

The SYK model: a window into non-Fermi liquids and black holes

Würzburg Seminar on
Quantum Field Theory and Gravity
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Subir Sachdev



PHYSICS



HARVARD

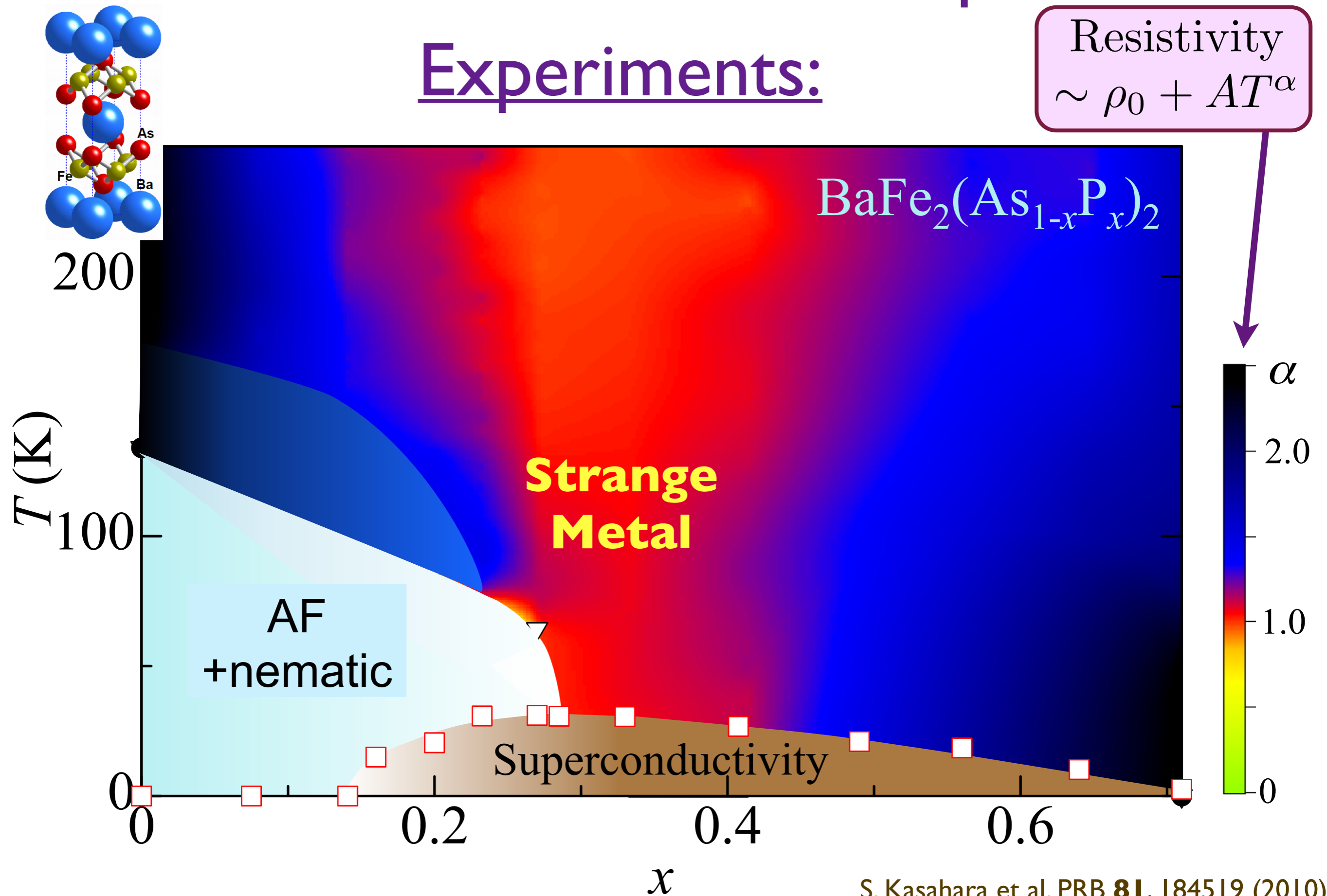
Talk online: sachdev.physics.harvard.edu

What is a non-Fermi liquid ?

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Experiments:

$$\text{Resistivity} \sim \rho_0 + AT^\alpha$$



What is a non-Fermi liquid ?

- **A compressible state of quantum matter:**

There is a global U(1) charge Q which commutes with the Hamiltonian, and upon applying a chemical potential change $\delta\mu$

$$\mathcal{H} \Rightarrow \mathcal{H} - \delta\mu Q$$
$$\frac{\delta\langle Q \rangle}{\delta\mu} \neq 0 \text{ as } T \rightarrow 0 \text{ in the thermodynamic limit}$$

Dirac/Weyl fermions, CFTs in spatial dimension $d > 1$ are *not* compressible.

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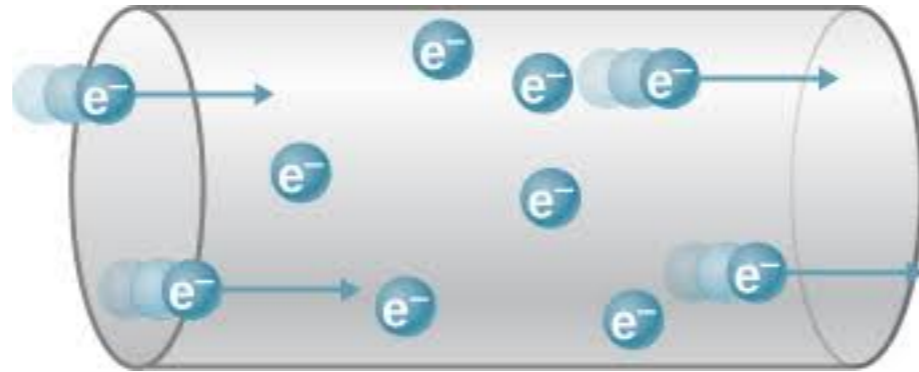
- **No quasiparticle excitations:**

In the presence of quasiparticles, the low-lying many-body energies E can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

Luttinger liquids, systems with an energy gap (TQFTs), do have quasiparticles.

Current flow with quasiparticles



Flowing quasiparticles scatter off each other in a typical scattering time τ

This time is much longer than a limiting
'Planckian time' $\frac{\hbar}{k_B T}$.

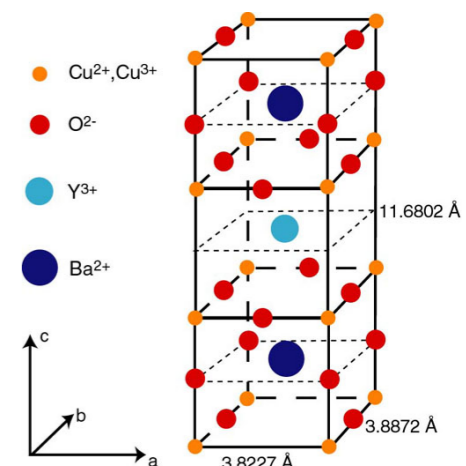
The long scattering time implies that quasiparticles are well-defined.

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11 \text{ kbar}$	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Electron scattering time τ in 7 different strange metals

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

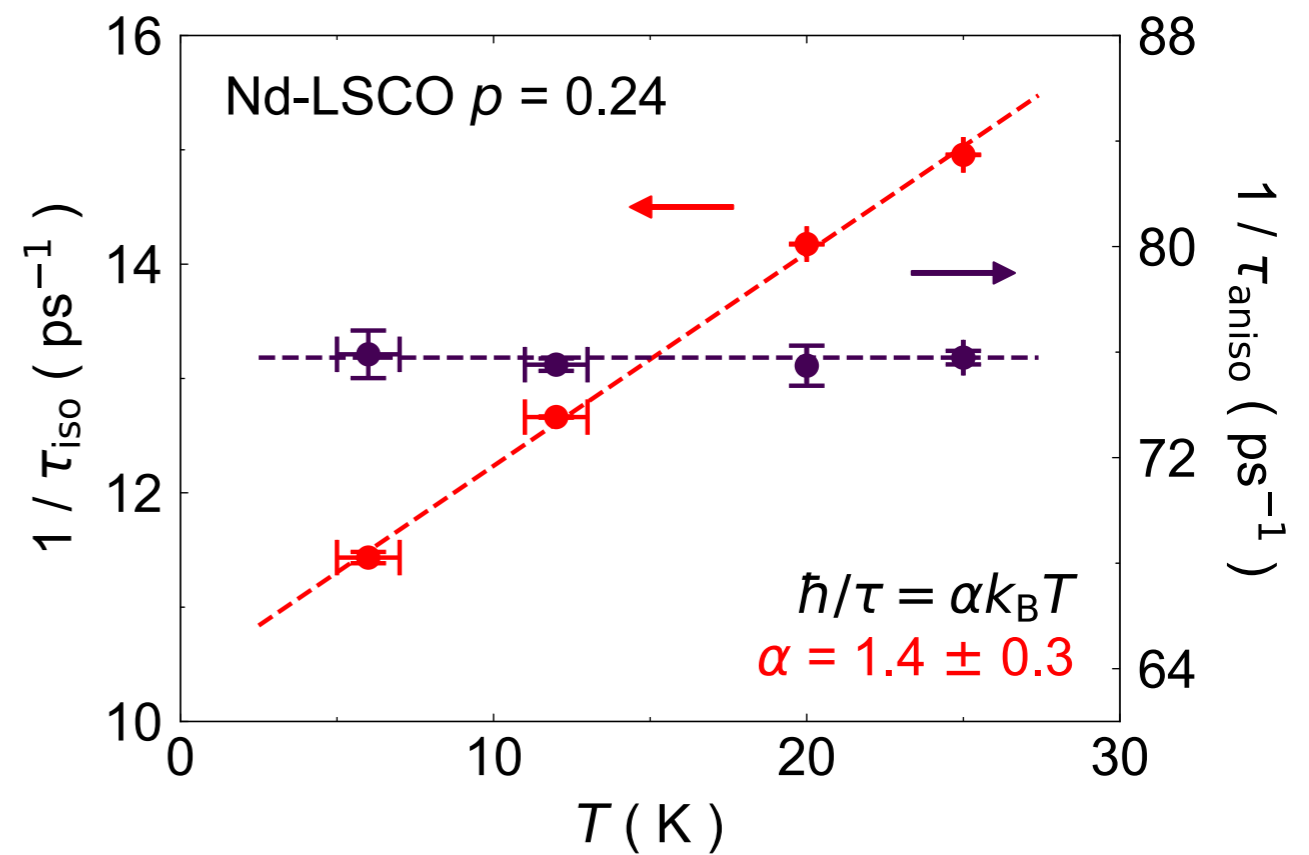
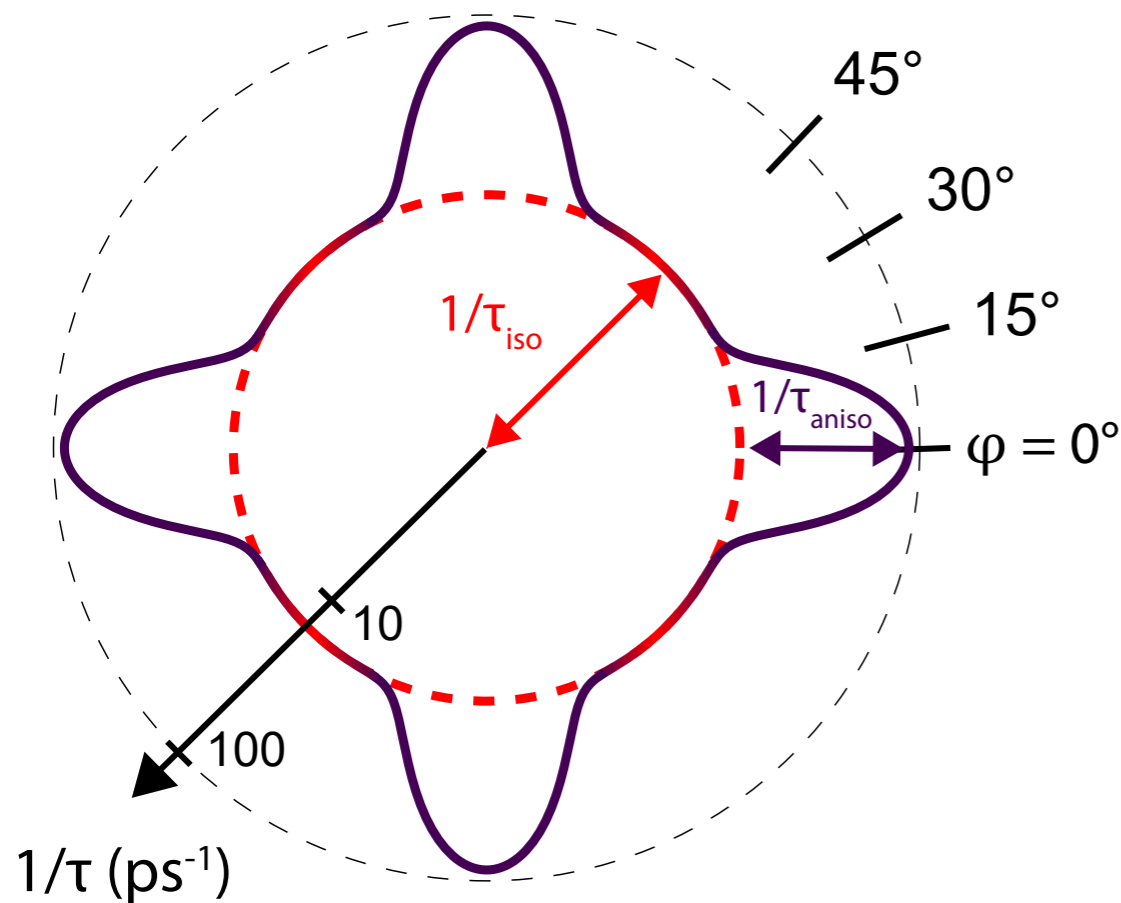
Current flow without quasiparticles



Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.



$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

1. SYK models

2. Time reparameterization soft mode
3. Charged black holes
4. Random t-J model
5. Critical Fermi surfaces: large N theory

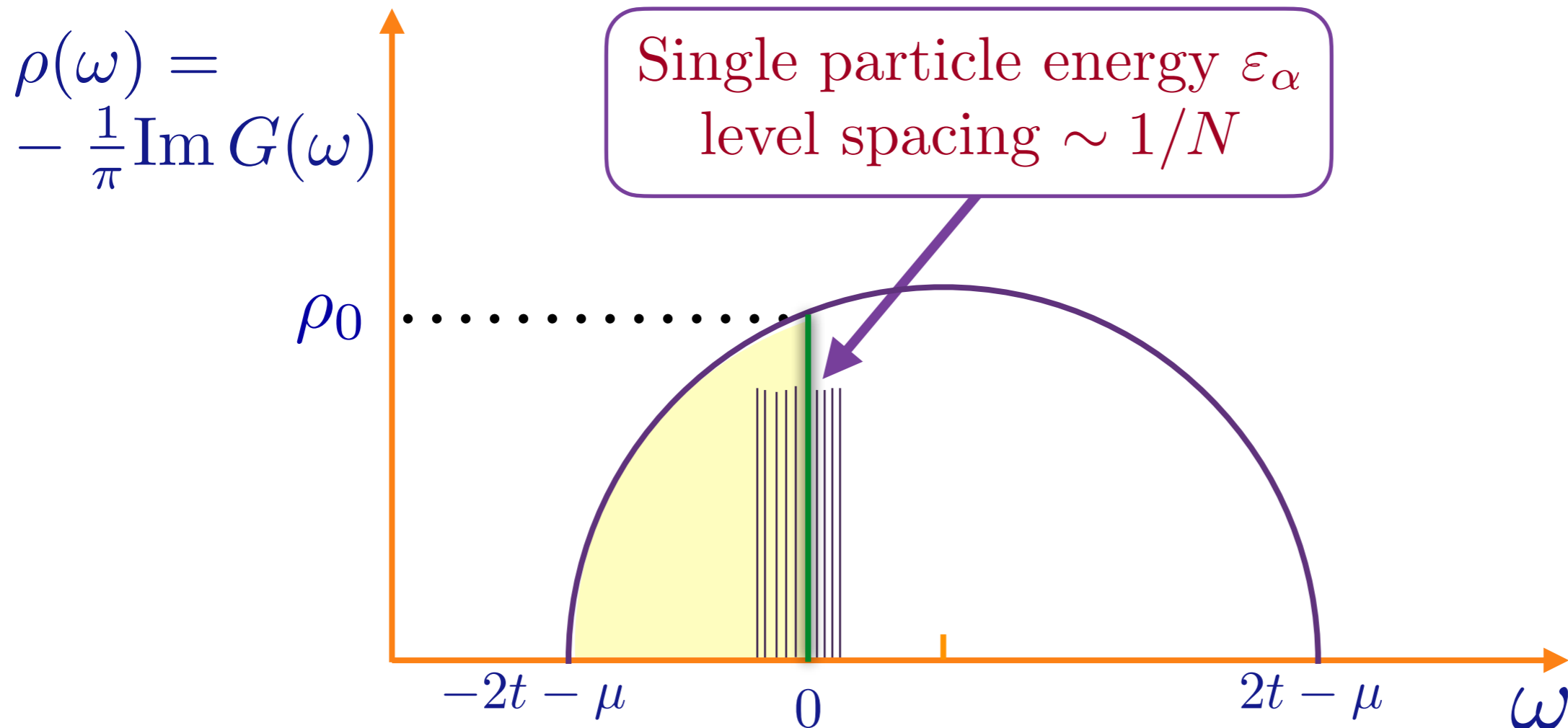
A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^N t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = Q$$

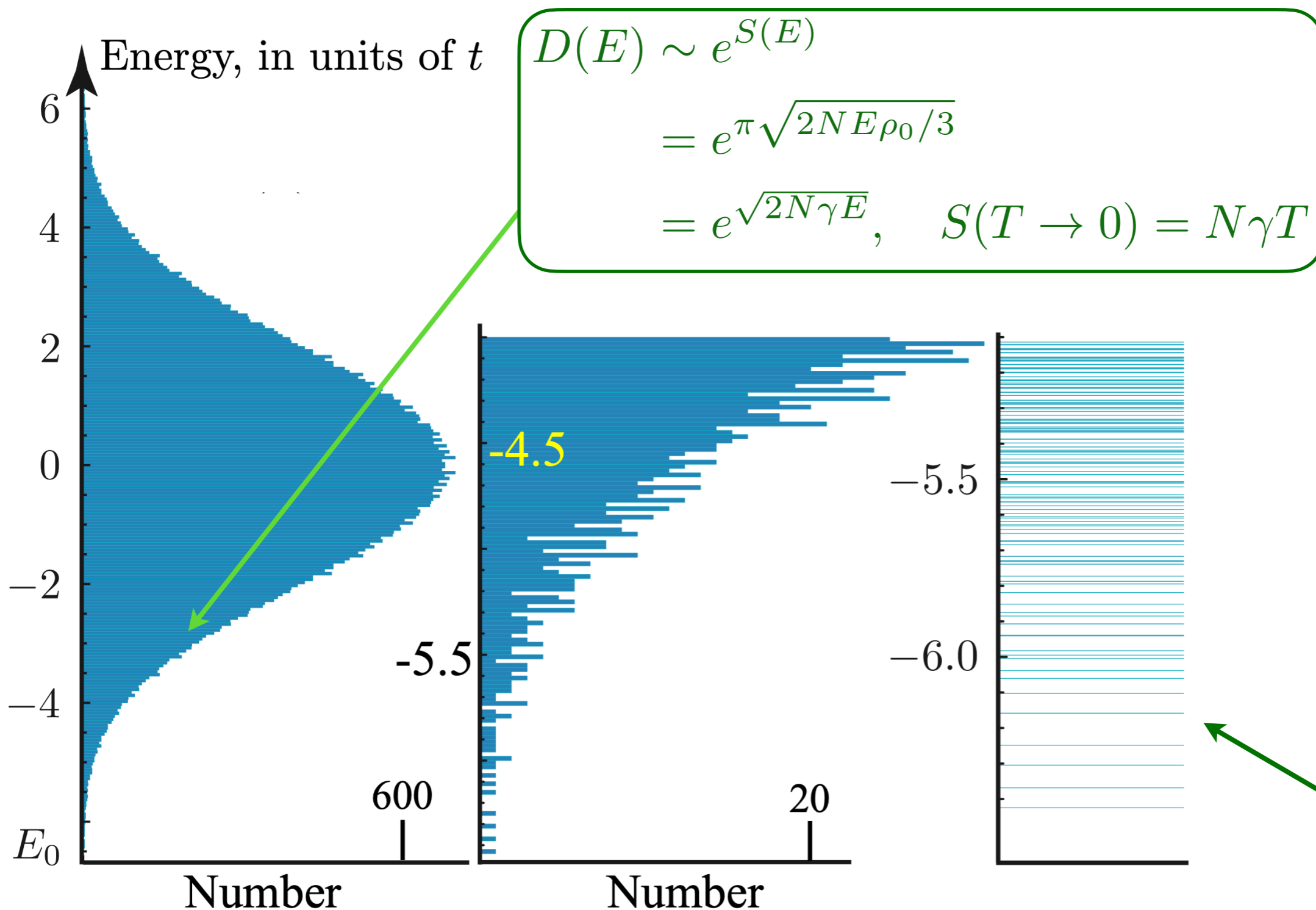
t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$



Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model:
 $E_0 + E_i = \sum_{\alpha} n_{\alpha} \epsilon_{\alpha}$
 $n_{\alpha} = 0, 1,$
 occupation number



Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

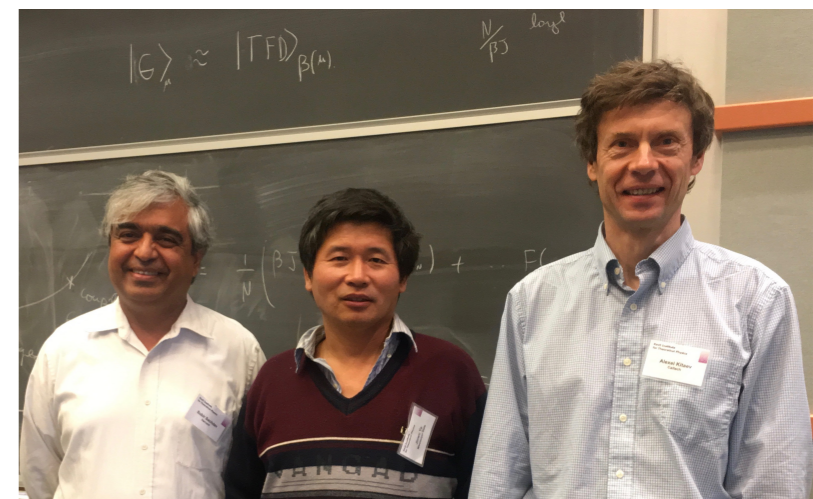
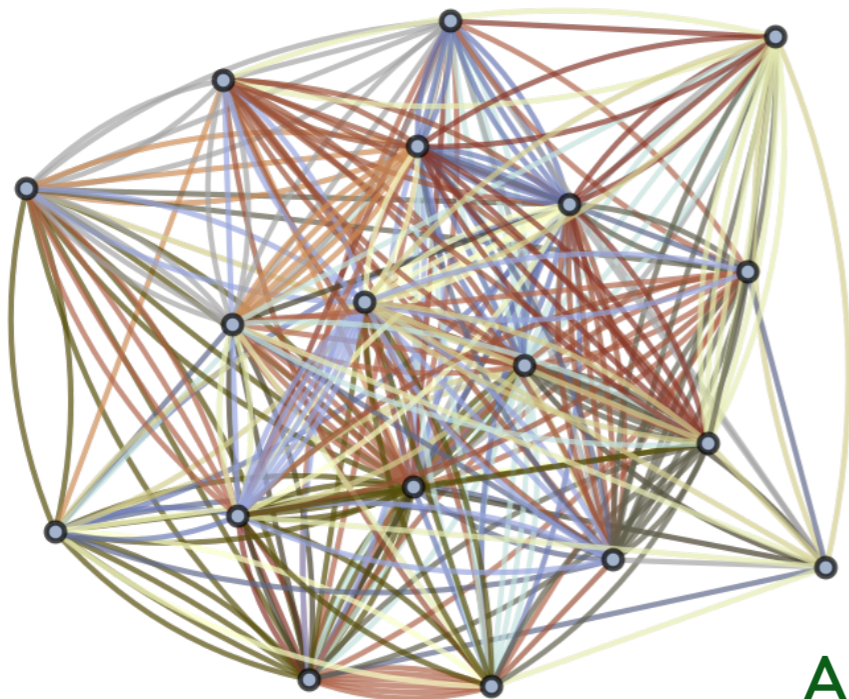
(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

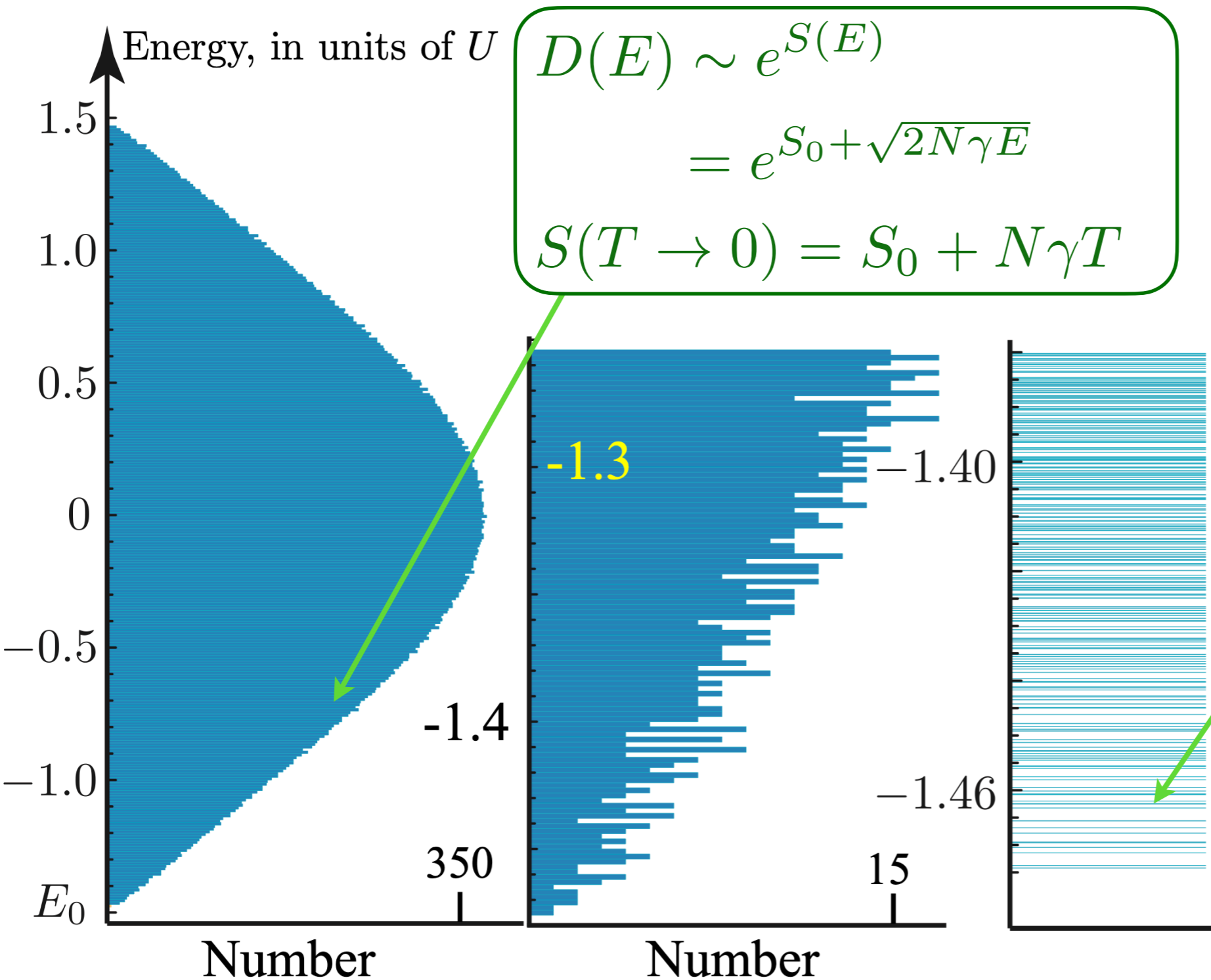


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim e^{S(E)}$$

$$= e^{S_0 + \sqrt{2N\gamma E}}$$

$$S(T \rightarrow 0) = S_0 + N\gamma T$$

No quasiparticle decomposition of many-body states

$$D(E) \sim 2 e^{S_0} \sinh(\sqrt{2N\gamma E})$$

$$S_0 = 0.464848 \dots N$$

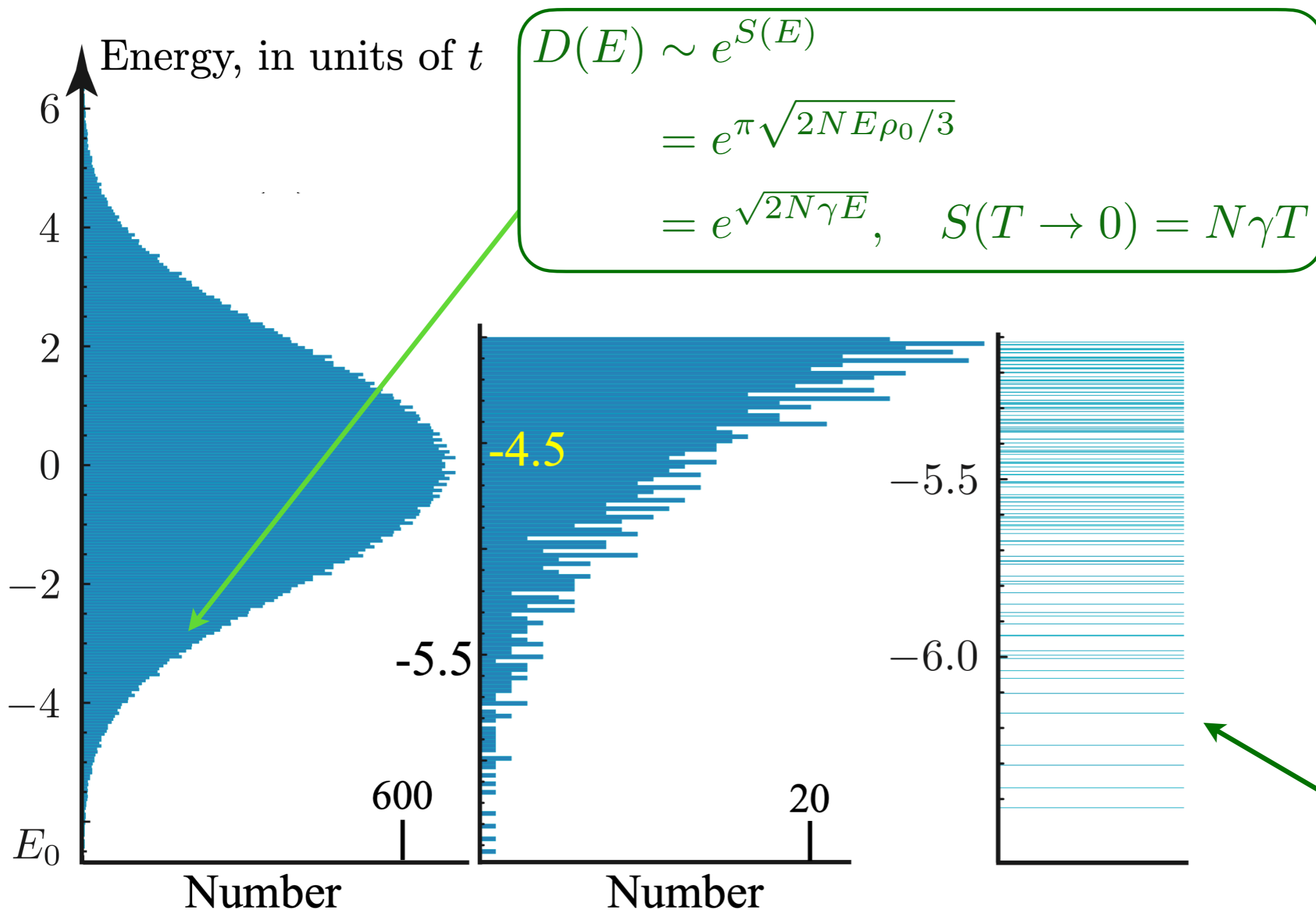
A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)

Complex SYK model

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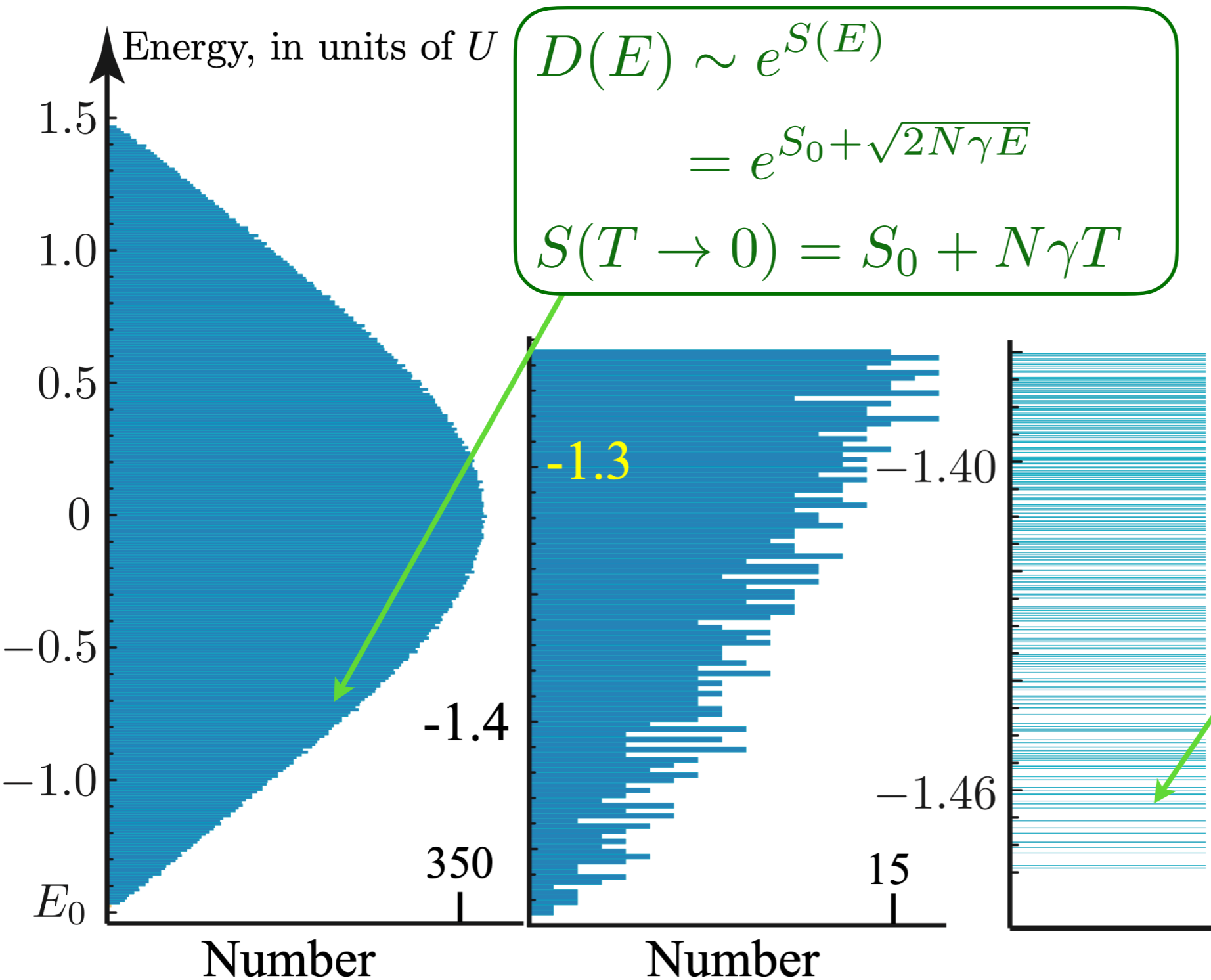
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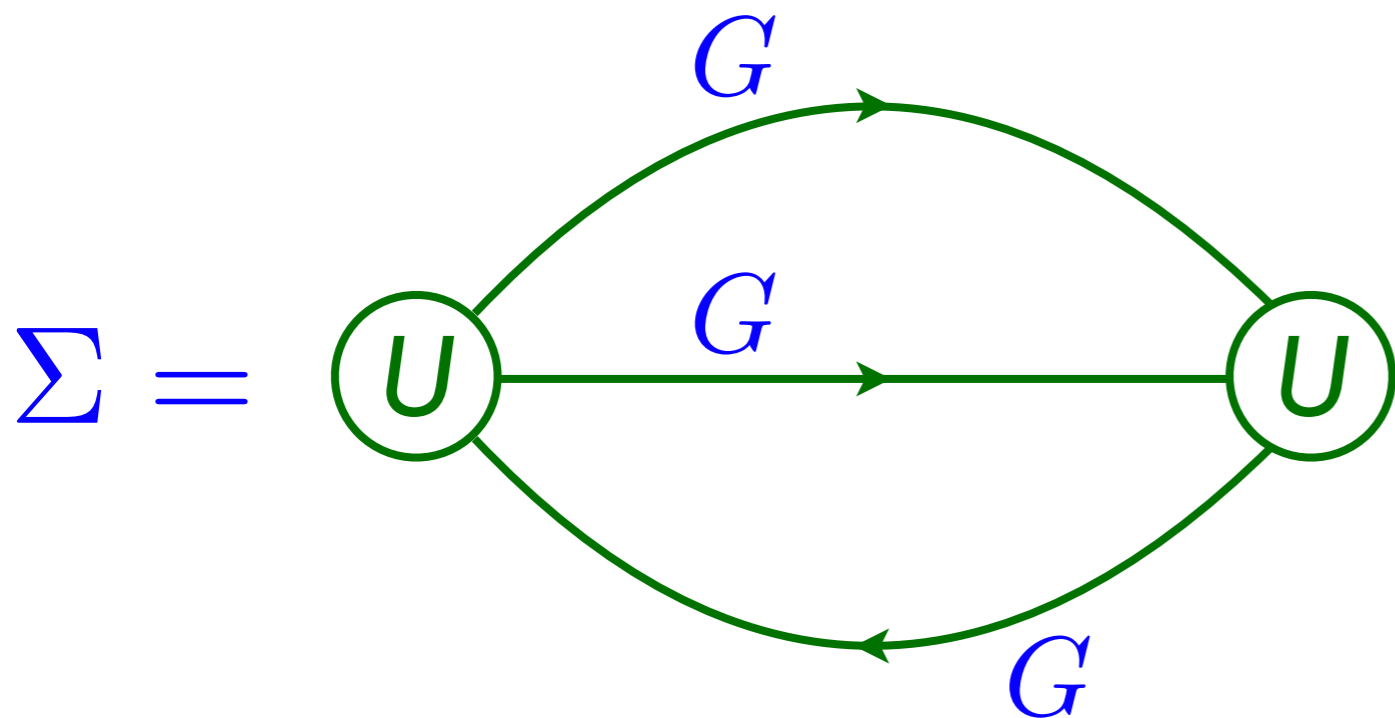
A. Georges, O. Parcollet, and S. Sachdev,
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Complex SYK model

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

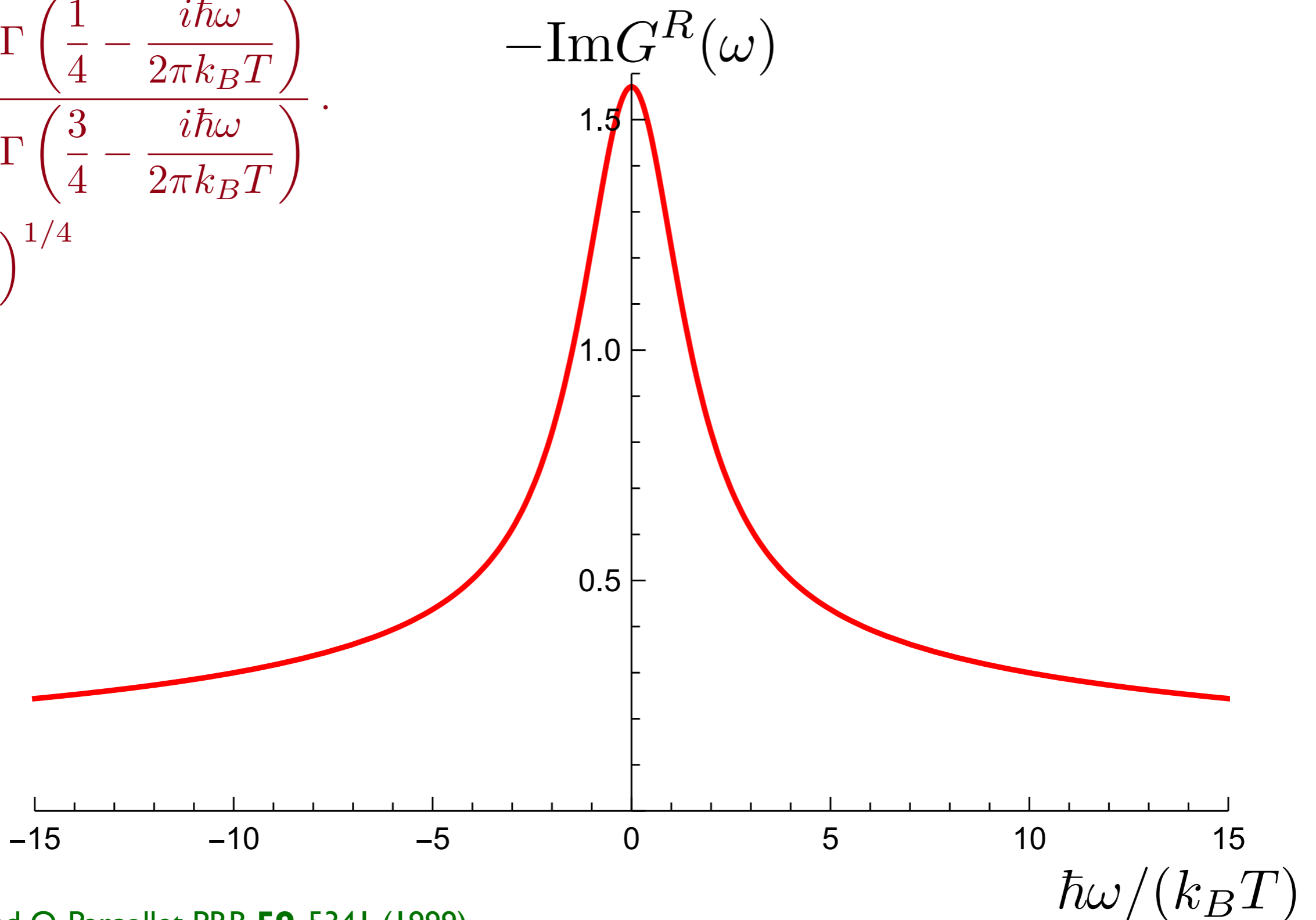


S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)



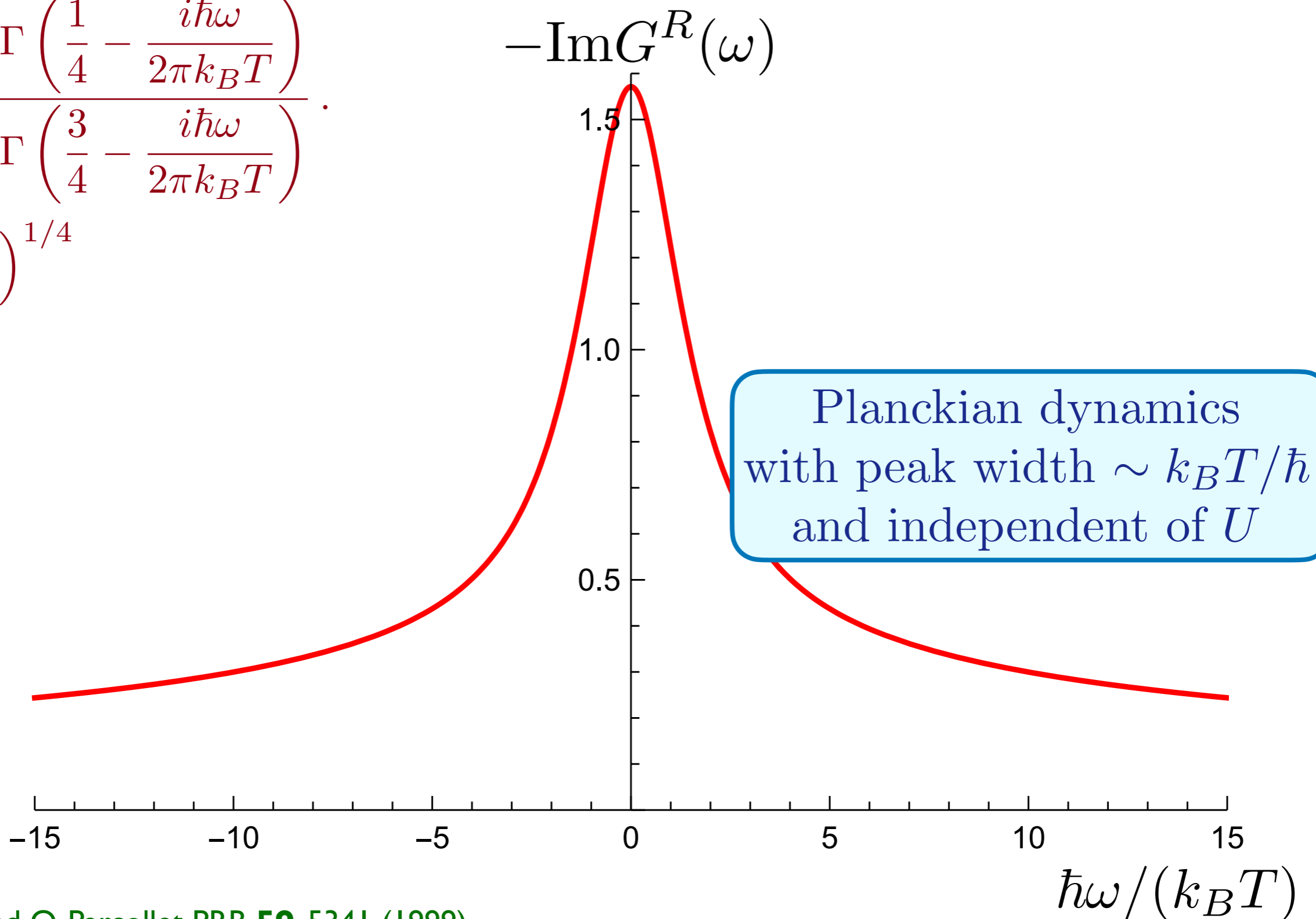
The SYK model

$$G_{\text{SYK}}^R(\omega) = \frac{-iC}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T}\right)}$$
$$C = \left(\frac{\pi}{U^2}\right)^{1/4}$$



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Time reparameterization symmetry and 2D gravity

After introducing replicas $a = 1 \dots n$, and integrating out the disorder, the partition function can be written as

$$Z = \int \mathcal{D}c_{\alpha a}(\tau) \exp \left[- \sum_{ia} \int_0^\beta d\tau c_{\alpha a}^\dagger \left(\frac{\partial}{\partial \tau} - \mu \right) c_{\alpha a} \right. \\ \left. - \frac{U^2}{4N^3} \sum_{ab} \int_0^\beta d\tau d\tau' \left| \sum_i c_{\alpha a}^\dagger(\tau) c_{\alpha b}(\tau') \right|^4 \right].$$

For simplicity, we neglect the replica indices, and introduce the identity

$$1 = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp \left[-N \int_0^\beta d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left(G(\tau_2, \tau_1) \right. \right. \\ \left. \left. + \frac{1}{N} \sum_i c_\alpha(\tau_2) c_\alpha^\dagger(\tau_1) \right) \right].$$

Time reparameterization symmetry and 2D gravity

Then the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)] \\ + \int d\tau_1 d\tau_2 [\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$

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At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \tilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

A. Kitaev, 2015

S. Sachdev, PRX **5**, 041025 (2015)

Time reparameterization symmetry and 2D gravity

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G, \Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-E_0/T + NS(E_0) - NS_{\text{eff}}[f, \phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv:1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Time reparameterization symmetry and 2D gravity

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f, \phi] = \frac{NK}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2.$$

Specifically, an argument constraining the effective at $T = 0$ is

$$S_{\text{eff}} \left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0 \right] = 0,$$

and this is origin of the Schwarzian.

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The same effective action is obtained for the boundary graviton of 2D gravity on AdS_2 .

- Exact evaluation of the path integral over $f(\tau)$ and $\phi(\tau)$ leads to the many-body density of states

$$D(E) \sim 2e^{S_0} \sinh(\sqrt{2N\gamma E})$$

- Saddle-point shift leads to a correction to the Green's function:

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} \left(1 + \frac{\alpha_G}{|\tau|} + \dots \right)$$

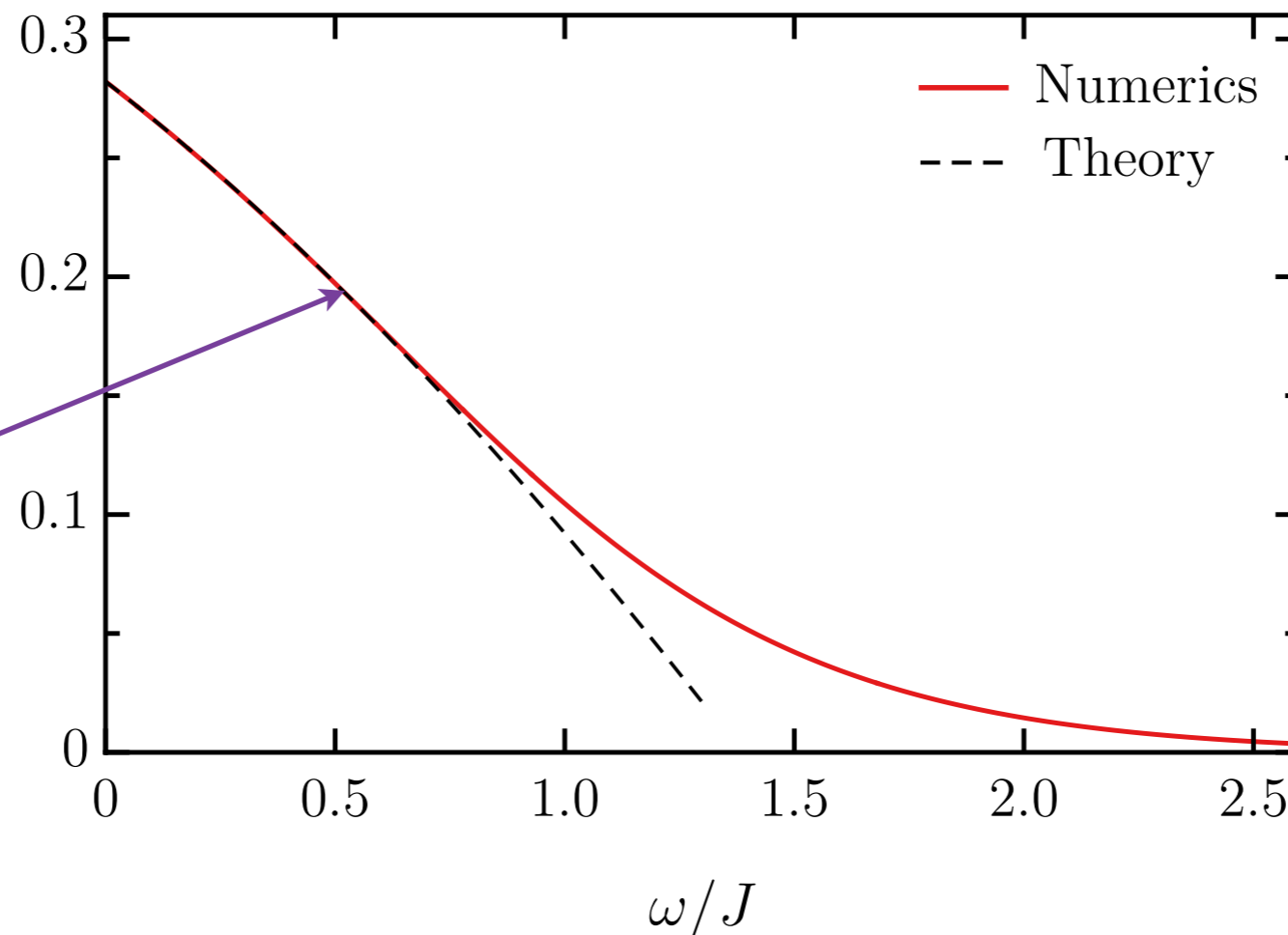
From this, we can compute the susceptibility $\chi(\tau) \sim G(\tau)G(-\tau)$

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0 | S_{+i} | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\text{Im}\chi_L(\omega) \sim \text{sgn}(\omega) \left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\dots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \dots \right]$$

Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory.
 \mathcal{C} is the co-efficient of the action for the ‘boundary graviton’ in holographic dual.



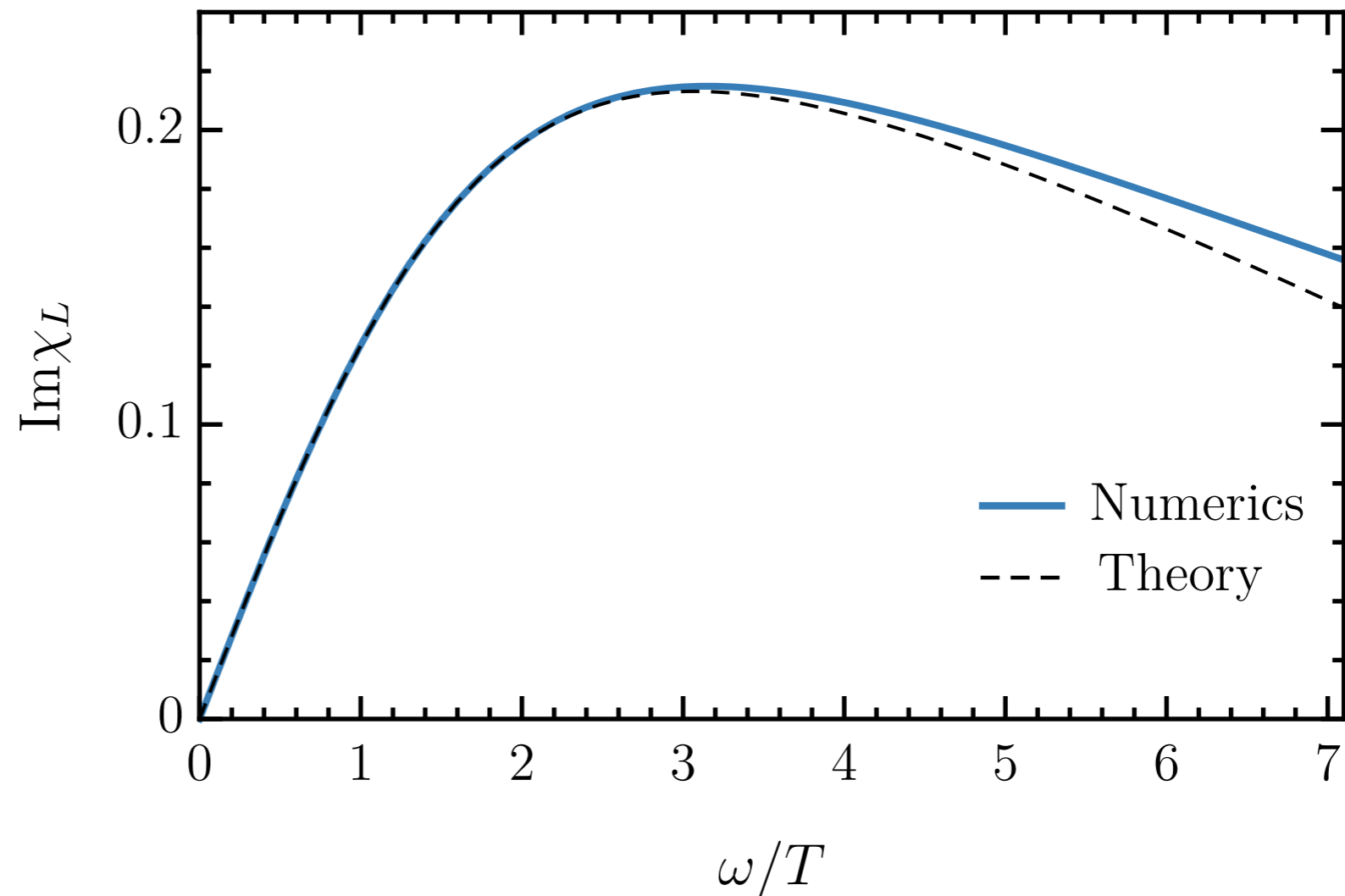
Correction
from the
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$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right) \left[1 - C\gamma\omega \tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \dots \right]$$



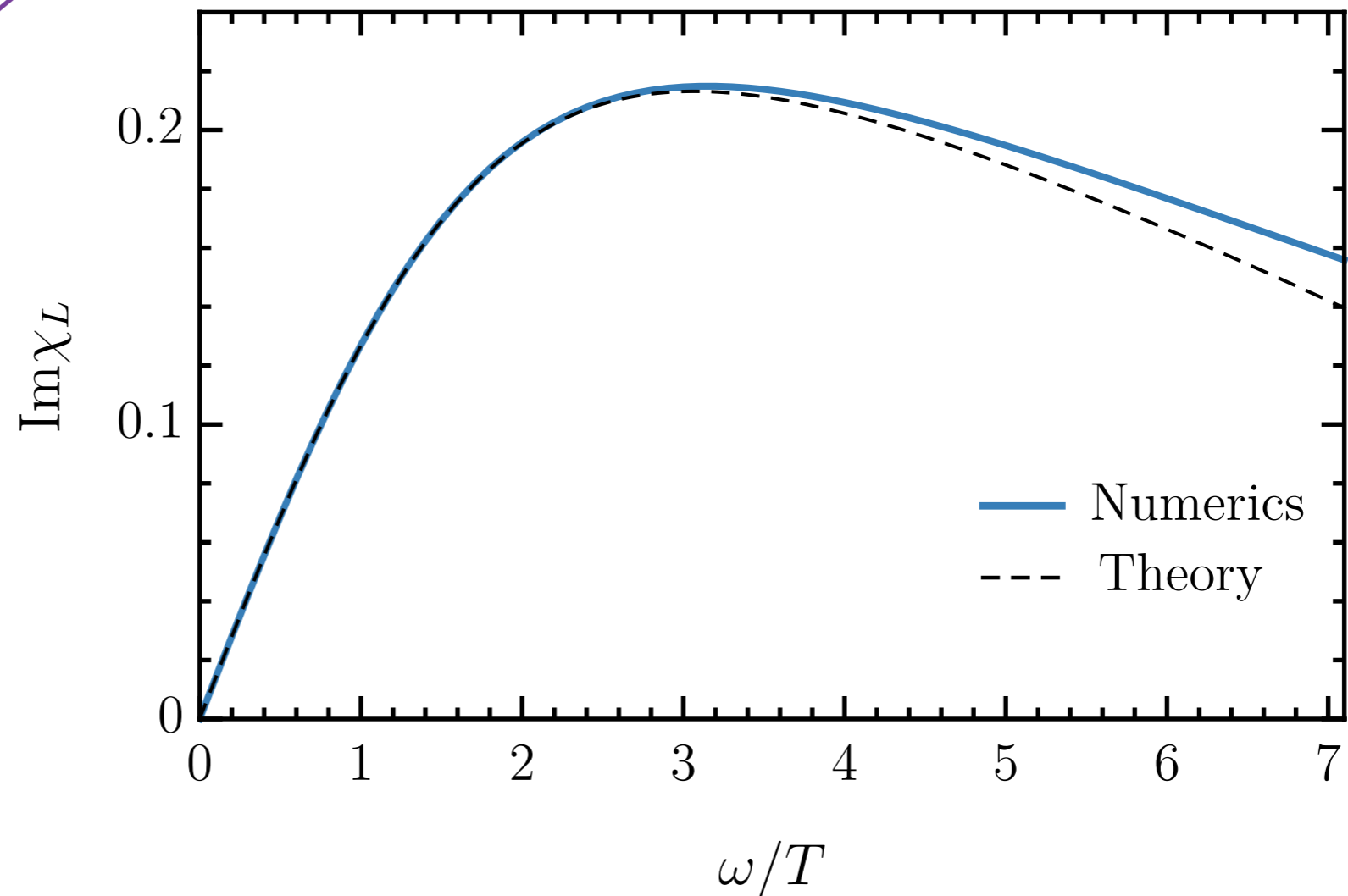
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Conformally (SL(2,R))
invariant result with
characteristic dissipative
time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

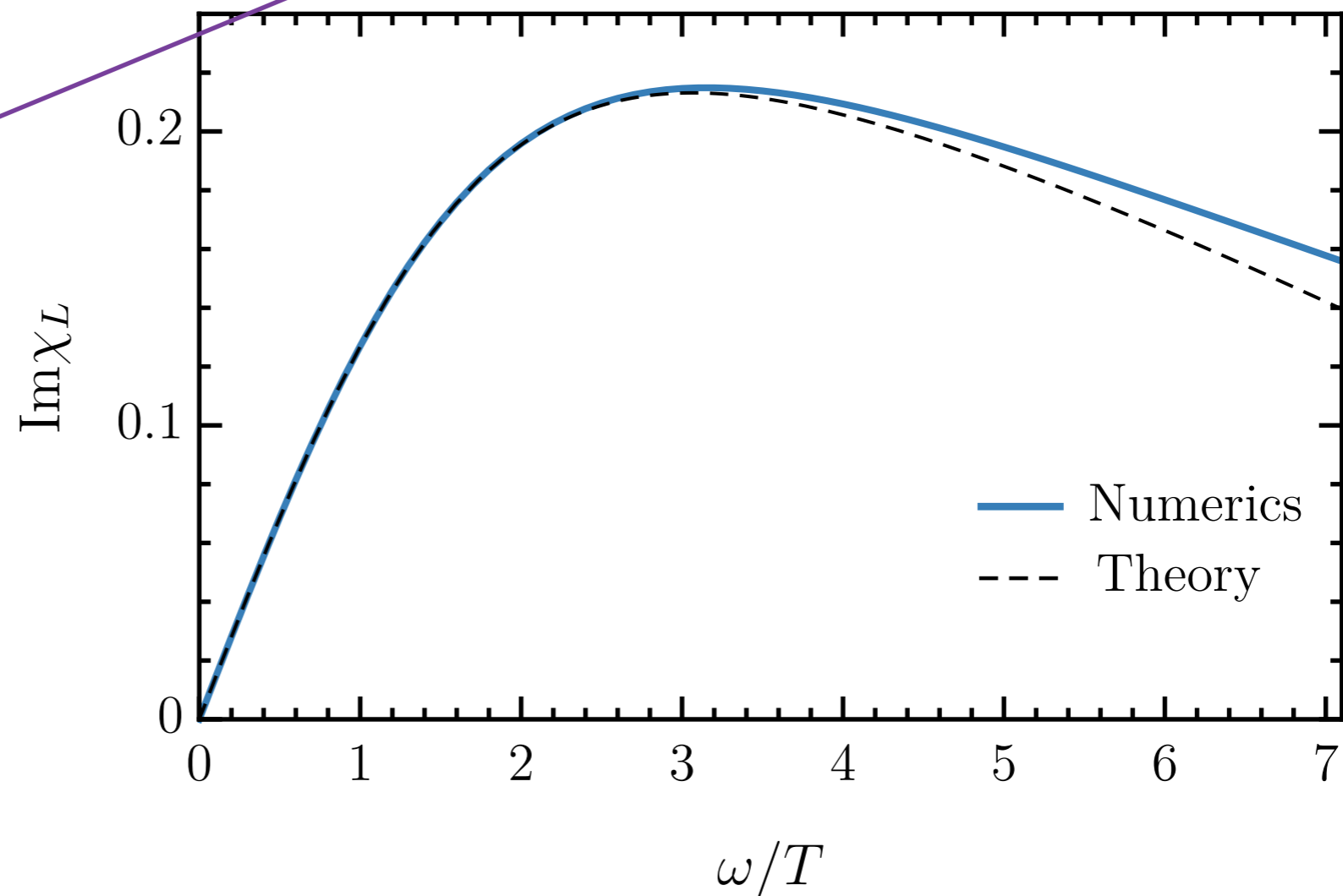


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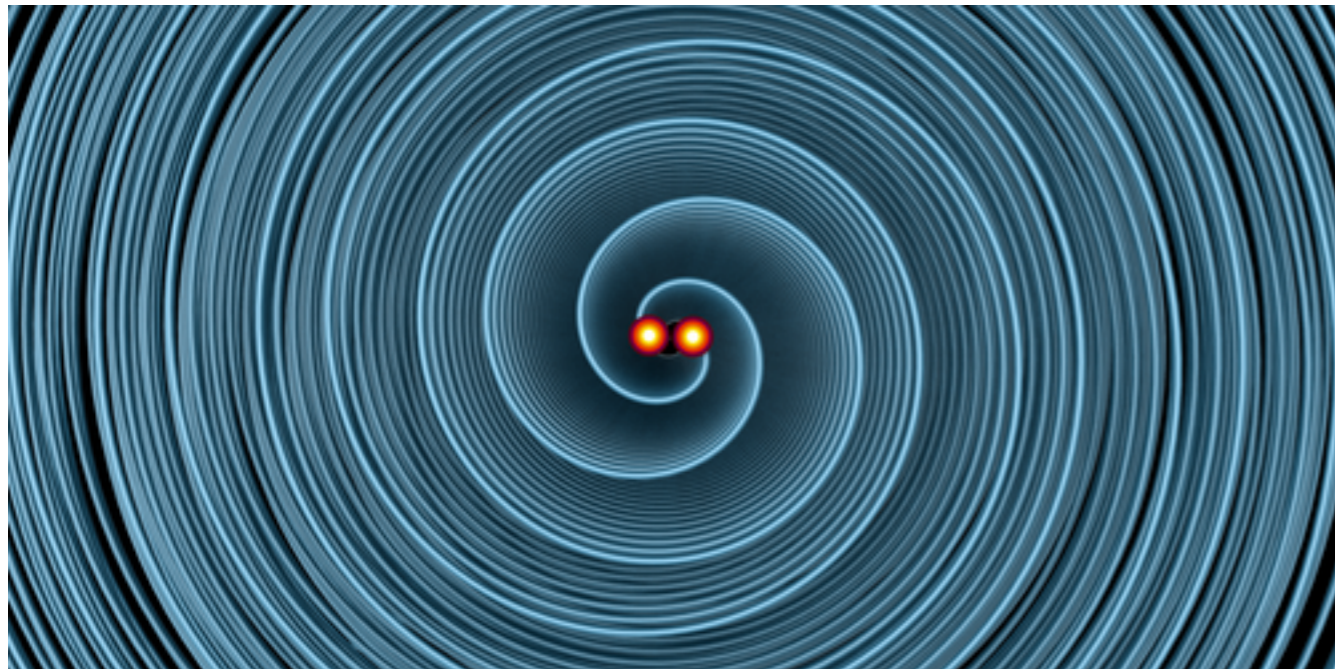
Black Holes Obey Information-Emission Limits

Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

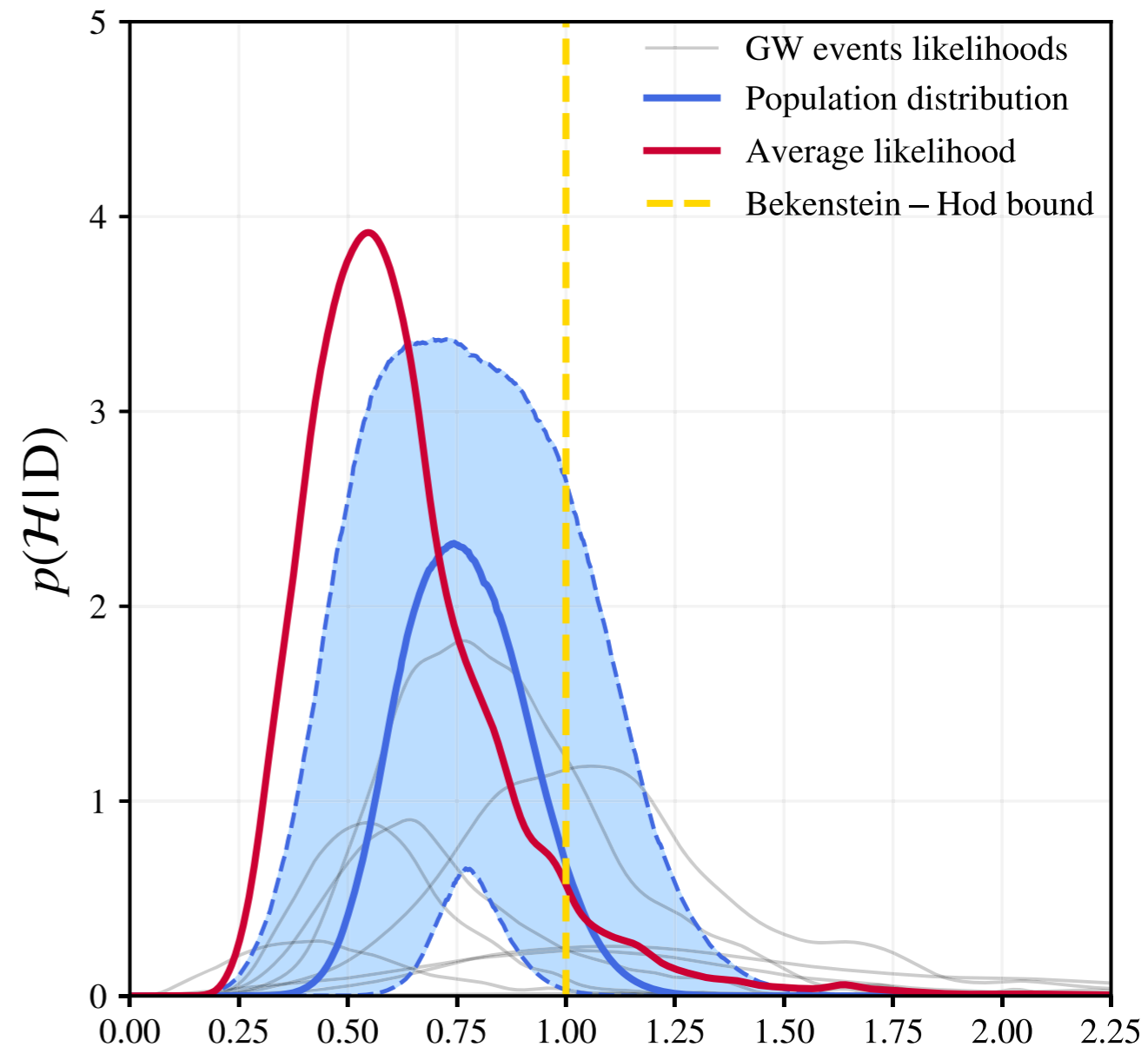
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, *Phys. Rev. Lett.* **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$




$$\mathcal{H} = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$

Thermodynamics of quantum black holes:

$$\int \mathcal{D}g_{\mu\nu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity}}^{(d+1)} [g_{\mu\nu}] \right)$$

Metric of
spacetime




Quantum gravity: a summation over all possible configurations of spacetime, each weighted by a factor which is the exponential of (the ‘action’ of Einstein gravity)/(Planck’s constant)

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Metric of
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In general, this summation is not well defined, because to the uncontrollably large number of spacetime configurations.

Thermodynamics of quantum black holes:

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity}}^{(d+1)}[g_{\mu\nu}]\right)$$
$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

Metric of
spacetime

Gibbons, Hawking (1977)

$$S_{BH} = \frac{Ac^3}{4G\hbar}$$

($\hbar/(k_B T)$ is the length of the Euclidean time circle)

A is the area of the black hole horizon.

Interpretation: Black hole entropy is
entanglement entropy across the horizon.

Holography and duality

Thermodynamics of quantum black holes:

$$\int \mathcal{D}g_{\mu\nu} \exp\left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein gravity}}^{(d+1)}[g_{\mu\nu}]\right)$$

$$= \exp(S_{BH}) \times \left(\text{Many body quantum theory} \right)$$

in $d - 1$ spatial dimensions *without* gravity

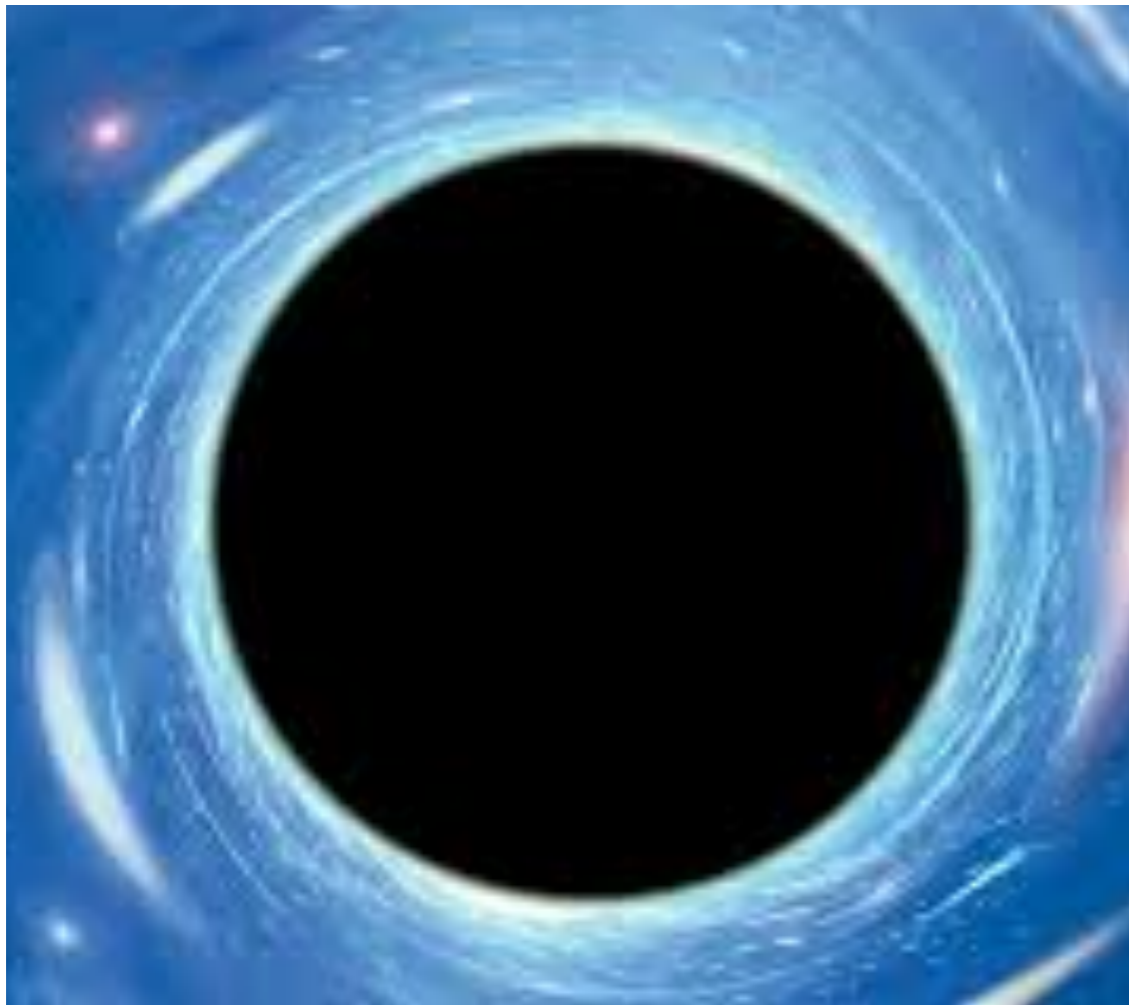
Metric of spacetime

Black holes are represented as a 'hologram' by a quantum many-body system in one lower dimension.

Duality: a 'change of variables' between the many-particle configurations and the metric of spacetime

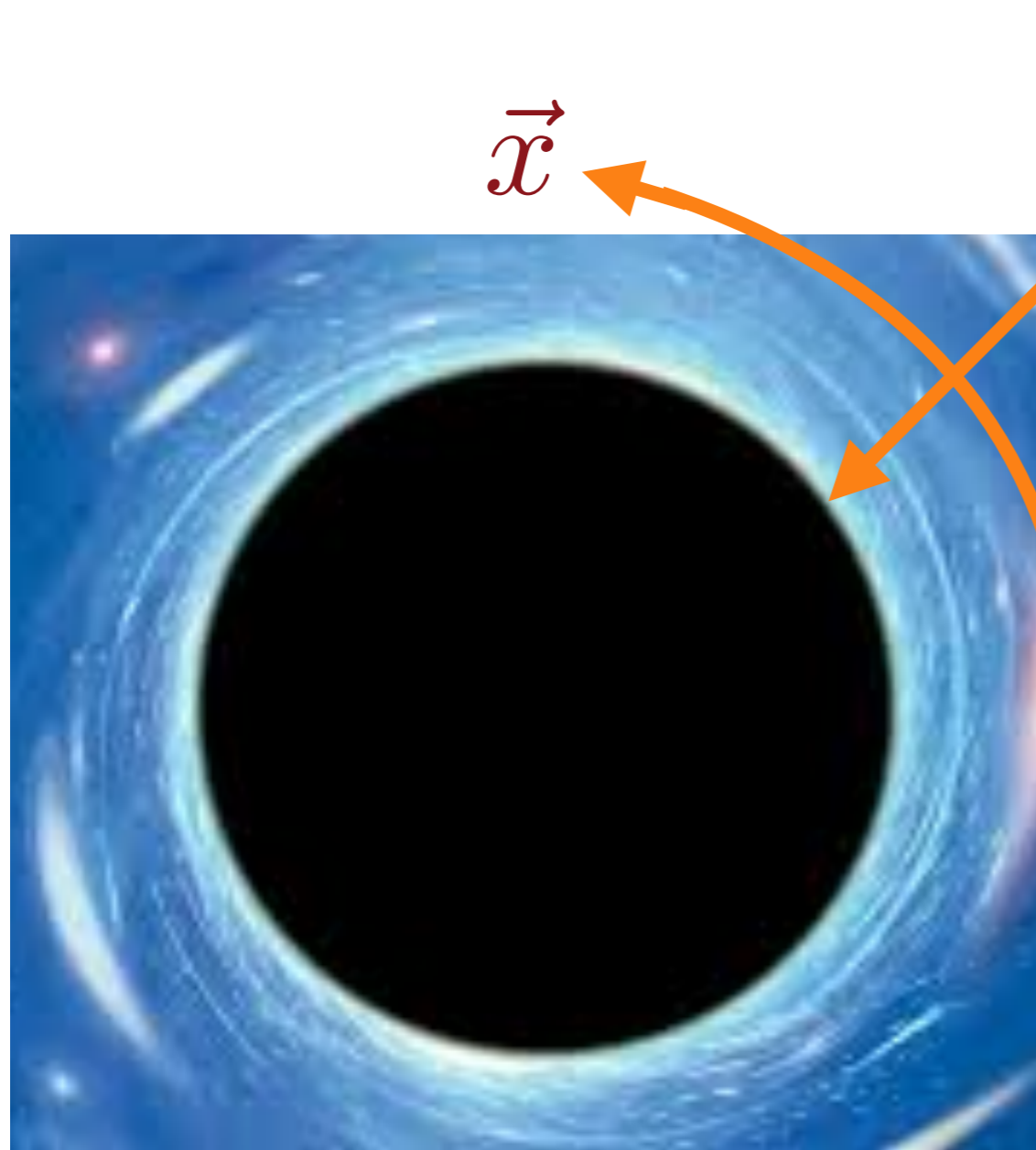


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





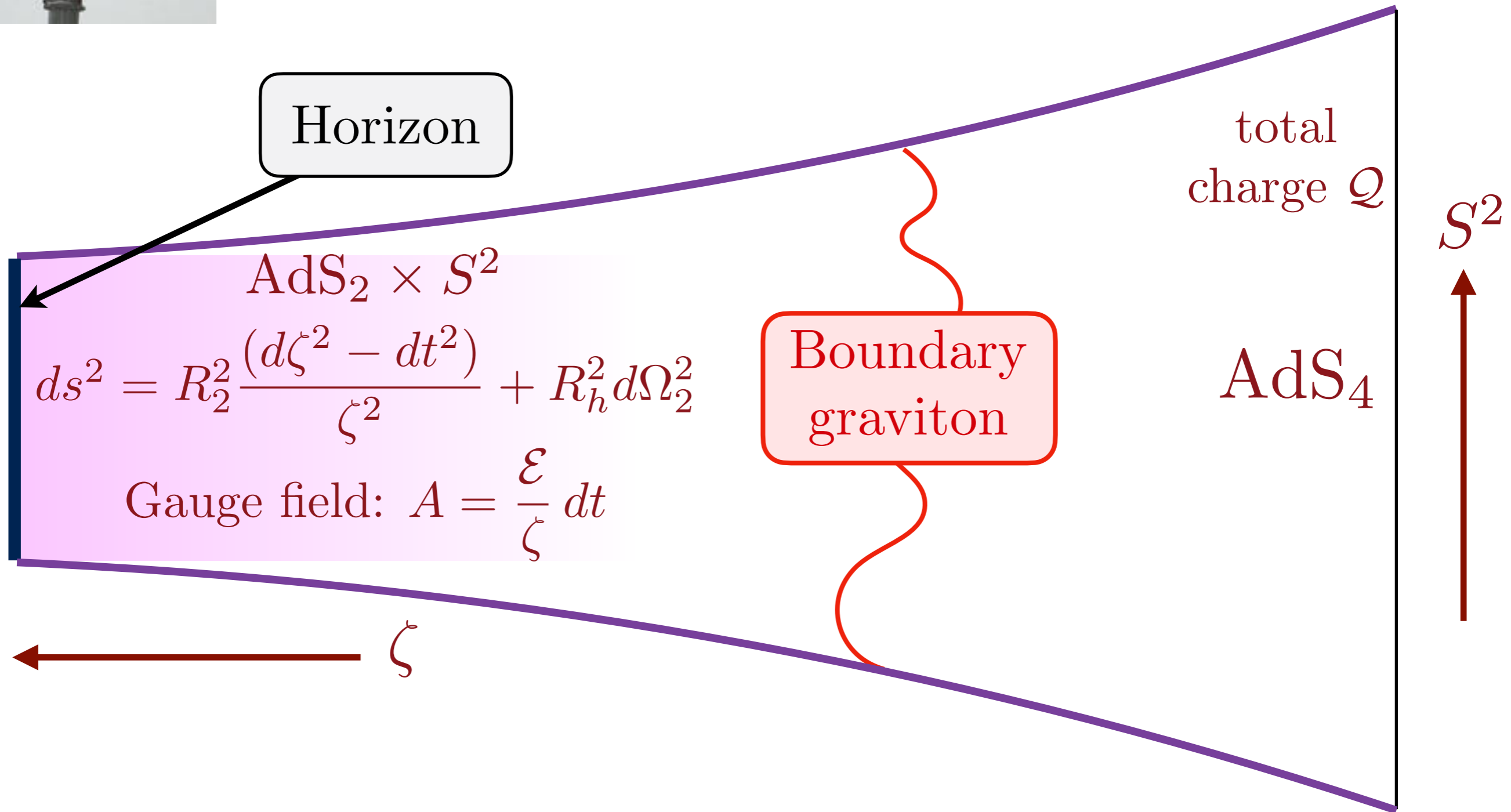
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension



Fluctuations about Reissner-Nordstrom black hole of Einstein-Maxwell theory



SYK model and charged black holes

Thermodynamics of charged quantum black holes

$$\int \mathcal{D}g_{\mu\nu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein-Maxwell theory}}^{(3+1)}[g_{\mu\nu}] \right) T \rightarrow 0,$$
$$\approx \int \mathcal{D}g_{\mu\nu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Gravity of AdS}_2 \text{ and boundary}}^{(1+1)}[g_{\mu\nu}] \right)$$

SYK model and charged black holes


Thermodynamics of charged quantum black holes

$$\begin{aligned} & \int \mathcal{D}g_{\mu\nu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Einstein-Maxwell theory}}^{(3+1)} [g_{\mu\nu}] \right) \quad T \rightarrow 0, \\ & \approx \int \mathcal{D}g_{\mu\nu} \exp \left(-\frac{1}{\hbar} \mathcal{S}_{\text{Gravity of AdS}_2 \text{ and boundary}}^{(1+1)} [g_{\mu\nu}] \right) \\ & = \exp \left(S_{BH} \right) \times \exp \left(-\frac{1}{T} \times \text{Free energy of SYK model} \right) \end{aligned}$$

The hologram of the 1+1 dimensional gravity near the horizon of a charged black hole is the 0+1 dimensional SYK model

SYK model and charged black holes

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Thermodynamics of charged quantum black holes

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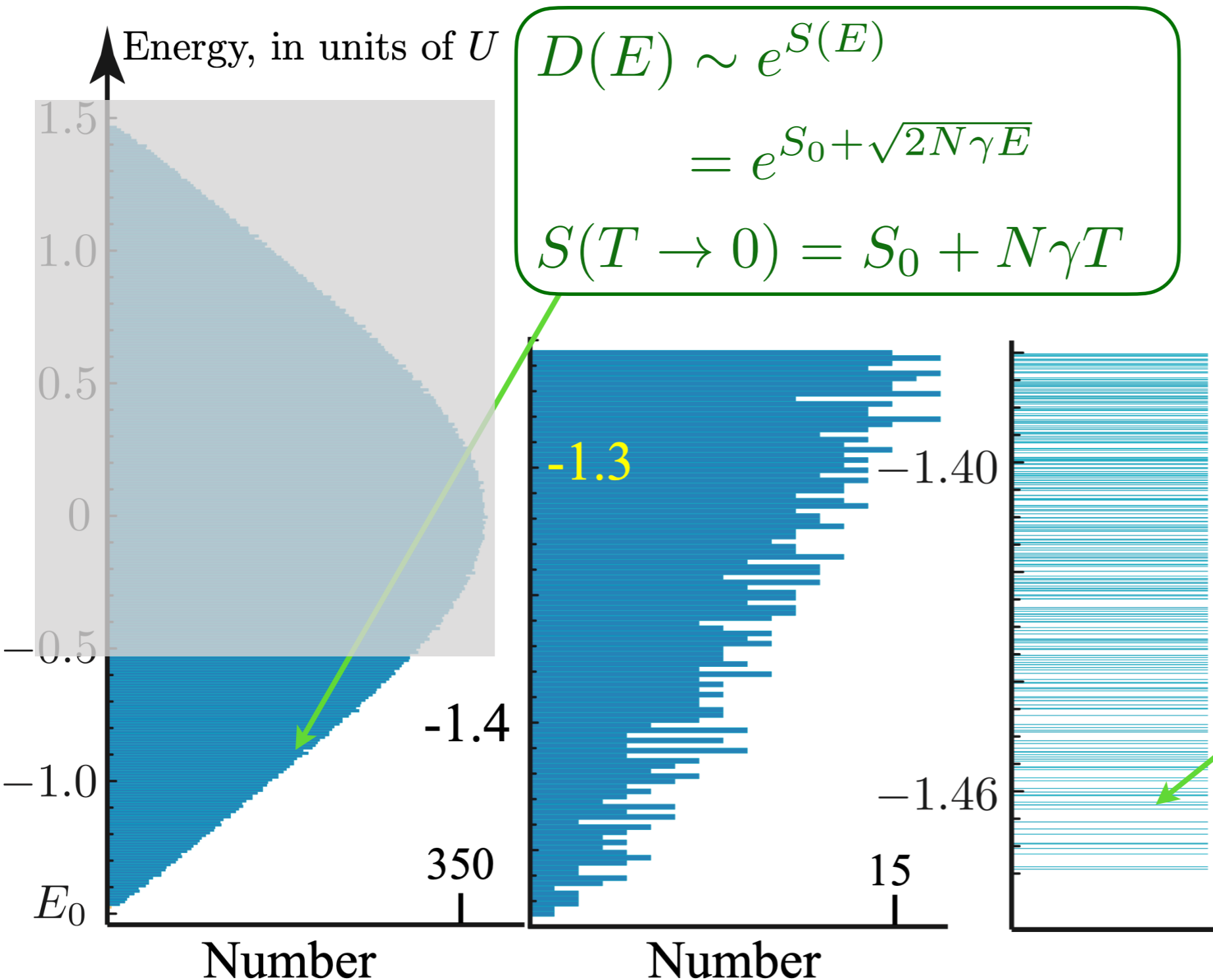
$$S(T \rightarrow 0, Q) = S_{BH} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{GT^2} \right)$$

$$S_{BH} = \frac{Ac^3}{4G\hbar} \left(1 + \frac{2(\pi A)^{1/2}T}{\hbar c} \right)$$

A is the area of the charged black hole horizon at $T = 0$, Q is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



No quasiparticle decomposition of many-body states

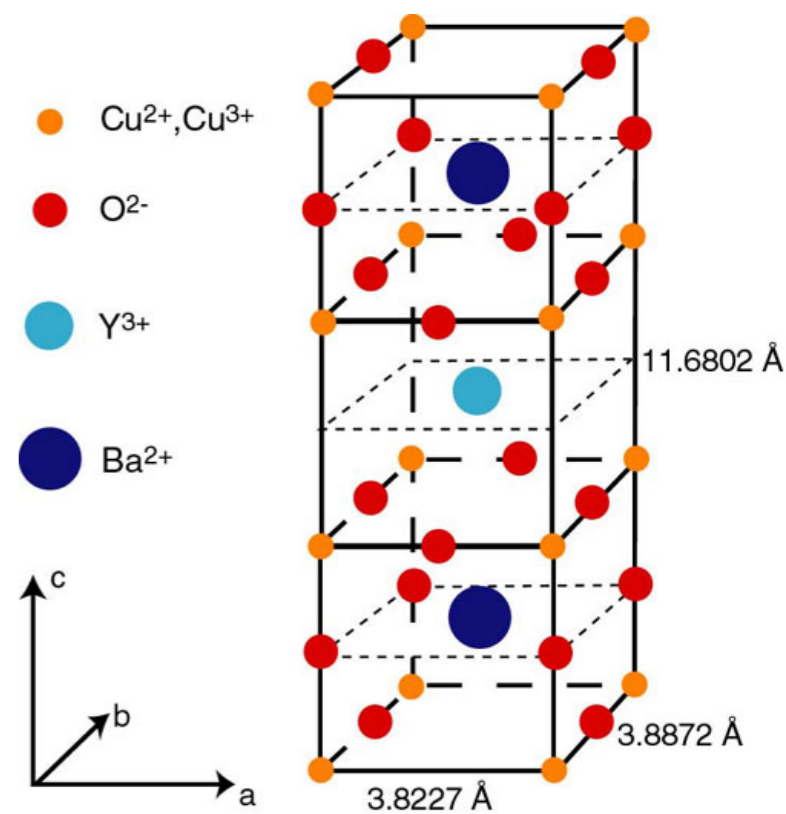
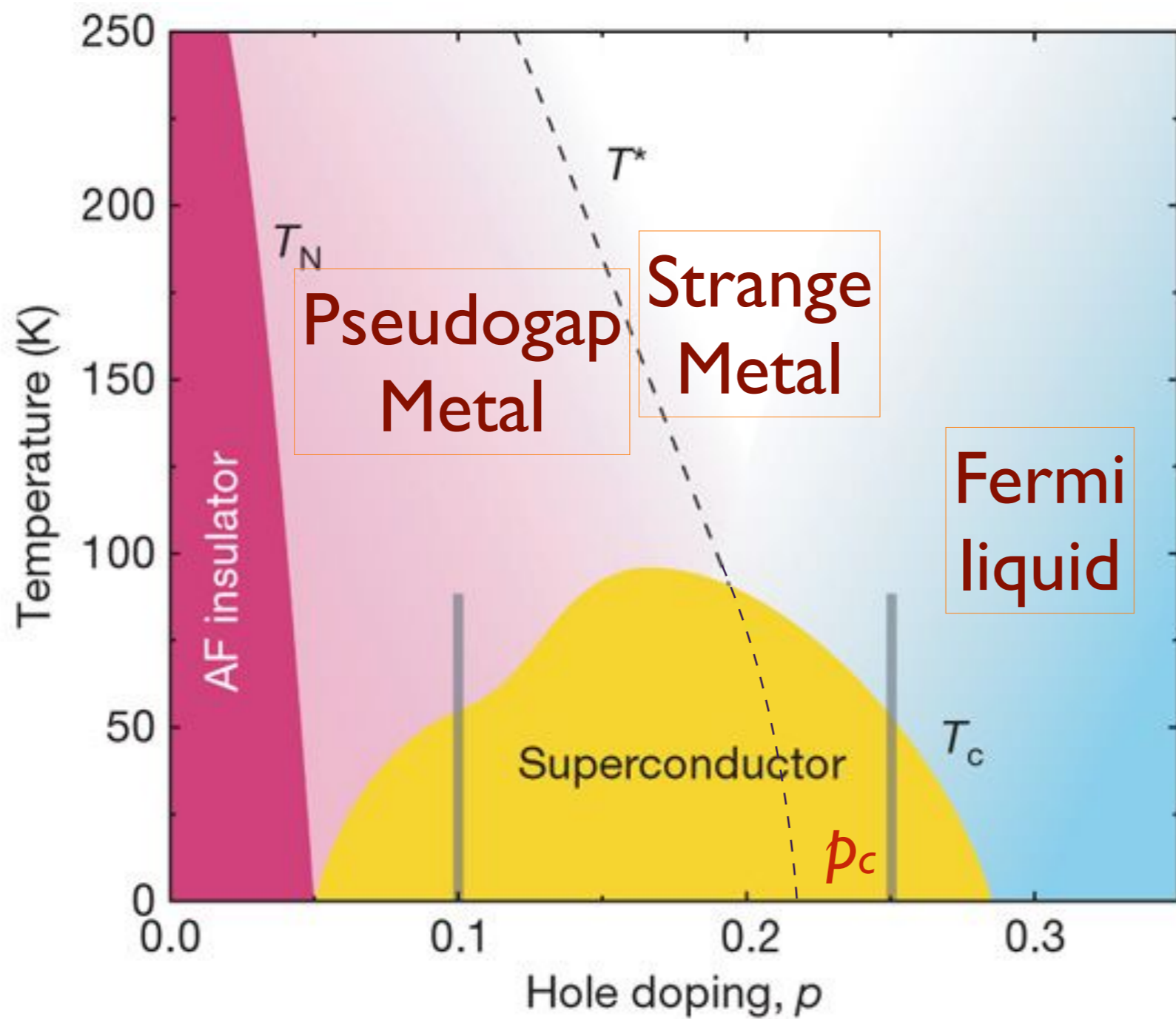
$$D(E) \sim 2 e^{S_0} \sinh(\sqrt{2N\gamma E})$$

$$S_0 = 0.464848 \dots N$$

A. Georges, O. Parcollet, and S. Sachdev,
 PRB **63**, 134406 (2001)

Complex SYK model and charged black holes !

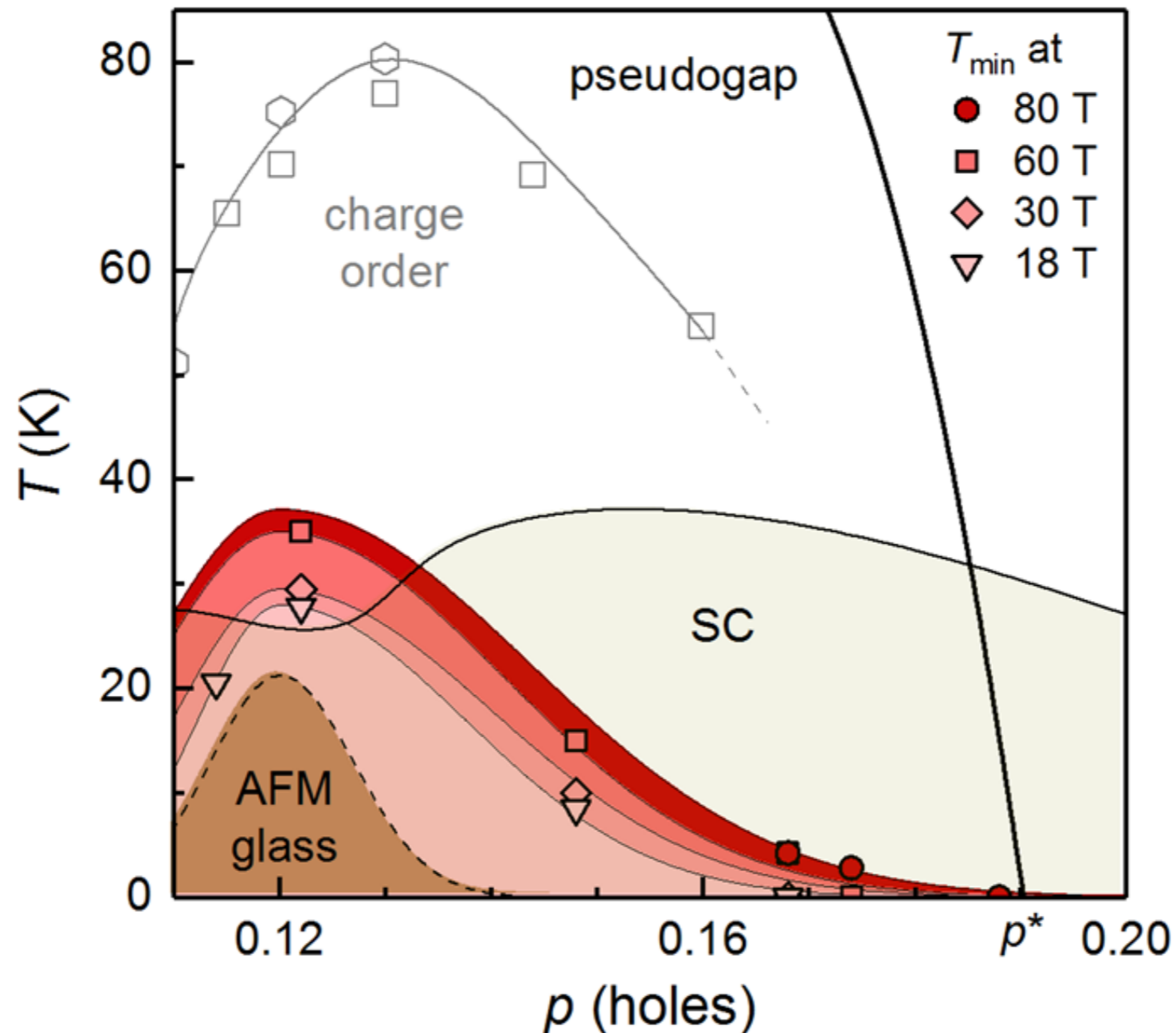
1. SYK models
2. Time reparameterization soft mode
3. Charged black holes
4. Random t-J model
5. Critical Fermi surfaces: large N theory



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet^{1†}, Igor Vinograd^{1†}, Rui Zhou^{1,2}, Siham Benhabib¹, Shangfei Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Sanath K. Ramakrishna³, Arneil P. Reyes³, Jérôme Debray⁴, Tohru Kurosawa⁵, Naoki Momono⁶, Migaku Oda⁵, Seiki Komiyama⁷, Shimpei Ono⁷, Masafumi Horio⁸, Johan Chang⁸, Cyril Proust¹, David LeBoeuf^{1*}, Marc-Henri Julien^{1*}



t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij} \delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$\text{---} \\ |0\rangle$$

$$\text{---} \uparrow \\ c_{\uparrow}^\dagger |0\rangle$$

$$\text{---} \downarrow \\ c_{\downarrow}^\dagger |0\rangle$$

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2$$

$$\text{---} \\ |0\rangle$$

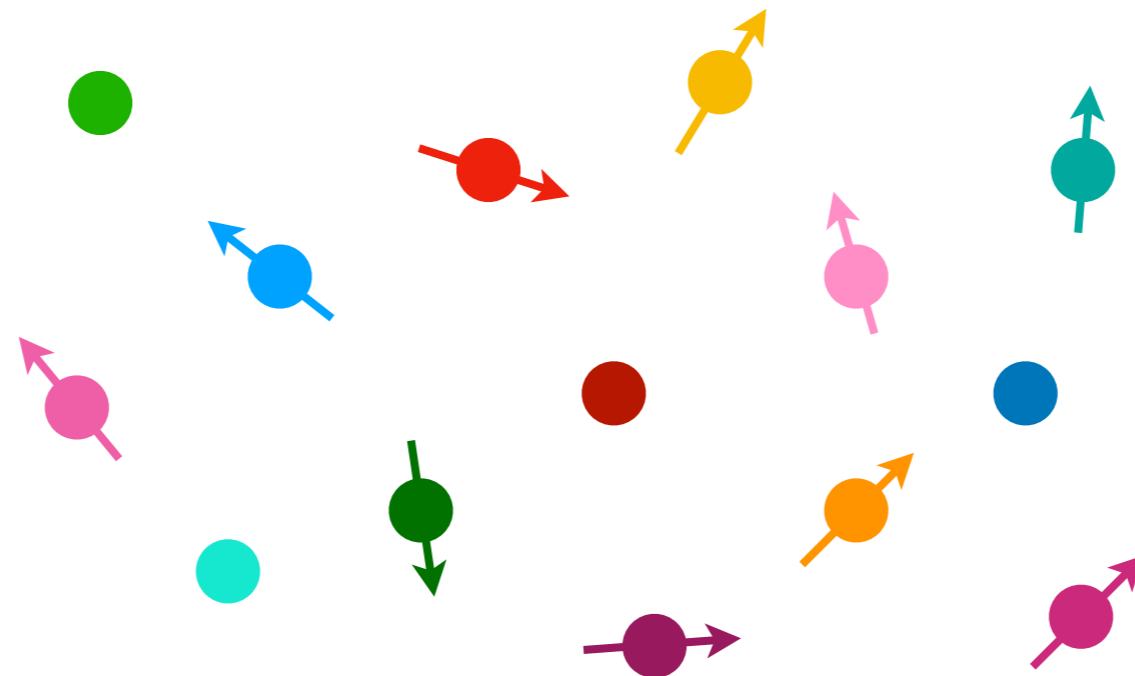
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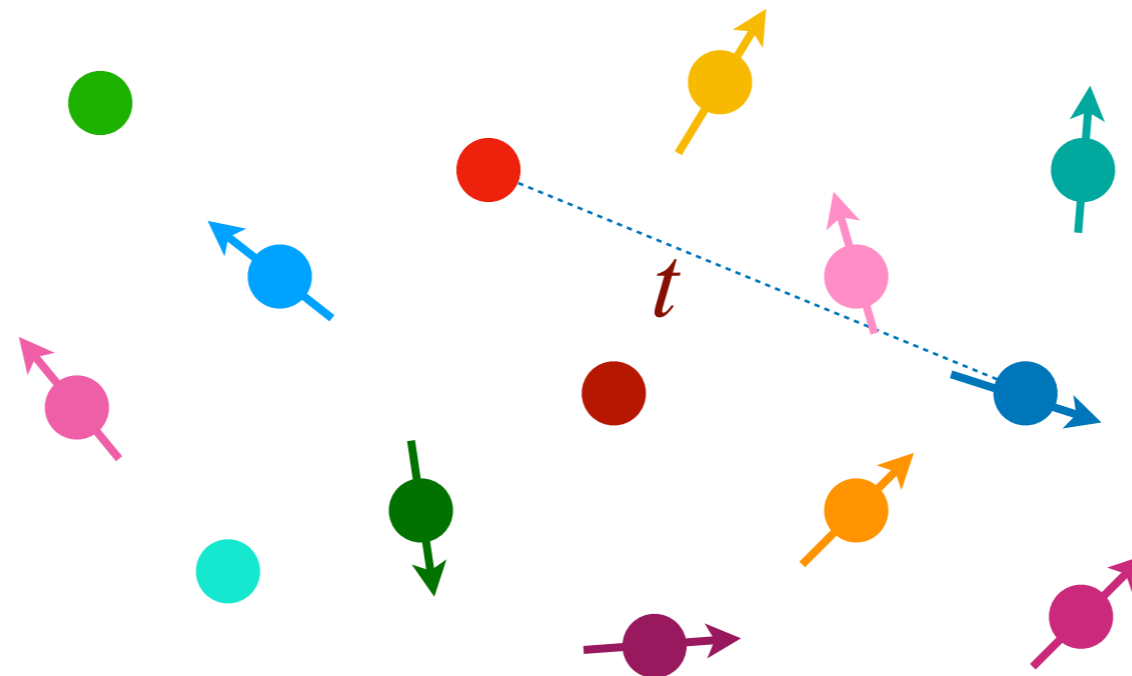
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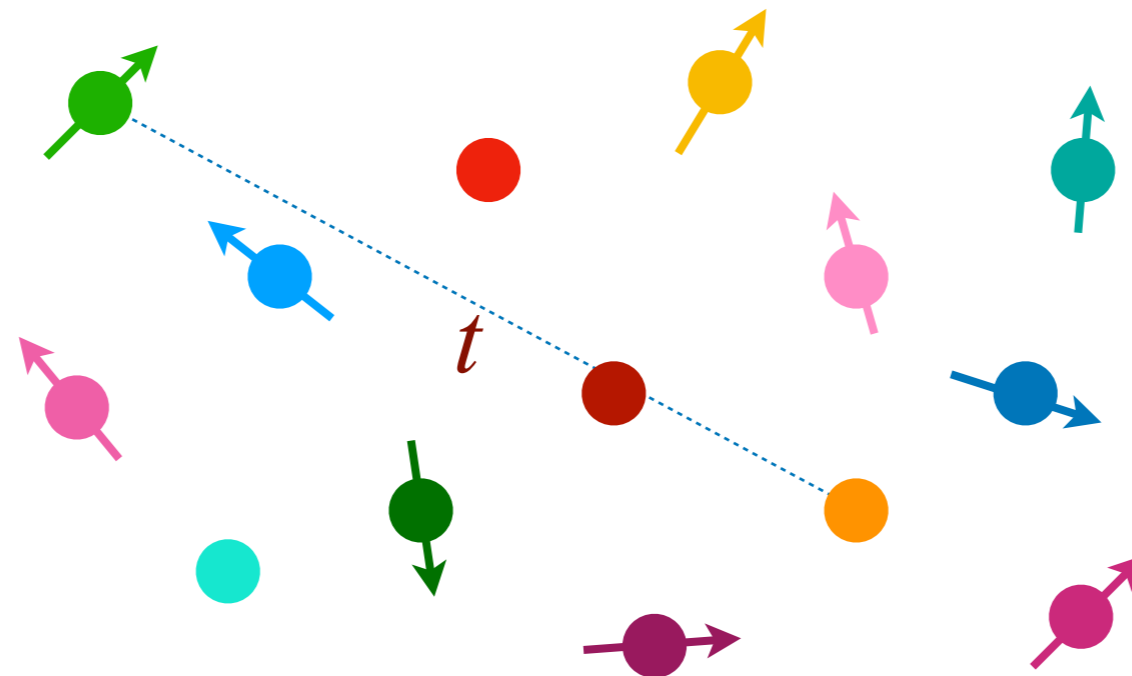
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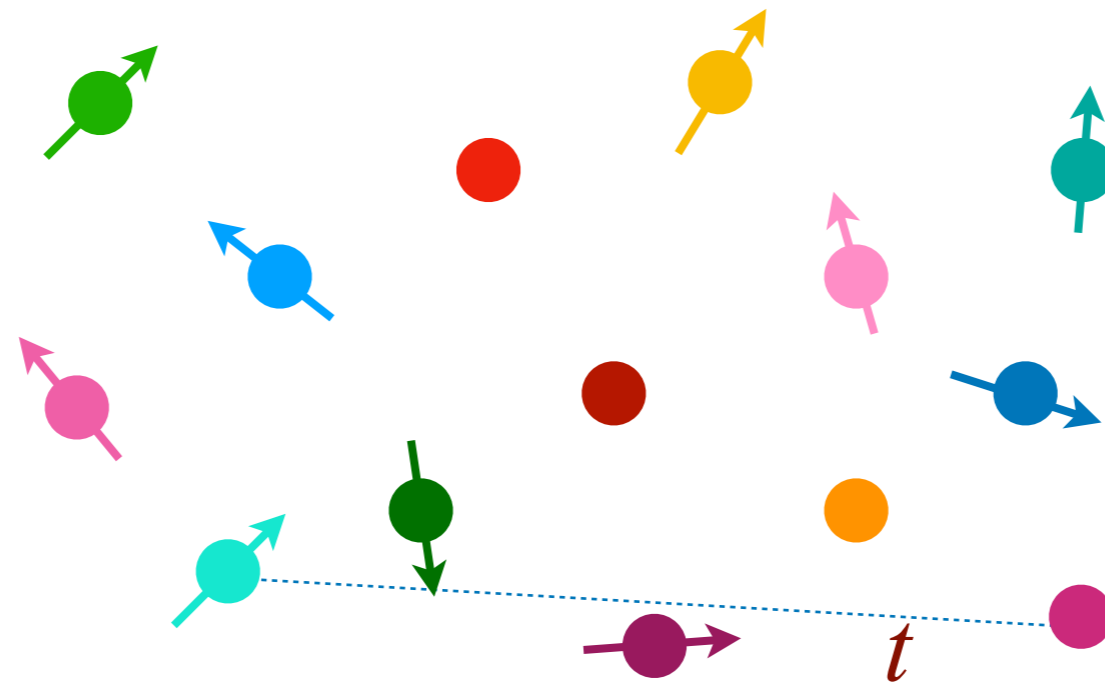
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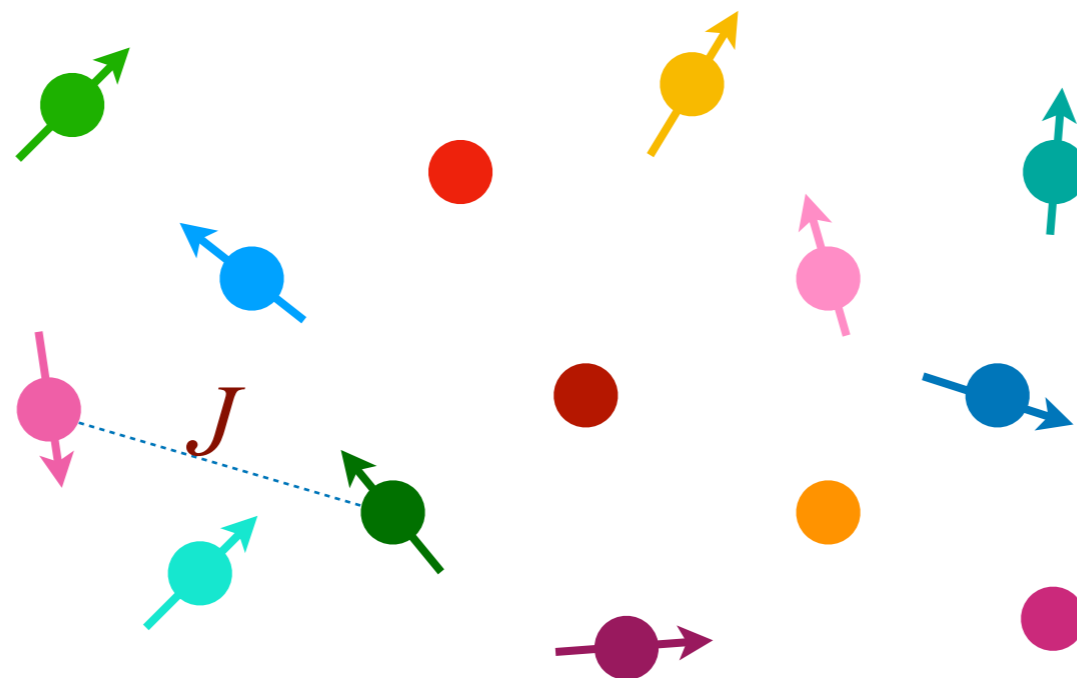
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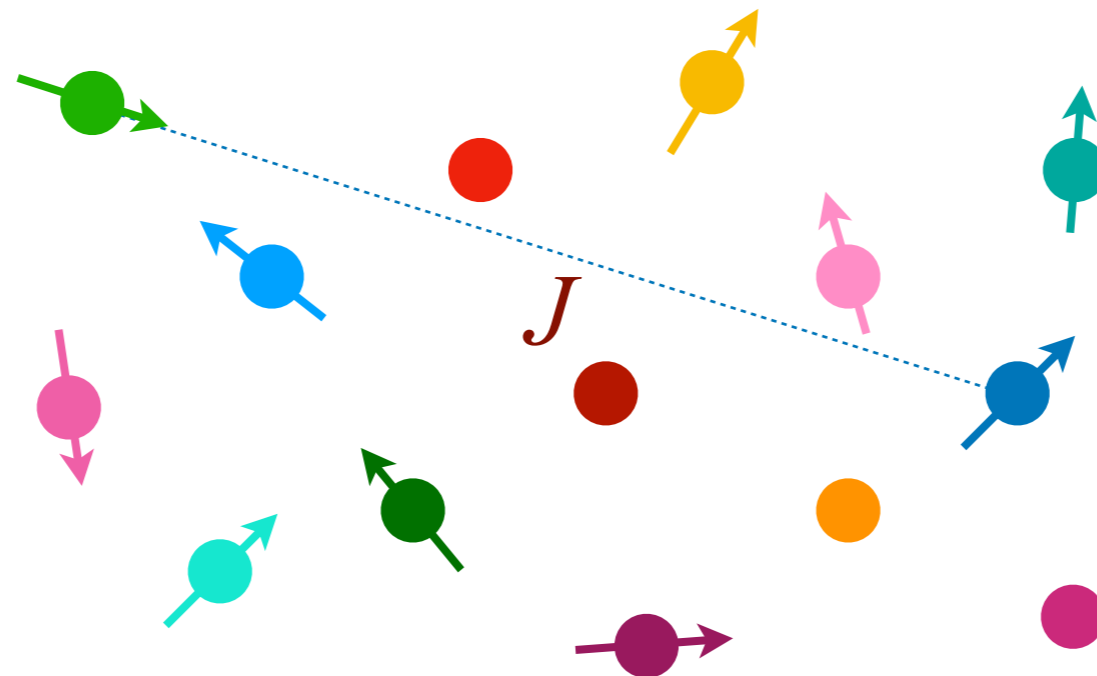
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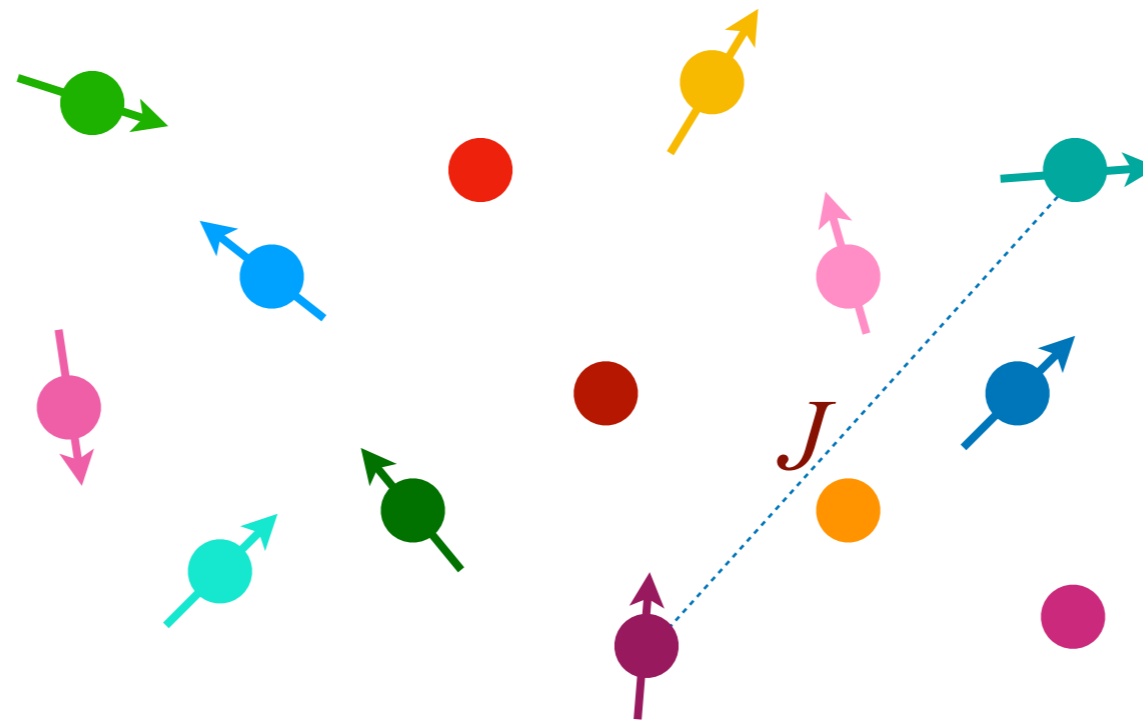
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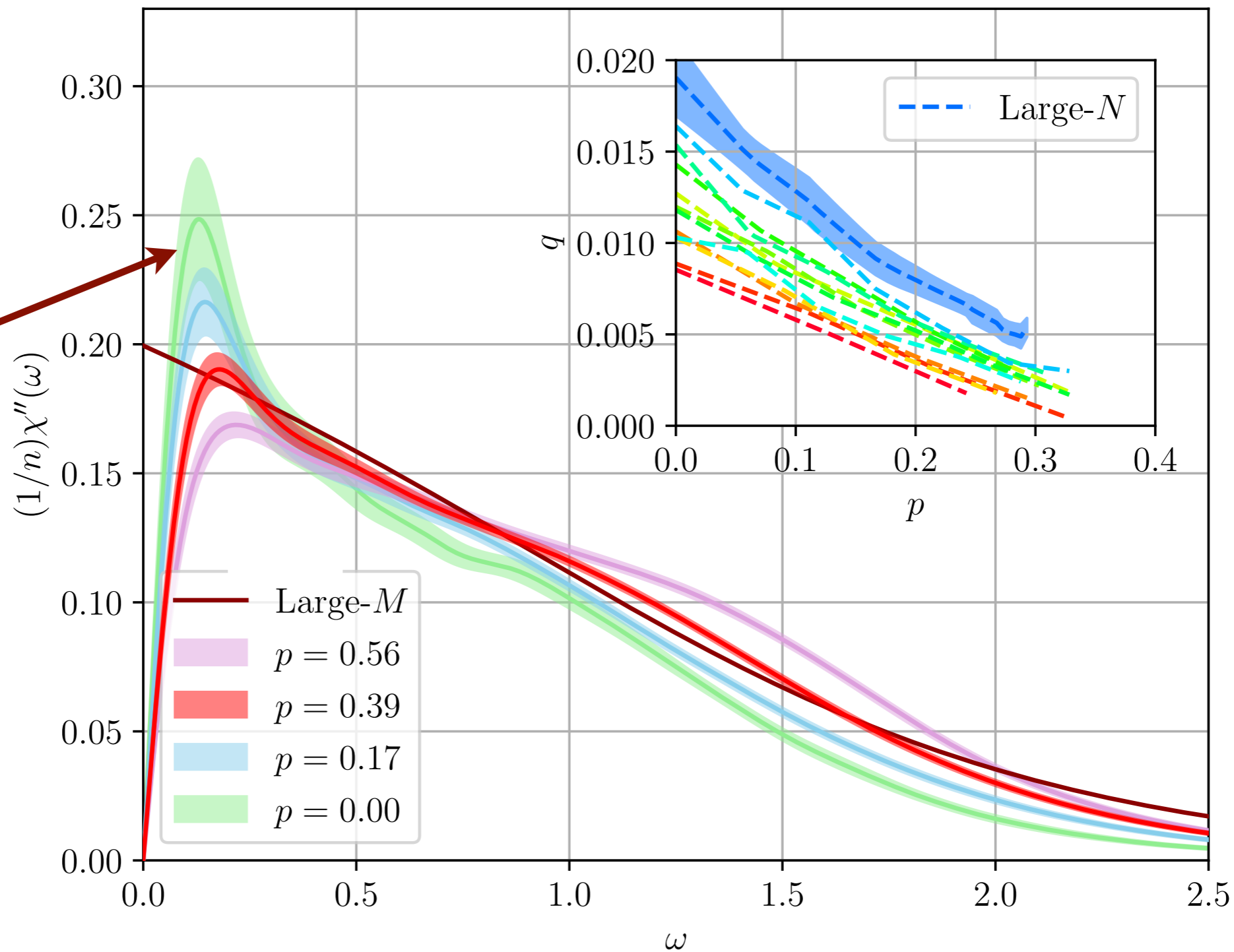
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Dynamic spin susceptibility

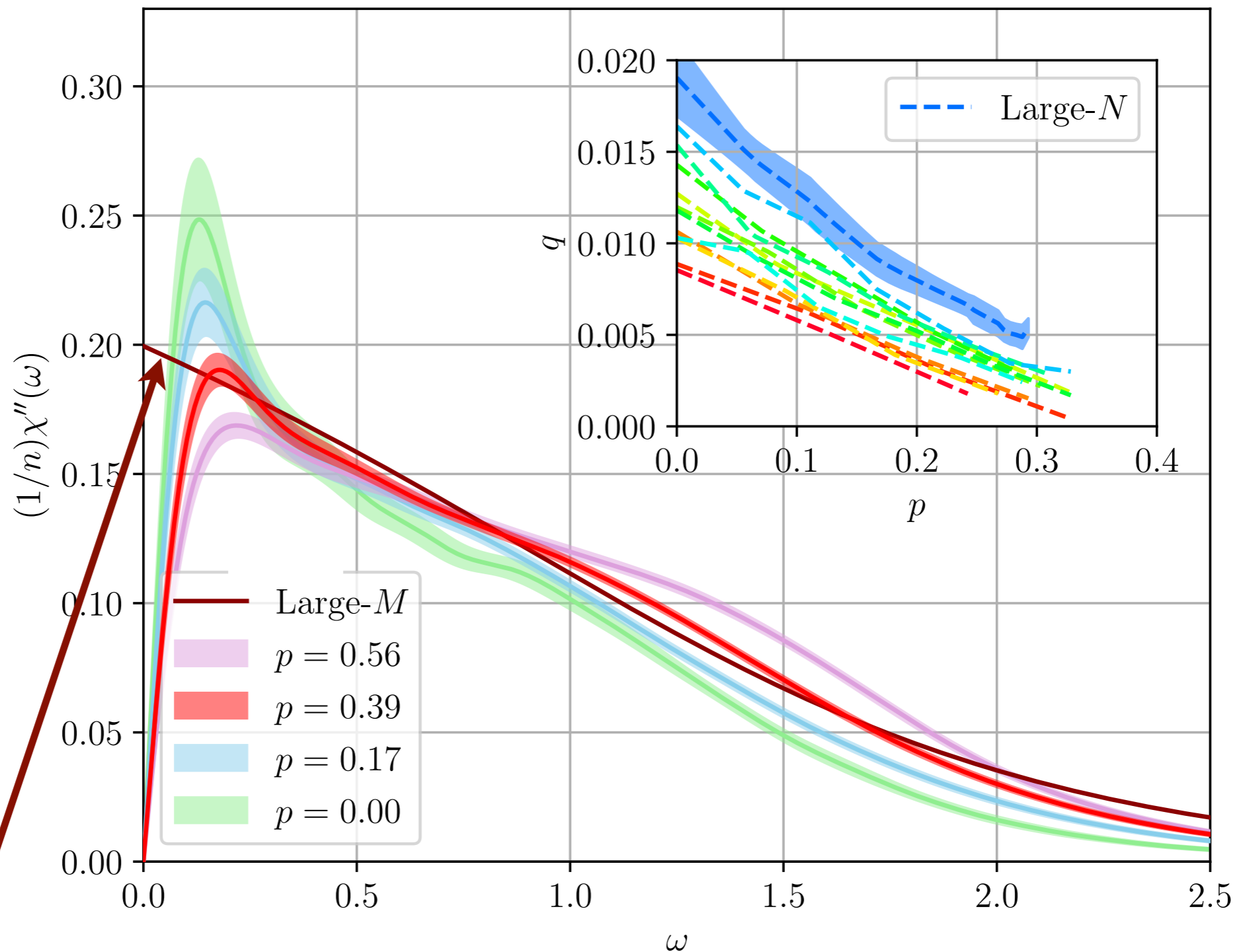
Spin glass order



H. Shackleton,
A. Wietek,
A. Georges, and
S. Sachdev,
PRL **126**,
136602 (2021)

Evidence for a quantum critical point at $p = p_c \approx 0.3$.
Spin glass order q non-zero for $p < p_c$

Dynamic spin susceptibility



H. Shackleton,
A. Wietek,
A. Georges, and
S. Sachdev,
PRL **126**,
136602 (2021)

Critical spin susceptibility matches the large M $SU(M)$ SYK model.

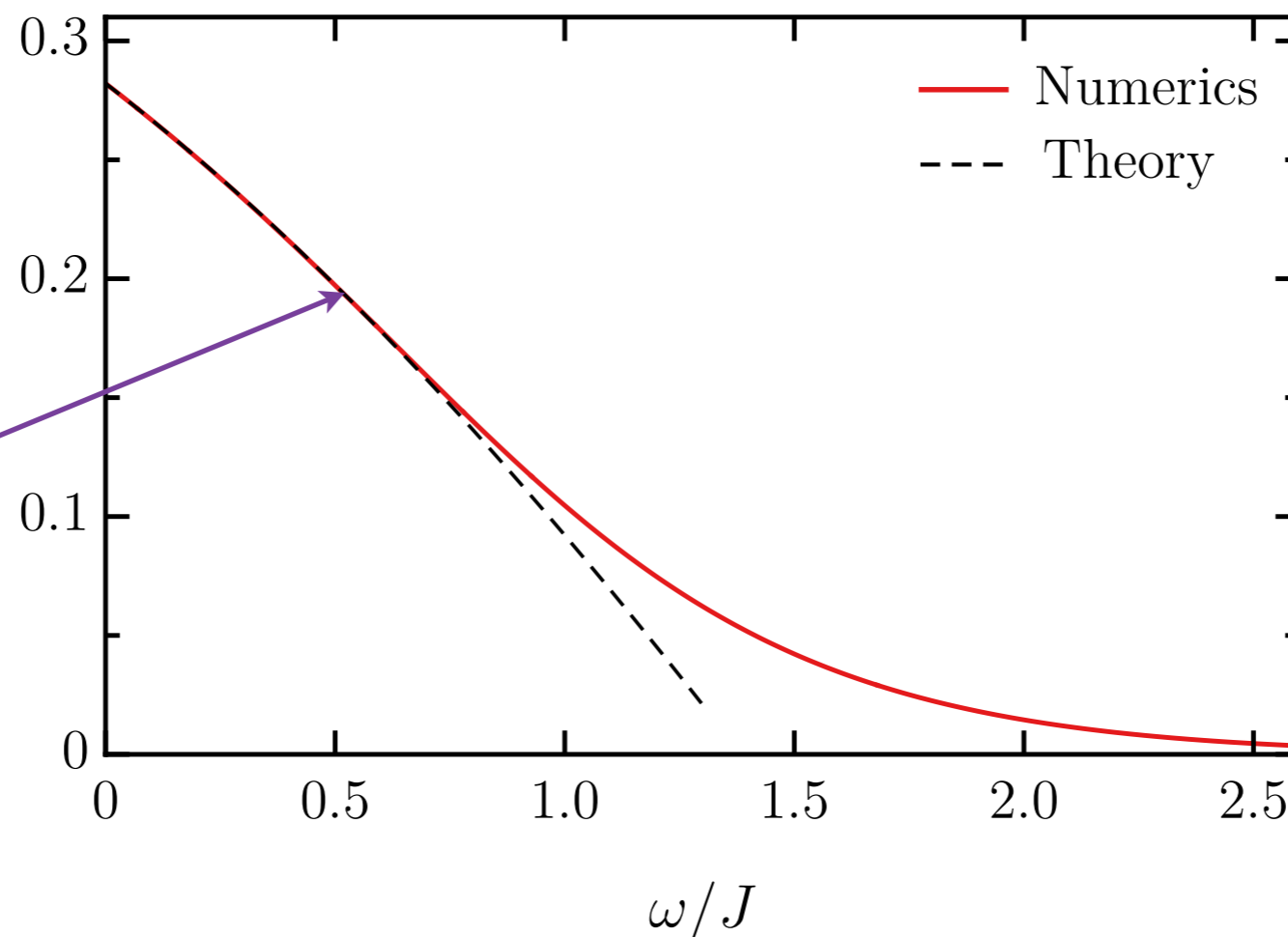
$\chi''(\omega) \sim \text{sgn}(\omega) [1 - \mathcal{C}\gamma|\omega| + \dots]$ has the ‘marginal’ $\text{sgn}(\omega)$ form, with a linear ω correction. Shown is the numerical solution of SYK equations (SY, PRL 1993), after rescaling J .

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0 | S_{+i} | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\text{Im}\chi_L(\omega) \sim \text{sgn}(\omega) \left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\dots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \dots \right]$$

Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory.
 \mathcal{C} is the co-efficient of the action for the ‘boundary graviton’ in holographic dual.



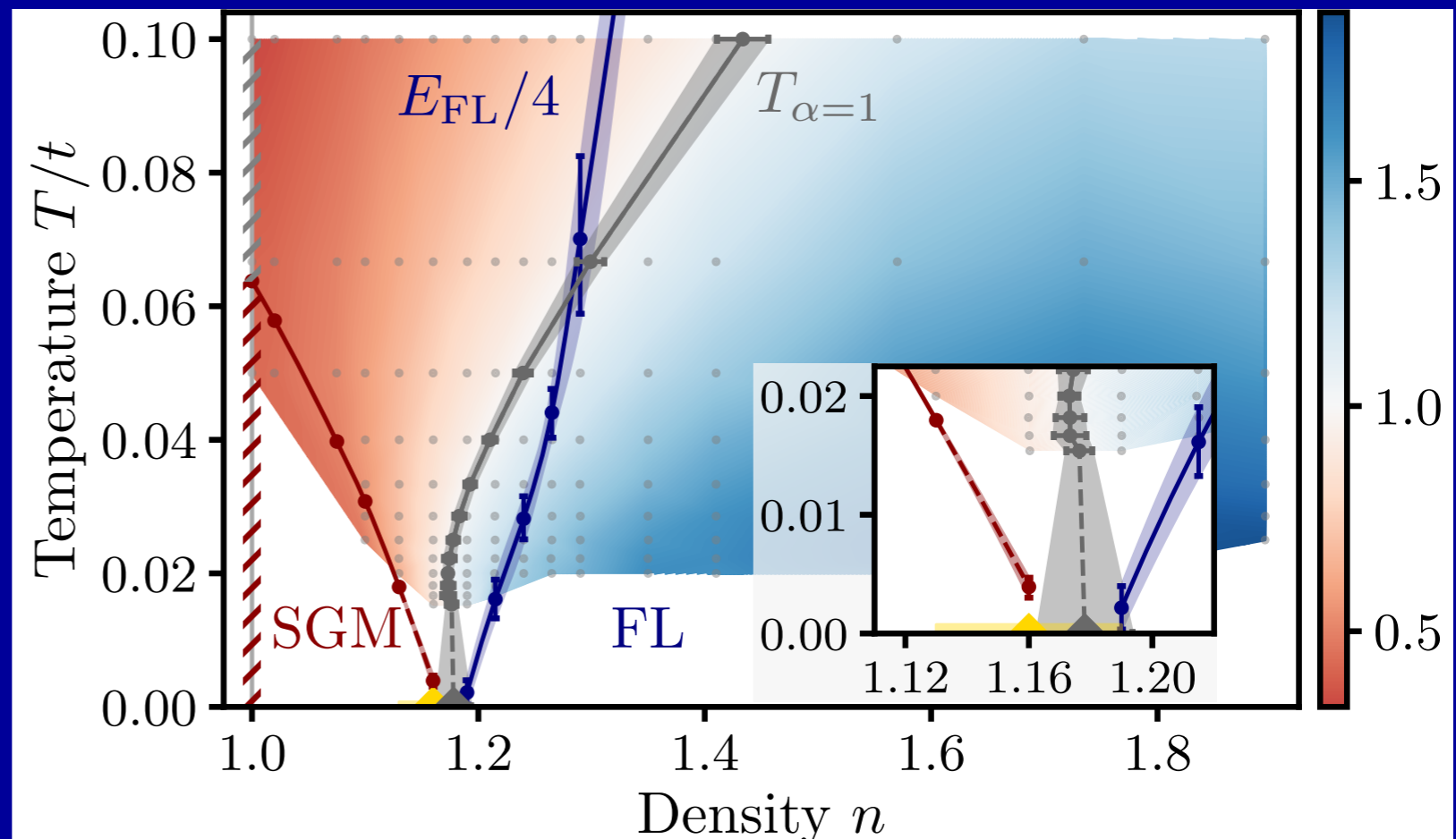
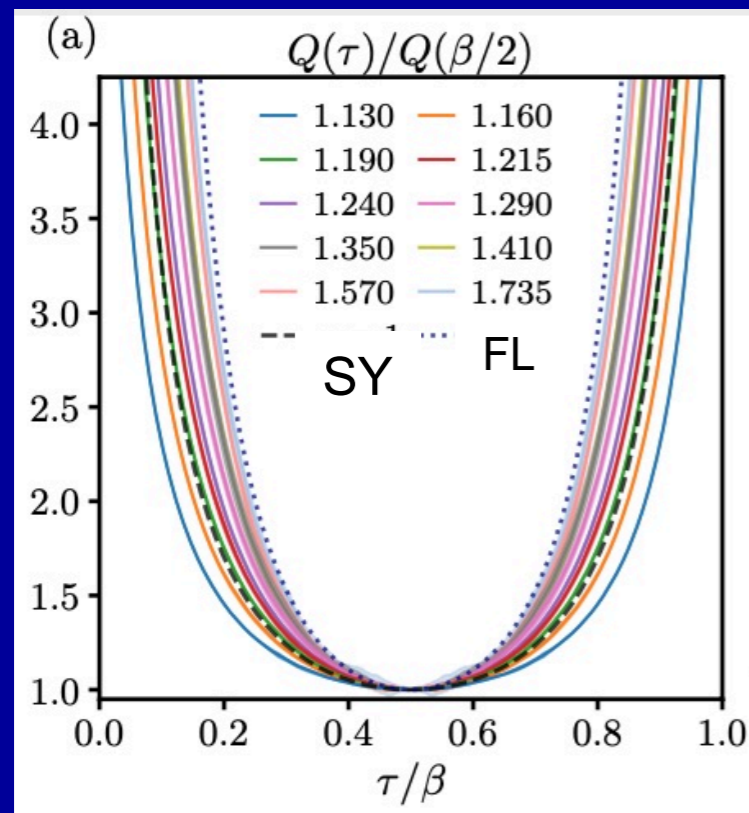
Correction
from the
boundary
graviton

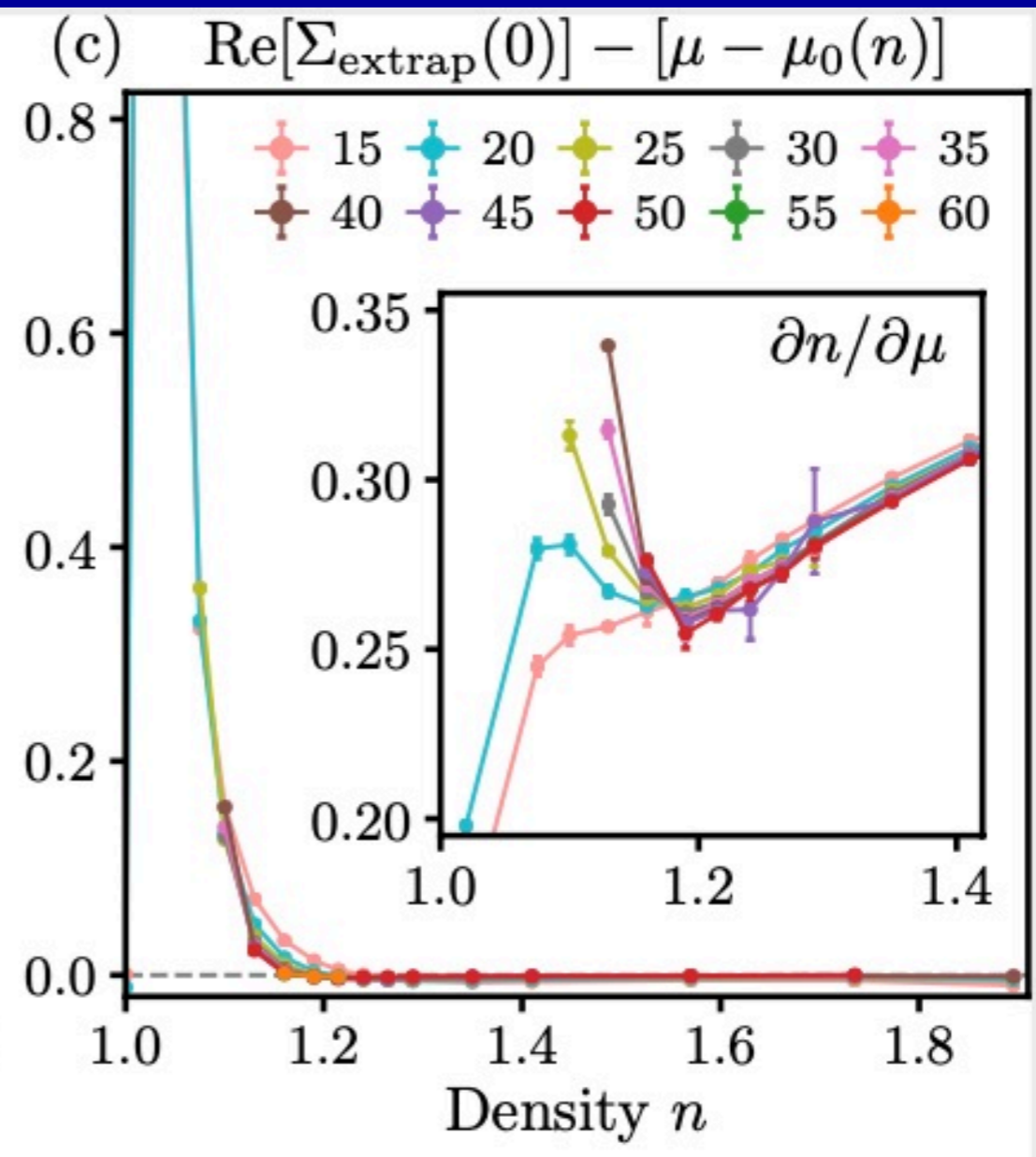
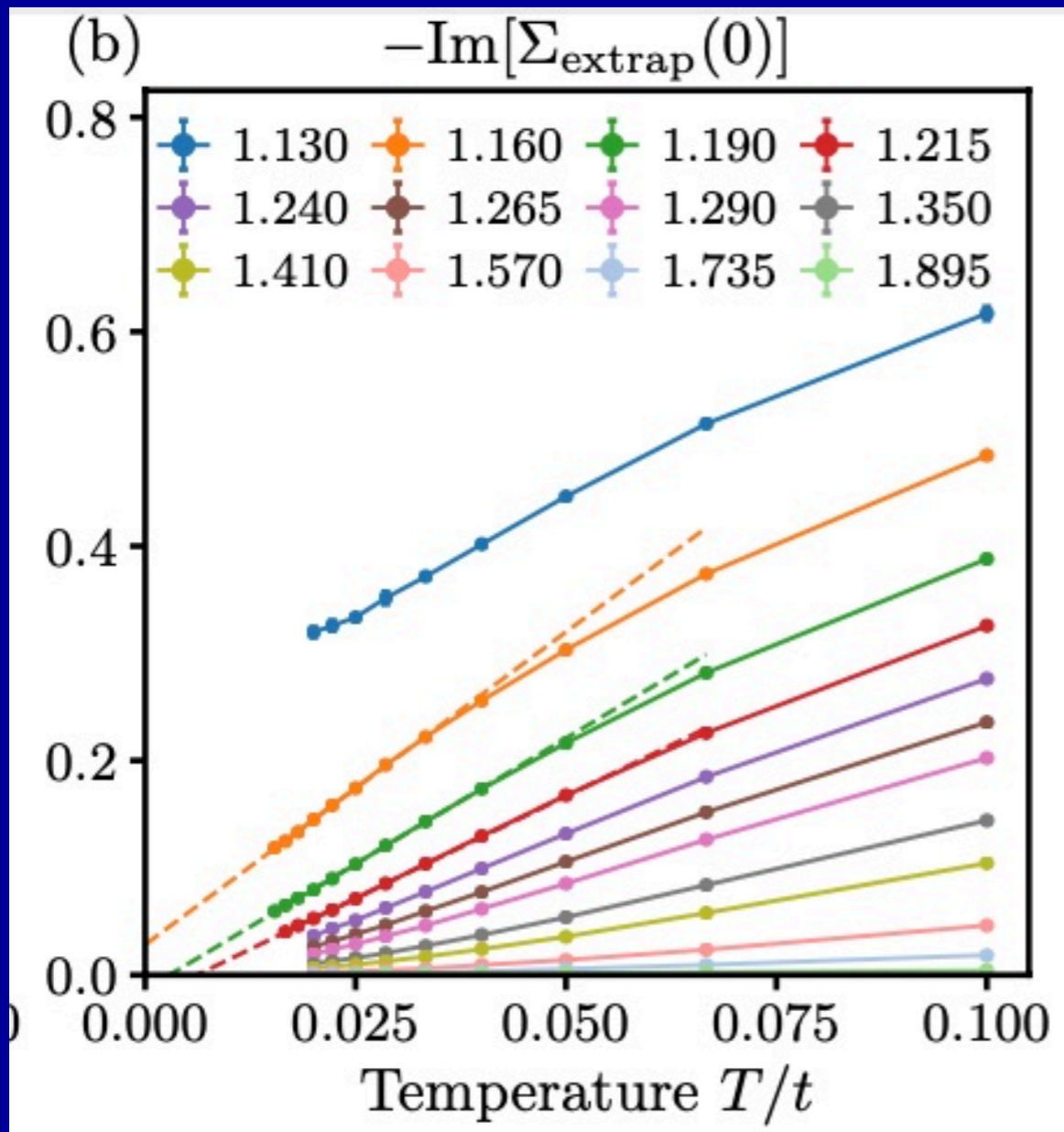


Doping the SU(2) t-J-U SY Model : An Intriguing Quantum Critical Point Solving the EDMFT Equations



P.T. Dumitrescu, N. Wentzell, A. Georges,
O. Parcollet, arXiv:2103.08607





Scattering rate
Linear in T
at QCP

$$\frac{1}{\tau^*} \approx 1.4 \frac{k_B T}{\hbar}$$

Luttinger volume of FS
Breaks down at QCP

P.T. Dumitrescu, N. Wentzell, A. Georges,
O. Parcollet, arXiv:2103.08607

Random t - J model: numerics

Metallic
spin glass.

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$$
$$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

SYK
criticality

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

Disordered
Fermi liquid.

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$
$$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

p_c

p

Random t - J model

Metallic
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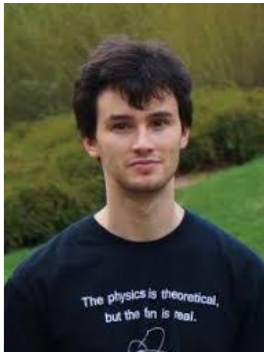
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p_c

p

$SU(M \rightarrow \infty)$ theory of SYK liquids
of fractionalized holons and spinons



Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the ‘*superspin*’ space of a boson b (the holon) and a fermion f_α (the spinon):

$$\begin{array}{ccc} \text{---} & \text{---}\uparrow & \text{---}\downarrow \\ b^\dagger |v\rangle & f_\uparrow^\dagger |v\rangle & f_\downarrow^\dagger |v\rangle \end{array}$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \end{aligned}$$

$$f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

$$\text{U(1) gauge invariance,} \quad b \rightarrow b e^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this SU(1|2) superspin space.

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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$$b_\alpha^\dagger b_\alpha + f^\dagger f = 1$$

$$\text{U(1) gauge invariance,} \quad f \rightarrow f e^{i\phi}, \quad b_\alpha \rightarrow b_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this SU(2|1) superspin space.

Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the ‘*superspin*’ space of a fermion f (the holon) and a boson b_α (the spinon):

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$$\text{SU}(1|2) \equiv \text{SU}(2|1)$$

$$\begin{aligned} c_\alpha &= b_\alpha f^\dagger \\ \vec{S} &= \frac{1}{2} b_\alpha^\dagger \sigma_{\alpha\beta} b_\beta \end{aligned}$$

$$b_\alpha^\dagger b_\alpha + f^\dagger f = 1$$

$$\text{U}(1) \text{ gauge invariance, } f \rightarrow f e^{i\phi}, \quad b_\alpha \rightarrow b_\alpha e^{i\phi}$$

The physical electron (c_α) and spin (\vec{S}) operators are rotations in this $\text{SU}(2|1)$ superspin space.

Random t - J model: large M limit

Both the t and the J terms involve four single-particle operators.

Consequently, in a large M limit, the saddle-point equations are very similar to the $q = 4$ SYK equations. These equations realize a critical phase with SYK criticality, provided none of the bosons condense.

$$\begin{aligned}G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\ \Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\ G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\ \Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau)\end{aligned}$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$\langle f^\dagger f \rangle = \frac{1}{2} - k\delta \quad , \quad \langle b^\dagger b \rangle = \delta .$$

Random t - j model: numerics



Metallic
spin glass.

Disordered
Fermi liquid.

SYK
criticality

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

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p_c

p

$SU(M \rightarrow \infty)$ theory of SYK liquids
of fractionalized holons and spinons

Random t - j model: numerics



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SYK
criticality

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$$

SU(1|2) theory

Disordered
Fermi liquid.
Condense holon b ,
 f_α carrier density $1 + p$

$$\begin{array}{cc} \text{---}\uparrow\text{---} & \text{---}\downarrow\text{---} \\ f_\uparrow^\dagger |v\rangle & f_\downarrow^\dagger |v\rangle \end{array}$$

$$\text{---} b^\dagger |v\rangle$$

$$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$$

$$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

p_c

p

Random t - j model: numerics

SU(2|1) theory

Metallic spin glass.

Condense spinon \mathbf{b}_α ,
 f carrier density p

$f^\dagger |v\rangle$

$\begin{array}{cc} \uparrow & \downarrow \\ \text{---} & \text{---} \\ \mathbf{b}_\uparrow^\dagger |v\rangle & \mathbf{b}_\downarrow^\dagger |v\rangle \end{array}$

$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \text{constant}$

$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$

SYK
 criticality

$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}$

SU(1|2) theory

Disordered Fermi liquid.

Condense holon b ,
 f_α carrier density $1 + p$

$\begin{array}{cc} \uparrow & \downarrow \\ \text{---} & \text{---} \\ f_\uparrow^\dagger |v\rangle & f_\downarrow^\dagger |v\rangle \end{array}$

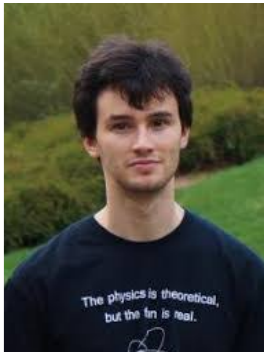
$b^\dagger |v\rangle$

$\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2}$

$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$

p_c

p



Random t - j model: numerics

SU(2|1) theory

Metallic spin glass.

Condense spinon \mathbf{b}_α ,
 f carrier density p

$f^\dagger |v\rangle$

$\begin{array}{c} \uparrow \\ \text{---} \\ \mathbf{b}_\uparrow^\dagger |v\rangle \end{array} \quad \begin{array}{c} \downarrow \\ \text{---} \\ \mathbf{b}_\downarrow^\dagger |v\rangle \end{array}$

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SYK
 criticality of
 fractionalized
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$\text{---} \quad \begin{array}{c} \uparrow \\ \text{---} \end{array} \quad \begin{array}{c} \downarrow \\ \text{---} \end{array}$

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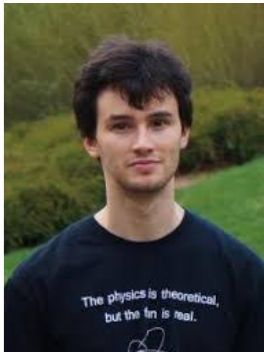
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$$\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

p_c

p

Zeroth order, $p_c = 1/3$



Time reparameterization soft mode and linear- T resistivity

The time reparameterization soft-mode now leads to corrections to the Green's functions of the partons

$$G_{f,b}(\tau) \sim \frac{\pm 1}{\sqrt{|\tau|}} \left(1 + \frac{\alpha_{f,b}}{|\tau|} + \dots \right)$$

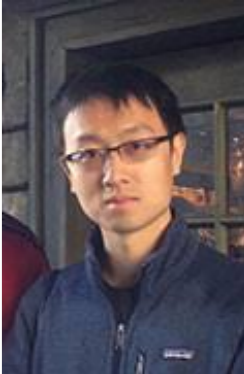
We can compute the resistivity from this in a large- d model, and find

$$\rho(T) = \rho(0) \left(1 + 8\alpha_G \frac{T}{J} + \dots \right) .$$

The α_G term arises from the contribution of the boundary graviton!

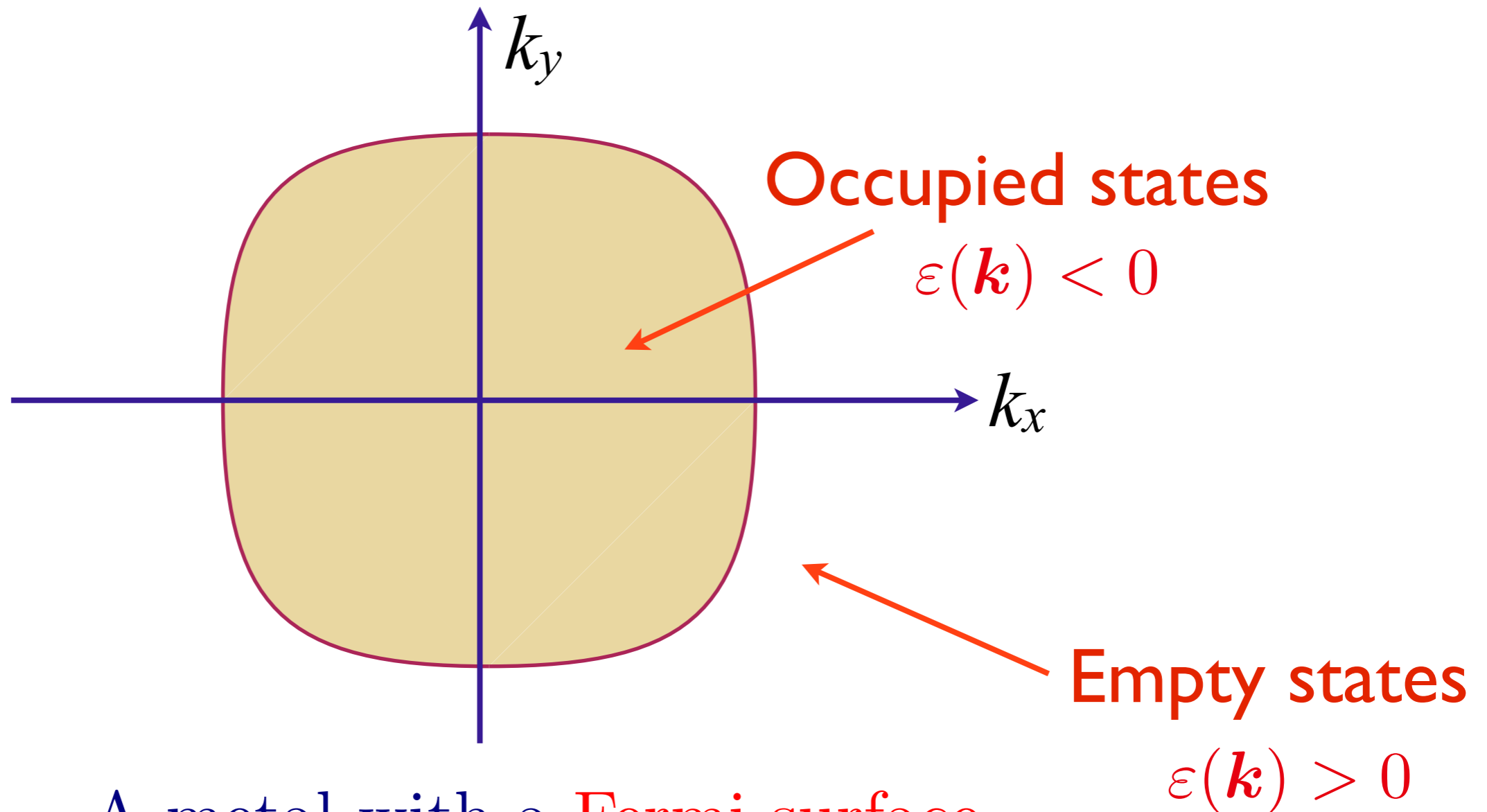
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)



1. SYK models
2. Time reparameterization soft mode
3. Charged black holes
4. Random t-J model
5. Critical Fermi surfaces: large N theory

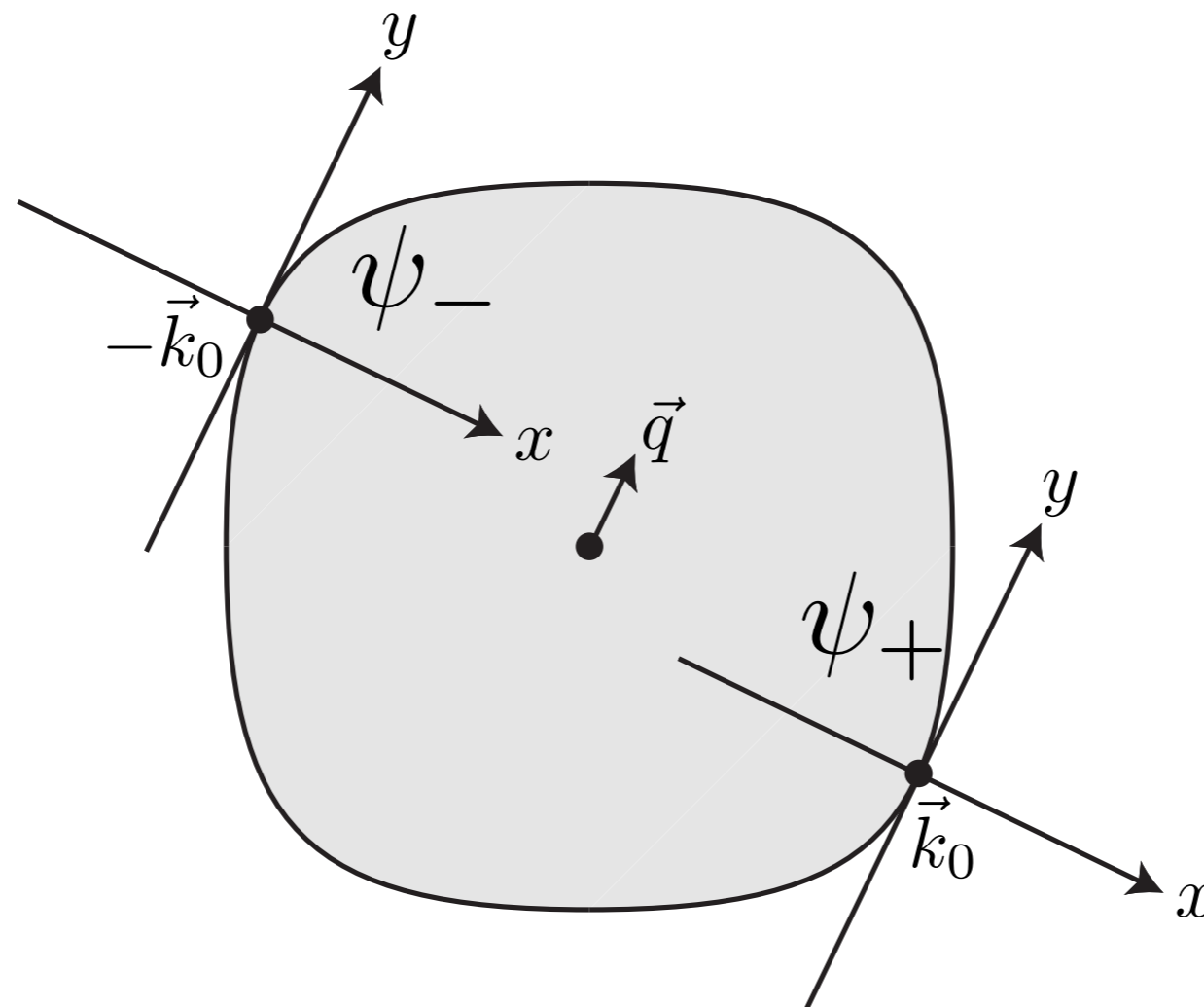
Fermi surface coupled to a gauge field



A metal with a Fermi surface
minimally coupled to a gauge field \mathbf{A}

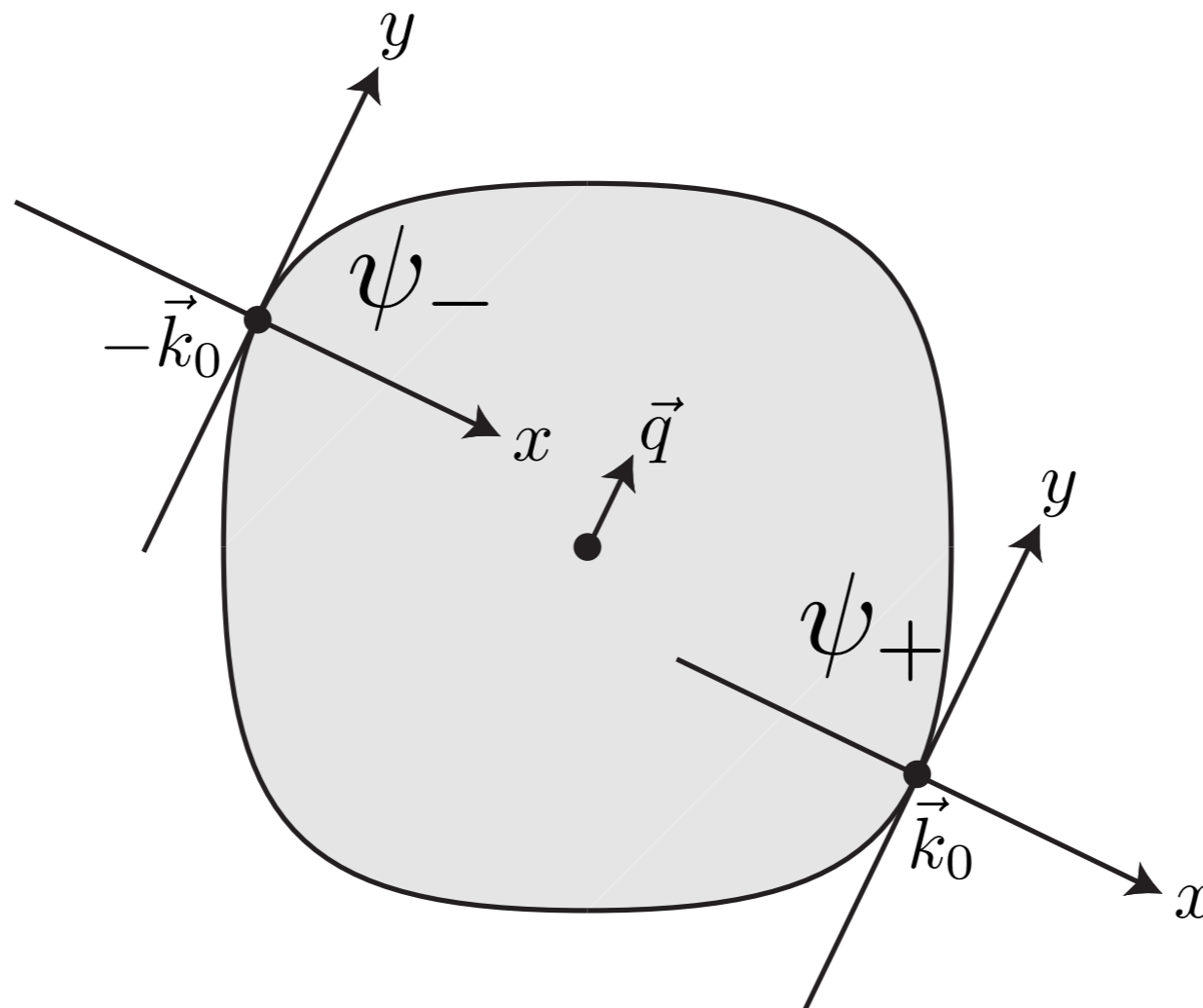
$$\mathcal{L} = c_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(-i\nabla - g\mathbf{A}) - \mu \right) c_{\mathbf{k}} + \frac{1}{2} (\nabla \times \mathbf{A})^2$$

Fermi surface coupled to a gauge field



- Gauge fluctuation at wavevector \mathbf{q} couples most efficiently to fermions near $\pm\mathbf{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\pm\mathbf{k}_0$. In Landau gauge $\mathbf{A} = (a, 0)$.

Fermi surface coupled to a gauge field



$$\mathcal{L}[\psi_{\pm}, a] = \psi_+^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - g a (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) + \frac{1}{2} (\partial_y a)^2$$

Large N theory of a critical Fermi surface



N flavors of fermions $\psi_{\pm\alpha}$,
 M flavors of a boson a_{α} , and
a “Yukawa coupling” $g_{\alpha\beta\gamma}$ which is a random function in
flavor space. Note: there is *no spatial randomness*. Take
the large N limit with M/N fixed.

$$\mathcal{L} = \psi_{+\alpha}^{\dagger} (\partial_{\tau} - i\partial_x - \partial_y^2) \psi_{+\alpha} + \psi_{-\alpha}^{\dagger} (\partial_{\tau} + i\partial_x - \partial_y^2) \psi_{-\alpha} \\ - \frac{g_{\alpha\beta\gamma}}{N} a_{\alpha} \left(\eta_{+\alpha} \psi_{+\beta}^{\dagger} \psi_{+\gamma} + \eta_{-\alpha} \psi_{-\beta}^{\dagger} \psi_{-\gamma} \right) + \frac{1}{2} (\partial_y a_{\alpha})^2$$

$\eta_{\pm\alpha} = \pm 1$ depending upon nature of a_{α} : gauge field, Higgs
field, order parameter

$$\overline{g_{\alpha\beta\gamma}} = 0 \quad , \quad \overline{|g_{\alpha\beta\gamma}|^2} = g^2$$

Large N theory of a critical Fermi surface

We can now proceed just as in the SYK model: we obtain a theory for Green's functions which are bilocal in both space and time. Using the spacetime coordinate $X \equiv (\tau, x, y)$, we can write the averaged partition function

$$\begin{aligned} \overline{\mathcal{Z}}_{\psi\phi} &= \int \mathcal{D}G(X_1, X_2) \mathcal{D}\Sigma(X_1, X_2) \mathcal{D}D(X_1, X_2) \\ &\quad \times \mathcal{D}\Pi(X_1, X_2) \exp[-NI(G, \Sigma, D, \Pi)] . \end{aligned}$$

The G - Σ - D - Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) &= \frac{g^2}{2} \text{Tr}(G \cdot [GD]) - \text{Tr}(G \cdot \Sigma) + \frac{1}{2} \text{Tr}(D \cdot \Pi) \\ &\quad - \ln \det [(\partial_{\tau_1} - i\partial_{x_1} - \partial_{y_1}^2) \delta(X_1 - X_2) + \Sigma(X_1, X_2)] \\ &\quad + \frac{1}{2} \ln \det [(-K\partial_{y_1}^2) \delta(X_1 - X_2) - \Pi(X_1, X_2)] . \end{aligned}$$

where we have introduced notation

$$\text{Tr}(f \cdot g) \equiv \int dX_1 dX_2 f(X_2, X_1) g(X_1, X_2) .$$

Large N theory of a critical Fermi surface

Saddle-point equations

$$G(\mathbf{k}, i\omega) = \frac{1}{i\omega - k_x - k_y^2 - \Sigma(\mathbf{k}, i\omega)}, \quad D(\mathbf{k}, i\omega) = \frac{1}{k_y^2 - \Pi(\mathbf{k}, i\omega)}$$
$$\Sigma(\mathbf{r}, \tau) = g^2 D(\mathbf{r}, \tau) G(\mathbf{r}, \tau), \quad \Pi(\mathbf{r}, \tau) = -g^2 G(-\mathbf{r}, -\tau) G(\mathbf{r}, \tau)$$

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Exact solution at low energies:

$$\Sigma(\mathbf{k}, \omega) = g^{4/3} T^{2/3} \Phi\left(\frac{\omega}{T}\right),$$

where $\Phi(z)$ is a universal scaling function, obtained by analytical continuation from imaginary Matsubara frequencies $\omega_n = (2n - 1)\pi T$

$$\Phi\left(\frac{i\omega_n}{T}\right) = -i \operatorname{sgn}(\omega_n) \frac{2^{5/3}}{3\sqrt{3}} H_{1/3}\left(\frac{|\omega_n| - \pi T}{2\pi T}\right)$$

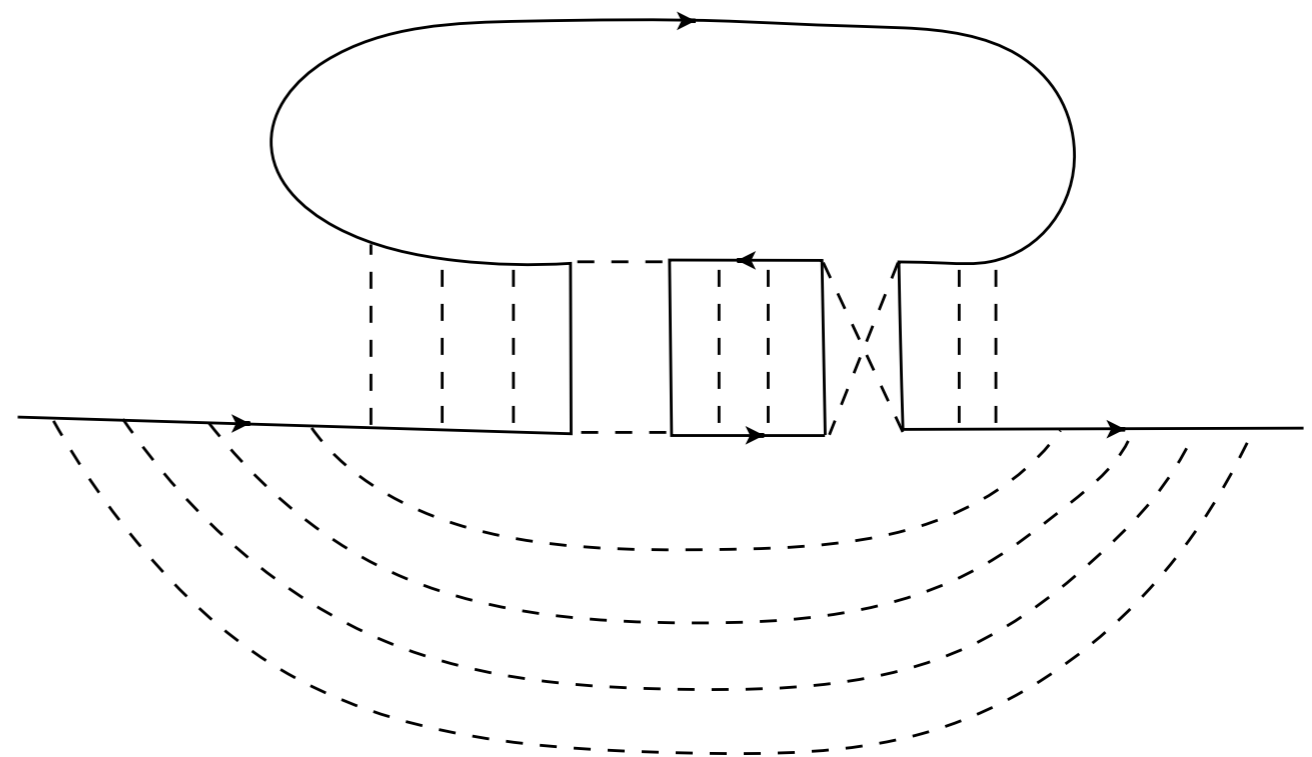
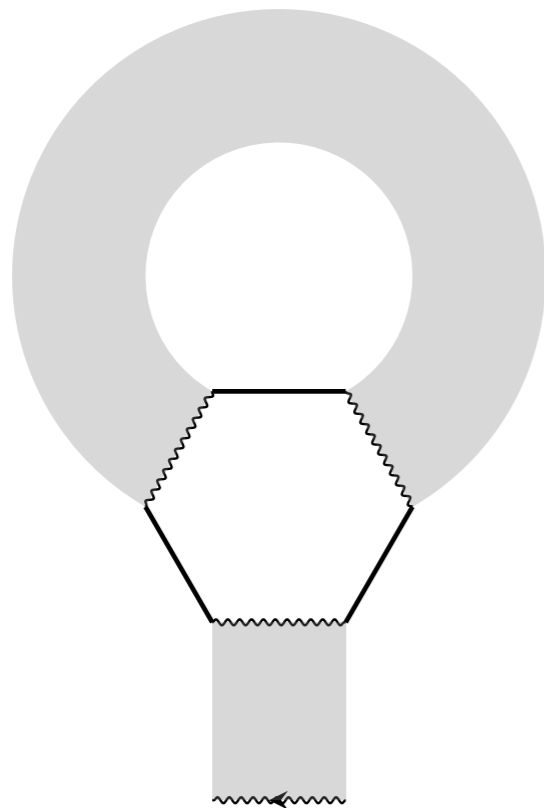
$$H_r (n \in \mathbb{Z}^+) = \sum_{j=1}^n \frac{1}{j^r}$$

Large N theory of a critical Fermi surface

- There is no time-reparameterization soft mode, and no associated violation of scaling: because of non-trivial action of time reparameterization on spatial co-ordinates.

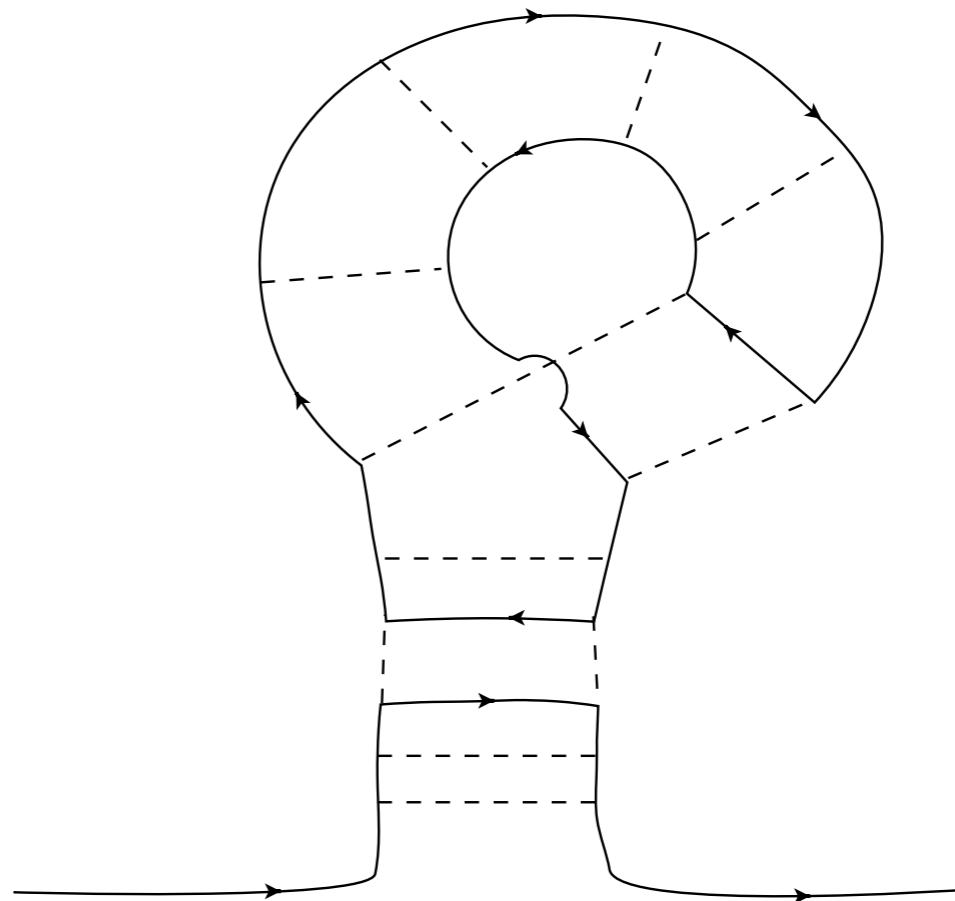
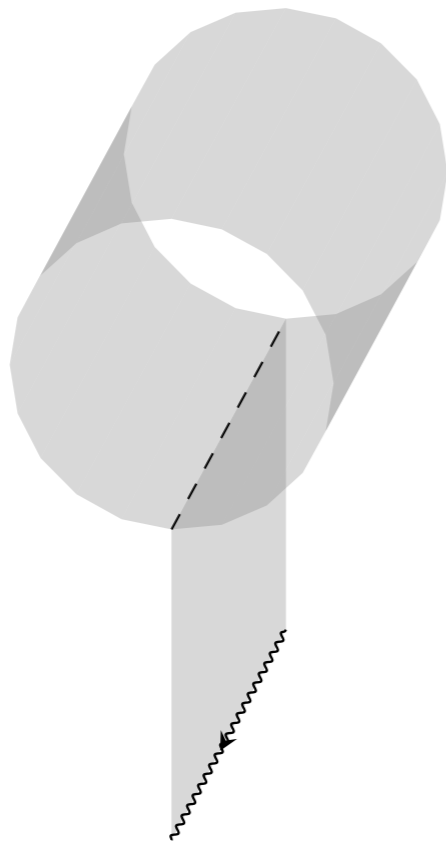
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- Large N theory of a critical Fermi surface is obtained by an average over theories with random Yukawa couplings: leads to a G - Σ theory of Green's functions which are bilocal in spacetime.