Planckian metals

Quantum Matter Frontier Seminars
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Subir Sachdev

INSTITUTE FOR ADVANCED STUDY

PHYSICS
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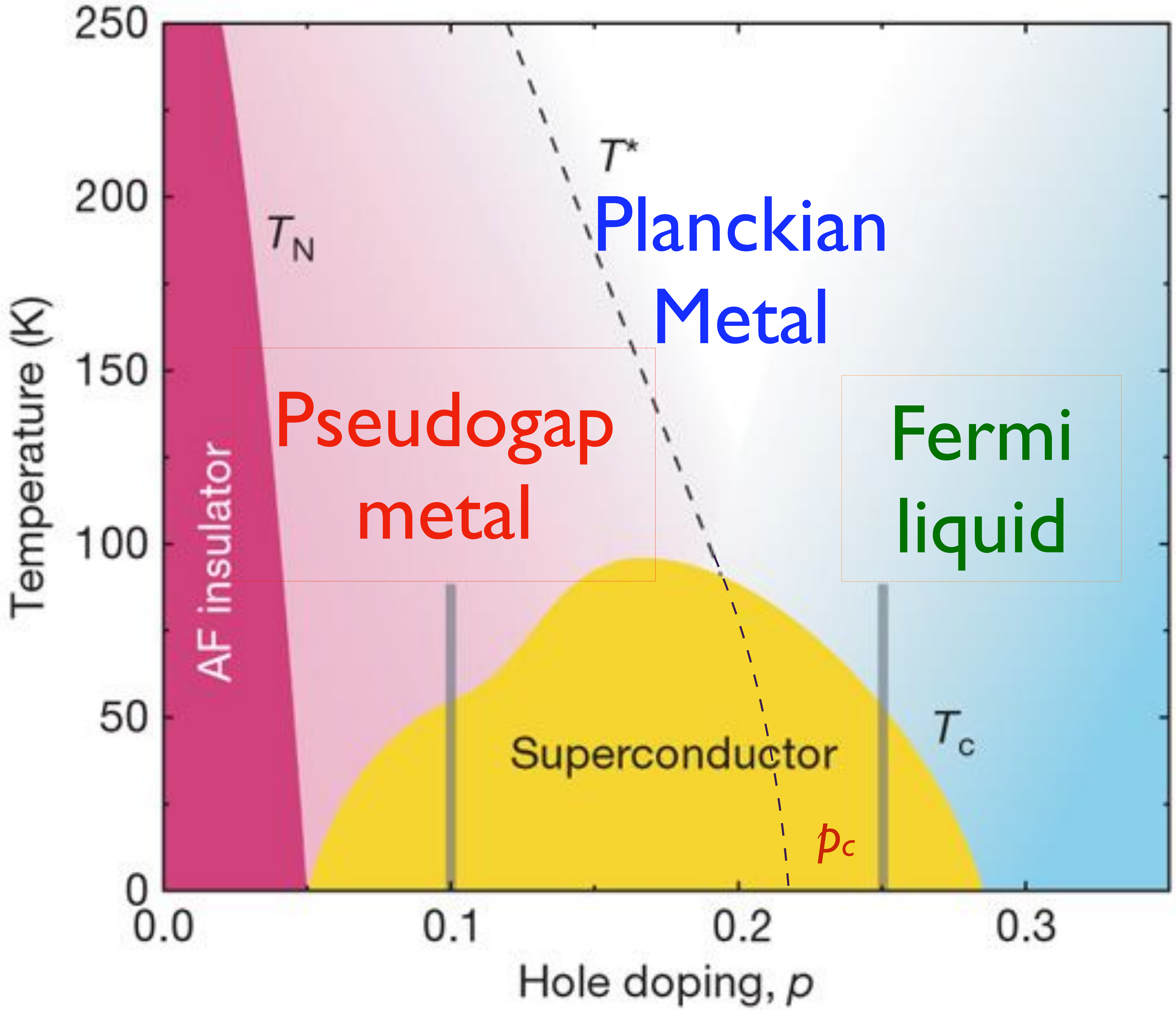
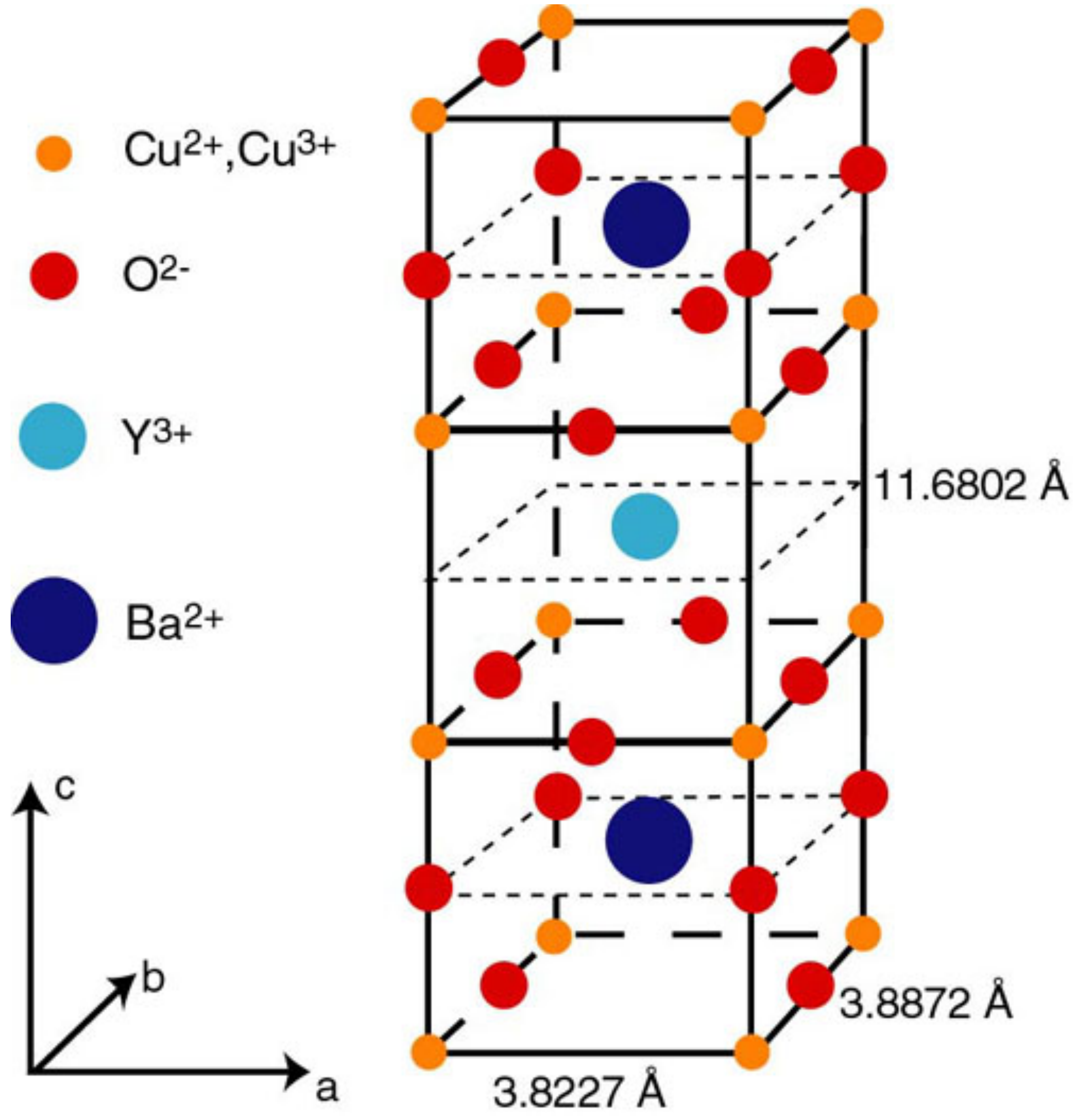
Talk online: sachdev.physics.harvard.edu

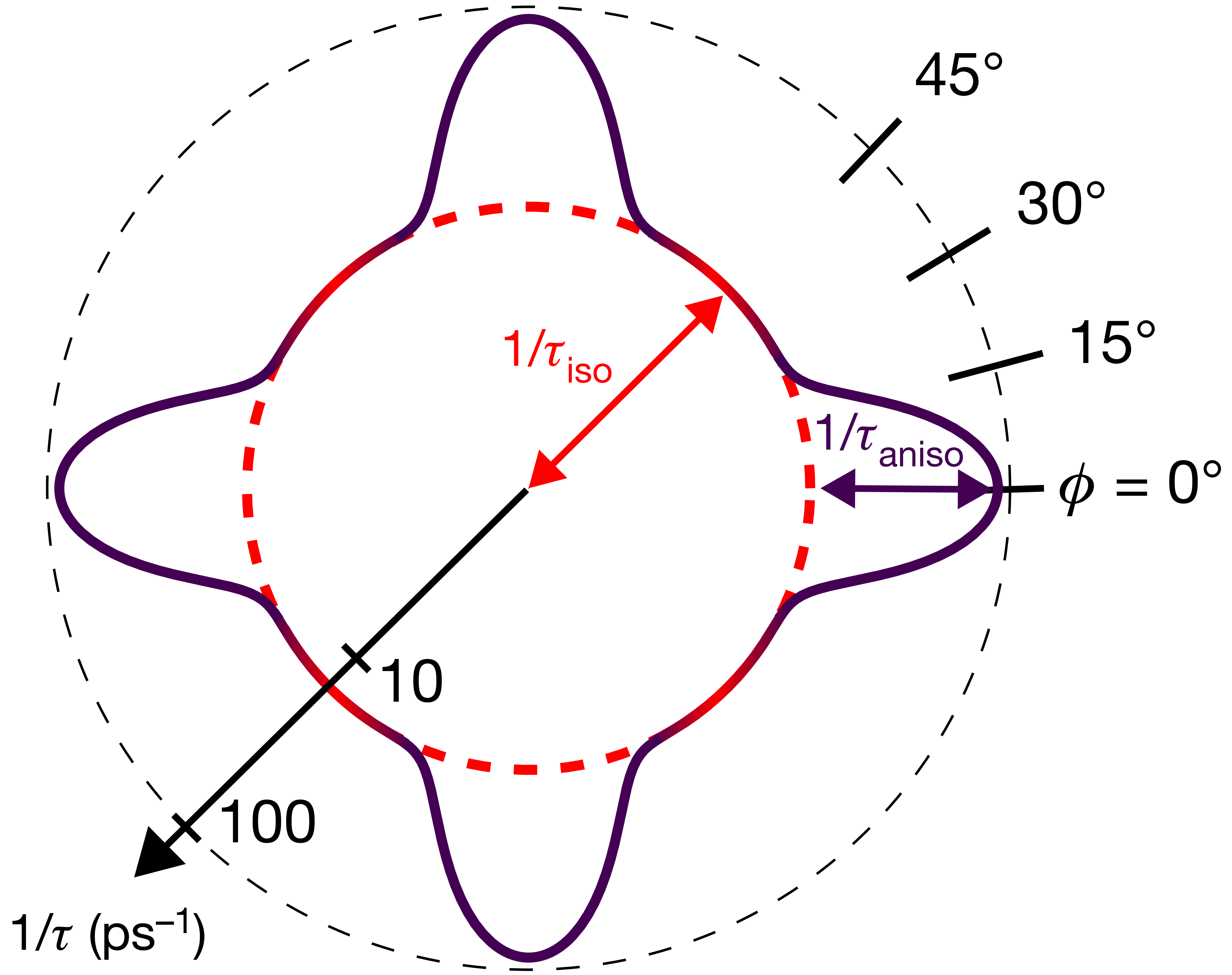

250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Planckian
Metal
Pseudogap
metal
Fermi
liquid
Superconductor
T_c
p_c

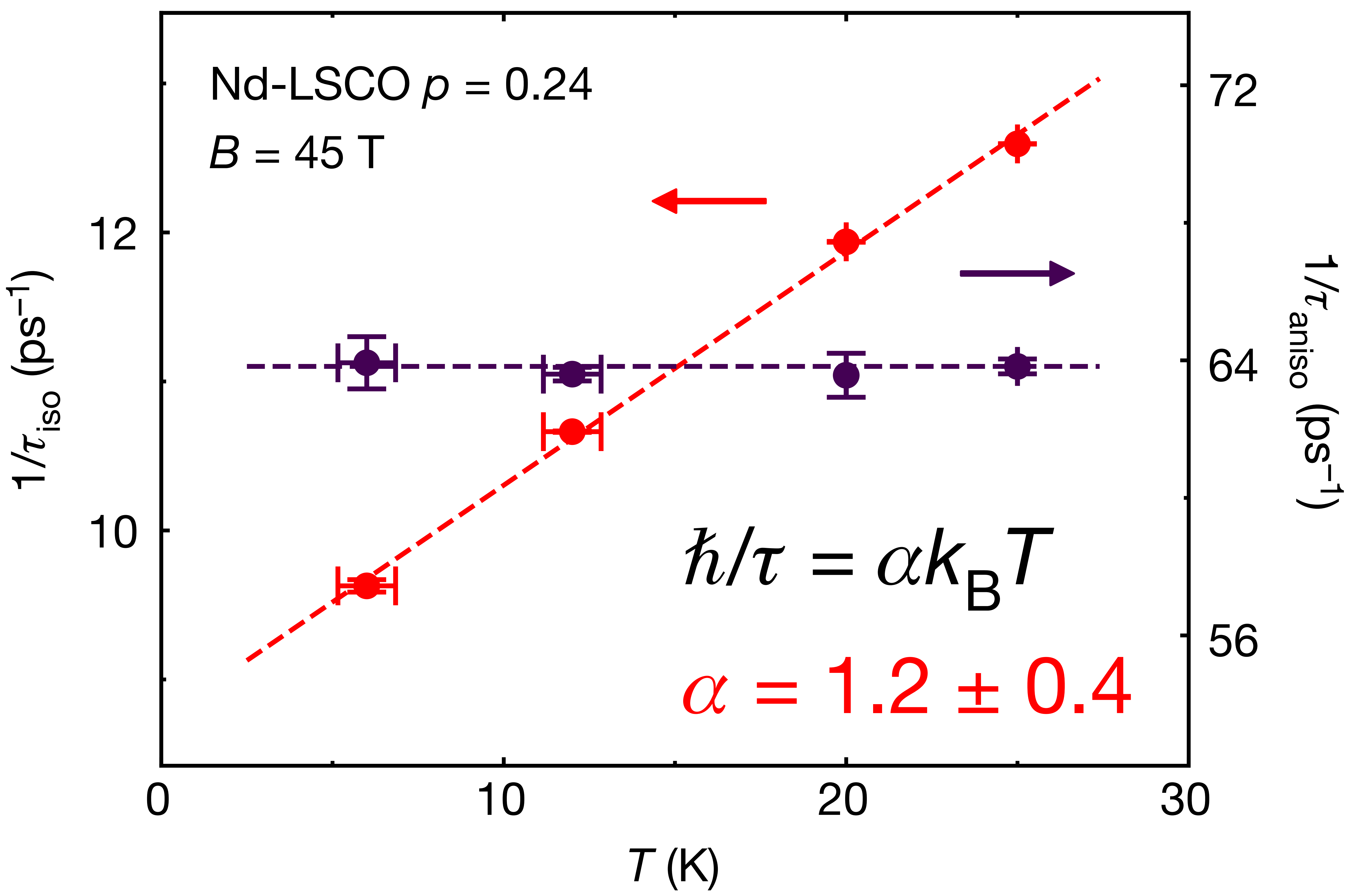

Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw

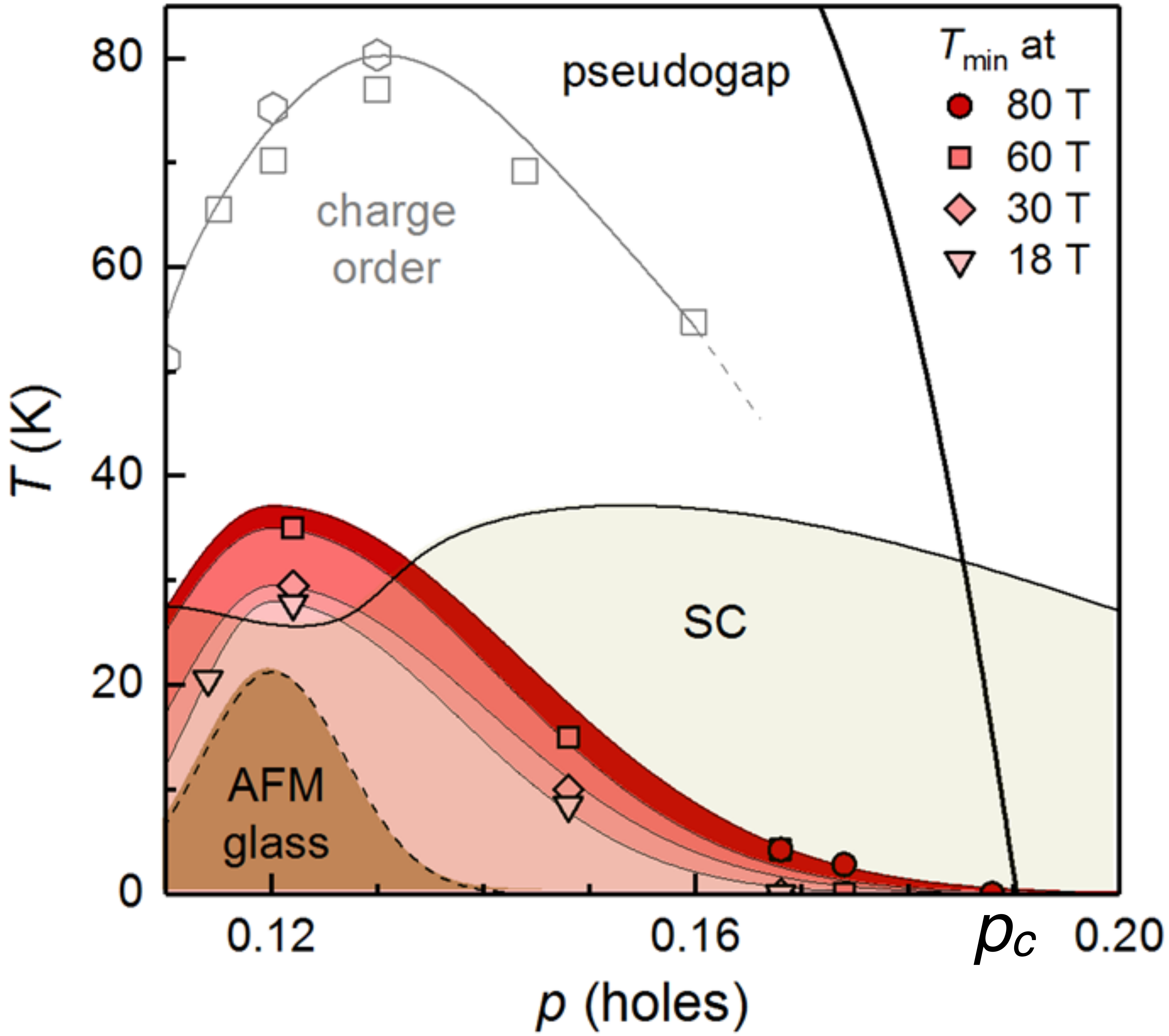




Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1]*, Marc-Henri Julien[1]*



1. SYK model

2. Random t-J model

3. Fermi surface coupled to a critical boson in 2 dimensions
   Large N expansion, maximal chaos, and transport

4. Black holes

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

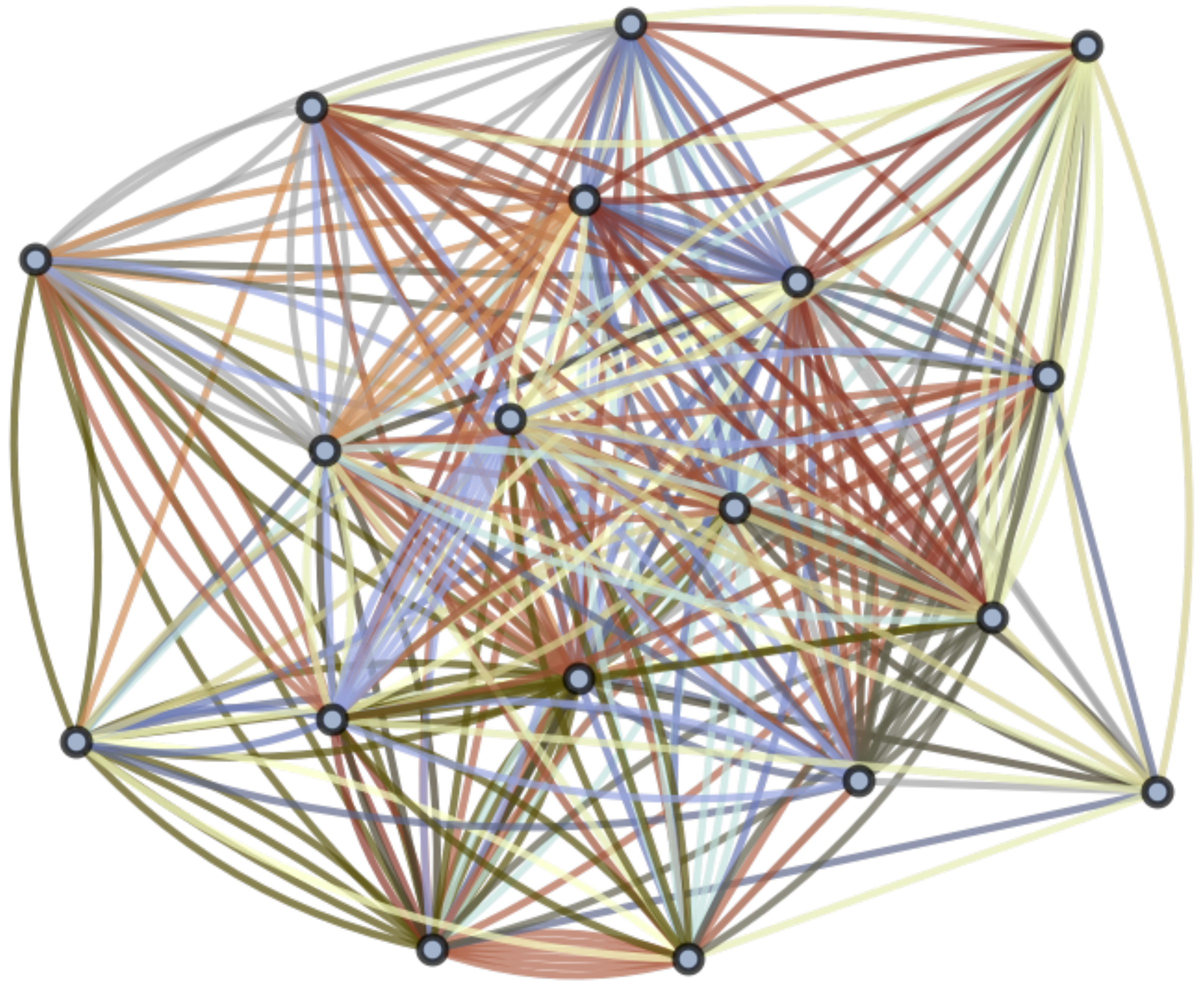
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

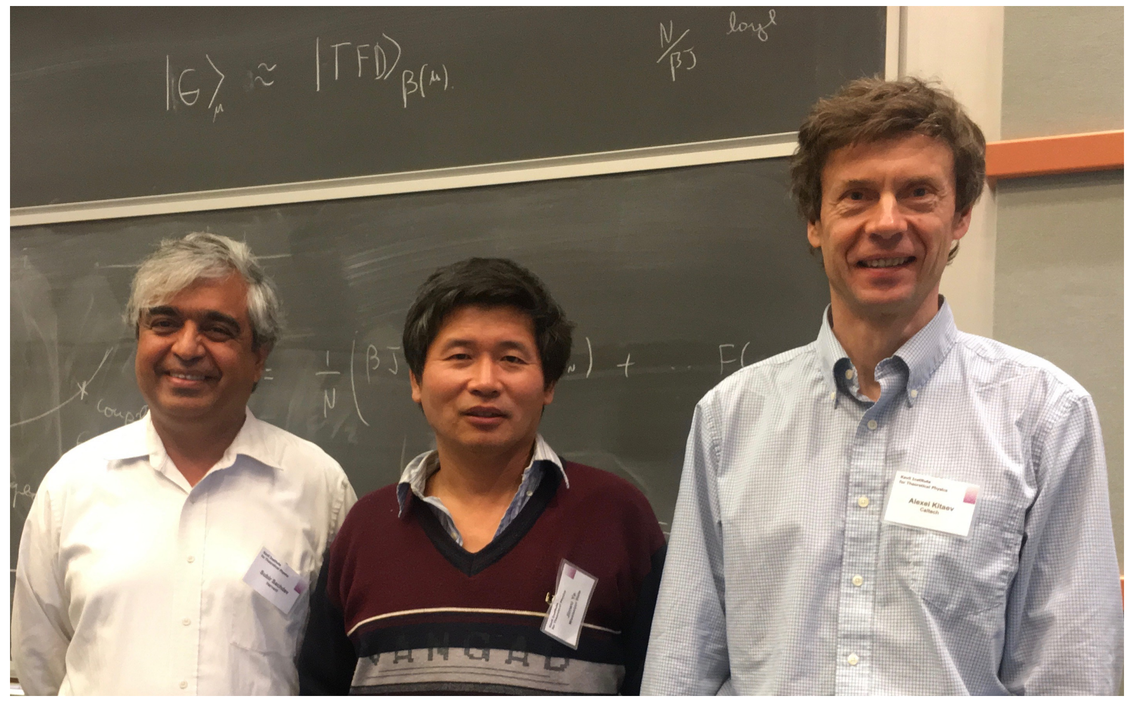
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The Sachdev-Ye-Kitaev (SYK) model

- Compressible quantum matter without quasiparticle excitations.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

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  $G(\omega, T) \sim \omega^{-1/2} F(\hbar\omega / k_B T)$.

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- Corrections to entropy from time reparameterization soft mode $S = N(s_0 + \gamma T) - (3/2)\ln(U/T)$.

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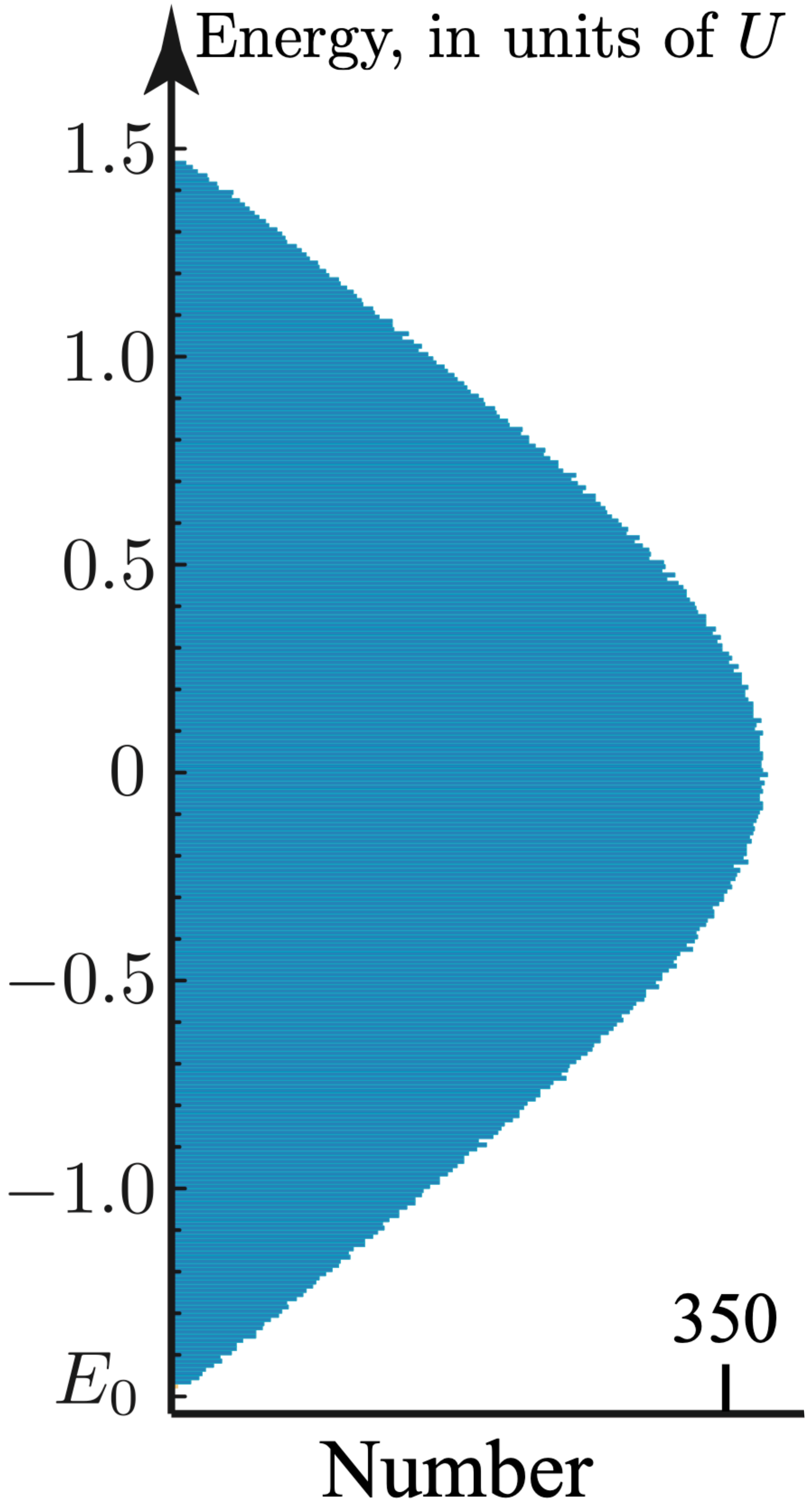
- Corrections to entropy from time reparameterization soft mode $S = N(s_0 + \gamma T) - (3/2)\ln(U/T)$.
- Time reparameterization mode also leads to maximal quantum chaos with out-of-time-order (OTOC) Lyapunov exponent $\lambda_L = 2\pi k_B T/\hbar$.

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

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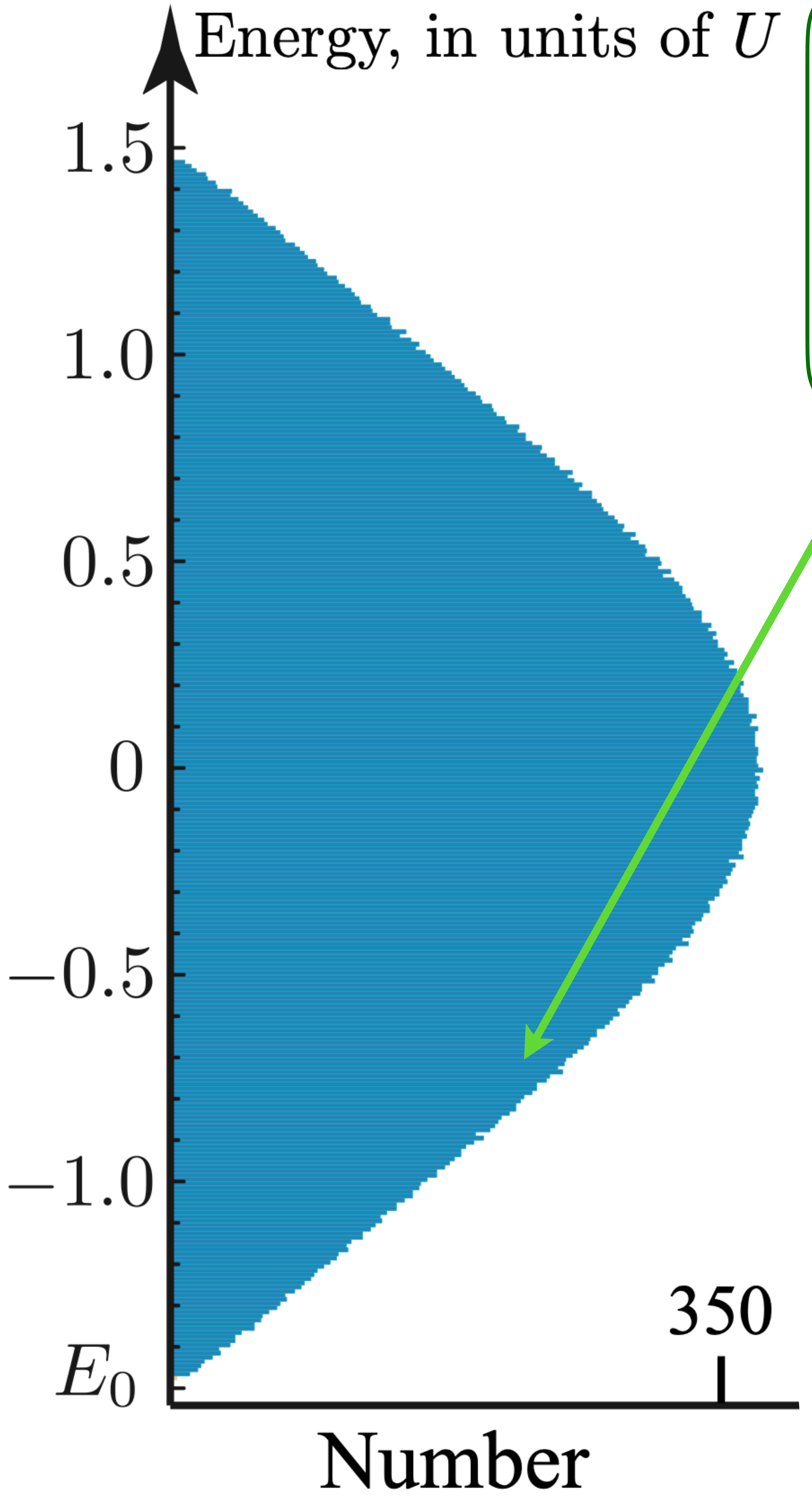

$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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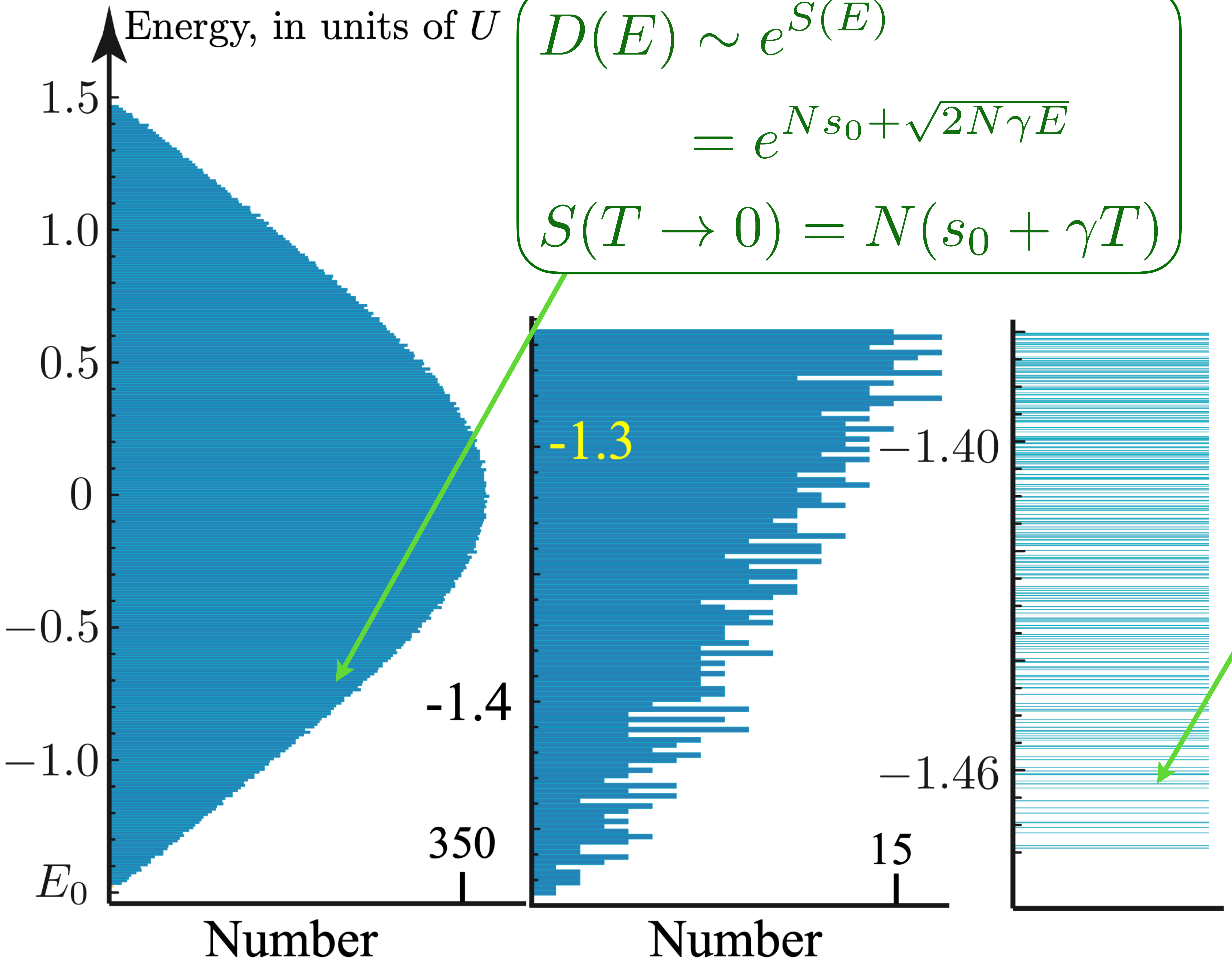

$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

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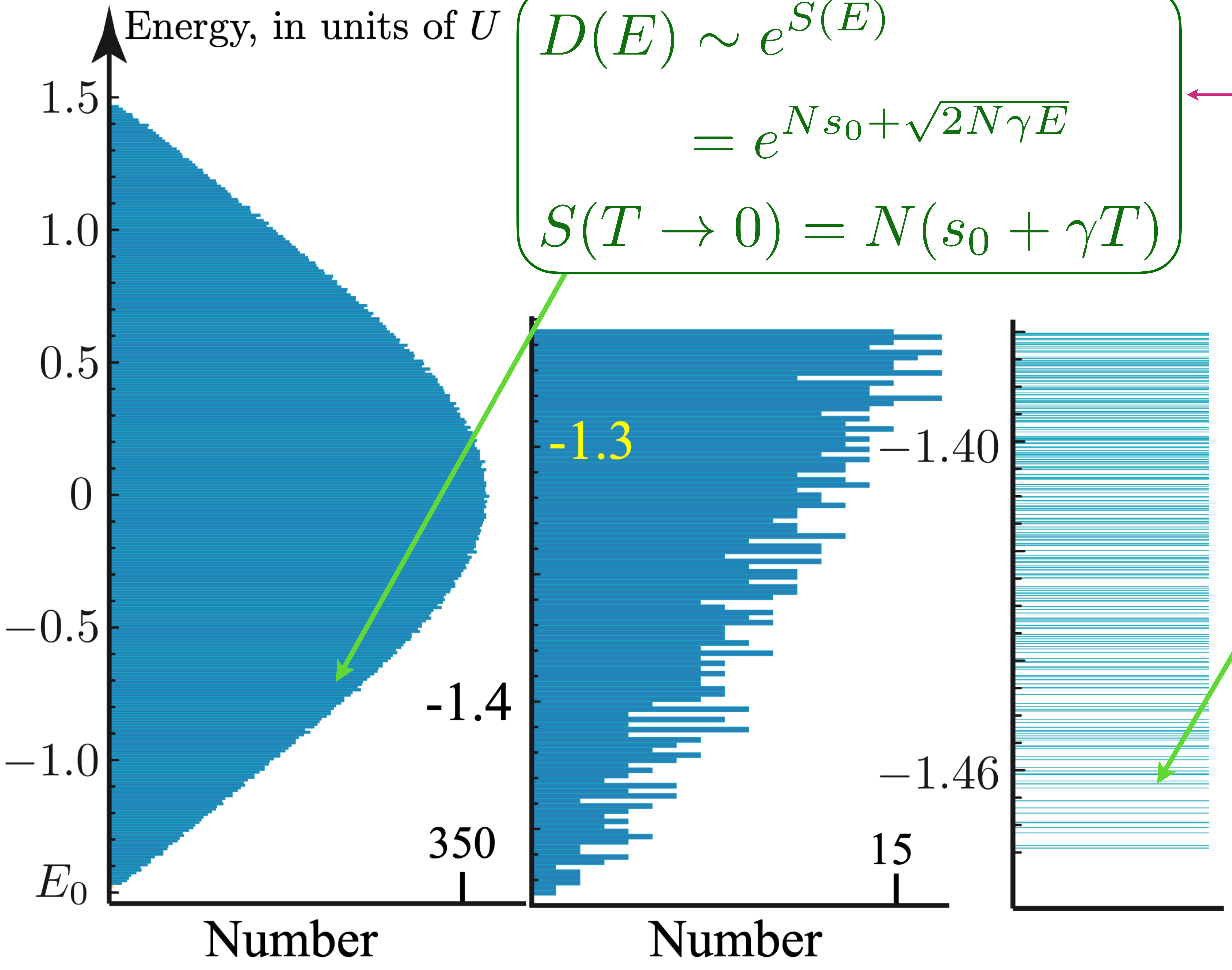

$$D(E) \sim e^{S(E)}$$
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$$D(E) \sim 2\, e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$
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Random *t-J* model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

$\mathcal{P}_d$ projects out doubly-occupied sites.

J_{ij} random, $\overline{J_{ij}} = 0$, $\overline{J_{ij}^2} = J^2$

t_{ij} random, $\overline{t_{ij}} = 0$, $\overline{t_{ij}^2} = t^2$

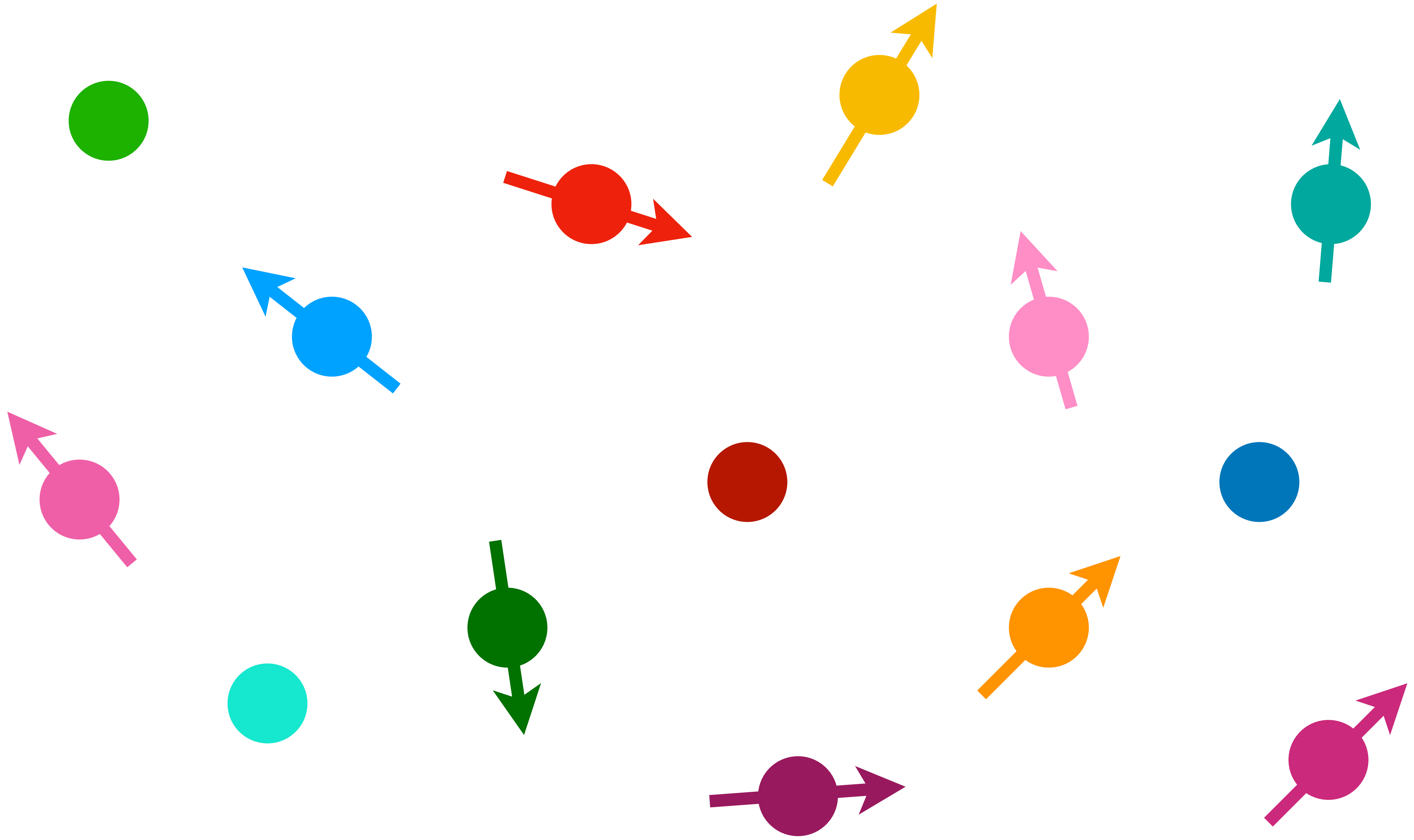
$J \quad \Rightarrow \quad$ two-particle interaction, as in SYK

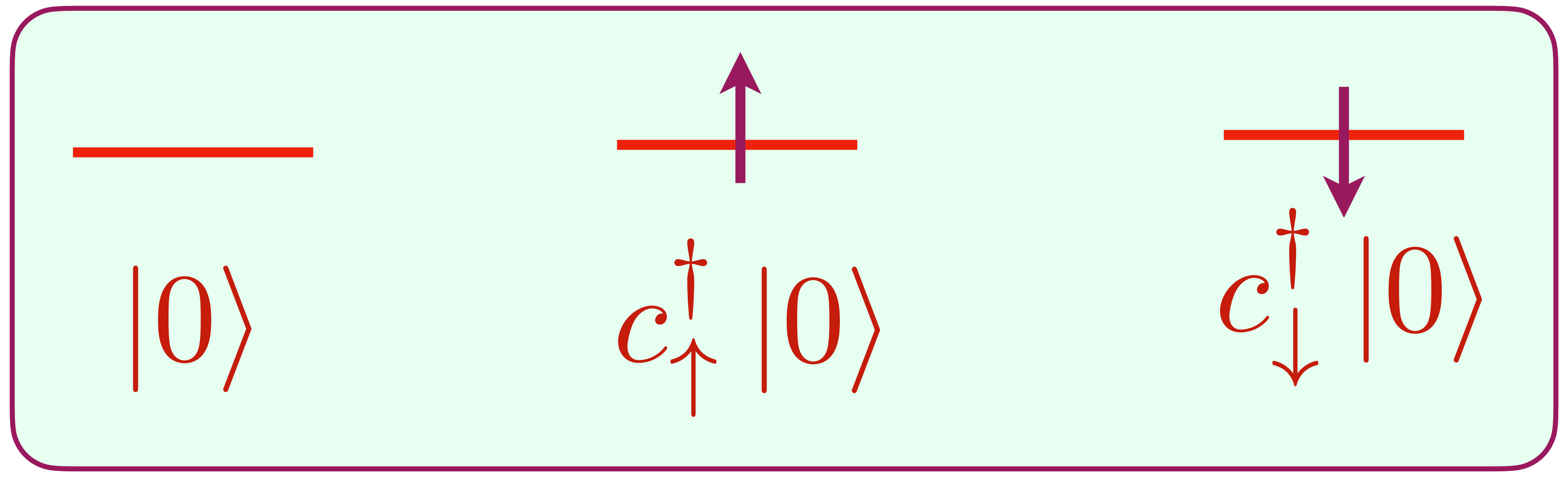
$t \quad \Rightarrow \quad$ one-particle hopping, as in random matrices

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





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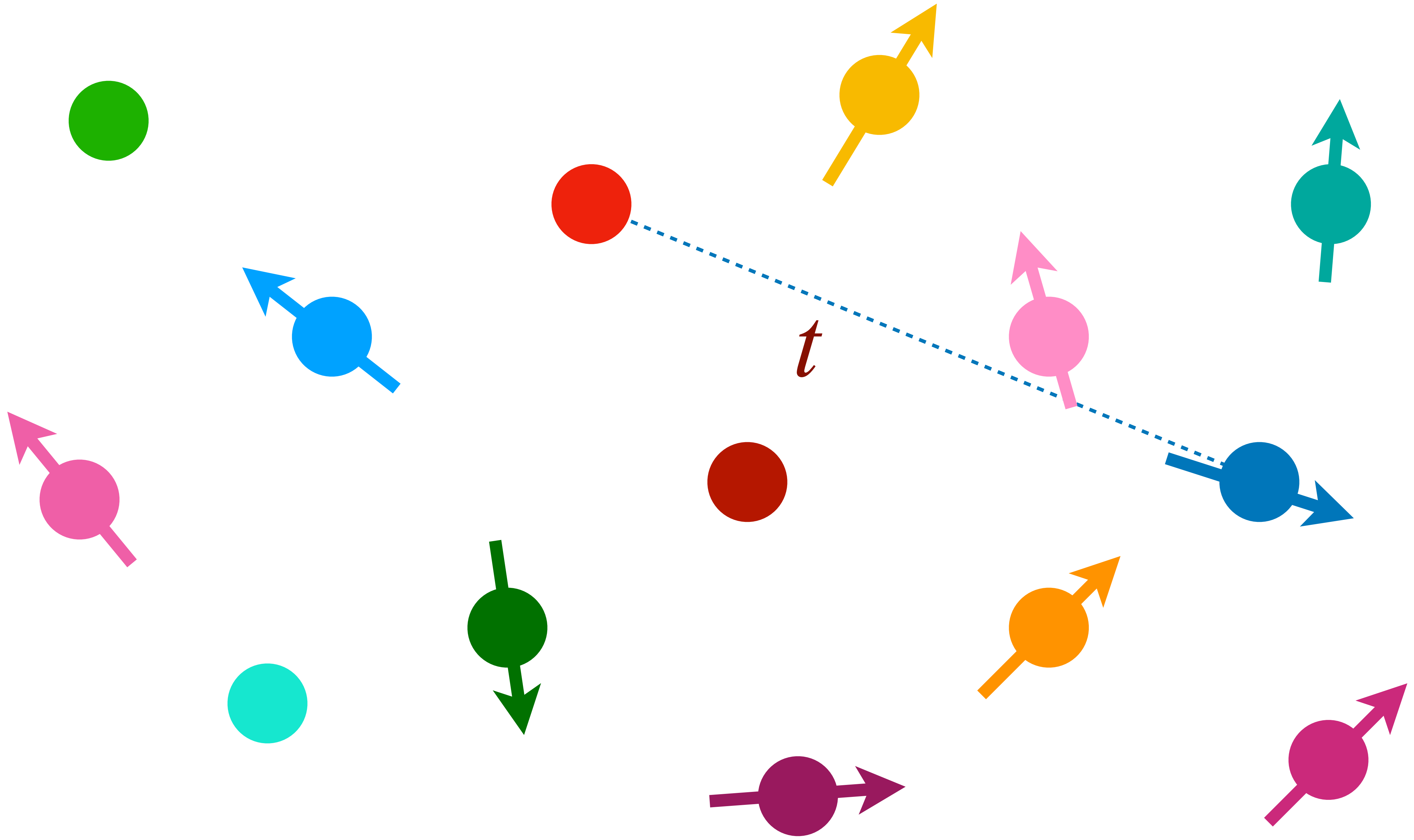

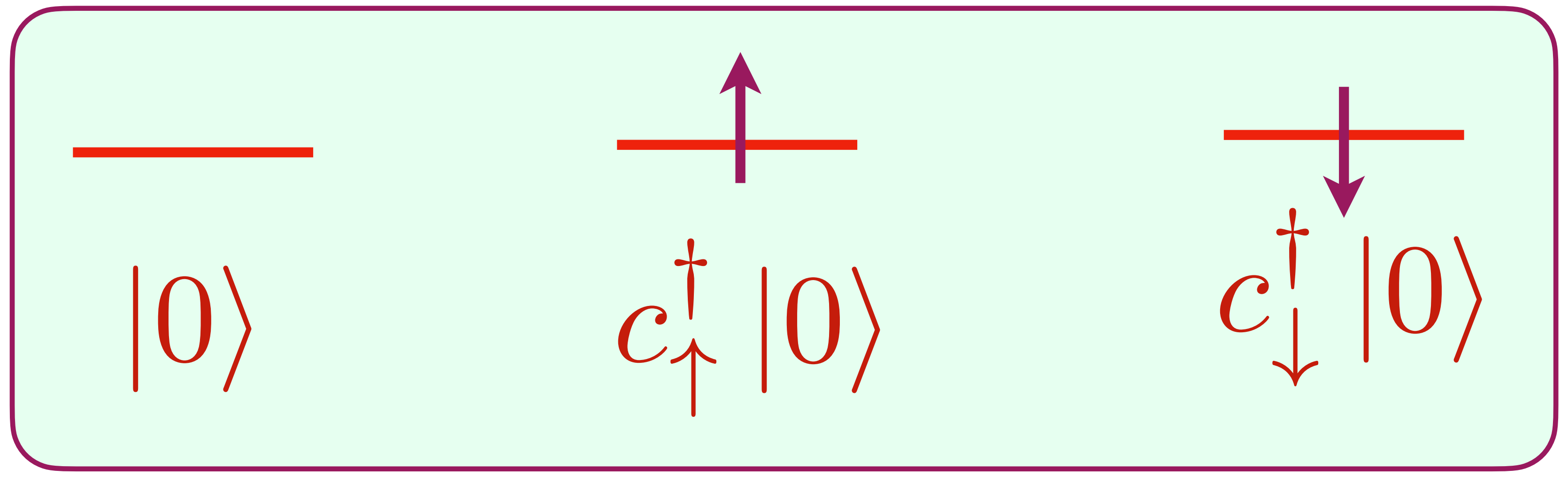

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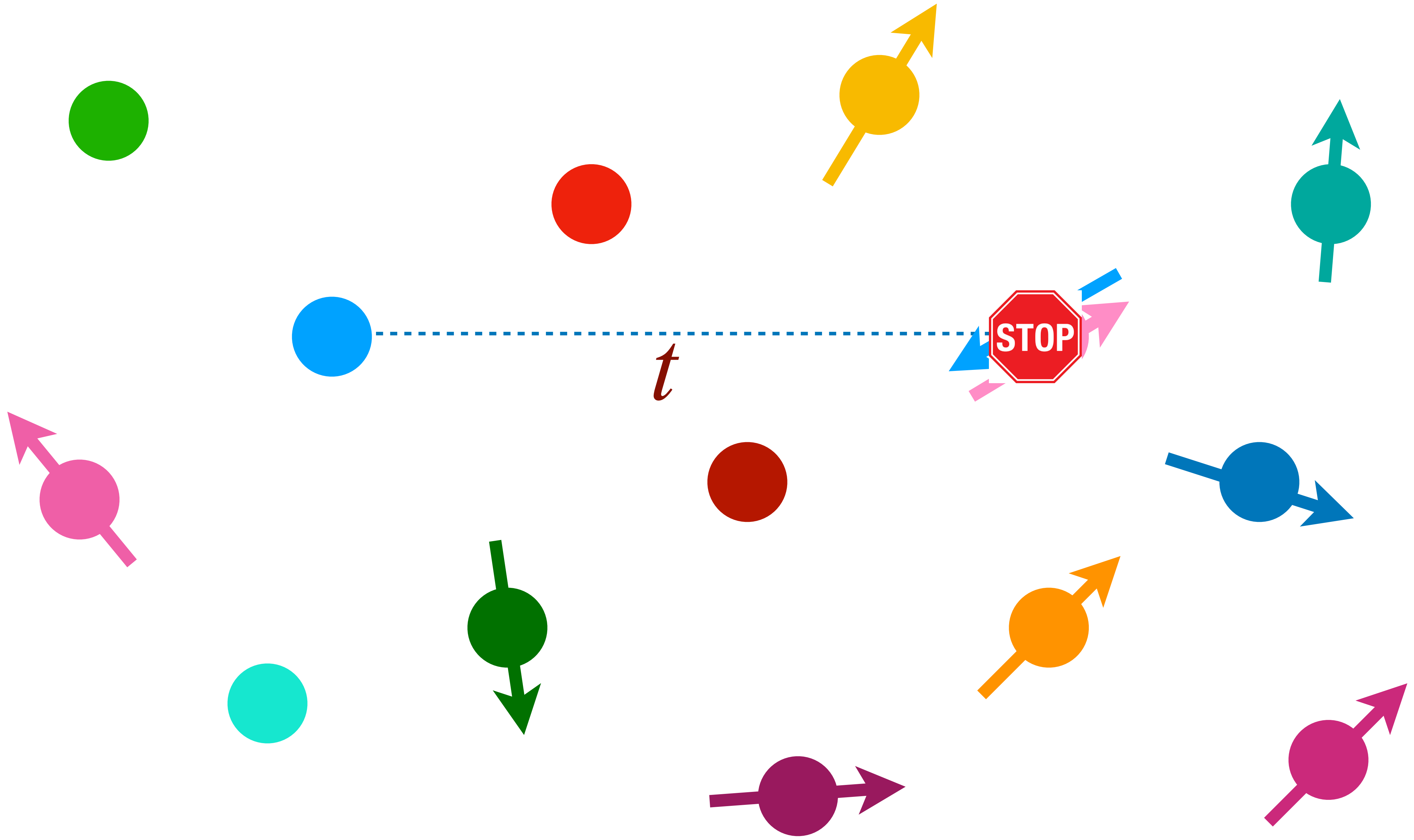

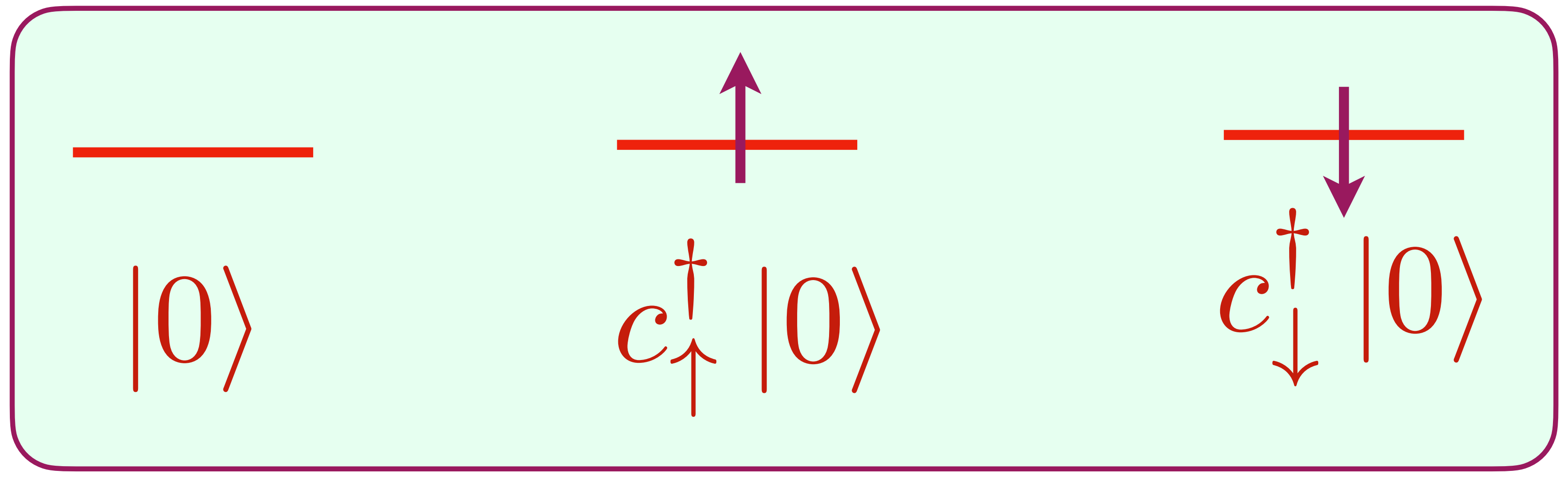

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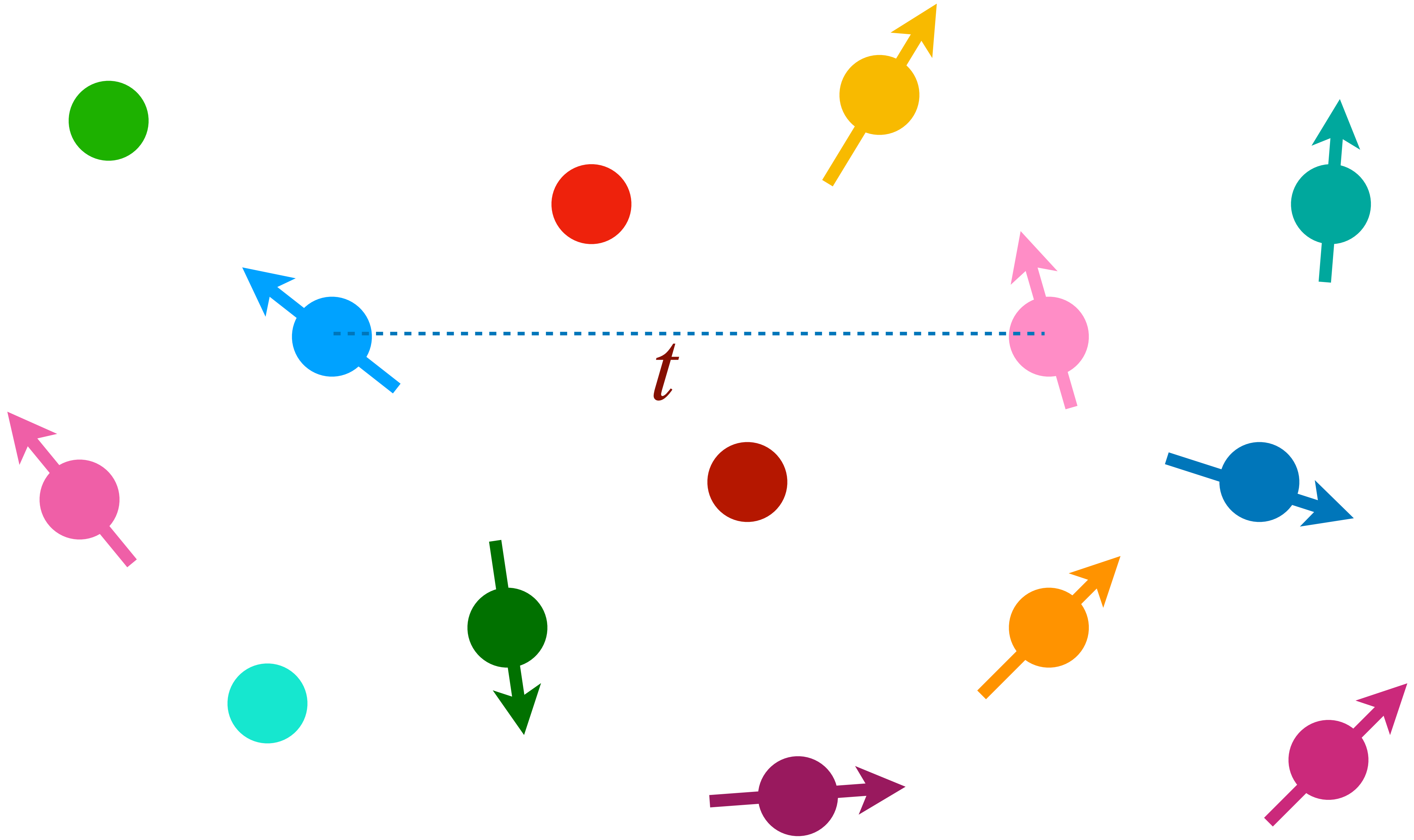

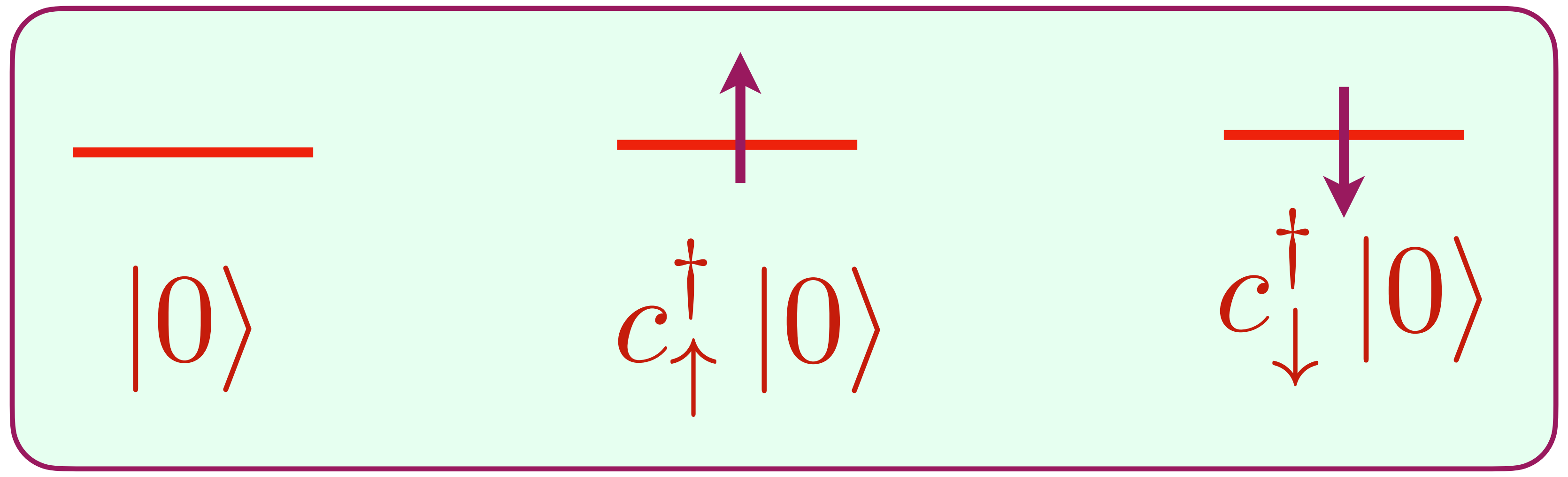

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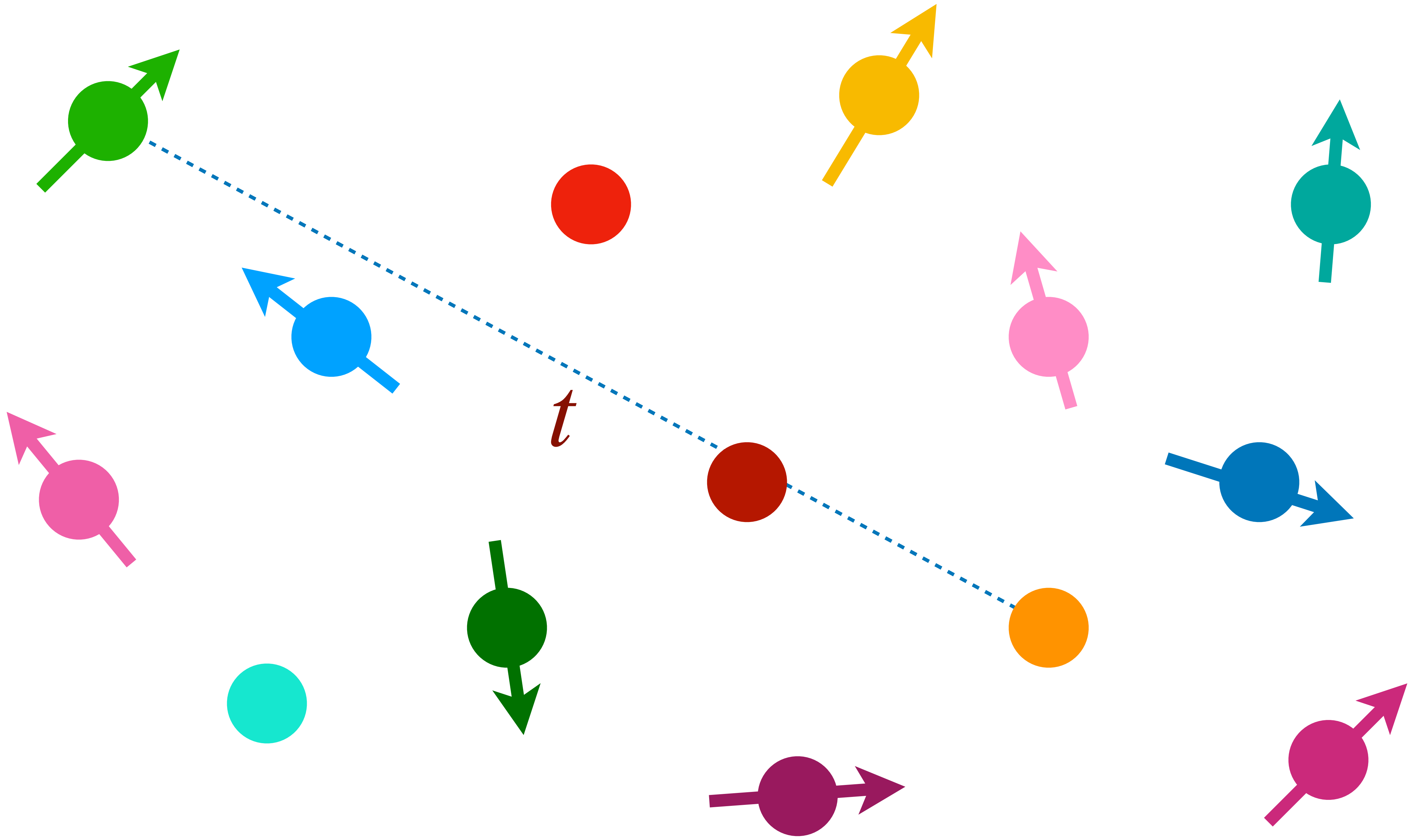

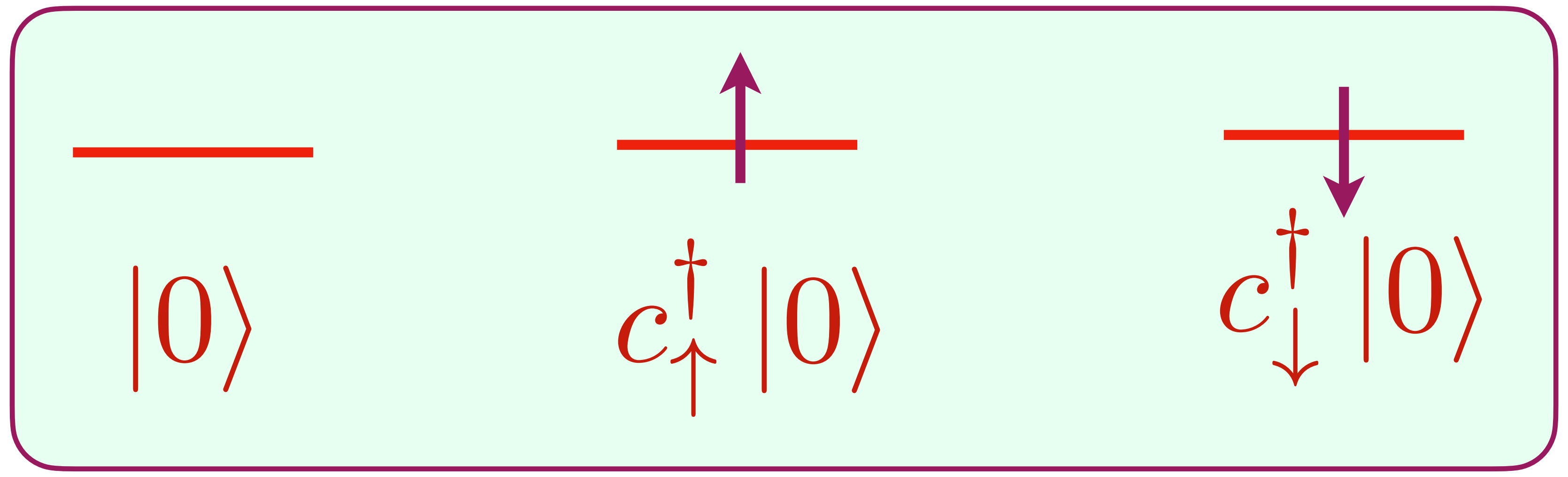

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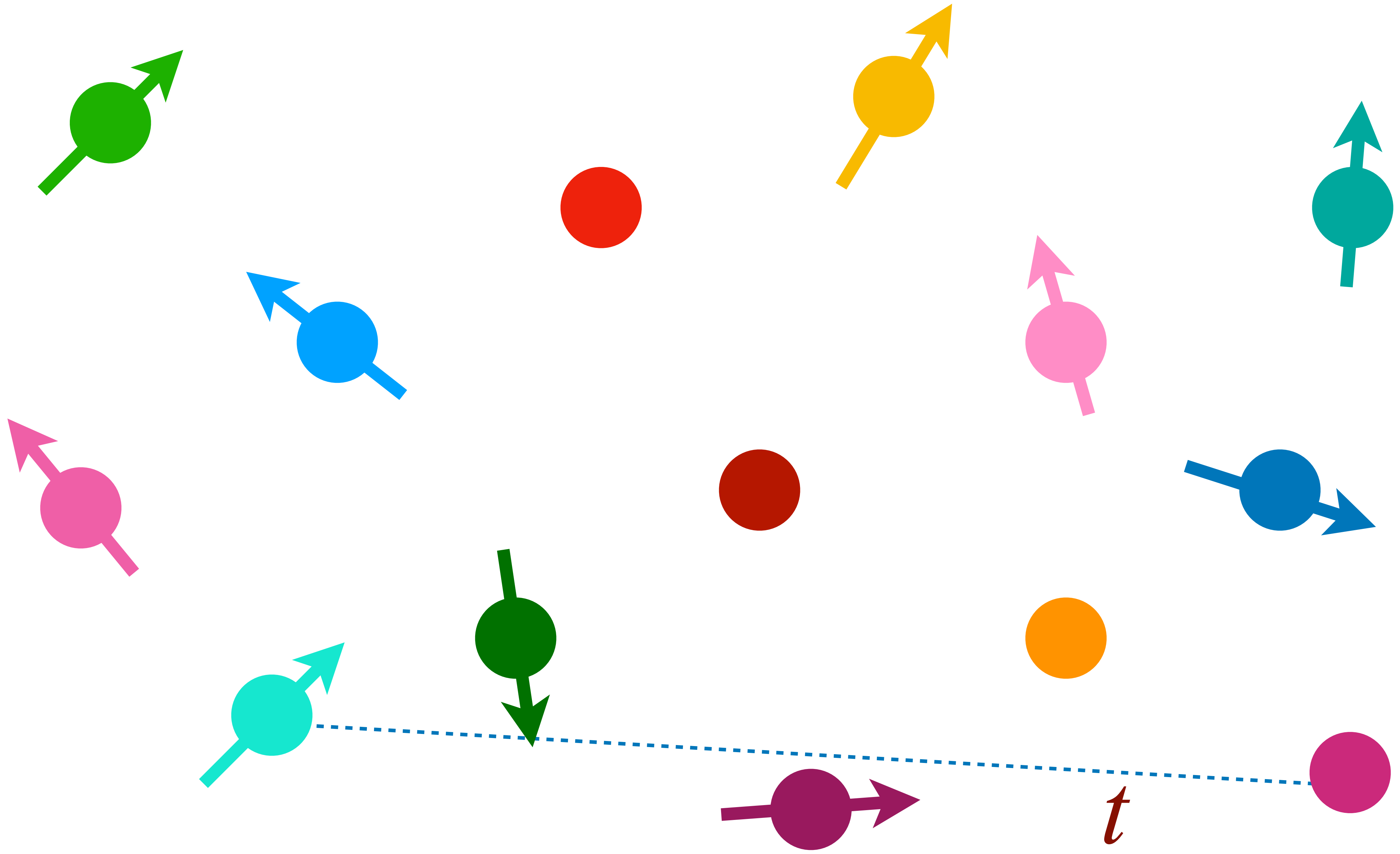

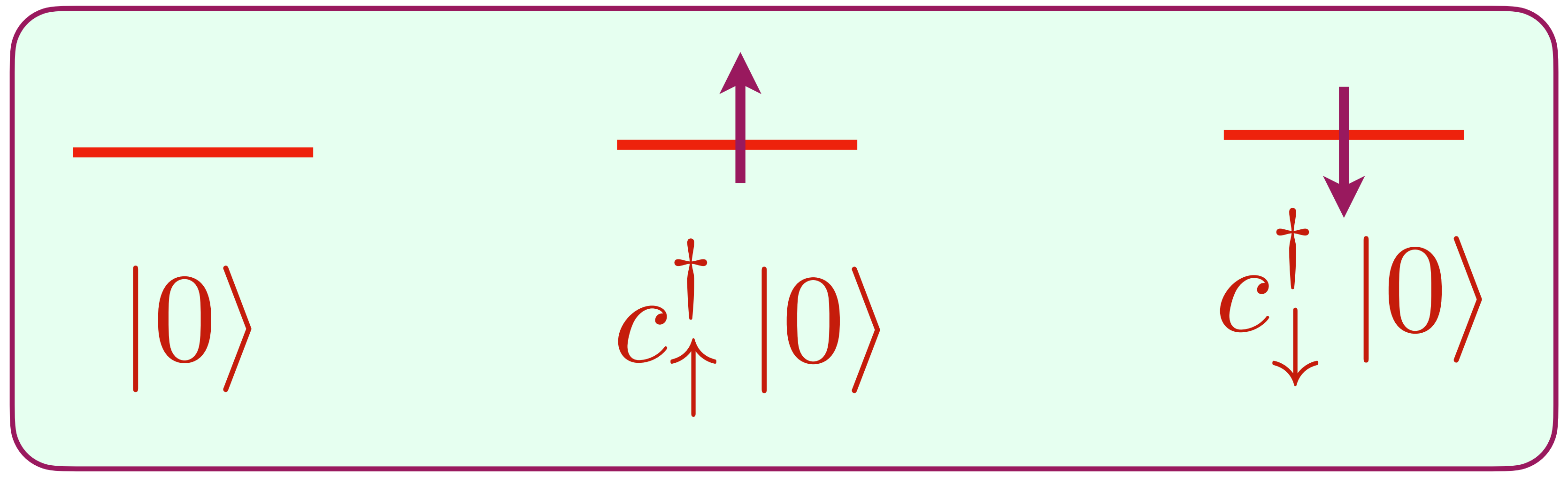

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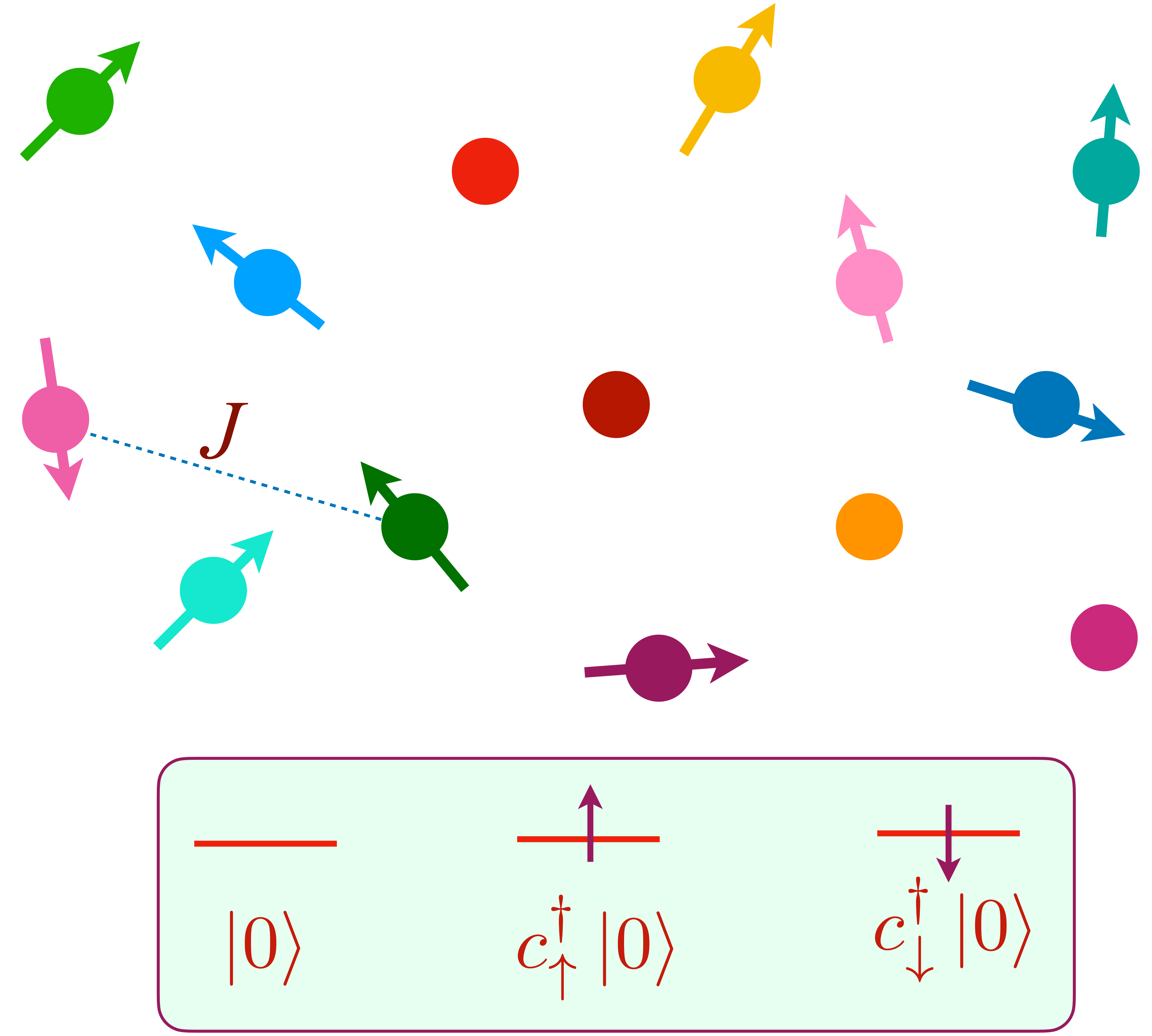

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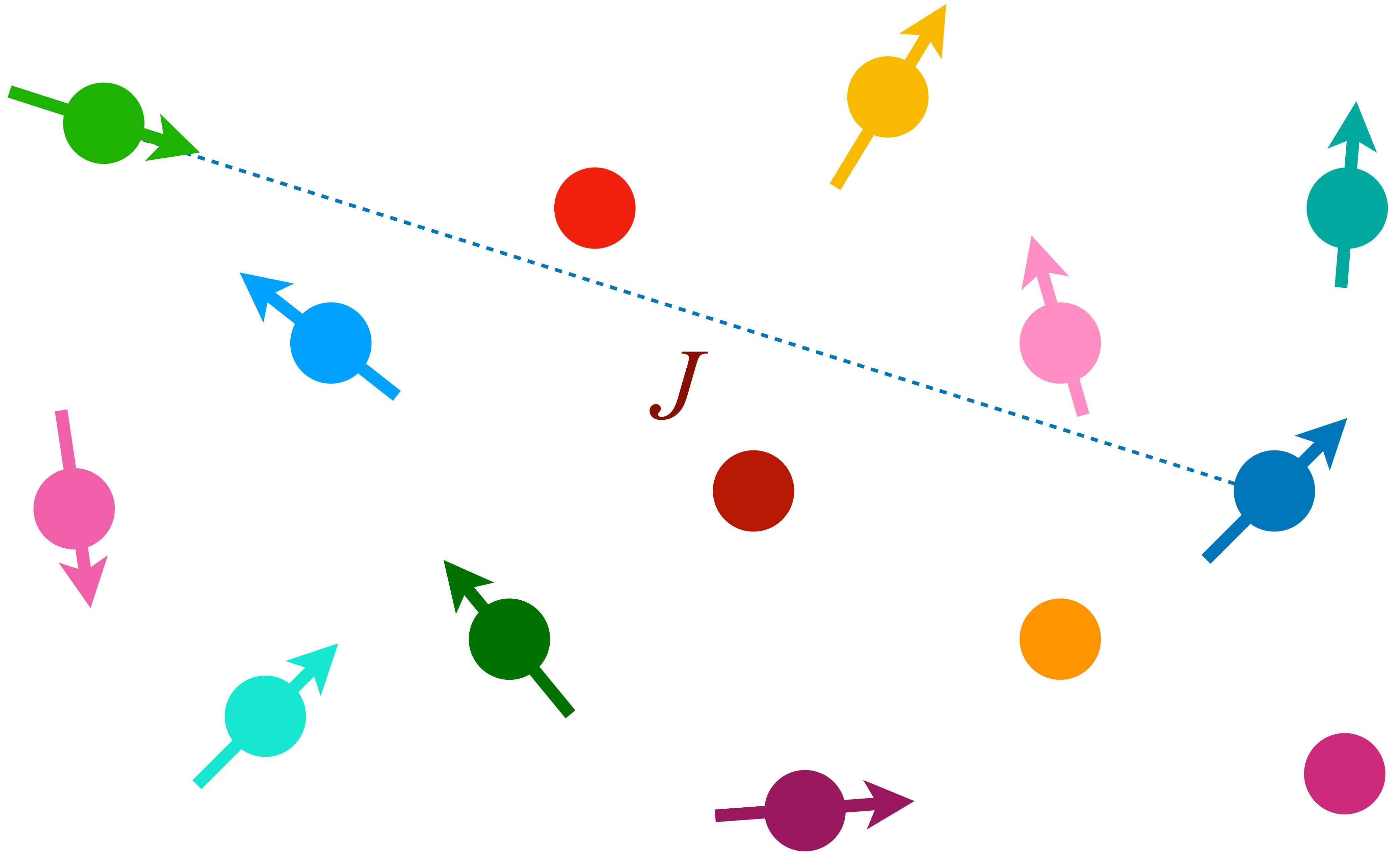

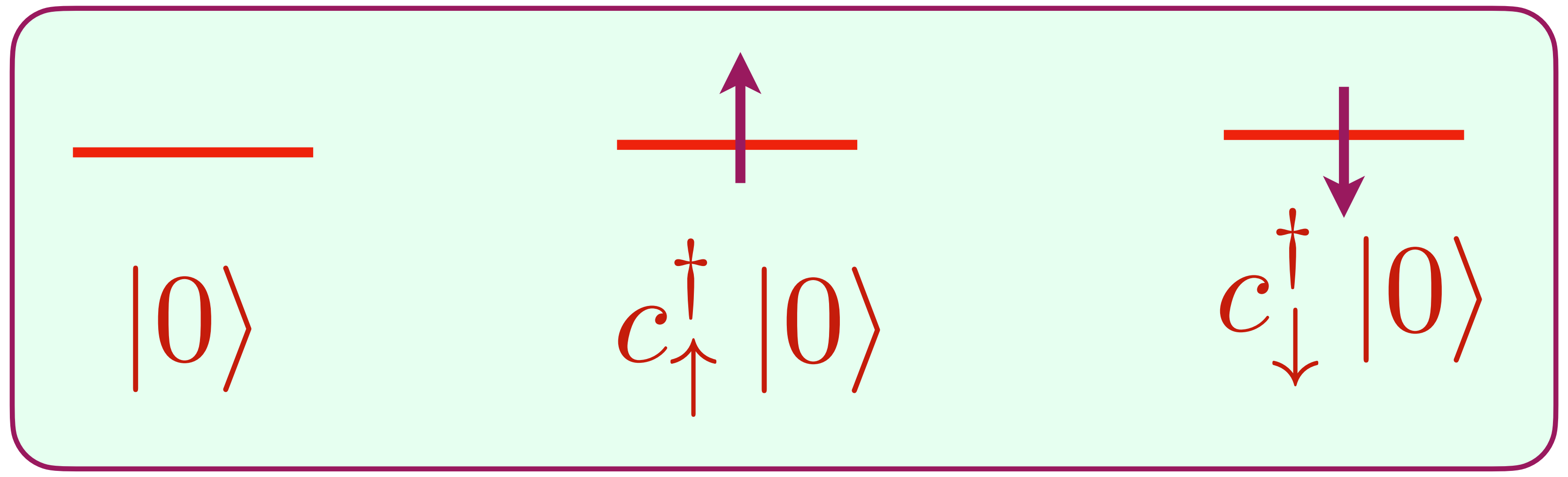

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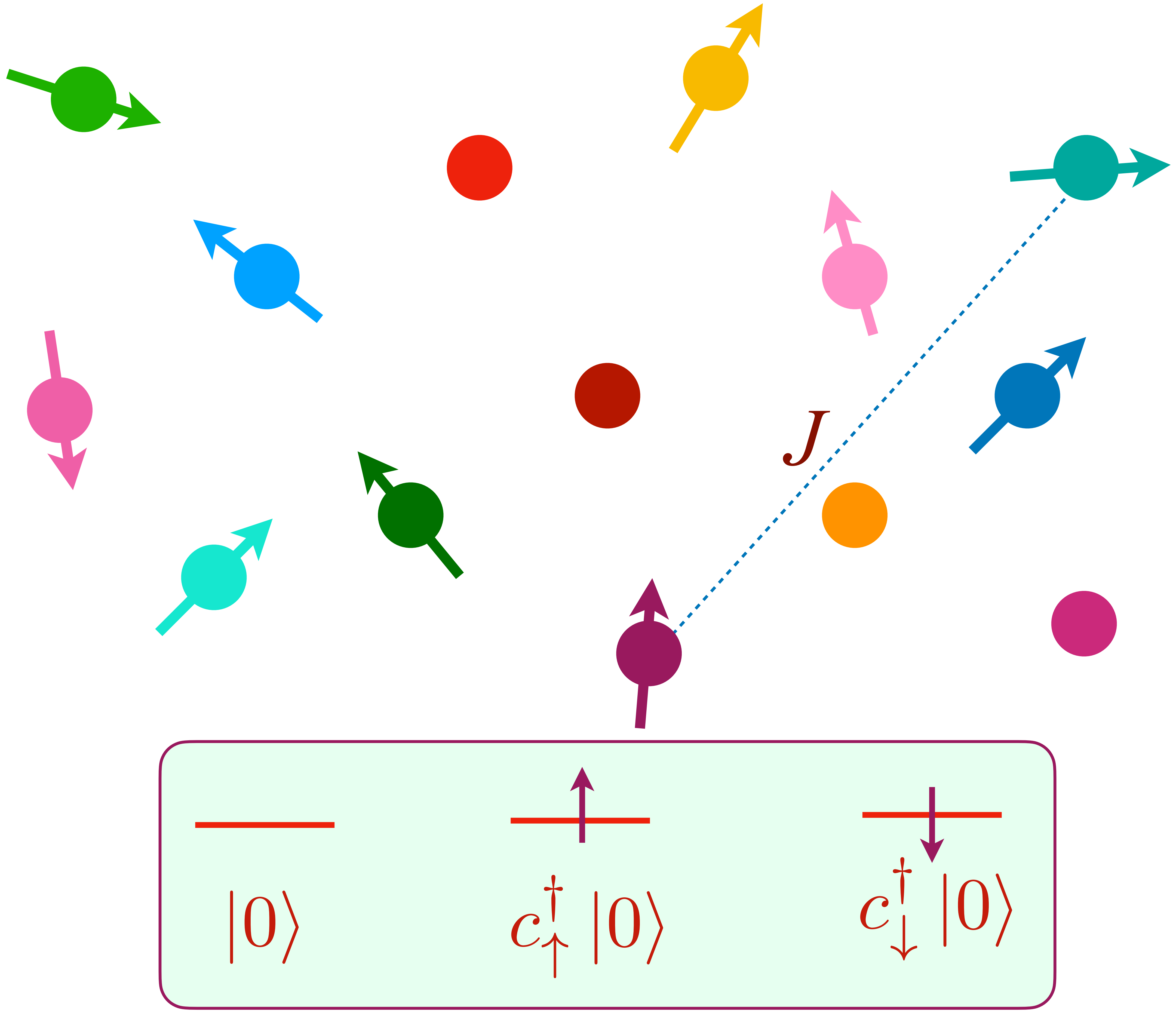
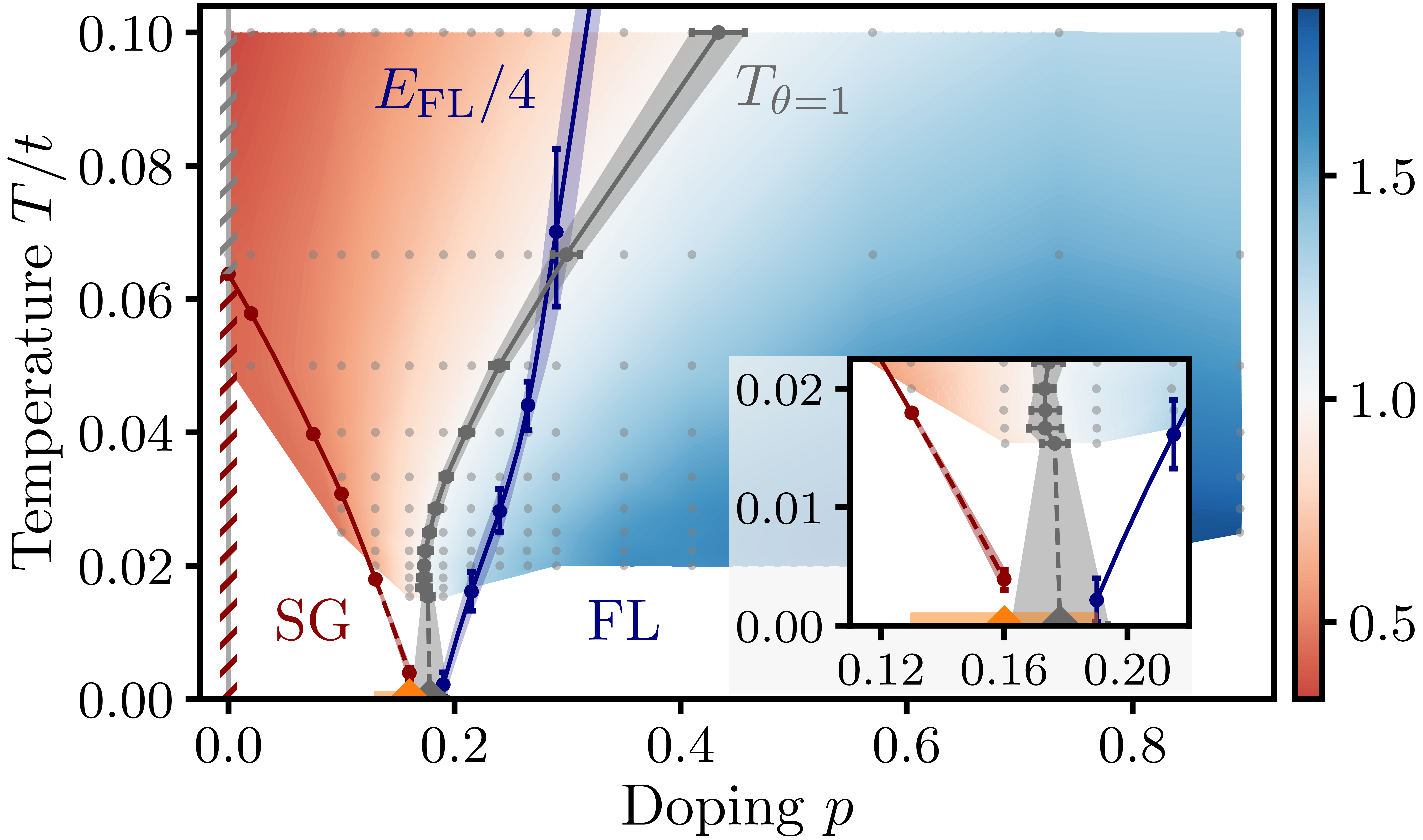


Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

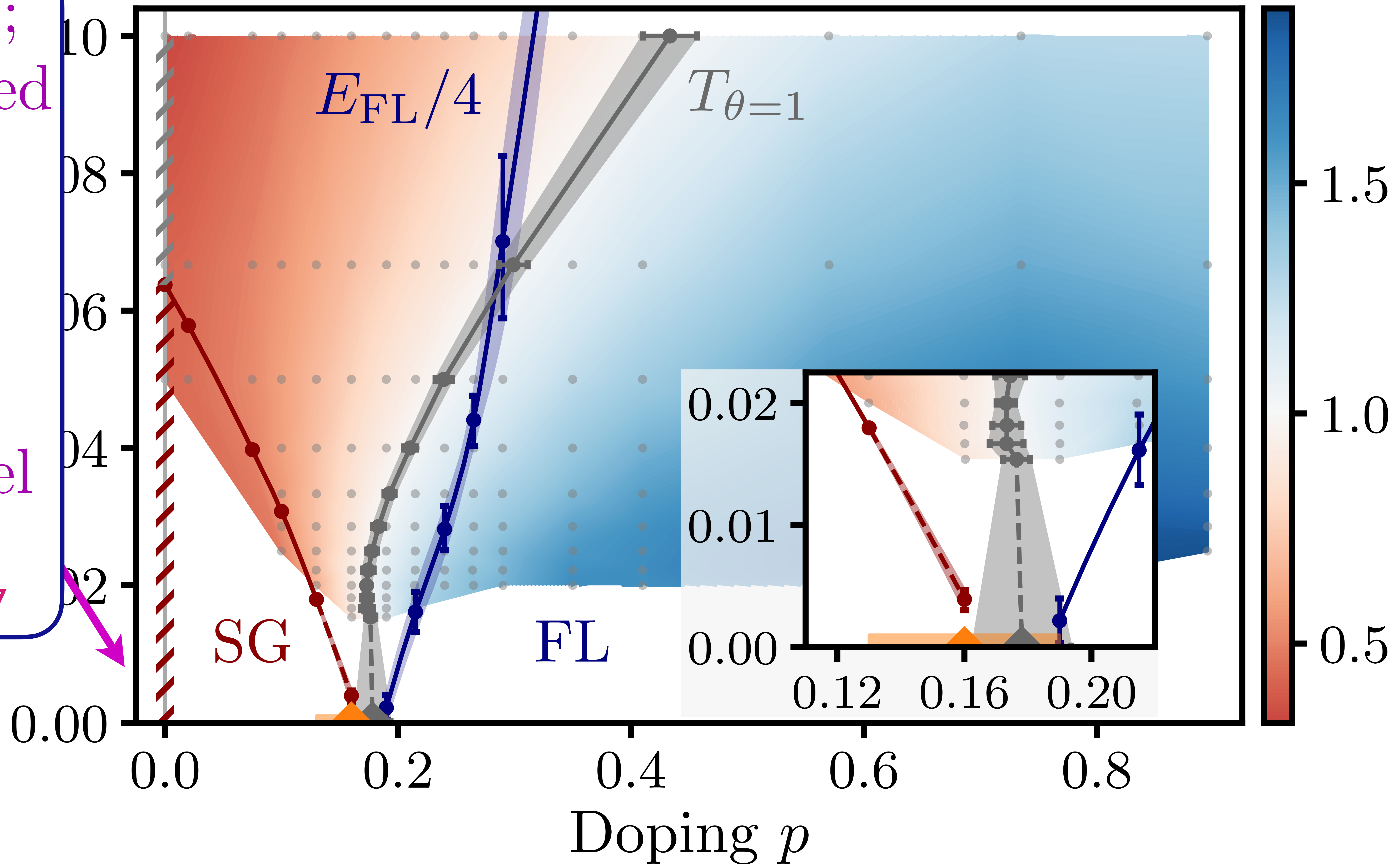
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}

Spin glass order; SYK fractionalized spin liquid for $\omega > Jq_{EA}$.

$T_c \sim Je^{-\sqrt{\pi M}}$ for SU(M) model

M. Christos, F. M. Haehl, and S. Sachdev, arXiv:2110.00007



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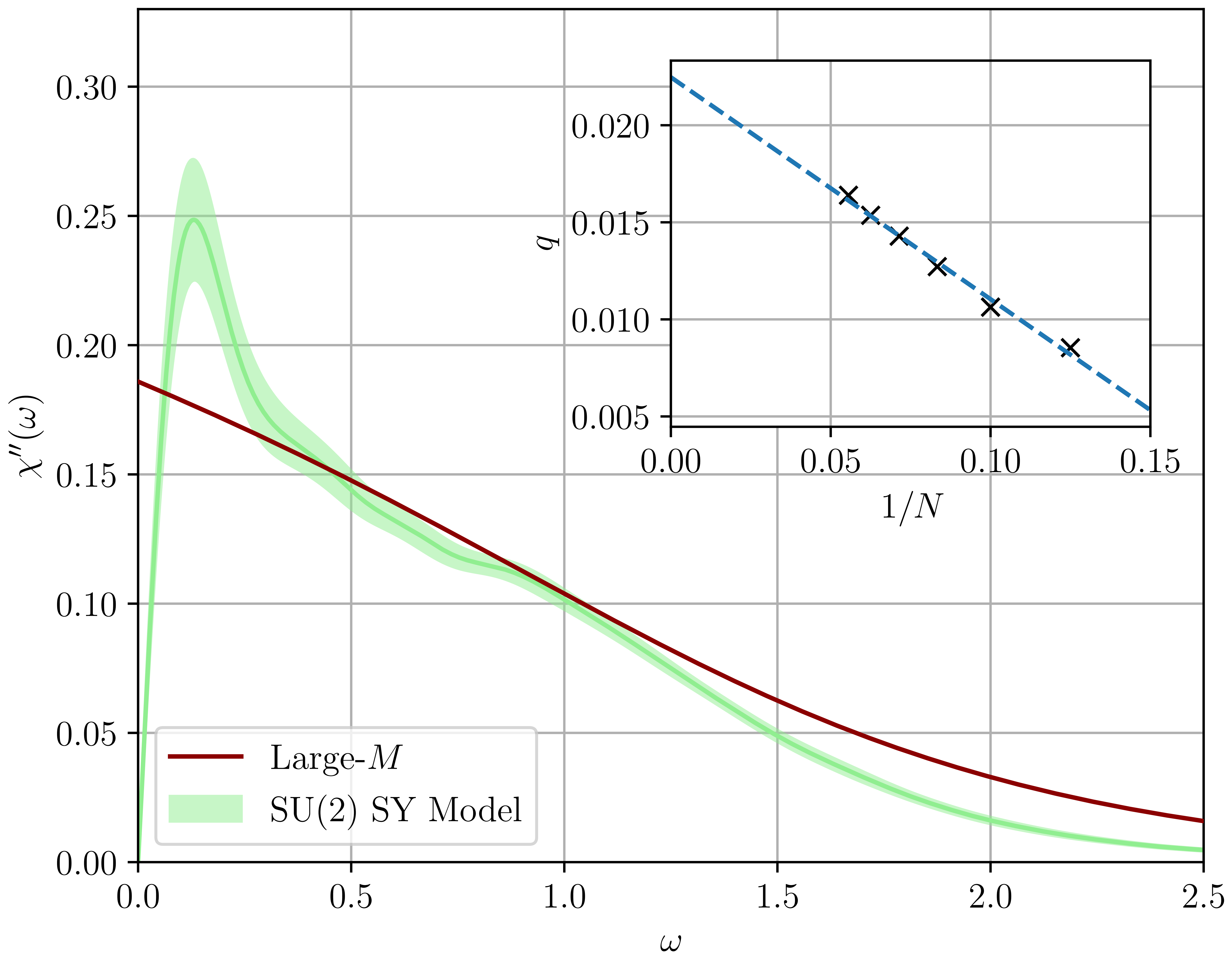
Parton theory of insulating J model

Generalize to SU(M) spins and introduce fermionic spinons f_α, $\alpha = 1, \ldots, M$

$$S_{\alpha\beta} = f_\alpha^\dagger f_\beta - \frac{\delta_{\alpha\beta}}{2}, \quad f_\alpha^\dagger f_\alpha = M/2 \,.$$

The large N limit, followed by the large M limit, leads to saddle-point equations for the spinons identical to those for the electrons in the SYK model.

Exact diagonalization of clusters of SU(2) spins



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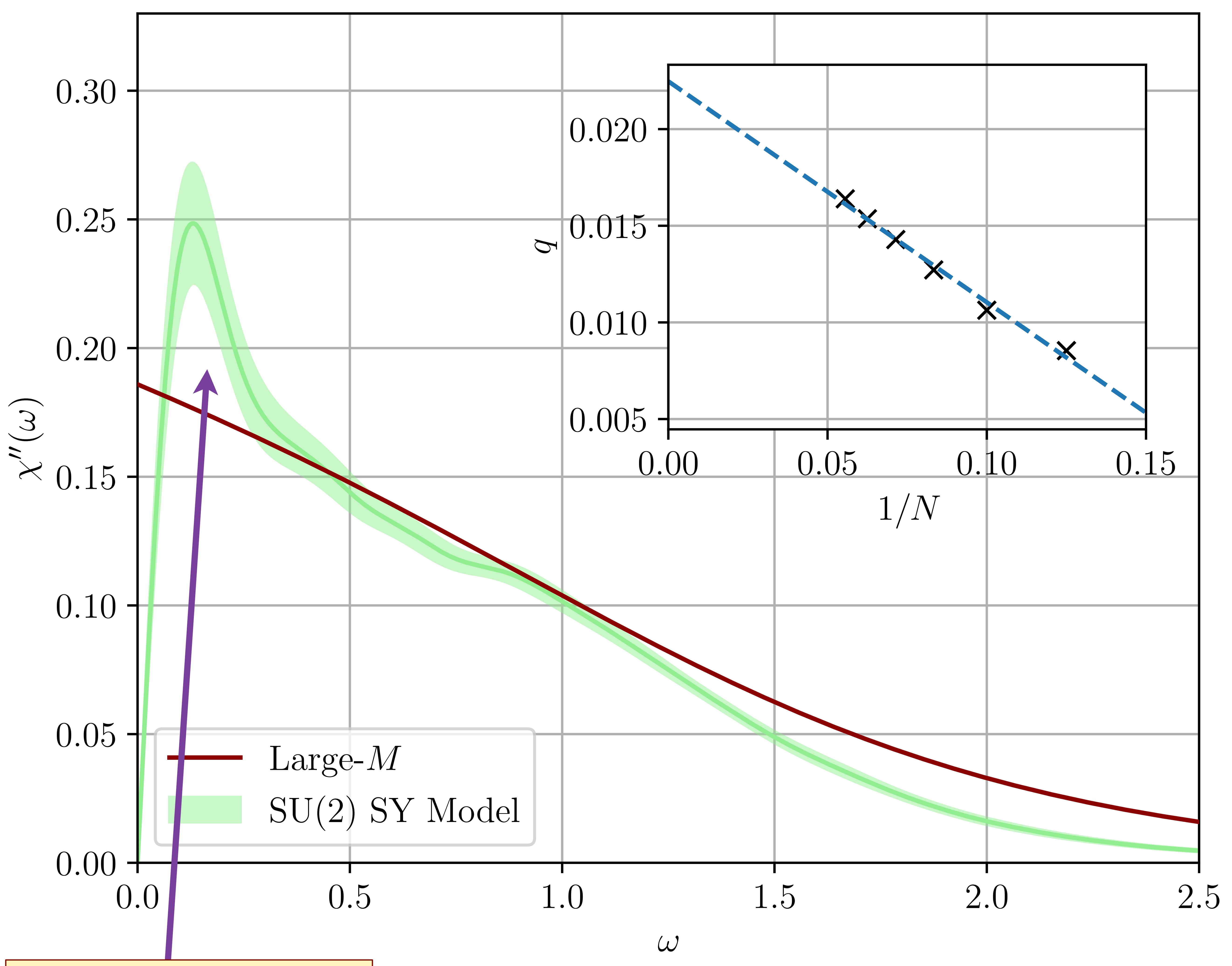

Spin glass

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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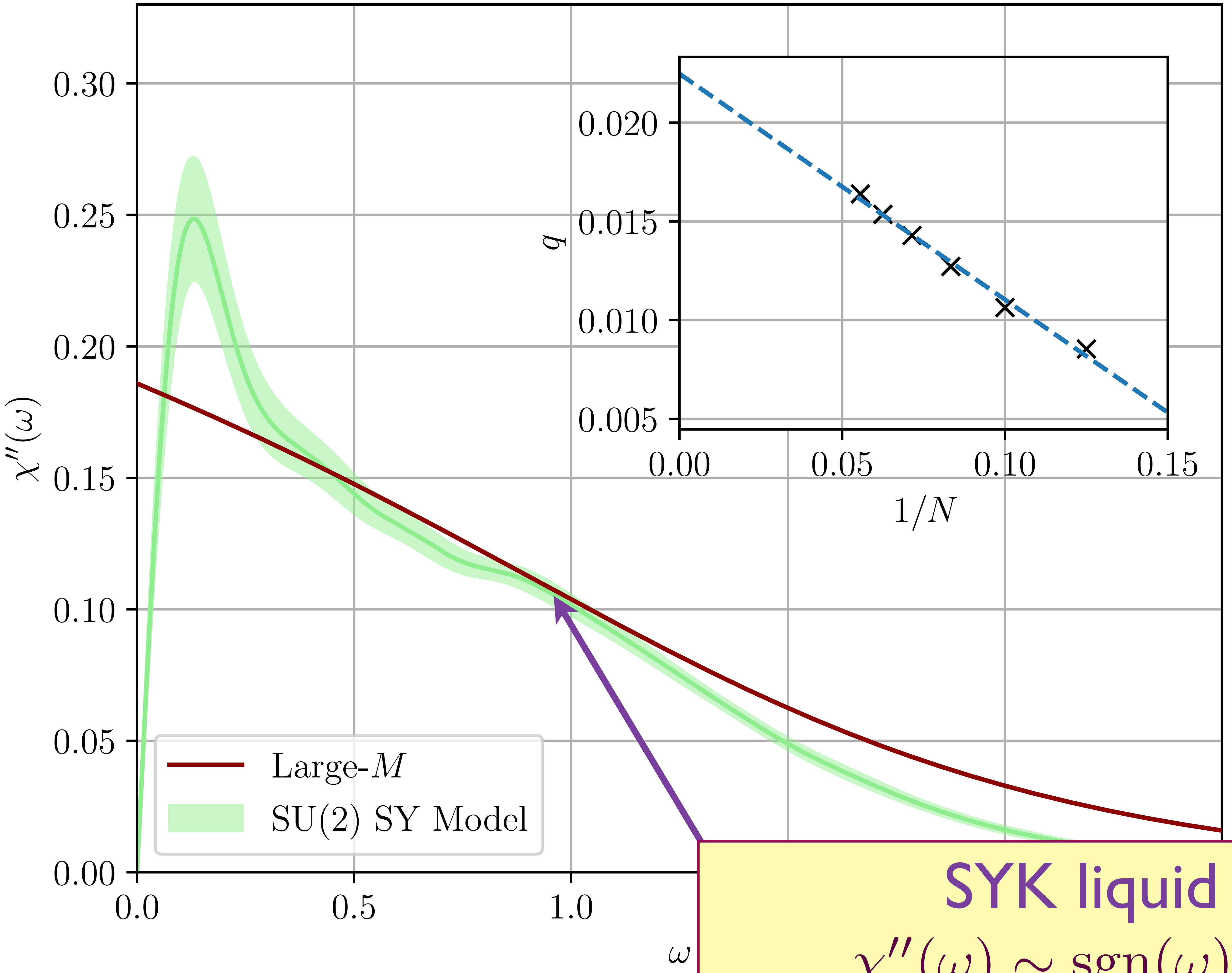
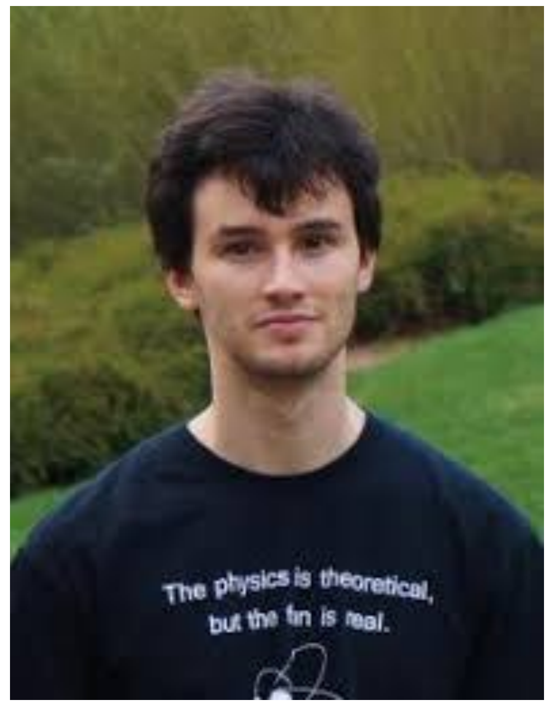


Maria Tikhanovskaya,
Haoyu Guo,
S. Sachdev,
G. Tarnopolsky,
arXiv: 2010.09742,
arXiv: 2012.14449



SYK liquid of spinons

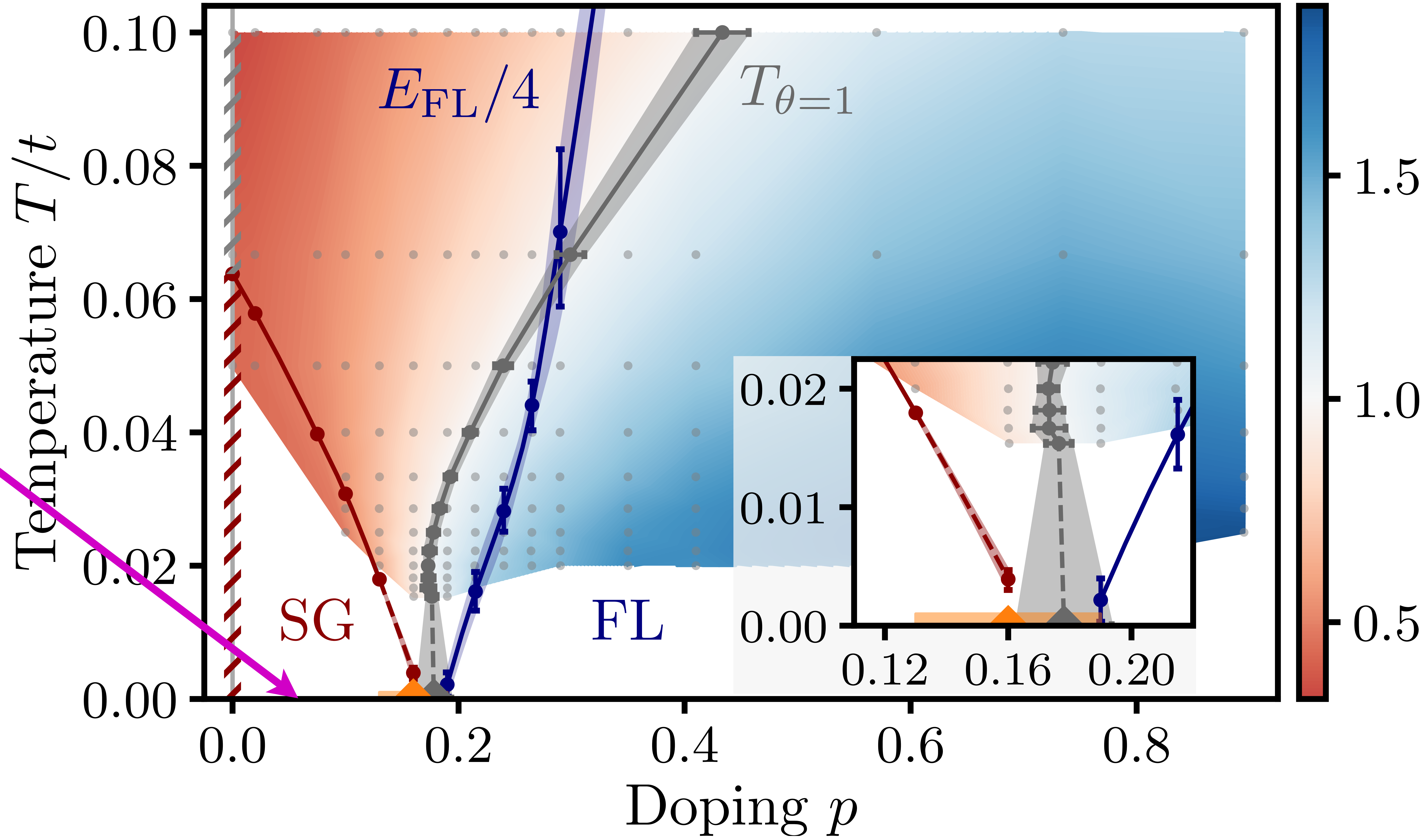
$$\chi''(\omega) \sim \mathrm{sgn}(\omega)\,[1 - c|\omega| + \ldots]$$

$|\omega|$ is from time reparameterization mode

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}

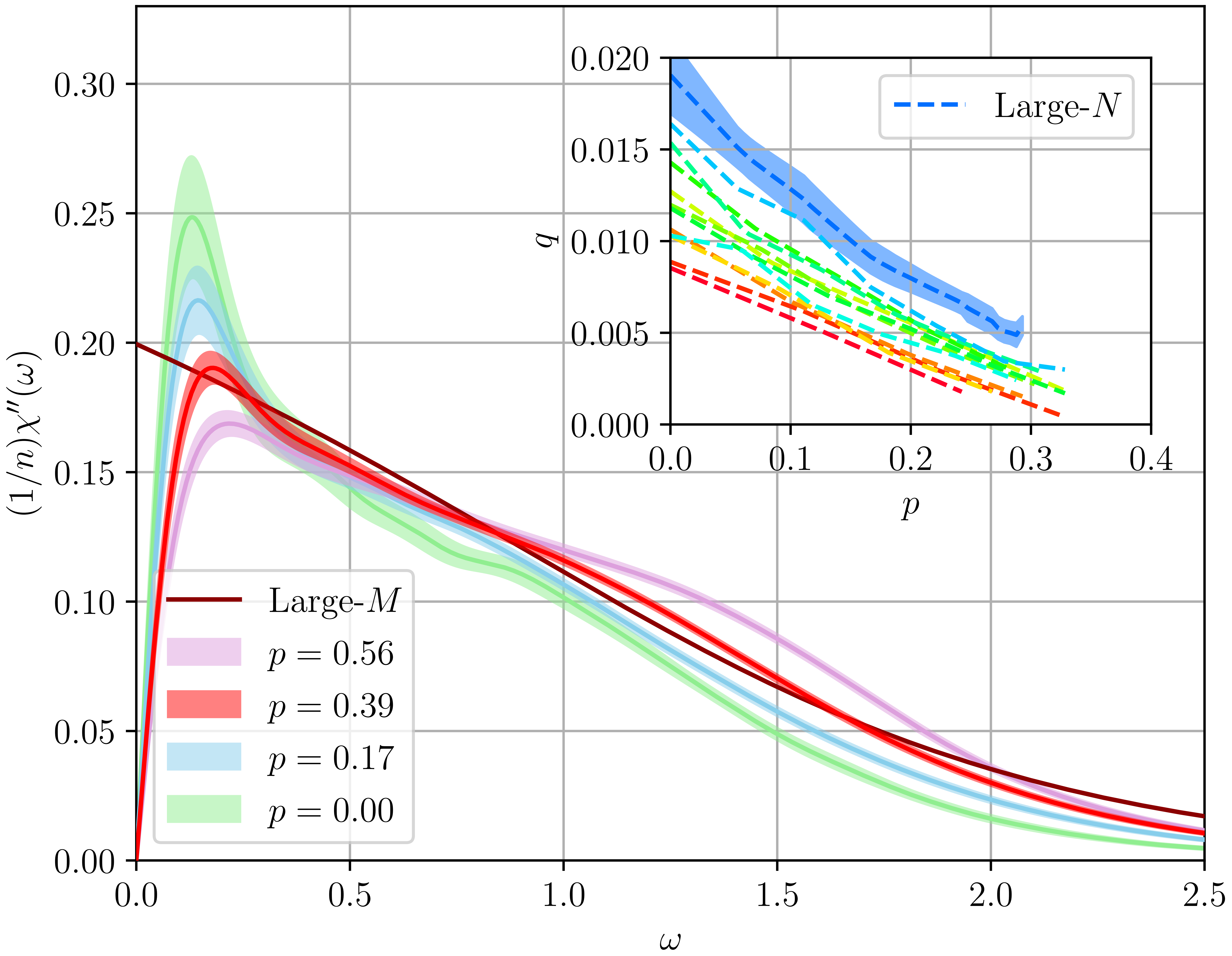
Spin glass order disappears with increasing p, but SYK liquid of spinons remains unchanged



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Numerical solution of t-J model on a fully-connected cluster with all-to-all and random t_{ij} and J_{ij}



Spin glass order disappears with increasing p, but SYK liquid of spinons remains unchanged

H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

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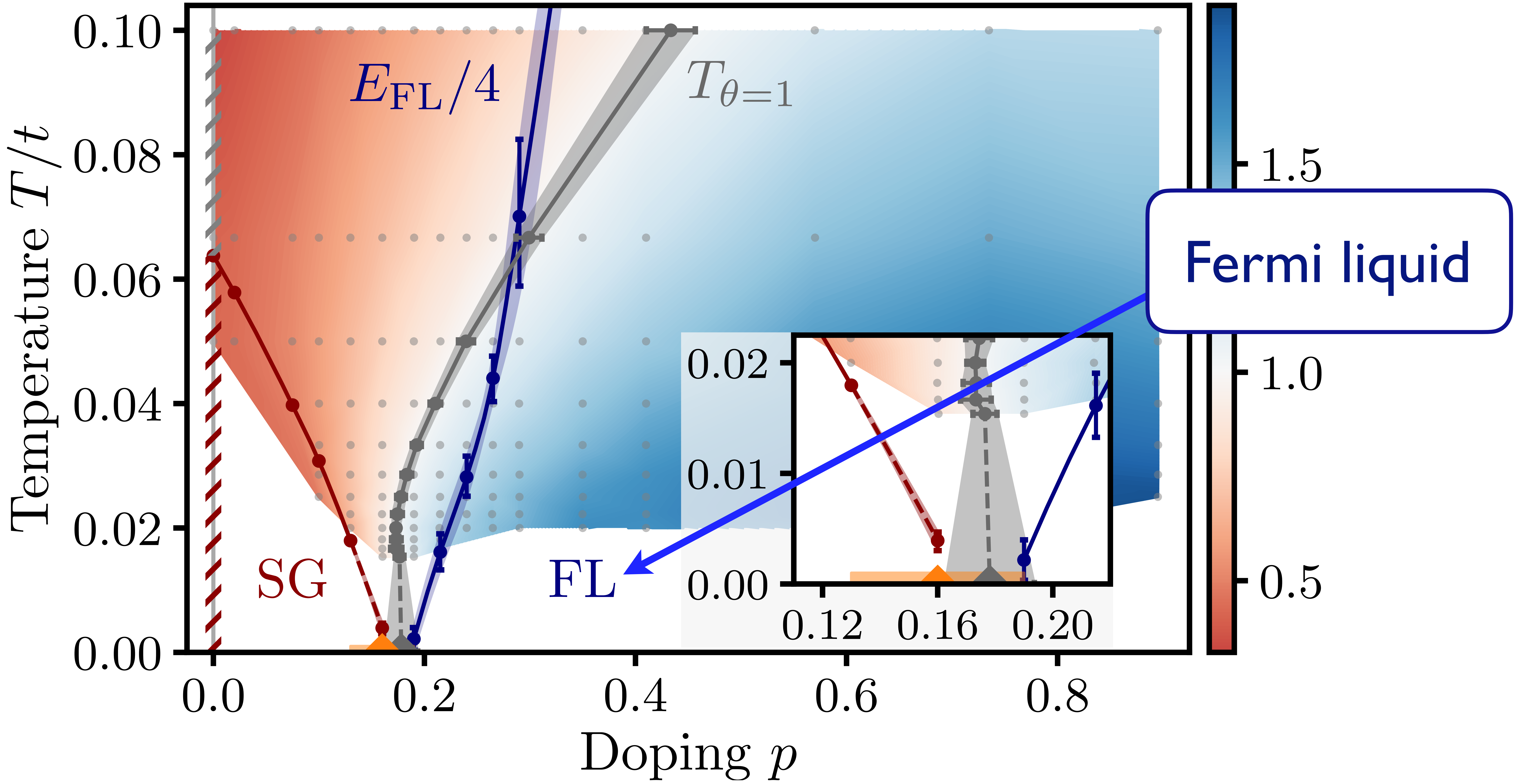

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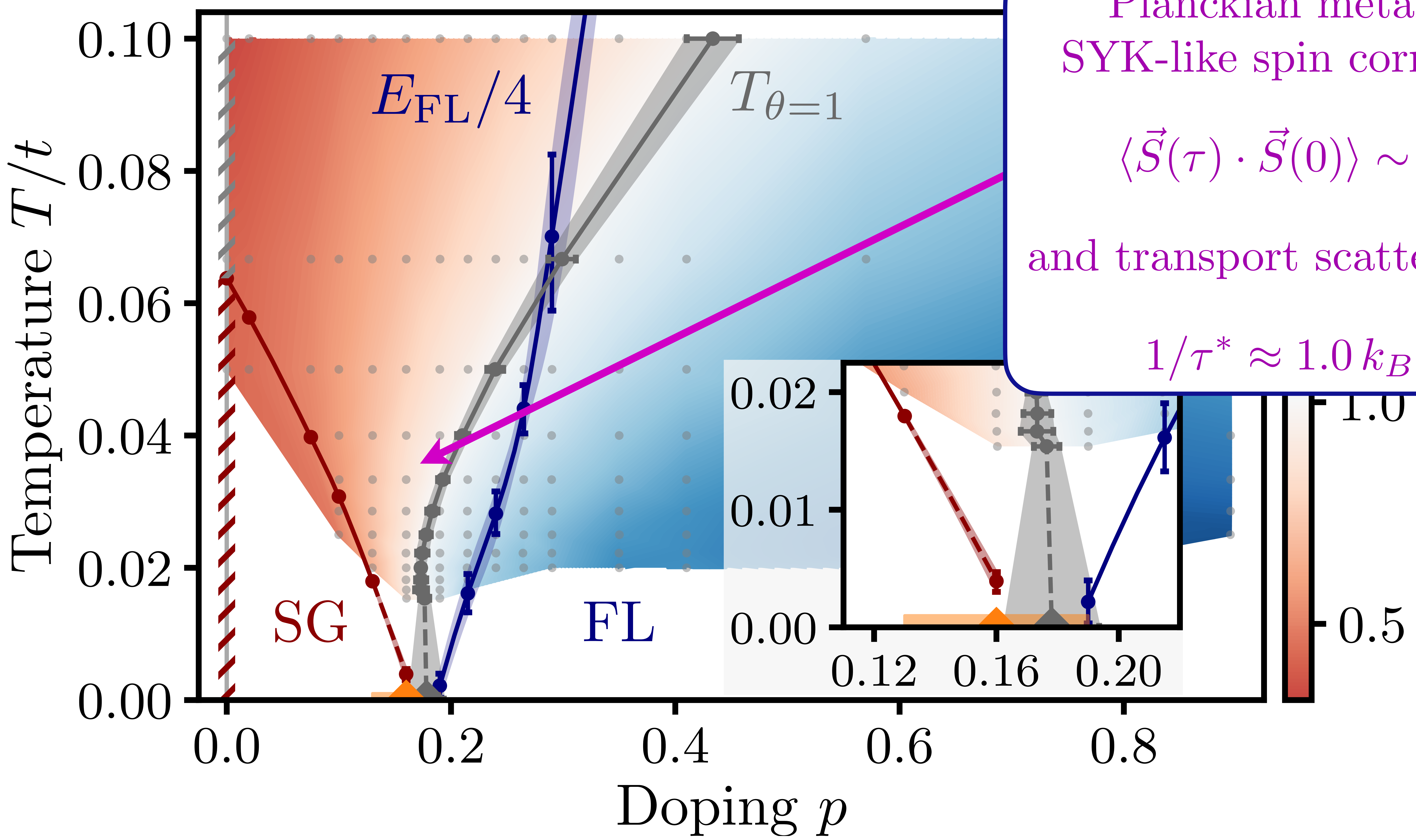


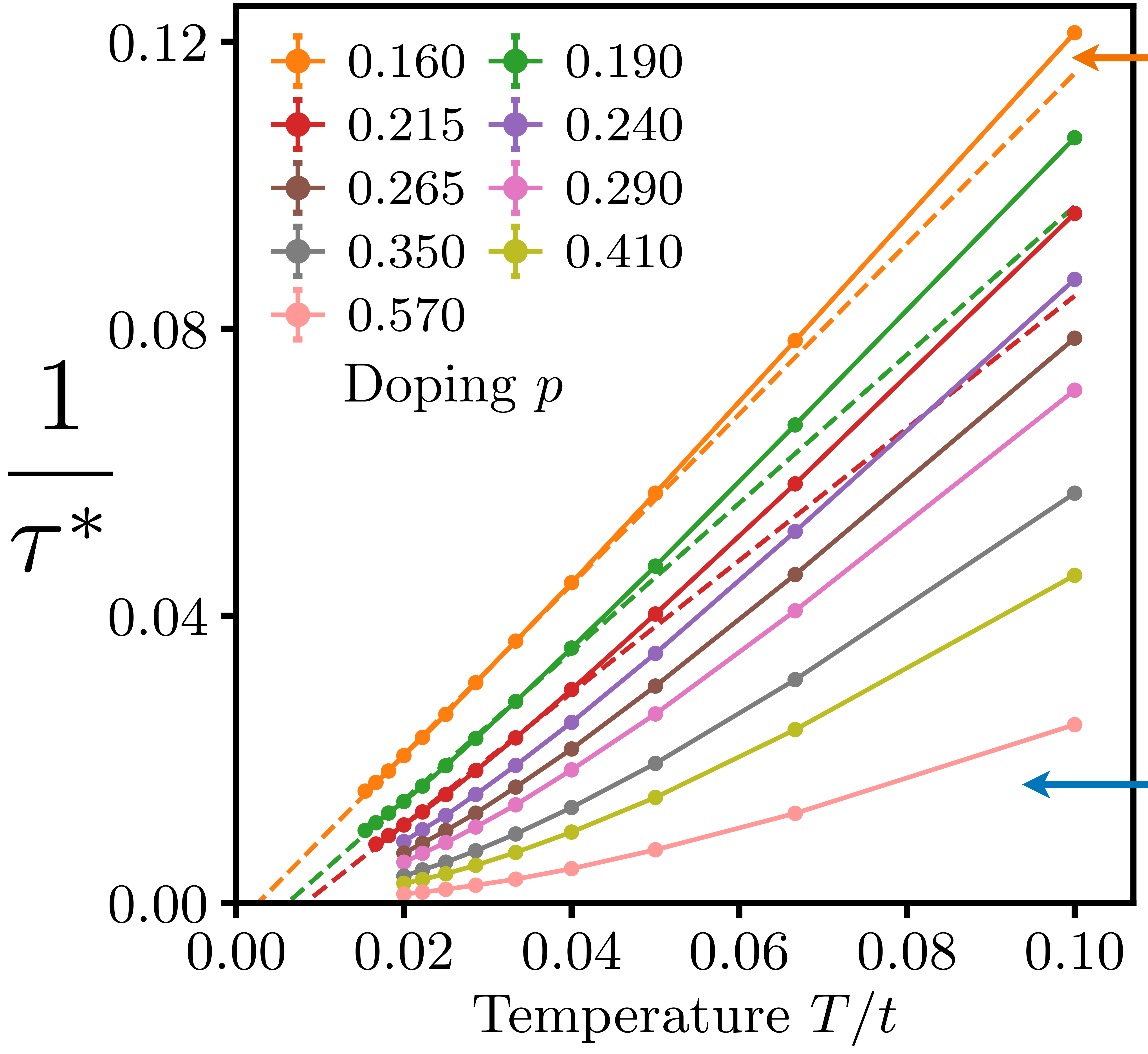
Planckian metal with SYK-like spin correlations

$\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim |\tau|^{-1}$

and transport scattering time

$1/\tau^* \approx 1.0\, k_B T/\hbar$

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)



$$\frac{1}{\tau^*} \simeq c\frac{k_B T}{\hbar}$$

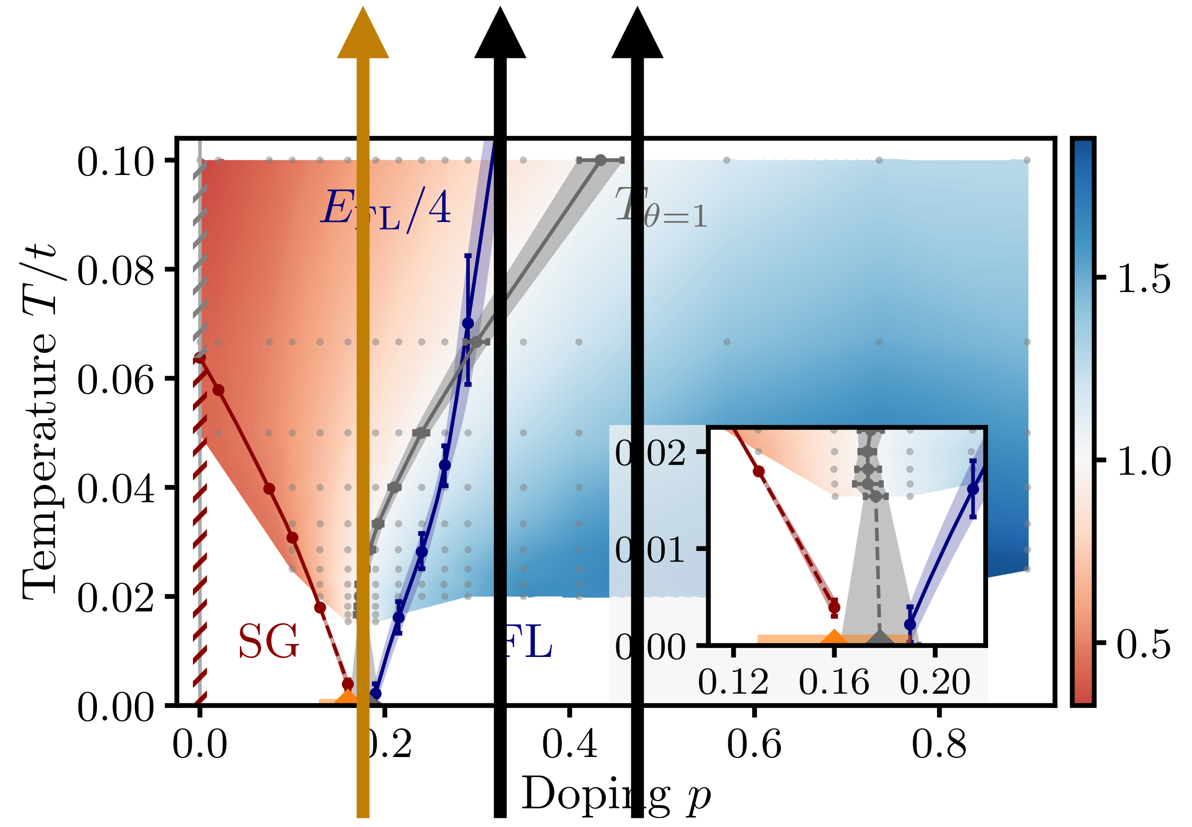
$$c \approx 1.0$$

Planckian metal
for $p \approx p_c$

Large M theory
Resistivity: $\rho(T) = \rho(0) + \tilde{c}\,T \ldots$
Linear T term is
correction to scaling
from time reparameterization mode.

Haoyu Guo, Yingfei Gu, and S. Sachdev 2020

$$\frac{1}{\tau^*} \propto T^2$$



P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607

1. SYK model

2. Random t-J model

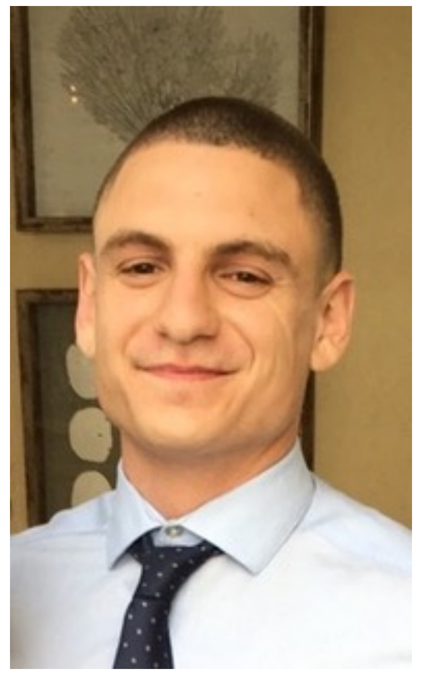
3. Fermi surface coupled to a critical boson in 2 dimensions
   Large N expansion, maximal chaos, and transport

4. Black holes

Large N theory of a critical Fermi surface

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.



Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv: 2103.08615

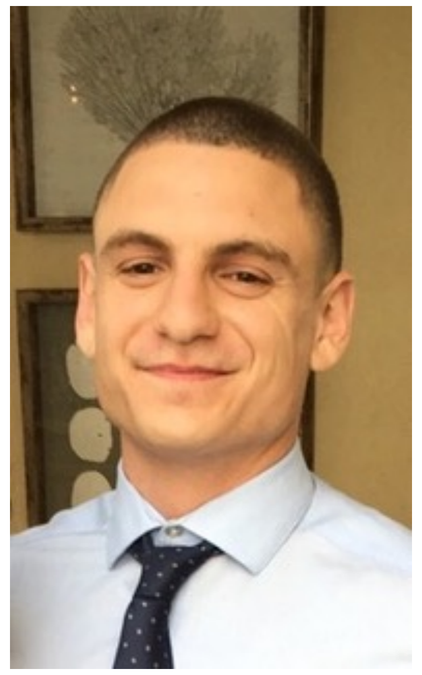
Large N theory of a critical Fermi surface

N flavors of fermions ψ_i,
N flavors of a boson ϕ_i, and
a "Yukawa coupling" g_{ijl} which is a random function in flavor space.
Note: there is *no spatial randomness*. In the large N limit

$$S = \int d\tau \sum_k \sum_{i=1}^{N} \psi_{ik}^\dagger(\tau) \left[\partial_\tau - 2t(\cos k_x + \cos k_y) - \mu\right] \psi_{ik}(\tau)$$

$$+ \frac{1}{2} \int d\tau \sum_q \sum_{i=1}^{N} \phi_{iq}(\tau) \left[-\partial_\tau^2 - 2J(\cos q_x + \cos q_y - 2) + m_b^2\right] \phi_{i,-q}(\tau)$$

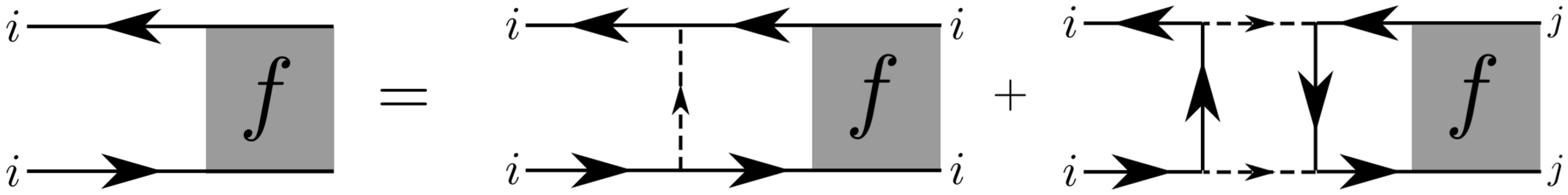
$$+ \int d\tau \sum_{k,q} \sum_{i,j,l=1}^{N} \left[\frac{g_{ijl}}{N} \psi_{i,k+q}^\dagger(\tau) \psi_{jk}(\tau) \phi_{lq}(\tau)\right] ,$$



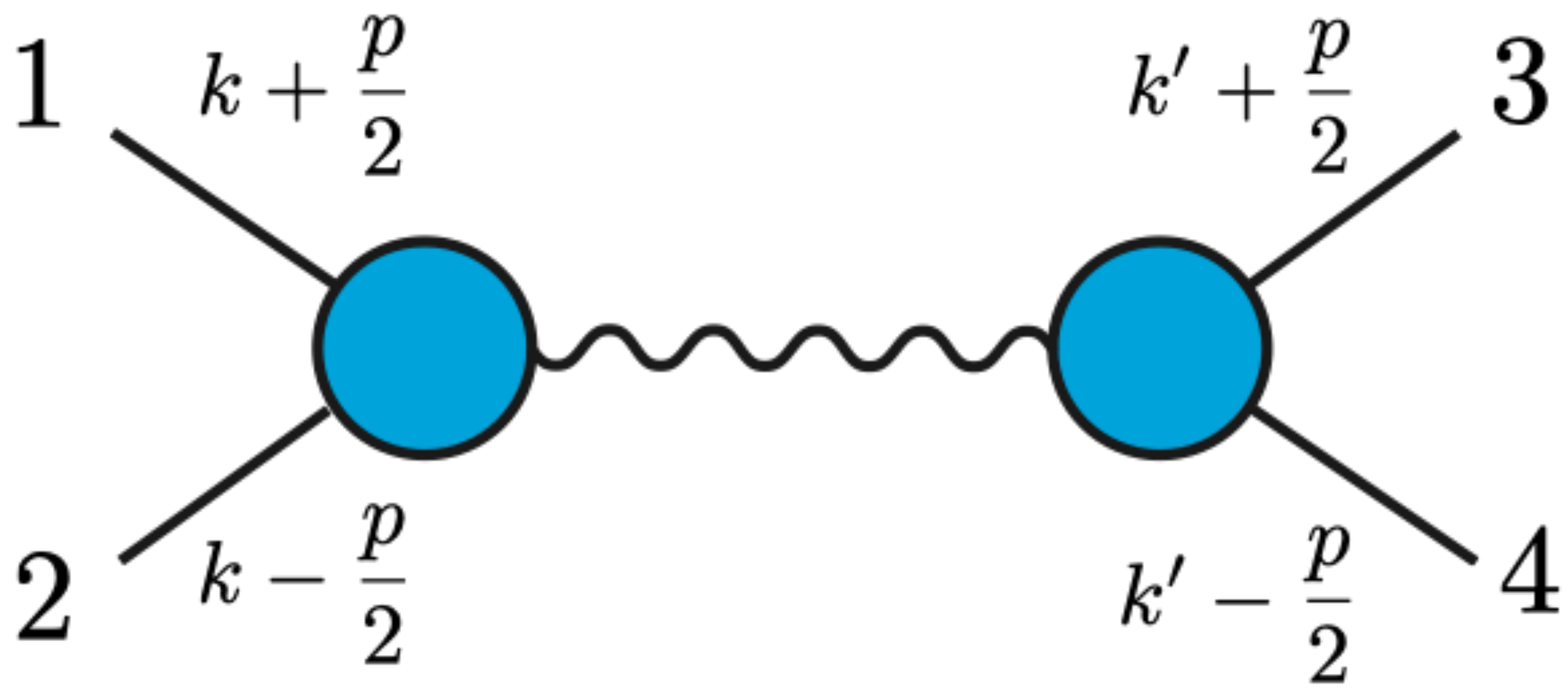
$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv: 2103.08615

Computation of fermion OTOC



Invariance under adding a ladder

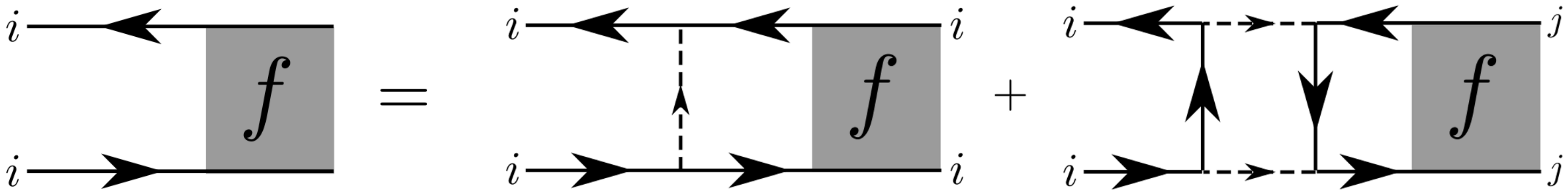


$$\mathrm{OTOC}_p(t_1,t_2,t_3,t_4;k,k') \approx \frac{e^{\lambda_L(p)(t_1+t_2-t_3-t_4)/2}}{C(p)}\Upsilon^R_p(t_{12},k)\Upsilon^A_p(t_{34},k')$$

$$\mathrm{OTOC}_{x,0}(t_1,t_2,t_3,t_4) = \int \frac{dp}{2\pi} e^{ipx}\mathrm{OTOC}_p(t_1,t_2,t_3,t_4)$$

$$\sim \frac{1}{N}u(x,t)\int_{k,k'}\Upsilon^R_p(t_{12},k)\Upsilon^A_p(t_{34},k') \qquad (4)$$

Computation of fermion OTOC



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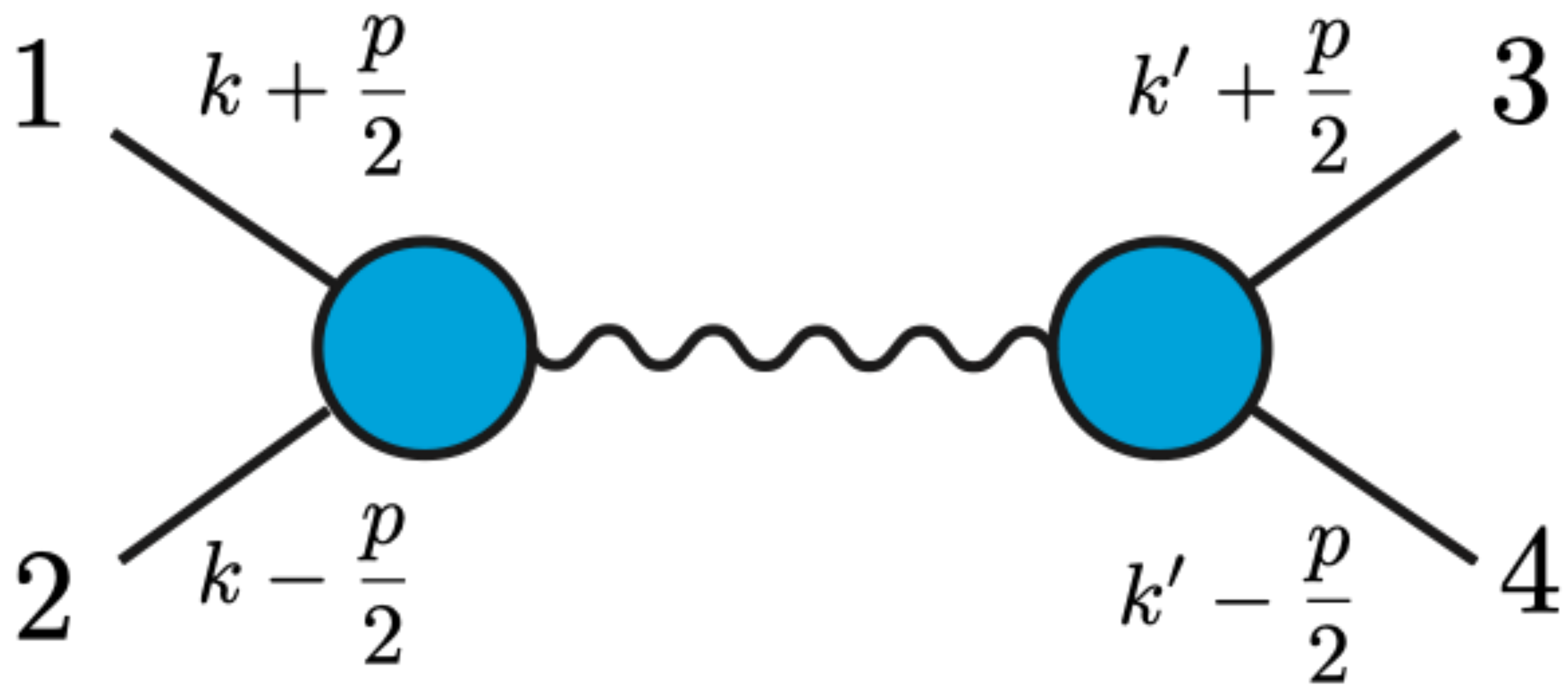

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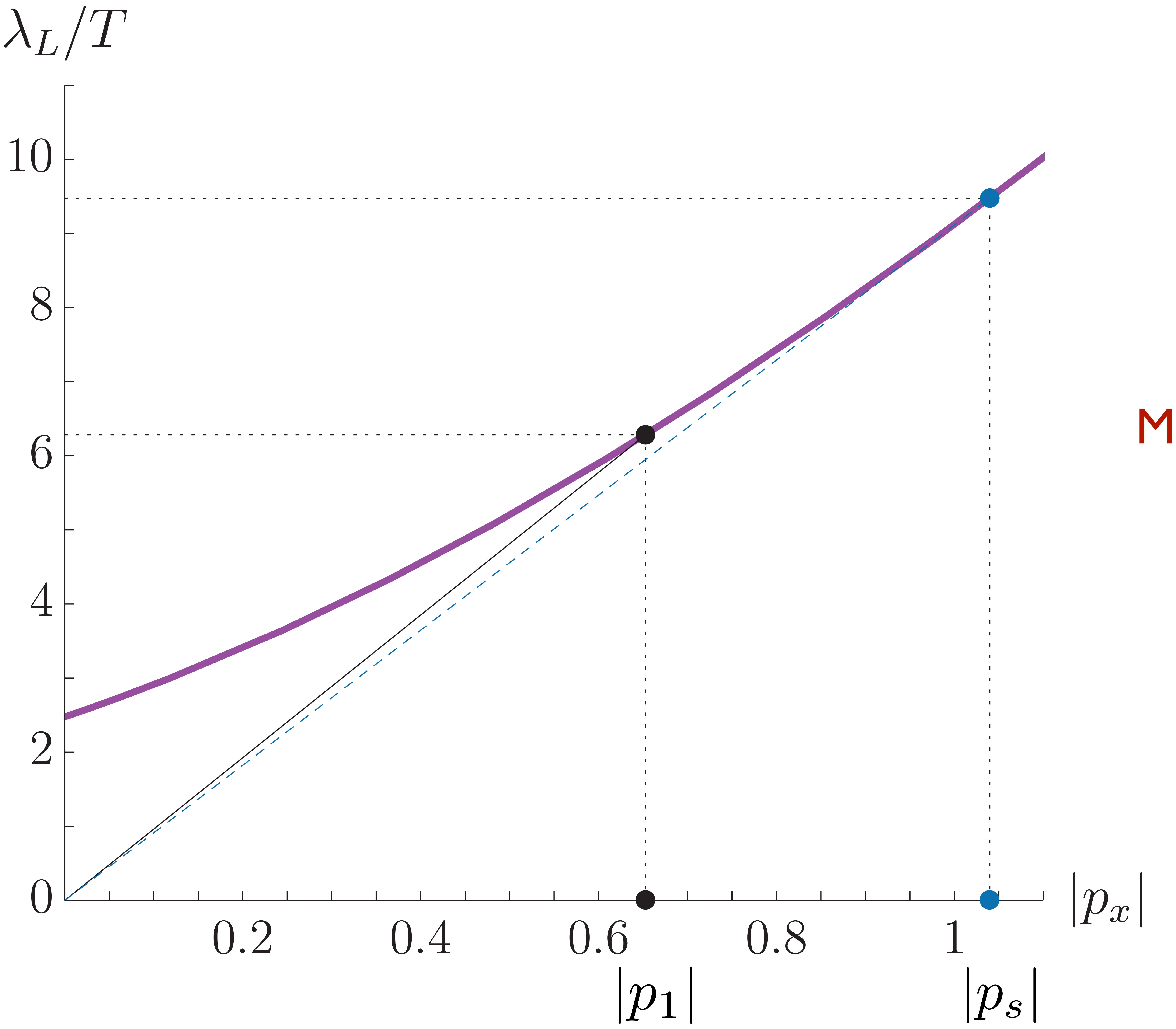
$$\sim \frac{1}{N}u(x,t)\int_{k,k'}\Upsilon_p^R(t_{12},k)\Upsilon_p^A(t_{34},k')$$

The "scramblon": $C(p) = \cos(\lambda_L(p)/(4T))$

$$u(x,t) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\frac{e^{\lambda_L(p)t+ipx}}{\cos(\lambda_L(p)/(4T))}$$

Gu and Kitaev (2019)



Maria Tikhanovskaya
Harvard

Aavishkar Patel
Berkeley

Gu and Kitaev (2019):
Compute λ_L for *imaginary* momentum: *i.e.* $p_x = i|p_x|$ and include contribution of pole

We find maximal chaos with $\lambda_L = 2\pi T$,
and butterfly velocity $v_1 = 2\pi/|p_1| \approx 9.67 g^{-4/3} T^{1/3}$

Transport of a critical Fermi surface

Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \mathrm{Re}\,\sigma_{\mathrm{reg}}(\omega)$$

A. Eberlein, I. Mandal, and S. S., PRB **94**, 045133 (2016)

$$\mathrm{Re}\,\sigma_{\mathrm{reg}}(\omega, T=0) \sim \frac{1}{\omega^{2/3}}$$

Confirmed in the large N theory.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. to appear

Have to include the effects of disorder or umklapp

Transport of a critical Fermi surface

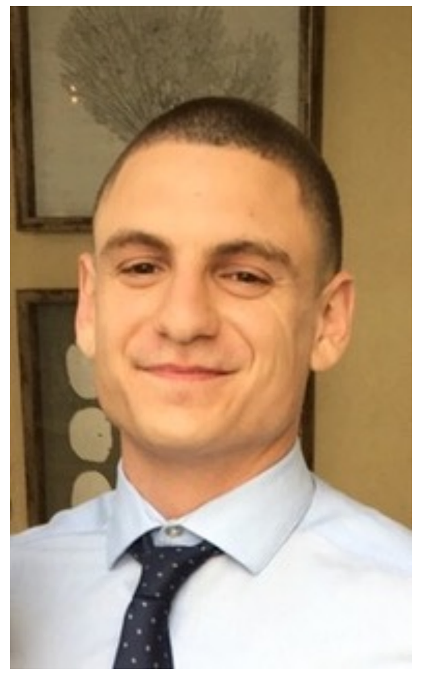
Random potential disorder

$$S_{\text{disorder},1} = \int d\tau \frac{1}{\sqrt{N}} \sum_r \sum_{ij=1}^{N} v_{ij}(r) \psi^\dagger_{ir}(\tau)\psi_{jr}(\tau)$$

where r labels lattice sites.
The potential $v_{ij}(r)$ is random both in position and flavor space

$$\overline{v^*_{ij}(r) v_{lm}(r')} = v^2\, \delta(r - r')\delta_{il}\delta_{jm}$$

In the low T scaling limit Leads to a non-zero d.c. resistivity.
This is similar to the scaling limit of the random t-J model.



Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. to appear

Transport of a critical Fermi surface

Random interaction disorder

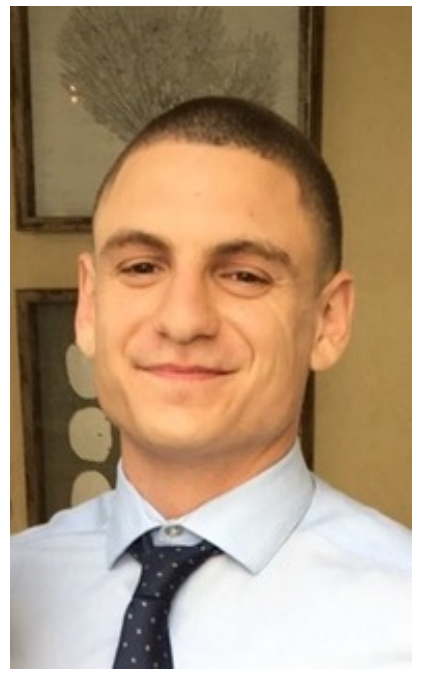
$$S_{\text{disorder},2} = \int d\tau \frac{1}{N} \sum_r \sum_{ilj=1}^{N} g'_{ijl}(r) \psi^\dagger_{ir}(\tau)\psi_{jr}(\tau)\phi_{lr}(\tau),$$

where r labels lattice sites.

The interaction $g'_{ijl}(r)$ is random both in position and flavor space

$$\overline{g'^*_{ijl}(r) g'_{abc}(r')} = g'^2\, \delta(r - r')\delta_{ia}\delta_{jb}\delta_{lc}.$$

A model with $g = v = 0$ and g' non-zero has been studied earlier, and yields Planckian transport with linear-in-T resistivity.



E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. arXiv: 2103.08615

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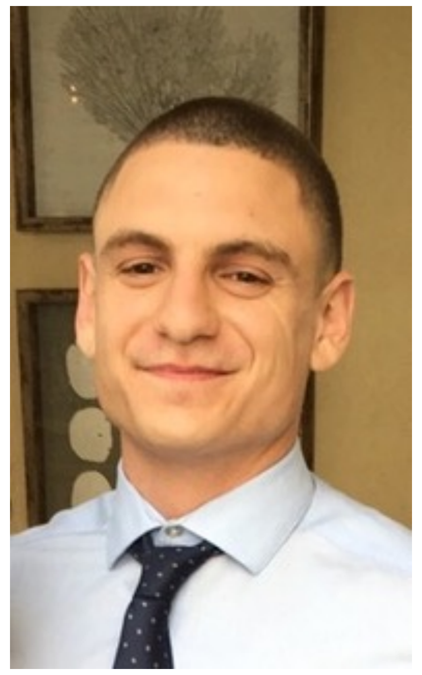

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With g, v, g' all non-zero, resistivity $\rho(T) = \rho(0) + \tilde{c}\,T \ldots$. $\rho(0)$ is determined by v, while $\tilde{c}$ is determined by a subleading operator, g', as in the random t-J model.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. to appear

1. SYK model

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4. Black holes

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_{\mu} \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_{\mu}]\right)$$

Metric of spacetime

Electromagnetic gauge field

In general, this integral is not well defined, because of an uncontrollably large number of spacetime configurations.

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$$= \exp\Big(S_{BH}\Big) \times \Big(\quad ....????... \quad \Big)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

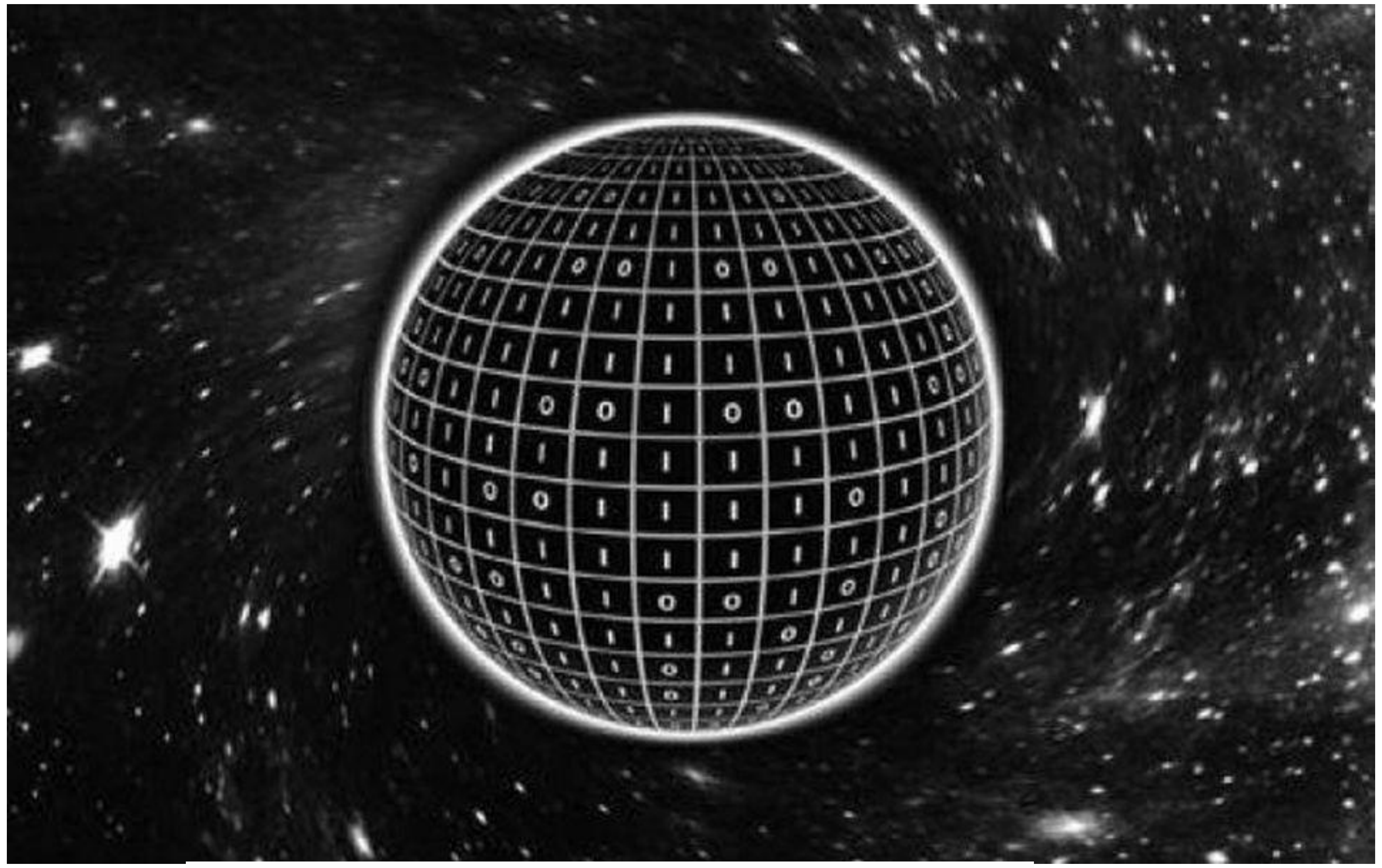
A_0 is the area of the charged black hole horizon at $T = 0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

Questions

- Is Einstein-Maxwell theory meaningful beyond the saddle point, and can we compute quantum fluctuation corrections to S_{BH}?

- Can the resulting entropy be understood as that of a unitary quantum system with a discrete spectrum ?

- Can we compute the evolution of the entropy as the black hole evaporates? Is it that of an evaporating unitary quantum system?



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

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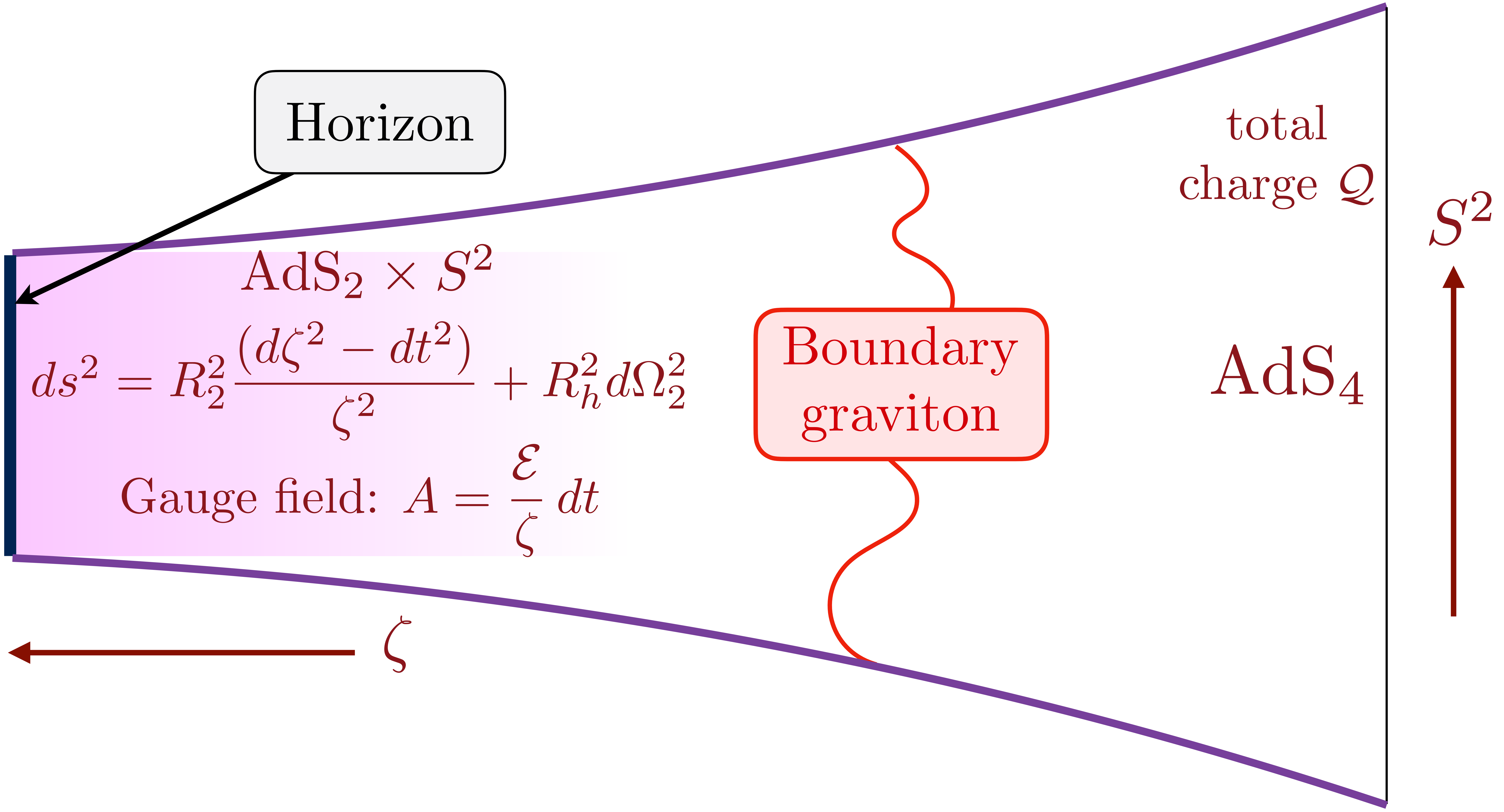
Note the similarity to the large N entropy of the SYK model !
(along with other similarities) Sachdev PRL 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory



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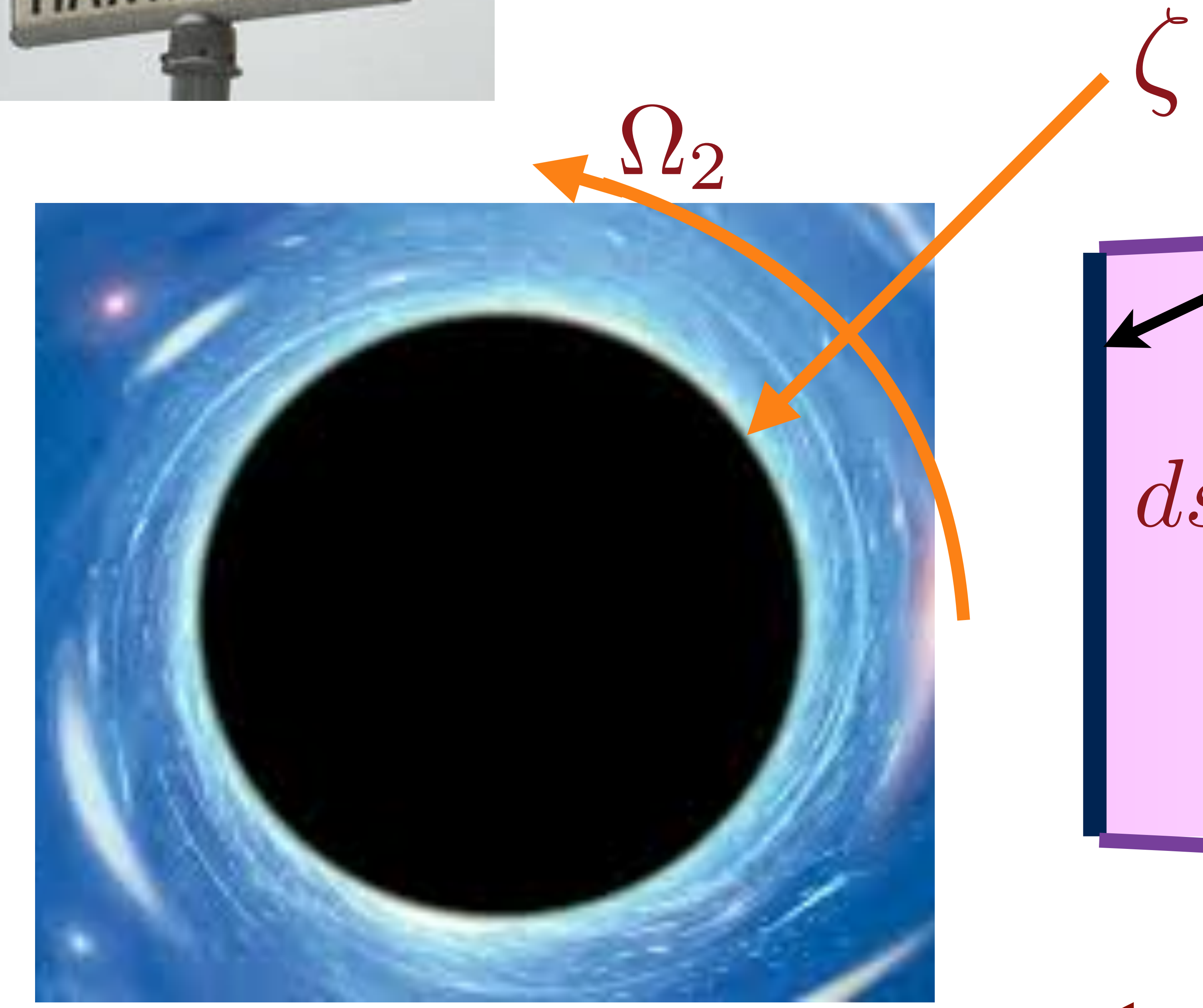

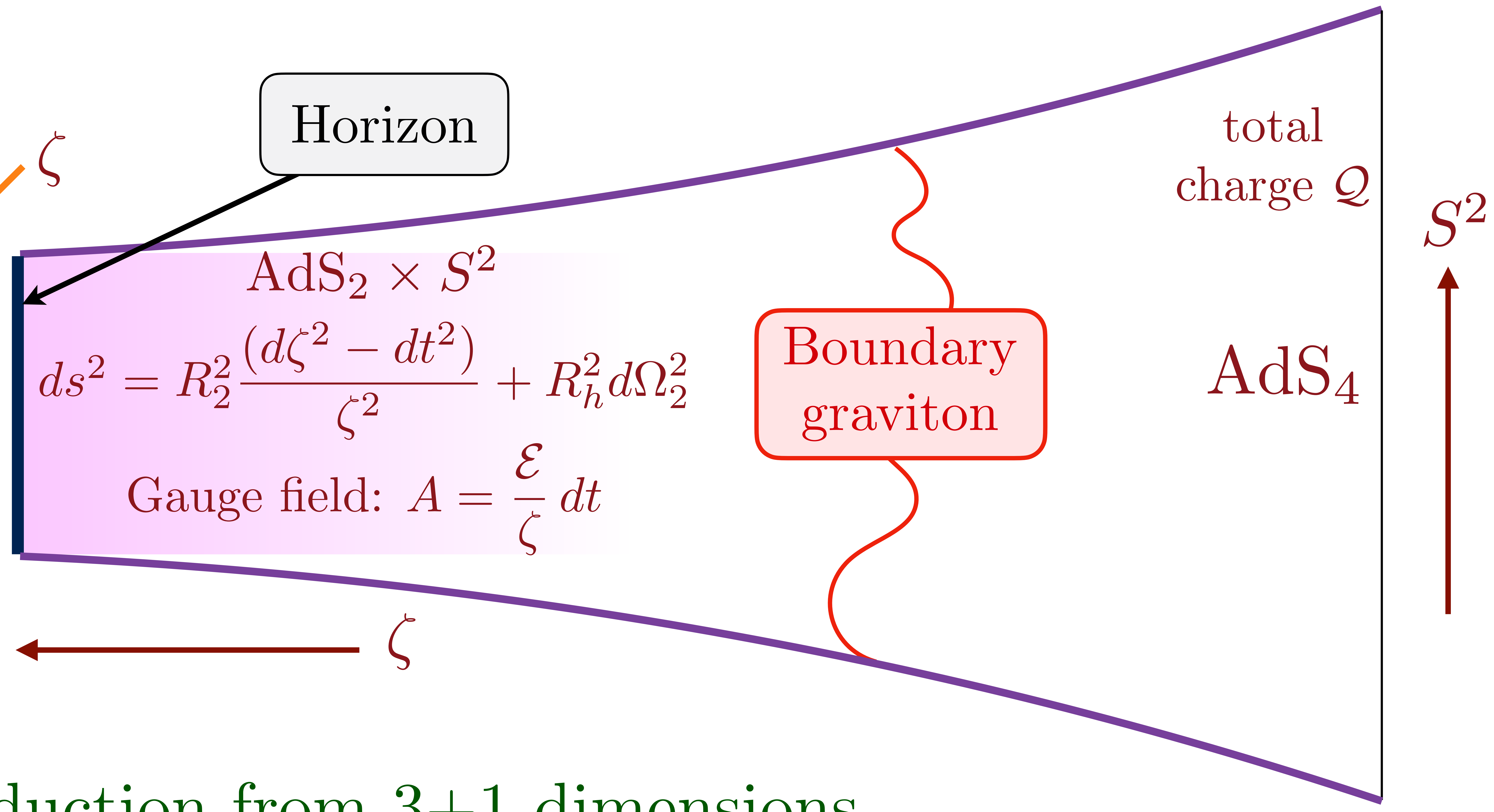

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

$$\approx \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu}, A_\mu]\right)$$

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

Maldacena, Stanford, Yang (2016)

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

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Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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$$S(T\to 0,\mathcal{Q}) = S_{BH} - \frac{3}{4}\ln\left(\frac{\hbar c^5}{GT^2}\right)$$

$$S_{BH} = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T=0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

(There is also a $-(241/45)\ln(A_0/G)$ correction at $T=0$ A. Sen 2011)

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Many-body density of states

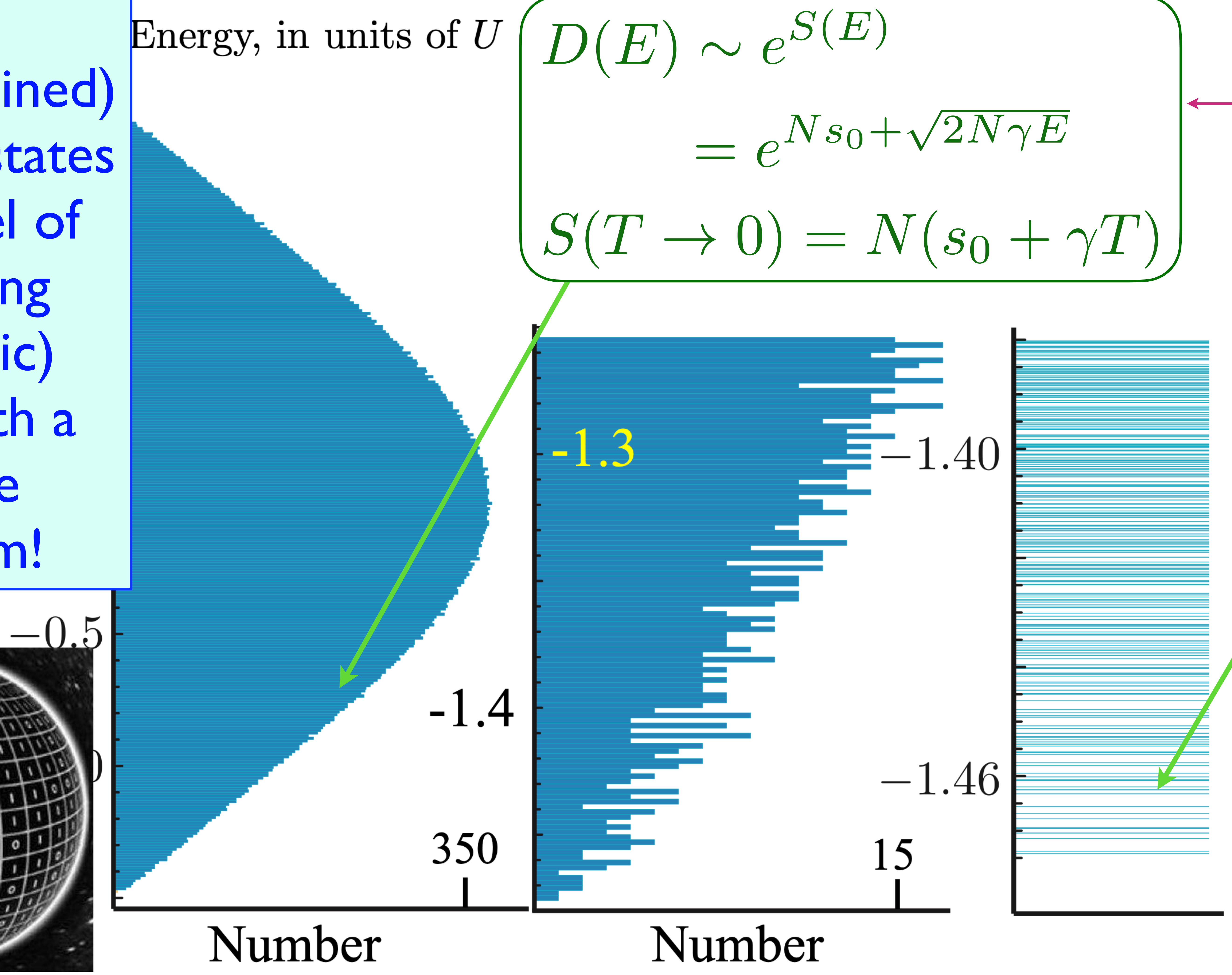
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

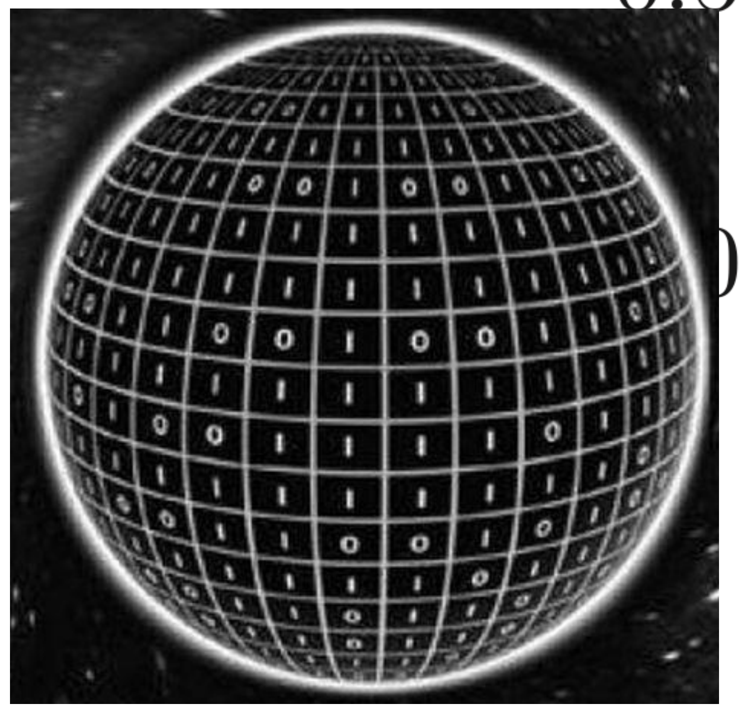
Same entropy and (coarse-grained) density of states in a model of interacting (fermionic) qubits with a discrete spectrum!

$$D(E) \sim e^{S(E)}$$
$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{Ns_0} \sinh(\sqrt{2N\gamma E})$$
$$S(T) = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$

$$D(E) \sim 2\, e^{Ns_0} \sqrt{2N\gamma E}$$





No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

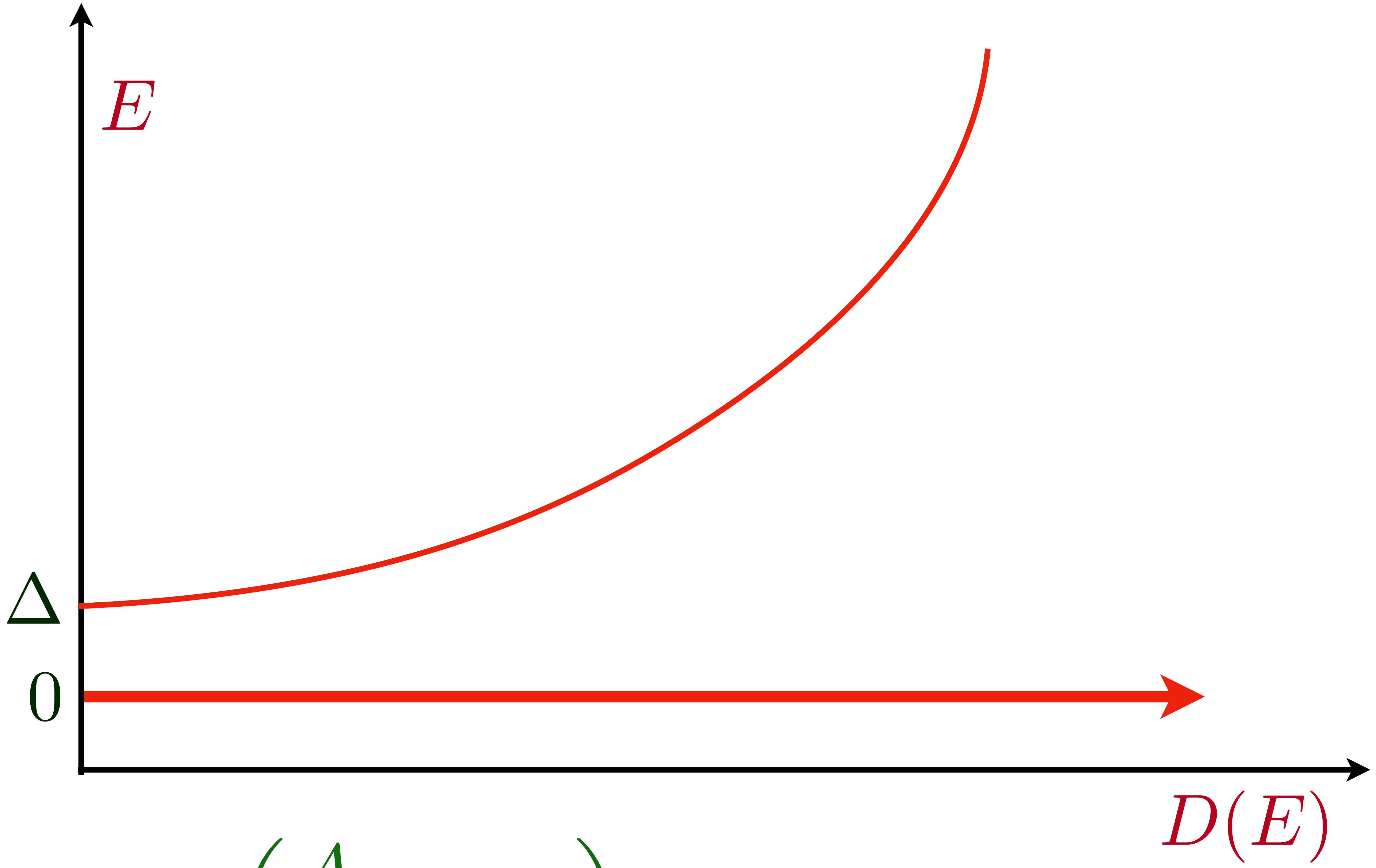
$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim \exp\left(\frac{A_0}{4G} + \dots\right)\delta(E) + f_{\text{reg}}(E - \Delta), \quad \Delta \sim R_h^{-1}$$

Supersymmetric black holes and SYK models

Fu, Gaiotto, Maldacena, Sachdev (2017); Stanford, Witten (2017); Heydeman, Iliesiu, Turiaci, Zhao (2020)

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