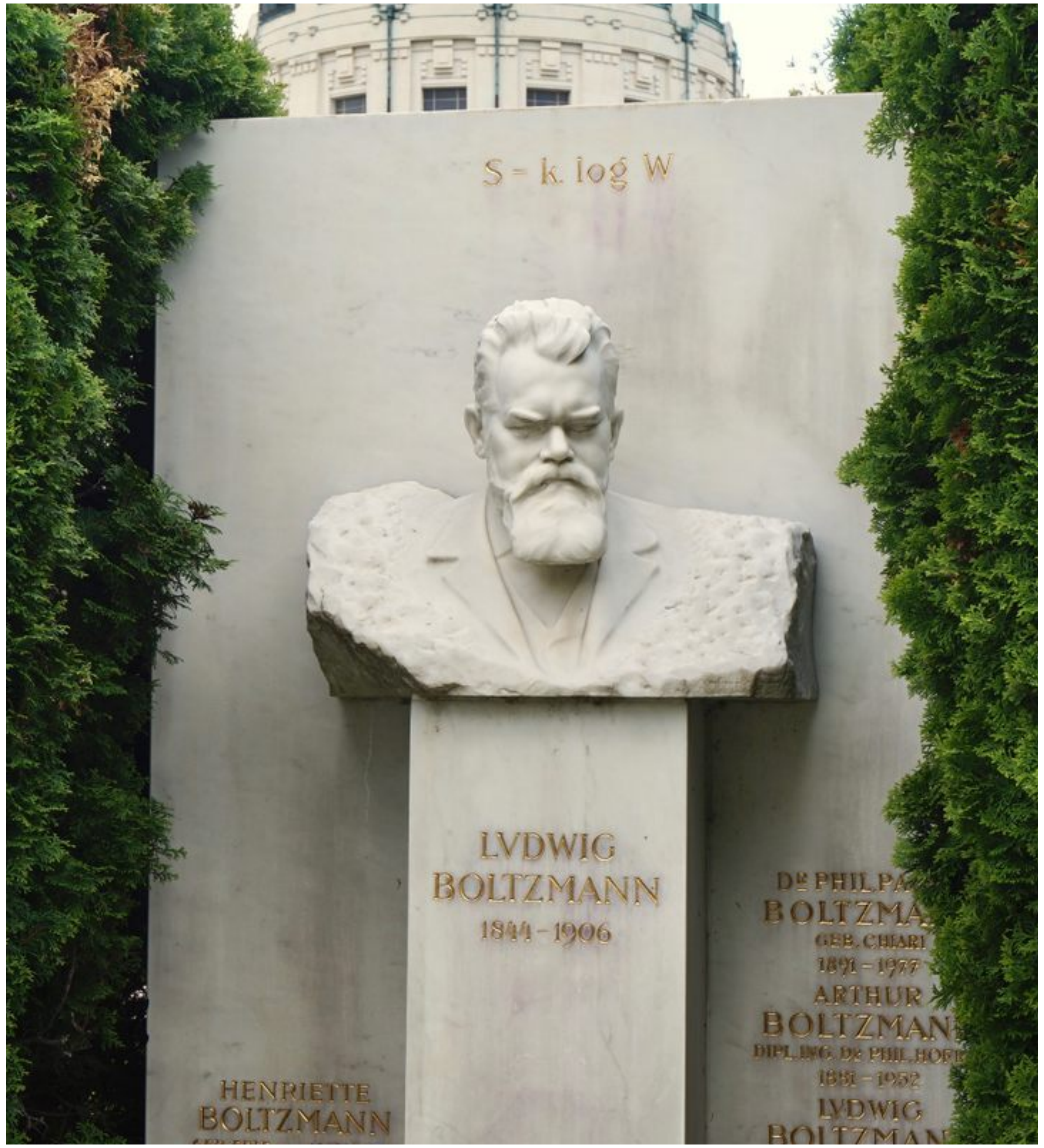
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



Quantum Boltzmann equation (Landau)
Dense gas of electrons



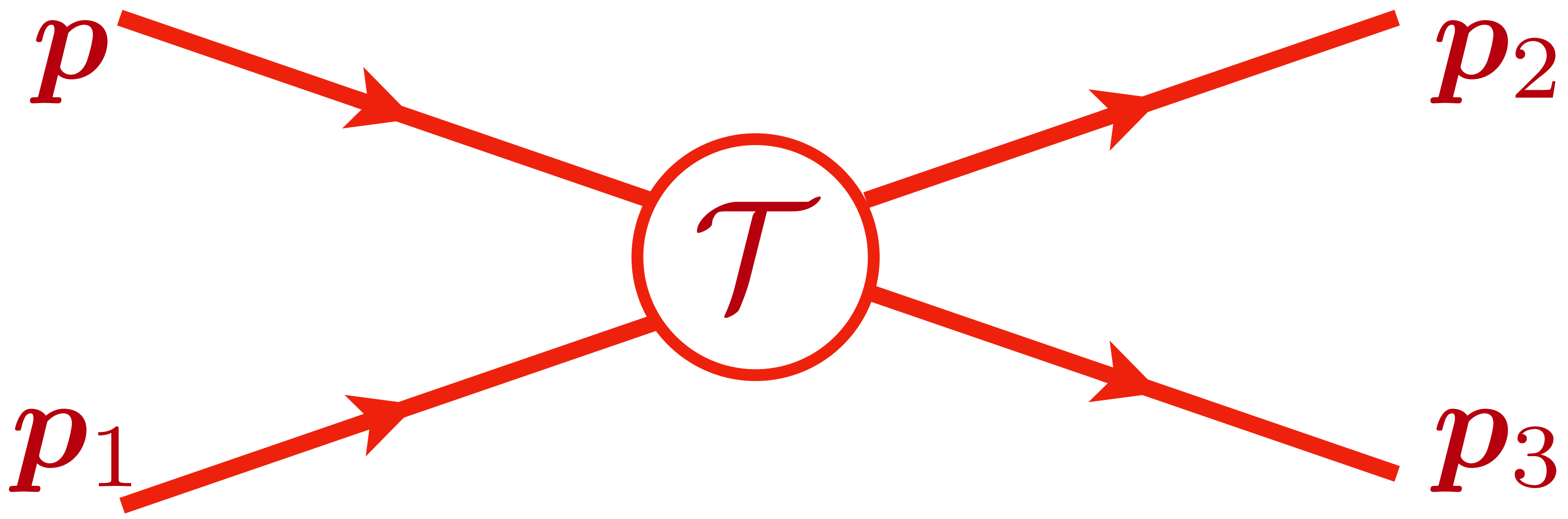
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

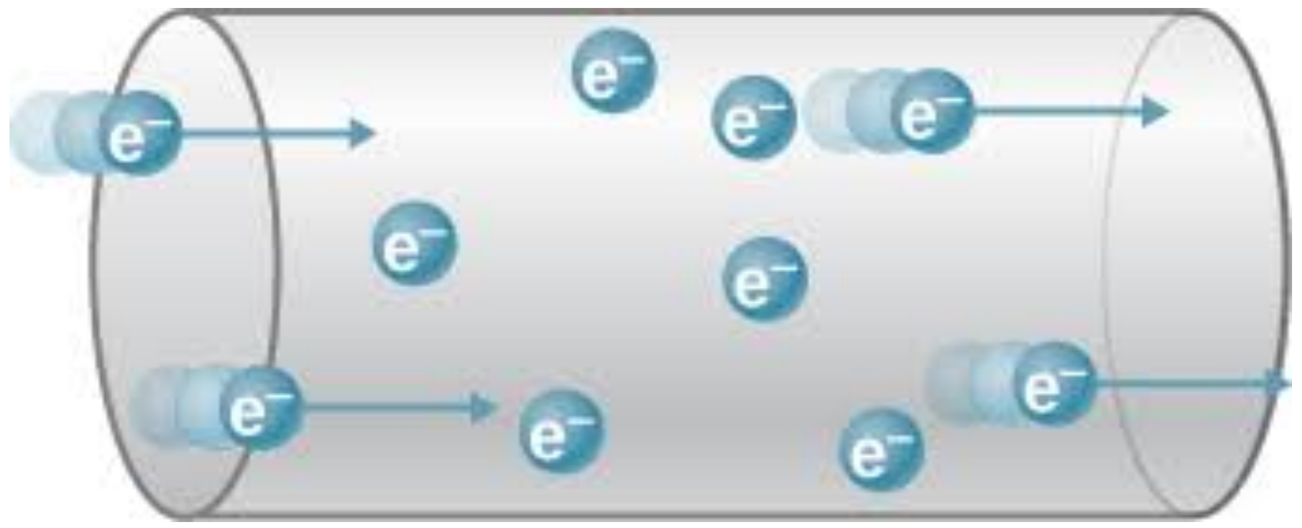
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in Copper



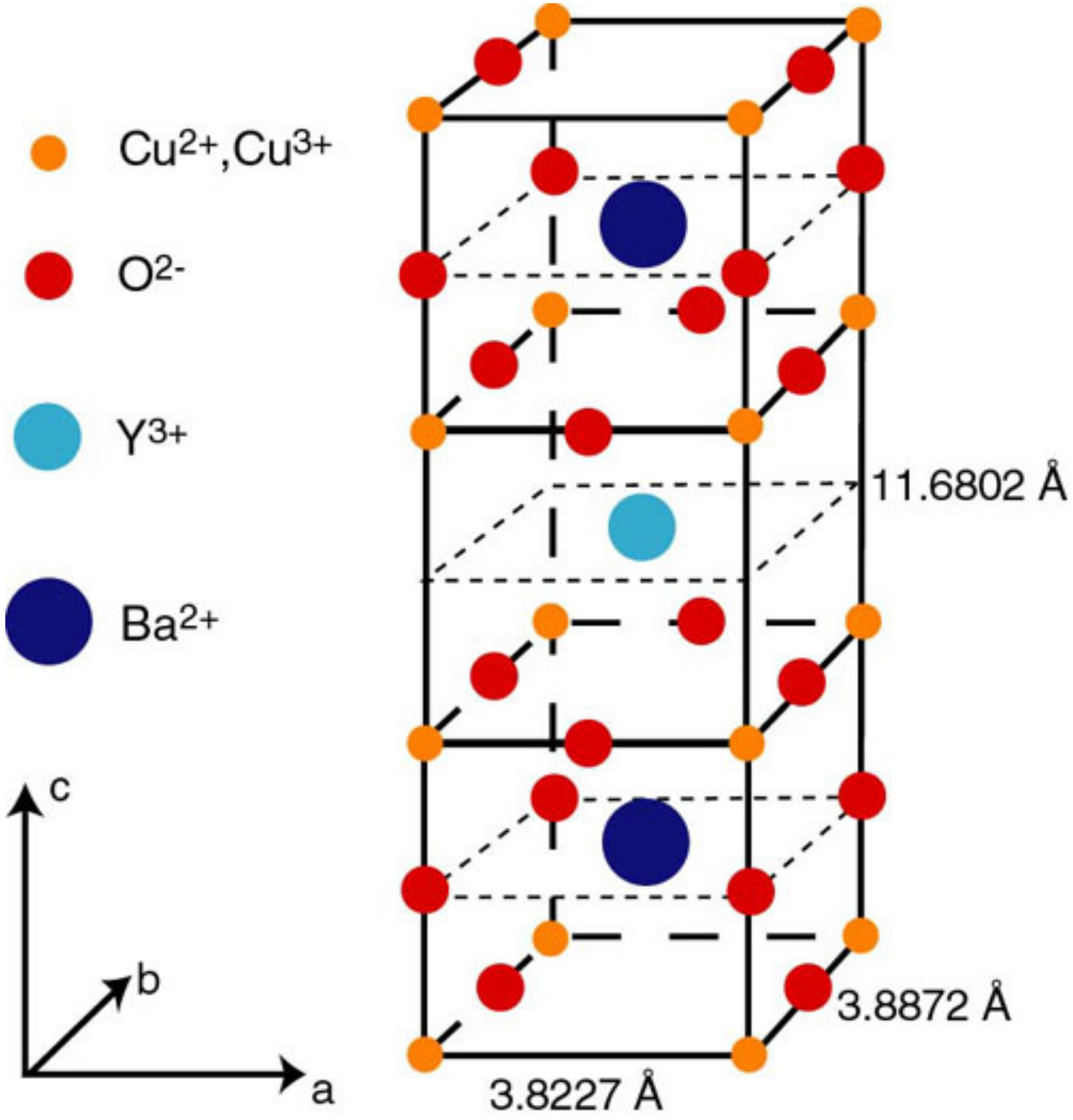
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

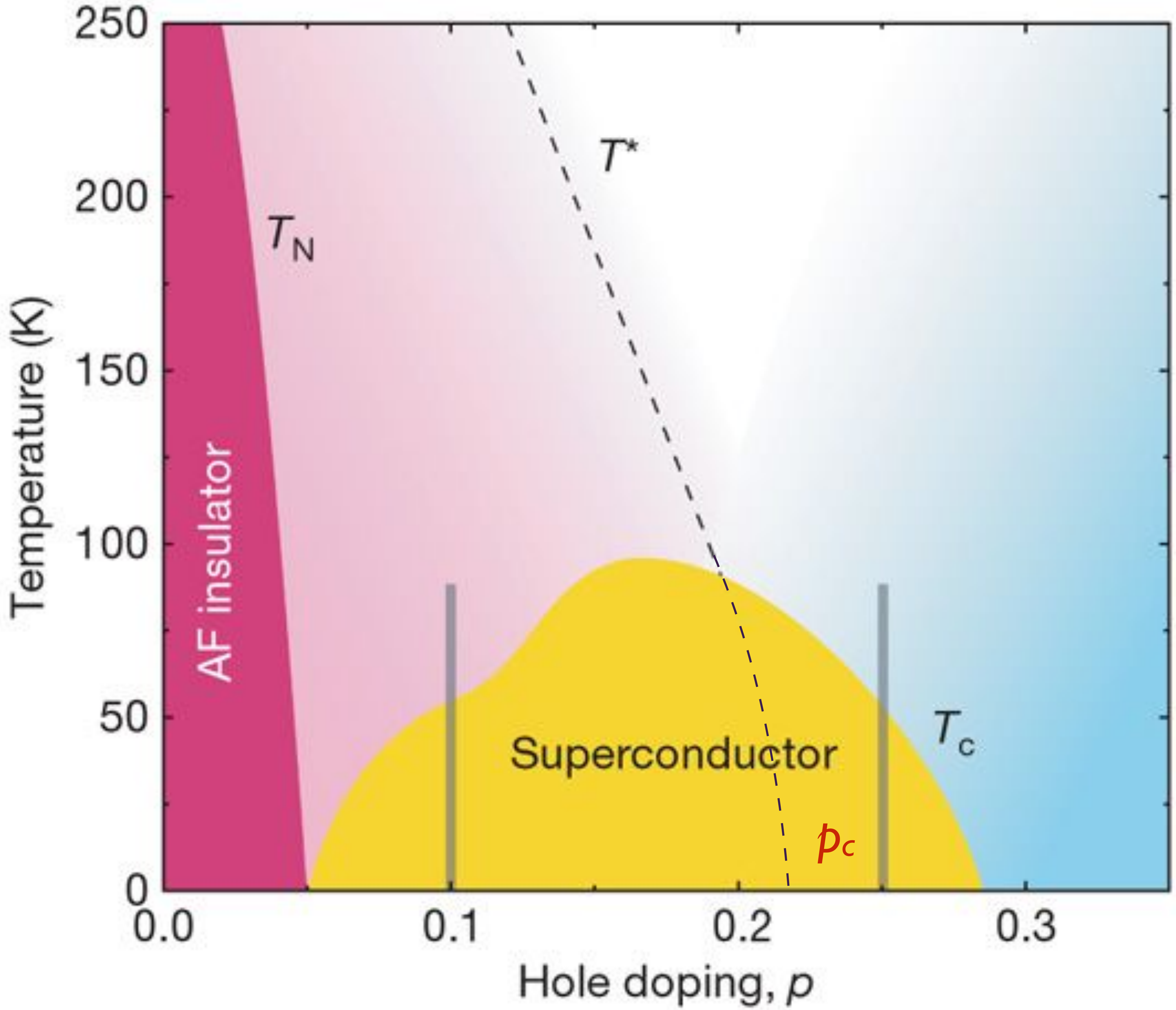
The long scattering time implies that individual electrons are well-defined.

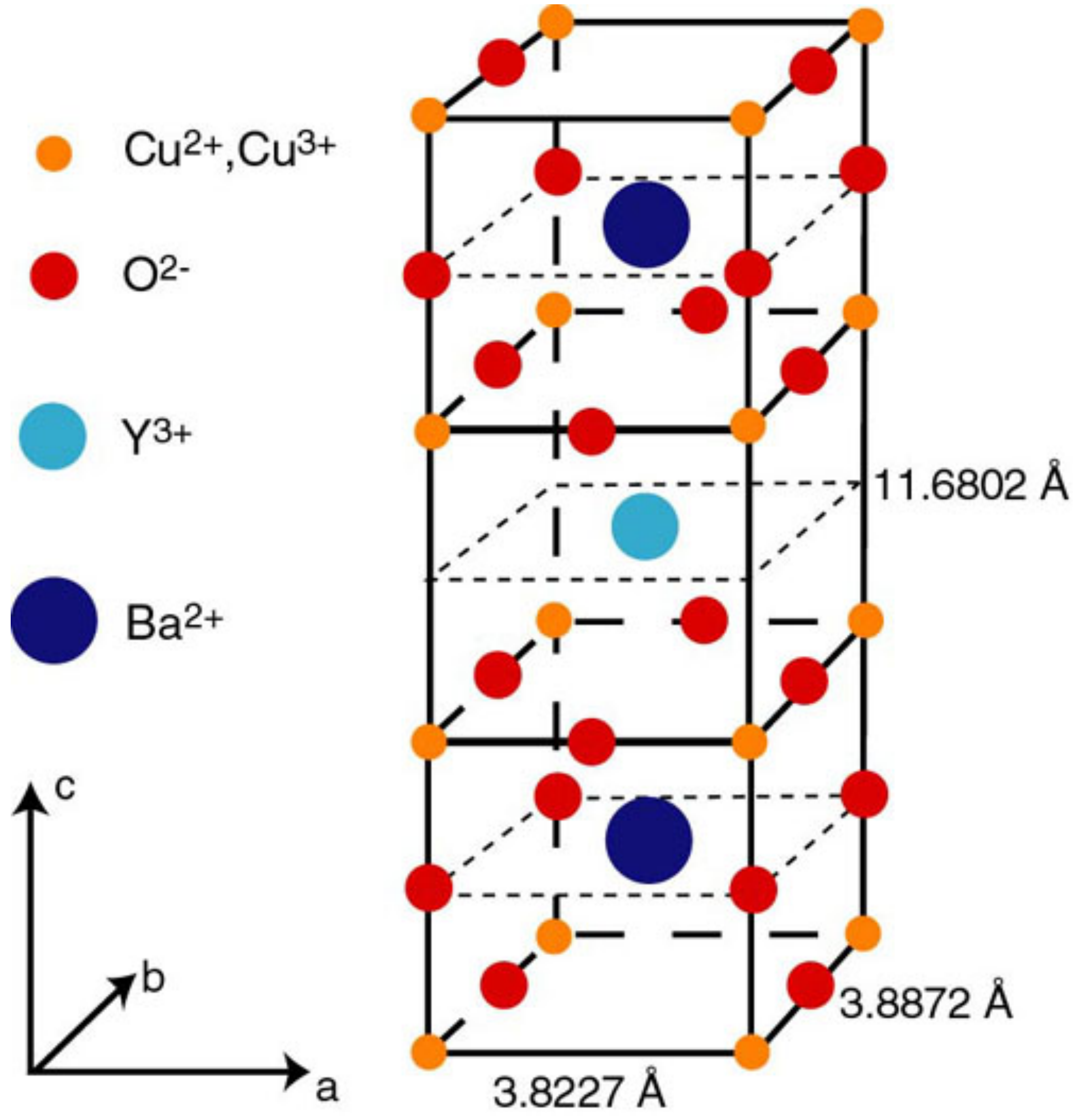
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

Cuprate high temperature superconductors

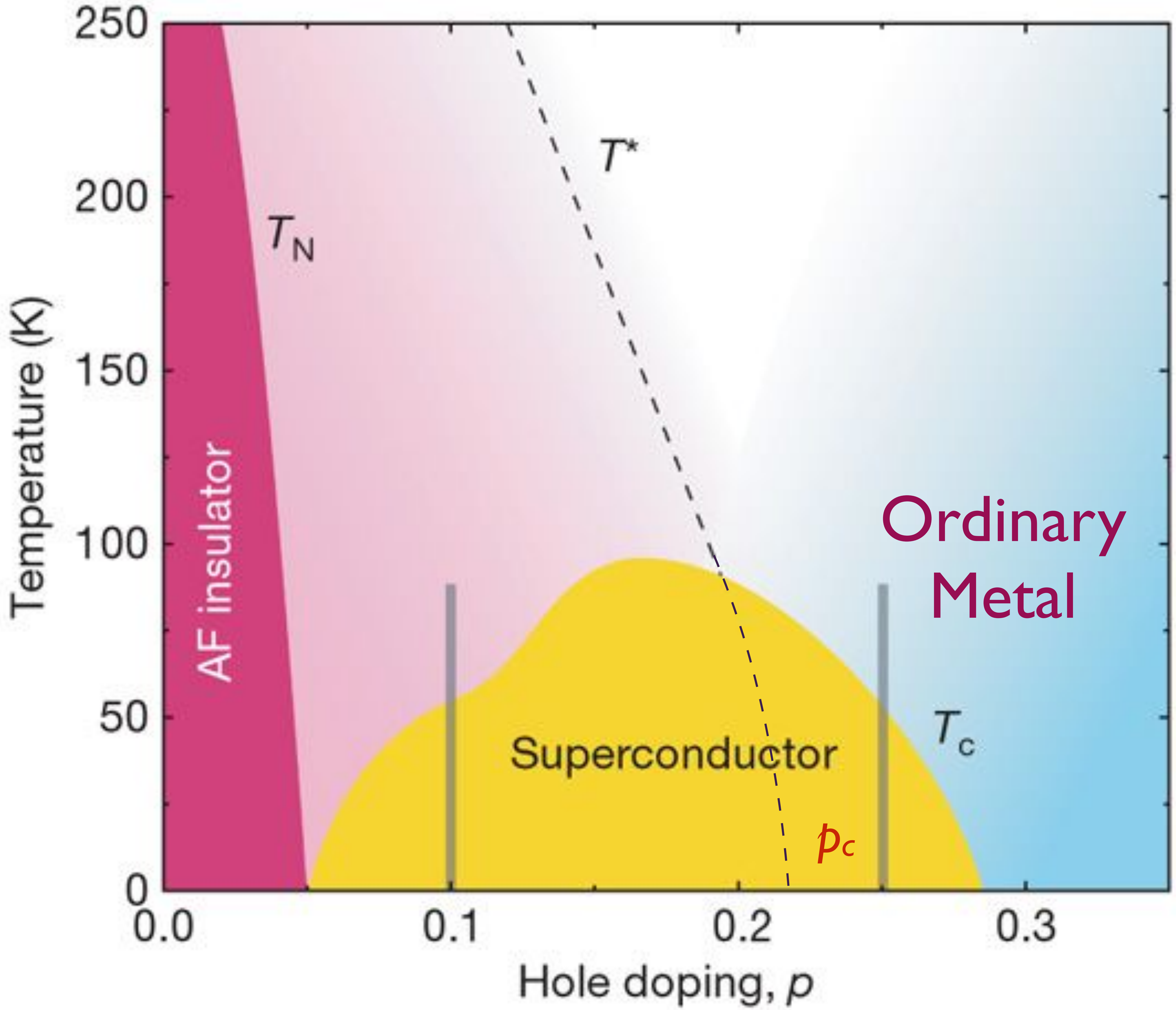


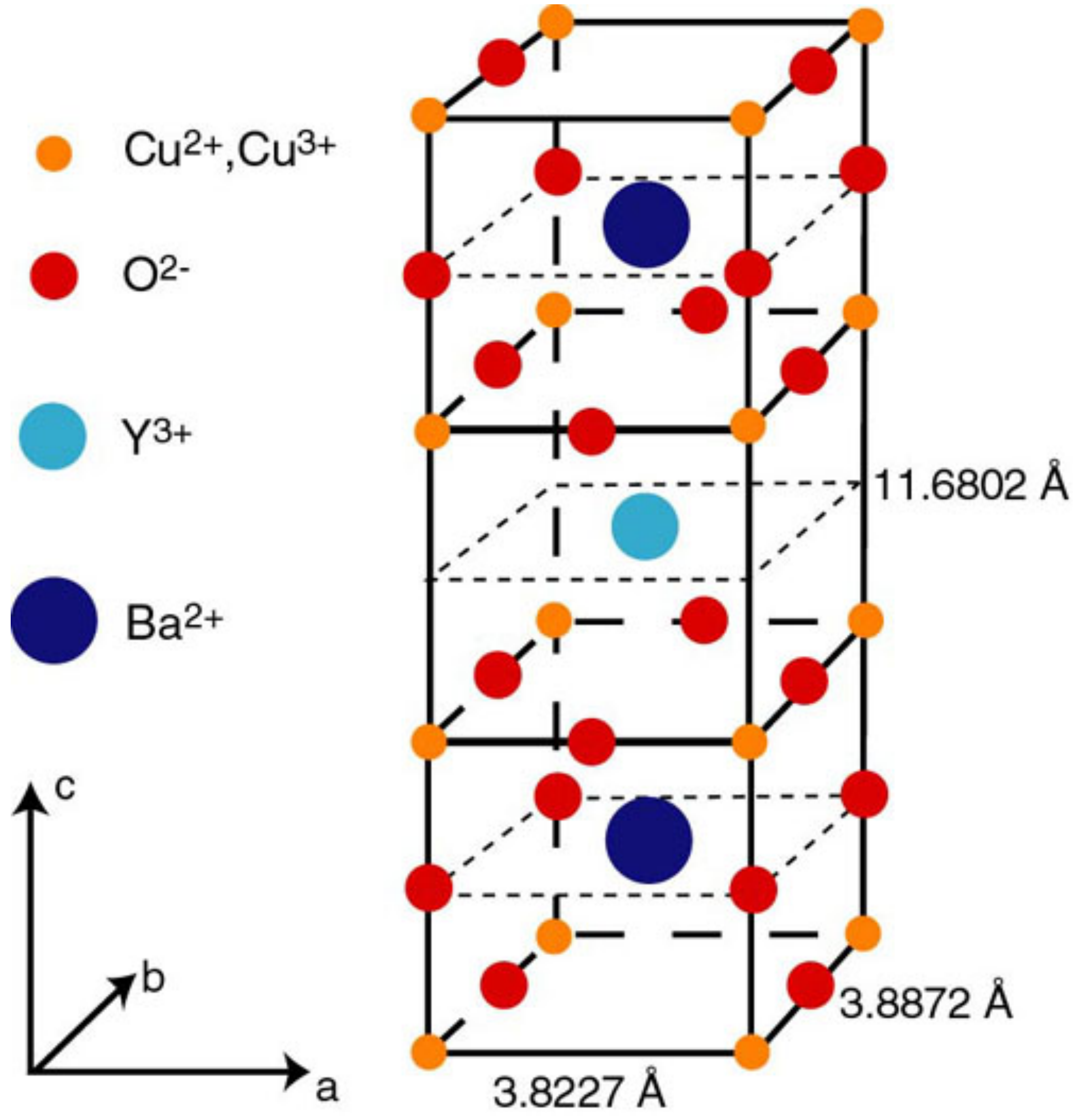
$$YBa_2Cu_3O_{6+x}$$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c

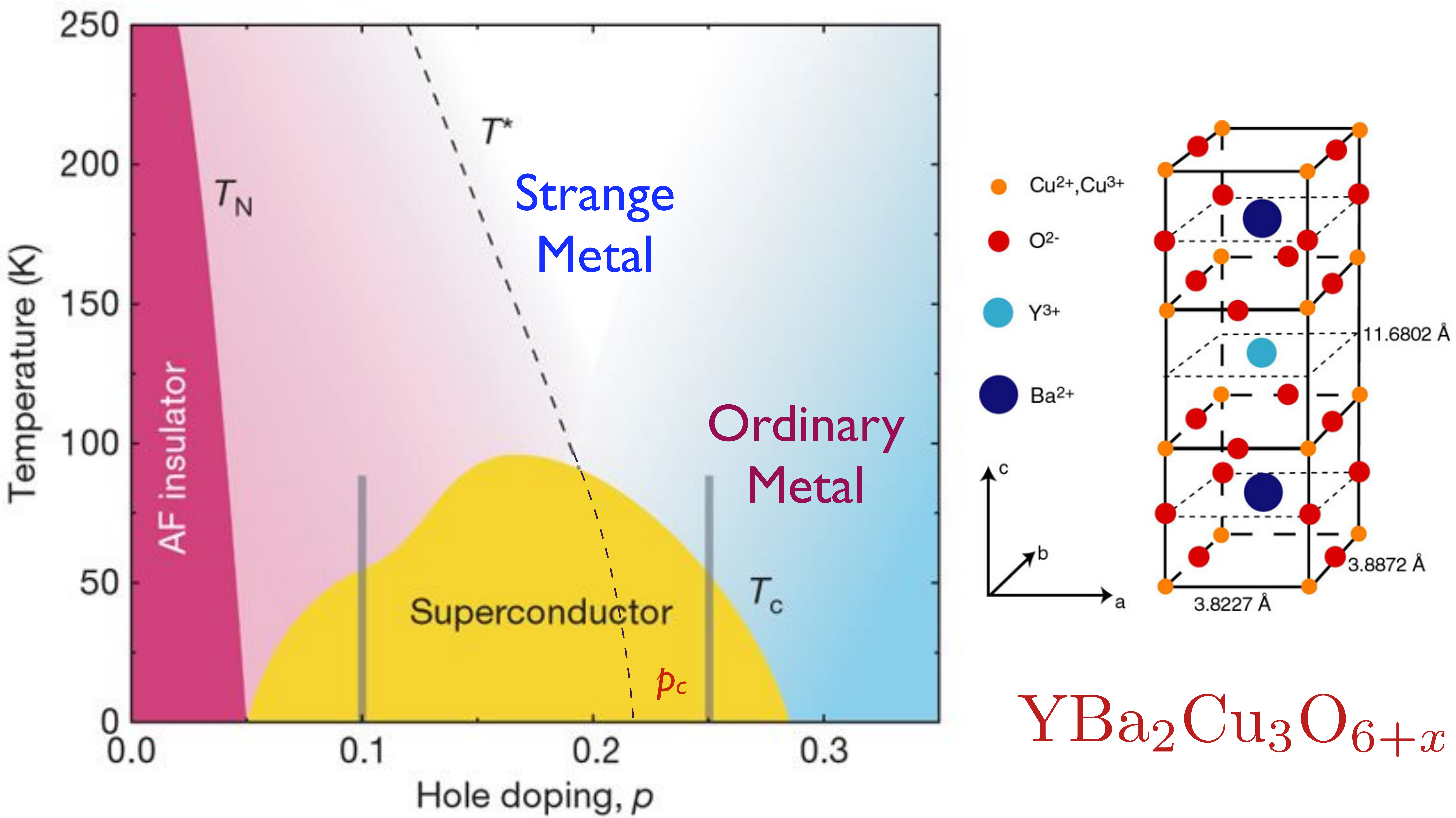

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$$YBa_2Cu_3O_{6+x}$$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Ordinary
Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c
Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
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c
b
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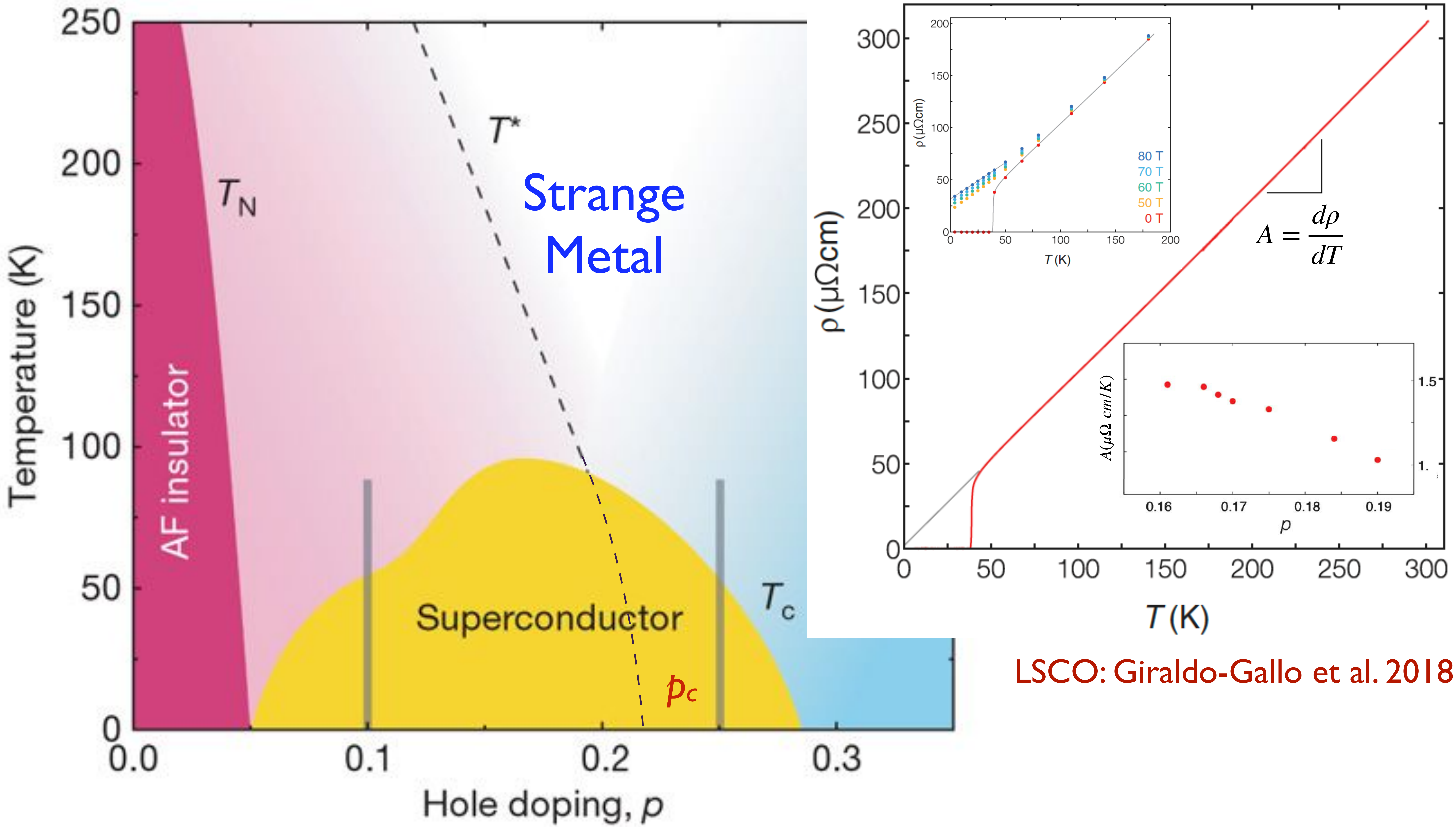
$YBa_2Cu_3O_{6+x}$

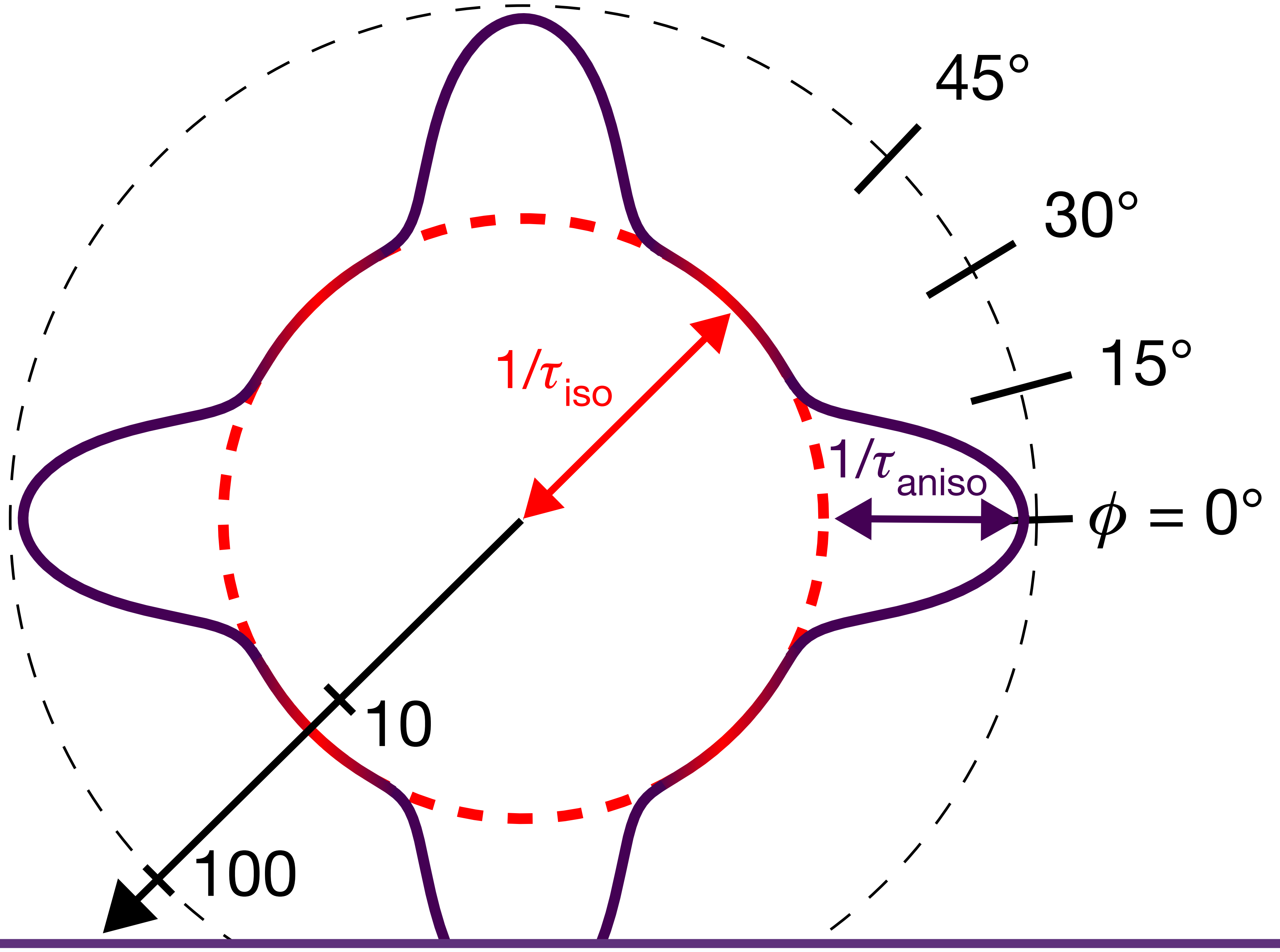


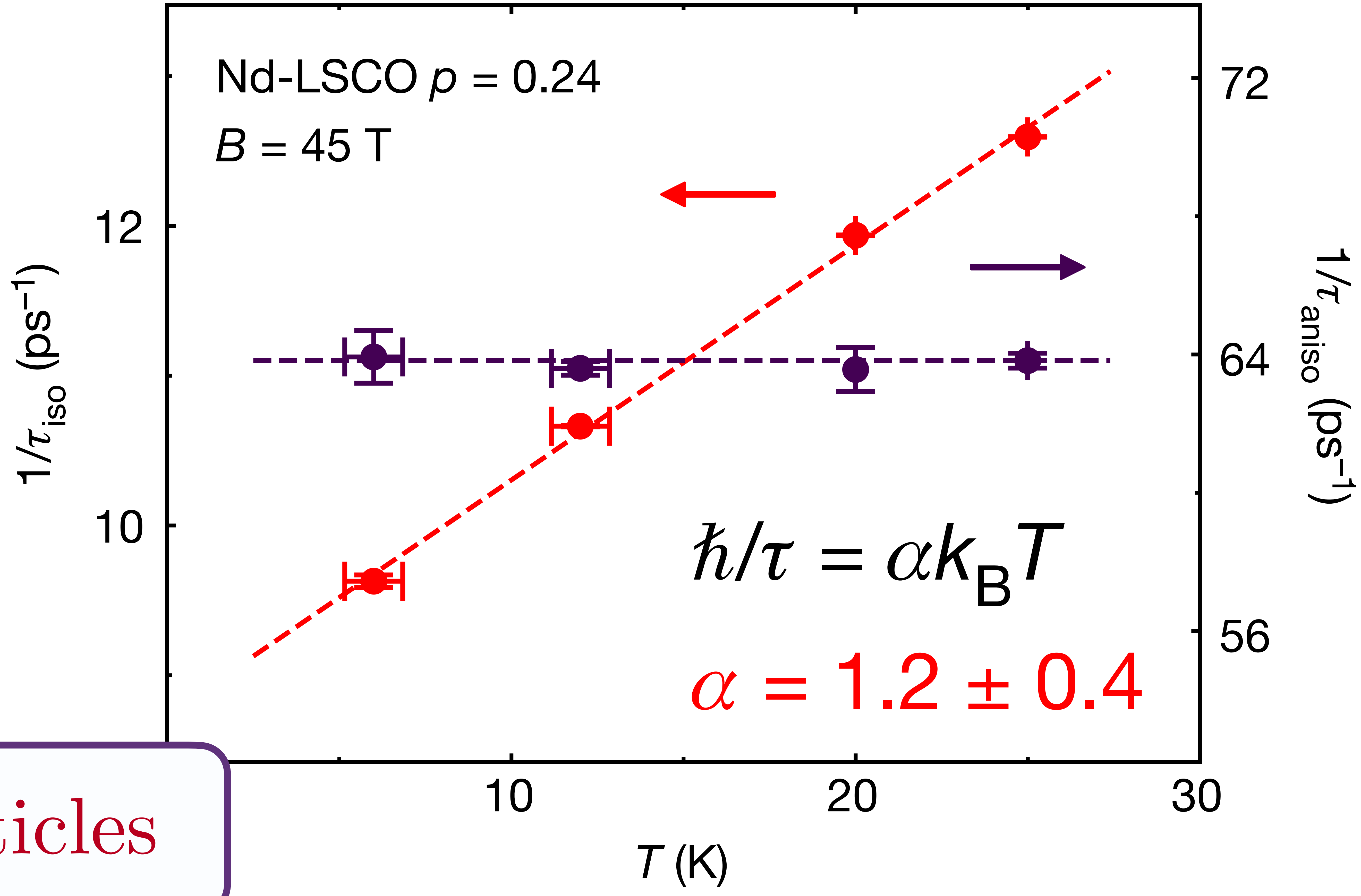
LSCO: Giraldo-Gallo et al. 2018

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description $\Rightarrow$
Many particle quantum entanglement
from quantum interference between "collisions"

Sachdev-Ye-Kitaev Model

The SYK model

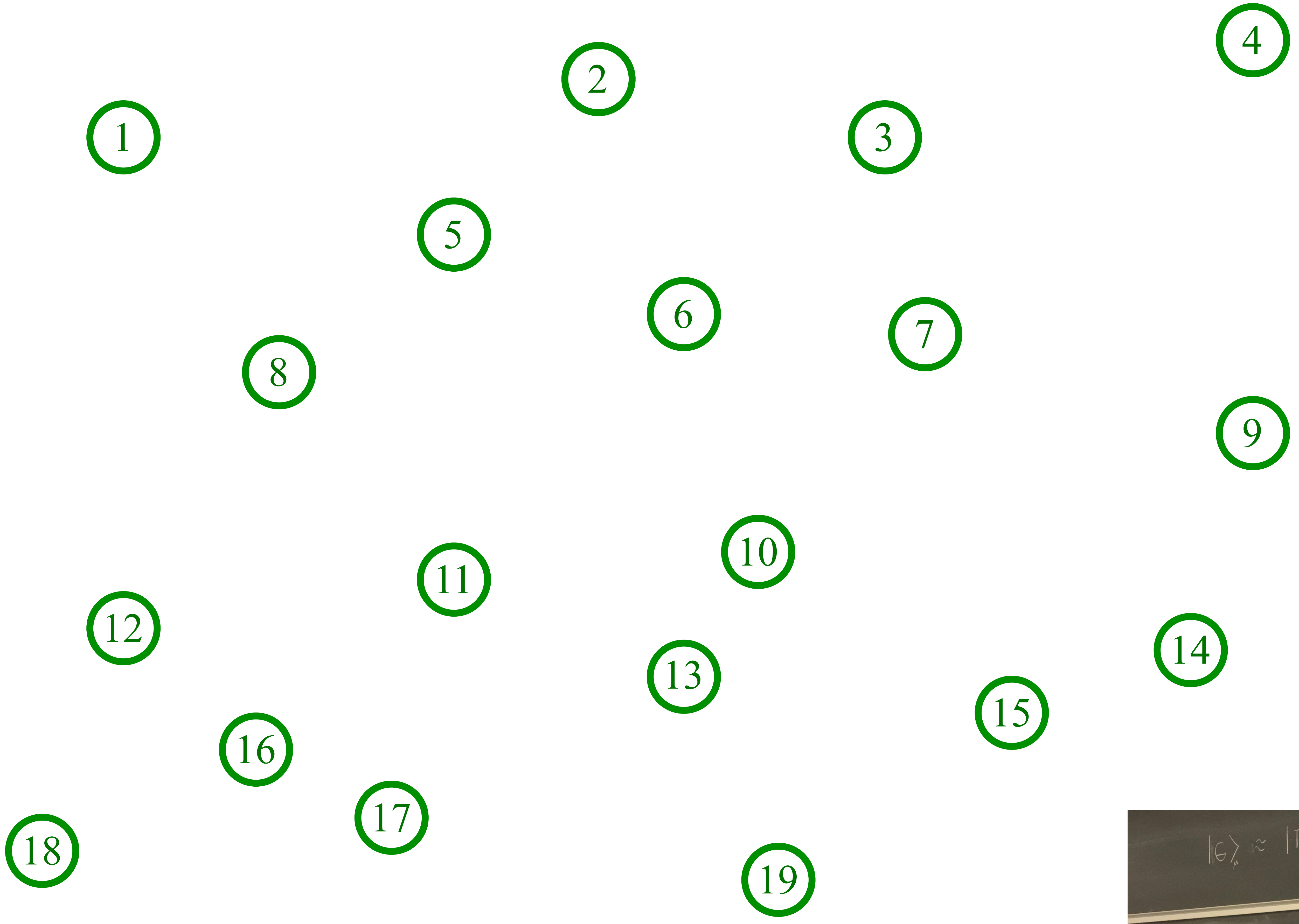
Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

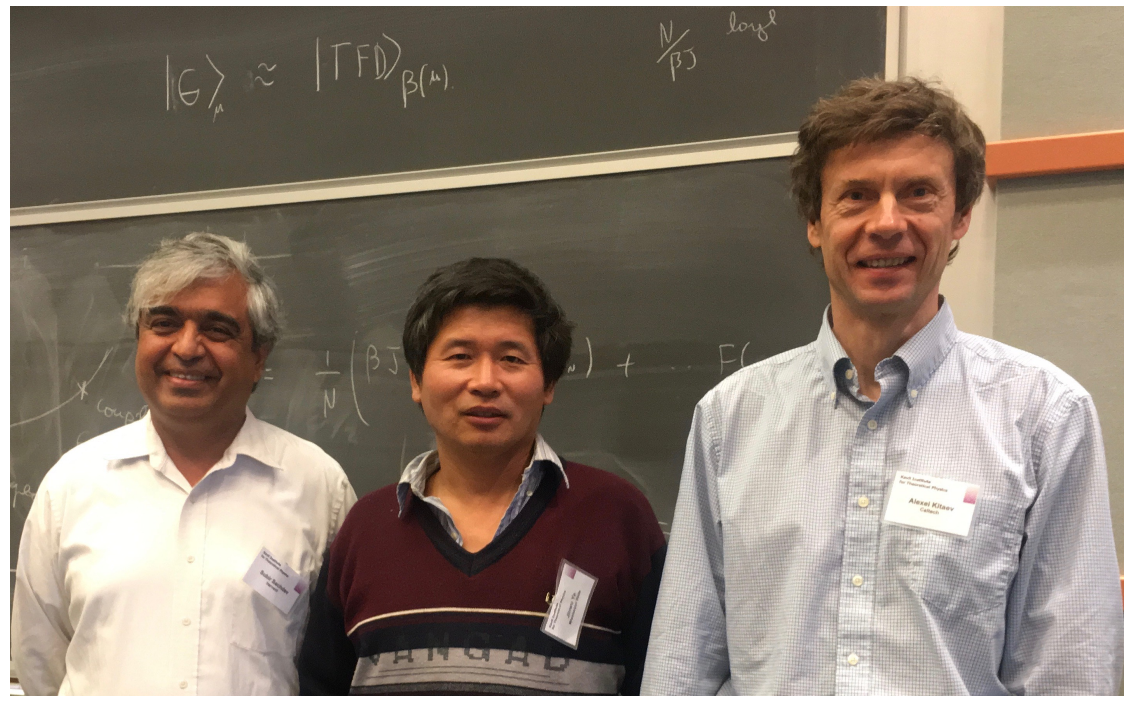
Yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

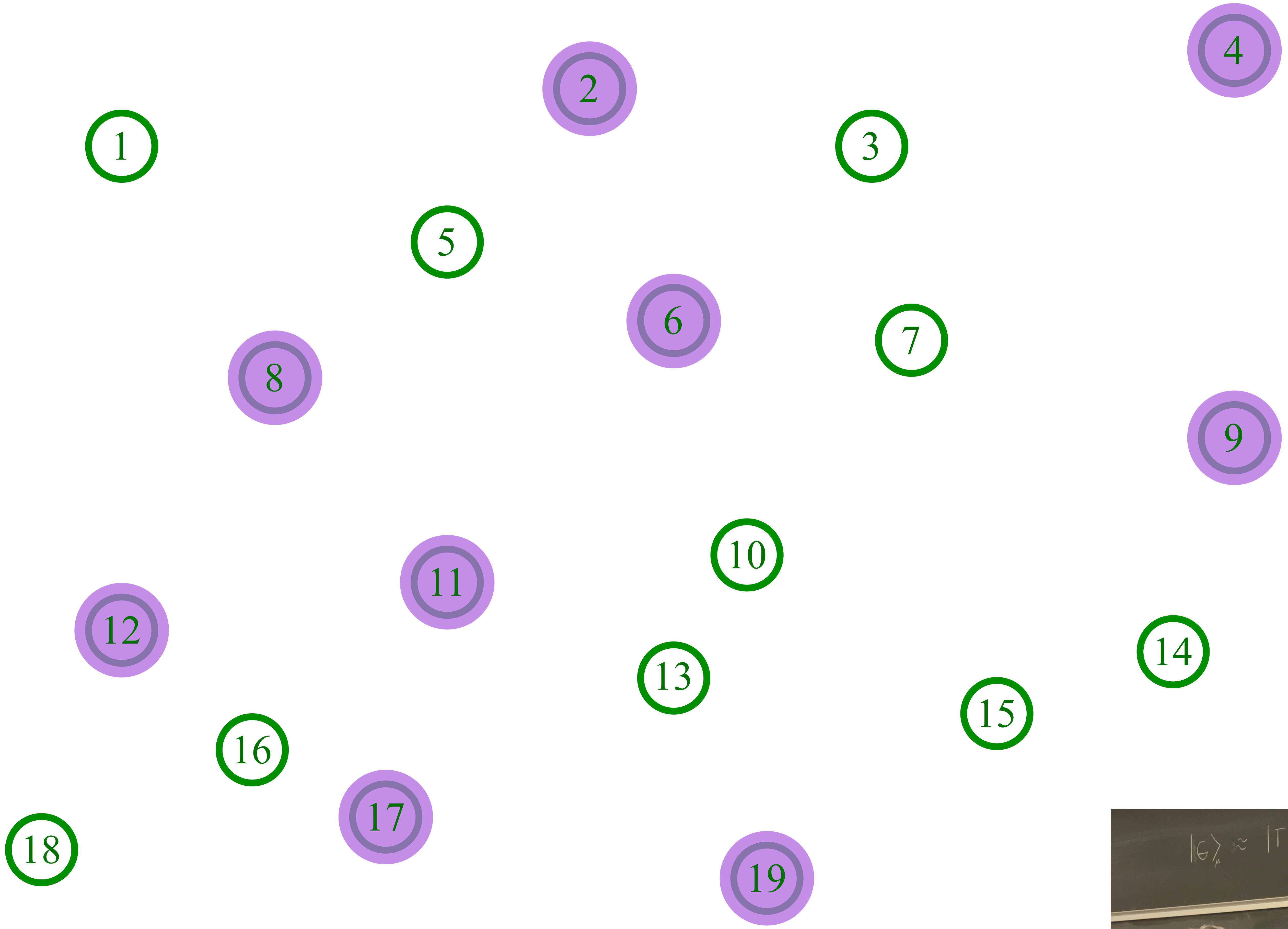


Pick a set of random positions

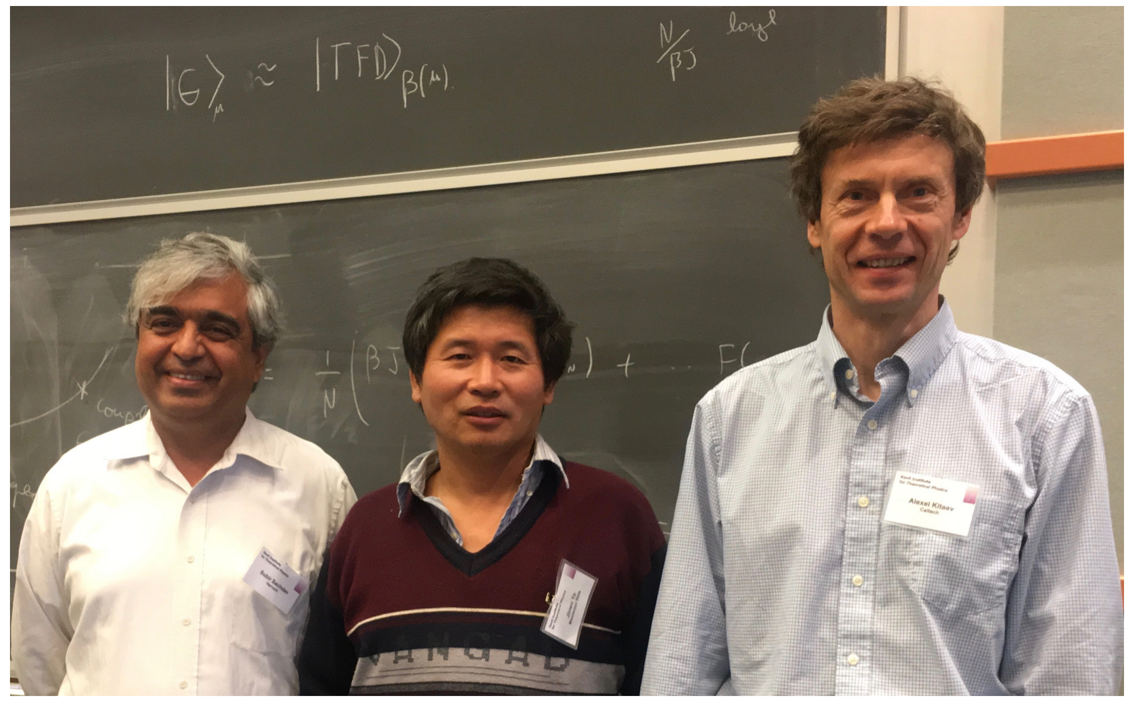


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



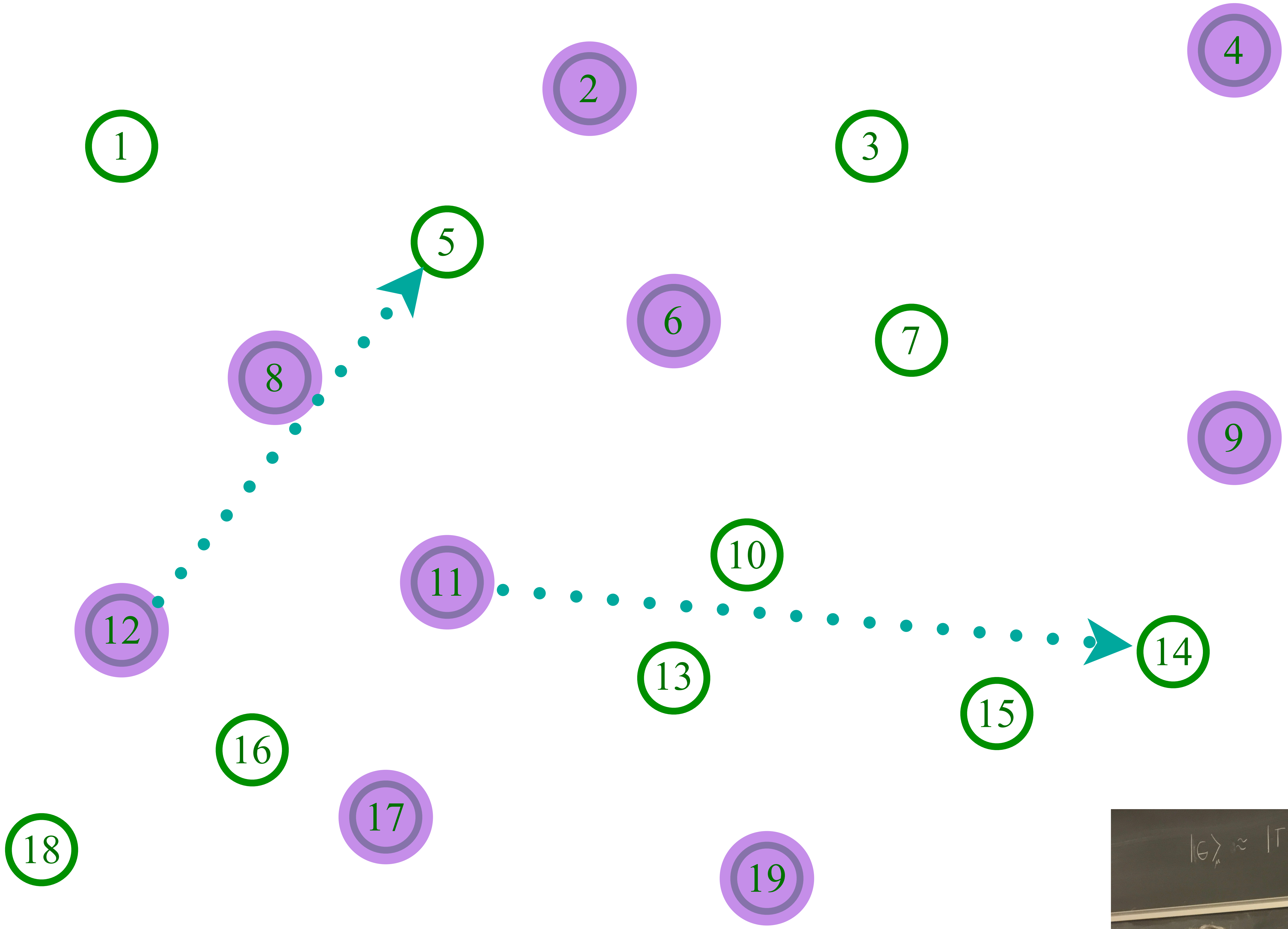
Place electrons randomly on some sites



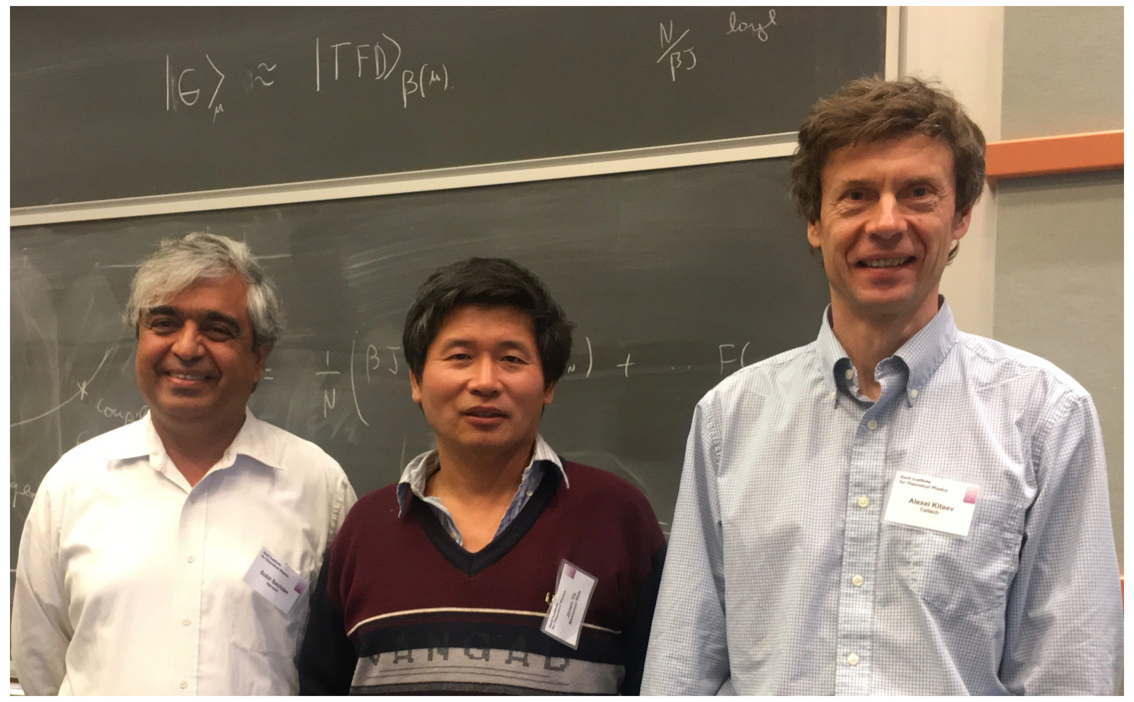
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



Place electrons randomly on some sites



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Sachdev, Ye (1993); Kitaev (2015)

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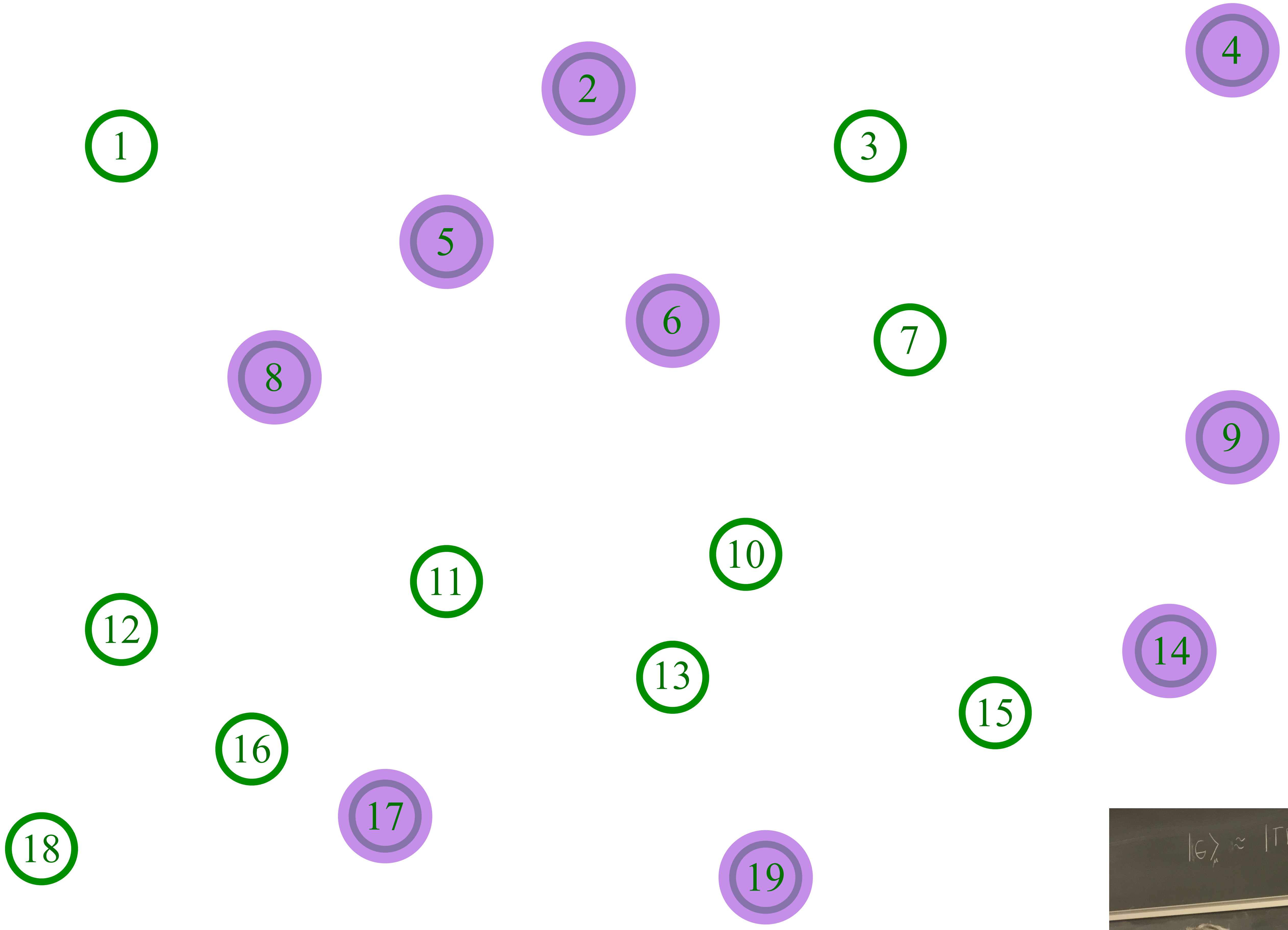
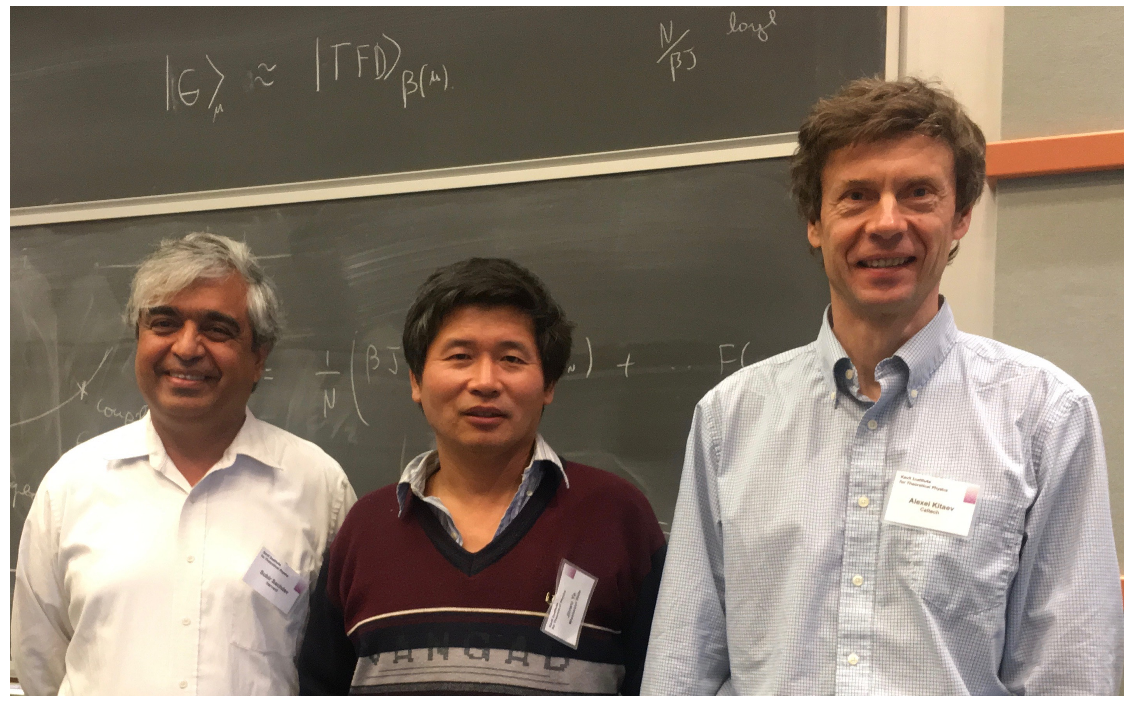


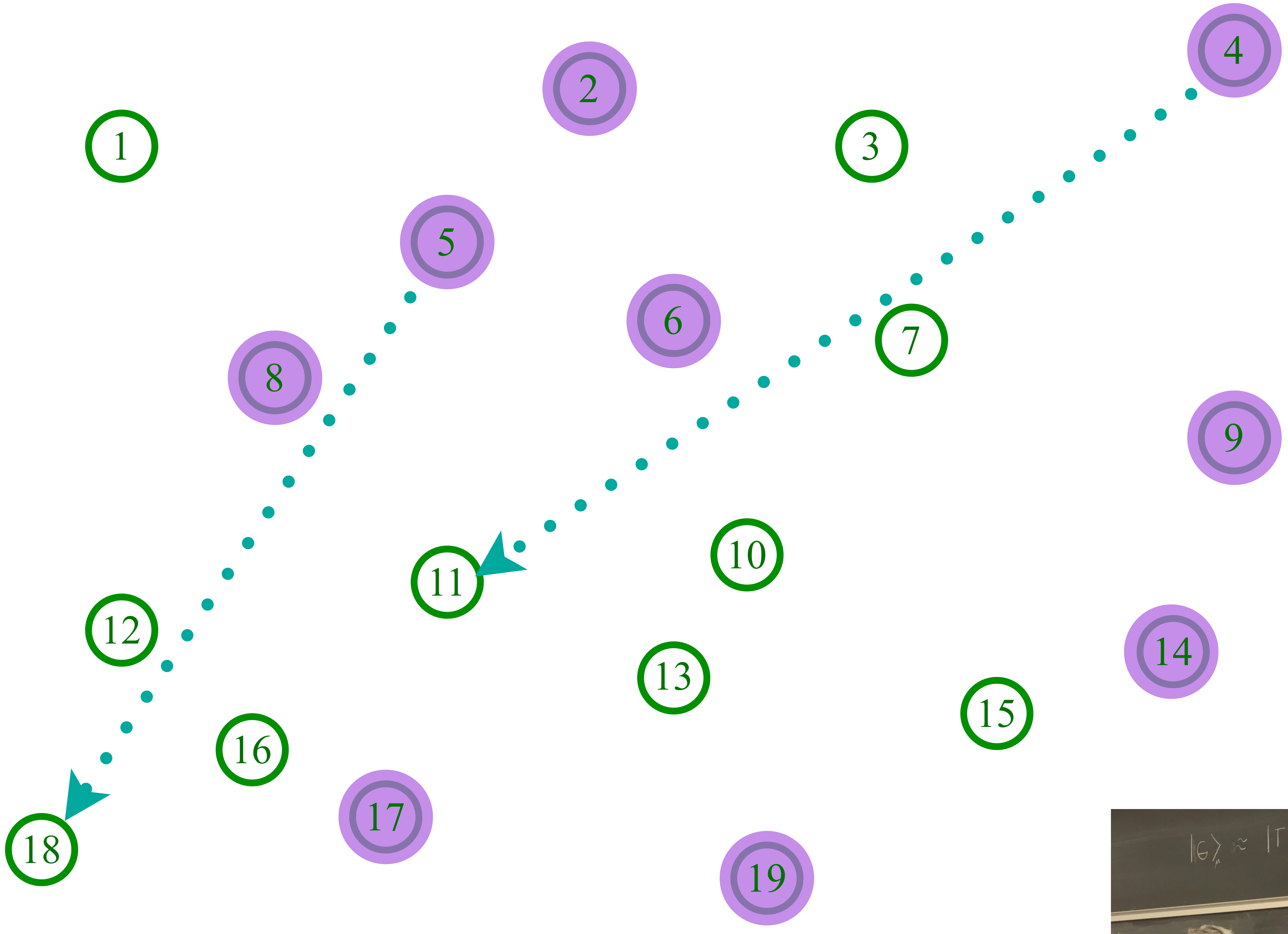
Entangle electrons pairwise randomly



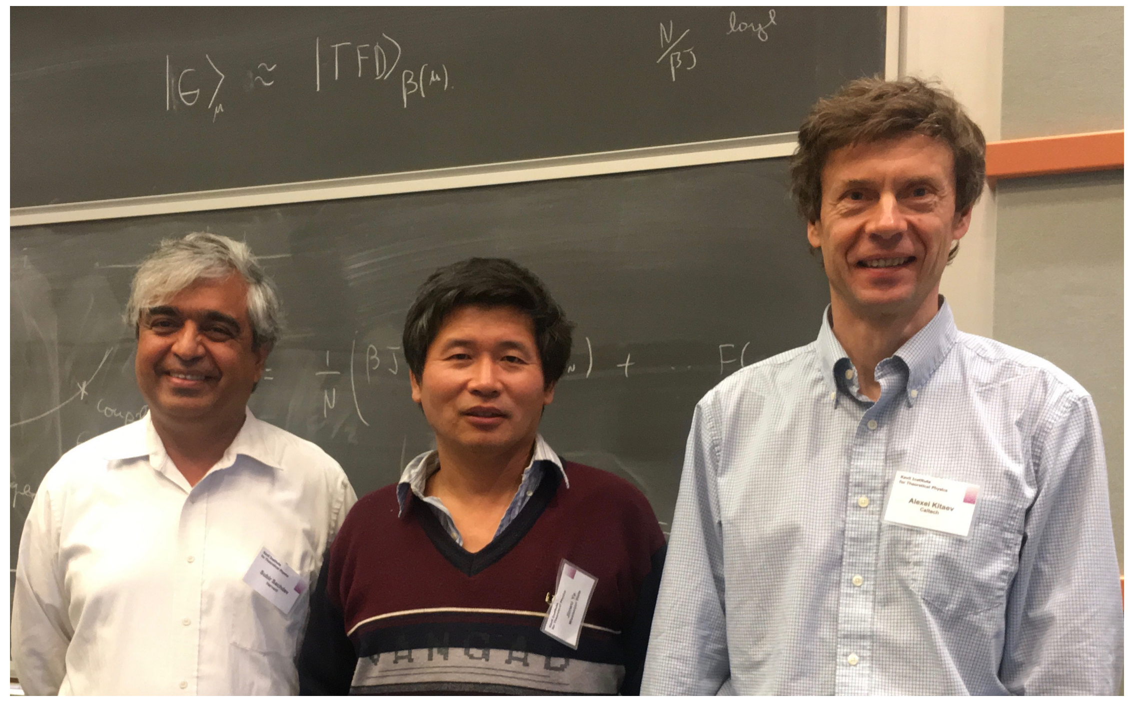
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



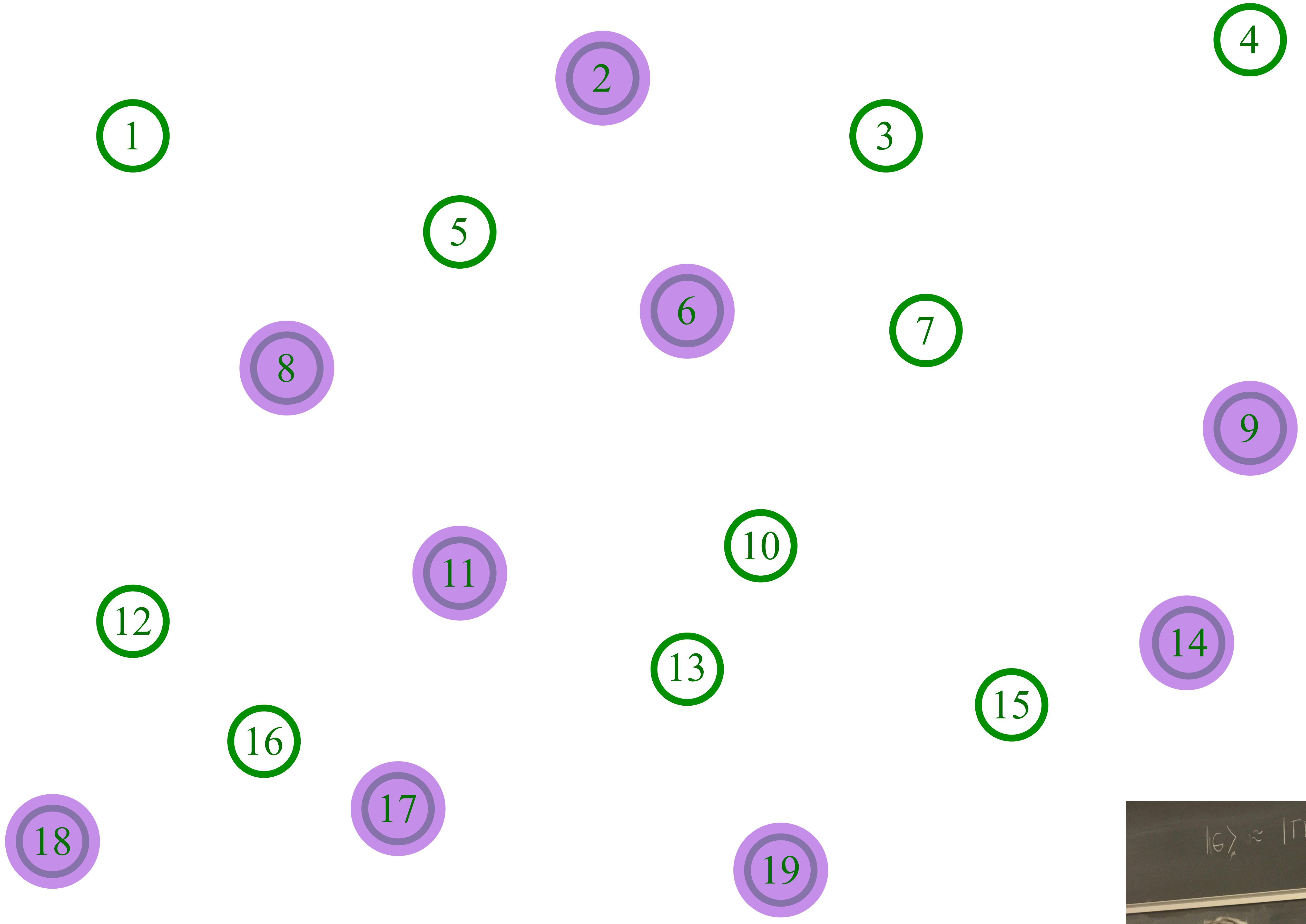
Entangle electrons pairwise randomly



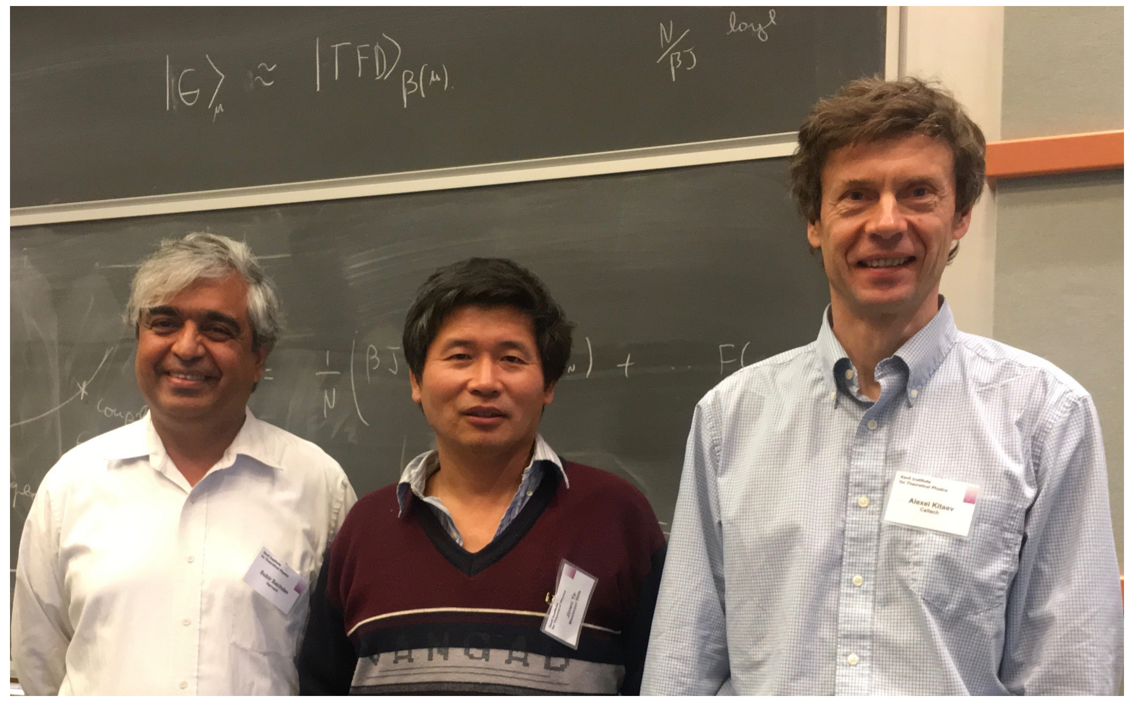
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



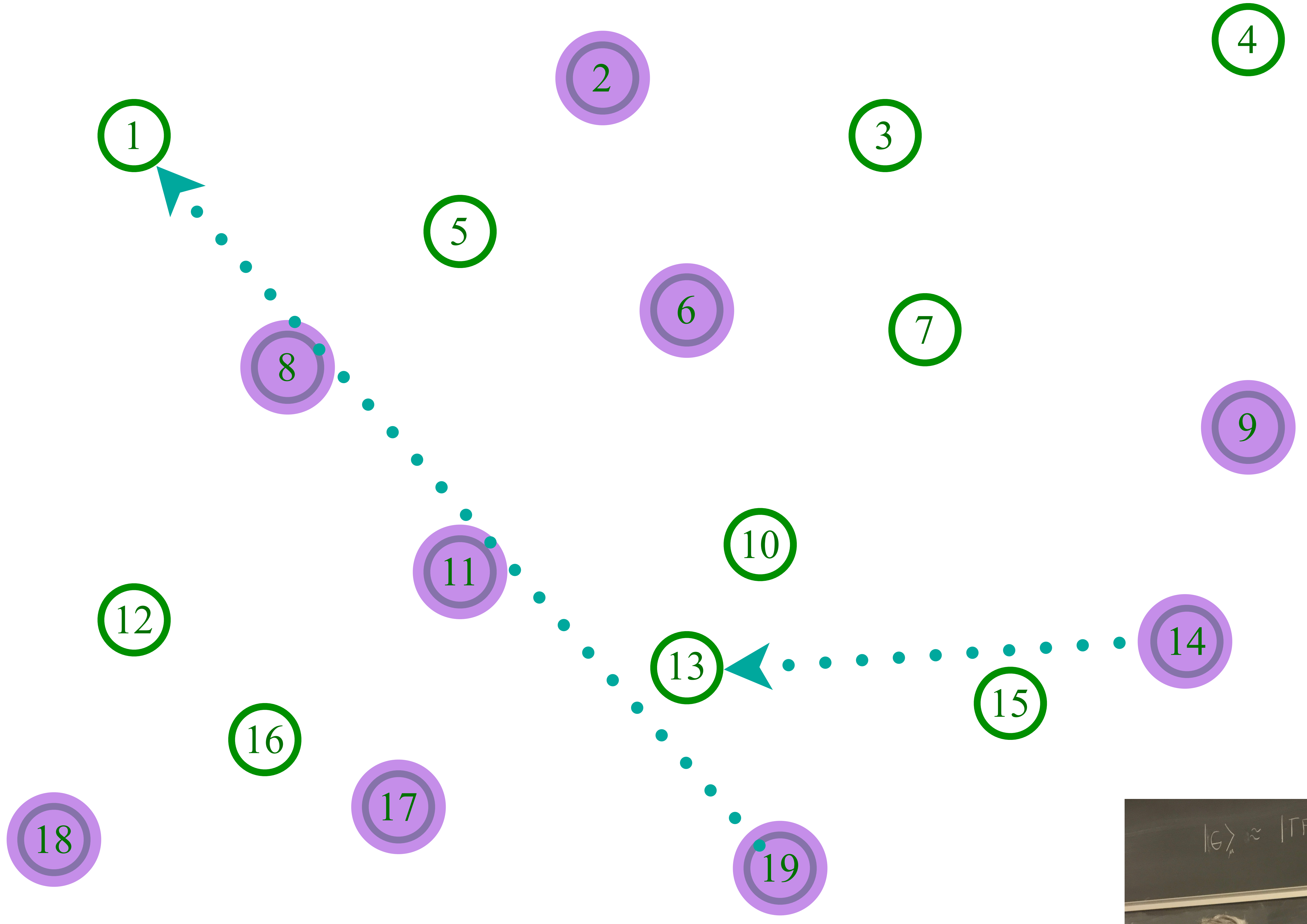
Entangle electrons pairwise randomly

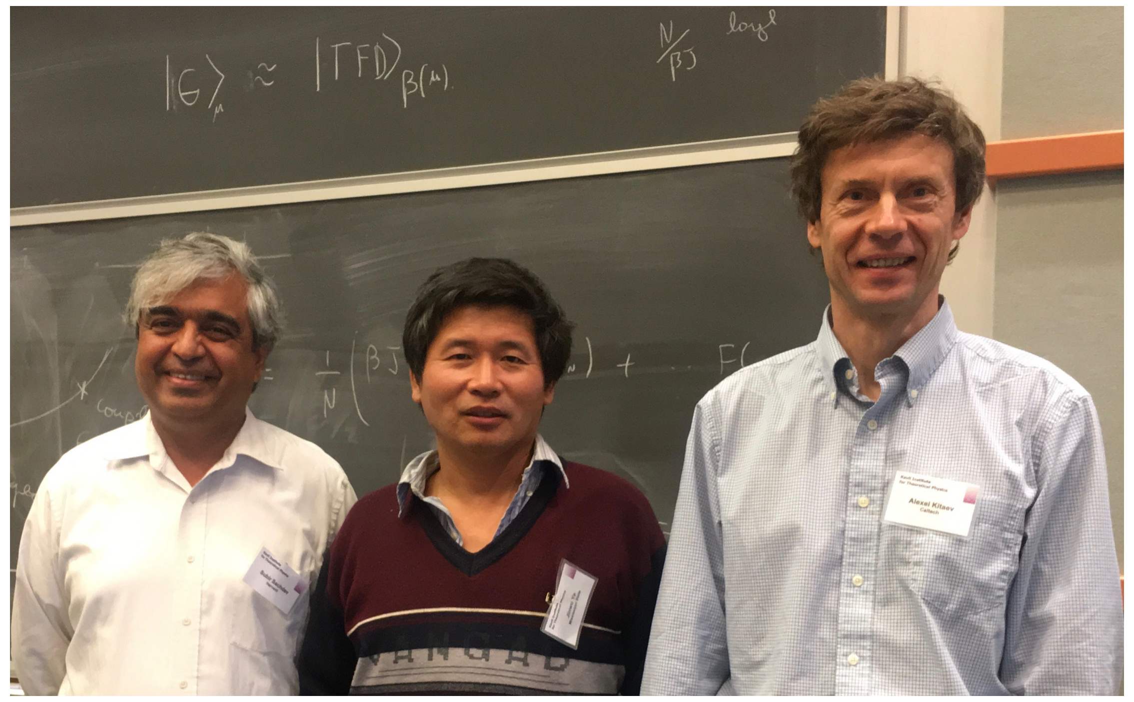


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



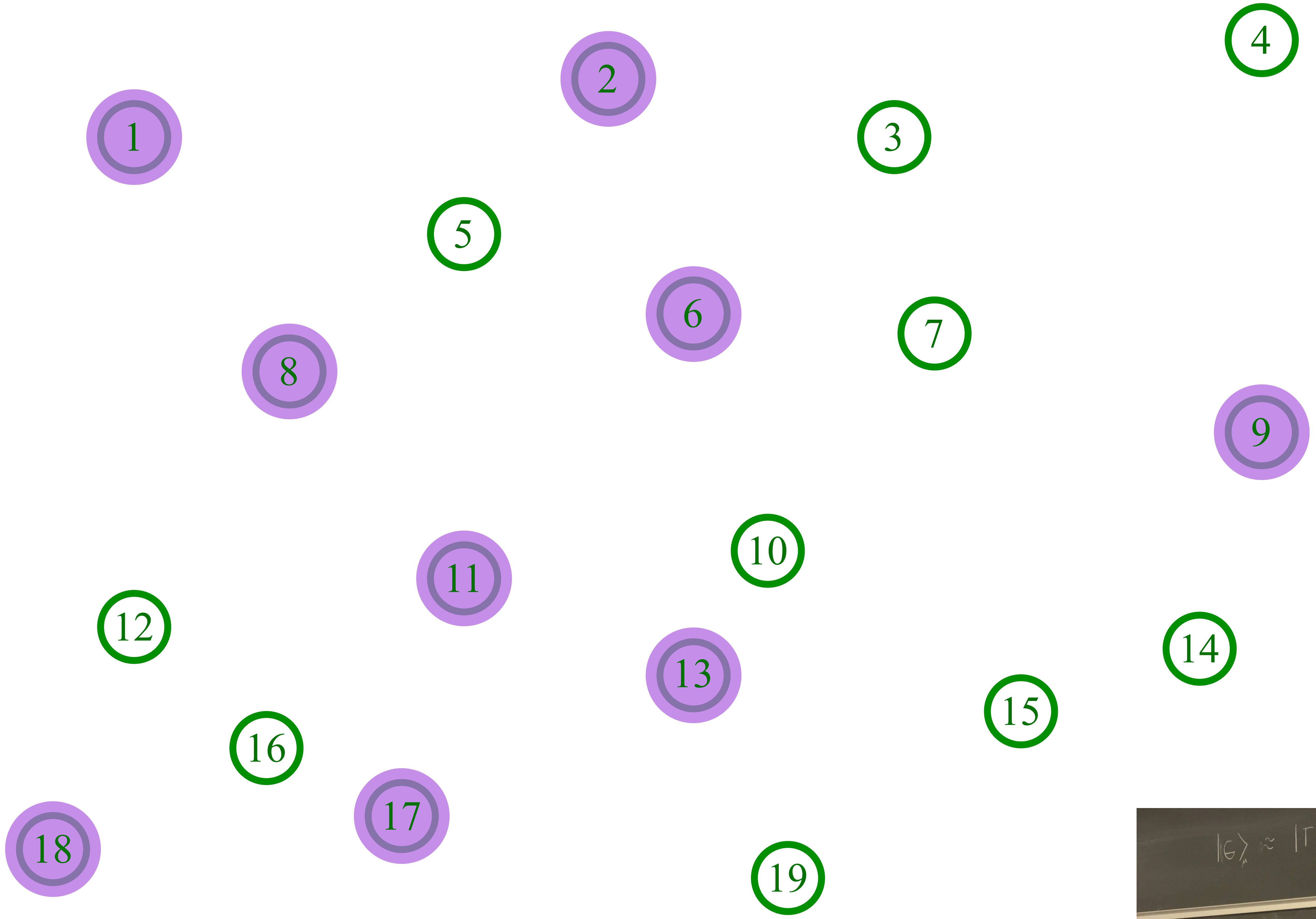


Entangle electrons pairwise randomly

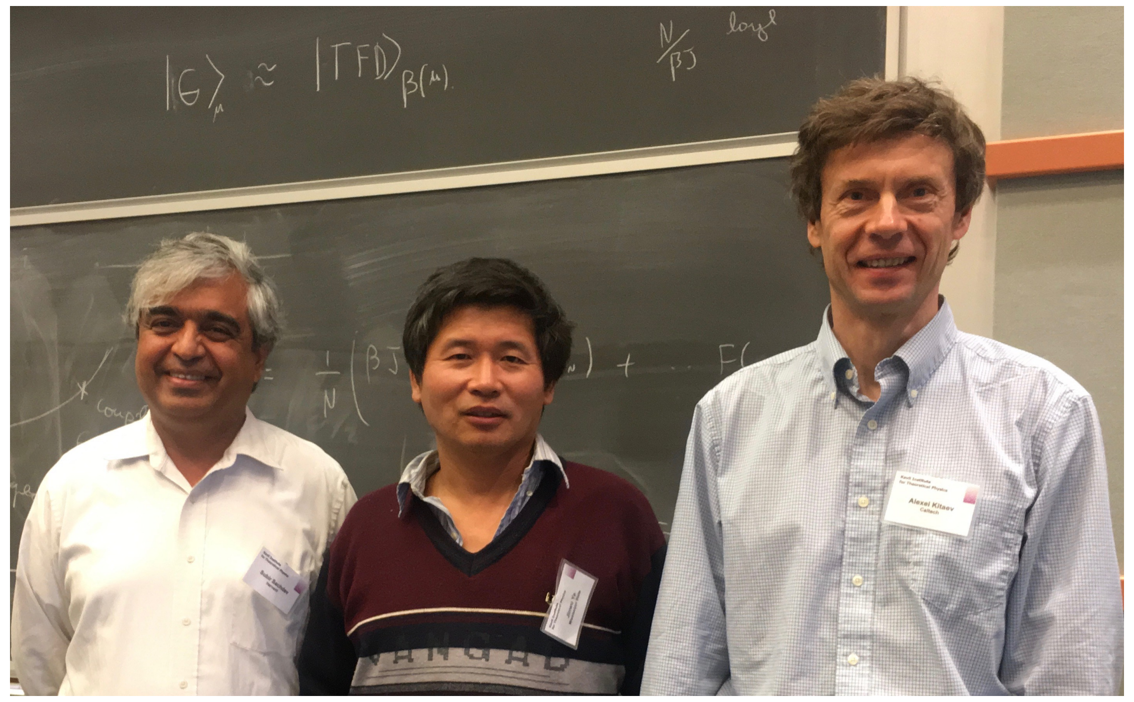
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



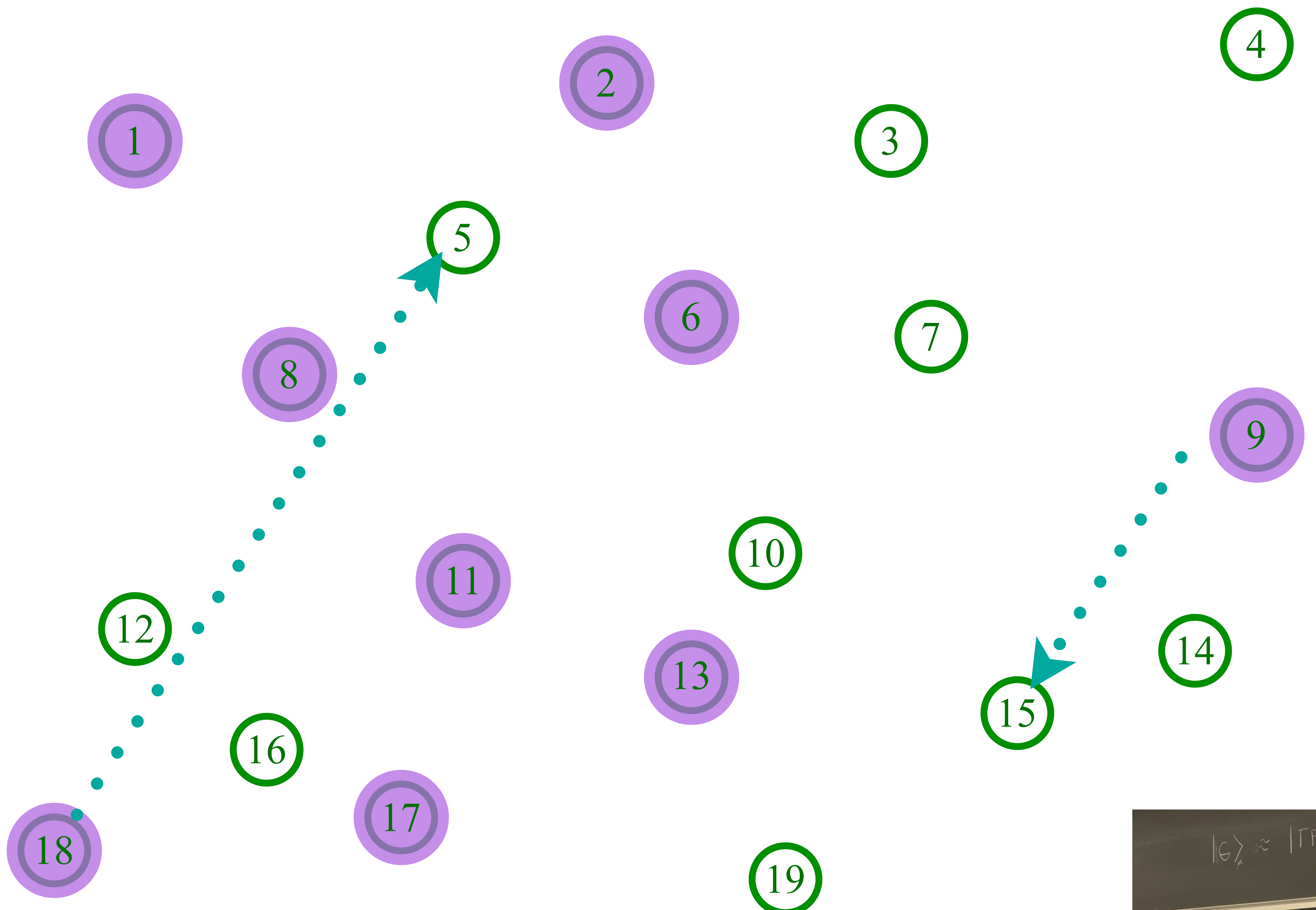
Entangle electrons pairwise randomly



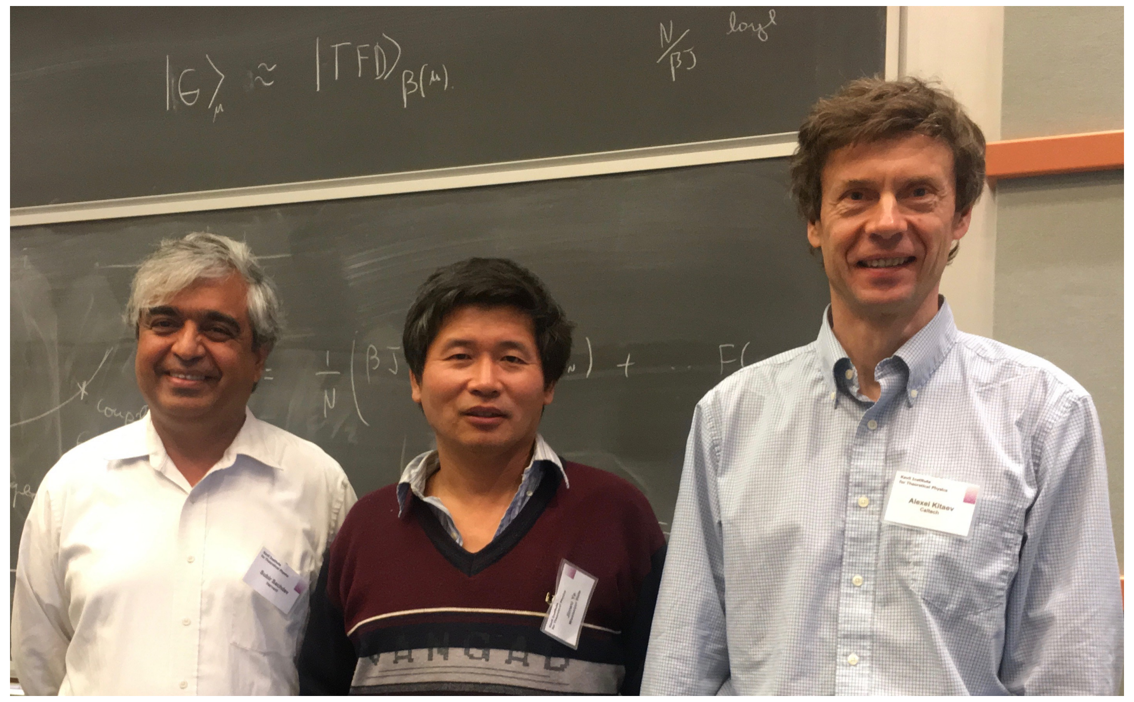
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



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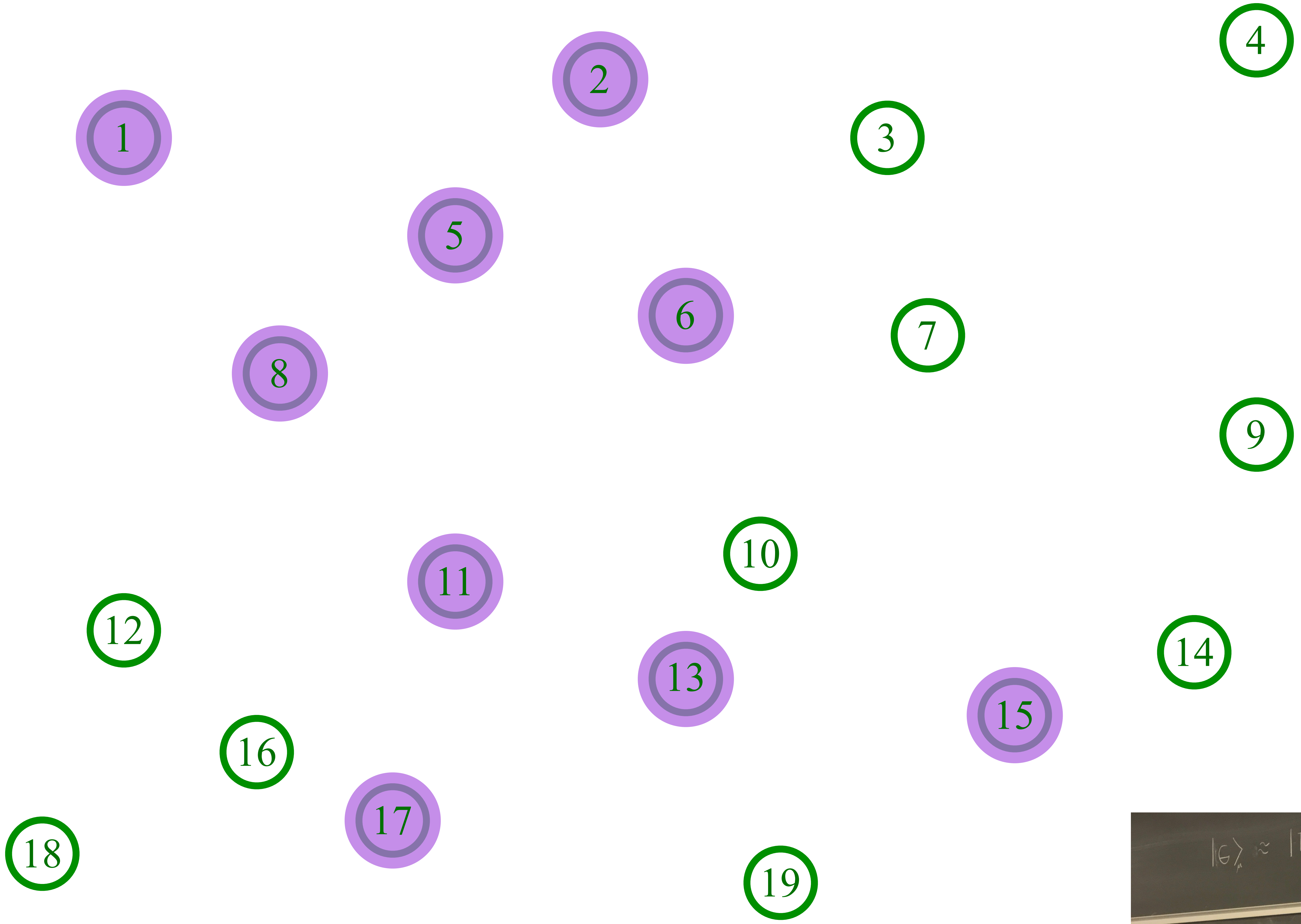
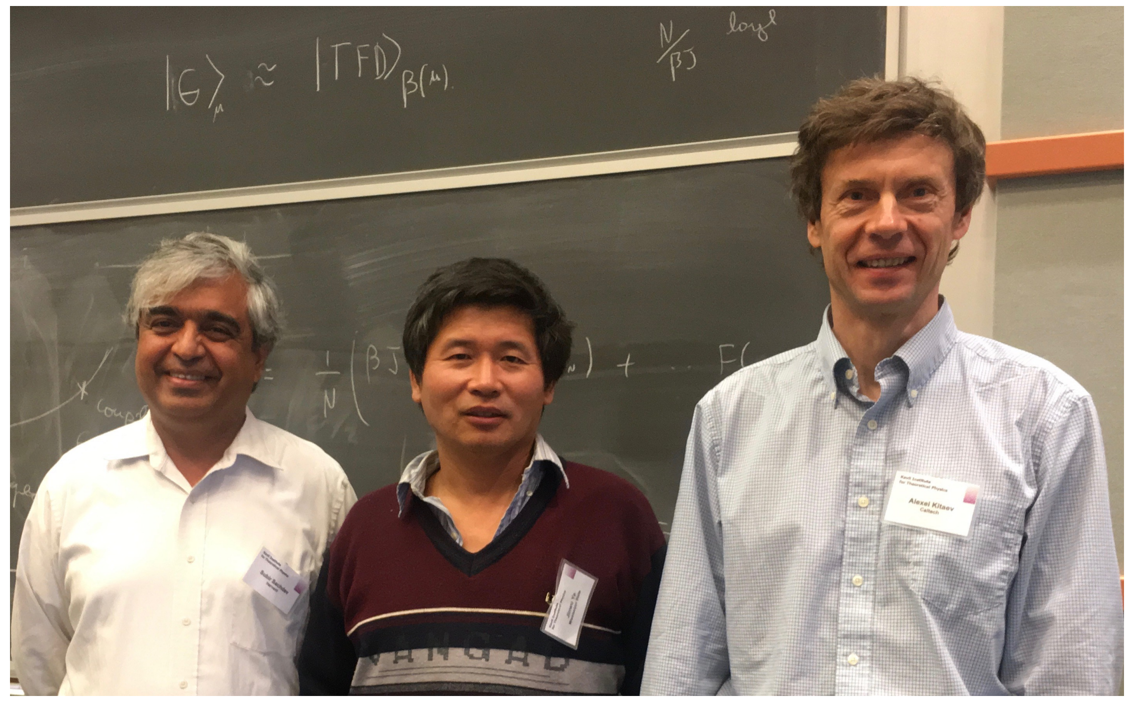


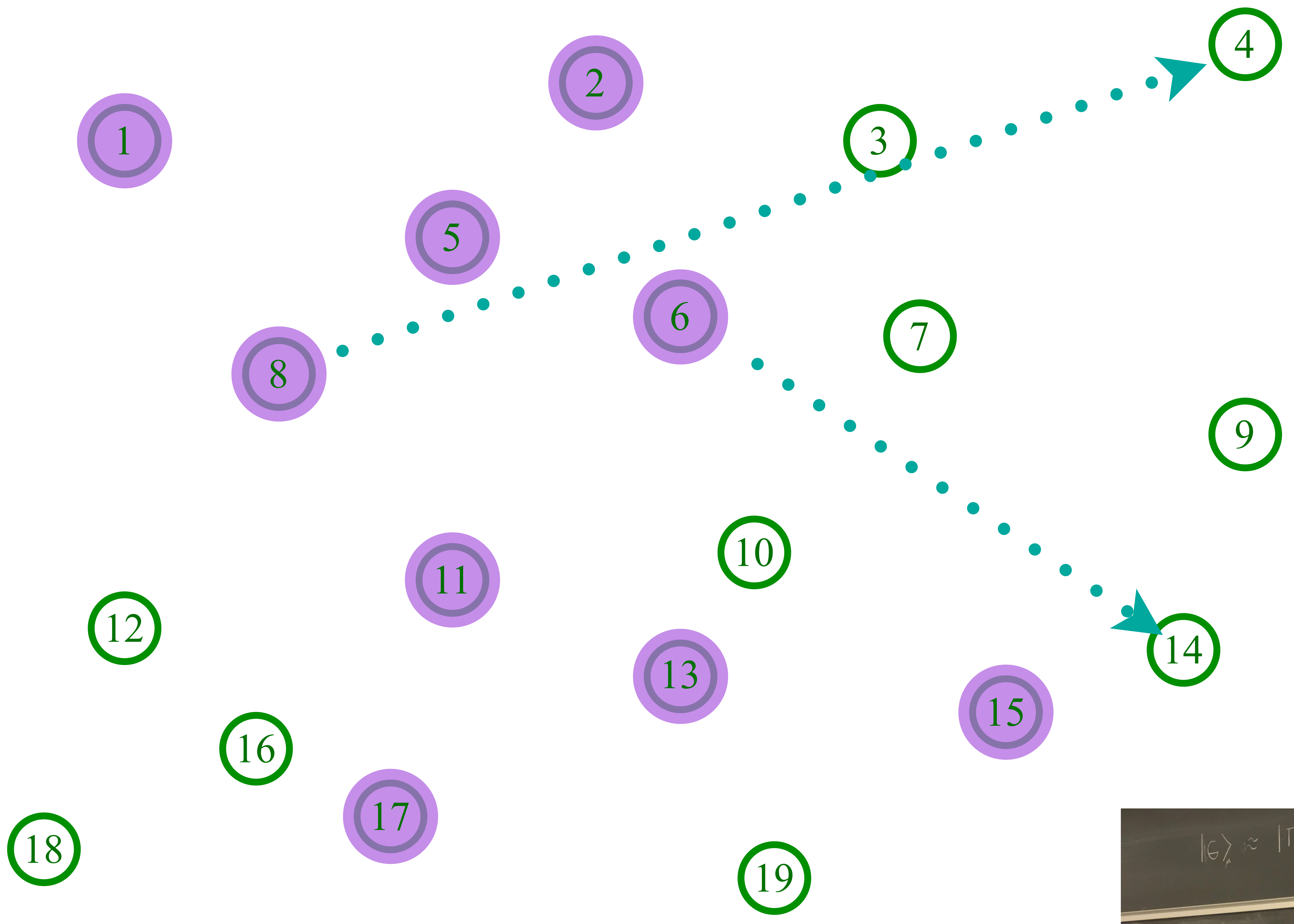
Entangle electrons pairwise randomly



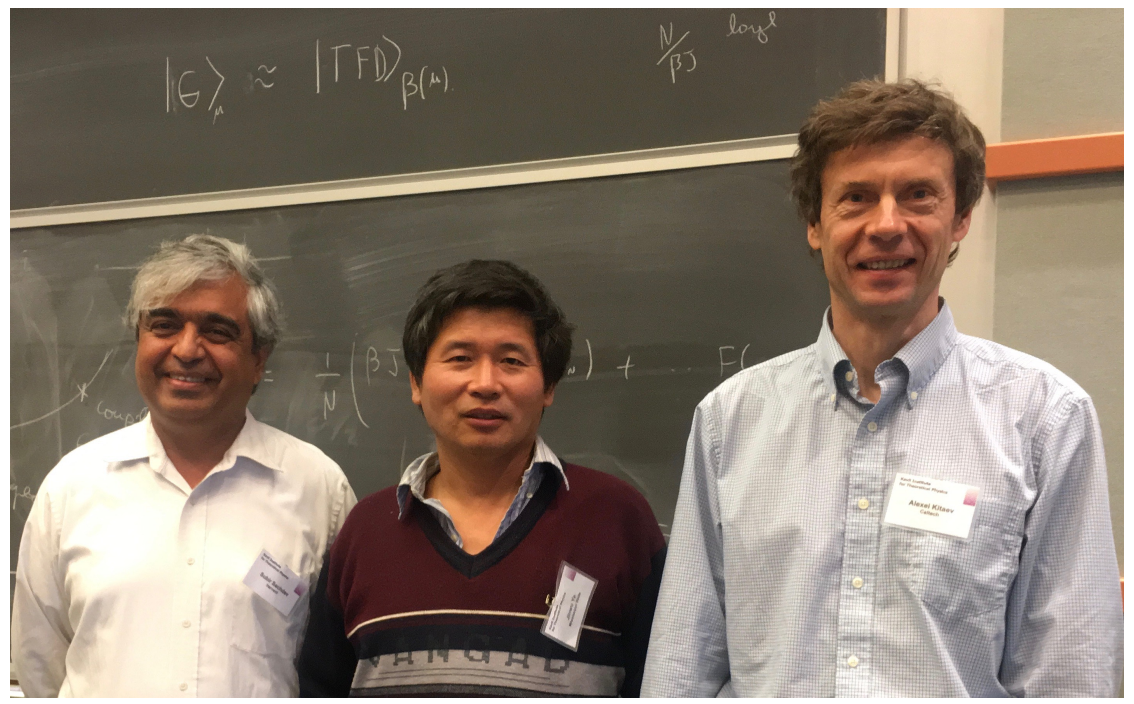
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



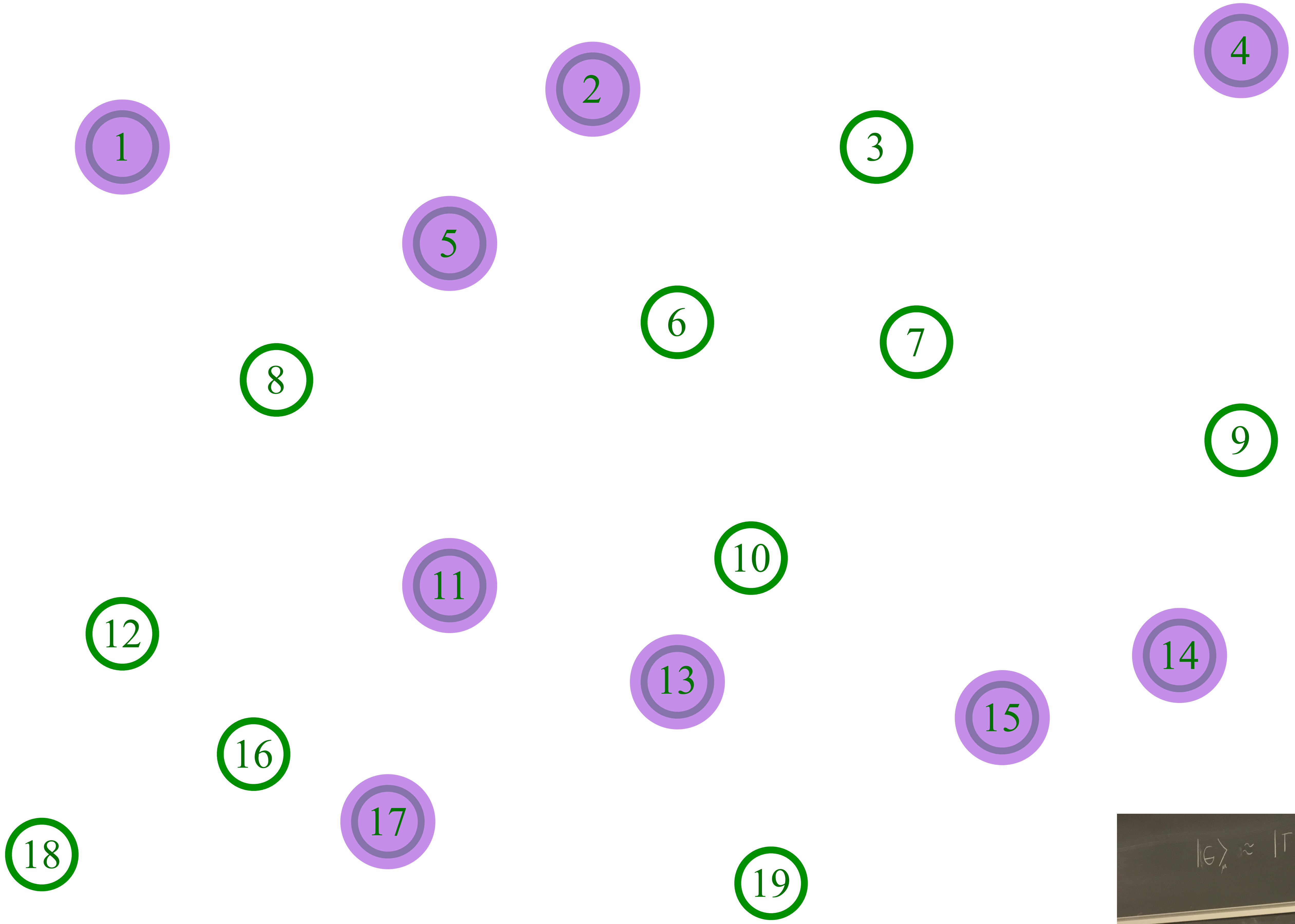
Entangle electrons pairwise randomly



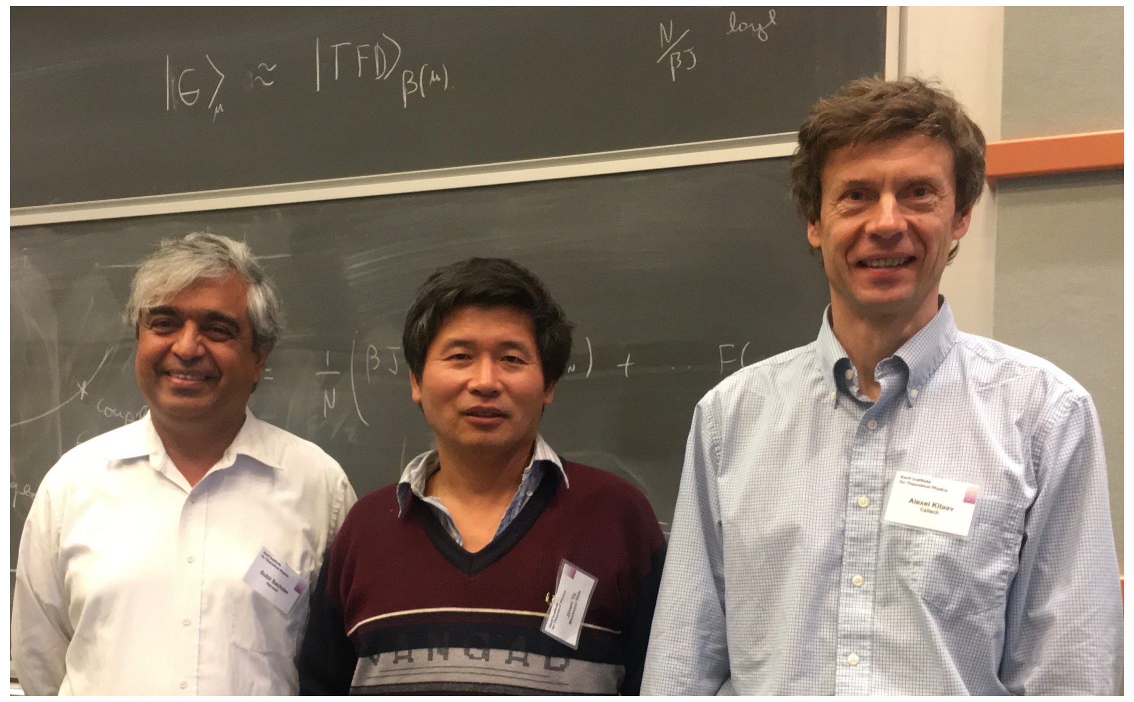
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

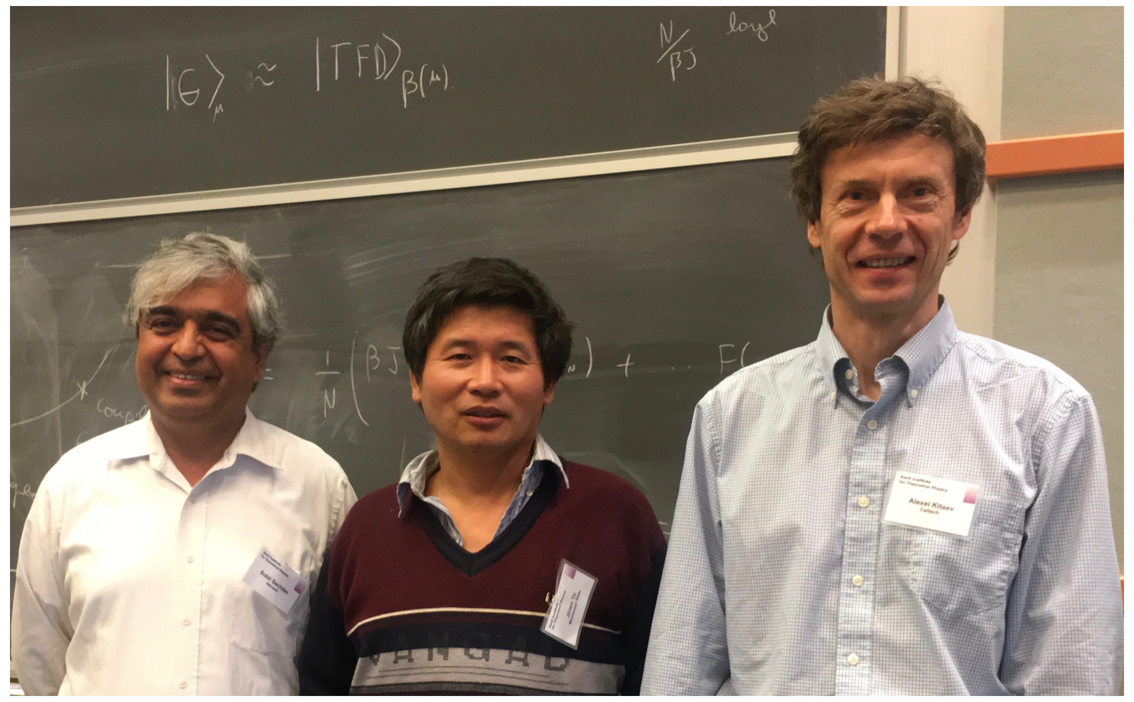
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

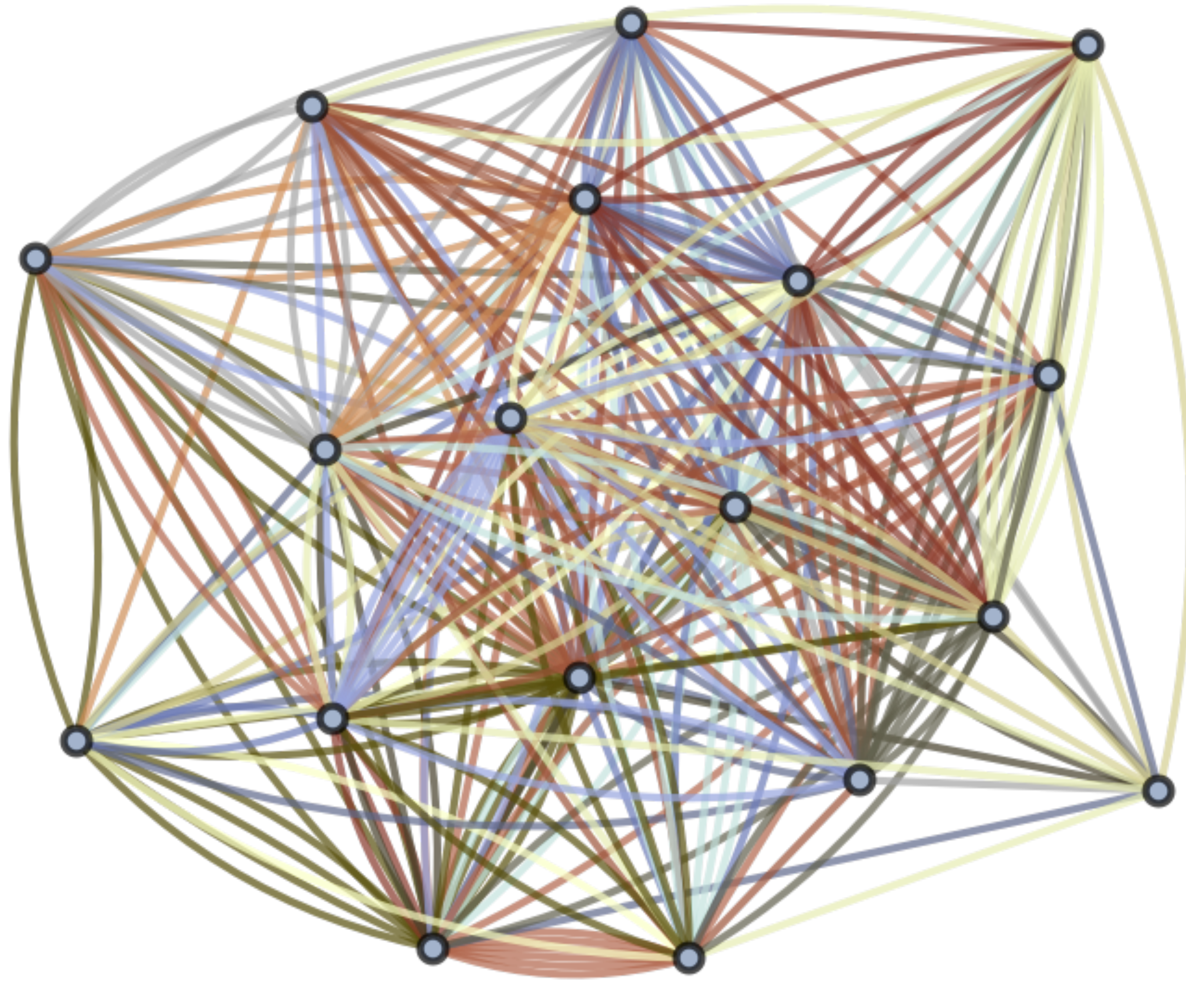
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

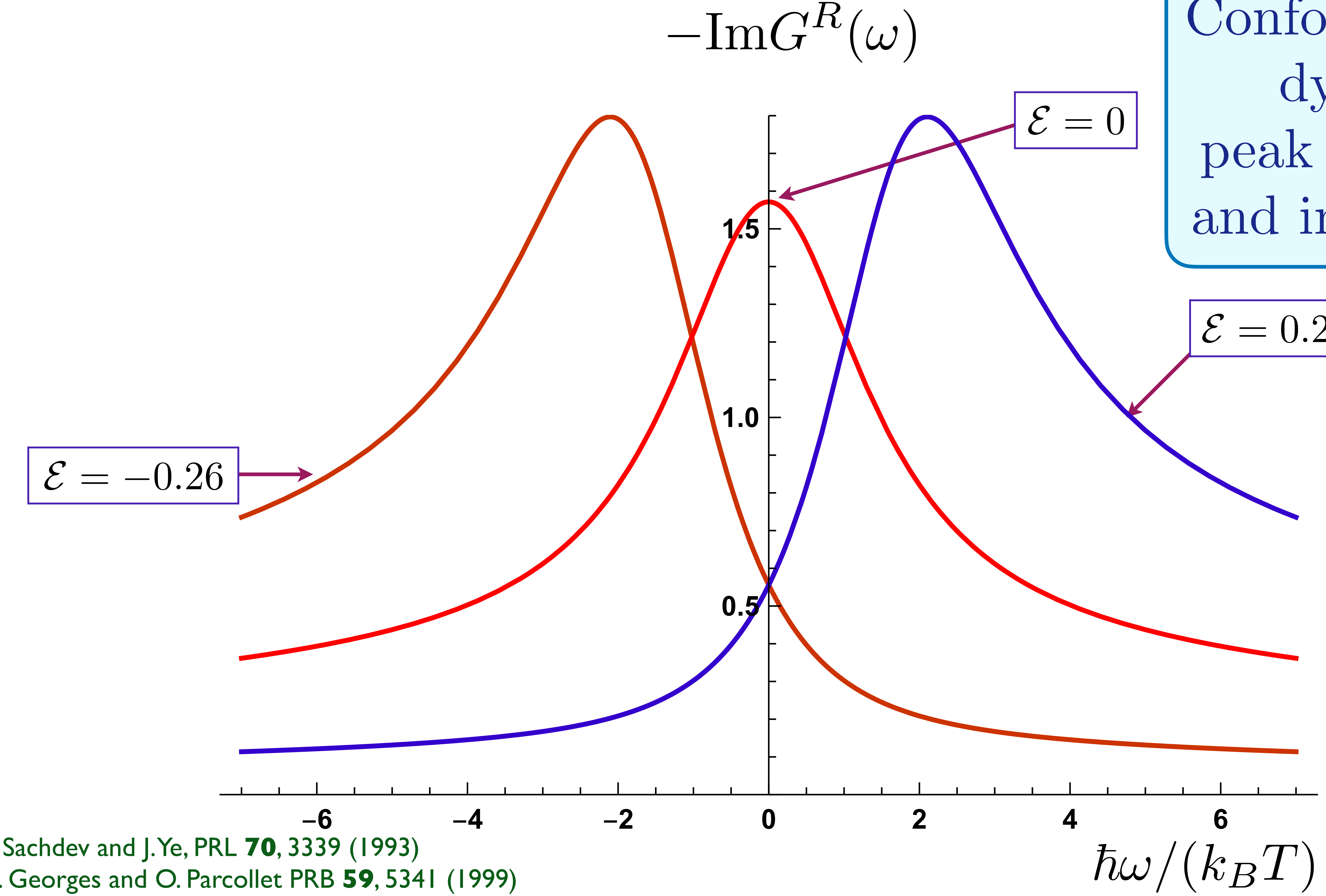
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The SYK model



Conformal 'Planckian' dynamics with peak width $\sim k_B T/\hbar$ and independent of U

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N} \sum_\alpha c_\alpha(\tau_1) c^\dagger_\alpha(\tau_2)$

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp\left(-N S_{\text{eff}}[G]\right)$$

The large N saddle point equation $\delta S_{\text{eff}} / \delta G = 0$ for $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ is

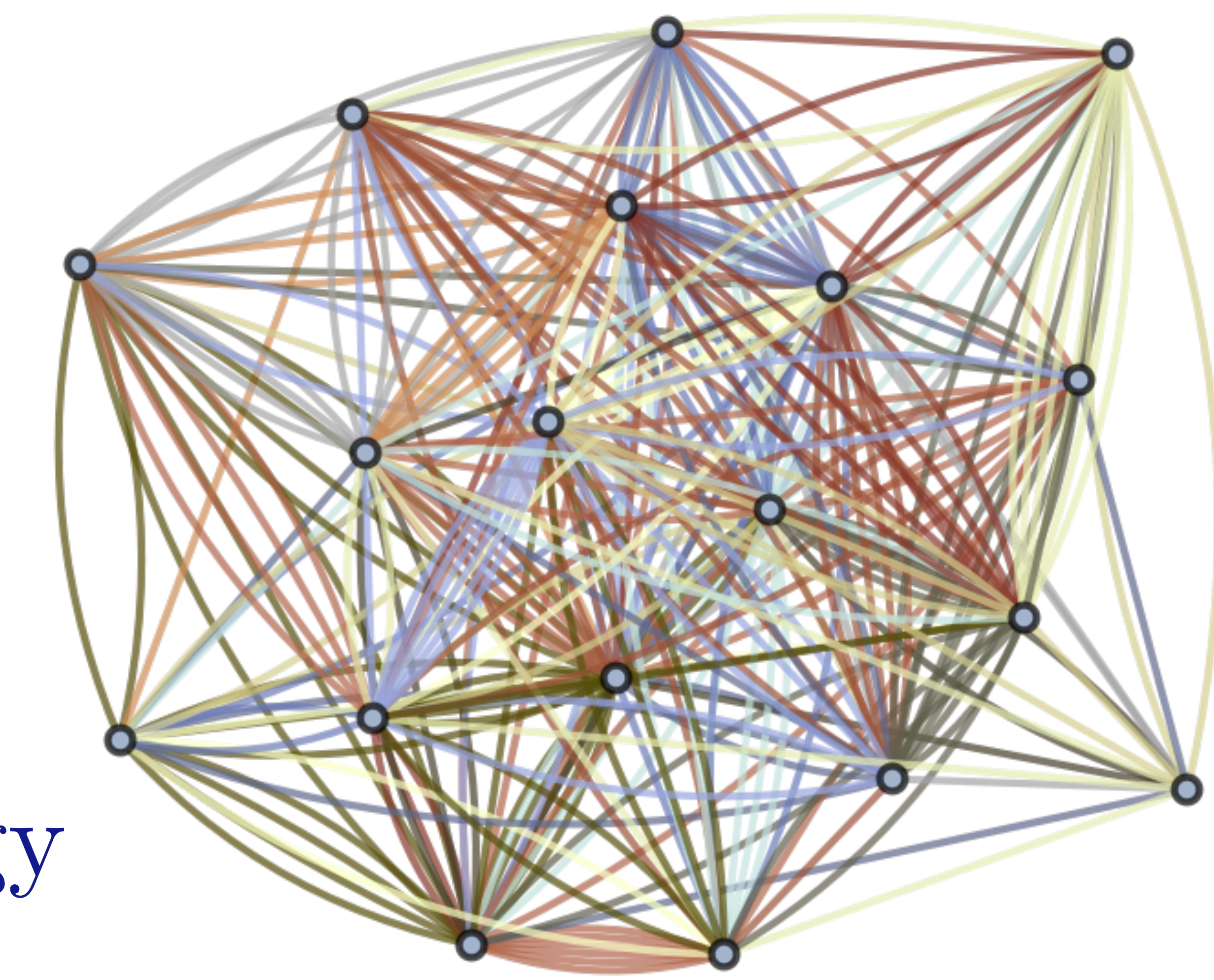
$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

Time reparameterization symmetry:

At frequencies $\ll U$, the path integral for is invariant under time reparametrization $f(\sigma)$

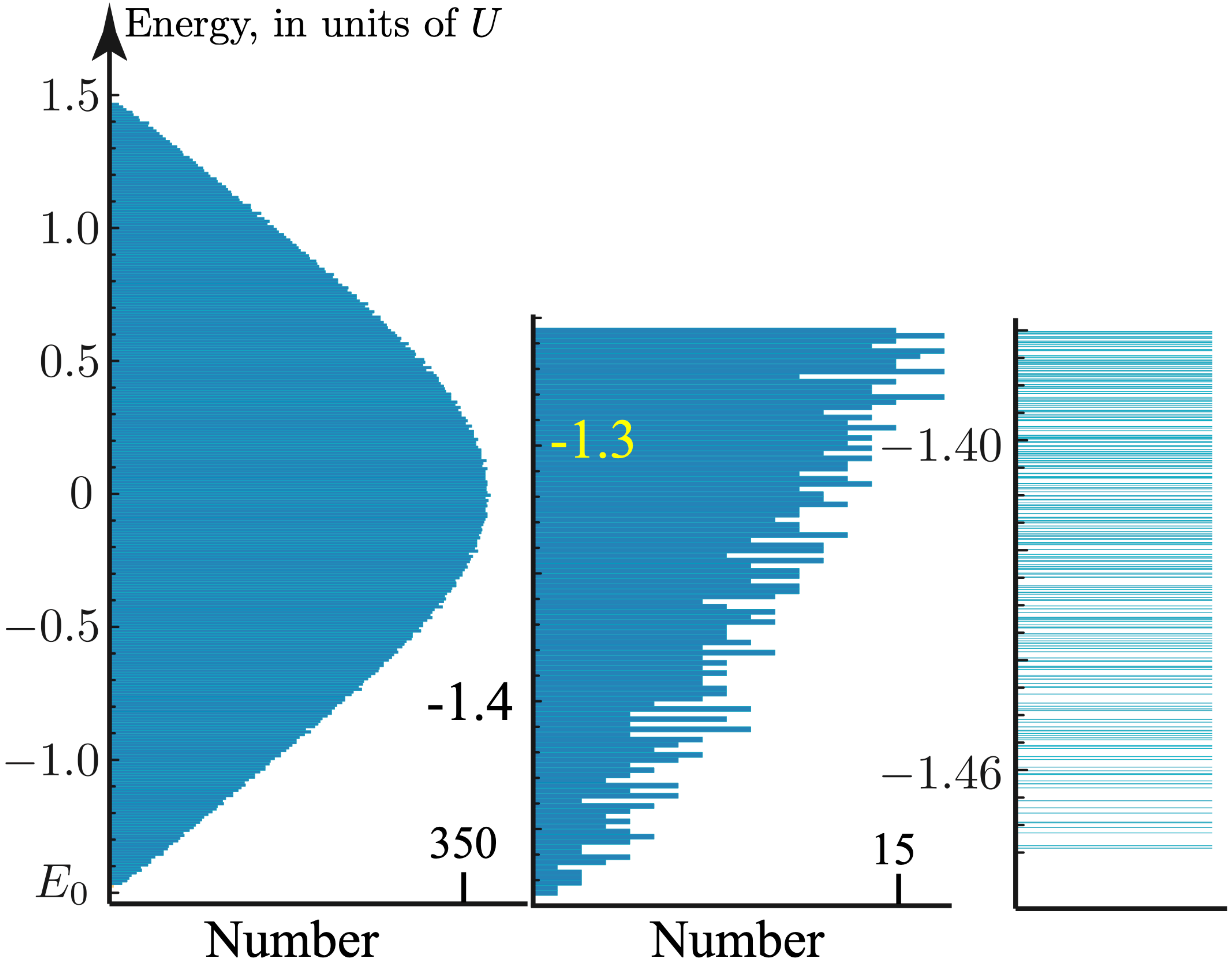
$$\tau = f(\sigma)$$
$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \widetilde{G}(\sigma_1, \sigma_2)$$



There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

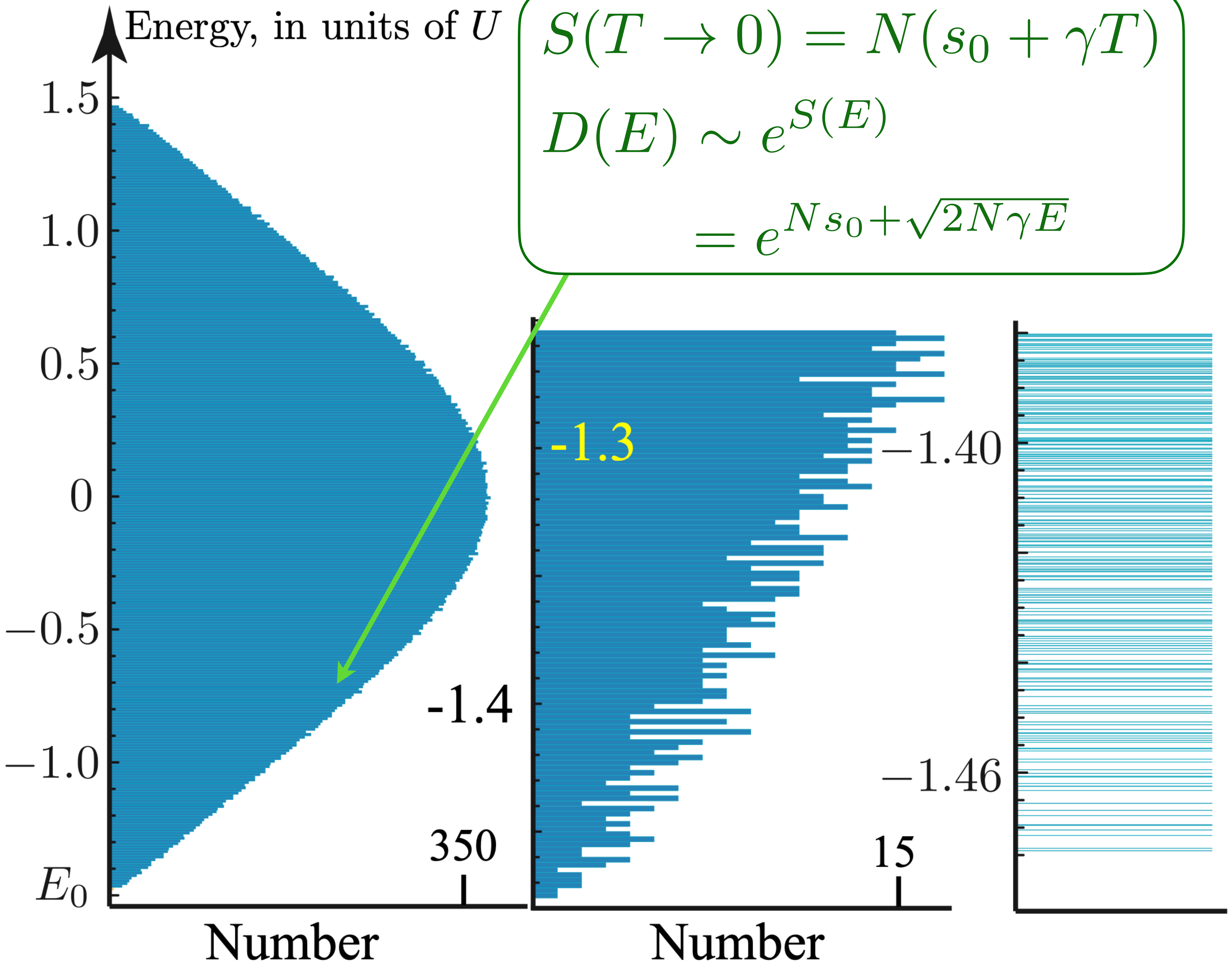
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.46484769917 \ldots .$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



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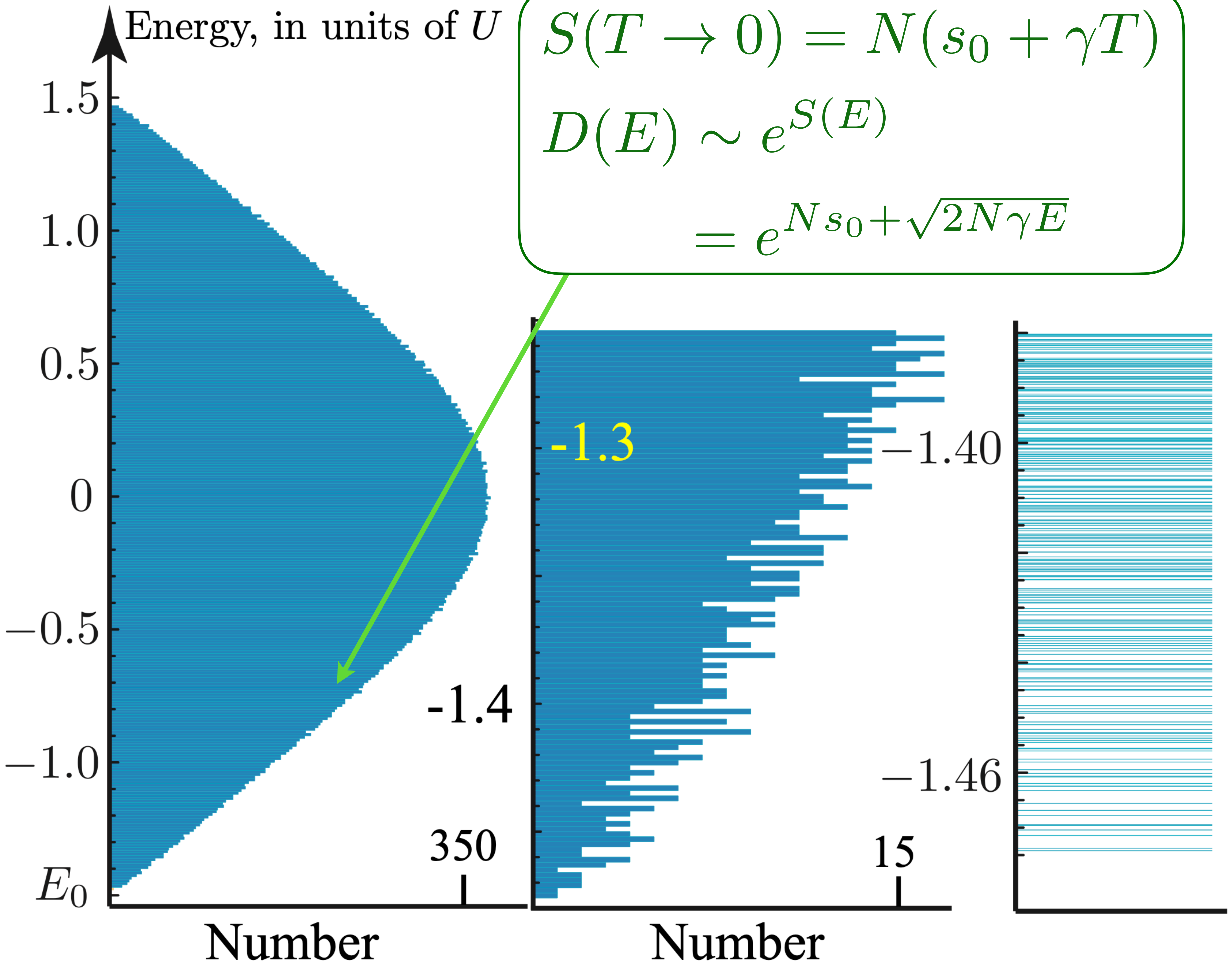
$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.46484769917\ldots.$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



Energy level spacing $\sim e^{-N s_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

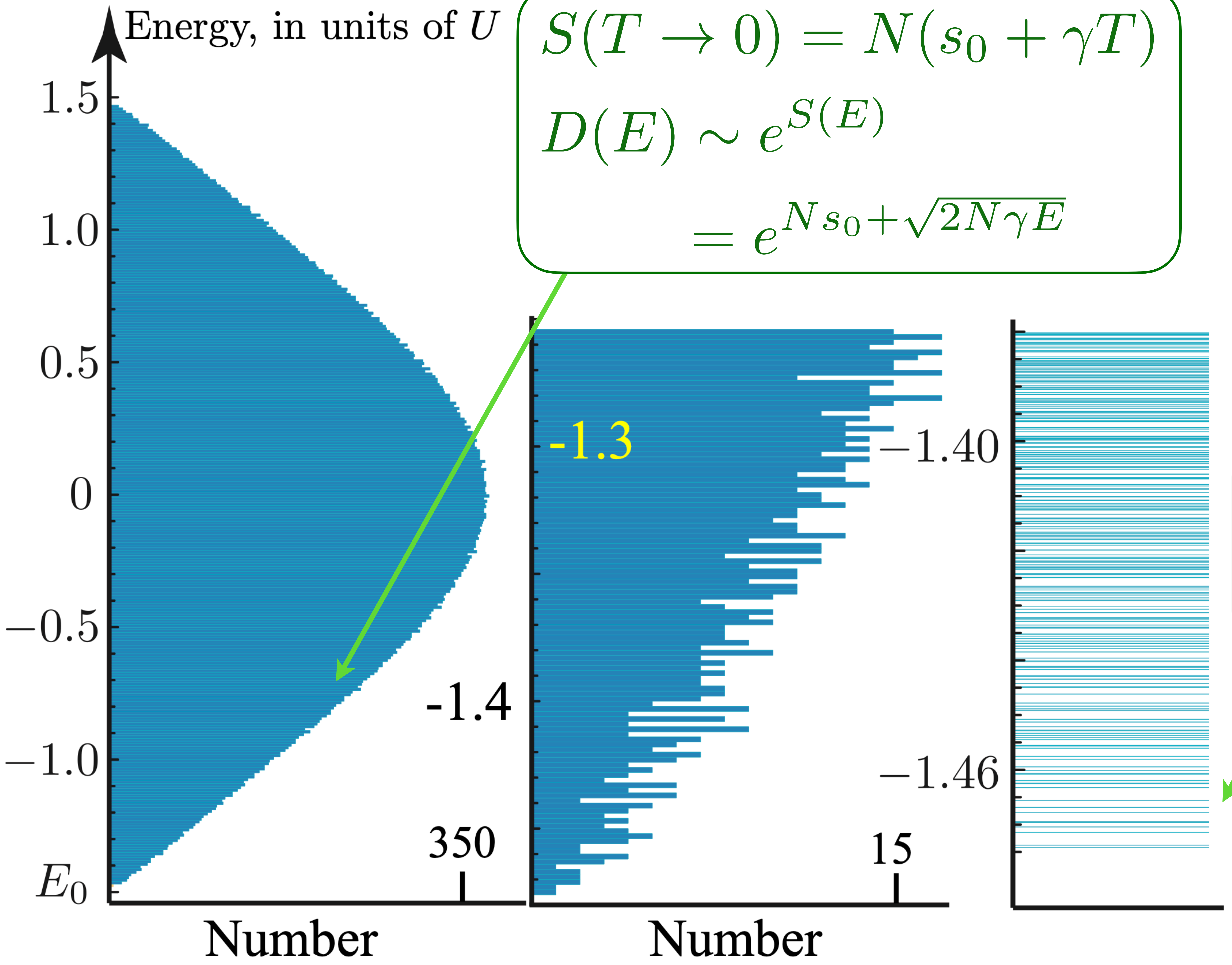
Beyond Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2 N \gamma E}}$$



At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.46484769917\ldots.$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2 N \gamma E})$$

J. S. Cotler et al., JHEP 05 (2017) 118

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

Complex SYK model

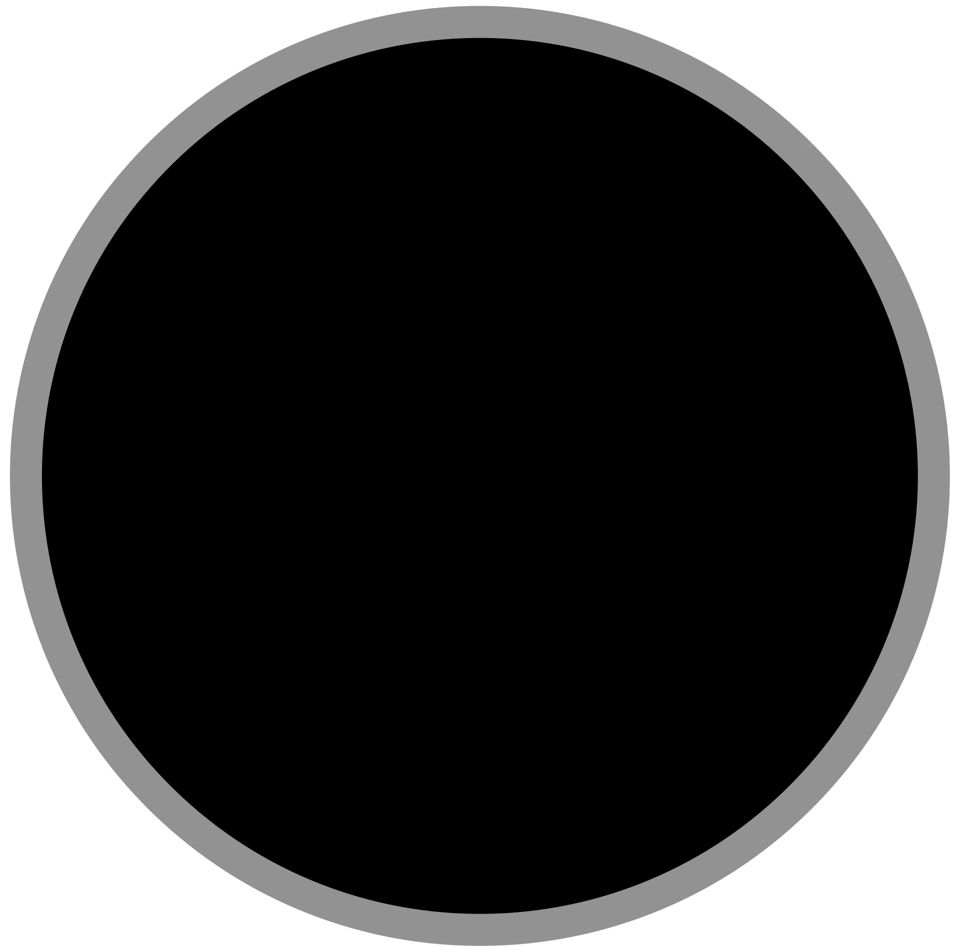
(Numerics: G. Tarnopolsky)

Quantum black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \dfrac{2GM}{c^2}$

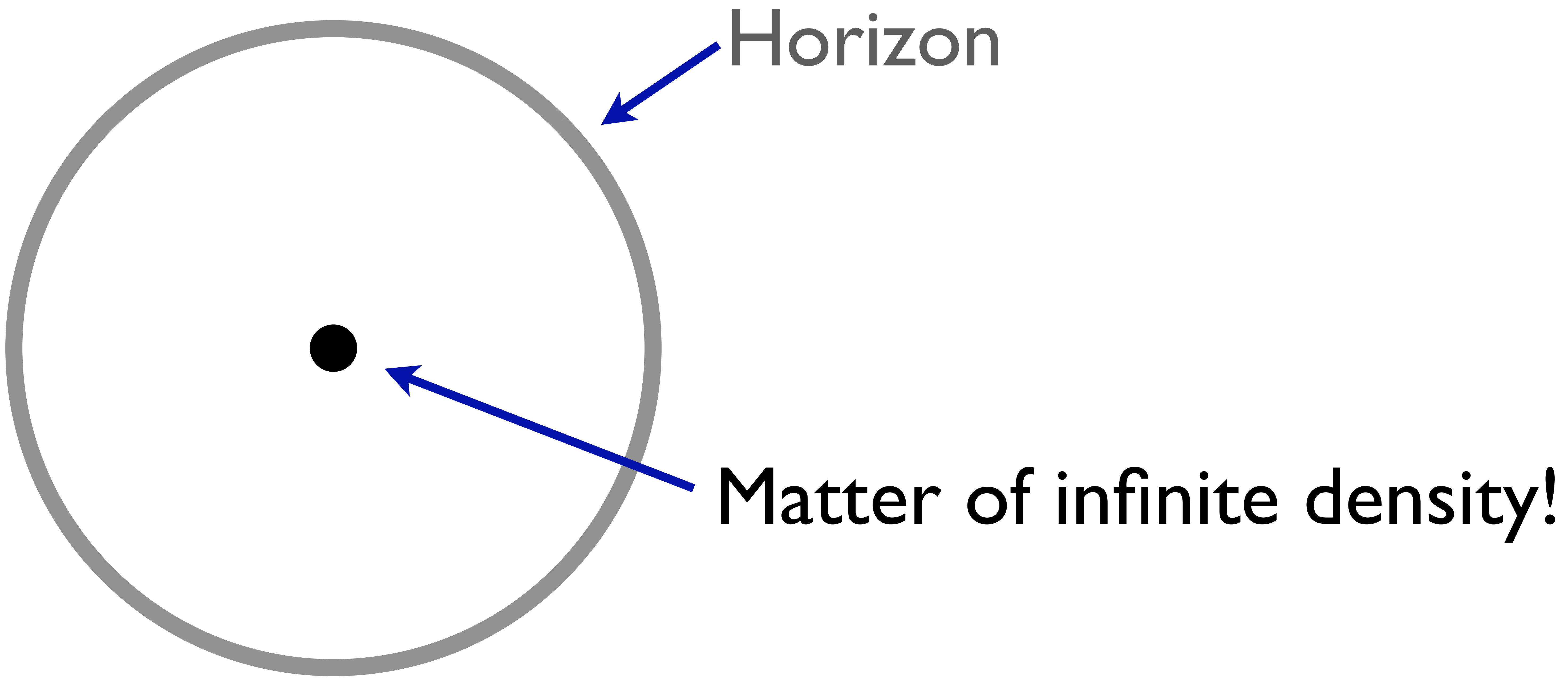


G Newton's constant, c velocity of light, M mass of black hole

For M = earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



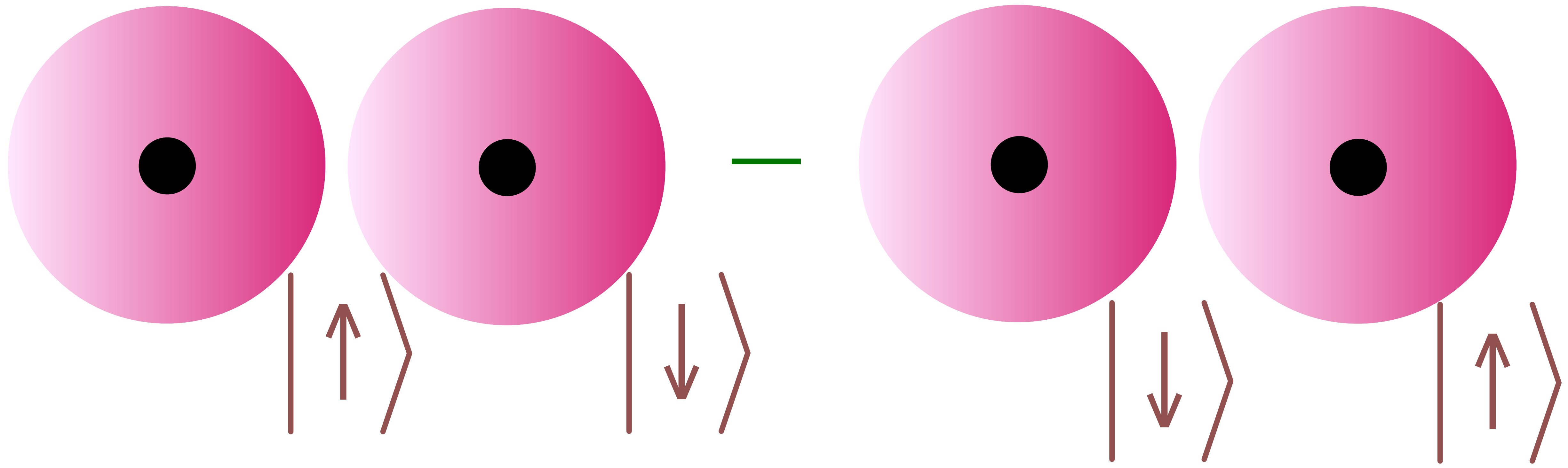
What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.

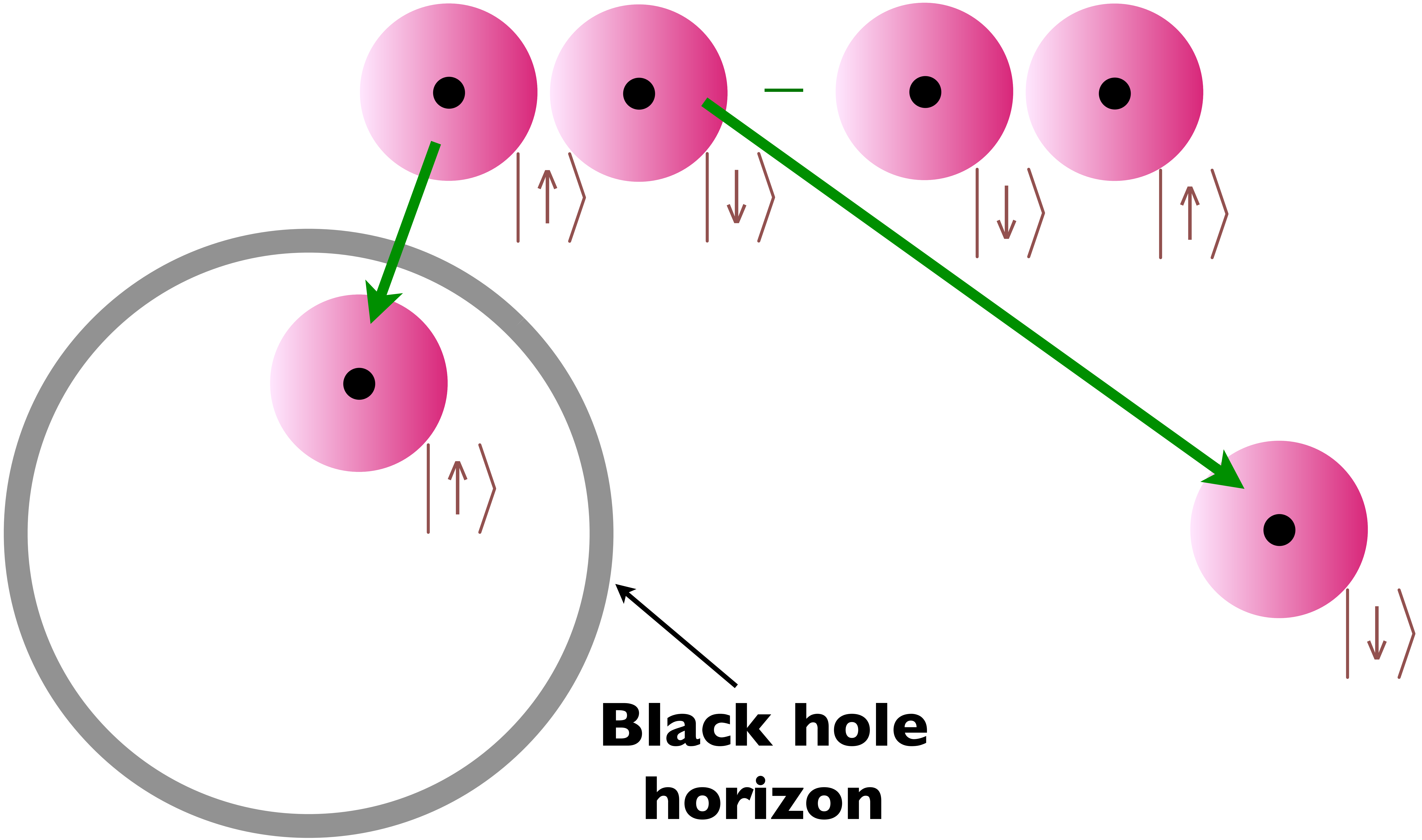
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

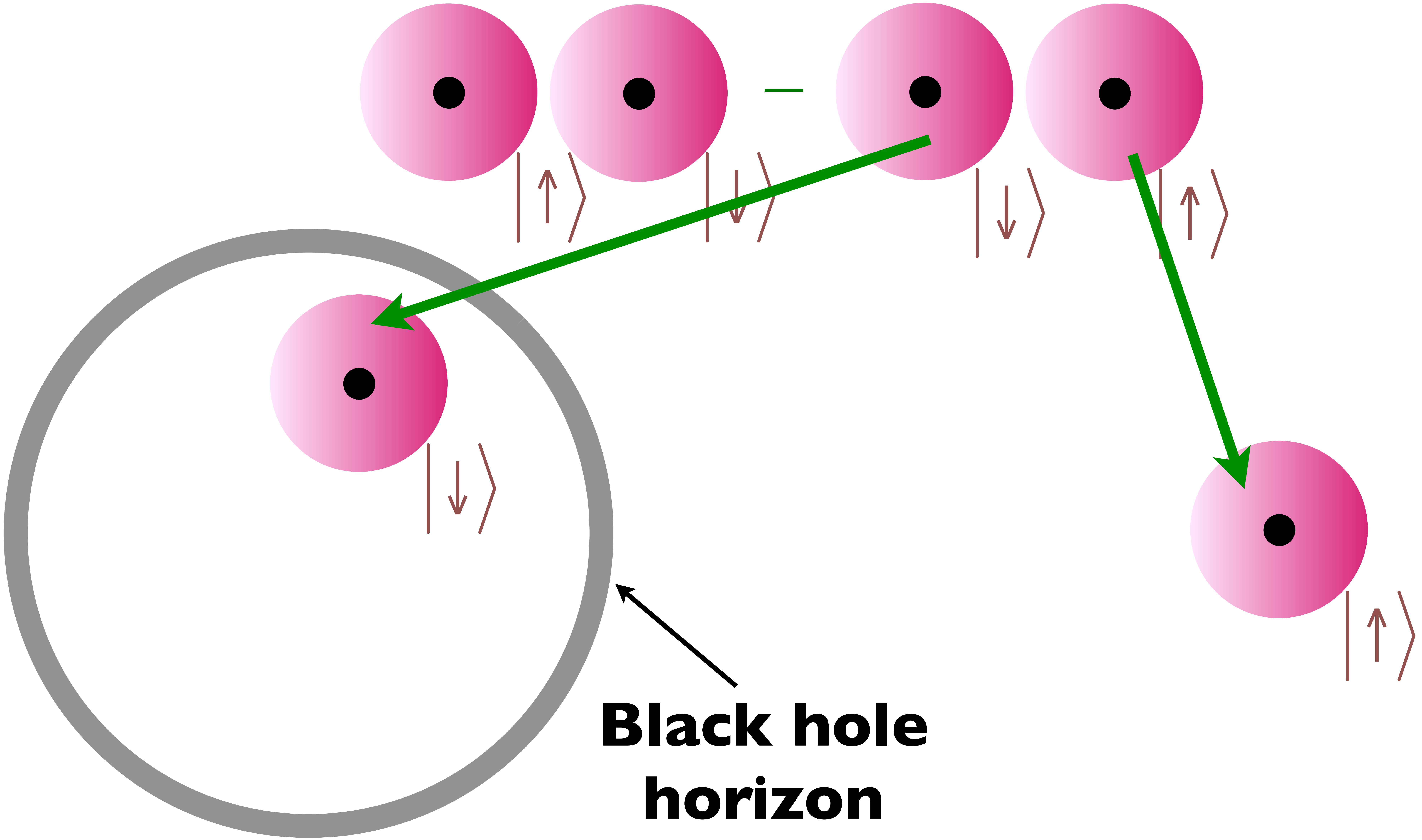
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



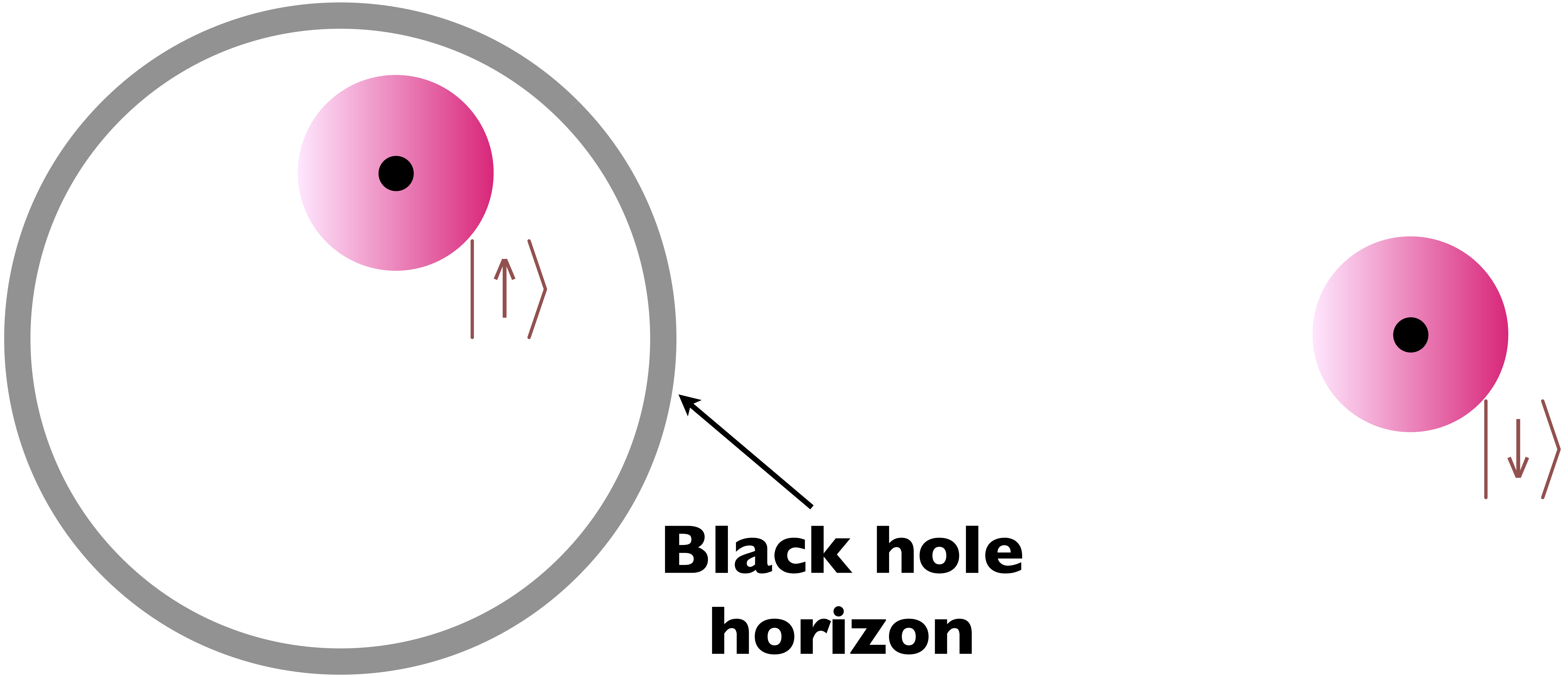
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



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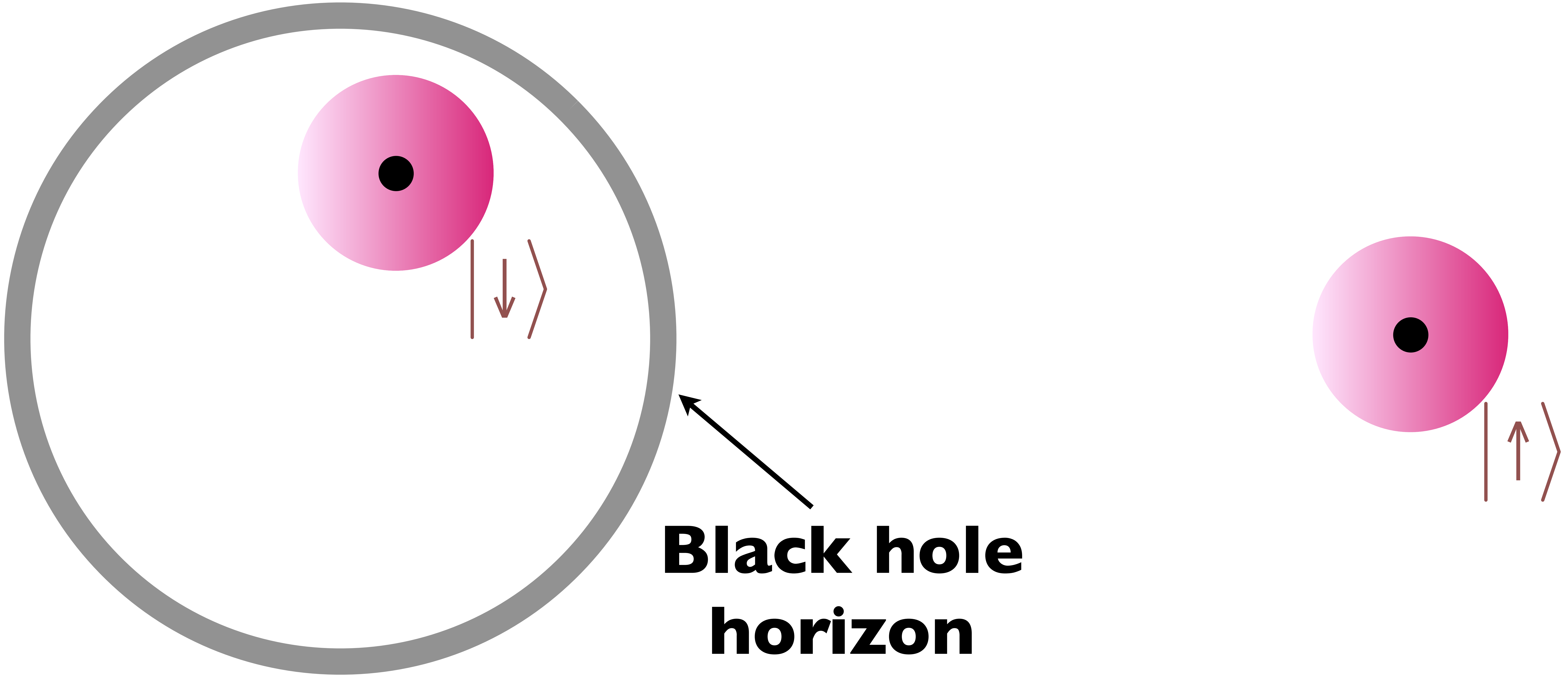
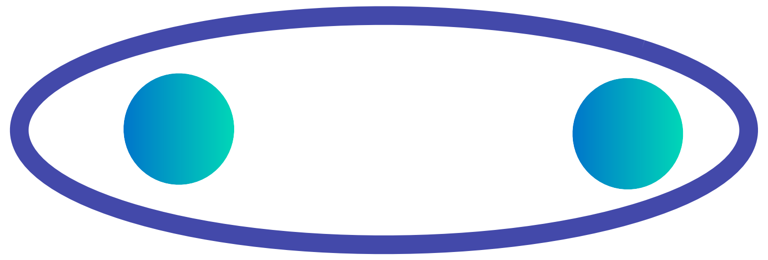
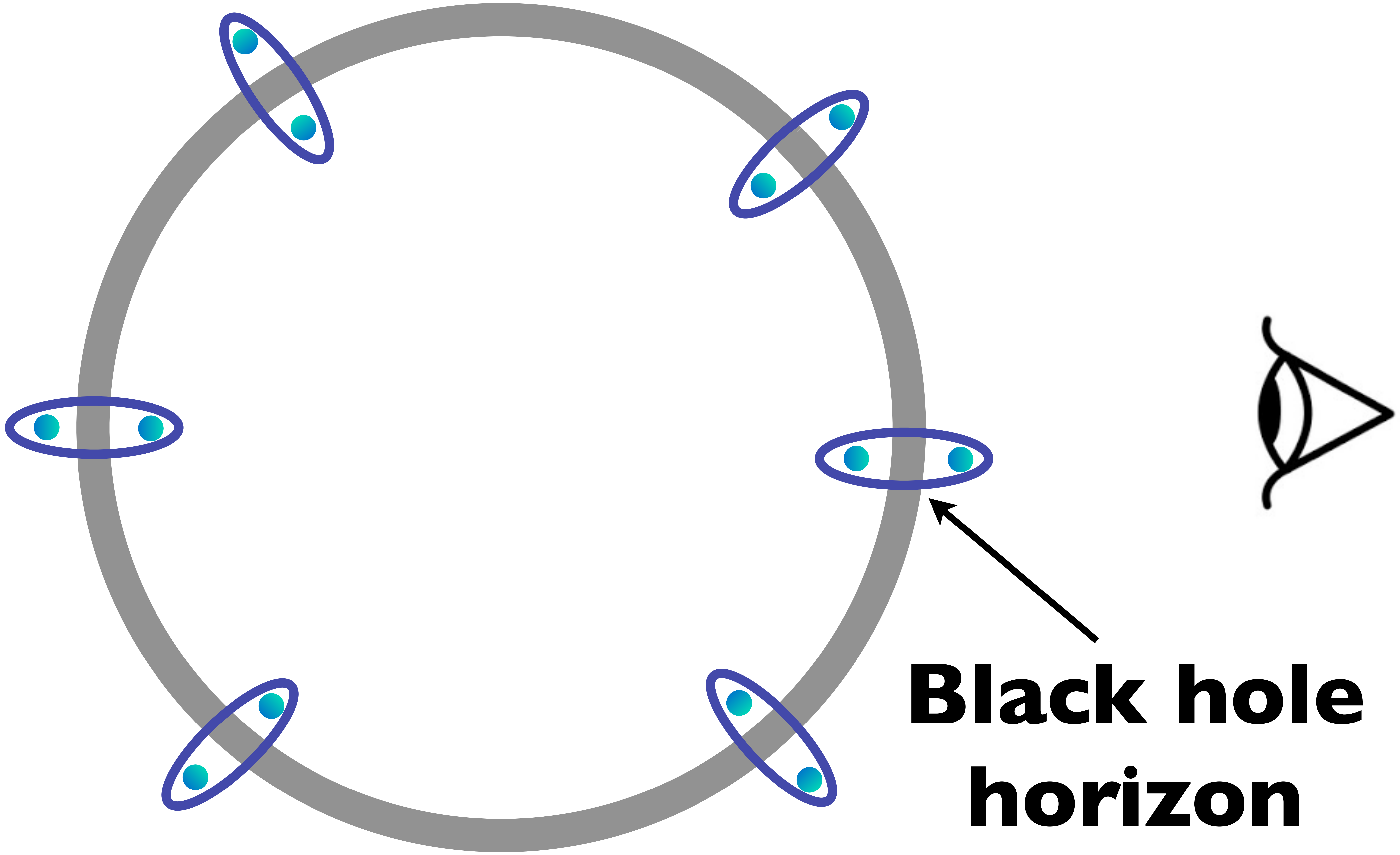


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside* the black hole, Hawking obtained

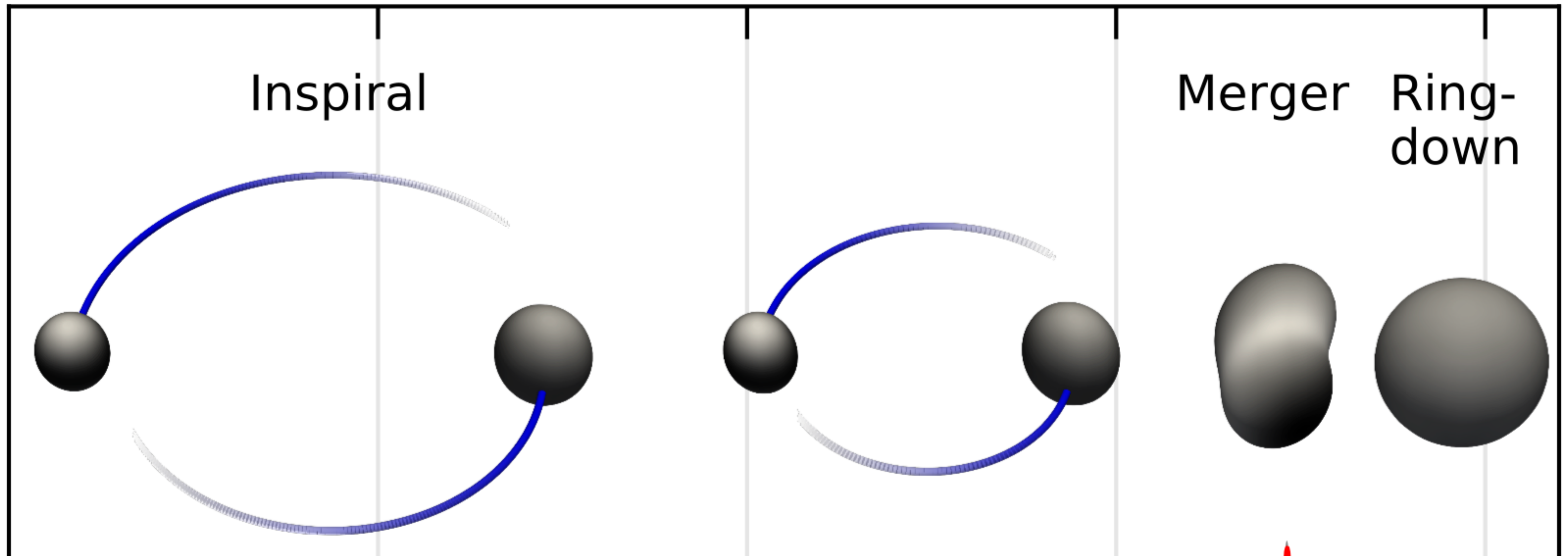
$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum black holes

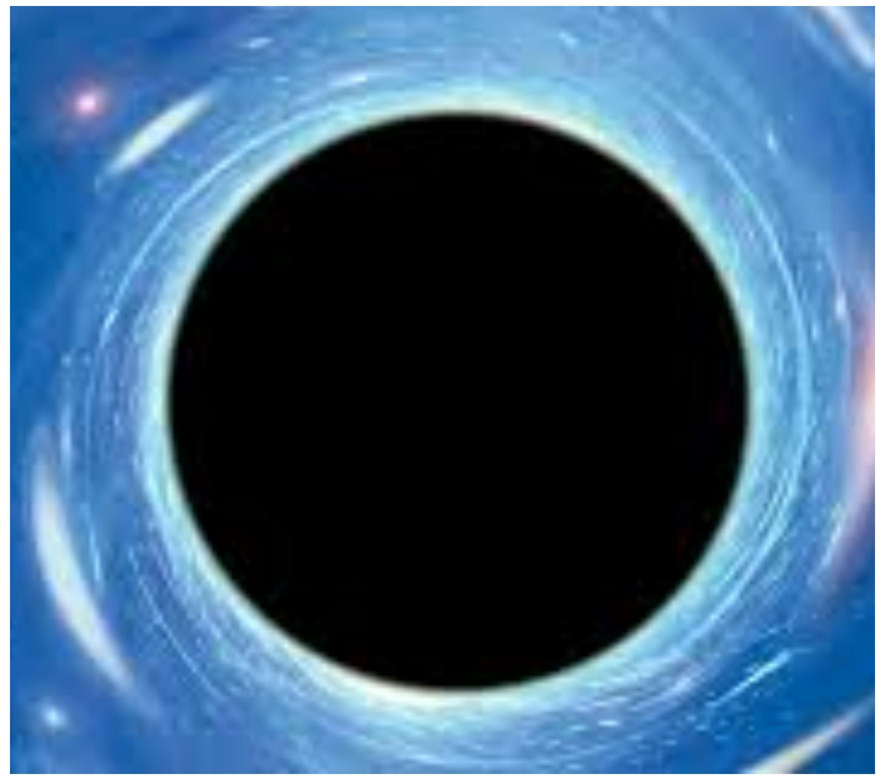
- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4 G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!

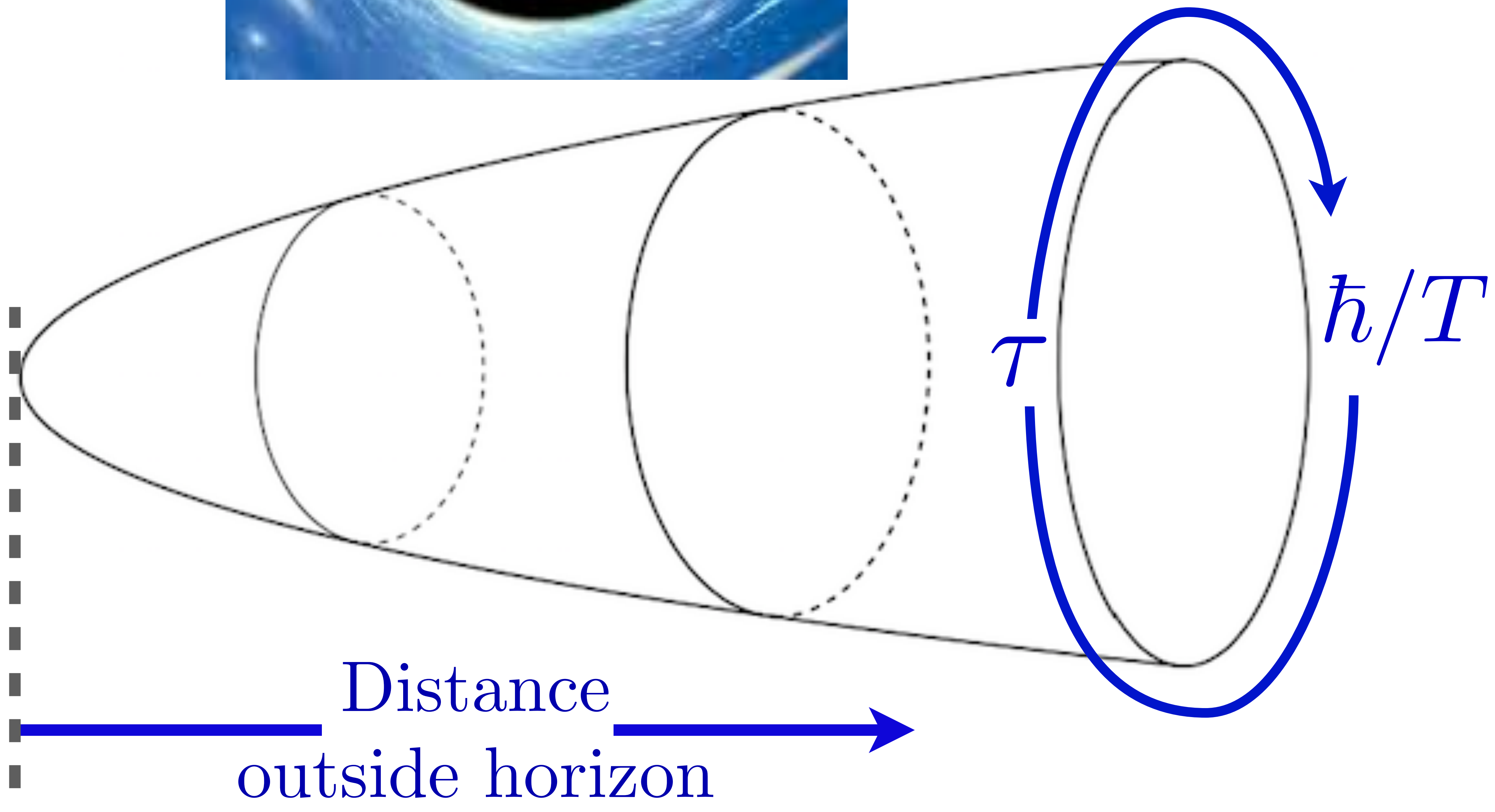


J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$





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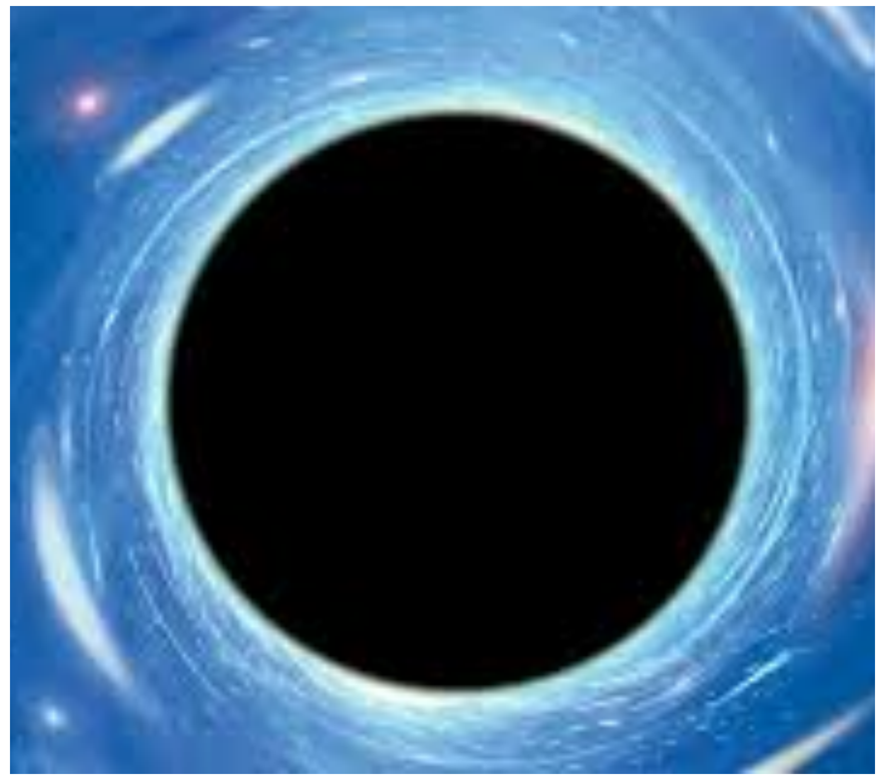
$$= \exp\left(S_{BH}\right) \times \left(\quad ....????... \quad \right)$$

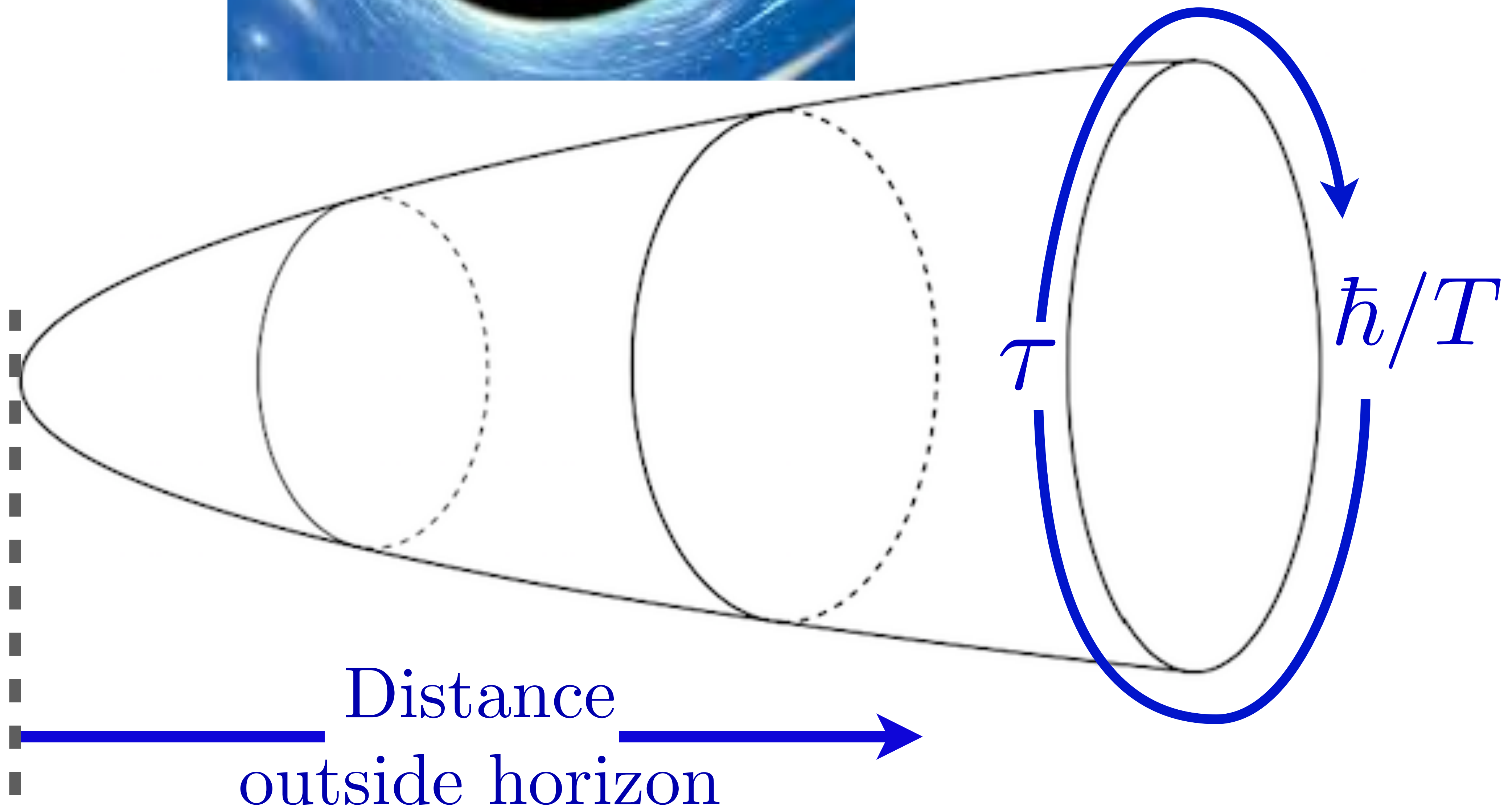
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$



$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



Quantum Black Holes

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model to a quantum theory of charged black holes

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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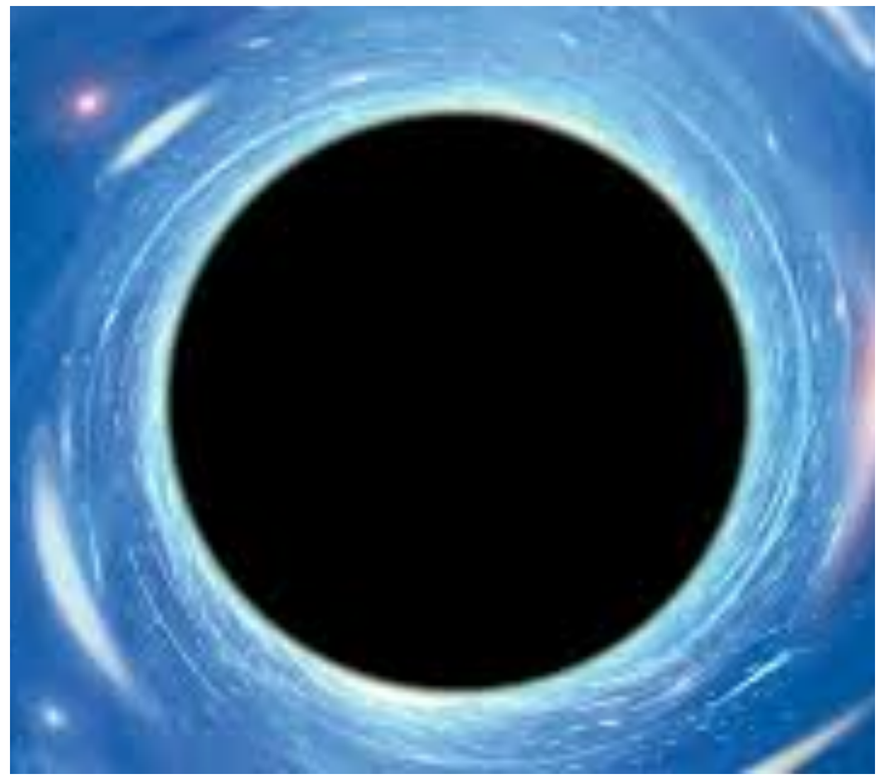

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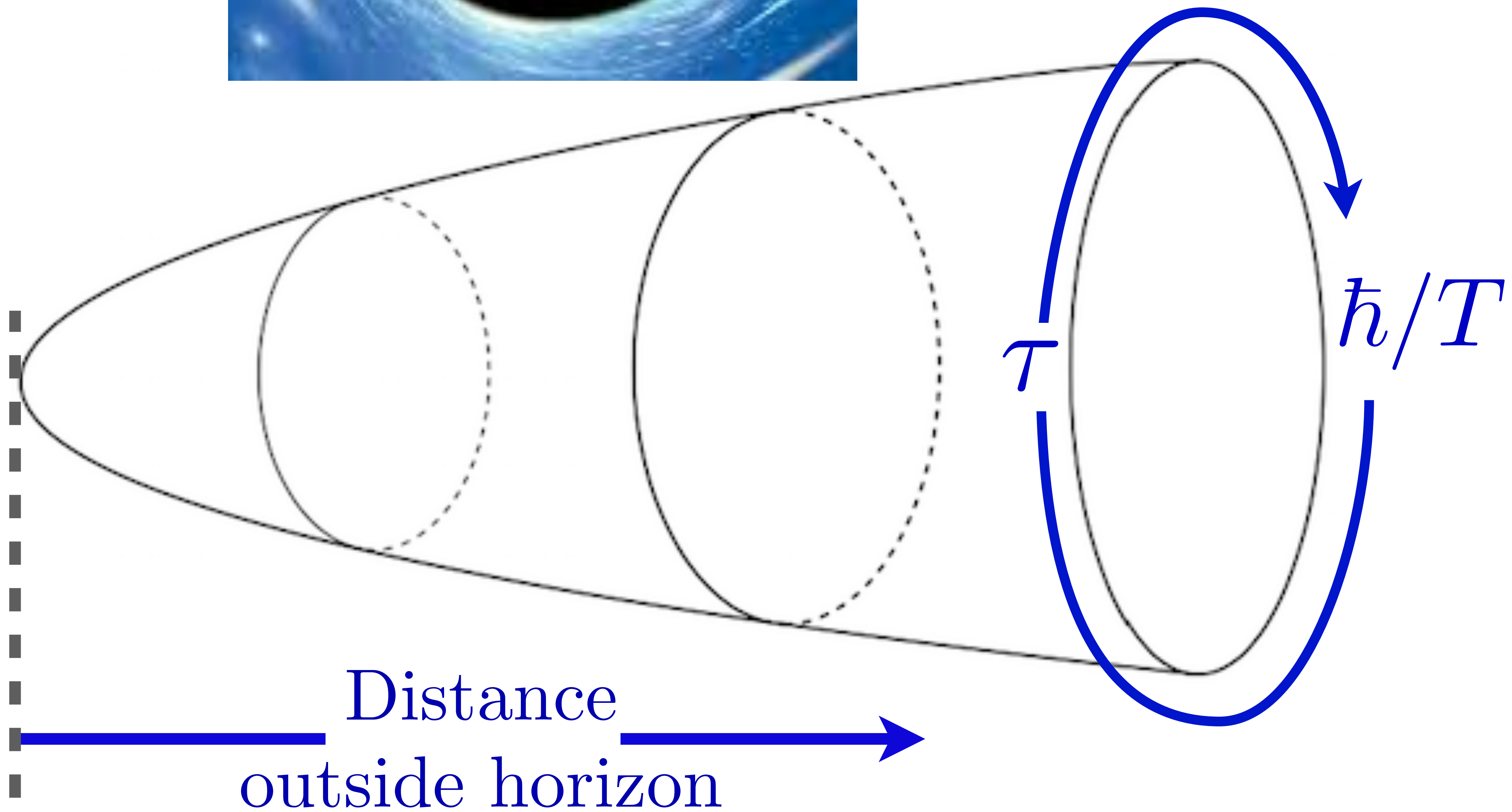

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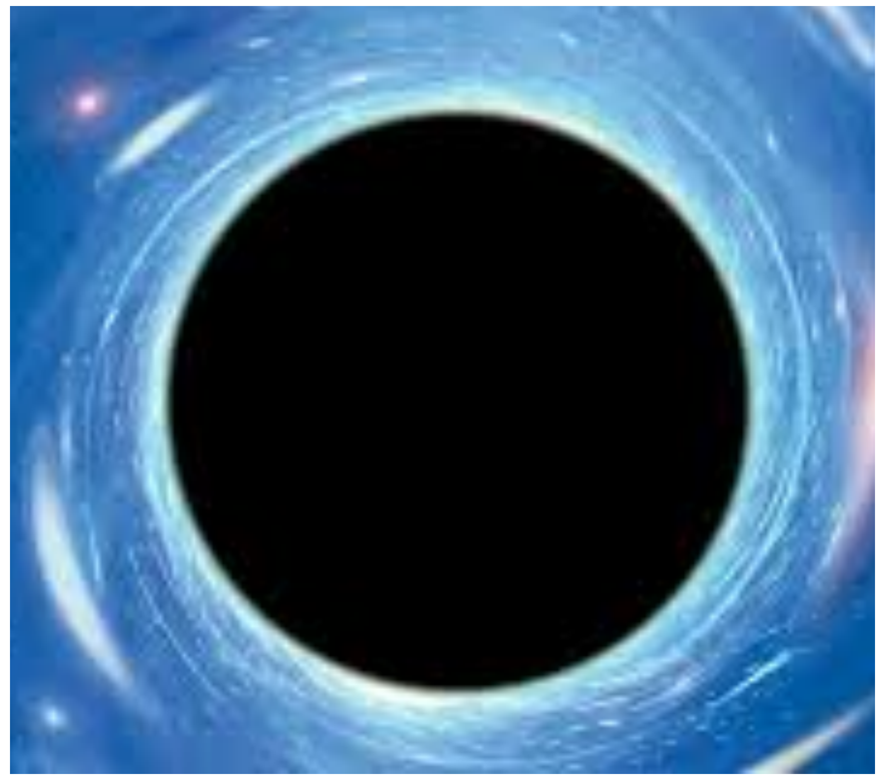

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PHYSICAL REVIEW LETTERS **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

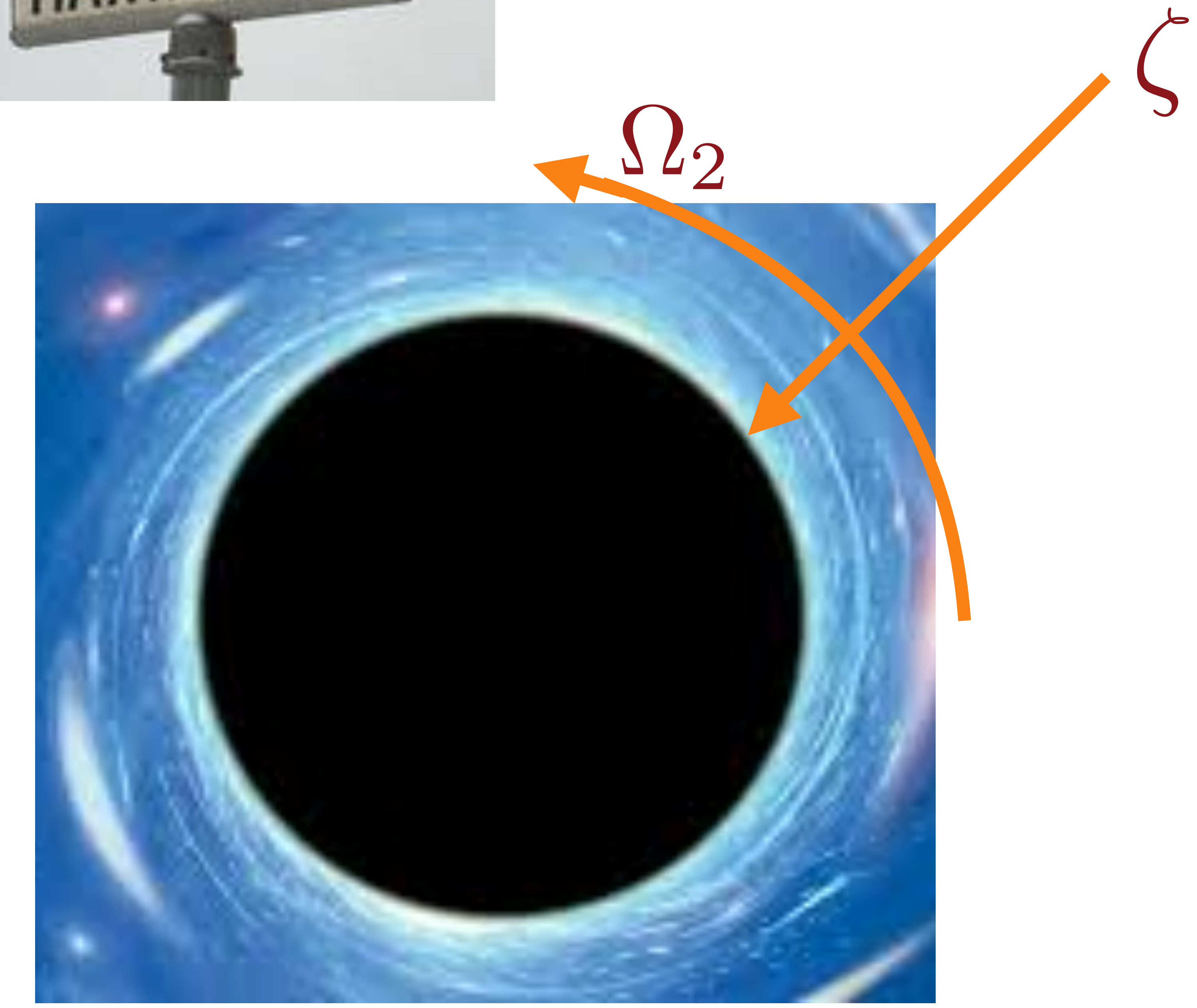
Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner, Liu, McGreevy, Vegh 2009; Sachdev 2010

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Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}+\ldots\right)$$

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Maldacena, Stanford, Yang (2016)

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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I_{\rm SYK}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} + \ldots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

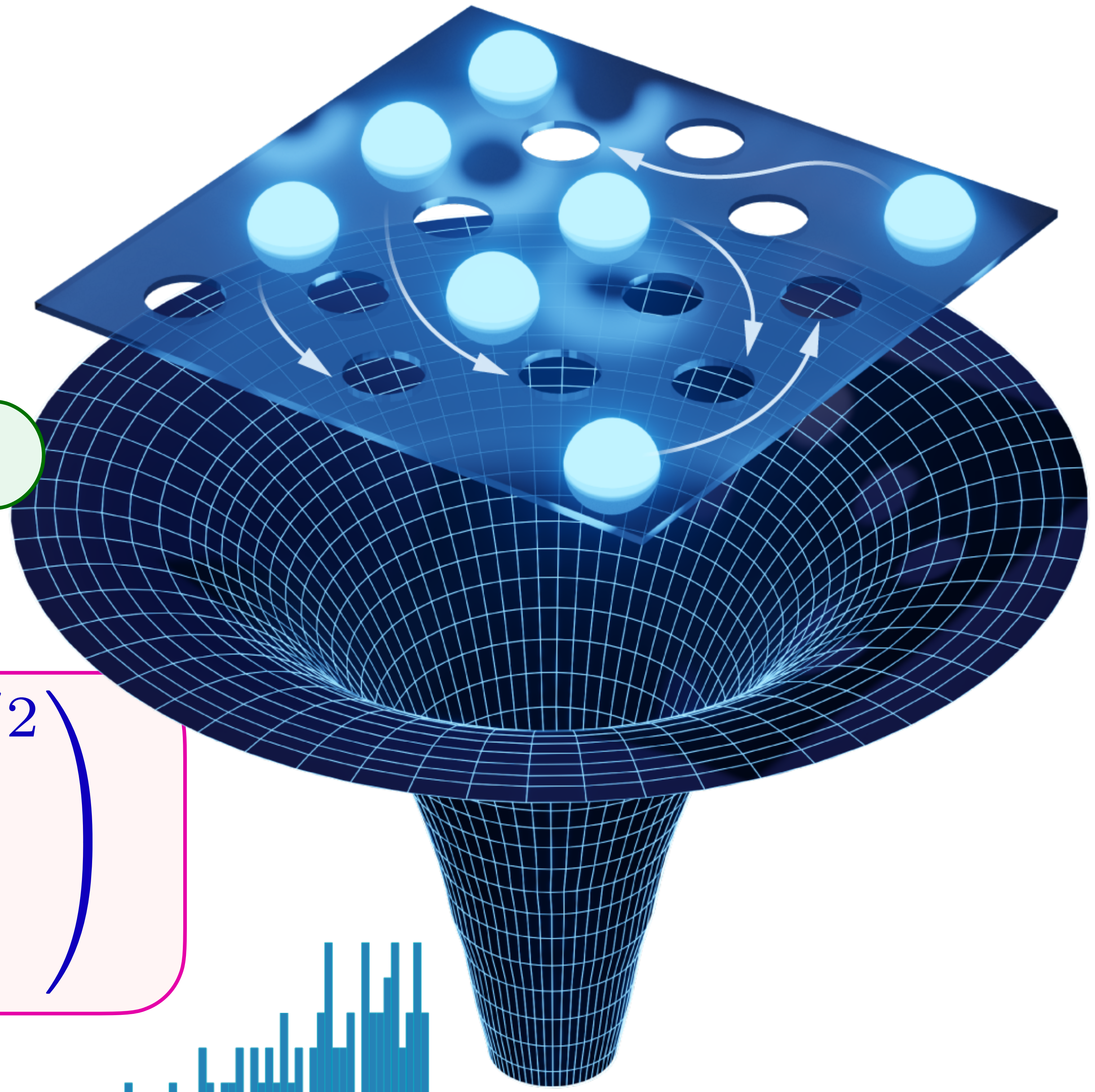
Quantum simulation of charged black holes by the SYK model

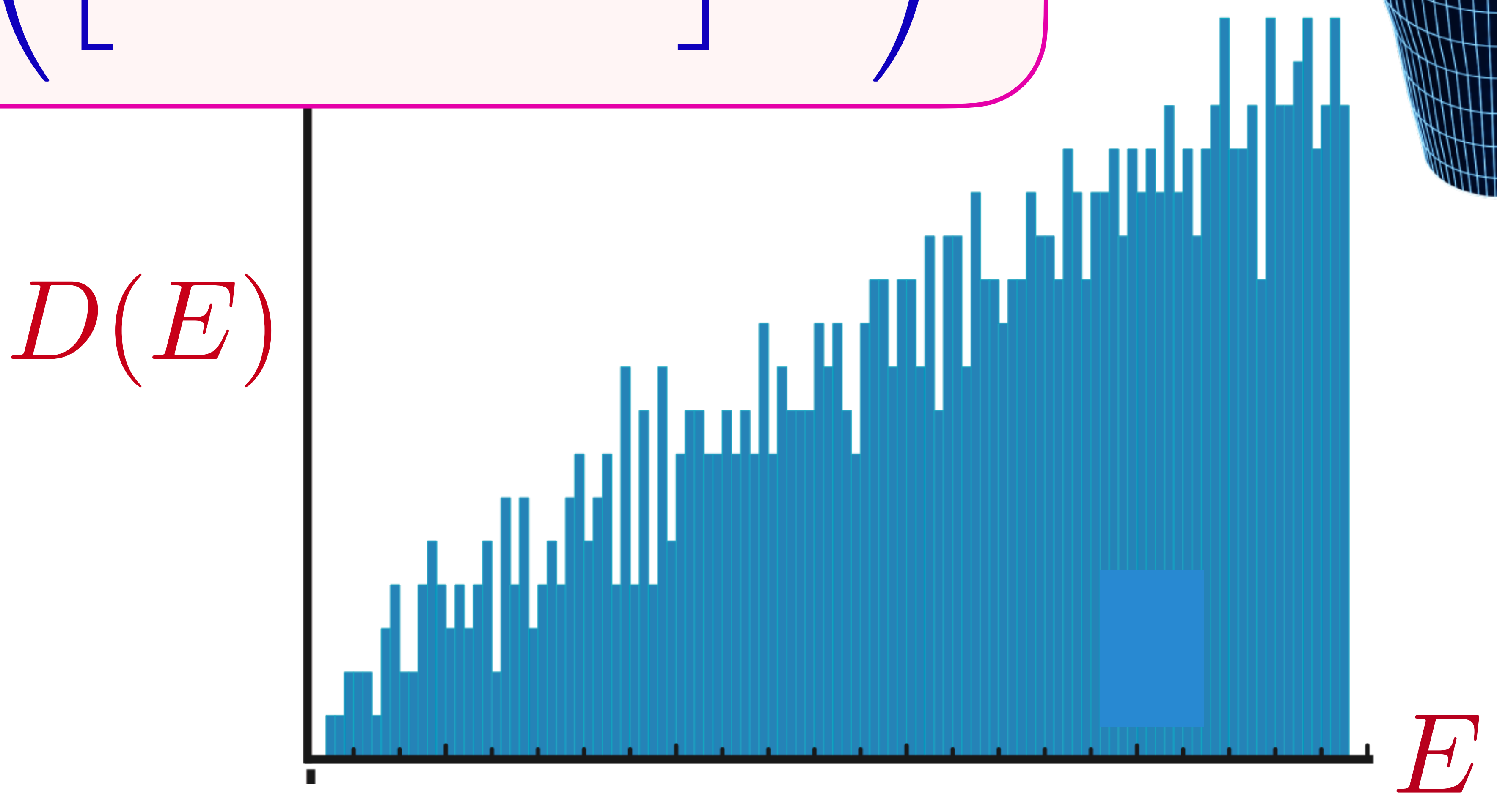
- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

Iliesiu, Murthy, Turiaci (2022) — Bekenstein-Hawking — Developments from the SYK model

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.





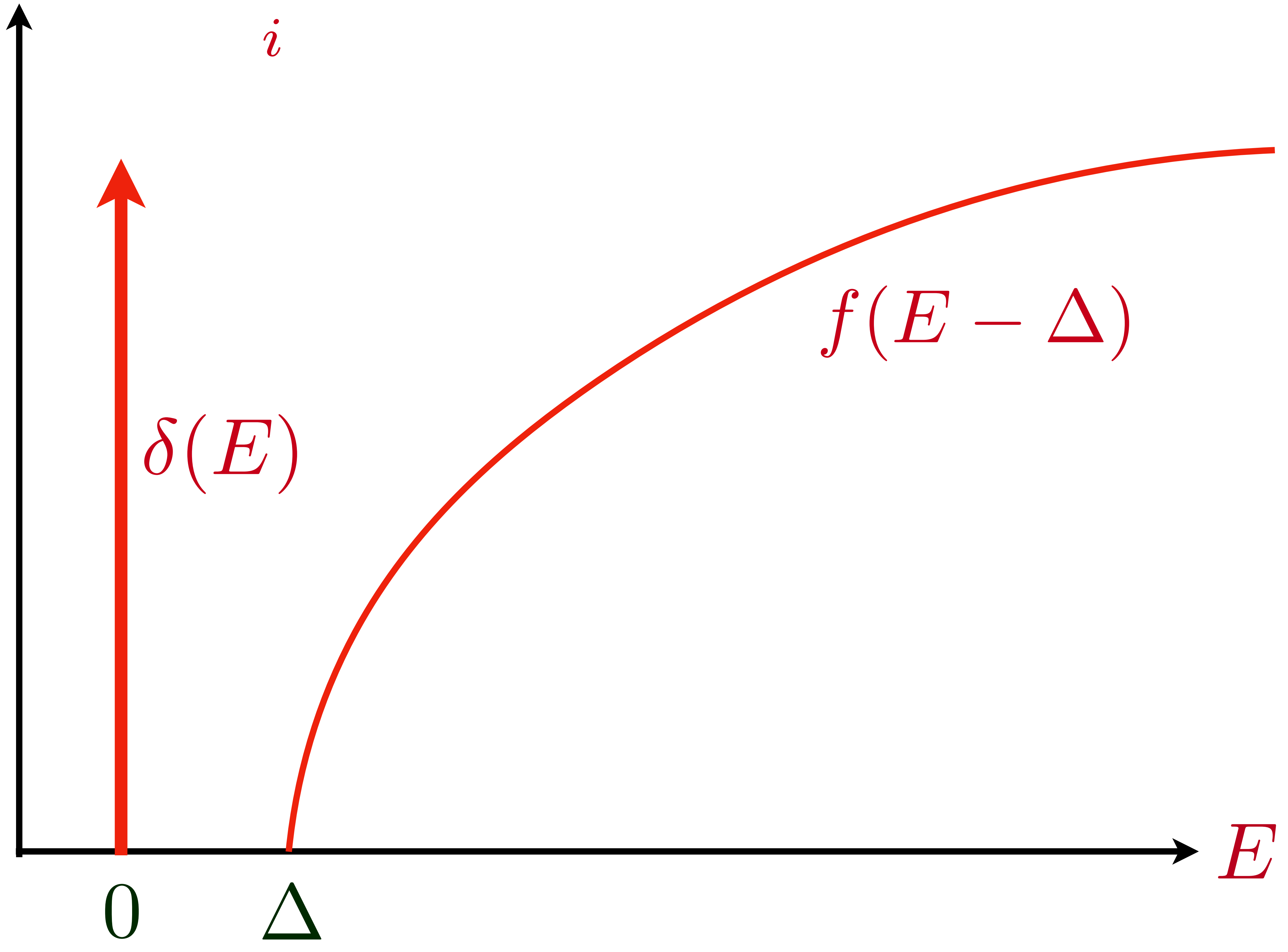
String theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots$$

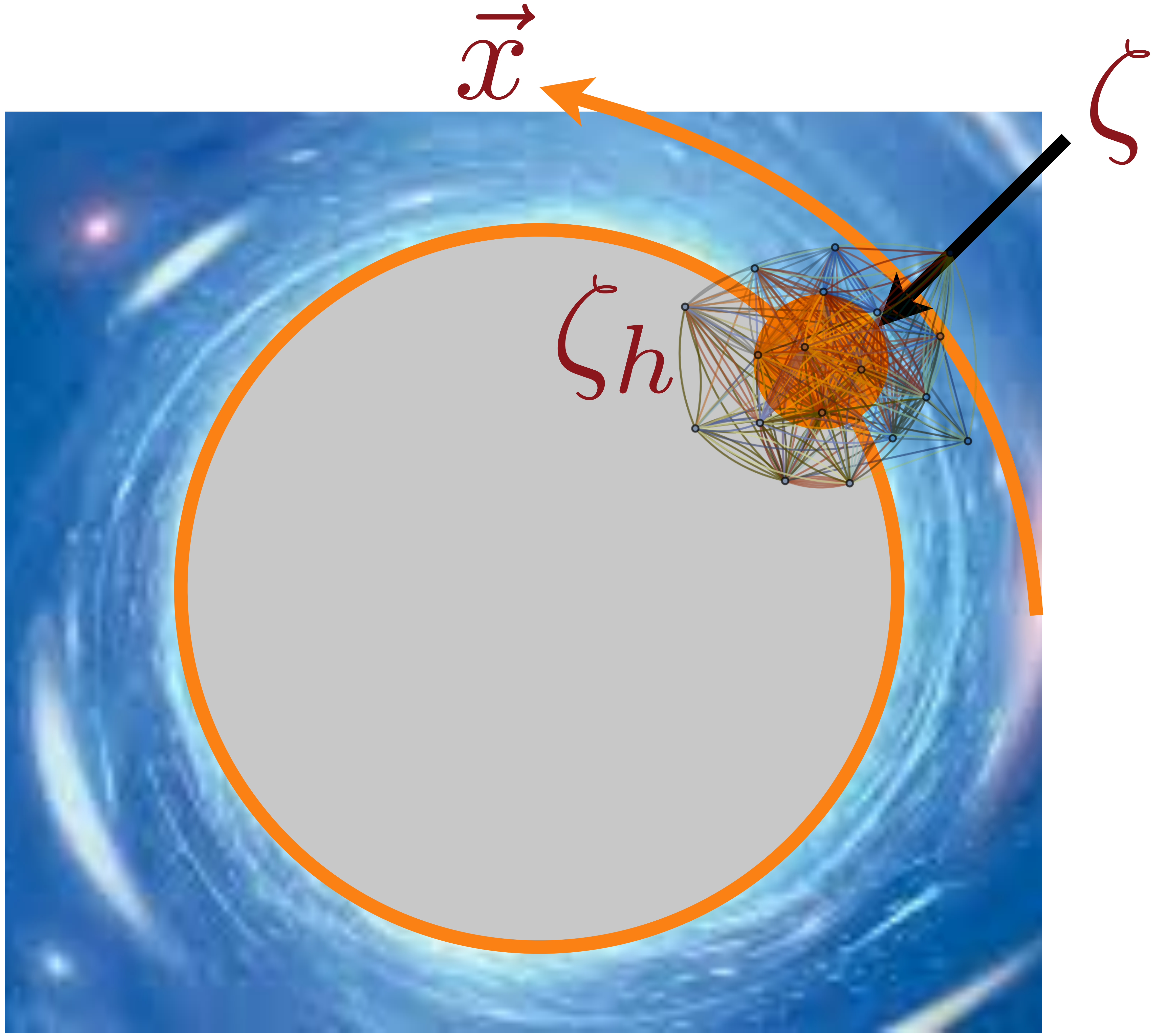
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

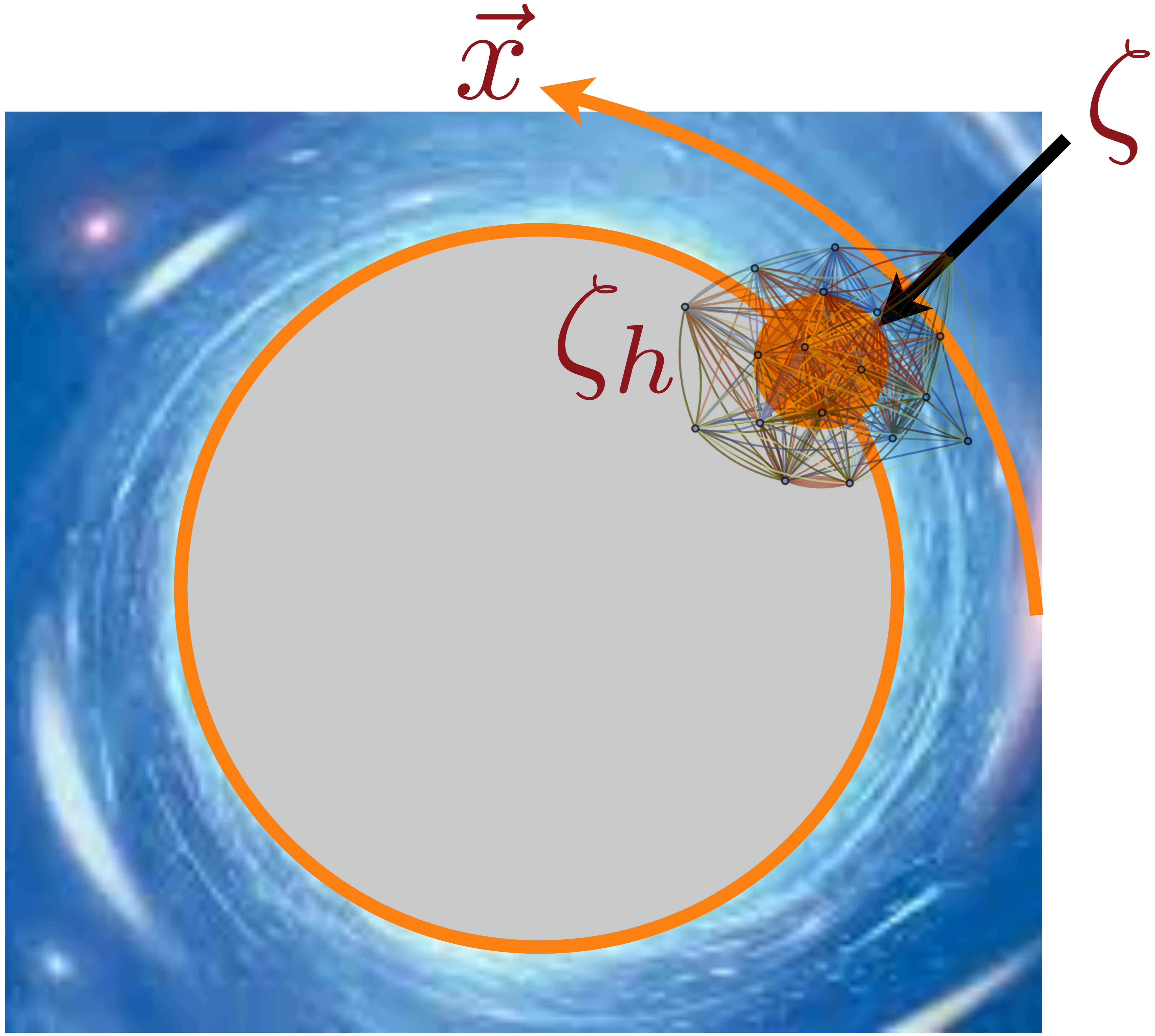
Quantum simulation of charged black holes by the SYK model



The SYK provides the needed realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model



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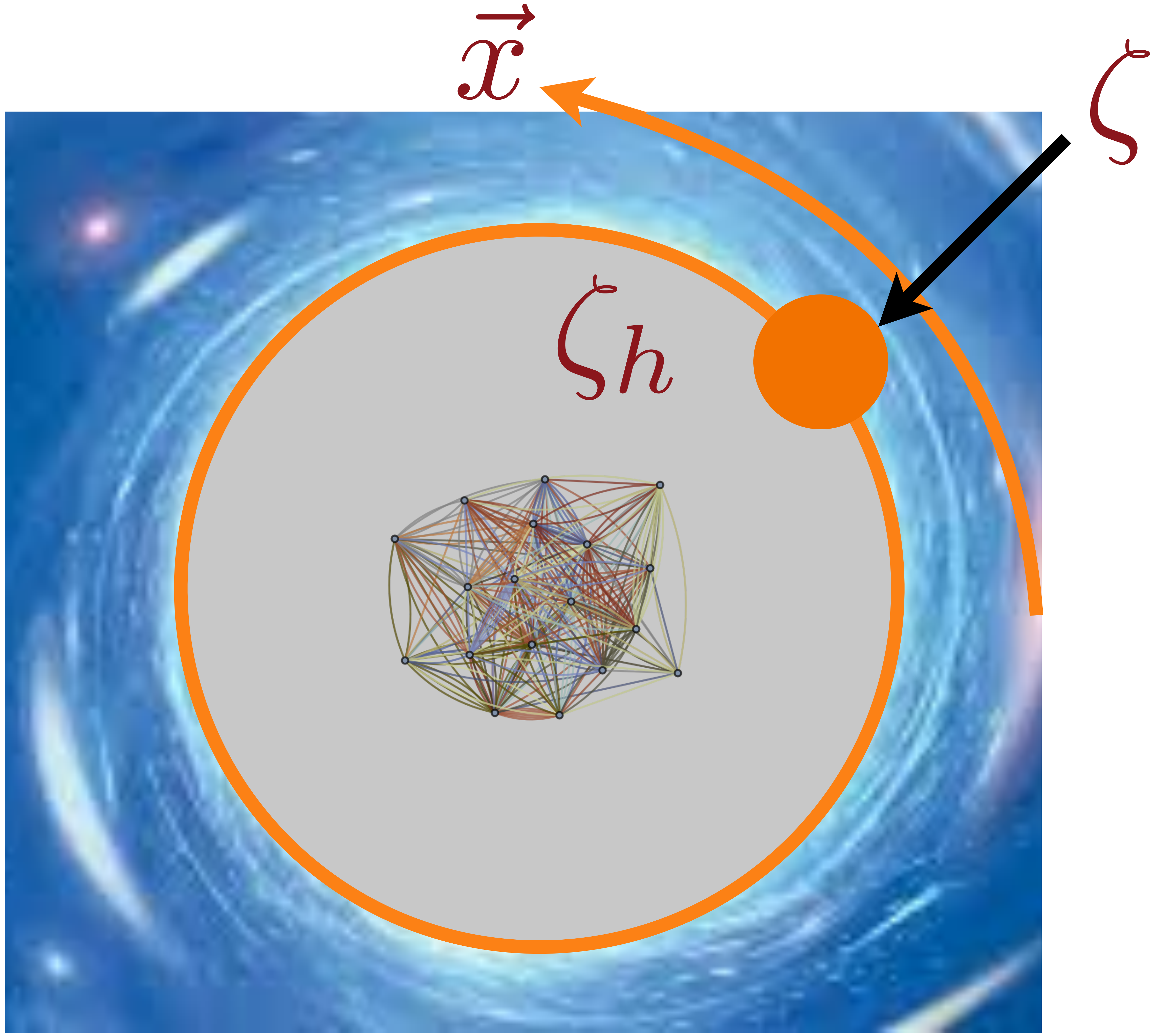

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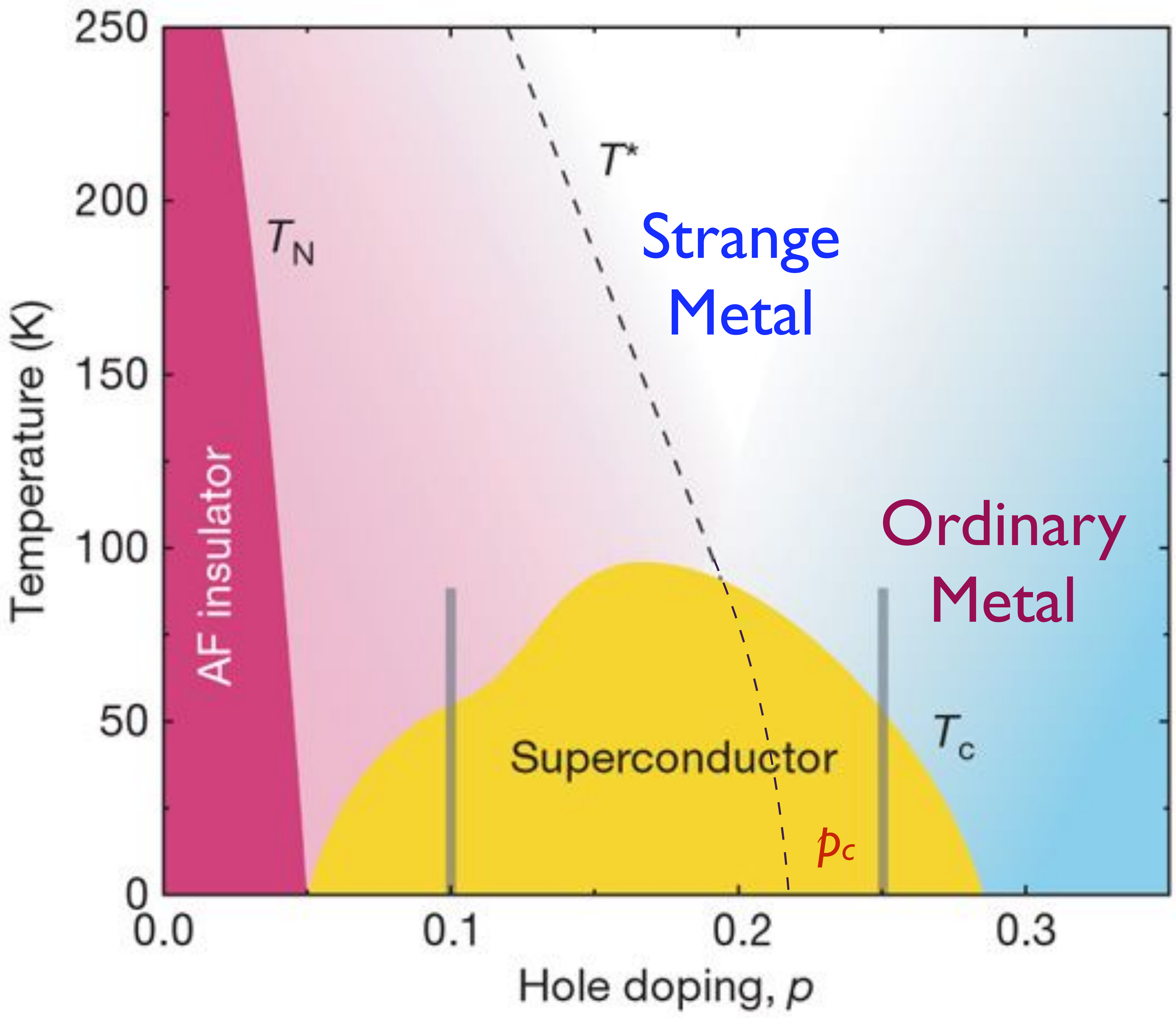
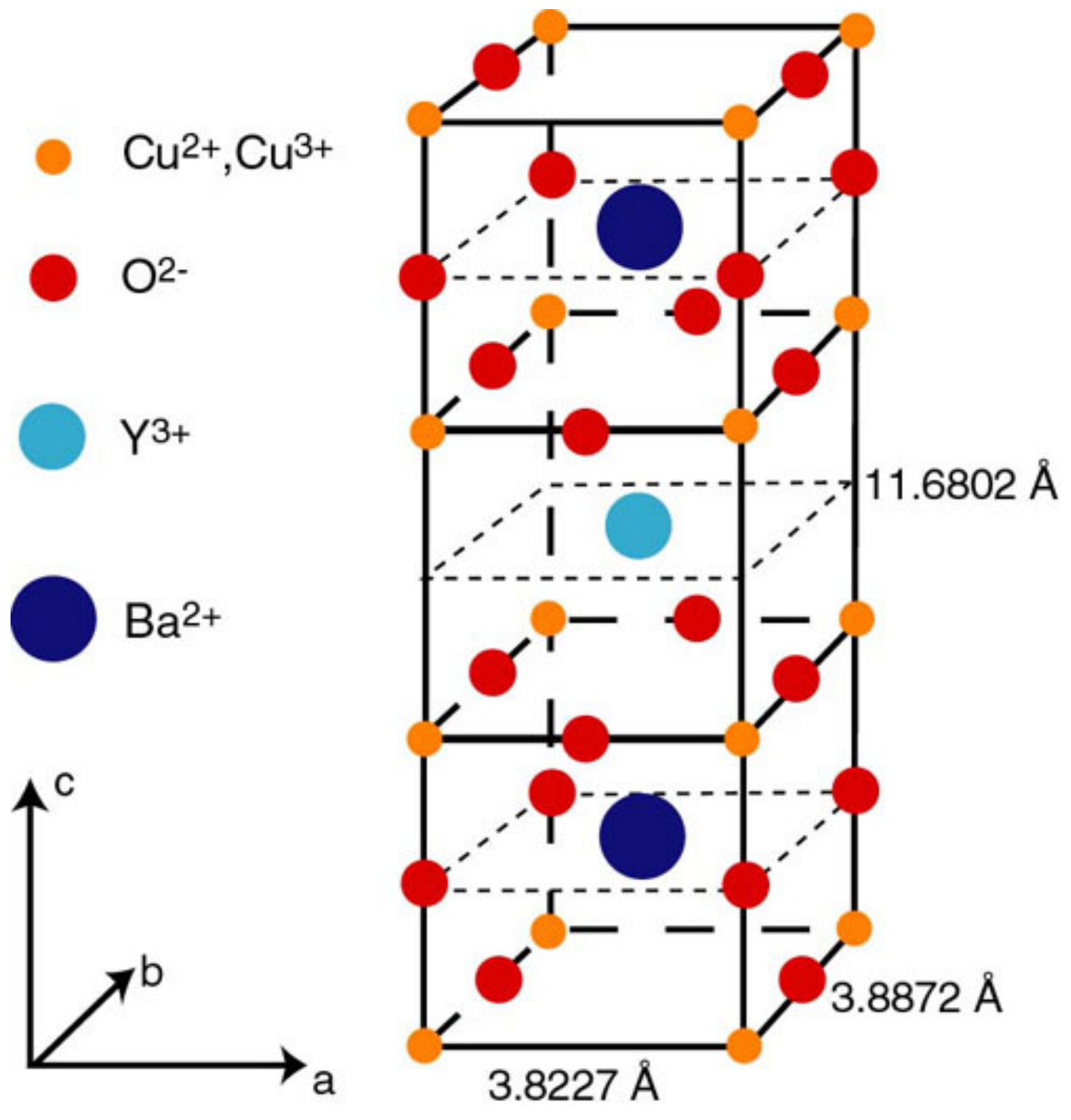
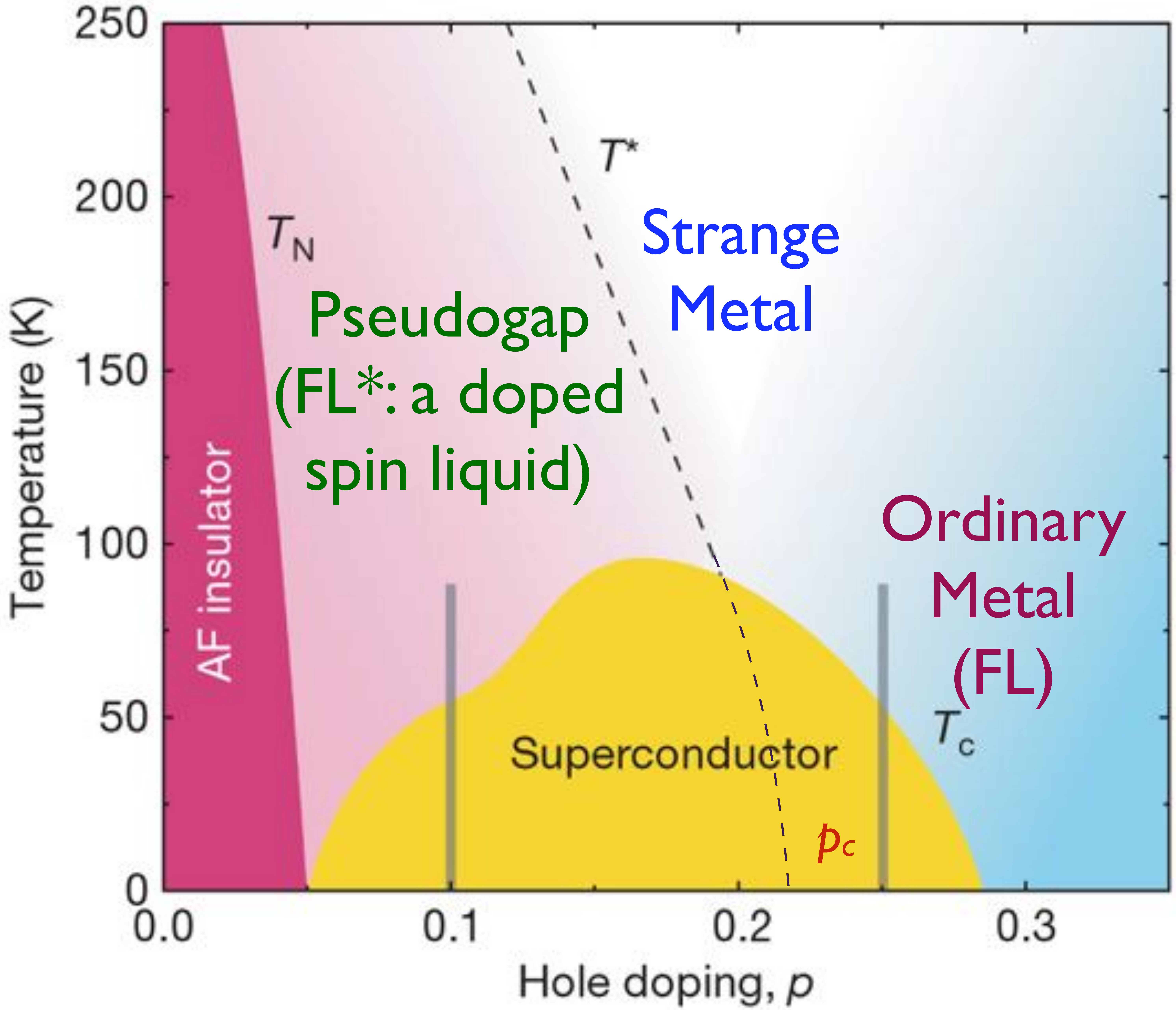
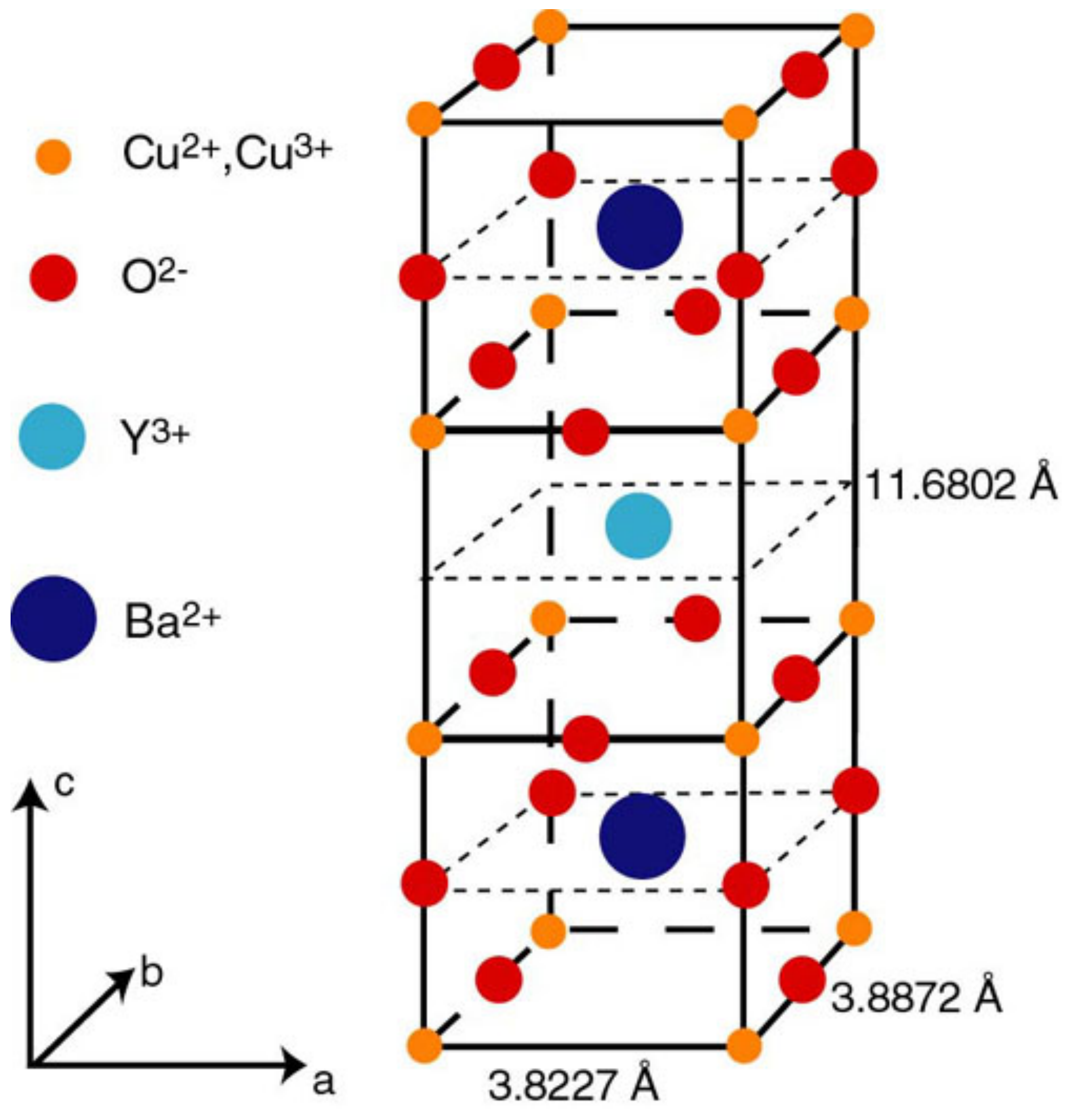
From the SYK model to a universal theory of strange metals


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

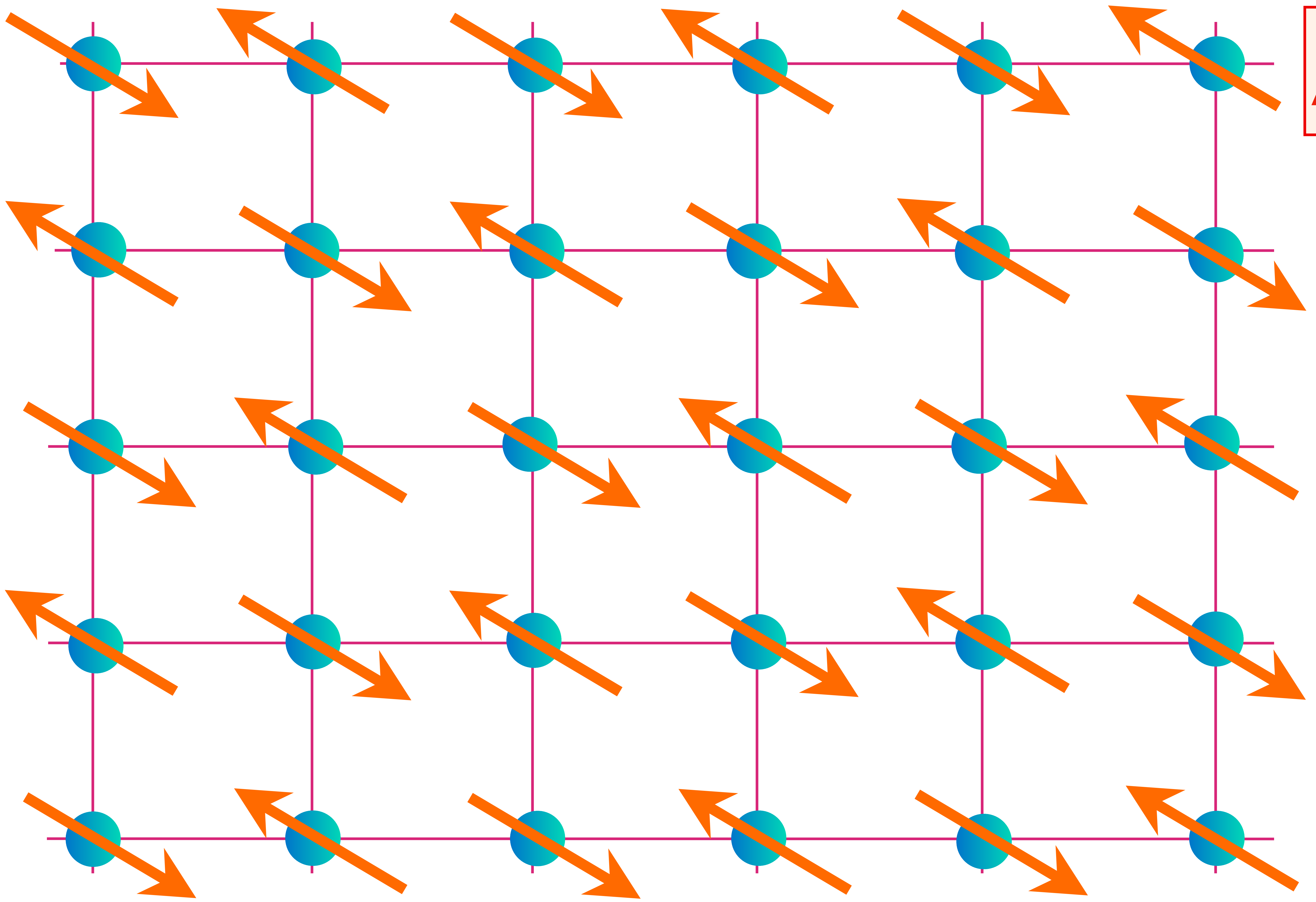
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Pseudogap
(FL*: a doped
spin liquid)
Strange
Metal
Ordinary
Metal
(FL)
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

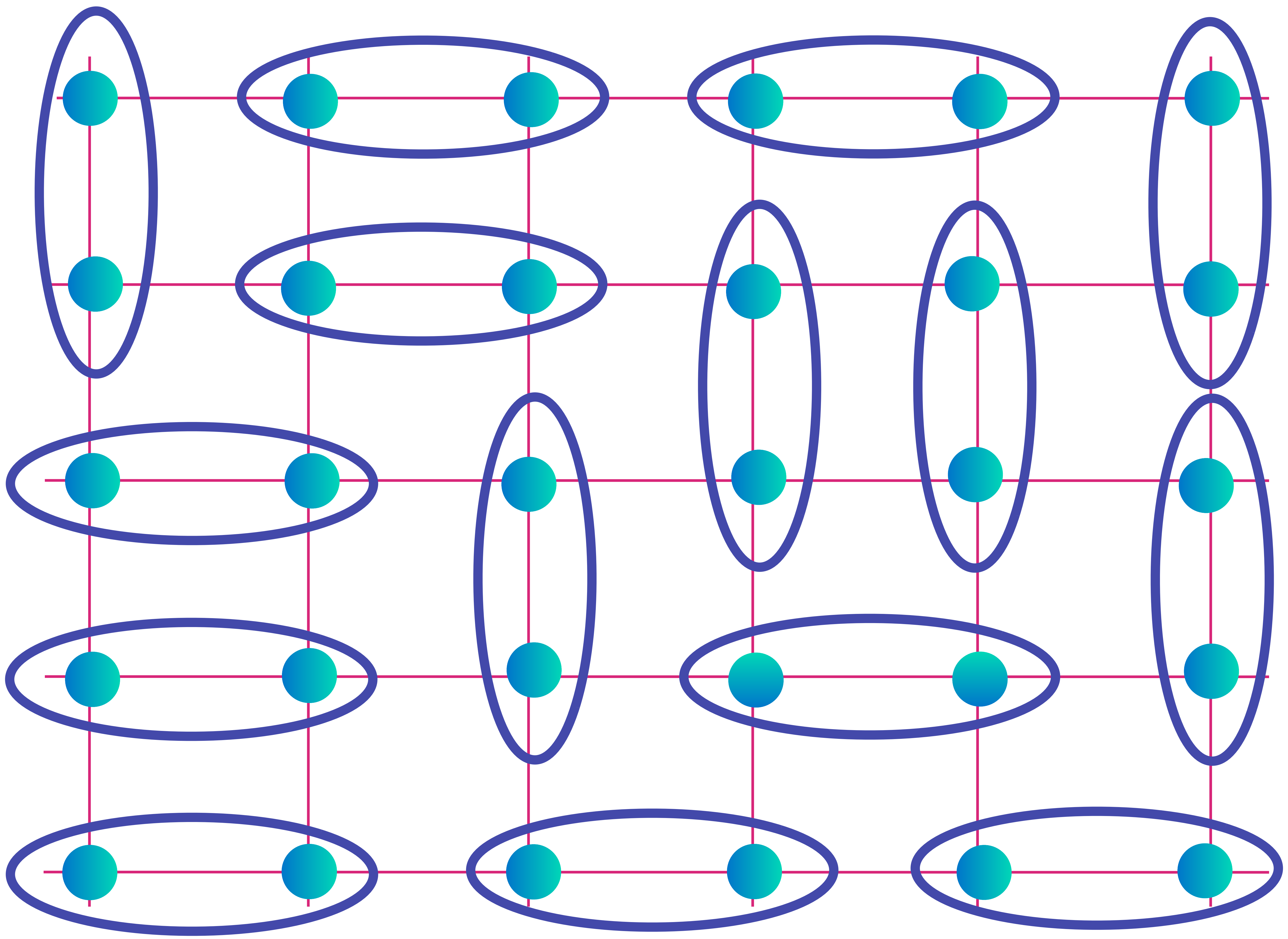
The dance of electrons on Cu atoms in YBCO



Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

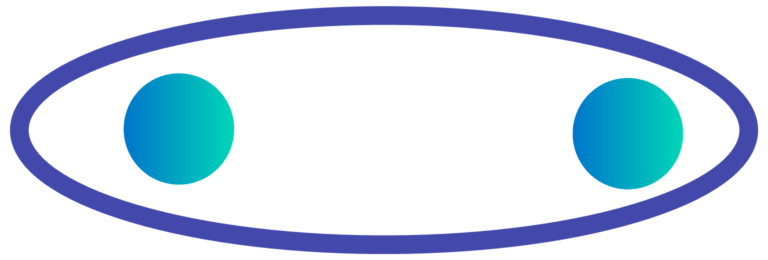
The dance of electrons on Cu atoms in YBCO



P.W. Anderson (1973)

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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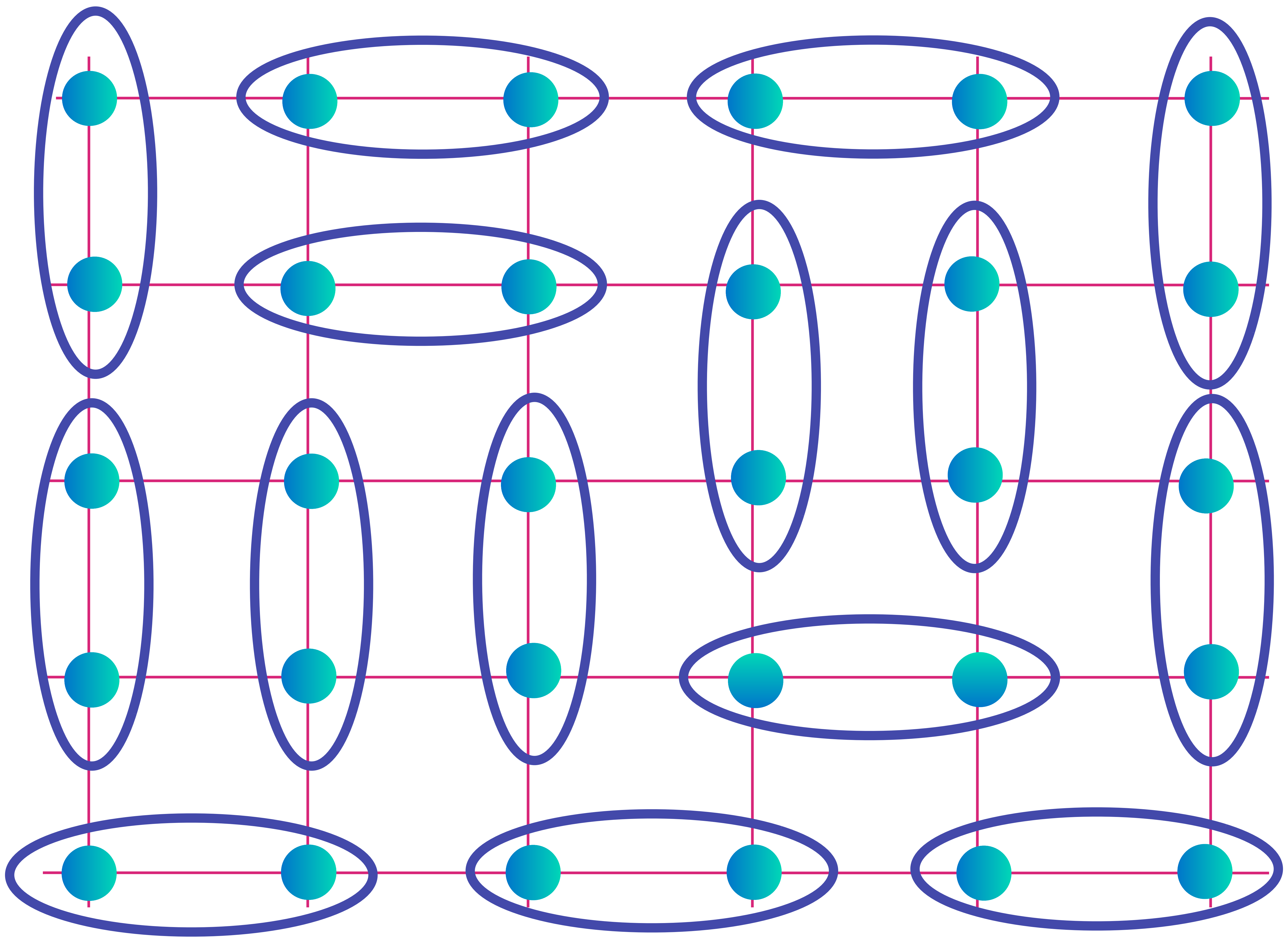

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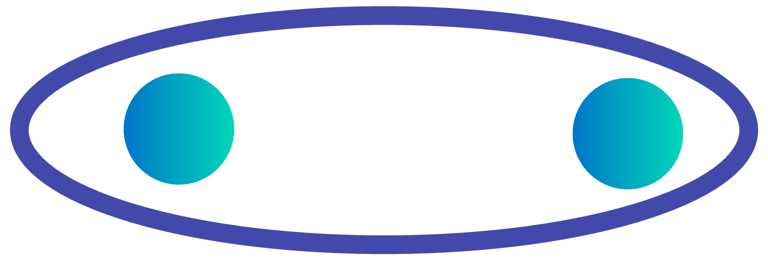

$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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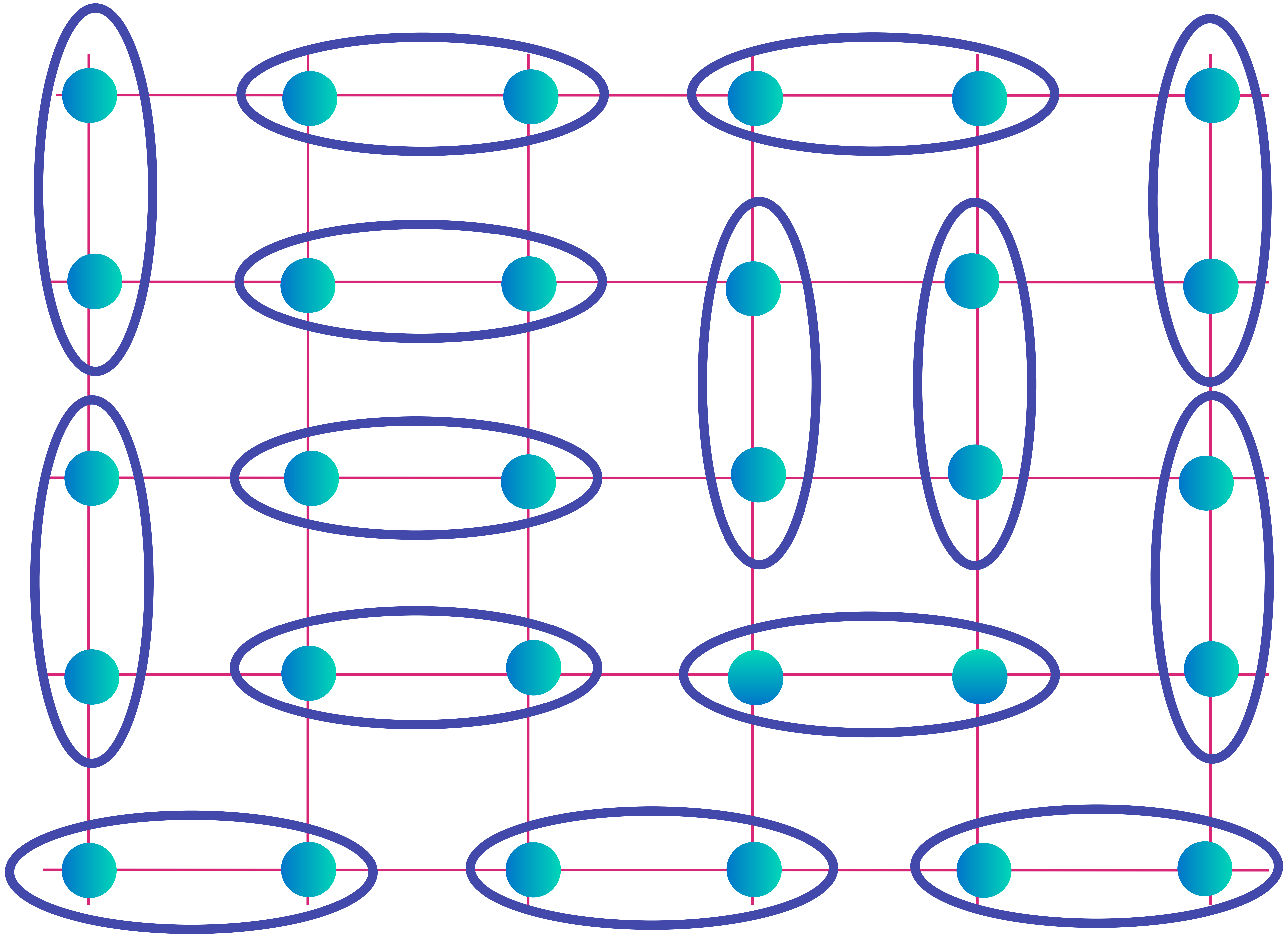
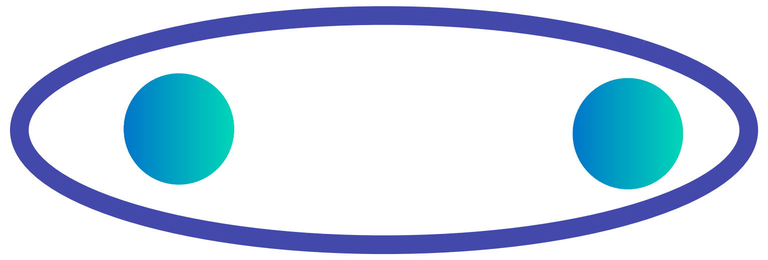


Electrons form entangled pairs, and the pairs entangle across the entire sample

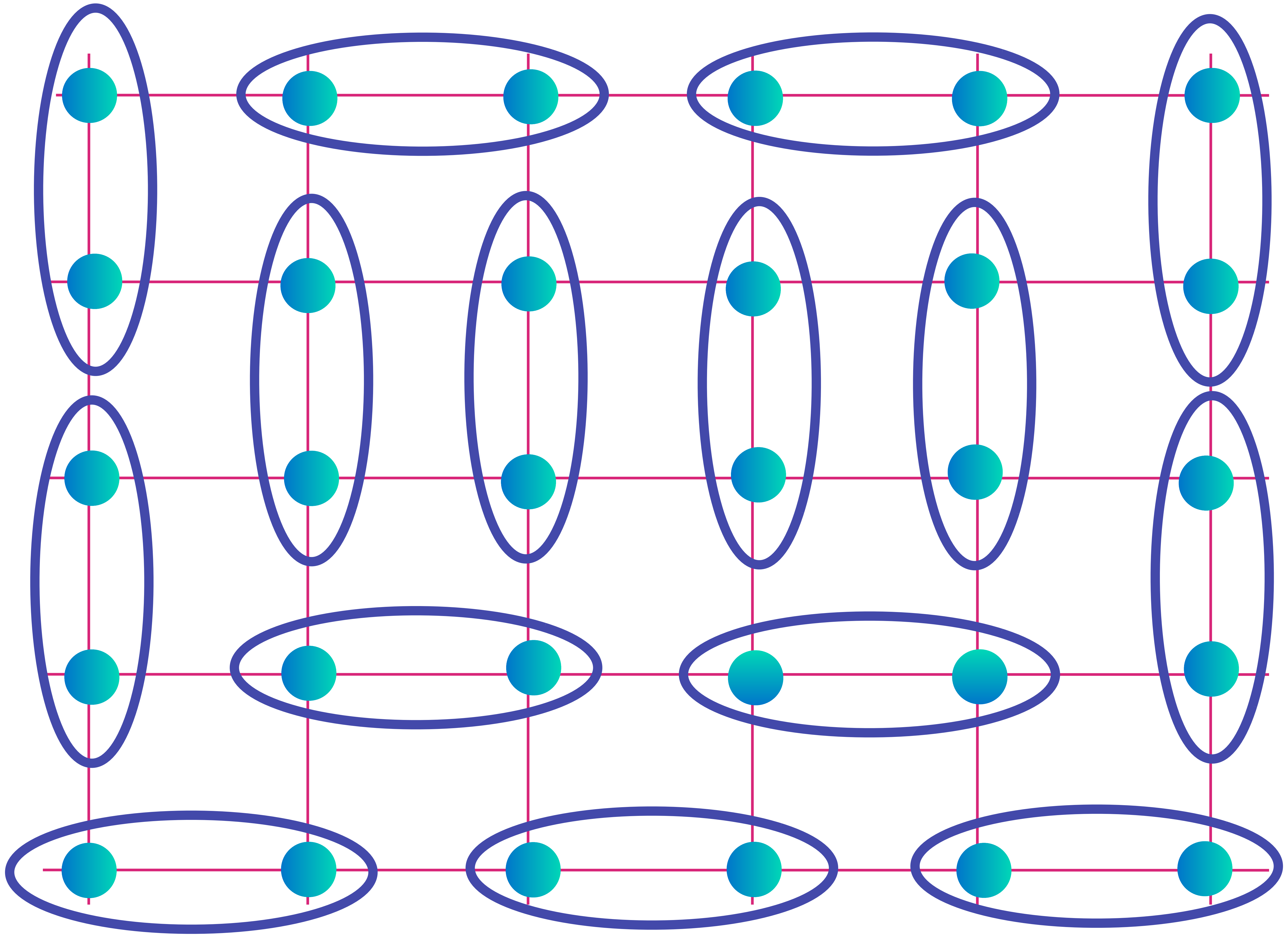


$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



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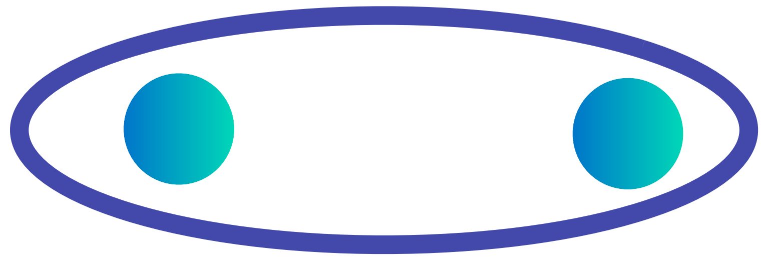

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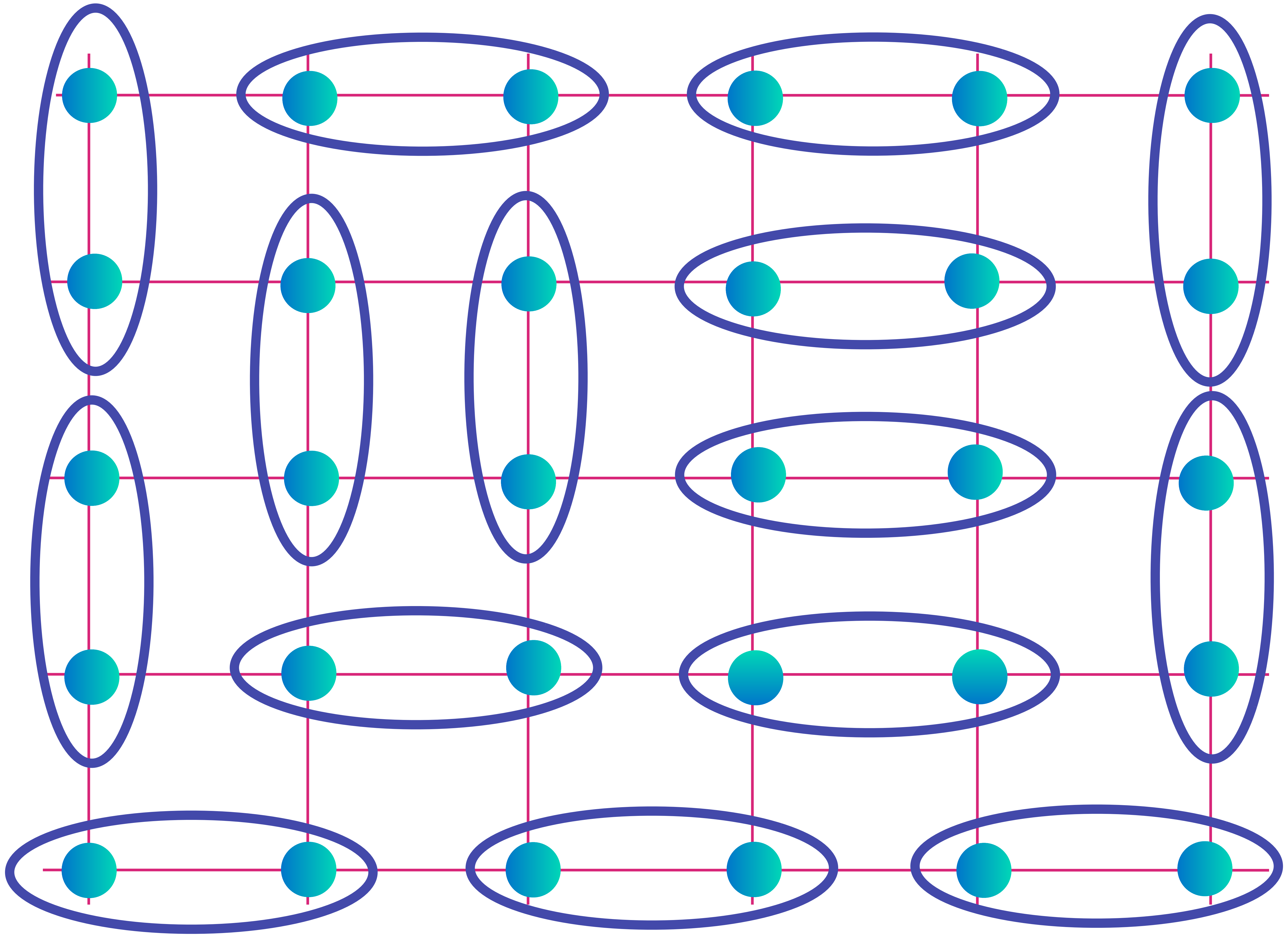

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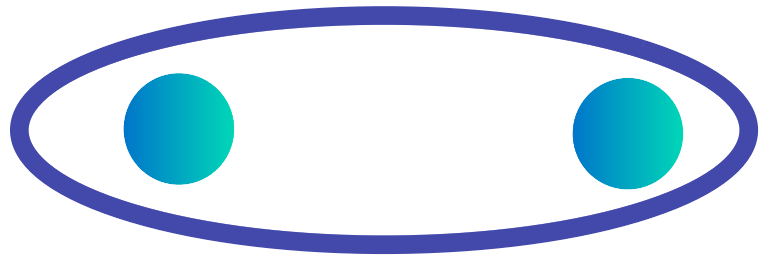

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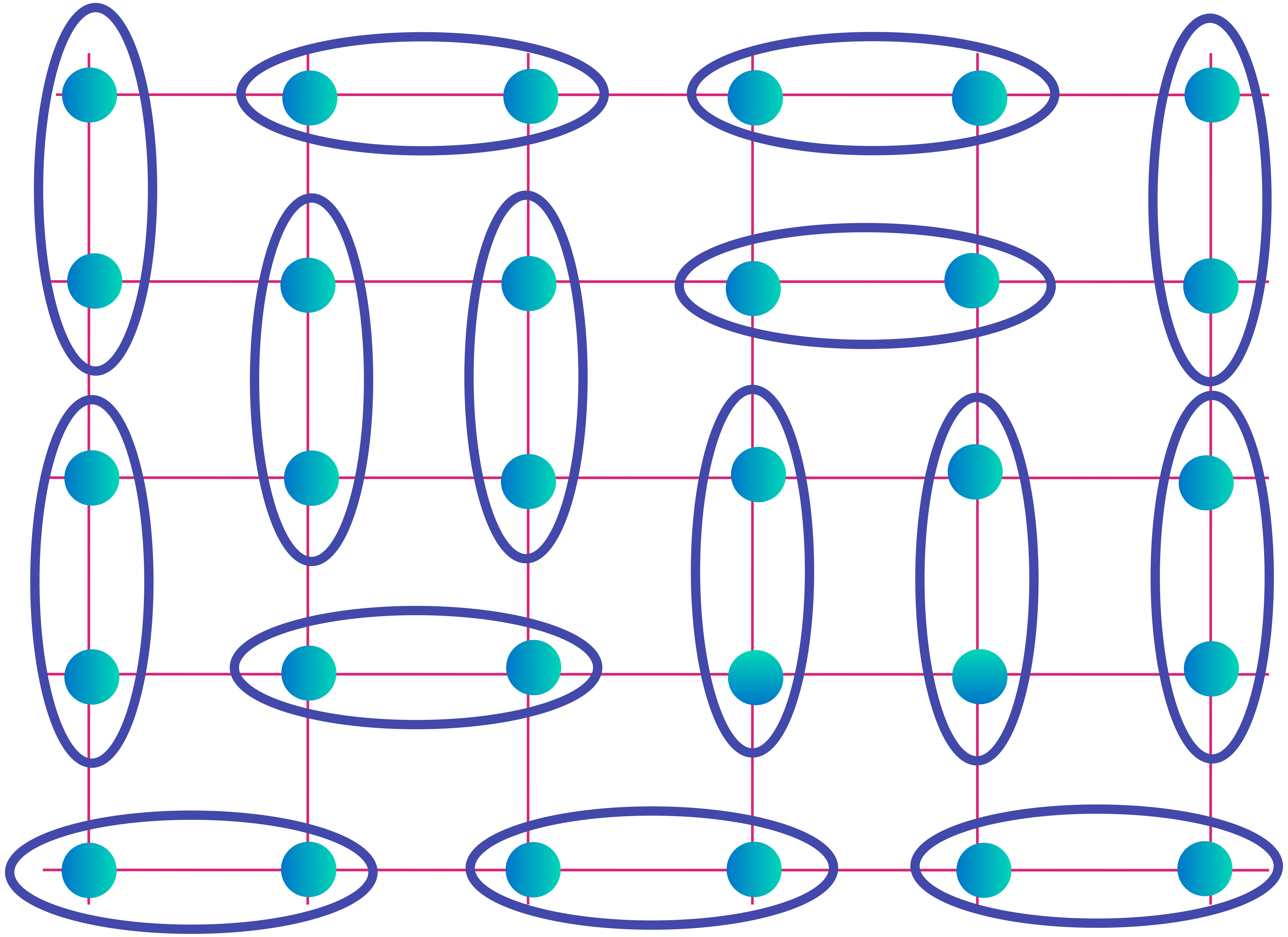

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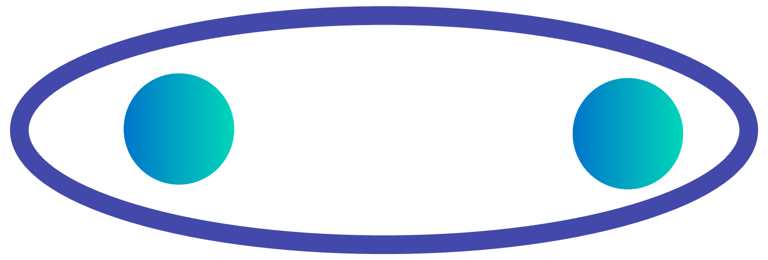

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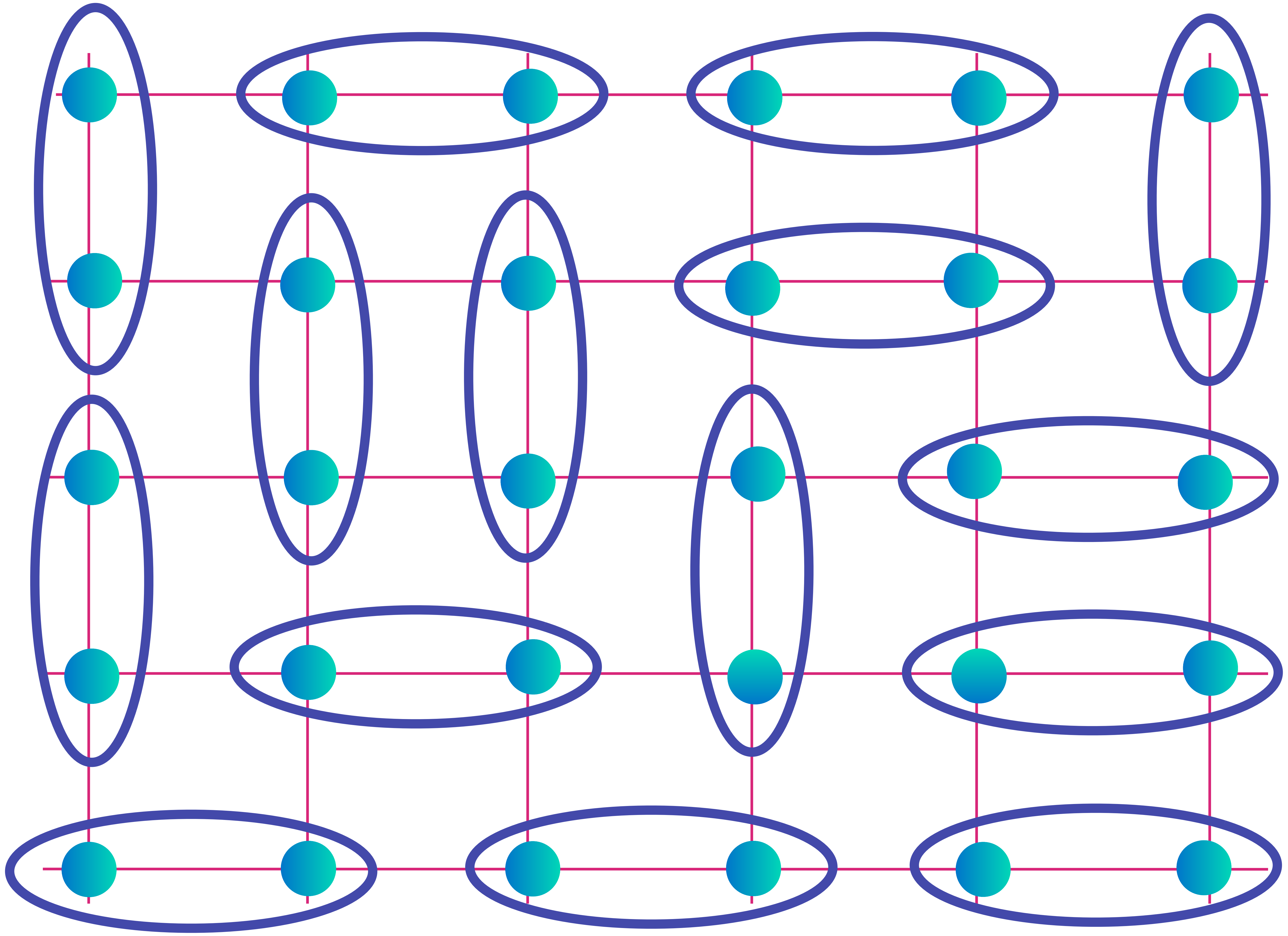

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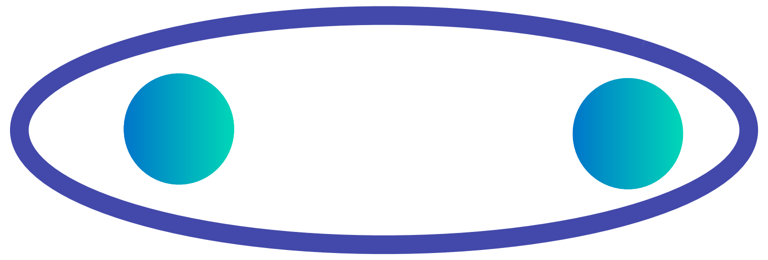

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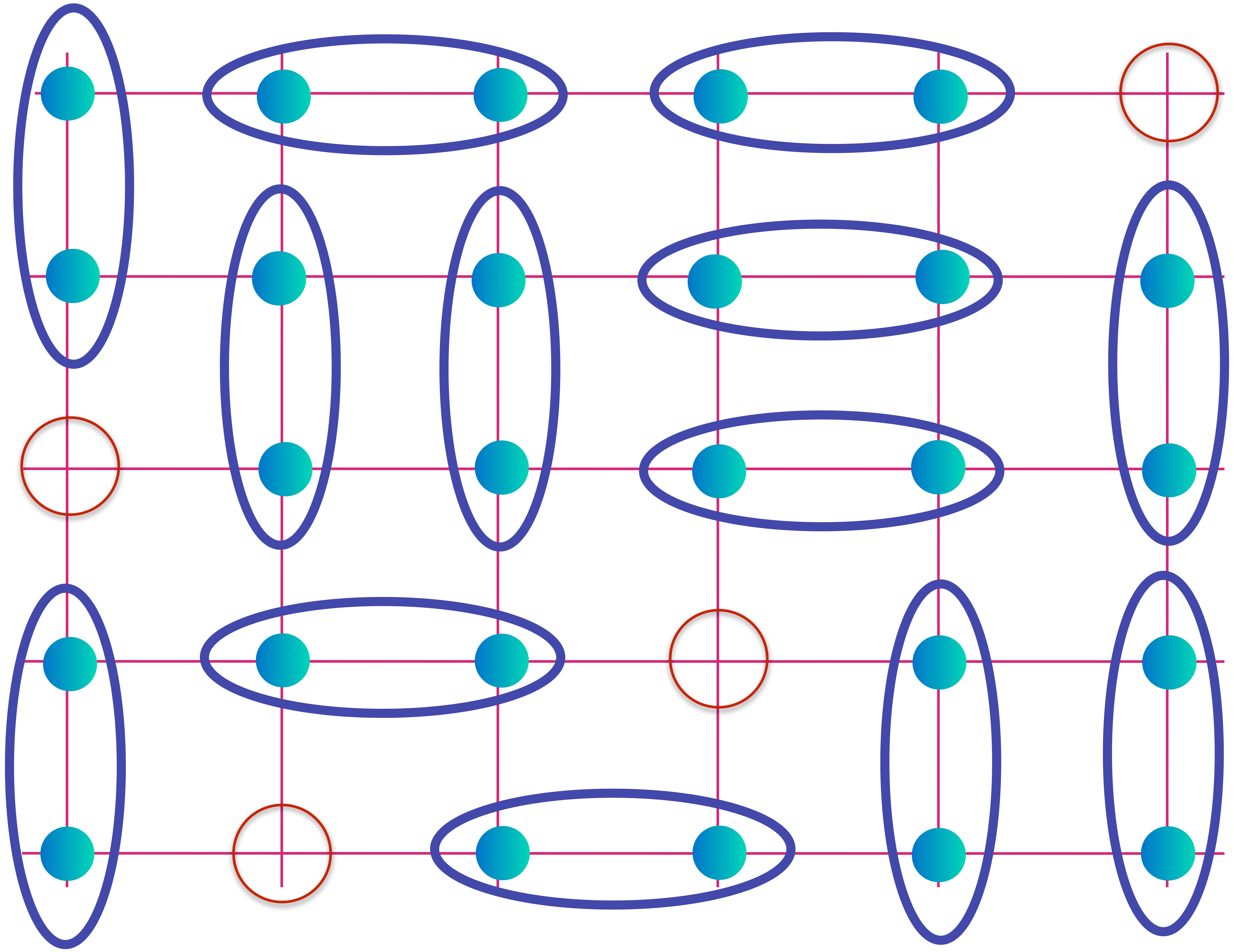
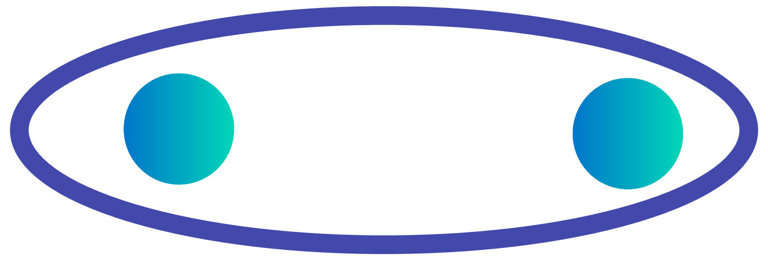


Holon metal

Spin liquid with density p of spinless, charge +e “holons”

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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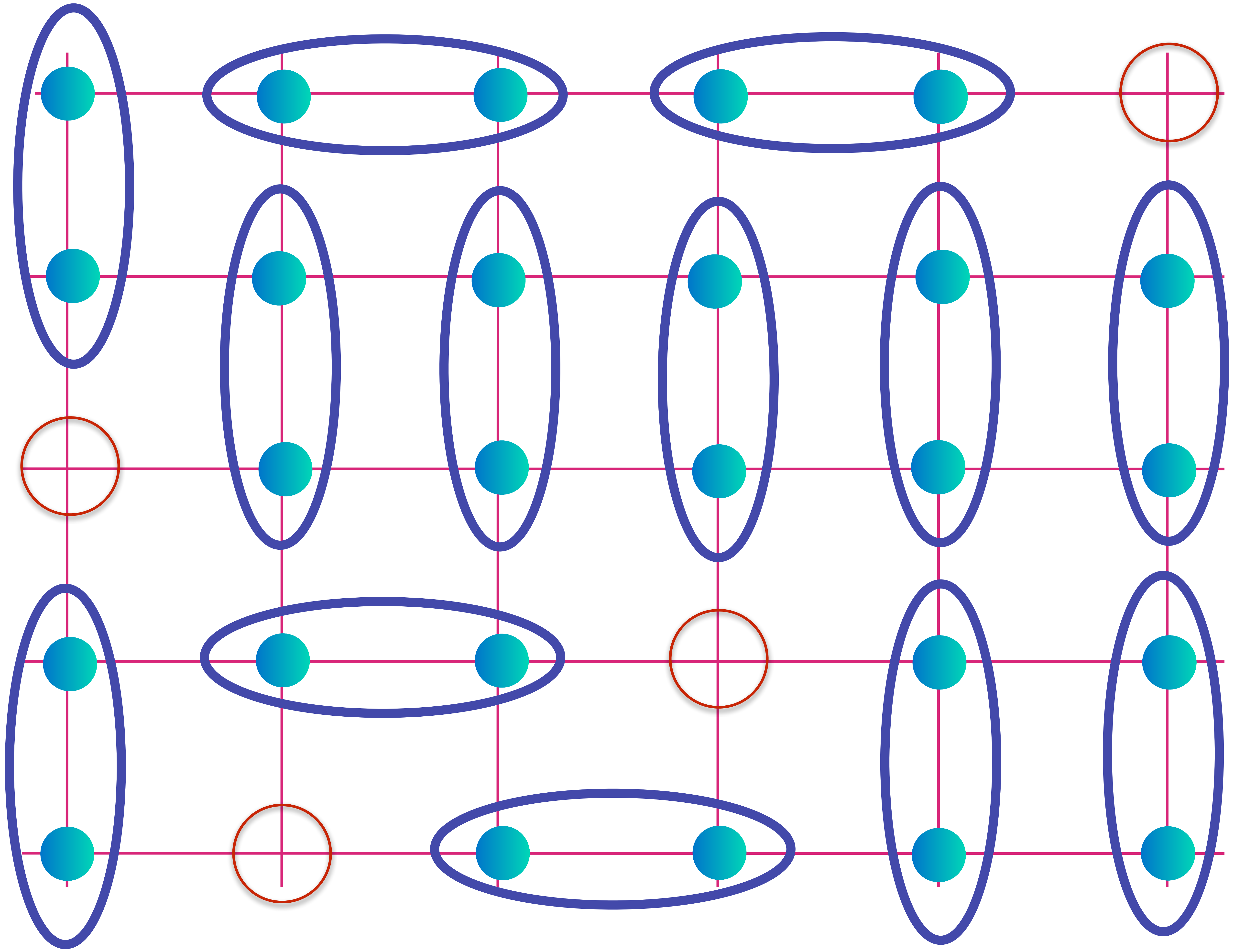
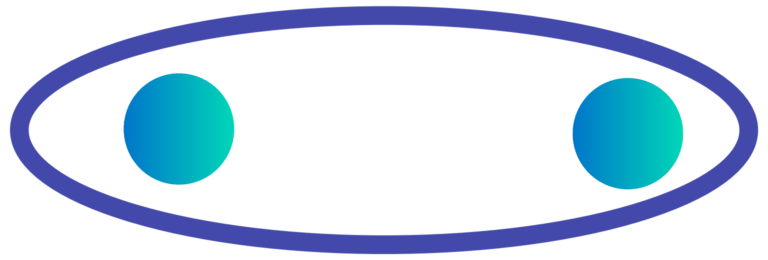


Holon metal

Spin liquid with density *p* of spinless, charge +e "holons"

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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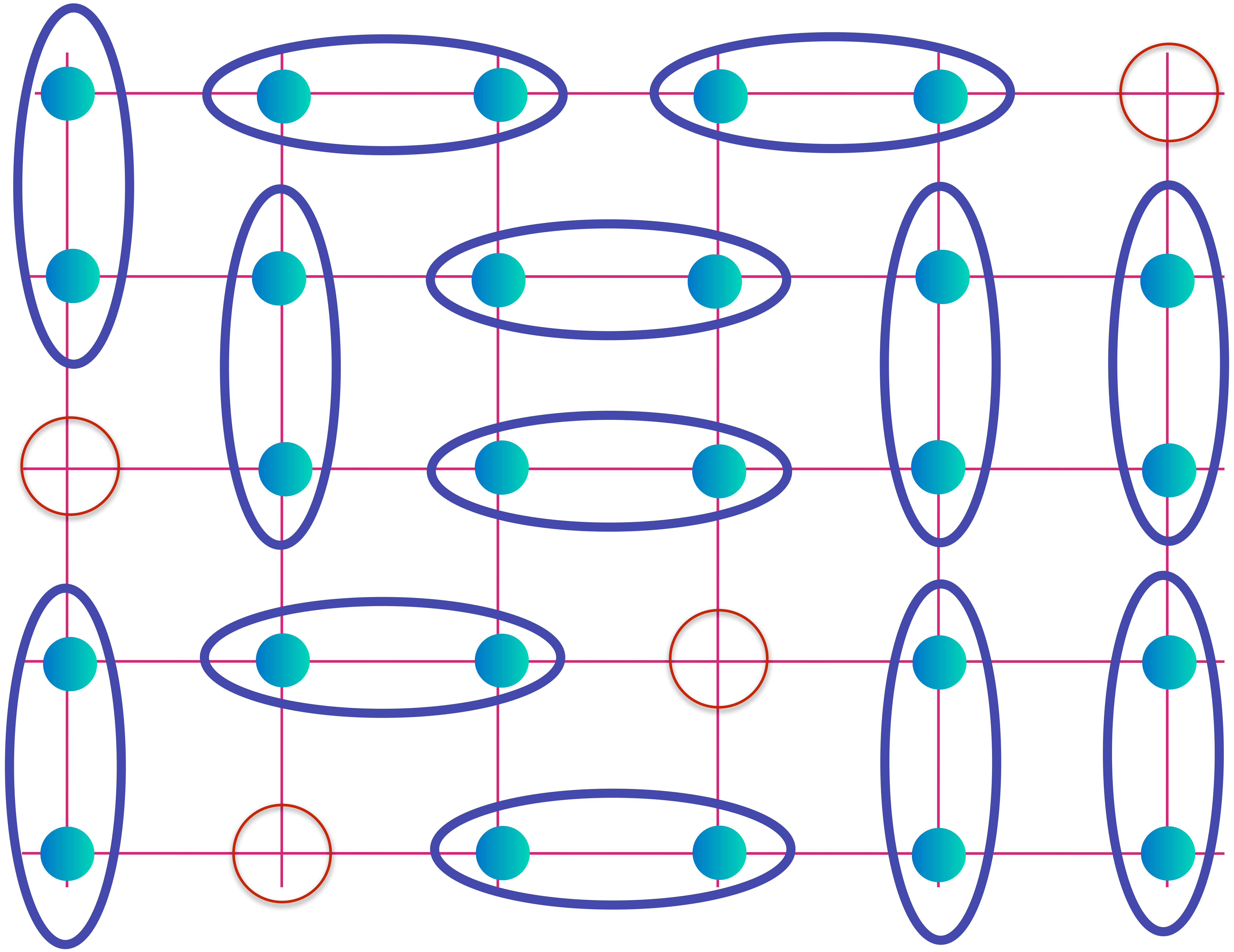

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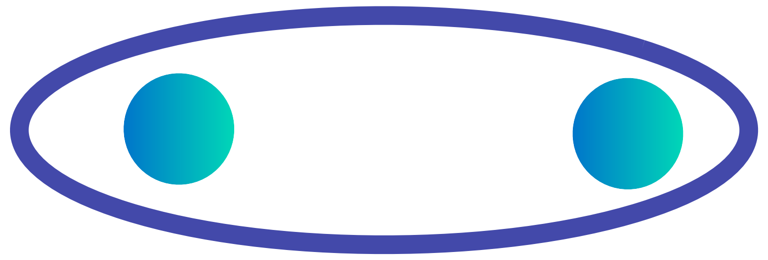 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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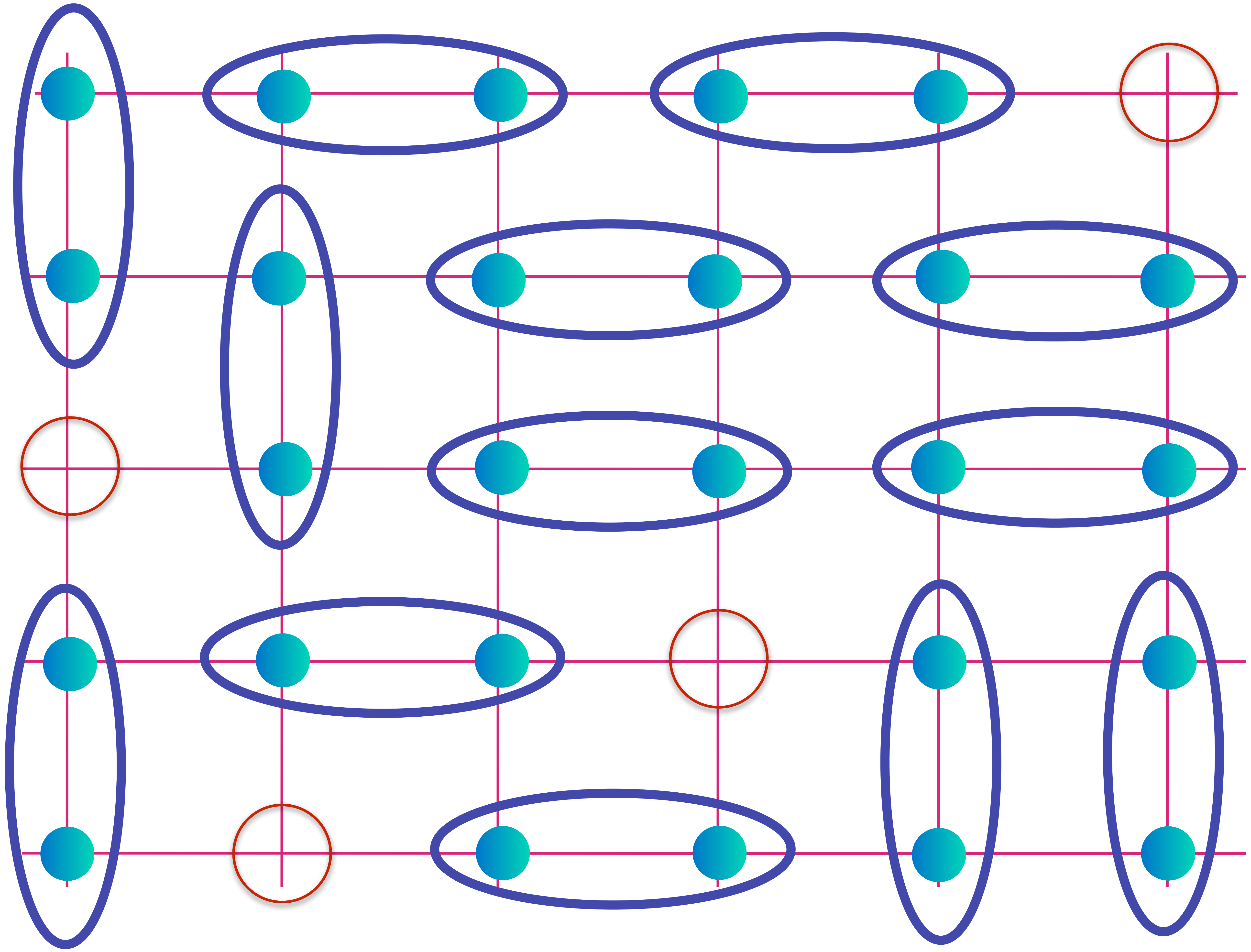

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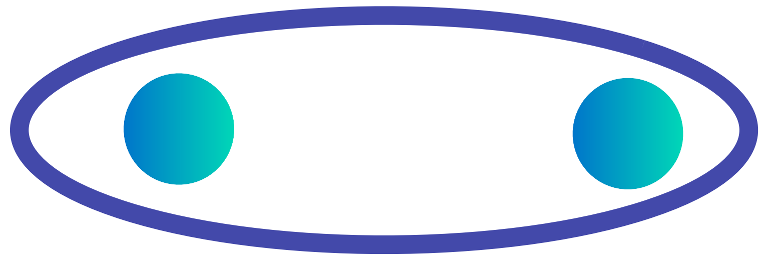 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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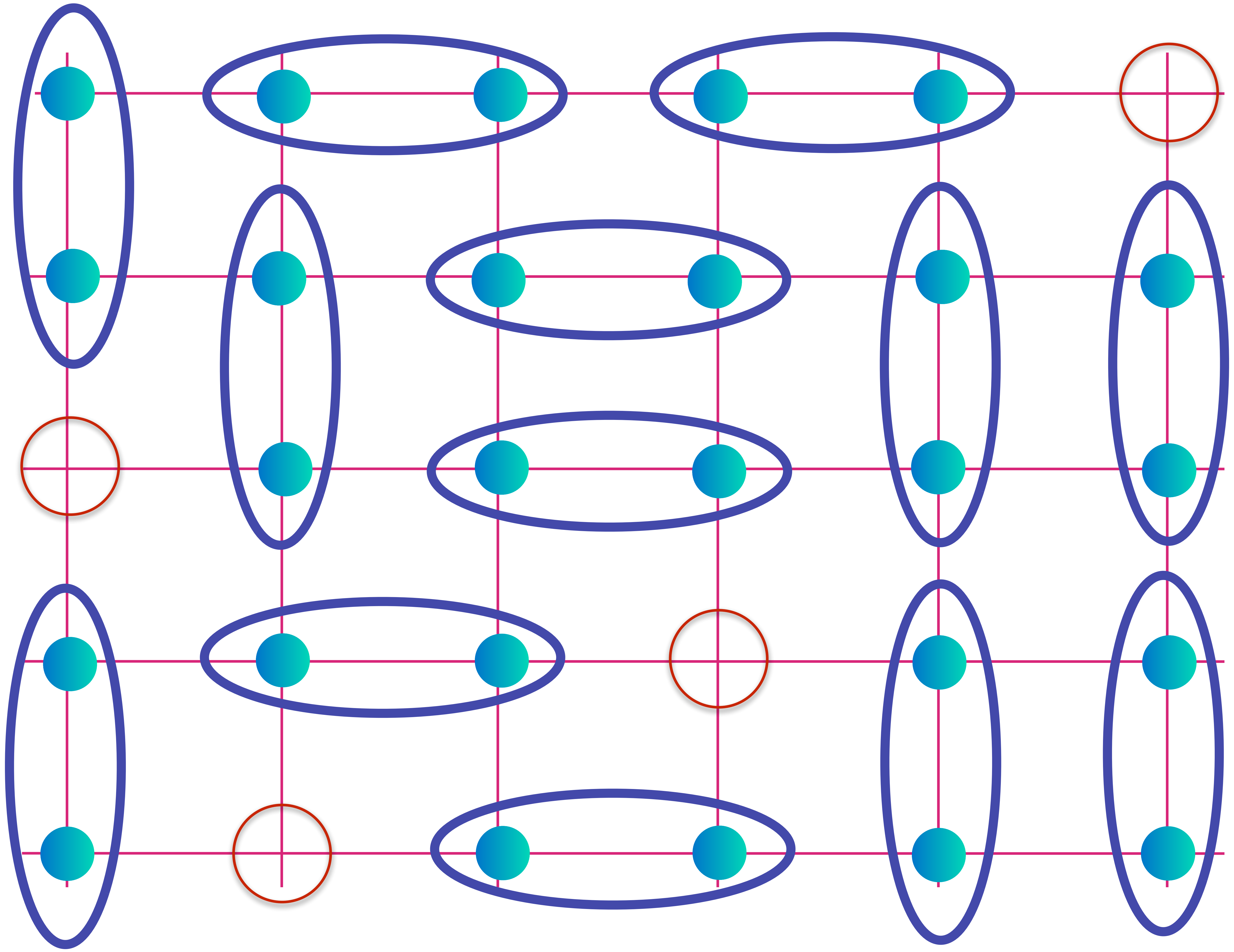

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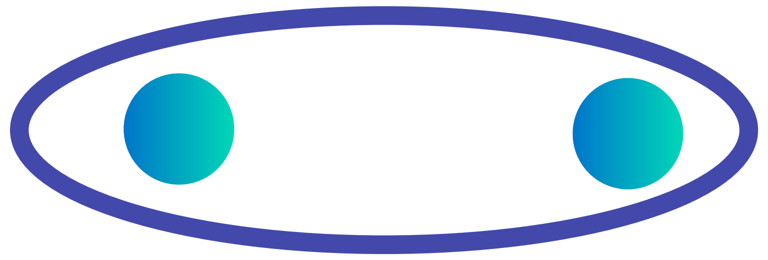 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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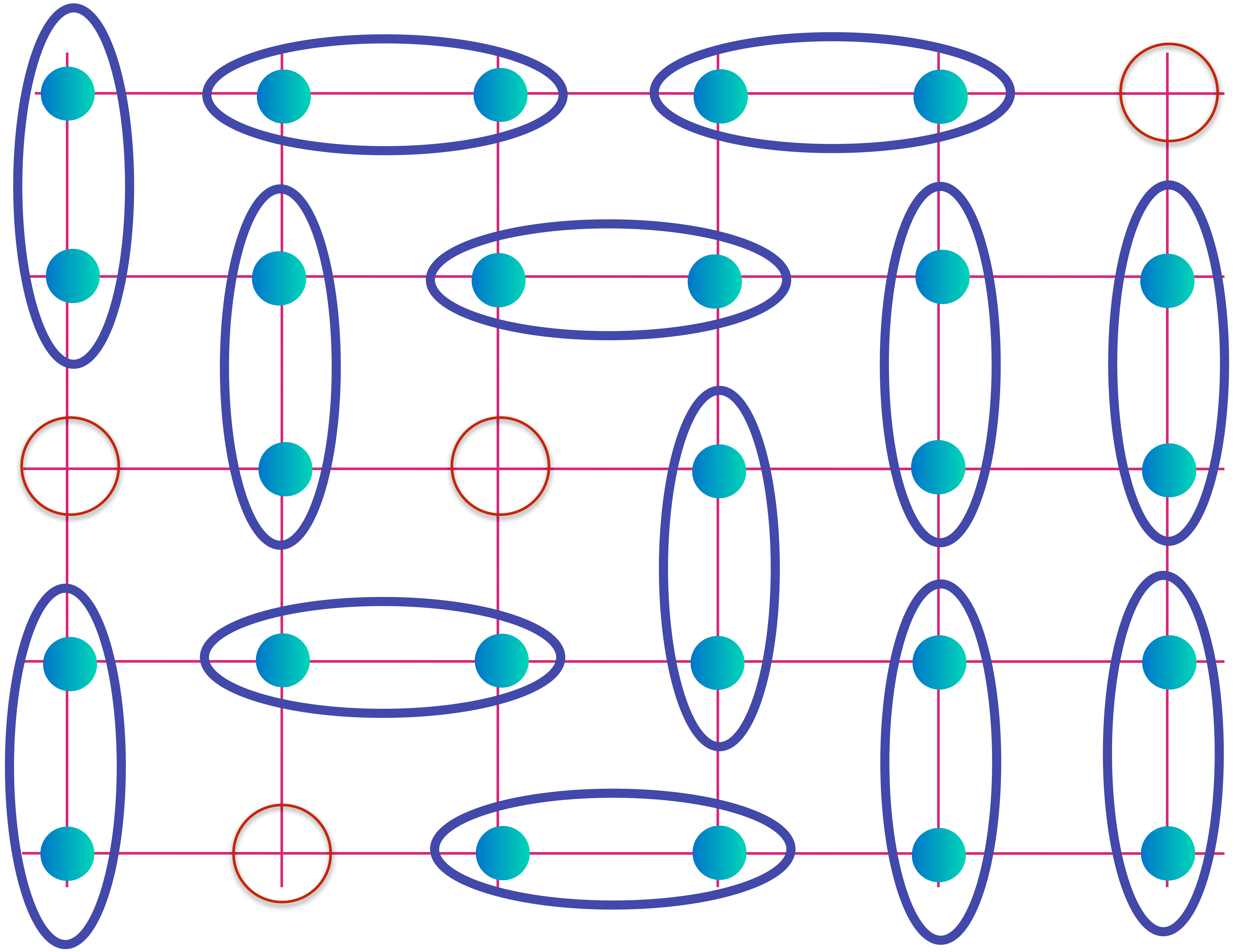

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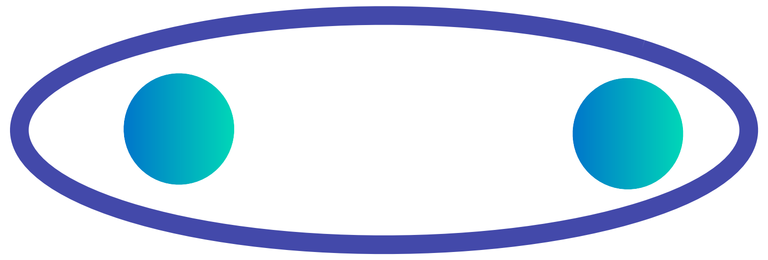

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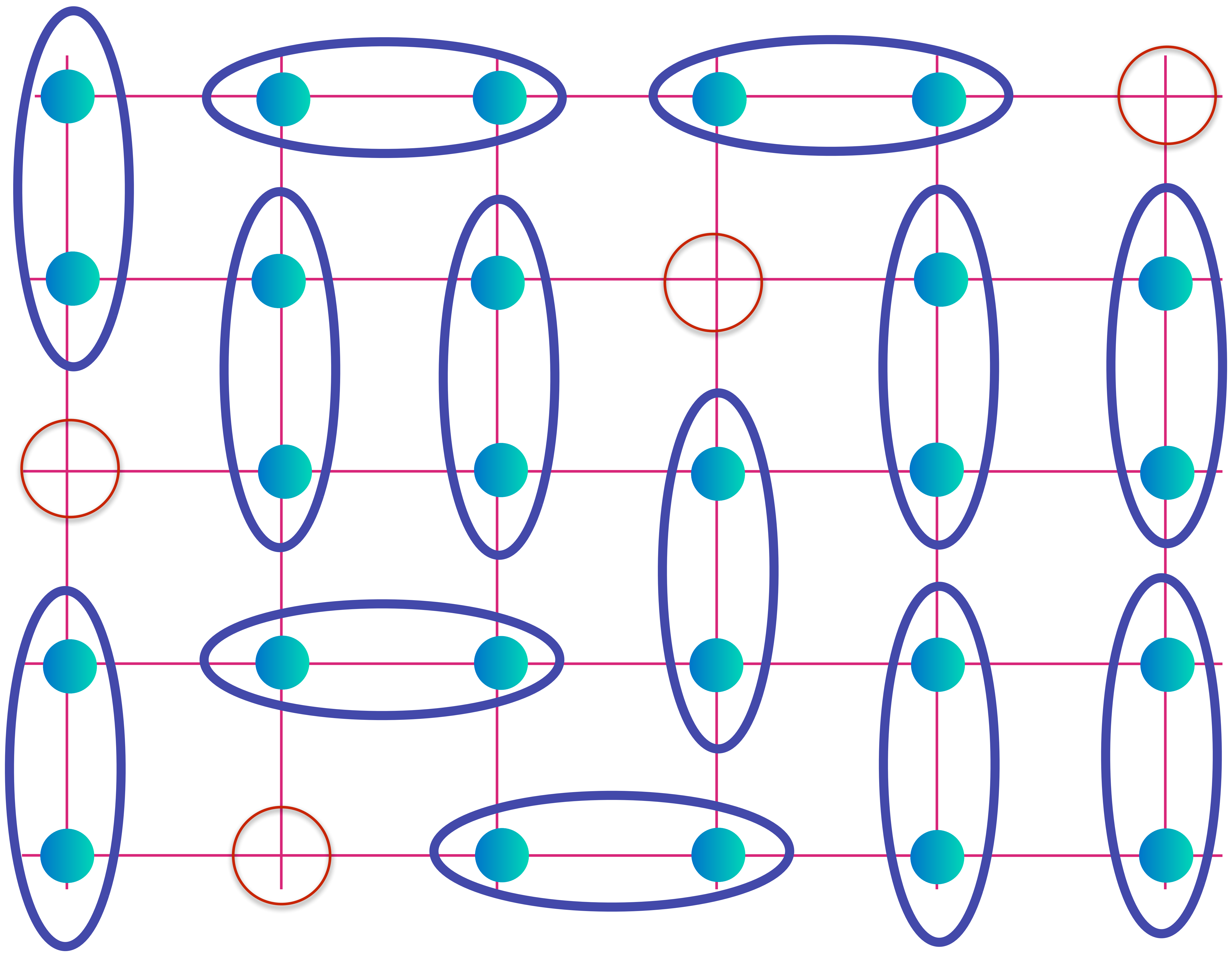

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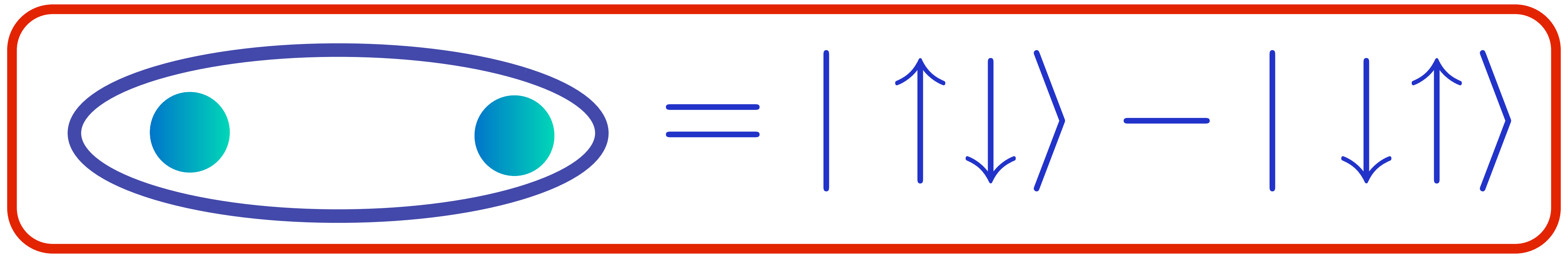

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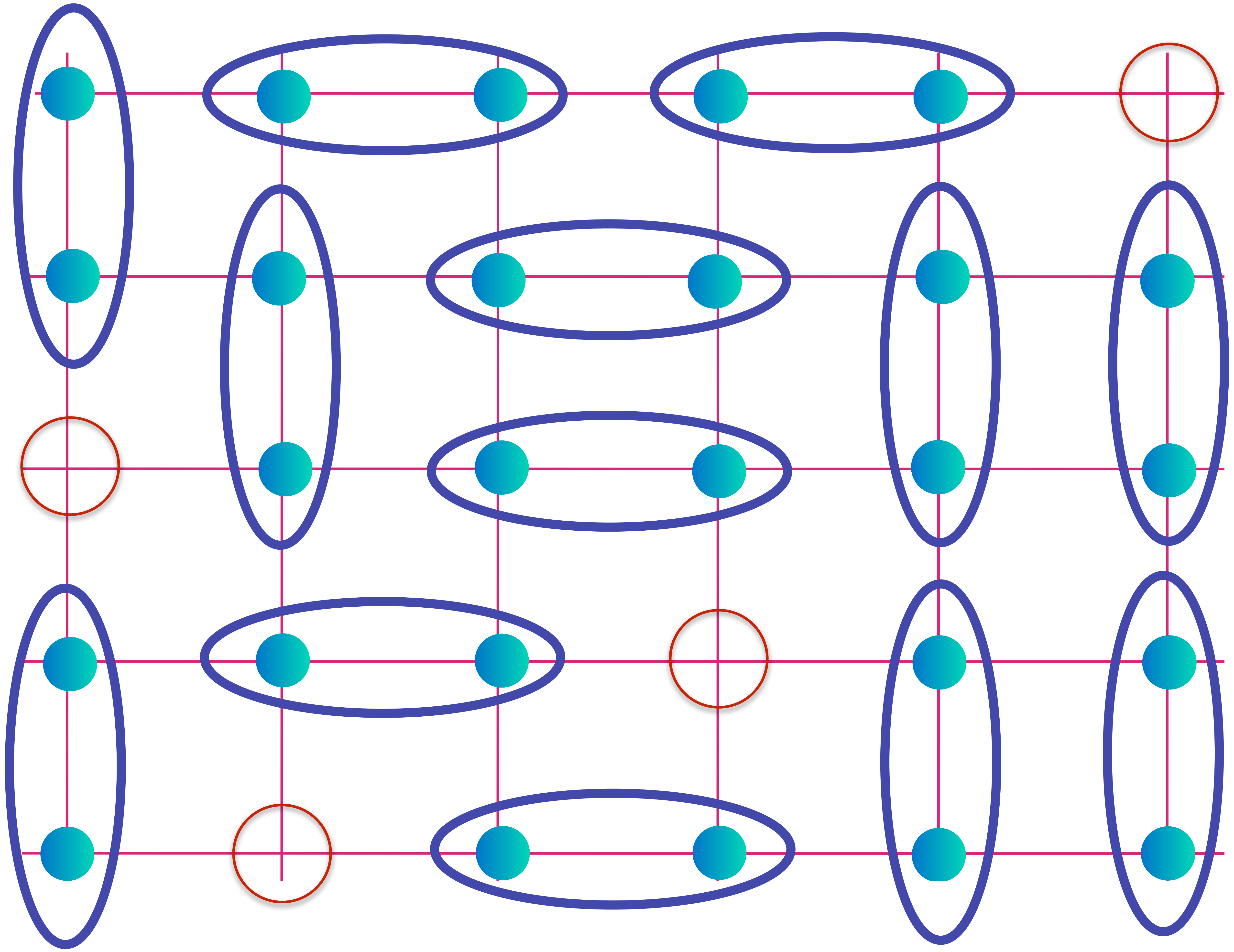

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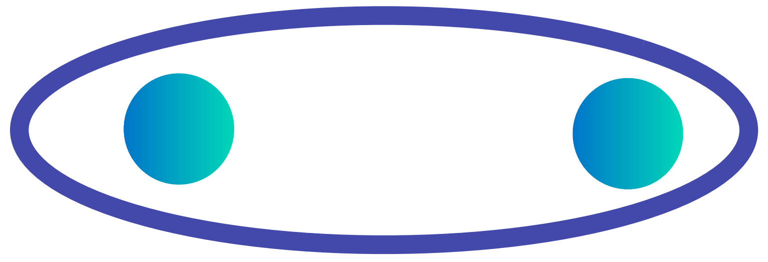
Spin liquid with density *p* of spinless, charge +e “holons”

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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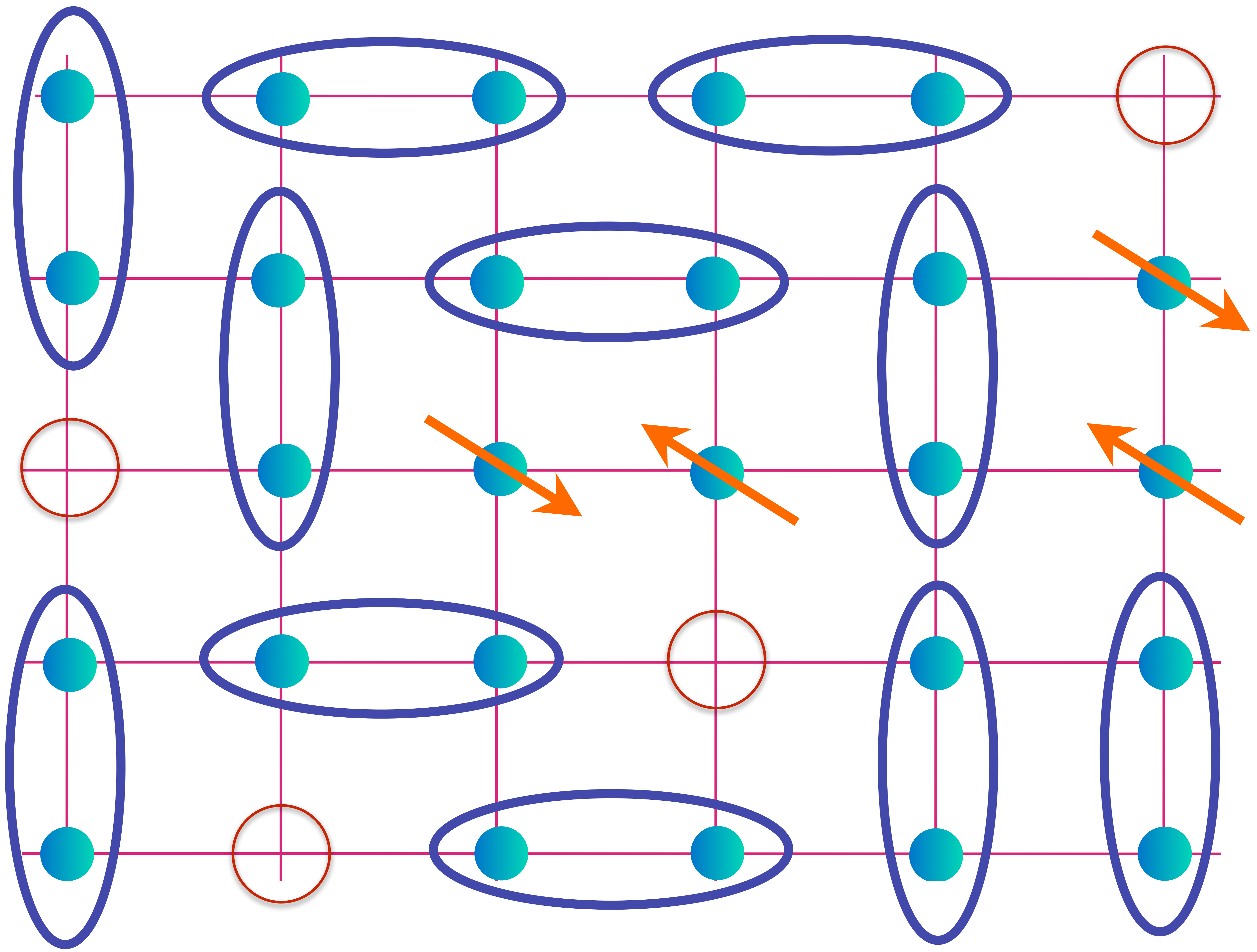

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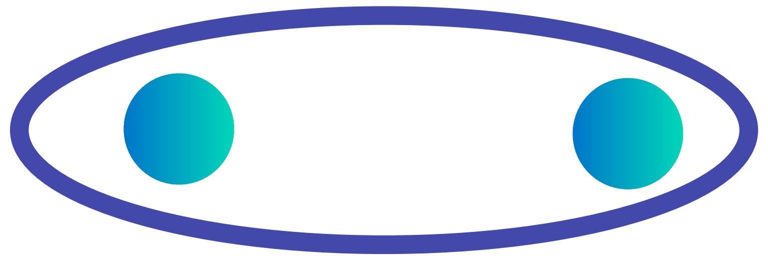
Spin liquid with density p of spinless, charge +e “holons” and charge 0, spin-1/2 “spinons”



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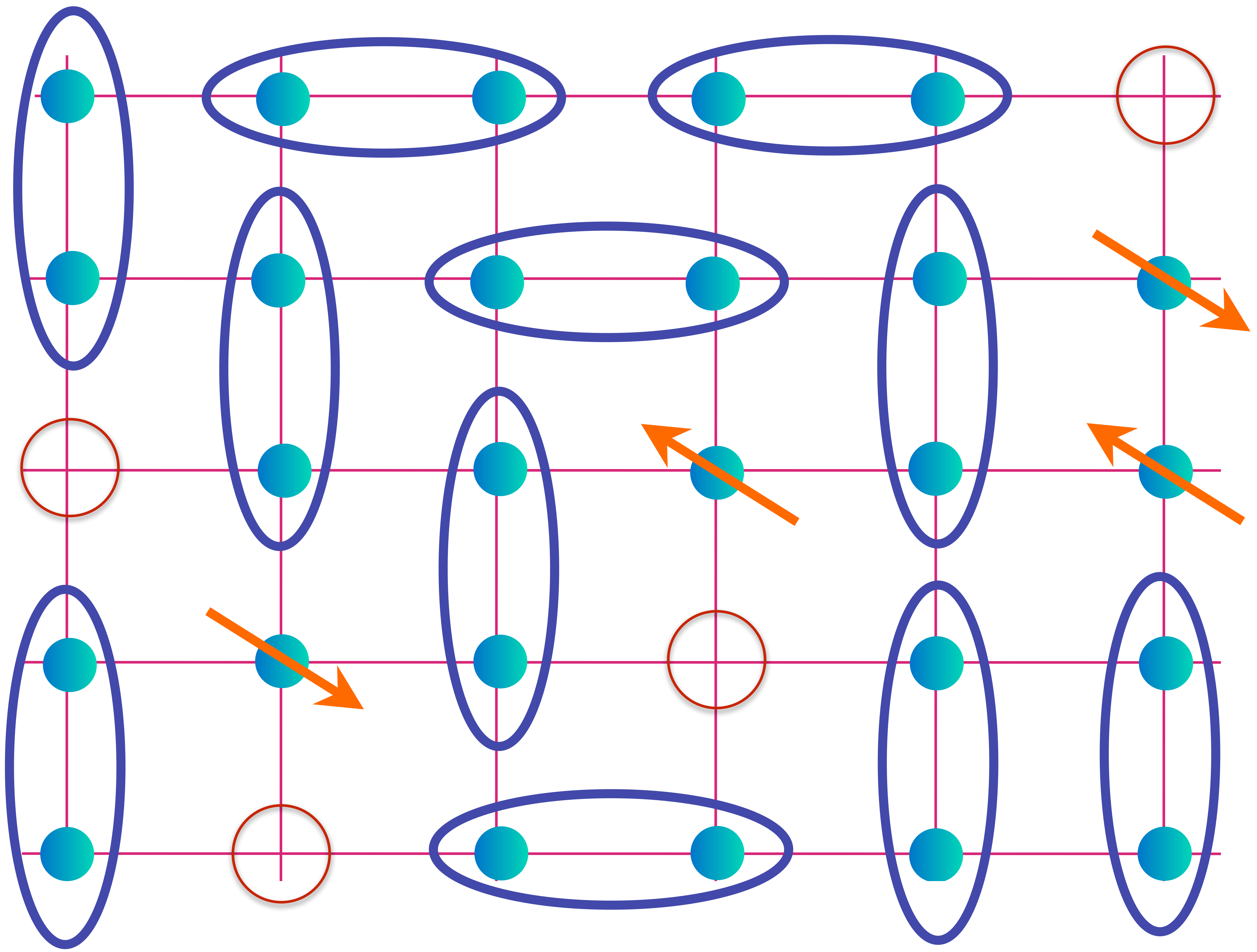

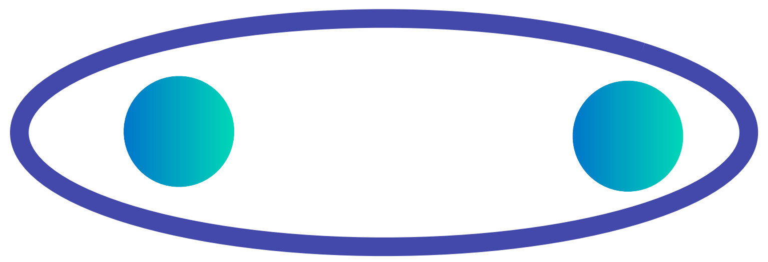 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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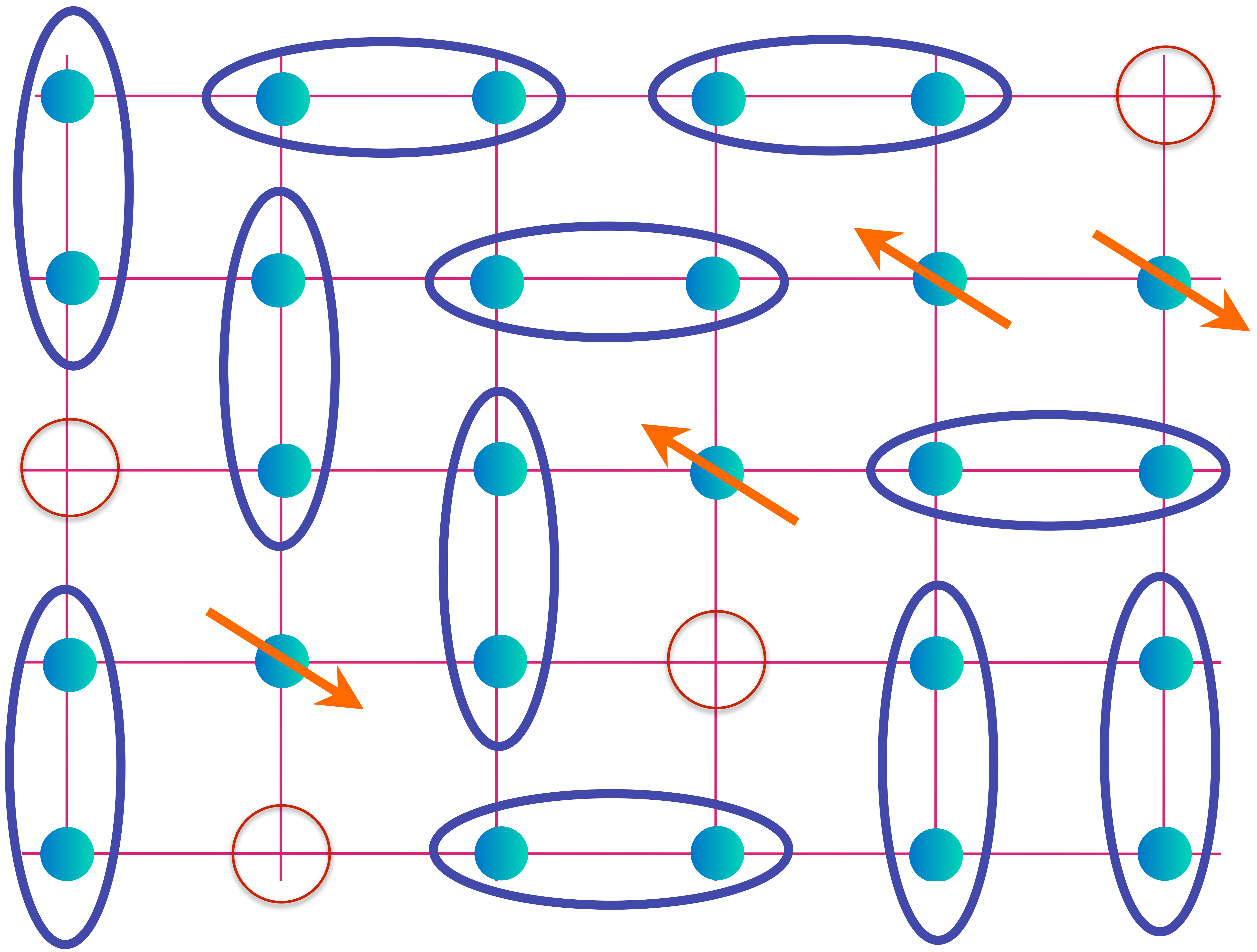

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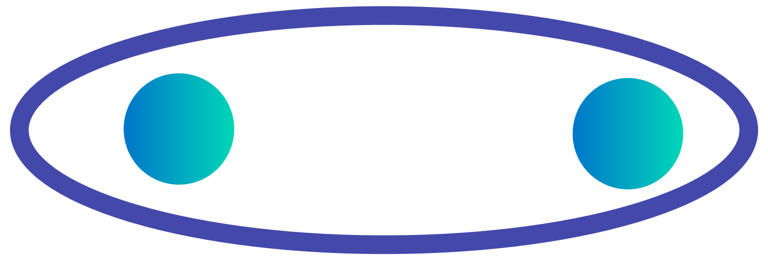

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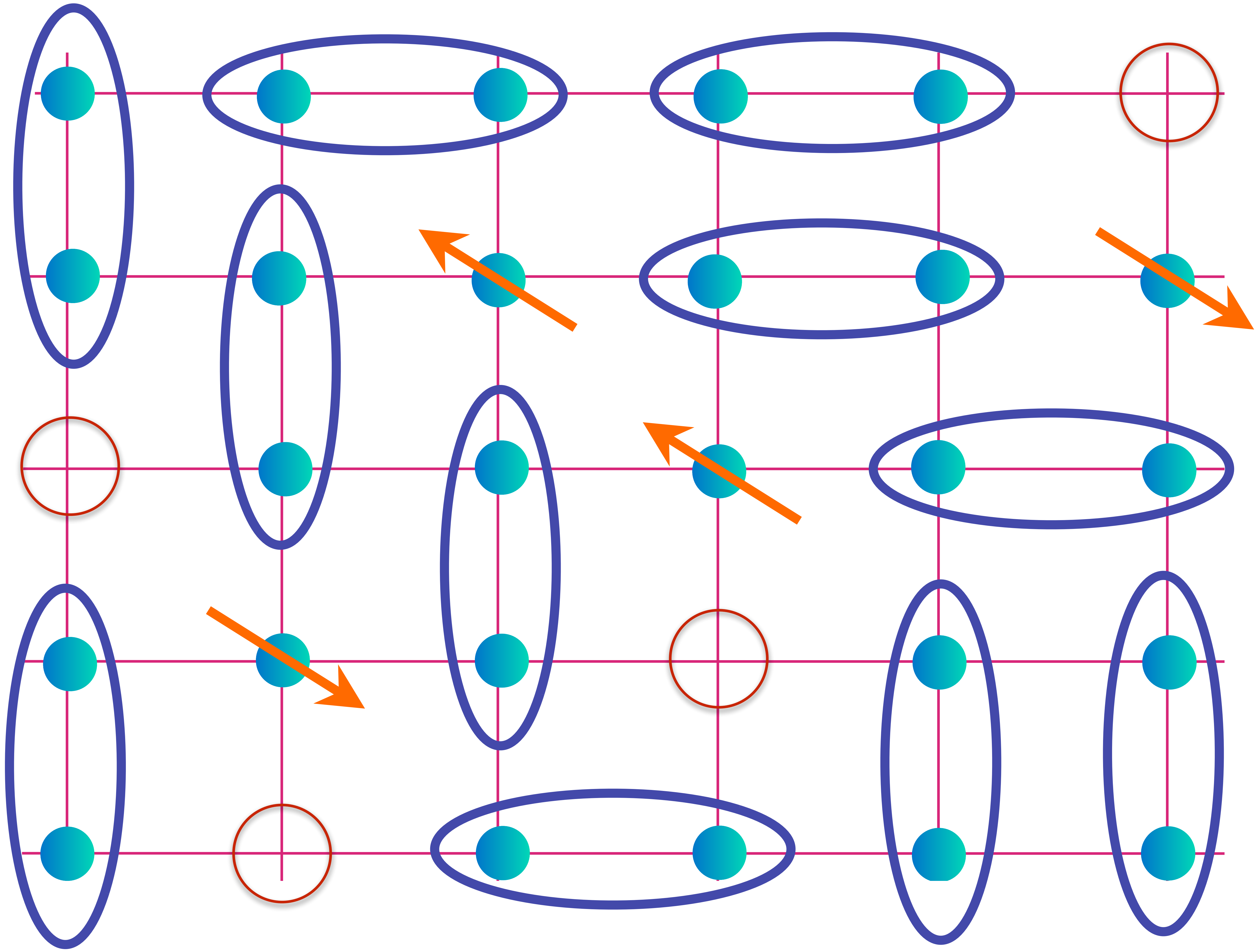

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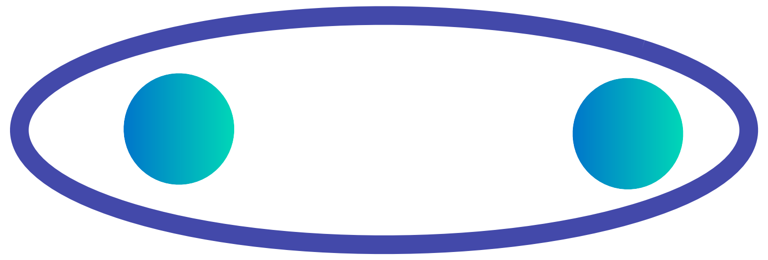 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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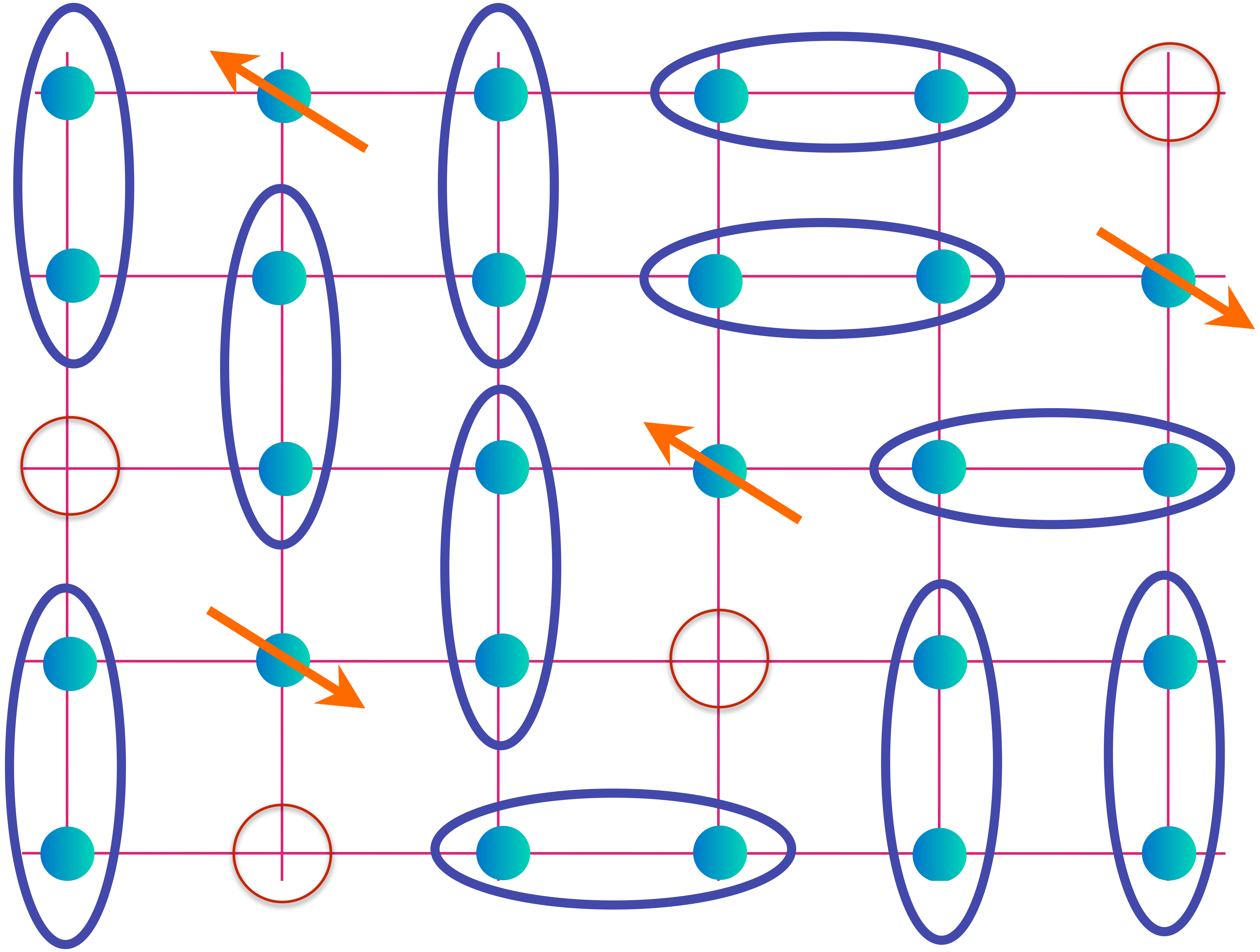

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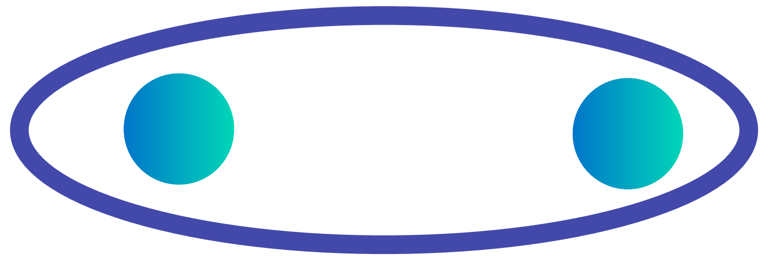 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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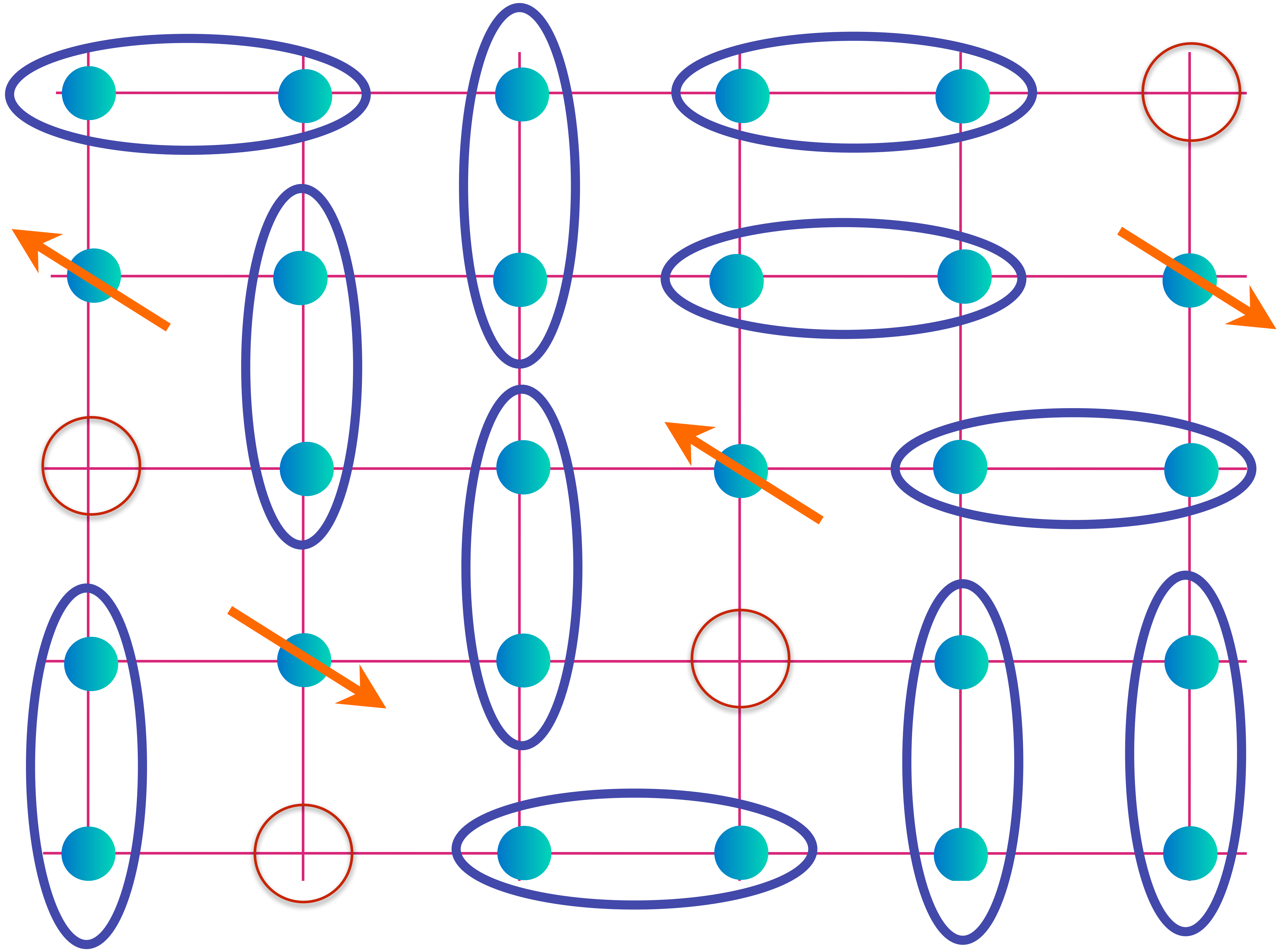

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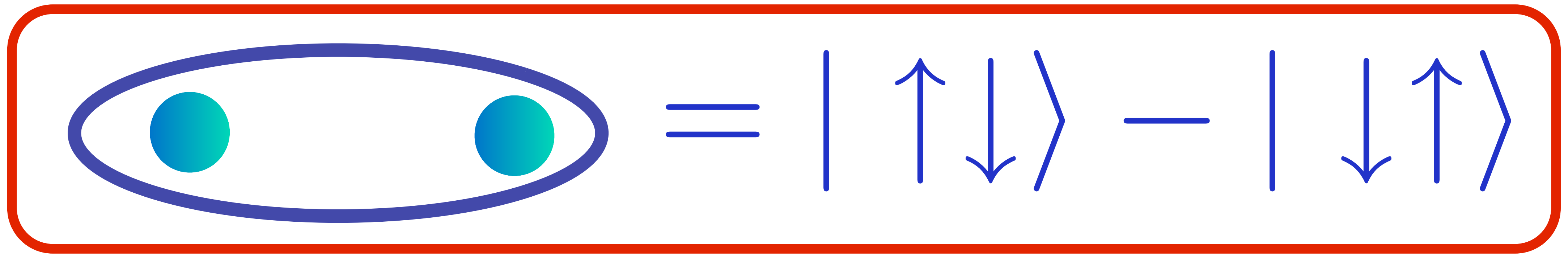

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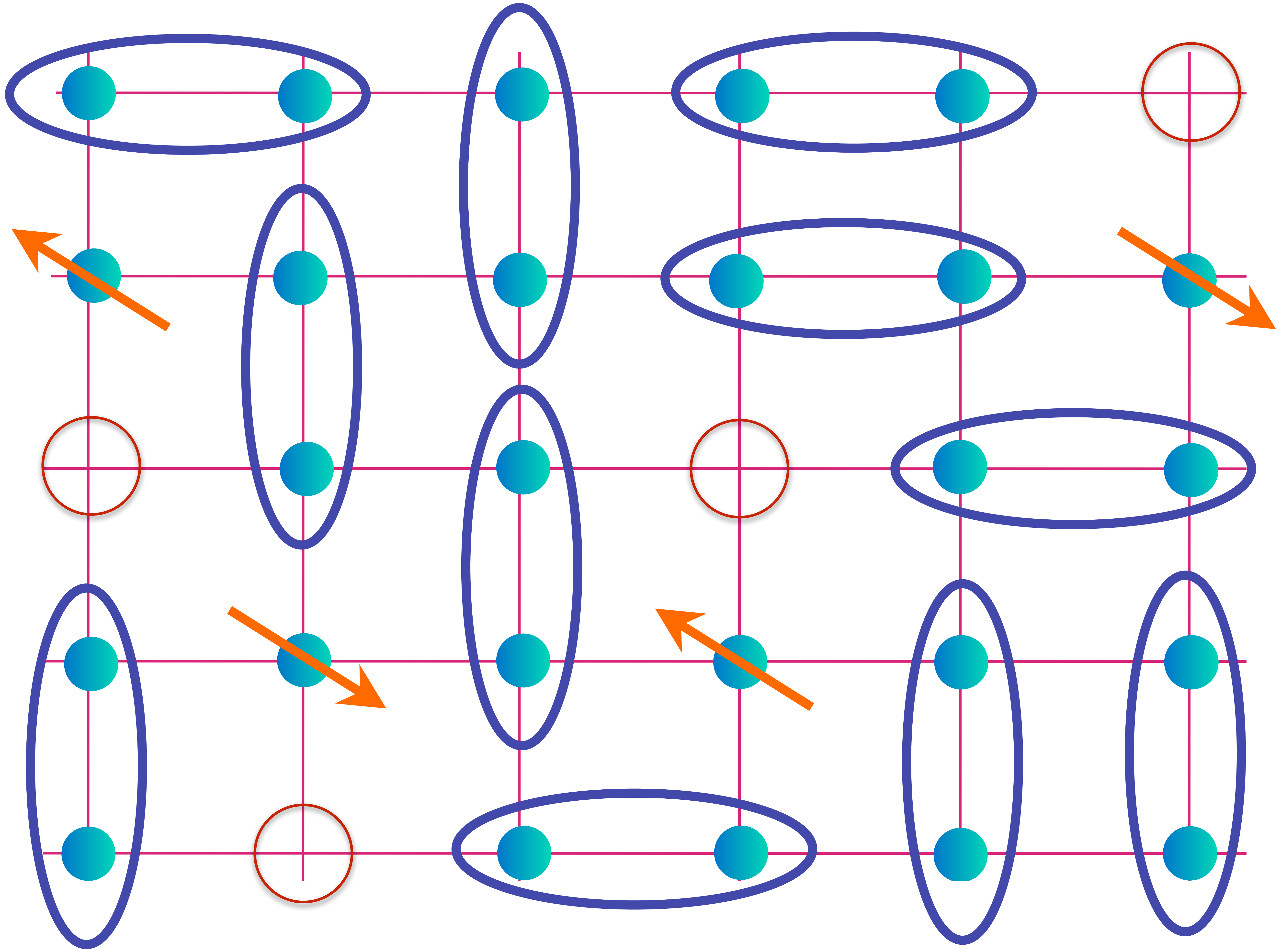

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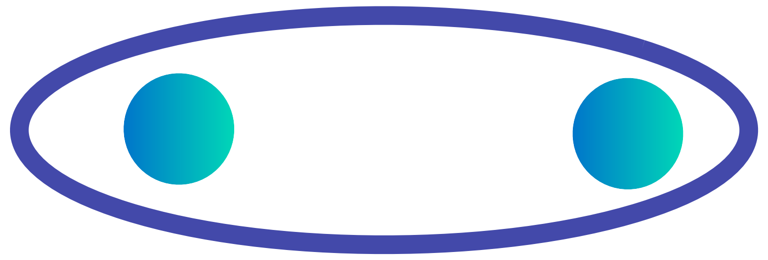 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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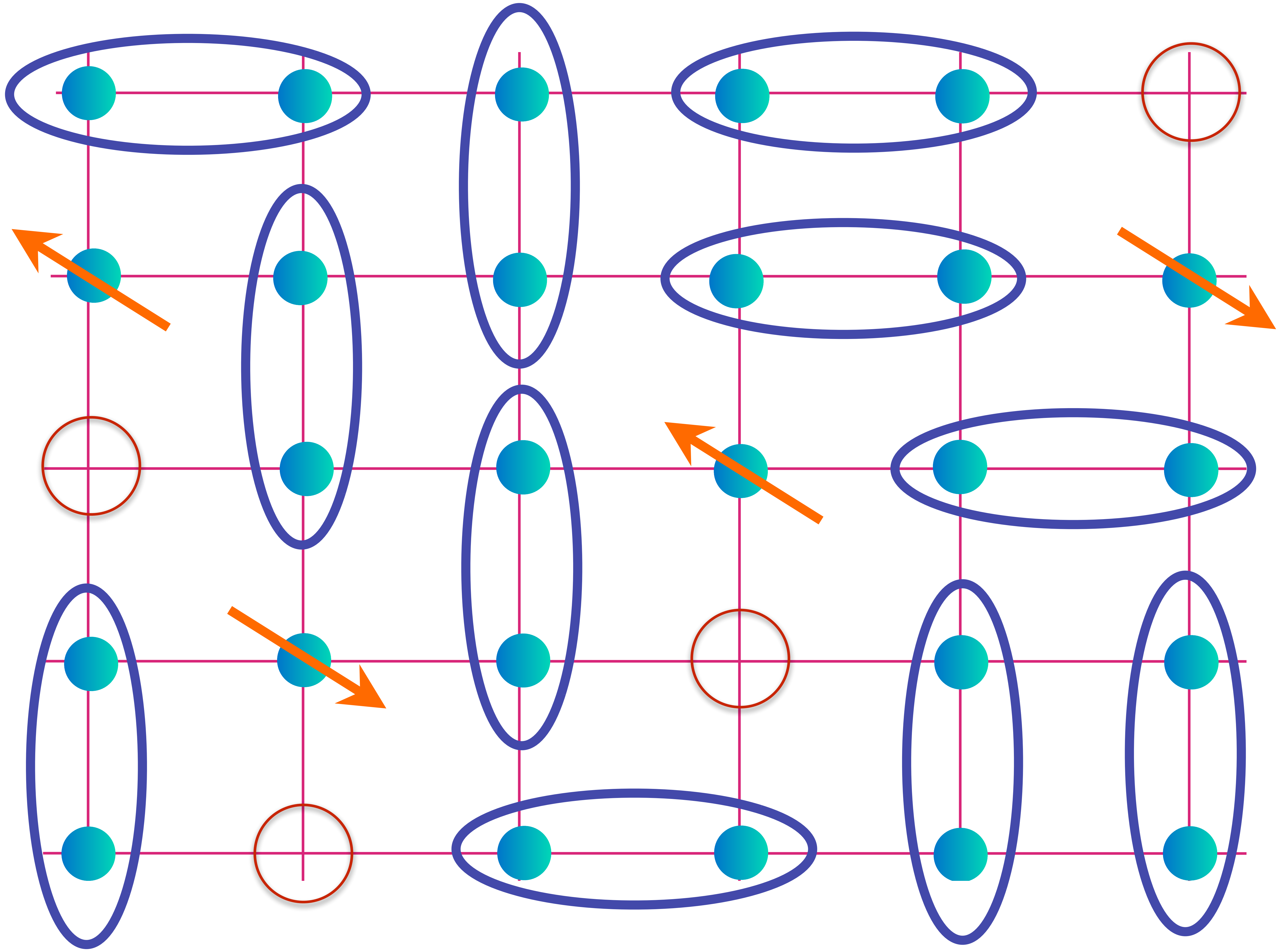

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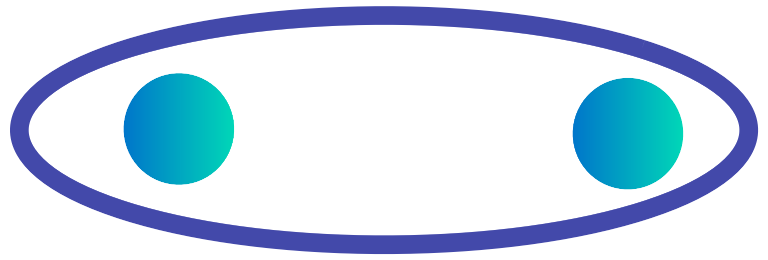 $= | \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle$

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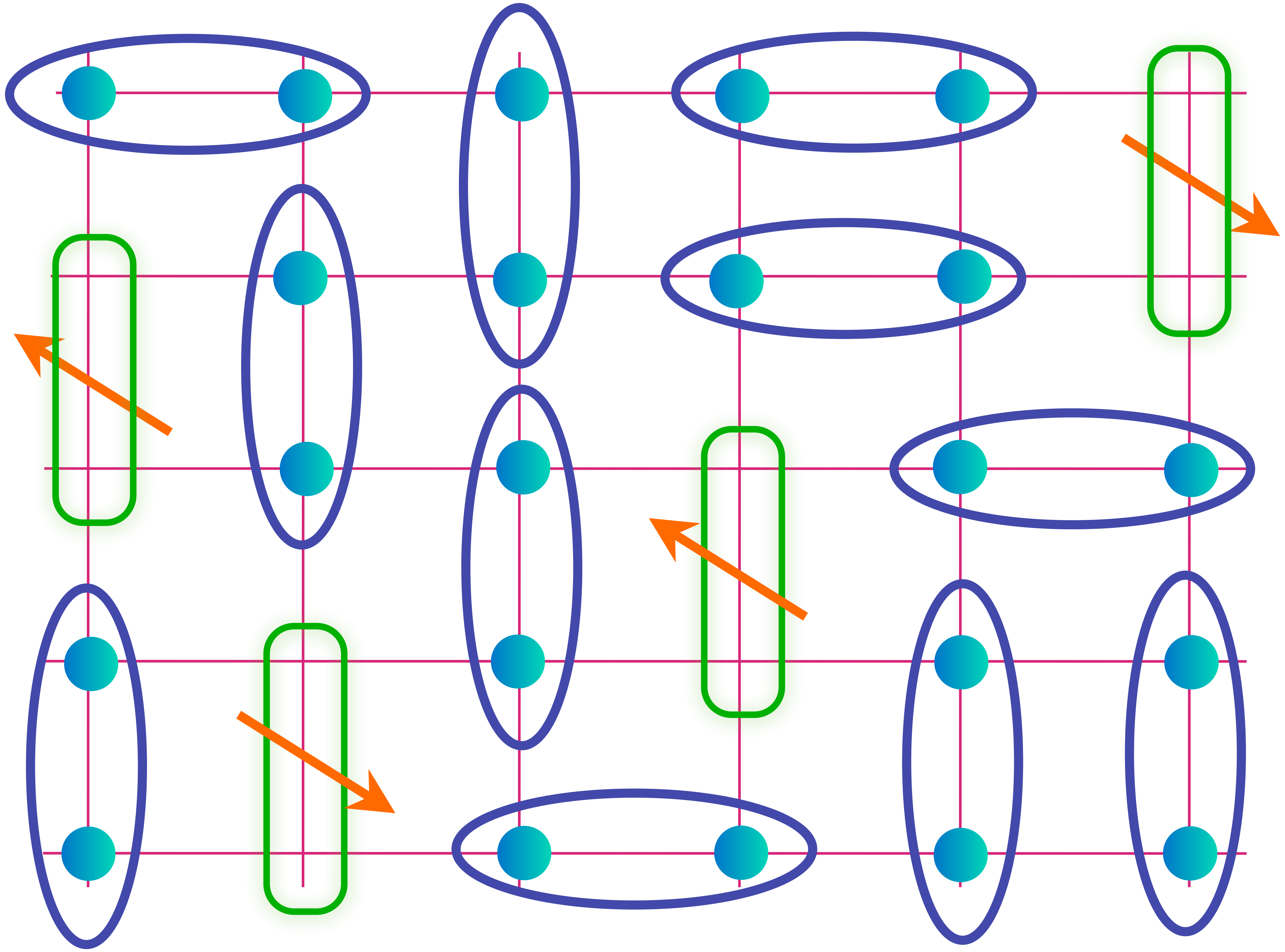


FL*

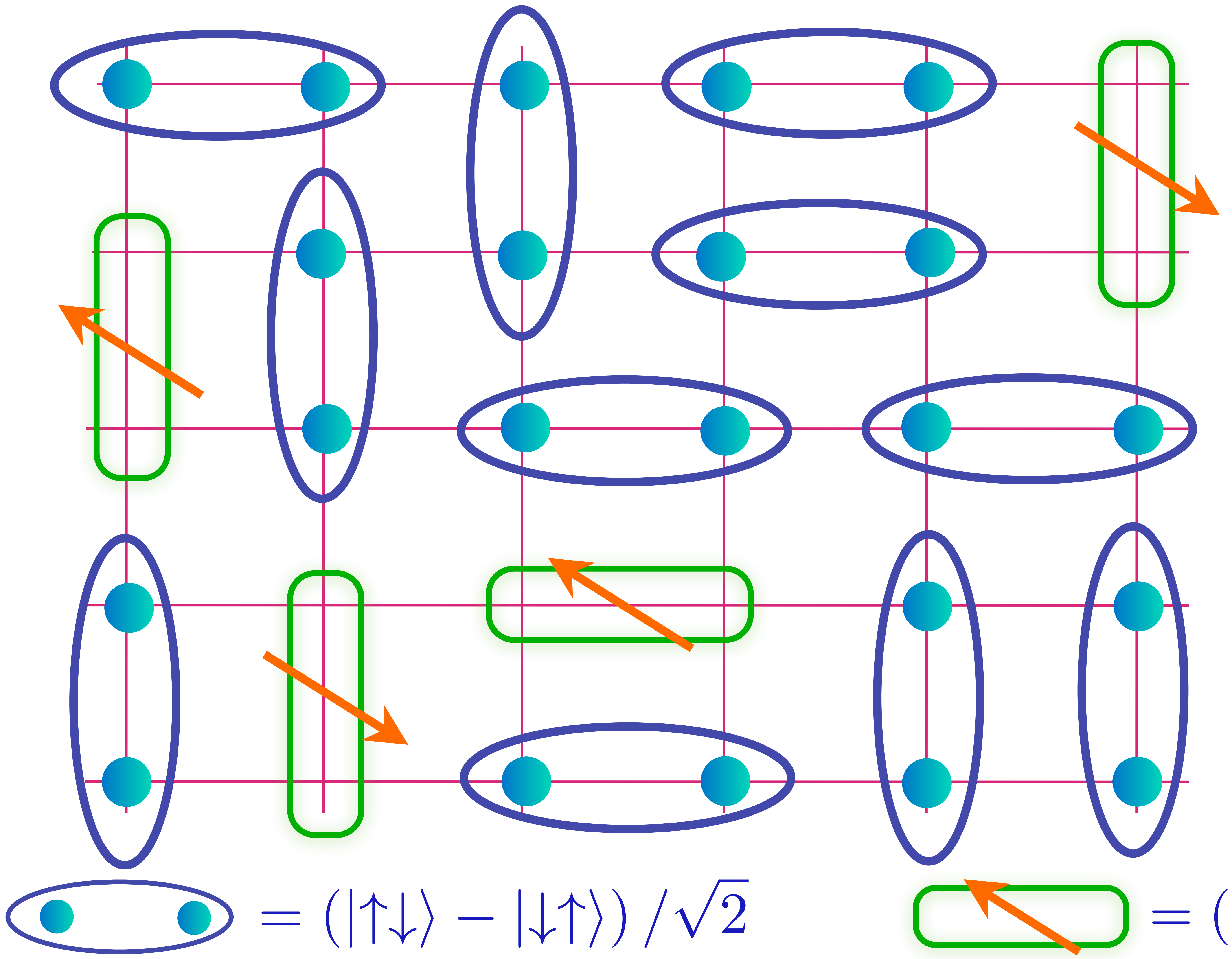
Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

The dance of electrons on Cu atoms in YBCO



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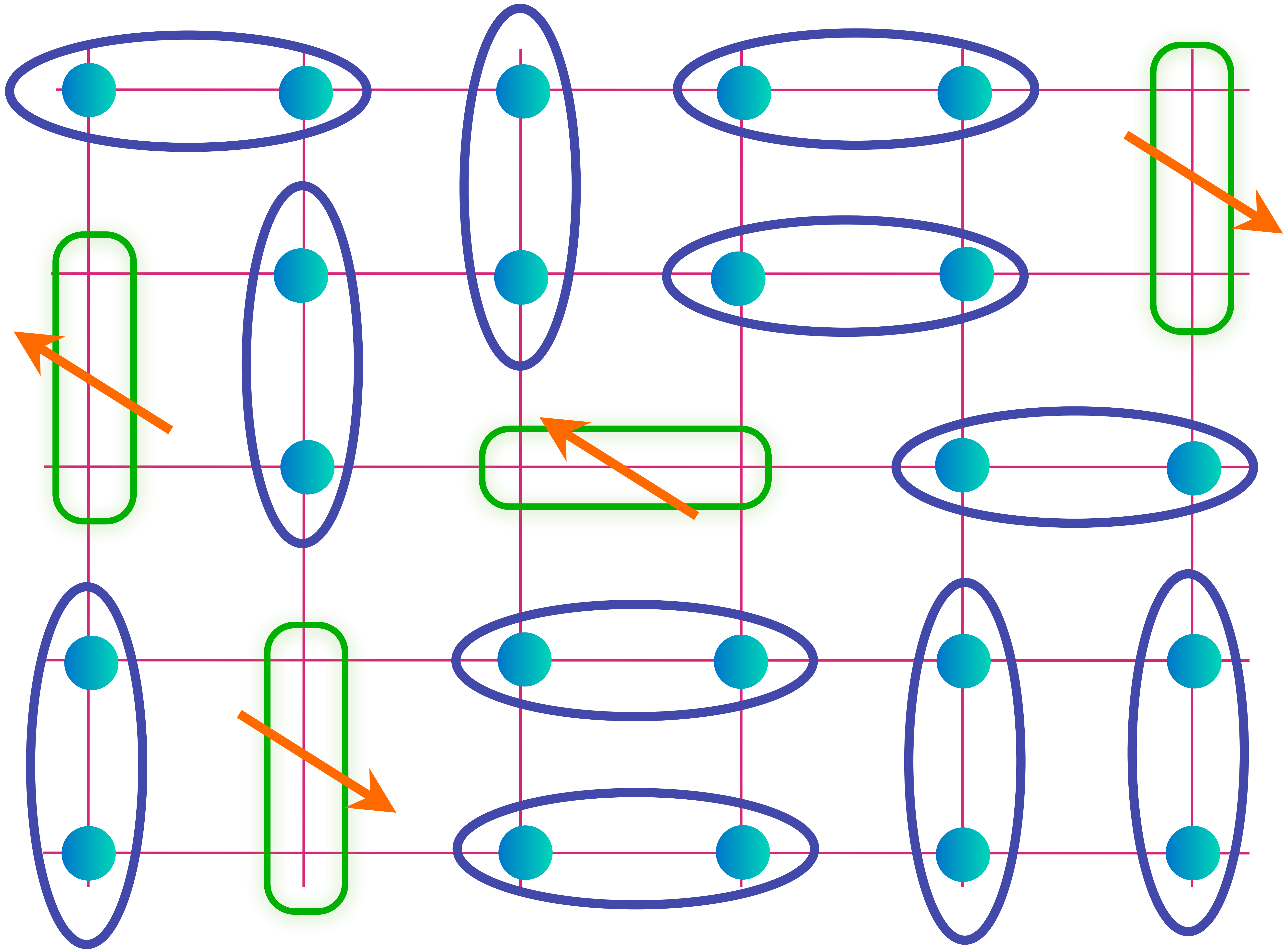

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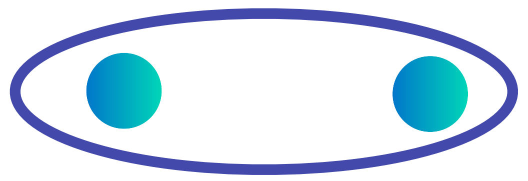
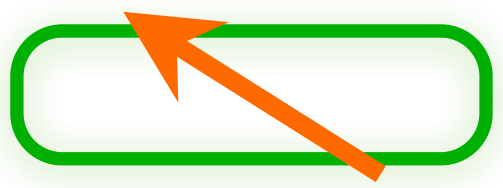
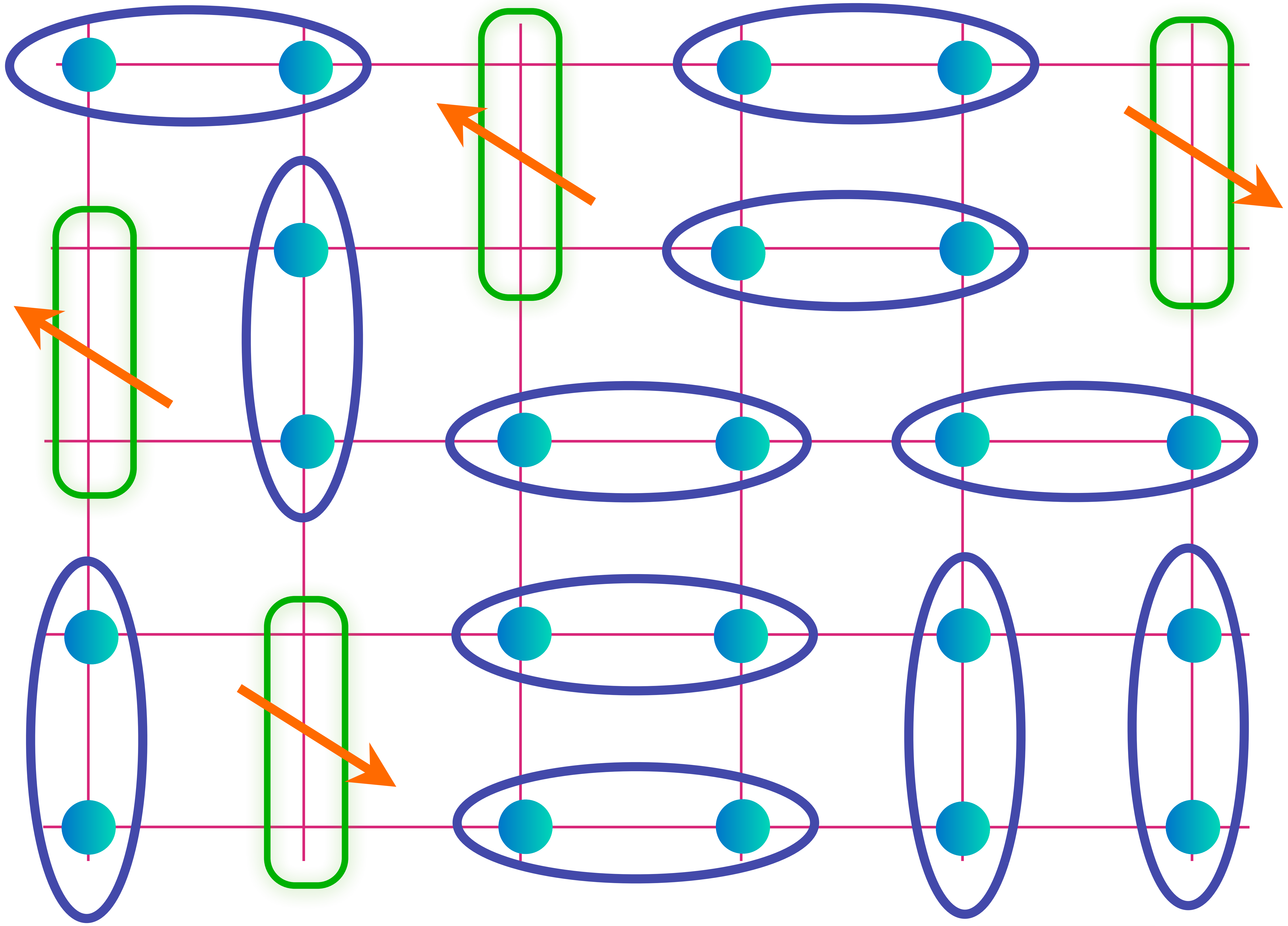
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

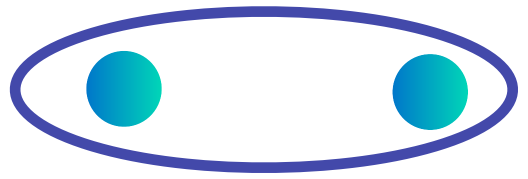
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

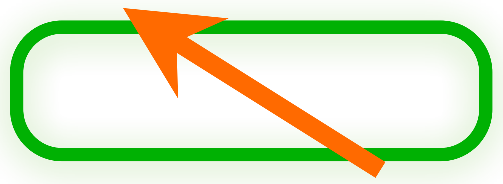
The dance of electrons on Cu atoms in YBCO



FL*

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

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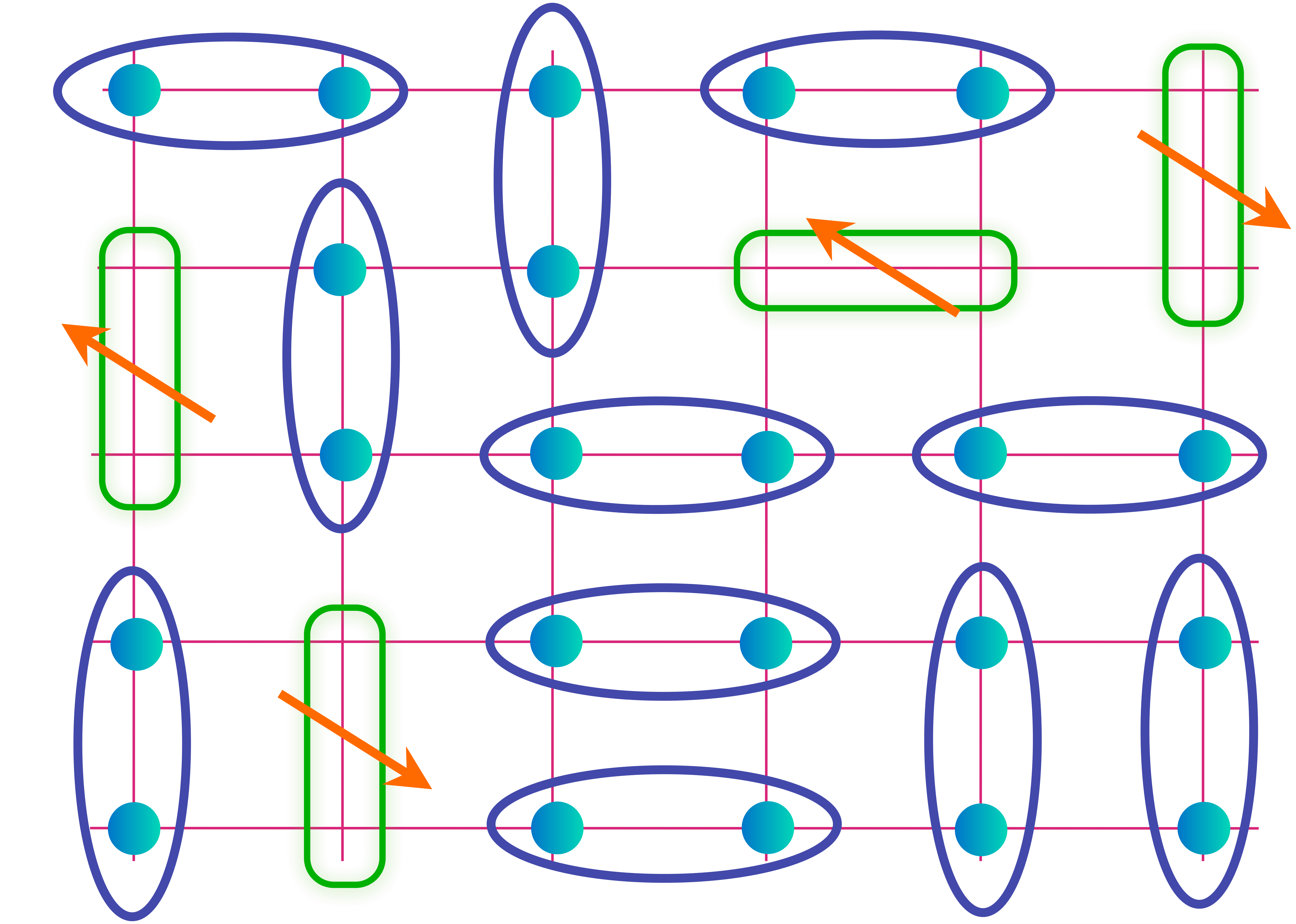
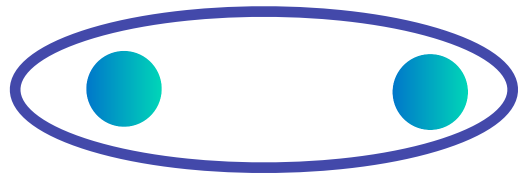
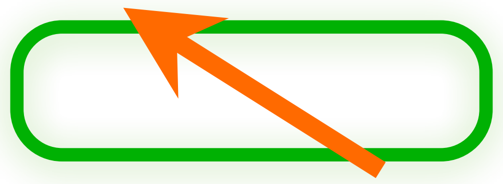


FL*

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

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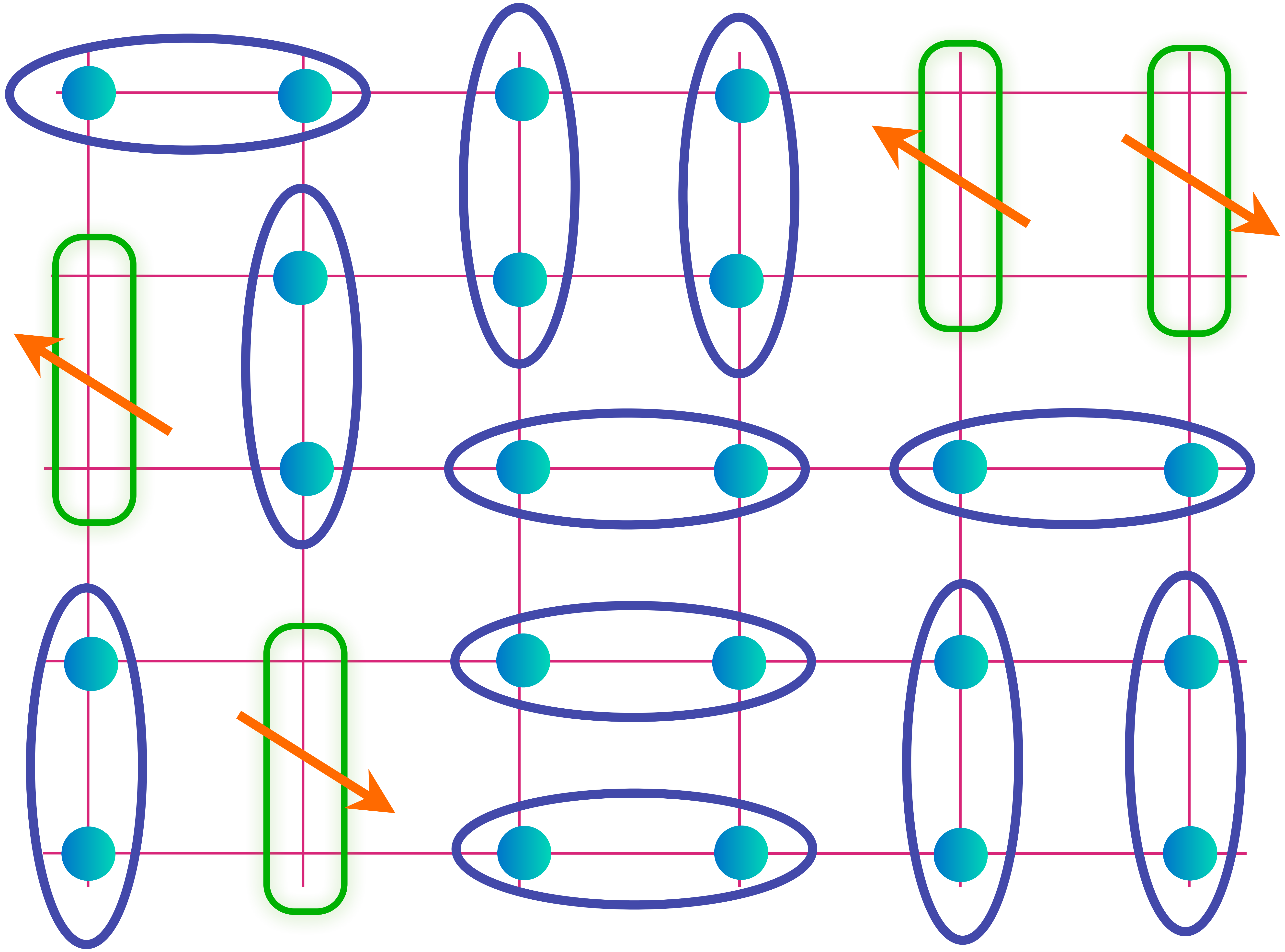

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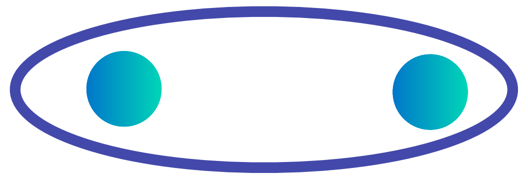
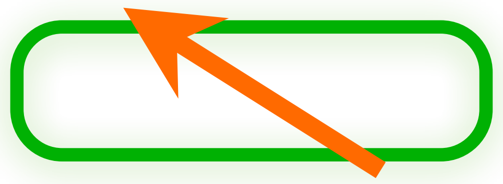
Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)

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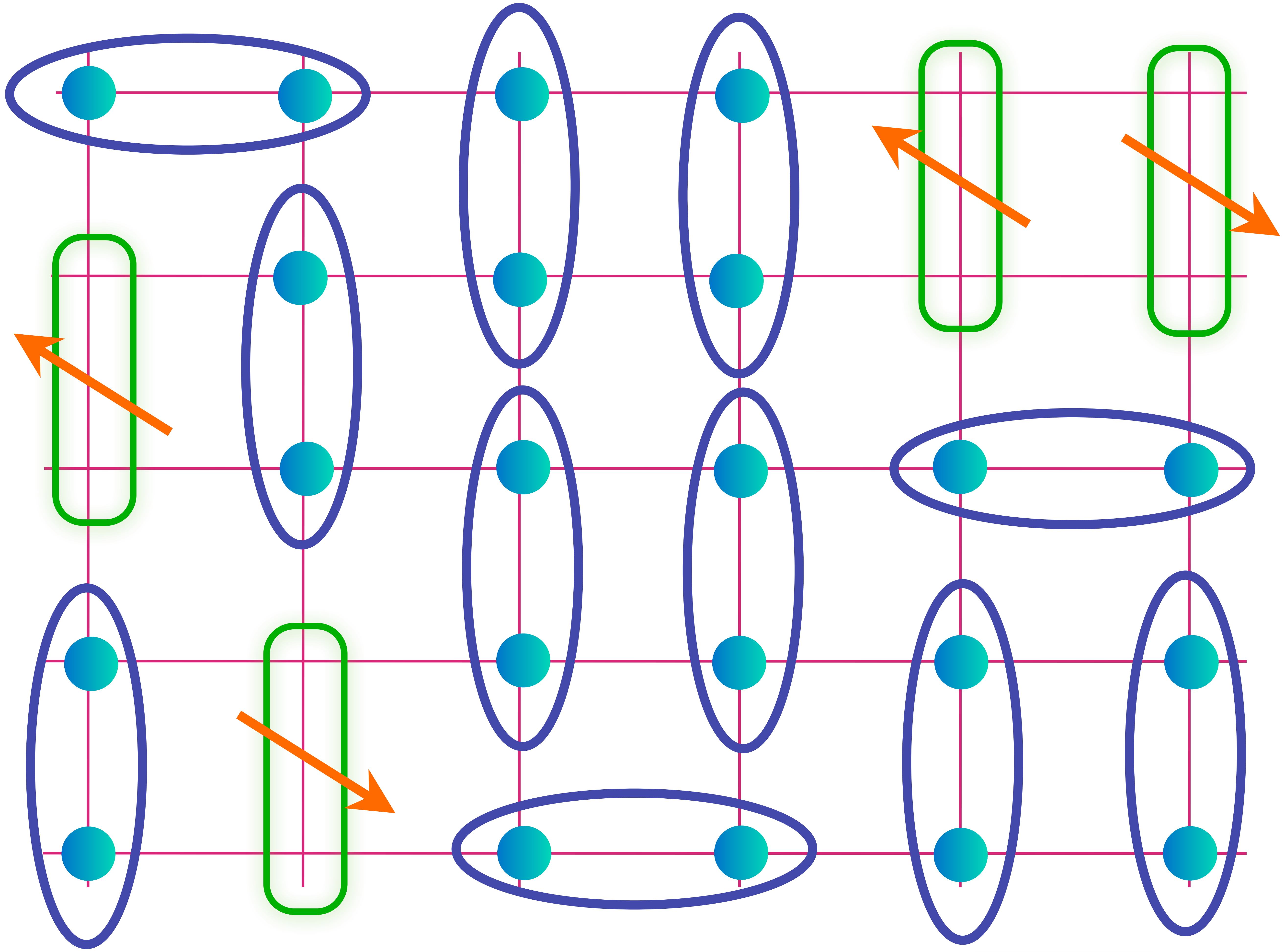
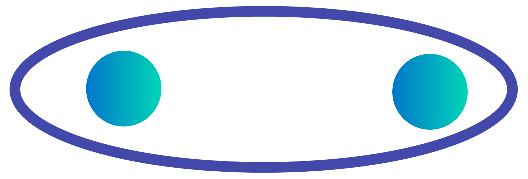
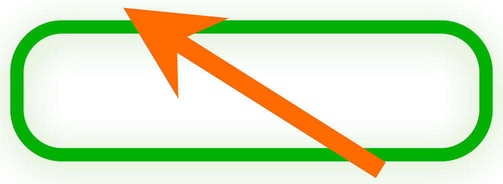


FL*

Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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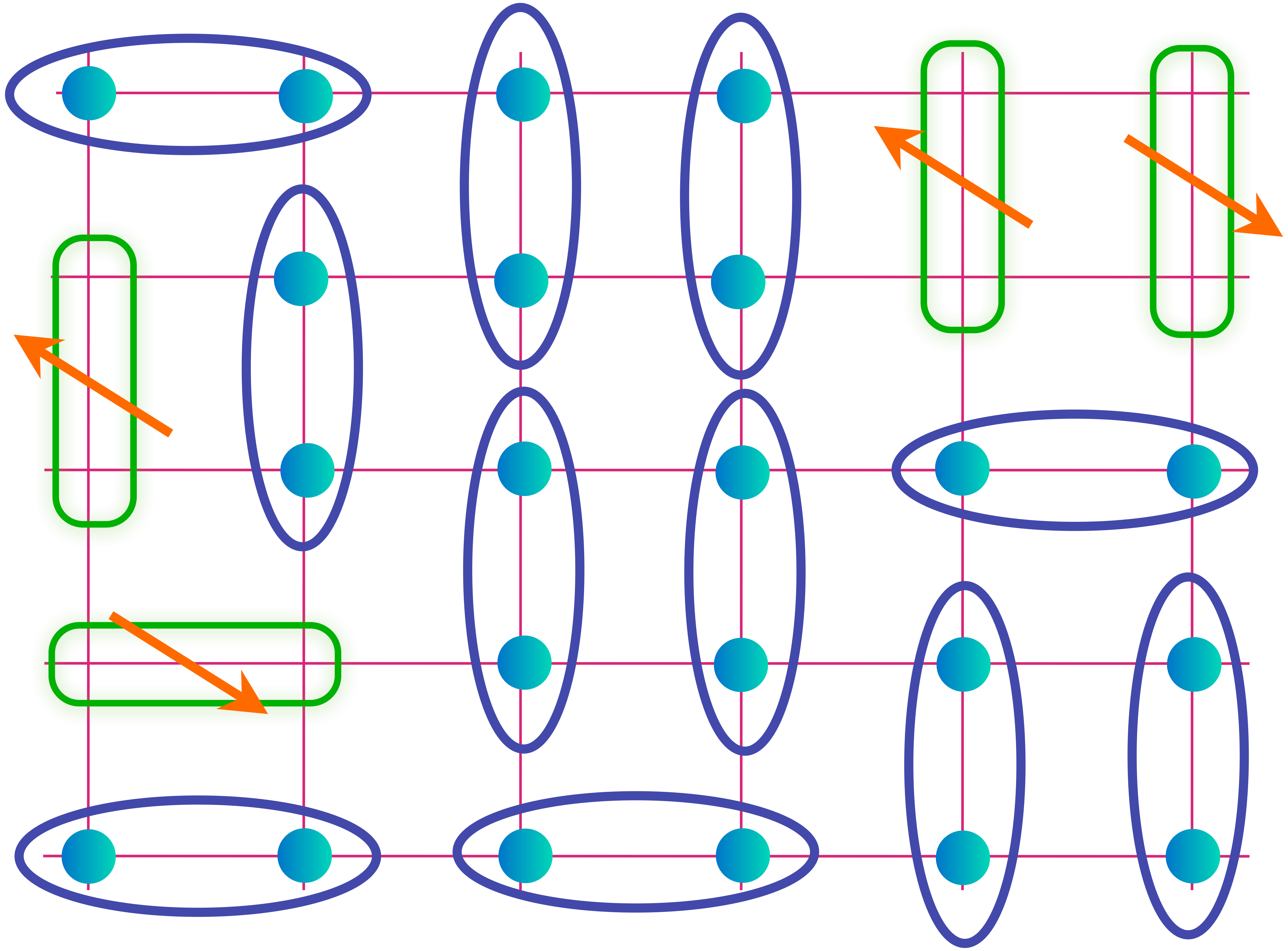

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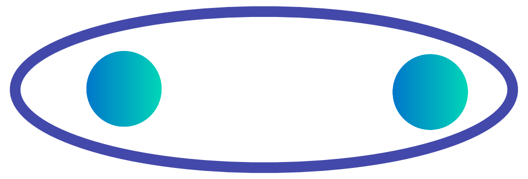
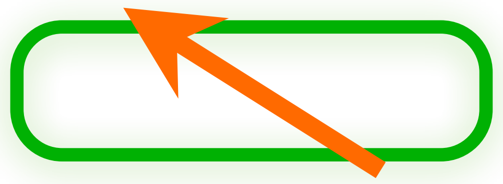
Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

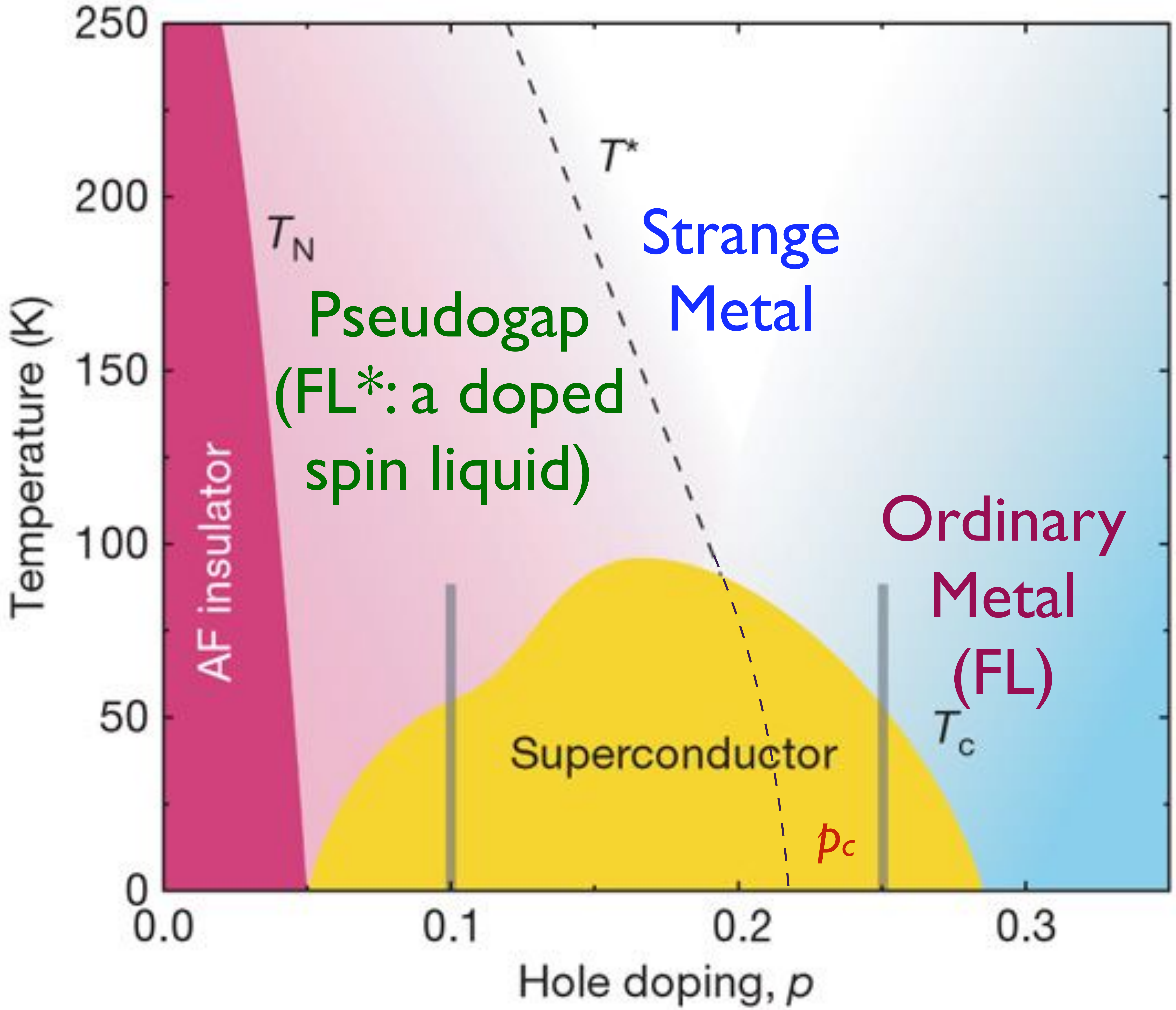


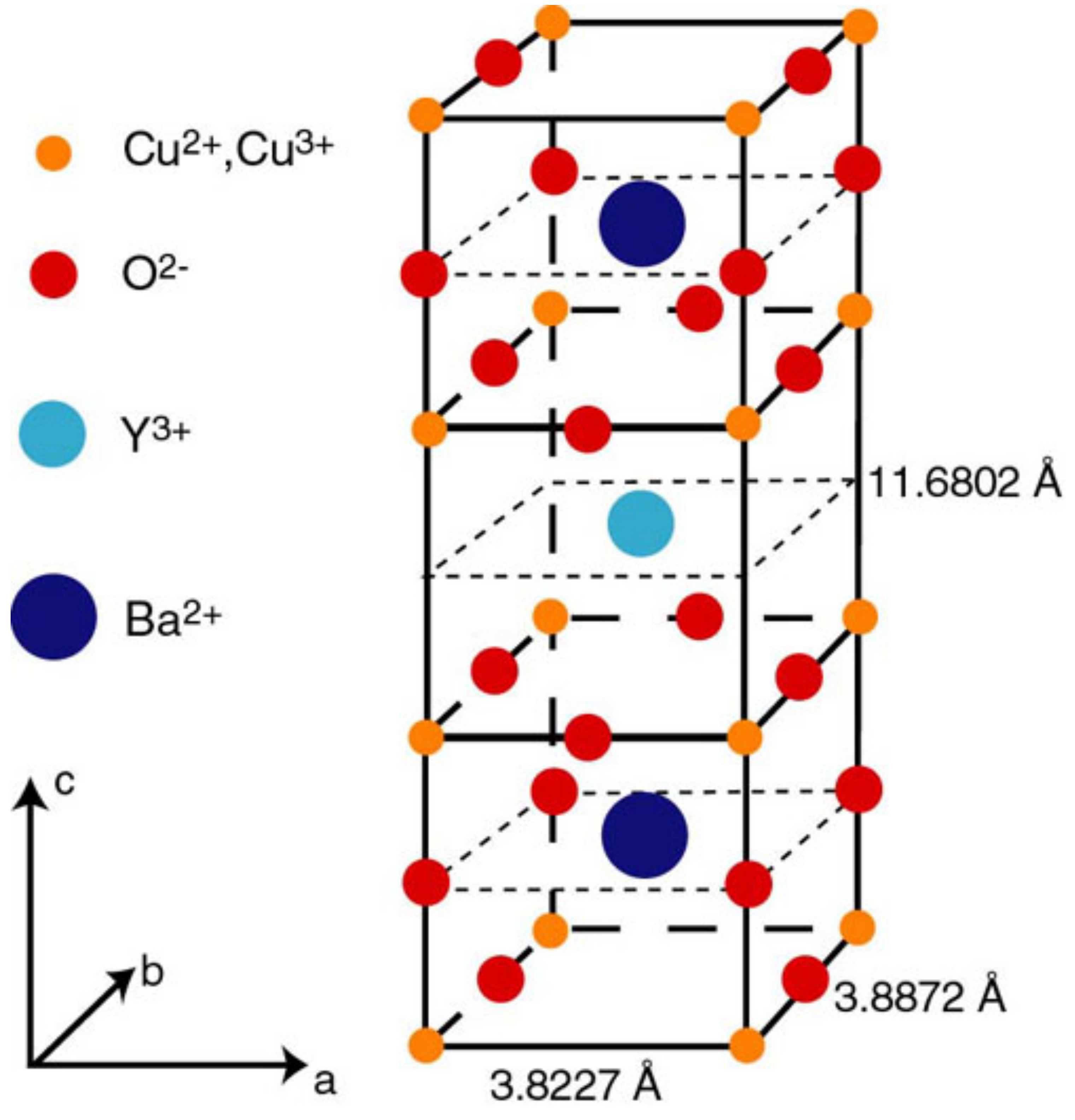
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



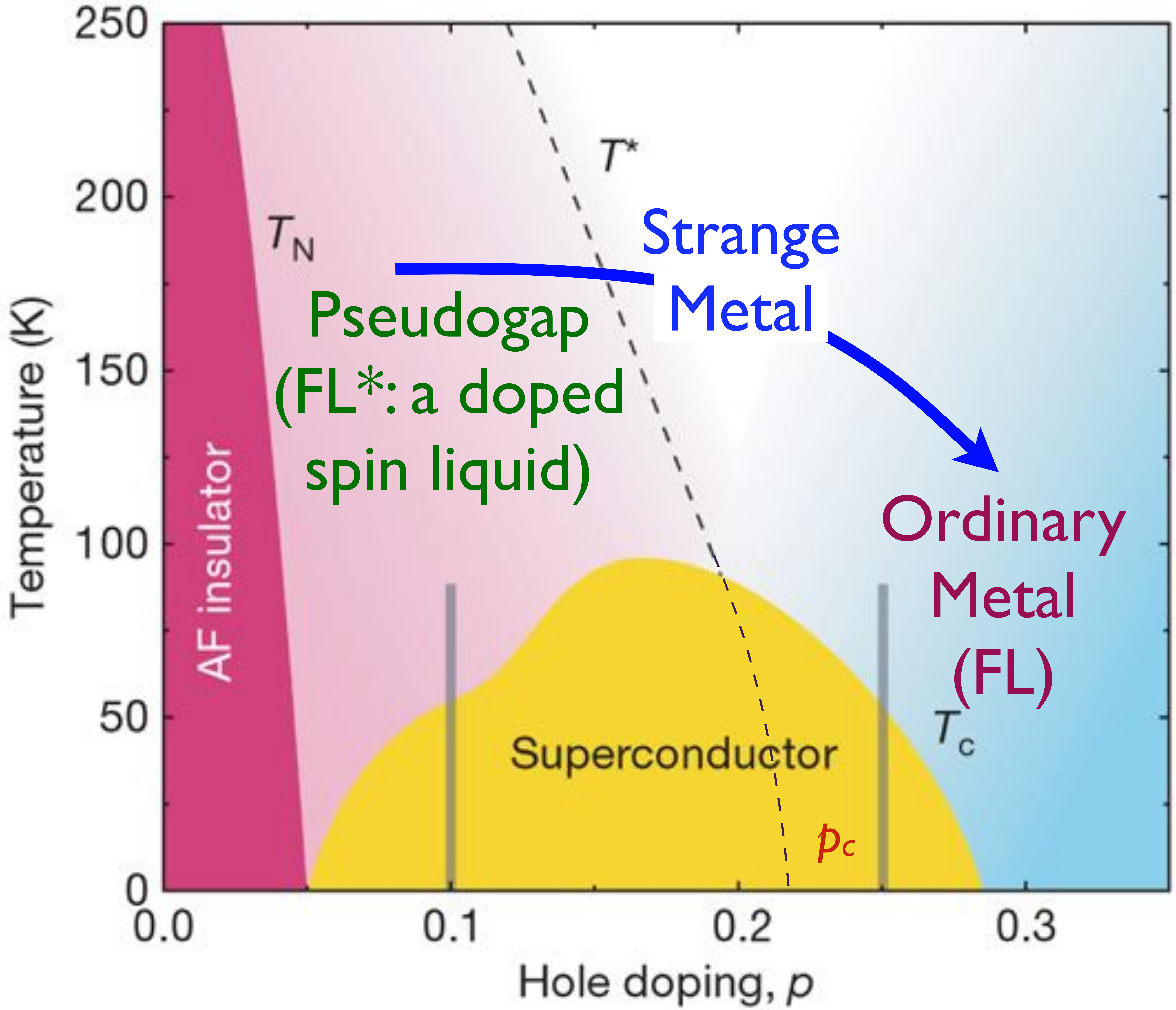
$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

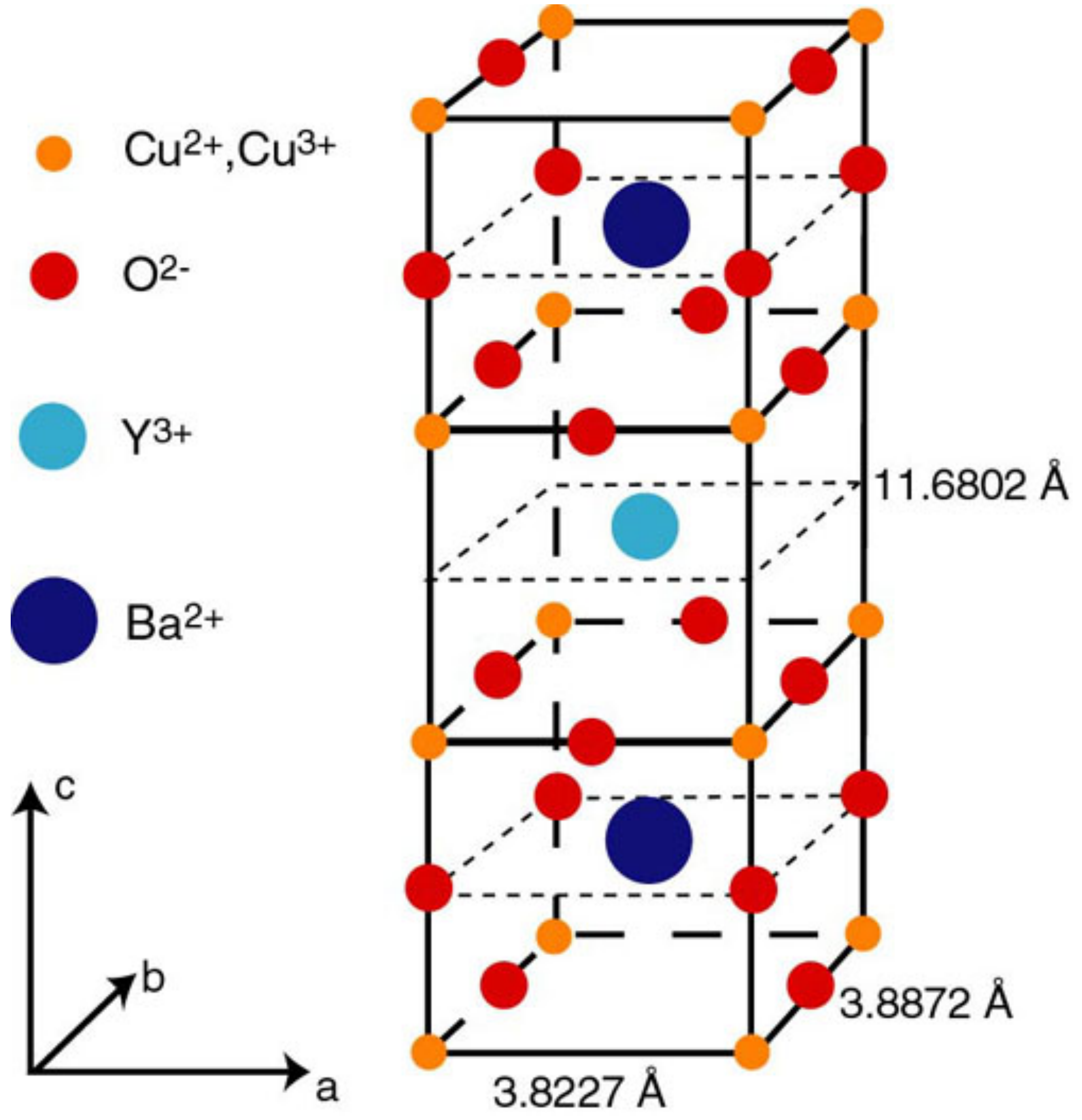
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996); T. Senthil, S. Sachdev, and M. Vojta, PRL **90**, 216403 (2003)


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Pseudogap
(FL*: a doped
spin liquid)
Strange
Metal
Ordinary
Metal
(FL)
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Pseudogap (FL*: a doped spin liquid)
Ordinary Metal (FL)
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
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c
b
a
11.6802 Å
3.8872 Å
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Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

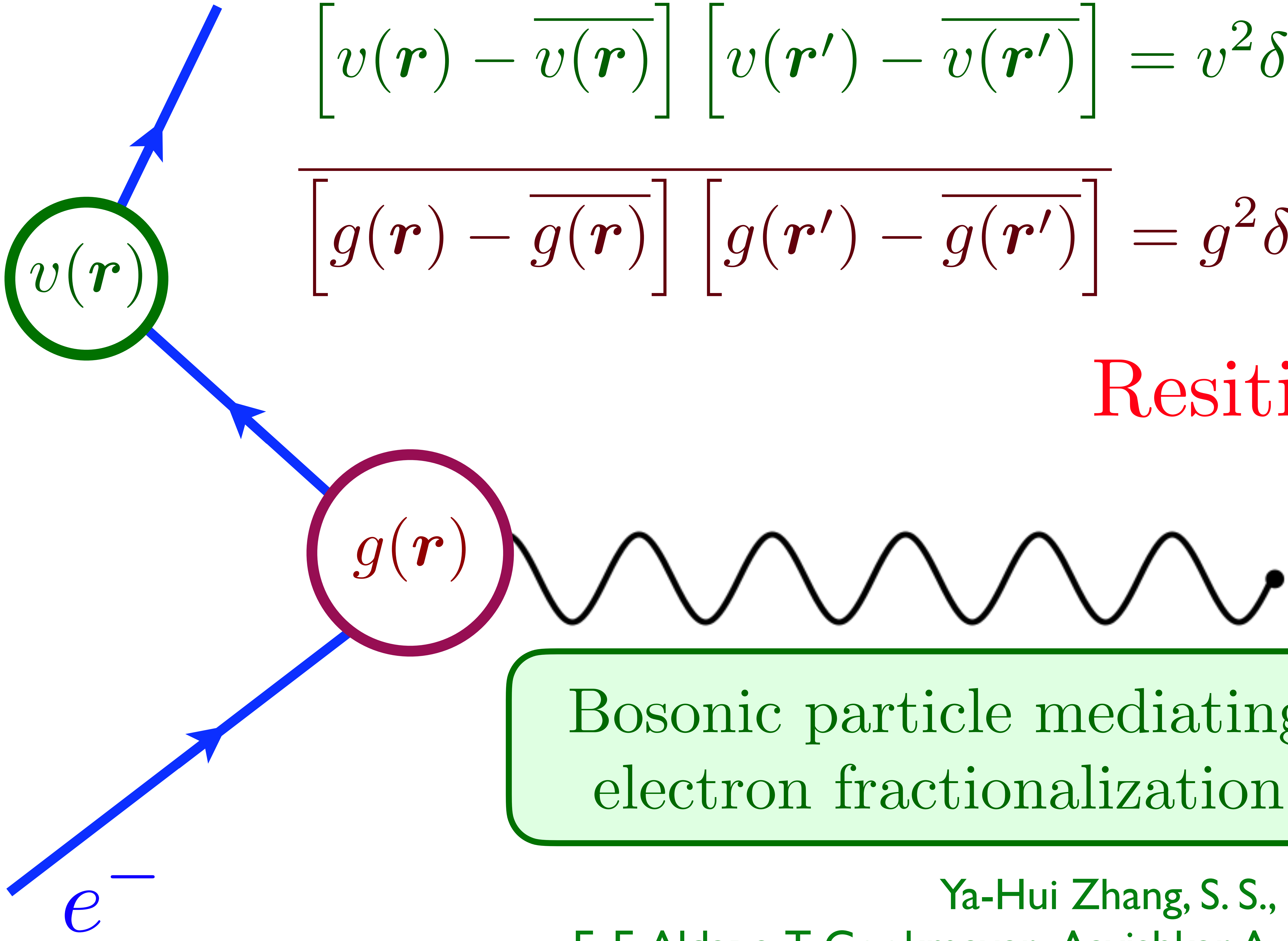
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

The dance of electrons on Cu atoms in YBCO

$$\overline{\left[v(\boldsymbol{r}) - \overline{v(\boldsymbol{r})}\right]\left[v(\boldsymbol{r}') - \overline{v(\boldsymbol{r}')}\right]} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

$$\overline{\left[g(\boldsymbol{r}) - \overline{g(\boldsymbol{r})}\right]\left[g(\boldsymbol{r}') - \overline{g(\boldsymbol{r}')}\right]} = g^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Random interactions as in SYK model



Resitivity $\rho(T) \sim v^2 + g^2 T$

Bosonic particle mediating electron fractionalization

Ya-Hui Zhang, S. S., PRR **2**, 023172 (2020); PRB **102** , 155124 (2020)
E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, PRB **105**, 235111 (2022)
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990, *Science*, to appear

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP **94**, 041002 (2002)

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, Nat. Commun. **14**, 3033 (2023)

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

The many faces of multi-particle entanglement

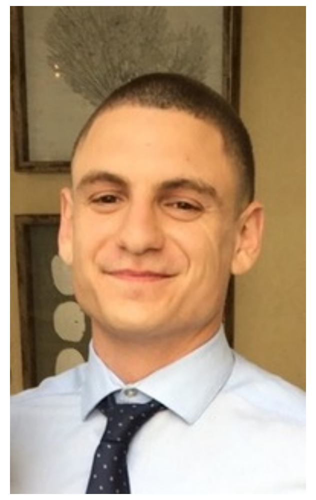
- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)
- A quantum theory of the interior of a black hole.

Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.





- Linear-T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S., Science to appear, arXiv: 2203.