SYK models, strange metals, and black holes

University of Illinois, Urbana, December 5, 2016

Subir Sachdev

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Talk online: sachdev.physics.harvard.edu

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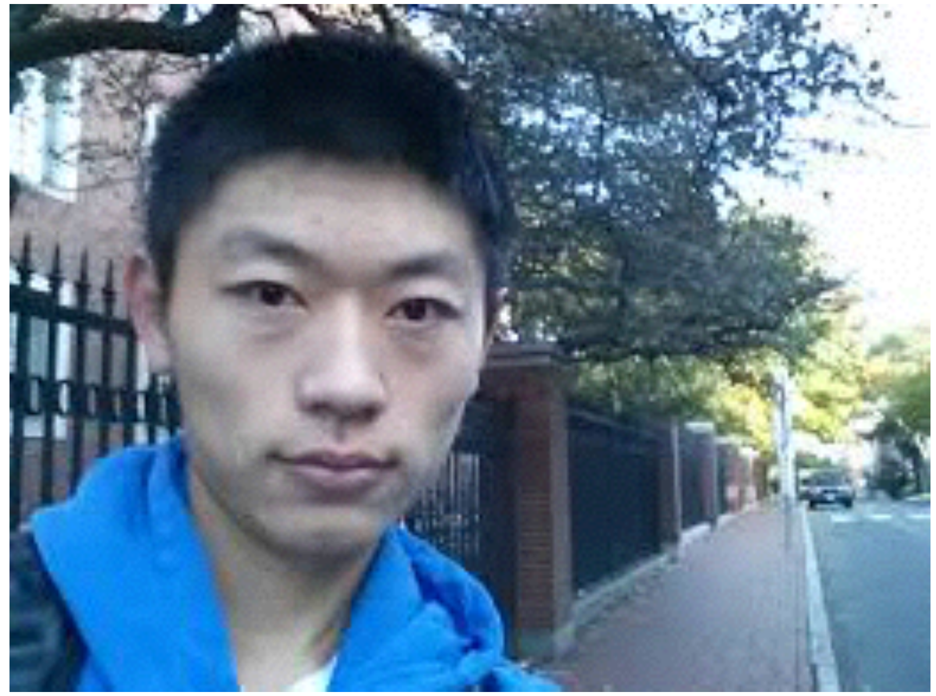
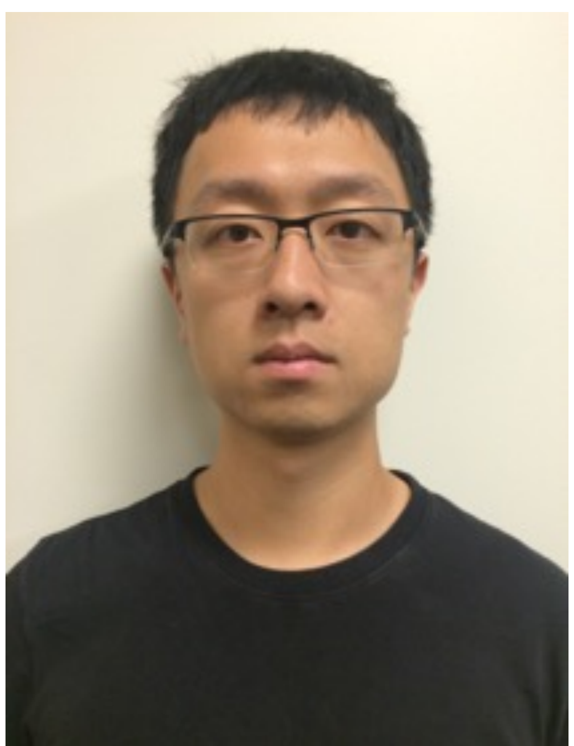
Aavishkar Patel, Harvard



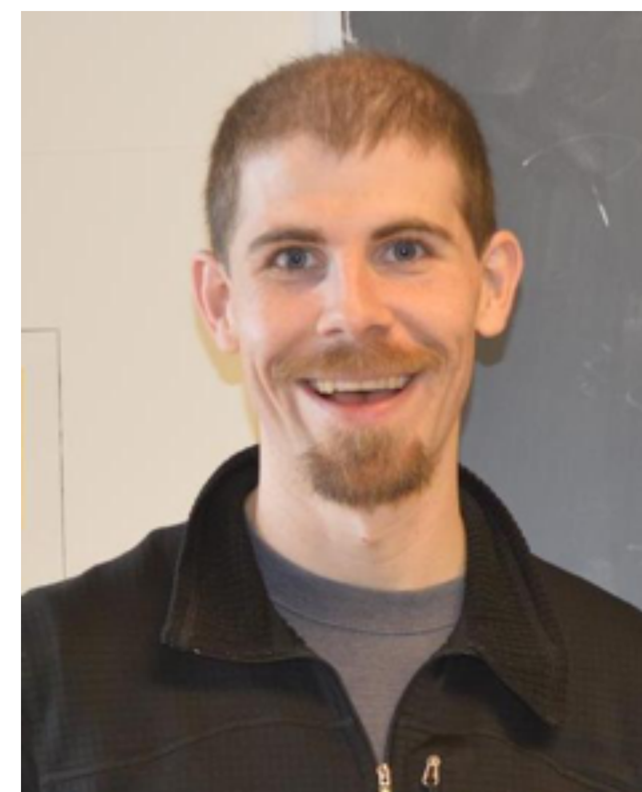
Wenbo Fu, Harvard

Antoine Georges, Paris



Yingfei Gu, Stanford

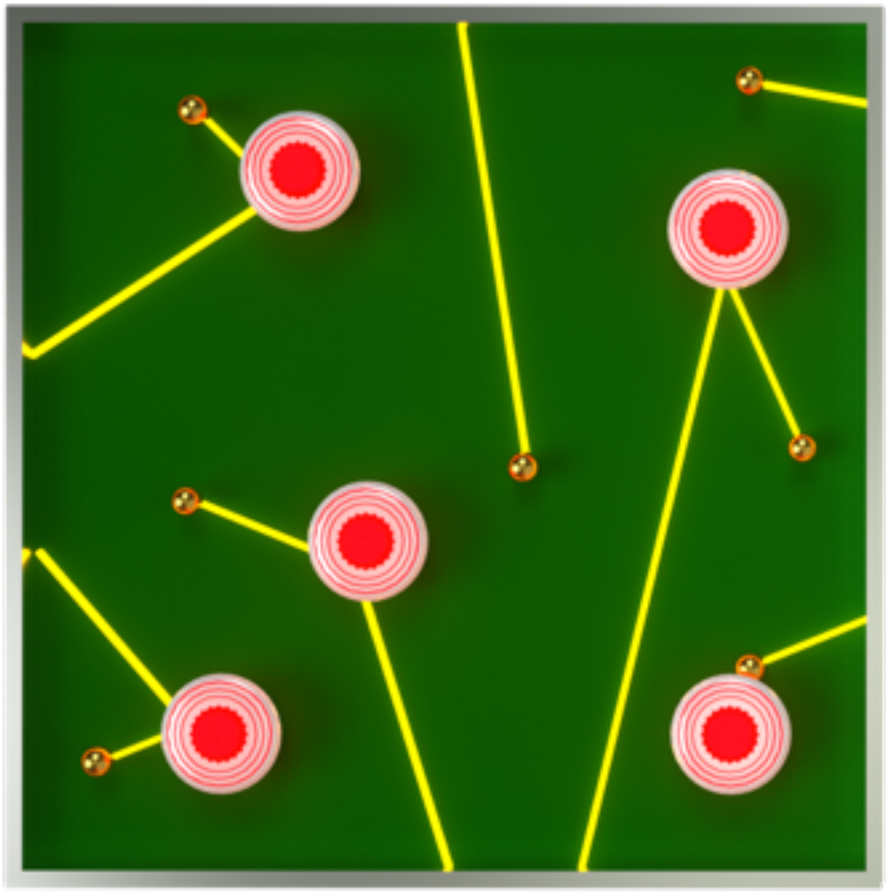
Richard Davison, Harvard



Kristan Jensen, SFSU

Quantum matter with quasiparticles:

- *Landau quasi-particles & holes*
- *Phonon*
- *Magnon*
- *Roton*
- *Plasmon*
- *Polaron*
- *Exciton*
- *Laughlin quasiparticle*
- *Composite fermion*
- *Bogoliubovon*
- *Anderson-Higgs mode*
- *Massless Dirac Fermions*
- *Weyl fermions*
- ….



Quantum matter <u>with</u> quasiparticles:

Most generally, a quasiparticle is an "additive" excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.

Quantum matter <u>without</u> quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

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Consider how rapidly the system loses "phase coherence", reaches local thermal equilibrium, or becomes "chaotic"

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

where C is a T-independent constant. Systems *without* quasiparticles have $\tau_\varphi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- By analogy with classical chaos, we define τ_L as the LYAPUNOV TIME over which the wavefunction of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by considering the magnitude-squared of the commutator of two observables, $\hat{A}$, $\hat{B}$ a time t apart:

$$\left\langle \left| [\hat{A}(t), \hat{B}(0)] \right|^2 \right\rangle \sim e^{t/\tau_L}$$

This time obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

A bound on quantum chaos:

- By analogy with classical chaos, we define τ_L as the Lyapunov time over which the wavefunction of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by considering the magnitude-squared of the commutator of two observables, $\hat{A}$, $\hat{B}$ a time t apart:

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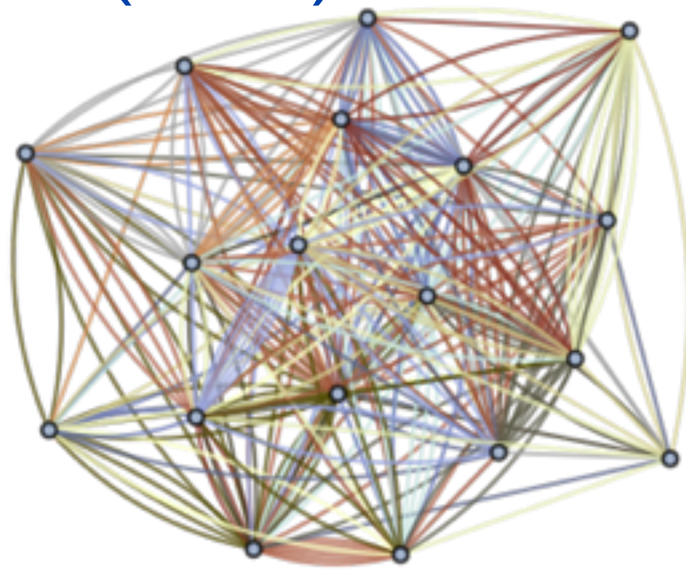
This time obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

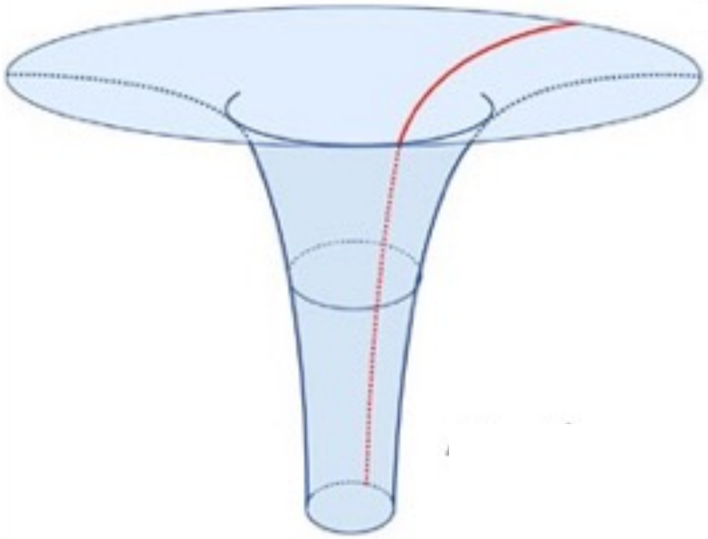
Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

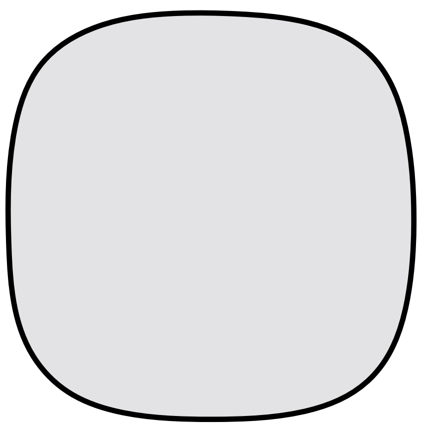
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



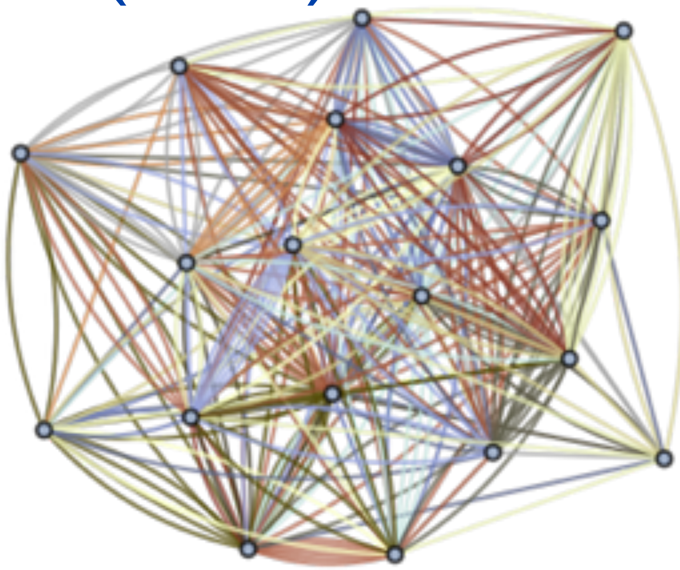


Fermi surface coupled to a gauge field

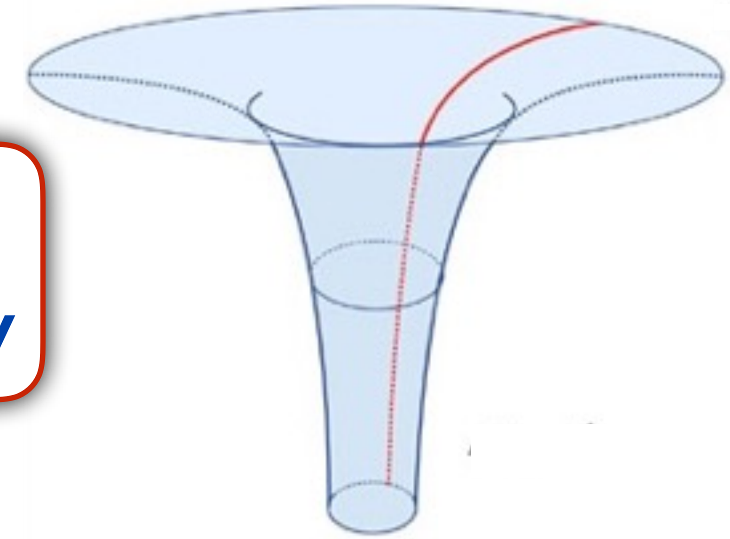
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

Quantum matter <u>without</u> quasiparticles:

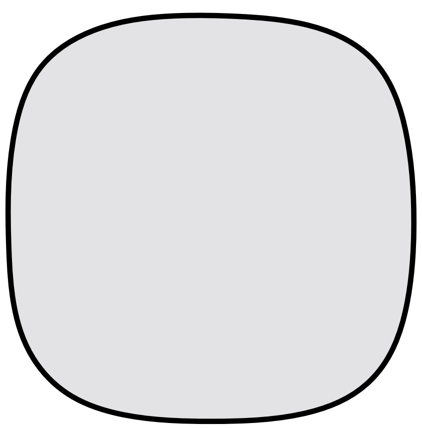
The Sachdev-Ye-Kitaev (SYK) models



Black holes with AdS_2 horizons



Same low energy theory



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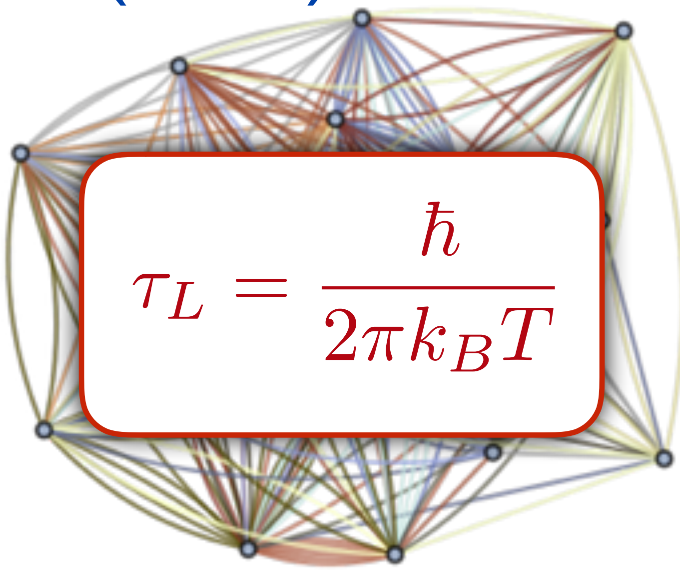
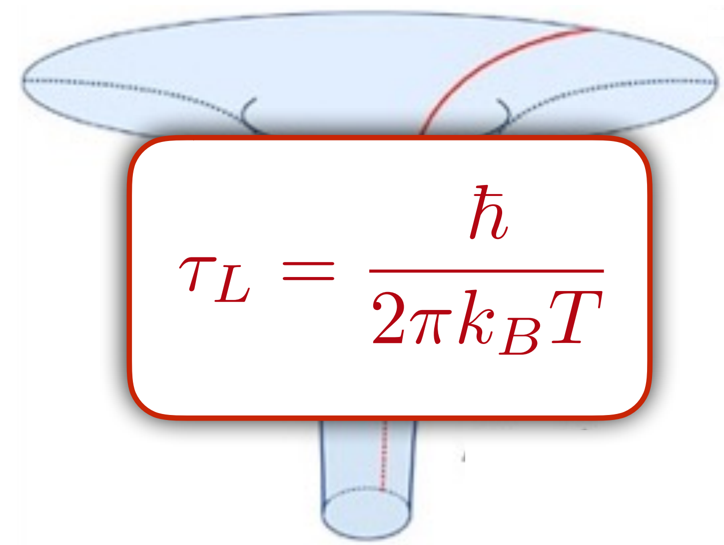


$$\tau_L = \frac{\hbar}{2\pi k_B T}$$

Black holes with AdS_2 horizons



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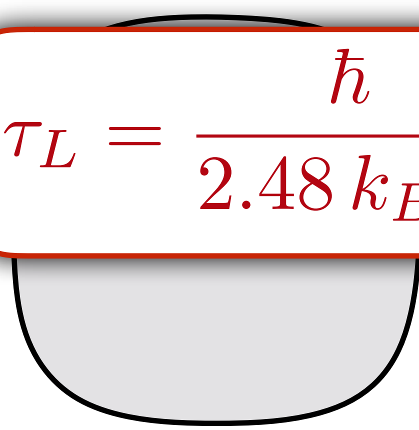


$$\tau_L = \frac{\hbar}{2.48\, k_B T}$$

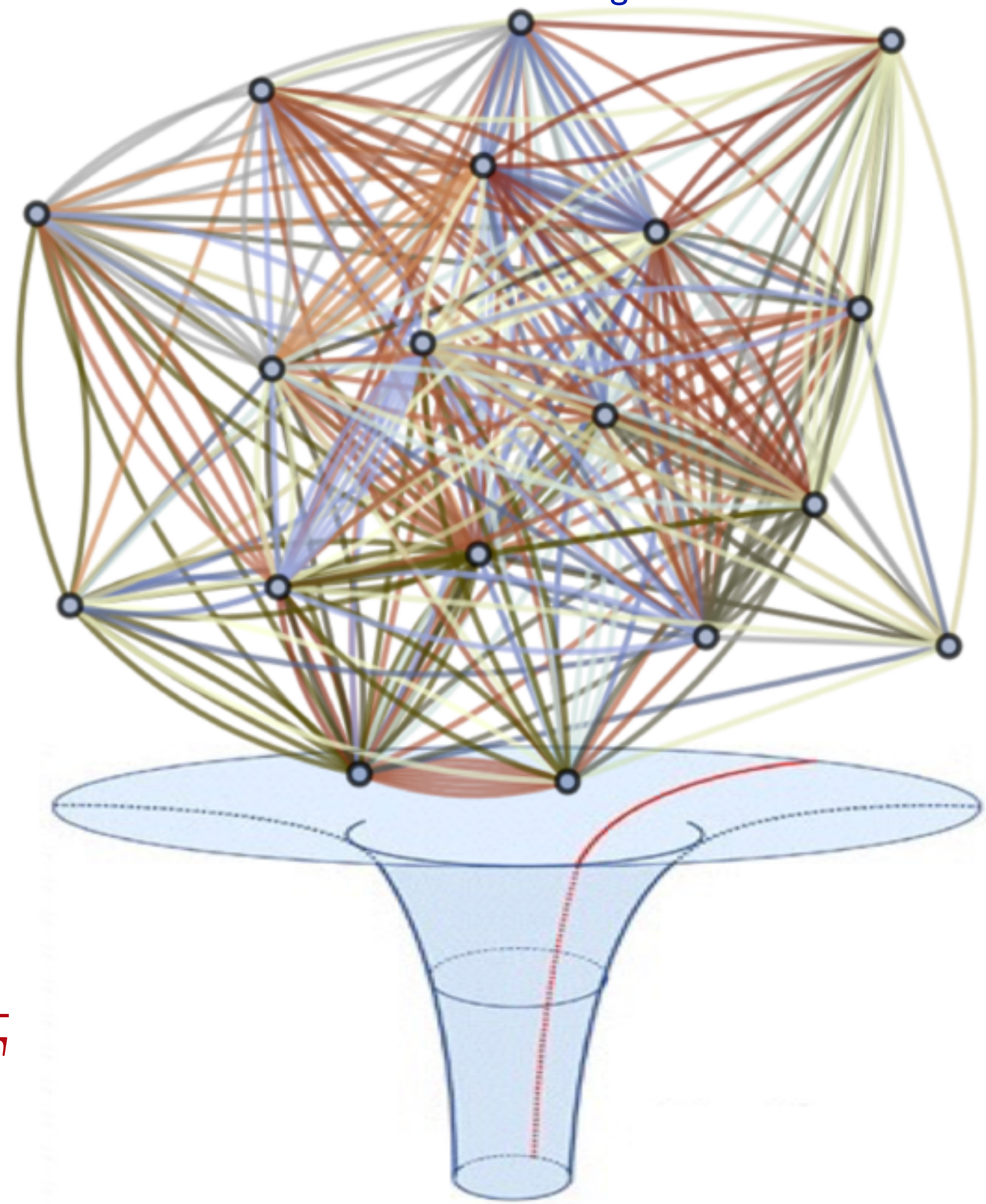
$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left(\partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

τ_L: the Lyapunov time to reach quantum chaos

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \dfrac{\hbar}{2\pi k_B T}$

Figure credit: L. Balents



Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

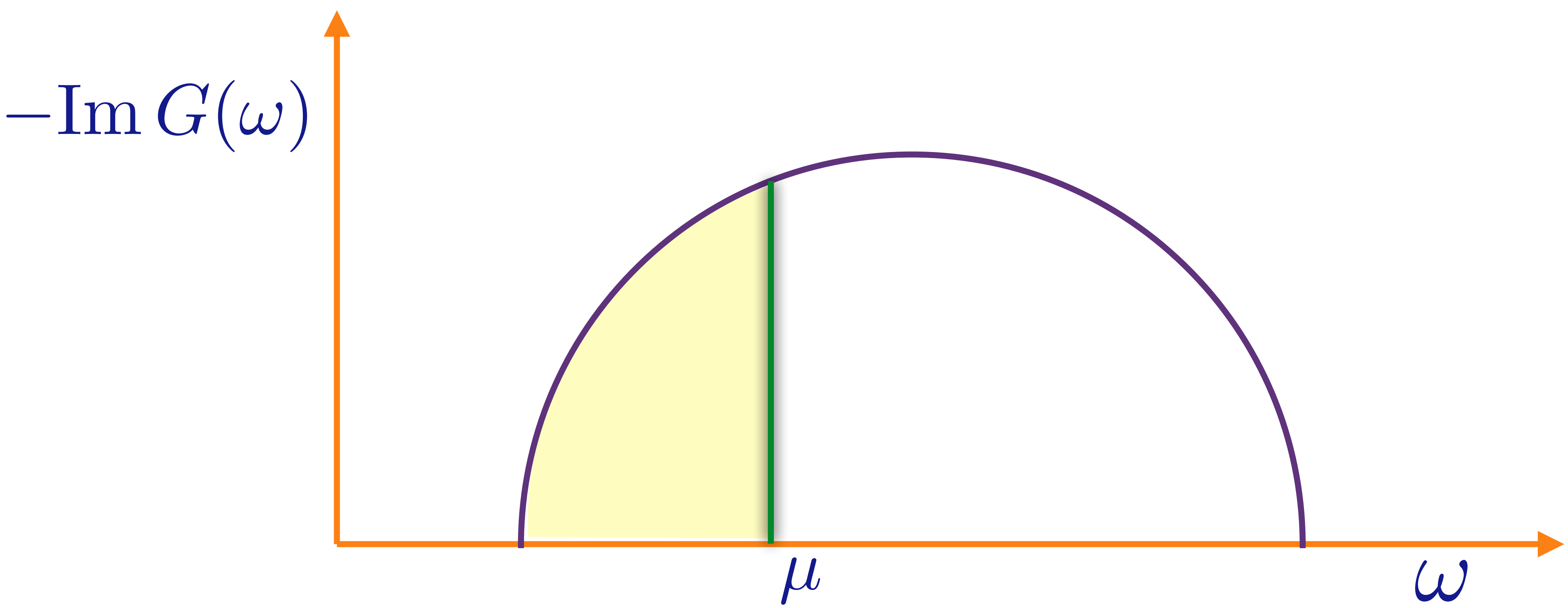
Fermions occupying the eigenstates of a *N* x *N* random matrix

Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Infinite-range model with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy E_α. By Fermi's Golden rule, for E_α at the Fermi energy

$$\frac{1}{\tau_\alpha} = \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta))\delta(E_\alpha + E_\beta - E_\gamma - E_\delta)$$
$$= \frac{\pi^3 J^2 \rho_0^2}{4} T^2$$

where ρ_0 is the density of states at the Fermi energy.

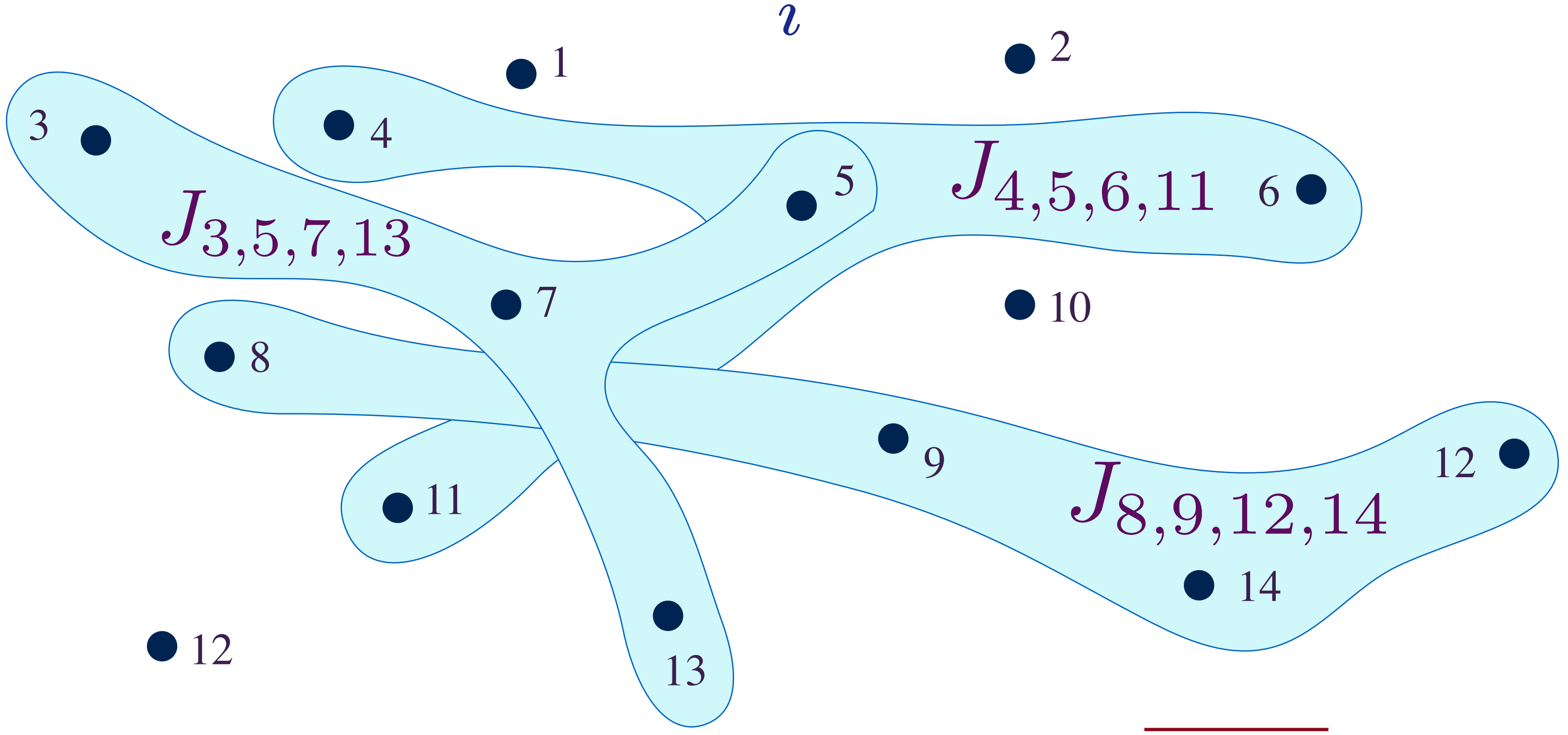
Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS_2

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$
  Not a ground state degeneracy: due to an exponentially small (in N) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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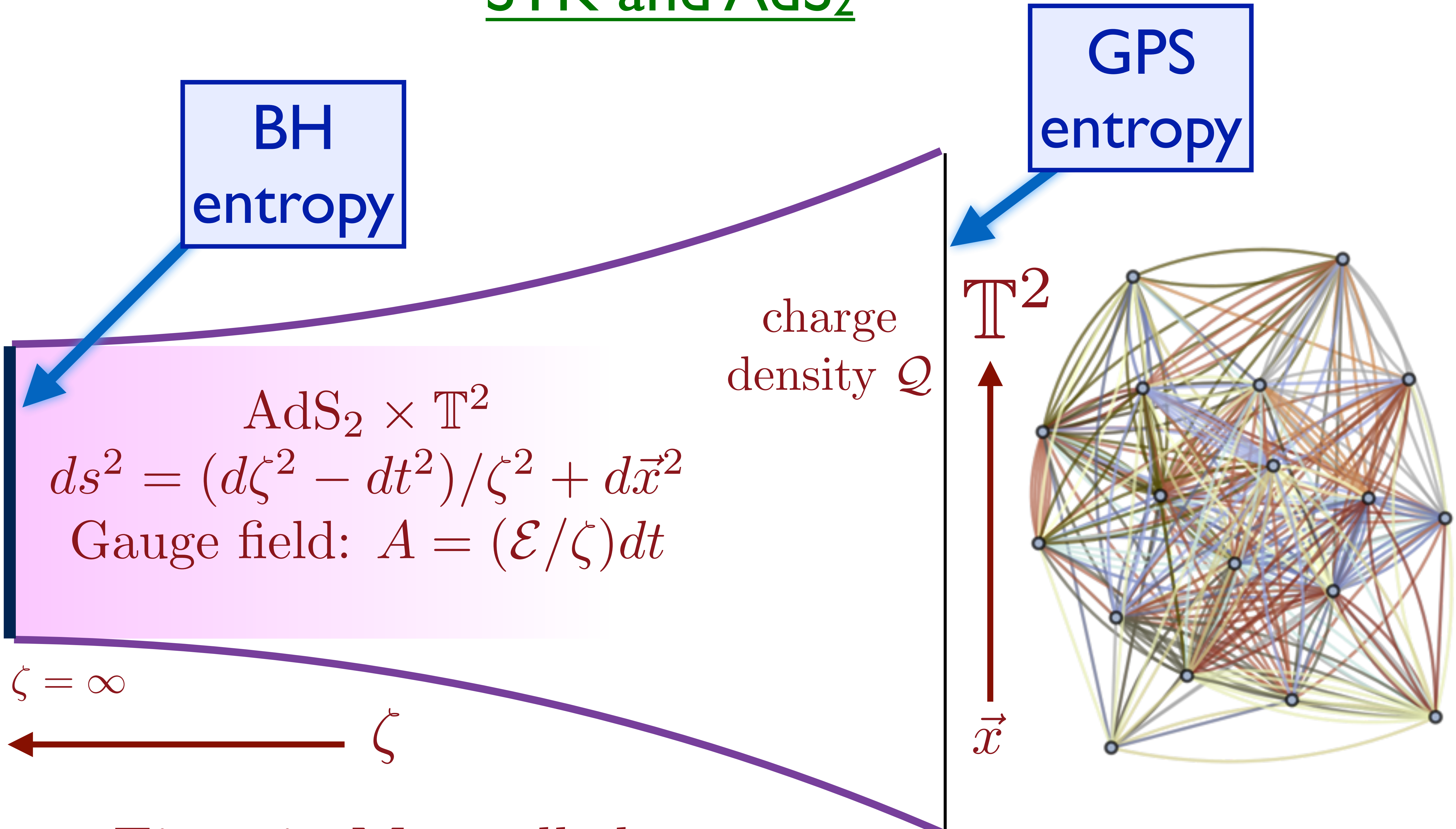
- The correlators and thermodynamics of SYK match those of quantum matter holographically dual to AdS_2 black hole horizons (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)). SYK models are "are states of matter at non-zero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nördstrom black holes". The Bekenstein-Hawking entropy is NS_0 (**GPS = BH**).

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



Einstein-Maxwell theory
$+$ cosmological constant

S. Sachdev, PRL **105**, 151602 (2010)

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SYK and AdS_2

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK and AdS$_2$

$$G(i\omega) = \frac{1}{\cancel{i\omega} + \cancel{\mu} - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \cancel{\mu} - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS_2

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

A. Kitaev, unpublished

SYK and AdS_2

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism
- SL(2,R) is the isometry group of AdS_2.

SYK and AdS$_2$

Reparametrization and phase zero modes

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action must vanish for $f(\tau) \in$ SL(2,R). The action for $\phi(\tau)$ and $f(\tau) = \dfrac{1}{\pi T}\tan(\pi T(\tau + \epsilon(\tau))$ fluctuations is

$$S_{\phi,f} = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{f, \tau\},$$

where $\{f, \tau\}$ is the Schwarzian:

$$\{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2.$$

J. Maldacena and D. Stanford, arXiv:1604.07818; R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

SYK and AdS_2

Reparametrization and phase zero modes

- The same effective action $S_{\phi,f}$ is also obtained from theories of gravity on AdS_2, after integrating out "bulk" modes for an effective theory on the boundary.

- The couplings, K, γ, $\mathcal{E}$ are all related to thermodynamic derivatives. The same relationship is obtained in the SYK and gravity theories.

J. Maldacena and D. Stanford, arXiv:1604.07818; R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Coupled SYK models

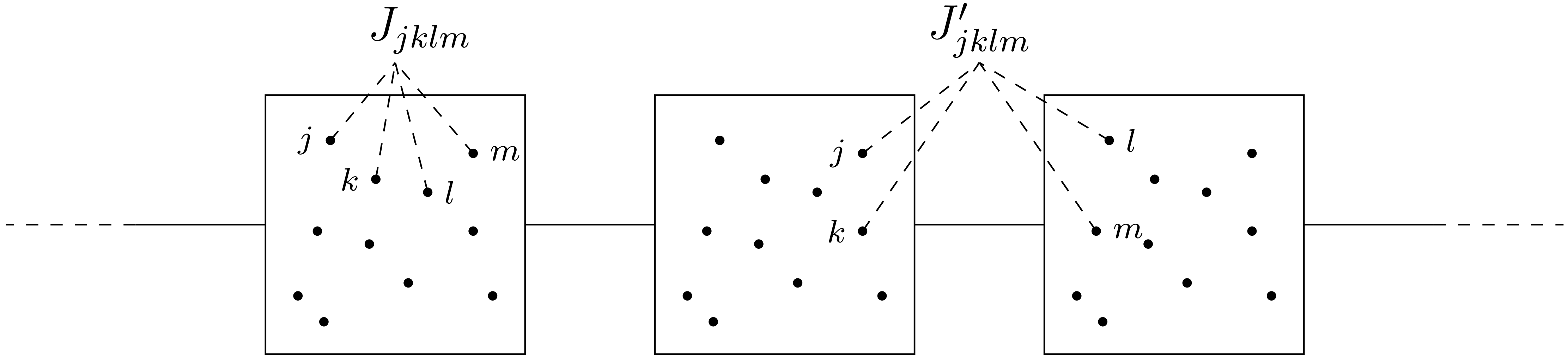


Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

Coupled SYK models

The response functions of the density, $\mathcal{Q}$, and the energy, E exhibit diffusion

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu\mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu\mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu\mathcal{Q}; E - \mu\mathcal{Q} \rangle_{k,\omega} / T \end{pmatrix} = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{pmatrix} \chi_s^{-1}.$$

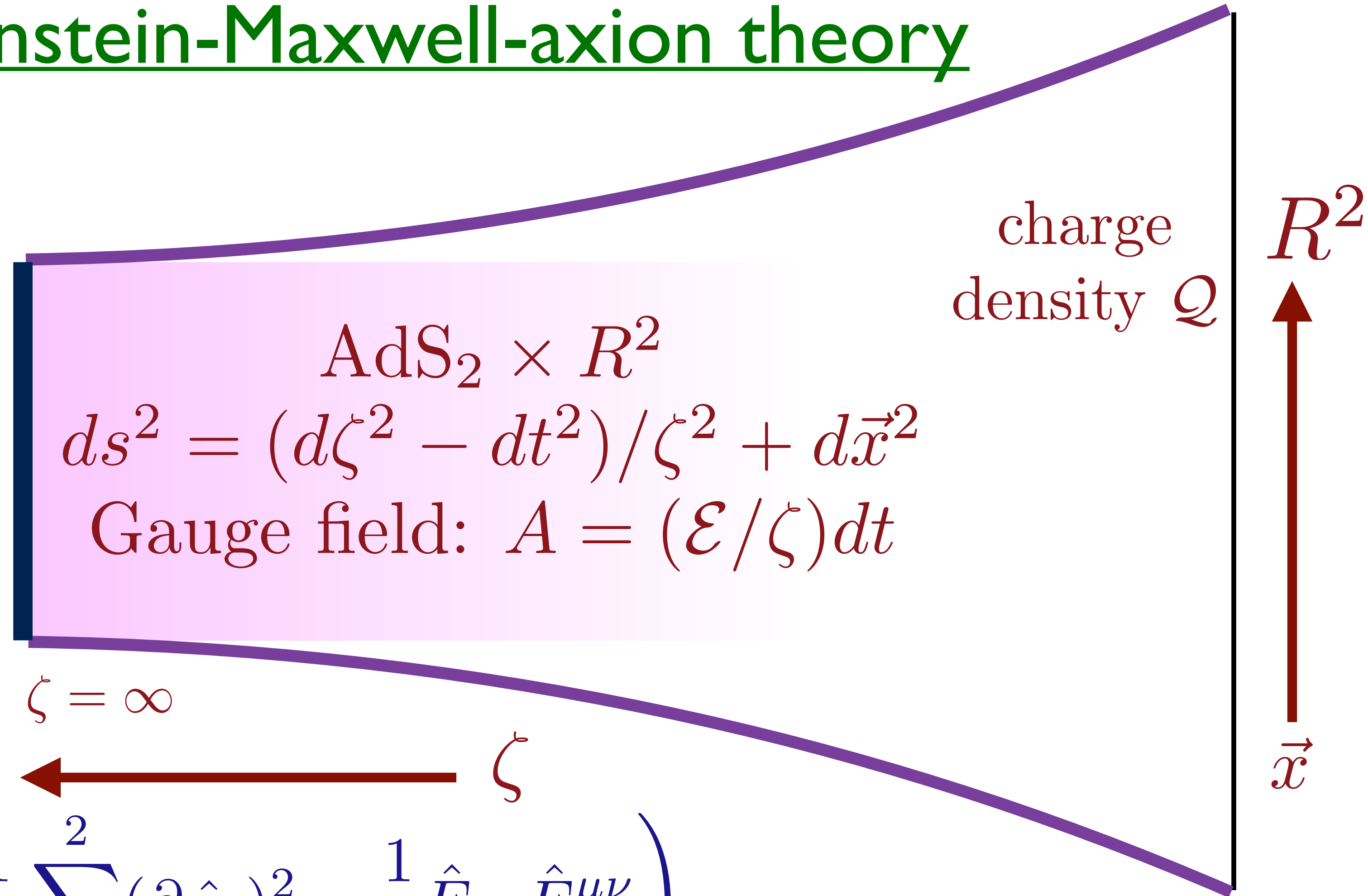
The Seebeck co-efficient (thermopower), α/σ, is given exactly by a thermodynamic derivative

$$\frac{\alpha}{\sigma} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

The coupled SYK models realize a disordered metal with no quasiparticle excitations.
(a "strange metal")

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

Holography:Einstein-Maxwell-axion theory



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

- For $\hat{\varphi}_i = 0$, we obtain the Reissner-Nördstrom-AdS charged black hole, with a near-horizon $\mathrm{AdS}_2 \times R^2$ near-horizon geometry.

- For $\hat{\varphi}_i = kx_i$, we obtain a similar solution but with momentum dissipation (a bulk massive graviton). This yields the same diffusive metal correlators as the coupled SYK models, and the same value of the Seebeck co-efficient.

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537;
R. A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004;
T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016);
R. Davison, Wenbo Fu, Yingfei Gu, A. Georges, K. Jensen, S. Sachdev, arXiv:1612.00849

Quantum chaos:

- In both the SYK and holographic models, the growth of chaos is characterized by

$$\left\langle \left|\{c(x,t), c(0,0)\}\right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left(t - \frac{|x|}{v_B}\right)\right)$$

where the Lyapunov time saturates the lower bound $\hbar/(2\pi k_B T)$ and the BUTTERFLY VELOCITY $v_B \sim T^{1/2}$.

- The thermal diffusivity, D_E is given exactly by

$$D_E = v_B^2 \tau_L .$$

There is no universal relationship between the charge diffusivity, D_c, and v_B.

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

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There is no universal relationship between the charge diffusivity, D_c, and v_B.

- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

Quantum matter without quasiparticles:

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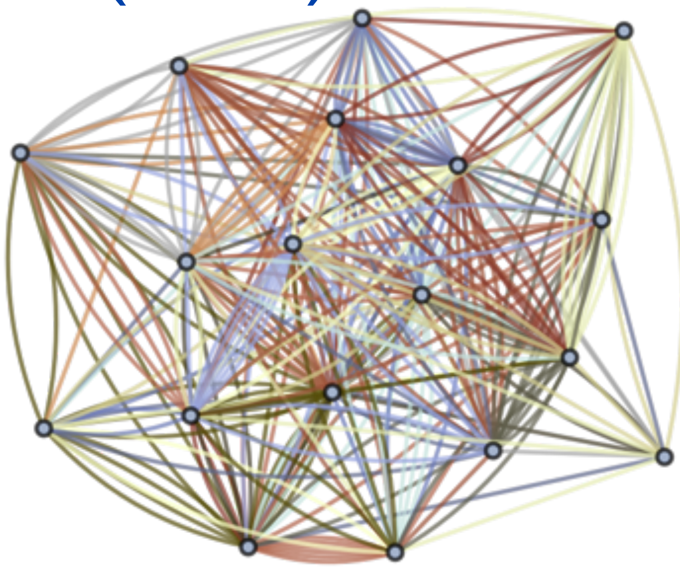

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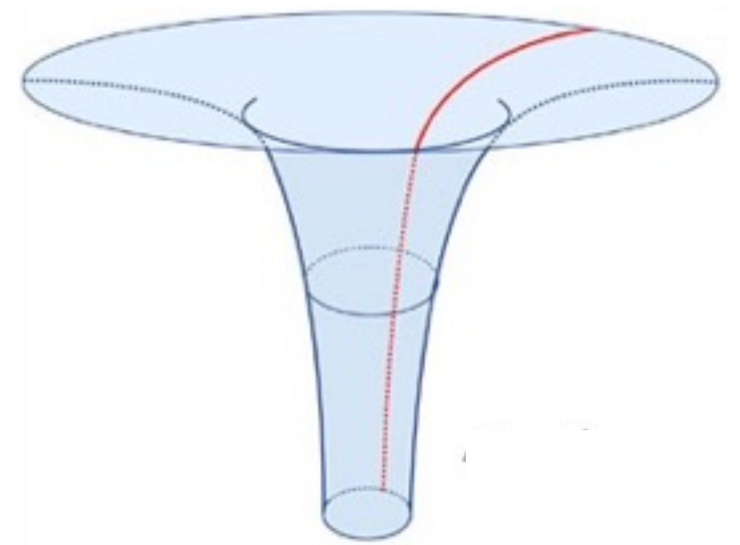

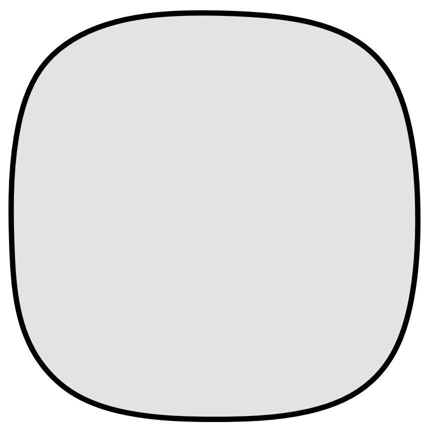

Fermi surface coupled to a gauge field

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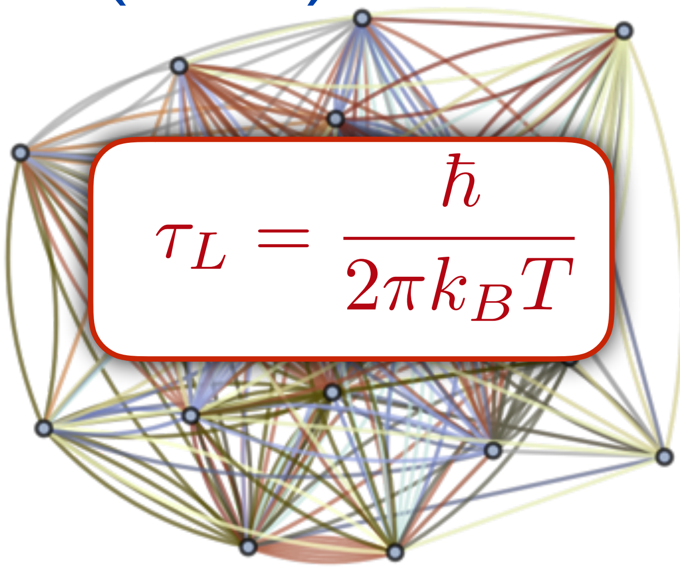
Quantum matter <u>without</u> quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models



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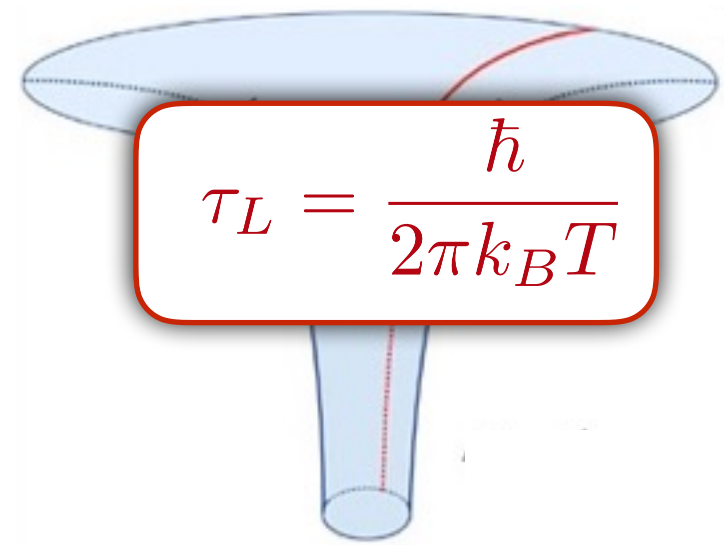

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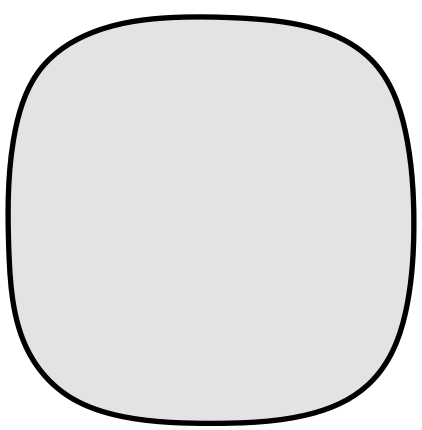

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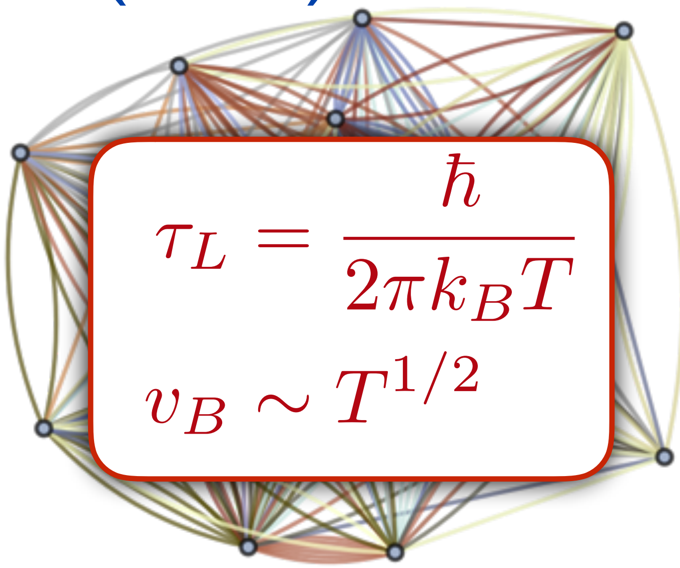

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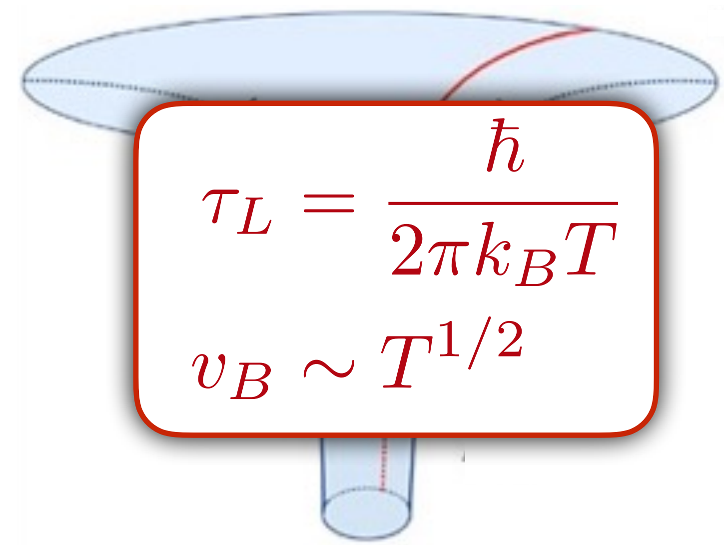

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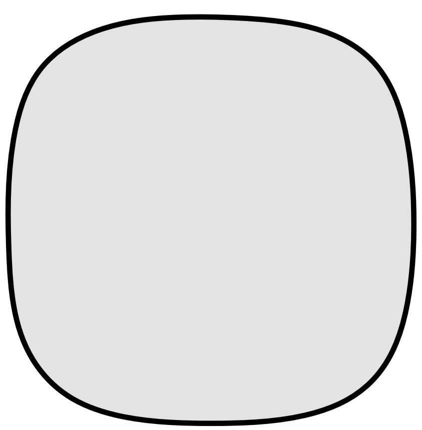

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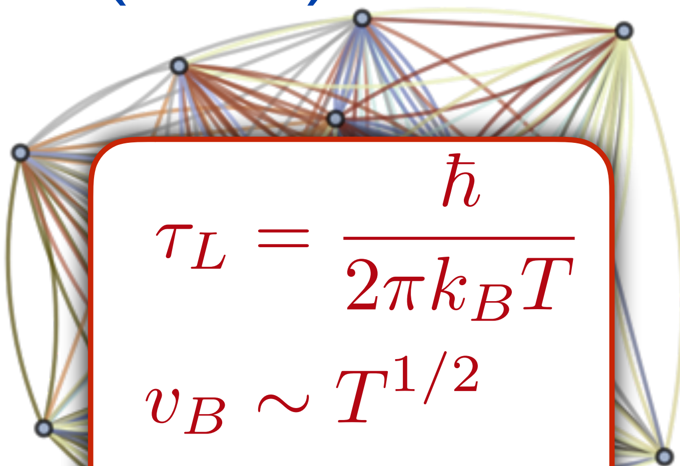

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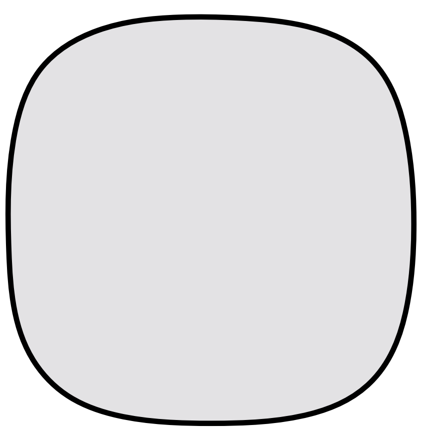

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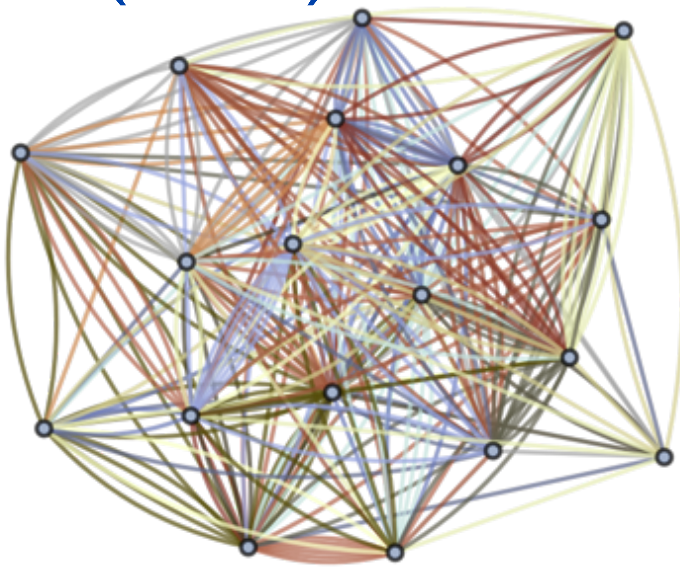

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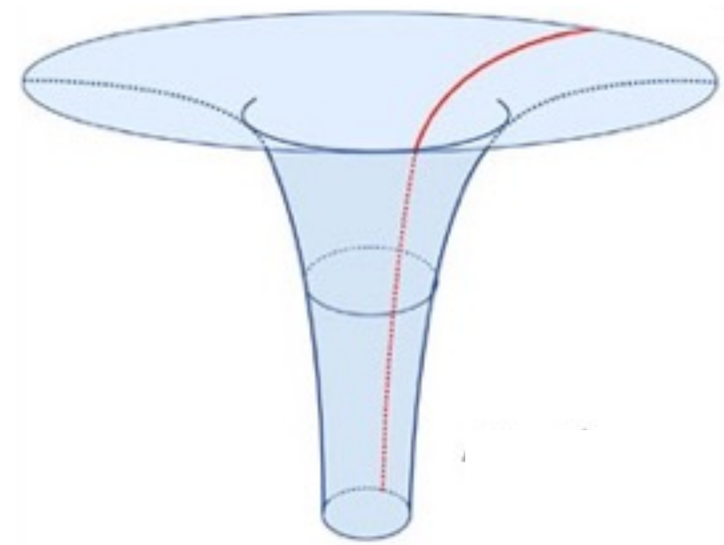

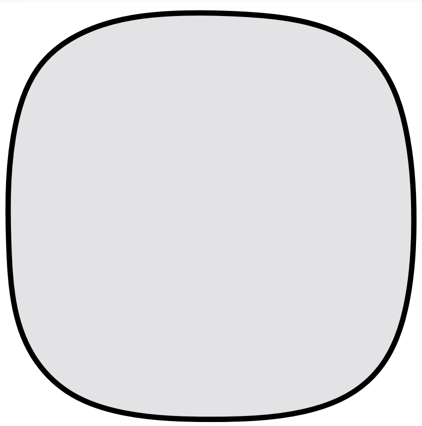

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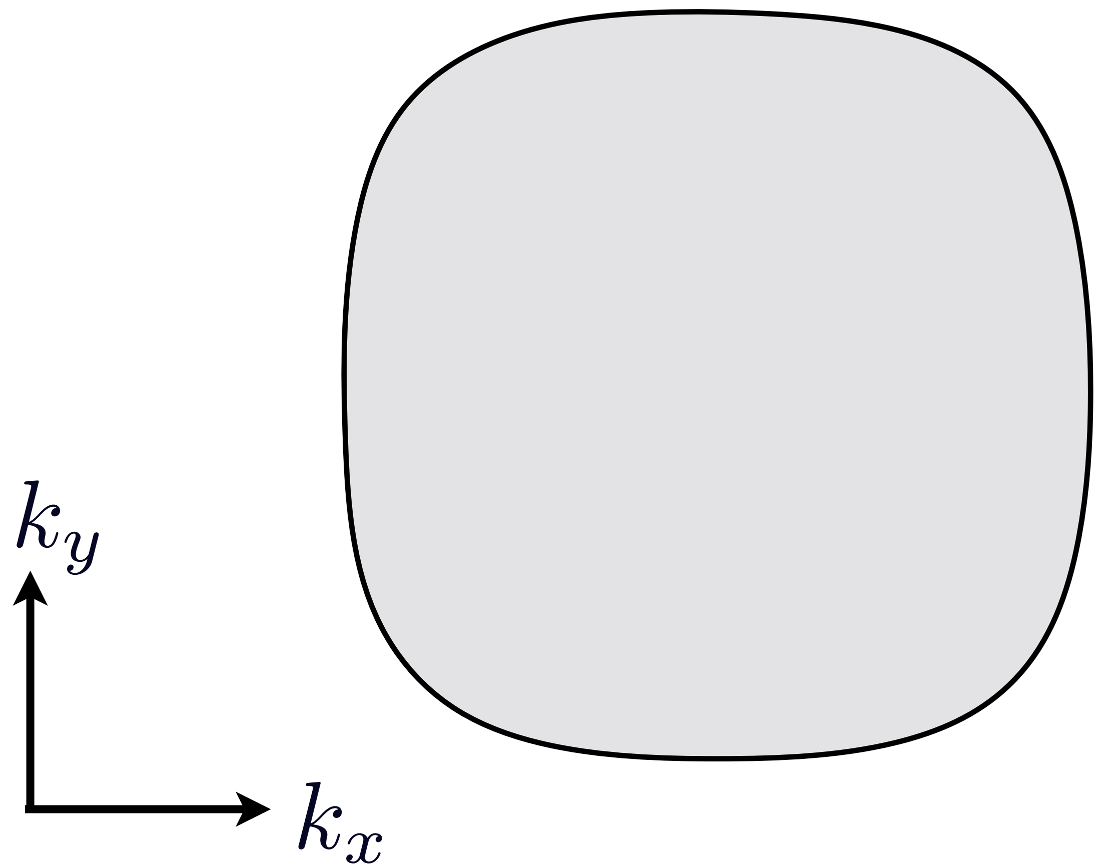

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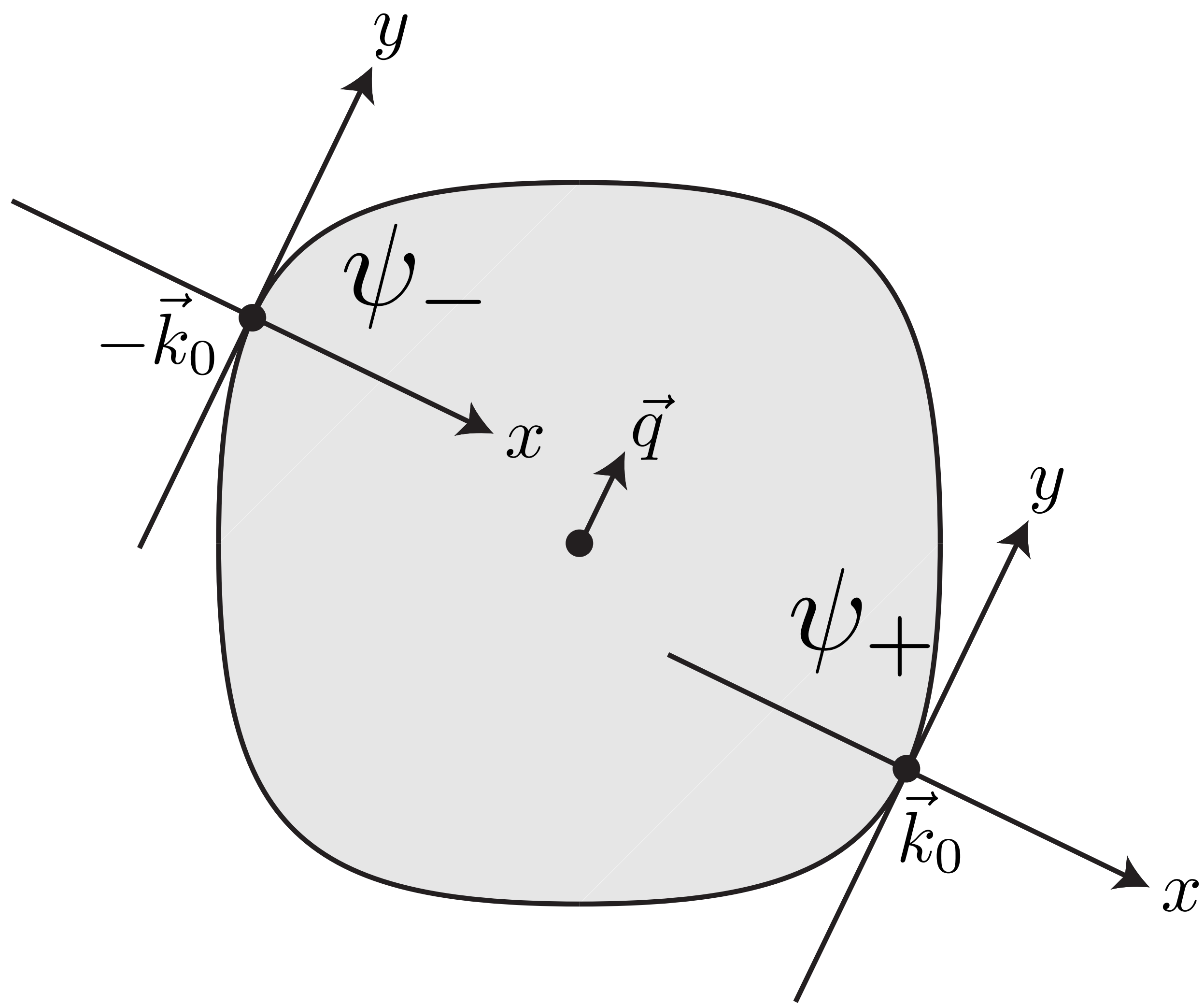


$$\mathcal{L}[\psi_\pm, a] =$$

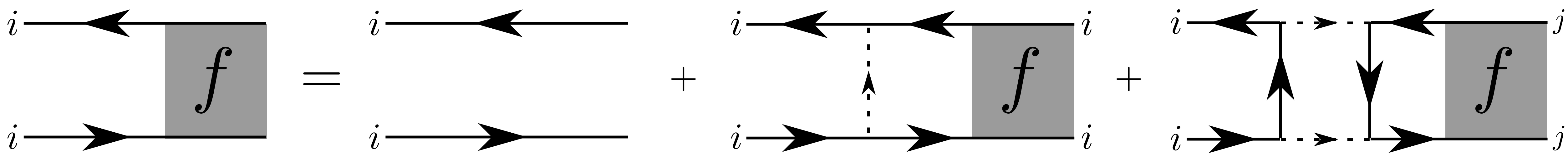
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos



$$f(t) = \frac{1}{N^2}\theta(t) \sum_{i,j=1}^{N} \int d^2x \; \mathrm{Tr}\Big[e^{-\beta H/2} \{\psi_i(x,t), \psi_j^\dagger(0)\} \times e^{-\beta H/2} \{\psi_i(x,t), \psi_j^\dagger(0)\}^\dagger \Big]$$

$$\sim \exp\Big((t - x/v_B)/\tau_L \Big)$$

A. A. Patel and S. Sachdev, arXiv:1611.00003

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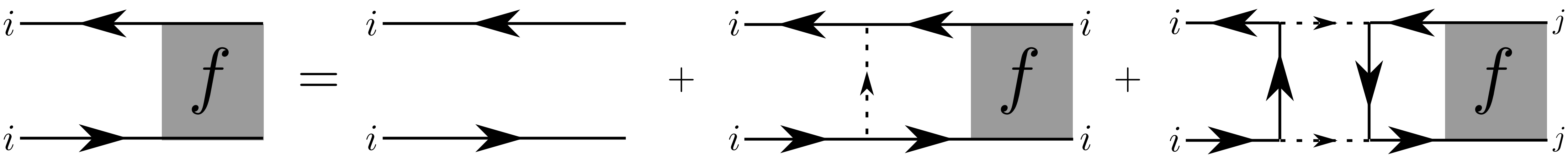

Strongly-coupled theory with no quasiparticles and fast scrambling:

$$\tau_L \approx \frac{\hbar}{2.48\, k_B T}$$

$$v_B \approx 4.1\, \frac{N T^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$D_E \approx 0.42\, v_B^2 \tau_L$$

N is the number of fermion flavors, v_F is the Fermi velocity, γ is the Fermi surface curvature, e is the gauge coupling constant.

A.A. Patel and S. Sachdev, arXiv:1611.00003

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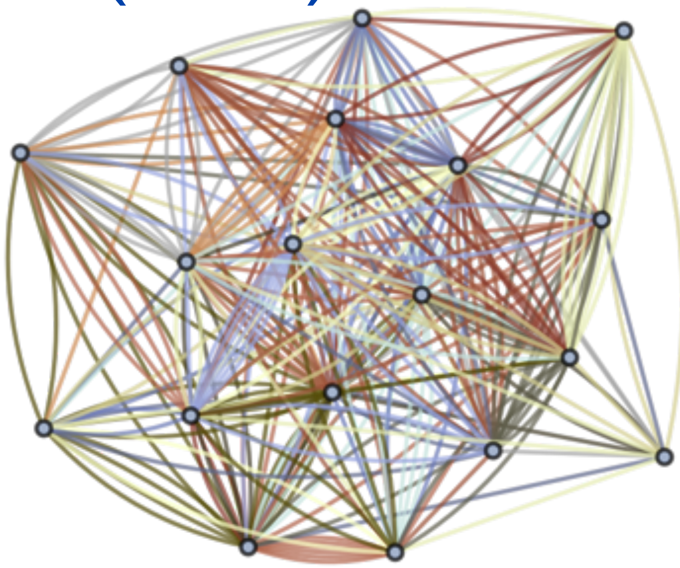

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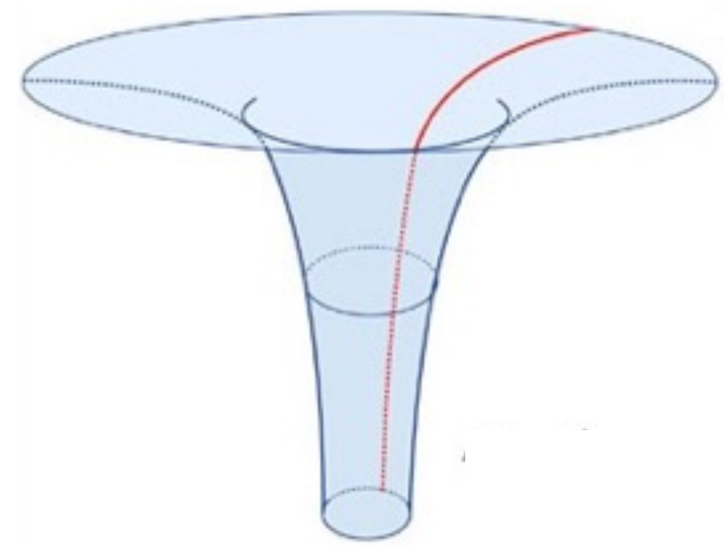

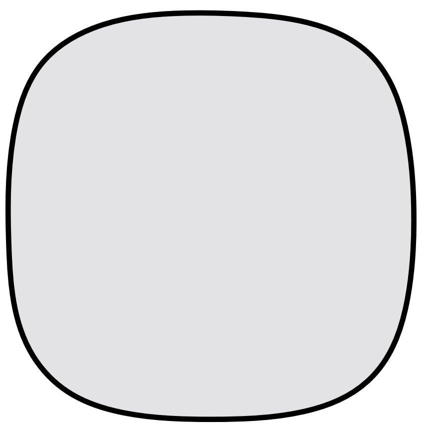

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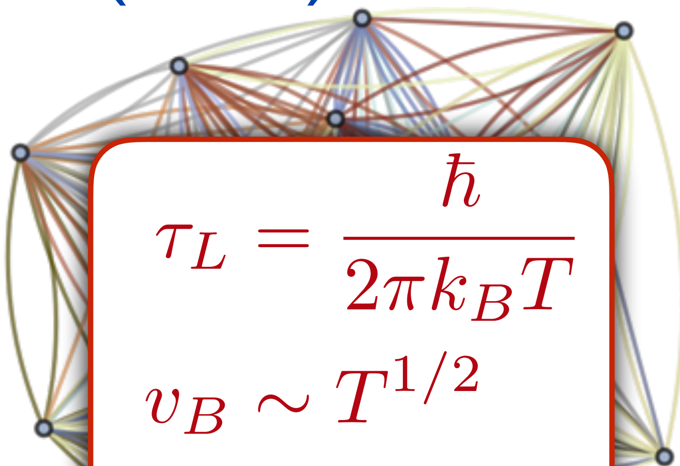

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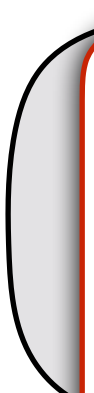

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