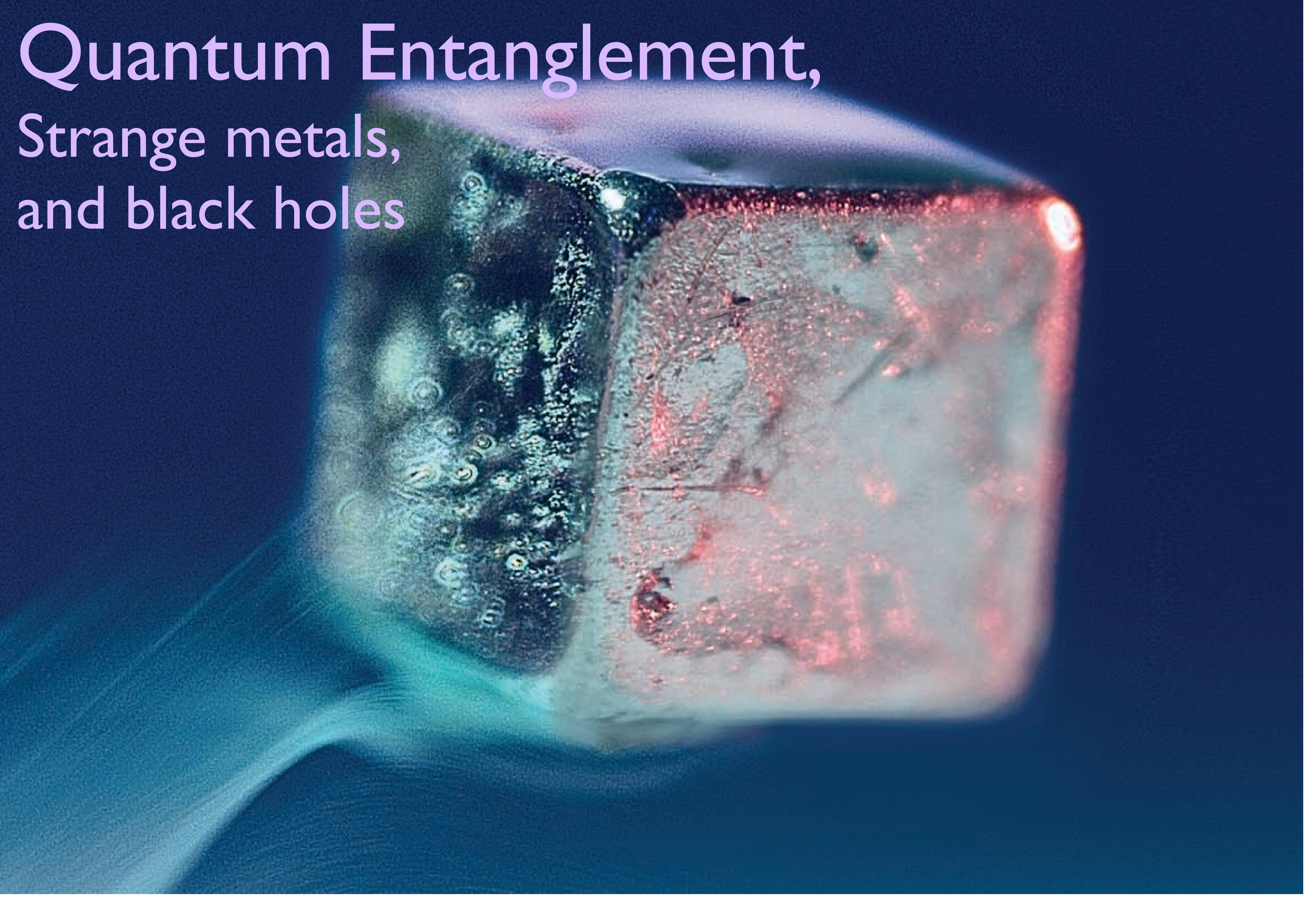
Quantum Entanglement,
Strange metals, and black holes

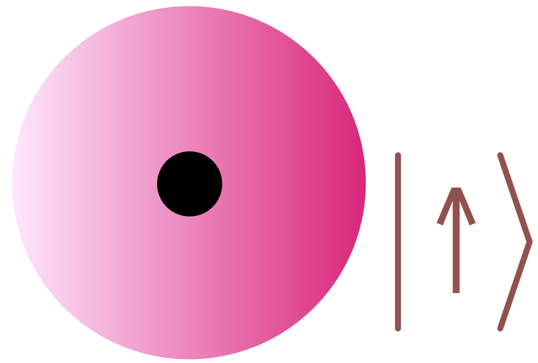


Subir Sachdev, Harvard University and Perimeter Institute

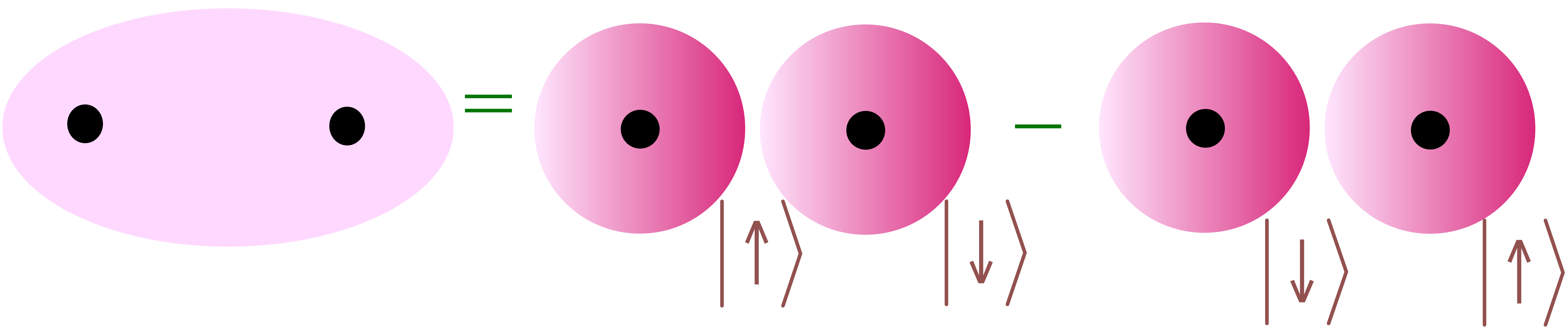
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

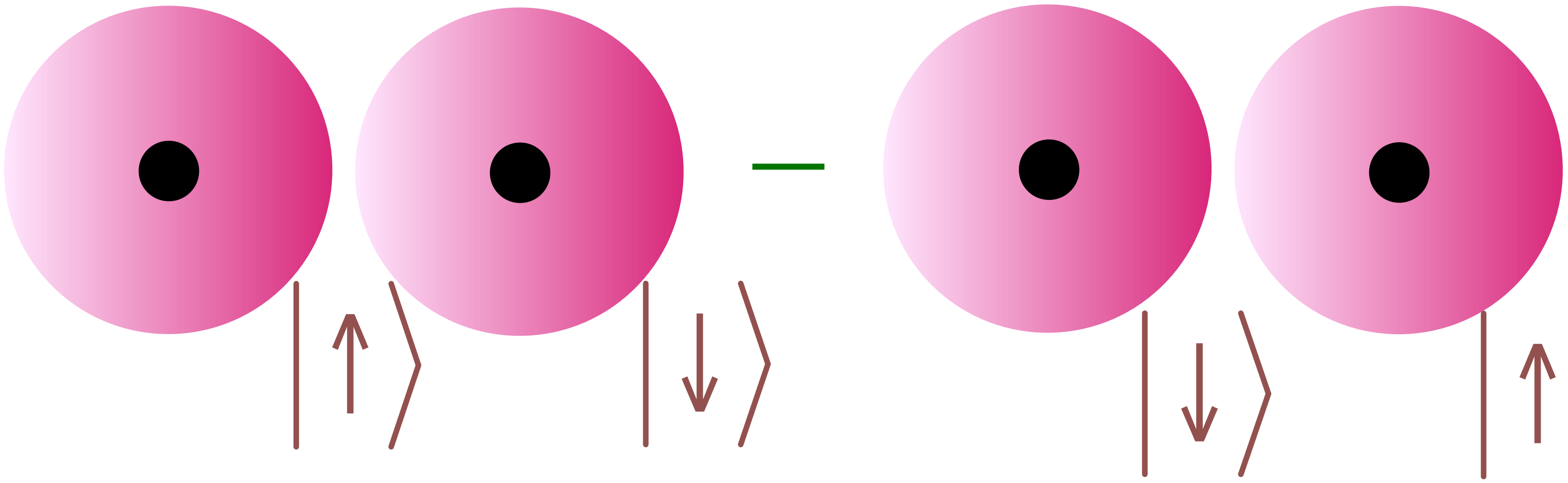


Hydrogen molecule:

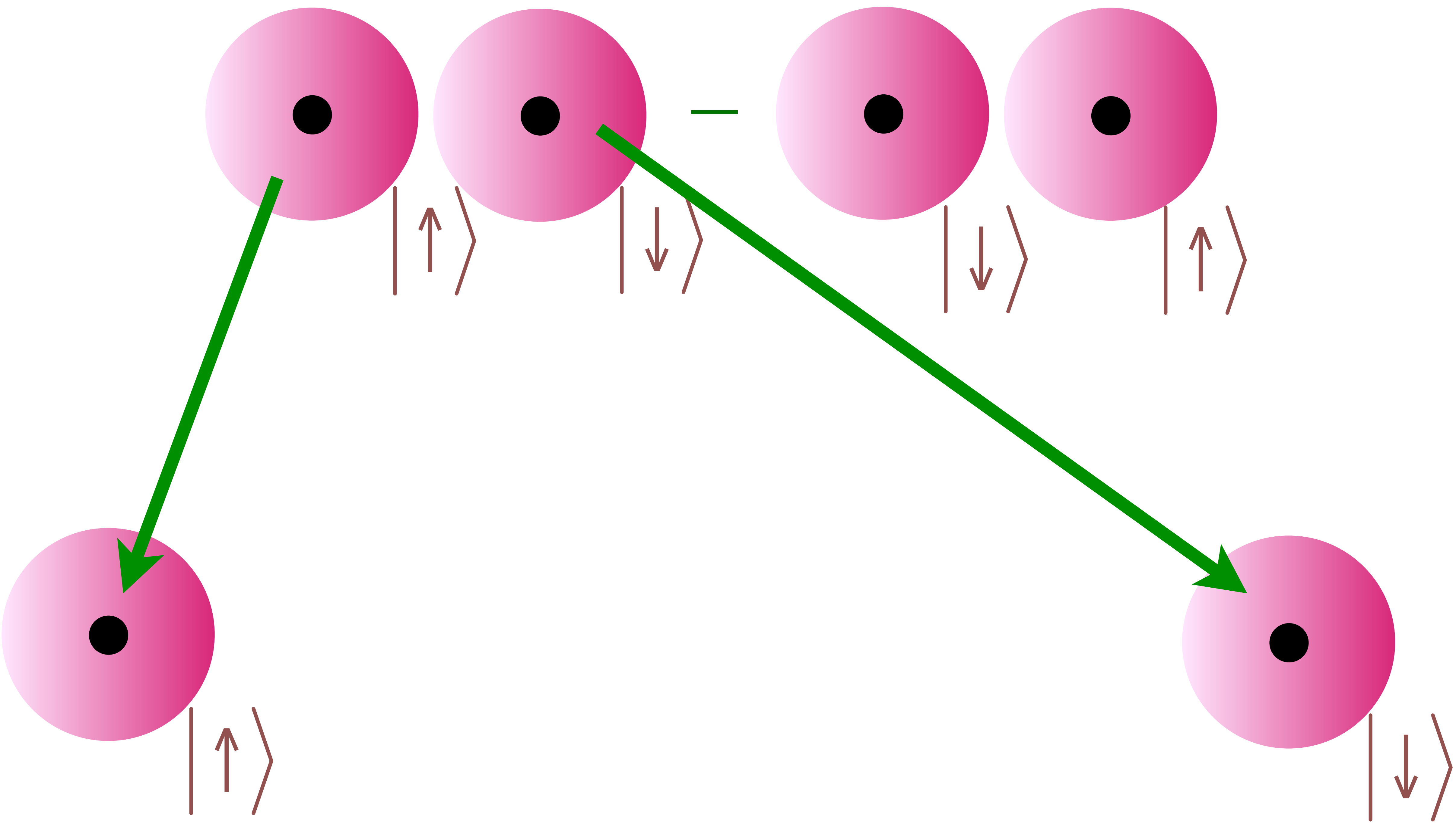


$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

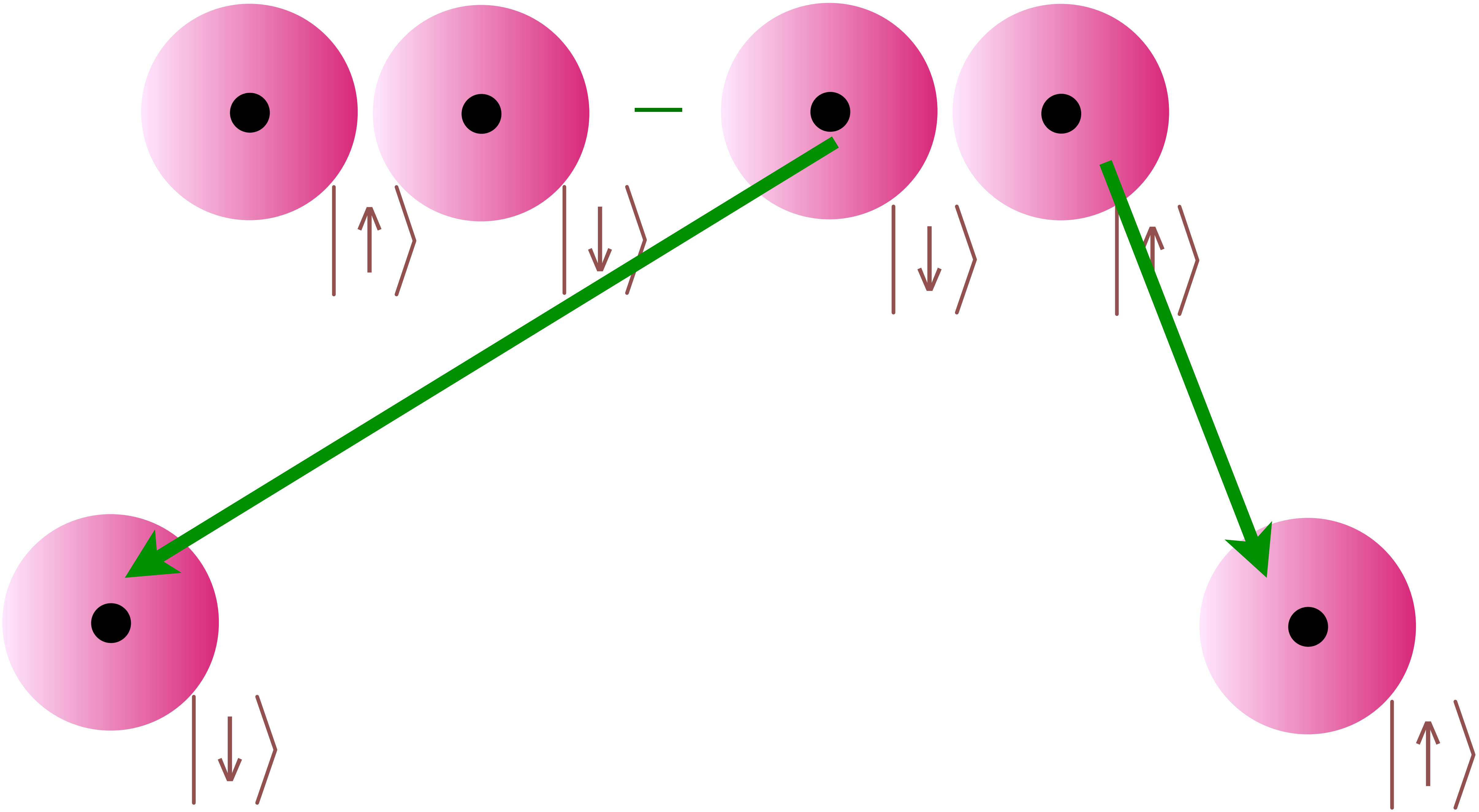
Quantum Entanglement: quantum superposition with more than one particle



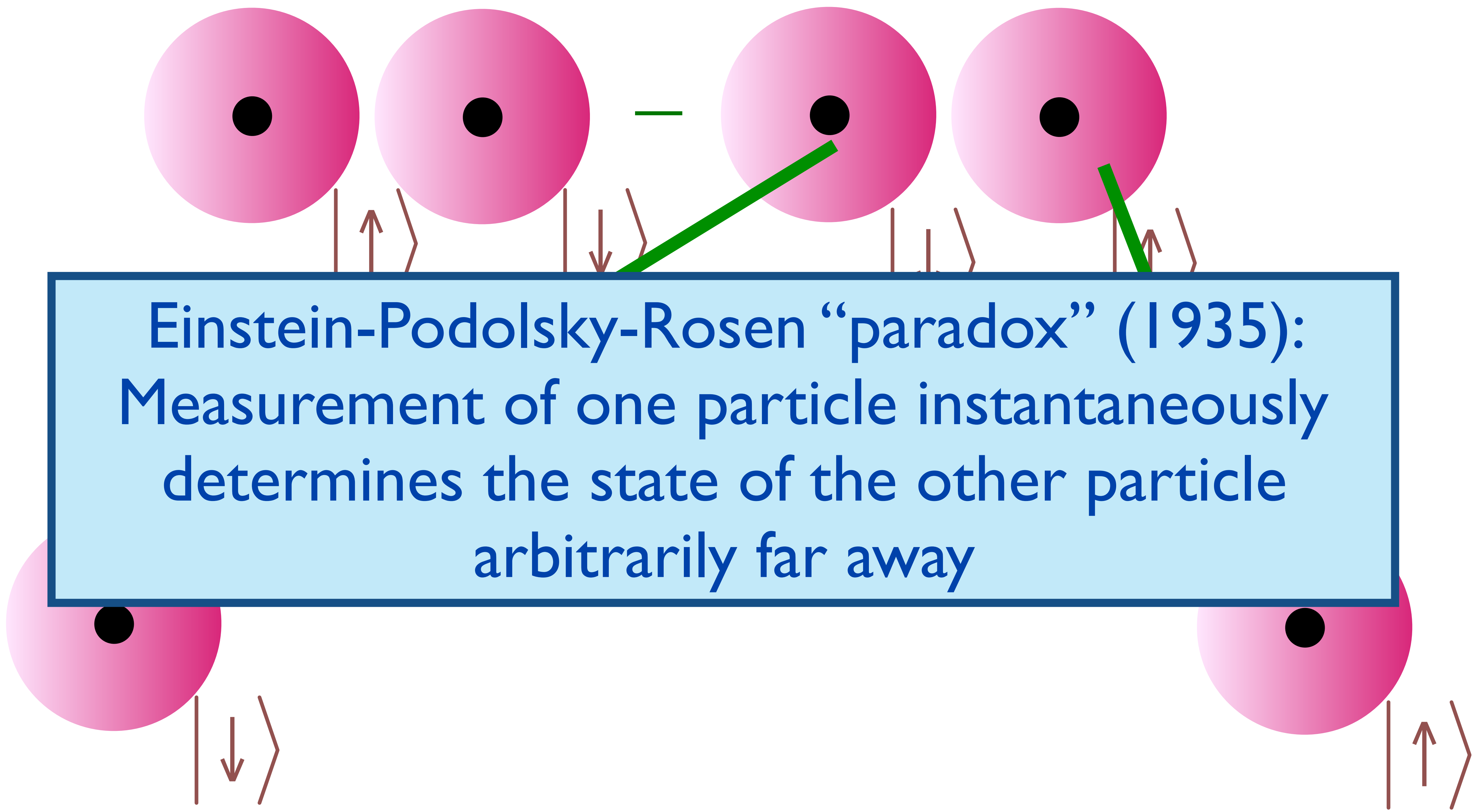
Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



Quantum Entanglement: quantum superposition with more than one particle



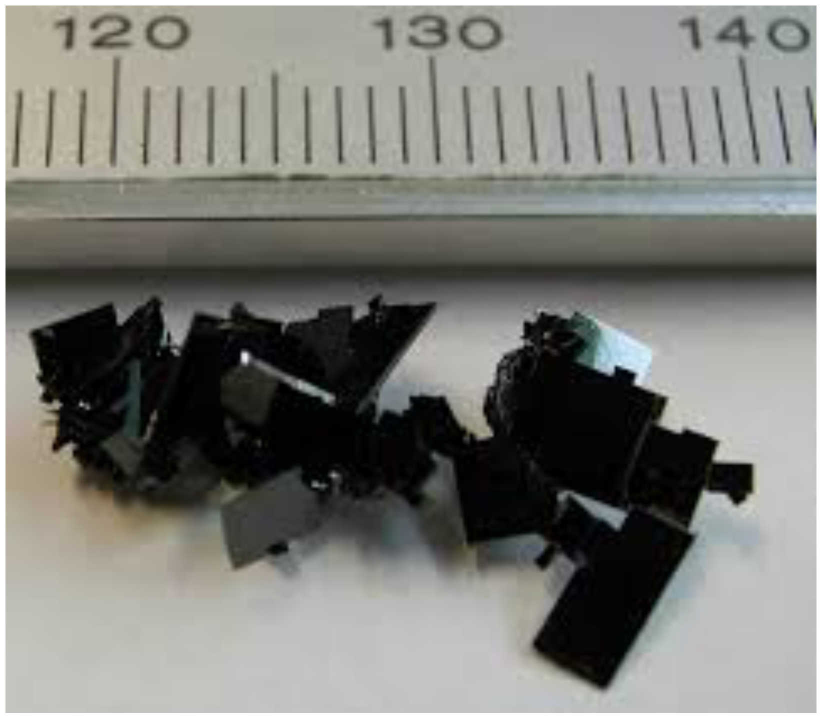
Einstein-Podolsky-Rosen "paradox" (1935):
Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

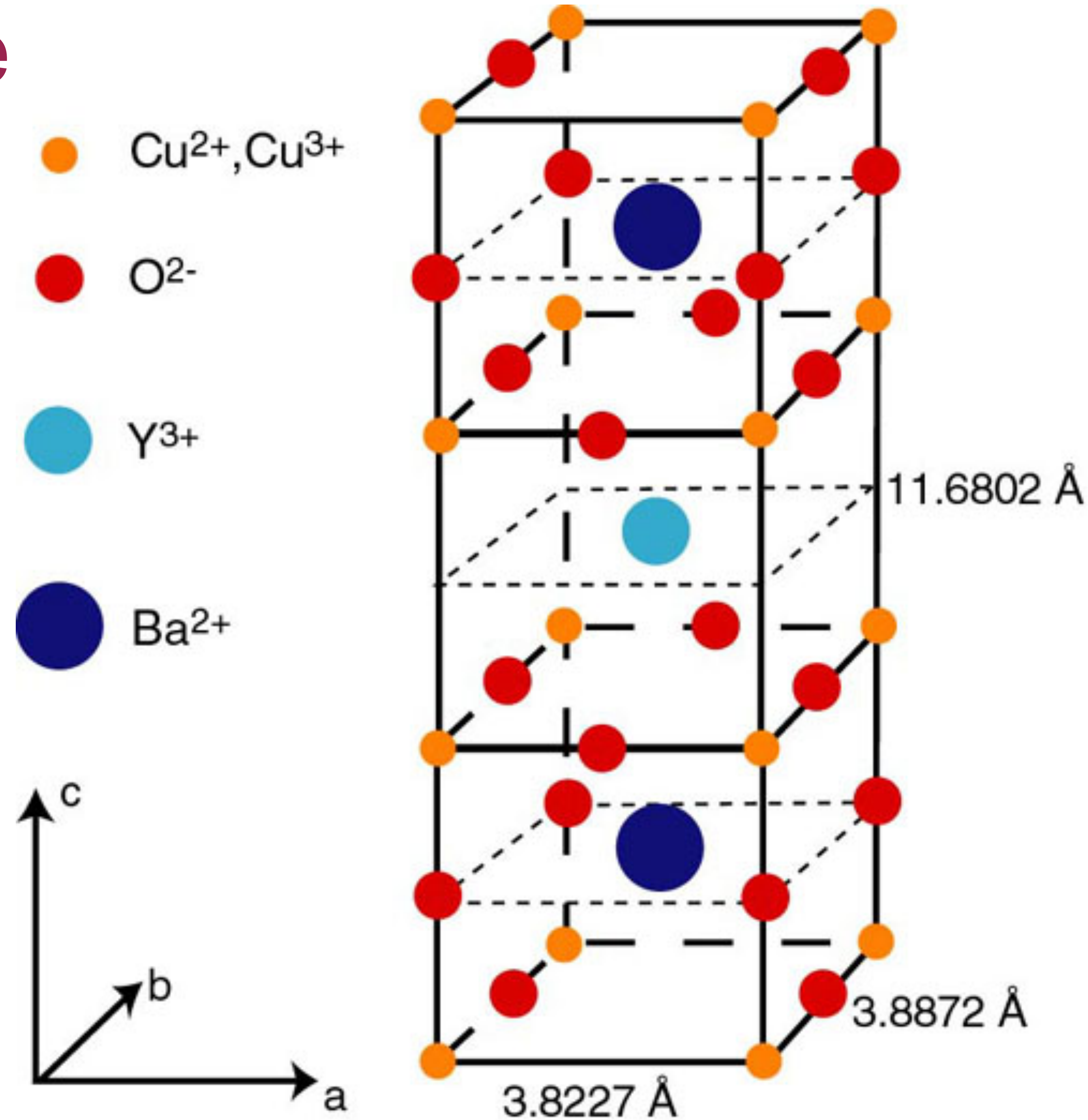
Quantum entanglement

Quantum entanglement

Strange metals

High temperature superconductors



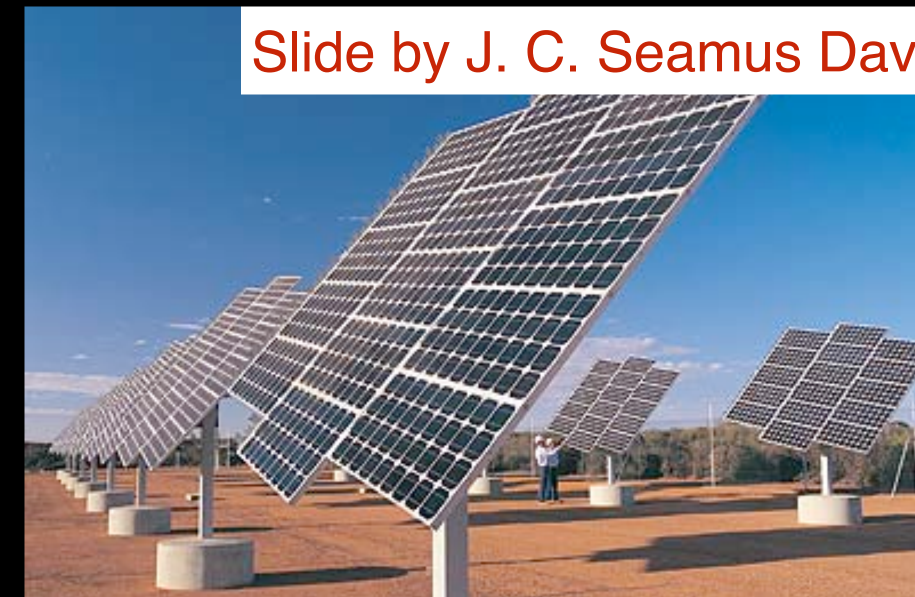


$$YBa_2Cu_3O_{6+x}$$

Slide by J. C. Seamus Davis

Power Efficiency/Capacity/Stability

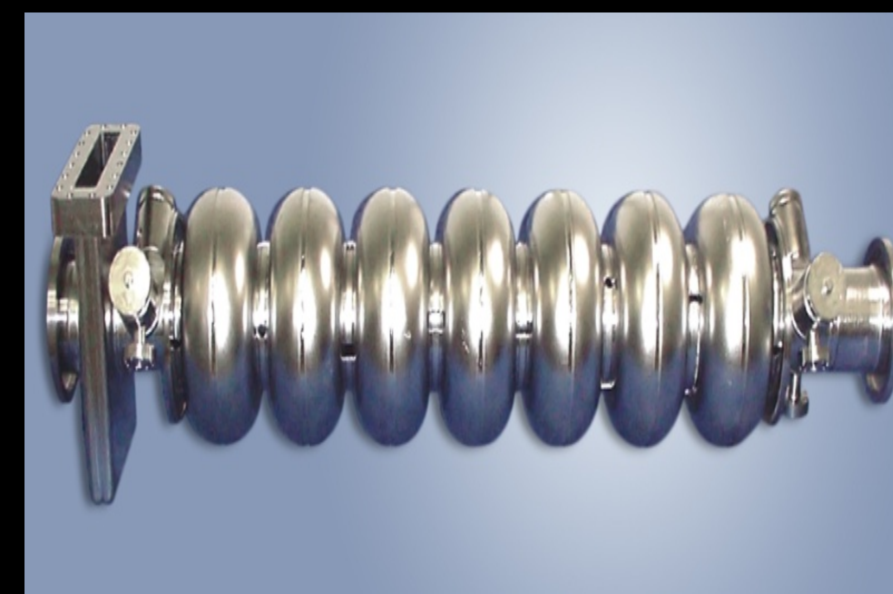
Power Bottlenecks



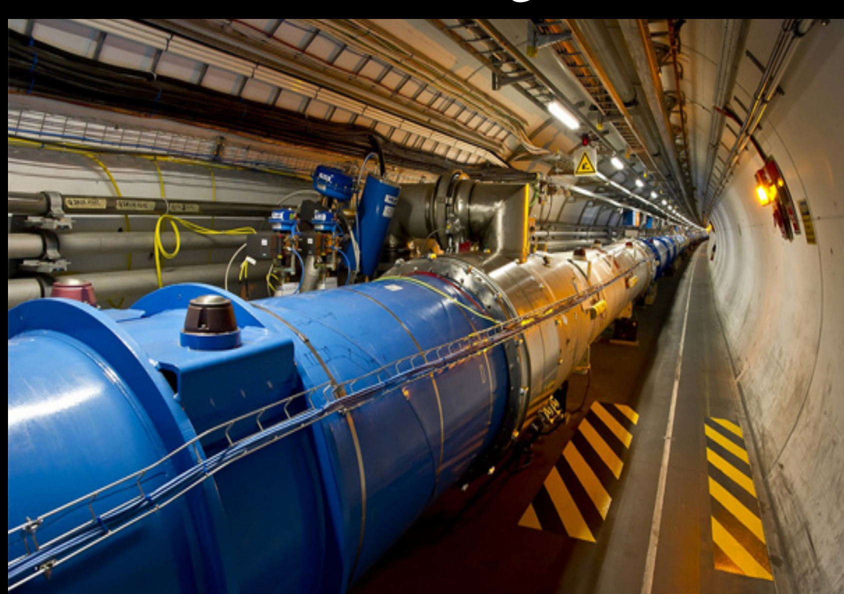
Accommodate Renewable Power

Efficient Rotating Machines

Information Technology



Next Generation HEP

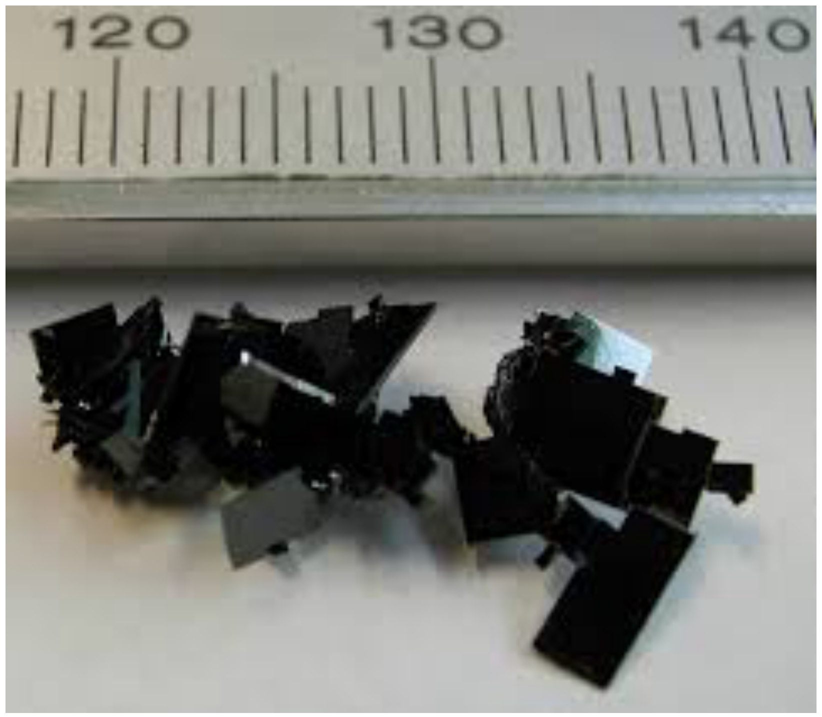


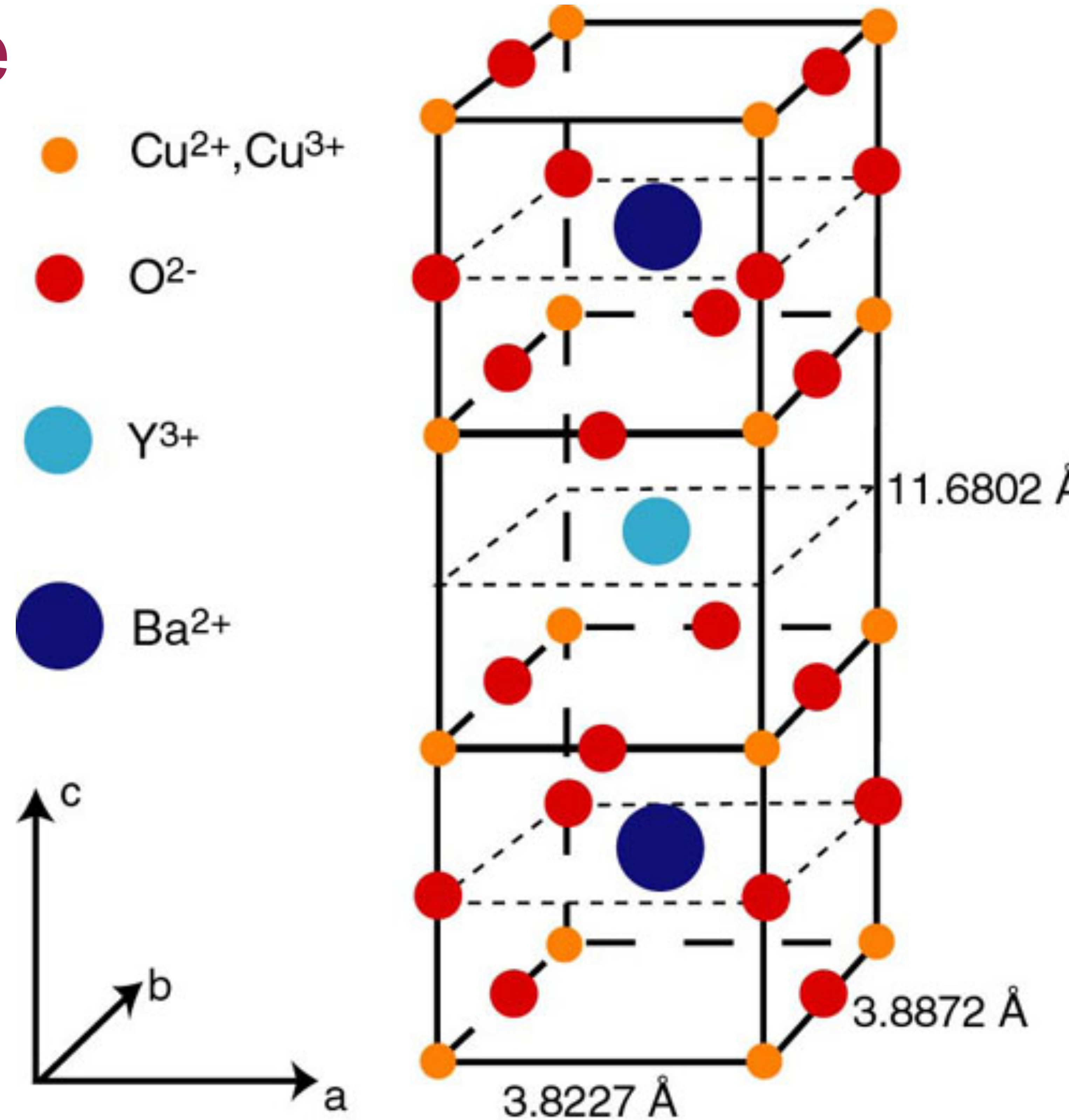
Ultra-High Magnetic Fields

Medical

Transport

High temperature superconductors





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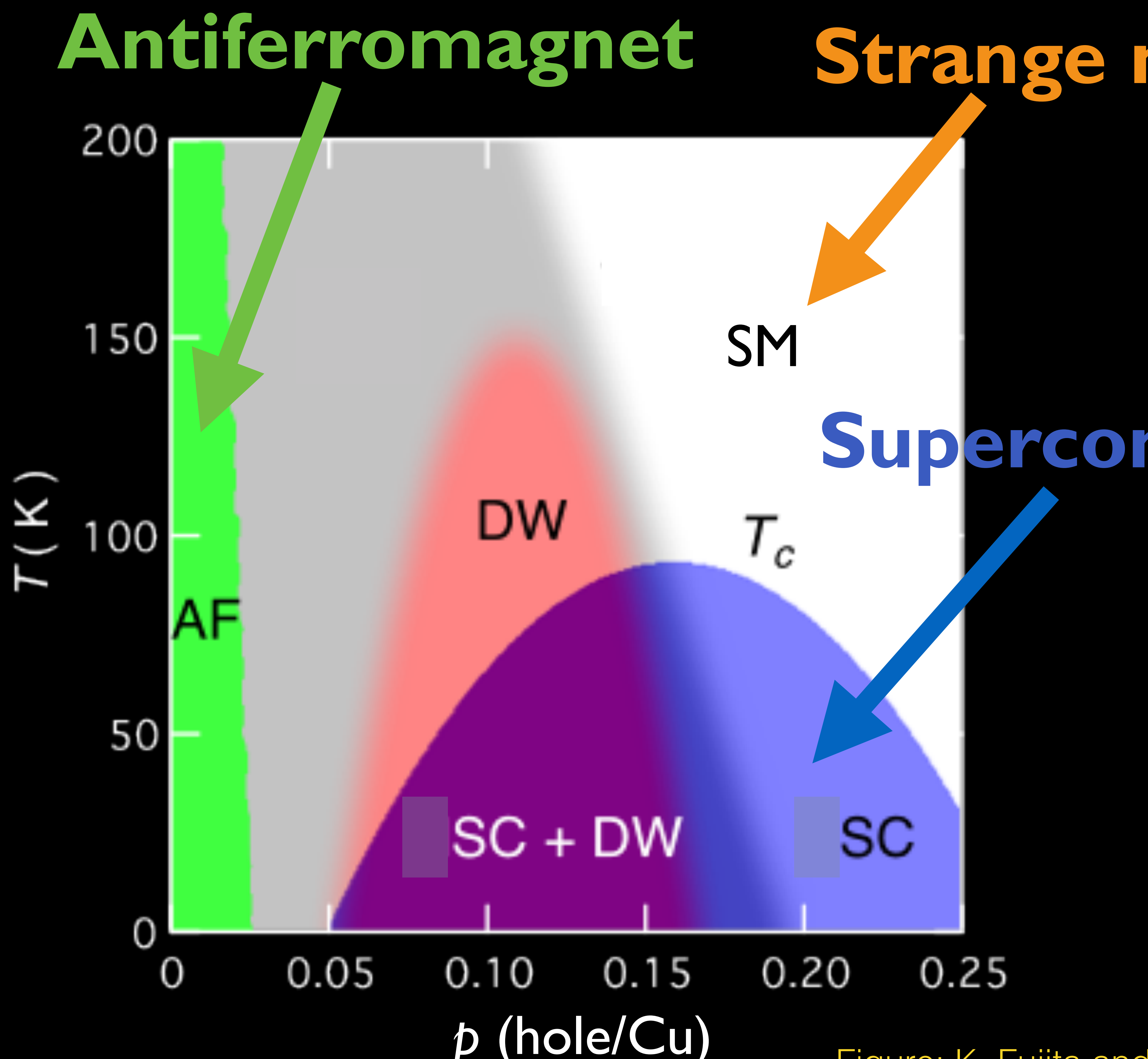


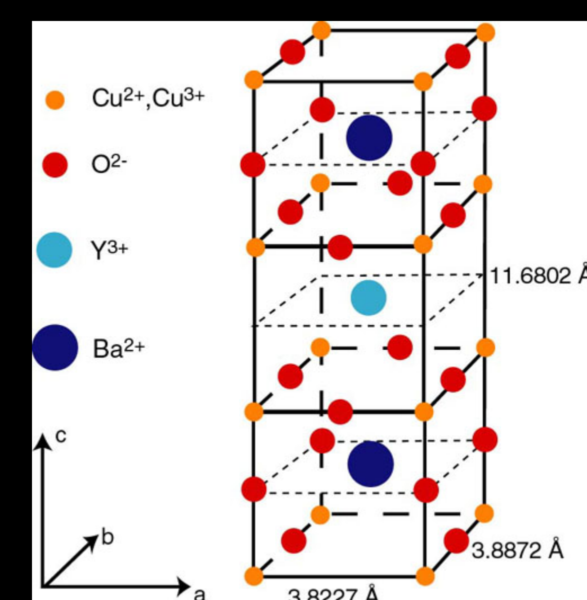
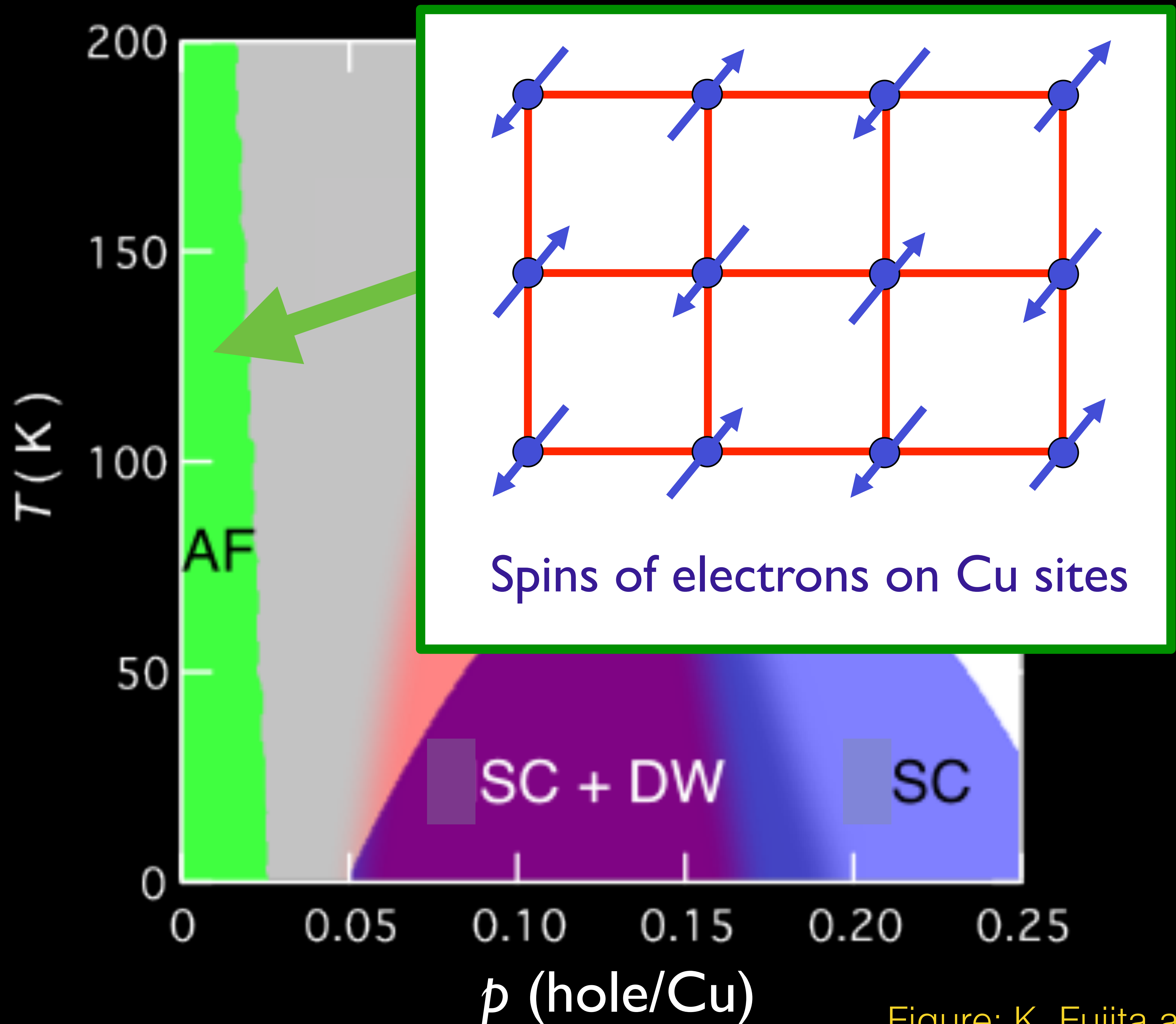


Figure: K. Fujita and J. C. Seamus Davis

Antiferromagnet



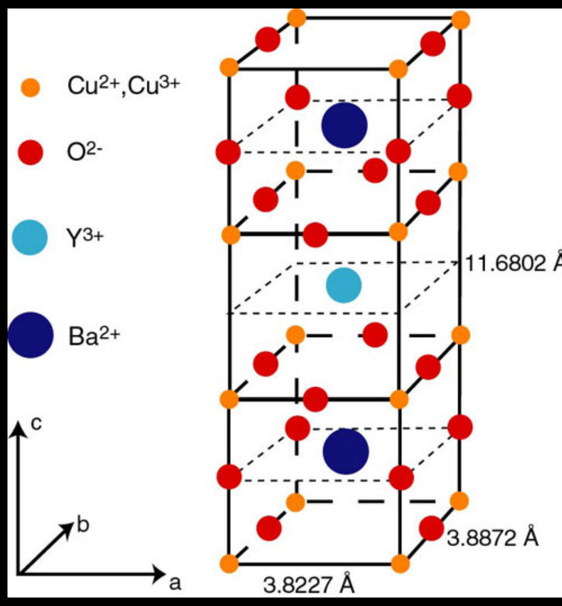
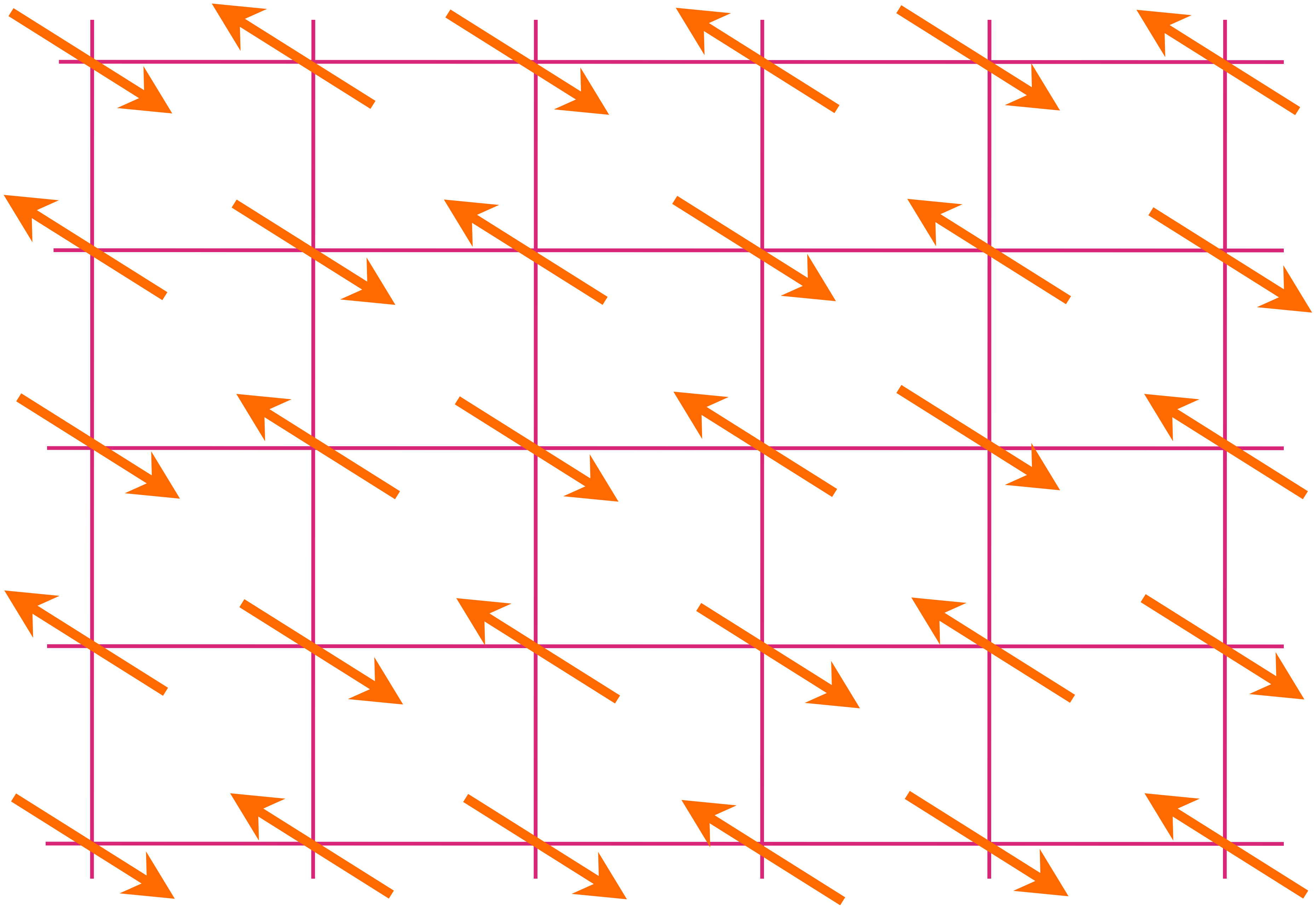


Figure: K. Fujita and J. C. Seamus Davis

Square lattice of Cu sites



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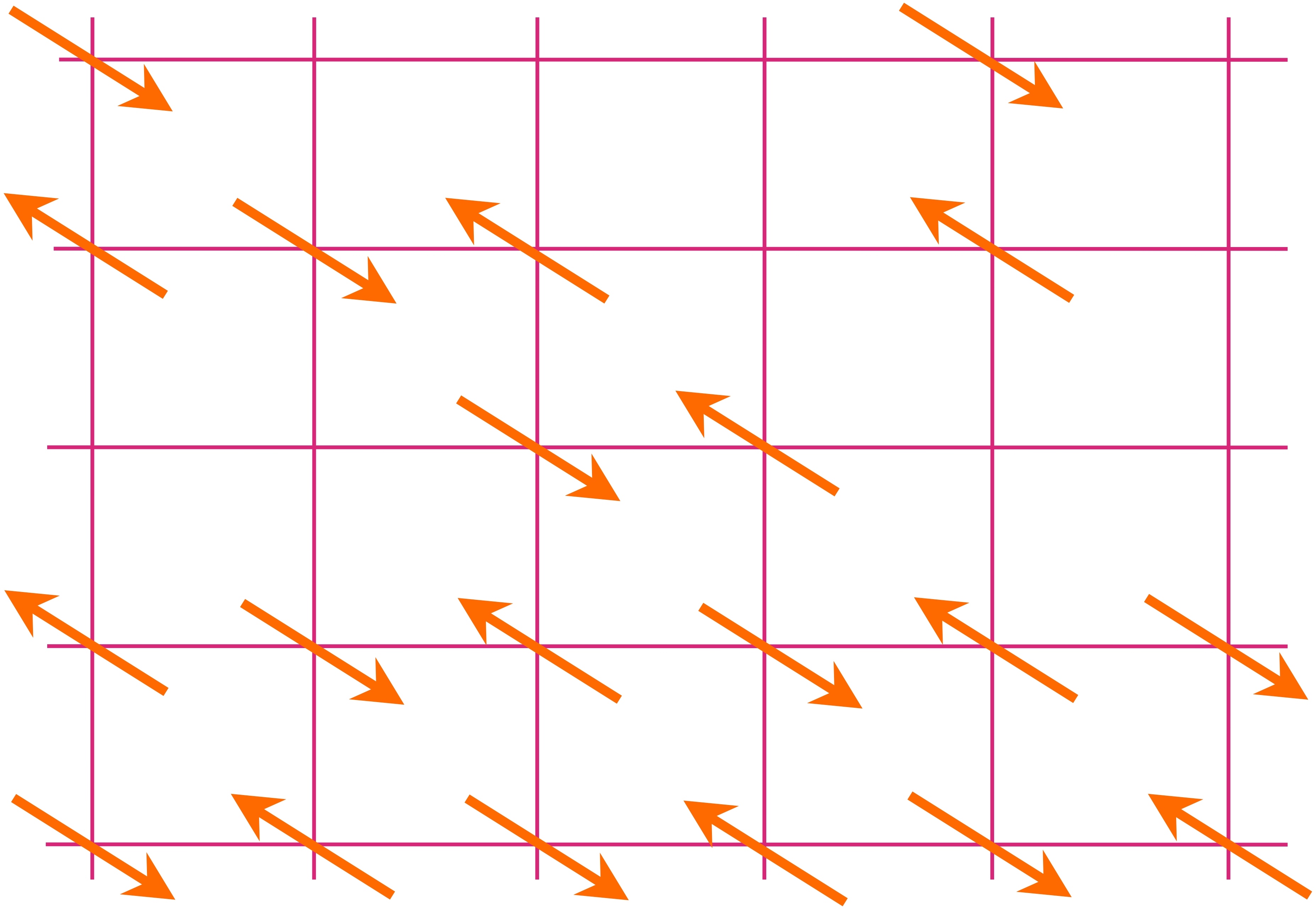
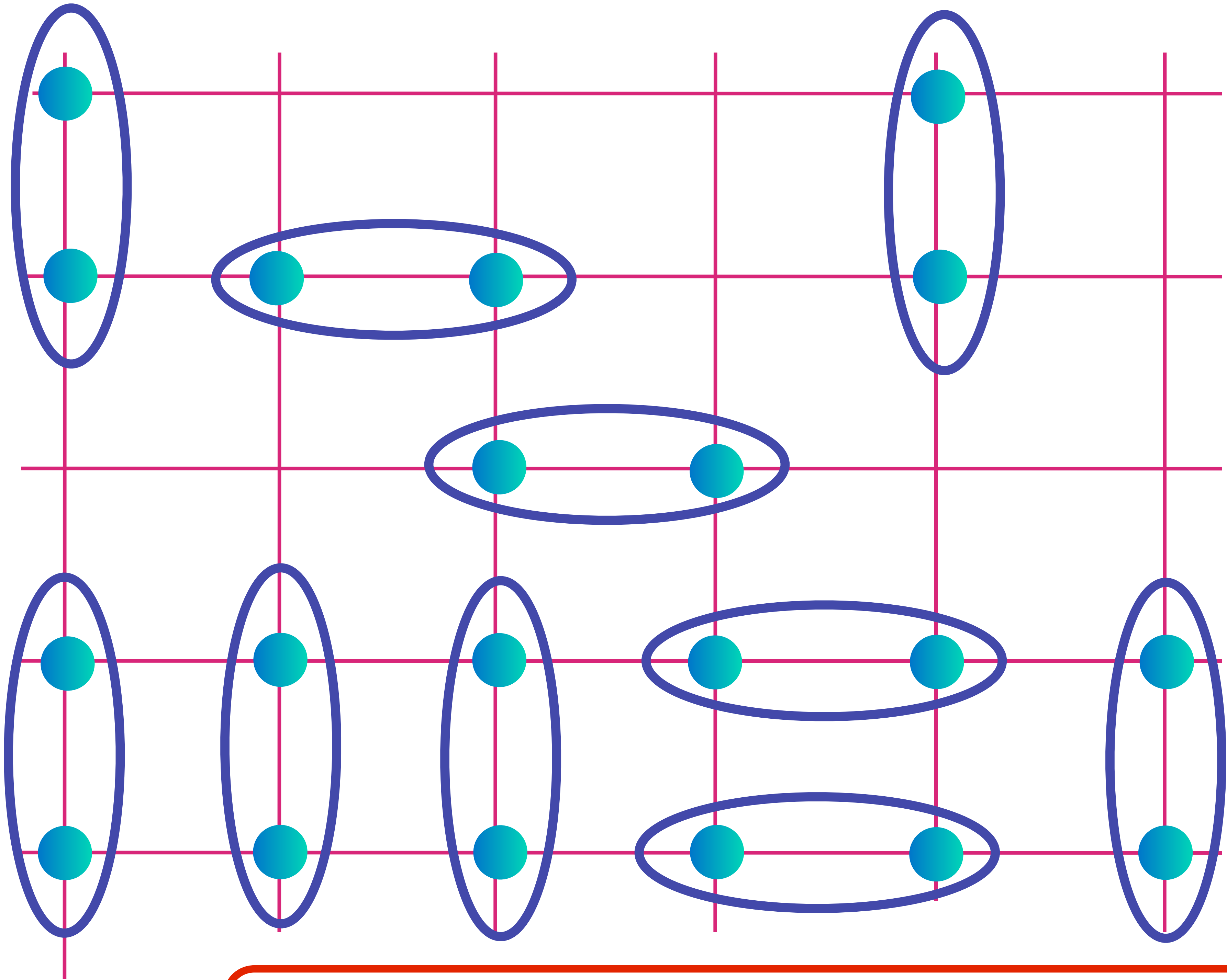
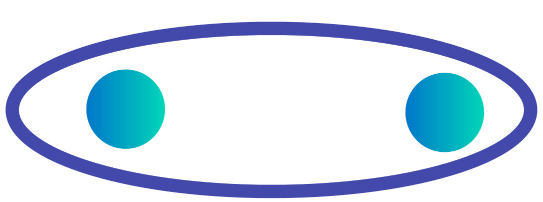


Remove density p electrons

Square lattice of Cu sites



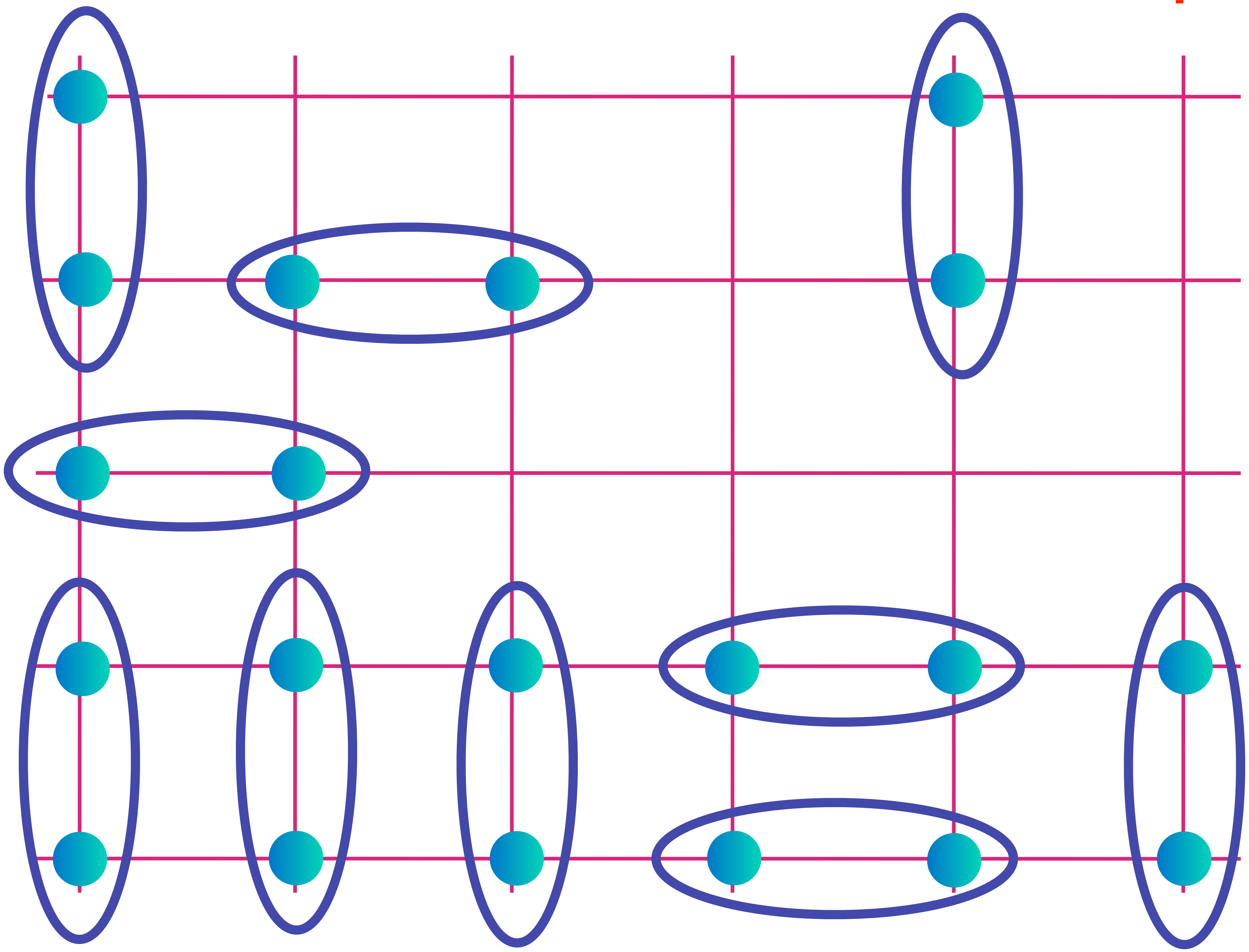
Electrons entangle in ("Cooper") pairs into chemical bonds



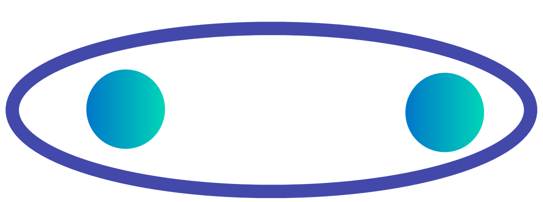
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

Superconductivity



Cooper pairs form quantum superpositions at different locations: "Bose-Einstein condensation" in which all pairs are "everywhere at the same time"

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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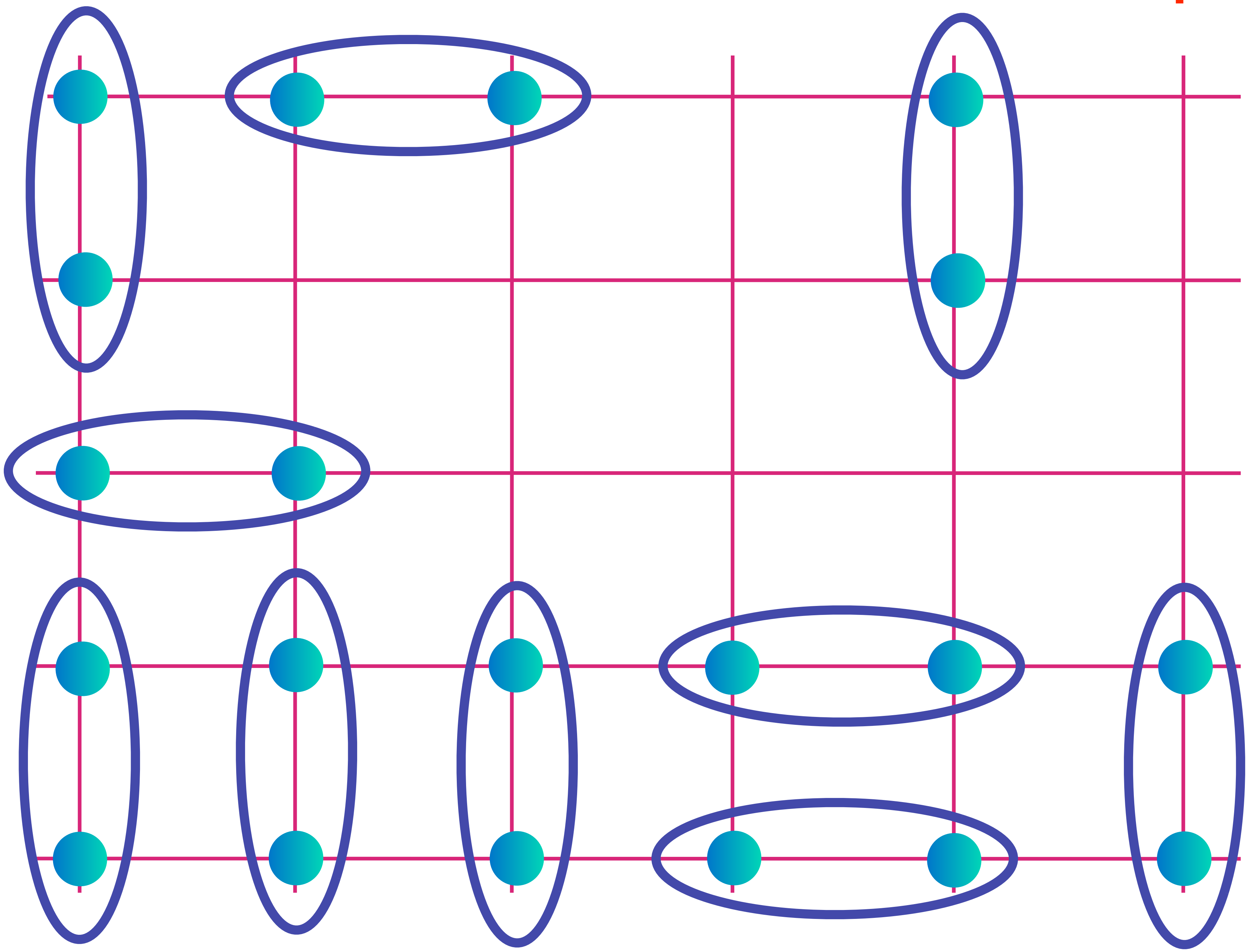

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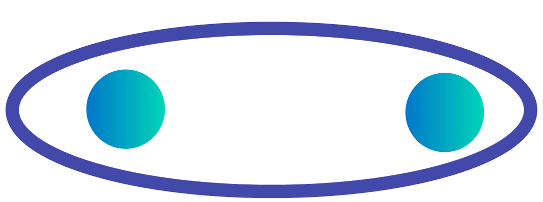

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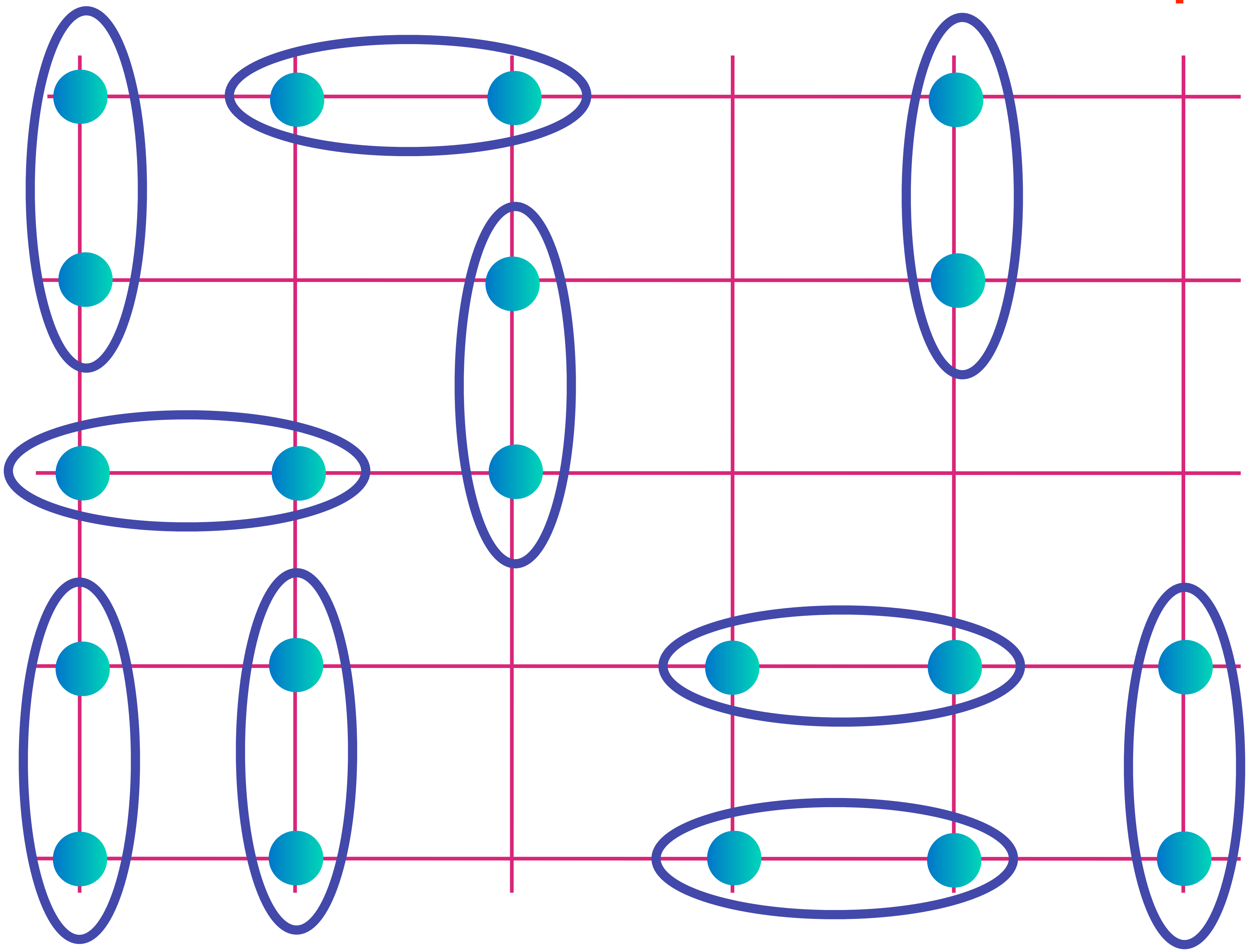

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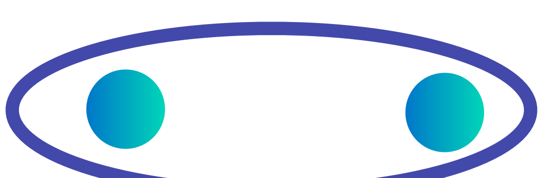 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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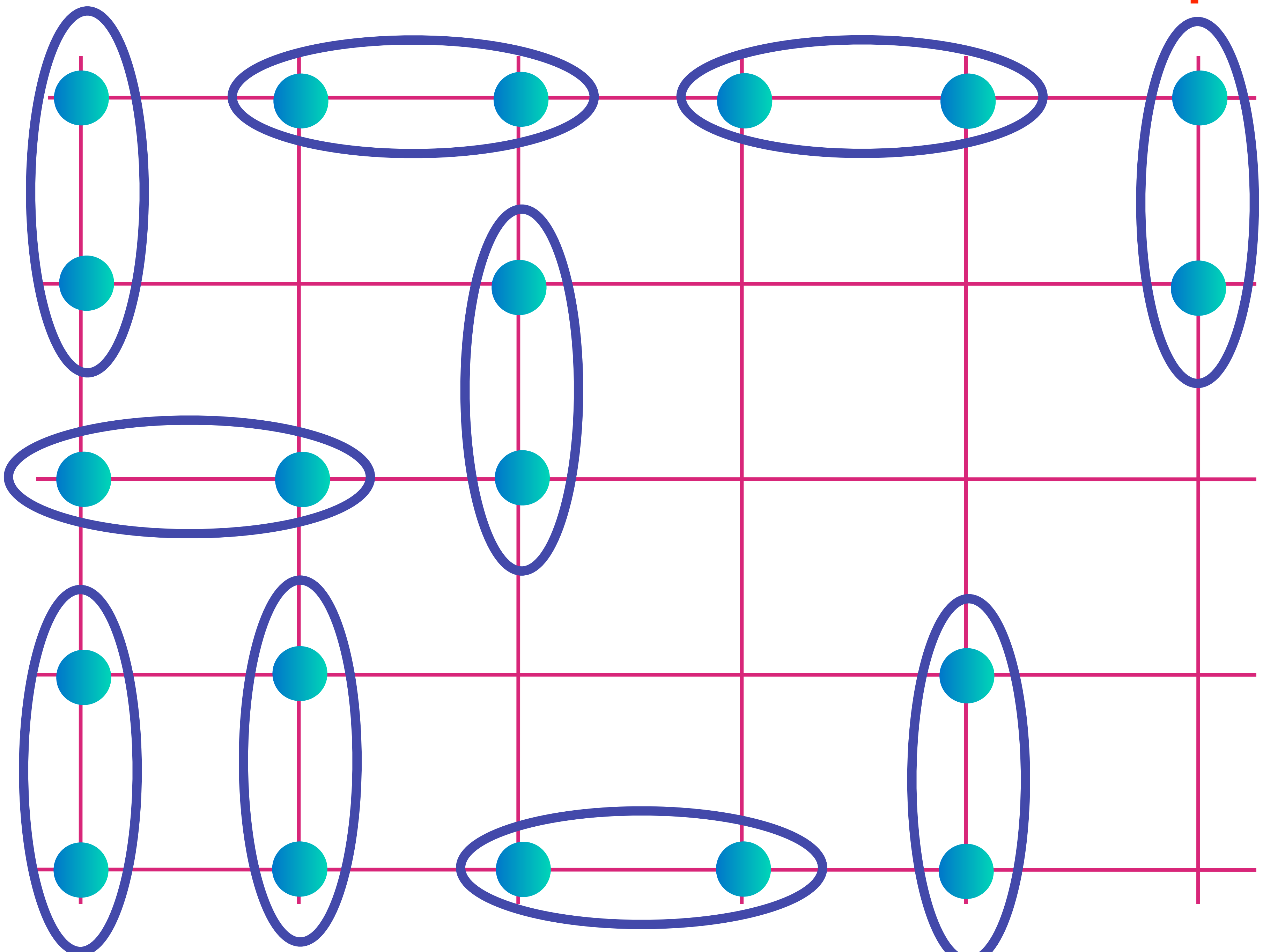

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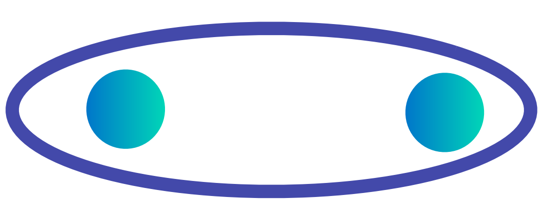
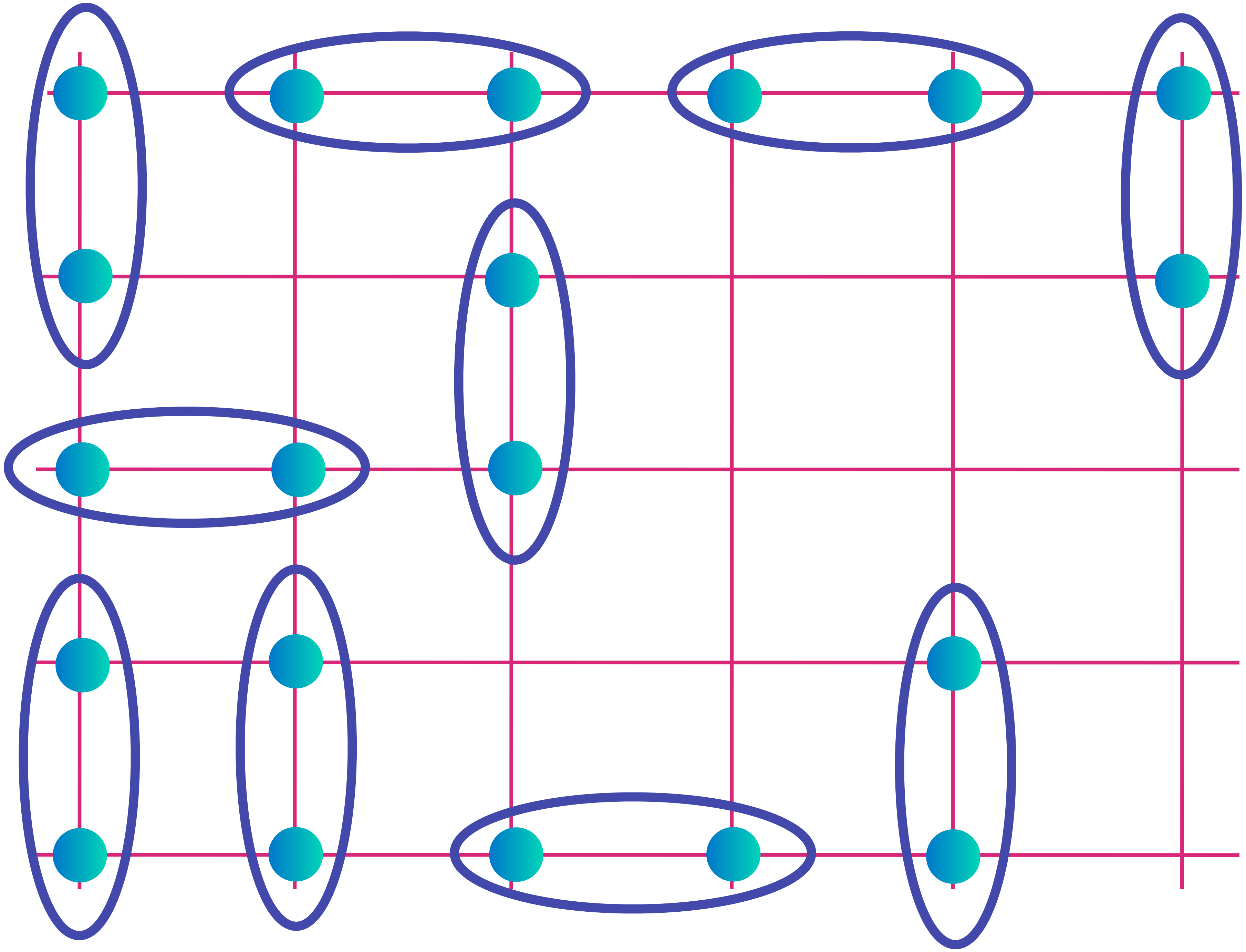
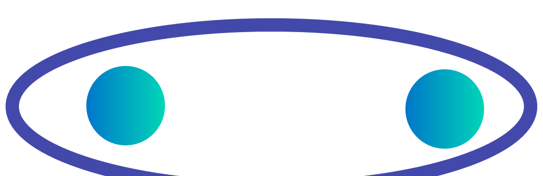
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Square lattice of Cu sites

High temperature superconductivity !



Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.



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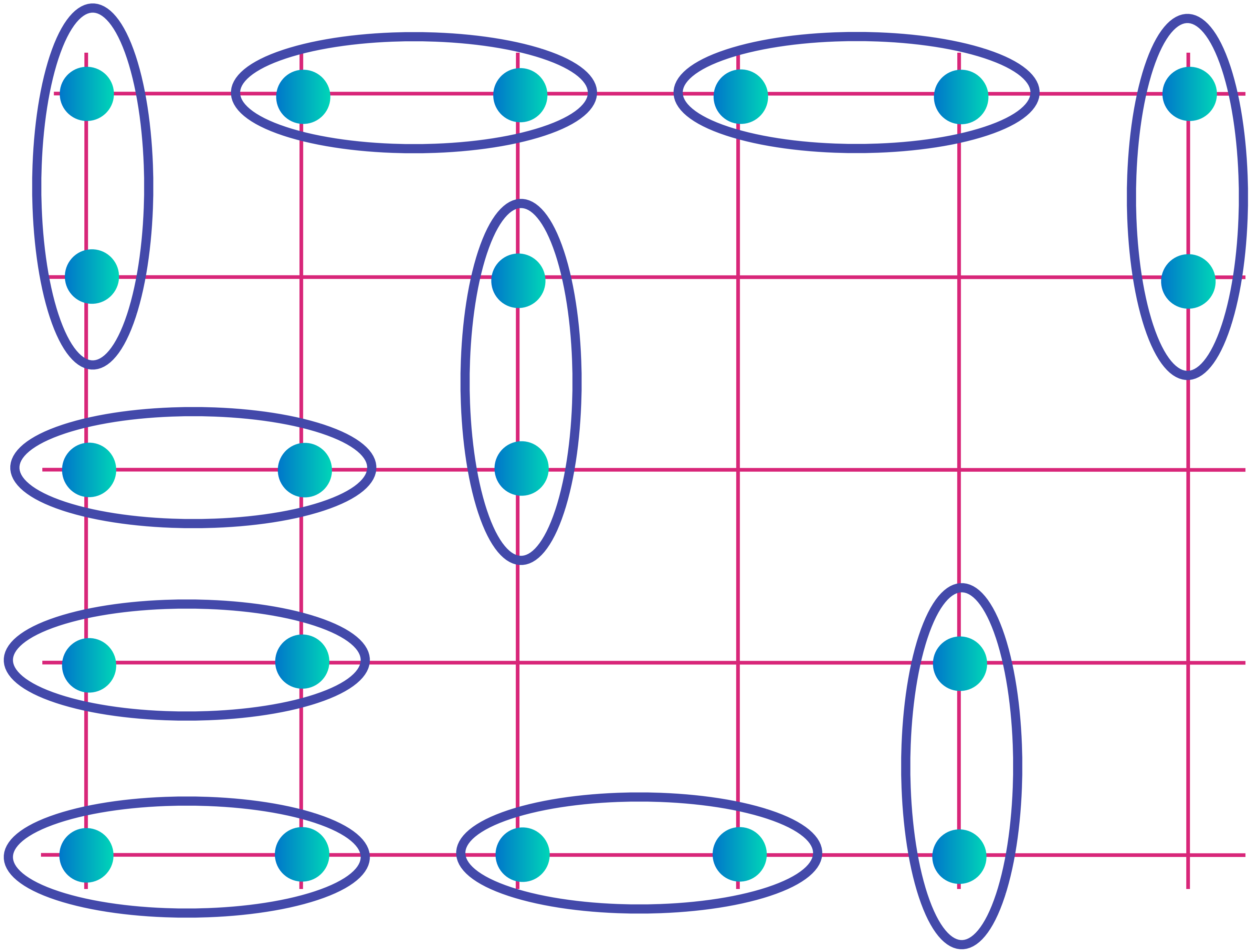

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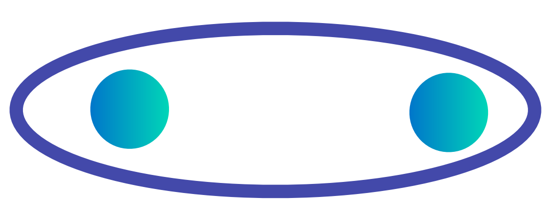

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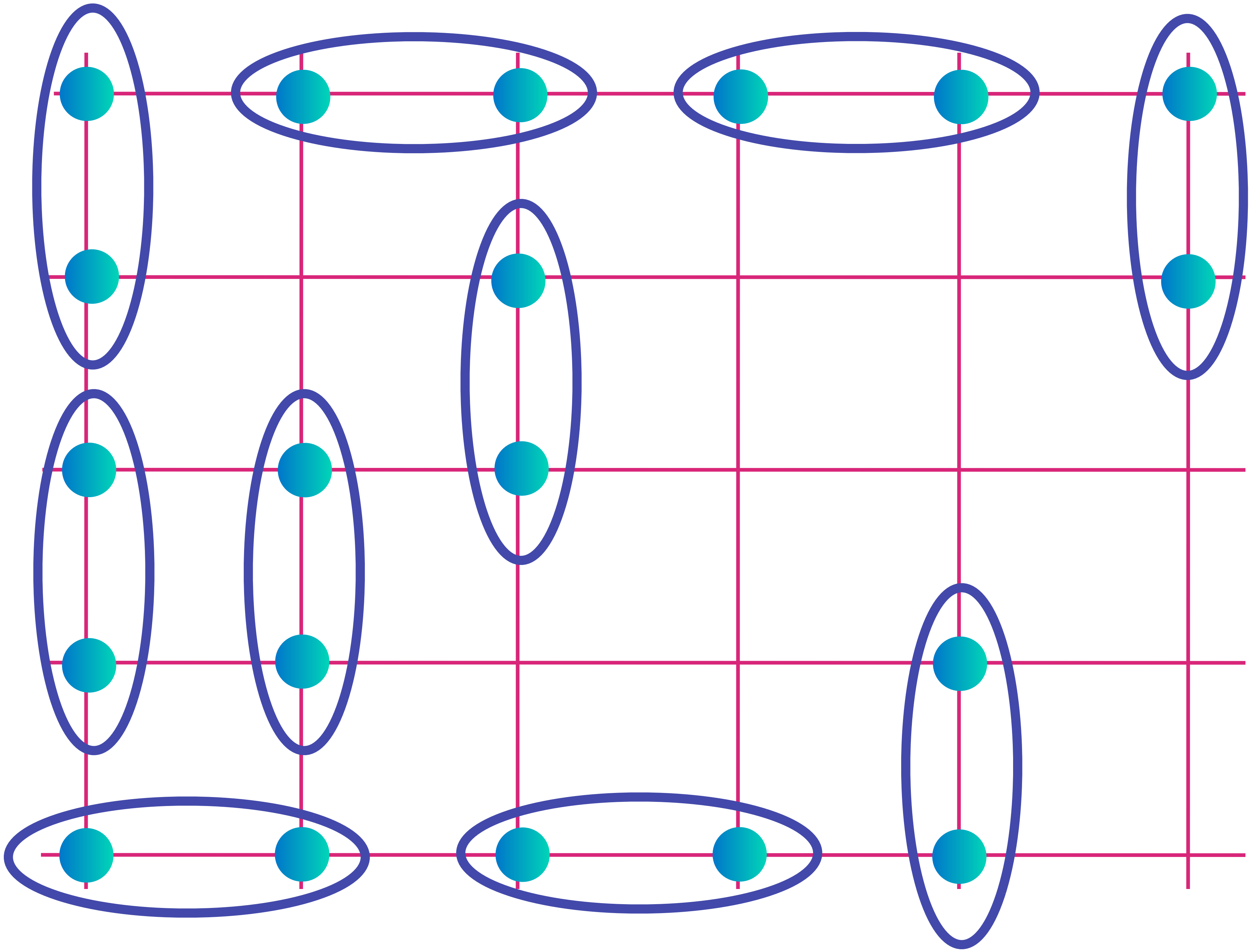

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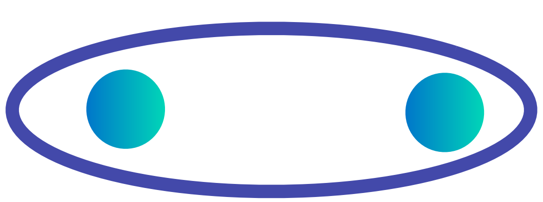

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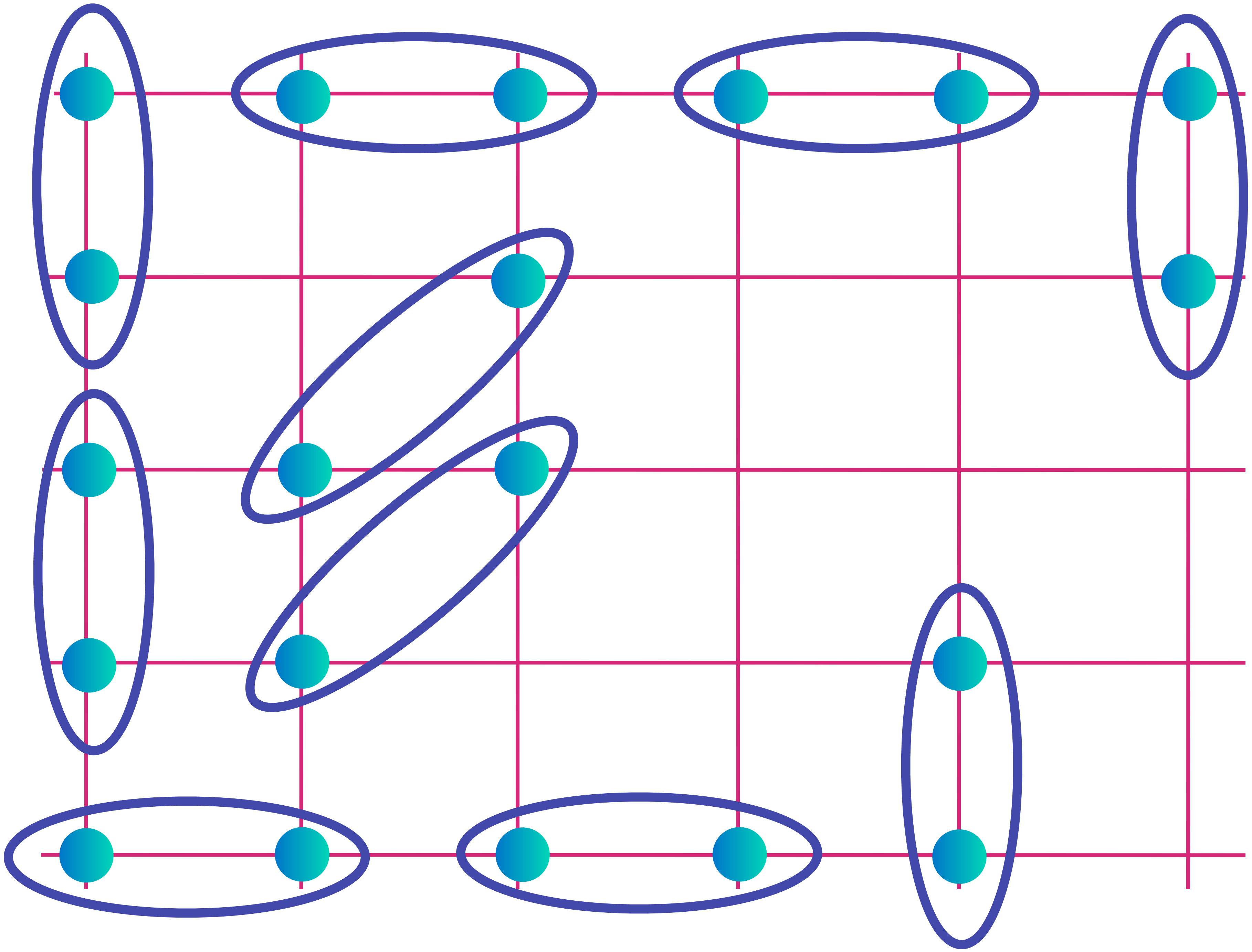

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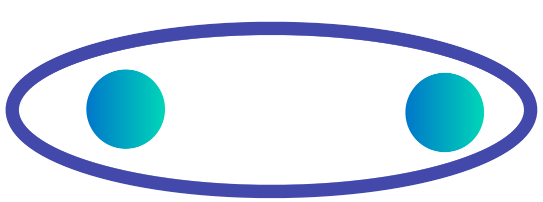

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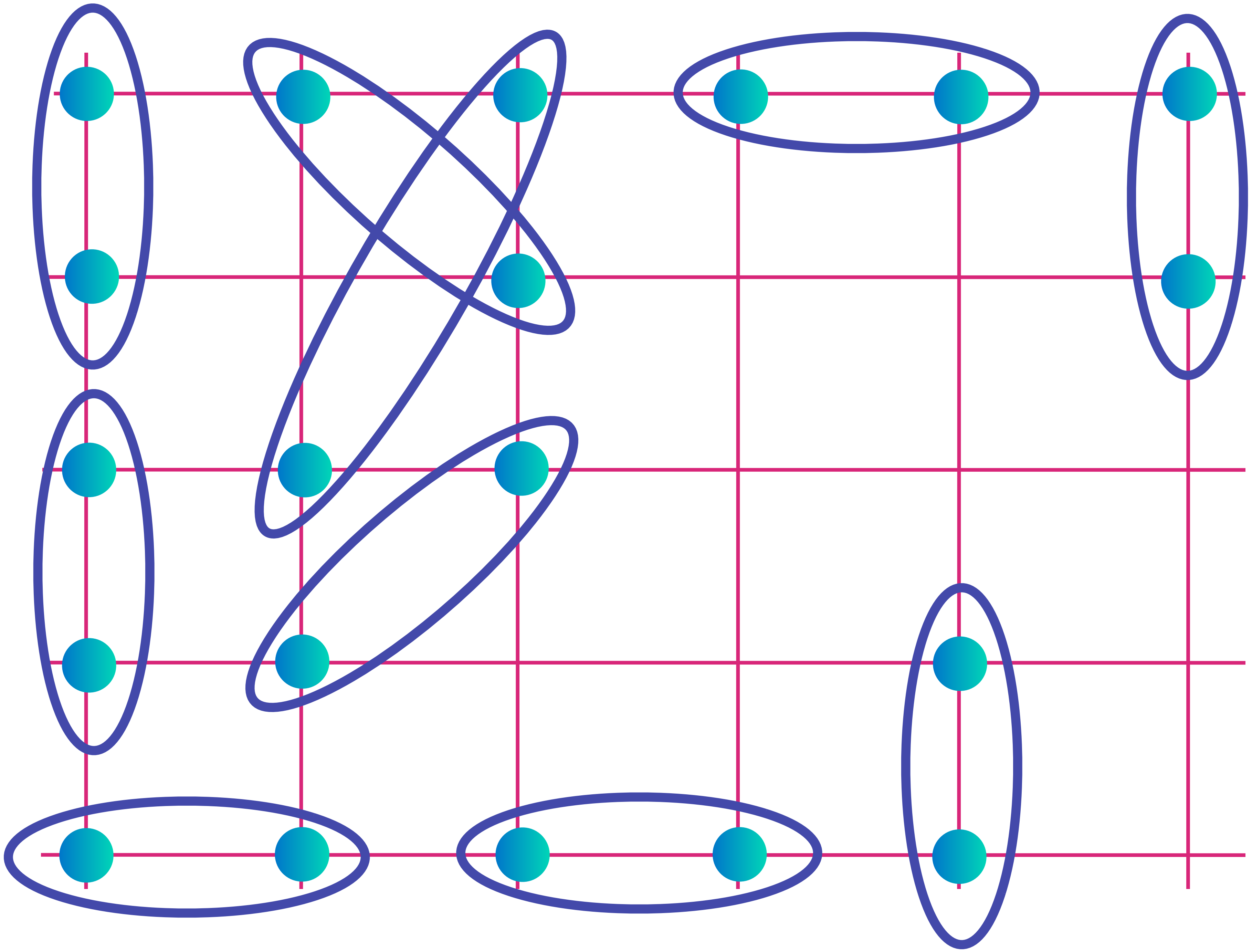

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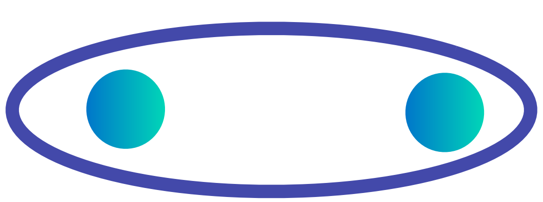

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Square lattice of Cu sites

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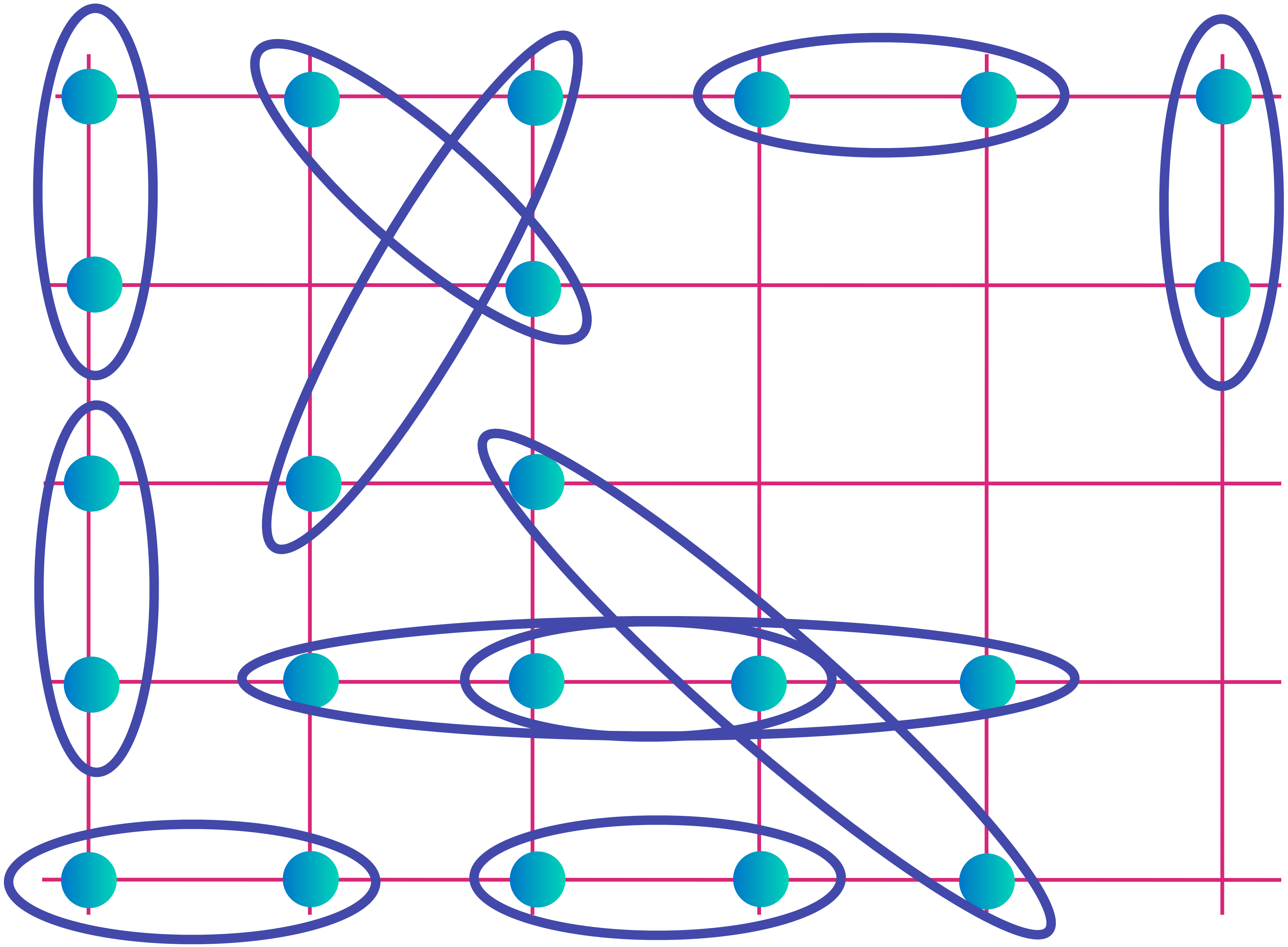

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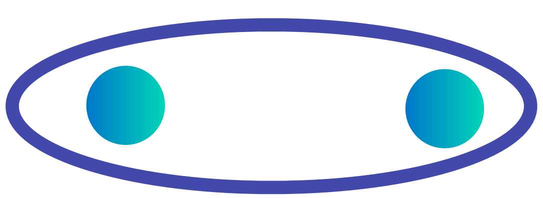

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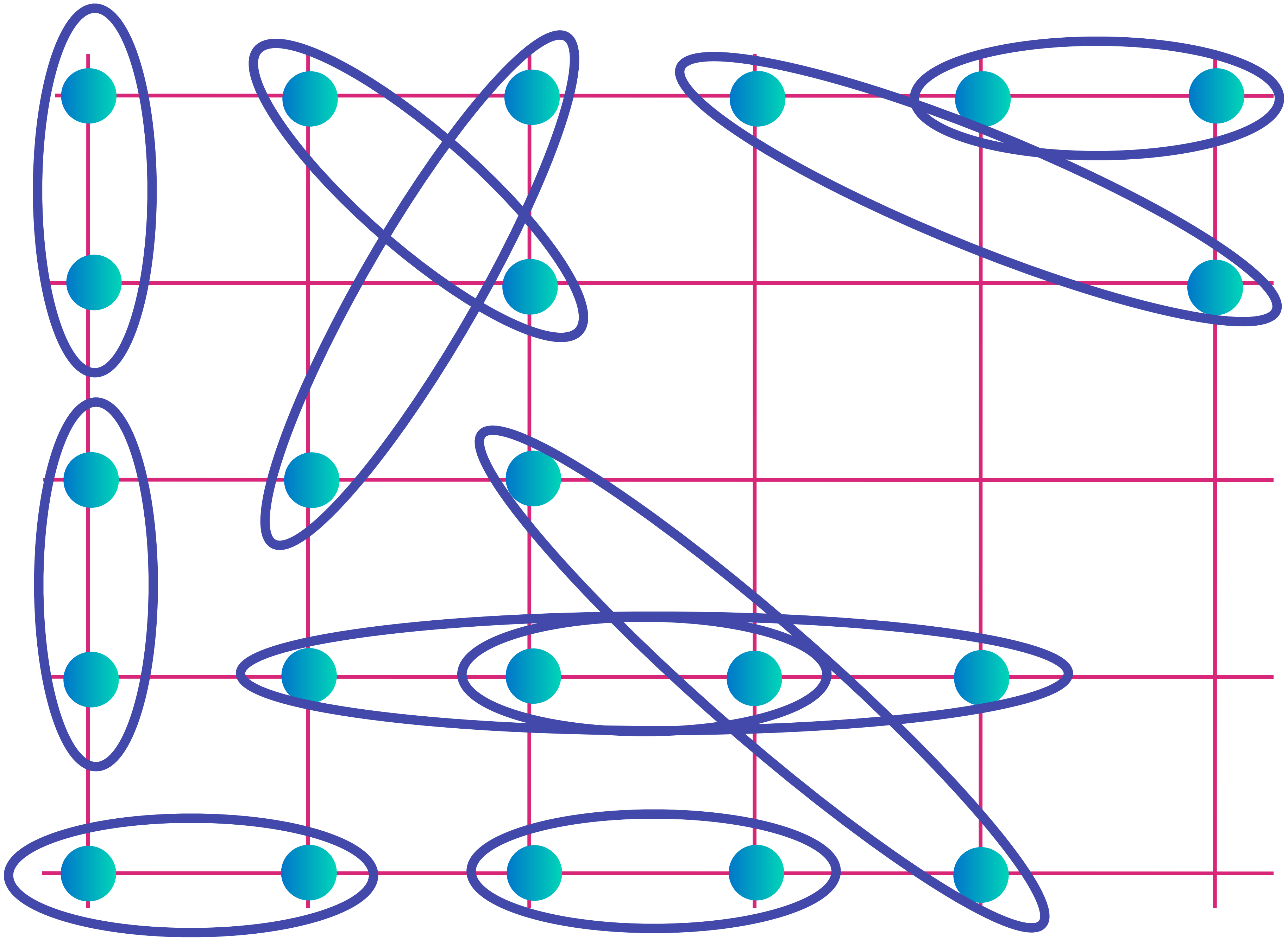

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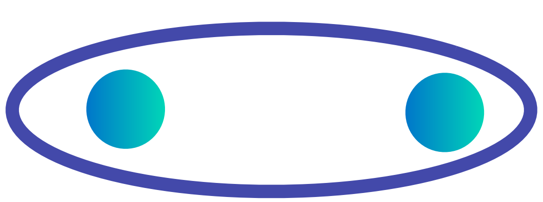

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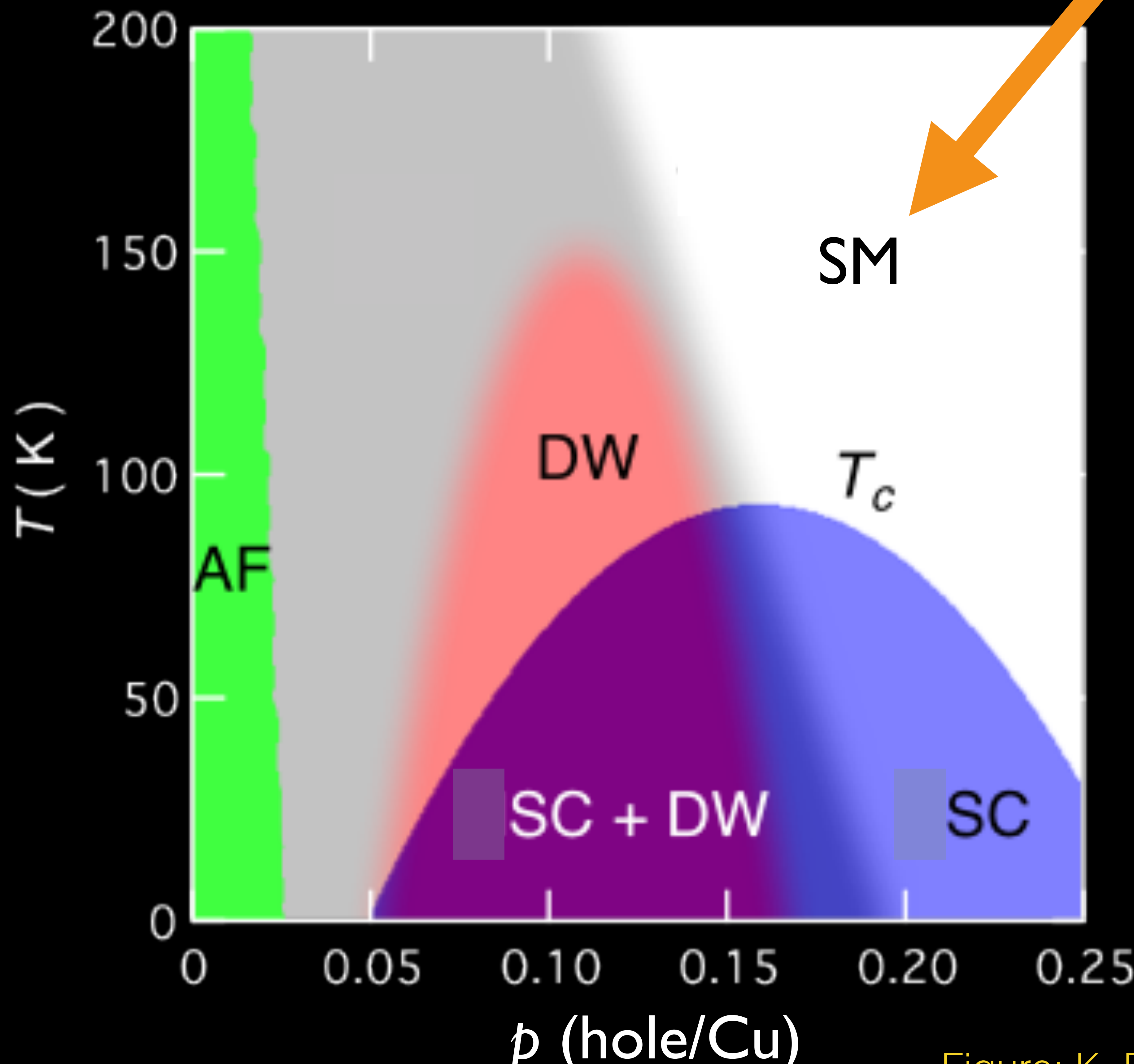
Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties

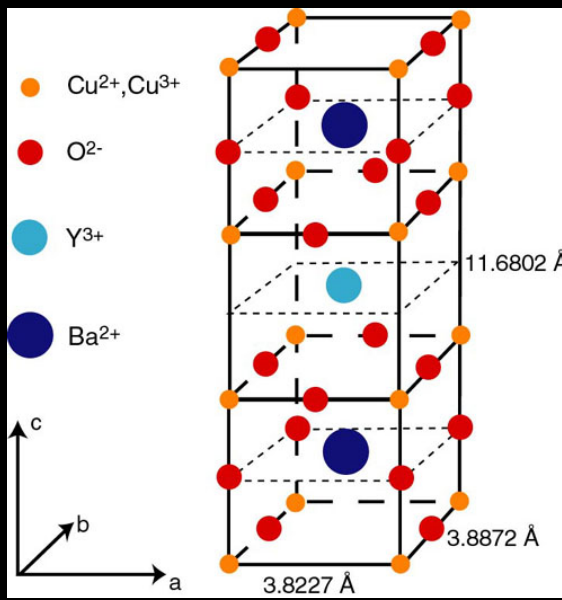
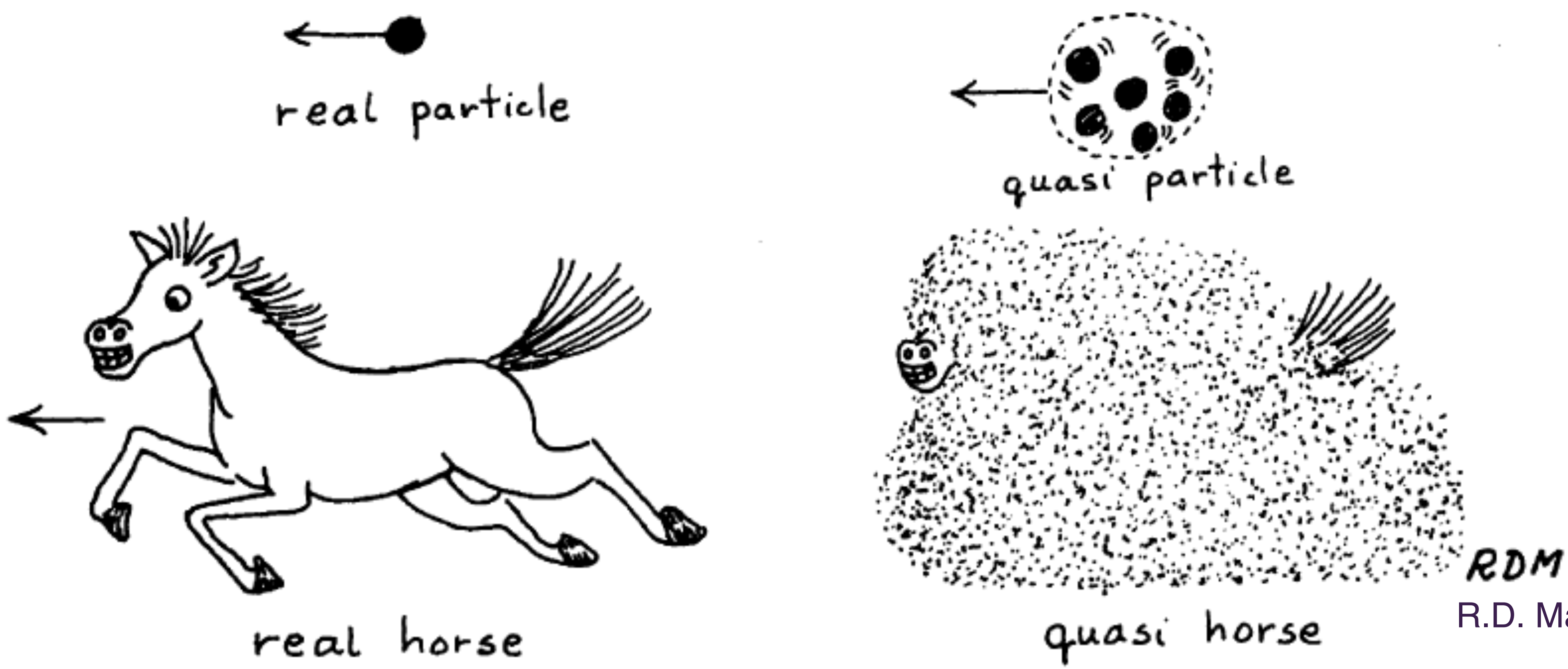


Figure: K. Fujita and J. C. Seamus Davis

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order $\hbar E_F/(k_B T)^2$ as $T \to 0$, where E_F is the Fermi energy.

Quantum matter without quasiparticles

The complex quantum entanglement in the strange metal does not allow for any quasiparticle excitations.

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- Systems *without* quasiparticles, like the strange metal, reach quantum chaos much more quickly than those with quasiparticles.

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- There is an *lower bound* on the phase coherence time (τ_φ), and the time to many-body quantum chaos (τ_L) in all many-body quantum systems as $T \to 0$:

$$\tau_\varphi \geq C \frac{\hbar}{k_B T} \qquad \text{(SS, 1999)}$$

$$\tau_L \geq \frac{\hbar}{2\pi k_B T} \qquad \text{(Maldacena, Shenker, Stanford, 2015)}$$

So *e.g.* we cannot have $\tau_\varphi \sim \hbar/\sqrt{J k_B T}$ where J is a microscopic coupling.

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  So *e.g.* we cannot have $\tau_\varphi \sim \hbar/\sqrt{J k_B T}$ where J is a microscopic coupling.

- In the strange metal the inequalities become equalities as $T \to 0$, and the time $\hbar/(k_B T)$ influences numerous observables.

Quantum entanglement

- Strange metals have no quasiparticle description.

- Their entropy is proportional to their volume.

- They relax to local thermal equilibrium in the fastest possible time $\sim \hbar/(k_B T)$.

Strange metals

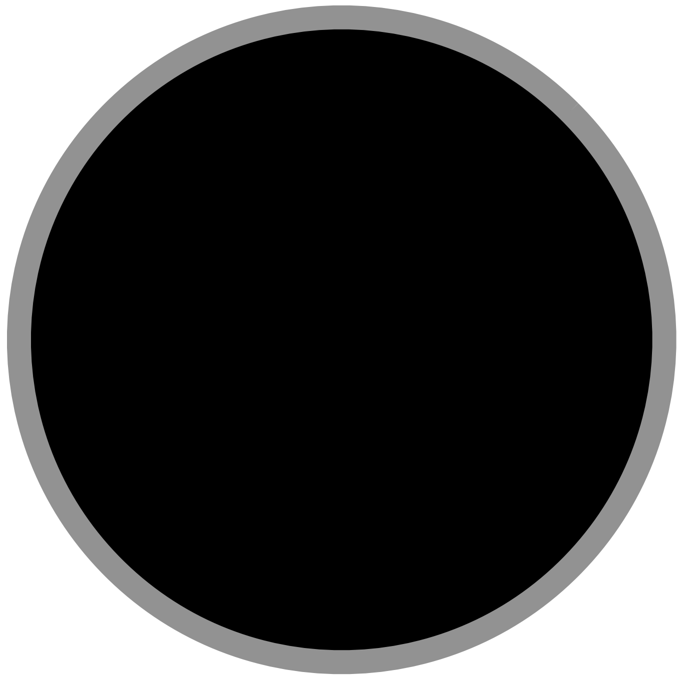
Quantum entanglement

Black holes

Black Holes

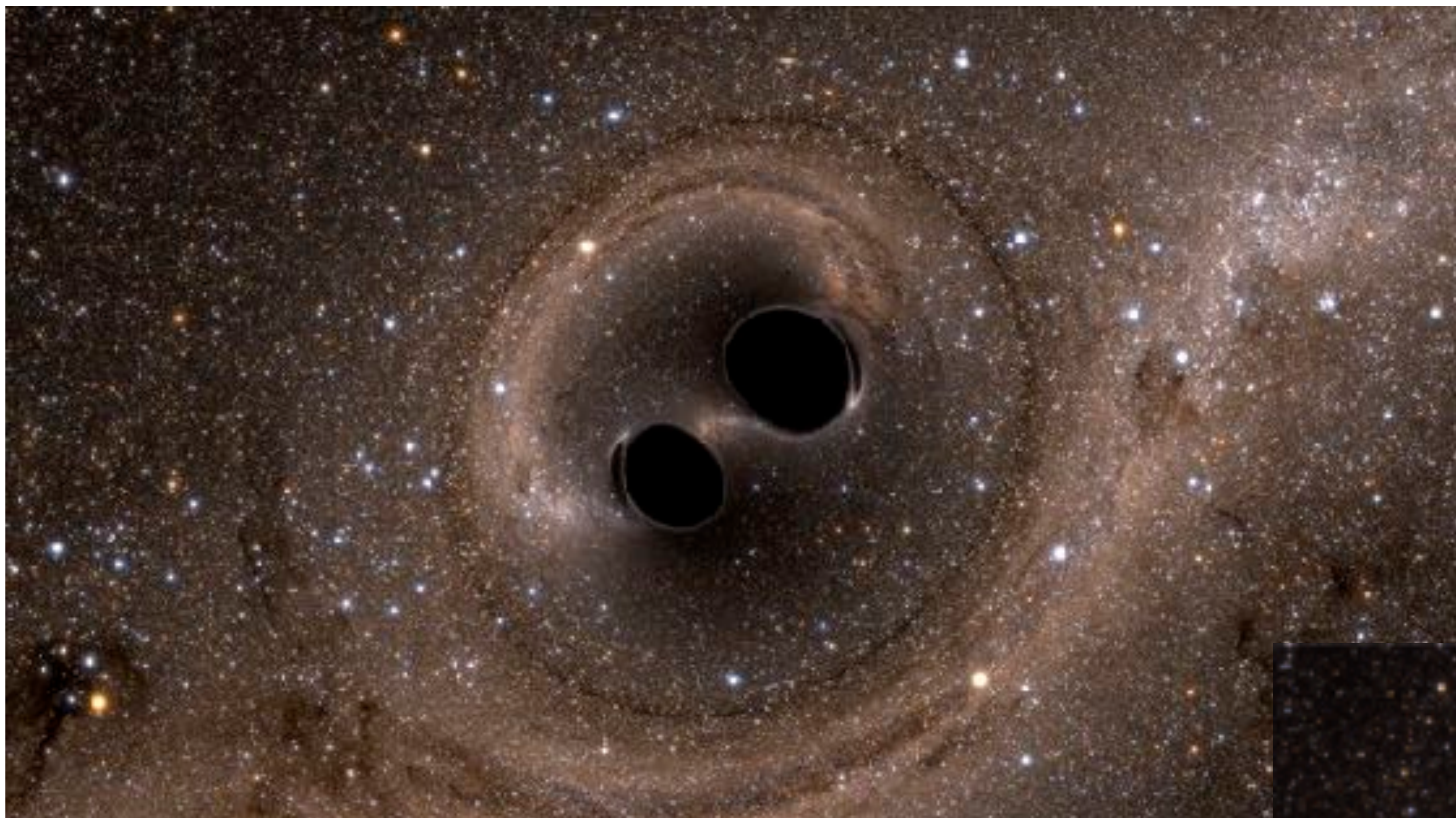
Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



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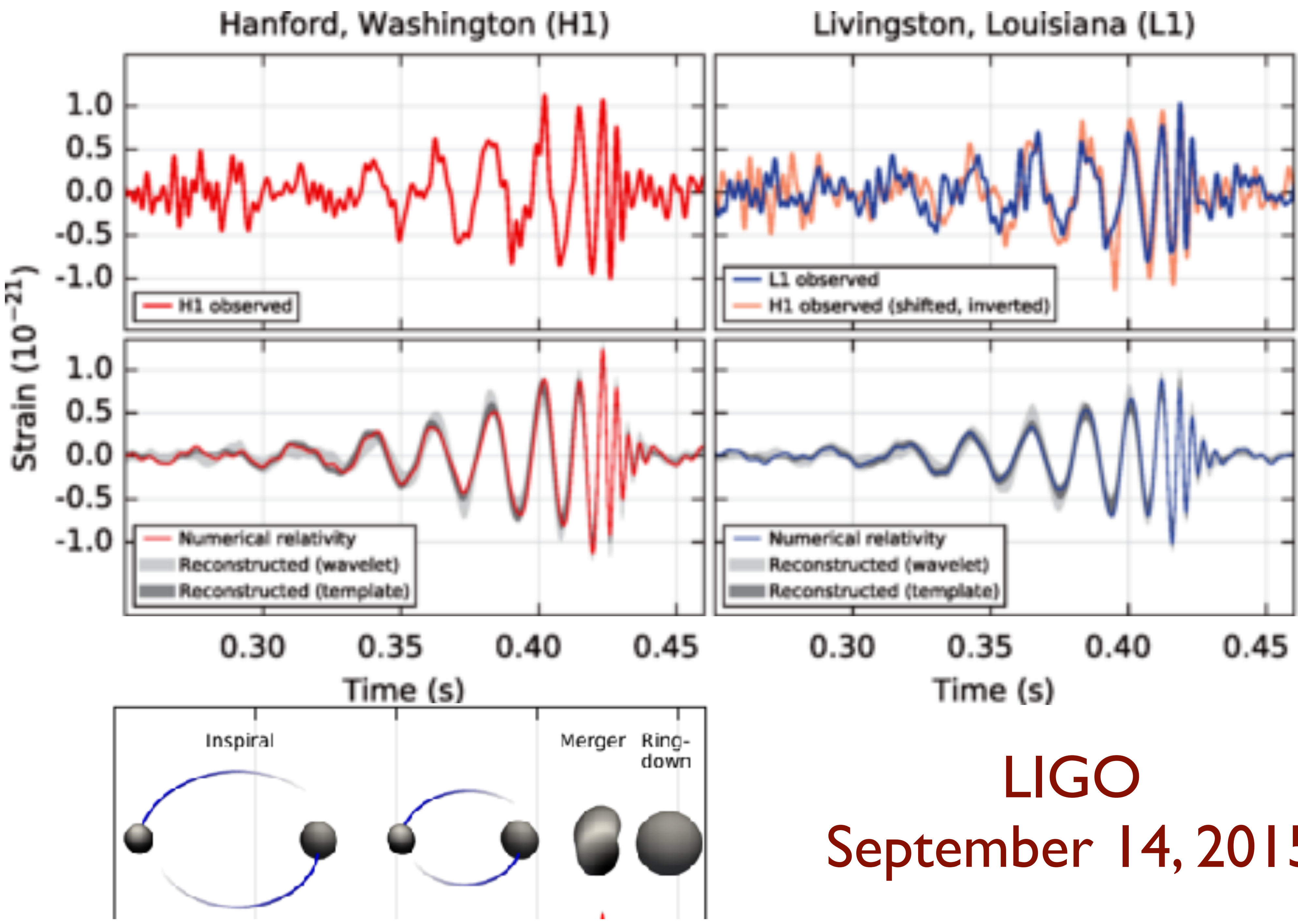


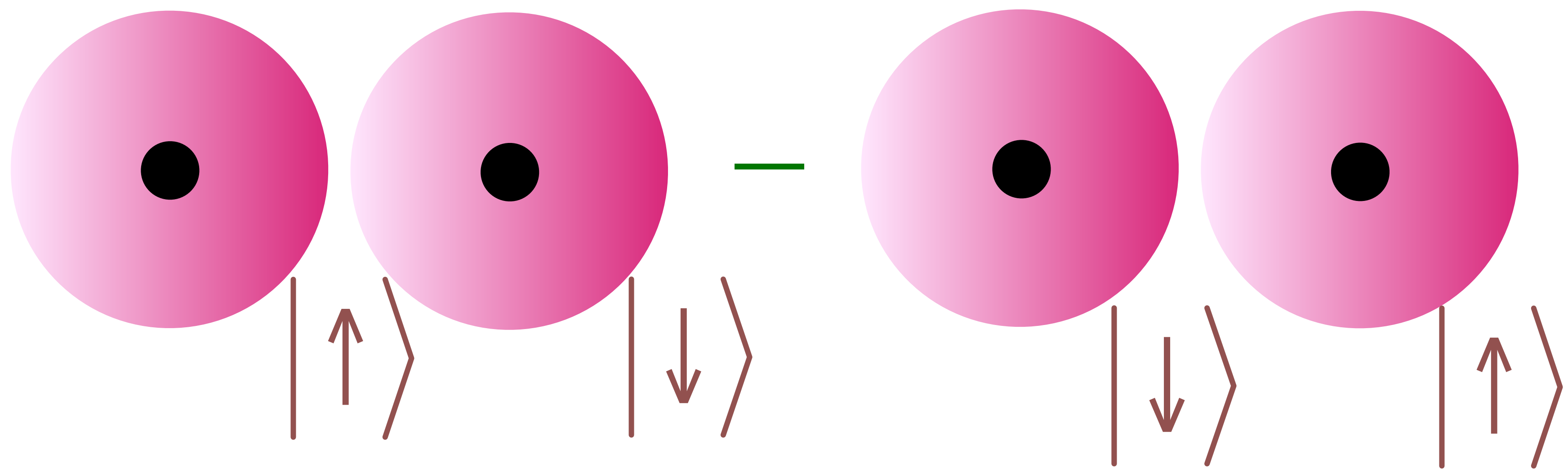
LIGO

September 14, 2015

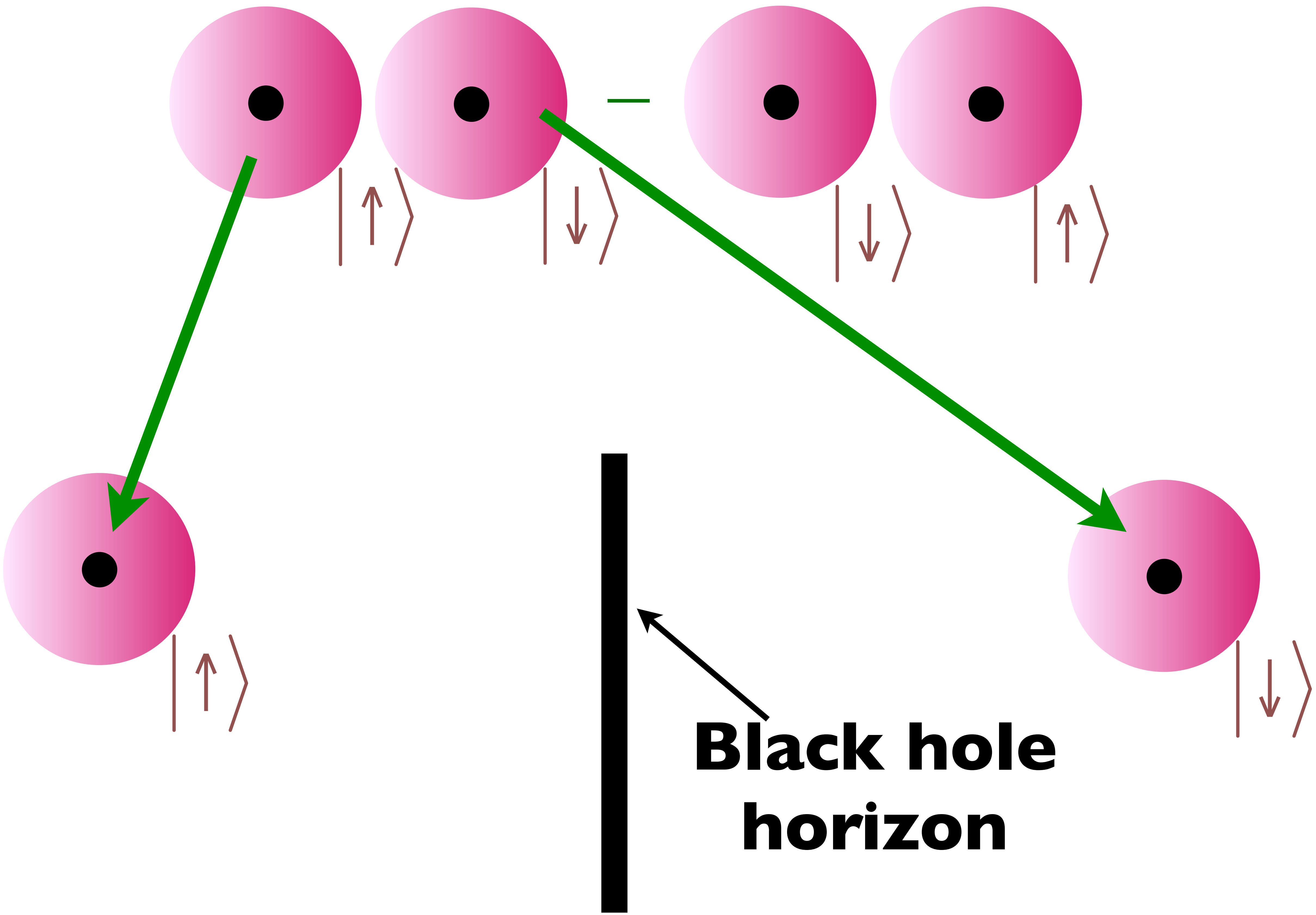
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



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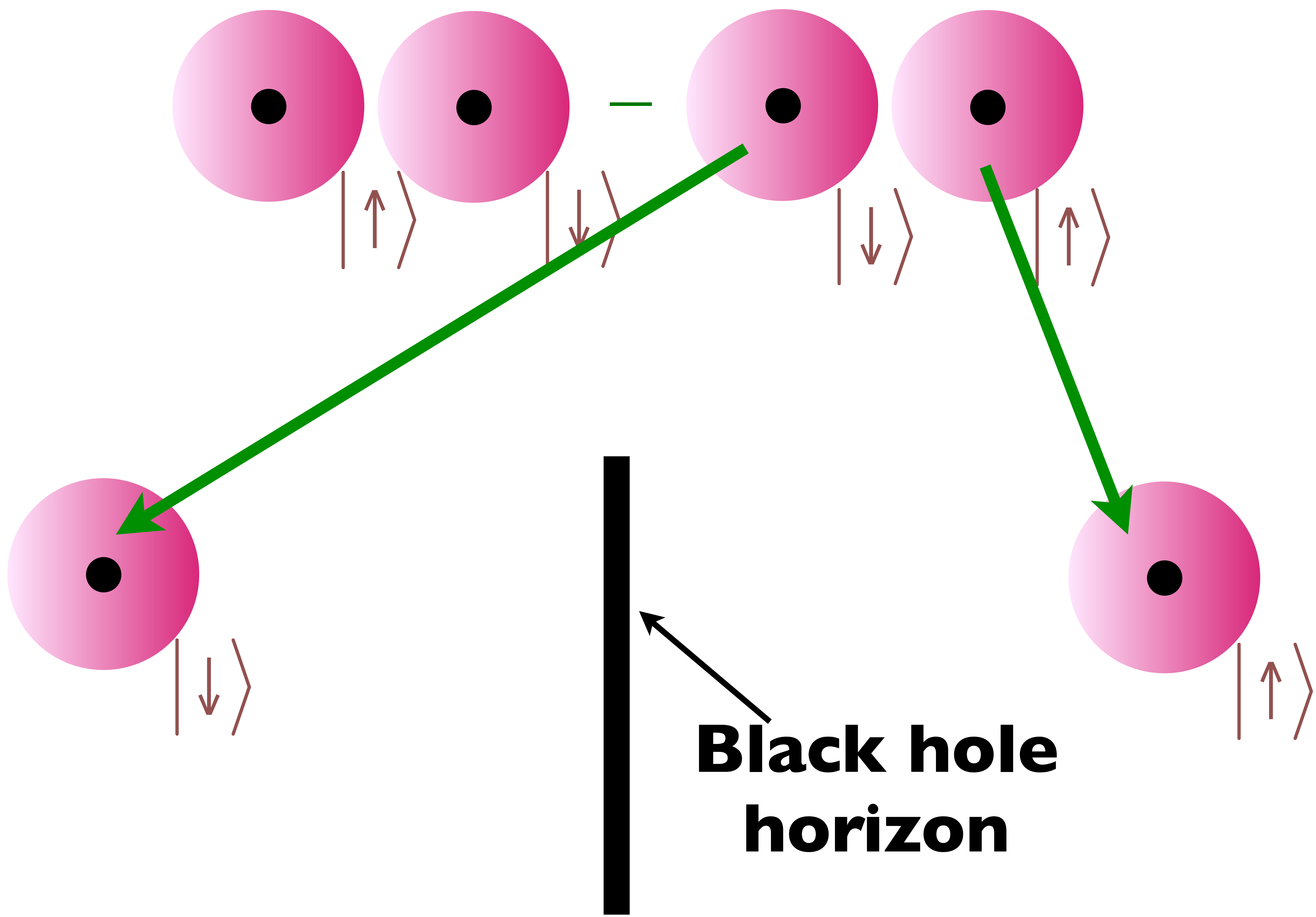
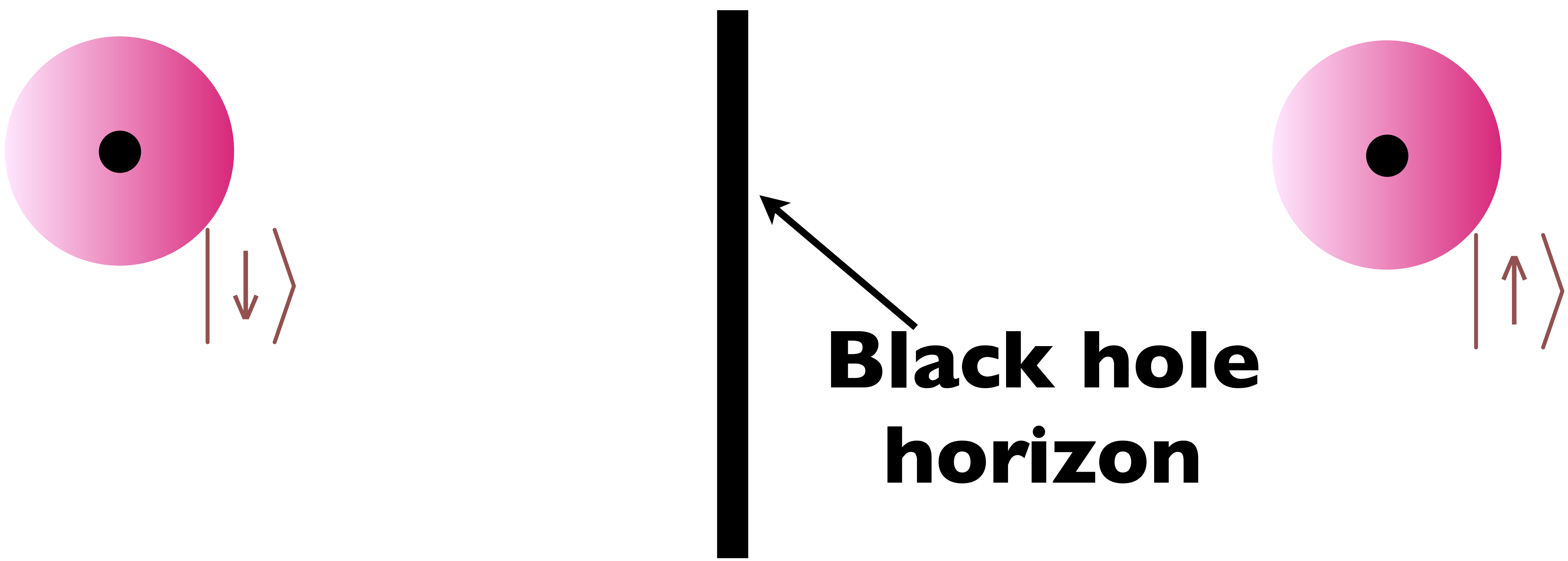


Quantum Entanglement across a black hole horizon

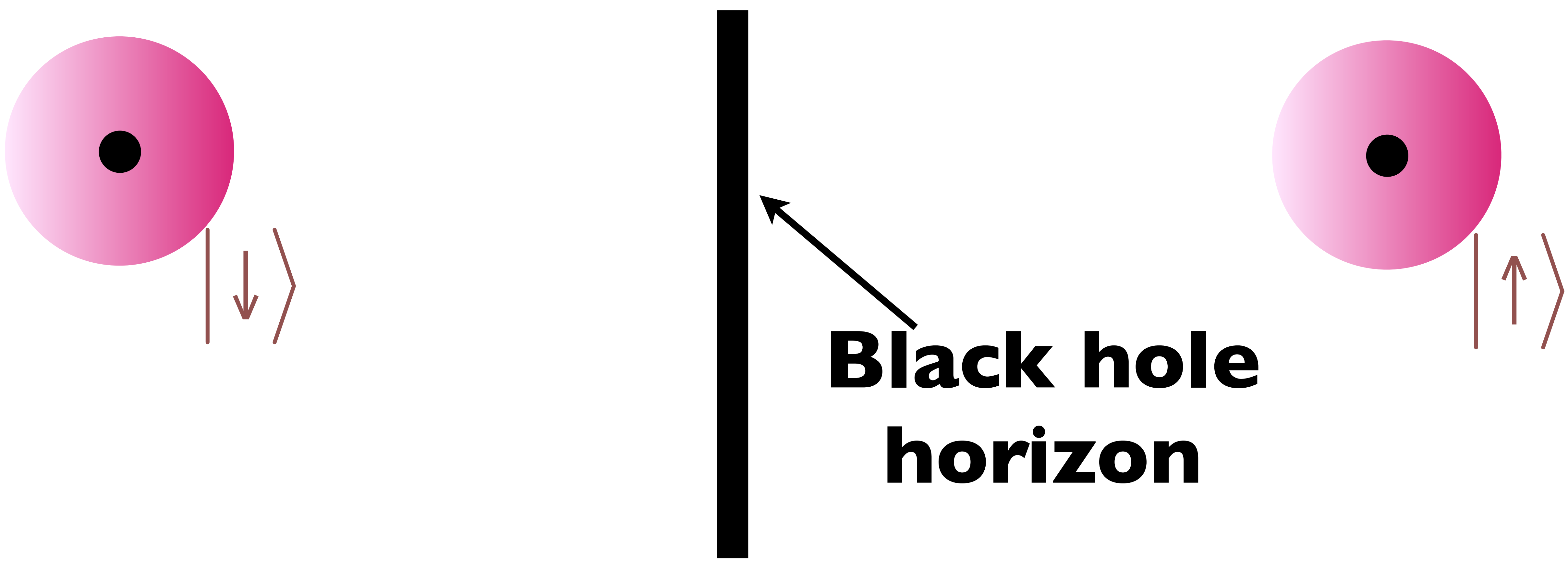
There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)



Quantum Entanglement across a black hole horizon

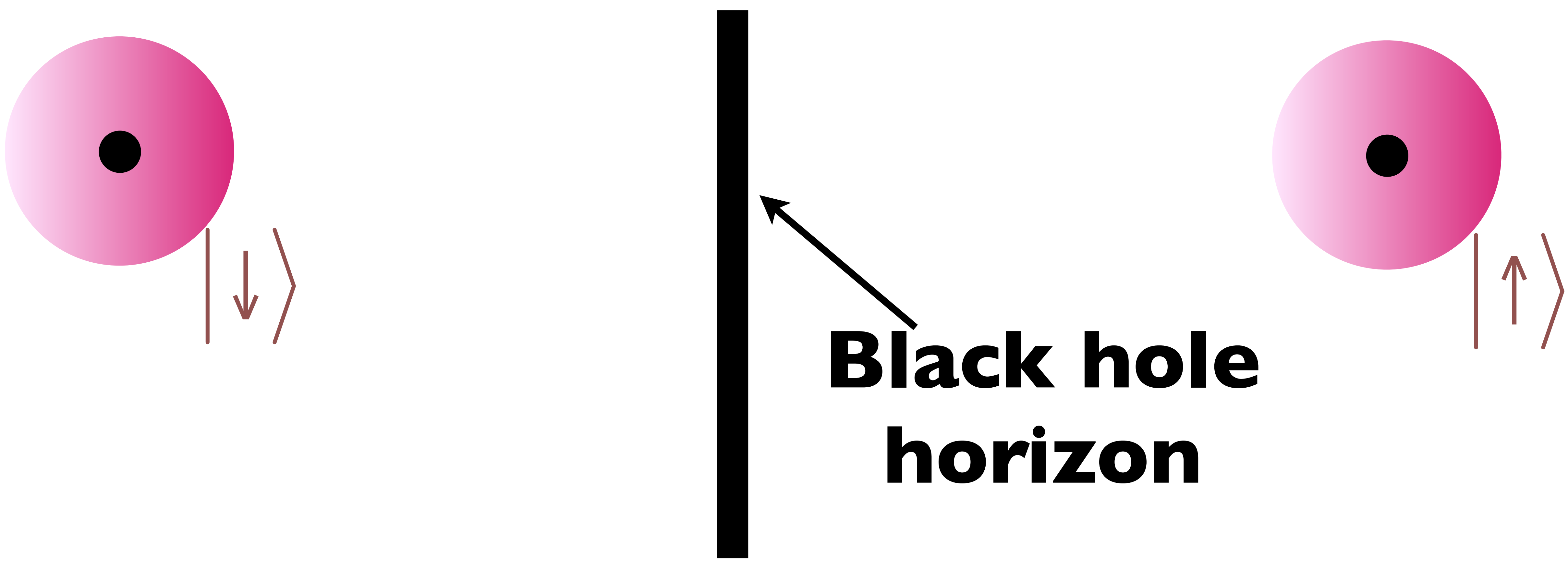
The Hawking temperature $k_B T_H = \dfrac{\hbar^2}{8\pi M \ell_P^2}$ and

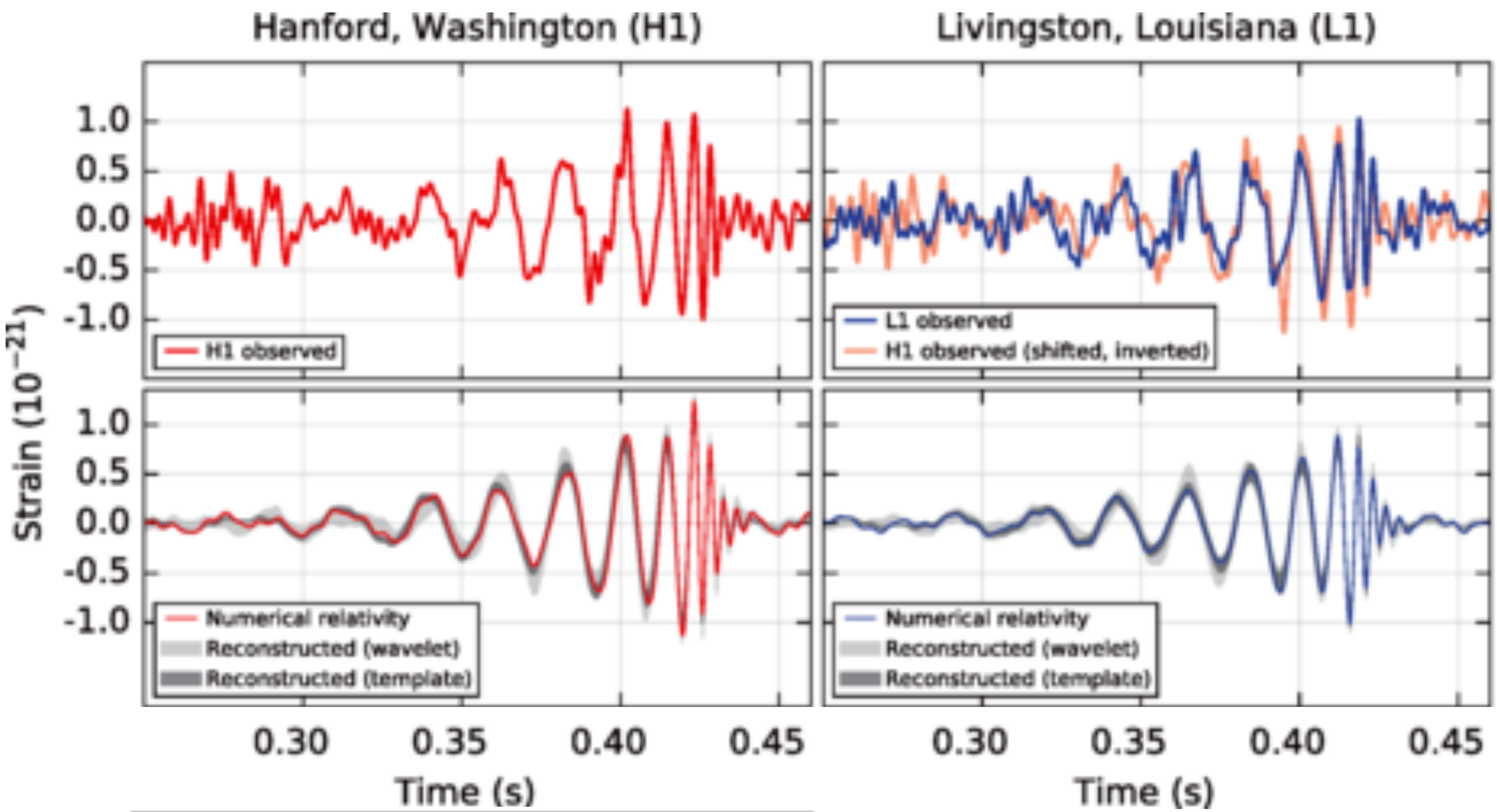
the Bekenstein-Hawking (BH) black hole entropy $\dfrac{S_{BH}}{k_B} = \dfrac{A}{4\ell_P^2}$

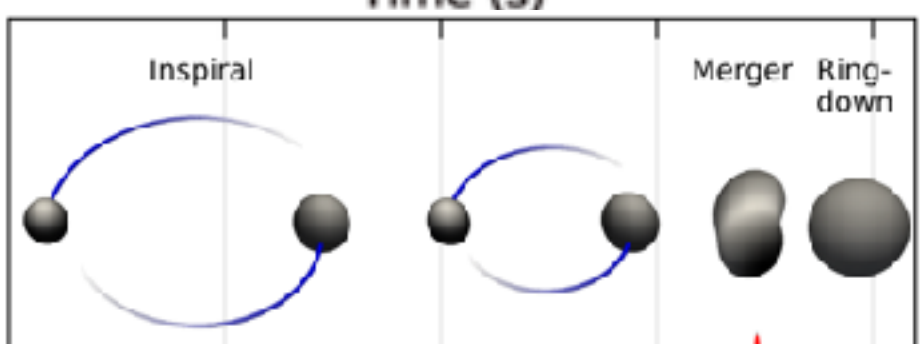
where $\ell_P = \sqrt{\hbar G/c^3}$ in the Planck length,

and A is the surface area of the black hole.

Note the entropy is proportional to the surface area rather than the volume.







LIGO
September 14, 2015

- The Hawking temperature, T_H influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \dfrac{\hbar}{k_B T_H} = \dfrac{8\pi G M}{c^3} \sim 8$ milliseconds.

Quantum entanglement

Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.

Quantum entanglement

- Strange metals have no quasiparticle description.
- Their entropy is proportional to their volume.
- They relax to local thermal equilibrium in the fastest possible time $\sim \hbar/(k_B T)$.

Strange metals

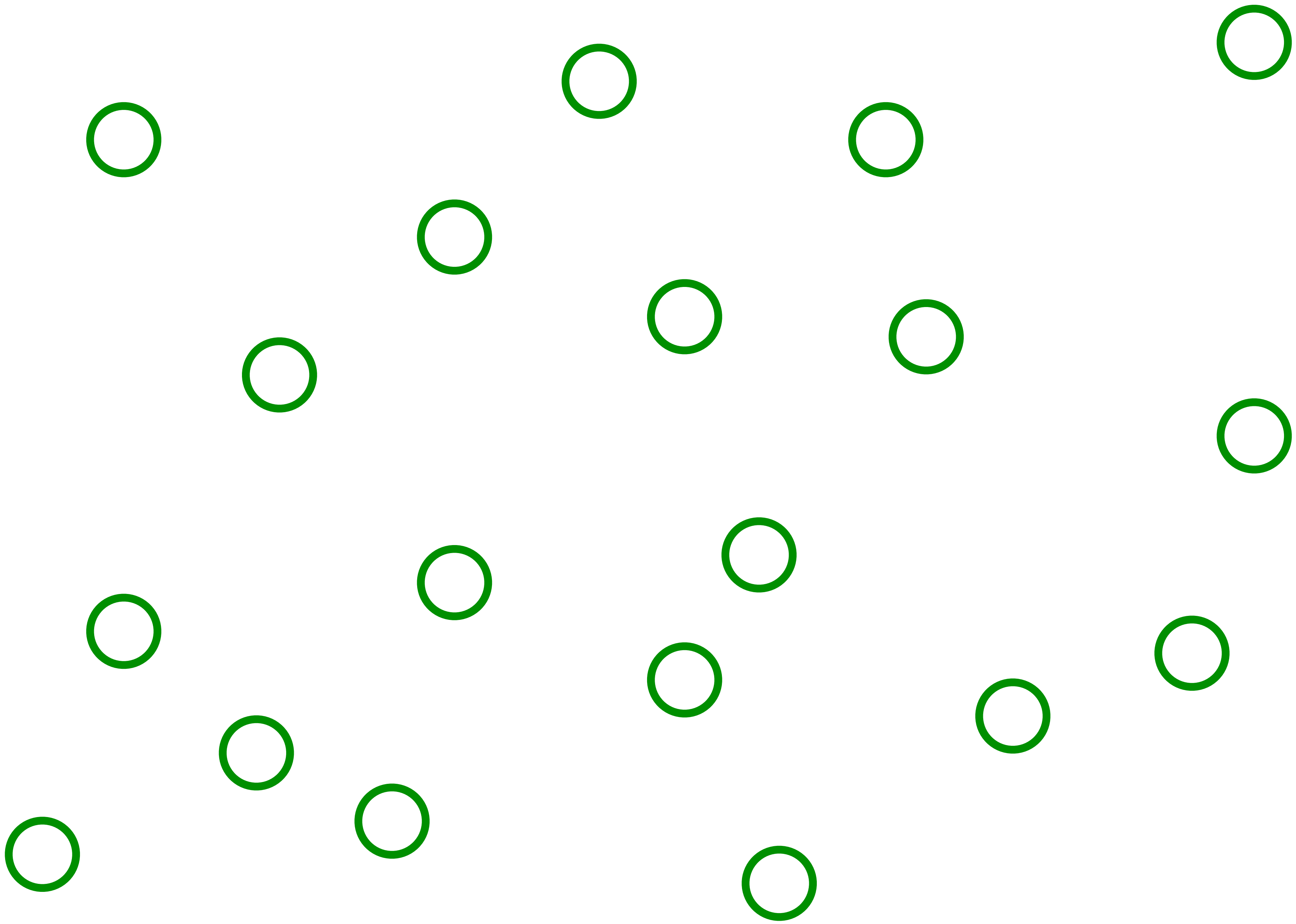
Quantum entanglement

Black holes

Strange metals

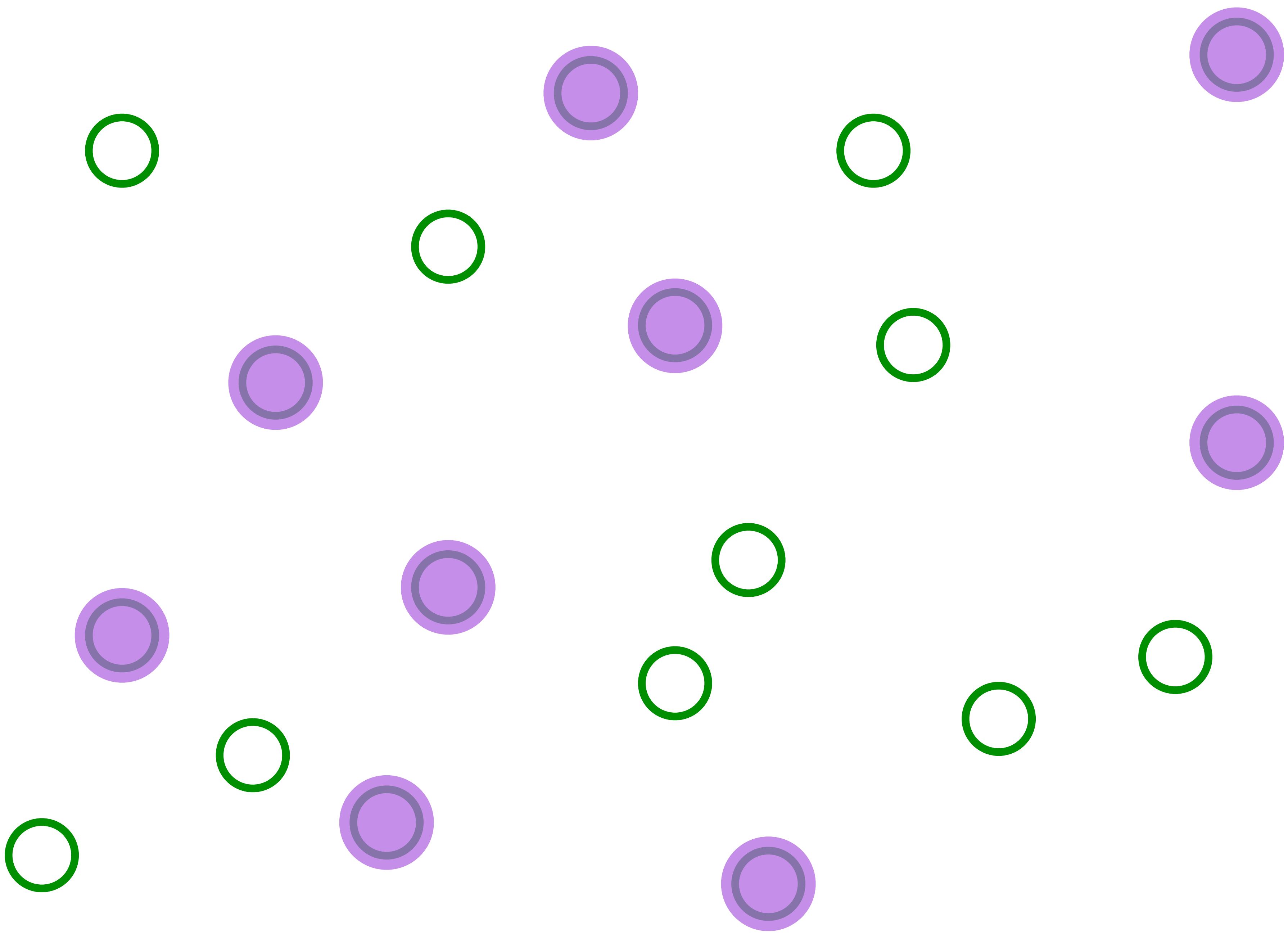
A "toy model" which is both a strange metal and a black hole!

A simple model of a metal with quasiparticles



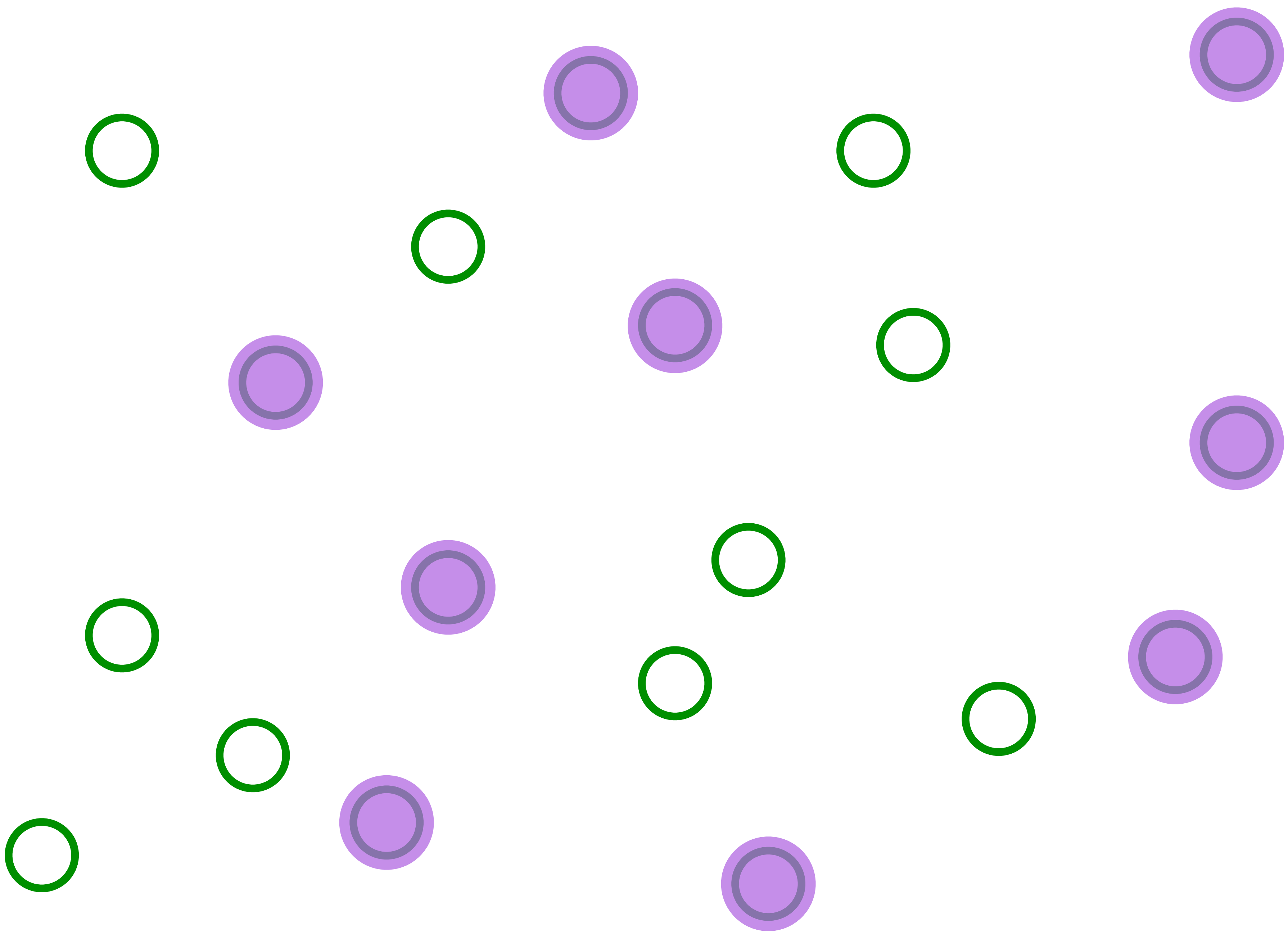
Pick a set of random positions

A simple model of a metal with quasiparticles



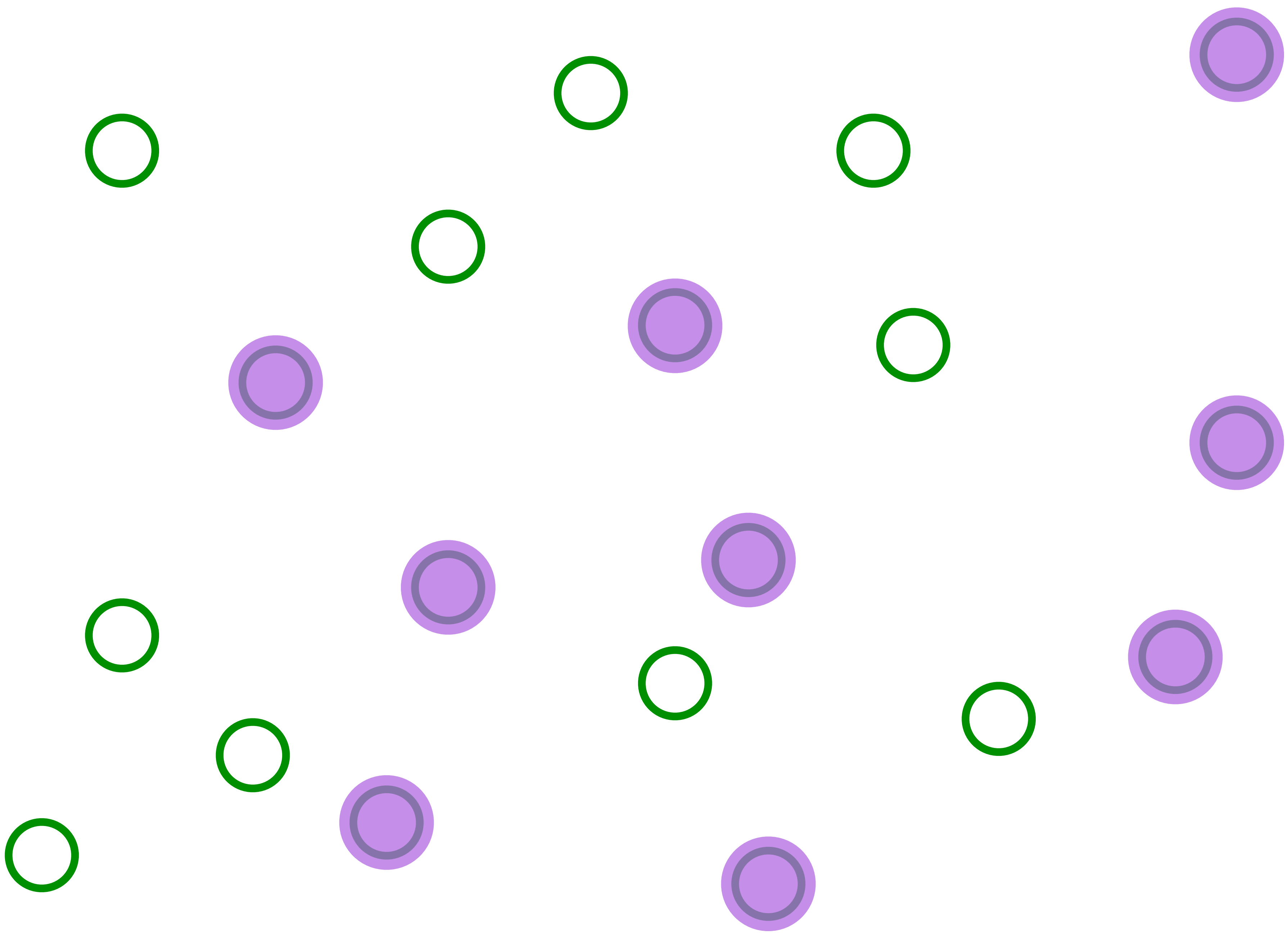
Place electrons randomly on some sites

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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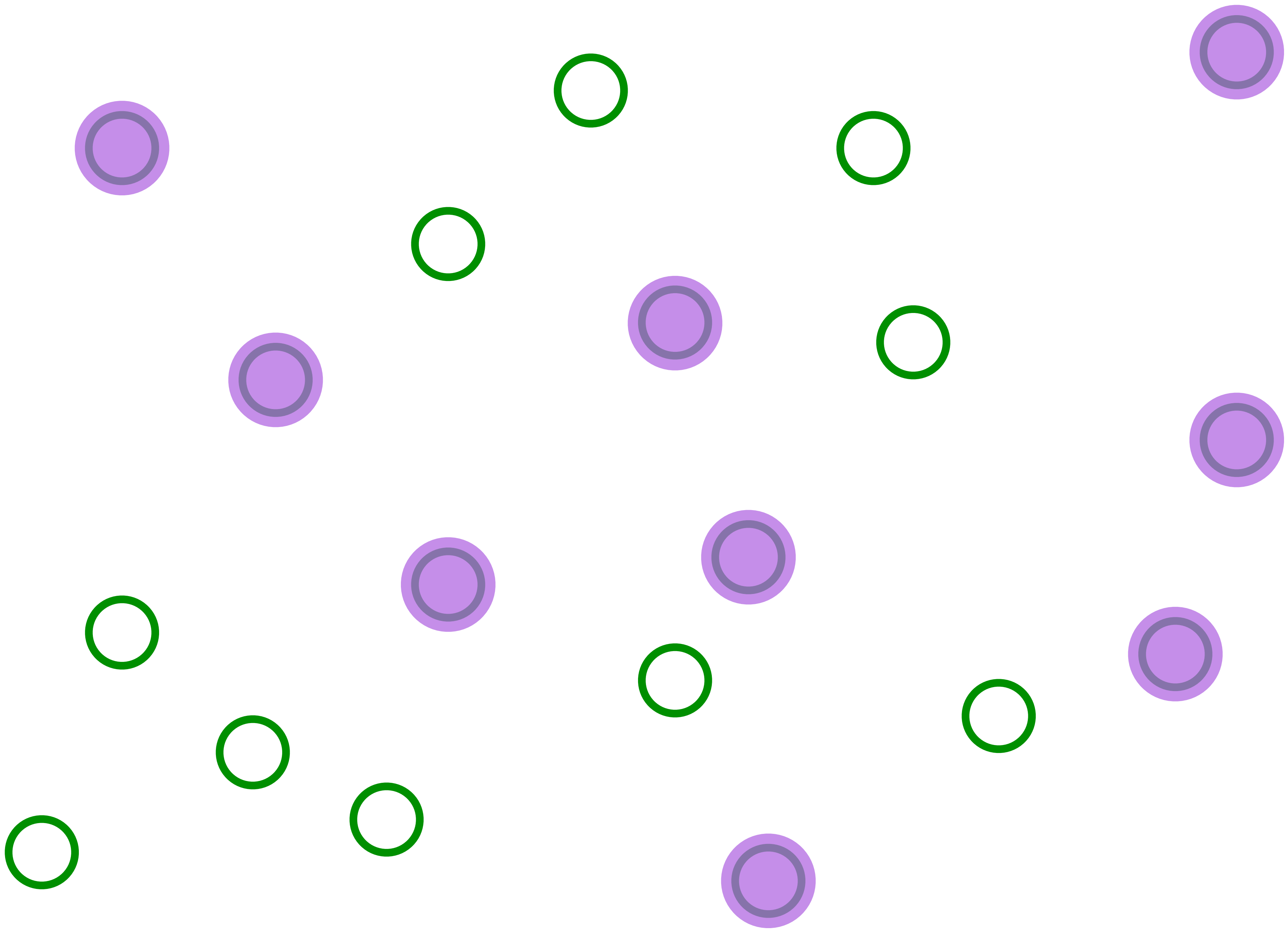
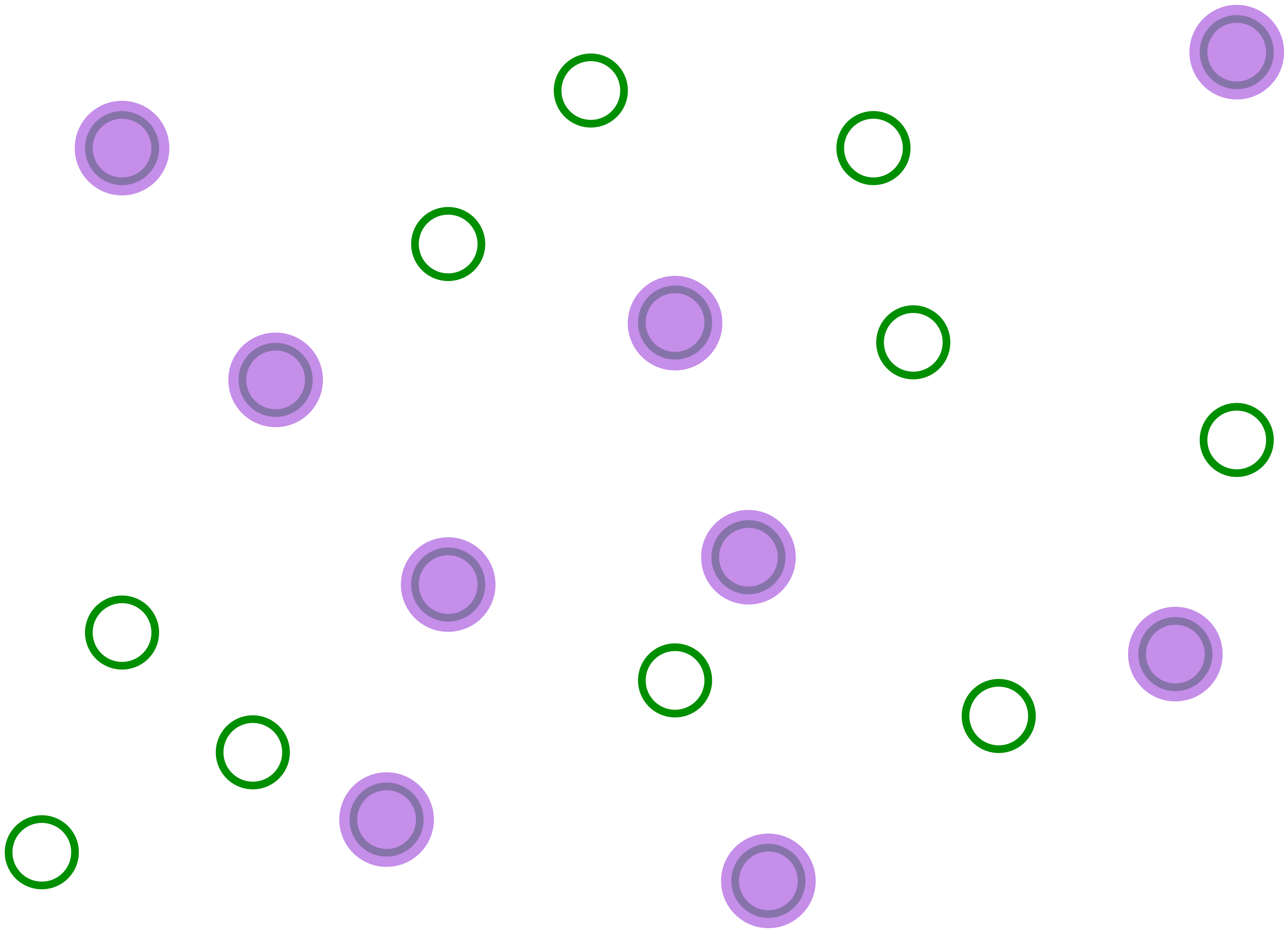
A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

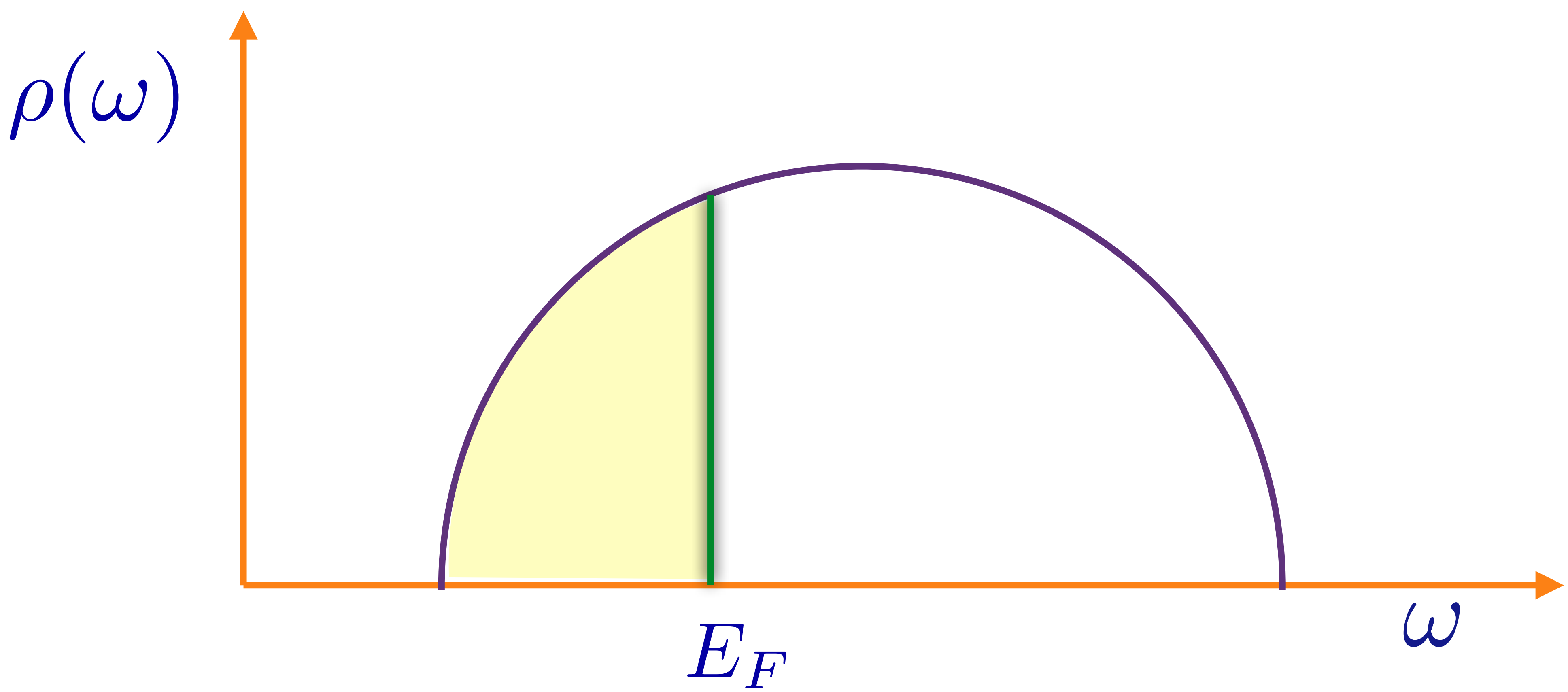
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

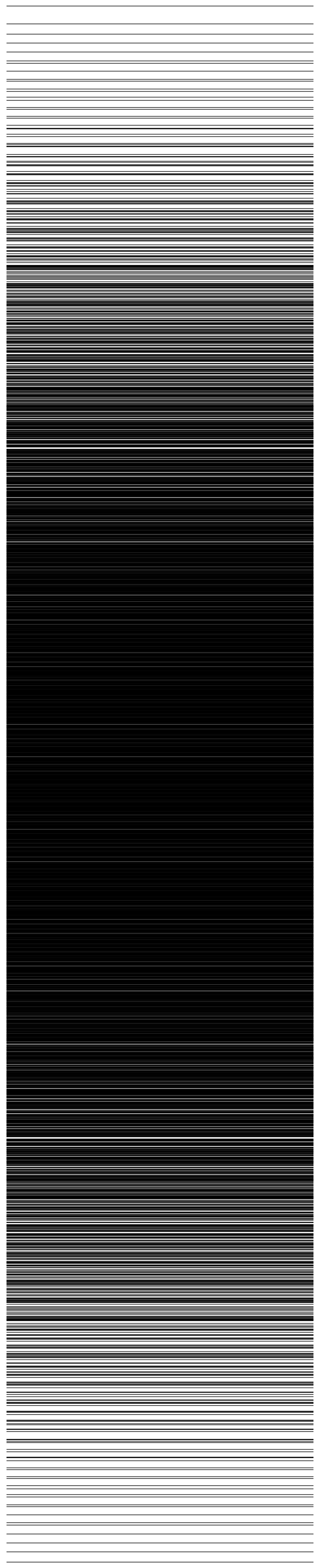
Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

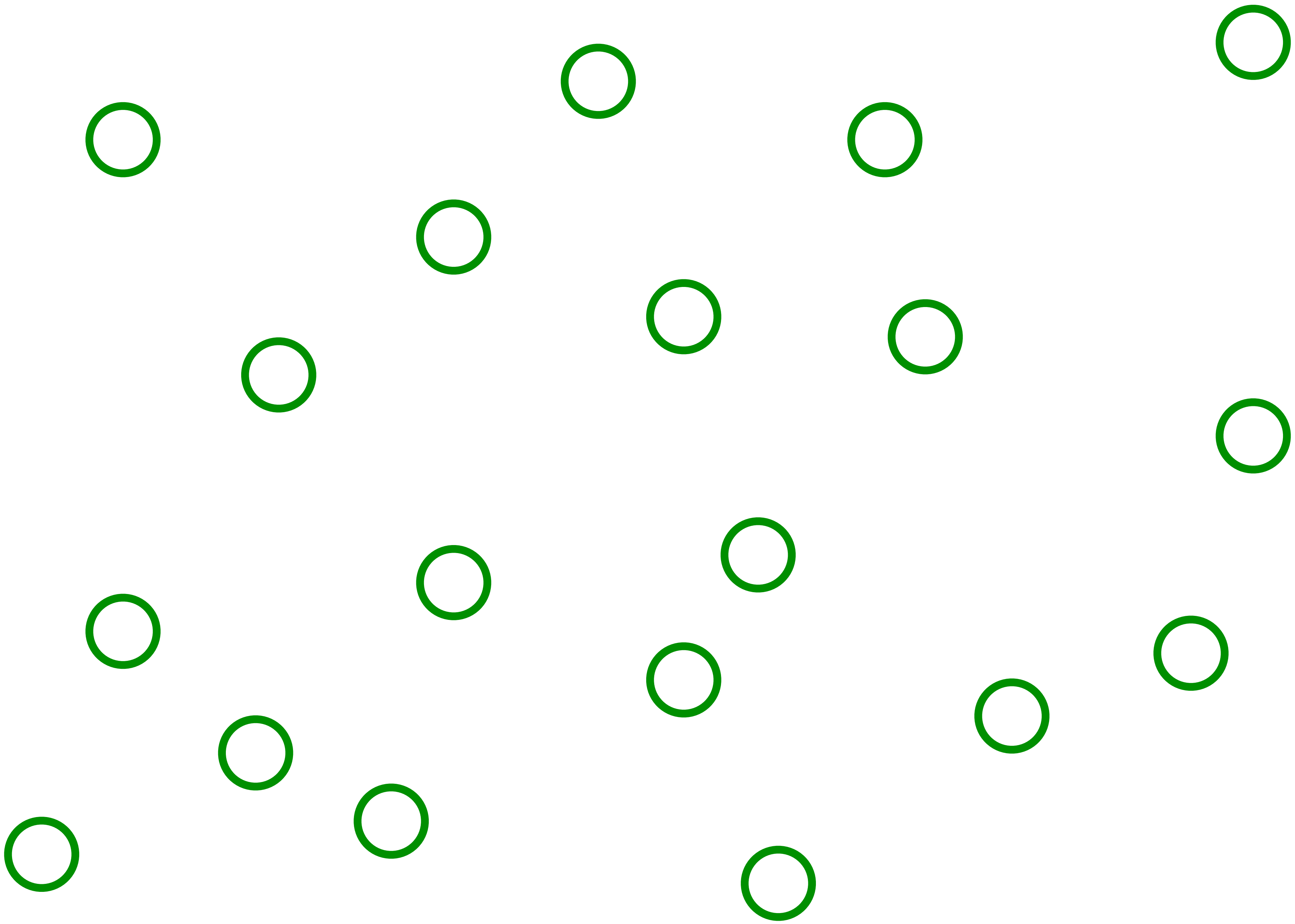
Quasiparticle excitations with spacing $\sim 1/N$

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

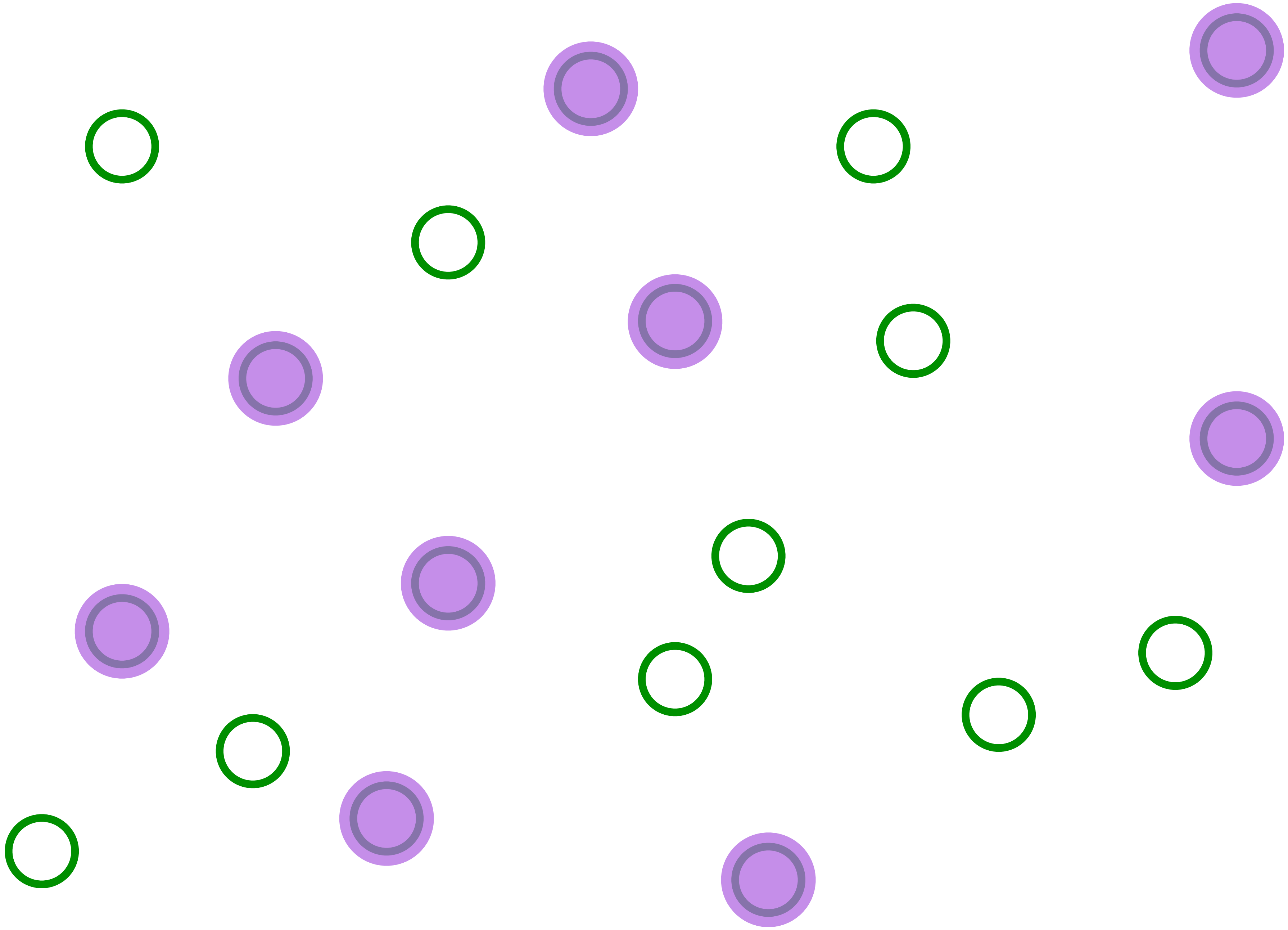
where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

The Sachdev-Ye-Kitaev (SYK) model



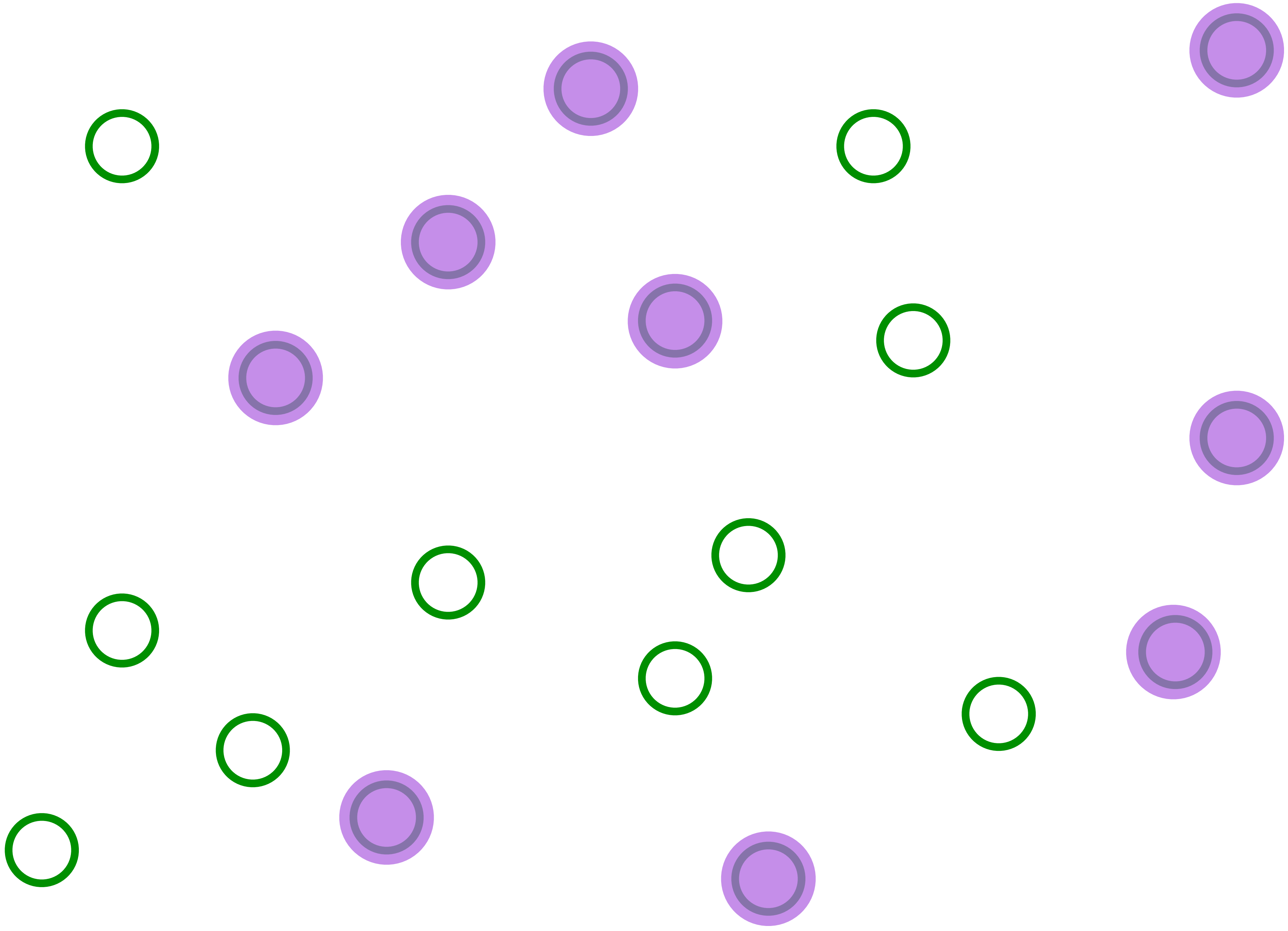
Pick a set of random positions

The Sachdev-Ye-Kitaev (SYK) model



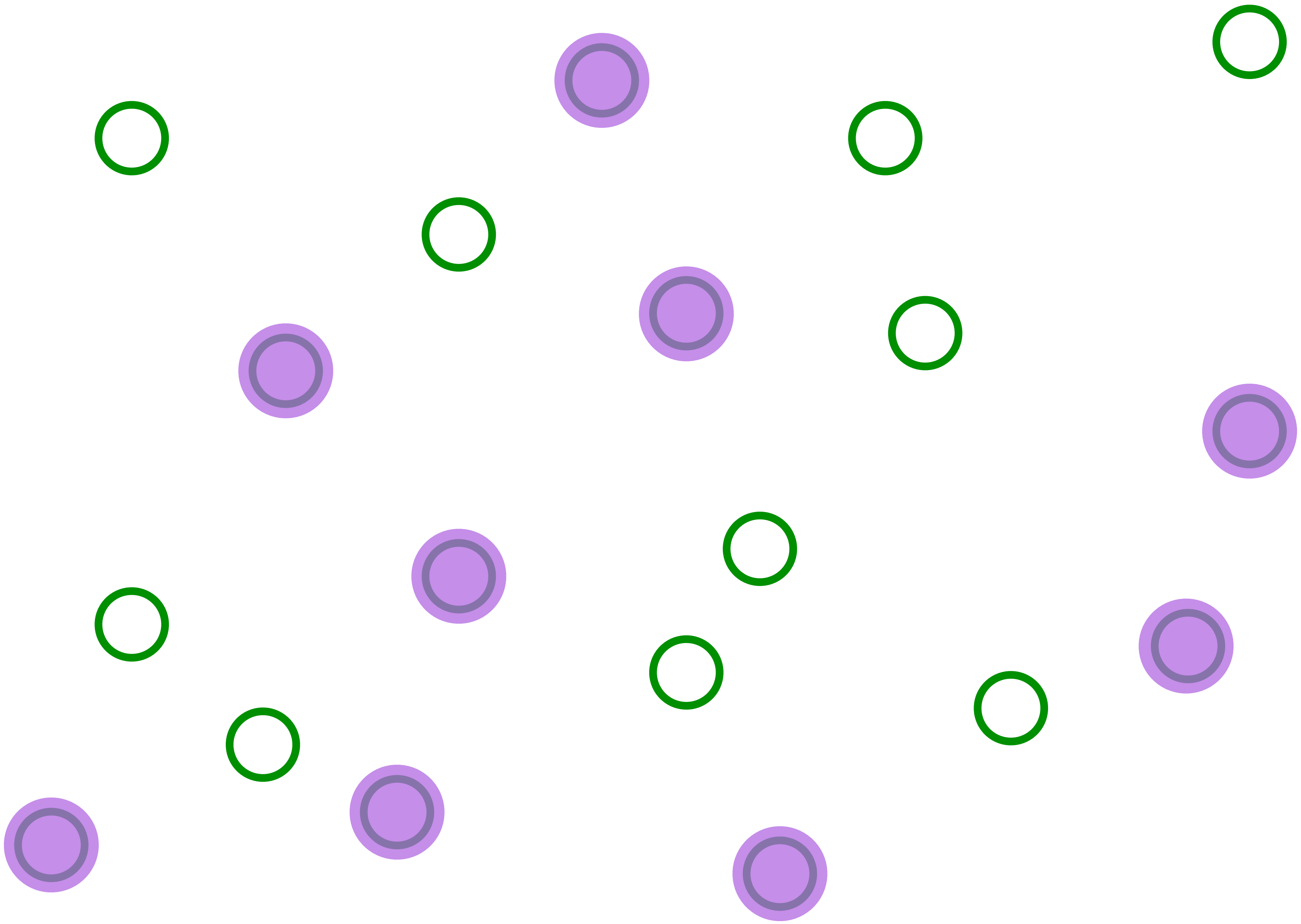
Place electrons randomly on some sites

The Sachdev-Ye-Kitaev (SYK) model



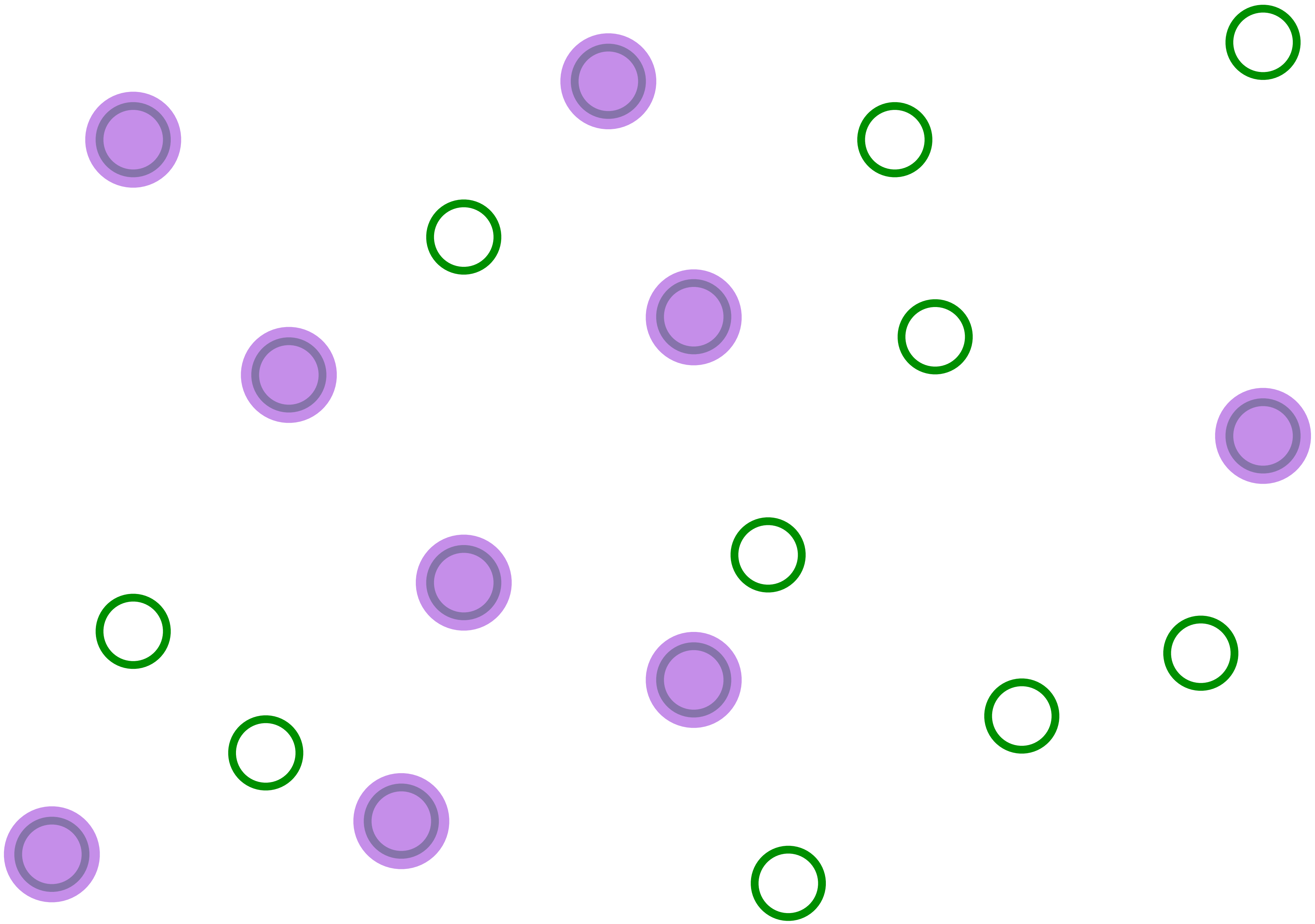
Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



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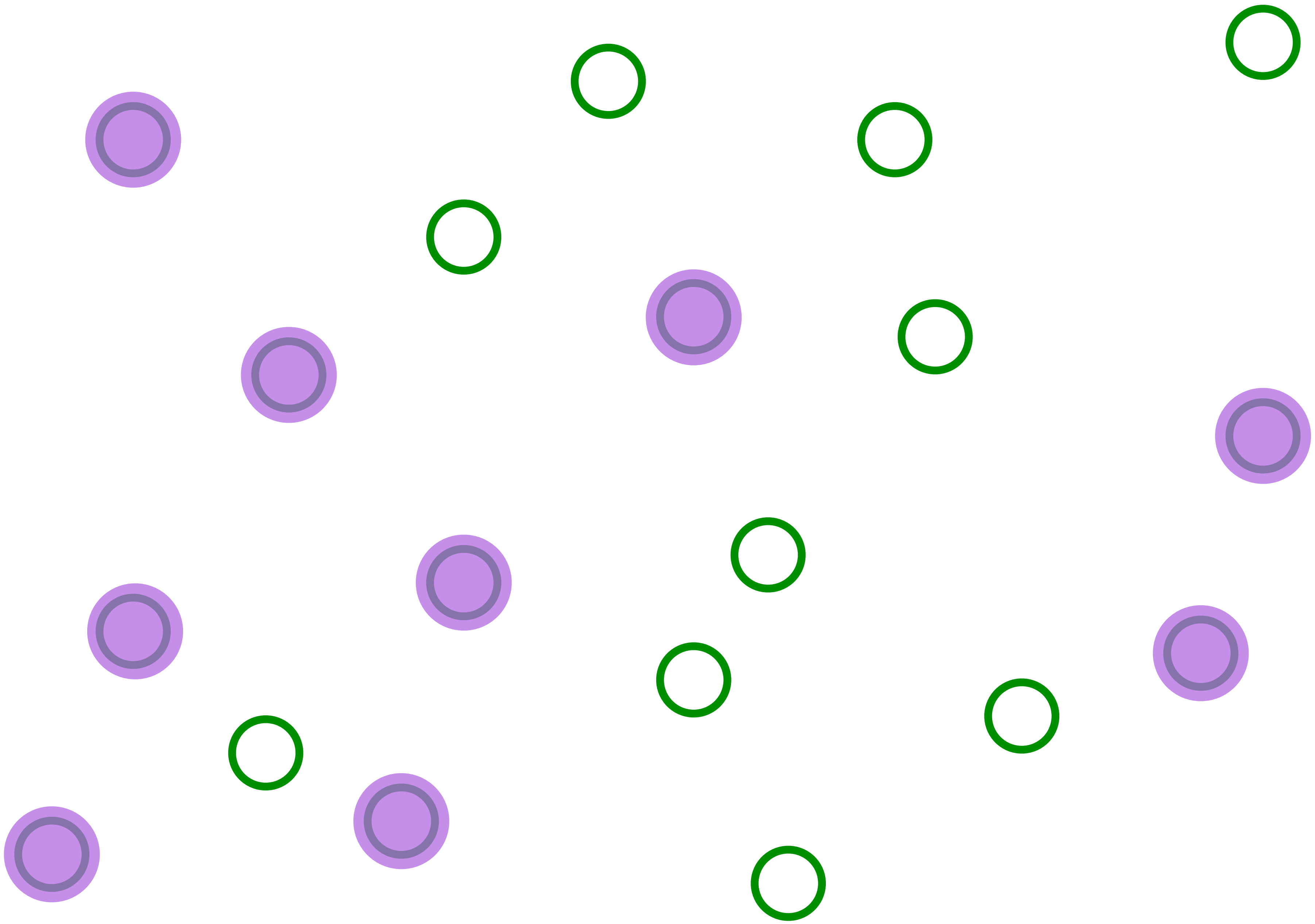
The Sachdev-Ye-Kitaev (SYK) model



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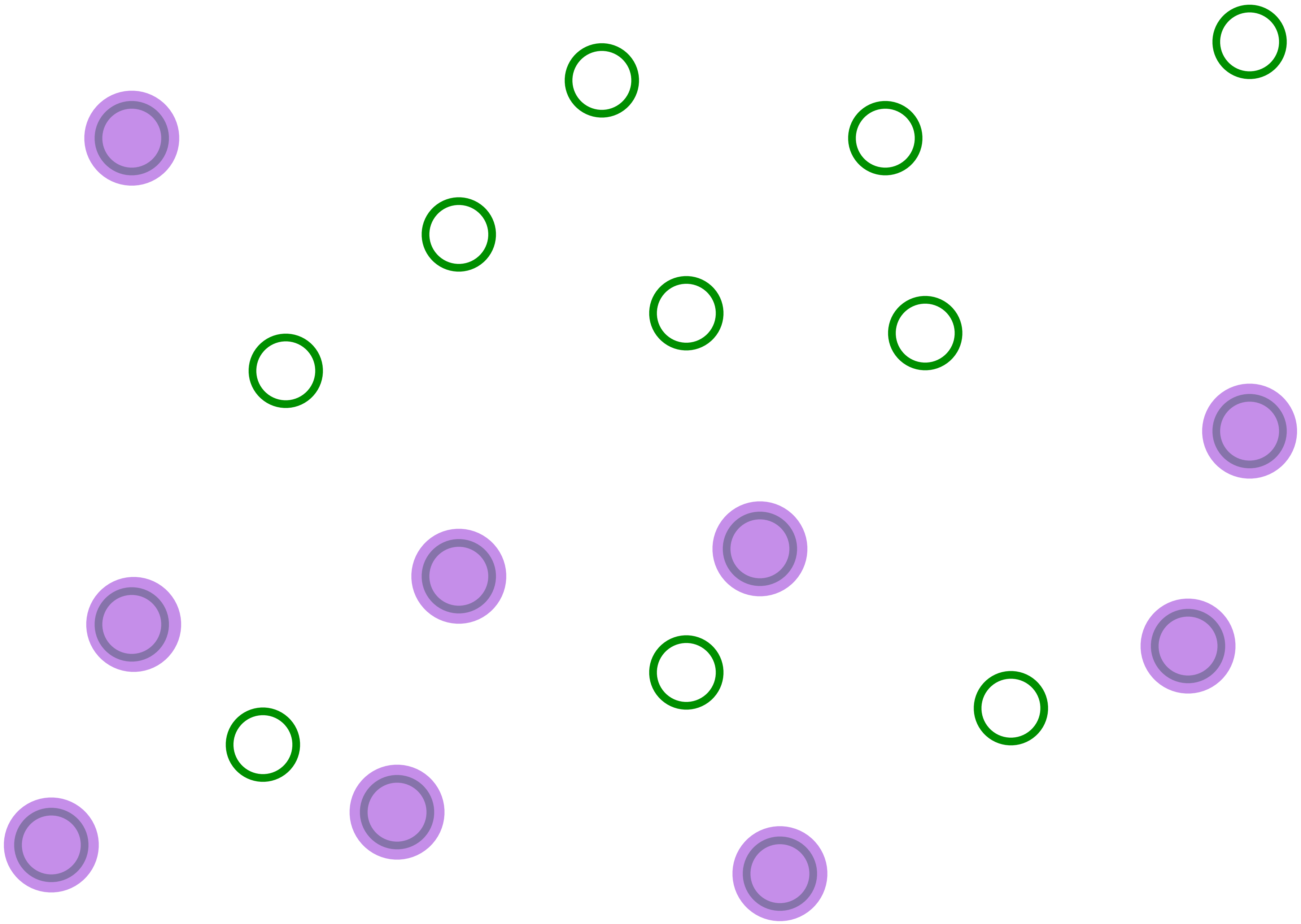

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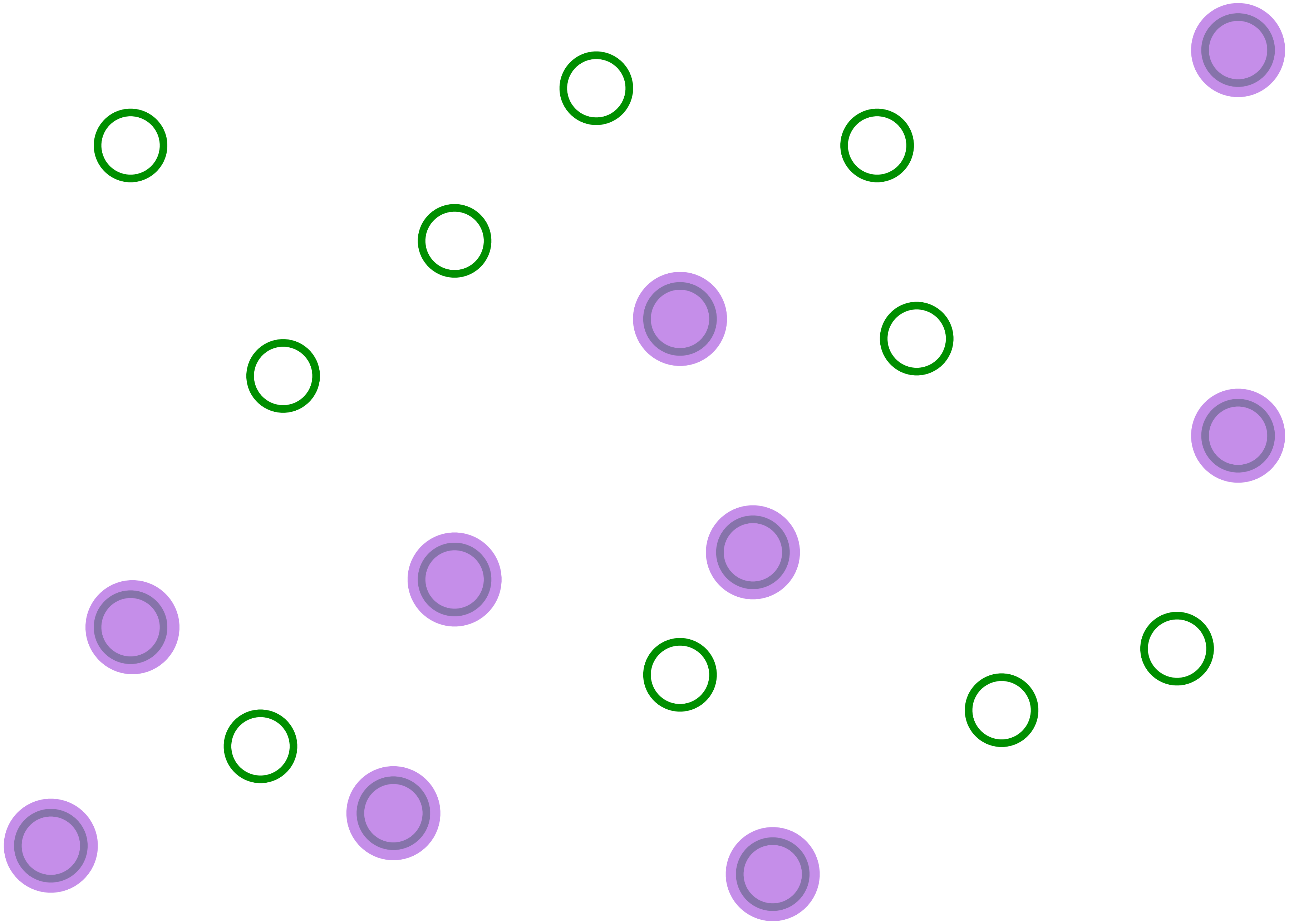
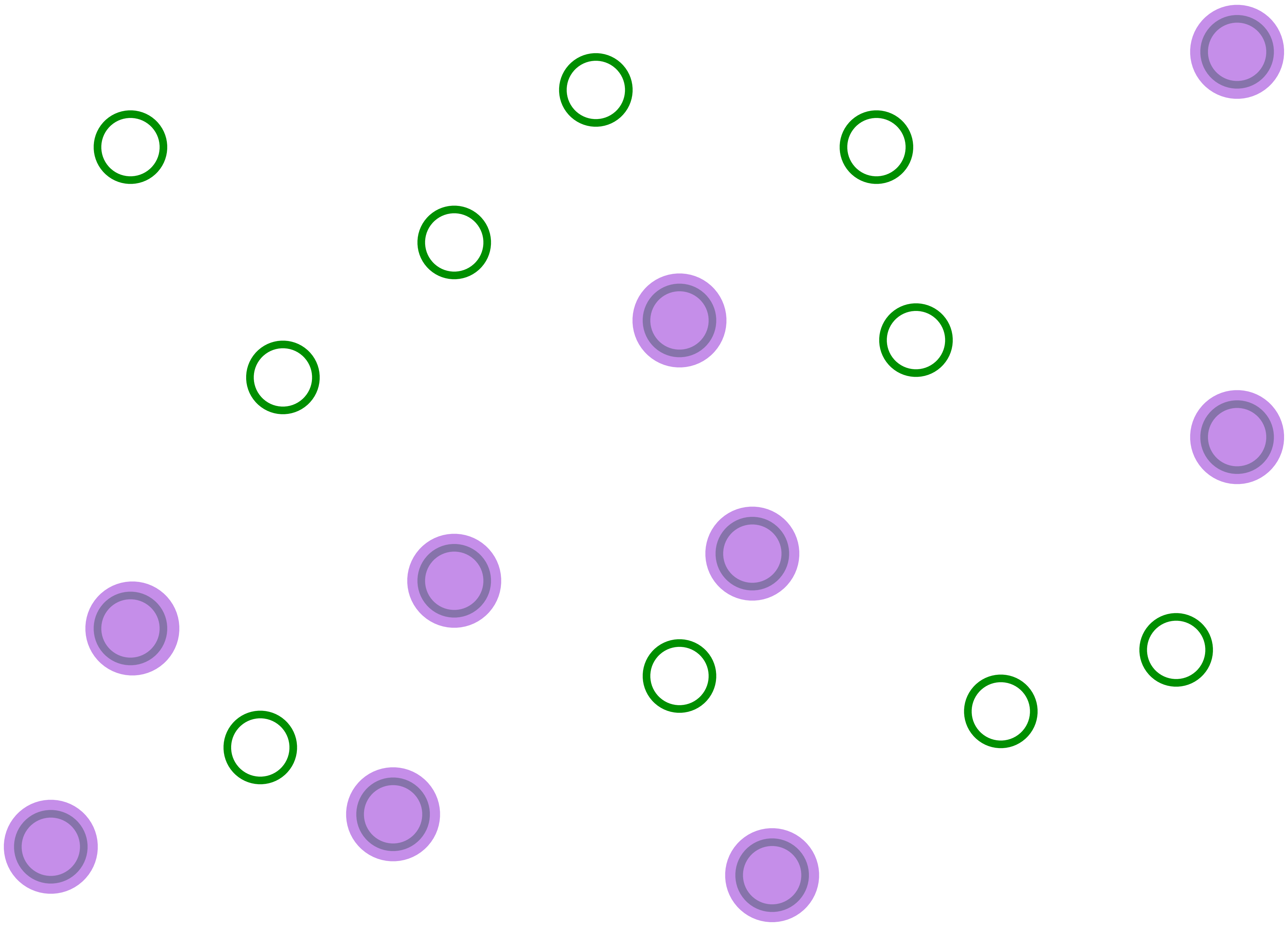
The Sachdev-Ye-Kitaev (SYK) model



Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model



This describes both a strange metal and a black hole!

The Sachdev-Ye-Kitaev (SYK) model

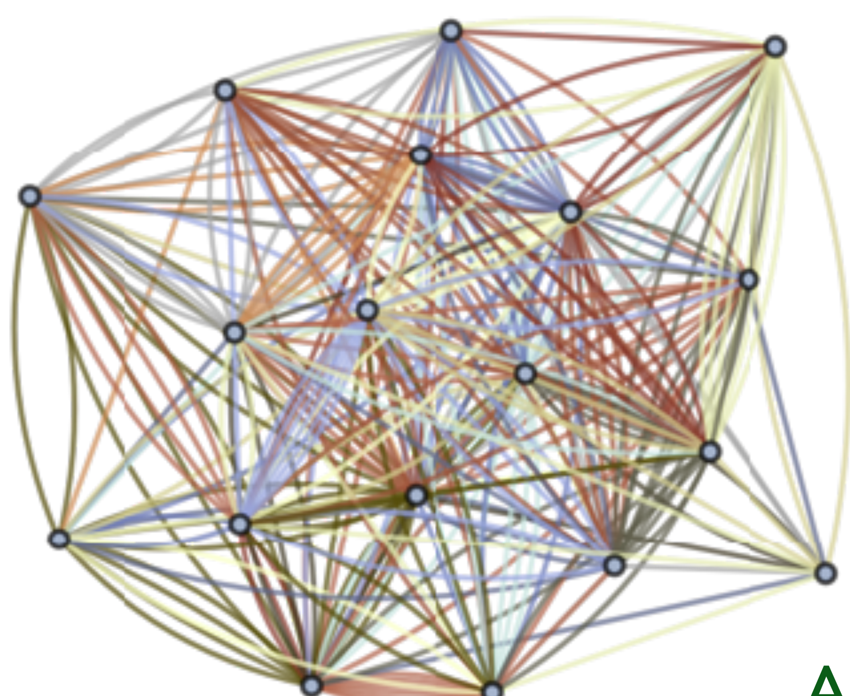
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

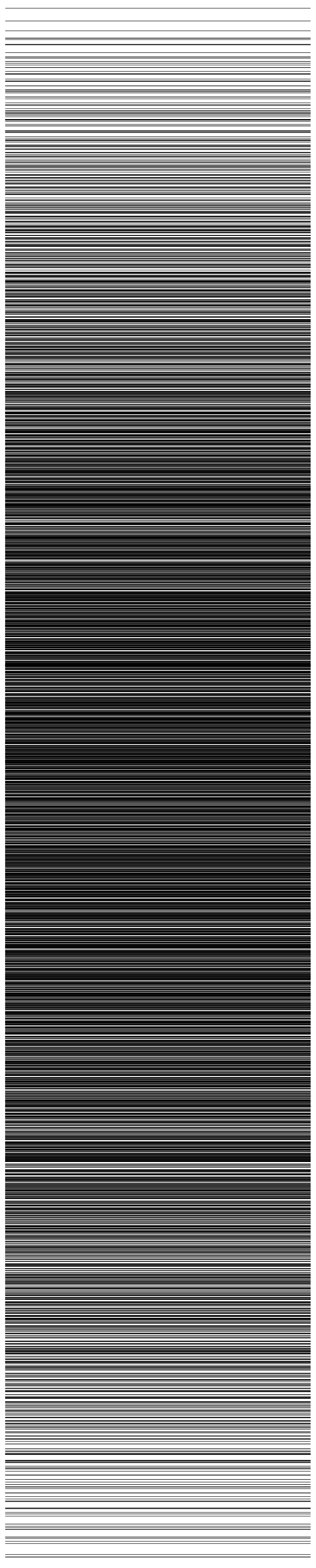
$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The Sachdev-Ye-Kitaev (SYK) model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-S_{GPS}}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy S_{GPS} with

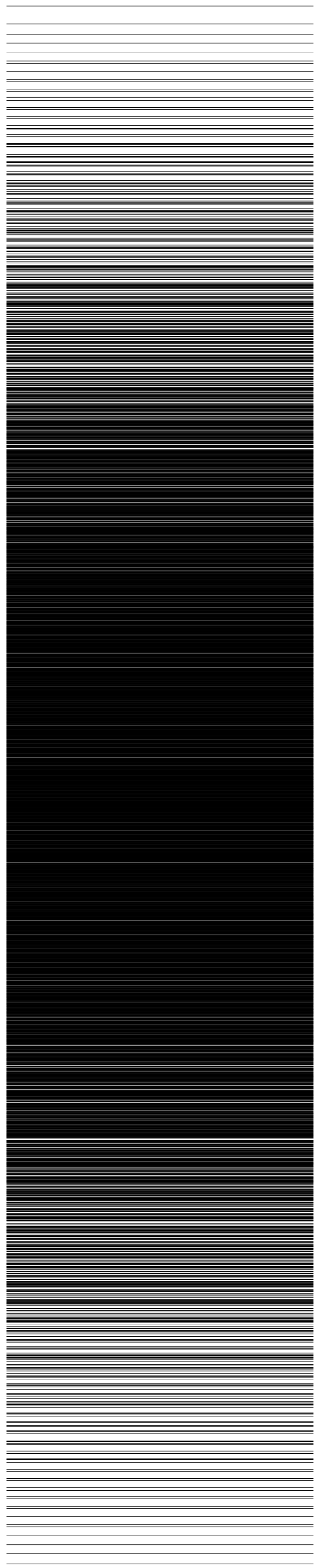
$$\begin{aligned} \frac{S_{GPS}}{N} &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\ &< \ln 2 \end{aligned}$$

where G is Catalan's constant, for the half-filled case $\mathcal{Q} = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

A simple model of a metal with quasiparticles



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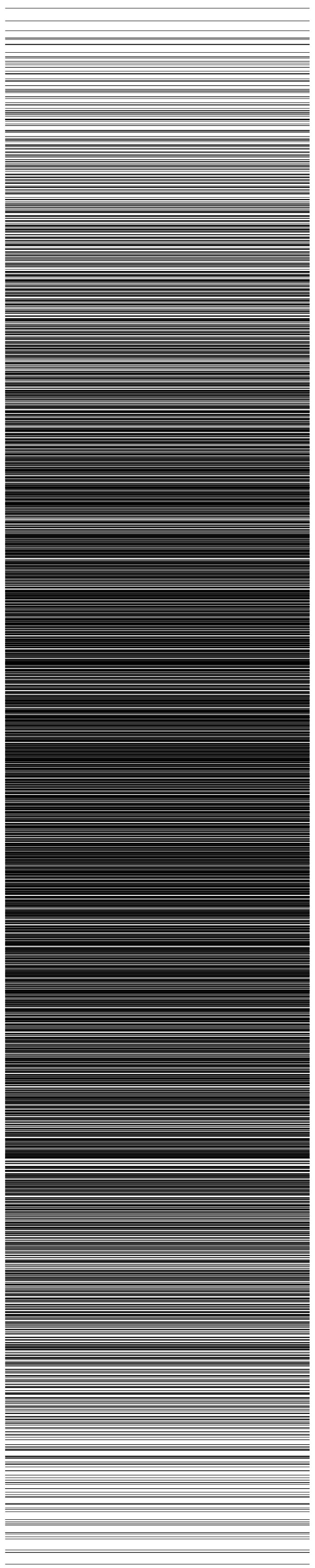

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W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

SYK and black holes

- The SYK model has a non-zero entropy, $S_{GPS} \propto N$ as $T \to 0$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

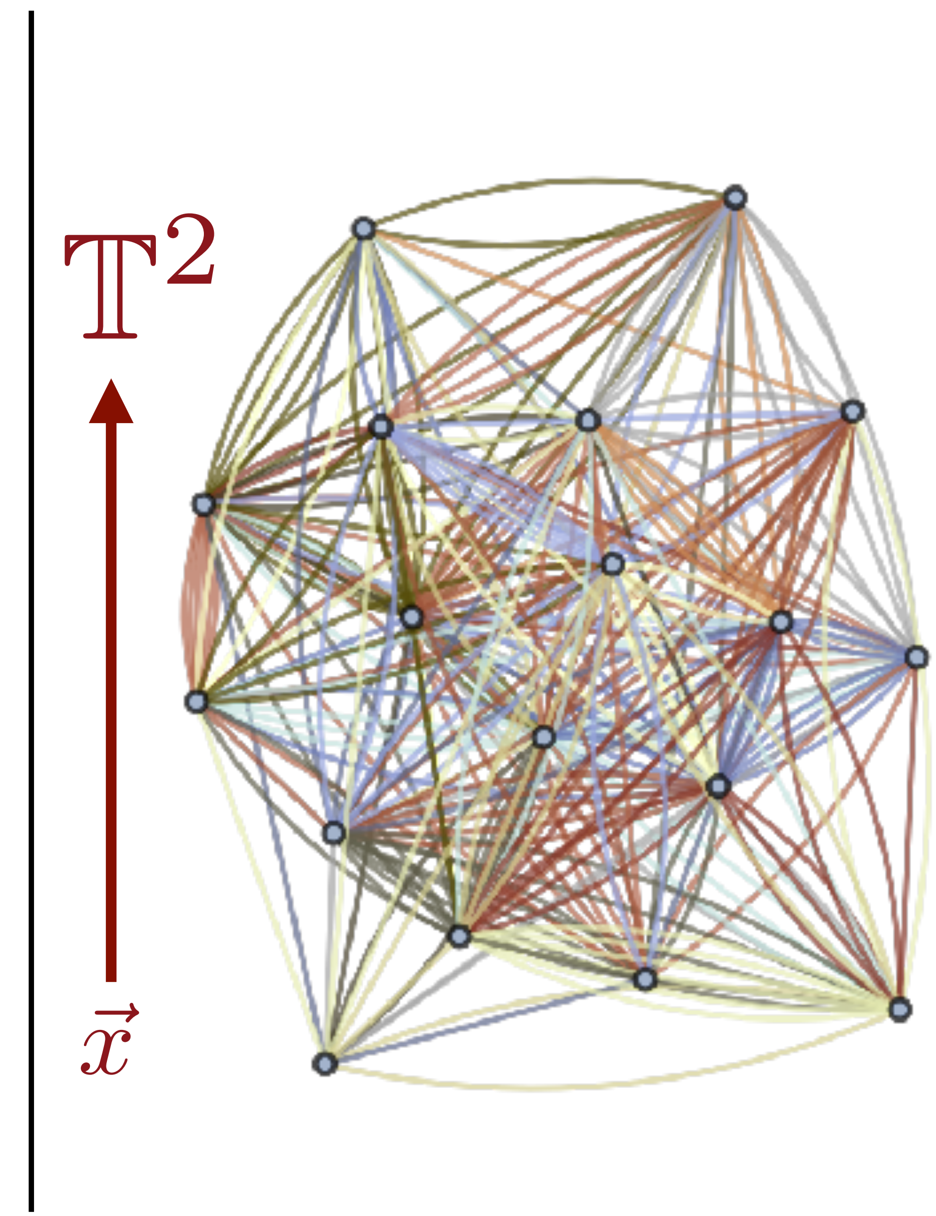
- The SYK model has a phase-coherence time $\tau_\varphi \sim \hbar/(k_B T)$

O. Parcollet and A. Georges, PRB **59**, 5341 (1999)

These properties indicate that SYK model 'holographically' realizes a black hole, and the black hole entropy $S_{BH} = S_{GPS}$.

S. Sachdev, PRL **105**, 151602 (2010)

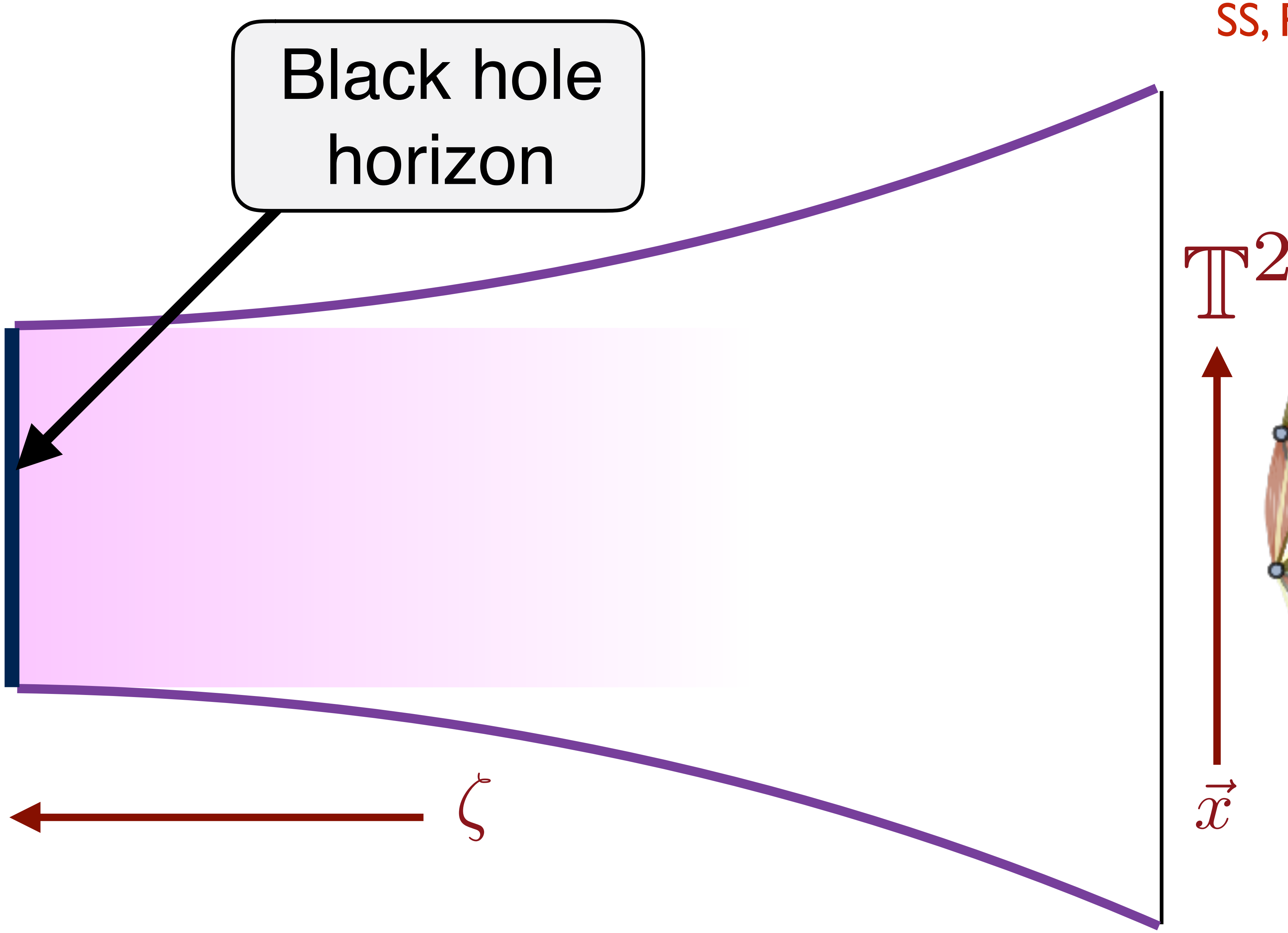
SYK and black holes

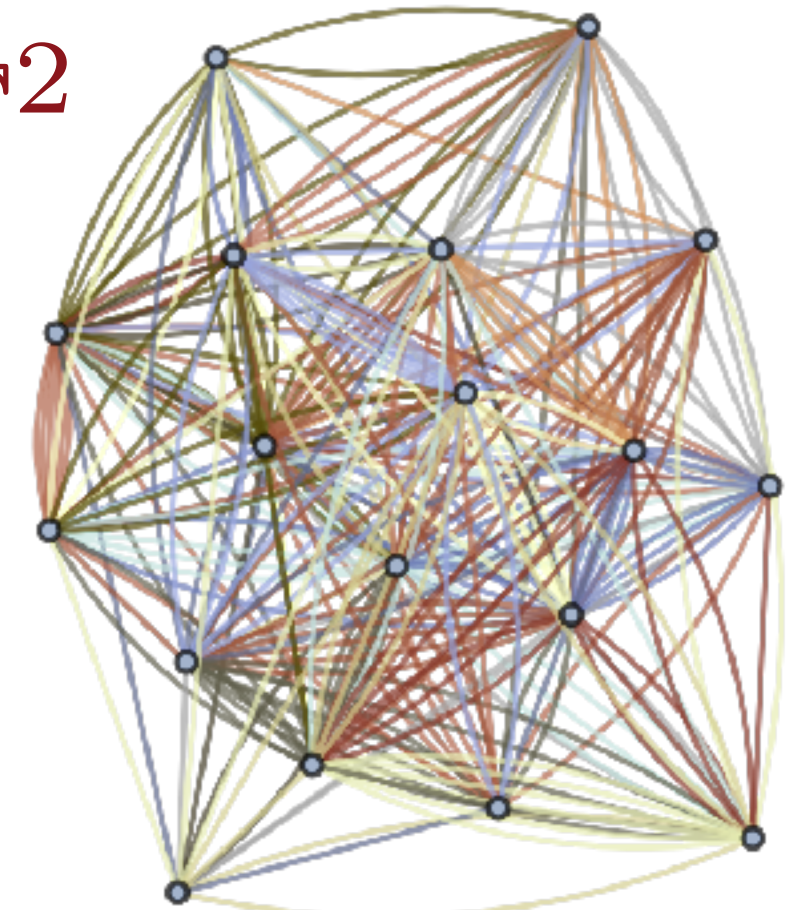


$\mathbb{T}^2 \Rightarrow$ two-dimensional torus

SYK and black holes

SS, PRL **105**, 151602 (2010)





The SYK model has "dual" description
in which an extra spatial dimension, ζ, emerges.
The curvature of this "emergent" spacetime is described
by Einstein's theory of general relativity

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished
S. Sachdev, PRX **5,** 041025 (2015)

SYK and AdS$_2$

Let us write the large N saddle point solutions of S as

$$\begin{aligned} G_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-1/2} \\ \Sigma_s(\tau_1 - \tau_2) &\sim (\tau_1 - \tau_2)^{-3/2}. \end{aligned}$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev, unpublished

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS_2.

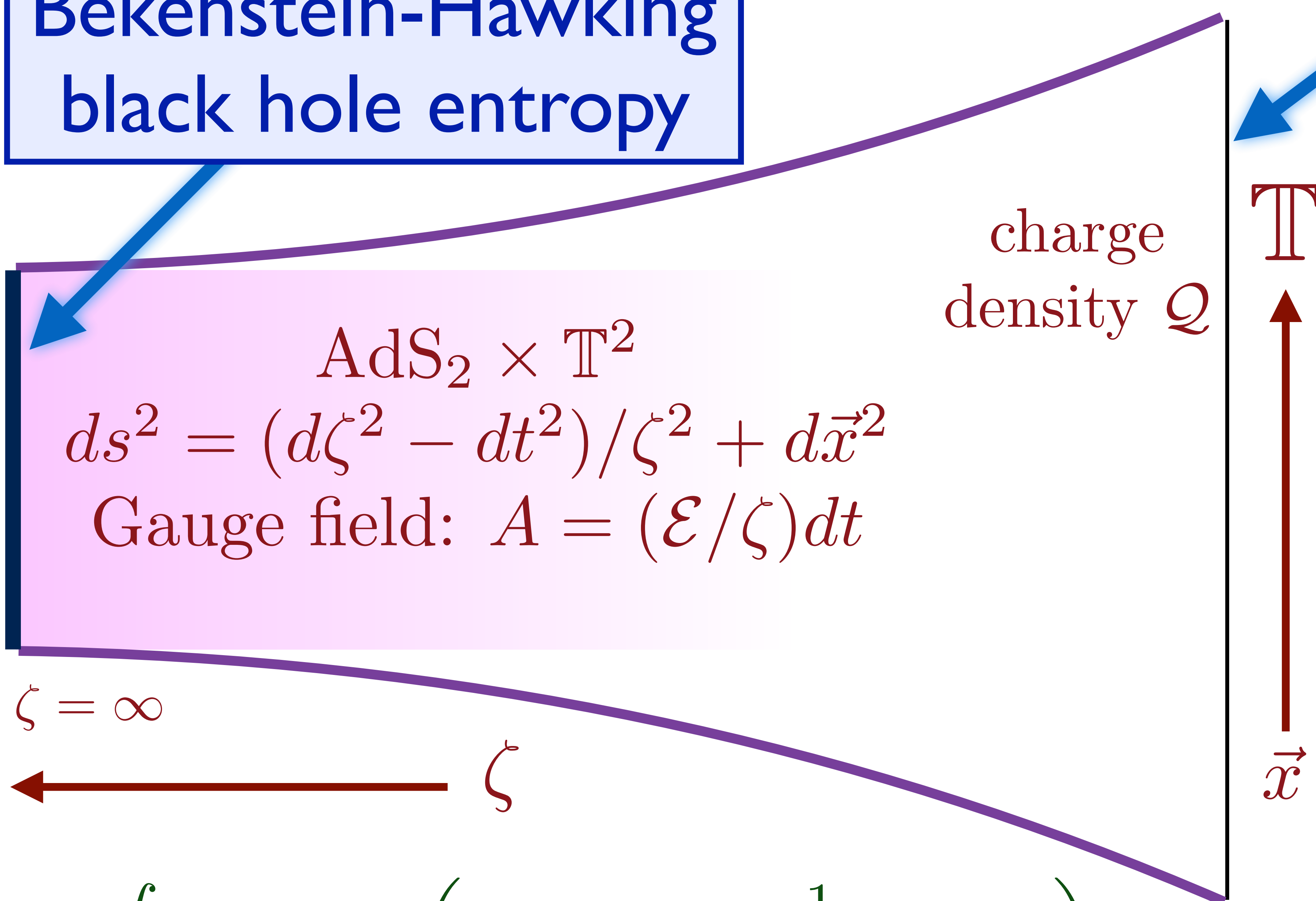
$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

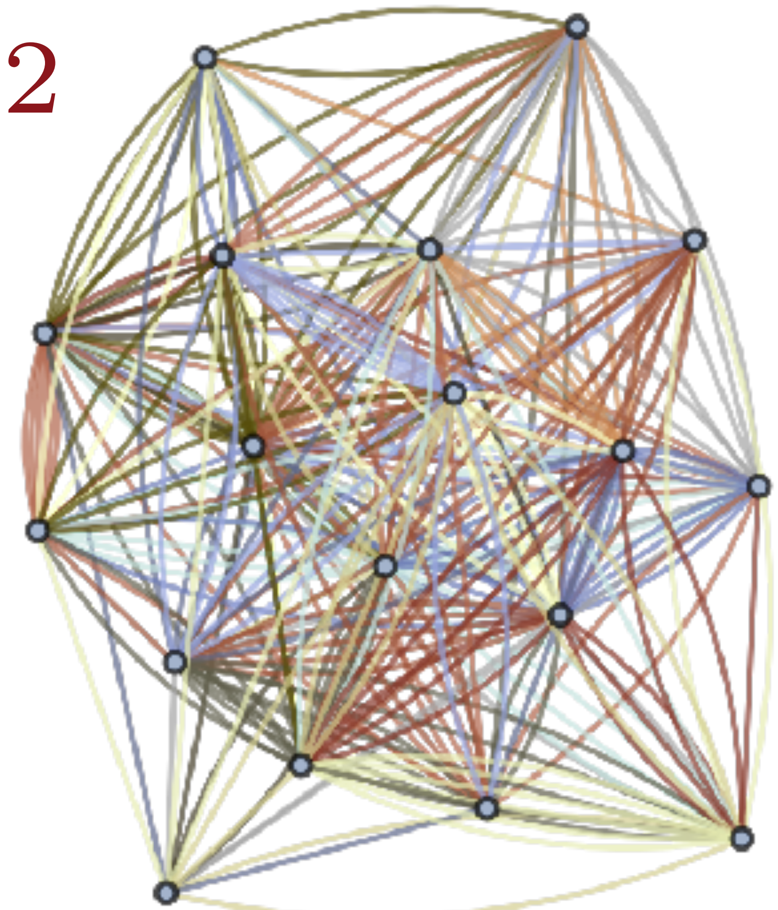
$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d}$$

with $ad - bc = 1$.

A. Kitaev, unpublished

SYK and black holes





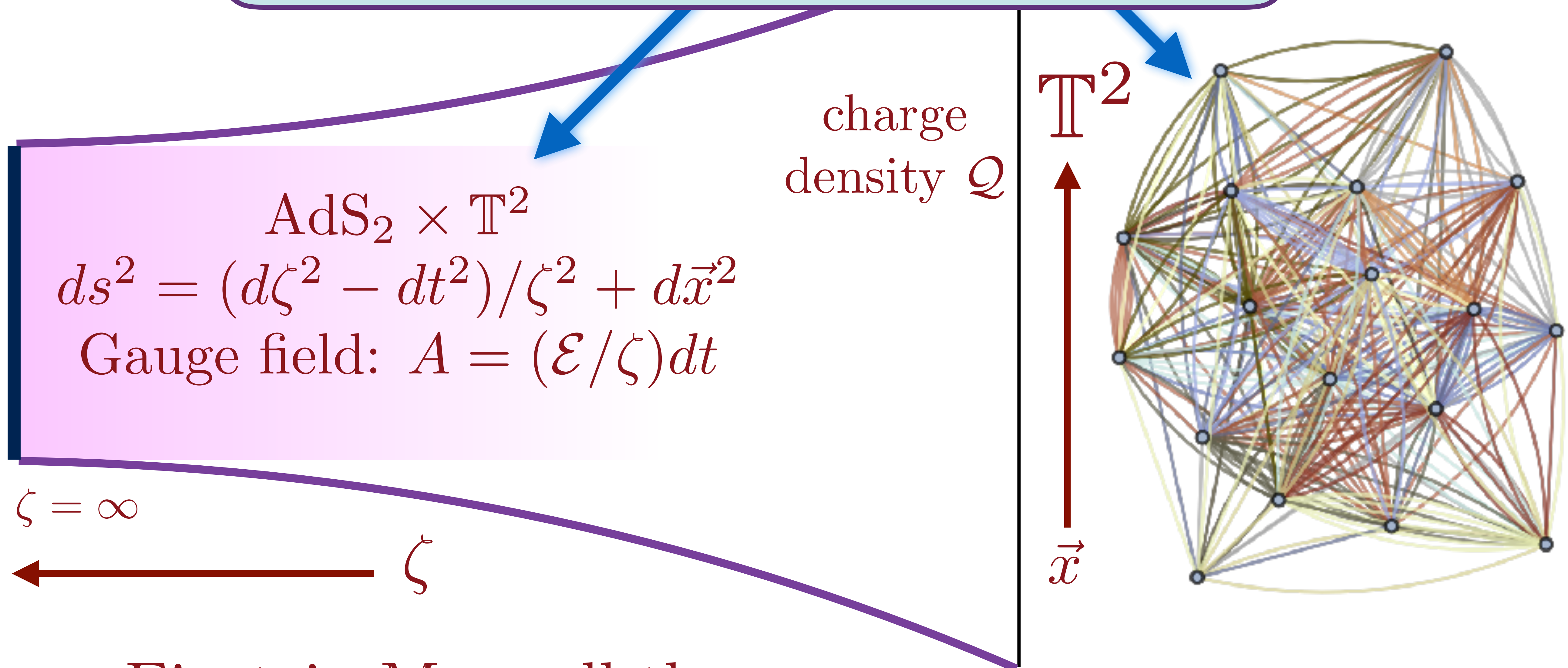
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \right)$$

SS, PRL **105**, 151602 (2010)

The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

SYK and black holes

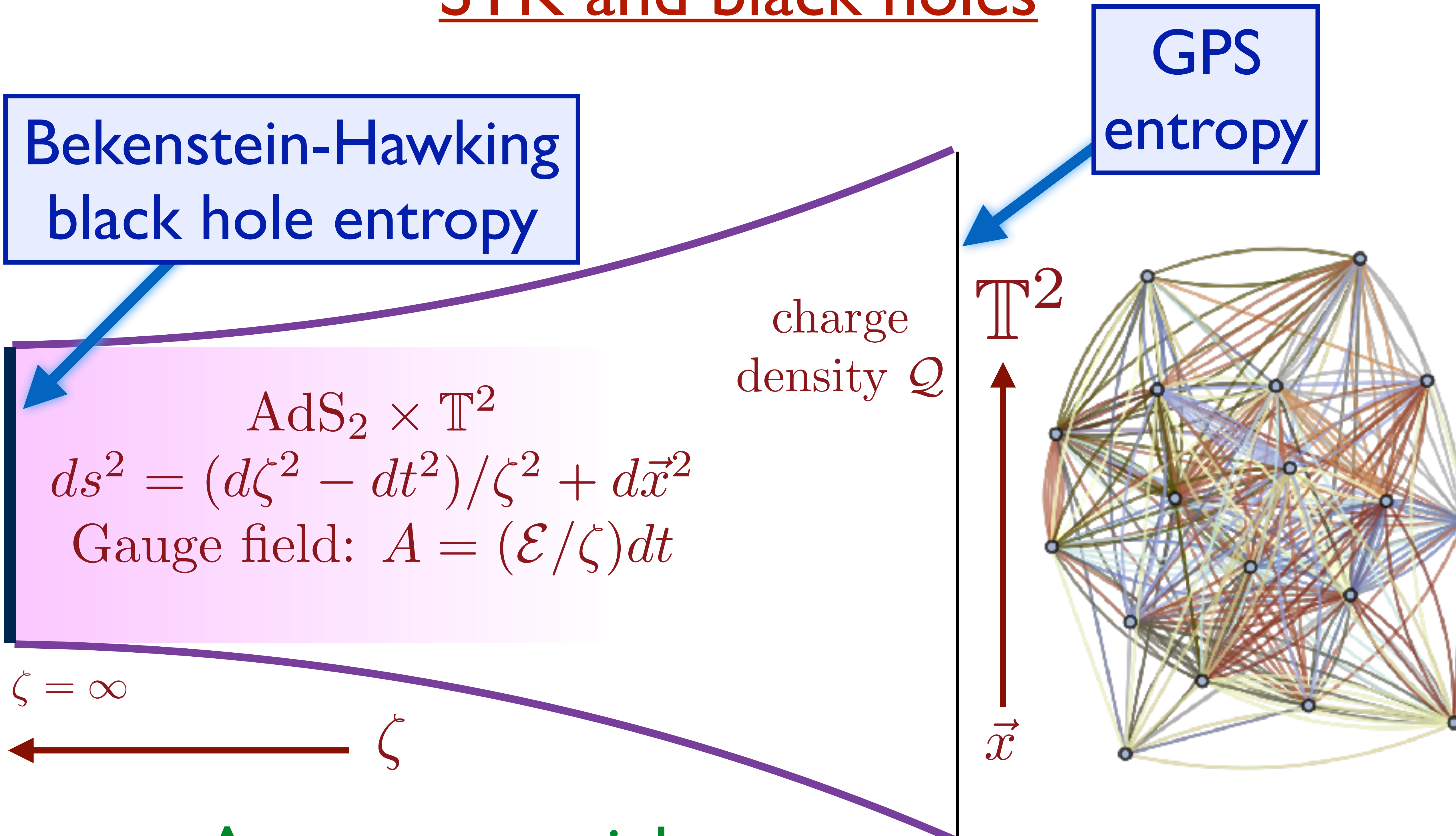
Same long-time effective action for energy and number fluctuations, involving Schwarzian derivatives of time reparameterizations $f(\tau)$.



Einstein-Maxwell theory
$+$ cosmological constant

A. Kitaev, unpublished, J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

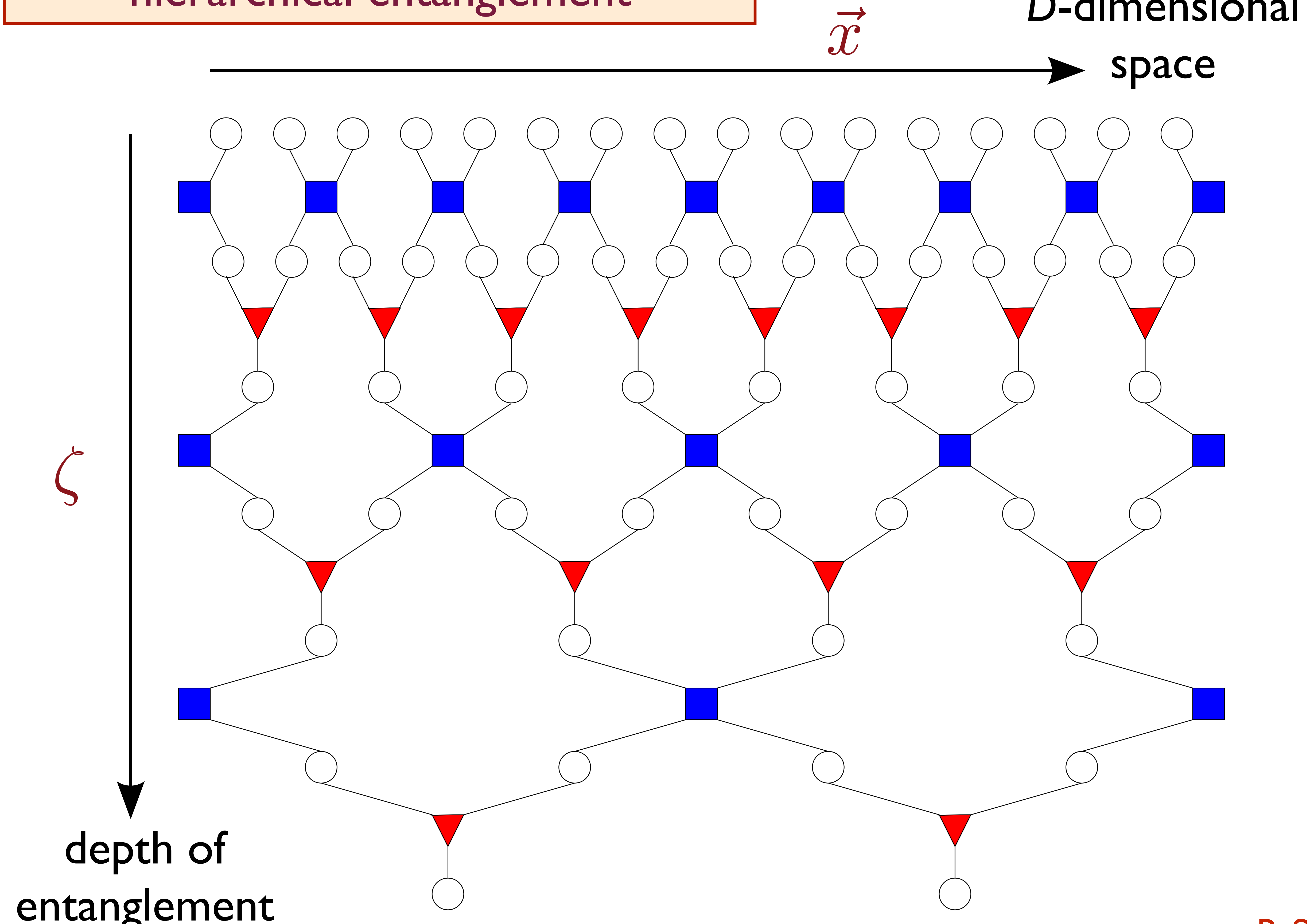
SYK and black holes



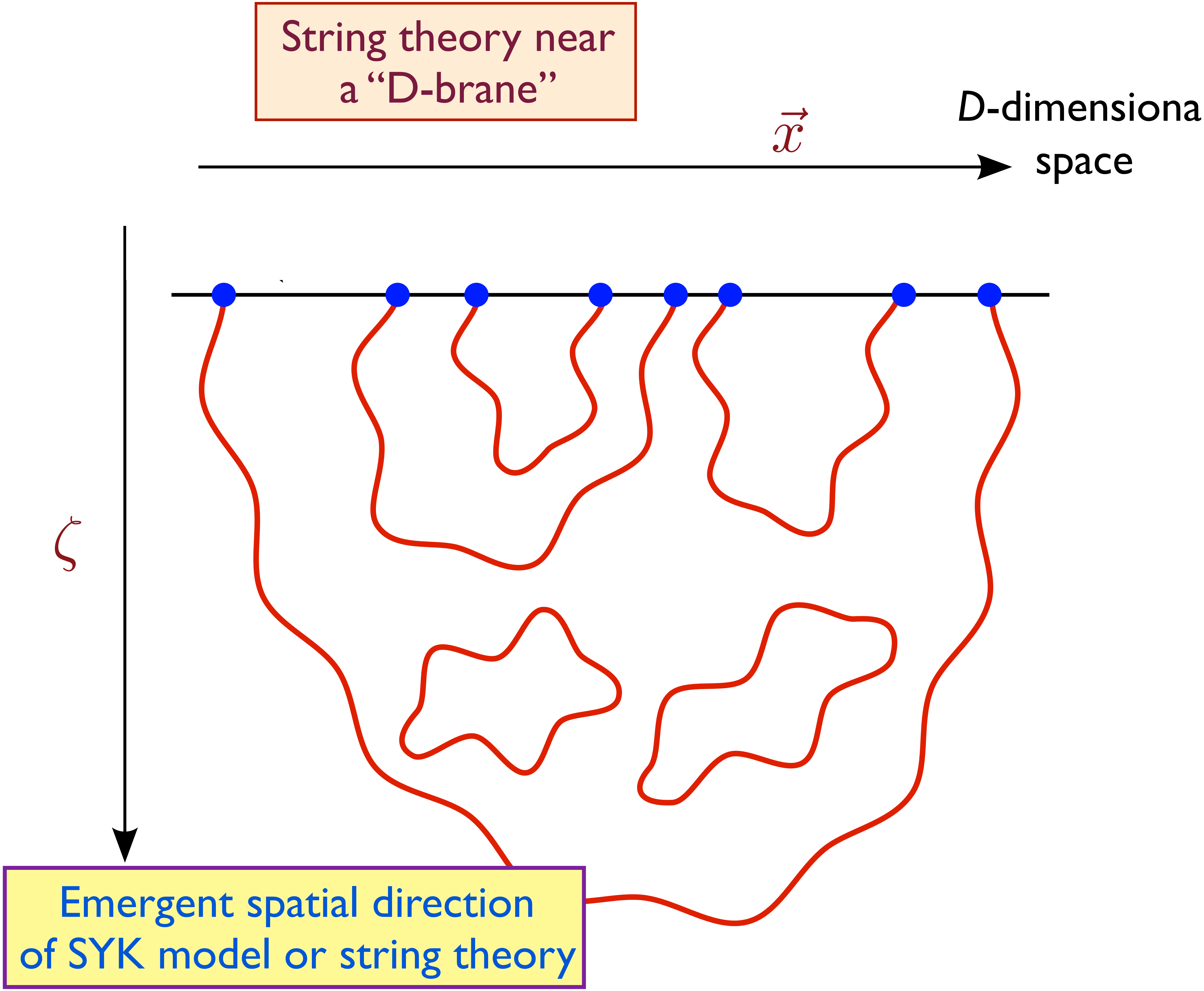
An extra spatial dimension emerges from quantum entanglement!

SS, PRL **105**, 151602 (2010)

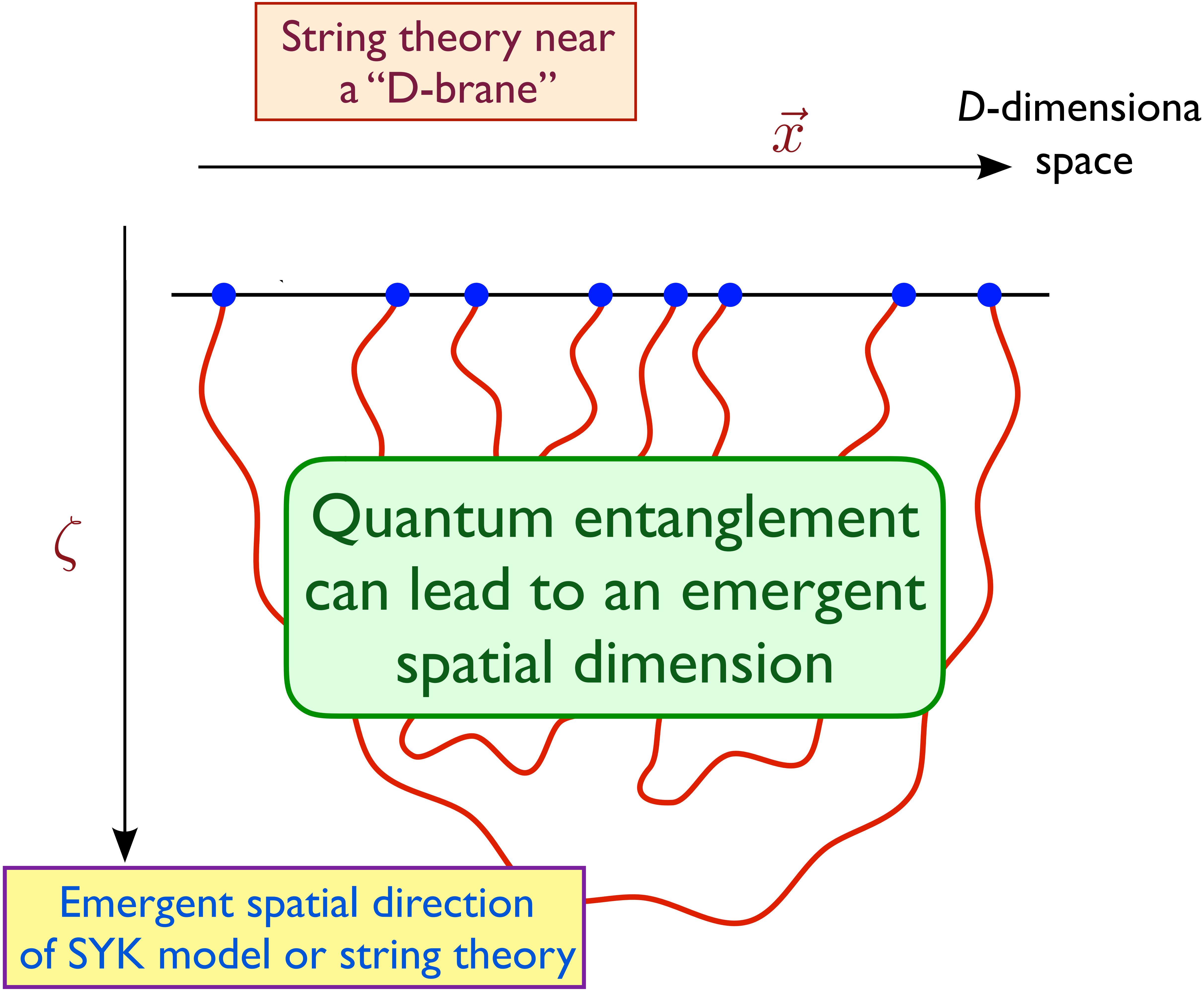
Tensor network of hierarchical entanglement



B. Swingle



String theory near a "D-brane"
$\vec{x}$
D-dimensional space
ζ
Emergent spatial direction of SYK model or string theory


String theory near a "D-brane"
$\vec{x}$
D-dimensional space
ζ
Quantum entanglement can lead to an emergent spatial dimension
Emergent spatial direction of SYK model or string theory

Coupled SYK models

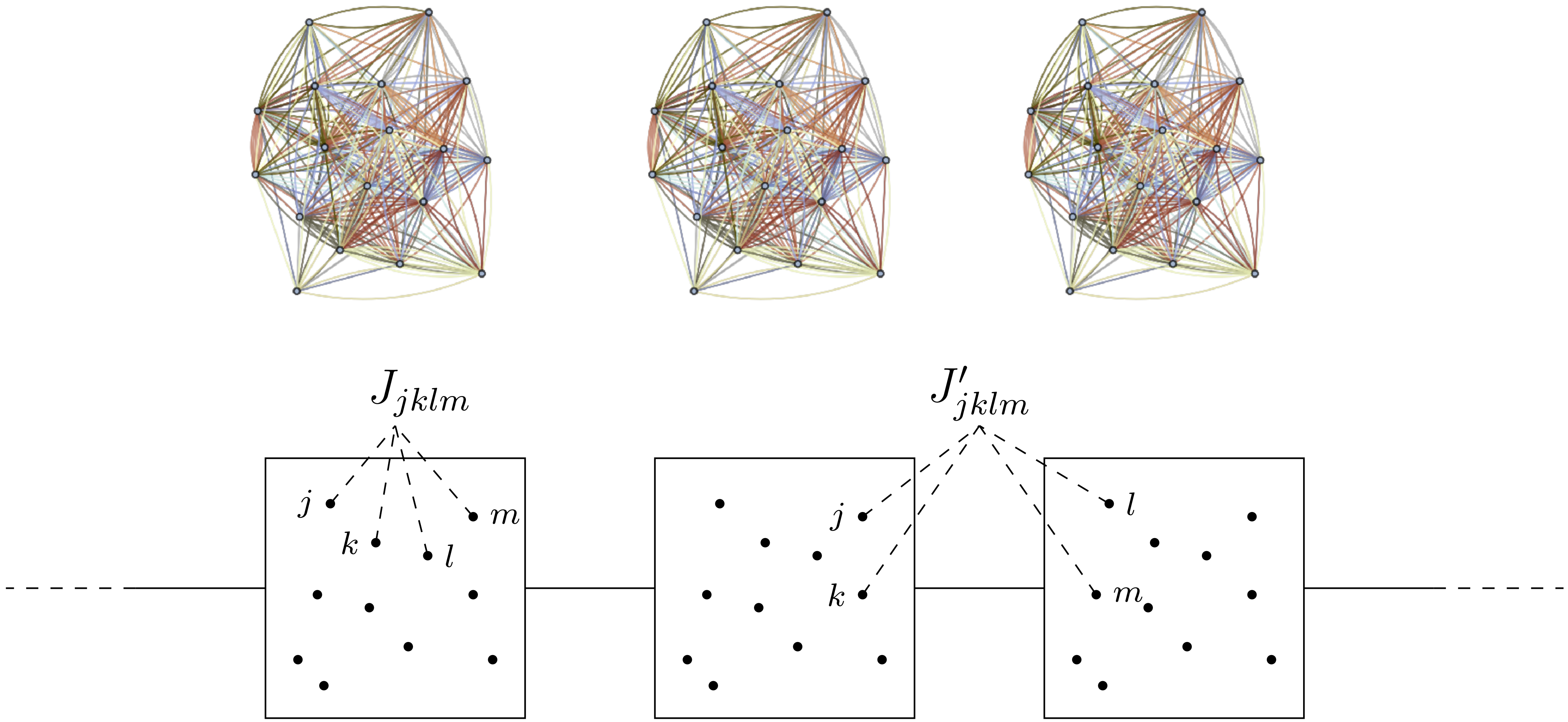
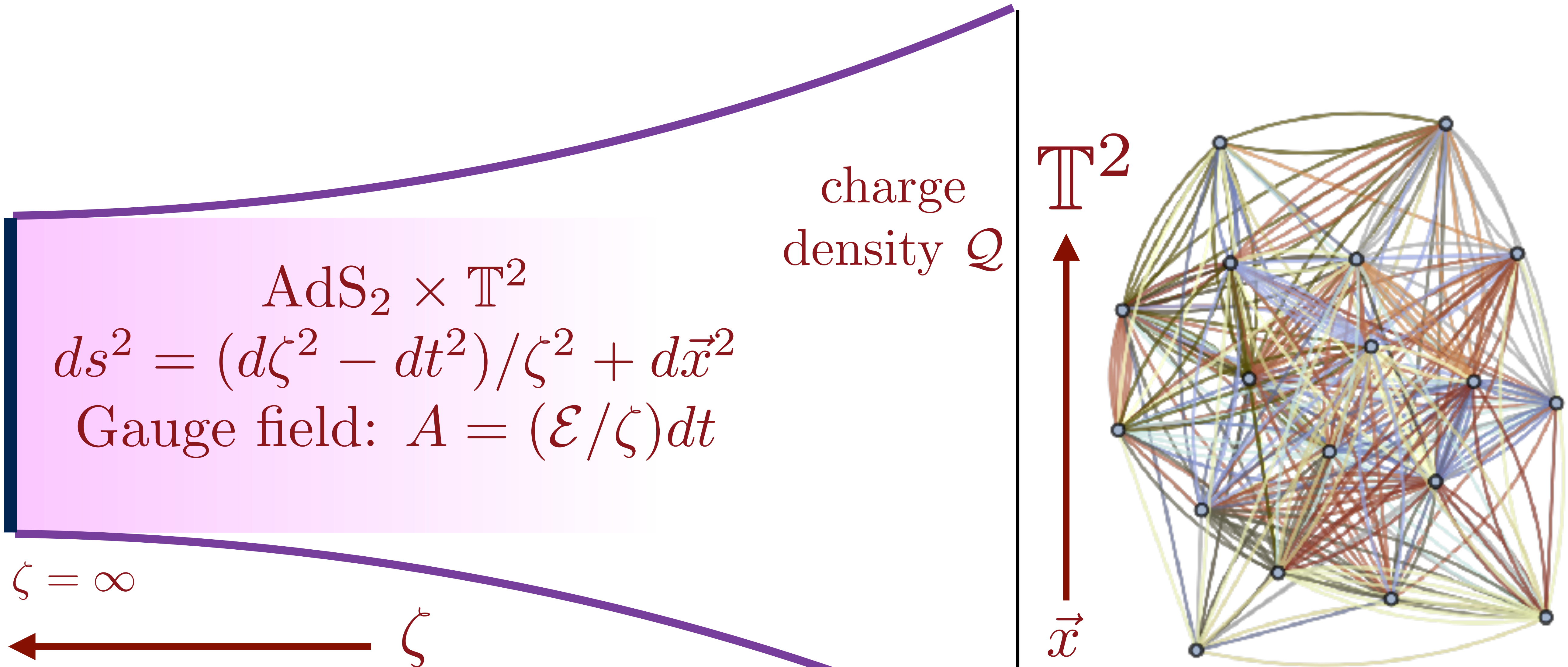


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

SYK and AdS_2



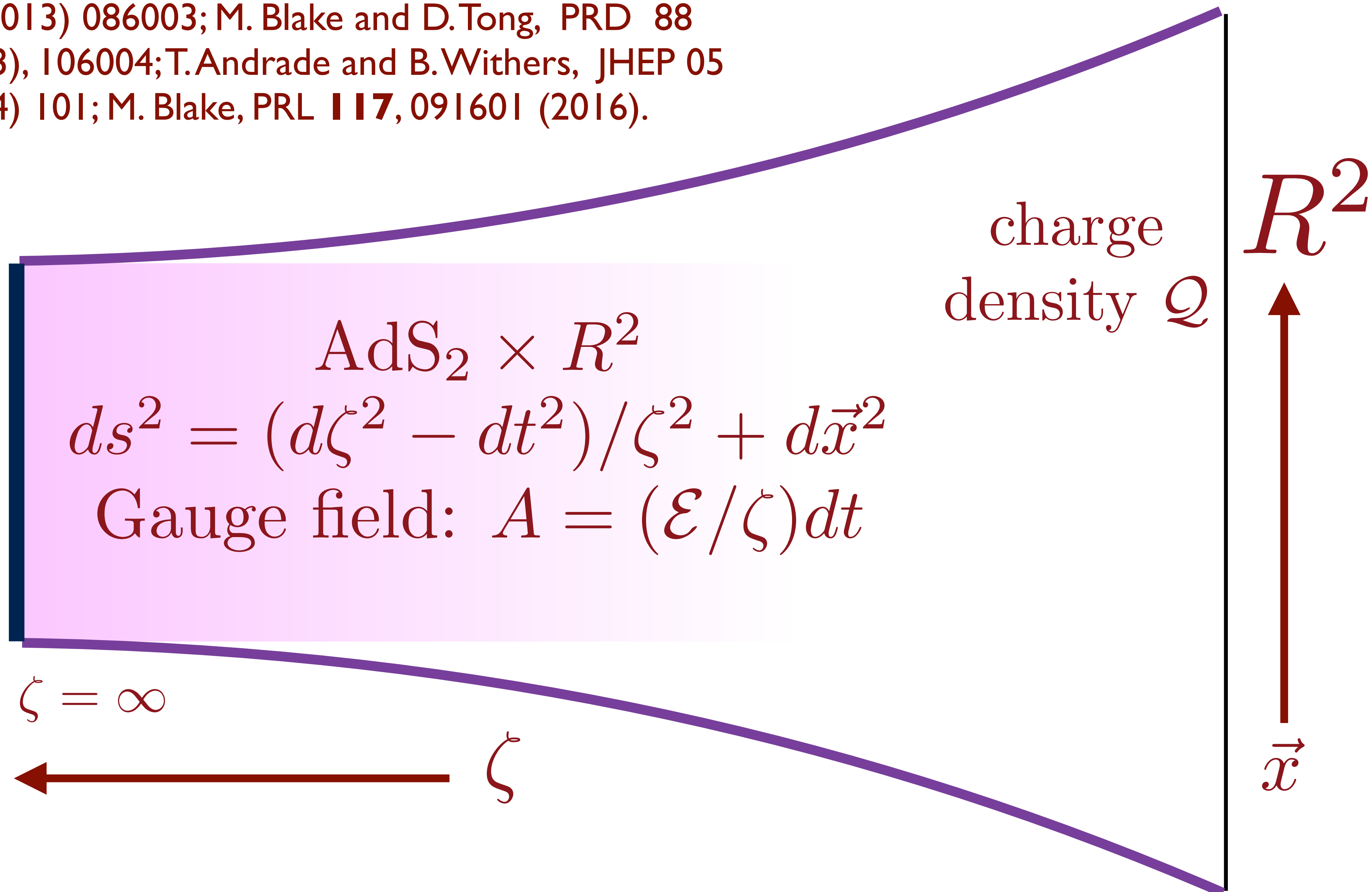
Einstein-Maxwell theory
+ cosmological constant

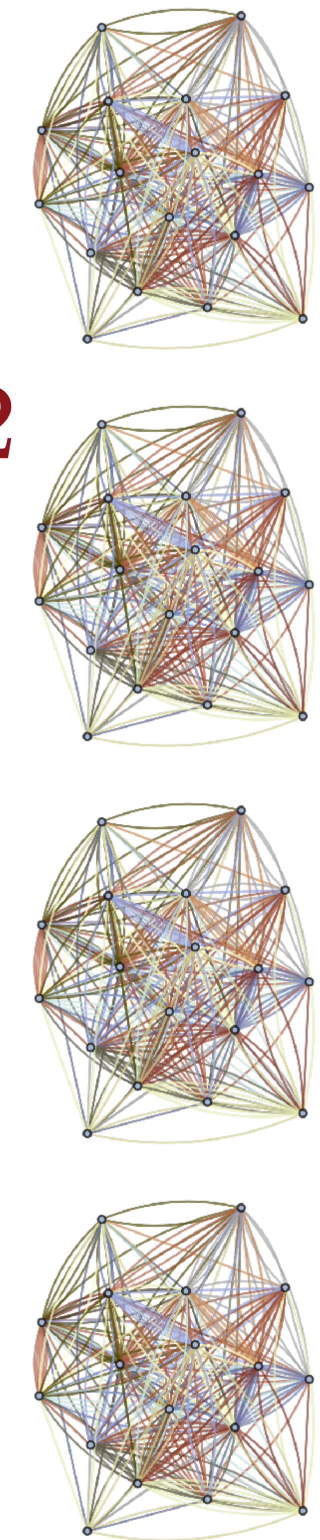
Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

S. Sachdev, PRL **105**, 151602 (2010)

Coupled SYK and AdS_4

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537; R. A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004; T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016).





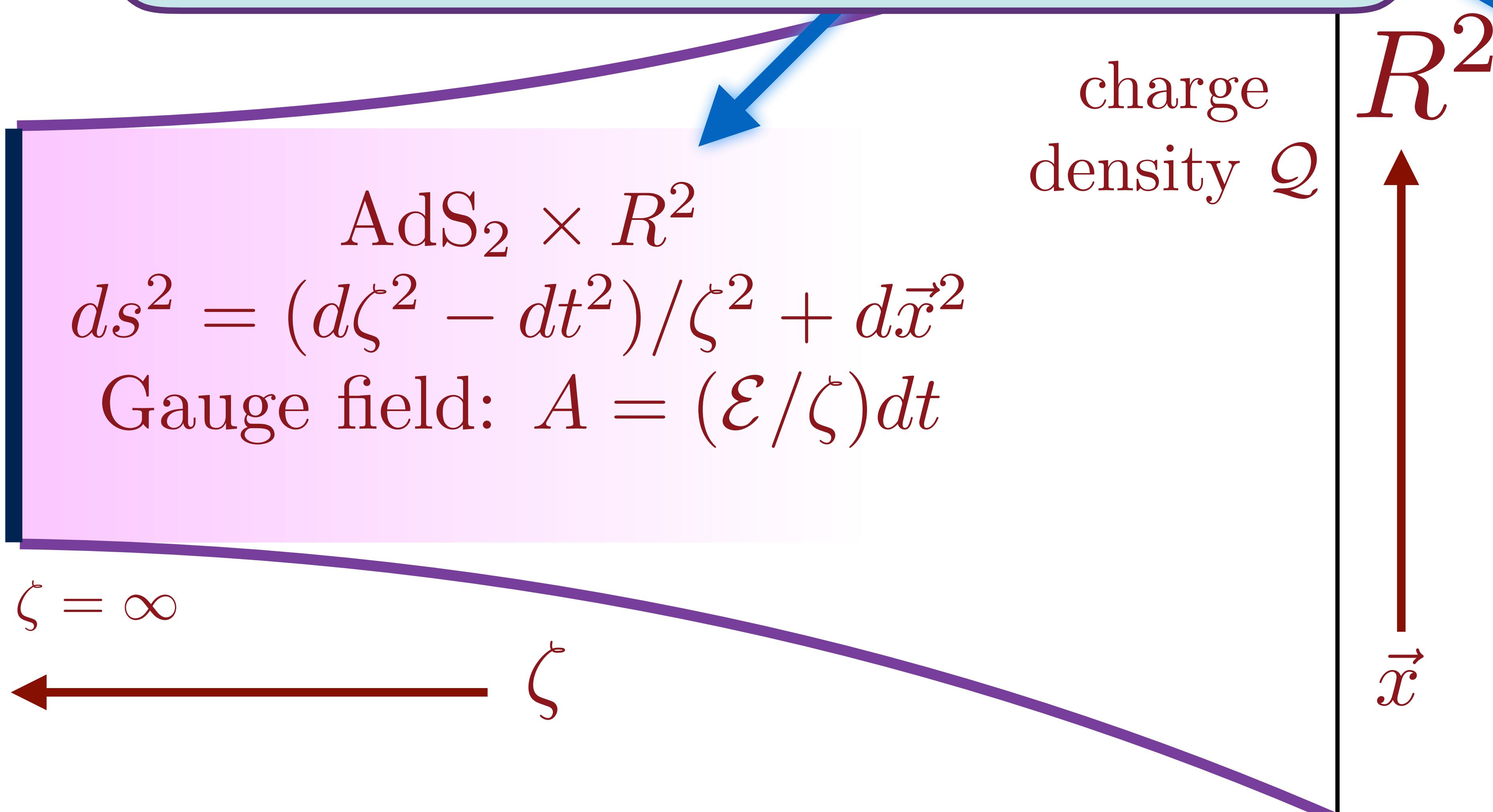
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv. 1612.00849

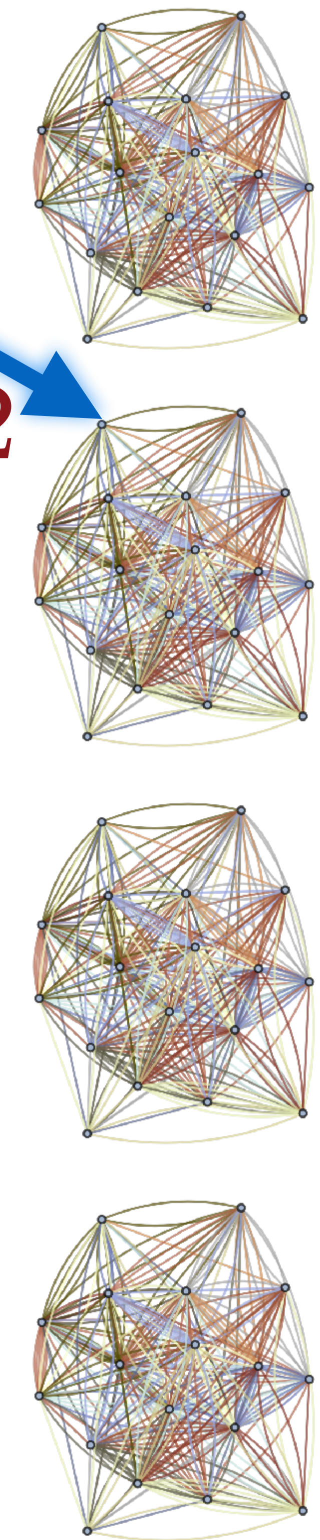
$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2} (\partial\hat{\varphi}_i)^2 - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

Coupled SYK and AdS$_4$

Matching correlators for thermoelectric diffusion, and quantum chaos
$\tau_L = \hbar/(2\pi k_B T)$, $v_B \sim T^{1/2}$,
and thermal diffusivity $D_E = v_B^2 \tau_L$





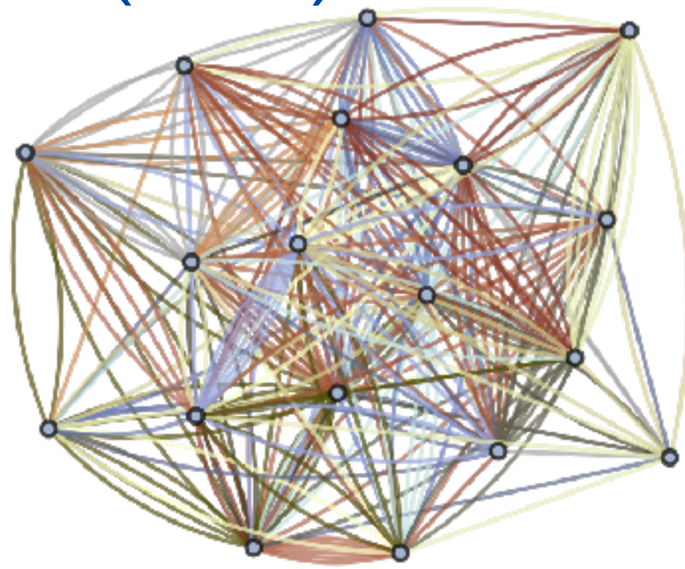
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv. 1612.00849

$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

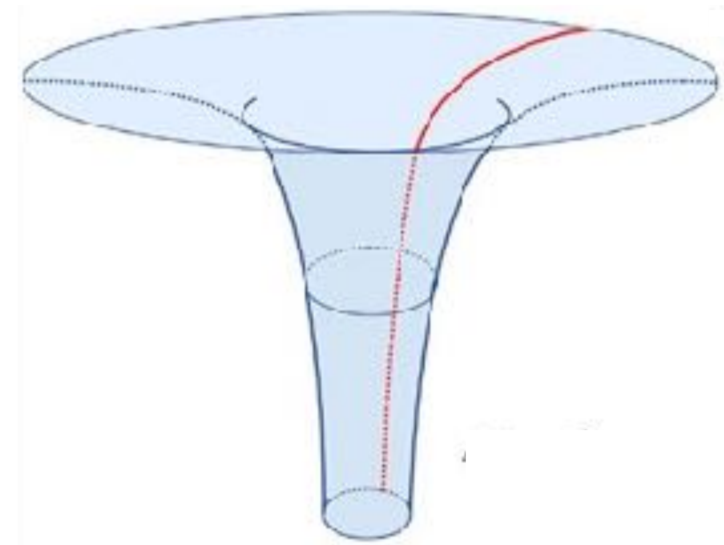
Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = k x_i$ leading to momentum disspation

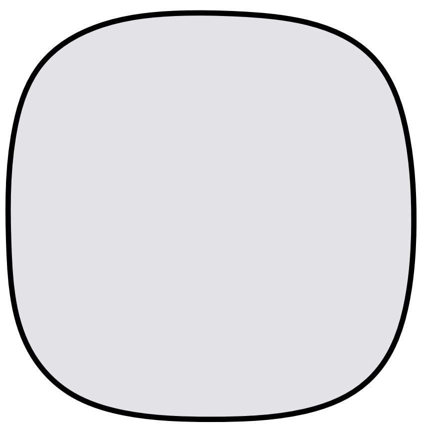
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



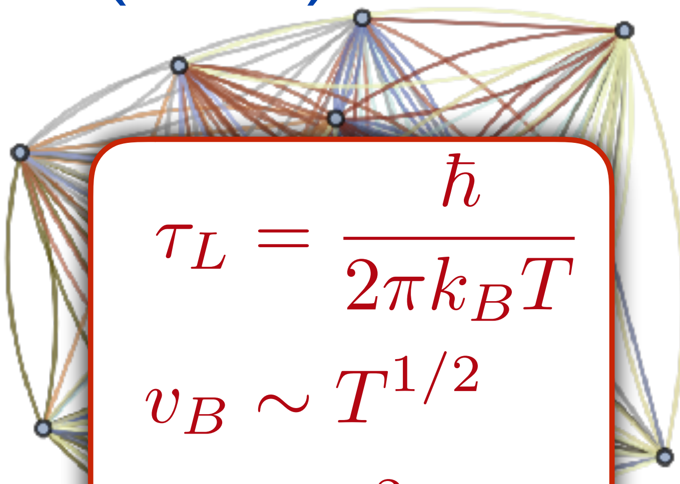


Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

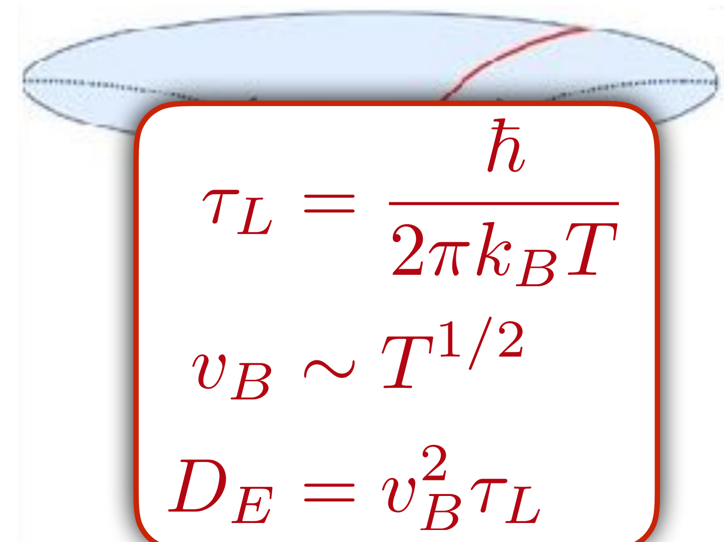


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

$$v_B \sim T^{1/2}$$

$$D_E = v_B^2 \tau_L$$

Black holes with AdS_2 horizons



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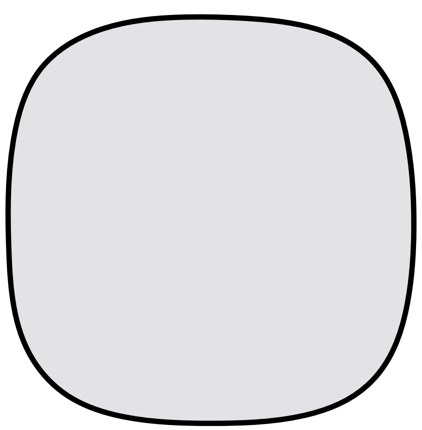


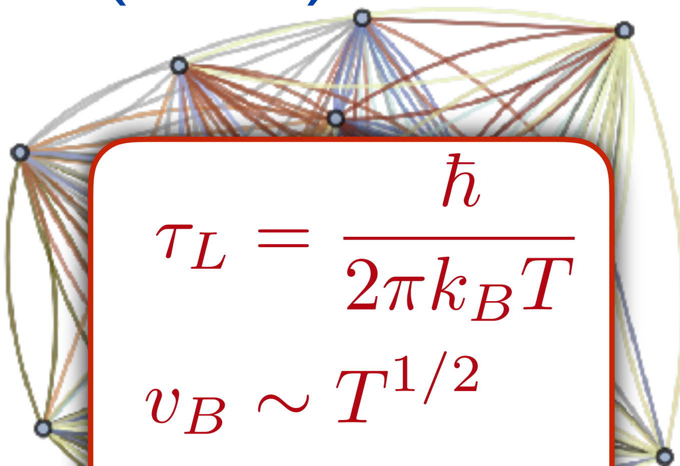
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



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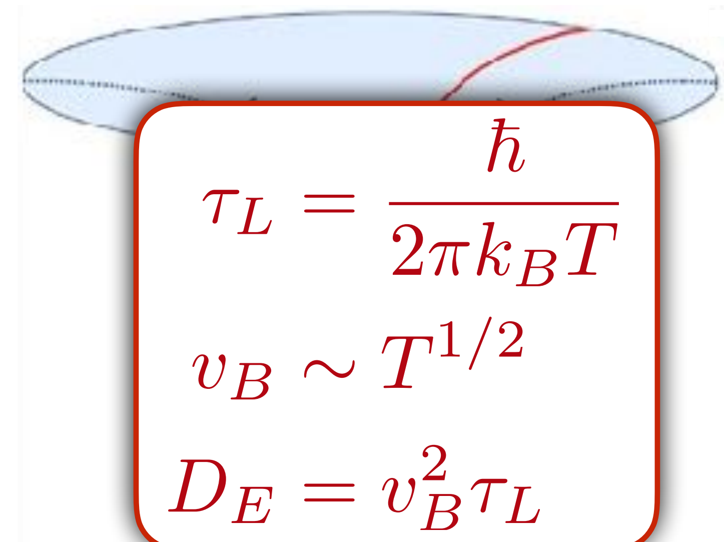

$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

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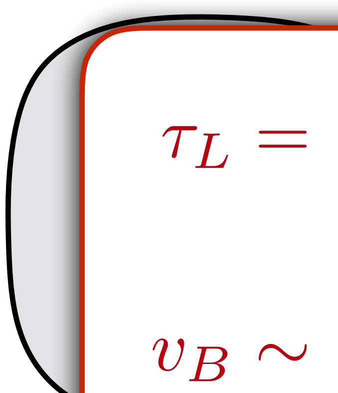
A. A. Patel and S. Sachdev, arXiv: 1611.00003

...rmi surface coupled to a gauge field



$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

$$v_B \sim \frac{N v_F^{5/3}}{e^{4/3}\gamma^{1/3}} T^{1/3}$$

$$D_E = 0.42 v_B^2 \tau_L$$

$$\mathcal{L}[\Psi \ldots \frac{(\nabla - i\vec{a})^2}{2m} - \mu \Big) \Psi + \frac{1}{2g^2}(\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

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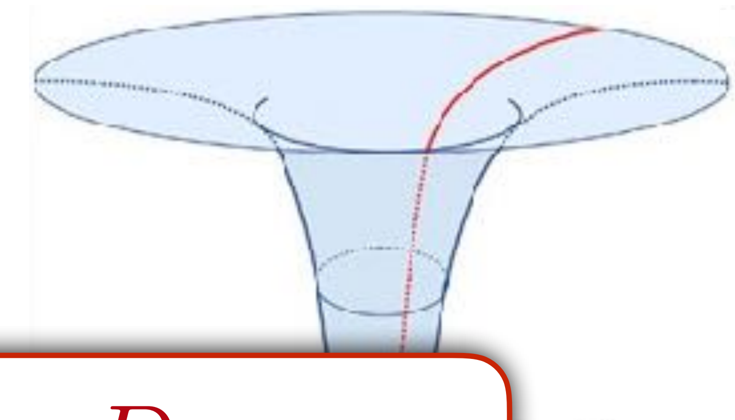
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

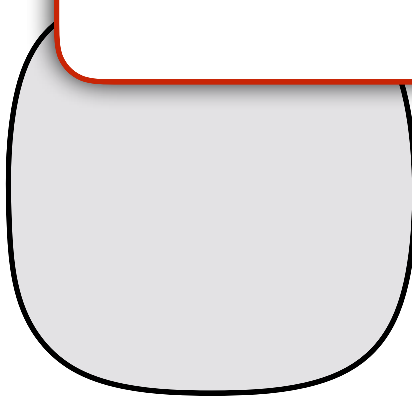
Black holes with AdS_2 horizons



Thermal diffusivity, D_E:

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$

in all three models



Fermi surface coupled to a gauge field

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τ_L: the Lyapunov time to reach quantum chaos

v_B: the "butterfly velocity" for the spatial propagation of chaos

Quantum entanglement

Black holes

Strange metals

A "toy model" which is both a strange metal and a black hole!

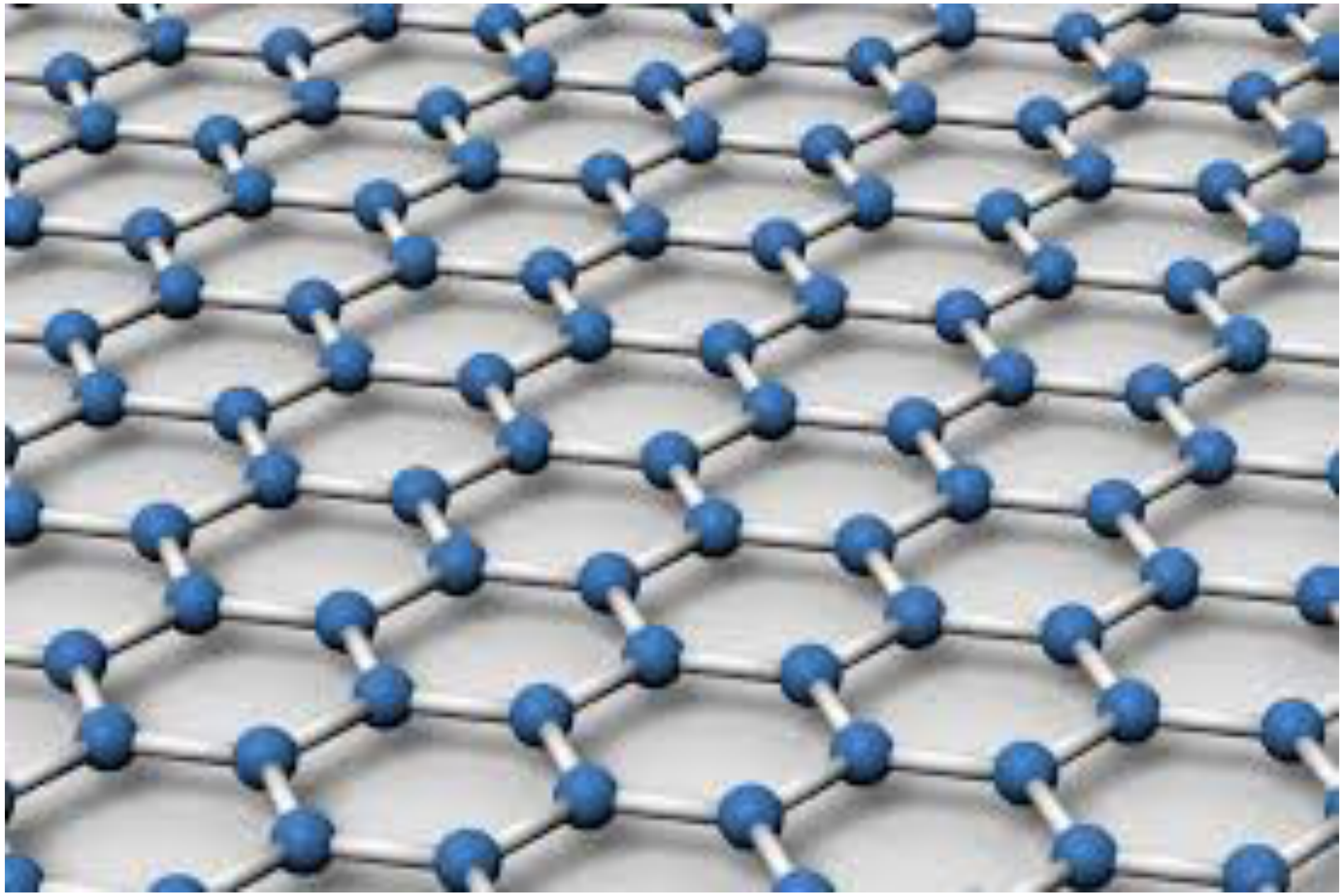
- Graphene

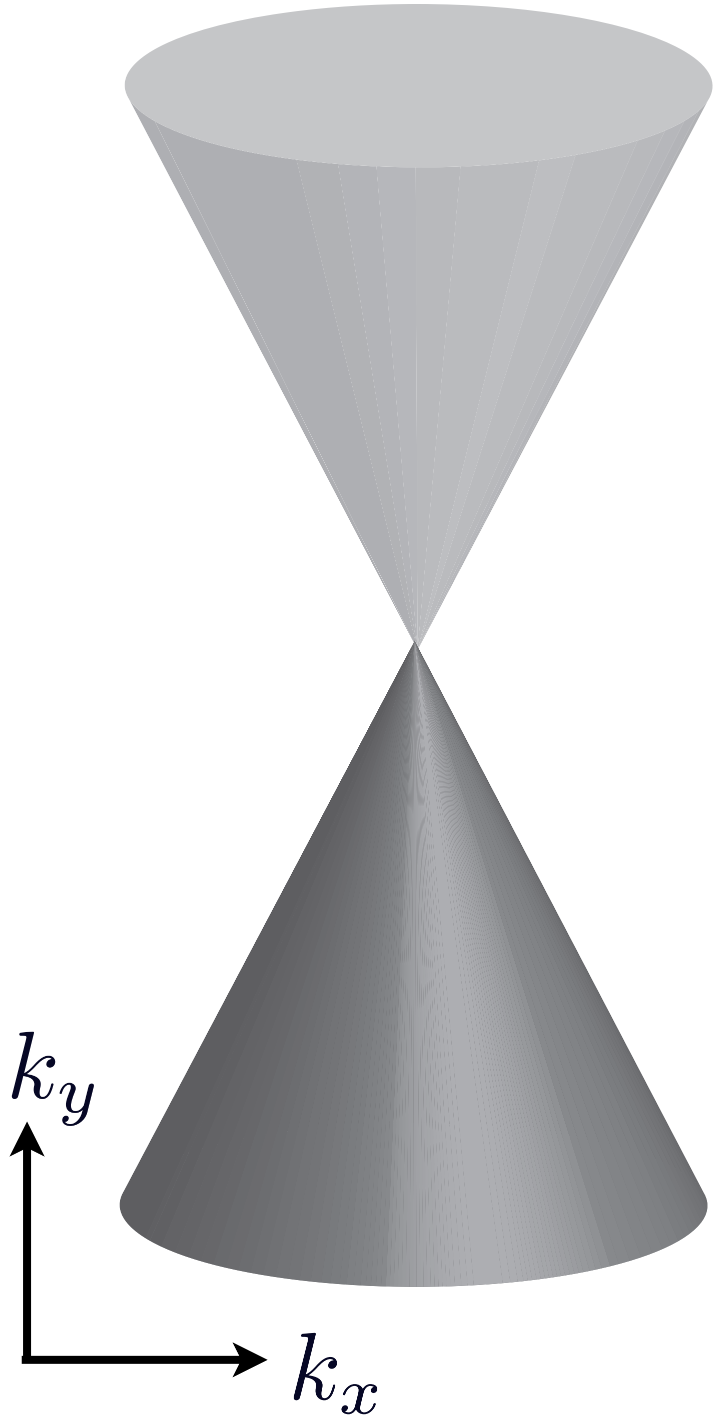
Non-quasiparticle “strange metal” transport

Theoretical predictions inspired by holography

Comparison with experiments

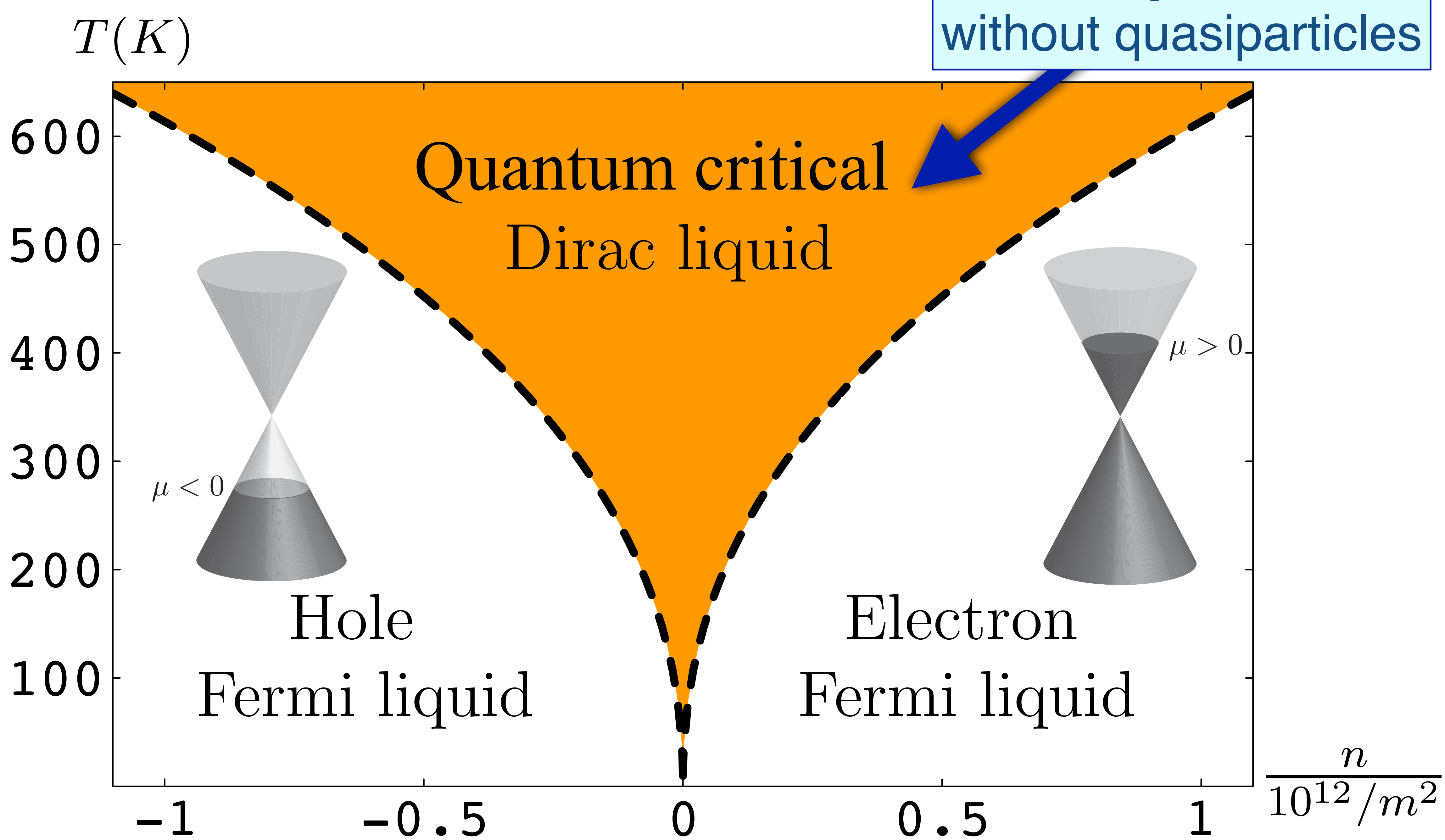
Graphene





Graphene

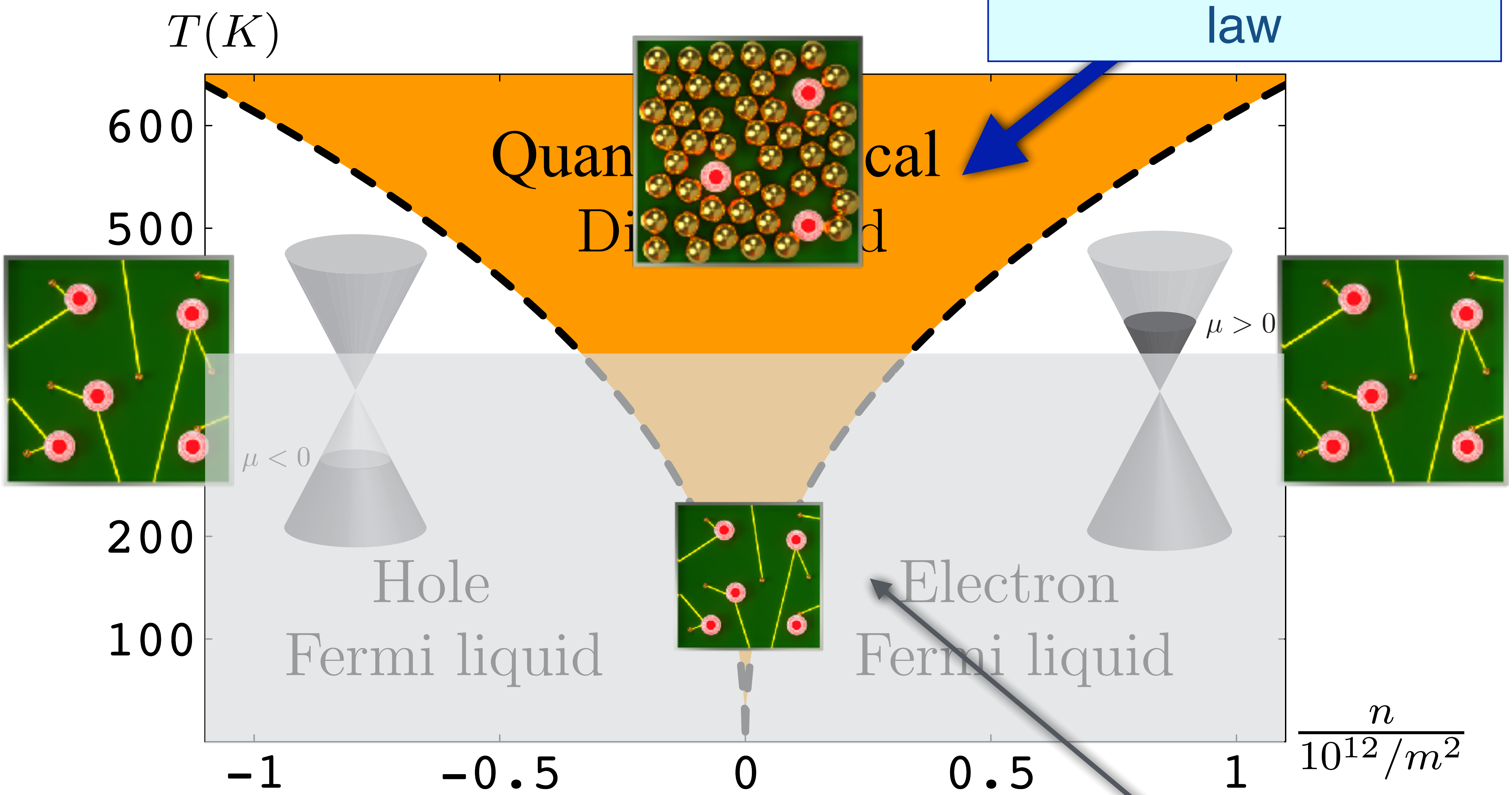
Predicted
“strange metal”
without quasiparticles



K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

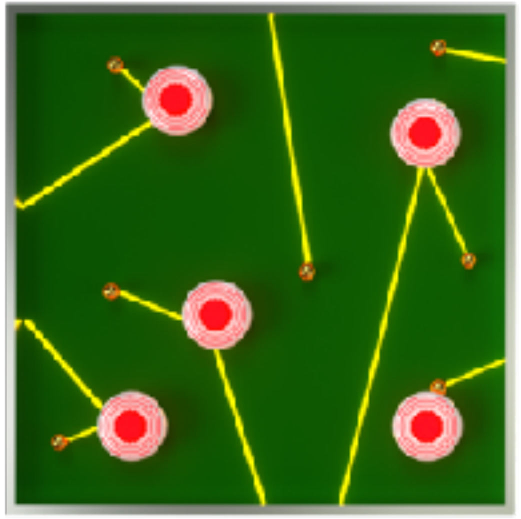
Large violations of the Wiedemann-Franz law



Impurity scattering dominates

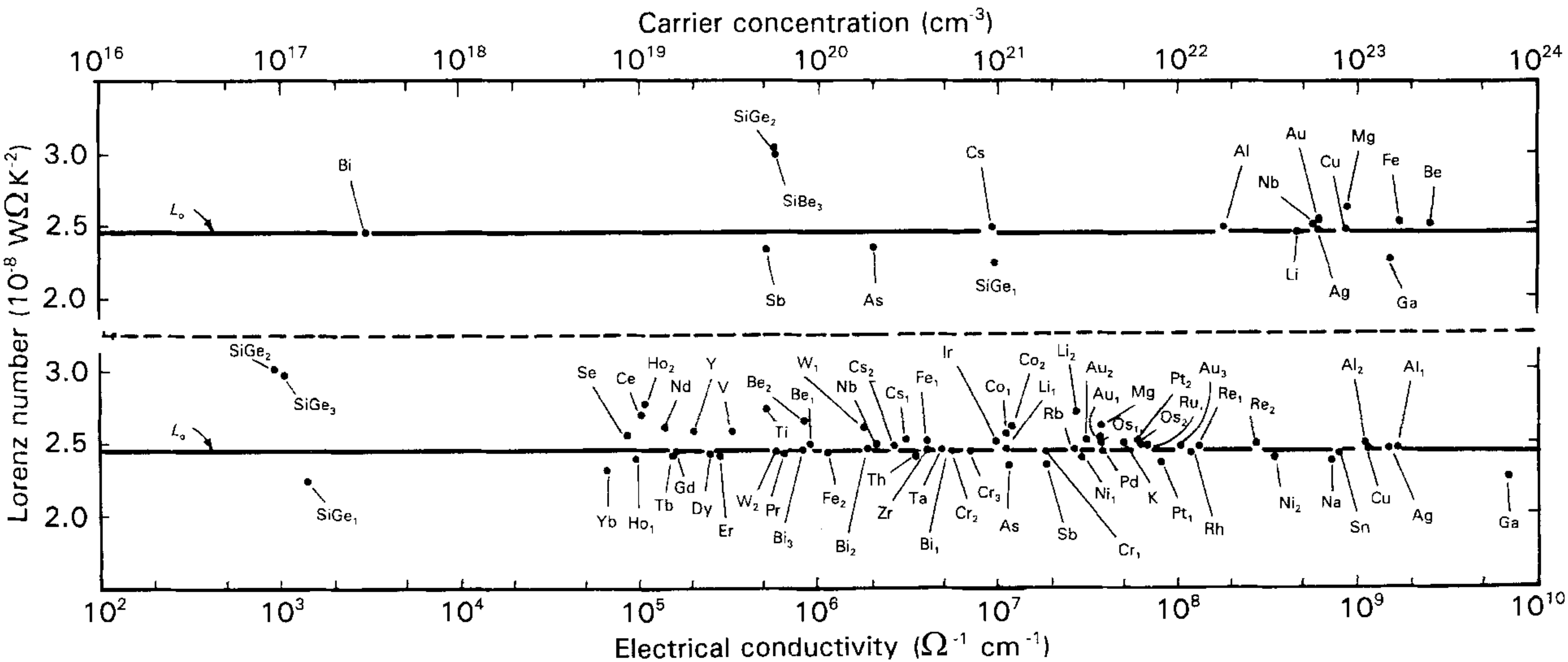
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal and electrical conductivity with quasiparticles



- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Prediction for transport in the graphene strange metal

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield for the Lorentz ratio $L = \kappa/(T\sigma)$

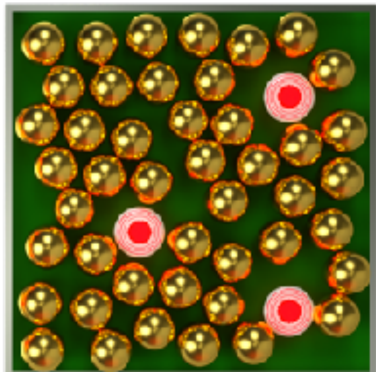
$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right)^{-2} ,$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

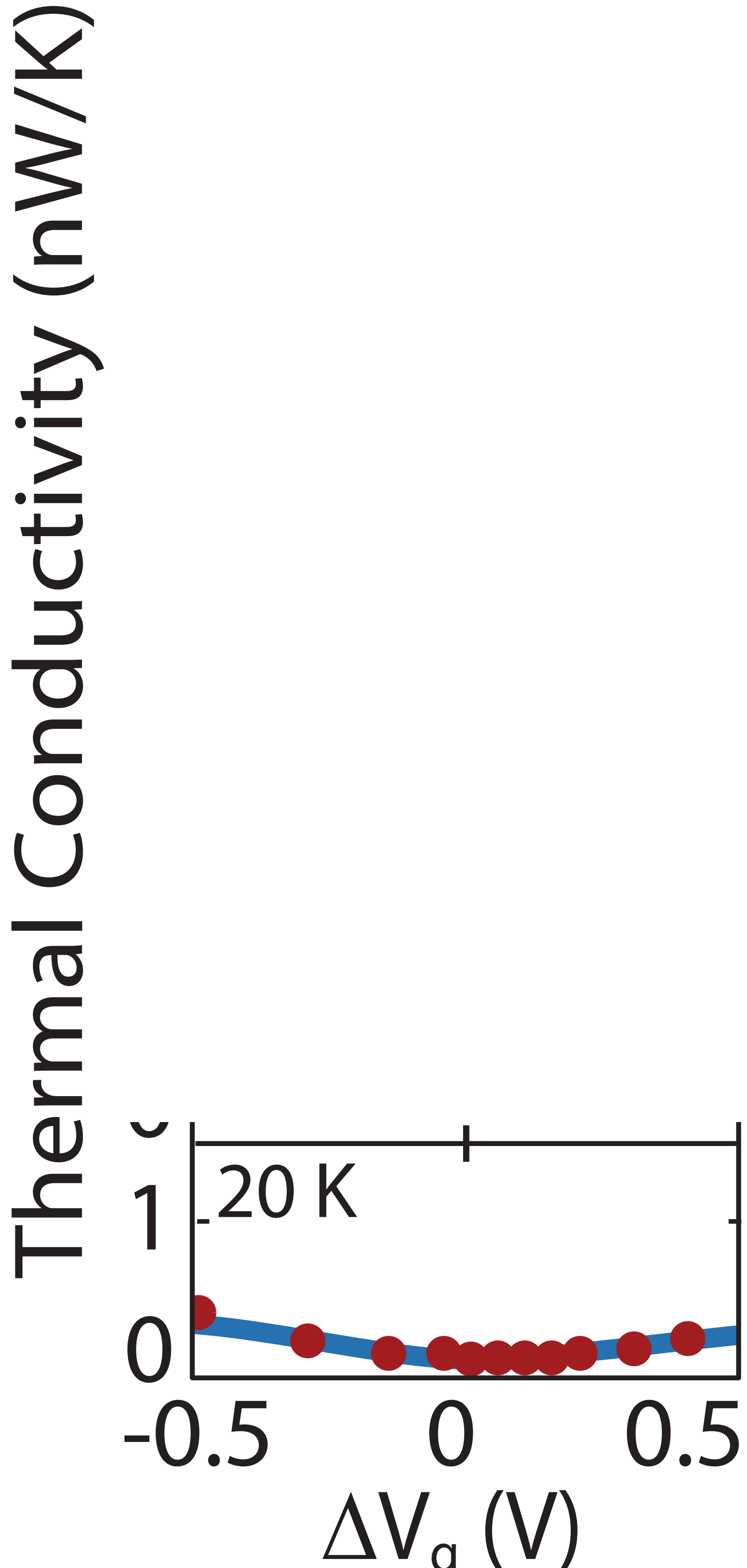
- For $\mathcal{Q} = 0$, as $\tau_{\rm imp} \to \infty$, σ remains finite, while $\kappa \to \infty$, and so $L \to \infty$.
- For $\mathcal{Q} \neq 0$, as $\tau_{\rm imp} \to \infty$, $\sigma \to \infty$, while κ remains finite, and so $L \to 0$.

Prediction: L diverges as $1/\mathcal{Q}^4$ near $\mathcal{Q} = 0$ in clean graphene.



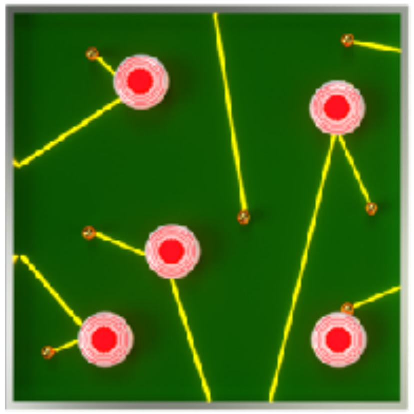
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

J. Crossno, Jing K. Shi, Ke Wang, Xiaomeng Liu, A. Harzheim, A. Lucas, S. Sachdev, Philip Kim, Takashi Taniguchi, Kenji Watanabe, T. A. Ohki, and Kin Chung Fong, Science **351**, 1058 (2016)



Red dots: data

Blue line: value for $L = L_0$



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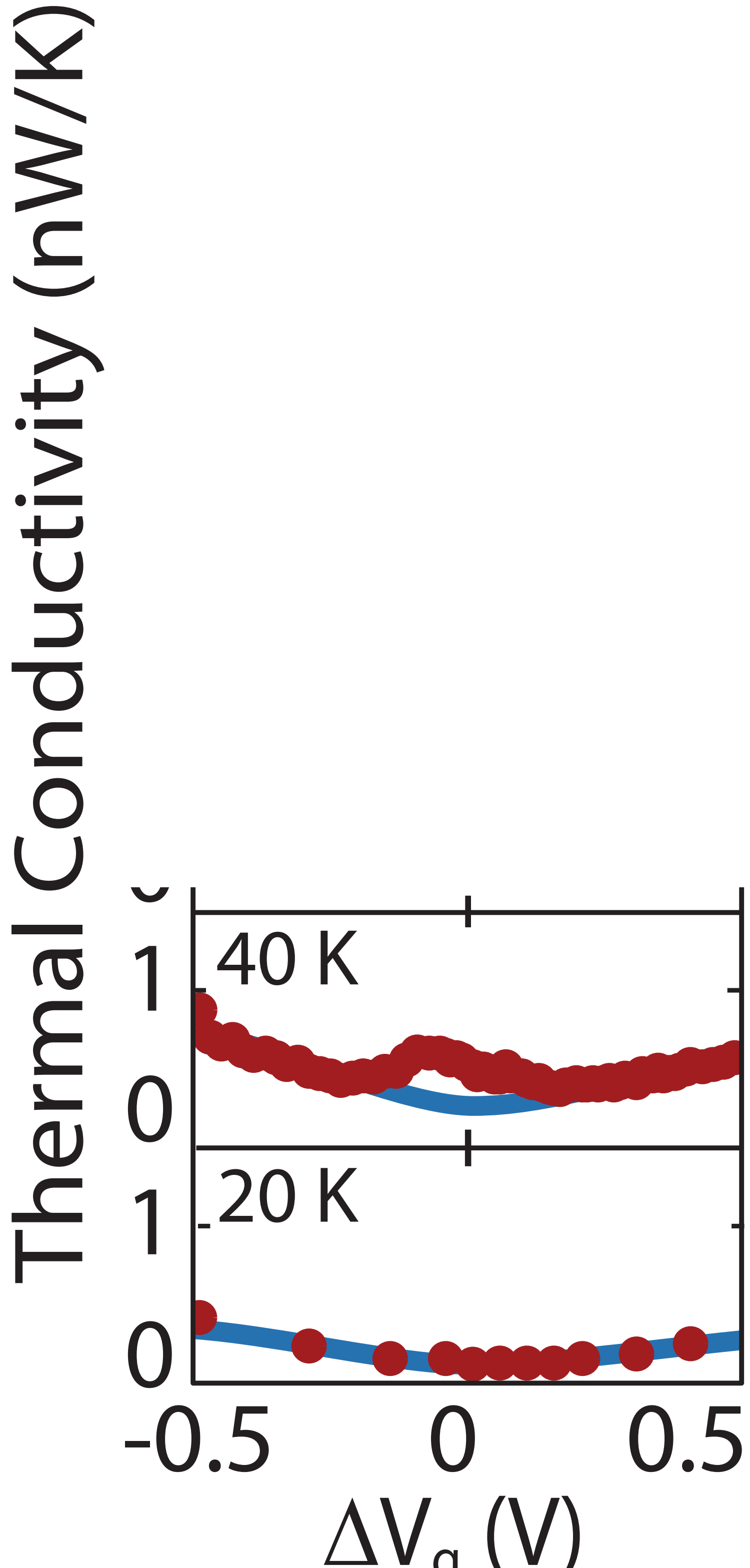

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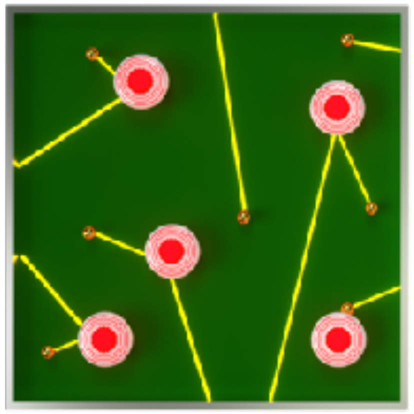

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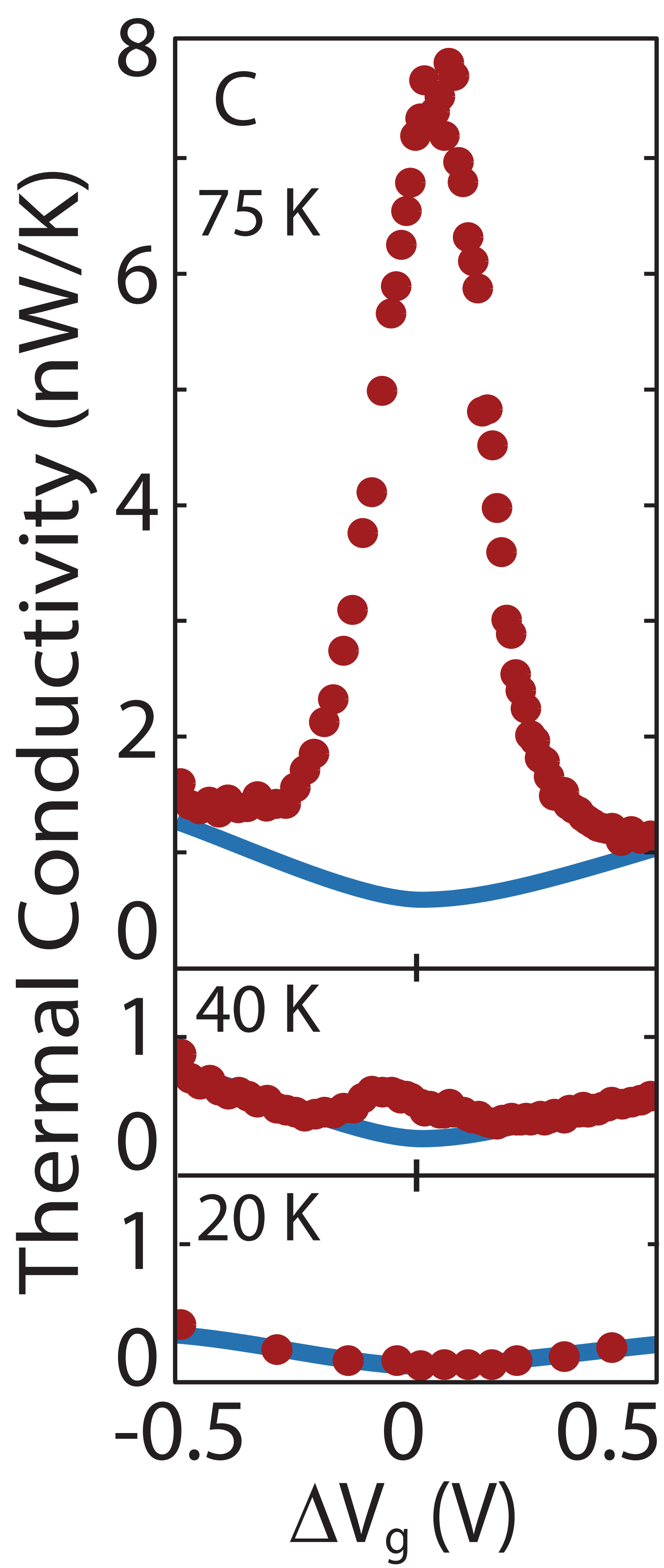

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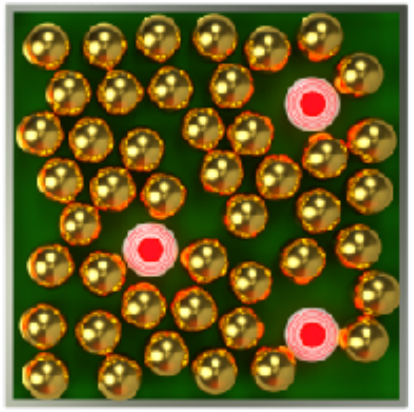

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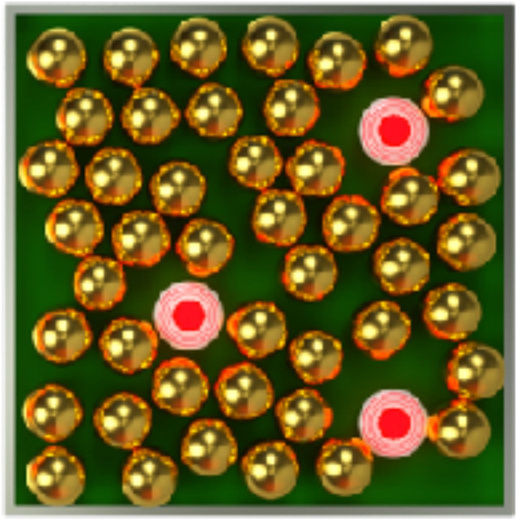
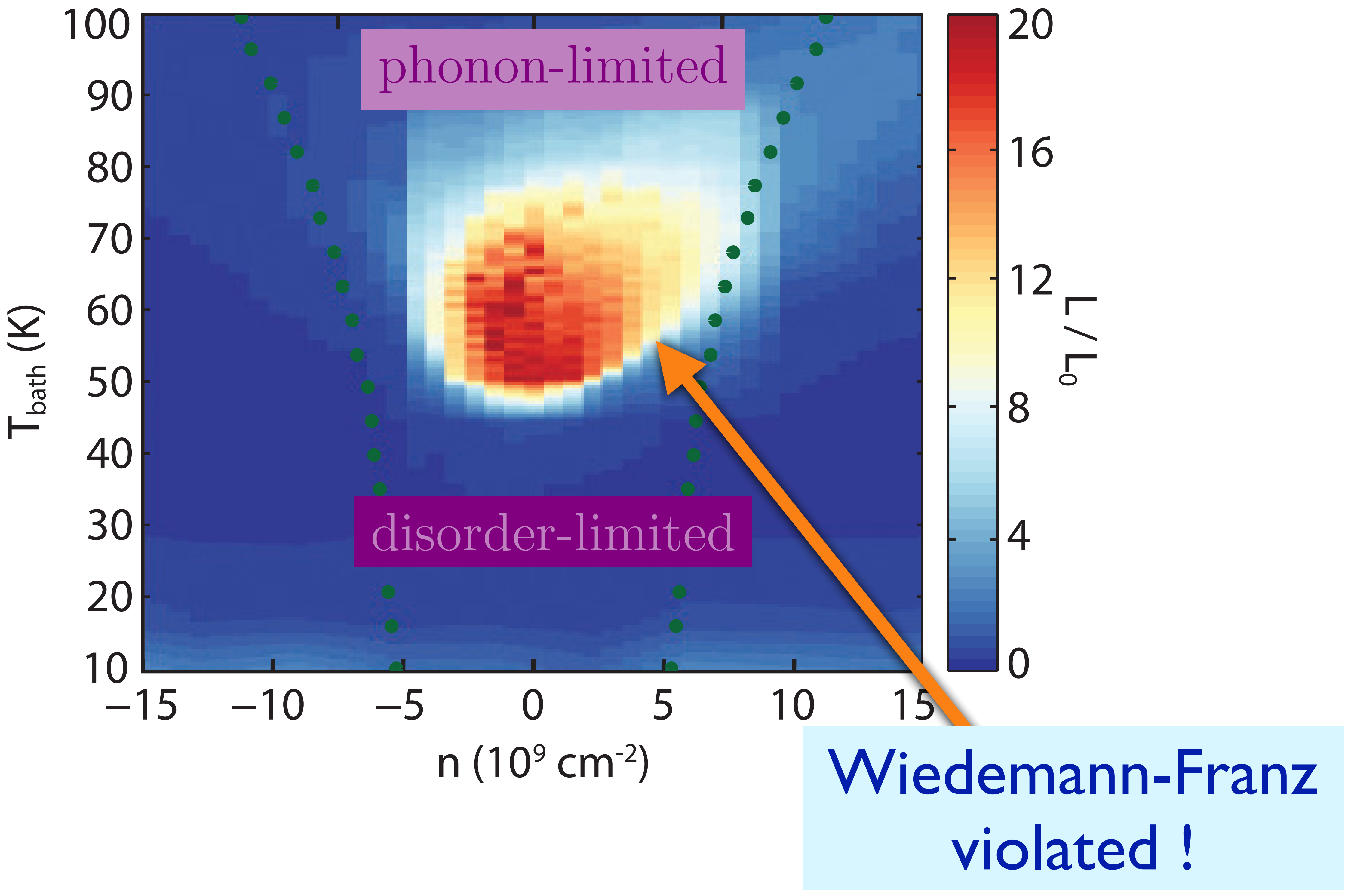
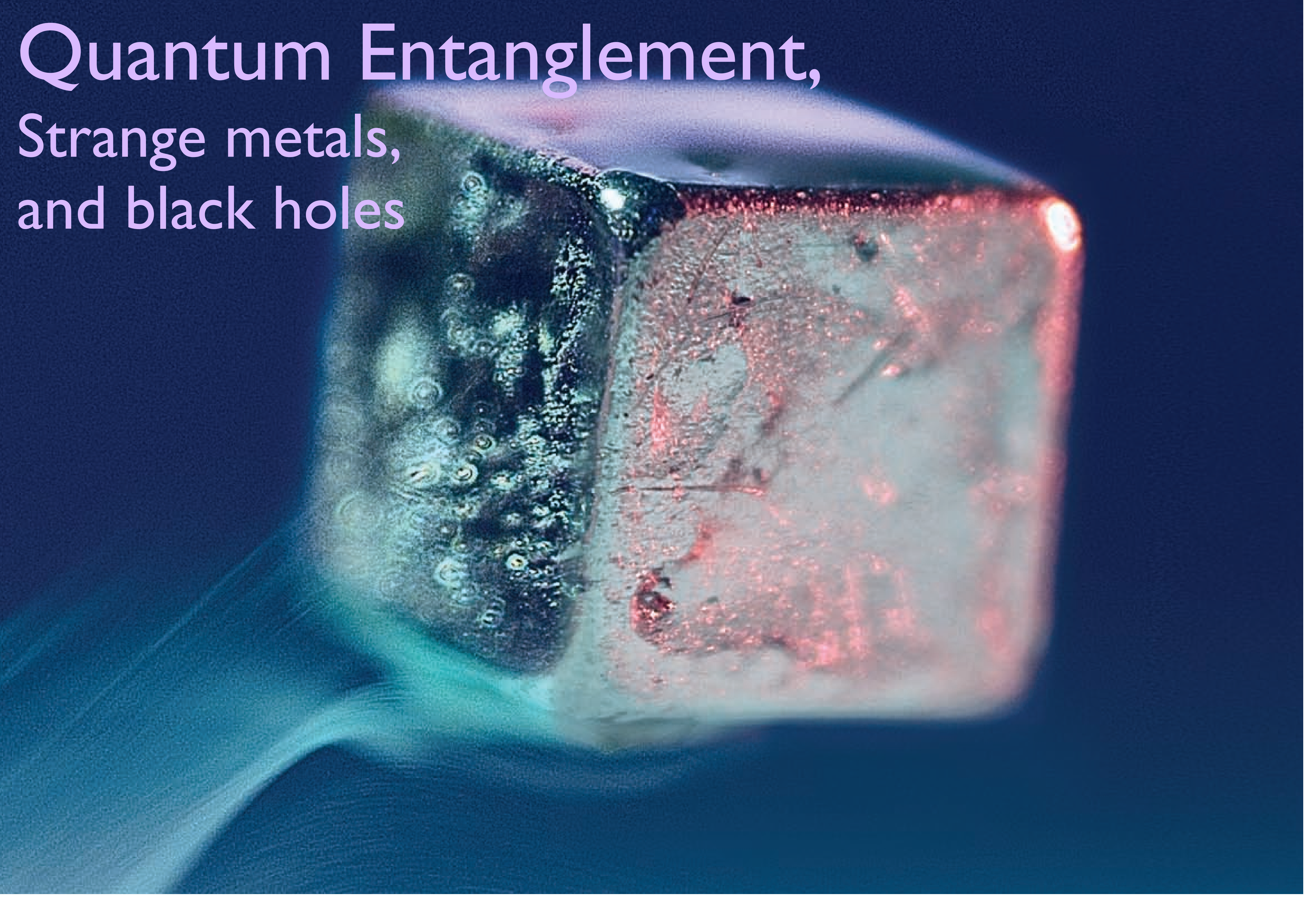


Strange metal in graphene



Quantum Entanglement,
Strange metals, and black holes



Subir Sachdev, Harvard University and Perimeter Institute