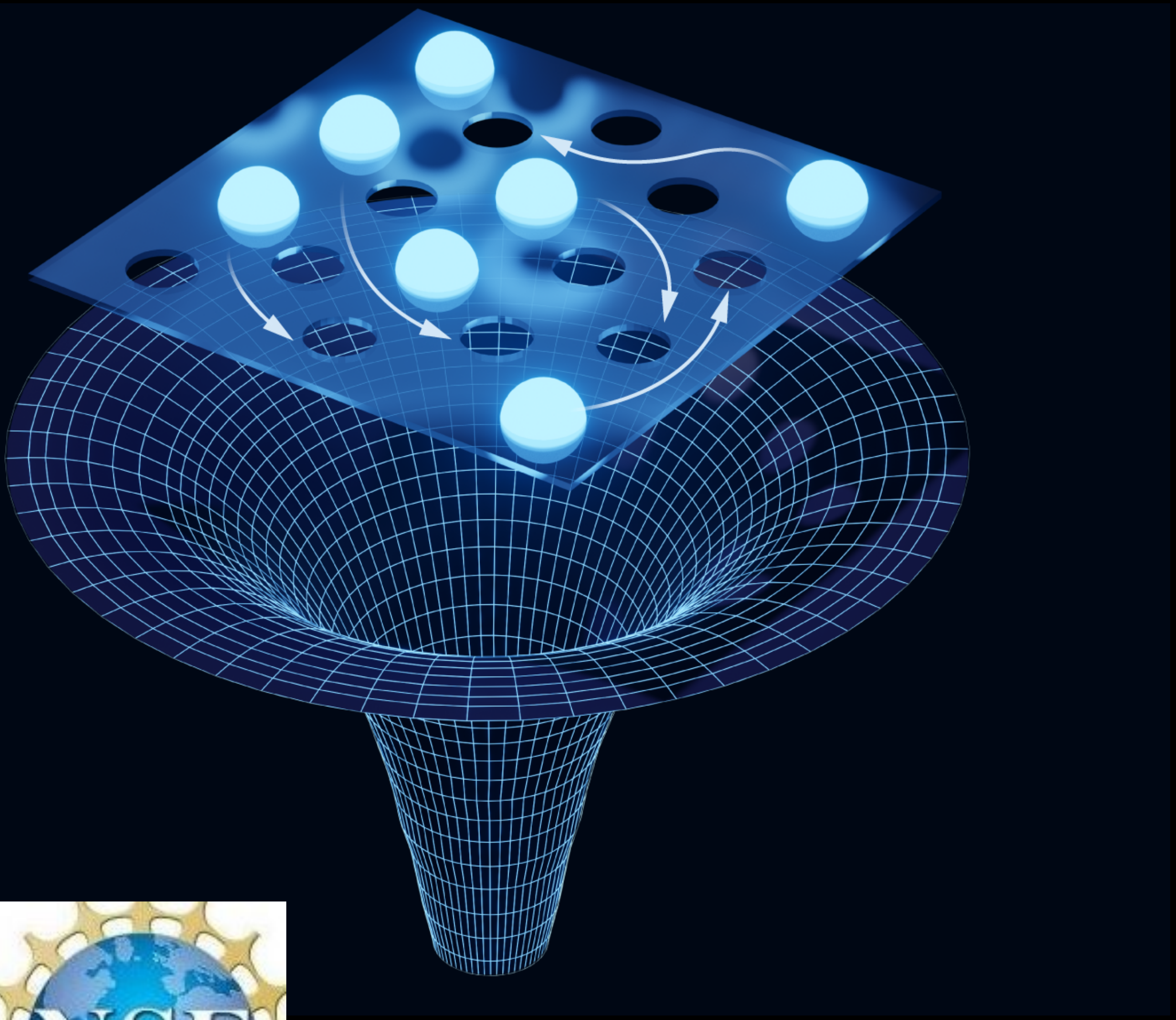
Quantum entanglement in nature: high temperature superconductors and black holes



Feenberg Lecture,
Washington University
in St. Louis
September 25, 2024

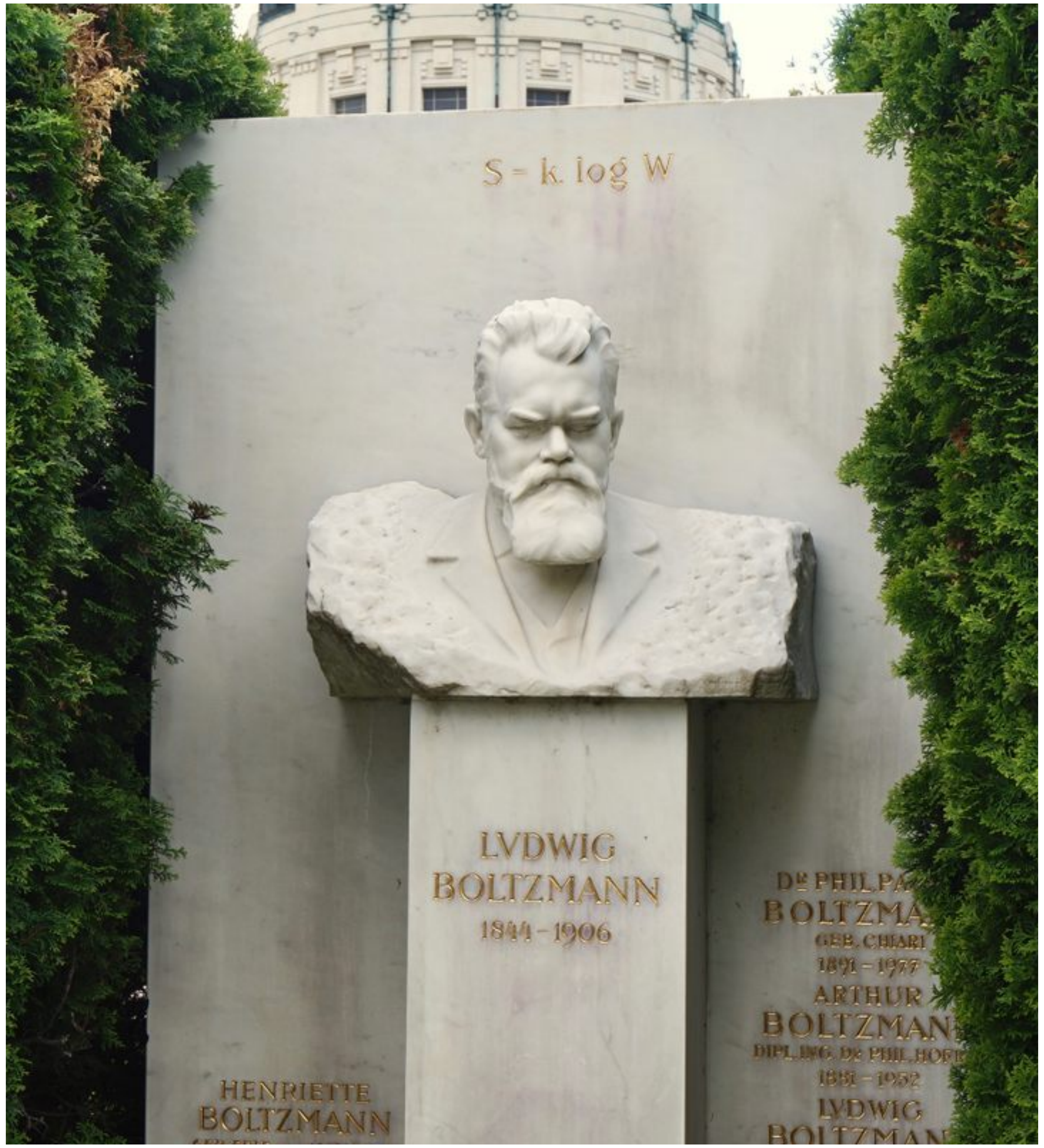
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD

Boltzmann-Landau theory of metals

Statistical interpretation of entropy (1870)

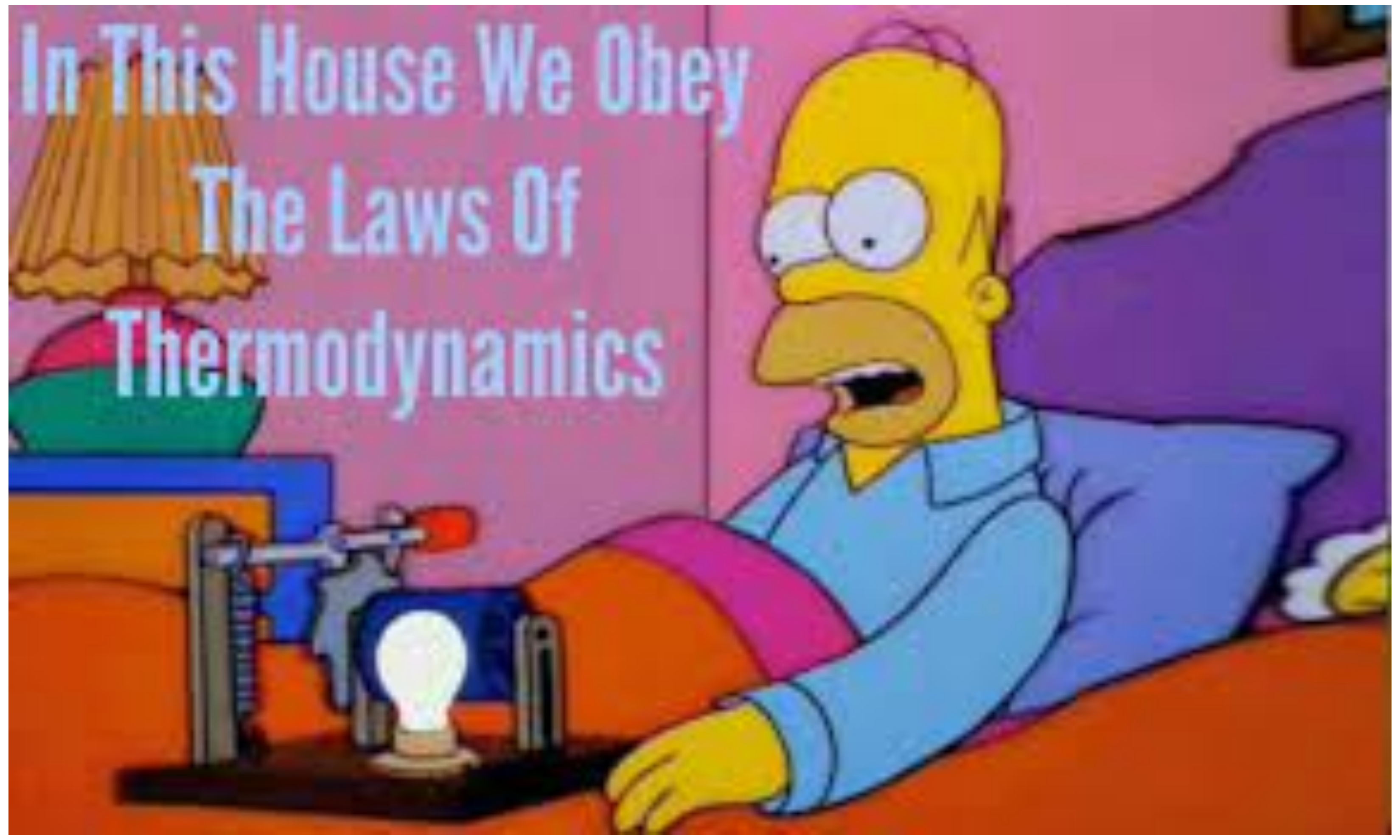


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

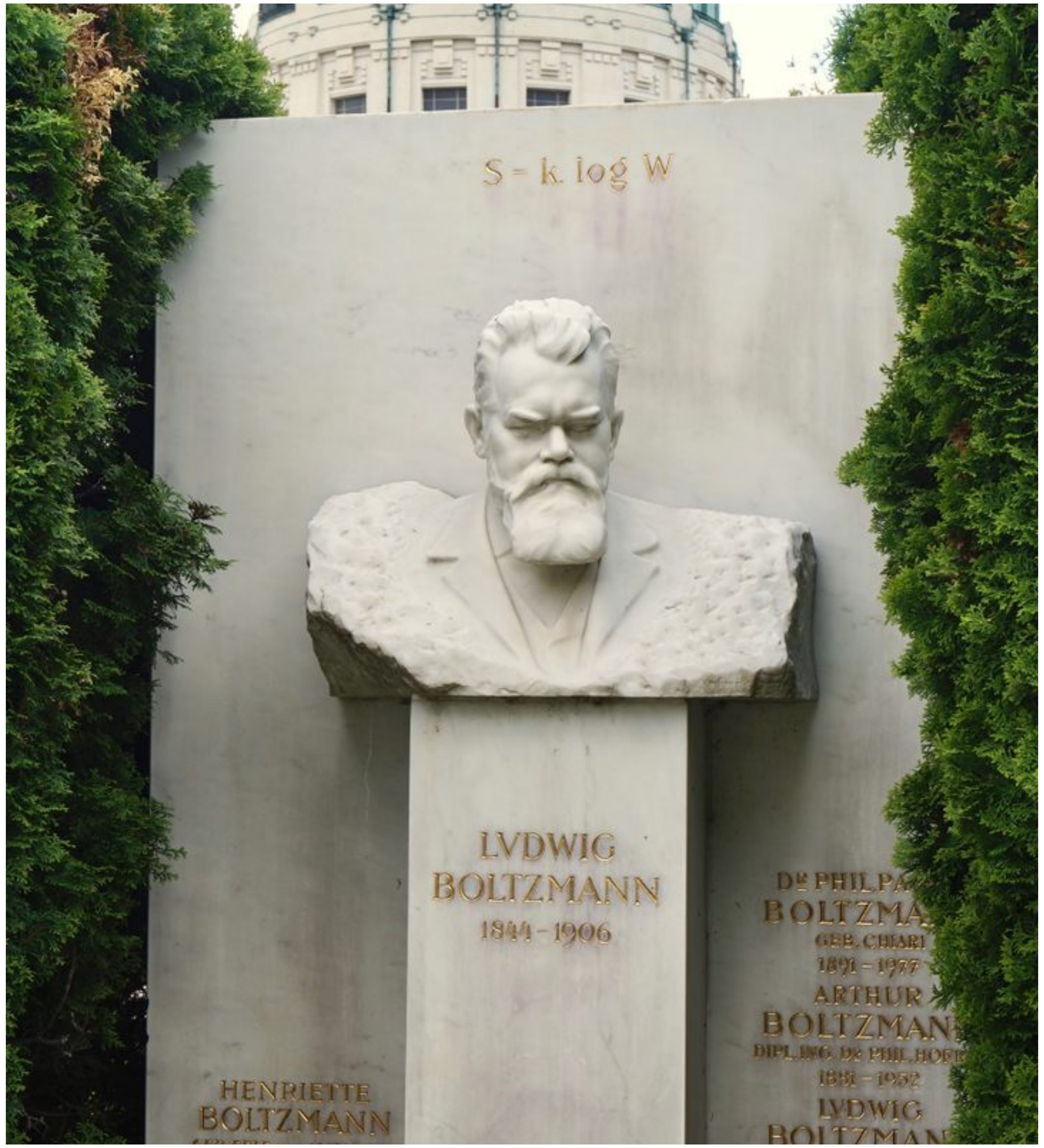
$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$

$$\frac{1}{T} = \frac{dS}{dE}$$



No perpetual motion machines!

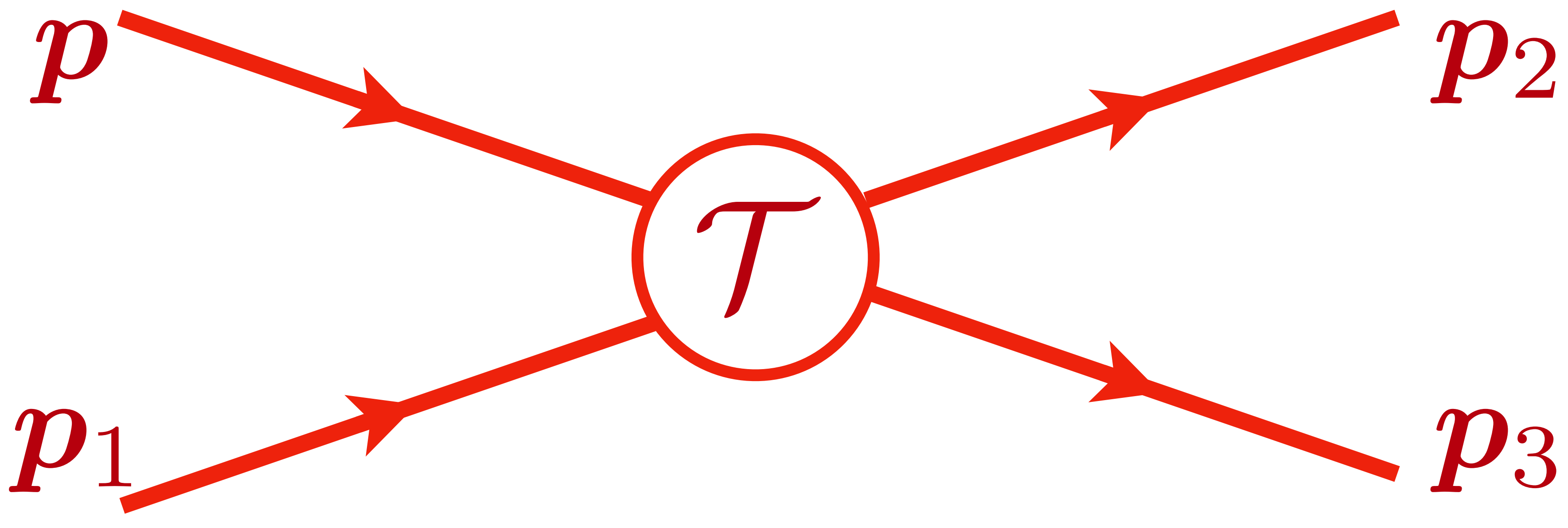


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

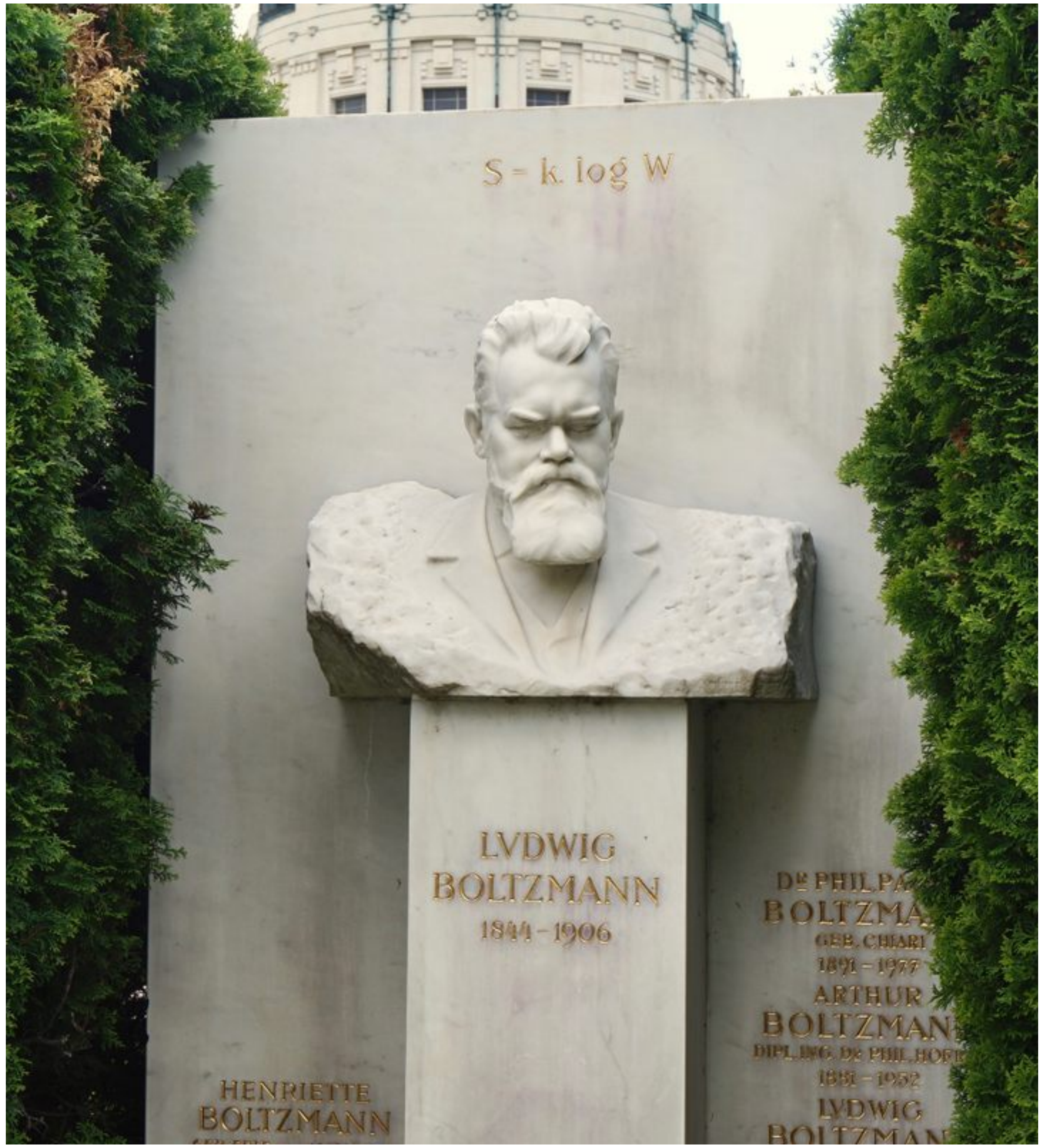
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



Quantum Boltzmann equation (Landau)
Dense gas of electrons



Ludwig Boltzmann
20 February 1844 - September 5, 1906
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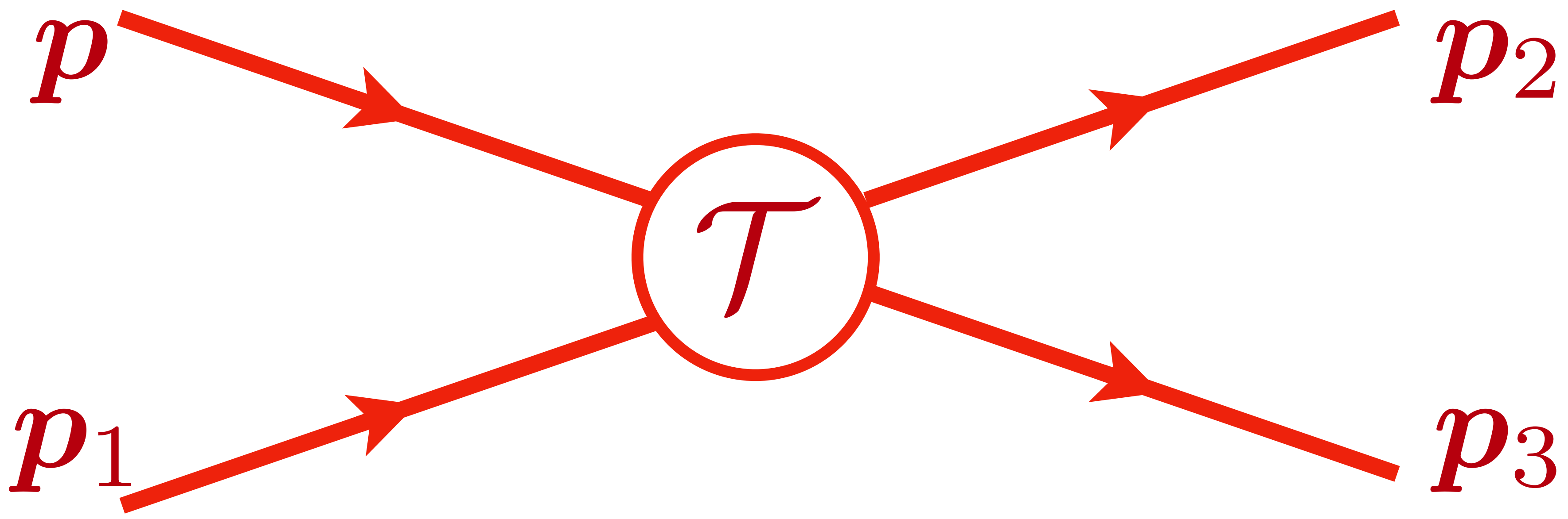
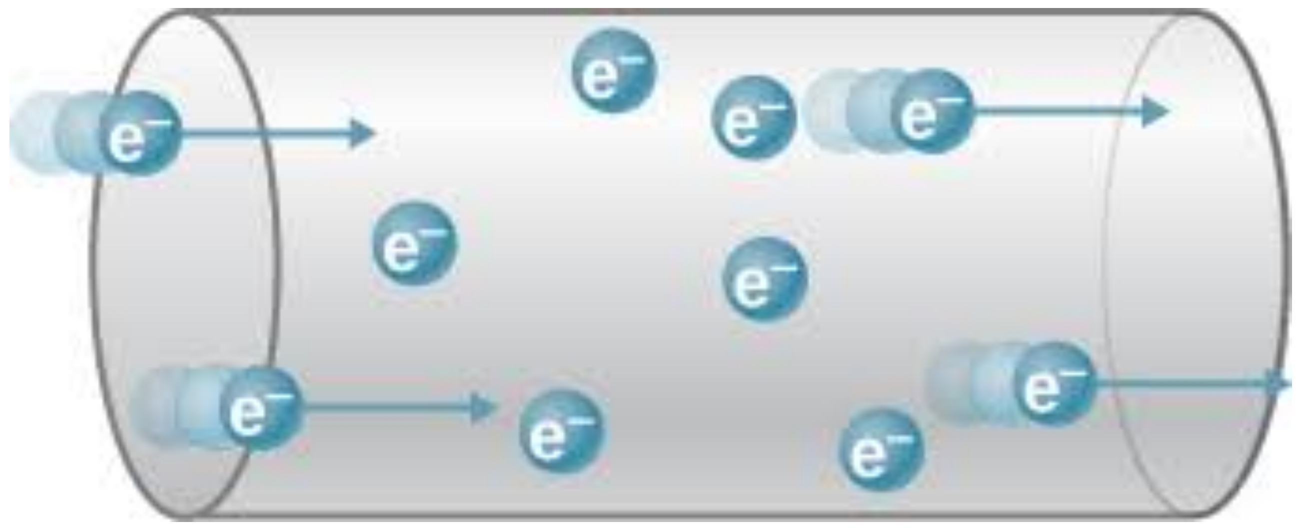
Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in ordinary metals



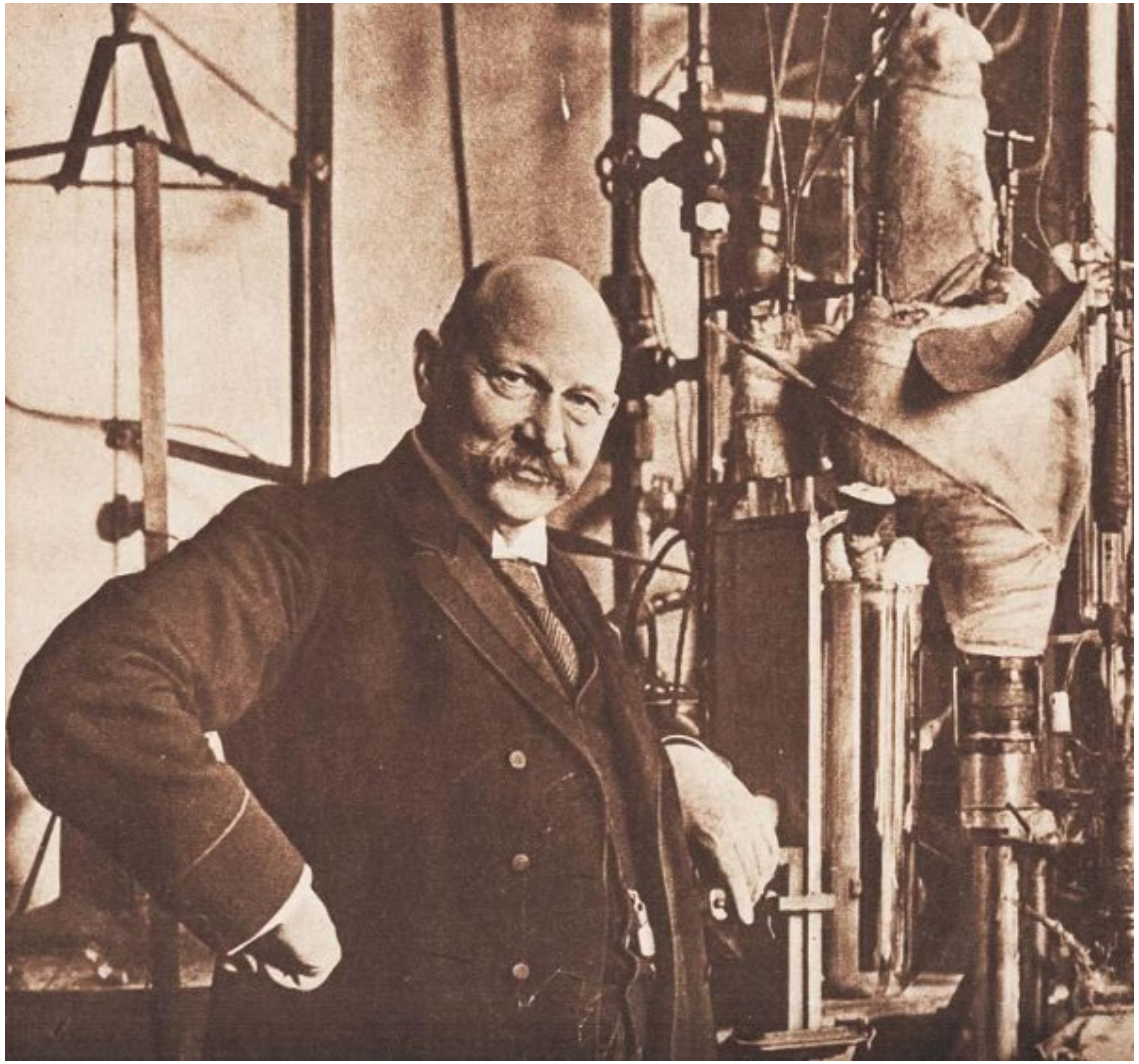
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

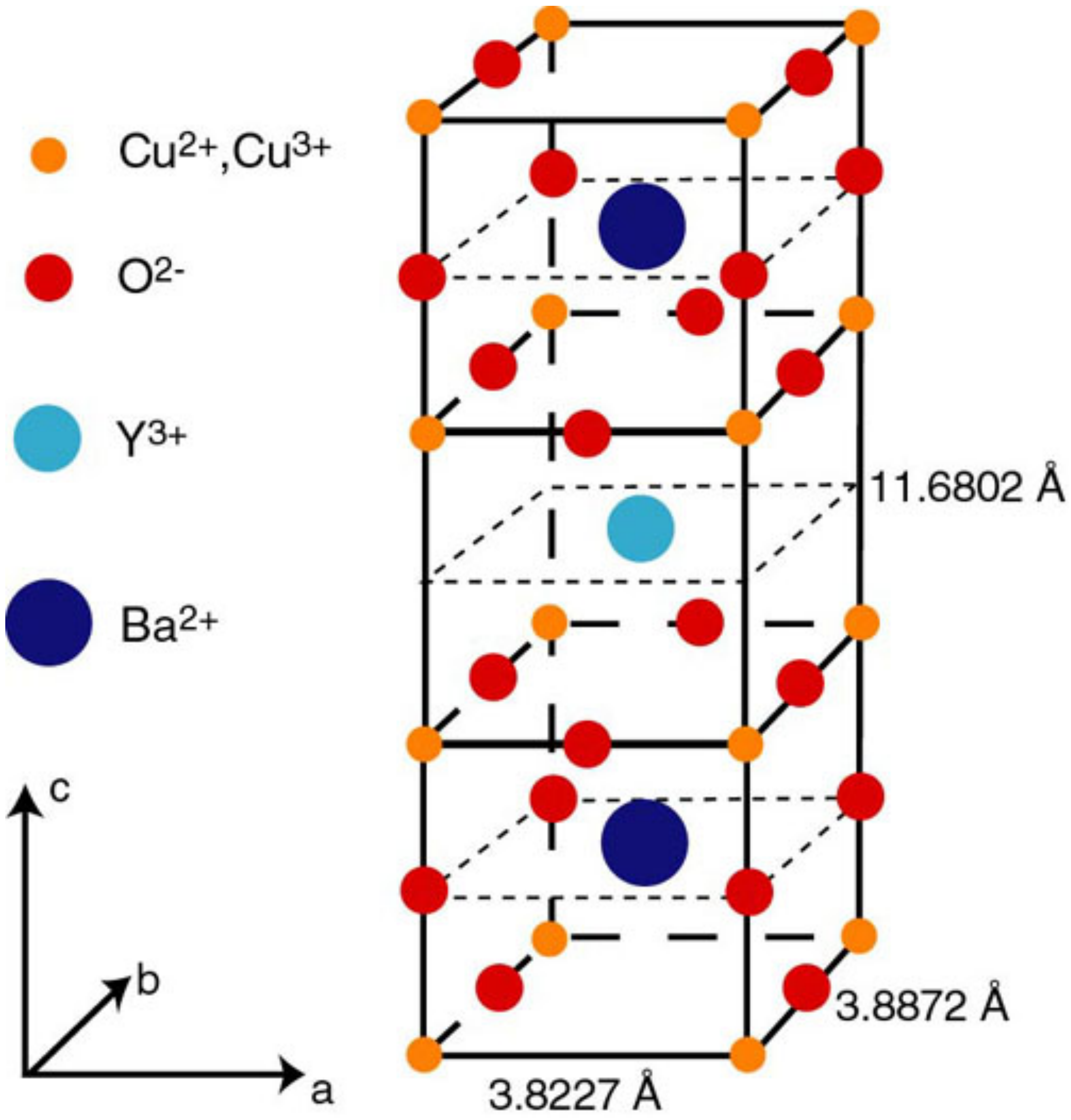
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

High temperature superconductivity

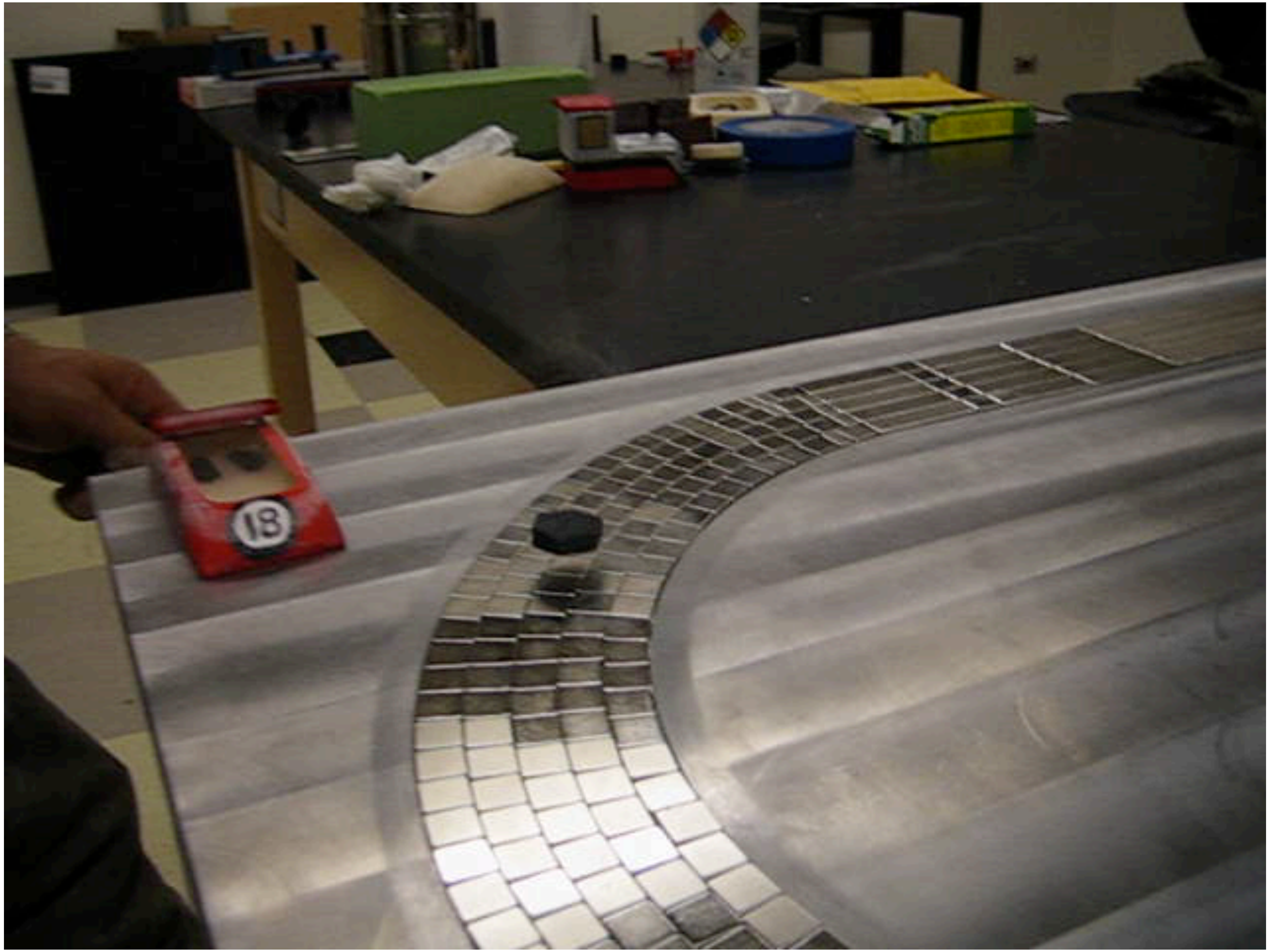


Kamerlingh Onnes 1911:
Mercury is a superconductor below -269 °C

Cuprate high temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

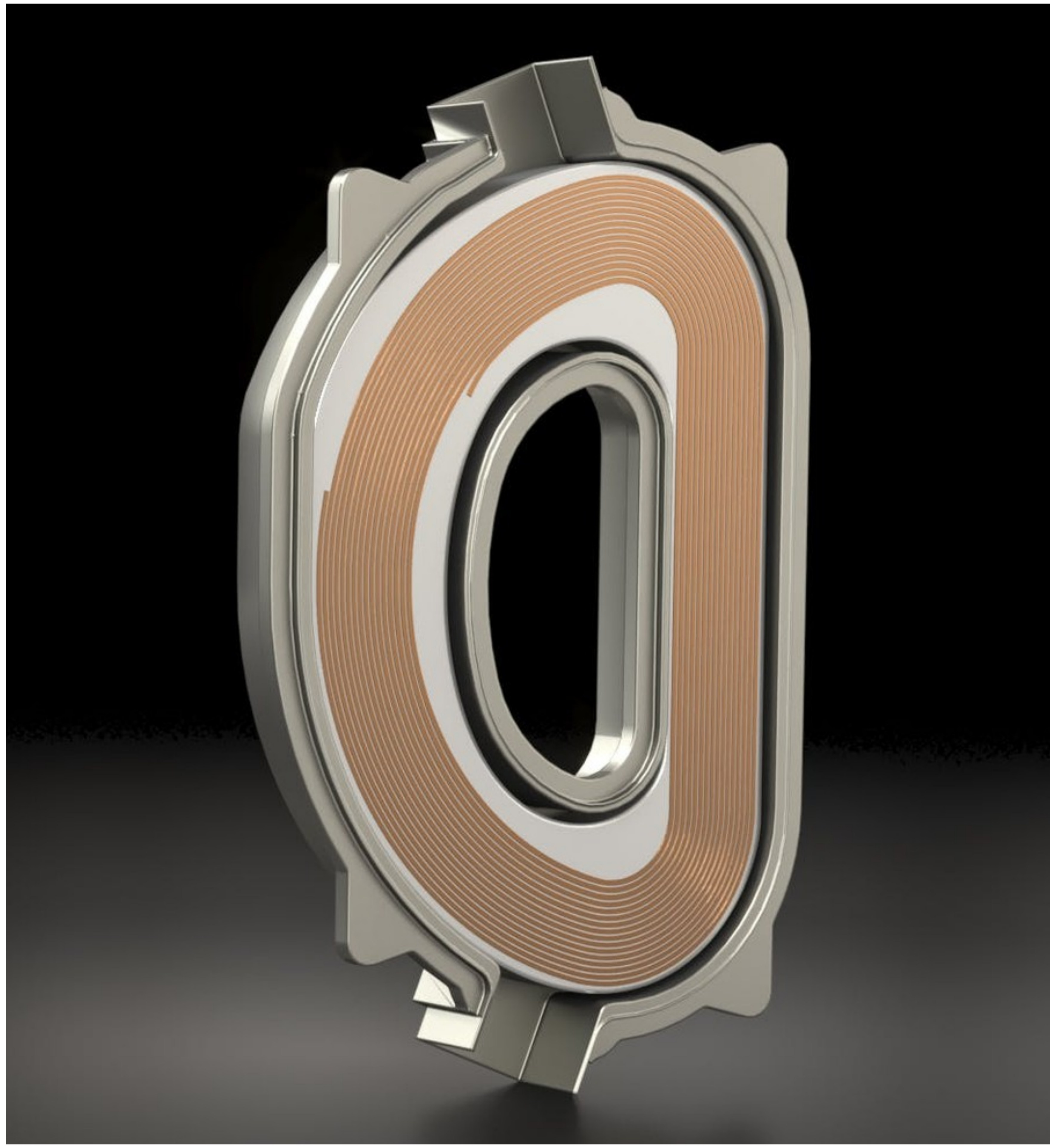
Julian Hetel and Nandini Trivedi, Ohio State University

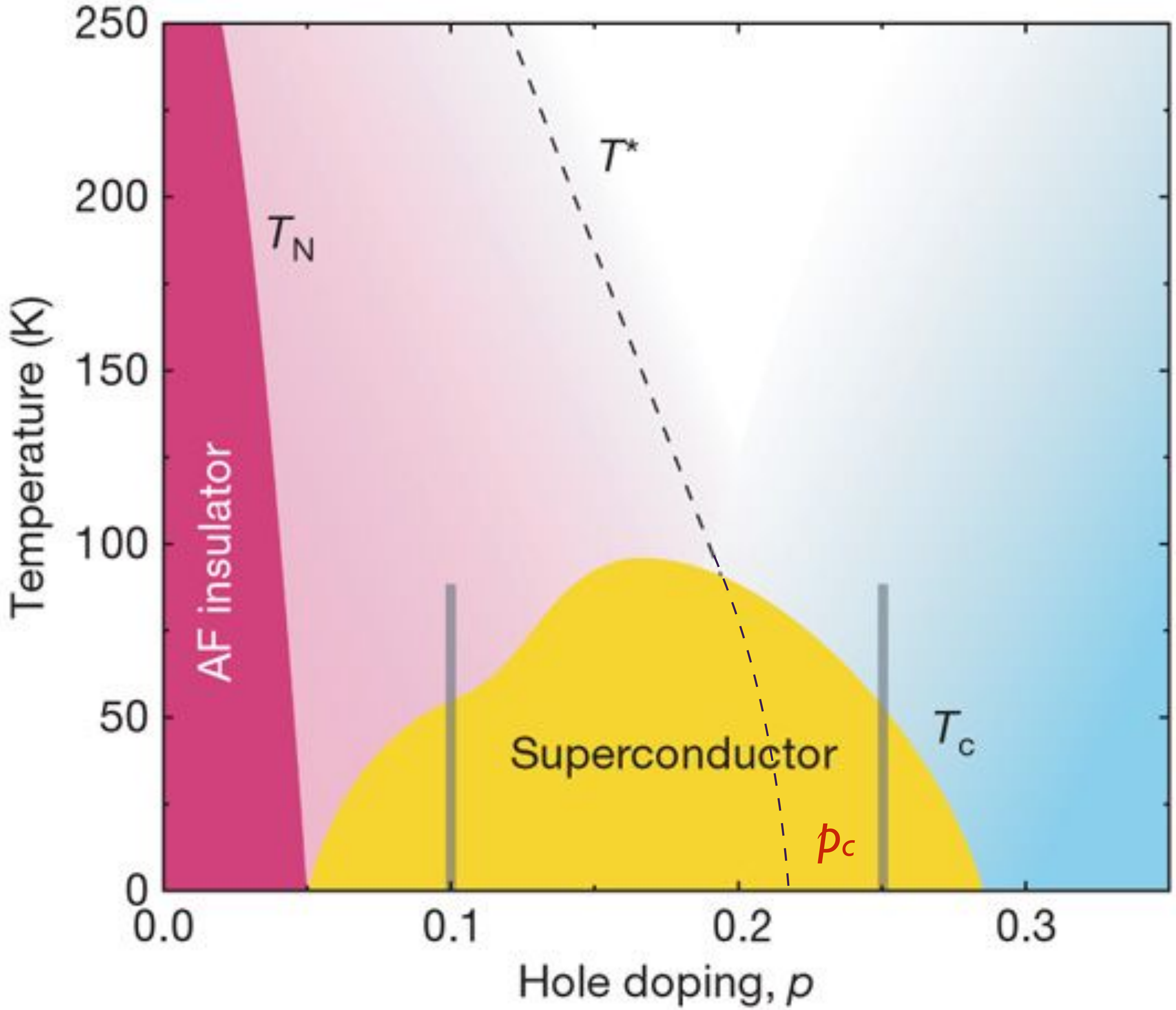
HTS Magnets: Enabling Technology

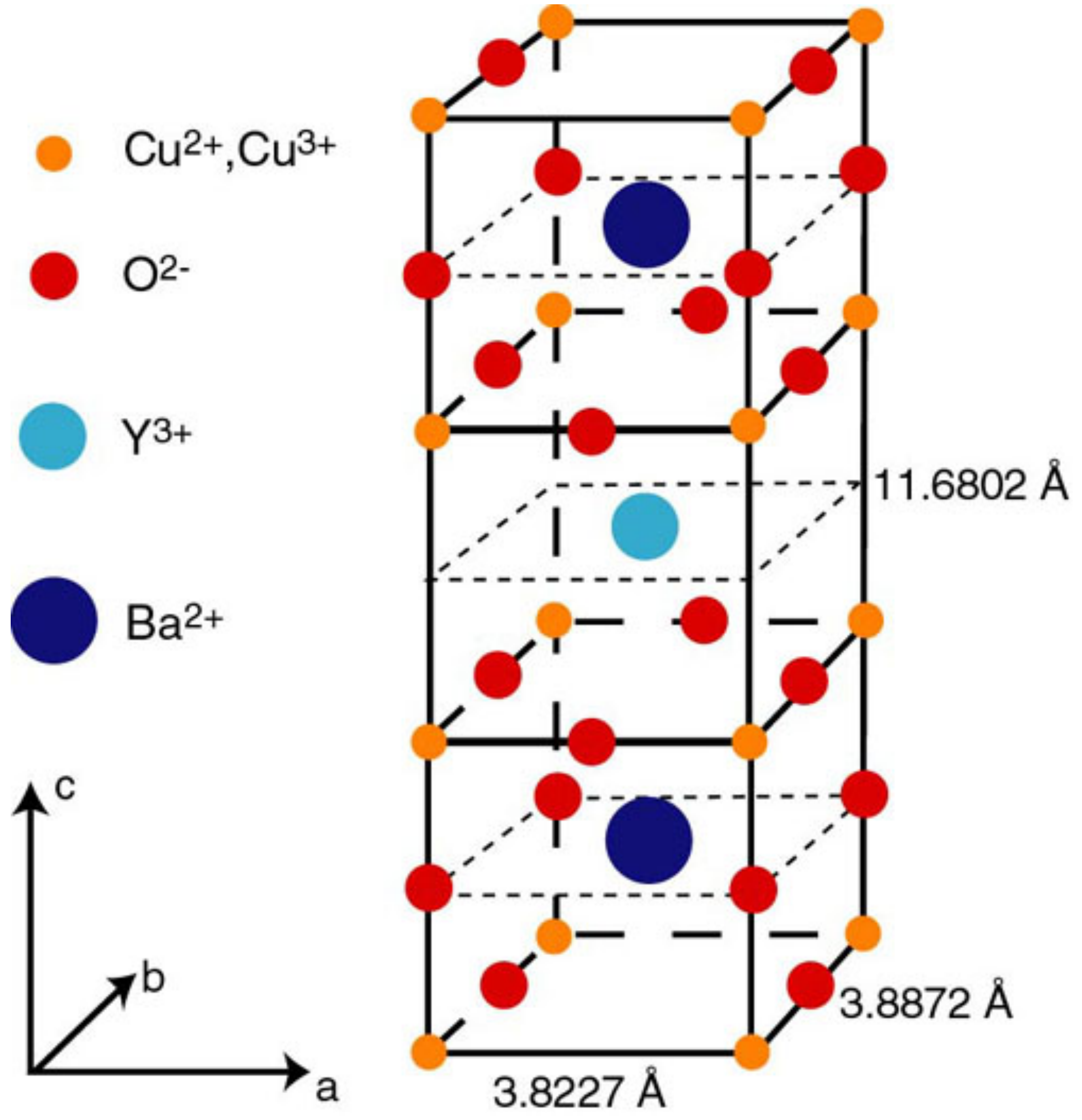
The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

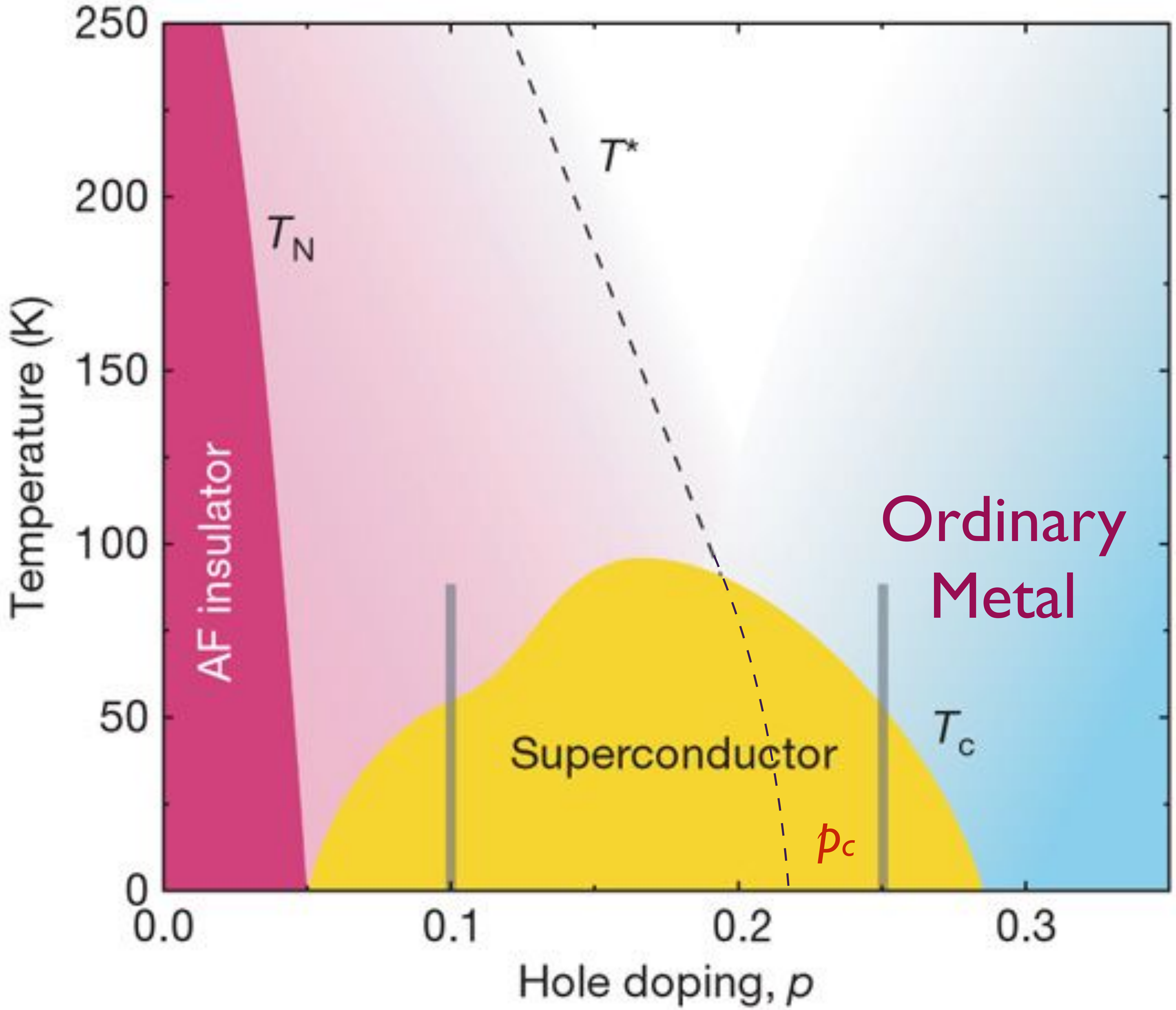
Commonwealth Fusion Systems




250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Ordinary Metal
Superconductor
T_c
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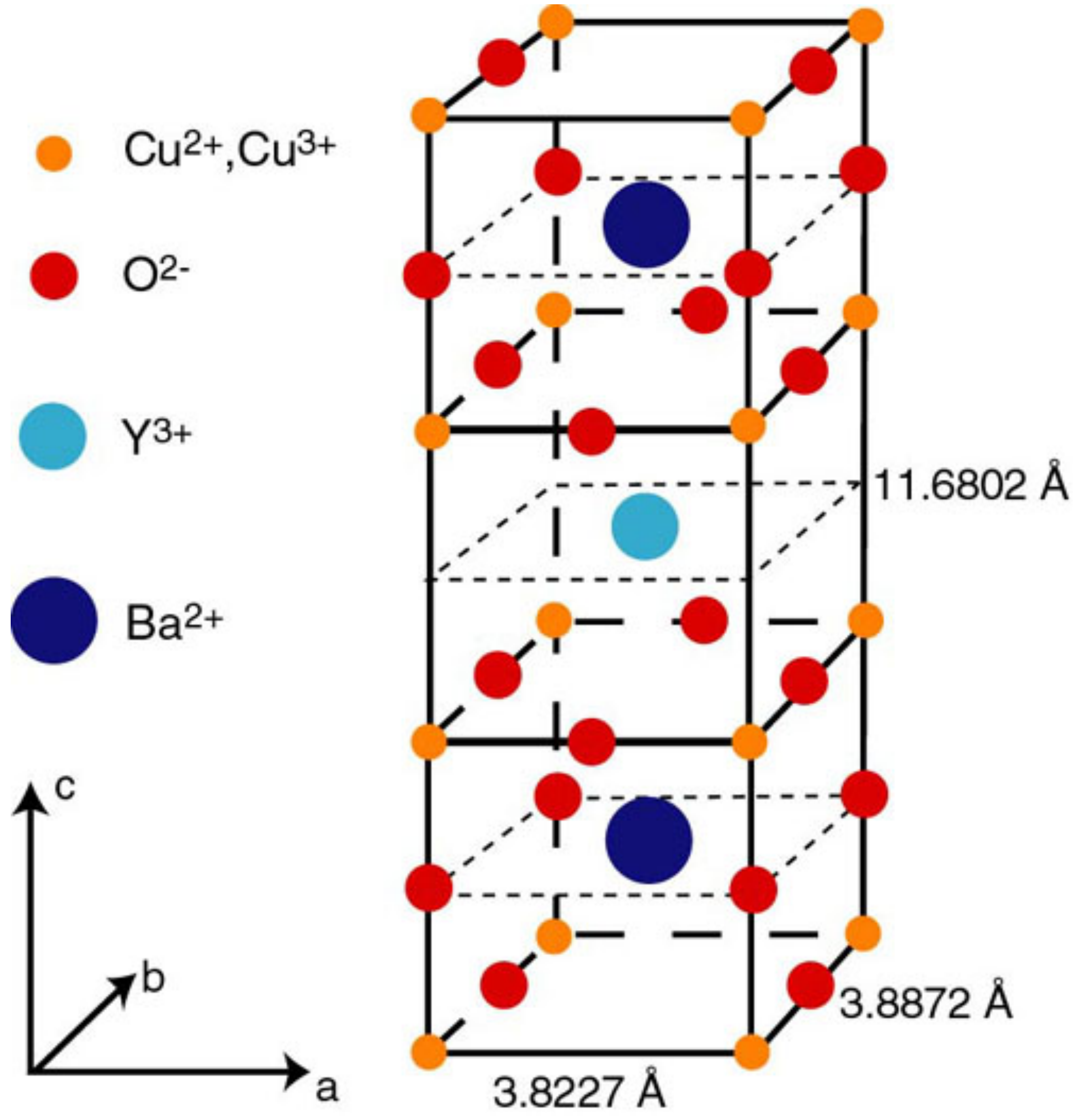
Cu^{2+},Cu^{3+}
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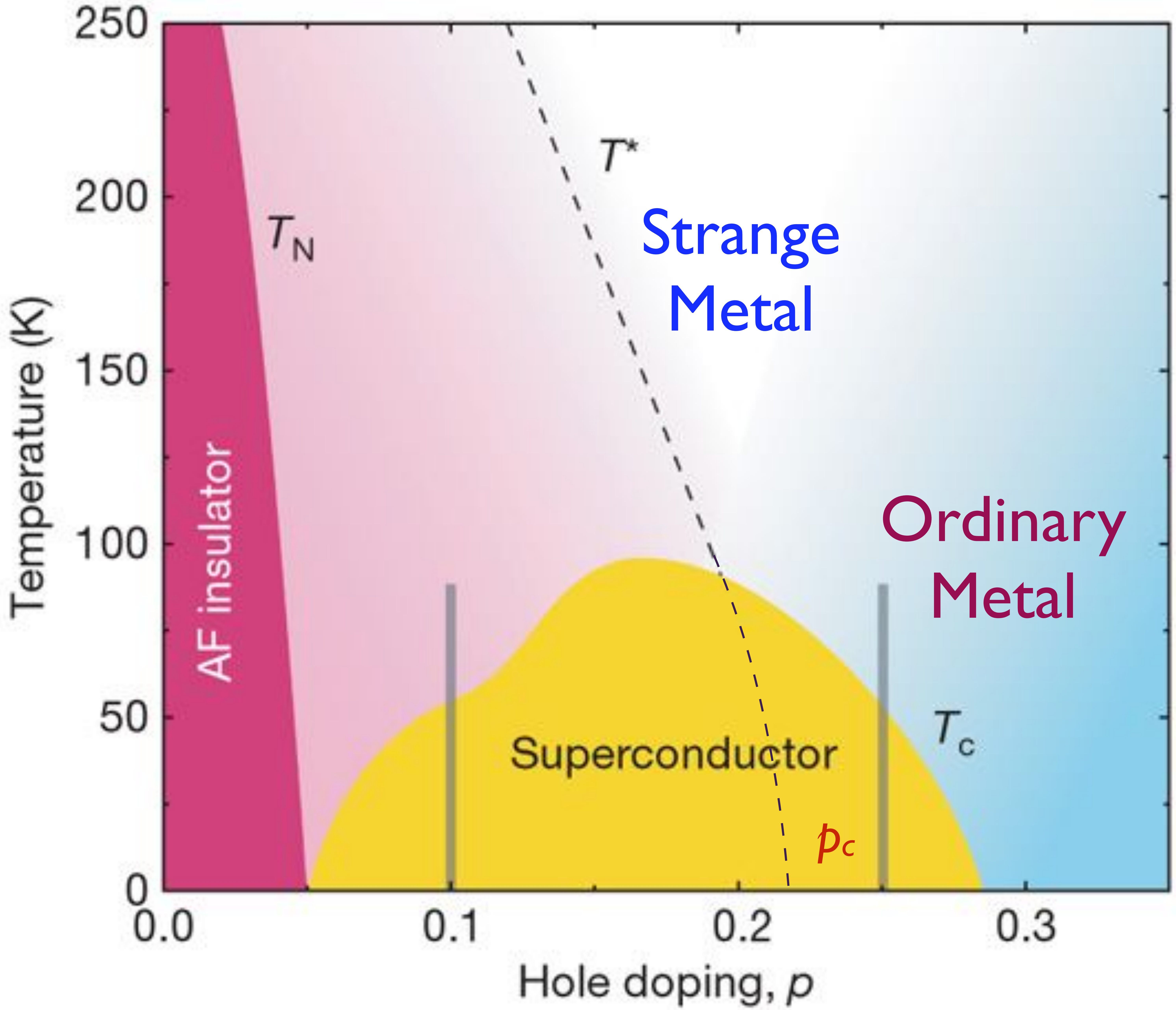
250
200
150
100
50
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0.0
0.1
0.2
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Temperature (K)
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AF insulator
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Strange Metal
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Superconductor
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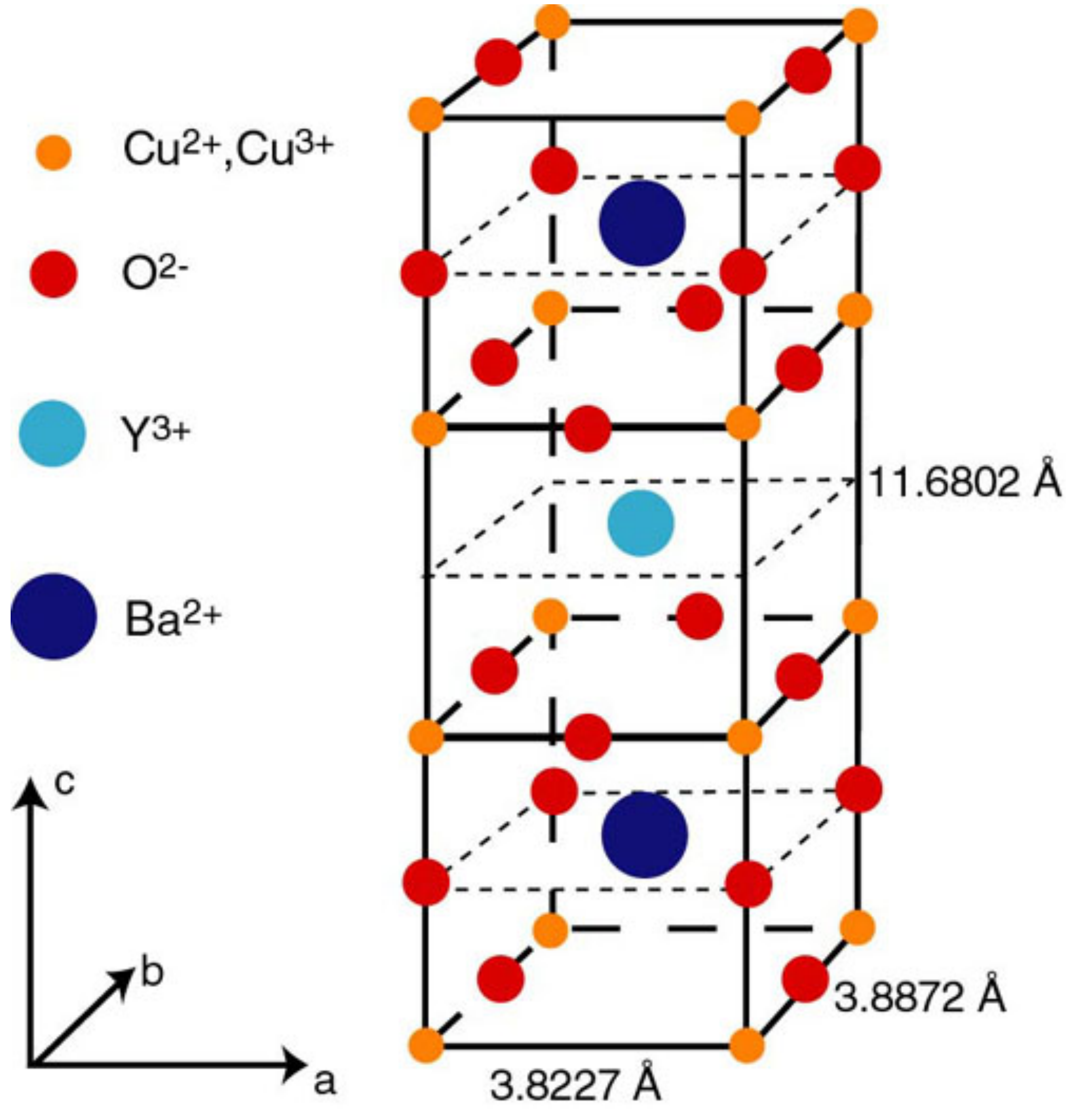
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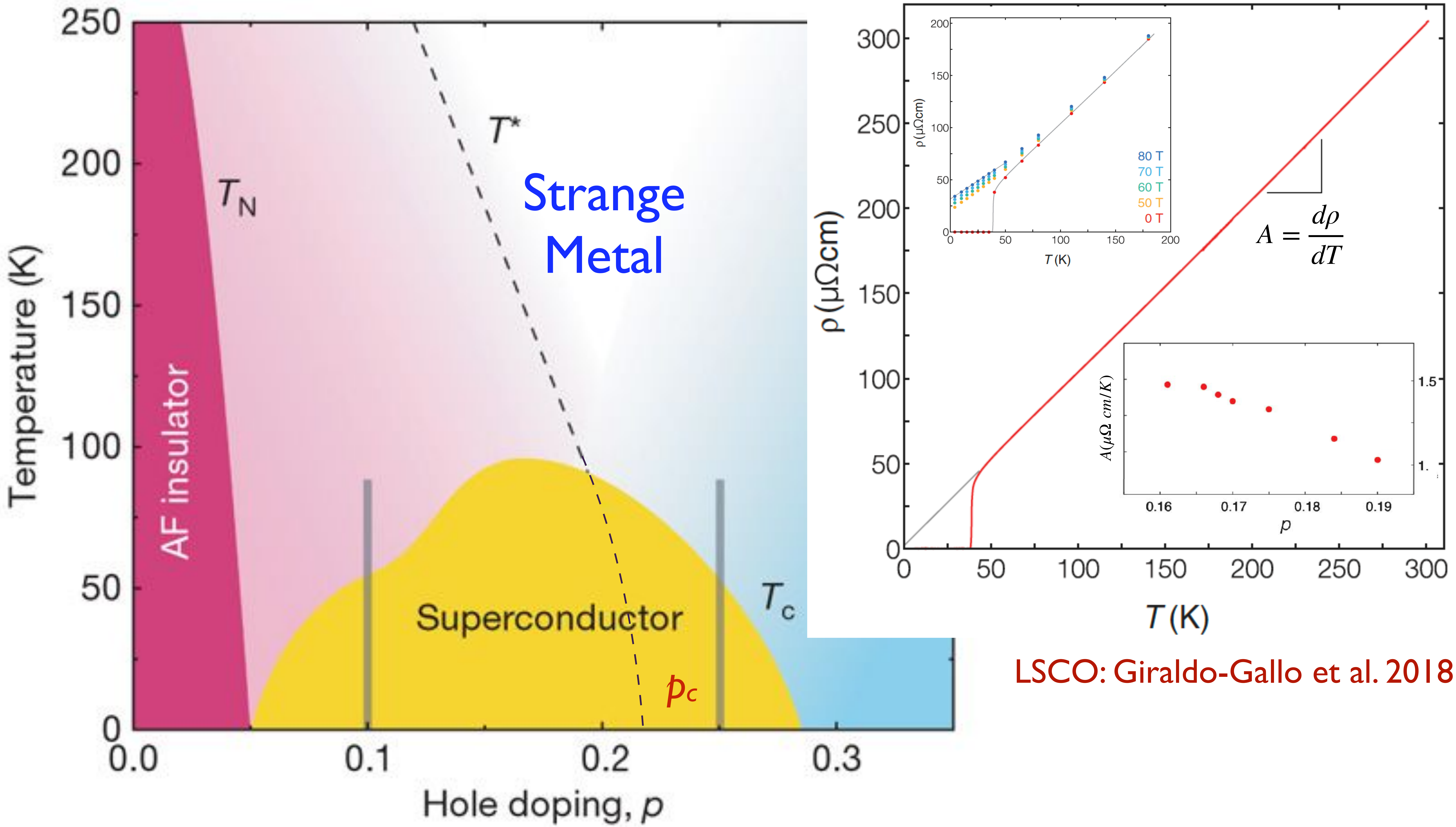


LSCO: Giraldo-Gallo et al. 2018

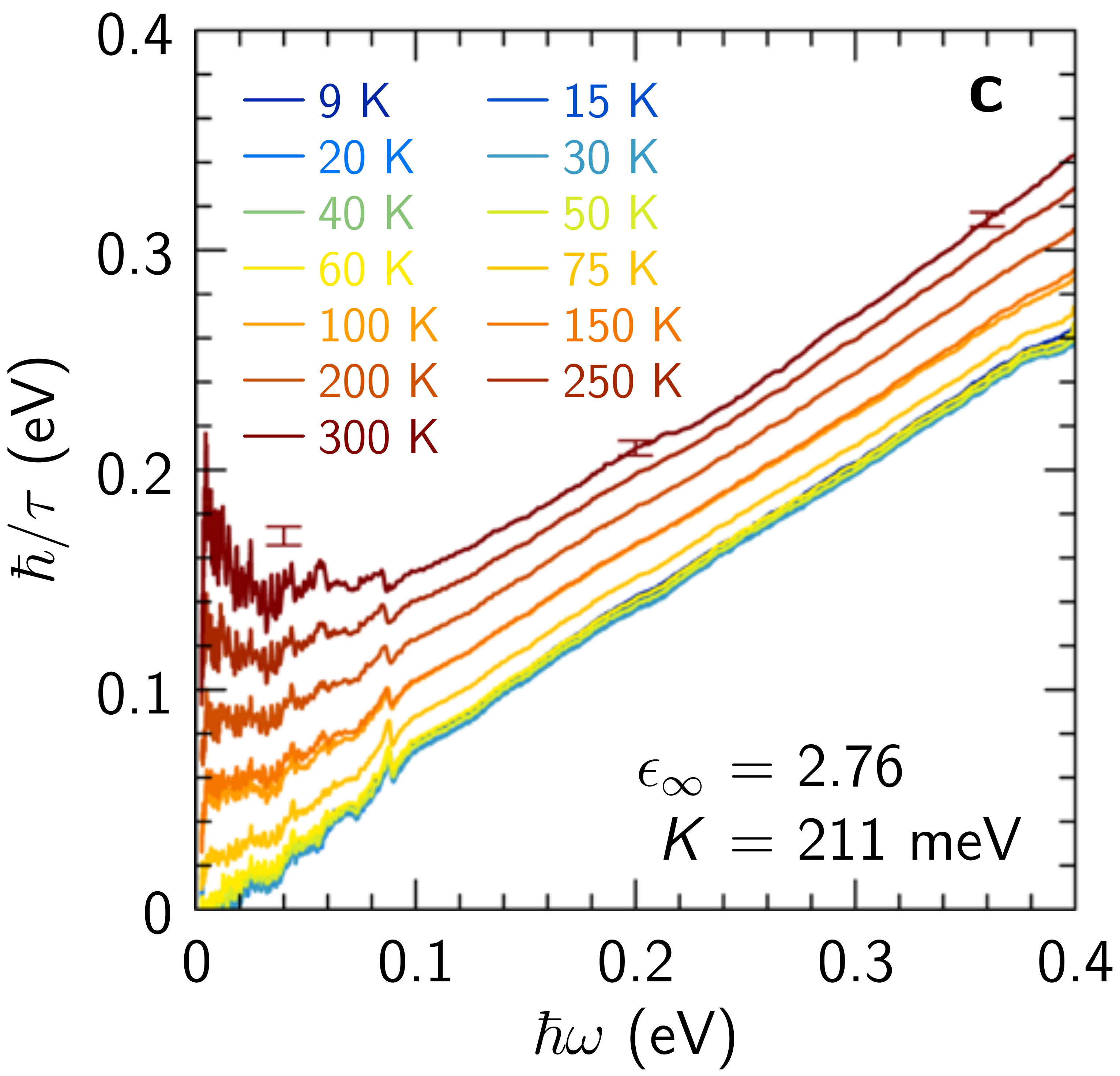
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

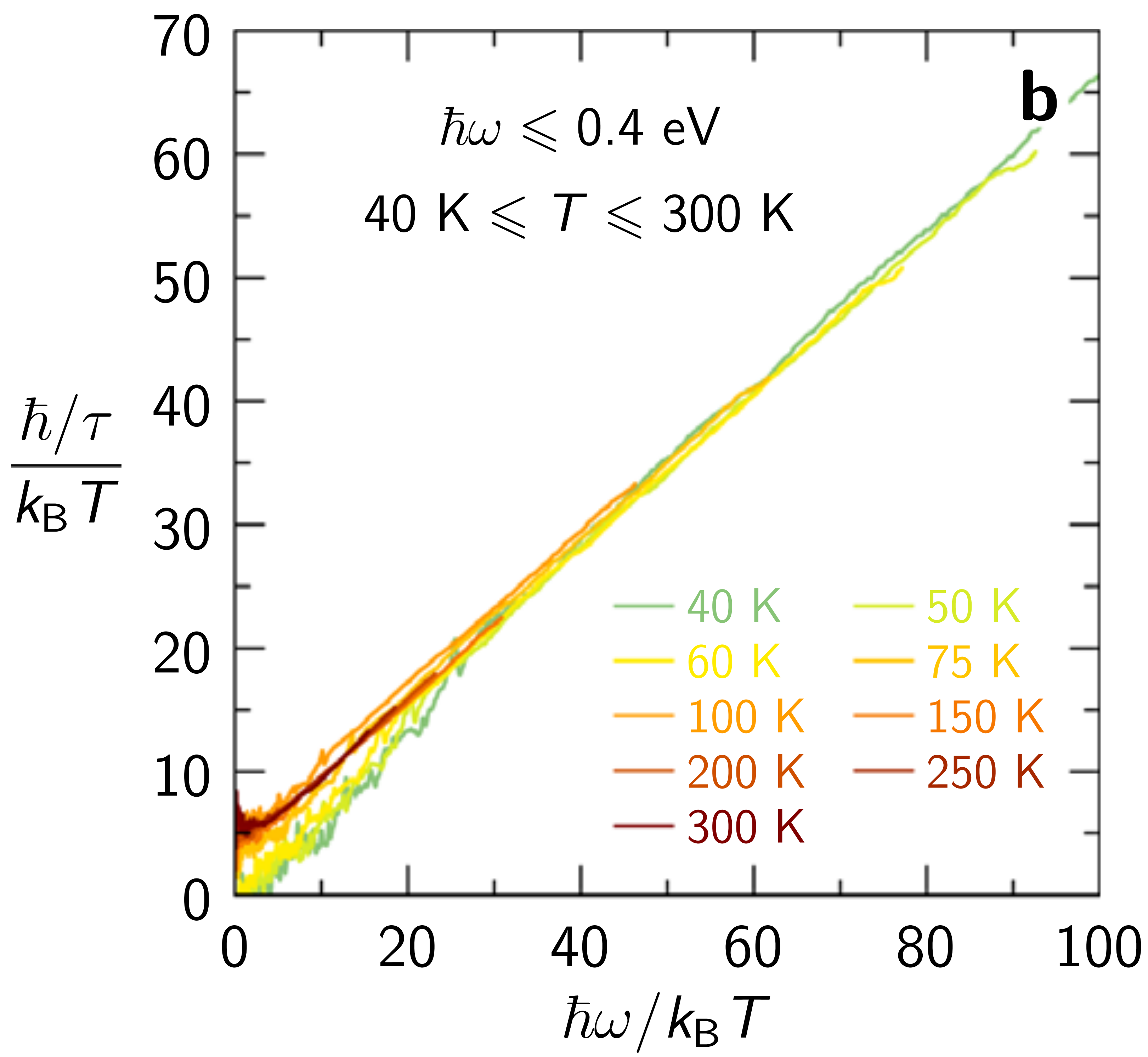
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!



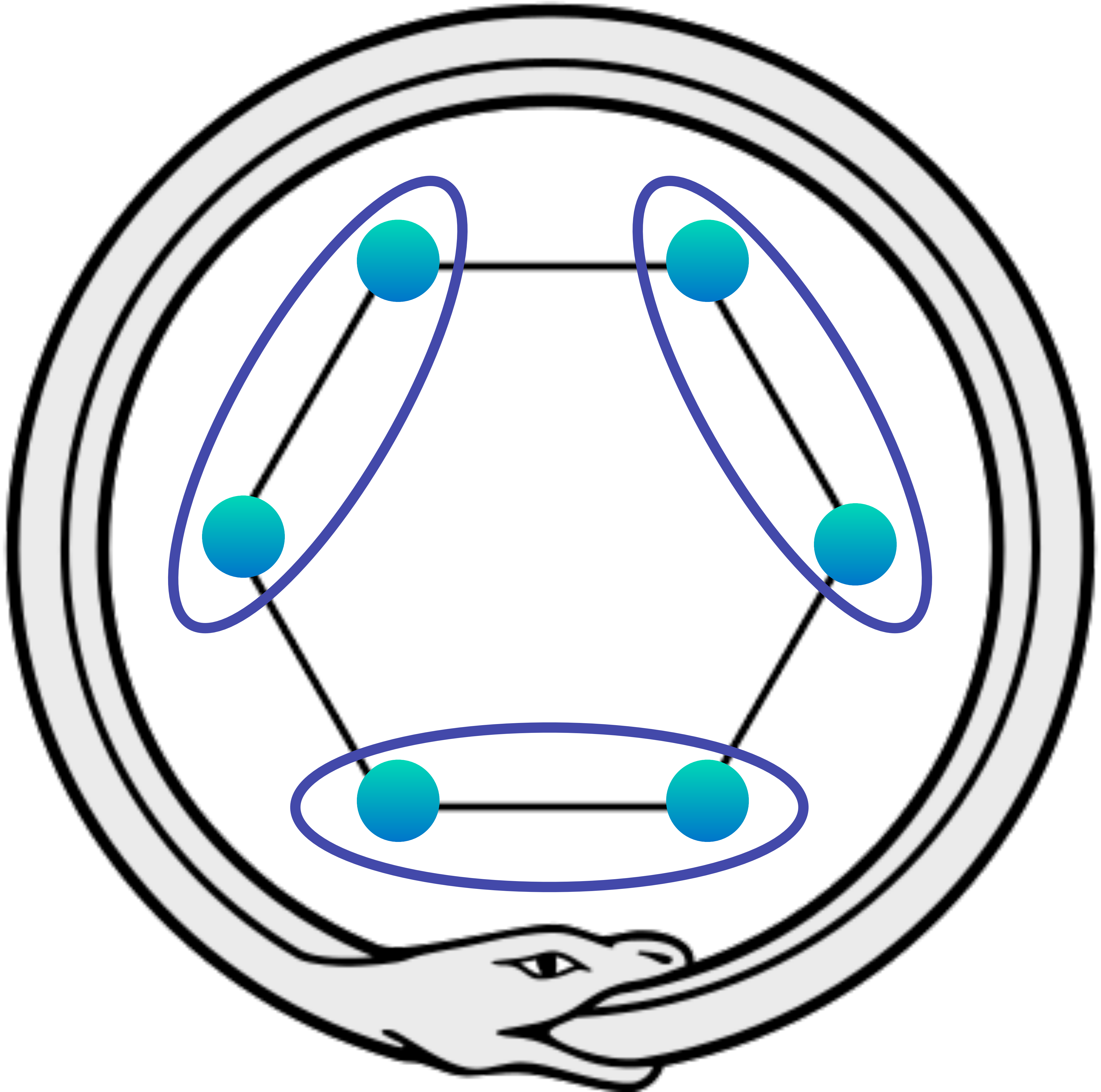


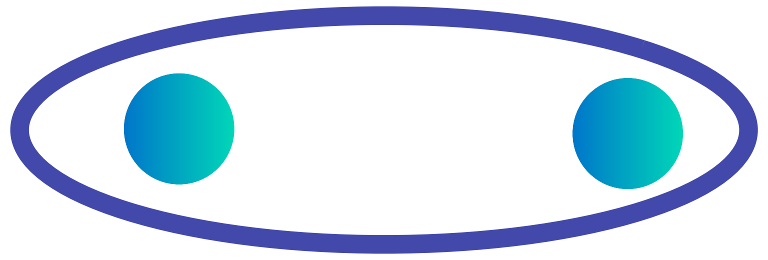
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

Quantum entanglement (1865)

Kekule’s spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail[*]





$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

[*]Wikipedia

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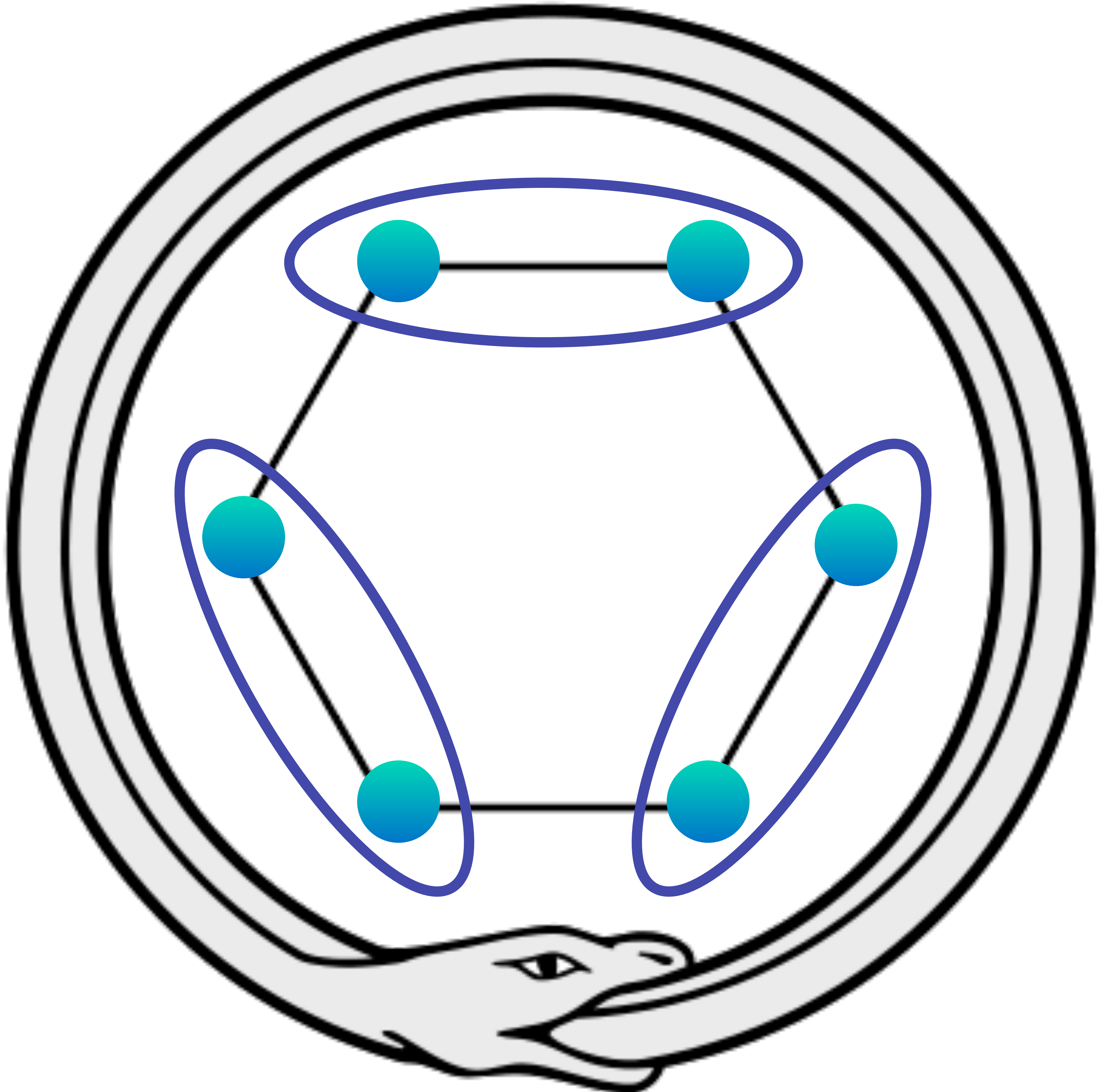
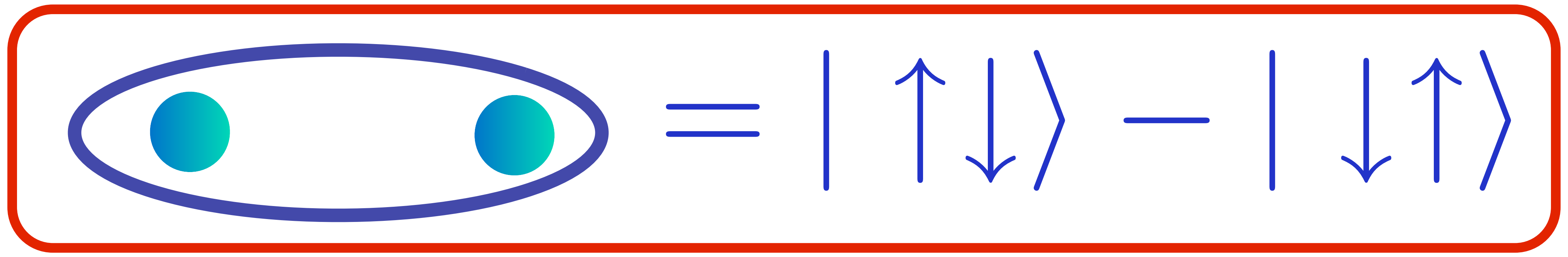




Benzene

*Wikipedia

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

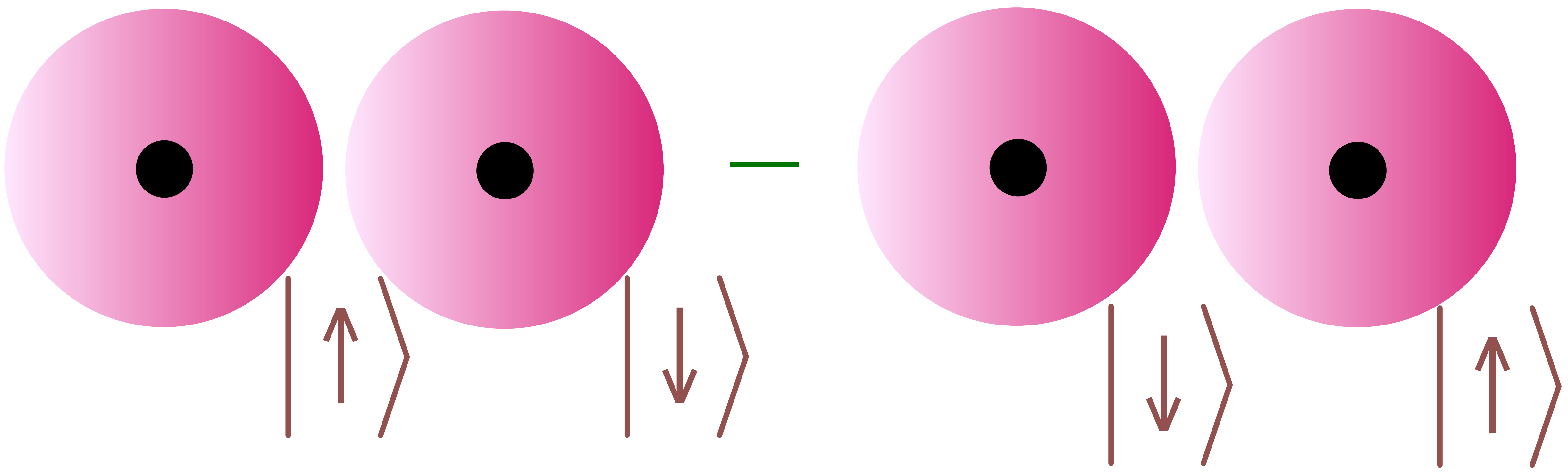
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
daß es wirklich mit der Theorie

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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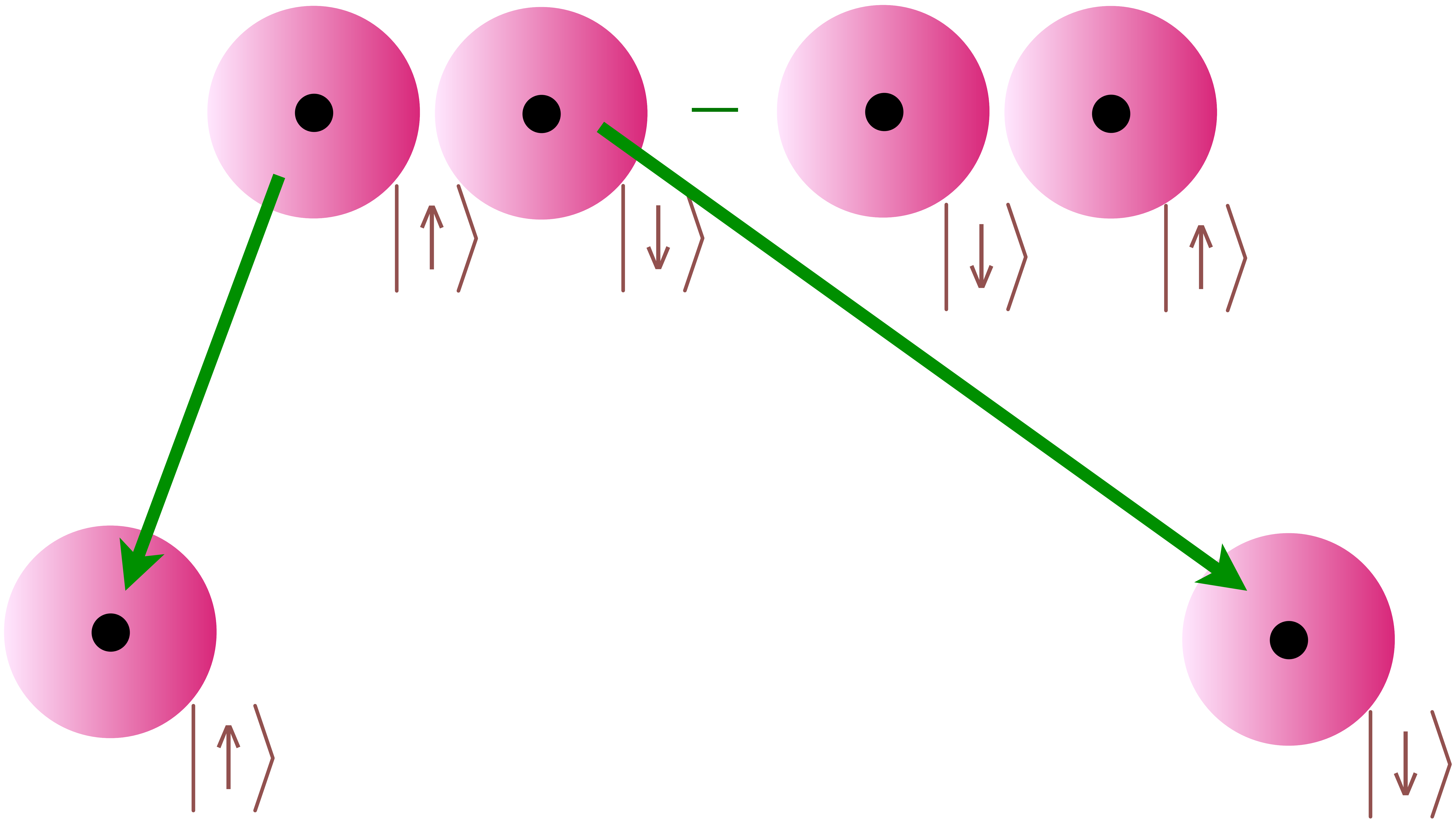

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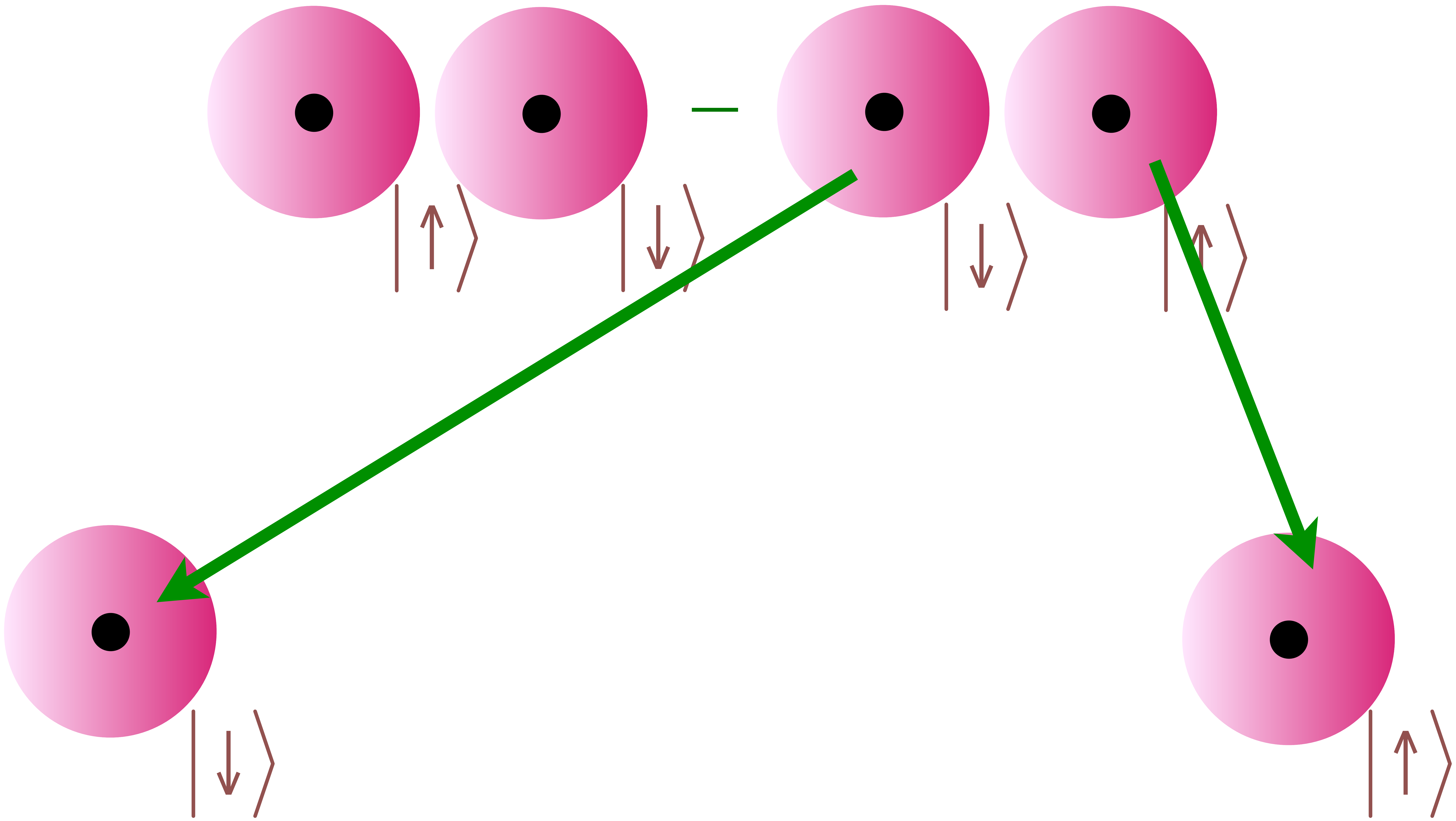

Quantum Entanglement

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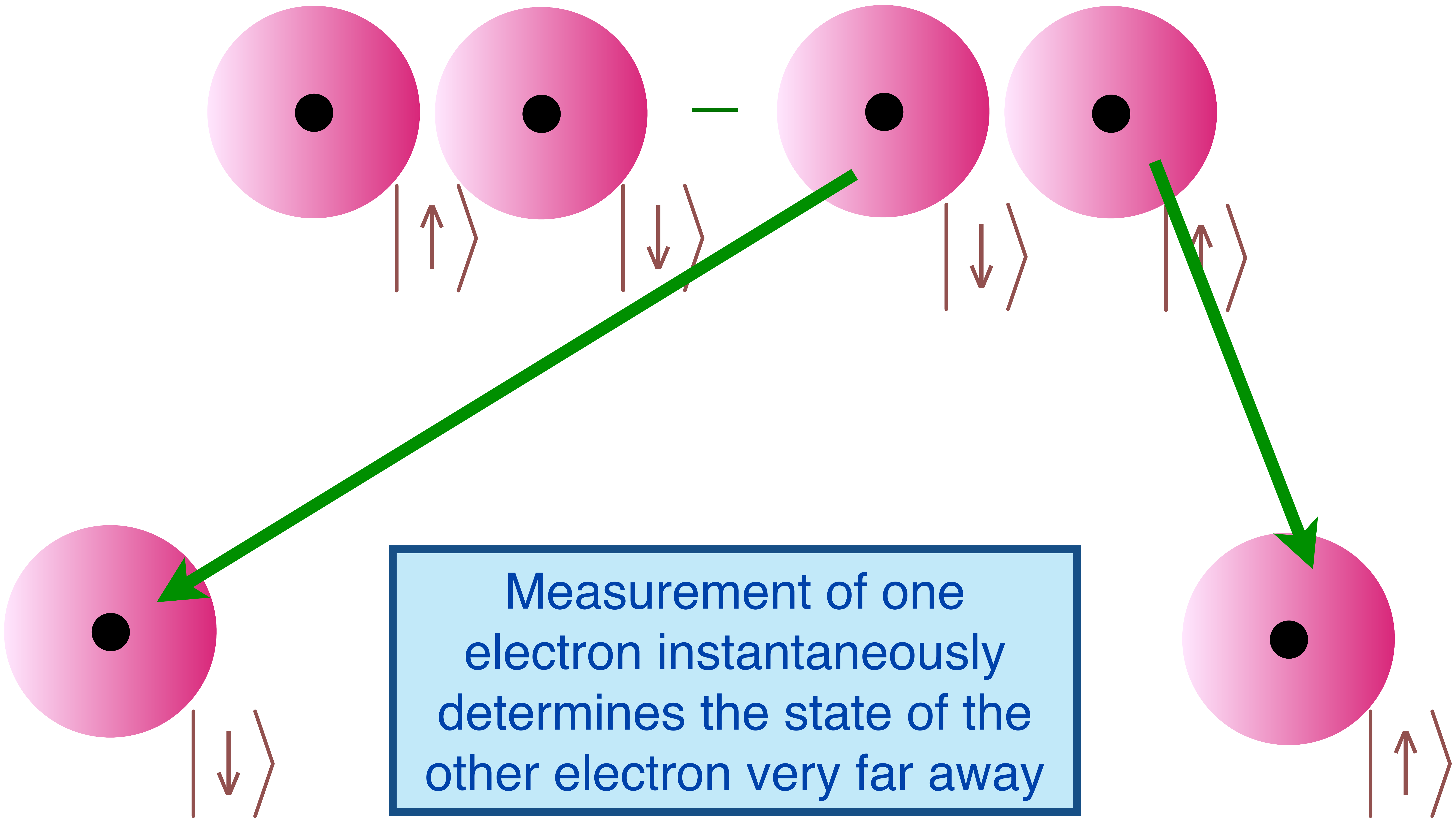
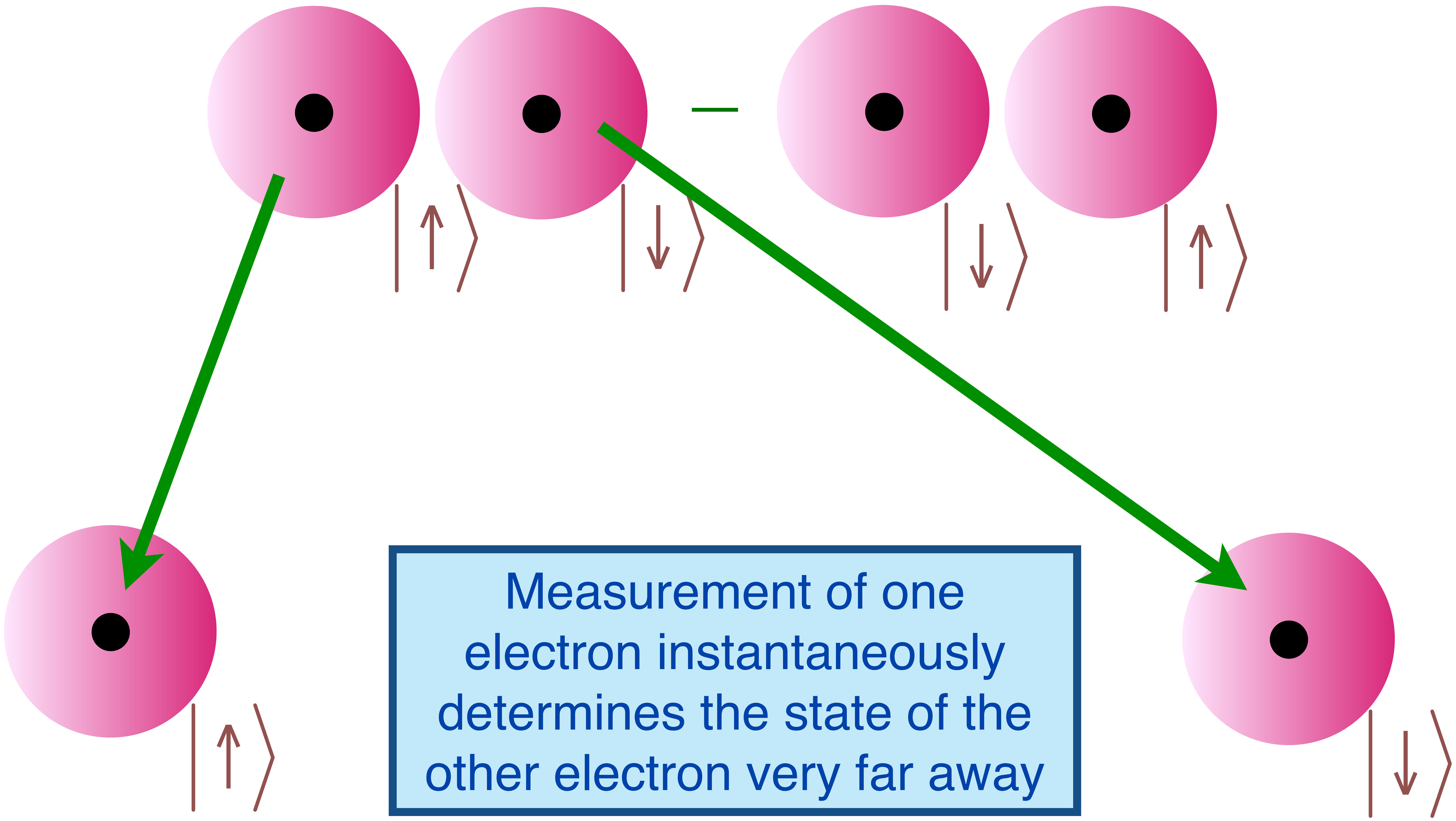


Measurement of one electron instantaneously determines the state of the other electron very far away

Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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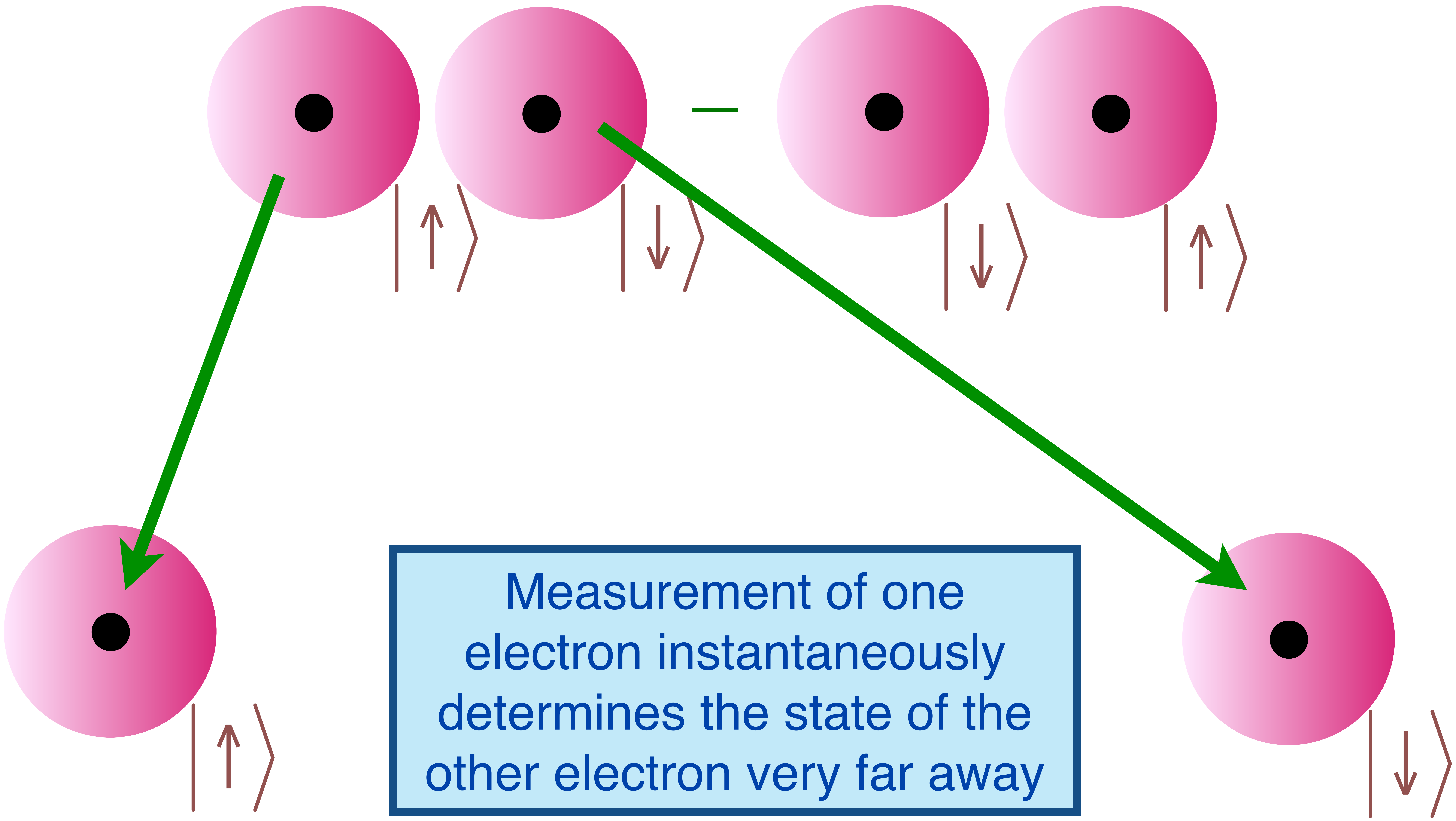

Spooky action at a distance !

Needed,
to solve open problems in the theory of superconductivity and black holes:

A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

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A solvable model of quantum entanglement of $3, 4, 5, \ldots \infty$ particles

The Sachdev-Ye-Kitaev model of many-particle entanglement

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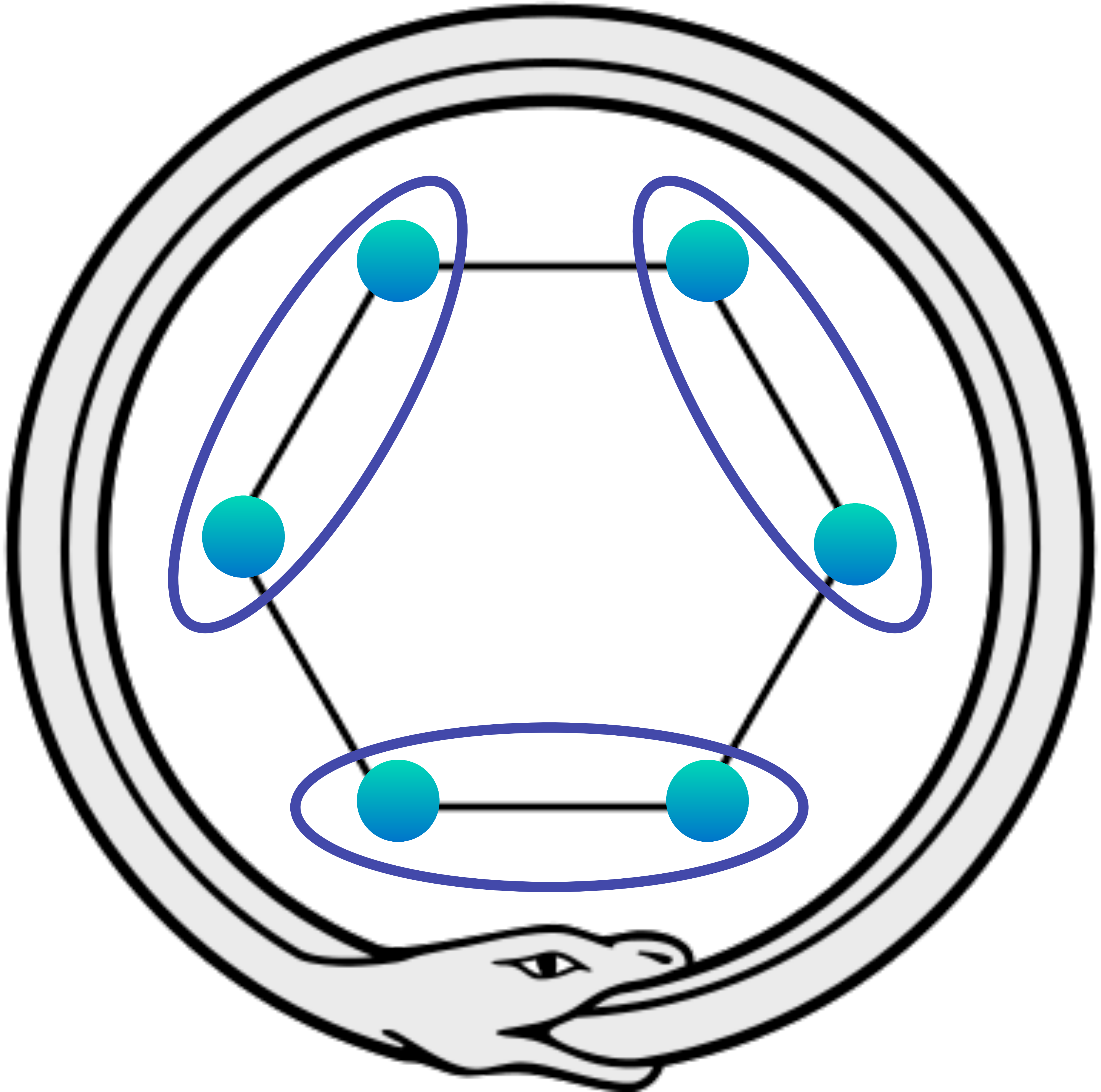
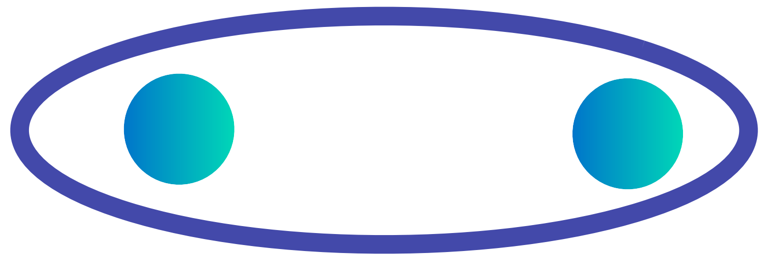
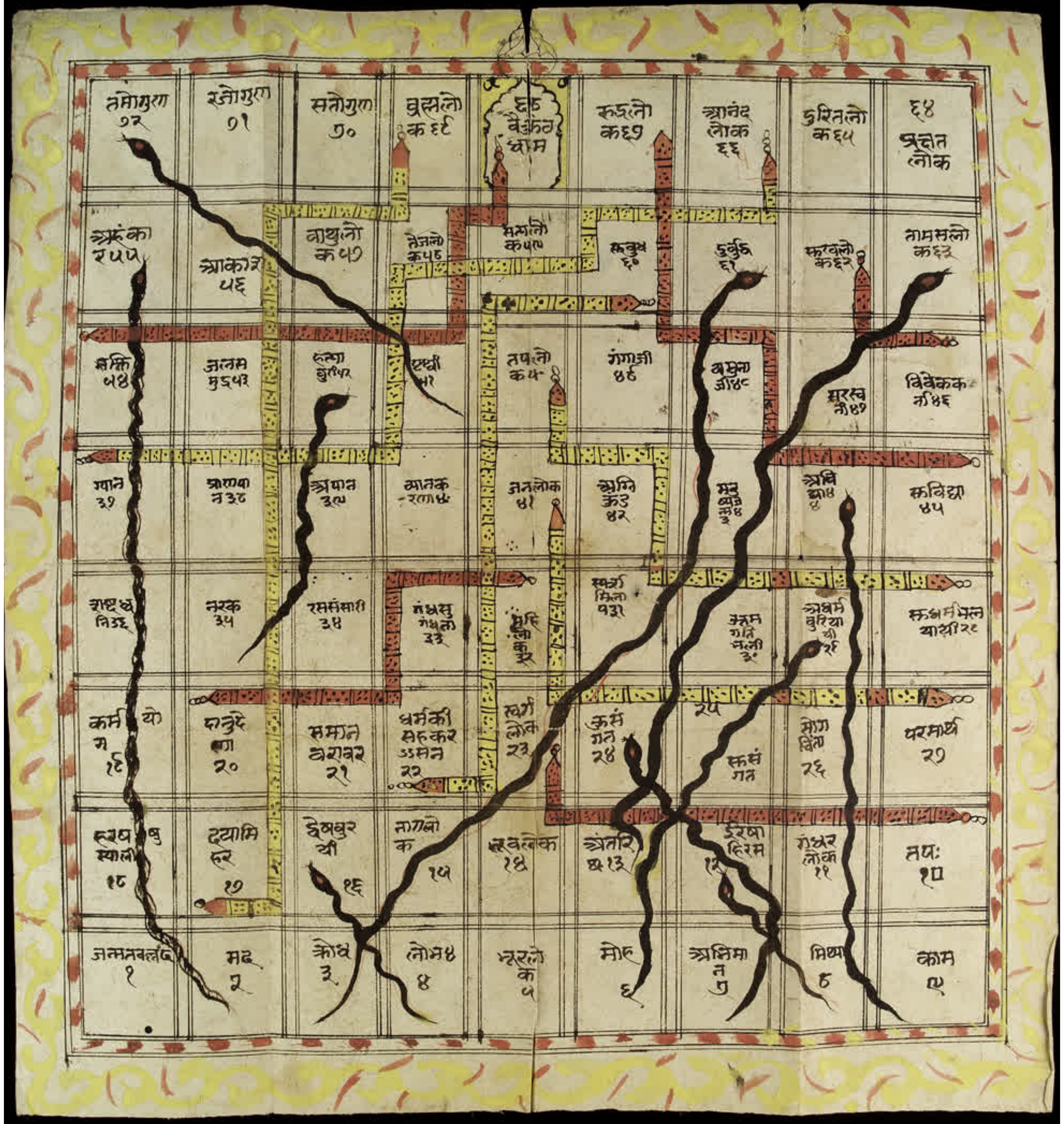


 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Benzene

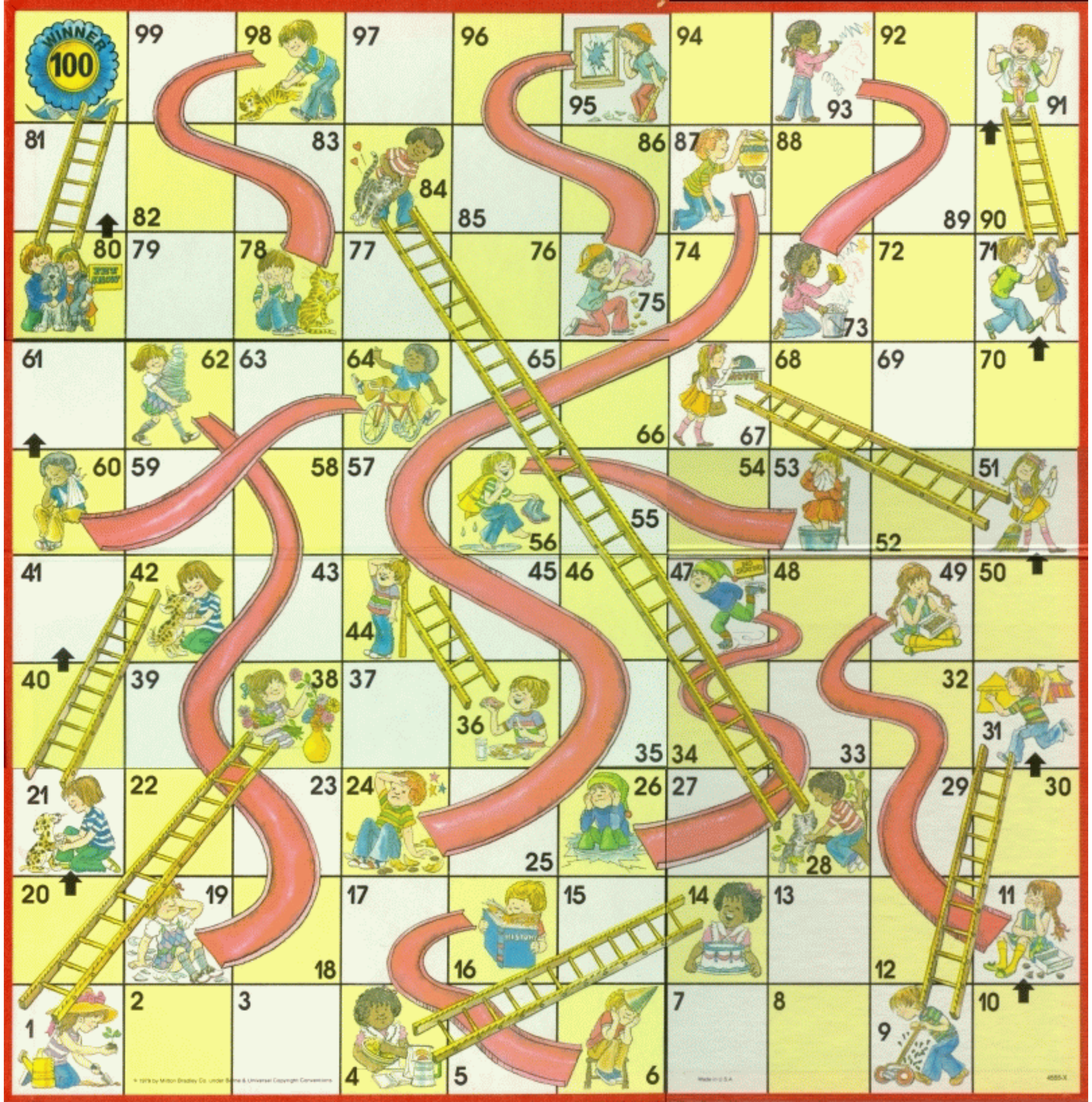
*Wikipedia



My spooky dream (1992)*

Ancient Indian game of Snakes and Ladders

*Not true



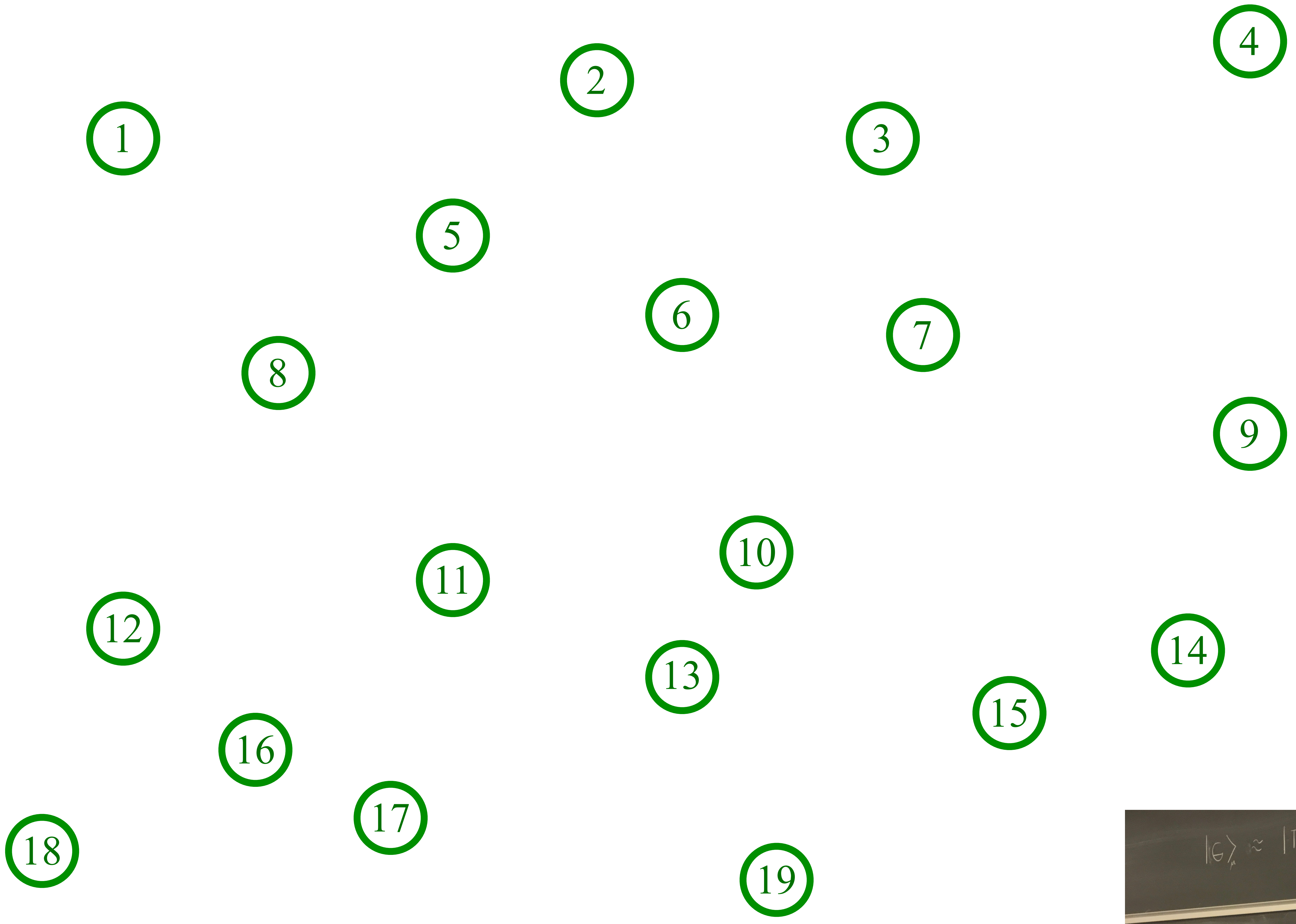
My spooky dream (1992)*

Hasbro game of Chutes and Ladders

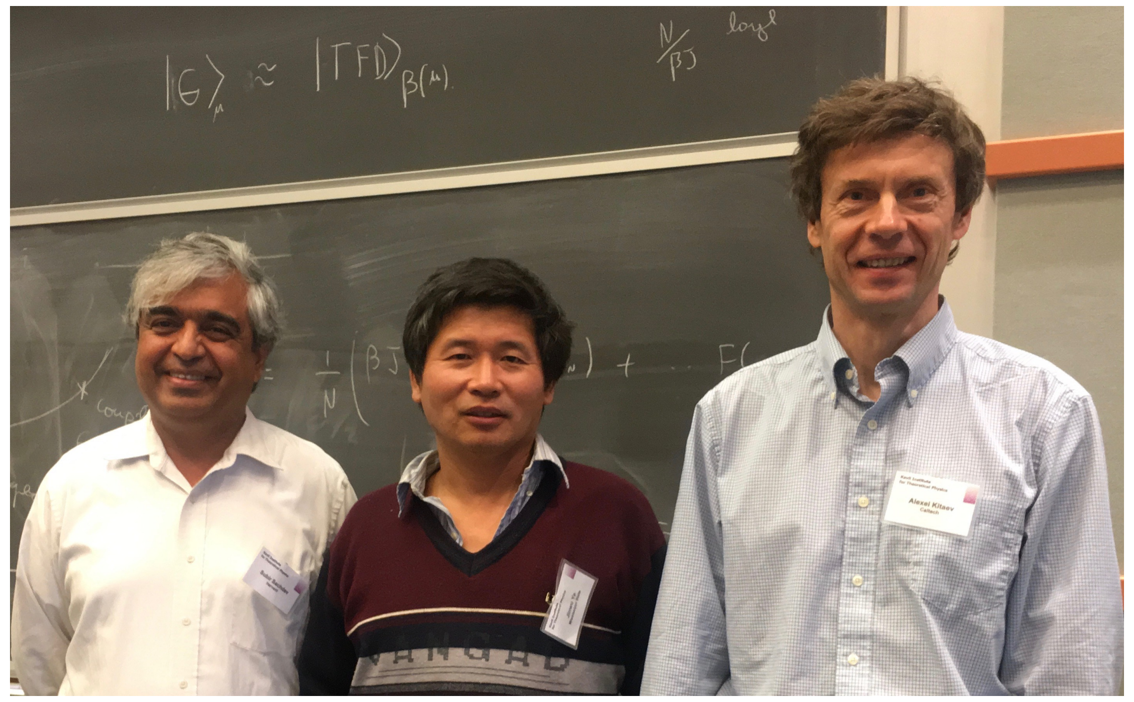
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



Pick a set of random positions



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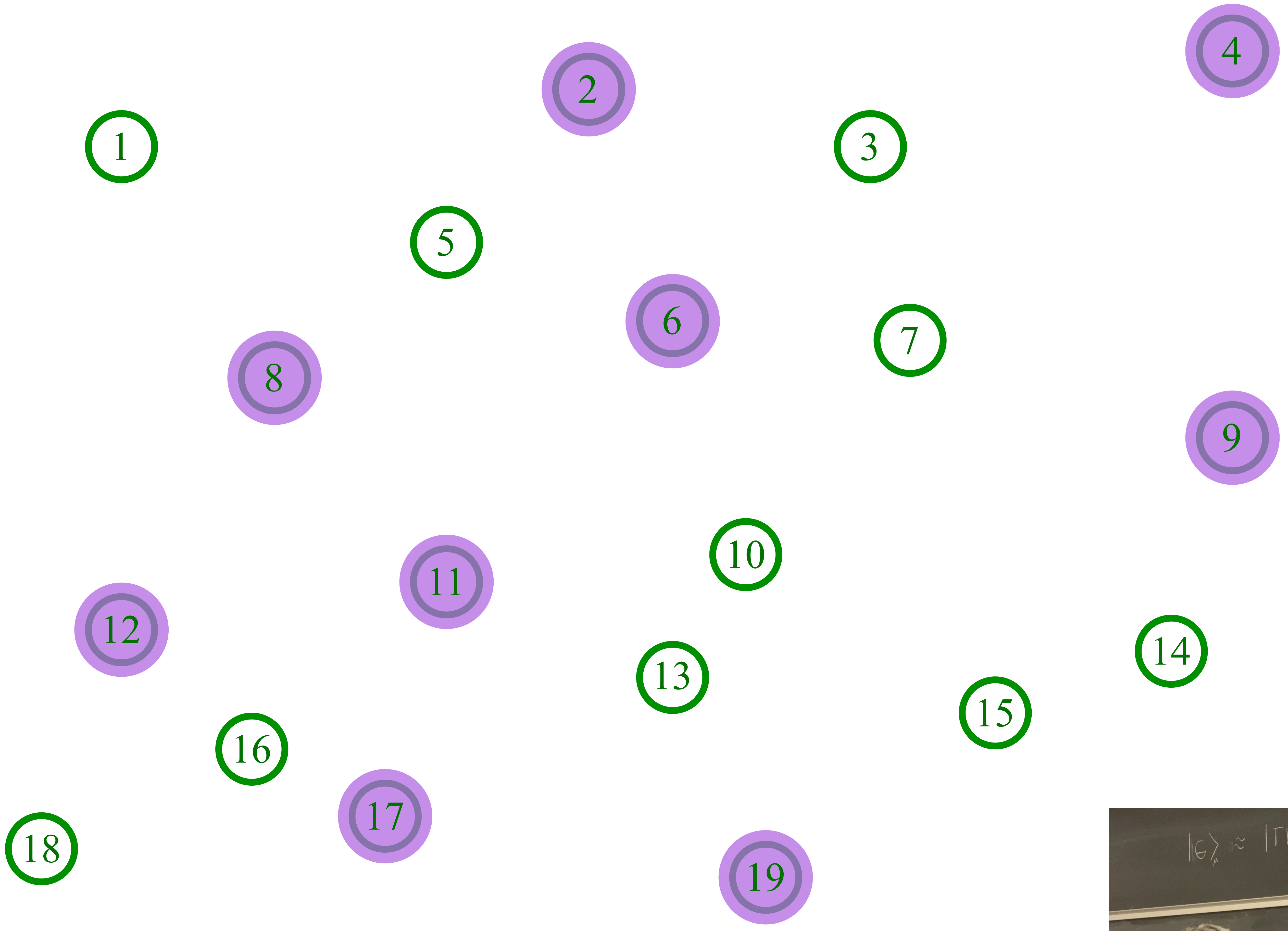
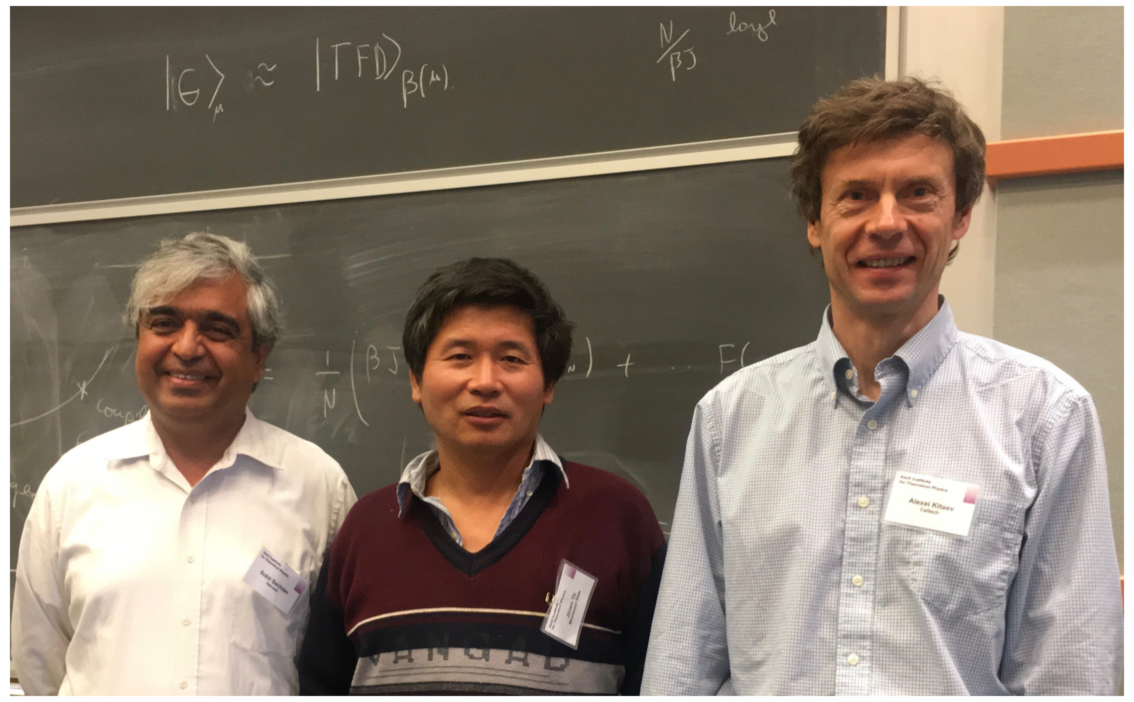
Sachdev, Ye (1993); Kitaev (2015)



Place electrons randomly on some sites



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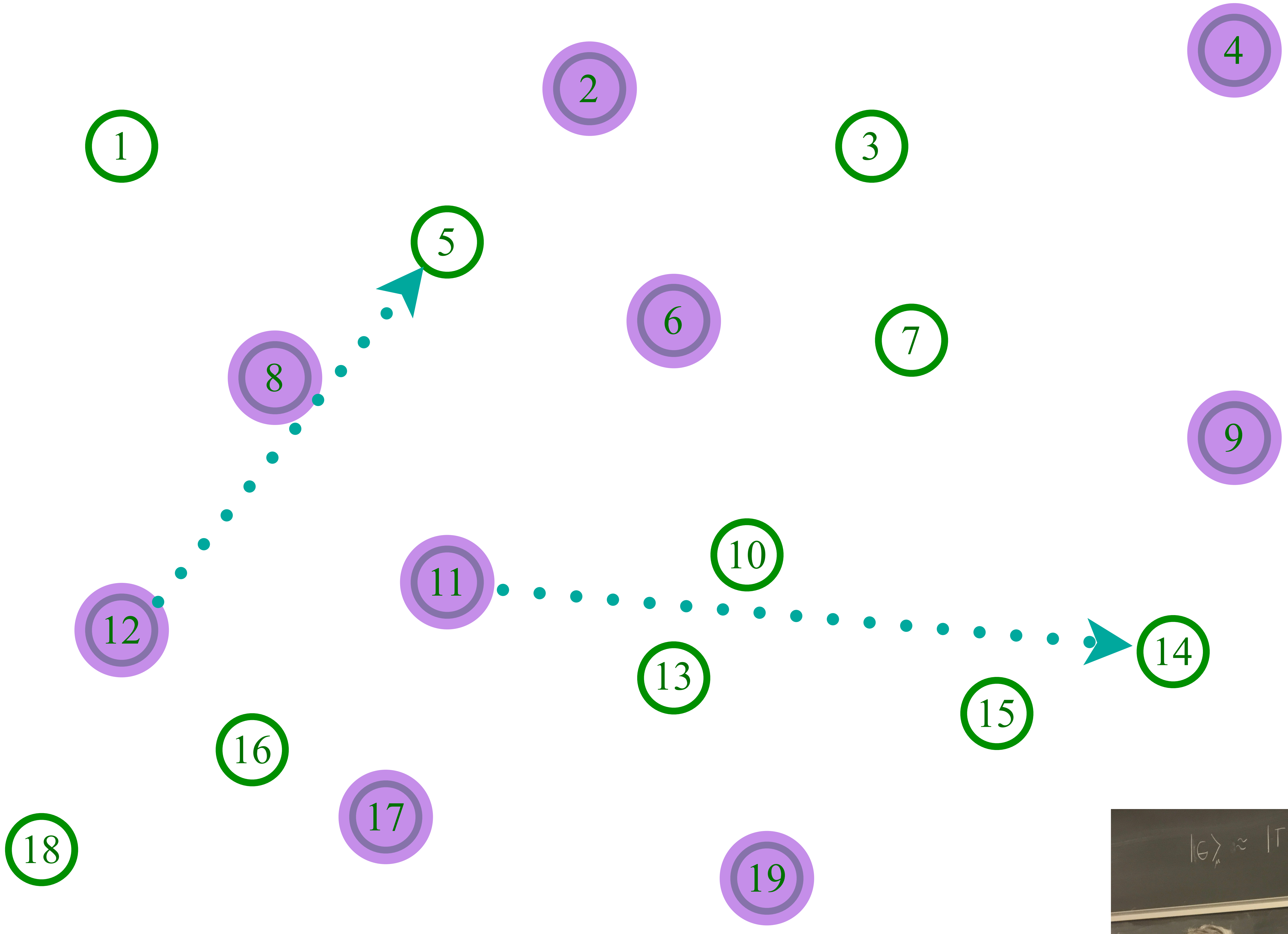
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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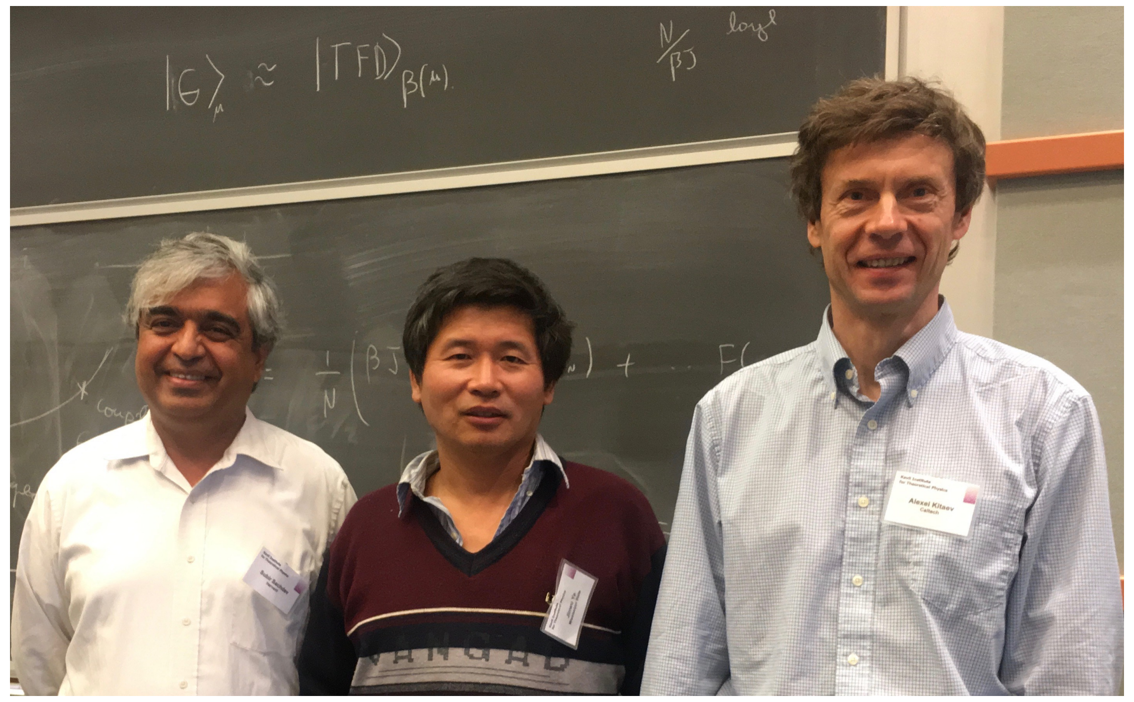

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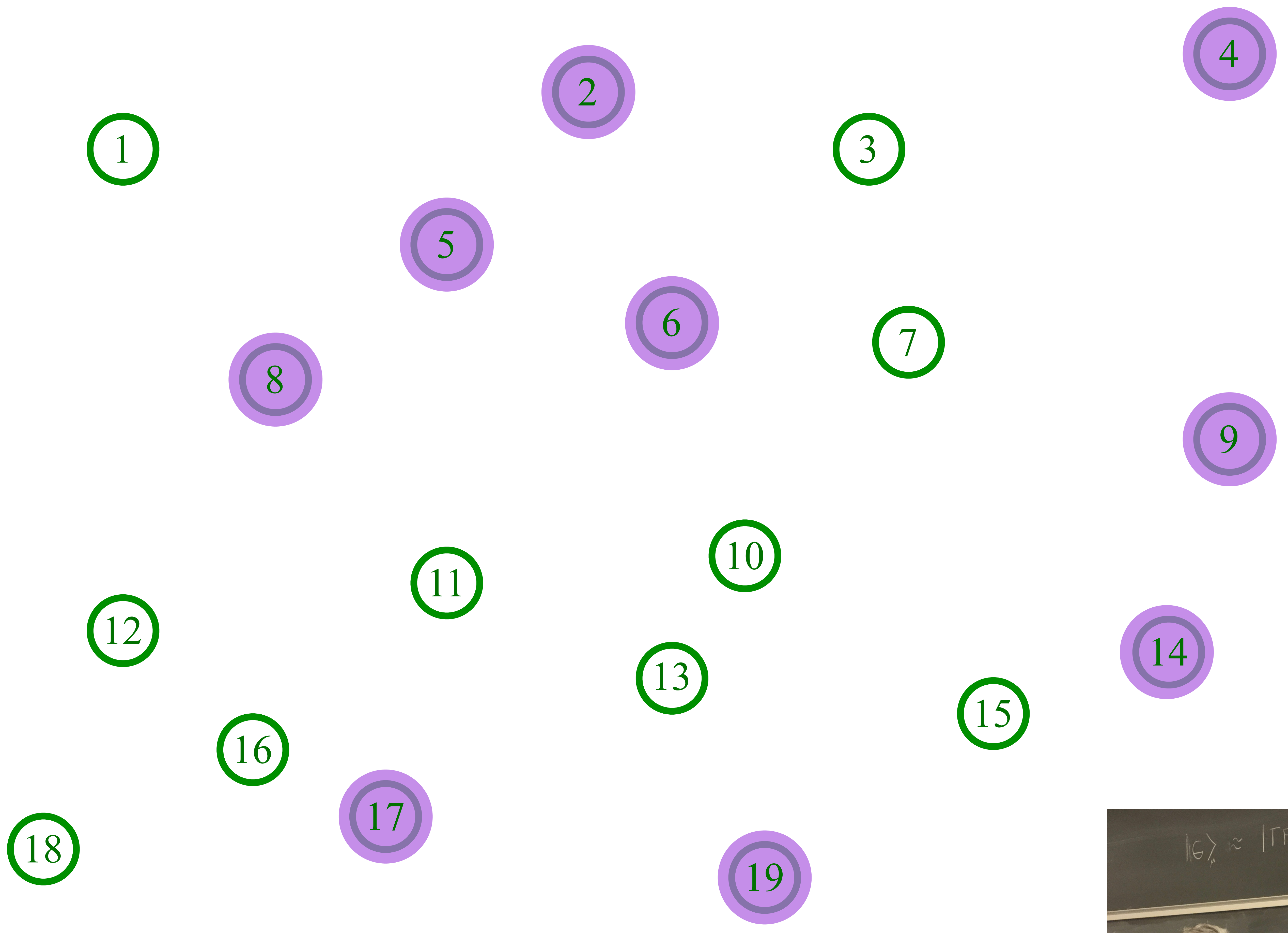
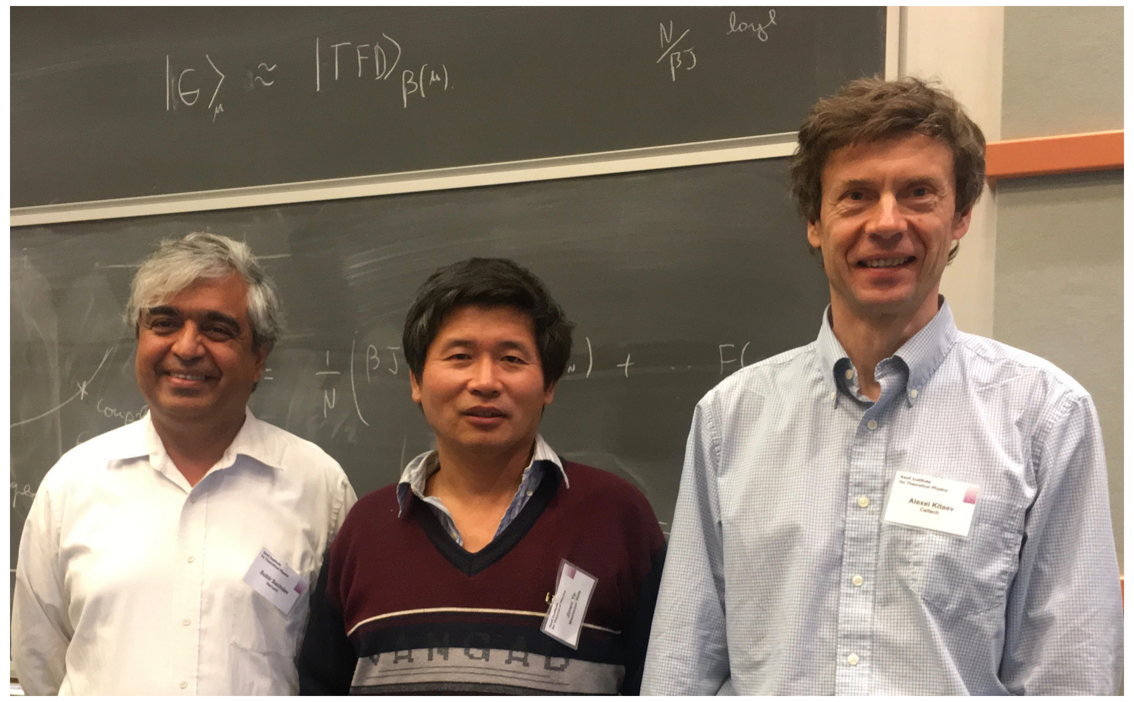


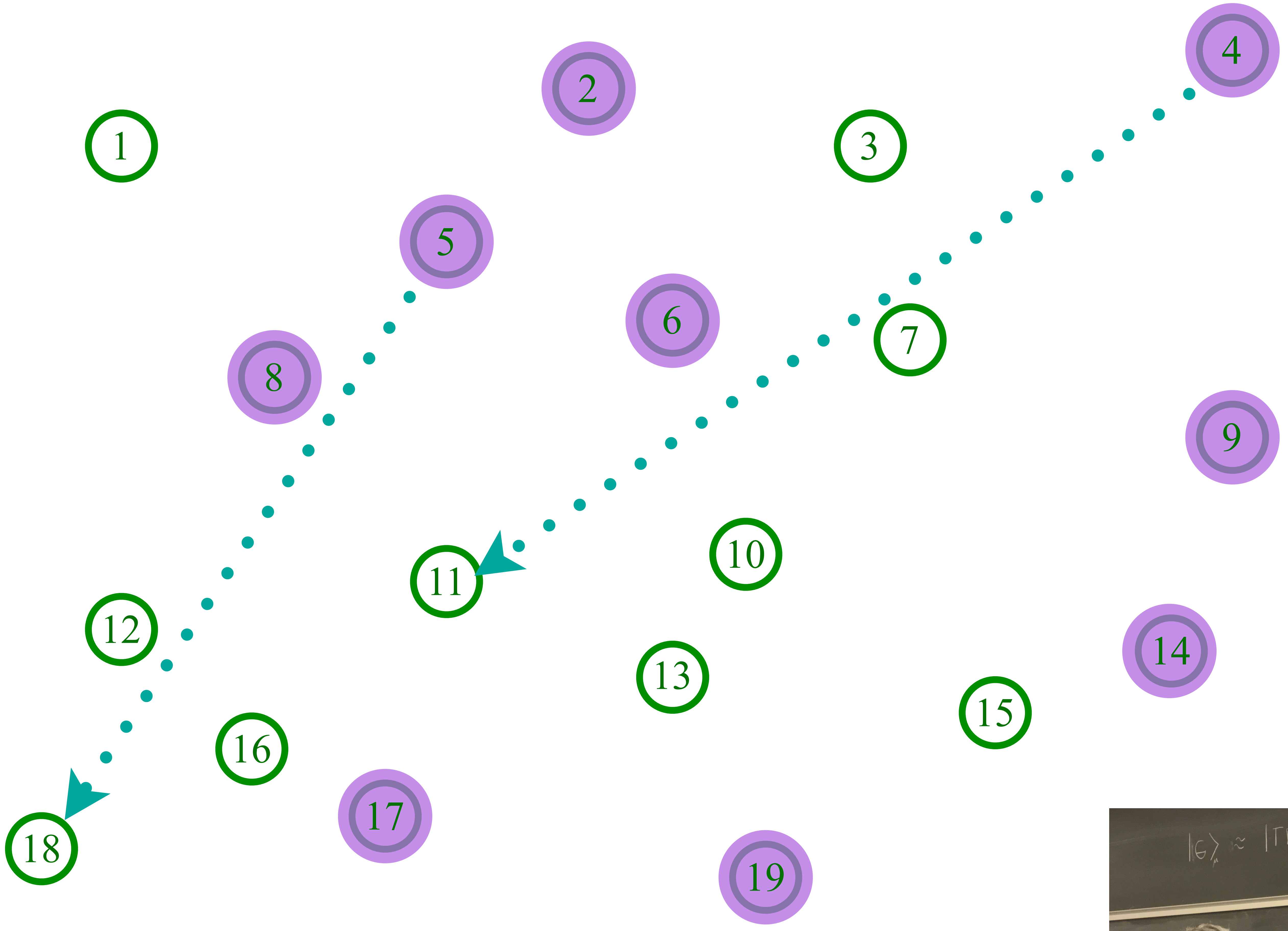
Entangle electrons pairwise randomly

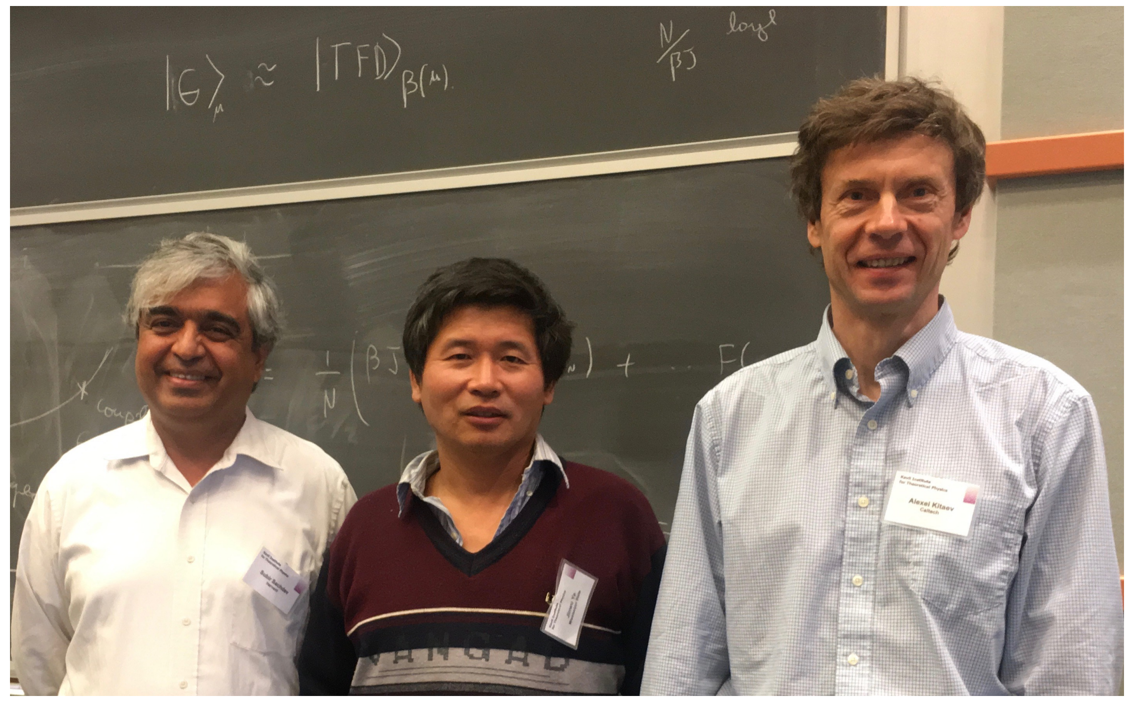


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$





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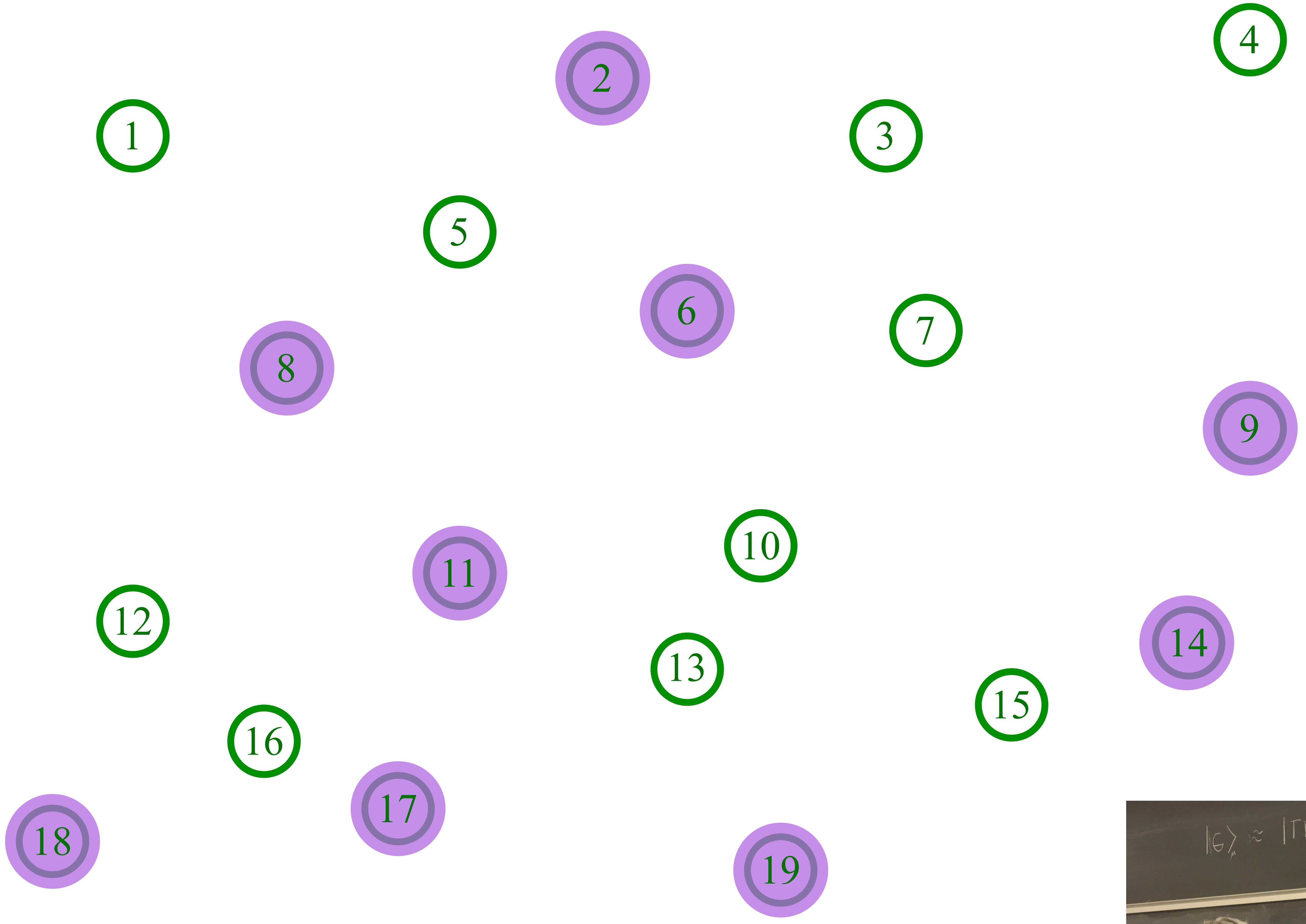

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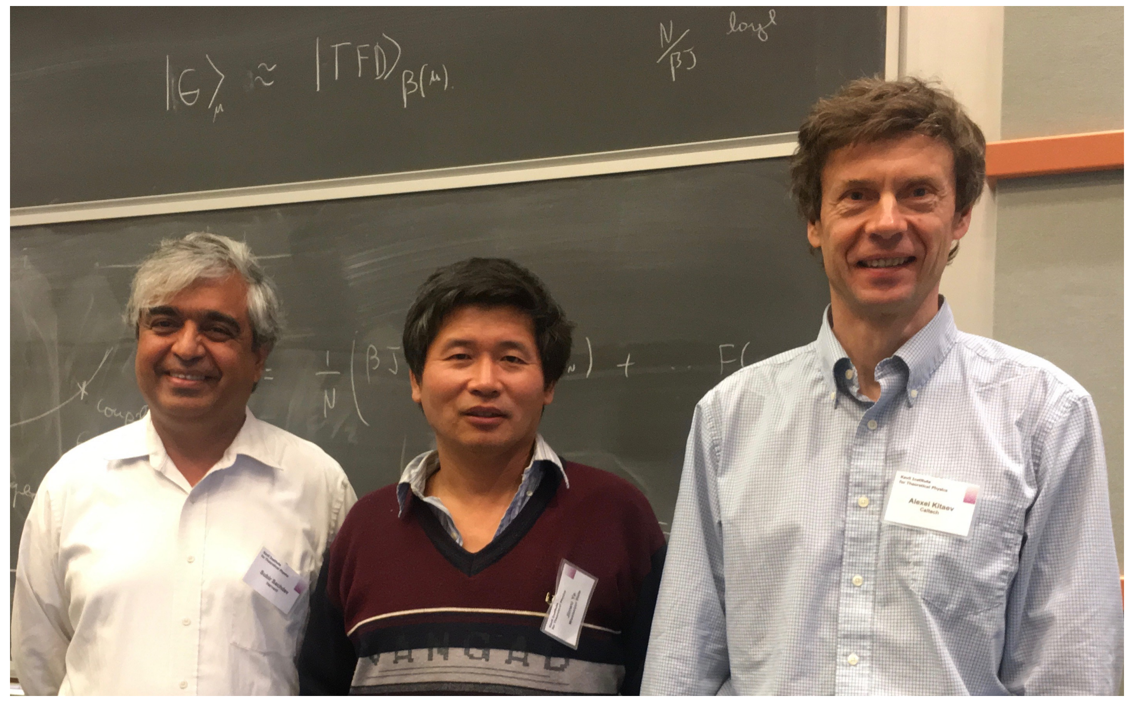

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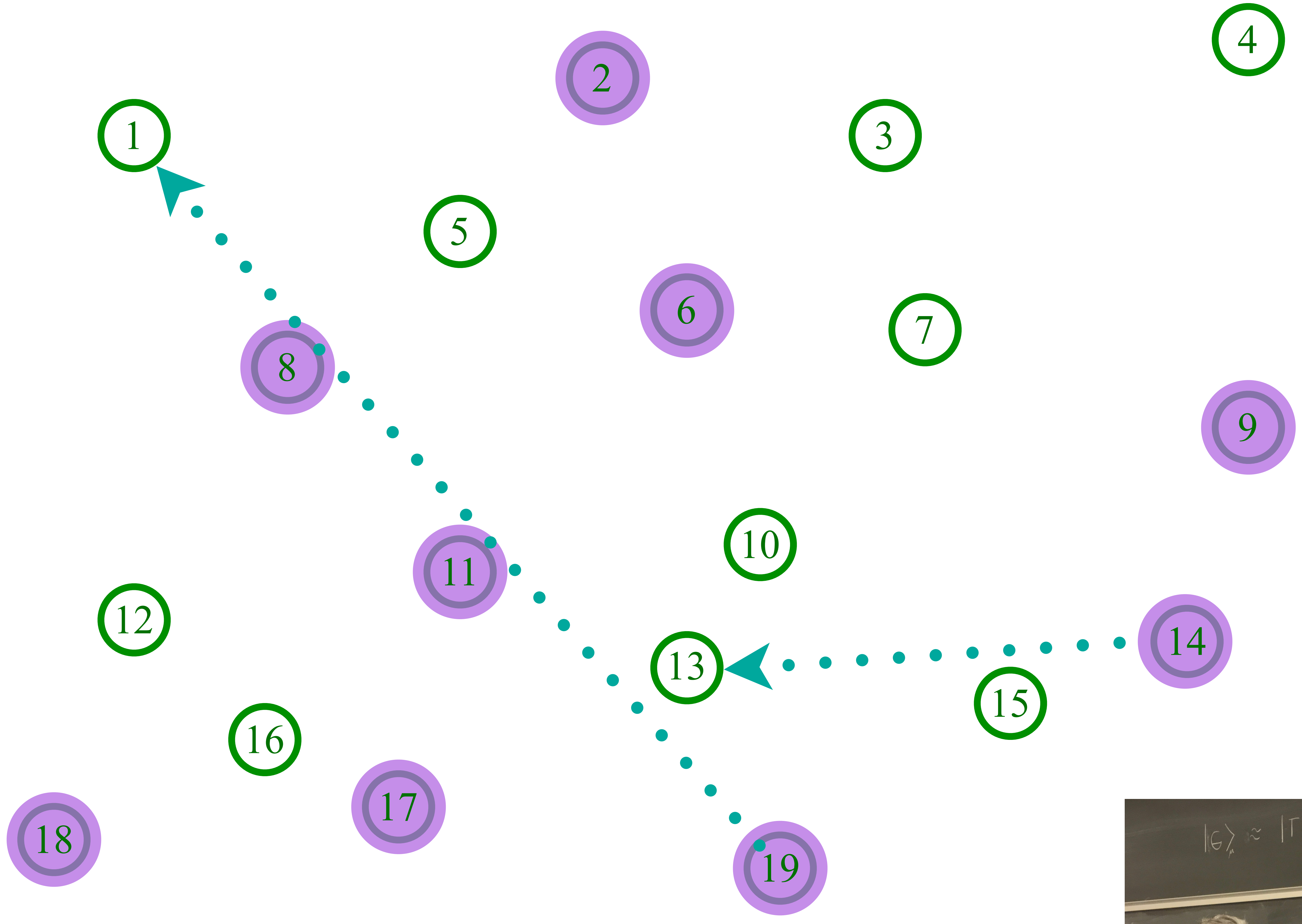
$$U_{14,19;1,13}$$



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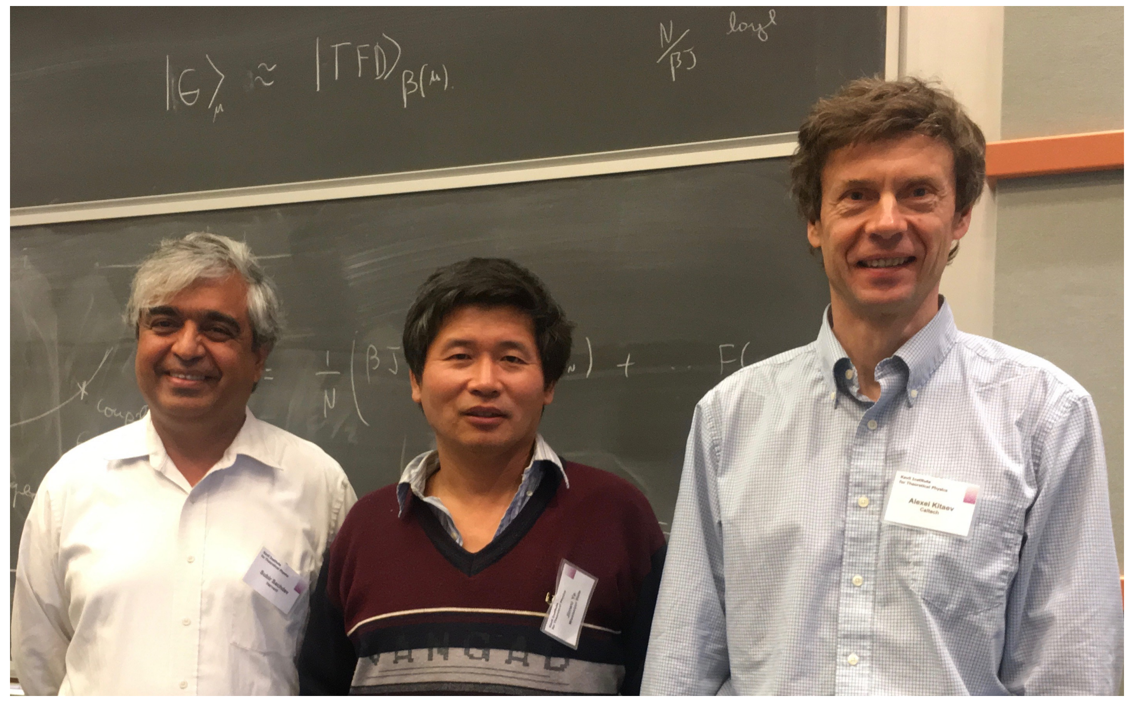

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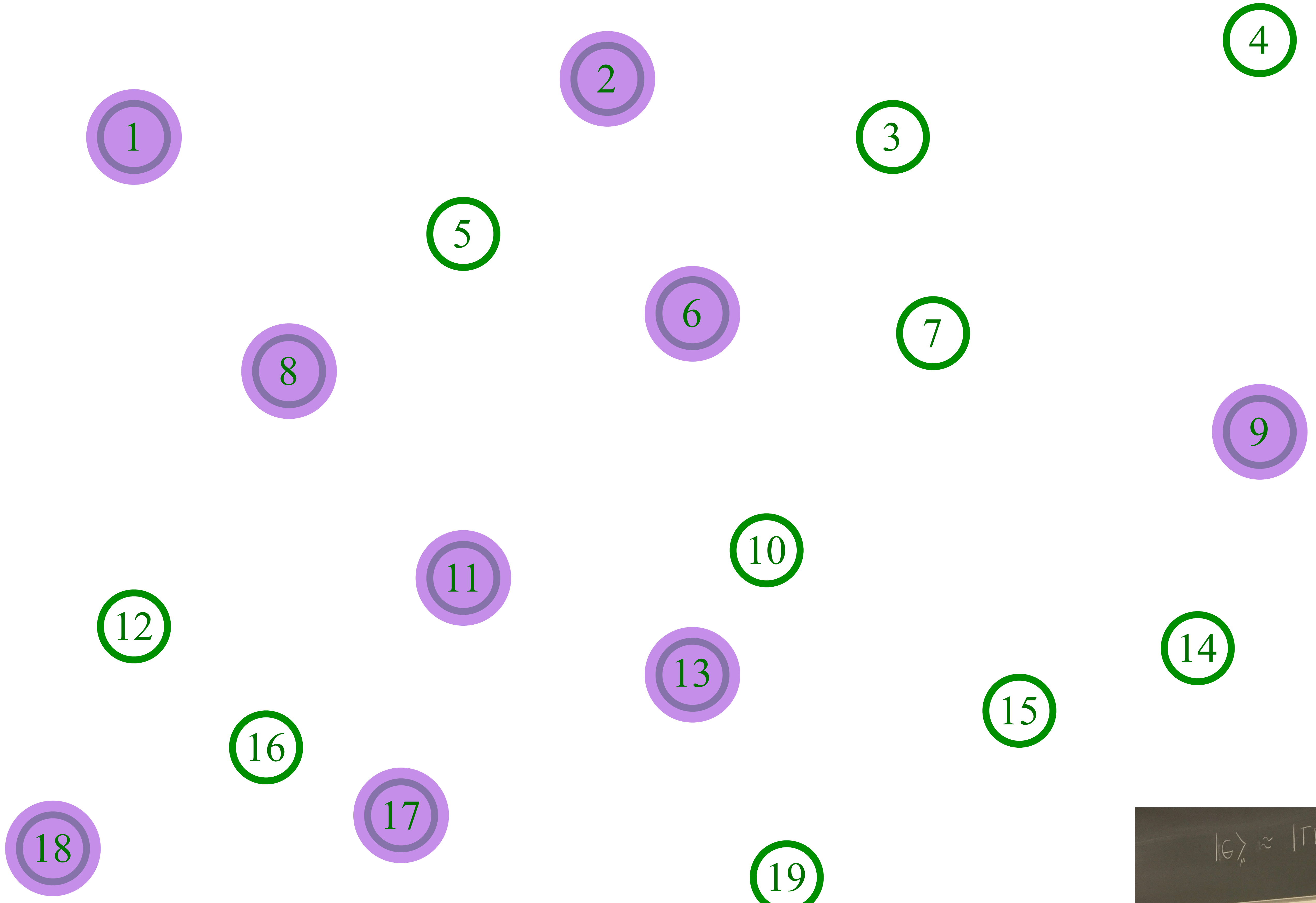

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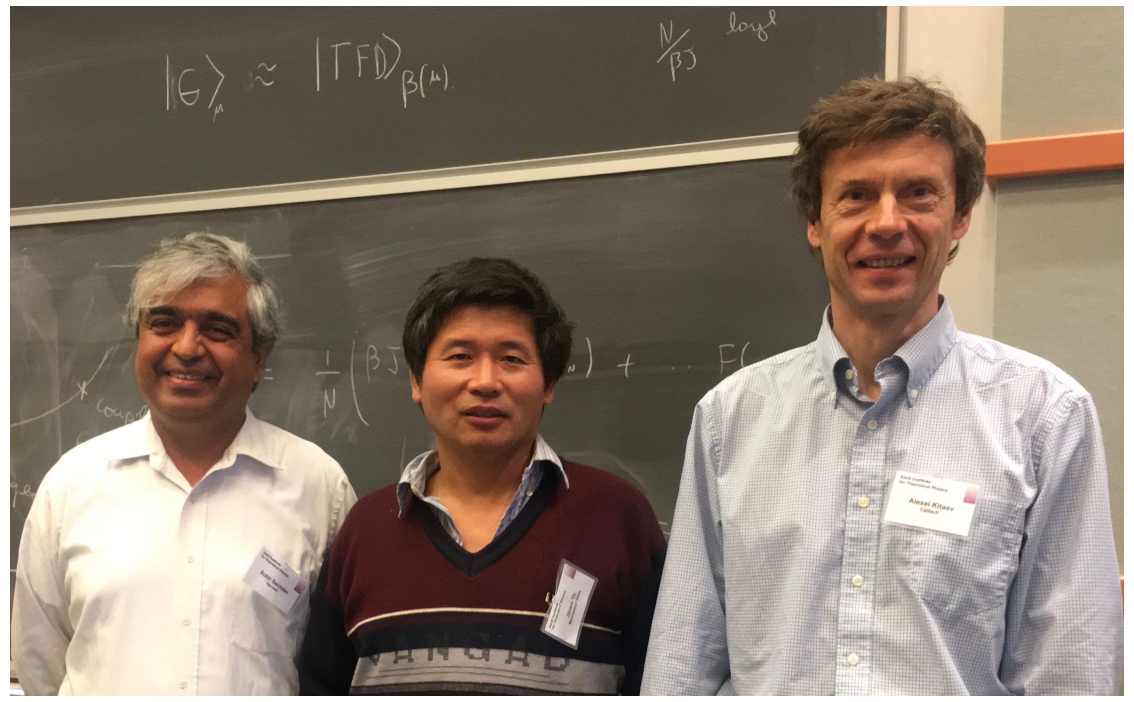
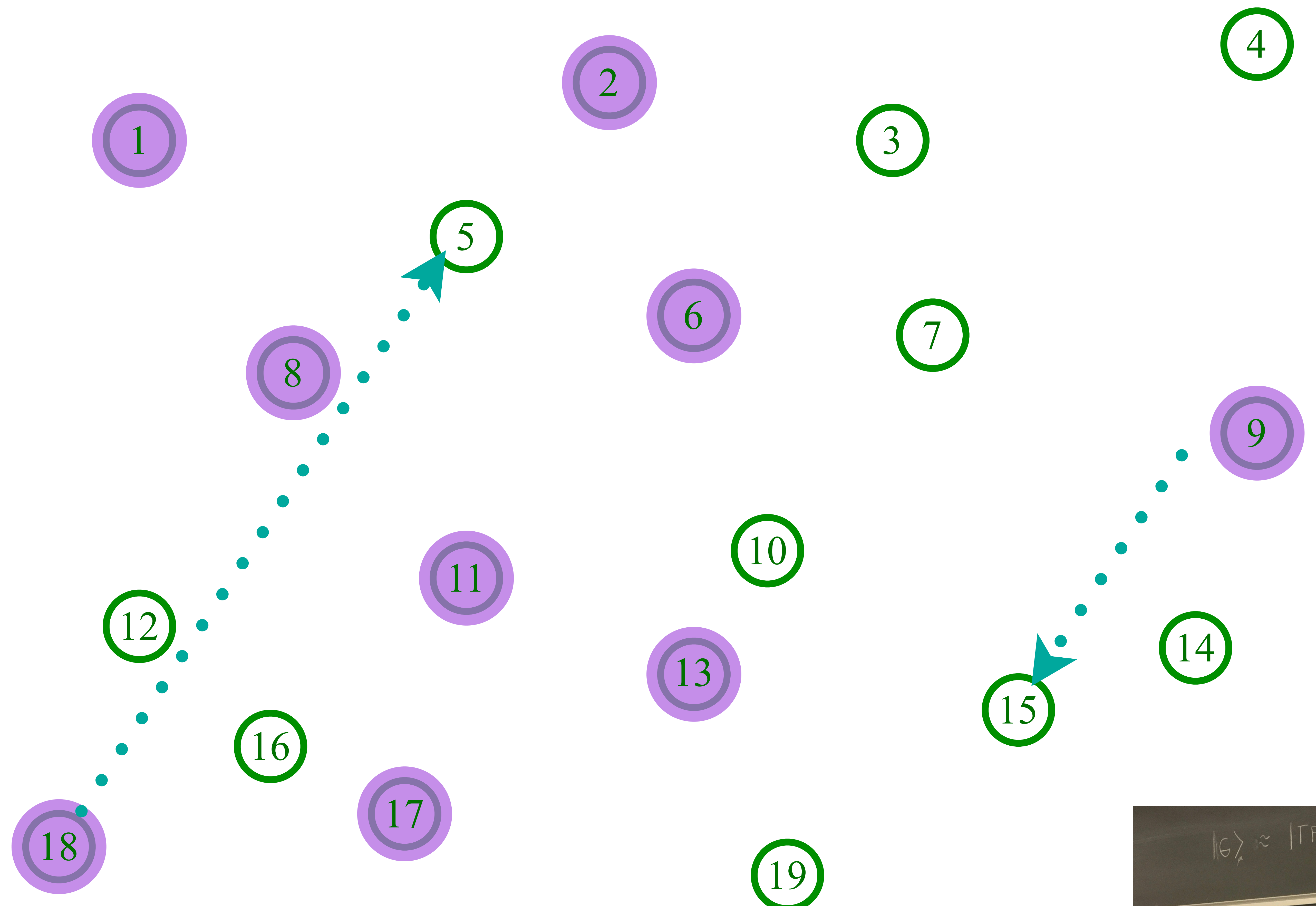


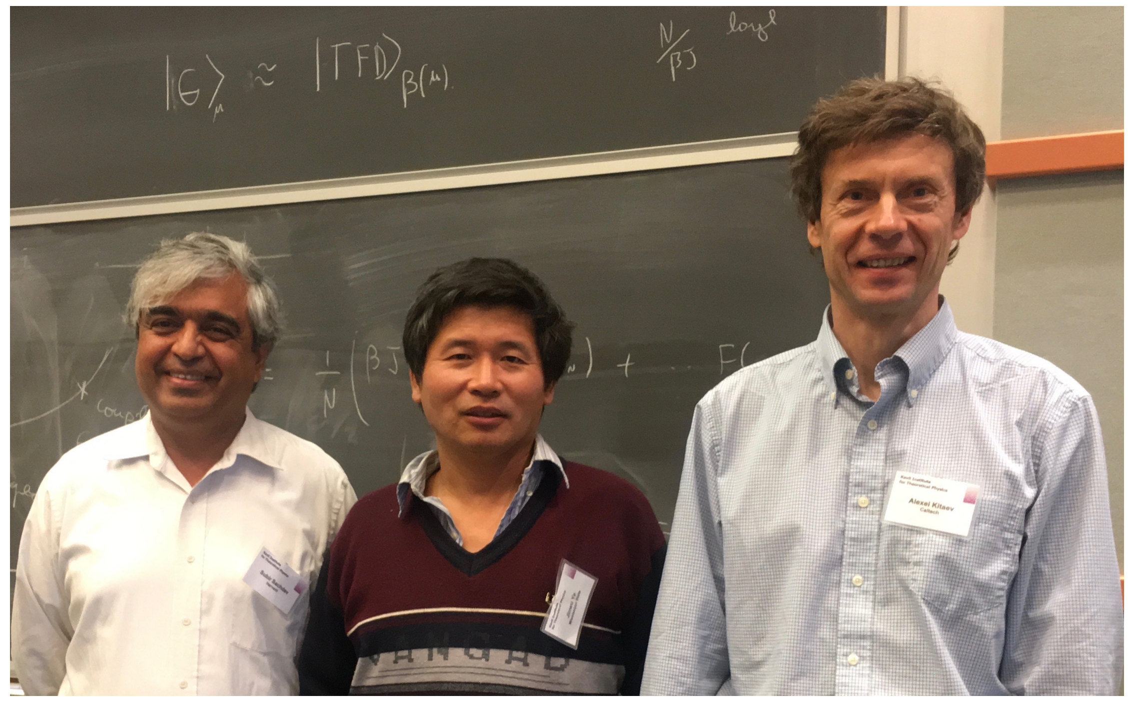
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



Entangle electrons pairwise randomly



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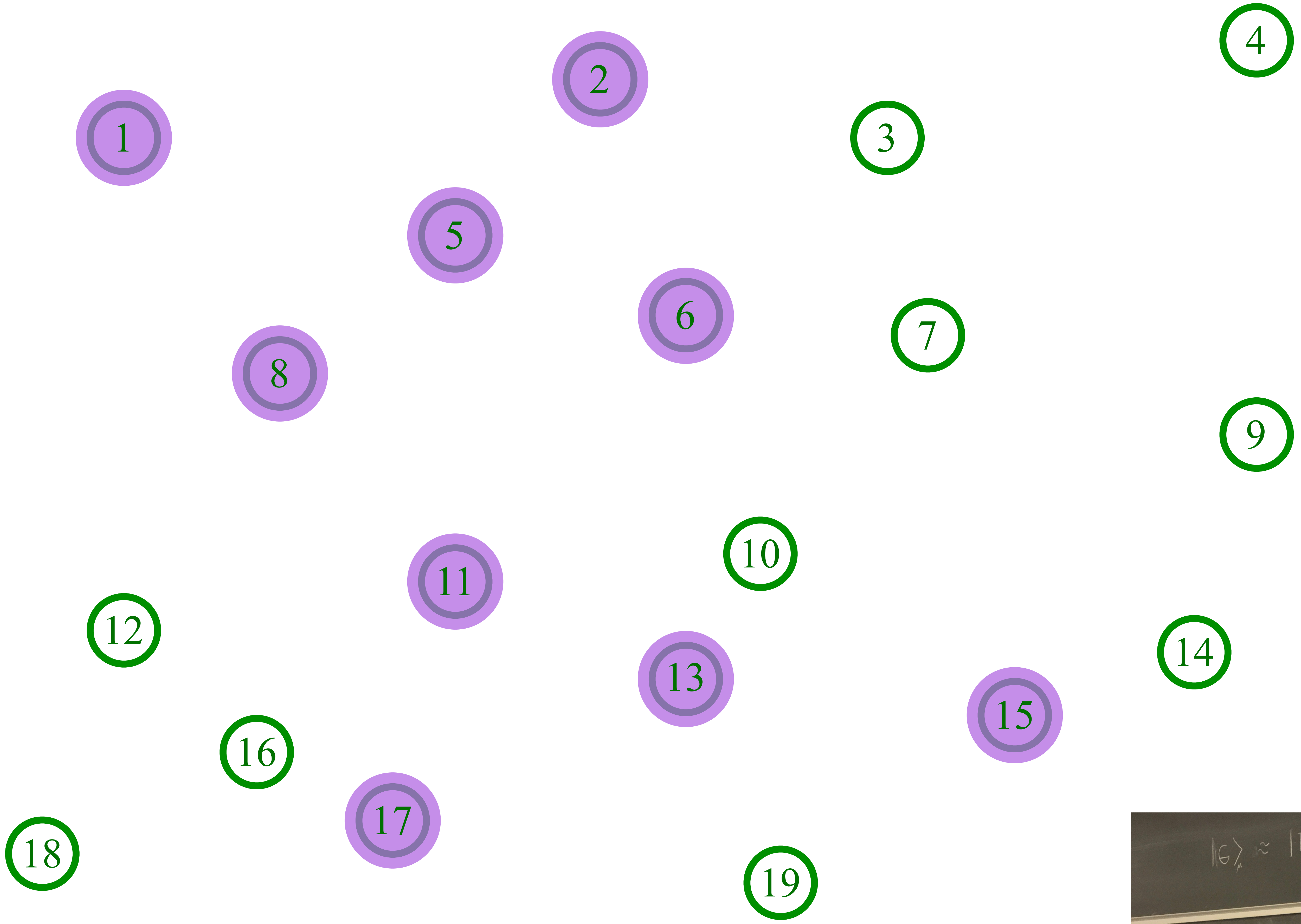

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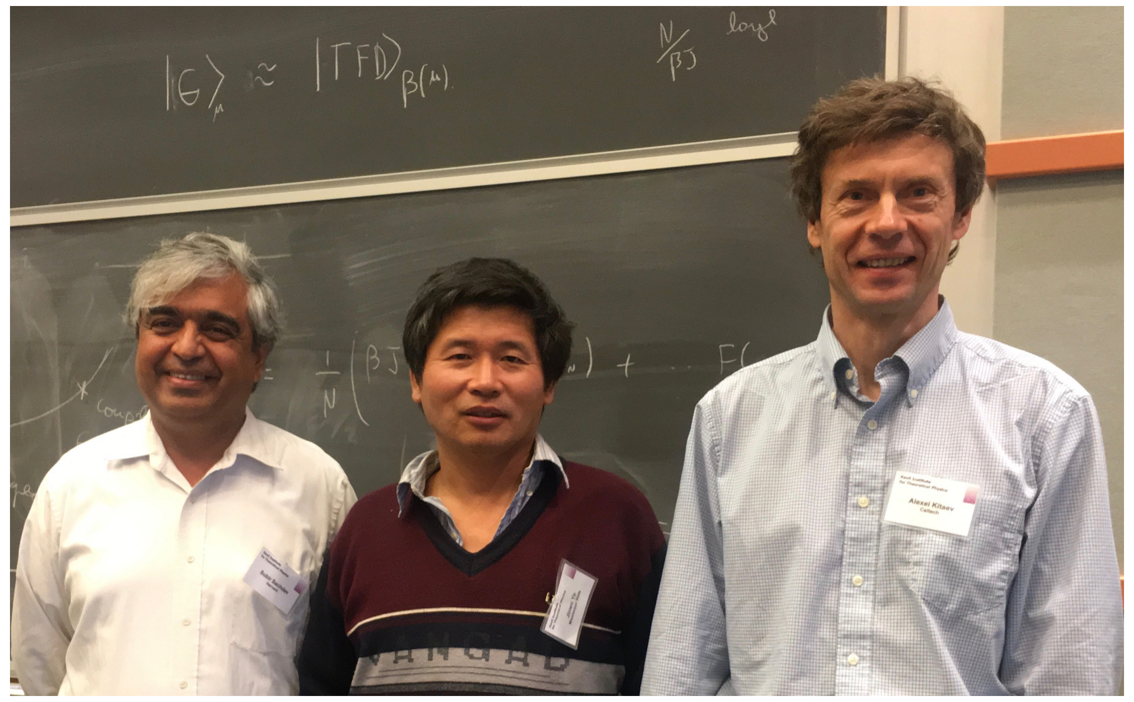

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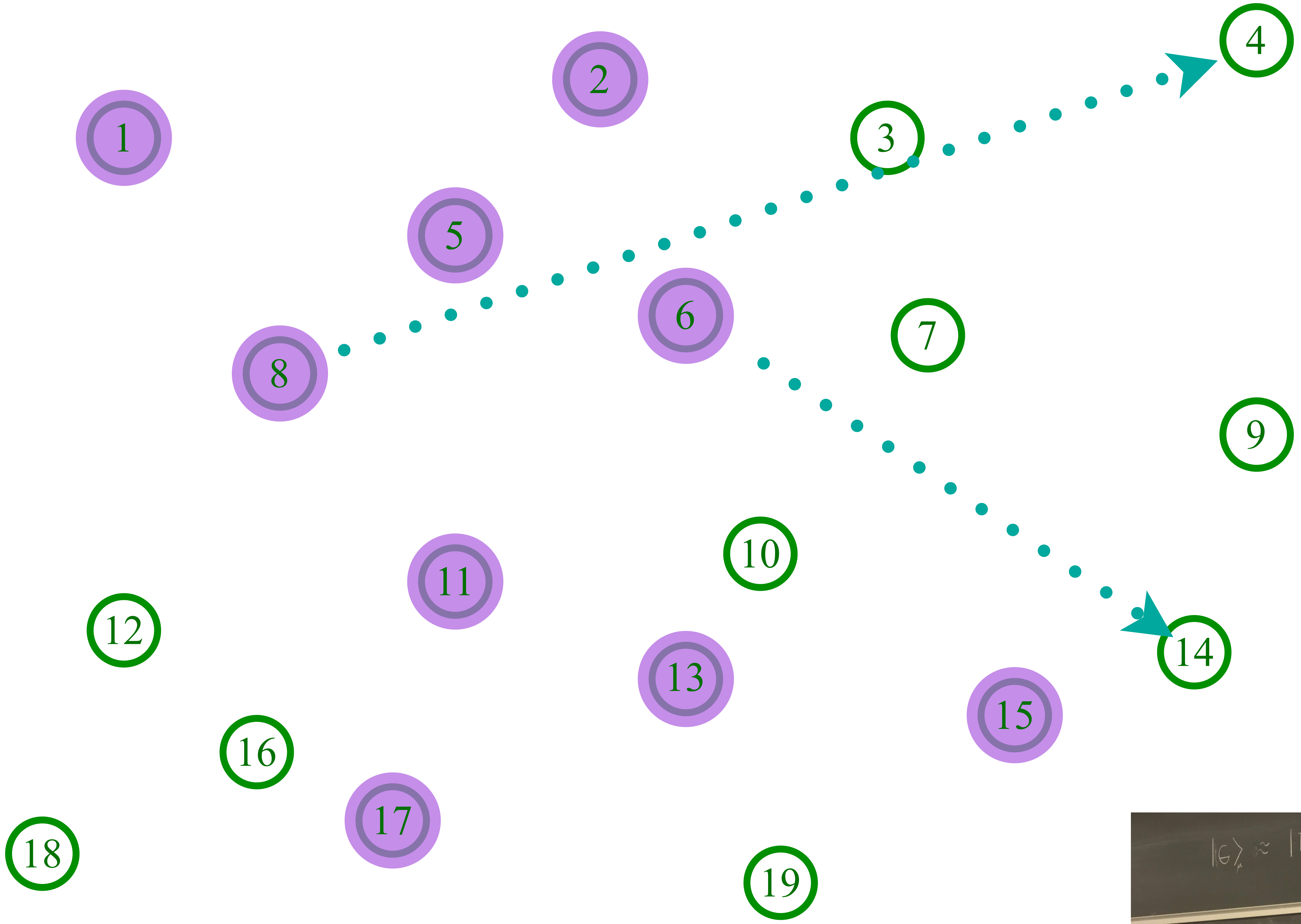

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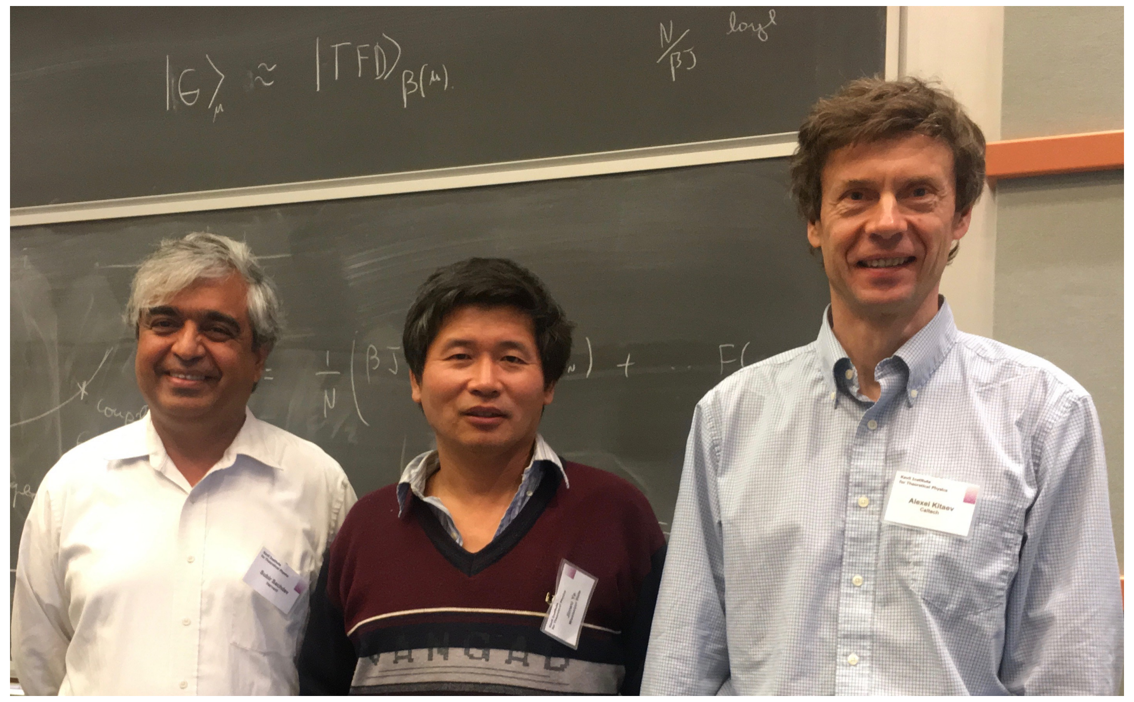

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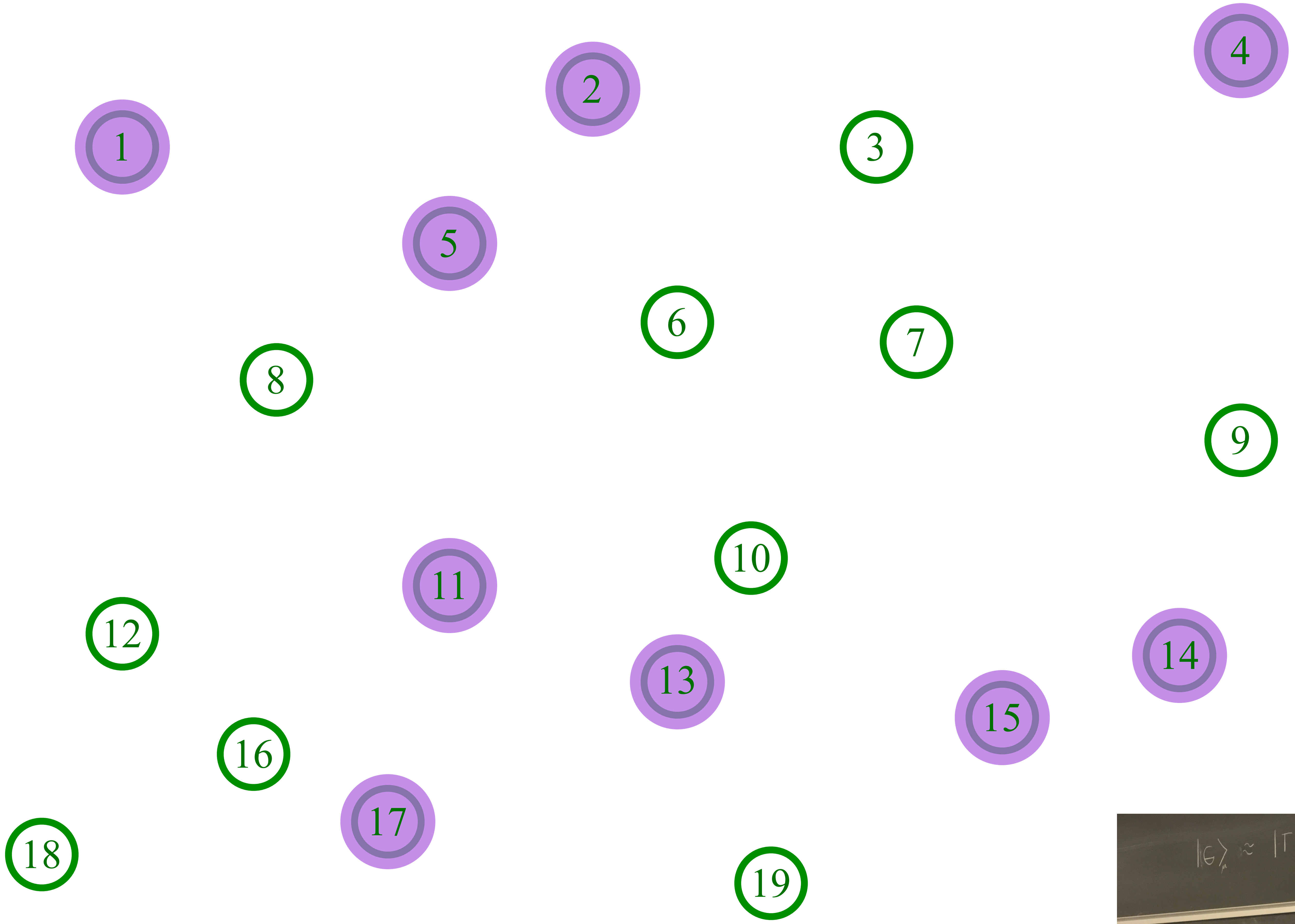

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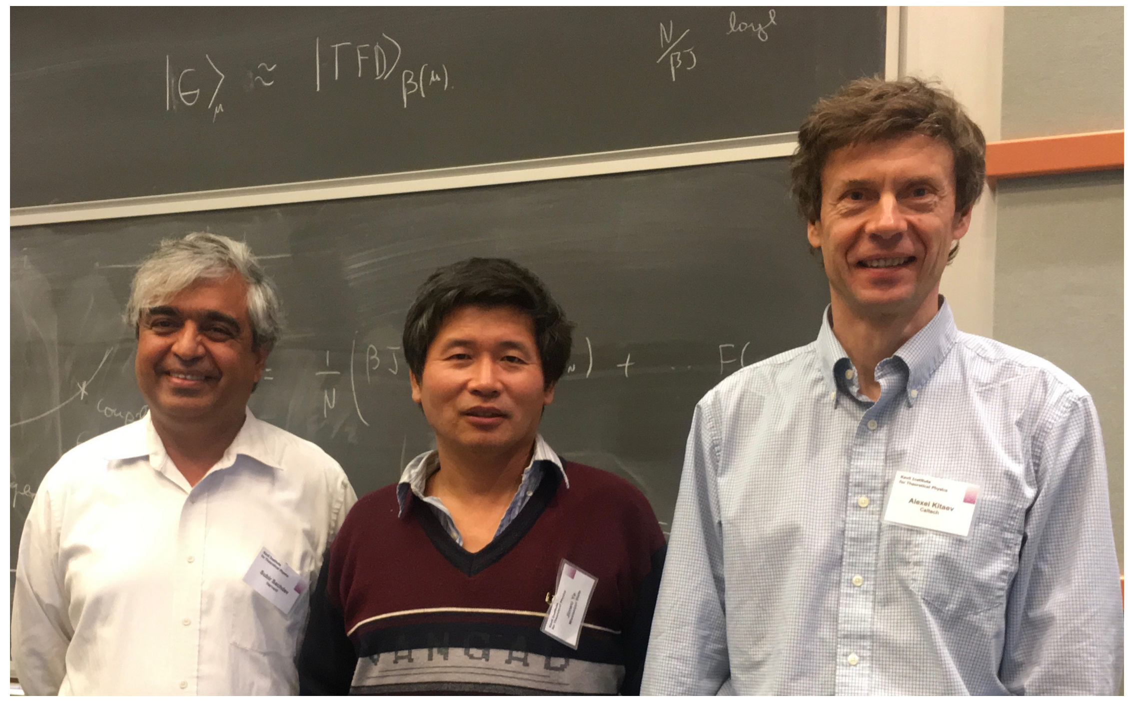


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

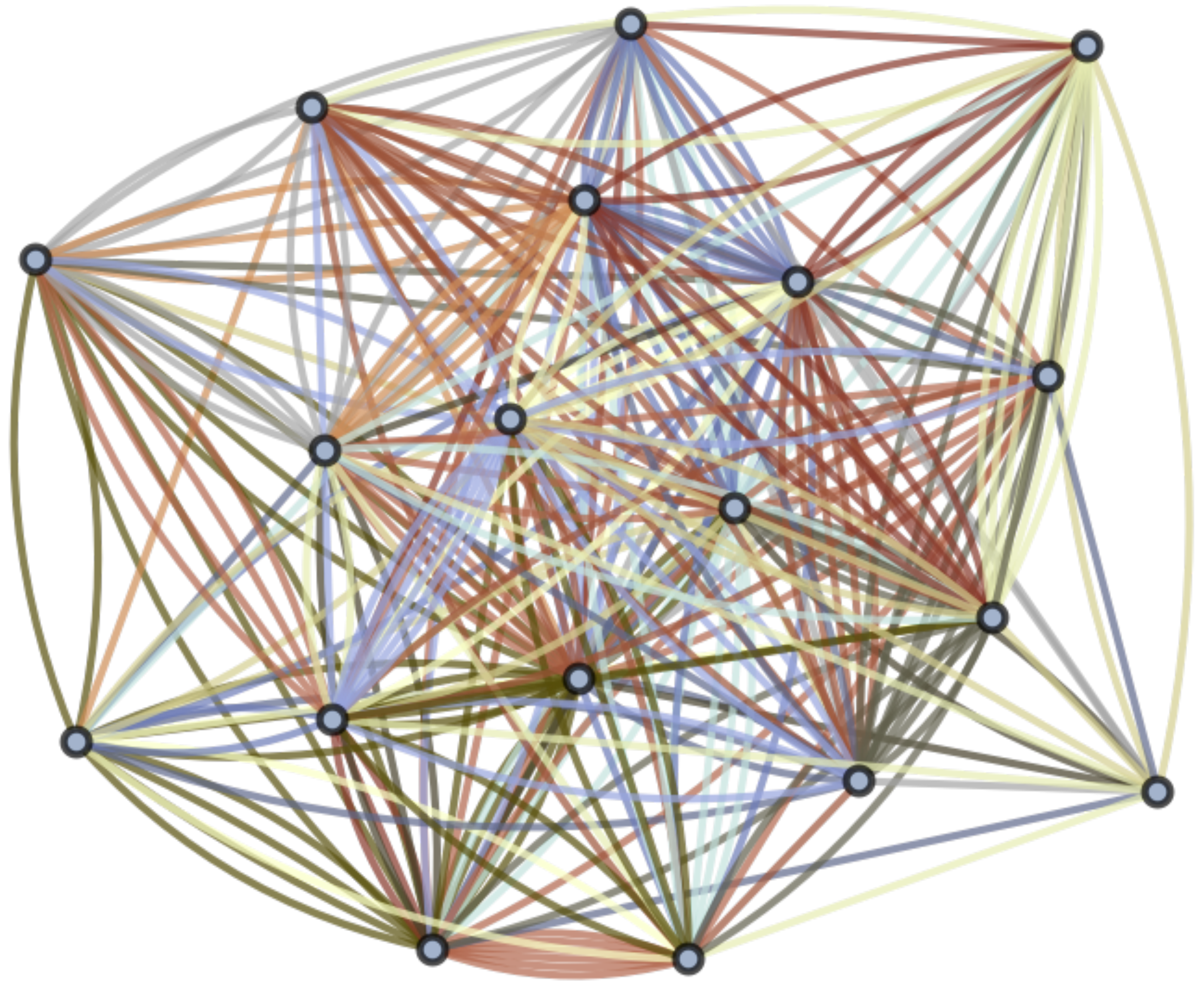
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

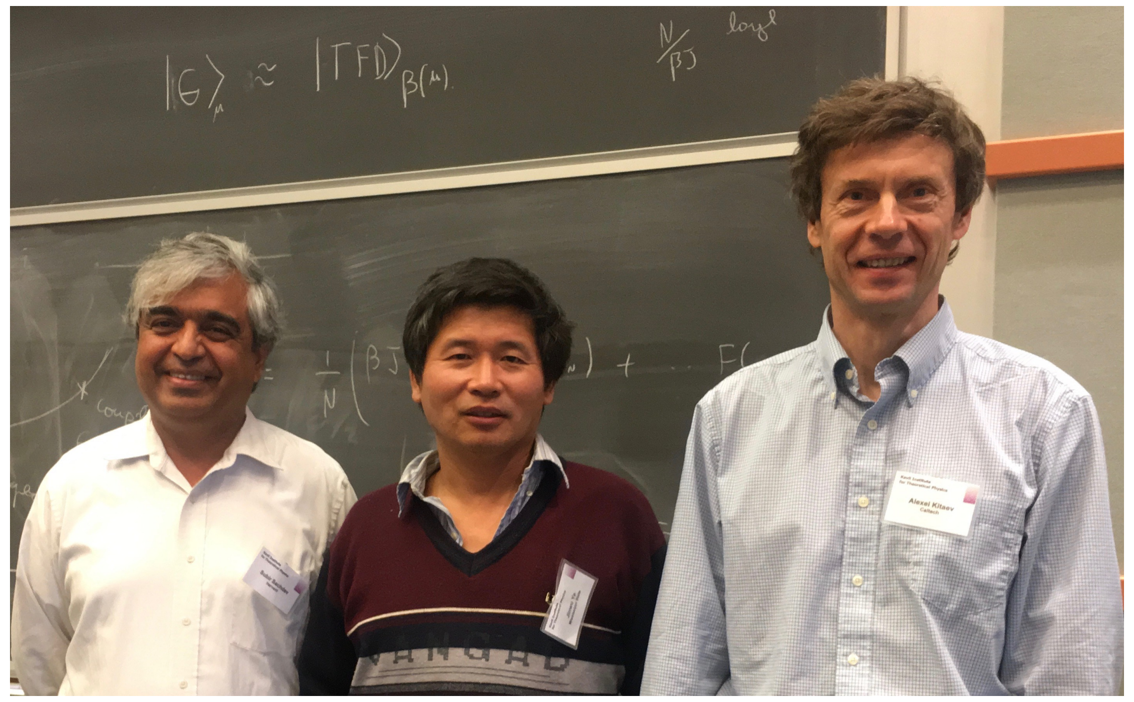
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \, ; \quad [\mathcal{H}, \mathcal{Q}] = 0 \, ; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



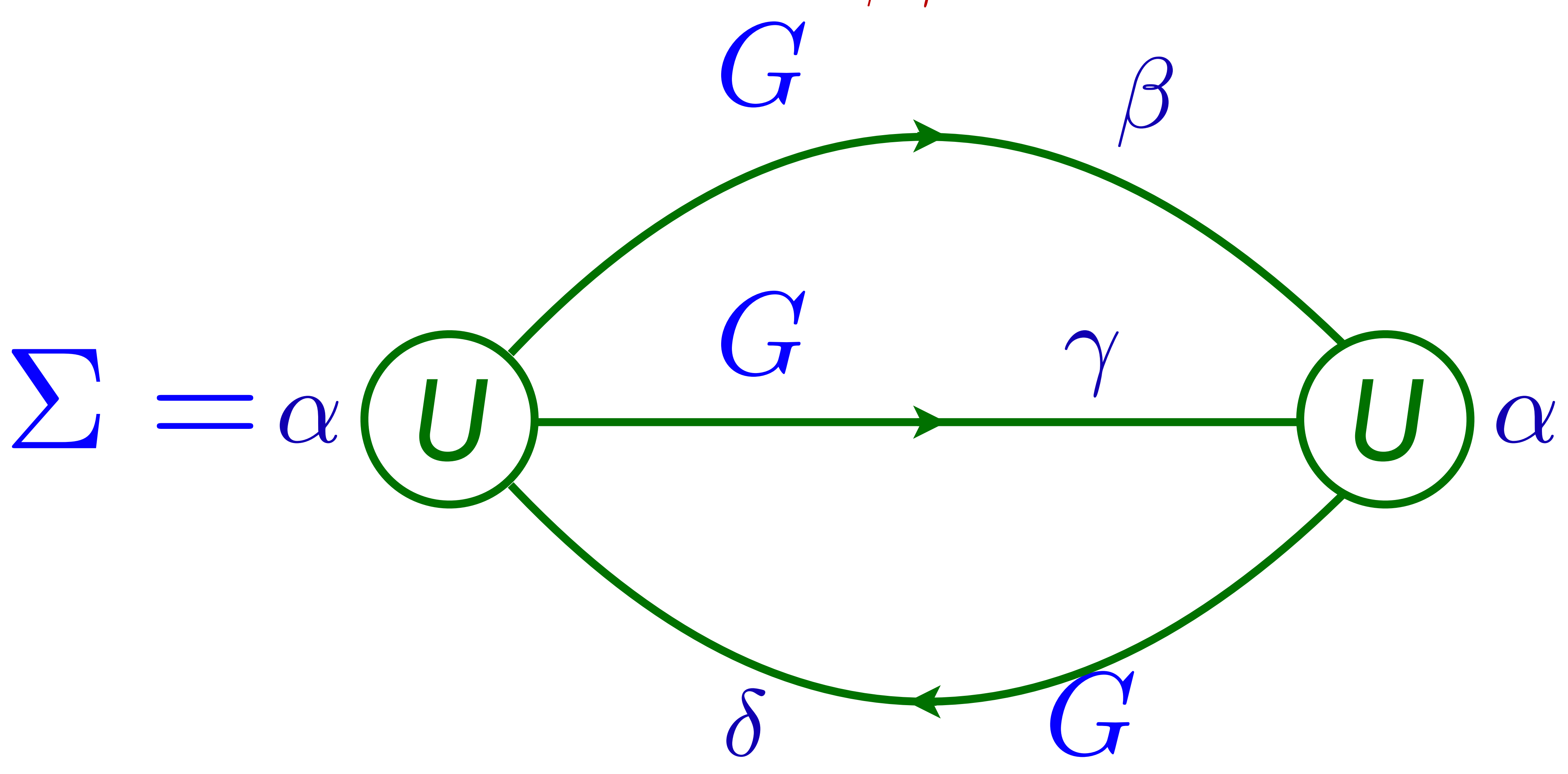


The Sachdev-Ye-Kitaev (SYK) model

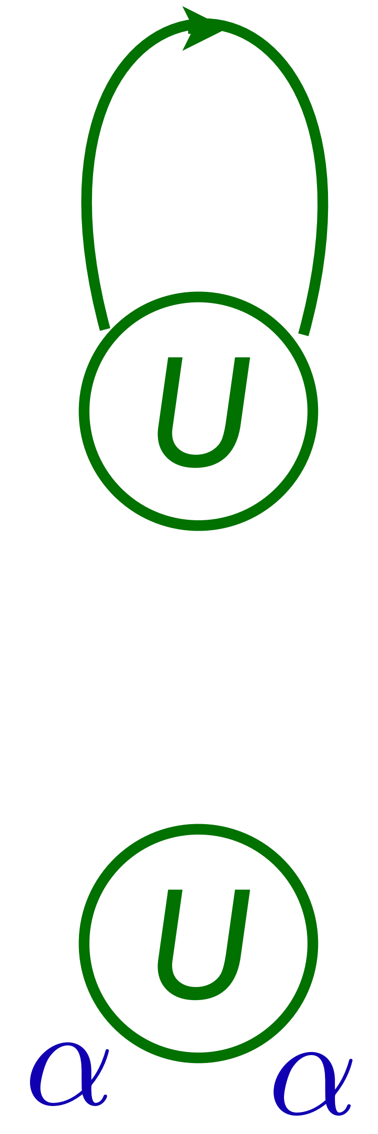
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$$\sum_{\beta\gamma\delta} \frac{U^2}{N^3} = U^2$$



$$\sum_{\beta\gamma} \frac{U^2}{N^3} = \frac{U^2}{N}$$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

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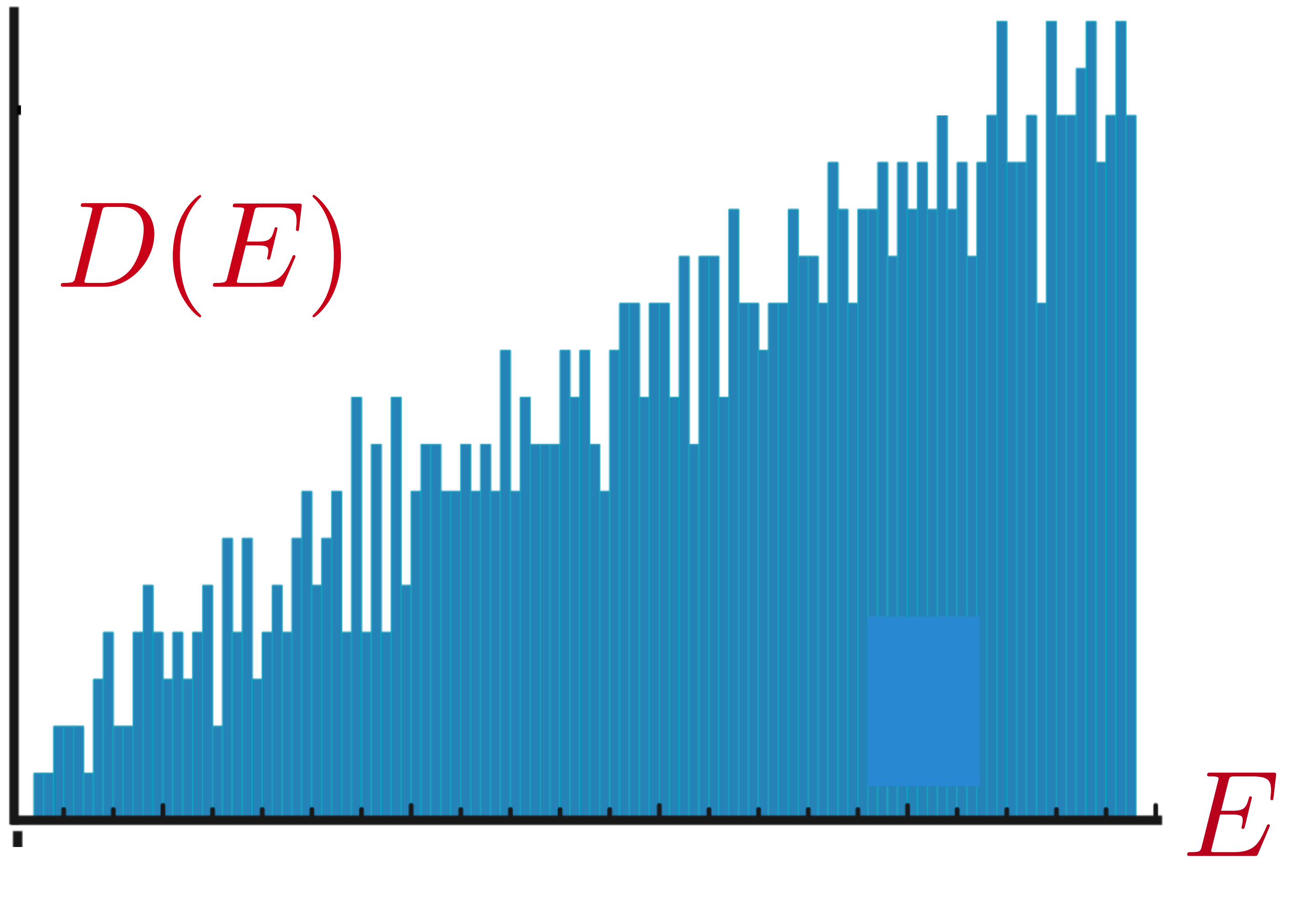
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

2. Zero temperature entropy without exponential ground state degeneracy!

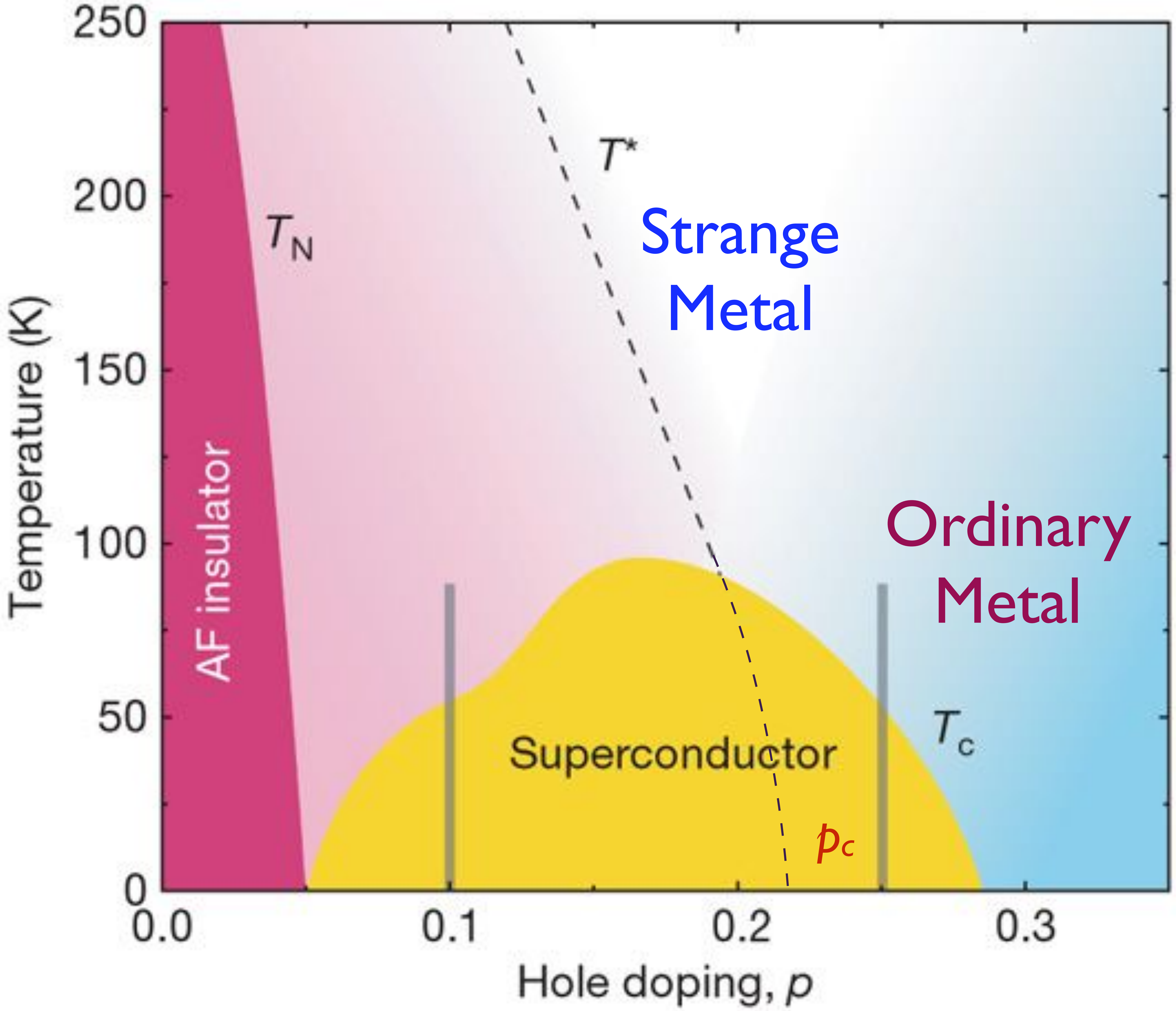
$$\lim_{T\to 0} \lim_{N\to\infty} \frac{1}{N} S(T) = s_0 \quad , \quad D(E \to 0) \sim e^{N s_0}$$

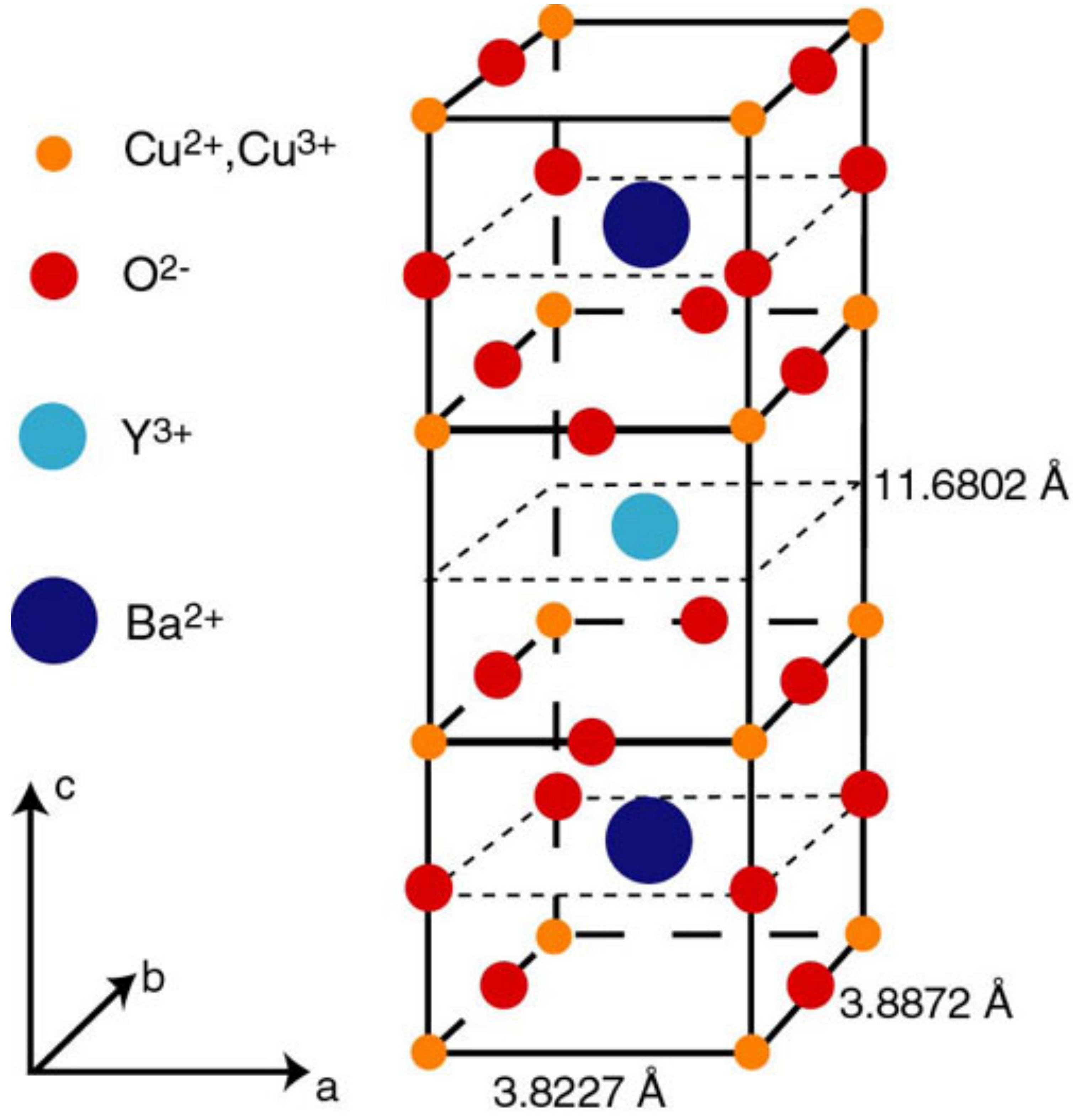
$$s_0 = 0.4648476991708051 0749.... \text{ for } \mathcal{Q} = 1/2.$$



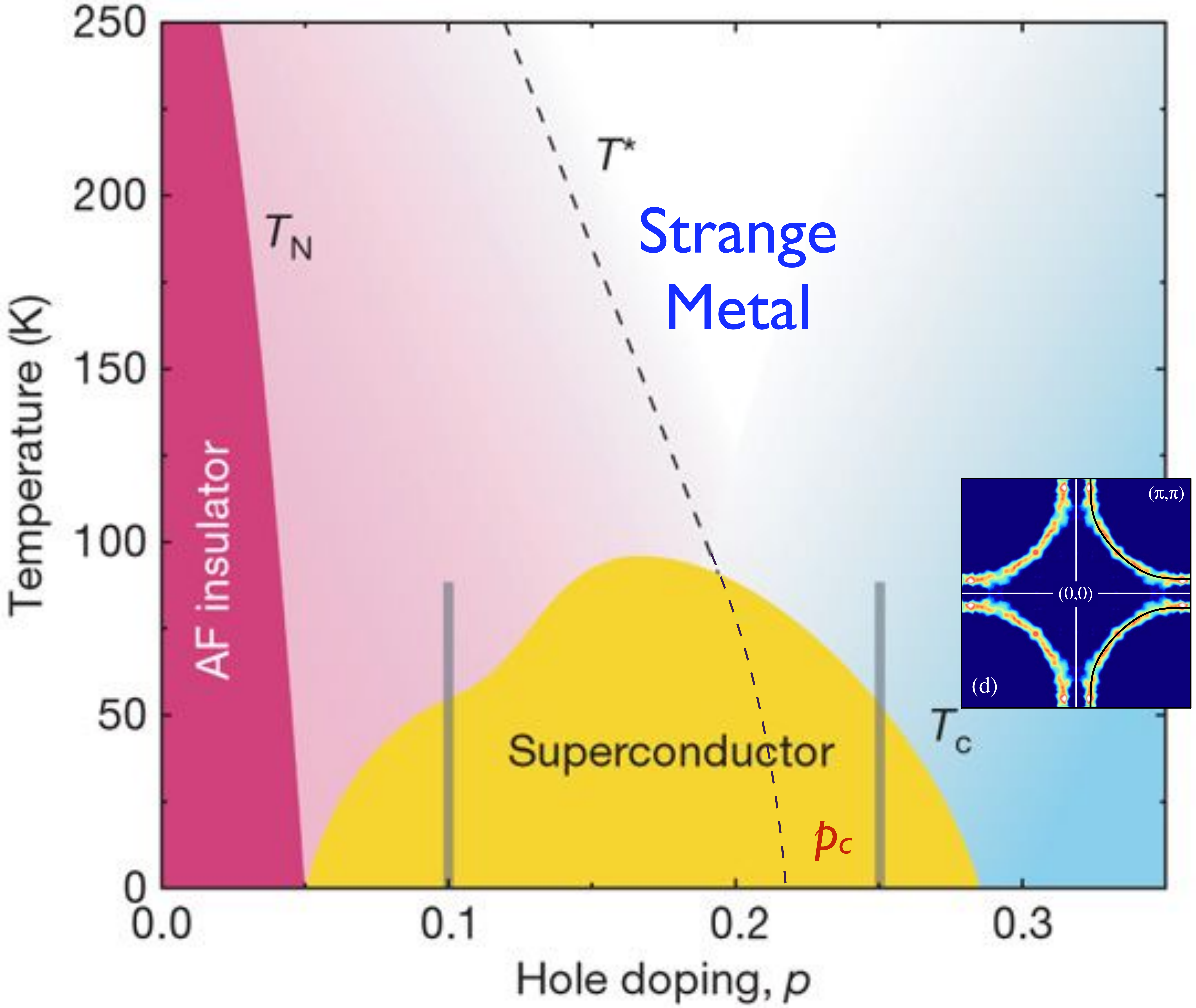
A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

The SYK model and strange metals

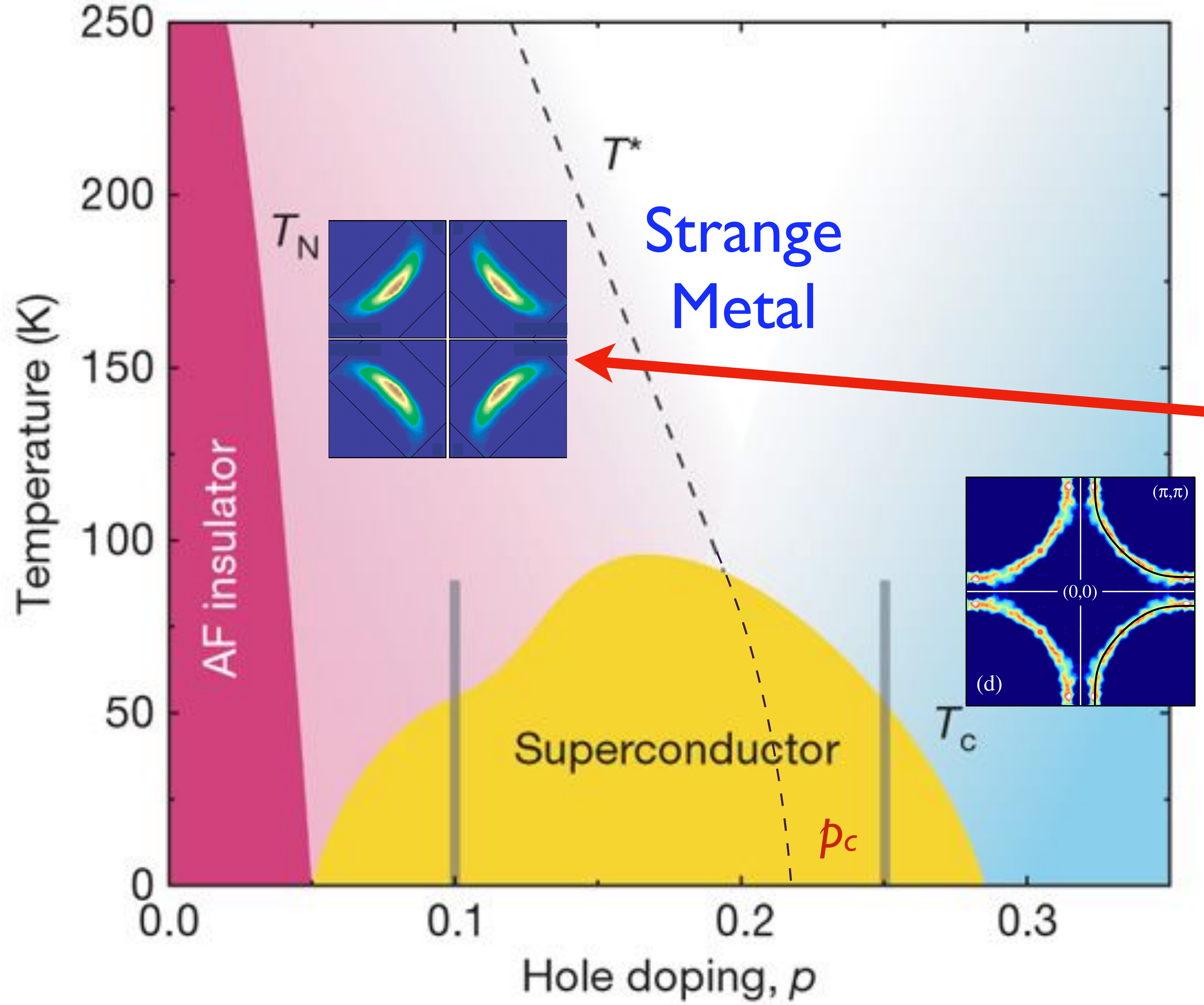

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

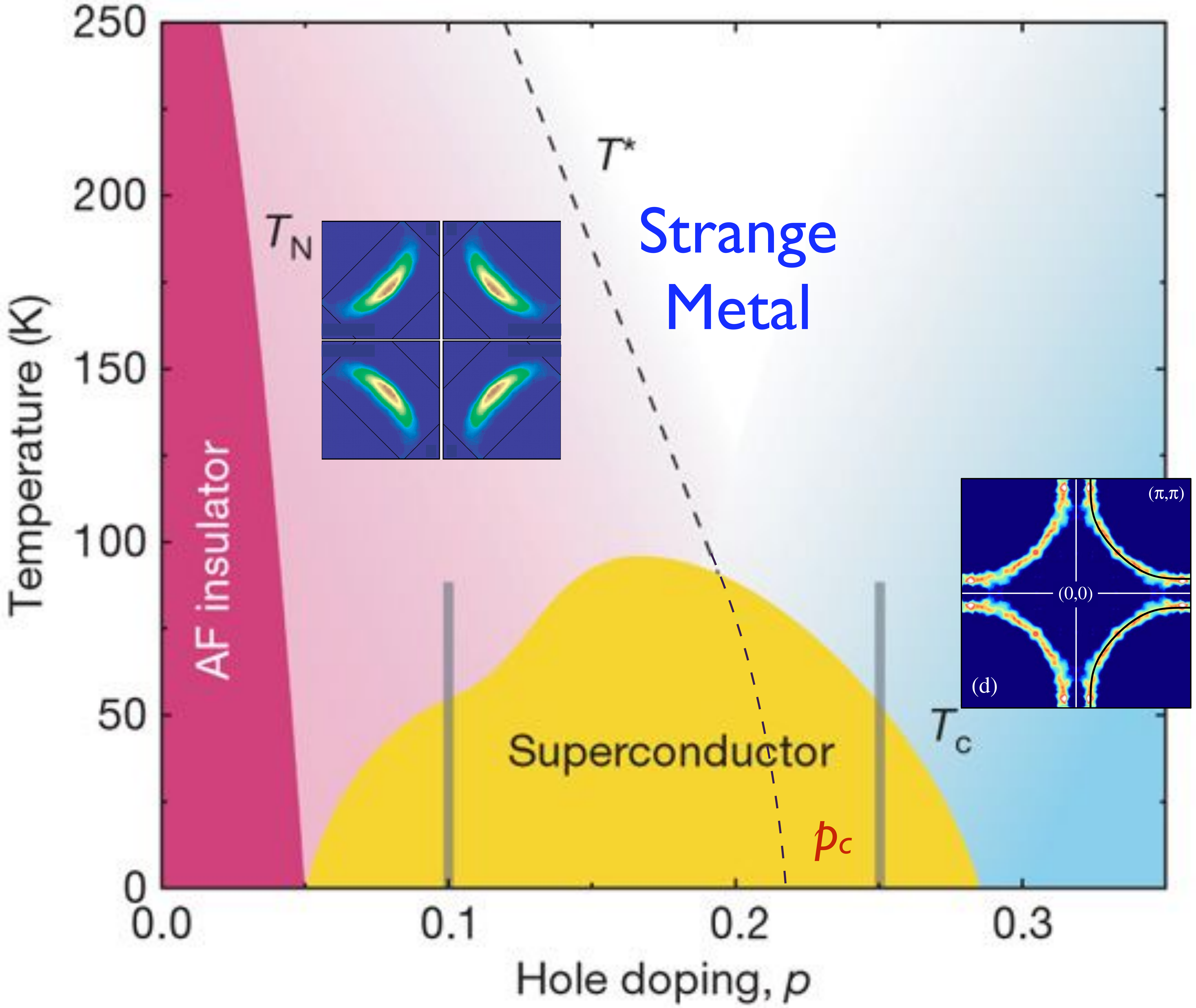
$YBa_2Cu_3O_{6+x}$



Fermi surface
as expected
in a model
of free electrons



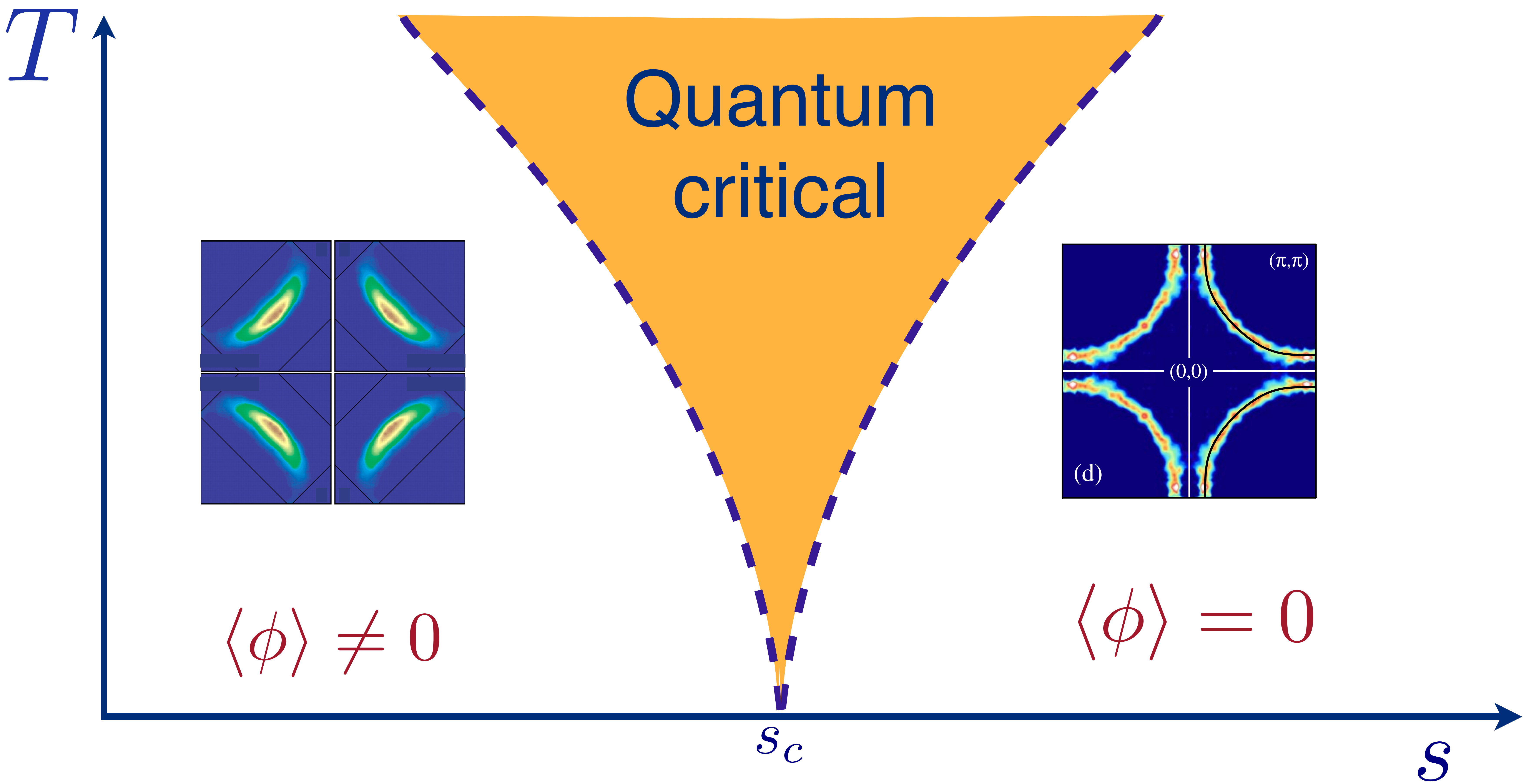
"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions



View the strange metal as a property of a $T = 0$ quantum phase transition involving change in the Fermi surface.

The onset of superconductivity may "hide" this quantum transition.

Quantum phase transitions of Fermi surface change



Fermi surface

$+$

a boson ϕ

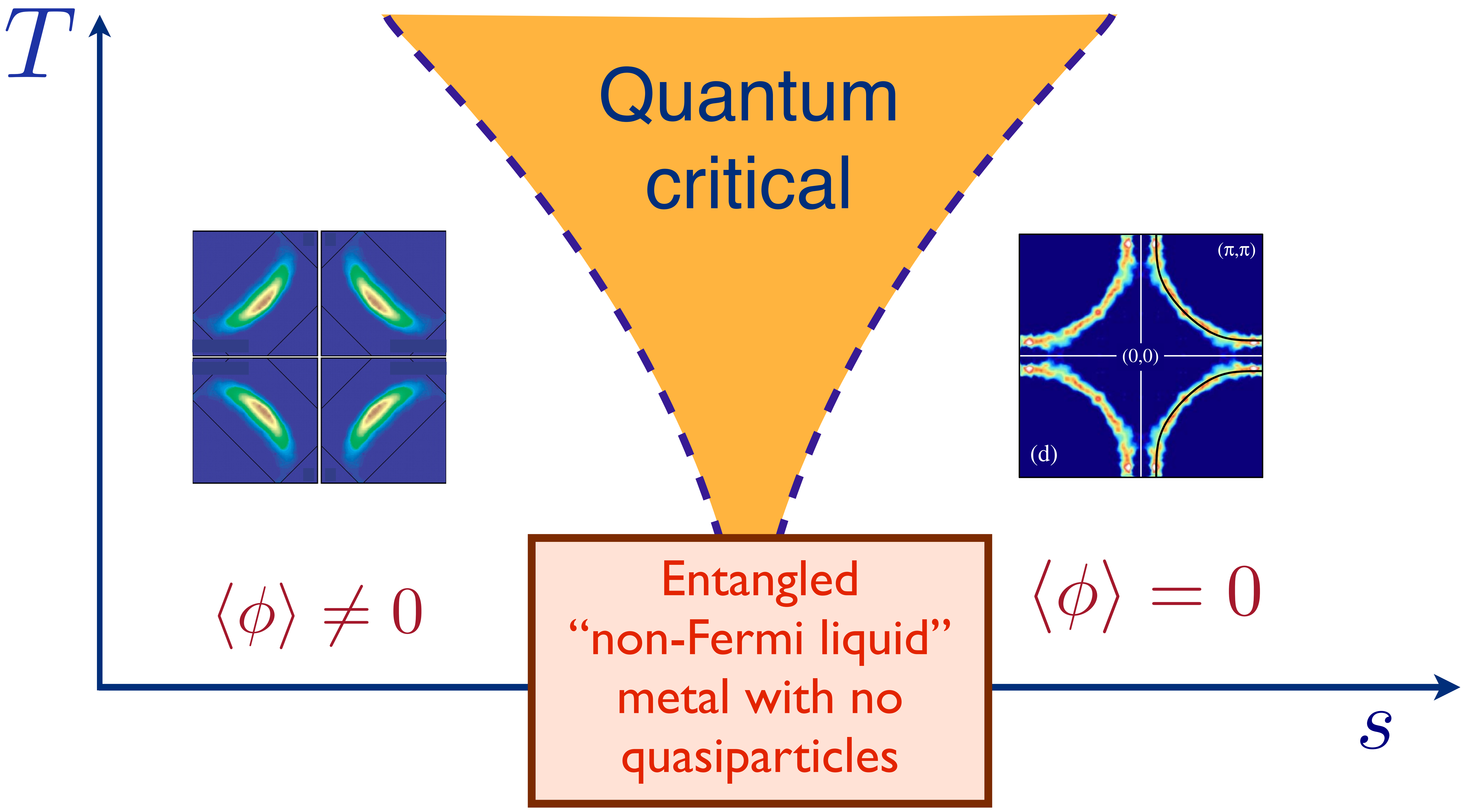
with a 'mass' s

and

a boson-fermion

Yukawa coupling g.

Quantum phase transitions of Fermi surface change



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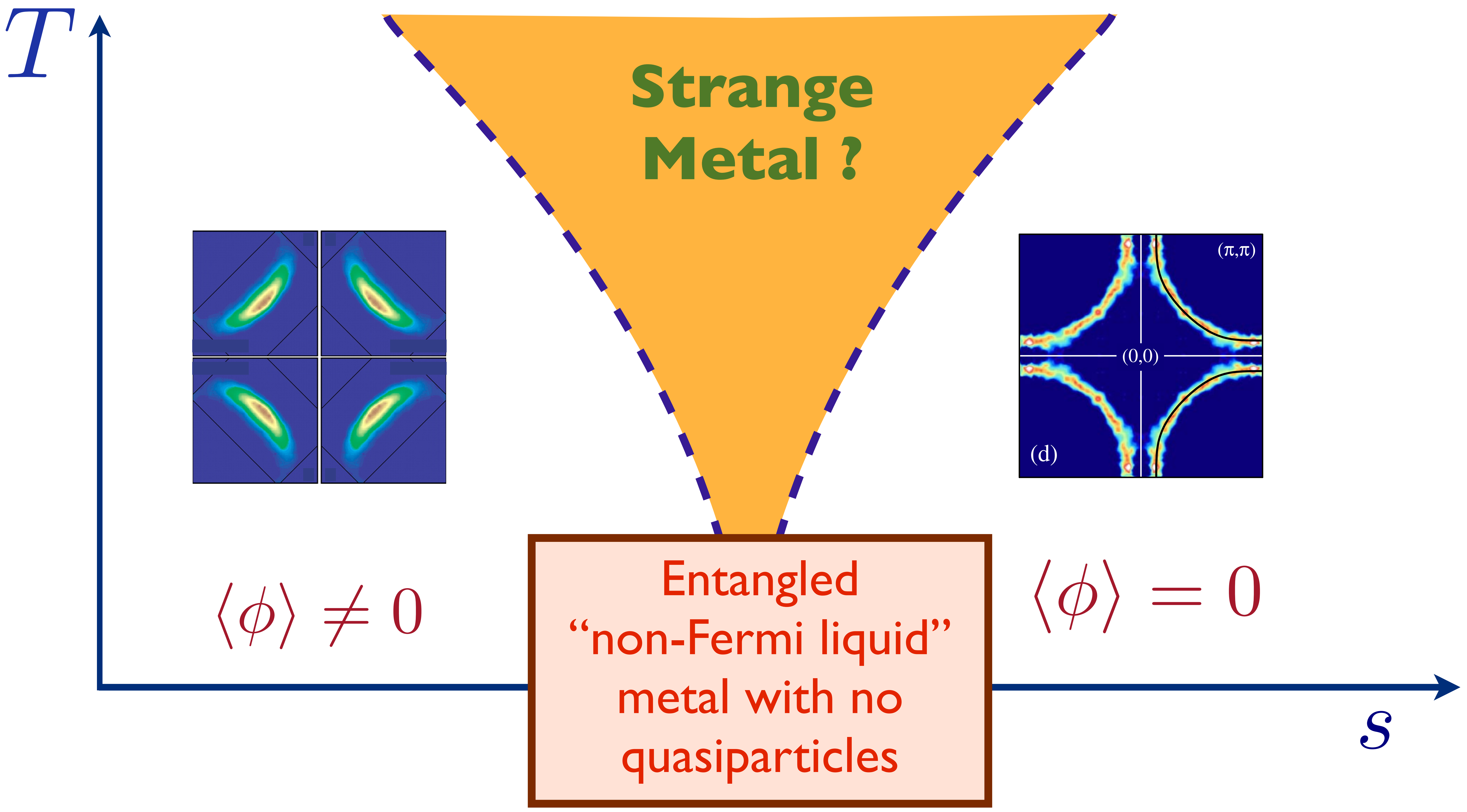

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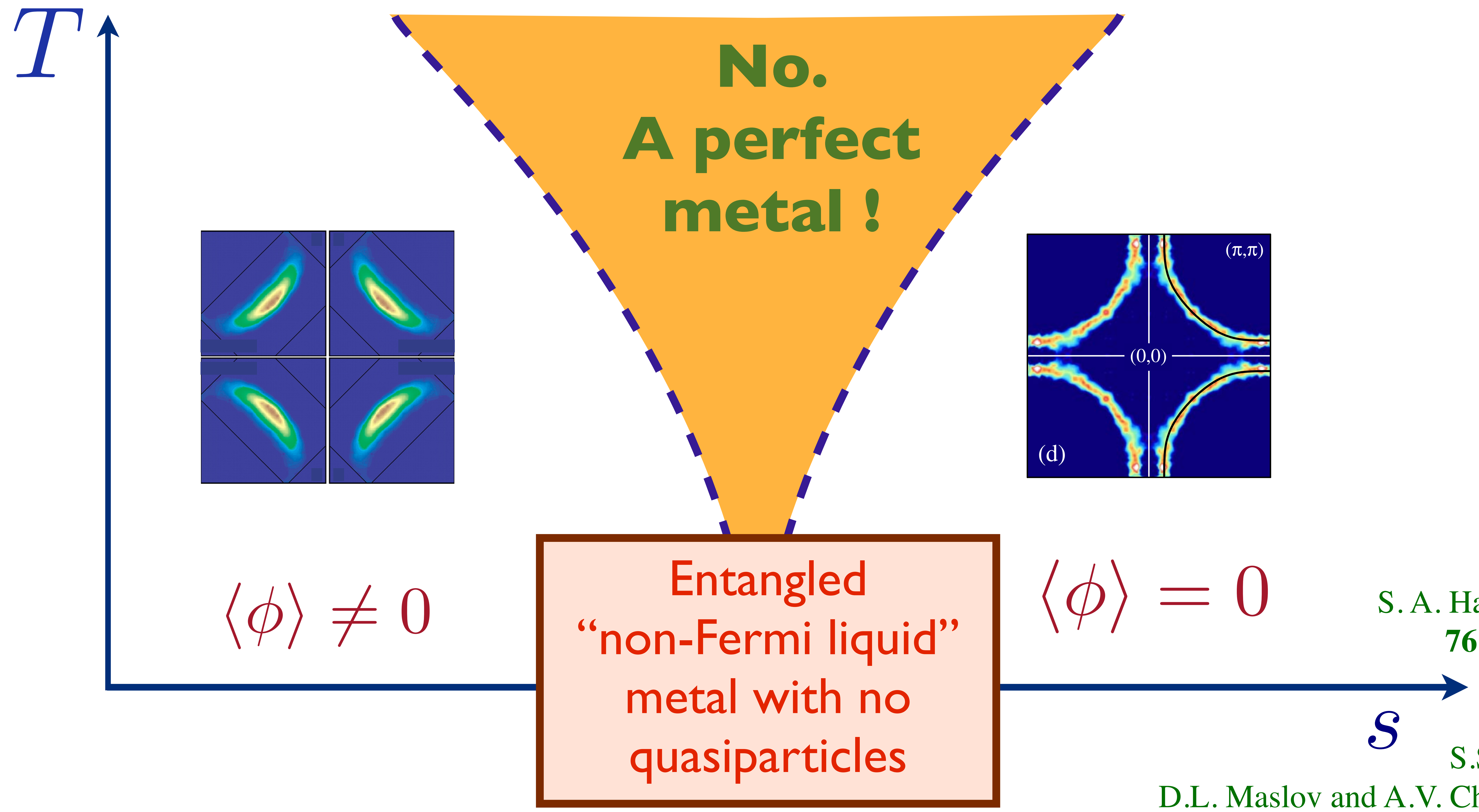

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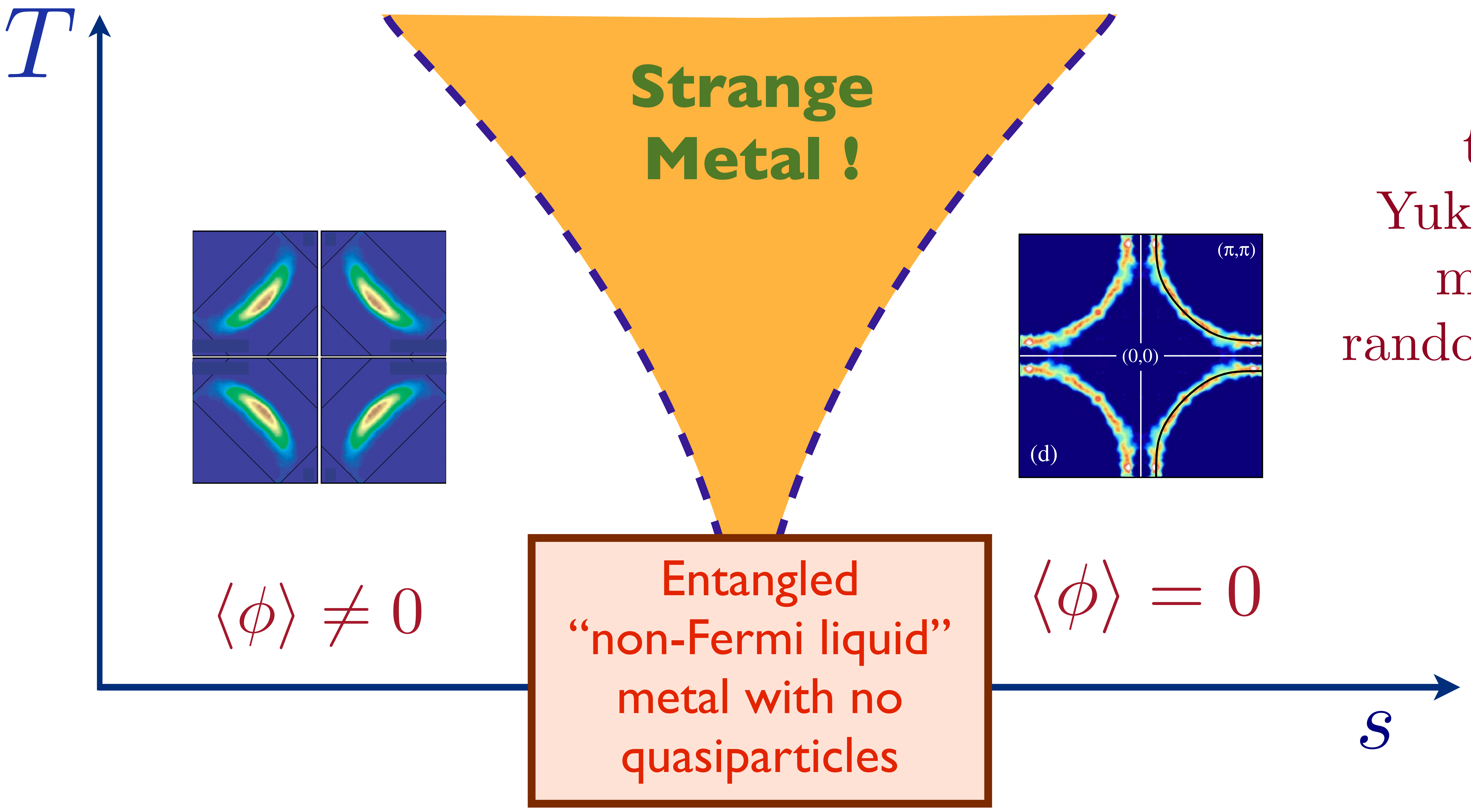
S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007); Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022); Haoyu Guo, Davide Valentinis, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024); D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017) ; Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)

Quantum phase transitions of Fermi surface change

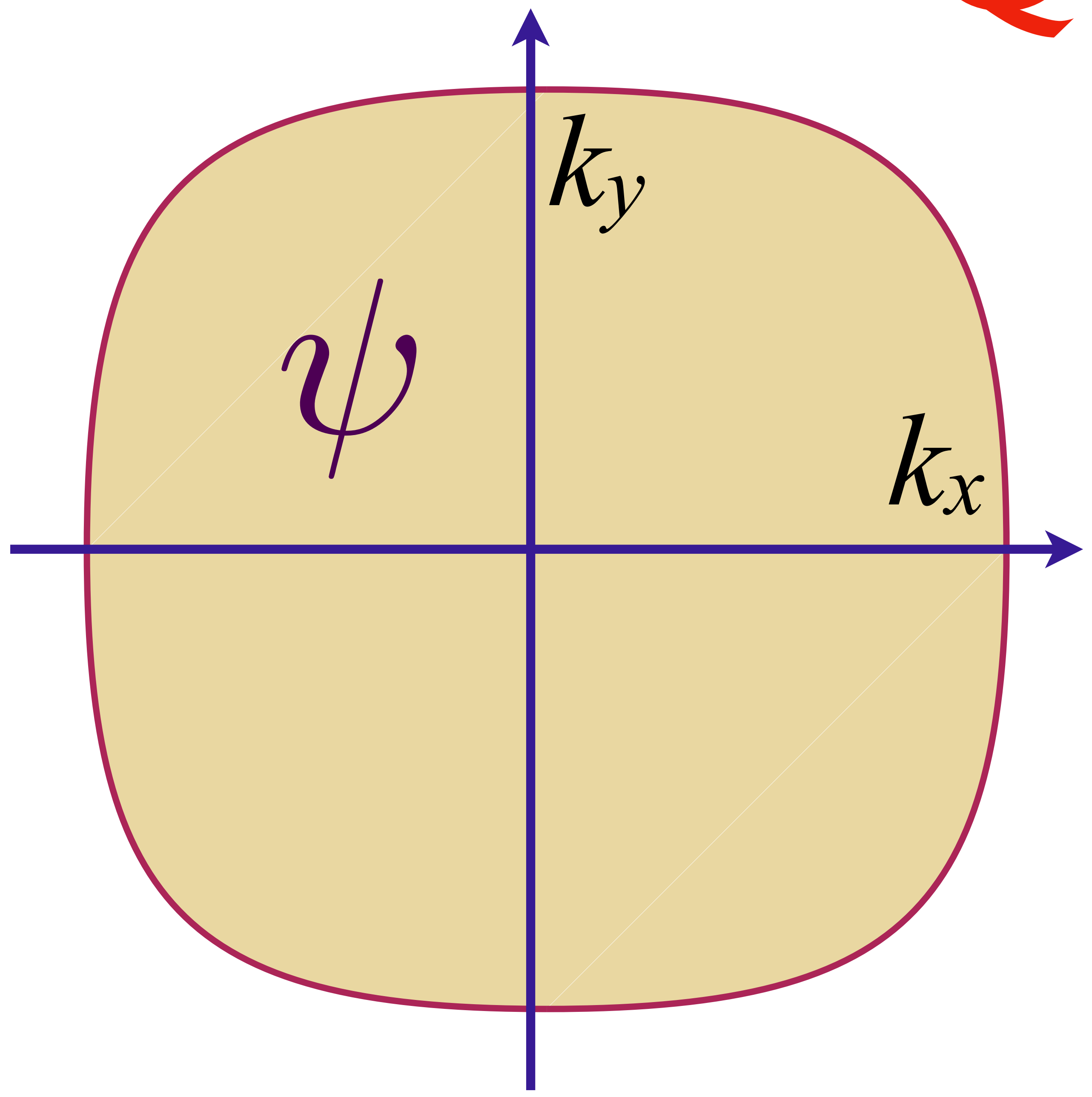


Universal theory of strange metals: the two-dimensional Yukawa-Sachdev-Ye-Kitaev model with a spatially random Yukawa coupling $g(\boldsymbol{r})$

spatial disorder in position of quantum critical point

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Quantum phase transitions of Fermi surface change



$$[g_{\alpha\beta\gamma} + g'_{\alpha\beta\gamma}(\boldsymbol{r})]\, \psi^\dagger_\alpha(\boldsymbol{r})\psi_\beta(\boldsymbol{r})\, \phi_\gamma(\boldsymbol{r})$$

Universal theory of strange metals: the two-dimensional Yukawa-Sachdev-Ye-Kitaev model with a spatially random Yukawa coupling $g(\boldsymbol{r})$

Spatially uniform Yukawa coupling g

with $\overline{g_{\alpha\beta\gamma}} = 0,\ \overline{g_{\alpha\beta\gamma} g_{abc}} = g^2 \delta_{\alpha a}\delta_{\beta b}\delta_{\gamma c}$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$

with $\overline{g'_{\alpha\beta\gamma}(\boldsymbol{r})} = 0,\ \overline{g'_{\alpha\beta\gamma}(\boldsymbol{r}) g'_{abc}(\boldsymbol{r}')} = g'^2 \delta_{\alpha a}\delta_{\beta b}\delta_{\gamma c}\delta(\boldsymbol{r} - \boldsymbol{r}')$

spatial disorder in position of quantum critical point

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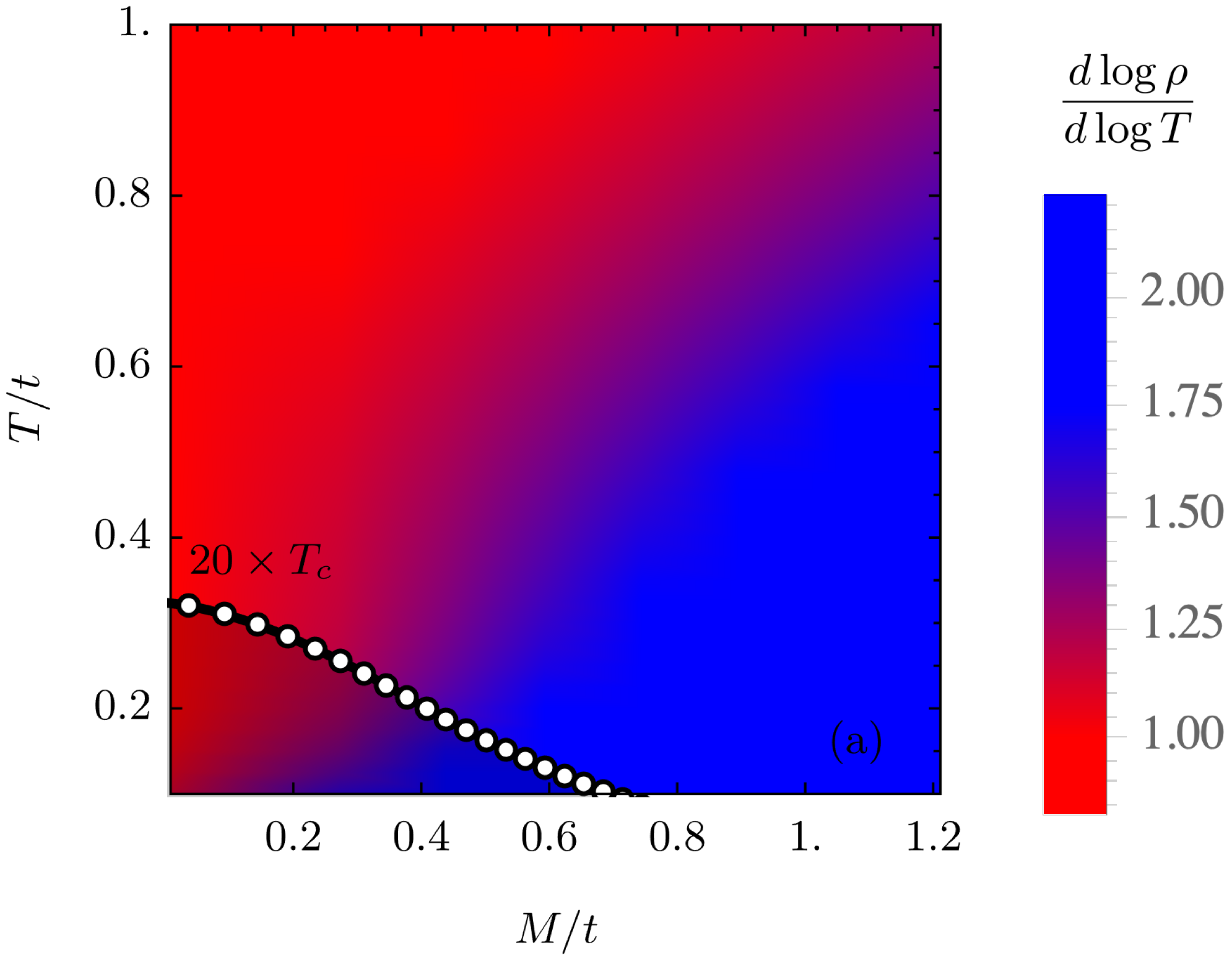
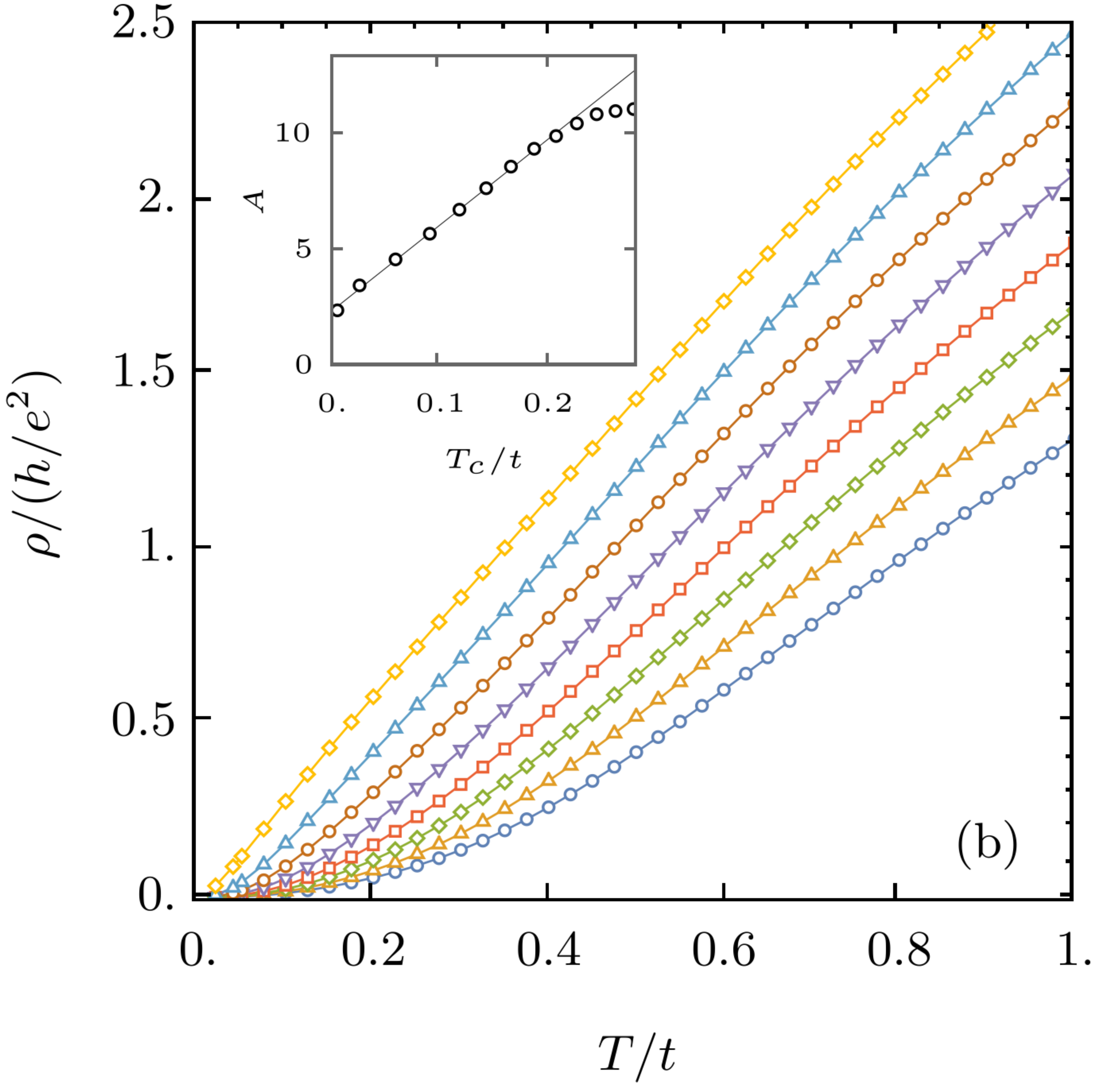
Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, arXiv:2406.07608



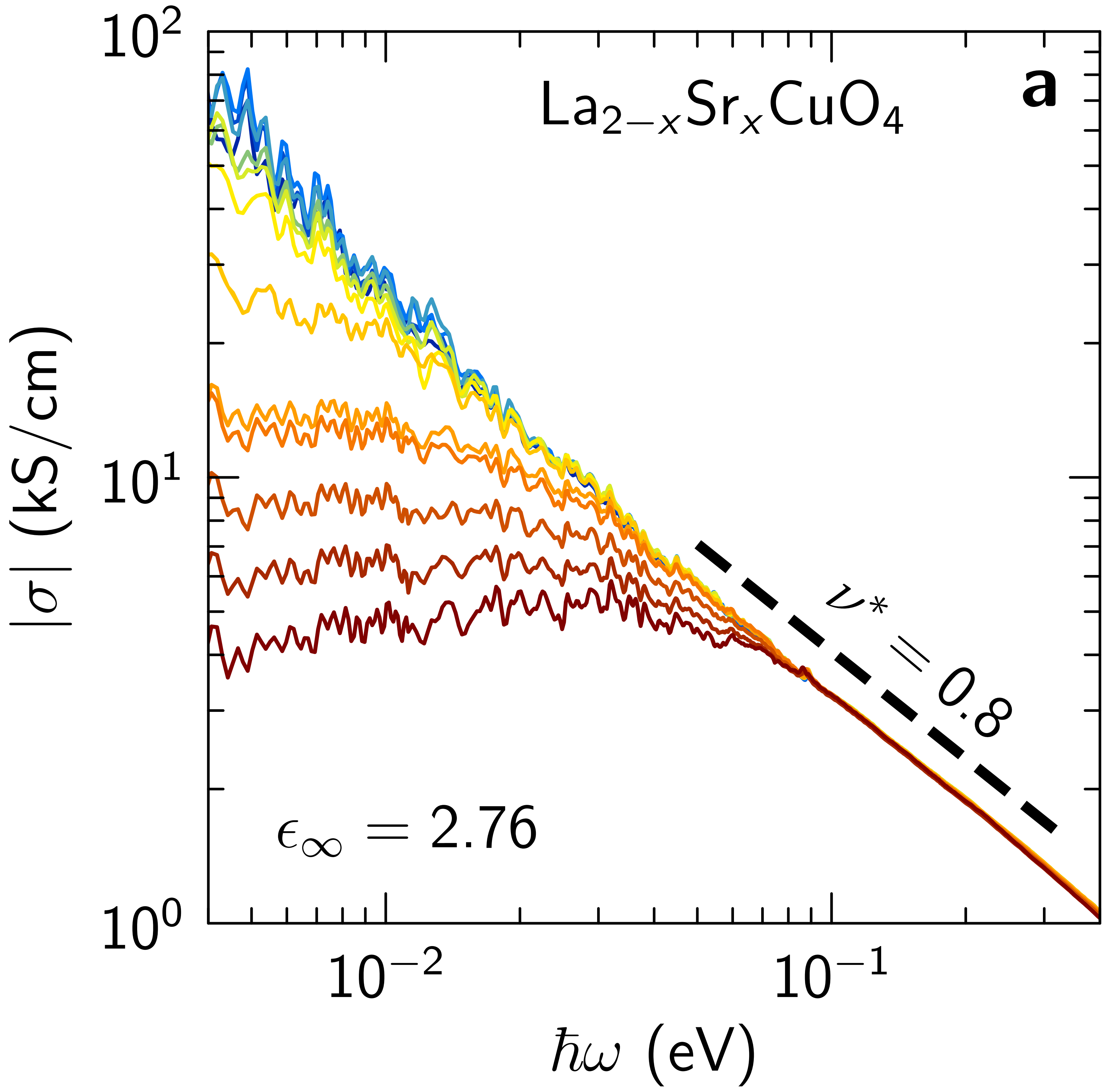


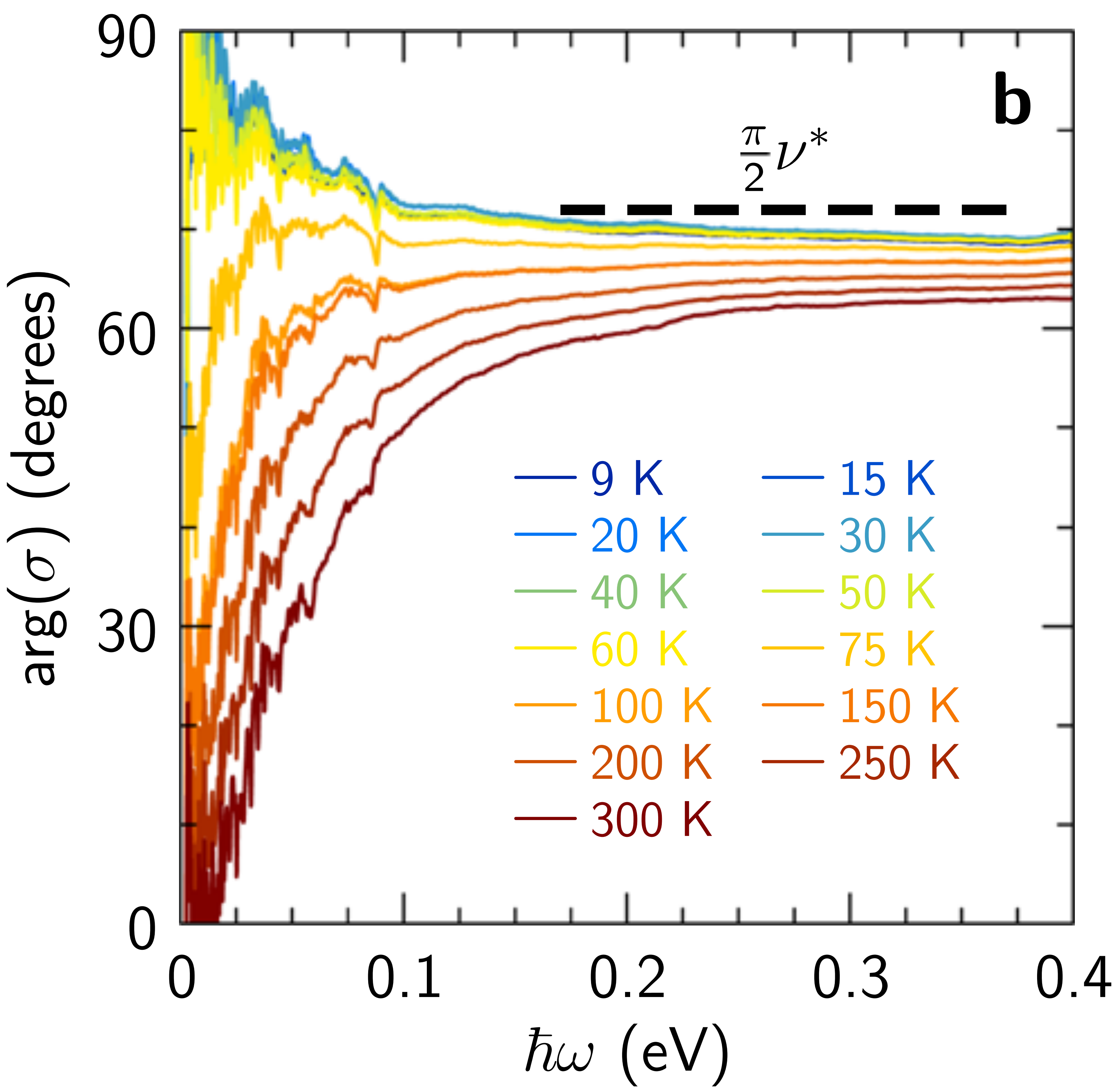
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





$La_{2-x}Sr_xCuO_4$

$p = 0.24$

$T_c = 19$ K

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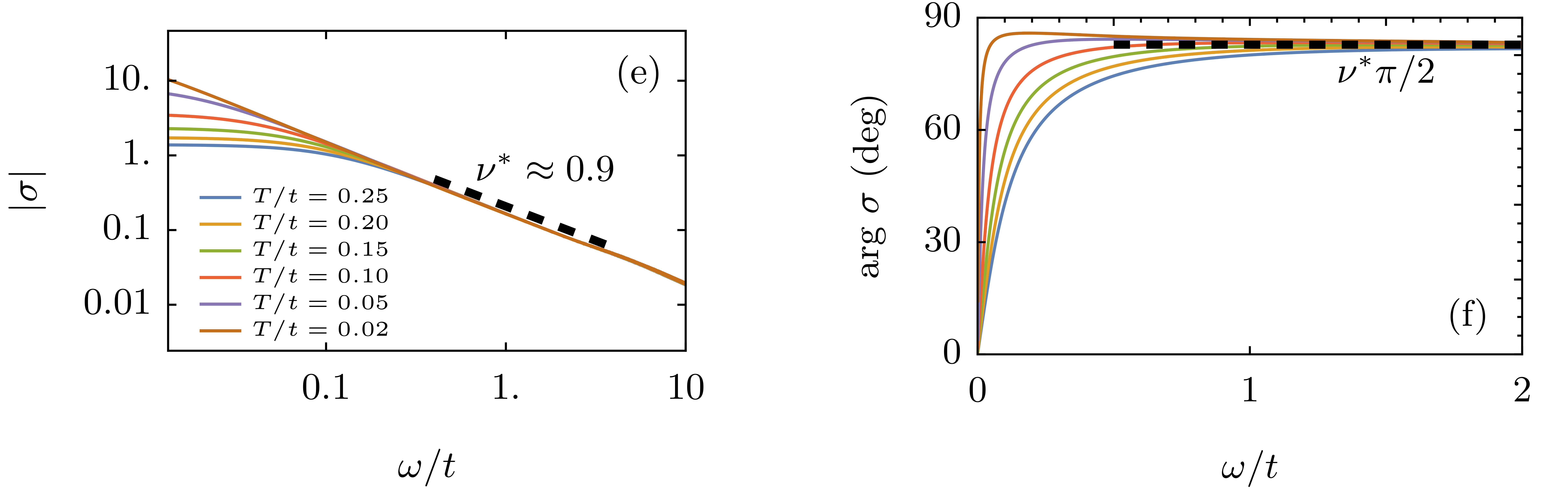

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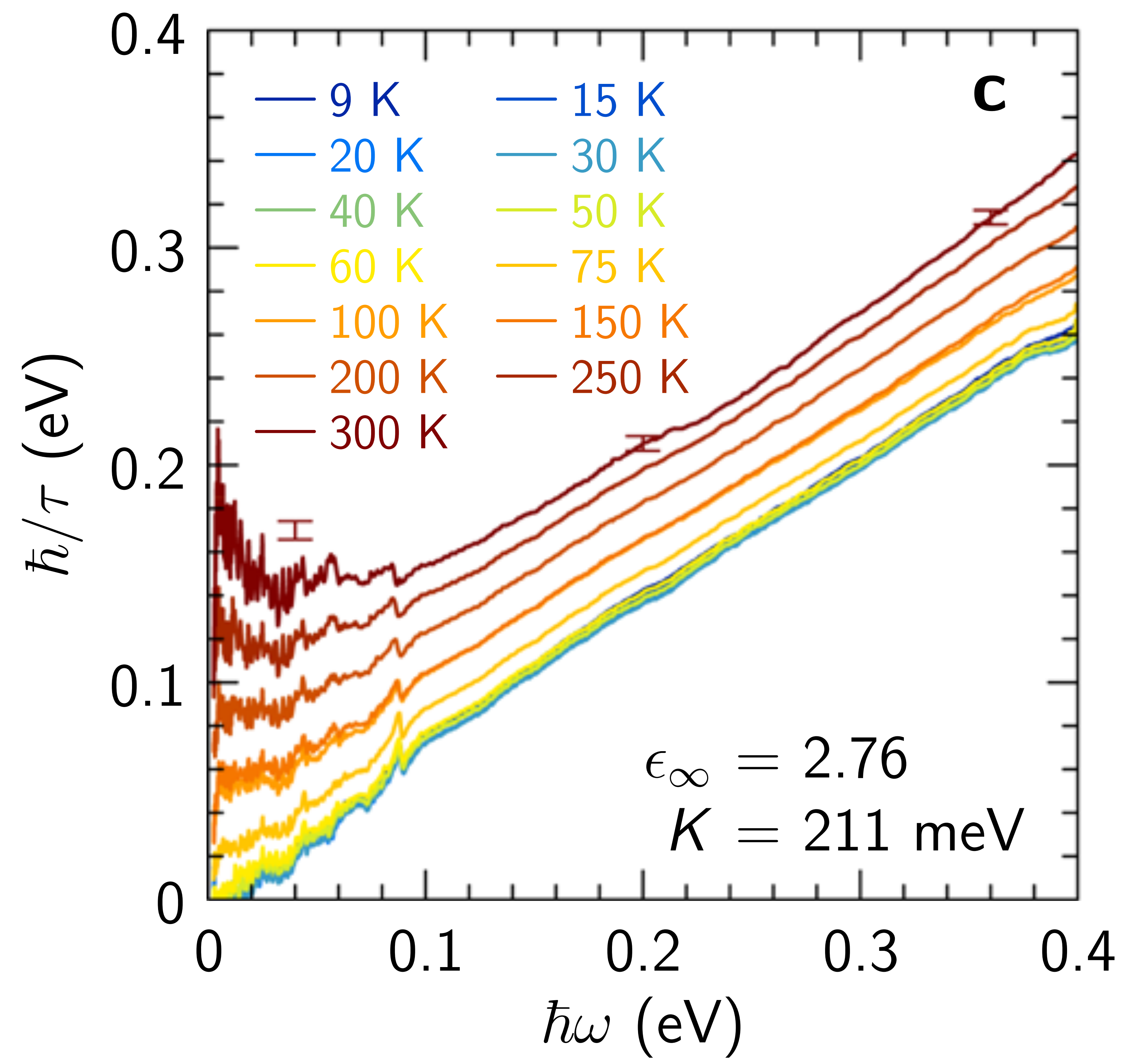

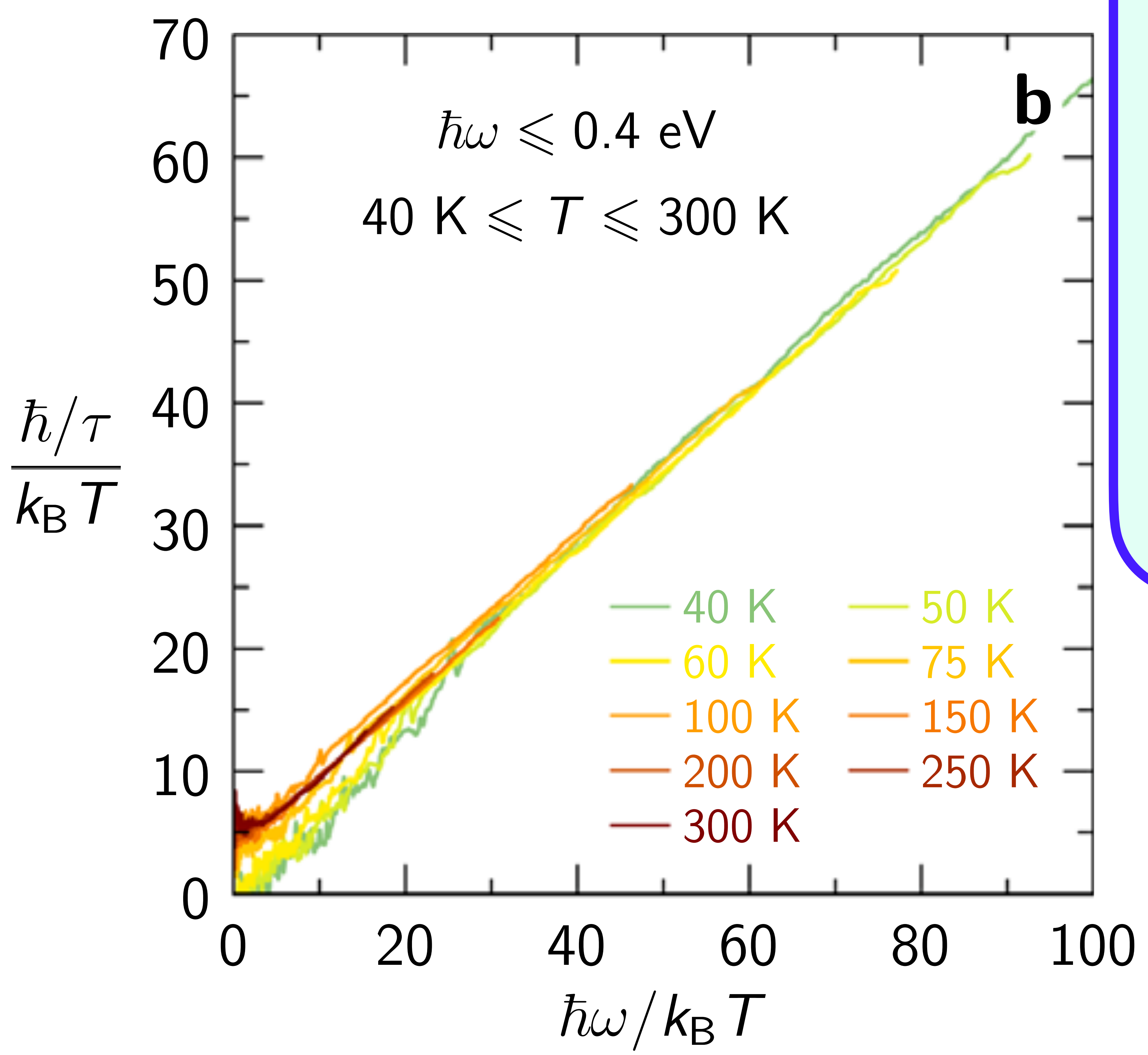

Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

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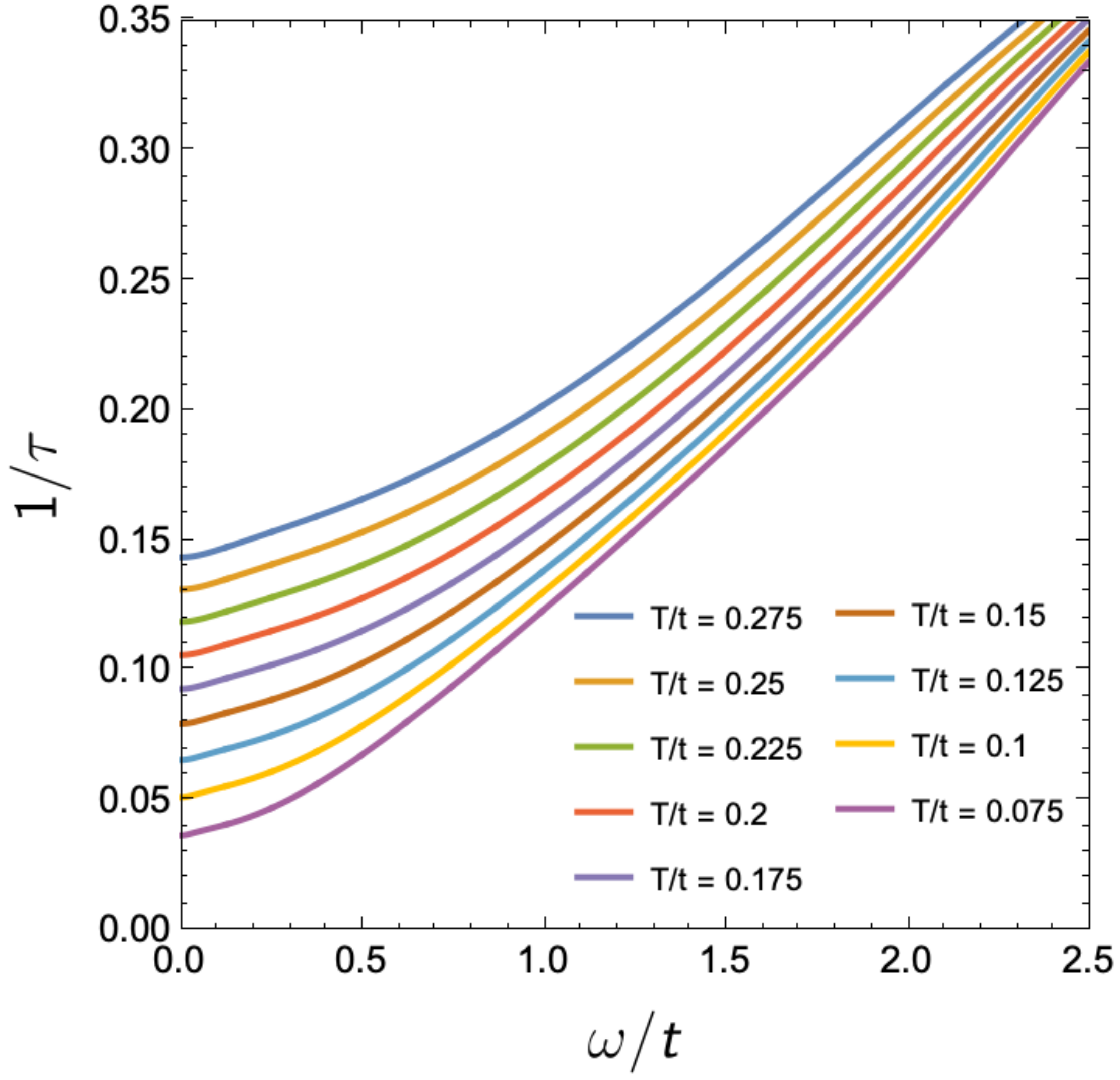
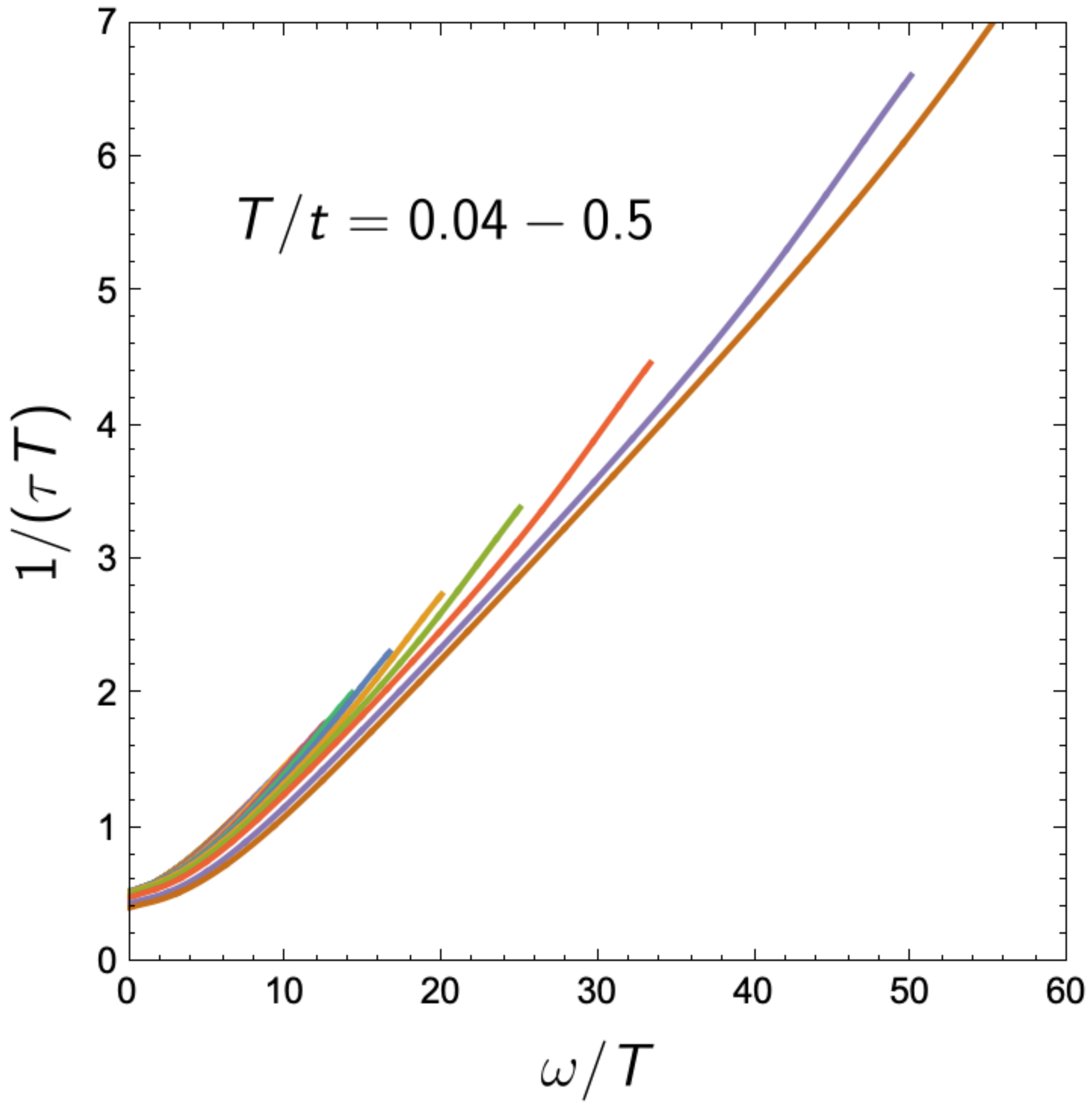




Planckian dynamics!

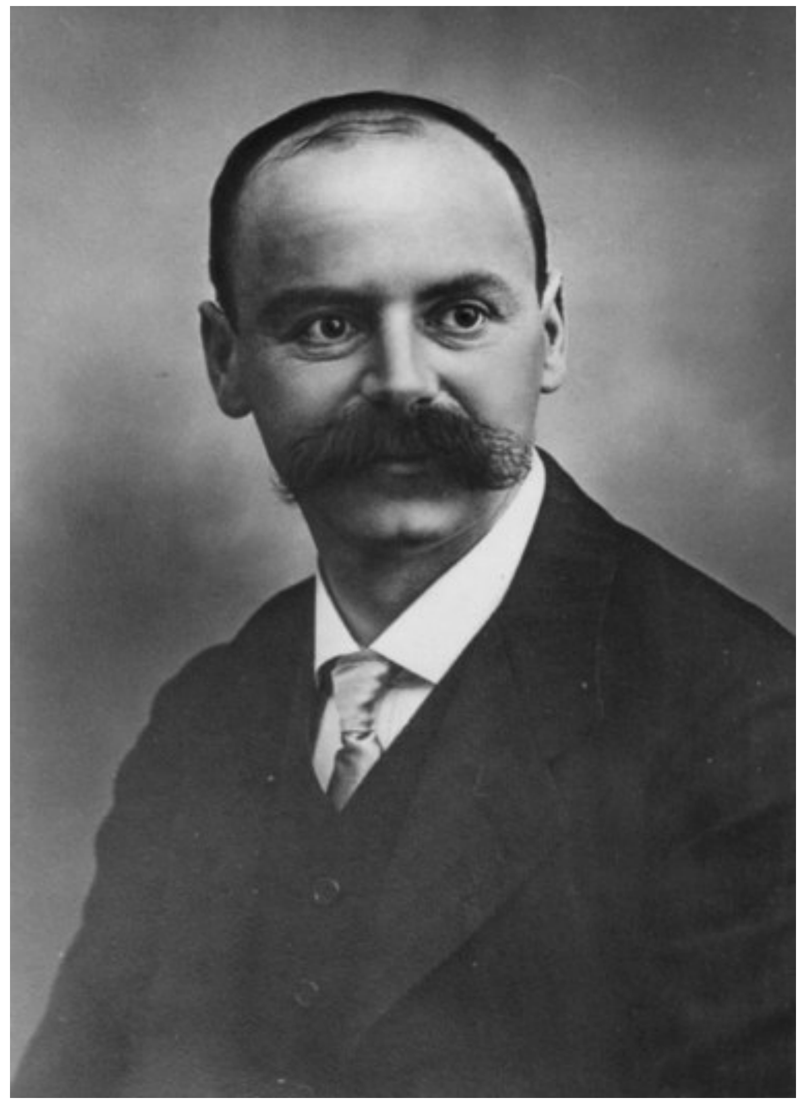
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Black holes
(1916)

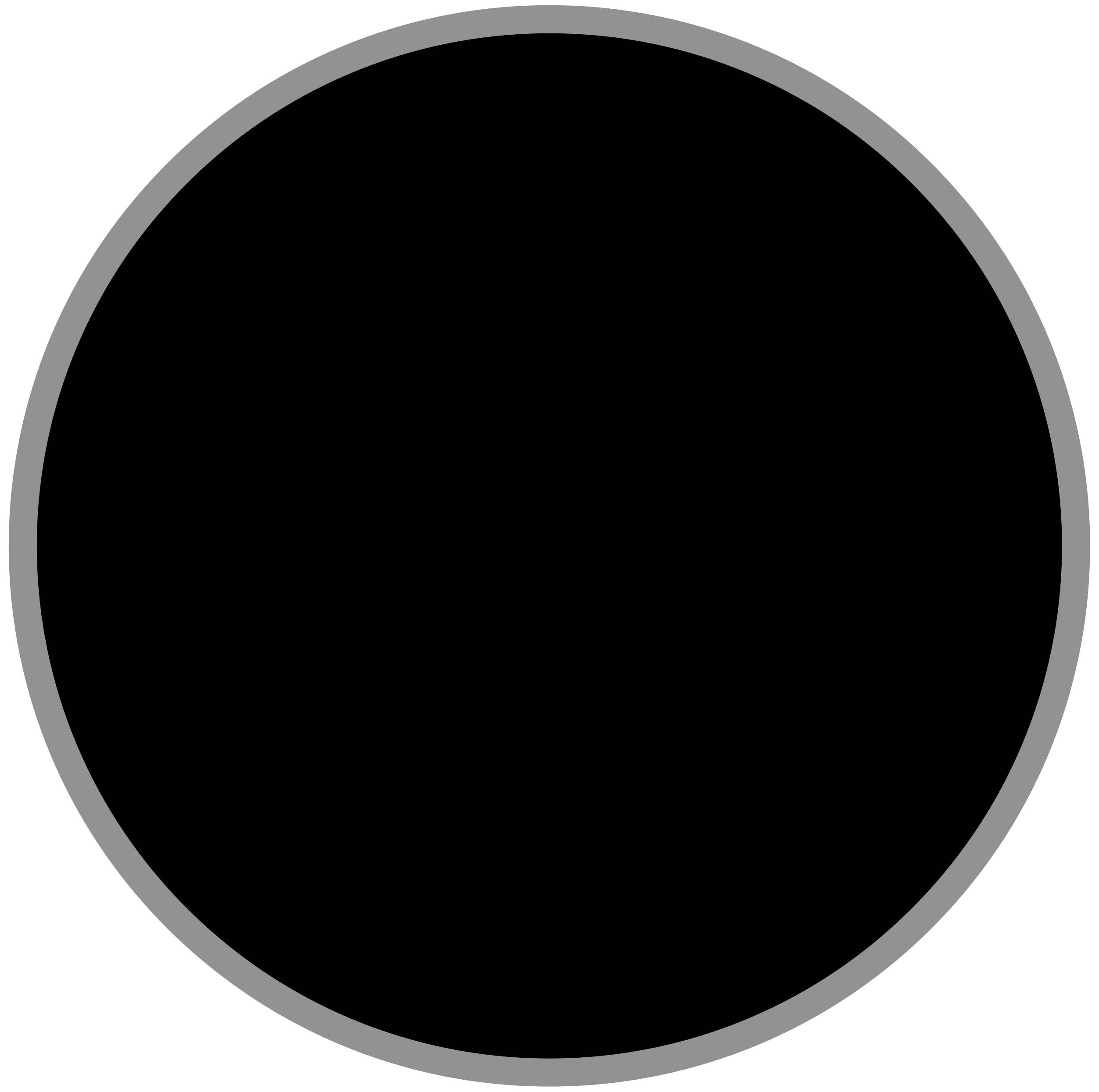
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

Horizon radius $R = \frac{2GM}{c^2}$

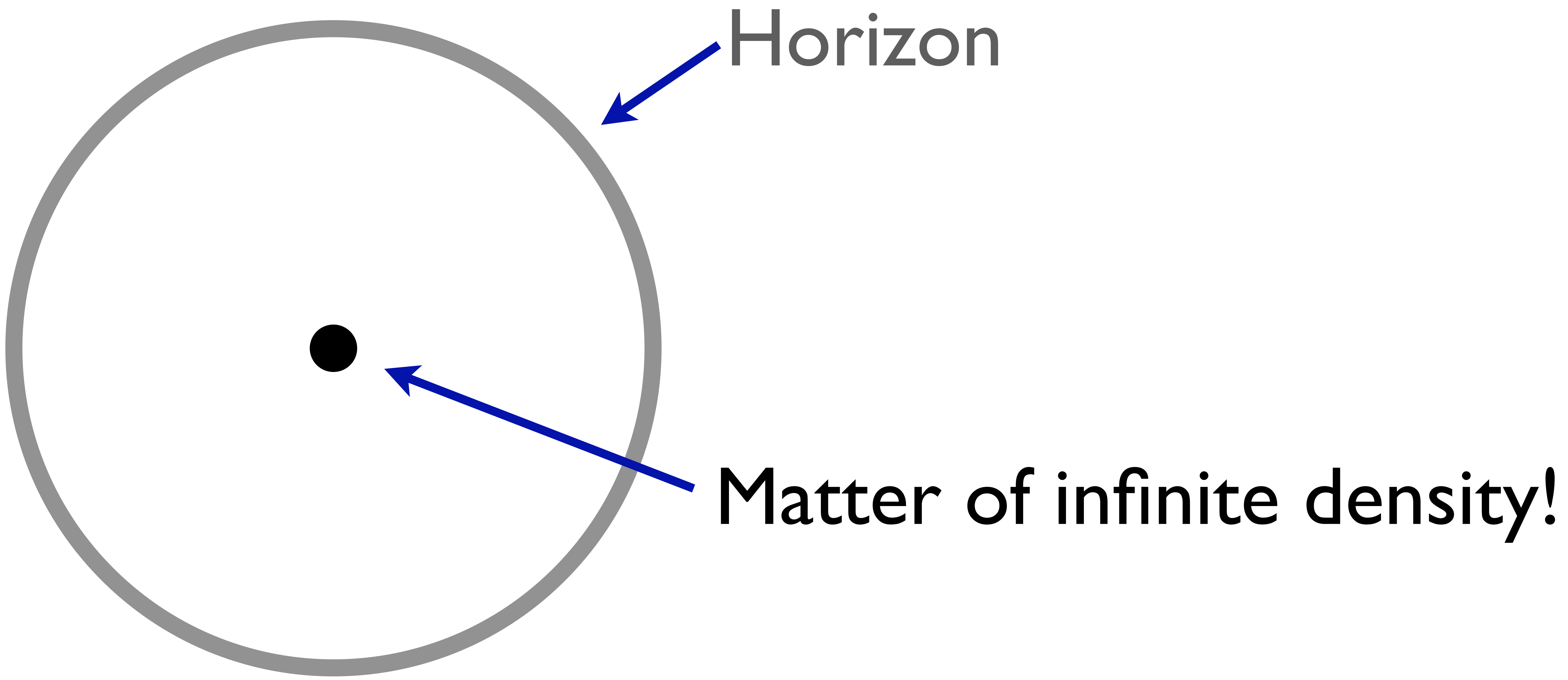


G Newton's constant, c velocity of light, M mass of black hole

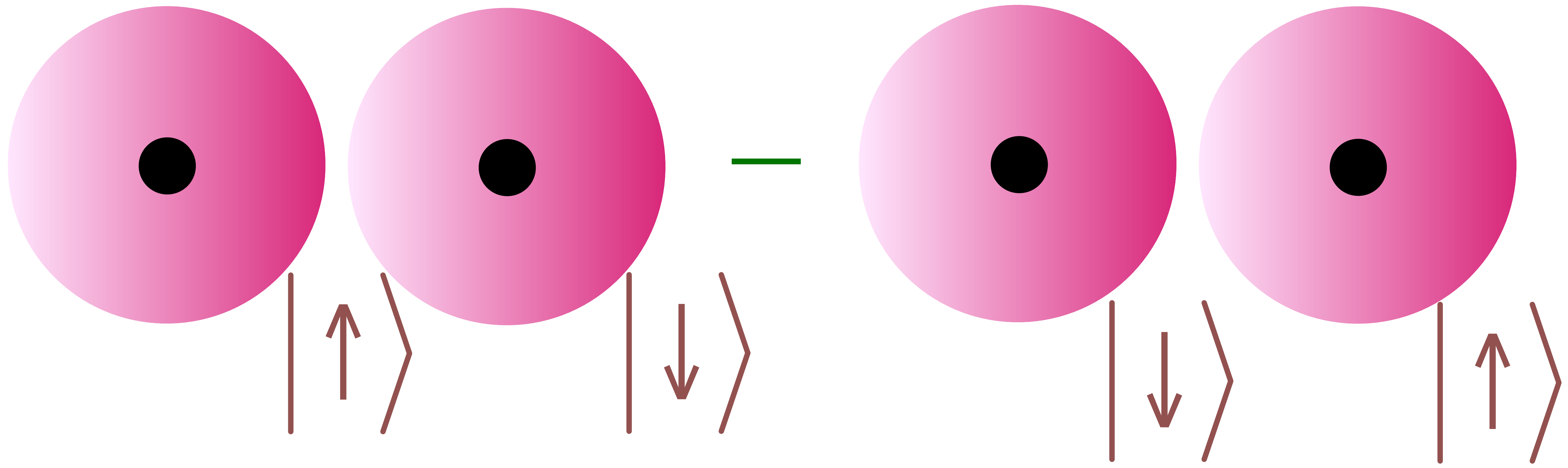
For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



Quantum Entanglement across a black hole horizon



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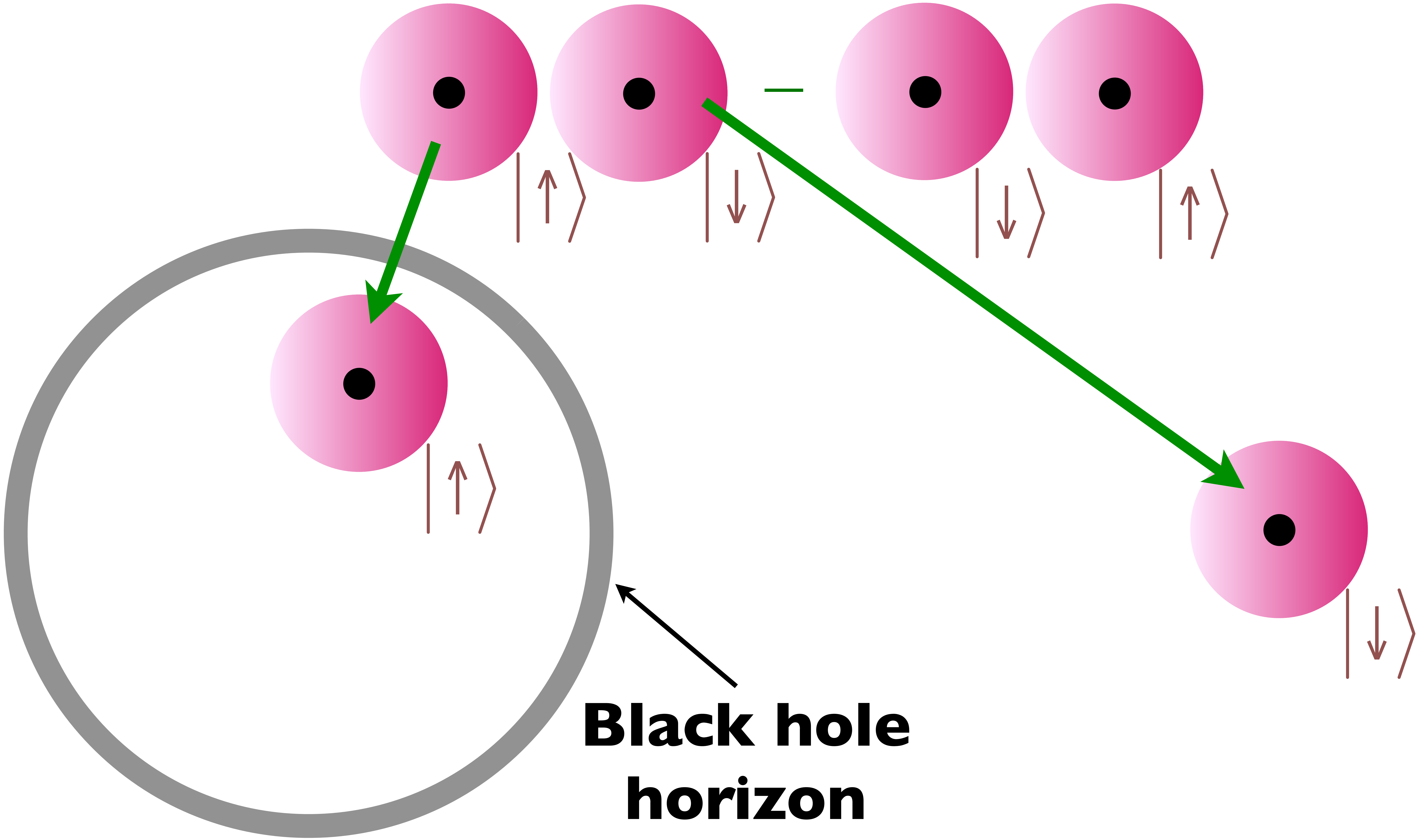

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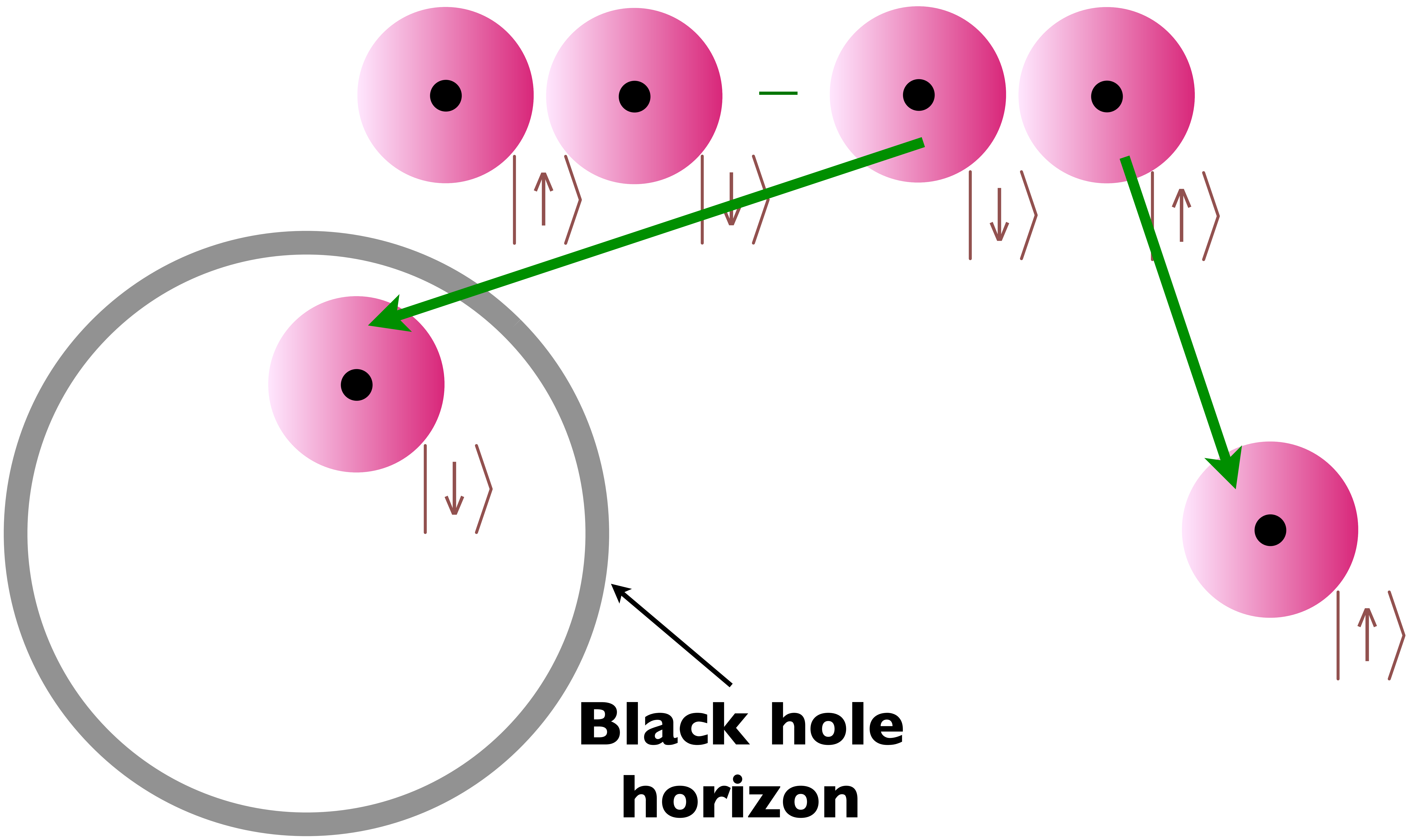
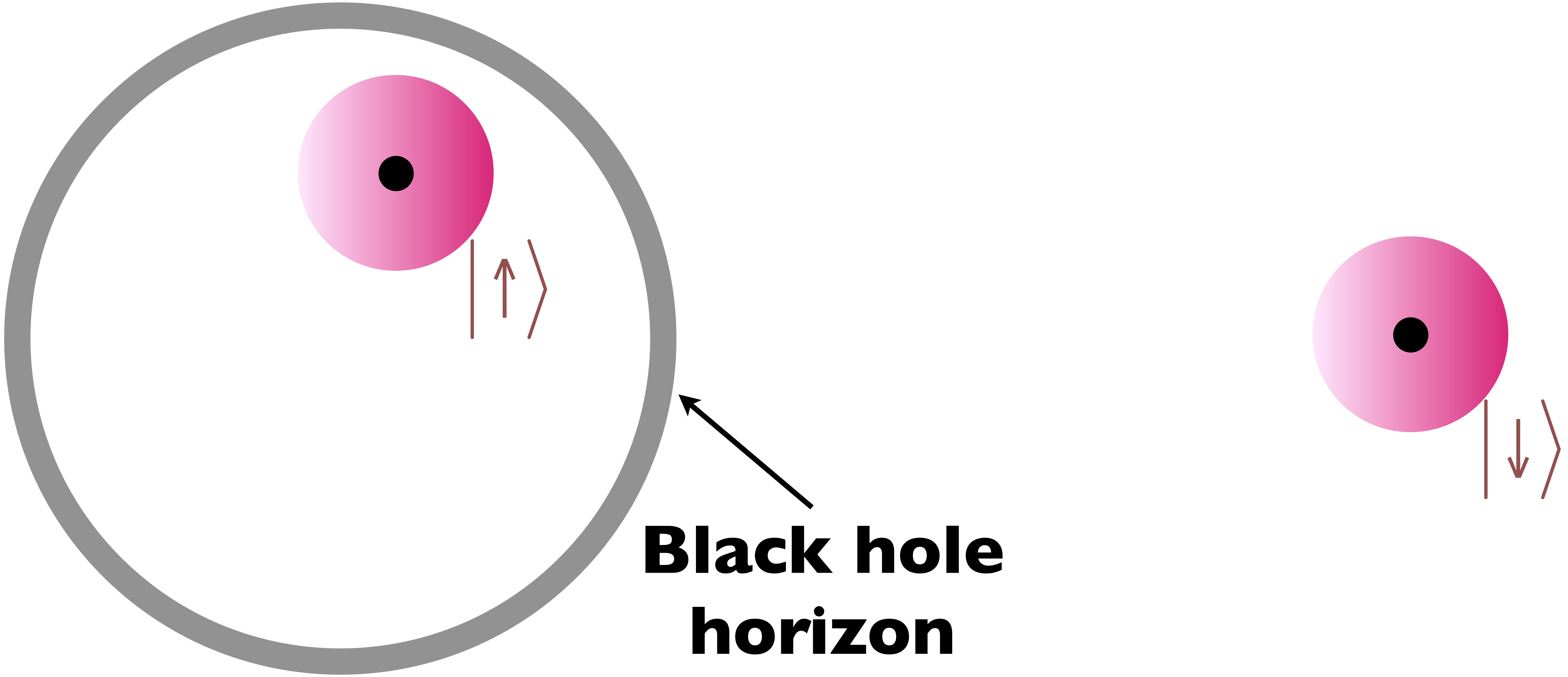


Quantum Entanglement across a black hole horizon

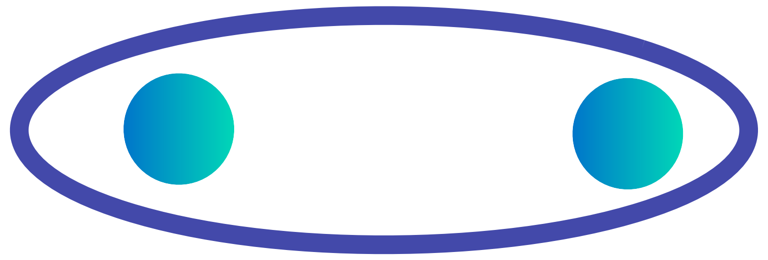
Bekenstein, Hawking: Black holes have a temperature and an entropy!

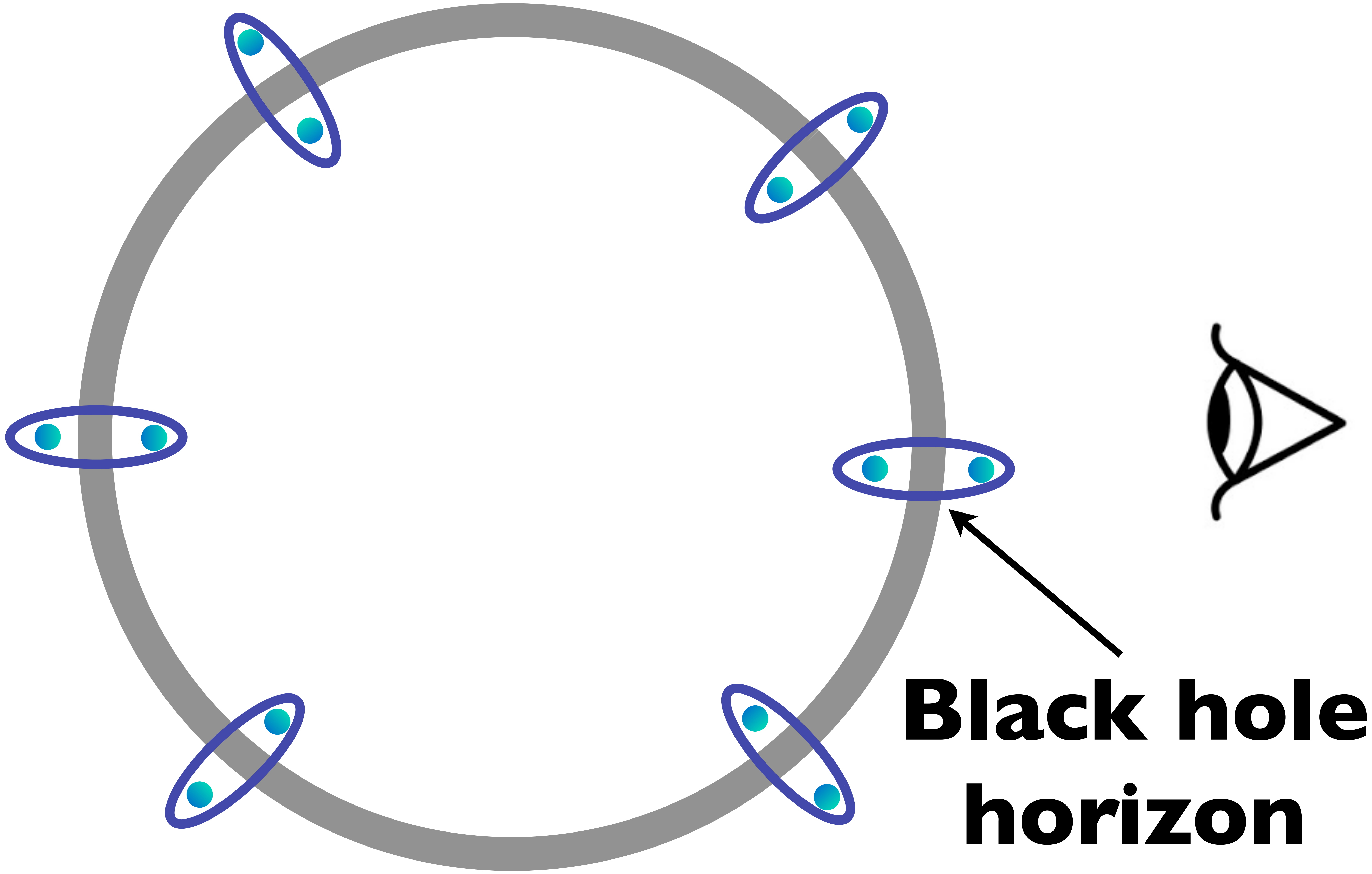
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



By computations *outside* the black hole, Bekenstein-Hawking obtained

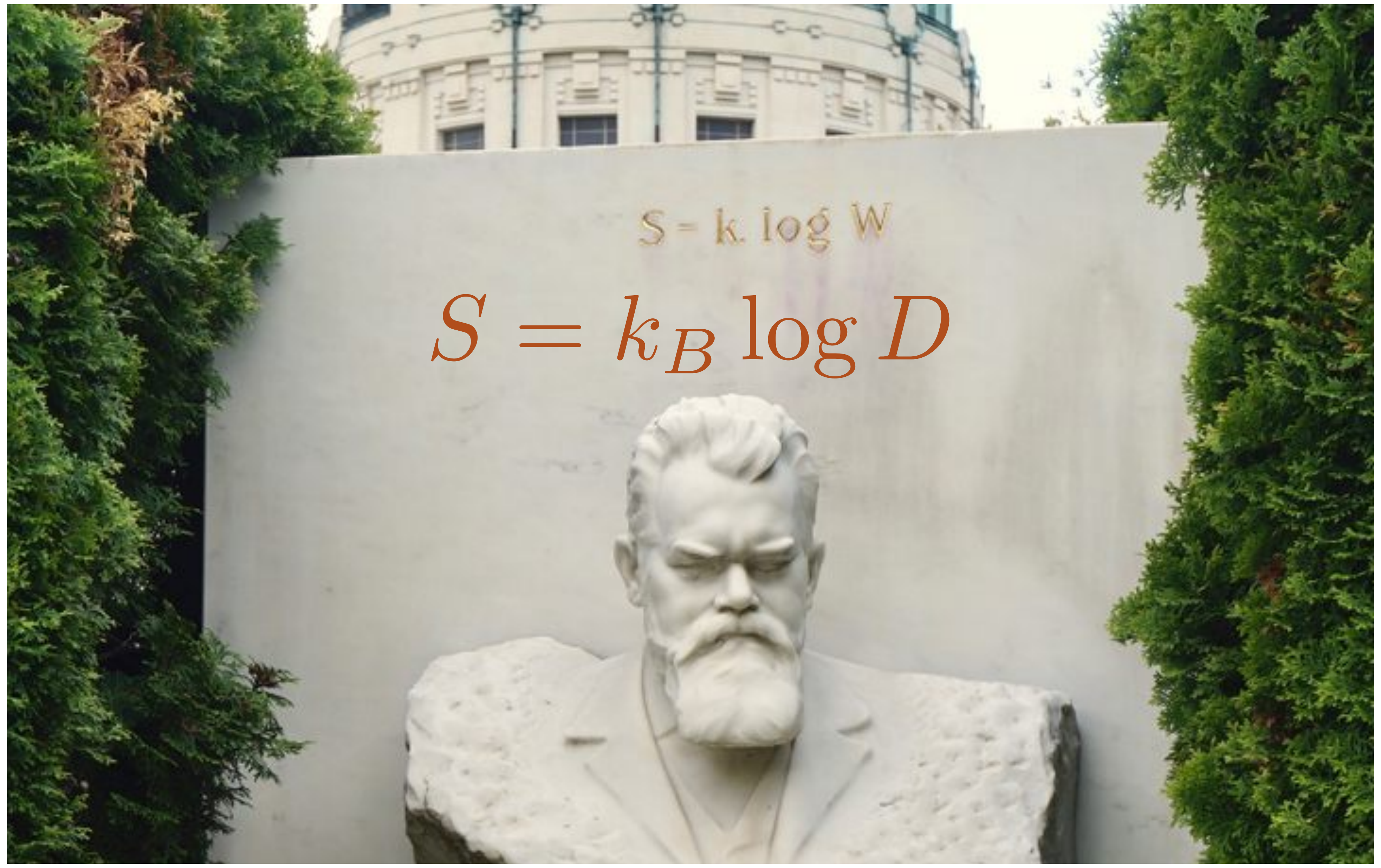
$$S = \frac{k_B A c^3}{4G\hbar}$$

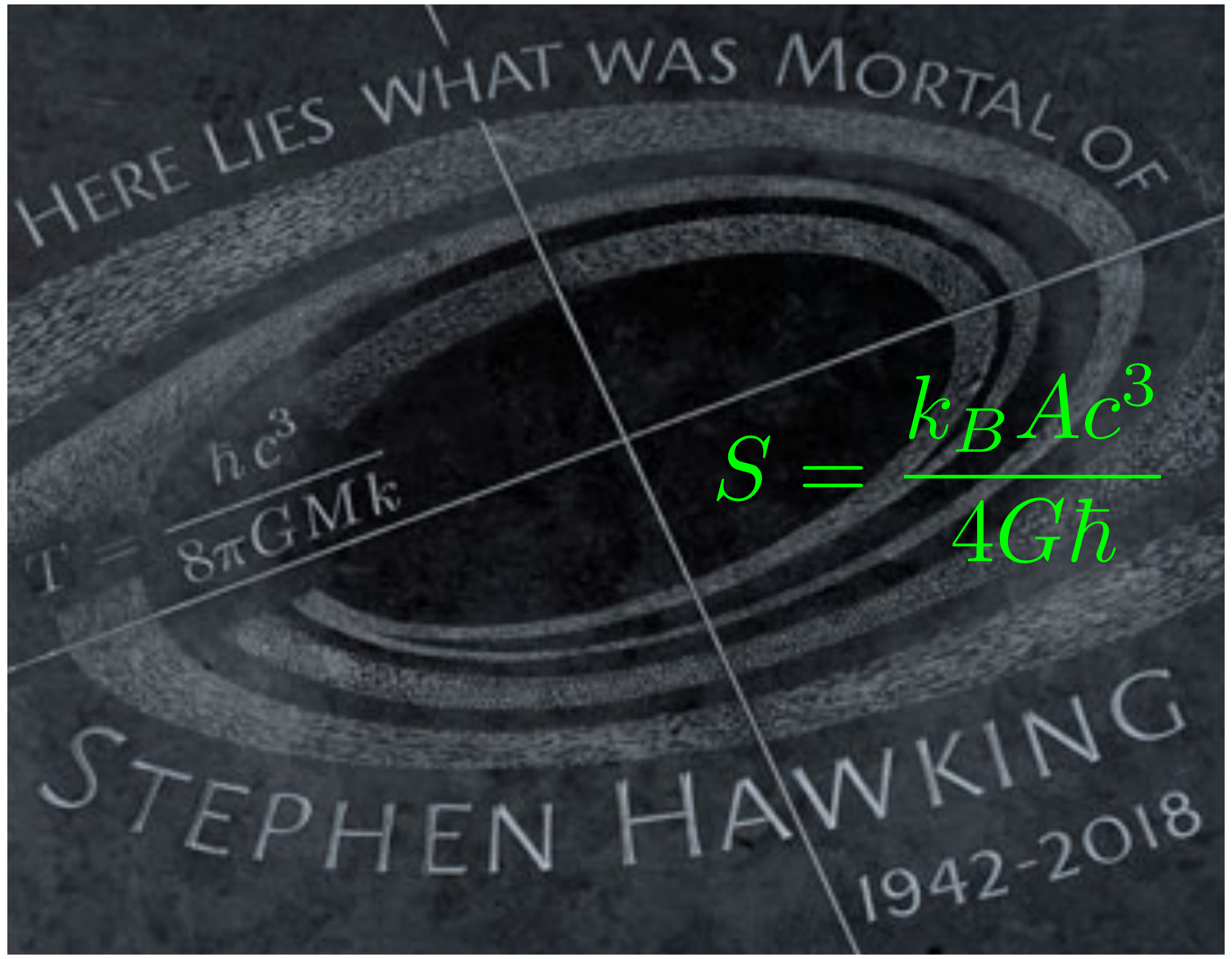
where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

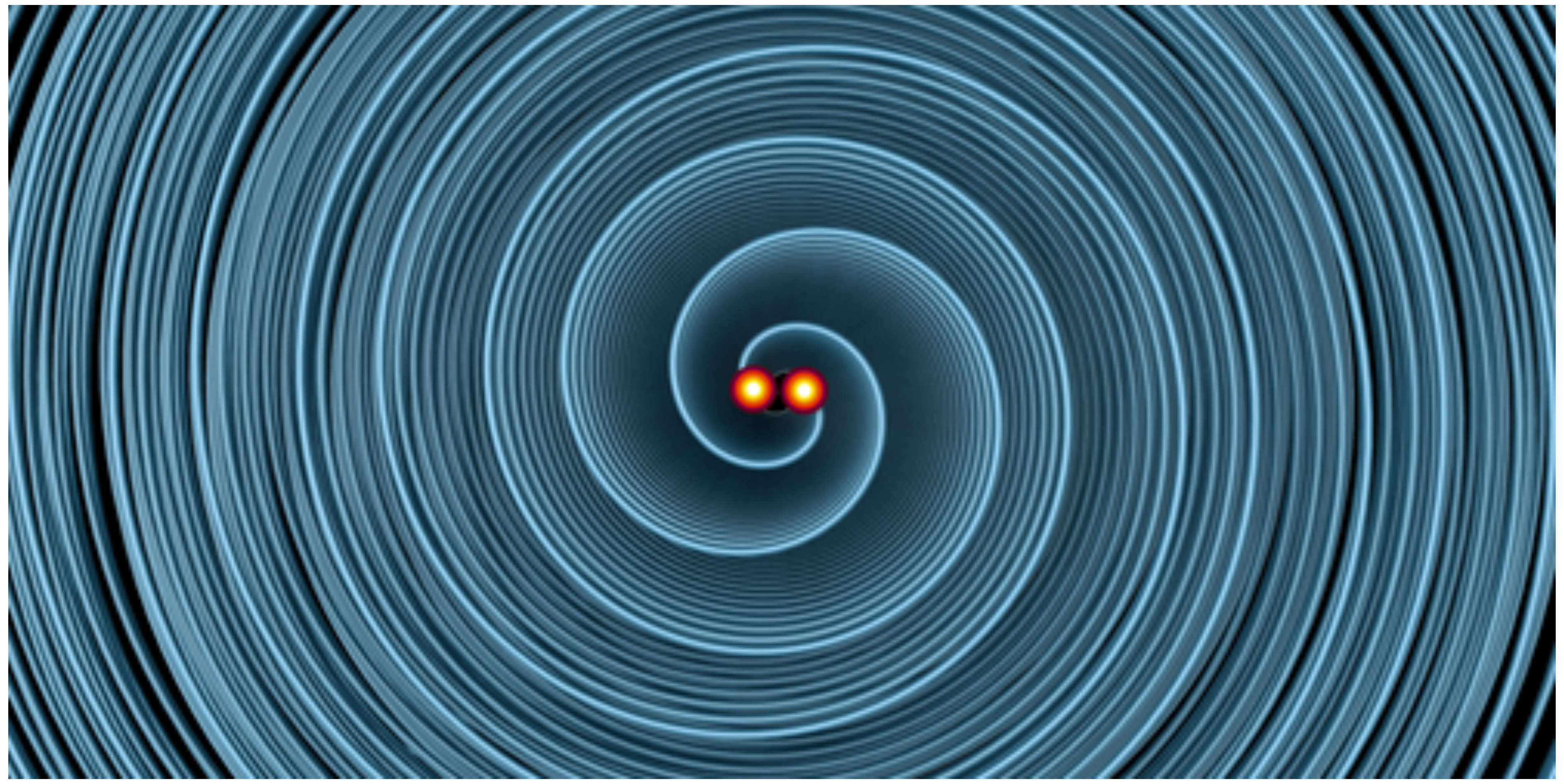
An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.

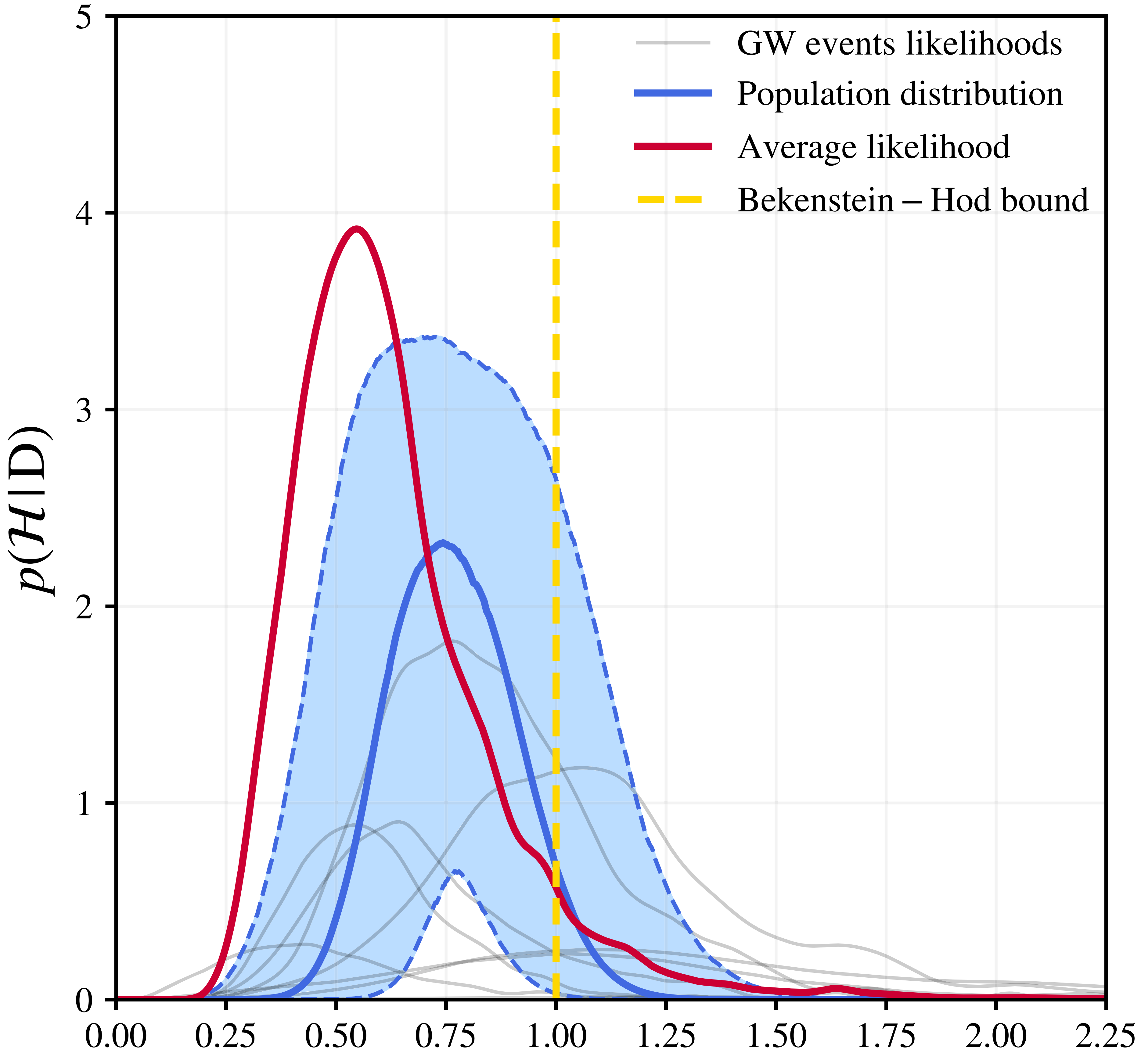
Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$





$$\mathcal{H} = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$

Quantum Black Holes

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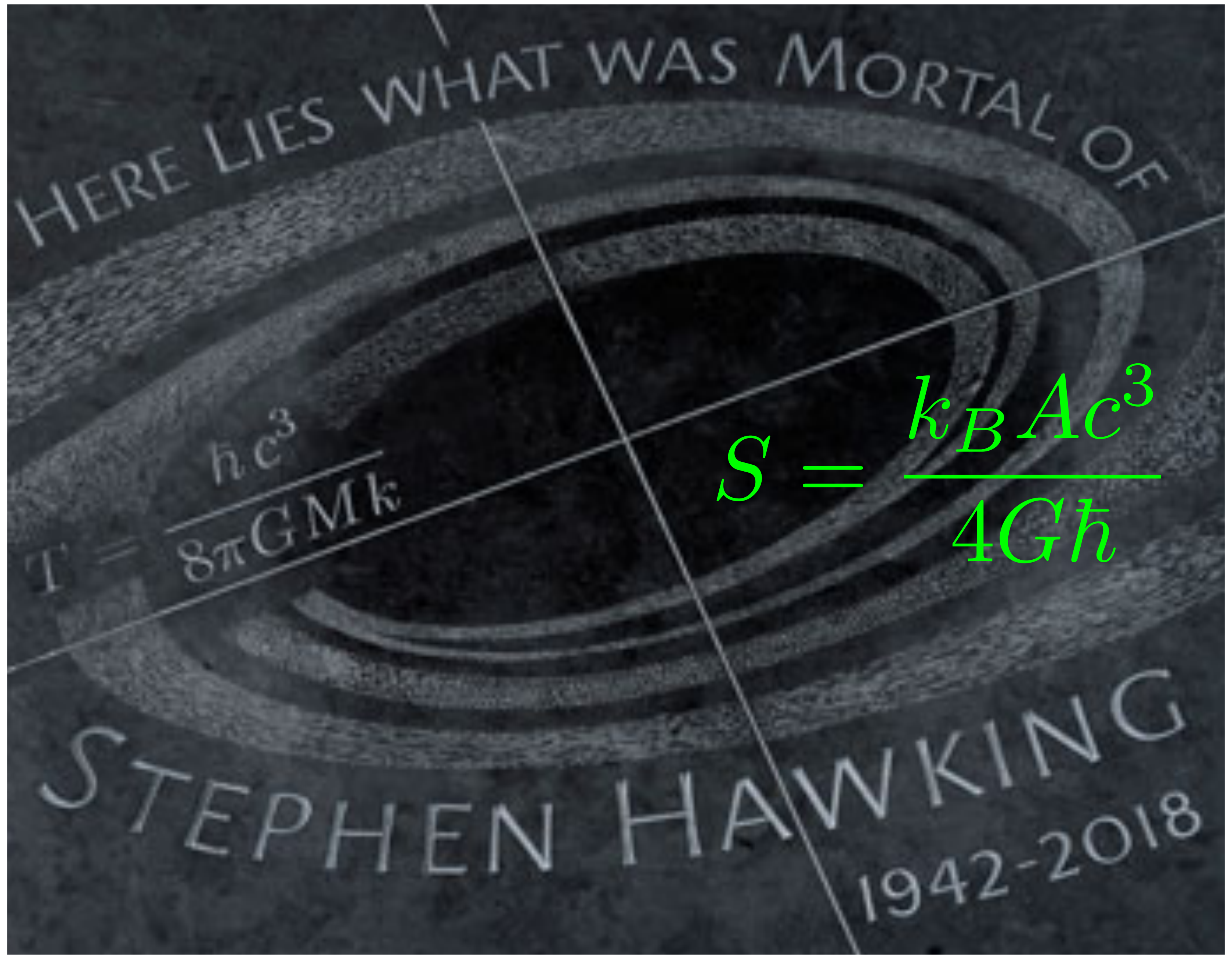
$$S = \frac{k_B A c^3}{4G\hbar} \qquad T = \frac{\hbar c^3}{8\pi G M k_B}$$

$$\tau_{\mathrm{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!



J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974); C.V. Vishveshwara, Nature **227**, 936 (1970)

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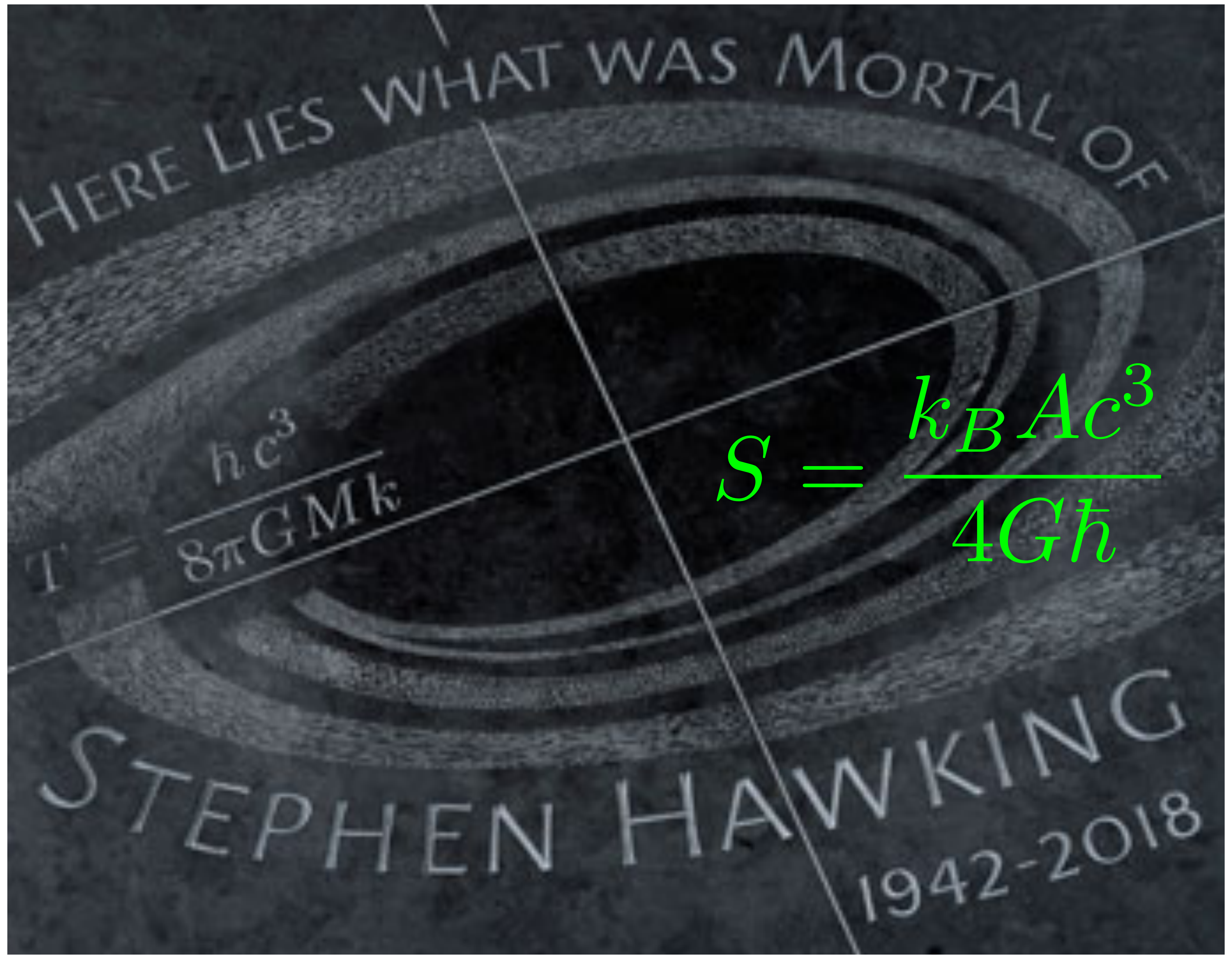
For a black hole with charge Q, the area $A_0 = 2GQ^2/c^4$ as $T \to 0$, and so $S(T \to 0) > 0$.



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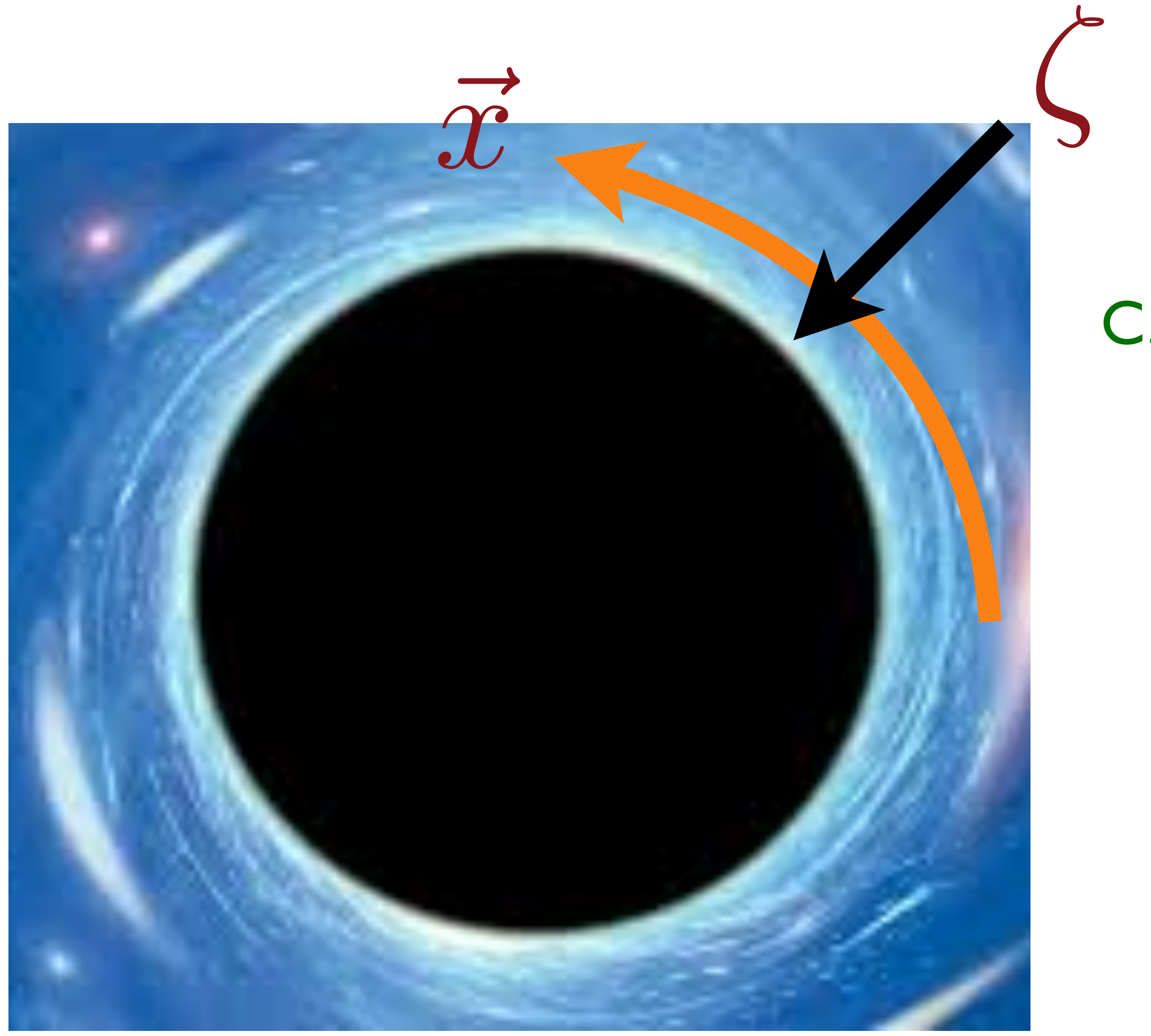
The SYK model and black holes

Semiclassical connection first proposed by S.S. in Physical Review Letters **105**, 151602 (2010): SYK model and charged black holes exhibit Planckian dynamics and zero temperature entropy.

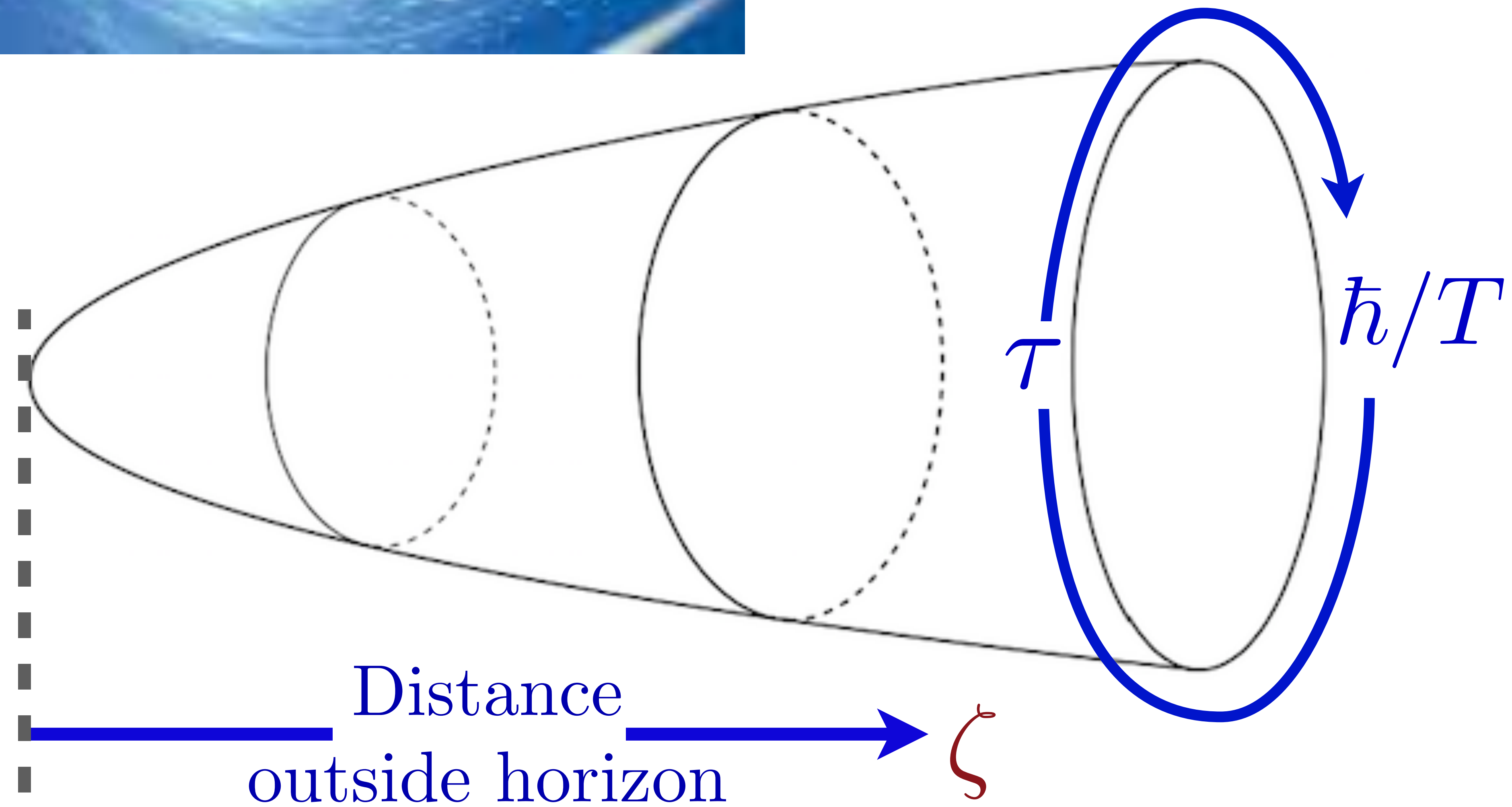
Fully quantum connection established in 2015 by A. Kitaev, J. Maldacena, D. Stanford....

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$



A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)



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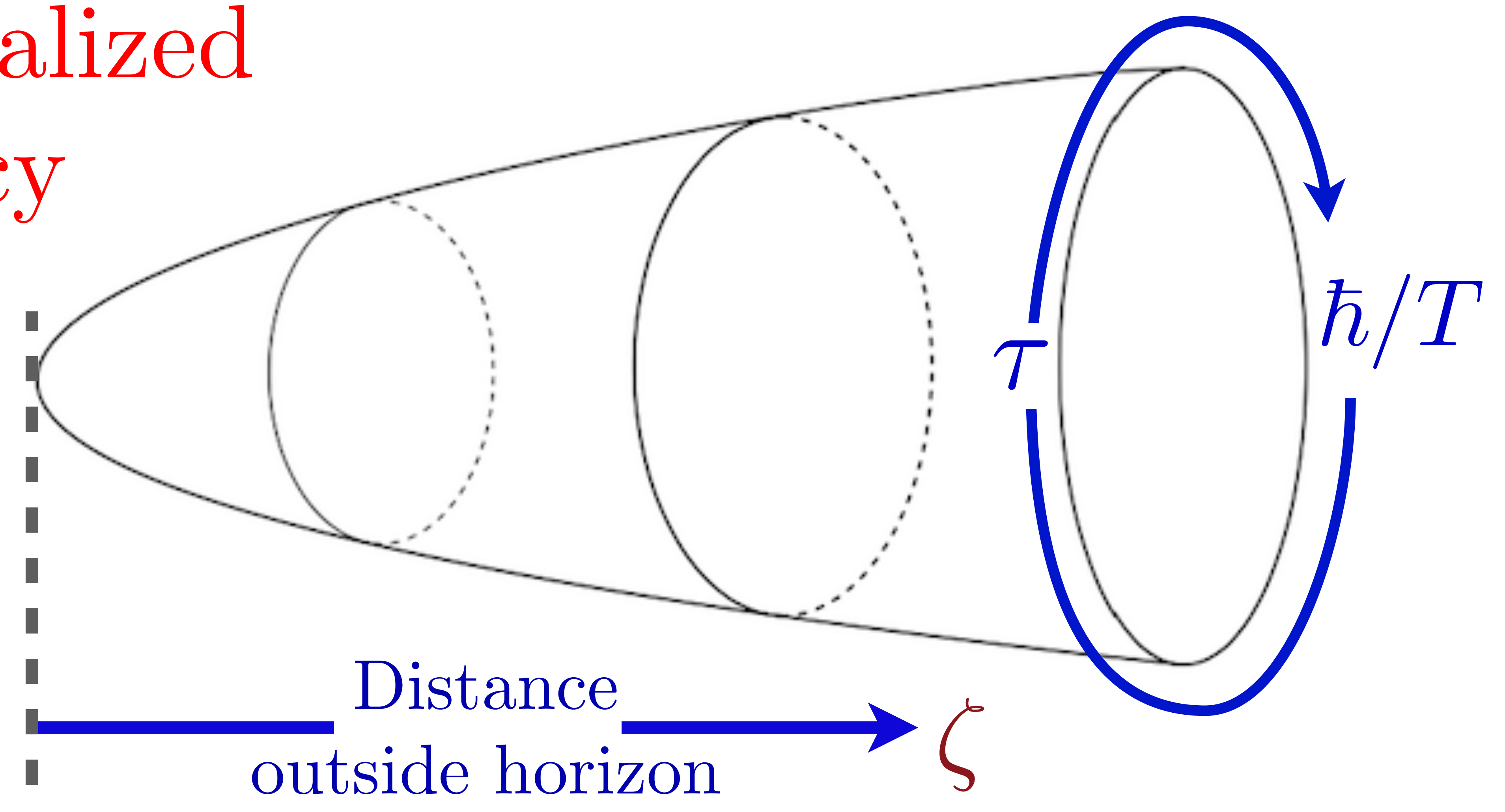
$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \quad \text{as } T \to 0$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

The Bekenstein-Hawking entropy $A_0c^3/(4\hbar G)$ is the analog of the $T \to 0$ GPS entropy, $N s_0$, of the SYK model.

This connection shows that the BH entropy is *not* realized by an exponentially large ground state degeneracy

Sachdev (2010)



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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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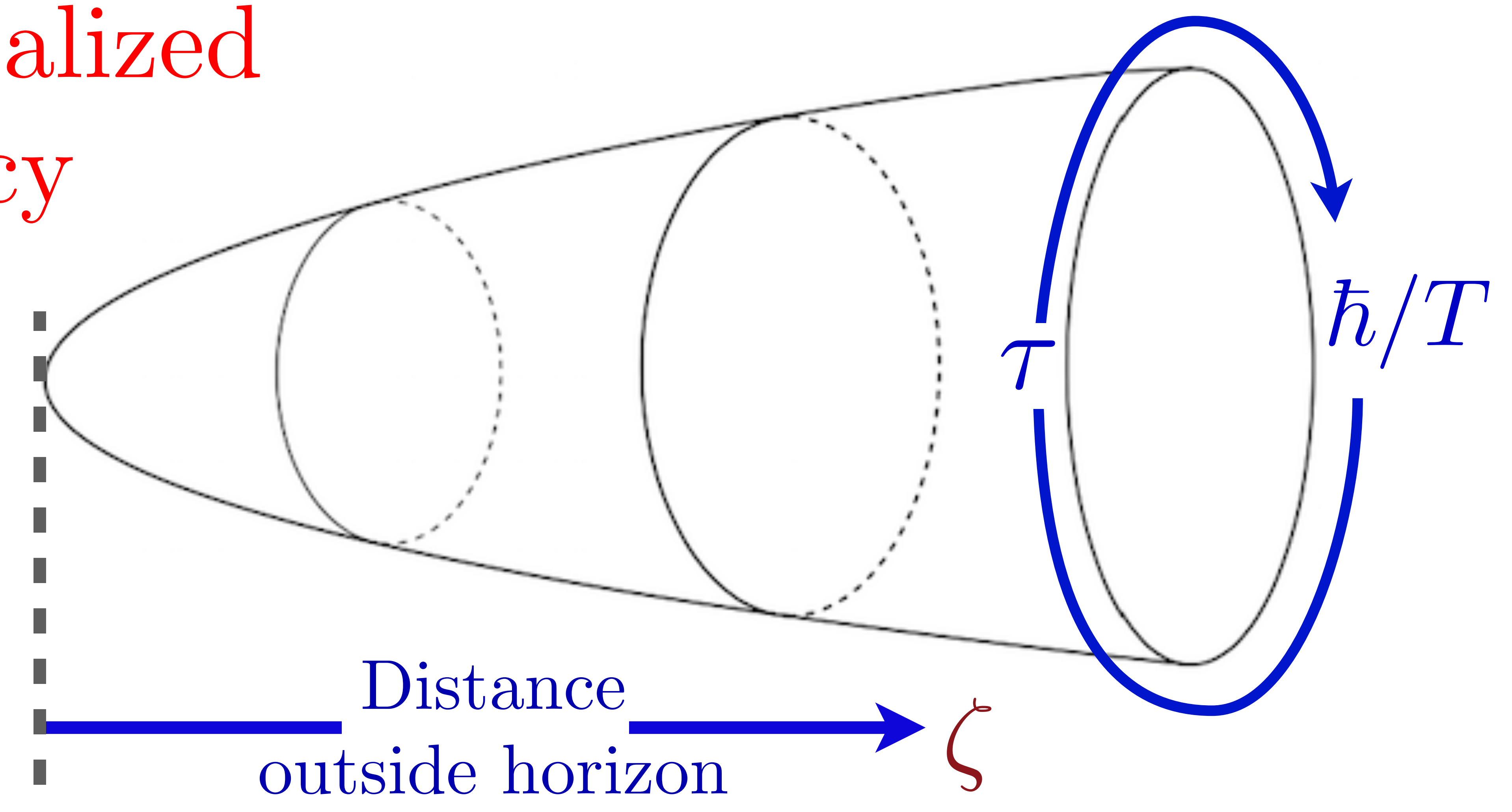

Kitaev (2015); Maldacena, Stanford, Yang (2016)

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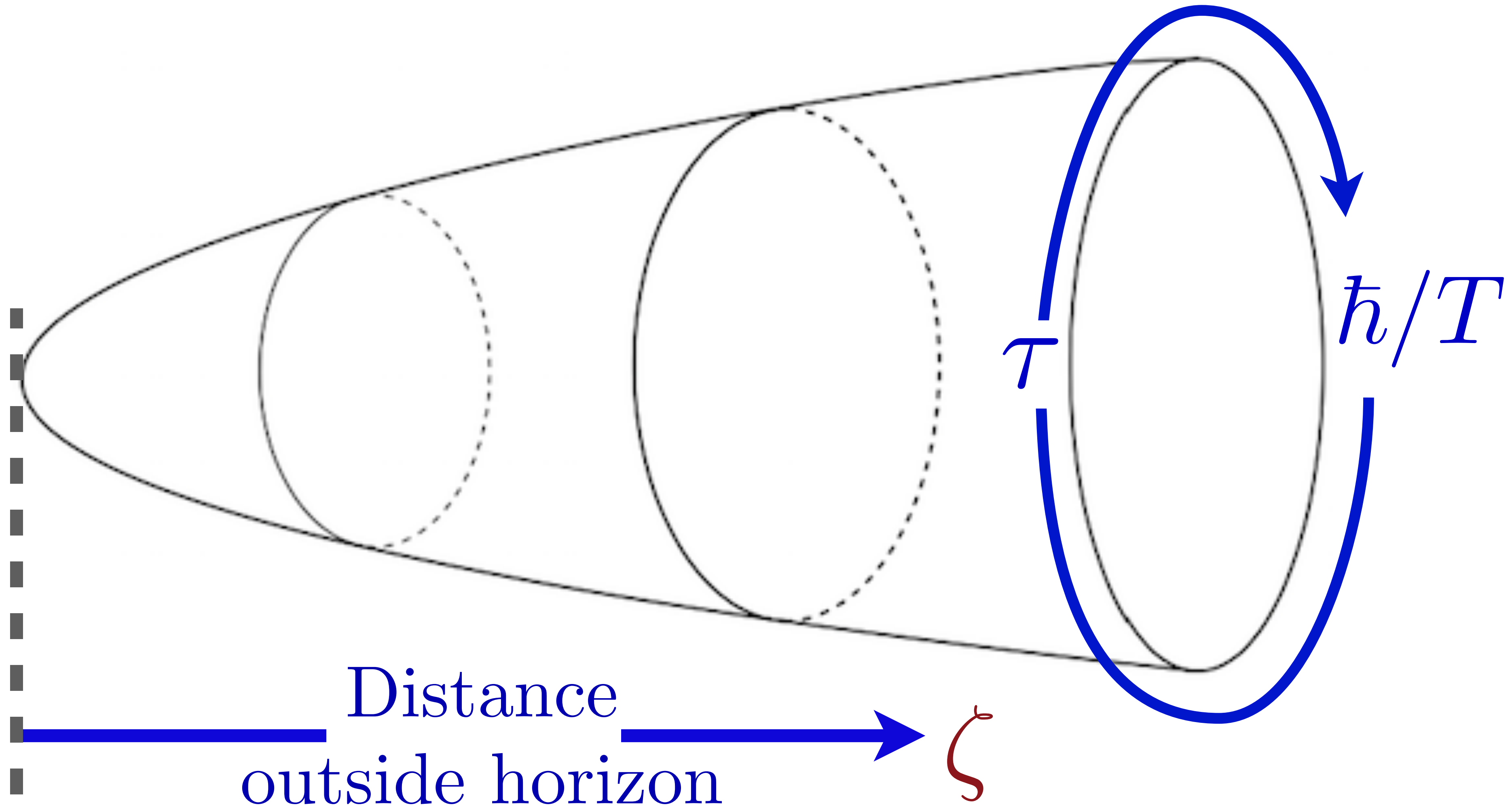
$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\frac{1}{\hbar} I^{(0+1)}_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

The path integral over the action $I^{(0+1)}_{SYK}$ can be evaluated exactly,
and leads to a computation of $D(E)$

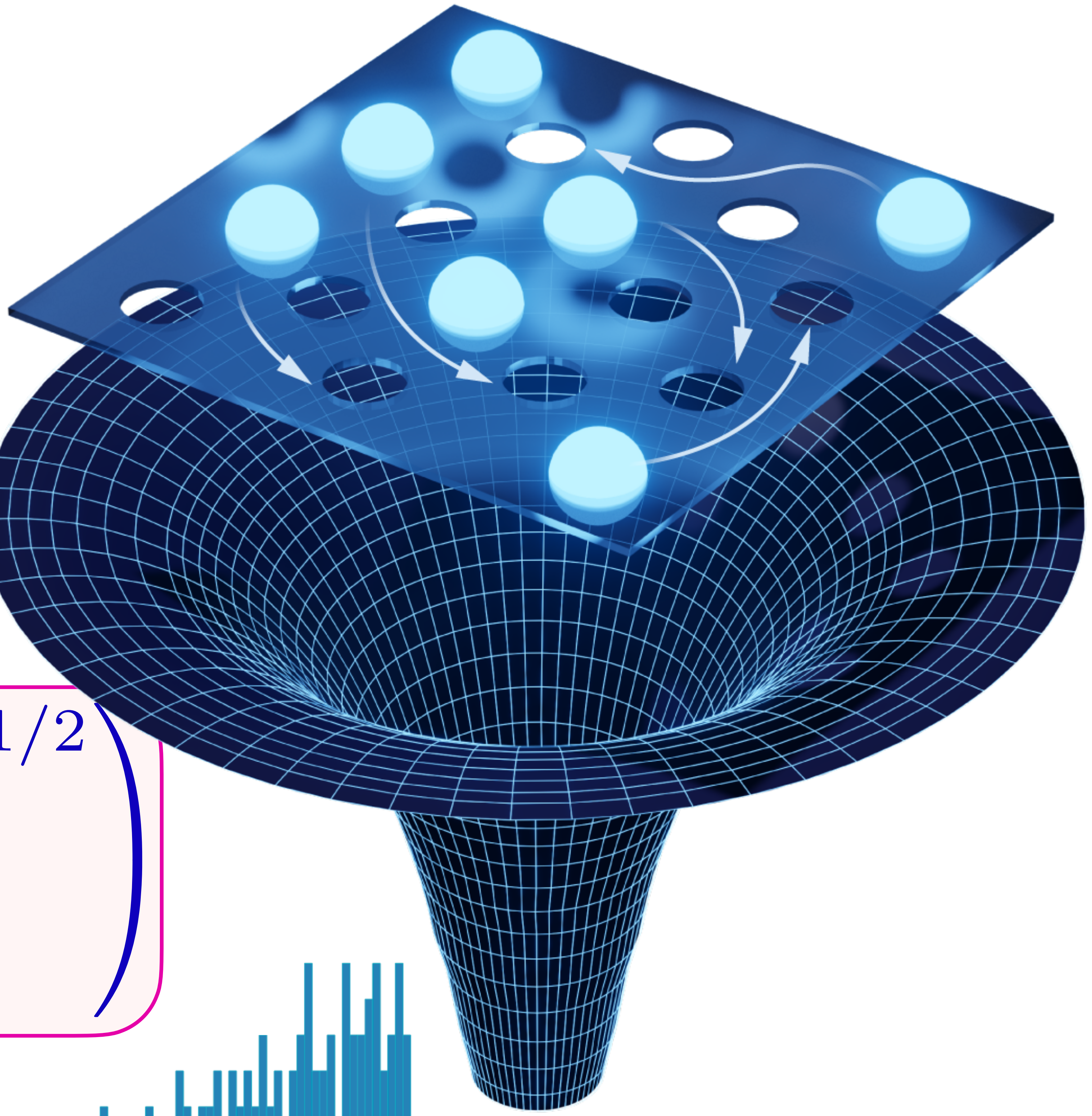
$$\mathcal{Z}(\mathcal{Q},T) = \int dE D(E) \exp\left(-\frac{E}{k_B T}\right)$$



Kitaev (2015); Maldacena, Stanford, Yang (2016); Cotler *et al.* (2017)

D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

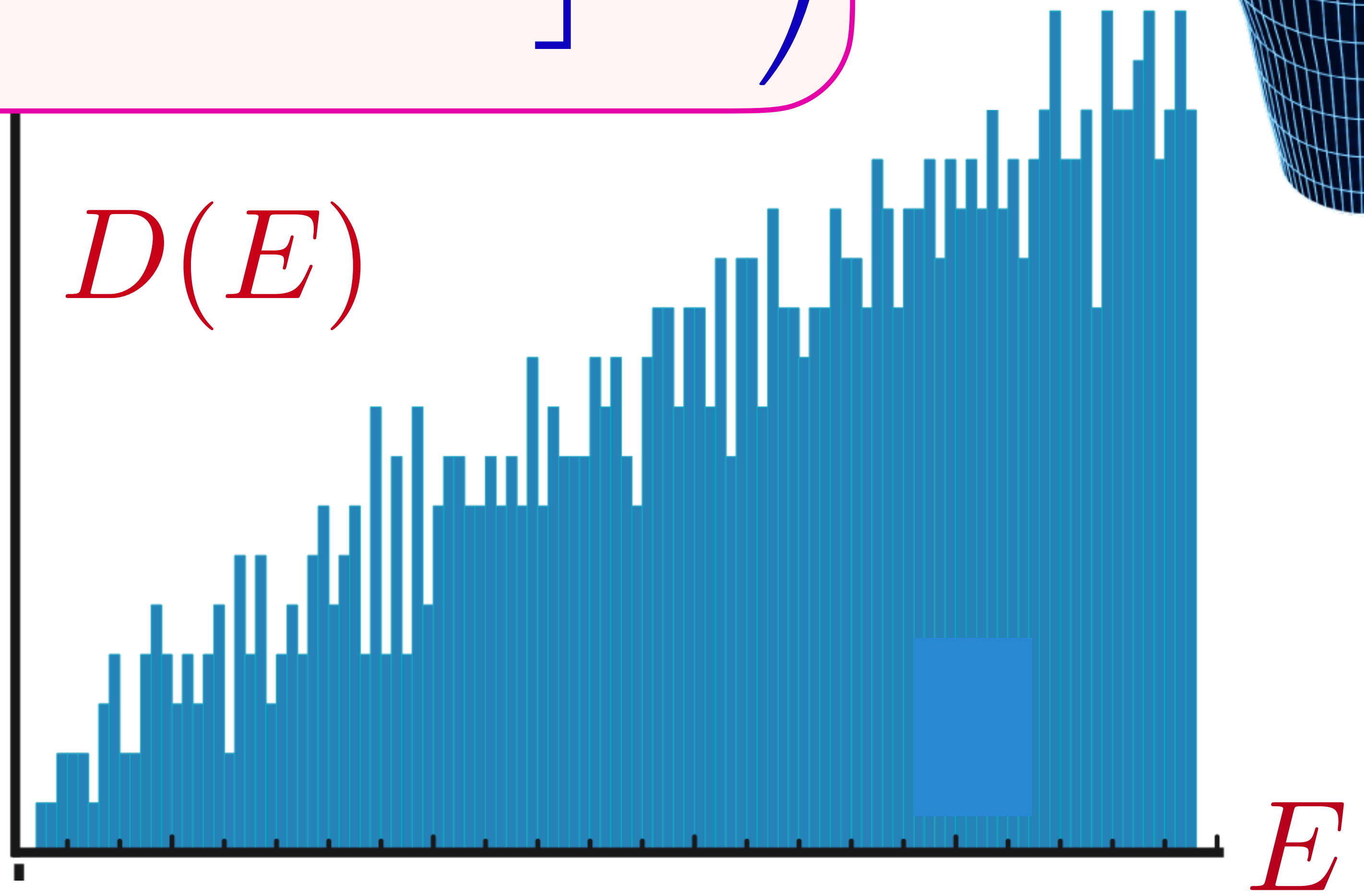


$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}\, {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

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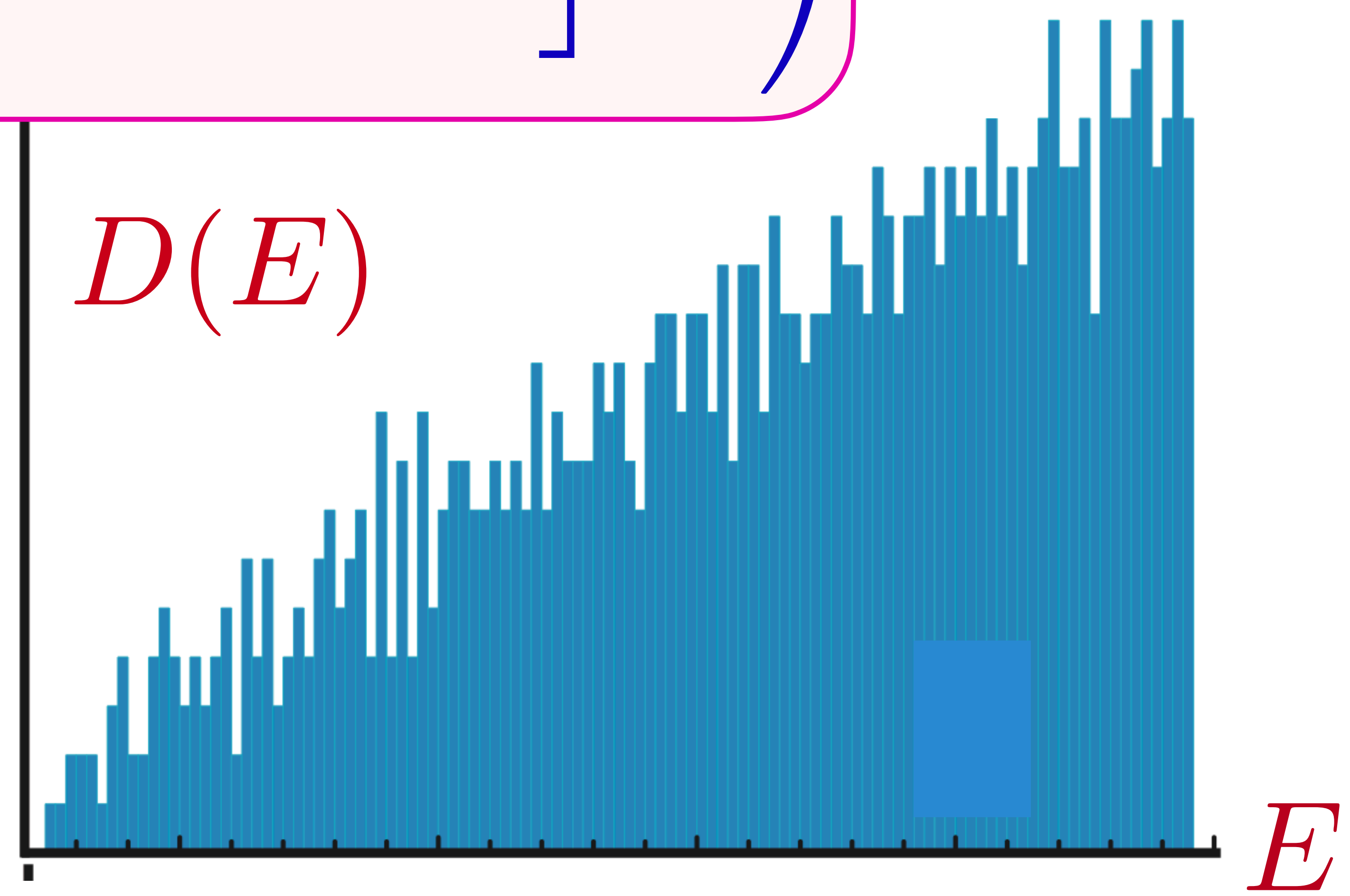
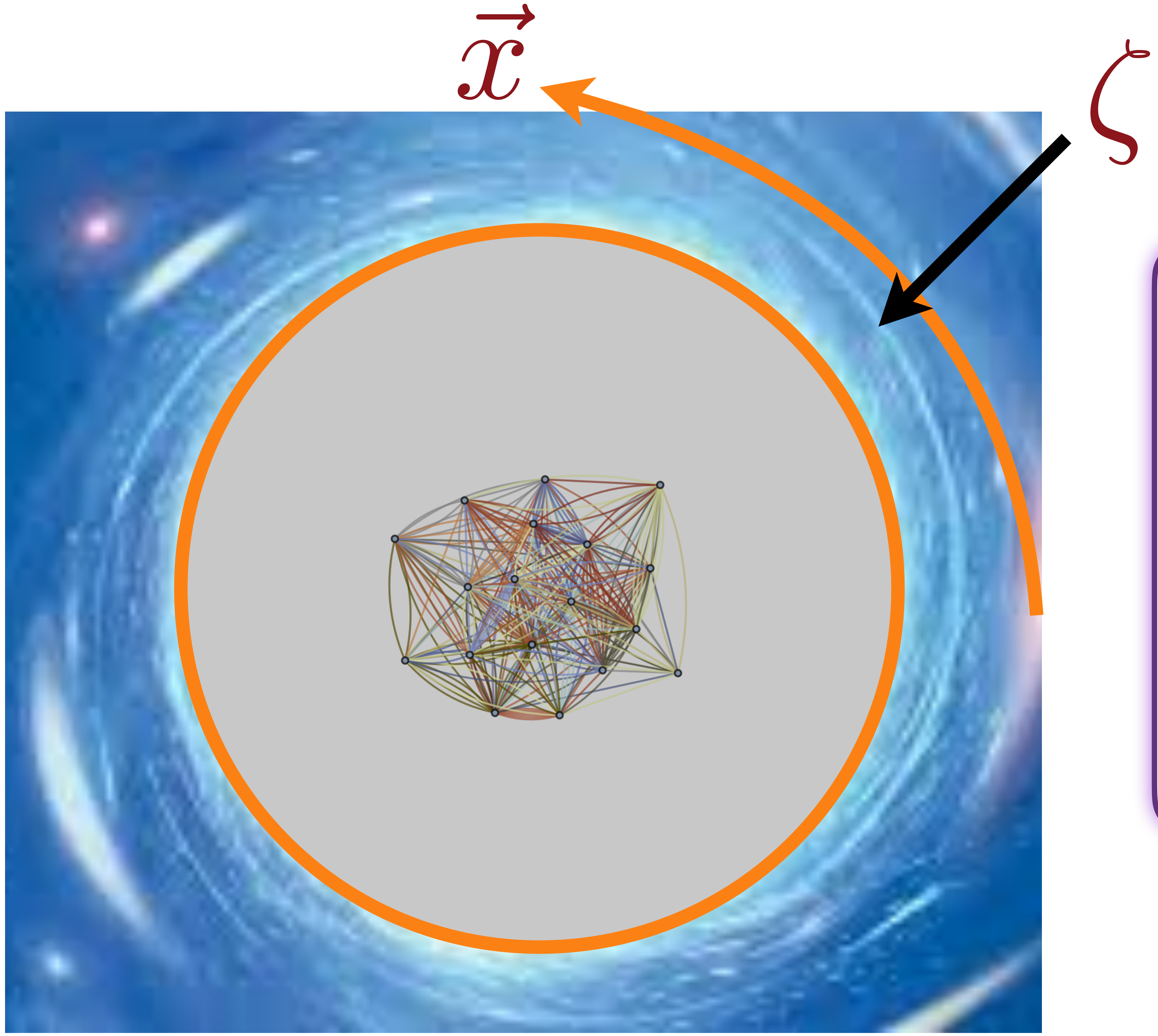
For supersymmetric charged black holes or SYK models: $D(E) = e^S \delta(E)$
i.e. exponential ground state degeneracy.



Quantum simulation of charged black holes by the SYK model

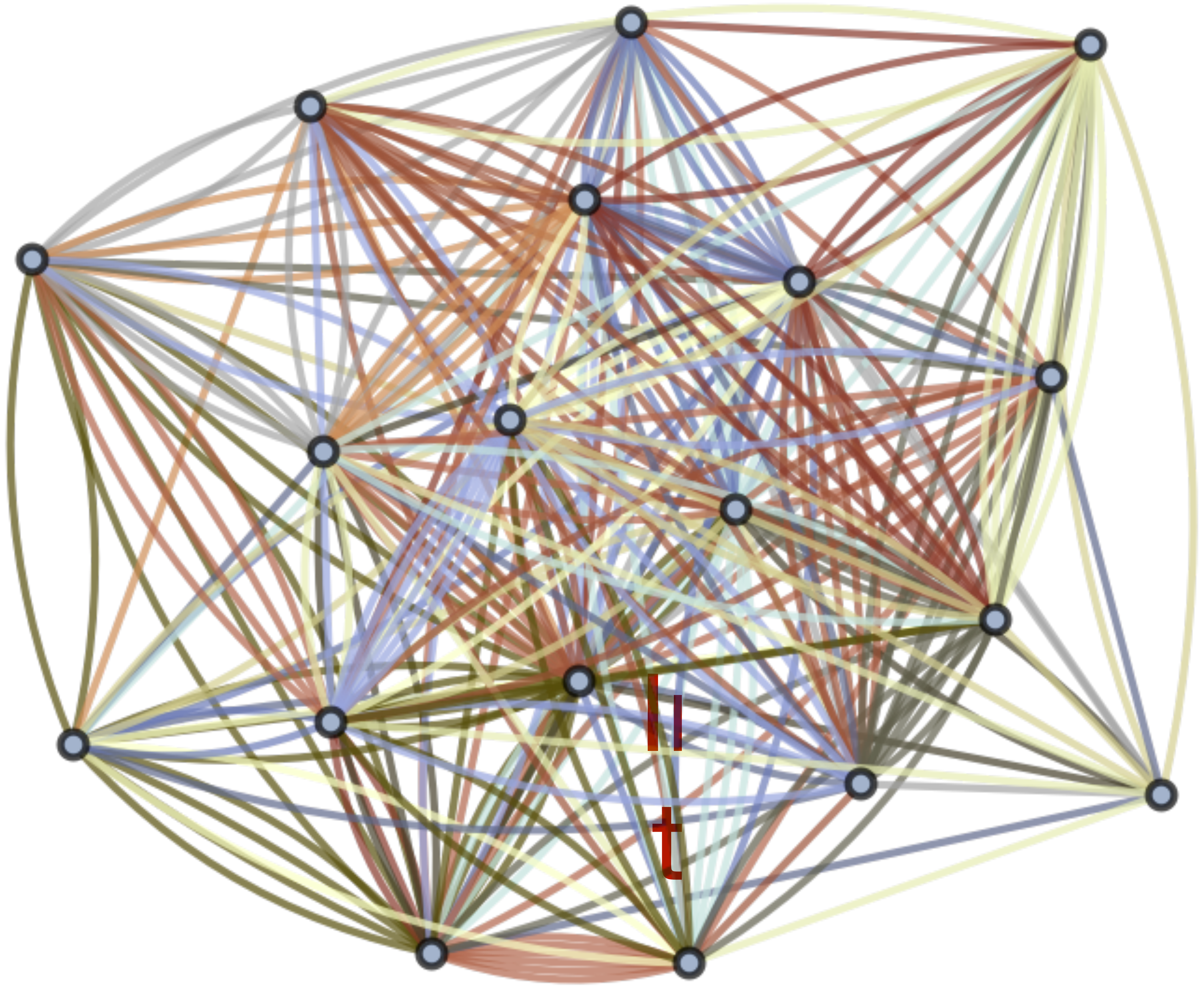


The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in ζ-τ co-ordinates.

Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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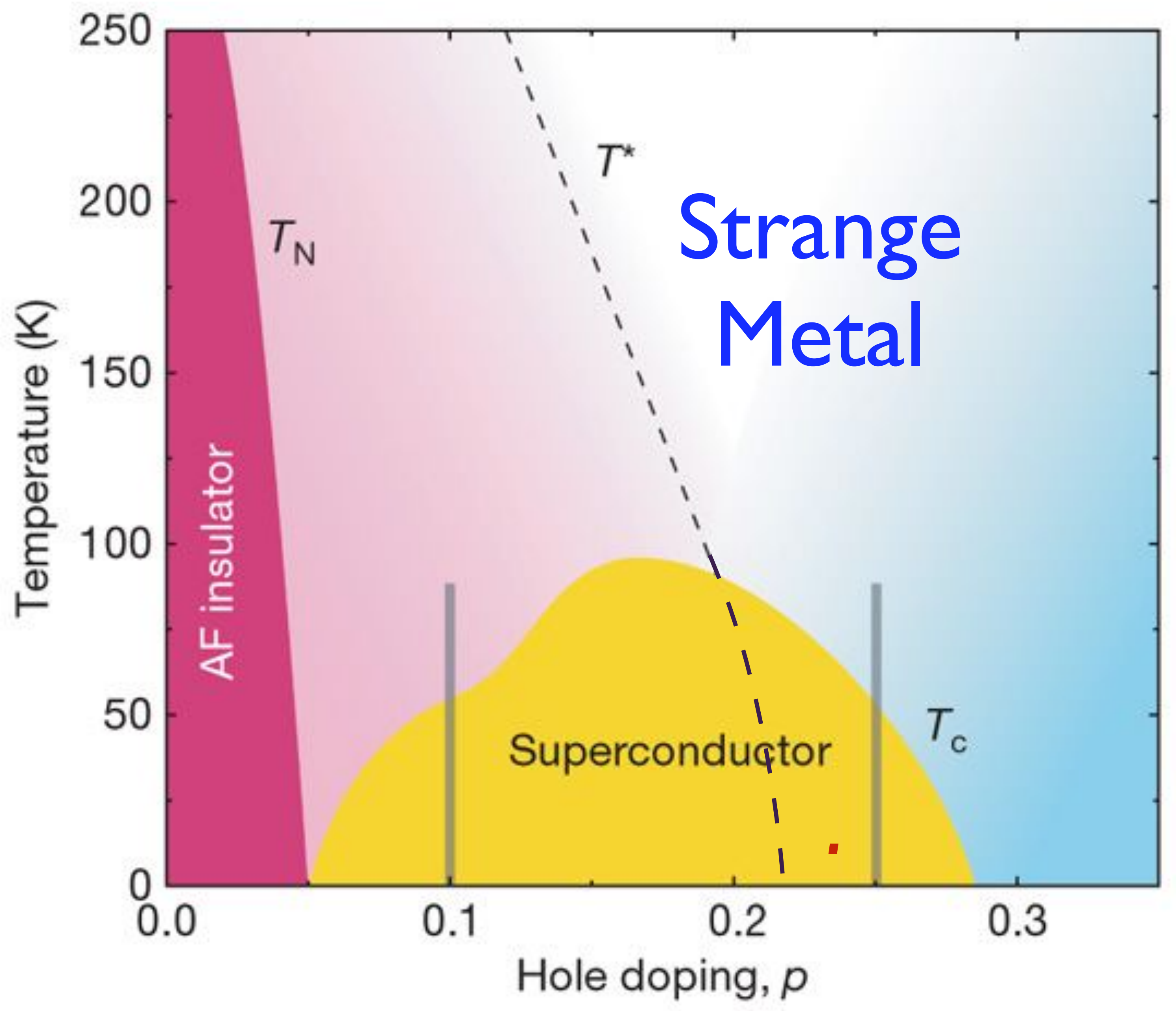
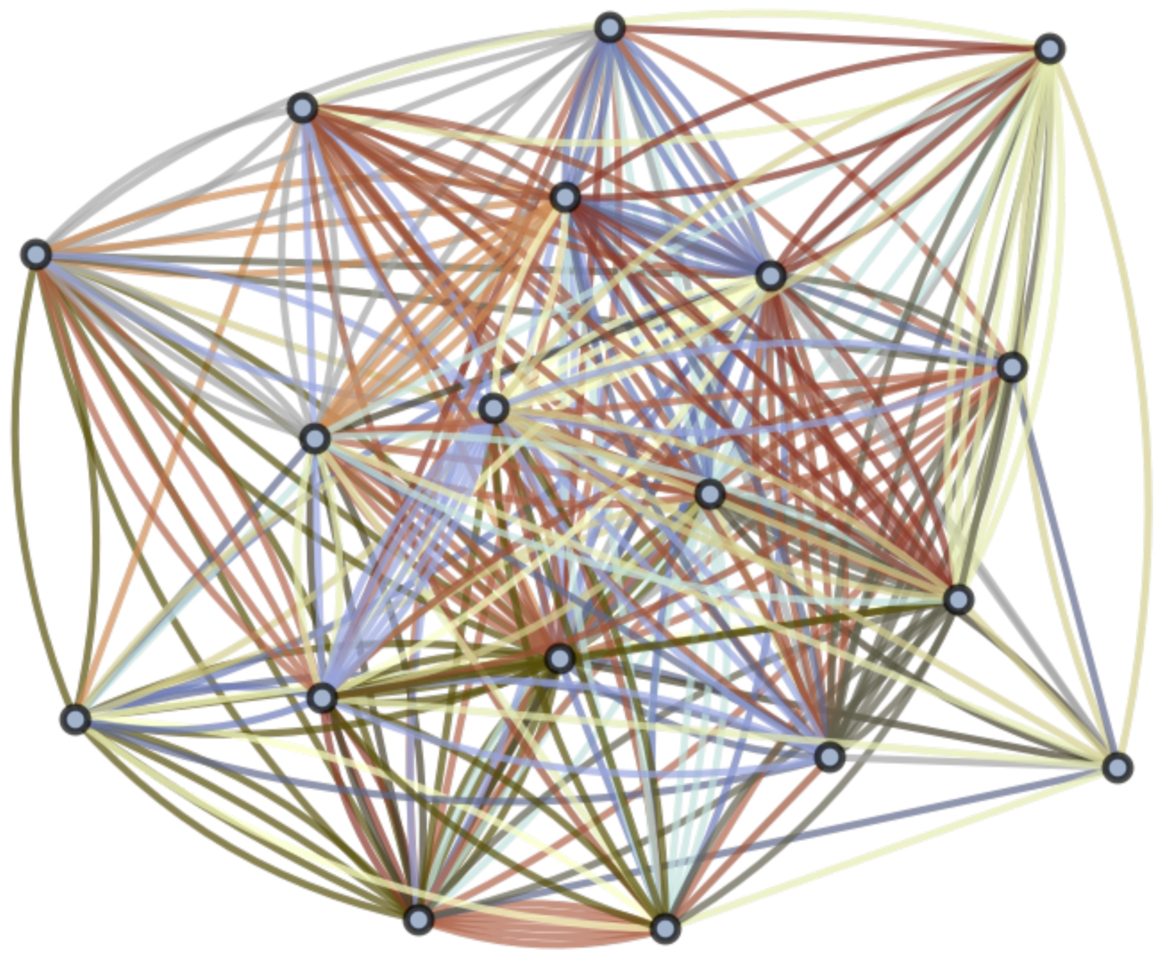


A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



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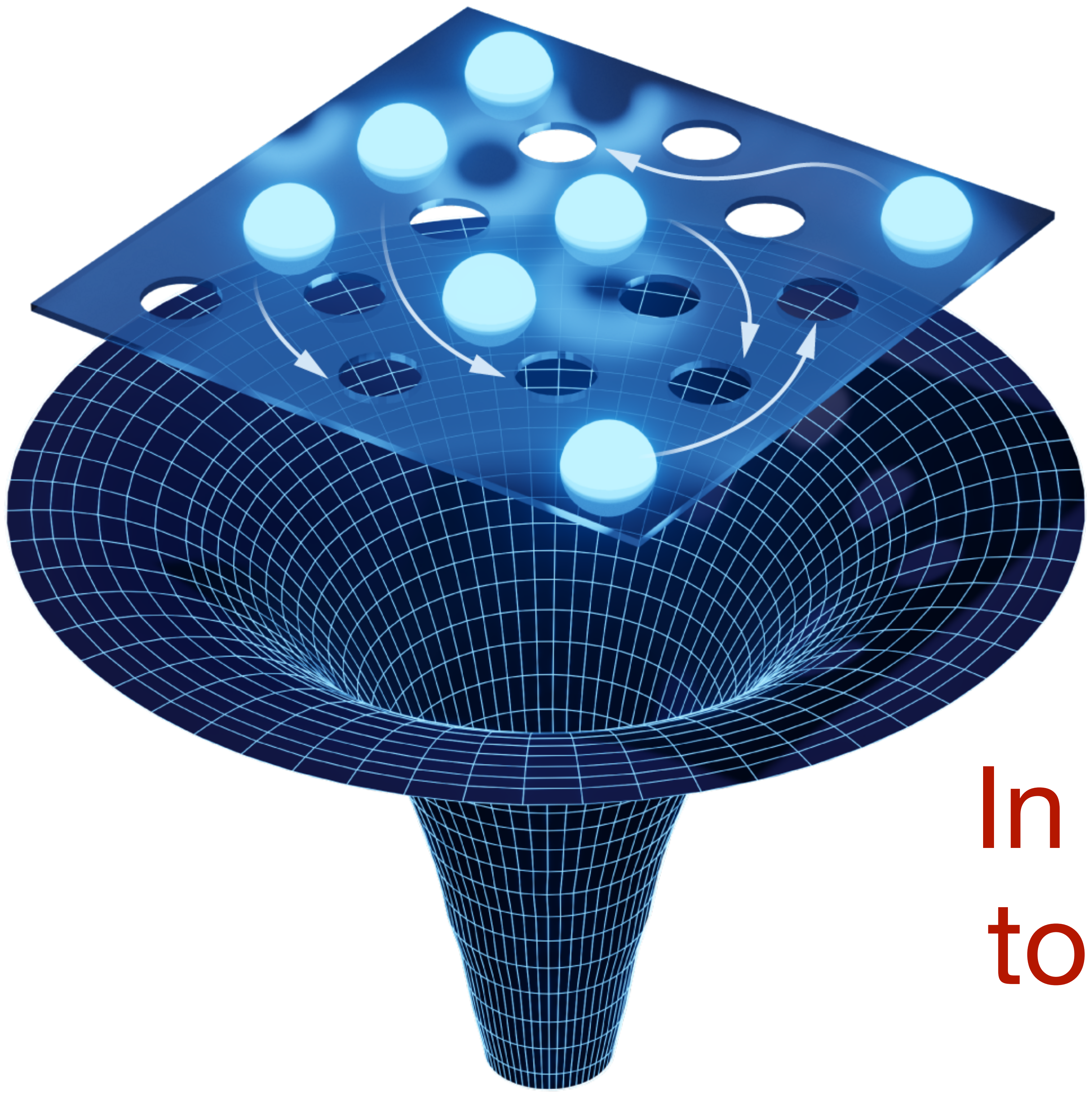

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In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged black holes***