Quantum criticality and high temperature superconductivity

University of Waterloo
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Talk online: sachdev.physics.harvard.edu

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Erik Sorensen
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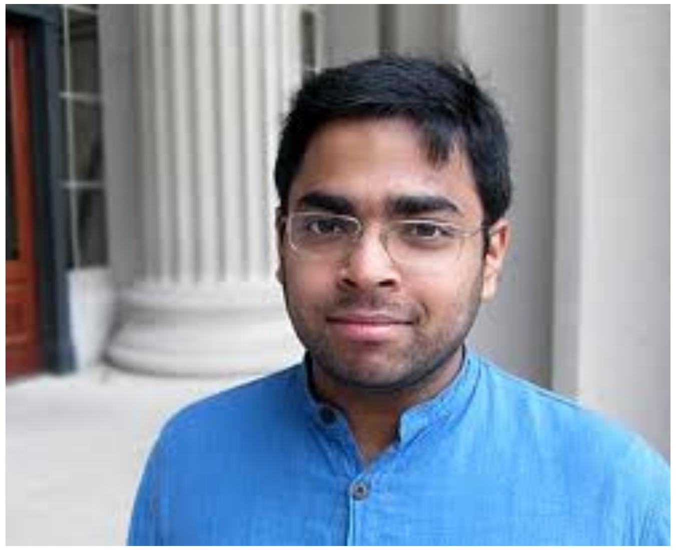
Stanford



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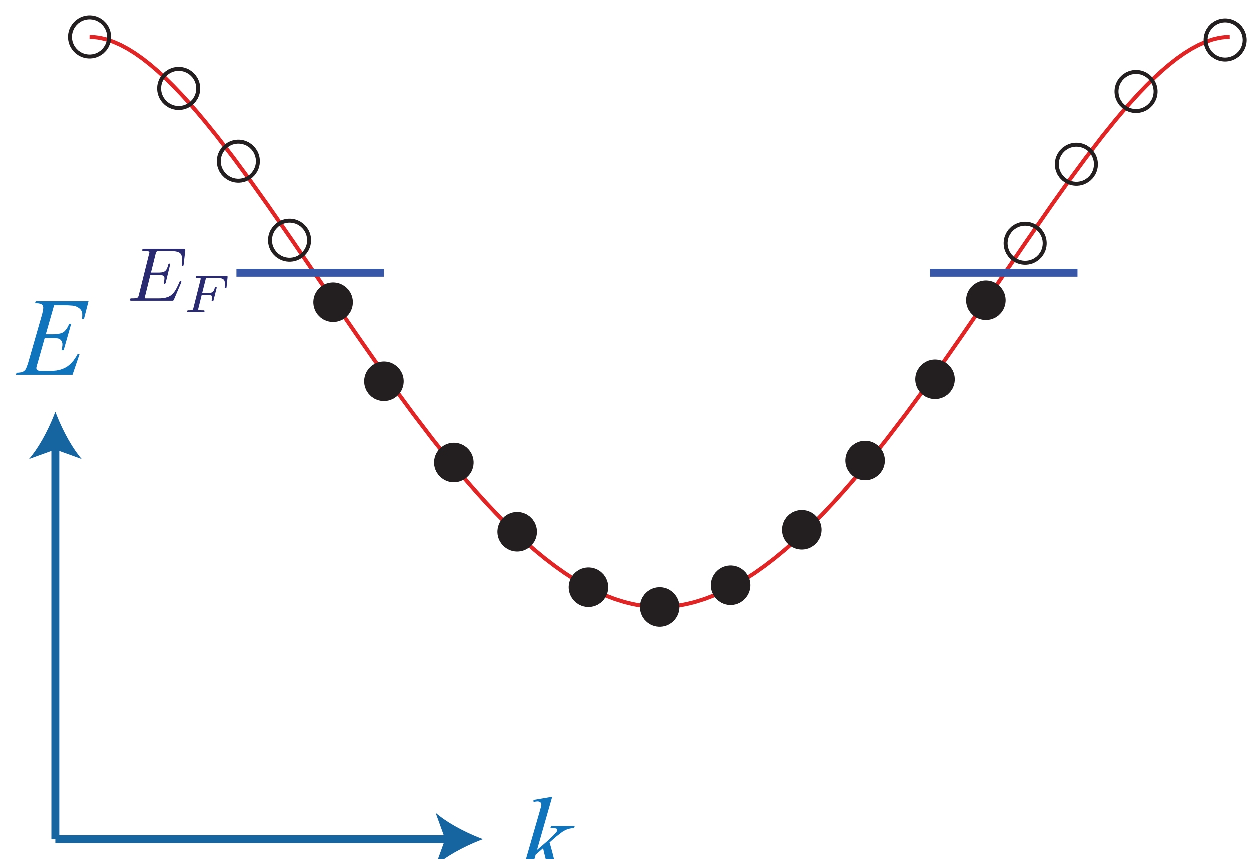
Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

Metals

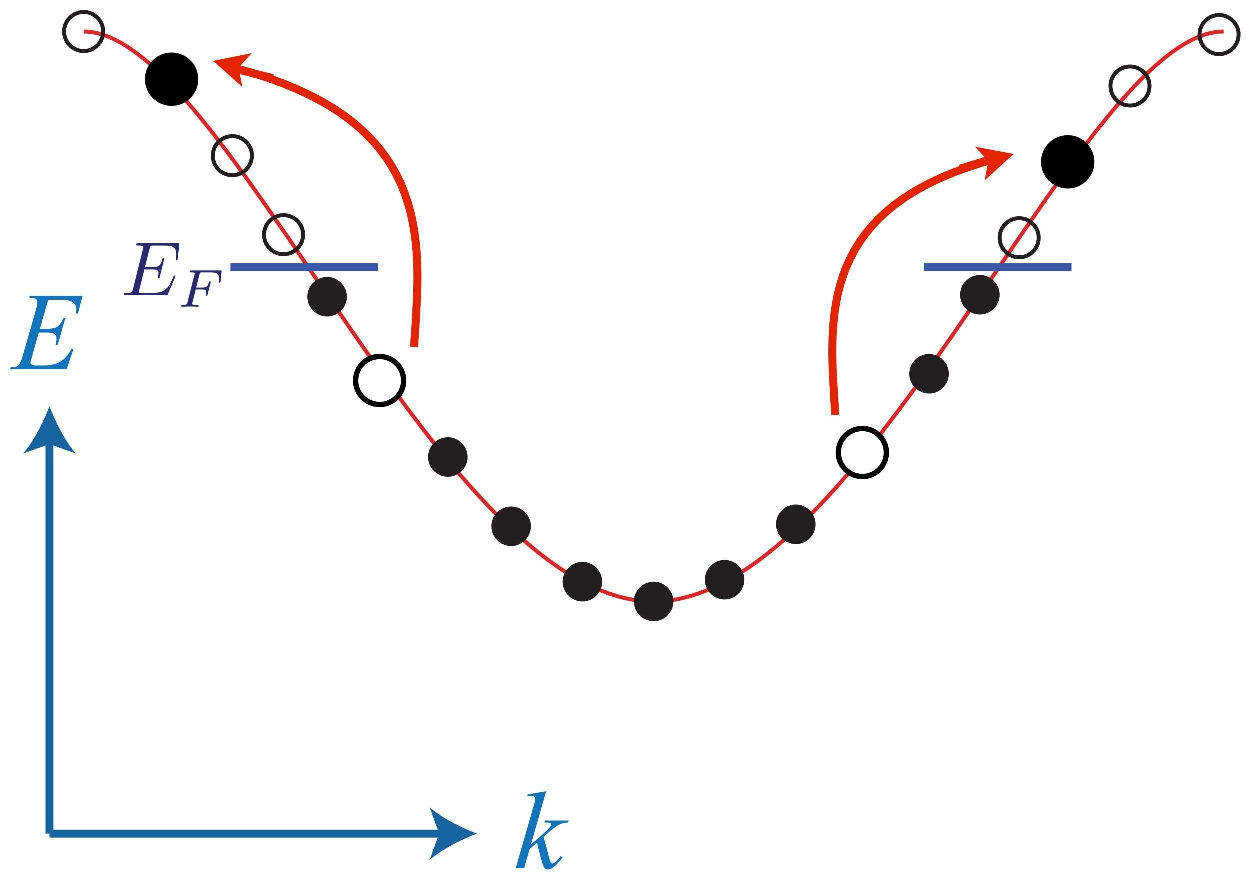


Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Quasiparticle structure of excited states

Metals



Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

1. *Ground states disconnected from independent electron states: many-particle entanglement*

2. **No quasiparticles**

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. *No quasiparticles*

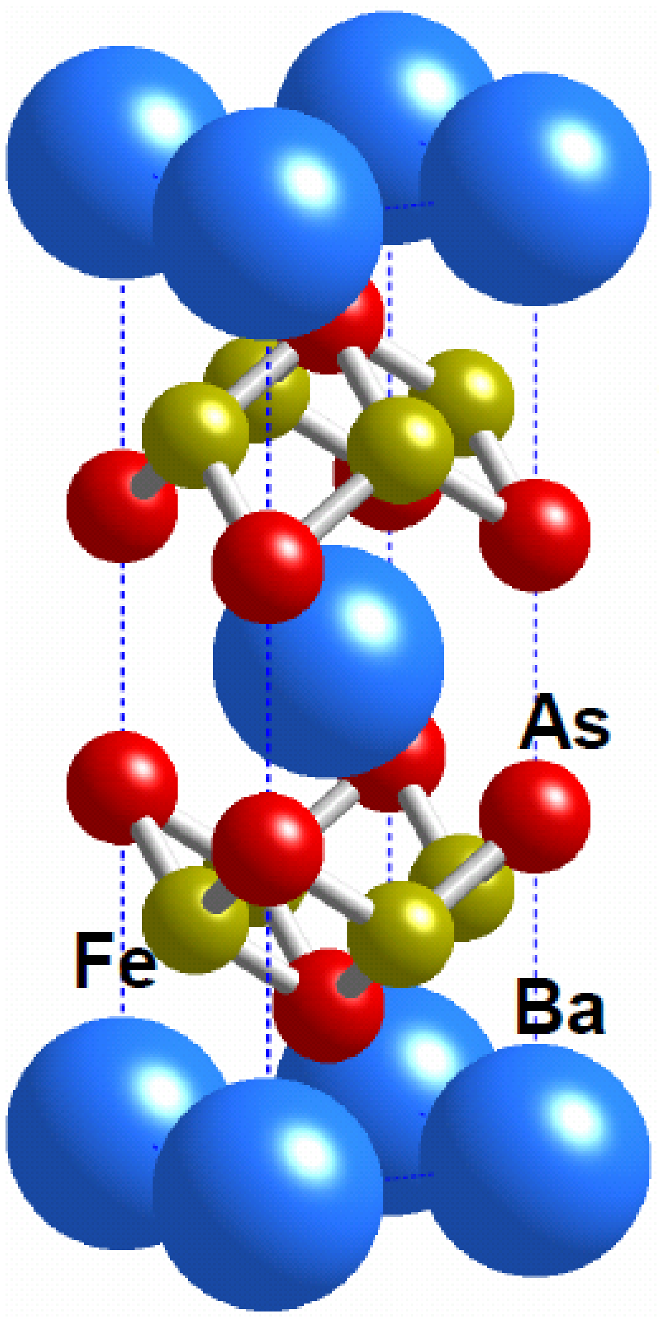
Only 2 examples:

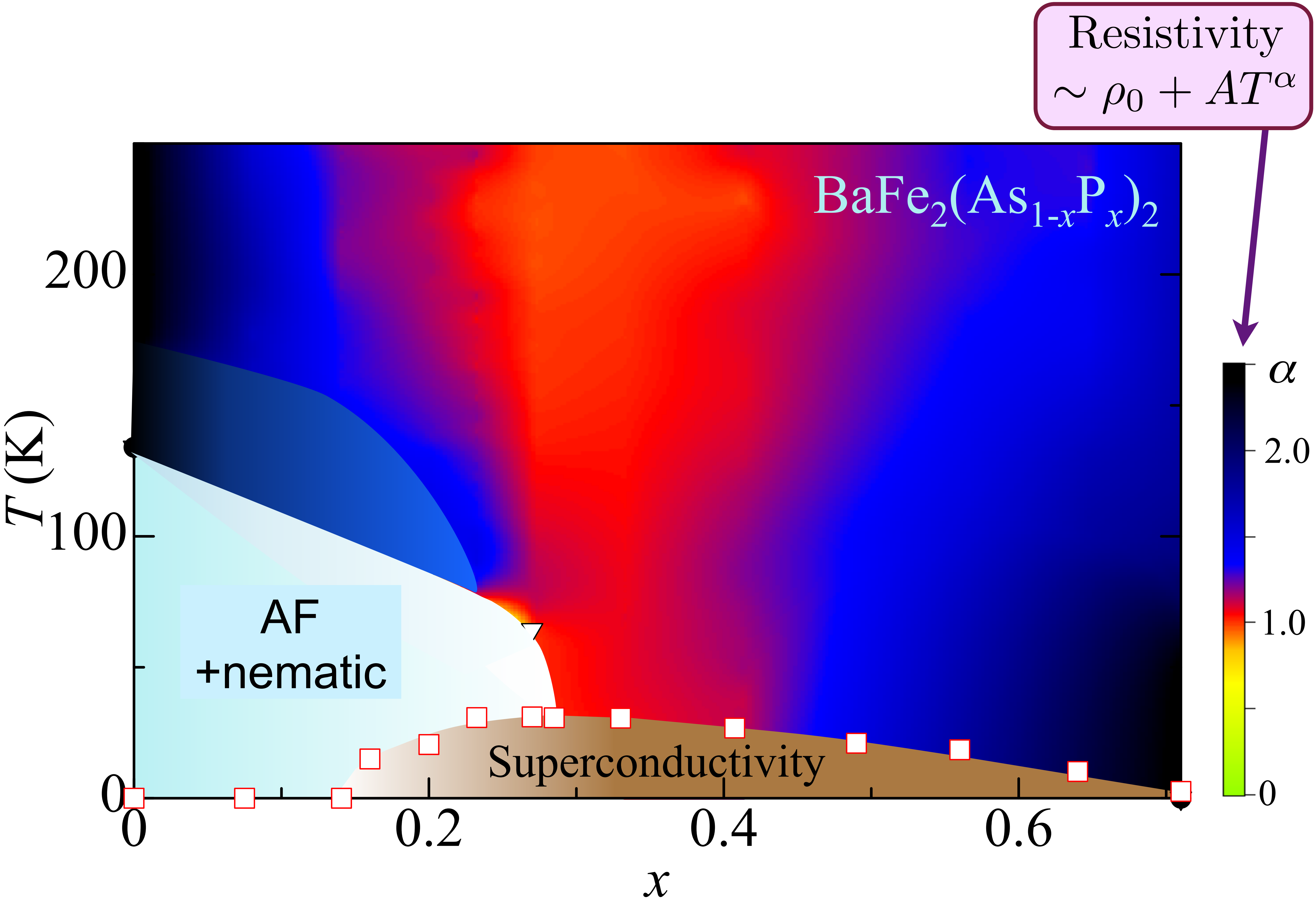
1. Conformal field theories in spatial dimension $d > 1$

2. Quantum critical metals in dimension $d=2$

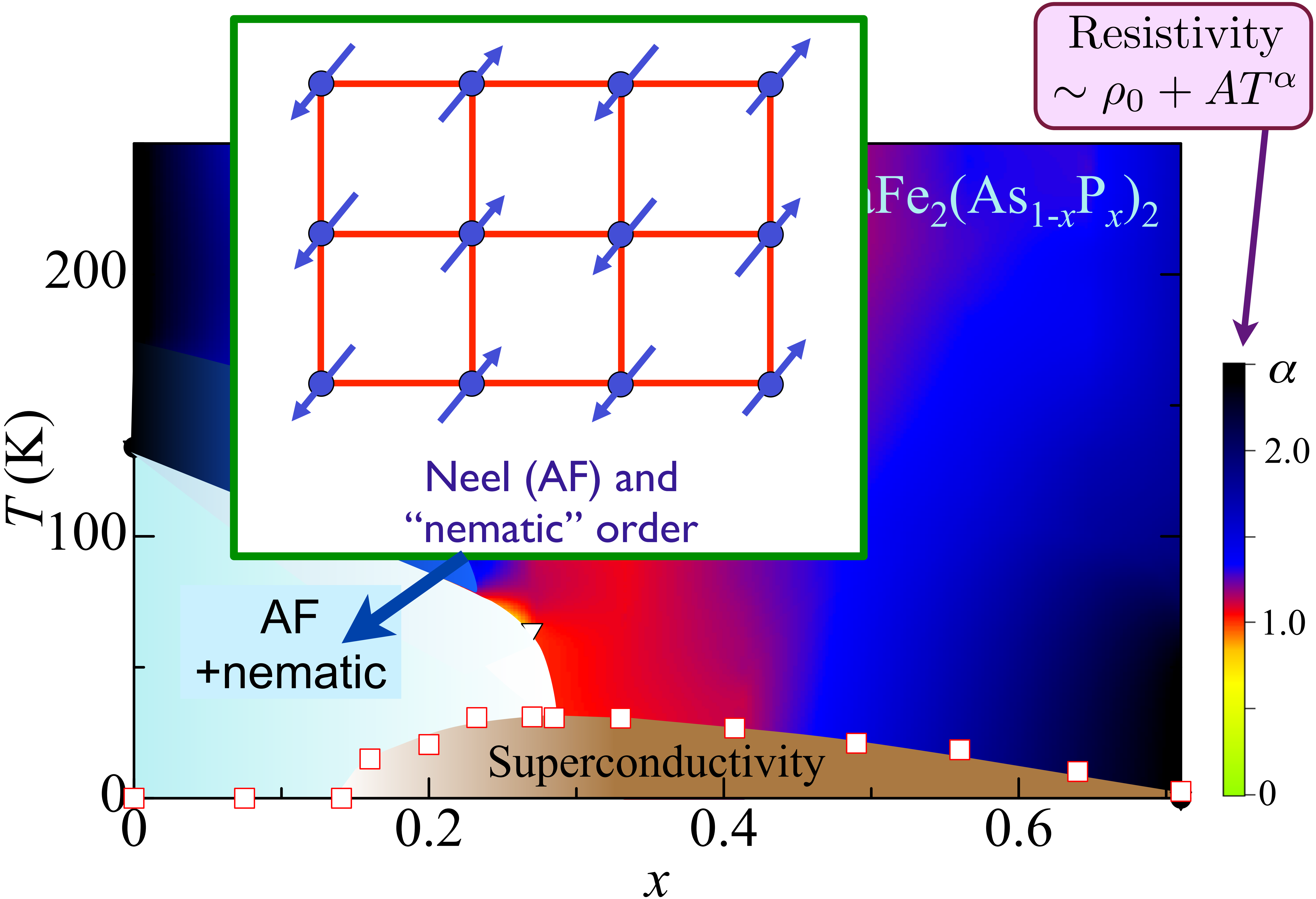
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)



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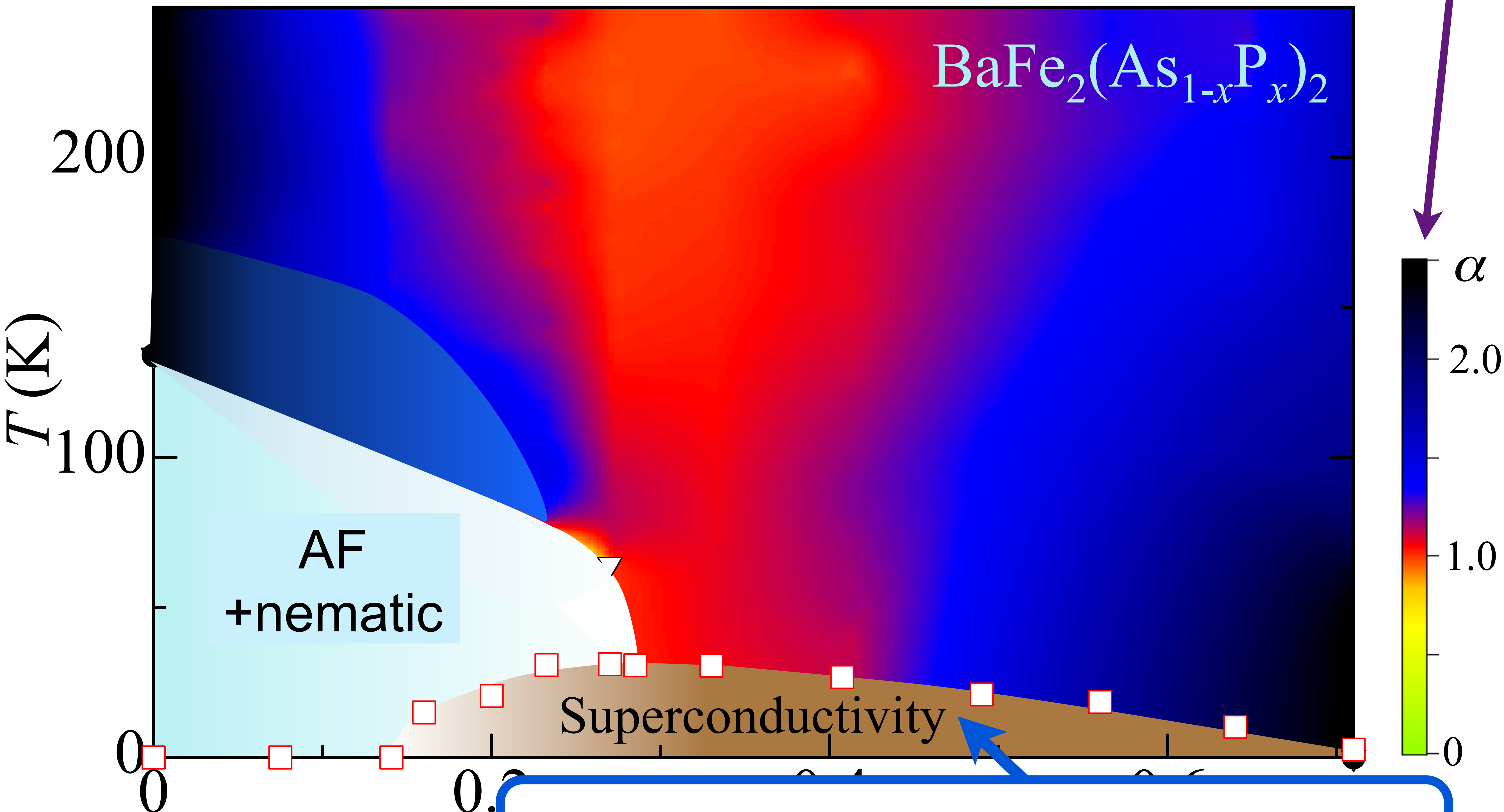
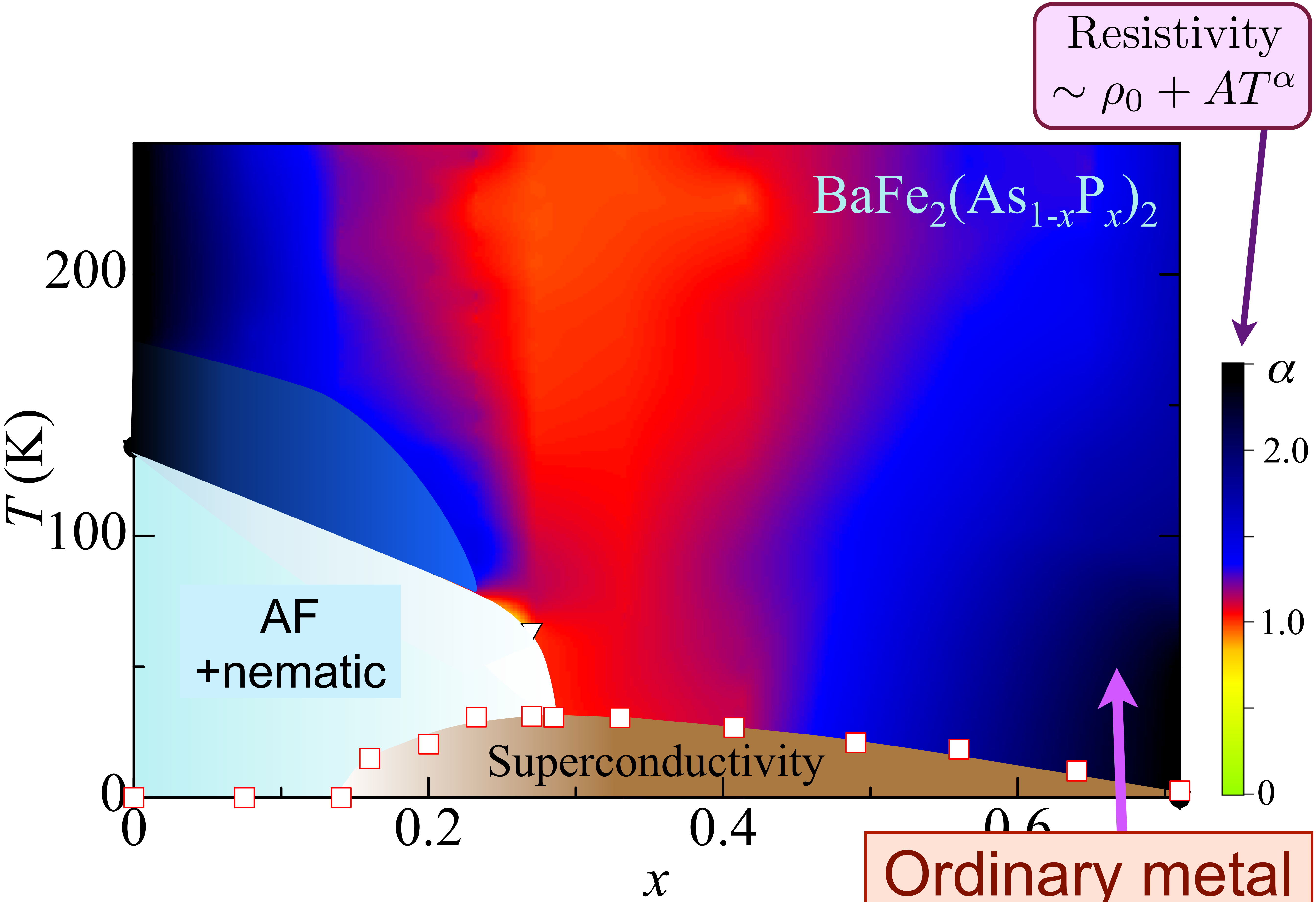
Resistivity
$\sim \rho_0 + AT^{\alpha}$



Superconductor
Bose condensate of pairs of electrons

S. Kasahara, T. Shiba
H. Ike



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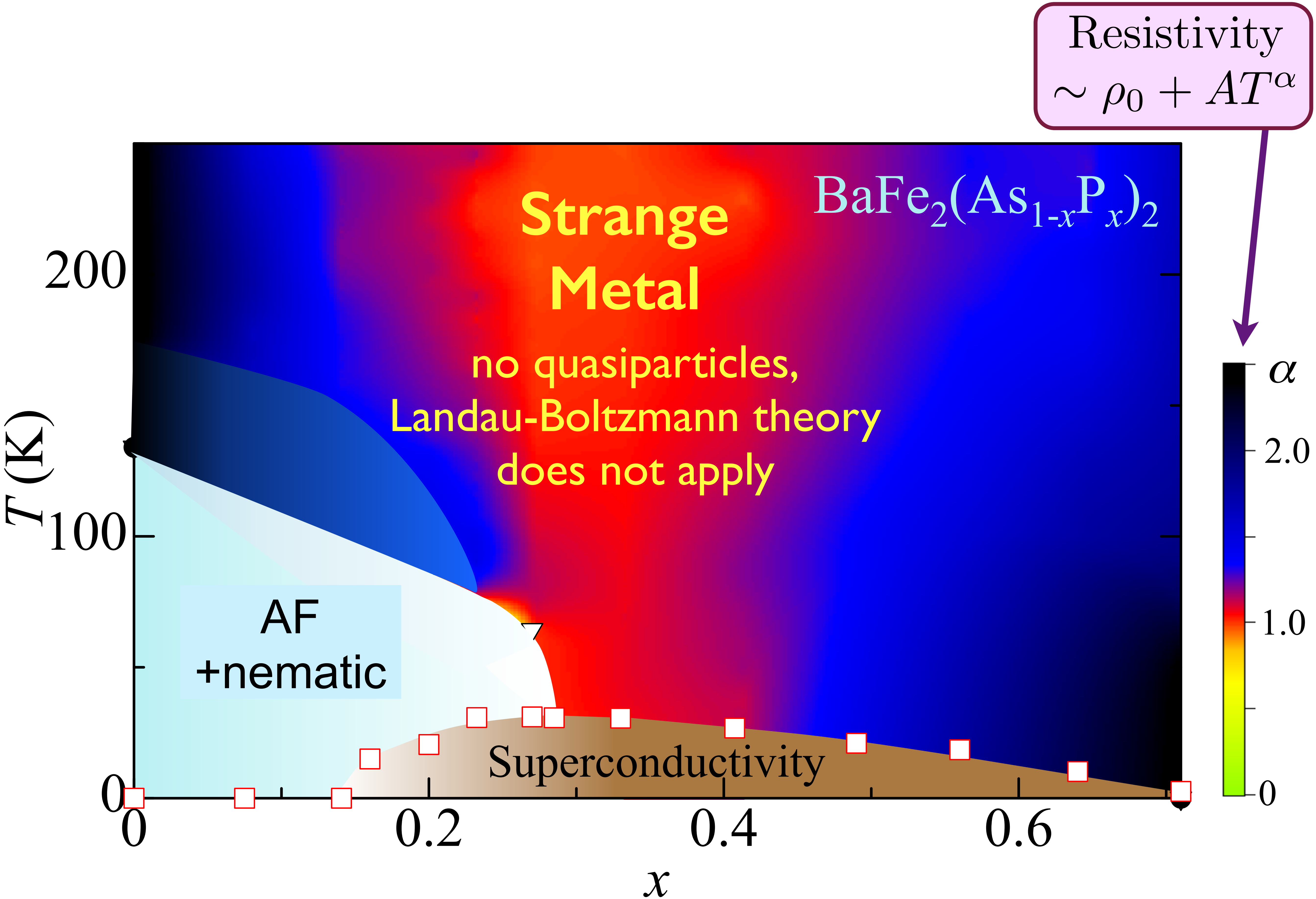

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Outline

1. The simplest model without quasiparticles

   A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice

   B. Conformal field theories in 2+1 dimensions, the AdS/CFT correspondence, and transport without quasiparticles.

2. Metals without quasiparticles

   A. The onset of antiferromagnetism in a metal

   B. Non-quasiparticle transport at the Ising-nematic quantum critical point

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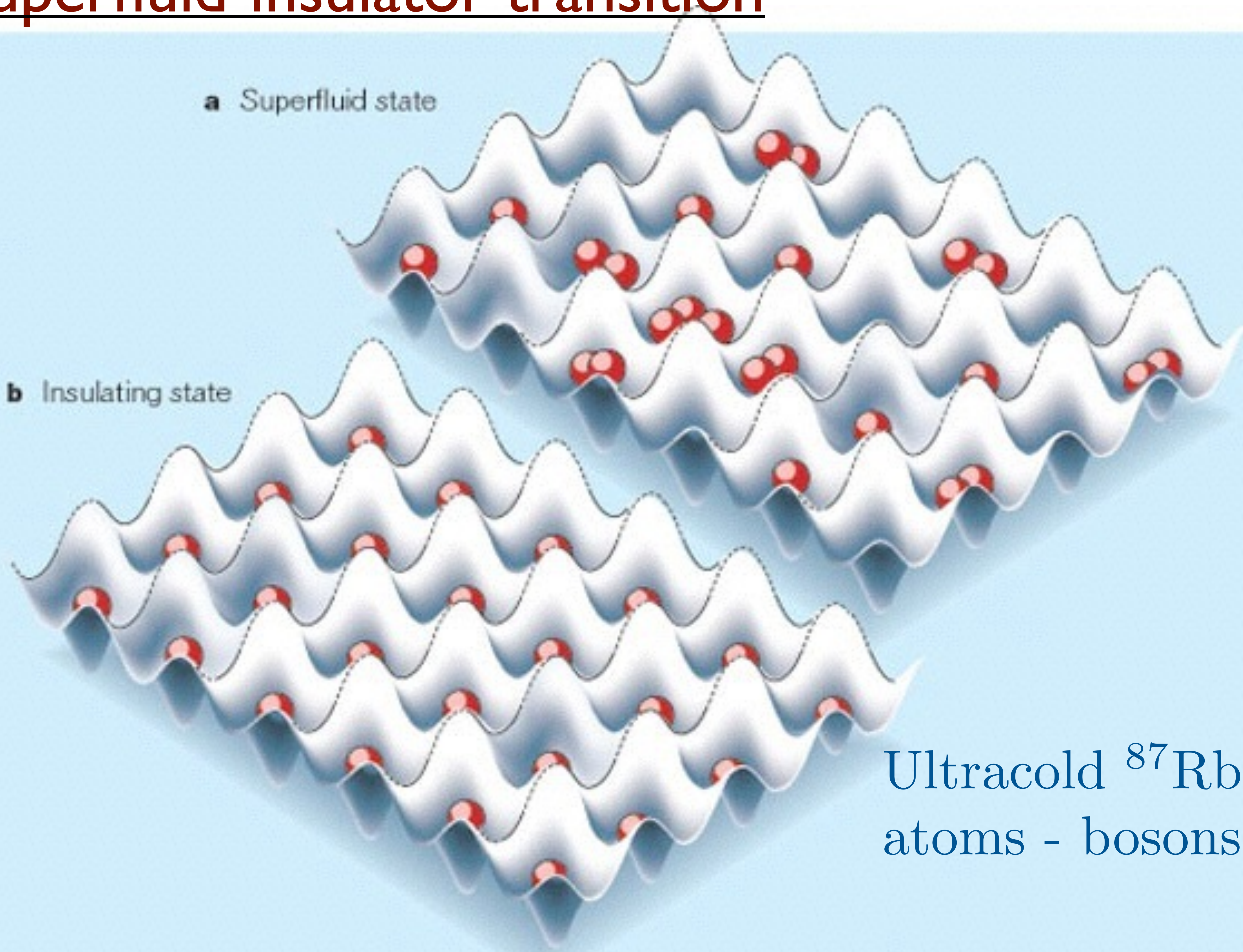
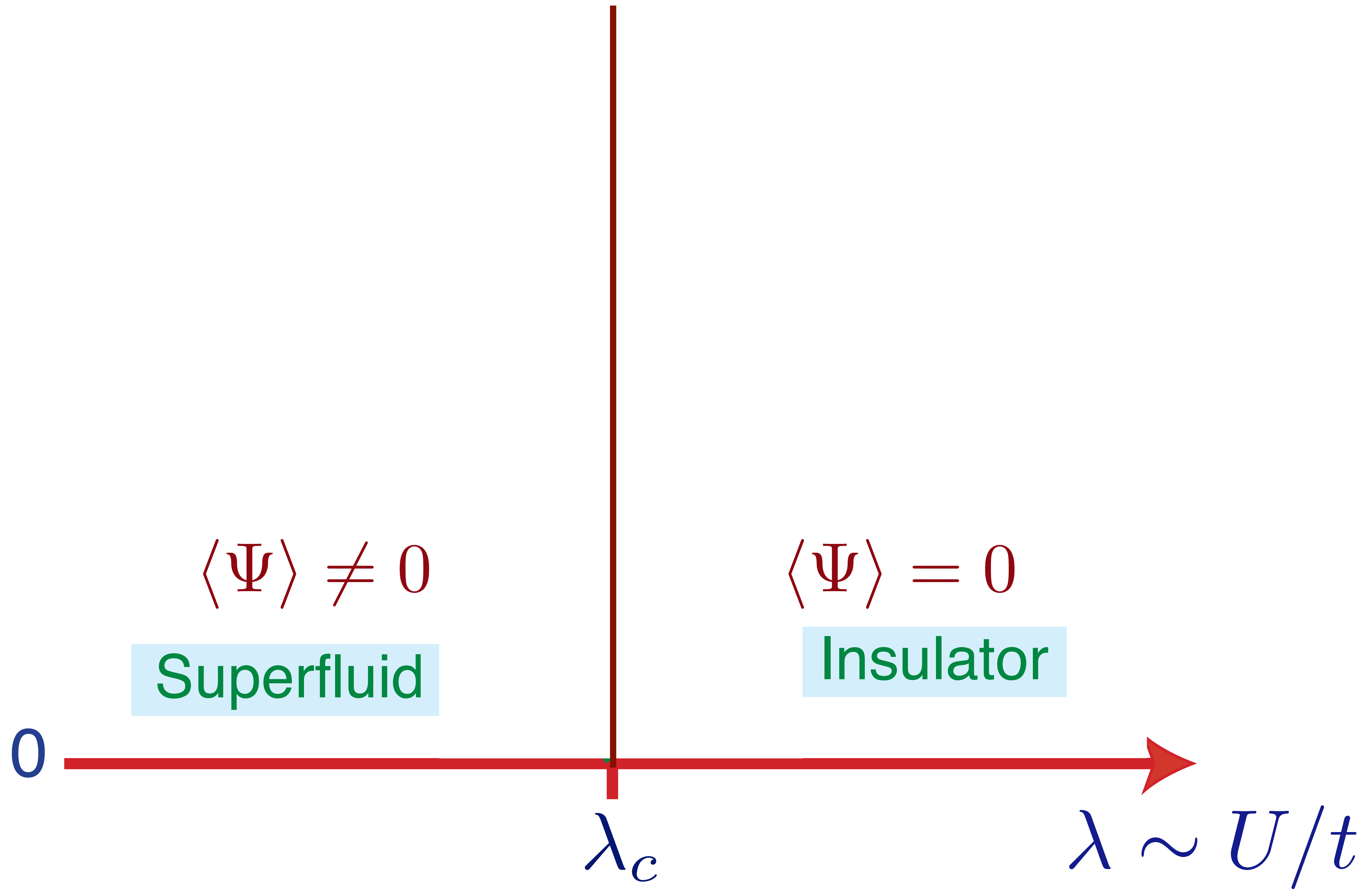
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

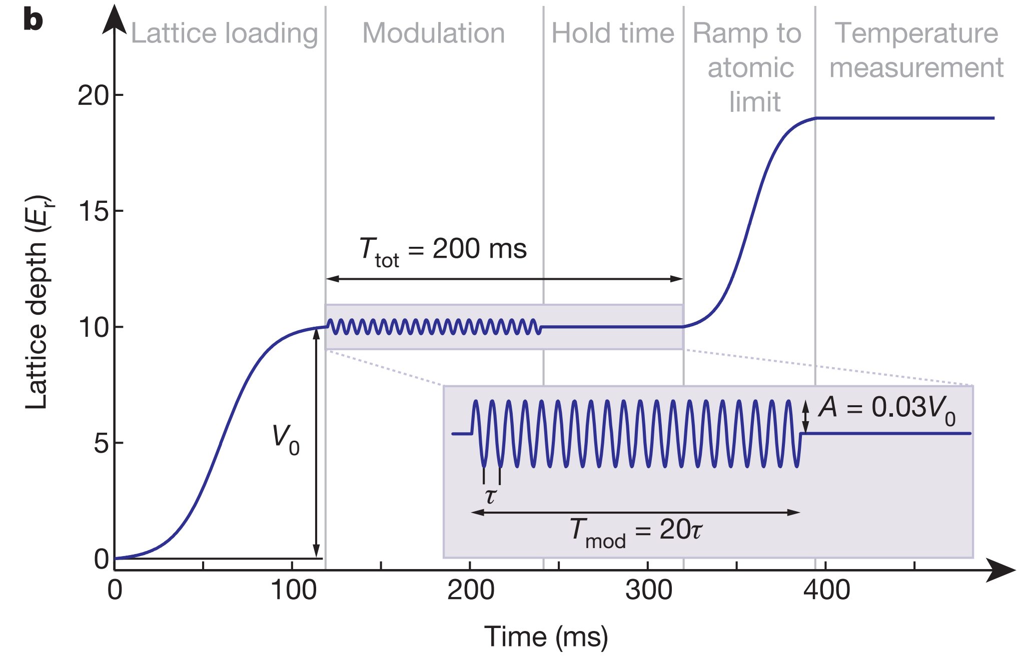
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

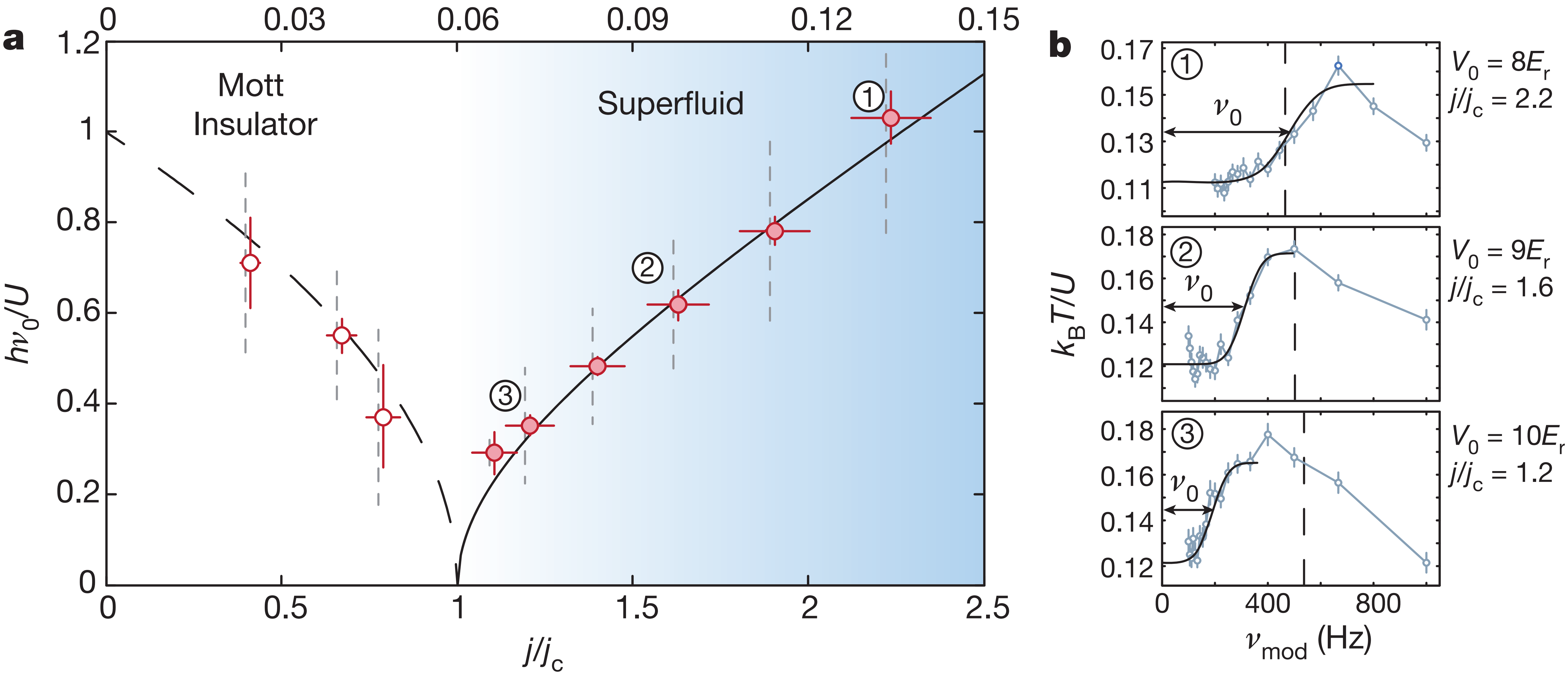
$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



Response to modulation of lattice depth:

Observation of quasiparticle excitations across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice

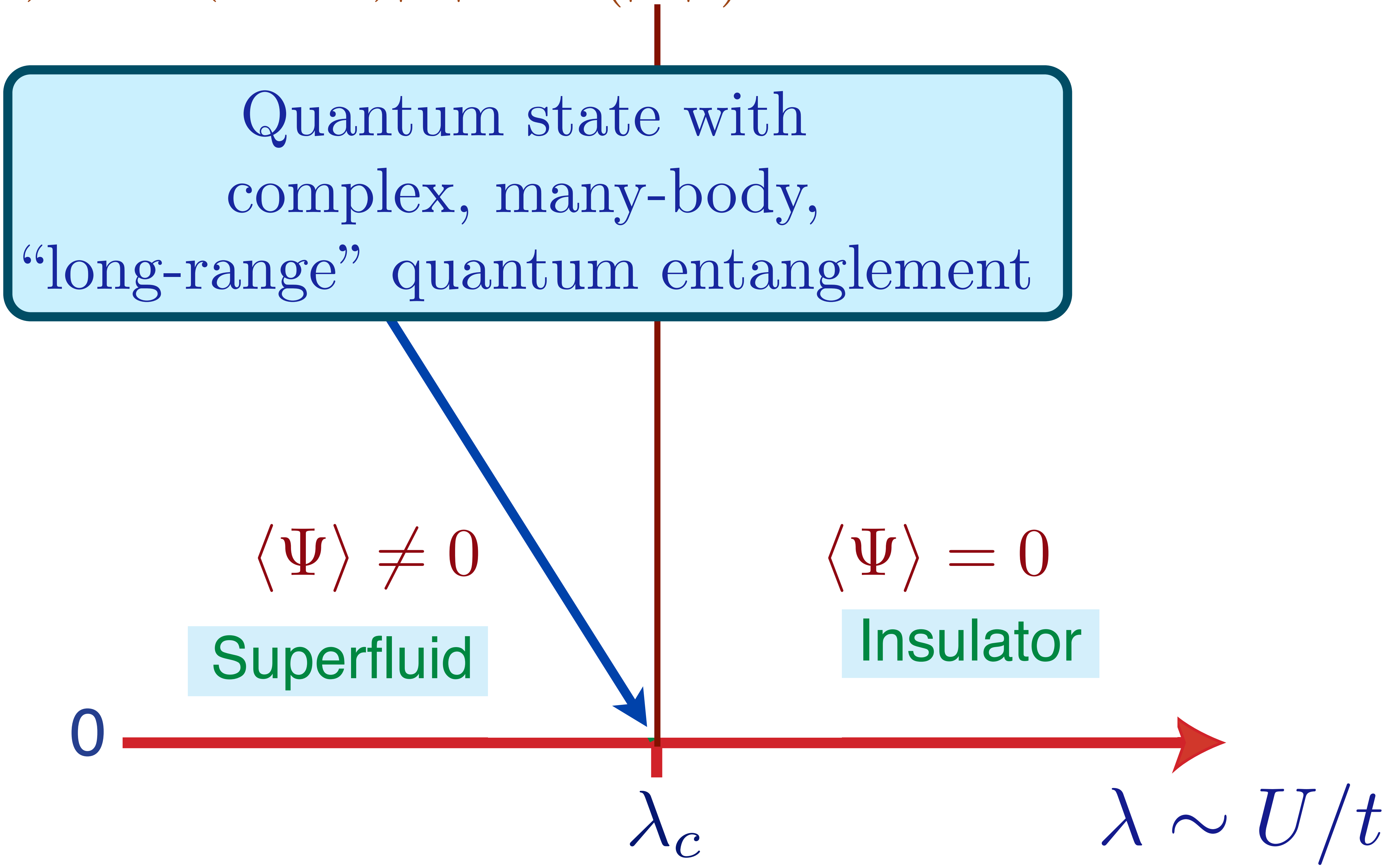




Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left(|\Psi|^2 \right)^2$$



Characteristics of quantum critical point

- Long-range entanglement

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- Long-range entanglement
- No quasiparticles - no simple description of excitations.
- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the sound velocity playing the role of the velocity of light.
- The theory of the critical point is strongly-coupled because the quartic-coupling u flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT3**

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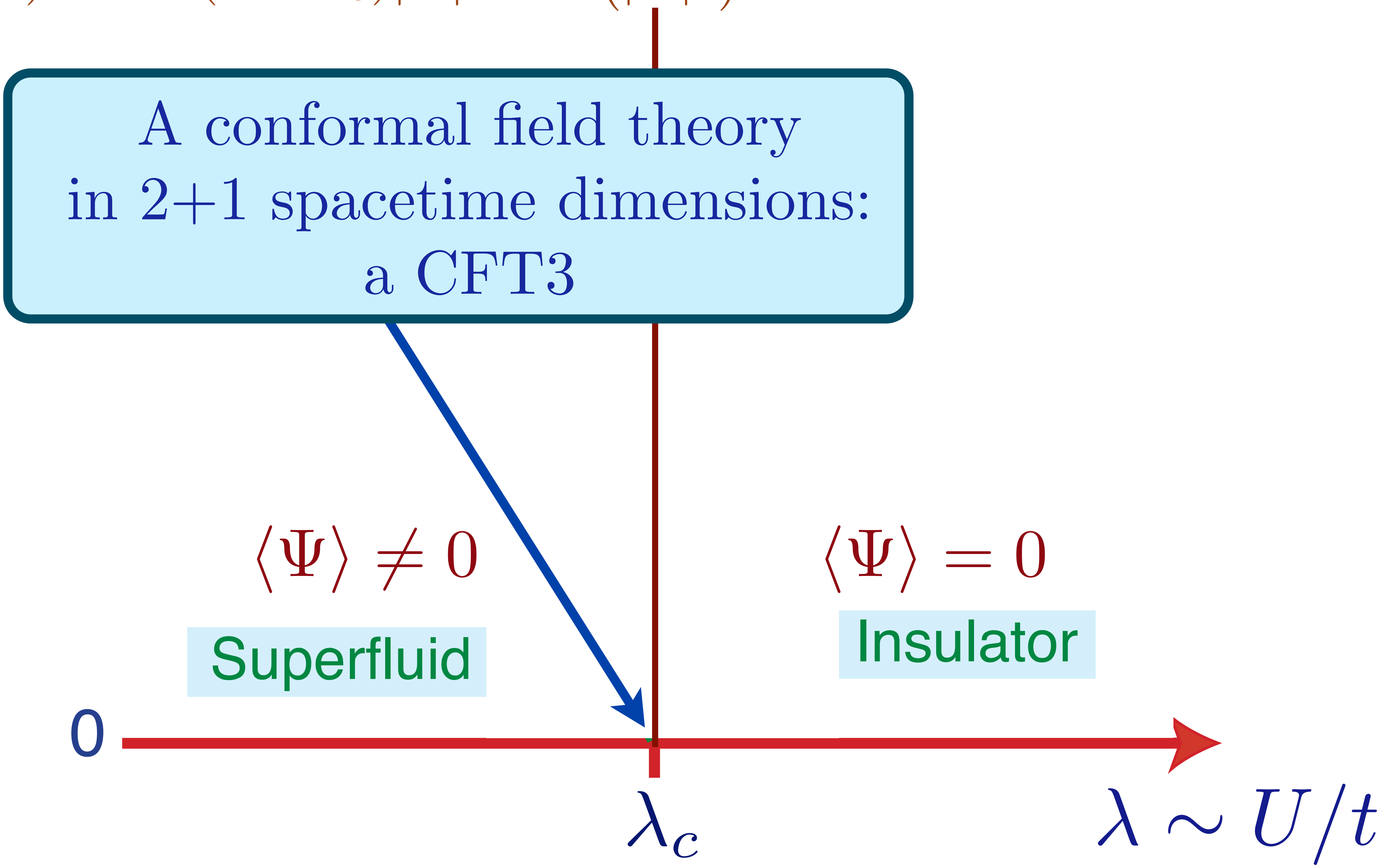
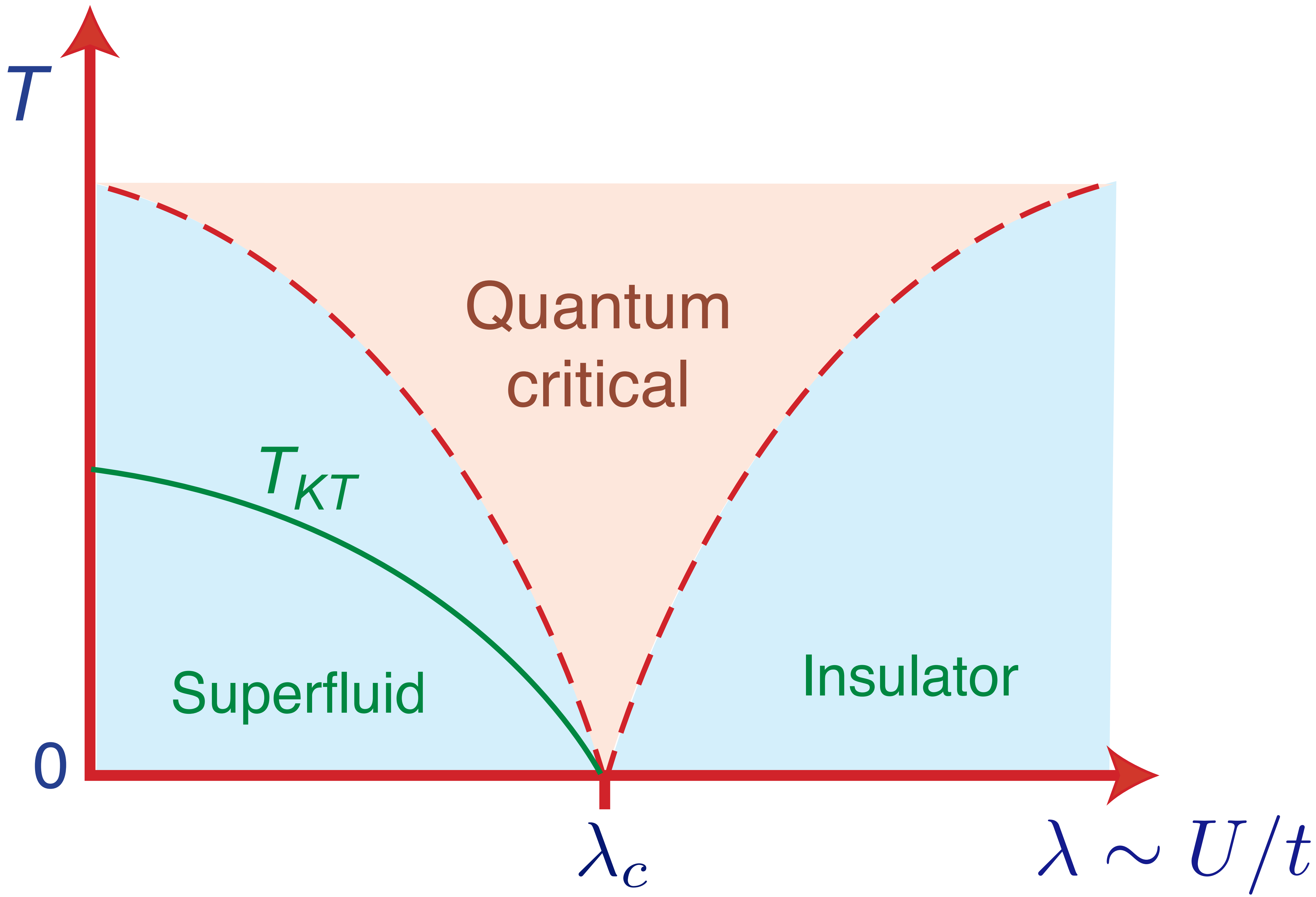
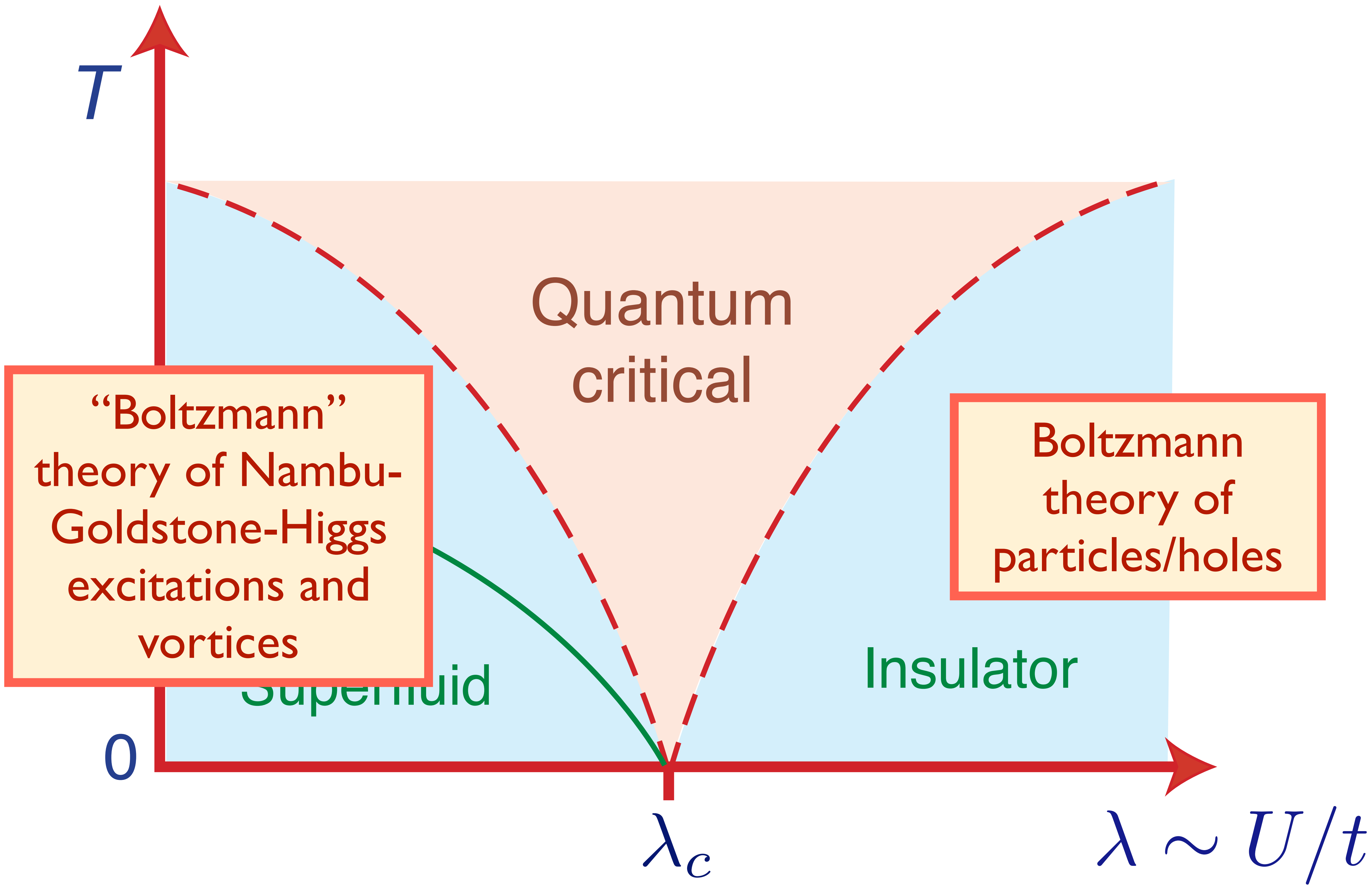




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$


T
0
Quantum
critical
“Boltzmann” theory of Nambu-Goldstone-Higgs excitations and vortices
Boltzmann theory of particles/holes
Superfluid
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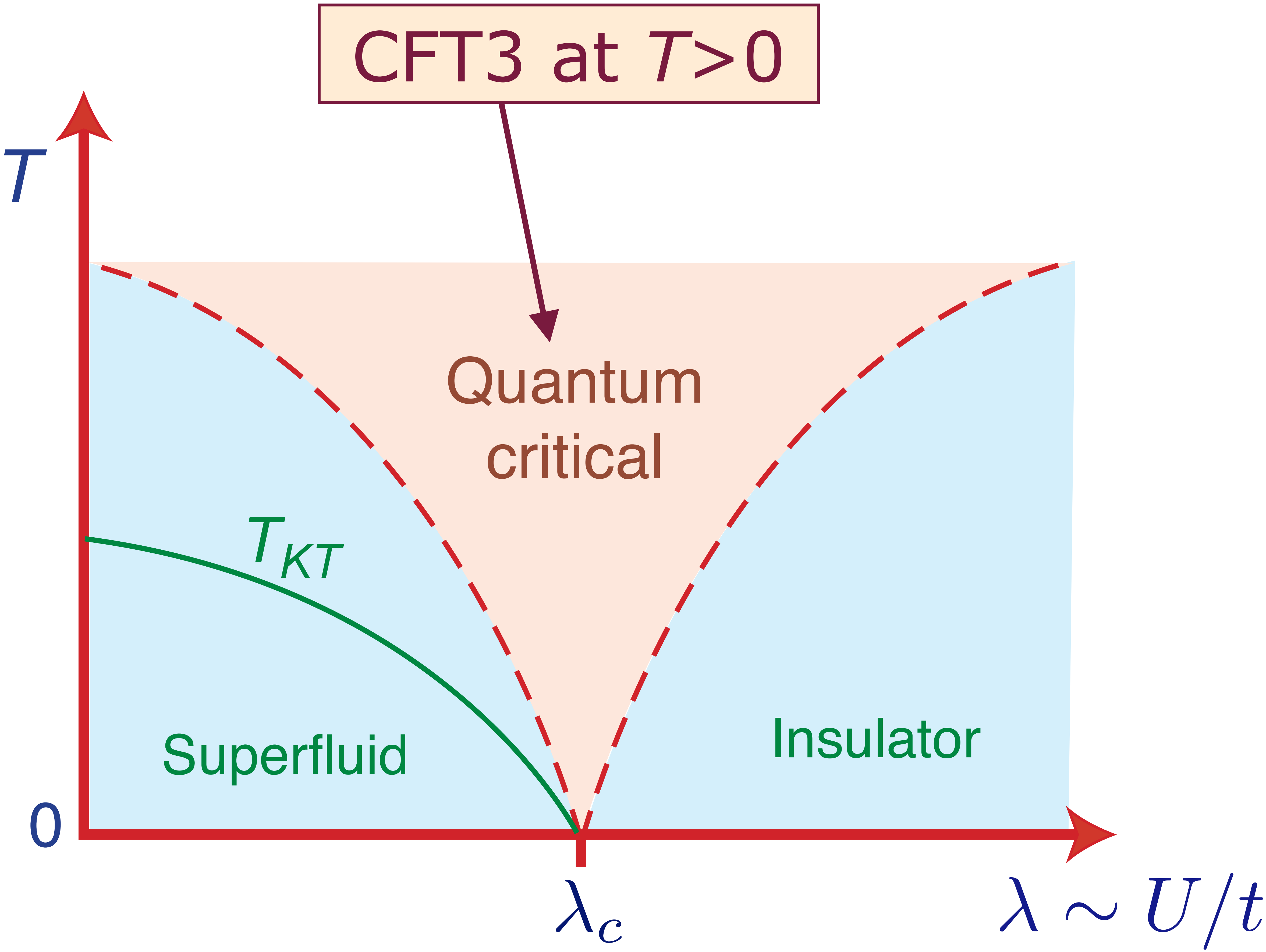
CFT3 at $T>0$
T
Quantum critical
T_{KT}
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0
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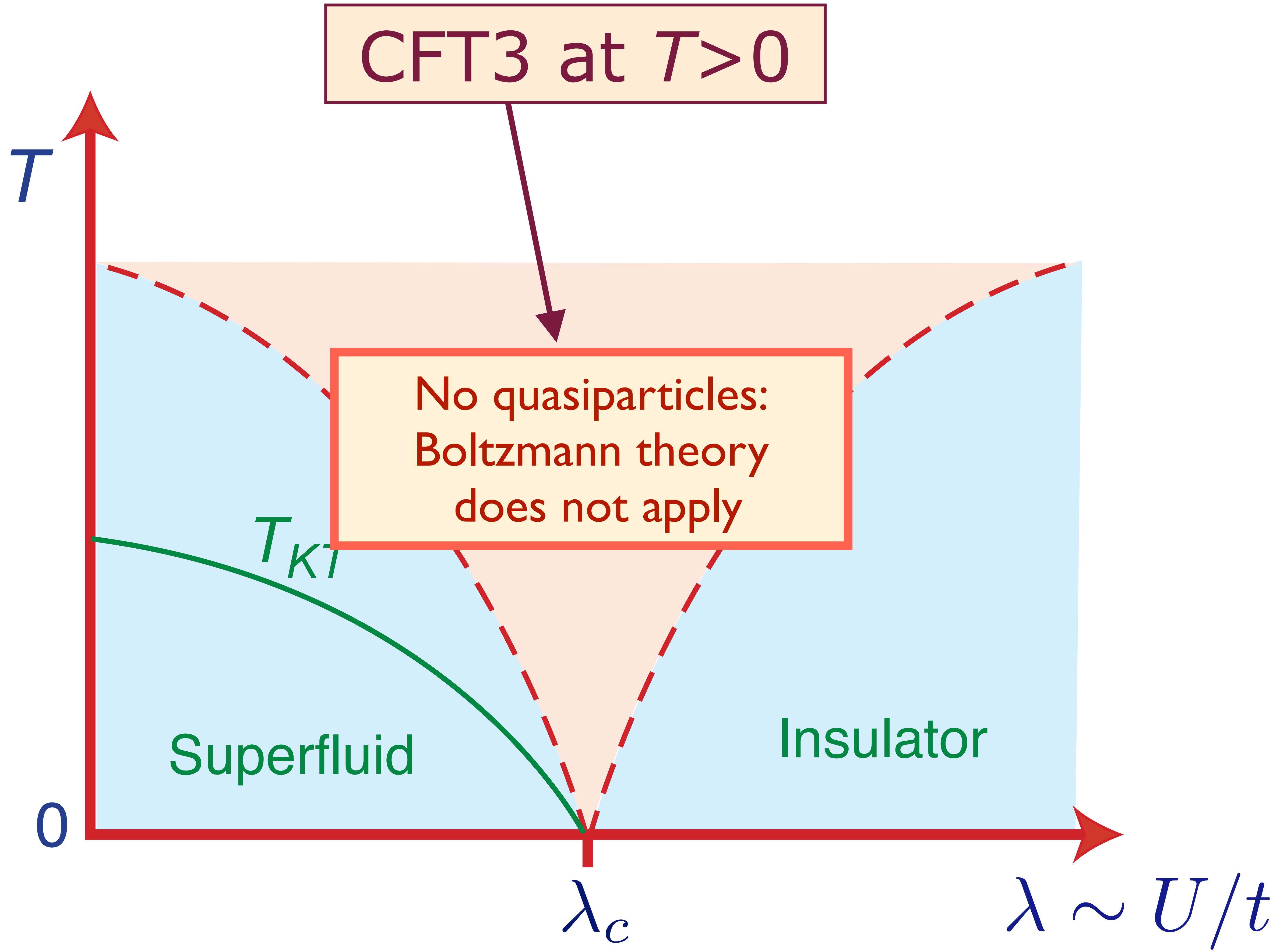

CFT3 at T>0
No quasiparticles:
Boltzmann theory
does not apply
T
T_{KT}
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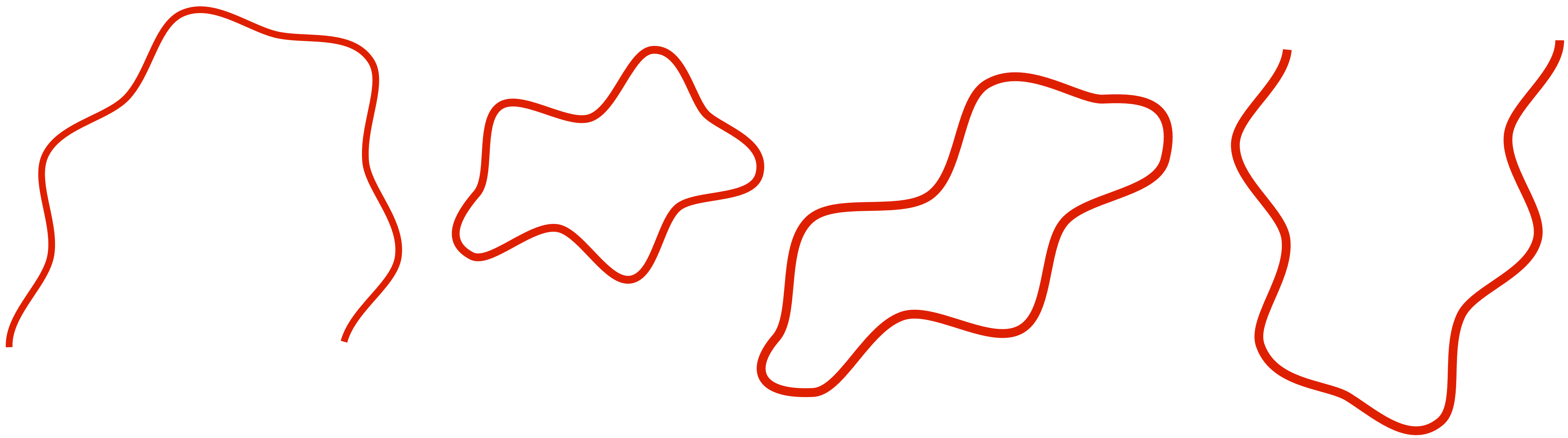
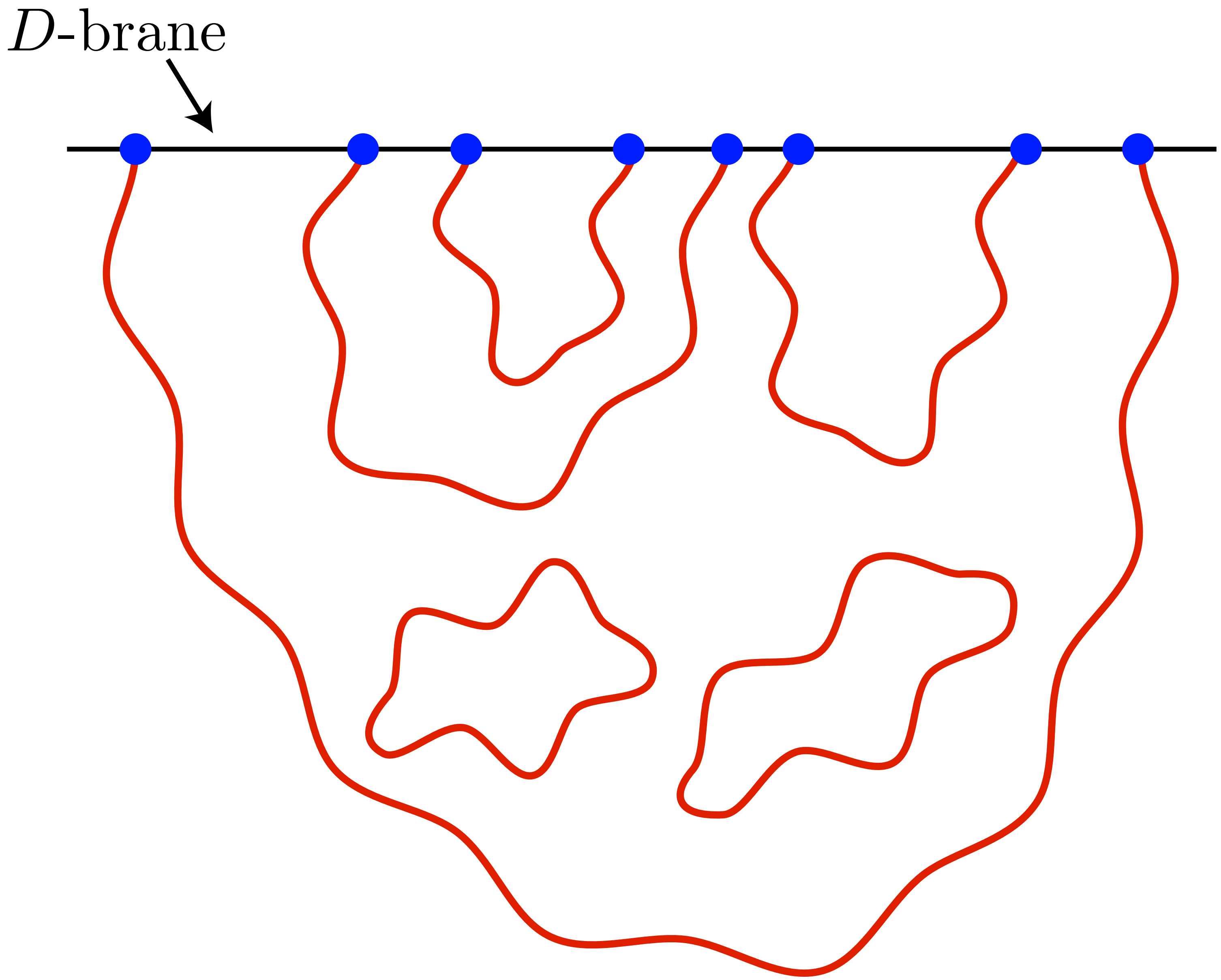
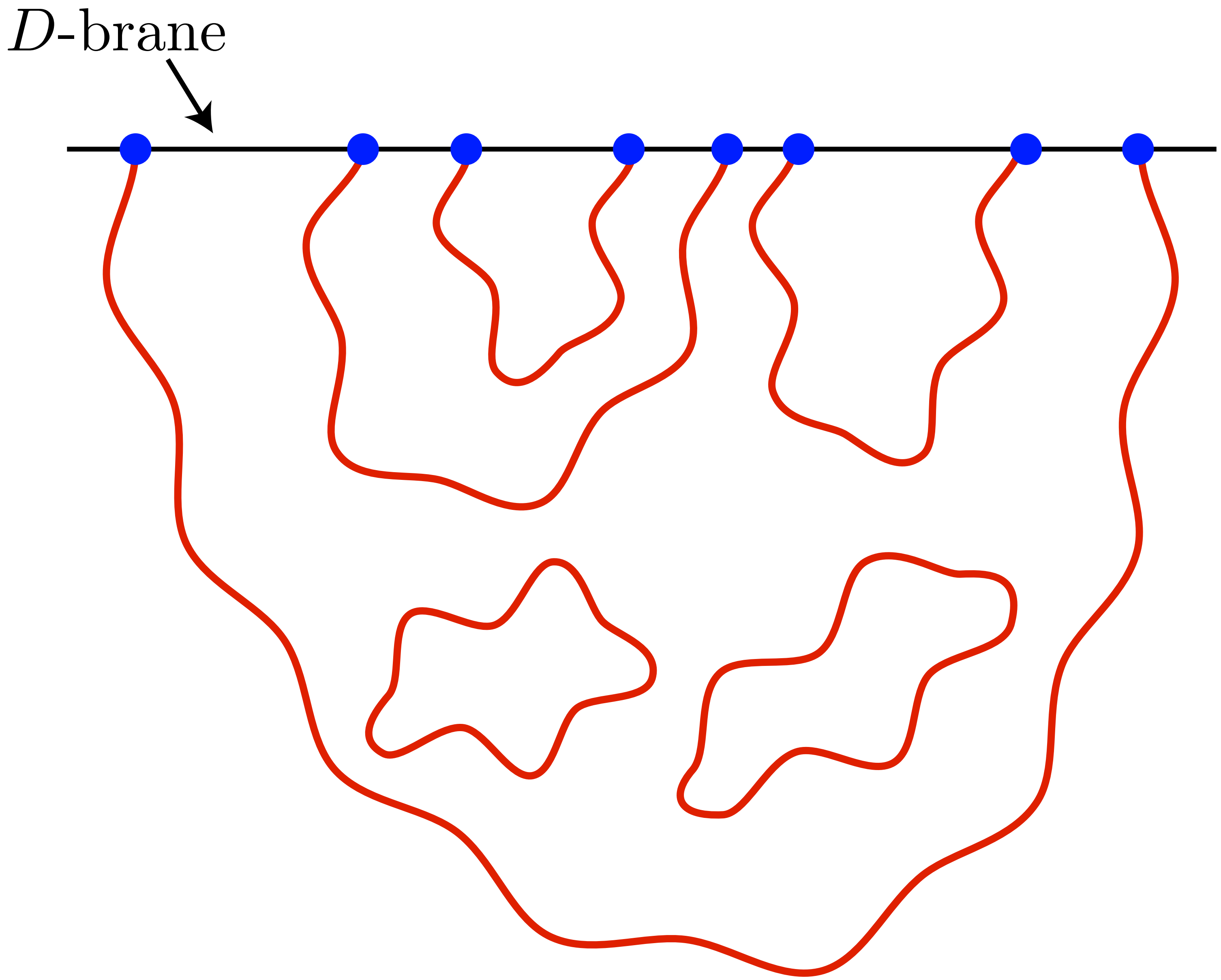
String theory



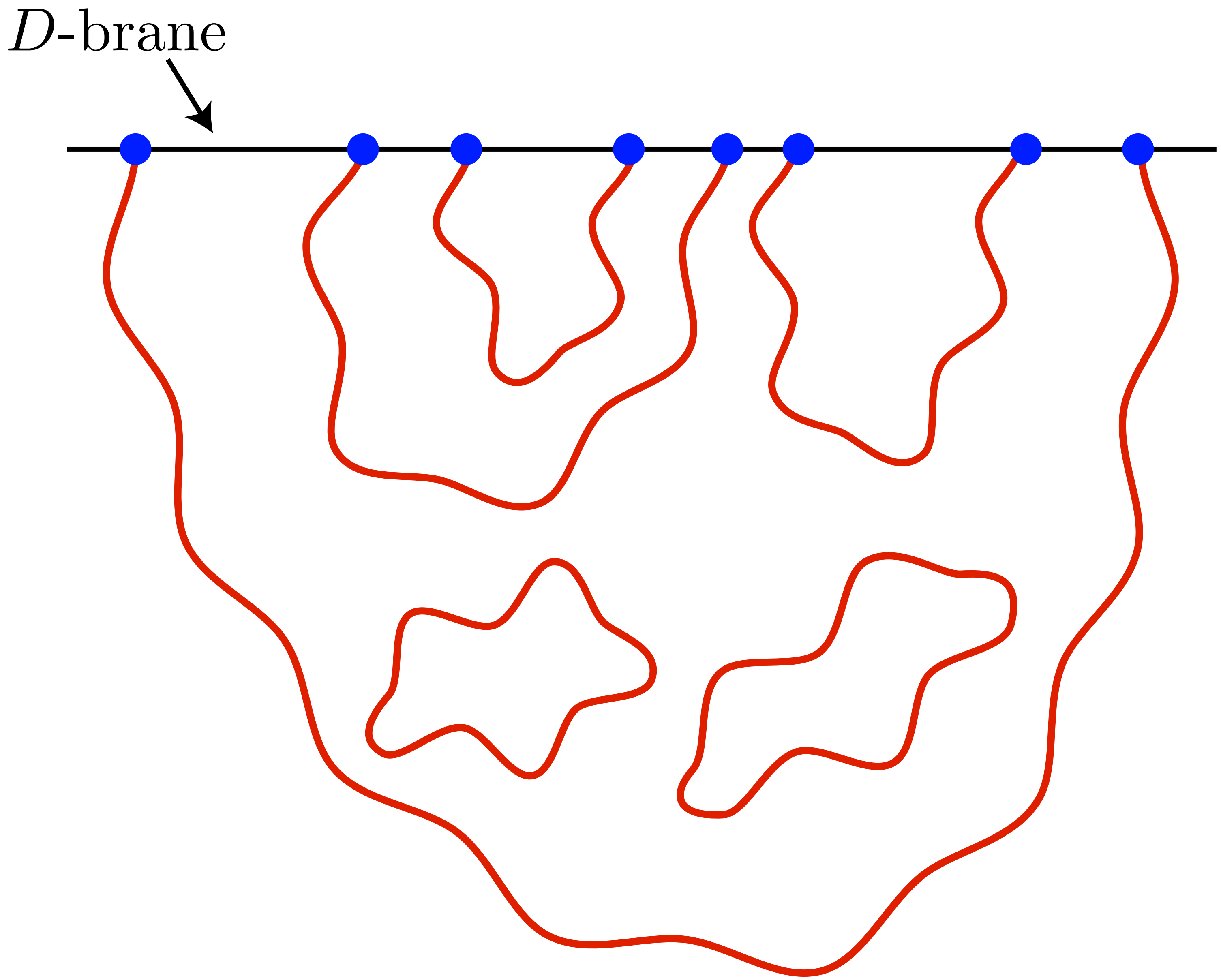
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .



- A D-brane is a d-dimensional surface on which strings can end.



- A D-brane is a d-dimensional surface on which strings can end.
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us: the strings connecting the particles on the D-brane encode the entanglement!



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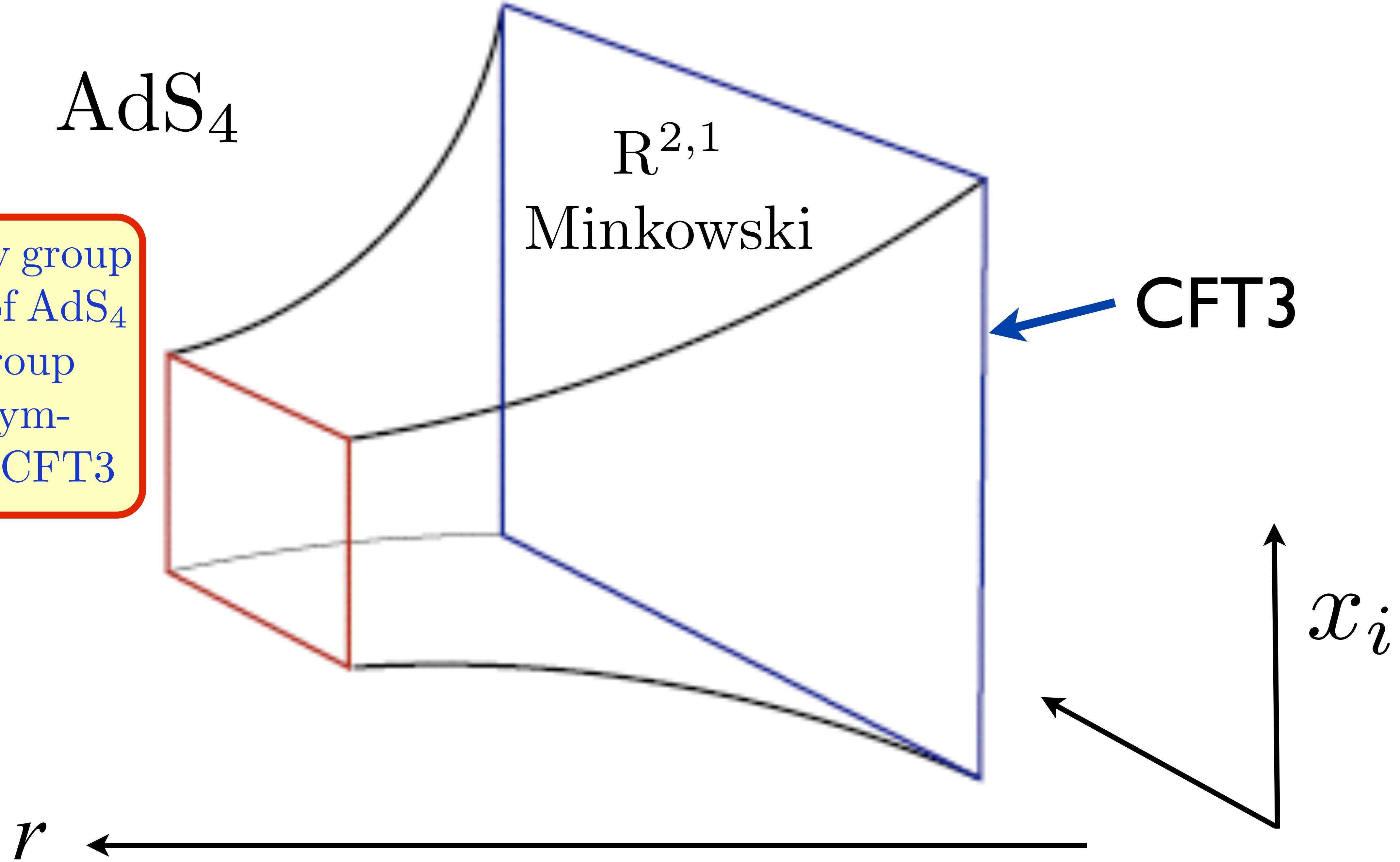
- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

AdS/CFT correspondence at zero temperature

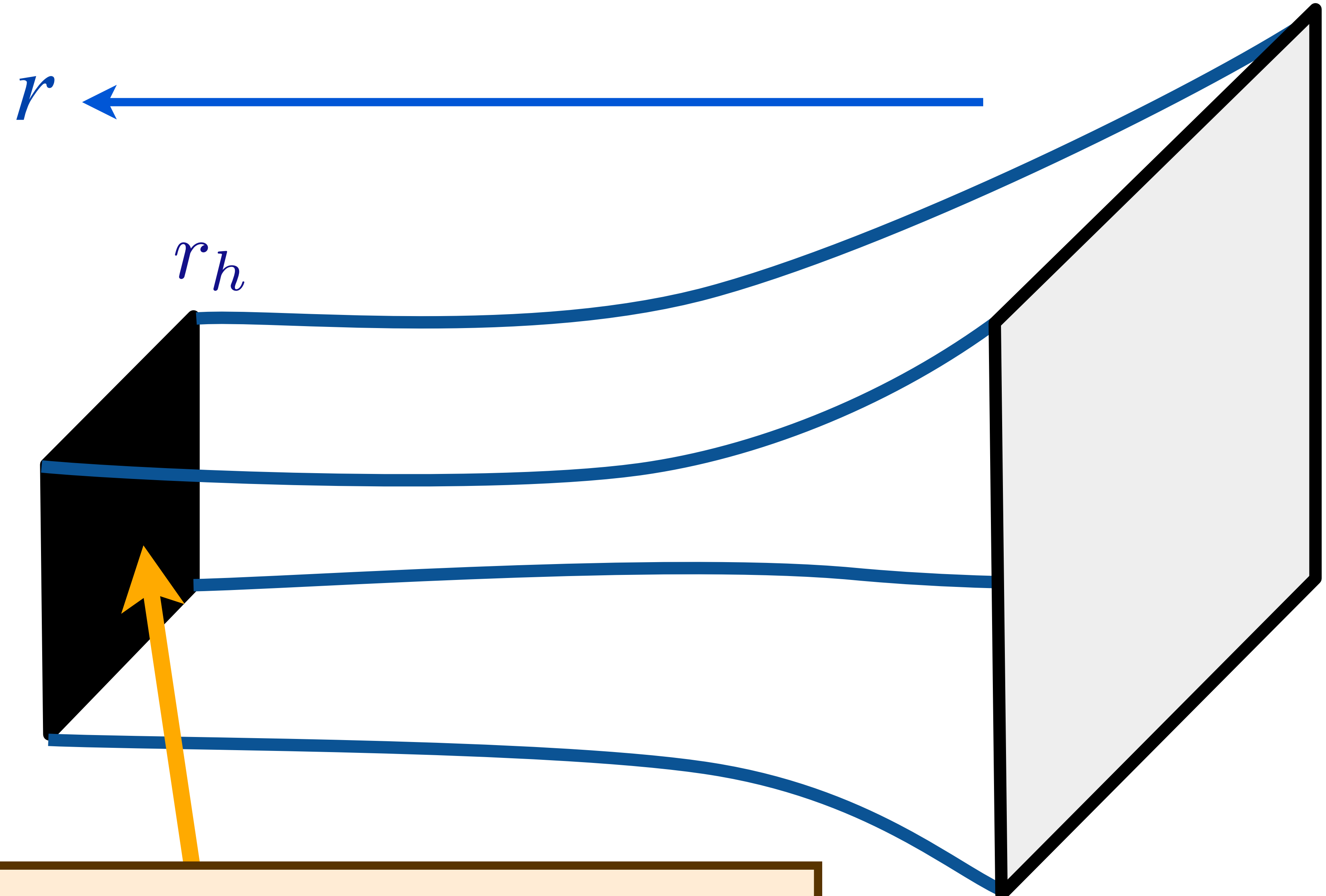


The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3

This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein's equations which are AdS_4 as $r \to 0$, but which have horizons at $r = r_h$.

A "horizon", similar to the surface of a black hole !

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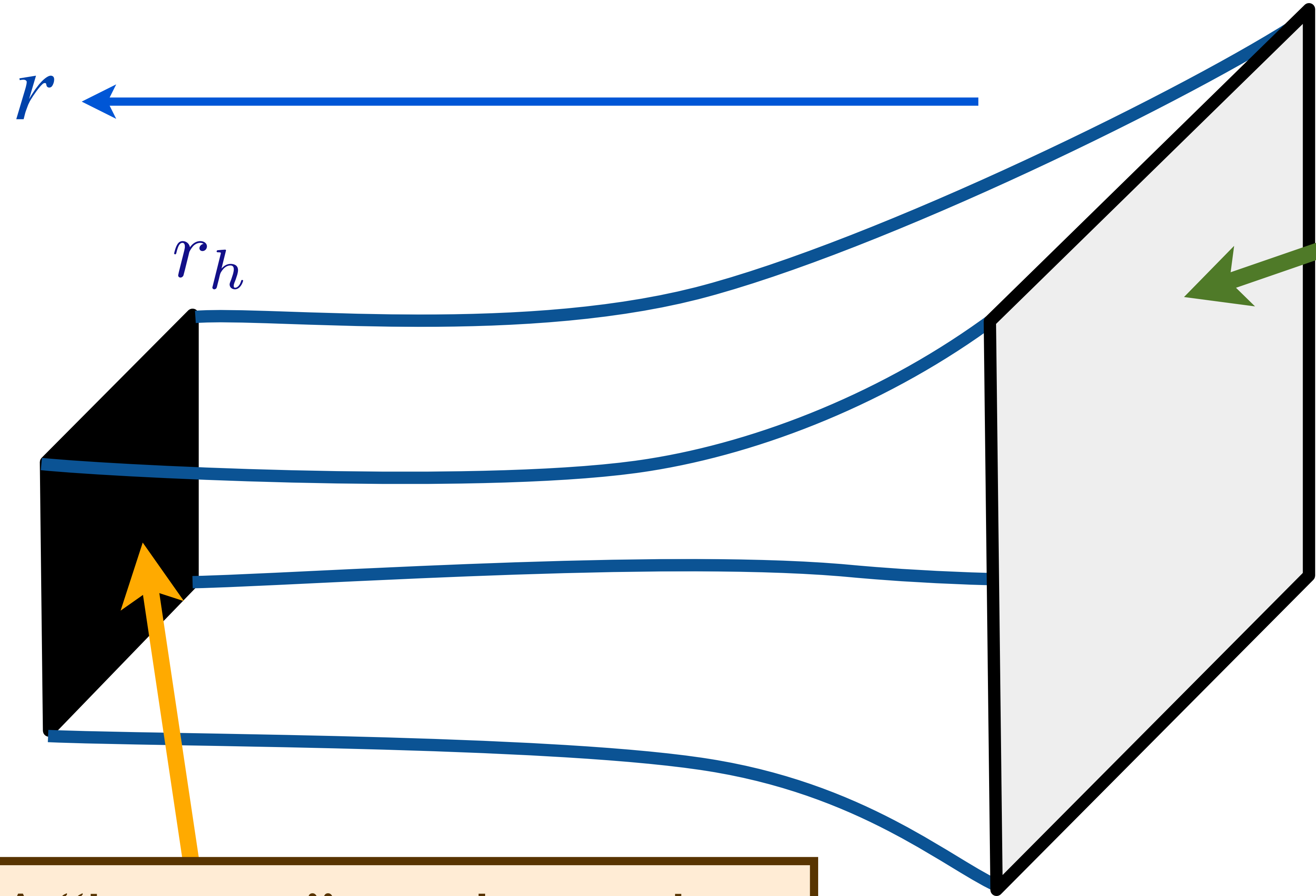

A CFT3 at a temperature $T \sim 1/r_h$ equal to the Hawking temperature of the horizon.

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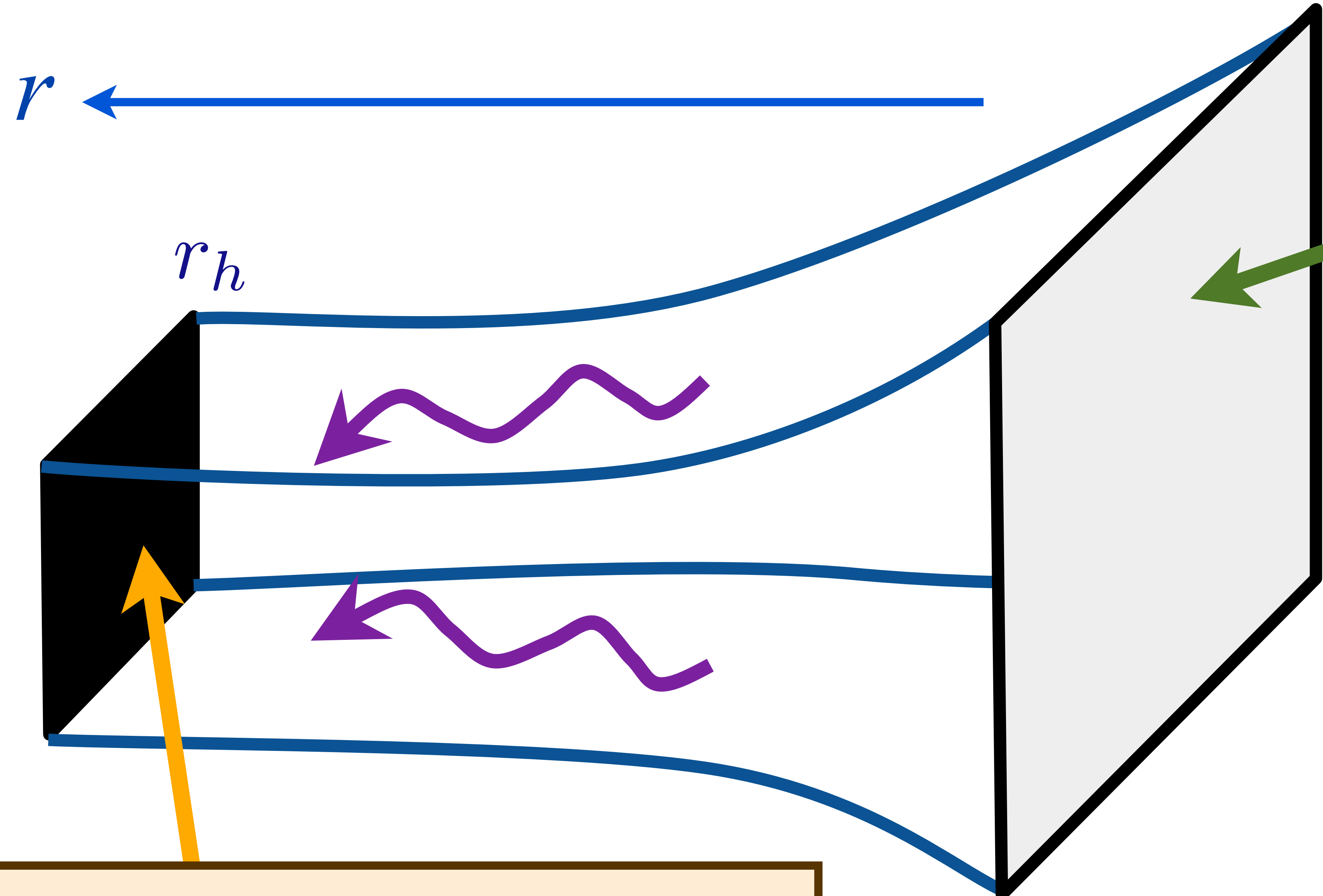

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A "horizon", similar to the surface of a black hole !

Dissipation and friction in the CFT3 = waves falling past the horizon

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

AdS$_4$ theory of quantum criticality

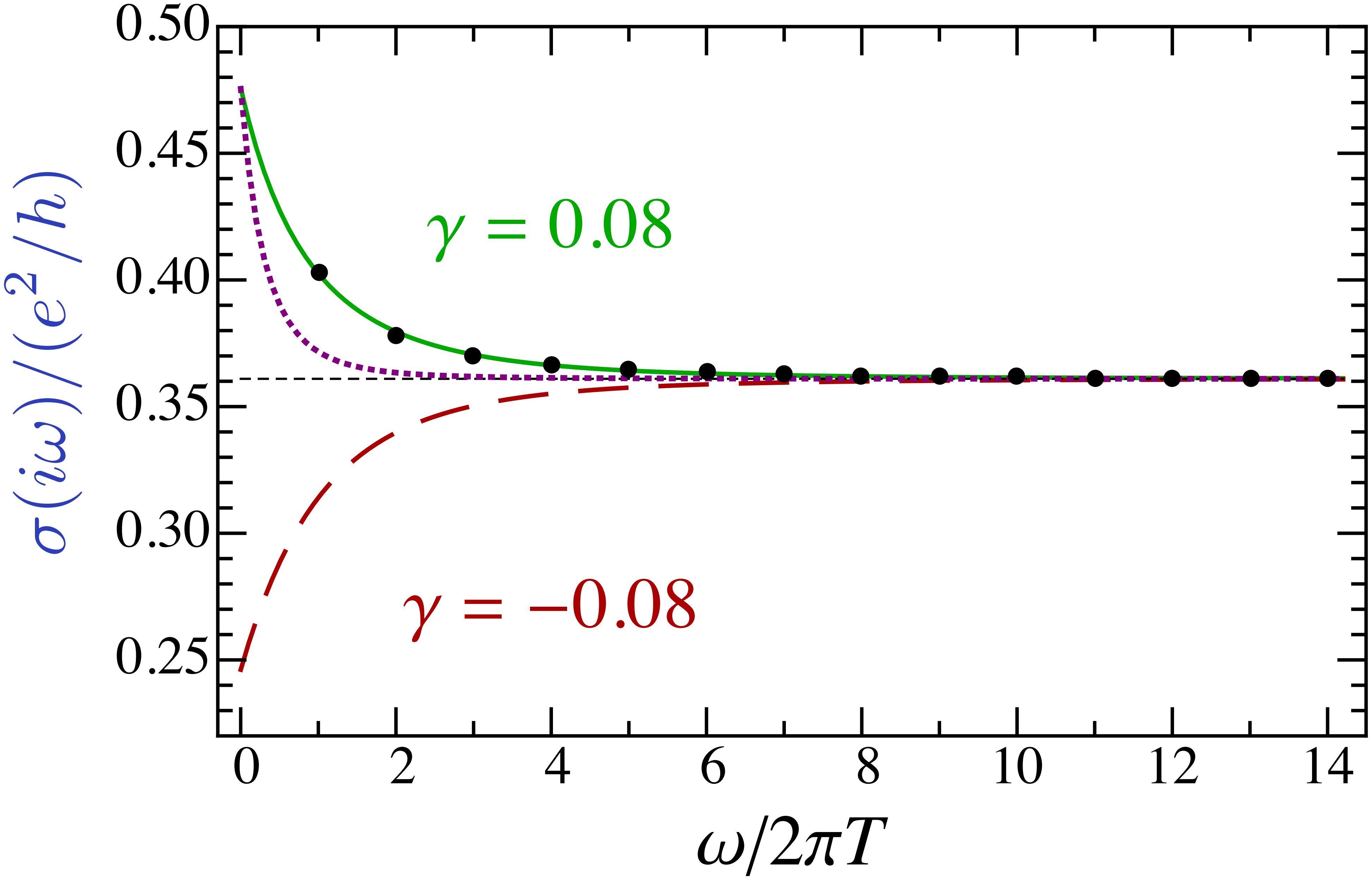
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\rm bulk} = \frac{1}{g_M^2}\int d^4x\,\sqrt{g}\left[\frac{1}{4}F_{ab}F^{ab} + \gamma L^2 C_{abcd}F^{ab}F^{cd}\right] + \int d^4x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833..$

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

Holography+quantum Monte Carlo



The holographic theory provides an excellent fit to imaginary-time quantum Monte Carlo on the boson Hubbard model with $\gamma = 0.08$, and we combine these methods to obtain absolute predictions for the frequency-dependent conductivity.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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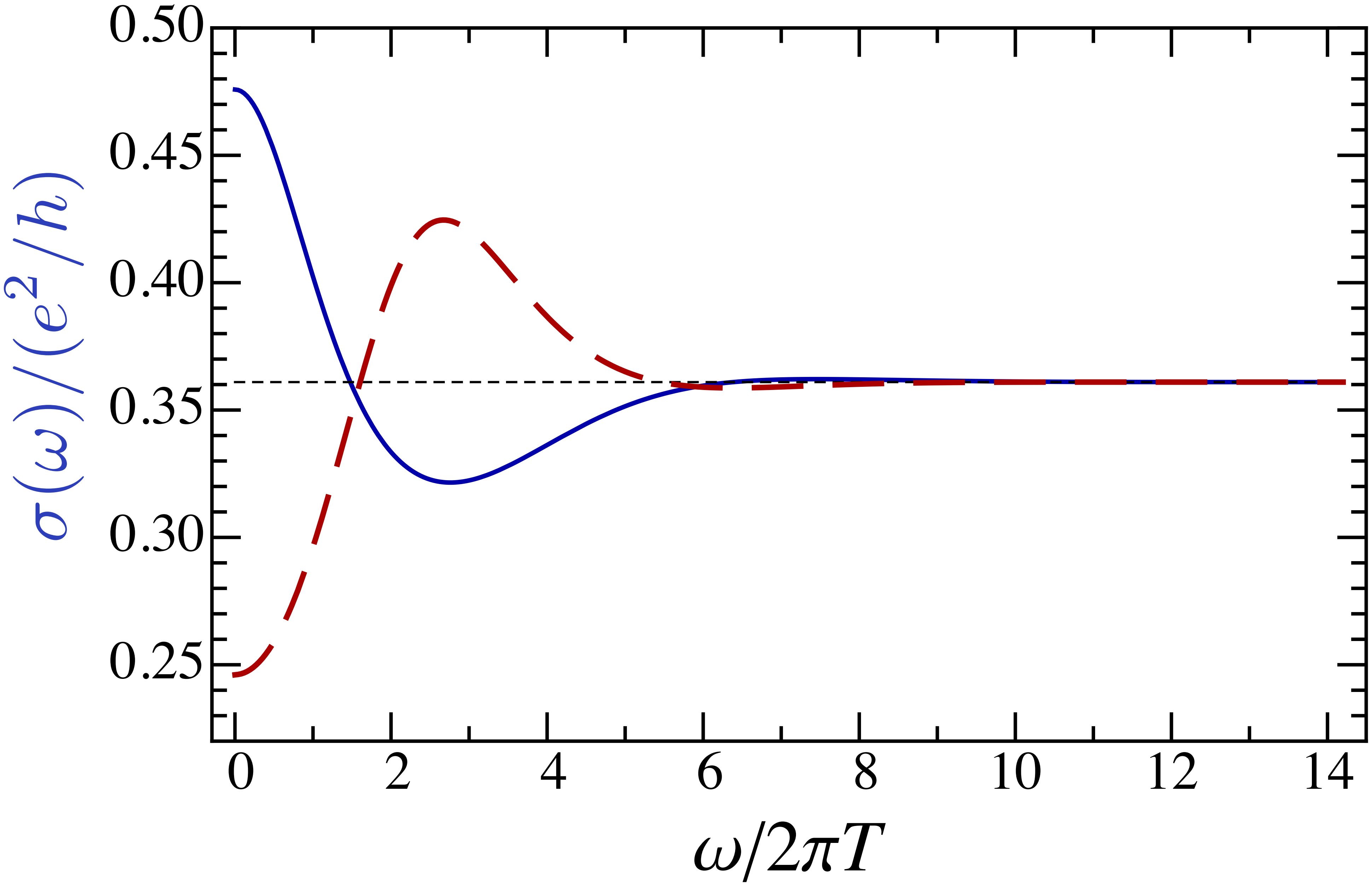

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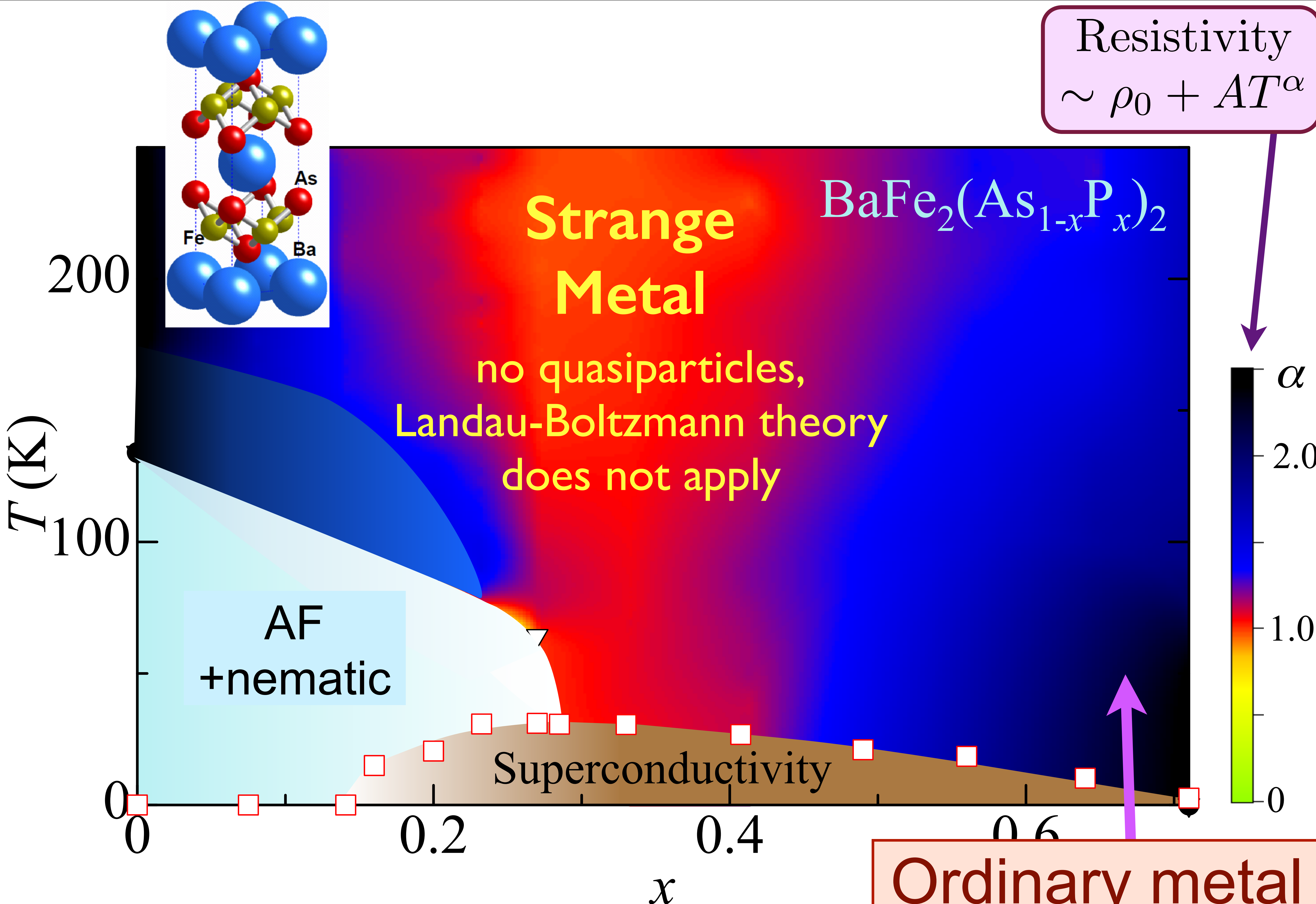

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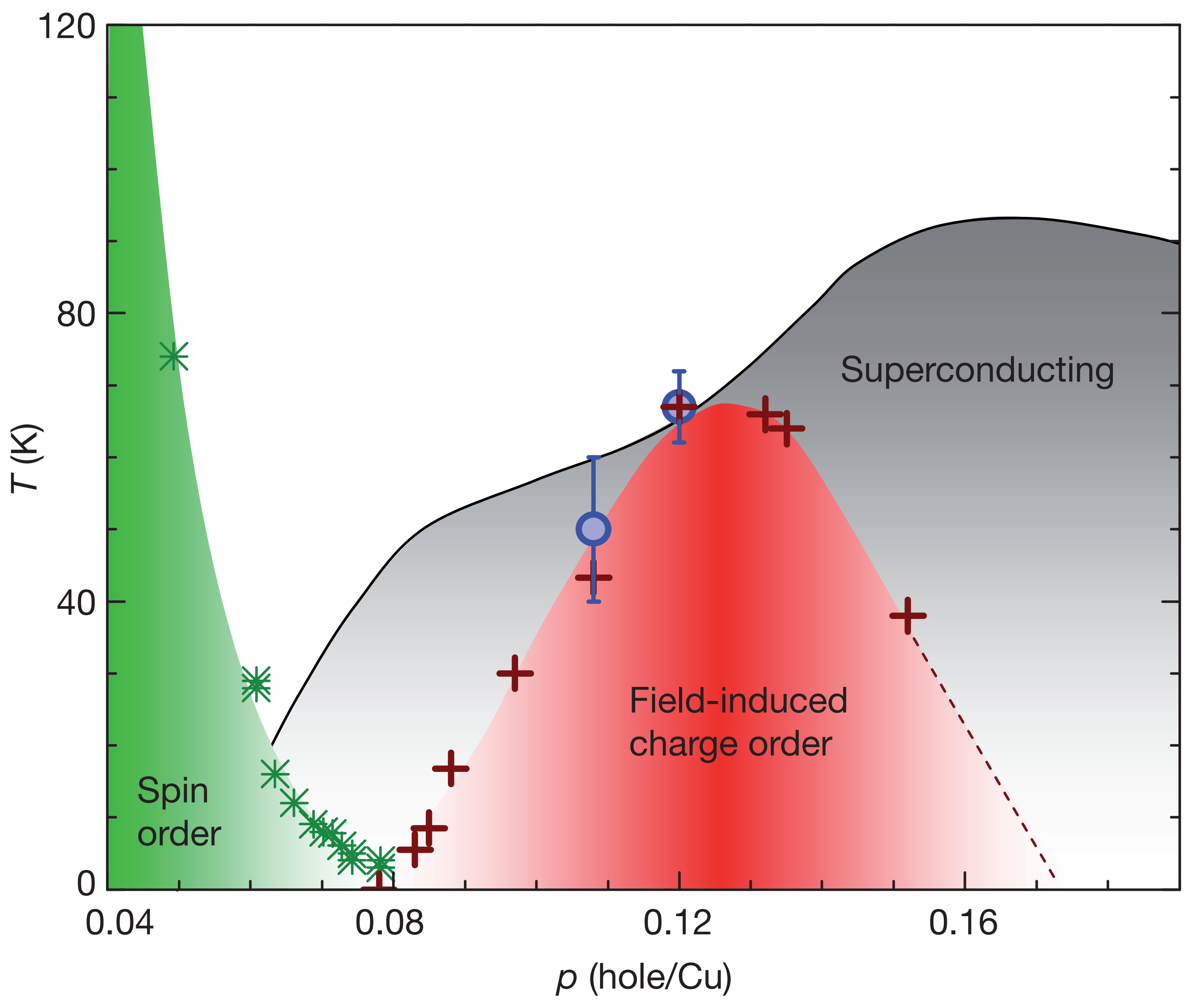
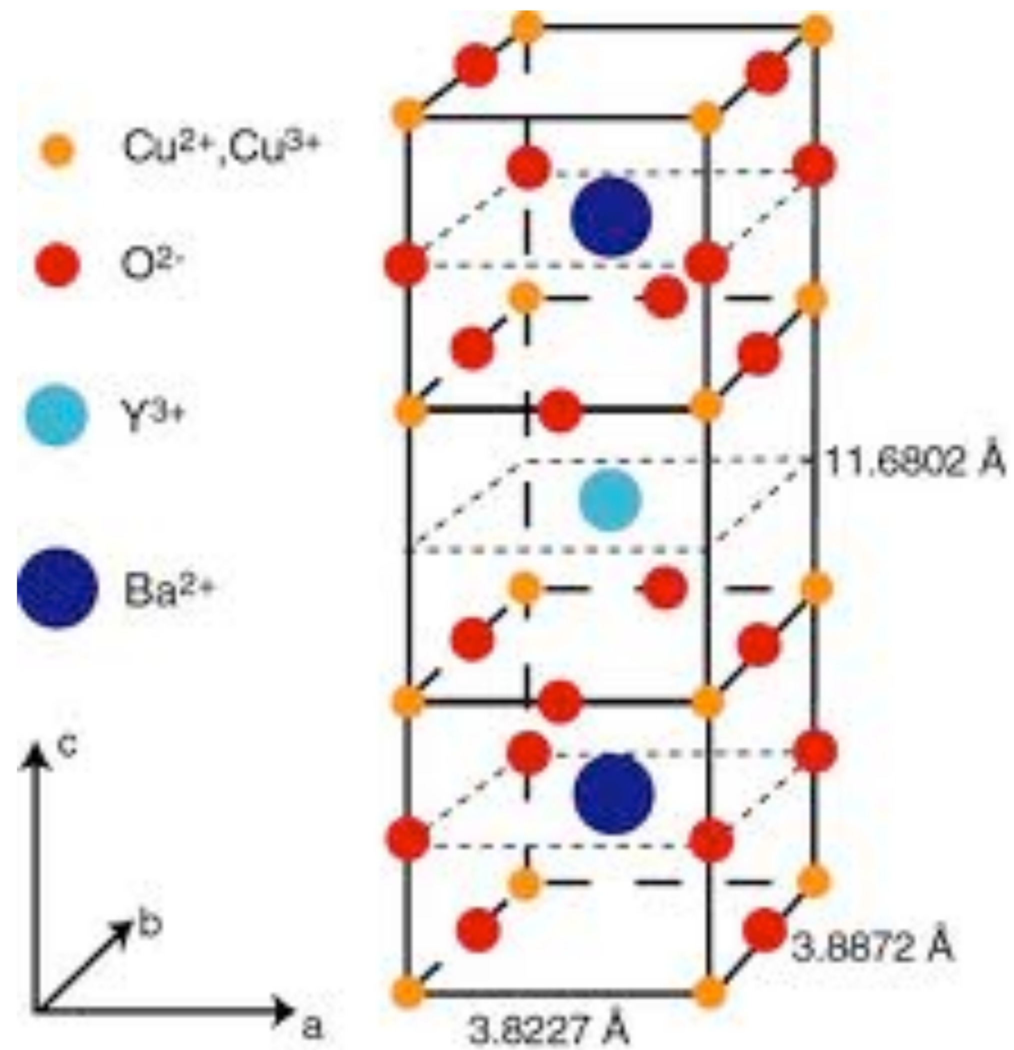
Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

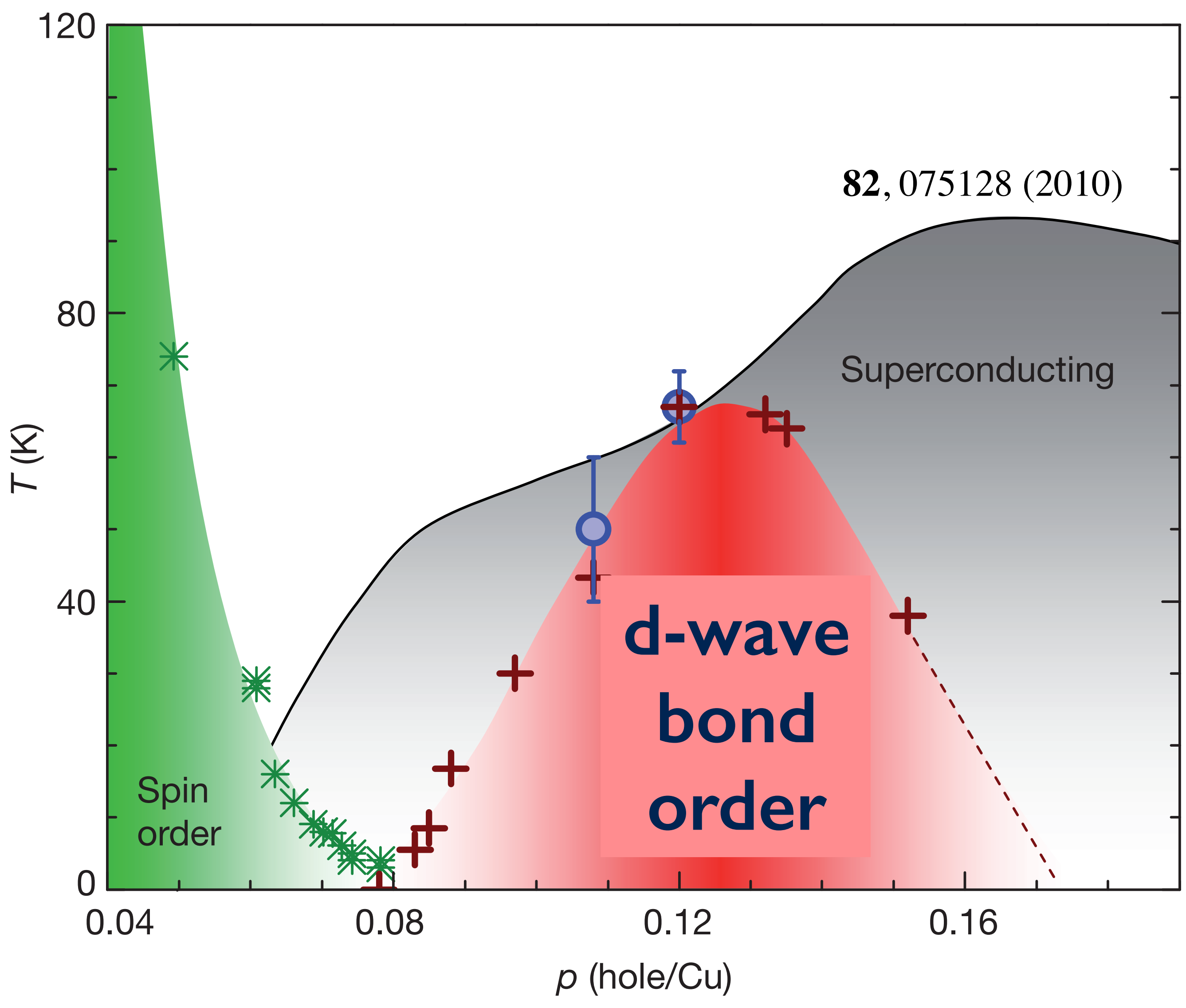
Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

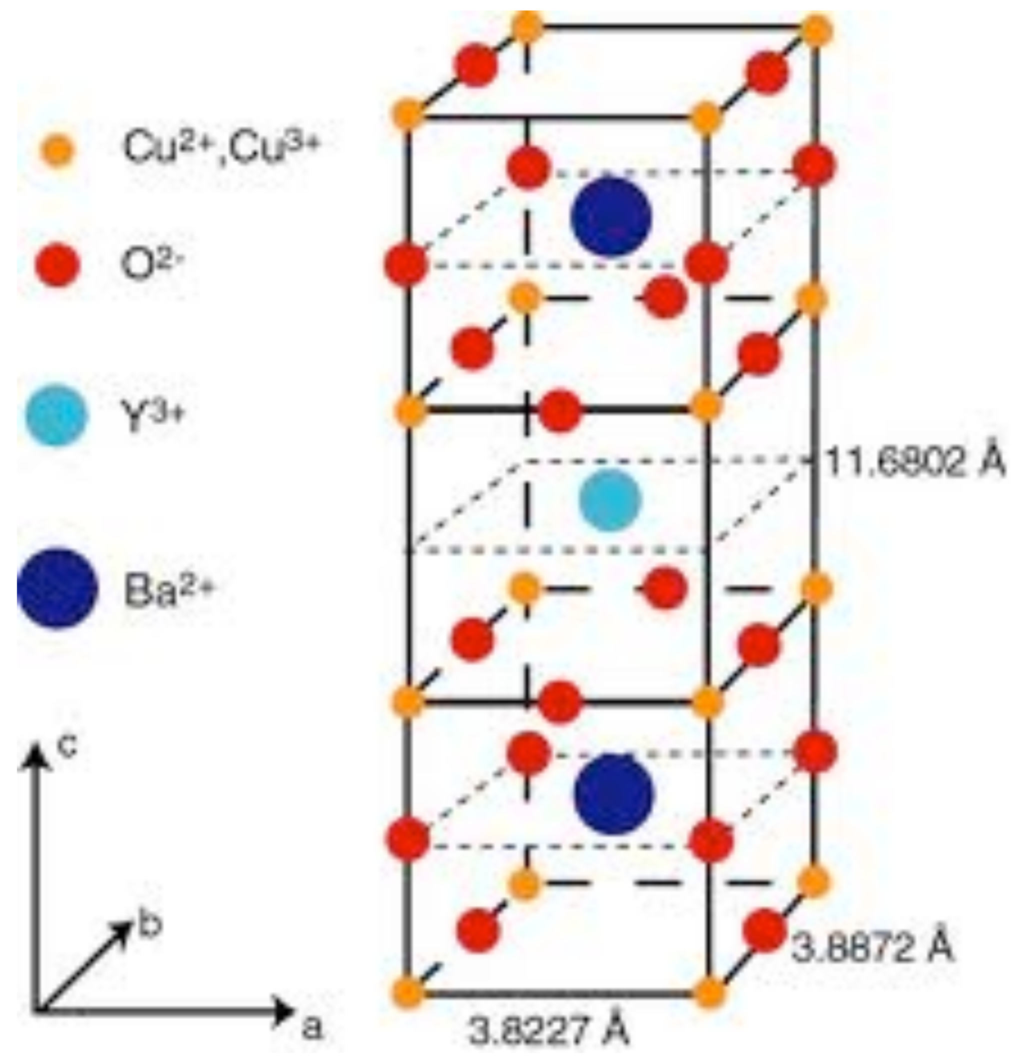
8 SEPTEMBER 2011 | VOL 477 | NATURE | 191





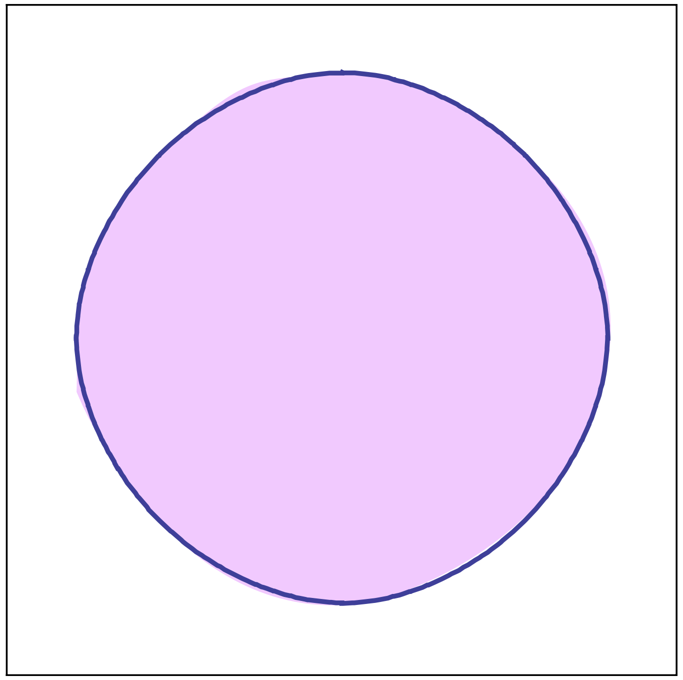
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B 77, 094504 (2008)
M. Metlitski and S. Sachdev, Physical Review B 82, 075128 (2010)
S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
A. Allais, J. Bauer, and S. Sachdev, to appear





Fermi surface+antiferromagnetism

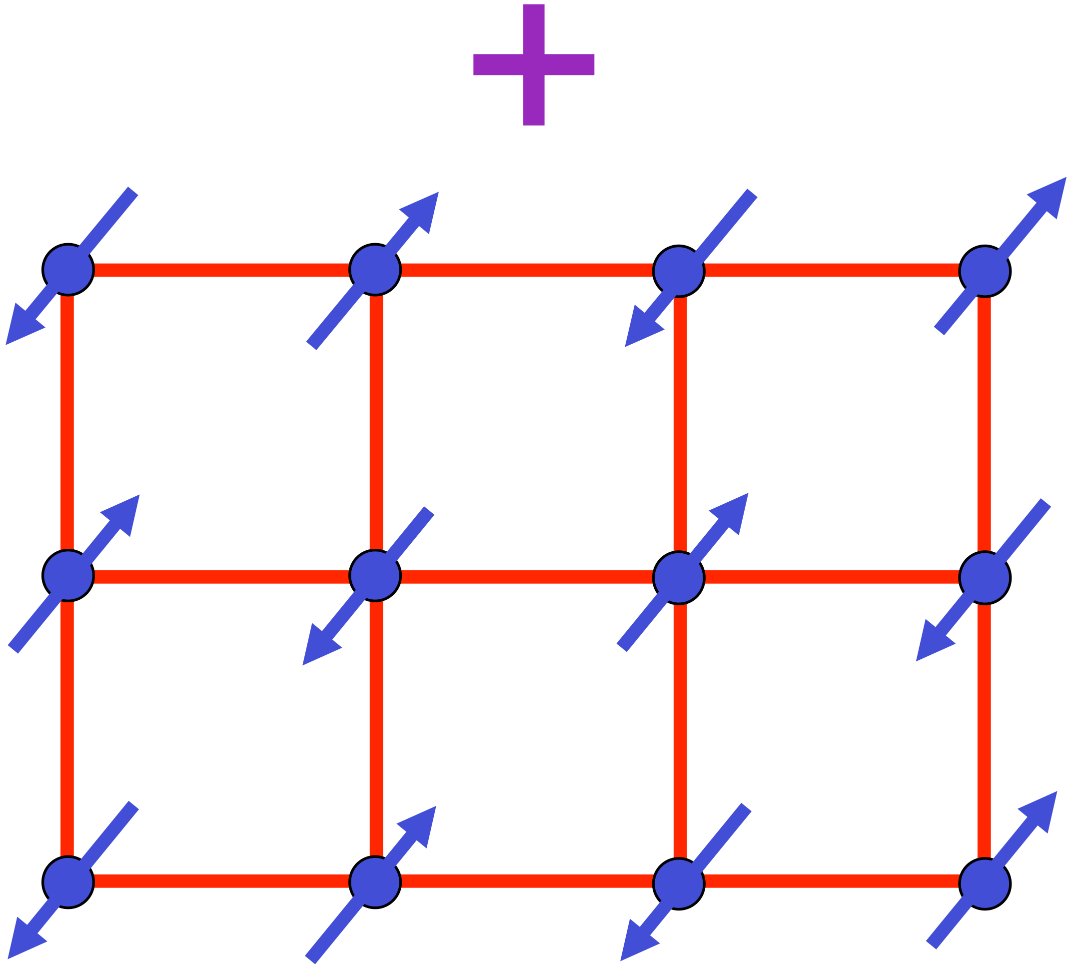
Metal with "large"
Fermi surface



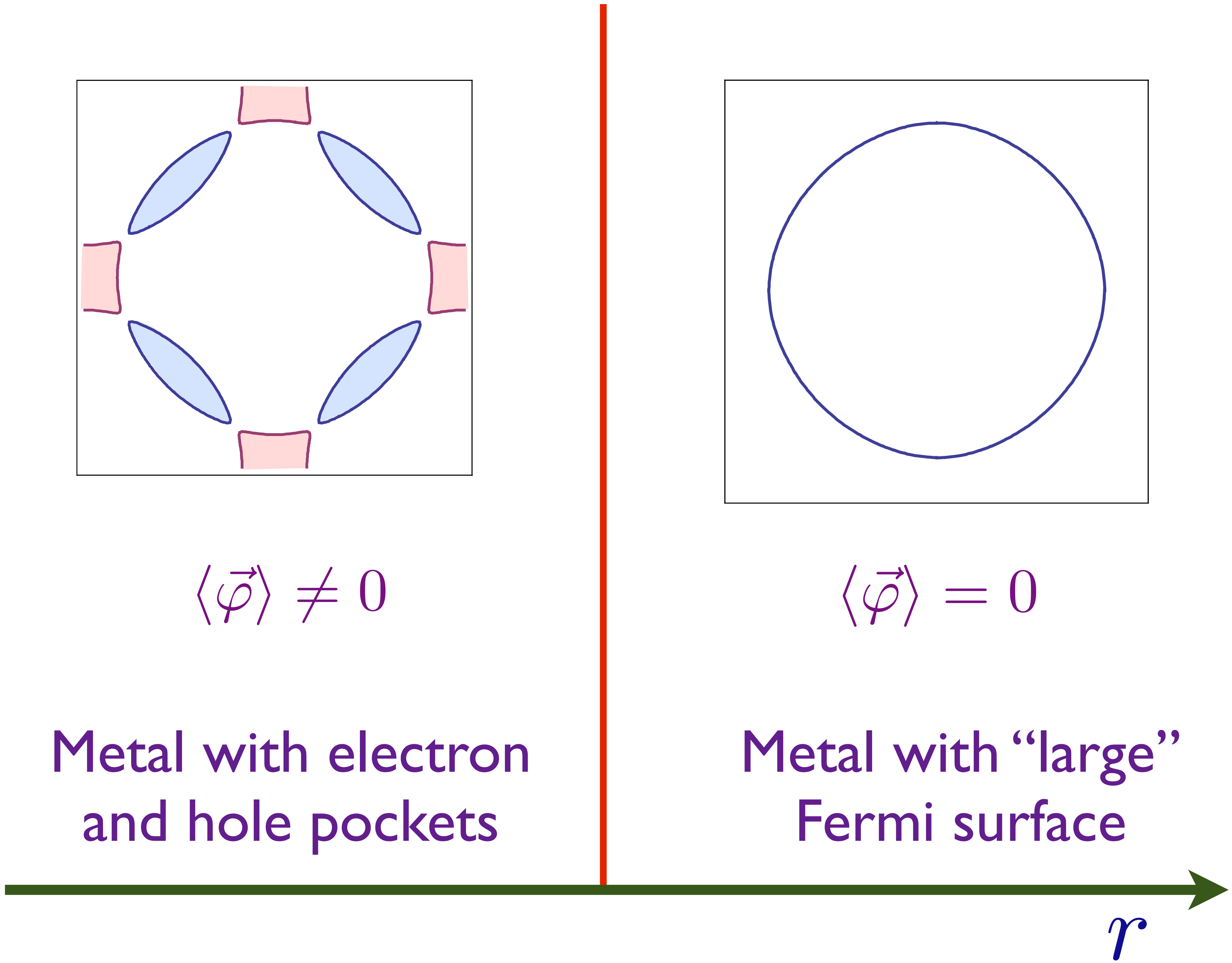
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.



Quantum phase transition with onset of antiferromagnetism in a metal



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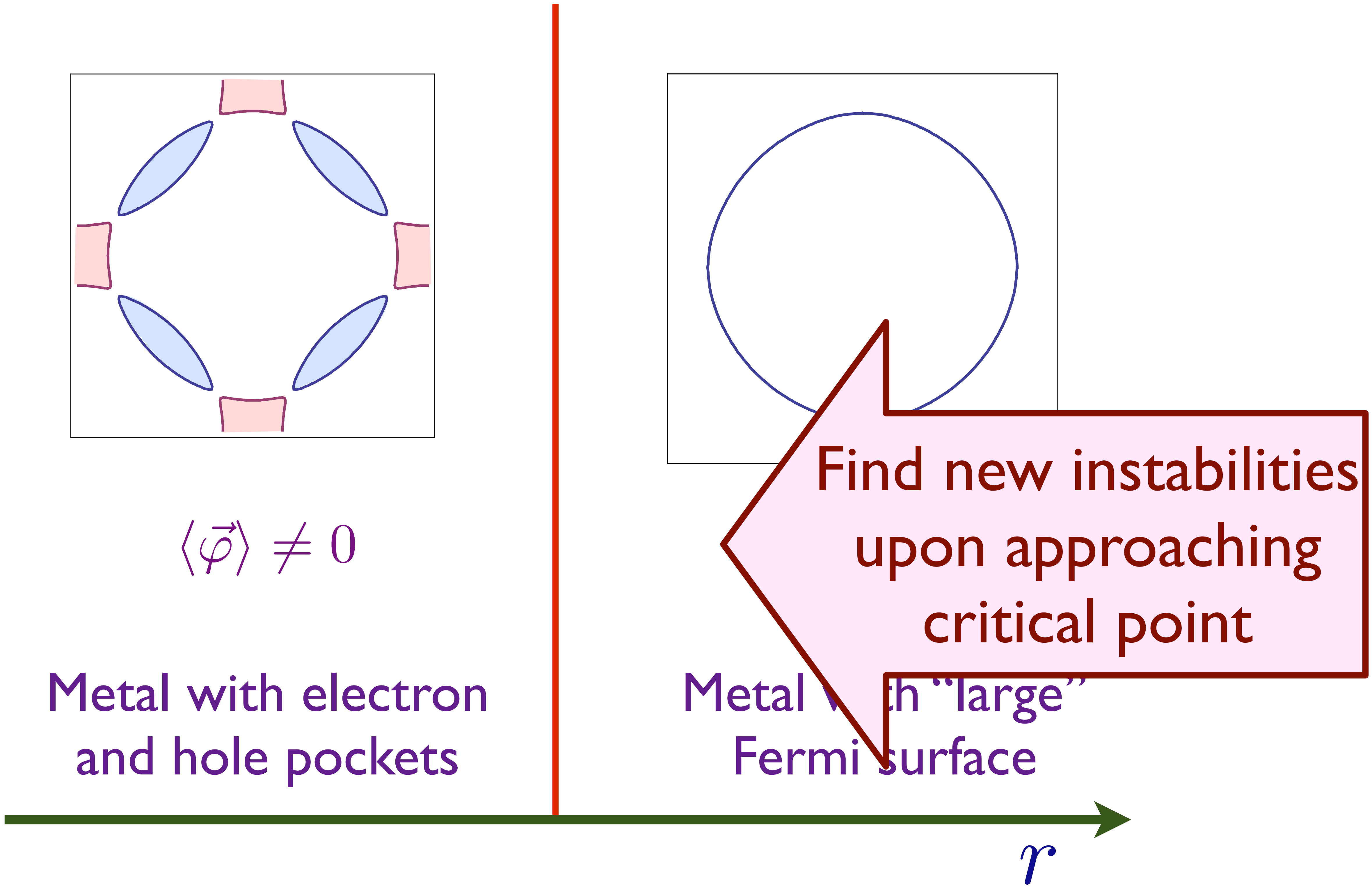

New physics in metals with antiferromagnetic correlation

- Weak-coupling instability to d-wave superconductivity, Ψ.

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
Ar. Abanov, A.V. Chubukov, and A.M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

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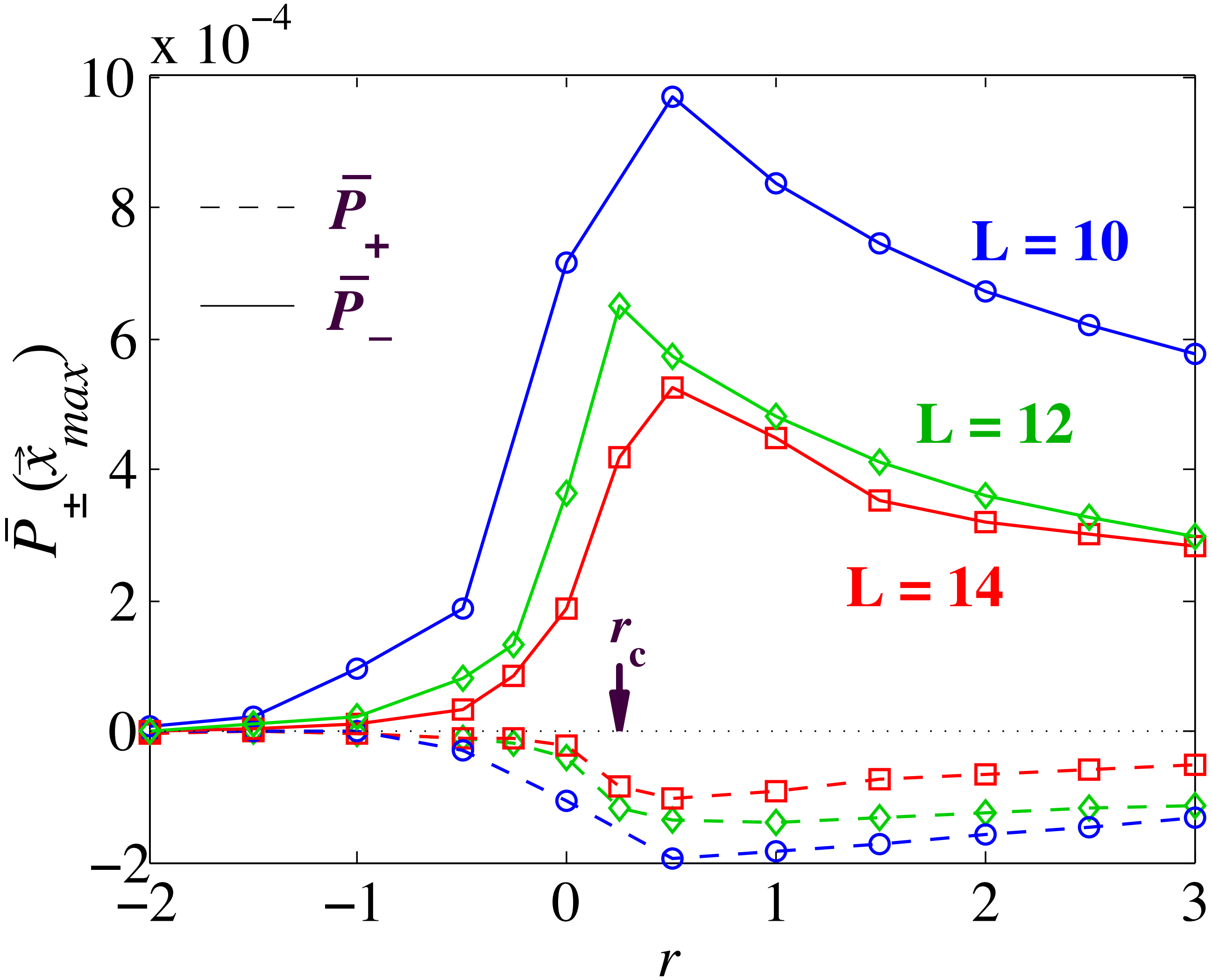
- Superconductivity survives at strong-coupling with critical antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

New physics in metals with antiferromagnetic correlation

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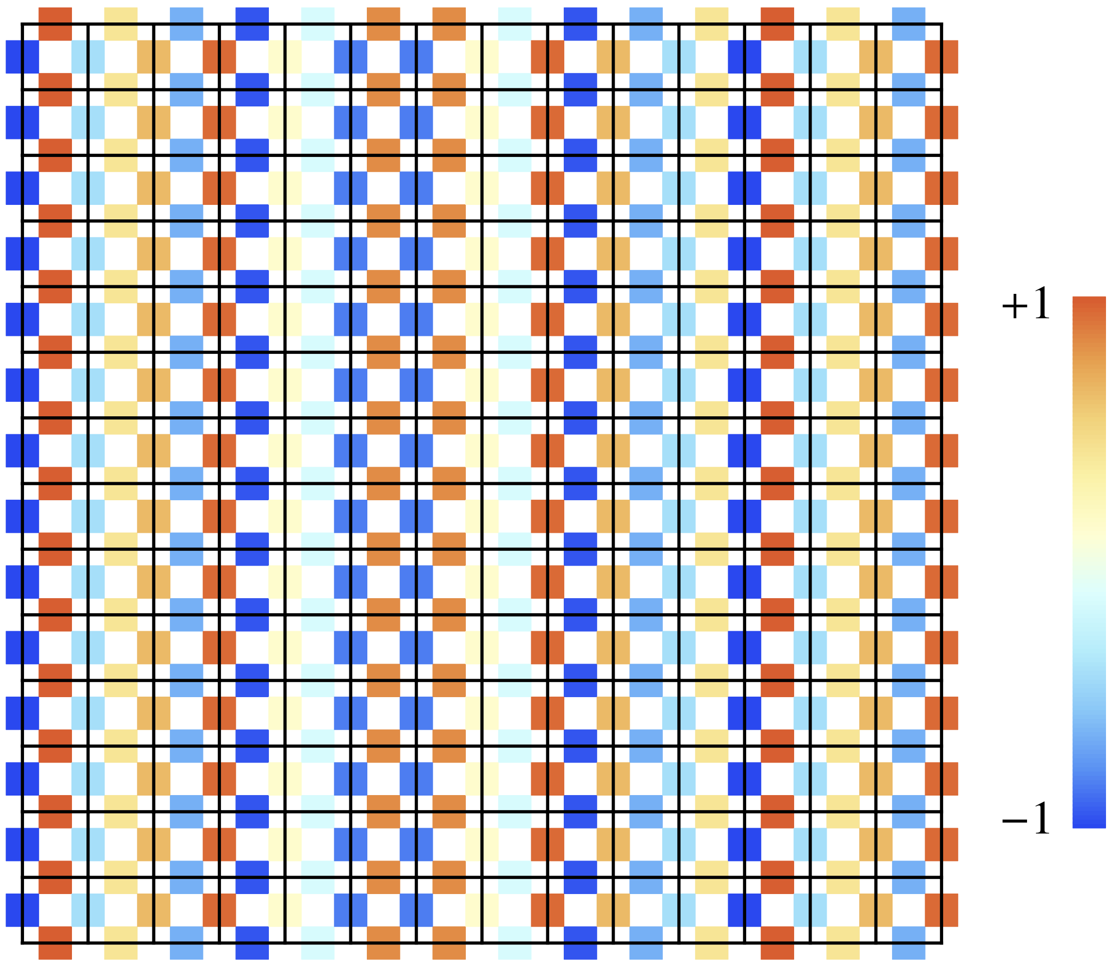

M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)
M. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)
S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
A. Allais, J. Bauer, and S. Sachdev, to appear

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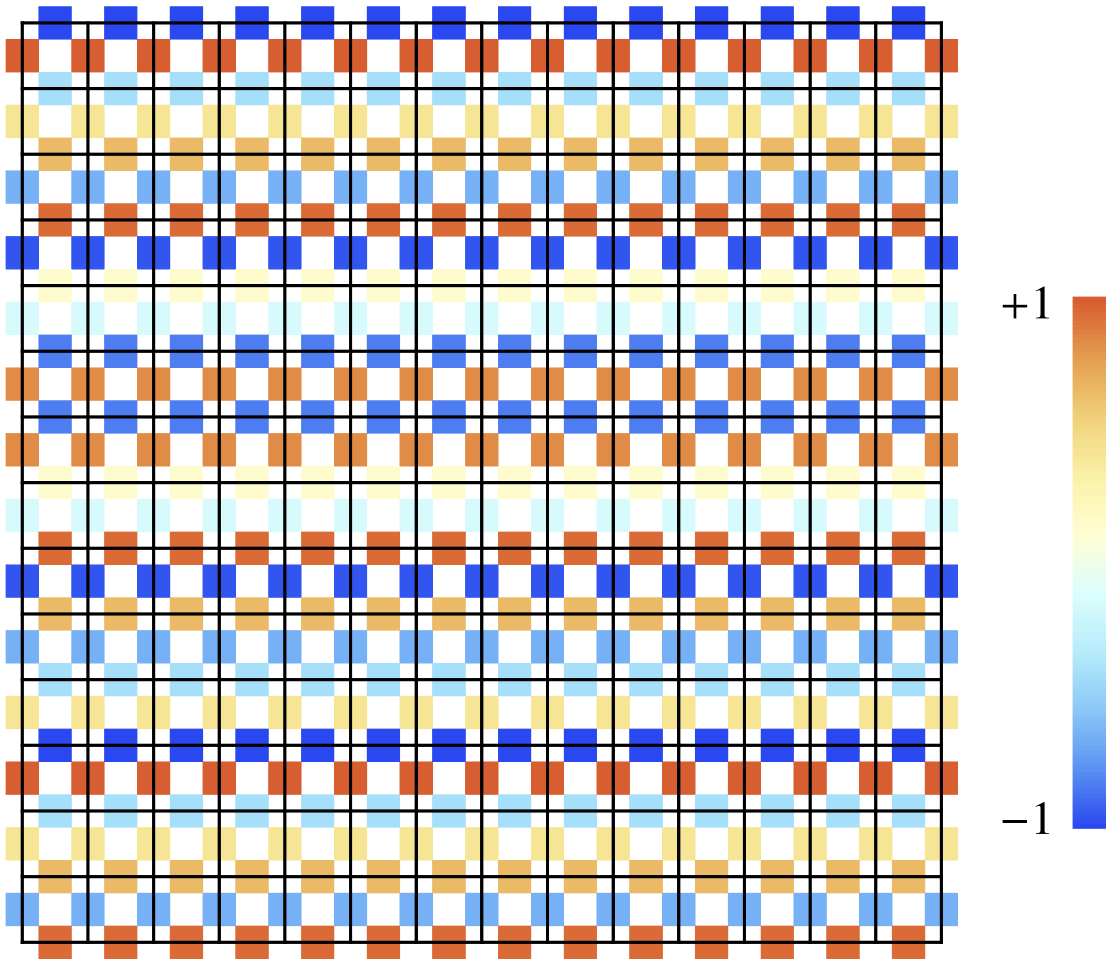

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M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)
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S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
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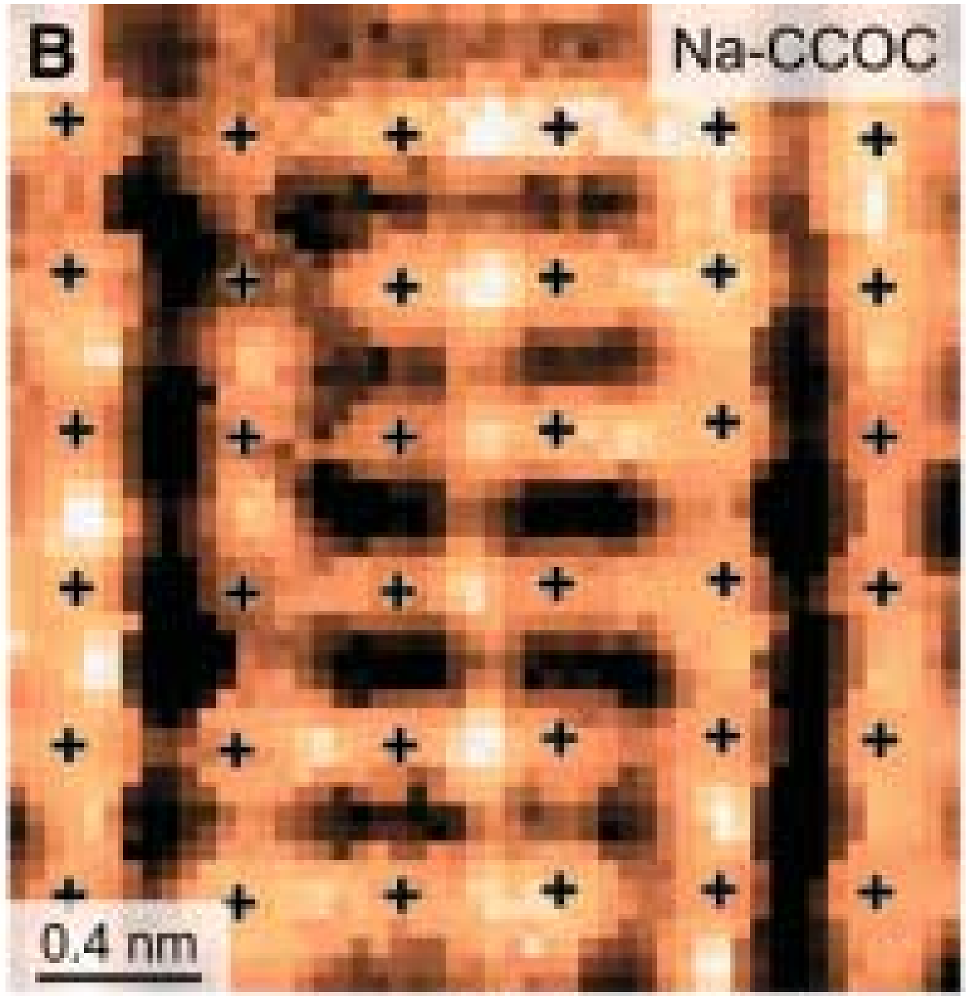
An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8*]

9 MARCH 2007 VOL 315 SCIENCE



Phys. Rev. Lett. **109**, 167001 (2012).

Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

In such a case, the energy shifts may in fact be a signature of a novel electronic state, such as a valence bond solid

New physics in metals with antiferromagnetic correlation

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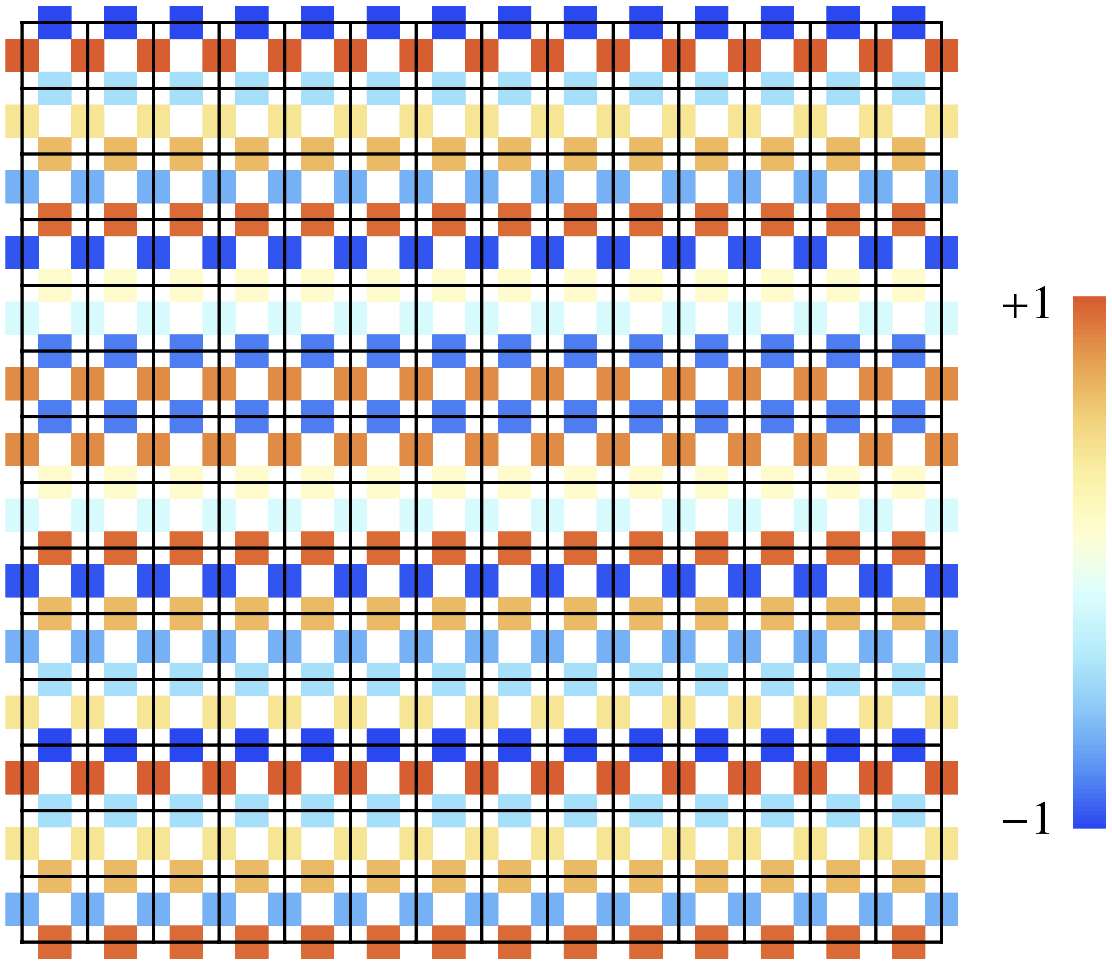

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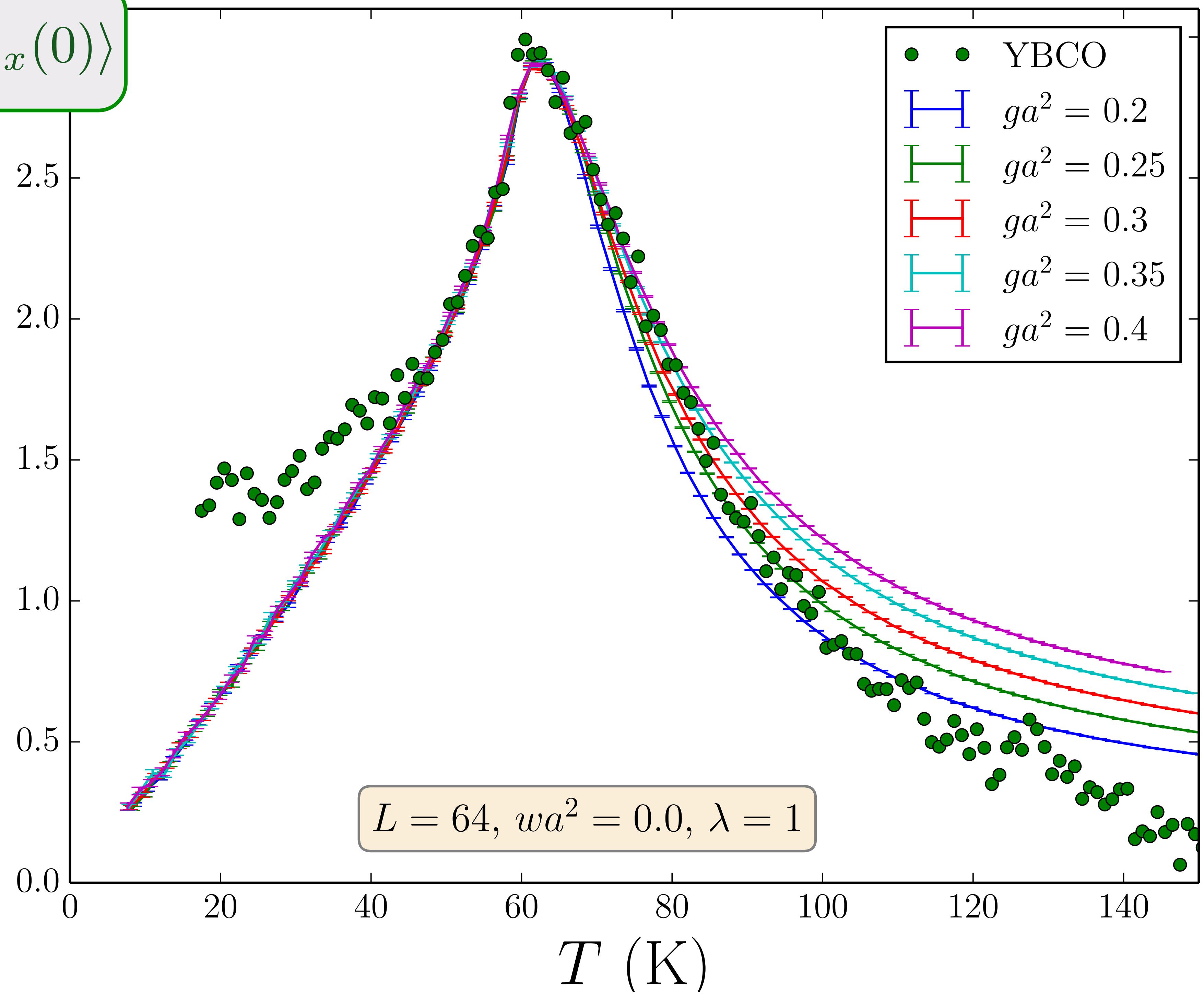
- X-ray scattering in underdoped YBCO is described by angular thermal fluctuations of a 6-component order parameter, Ψ, Φ_x, Φ_y.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

Comparison of Monte Carlo with experiments

$$S_{\Phi_x} = \int d^2 r \, \langle \Phi_x(\mathbf{r}) \Phi_x(0) \rangle$$

Charge order structure factor S_{Φ_x}



For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160$K.
The height was also rescaled to make the peak heights match.

L. E. Hayward, D. G. Hawthorn, R. G. Melko, and S. Sachdev, arXiv:1309.6639

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- Possible quantum critical point at optimal doping involving onset of Ising-nematic order $\phi = |\Phi_x|^2 - |\Phi_y|^2$

Outline

1. The simplest model without quasiparticles

   A. Superfluid-insulator transition
   of ultracold bosonic atoms in an optical lattice

   B. Conformal field theories in 2+1 dimensions,
   the AdS/CFT correspondence, and transport
   without quasiparticles.

2. Metals without quasiparticles

   A. The onset of antiferromagnetism in a metal

   B. Non-quasiparticle transport at the
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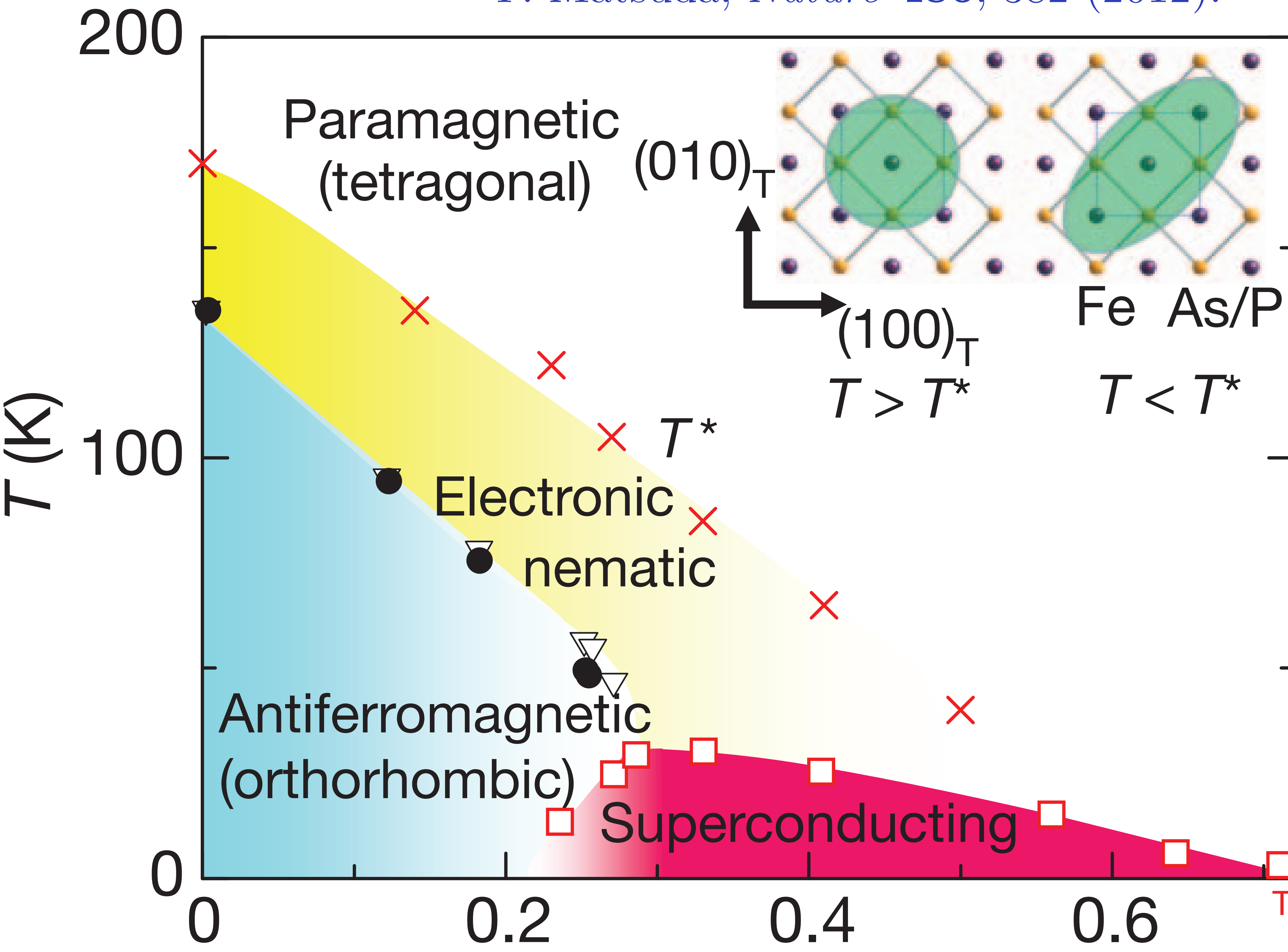
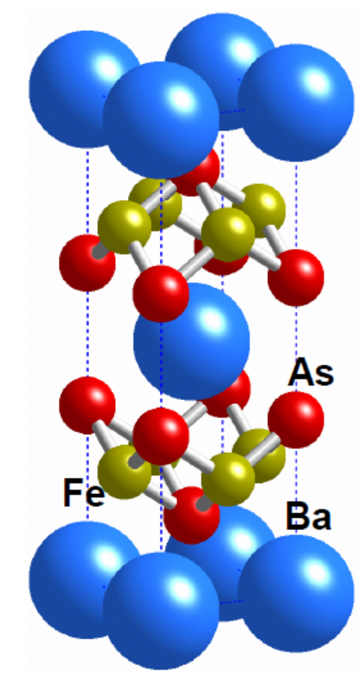
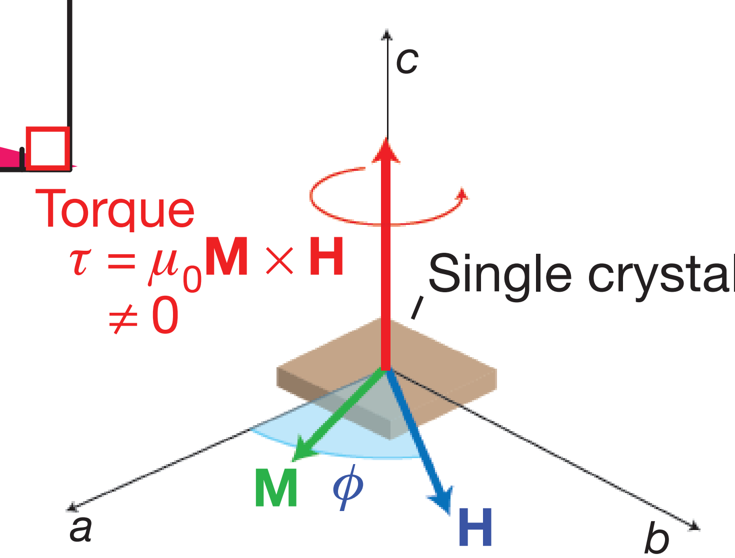
S. Kasahara, H.J. Shi, K. Hashimoto, S. Tonegawa, Y. Mizukami, T. Shibauchi, K. Sugimoto, T. Fukuda, T. Terashima, A.H. Nevidomskyy, and Y. Matsuda, *Nature* **486**, 382 (2012).



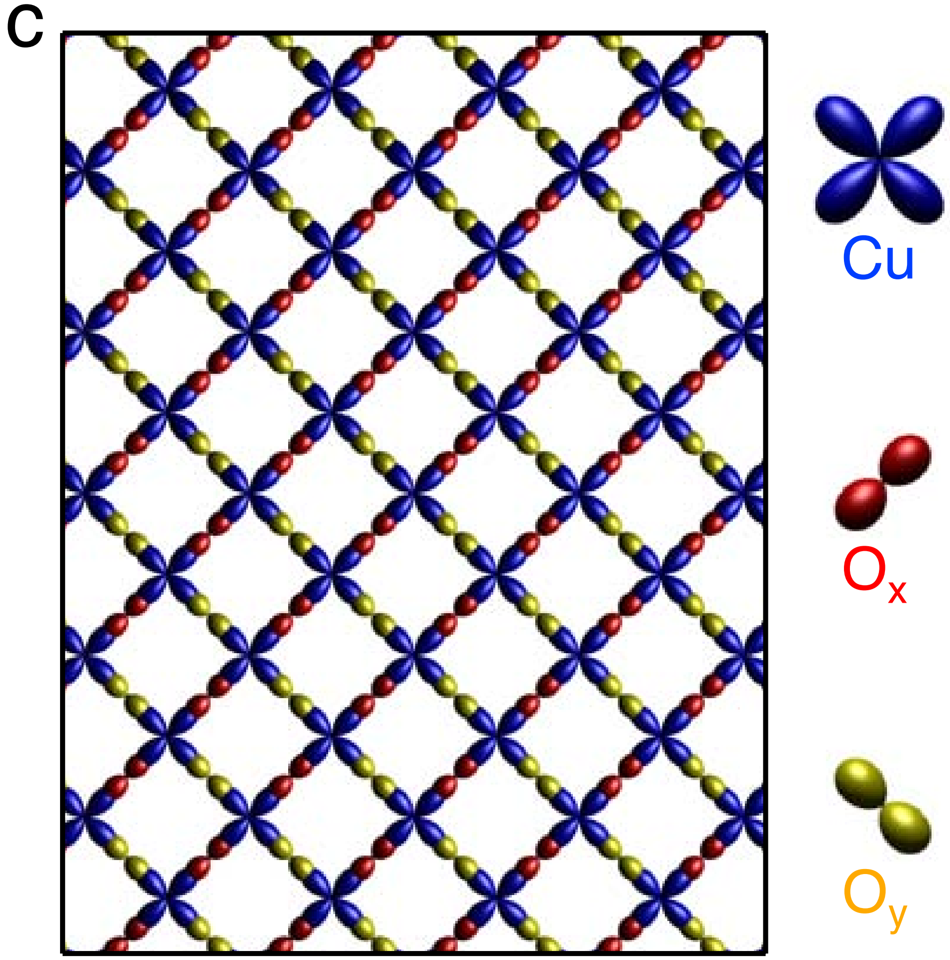


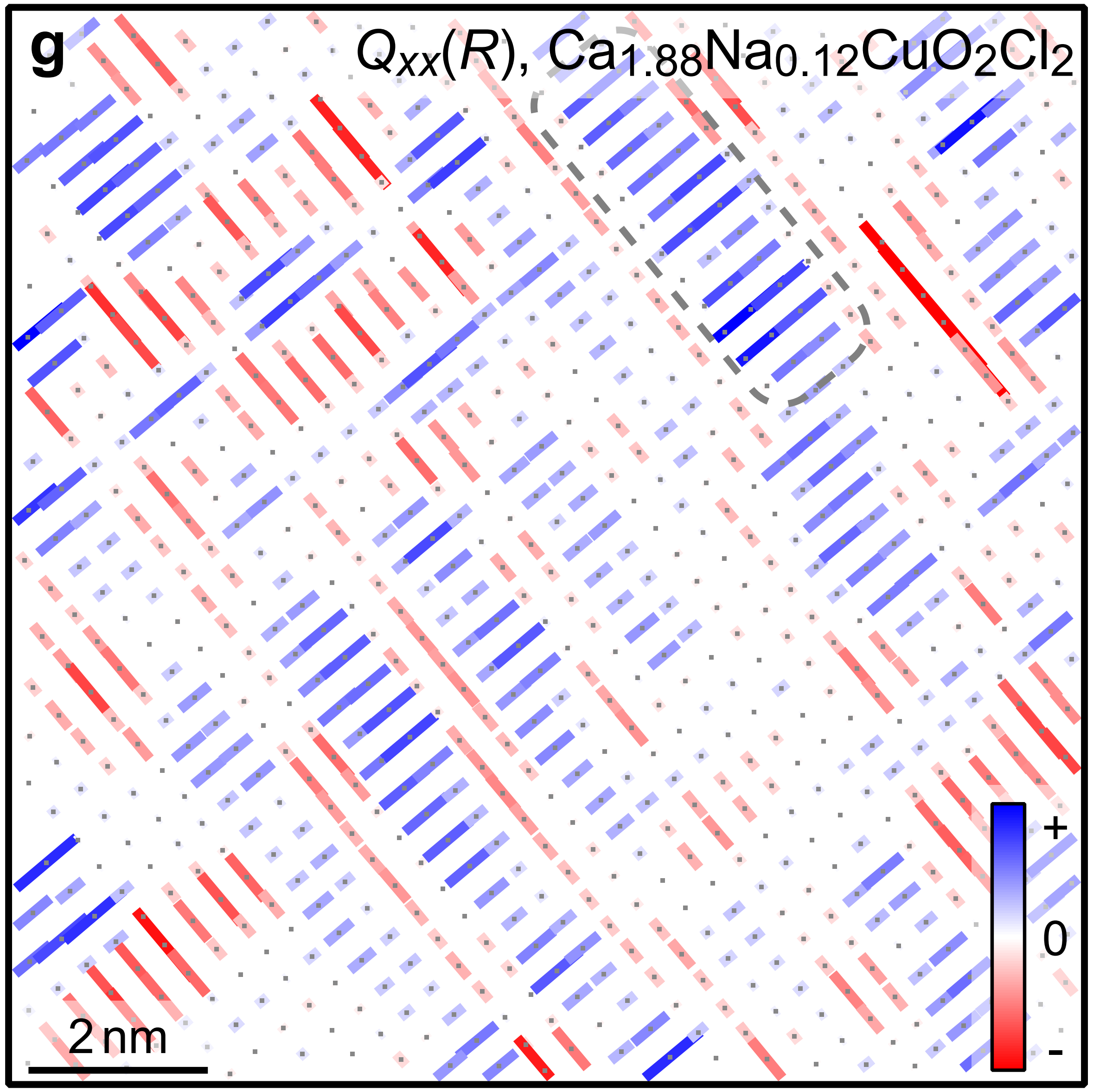


$BaFe_2(As_{1-x}P_x)_2$

Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

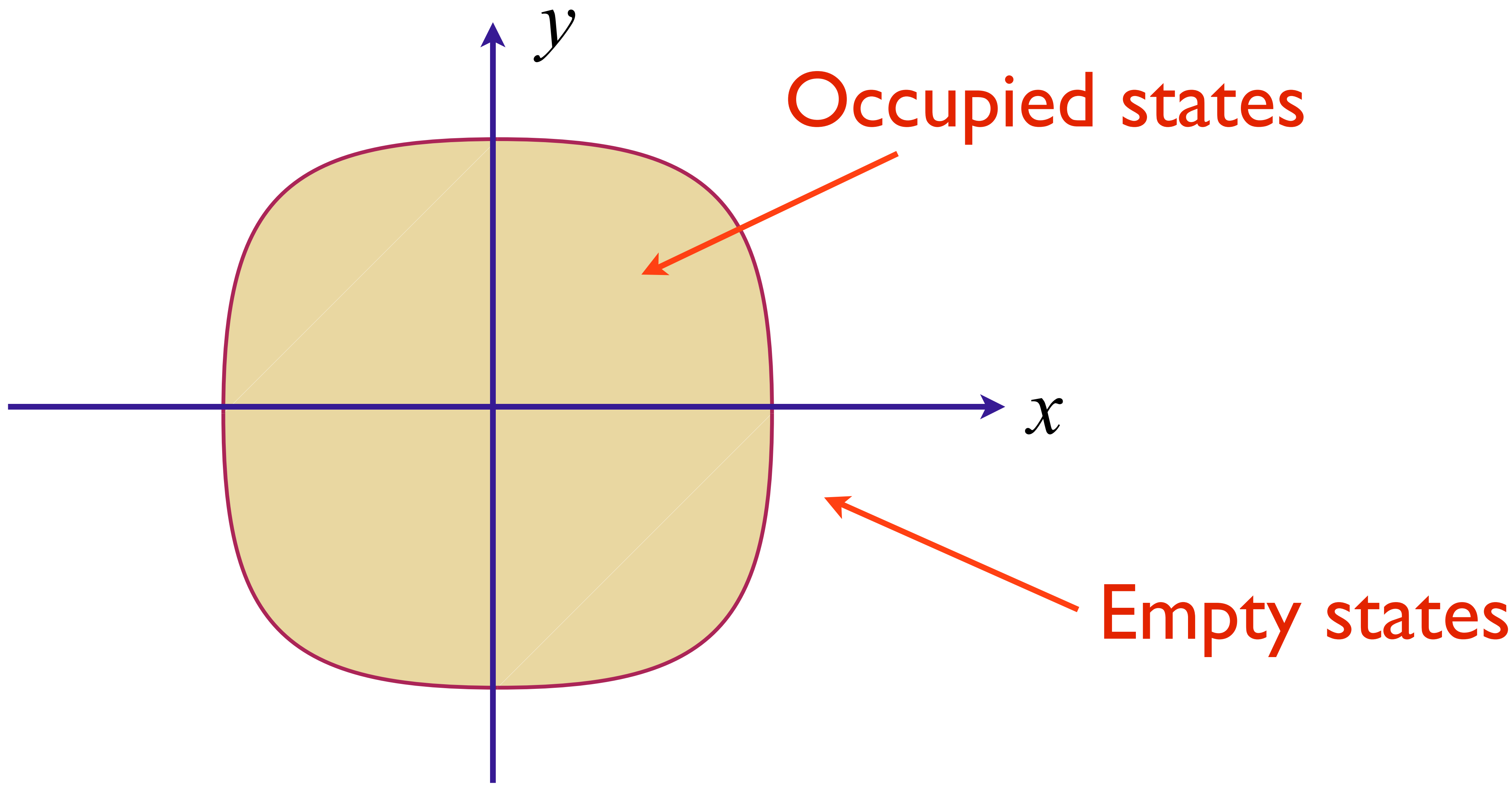
Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi
Nature Physics, **8**, 534 (2012).





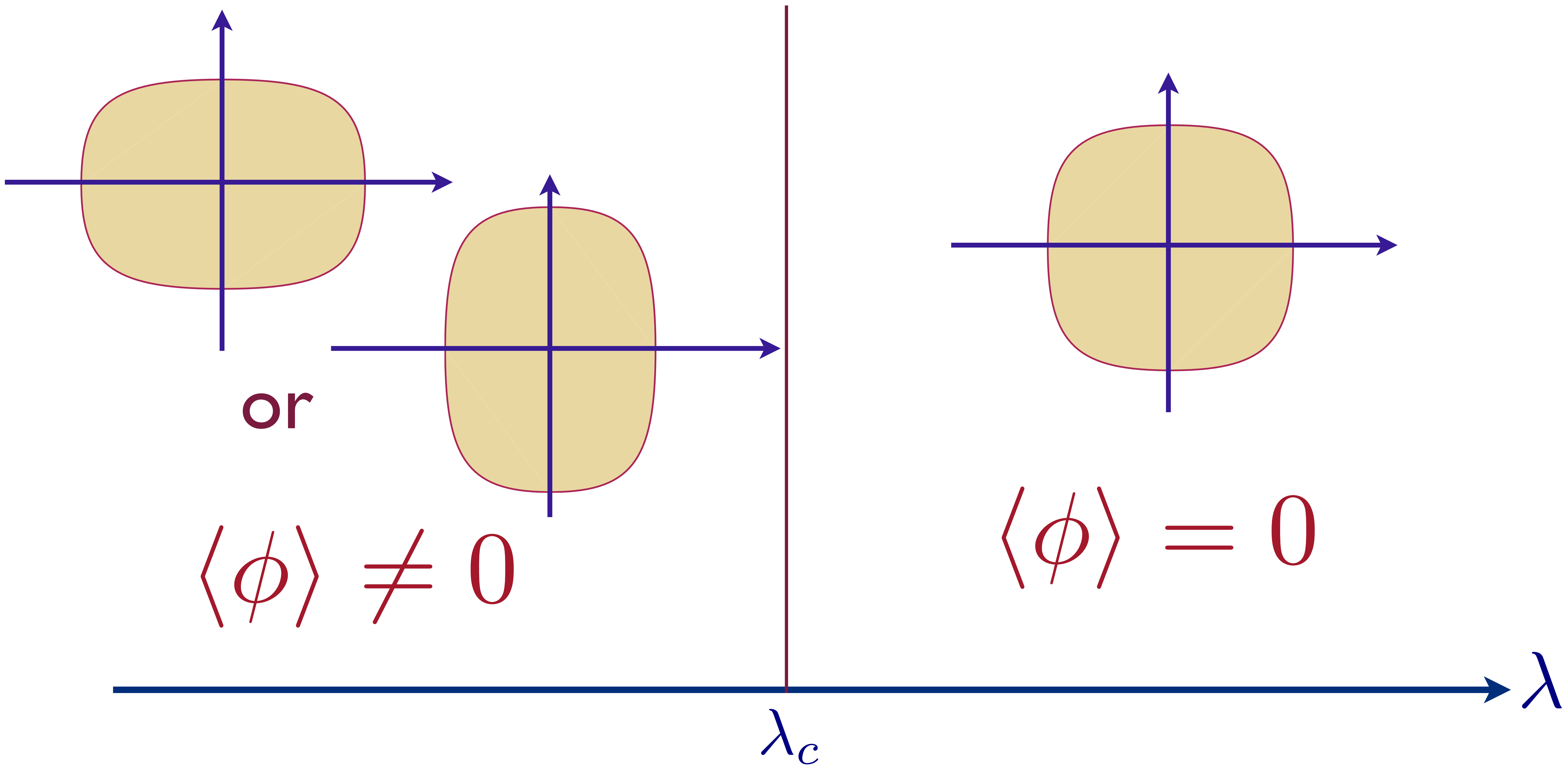
Evidence for "nematic" order
(*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

Quantum criticality of Ising-nematic ordering in a metal



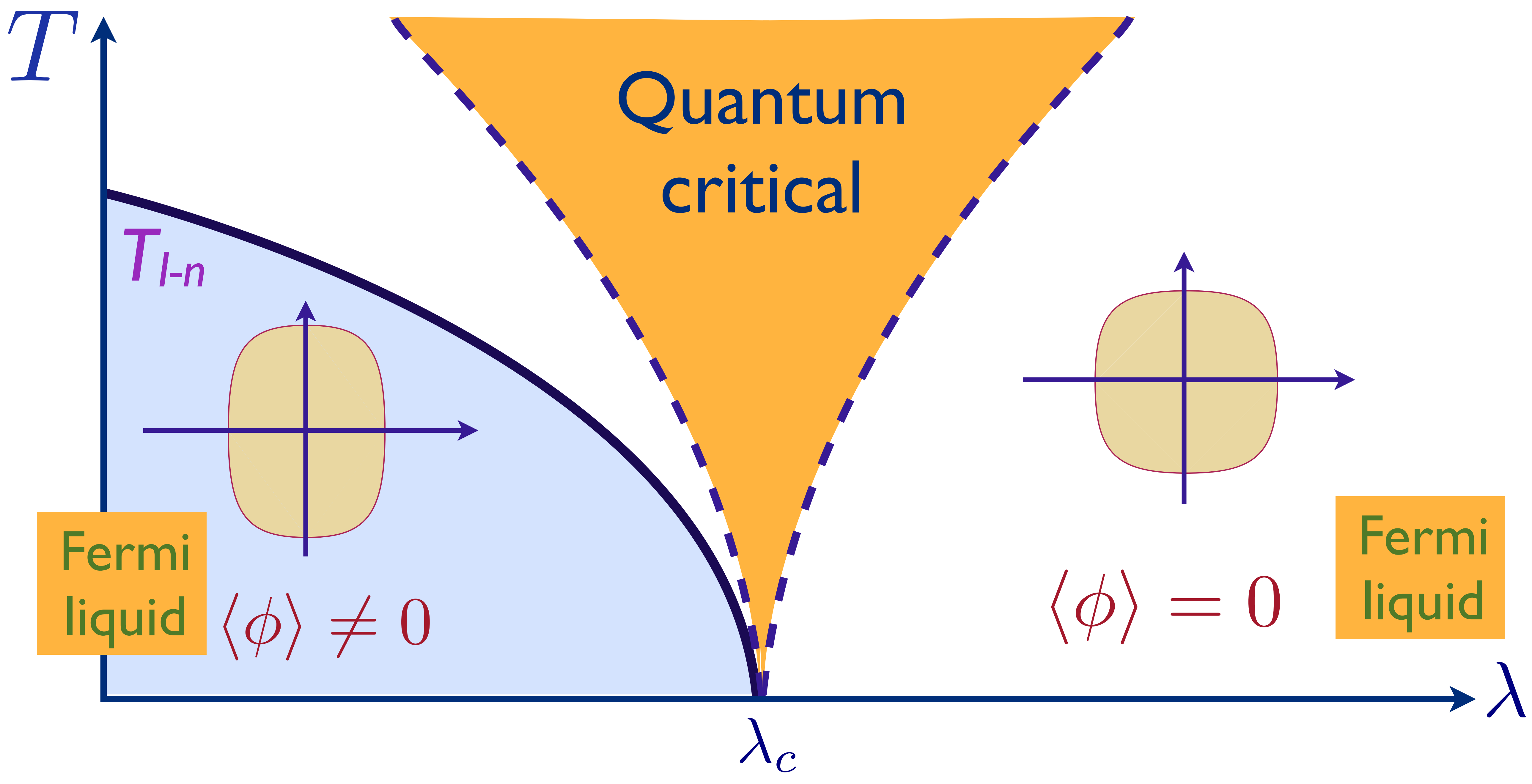
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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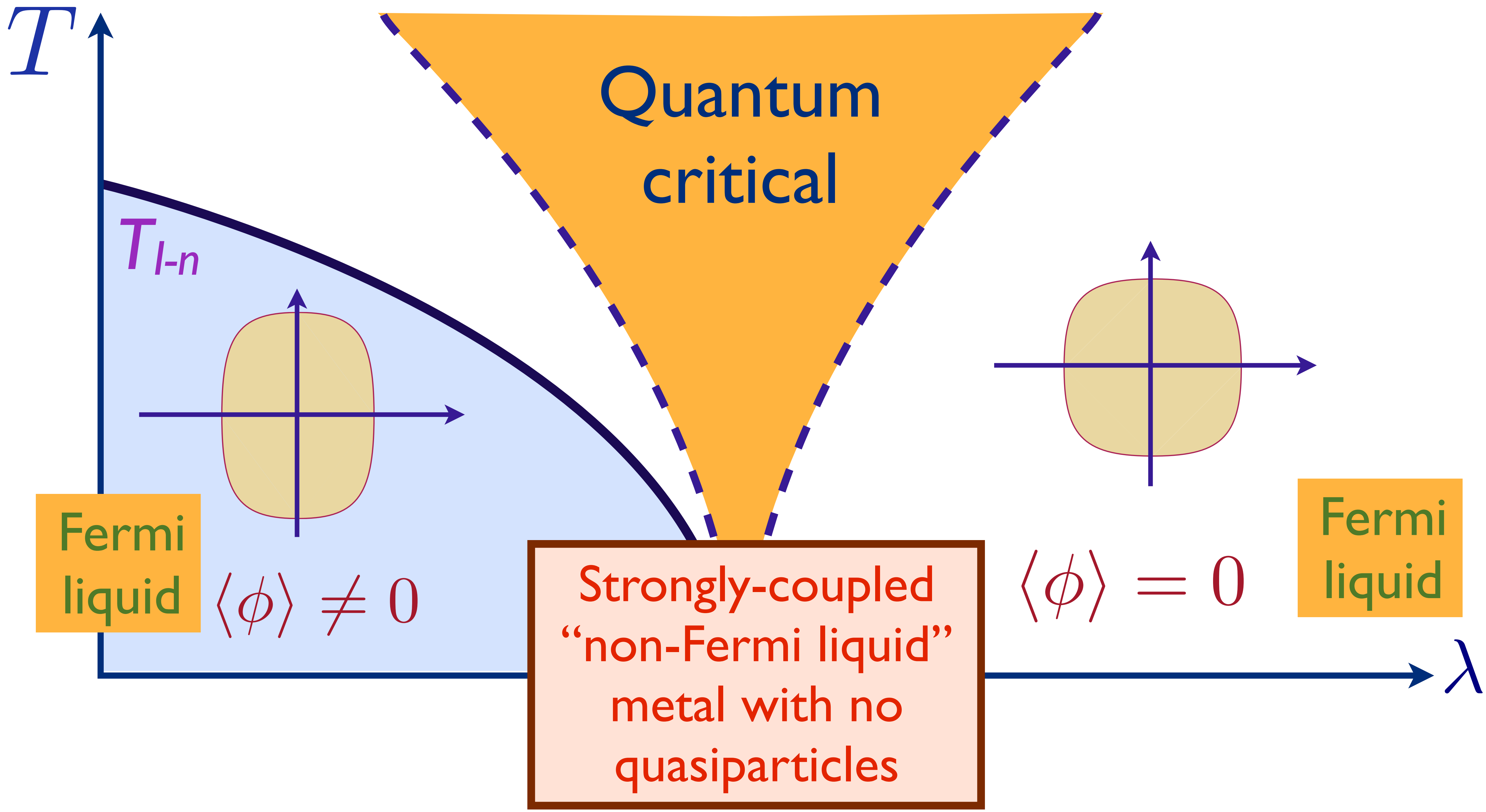

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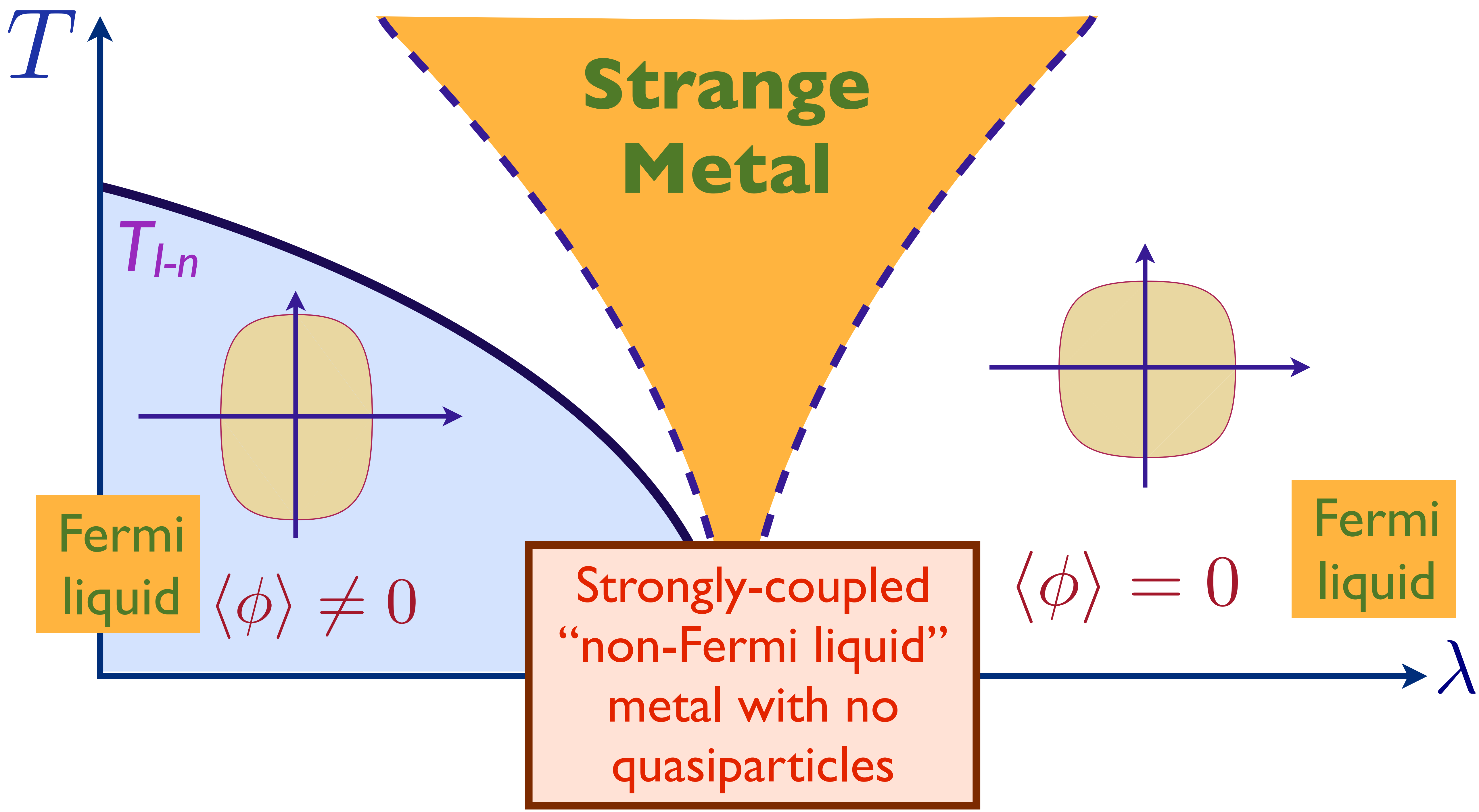

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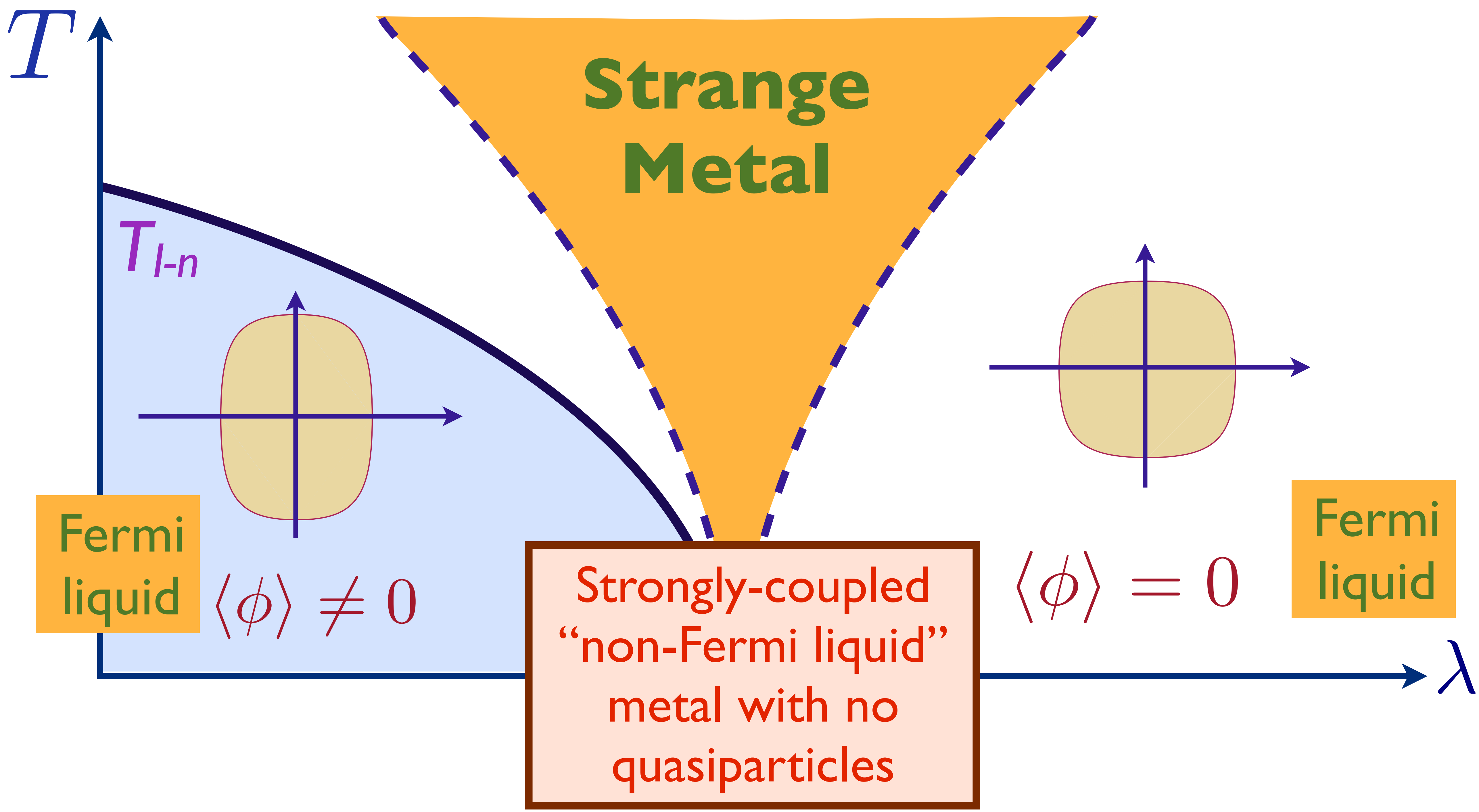

Common theoretical belief from an analysis of scattering of charged electronic quasiparticles off bosonic ϕ fluctuations: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c^\dagger_{\mathbf{k}\alpha} \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = - g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k \right.$$

Field theory of bosonic order parameter

Quantum criticality of Ising-nematic ordering in a metal

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$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Electrons with a Fermi surface: $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu \ldots$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

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"Yukawa" coupling between bosons and fermions

Quantum criticality of Ising-nematic ordering in a metal

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$$\mathcal{S}_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2) c_\alpha^\dagger \right\} c_\alpha \right]$$

This continuum theory has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$, and so the resistivity $\rho(T) = 0$.

Quantum criticality of Ising-nematic ordering in a metal

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The resistivity of the strange metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} = \int d^2r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] ,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2\, \delta(\mathbf{r} - \mathbf{r}') ,$$

$$\overline{h(\mathbf{r})} = 0 \quad ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2\, \delta(\mathbf{r} - \mathbf{r}') ,$$

we obtain the resistivity for current along angle ϑ

$$\rho(T) = \frac{1}{\chi^2_{\mathbf{J},\mathbf{P}}} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2}\, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2\, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2\, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right)$$

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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<u>Bosonic term</u>: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim T$ at higher T,
to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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<u>Bosonic term</u>: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Also obtained in holographic theory of a generalized compressible quantum state (A. Lucas, S. Sachdev, and K. Schalm, `arXiv:1401.7933`).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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