

Quantum criticality and the phase diagram of the cuprates

Talk online: sachdev.physics.harvard.edu



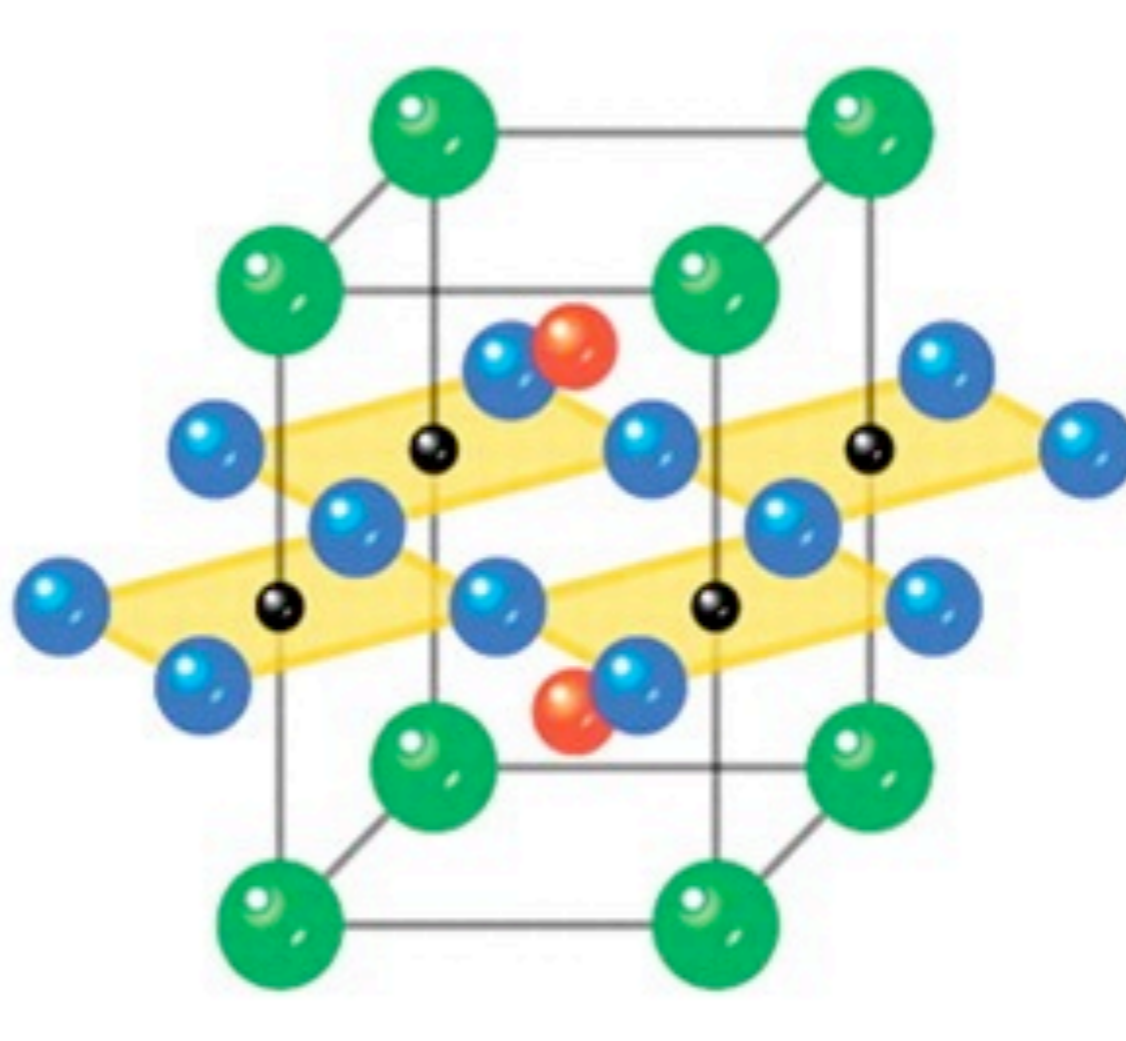
Victor Galitski, Maryland
Ribhu Kaul, Harvard → Kentucky
Max Metlitski, Harvard
Eun Gook Moon, Harvard
Yang Qi, Harvard
Cenke Xu, Harvard → Santa Barbara



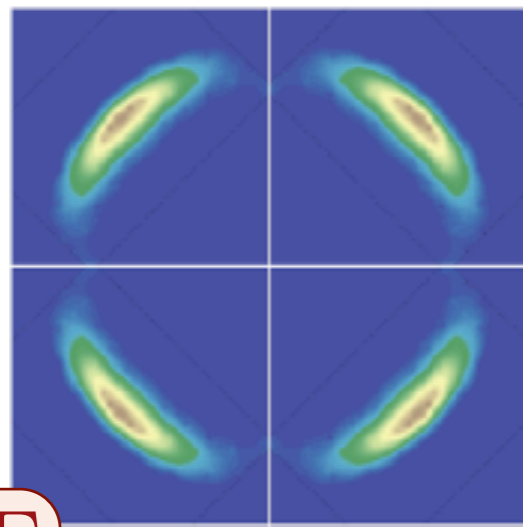
The cuprate superconductors

Na-CCOC

- Cu
- Ca/Na
- O
- Cl

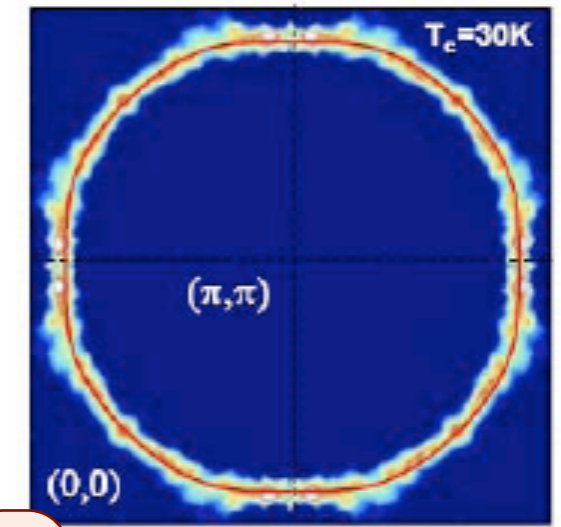
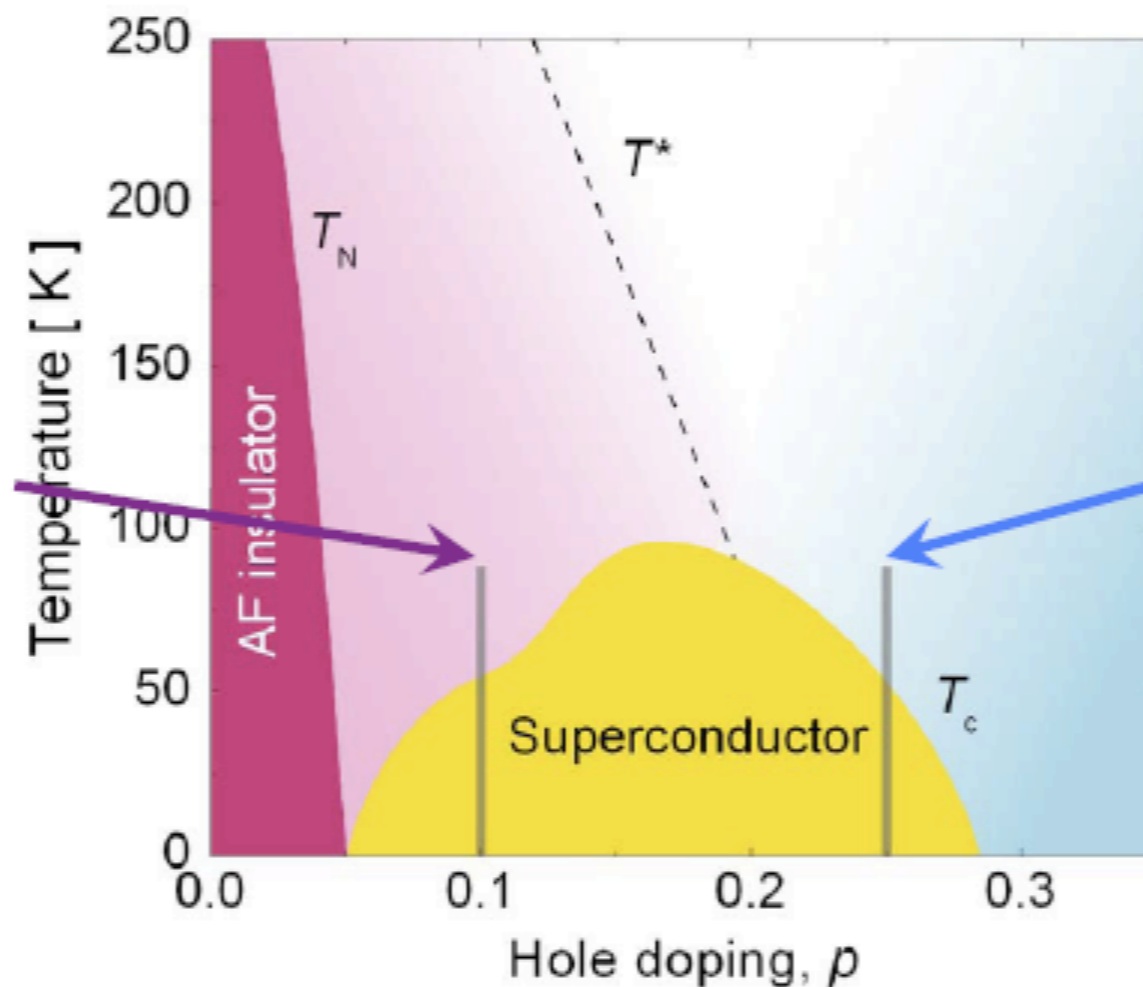


Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



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K.M. Shen et al., Science 2005



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M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

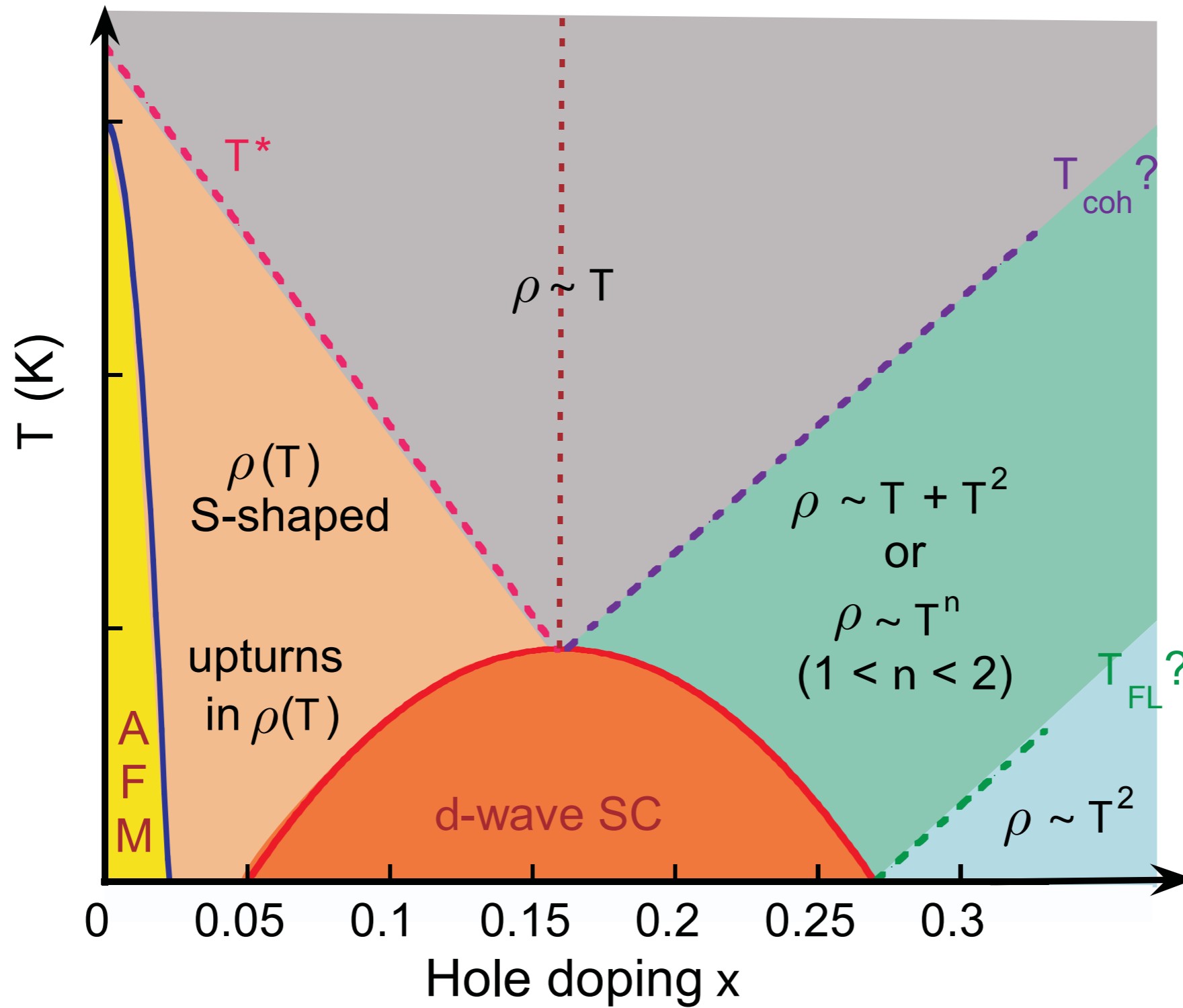
Large hole
Fermi surface

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

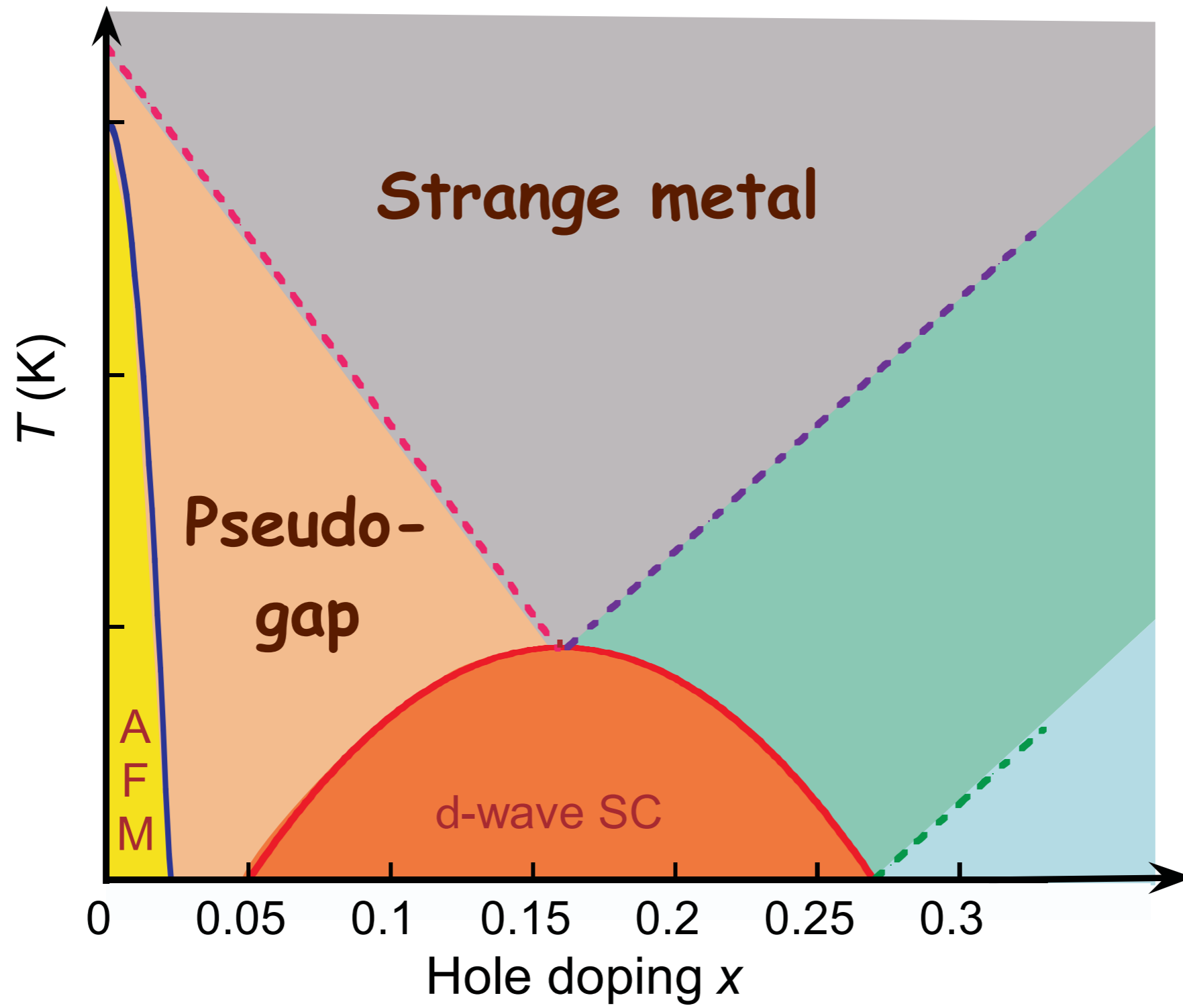
**Fermi
surface**

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



**Antiferro-
magnetism**

**d-wave
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**Fermi
surface**

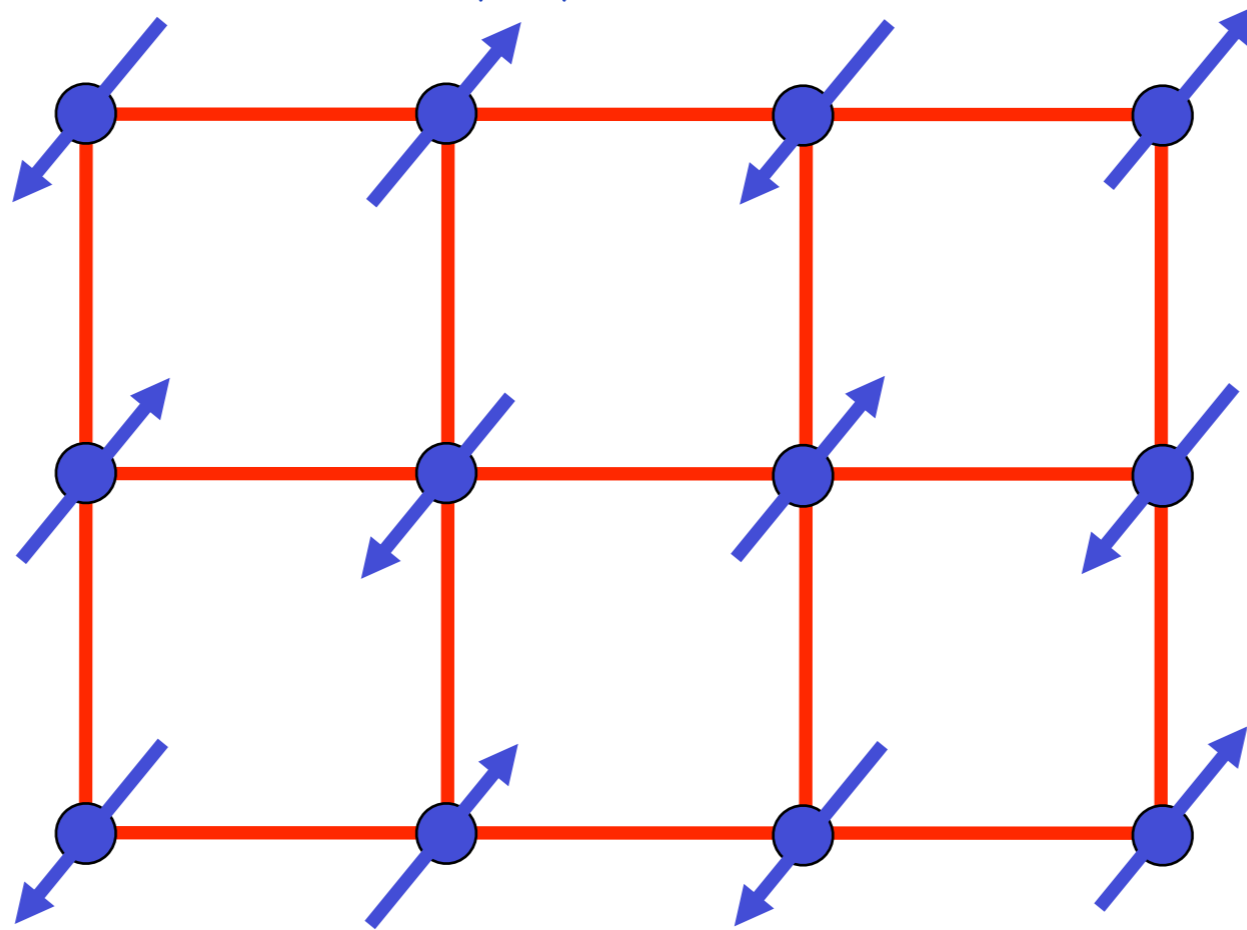
**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

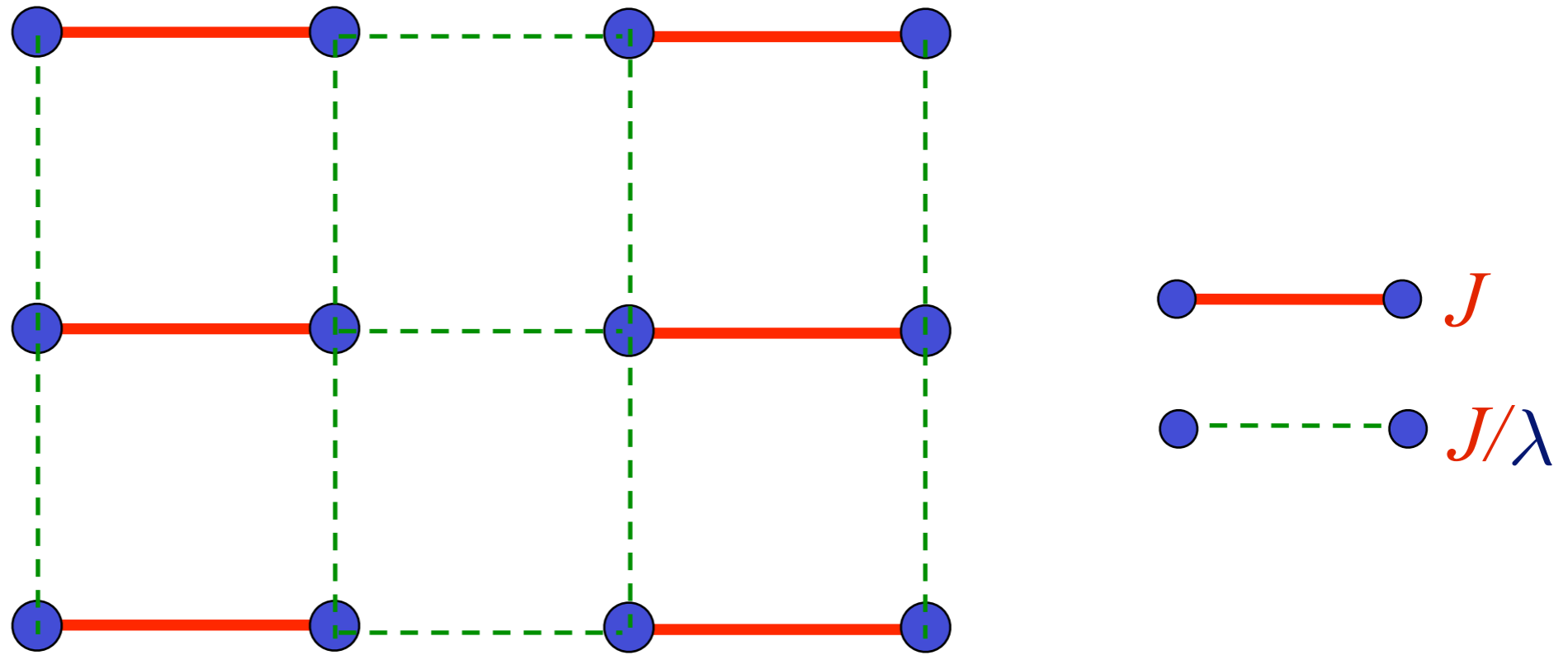
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

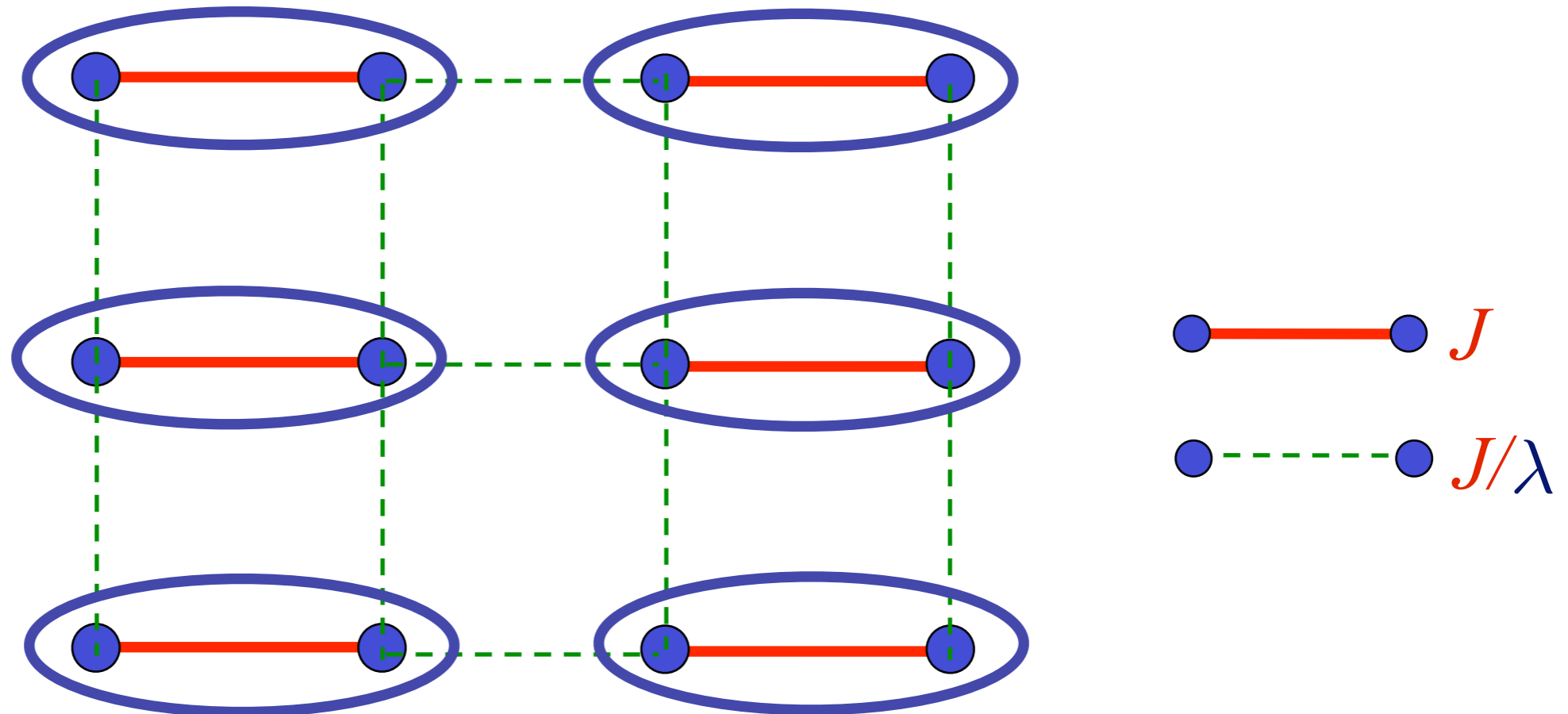
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

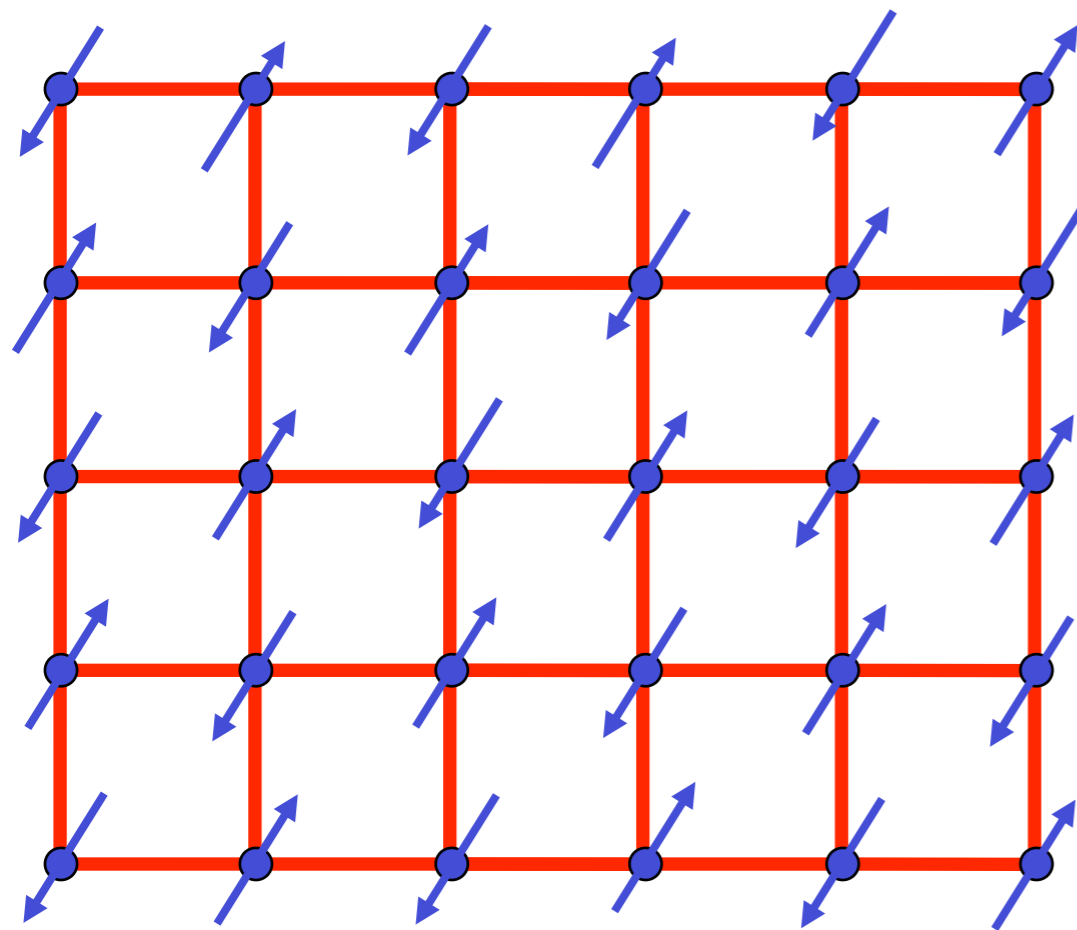
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

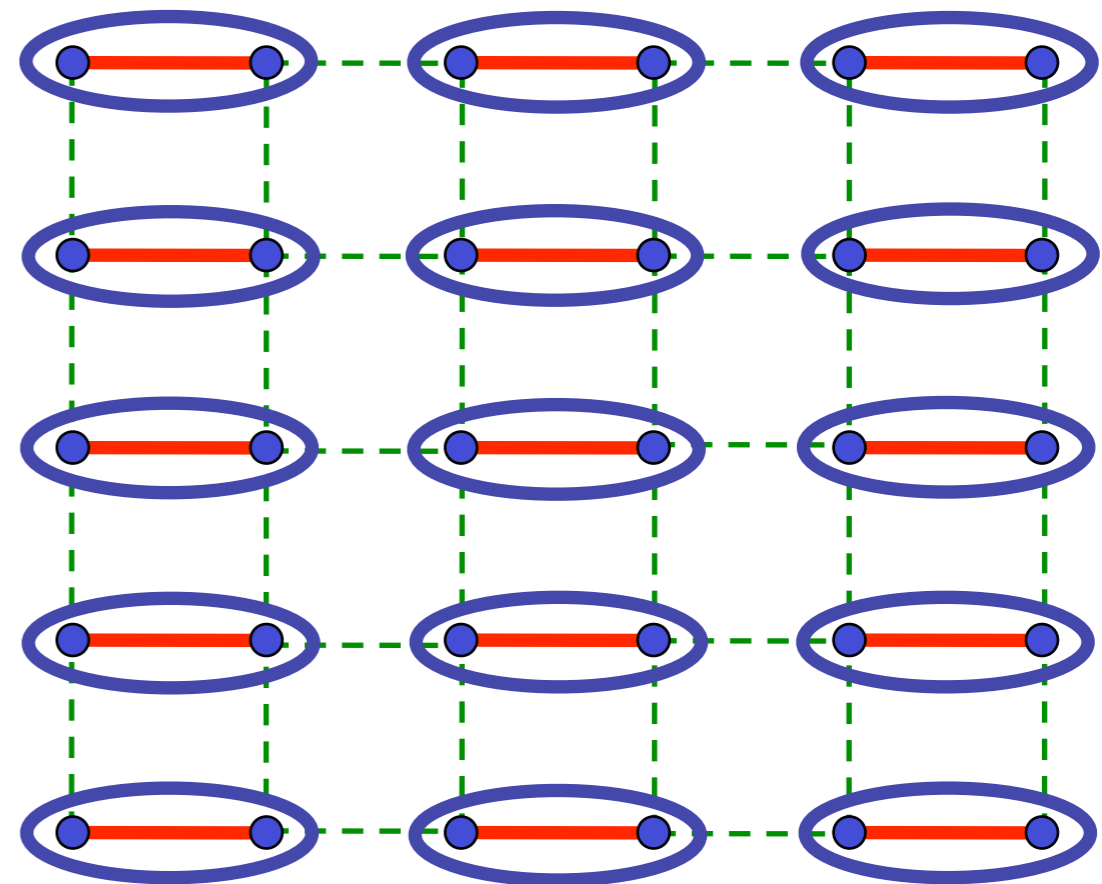


Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



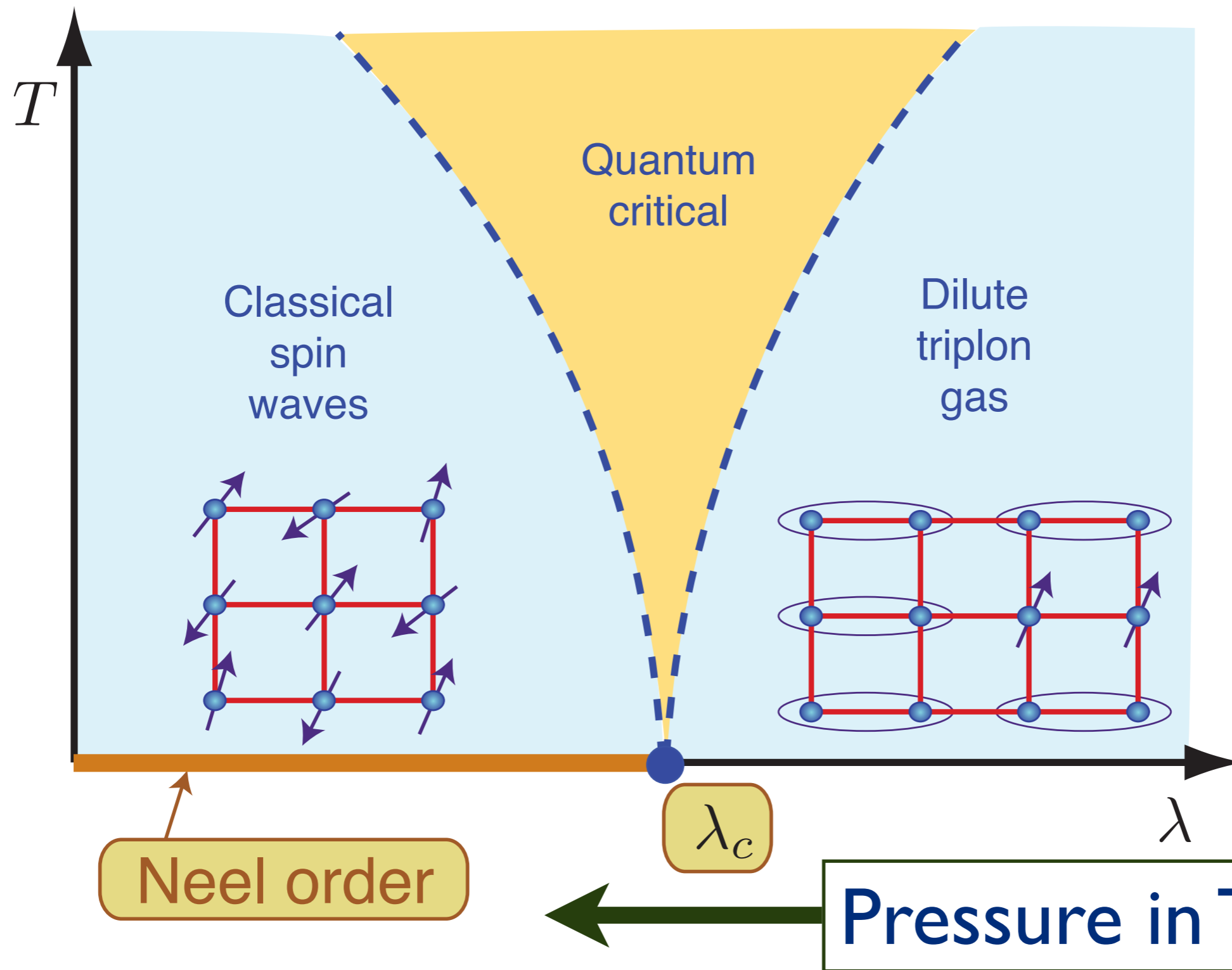
$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



Pressure in TlCuCl_3

Christian Ruedg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

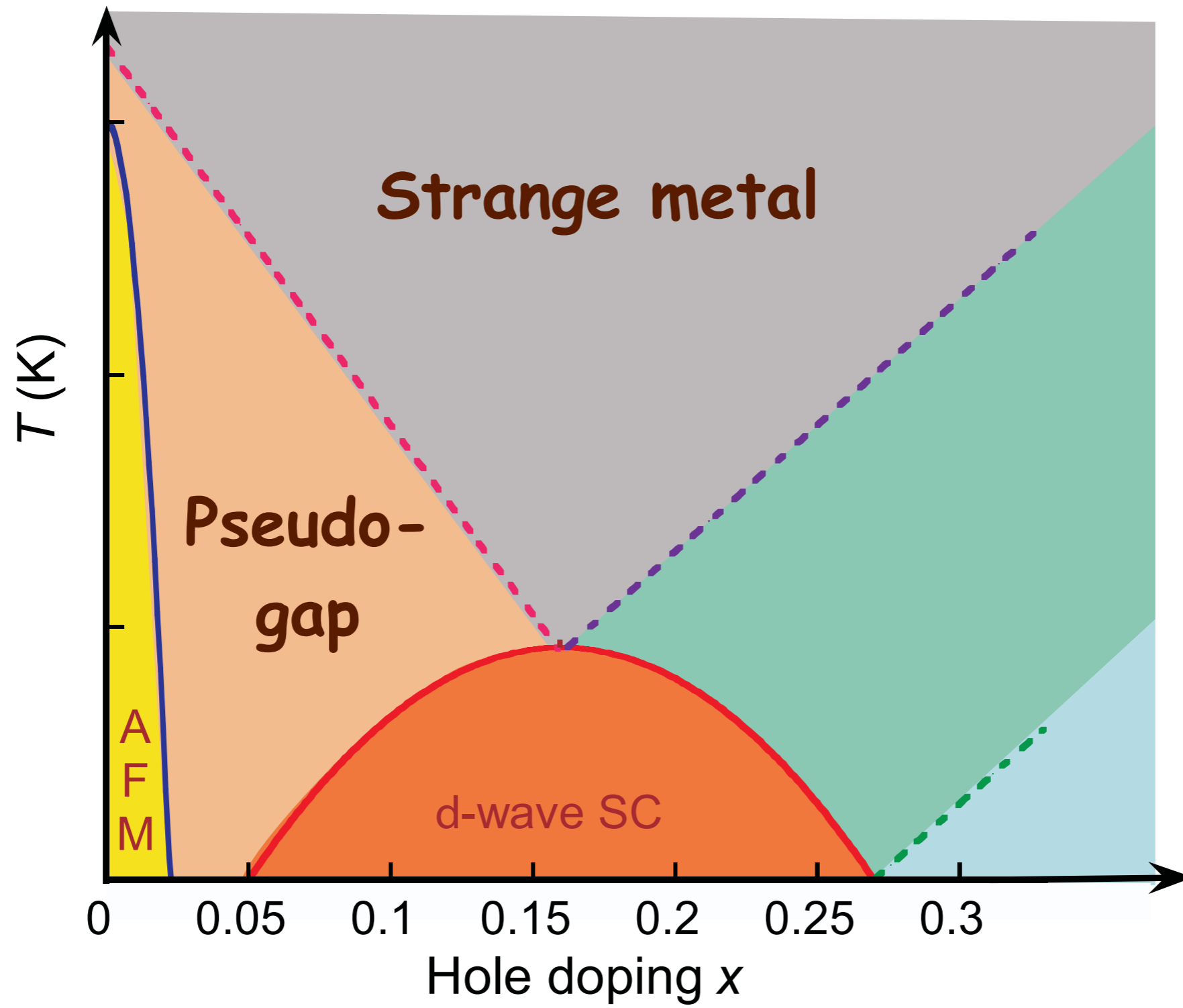
Canonical quantum critical phase diagram of coupled-dimer antiferromagnet



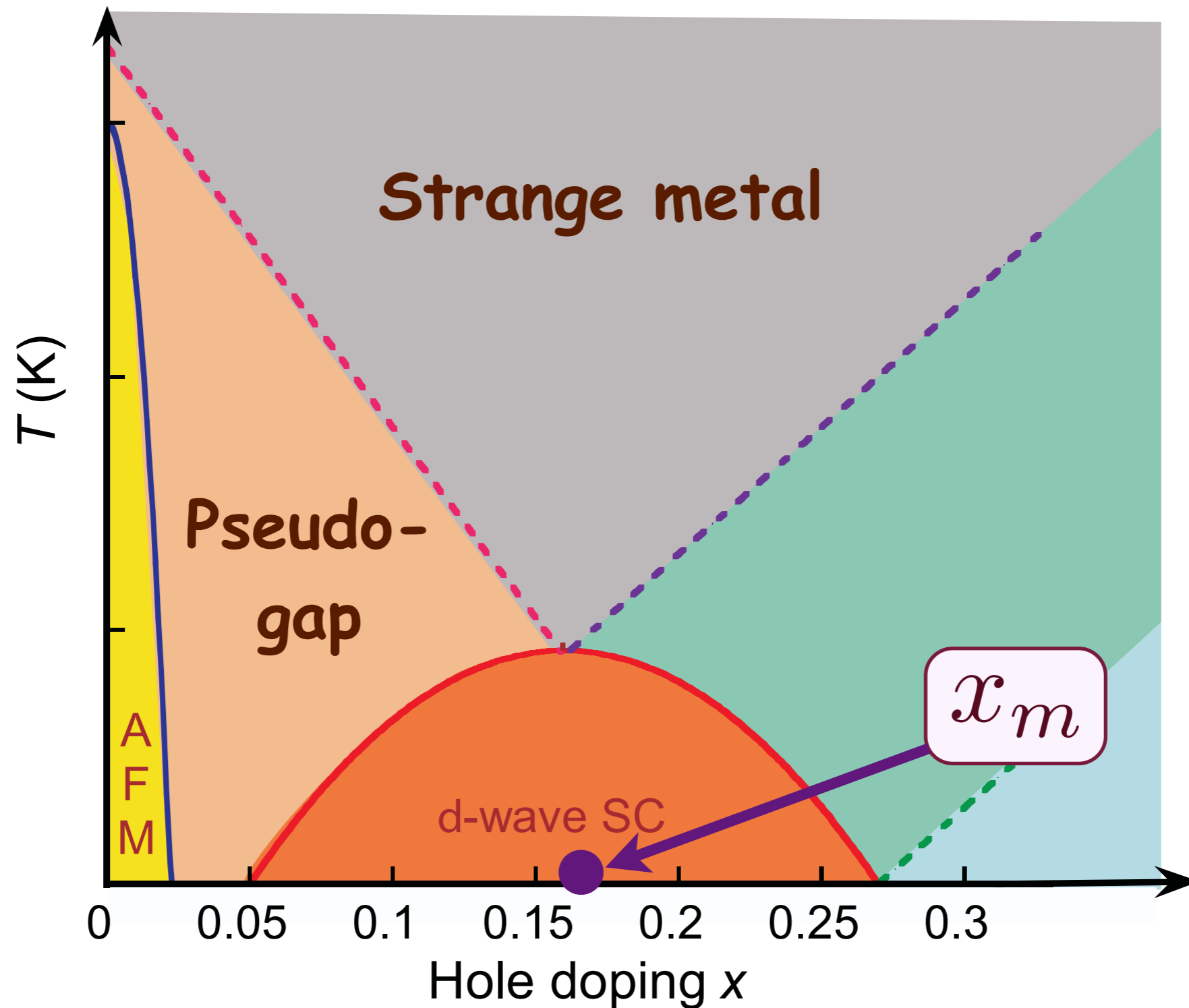
S. Sachdev and
J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).

Christian Rugg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates



Crossovers in transport properties of hole-doped cuprates



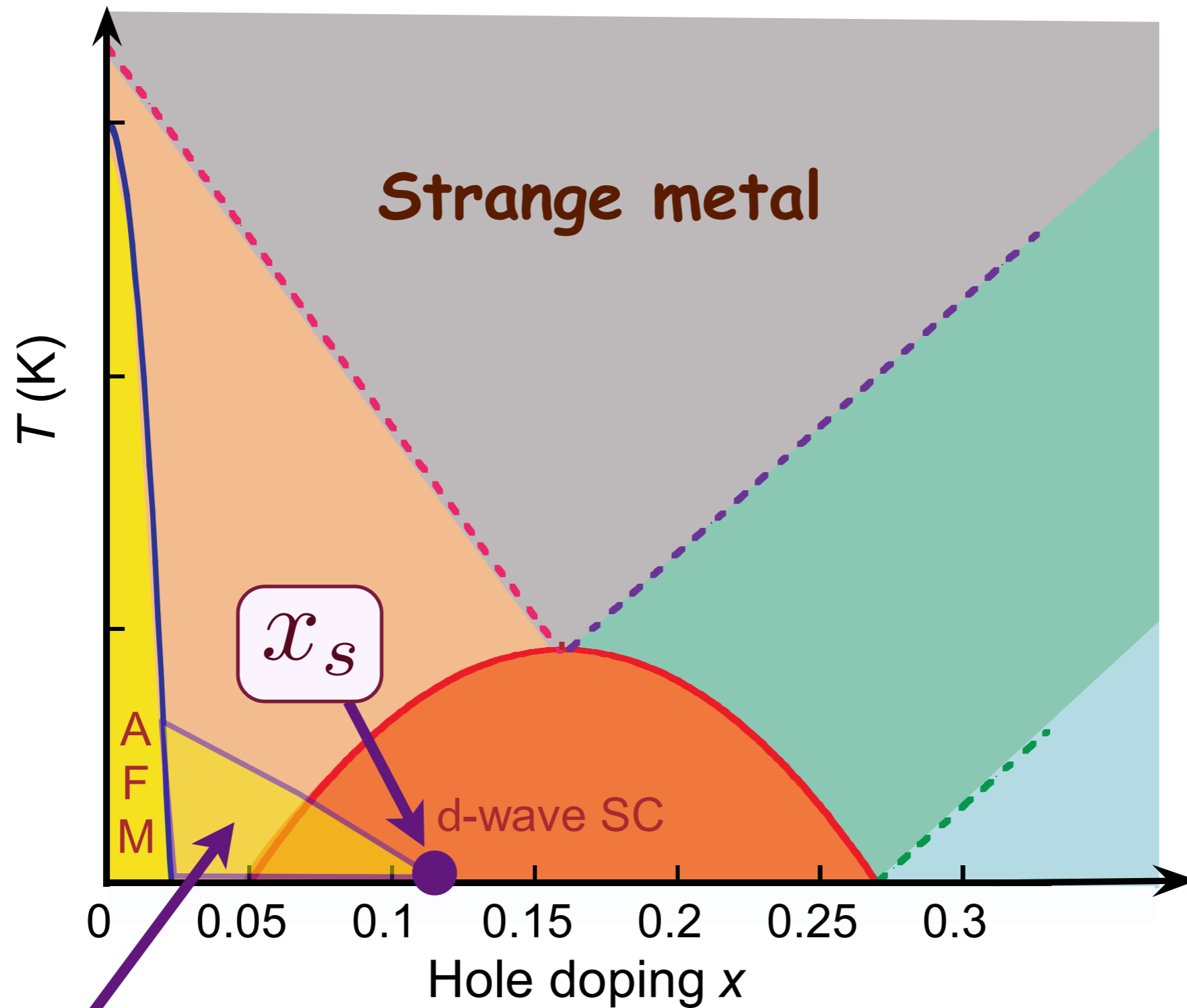
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev. B* **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

**Fermi
surface**

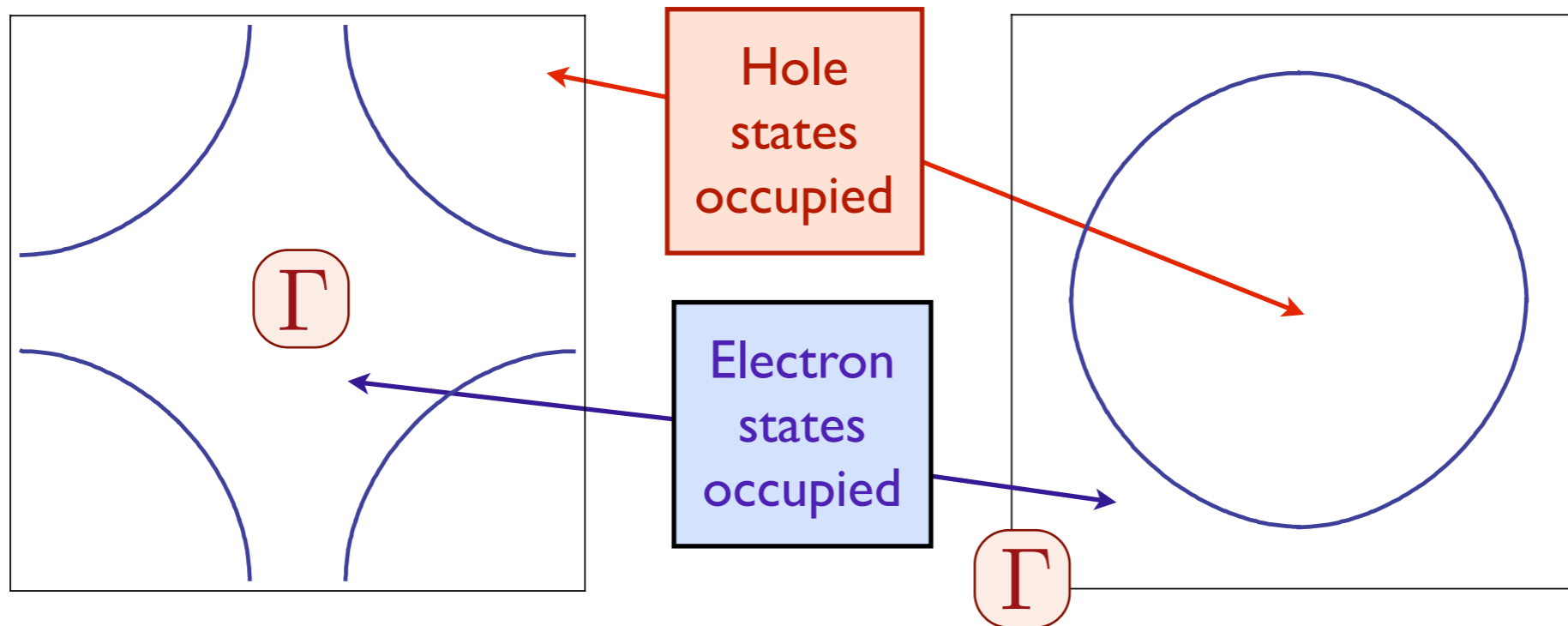


**Antiferro-
magnetism**

**d-wave
supercon-
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**Fermi
surface**

“Large” Fermi surfaces in cuprates



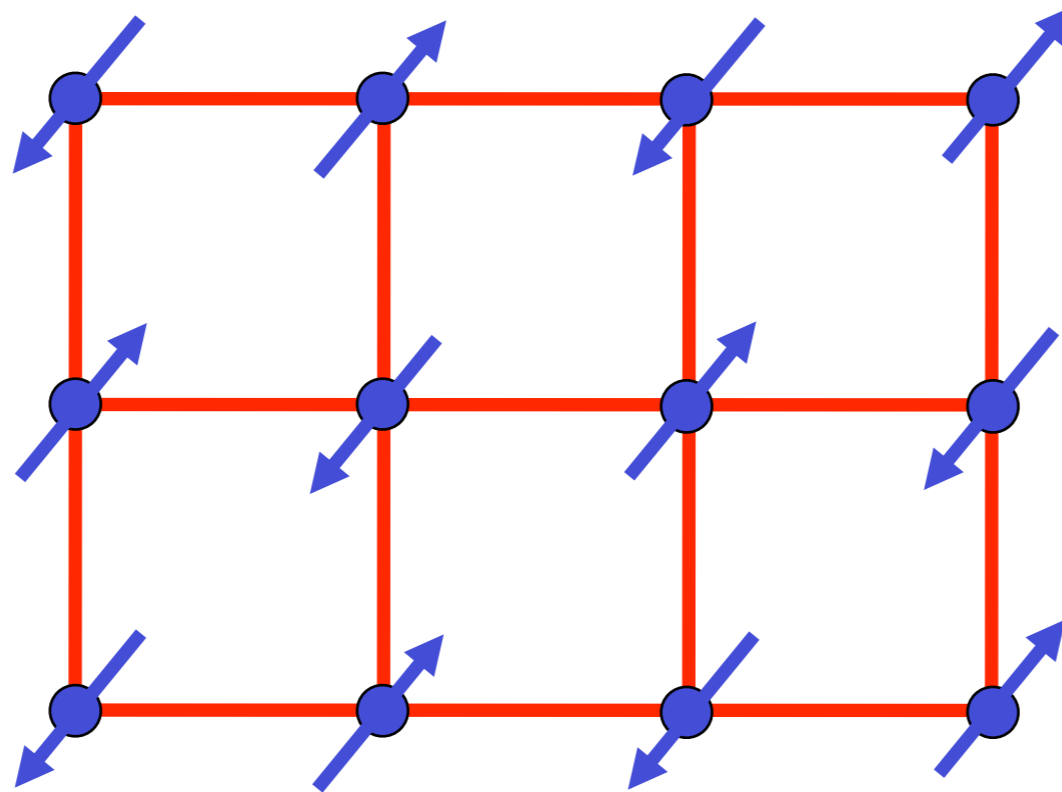
$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$A_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$A_h = 4\pi^2 - A_e$$

Spin density wave theory

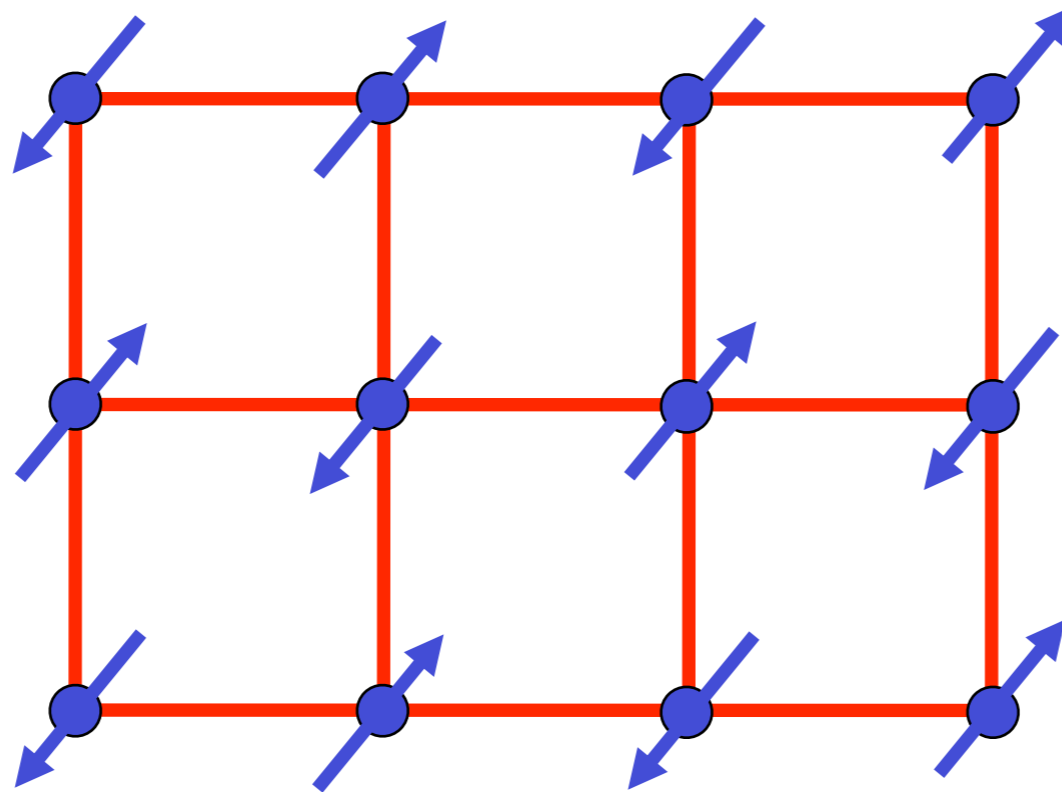


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and \mathbf{K} is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



Spin density wave Hamiltonian

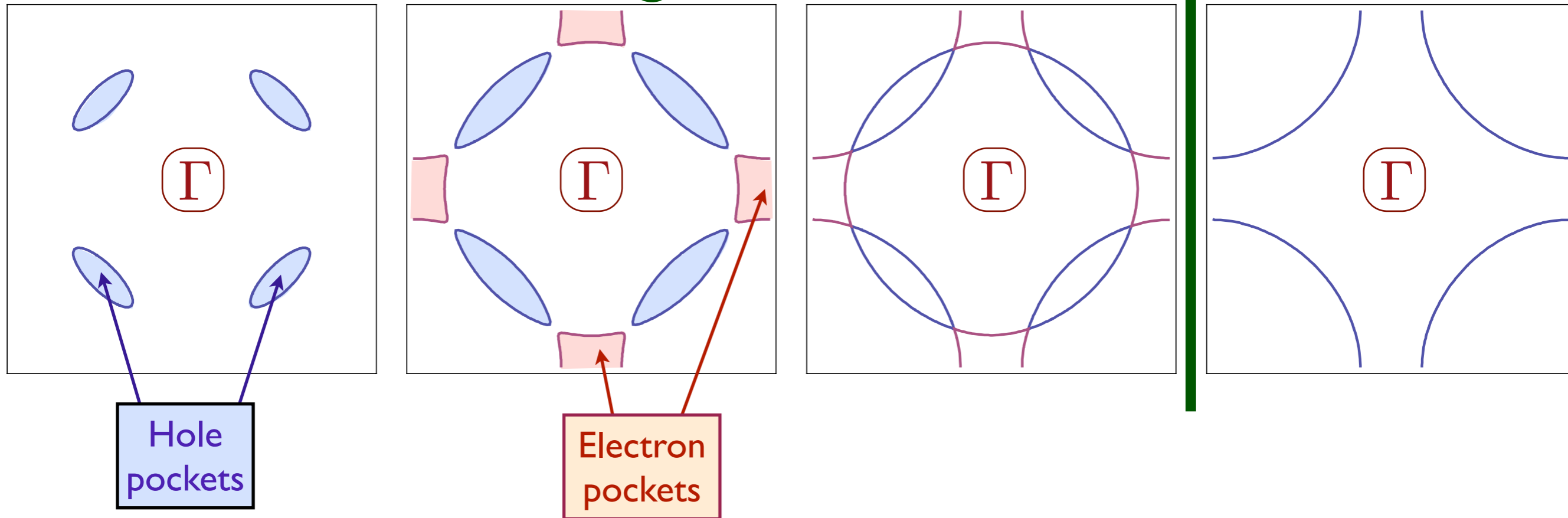
$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

Diagonalize $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \varphi^2}$$

Hole-doped cuprates

← Increasing SDW order →

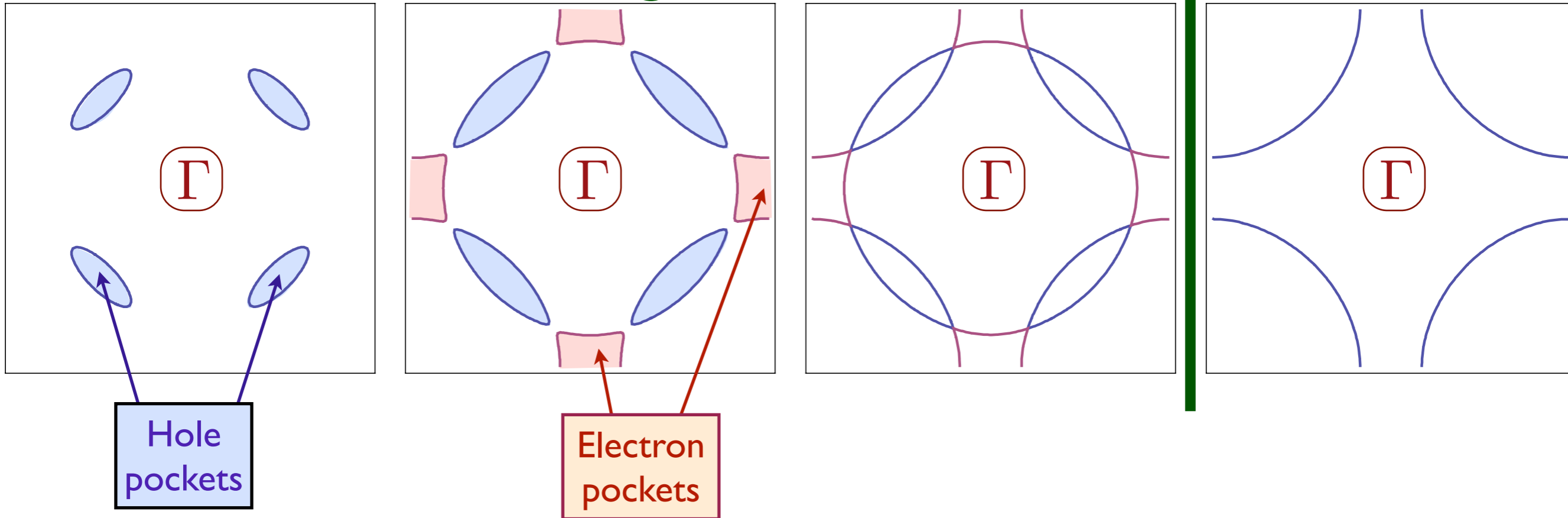


Large Fermi surface breaks up into
electron and hole pockets

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates

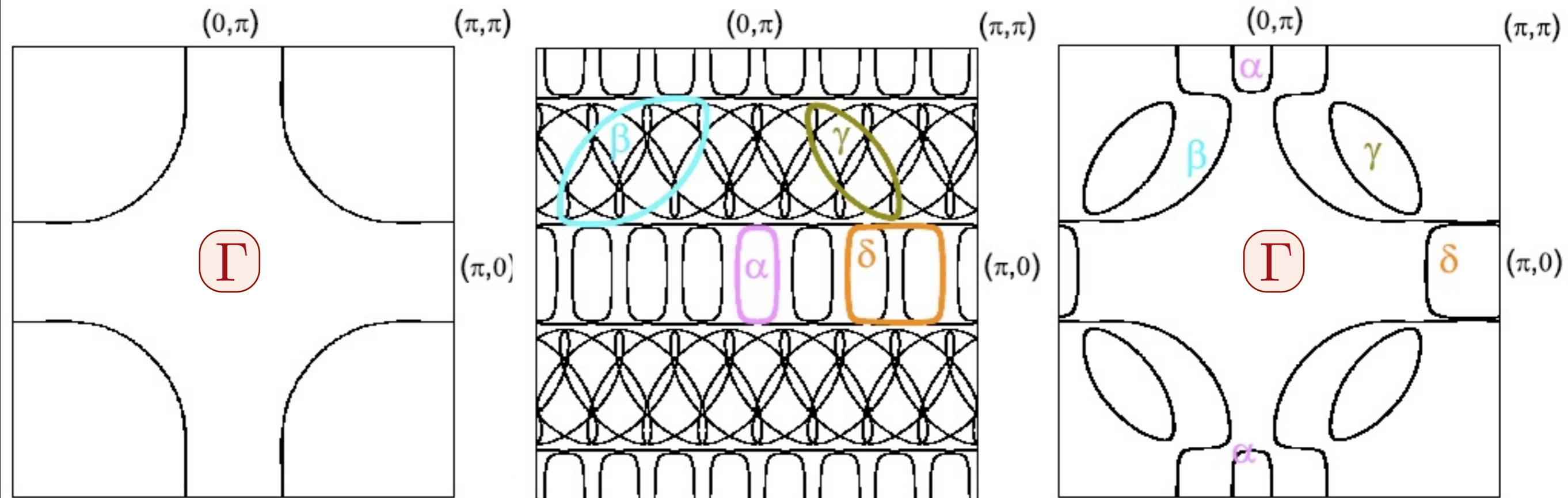
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Spin density wave theory in hole-doped cuprates



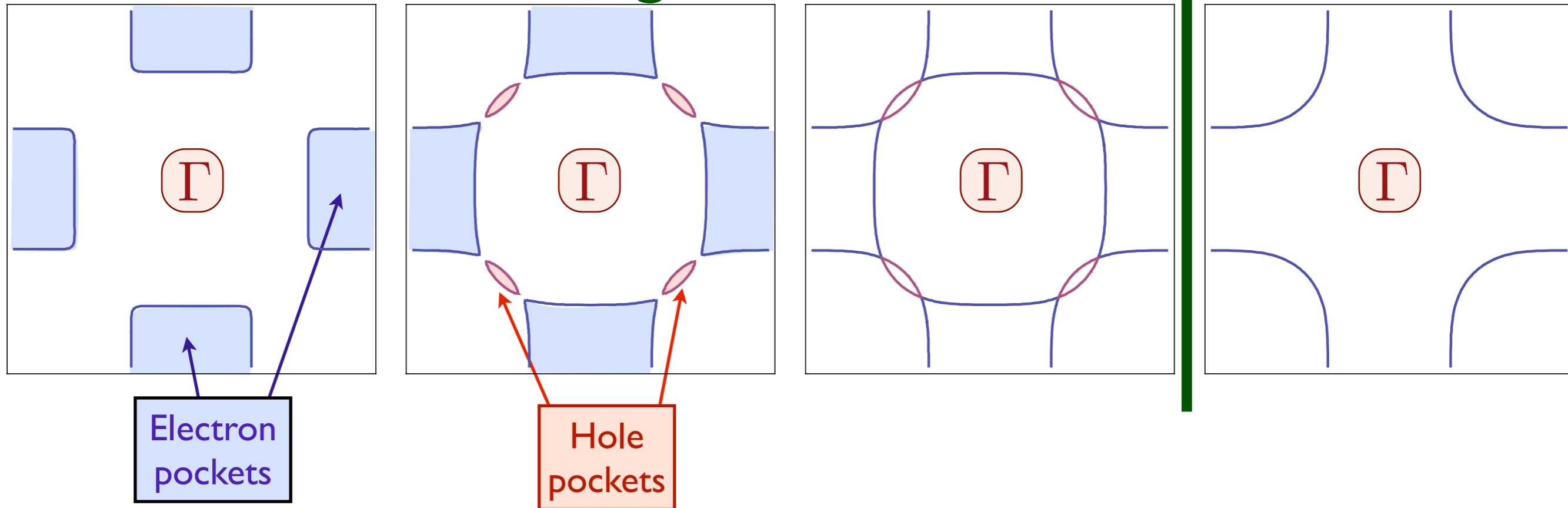
Incommensurate order in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).

N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Electron-doped cuprates

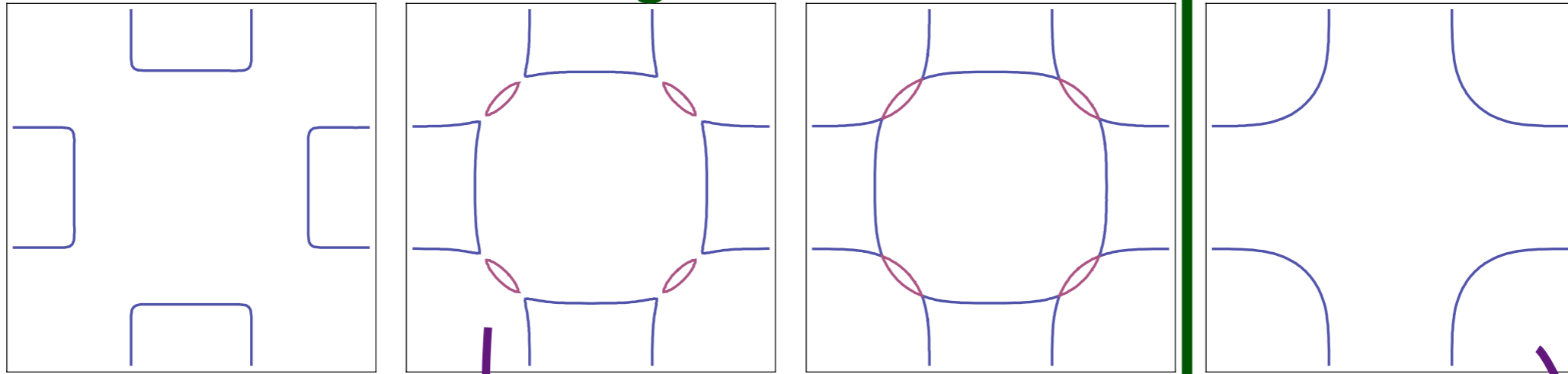
← Increasing SDW order →



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electron and hole pockets

D. Senechal and A.-M. S. Tremblay, *Physical Review Letters* **92**, 126401 (2004)
J. Lin, and A. J. Millis, *Physical Review B* **72**, 214506 (2005).

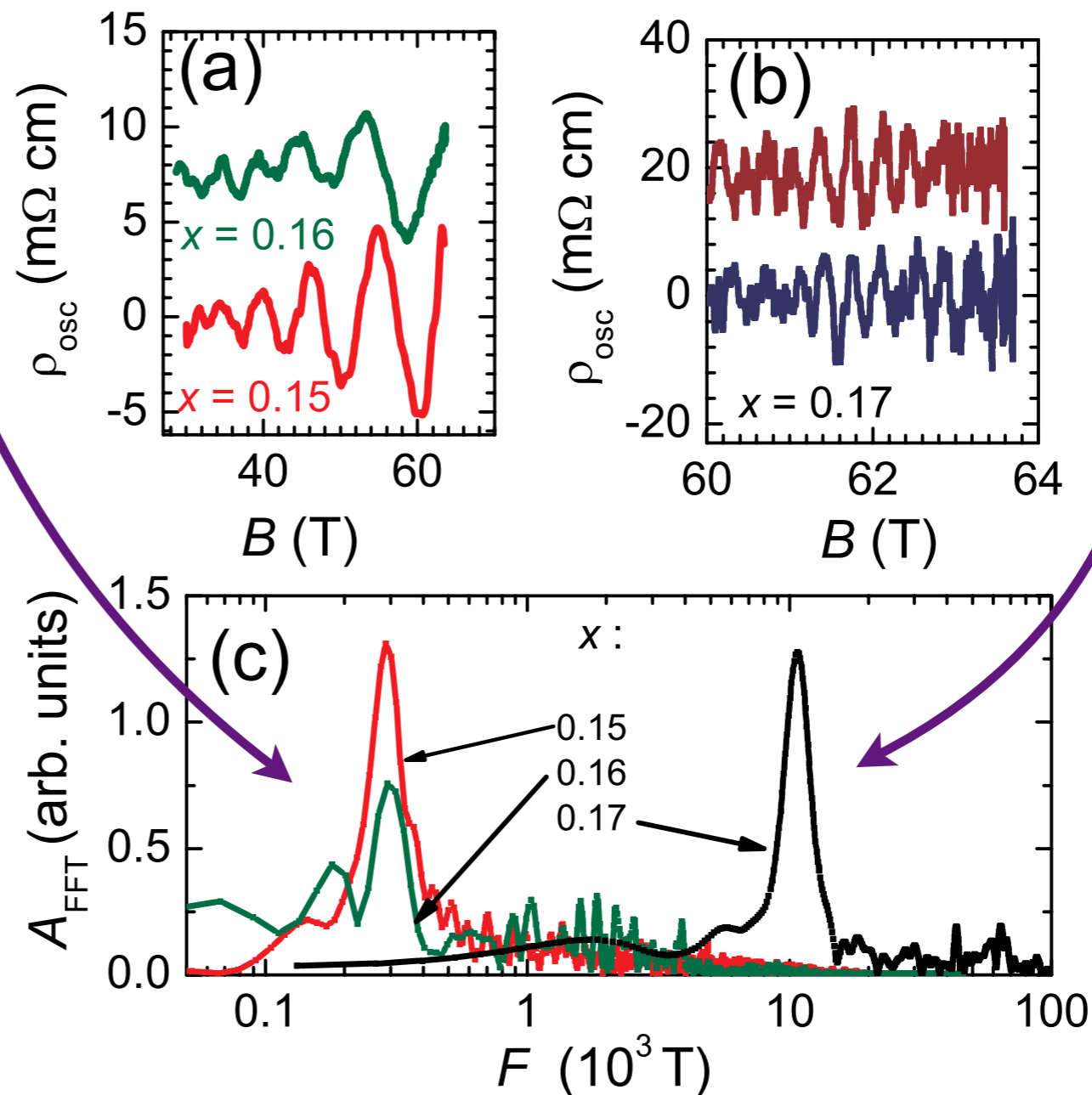
← Increasing SDW order →



Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

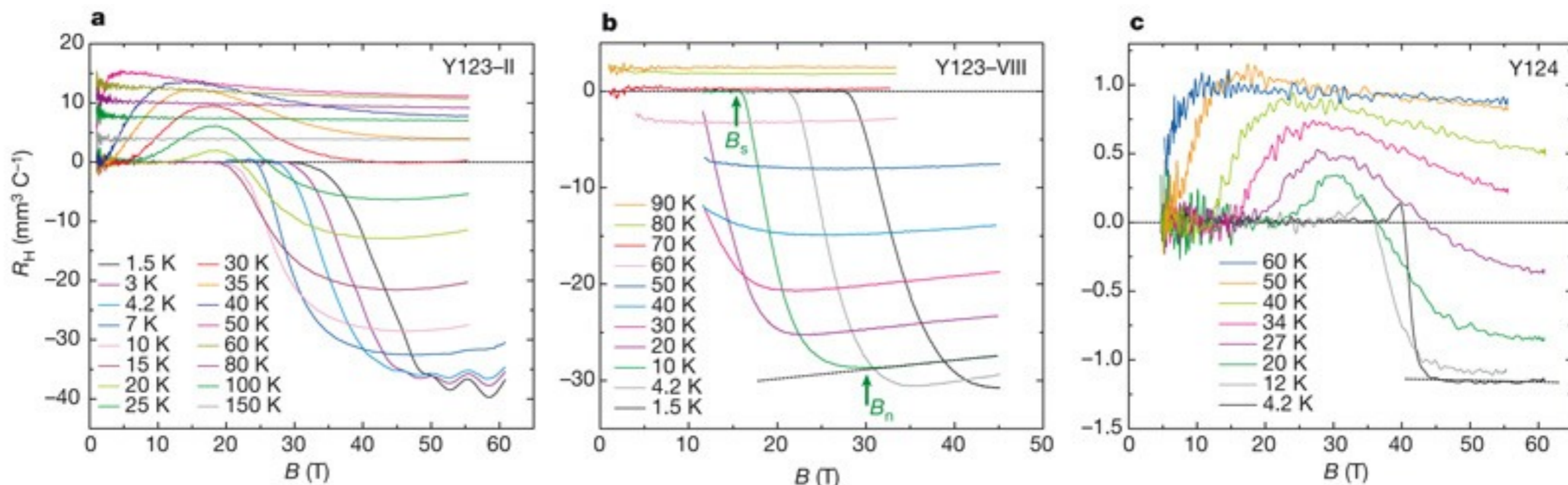


Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high- T_c superconductors

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Nature **450**, 533 (2007)

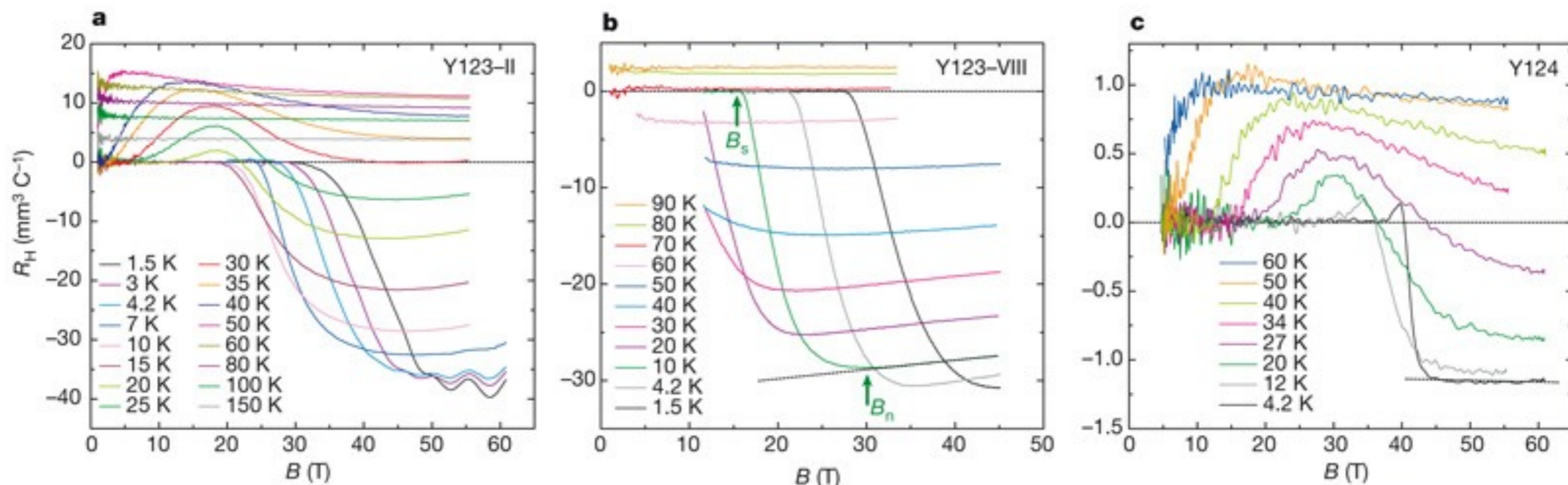


Quantum oscillations

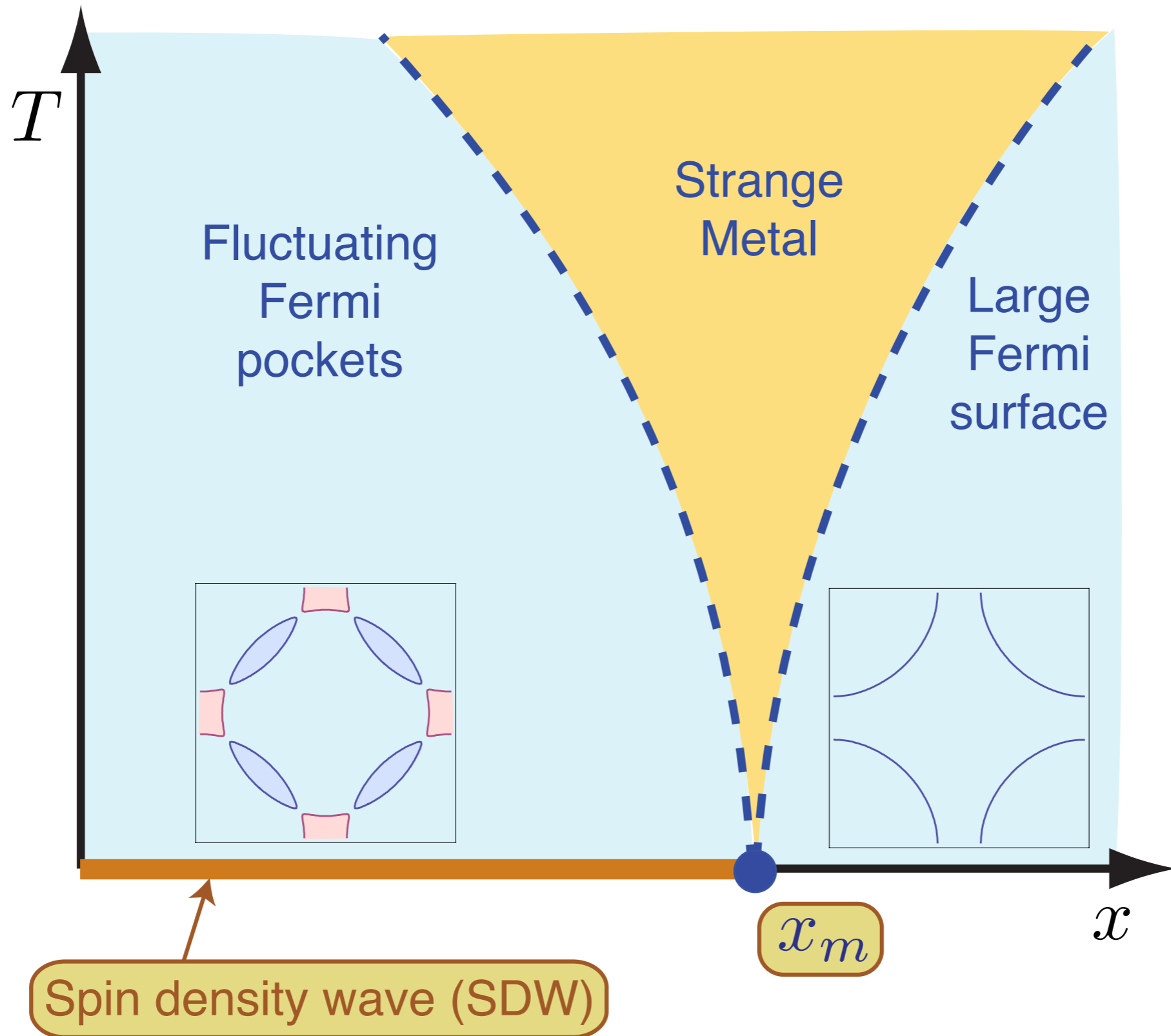
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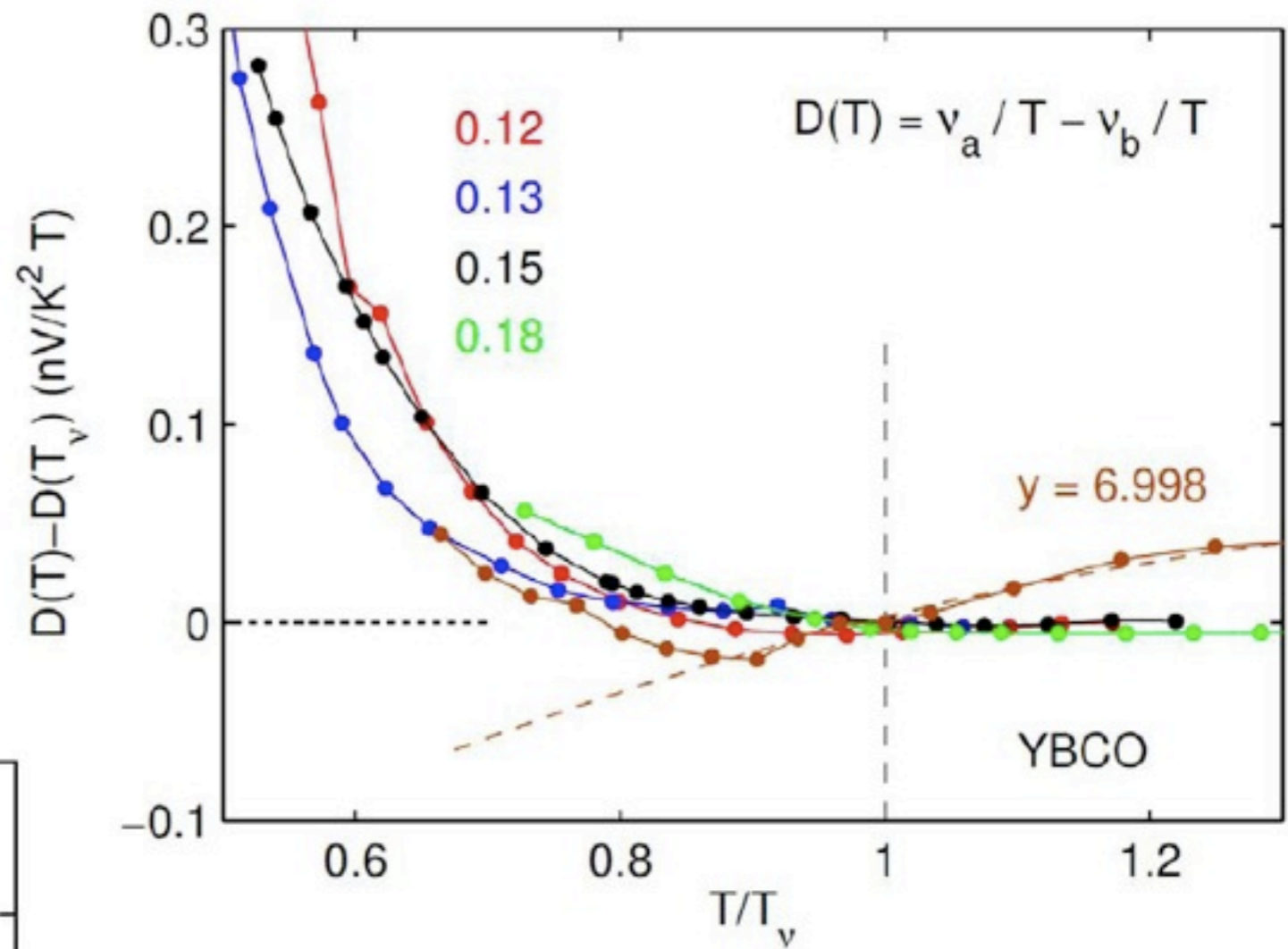
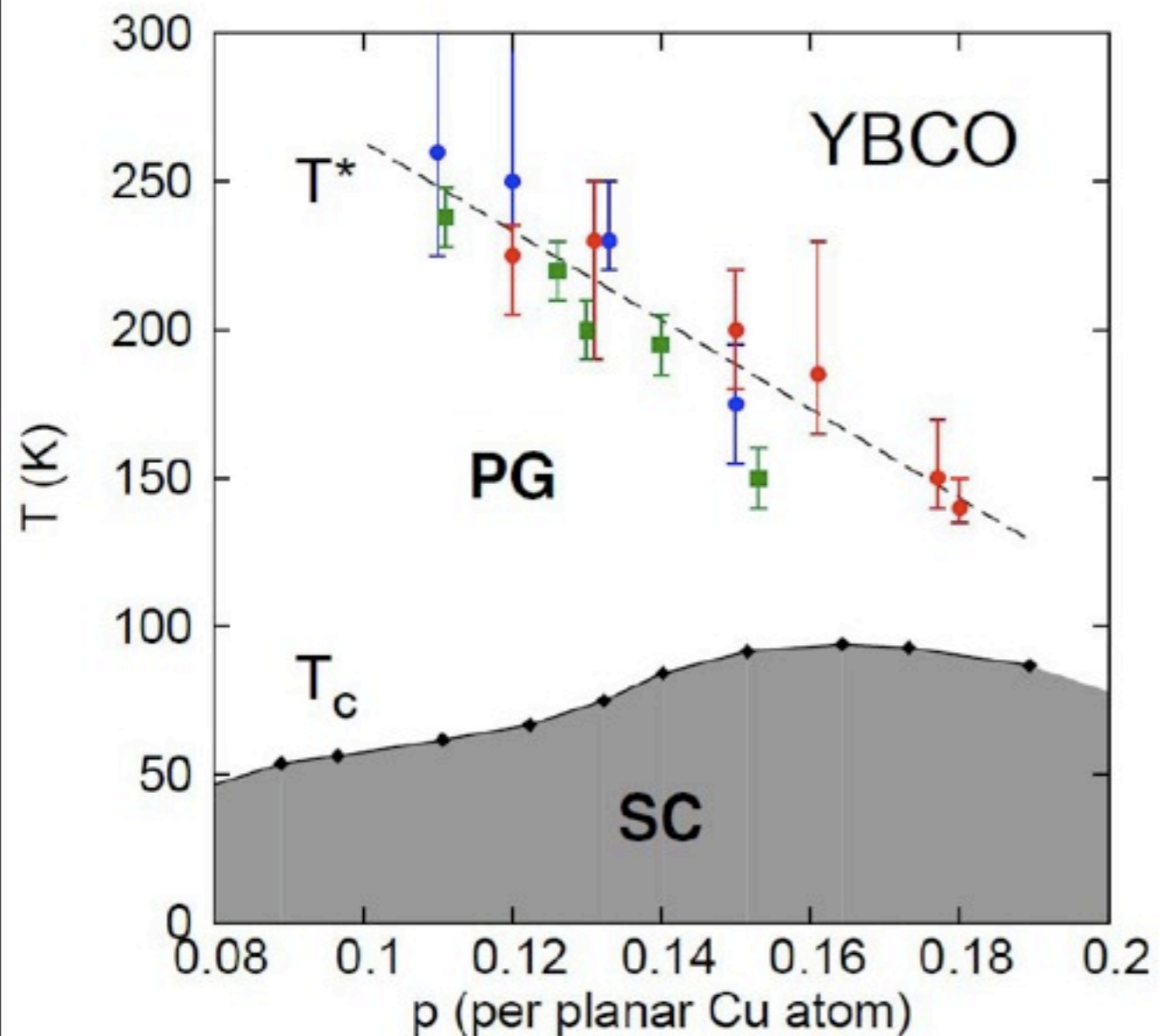
Theory of quantum criticality in the cuprates



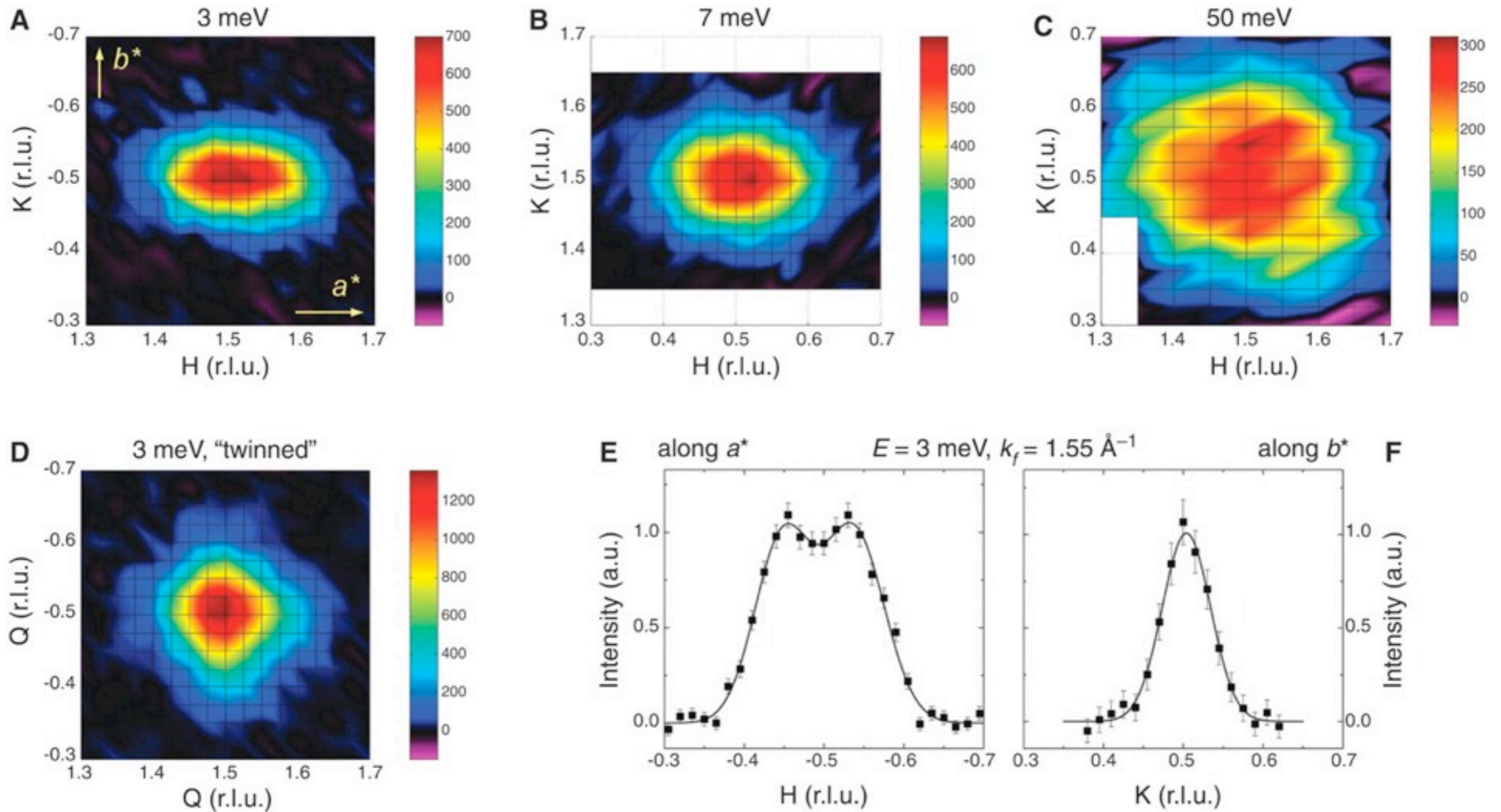
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Broken rotational symmetry in the pseudogap phase of a high- T_c superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430



S.A. Kivelson, E. Fradkin, and V.J. Emery, *Nature* **393**, 550 (1998).



Nematic order in YBCO

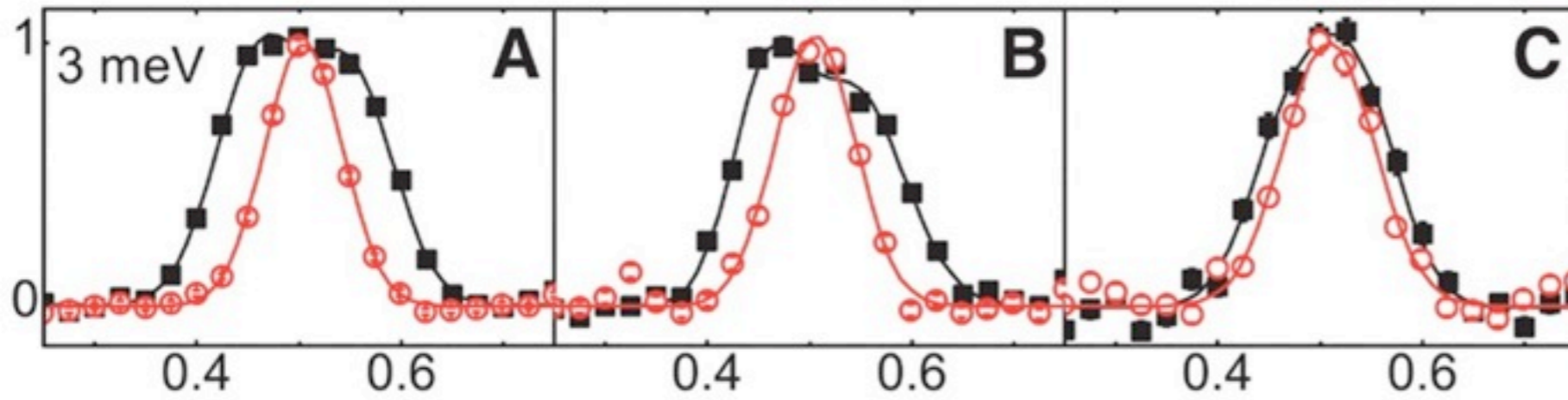
V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer, *Science* **319**, 597 (2008)

■ along a^* ○ along b^*

5 K

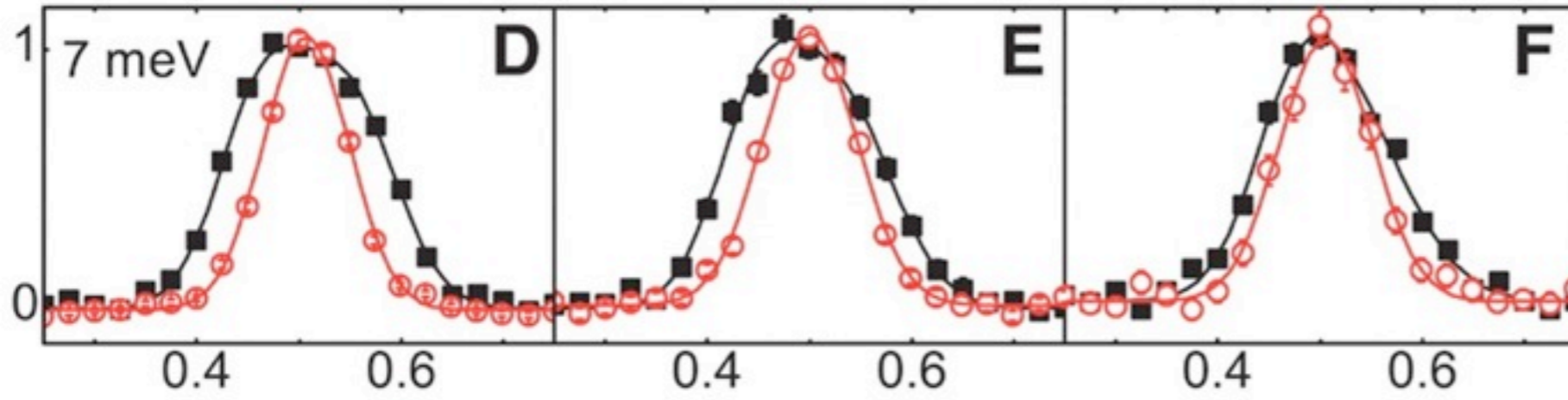
40 K

100 K

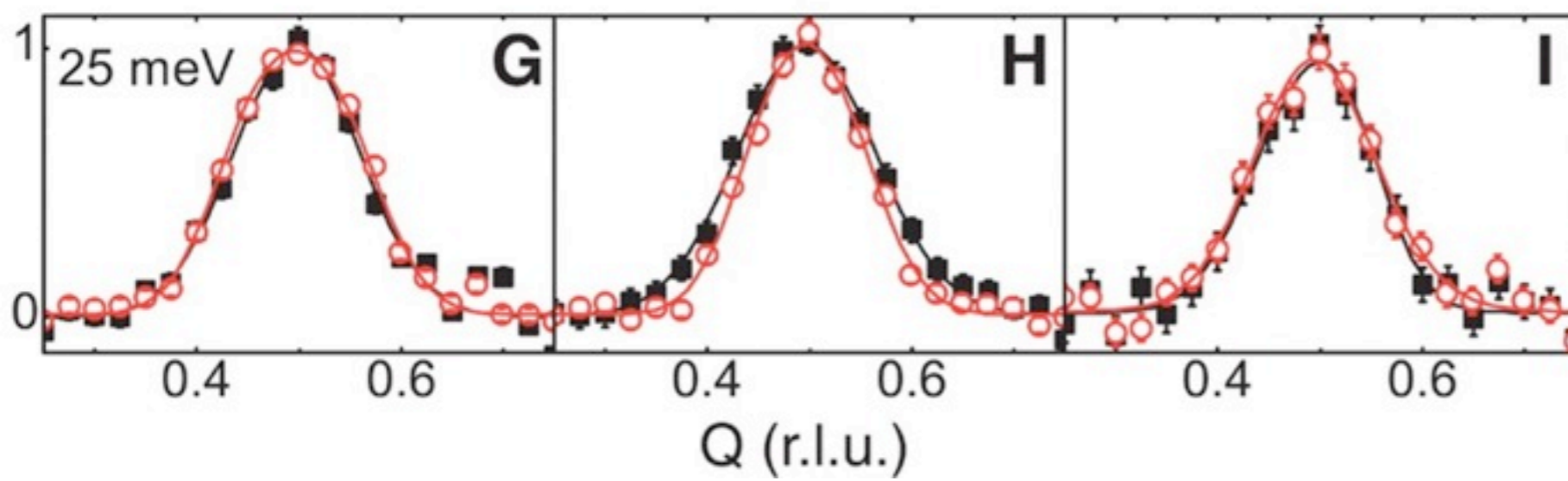


Intensity (a.u.)

7 meV



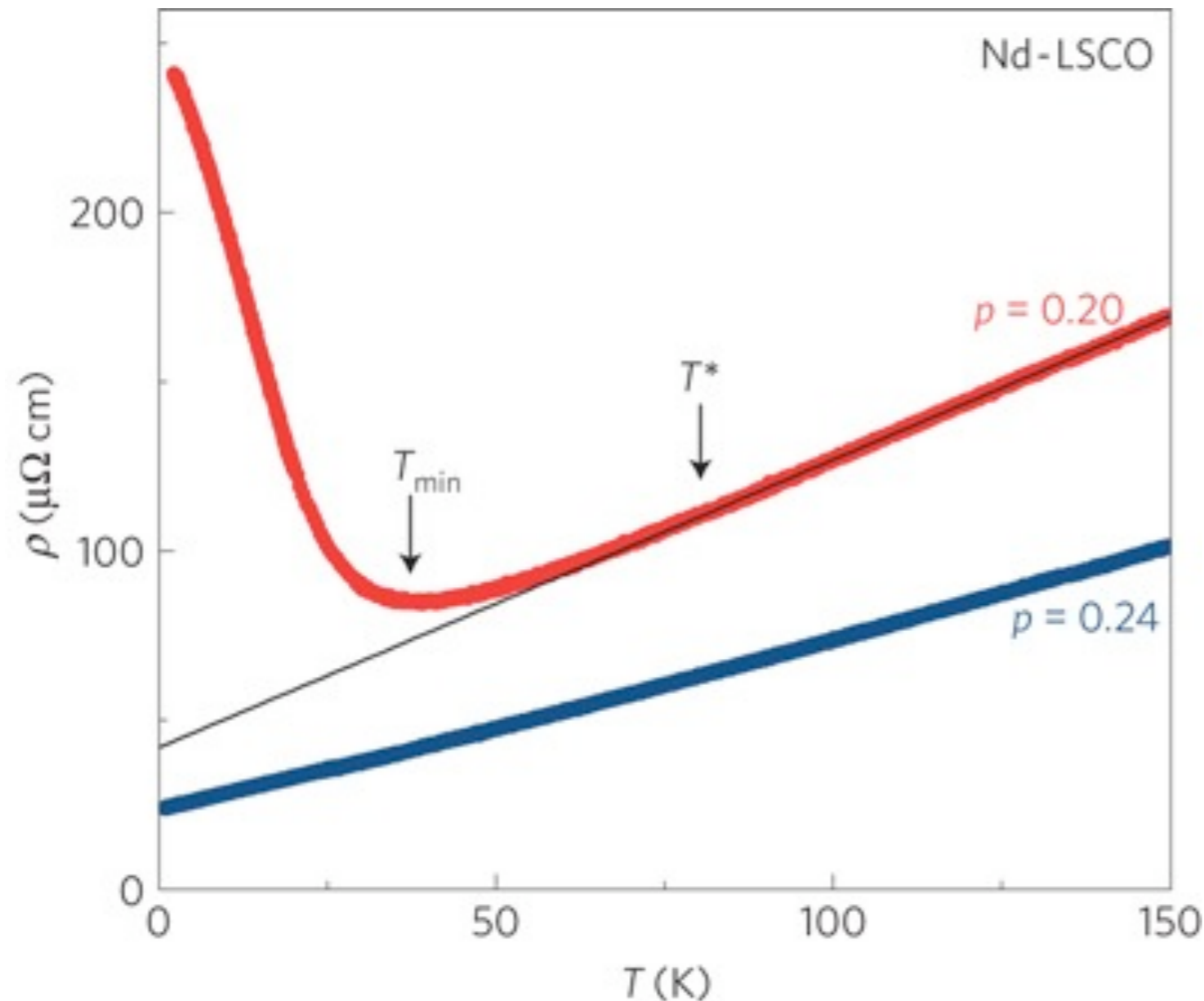
25 meV



V. Hinkov, D. Haug,
B. Fauqué, P. Bourges,
Y. Sidis, A. Ivanov,
C. Bernhard, C. T. Lin,
and B. Keimer ,
Science **319**, 597
(2008)

Nematic order in YBCO

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c

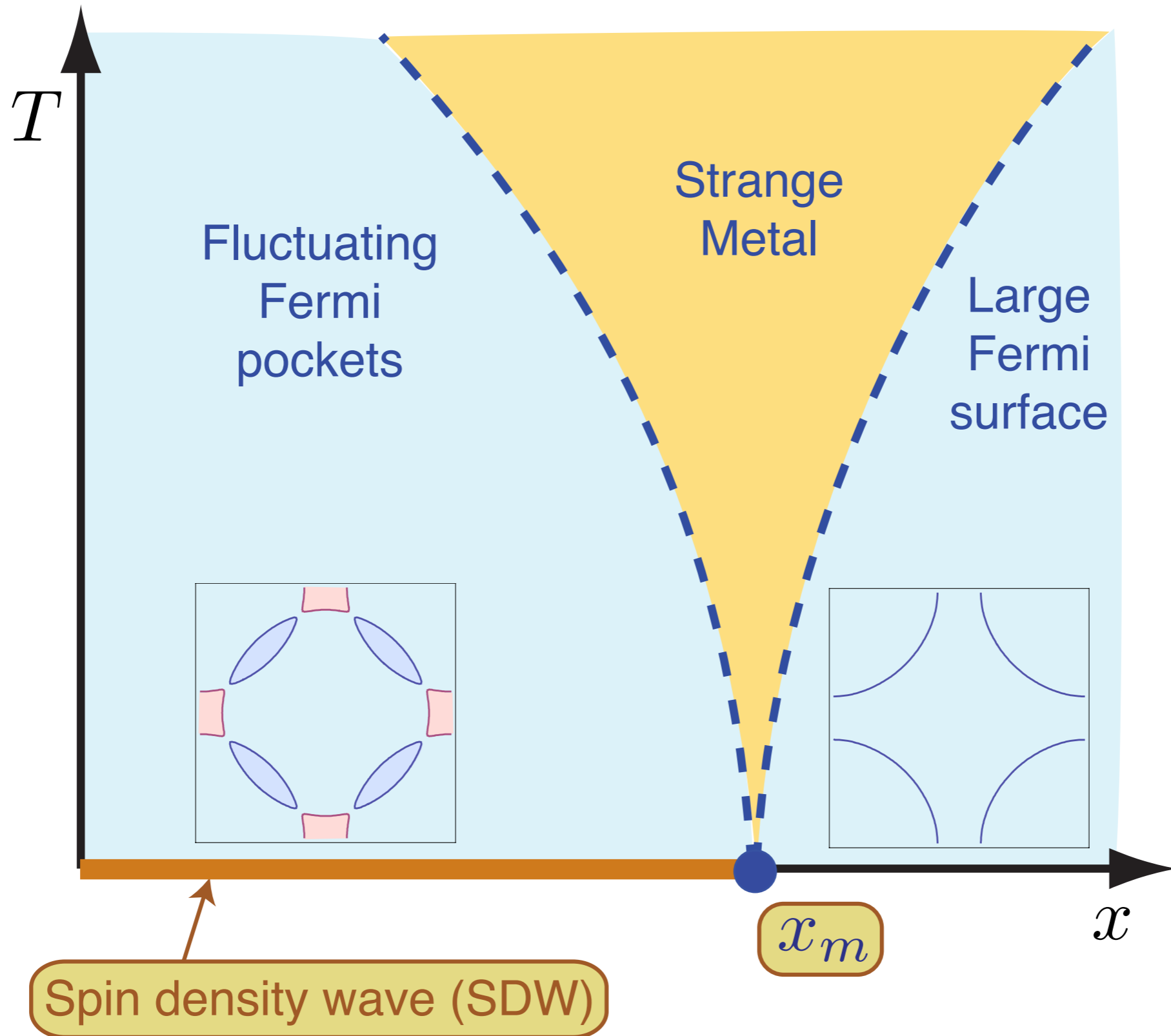


Magnetic field of
upto 35 T
used to suppress
superconductivity

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high- T_c superconductor

R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point
in metal at $x = x_m$



**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

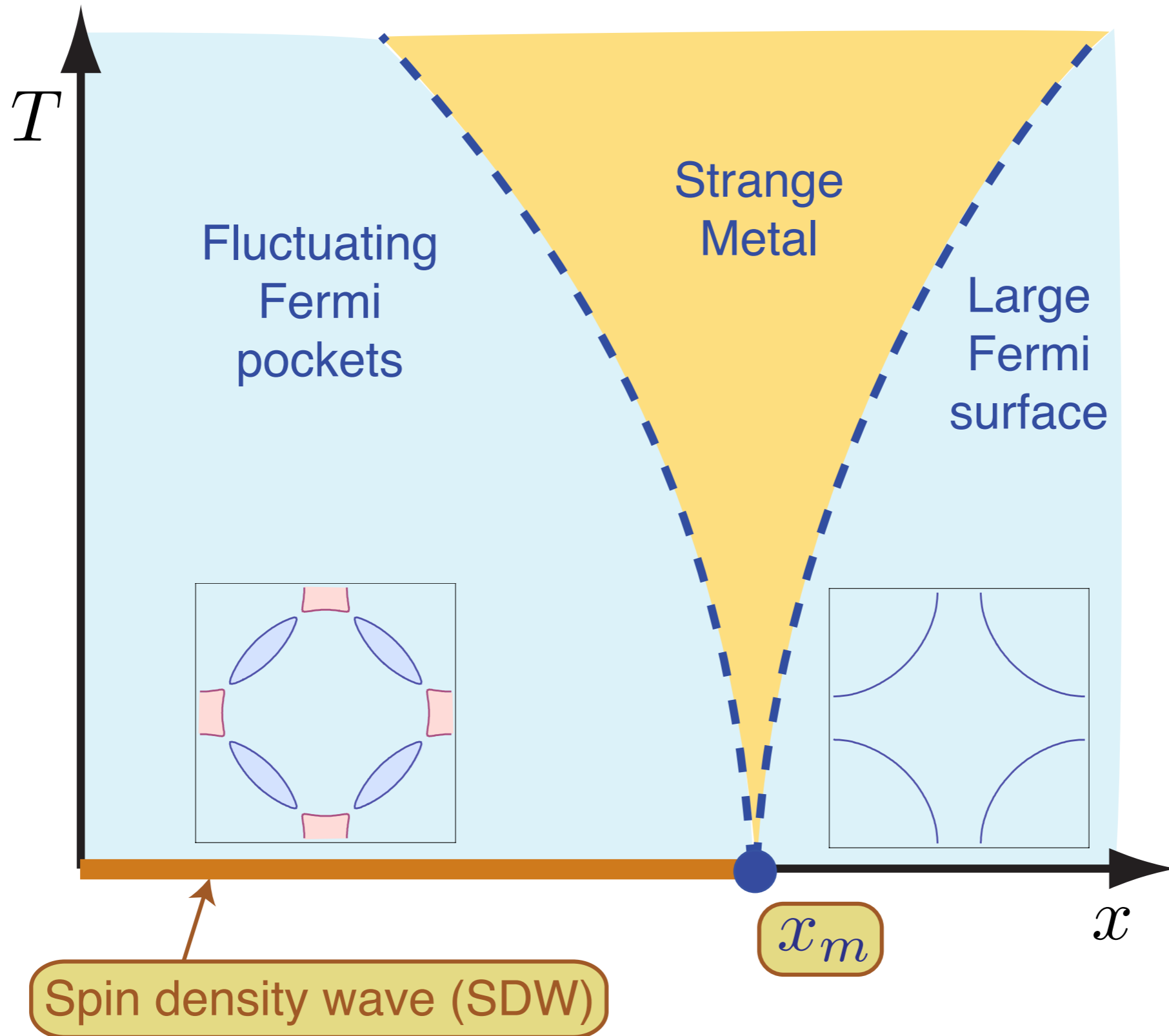
**Fermi
surface**

**Antiferro-
magnetism**

**d-wave
supercon-
ductivity**

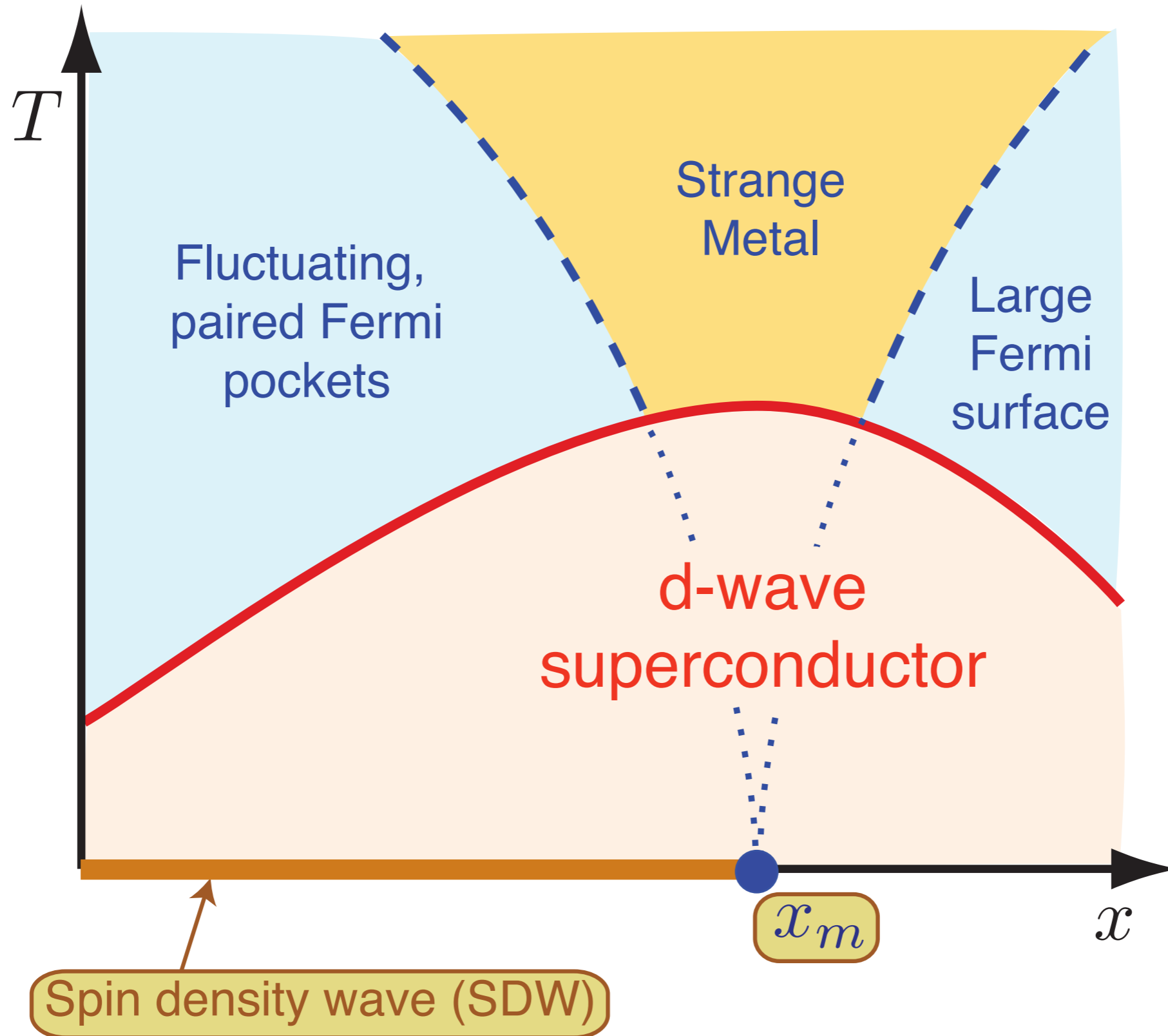
**Fermi
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Theory of quantum criticality in the cuprates



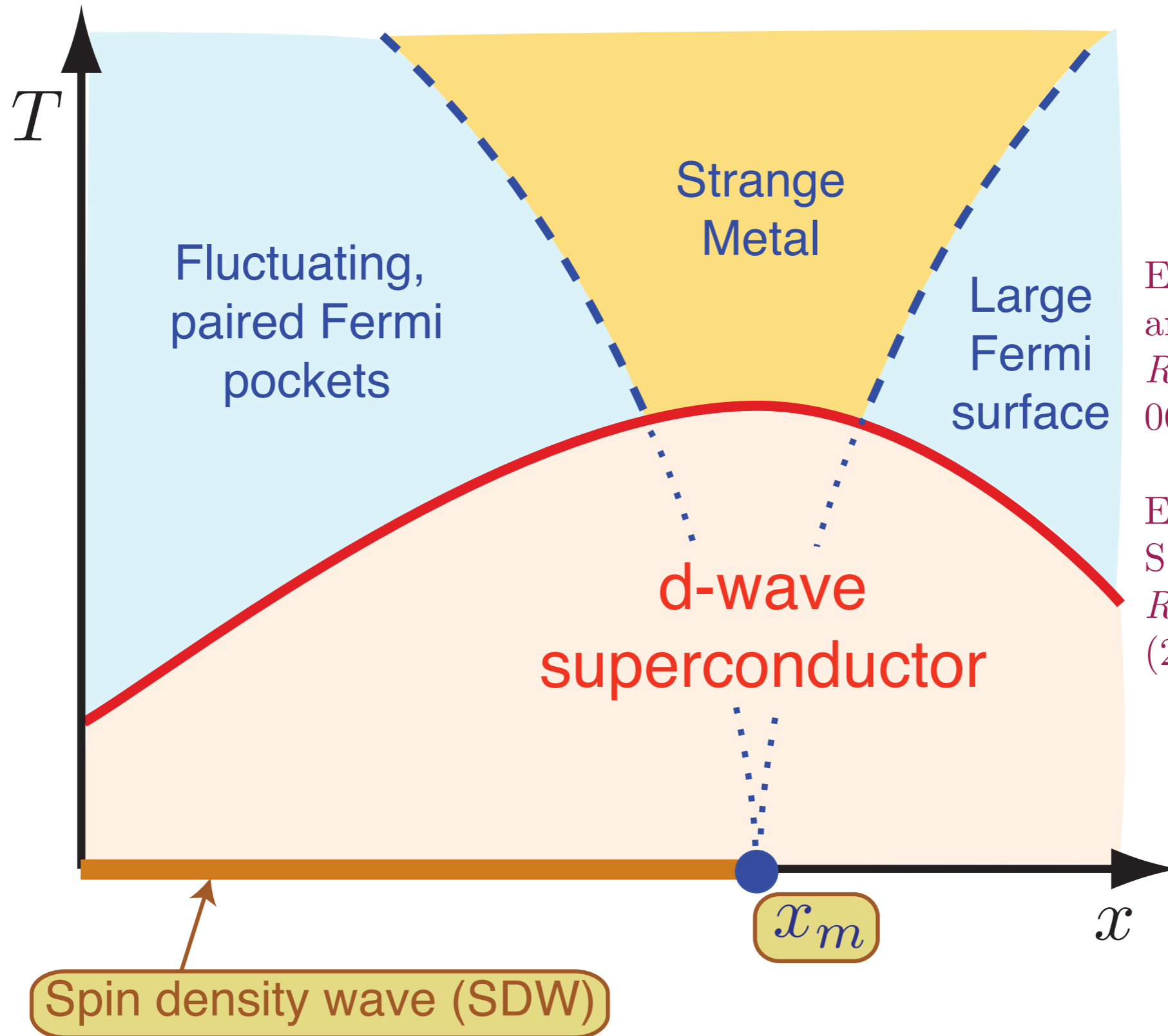
Underlying SDW ordering quantum critical point
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Theory of quantum criticality in the cuprates



Onset of d -wave superconductivity
hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates

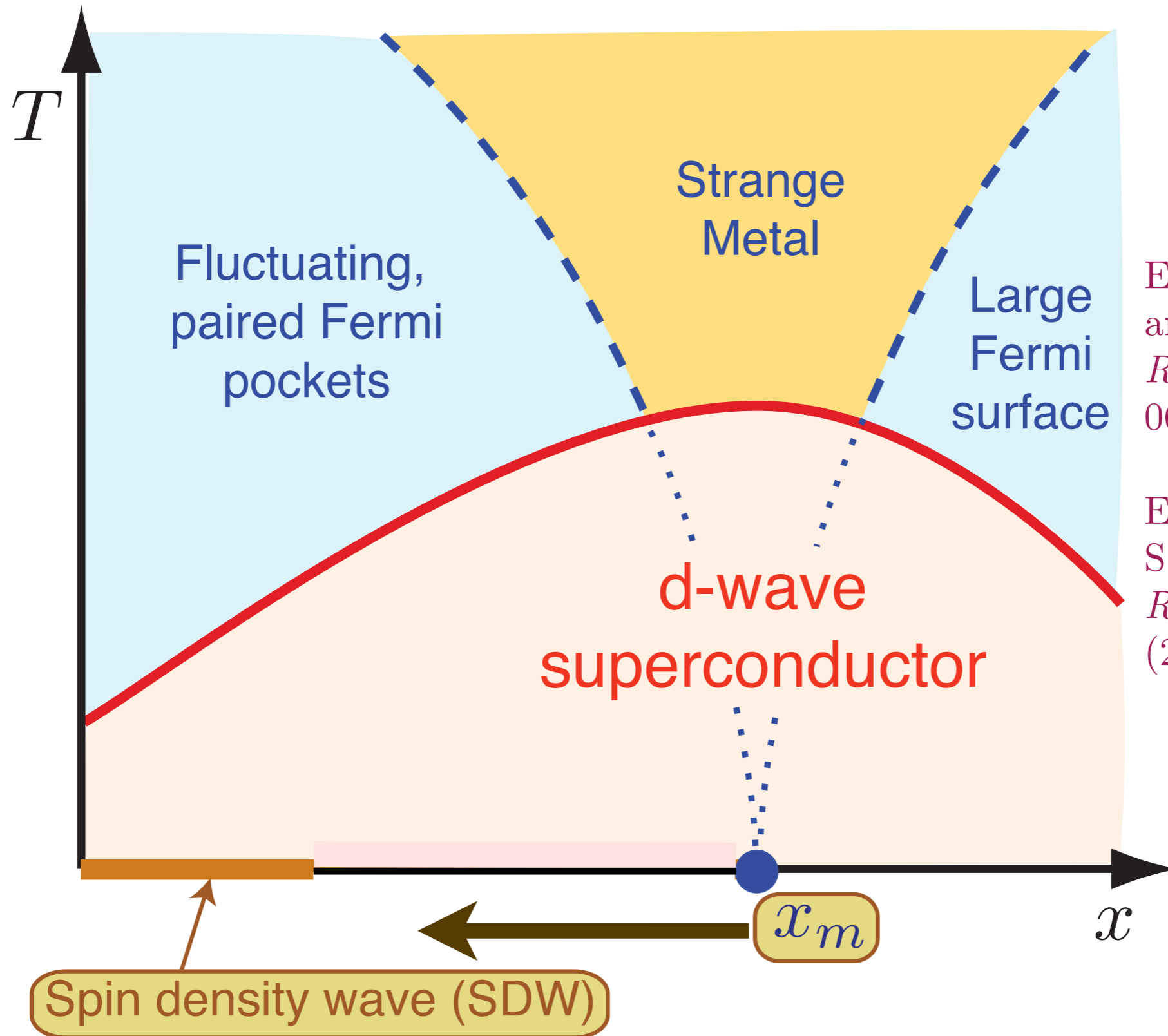


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

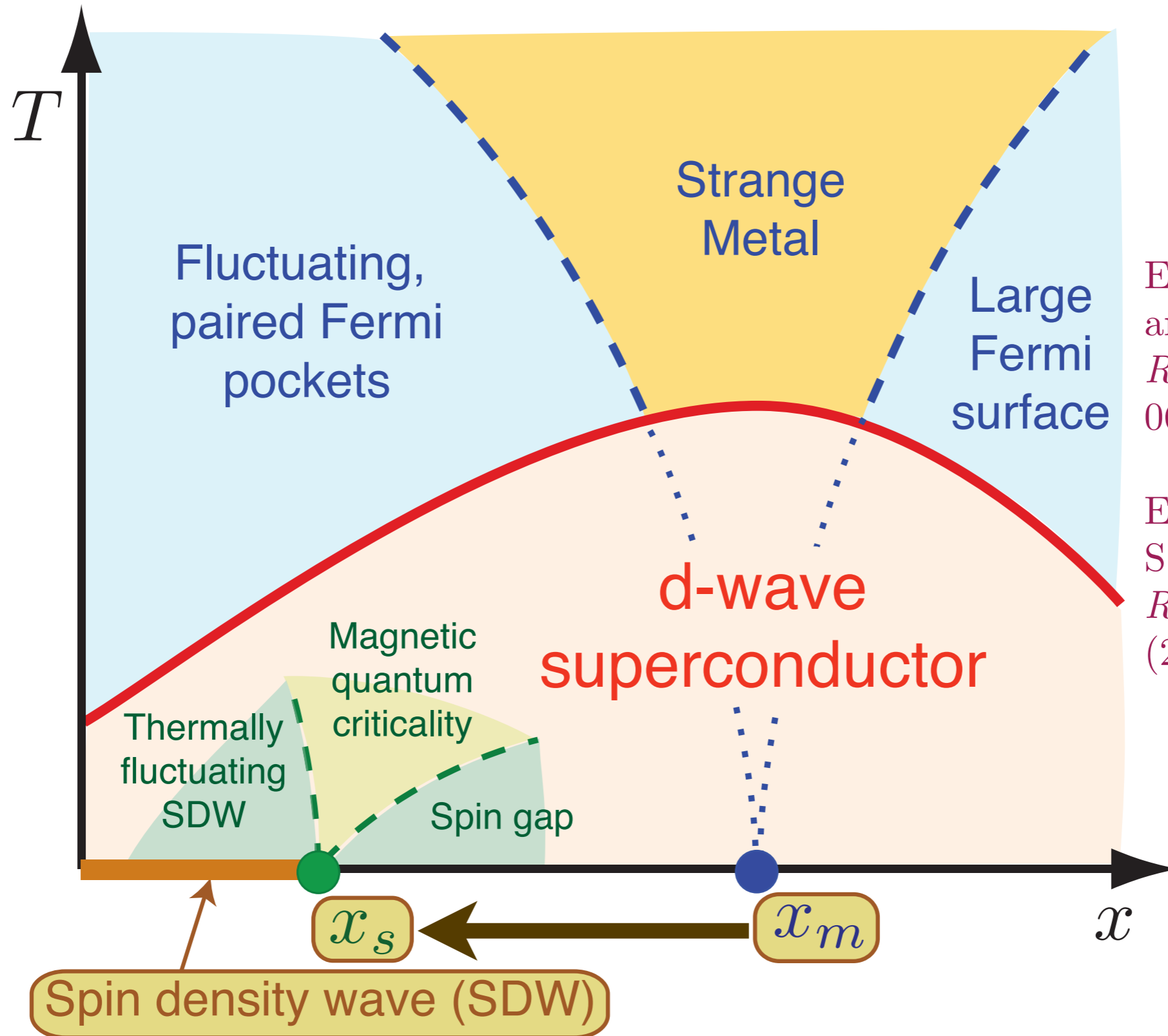


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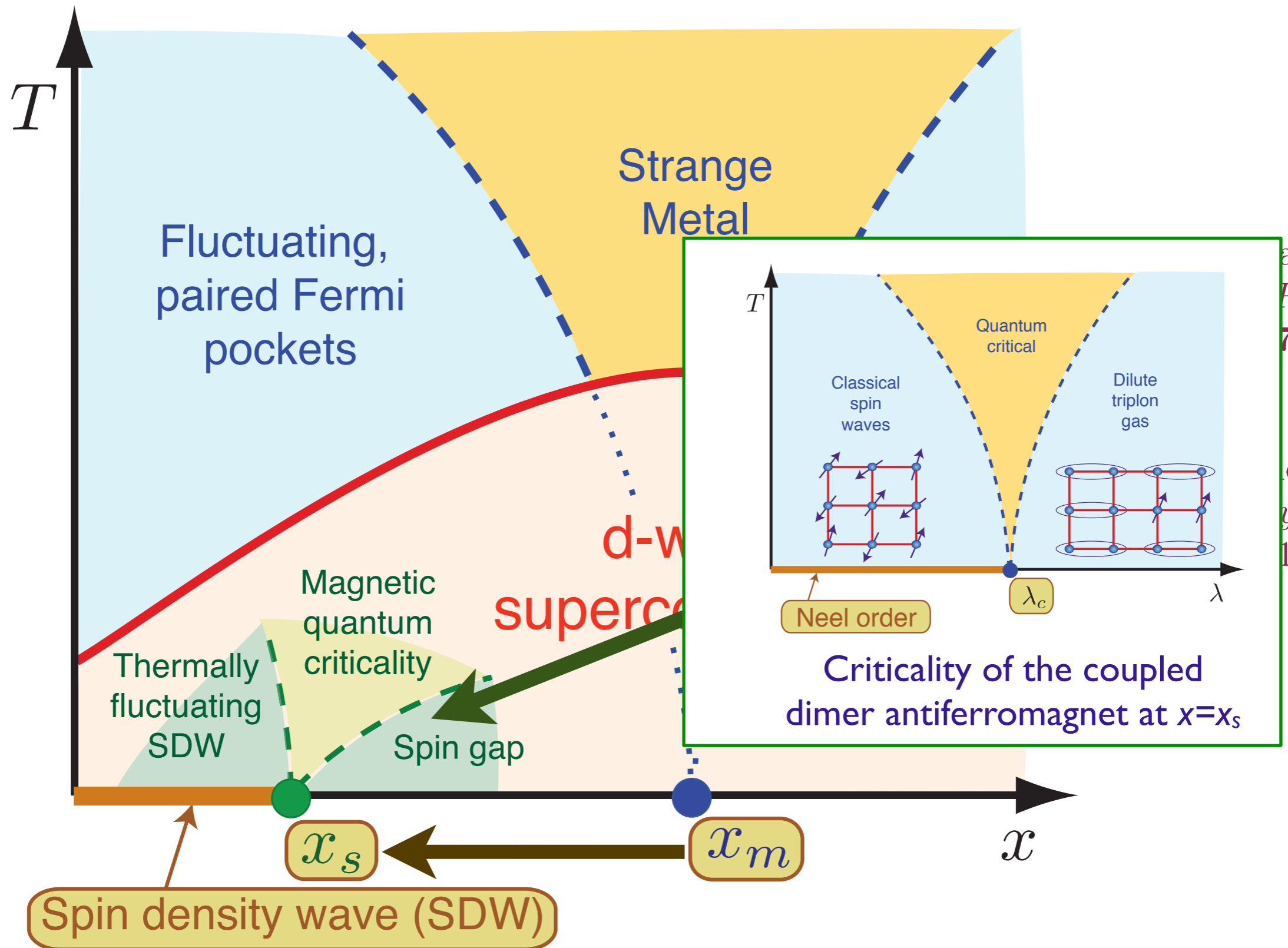


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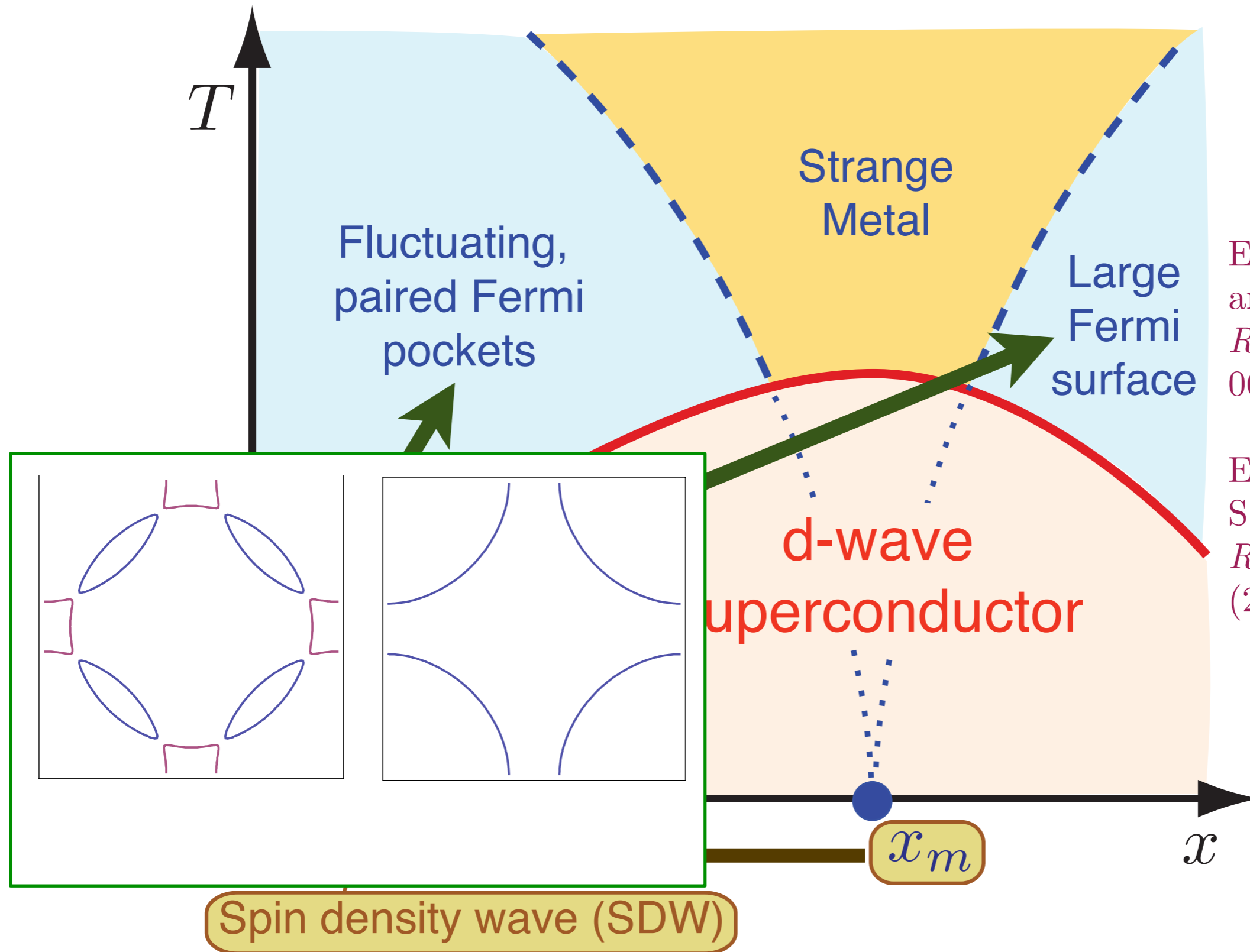
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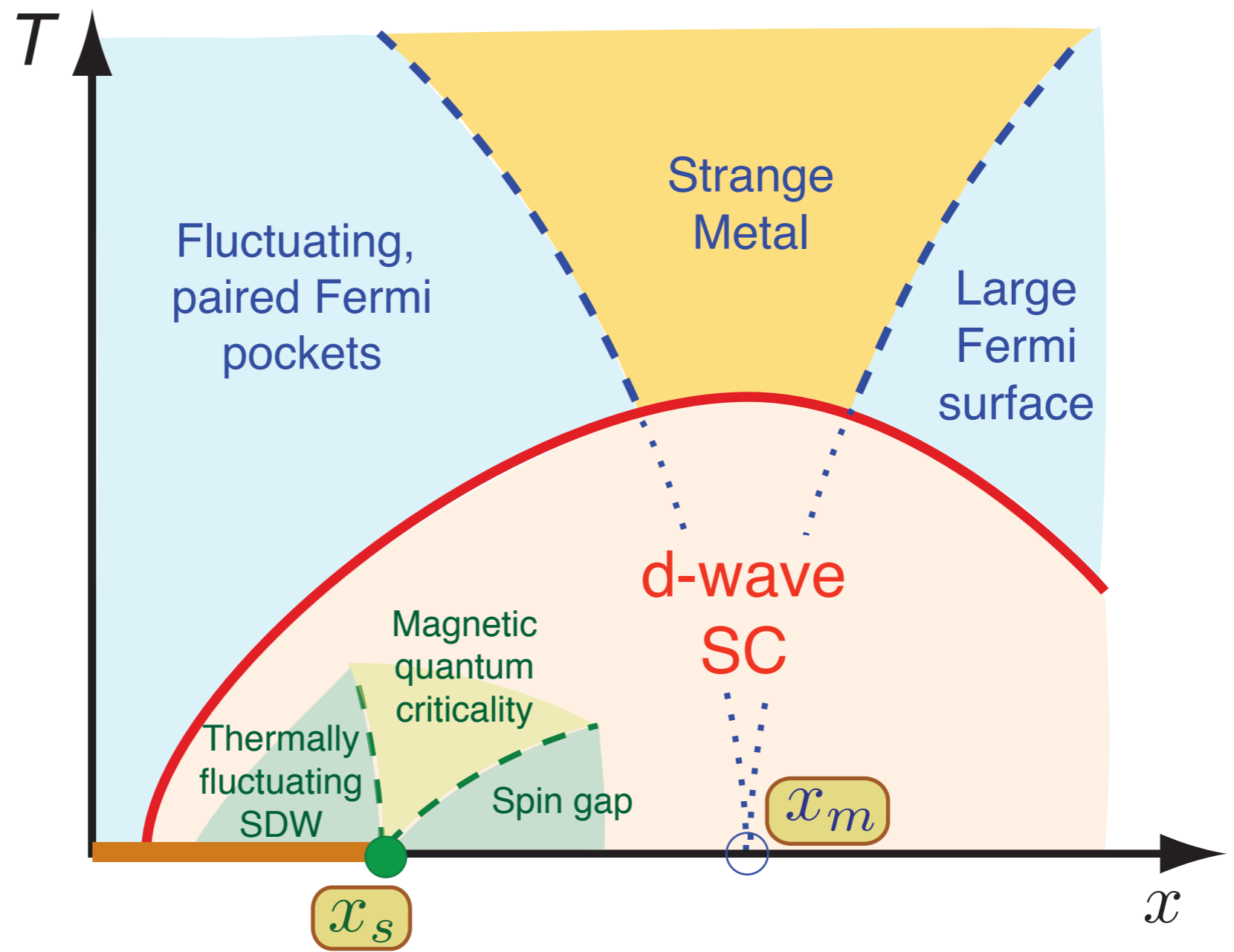
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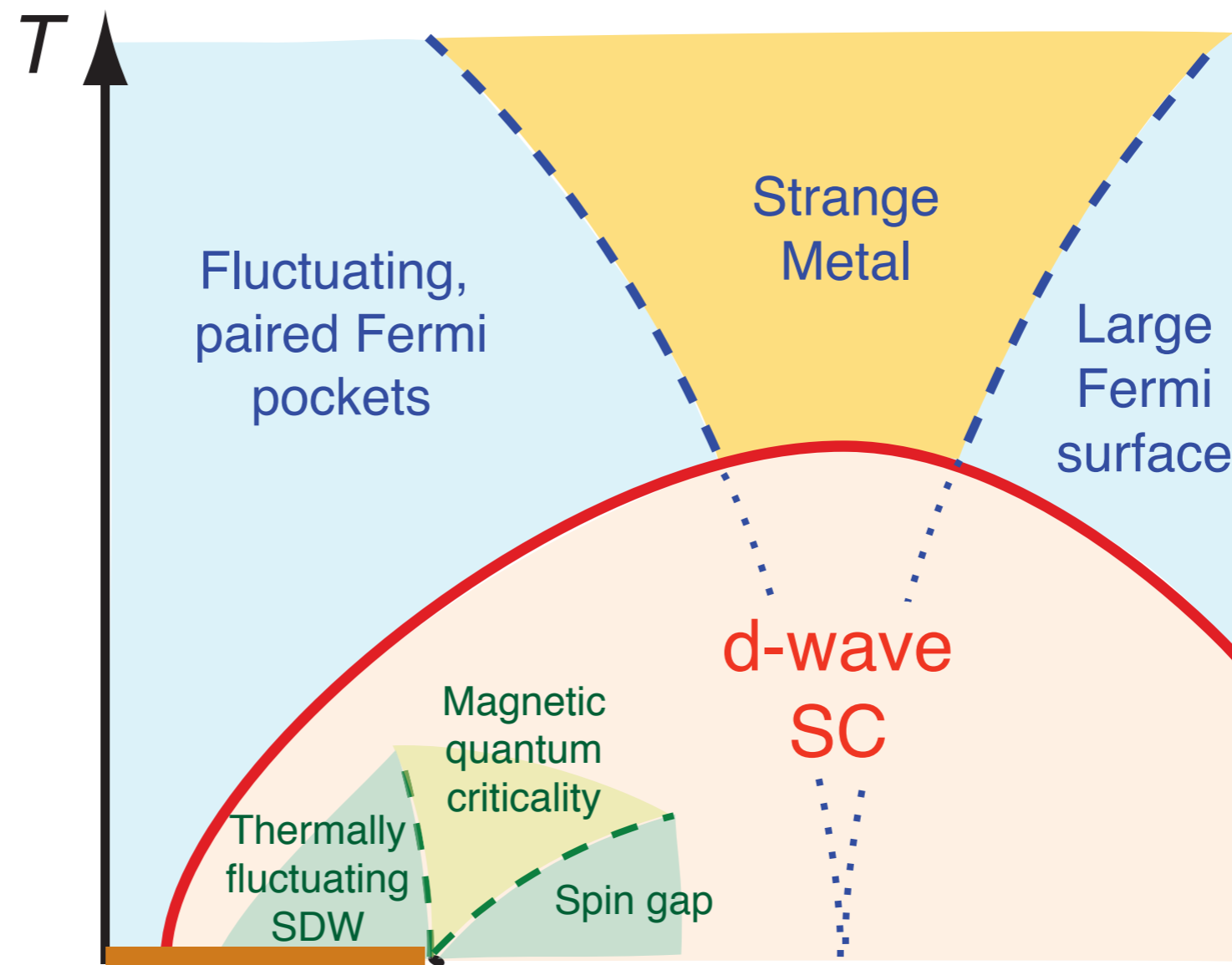


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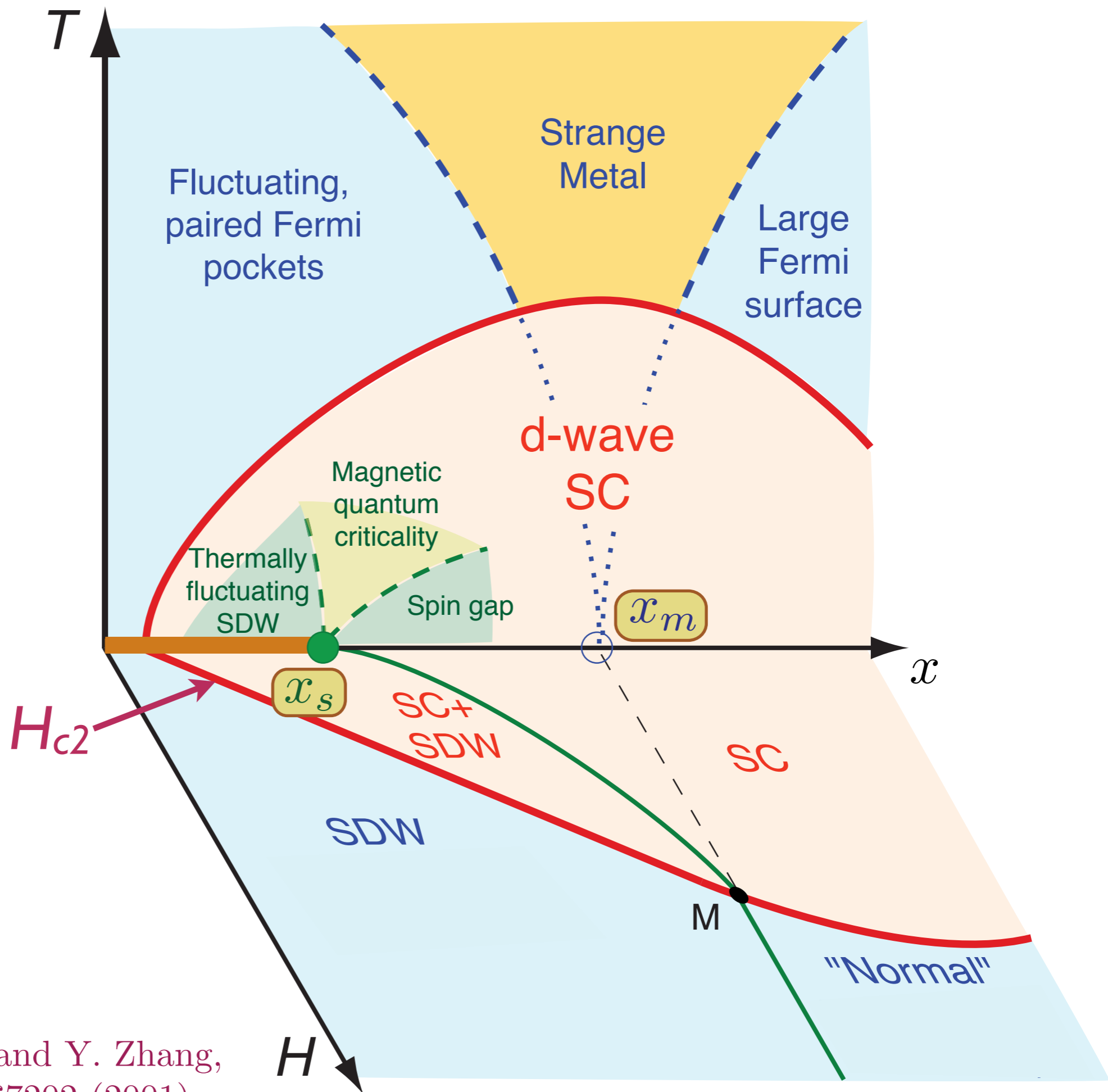
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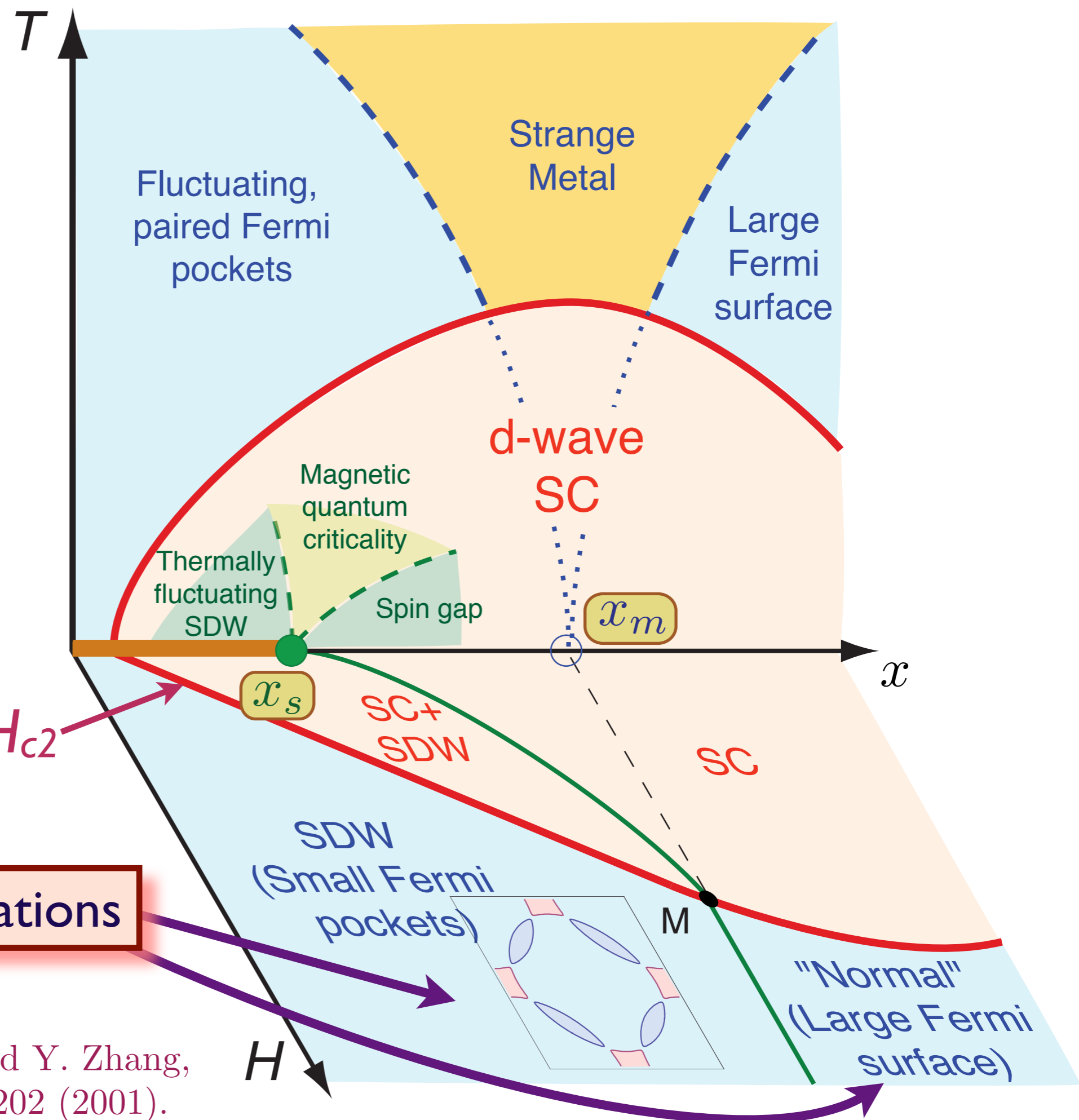




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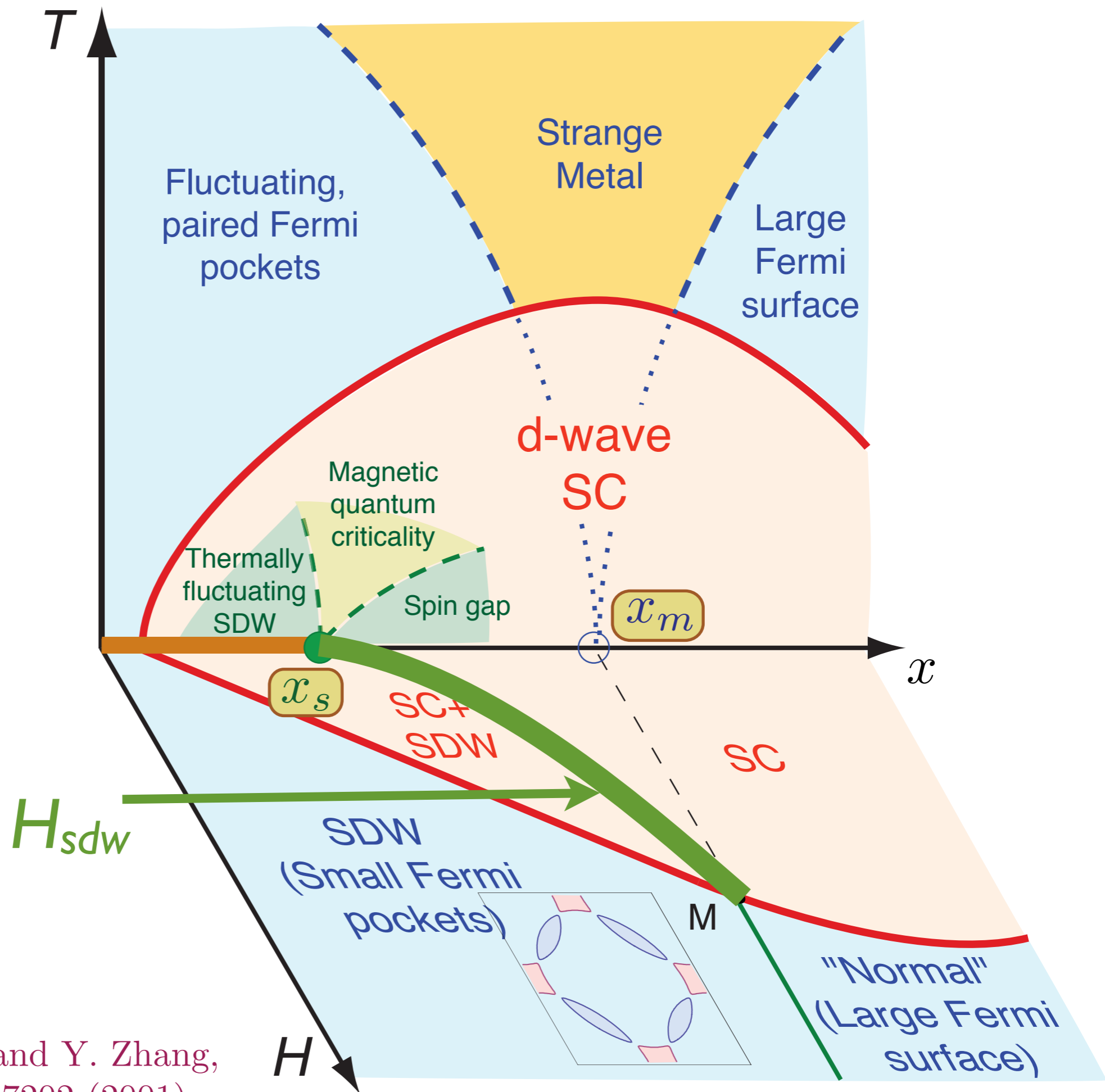


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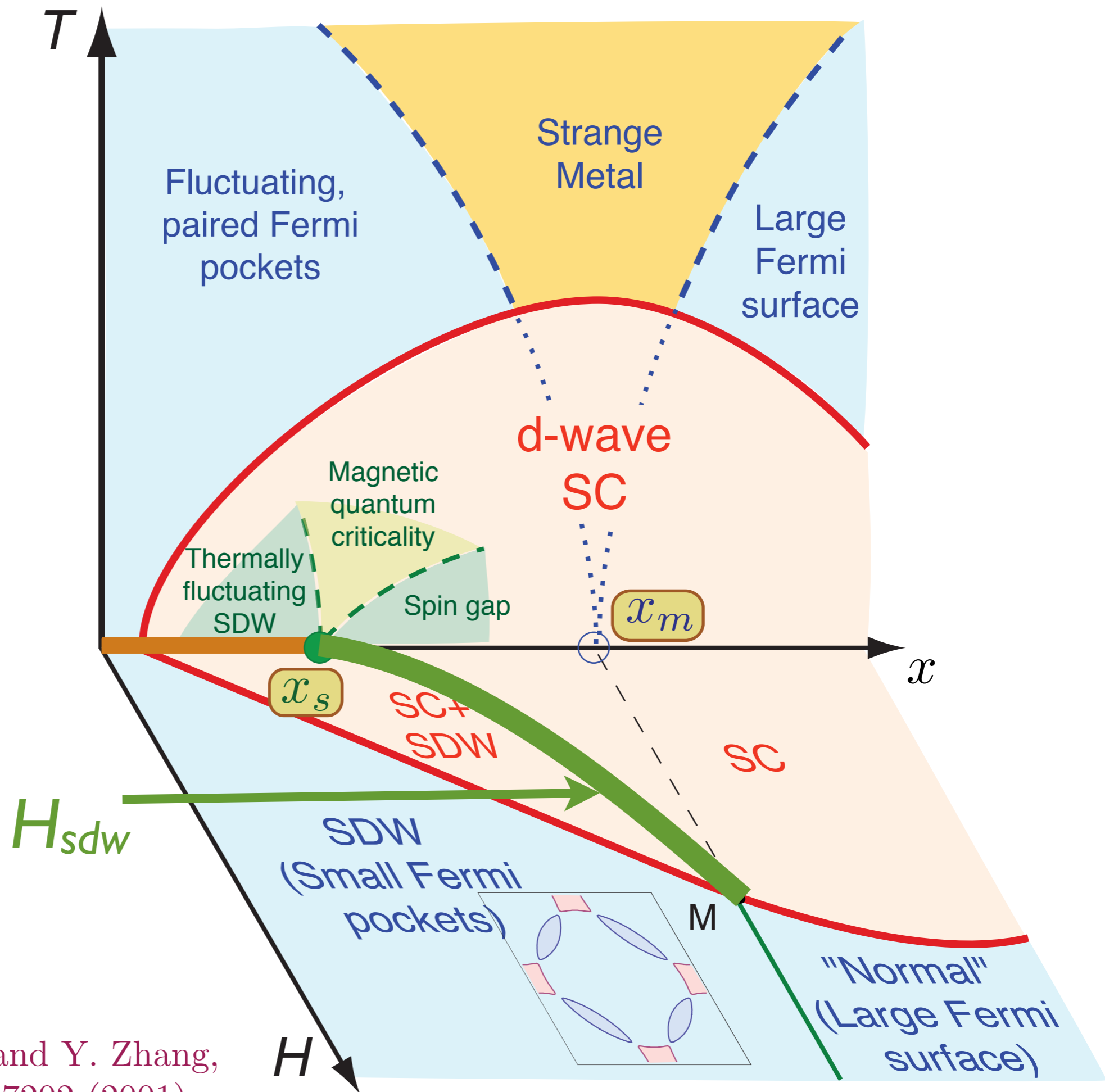


Quantum oscillations

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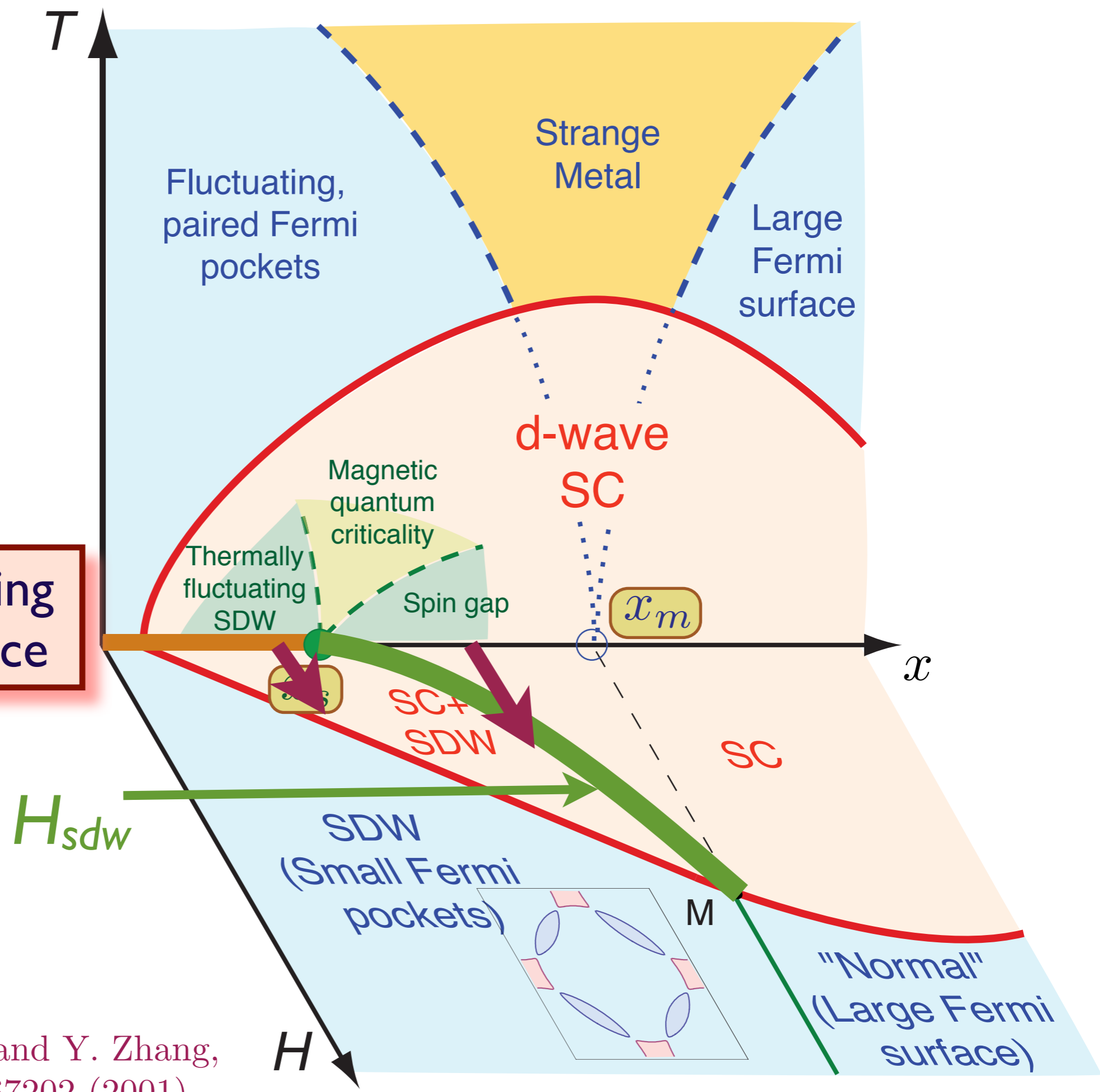


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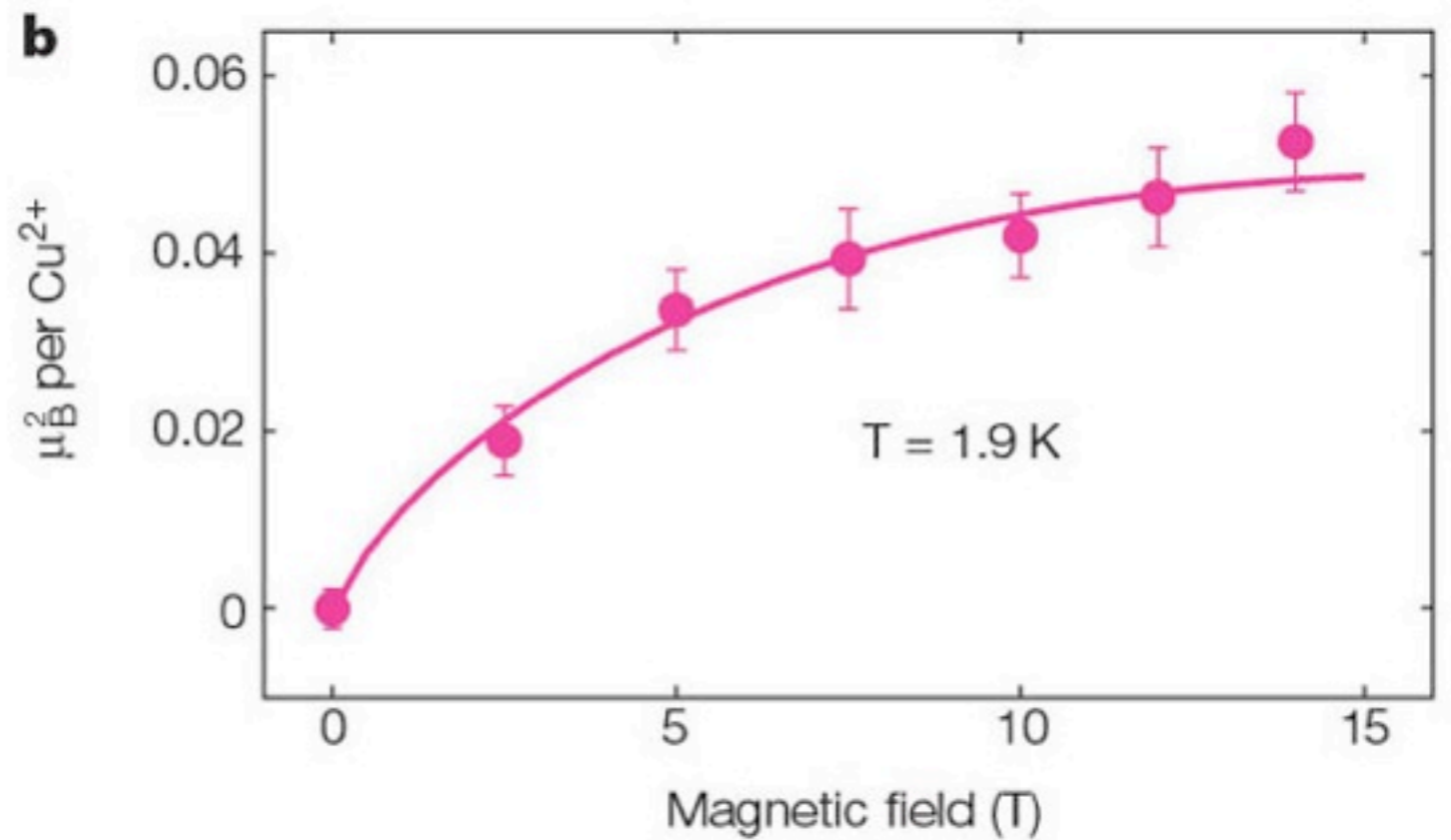
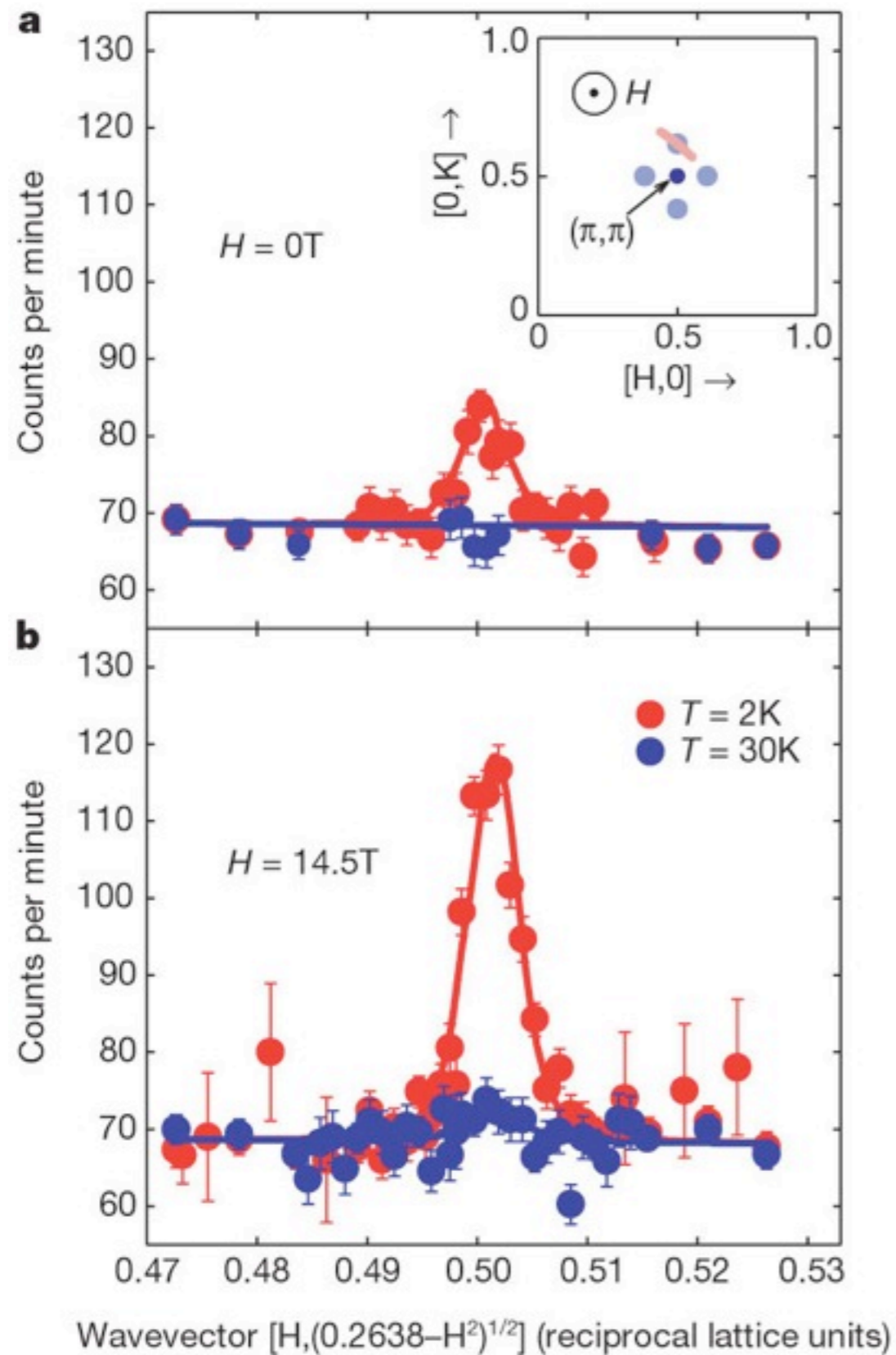


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Phys. Rev. Lett. **87**, 067202 (2001).

Neutron scattering & muon resonance



E. Demler, S. Sachdev and Y. Zhang,
Phys. Rev. Lett. **87**, 067202 (2001).



*B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature **415**, 299 (2002)*

*B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder Science **291**, 1759 (2001).*

Field-induced transition between magnetically disordered and ordered phases in underdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

B. Khaykovich,¹ S. Wakimoto,² R. J. Birgeneau,³ M. A. Kastner,¹ Y. S. Lee,¹ P. Smeibidl,⁴ P. Vorderwisch,⁴ and K. Yamada⁵

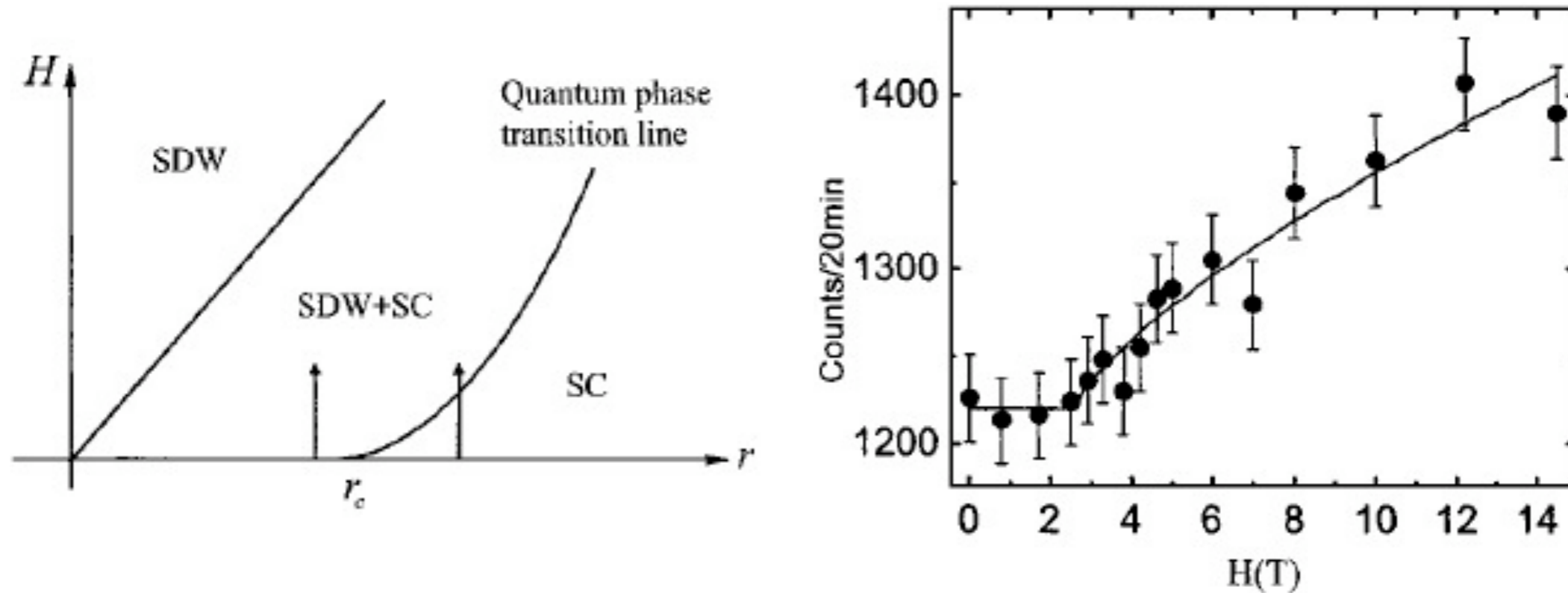
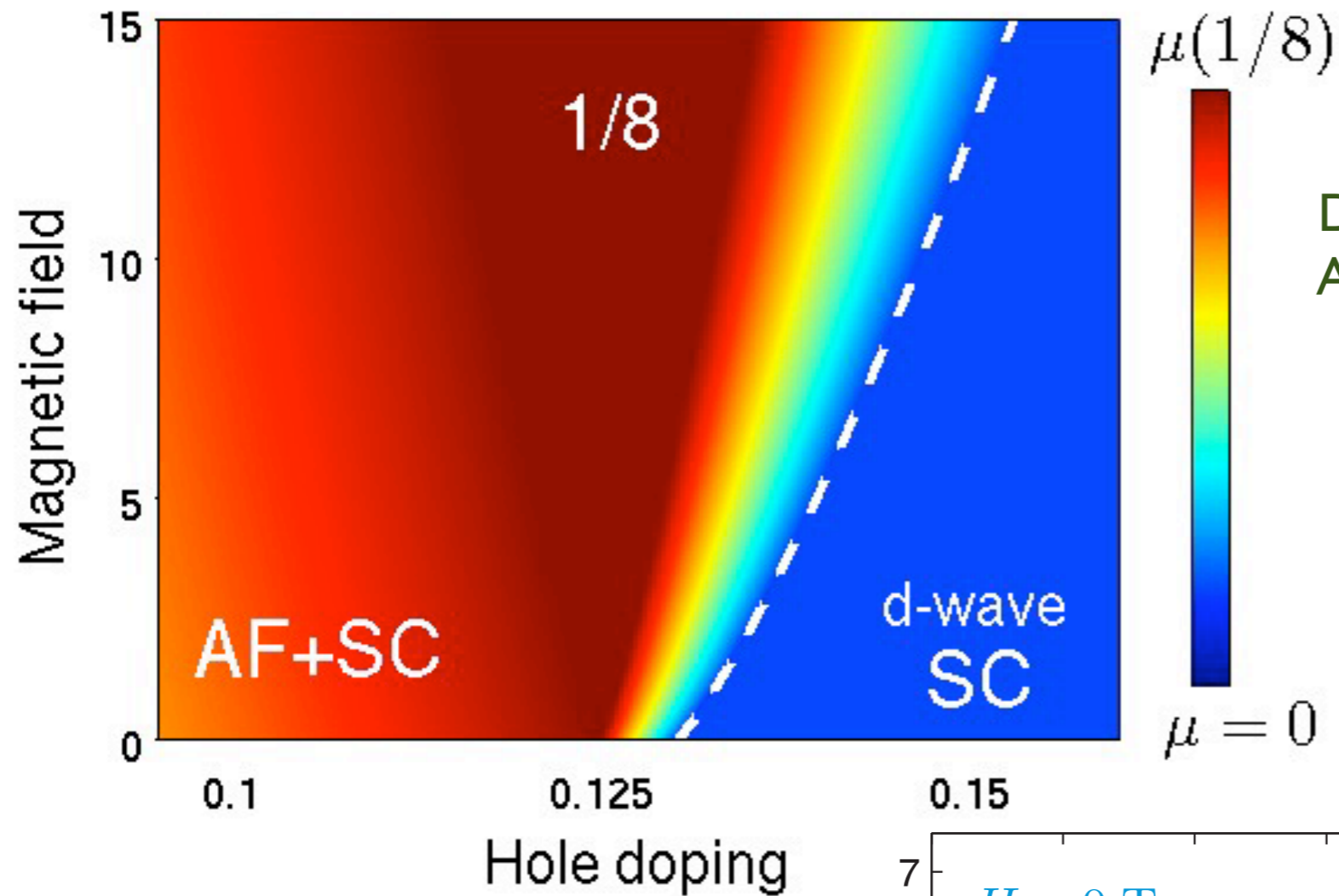
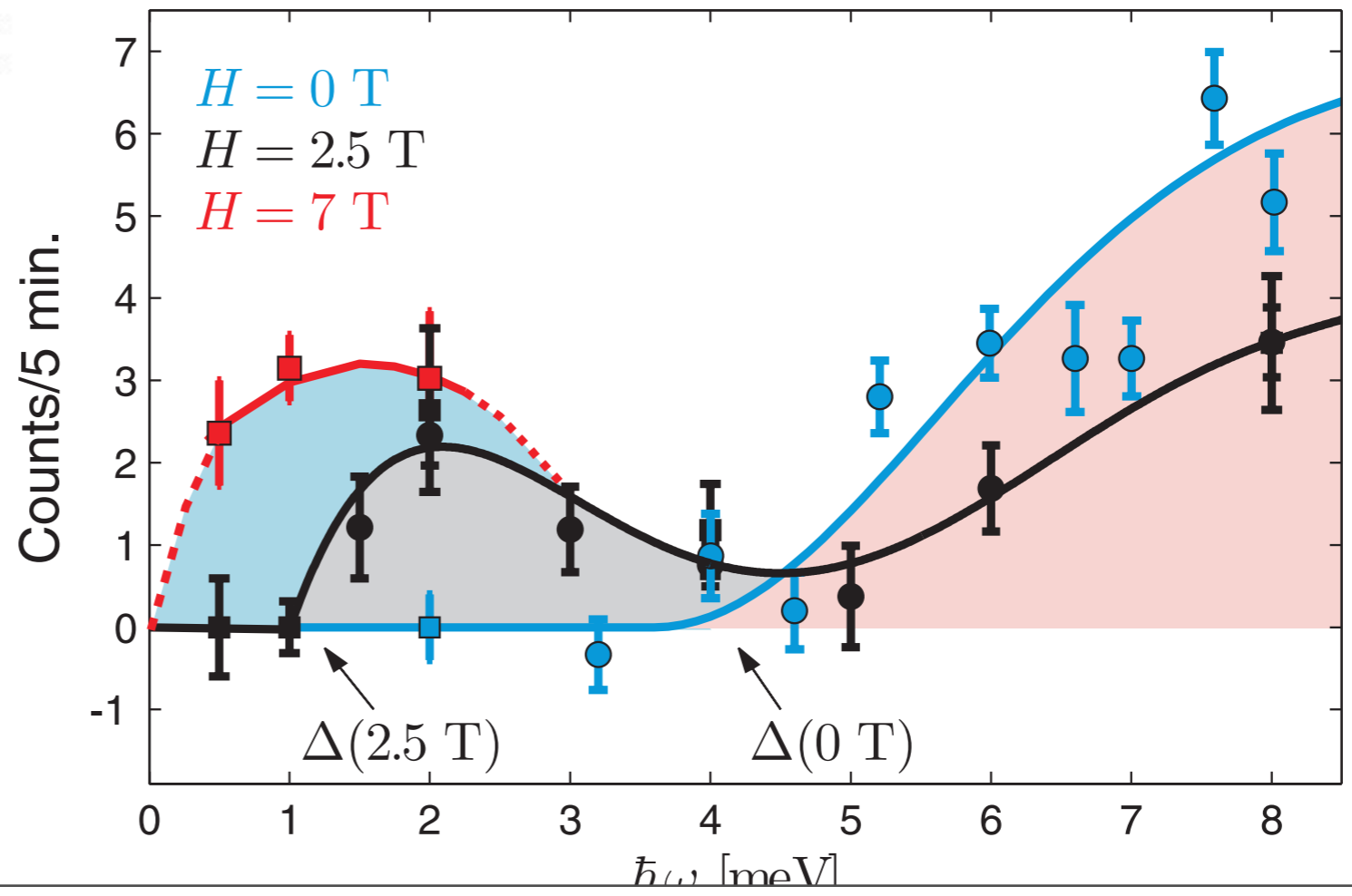


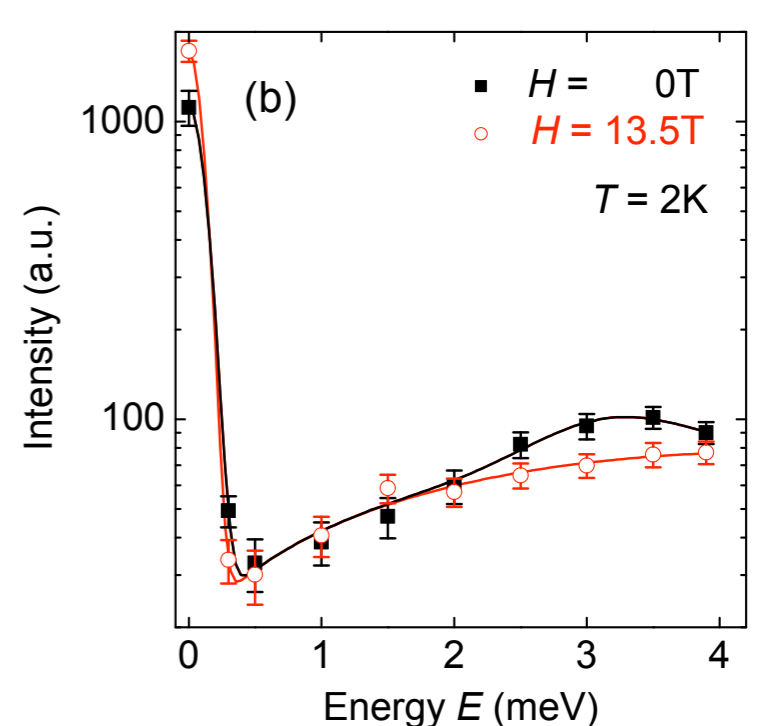
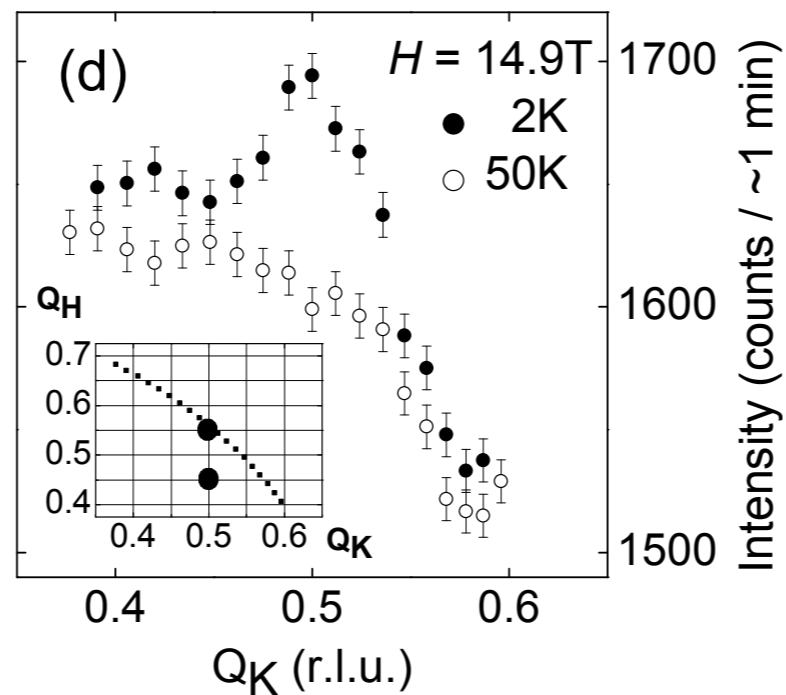
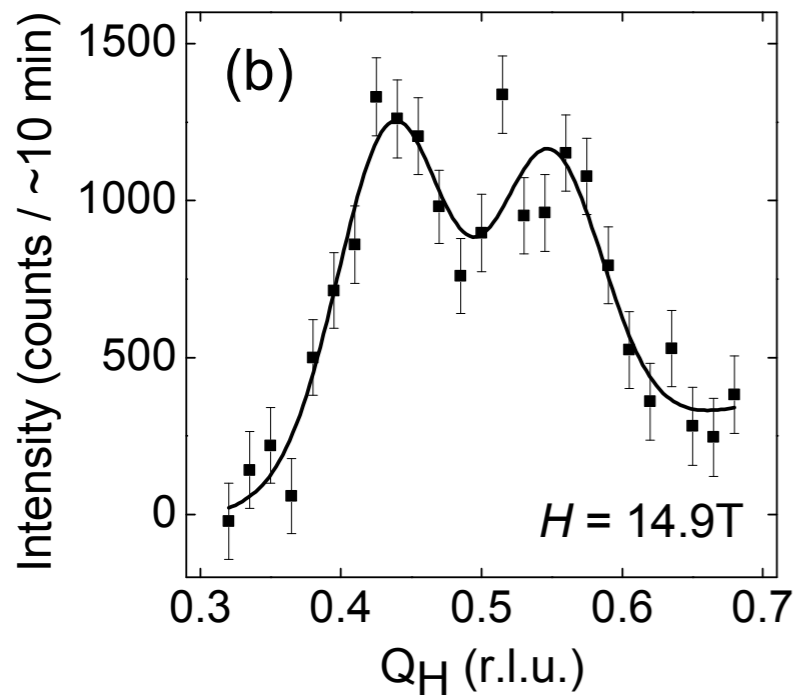
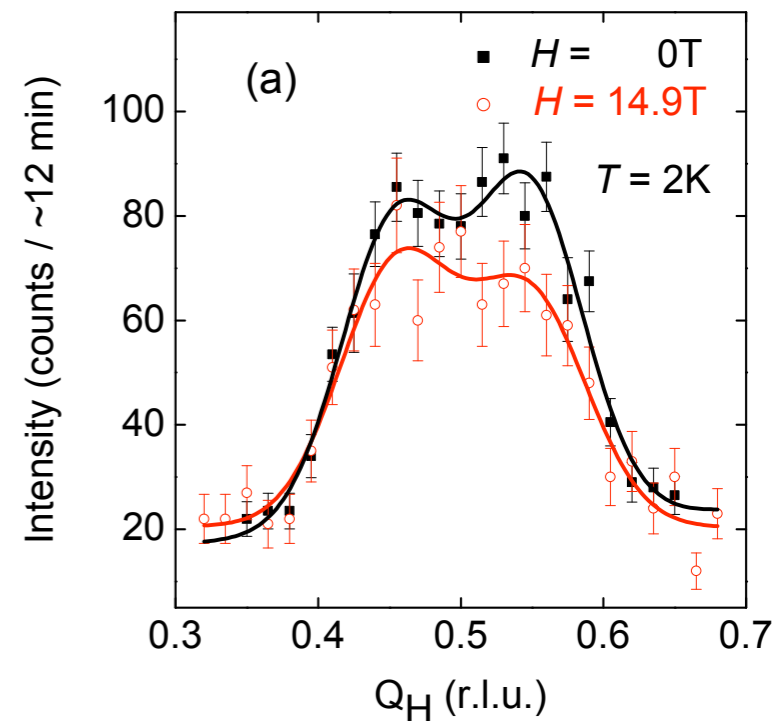
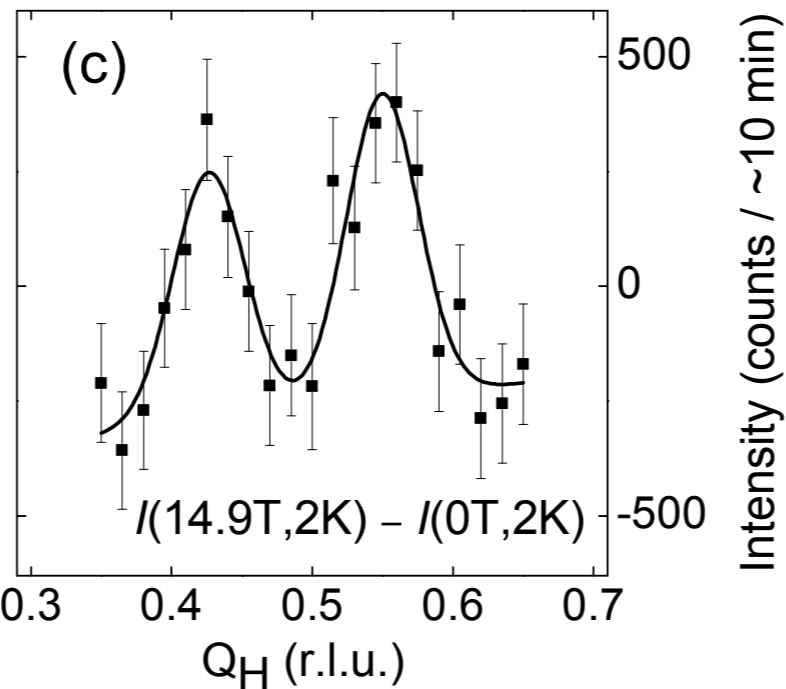
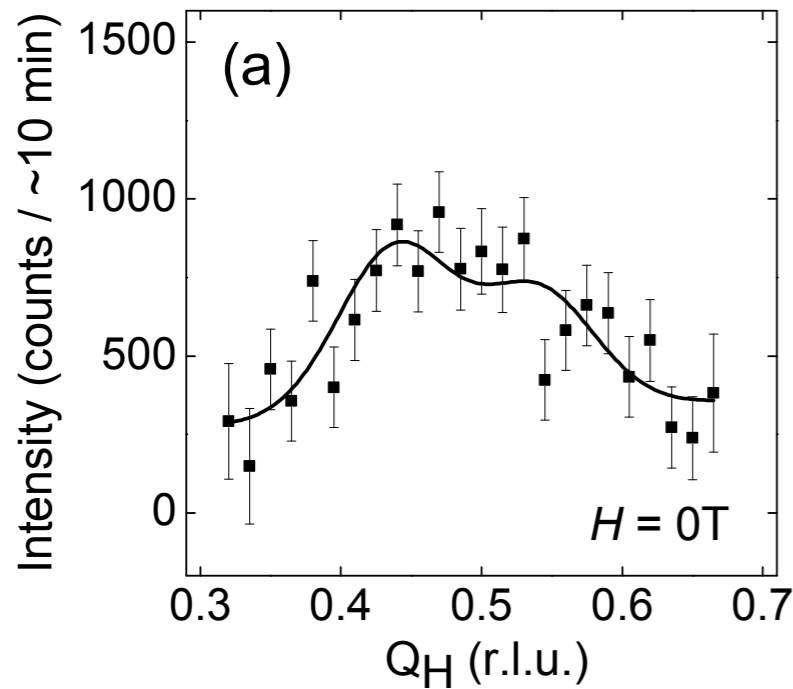
FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125, 0.125, 0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.



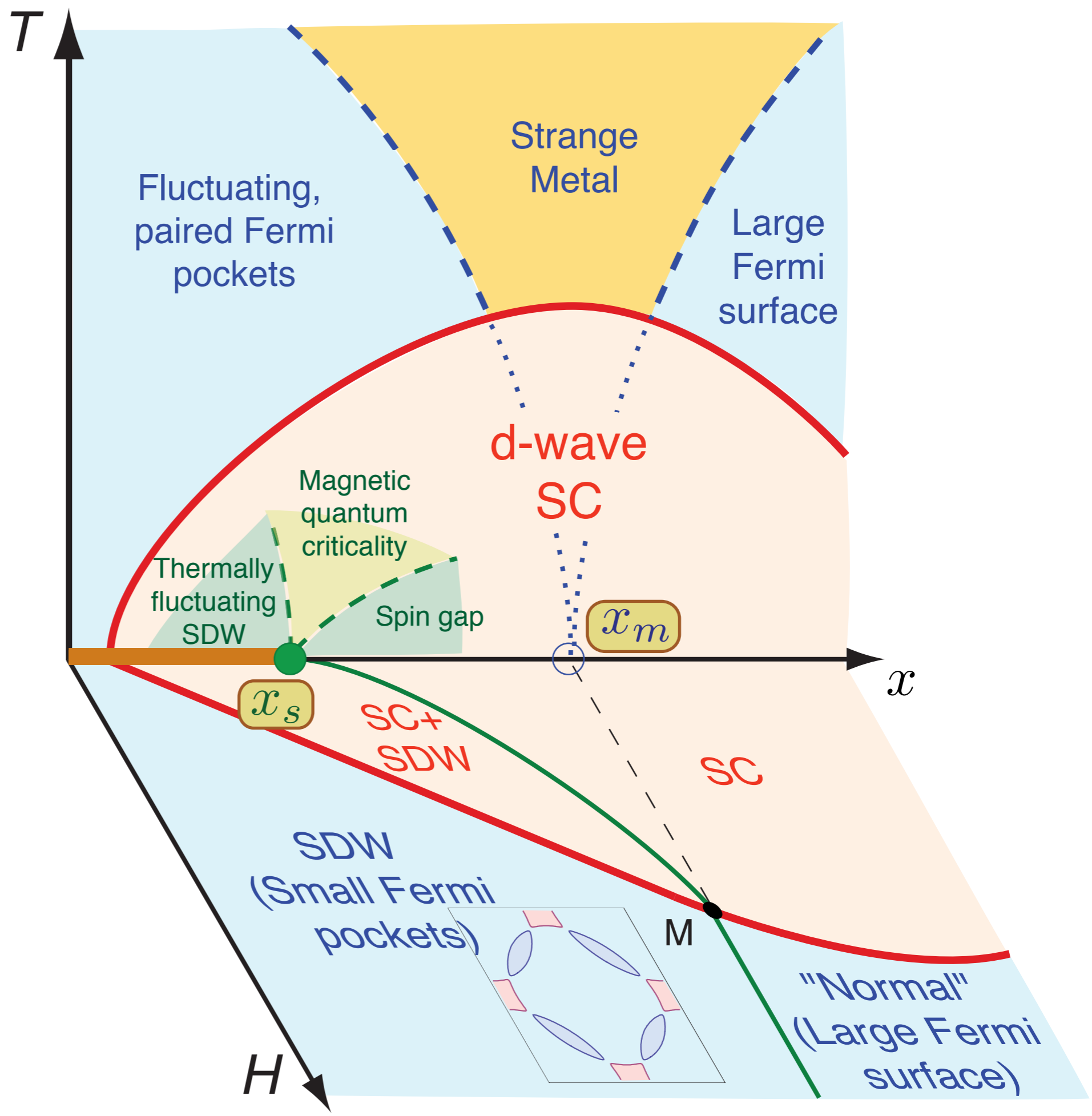
J. Chang, Ch. Niedermayer, R. Gilardi,
 N.B. Christensen, H.M. Ronnow,
 D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev,
 A. Hiess, S. Pailhes, C. Baines, N. Momono,
 M. Oda, M. Ido, and J. Mesot,
Physical Review B **78**, 104525 (2008).

J. Chang, N. B. Christensen,
 Ch. Niedermayer, K. Lefmann,
 H. M. Roennow, D. F. McMorrow,
 A. Schneidewind, P. Link, A. Hiess,
 M. Boehm, R. Mottl, S. Pailhes,
 N. Momono, M. Oda, M. Ido, and
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Phys. Rev. Lett. **102**, 177006
 (2009).

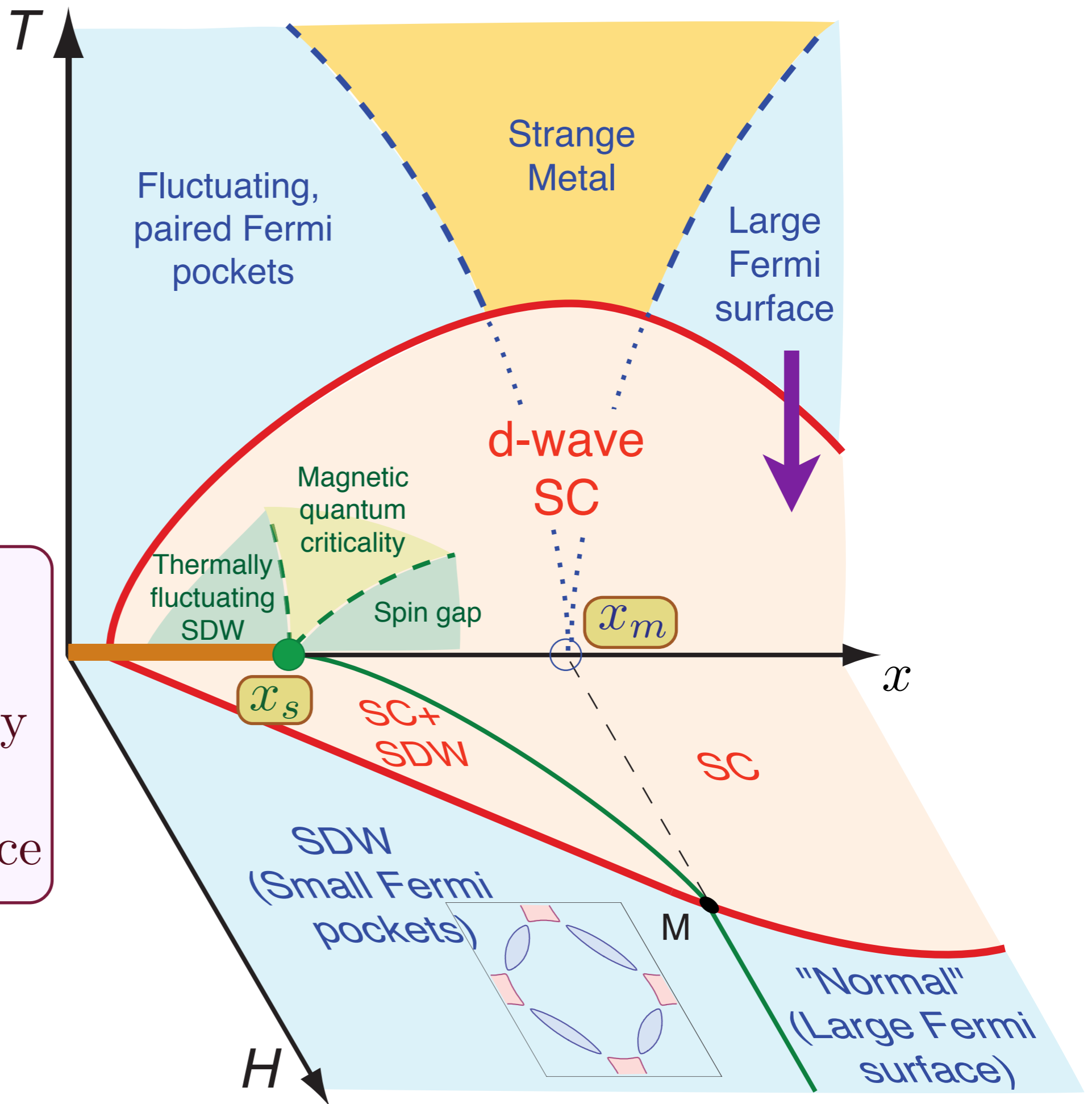




D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

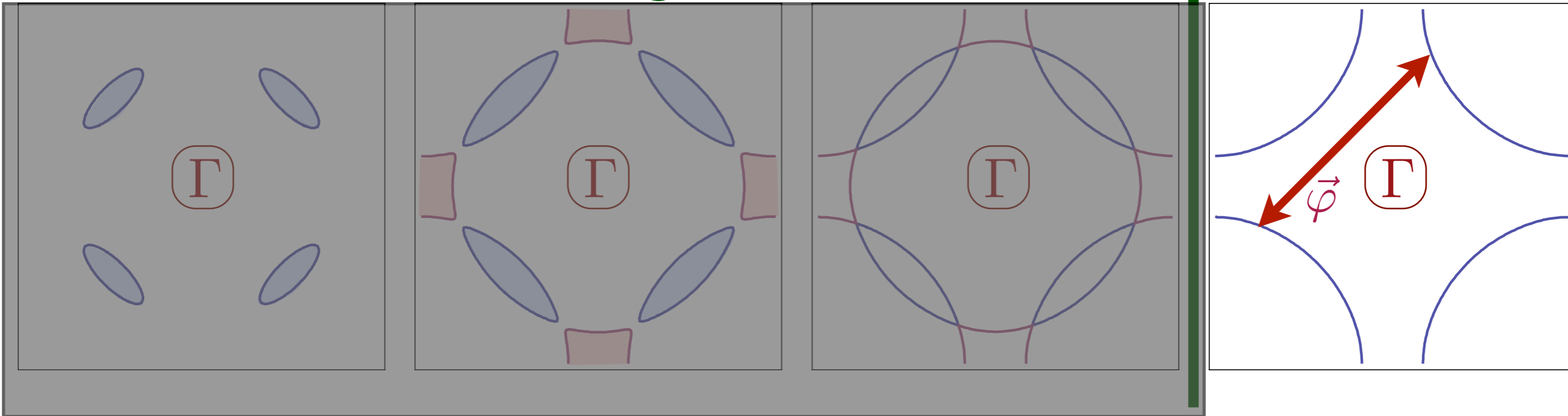


Theory of the onset of *d*-wave superconductivity from a large Fermi surface



Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates

← Increasing SDW order →

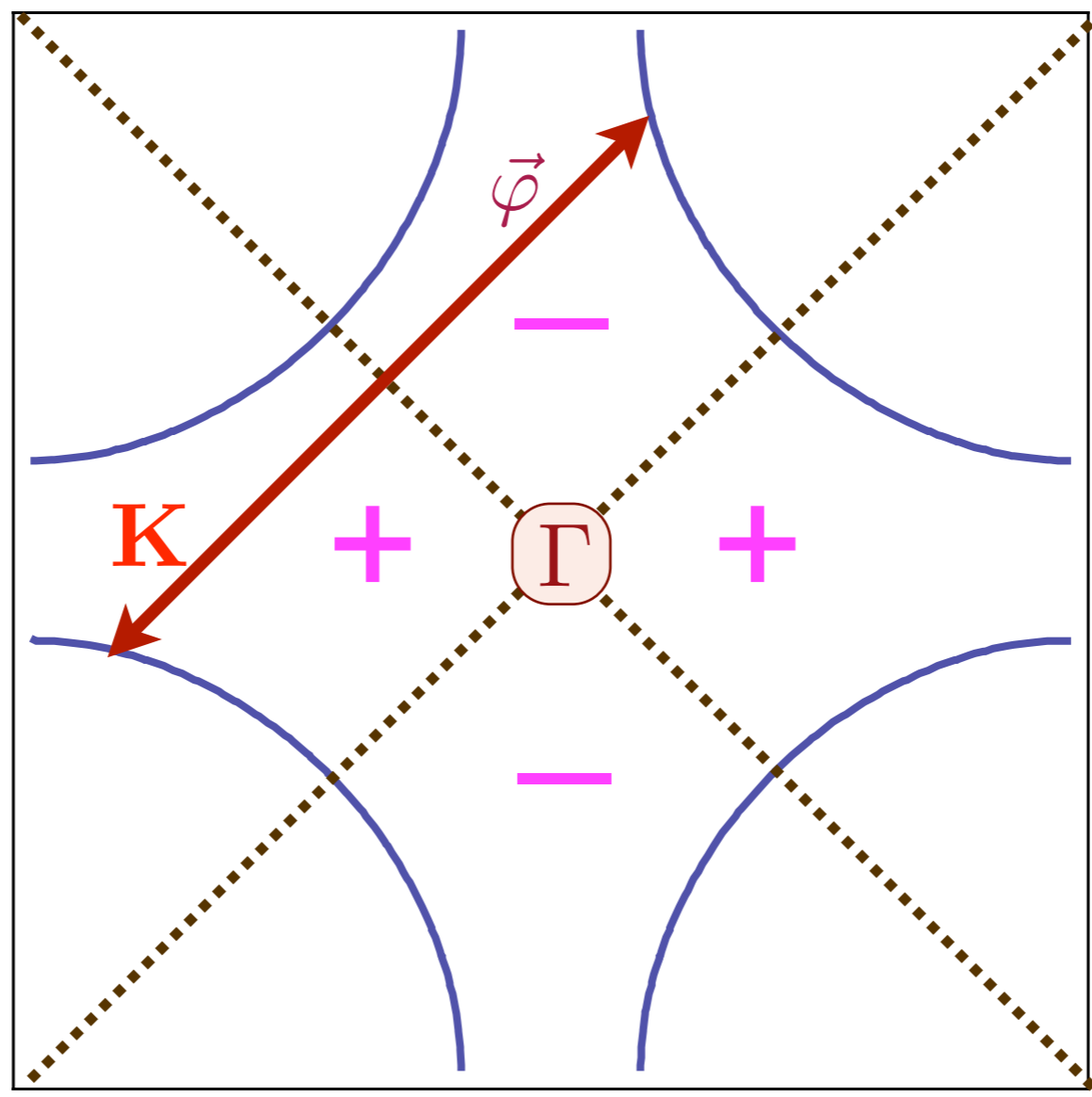


Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

d -wave pairing of the large Fermi surface

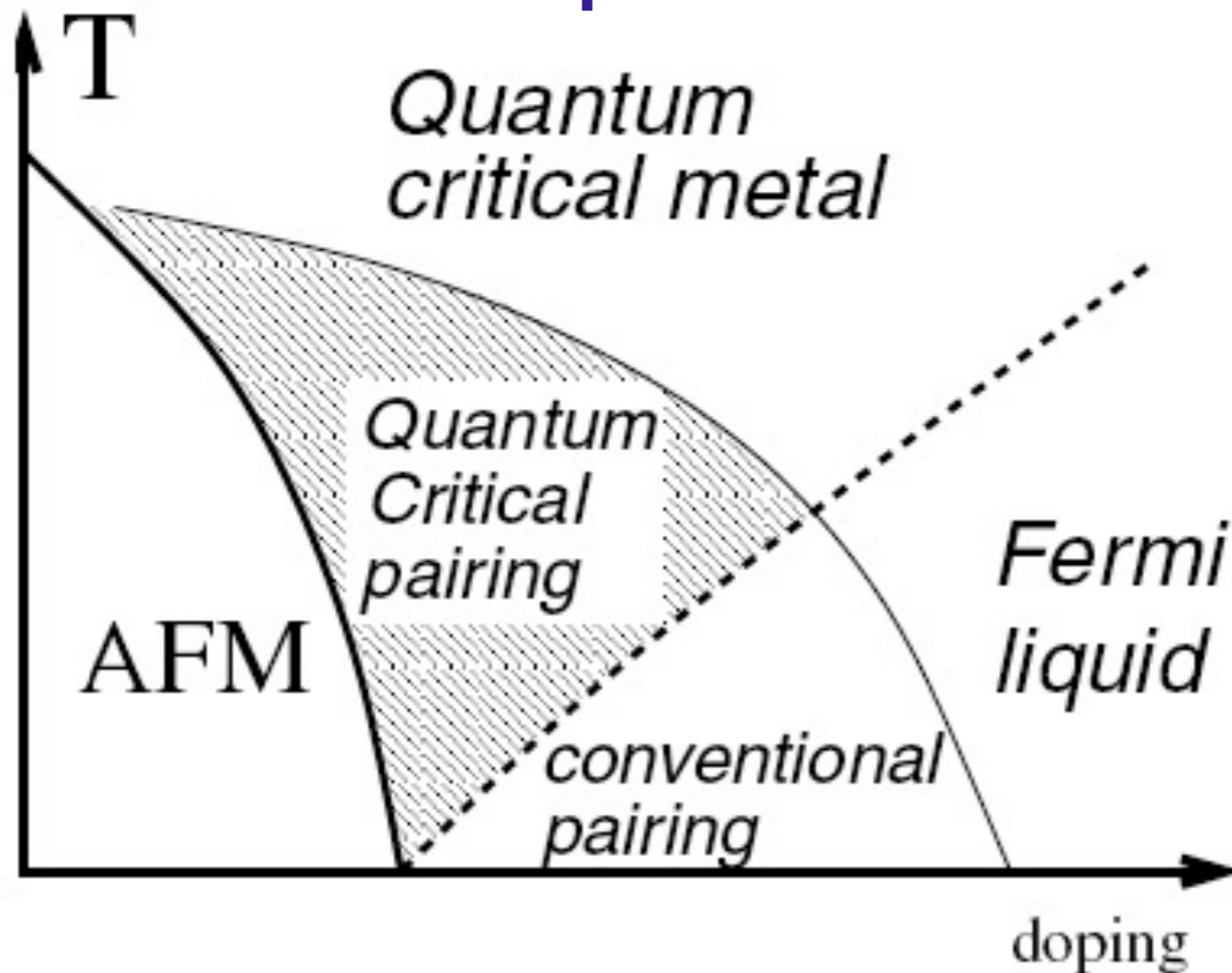


$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

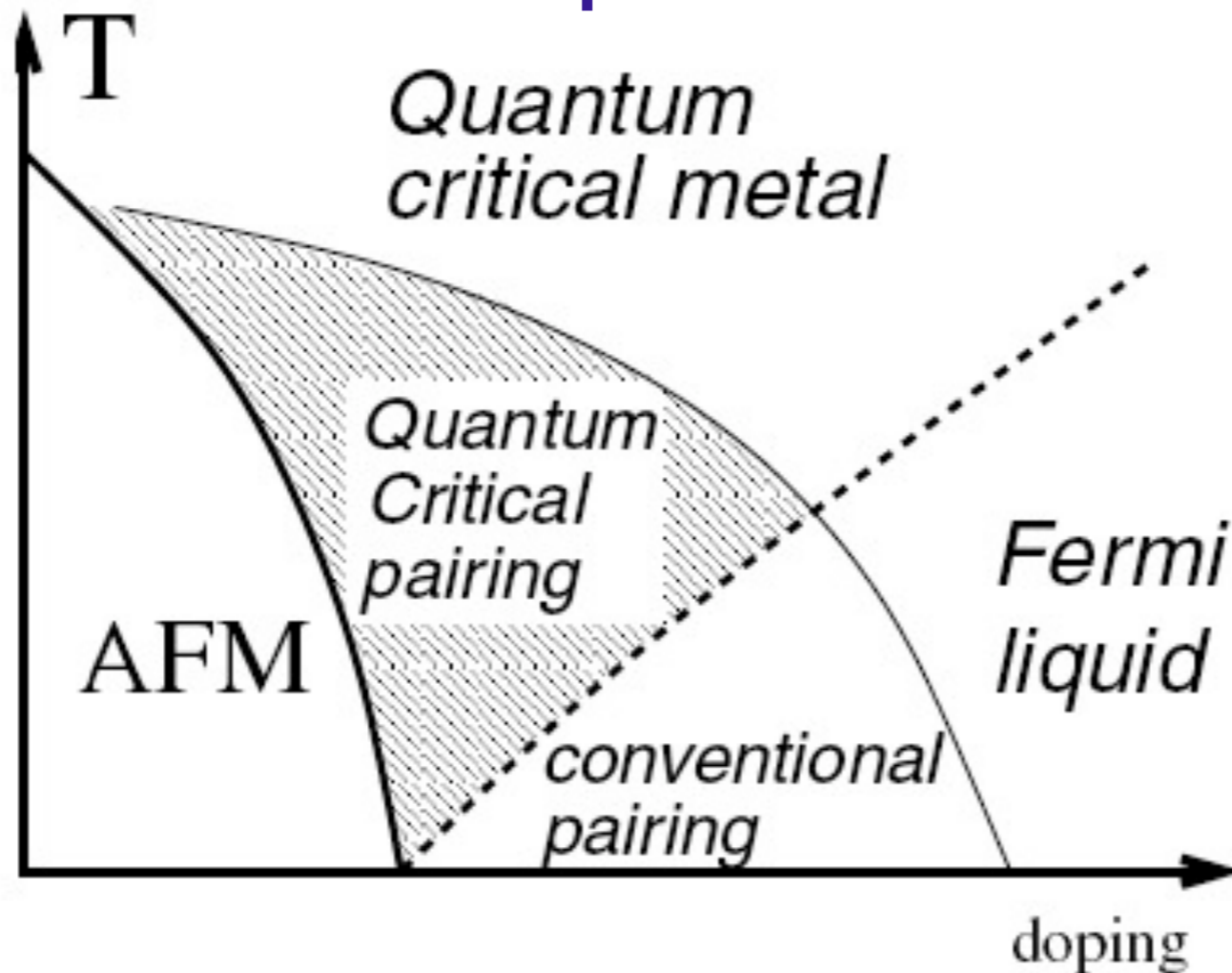
Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c increases upon approaching the SDW transition.

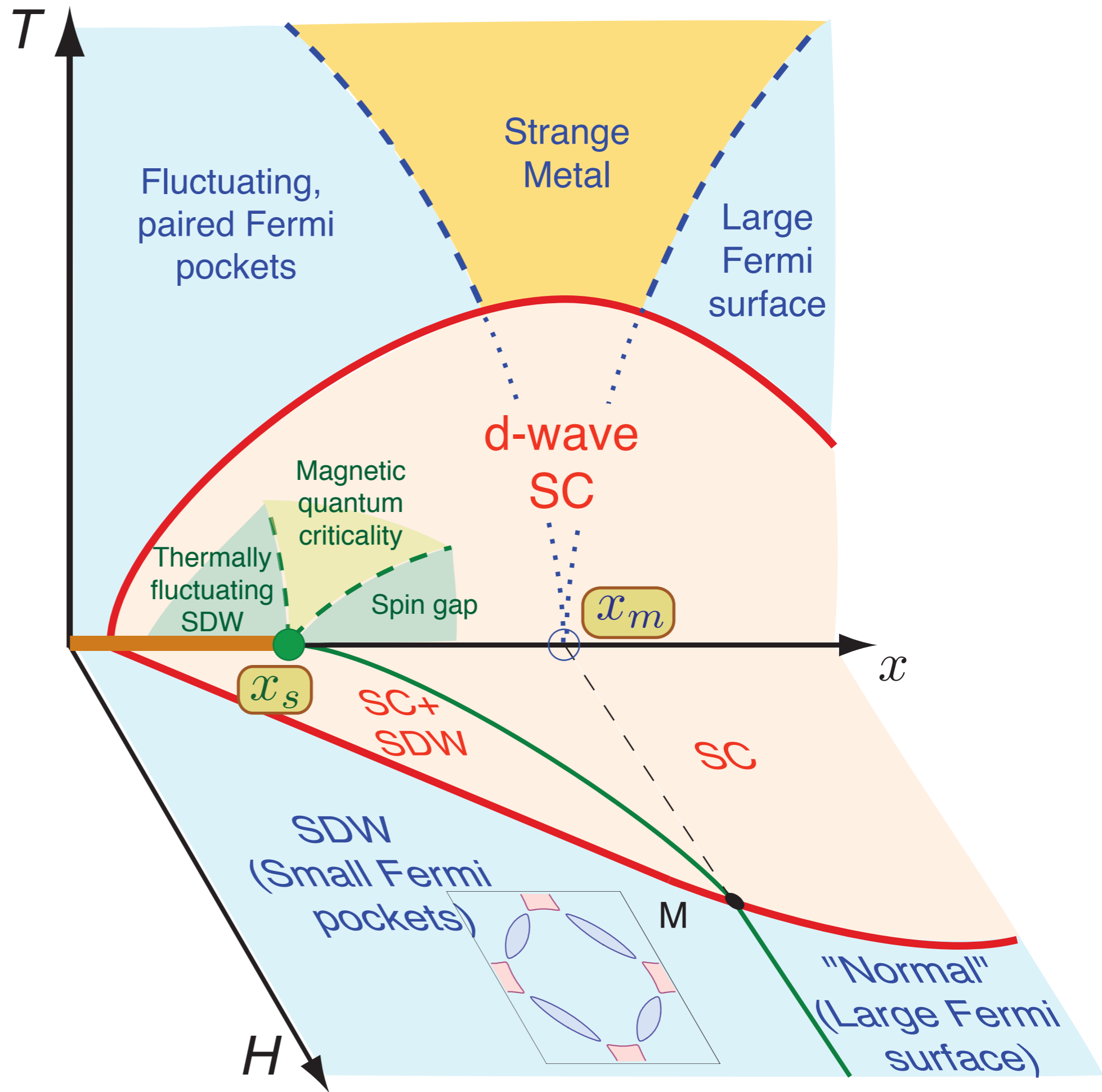
Ar. Abanov, A. V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory

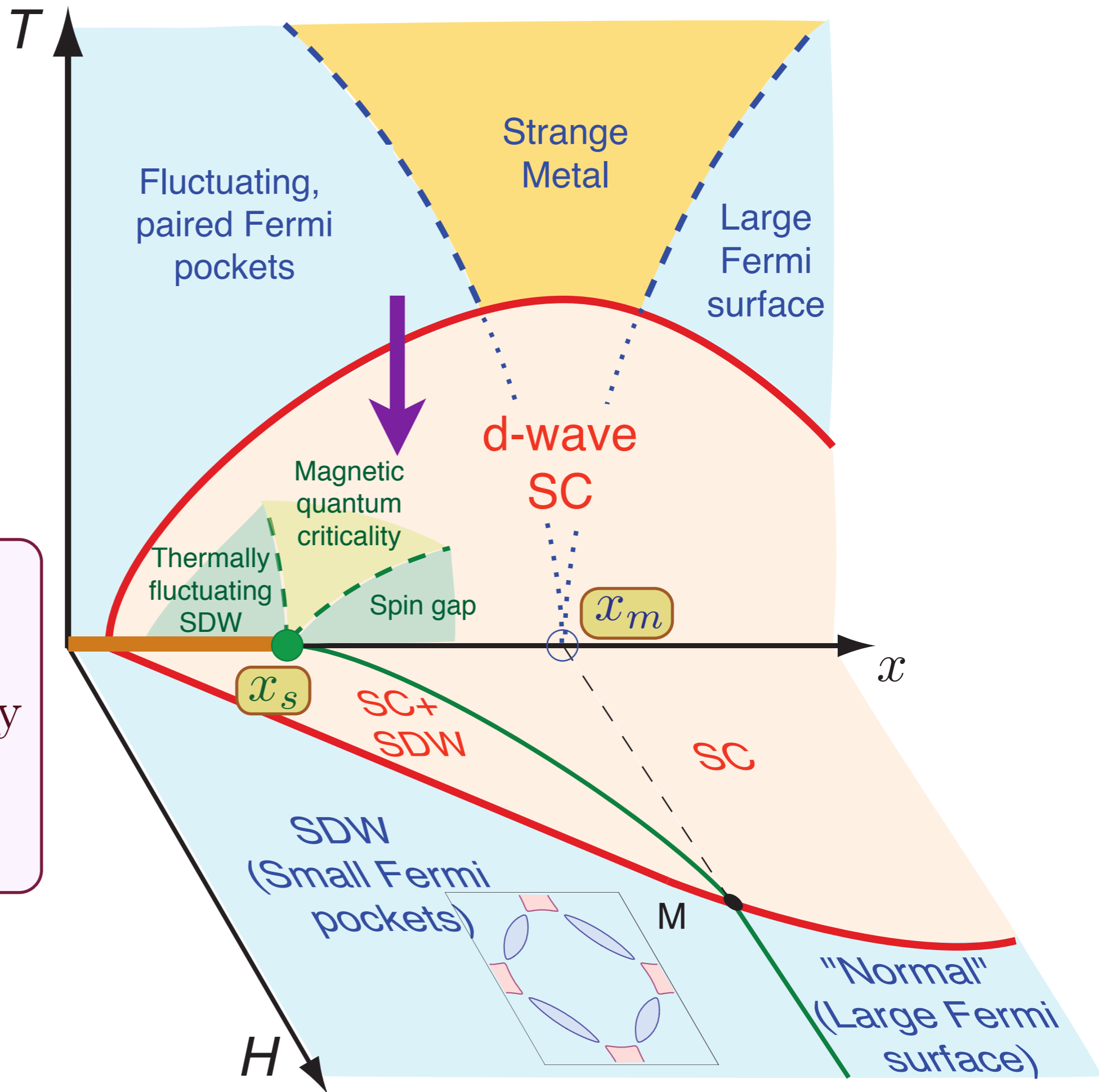


- T_c increases upon approaching the SDW transition.
- Pairing from SDW fluctuations: SDW and SC orders do not compete, but attract each other.

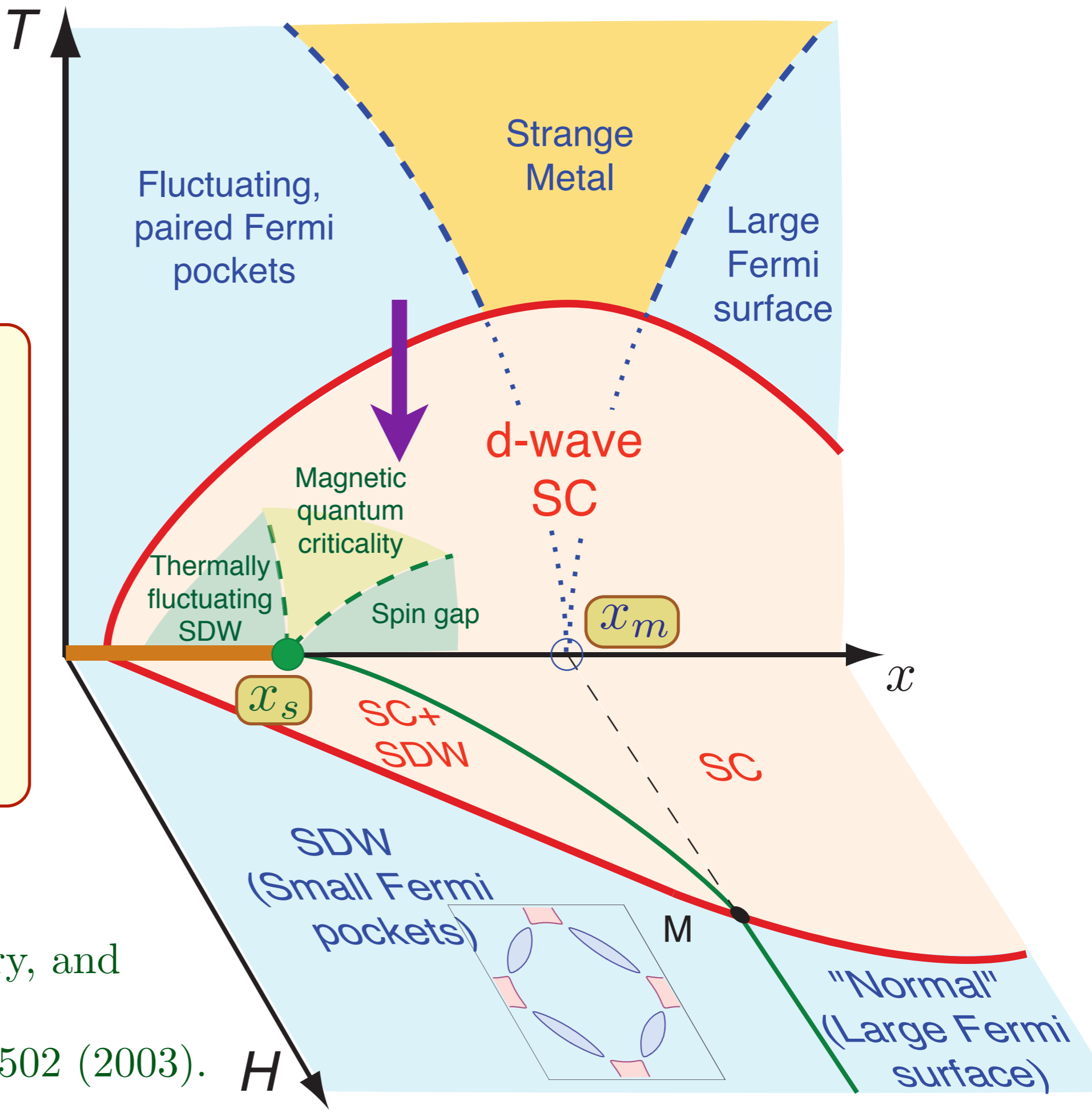
Ar. Abanov, A. V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).



Theory of the onset of *d*-wave superconductivity from small Fermi pockets



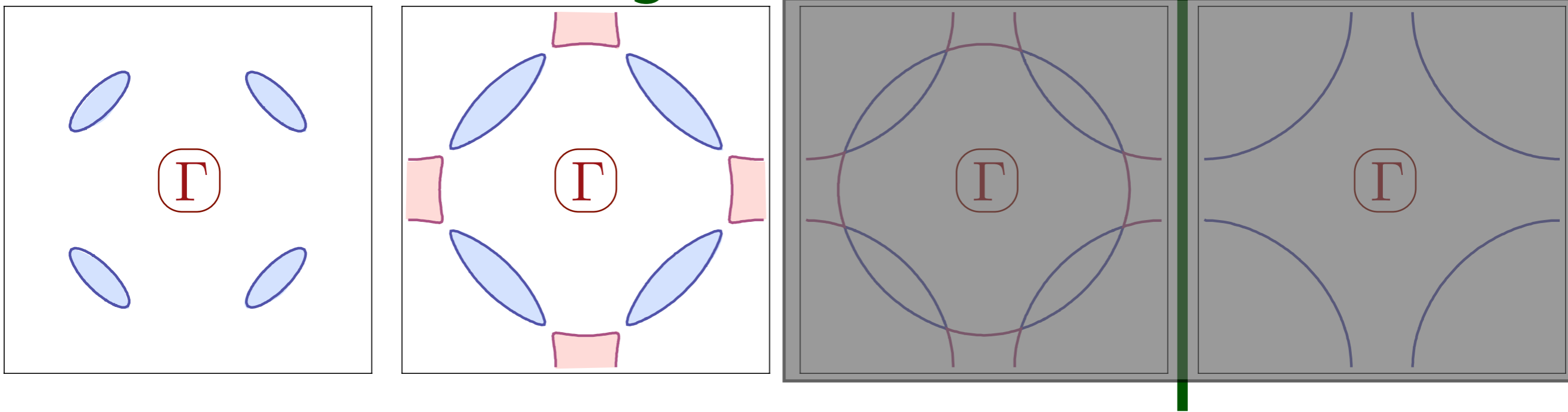
Physics of competition:
d-wave SC and SDW “eat up” same pieces of the large Fermi surface.



B. Kyung, J.-S. Landry, and A.-M. S. Tremblay, Phys. Rev. B **68**, 174502 (2003).

Theory of underdoped cuprates

← Increasing SDW order →



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\varphi}$

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} ; \quad R^{\dagger} \hat{\varphi} \cdot \vec{\sigma} R = \sigma^z ; \quad R^{\dagger} R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)

B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).

J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)

Theory of underdoped cuprates

$$\text{With } R = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix} \text{ or } \hat{\vec{\varphi}} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

the theory is invariant under

$$z_{\alpha} \rightarrow e^{i\theta} z_{\alpha} ; \psi_{+} \rightarrow e^{-i\theta} \psi_{+} ; \psi_{-} \rightarrow e^{i\theta} \psi_{-}$$

We obtain a U(1) gauge theory of

- bosonic neutral spinons z_{α} ;
- spinless, charged fermions ψ_{\pm} ;
- an emergent U(1) gauge field A_{μ} .

X.-G. Wen, *Phys. Rev. B* **39**, 7223 (1989).

P. A. Lee, *Phys. Rev. Lett.* **63**, 680 (1989).

R. Shankar, *Phys. Rev. Lett.* **63**, 203 (1989).

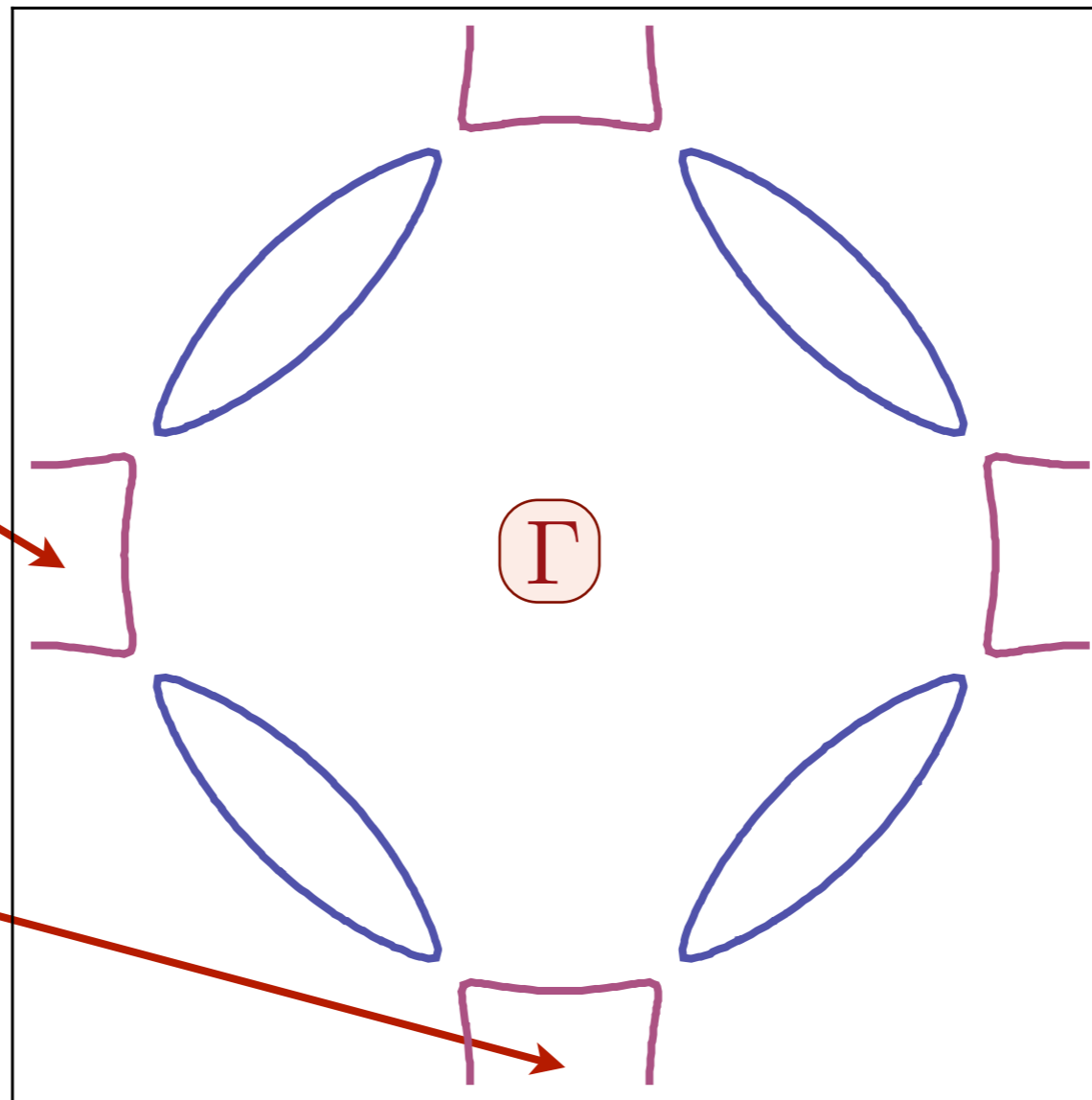
L. B. Ioffe and P. B. Wiegmann, *Phys. Rev. Lett.* **65**, 653 (1990).

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007).

Pairing across T_c

Electron $c_{2\alpha}$,
spinless fermion g_{\pm}

Electron $c_{1\alpha}$,
spinless fermion g_{\pm}



Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_{\pm} \equiv g_{\pm}$,
and the electron operators are

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

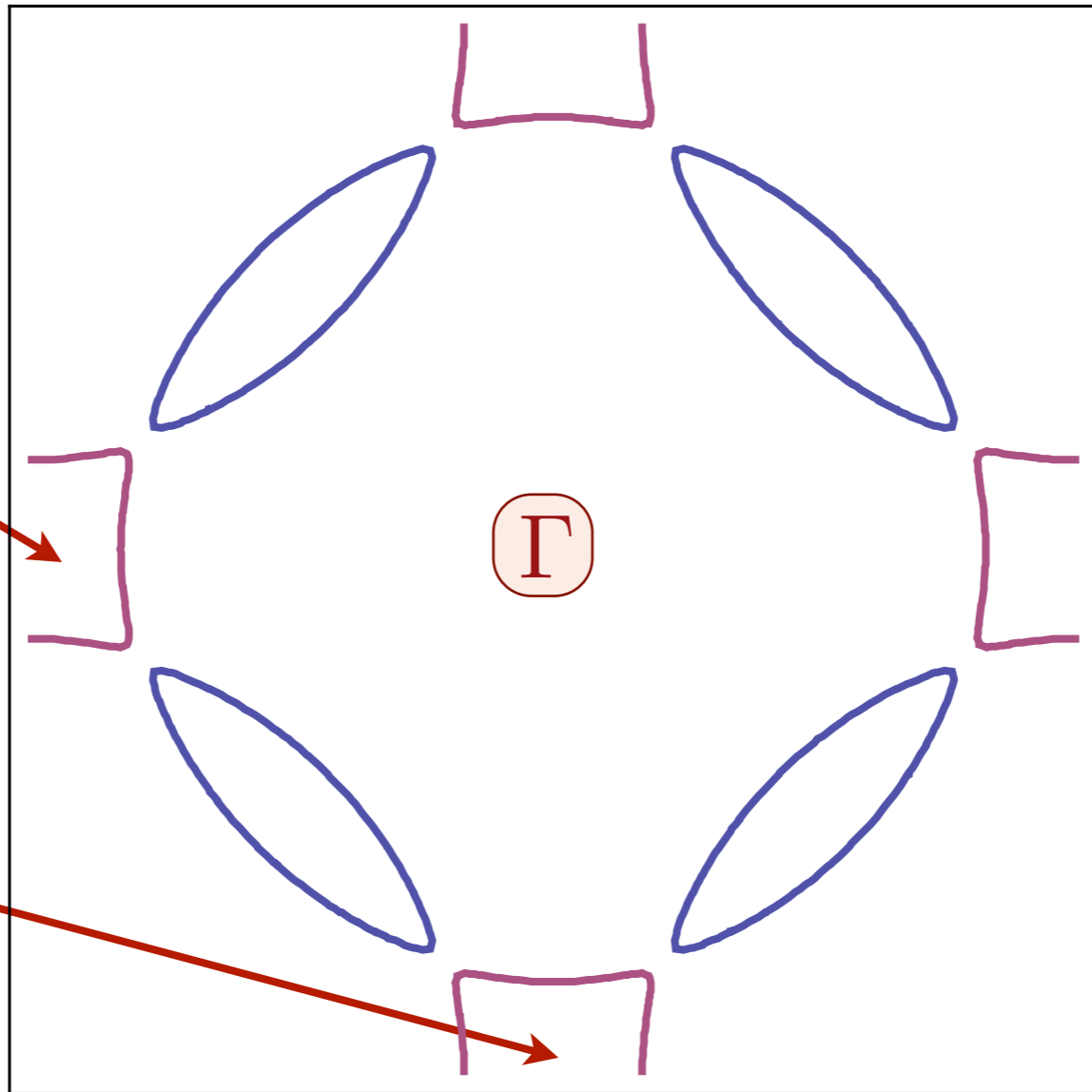
$$\mathcal{R}_z \equiv \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix}.$$

R. K. Kaul, M. Metlitski, S. Sachdev,
and Cenke Xu,
Phys. Rev. B **78**, 045110 (2008).

Pairing across T_c

Electron $c_{2\alpha}$,
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R. K. Kaul, M. Metlitski, S. Sachdev,
and Cenke Xu,
Phys. Rev. B **78**, 045110 (2008).

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Attractive gauge forces lead to simple s -wave pairing of the g_{\pm}

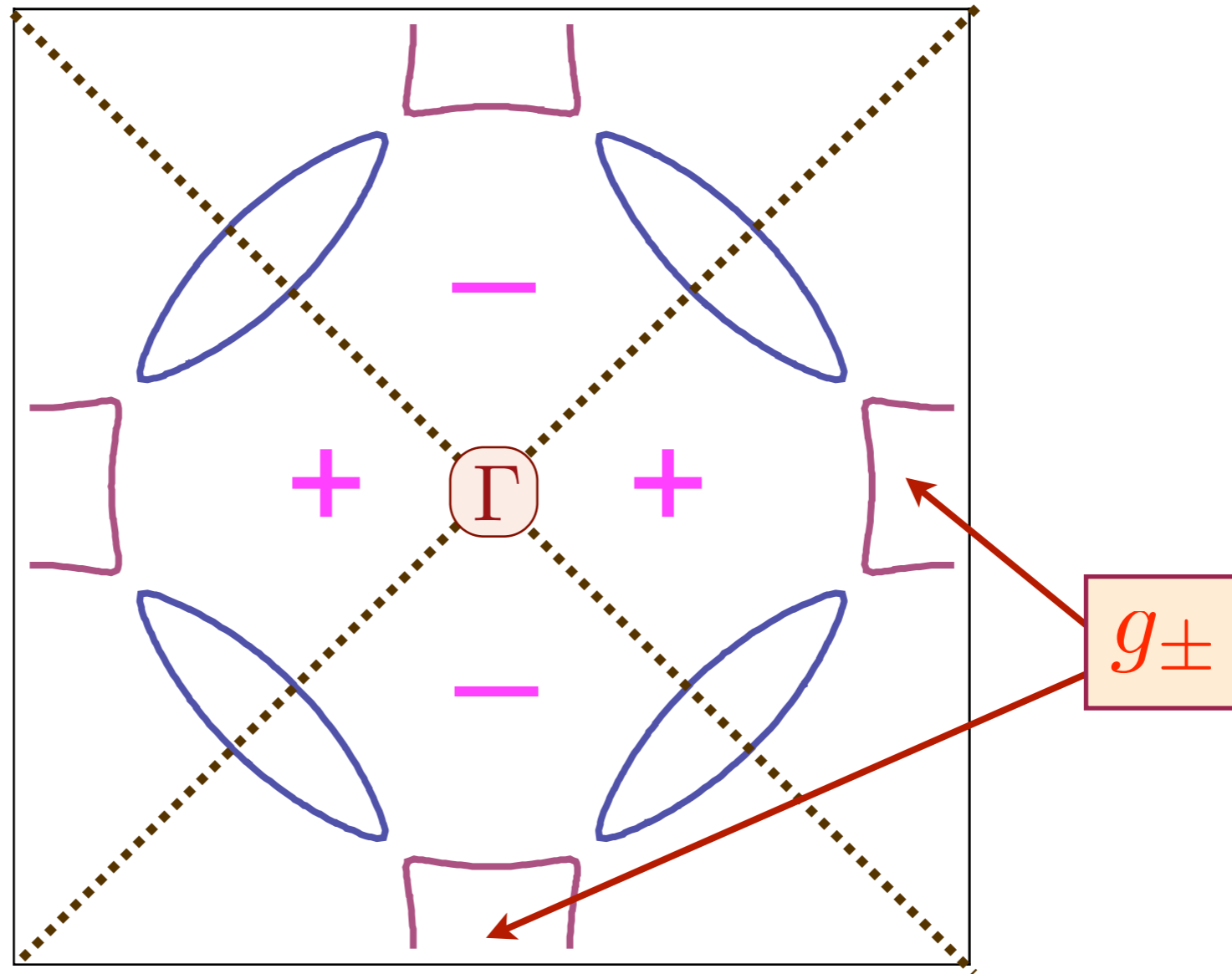
$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

$$\begin{aligned}\langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \langle |z_{\alpha}|^2 \rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \langle |z_{\alpha}|^2 \rangle\end{aligned}$$

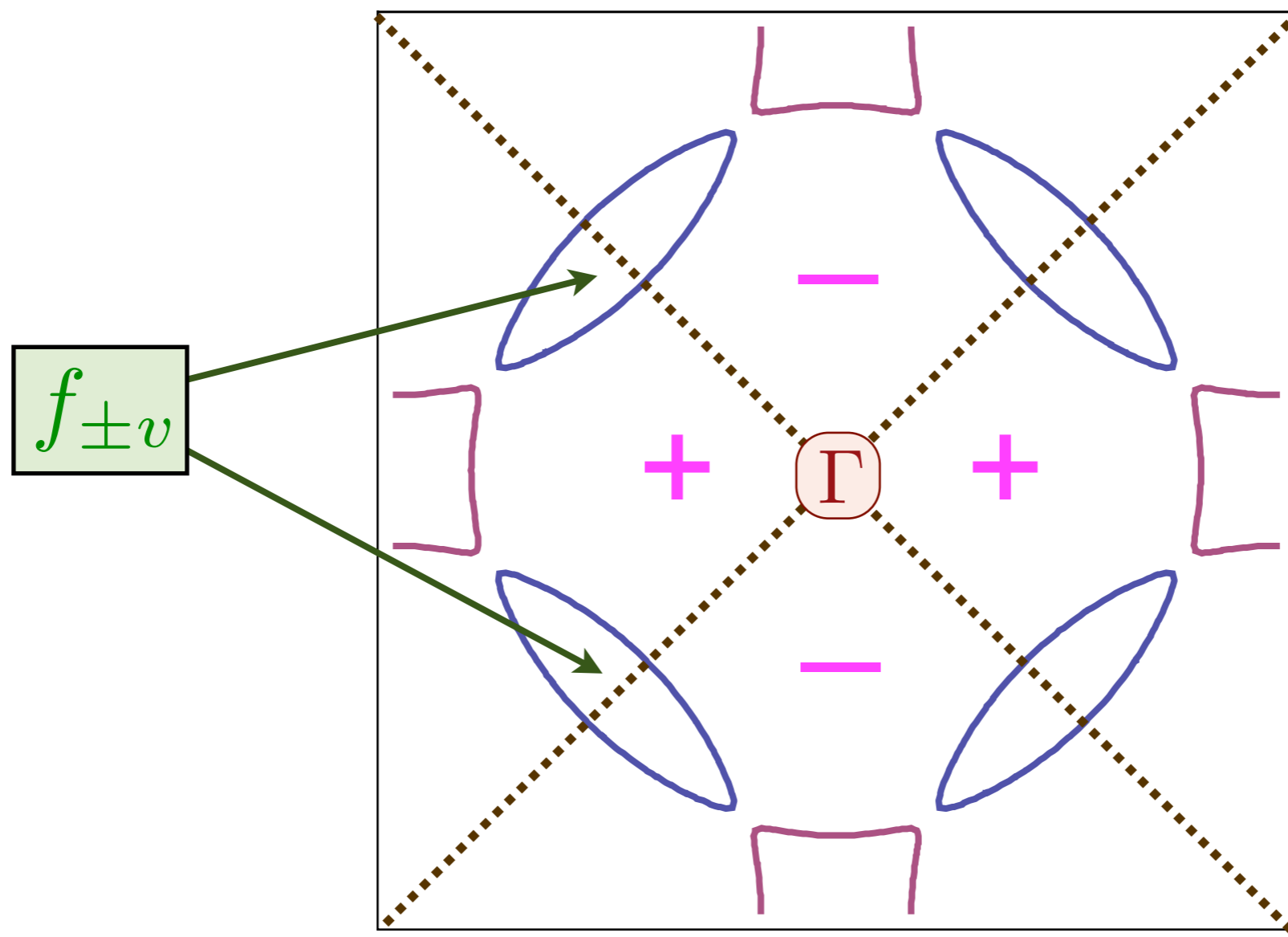
i.e. d -wave pairing !

Strong pairing of the g_{\pm} electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

Weak pairing of the f_{\pm} hole pockets



$$\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle \sim (k_x - k_y) J \langle g_+ g_- \rangle;$$

$$\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle \sim (k_x + k_y) J \langle g_+ g_- \rangle;$$

$$\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle = 0,$$

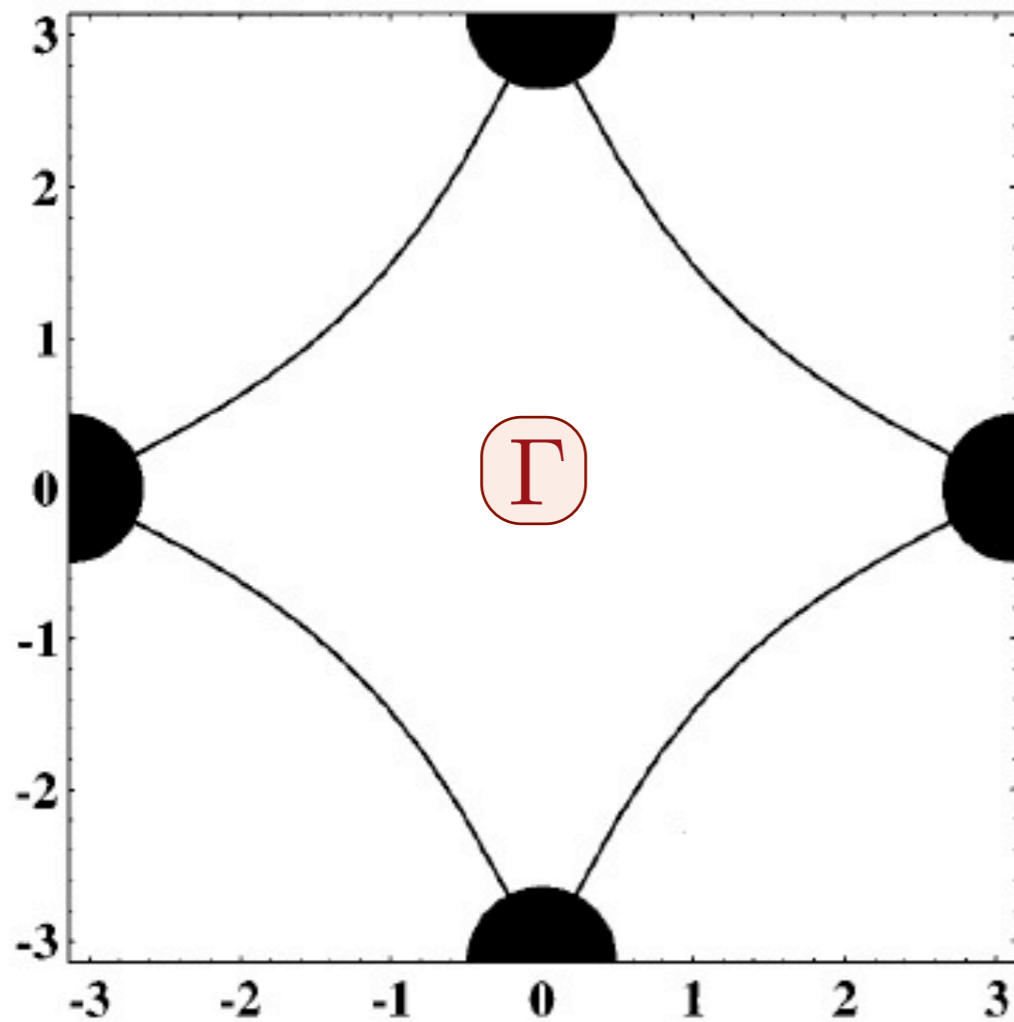


FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}' V_{p,q} (b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \text{H.c.})$$

$$+ \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma};$$

$$V_{p,q} = Va^2(p_x^2 - p_y^2)$$

-2e bosons at antinodes,
 +e fermion “arcs” at nodes,
 and proximity “Josephson”
 coupling

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, *Phys. Rev. B* **55**, 3173 (1997).

Similar features in our theory

V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Features of superconductivity

- d -wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.

V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Features of superconductivity

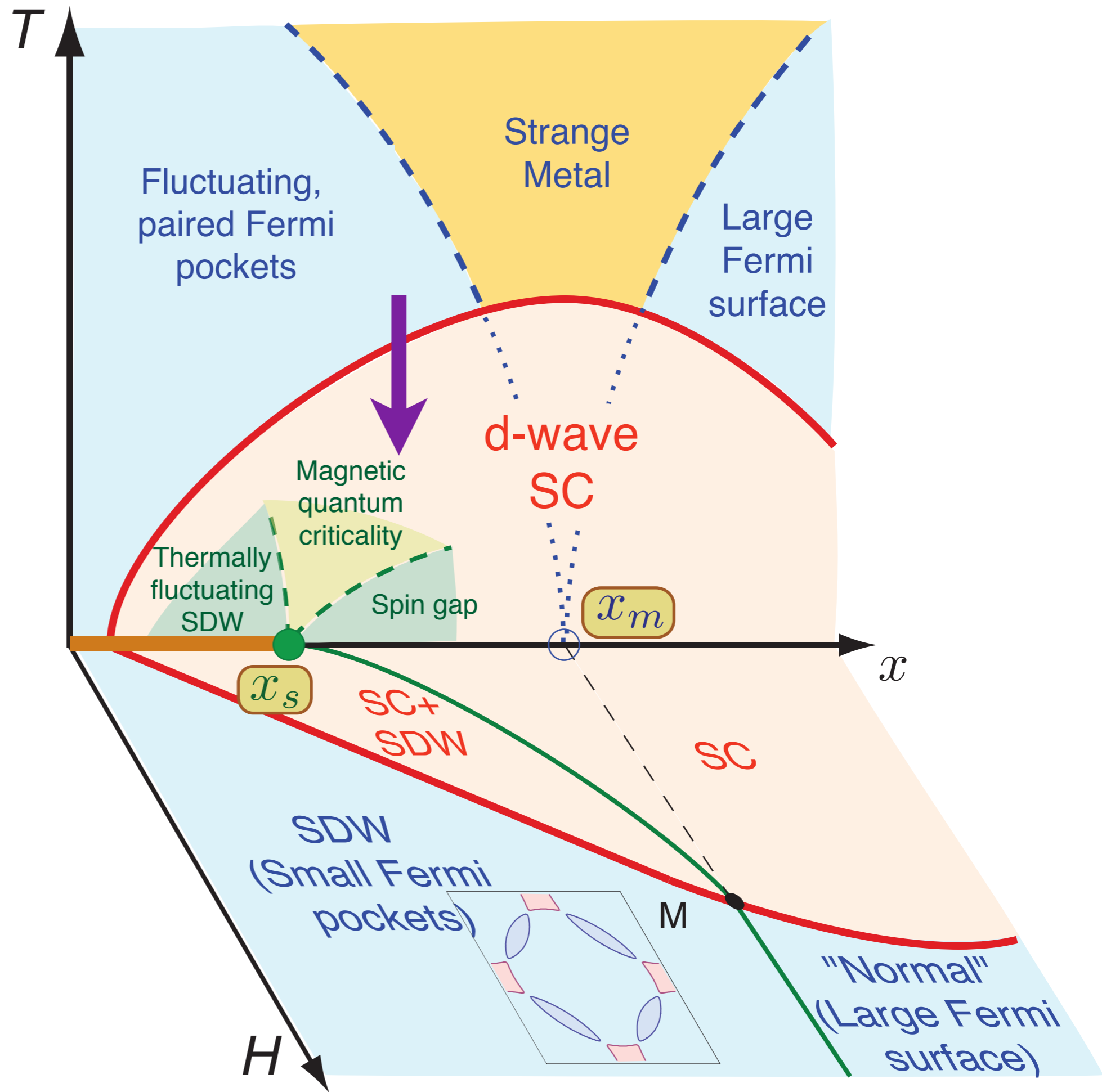
- d -wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.

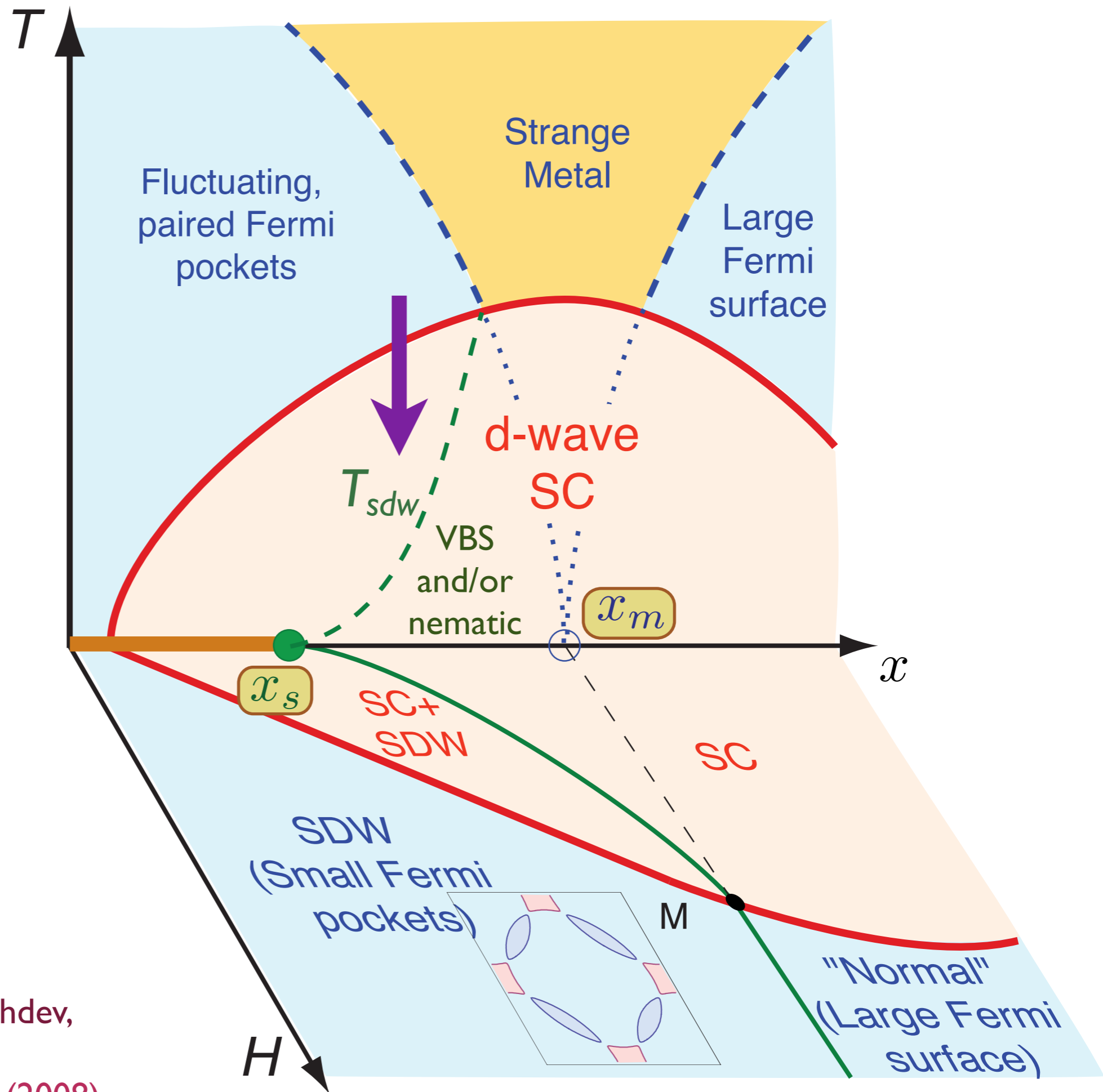
Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

Features of superconductivity

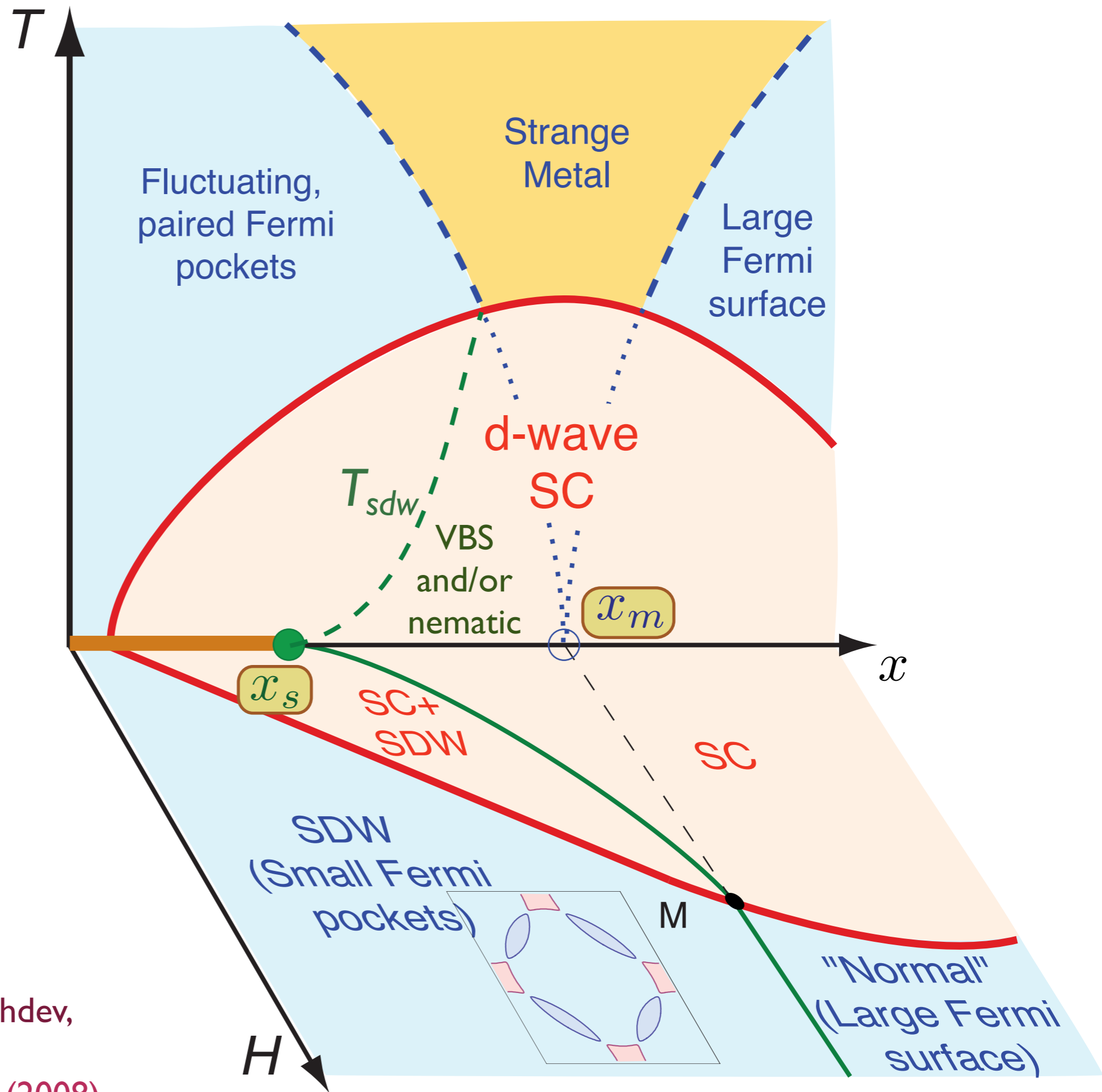
- d -wave superconductivity.
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- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.
- After onset of superconductivity, monopoles condense and lead to confinement and **nematic** and/or **valence bond solid (VBS)** order.

R. K. Kaul, M. Metlitski, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

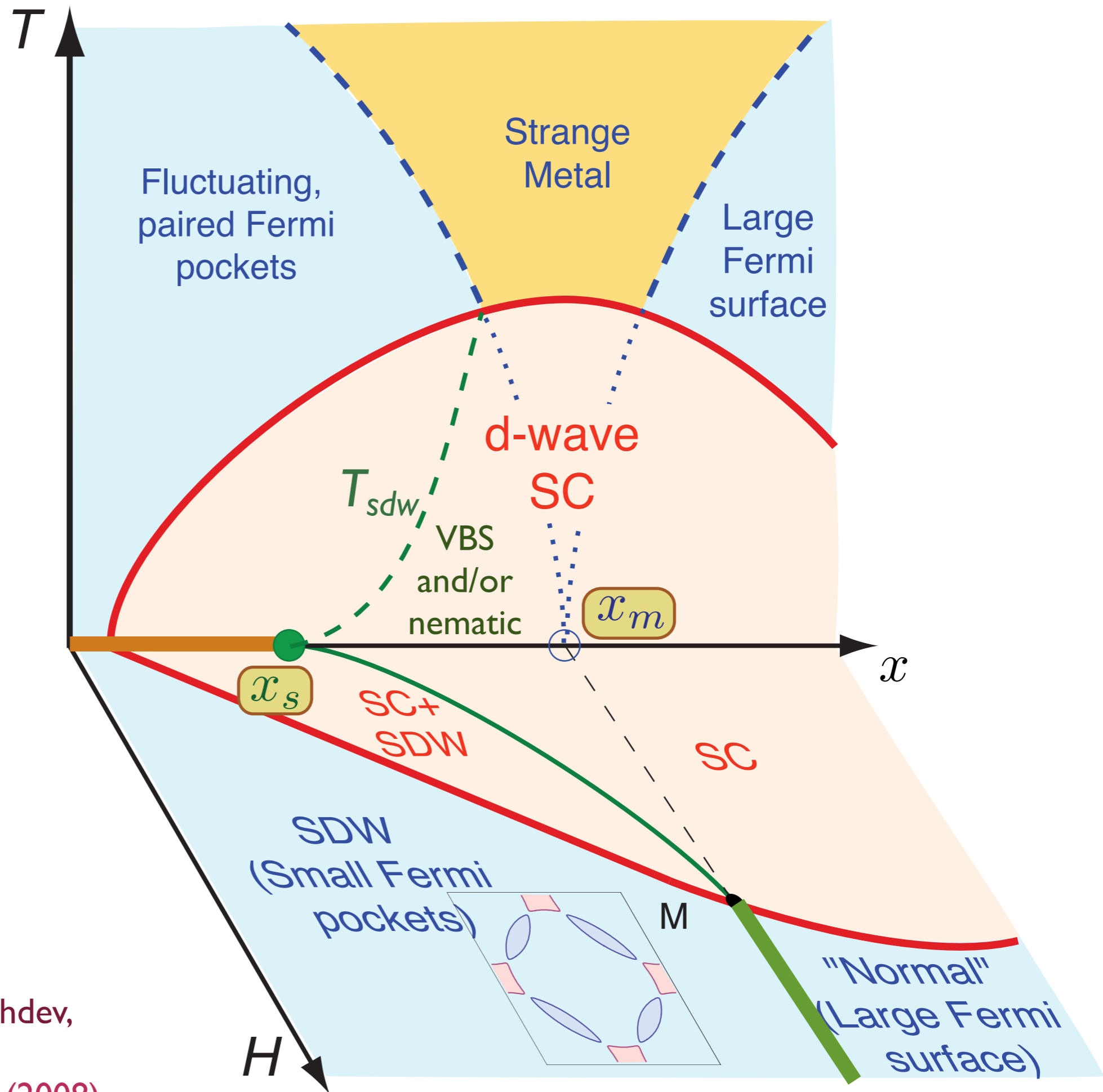




R. K. Kaul, M. Metlitski, S. Sachdev,
 and Cenke Xu,
Physical Review B **78**, 045110 (2008).



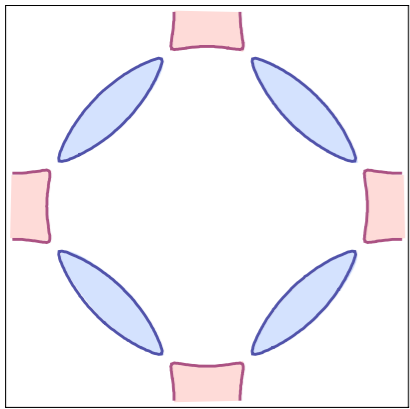
R. K. Kaul, M. Metlitski, S. Sachdev,
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Physical Review B **78**, 045110 (2008).



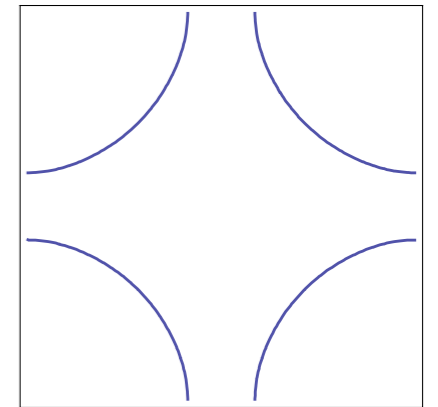
R. K. Kaul, M. Metlitski, S. Sachdev,
 and Cenke Xu,
Physical Review B **78**, 045110 (2008).

Quantum phase transitions in metal

$\langle z \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



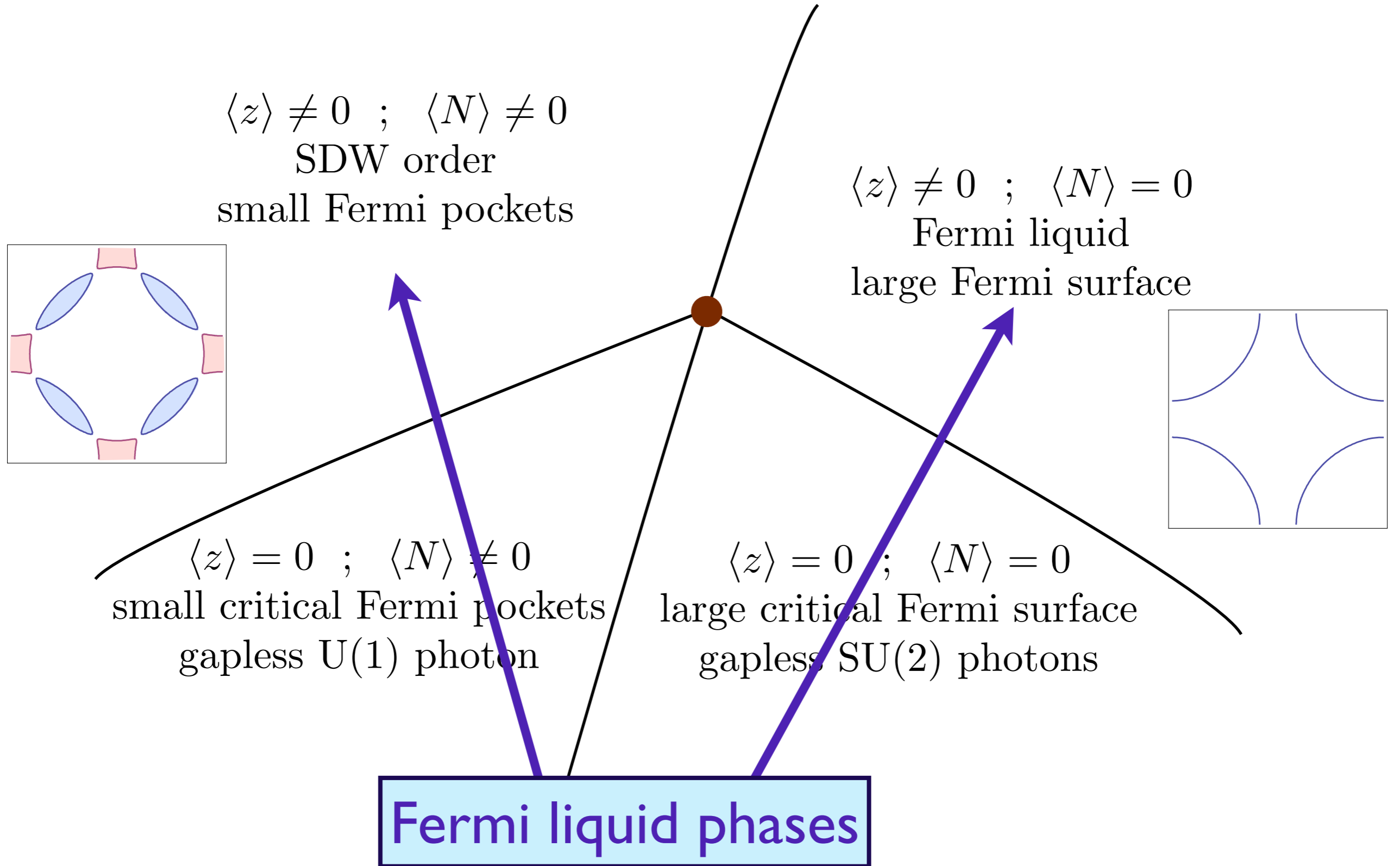
$\langle z \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



$\langle z \rangle = 0$; $\langle N \rangle \neq 0$
small critical Fermi pockets
gapless U(1) photon

$\langle z \rangle = 0$; $\langle N \rangle = 0$
large critical Fermi surface
gapless SU(2) photons

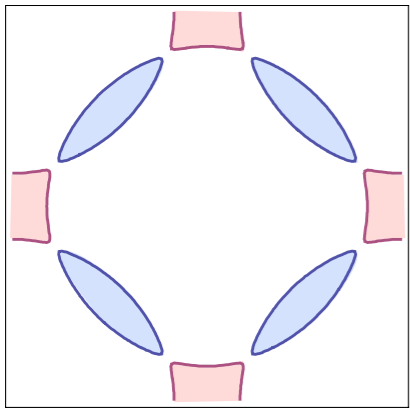
Quantum phase transitions in metal



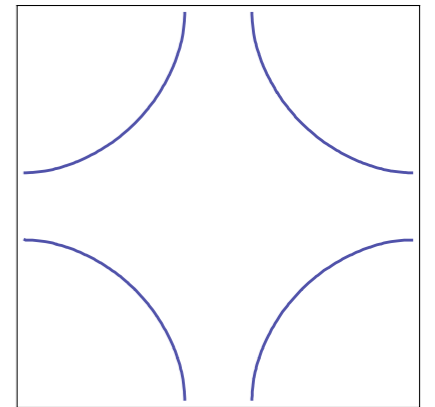
S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Quantum phase transitions in metal

$\langle z \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



$\langle z \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



$\langle z \rangle = 0$; $\langle N \rangle \neq 0$
small critical Fermi pockets
gapless U(1) photon

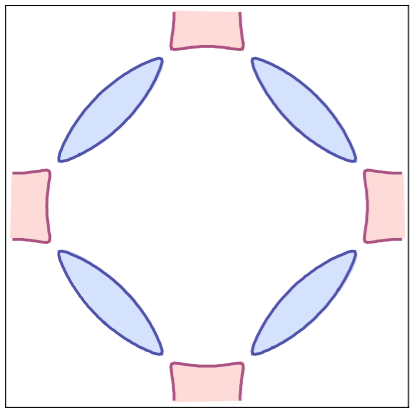
$\langle z \rangle = 0$; $\langle N \rangle = 0$
large critical Fermi surface
gapless SU(2) photons

non Fermi liquid phases

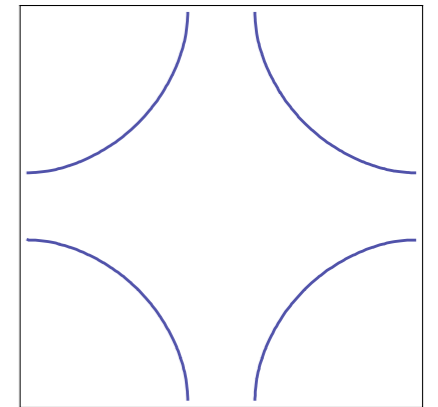
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Quantum phase transitions in metal

$\langle z \rangle \neq 0$; $\langle N \rangle \neq 0$
SDW order
small Fermi pockets



$\langle z \rangle \neq 0$; $\langle N \rangle = 0$
Fermi liquid
large Fermi surface



$\langle z \rangle = 0$; $\langle N \rangle \neq 0$
small critical Fermi pockets
gapless U(1) photon

$\langle z \rangle = 0$; $\langle N \rangle = 0$
large critical Fermi surface
gapless SU(2) photons

Hertz-Millis-Moriya theory

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)



Max Metlitski

M. Metlitski and S. Sachdev, *to appear*

Ar. Abanov, A.V. Chubukov, and J. Schmalian,
Advances in Physics **52**, 119 (2003)

Sung-Sik Lee, arXiv:0905.4532.

Hertz-Millis-Moriya (HMM) theory:
mean field theory

+

Gaussian fluctuations of overdamped paramagnons.

Theory for the onset
of spin density wave
order in metals is
strongly coupled in
two dimensions

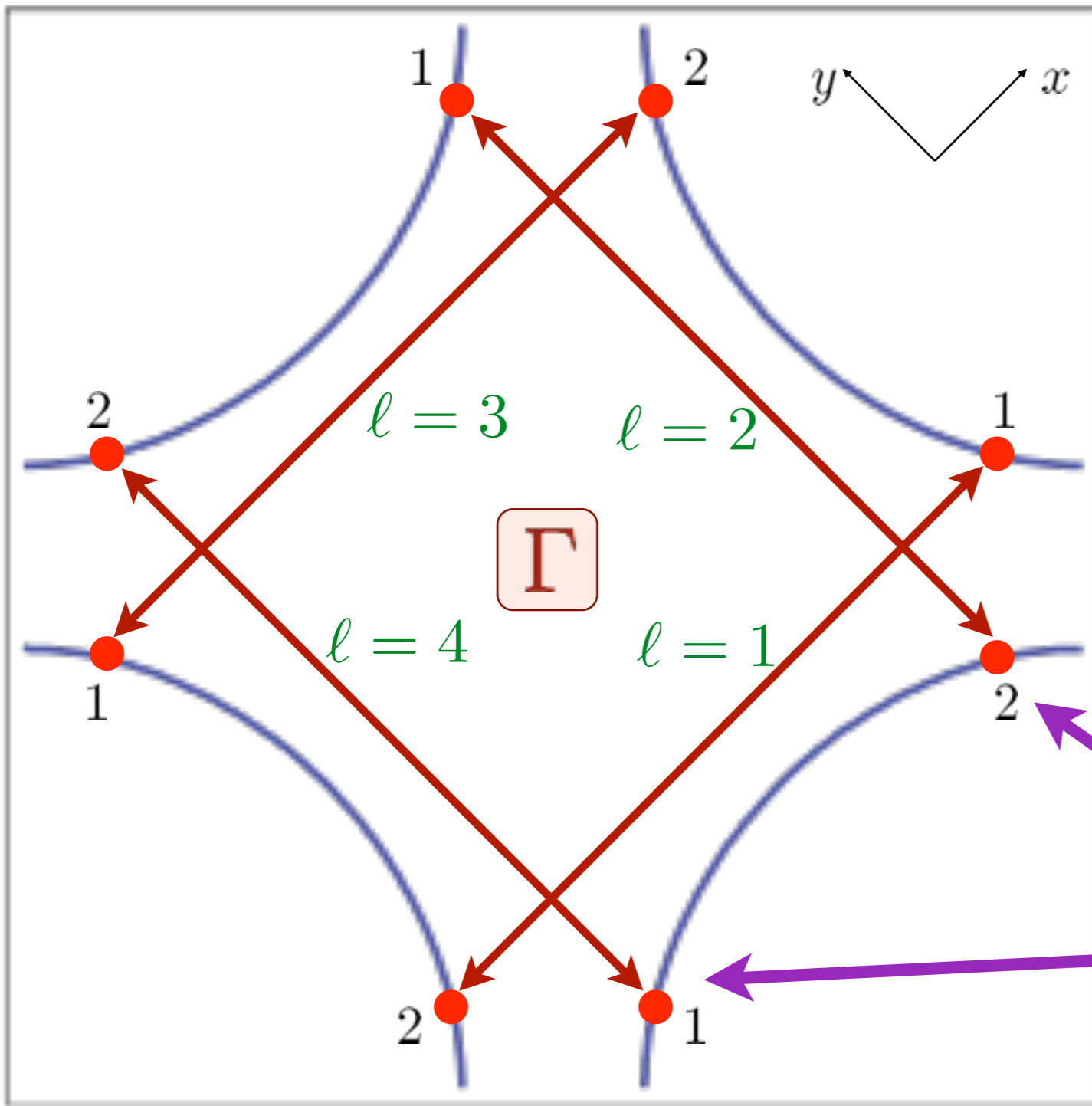
Start from the “spin-fermion” model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp(-\mathcal{S})$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2r \left[(\partial_r \vec{\varphi})^2 + \frac{1}{c^2} (\partial_\tau \vec{\varphi})^2 \right]$$



Low energy fermions

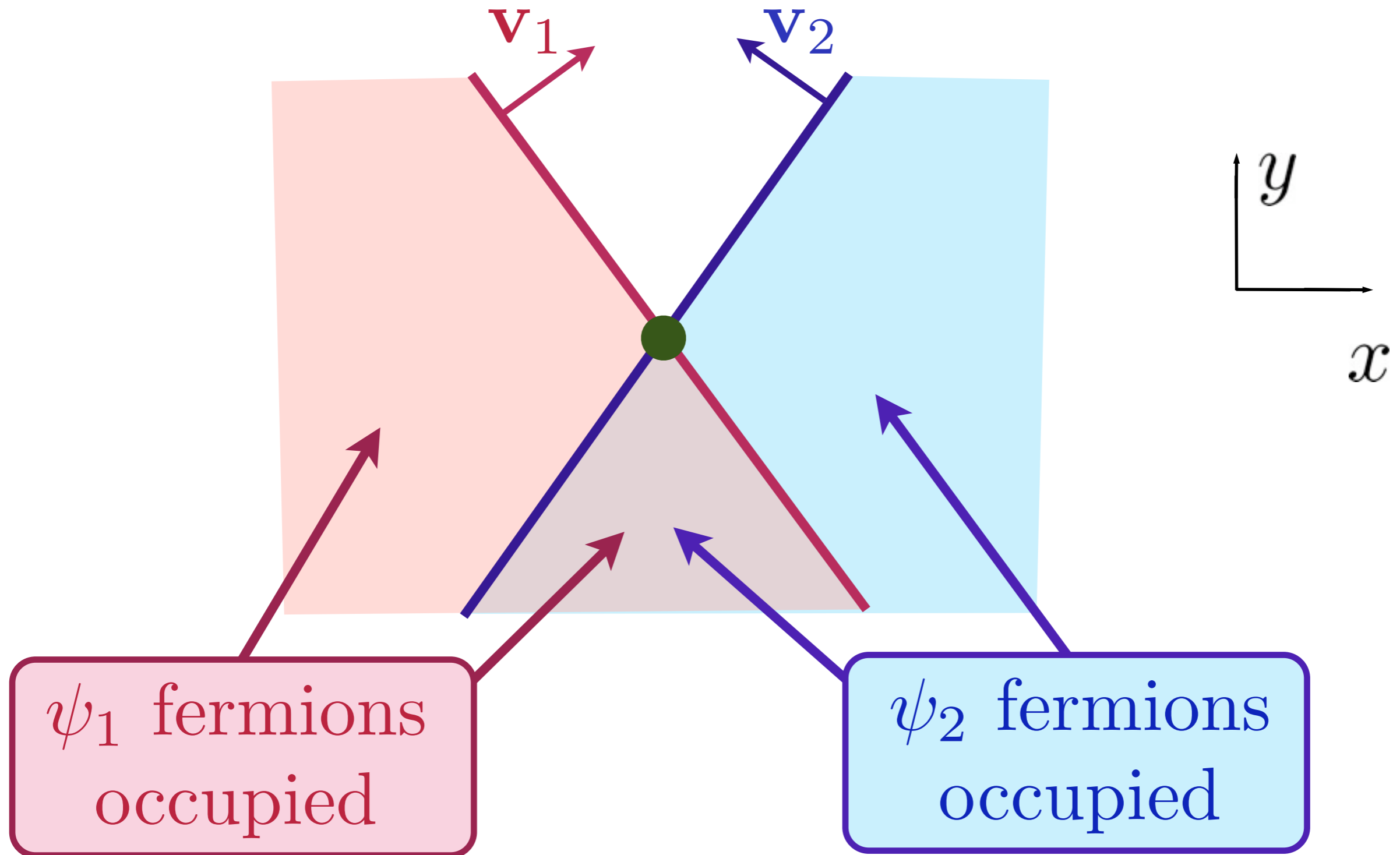
$$\psi_{1\alpha}^l, \psi_{2\alpha}^l$$

$$l = 1, \dots, 4$$

$$\mathcal{L}_f = \psi_{1\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^l \cdot \nabla_r) \psi_{1\alpha}^l + \psi_{2\alpha}^{l\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^l \cdot \nabla_r) \psi_{2\alpha}^l$$

$$\mathbf{v}_1^{l=1} = (v_x, v_y), \quad \mathbf{v}_2^{l=1} = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$



$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling:
$$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

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HMM theory

Integrate out fermions and obtain non-local corrections to \mathcal{L}_φ

$$\mathcal{L}_\varphi = \frac{1}{2} \vec{\varphi}^2 [\mathbf{q}^2 + \gamma |\omega|] / 2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings

Ar.Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

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$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

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$$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

Perform RG on both fermions and $\vec{\varphi}$,
using a *local* field theory.

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i \frac{1}{N\sqrt{\gamma}v} \sqrt{\omega} F \left(\frac{v^2 q^2}{\omega} \right)}$$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

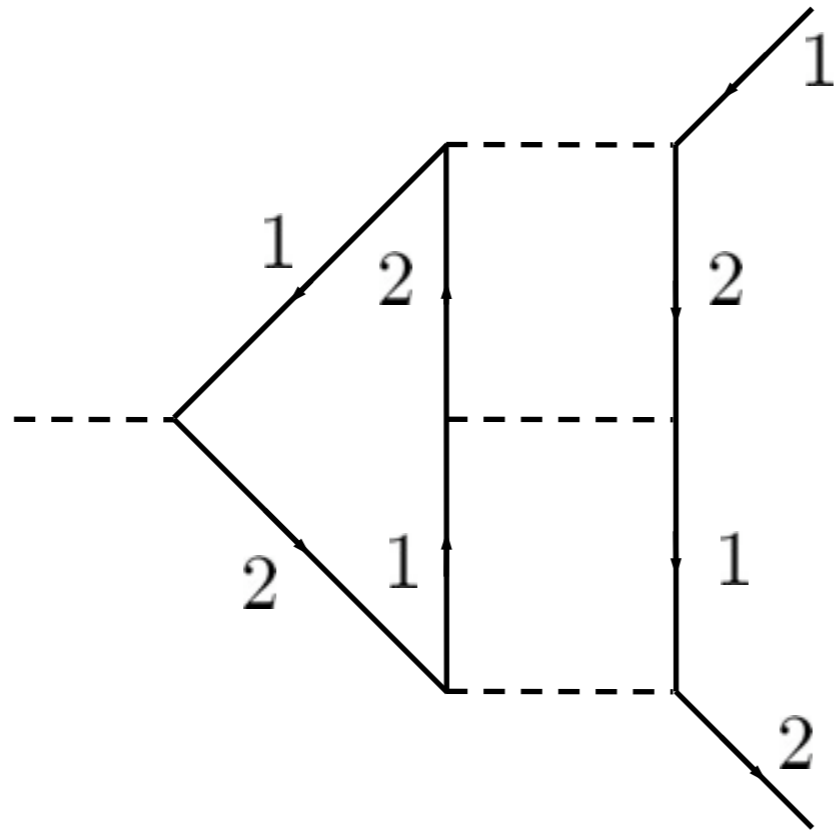
$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i \frac{1}{N\sqrt{\gamma}v} \sqrt{\omega} F \left(\frac{v^2 q^2}{\omega} \right)}$$

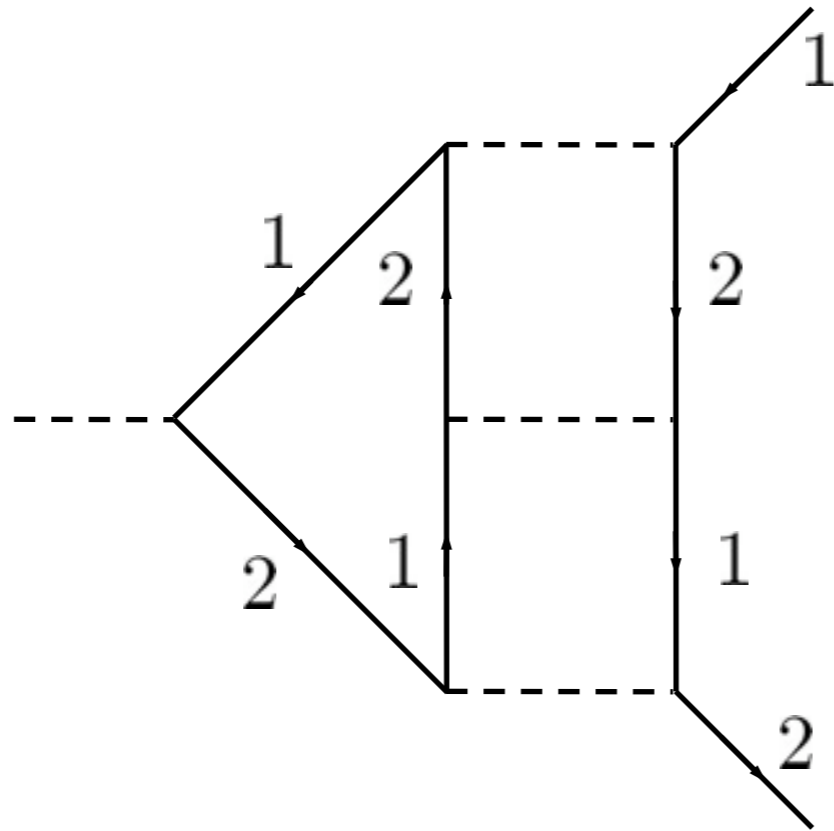
 **Dangerous**

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops
 (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

Actual order $\sim \frac{1}{N^0}$

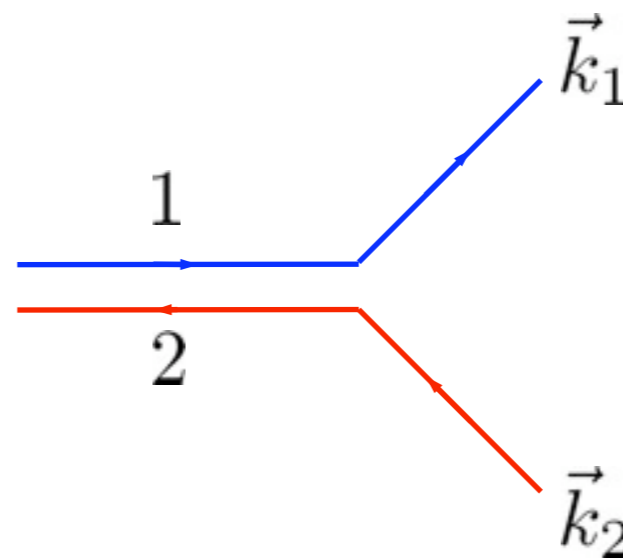
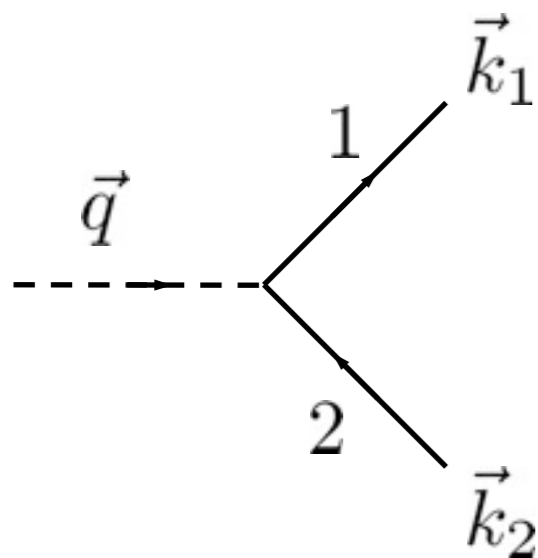
Double line representation

- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \quad \Sigma_1 \sim \frac{1}{N}$$

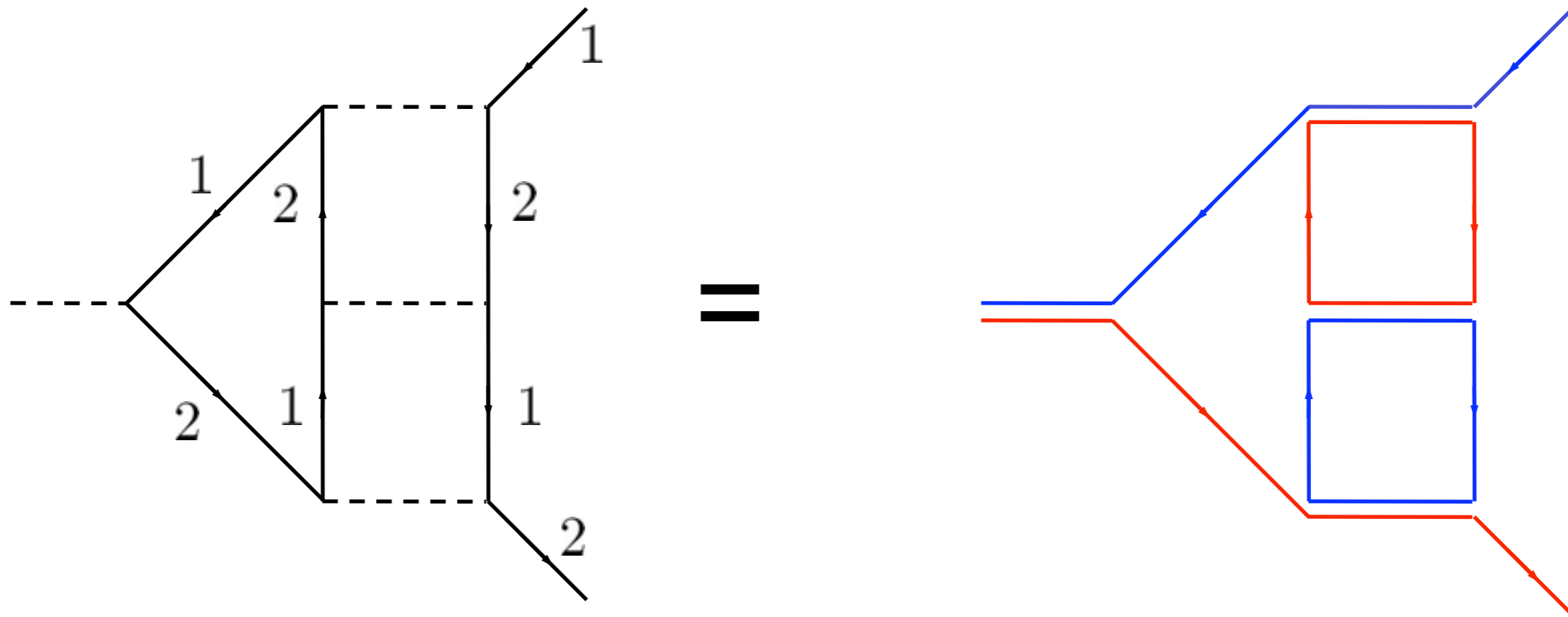
- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

$$\vec{q} = \vec{k}_1 - \vec{k}_2 \quad \vec{k}_1 \in \text{FS of } \psi_1 \quad \vec{k}_2 \in \text{FS of } \psi_2$$



R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
 S.W.Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

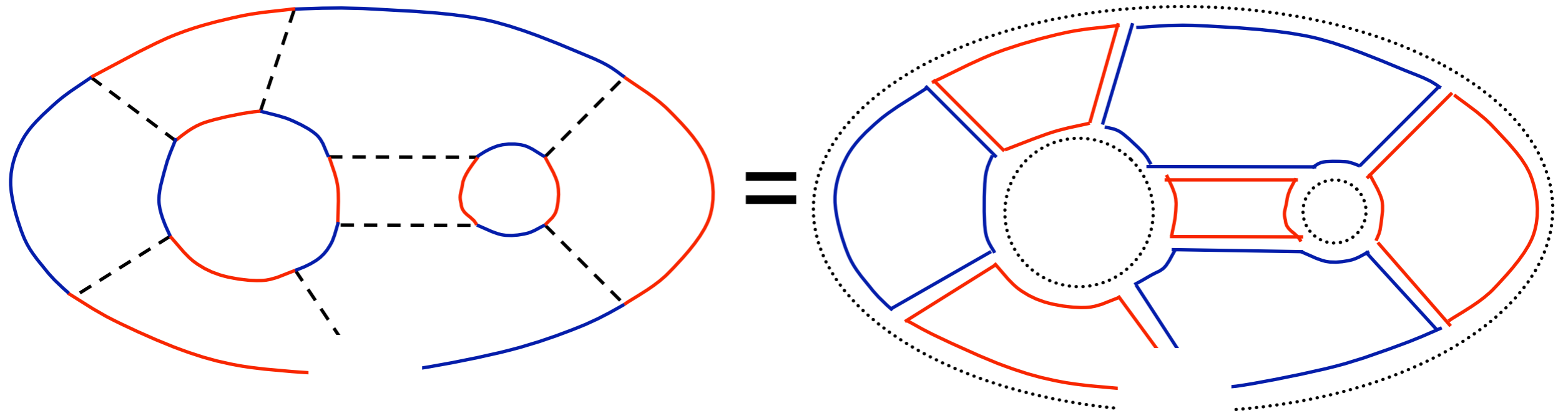
New infra-red singularities as $\zeta \rightarrow 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \rightarrow 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops
 (Breakdown of Migdal-Eliashberg)

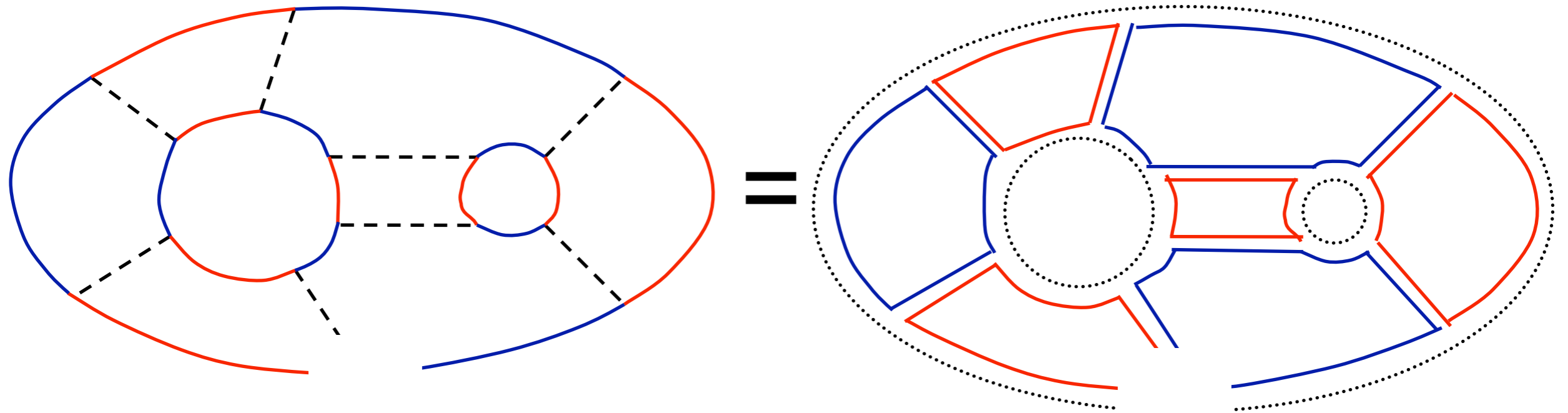


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines
 by drawing additional dotted single line loops for each fermion loop

Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \rightarrow 0$ at higher loops
(Breakdown of Migdal-Eliashberg)

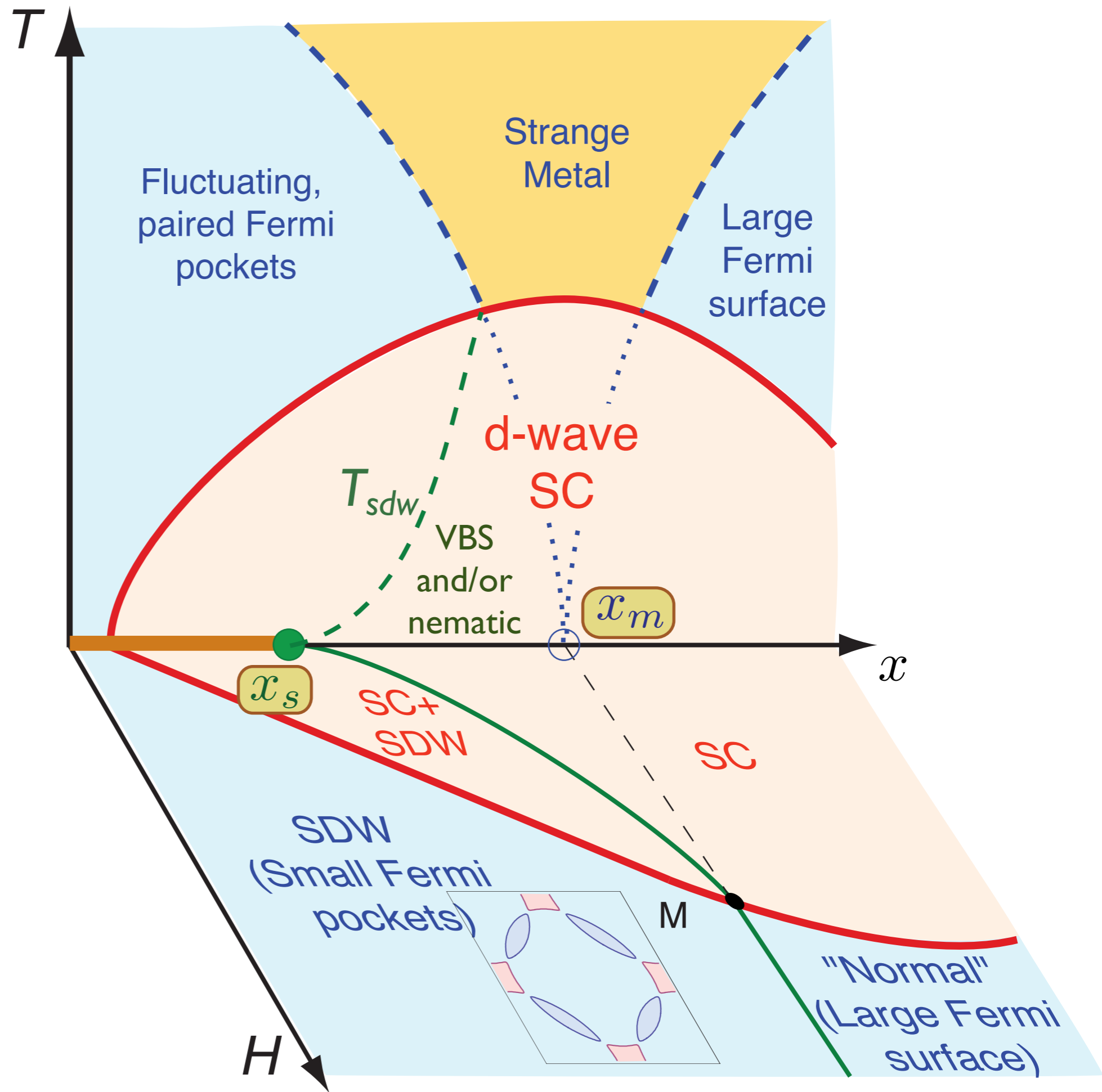


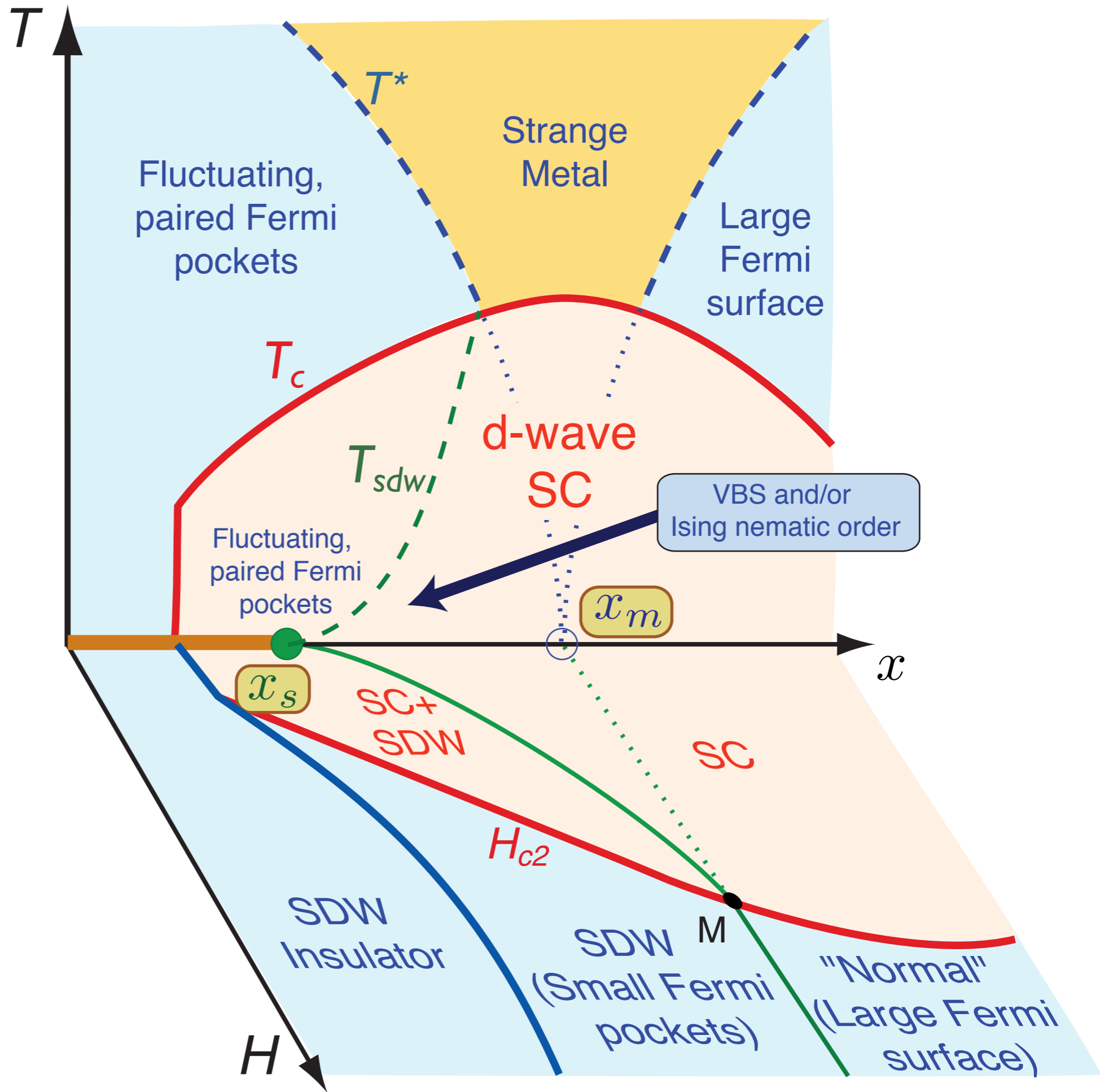
$$\text{Actual order} \sim \frac{1}{N^0}$$



A consistent analysis requires
resummation of all planar graphs







Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theory for the onset of spin density wave order in metals is strongly coupled in two dimensions