Deconfined Z_2 gauge theory in Rydberg atom arrays

InQubator for Quantum Simulation
University of Washington

April 11, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS
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1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

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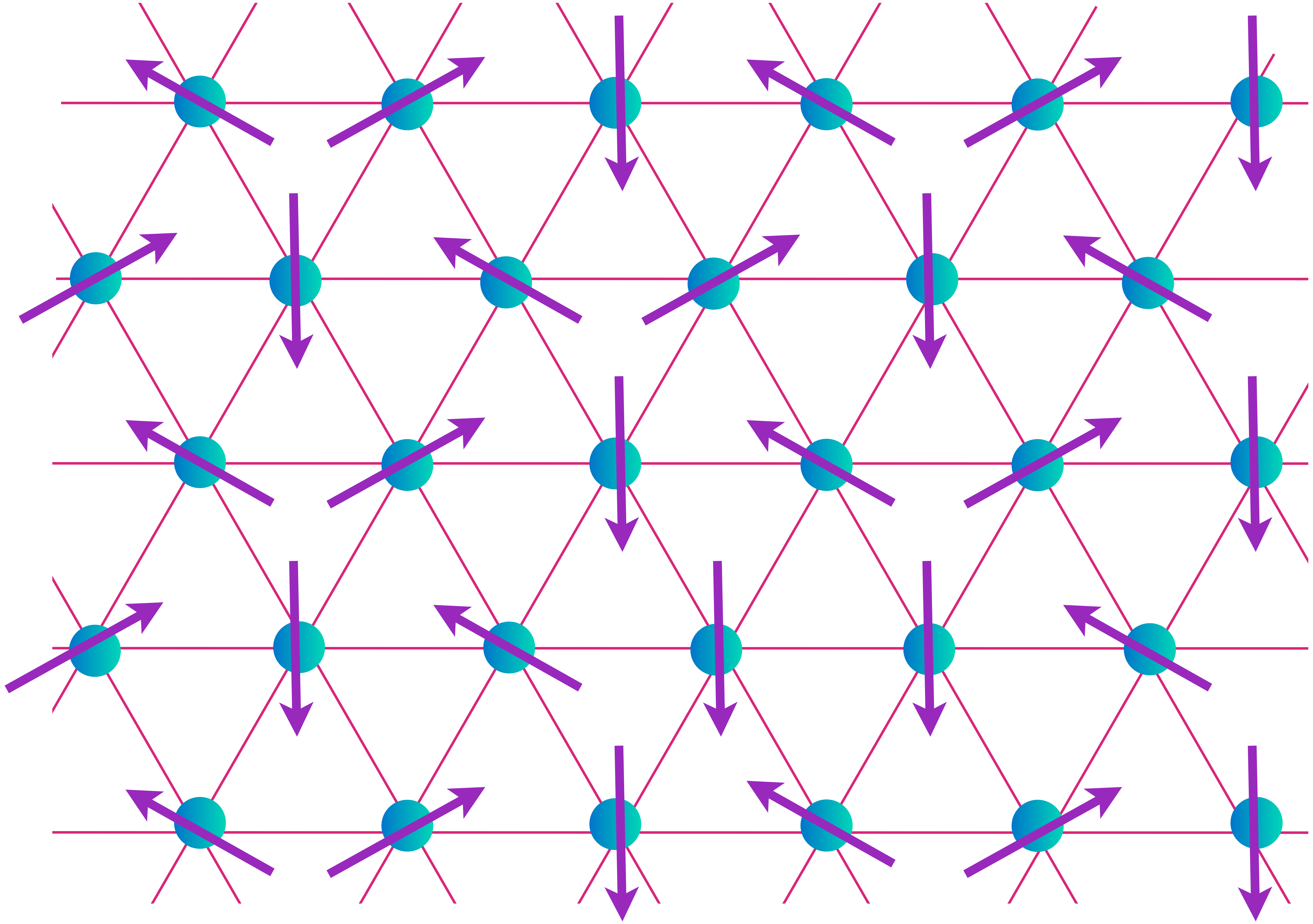
2. Rydberg atoms as a Z_2 gauge theory

Probing topological spin liquids

Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mapping of bosons and spins

Boson B

$B^\dagger B \leq 1$

States $|0\rangle$, $B^\dagger|0\rangle$

Operators

- $B^\dagger \longleftrightarrow S_+ = S_x + iS_y$
- $B \longleftrightarrow S_- = S_x - iS_y$
- $B^\dagger B - \frac{1}{2} \longleftrightarrow S_z$

Spin $\vec{S}$

$\vec{S}^2 = 3/4$

States $|\uparrow\rangle$ $|\downarrow\rangle$

Operators

- $S_+ = S_x + iS_y$
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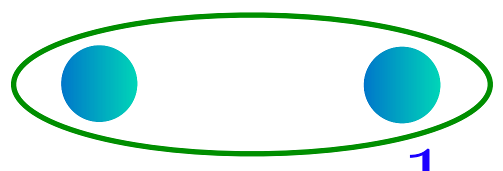
- S_z

Spin liquid: resonating valence bonds

Bosons at half-filling,
or a spin model with S=1/2 per unit cell



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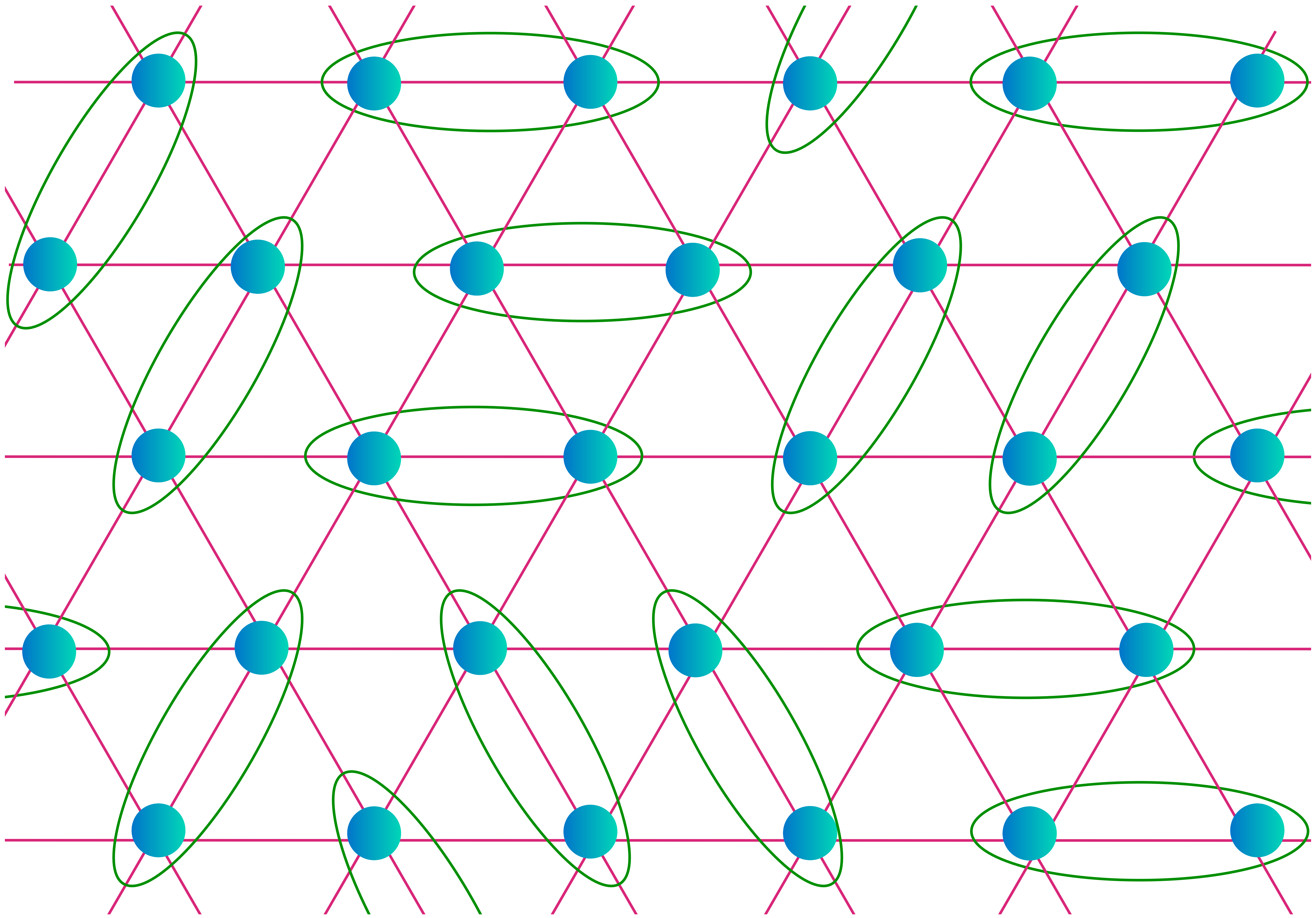

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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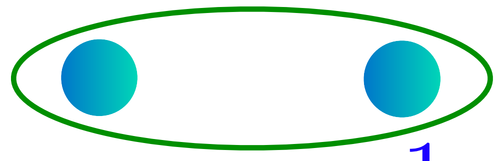

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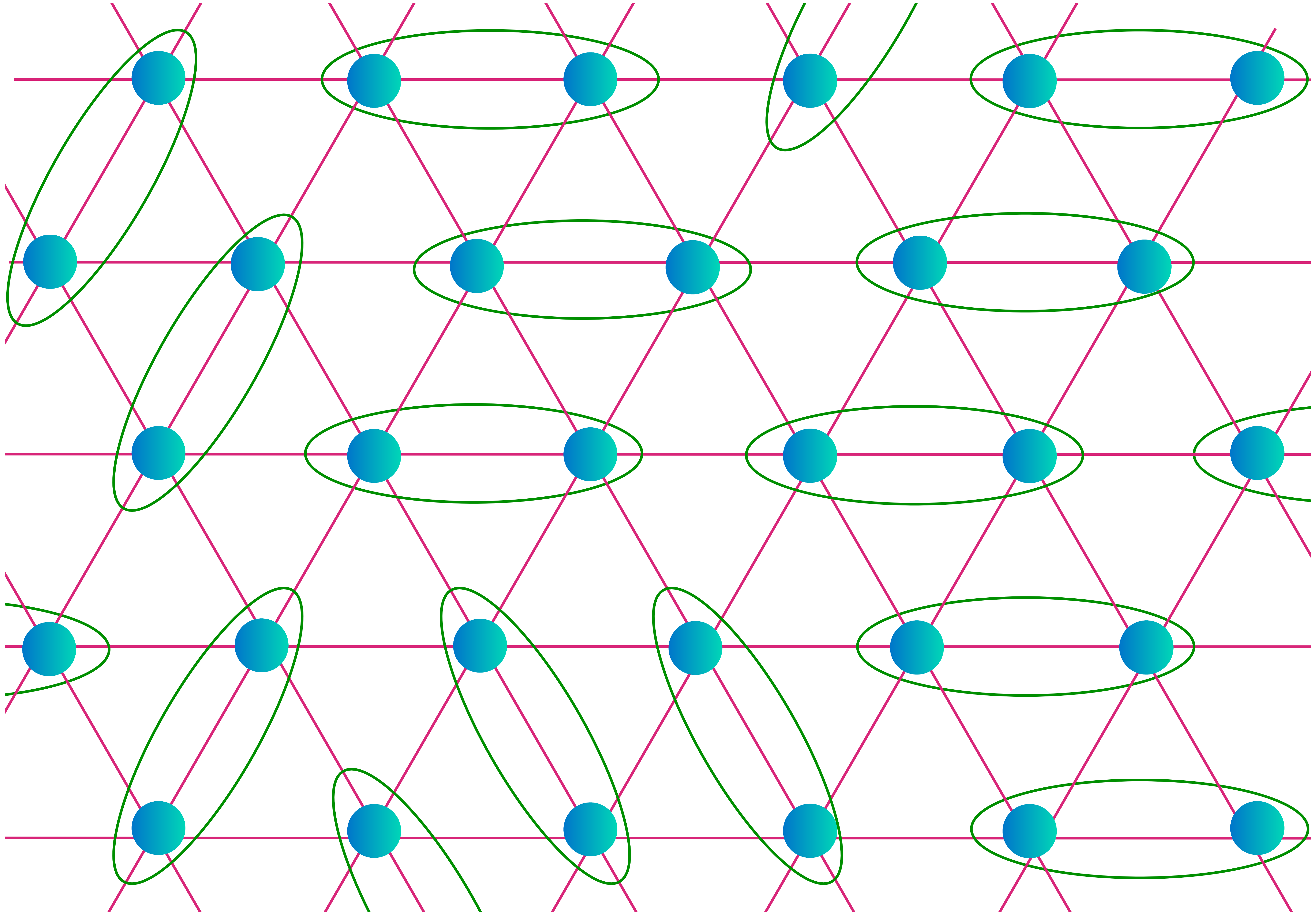

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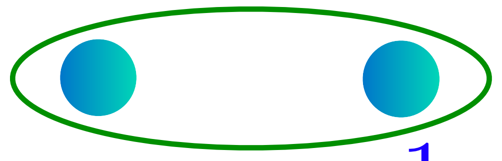

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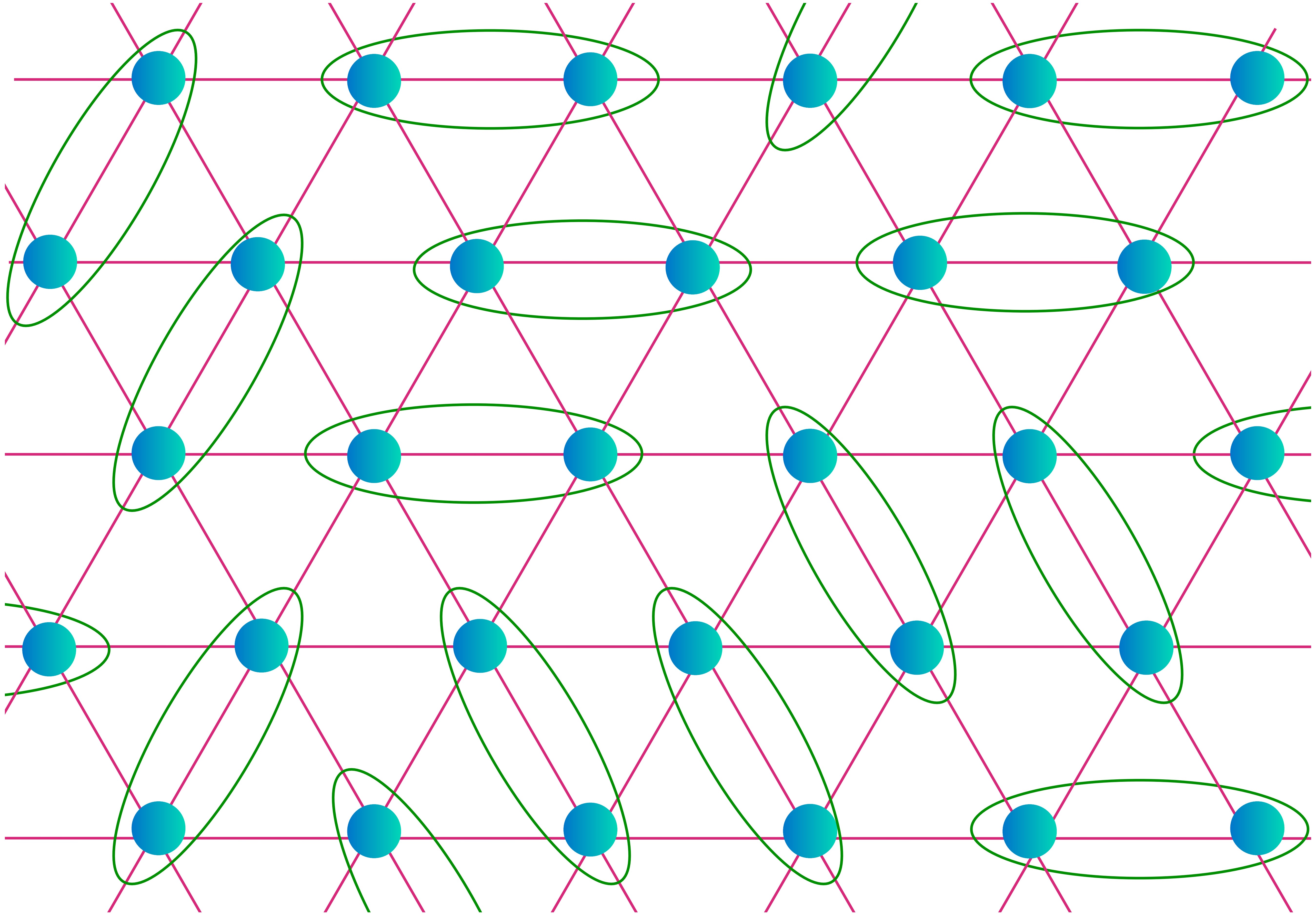

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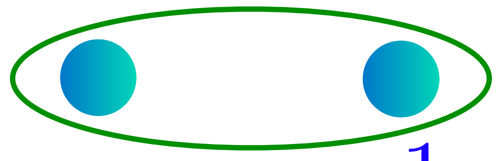

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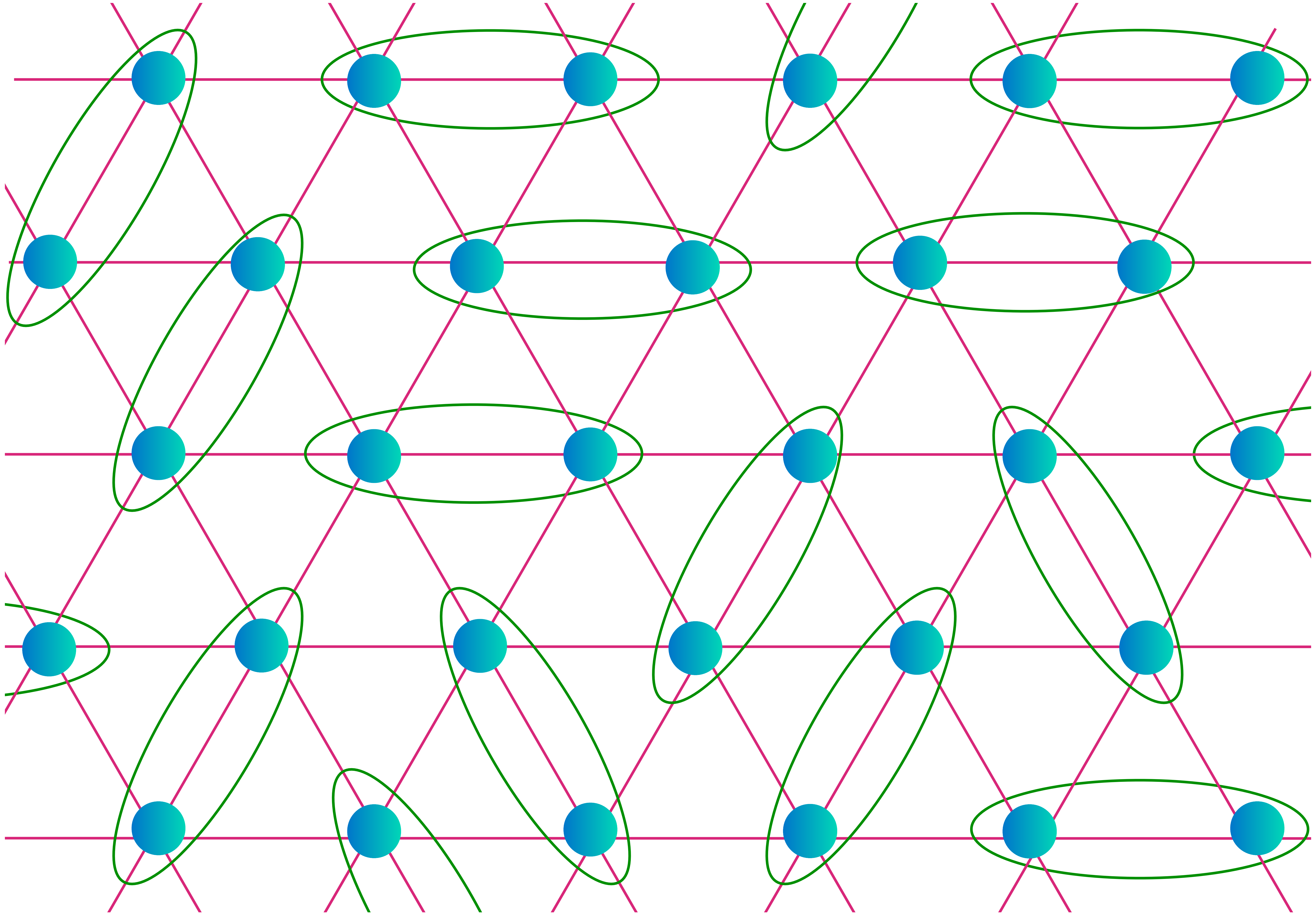

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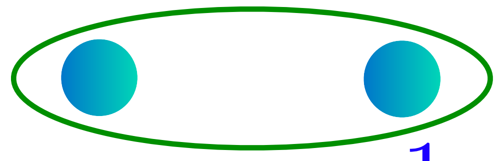

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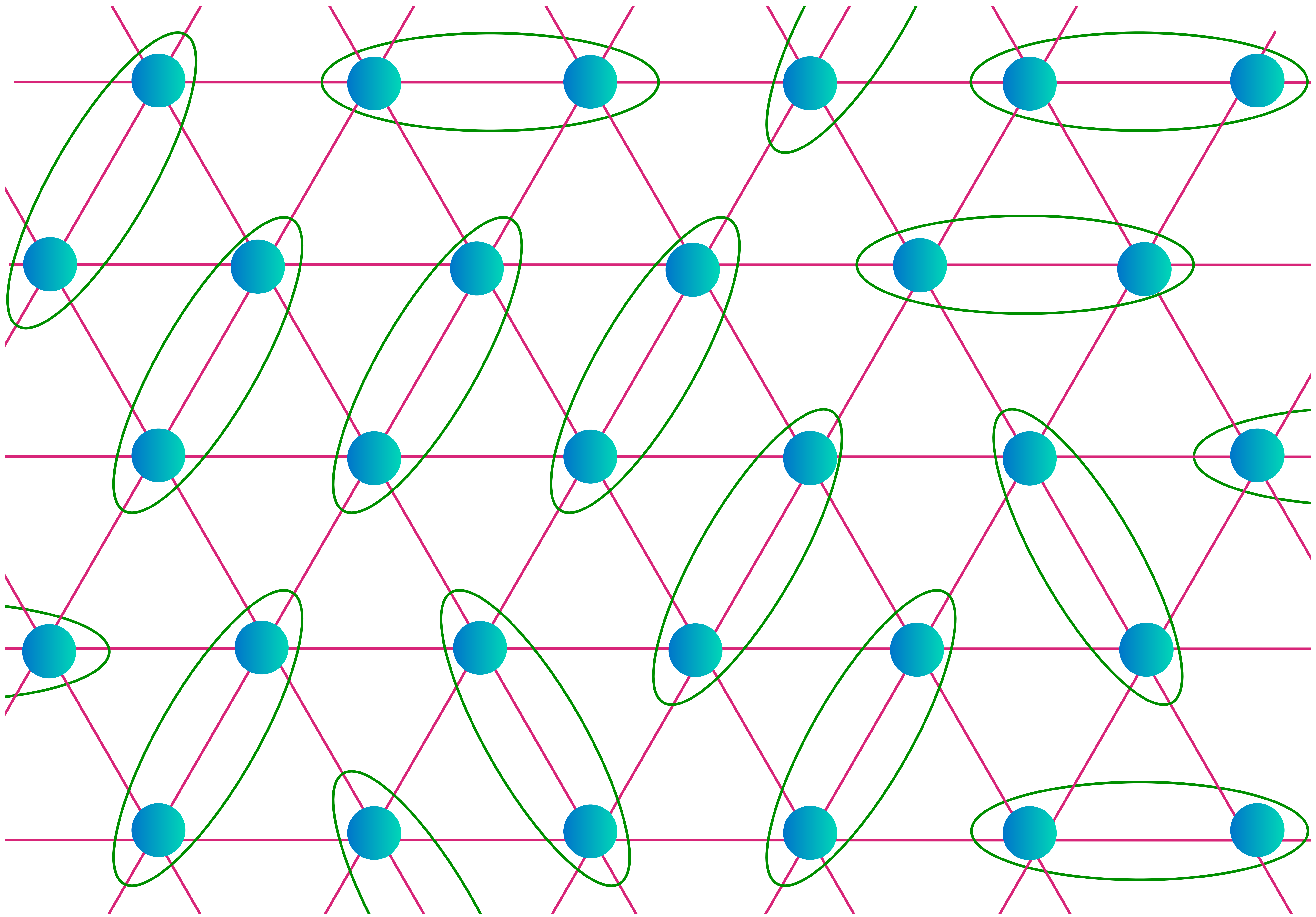

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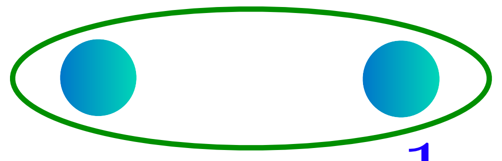

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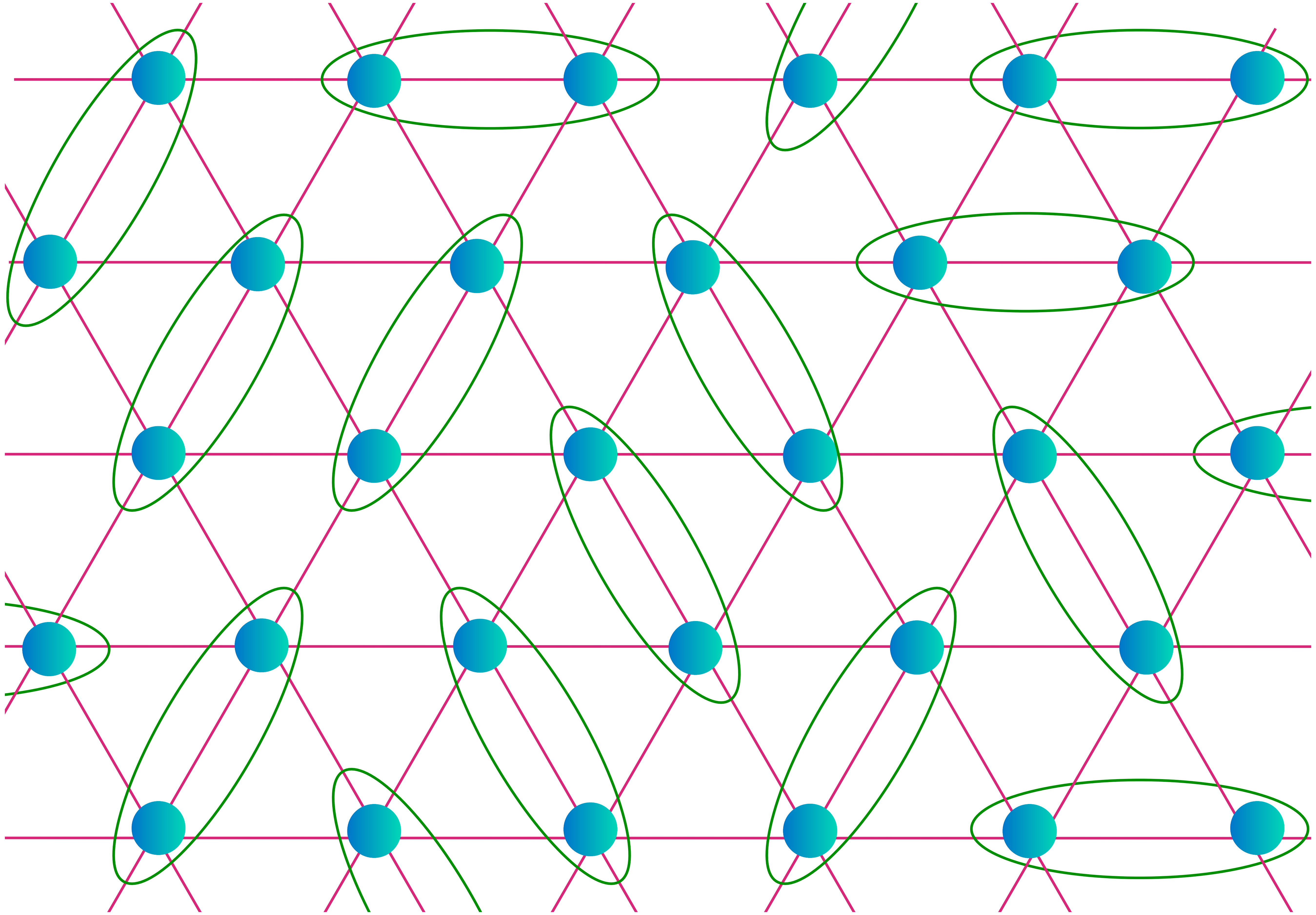

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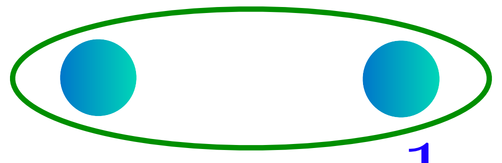
RVB: Z_2 spin liquid

Excitations with boson number 1/2



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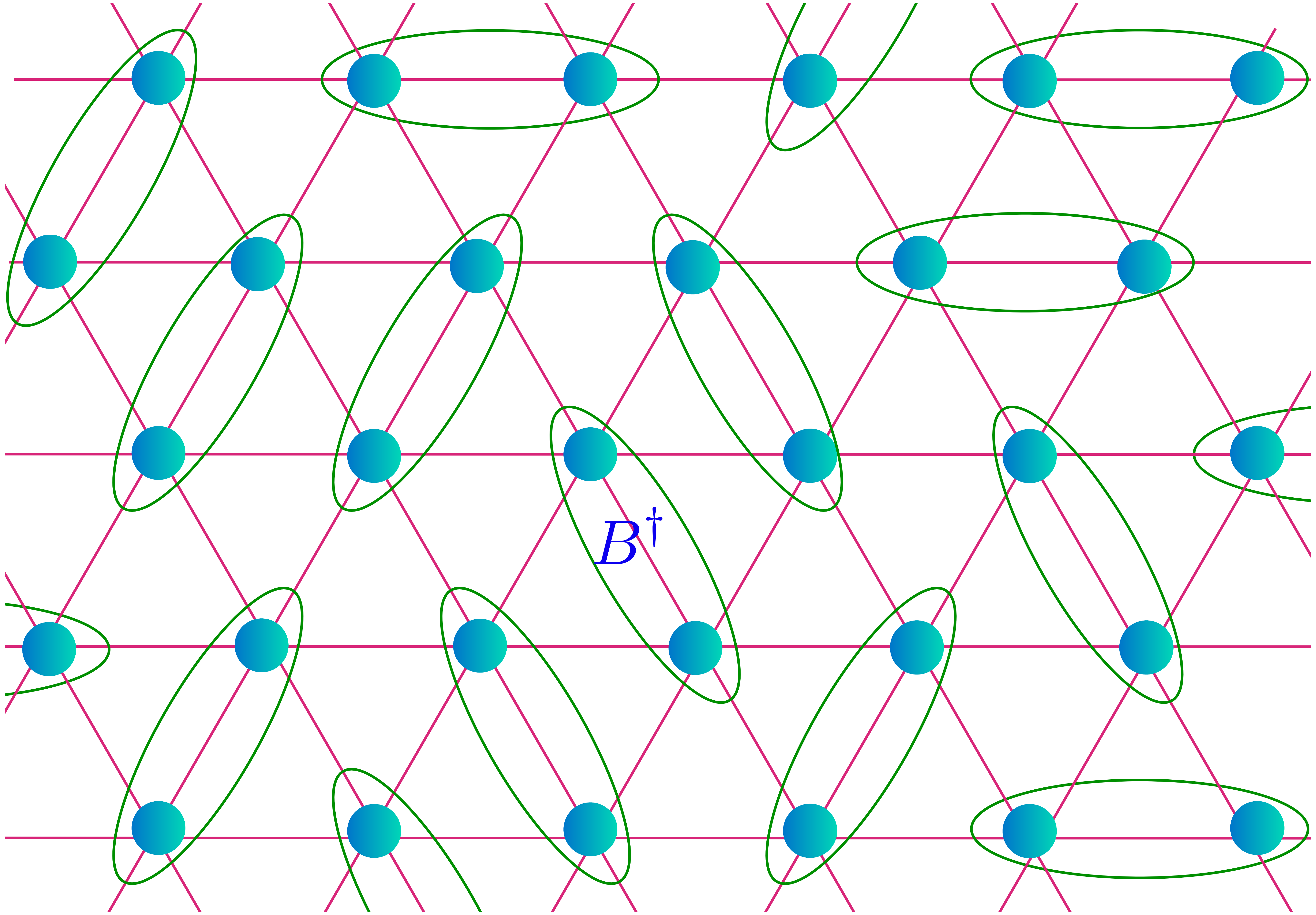
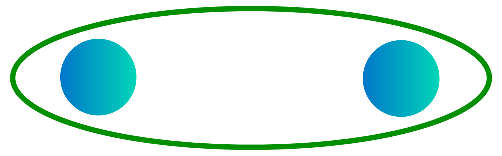


$B_2^\dagger$ 

$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} |\uparrow\uparrow\rangle$$

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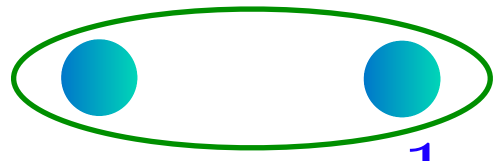
Excitations with boson number 1/2
a "spinon"



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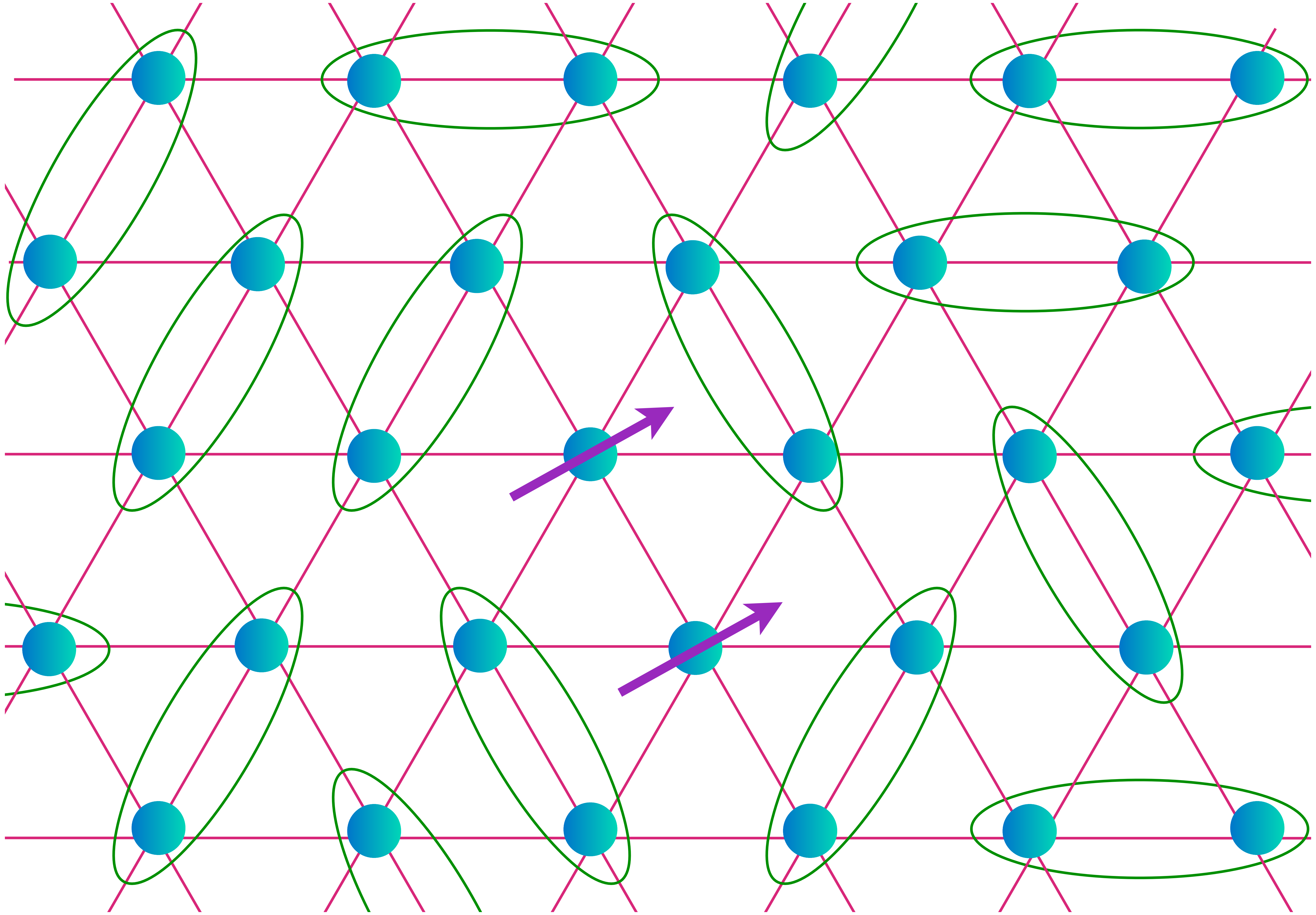

- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

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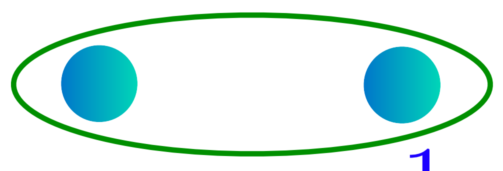

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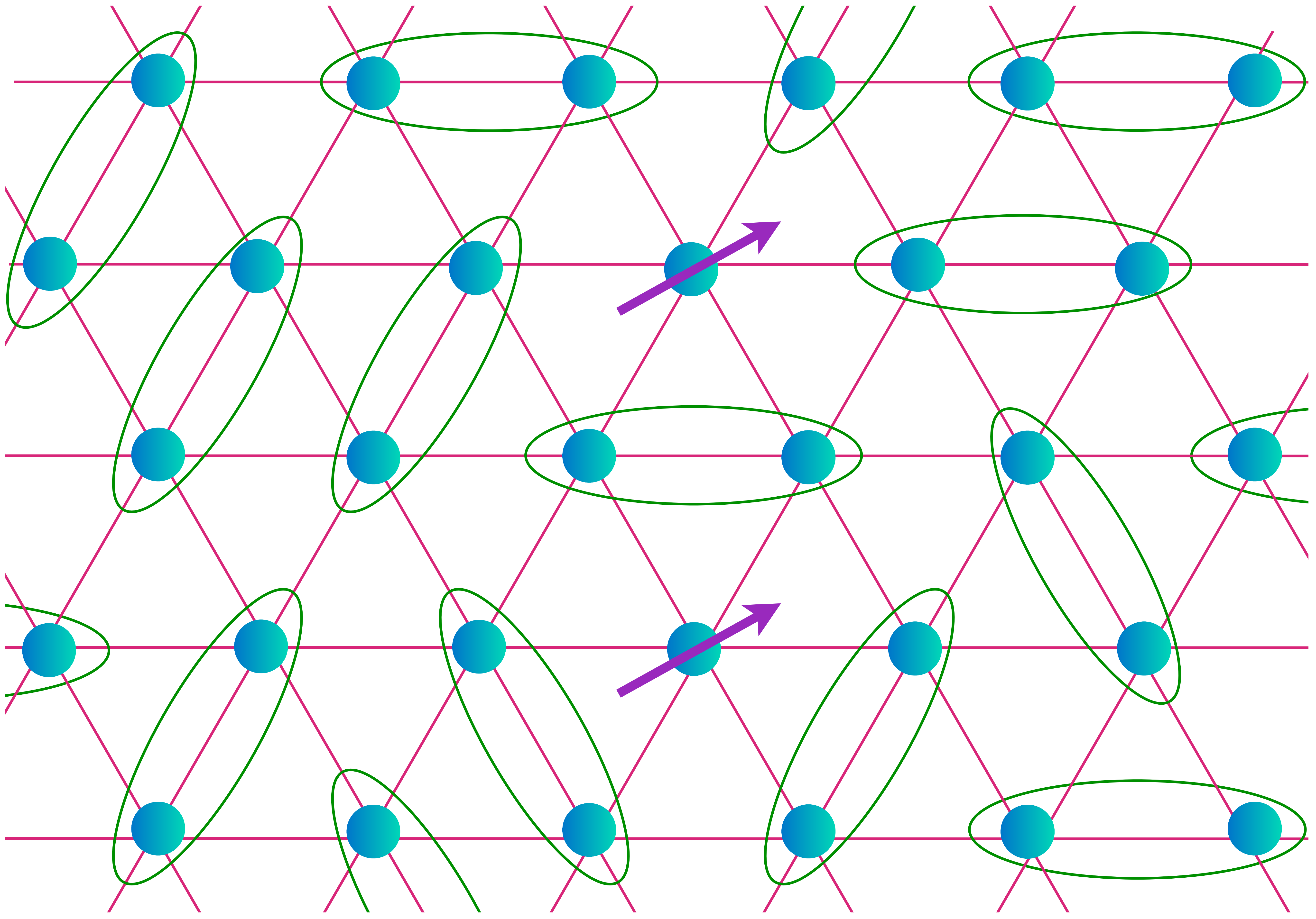

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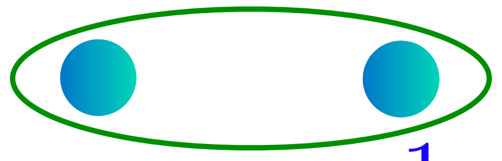

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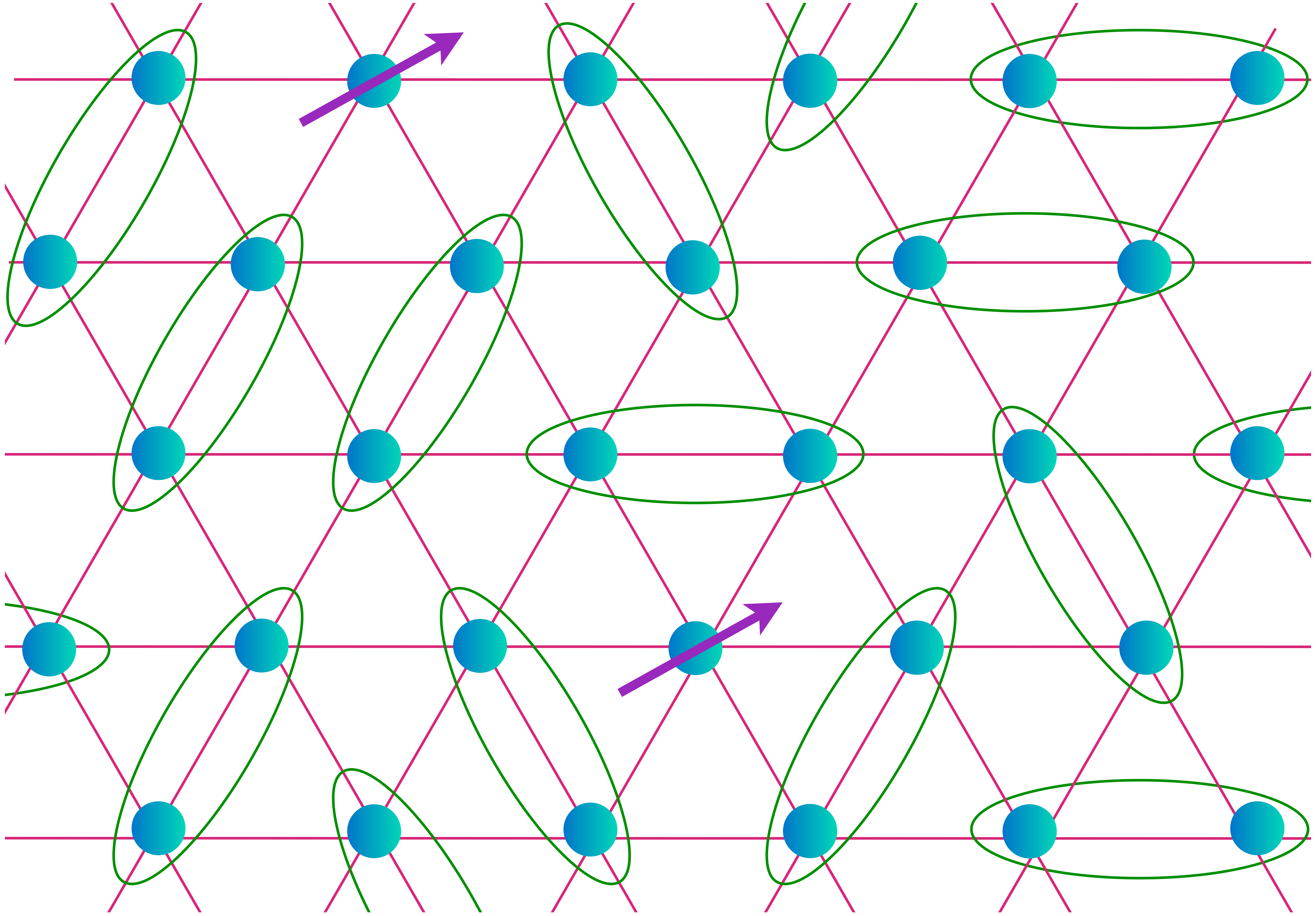

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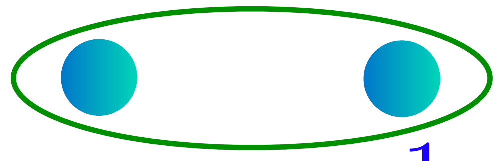

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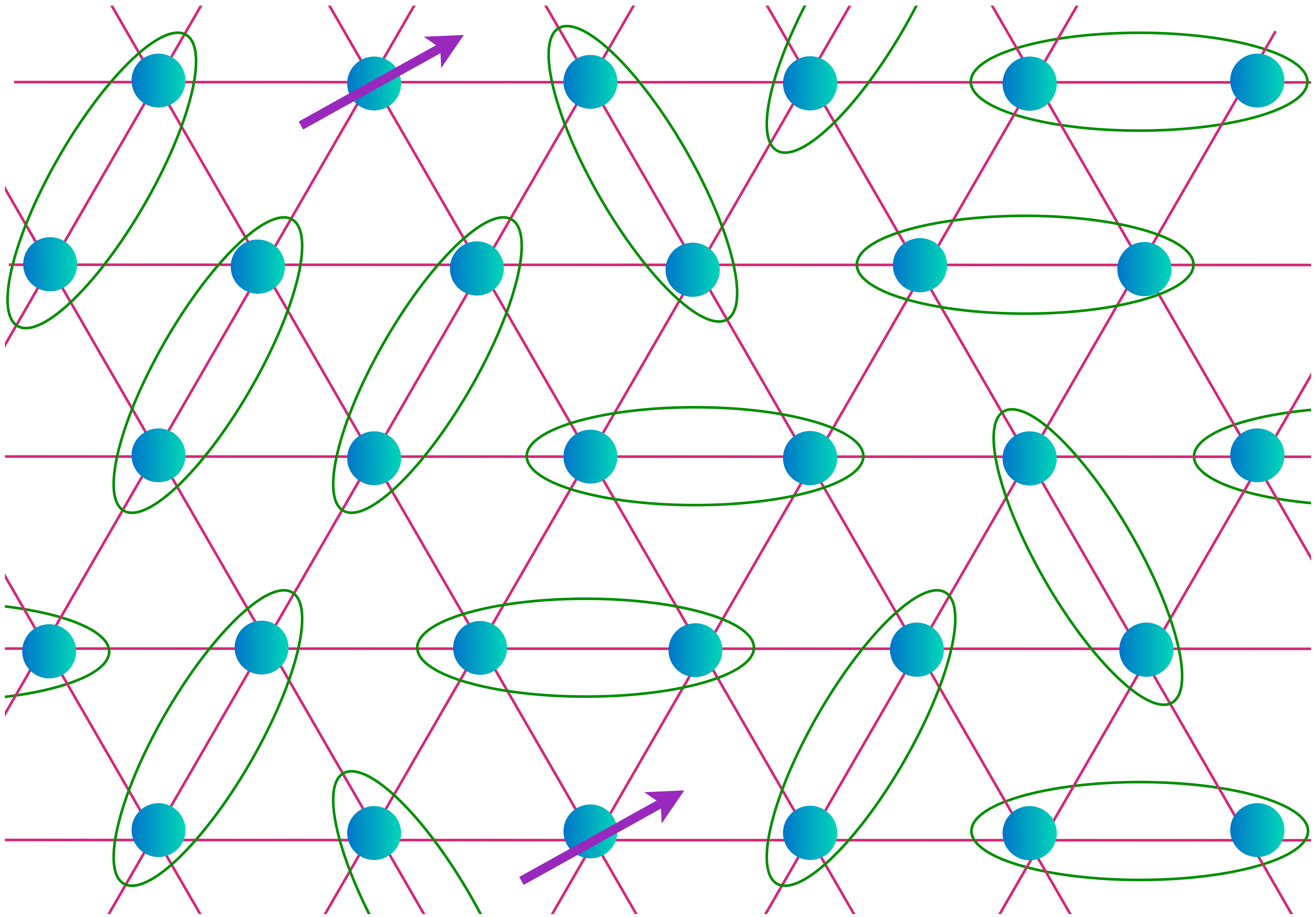

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Excitations with boson number 0
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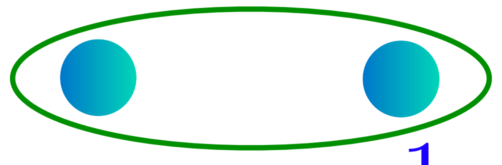

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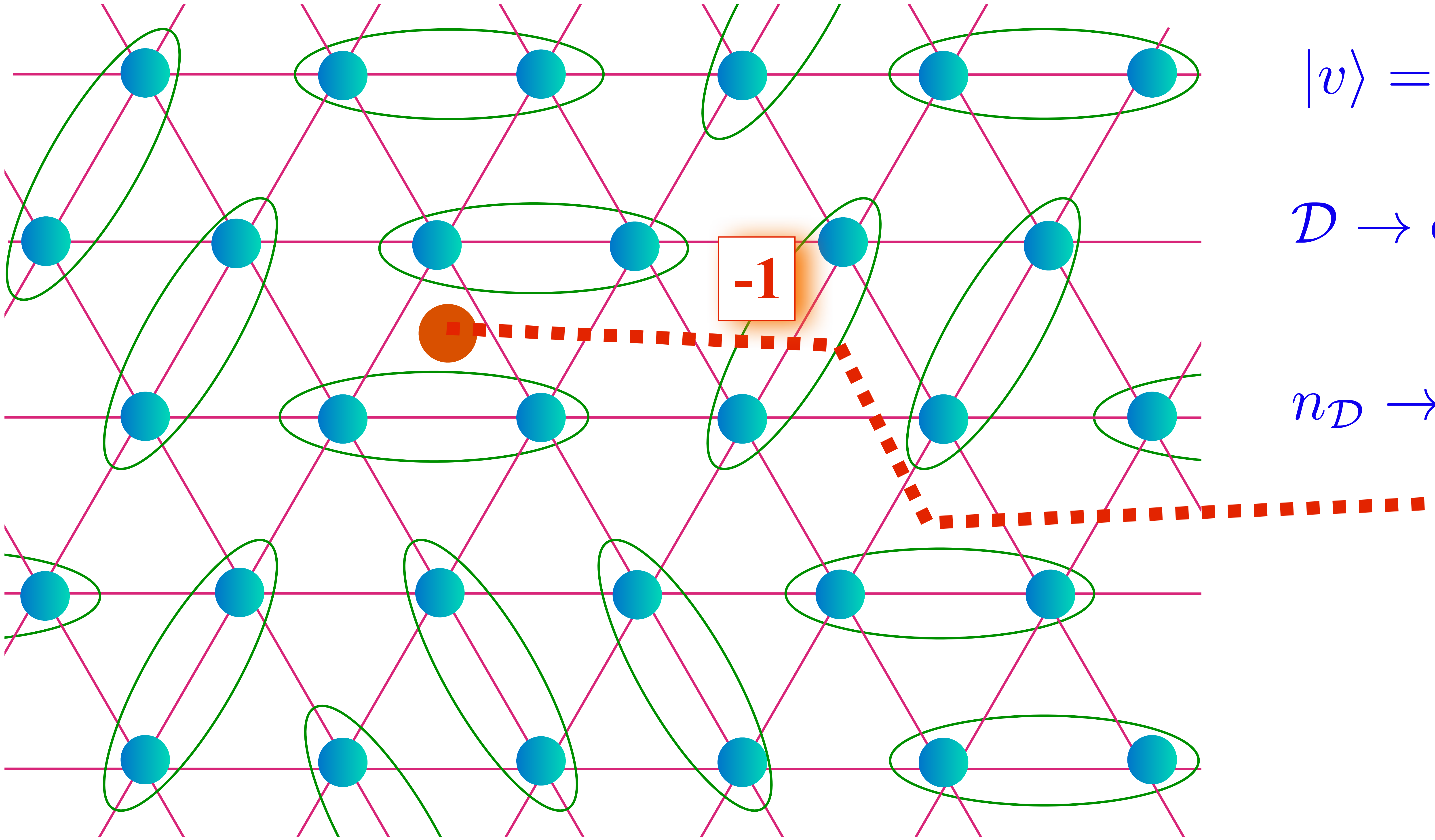

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N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

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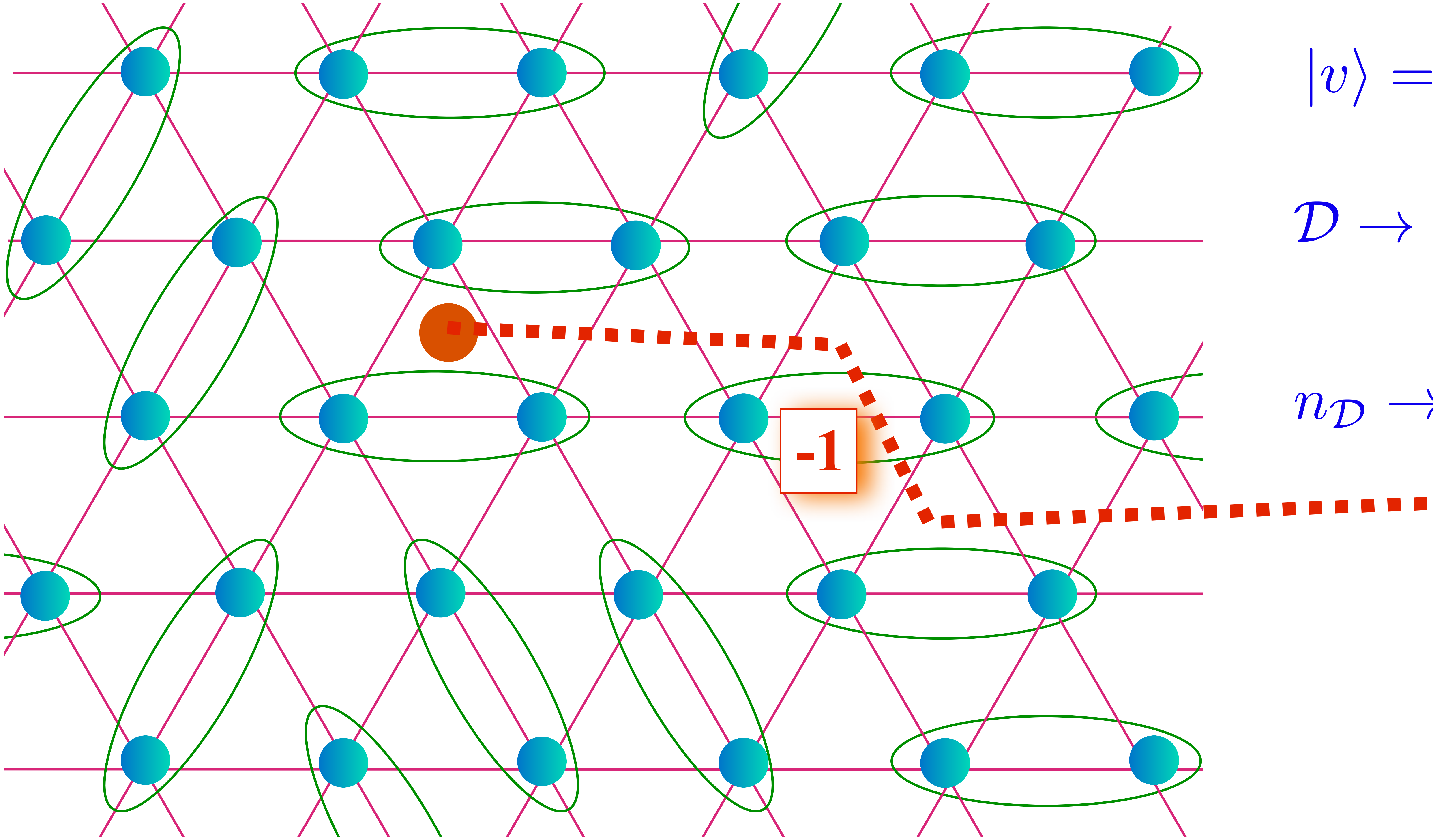

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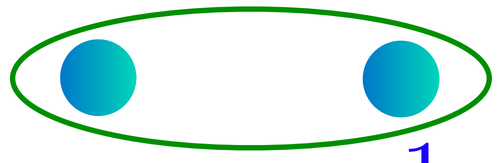

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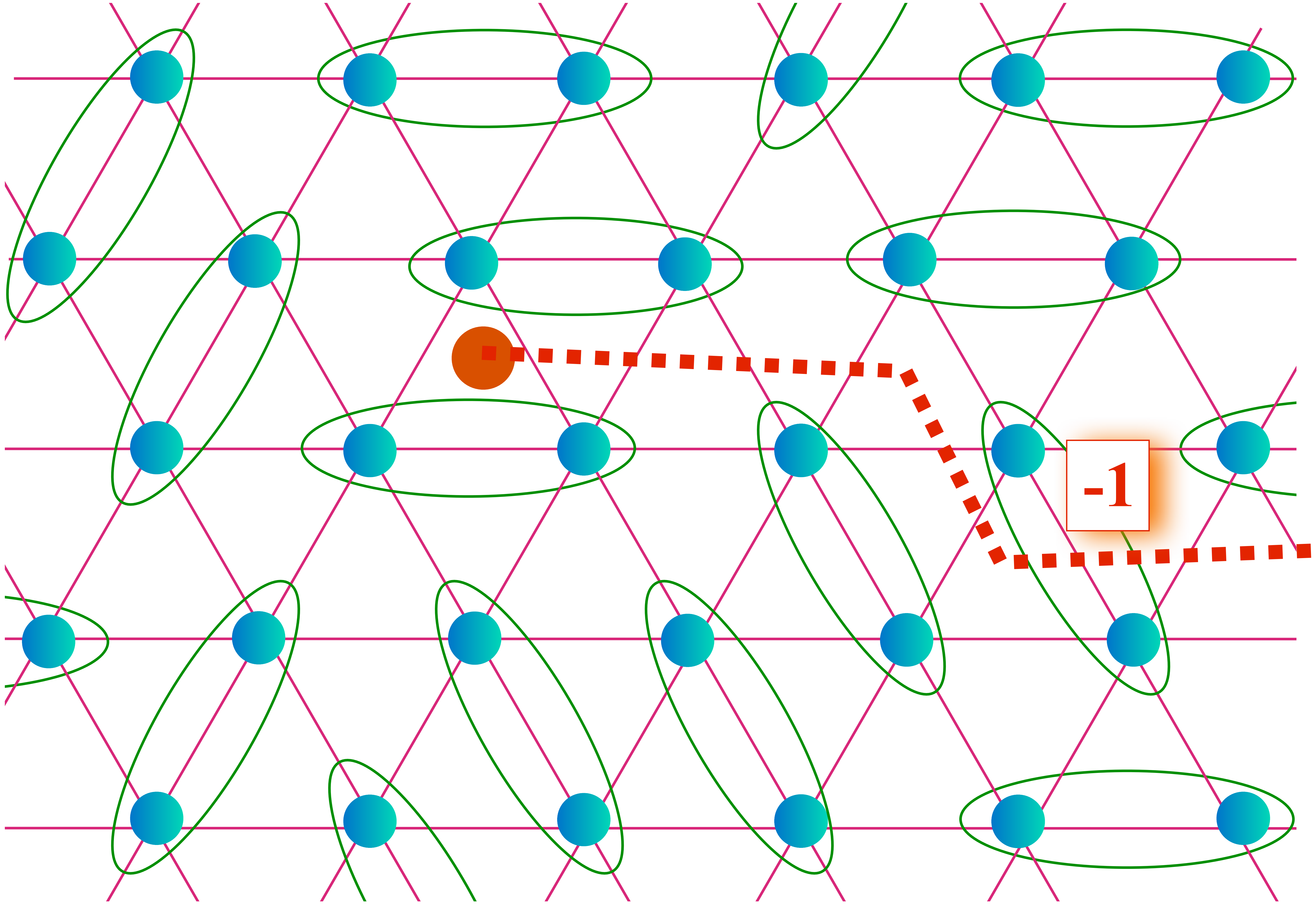

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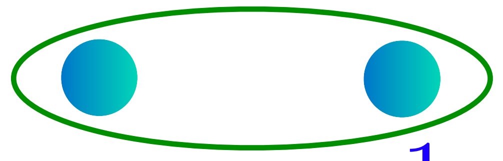

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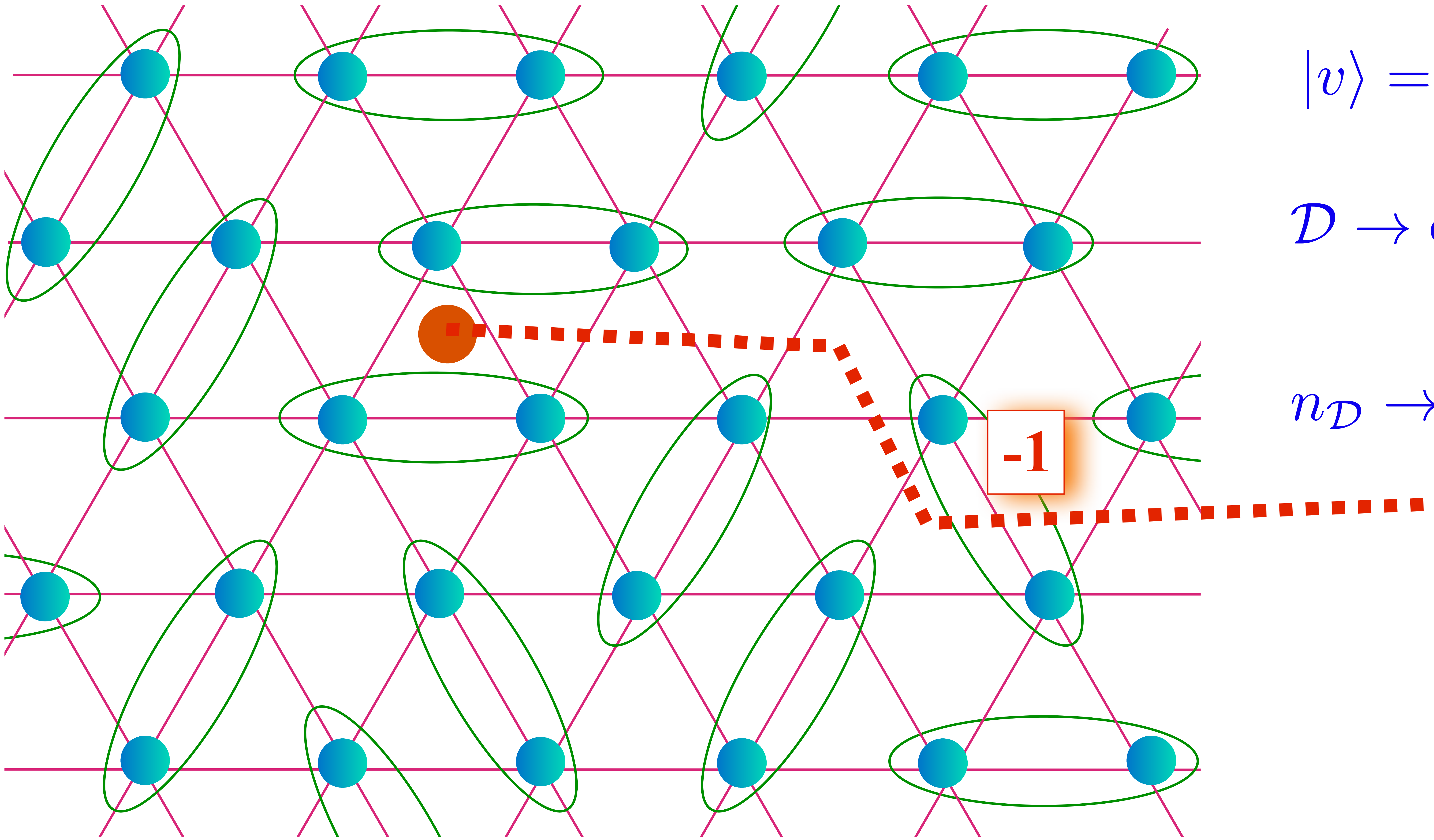

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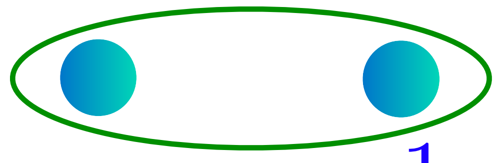

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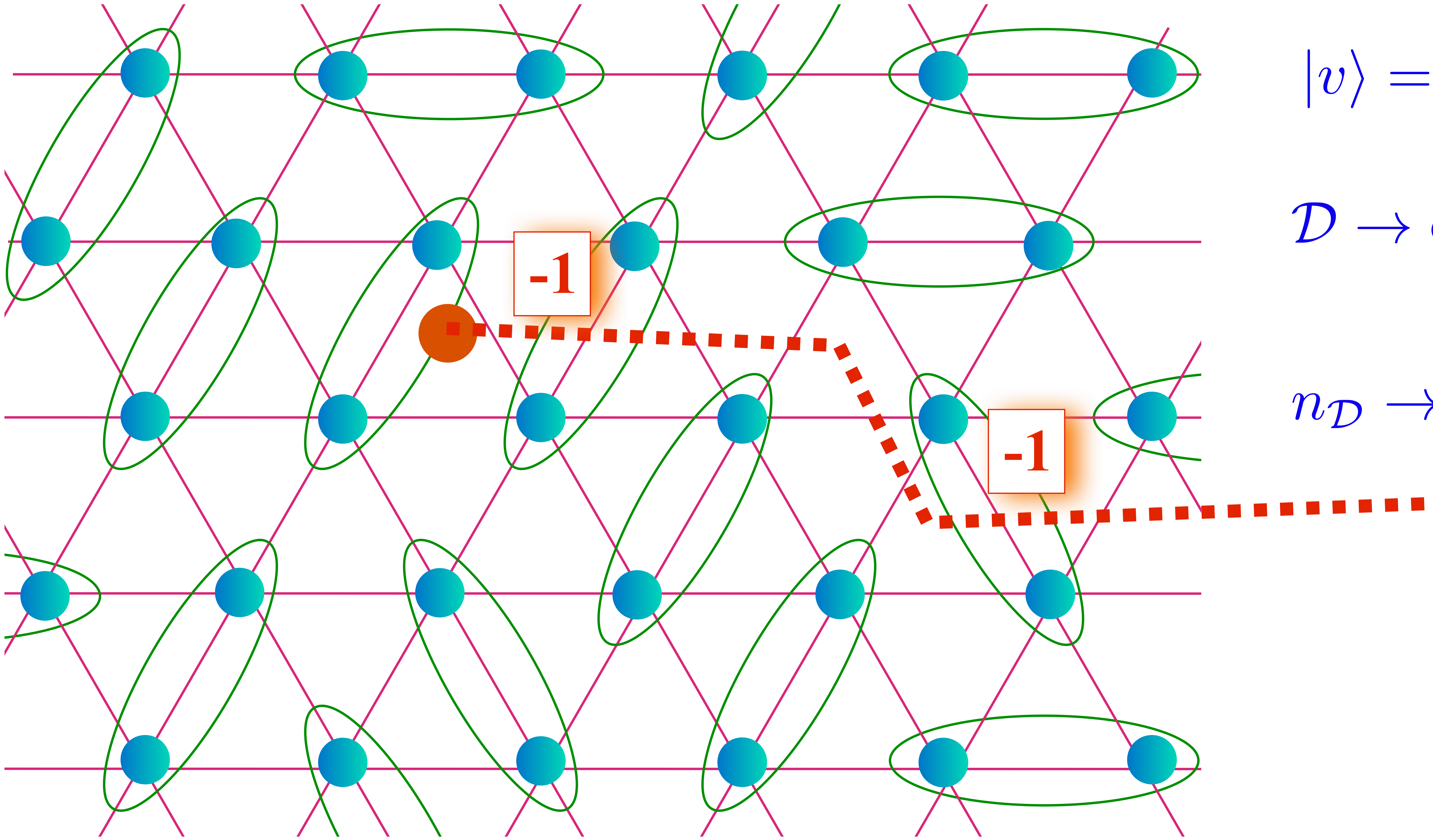

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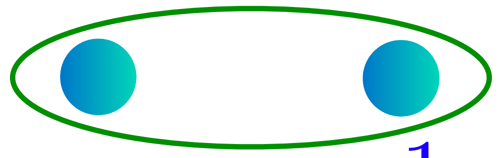

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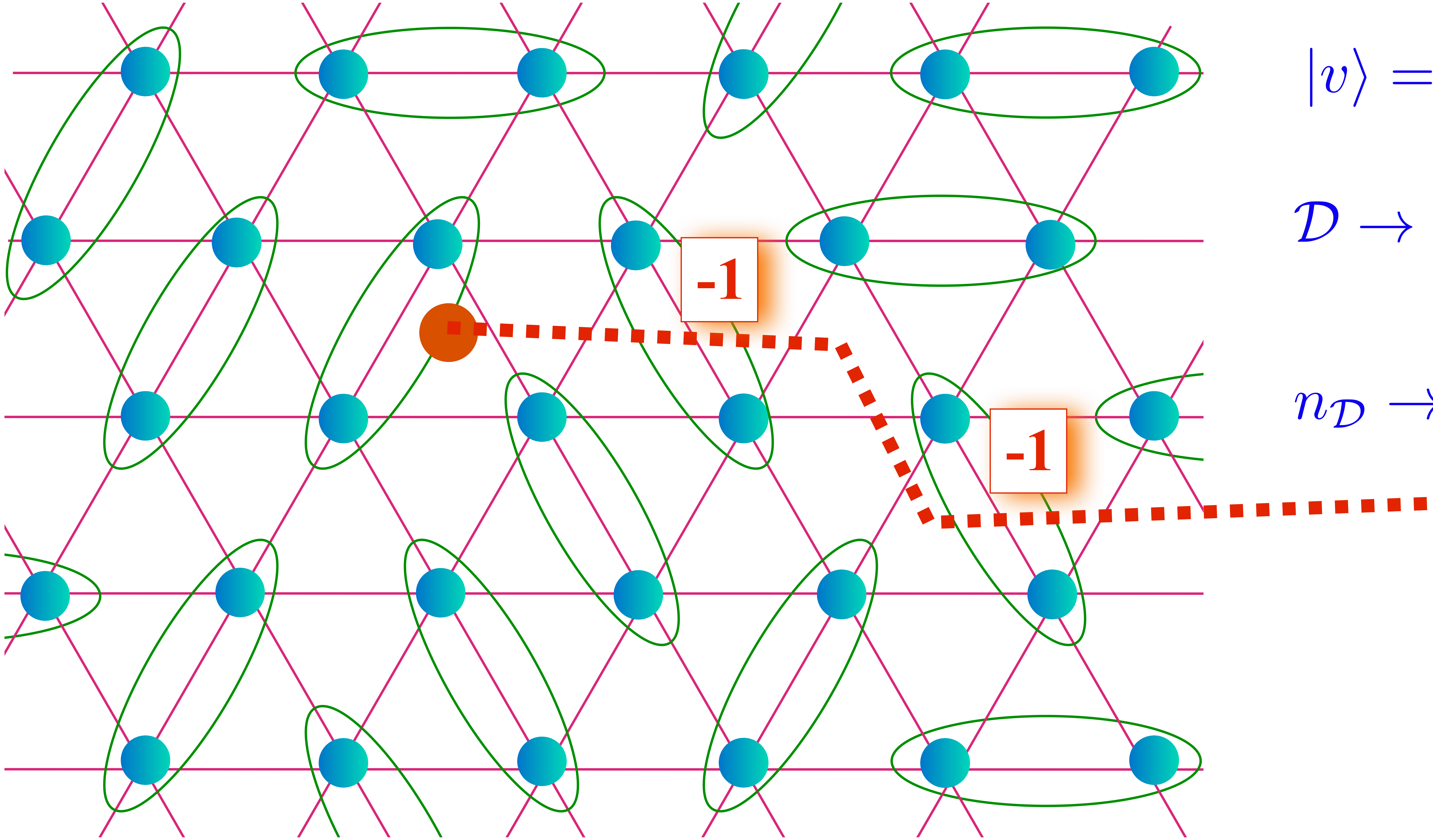

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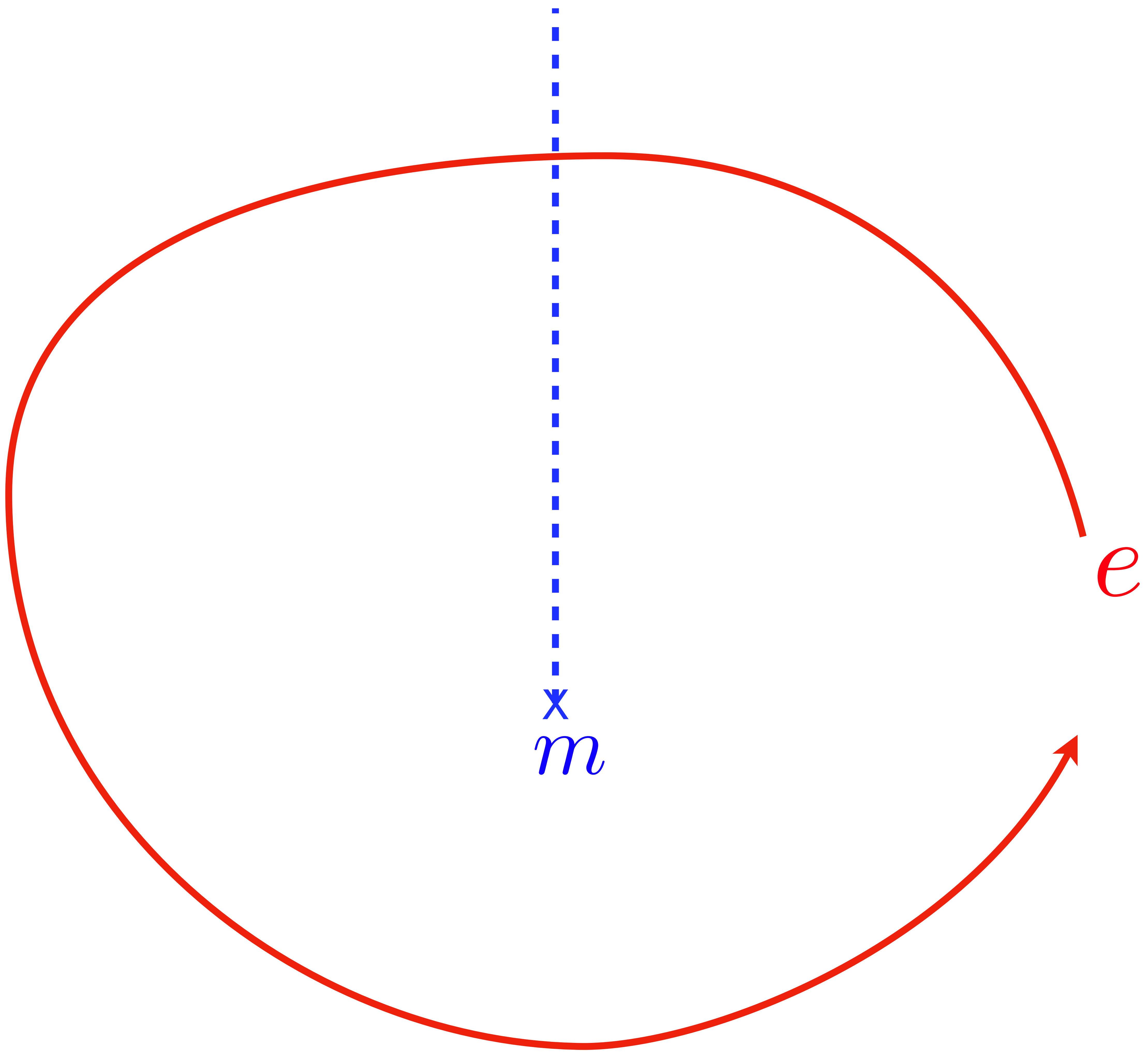


The e spinon and the m vison are *mutual* semions

RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

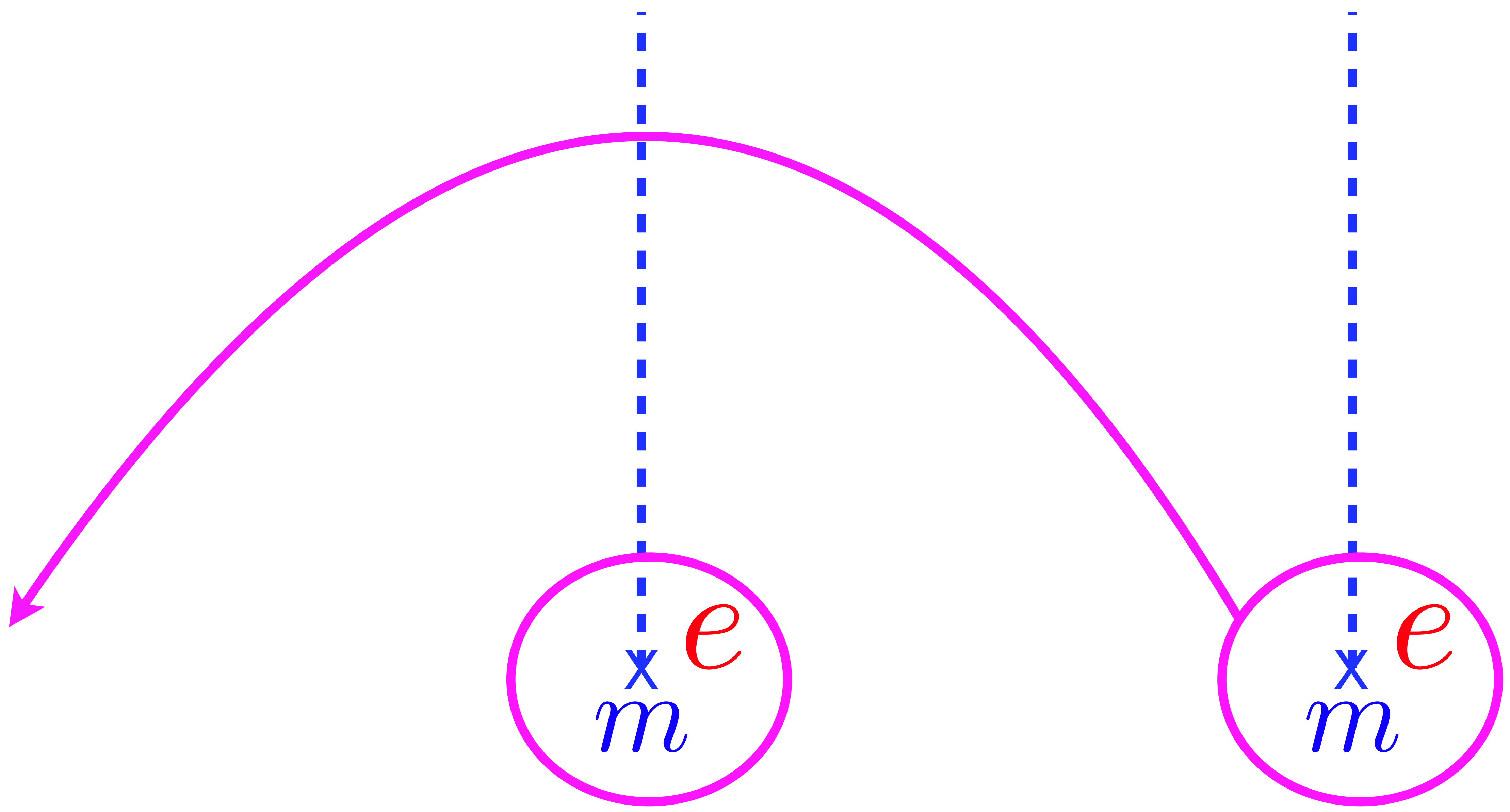
The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are 'spinon' excitations which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	$1/2$	$1/2$	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.

These anyons are 'topological': they cannot be created individually by any local operator, and their existence implies a four-fold ground state degeneracy on a large torus.

RVB: Z_2 spin liquid



The ϵ spinon is a fermion.

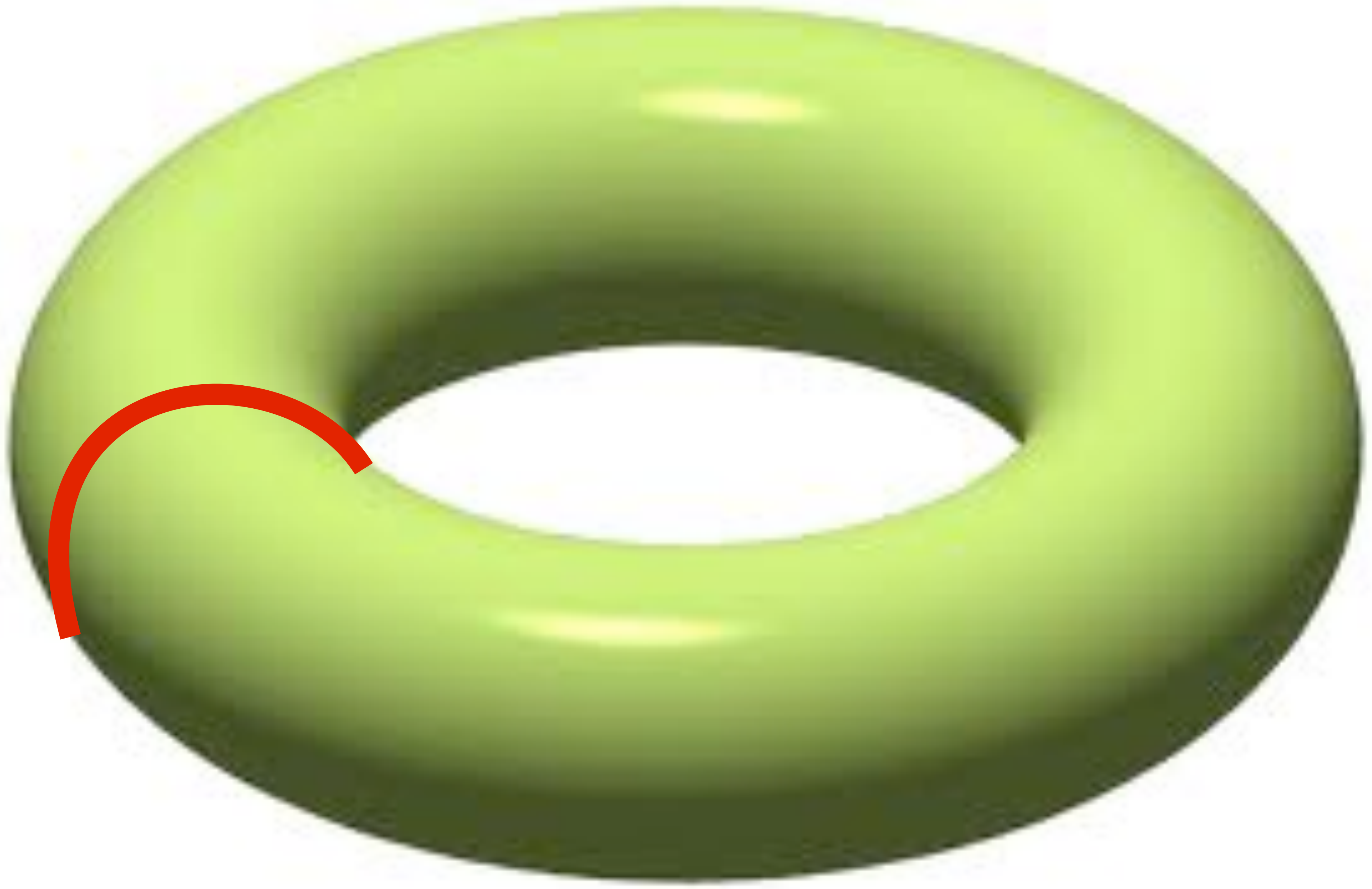
RVB: Z_2 spin liquid

Ground state degeneracy on the torus

Place insulator on a torus:

RVB: Z_2 spin liquid

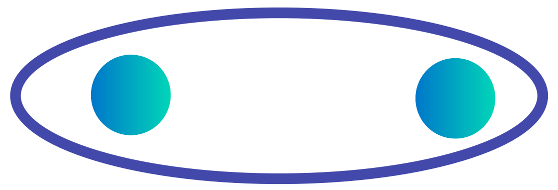
Ground state degeneracy on the torus



Place insulator on a torus:

Obtain a degenerate orthogonal state by modifying the wavefunction on a "branch-cut" encircling the torus.

RVB: Z_2 spin liquid



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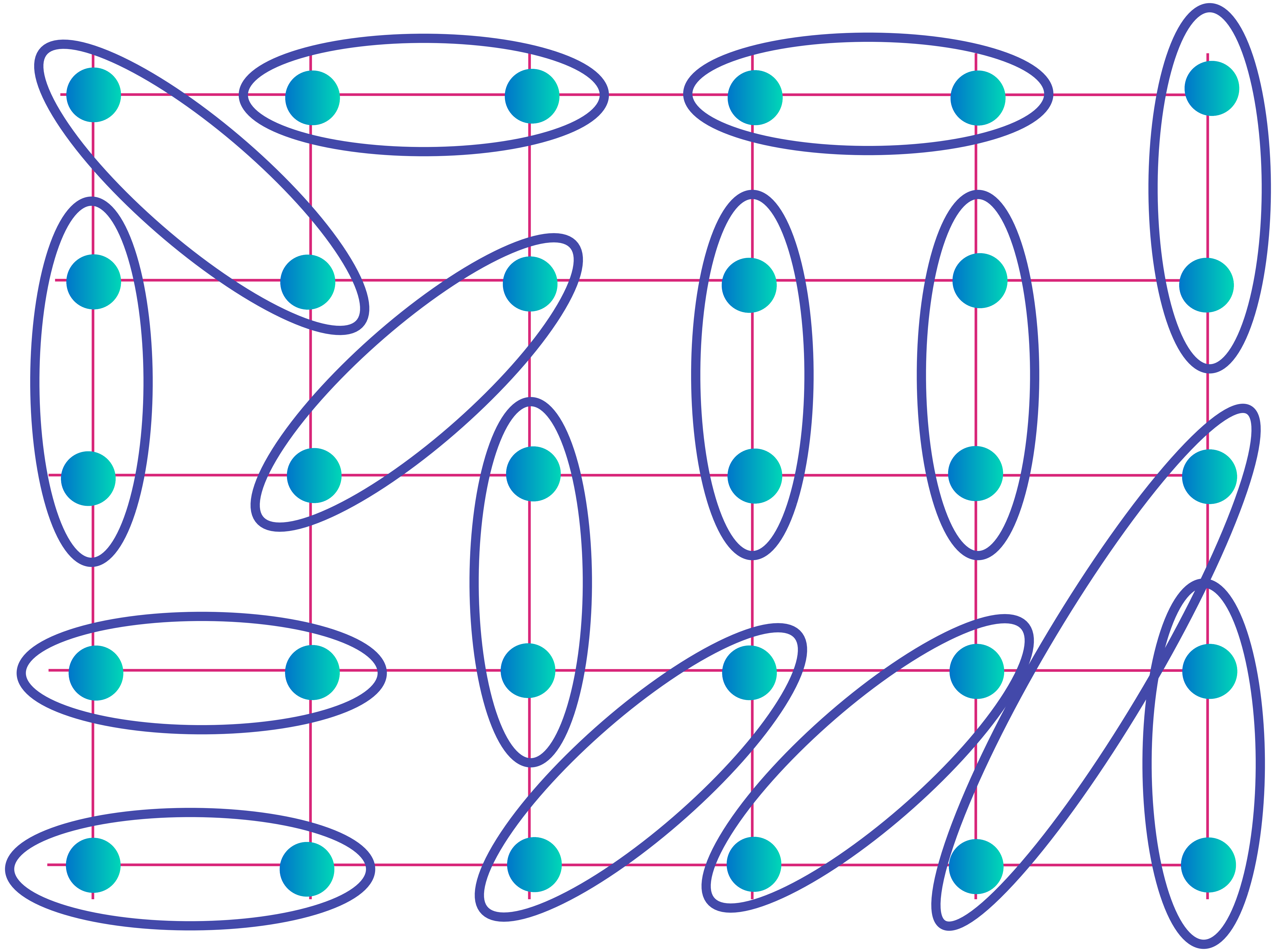

Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

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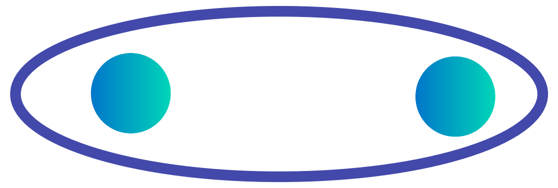

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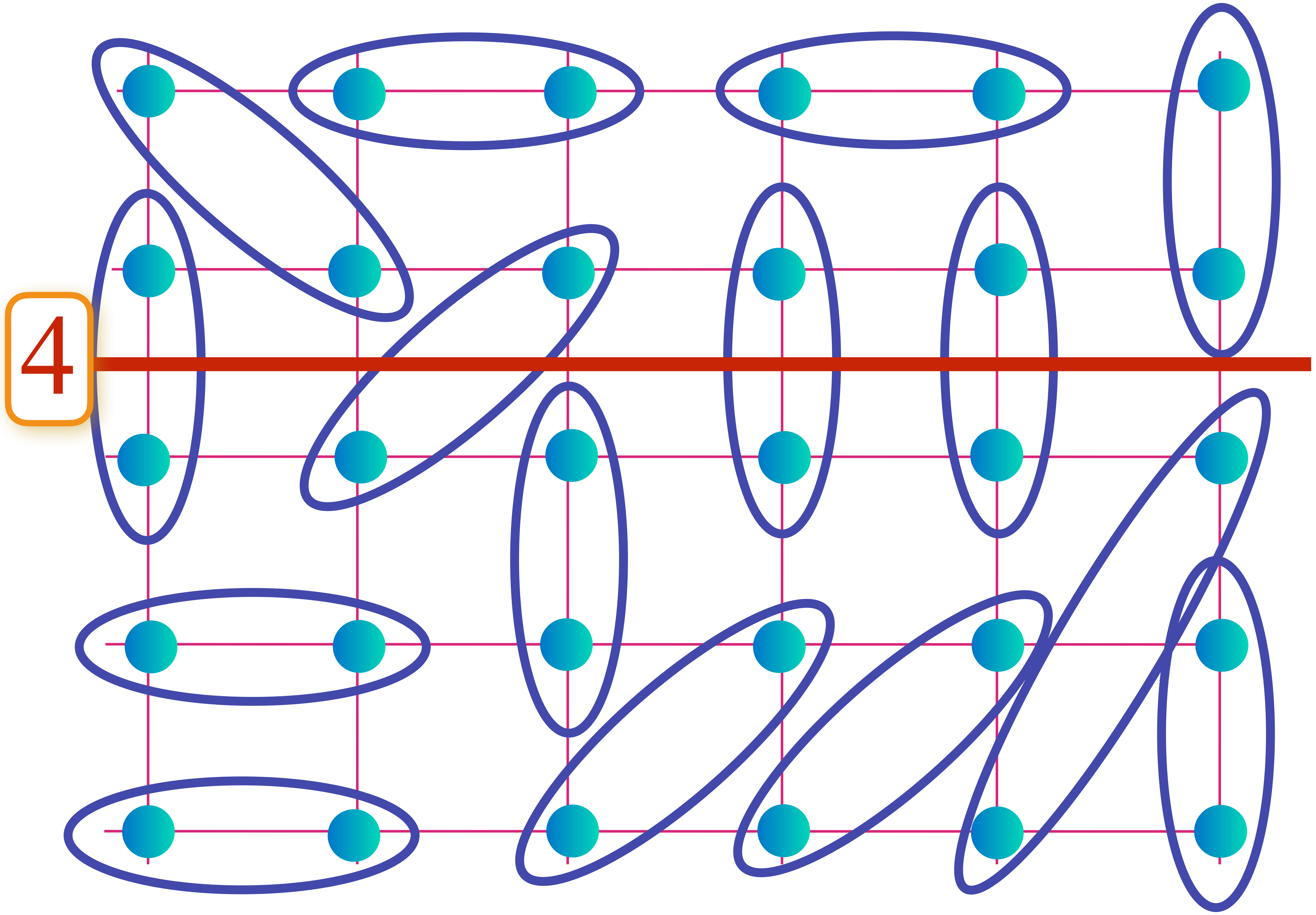

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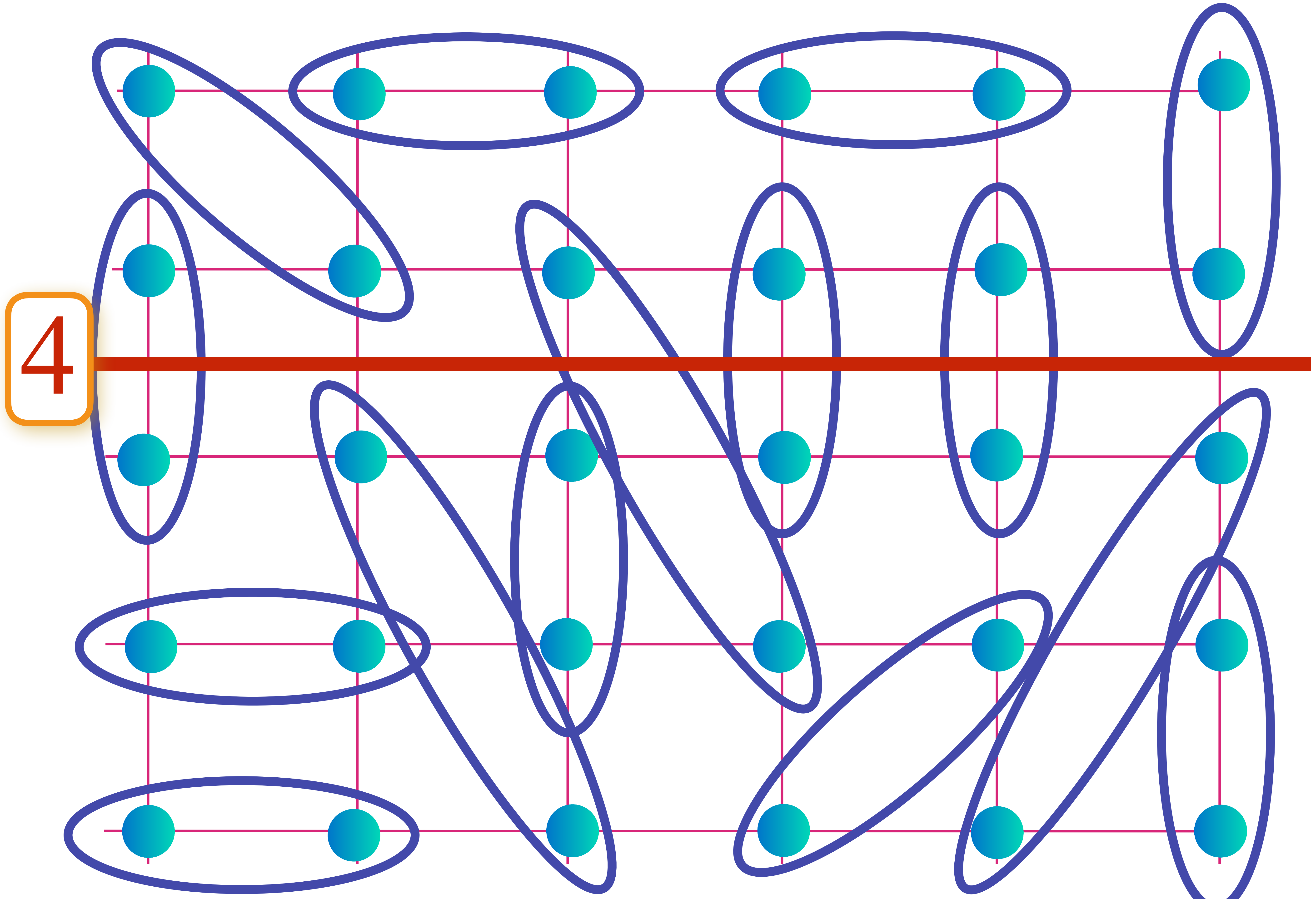
$$\text{(dimer)} = \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

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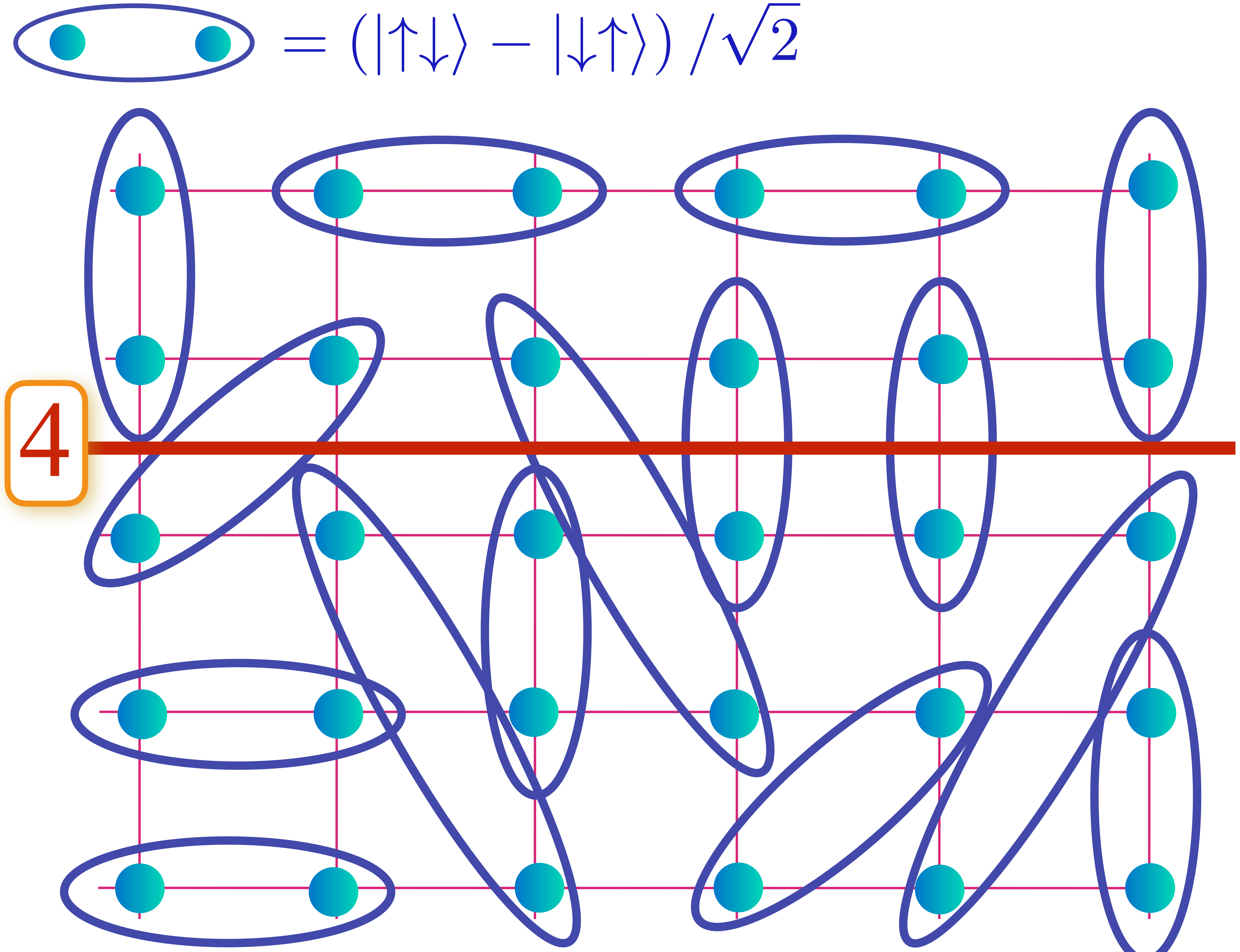

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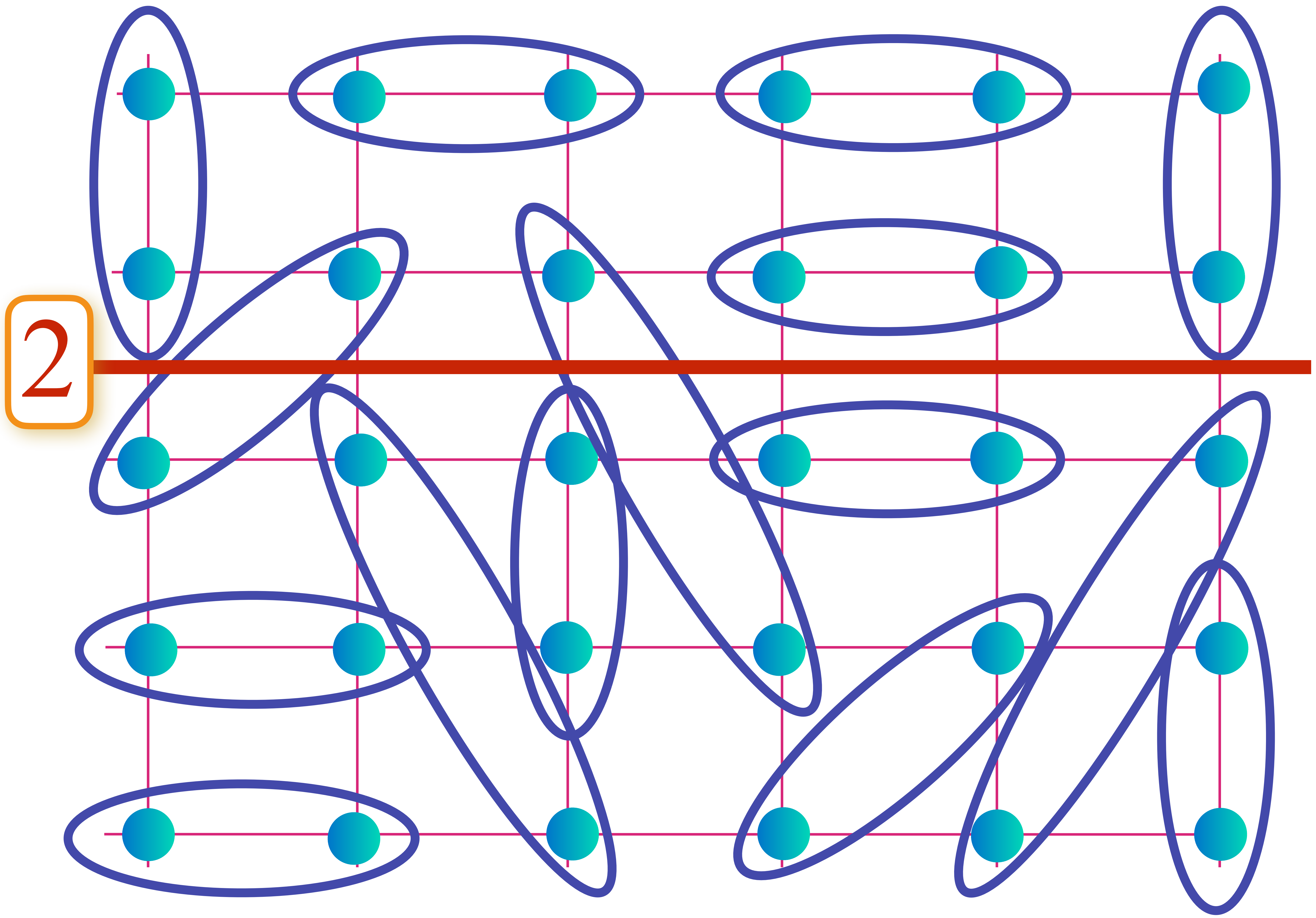

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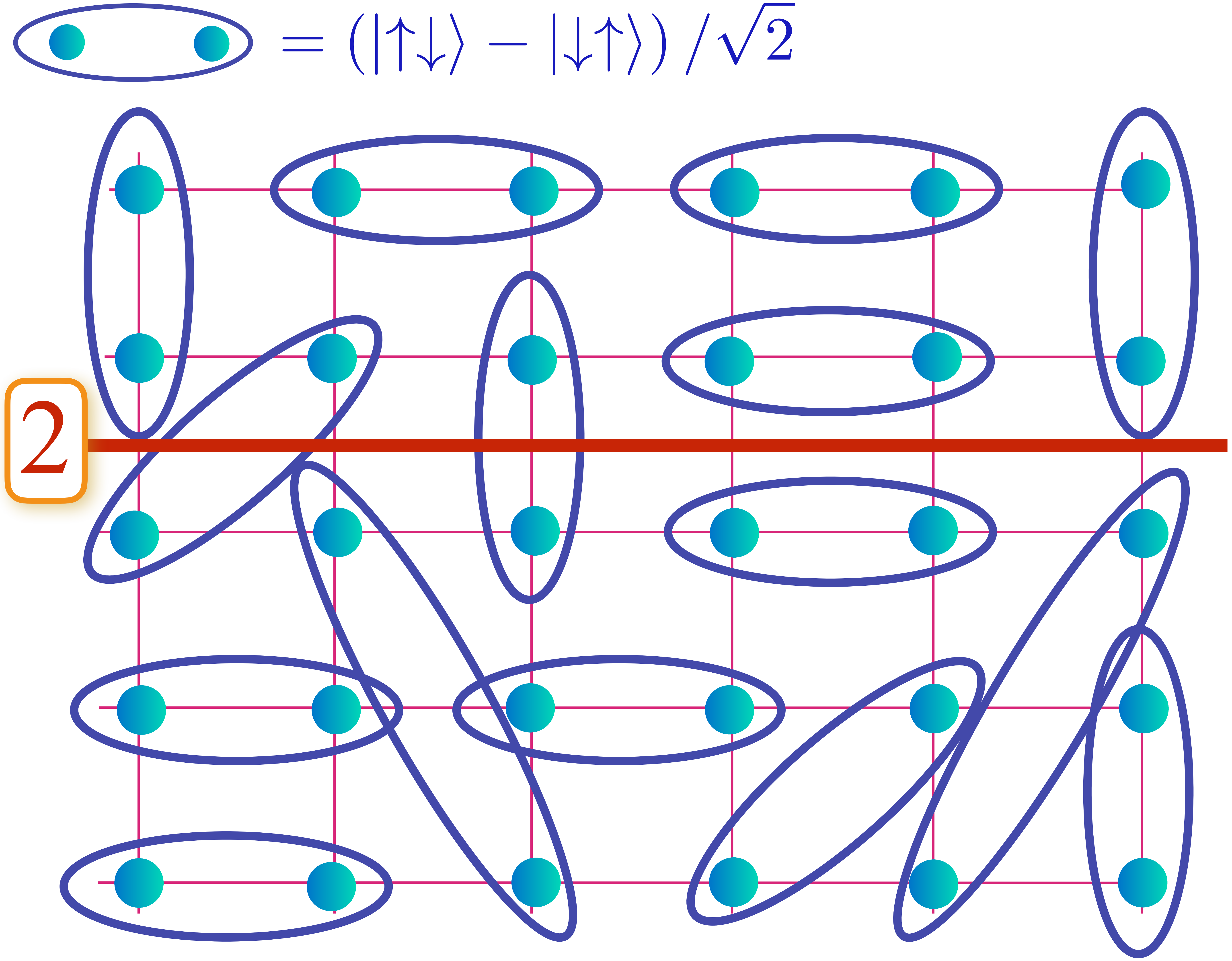

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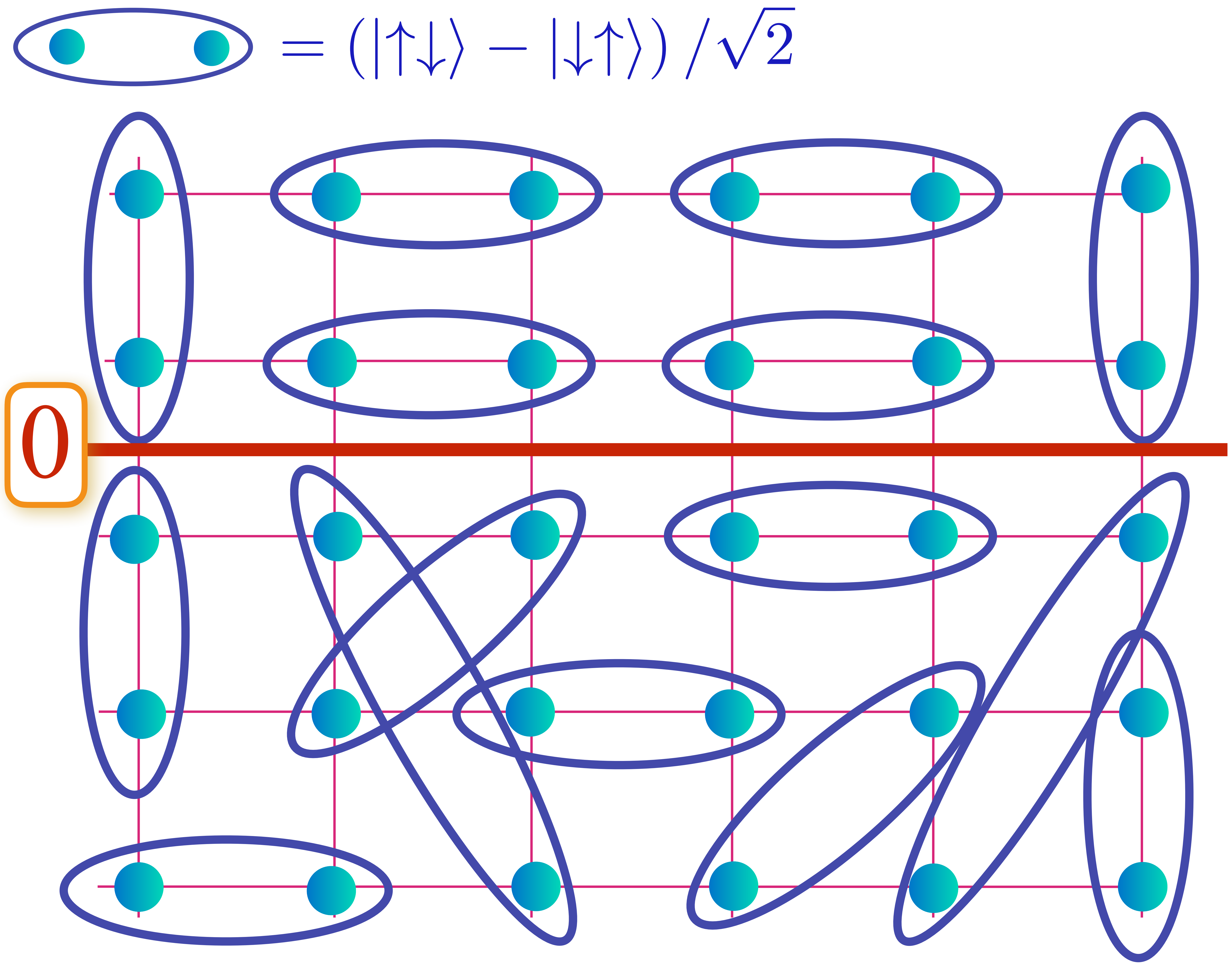

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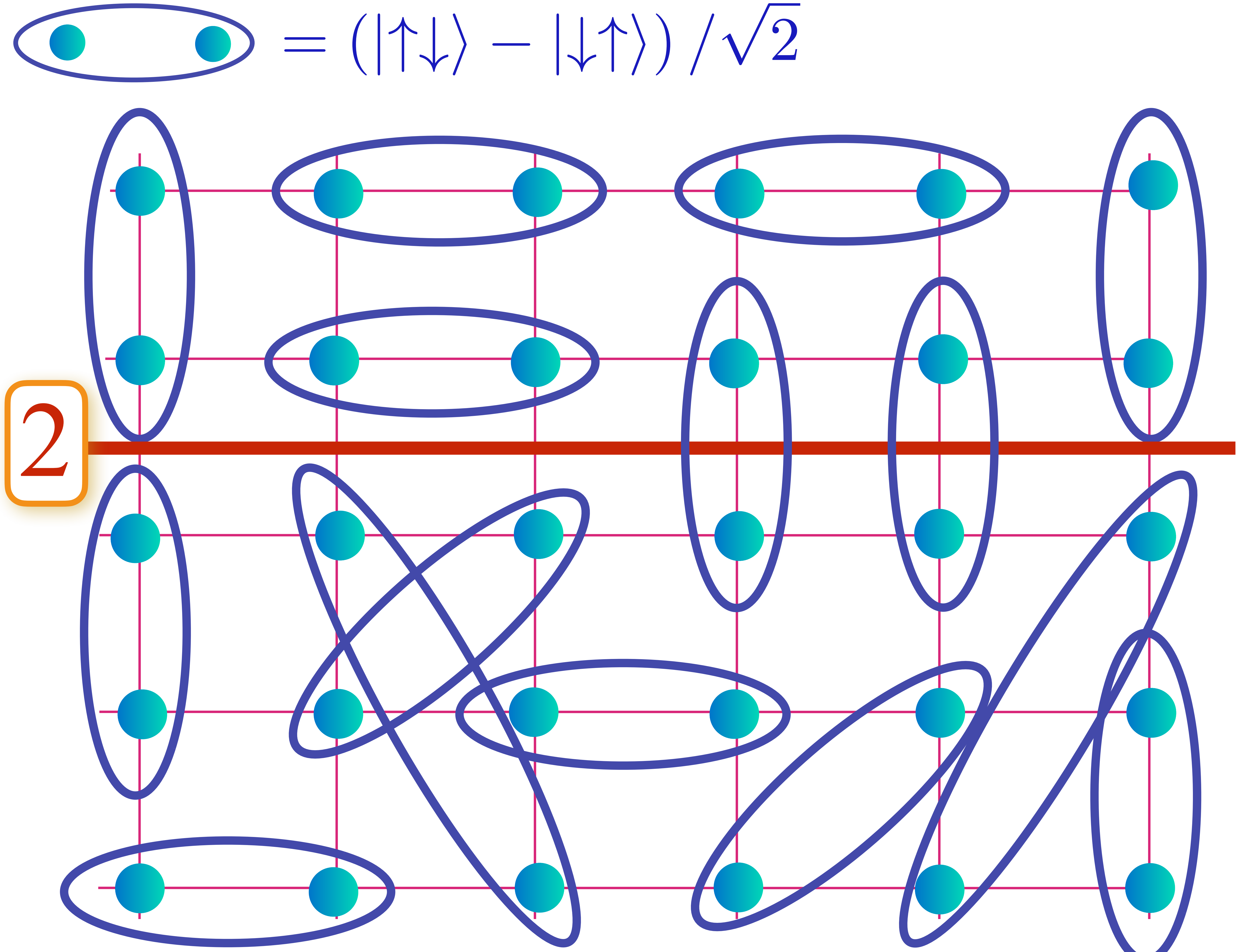

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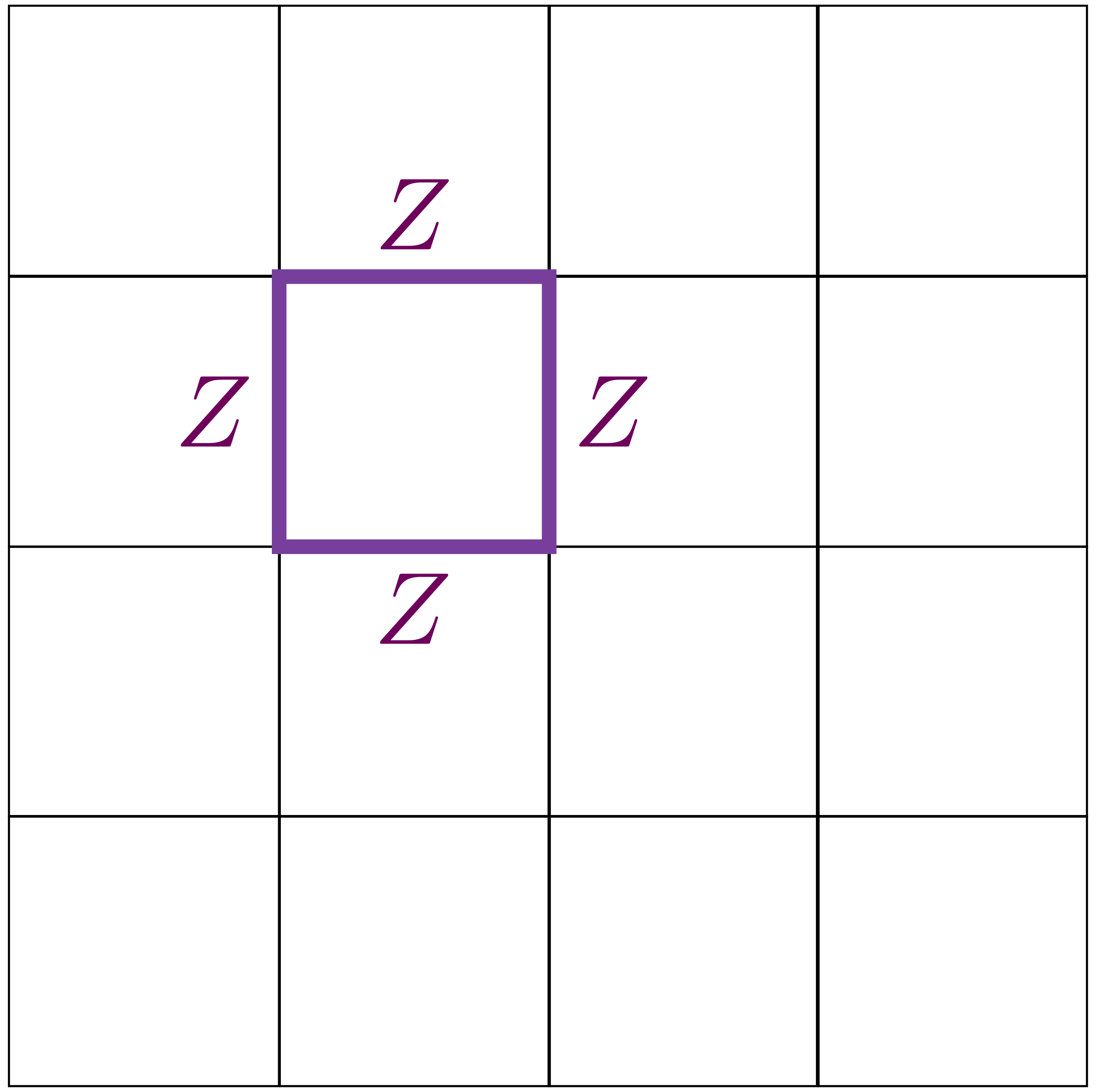
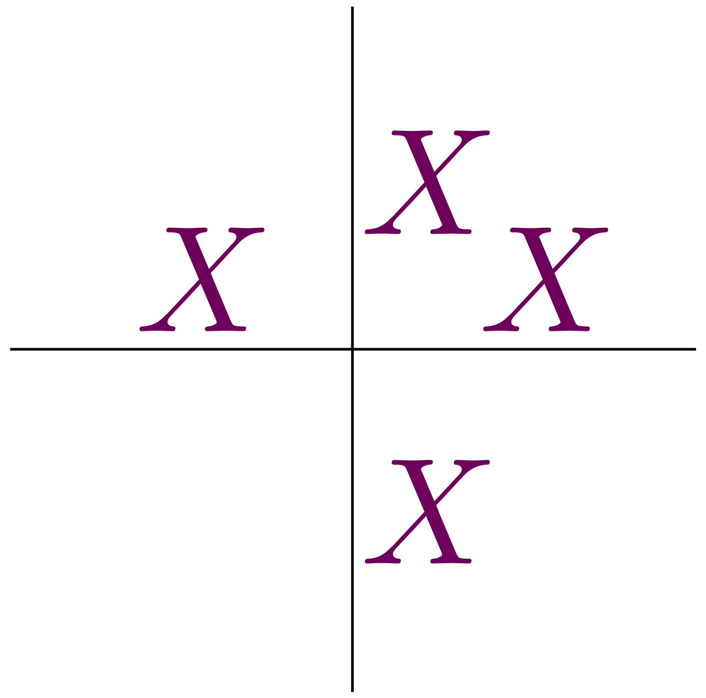
Pure Z_2 gauge theory

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$



$G_i =$ 

$$[\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0 \quad , \quad G_i = (-1)^{2S} \text{ for spin } S \text{ antiferromagnets}$$

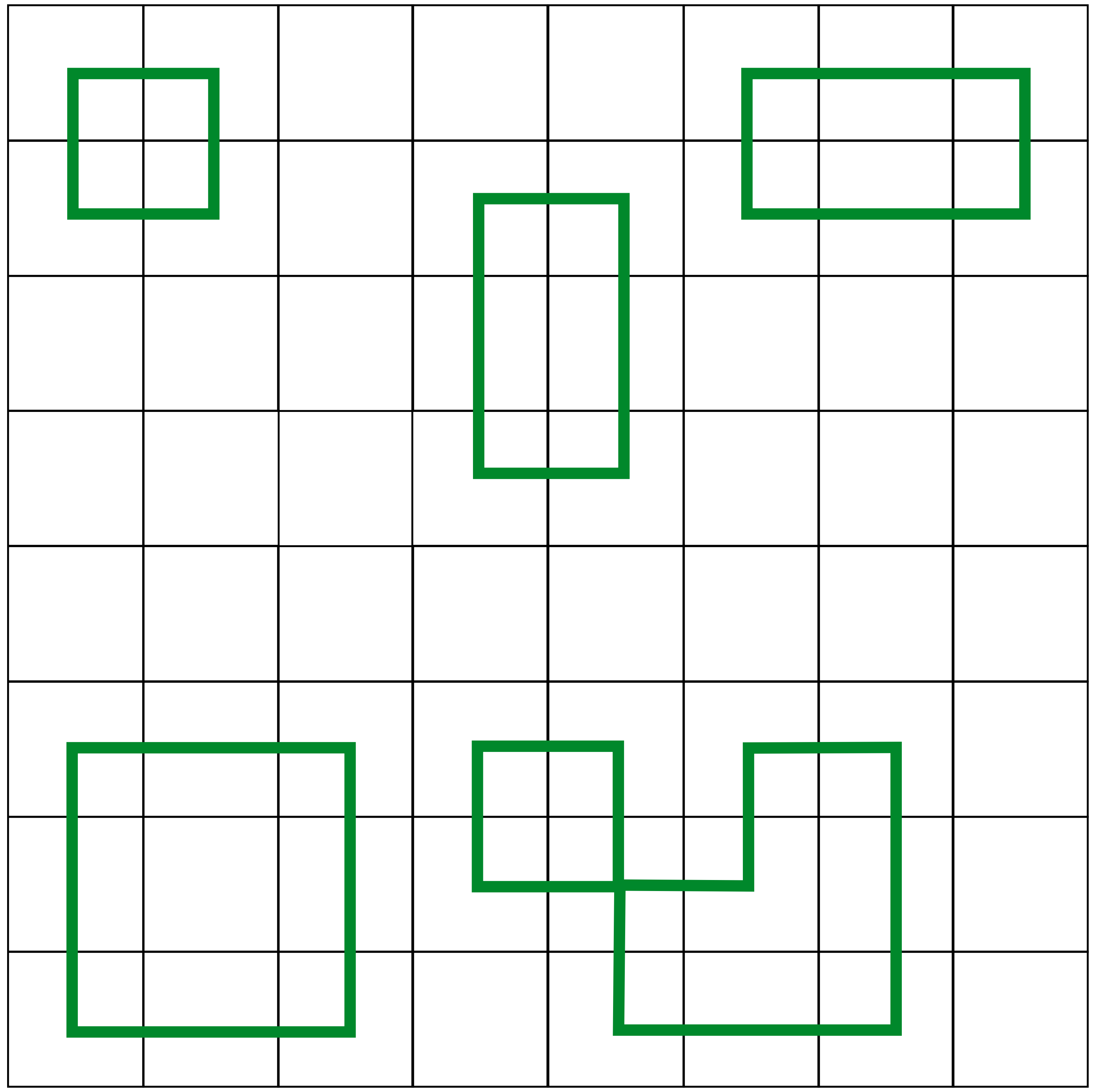
Pure Z_2 gauge theory

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$

Ground state as $g \to 0$:
sum over all closed loops
on the dual lattice.

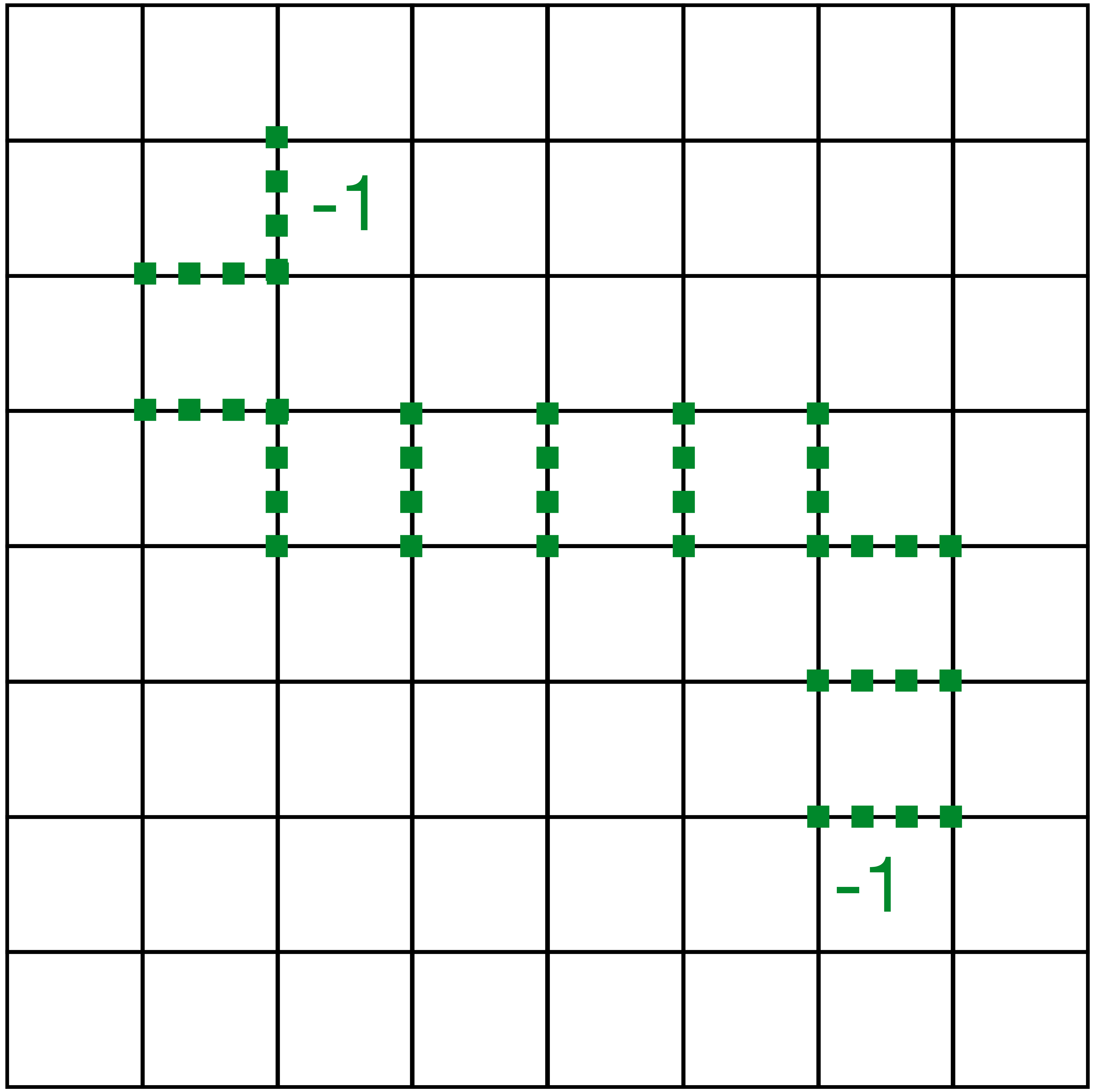
$$|0\rangle = \prod_i \left(1 + (-1)^{2S} G_i\right) |\Uparrow\rangle \; .$$

Every link crossing a green line is in state $|\Downarrow\rangle$.
Gauge-invariant state
with zero flux is every plaquette.



$|0\rangle$

Pure Z_2 gauge theory



Two visons connected by an invisible string. The dashed lines indicate the links, ℓ, on which $Z_\ell = -1$. The plaquettes with an even number of dashed lines on their edges carry no $\mathbb{Z}_2$ fluxes, and so are 'invisible'.

Implies $|\downarrow\rangle$ can be interpreted as a valence bond in the antiferromagnet

Z_2 gauge theory with matter

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$

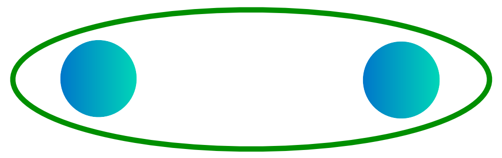
$$- J \sum_{\ell \in (i,j)} \tau_i^z Z_\ell \tau_j^z - h \sum_i \tau_\ell^x$$

$$G_i = \tau_i^x \prod_{\ell \in i} X_\ell \quad , \quad [\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

Now we choose $G_i = 1$ and $\text{sgn}(h) = (-1)^{2S}$.

The τ_i^x operator creates a $\mathbb{Z}_2$ electric charge – a ‘spinon’ which has mutual semionic statistics with a vison.

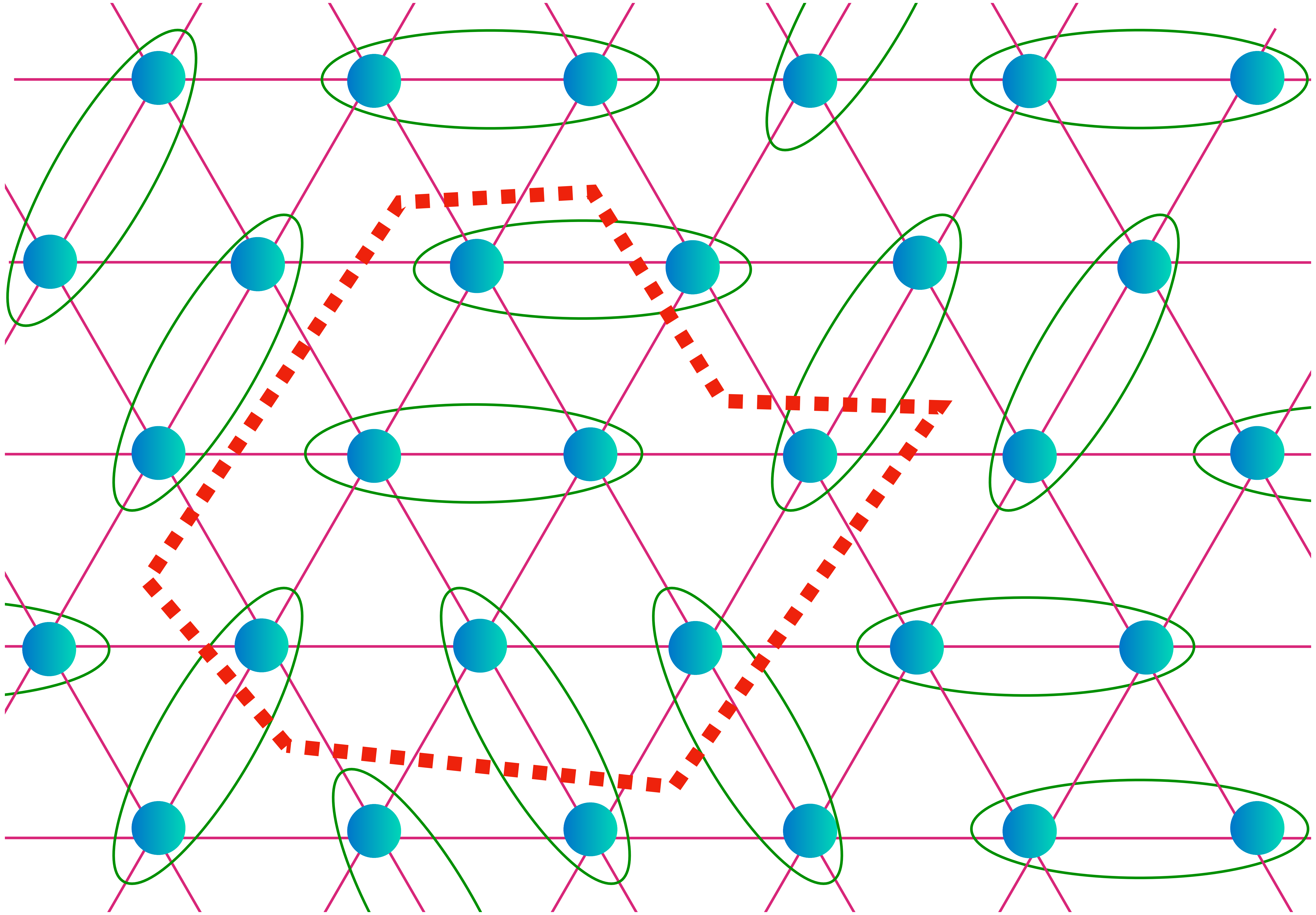
RVB: Z_2 spin liquid



$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Topological X operator of the Z_2 Spin liquid

5



X = product of G_i for i inside red line

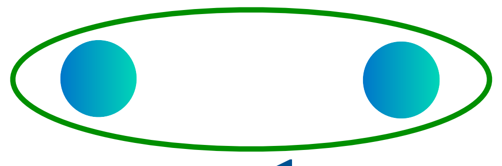
= parity of sites enclosed by red line for $S = 1/2$

$= (-1)^{n_{\mathcal{D}}}$

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line

RVB: Z_2 spin liquid

Topological X operator of the Z_2 Spin liquid



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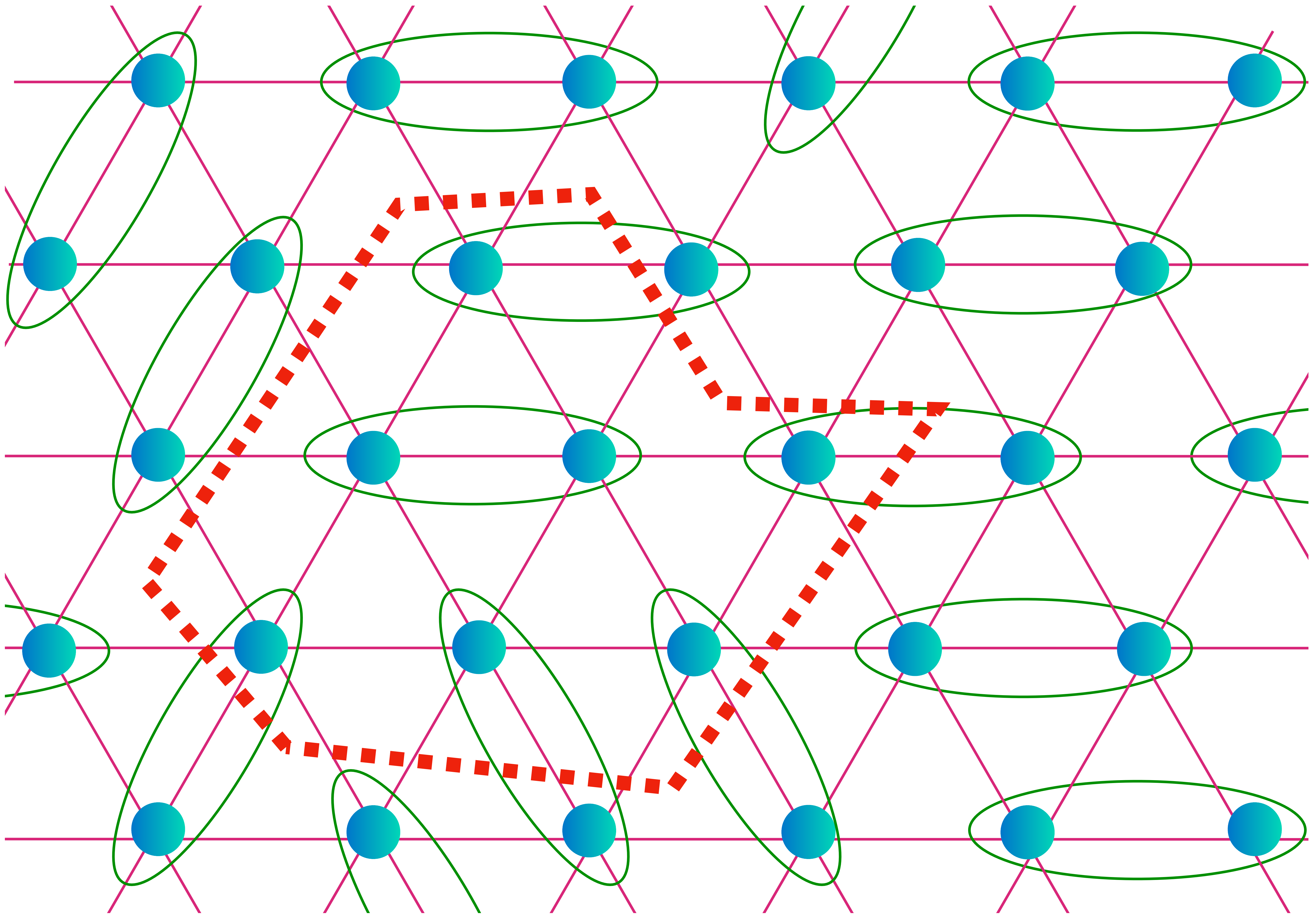

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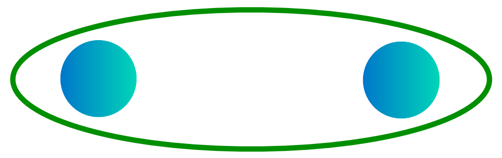

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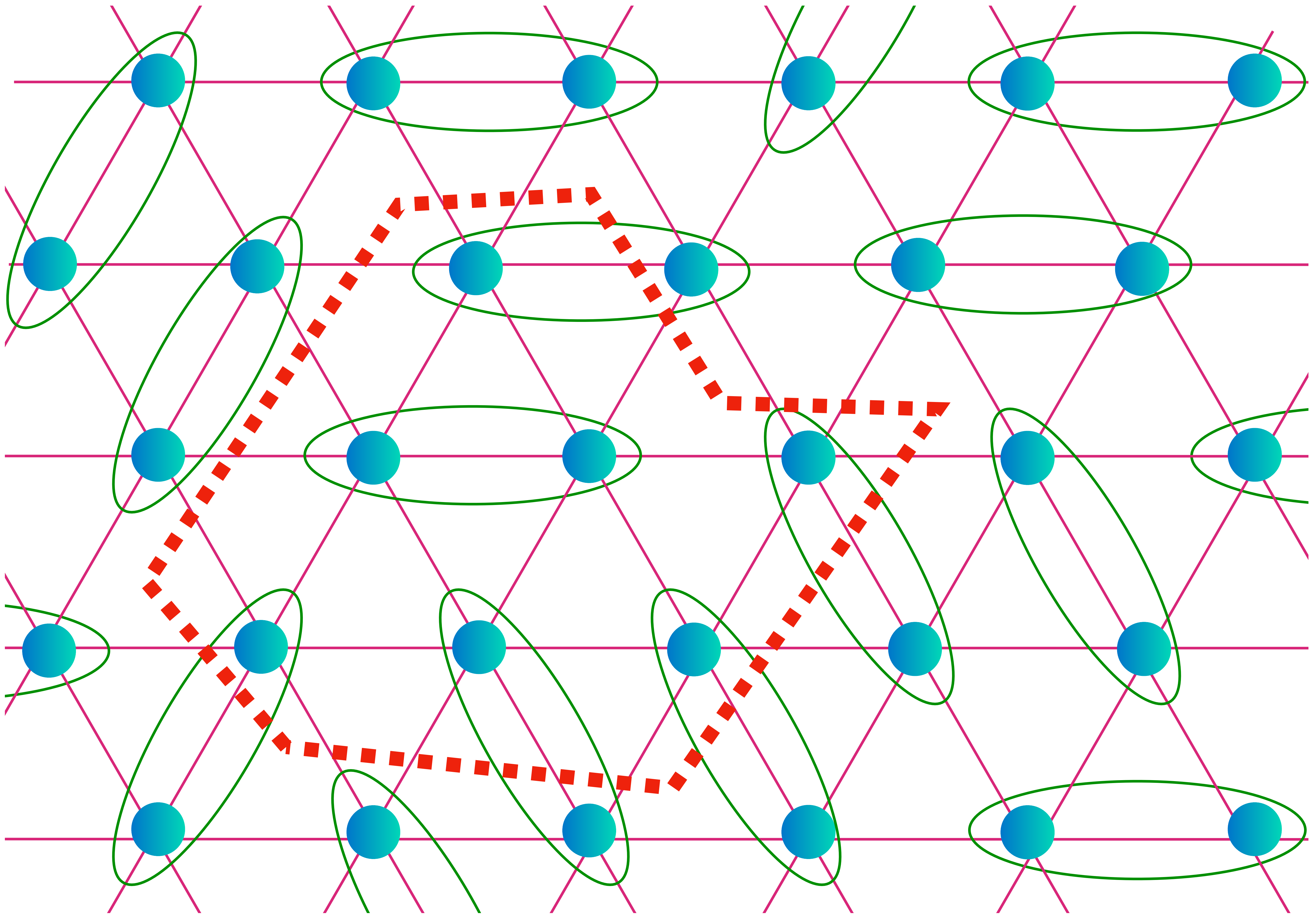

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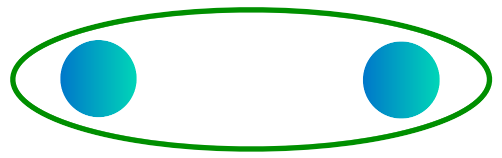

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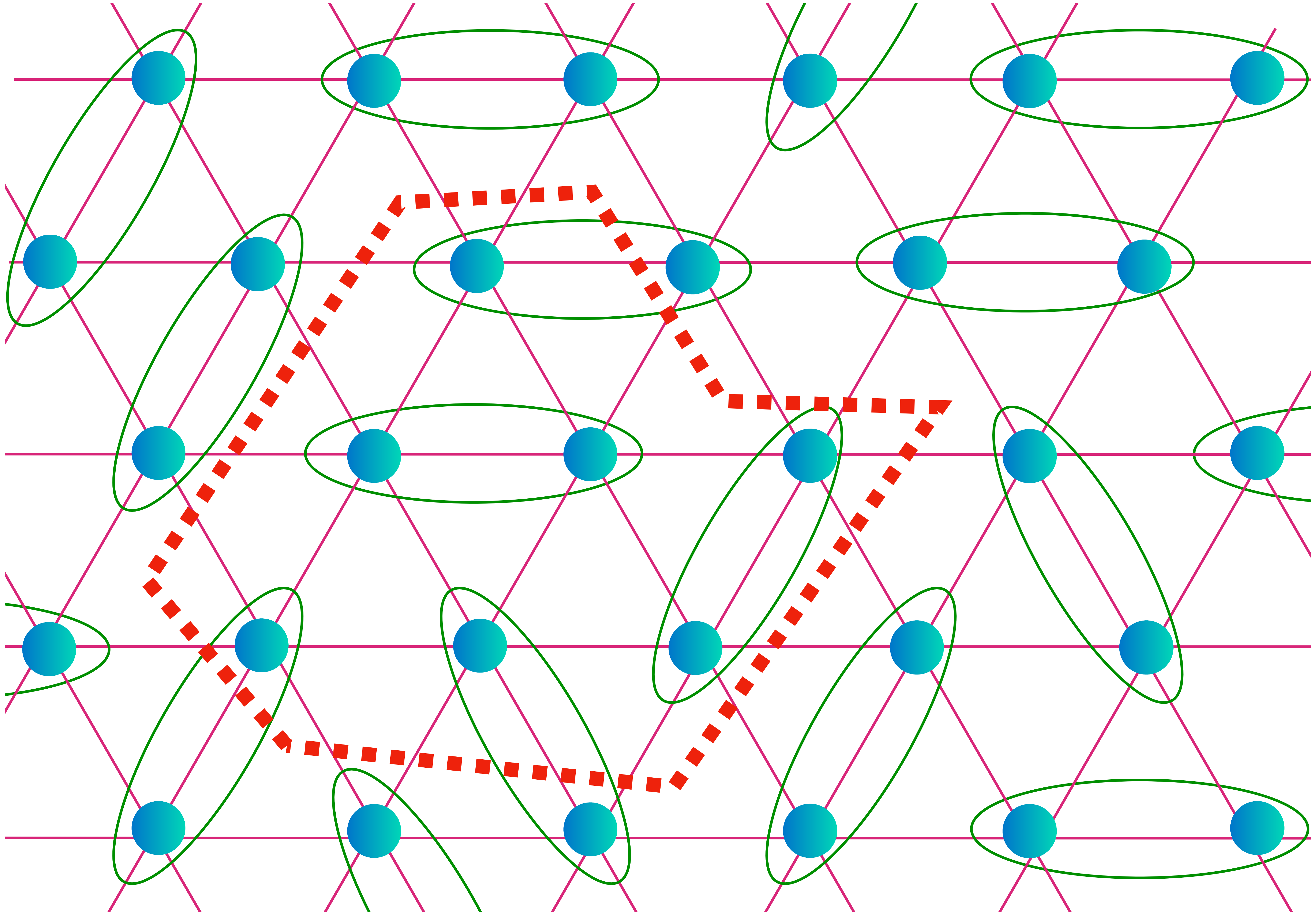

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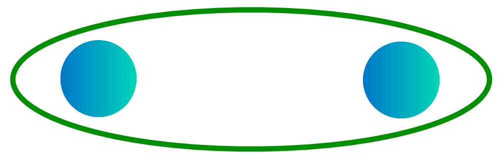

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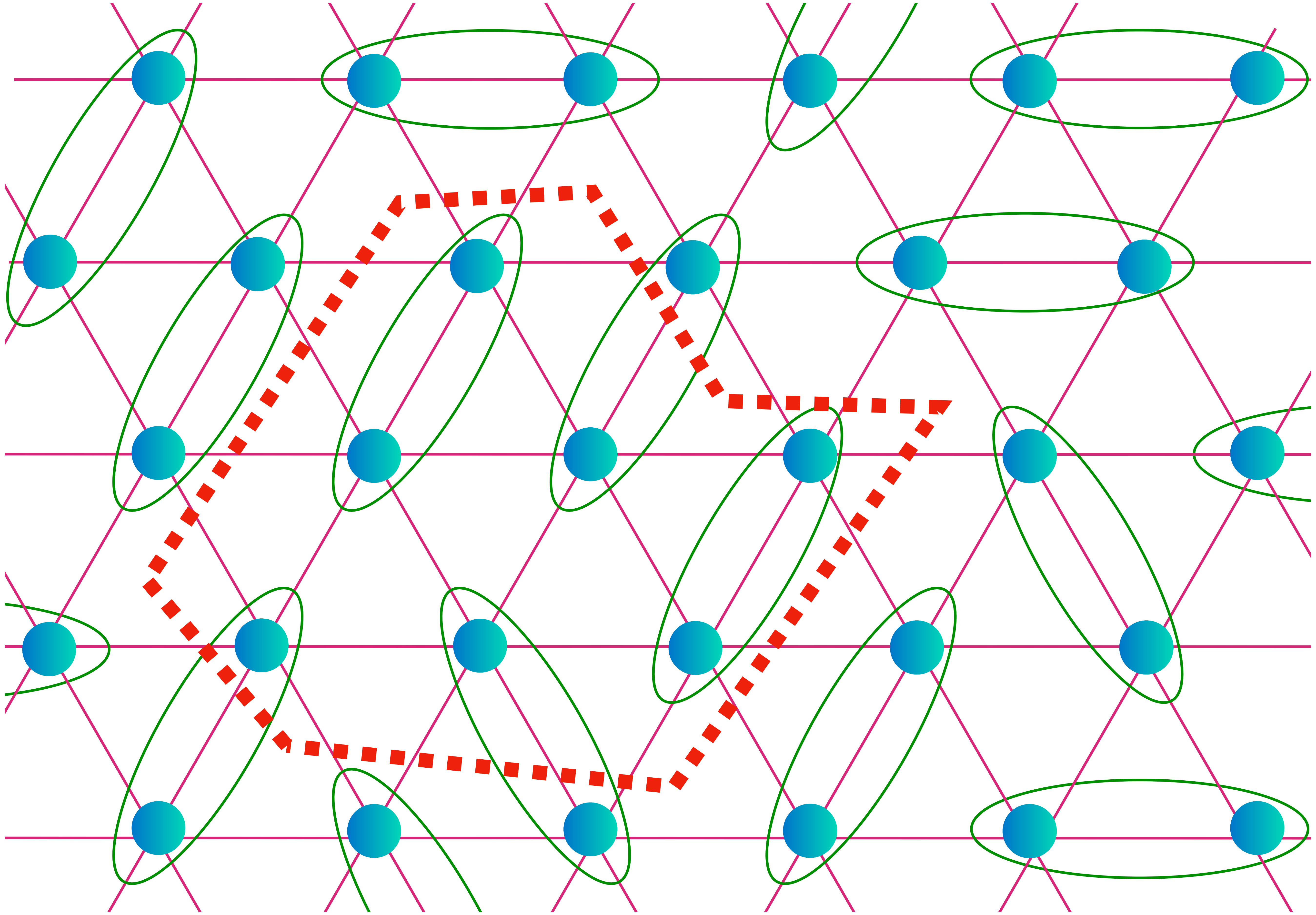

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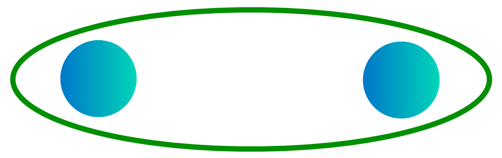

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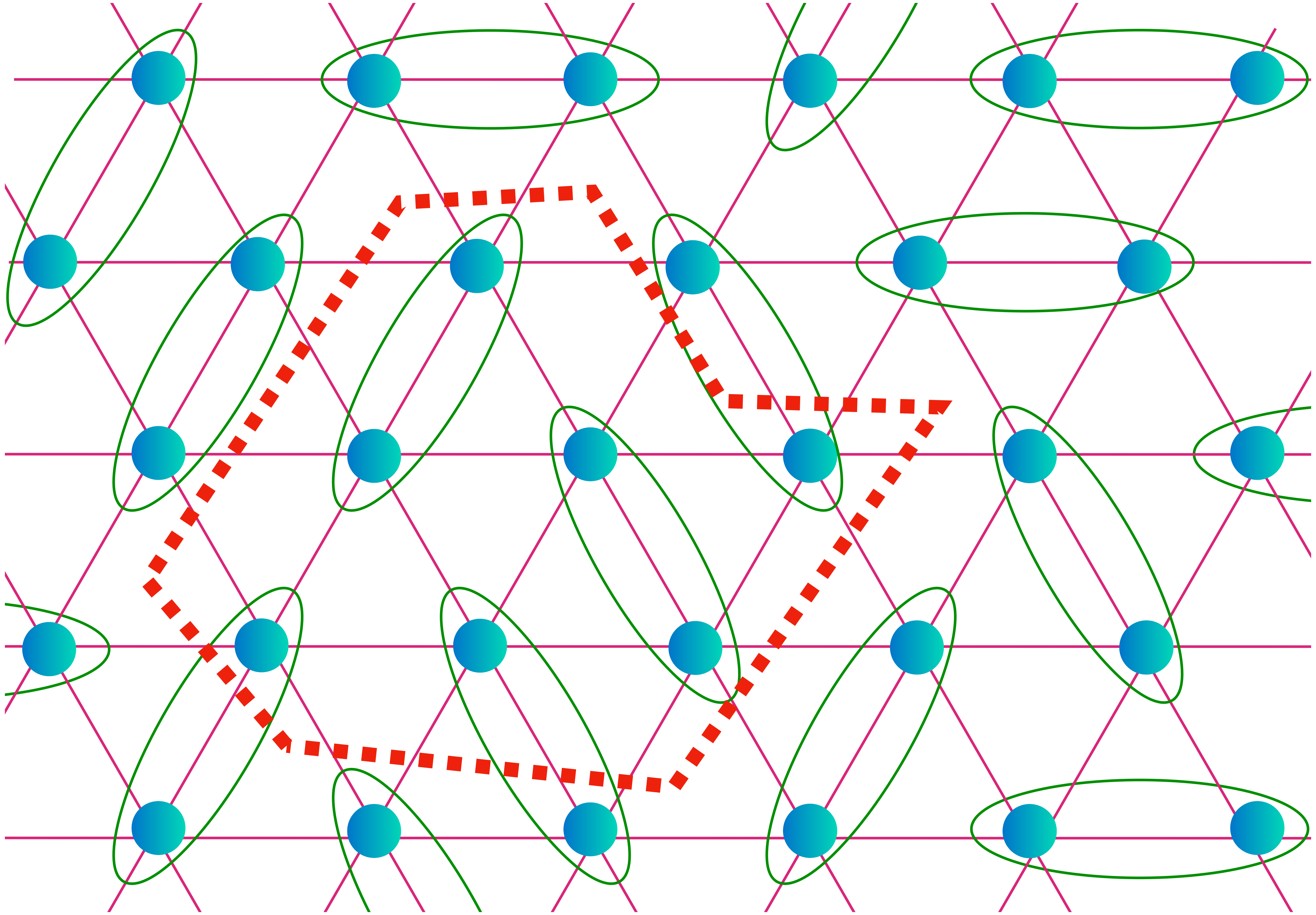

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1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

Probing topological spin liquids

Quantum phases of Rydberg atoms on a kagome lattice,
Rhine Samajdar, Wen Wei Ho, Hannes Pichler, M. D. Lukin, and S. S.,
Proceedings of the National Academy of Sciences **118**, e2015785118 (2021); arXiv:2011.12295

Emergent Z_2 gauge theories and topological excitations in Rydberg atom arrays,
Rhine Samajdar, Darshan G. Joshi, Yanting Teng, and S. S., arXiv:2204.00632

Rhine Samajdar

Wen Wei Ho

Mikhail Lukin

Hannes Pichler

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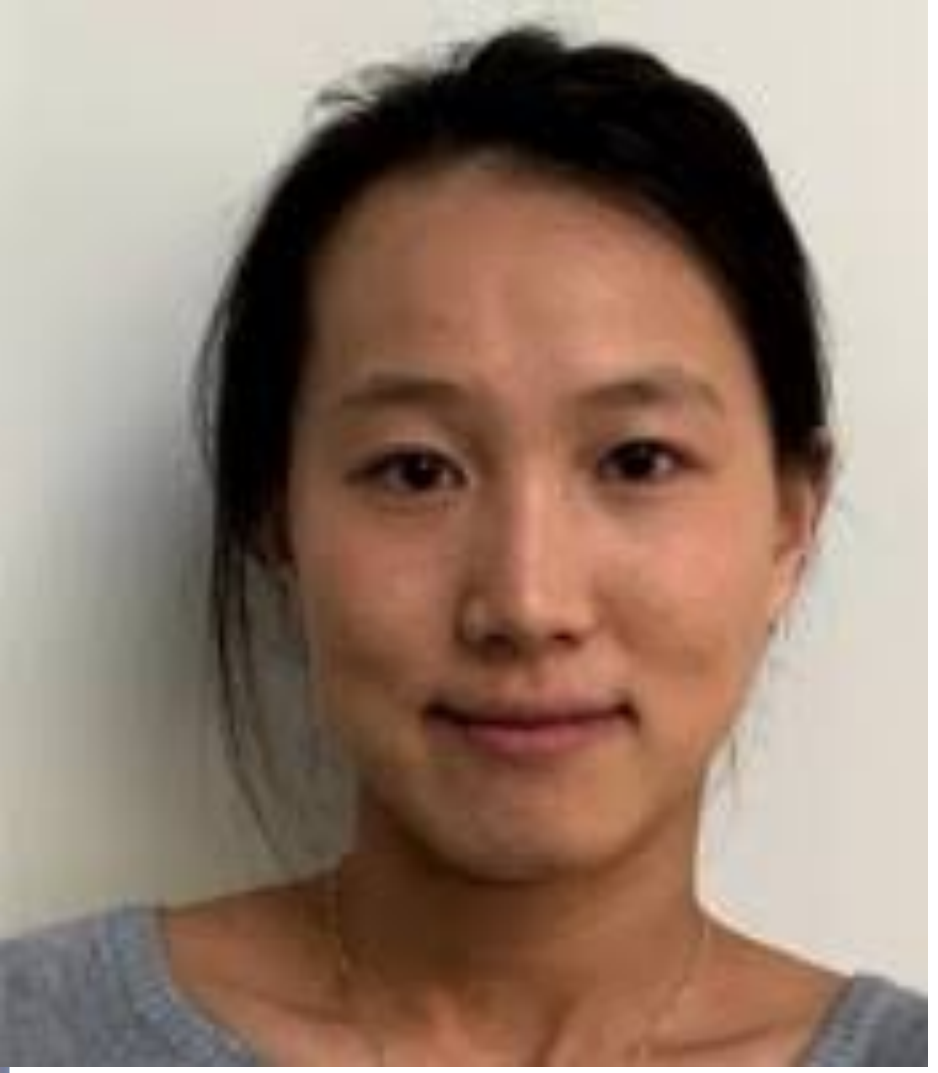

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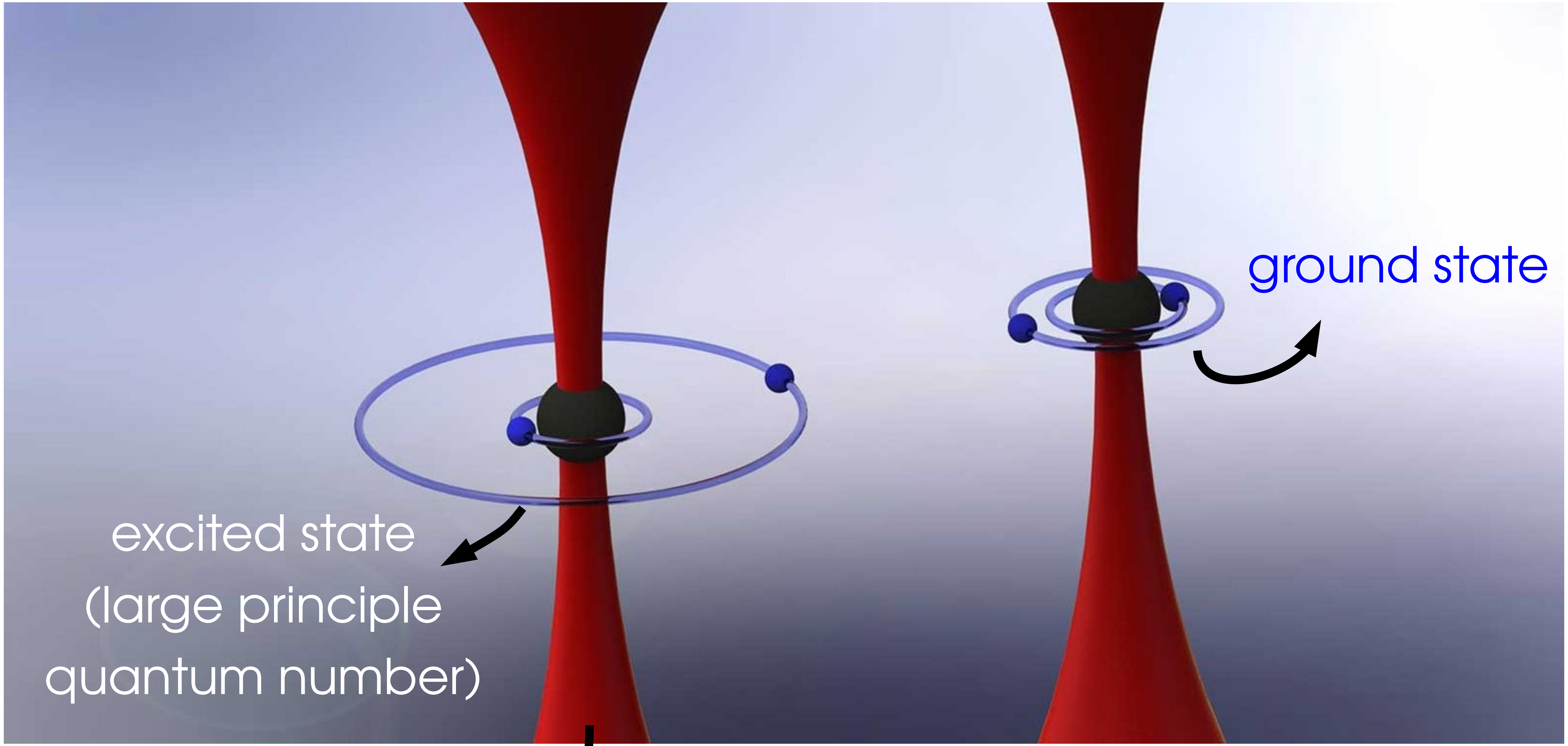


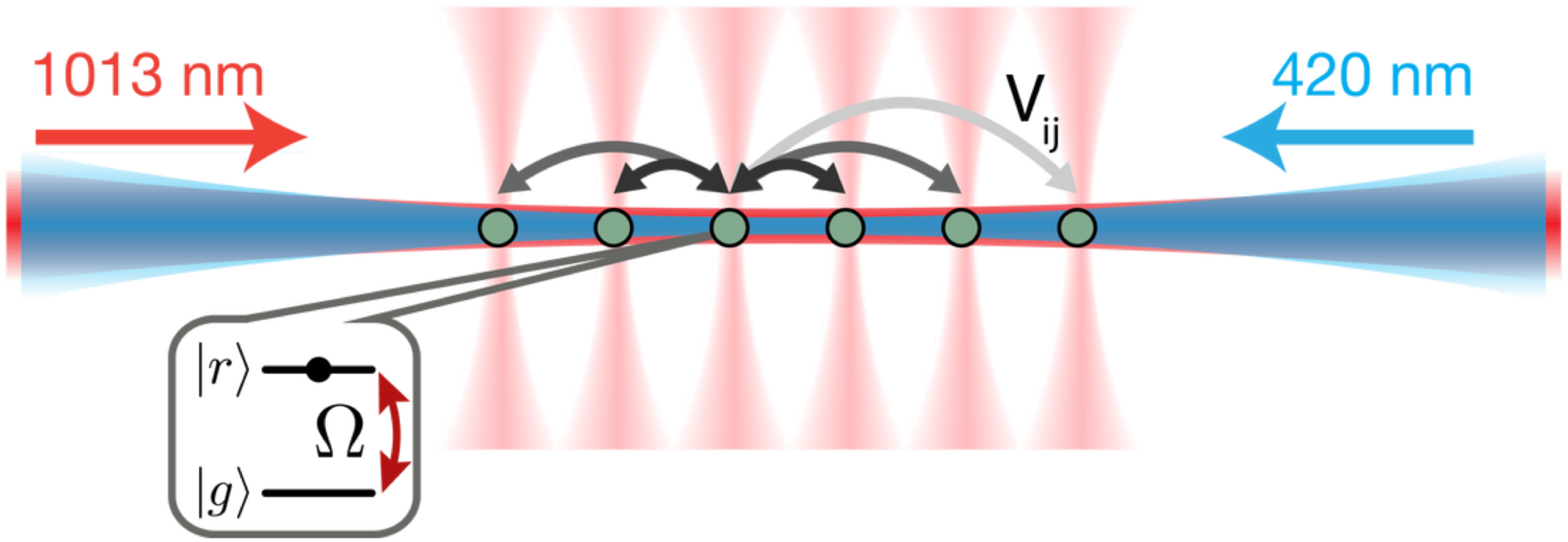
$$V_{|\ell-\ell'|} \sim \frac{1}{|\ell-\ell'|^6}$$

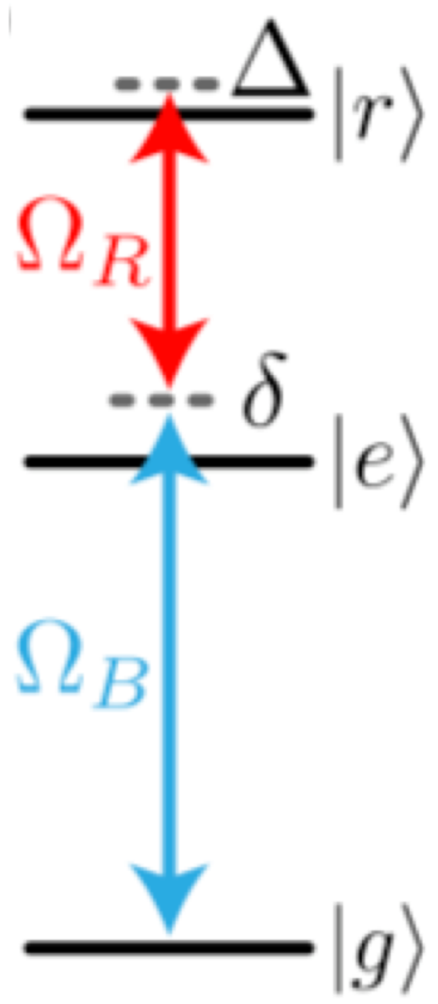
optical tweezer (traps atom)

Fig: https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms

$$H_{\mathrm{Ryd}} = \sum_{\ell} \left[\frac{\Omega}{2} \left(|g\rangle\langle r| + |r\rangle\langle g| \right)_{\ell} - \Delta |r\rangle\langle r| \right] + \sum_{(\ell,\ell')} V_{|\ell-\ell'|} \Big(|r\rangle\langle r|_{\ell} \otimes |r\rangle\langle r|_{\ell'} \Big)$$

QPTs in a Rydberg quantum simulator





$$|g\rangle \equiv |0\rangle$$

$$|r\rangle \equiv b^\dagger |0\rangle$$

$$\mathcal{H} = \sum_\ell \left[\frac{\Omega}{2} \left(b_\ell + b_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

$$V_{|\ell - \ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

$$n_\ell \equiv b_\ell^\dagger b_\ell$$

$n_\ell = 0, 1$ 'hard core' bosons

FSS model

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

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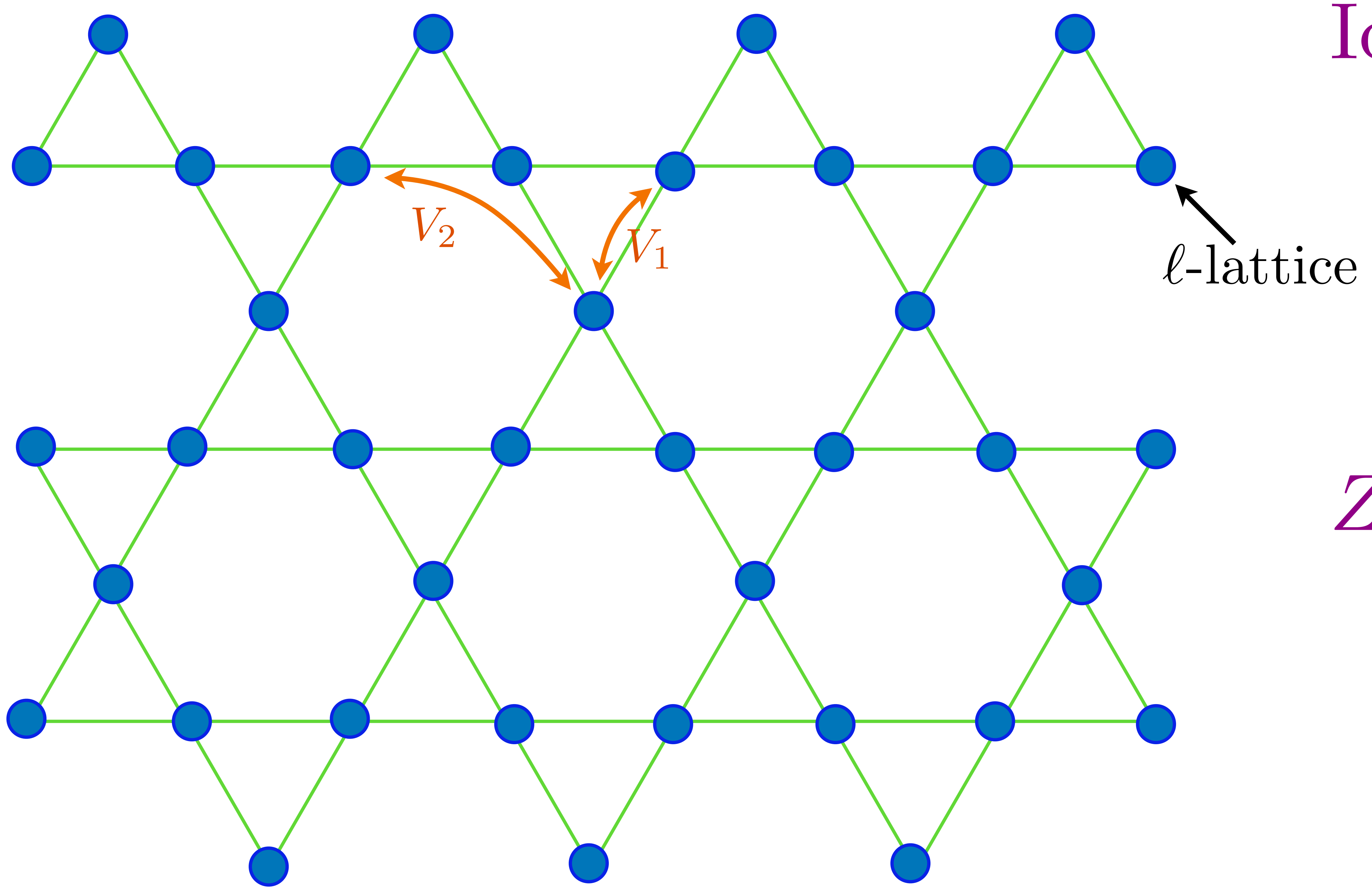

Identify hard core bosons with a qubit X, Y, Z

$$b_\ell + b_\ell^\dagger \Leftrightarrow Z_\ell$$
$$n_\ell \Leftrightarrow (1 - X_\ell)/2$$

Z will become the $\mathbb{Z}_2$ gauge field

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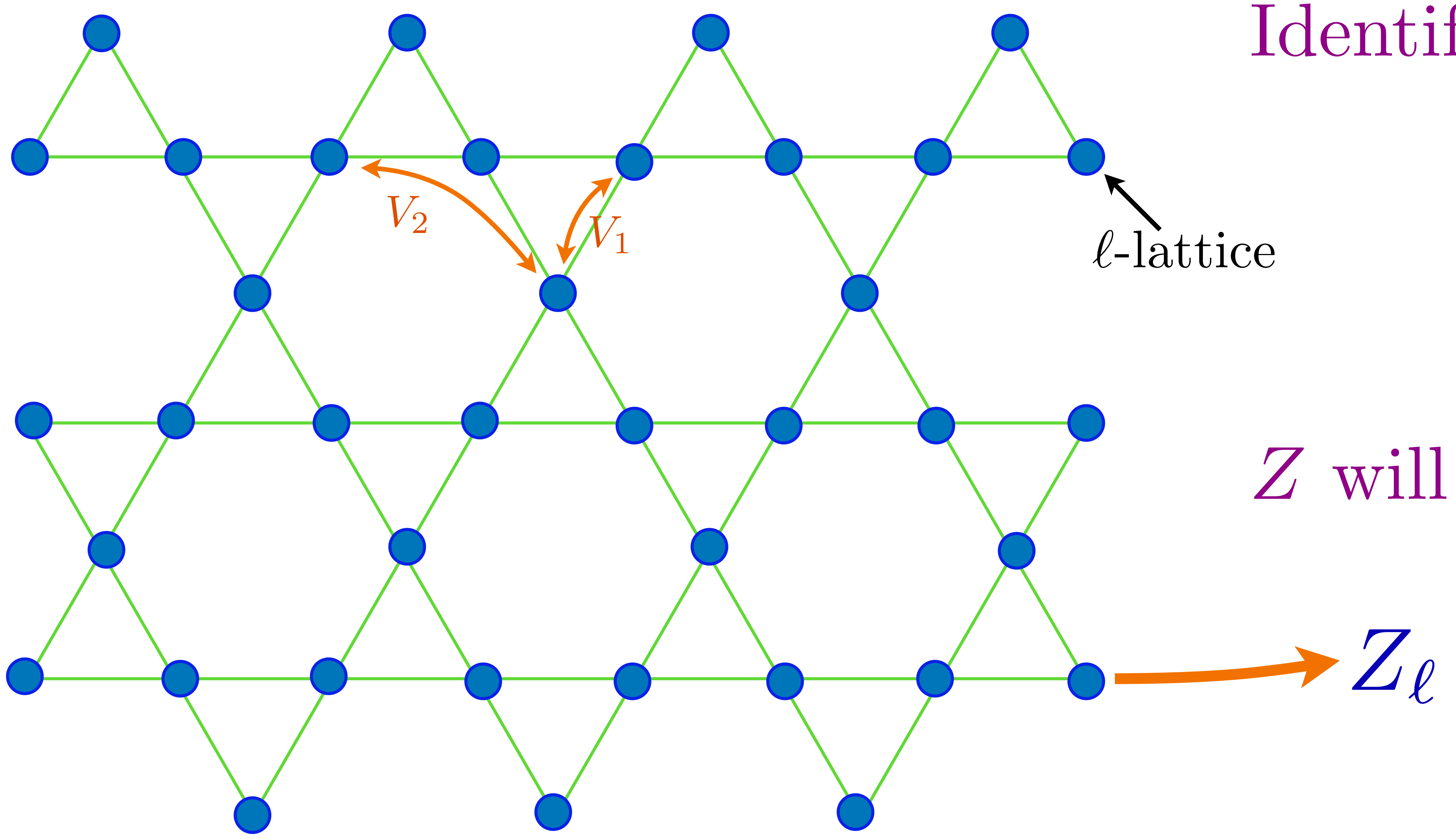
$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} Z_\ell + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell<\ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$



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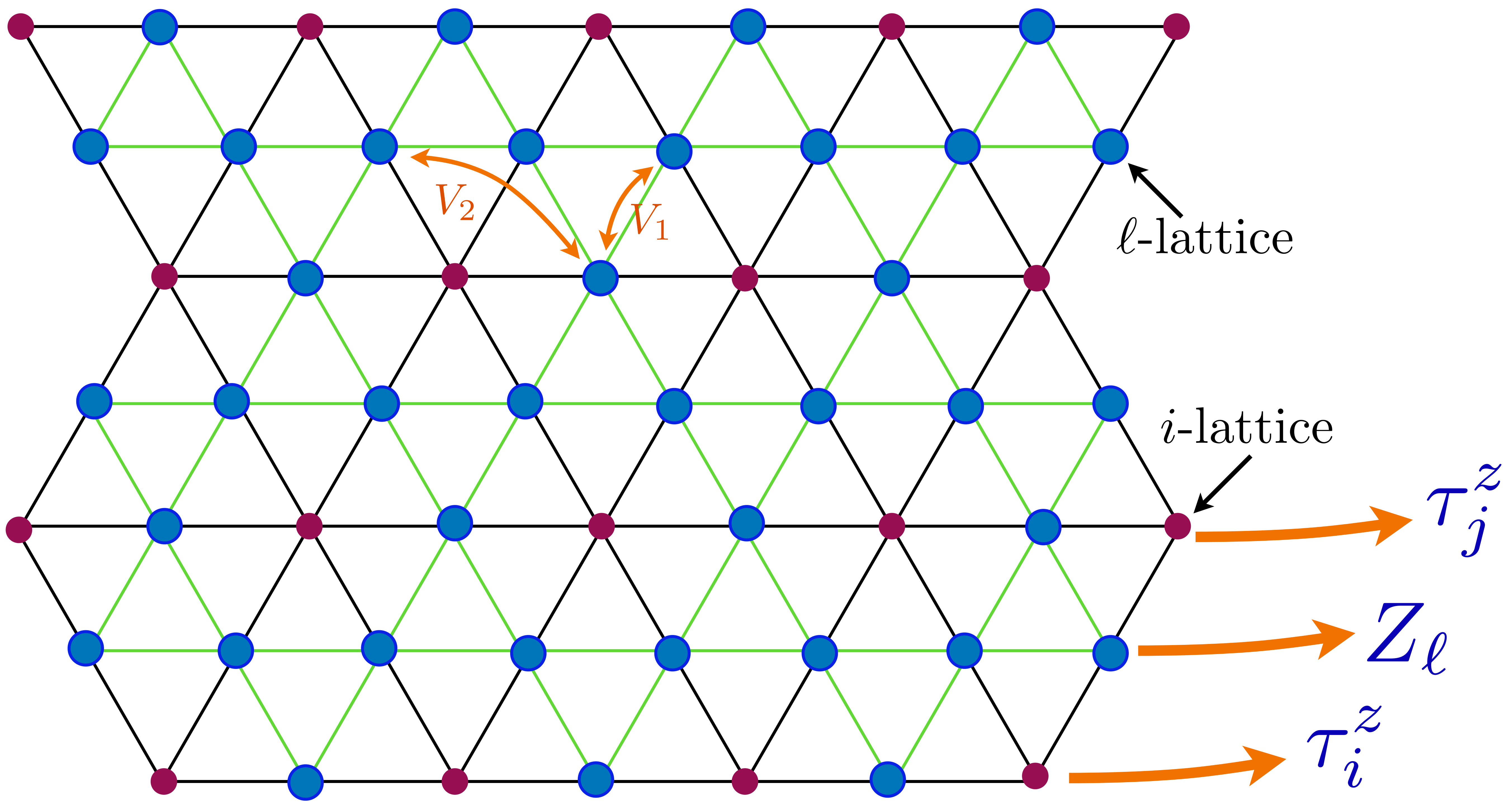
$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[\frac{\Omega}{2} \tau_i^z \, Z_\ell \, \tau_j^z + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$

Introduce $\mathbb{Z}_2$ matter fields on 'i sites'. Gauge invariance: $\tau_i^z \to \varrho_i \tau_i^z$, $Z_{ij} \to \varrho_i Z_{ij} \varrho_j$, $\tau_i^x \to \tau_i^x$, $X_\ell \to X_\ell$, $\varrho_i = \pm 1$. Gauss law constraint: $G_i = \tau_i^x \prod_{\ell \in i} X_\ell = 1$.



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021)

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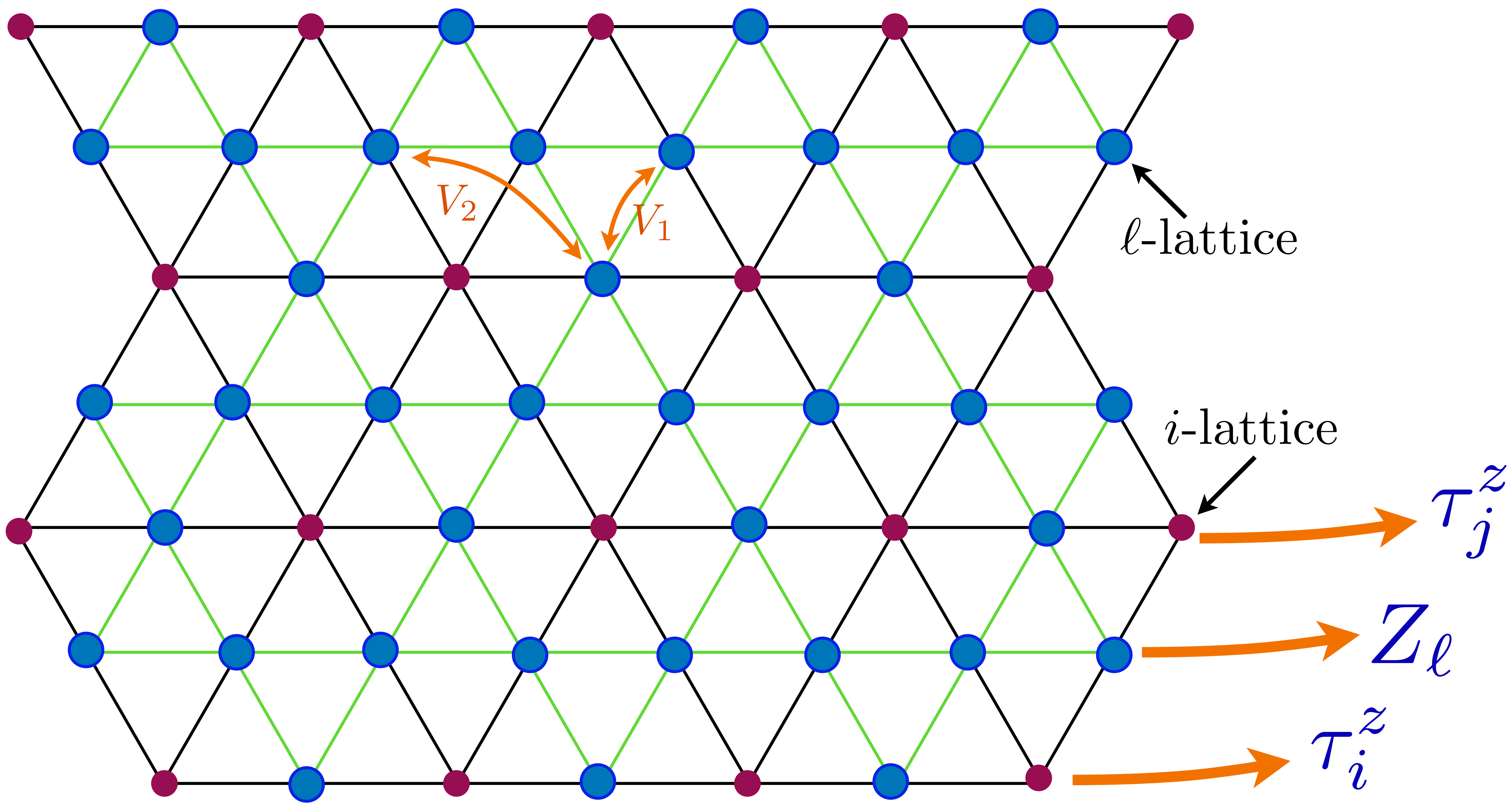
The K_{loop} terms are generated in a large V expansion: ‘resonance’ between Rydberg states can stabilize a phase with deconfined $\mathbb{Z}_2$ gauge charges *i.e.* a $\mathbb{Z}_2$ spin liquid



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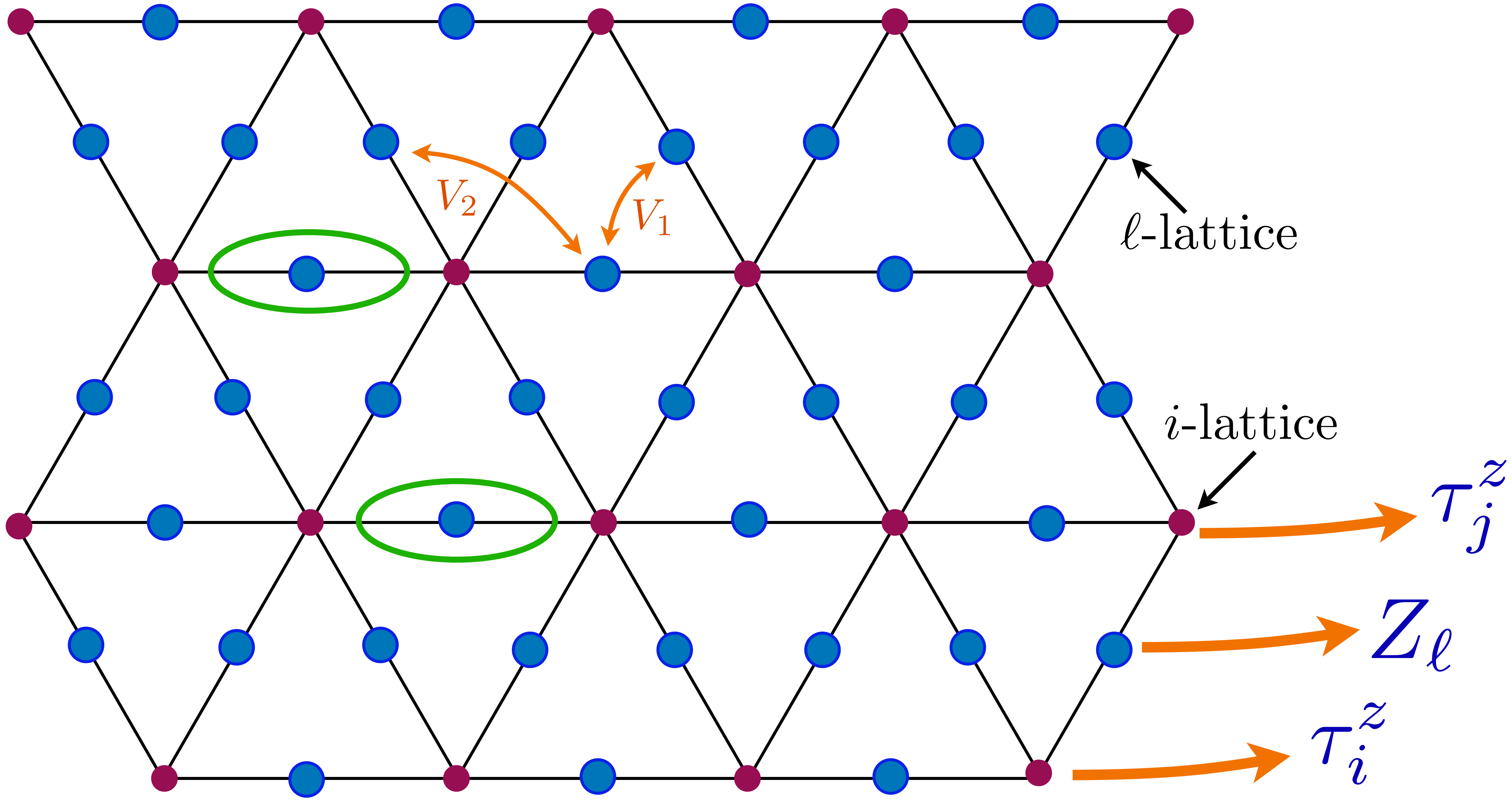

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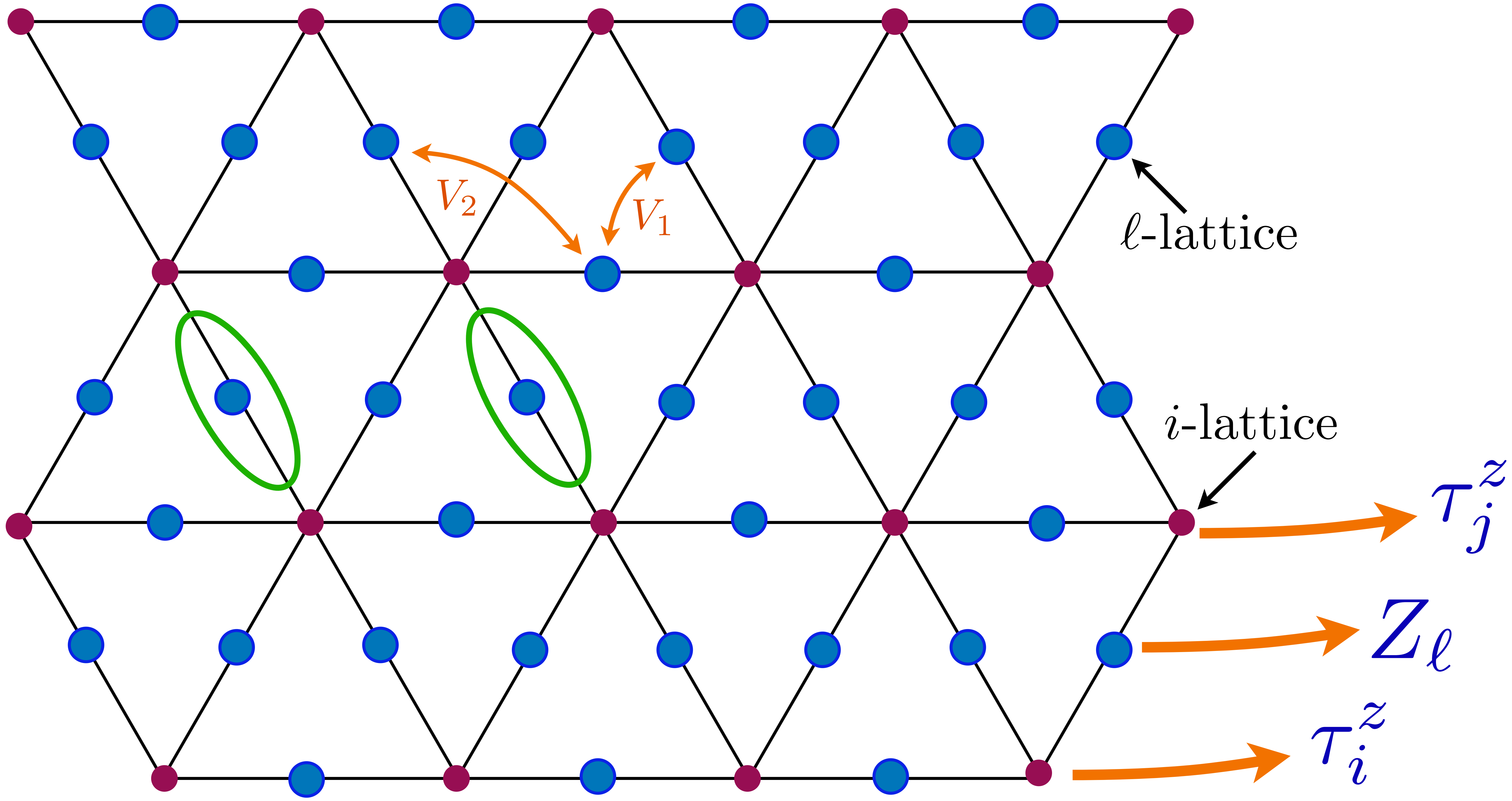

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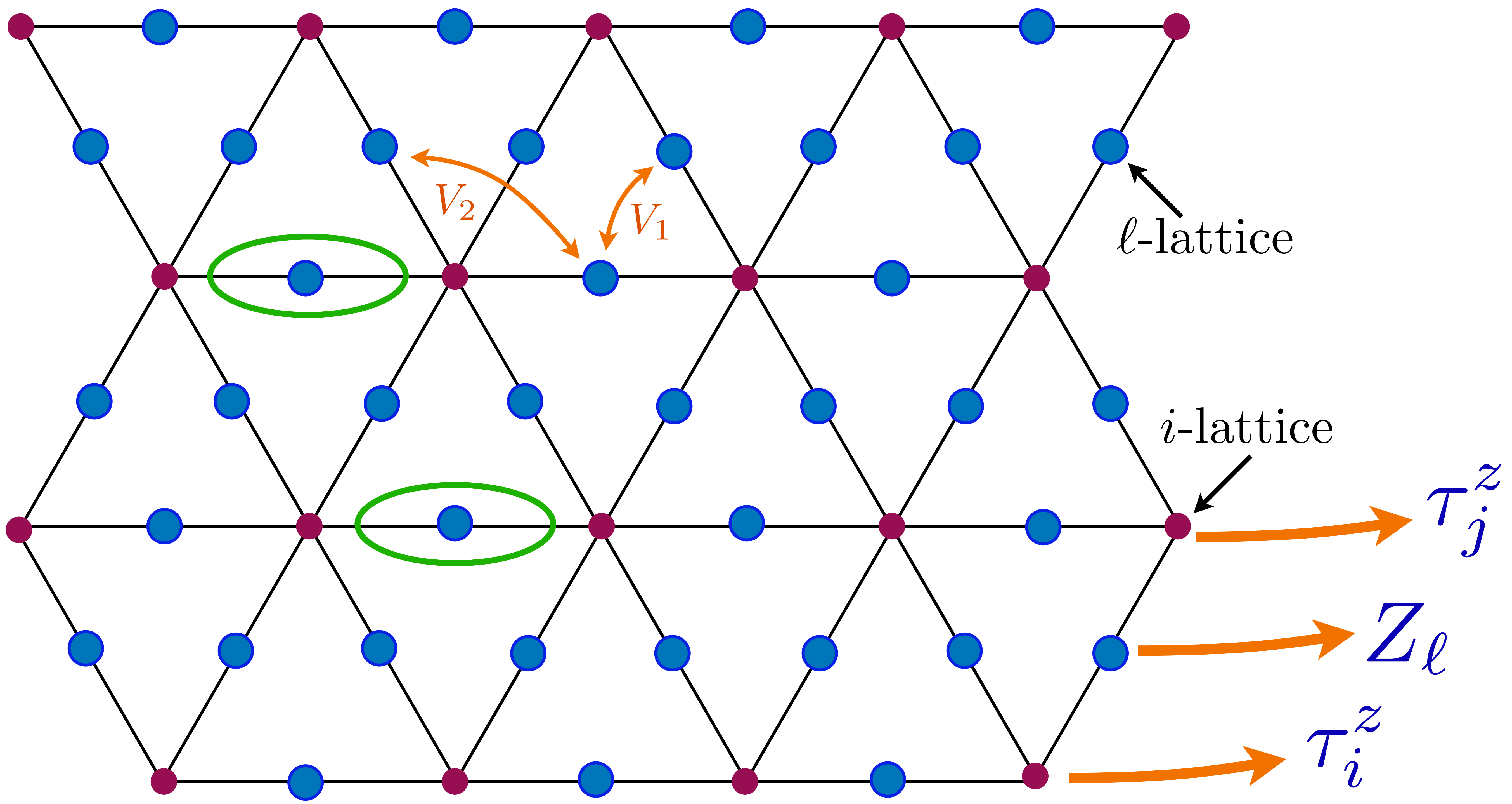

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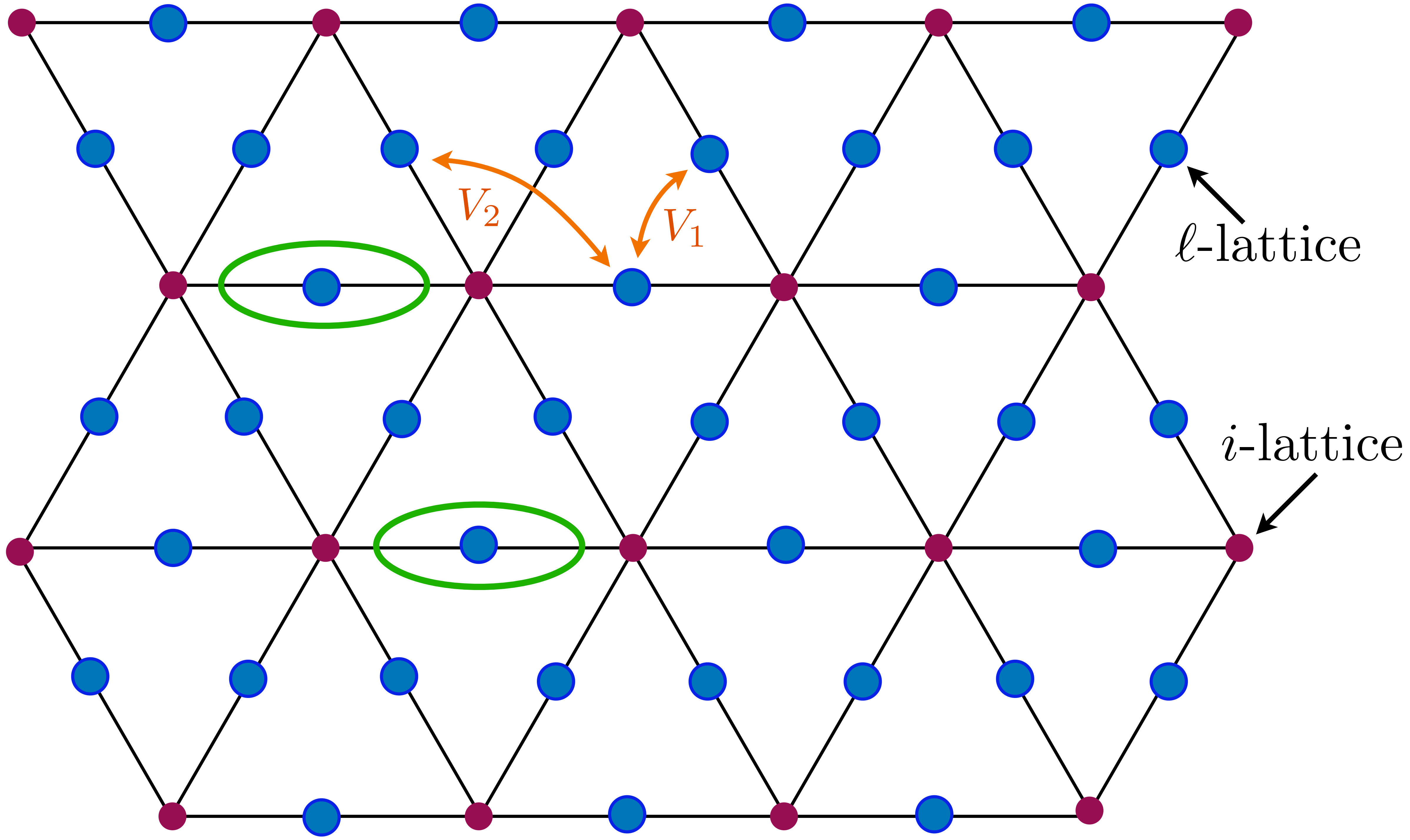

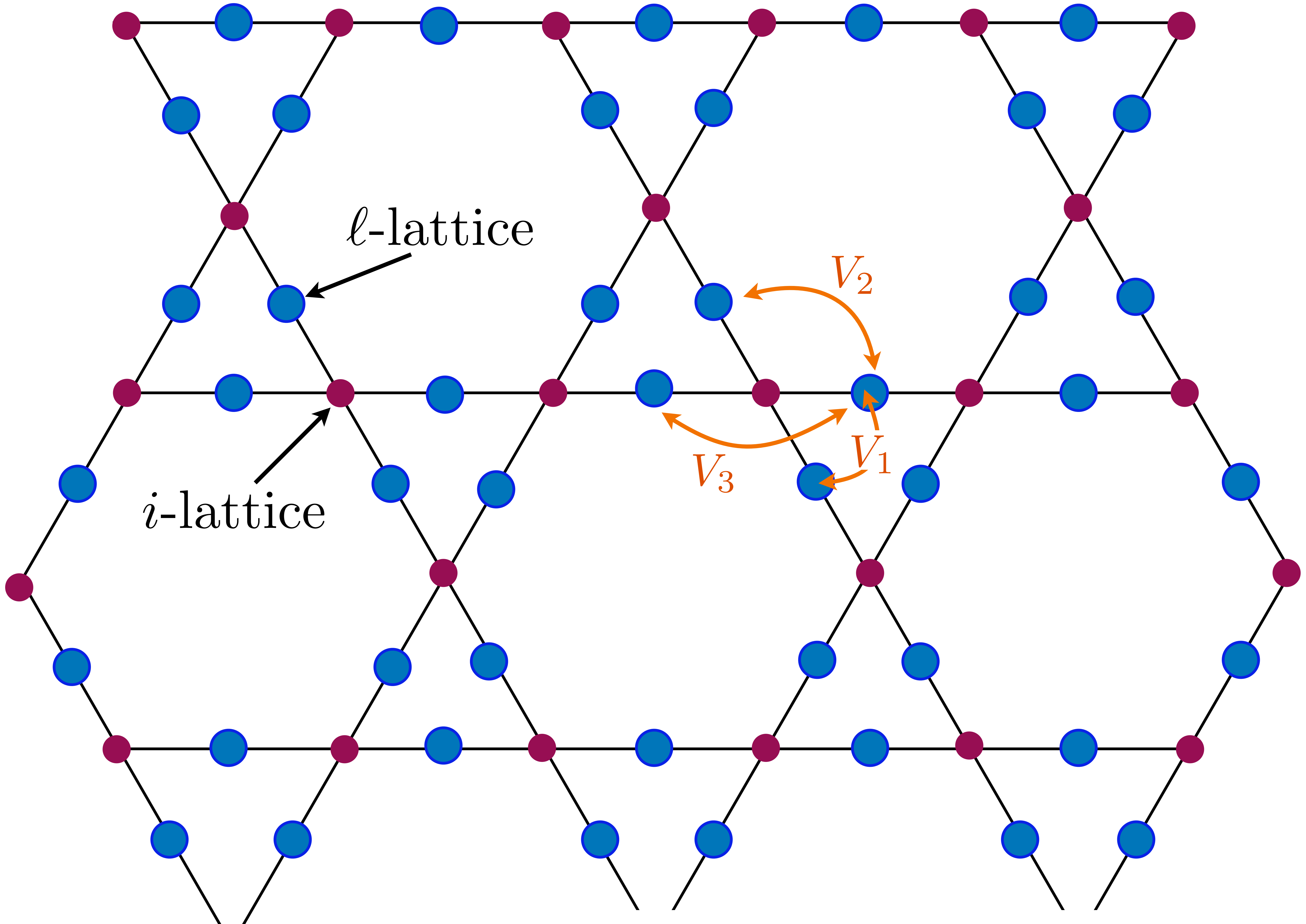

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021) R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

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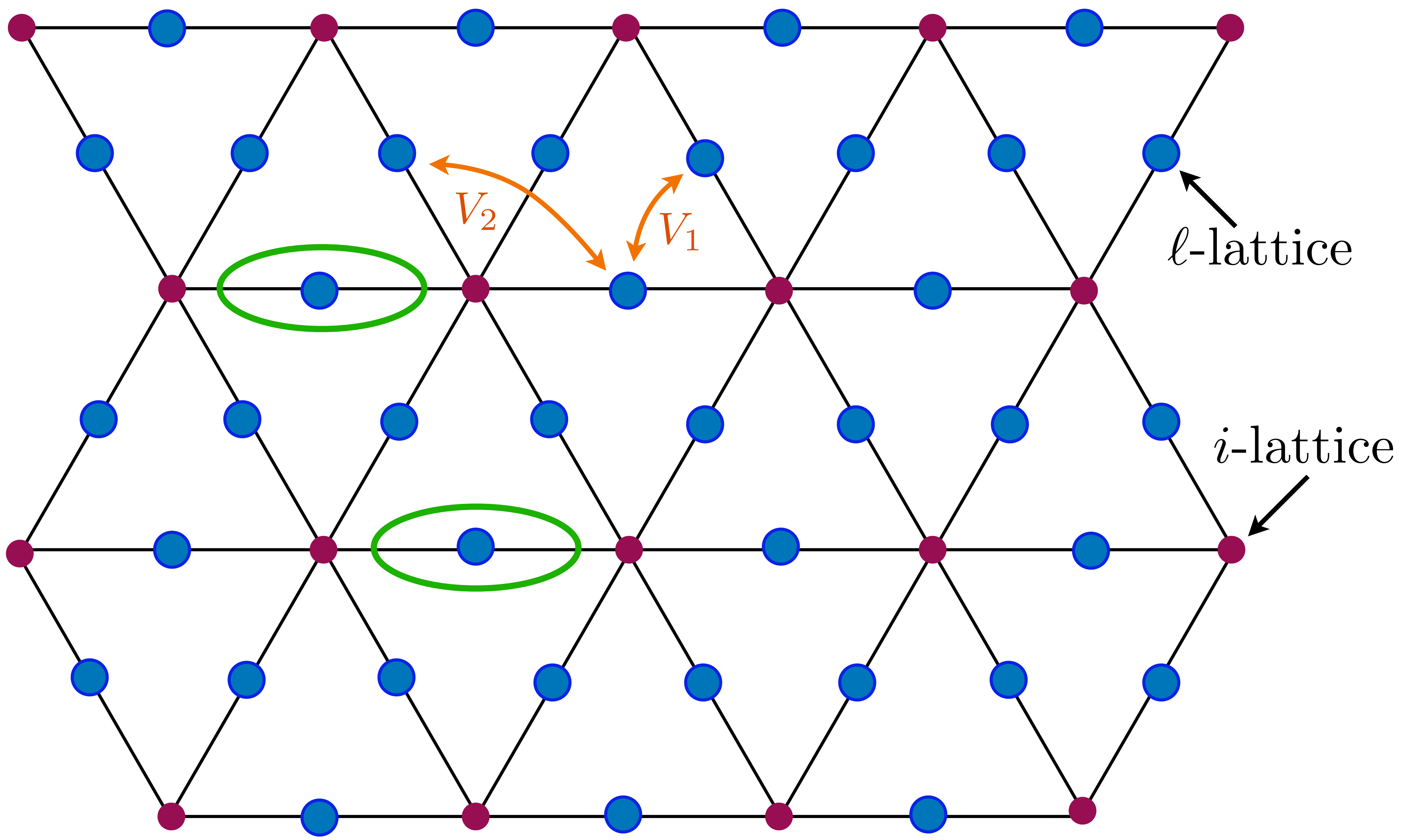

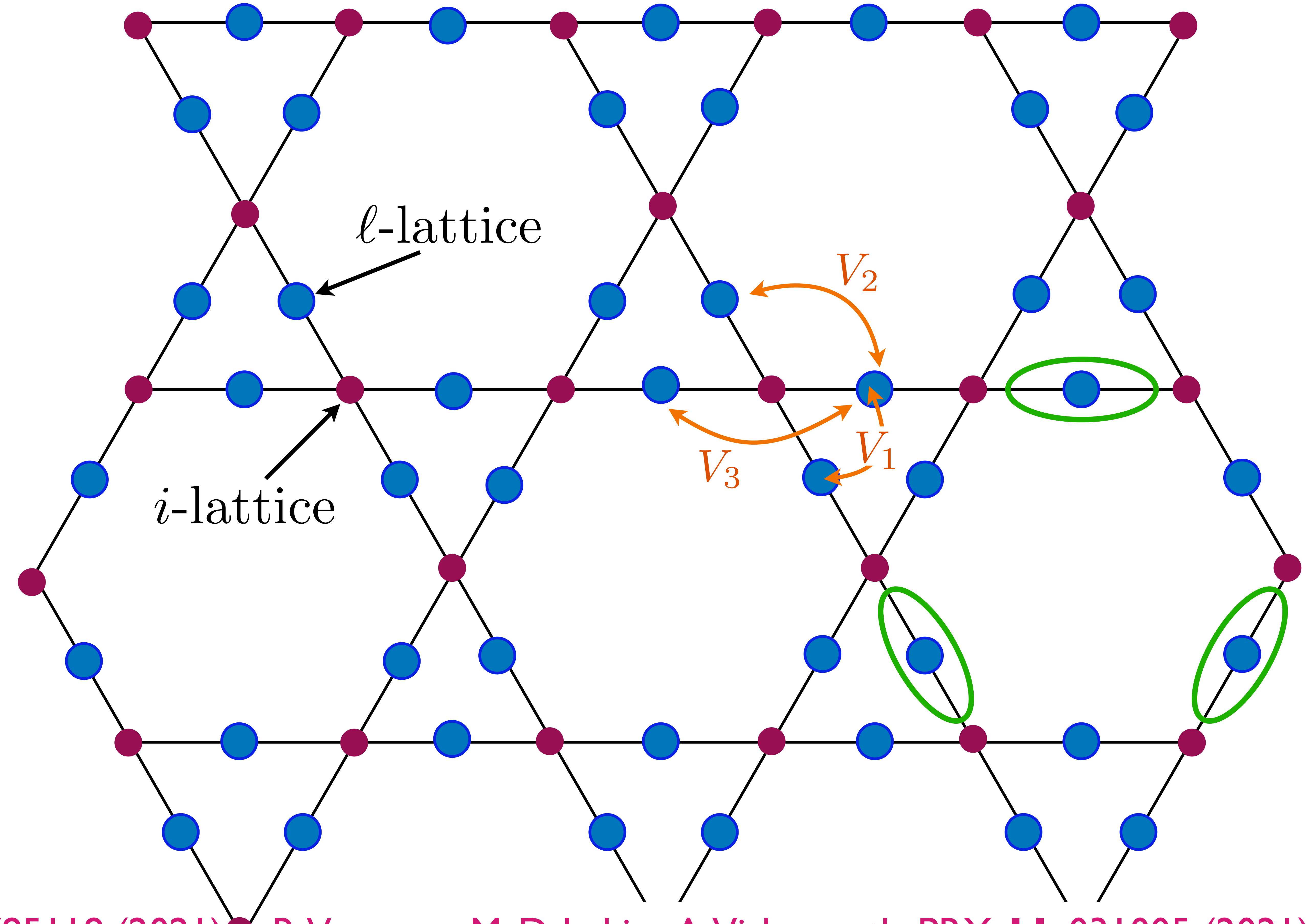

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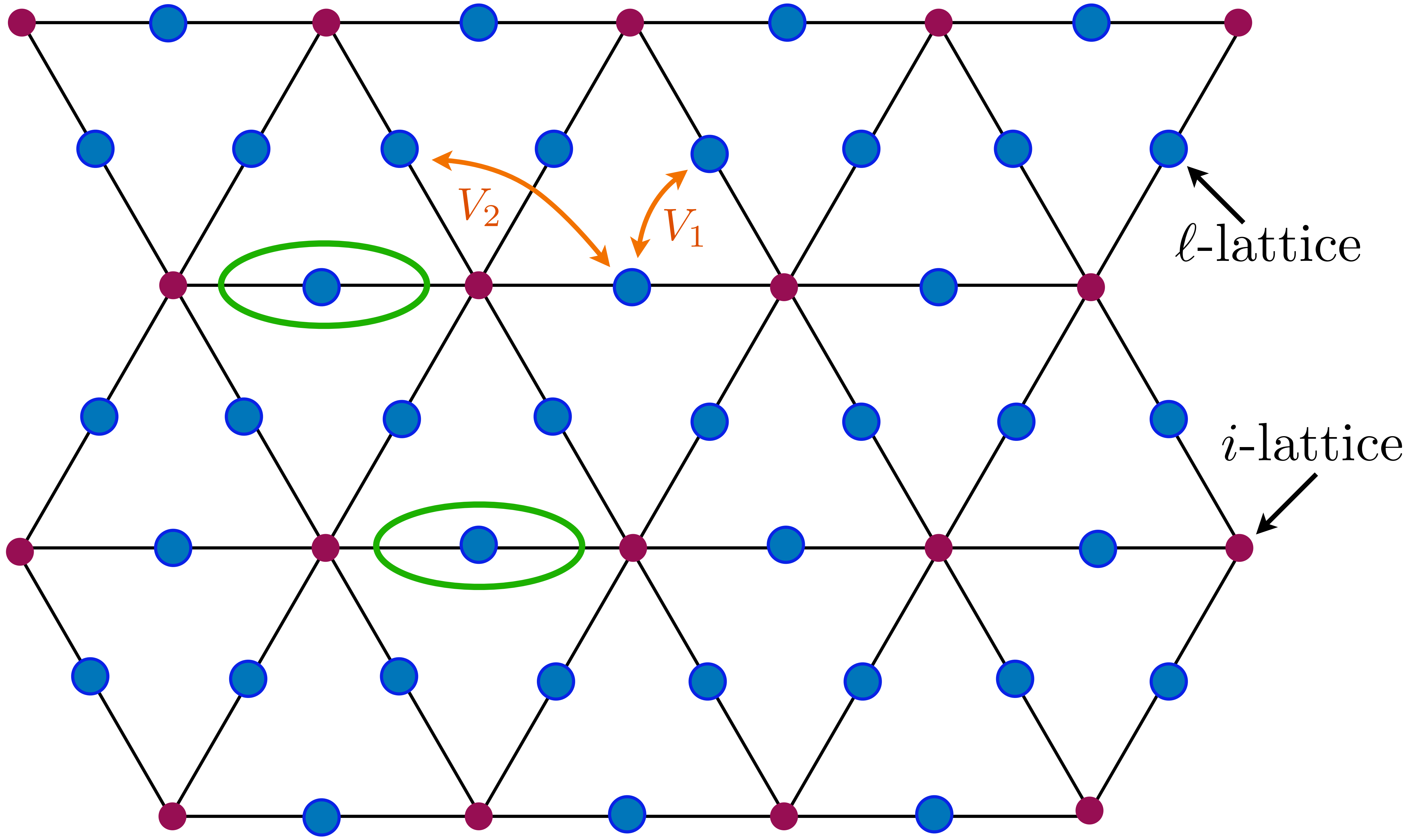

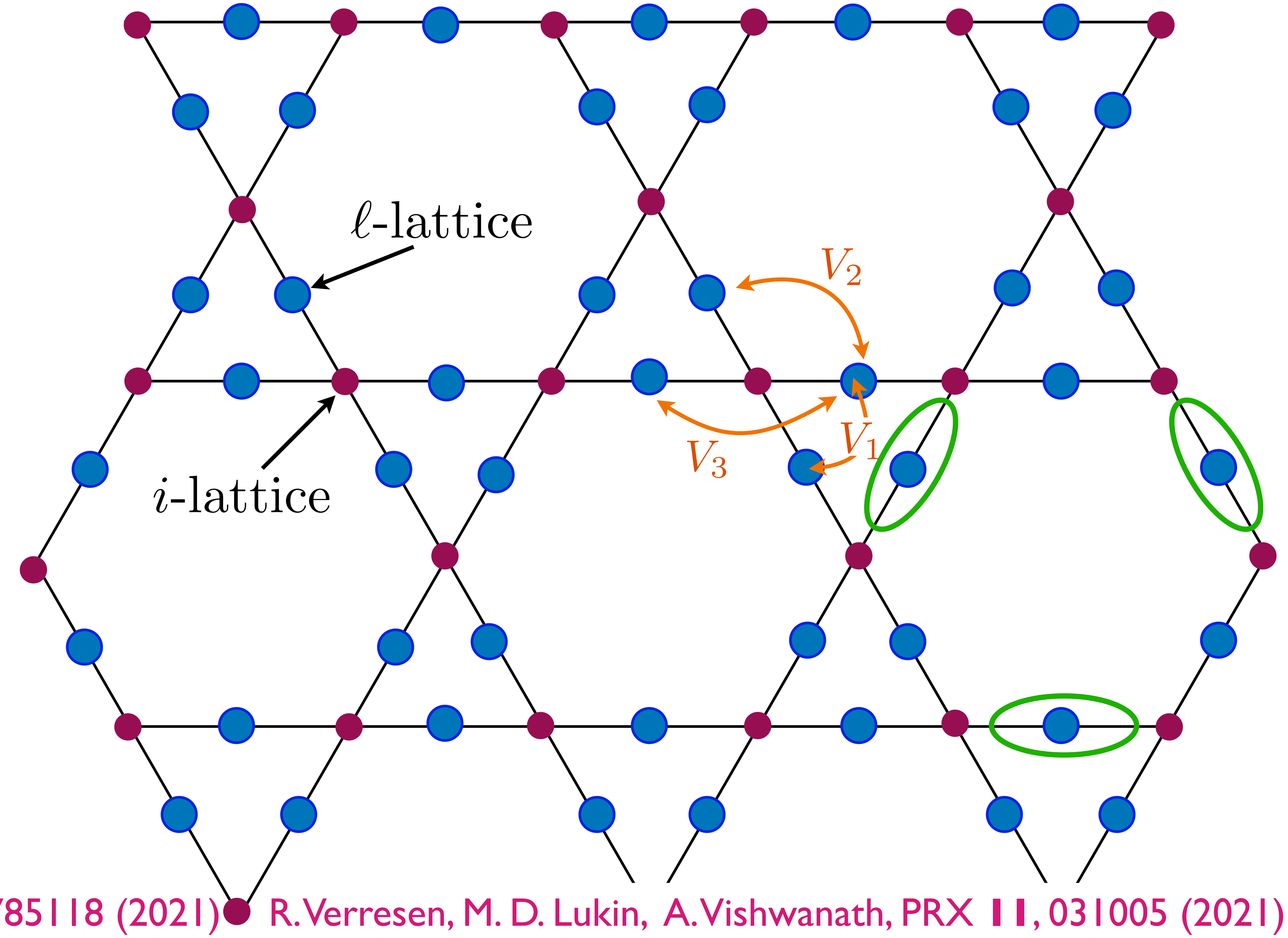

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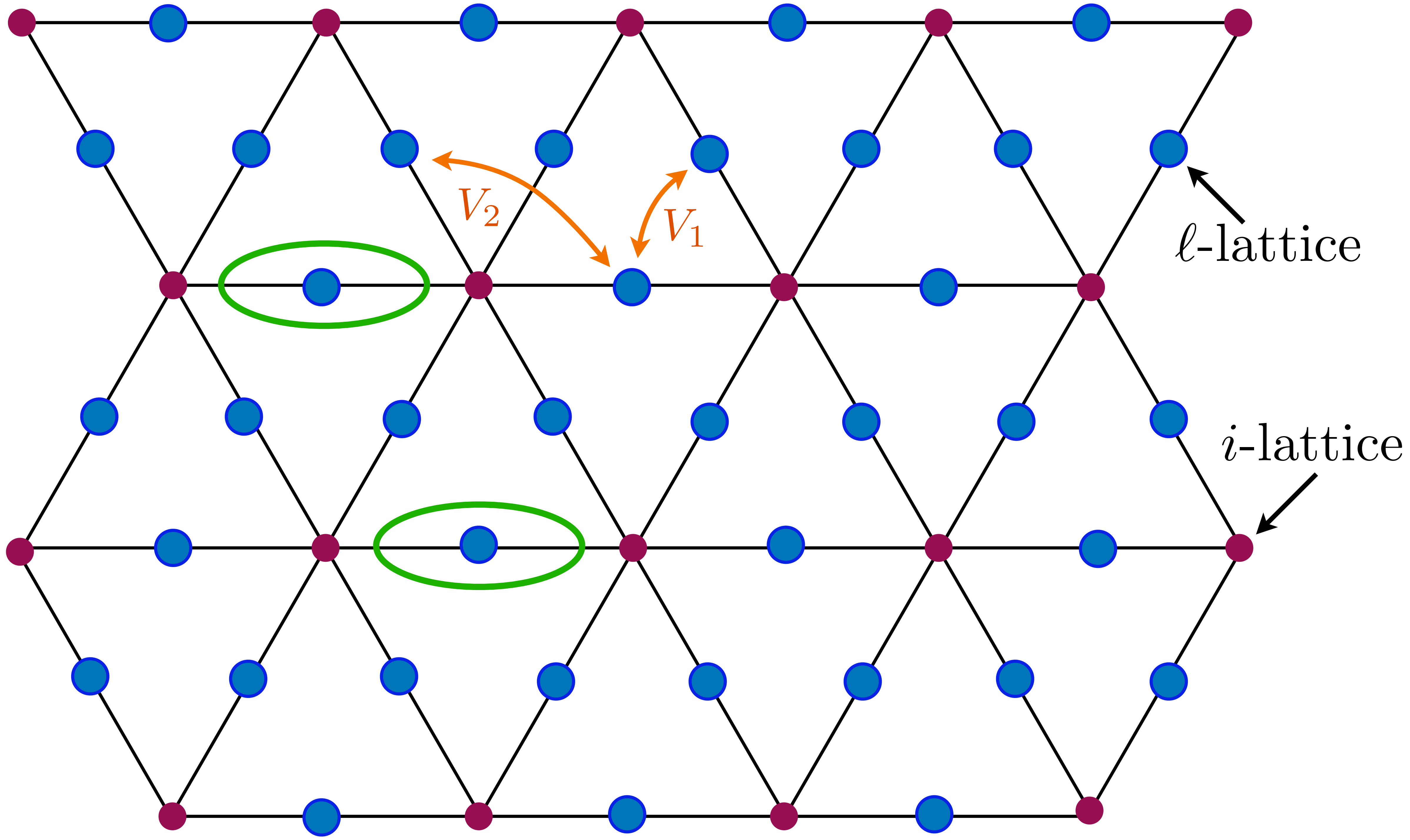

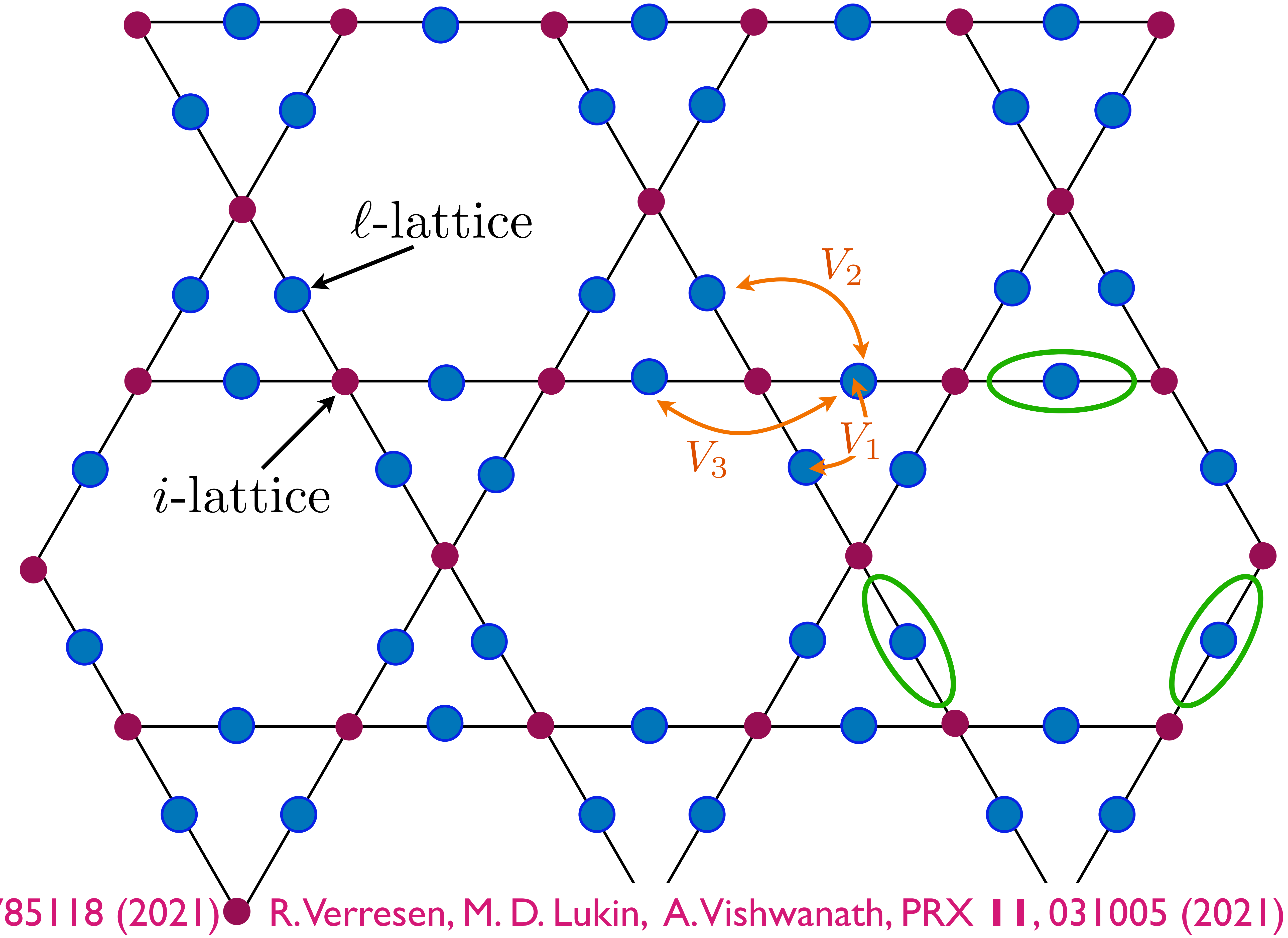

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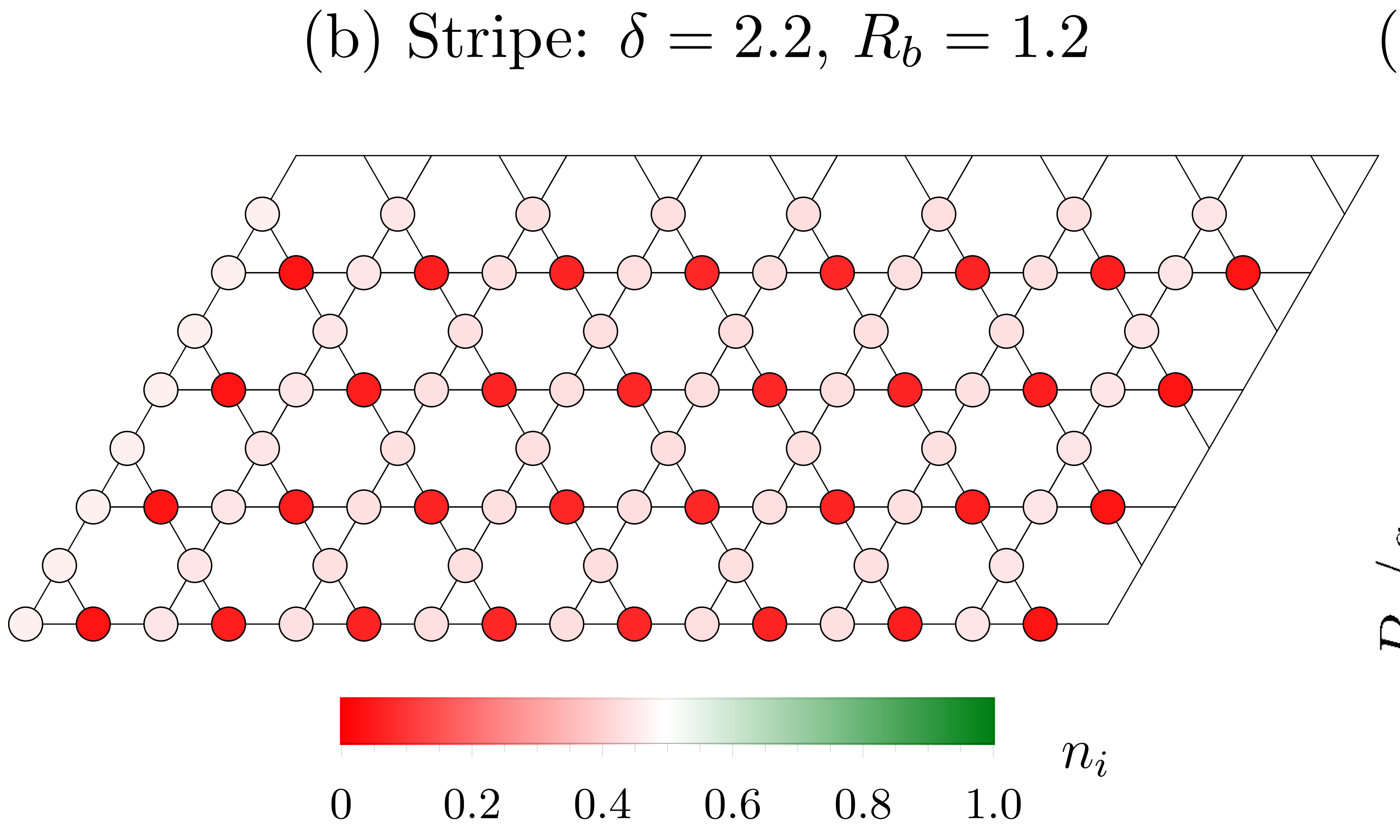
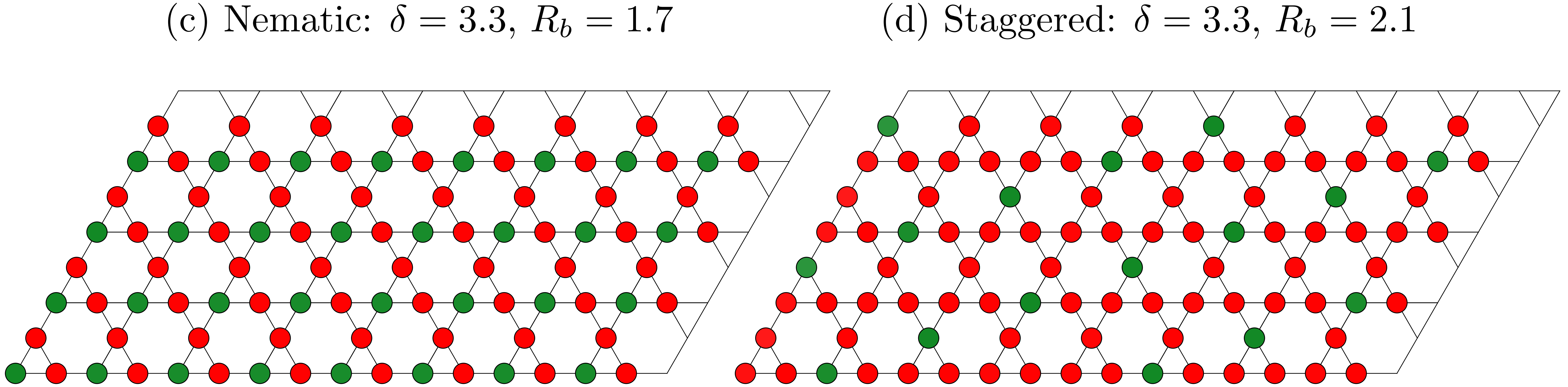
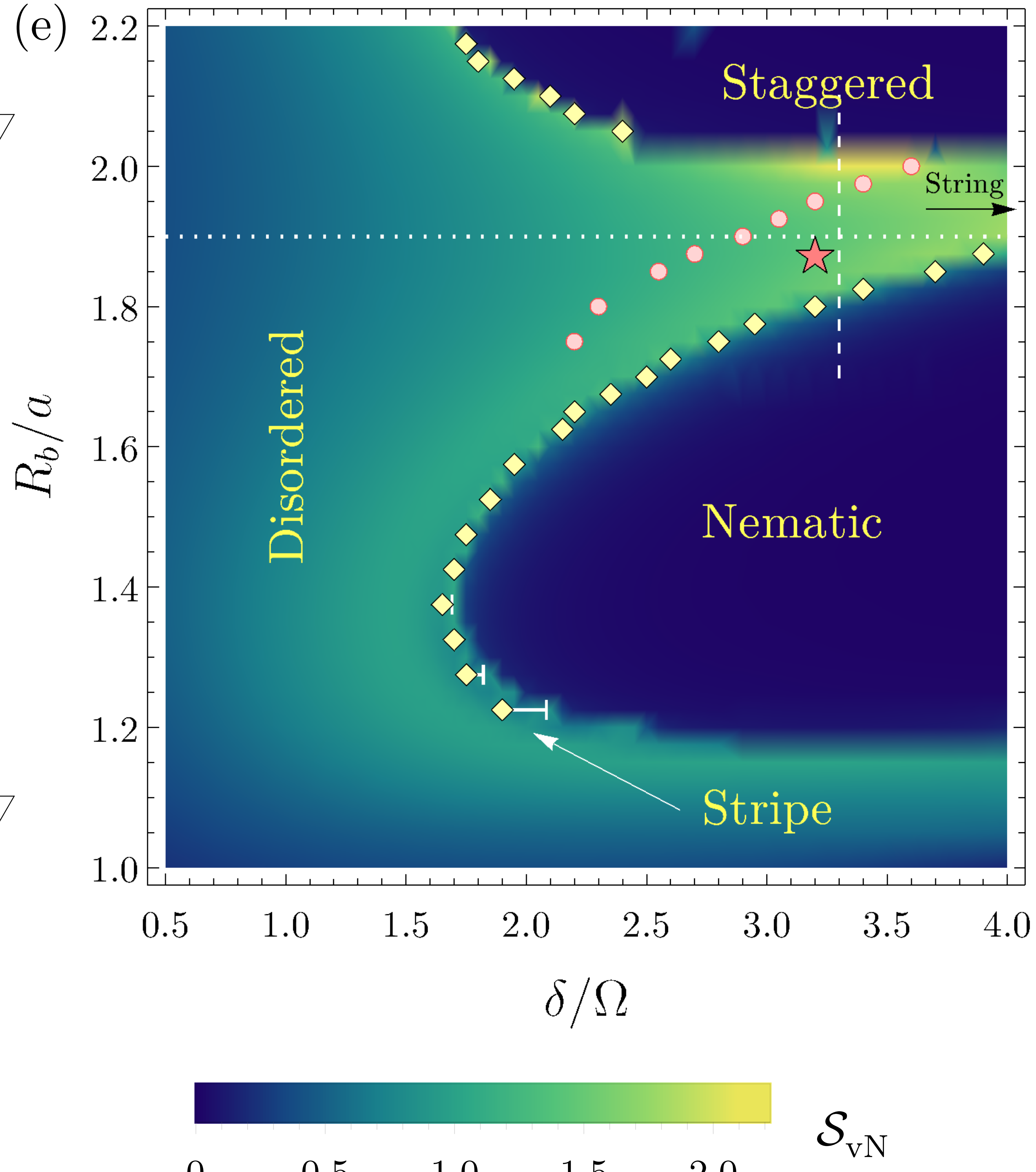
Rydberg atoms on site-kagome lattice: theory







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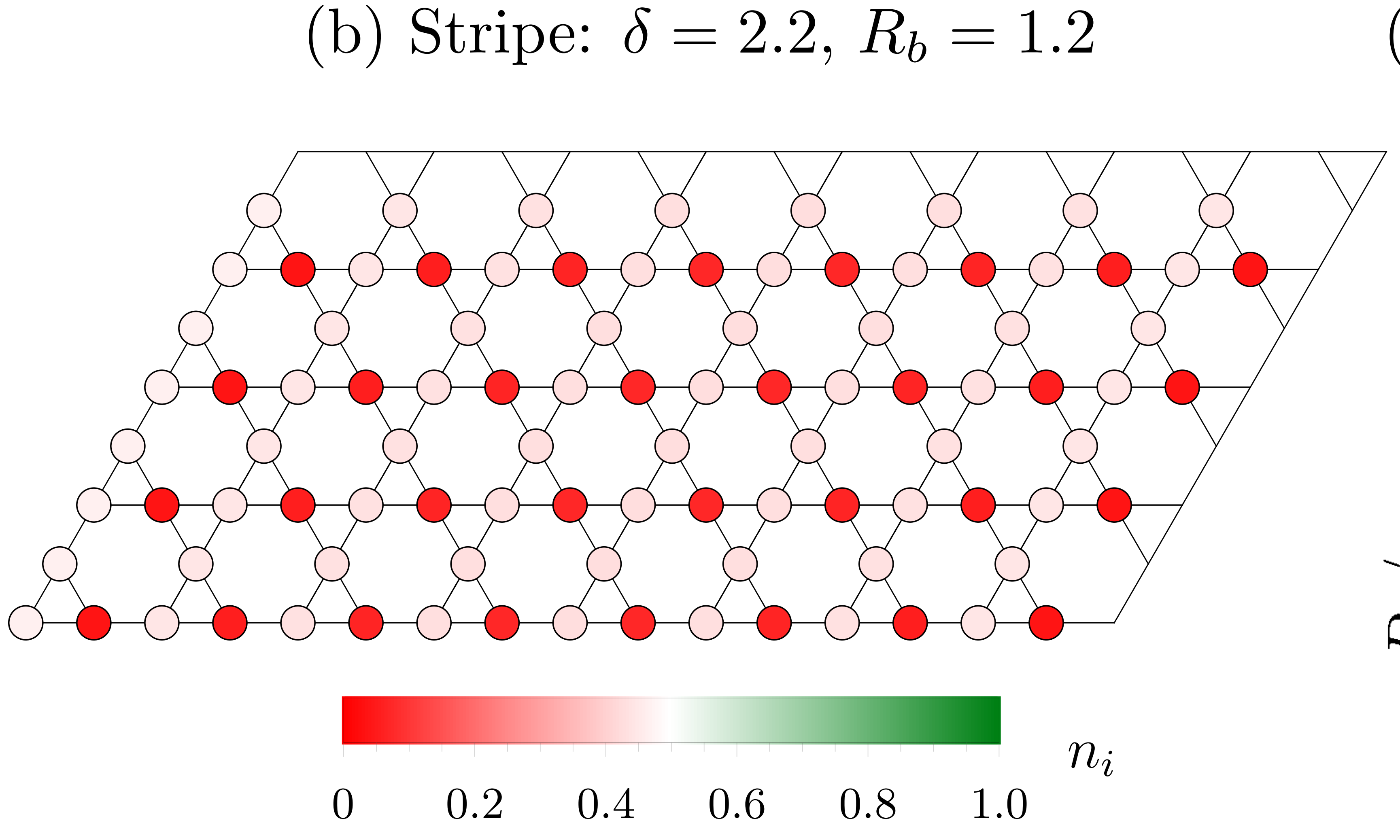
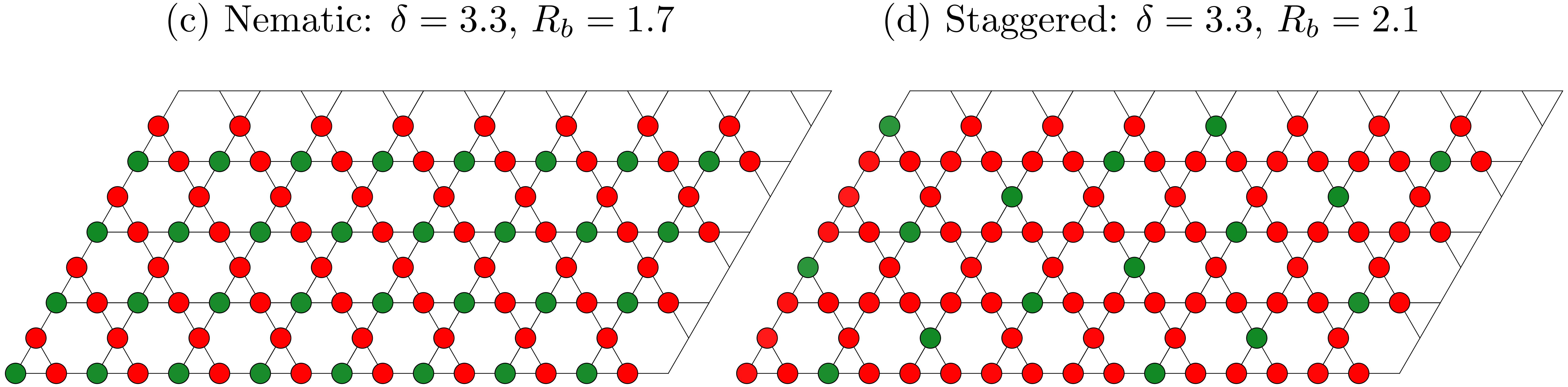
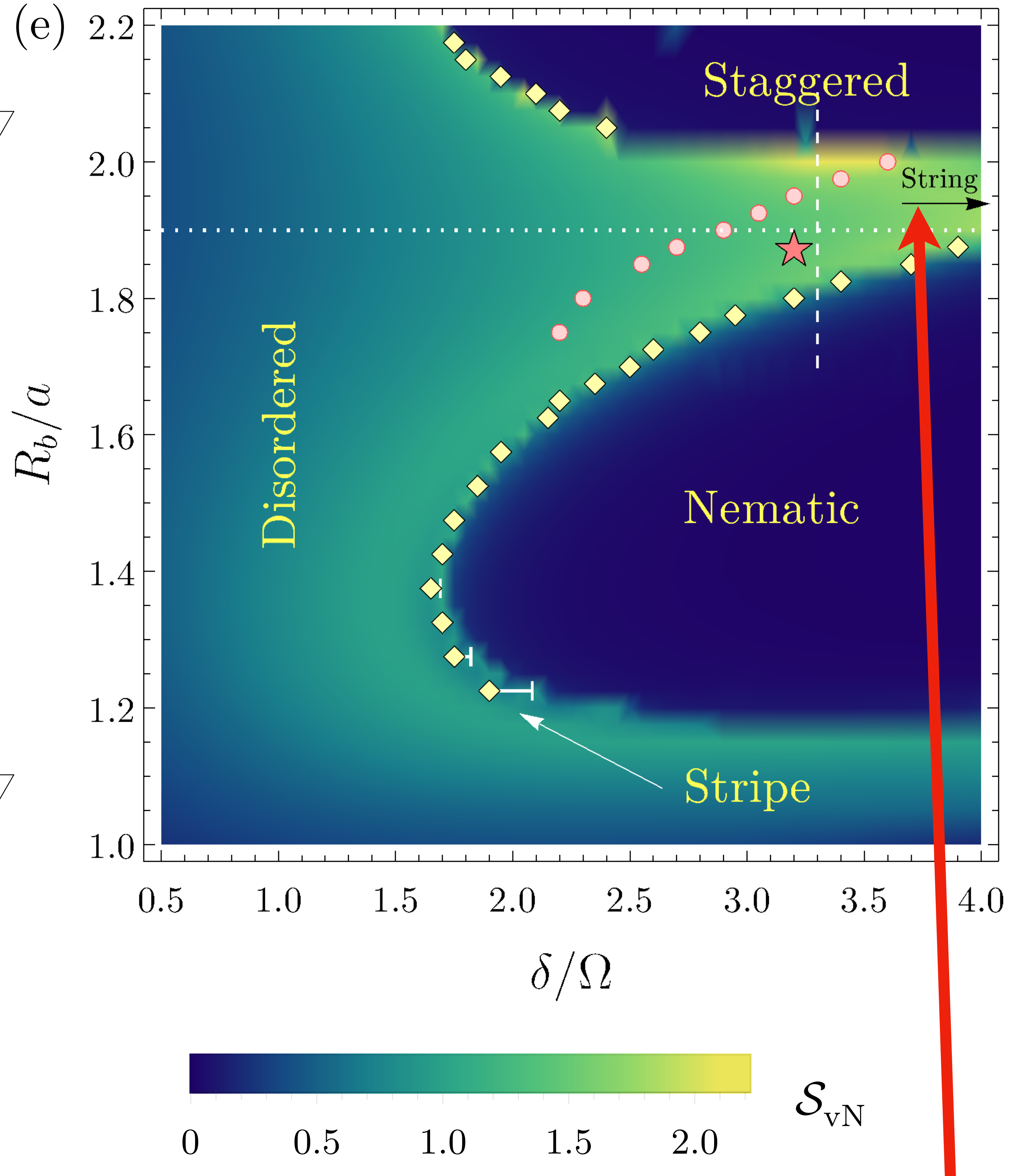






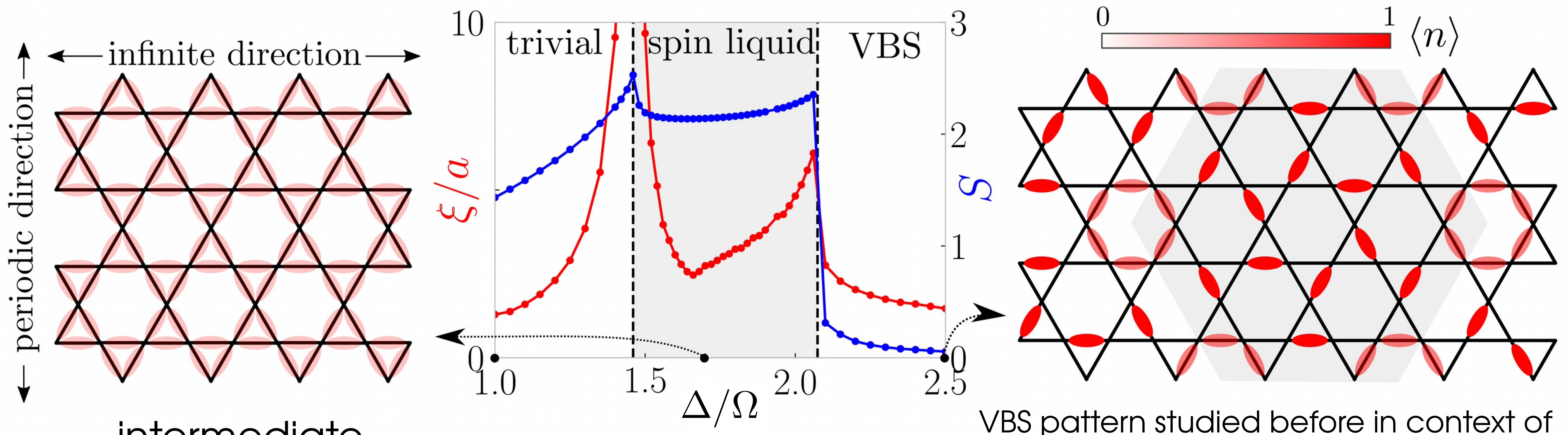
Topological spin liquid described by emergent $\mathbb{Z}_2$ gauge theory ?

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

Rydberg atoms on link-kagome lattice: theory

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i<j} V_{|i-j|} n_i n_j \, , \quad n_j \equiv b_j^\dagger b_j = 0, 1.$$

The sites j are on the links of the kagome lattice.
Examine the PXP model, $V_{\text{nearest neighbor}} = \infty$, other $V_k = 0$.



intermediate featureless phase

DMRG

VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)

Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

Rydberg atoms
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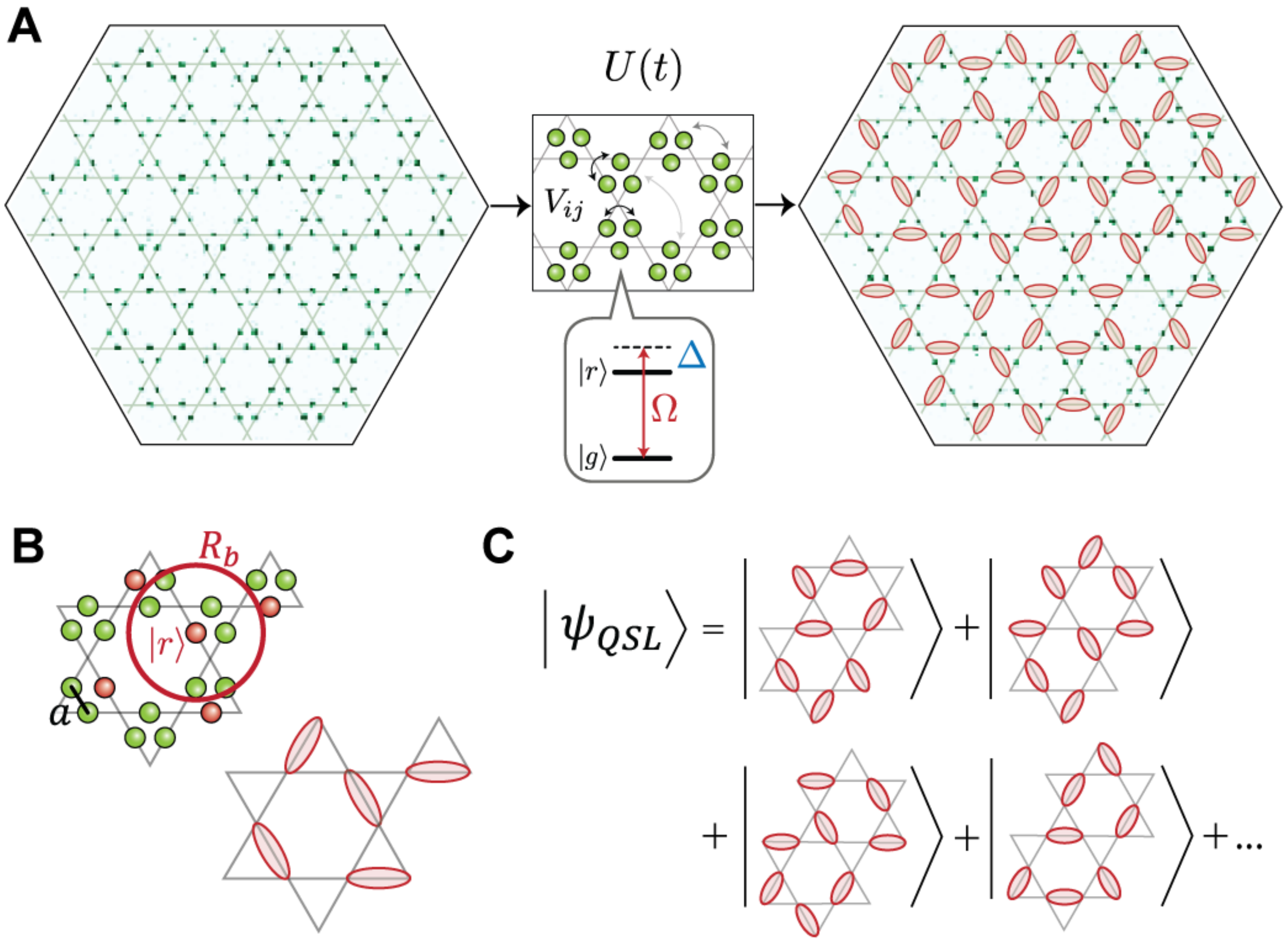

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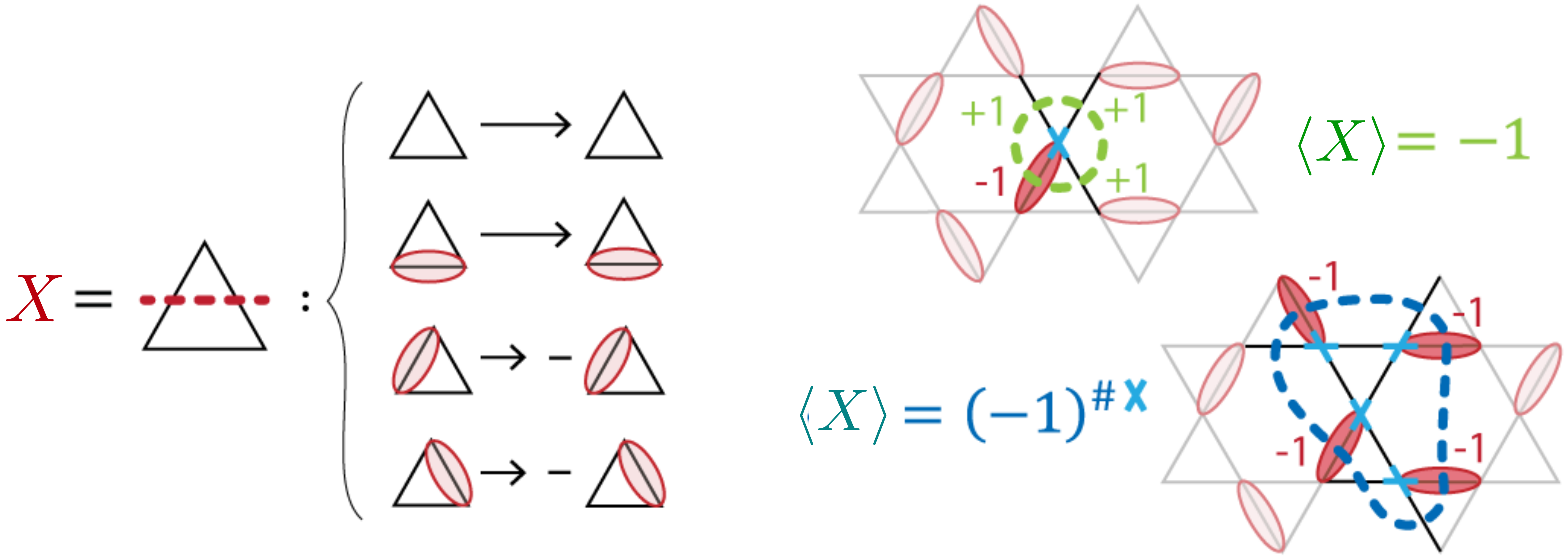

Measurement of
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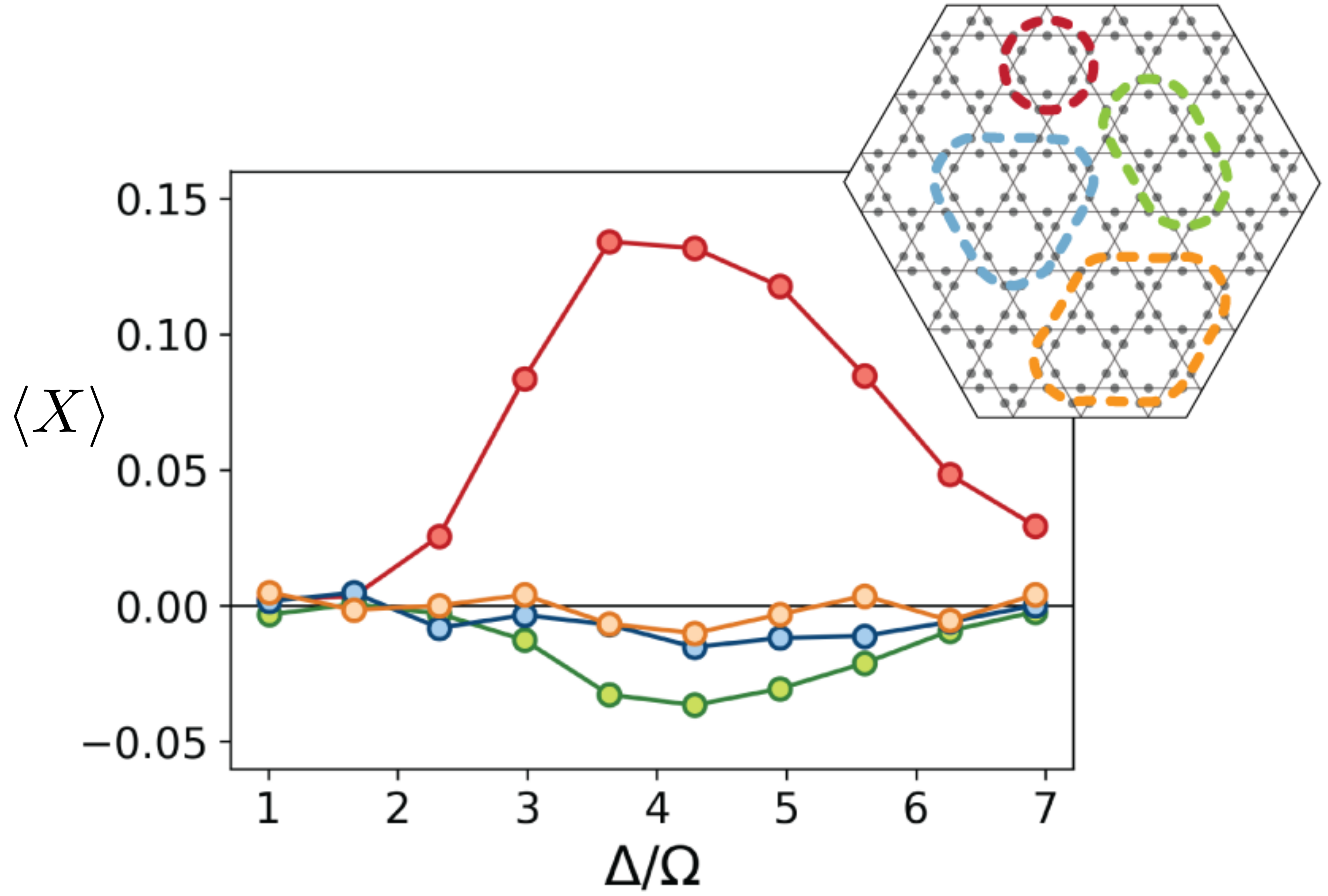
X operator
$= \prod_{\text{loop}} X_\ell$.
Detects close-packed dimers.



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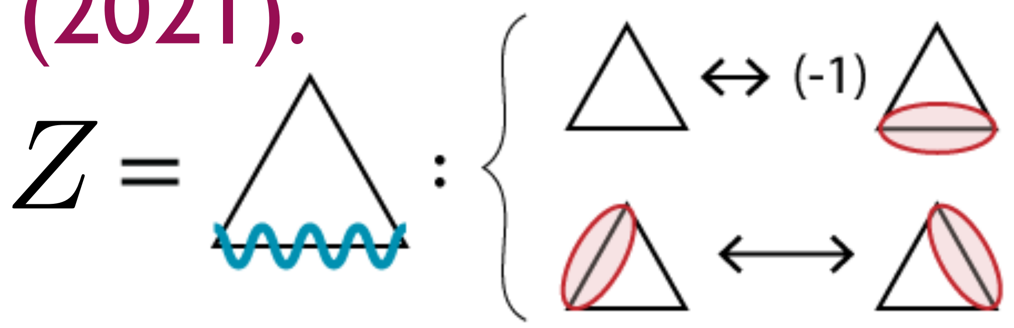

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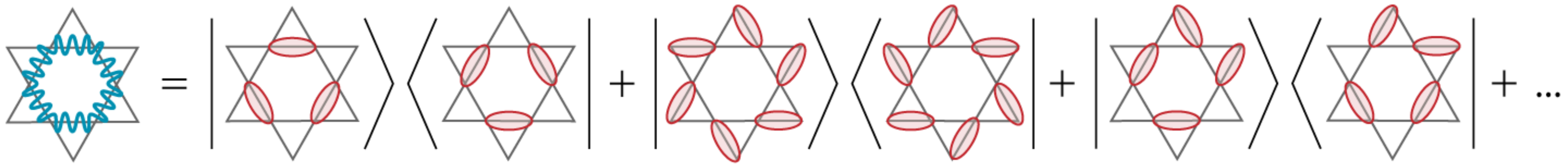
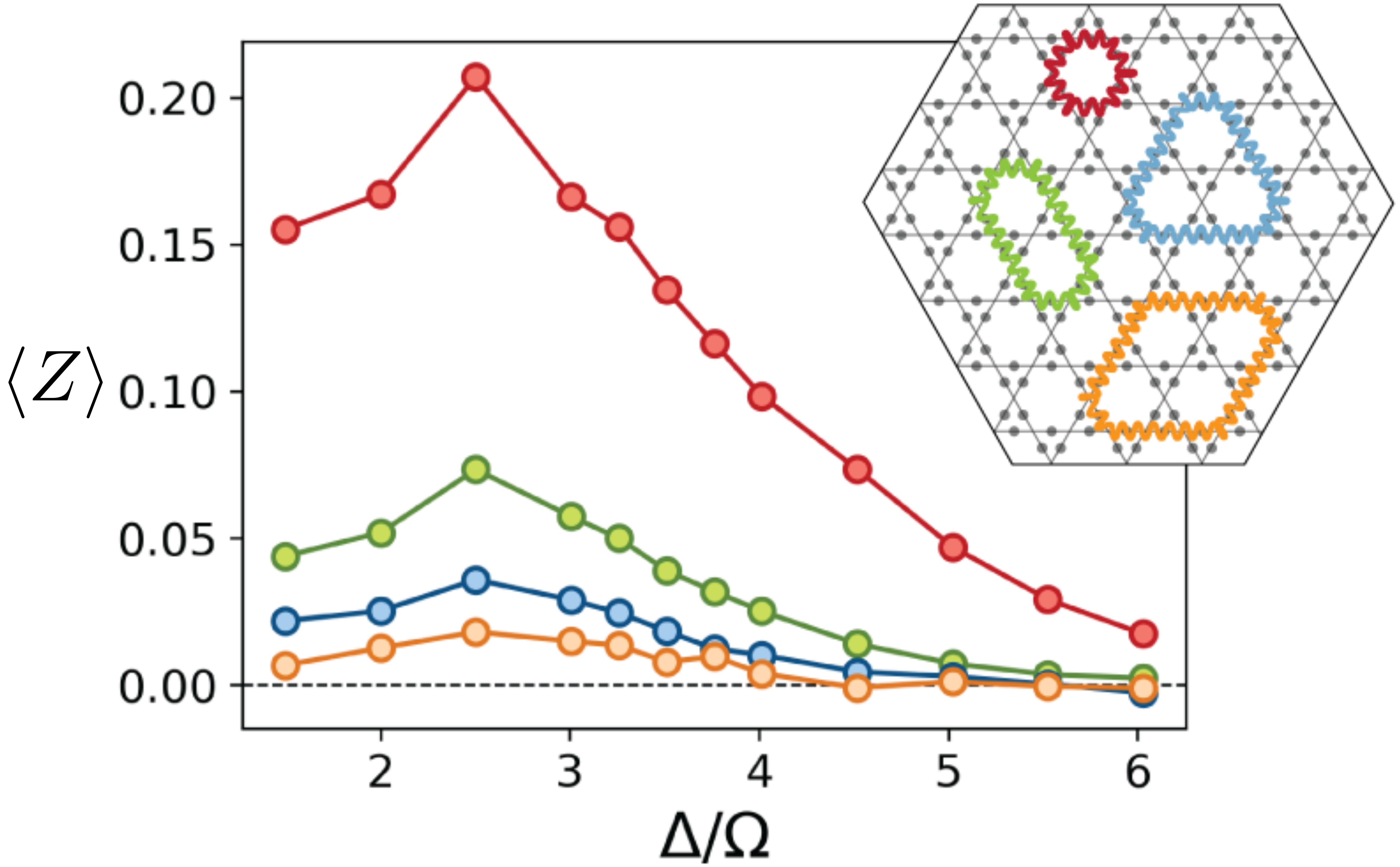


Measurement of
the topological
Z operator.
Detects resonance
between dimer loops.



Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for deconfined phase of $\mathbb{Z}_2$ gauge theory on the ruby lattice.