

Unveiling the order of the high temperature superconductors

Weizmann Institute of Science
June 5, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Cornell



Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL



Stephen Edkins
Cornell / St Andrews



Michael Lawler



J. C. Seamus Davis



Eun-Ah Kim

Theorists at Harvard



Max Metlitski



Rolando
La Placa



Andrea Allais



Johannes
Bauer

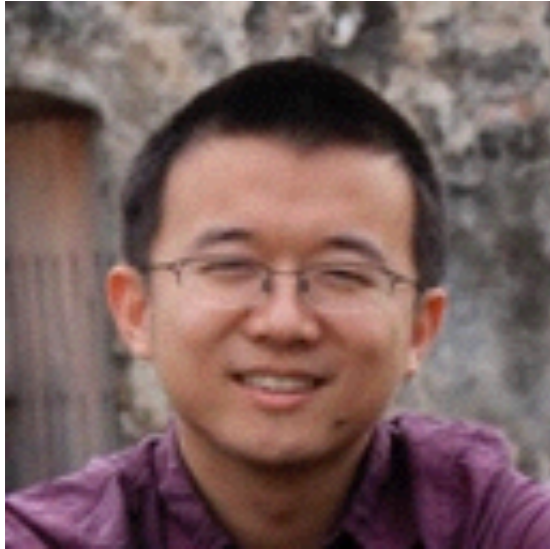


Debanjan
Chowdhury



Jay Deep
Sau

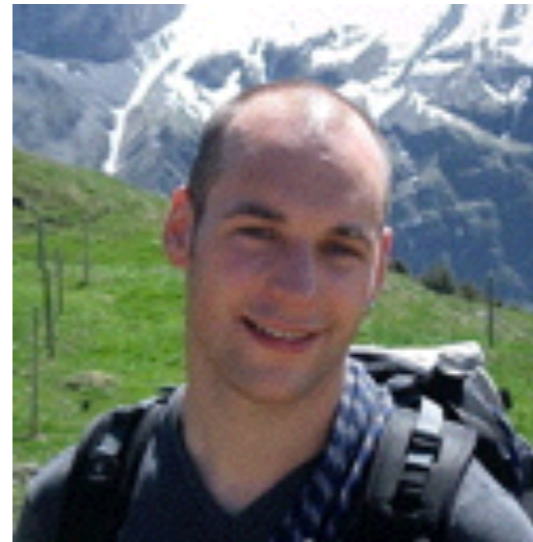
Experimentalists at Harvard



Yang He



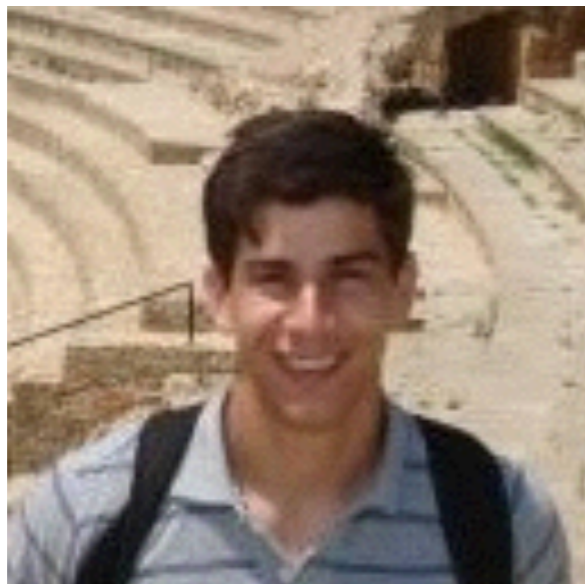
Yi Yin



Martin Zech



Anjan
Soumyanarayanan



Ilija Zelkovic

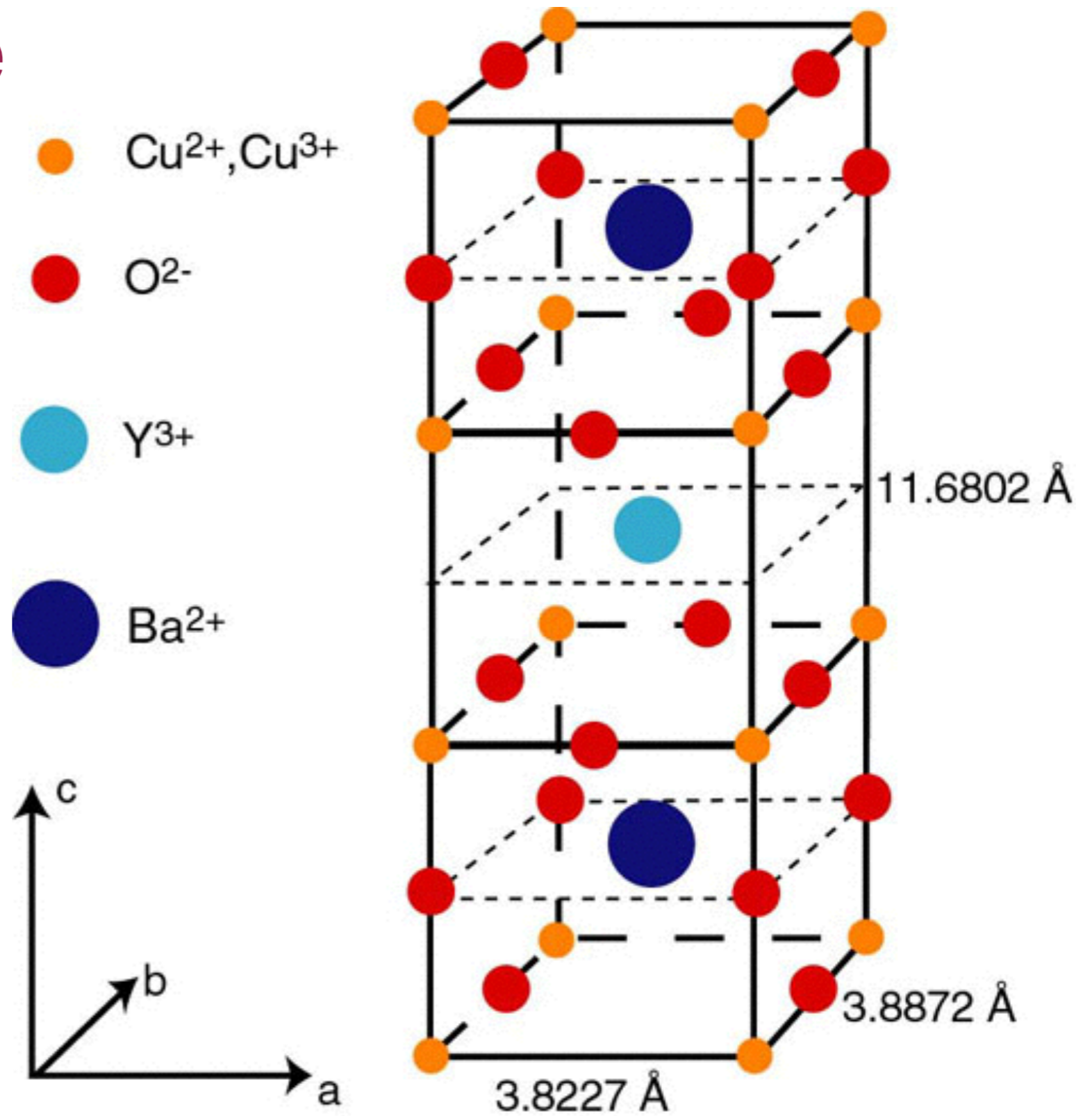


Michael Yee

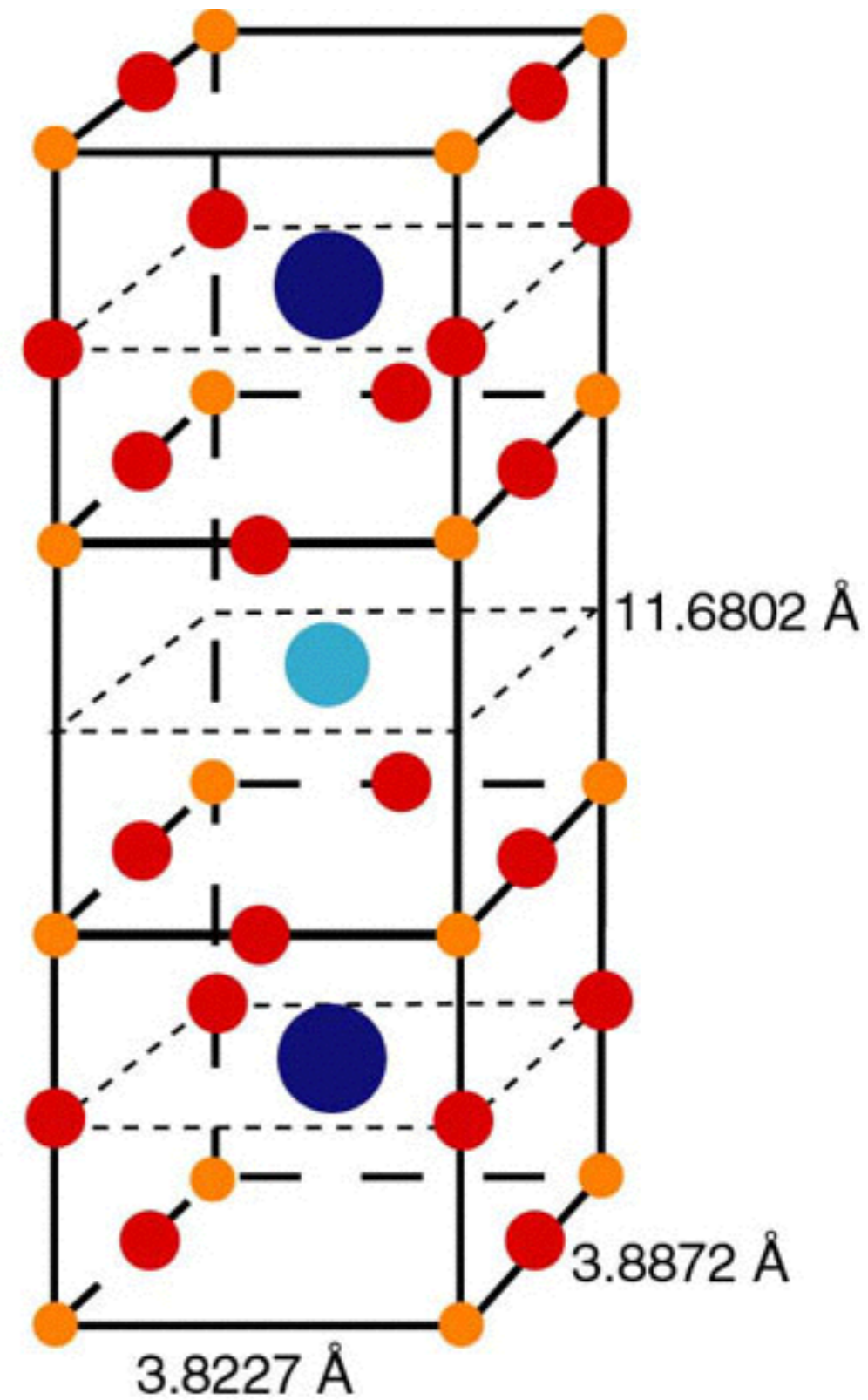
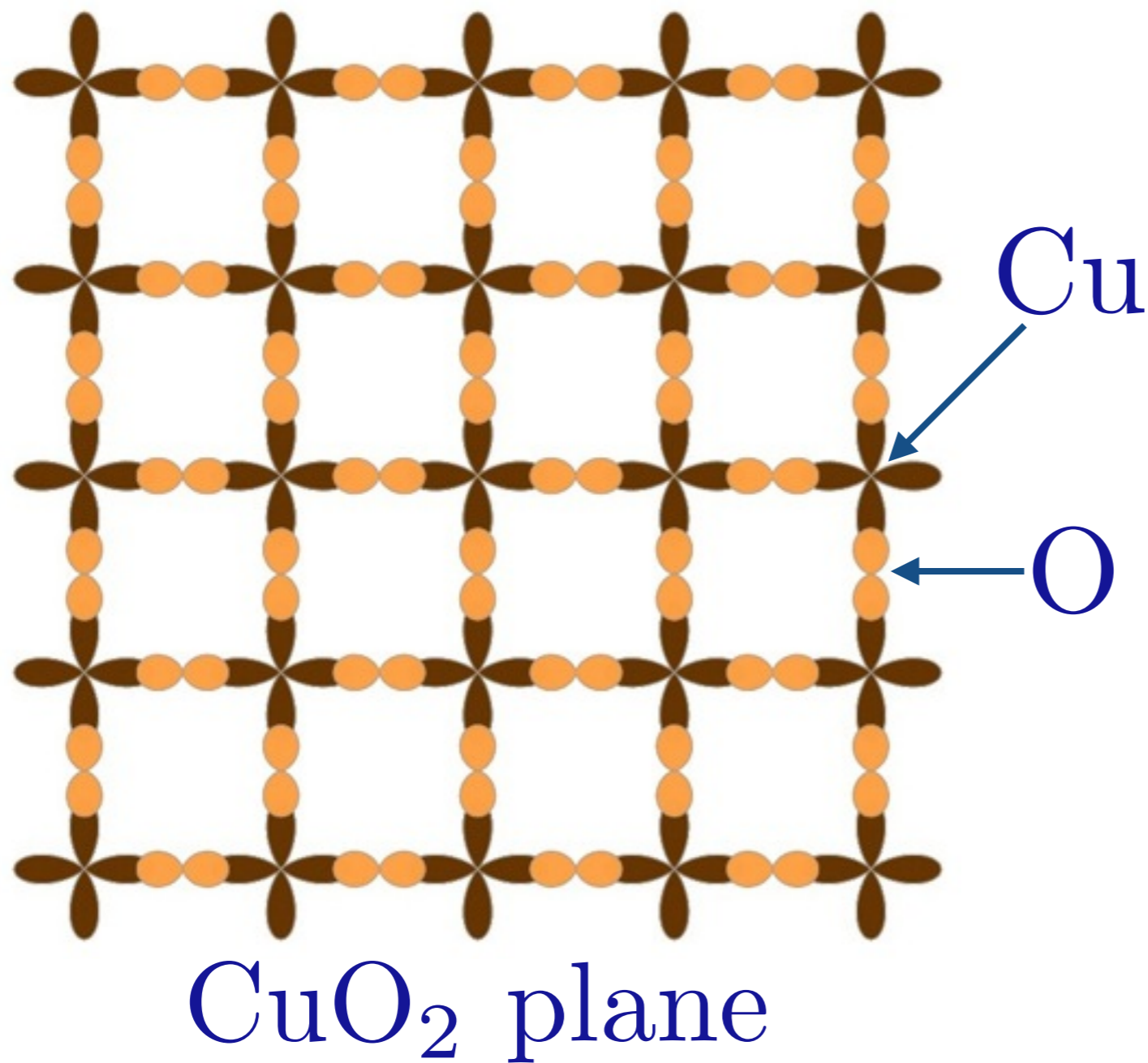


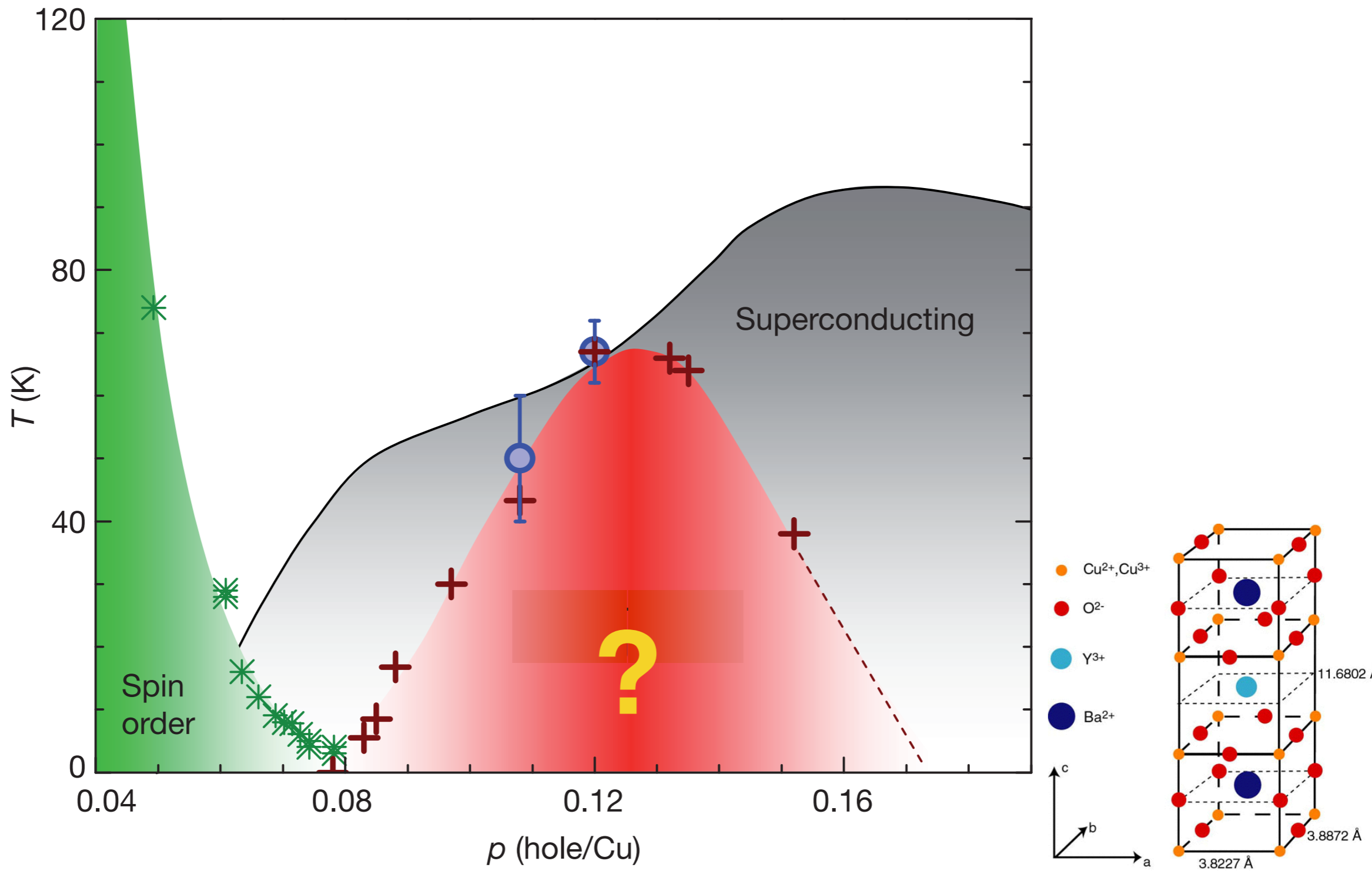
Jennifer Hoffman

High temperature superconductors

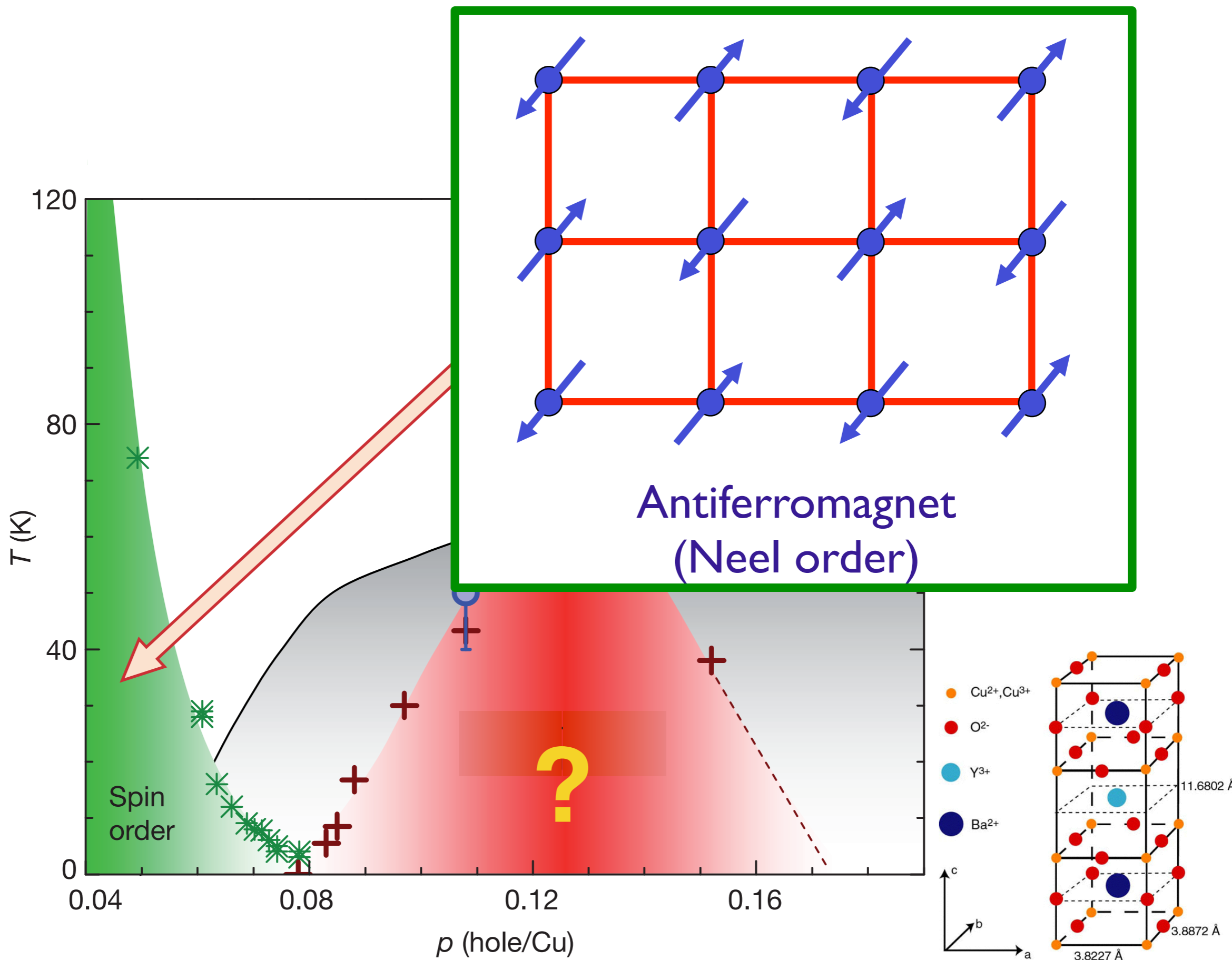


High temperature superconductors

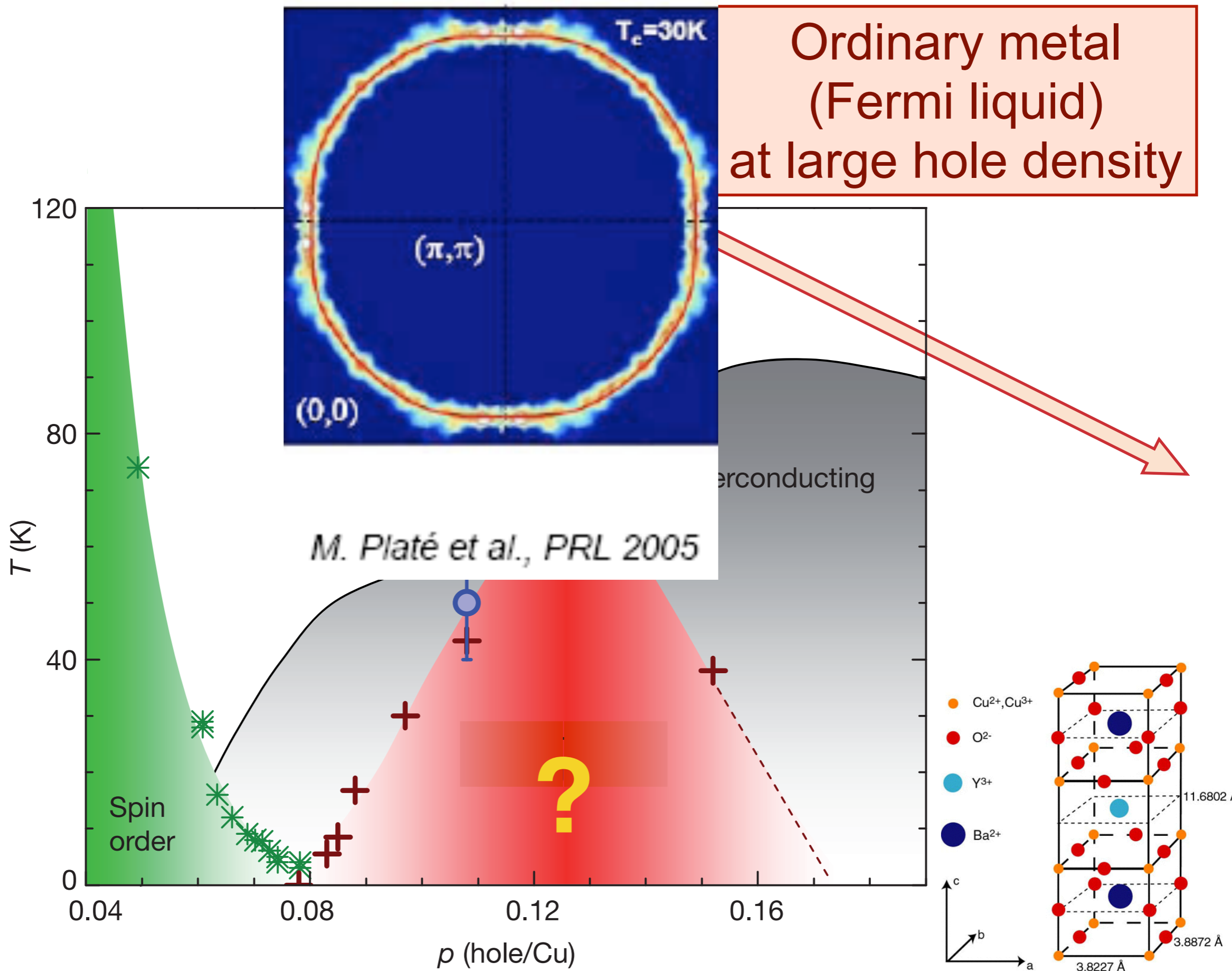




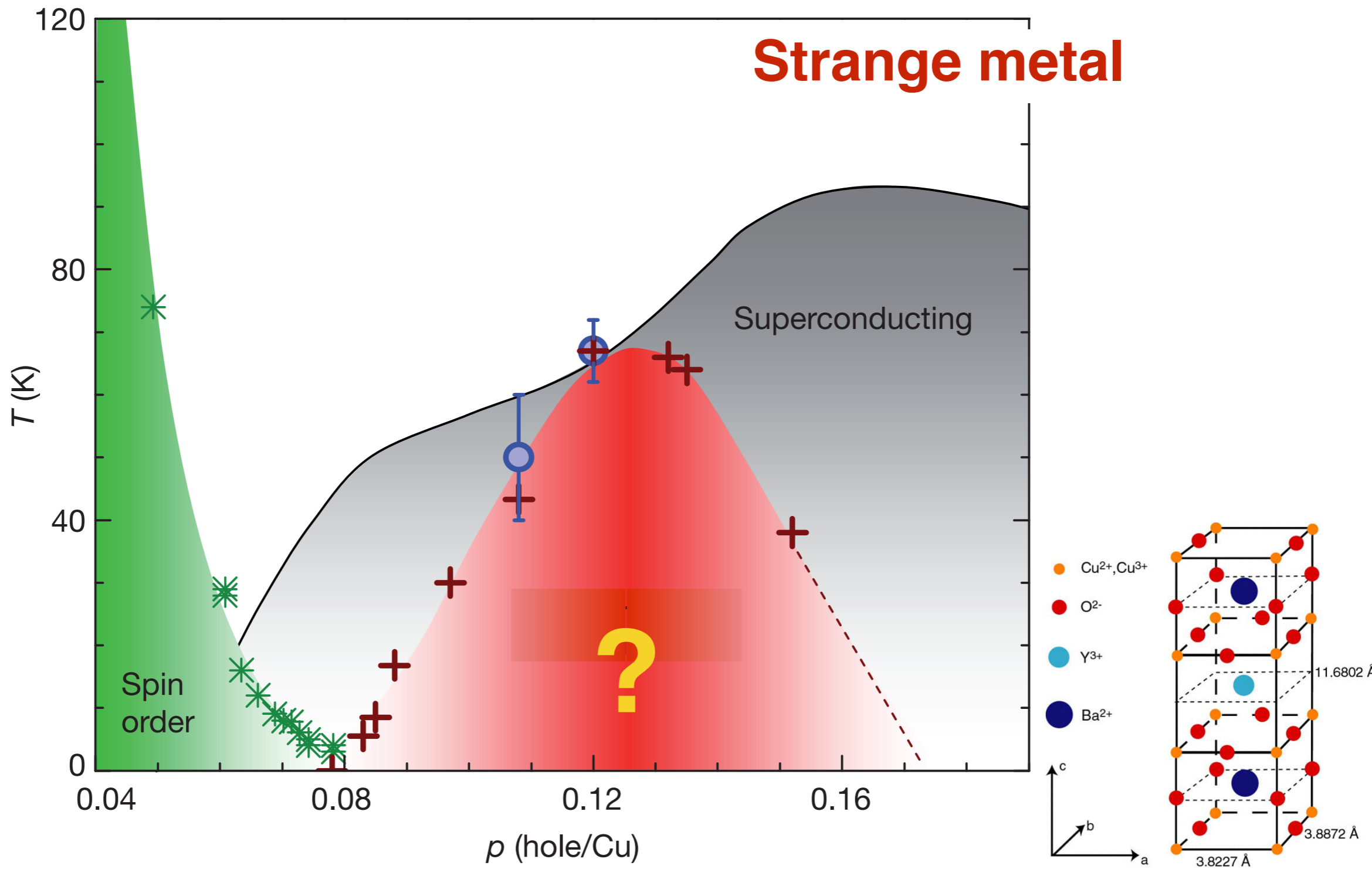
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



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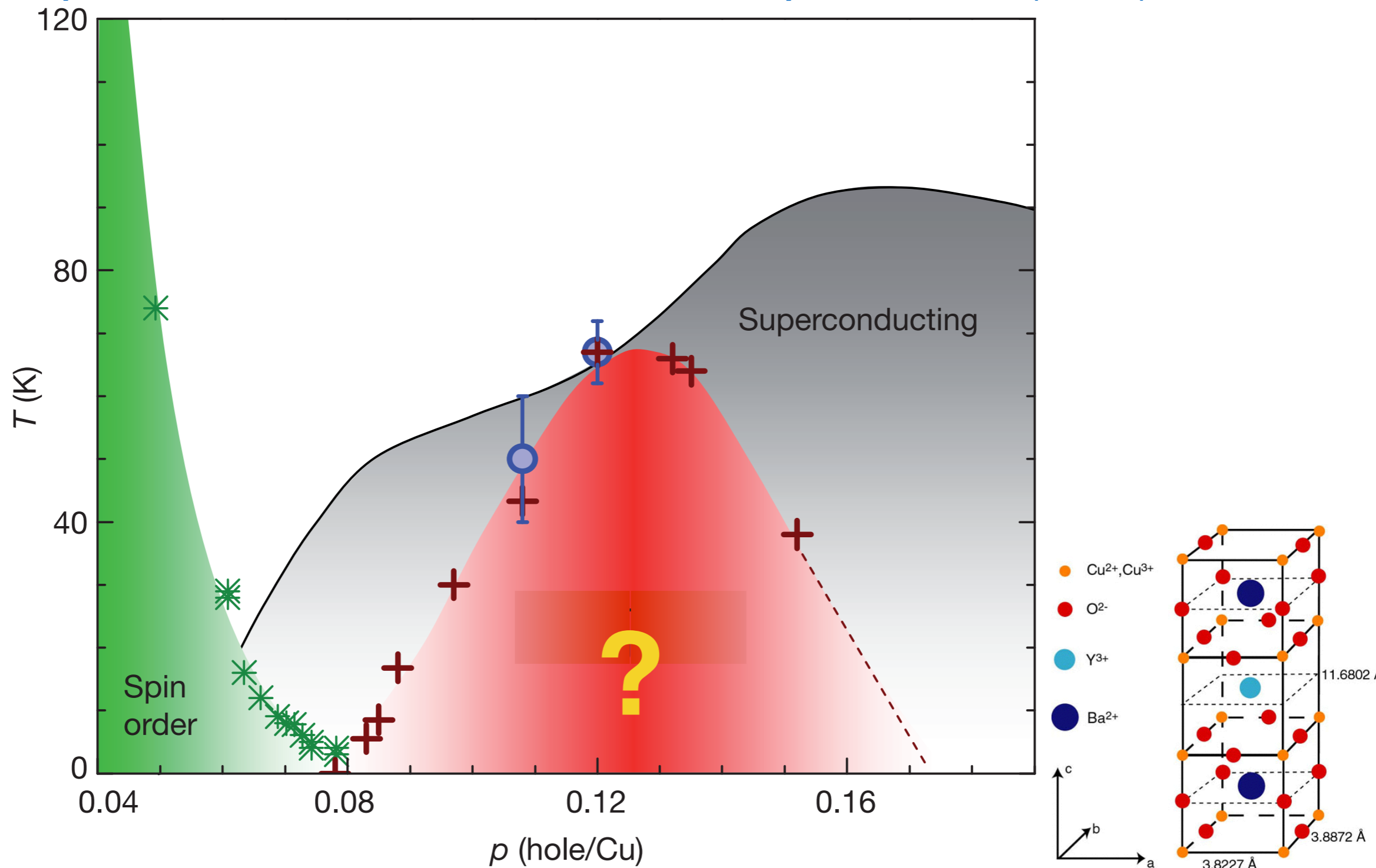


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In the La-based superconductors, the region with magnetic order overlaps with the red region - I will not discuss these materials: see S.A. Kivelson, I.P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, *Rev. Mod. Phys.* **75**, 1201 (2003)



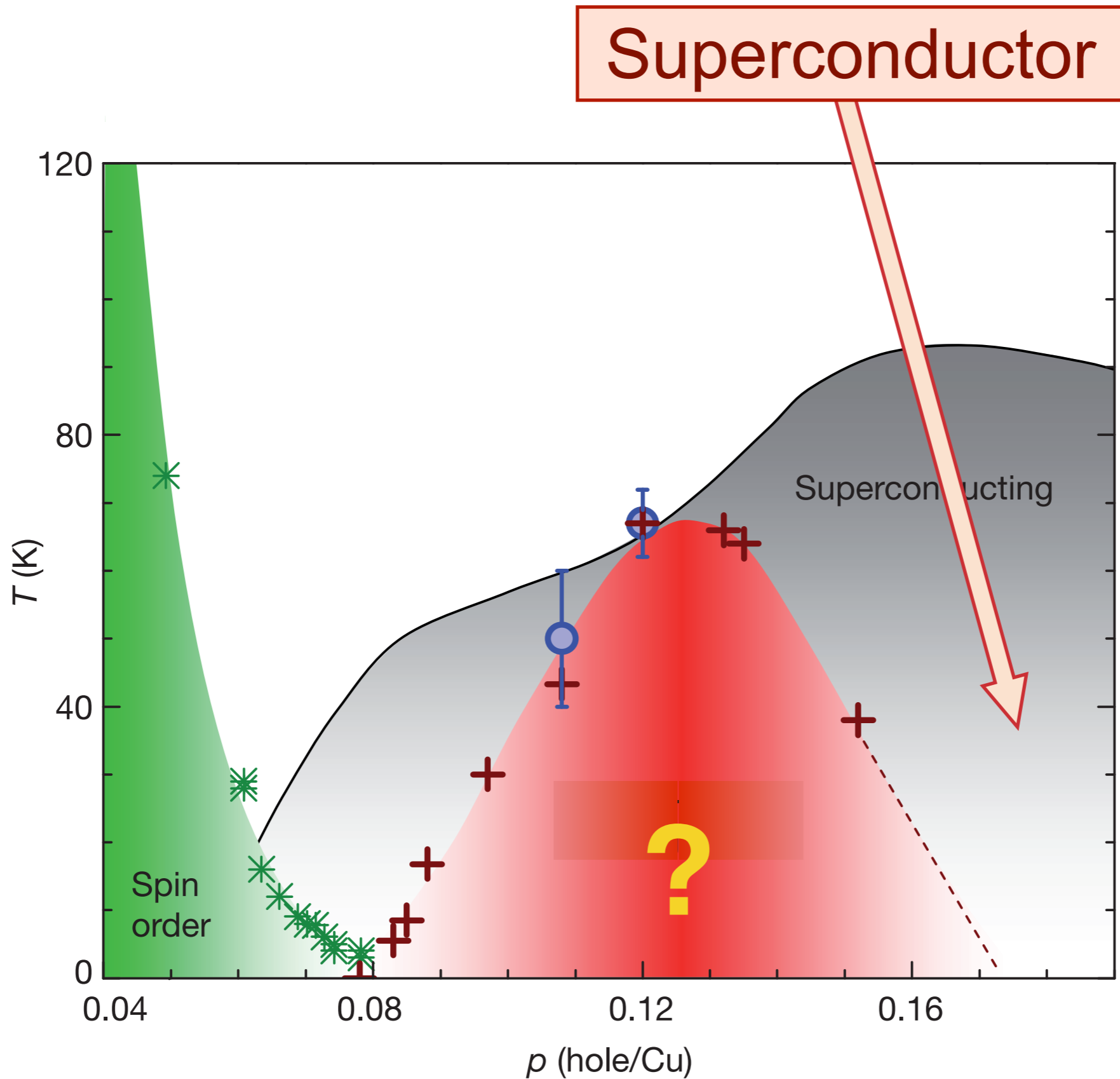
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Outline

1. Unconventional d -wave superconductivity
2. Low hole density state:
An unconventional density wave
3. Outline of theoretical prediction
4. Evolution of Fermi surface

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Superconductivity: Bose condensation of Cooper pairs of electrons

$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

Superconductivity: Bose condensation of Cooper pairs of electrons

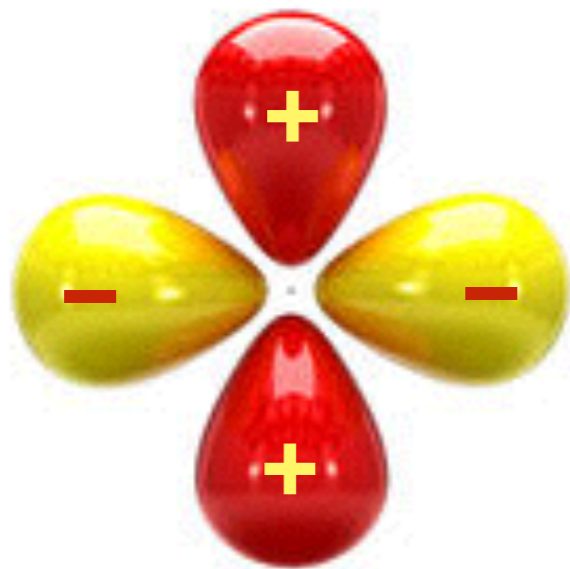
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Nearly constant condensate wavefunction
(superconducting order parameter)

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Superconductivity: Bose condensation of Cooper pairs of electrons

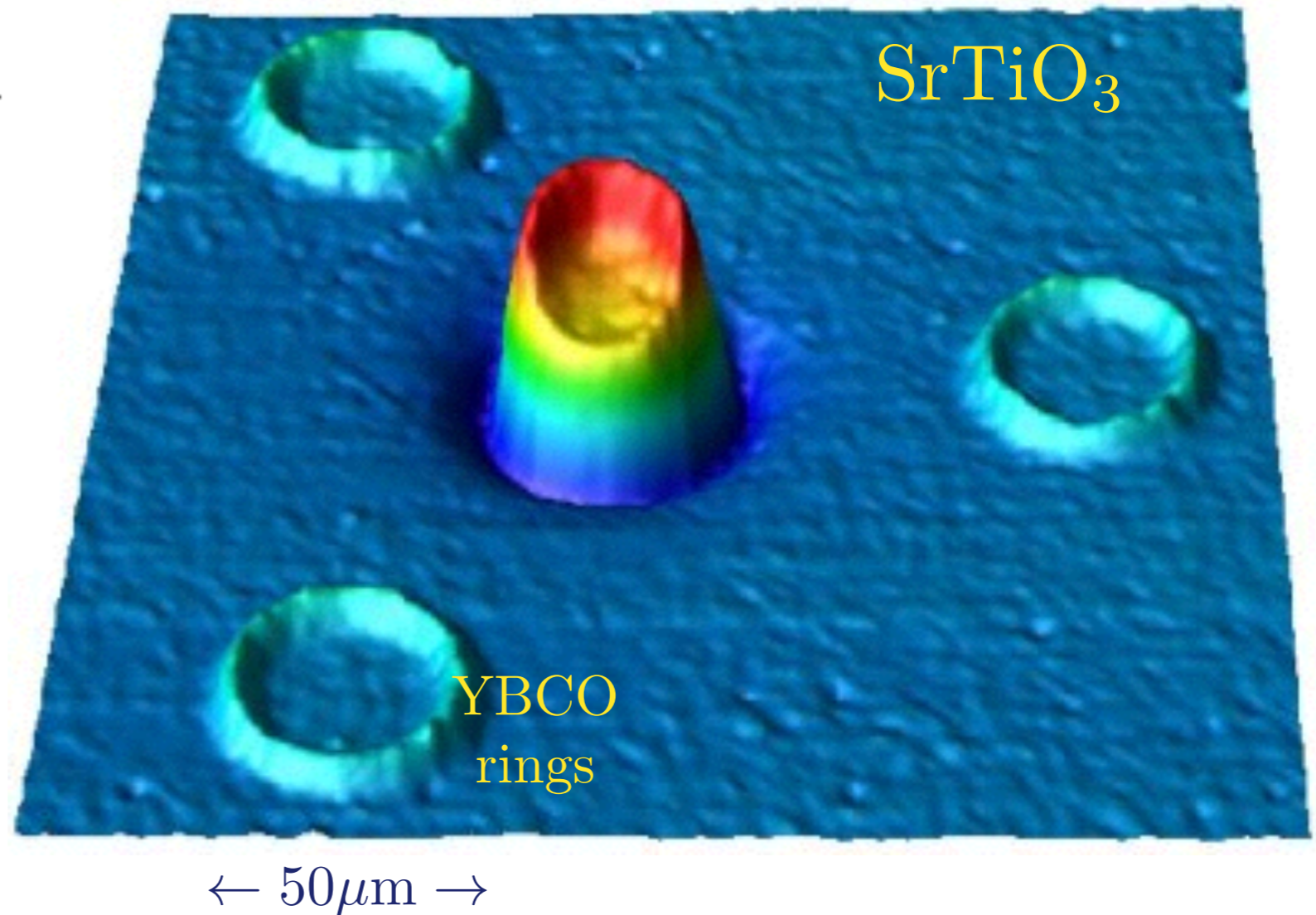
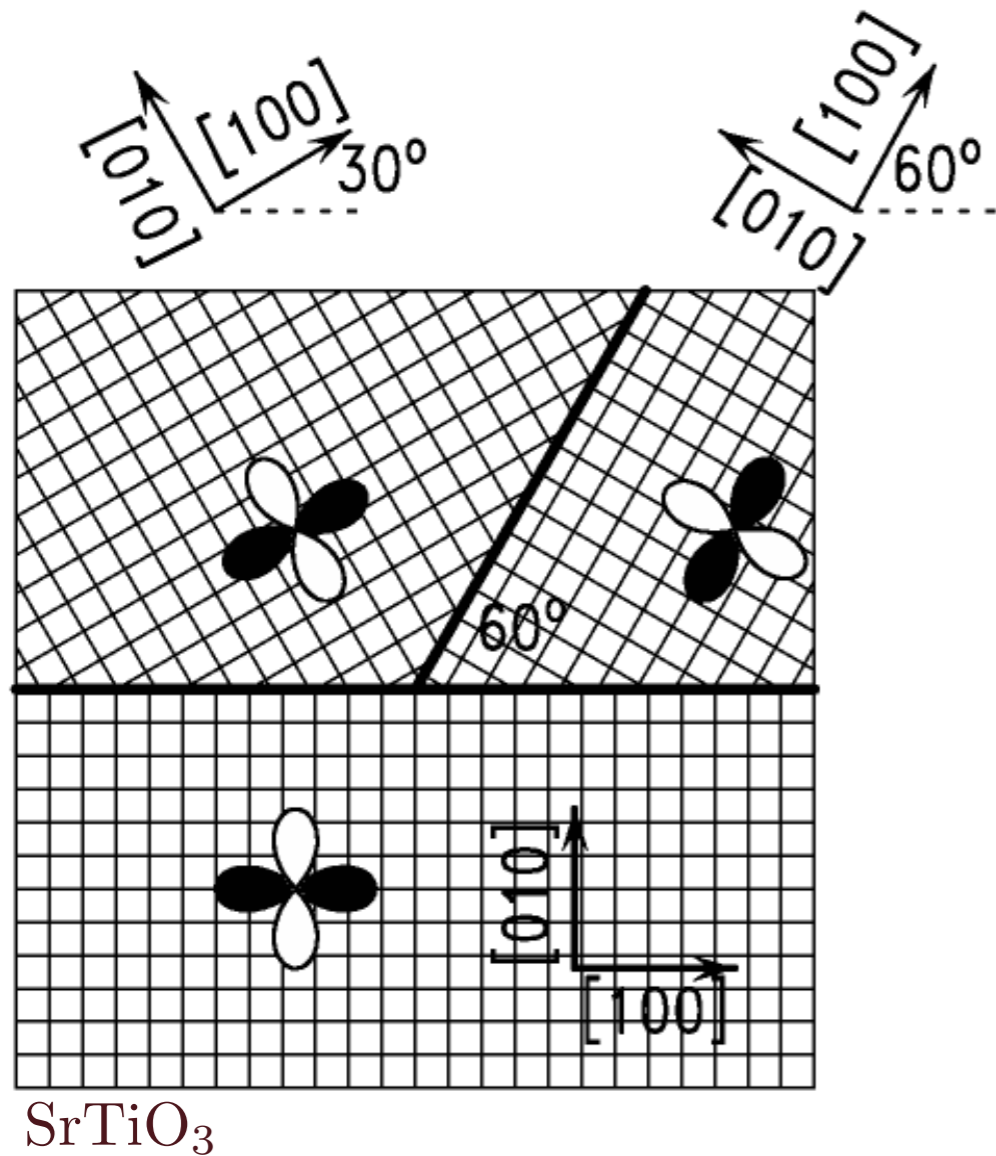
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Internal Cooper-pair wavefunction
has *d*-wave form in cuprates:
Unconventional superconductivity

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

Phase-sensitive measurement of the d -wave symmetry of Cooper pairs



Pairing Symmetry and Flux Quantization in a Tricrystal Superconducting Ring of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

C. C. Tsuei, J. R. Kirtley, C. C. Chi,* Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen
IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

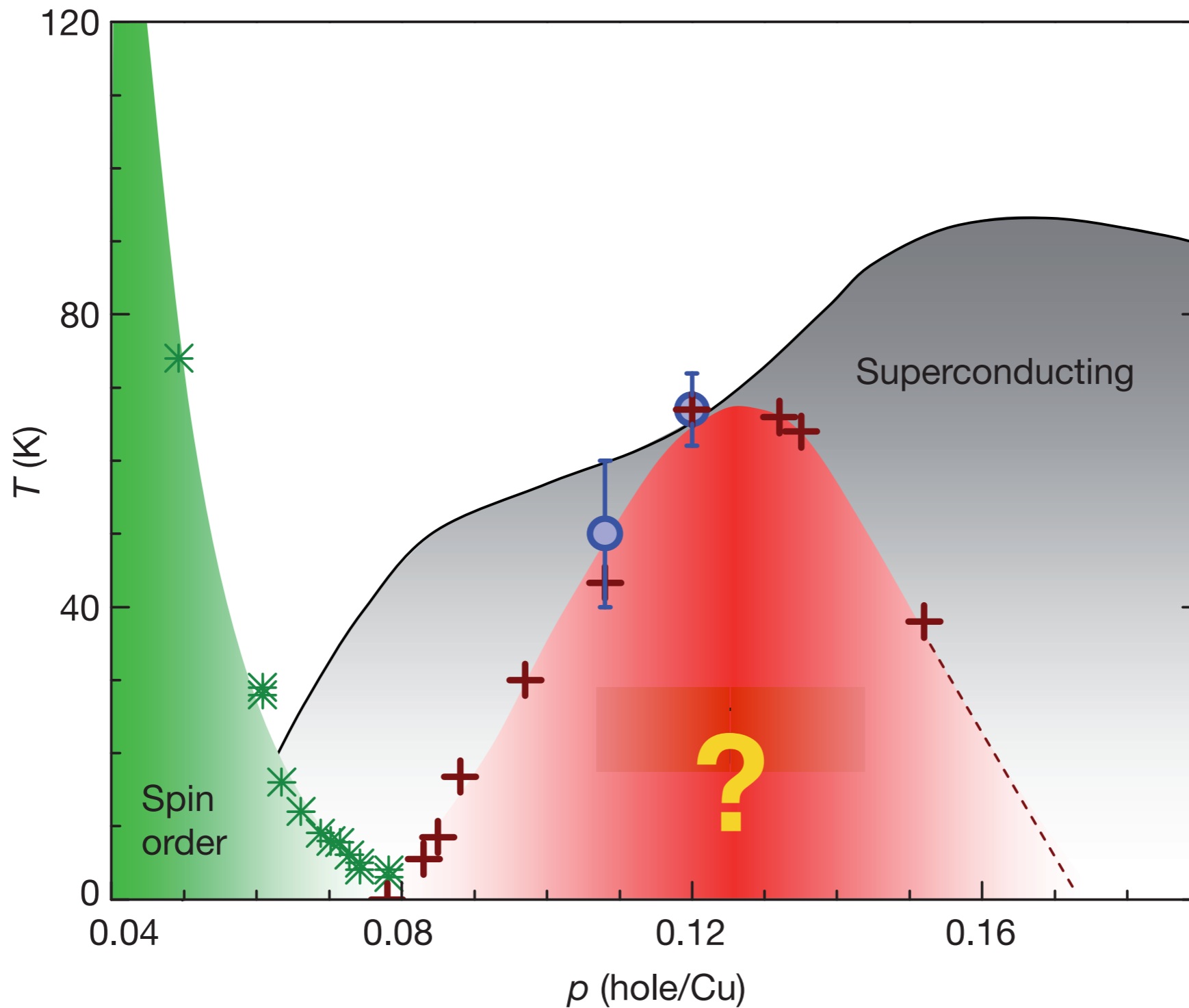
Phys. Rev. Lett. **73**, 593 (1994)

Outline

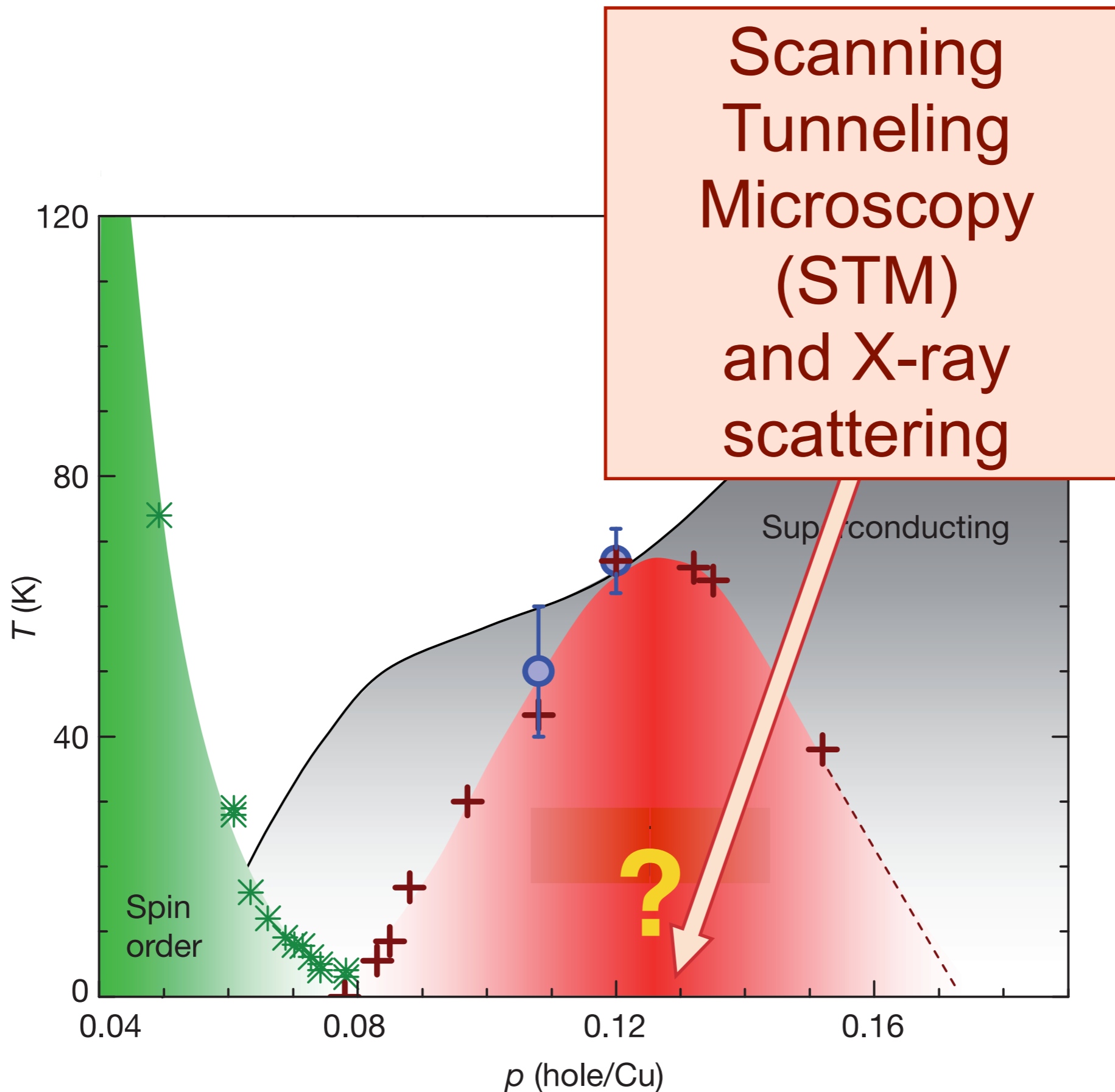
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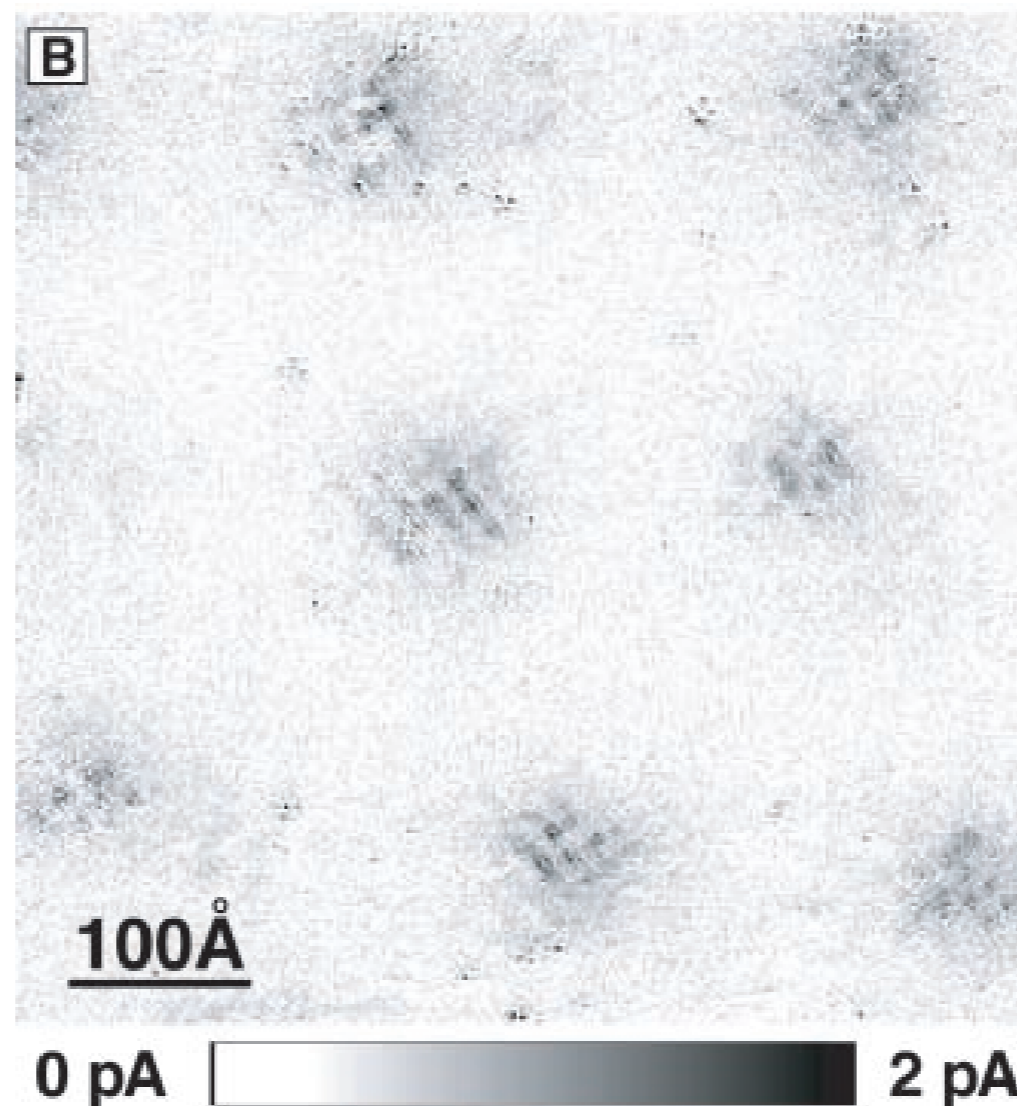
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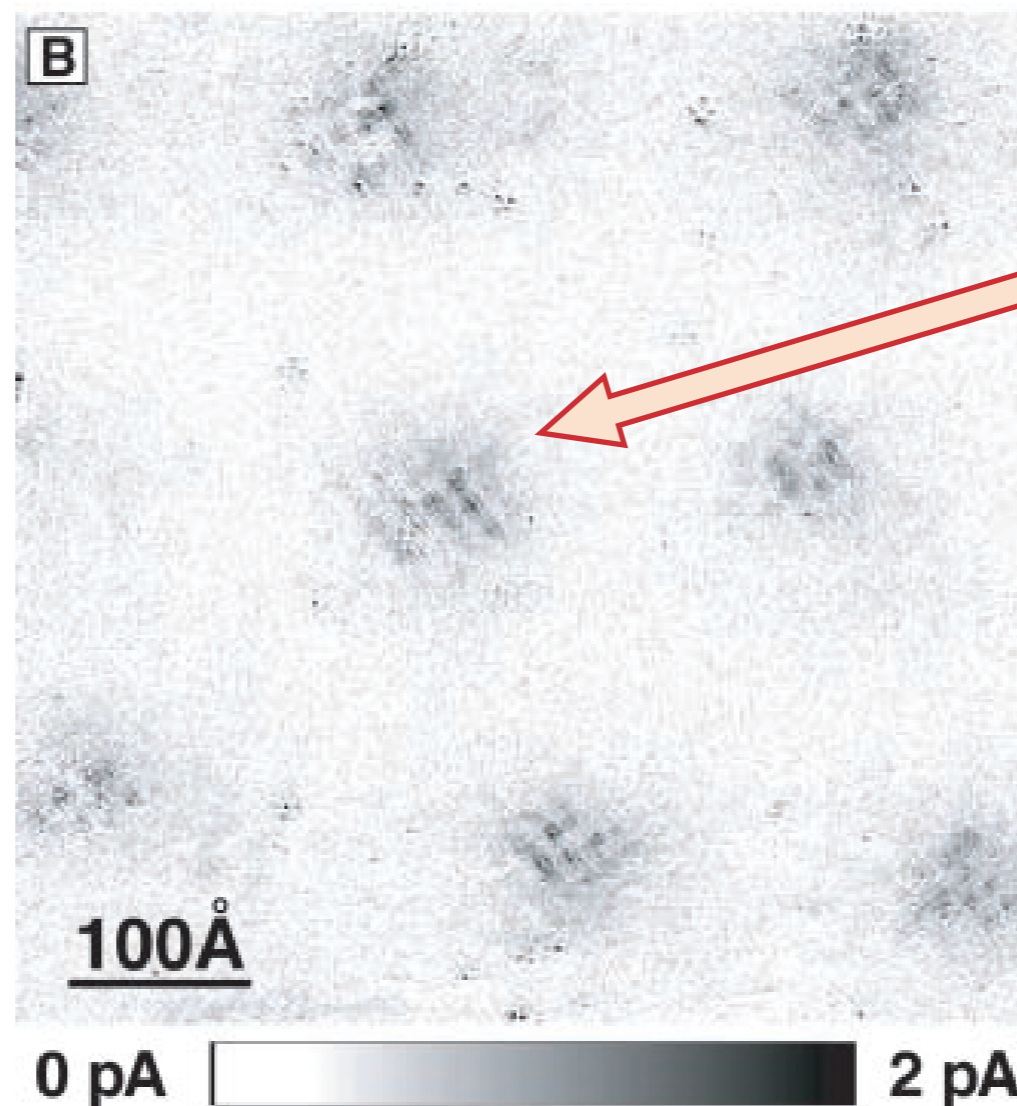
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).



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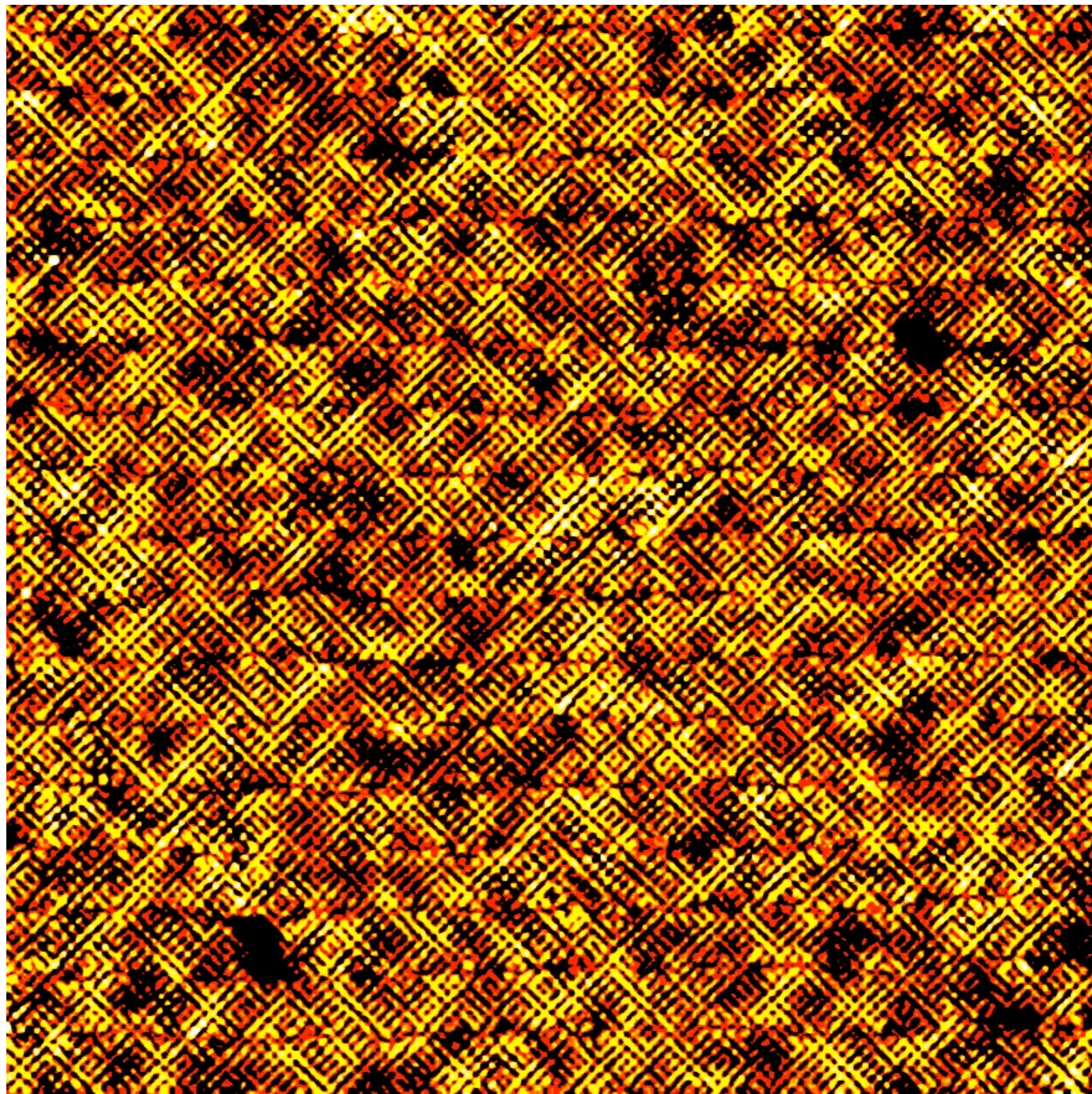
A density wave
with wavelength \approx
4 lattice spacings
around vortex
cores ?

See also

C. Howald, H. Eisaki,
N. Kaneko, M. Greven,
and A. Kapitulnik,
Phys. Rev. B **67**,
014533 (2003);

M. Vershinin, S. Misra,
S. Ono, Y. Abe, Yoichi
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303, 1995 (2004).

W. D. Wise, M. C. Boyer,
K. Chatterjee, T. Kondo,
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Nature Phys. **4**, 696
(2008).



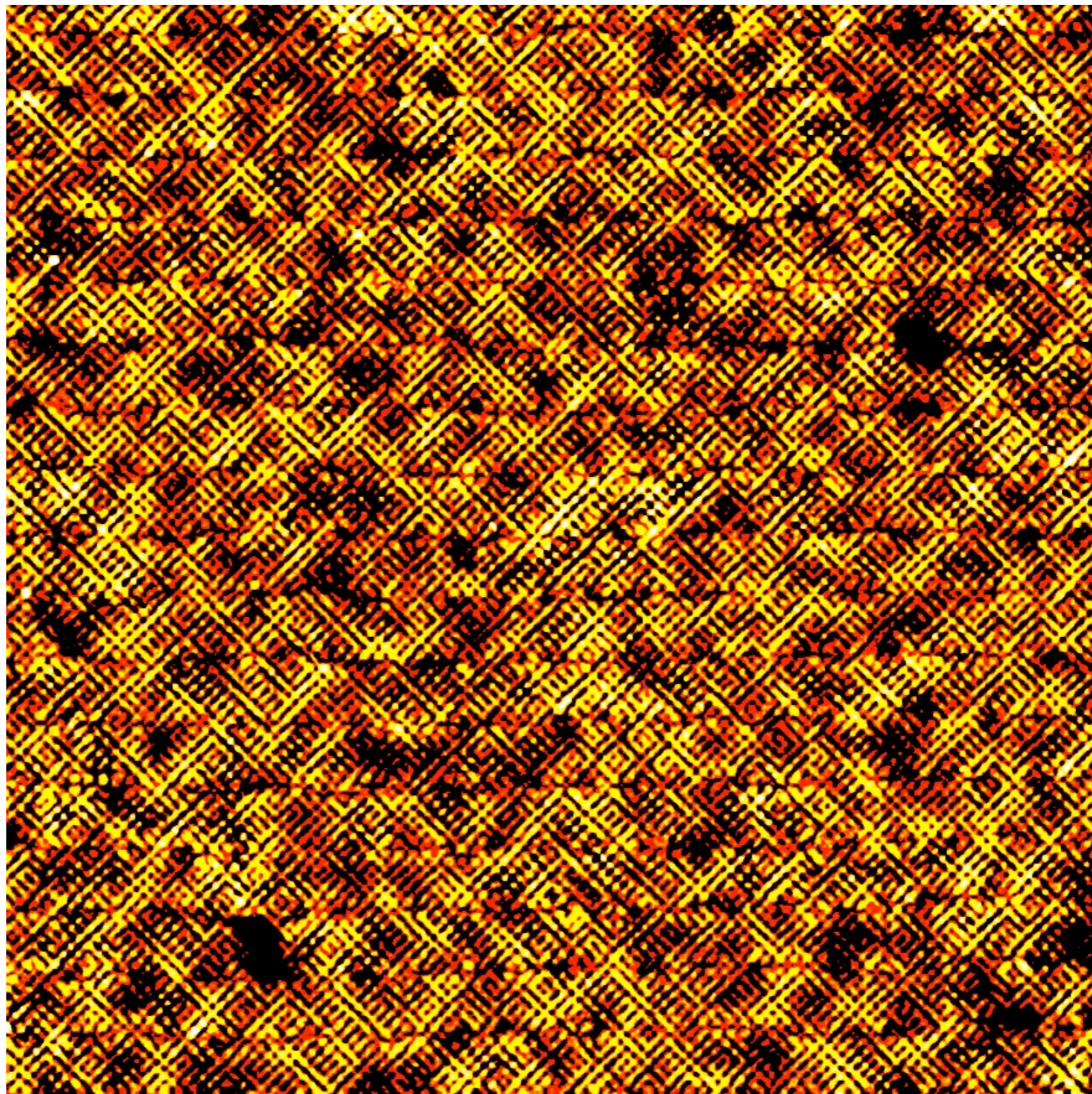
“R-map” of BSCCO in zero magnetic field, similar to those published in
Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri,
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Science **315**, 1380 (2007).

See also

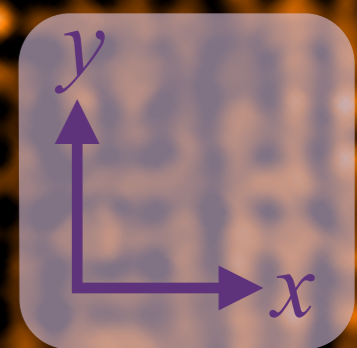
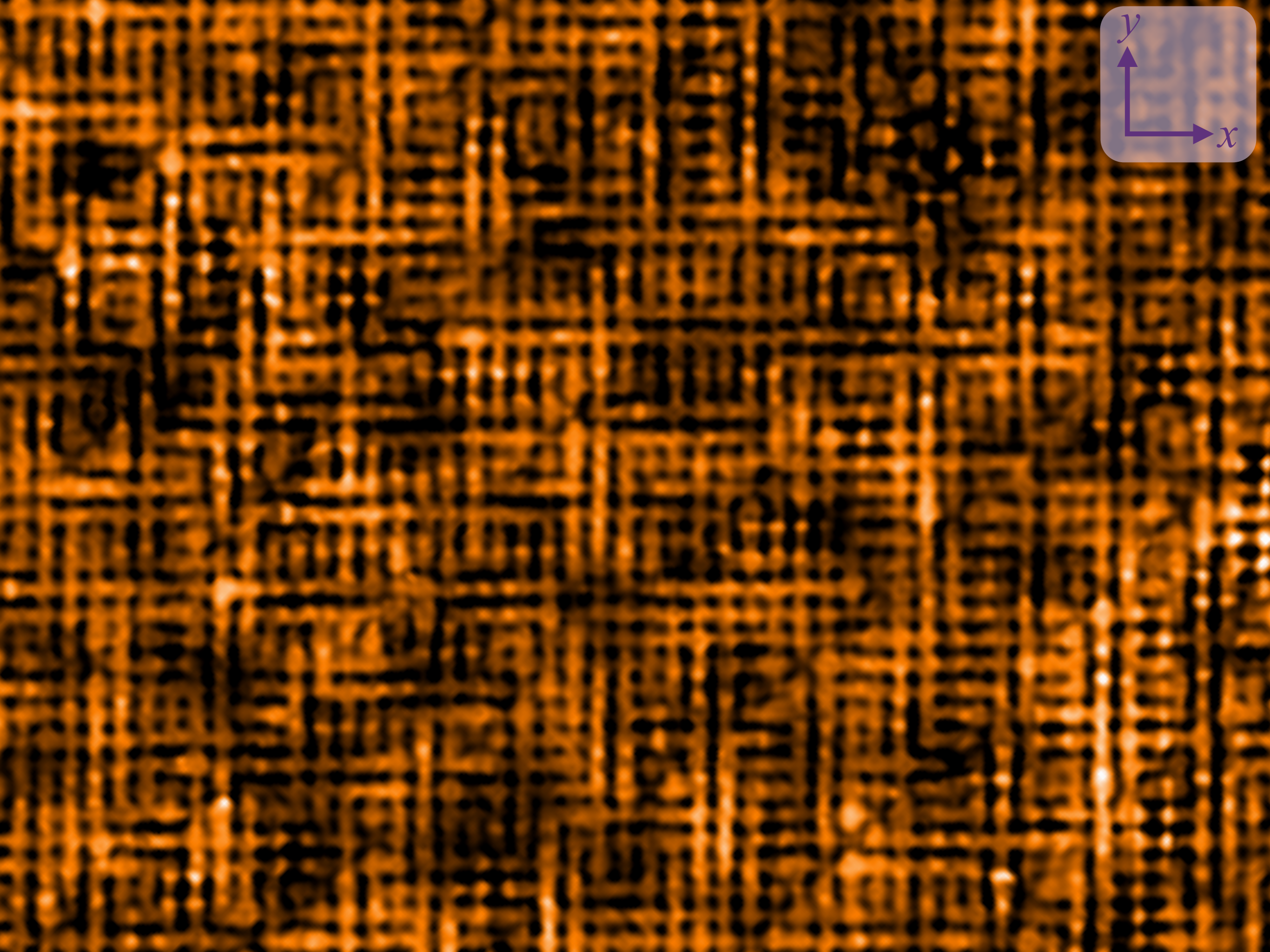
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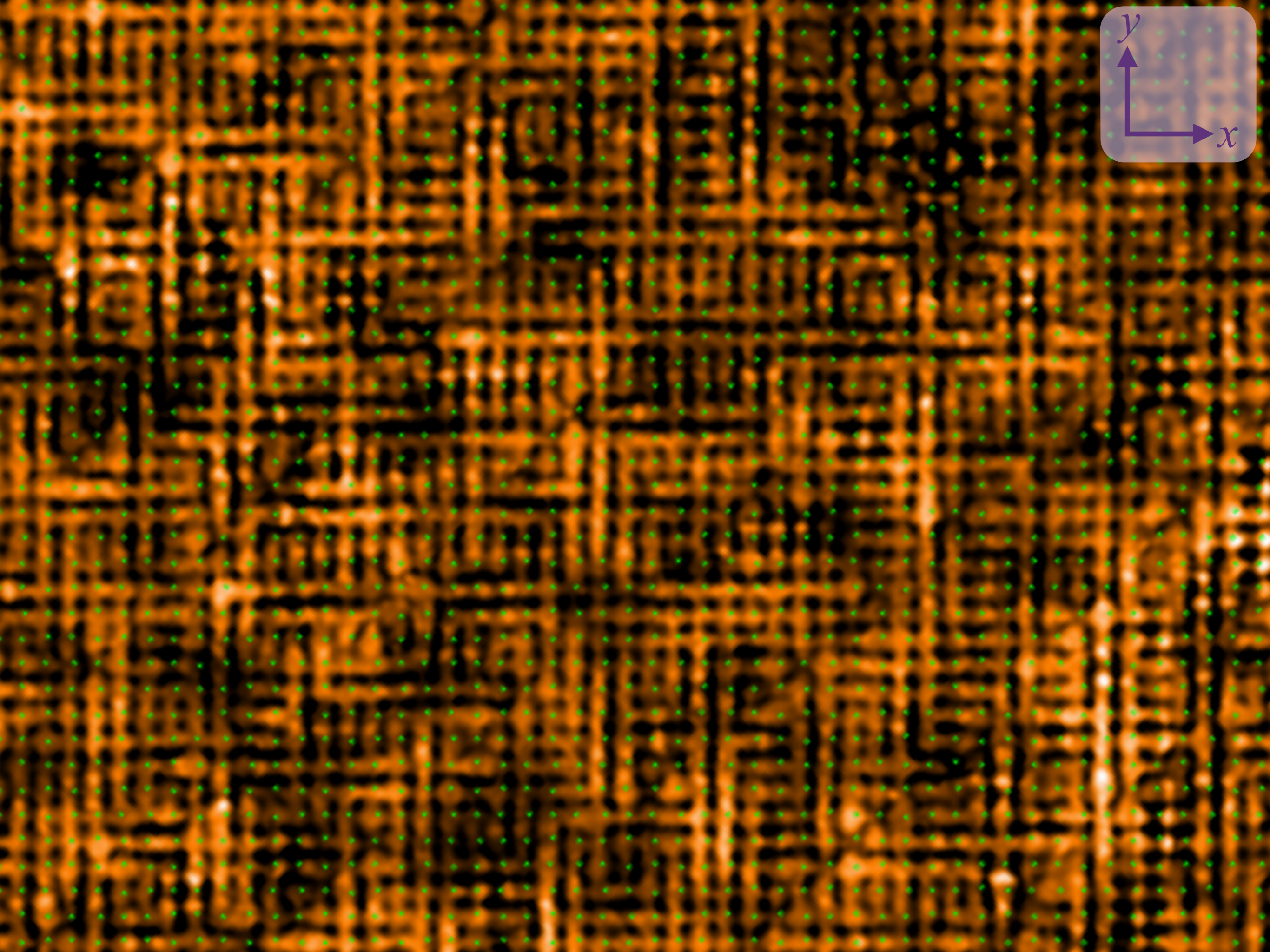
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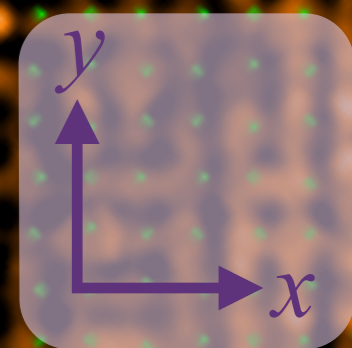


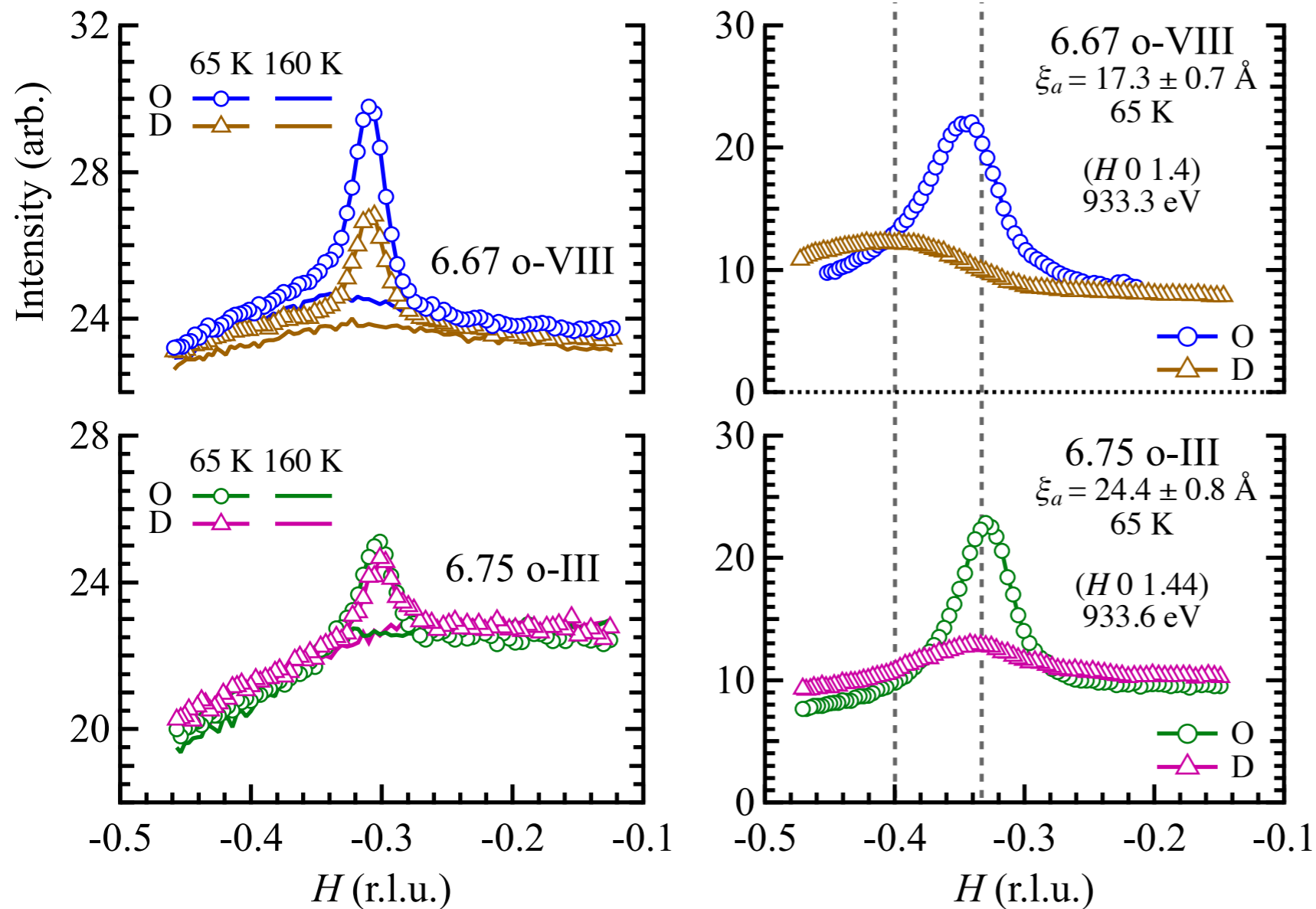
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A density wave with
wavelength ≈ 4 lattice sites ?





A. J. Achkar, X. Mao, Christopher McMahon, R. Sutarto, F. He, Ruixing Liang, D. A. Bonn, W. N. Hardy, and D. G. Hawthorn, arXiv:1312.6630

Many groups have observed “charge” density peaks in X-ray scattering over the past 2 years.

The X-ray wavevector is found to agree with the wavevector obtained from STM.

R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Yang He, M. Le Tacon, I.S. Elfimov, J. E. Hoffman, G.A. Sawatzky, B. Keimer, A. Damascelli, Science **343**, 390 (2014).

Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

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CDW wavevector \mathbf{Q}

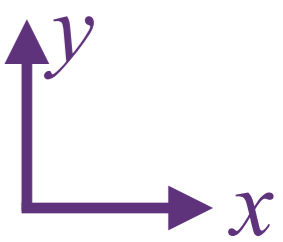
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Nearly constant CDW order parameter

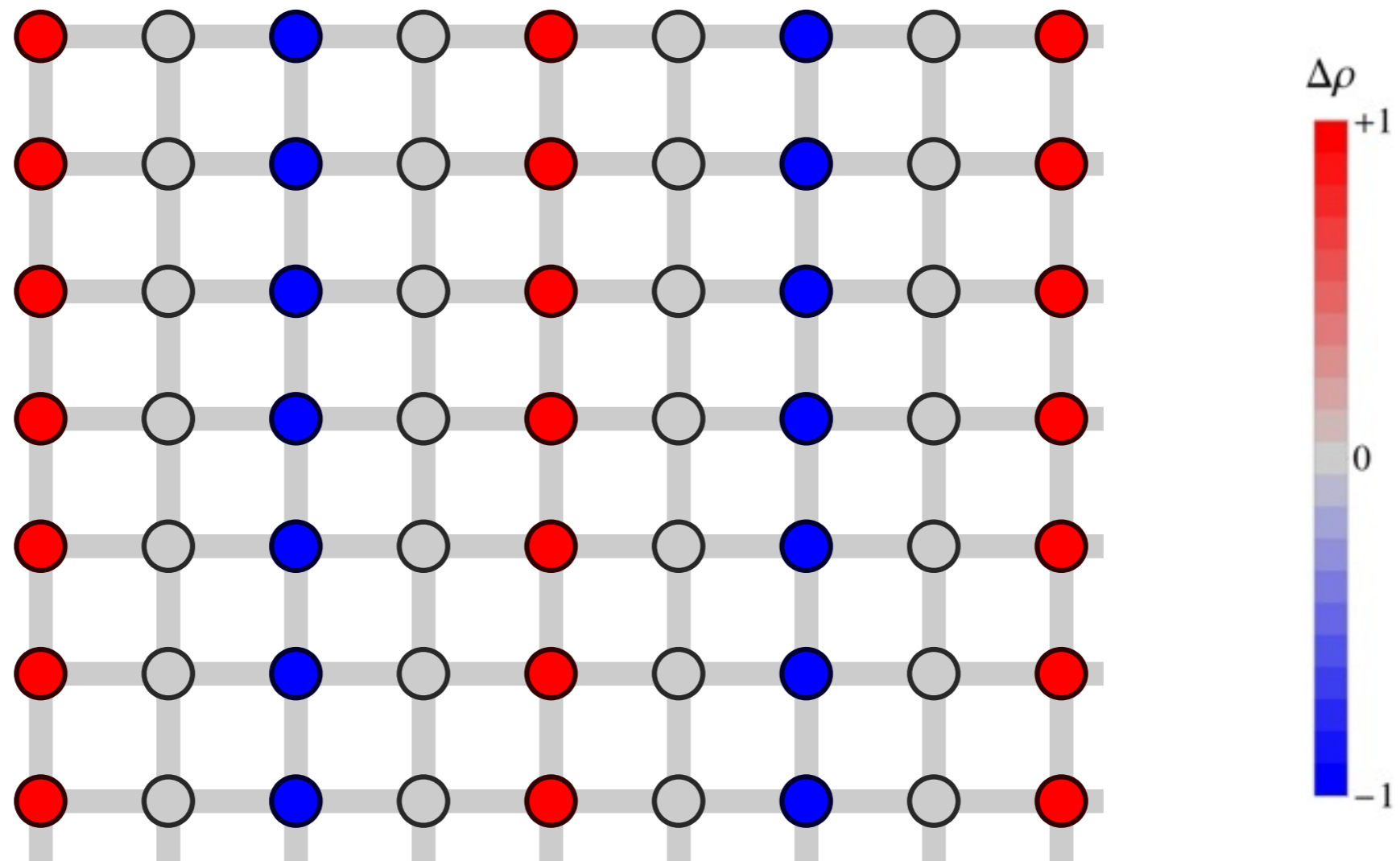
CDW order.



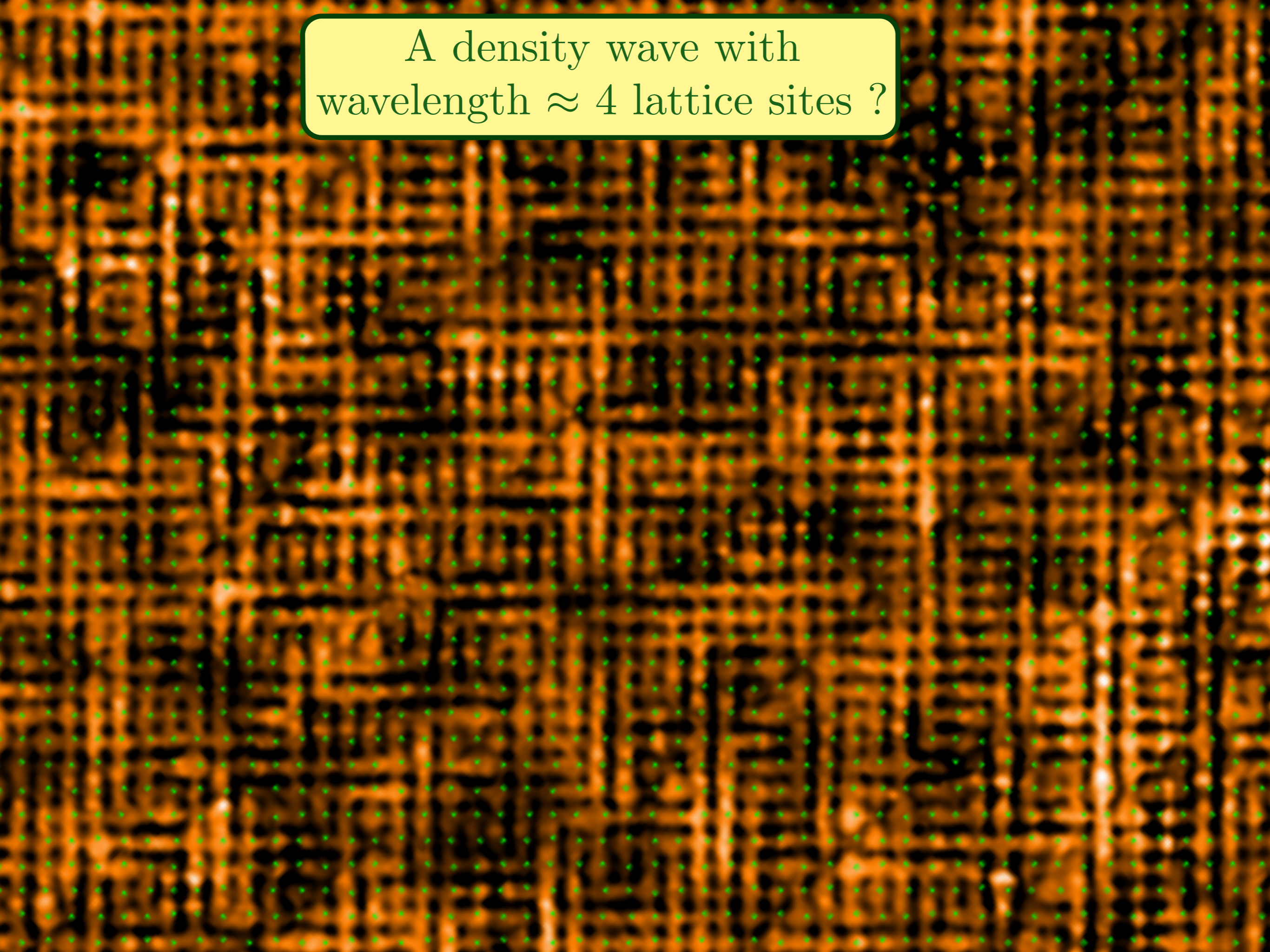
Plot of $P_{ii} = \langle c_{i\alpha}^\dagger c_{i\alpha} \rangle$ with

$$P_{ii} = e^{i\mathbf{Q}\cdot\mathbf{r}_i} + \text{c.c.}$$

with $\mathbf{Q} = 2\pi(1/4, 0)$



A density wave with
wavelength ≈ 4 lattice sites ?



Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

Unconventional density wave (DW) :
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

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DW wavevector \mathbf{Q}

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Nearly constant DW order parameter

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Internal particle-hole pair wavefunction

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand (using reflection symmetry for \mathbf{Q} along axes or diagonals)

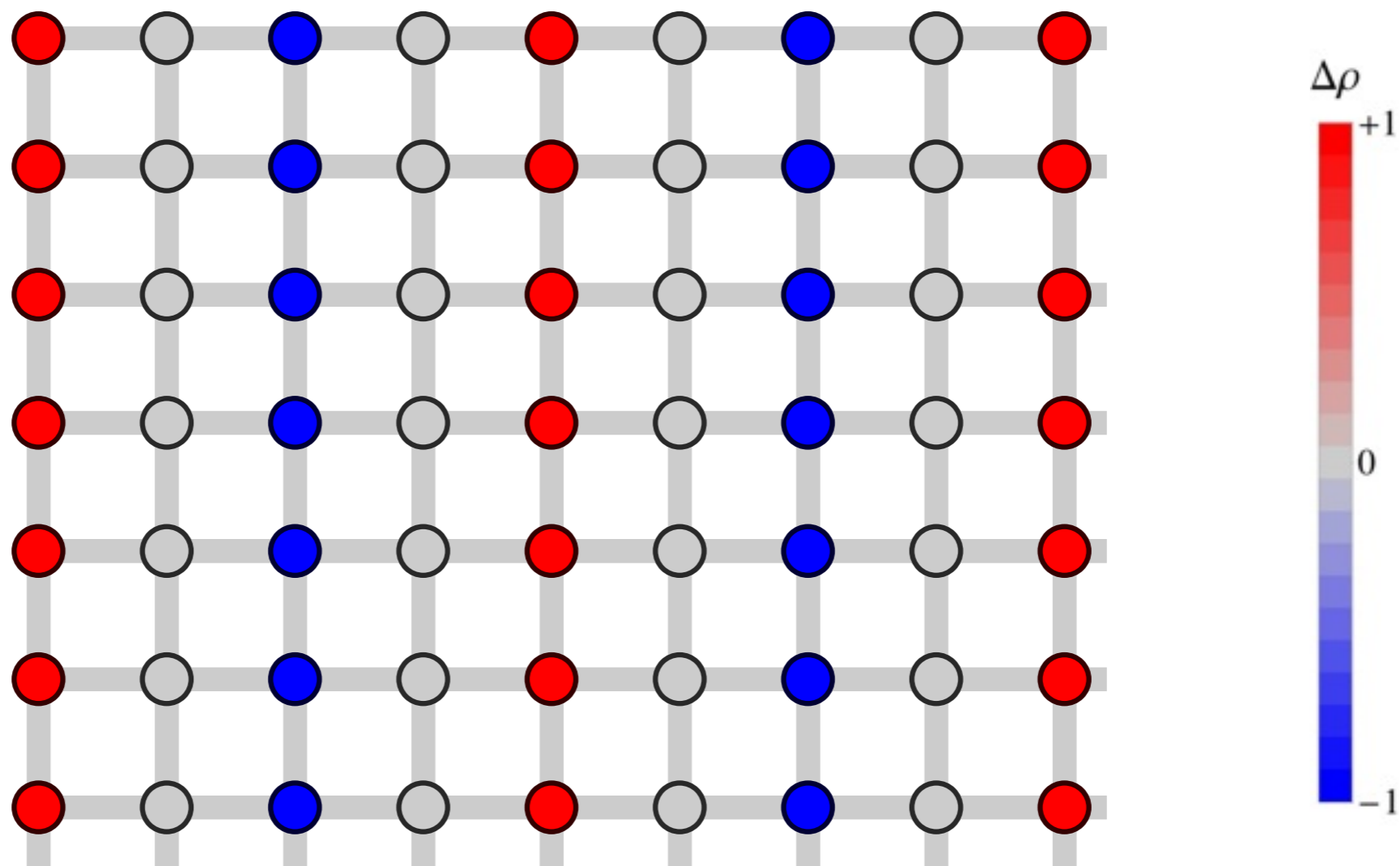
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

Conventional CDW order: *s*-wave

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

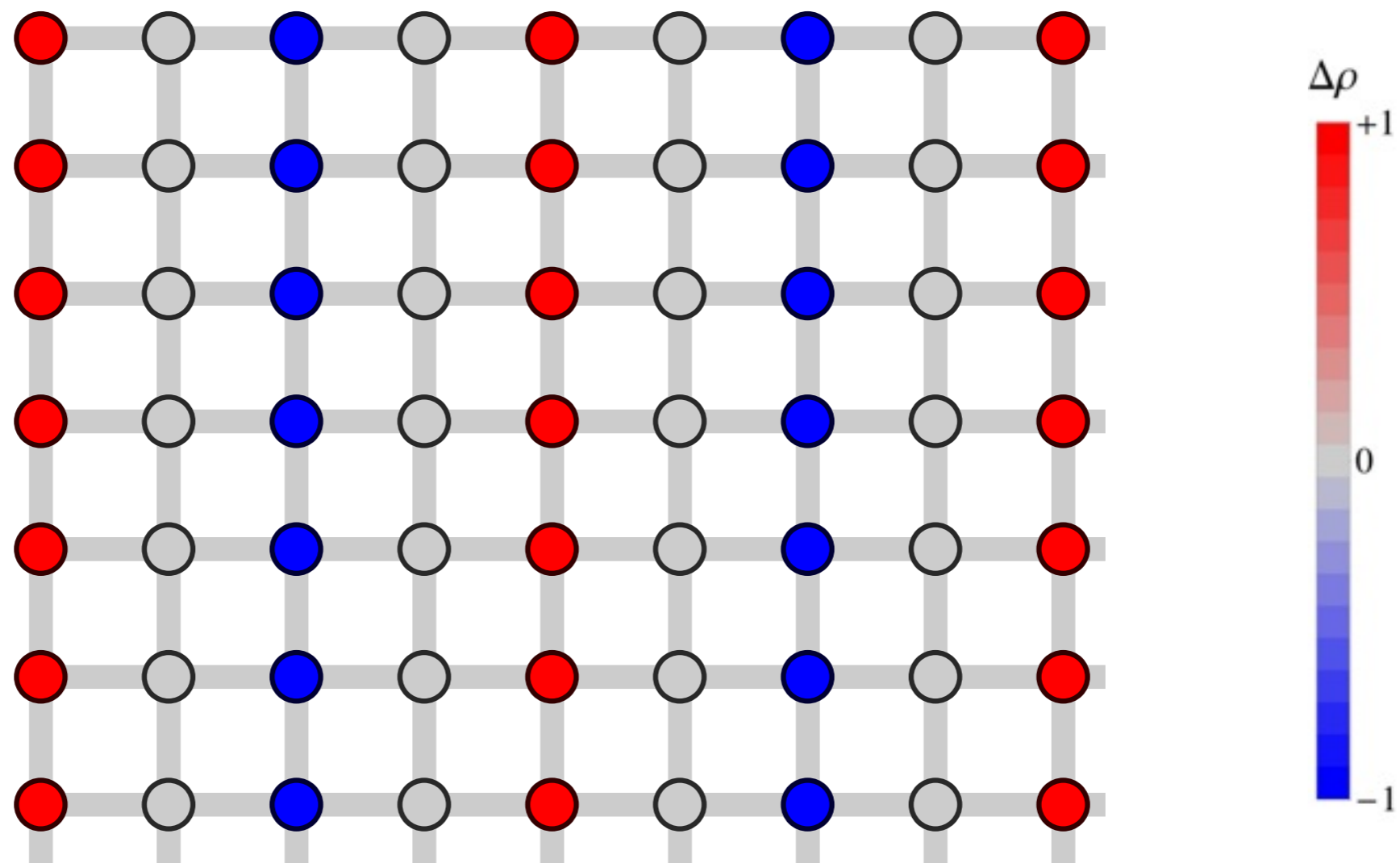
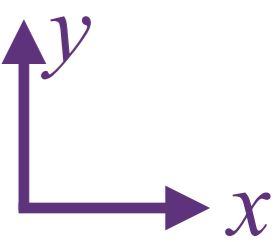


Conventional CDW order: *s*-wave

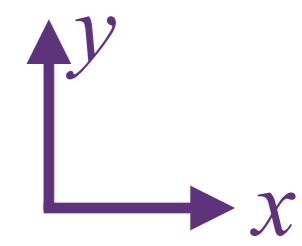
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$$\text{with } \mathbf{Q} = 2\pi(1/4, 0)$$



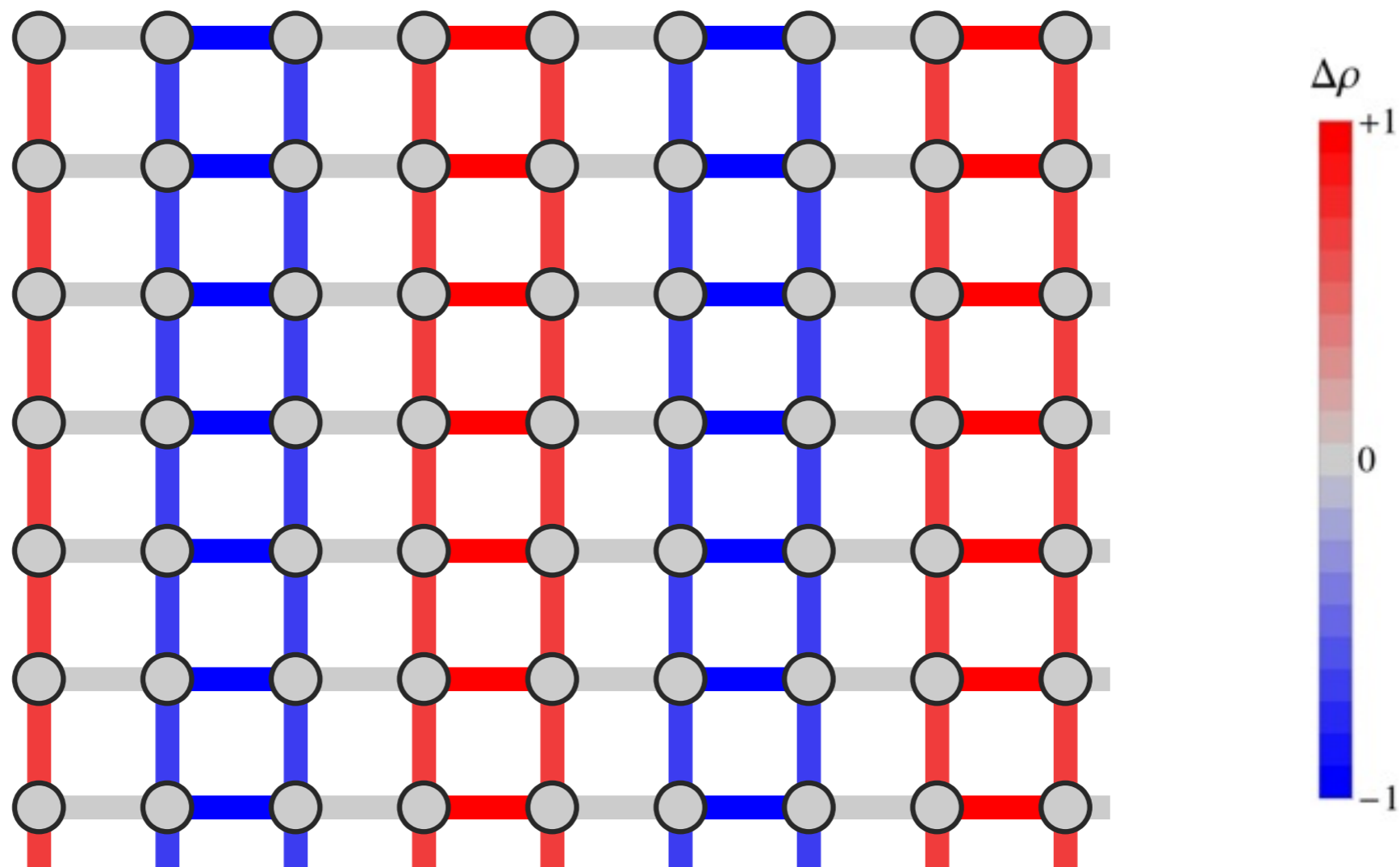
Unconventional DW order: s' -wave



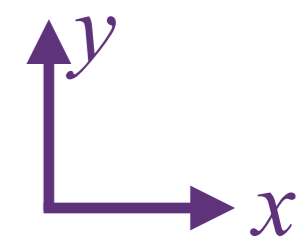
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$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



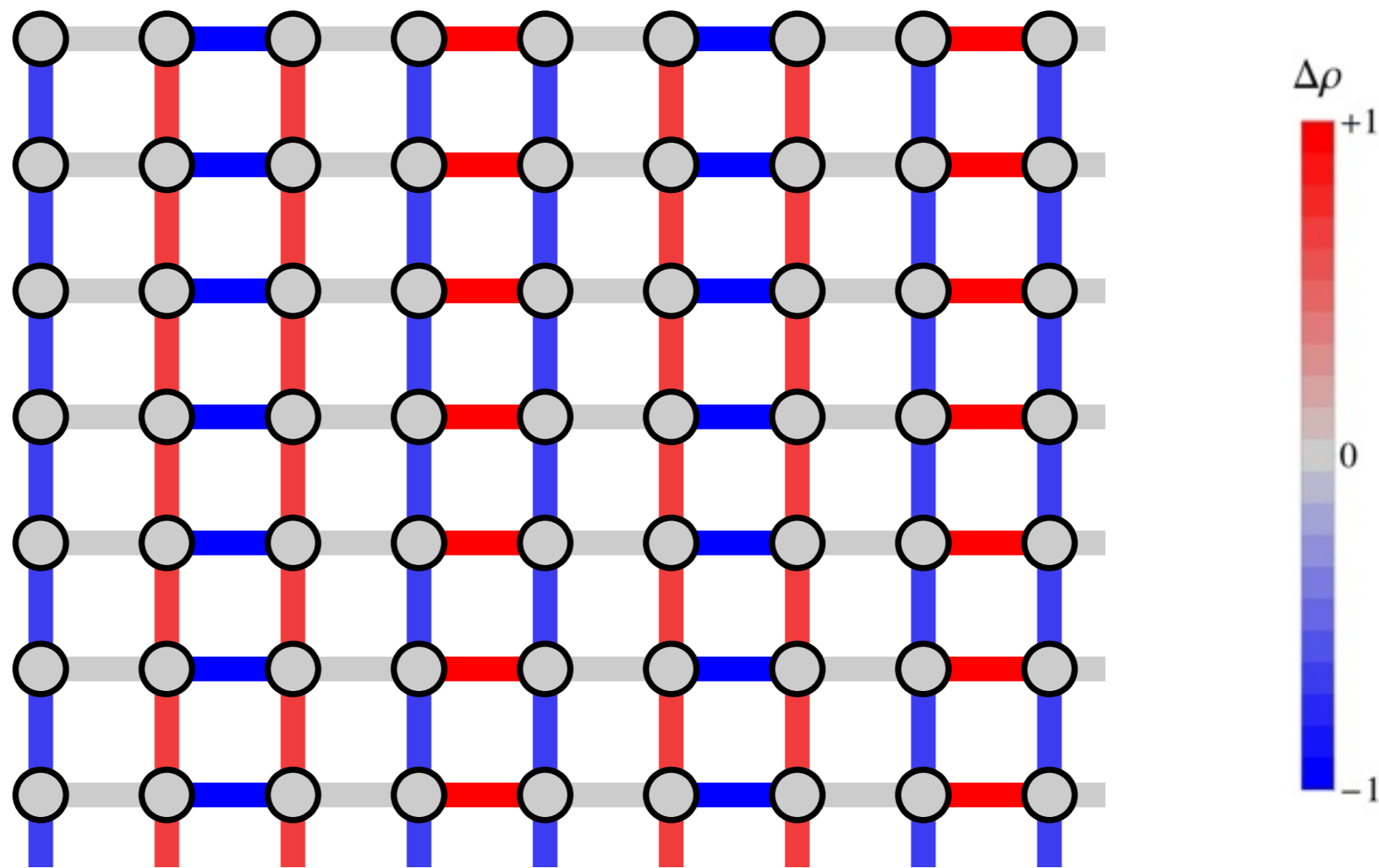
Unconventional DW order: d -wave



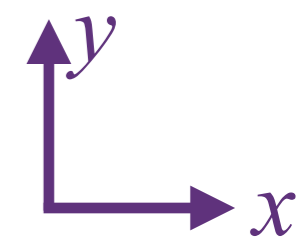
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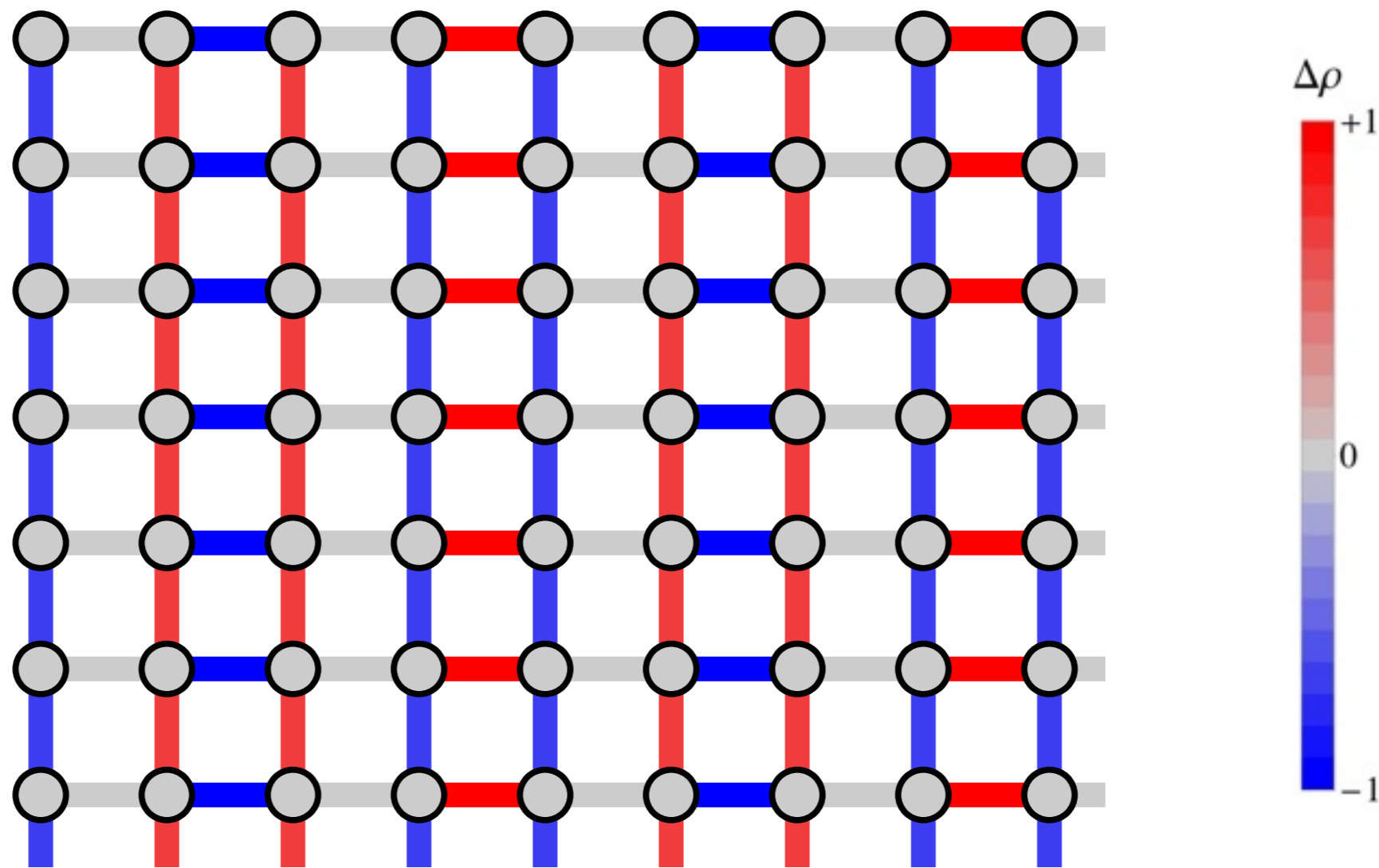
Unconventional DW order: d -wave



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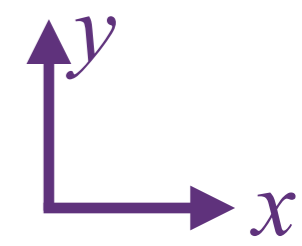
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$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



A DW with d -wave predominant is nearly degenerate with d -wave SC near the quantum-critical point for antiferromagnetism in a metal.

Unconventional DW order: d -wave

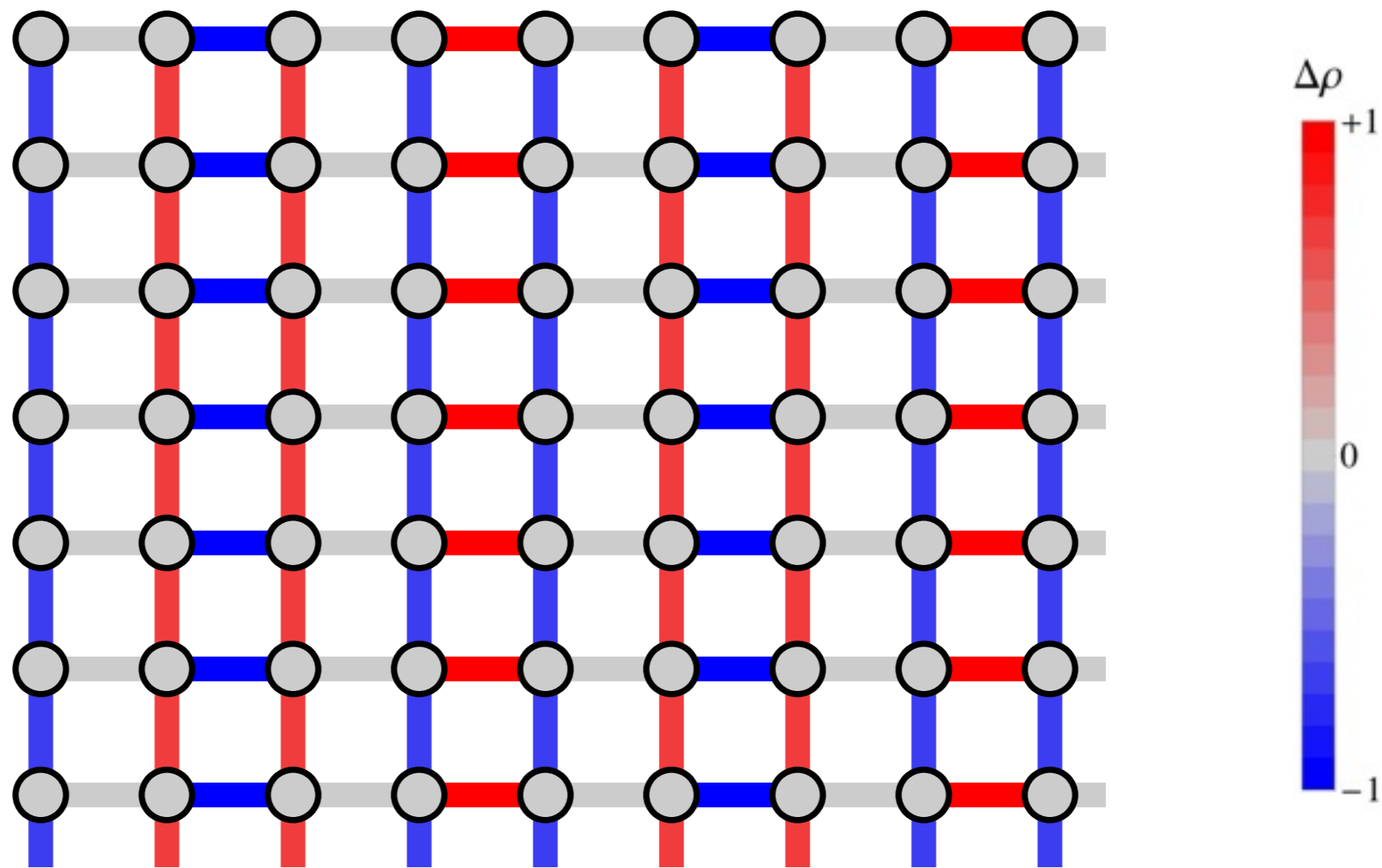


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

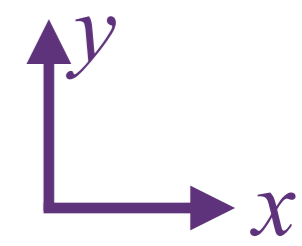
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific d -wave bond order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional CDW order: *d*-wave

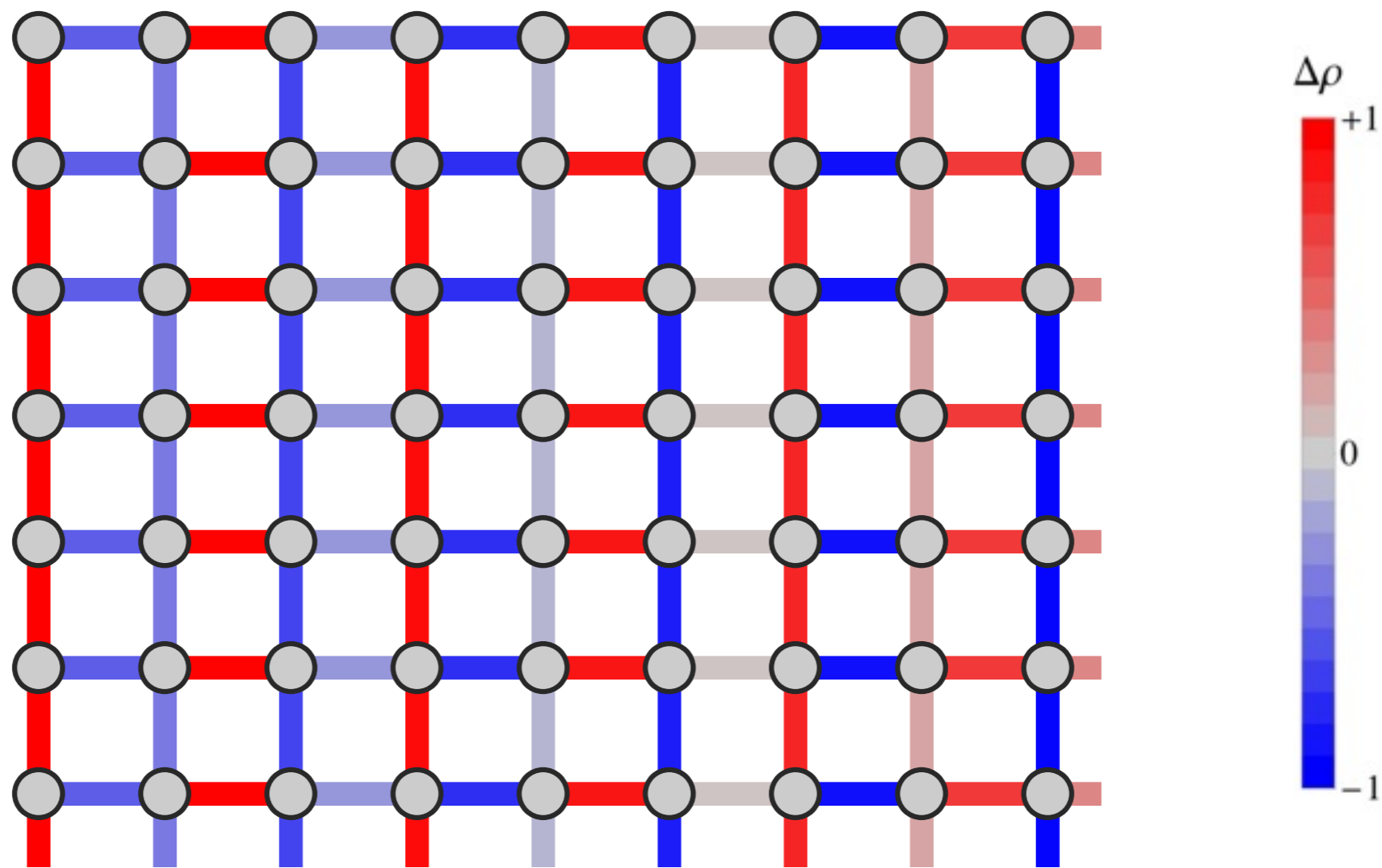


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

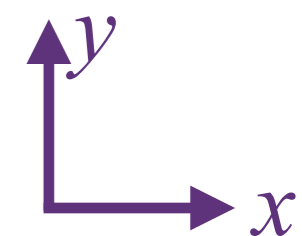
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



This specific *d*-wave bond order (with \mathbf{Q} along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

Unconventional DW order: $d + s$ -wave

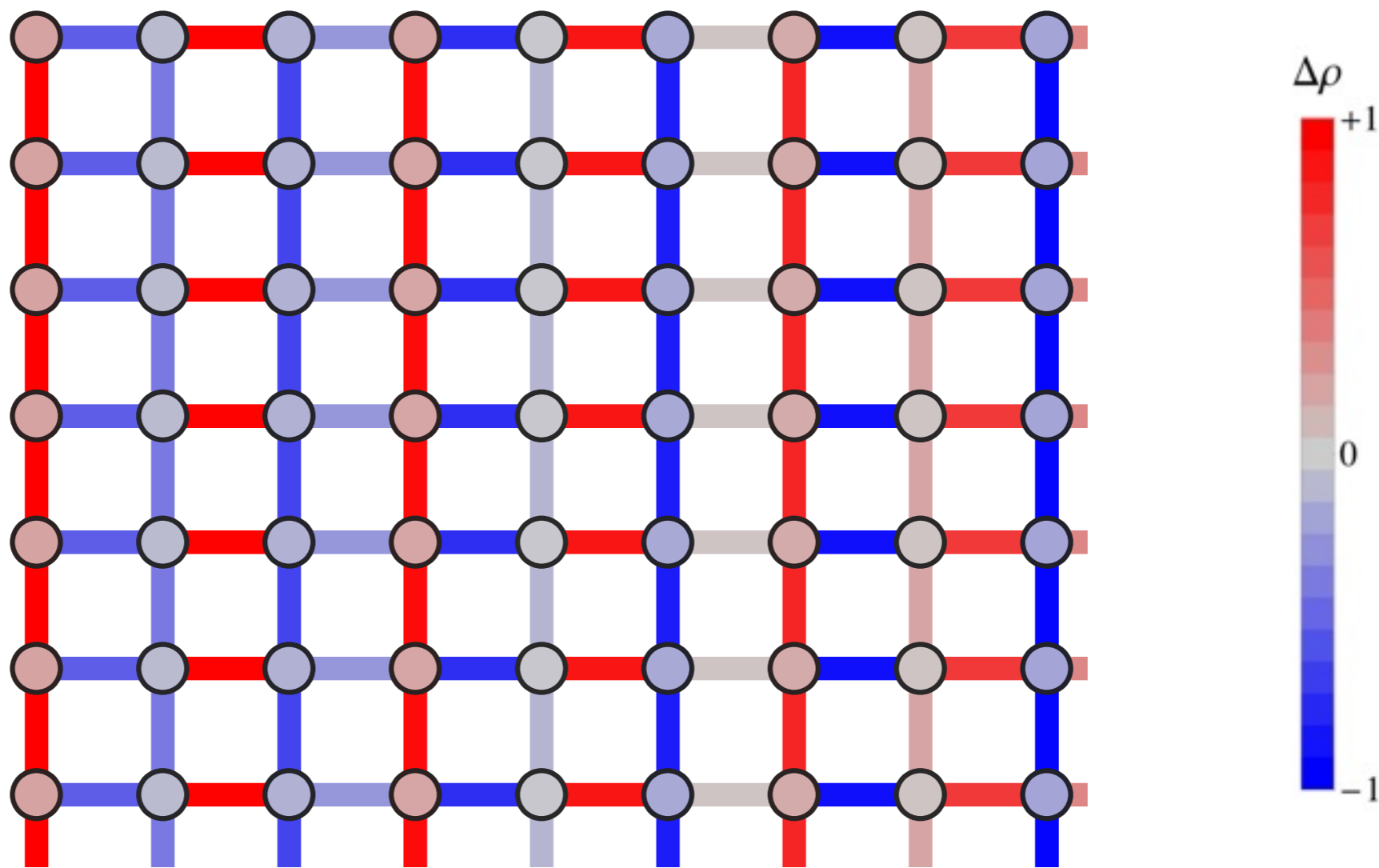


Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [0.2 + \cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(0.317, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



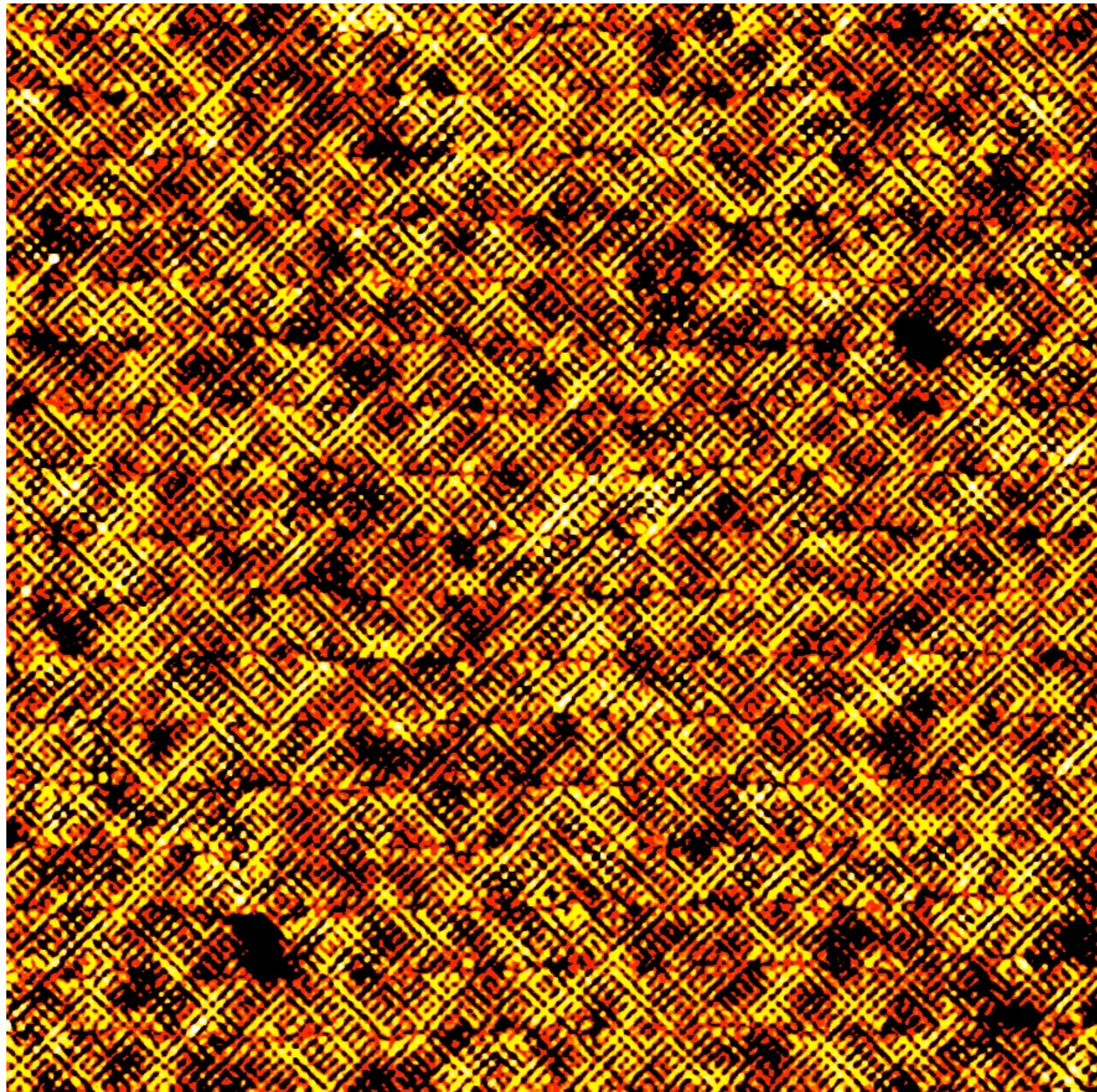
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See also

C. Howald, H. Eisaki,
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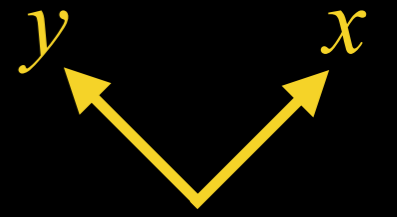
W. D. Wise, M. C. Boyer,
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Nature Phys. **4**, 696
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“R-map” of BSCCO in zero magnetic field, similar to those published in
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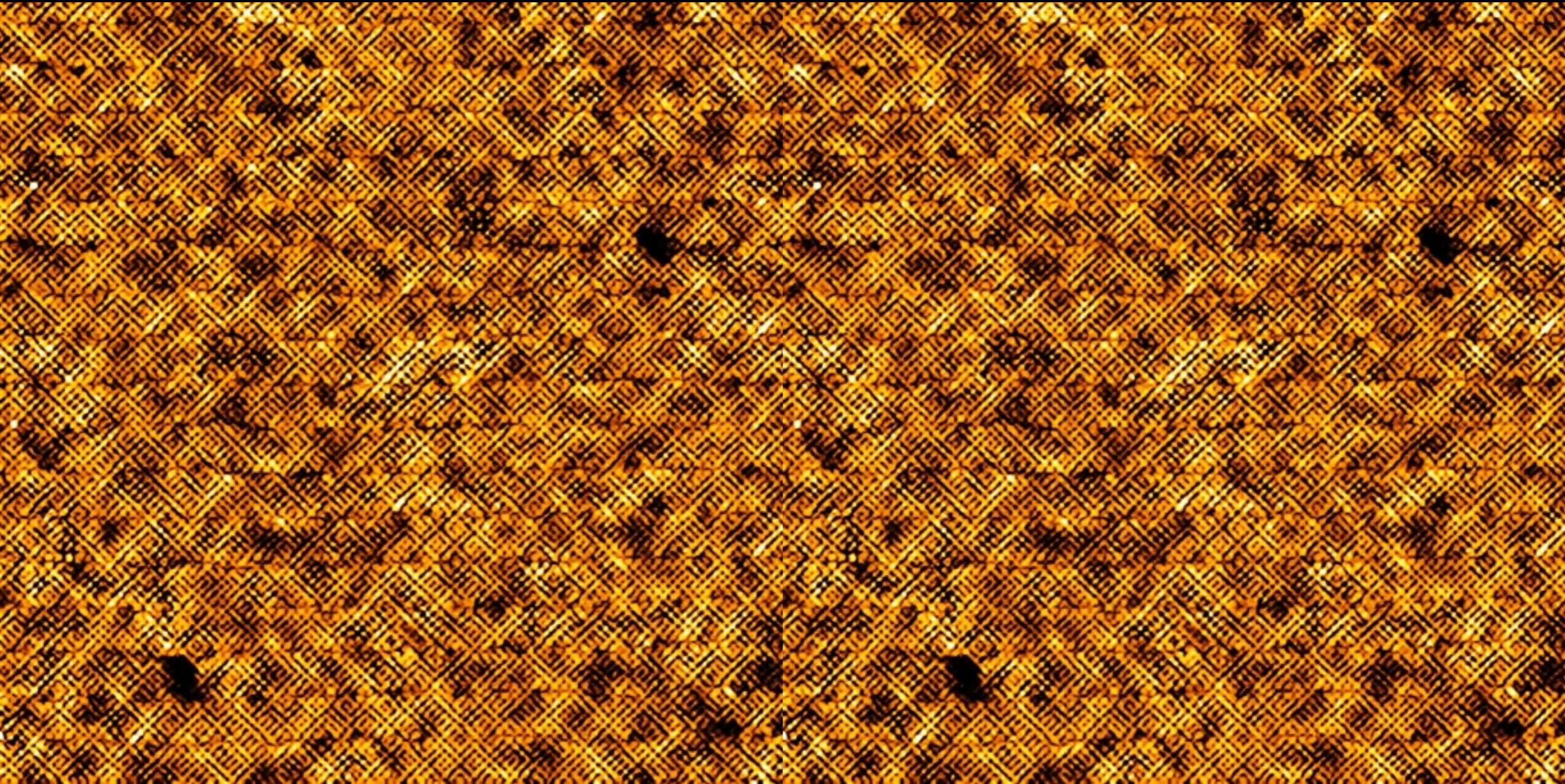
UD45K
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



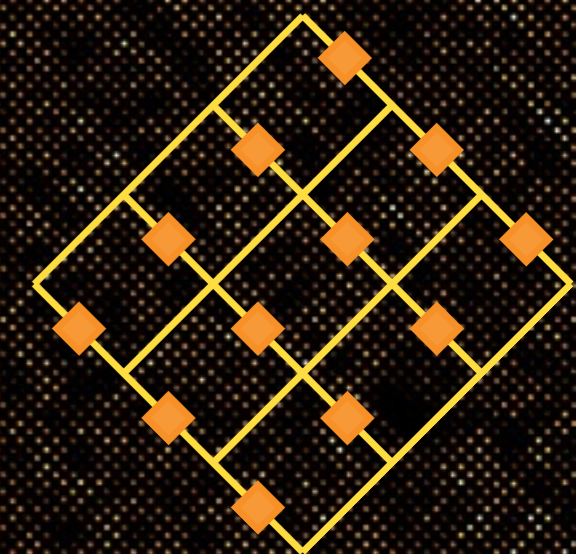
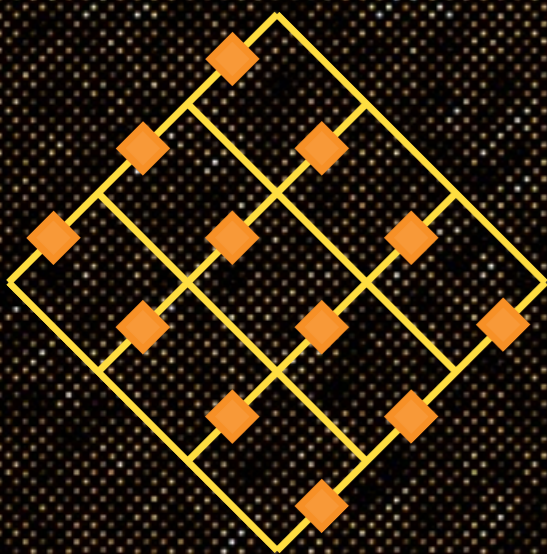
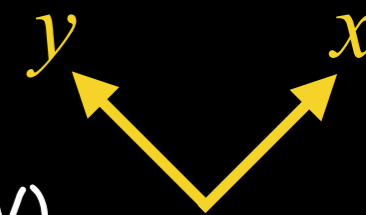
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

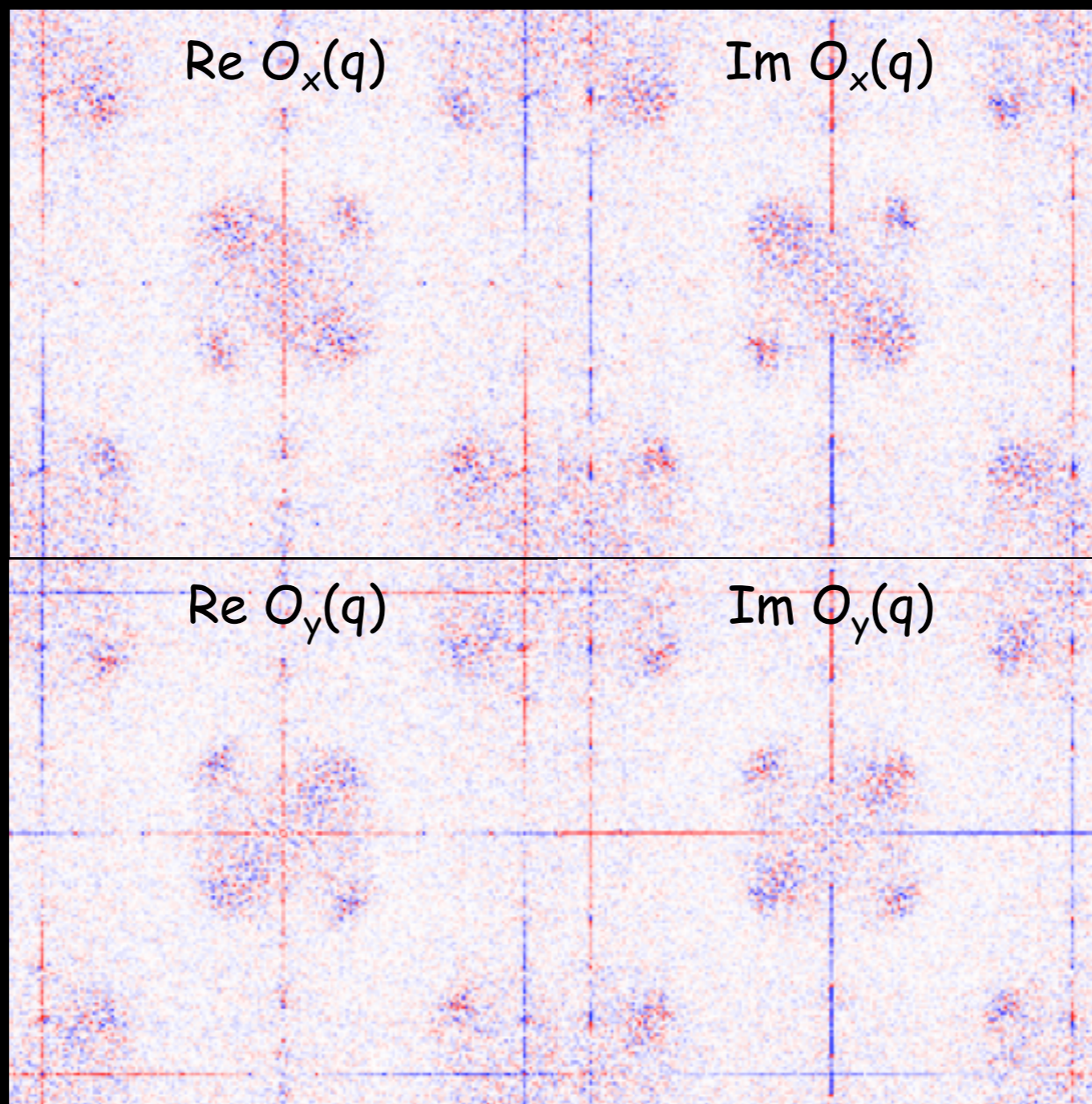
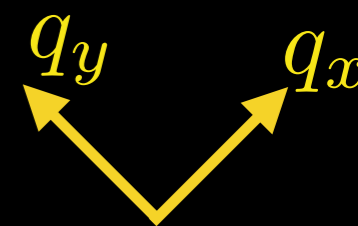
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

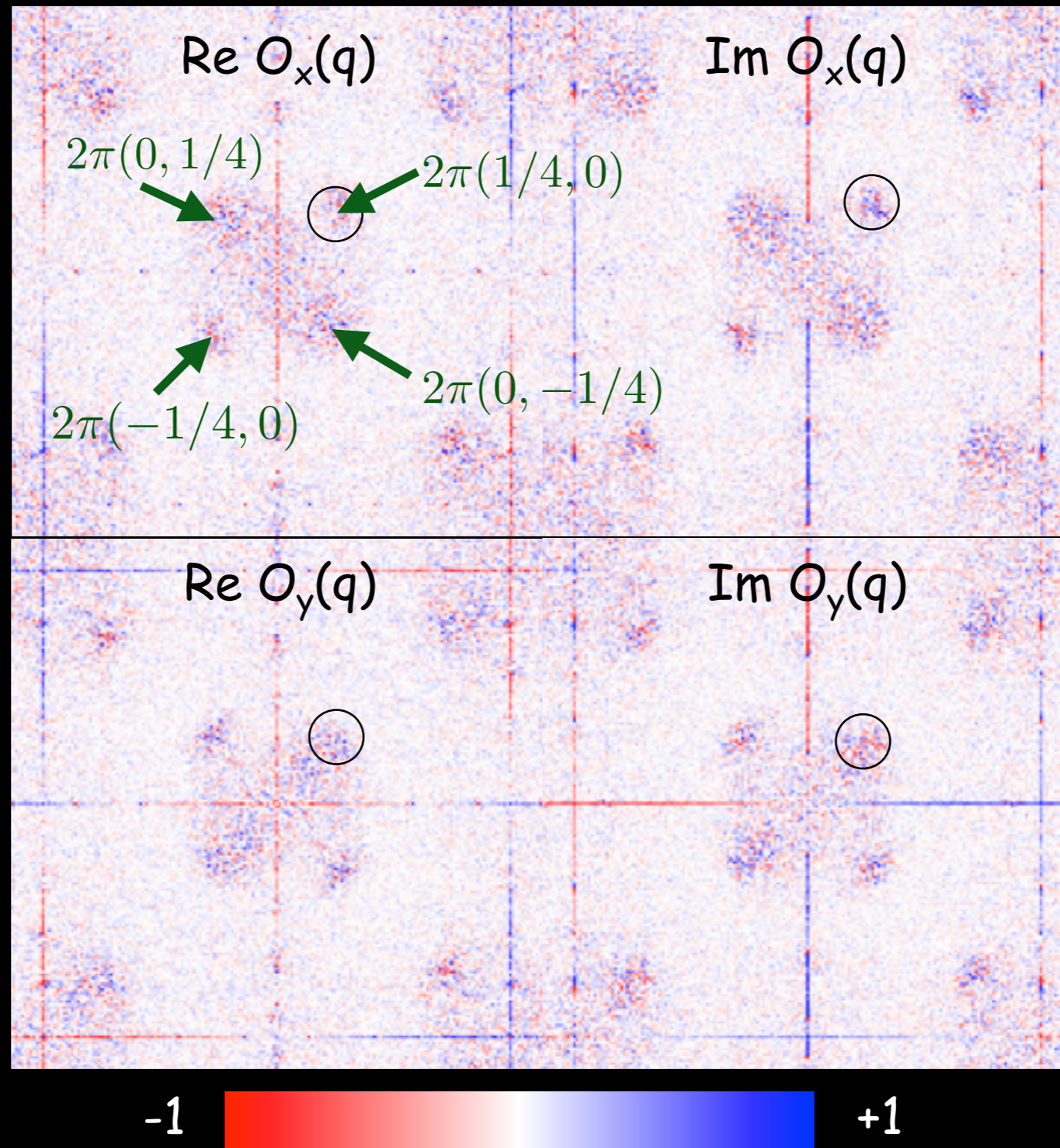
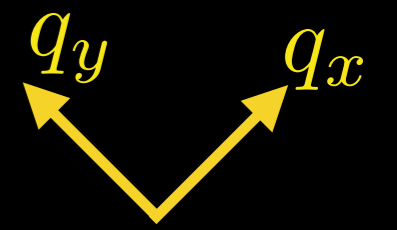


UD45K

Broad (0,Q) and (Q,0) DW Features

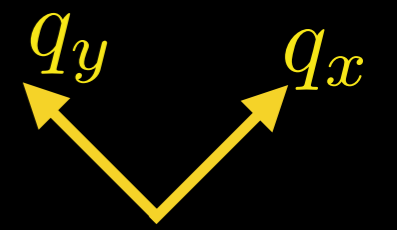


Broad (0,Q) and (Q,0) DW Features

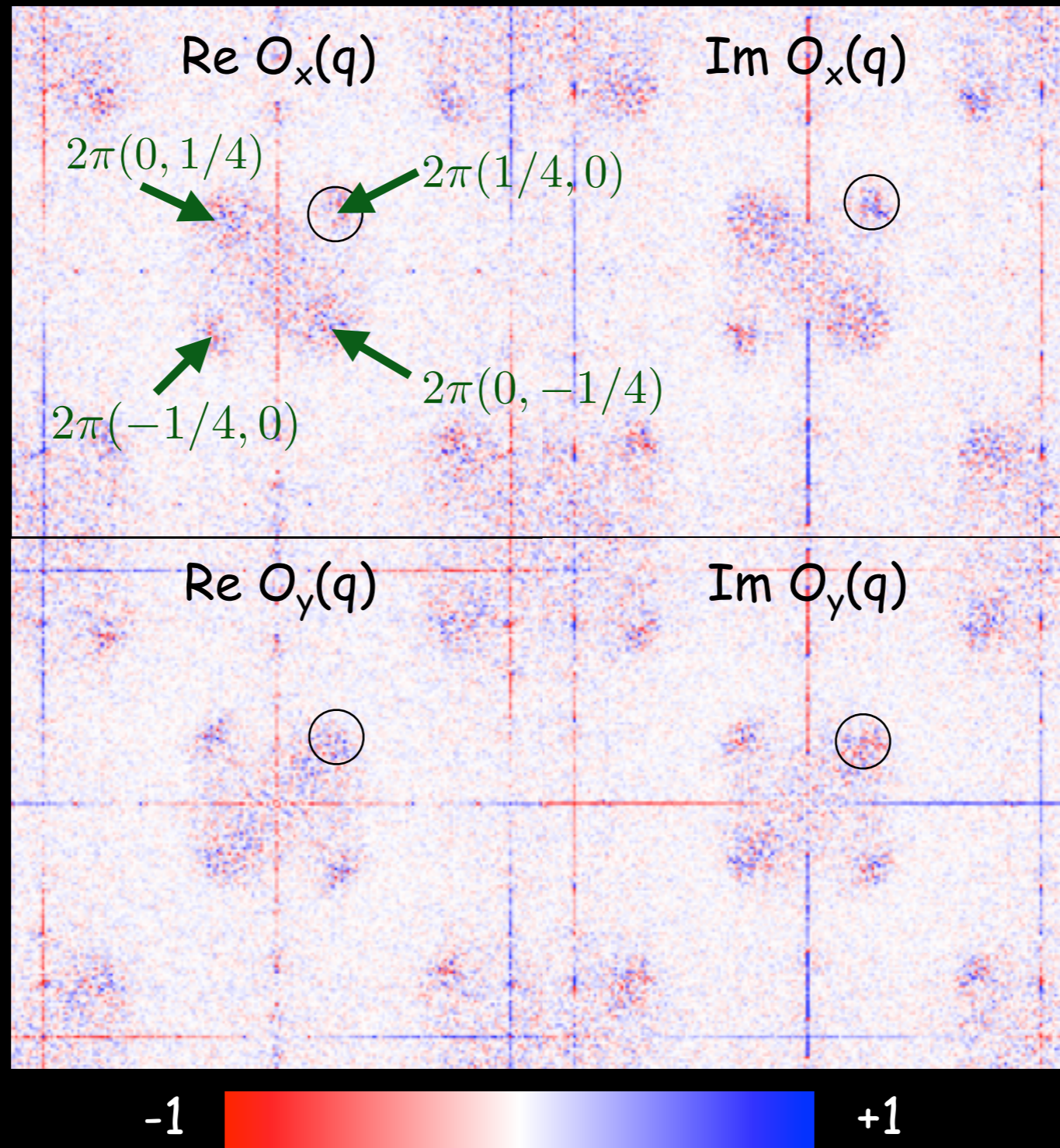


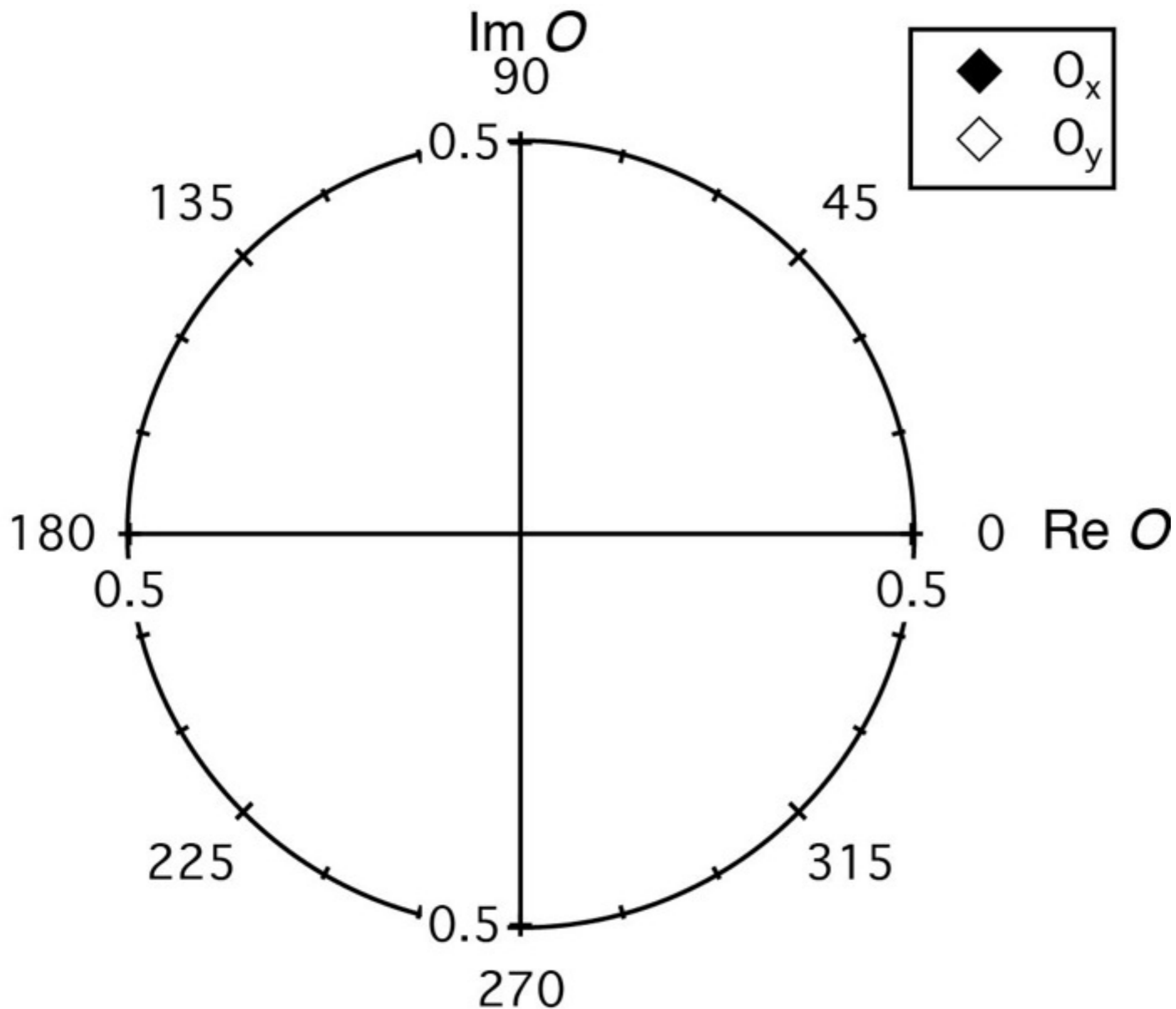
UD45K

Broad (0,Q) and (Q,0) DW Features

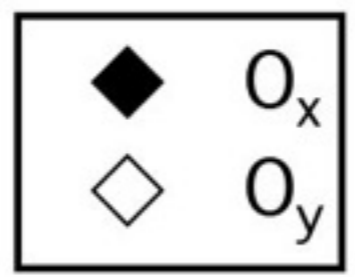
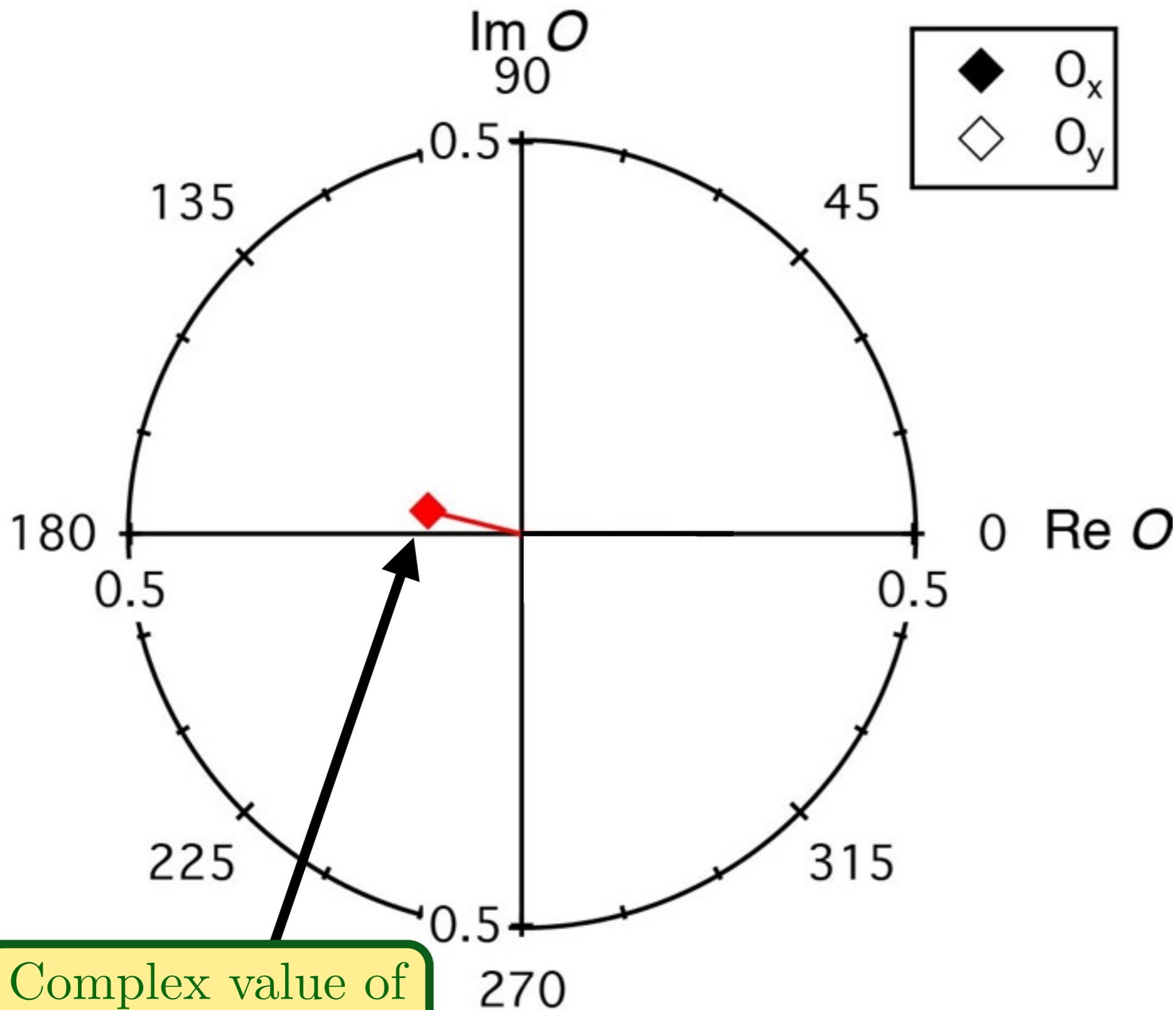


For each pixel in the circles, we obtain 2 complex numbers, $O_x(q)$ and $O_y(q)$.



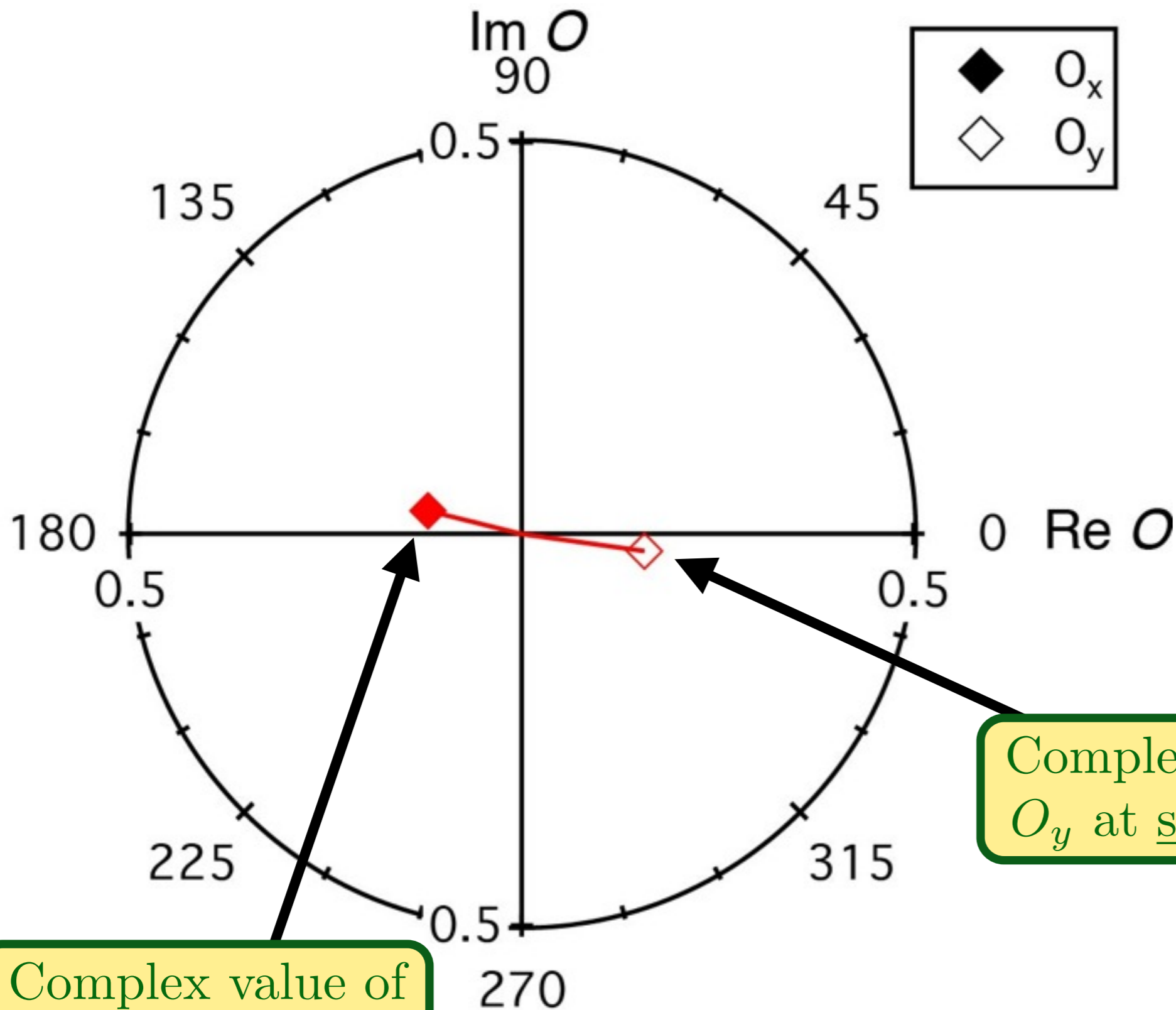


Phase-sensitive measurement of the d symmetry of density wave order



Phase-sensitive measurement of the *d* symmetry of density wave order

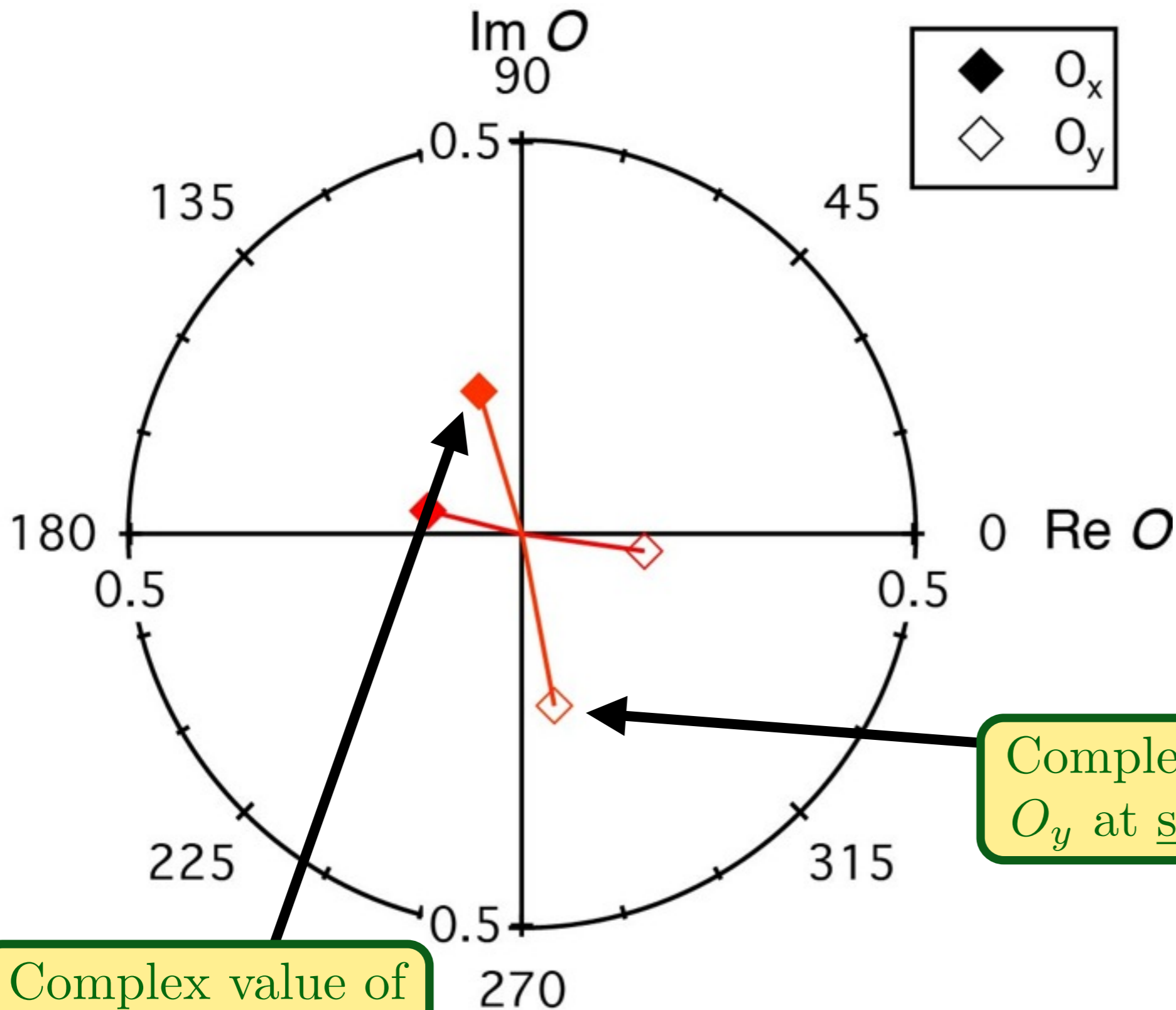
Complex value of O_x at a pixel



Phase-sensitive measurement of the *d* symmetry of density wave order

Complex value of O_x at a pixel

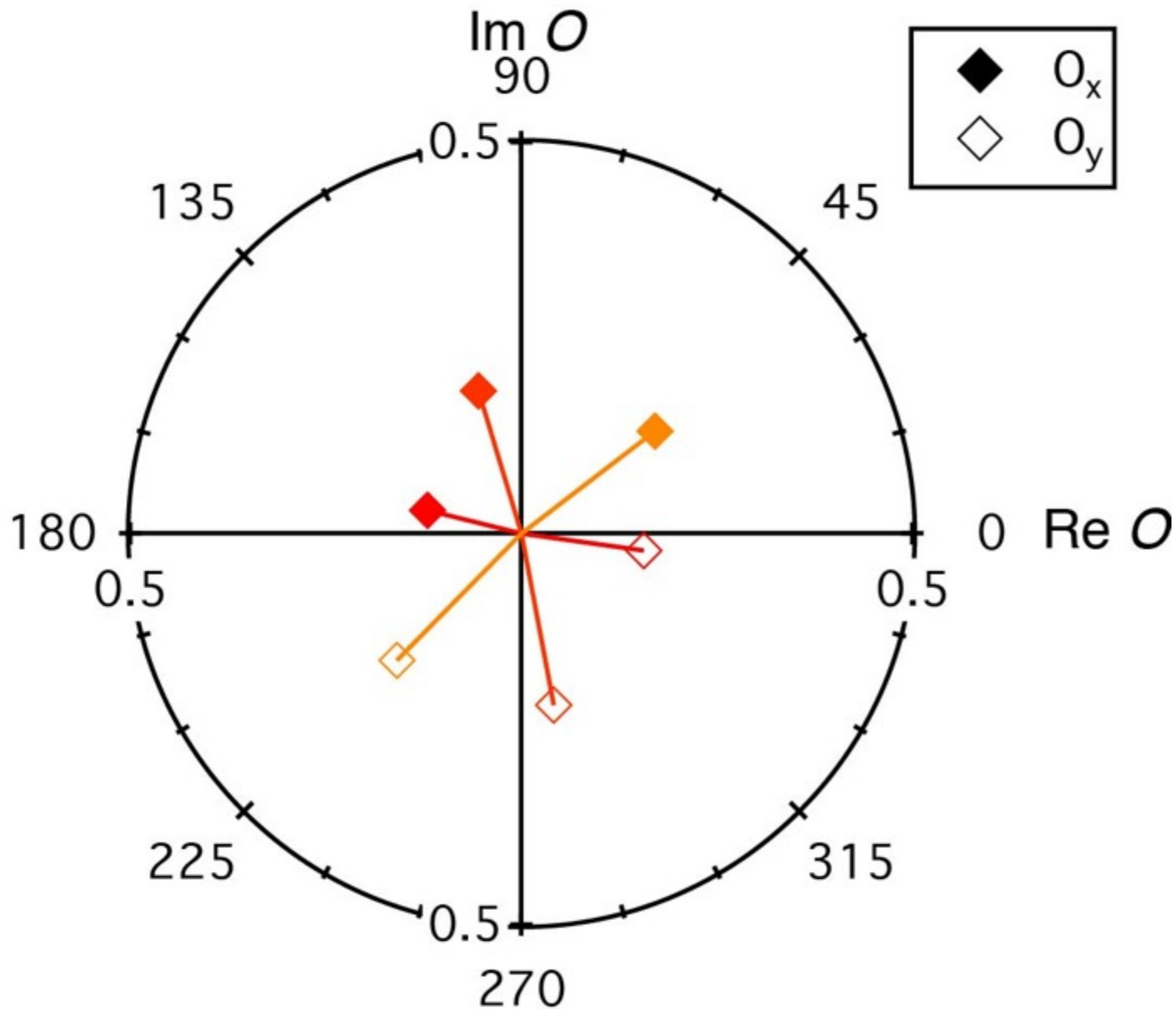
Complex value of O_y at same pixel



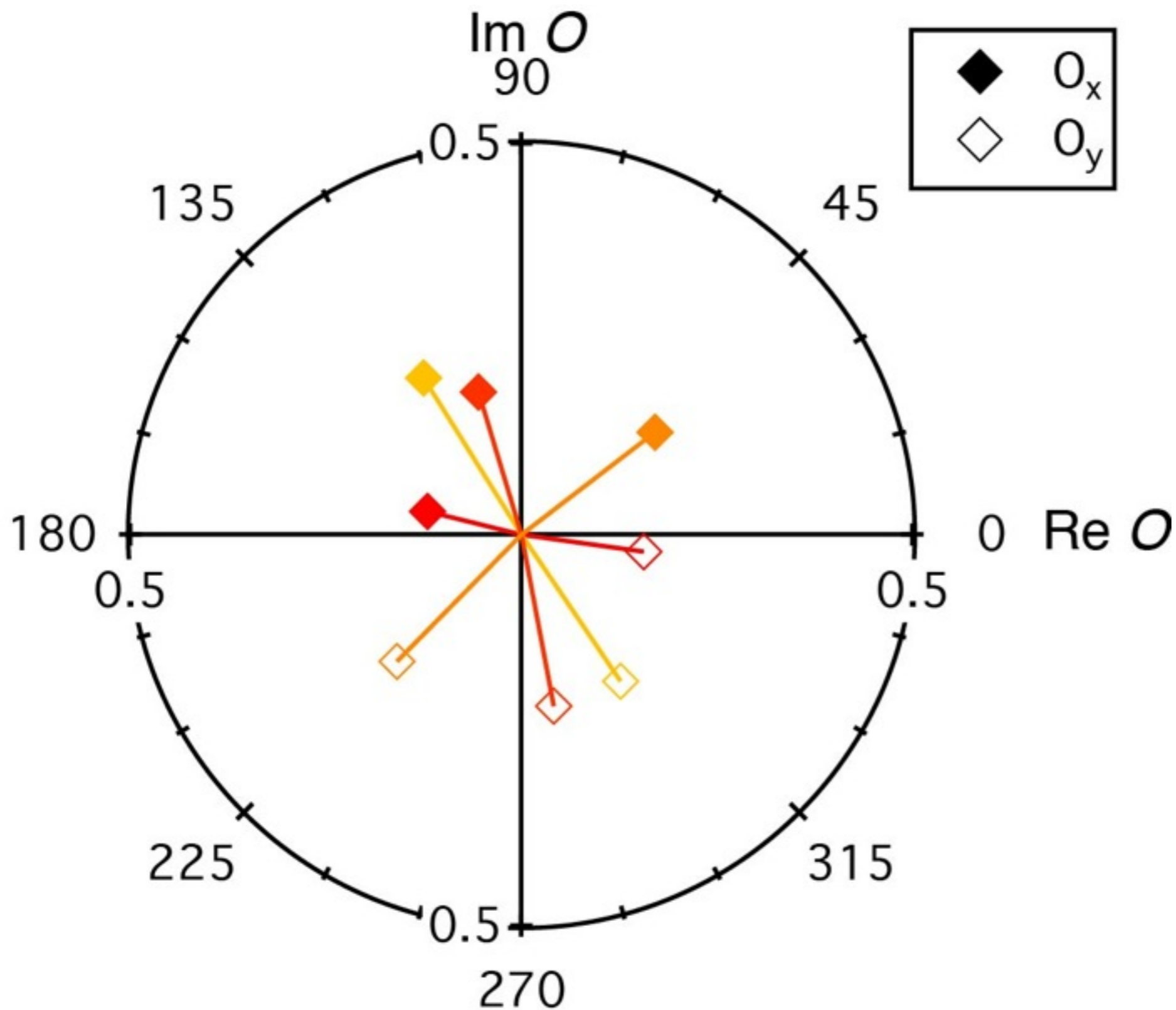
Phase-sensitive measurement of the *d* symmetry of density wave order

Complex value of O_x at a pixel

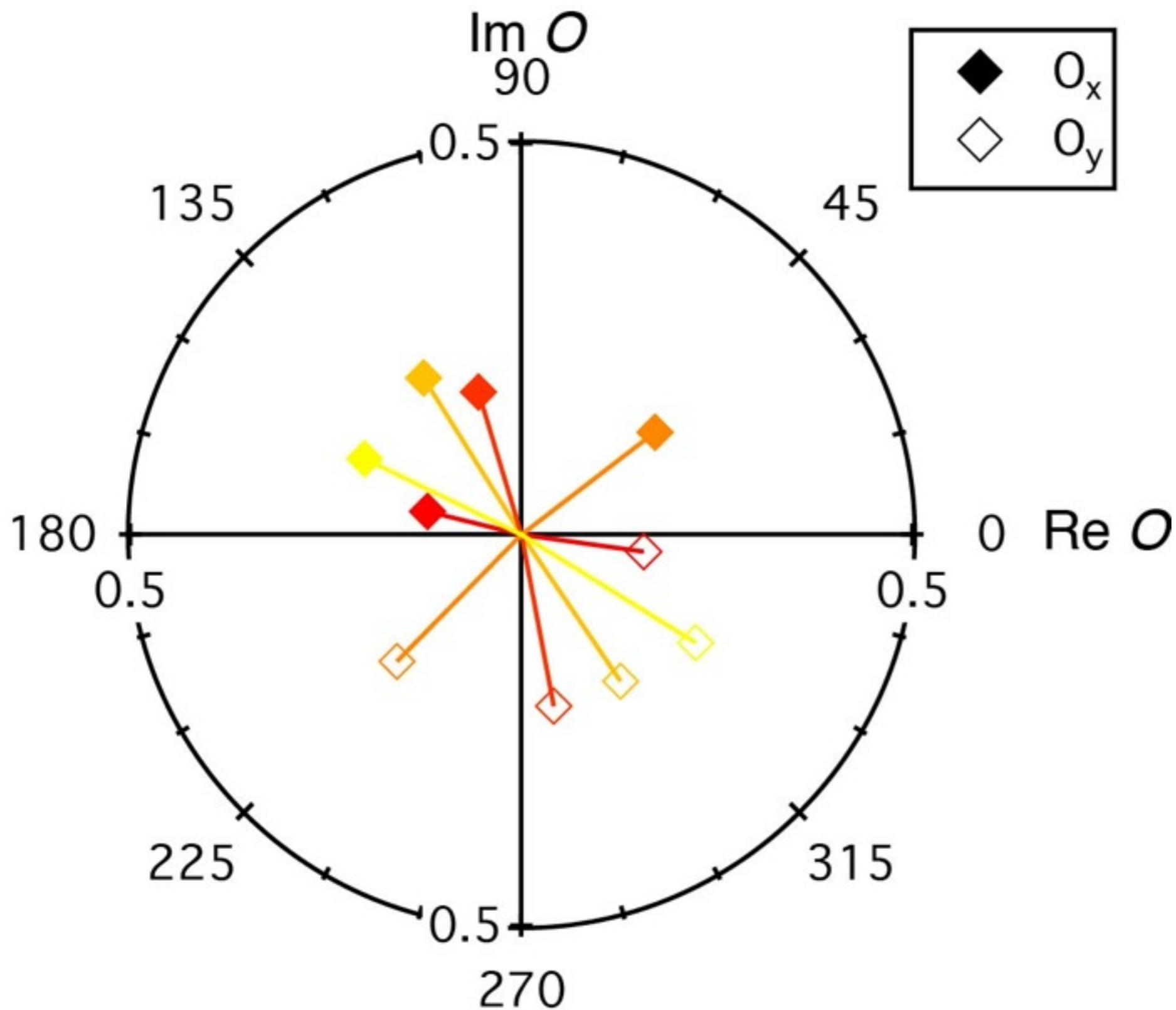
Complex value of O_y at same pixel



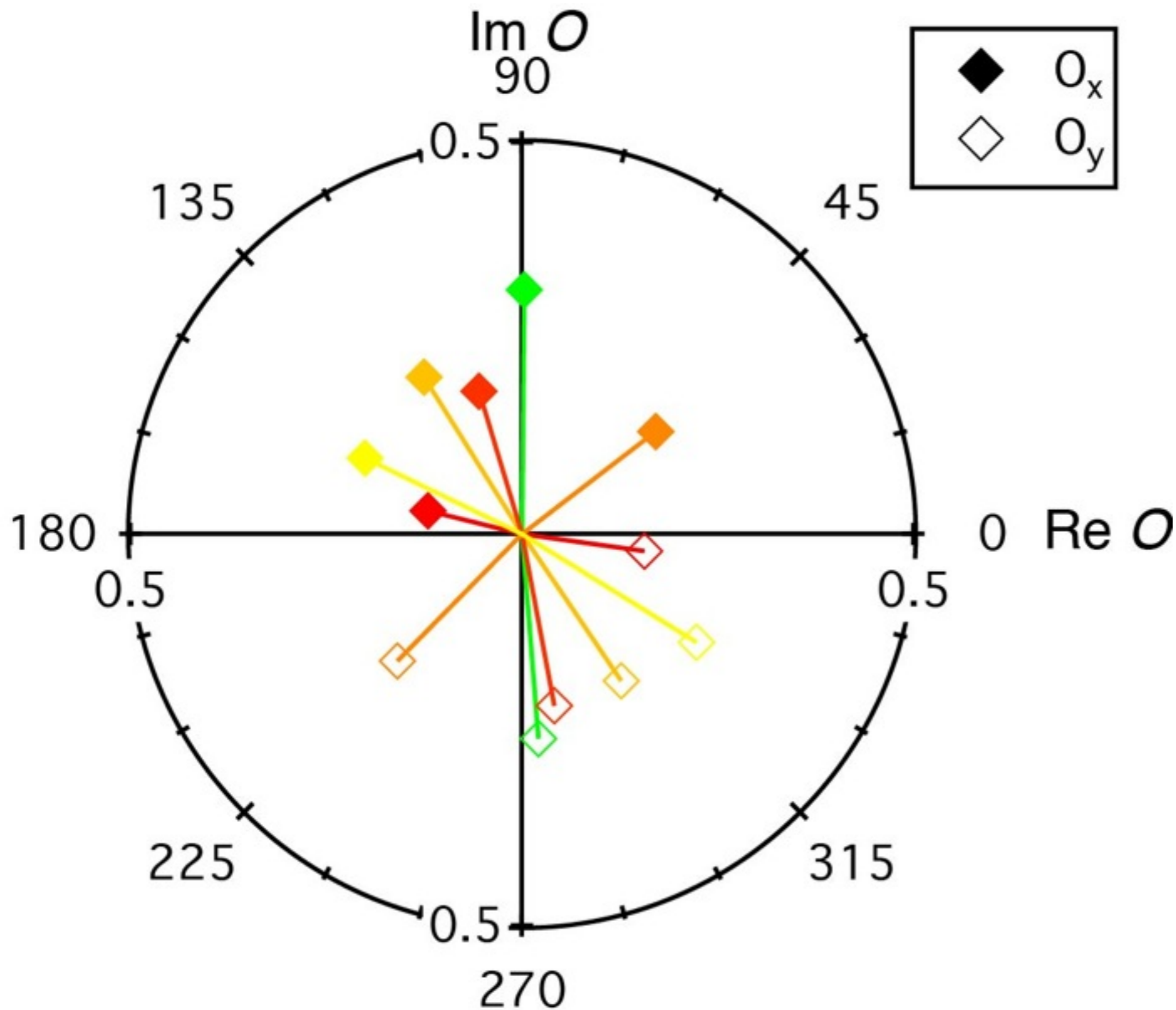
**Phase-sensitive
measurement of
the d symmetry
of density wave
order**



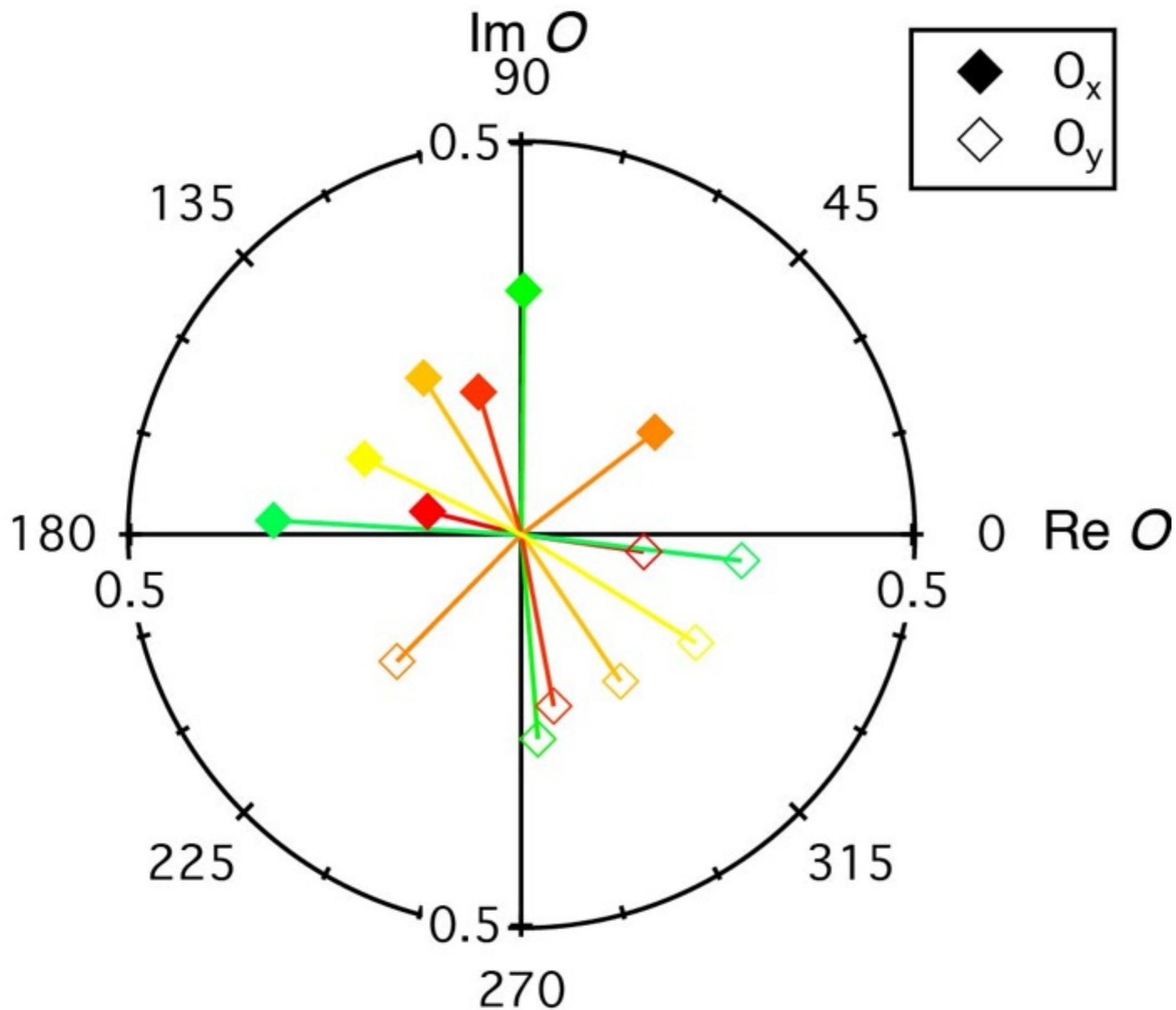
Phase-sensitive measurement of the d symmetry of density wave order



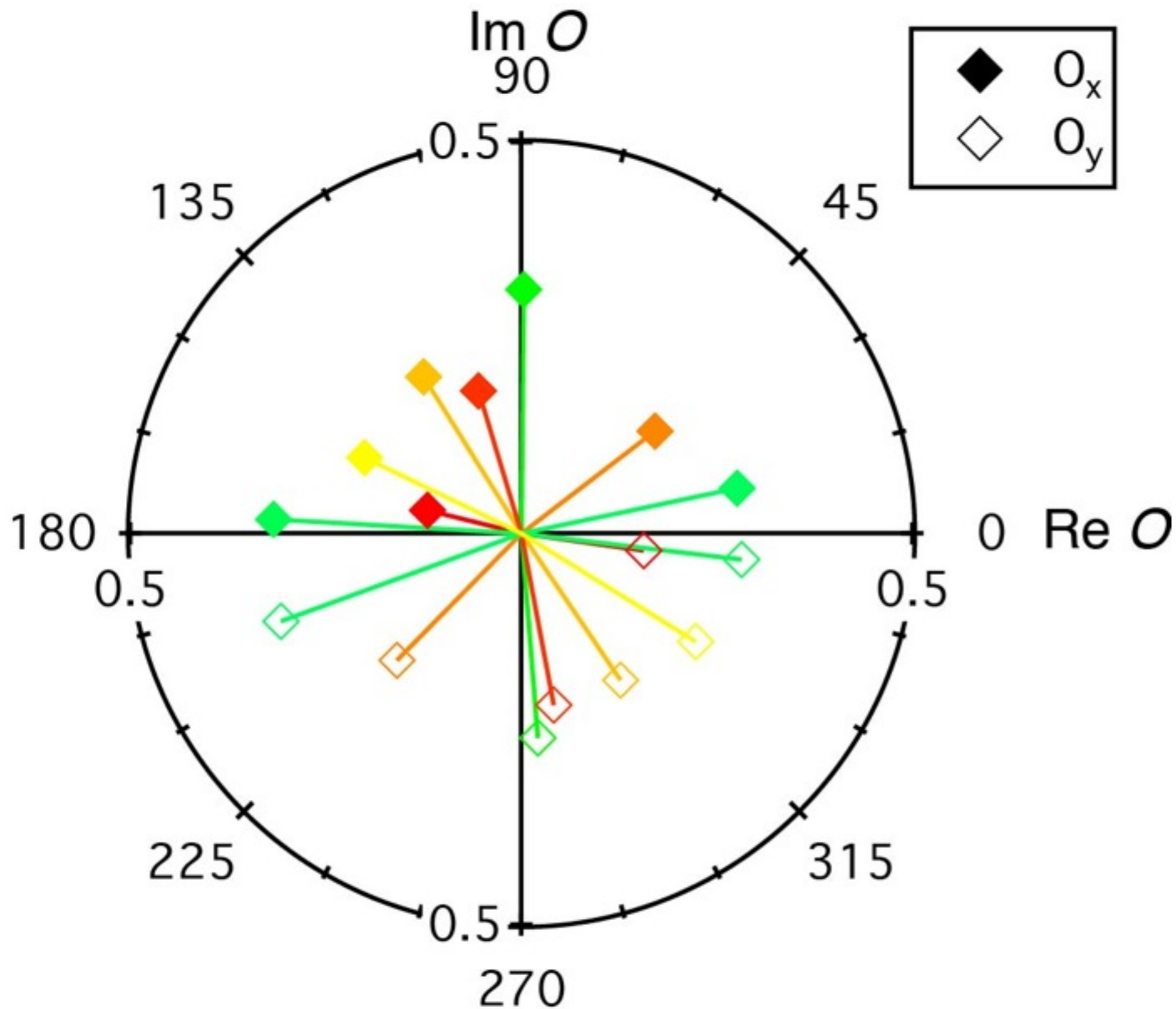
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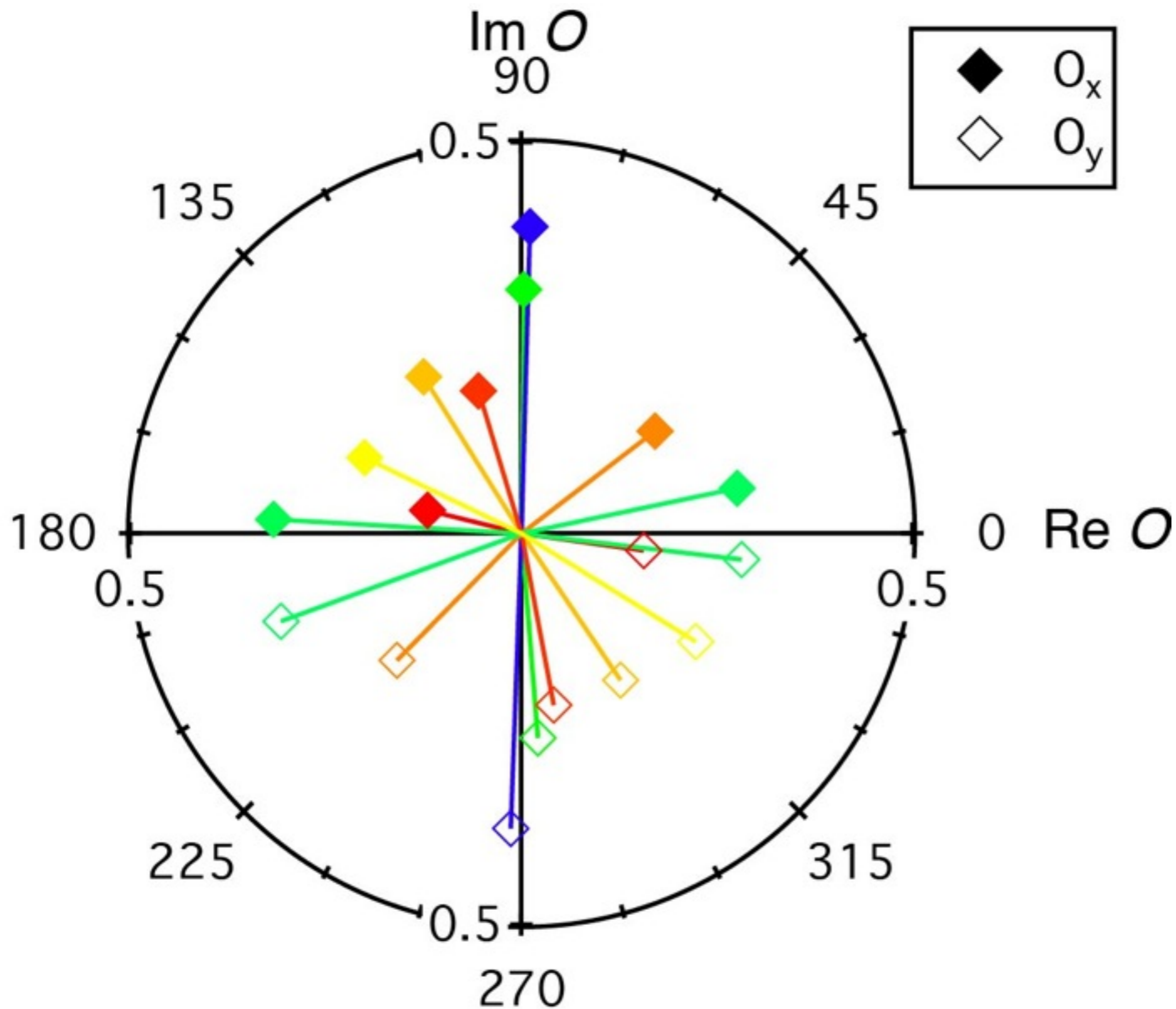
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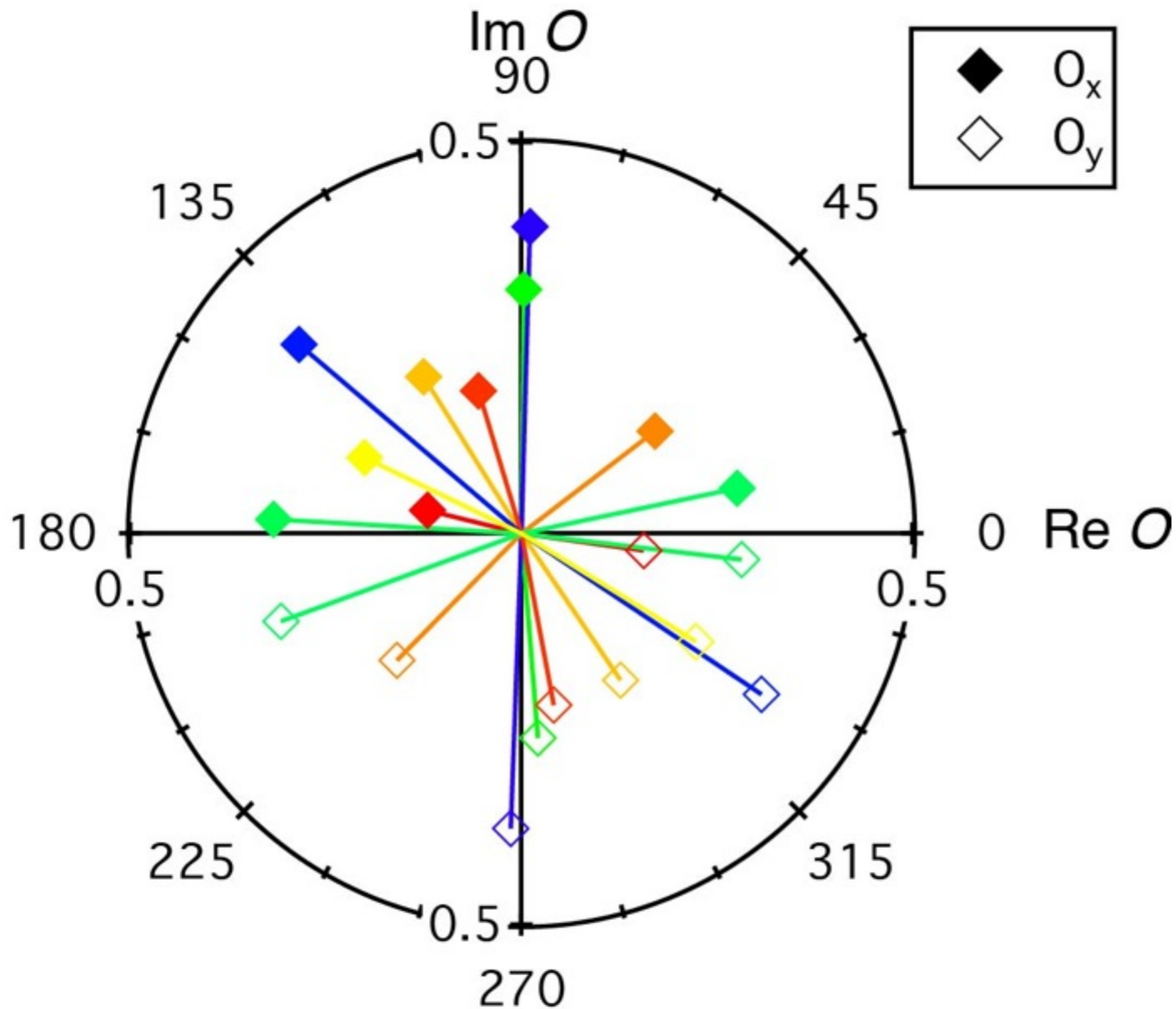
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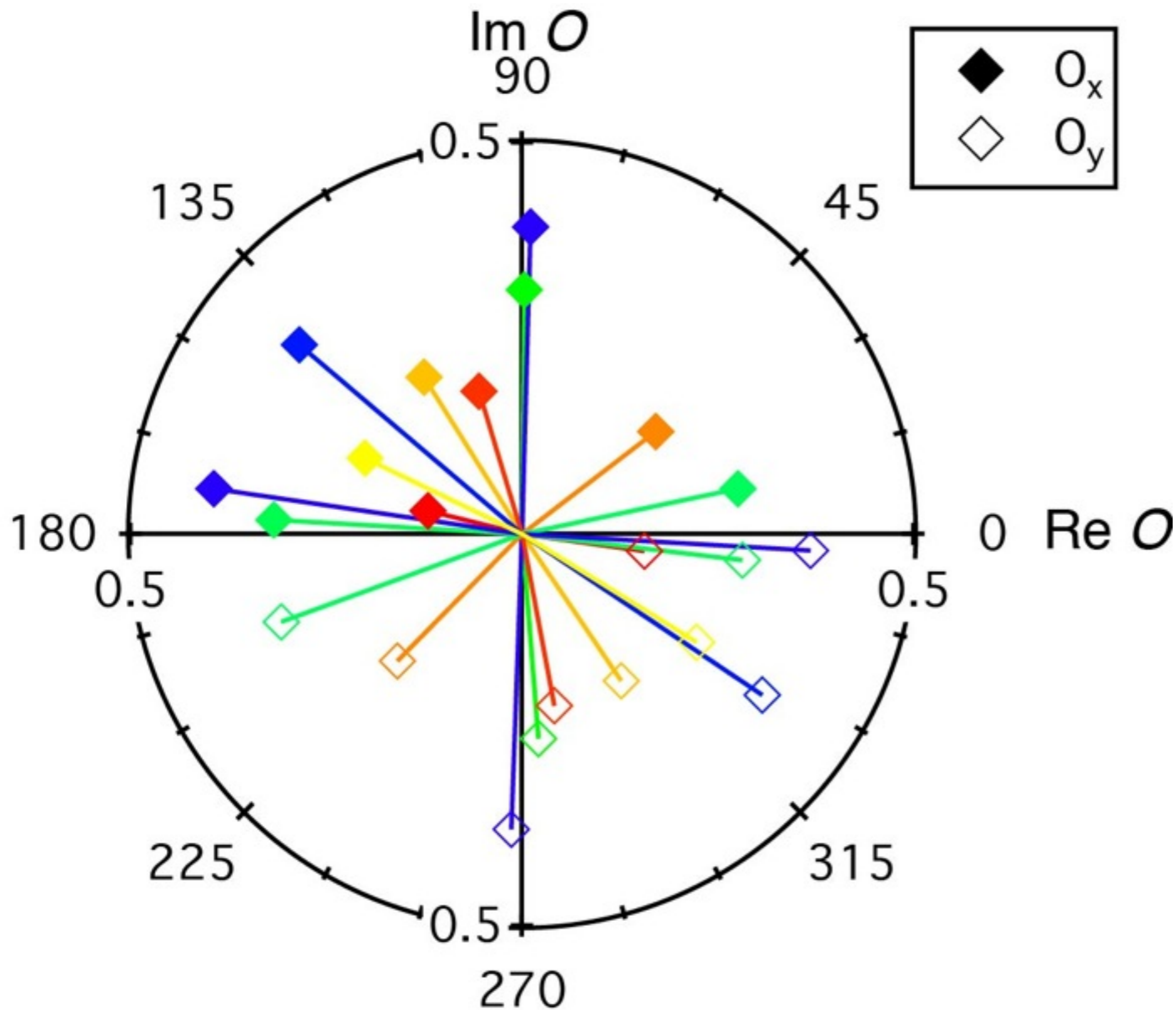
Phase-sensitive measurement of the d symmetry of density wave order



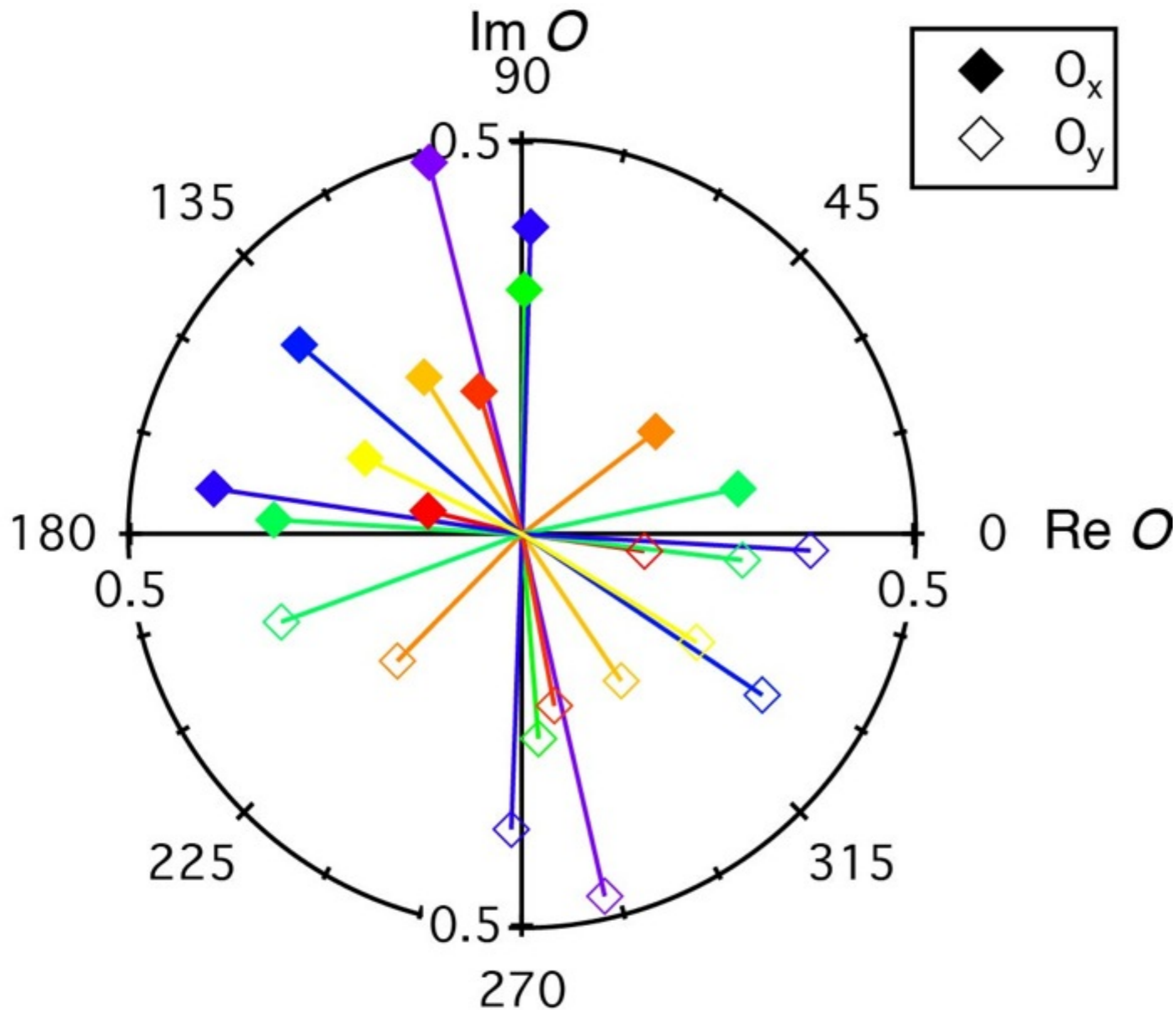
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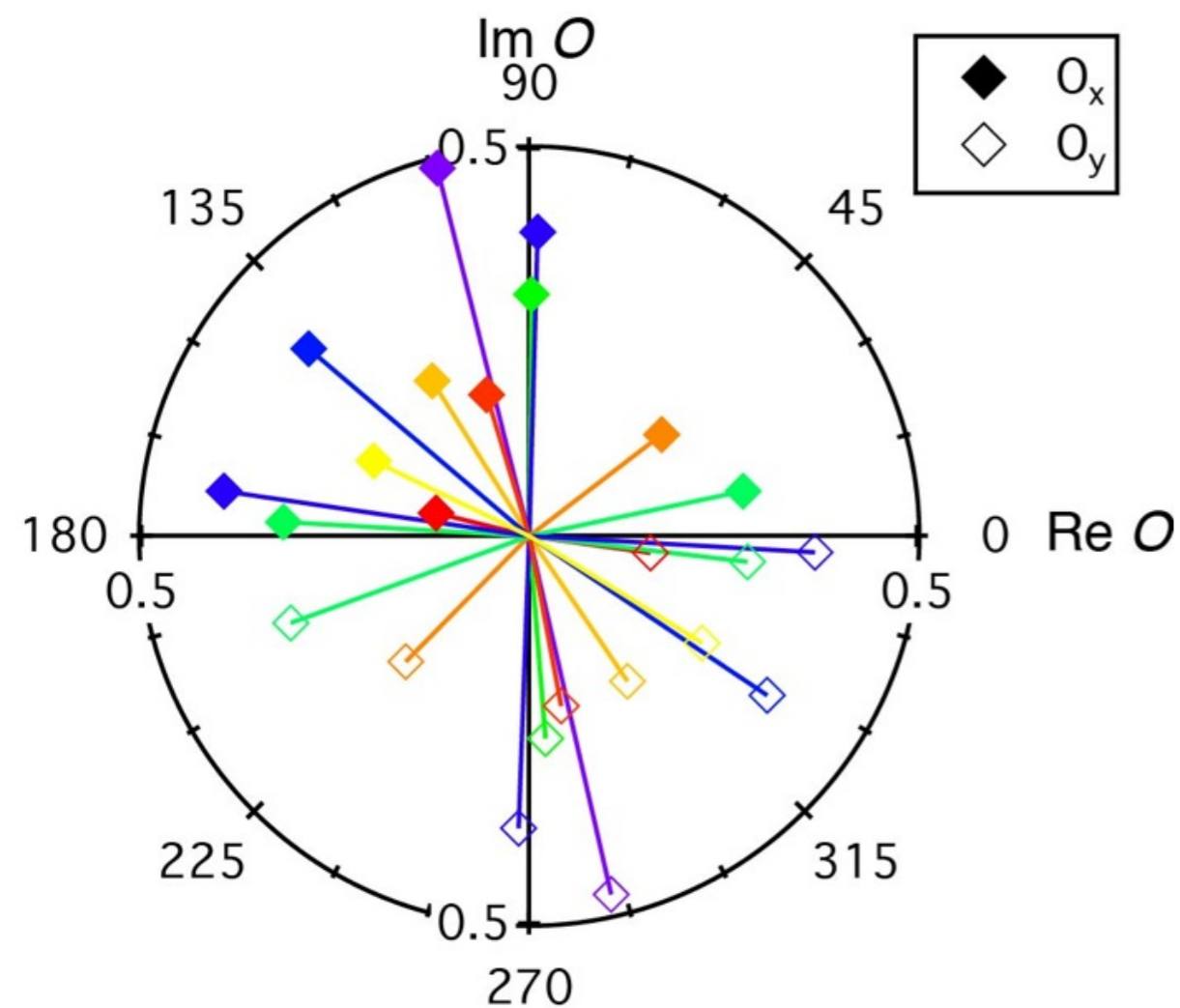
Phase-sensitive measurement of the *d* symmetry of density wave order



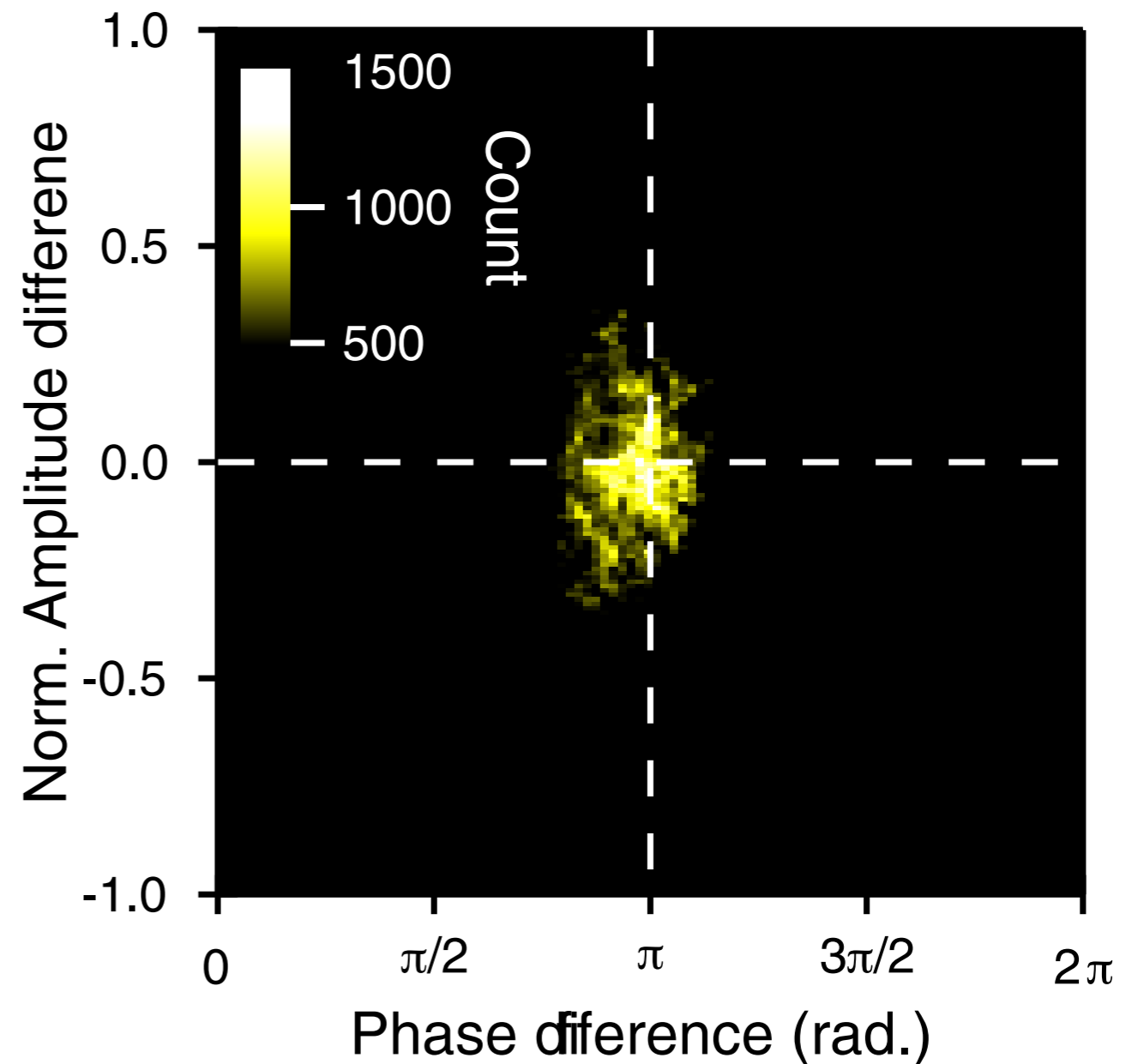
Phase-sensitive measurement of the d symmetry of density wave order

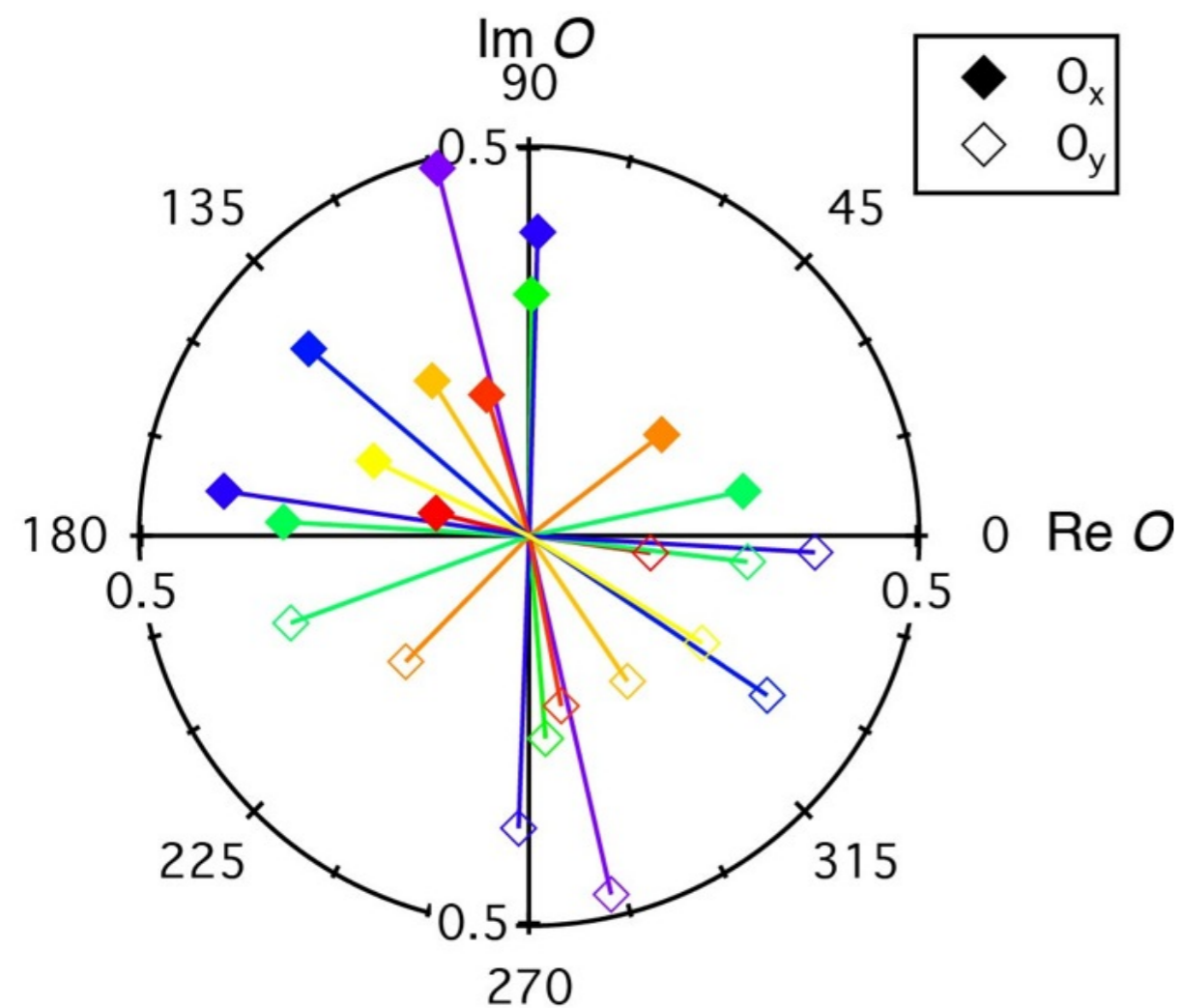


Phase-sensitive measurement of the *d* symmetry of density wave order

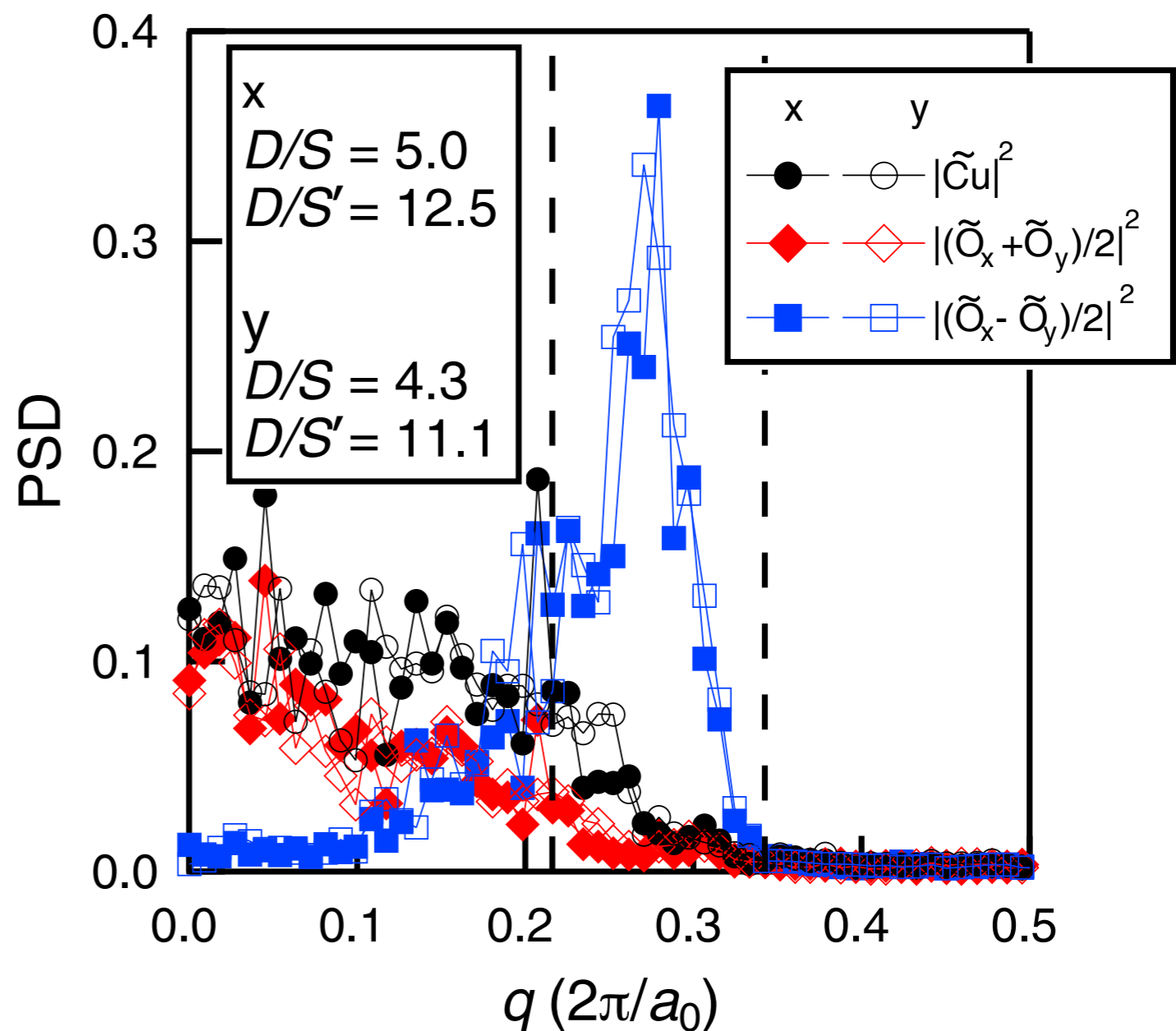


Phase-sensitive measurement of the d symmetry of density wave order





Phase-sensitive measurement of the d symmetry of density wave order

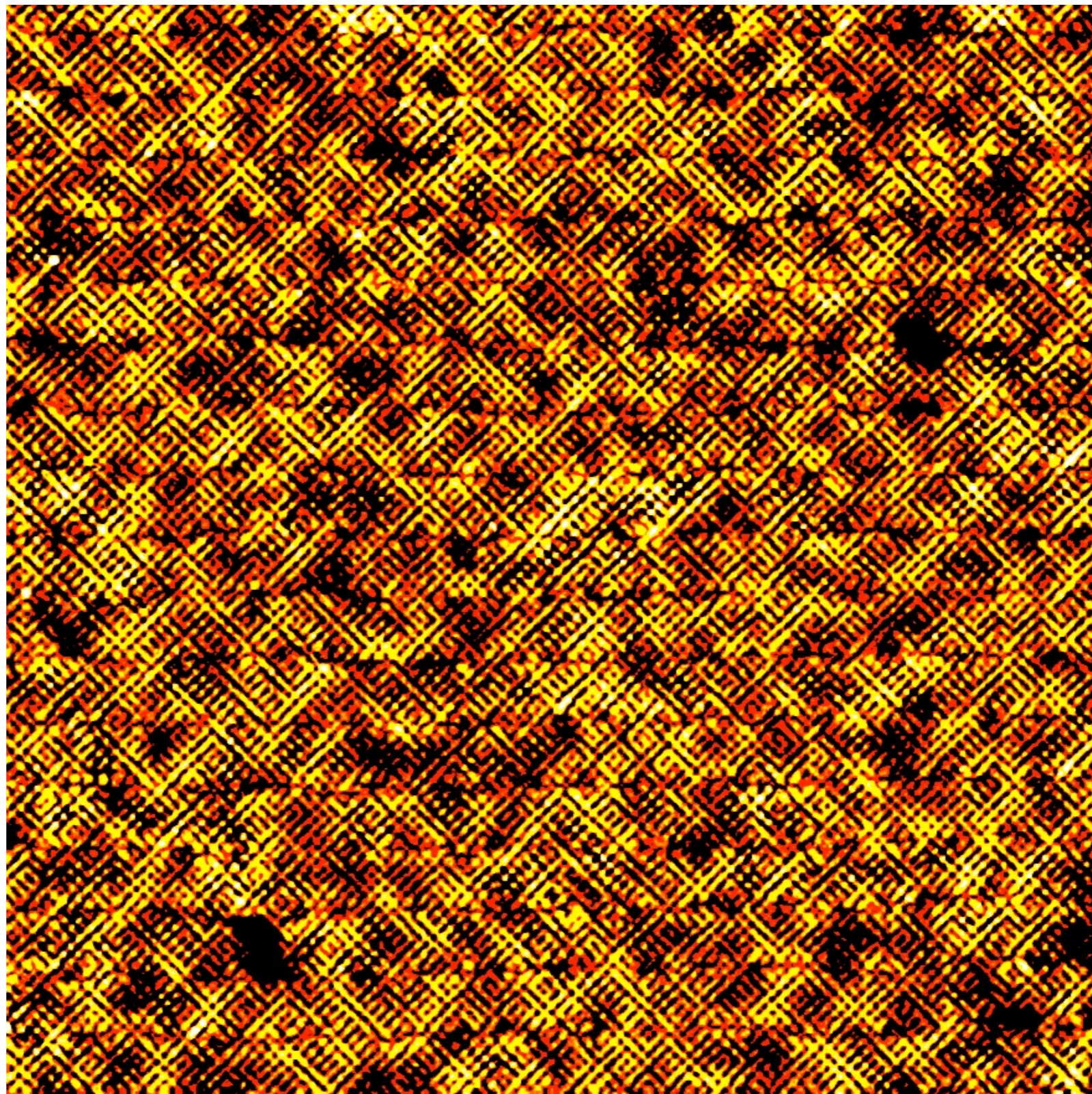


See also

C. Howald, H. Eisaki,
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and A. Kapitulnik,
Phys. Rev. B **67**,
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M. Vershinin, S. Misra,
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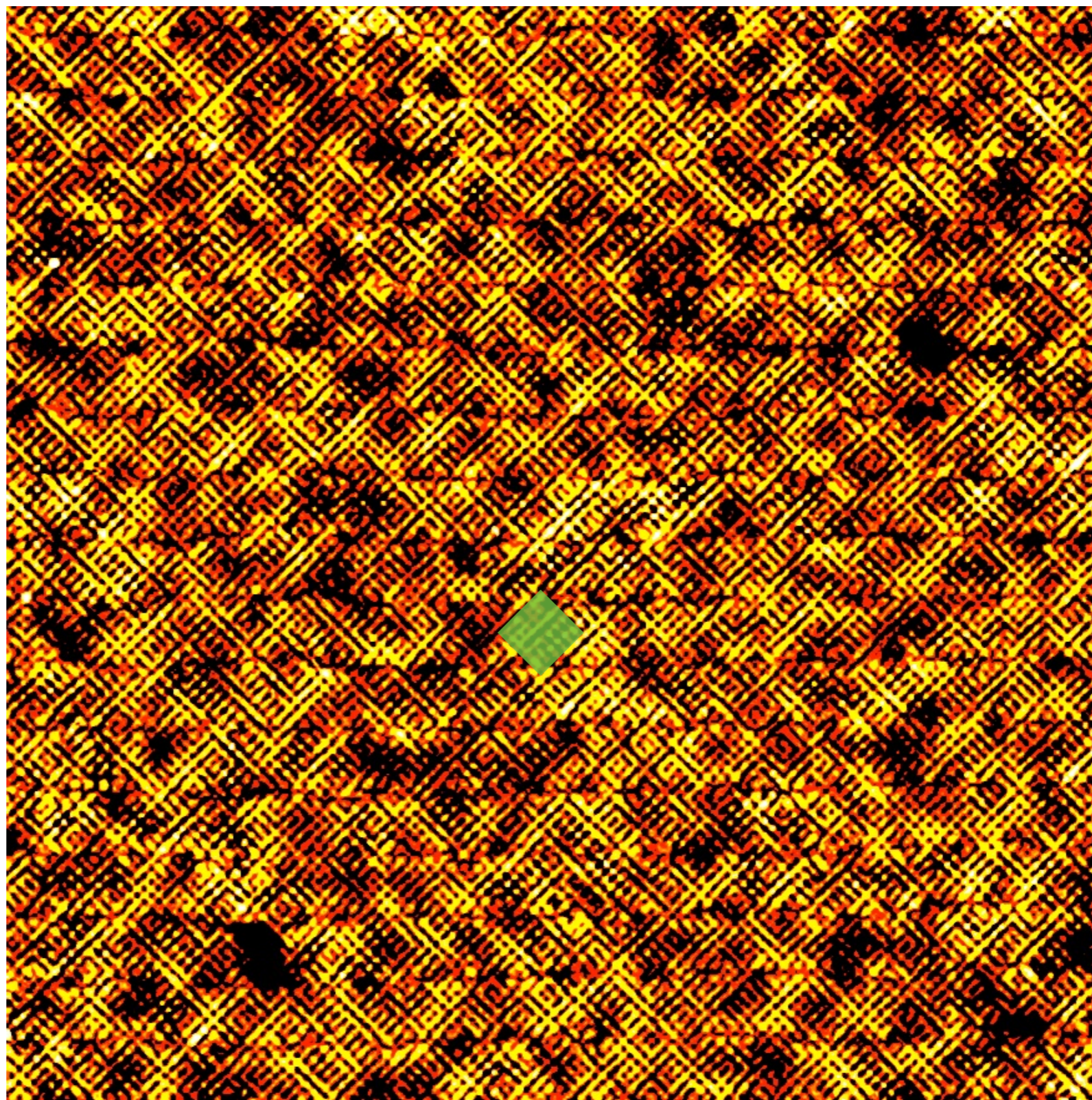
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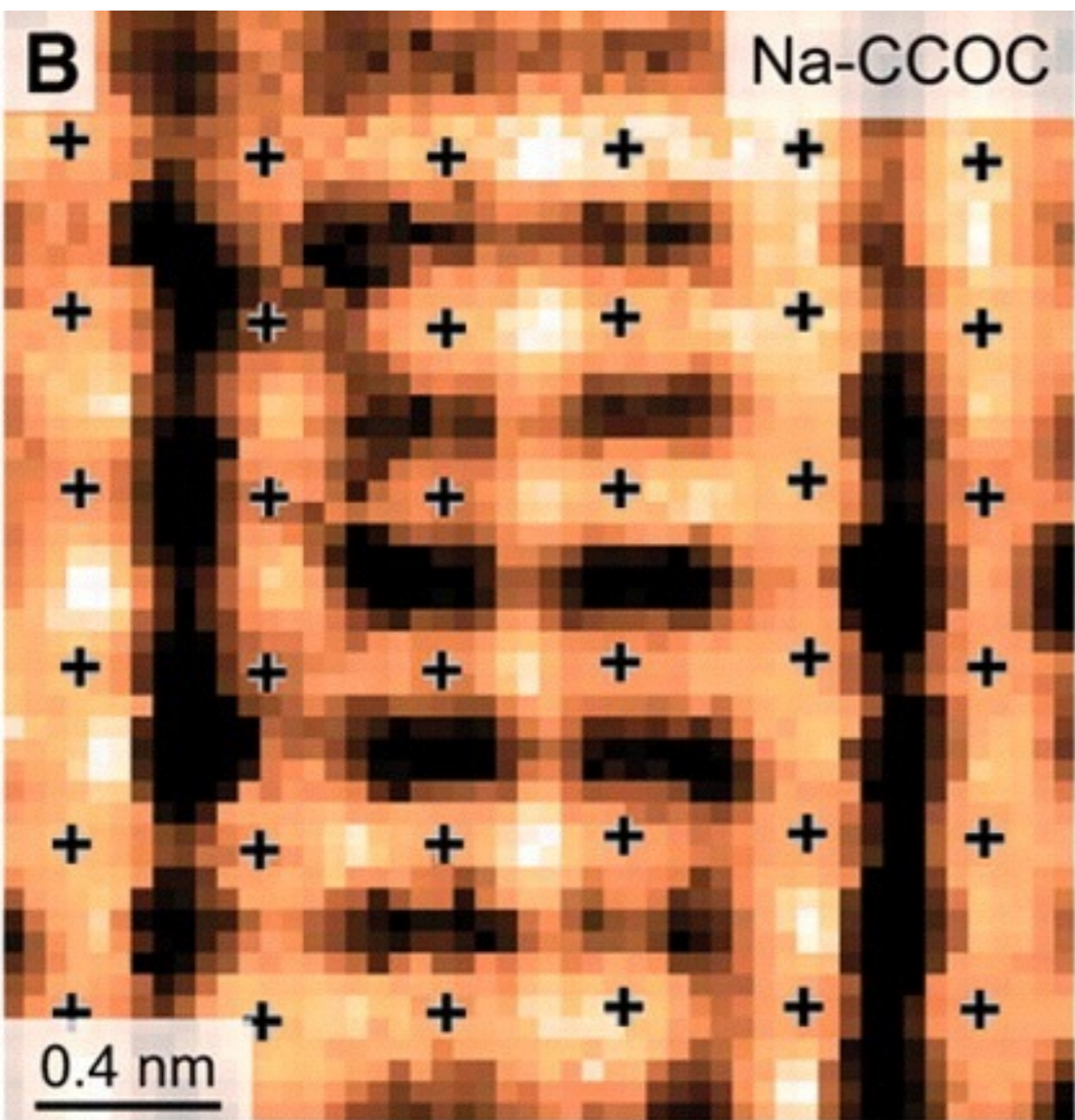
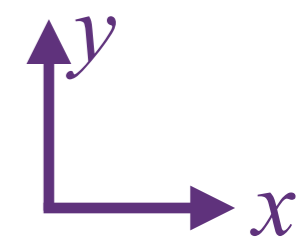
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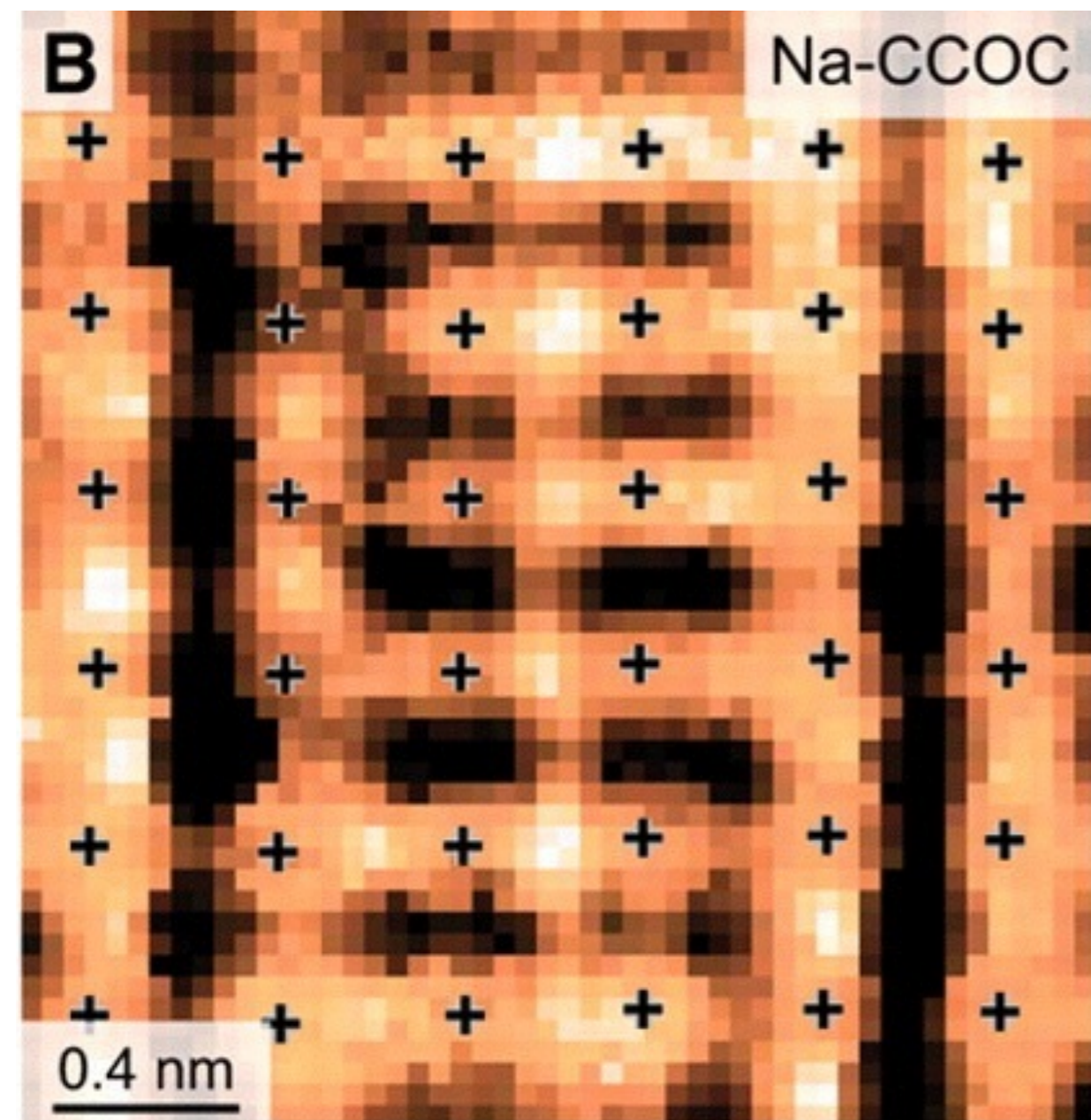
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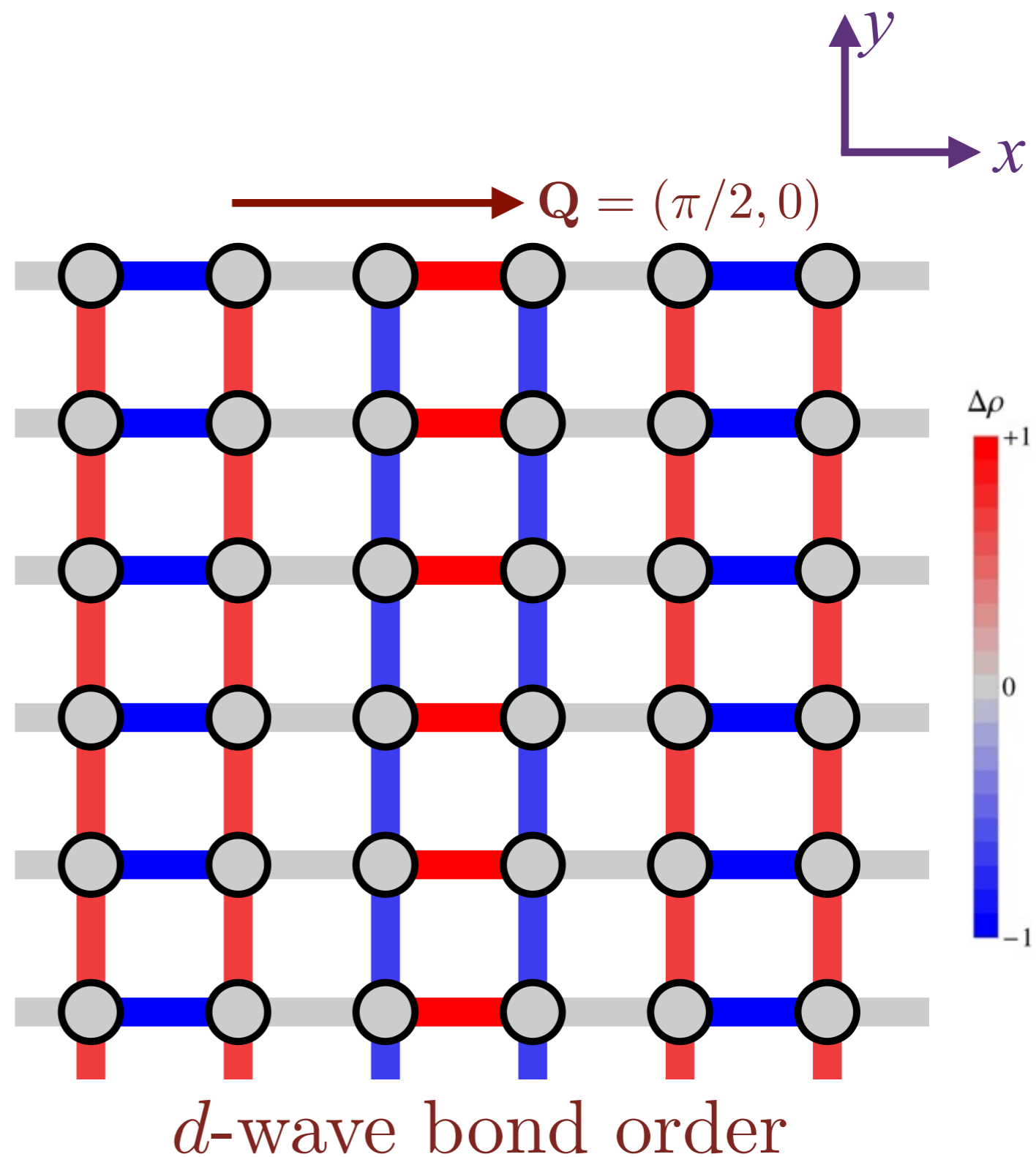
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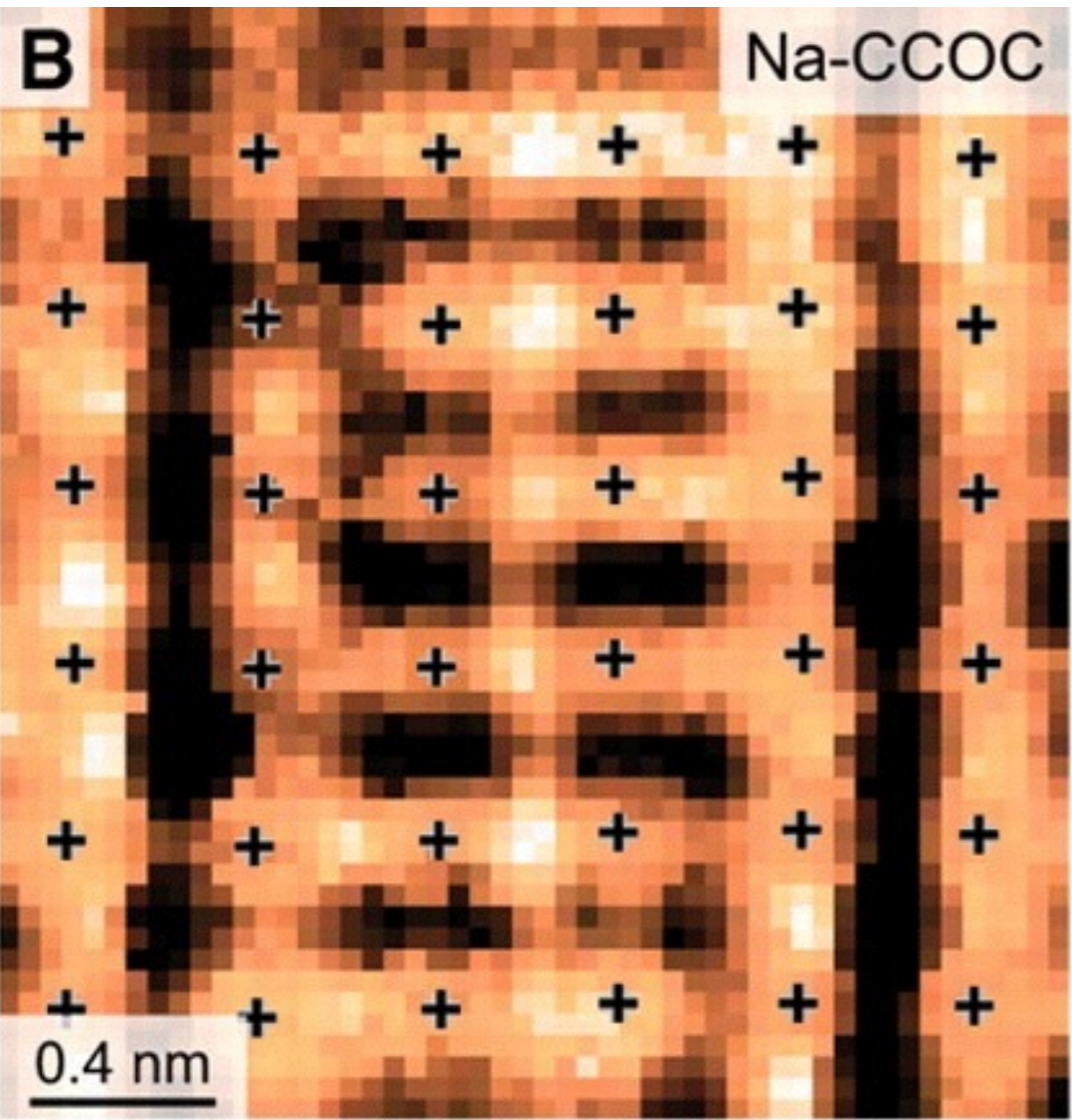
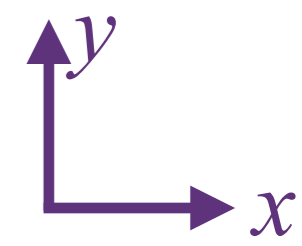
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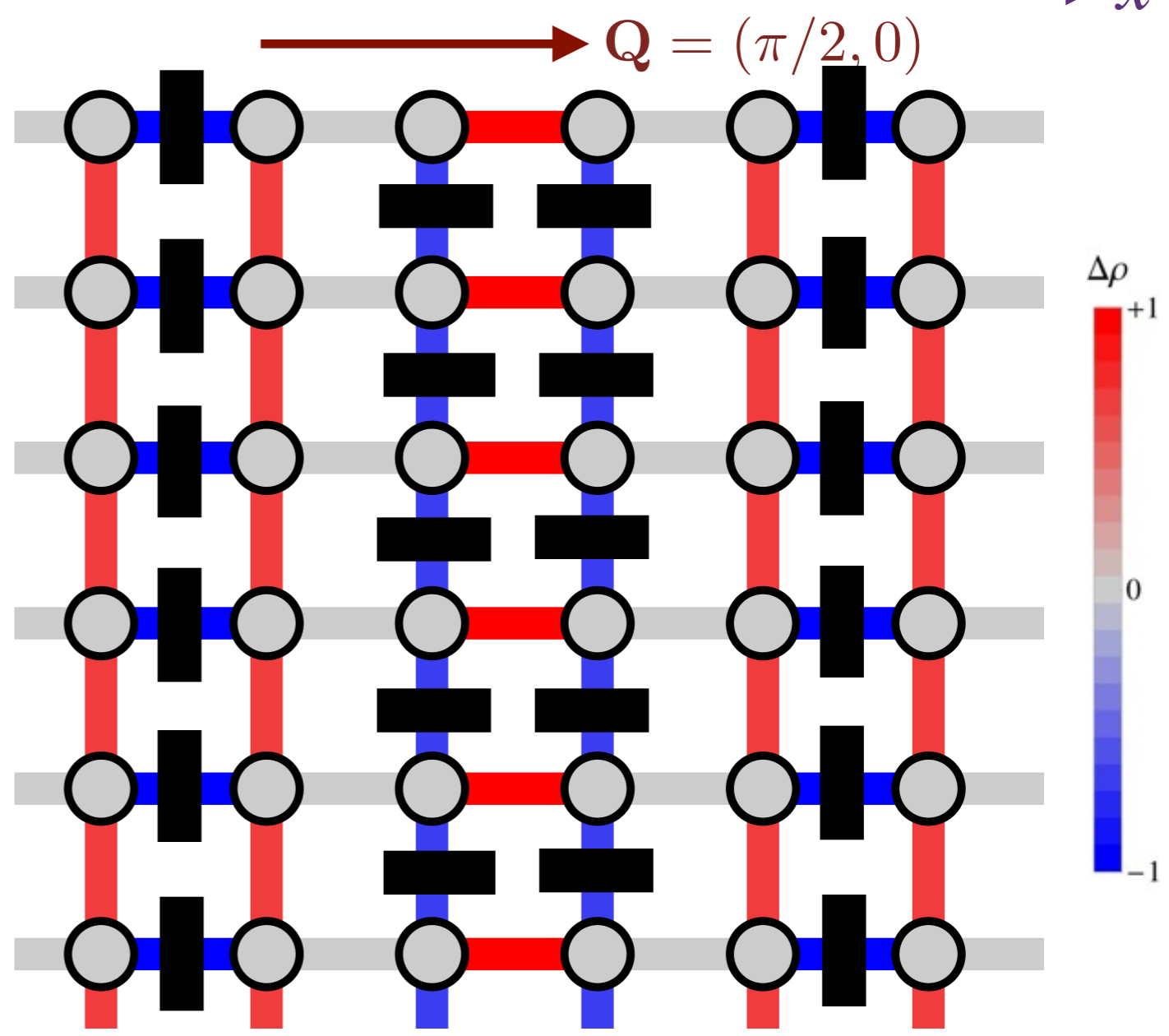
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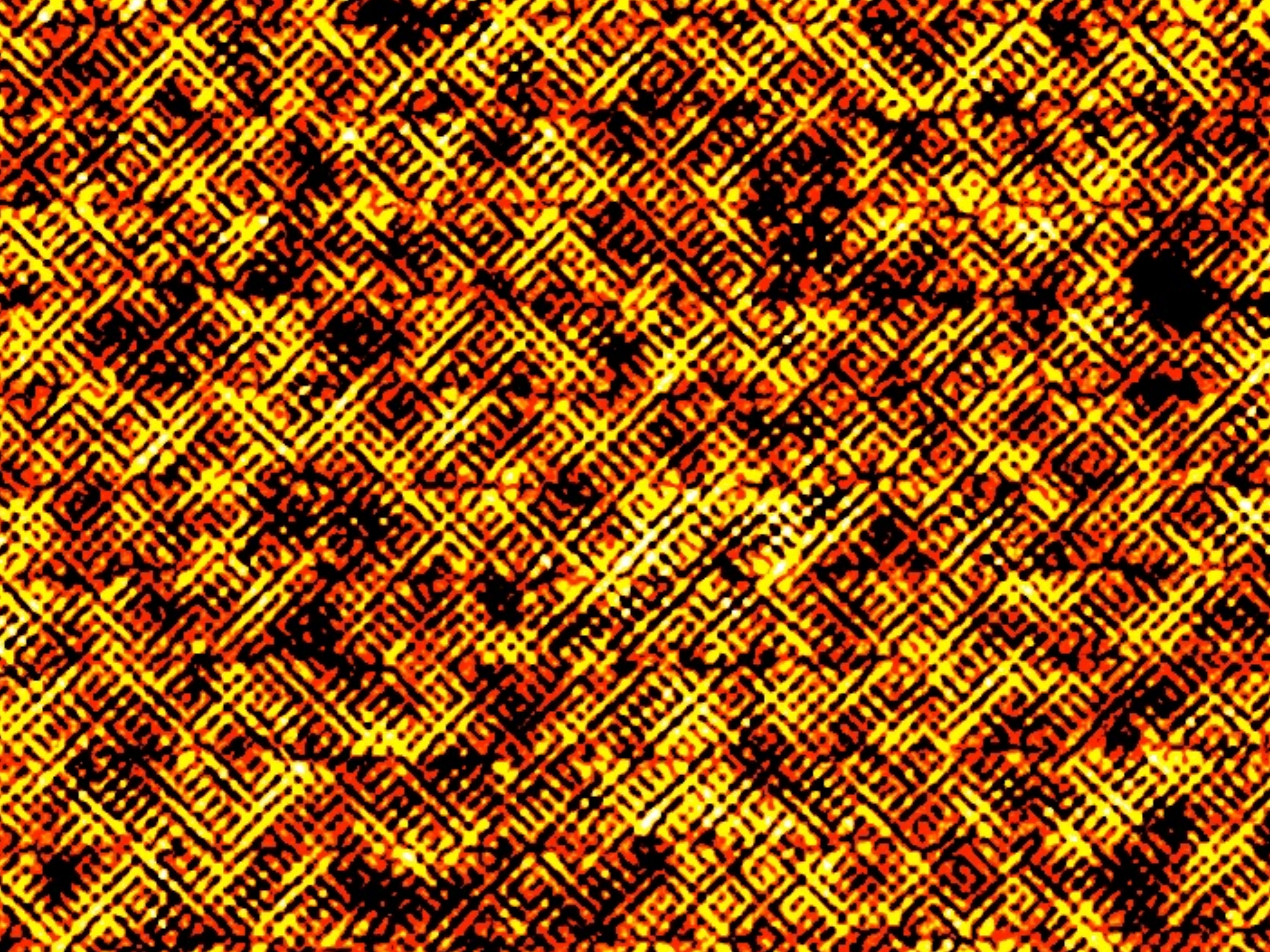


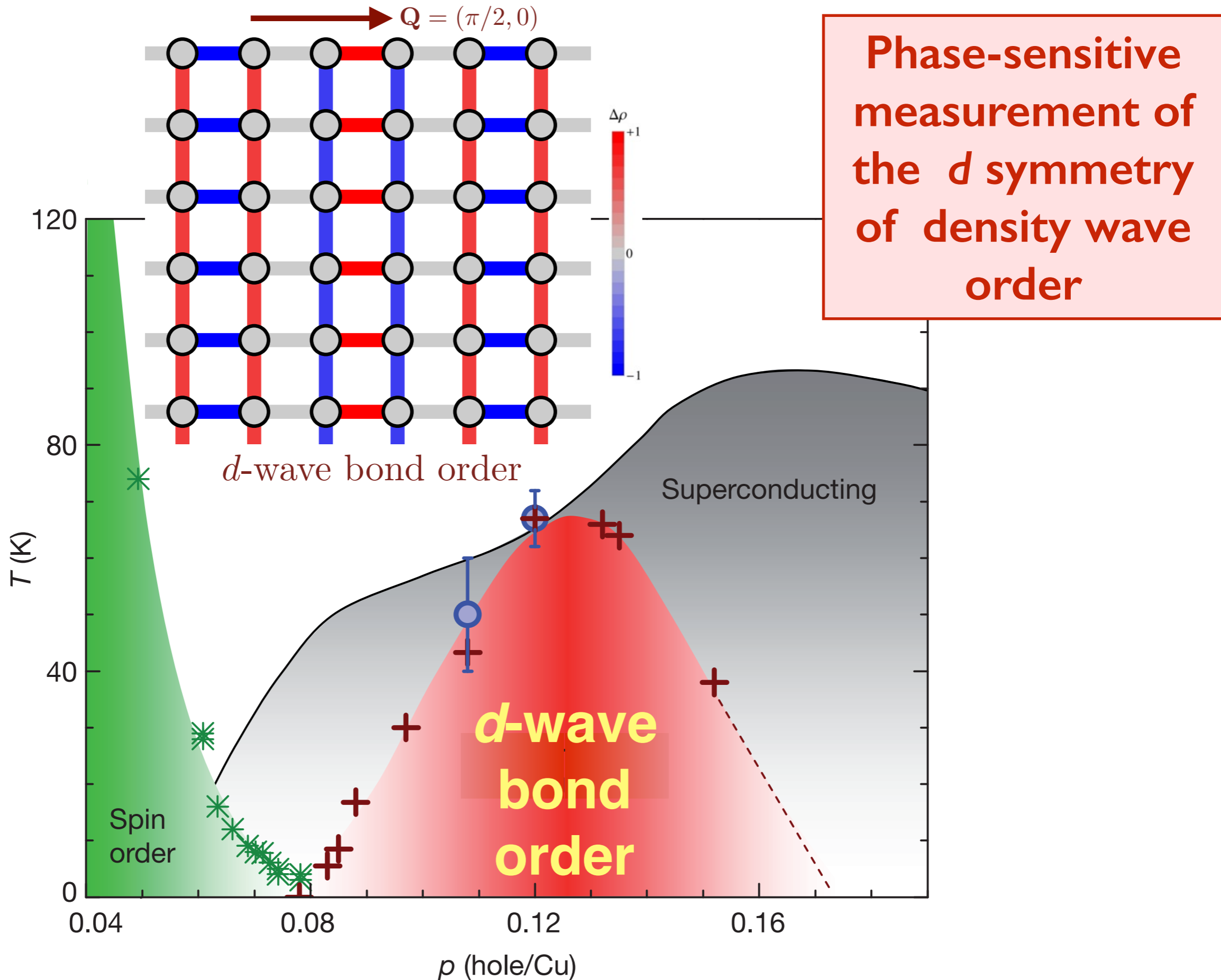
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d-wave bond order

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Outline

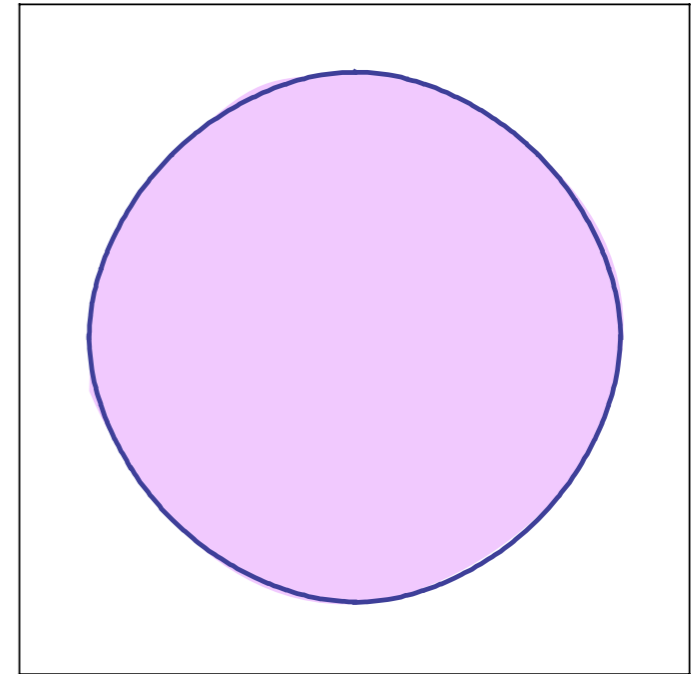
1. Unconventional d -wave superconductivity
2. Low hole density state:
An unconventional density wave
3. Outline of theoretical prediction
4. Evolution of Fermi surface

Outline

1. Unconventional d -wave superconductivity
2. Low hole density state:
An unconventional density wave
3. Outline of theoretical prediction
4. Evolution of Fermi surface

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

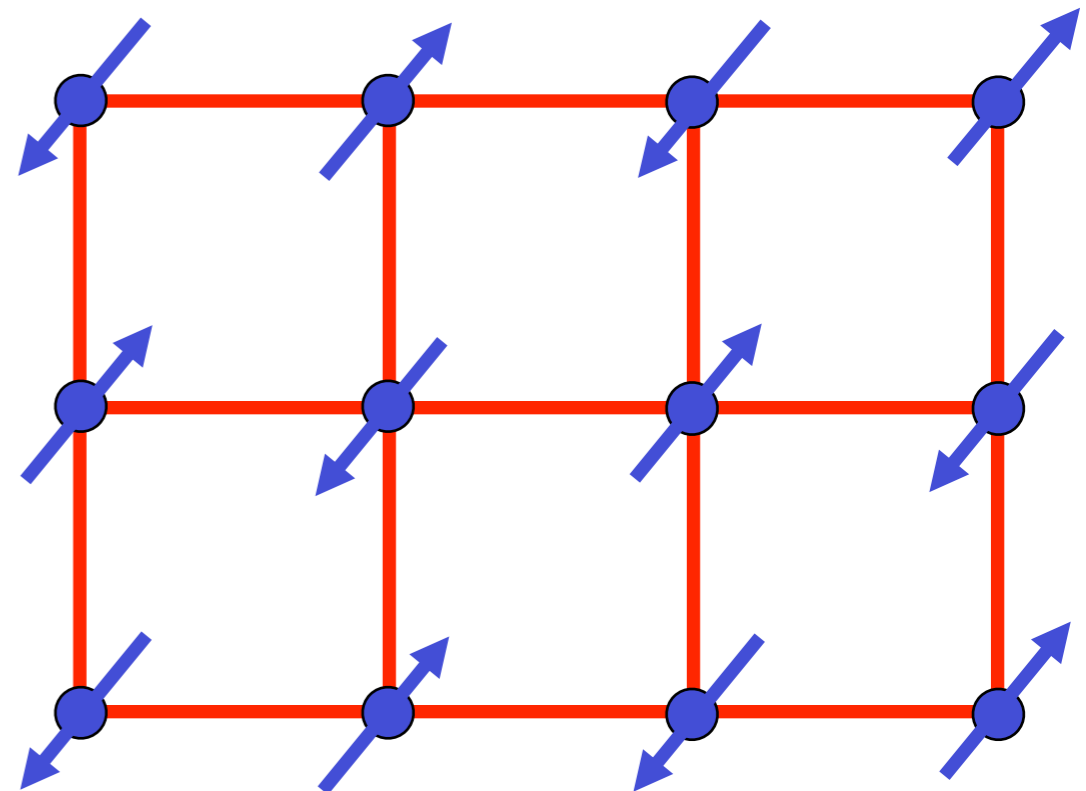


+

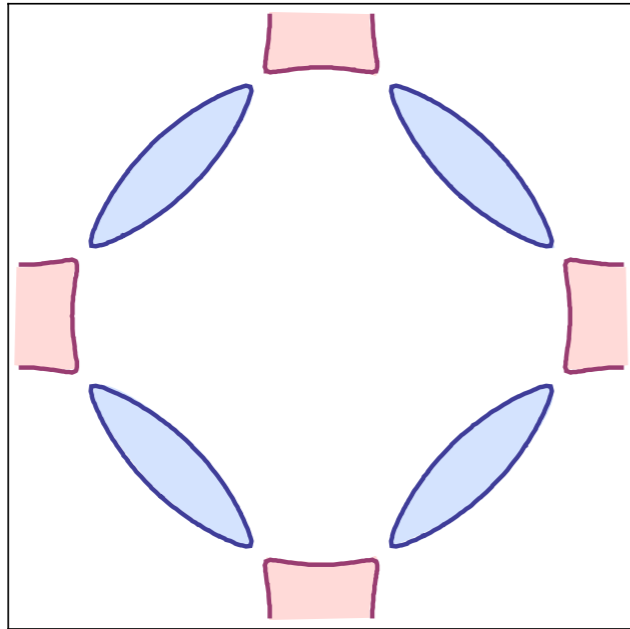
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering
wavevector.

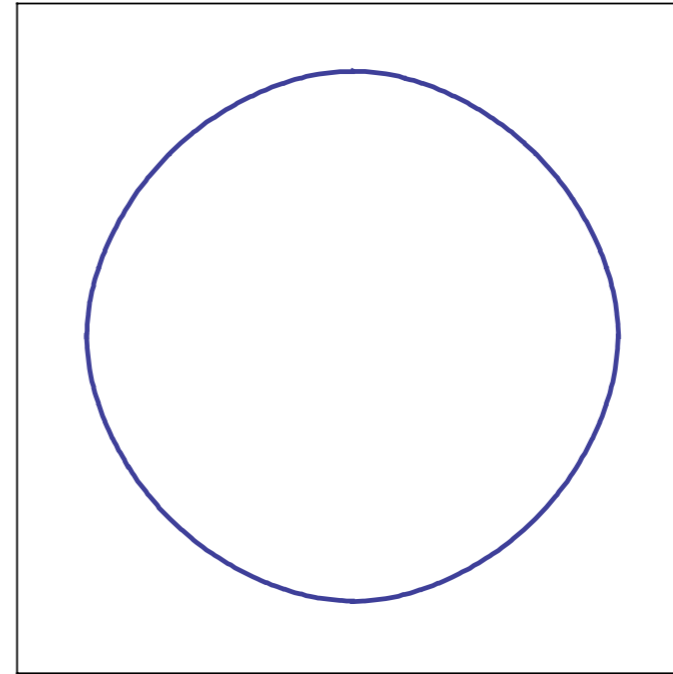


Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

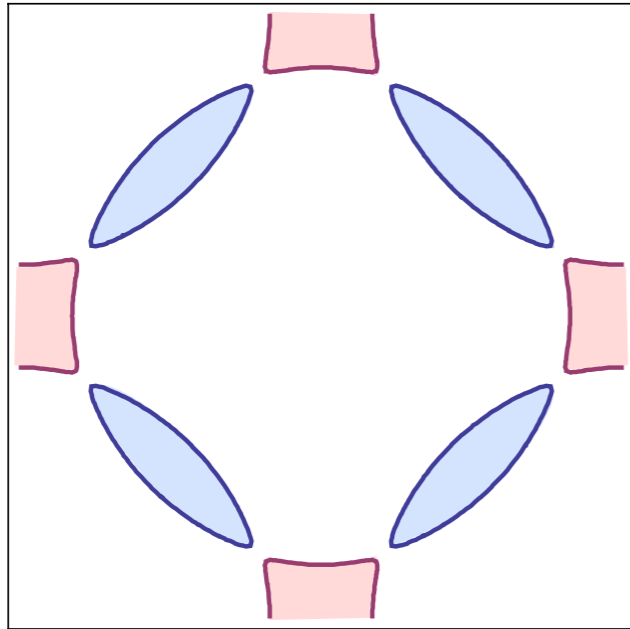


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

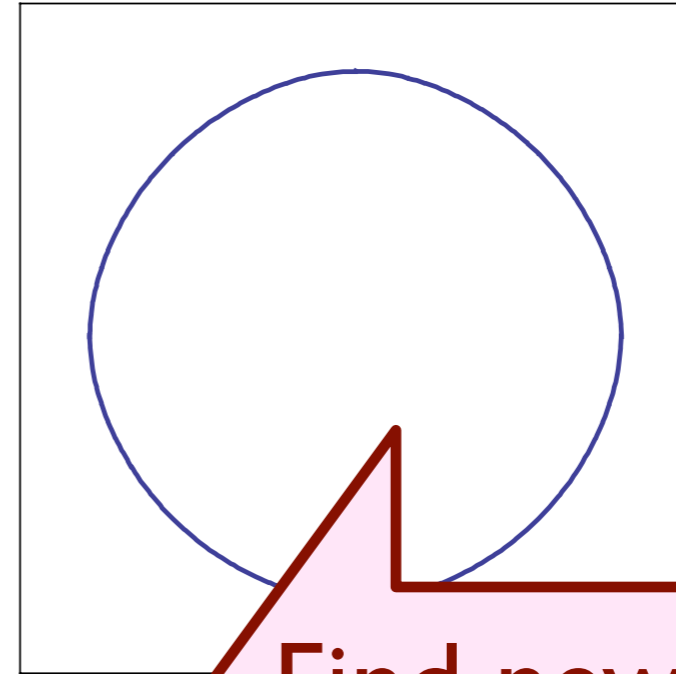
r

Quantum phase transition with onset of antiferromagnetism in a metal



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Metal with electron
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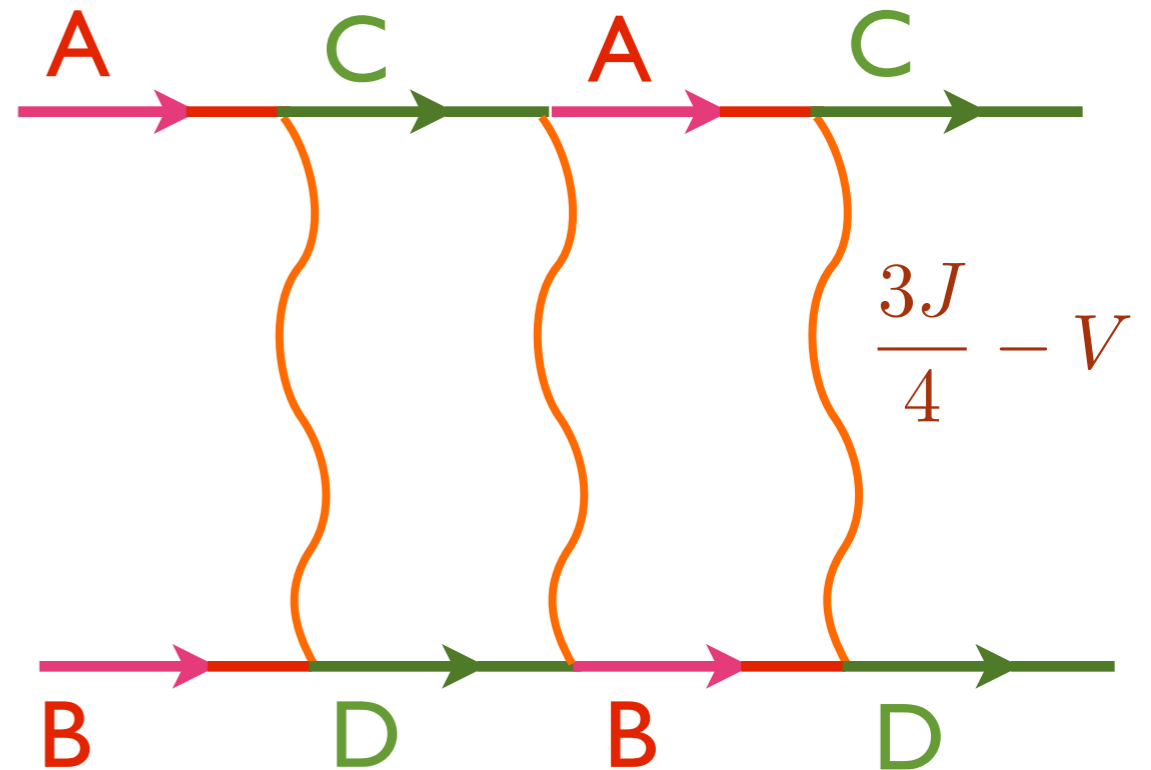
Metal with "large"
Fermi surface

Find new instabilities
upon approaching
critical point

r

Pairing “glue” from antiferromagnetic fluctuations

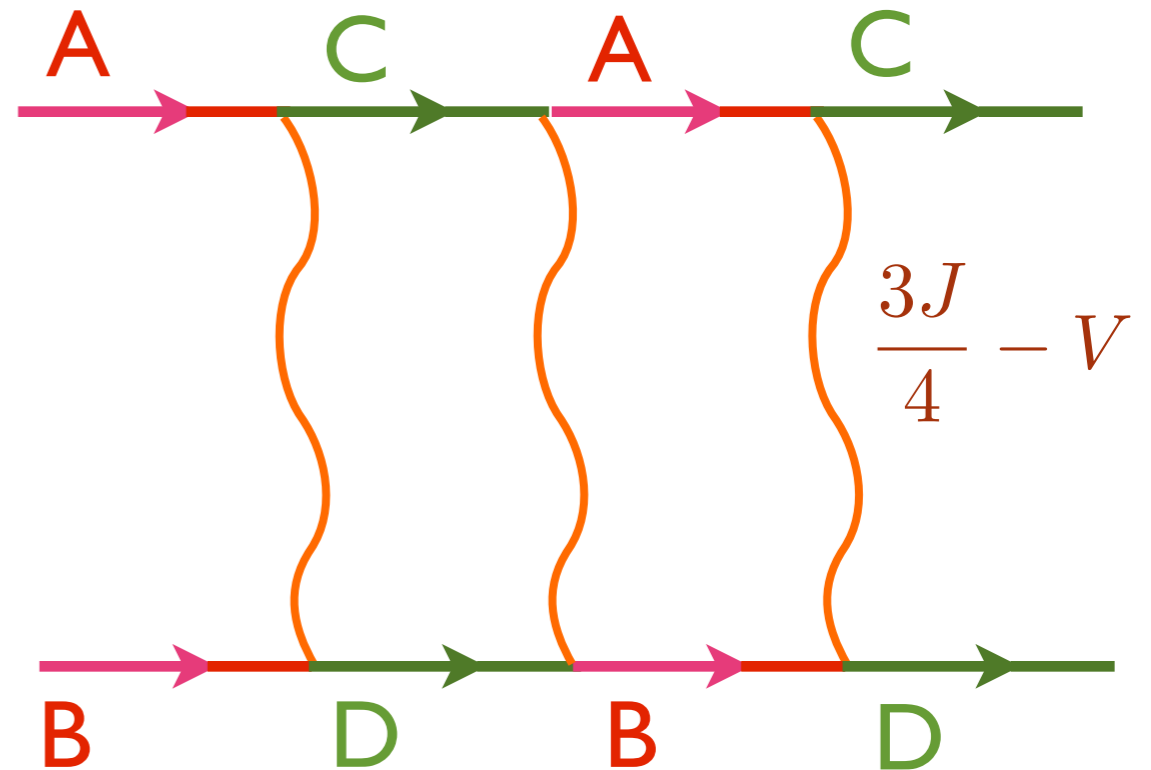
$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



Pairing “glue” from antiferromagnetic fluctuations

Electron hopping

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)

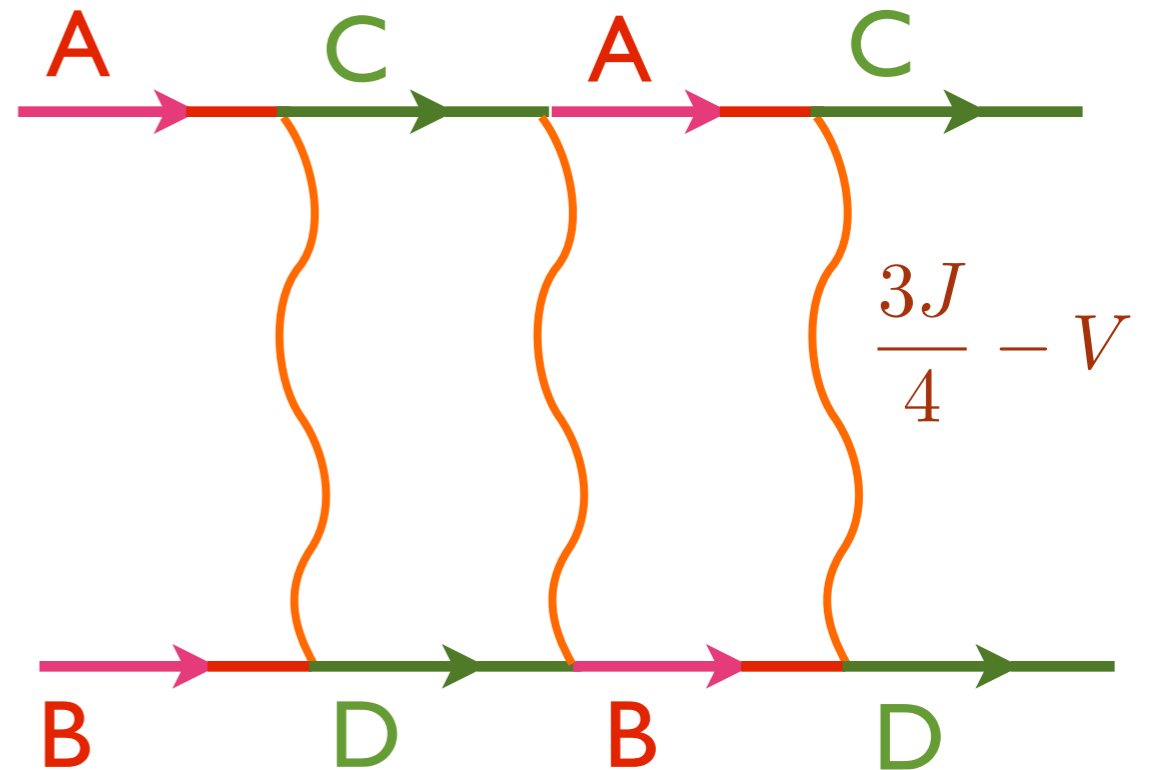
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

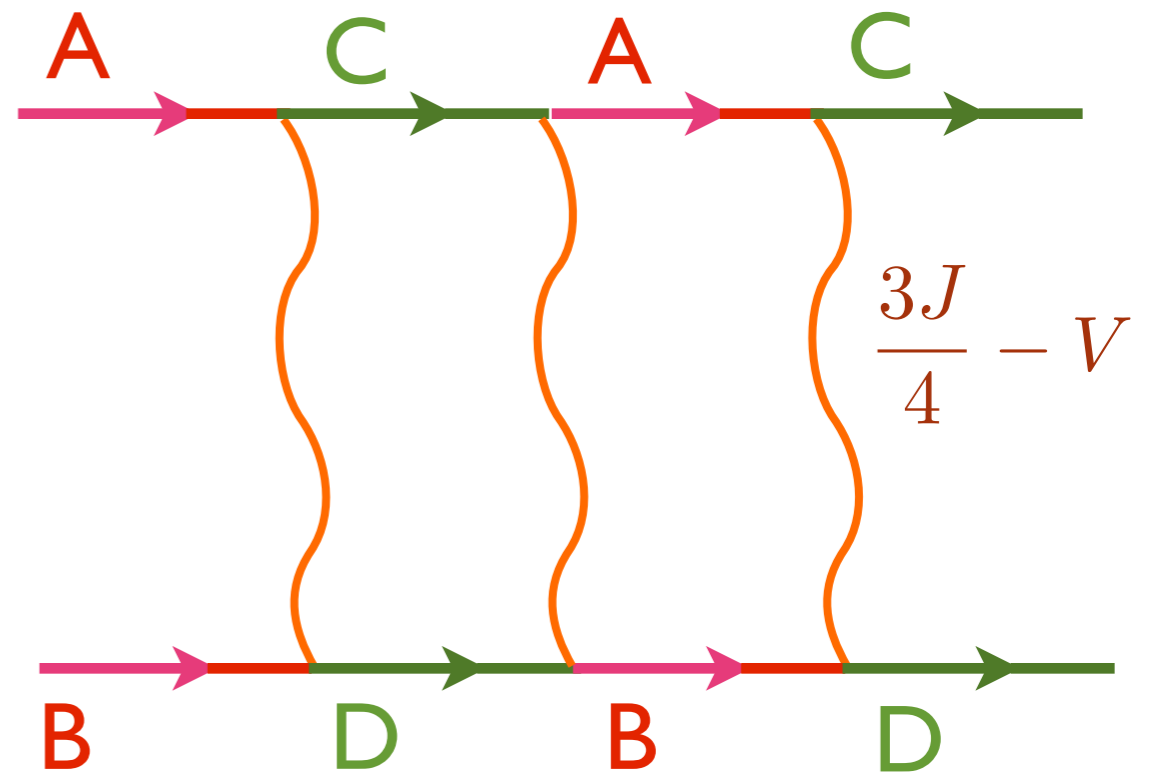
Antiferromagnetic
exchange
interaction



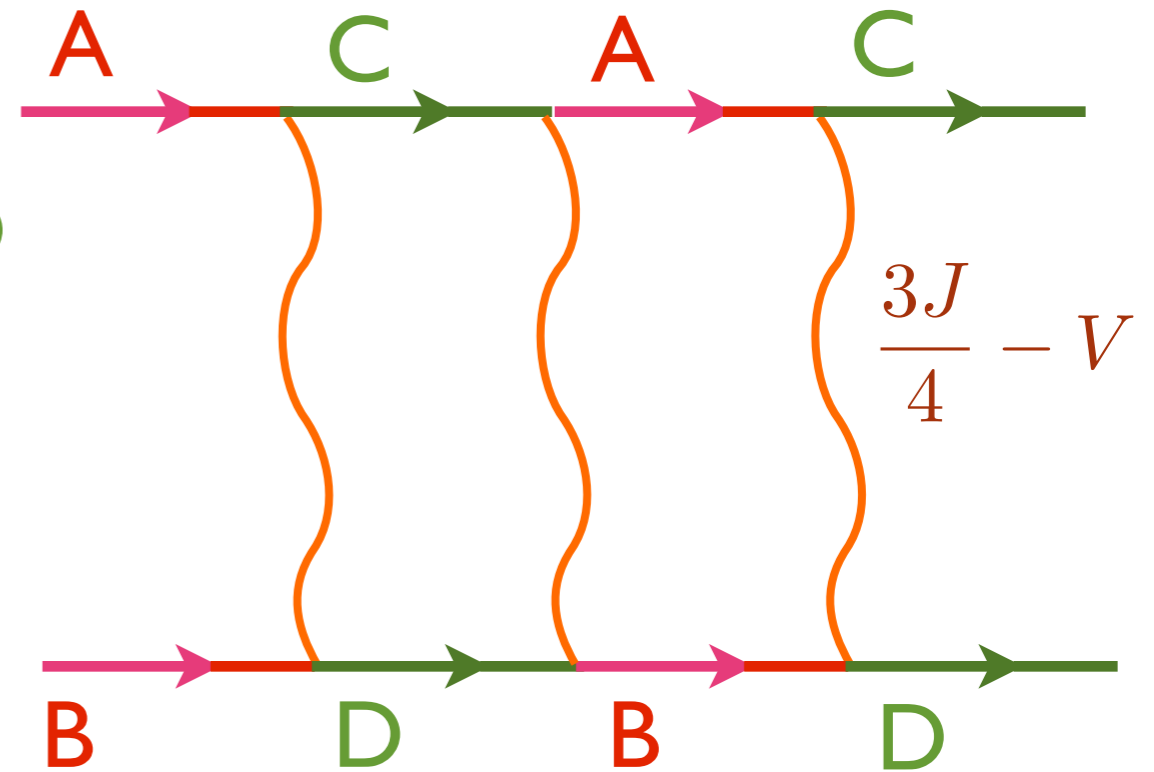
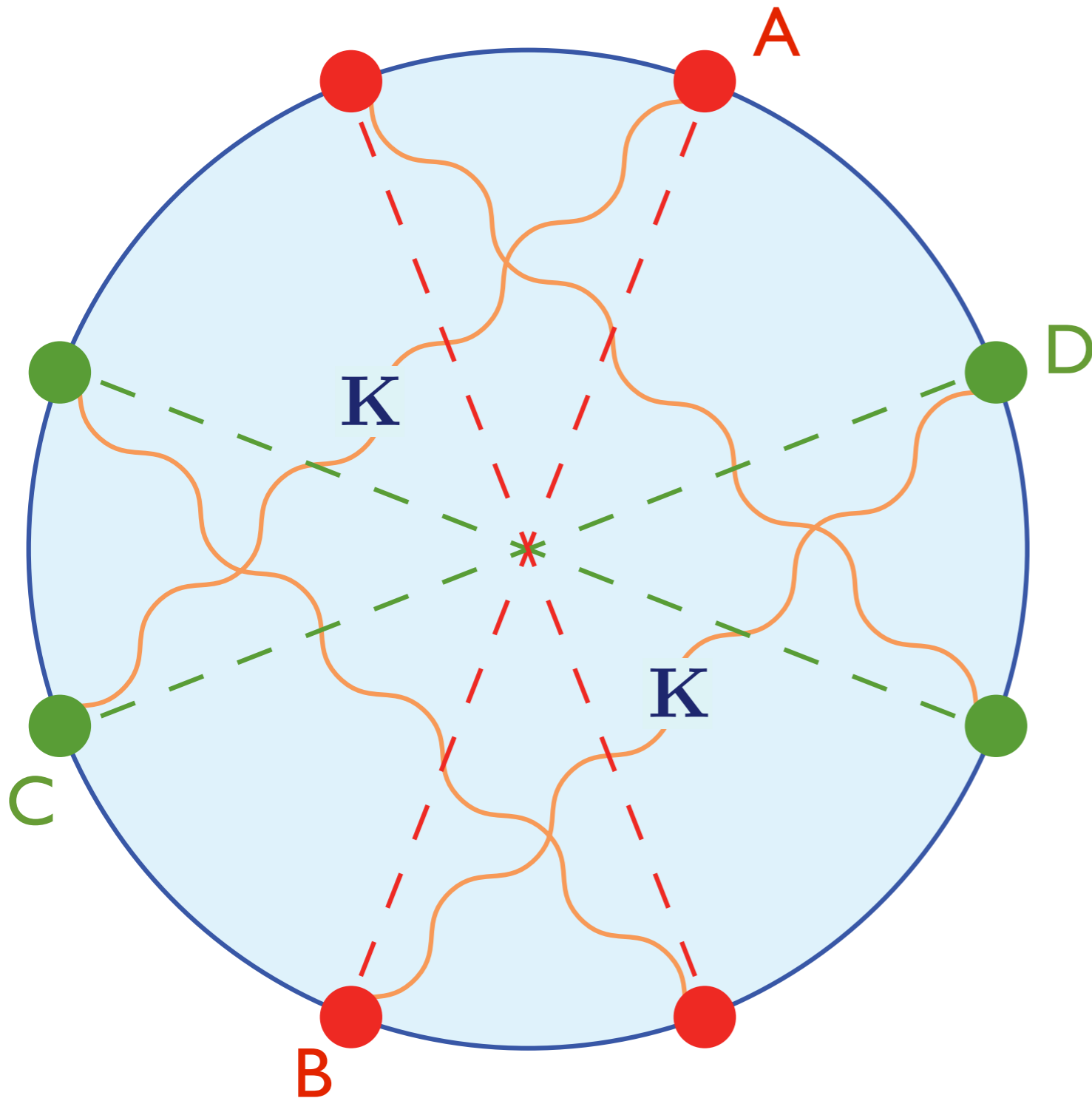
Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

Coulomb
repulsion

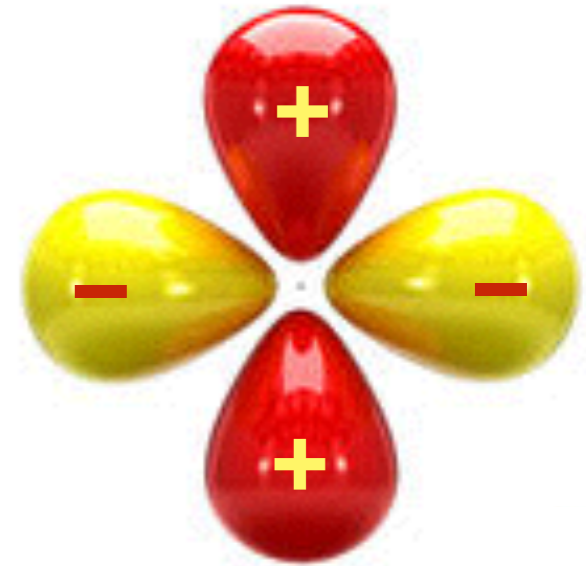
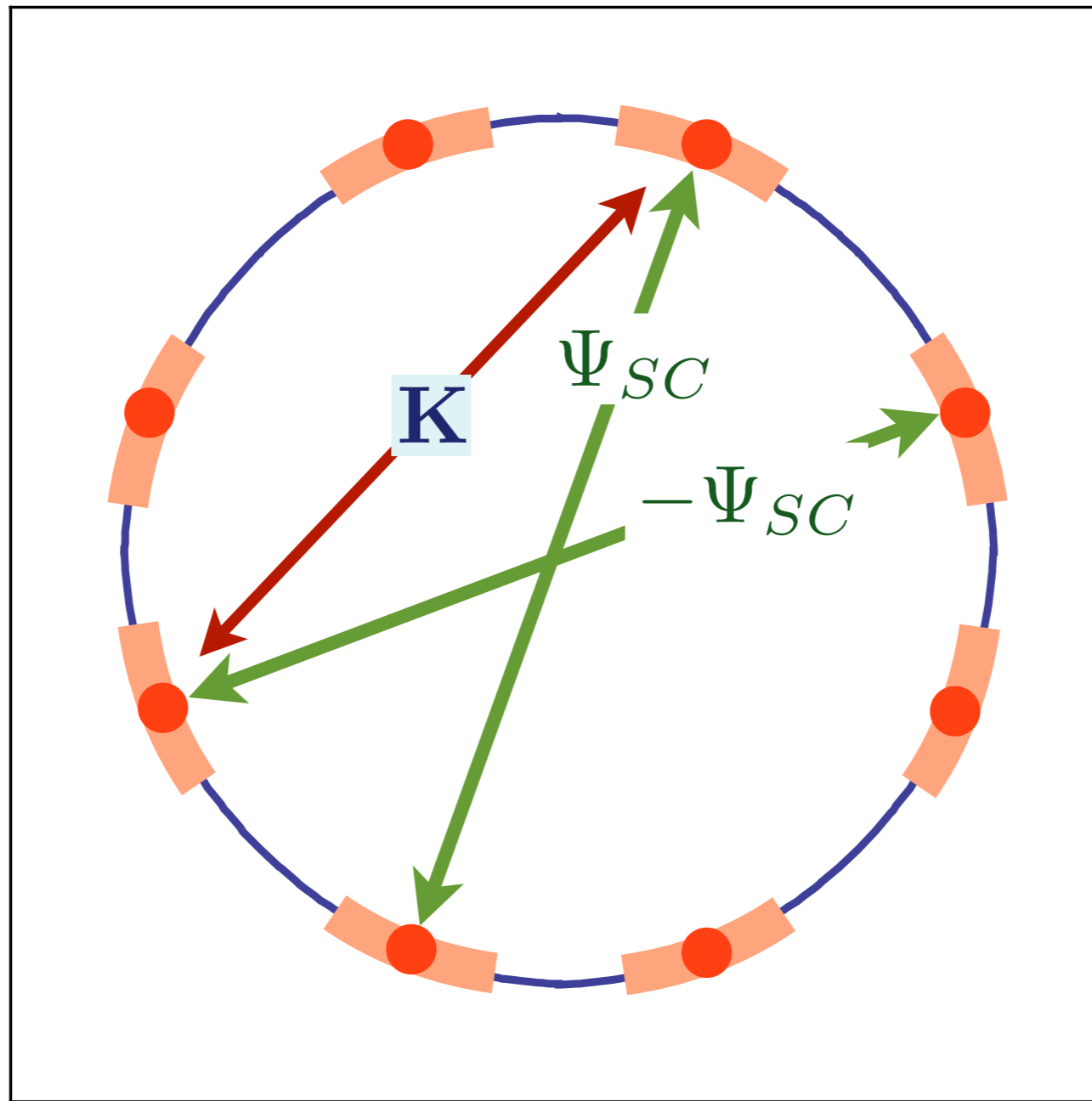


Pairing “glue” from antiferromagnetic fluctuations



V.J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} (\cos k_x - \cos k_y) \Psi_{SC}$$



**d-wave superconductor:
sign-changing pairing amplitude**

V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

Pseudospin symmetry of the exchange interaction

$$H_J = J \vec{S}_1 \cdot \vec{S}_2$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} J \left(\Psi_{1\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{1\beta a} \right) \cdot \left(\Psi_{2\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{2\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site $\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$

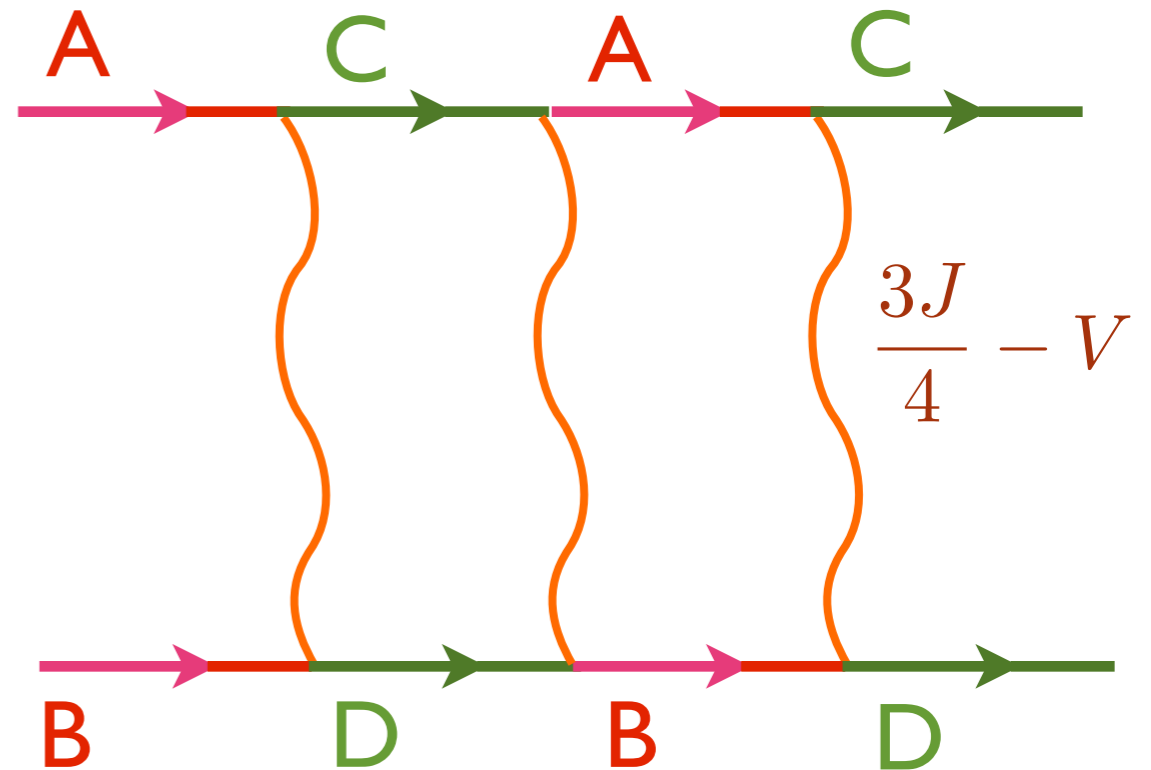
I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)

E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)

P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

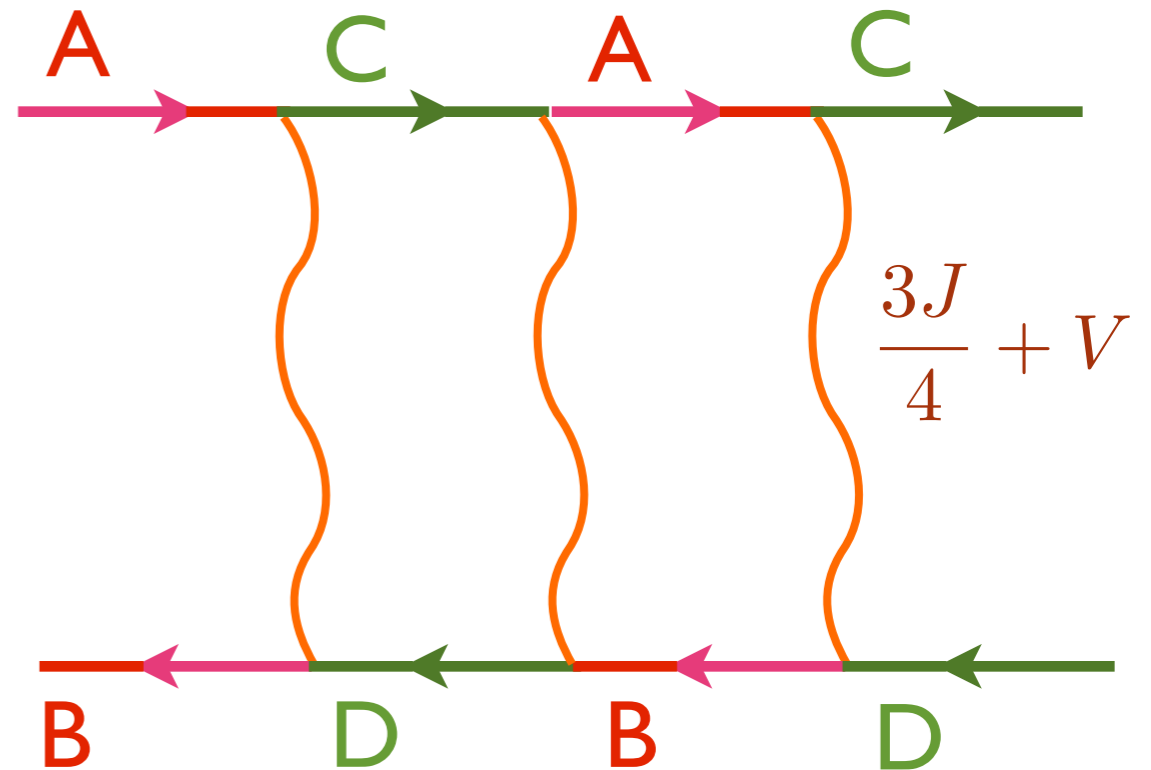
Pairing “glue” from antiferromagnetic fluctuations

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 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



Same “glue” leads to *d*-wave particle-hole pairing !

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

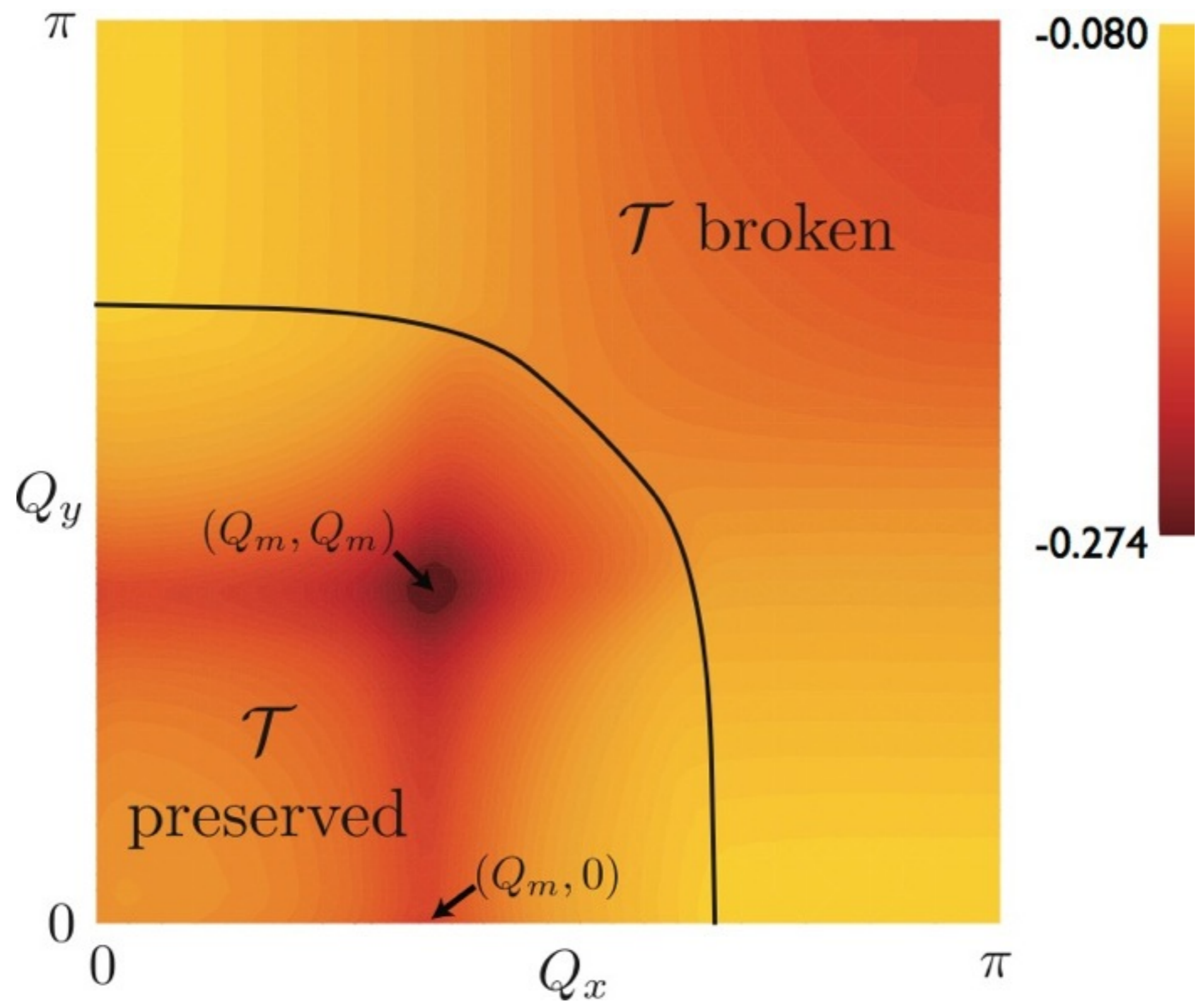


M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

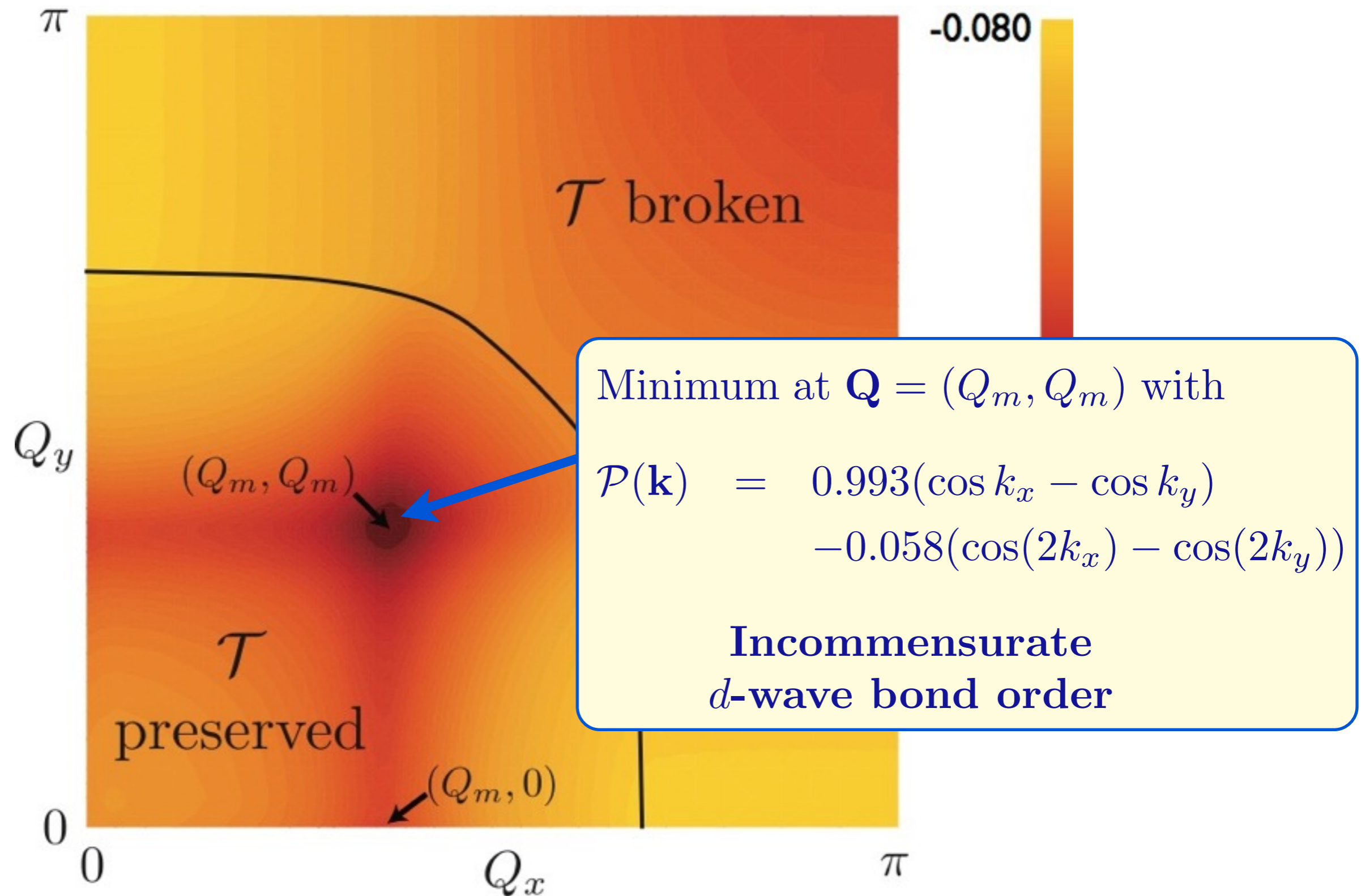
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)

J. C. Davis and Dung-Hai Lee, *Proc. Natl. Acad. Sci.* **110**, 17623 (2013)

J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)

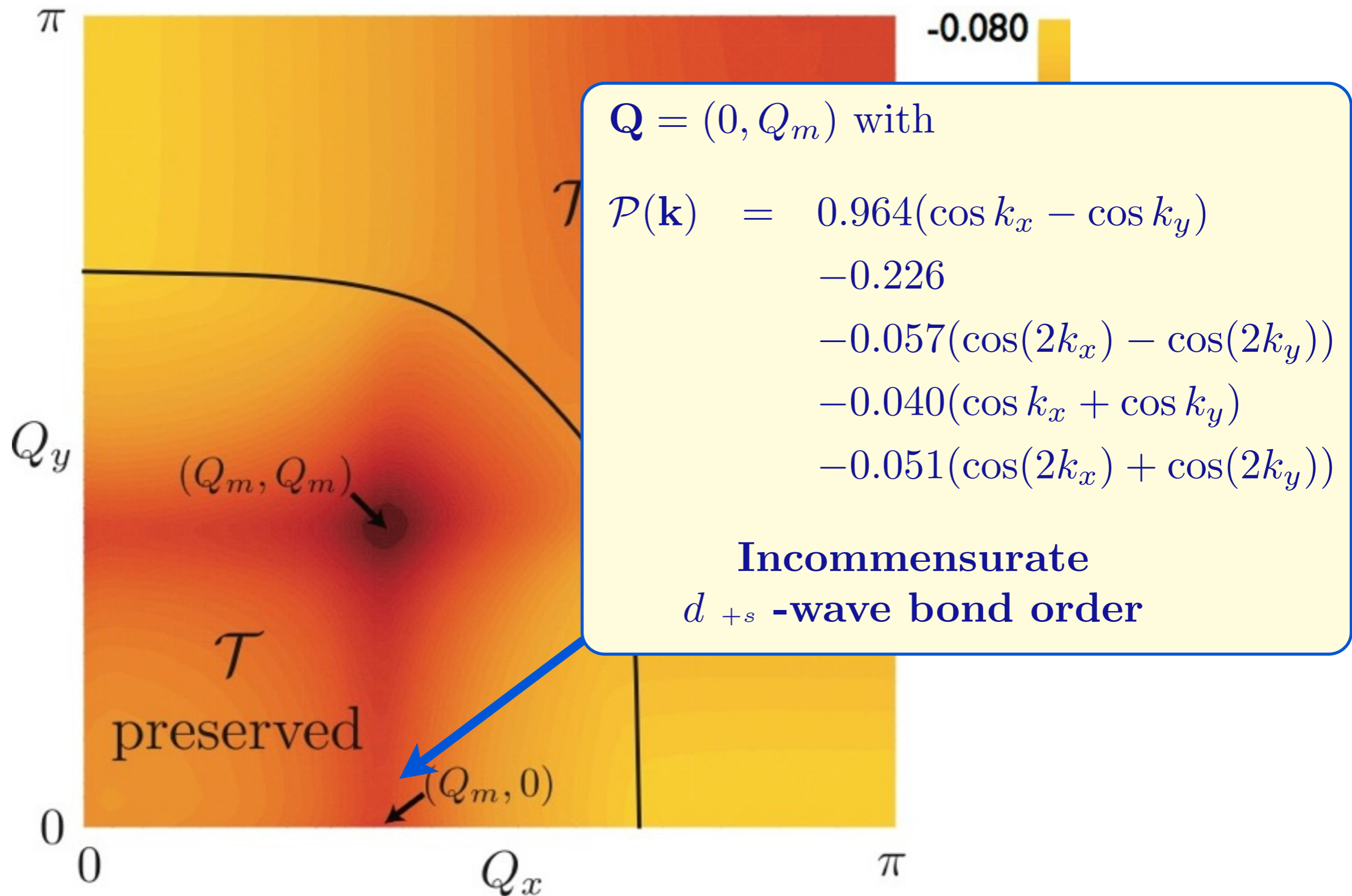


Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator. The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

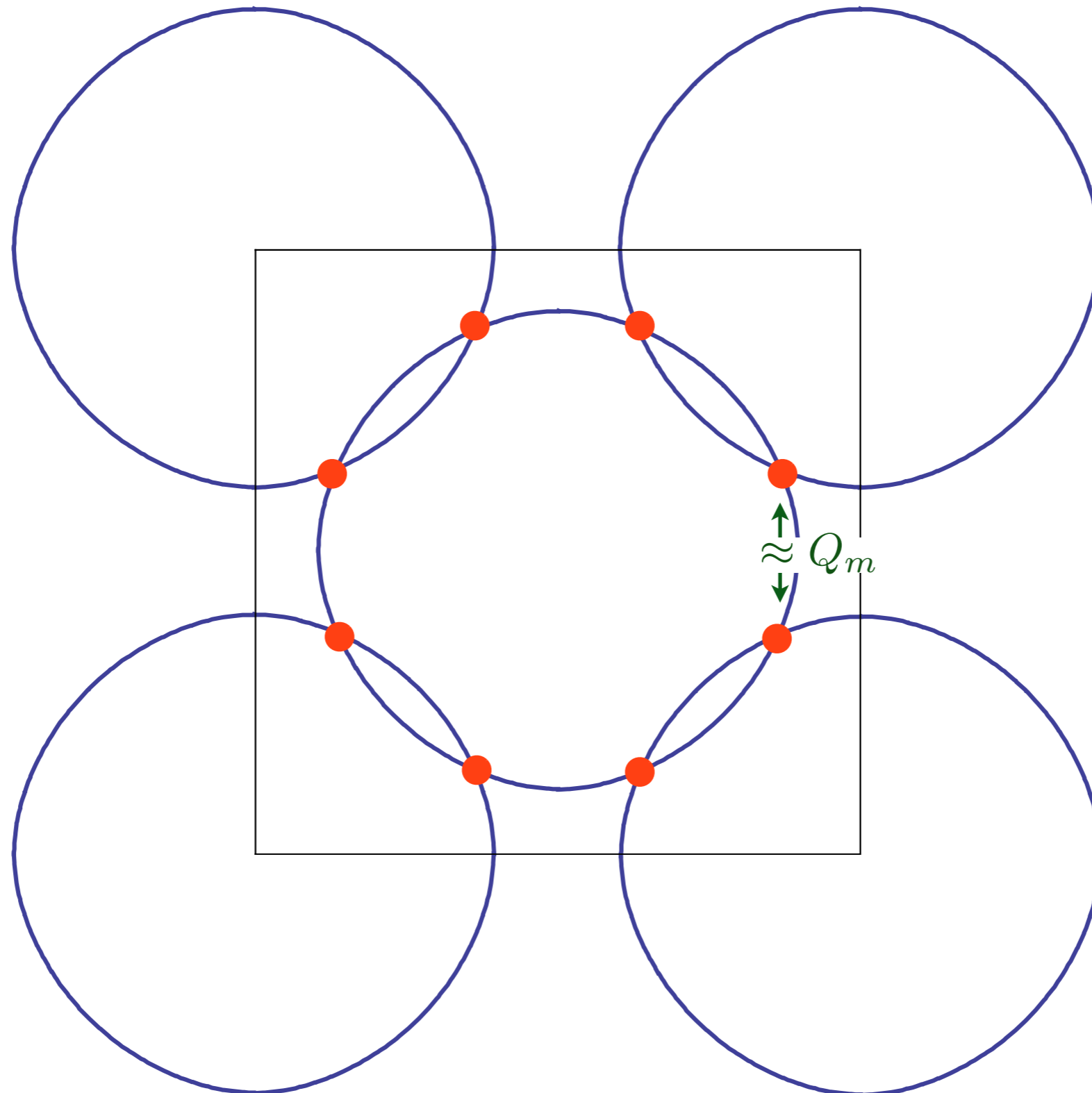
$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$



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Fermi surface+antiferromagnetism



Q_m is approximately the separation between hotspots.

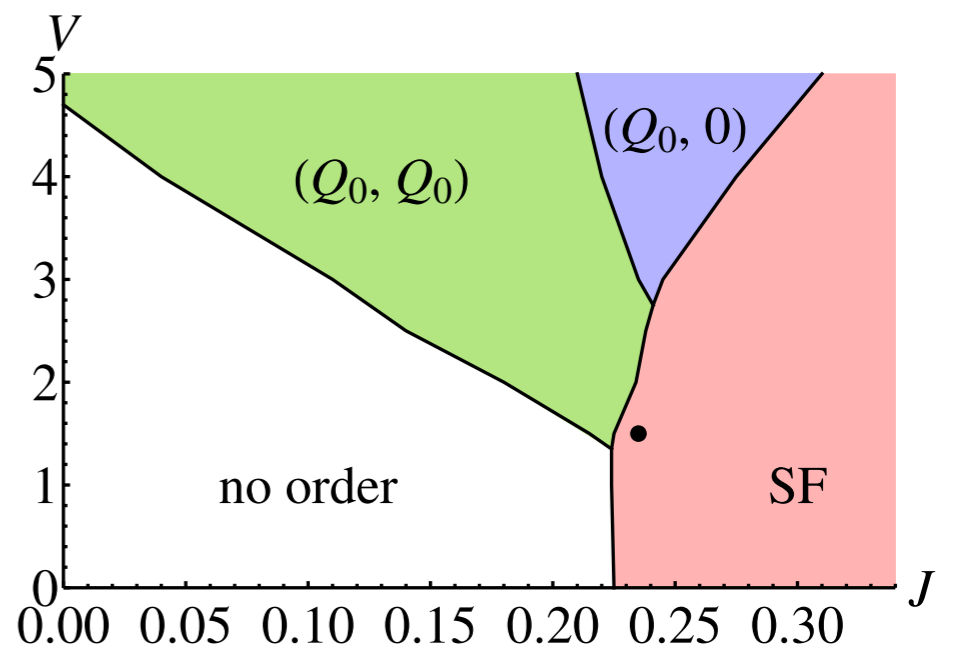
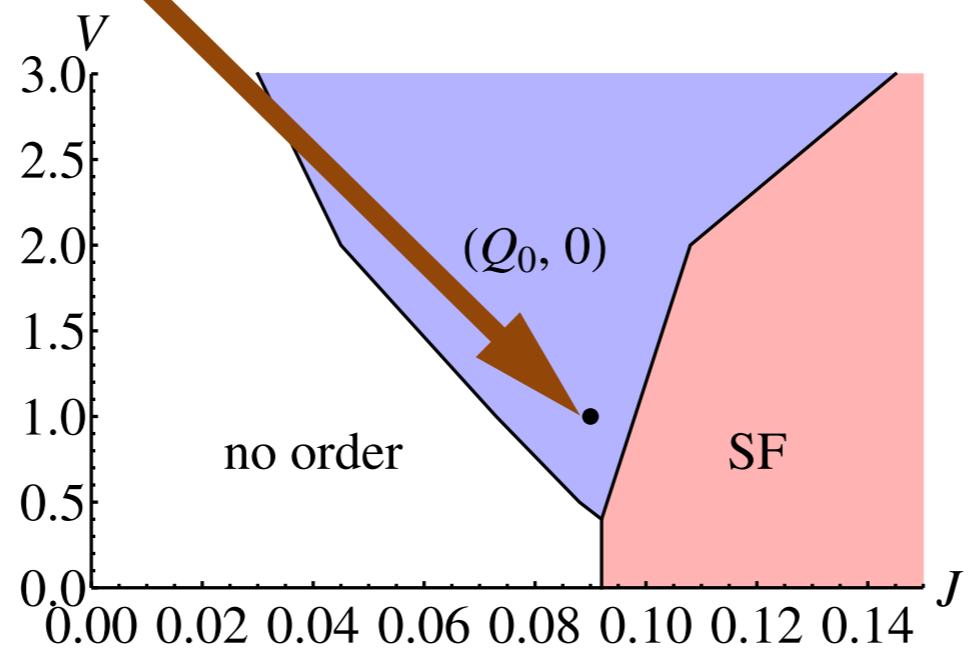
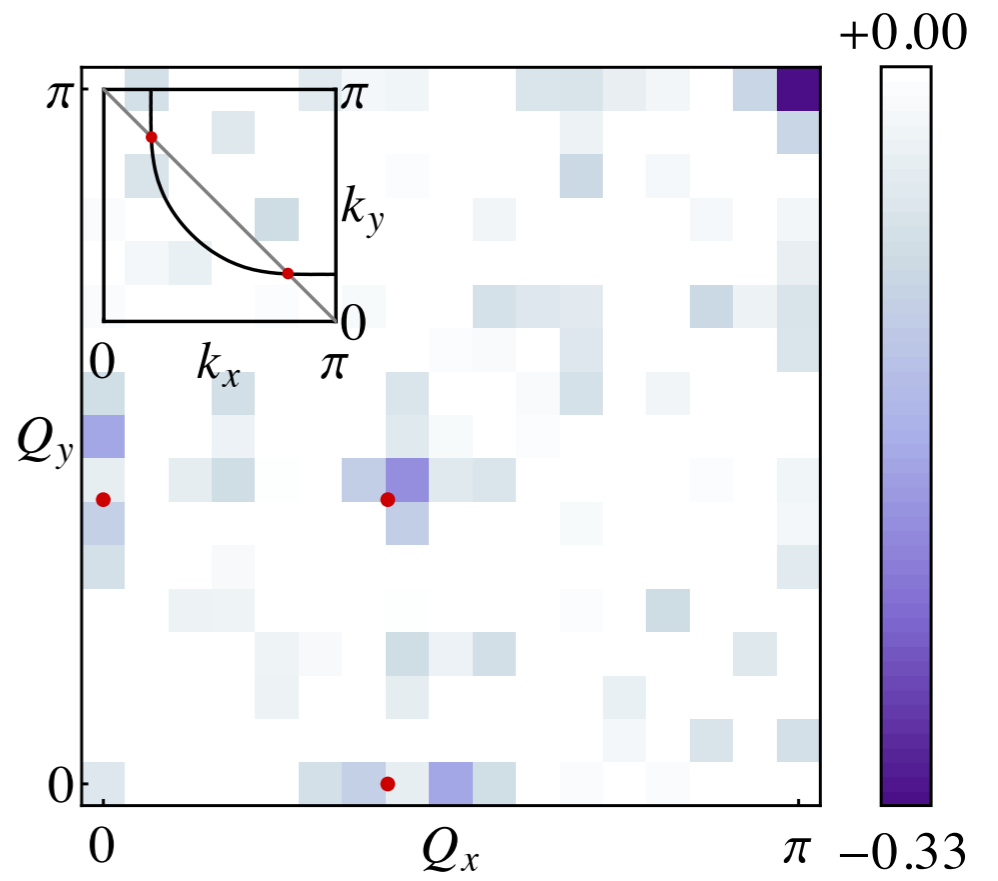
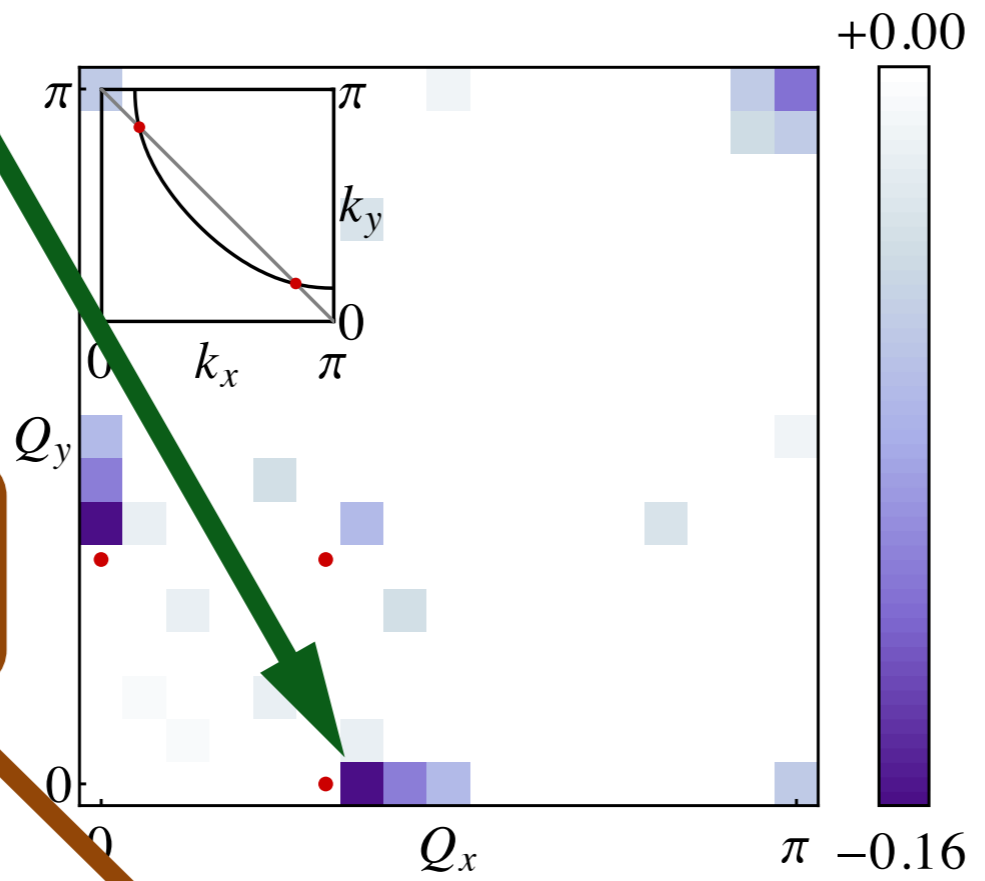
Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
 A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807

Q of the lowest energy state. Predominantly *d* wave

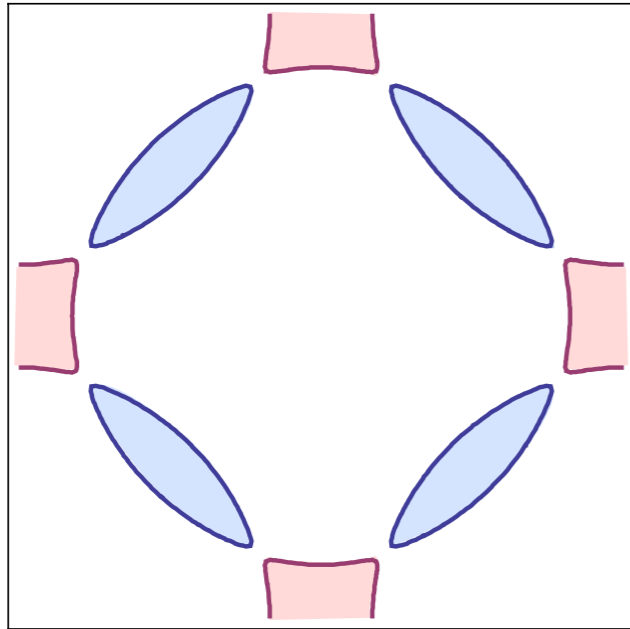
Q-plot above at this value of J, V



Andrea Allais

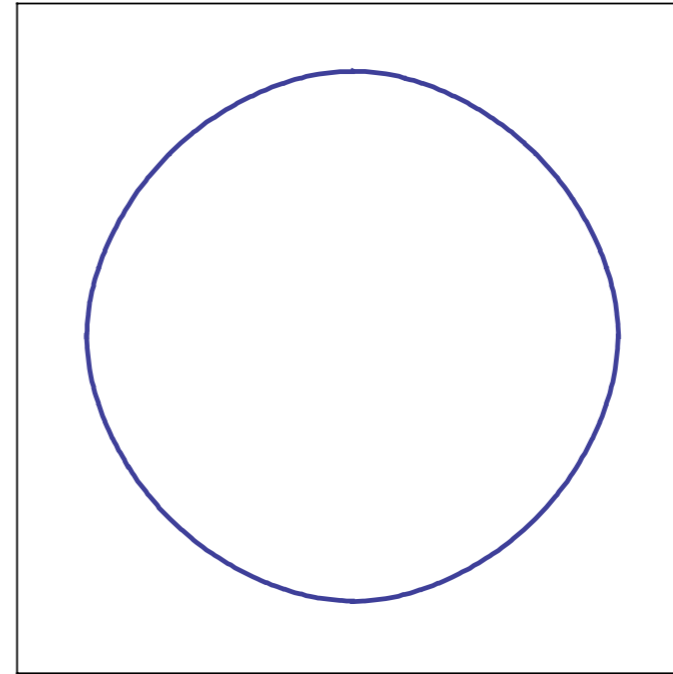


Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

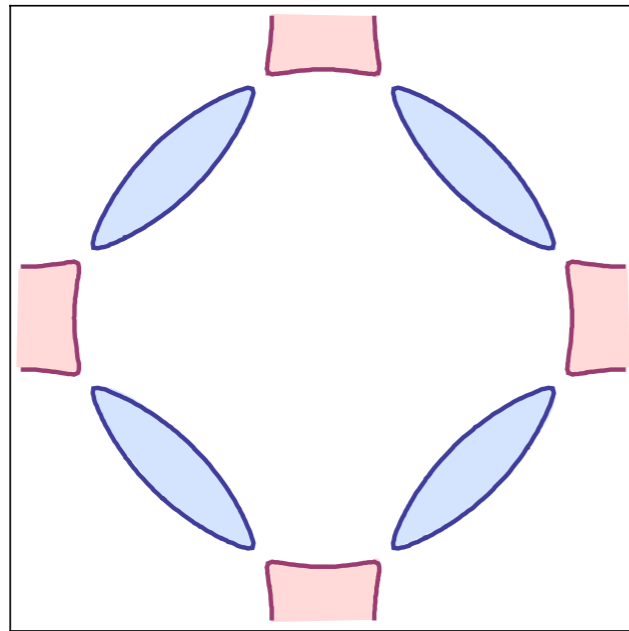


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

r

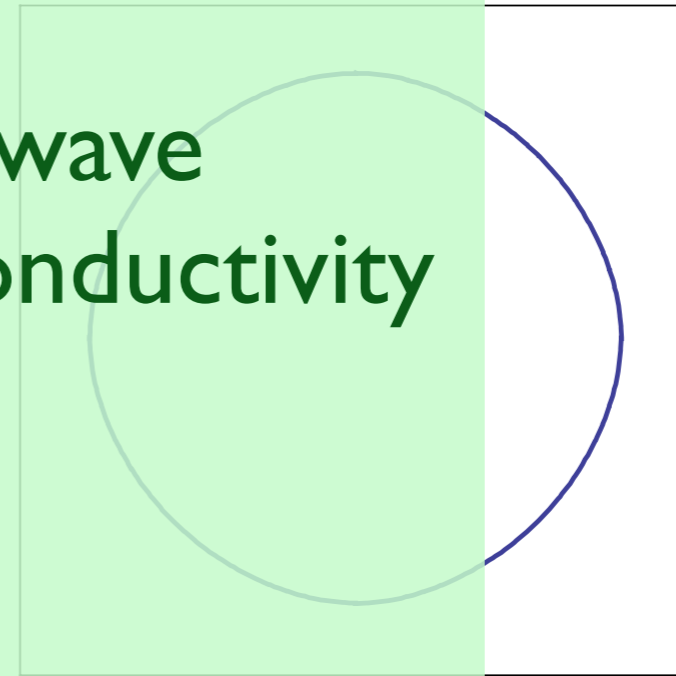
Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

d-wave
superconductivity

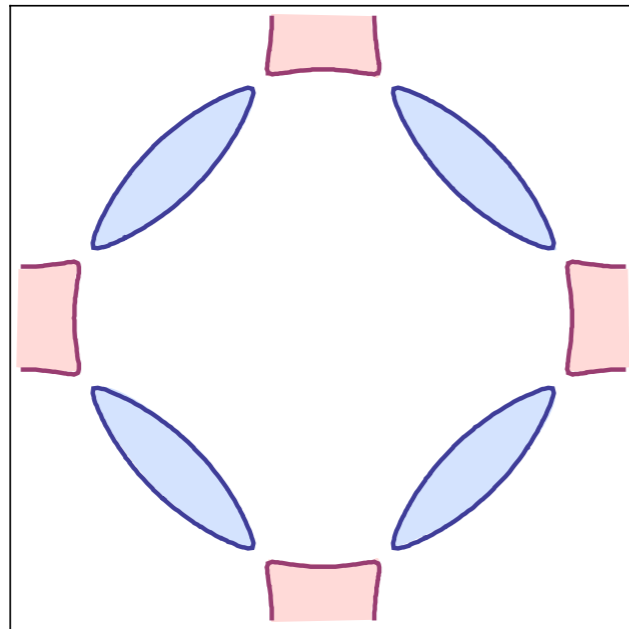


$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large"
Fermi surface

r

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

d-wave
superconductivity
and
an unconventional
density wave
with a
d-wave form factor

$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

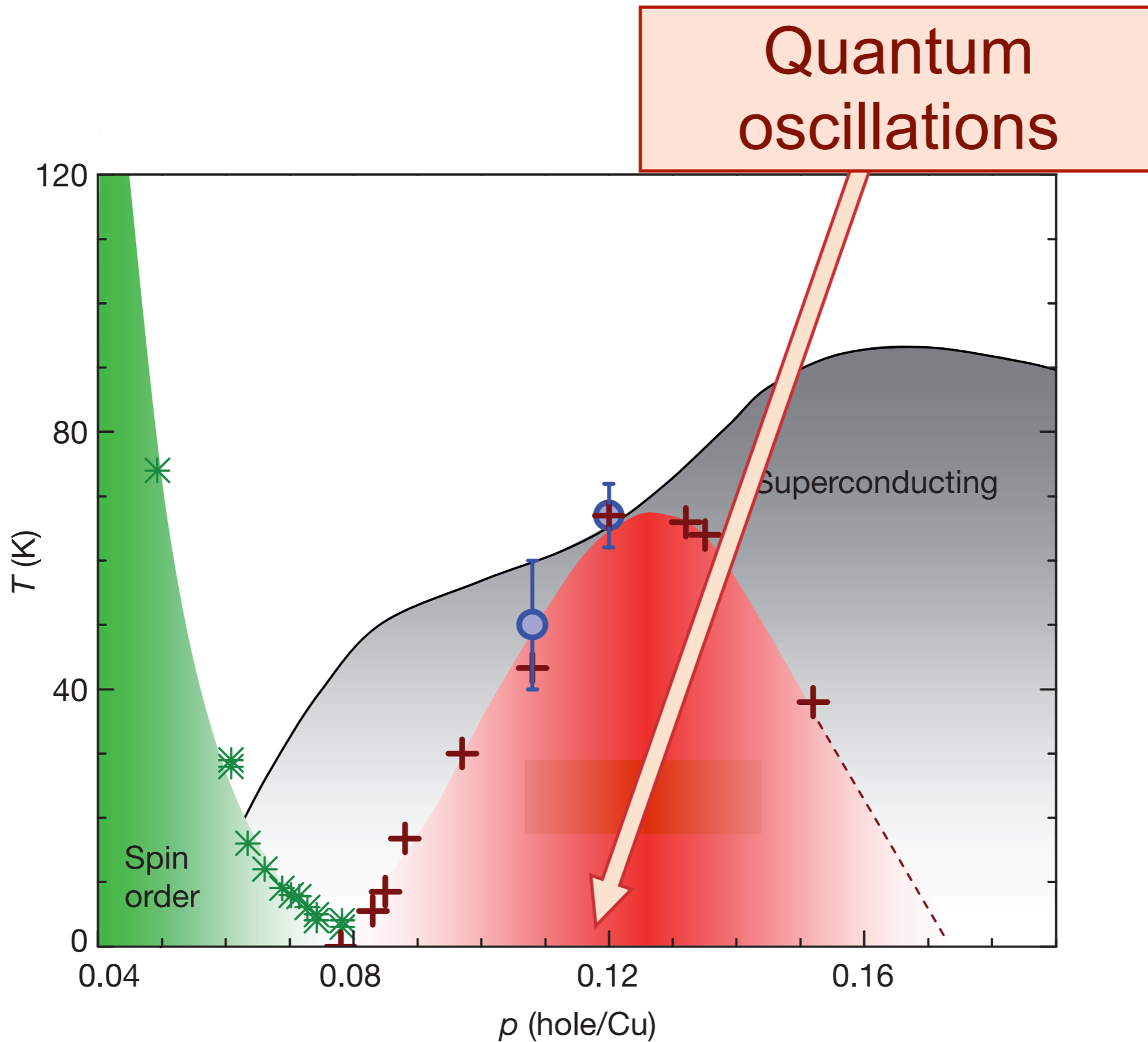
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Outline

1. Unconventional d -wave superconductivity
2. Low hole density state:
An unconventional density wave
3. Outline of theoretical prediction
4. Evolution of Fermi surface

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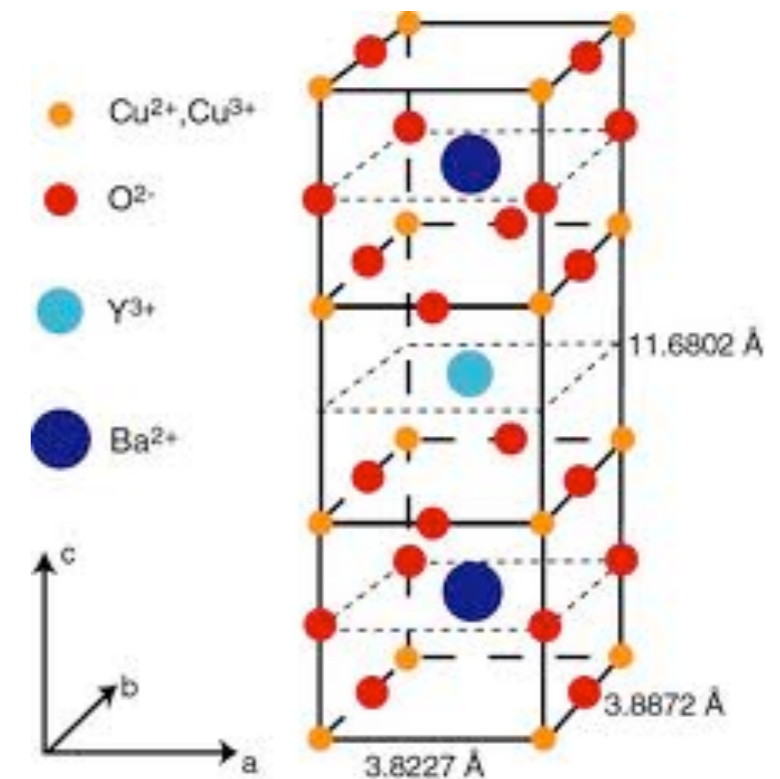
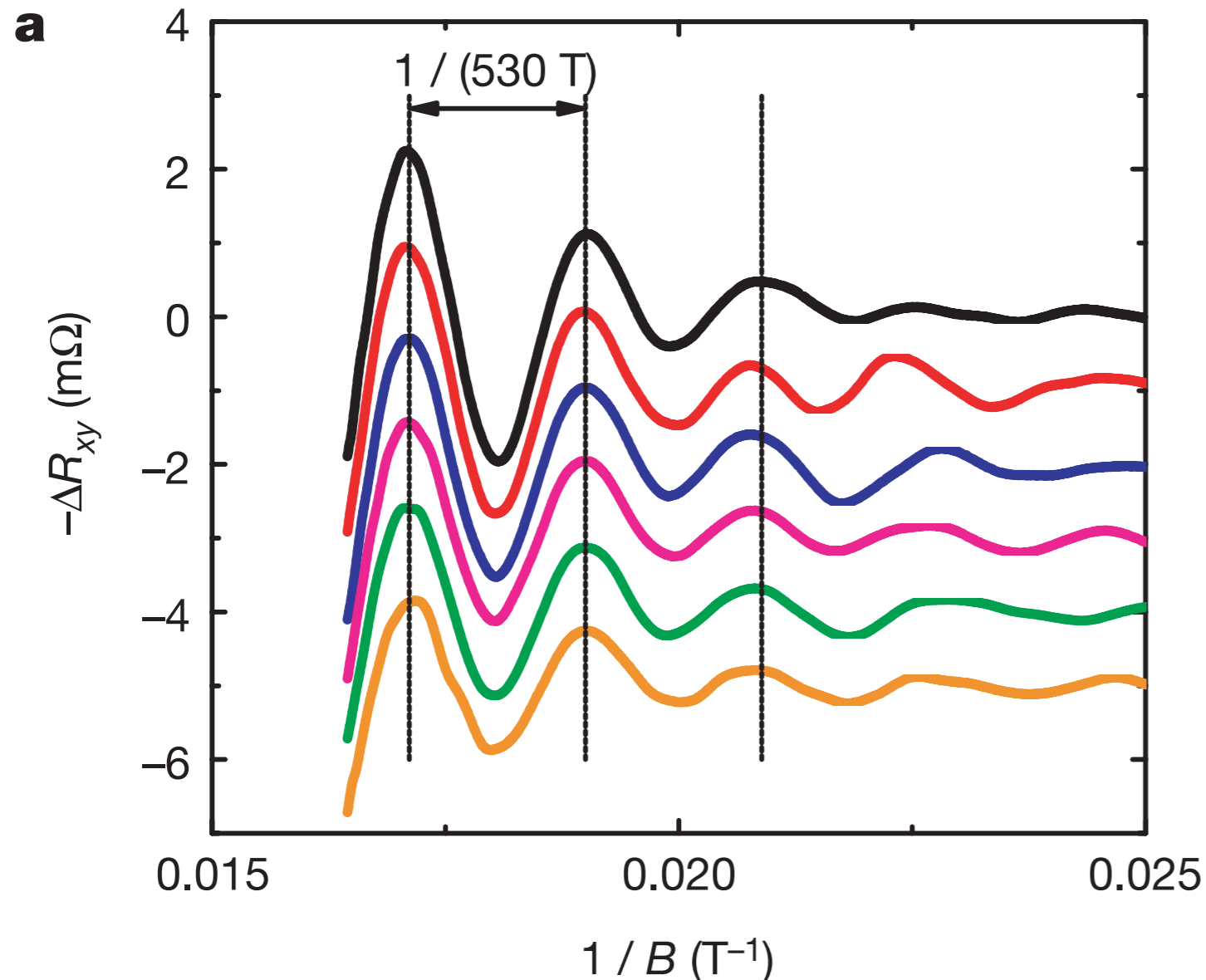


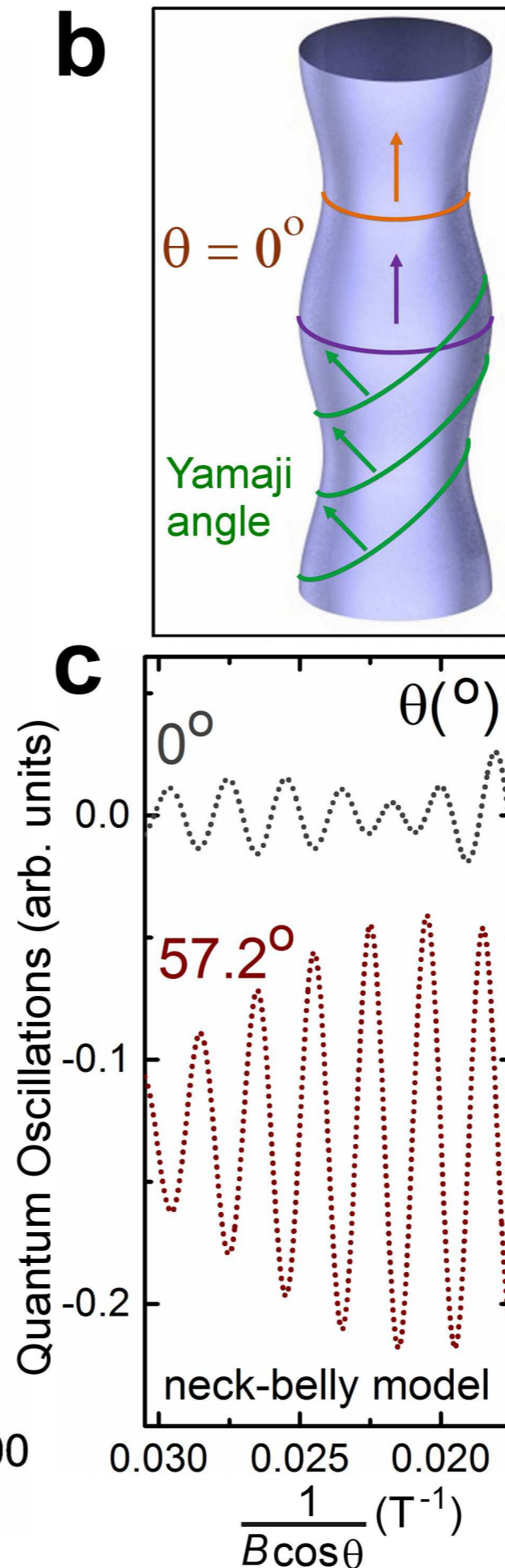
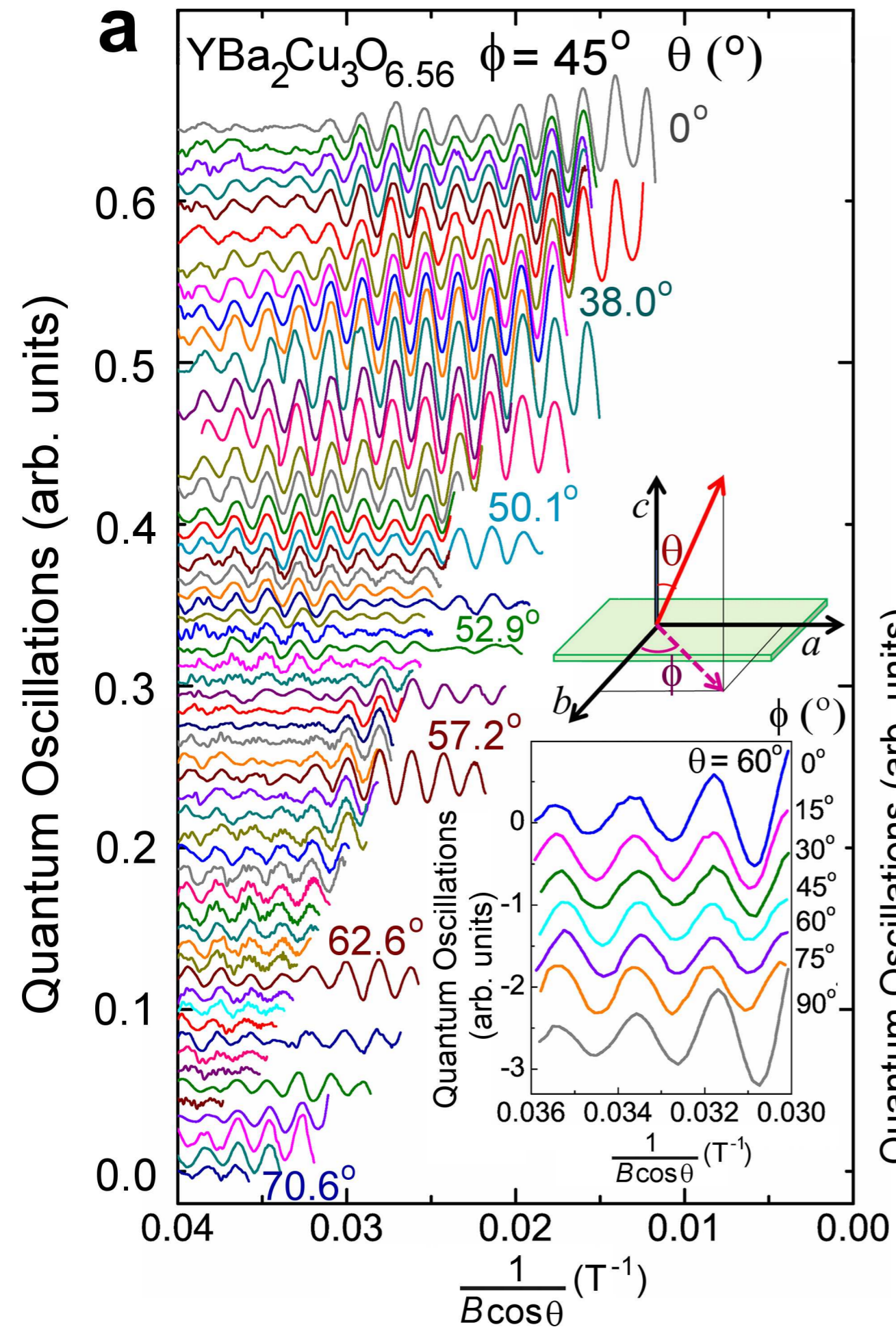
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).

Quantum oscillations and the Fermi surface in an underdoped high- T_c superconductor

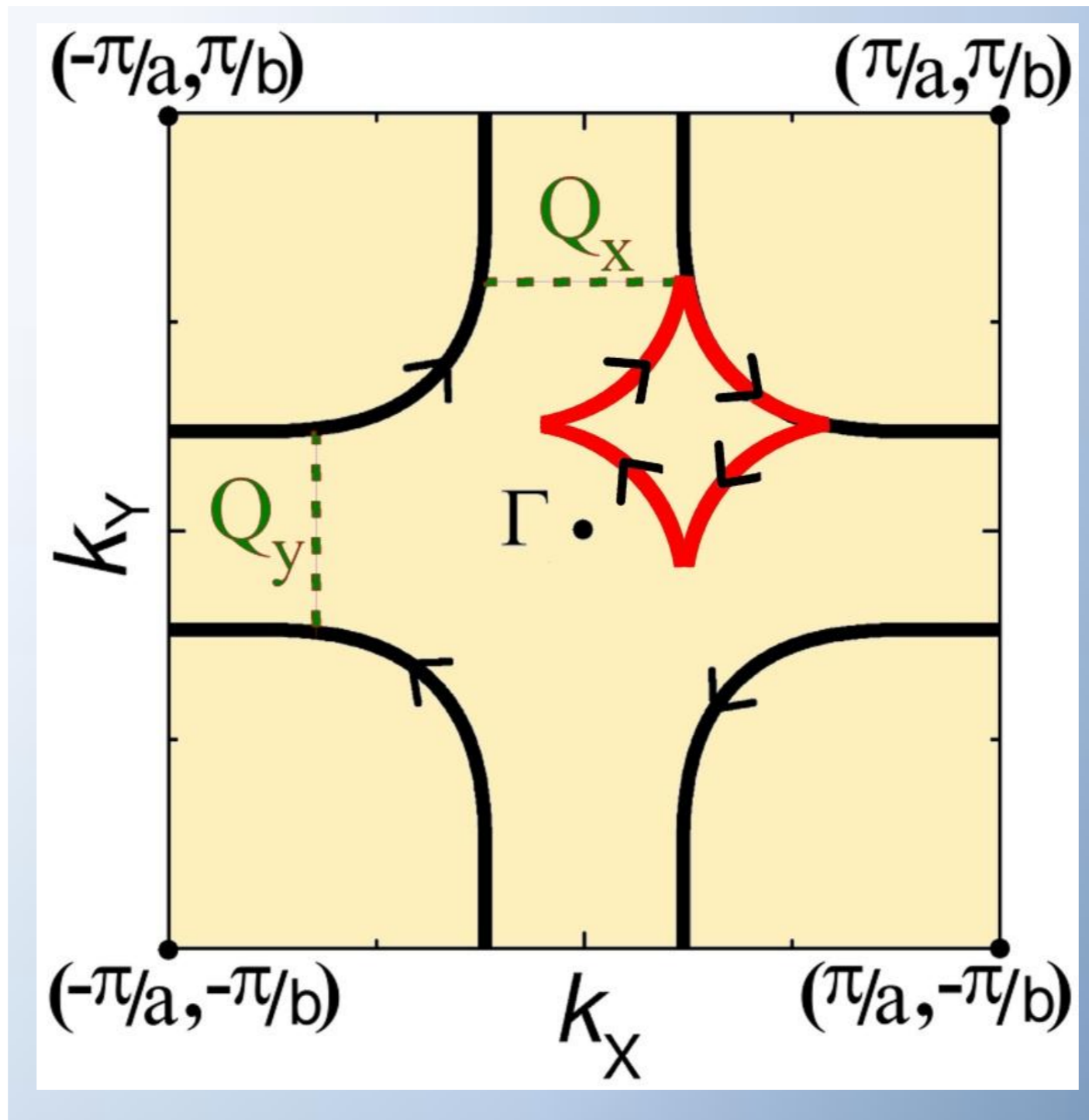
Nicolas Doiron-Leyraud¹, Cyril Proust², David LeBoeuf¹, Julien Levallois², Jean-Baptiste Bonnemaïson¹, Ruixing Liang^{3,4}, D. A. Bonn^{3,4}, W. N. Hardy^{3,4} & Louis Taillefer^{1,4}

Nature 447, 565 (2007)





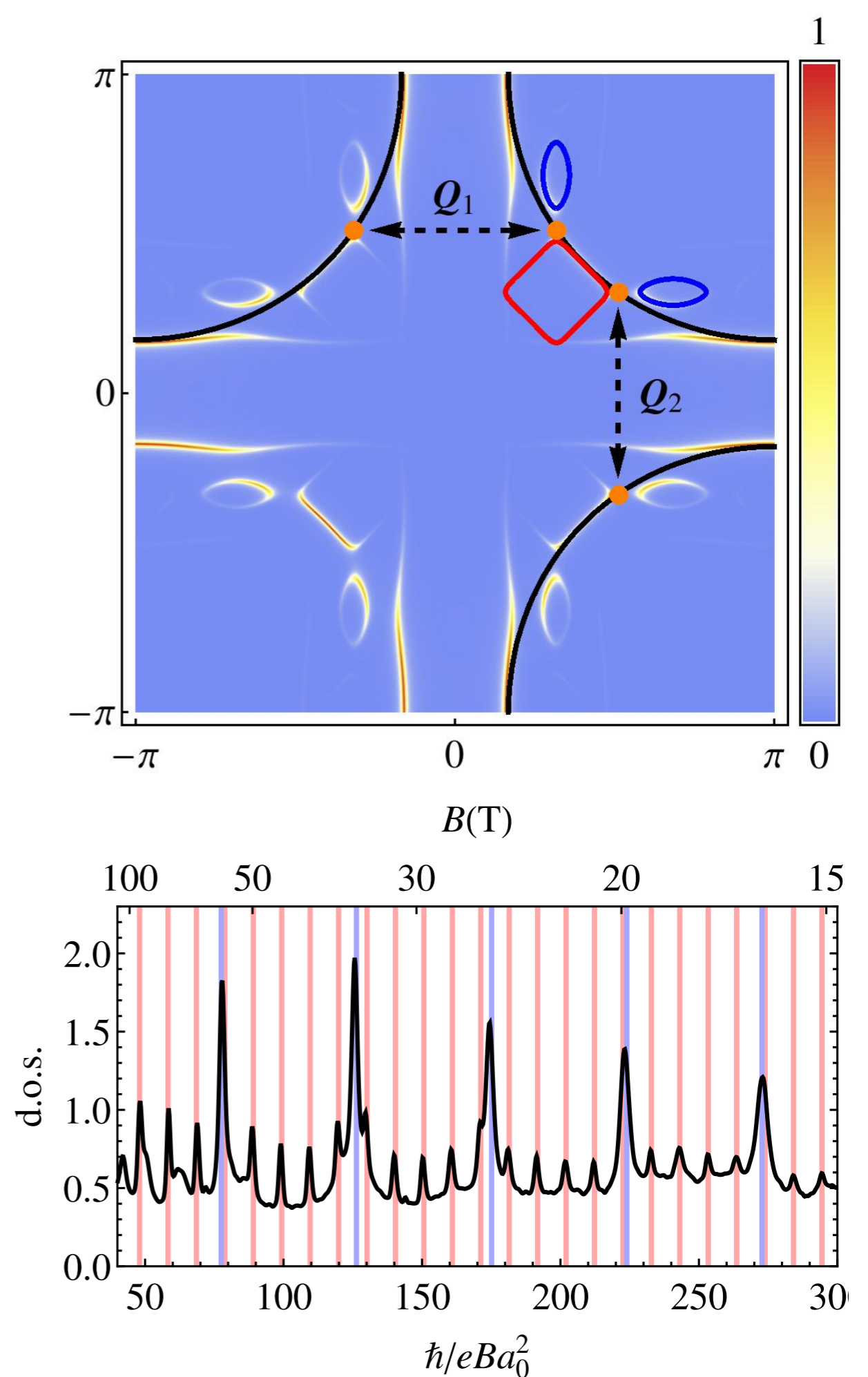
Normal-state nodal
electronic structure
in underdoped
high- T_c copper oxides
Suchitra
E. Sebastian,
N. Harrison,
F. F. Balakirev,
M. M. Altarawneh,
P. A. Goddard,
Ruixing Liang,
D. A. Bonn,
W. N. Hardy, and
G. G. Lonzarich
Nature, to appear



N. Harrison and S. E. Sebastian, Phys. Rev. Lett. **106**, 226402 (2011).

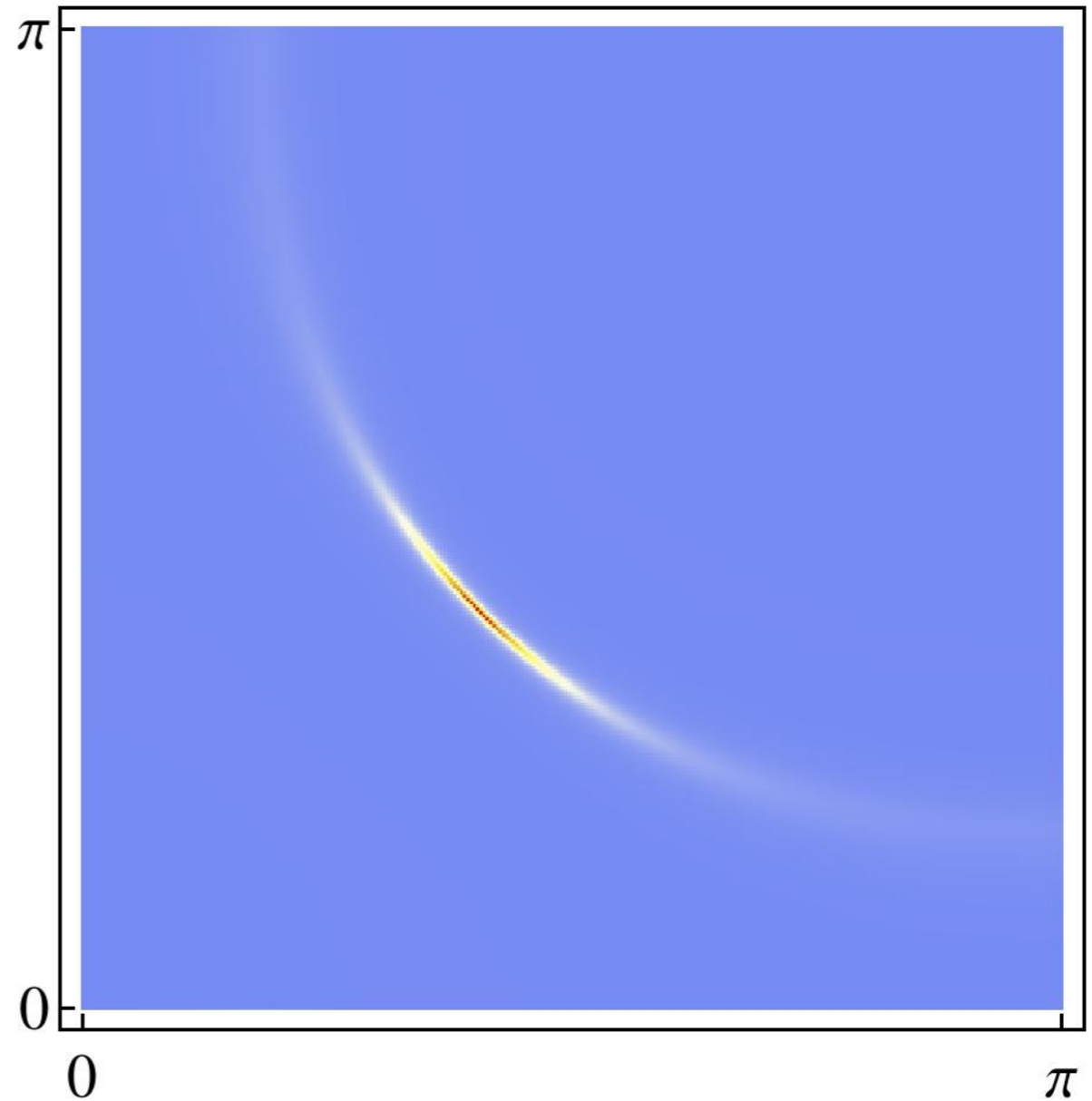
Electron spectral density
and
quantum oscillations
from bi-directional
density wave order

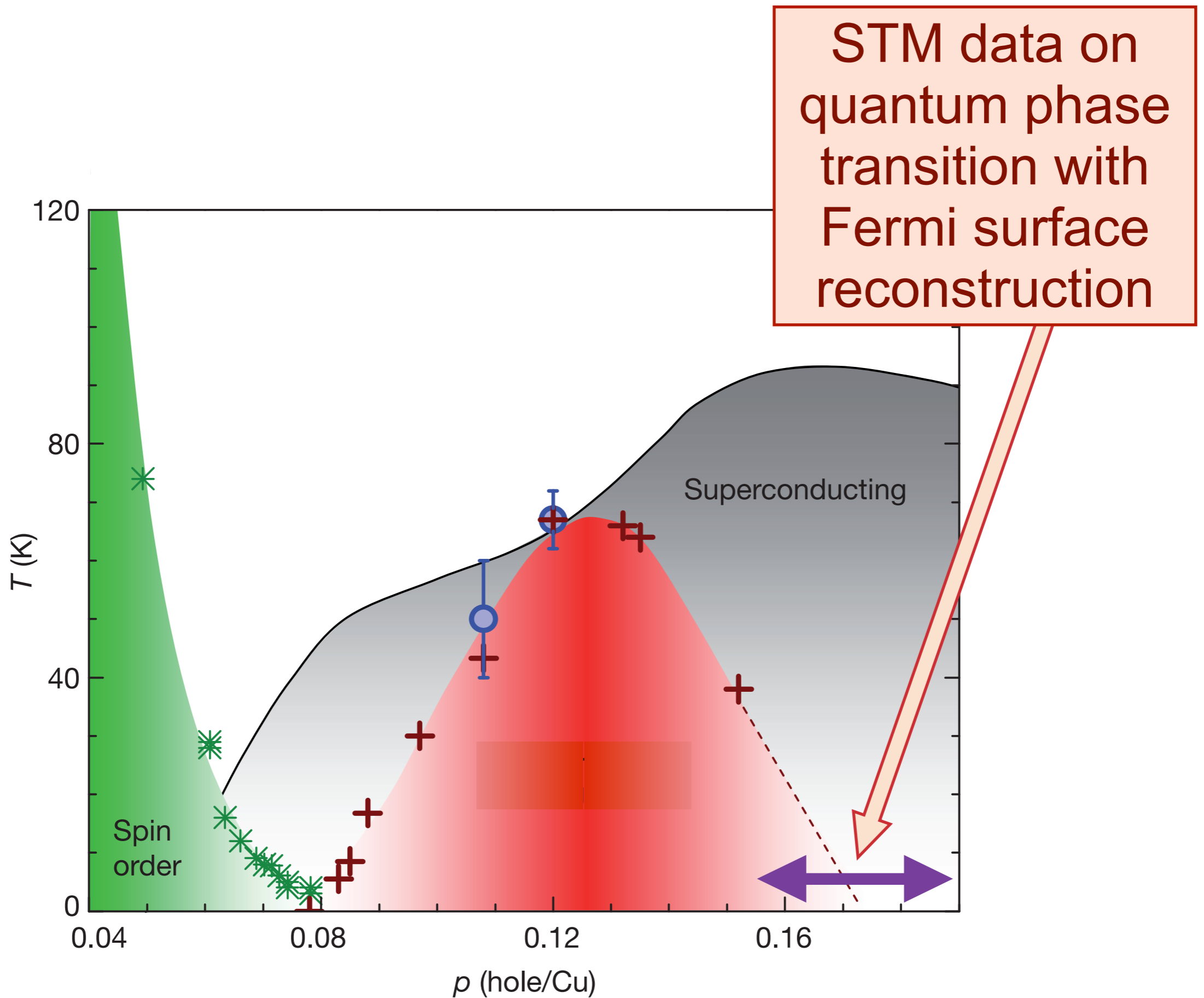
Andrea Allais,
Debanjan Chowdhury,
and Subir Sachdev,
arXiv:1406.0503



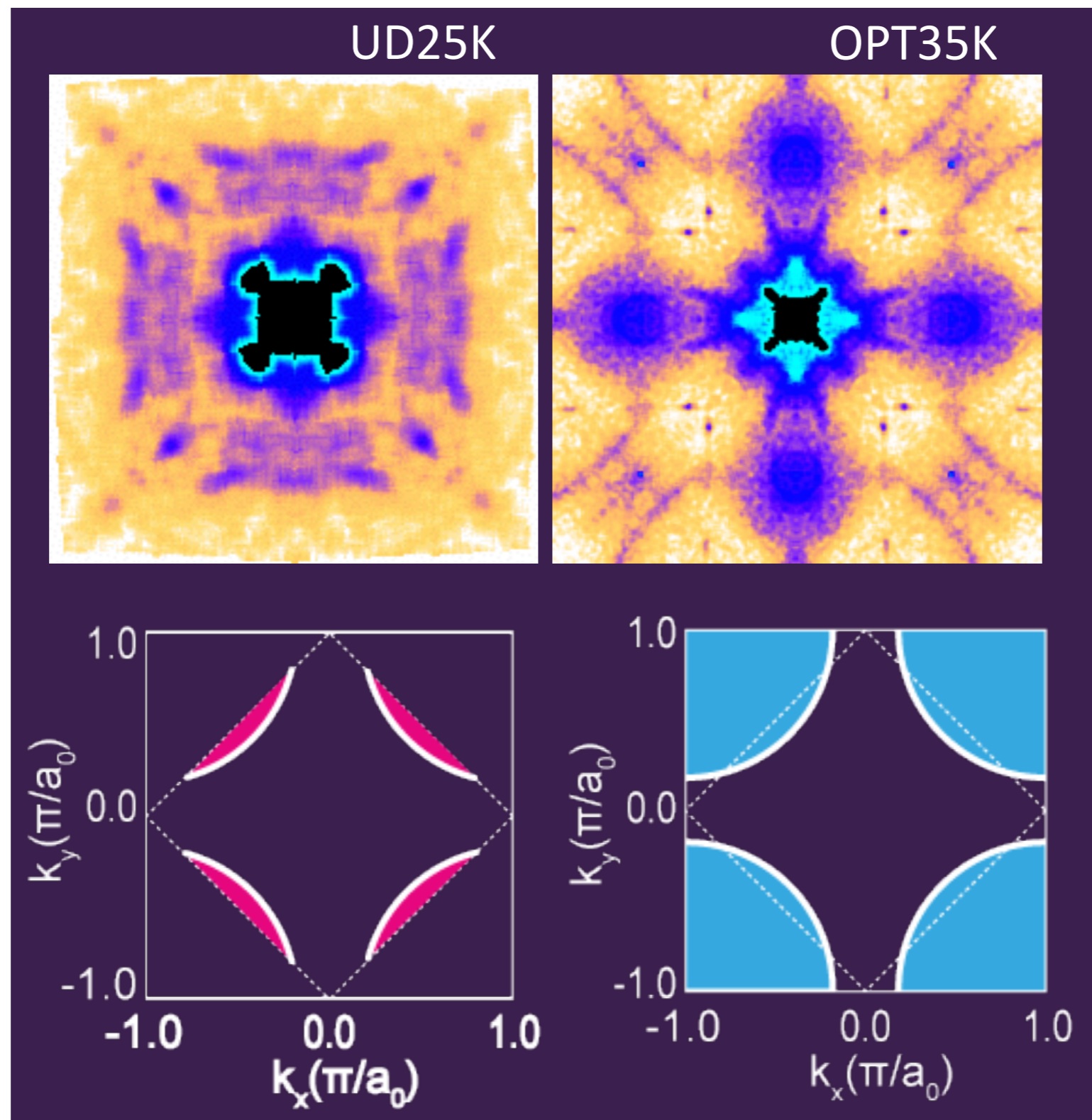
Electron spectral density
from fluctuating
superconducting
and density wave
orders

Andrea Allais,
Debanjan Chowdhury,
and Subir Sachdev,
arXiv:1406.0503

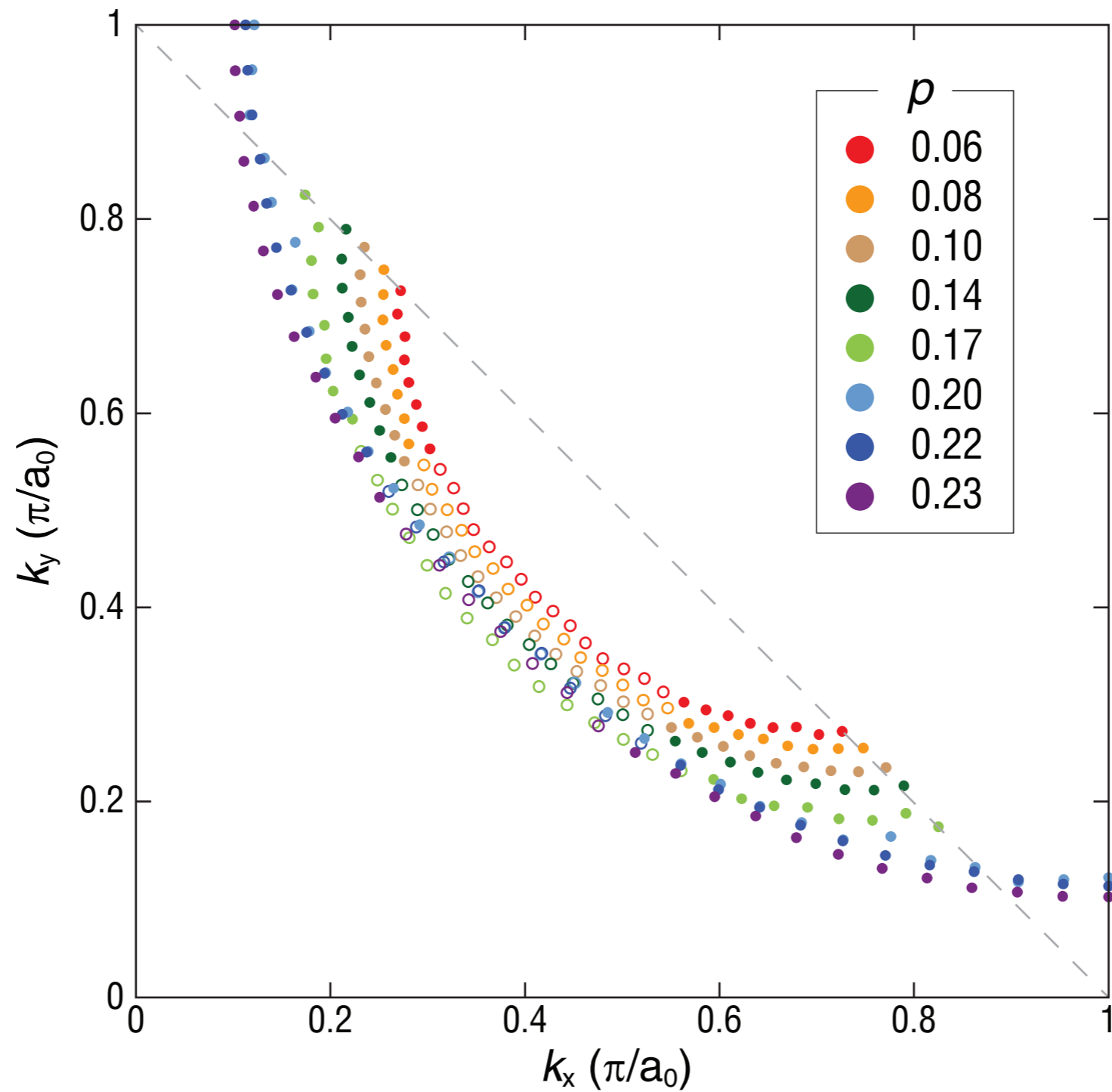




T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



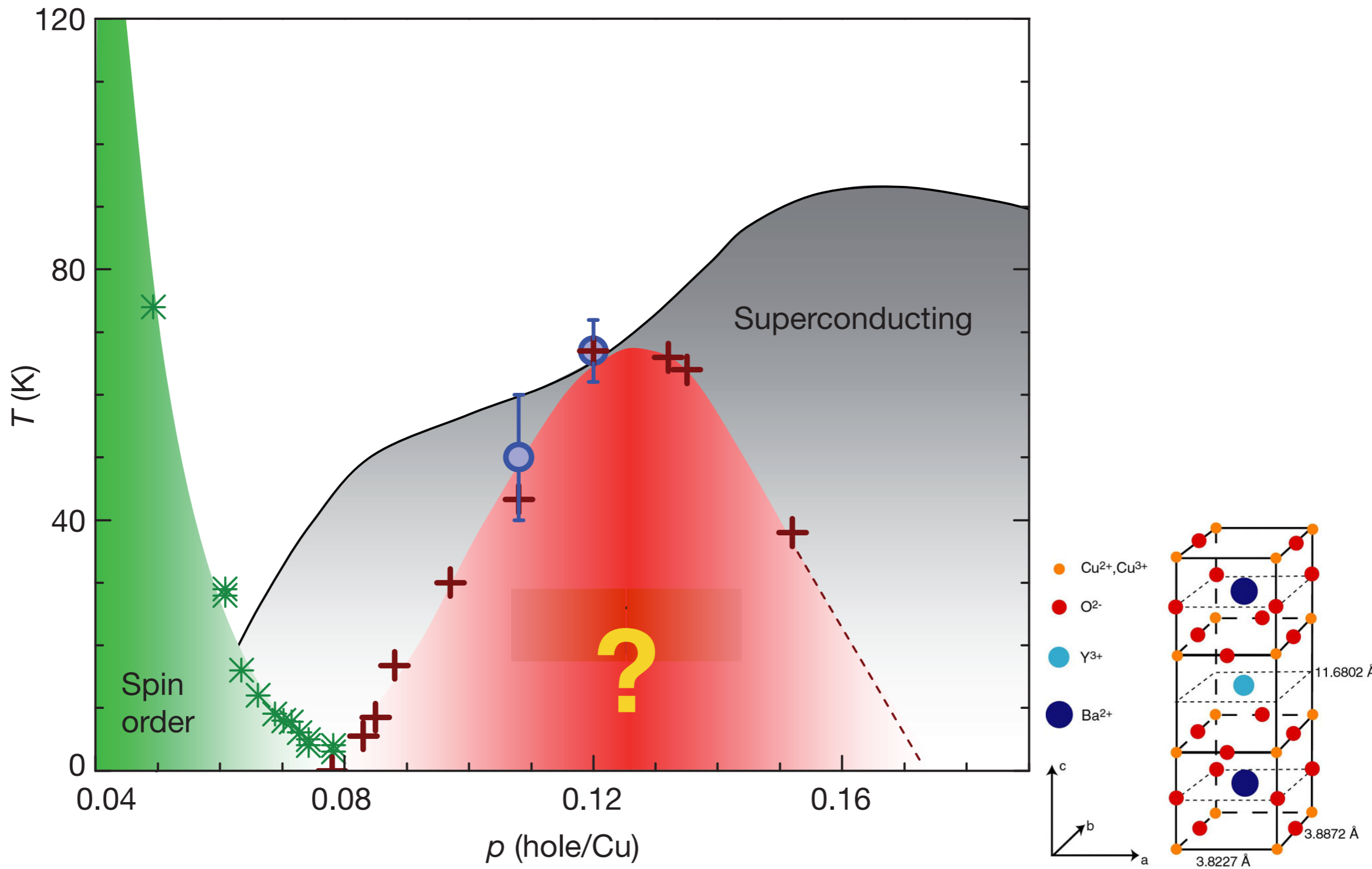
Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, J. E. Hoffman, *Science* **344**, 608 (2014).



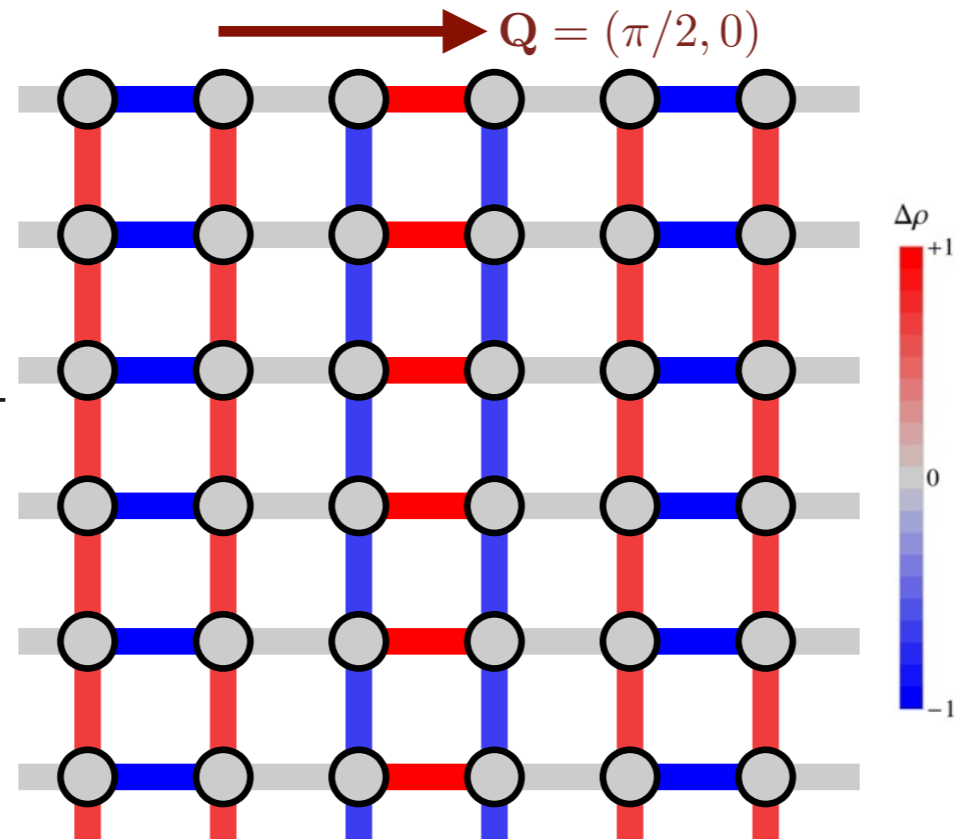
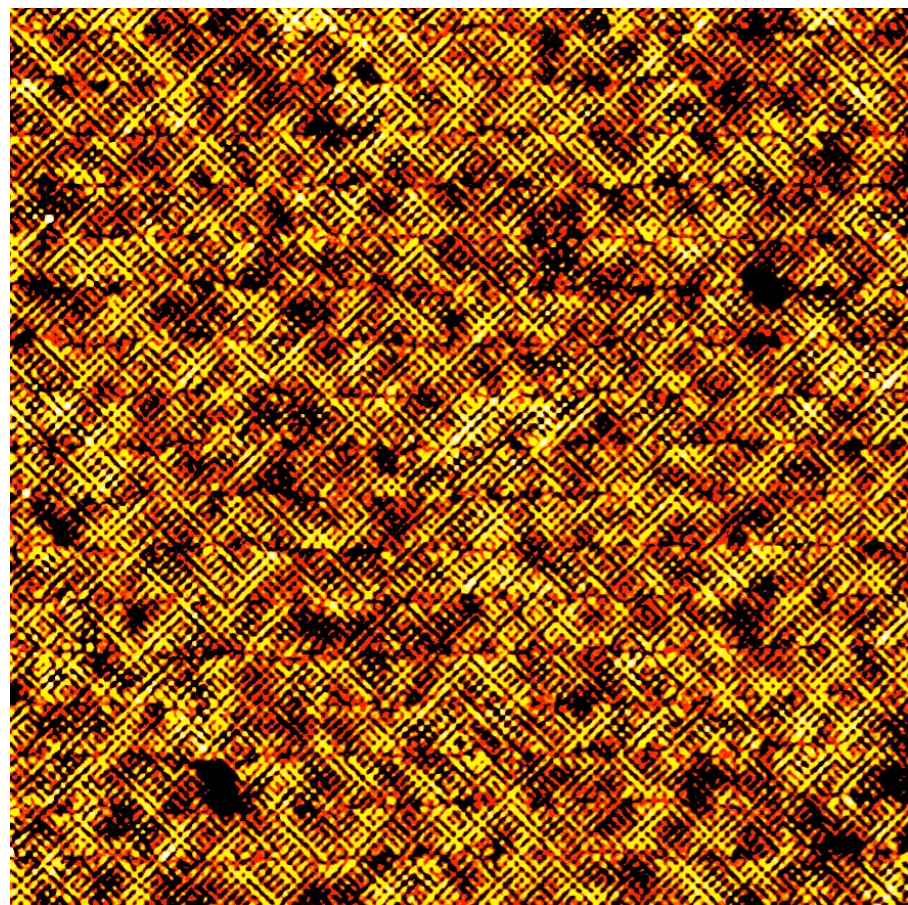
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, and J. C. Davis, *Science* **344**, 612 (2014).

Conclusions

1. *d*-wave superconductivity
2. Low hole density state:
*An unconventional density wave:
d-wave bond order*
3. Theoretical background
4. Evolution of Fermi surface



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



Phase-sensitive measurement of the *d* symmetry of density wave order

