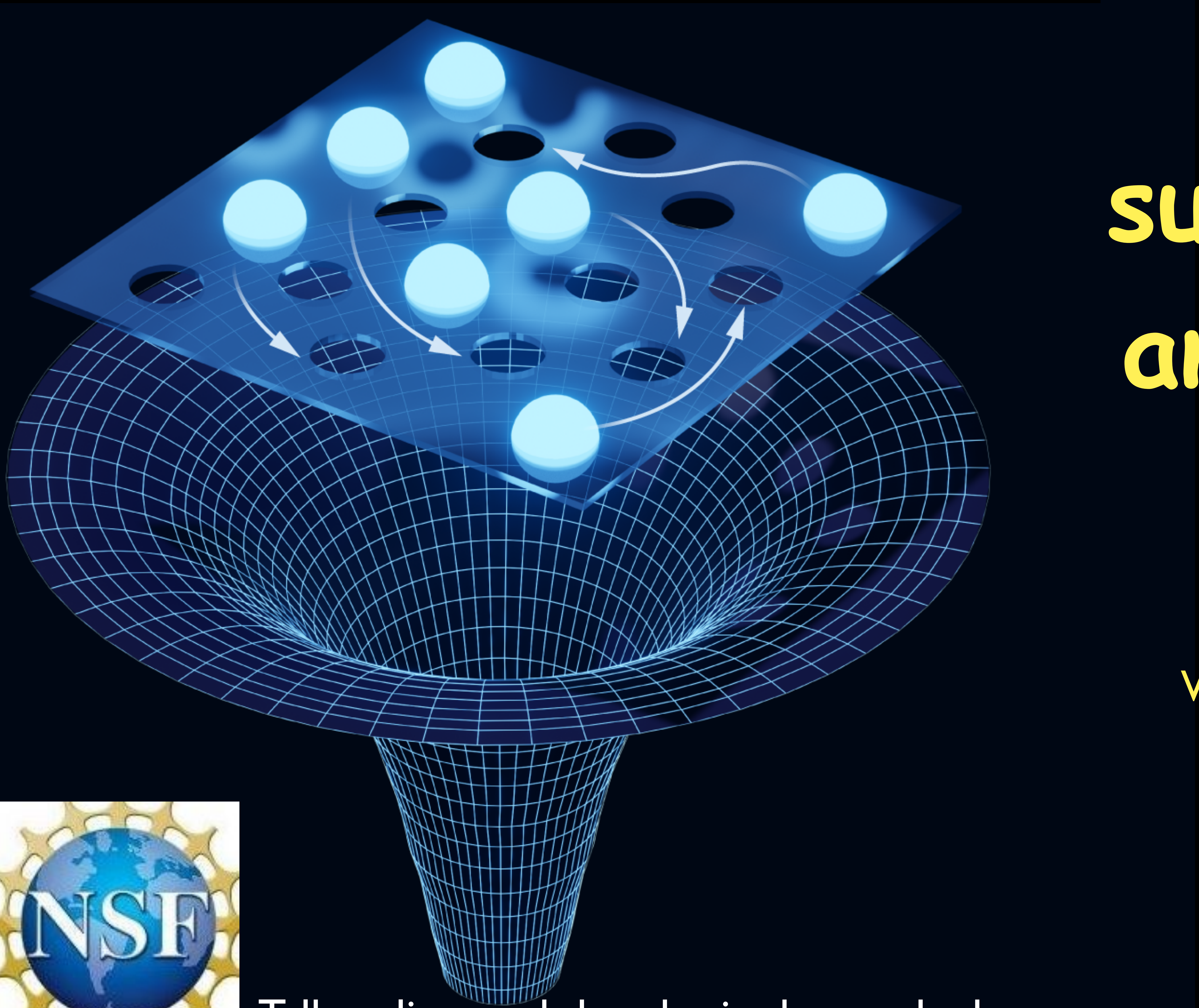
Quantum entanglement in nature: superconductors and black holes



Symposium on Condensed Matter
Weizmann Institute of Science, Israel
June 30, 2024

Subir Sachdev

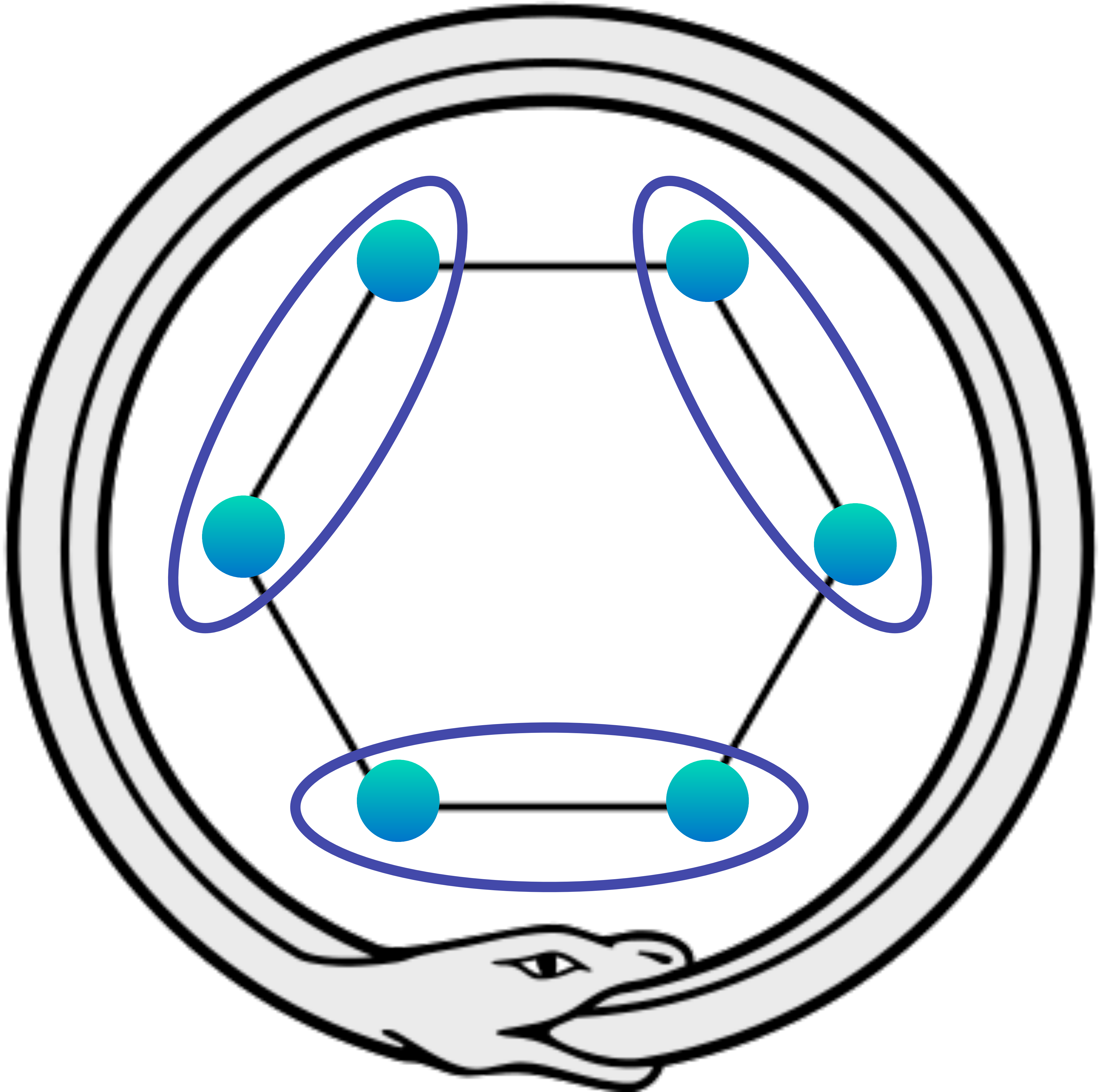
PHYSICS
VE RI TAS
HARVARD

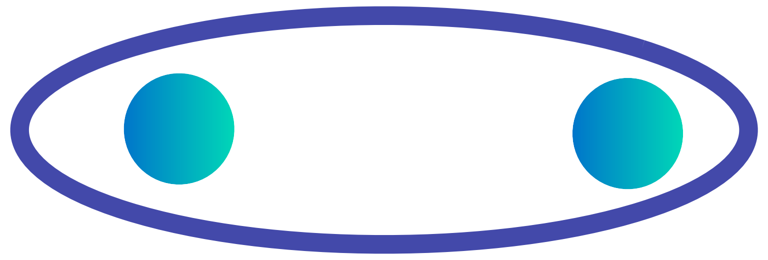
Talk online: sachdev.physics.harvard.edu

Quantum entanglement (1865)

Kekule’s spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



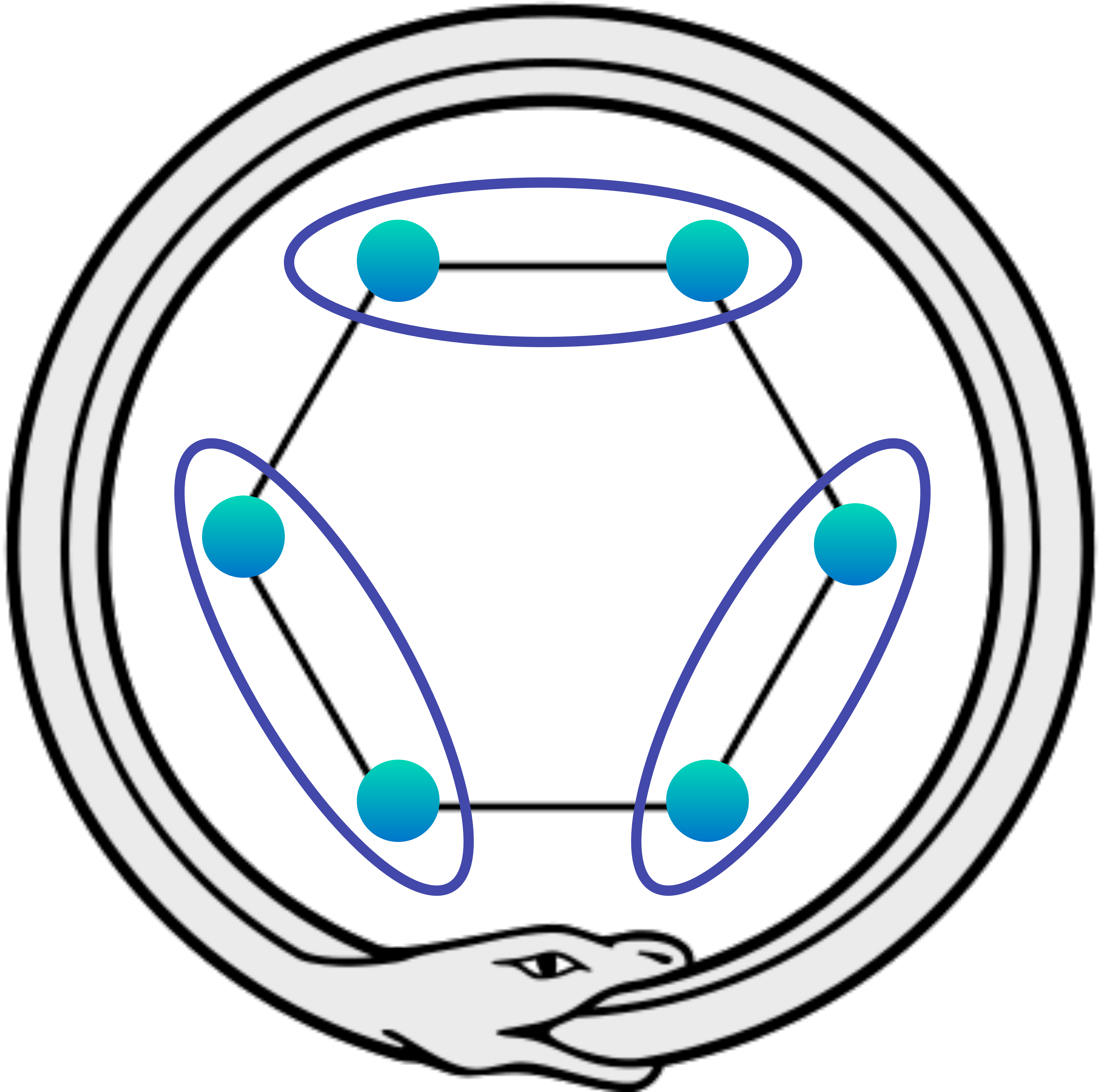
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

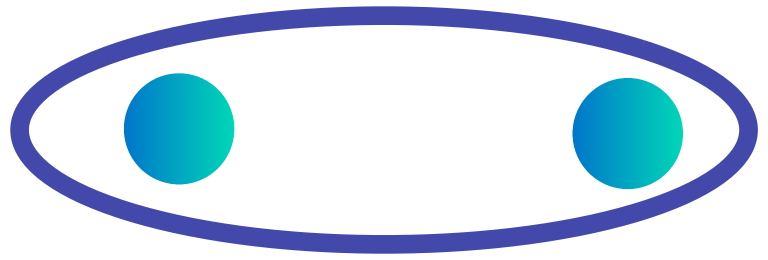
Benzene

*Wikipedia

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*





$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

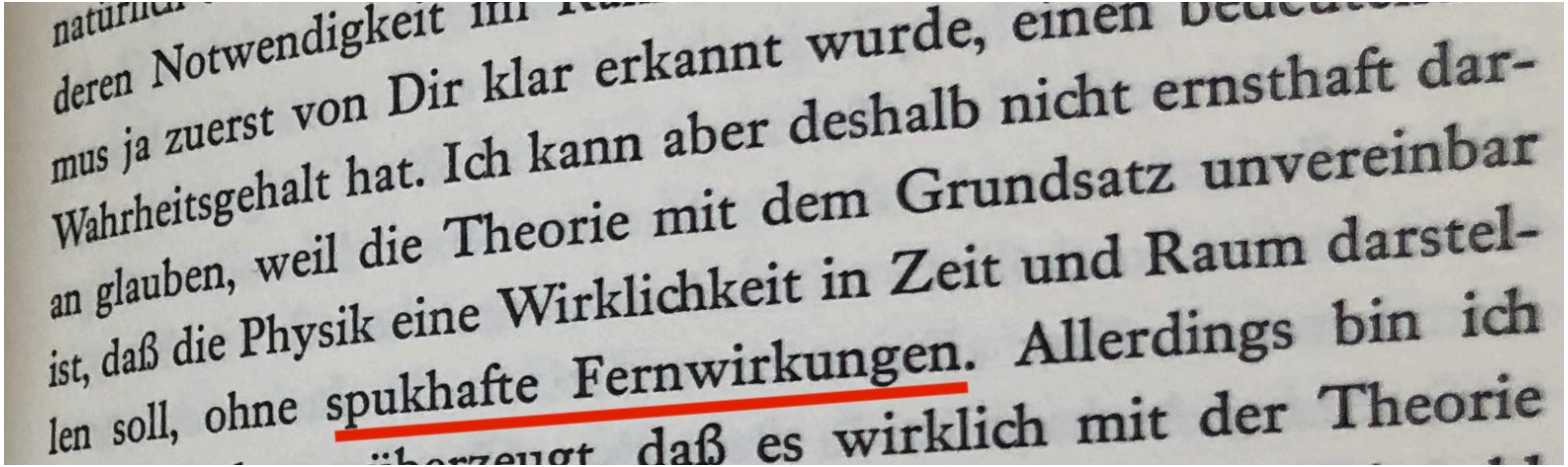
*Wikipedia

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

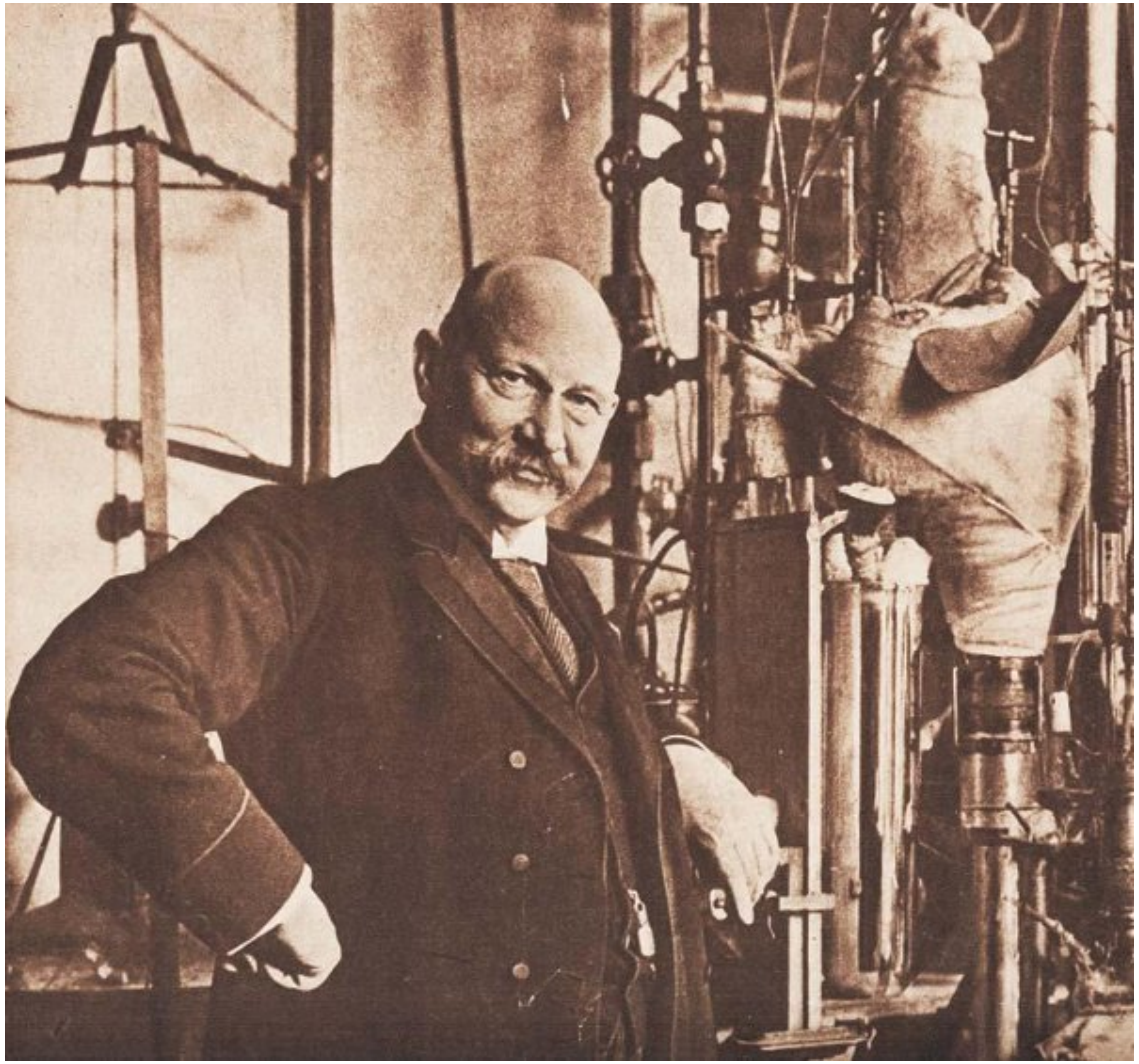
(Received March 25, 1935)


deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

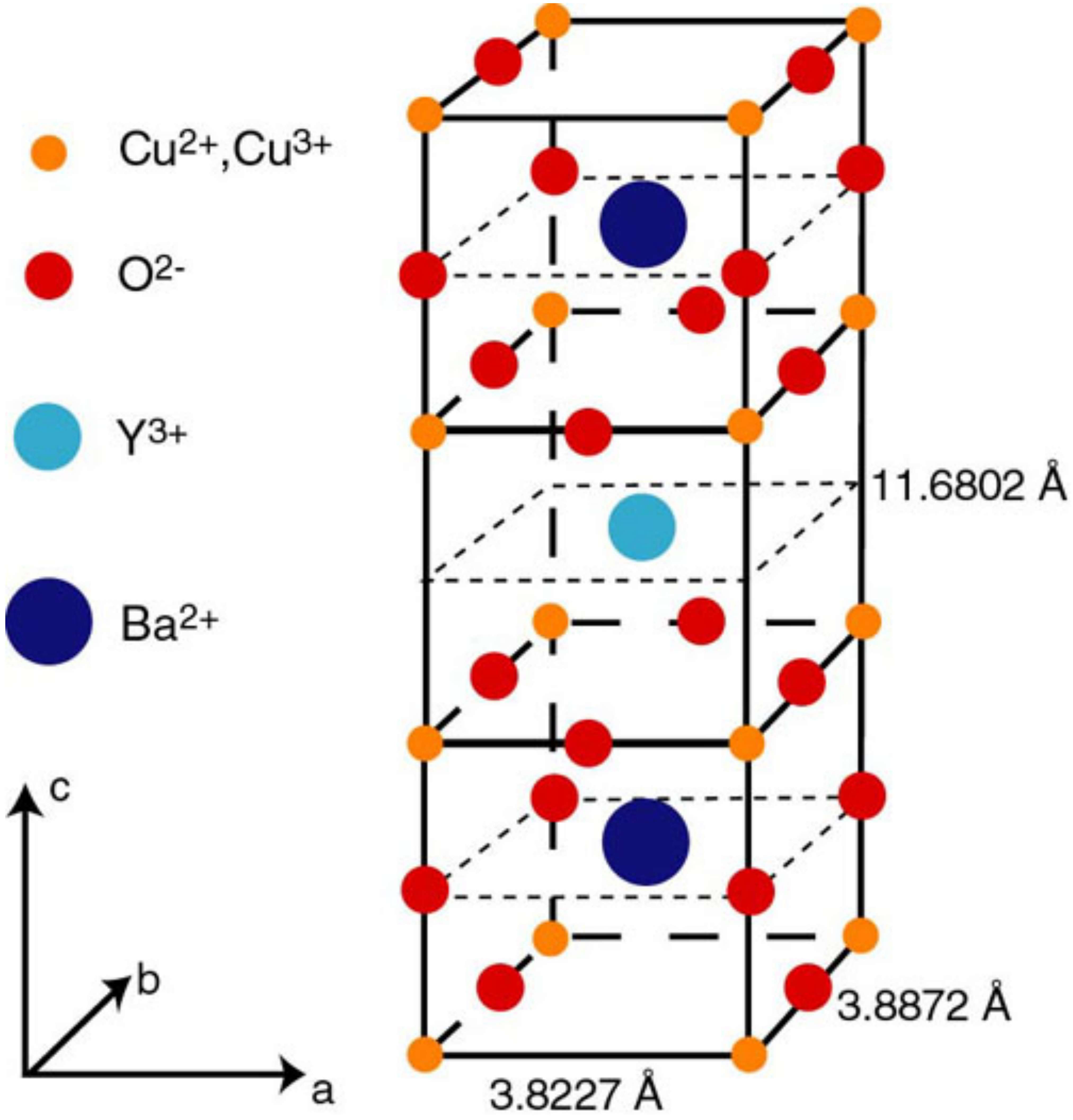
Albert Einstein to Max Born, 3 March 1947

Superconductors (1911)



Kamerlingh Onnes 1911:
Mercury is a superconductor below -269 °C

Cuprate high temperature superconductors



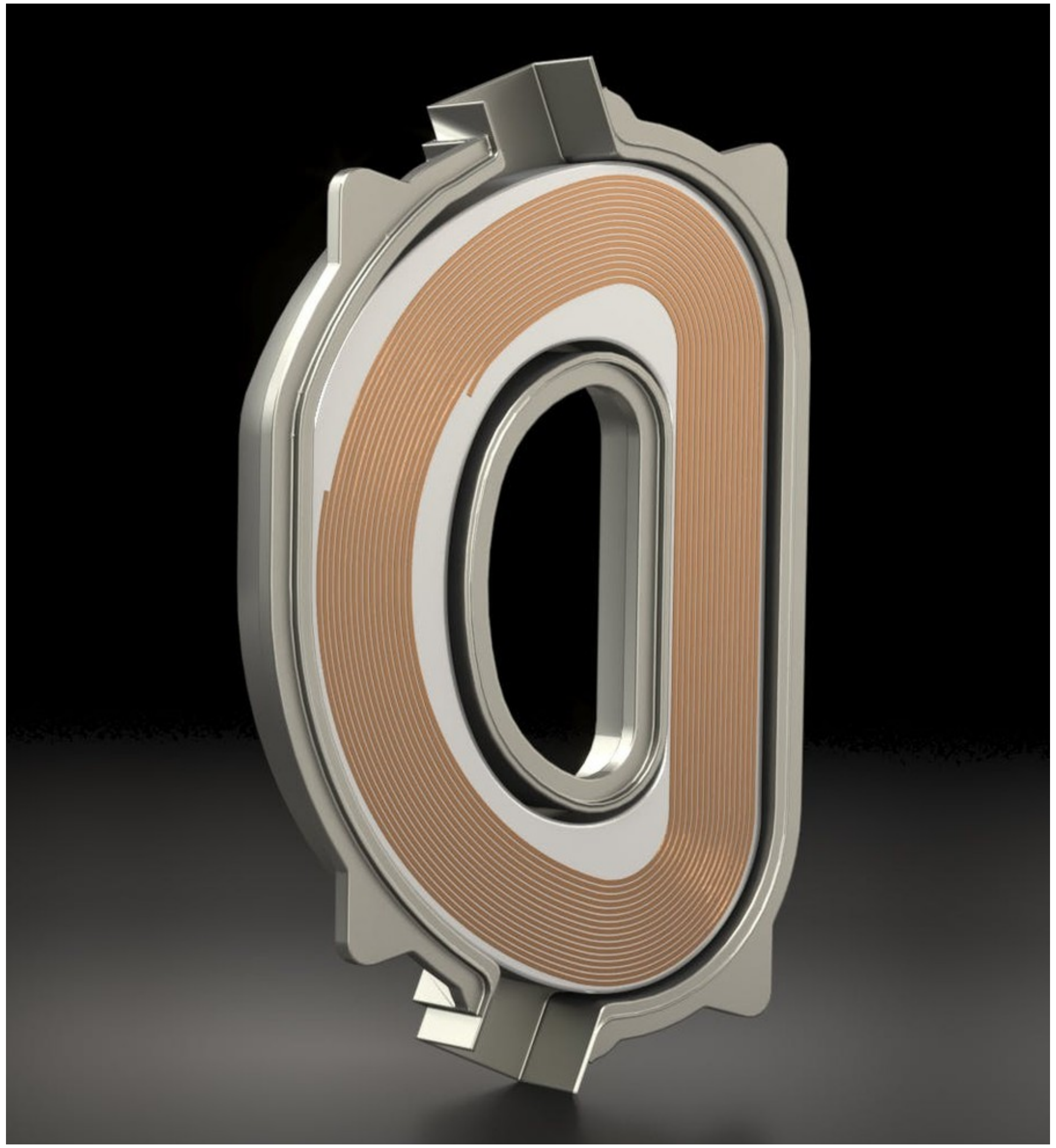
$YBa_2Cu_3O_{6+x}$

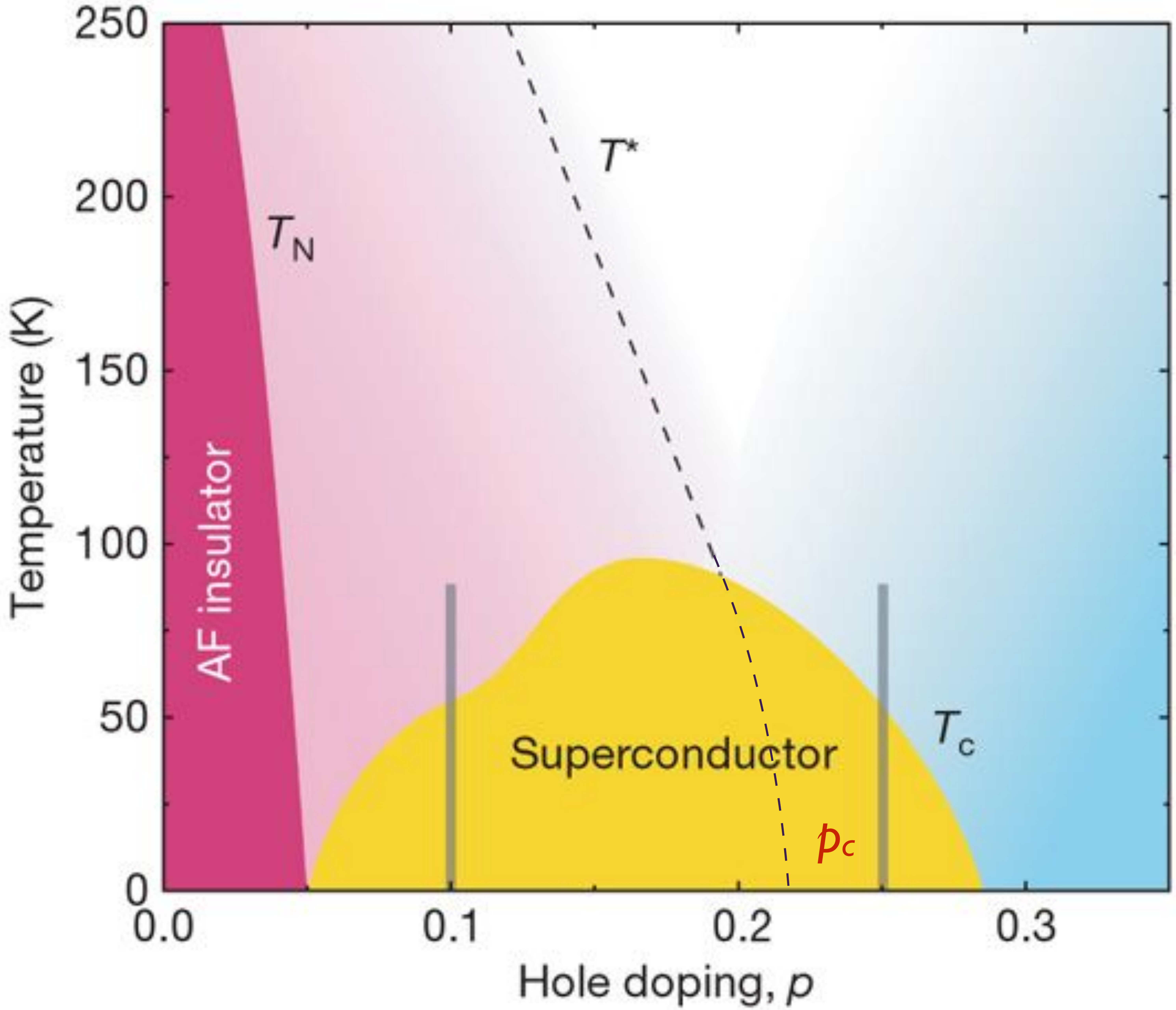
HTS Magnets: Enabling Technology

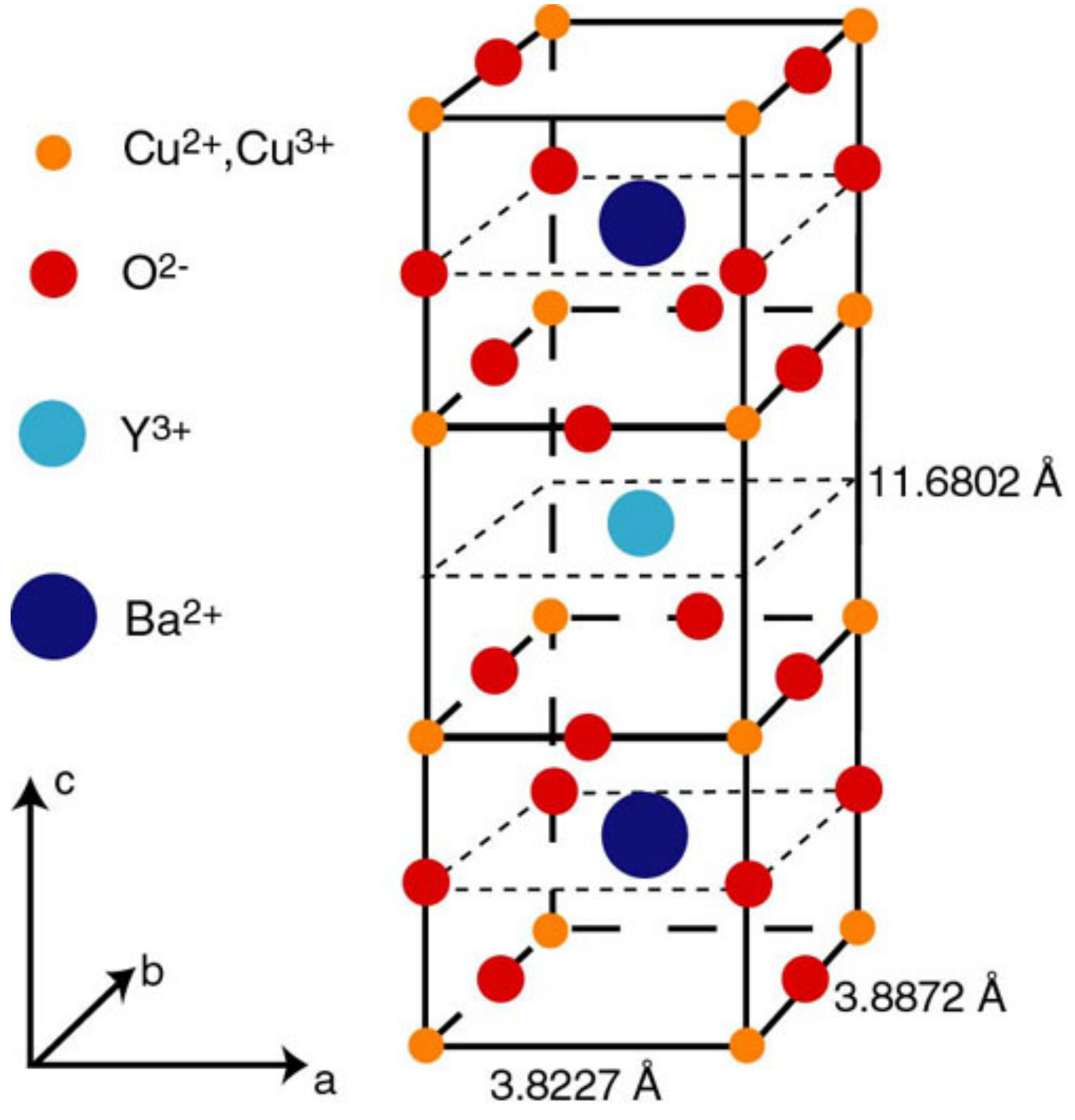
The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

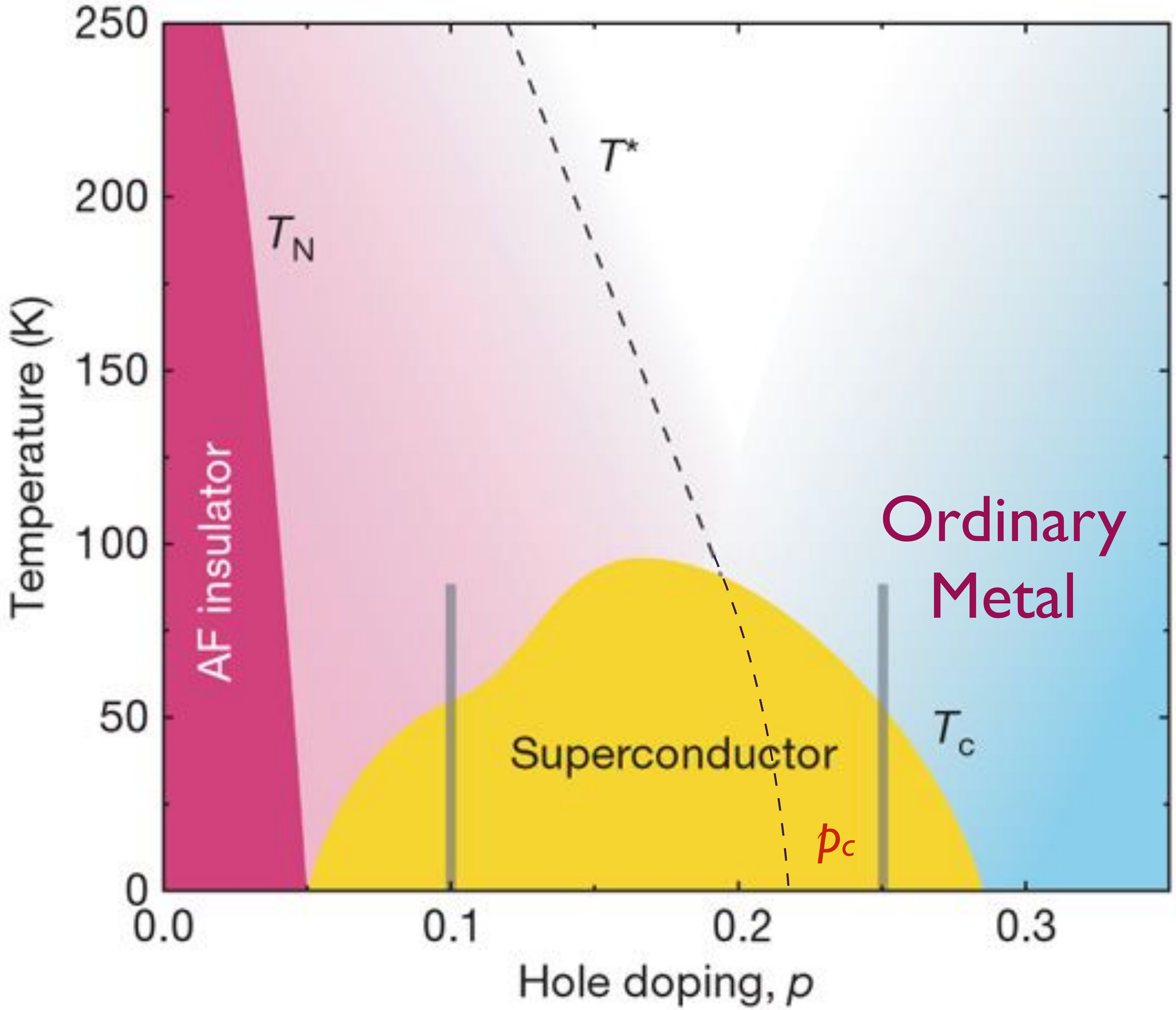
Commonwealth Fusion Systems

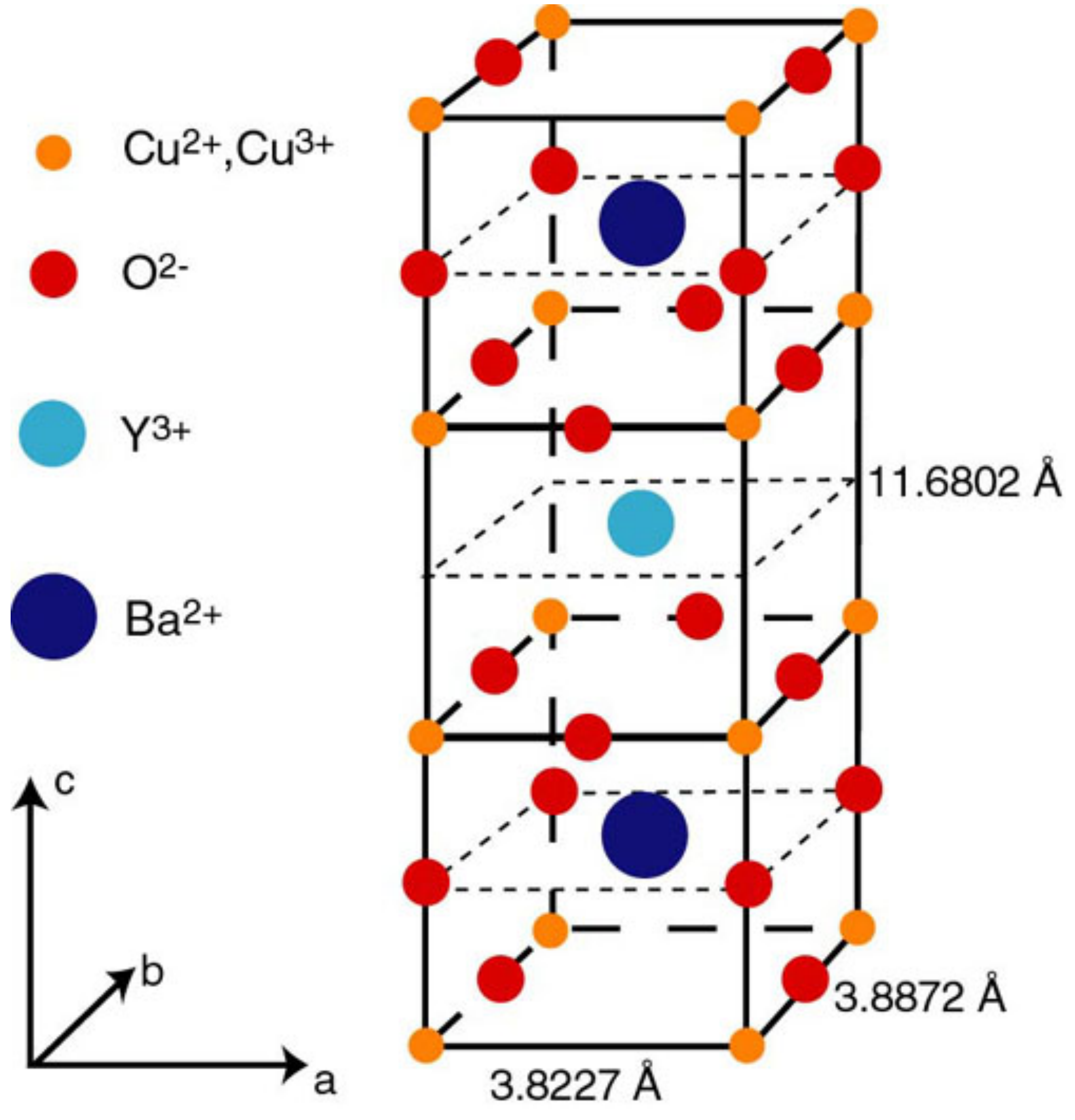



250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
p_c

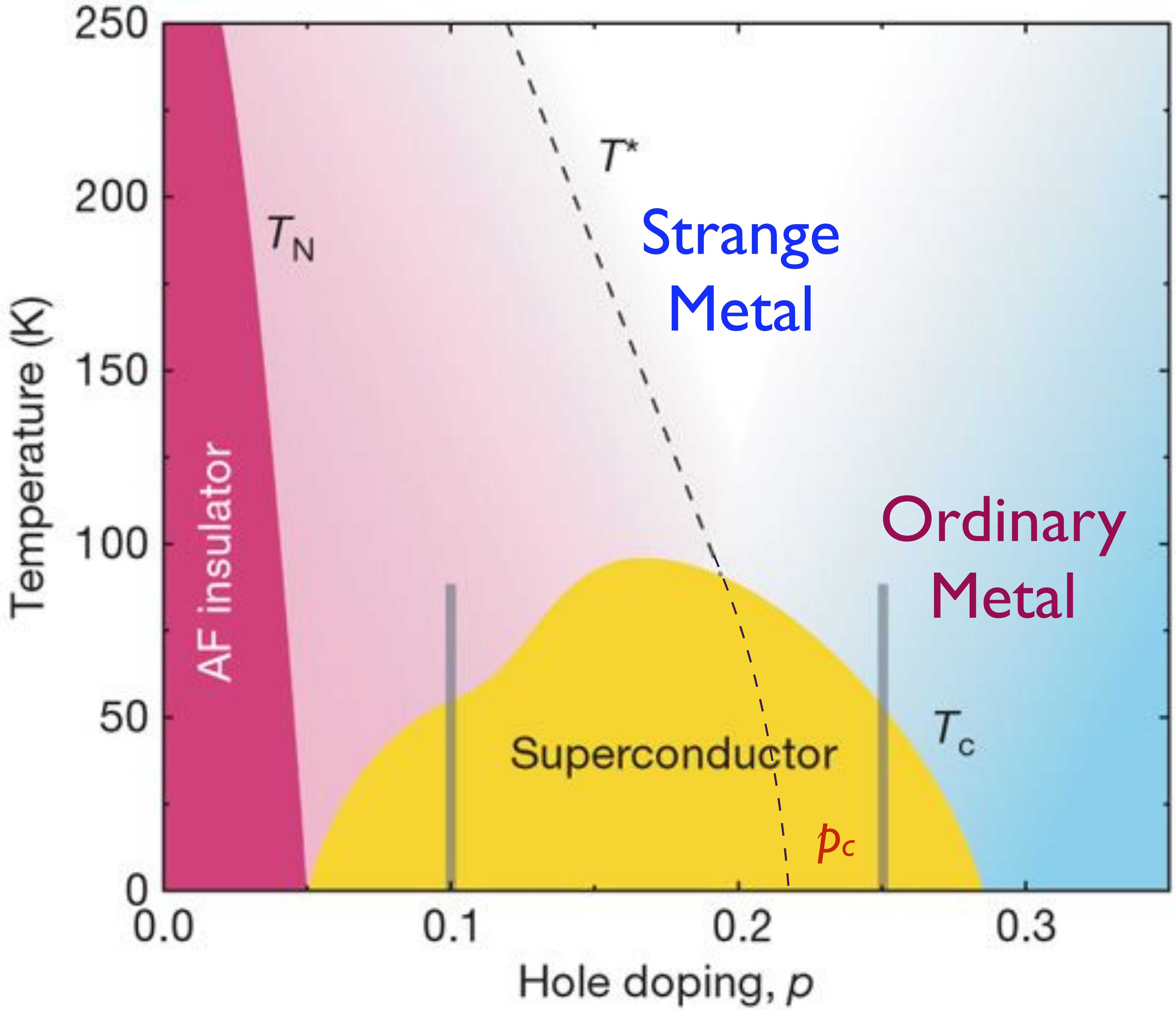

Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

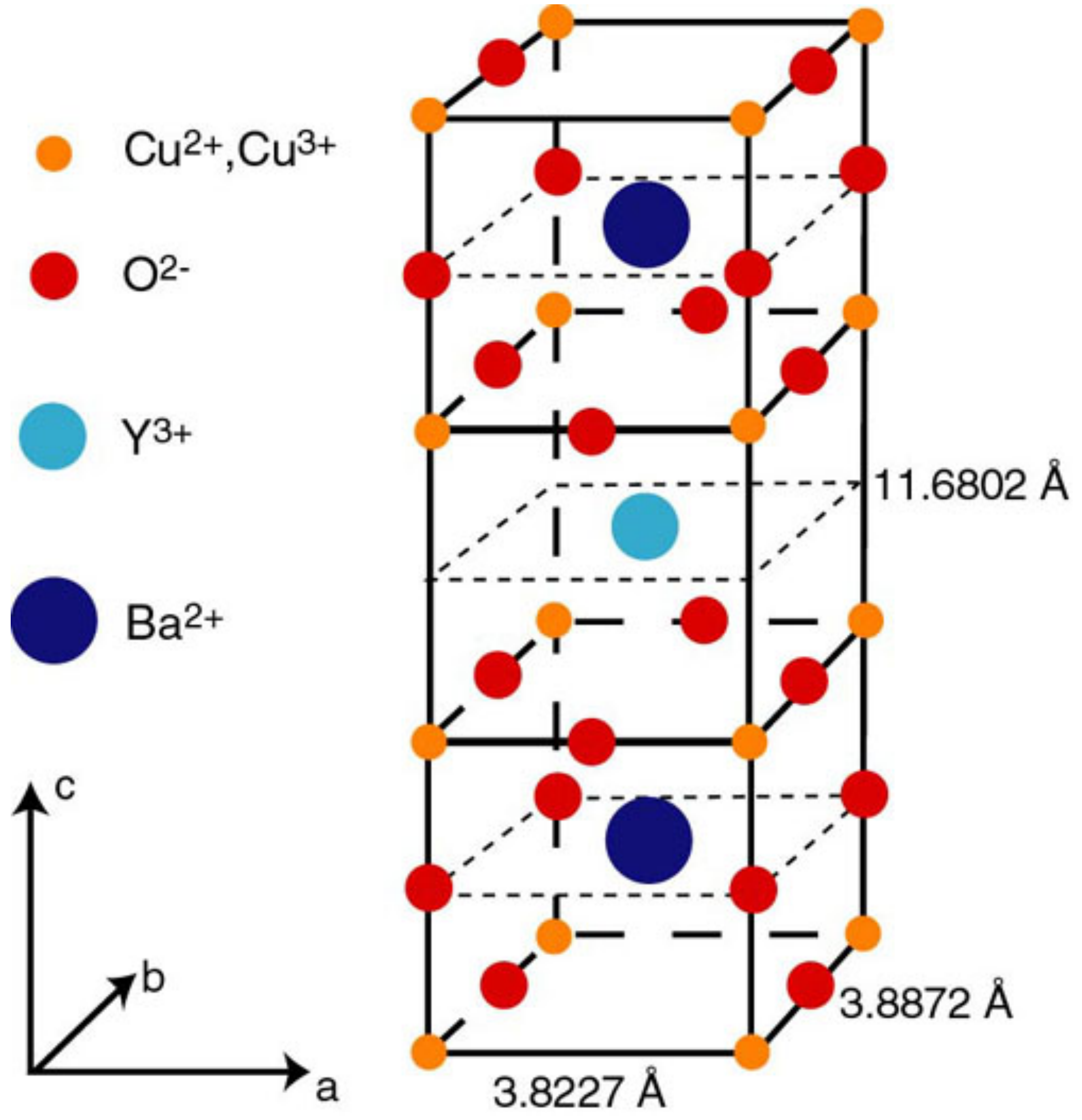
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Ordinary
Metal
Superconductor
T_c
p_c

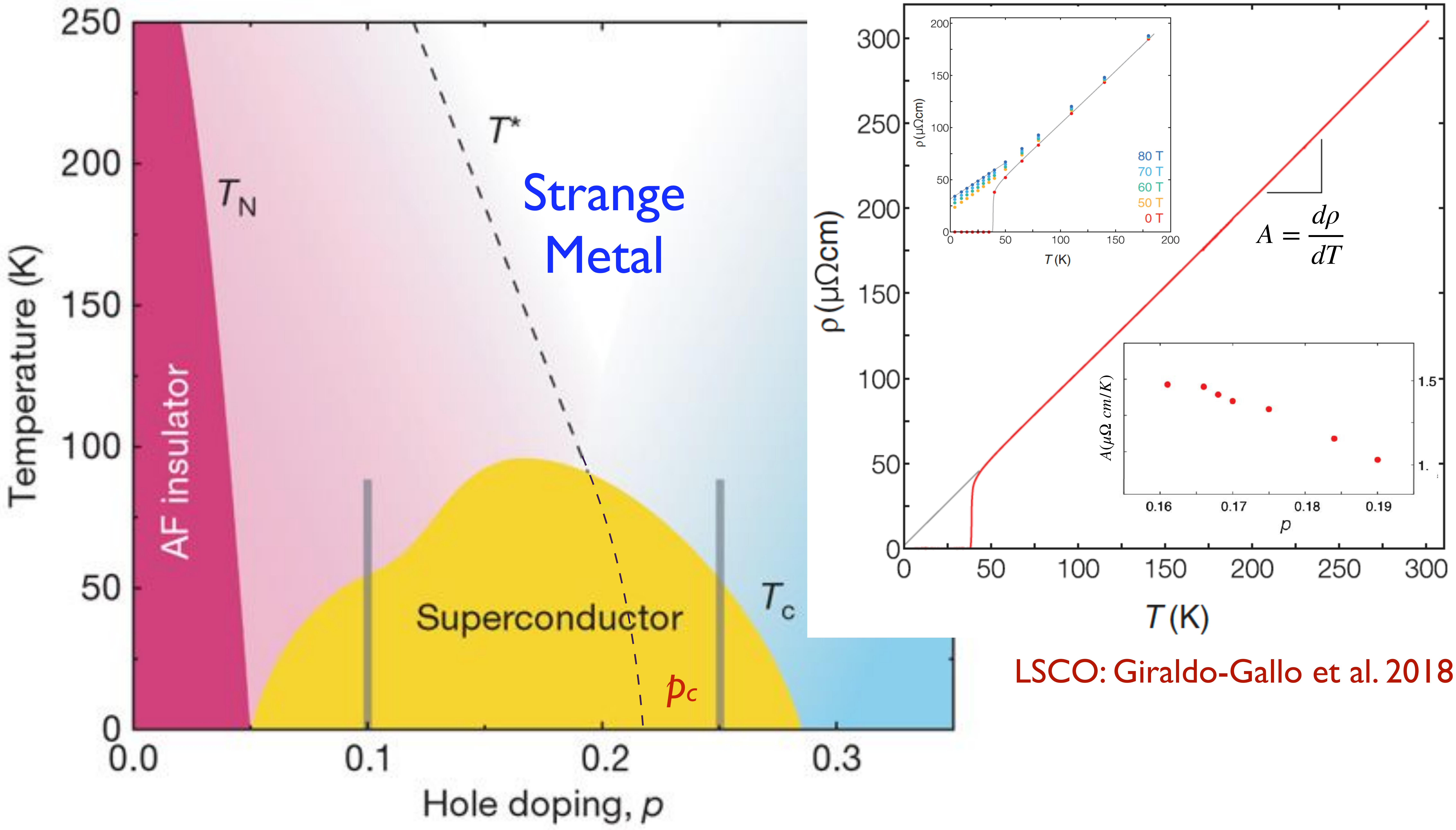

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

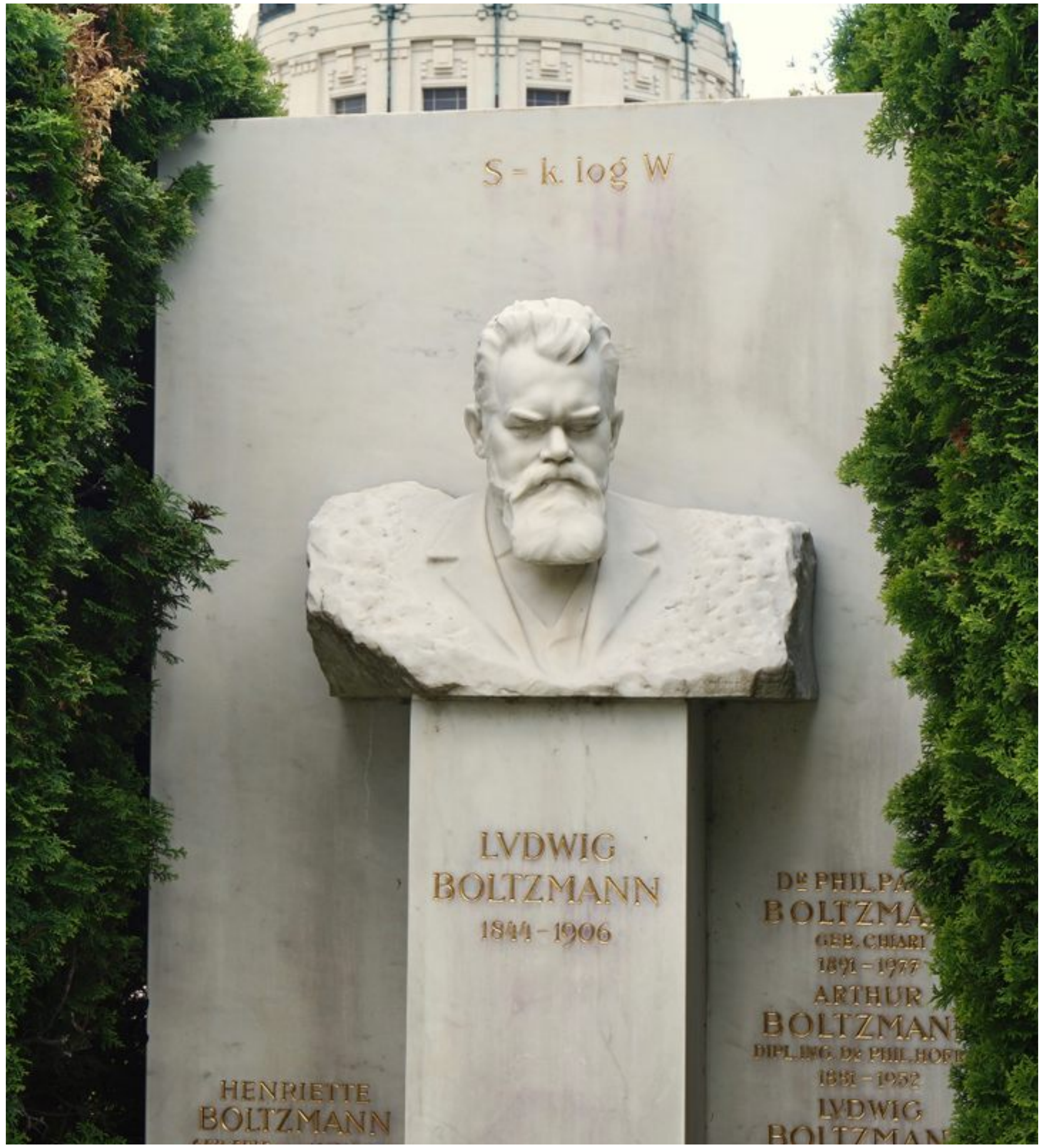

250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



LSCO: Giraldo-Gallo et al. 2018



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

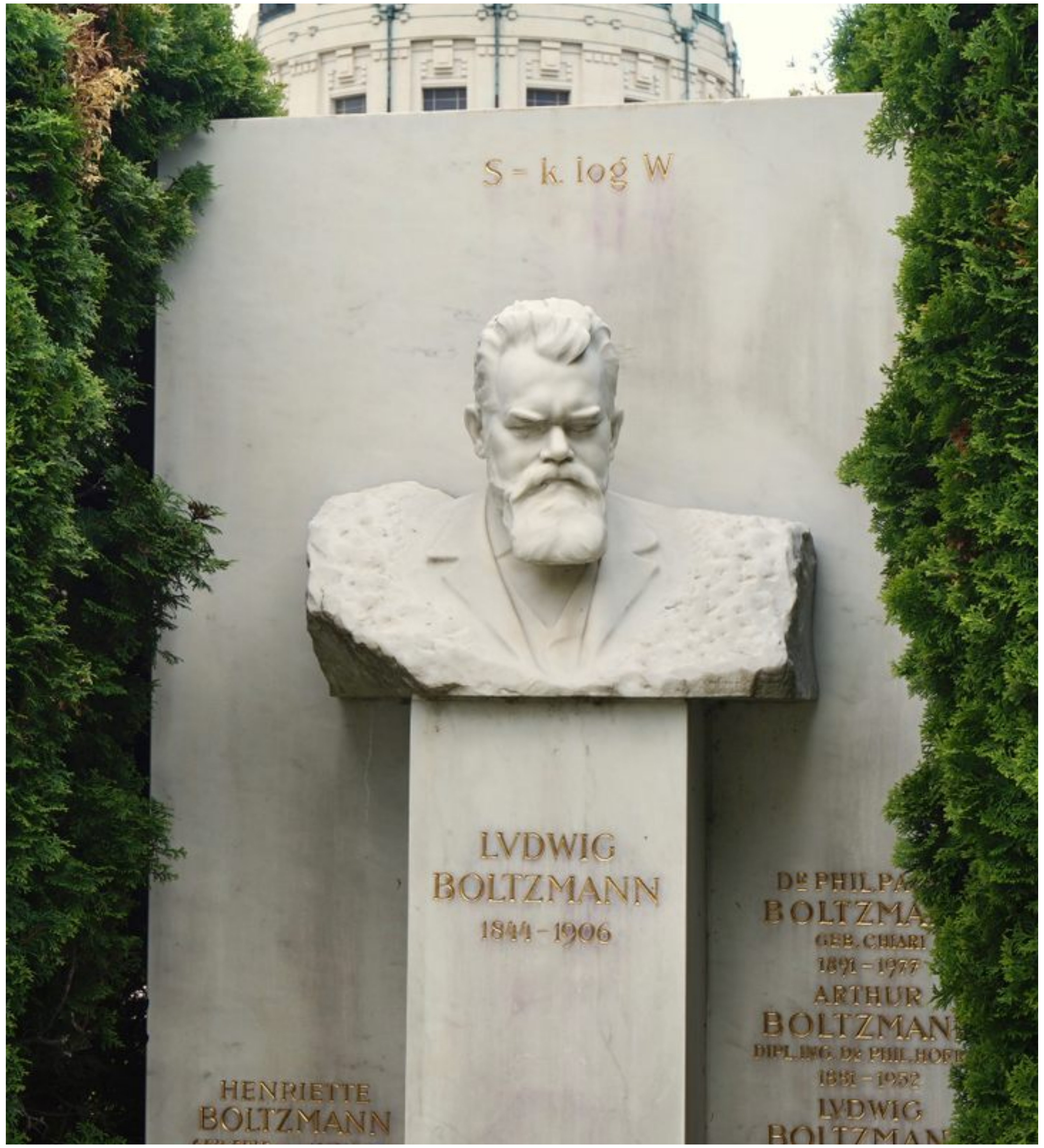
Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

$$\frac{1}{T} = \frac{dS}{dE}$$

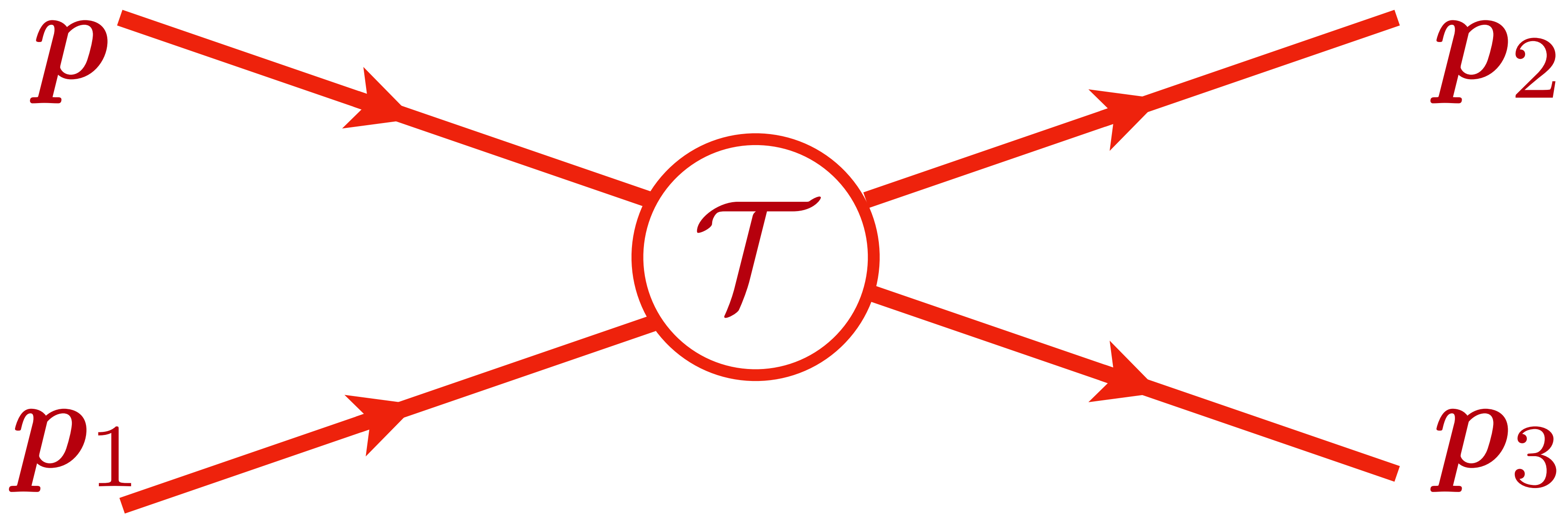
Quantum Boltzmann equation (Landau)
Dense gas of electrons



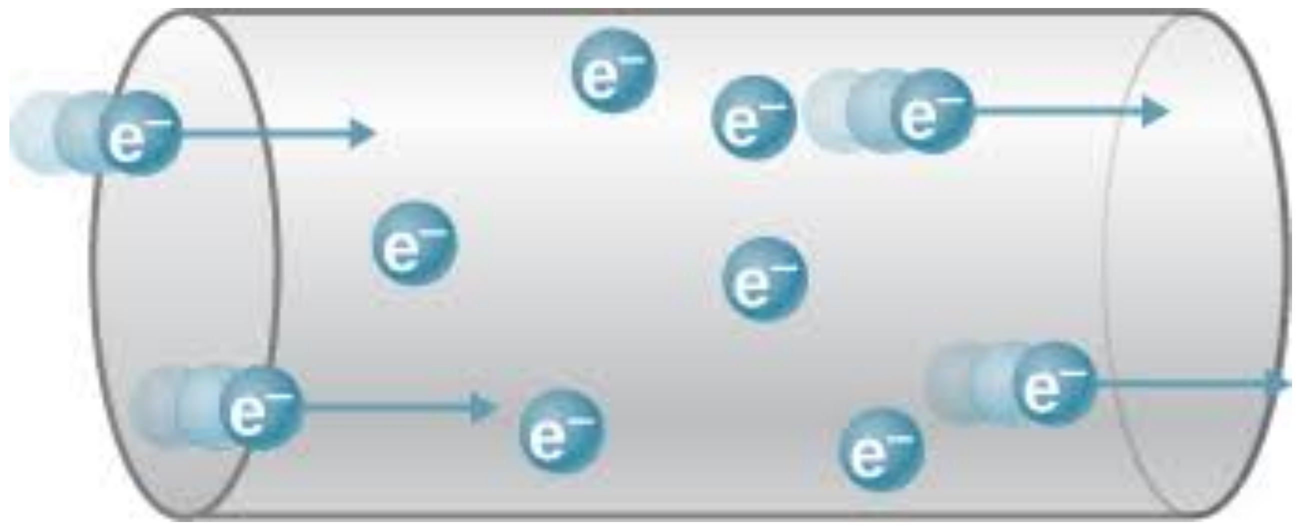
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

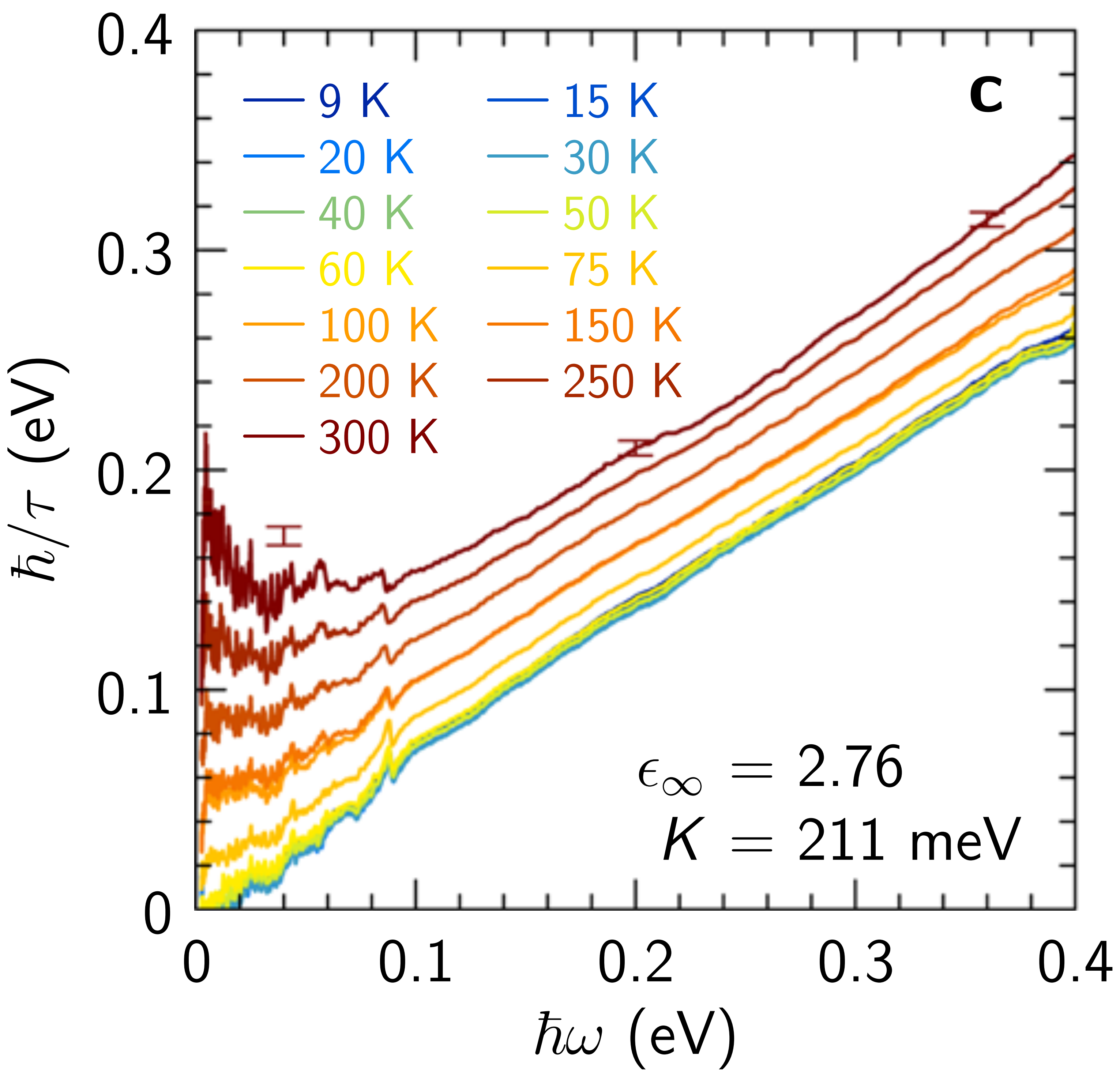
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

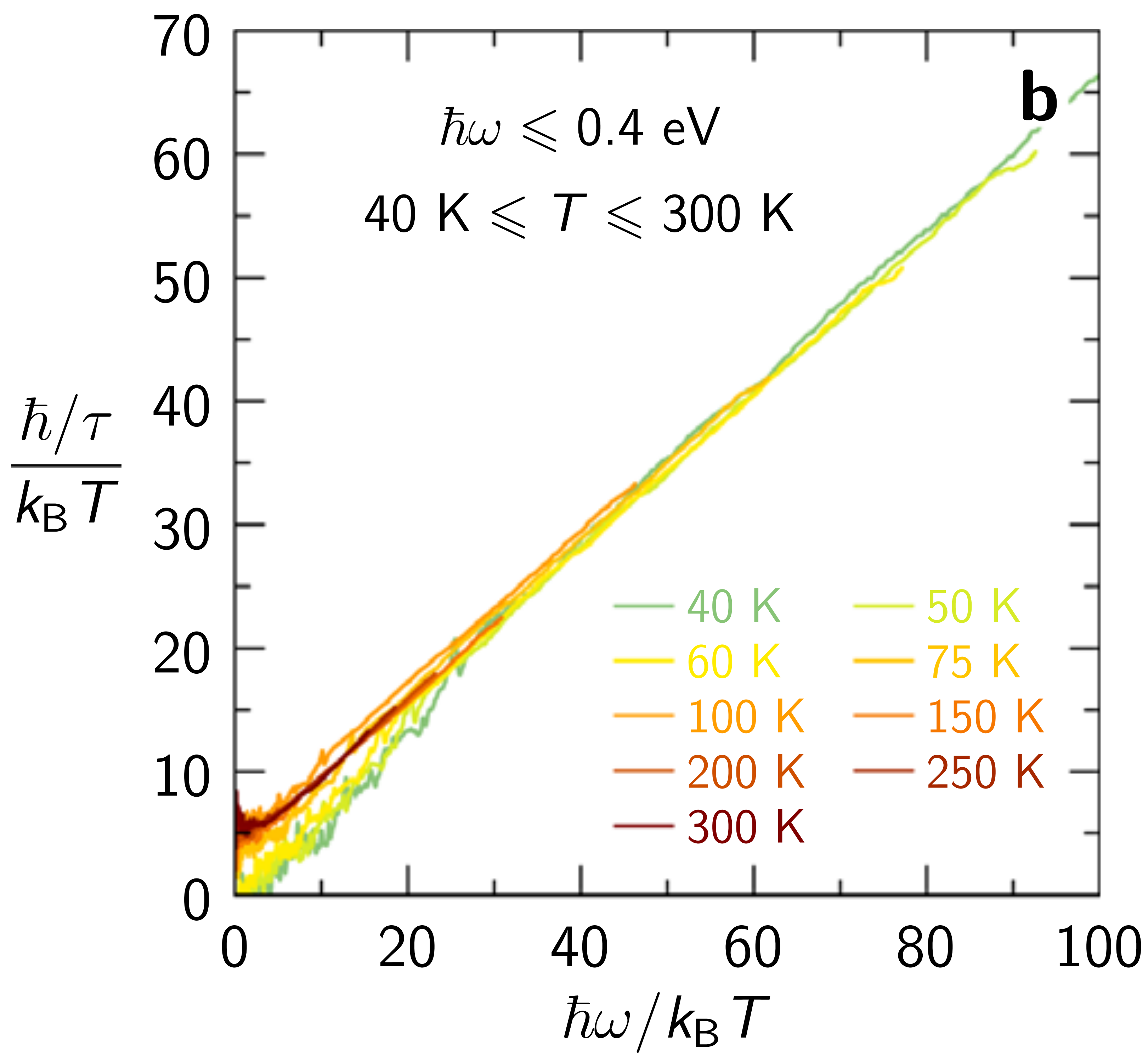
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!



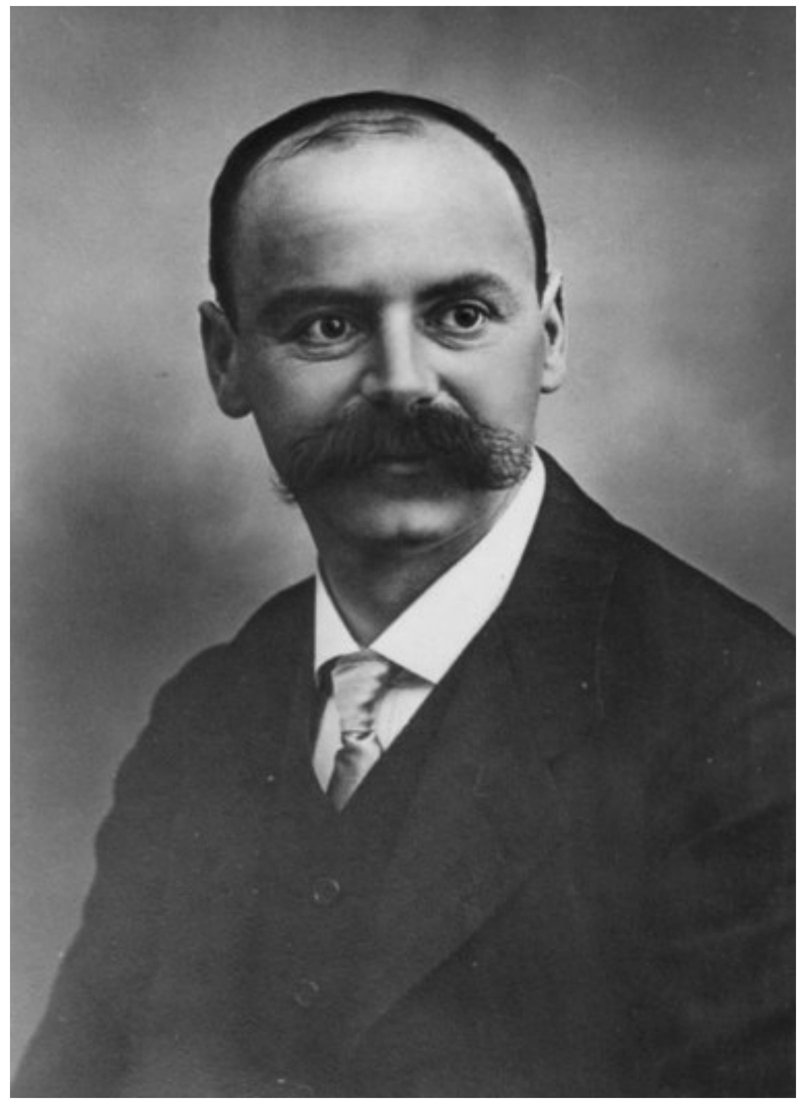


$$\tau(\omega) = \frac{\hbar}{k_B T}F\left(\frac{\hbar\omega}{k_B T}\right)$$

Black holes
(1916)

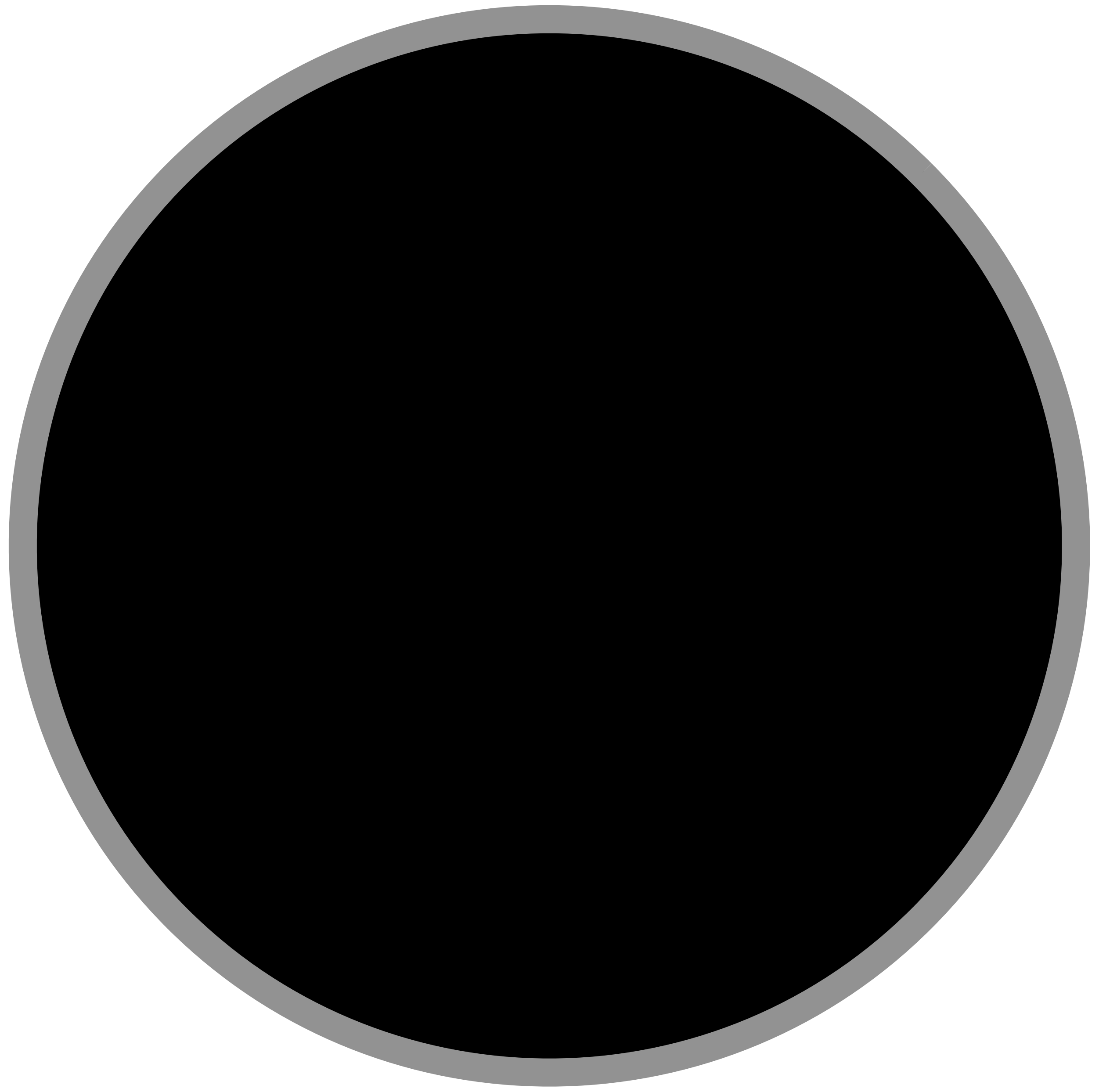
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

Horizon radius $R = \dfrac{2GM}{c^2}$

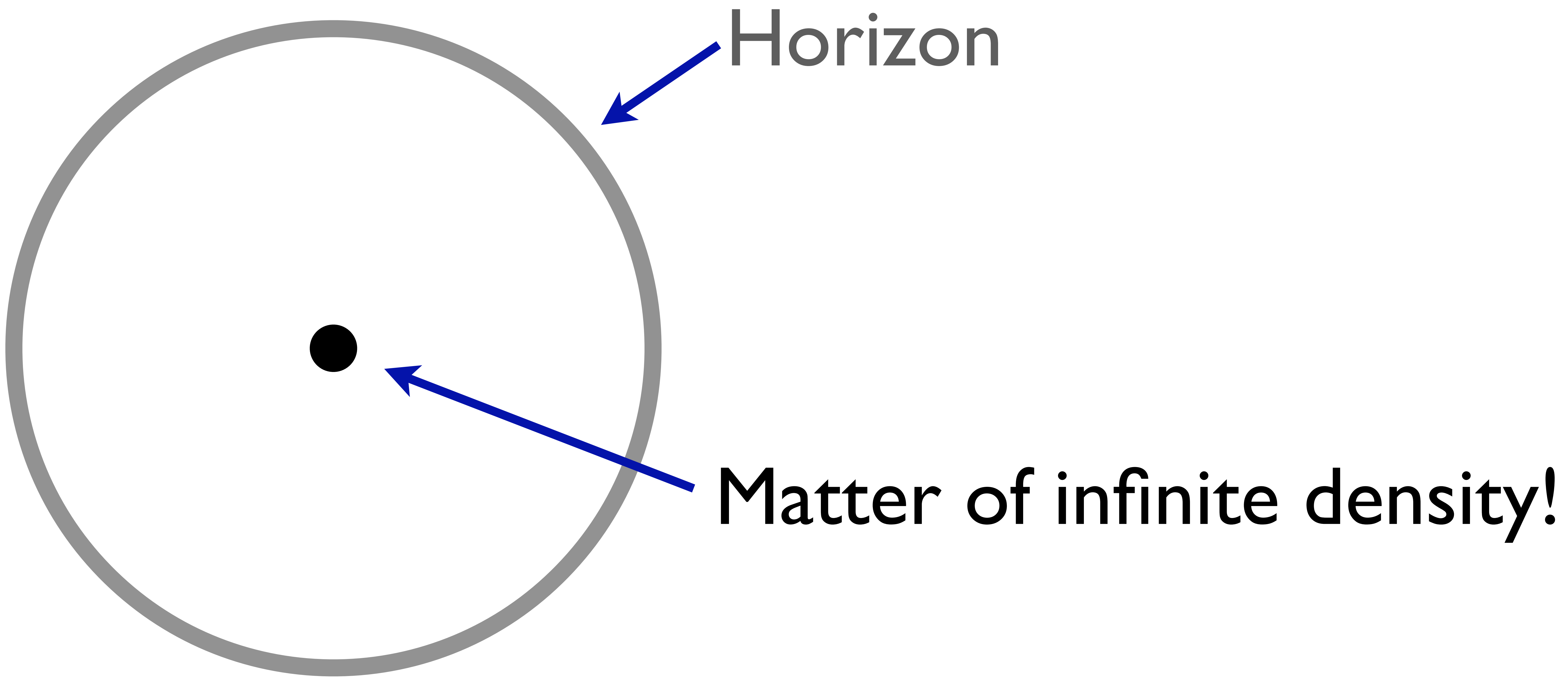


G Newton's constant, c velocity of light, M mass of black hole

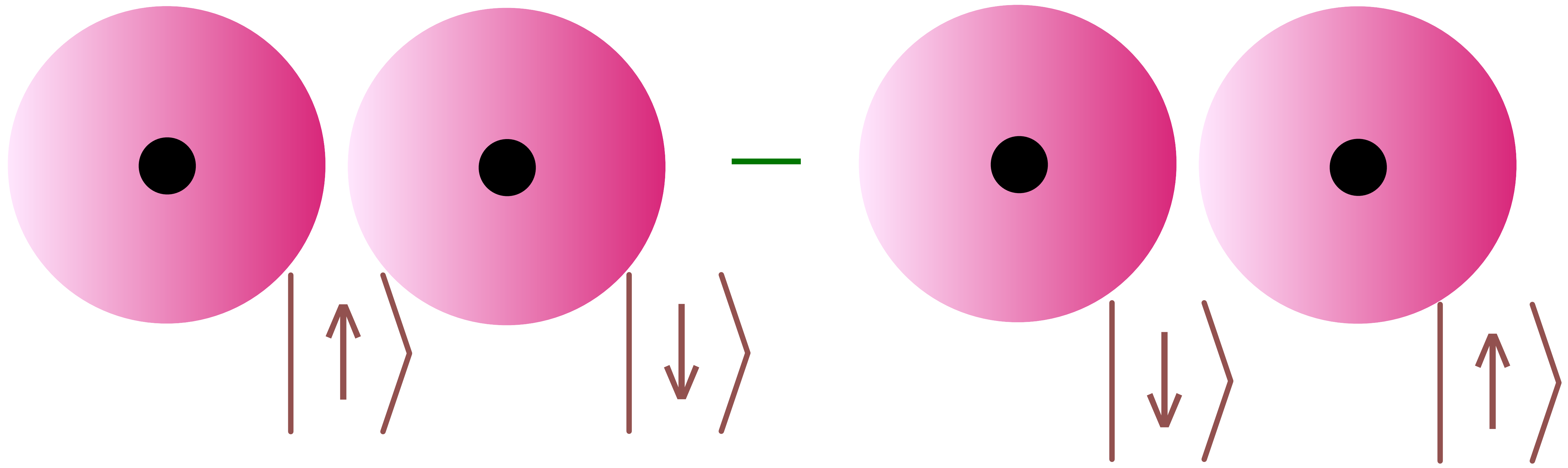
For $M =$ earth's mass, $R \approx 9\, mm$!

What is inside a black hole ???

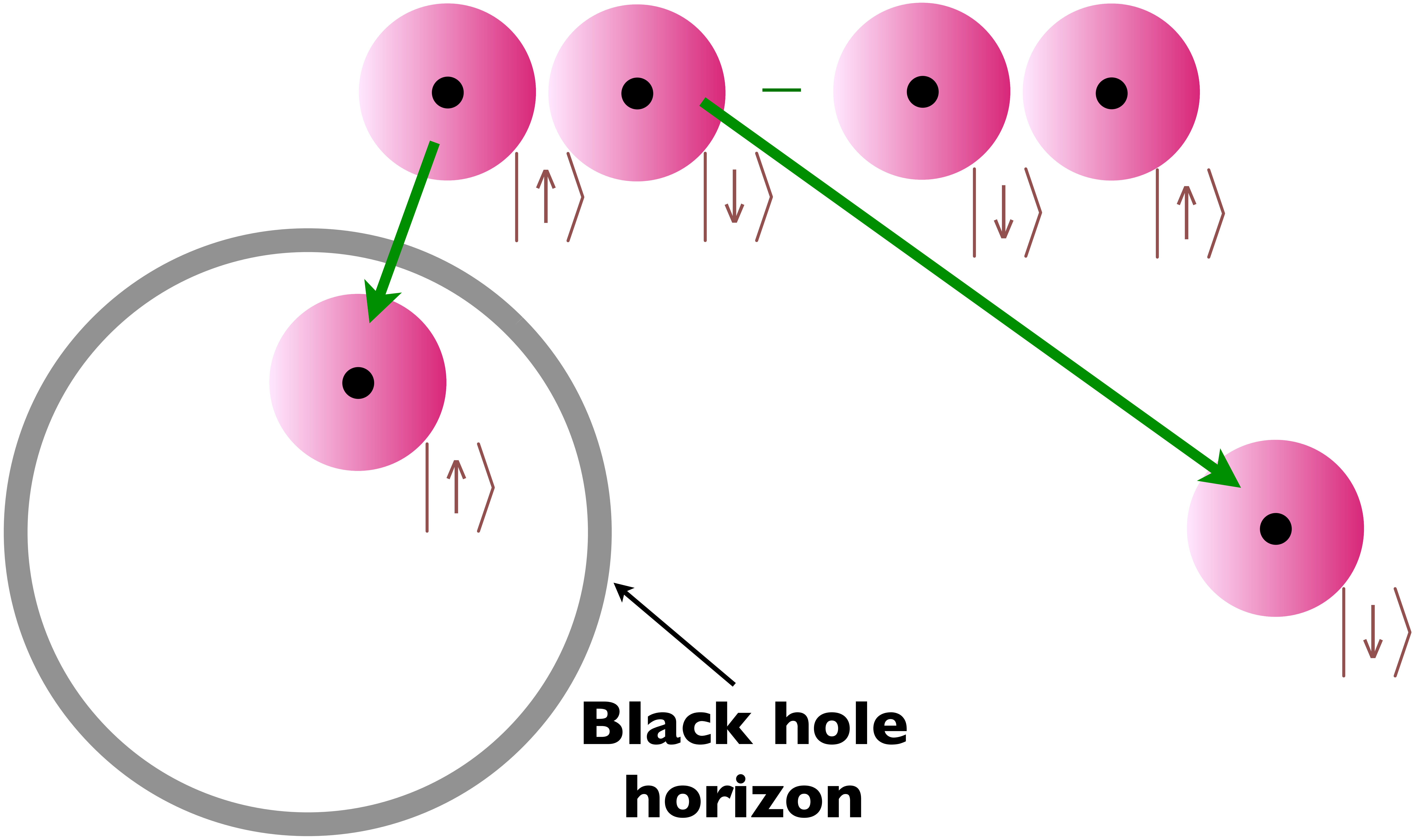
In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



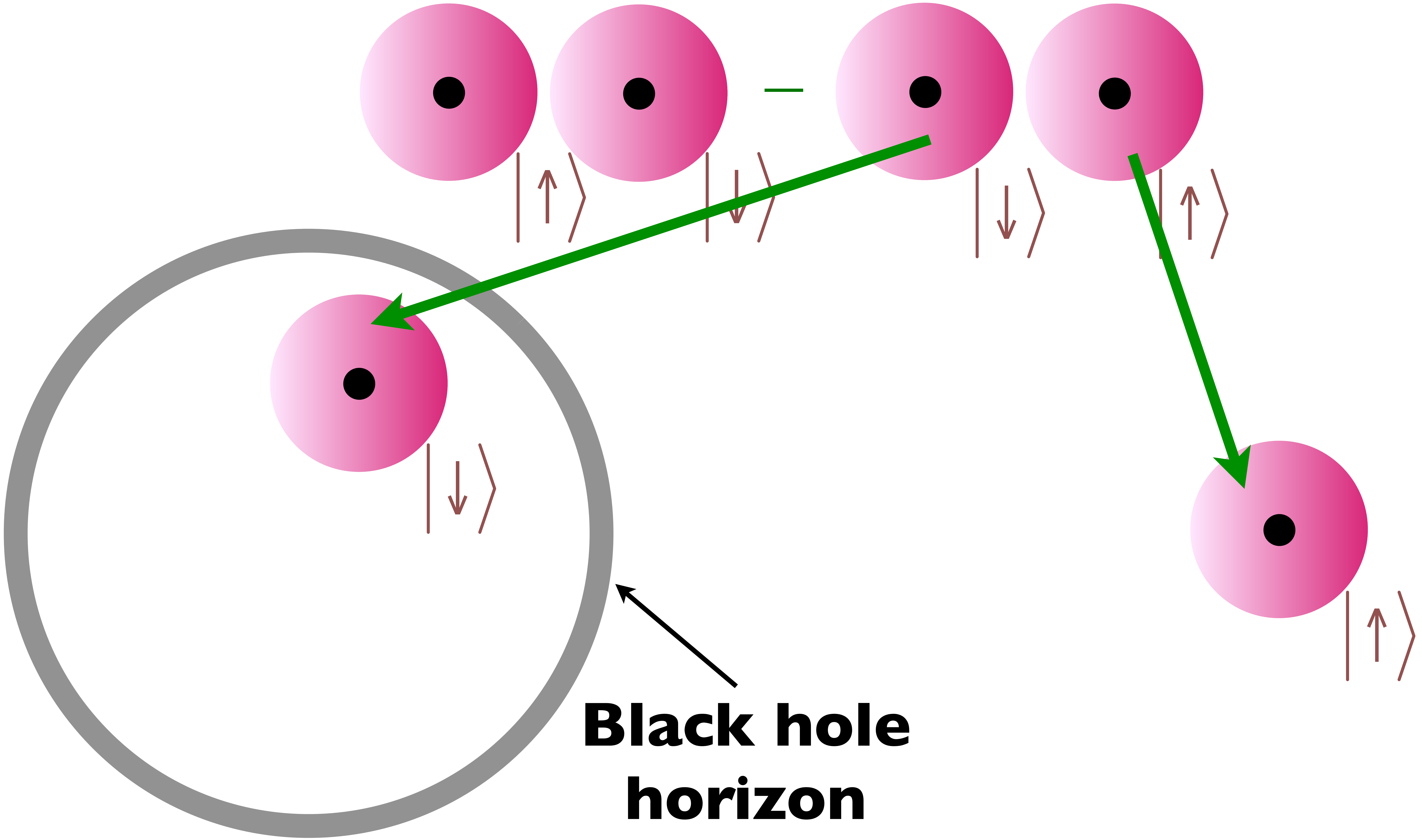
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



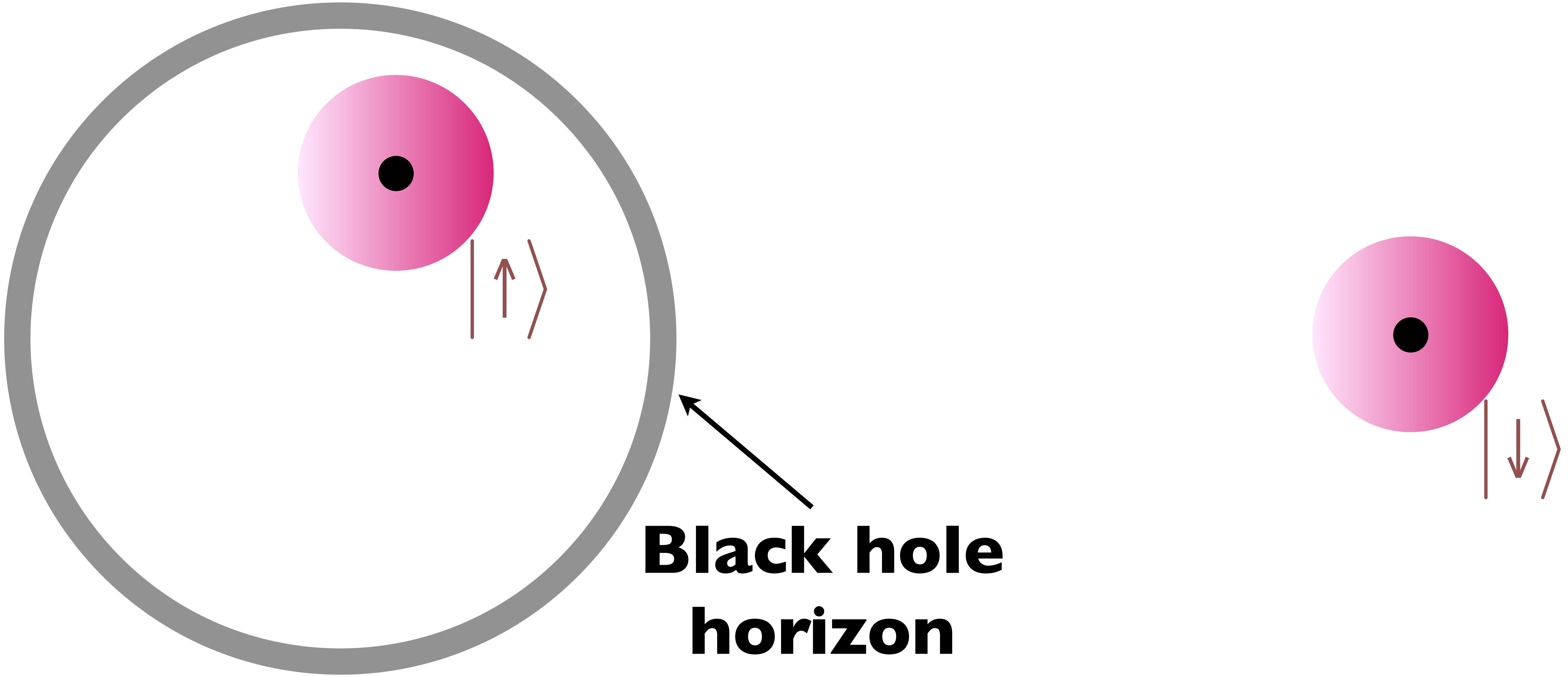
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

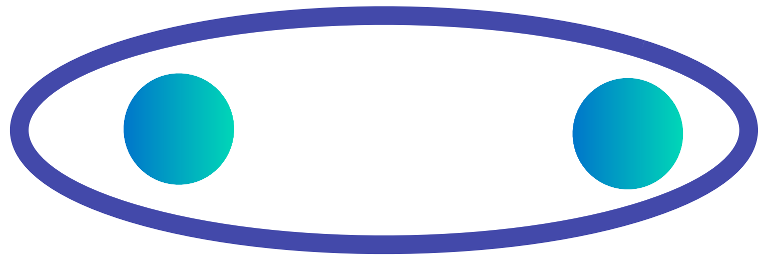
Bekenstein, Hawking: Black holes have a temperature and an entropy!

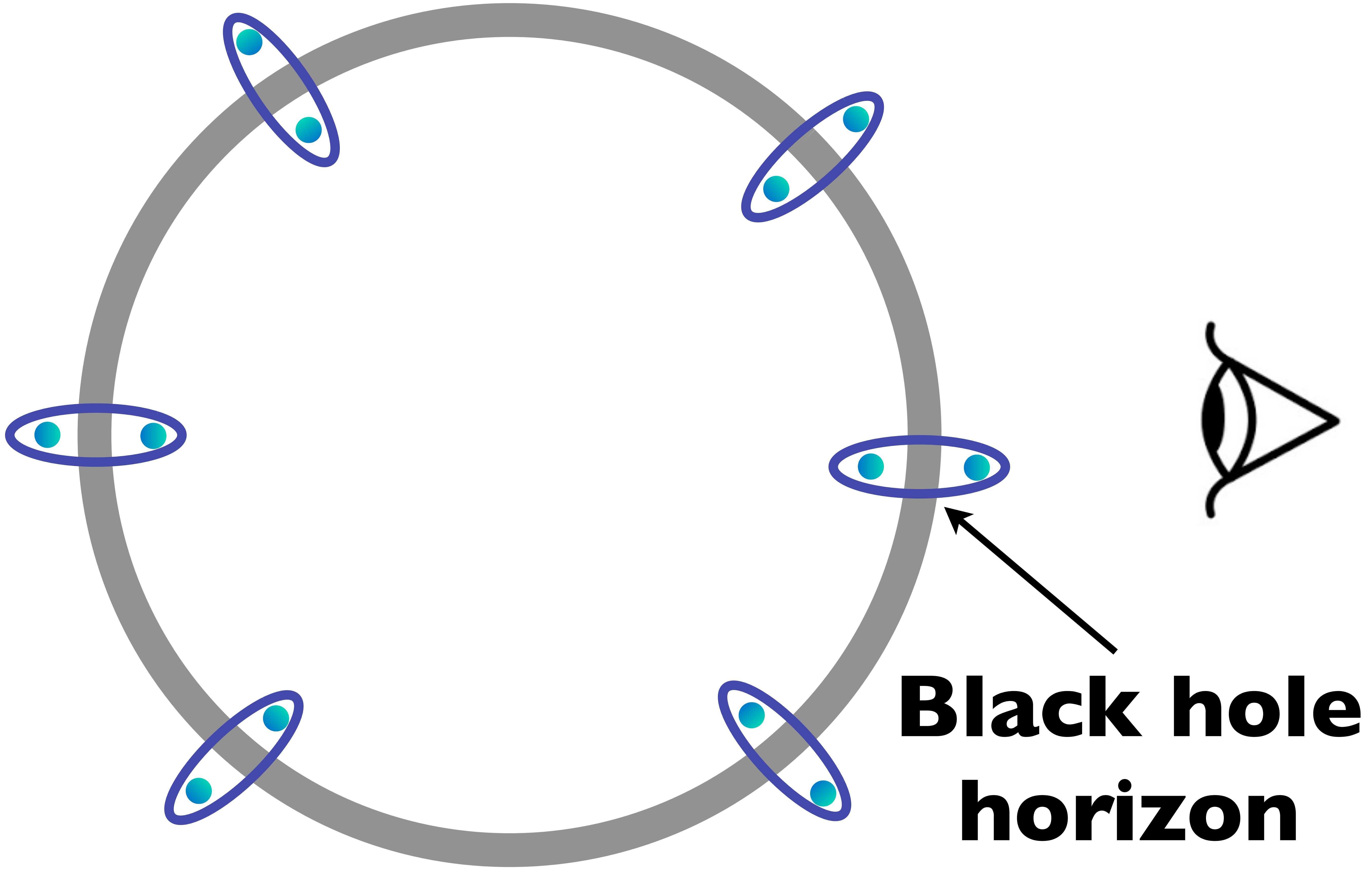
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Bekenstein-Hawking obtained

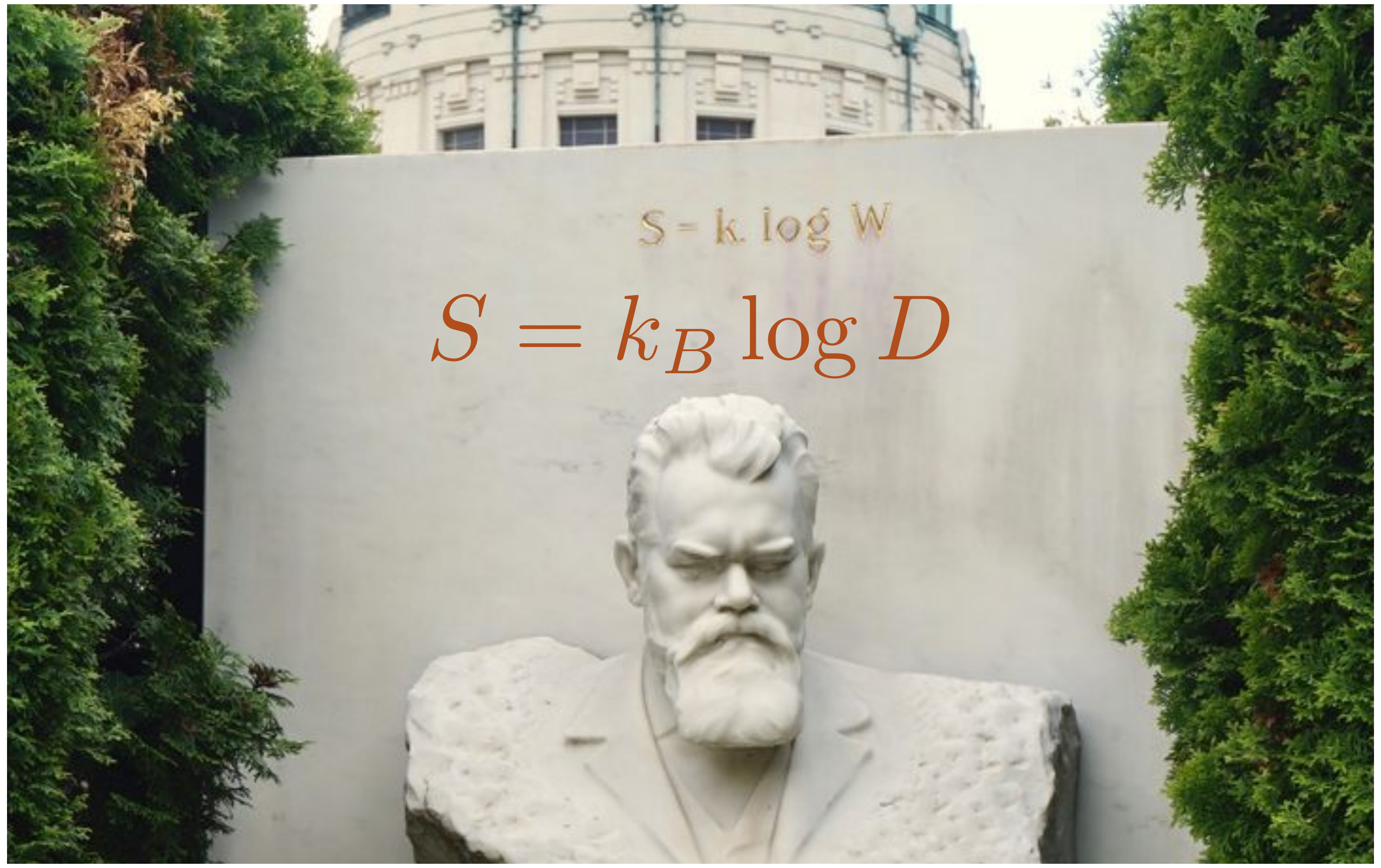
$$S = \frac{k_B A c^3}{4G\hbar}$$

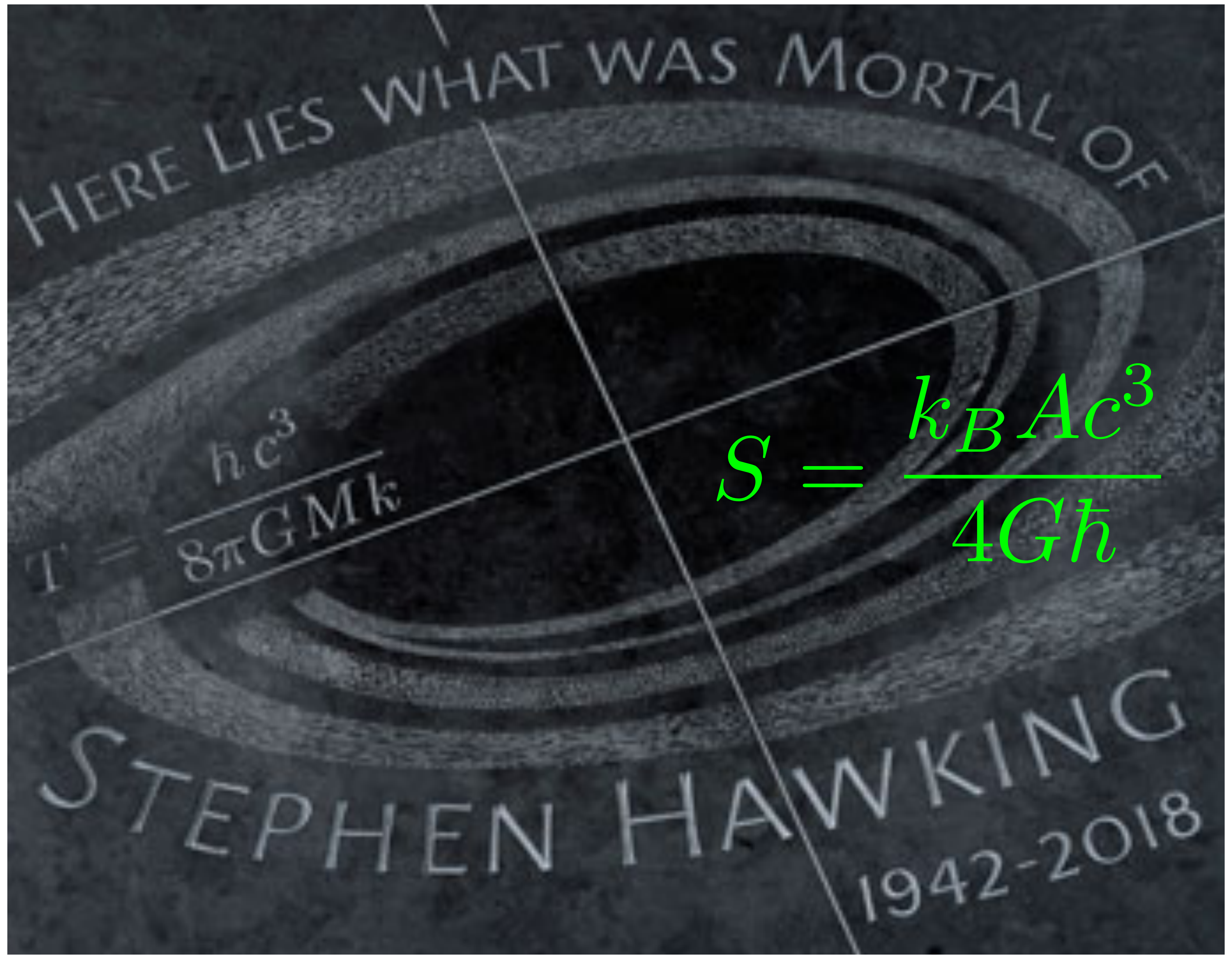
where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





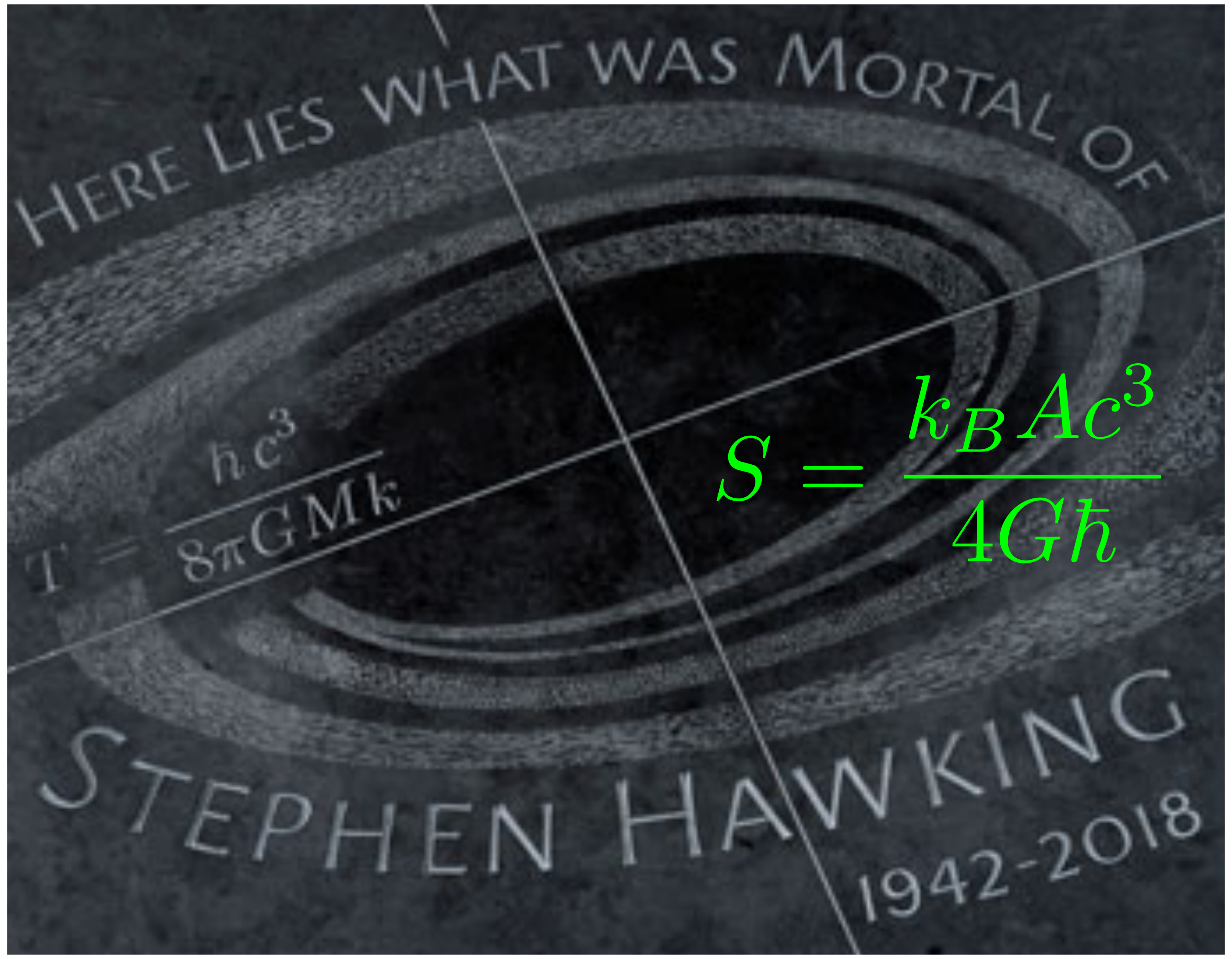
Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?

$$S = \frac{k_B A c^3}{4G\hbar} \qquad T = \frac{\hbar c^3}{8\pi G M k_B}$$

$$\tau_{\rm ring-down} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

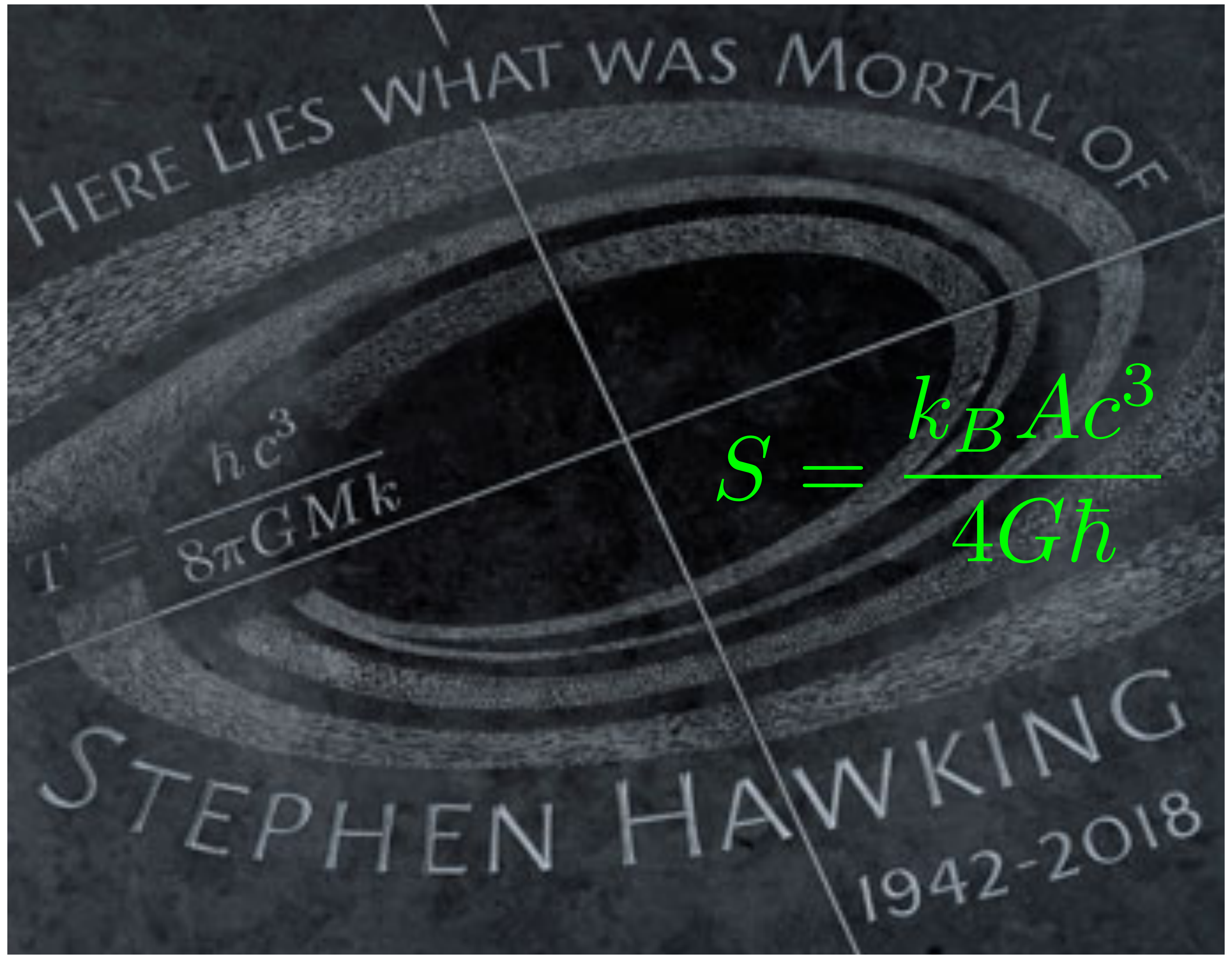


J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974); C.V. Vishveshwara, Nature **227**, 936 (1970)

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?

For a black hole with charge Q, the area $A_0 = 2GQ^2/c^4$ as $T \to 0$, and so $S(T \to 0) > 0$.



J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974); C.V. Vishveshwara, Nature **227**, 936 (1970)

Needed,
to solve open problems in the theory of superconductivity and black holes:

A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

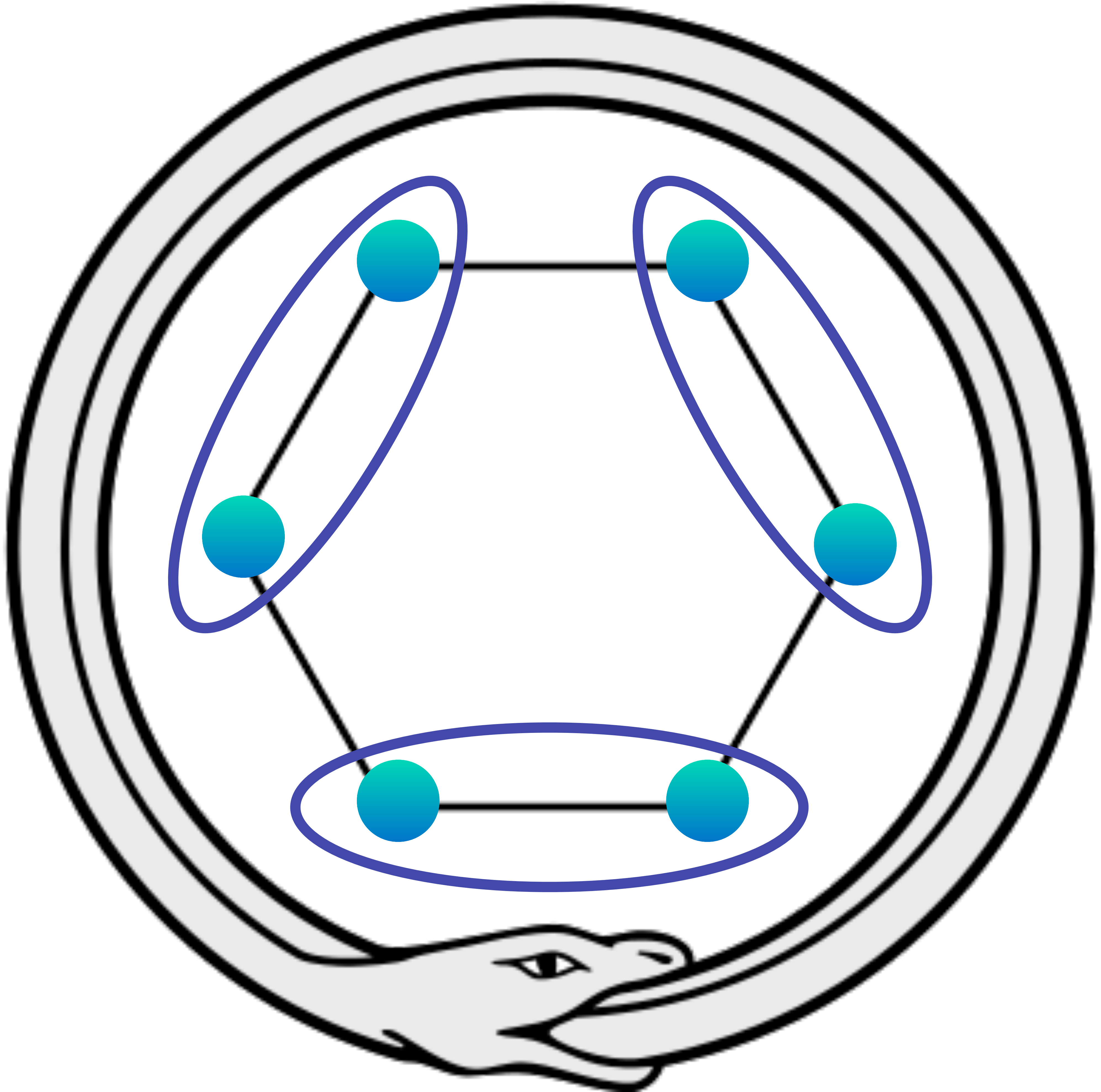
Needed,
to solve open problems in the theory of superconductivity and black holes:

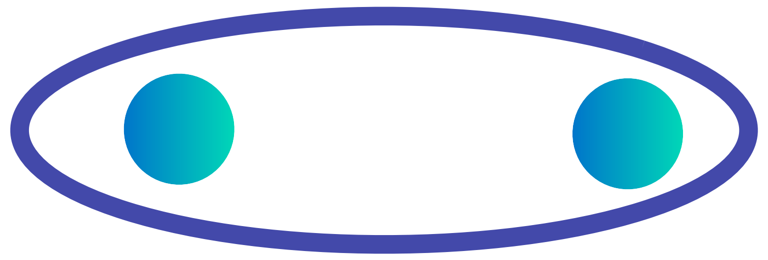
A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

The Sachdev-Ye-Kitaev model of many-particle entanglement

Kekule's spooky dream (1865)

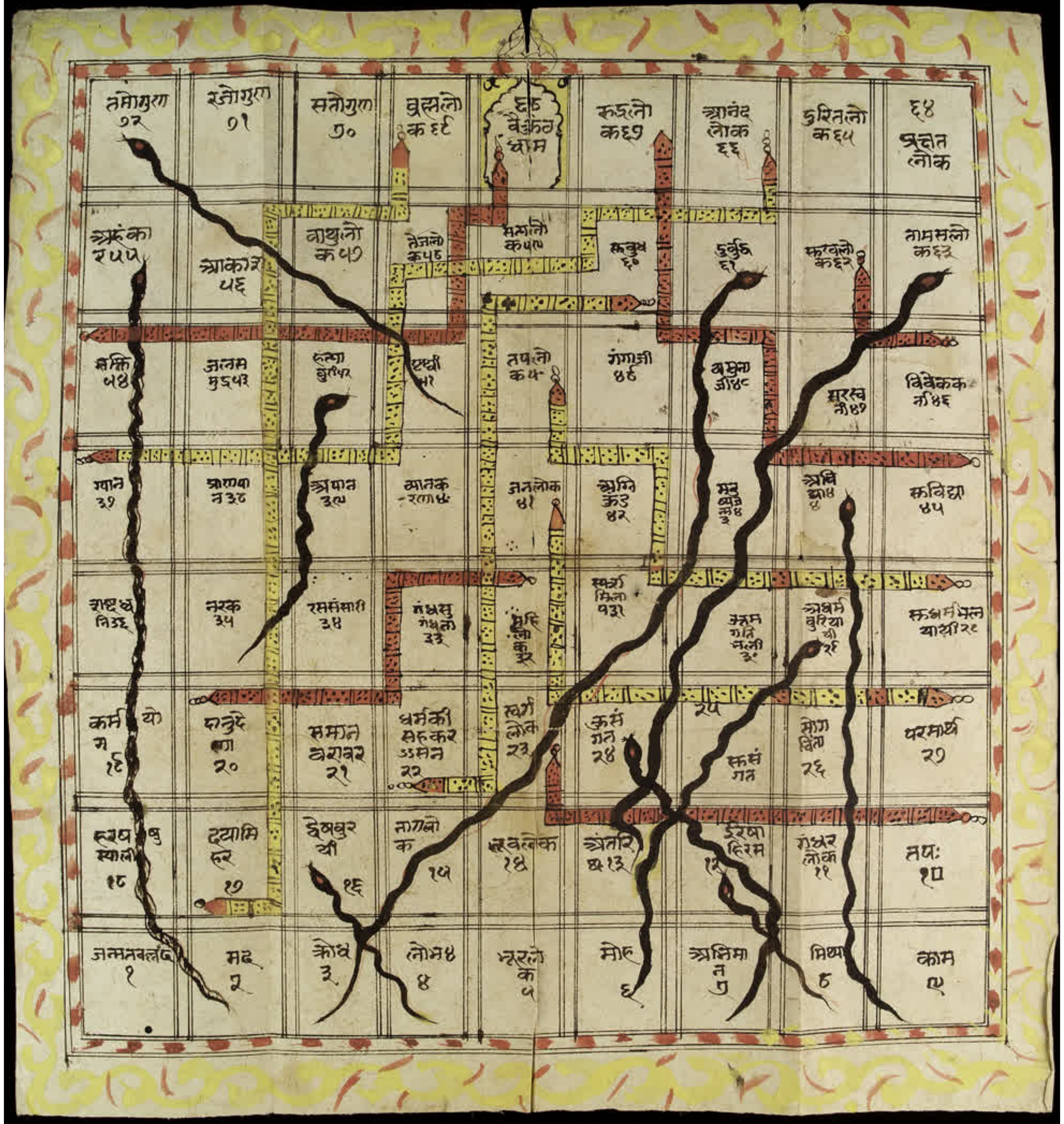
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

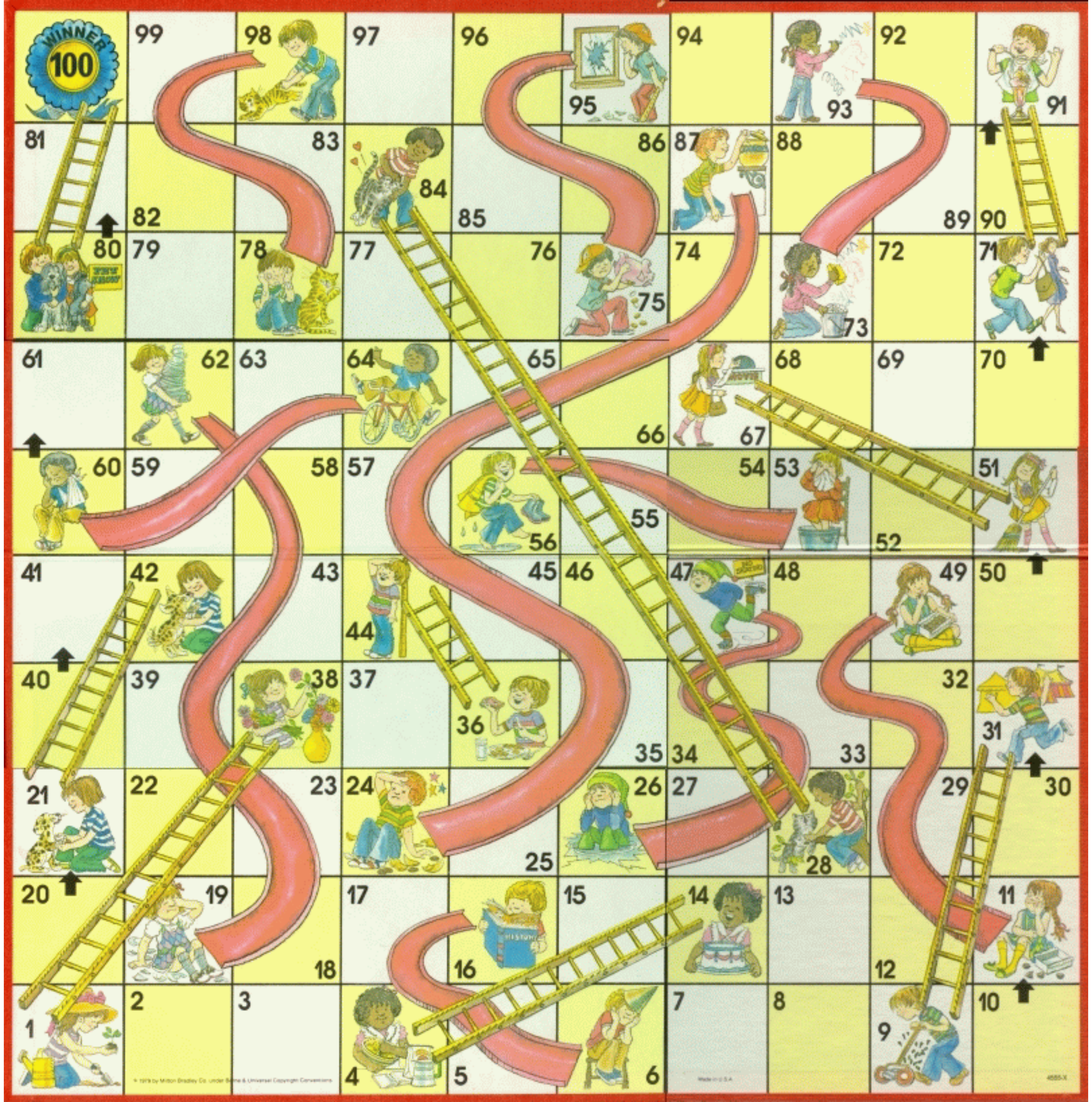
*Wikipedia



My spooky dream (1992)*

Ancient Indian game of Snakes and Ladders

*Not true



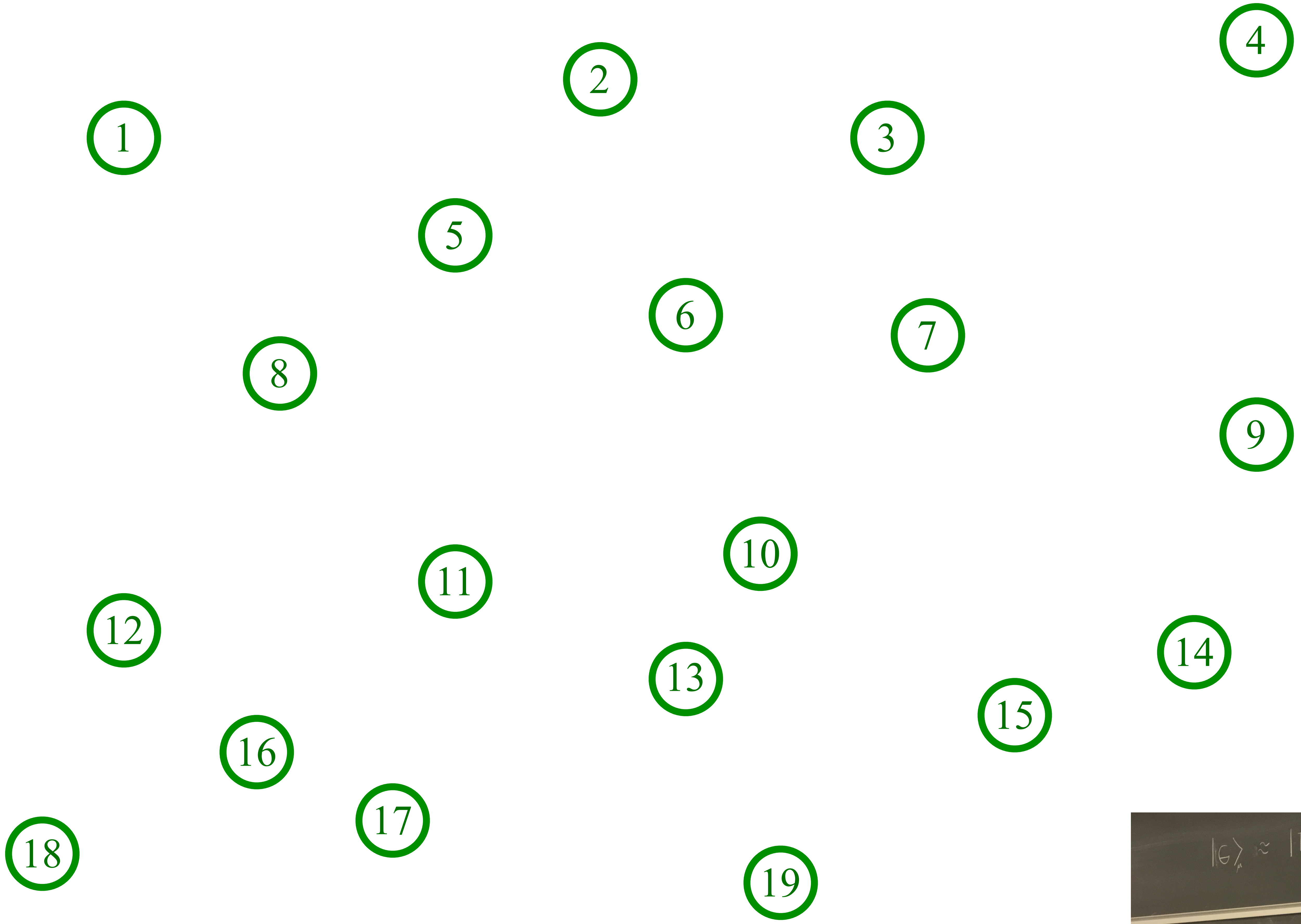
My spooky dream (1992)*

Hasbro game of Chutes and Ladders

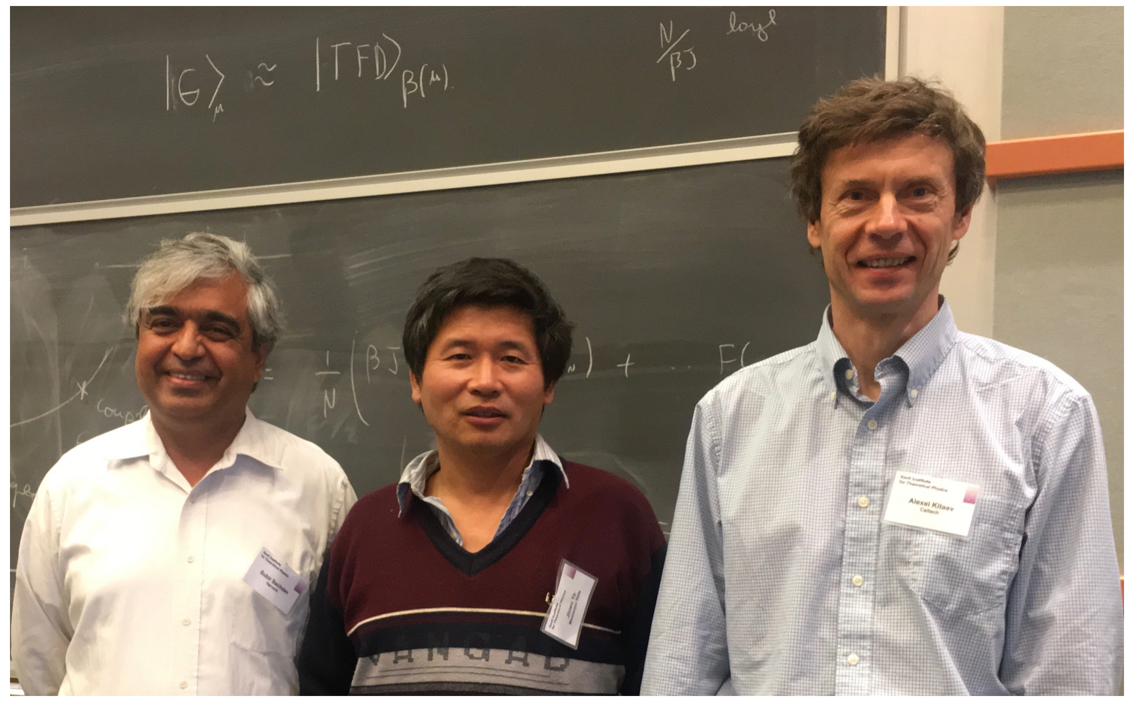
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

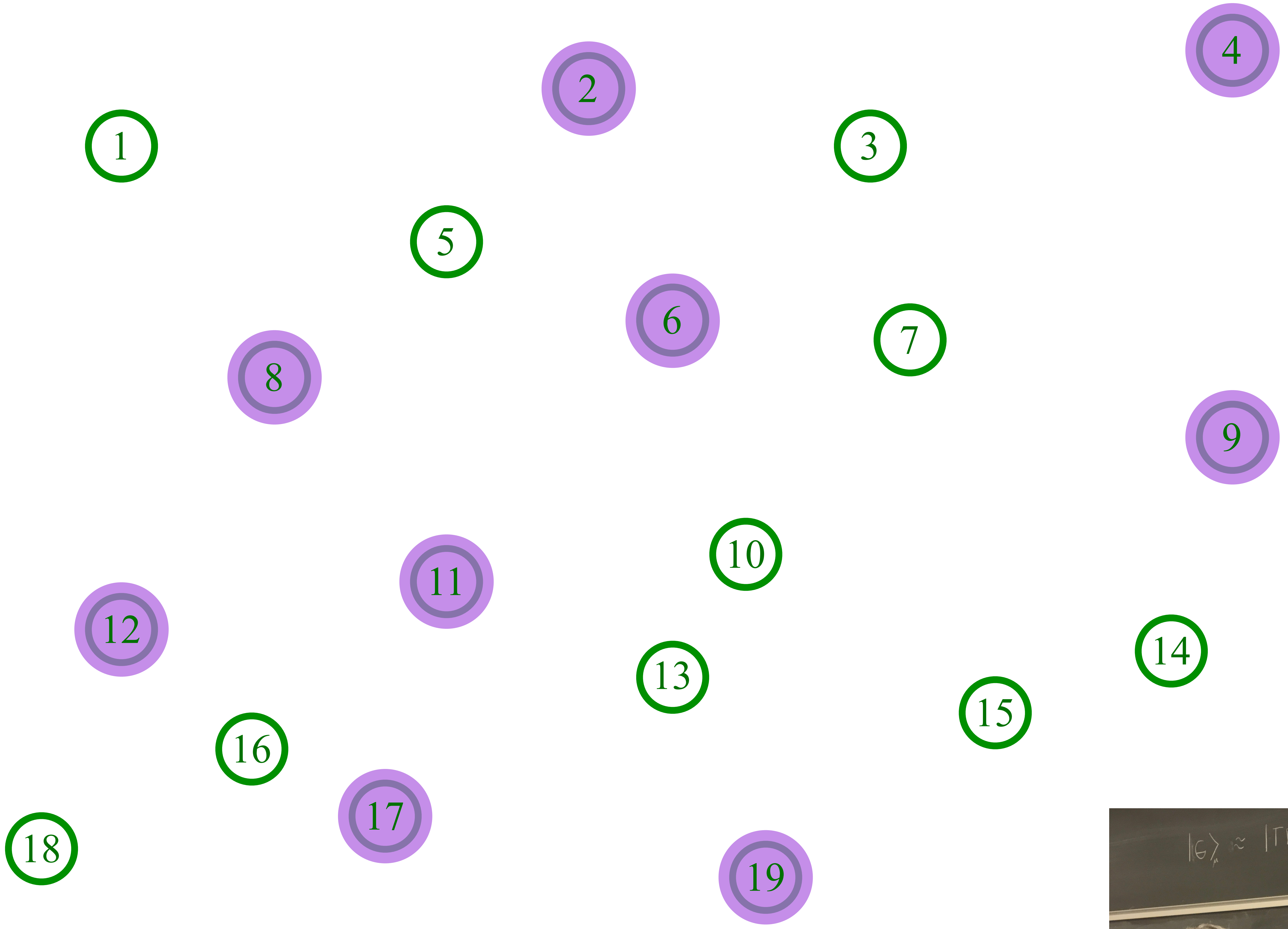


Pick a set of random positions

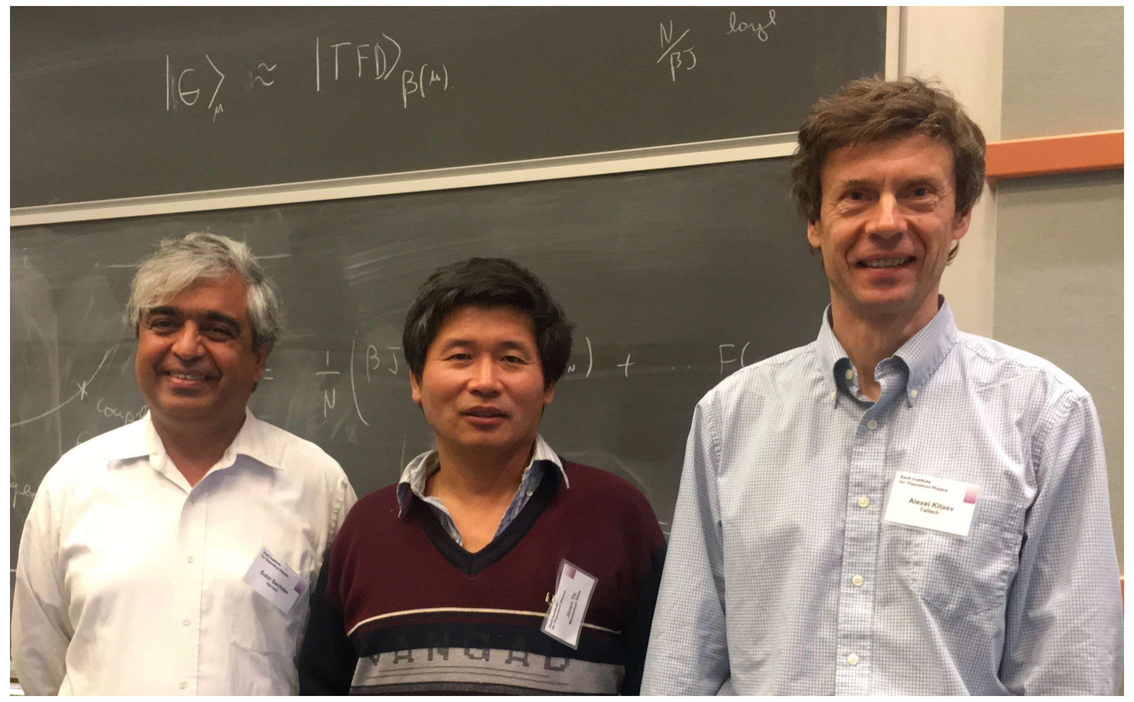


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



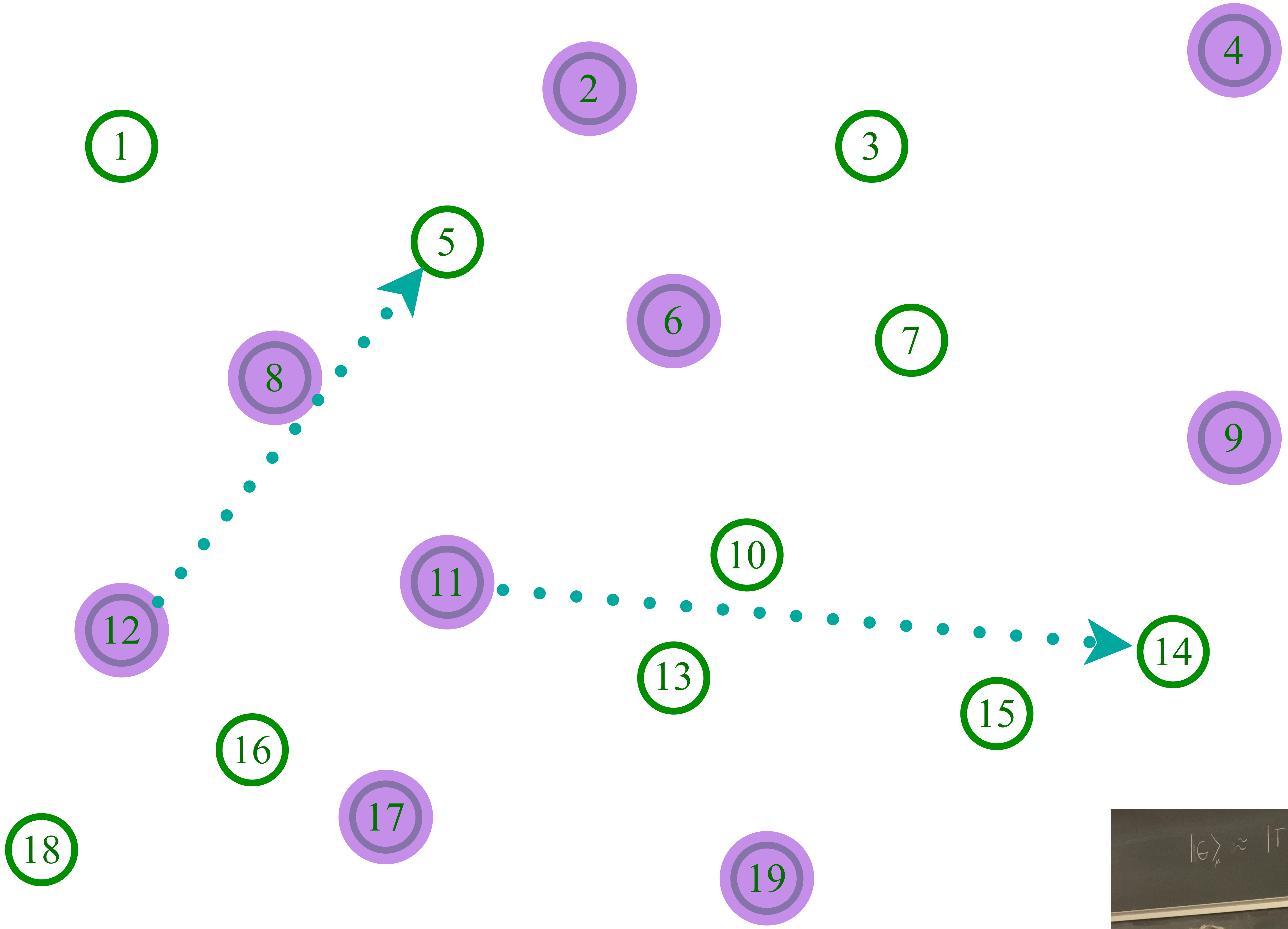
Place electrons randomly on some sites



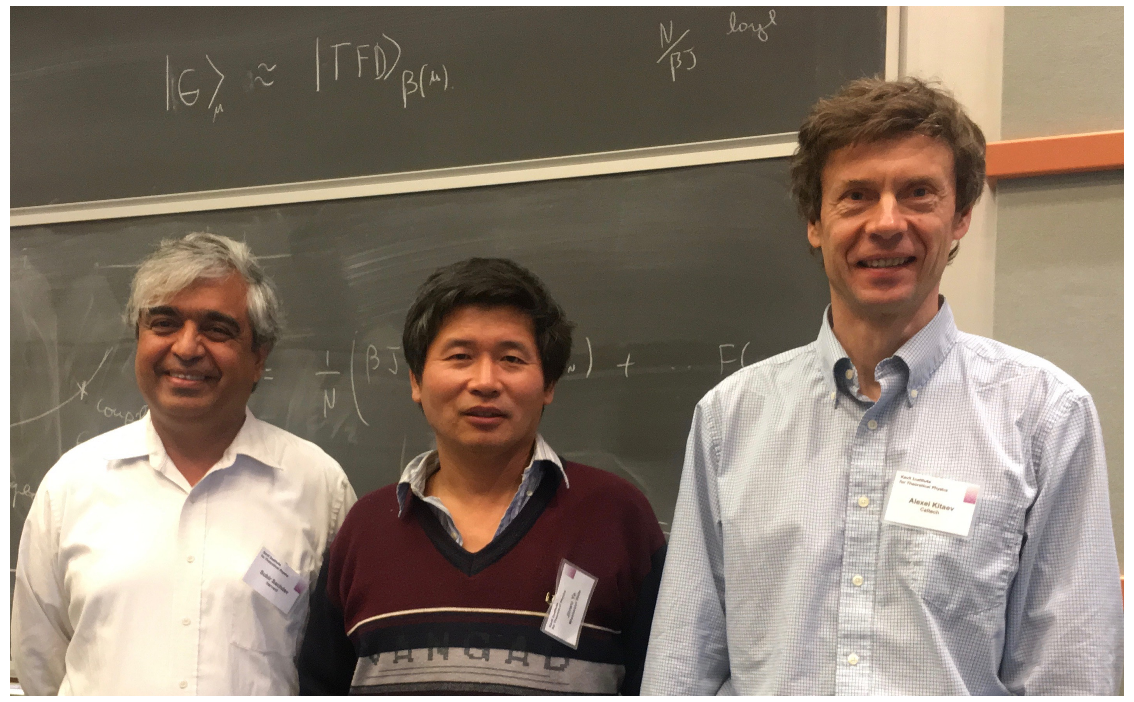
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



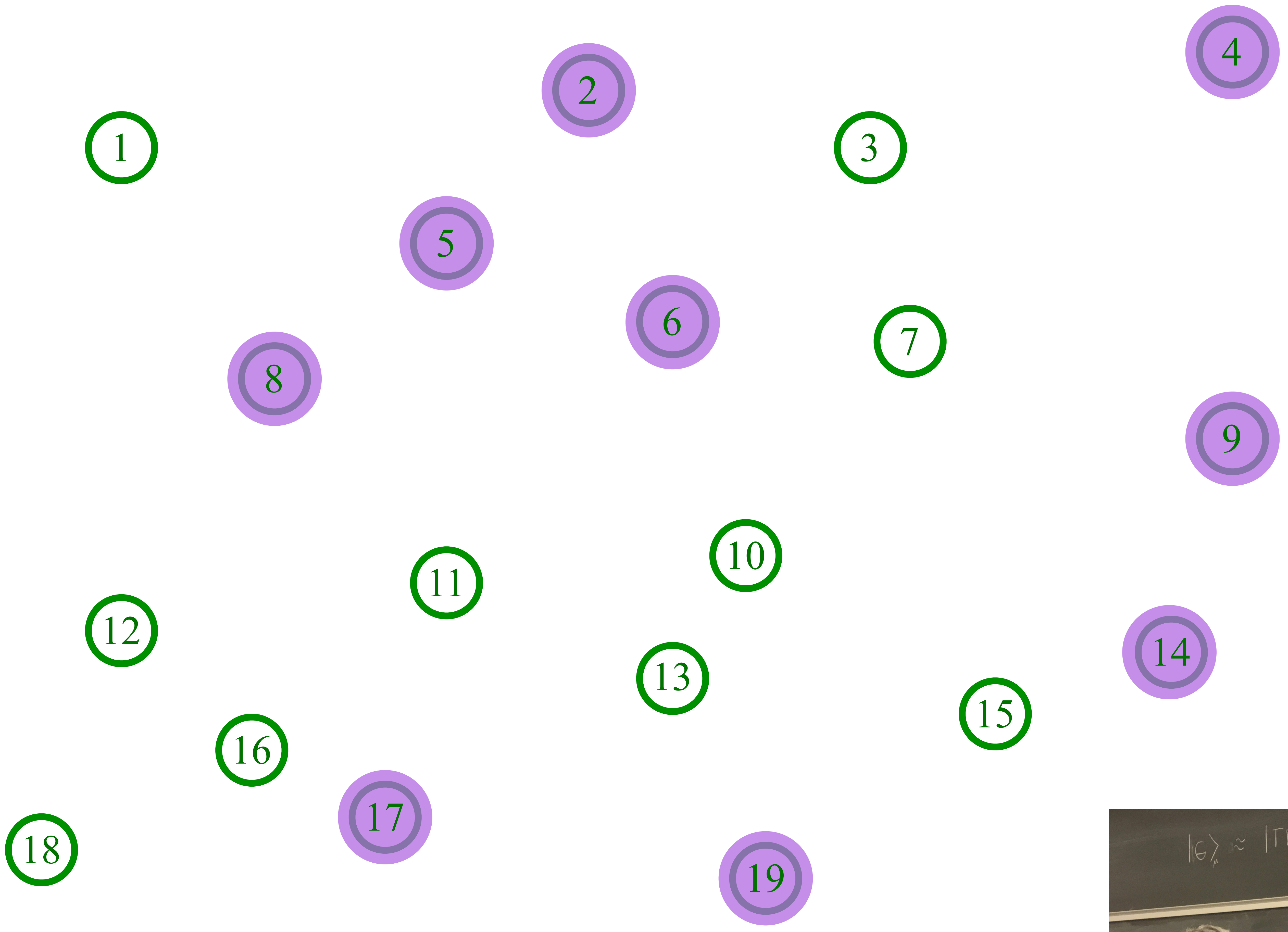
Place electrons randomly on some sites



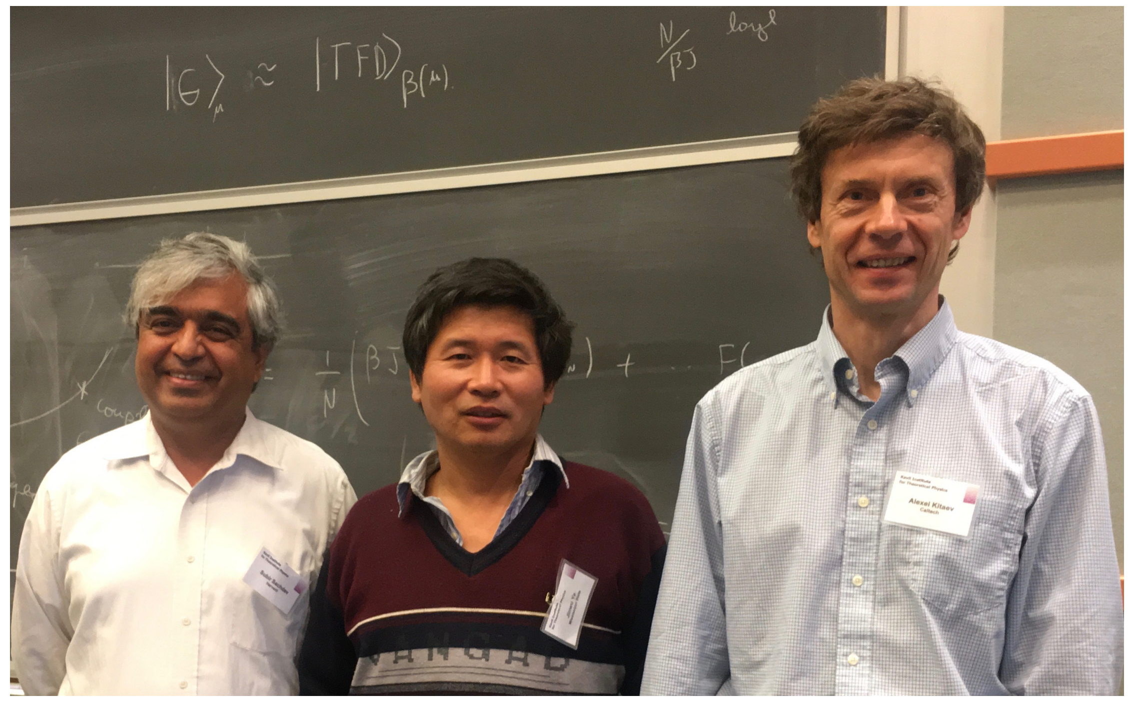
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



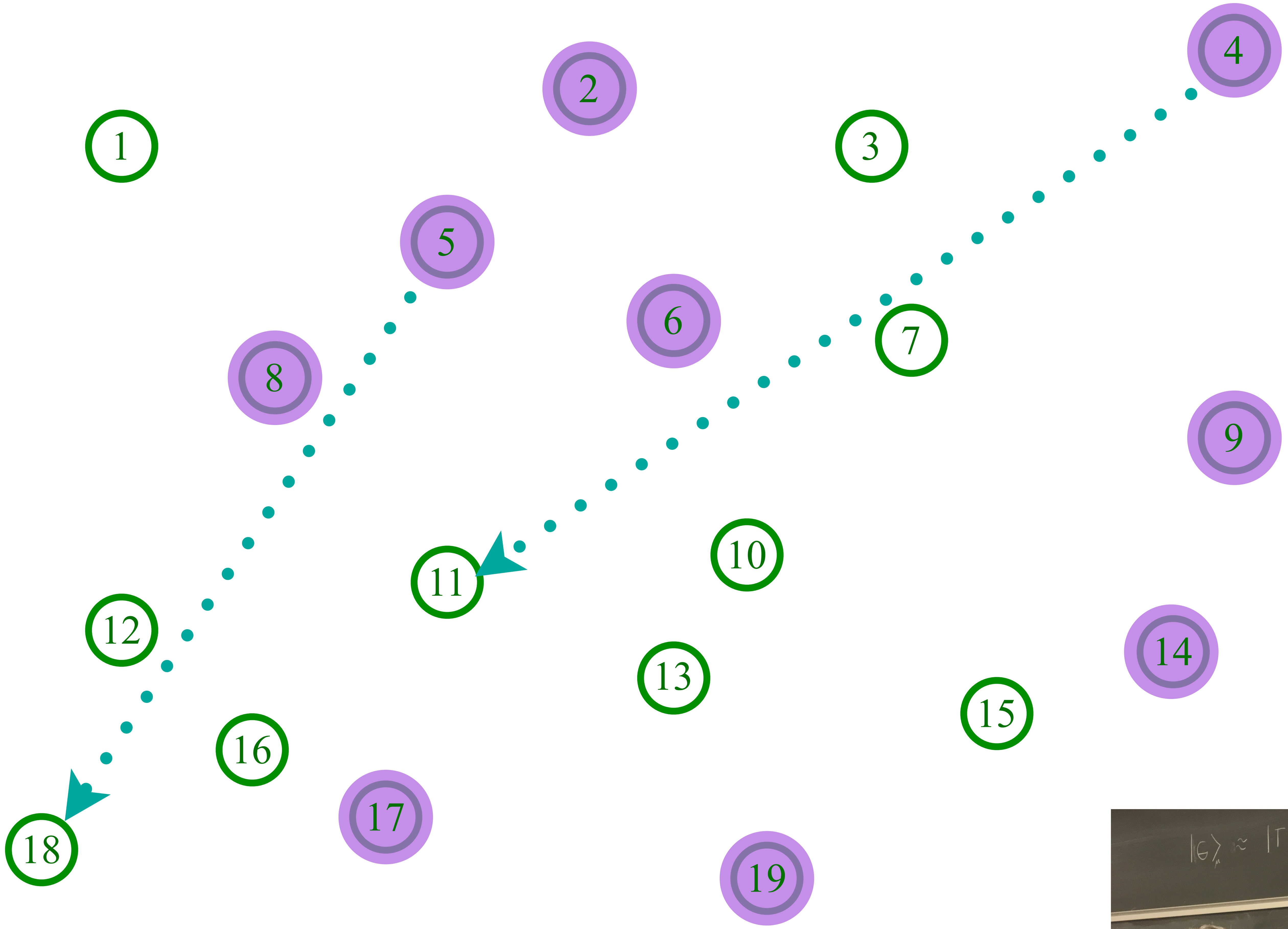
Entangle electrons pairwise randomly



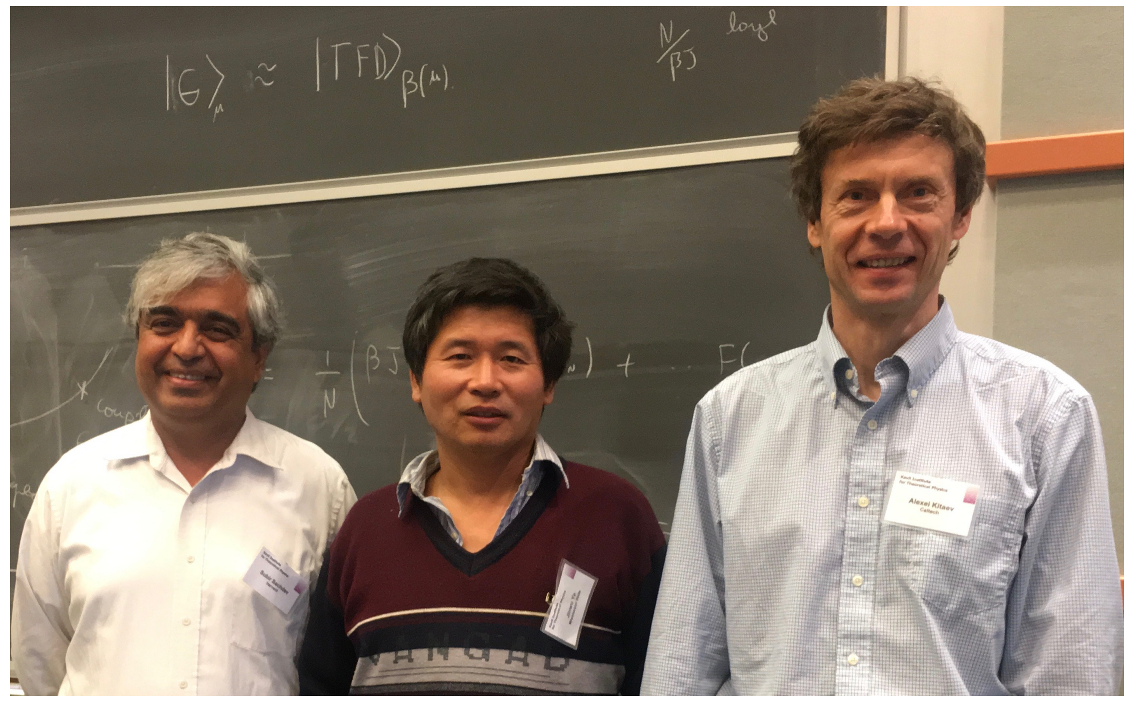
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



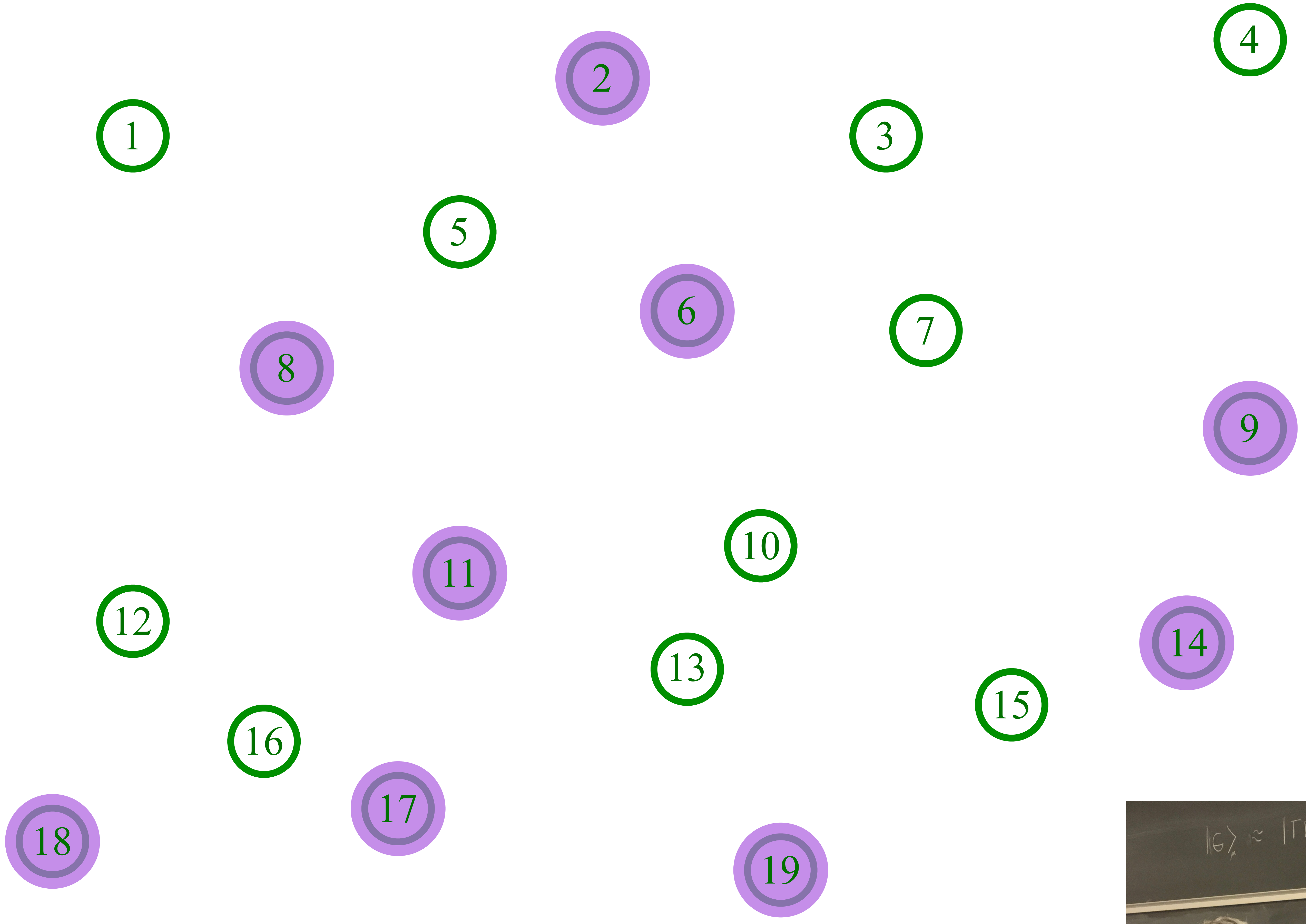
Entangle electrons pairwise randomly



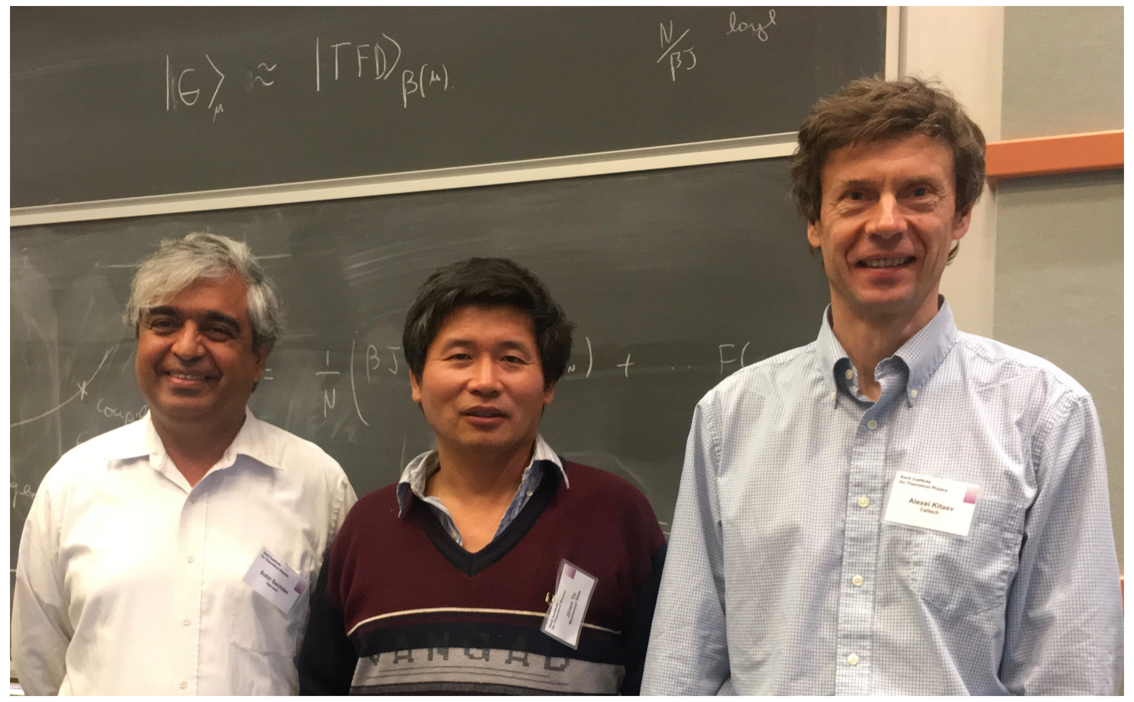
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



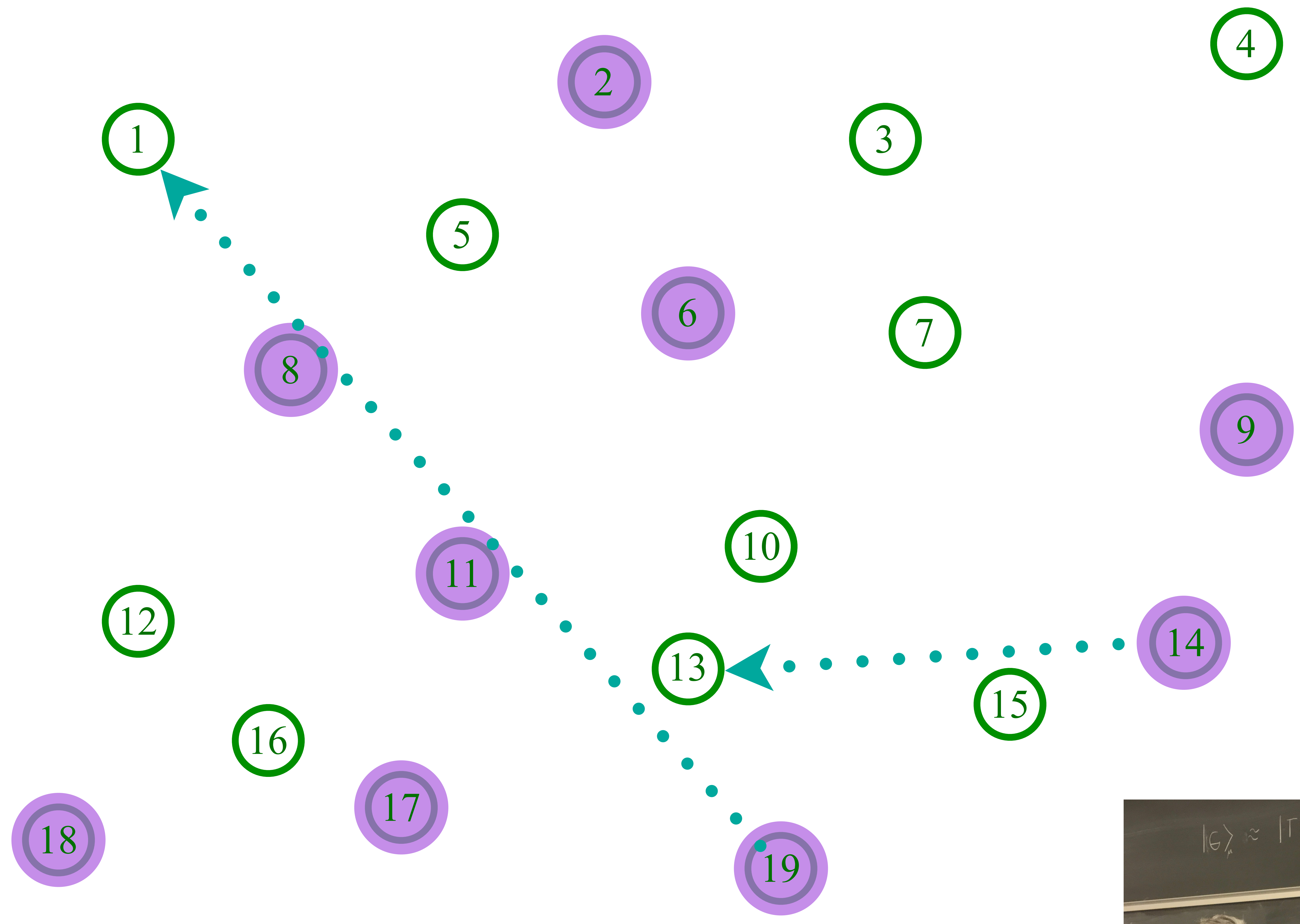
Entangle electrons pairwise randomly



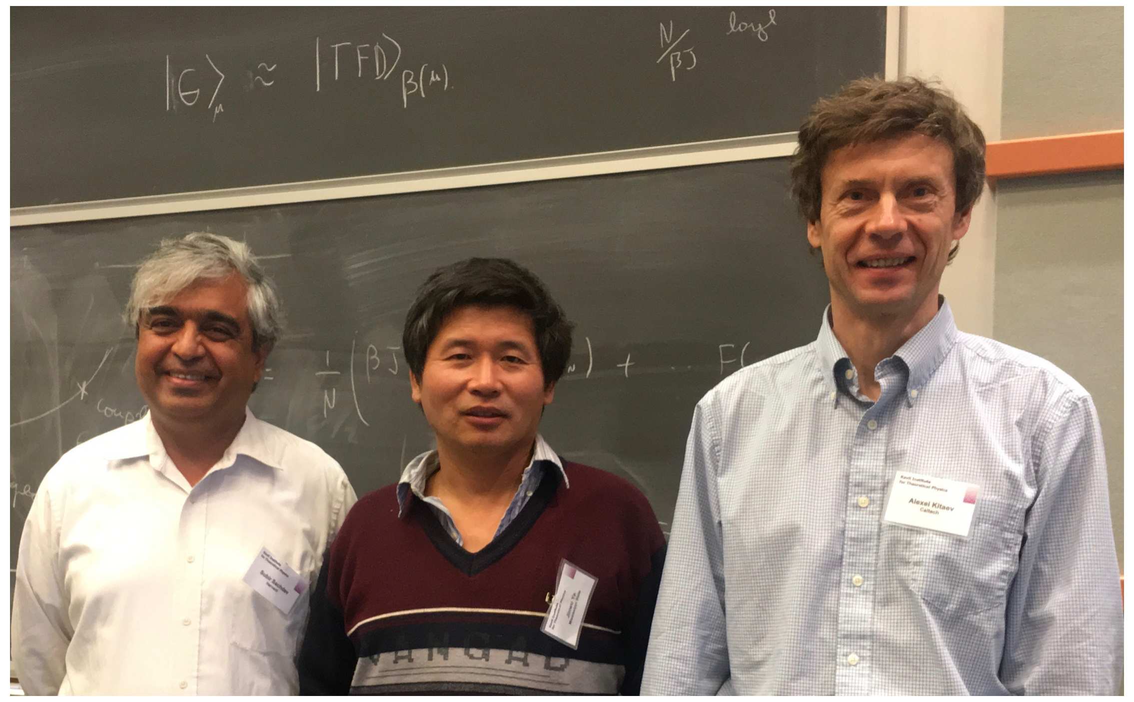
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



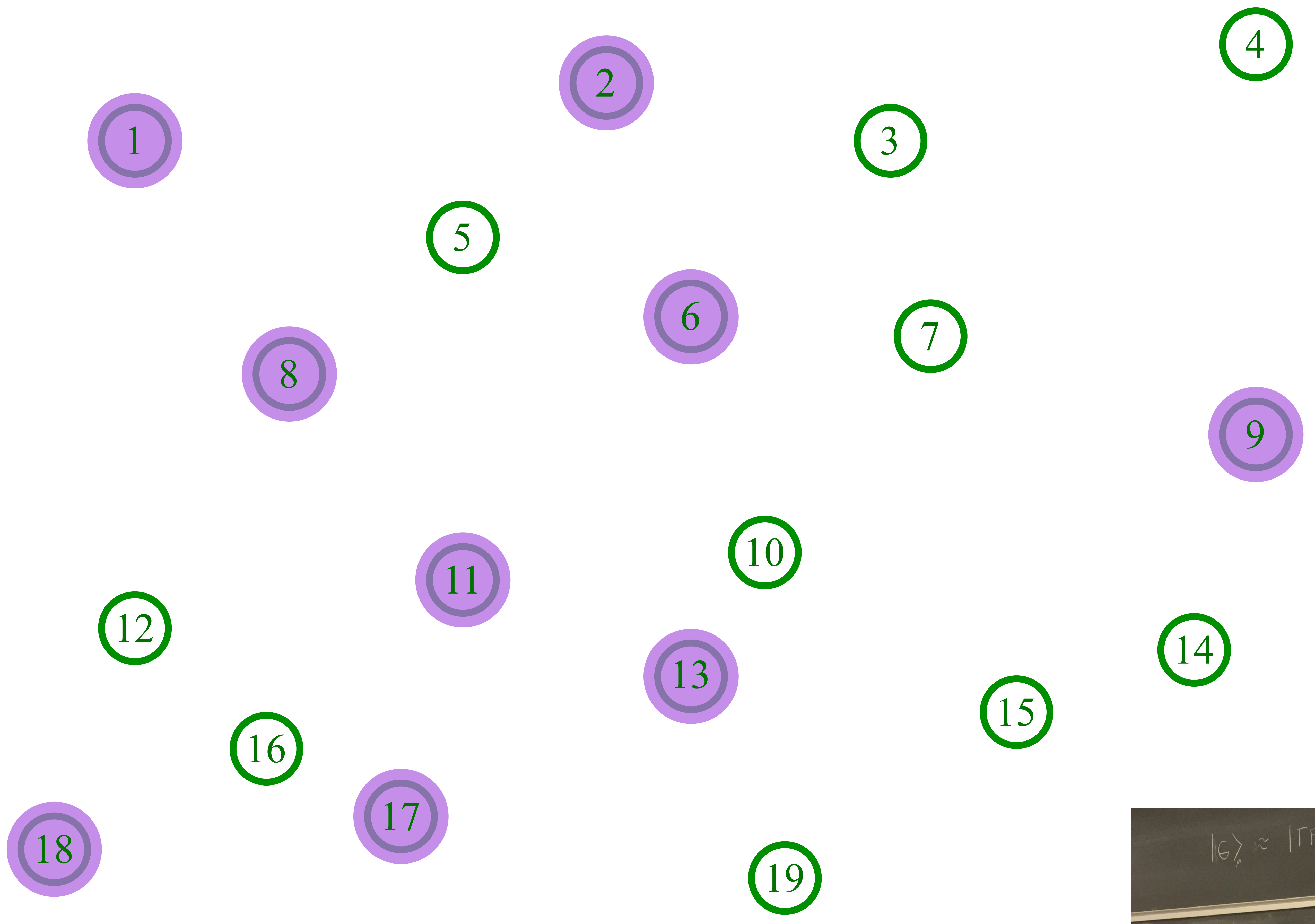
Entangle electrons pairwise randomly



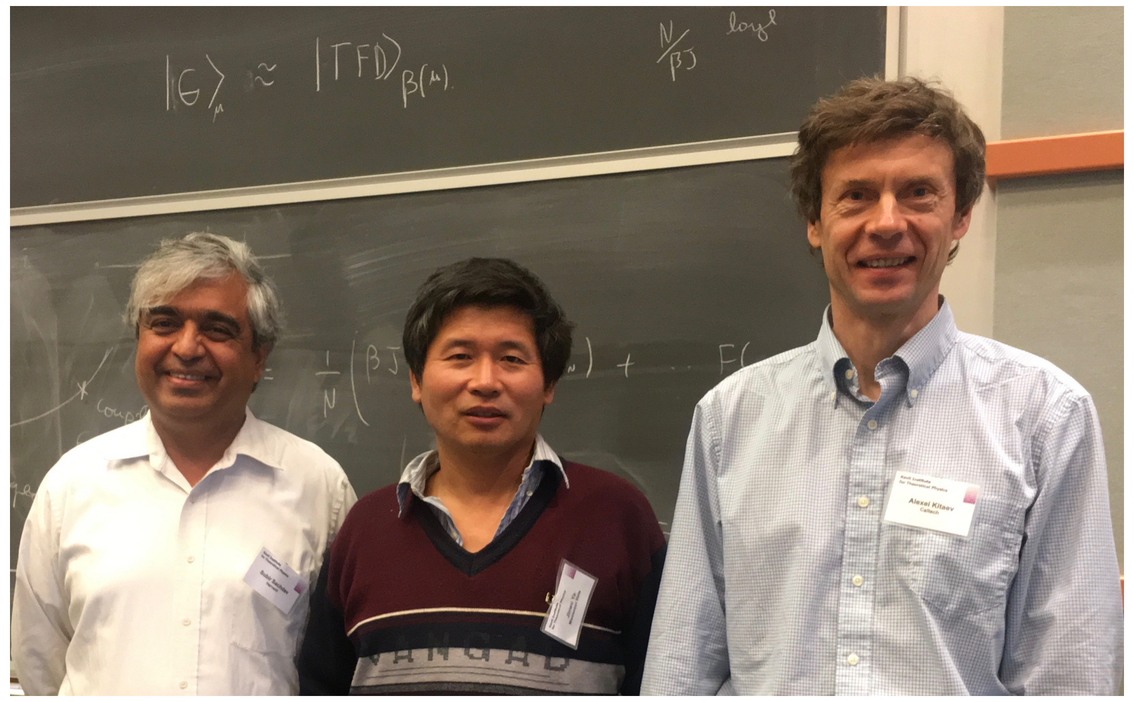
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



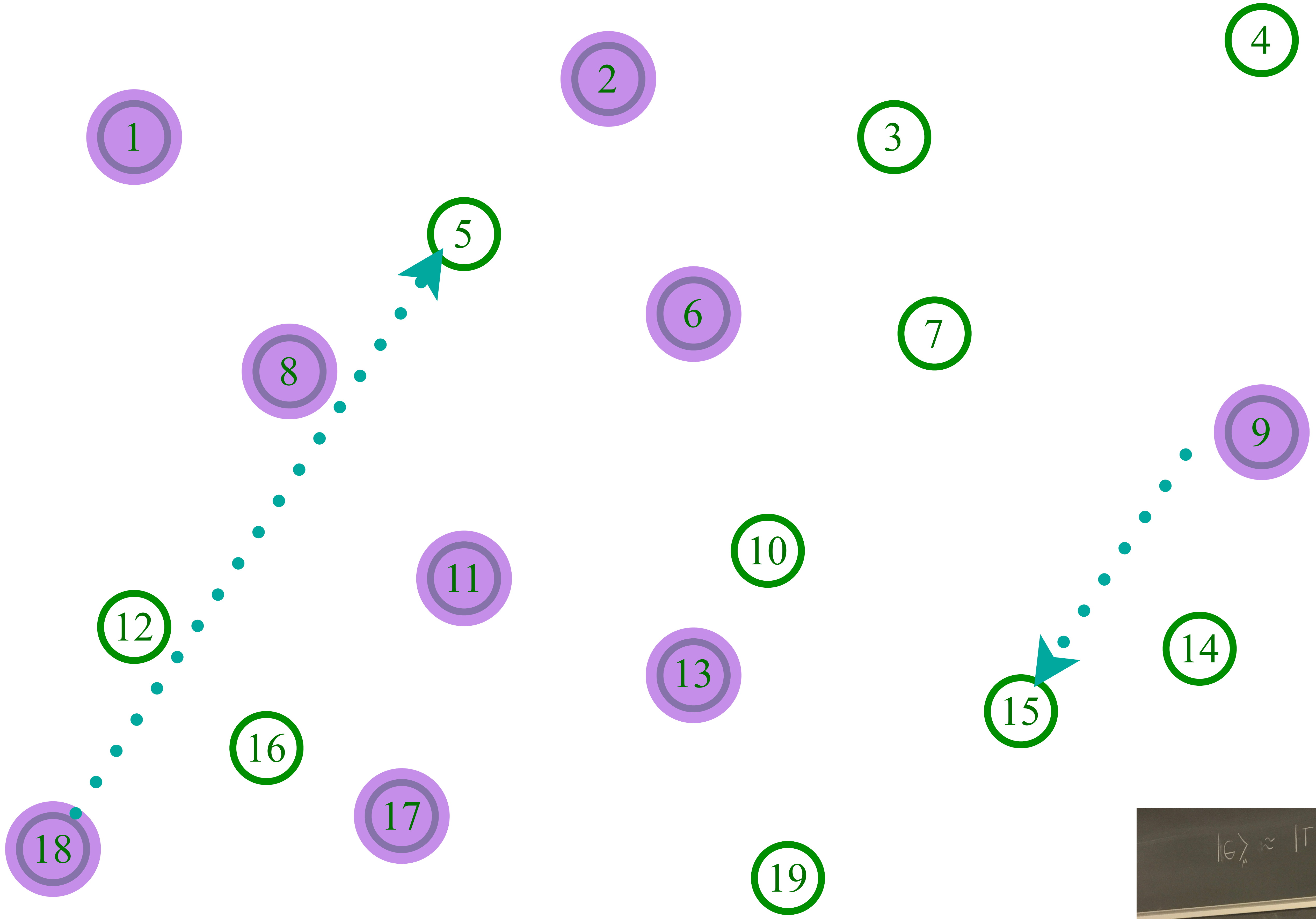
Entangle electrons pairwise randomly



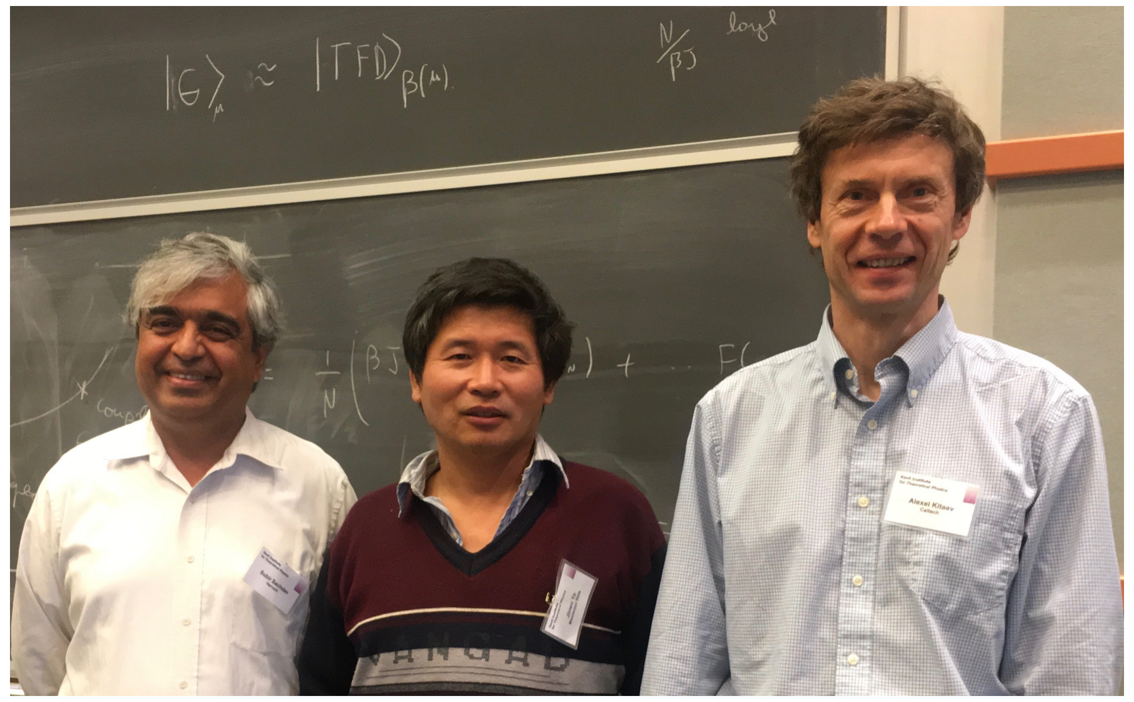
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



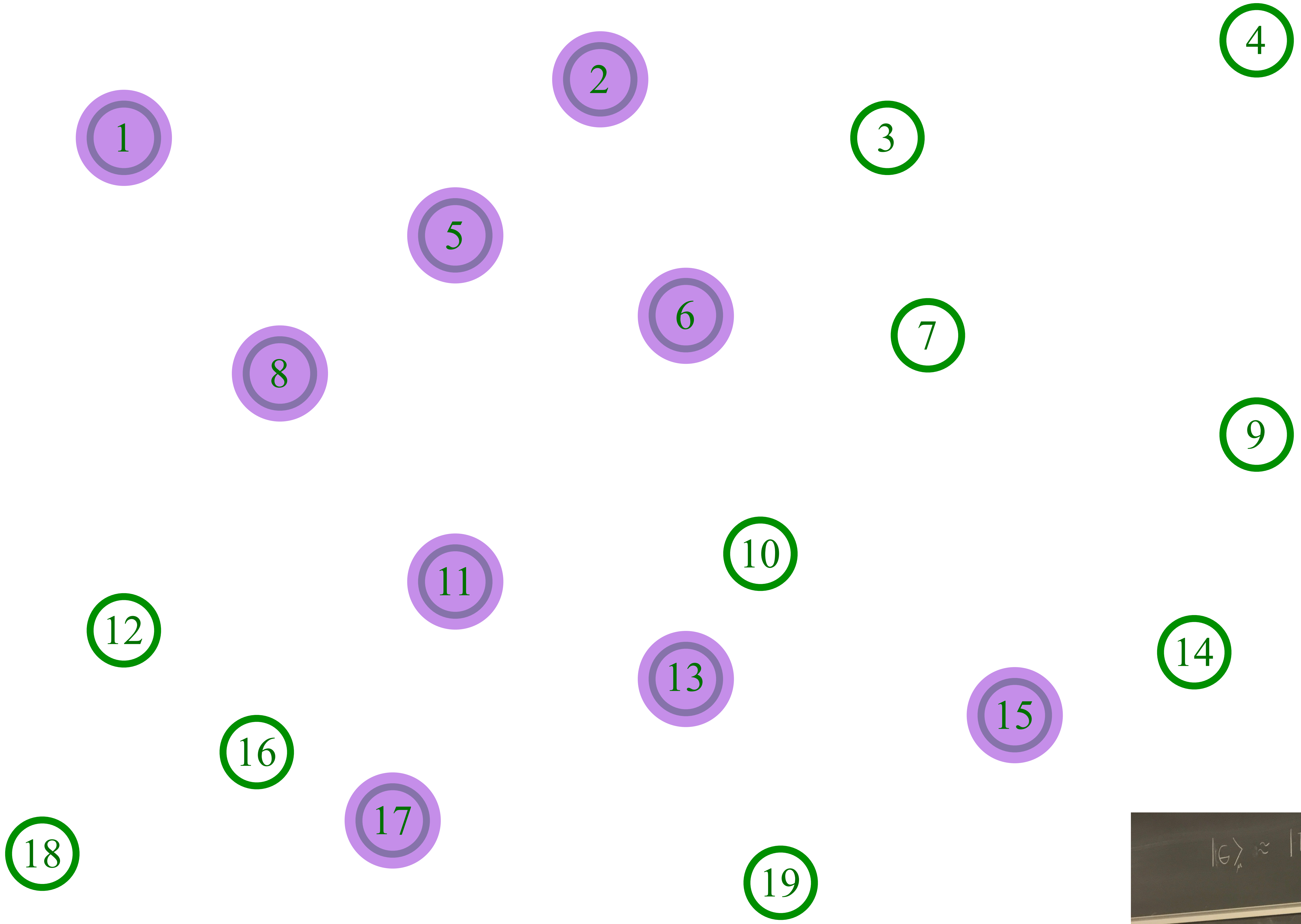
Entangle electrons pairwise randomly



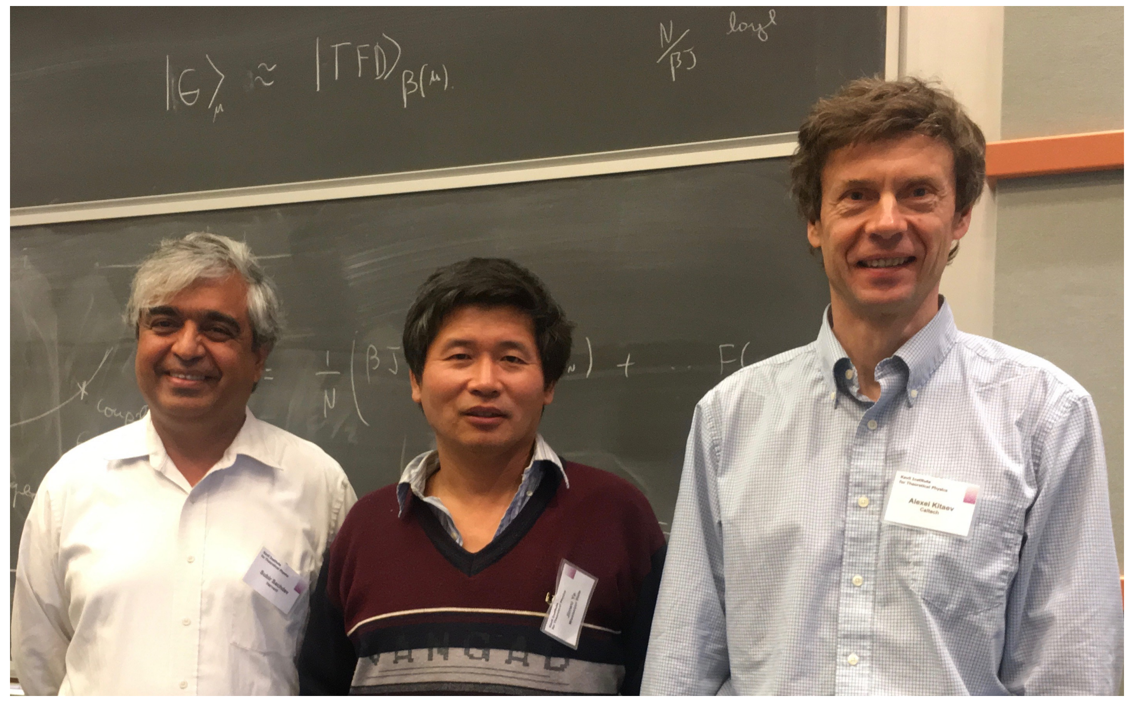
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



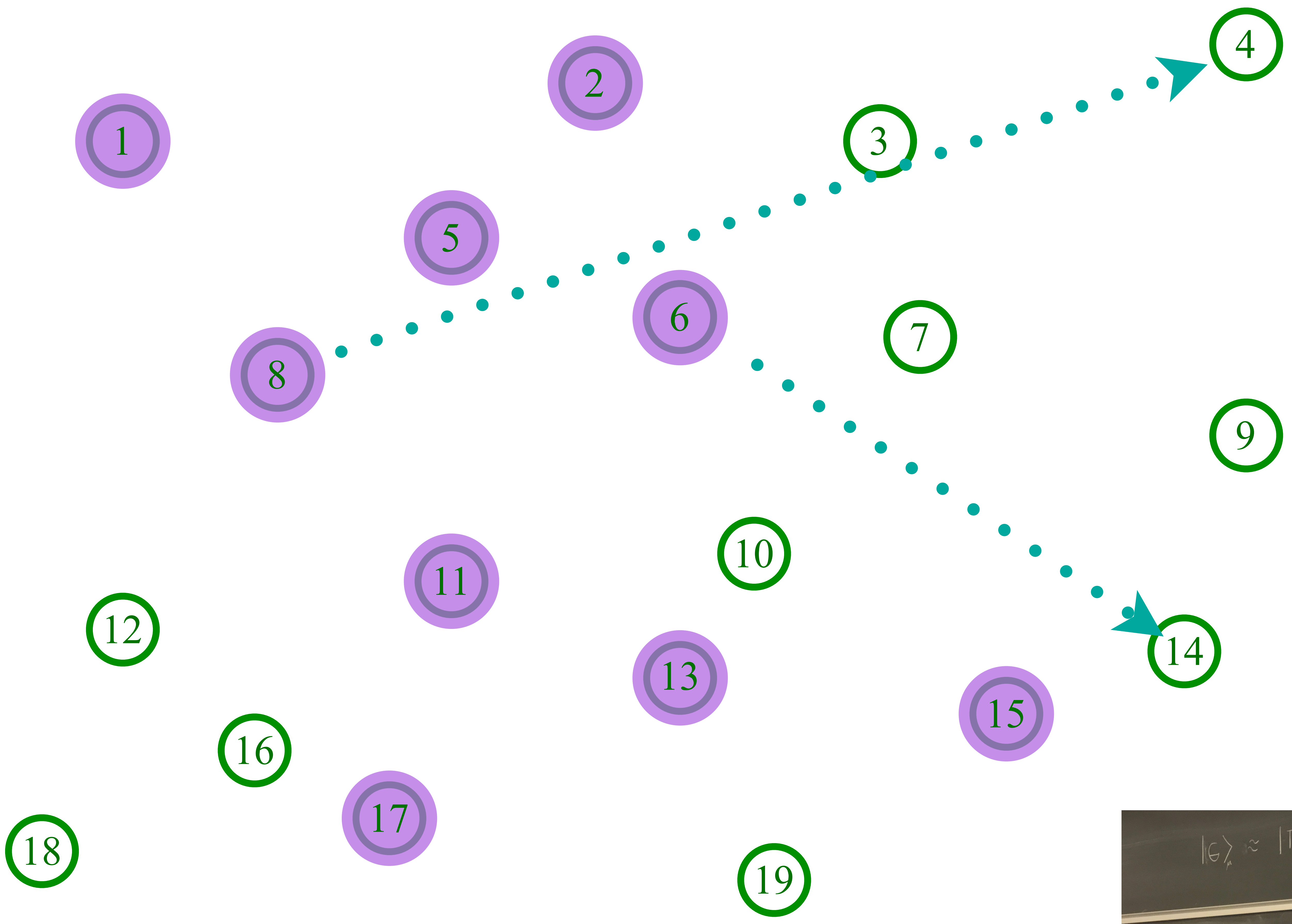
Entangle electrons pairwise randomly



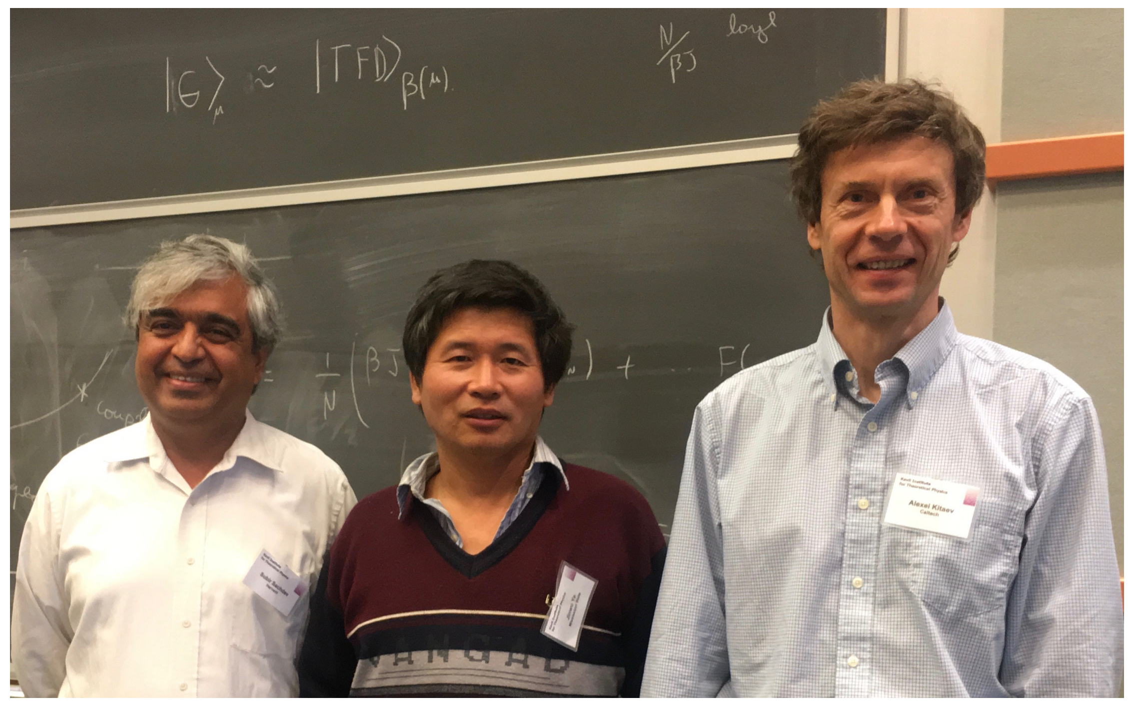
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



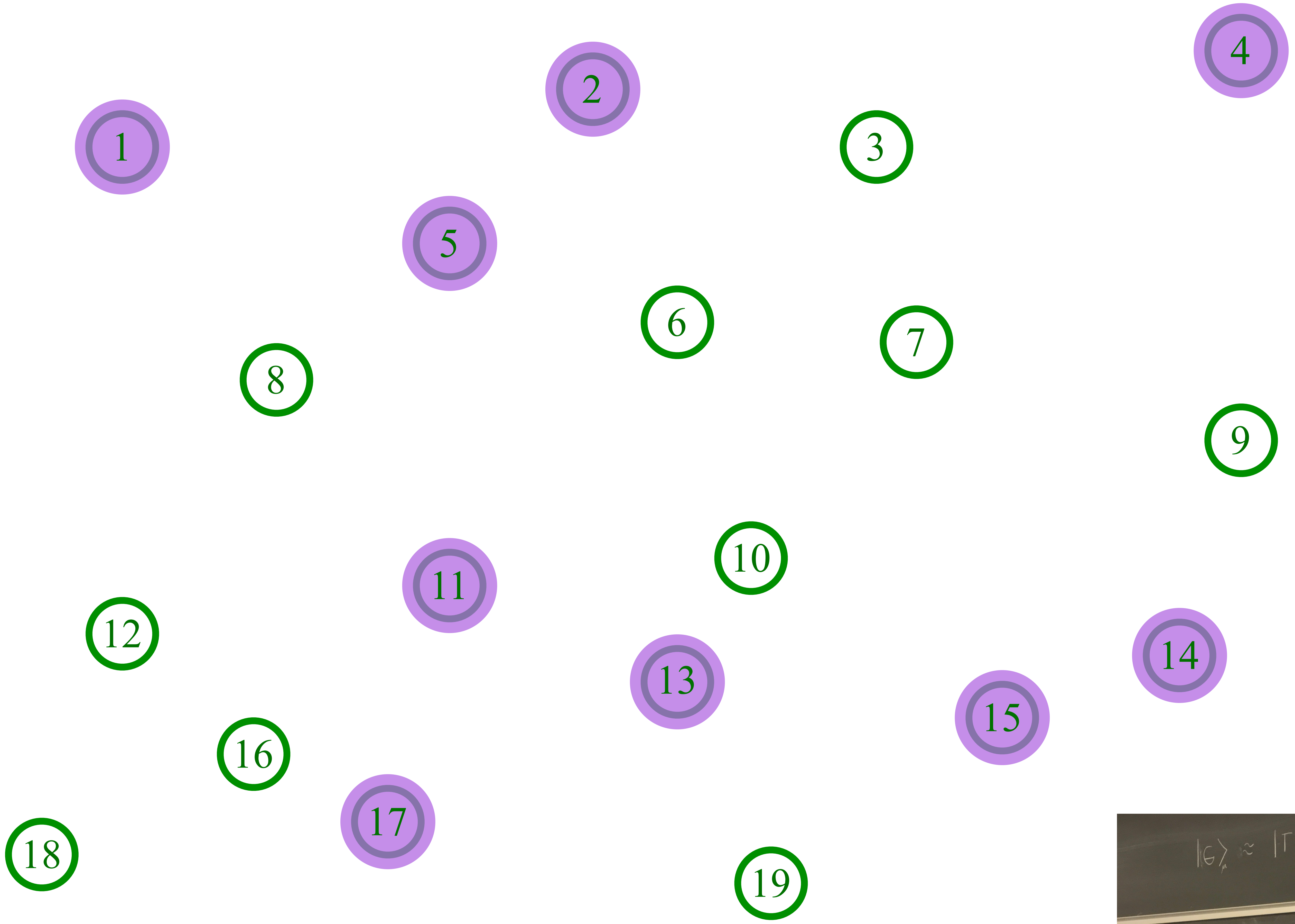
Entangle electrons pairwise randomly

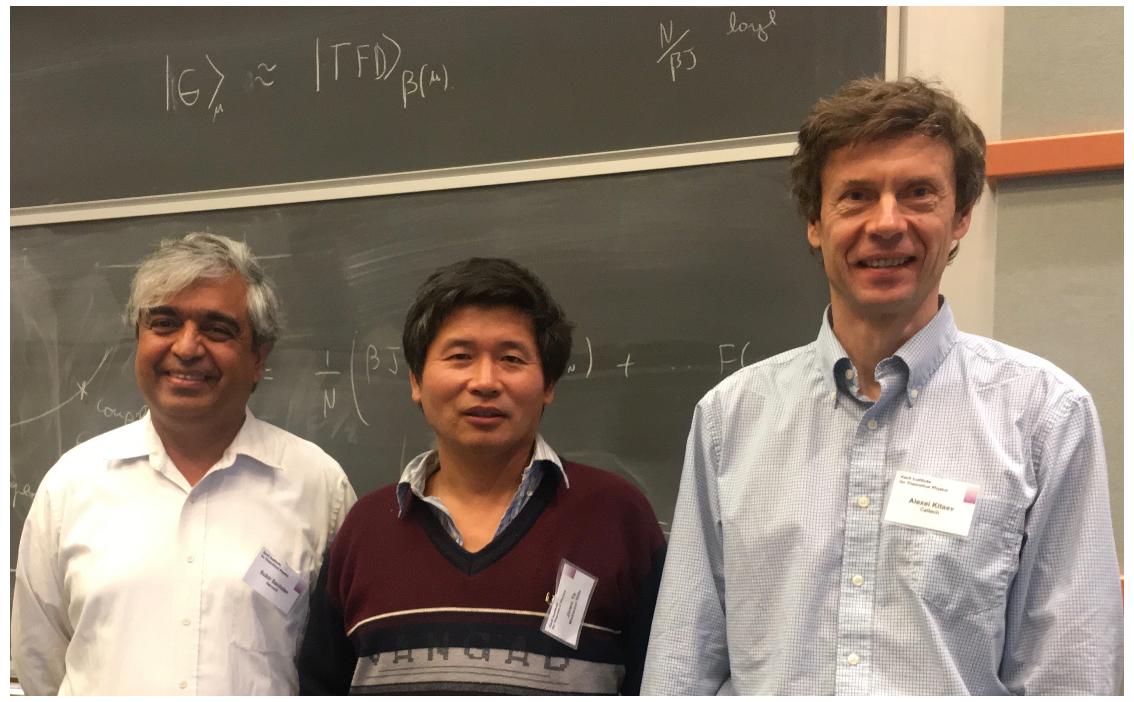


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$





Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

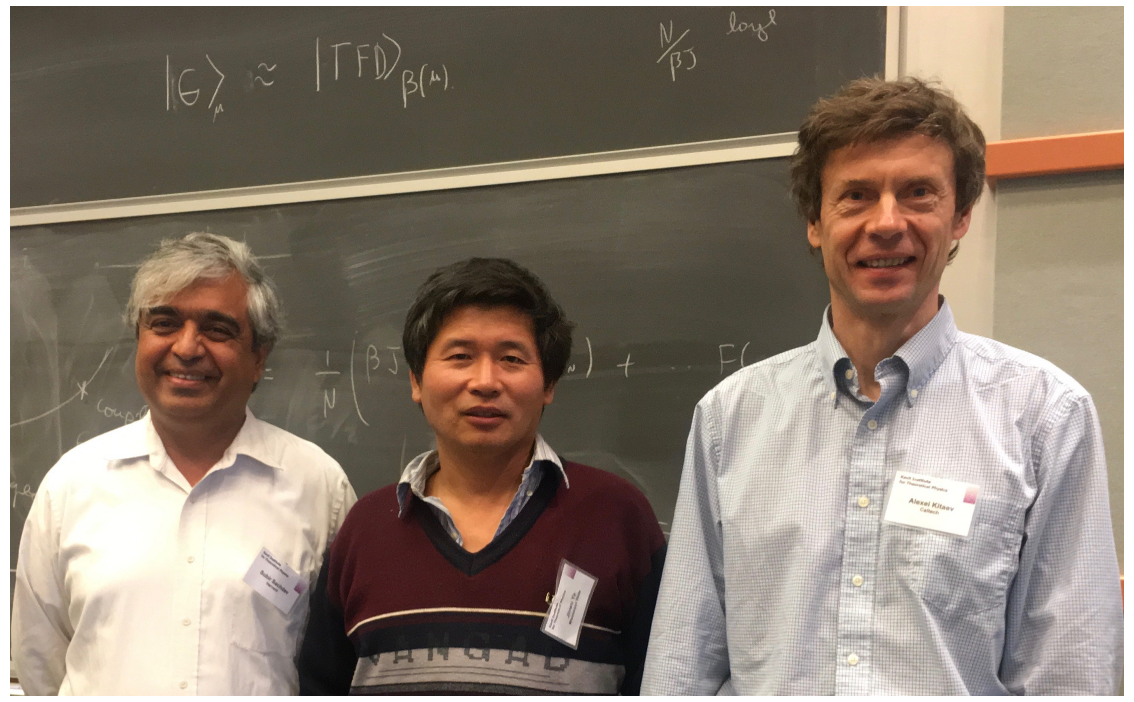
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

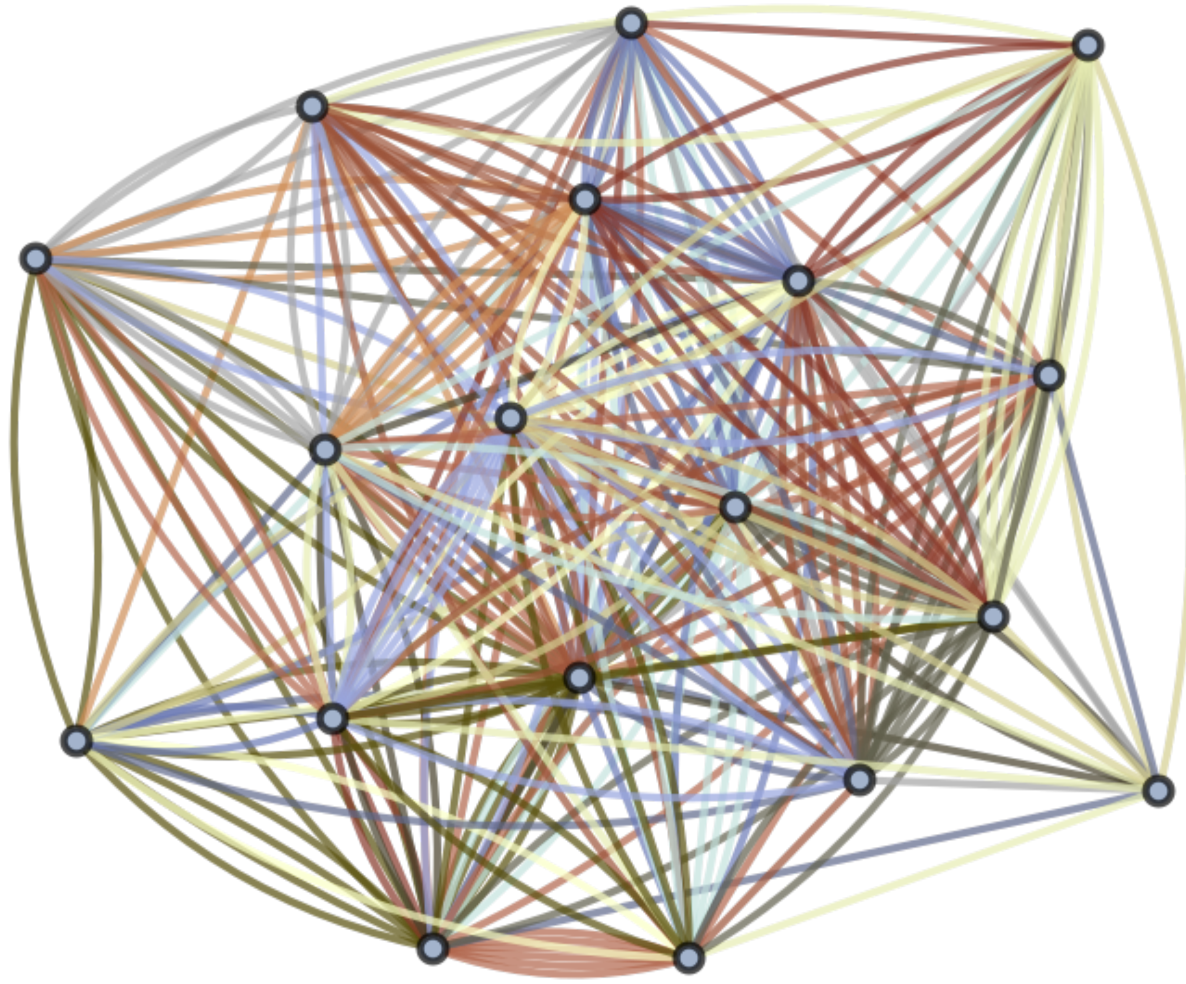
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



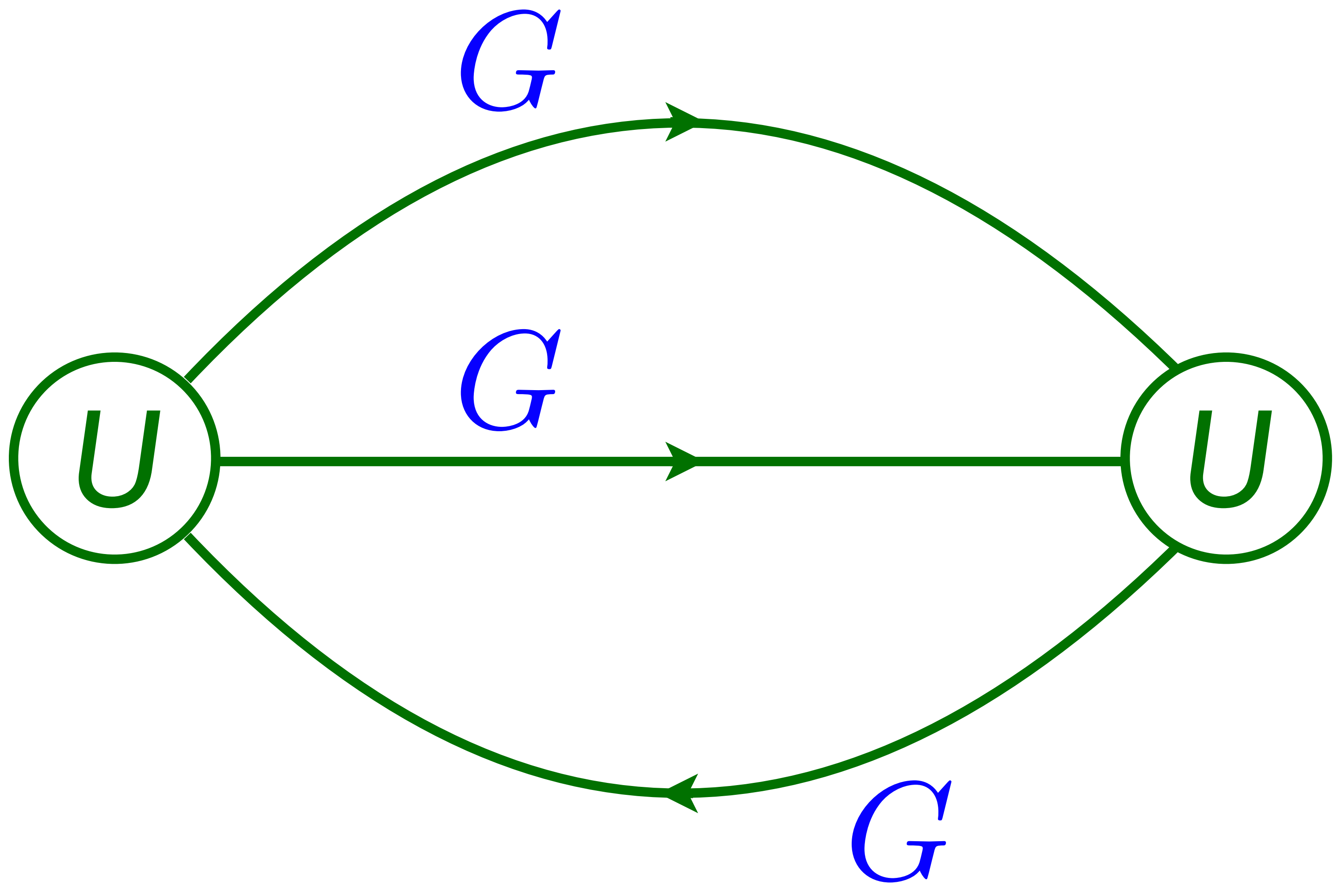


The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G_s(\tau) = -\sum_\alpha \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle / N$ in the large N limit:

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

$$G_s(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G_s(\tau) = -\sum_\alpha \left\langle c_\alpha(\tau) c^\dagger_\alpha(0) \right\rangle / N$ in the large N limit:

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

$$G_s(\tau = 0^-) = \mathcal{Q}.$$

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

The SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G_s(\tau) = -\sum_\alpha \langle c_\alpha(\tau) c_\alpha^\dagger(0)\rangle / N$ in the large N limit:

$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

$$G_s(\tau = 0^-) = \mathcal{Q}.$$

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model

A. Georges, O. Parcollet, and S. Sachdev,
Physical Review B **63**, 134406 (2001)

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G_s(\tau) = -\sum_\alpha \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle / N$ in the large N limit:

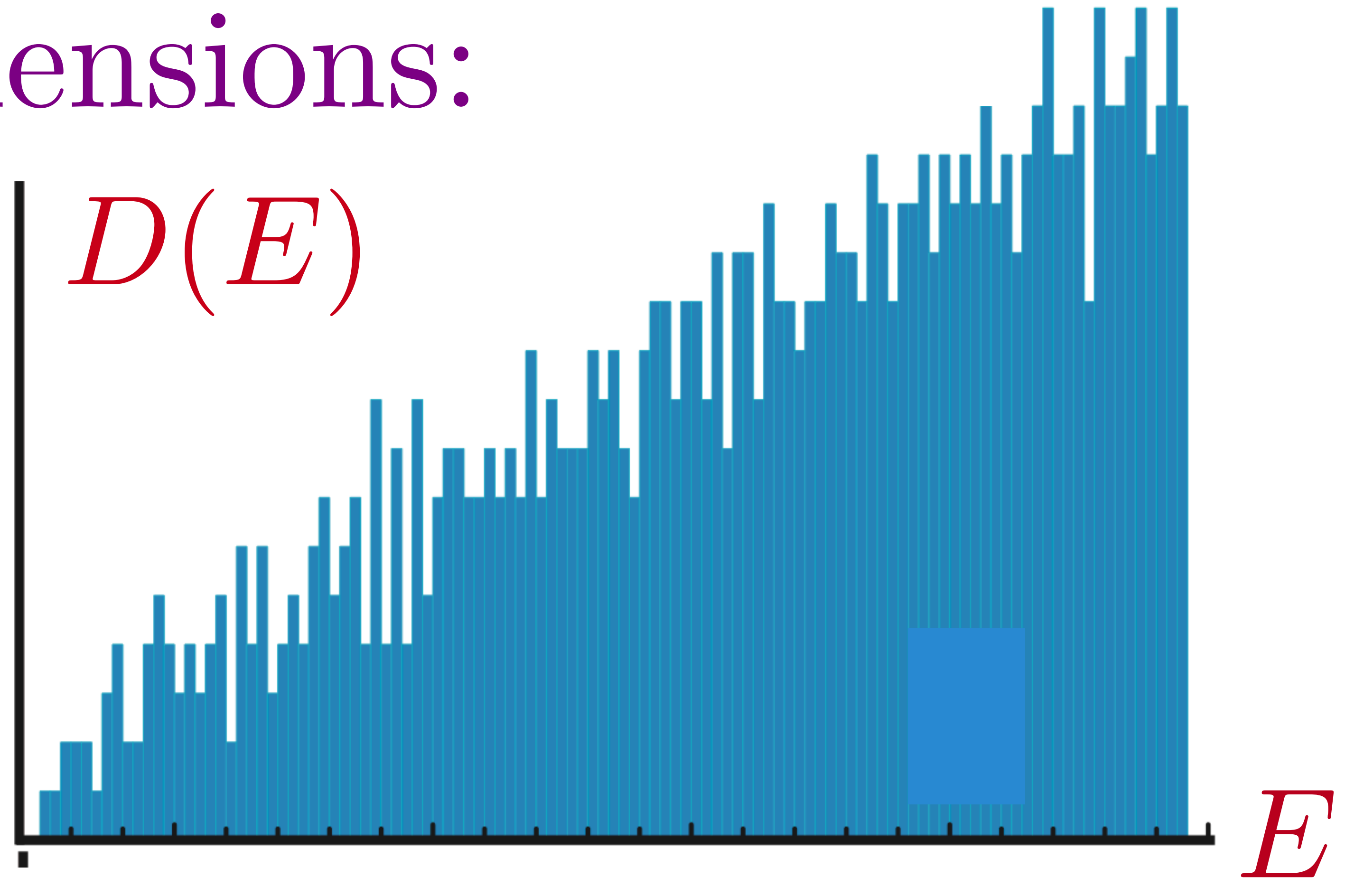
$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma_s(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$
$$G_s(\tau = 0^-) = \mathcal{Q}.$$

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \to 0) \sim e^{N s_0}$$

$s_0 = 0.464847699170805107 49....$ for $\mathcal{Q} = 1/2$.



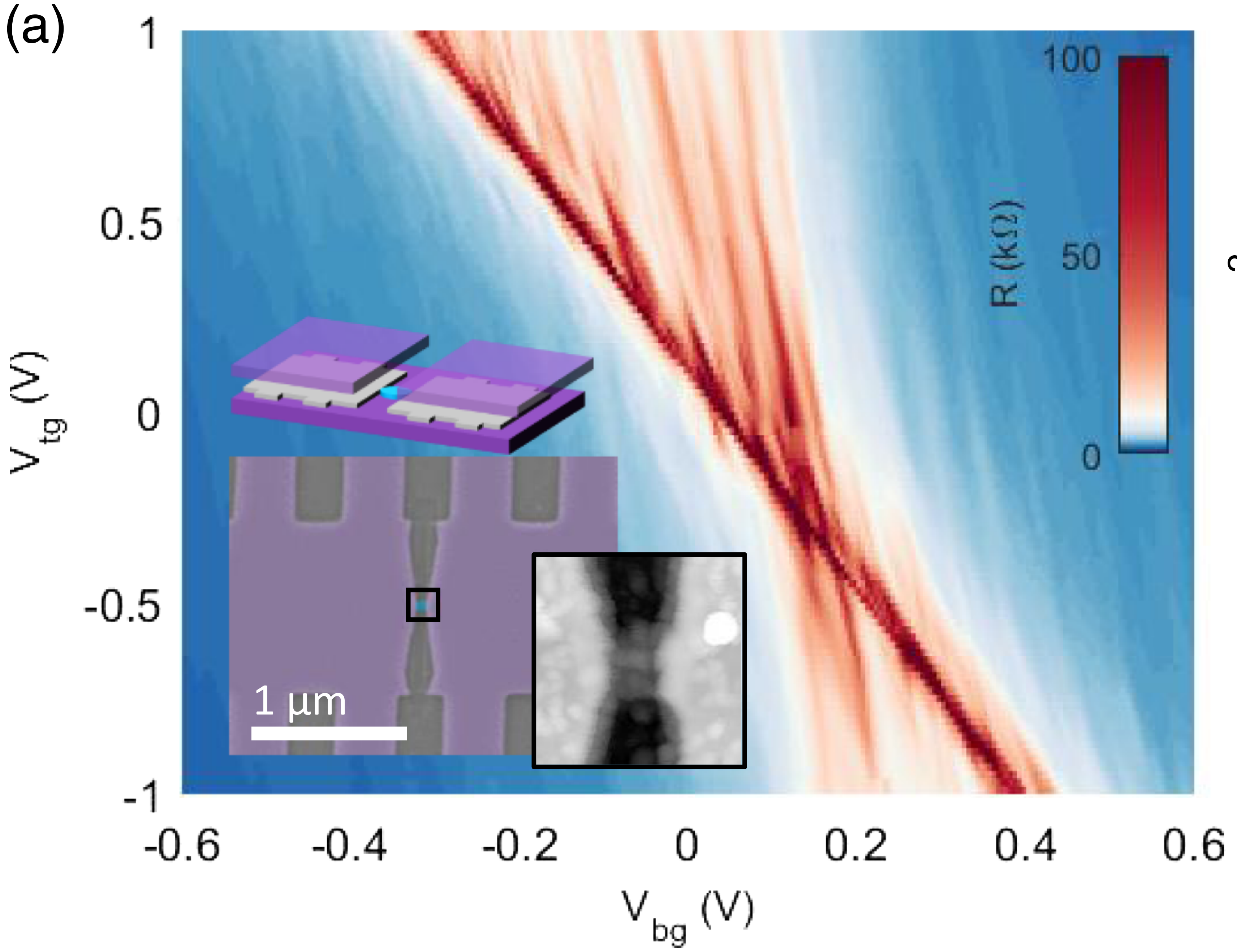
PHYSICAL REVIEW LETTERS **132**, 246502 (2024)

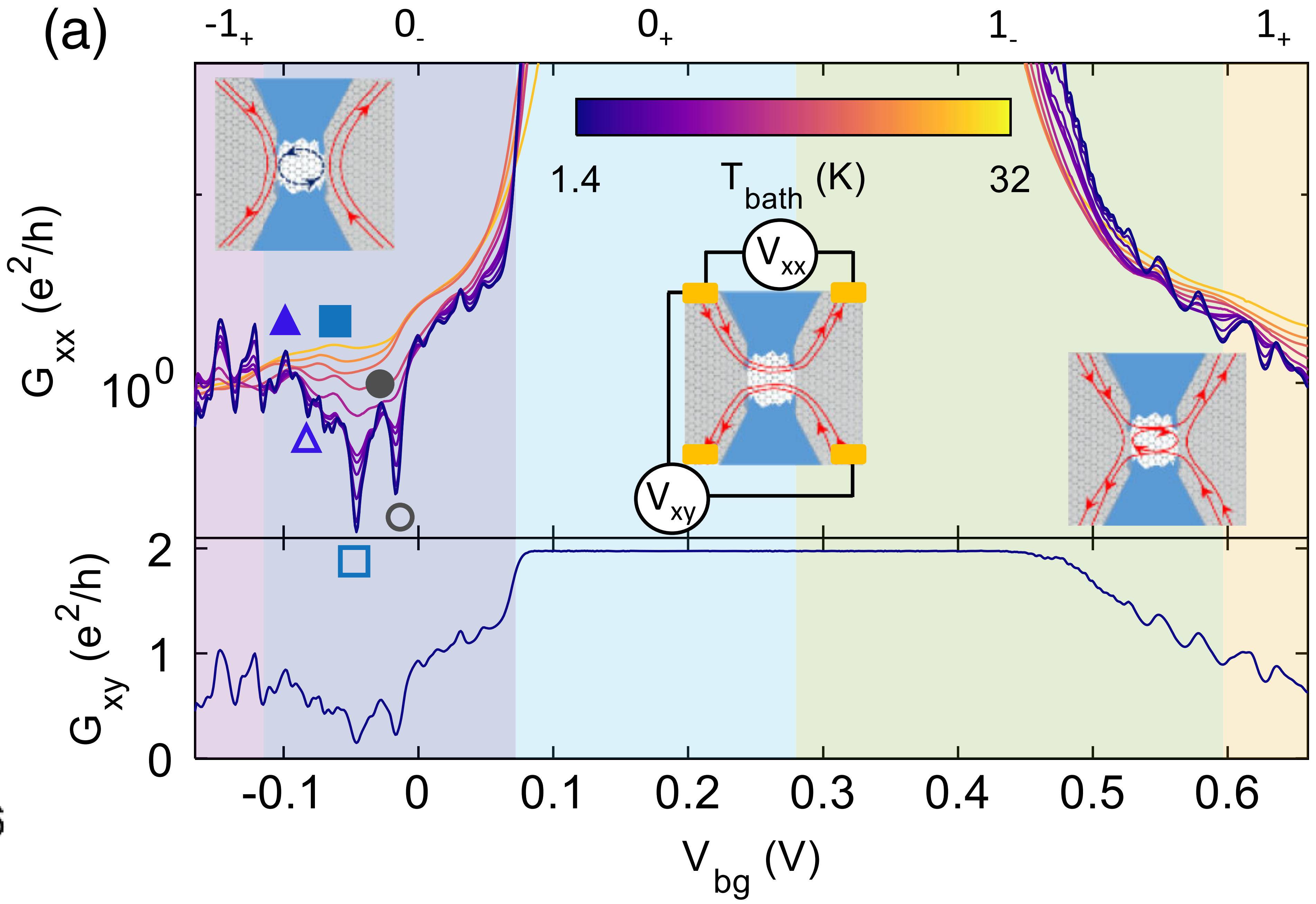
Editors' Suggestion

Magneto-Thermoelectric Transport in Graphene Quantum Dot with Strong Correlations

Laurel E. Anderson,[1] Antti Laitinen,[1] Andrew Zimmerman,[1] Thomas Werkmeister,[2] Henry Shackleton,[1]
Alexander Kruchkov,[1,3,4] Takashi Taniguchi,[5] Kenji Watanabe,[6] Subir Sachdev,[1] and Philip Kim[1,2]

In specific temperature ranges, we observe a suppression of electric conductance fluctuations and slowly decreasing thermoelectric power across the GQD with increasing temperature, consistent with recent theory for the SYK regime.





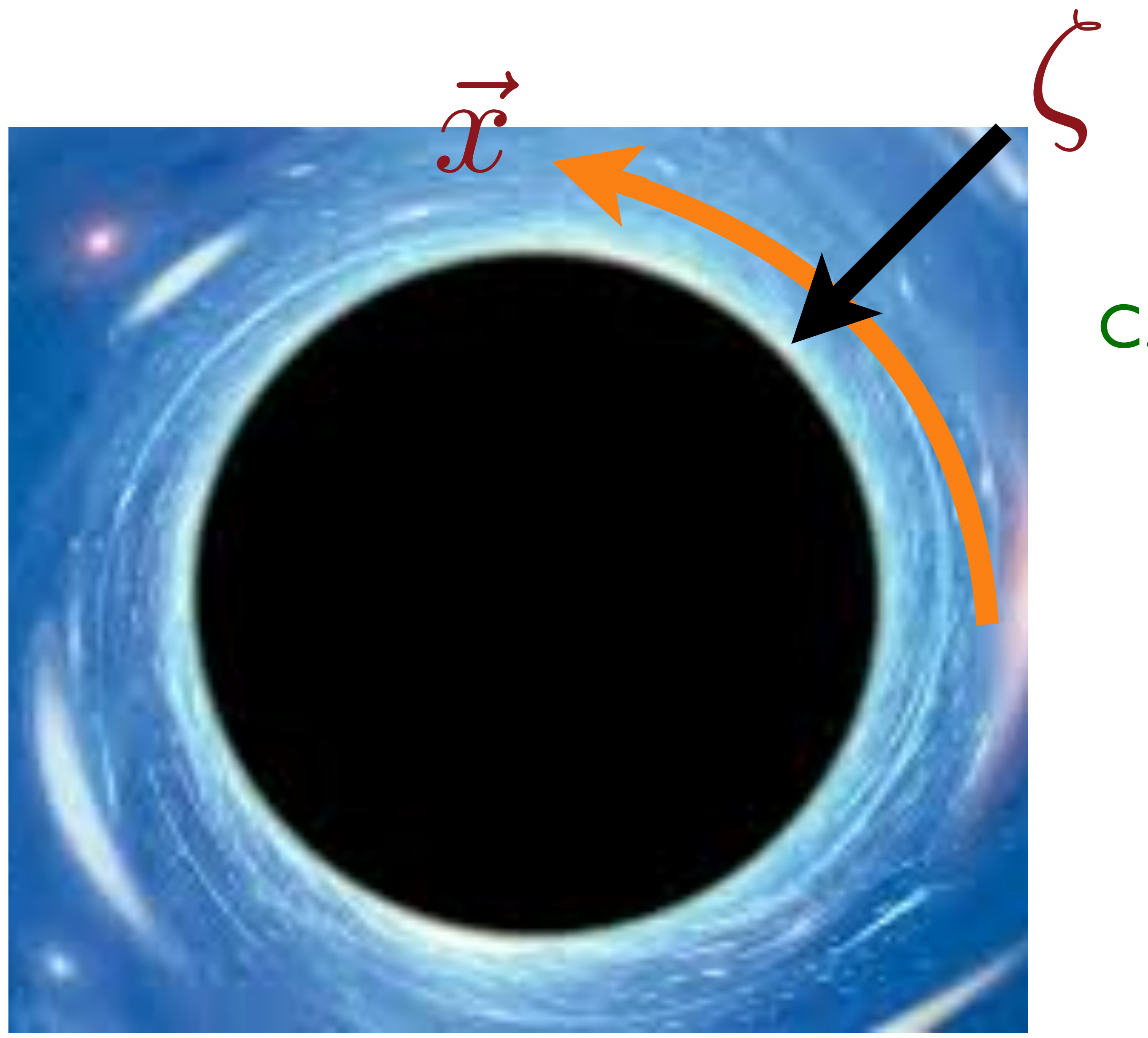
The SYK model and black holes

Semiclassical connection first proposed by S.S. in Physical Review Letters **105**, 151602 (2010): SYK model and charged black holes exhibit Planckian dynamics and zero temperature entropy.

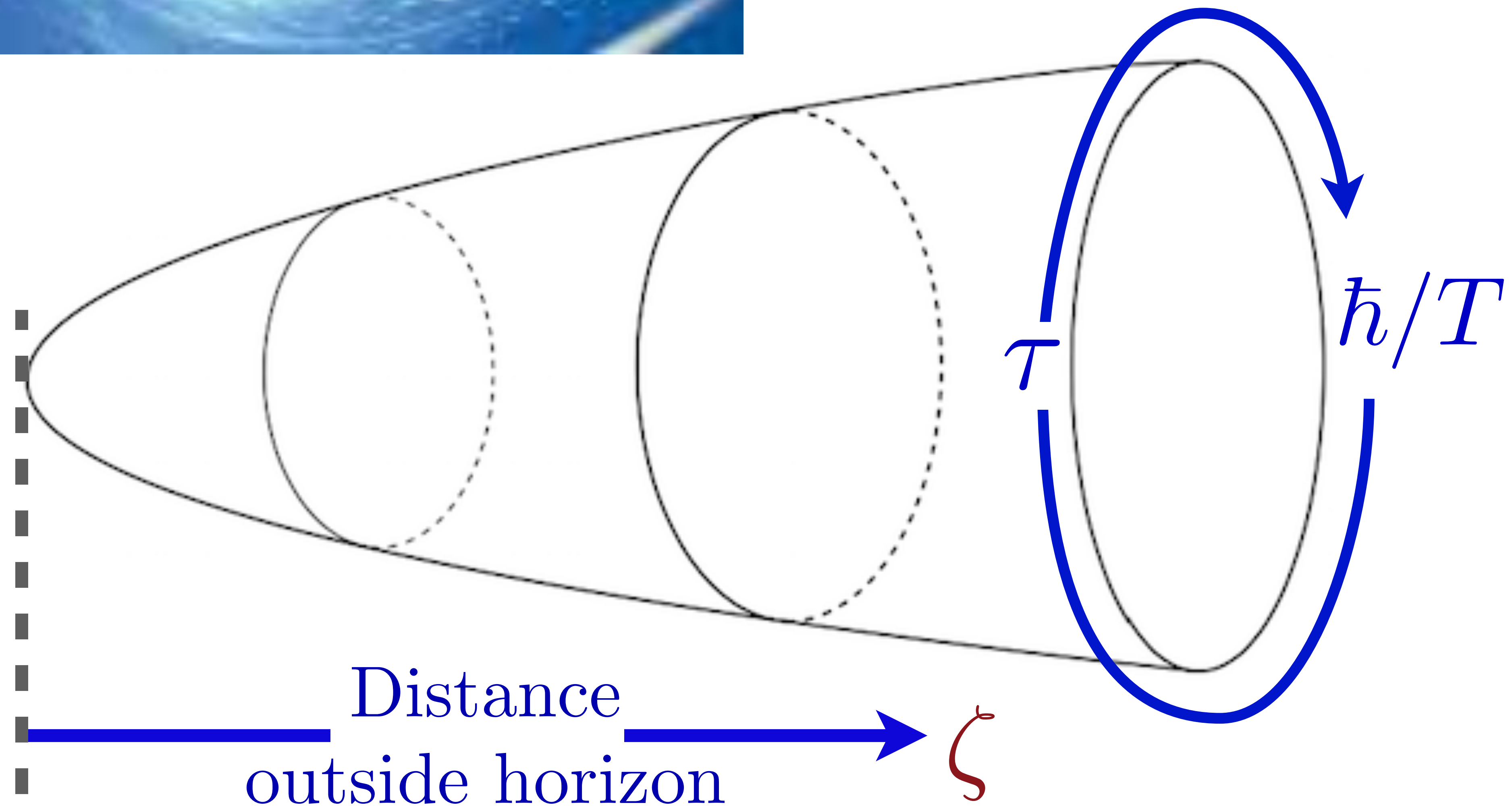
Fully quantum connection established in 2015 by A. Kitaev, J. Maldacena, D. Stanford....

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$



A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

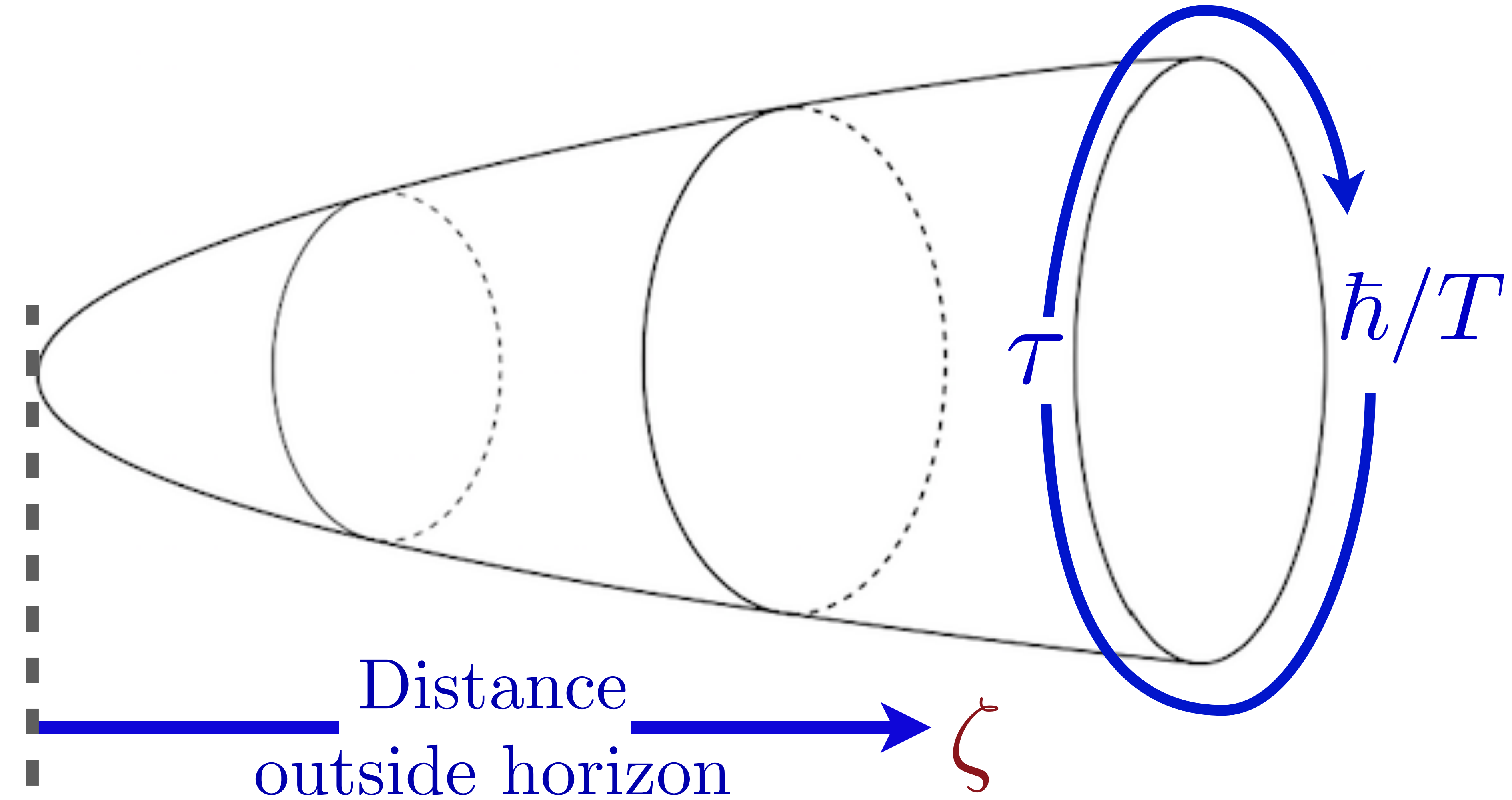


Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q}, T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu}, A_\mu]\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.



Maldacena, Stanford, Yang (2016)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

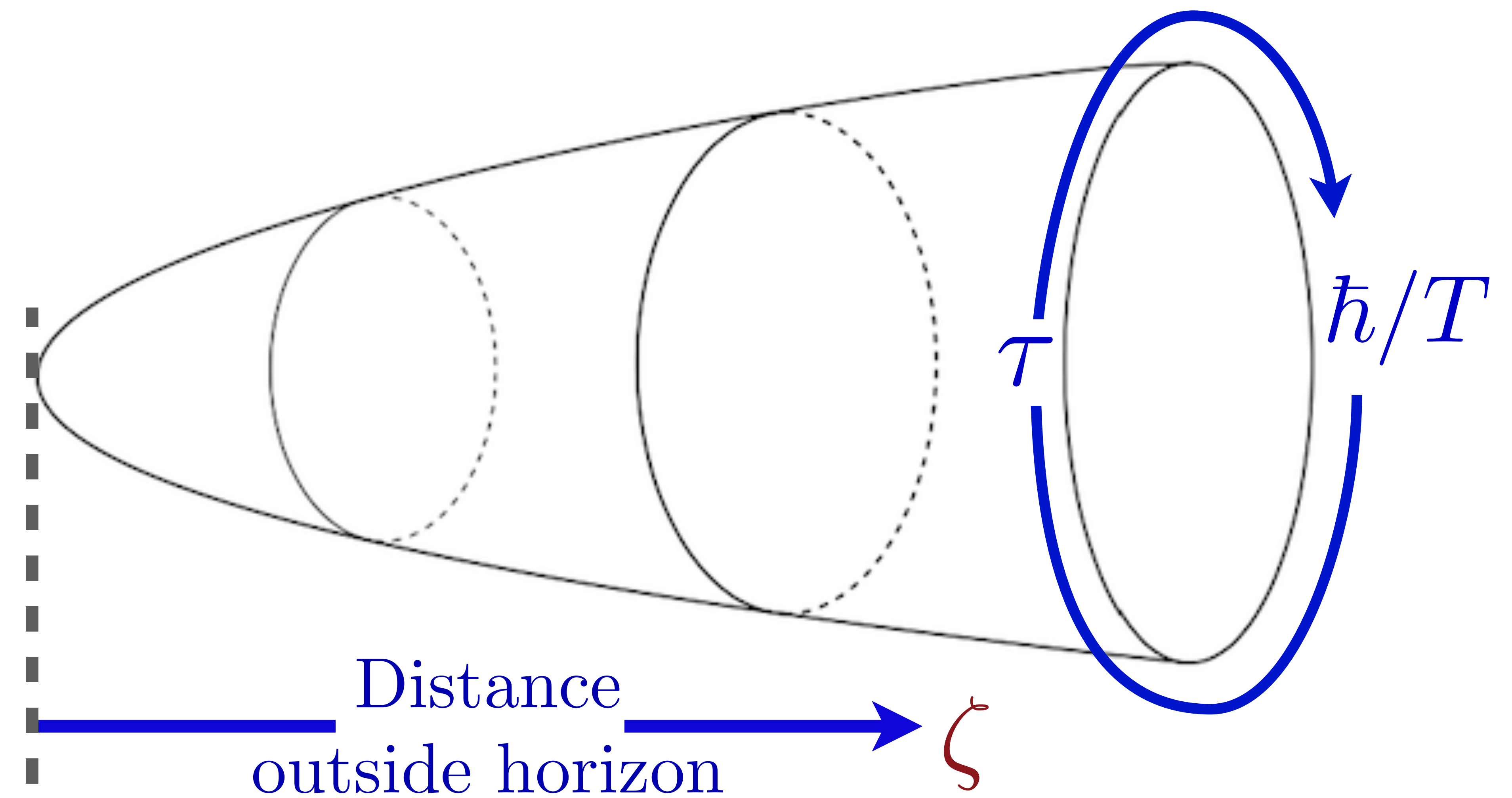
$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu},A_\mu]\right)$$

$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar} I^{(0+1)}_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

The path integral over the action $I^{(0+1)}_{SYK}$ can be evaluated exactly, and leads to a computation of $D(E)$

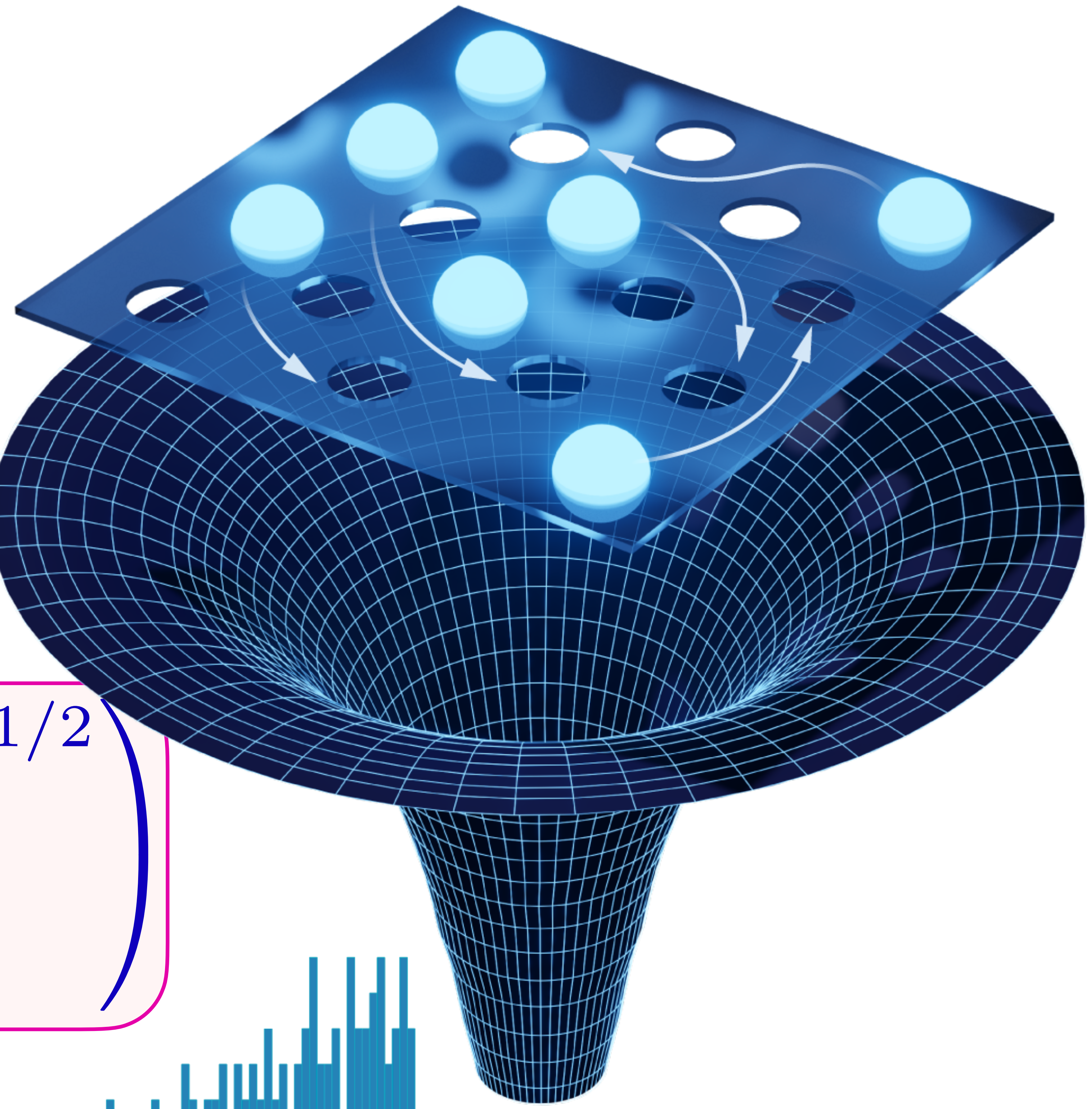
$$\mathcal{Z}(\mathcal{Q},T) = \int dE D(E)\exp\left(-\frac{E}{k_B T}\right)$$



Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

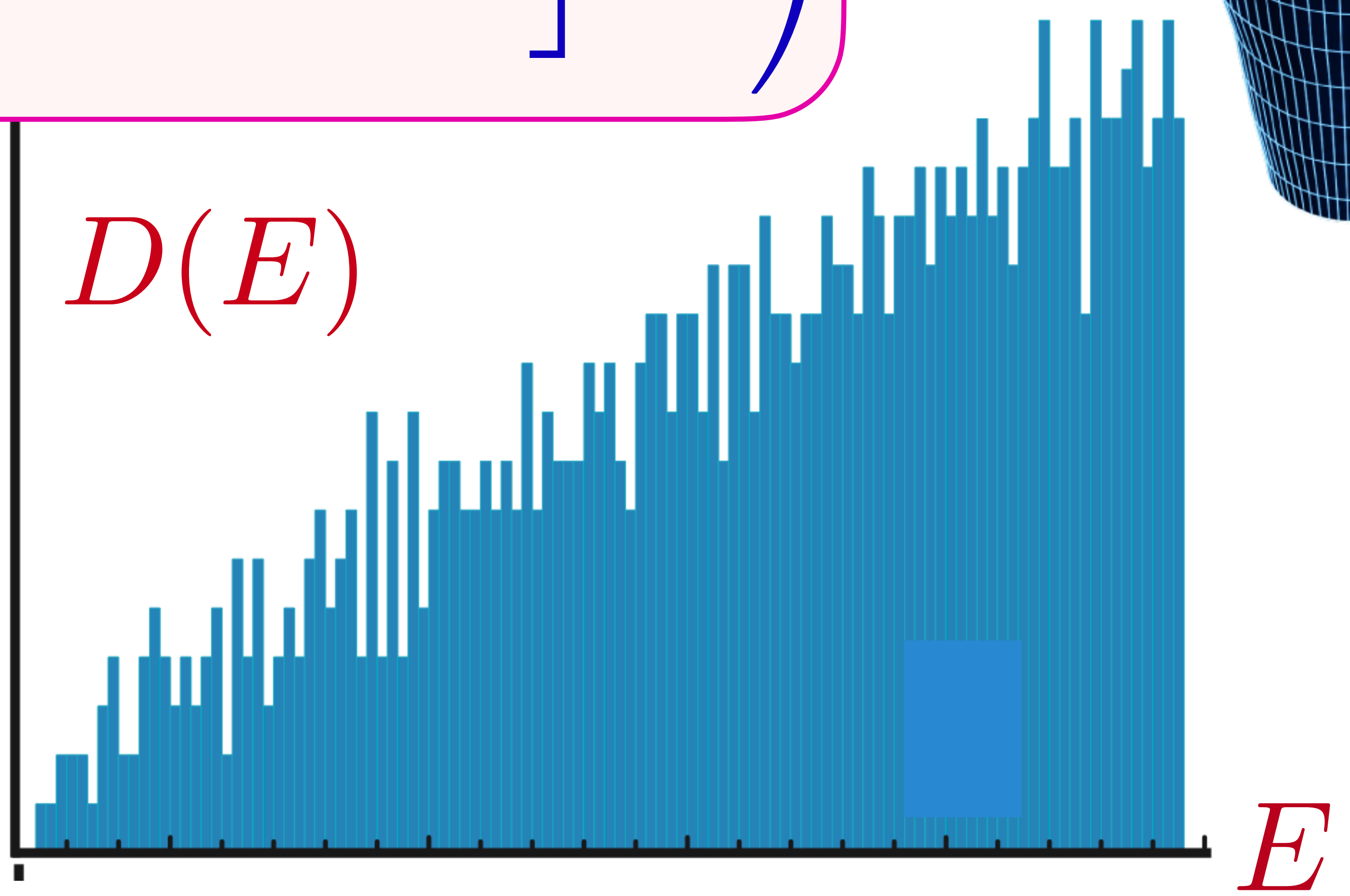


Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

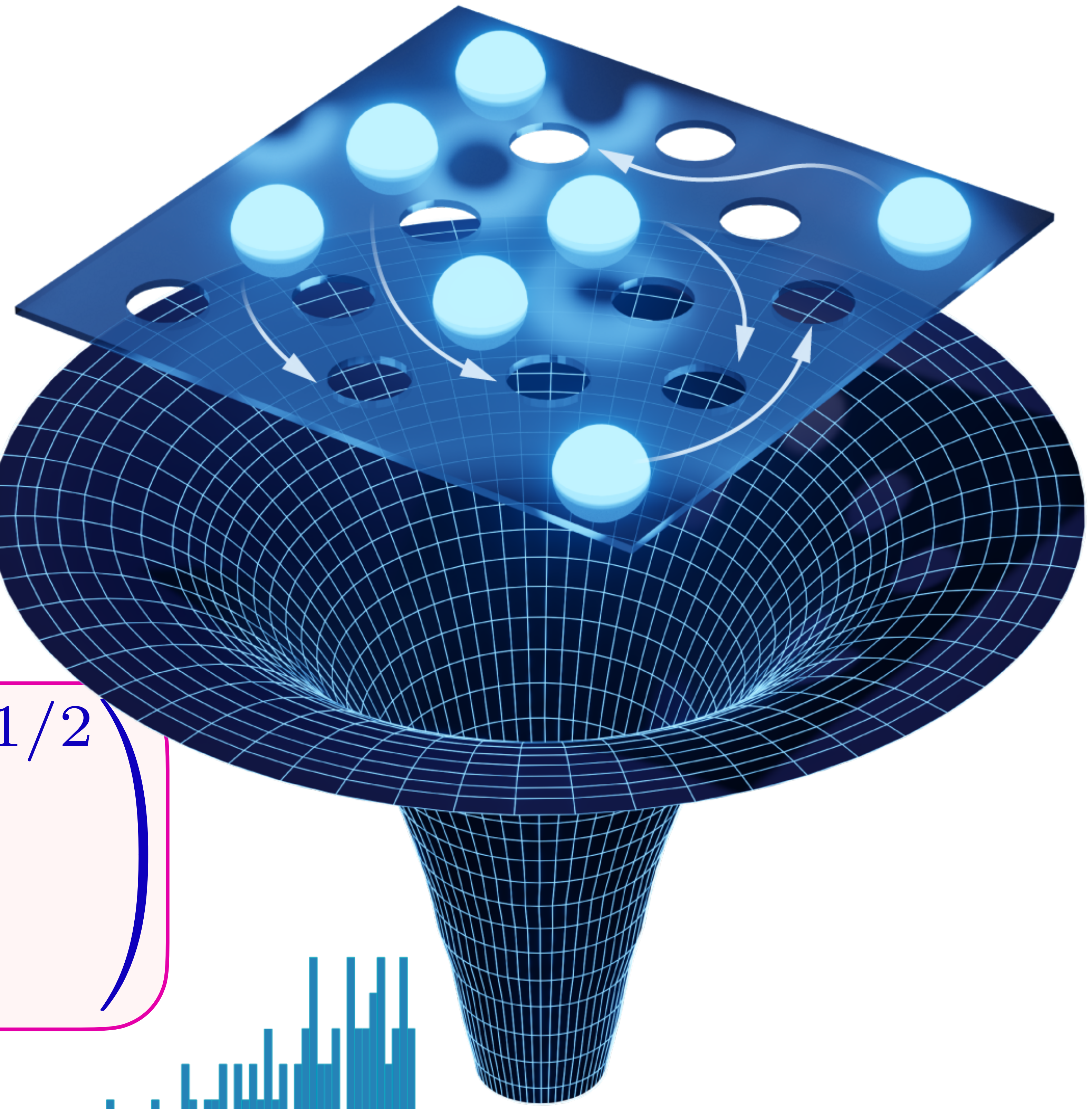
Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

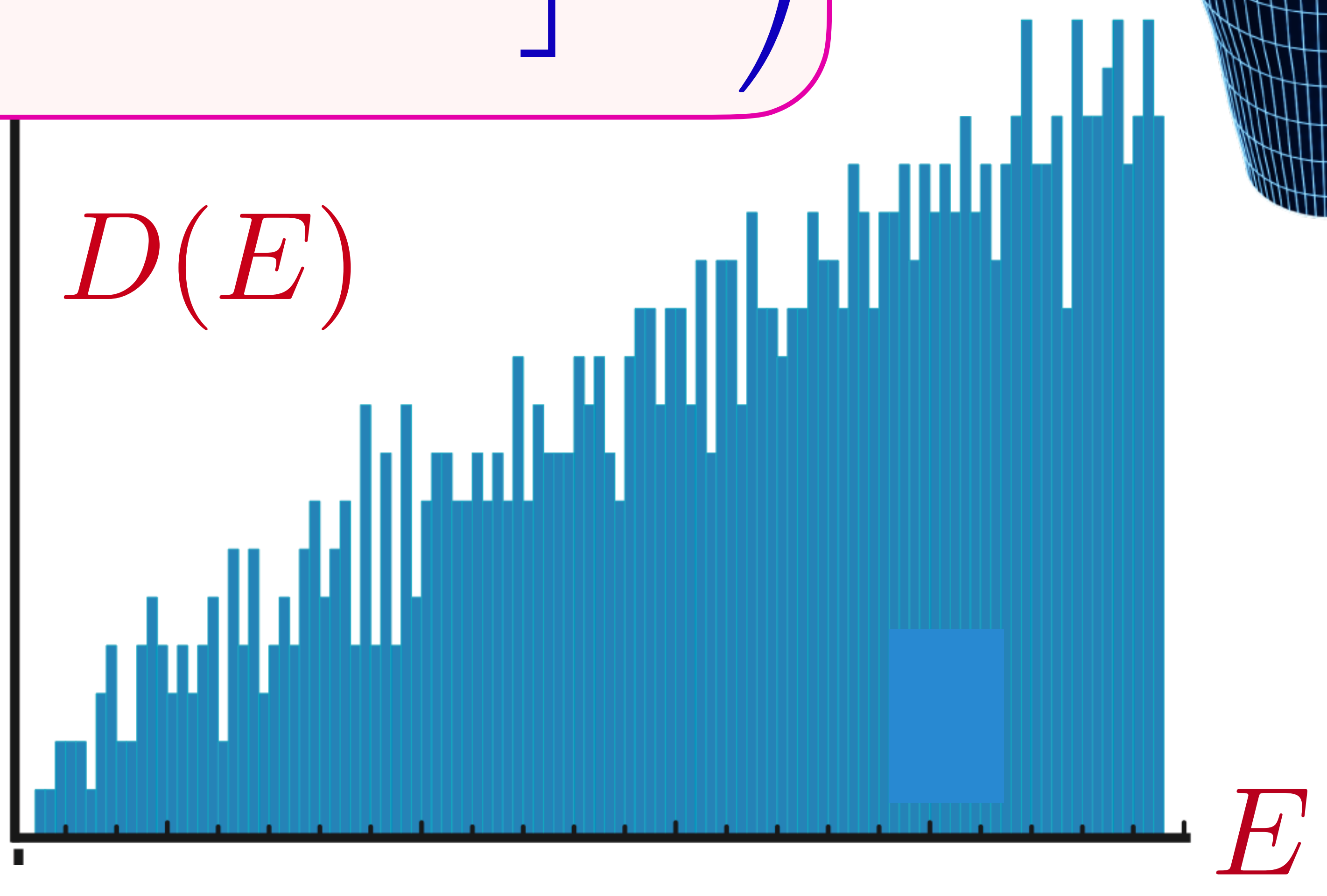
D(E) of charged black holes from the SYK model



- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T=0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

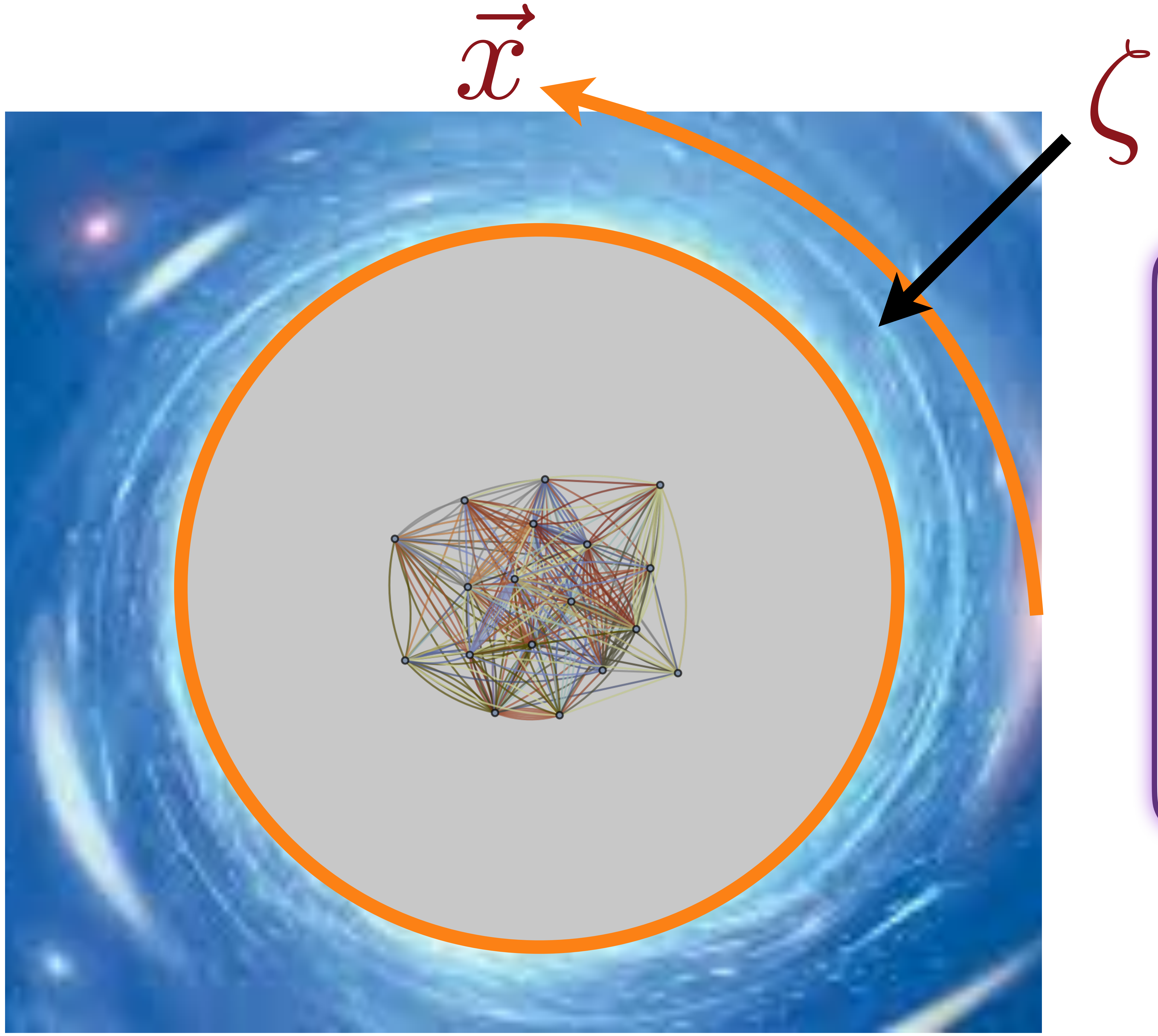
$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Bekenstein-Hawking entropy have exactly the same energy (Strominger, Vafa (1996)).



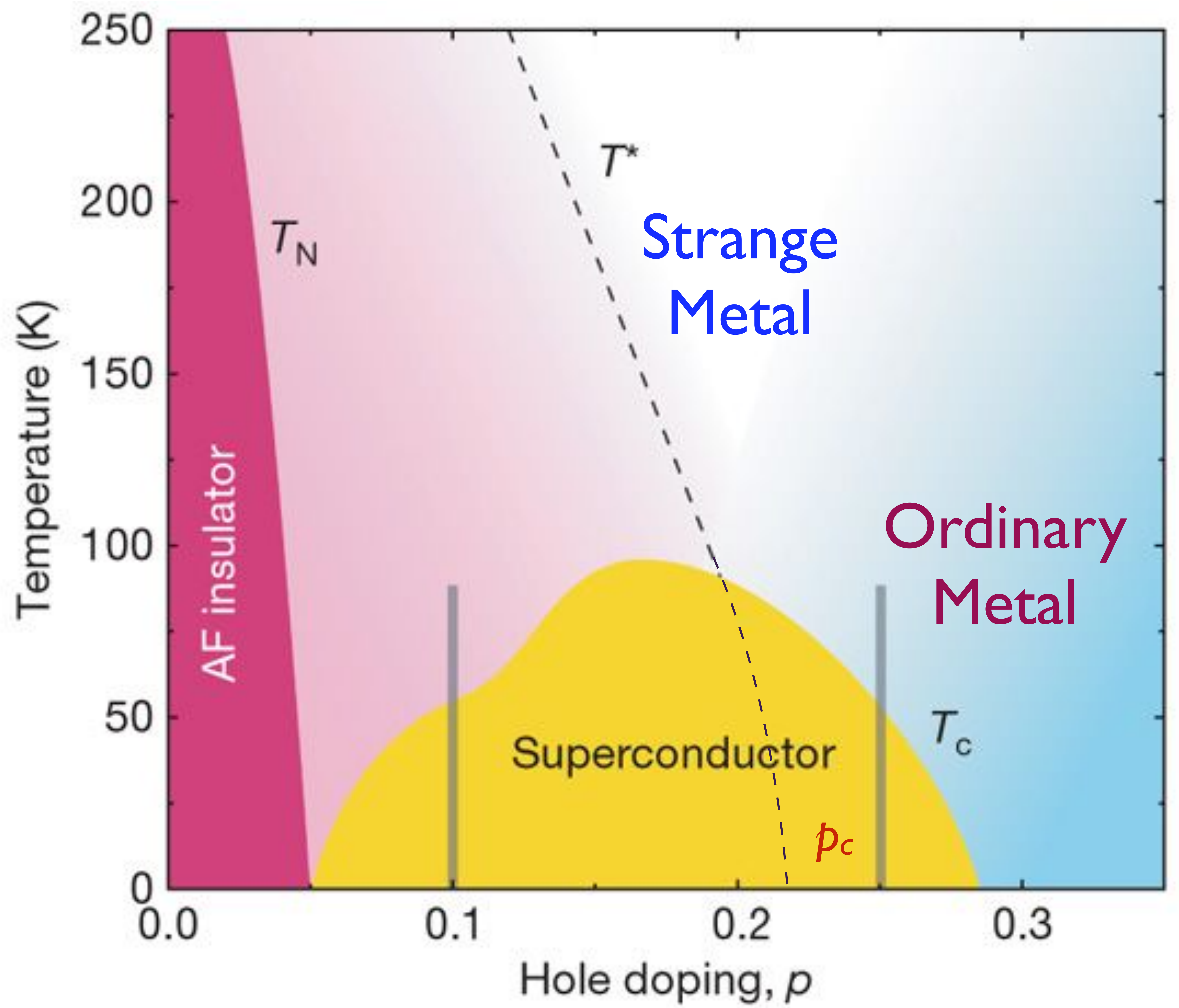
D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model

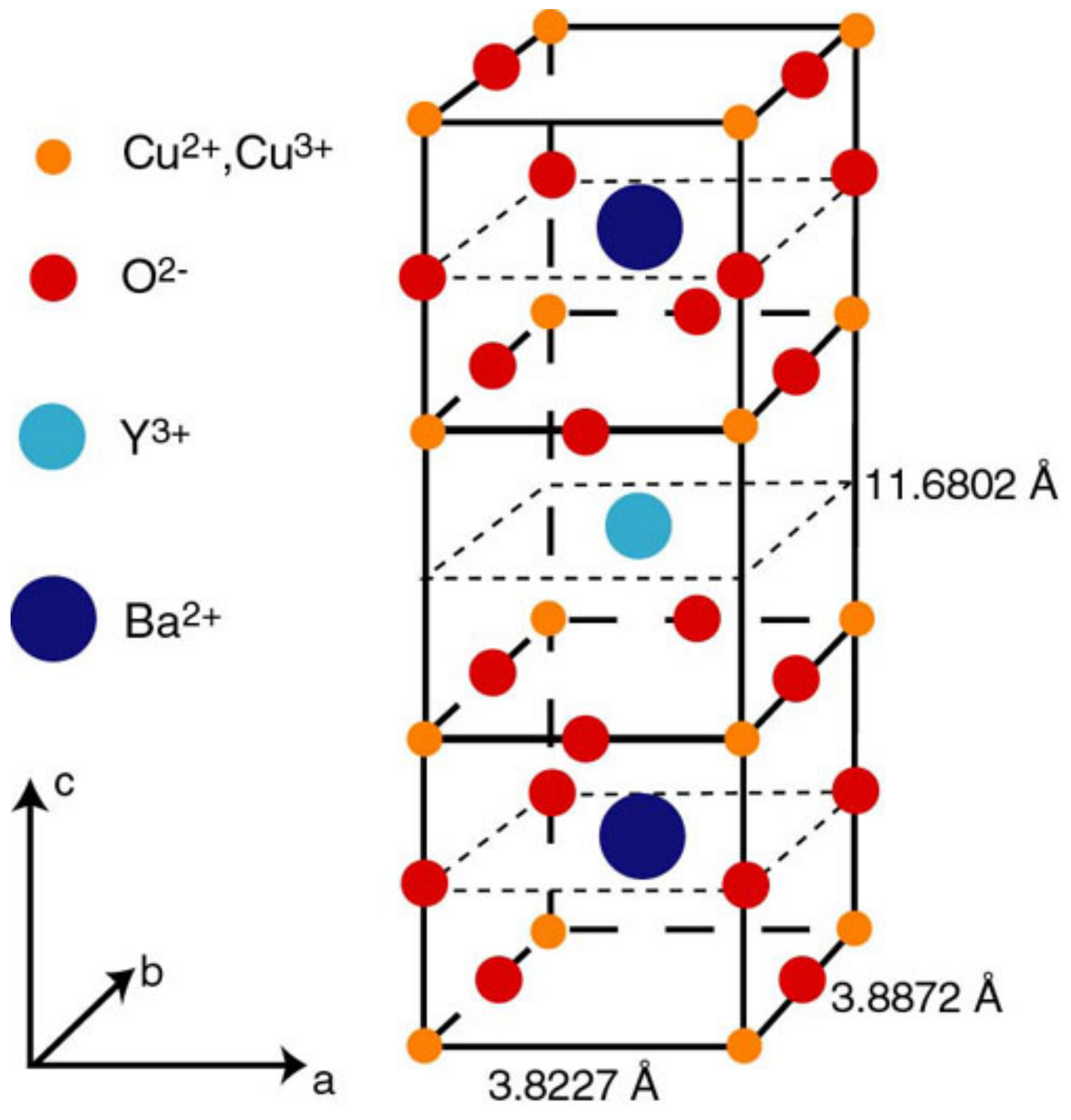


The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in ζ-τ co-ordinates.

The SYK model and superconductors

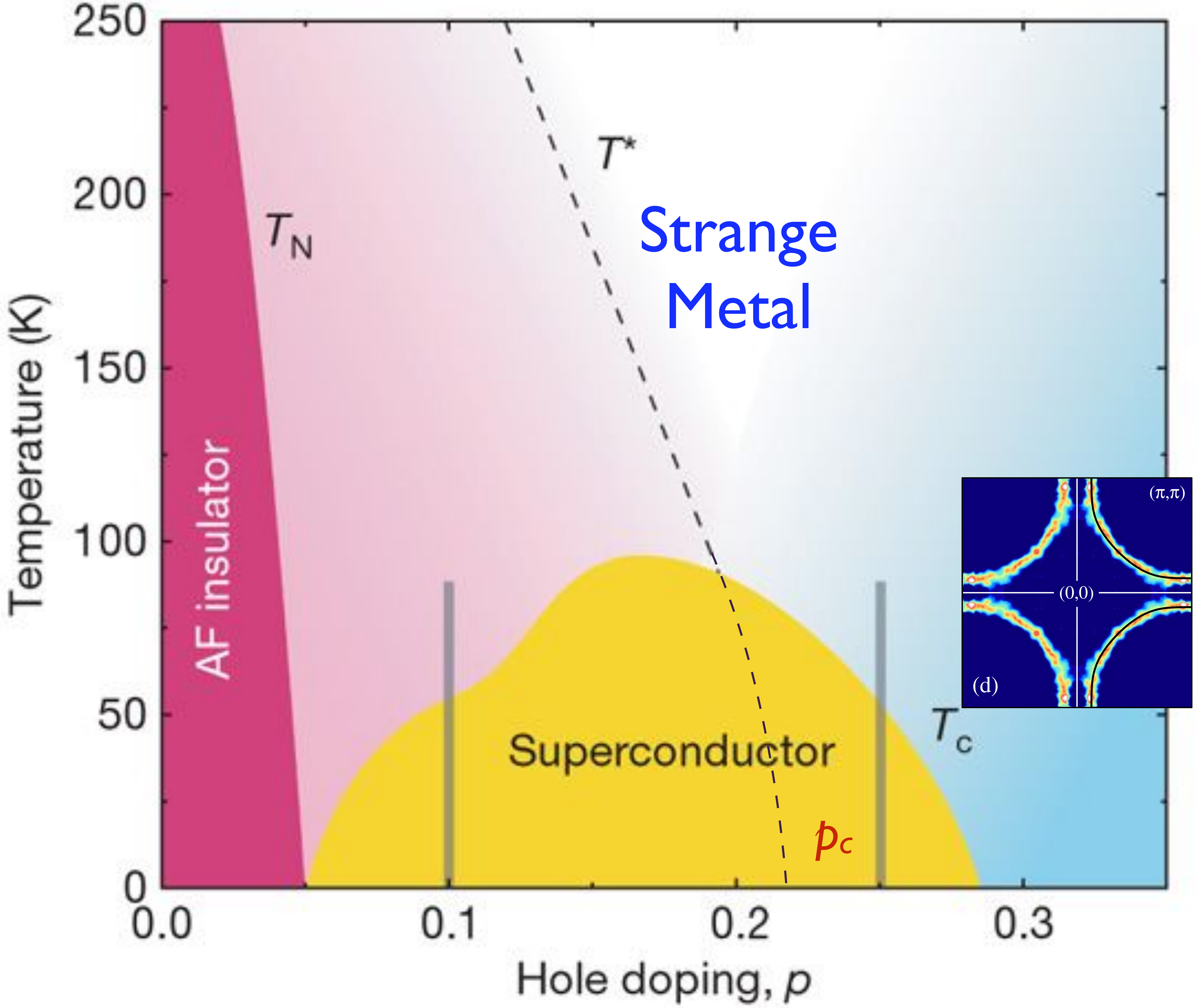


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c

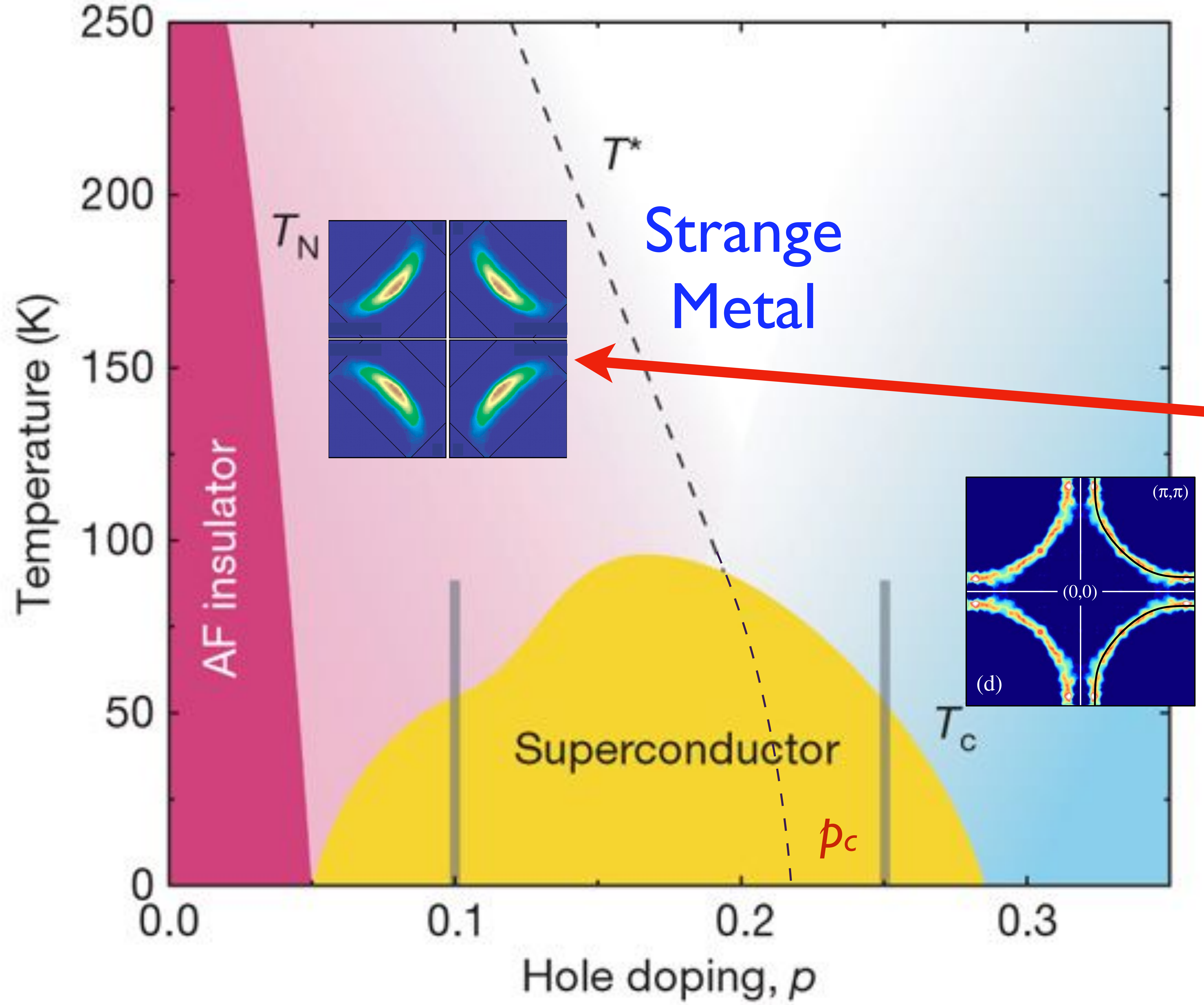


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

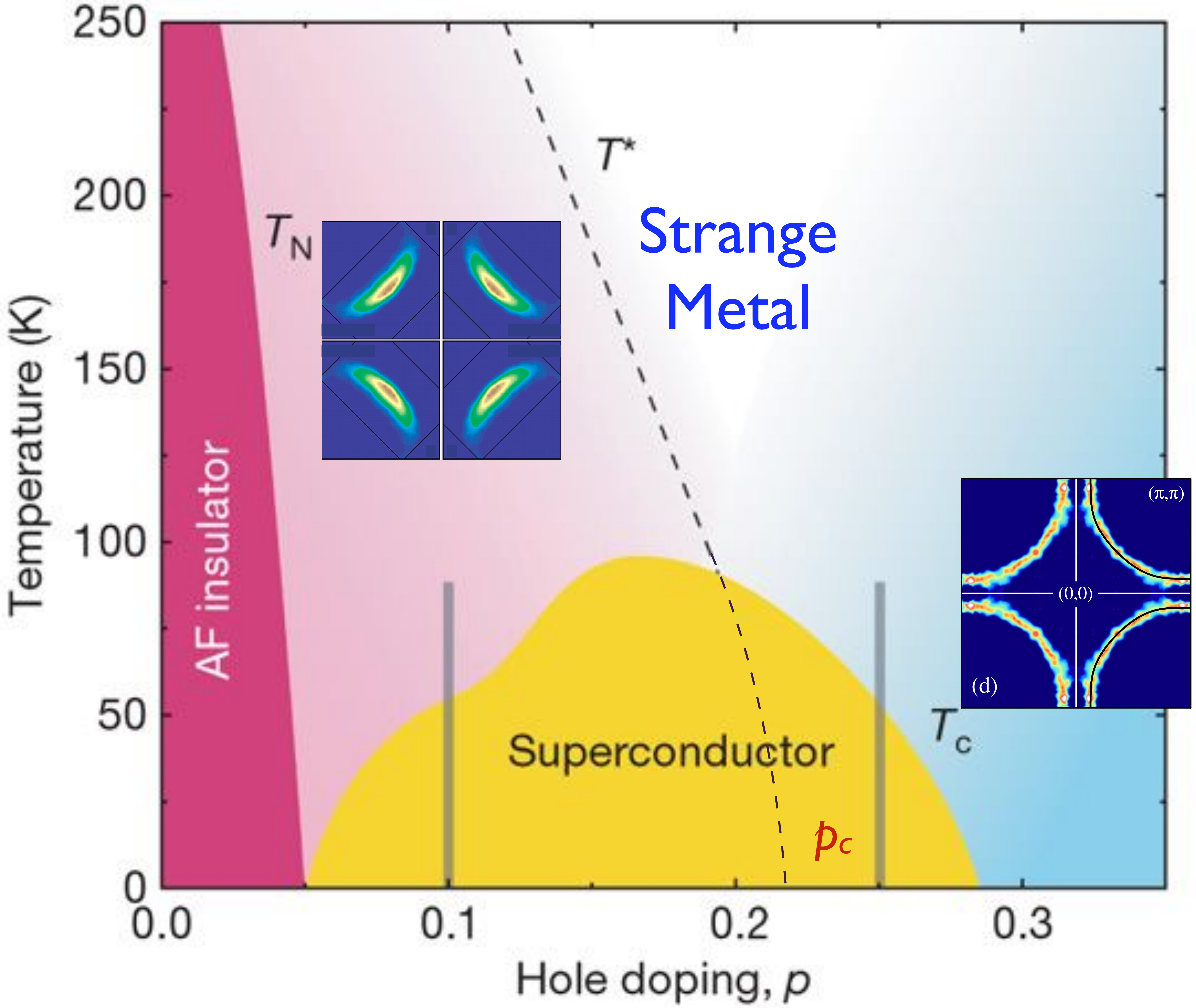
$YBa_2Cu_3O_{6+x}$



Fermi surface
as expected
in a model
of free electrons



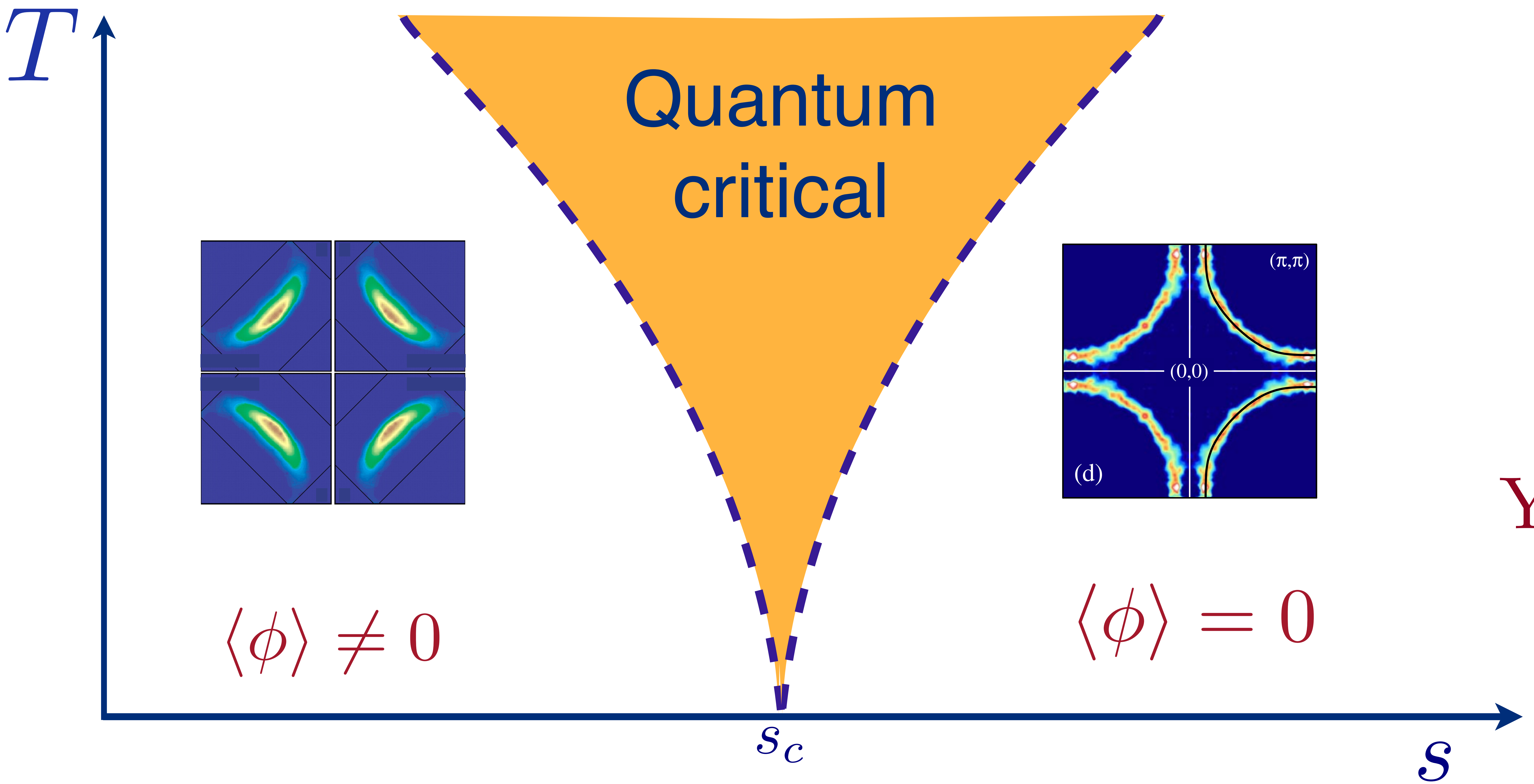
"Pseudogap metal" Fermi surface modified by electron-electron interactions



View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

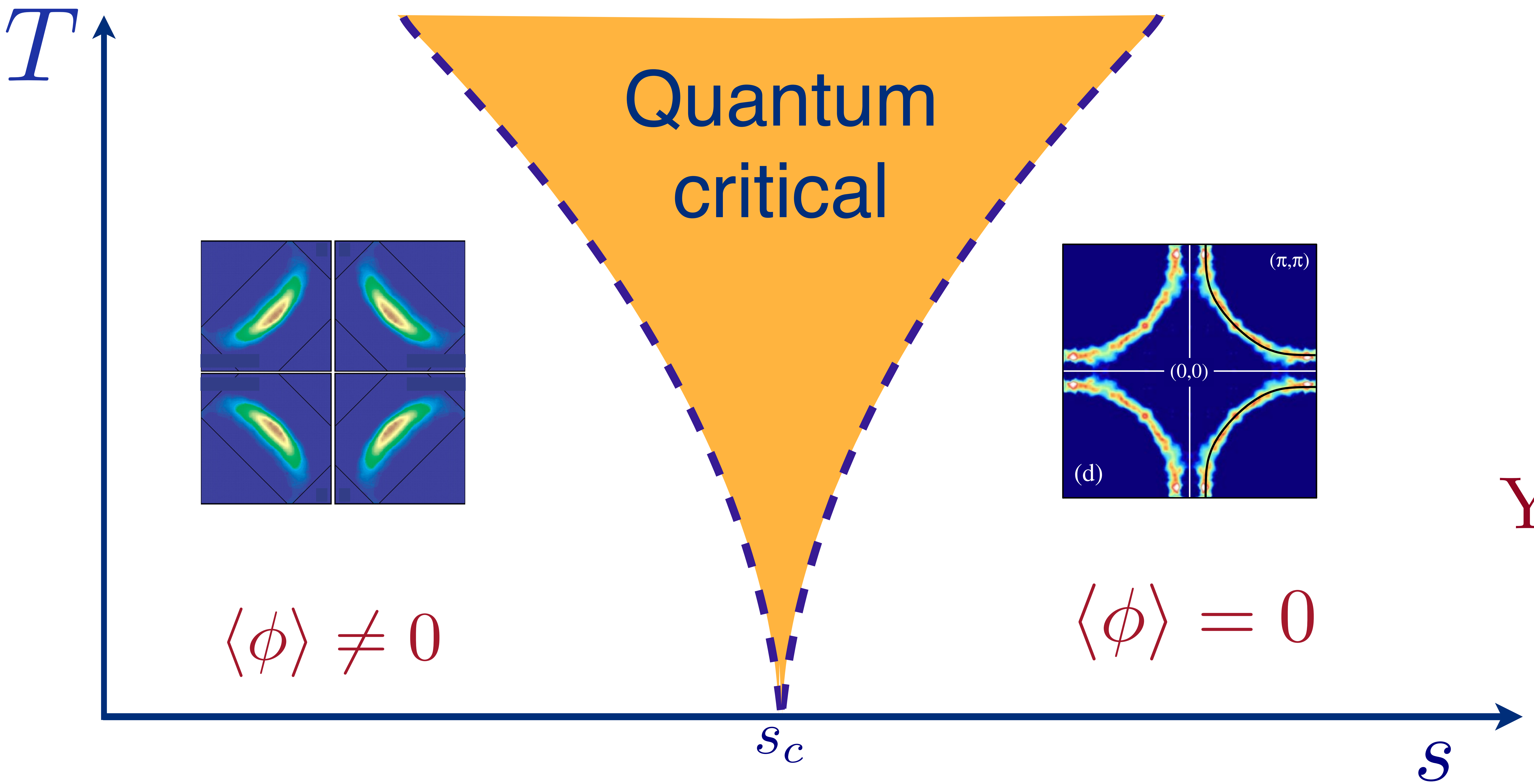
The onset of superconductivity may "hide" this quantum transition.

Quantum phase transitions of Fermi surface change



Fermi surface
$+$
scalar field ϕ
with a 'mass' s
and
a boson-fermion
Yukawa coupling g.

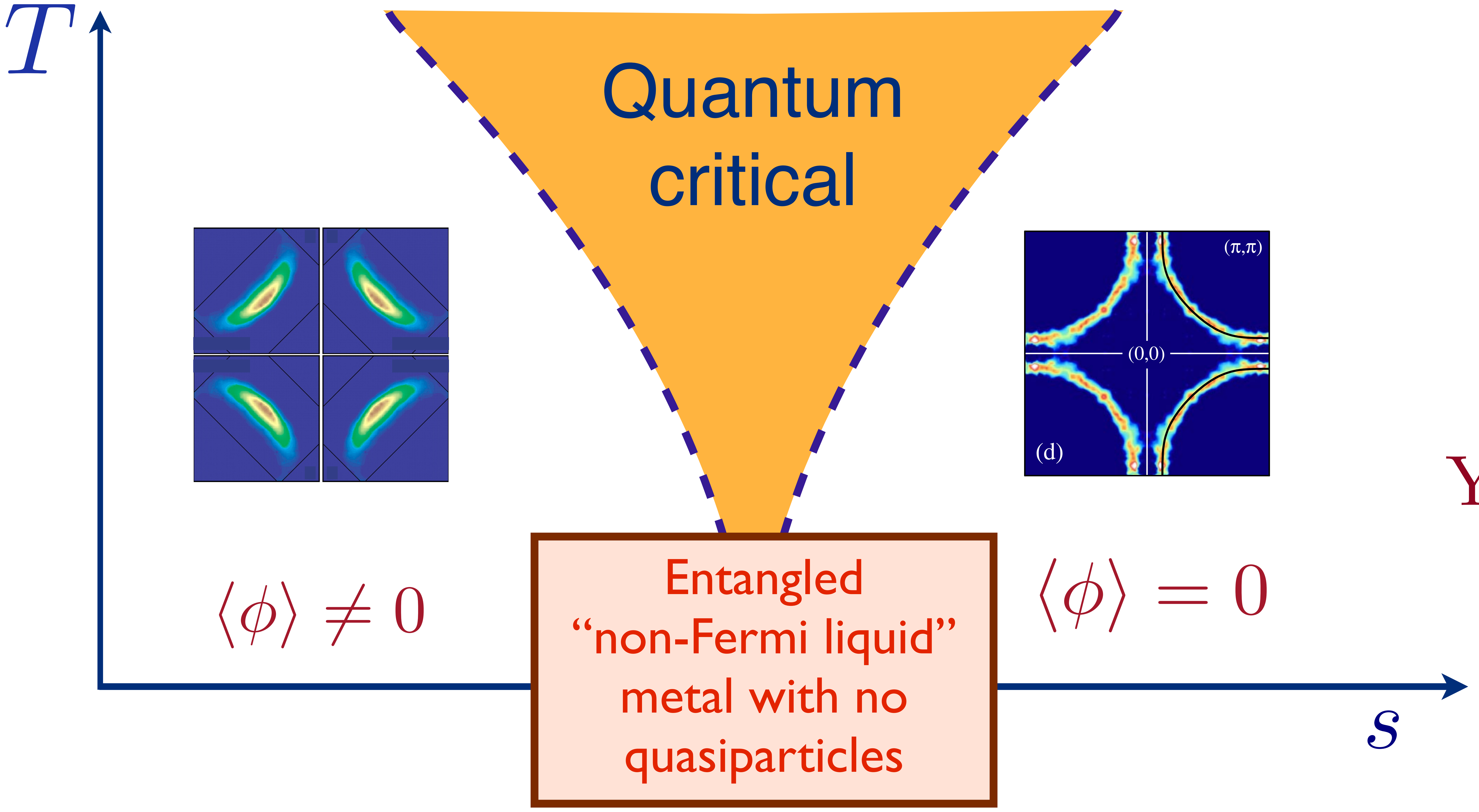
Quantum phase transitions of Fermi surface change



Fermi surface
$+$
scalar field ϕ
with a 'mass' s
and
a boson-fermion
Yukawa coupling g.

ϕ could represent
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume
changing transition

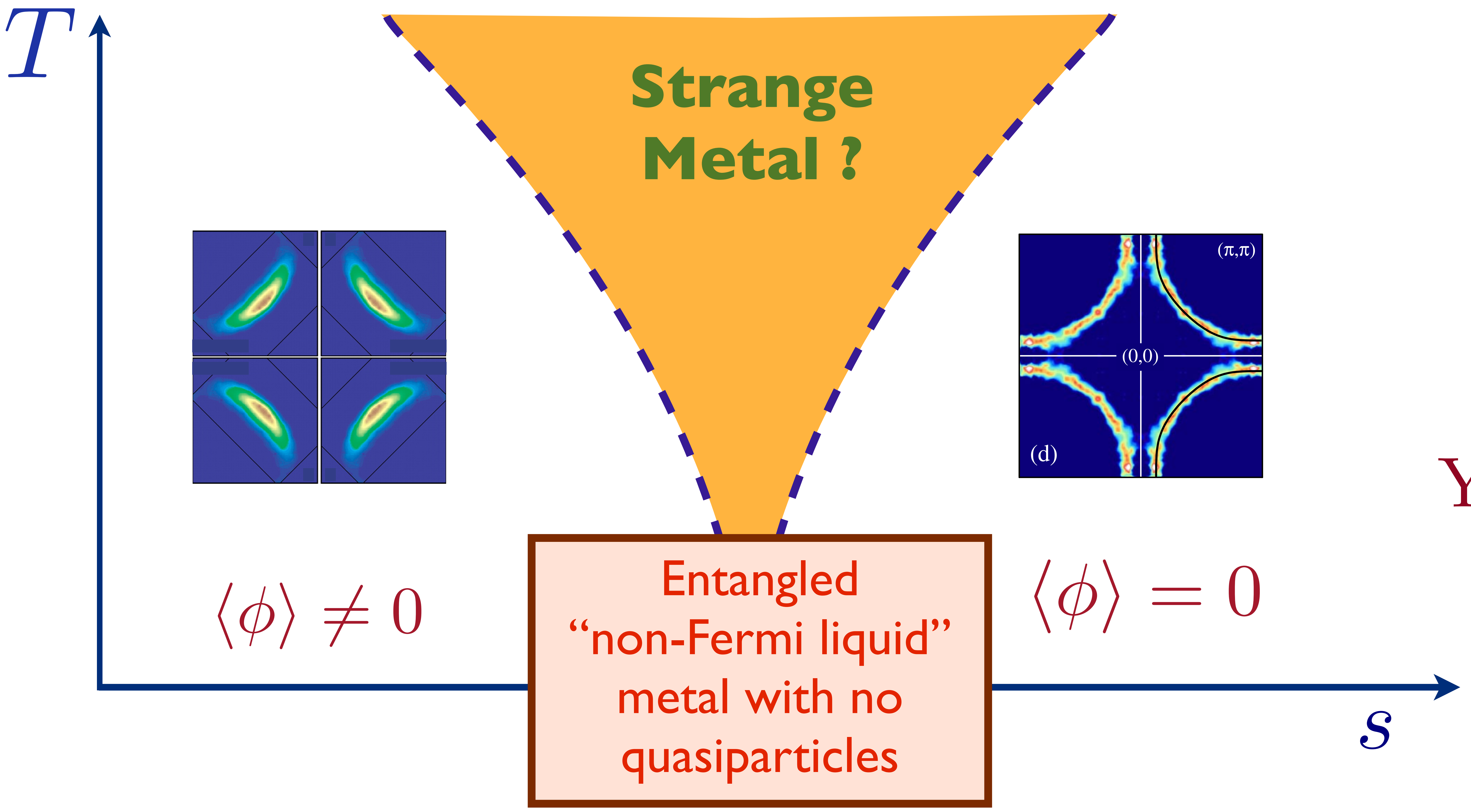
Quantum phase transitions of Fermi surface change



Fermi surface
+
scalar field ϕ
with a 'mass' s
and
a boson-fermion
Yukawa coupling g.

ϕ could represent
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume
changing transition

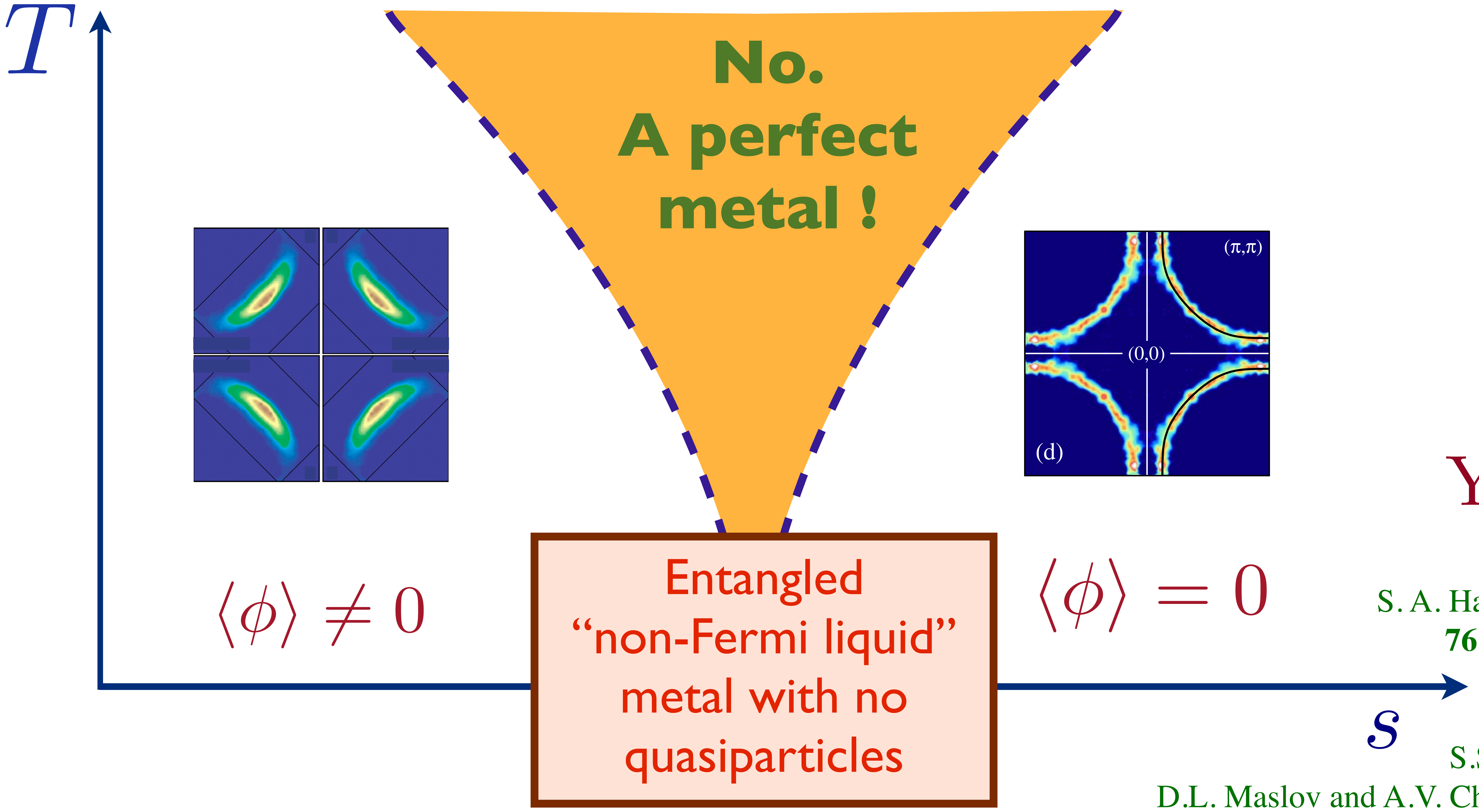
Quantum phase transitions of Fermi surface change



Fermi surface
$+$
scalar field ϕ
with a 'mass' s
and
a boson-fermion
Yukawa coupling g.

ϕ could represent
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume
changing transition

Quantum phase transitions of Fermi surface change

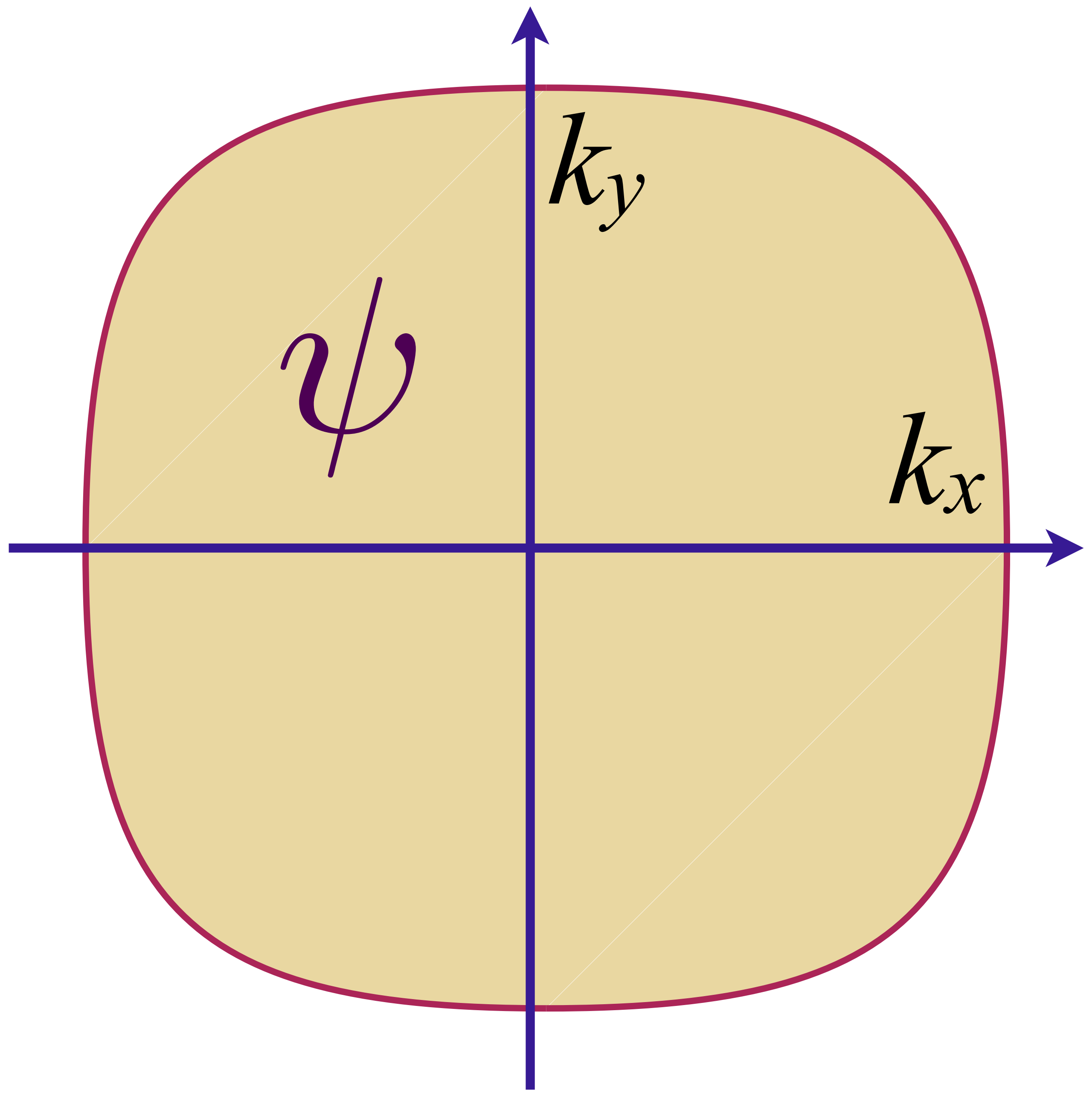


Fermi surface
$+$
scalar field ϕ
with a 'mass' s
and
a boson-fermion
Yukawa coupling g.

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007); Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022); Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024); D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017) ; Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



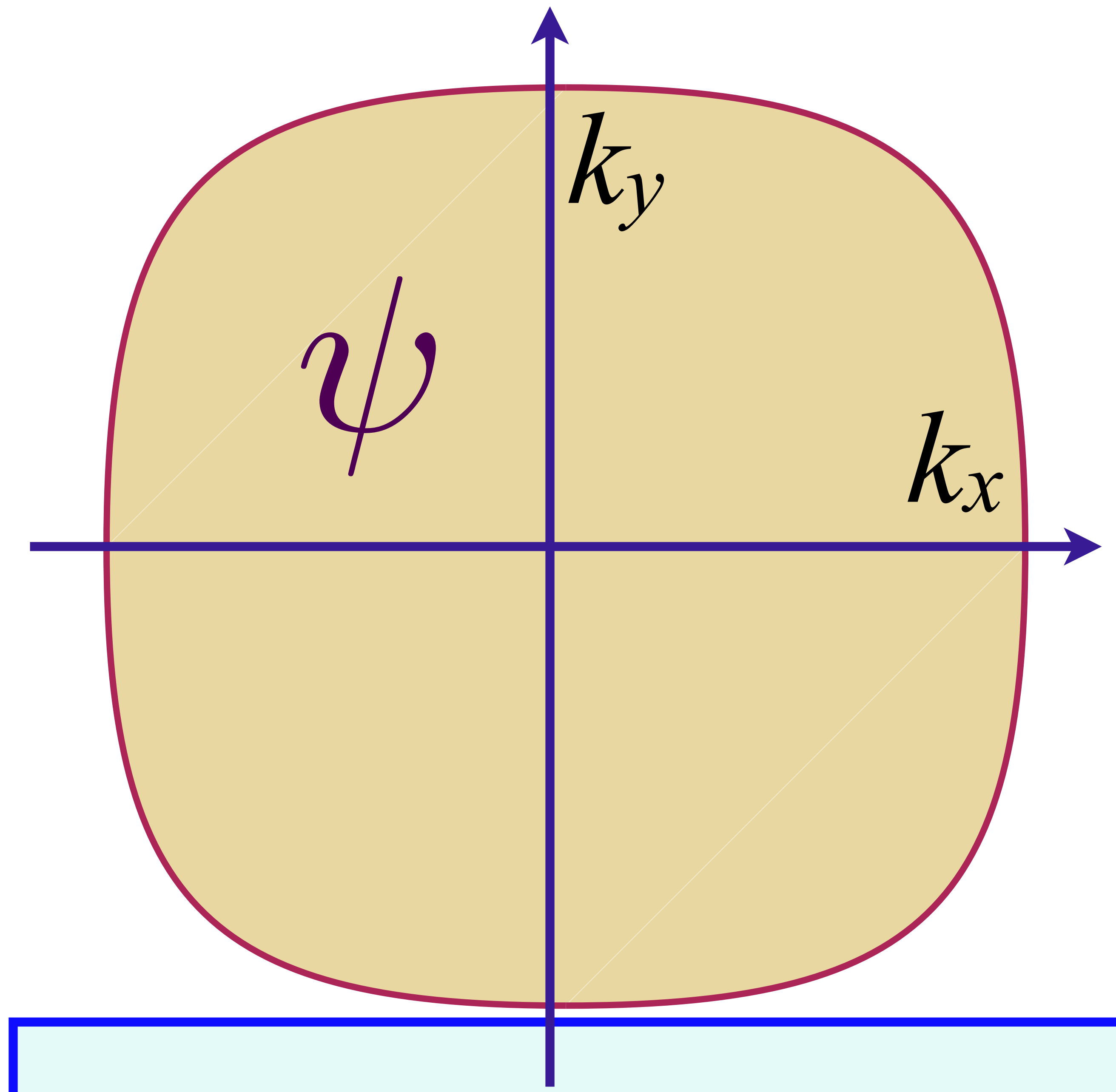
ϕ could represent
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume
changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4$$

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



ϕ could represent
e.g. Ising-nematic order,
spin-density wave order,
Higgs boson for Fermi-volume
changing transition

$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 \; + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

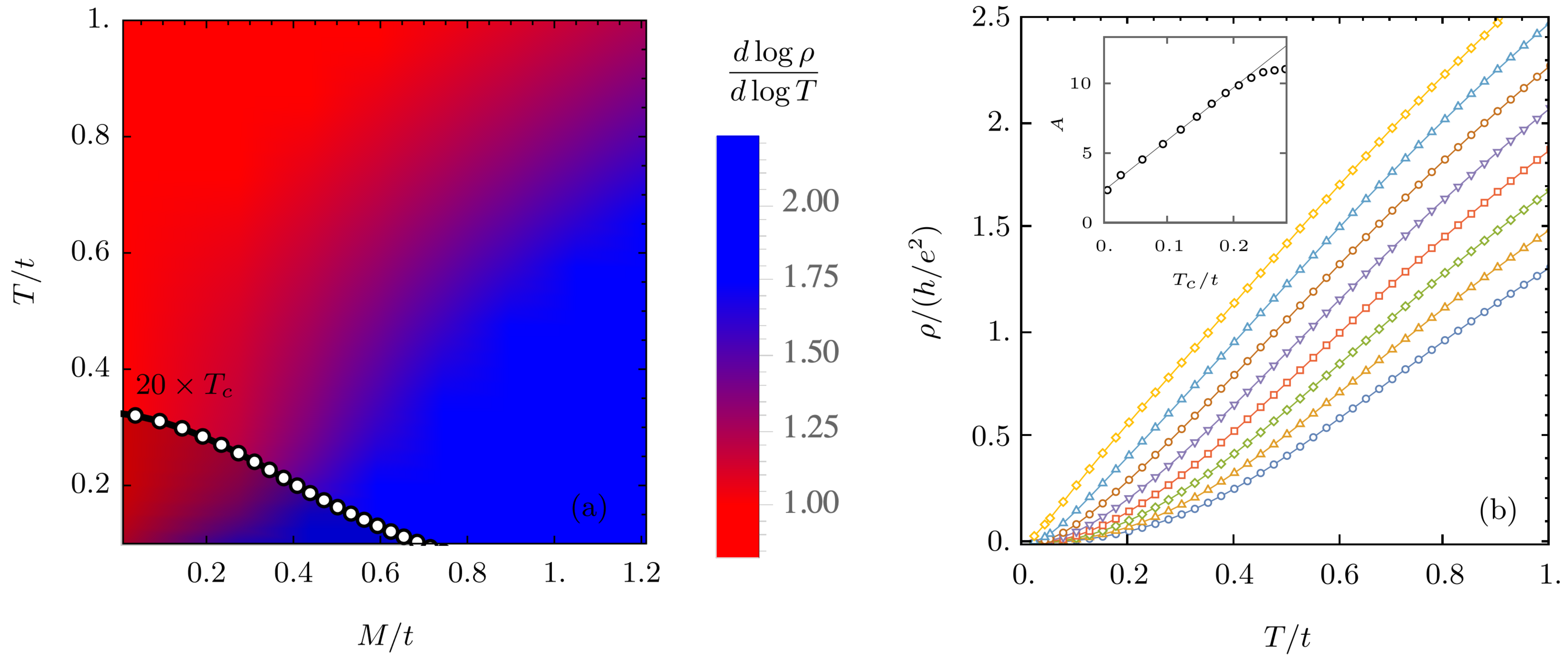
Spatial randomness in position of quantum critical point,
$g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Solve with the self-averaging methods of the SYK model

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608

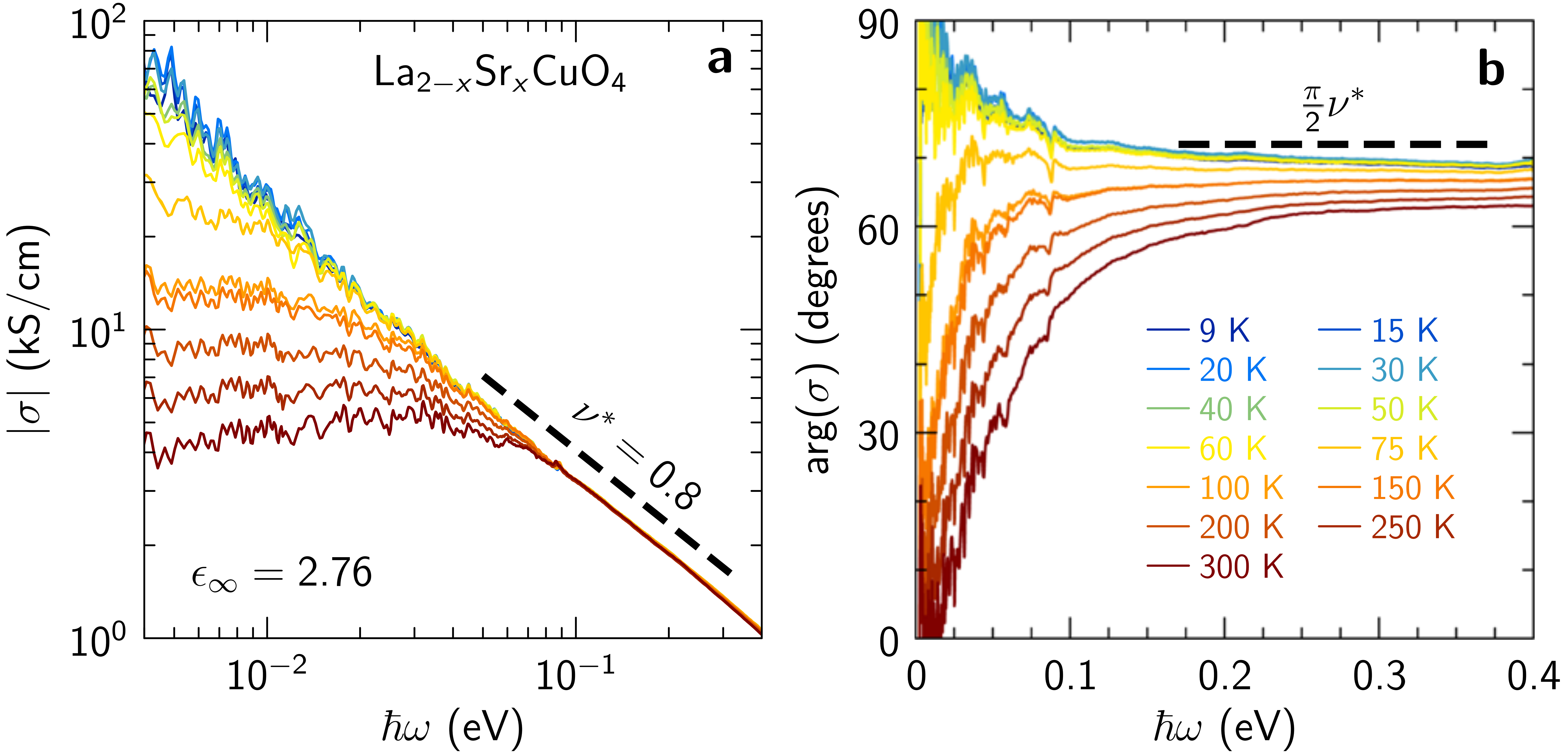


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$



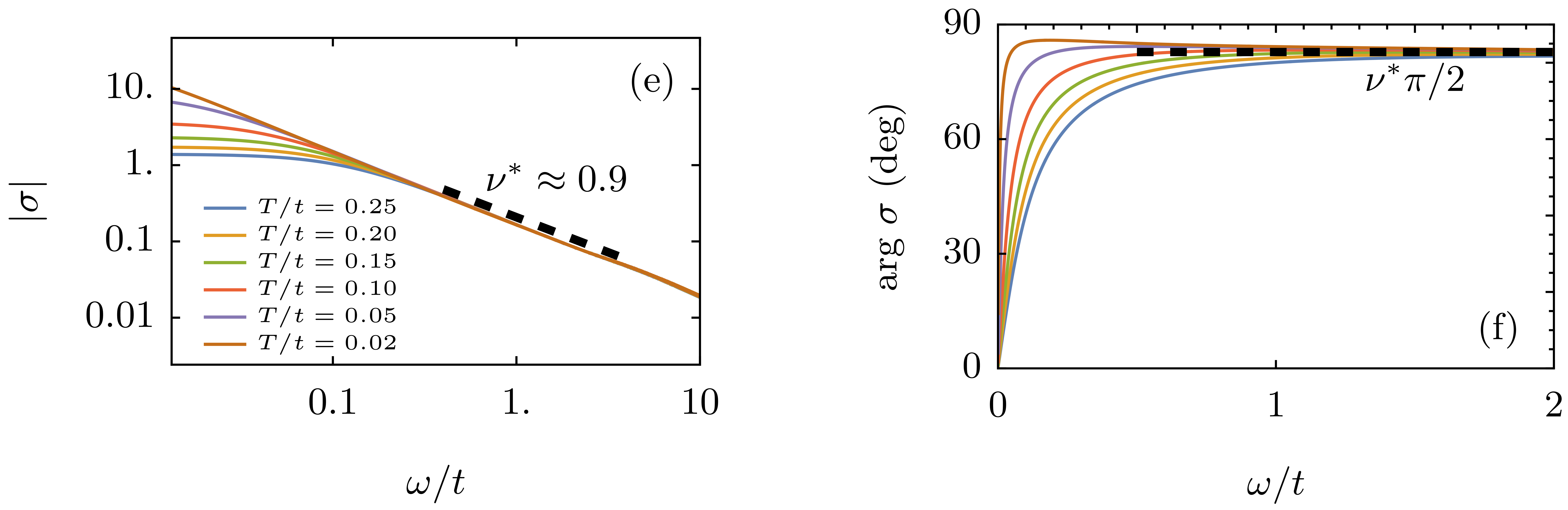
$La_{2-x}Sr_xCuO_4$
$p = 0.24$
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

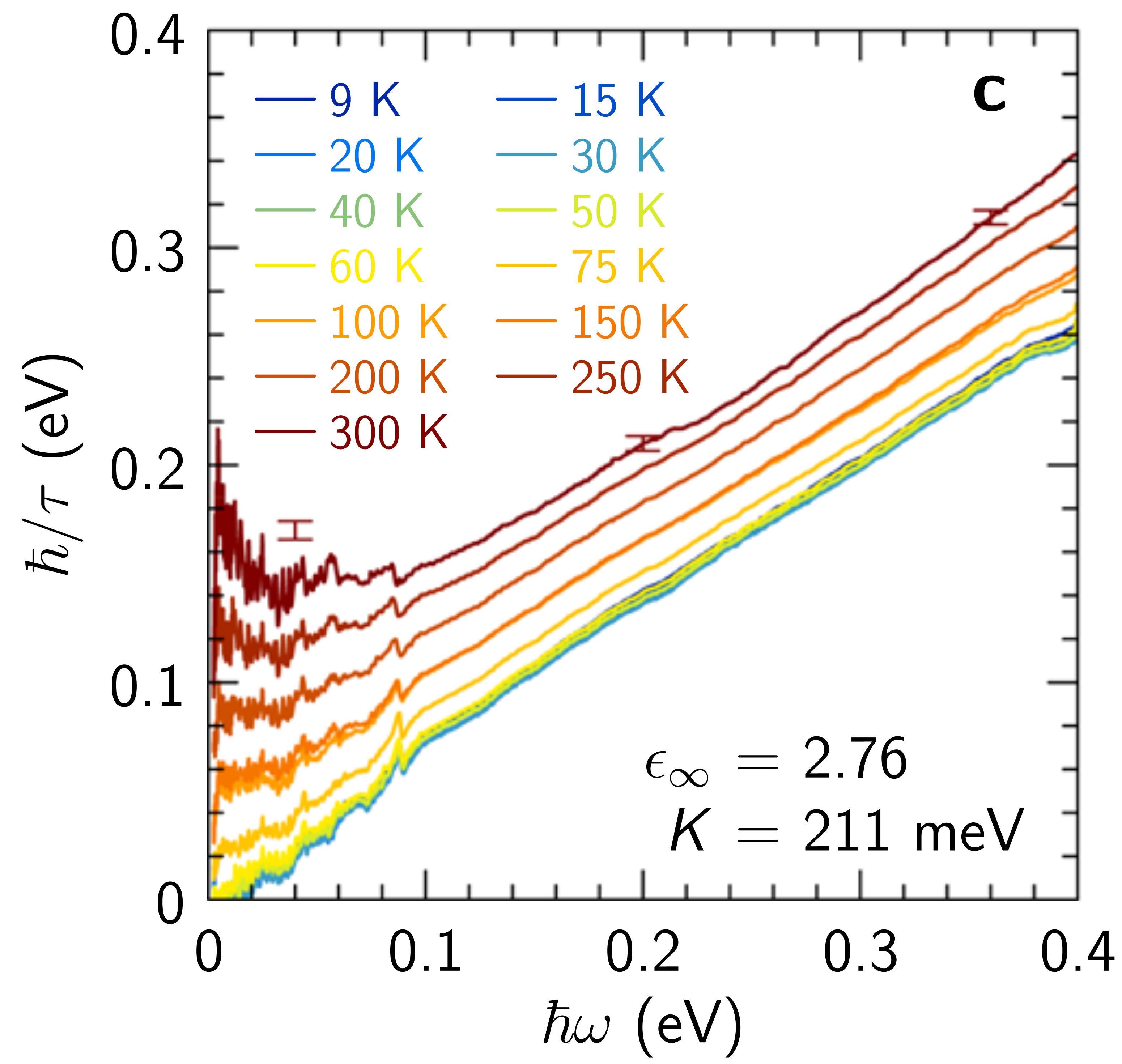


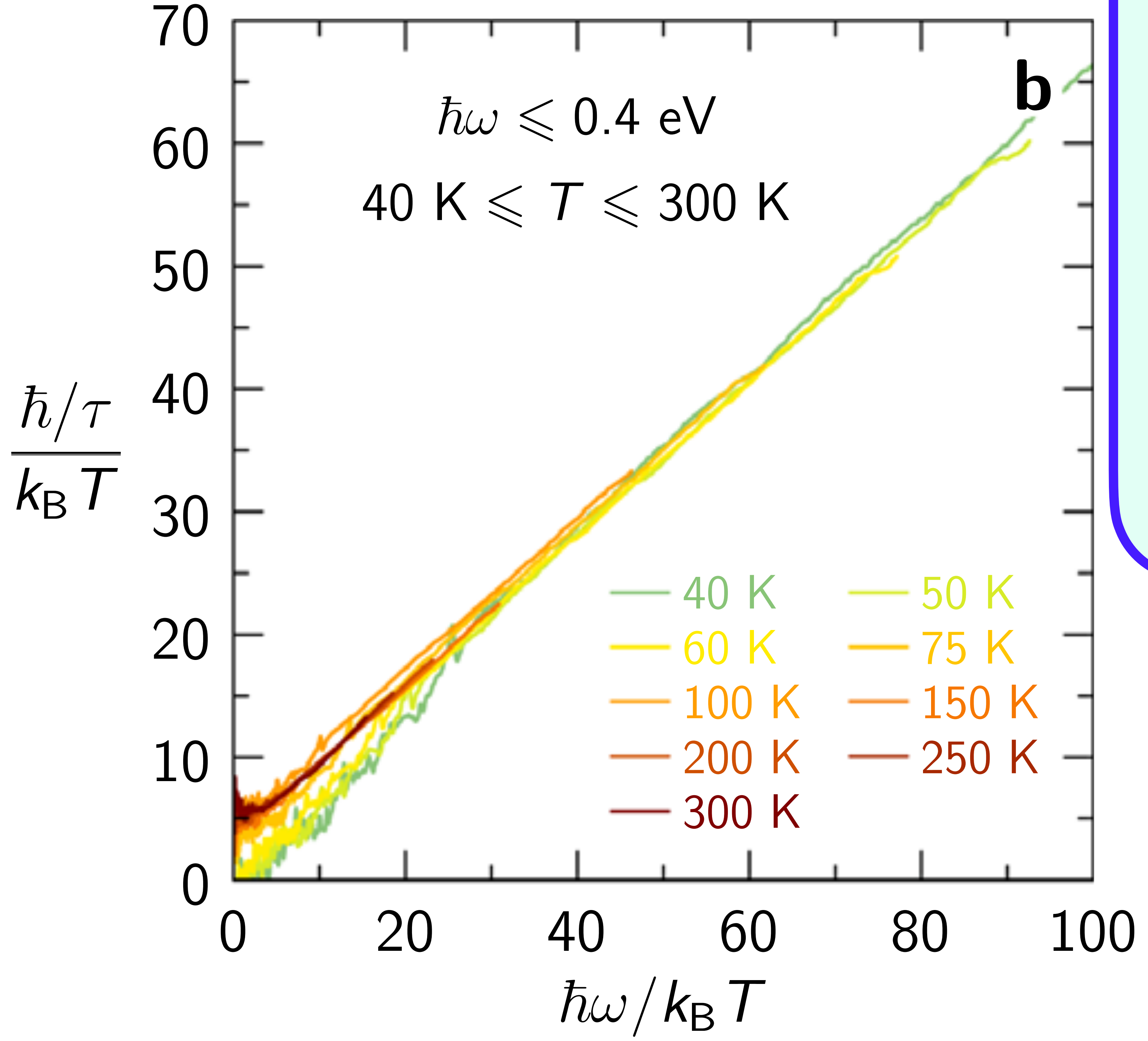
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$$S(T \to 0) \sim T\ln(1/T).$$

$La_{2-x}Sr_xCuO_4$

$p = 0.24$

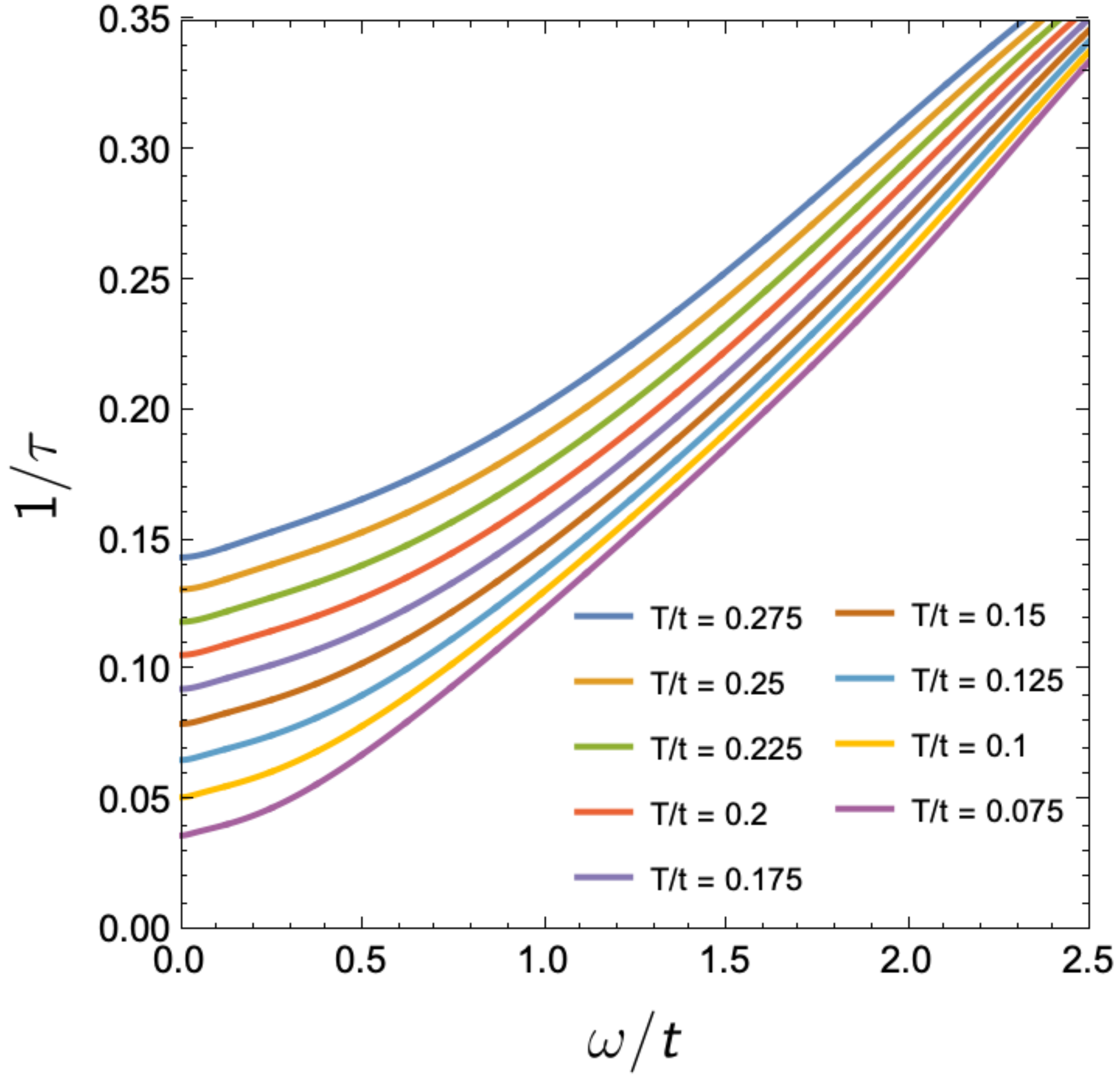
$T_c = 19$ K

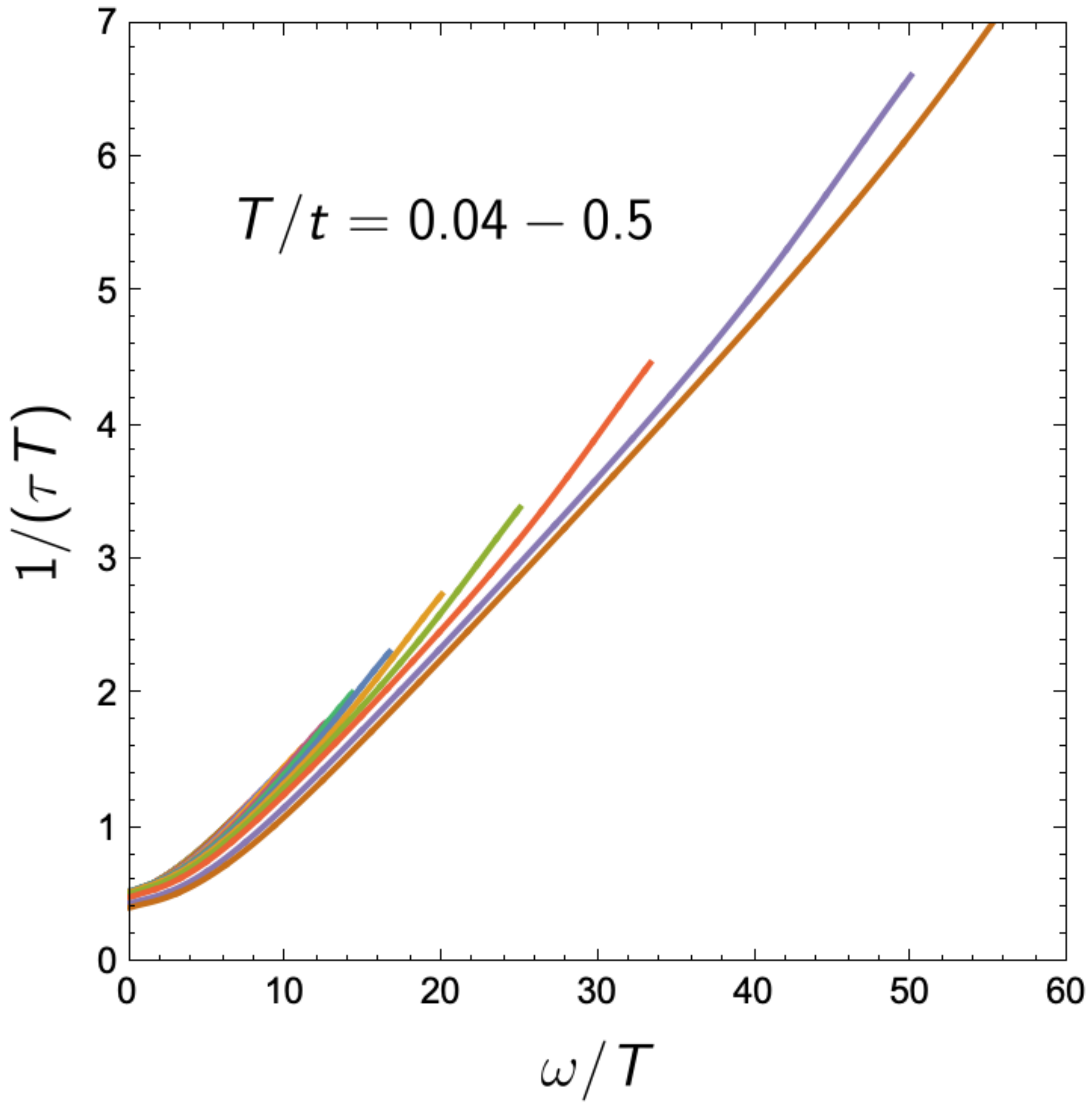
Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

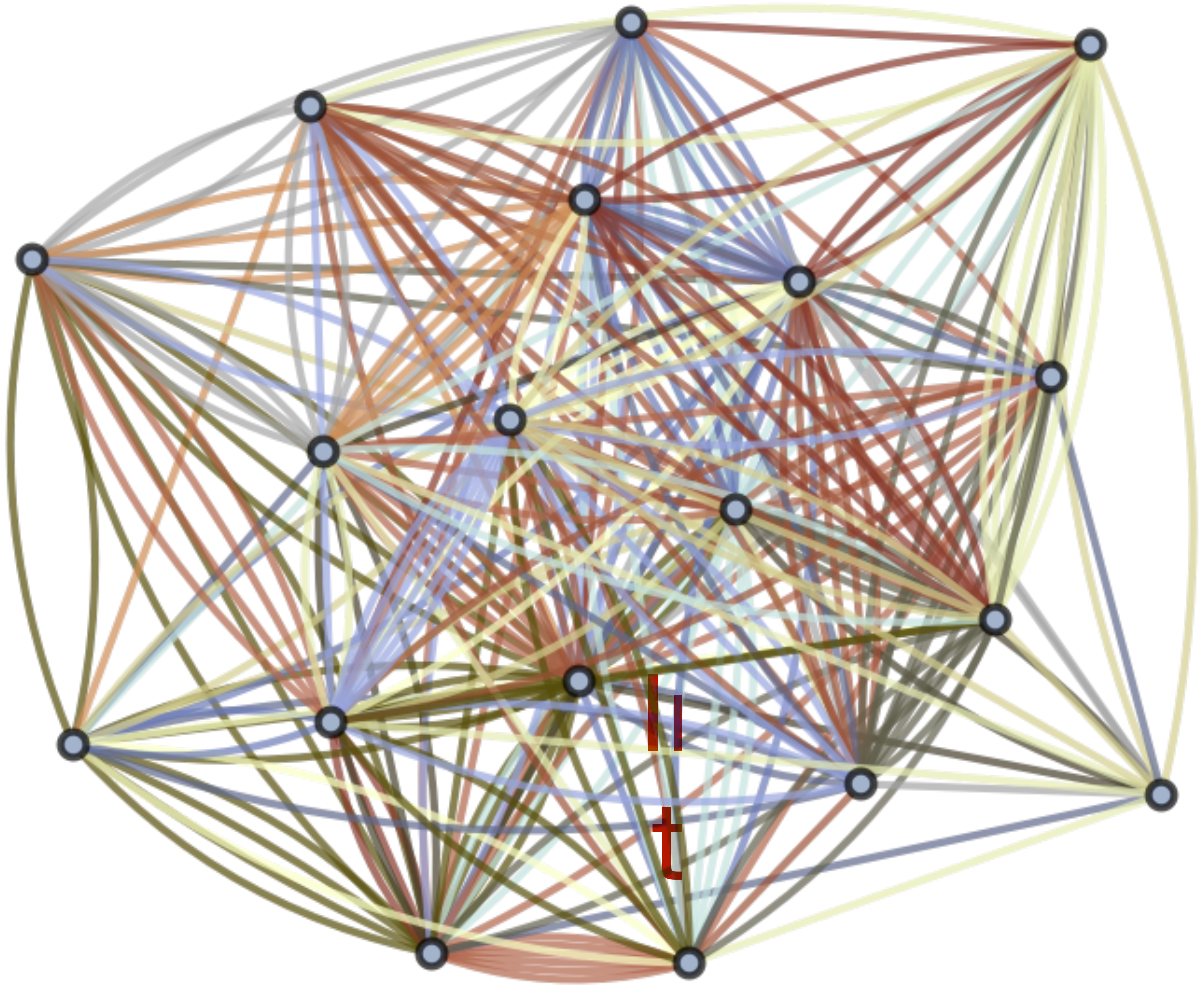
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Recap

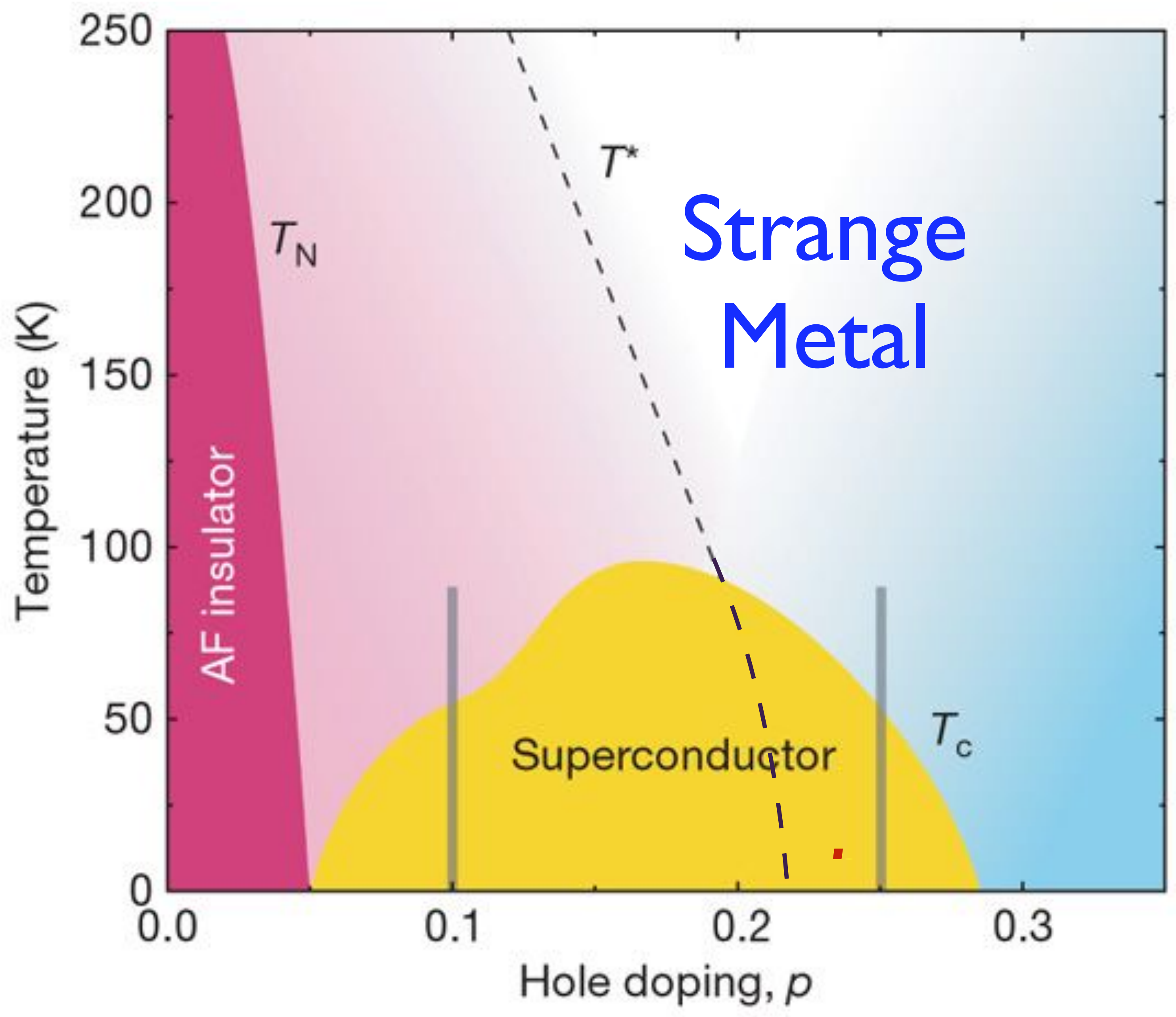
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

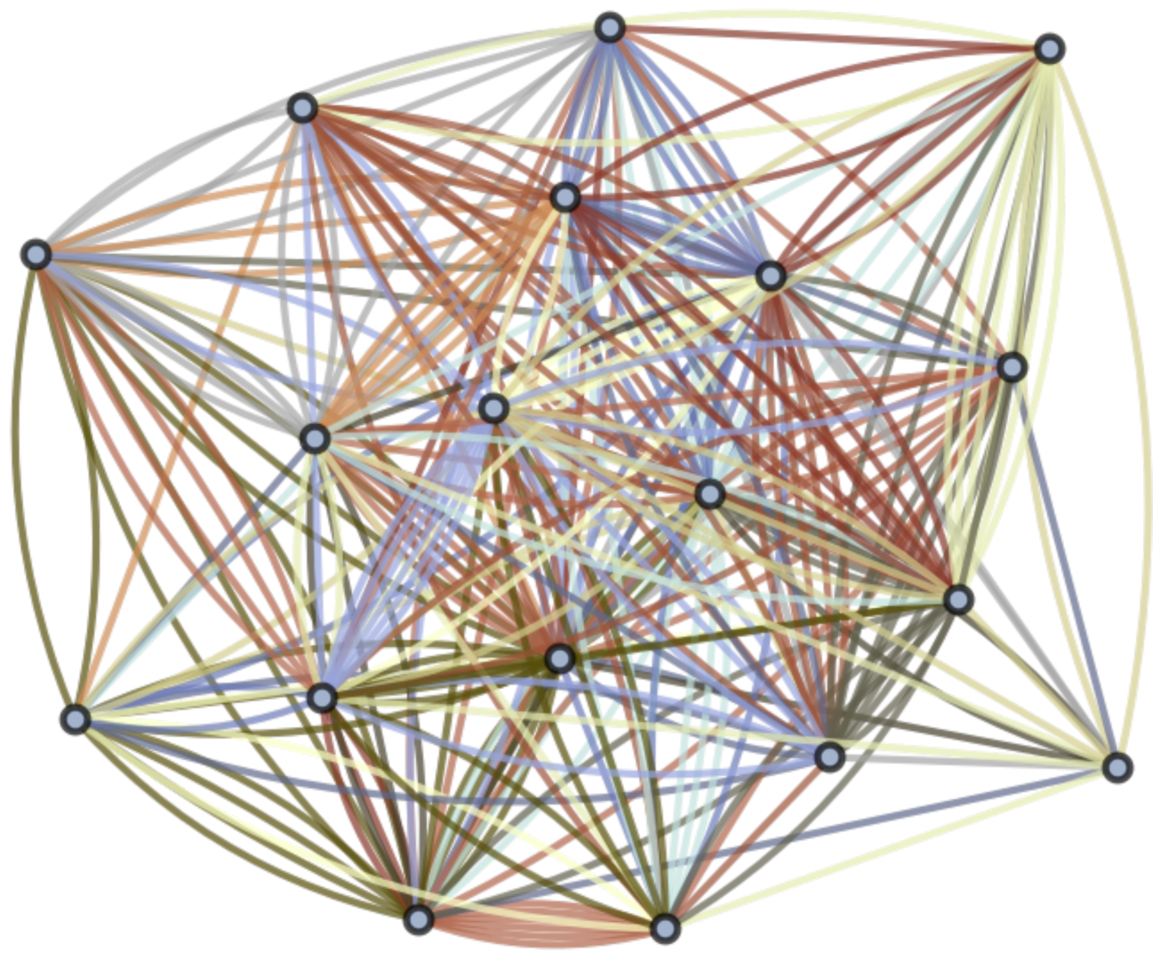


The Sachdev-Ye-Kitaev (SYK) model

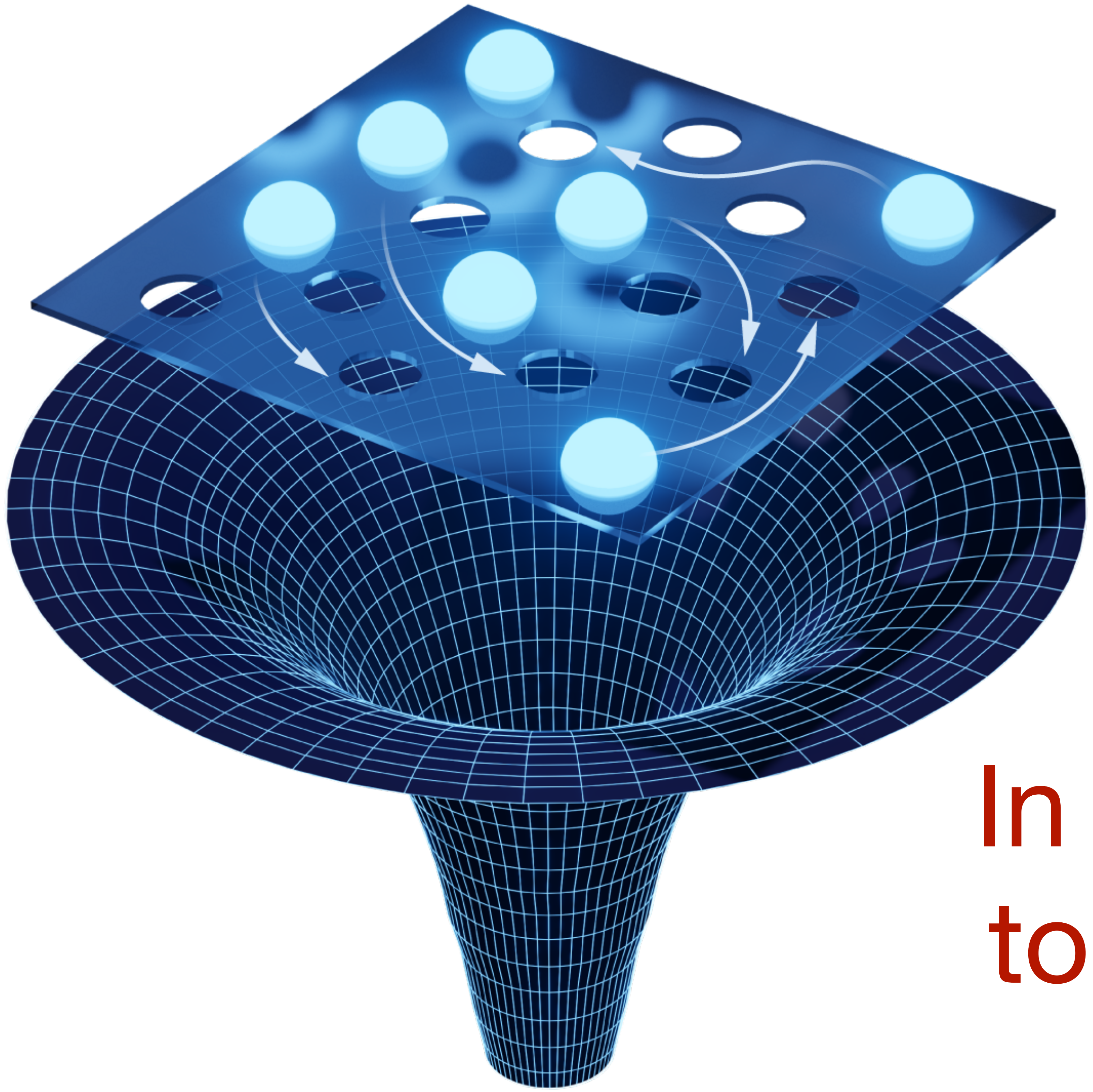
The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials

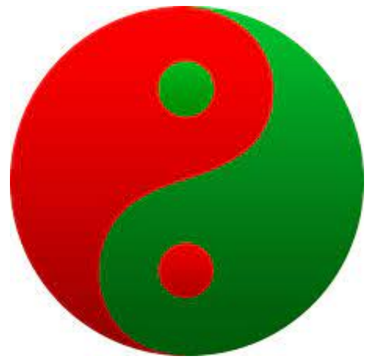


The Sachdev-Ye-Kitaev (SYK) model



The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged black holes***