Universal theory of strange metals from spatially random interactions

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Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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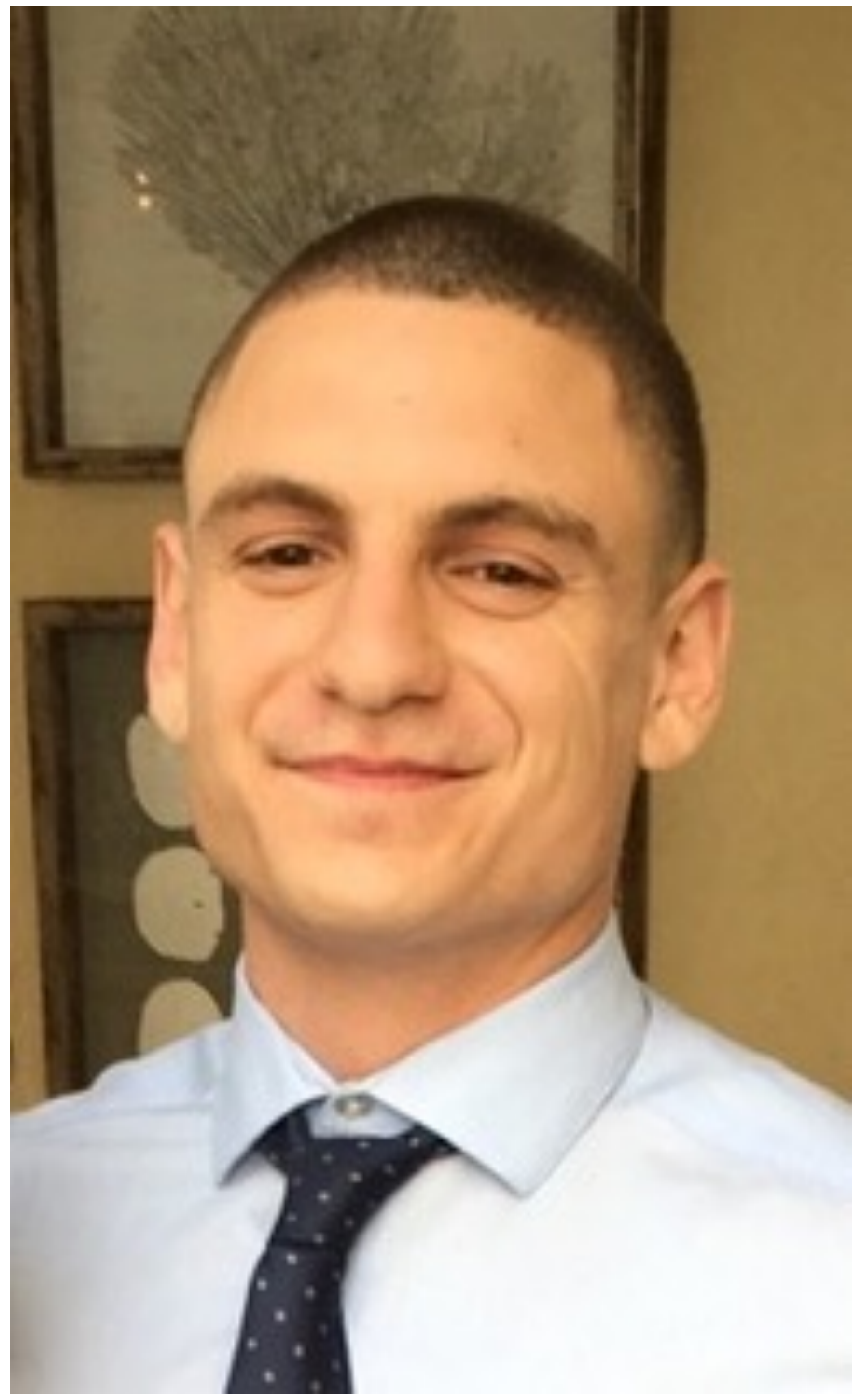

Aavishkar Patel
Flatiron Institute, NYC

Haoyu Guo
Harvard

Ilya Esterlis
Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

Fermi liquids and their cousins: (defined by single-particle properties)

- Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \ll |\varepsilon|$ and a density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

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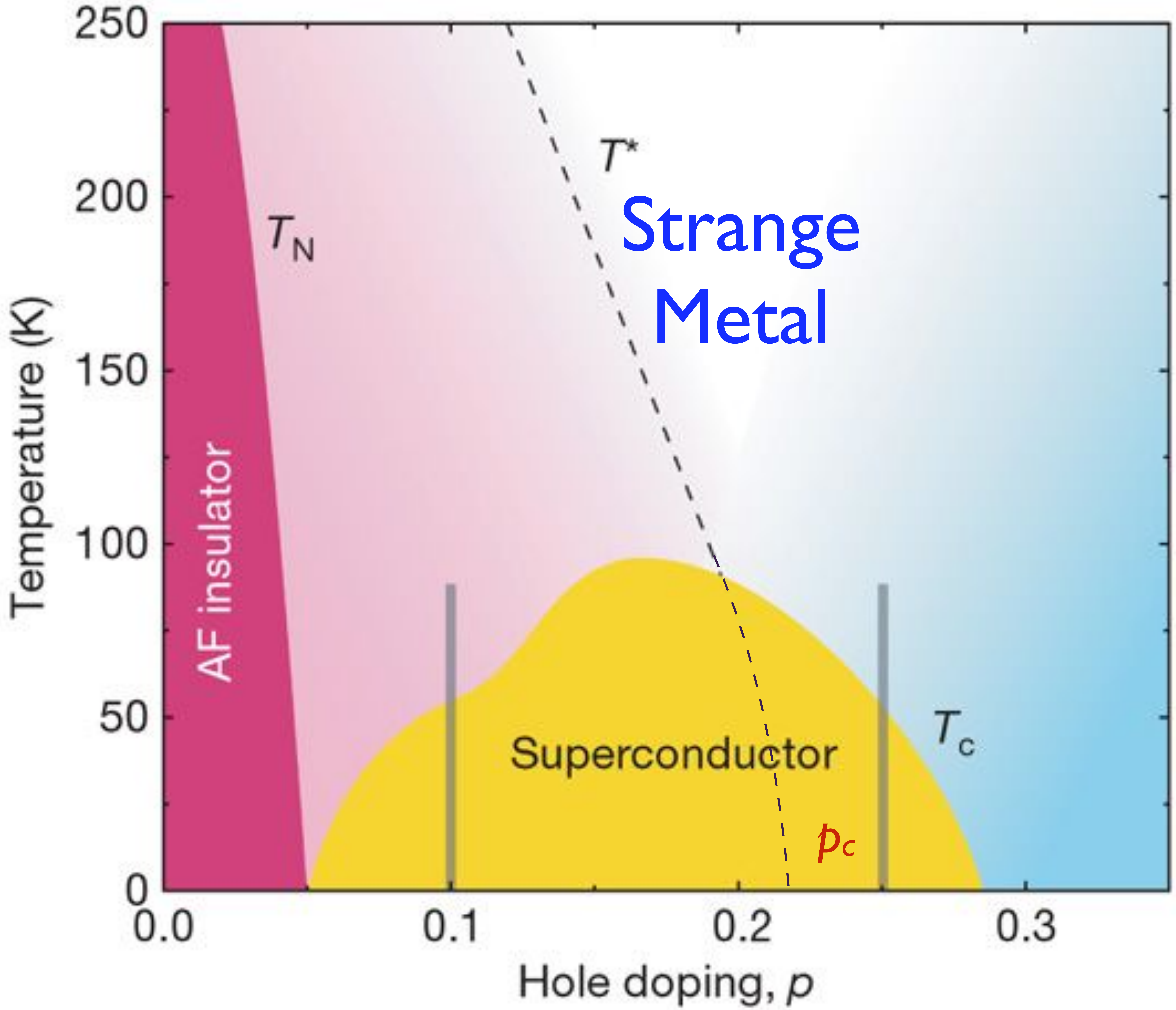
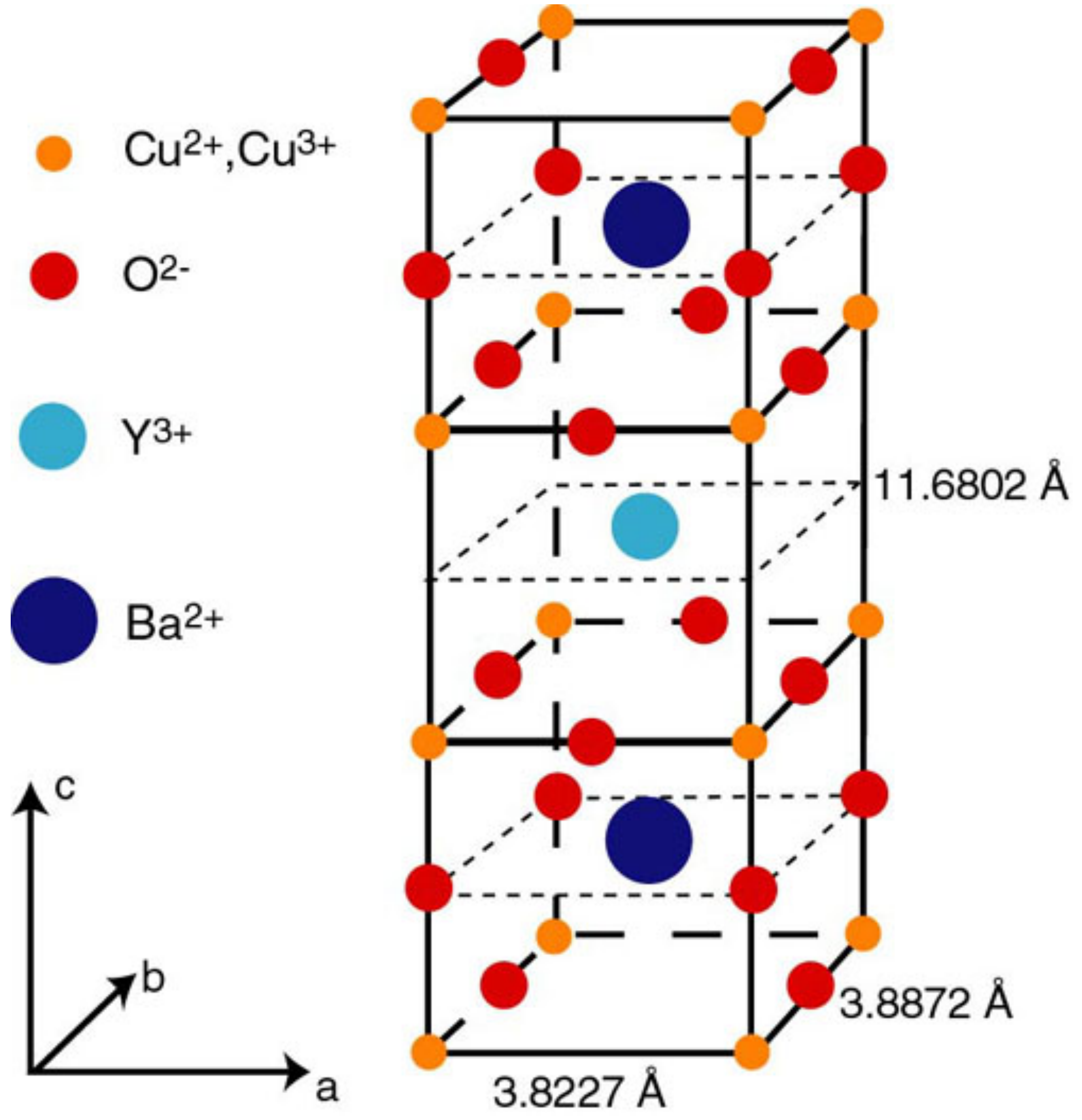
- Marginal Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \sim |\varepsilon|$ and a density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

Strange metals

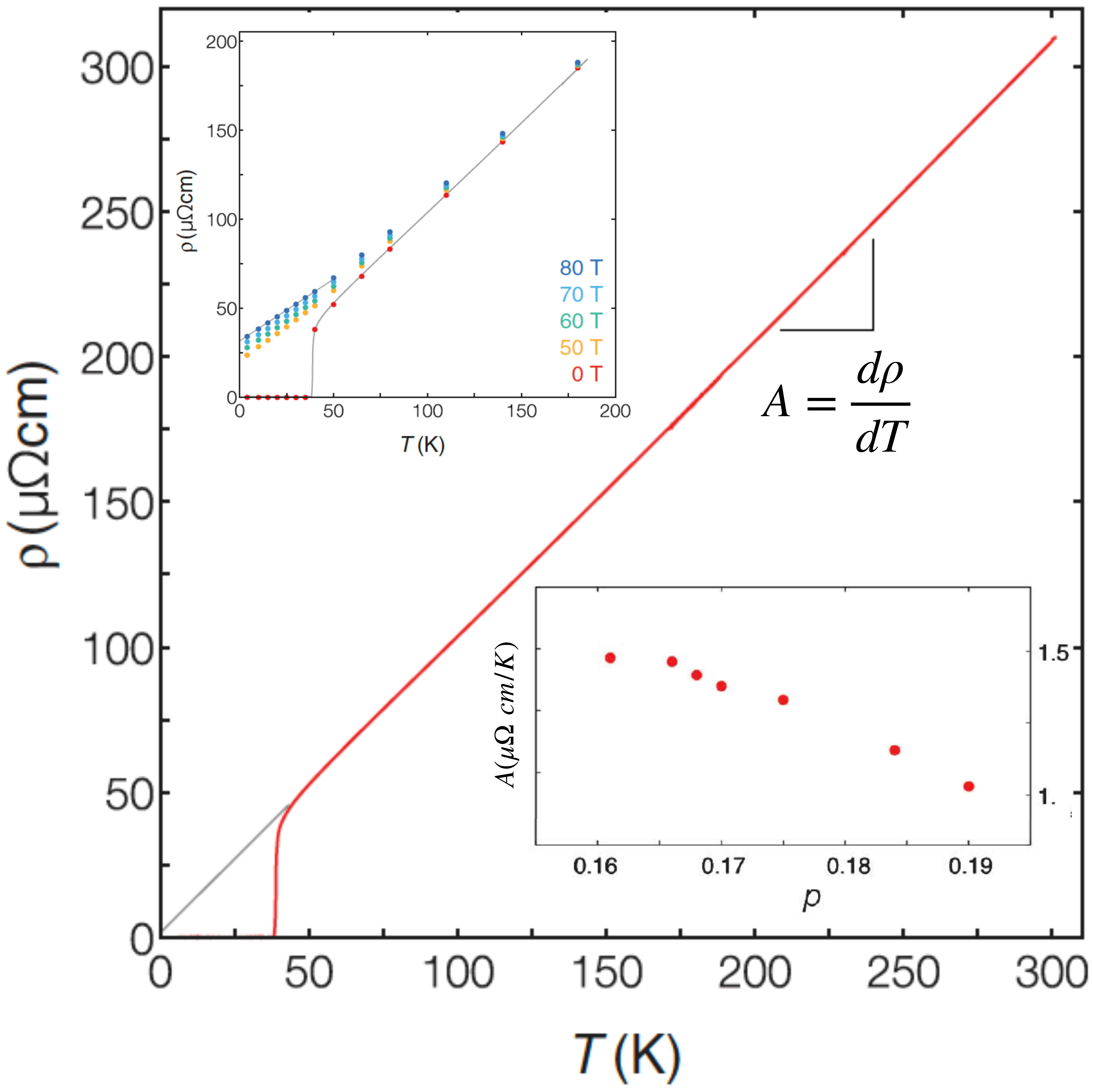
(defined by transport properties)



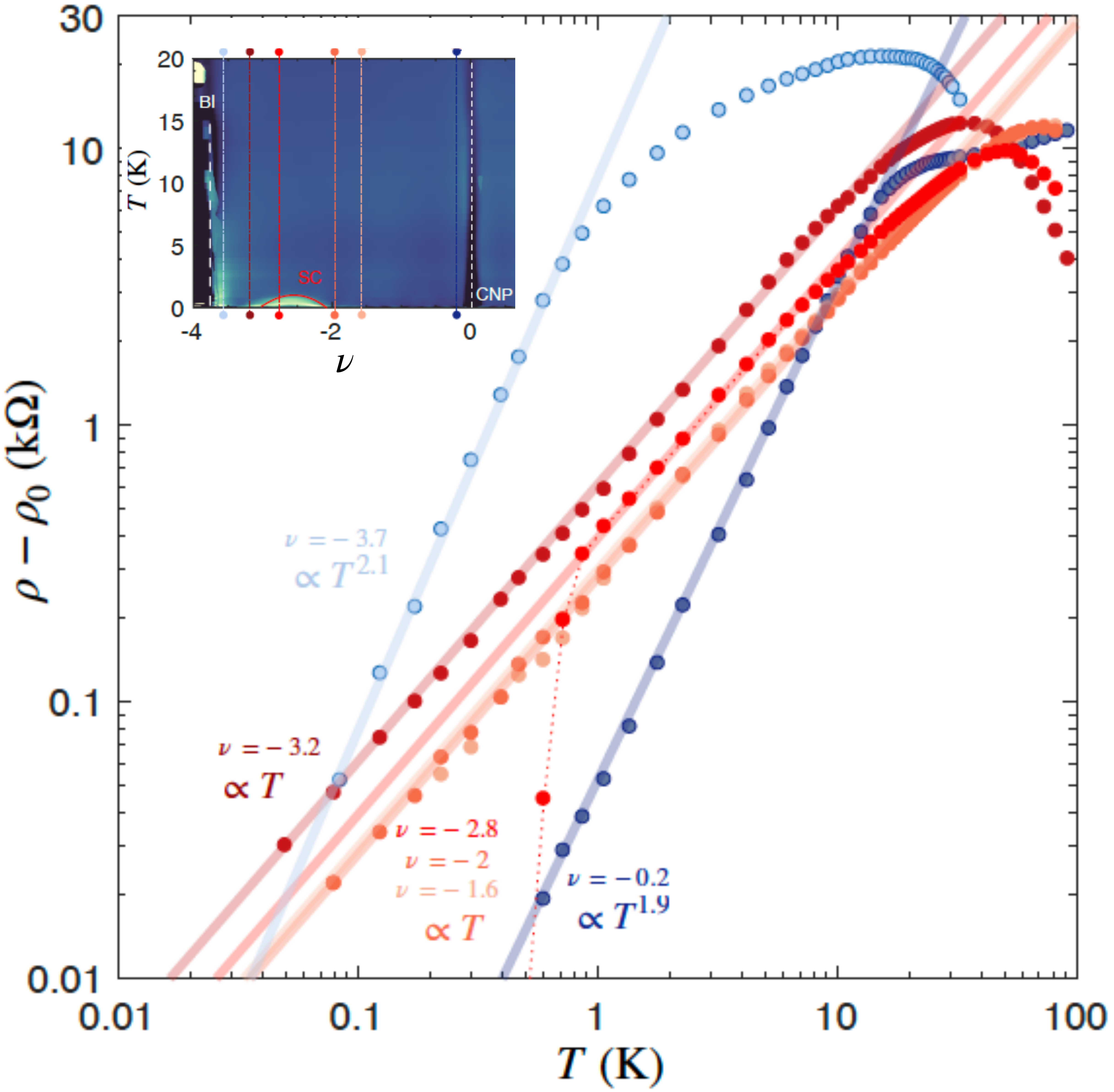
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018

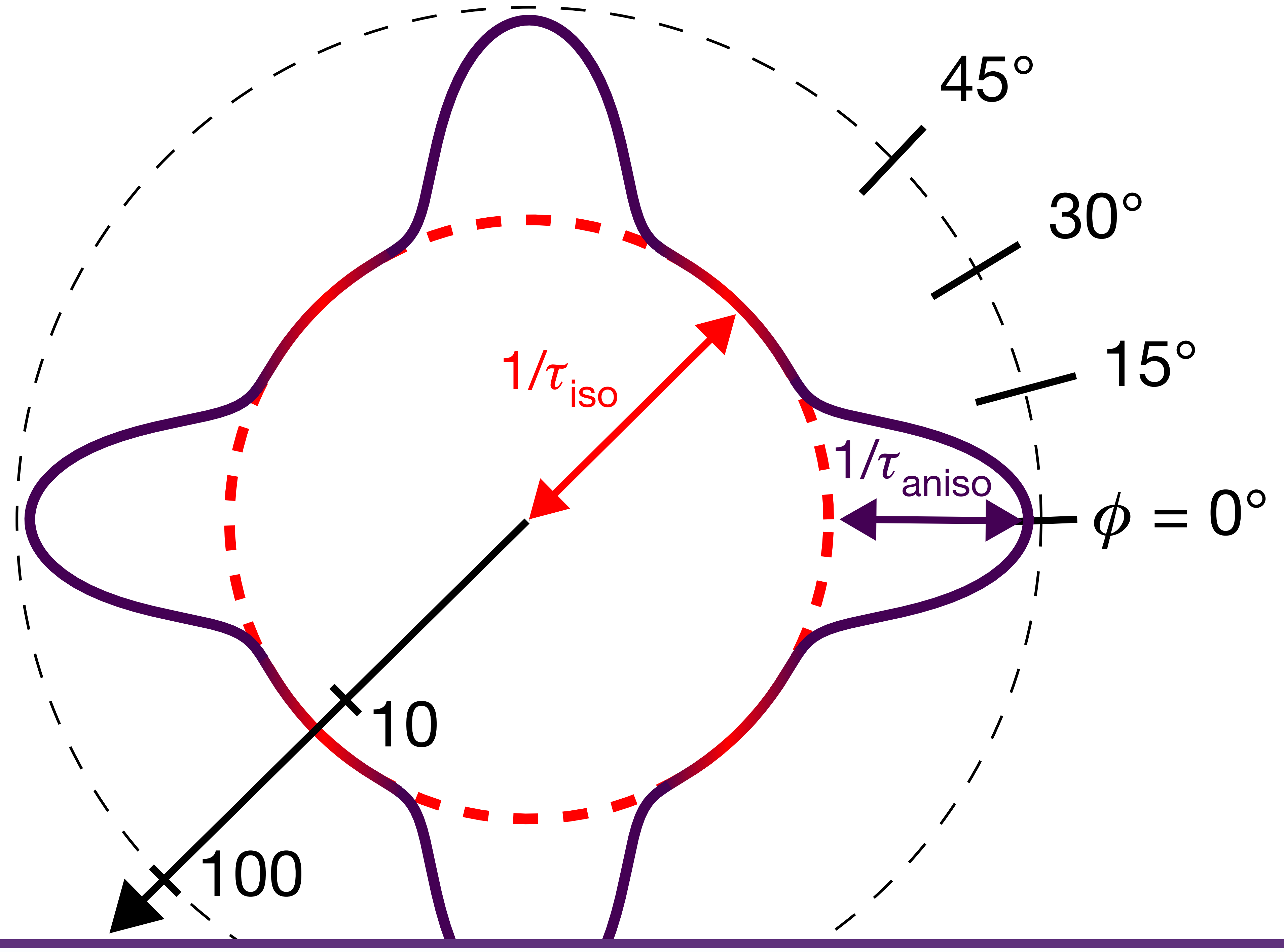


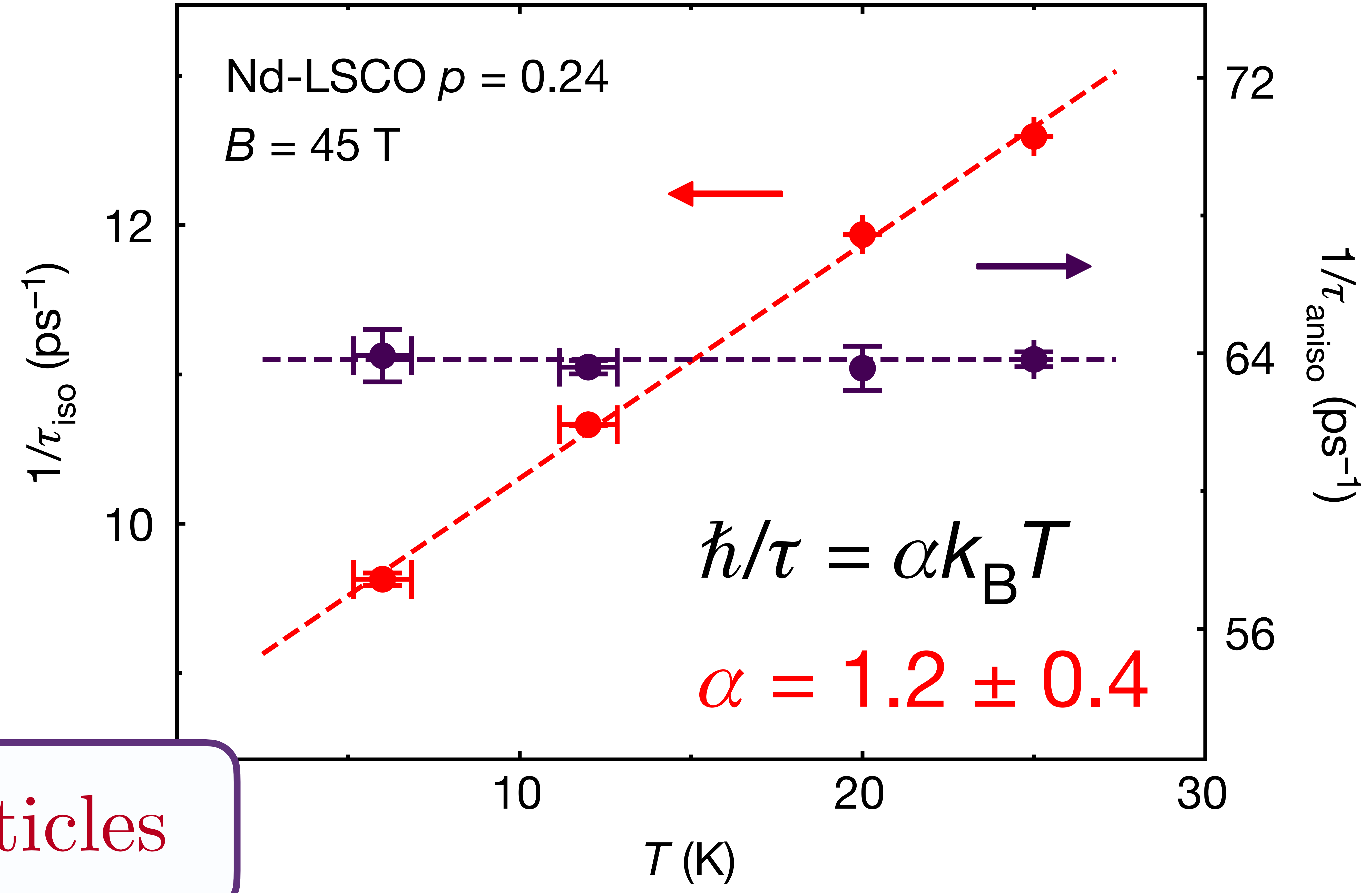
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Current flow without quasiparticles

Properties of a strange metal:

- Resitivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$ and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

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S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

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- Optical conductivity

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

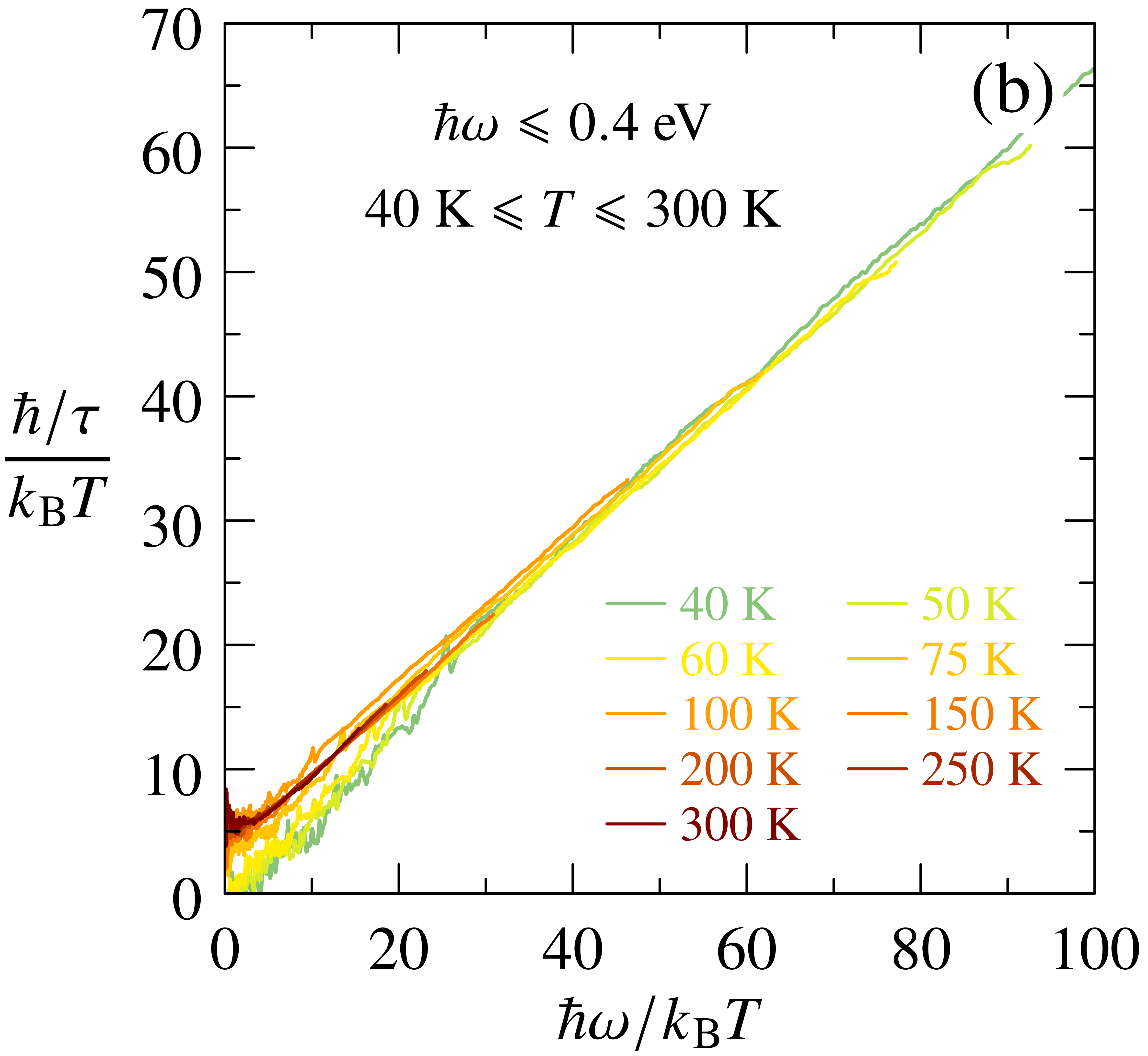
$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau(\omega)} = \frac{k_B T}{\hbar} G\left(\frac{\hbar\omega}{k_B T}\right)$$

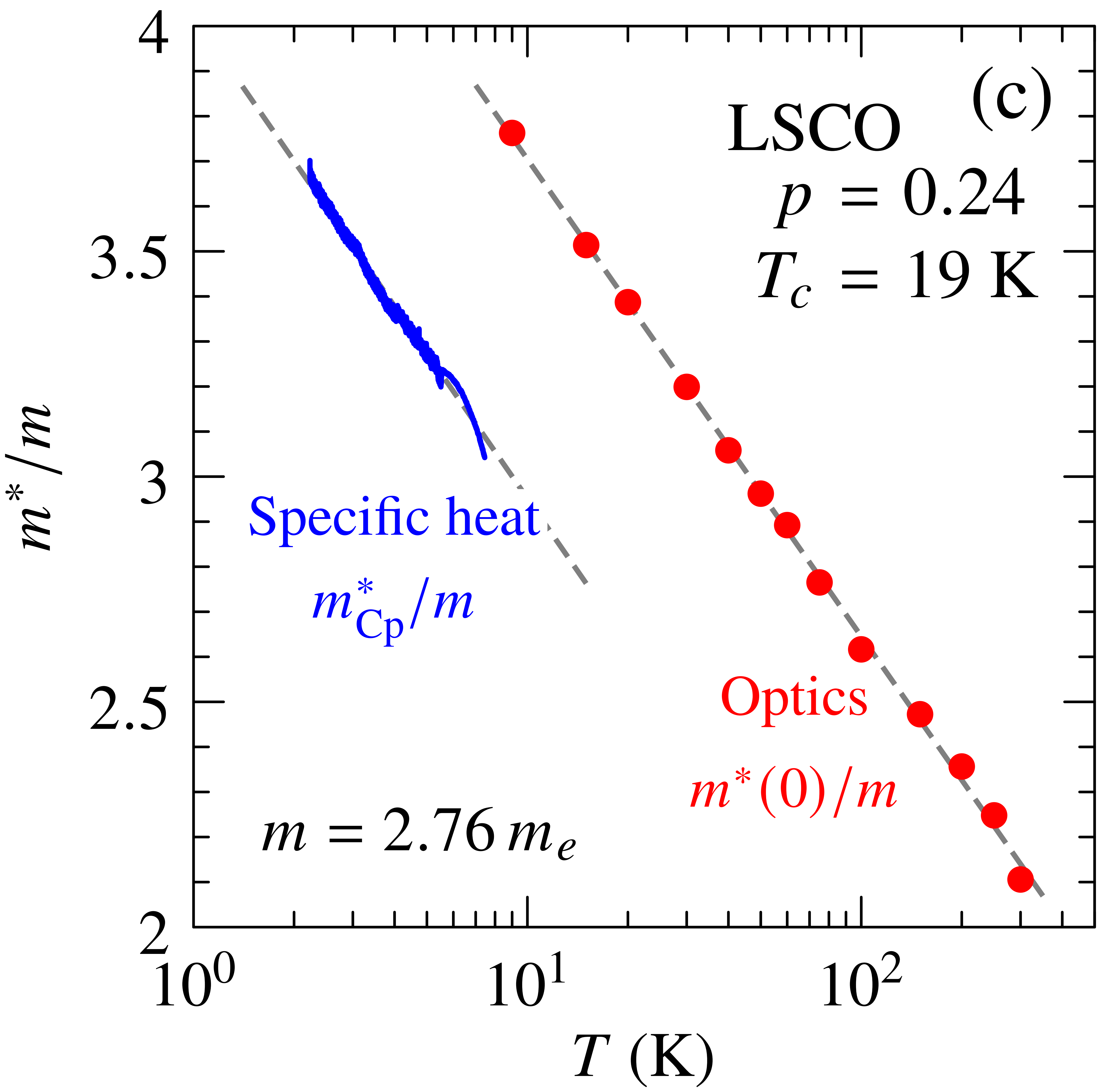
B. Michon……A. Georges, arXiv:2205.04030

Properties of a strange metal:

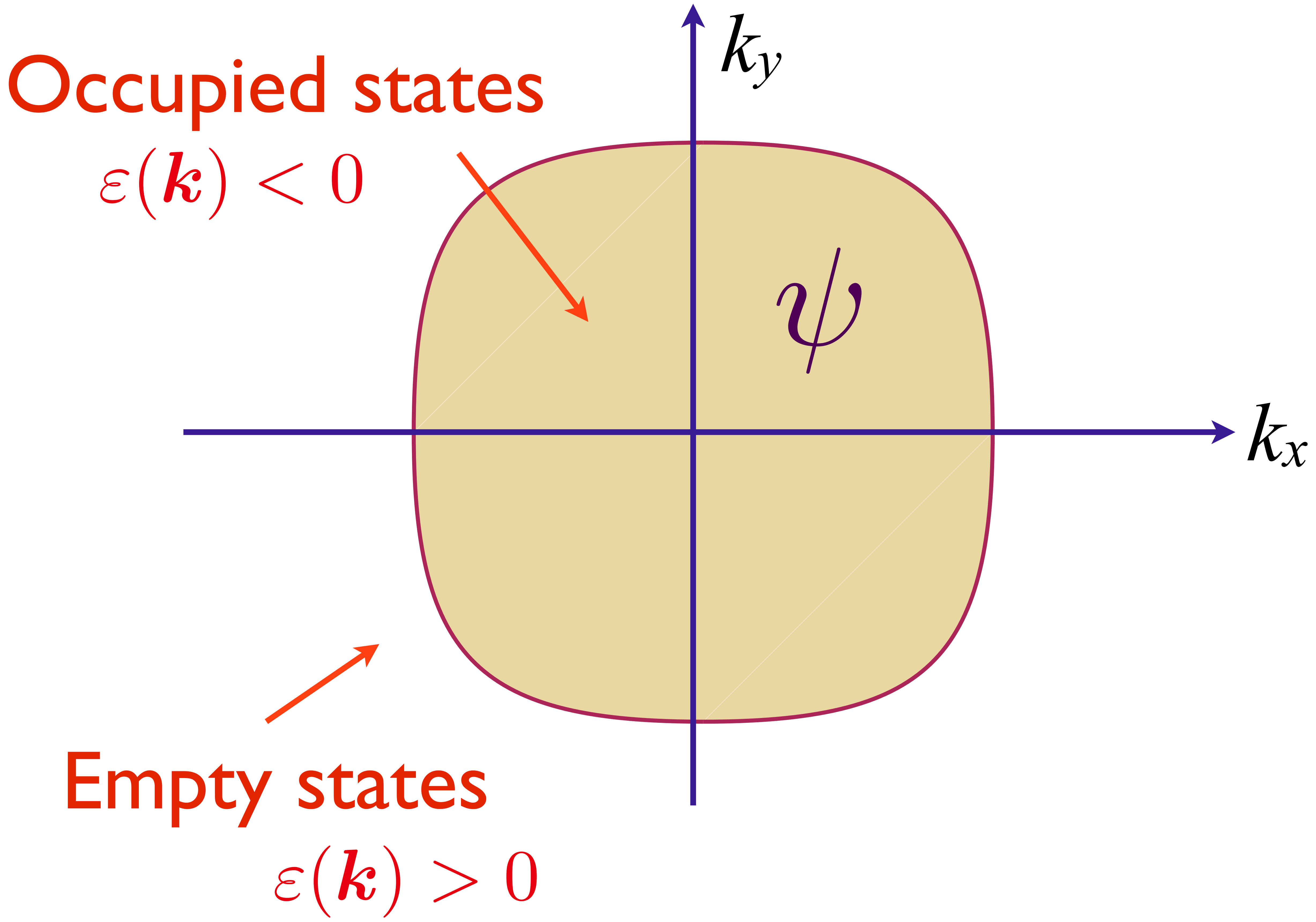
- Optical conductivity

B. Michon……A. Georges, arXiv:2205.04030





3 key ingredients of our universal theory of strange metals:



+

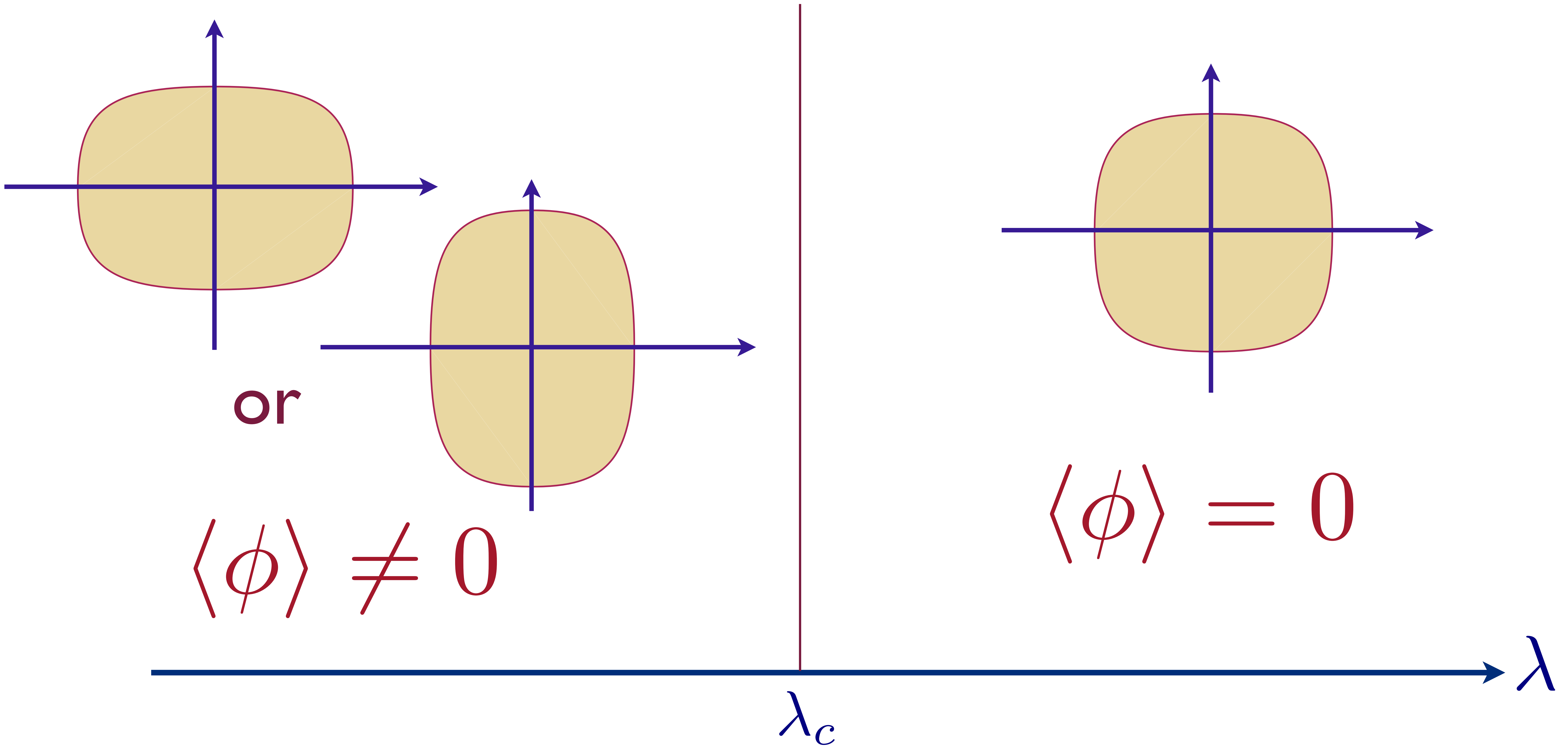
1. a critical boson

$$\phi$$

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order...

3 key ingredients of our universal theory of strange metals:

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

3 key ingredients of our universal theory of strange metals:

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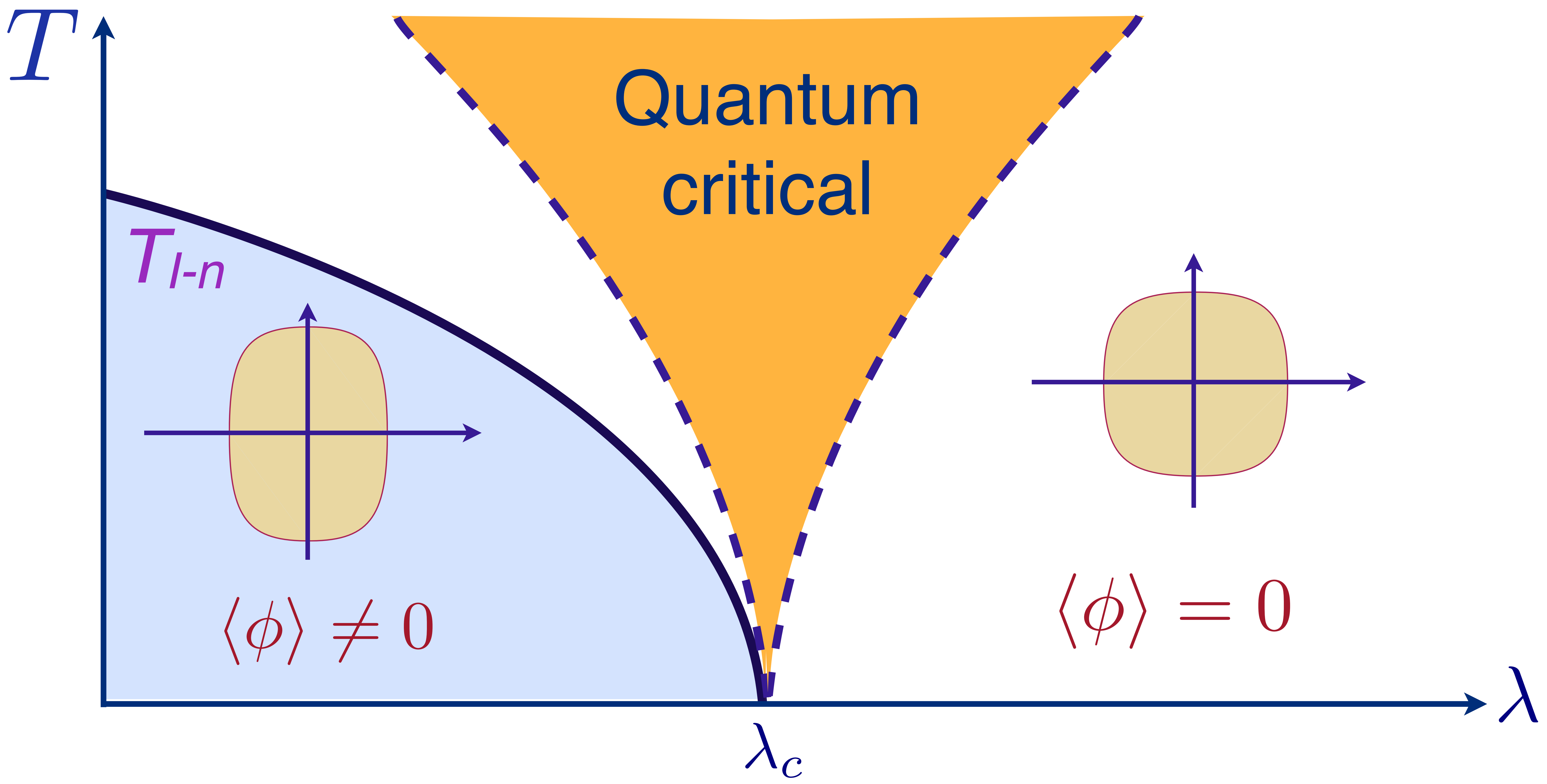


Phase diagram as a function of T and λ

3 key ingredients of our universal theory of strange metals:

2. Spatially random interactions: *e.g.* randomness in hopping t_{ij}, leads to randomness in exchange interactions t_{ij}^2/U. More generally we have fermion (ψ) and boson Yukawa coupling of the form

$$\int d^2r d\tau \left[g + g'(r)\right] \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)\,,$$

where g is spatially uniform and $g'(r)$ is spatially random.

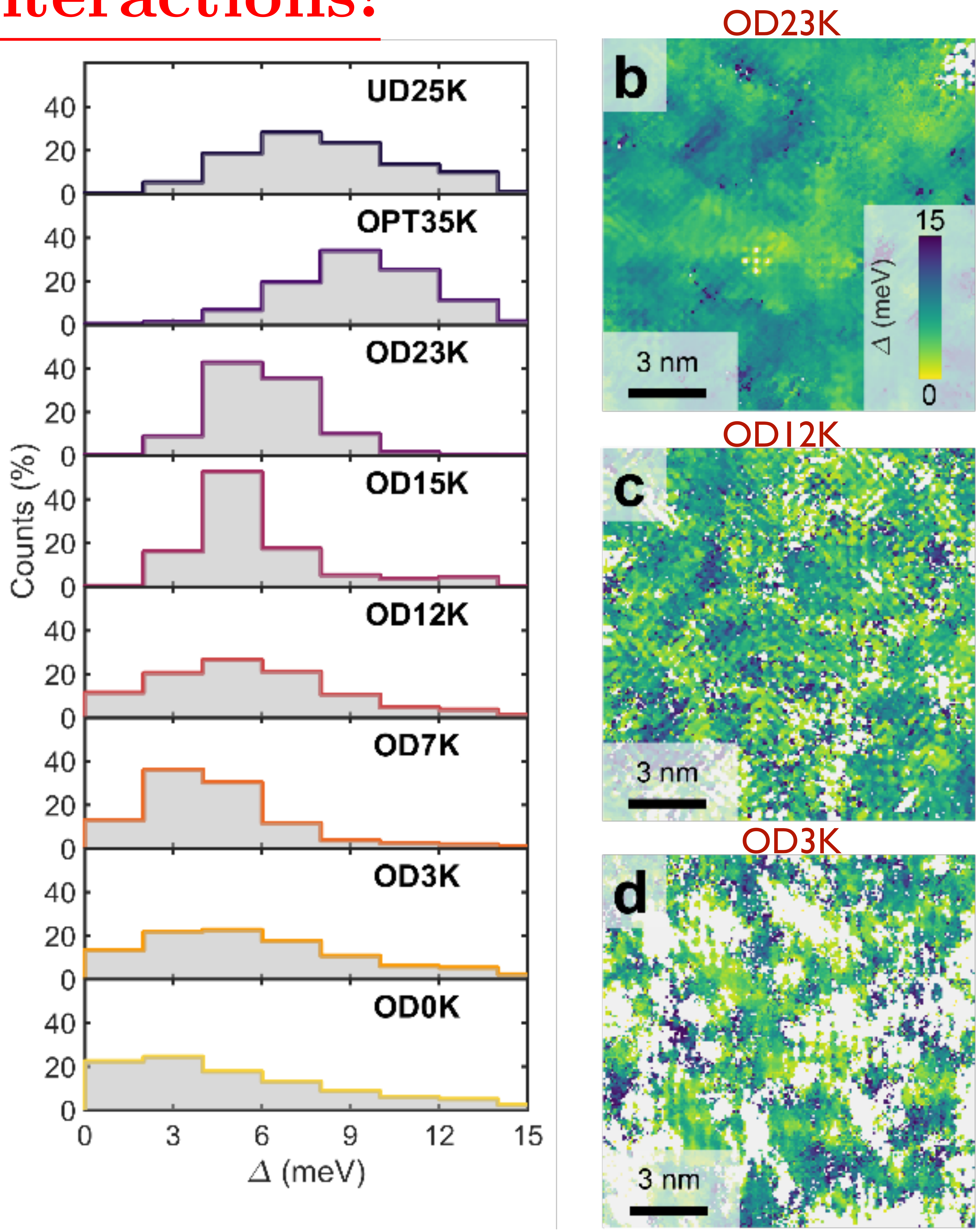
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

arXiv:2205.09740



3 key ingredients of our universal theory of strange metals:

3. Fermion-boson drag:

- For electron-phonon scattering in metals, we have "Bloch's law" (1931): a resistivity $\rho(T) \sim T^5$.

  However, Bloch's law ignores conservation of total momentum, or **phonon drag**. Peierls (1932) pointed out that the conservation of total momentum implies that an electrical current cannot decay, and so the resistance is practically zero in a pure sample. But because of the weak electron-phonon coupling, Bloch's law applies except in ultrapure crystals.

3 key ingredients of our universal theory of strange metals:

3. Fermion-boson drag:

In a non-Fermi liquid, we cannot separate the momenta carried by the fermions and the bosons, because neither of them exists at low energies! We must treat the combined system together: *extreme drag*. The analog of Bloch's law does not apply.

3 key ingredients of our universal theory of strange metals:

Needed: a systematic method
to include these key ingredients!

Fermi surface coupled to a critical boson: No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

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Fermi surface coupled to a critical boson:

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A marginal Fermi liquid but NOT a strange metal

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A non-Fermi liquid but NOT a strange metal

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Fermi surface coupled to a critical boson:

Interaction disorder

A marginal Fermi liquid AND a strange metal

Needed: a systematic method
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Yukawa-SYK "toy" models

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

Dionysios Anninos, Tarek Anous, Paul de Lange, George Konstantinidis, JHEP 03, 066 (2015)
W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

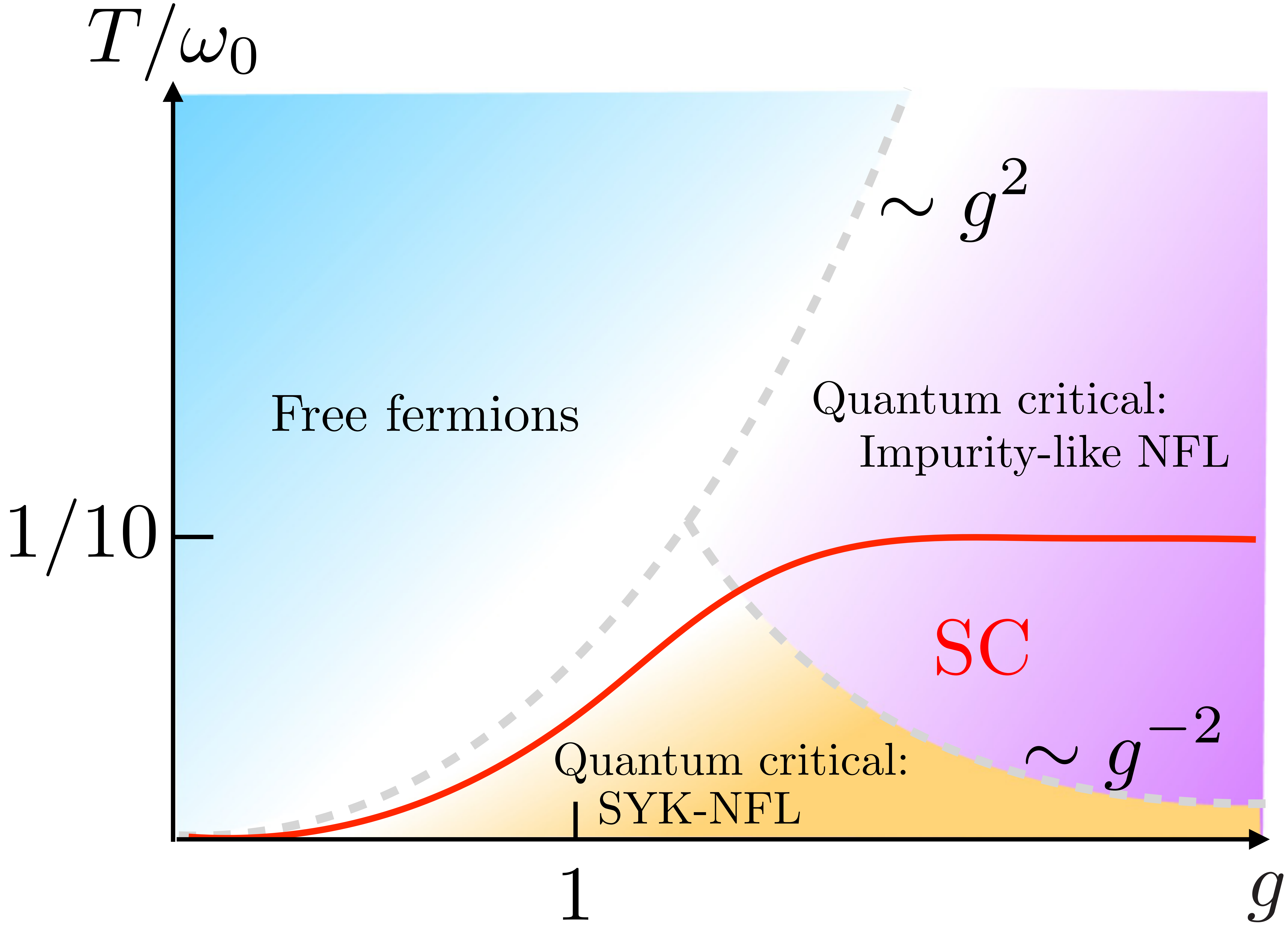
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Needed: a systematic method to include these key ingredients!

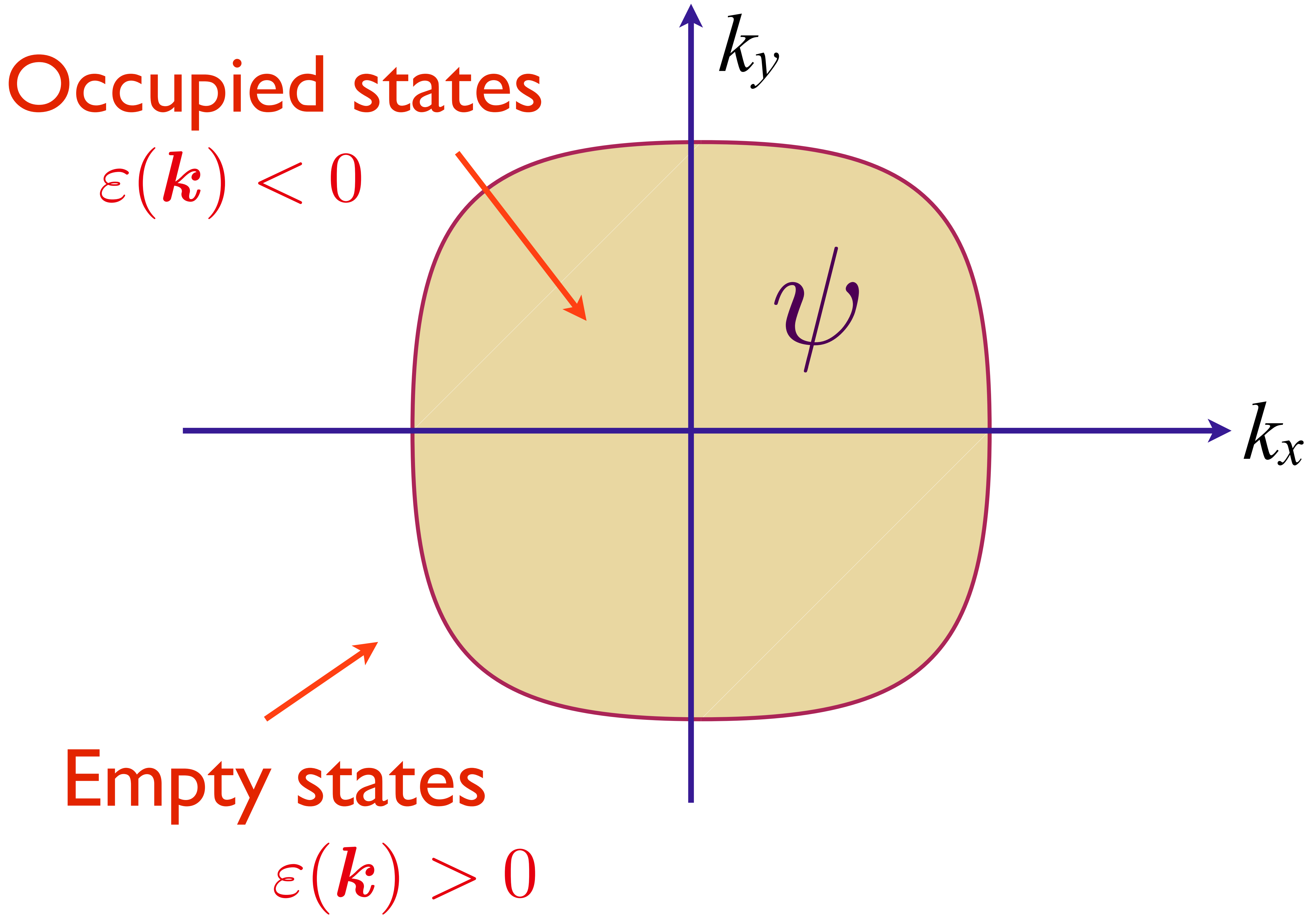
Beyond "toy" models....

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson



$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

+

a critical boson

ϕ

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order...

$$\mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + s\phi^2 \right]$$

Fermi surface coupled to a critical boson

"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

<u>Application of Yukawa-SYK approach:</u>

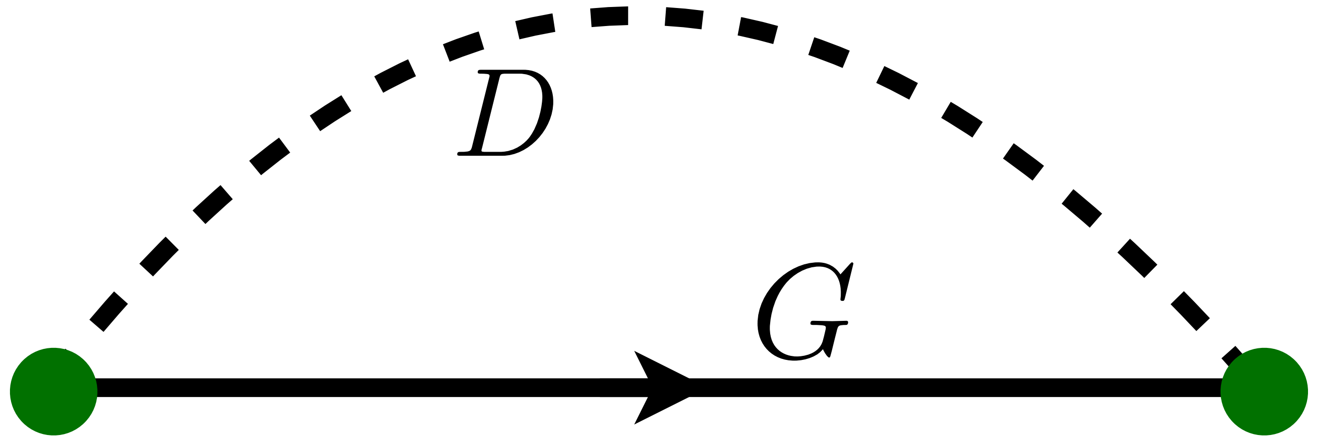
Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

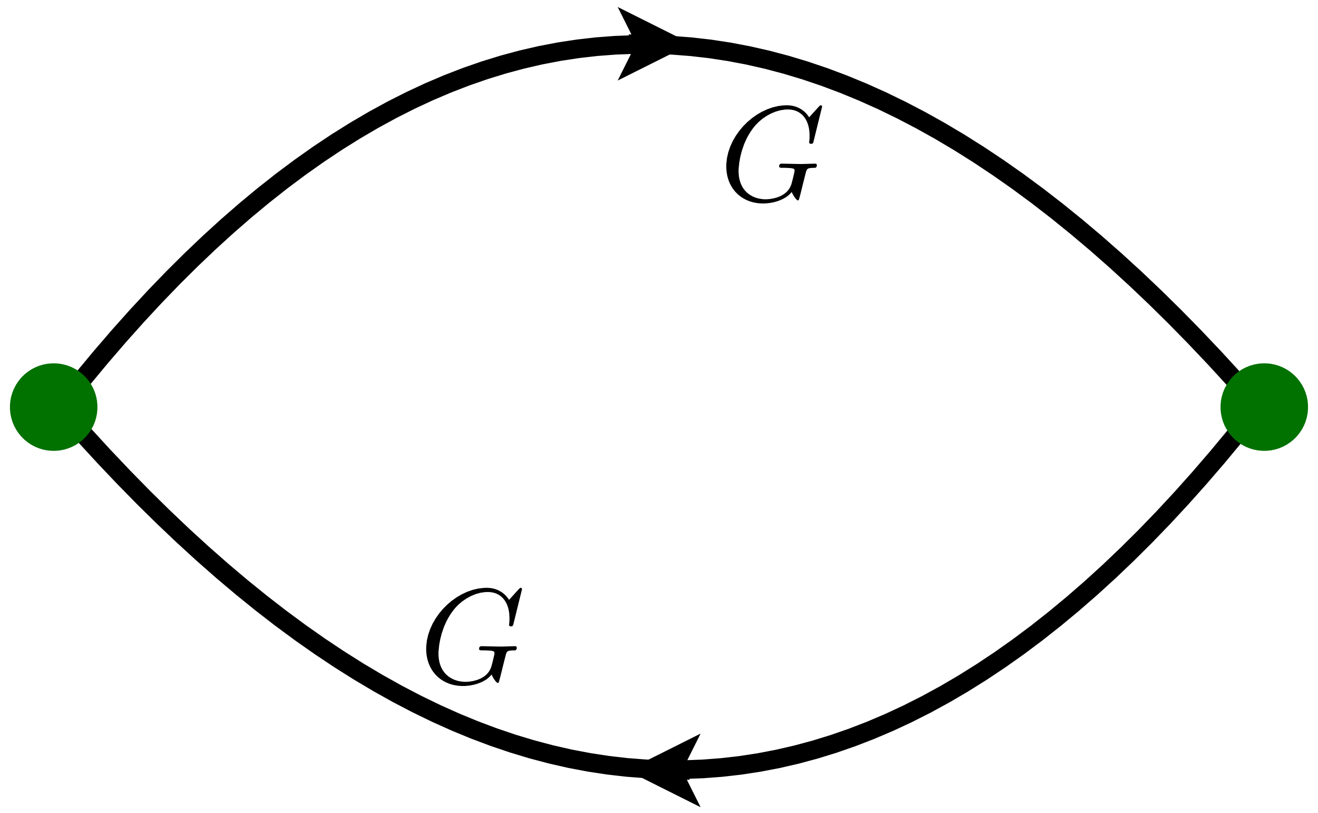
Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

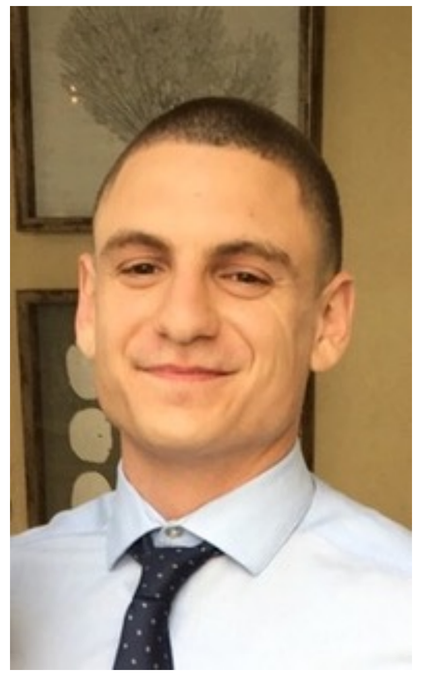
G-Σ-D-Π Theory

The saddle point equations are

$$\Sigma(\boldsymbol{r},\tau) = g^2 \lambda D(\boldsymbol{r},\tau) G(\boldsymbol{r},\tau),$$
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau) G(\boldsymbol{r},\tau),$$
$$G(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n)},$$
$$D(\boldsymbol{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q}, i\Omega_m)}.$$

$\Sigma =$ 

$\Pi =$ 



Exact Solution at small ω:

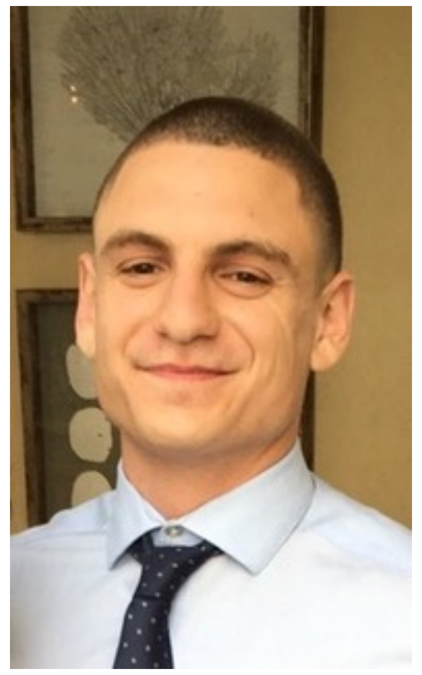
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

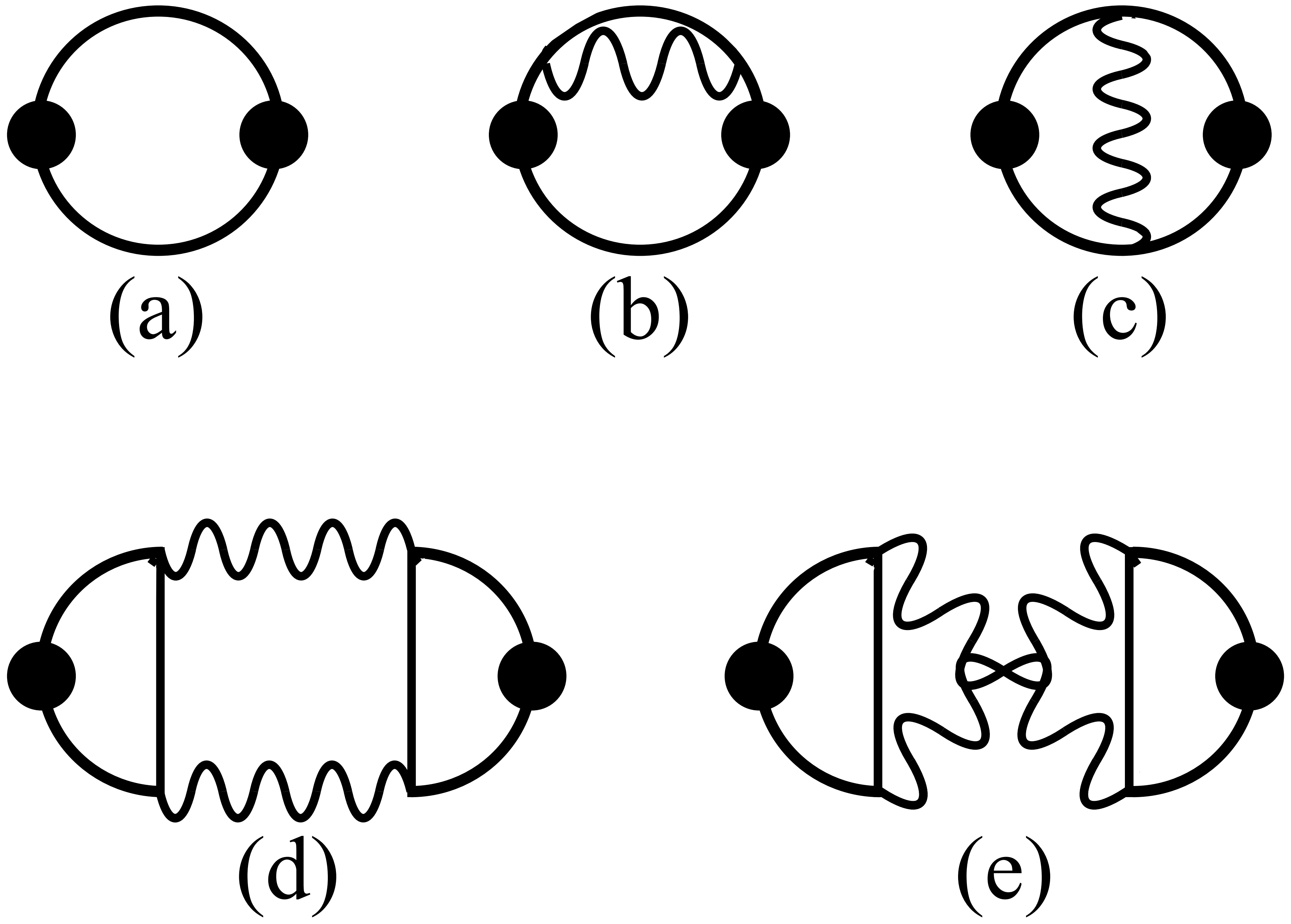
Fermi surface coupled to a critical boson



"Yukawa" coupling: $$\frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Transport:

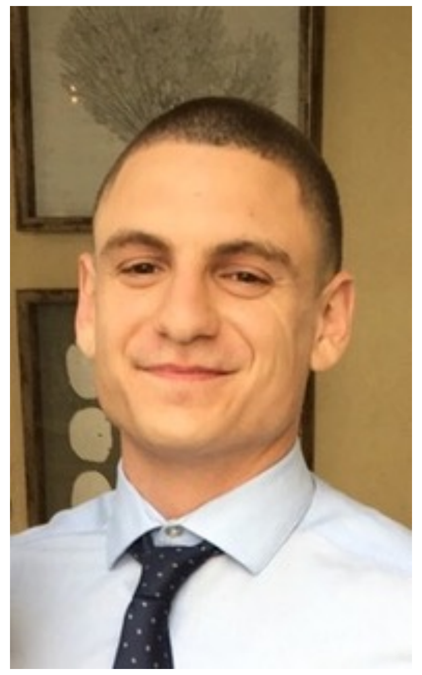


+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ (analog of Bloch's law) and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

These conclusions are not consistent with conservation of total momentum *i.e.* 'boson drag'.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990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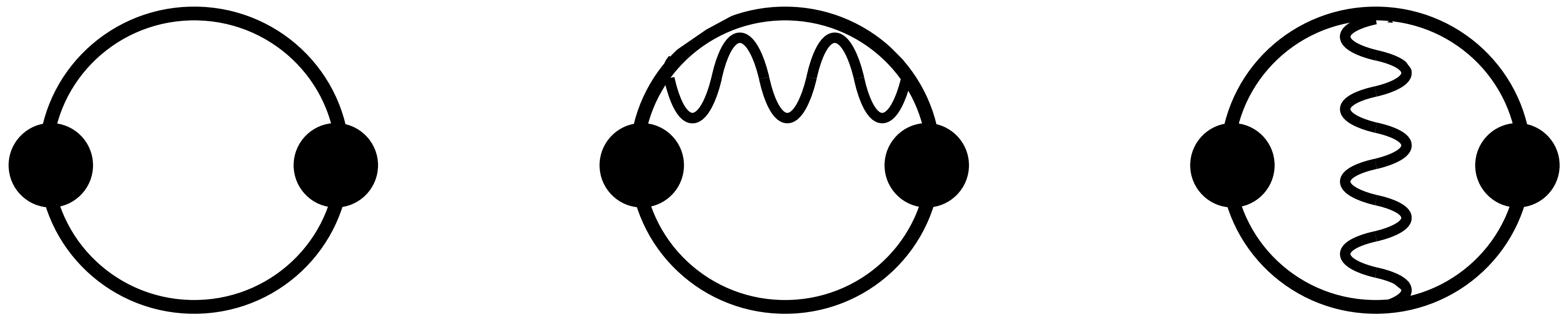
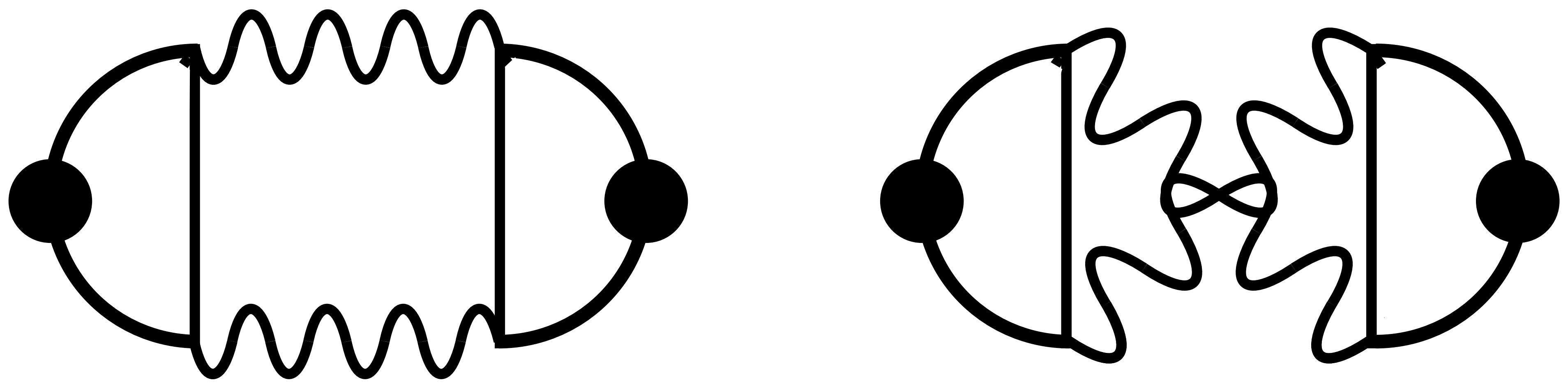


(a) (b) (c)



(d) (e)

+ all ladders and bubbles…..

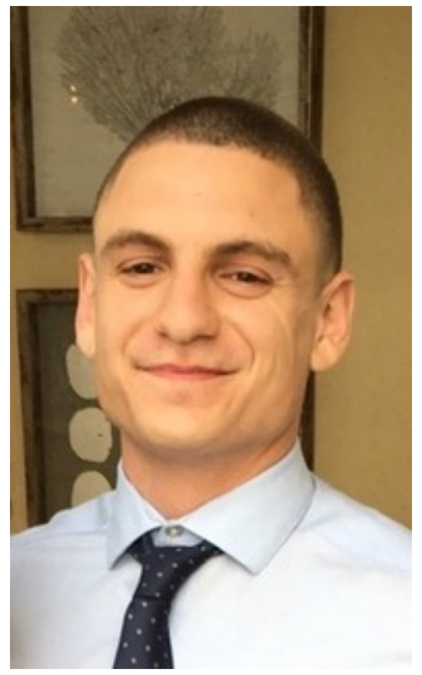
Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson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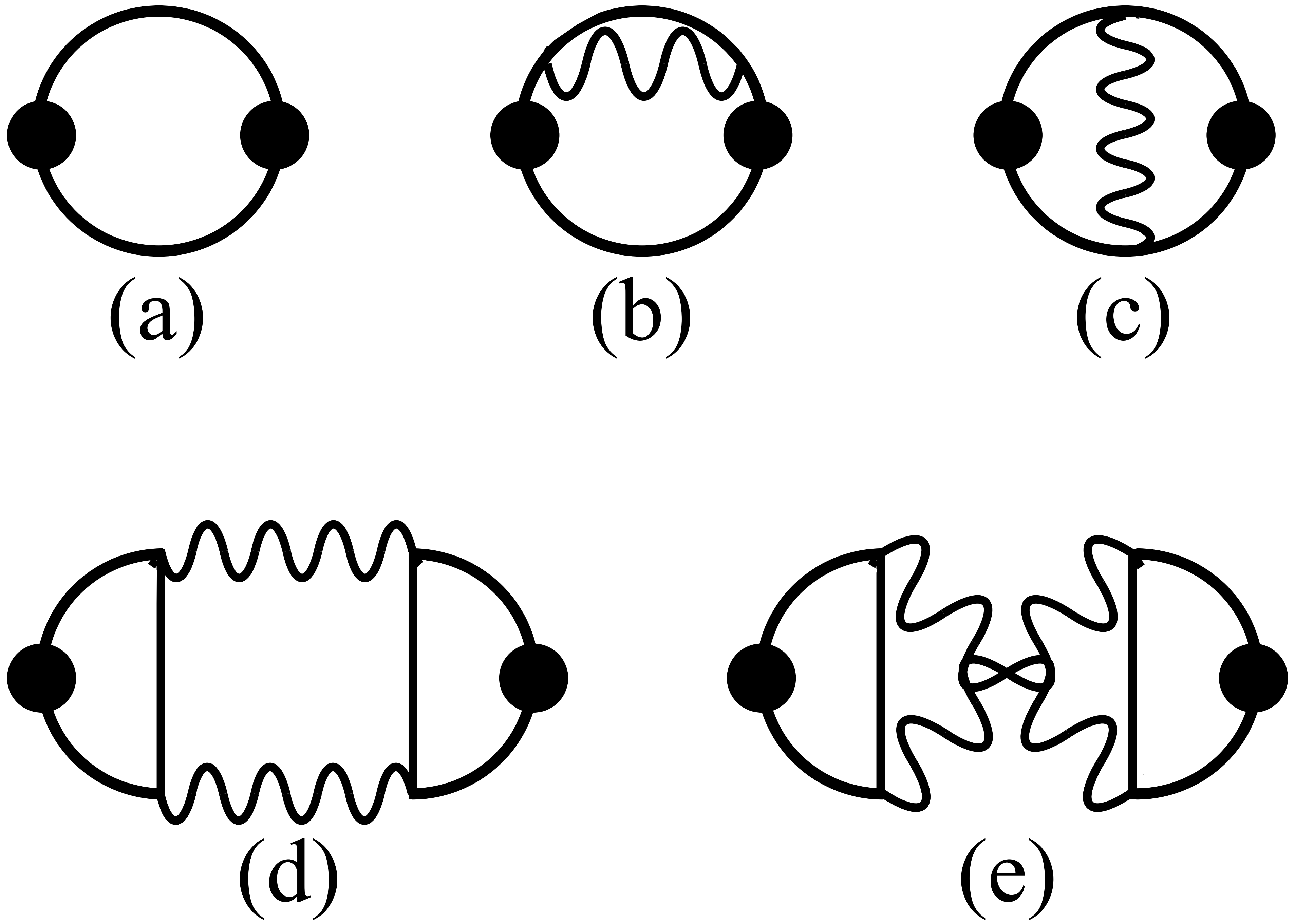

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A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \ldots$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson:

No spatial disorder

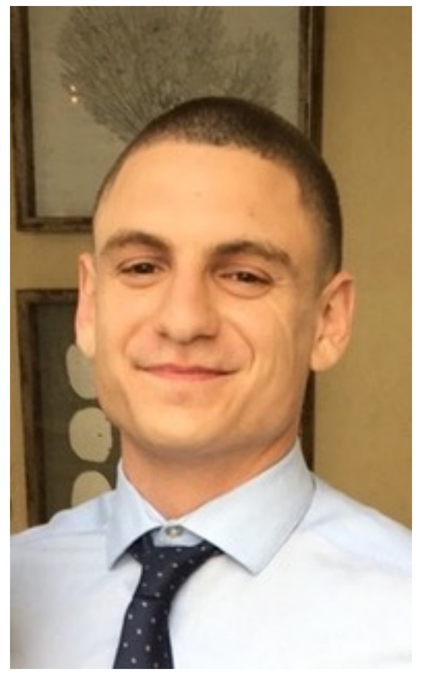
A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with spatial disorder

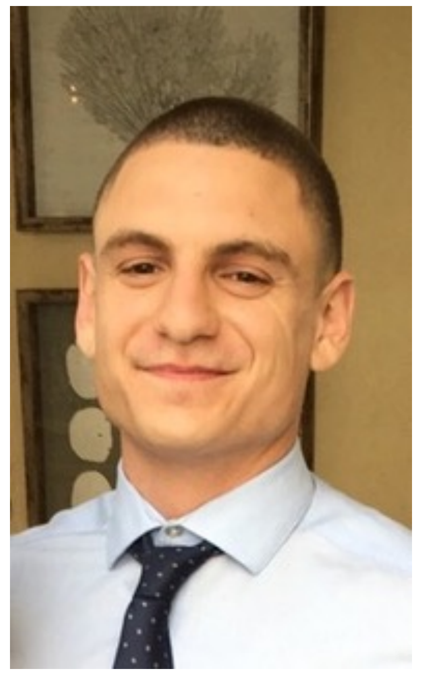

"Yukawa" coupling: $$\frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random potential: $$+\frac{1}{\sqrt{N}}\int d^2r d\tau\, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl} g_{abc}} = g^2\, \delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r) v_{lm}(r')} = v^2\, \delta(r-r')\delta_{il}\delta_{jm}$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder


"Yukawa" coupling: $\frac{g_{ij\ell}}{N} \int d^2r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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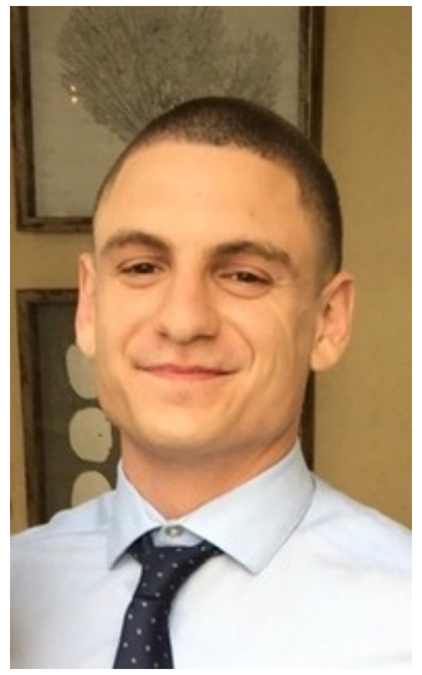
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|)$

Marginal Fermi liquid self energy and $T \log T$ specific heat

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder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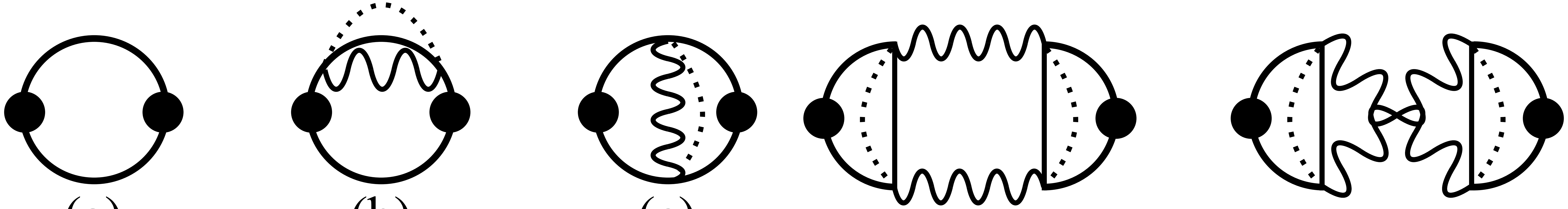

(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$

(c) $\sigma_{V,g}$

(d)

(e)

+ all ladders and bubbles.....

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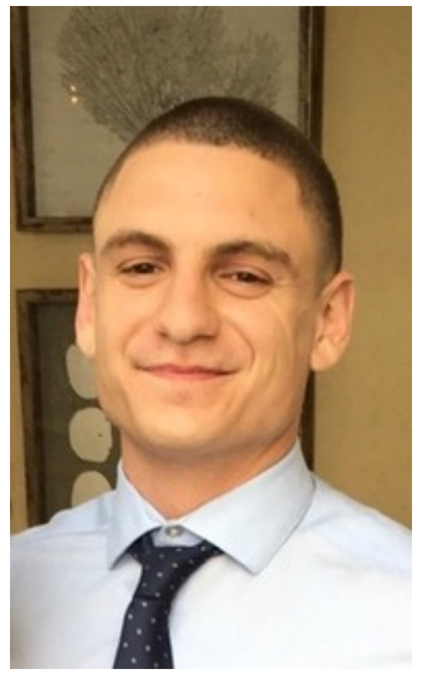

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The g^2log term does not contribute to transport:
With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots .$$

with $1/\rho(0) \sim 1/\tau_{\rm trans} \sim v^2$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson:

Potential disorder

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Interaction disorder

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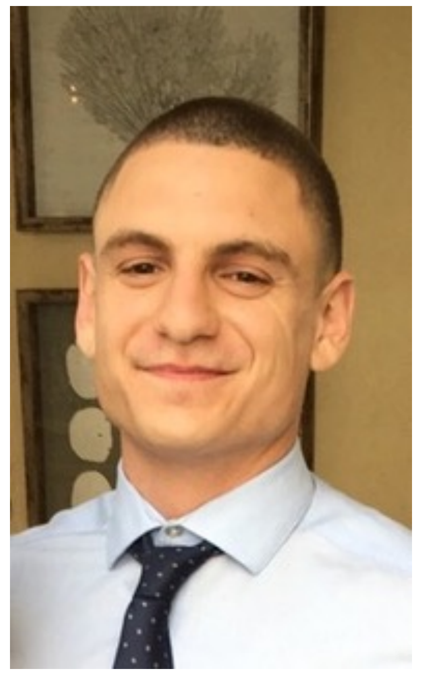


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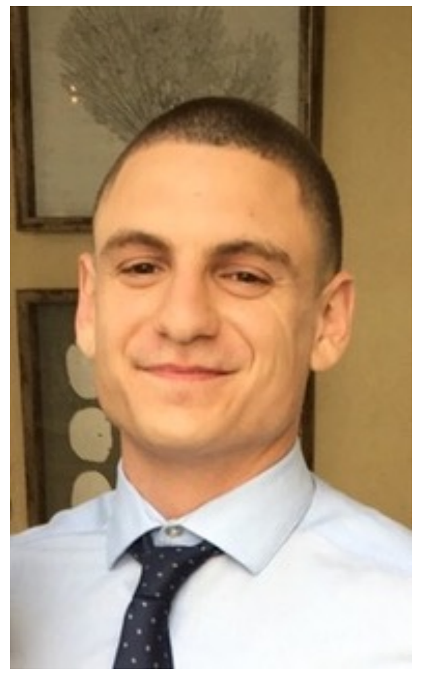
Fermi surface coupled to a critical boson with spatial disorder


Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \qquad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|, \qquad D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|}$$

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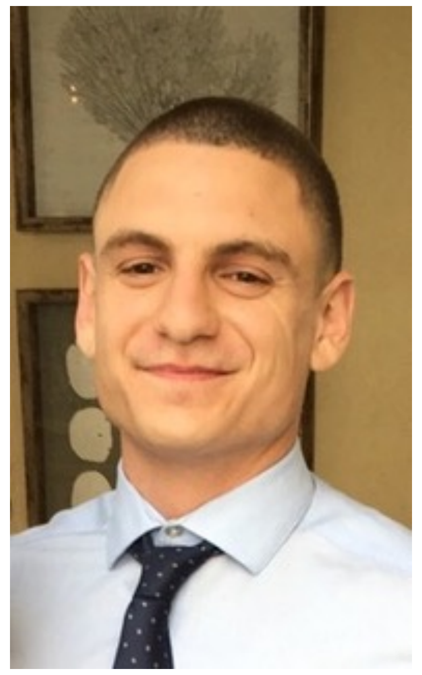


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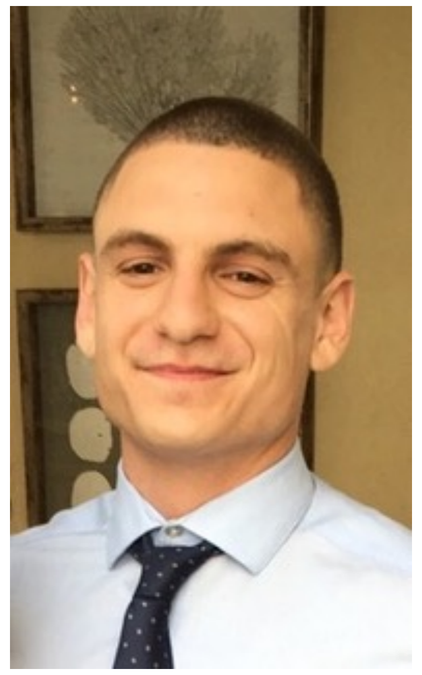

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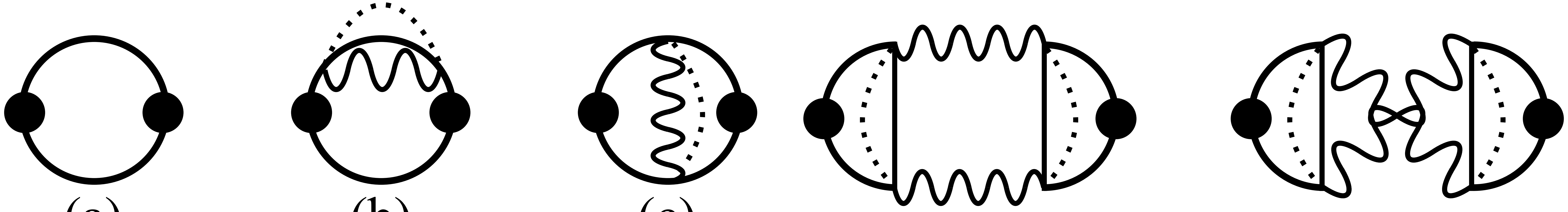

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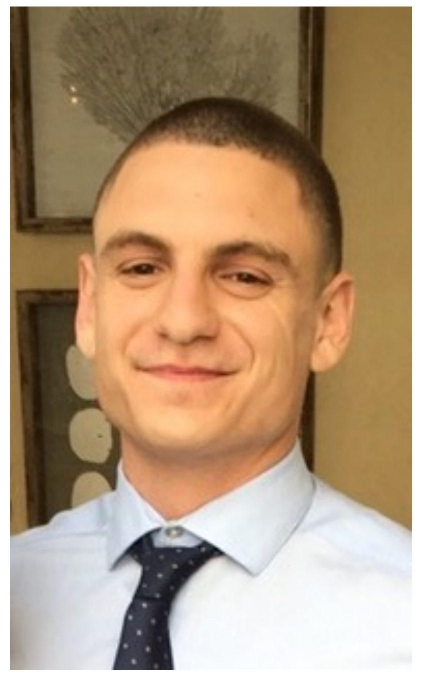


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Conductivity:

The g^2 log term does not contribute to transport but the g'^2 log term does!

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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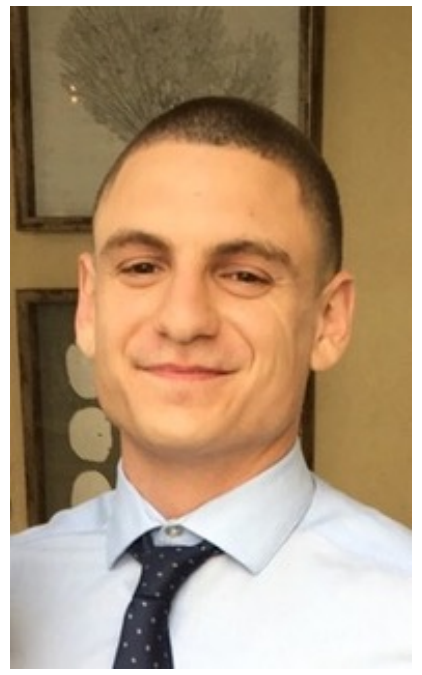

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Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega\, m^*(\omega)/m]^{-1}$

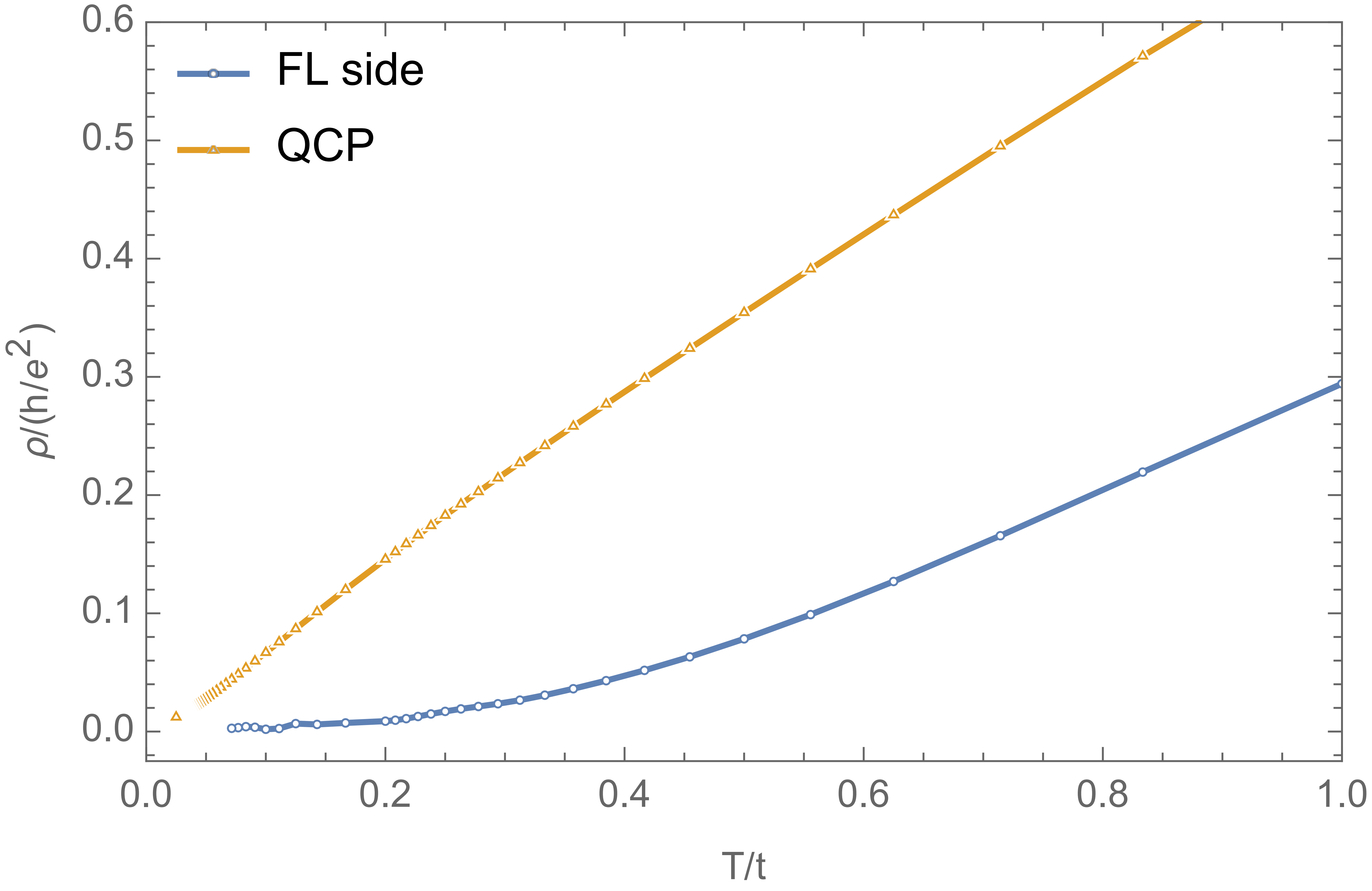
$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

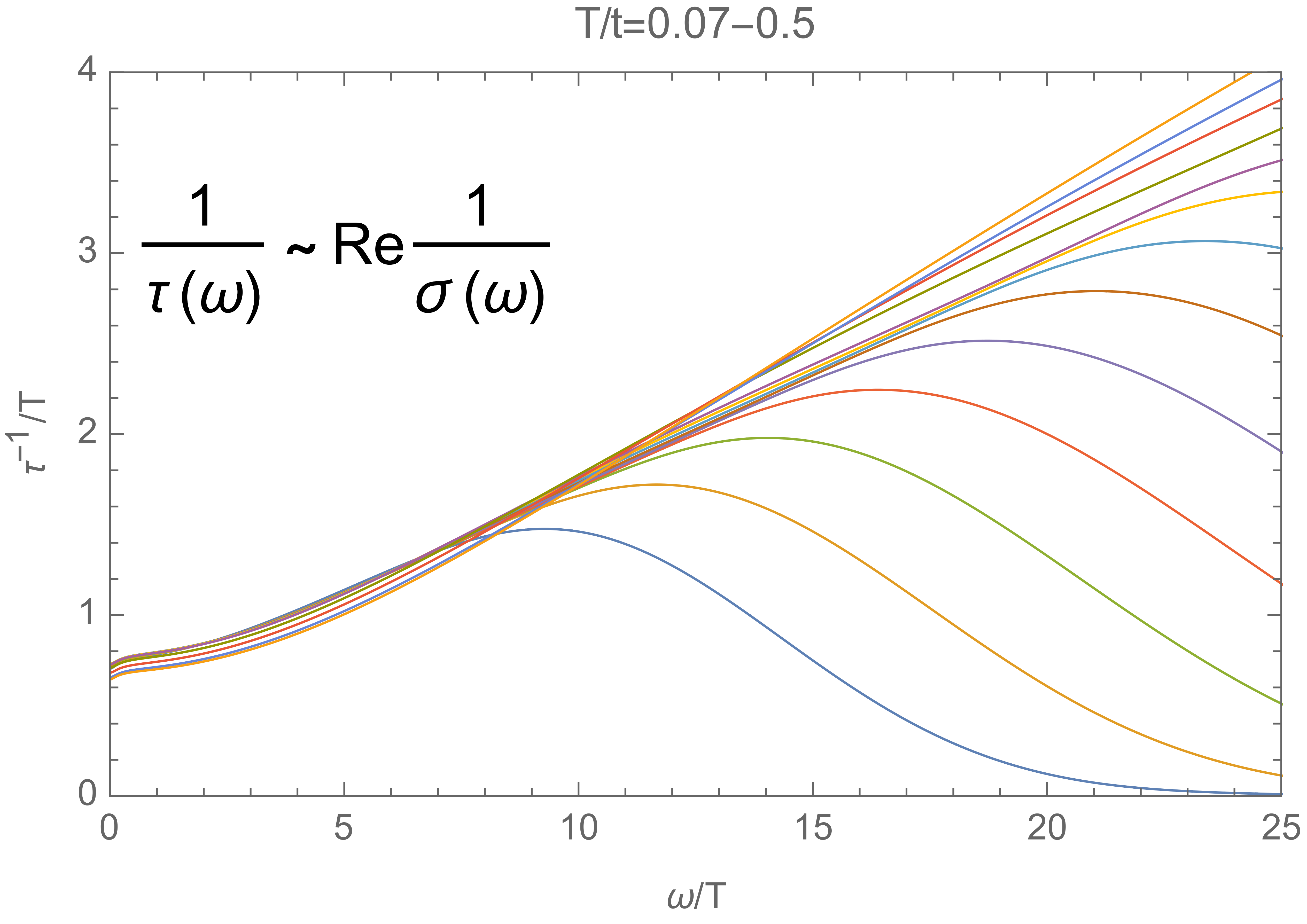
Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Strange metal from a Yukawa-SYK model

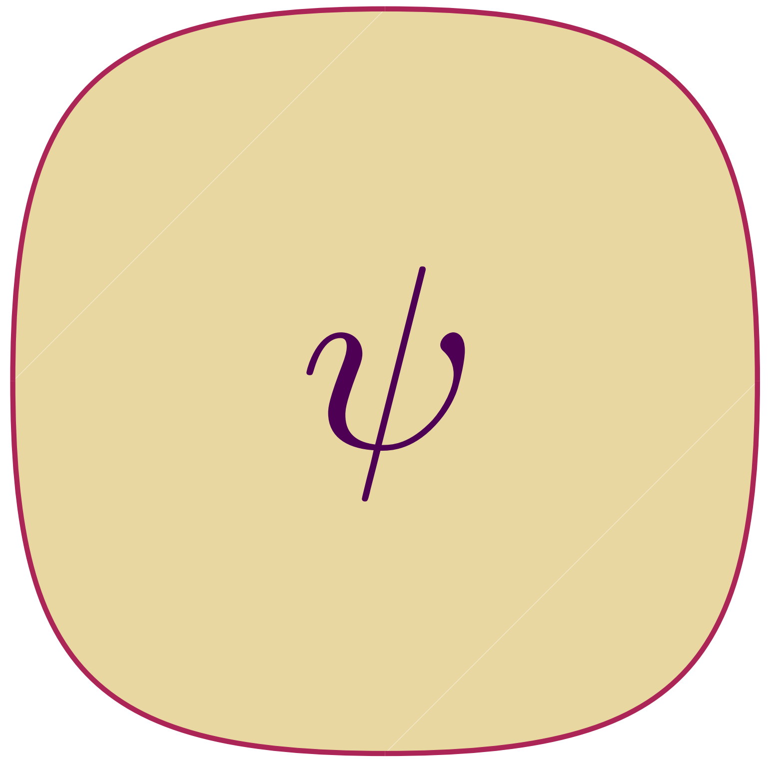
Full numerical solution of large N limit at $g' \neq 0$, $g = 0$
(the singular corrections from a non-zero g are expected to vanish in the conductivity).





Ilya Esterlis, Aavishkar Patel, Haoyu Guo,, S.S. to appear

Strange metal from a Yukawa-SYK model



$+$ a critical boson ϕ

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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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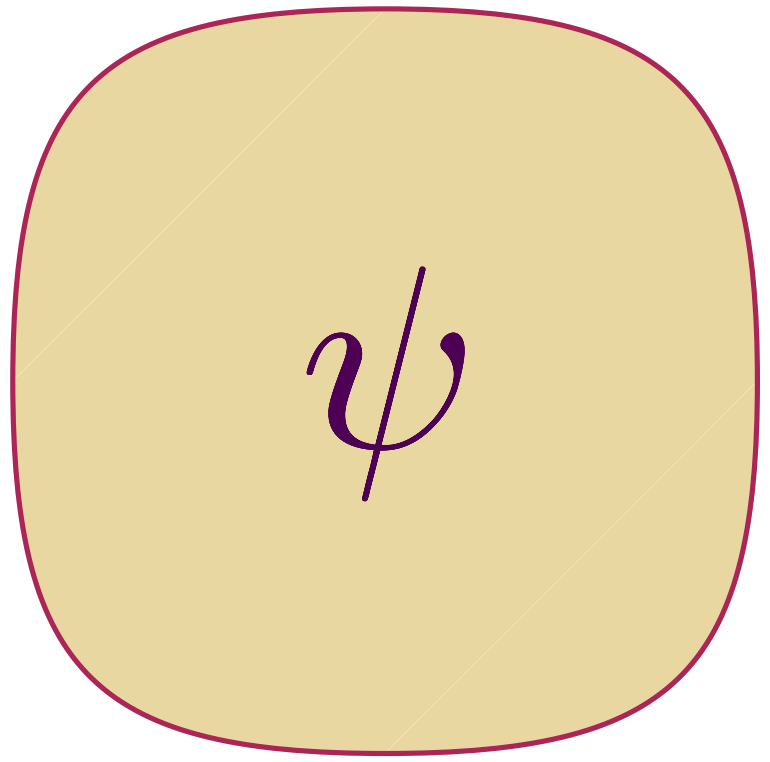

$+$ a critical boson ϕ

Non-Fermi liquid with $T^{2/3}$ specific heat, but conductivity $\sigma(\omega) \sim \delta(\omega)$

"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N} \displaystyle\int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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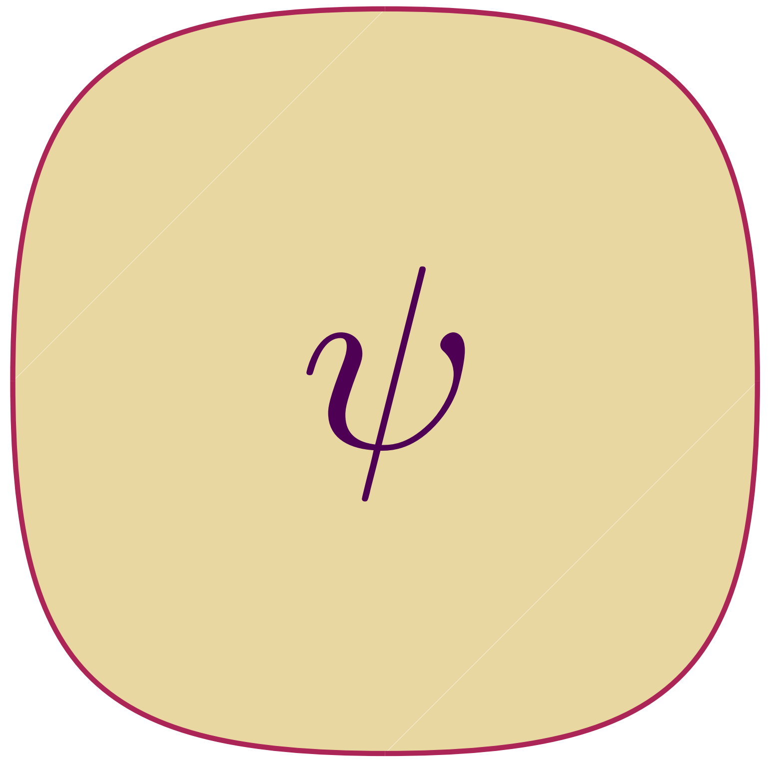

$+$ a critical boson ϕ

MFL self-energy, $T\ln(1/T)$ specific heat,
but T-independent 'residual' resistivity,
and negligible optical conductivity

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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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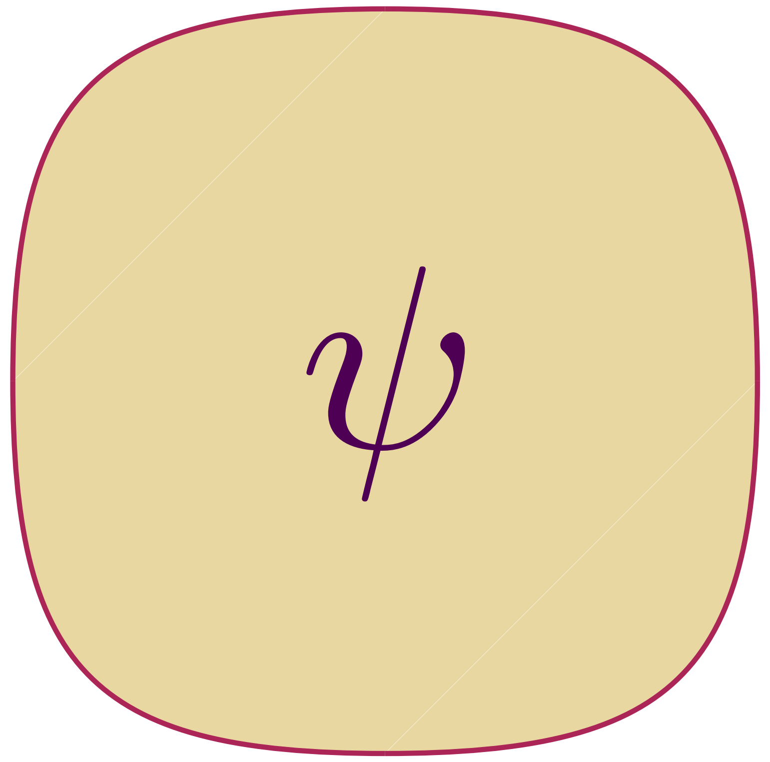

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MFL self-energy, $T\ln(1/T)$ specific heat,
linear-T resistivity and
$1/[\omega - i(2\omega/\pi)\ln(\Lambda/\omega)]$ optical conductivity

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Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

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Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev,
arXiv: 2109.05037, Reviews of Modern Physics

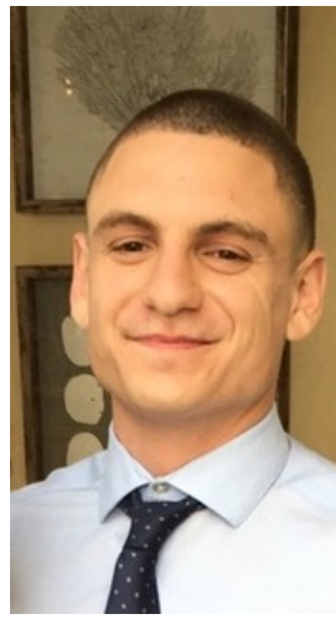
Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Universal theory of a marginal Fermi liquid and a strange metal (including linear-T resistivity): spatially random interactions in a two-dimensional quantum-critical metal, solvable in a Yukawa-SYK-like large N limit.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, Reviews of Modern Physics

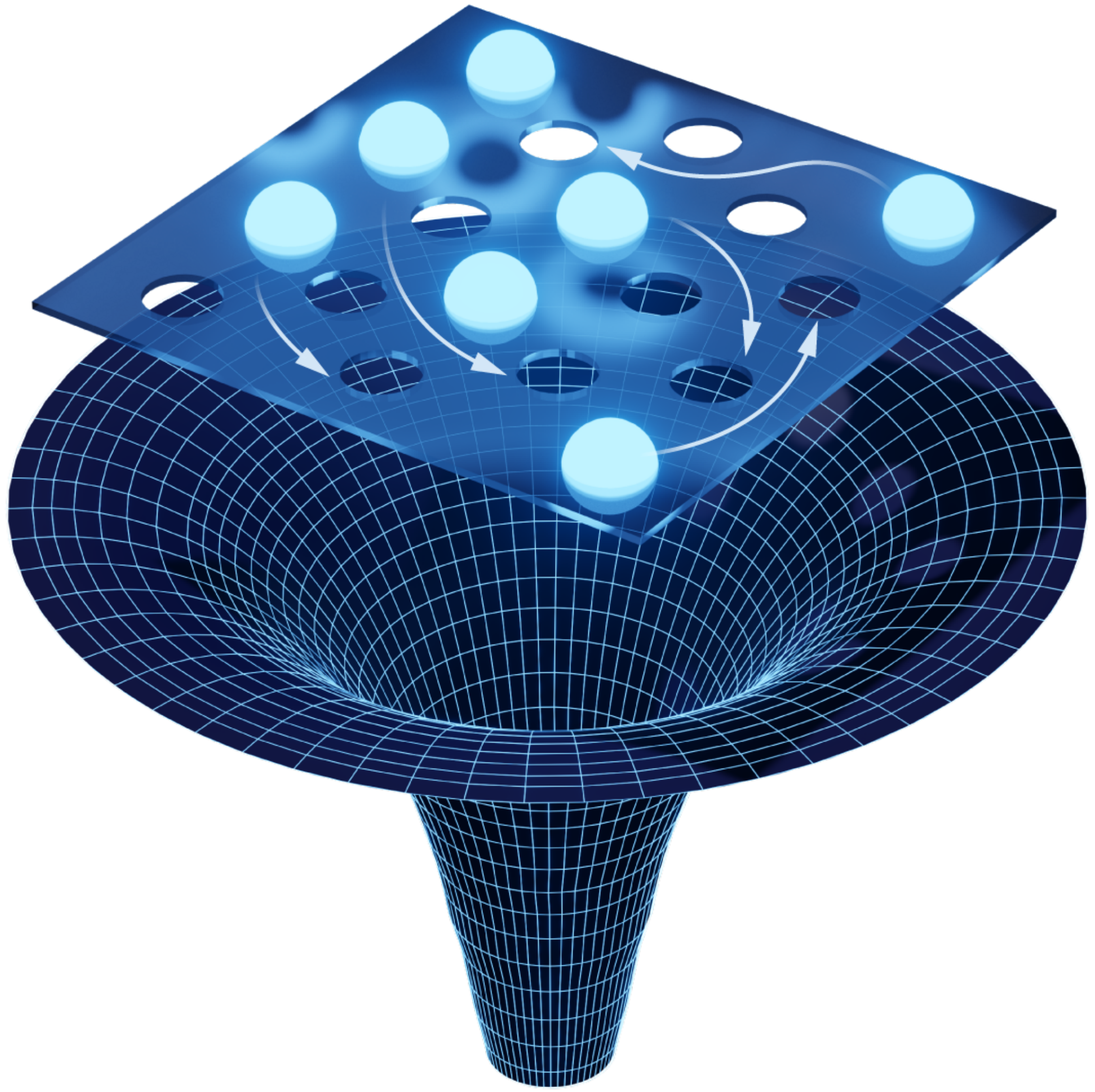




Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990

Summary

- Black holes with a net charge in asymptotically Minkowski space have a near horizon $\mathrm{AdS}_2 \times S^2$ geometry: this geometry has an emergemt time-reparameterization soft mode with an action identical to that of the SYK model. In other words, the SYK model is a quantum simulation of the low energy physics of charged black holes in Einstein-Maxwell theory.



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