Quantum spin liquids and the phases of the cuprates

Ultra Quantum Matter meeting
Harvard University
September 15, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.Sachdev, *Phys. Rev. B* **108**, 045123 (2023)
Maine Christos and S.Sachdev, arXiv:2308.03835

PHYSICS
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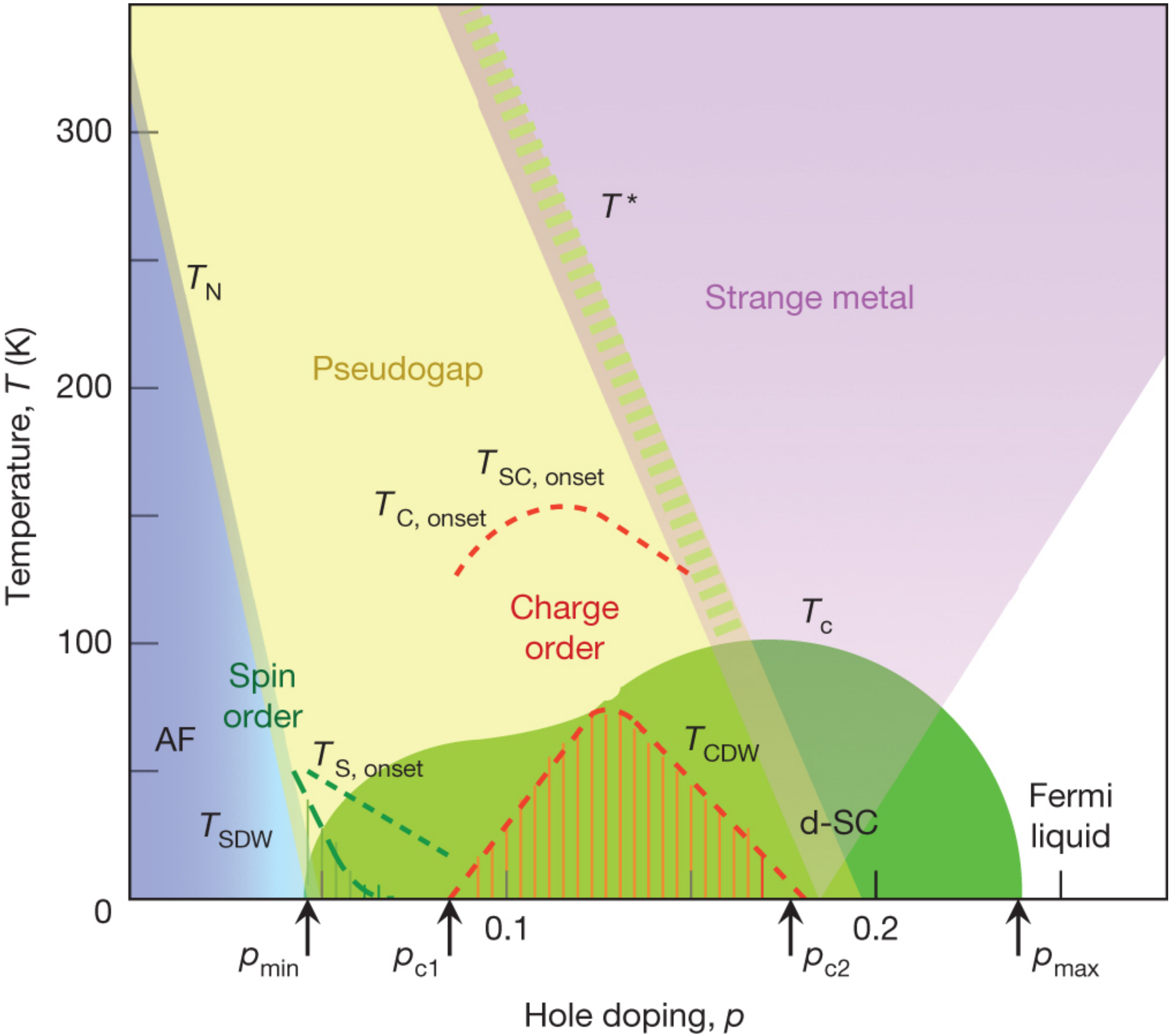
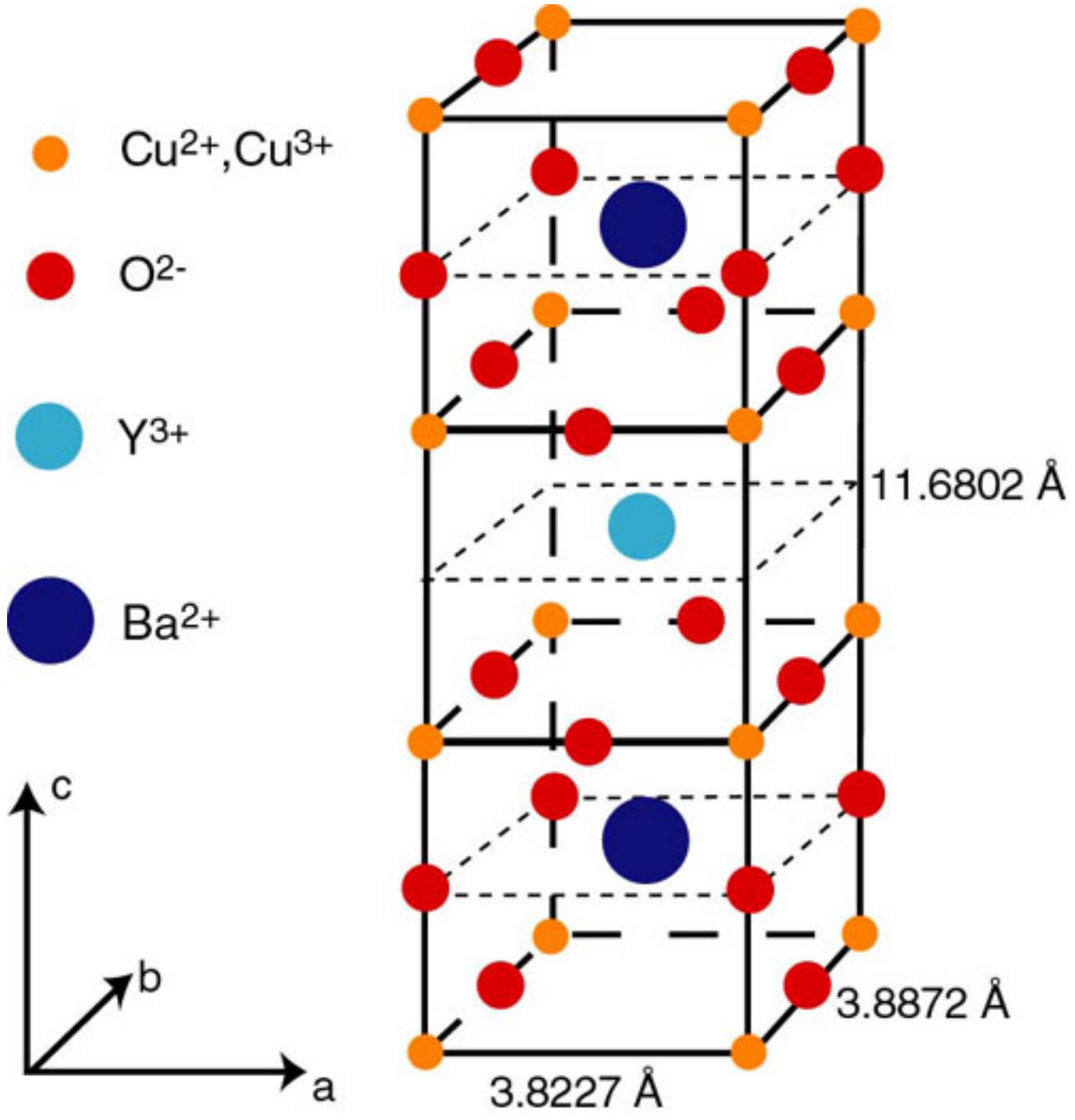
HARVARD

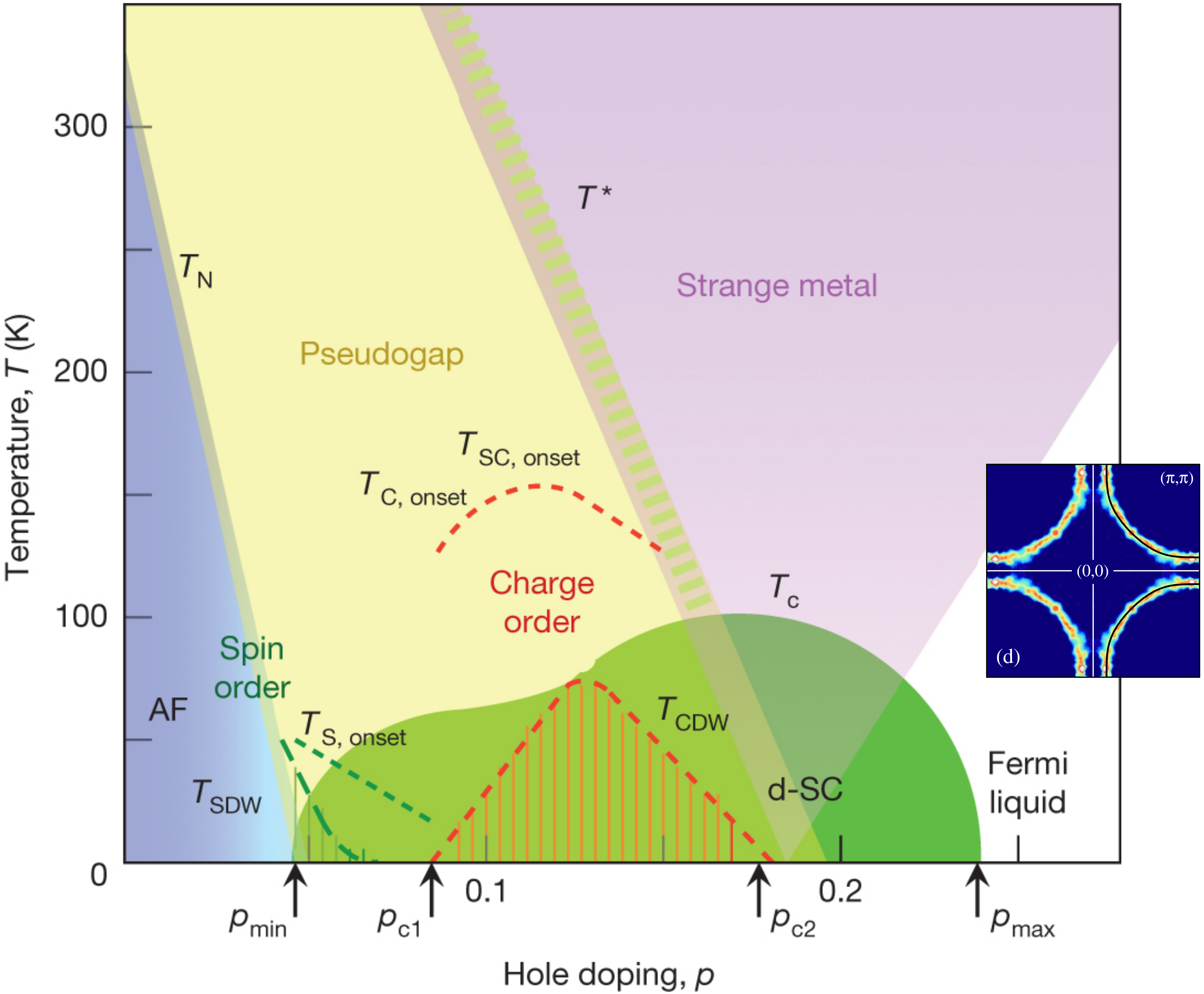
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





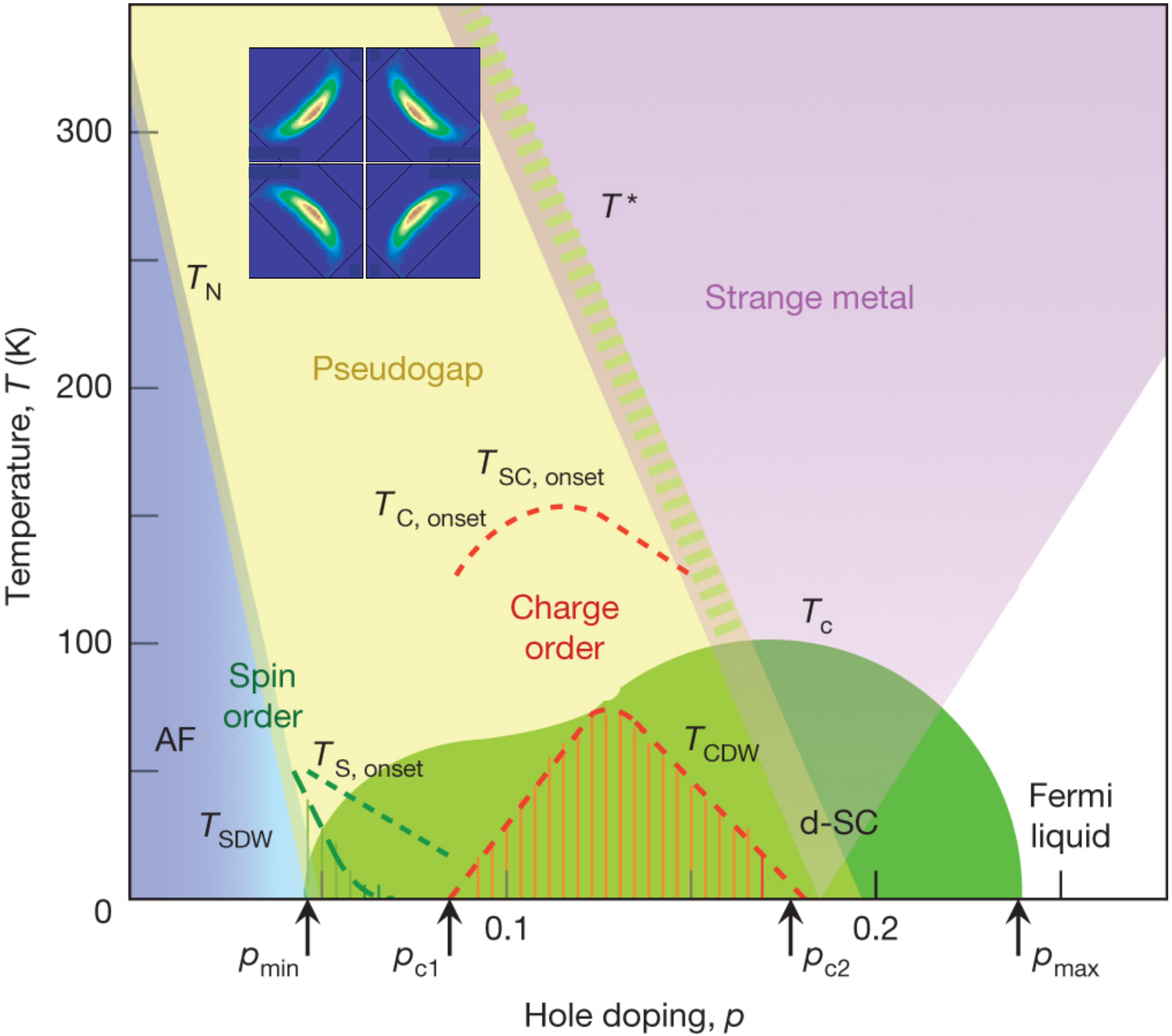
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



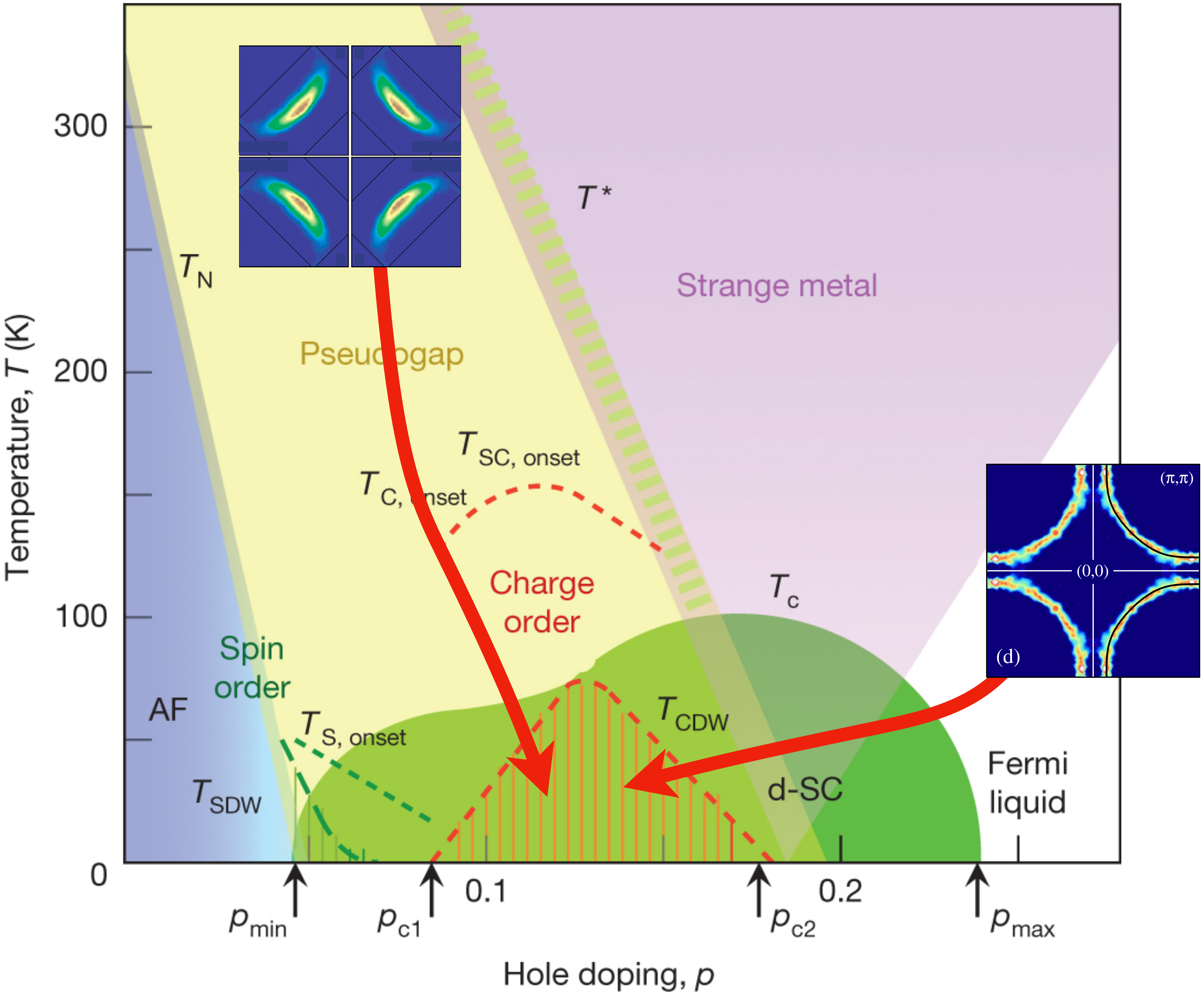
Fermi liquid
in the
overdoped metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Theory for "pseudogap metal" with "Fermi arcs"?

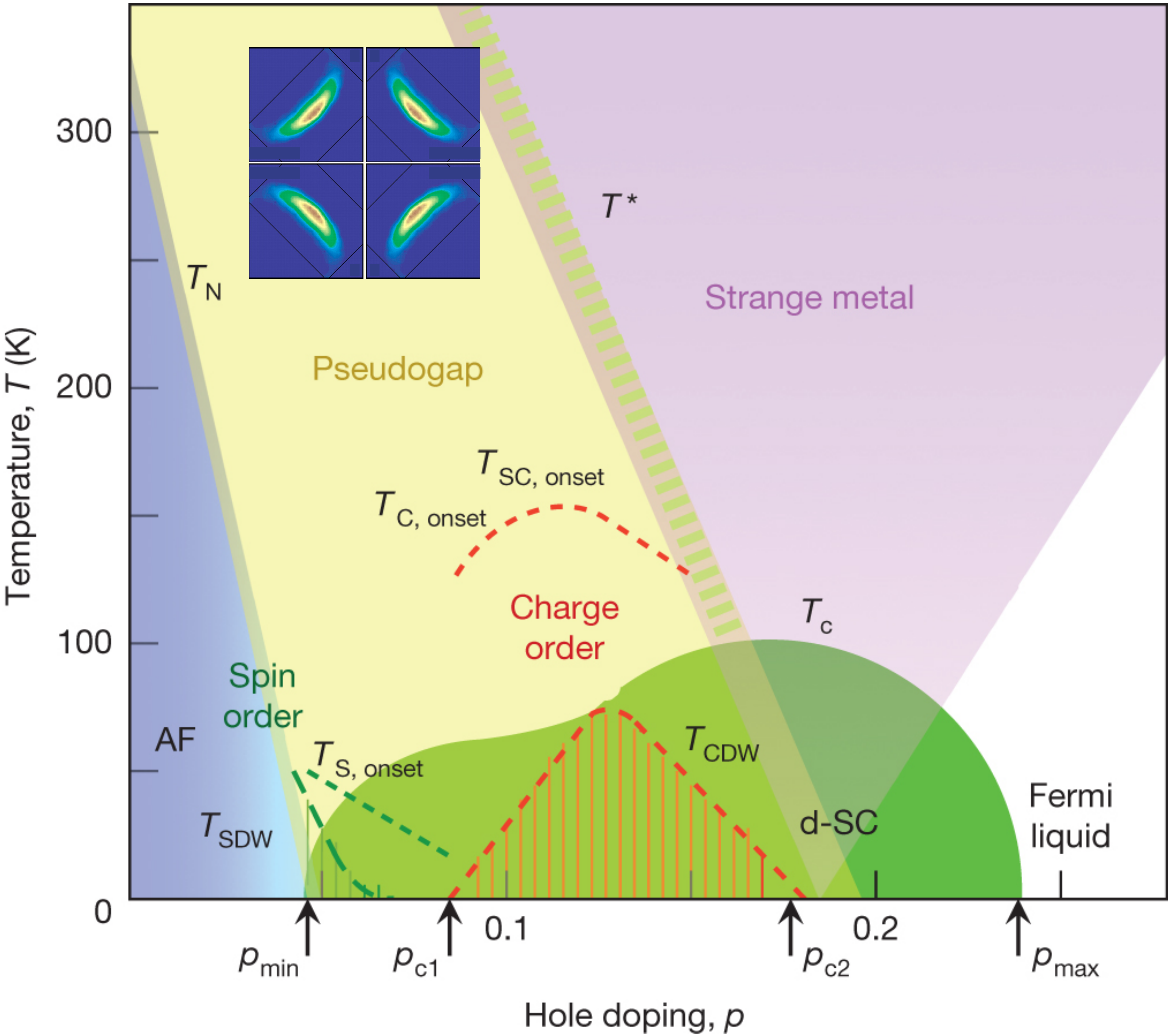
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

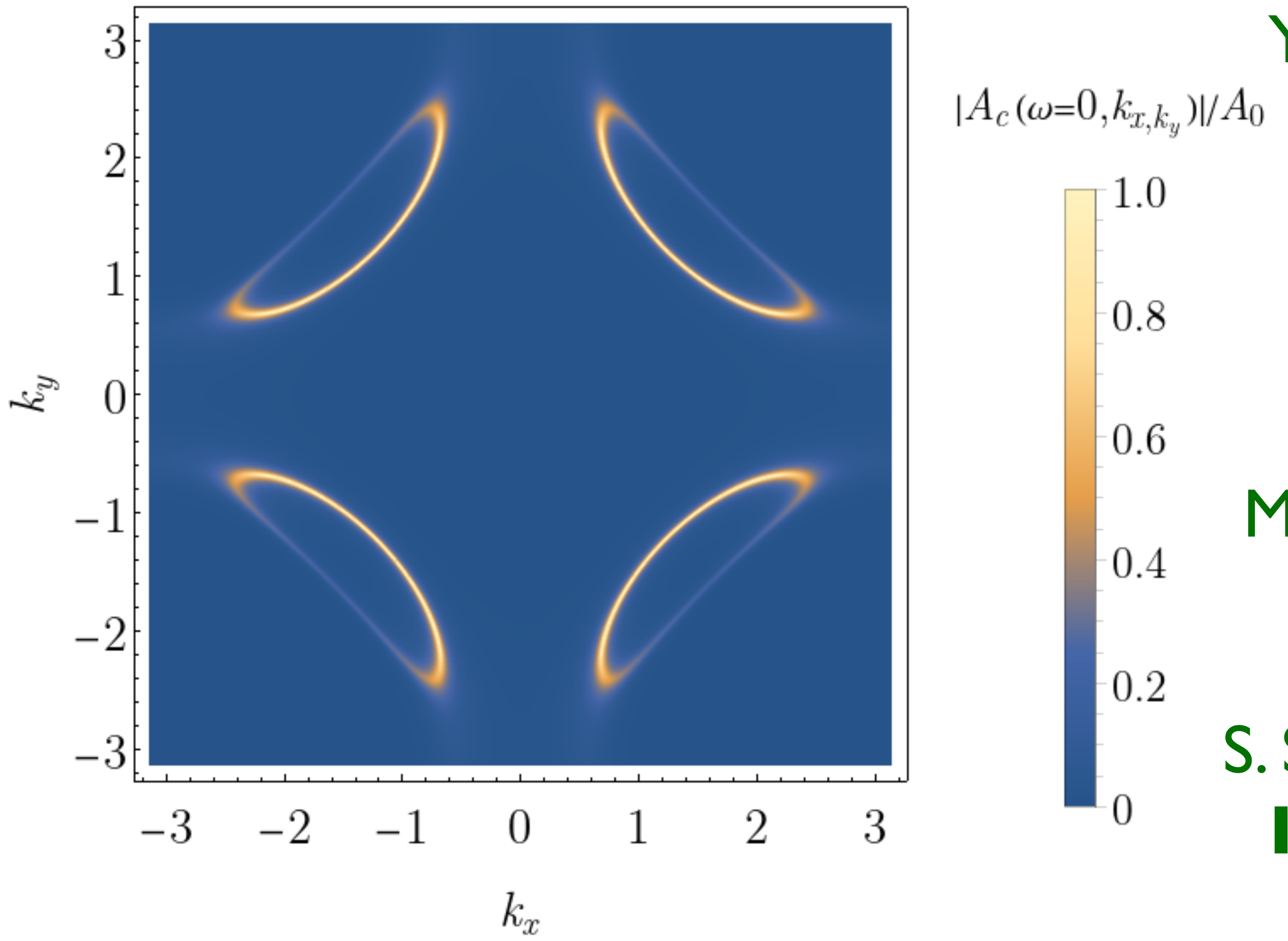


Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





Y. Qi and S. Sachdev *Phys. Rev. B* **81**, 115129 (2010)

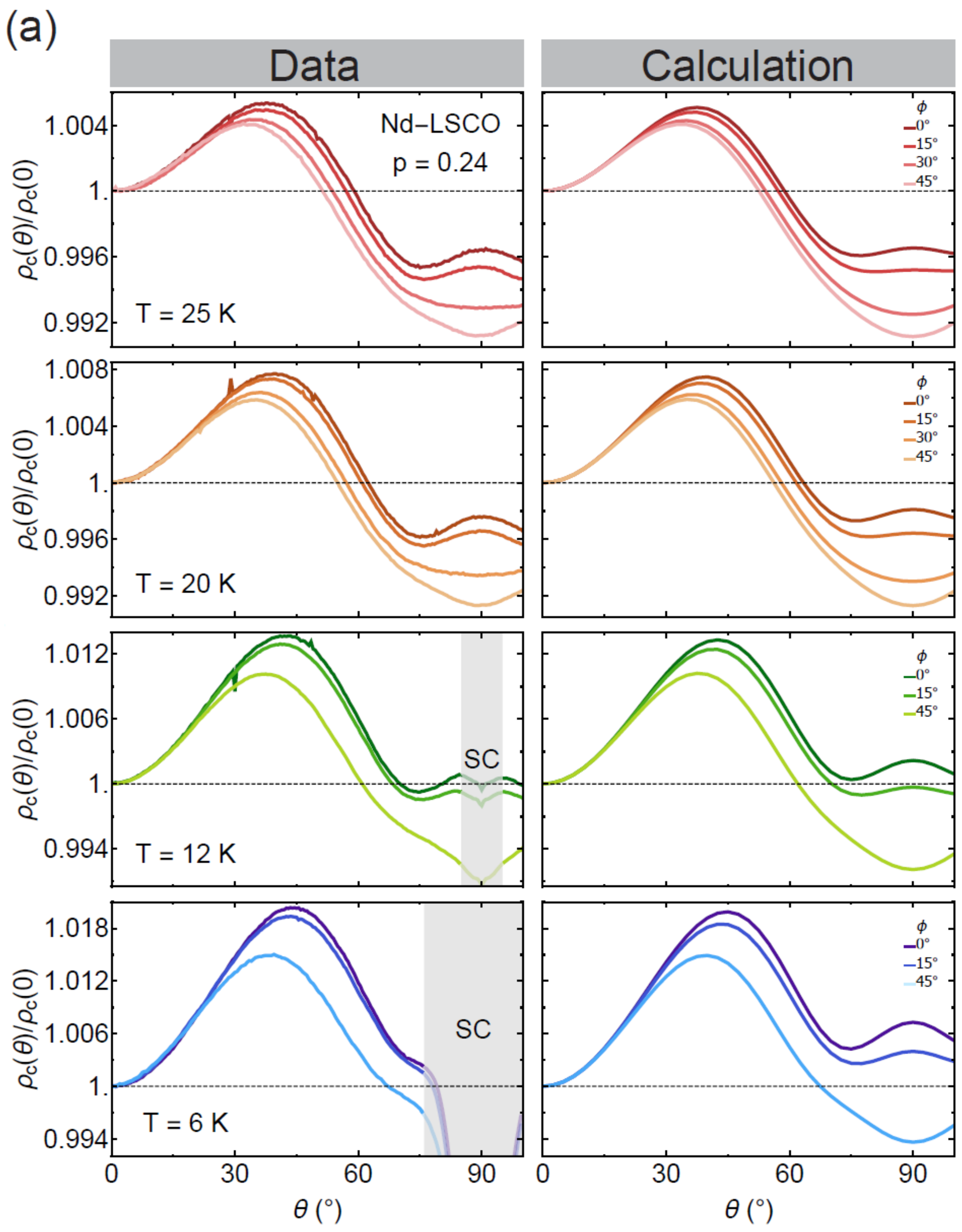
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

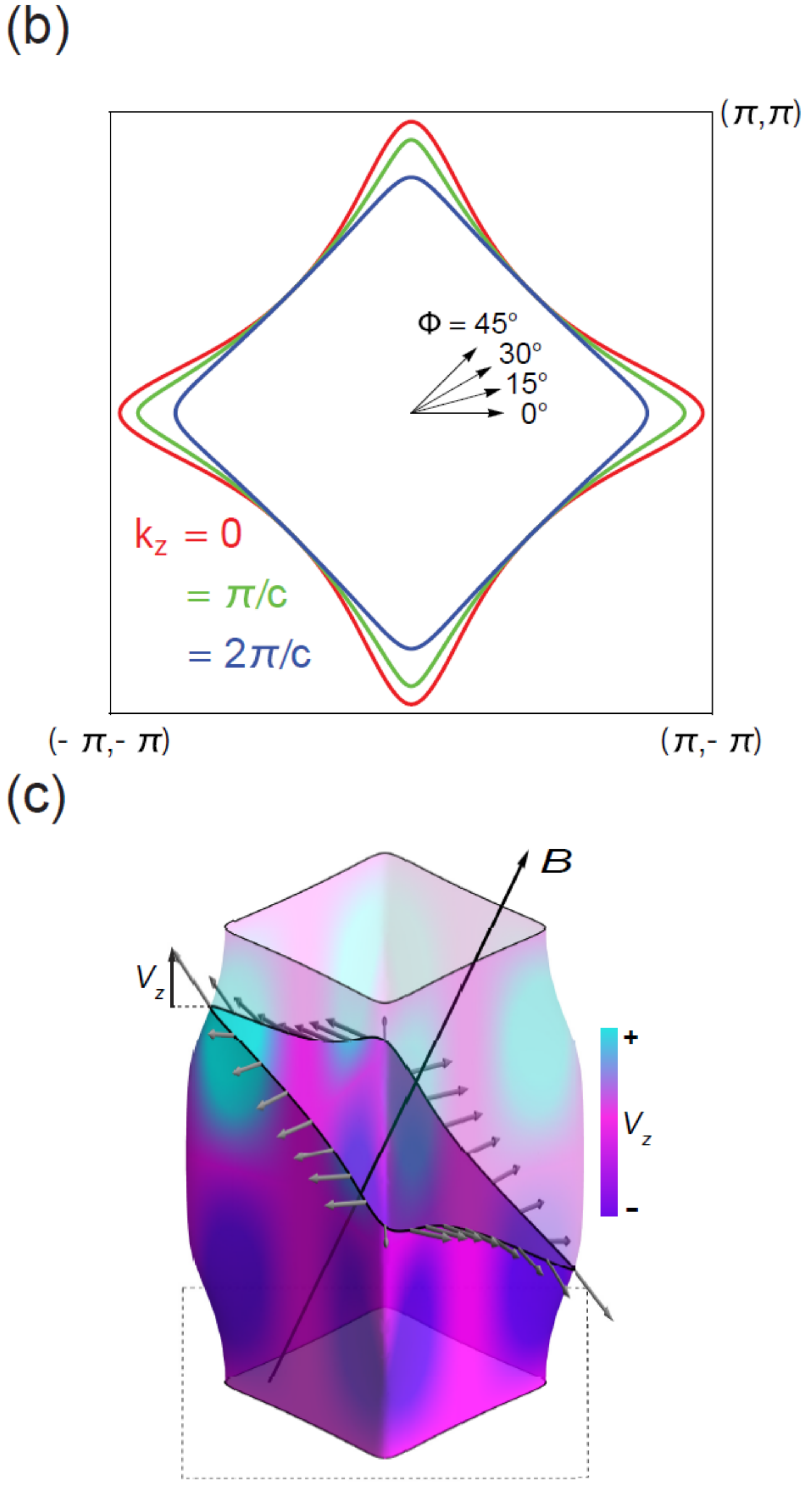
Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *Phys. Rev. B* **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, *Phys. Rev. B* **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *Phys. Rev. Lett.* **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, *Phys. Rev. Lett.* **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022).
Jinchao Zhao, Gabriele La Nave, Philip Phillips, arXiv:2304.04787.

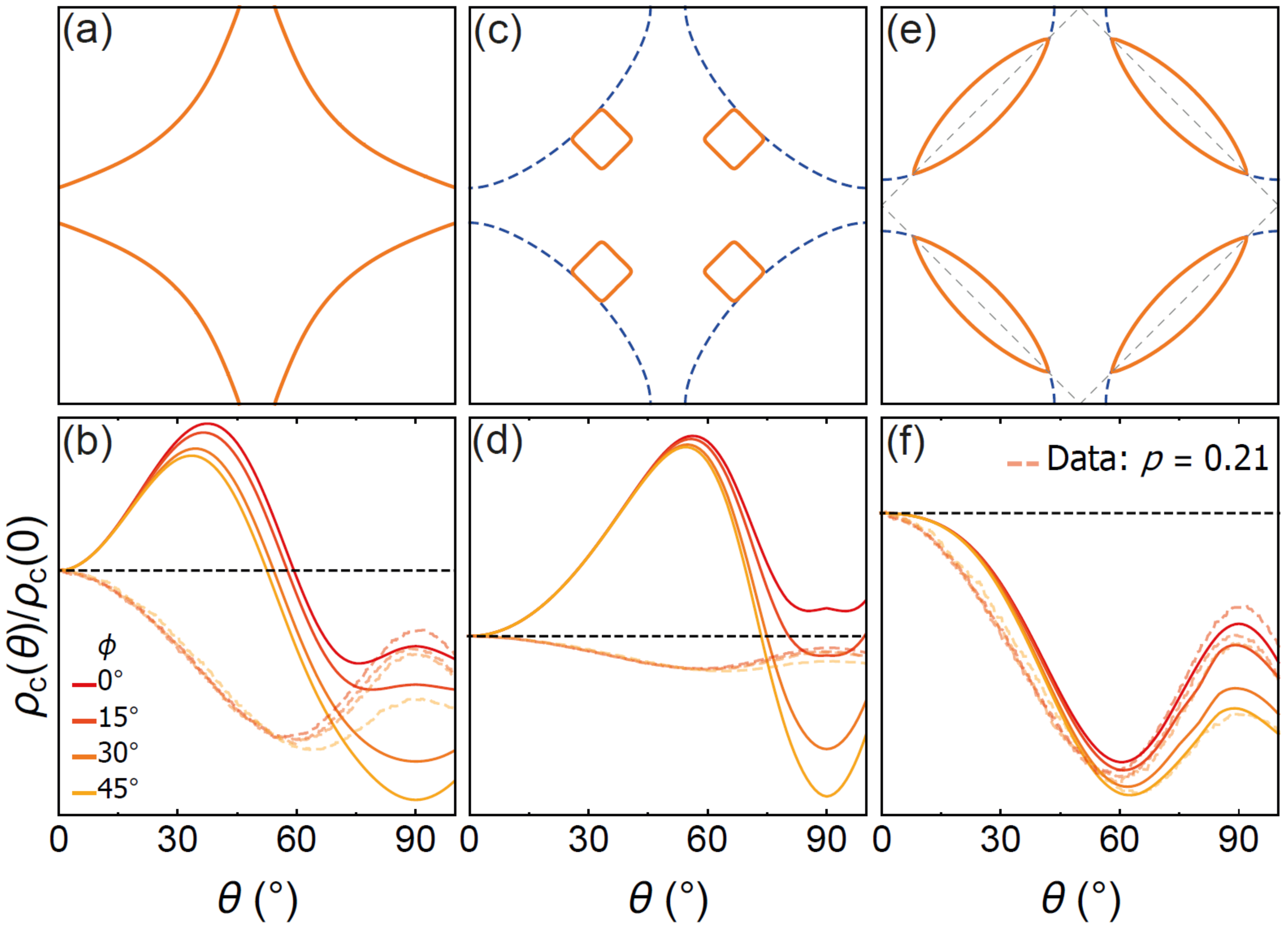
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)



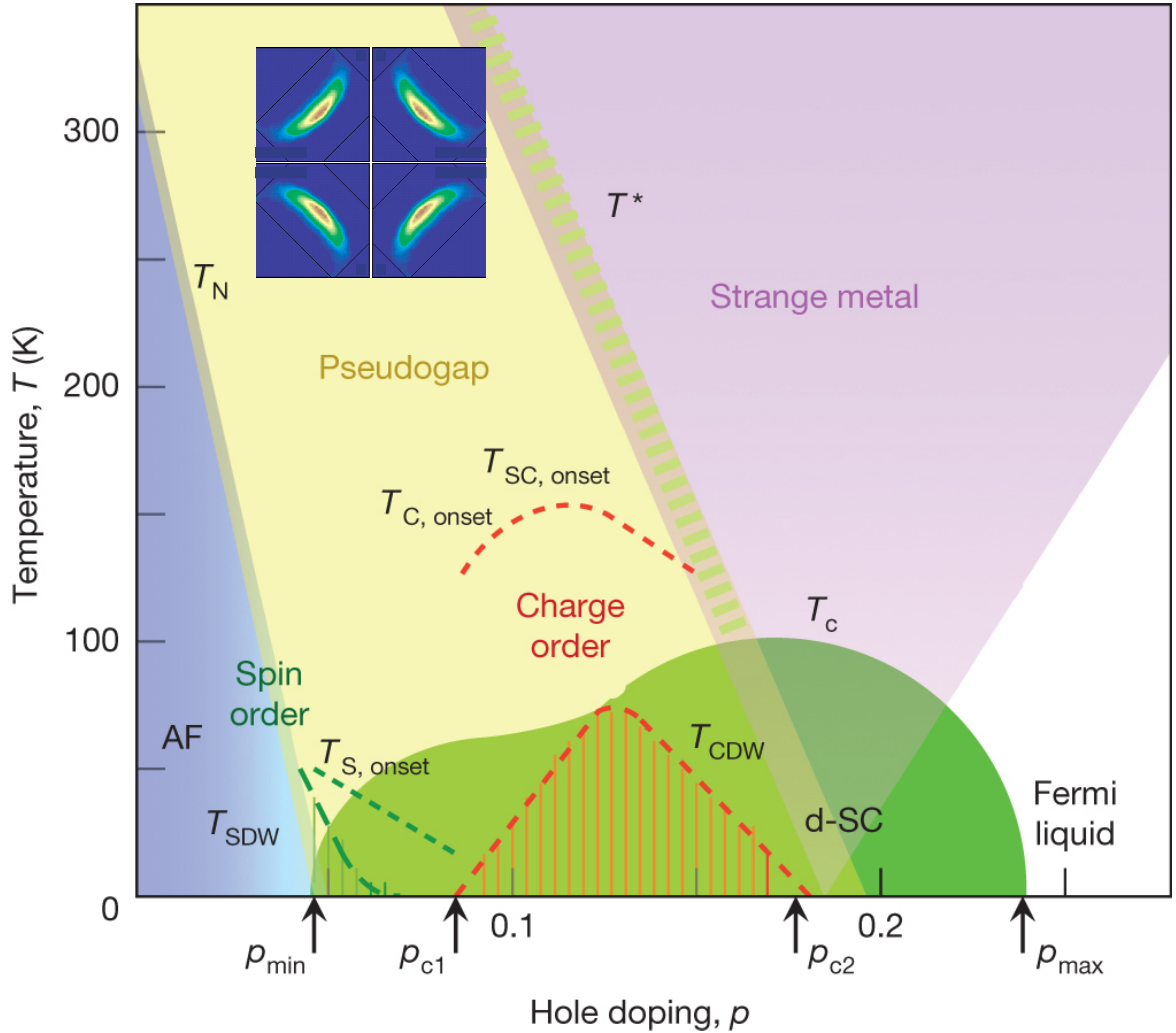


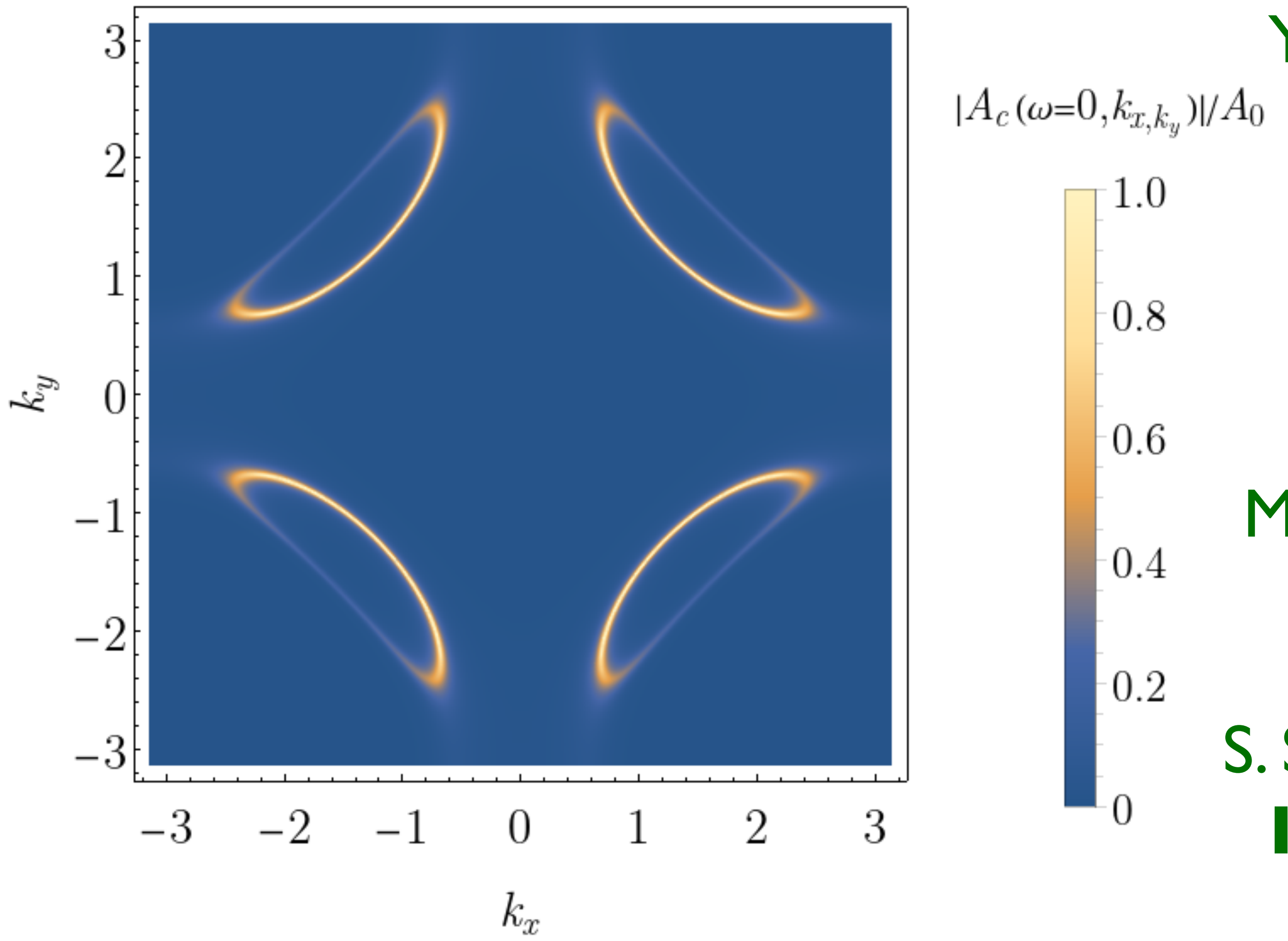
$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





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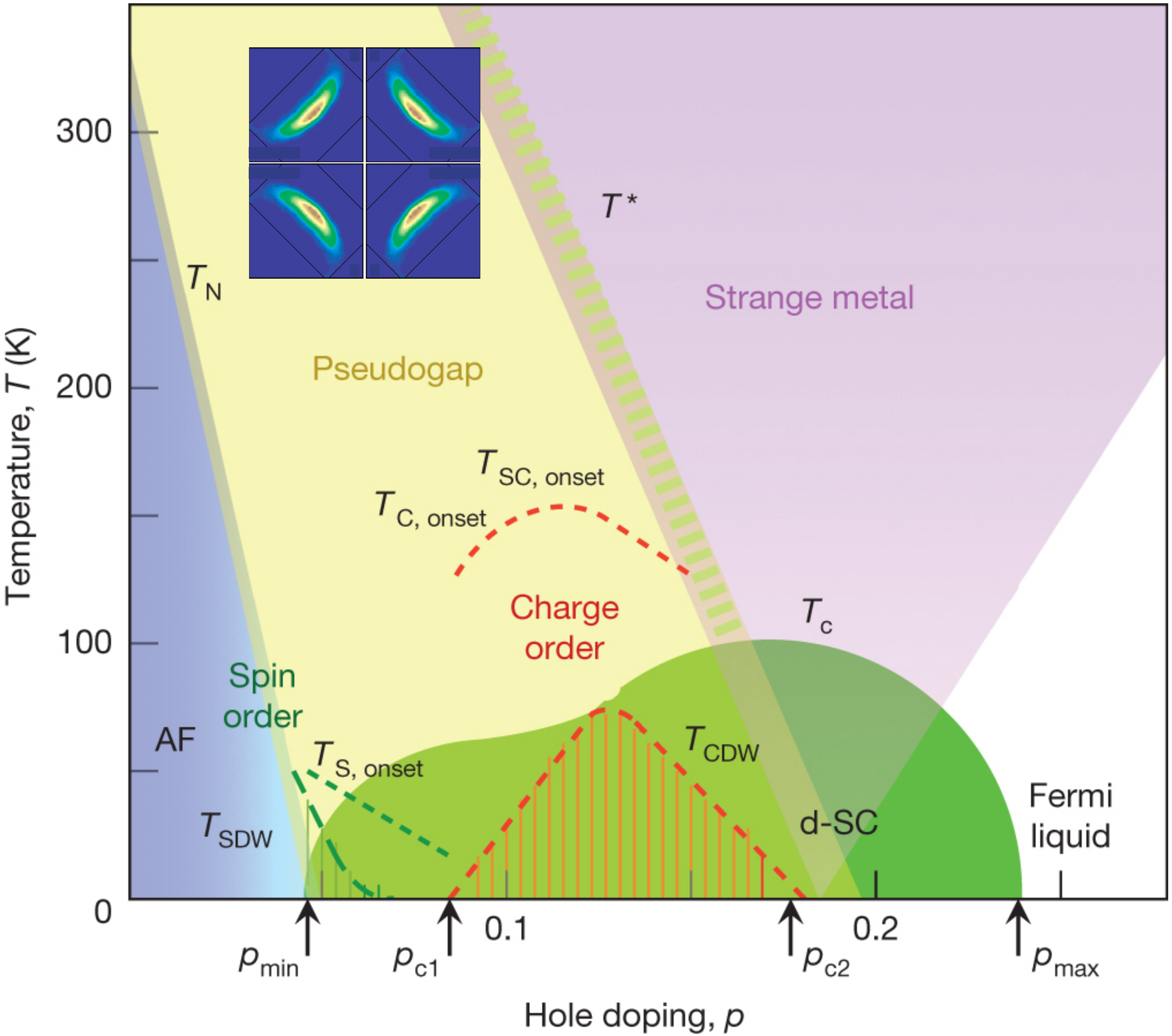

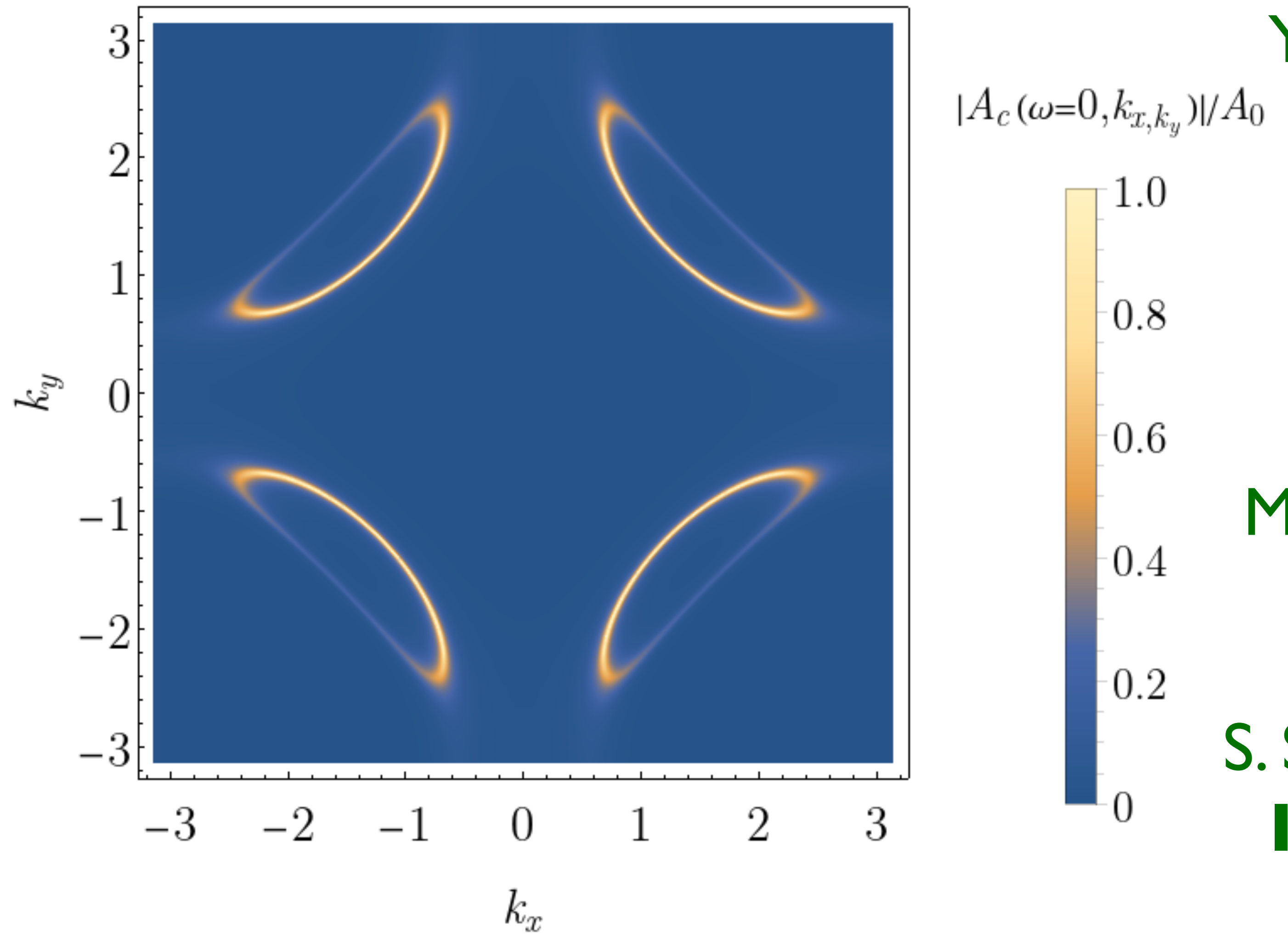

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Green function zeros….

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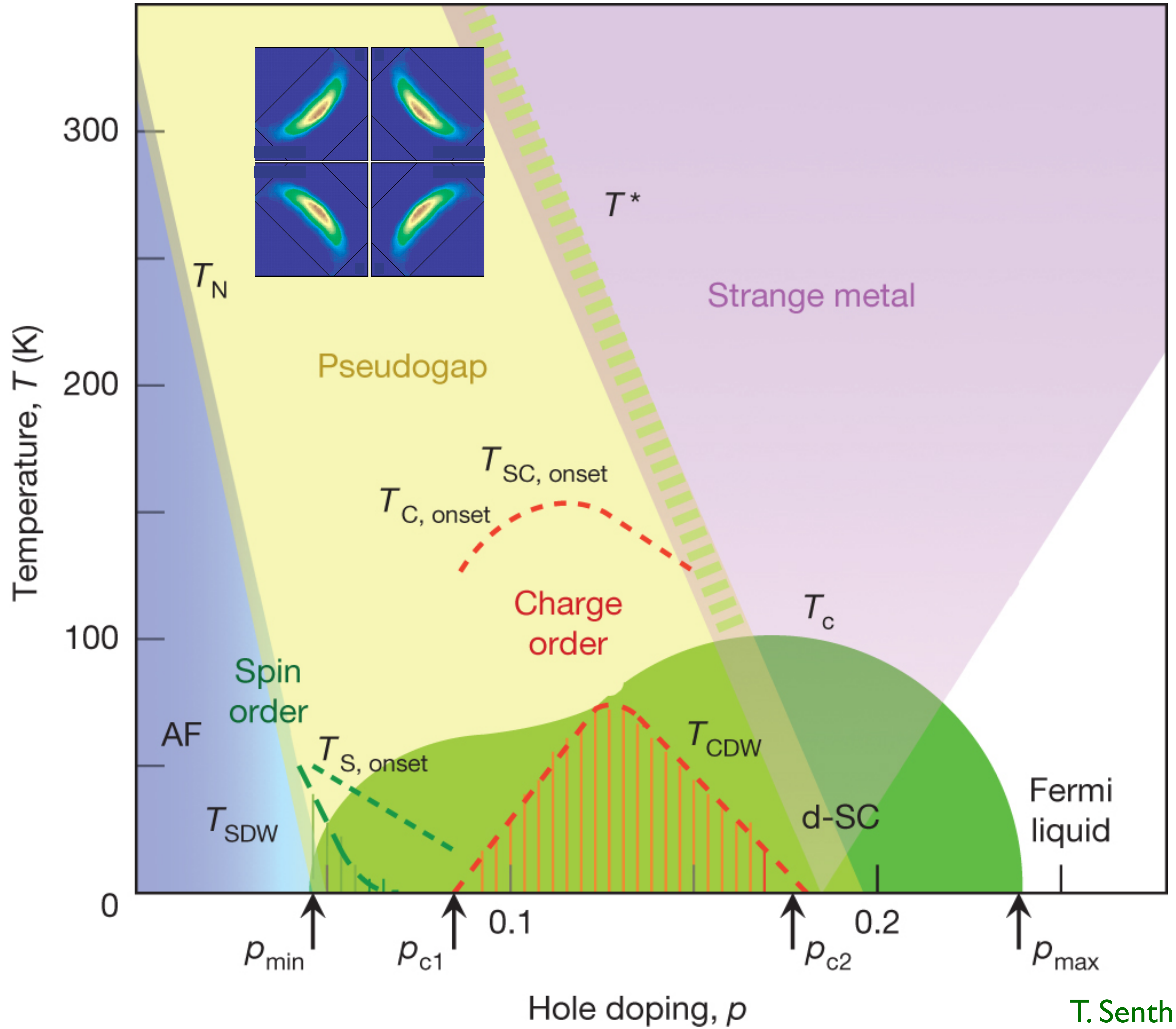

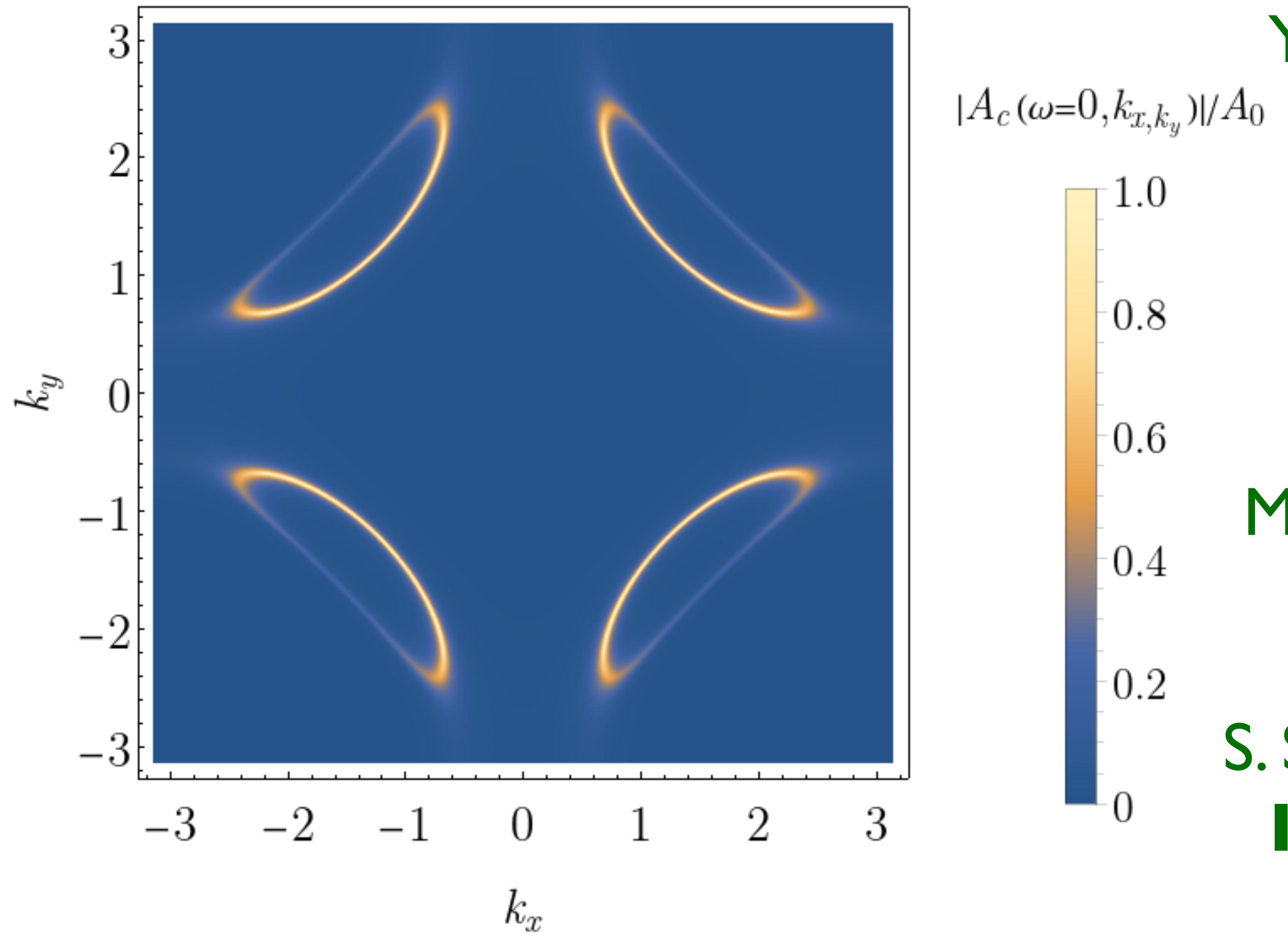

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Hole pocket Fermi surfaces
of size p with
charge e, spin-$1/2$ quasiparticles
$+$
'spectator'
square lattice spin liquid
at half-filling.

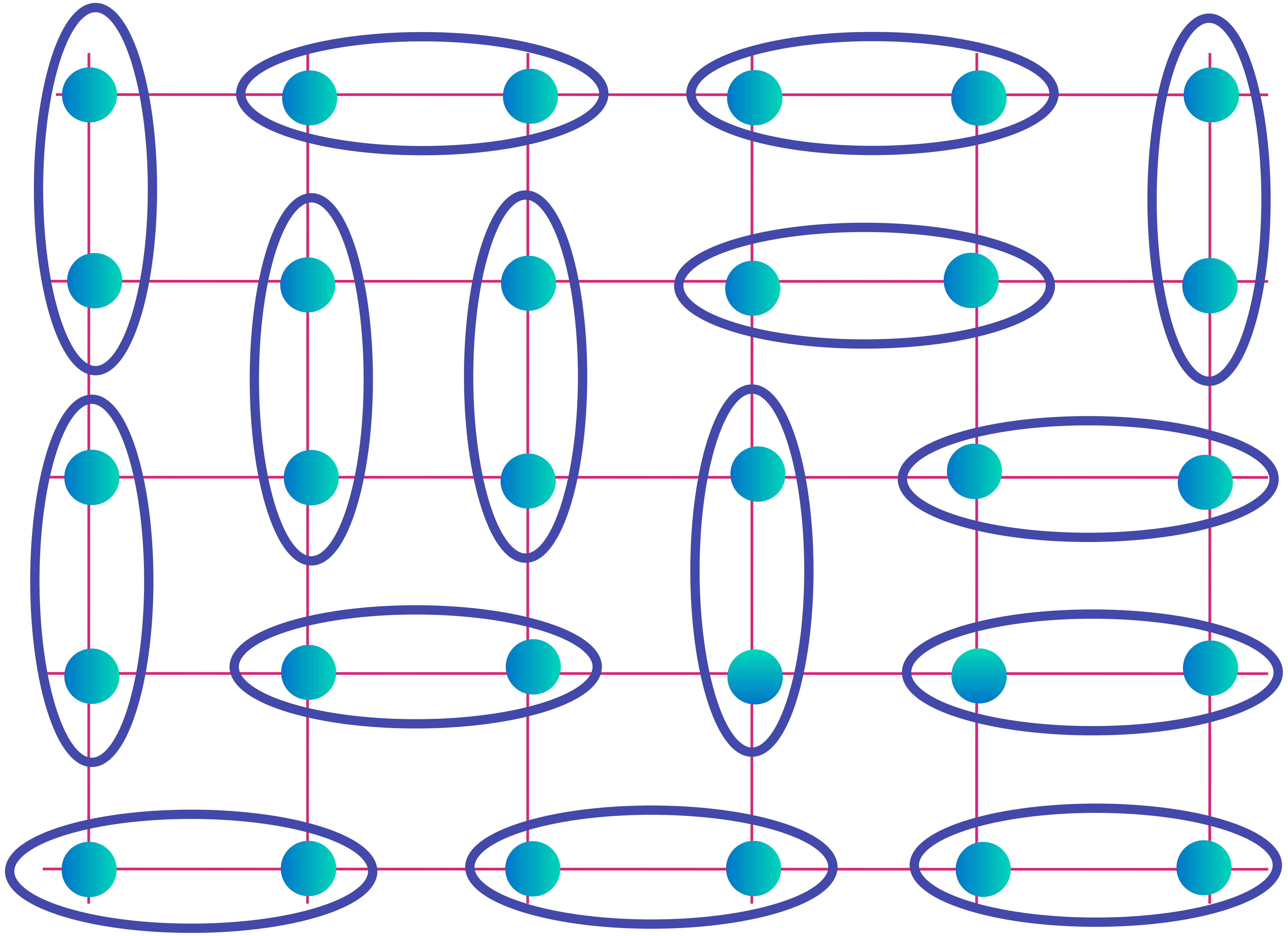
FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1 + p)$.

T. Senthil, M. Vojta, and S. Sachdev , *Phys. Rev. B* **69**, 035111 (2004)

From FL*
to
a cuprate phase diagram

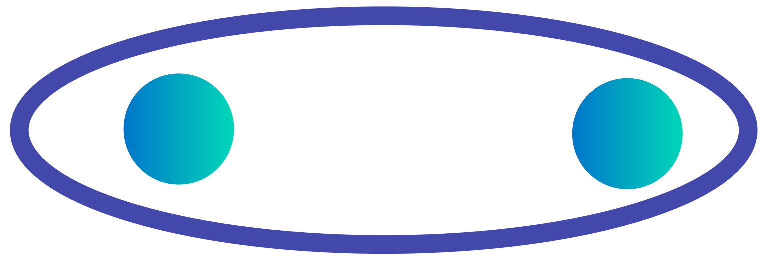
The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)



Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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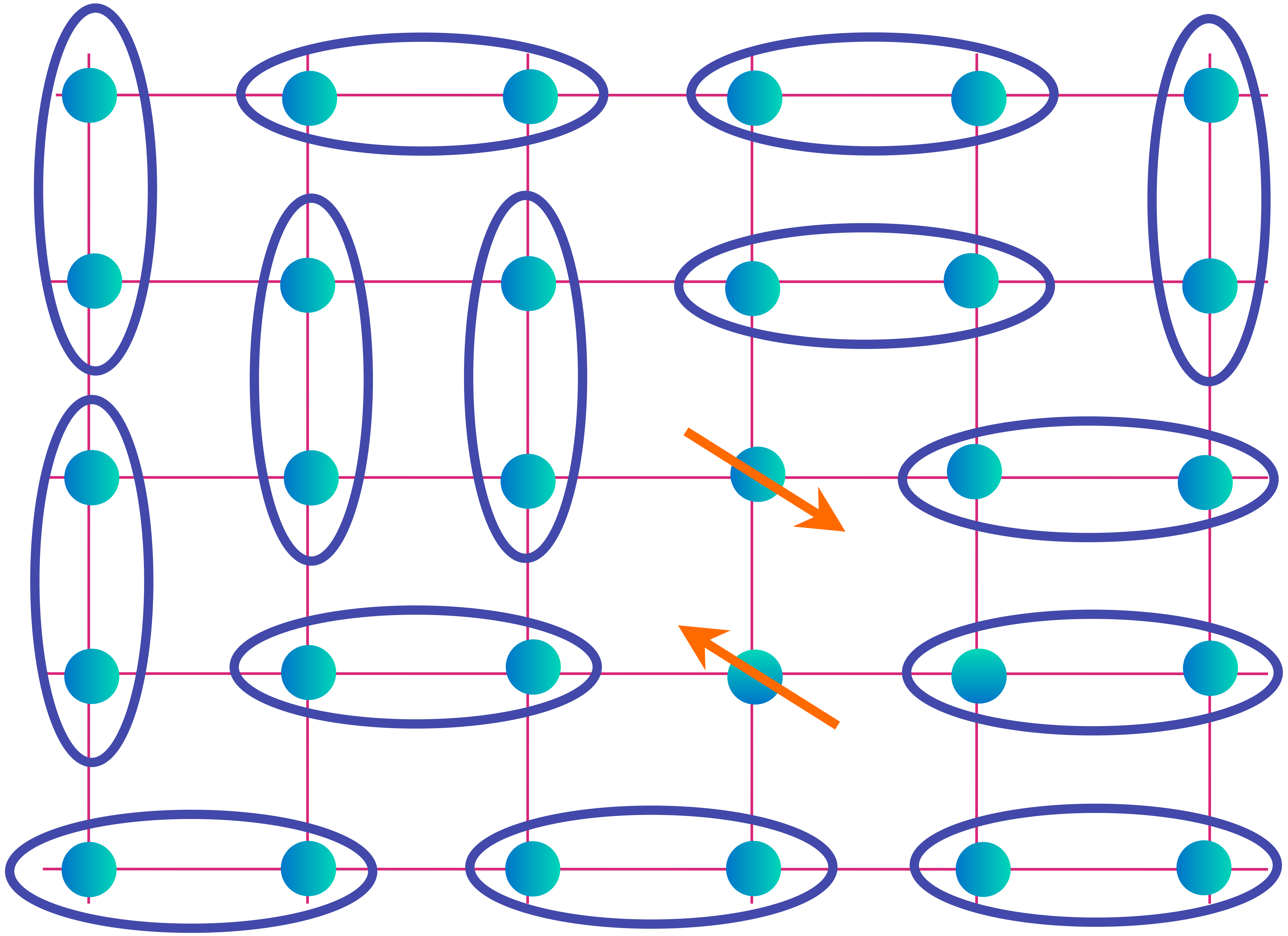
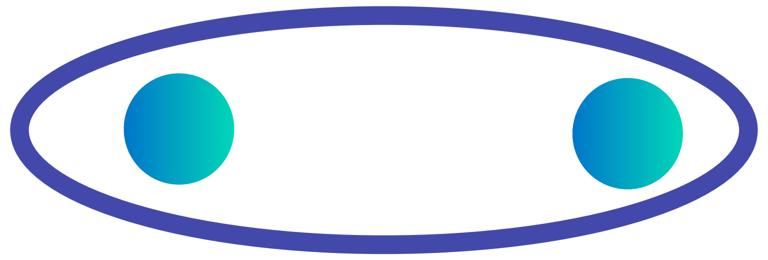
The dance of electrons on Cu atoms in YBCO



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

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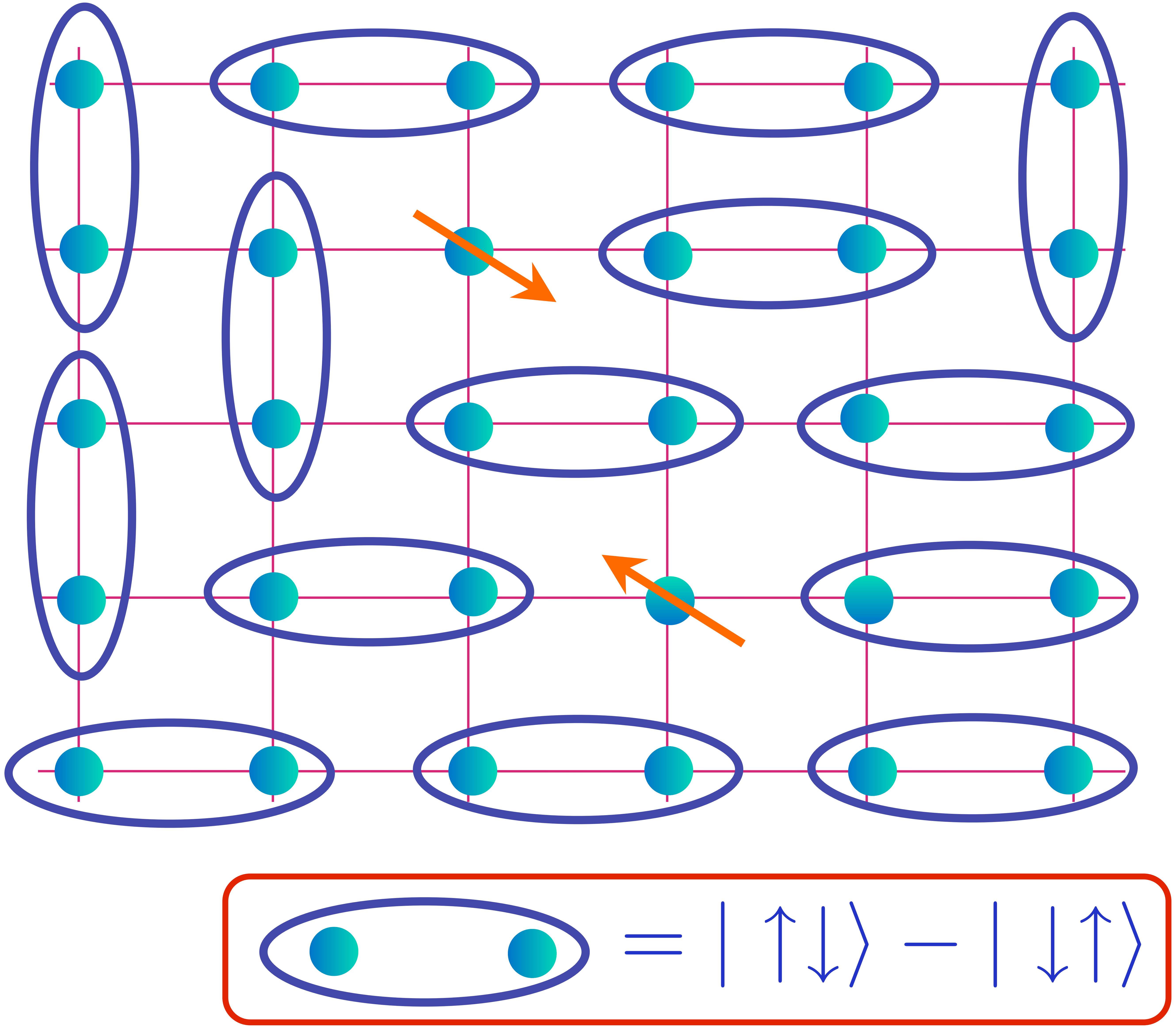

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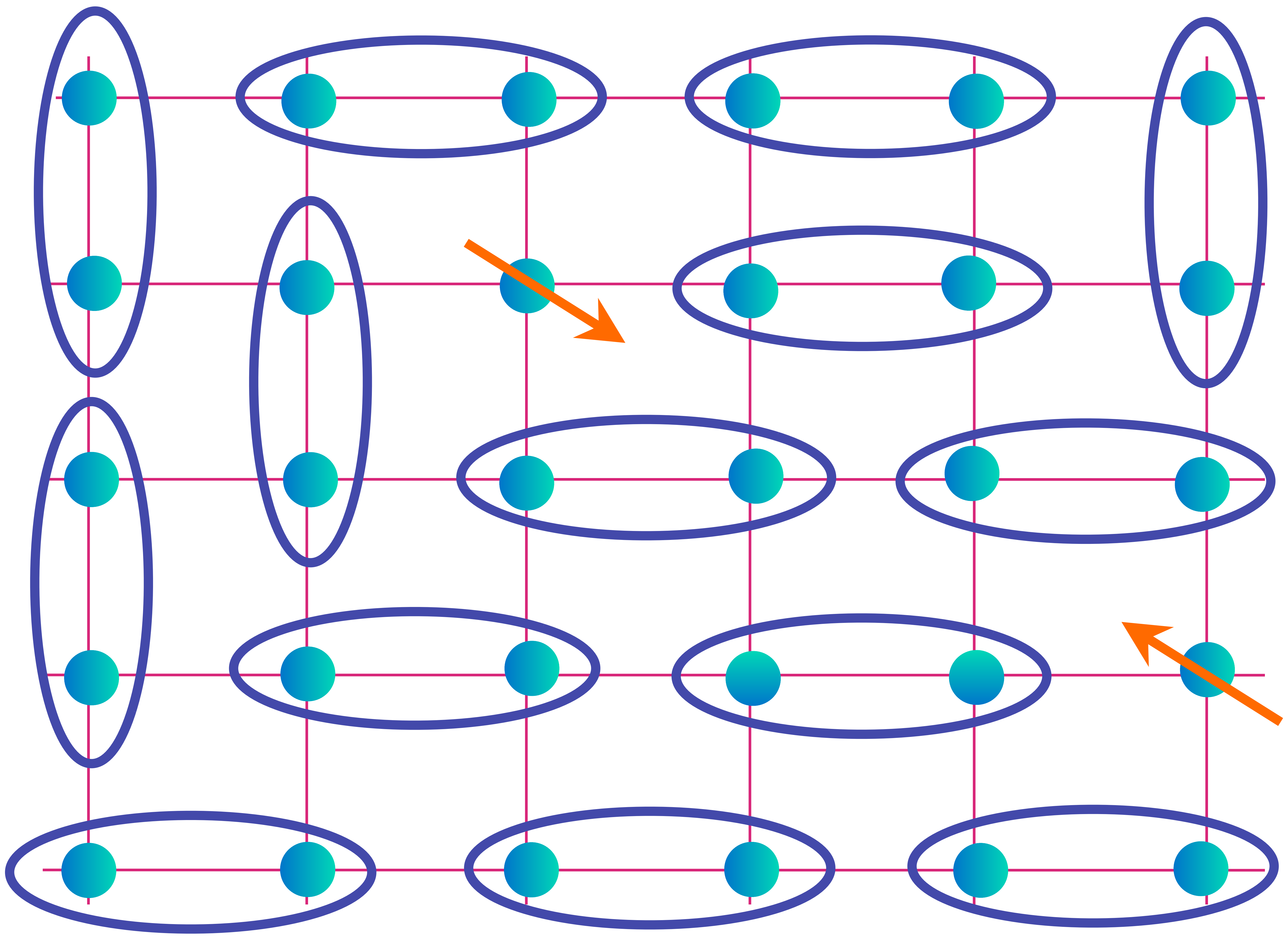

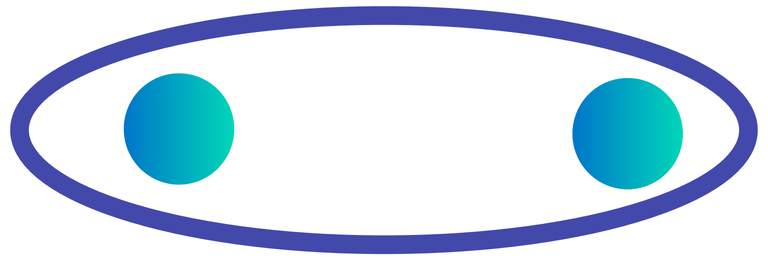

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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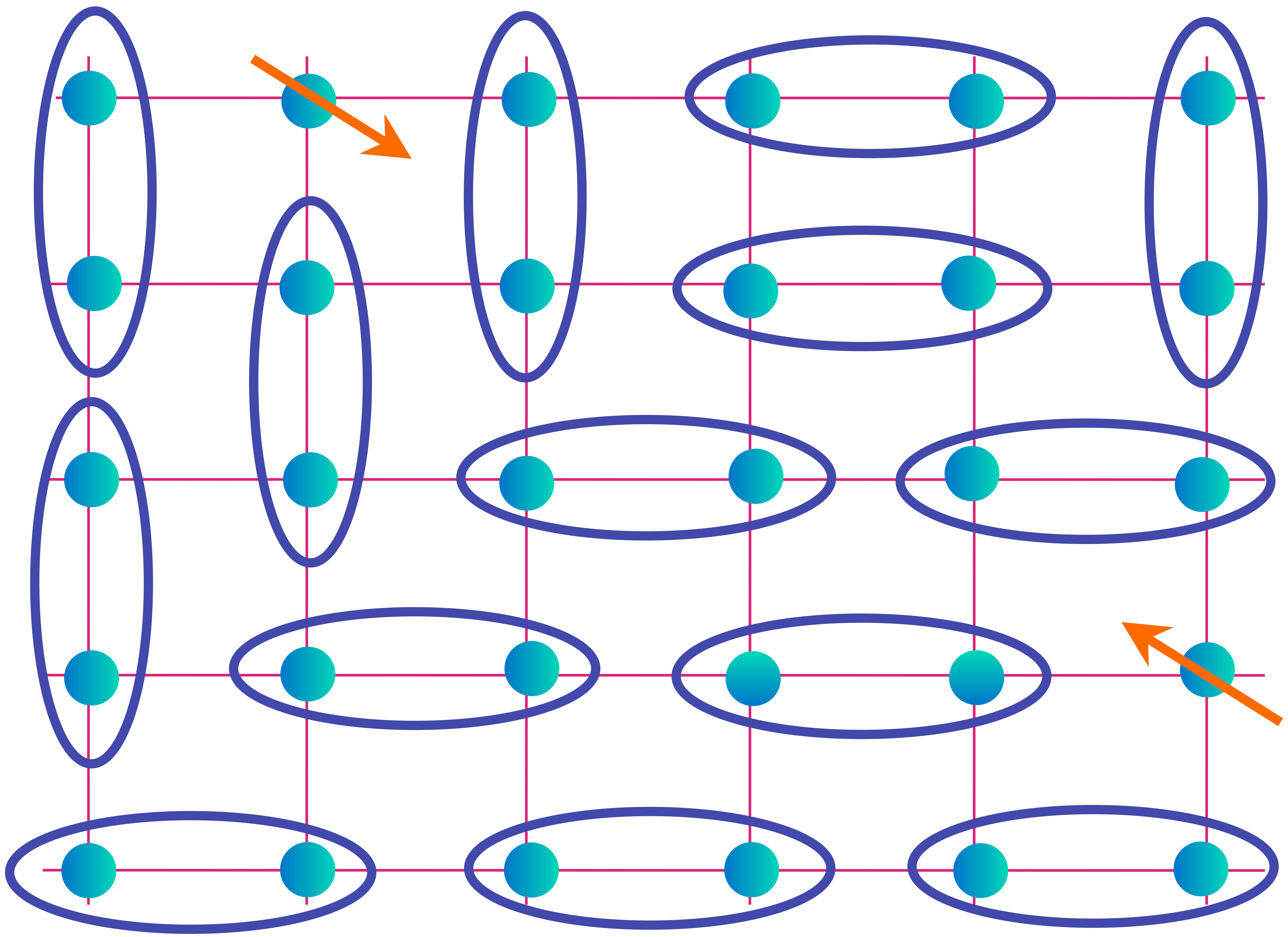

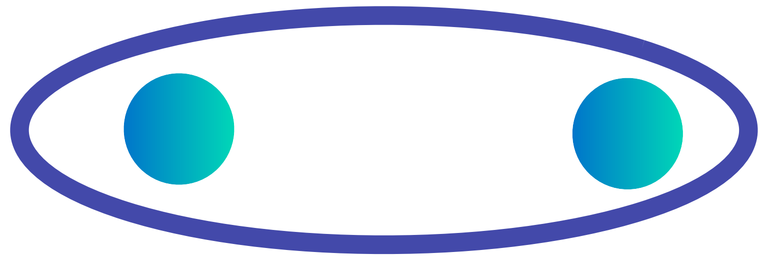

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Holons

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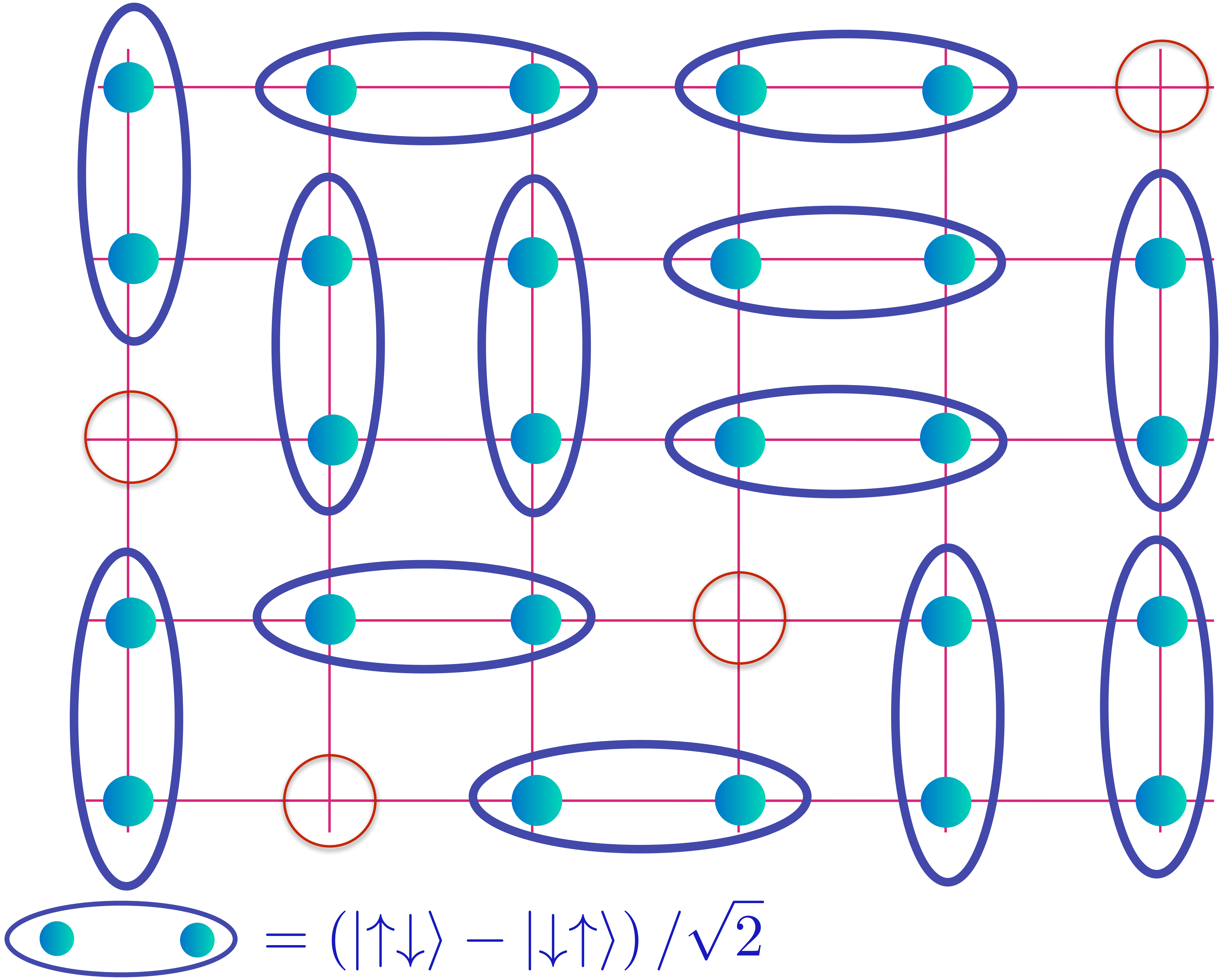

Spin liquid

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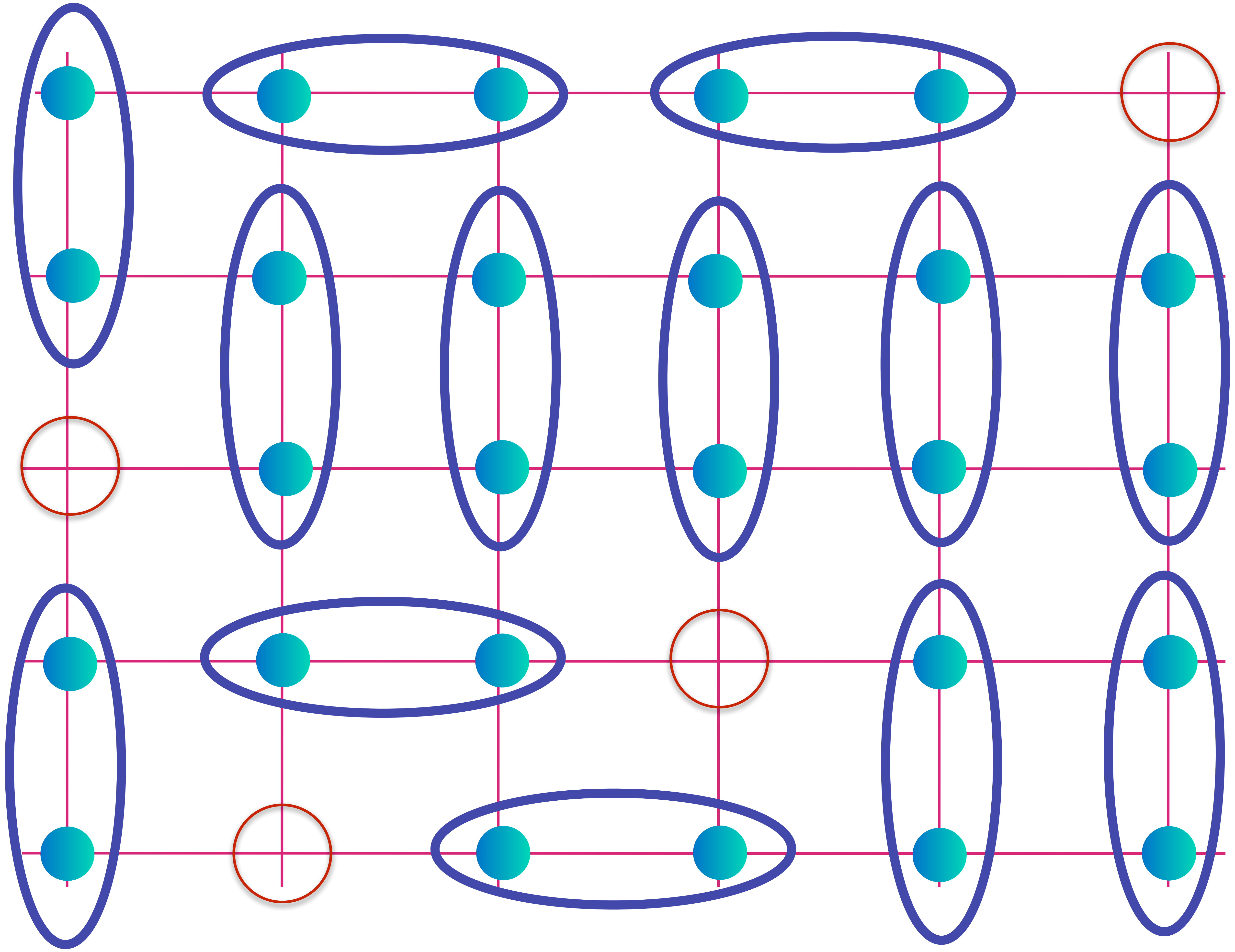

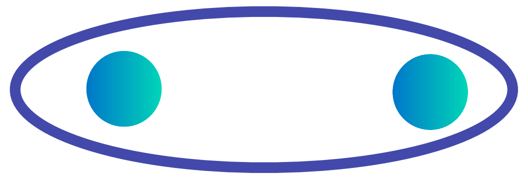 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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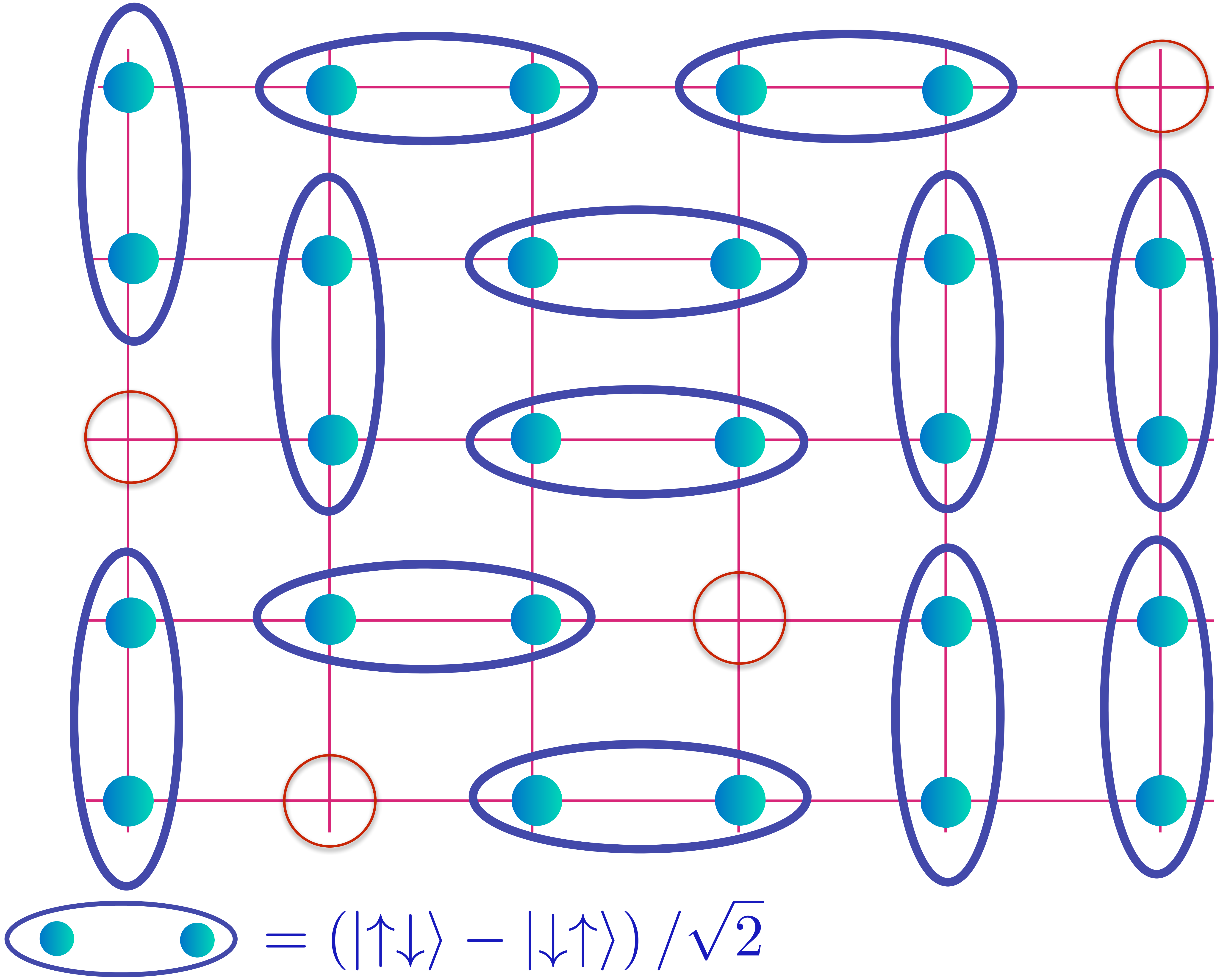

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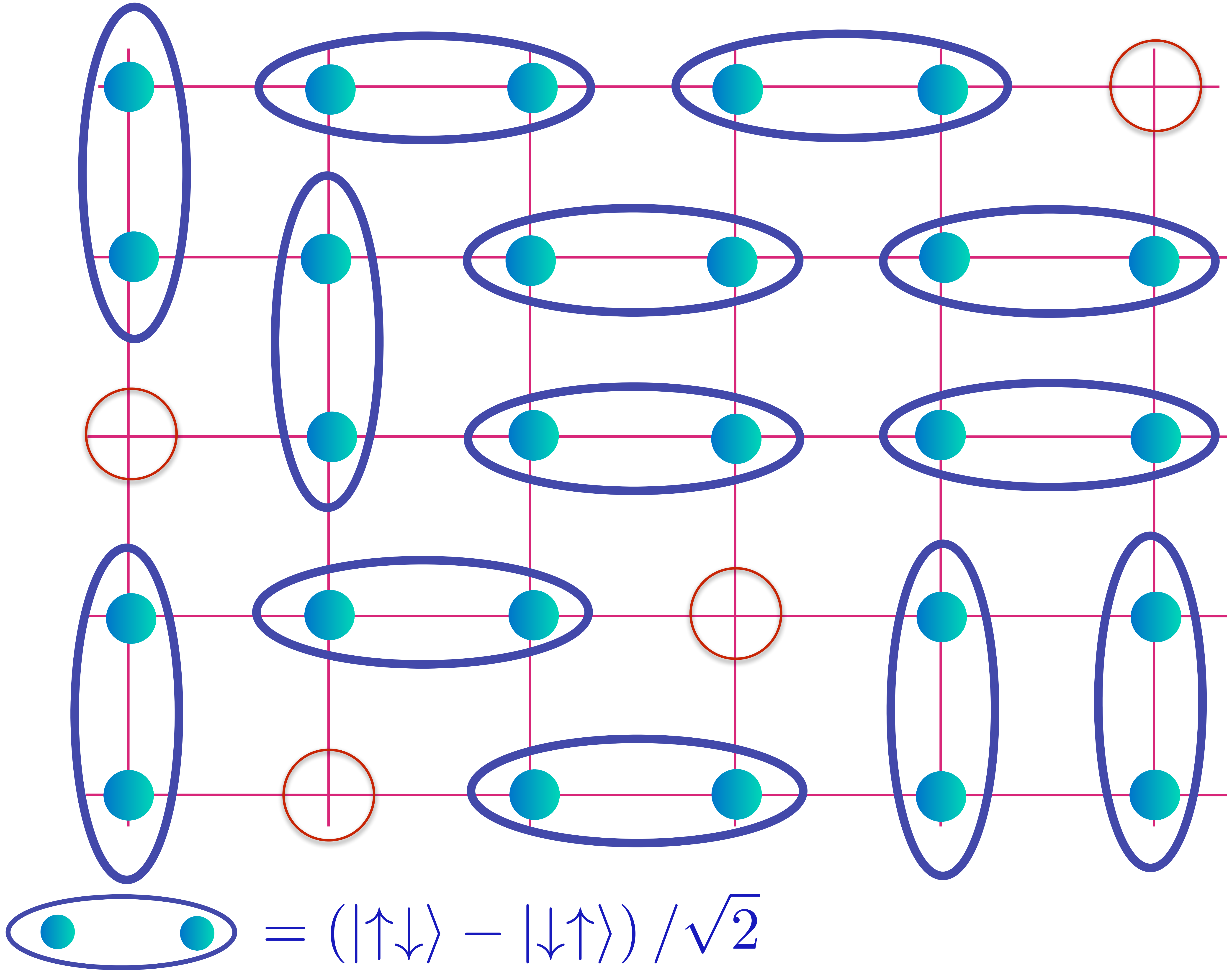

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Holon metal

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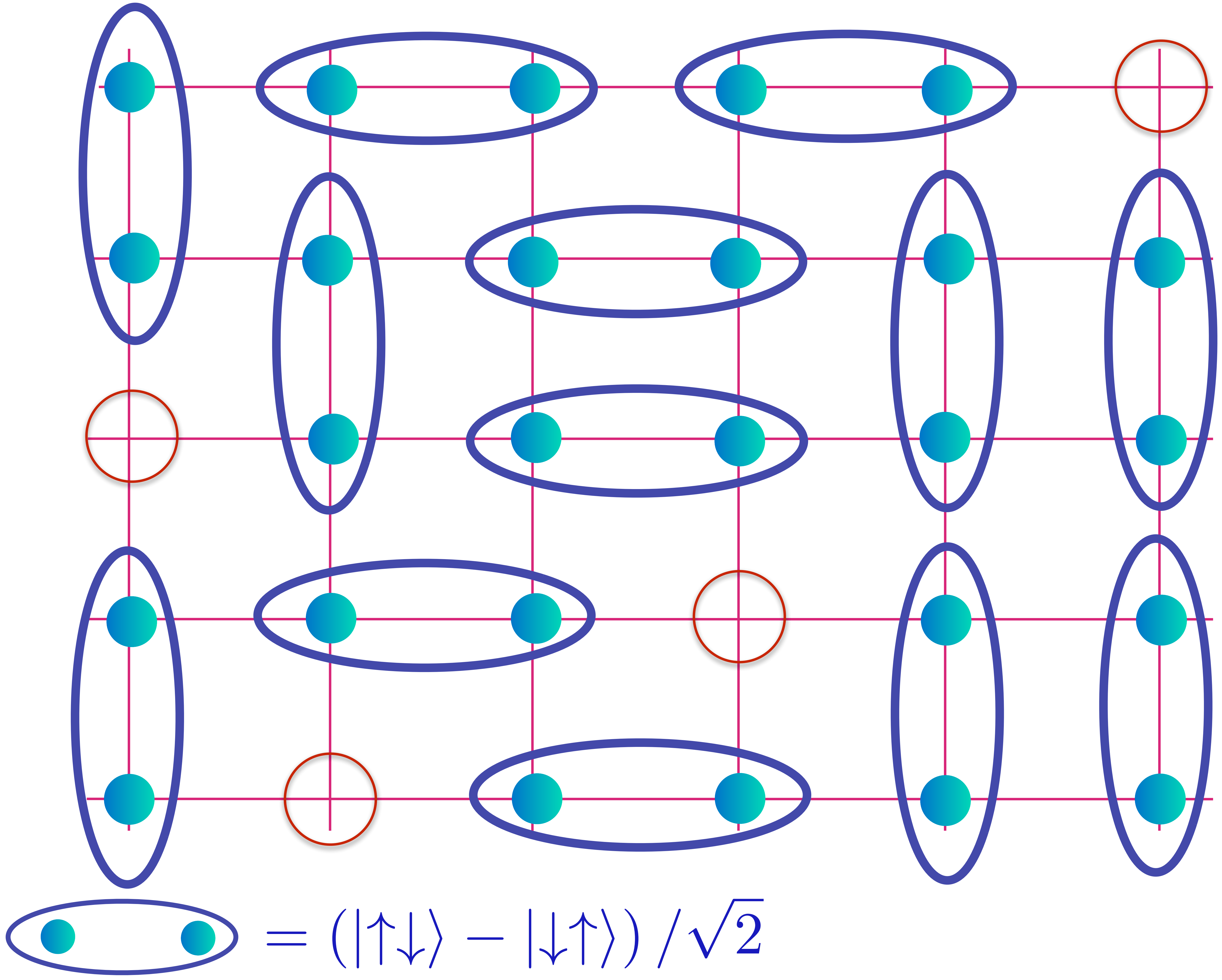

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

If each holon is
a fermion,
we obtain
a Fermi surface
of holons
of size p

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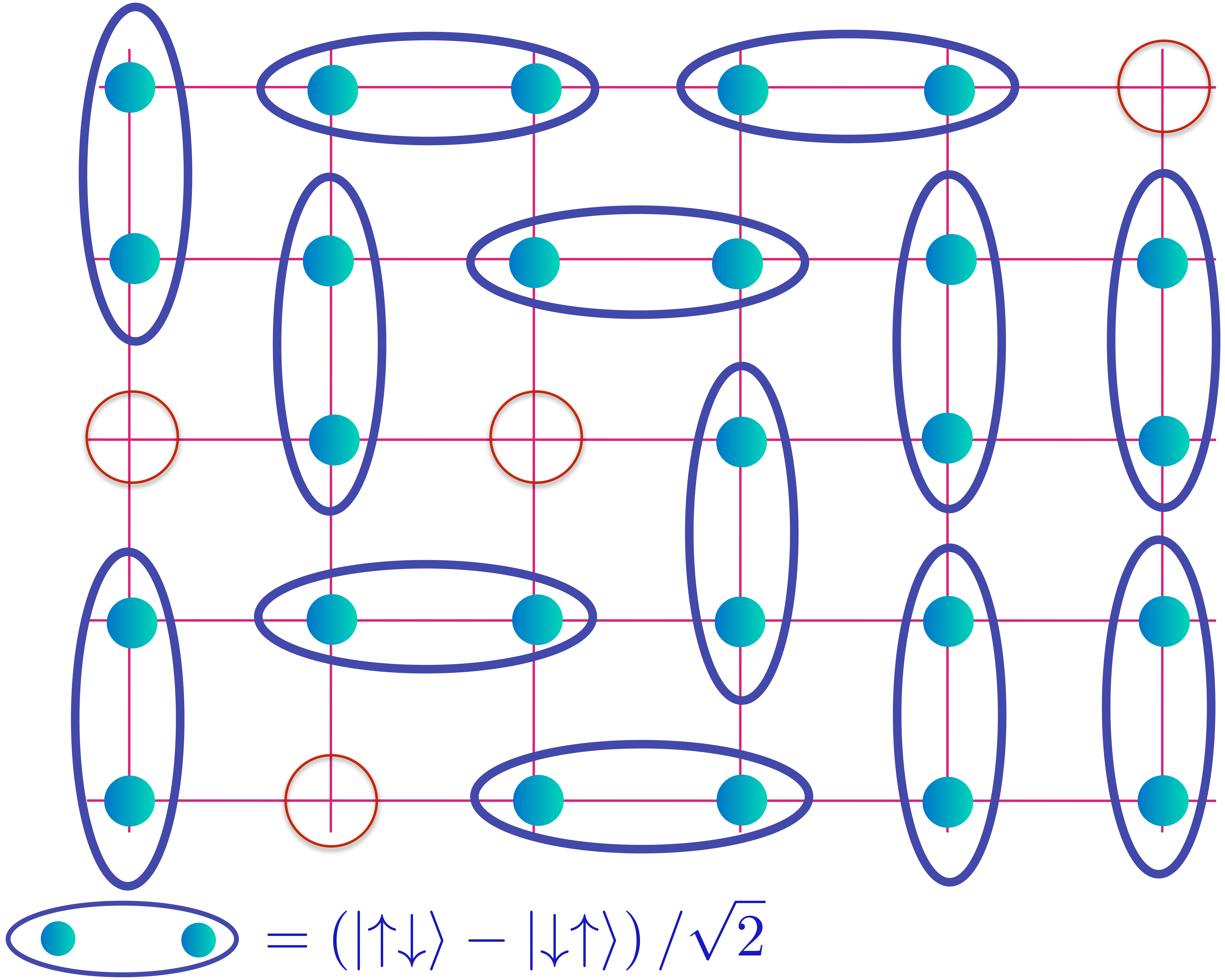

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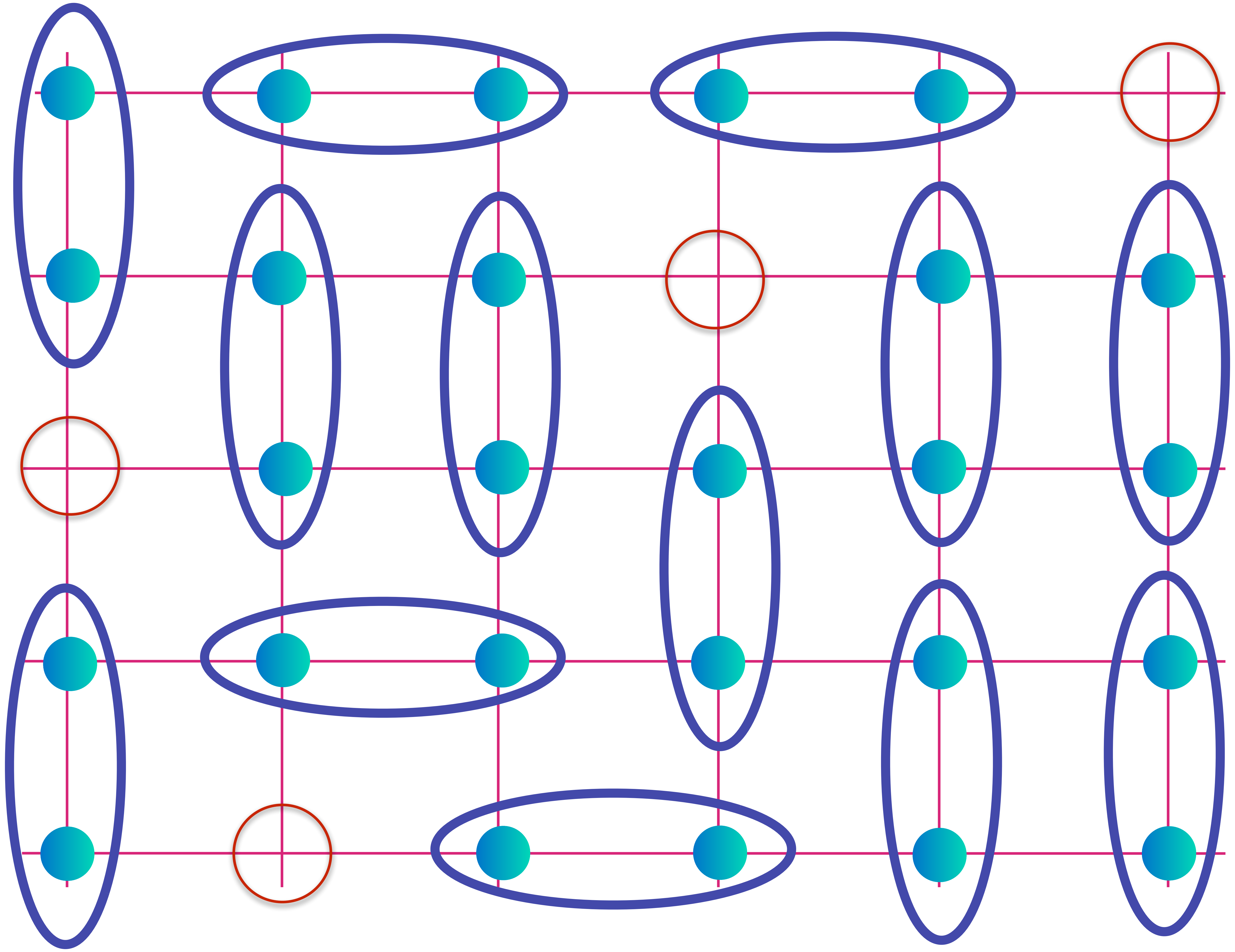

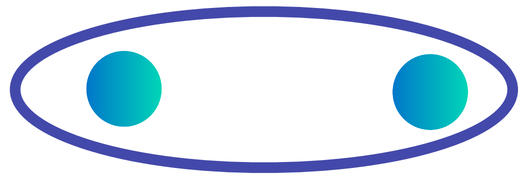 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

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Cannot explain pseudogap metal

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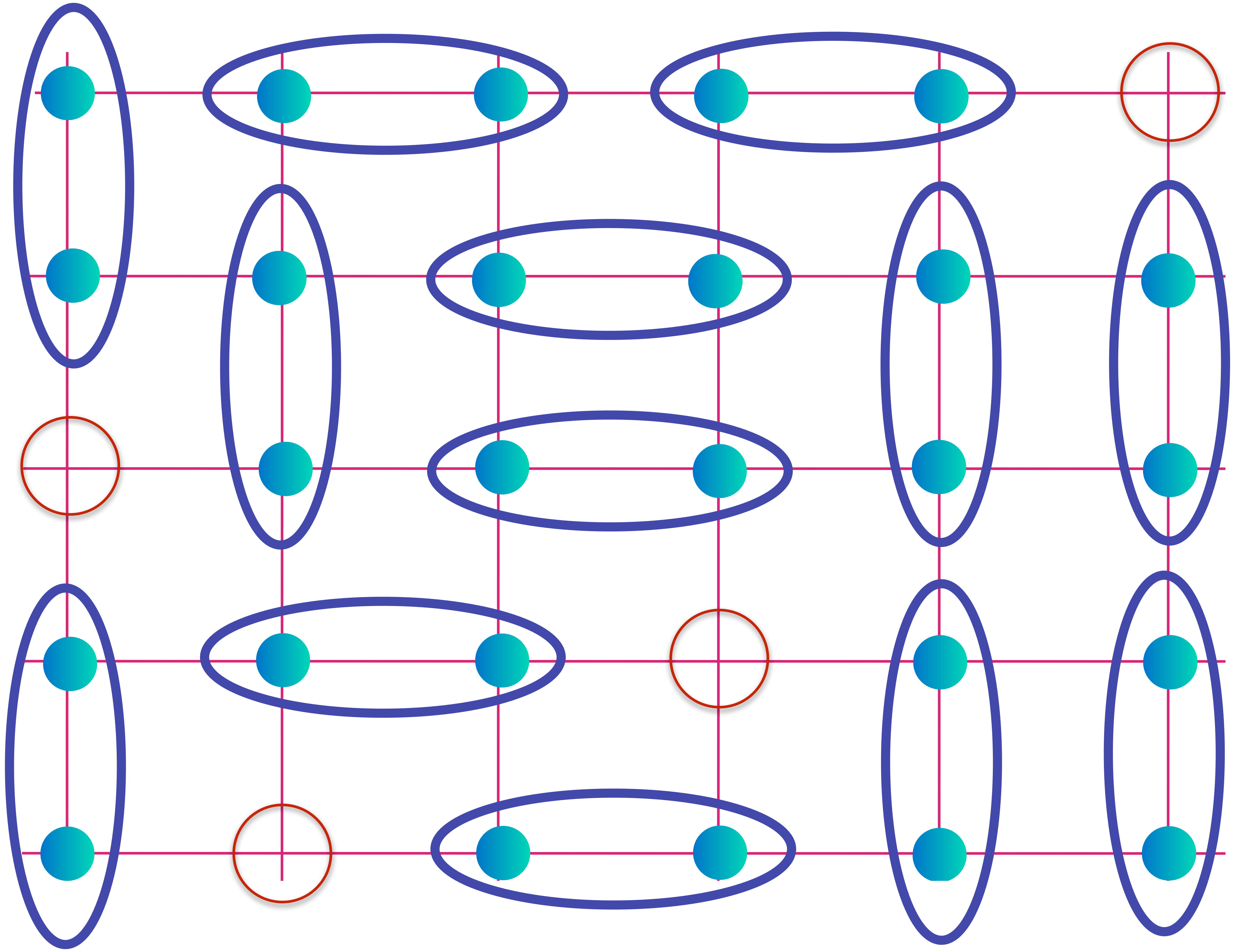
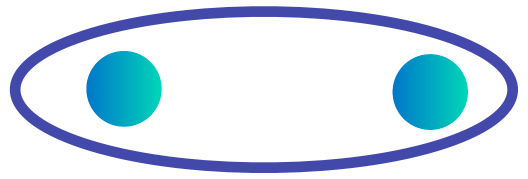




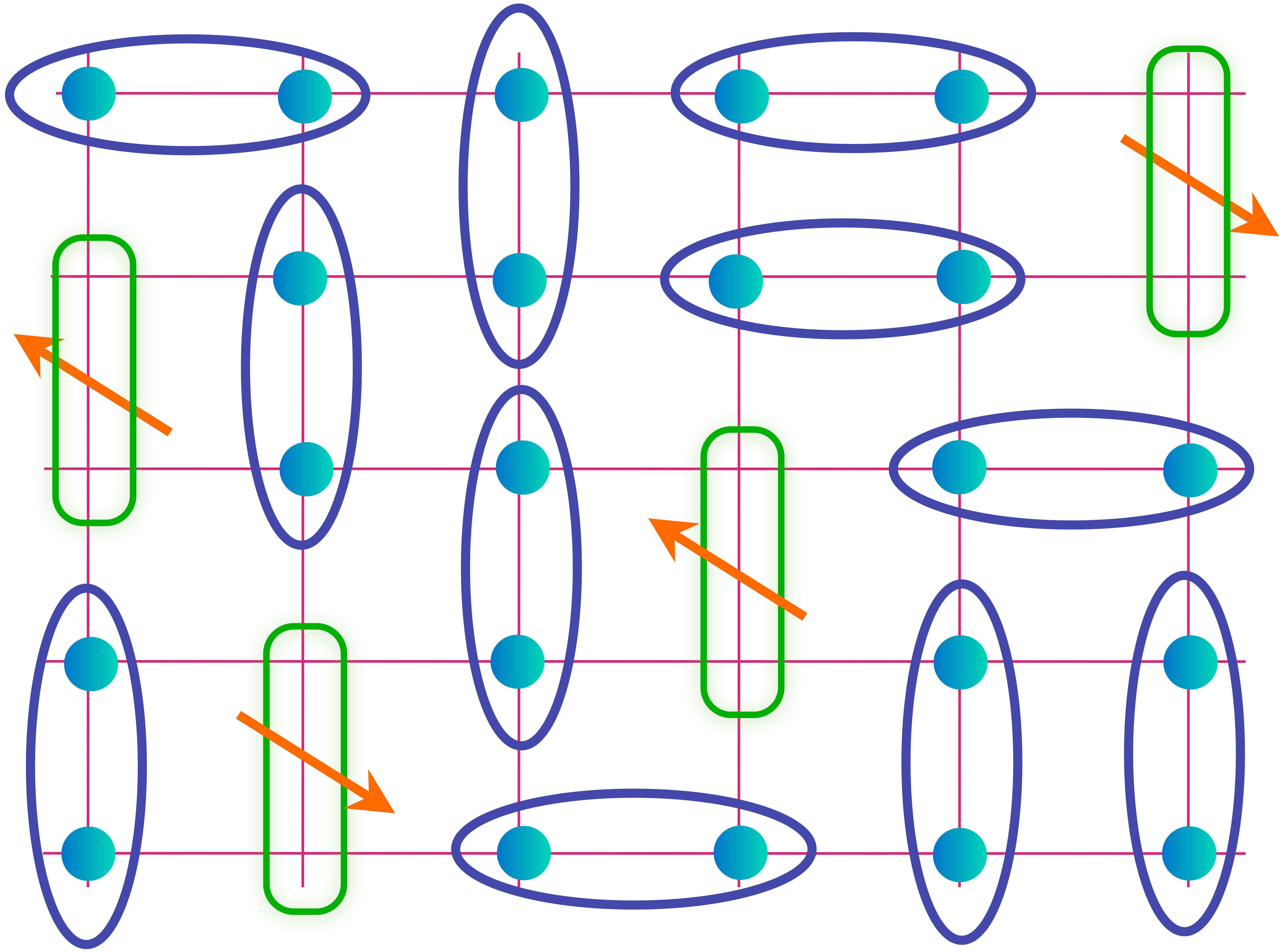
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

Cannot explain pseudogap metal

FL* in a ***one-band*** model

S. Sachdev *Phys. Rev. B* **49**, 6770 (1994); X.-G. Wen and P.A. Lee *Phys. Rev. Lett.* **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

E. G. Moon and S. Sachdev, *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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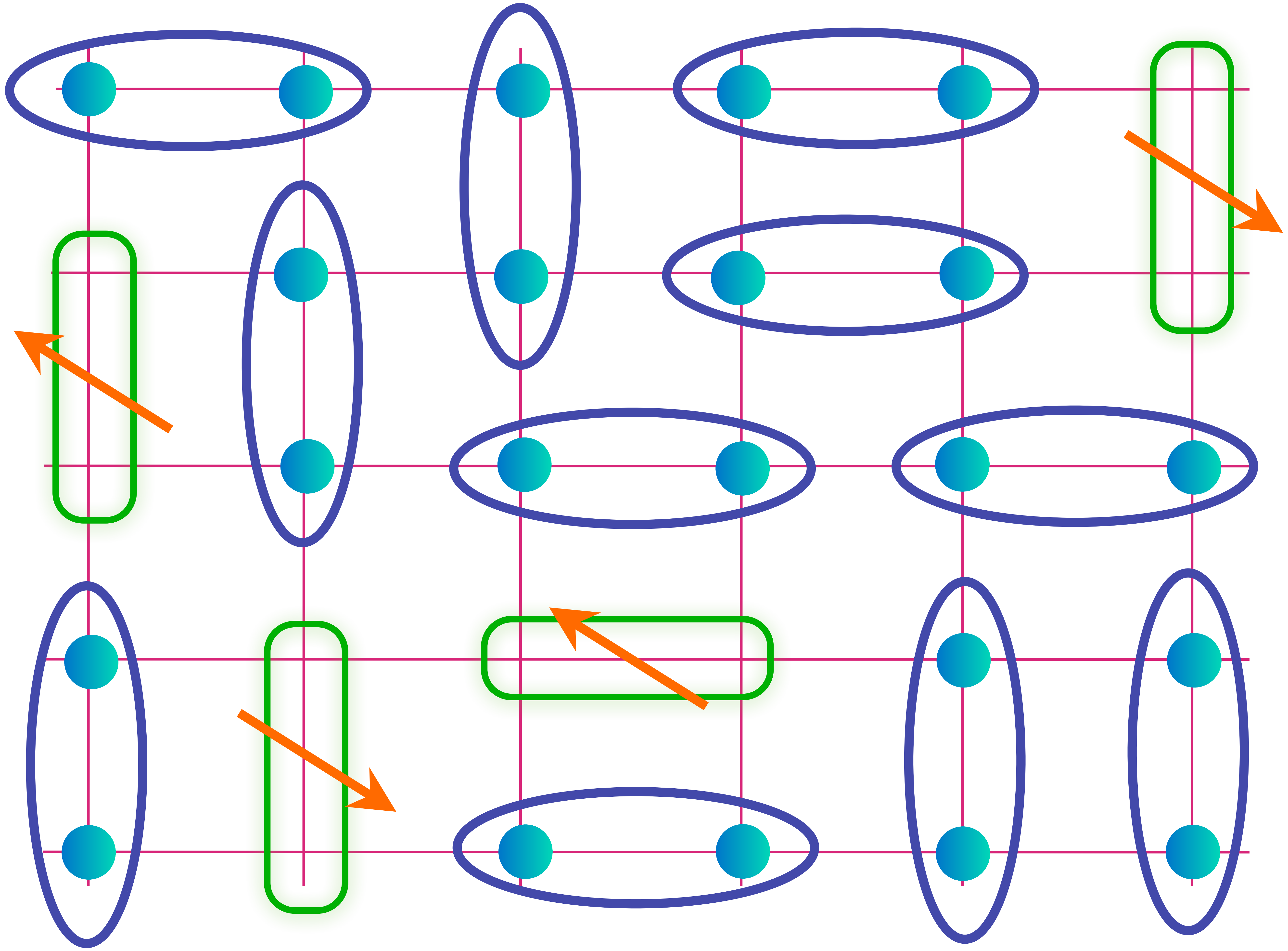

Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

Spinon-holon bound state

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

$$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

E. G. Moon and S. Sachdev, *Phys. Rev. B* **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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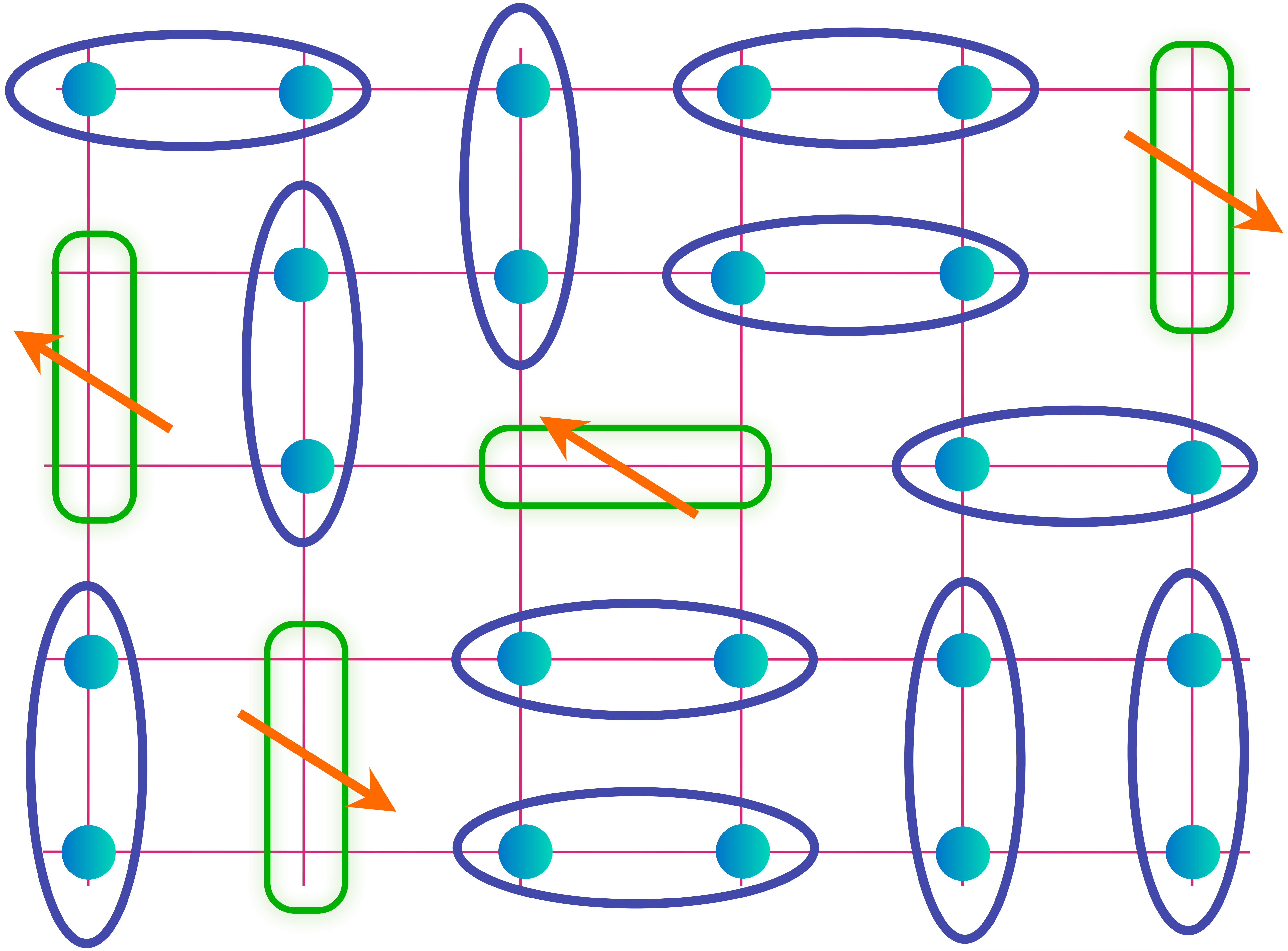

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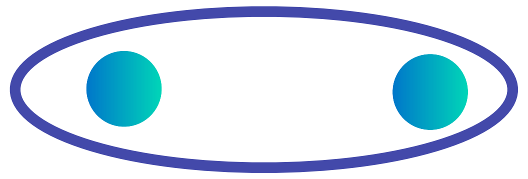
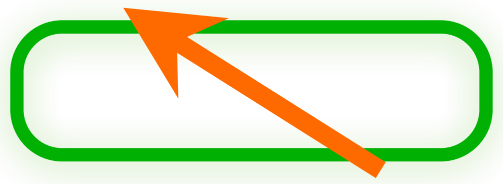
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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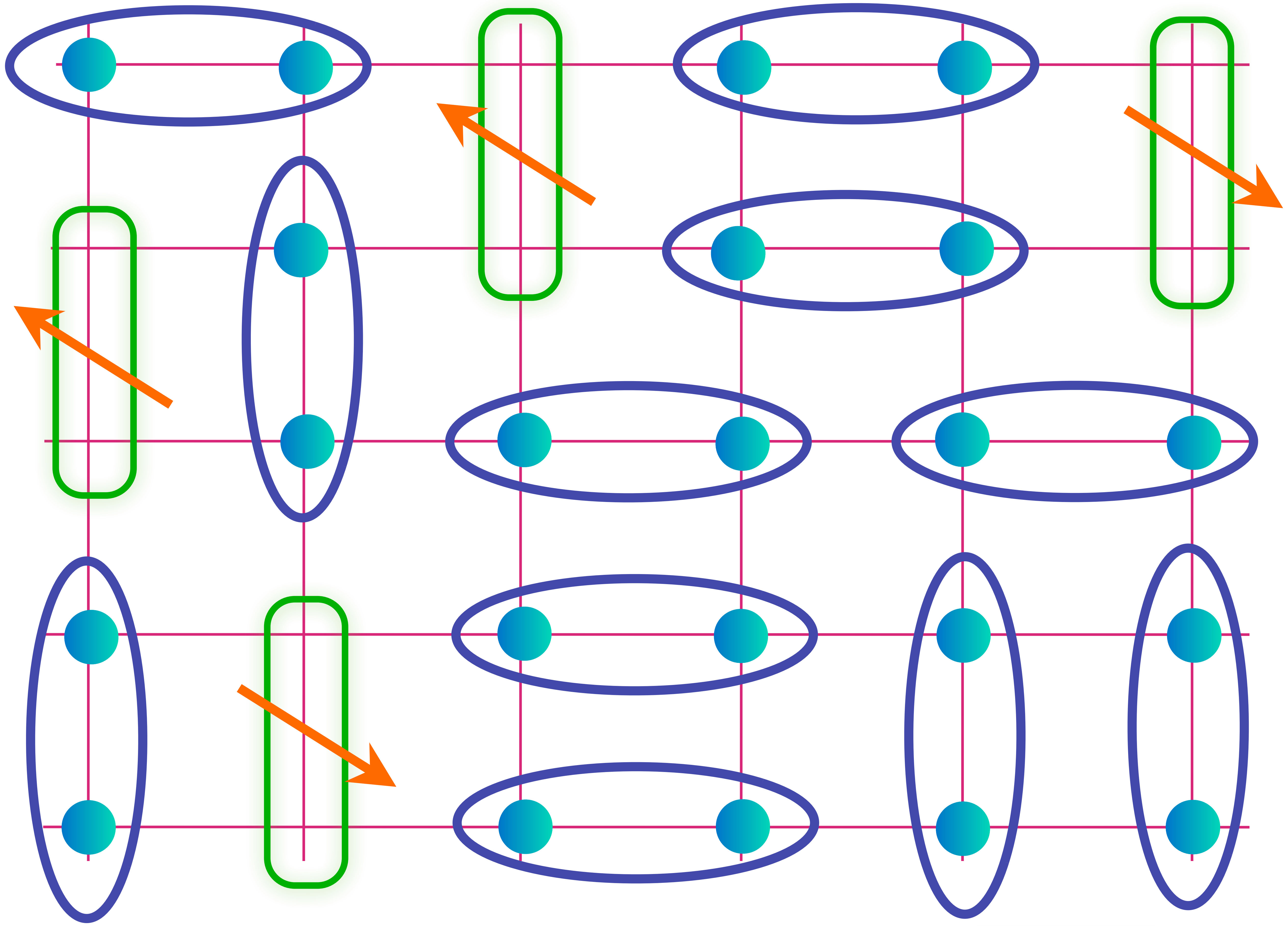

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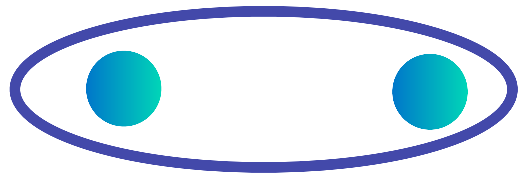
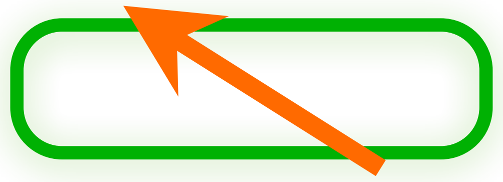
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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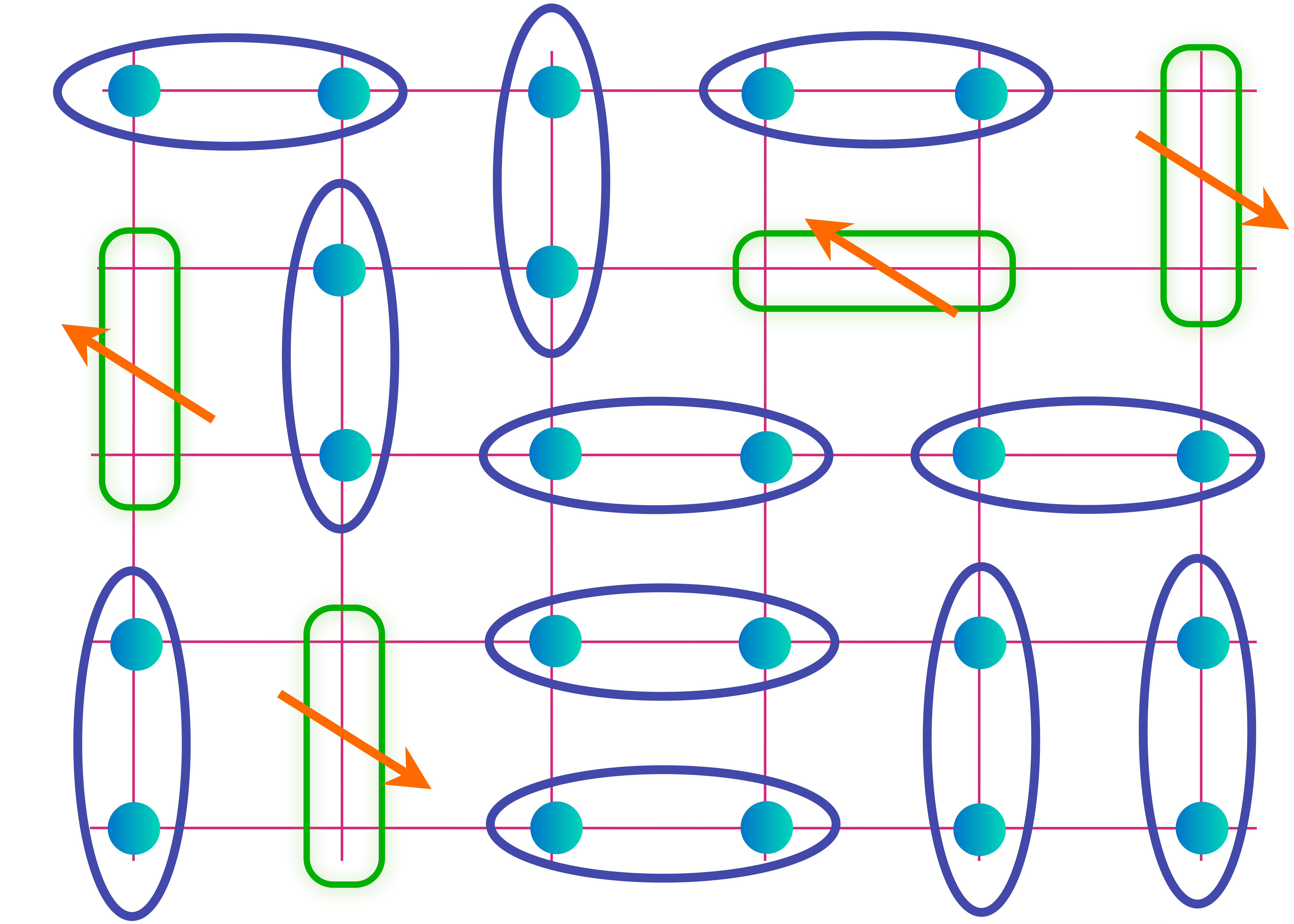

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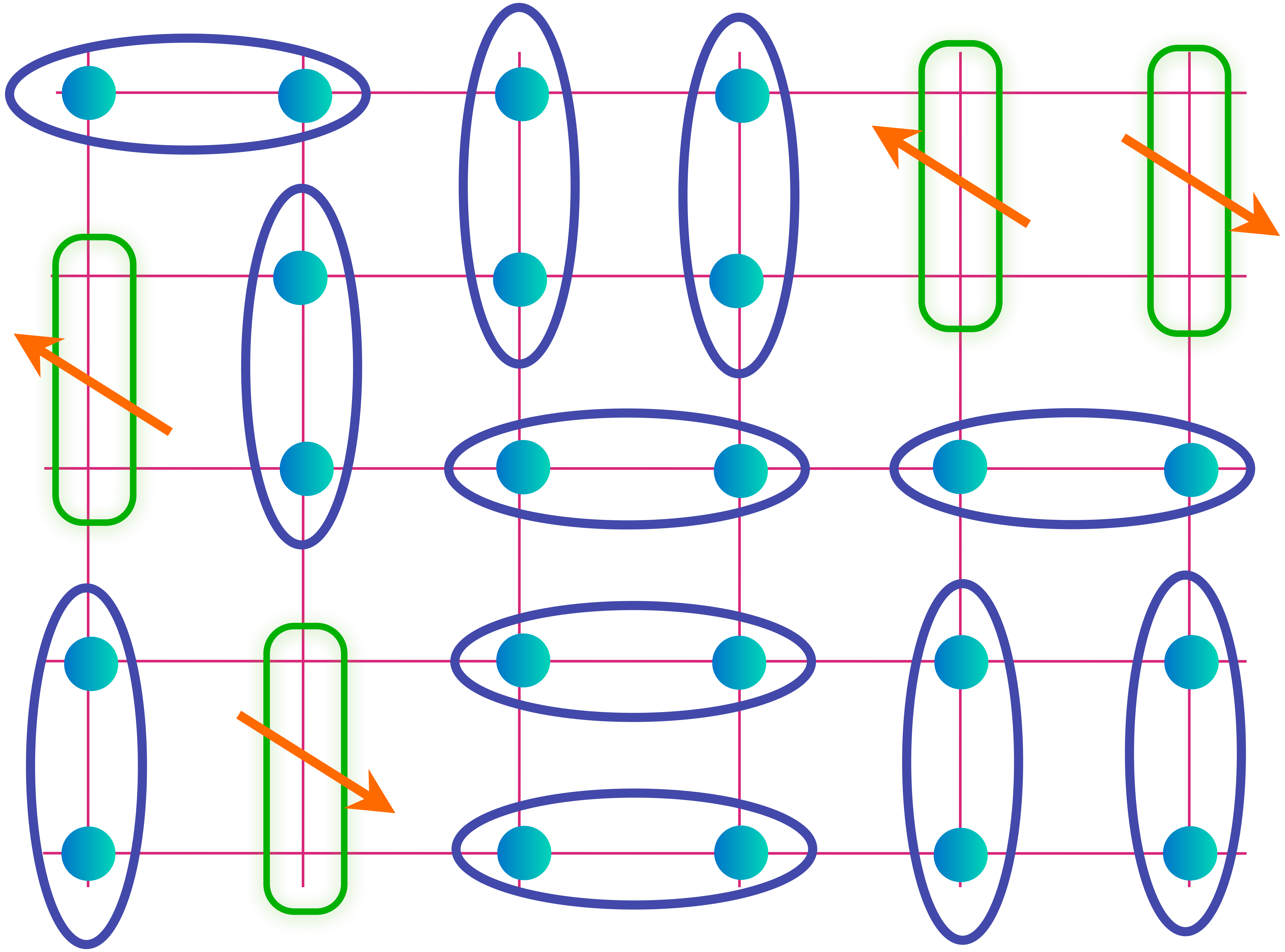

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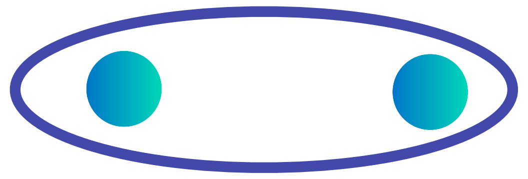

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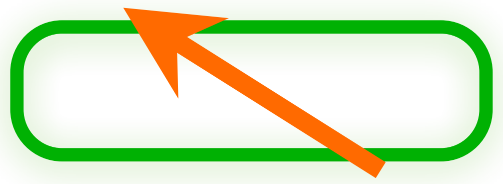

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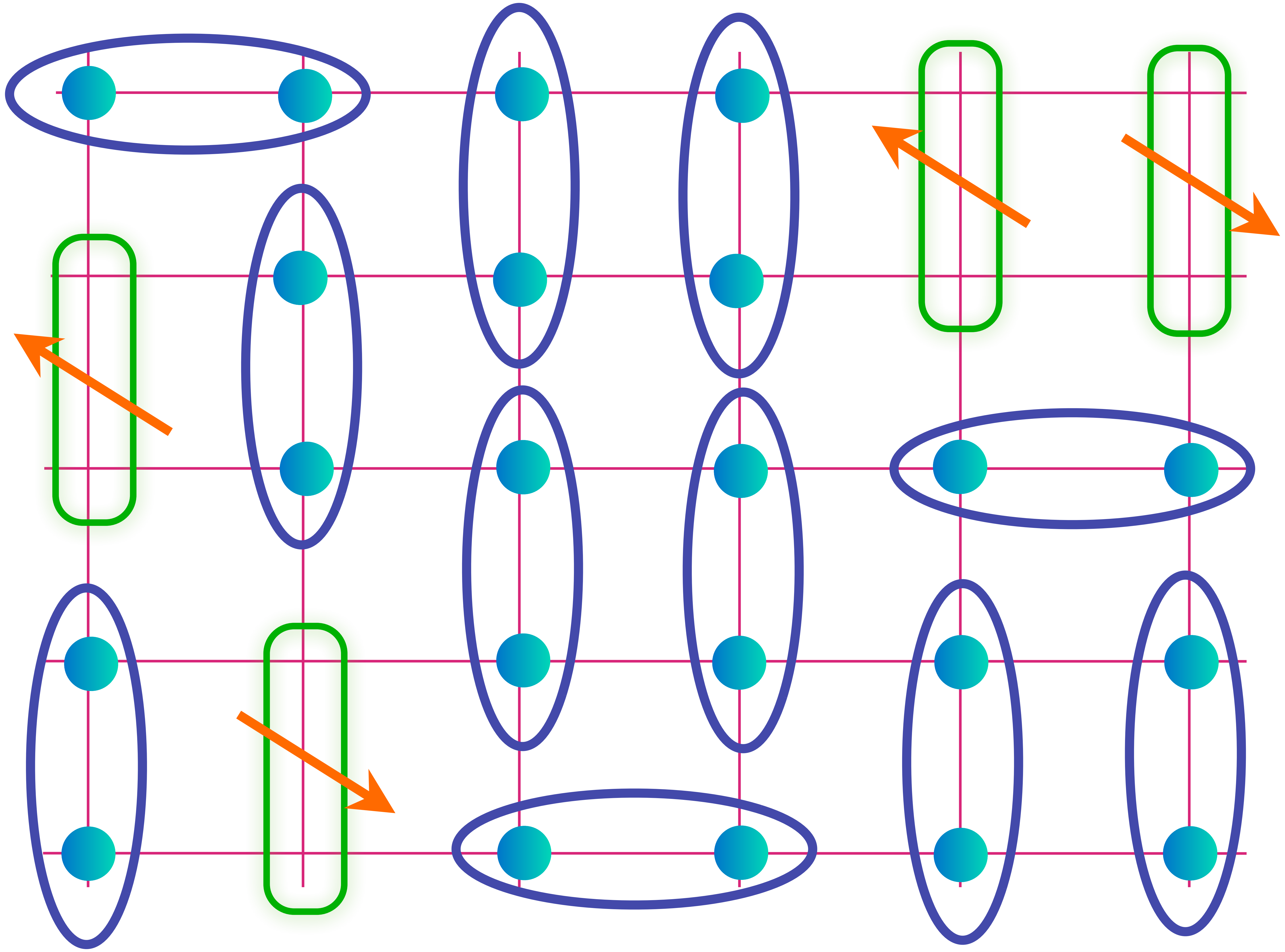

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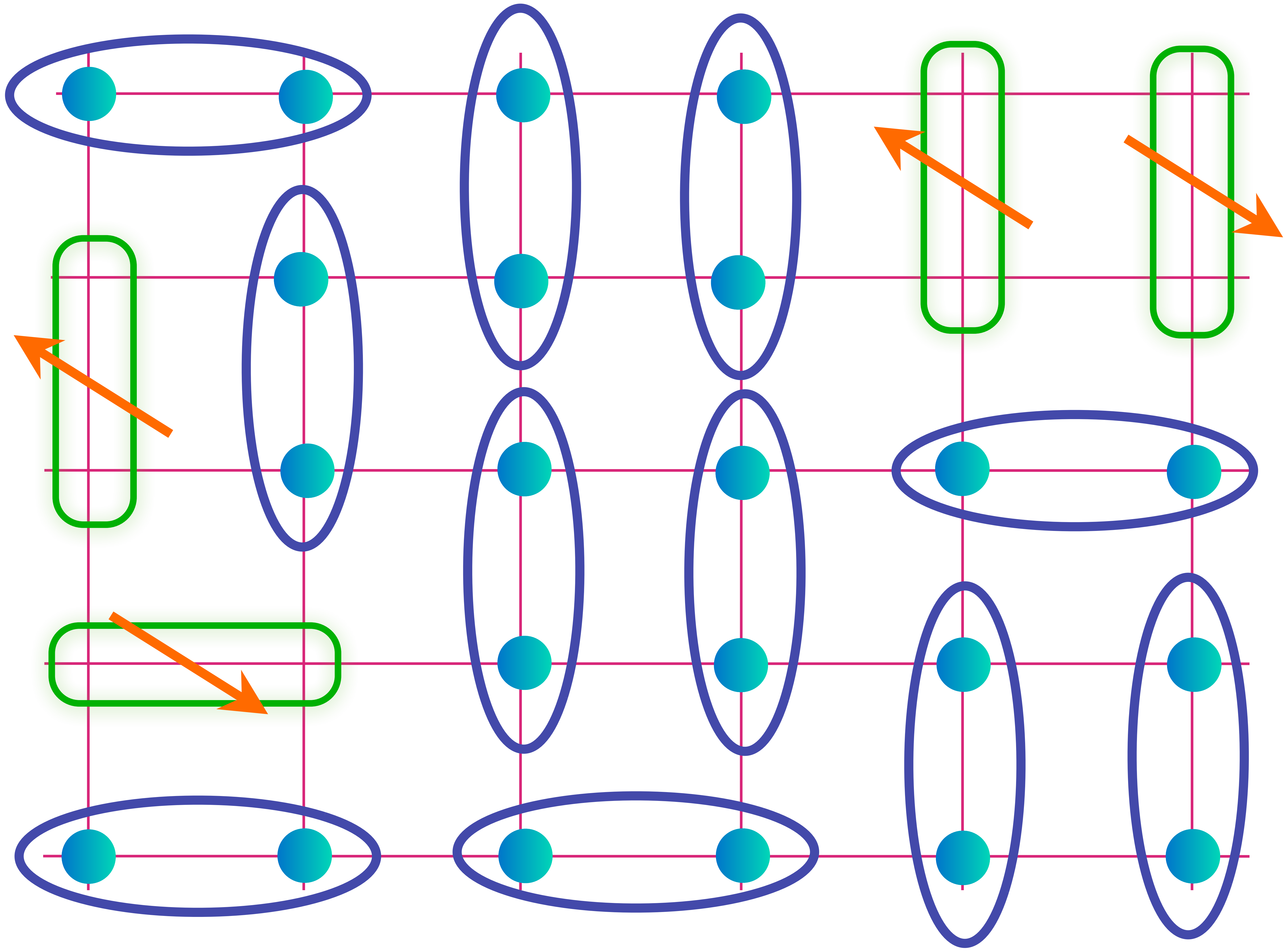

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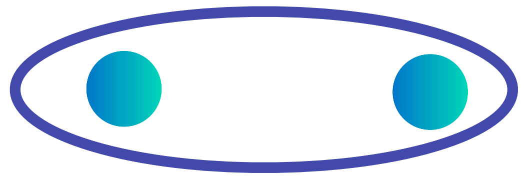
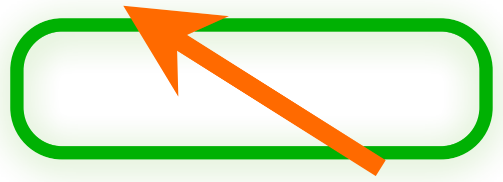
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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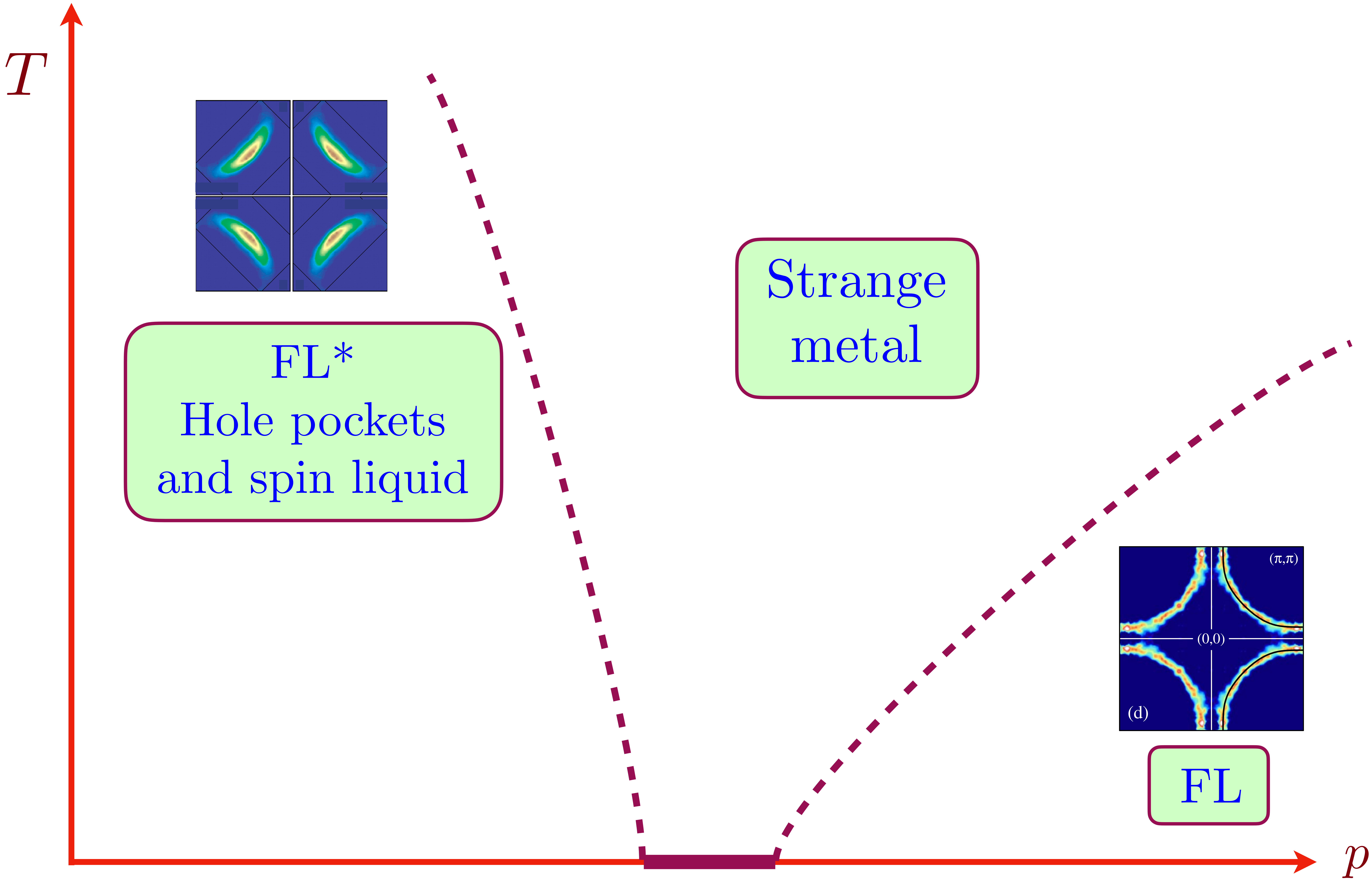
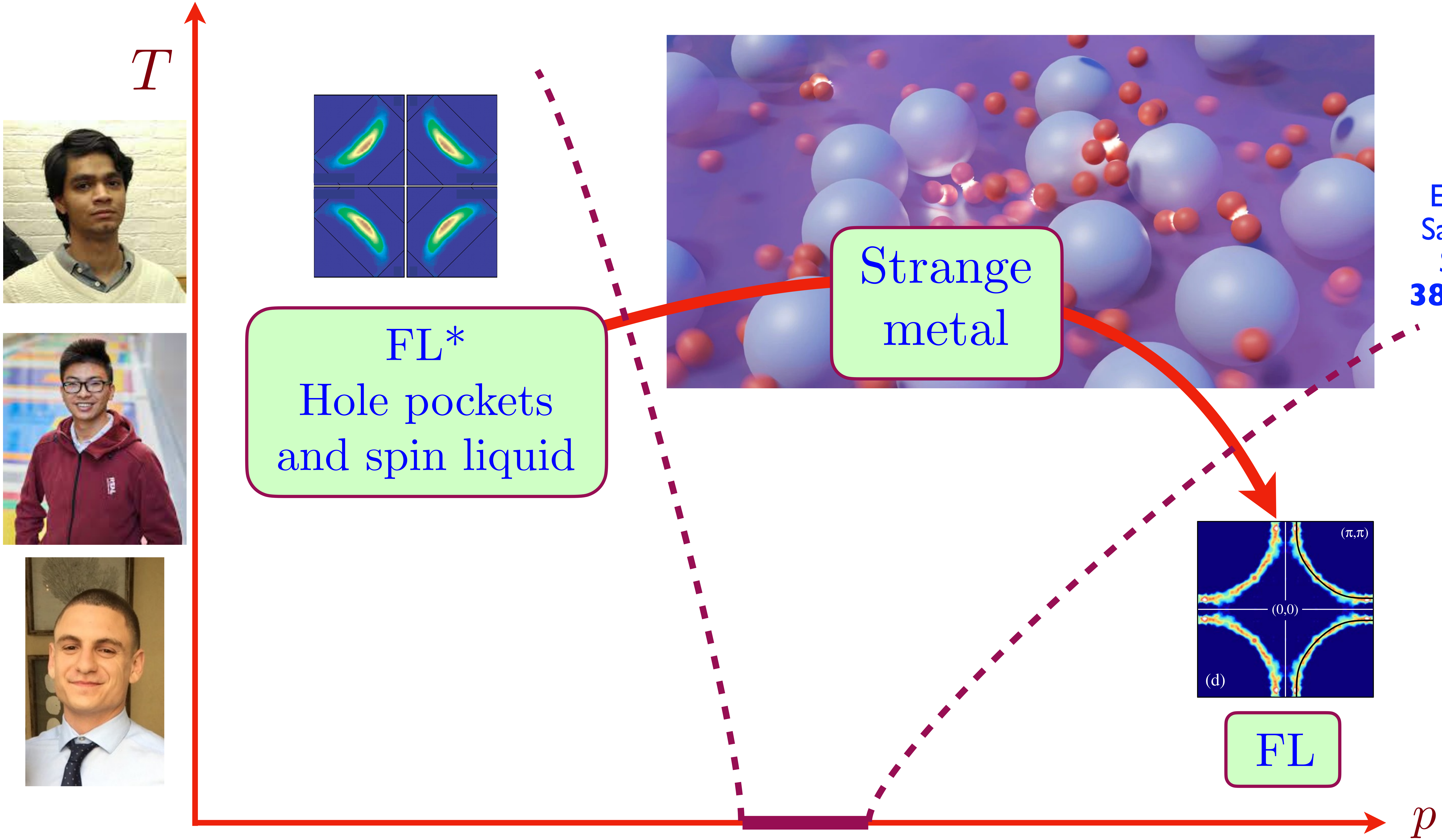
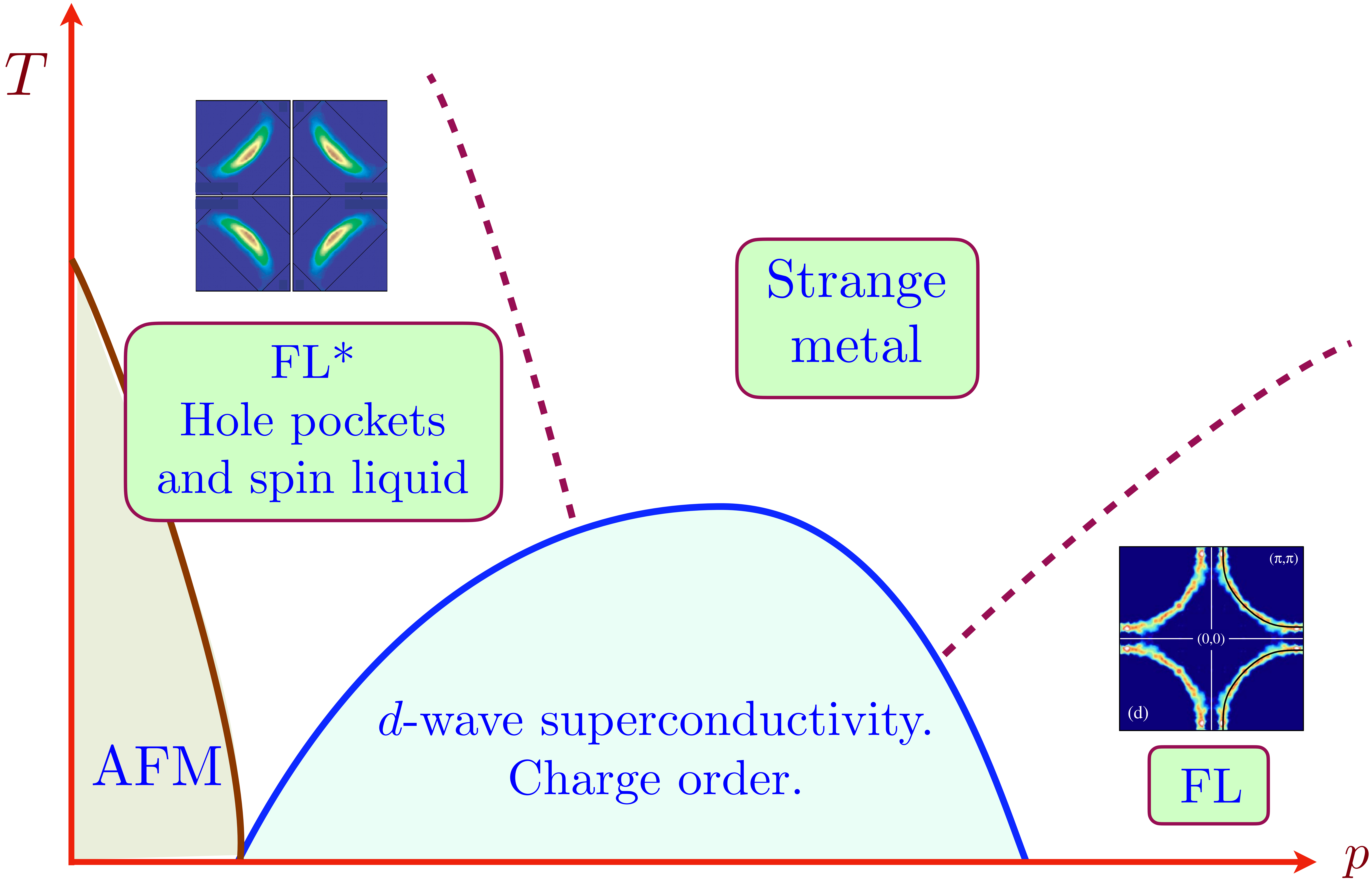

T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
p



Patel, Haoyu Guo, Esterlis, Sachdev, *Science* **381**, 790 (2023)


T
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p


The onset of conventional order is a *confinement transition* for the gauge theory describing the fractionalized excitations of the FL* spin liquid
T
Strange metal
FL*
Hole pockets and spin liquid
AFM
d-wave superconductivity.
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(π,π)
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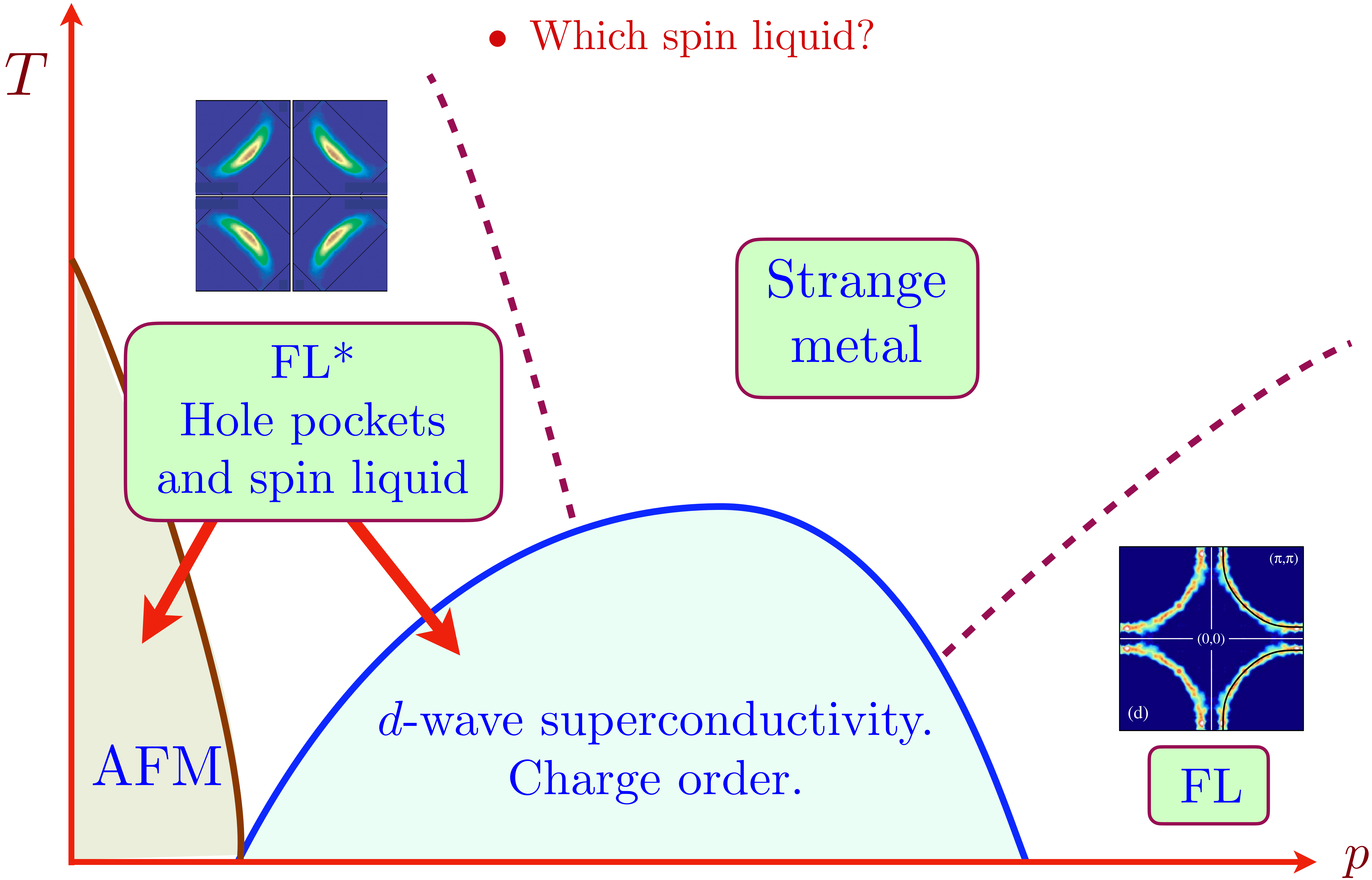
(d)
FL
p



• Which spin liquid?
T
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
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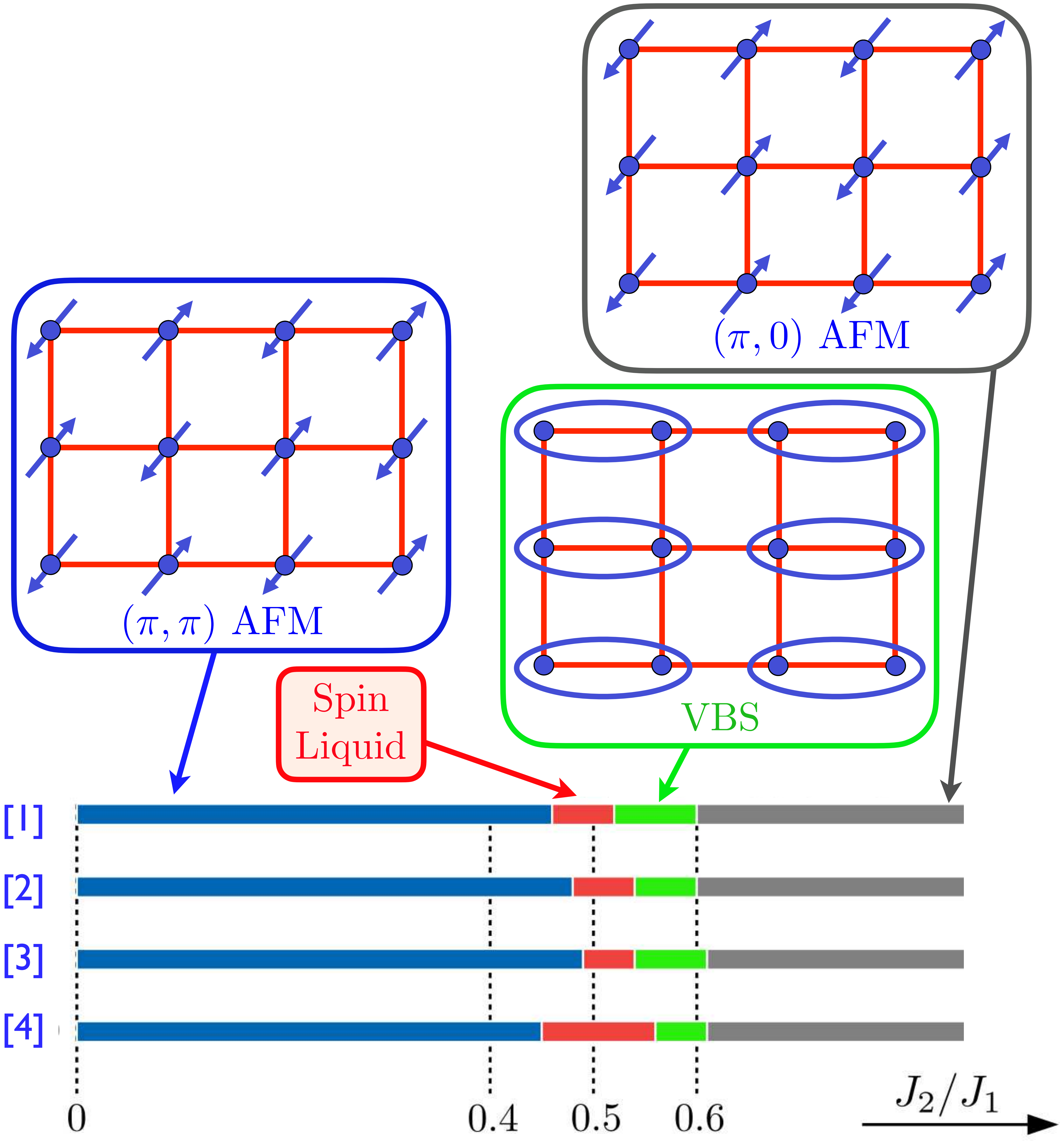
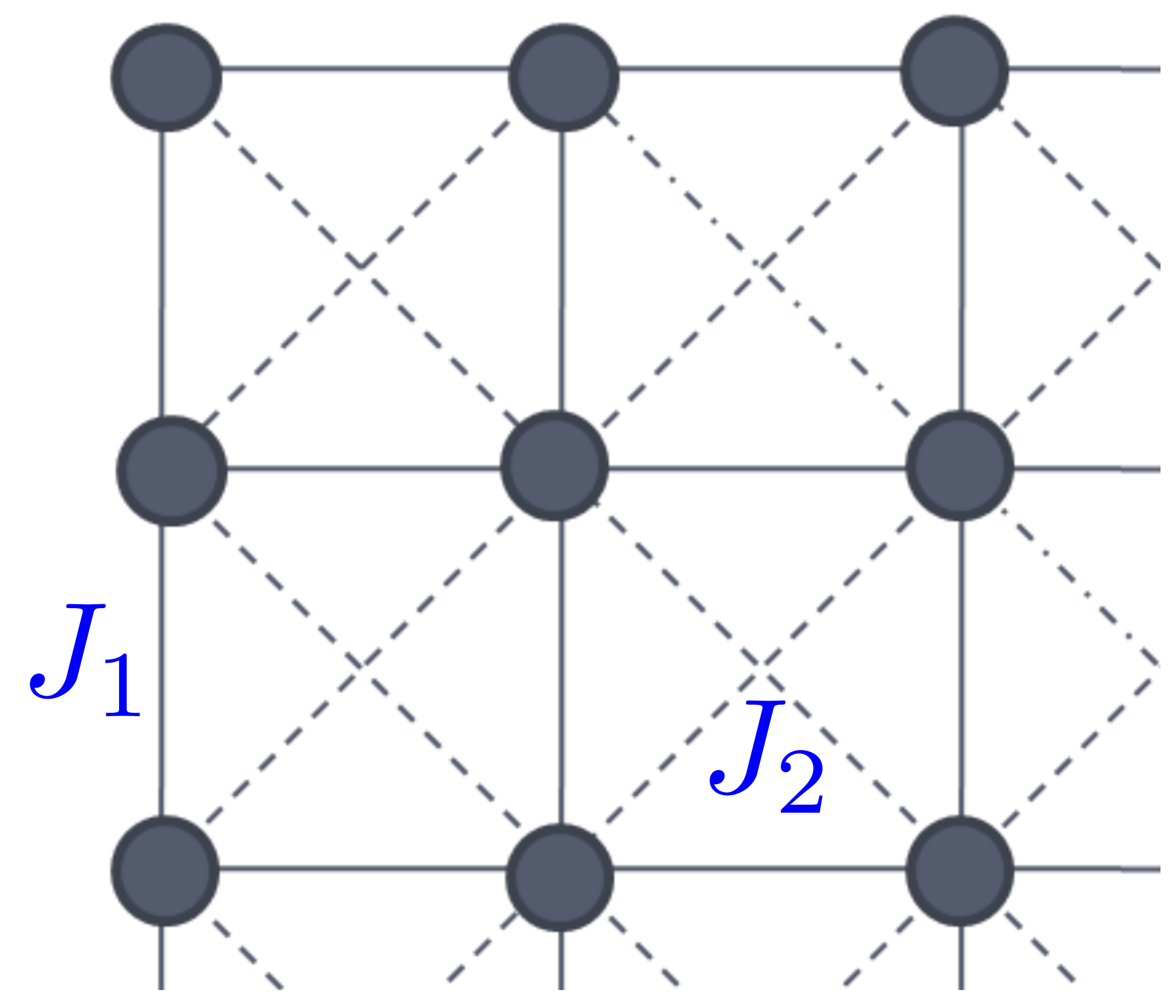
Charge order.
(π,π)
(0,0)
(d)
FL
p

Which spin liquid?
Mean-field theory for FL* in a single-band model?
T
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

The CP^1/π-flux spin liquid

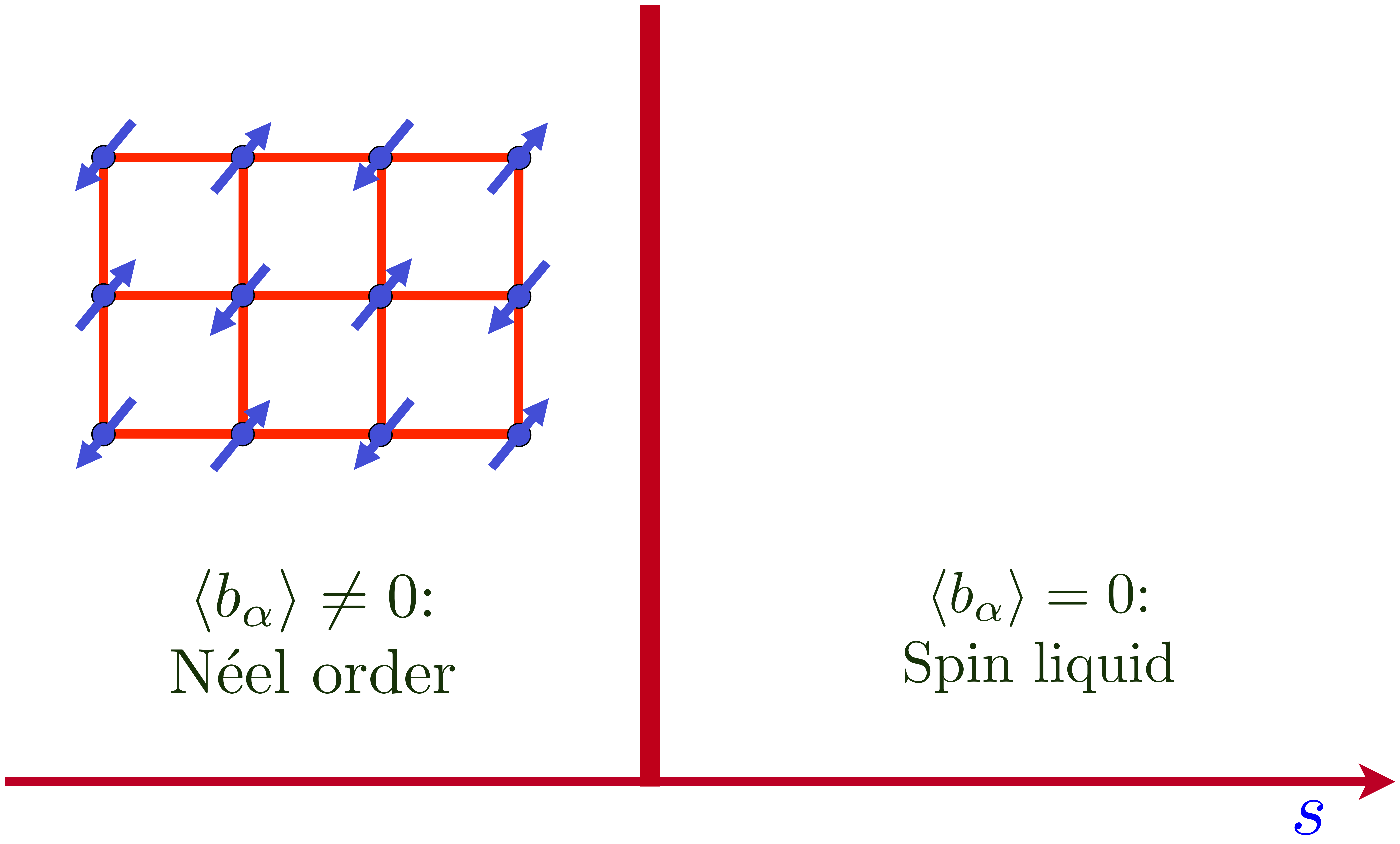
$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67,** 1034 (2022)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988)

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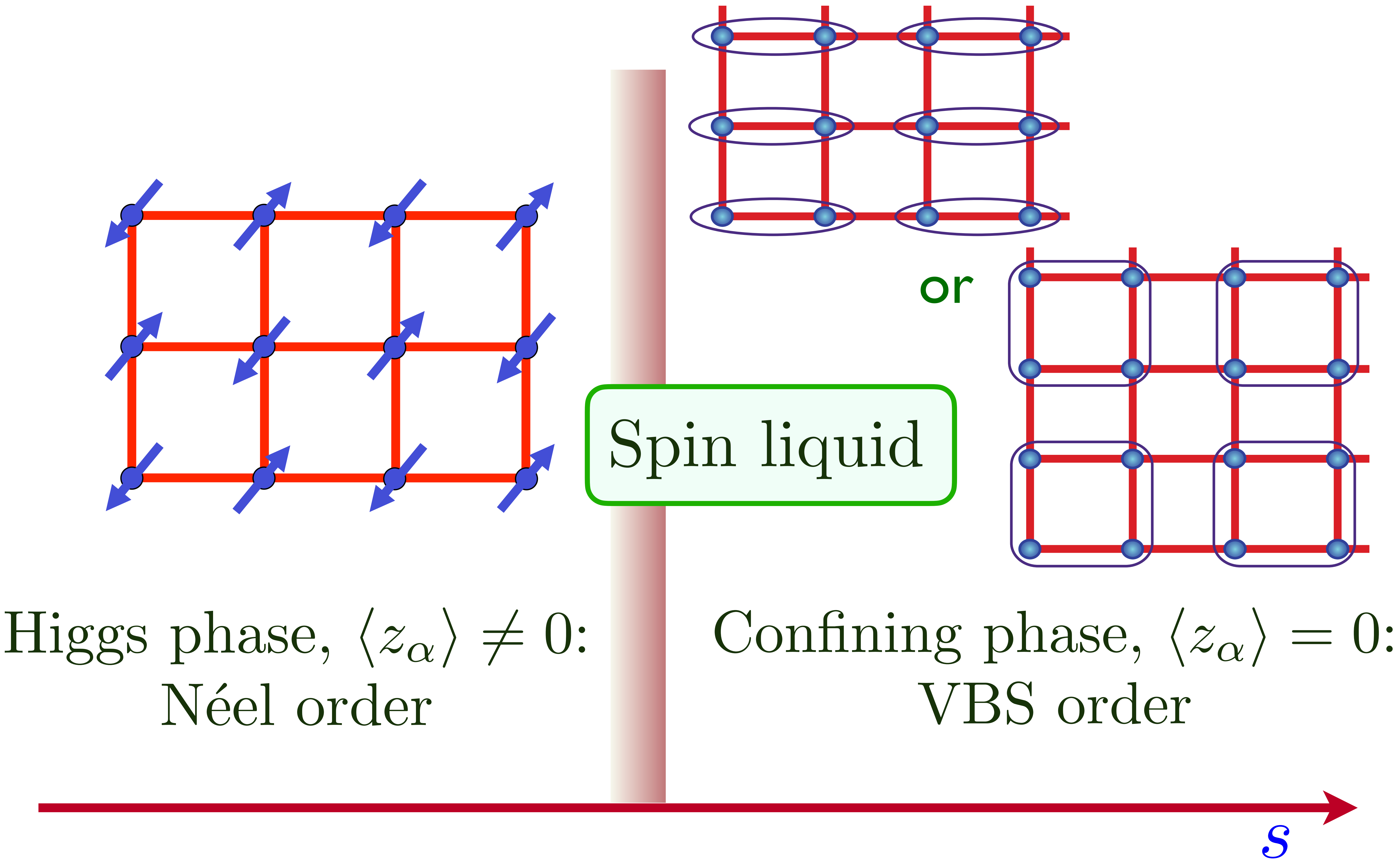

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Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2
square
lattice anti-
ferromagnet

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2 square lattice anti-ferromagnet

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

I. Affleck and J.B. Marston, *Phys. Rev. B* **37**, 3774 (1988)

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620; C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)

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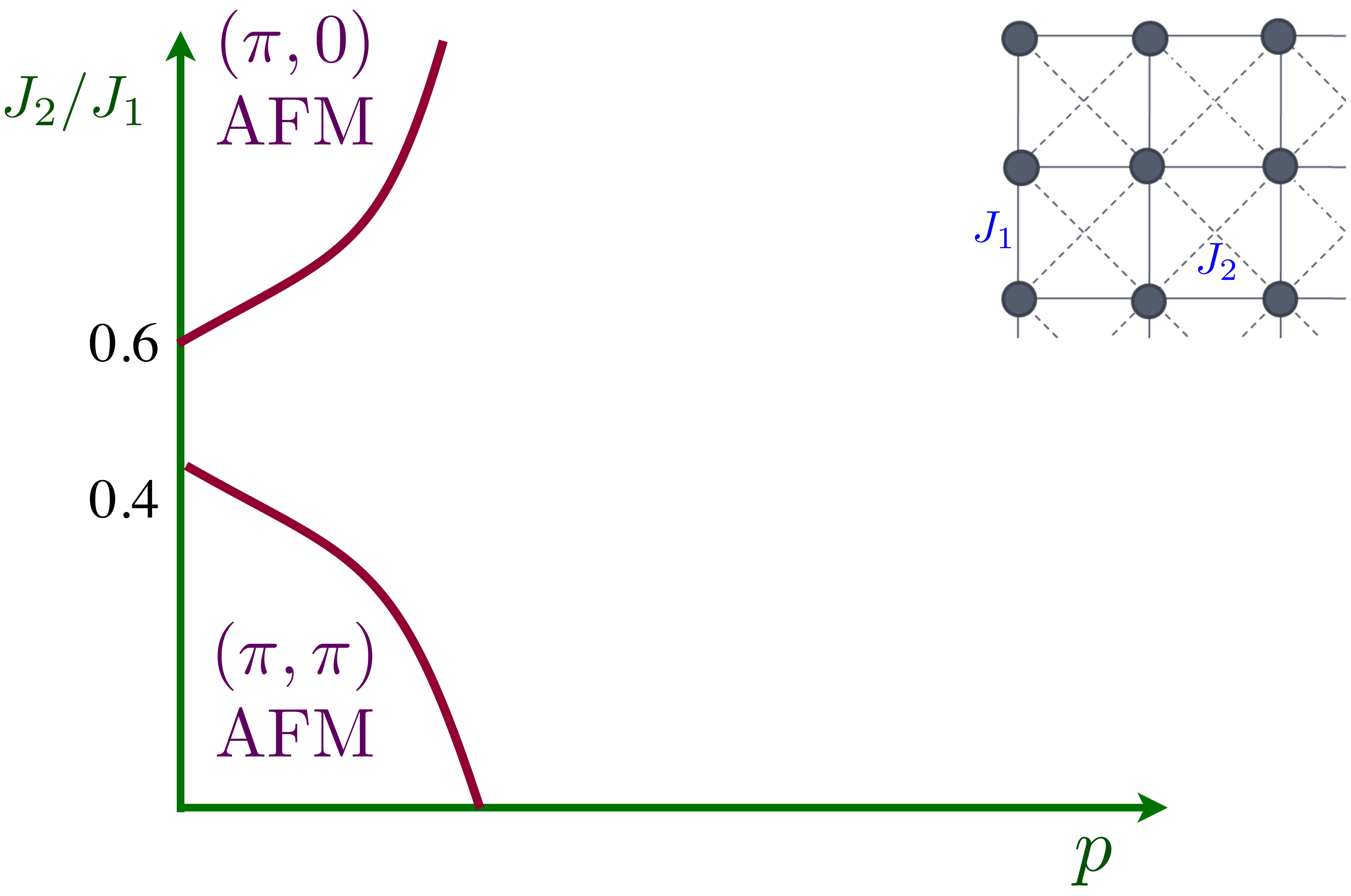
Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Anders W. Sandvik *Phys. Rev. Lett.* **98**, 227202 (2007)
Adam Nahum, P. Serna, J. T. Chalker, M. Ortuño, and A. M. Somoza, *Phys. Rev. Lett.* **115**, 267203 (2015)
Z. Zhou, L. Hu, W. Zhu, and Yin-Chen He, arXiv:2306.16435

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

Phys. Rev. Lett. **127**, 097002 (2021)



Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any “competing” charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

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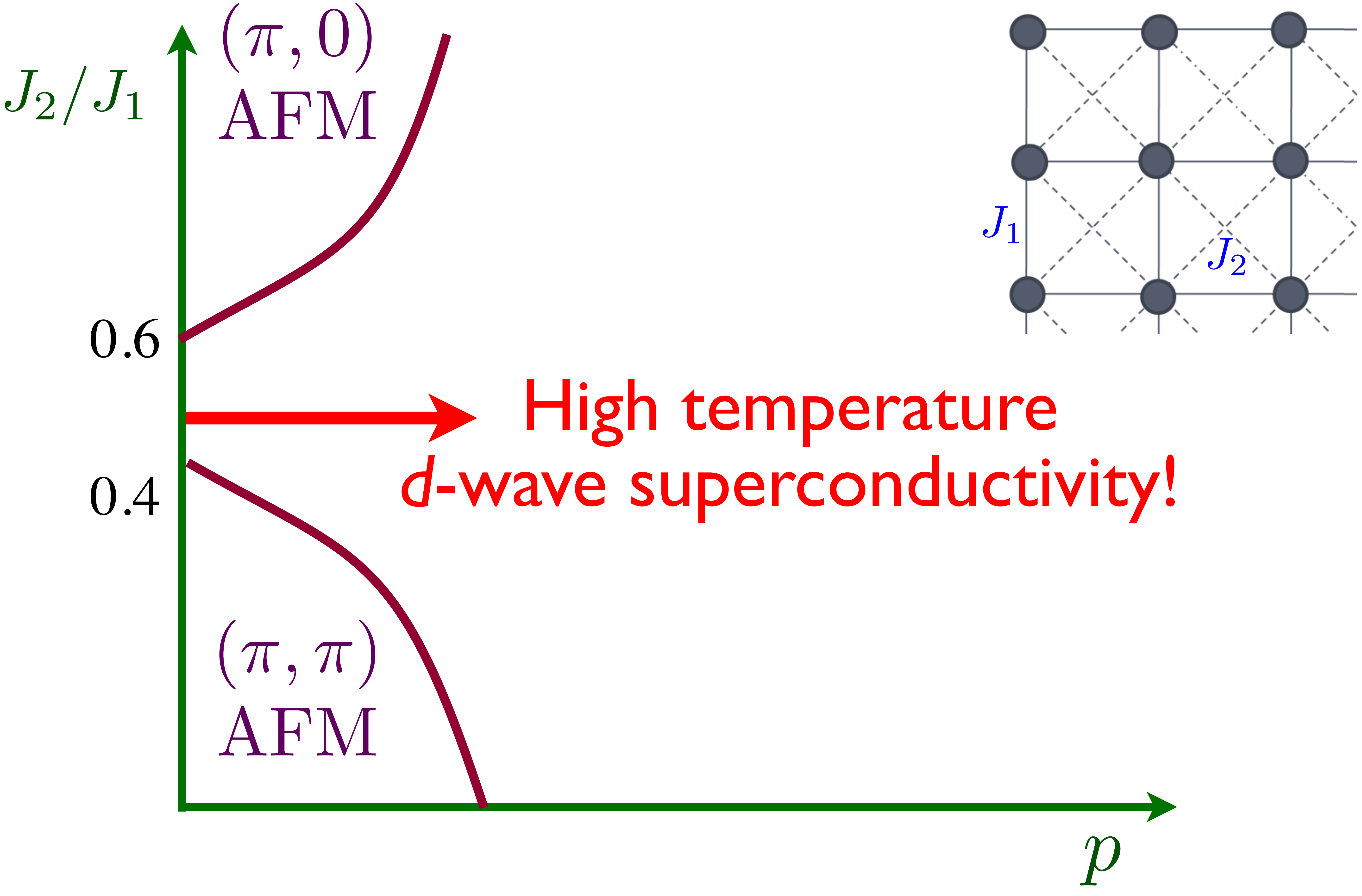

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Ancilla theory of FL*
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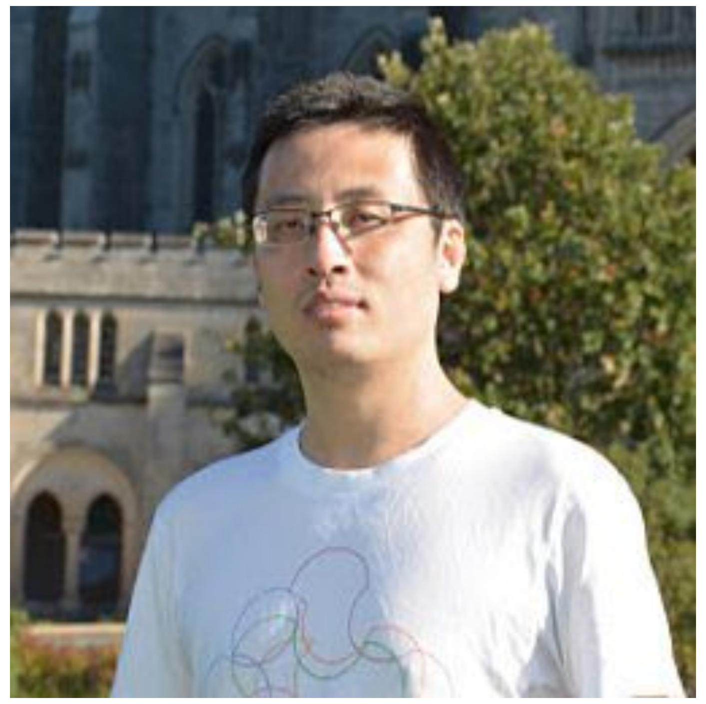


Ya-Hui Zhang

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

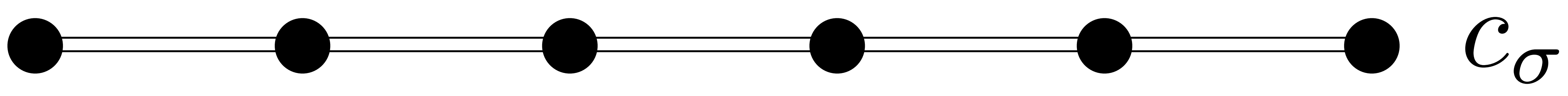
Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

$\boldsymbol{\Phi}_i$ is the creation/annihilation operator for charge 0, spin $S = 1$ particle.

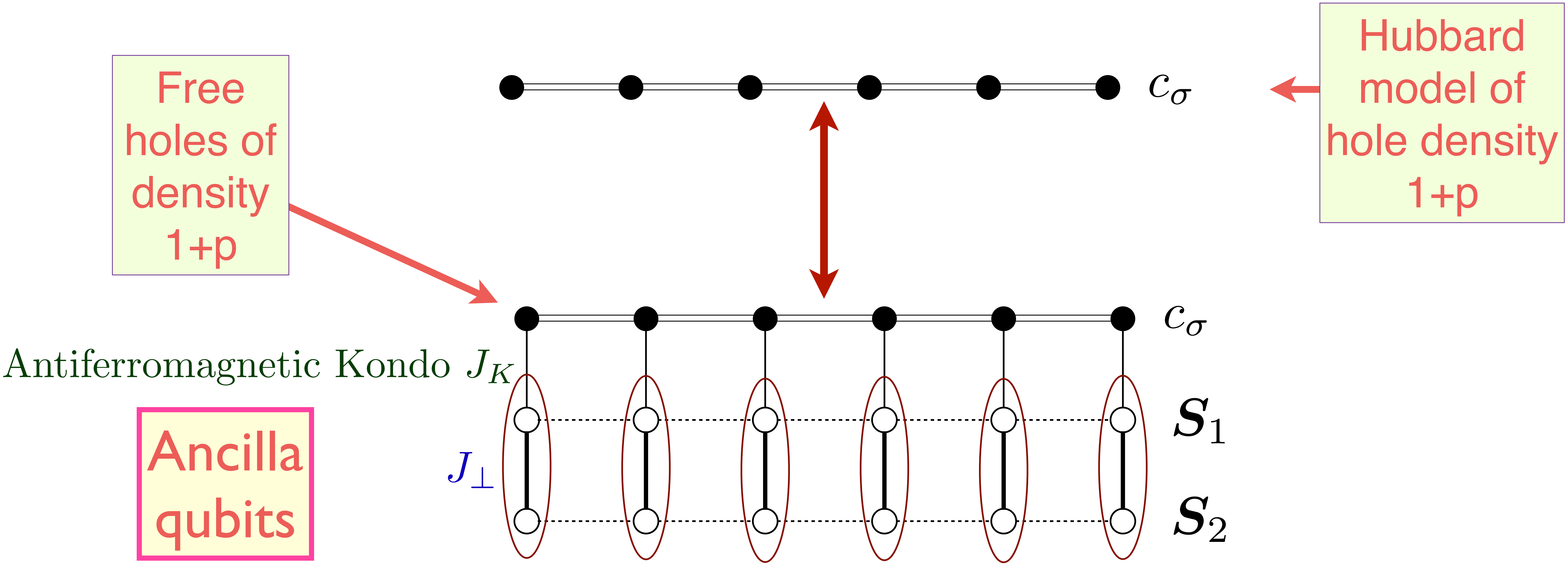
Ancilla theory of the Hubbard model



Hubbard model of hole density 1+p

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

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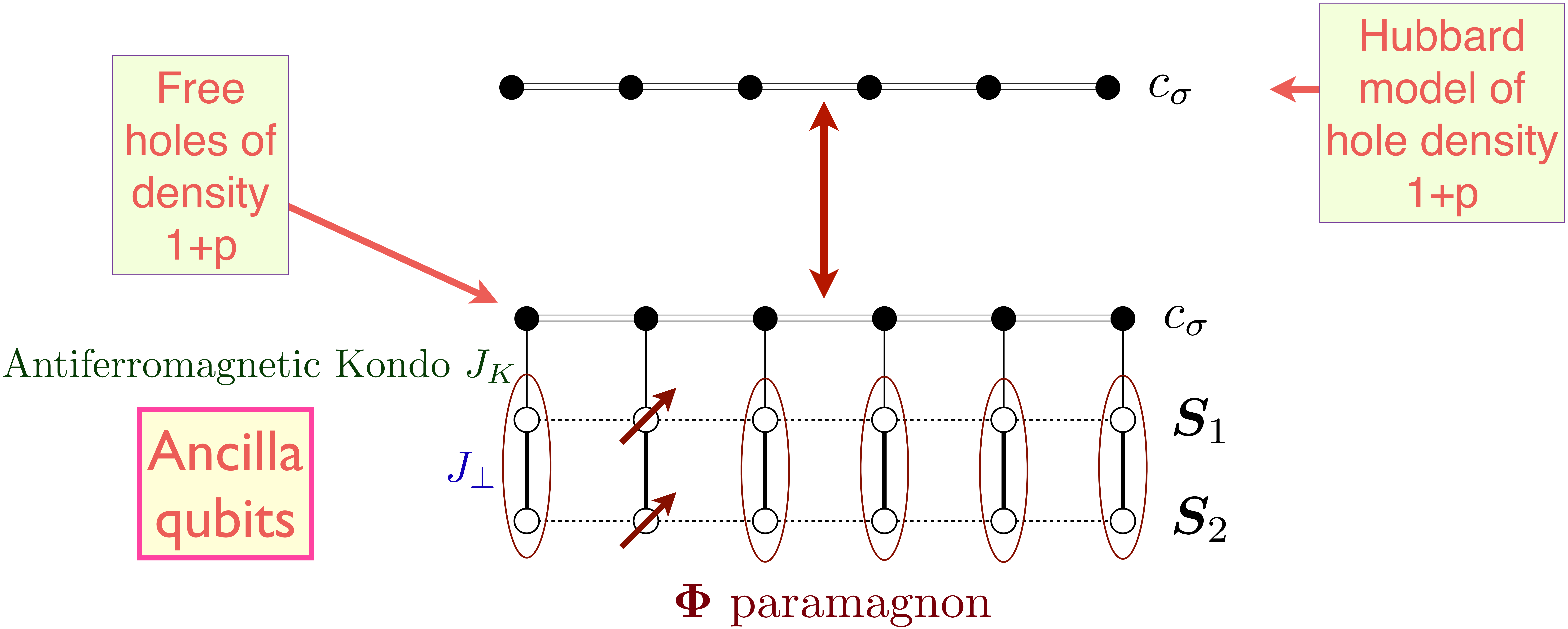
Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)



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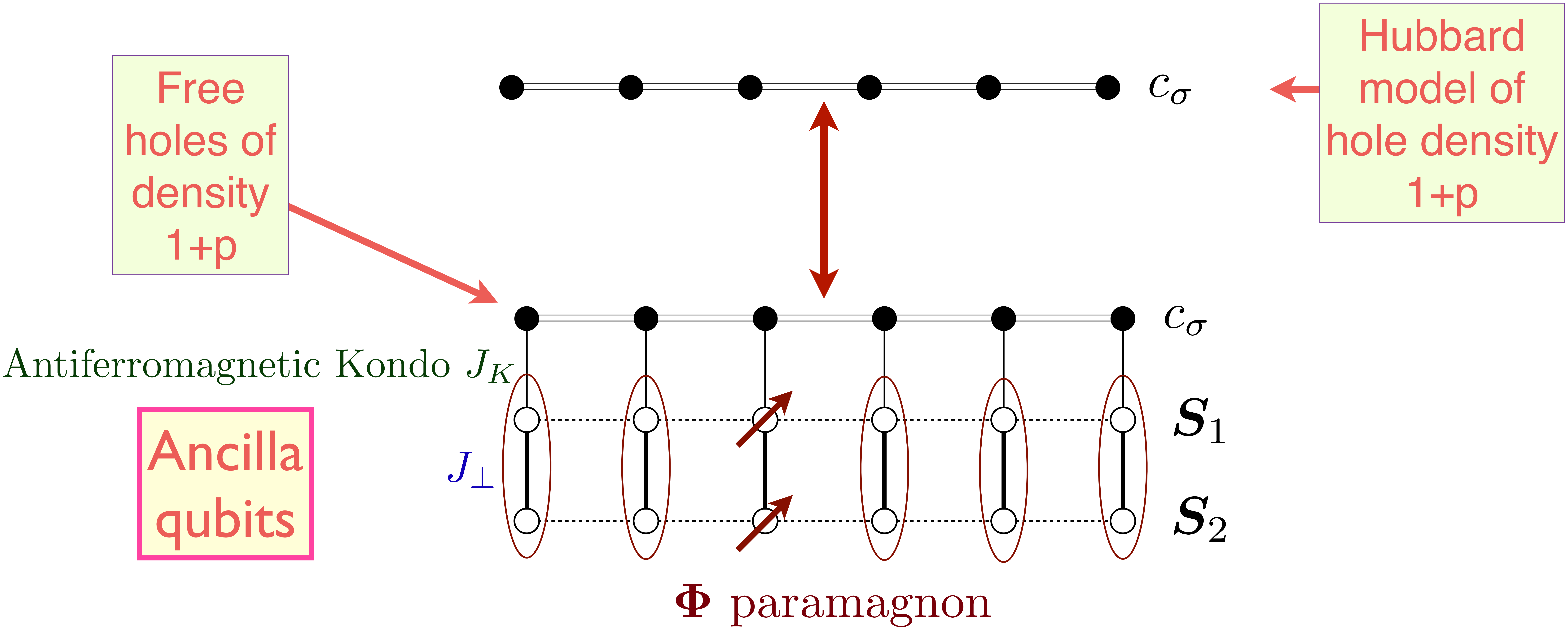

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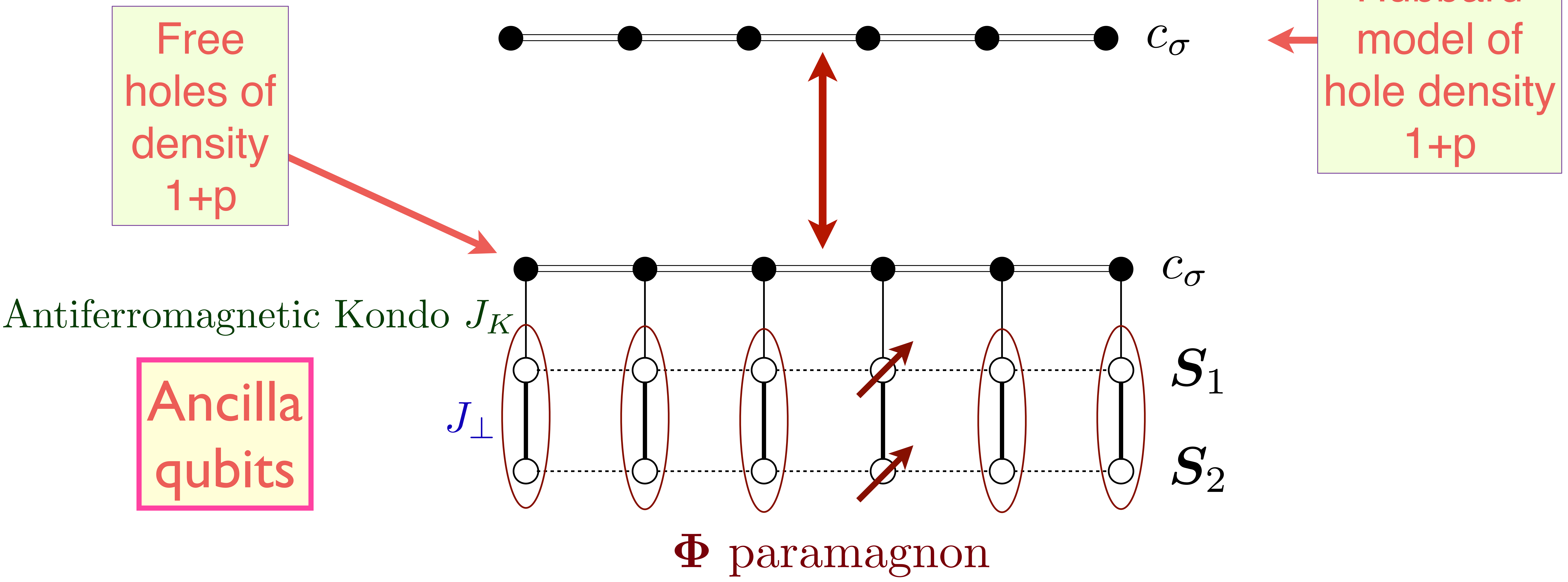

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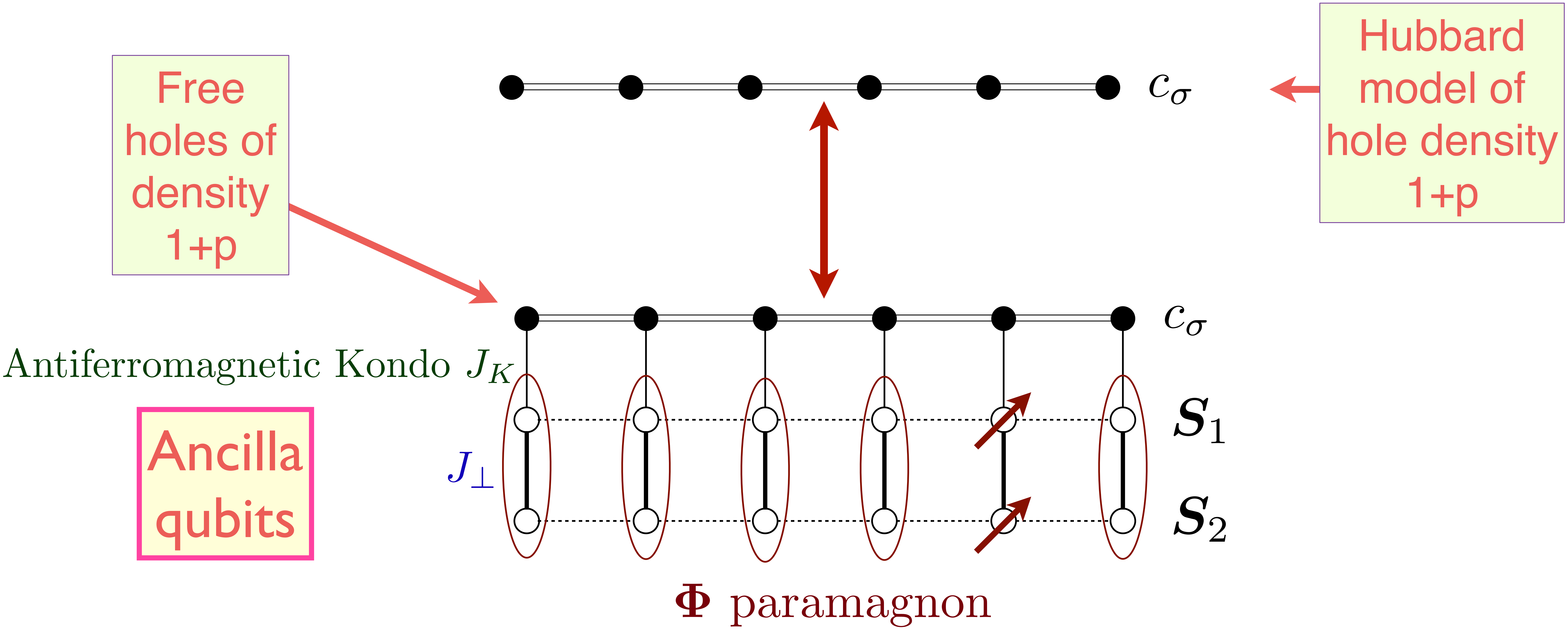
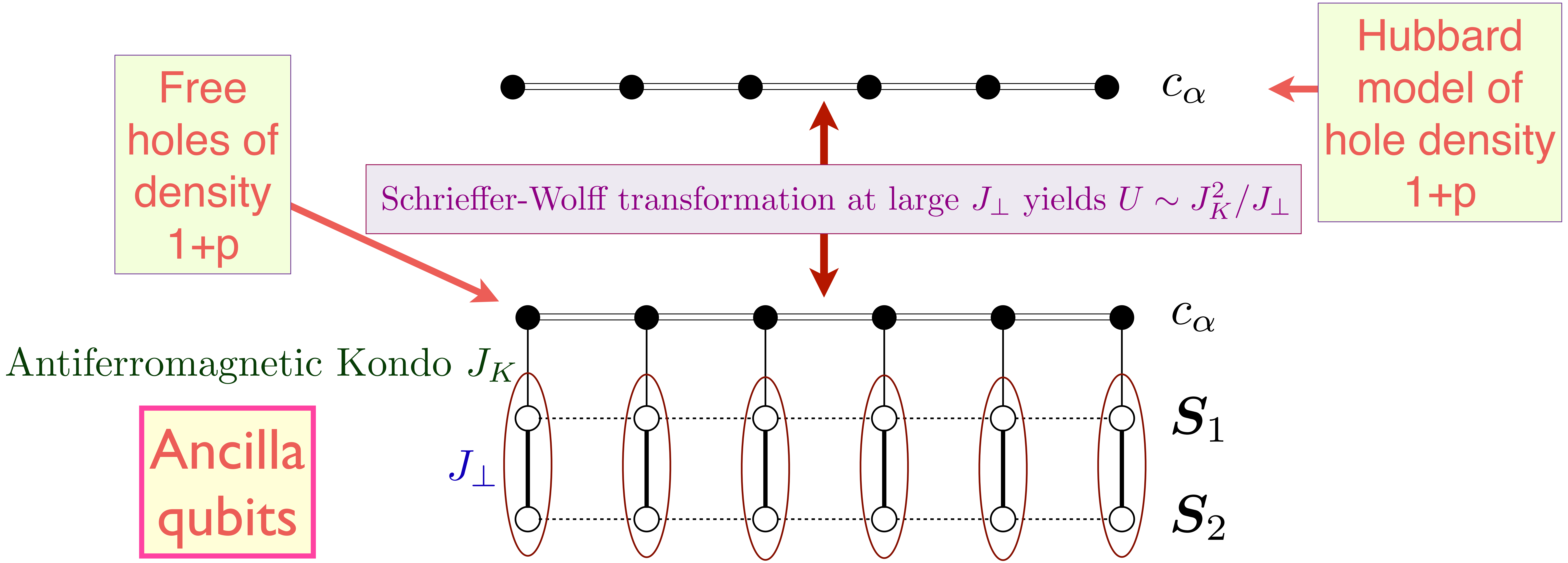
Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

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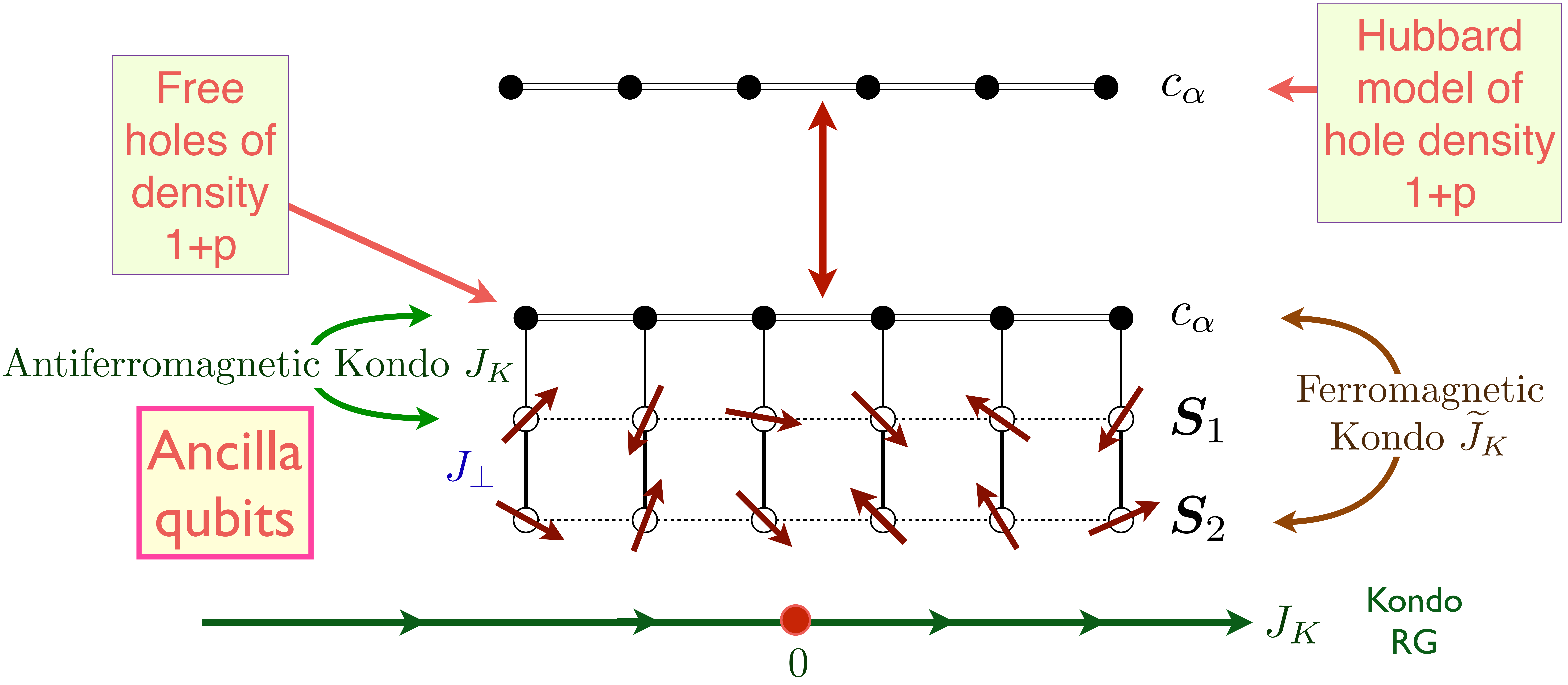
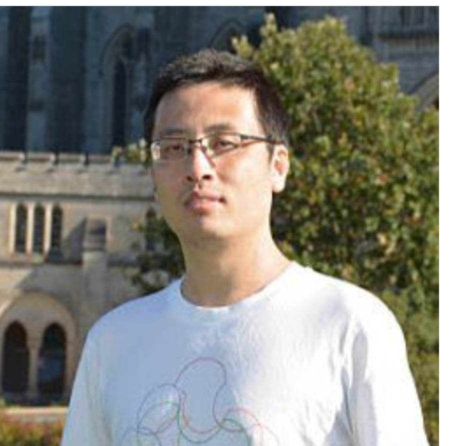


$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)

Ya-Hui Zhang

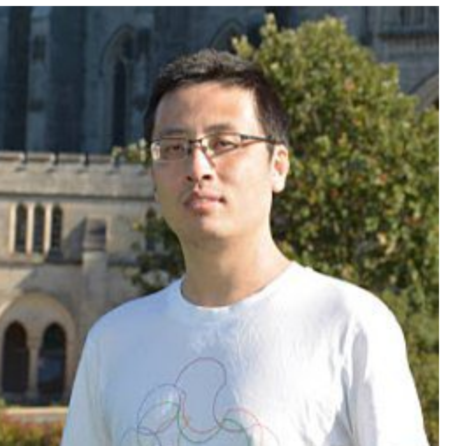


Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

Ya-Hui Zhang



Yahui Zhang

Alexander Nikolaenko

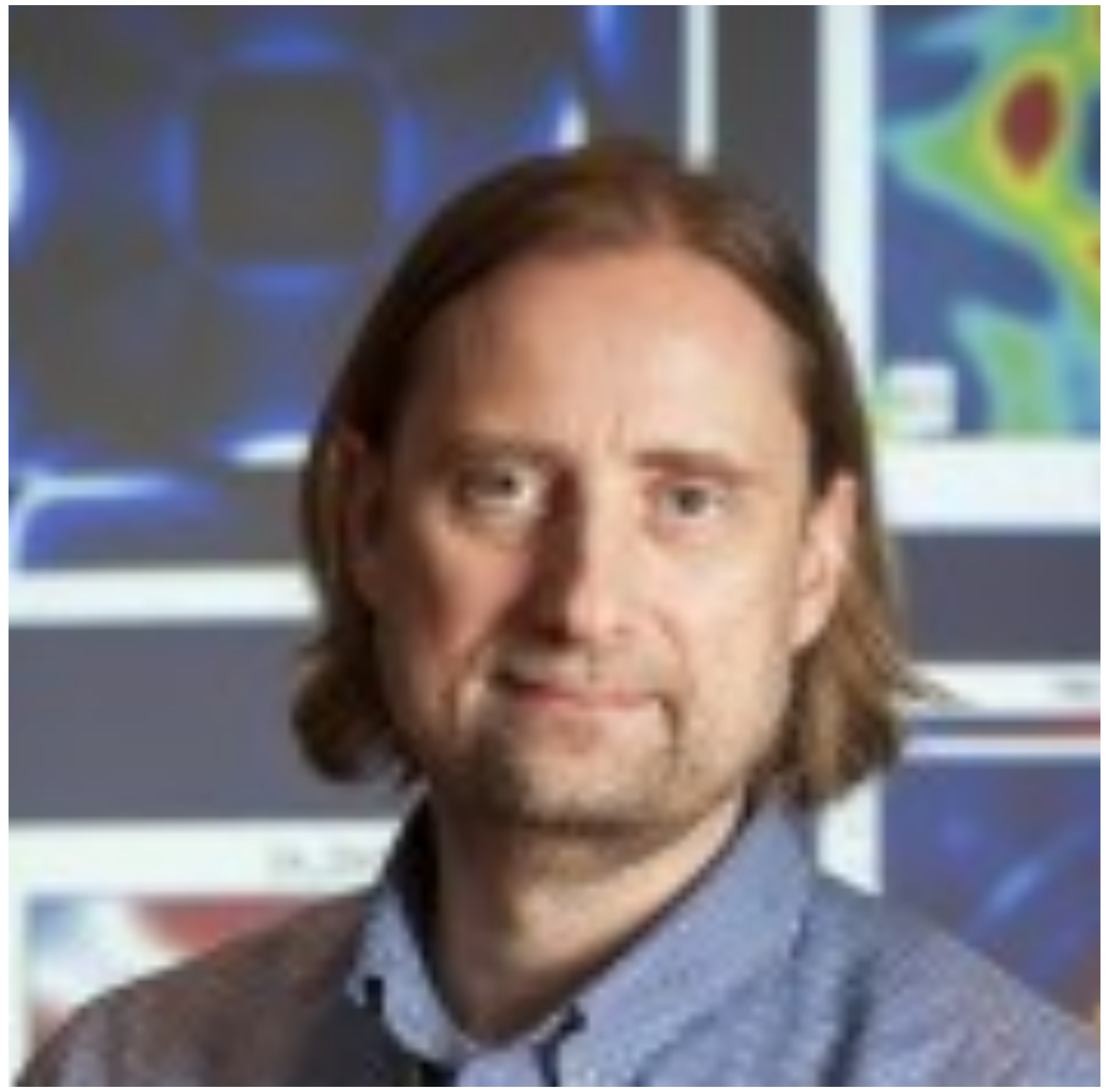
Maria Tikhanovskaya

Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

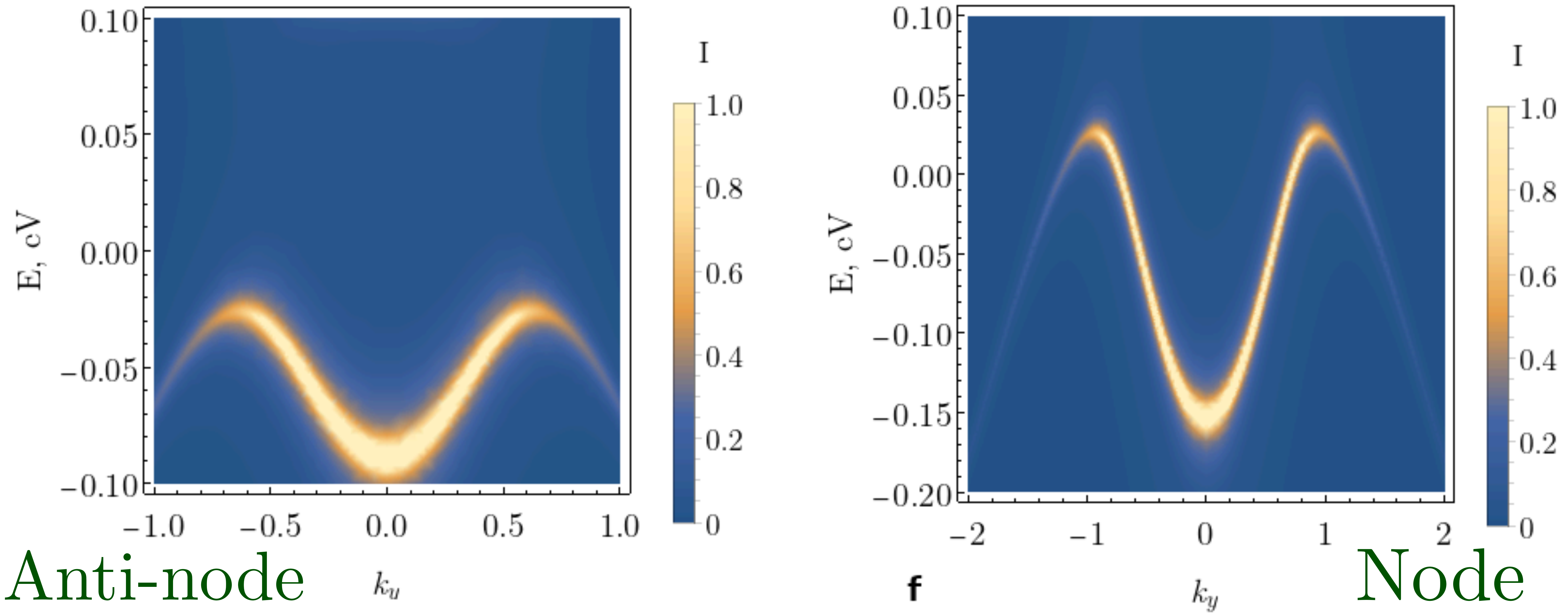
arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



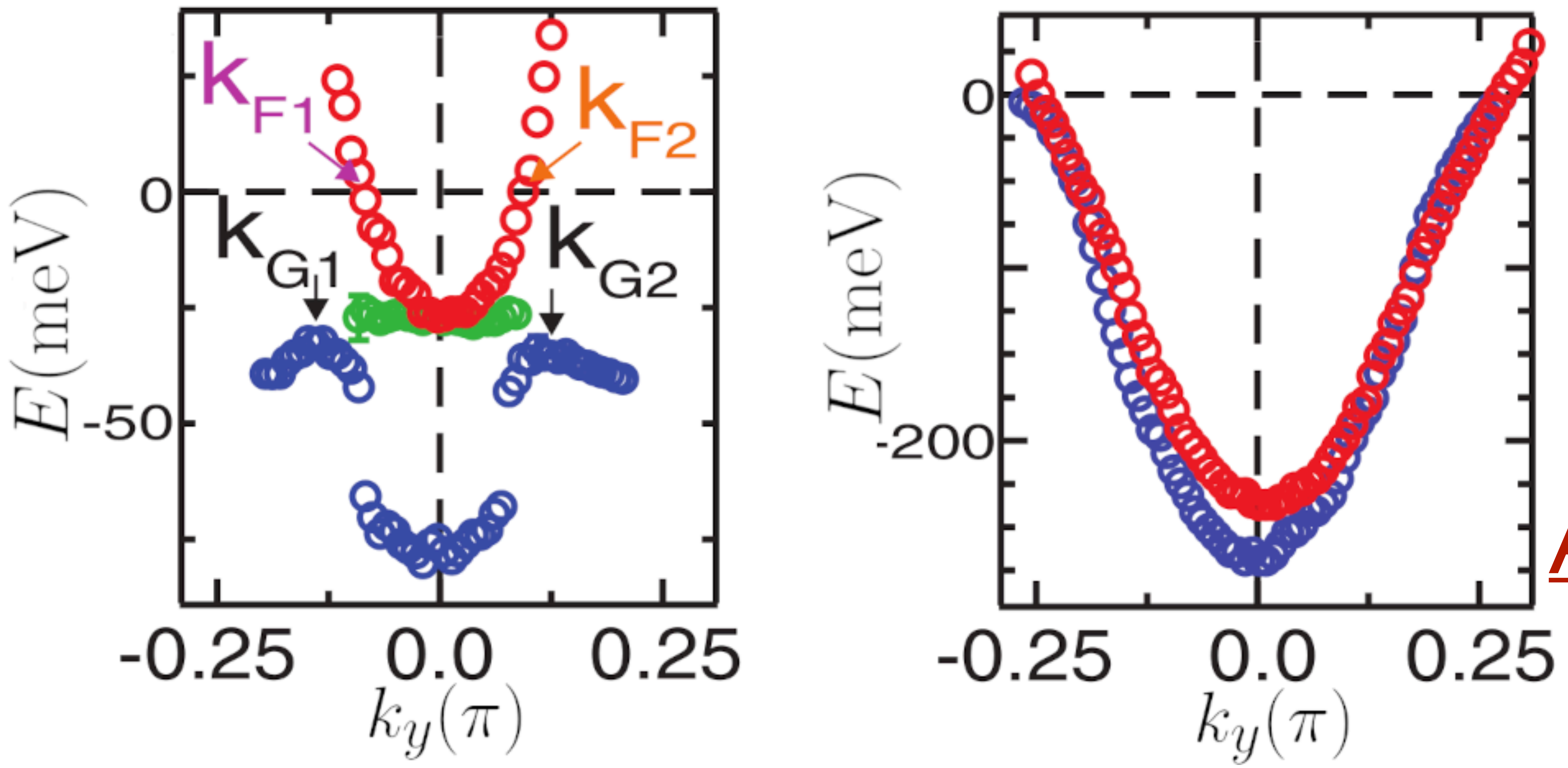
Mascot, Nikolaenko, Tikhanovskaya, Zhang, Morr, and Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

FL* in a ***one-band*** model



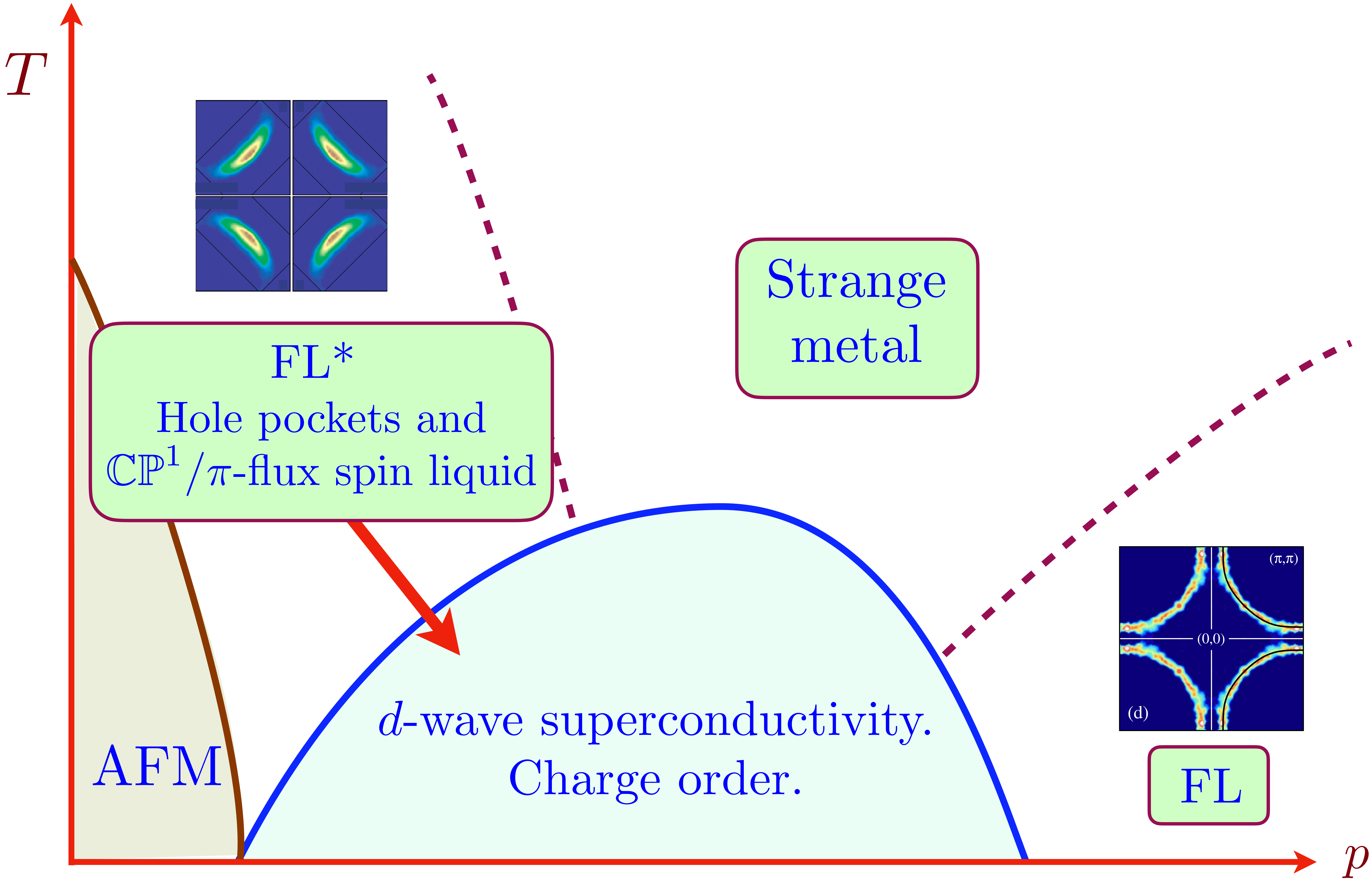
Broadening by second ancilla layer is needed to describe MDC and EDC

He, Hashimoto, Karapetyan, Koralek, Hinton, Testaud, Nathan, Yoshida, Yao, Tanaka, Meevasana, Moore, Lu, Mo, Ishikado, Eisaki, Hussain, Devereaux, Kivelson, Orenstein, Kapitulnik, and Shen, *Science* **331**, 1579 (2011)



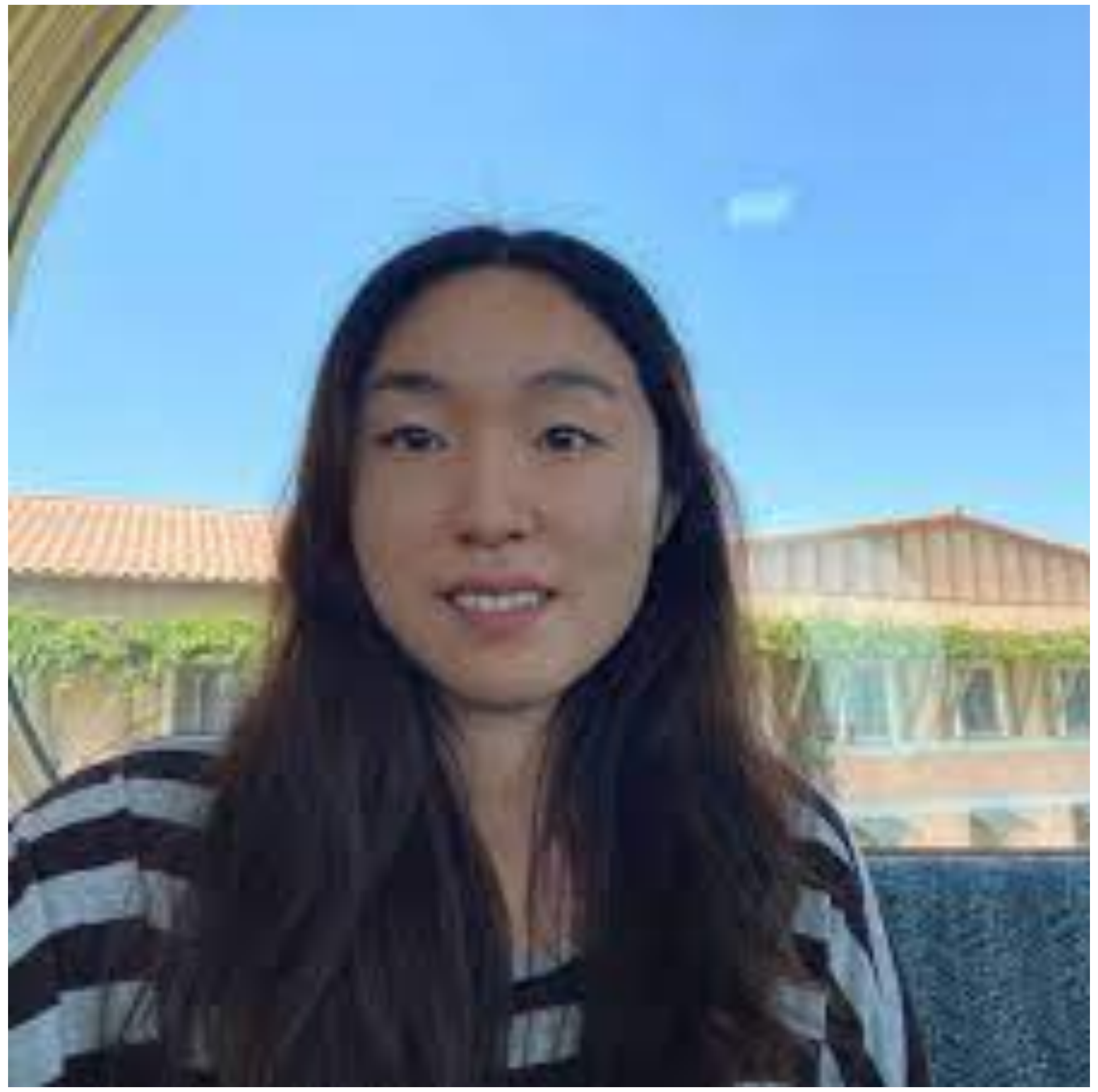
ARPES on Bi2201

From CP^1/π-flux FL*
to
d-wave superconductivity



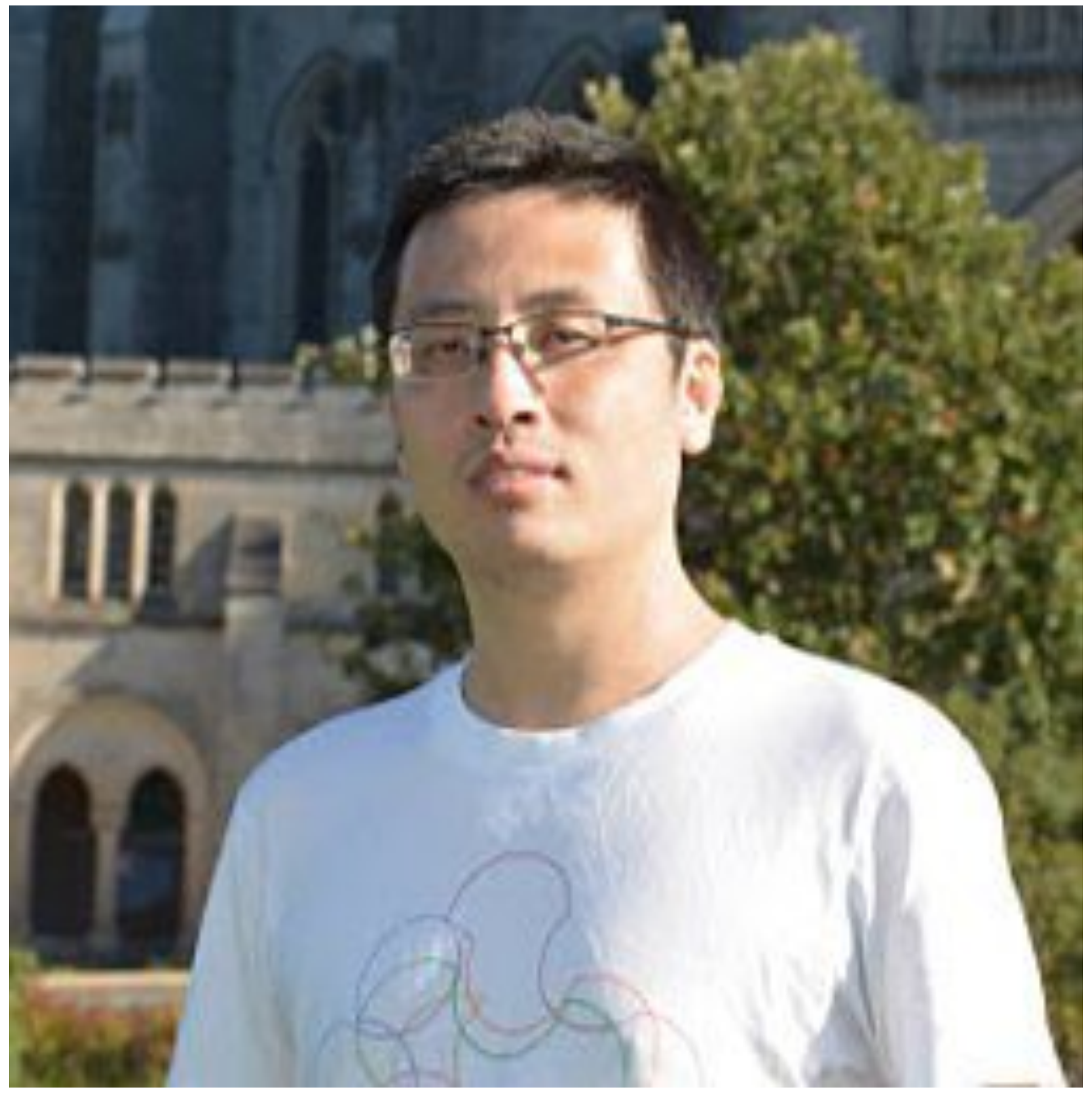
T
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

Maine Christos



Zhu-Xi Luo

Mathias Scheurer



Ya-Hui Zhang

Henry Shackleton

Ancilla theory of the Hubbard model

Ancilla theory of the Hubbard model

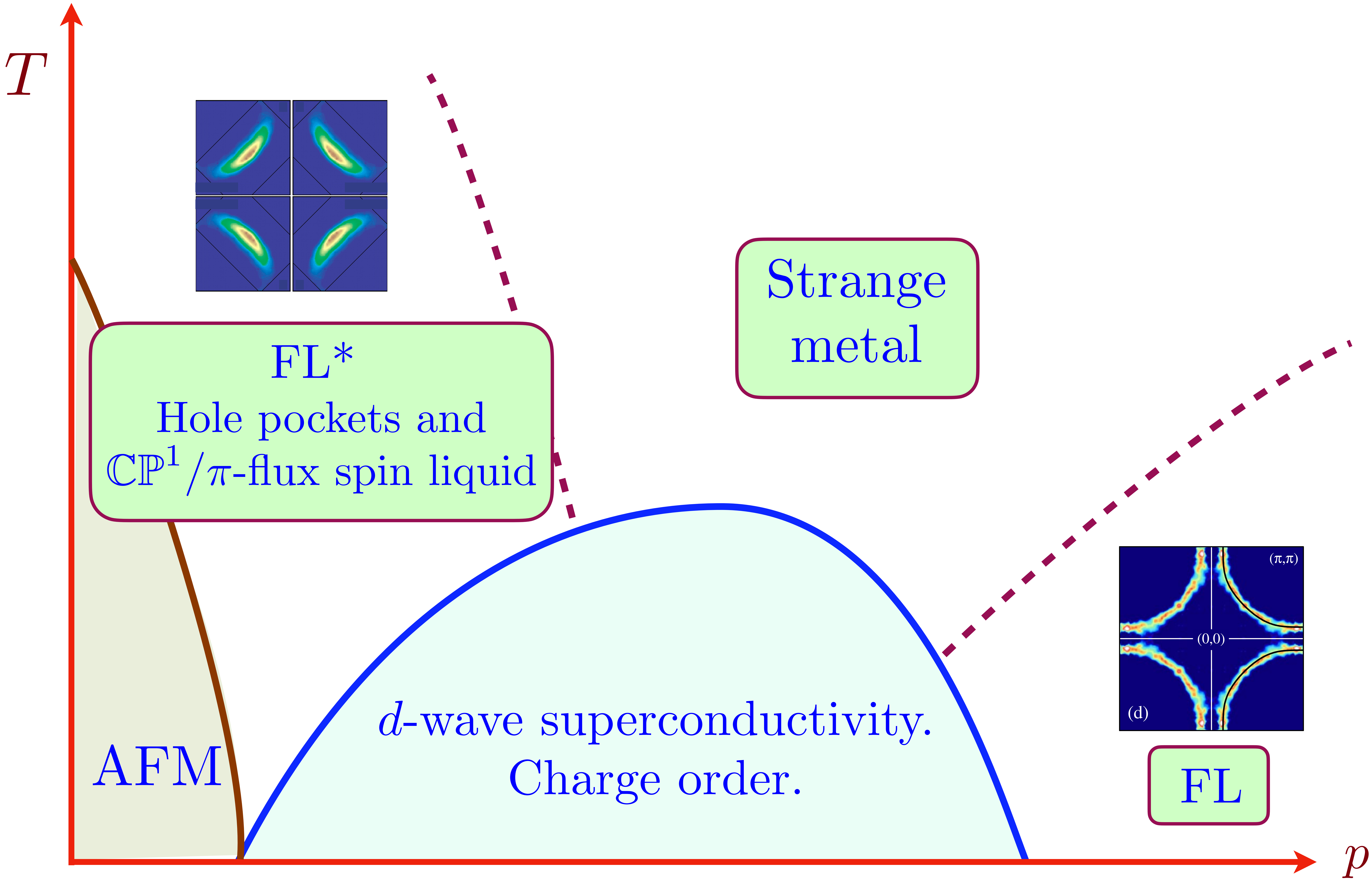
Higgs field Φ drives FL*-strange metal-FL

Ancilla theory of the Hubbard model

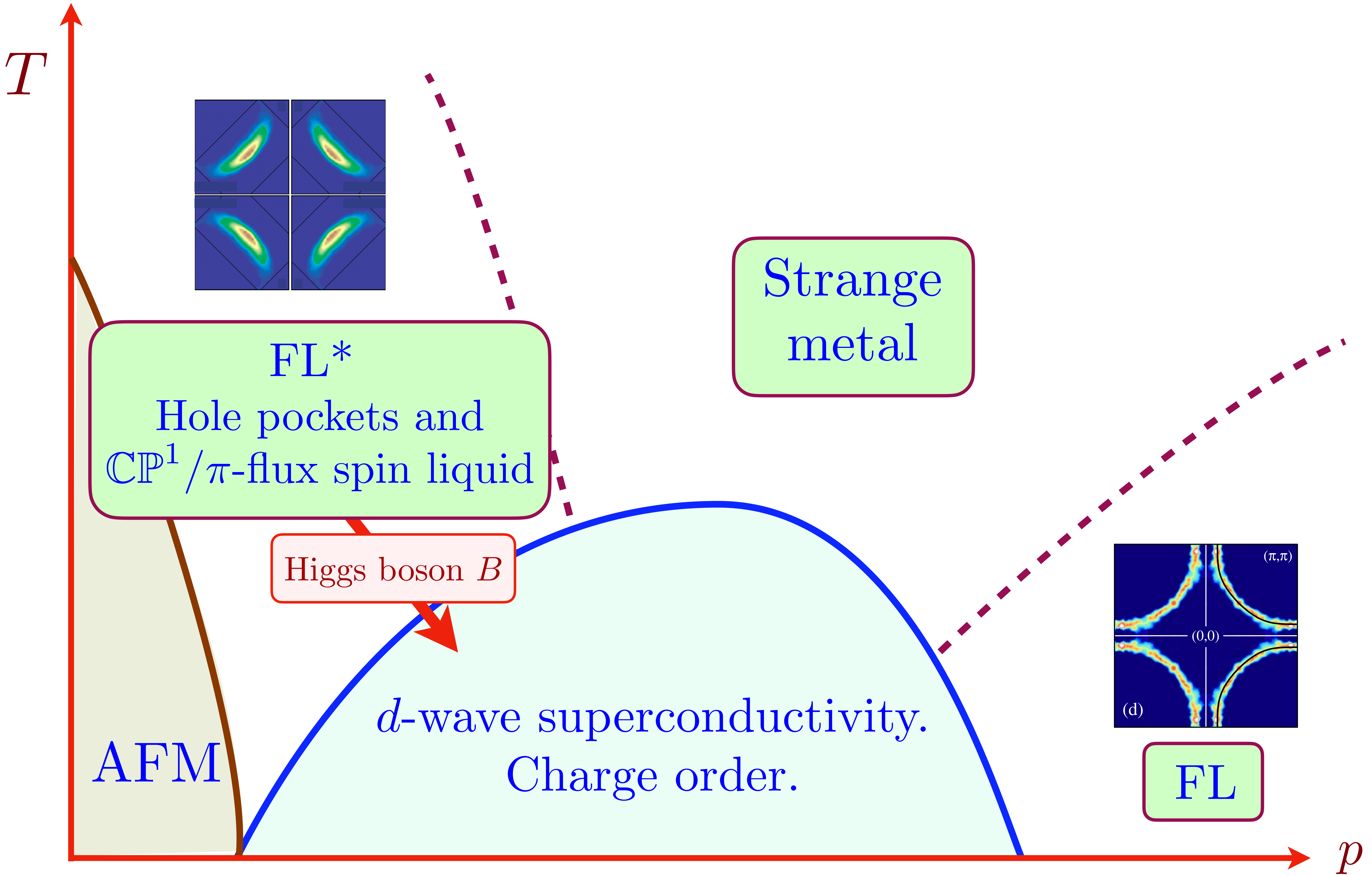
B has same quantum numbers as boson in X.-G. Wen and P.A. Lee, *Phys. Rev. Lett.* **76**, 503 (1996)

Ancilla theory of the Hubbard model

Higgs field B drives FL*-dSC/CDW



T
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Strange
metal
d-wave superconductivity.
Charge order.
AFM
(π,π)
(0,0)
(d)
FL
p



T
Strange metal
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Higgs boson B
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

Spinons move in π flux $\Rightarrow$ B moves in π flux $\Rightarrow$ Near-degenerate instabilities to dSC with 4 nodal points and to charge order.
T
Strange metal
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Higgs boson B
(π,π)
(0,0)
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d-wave superconductivity.
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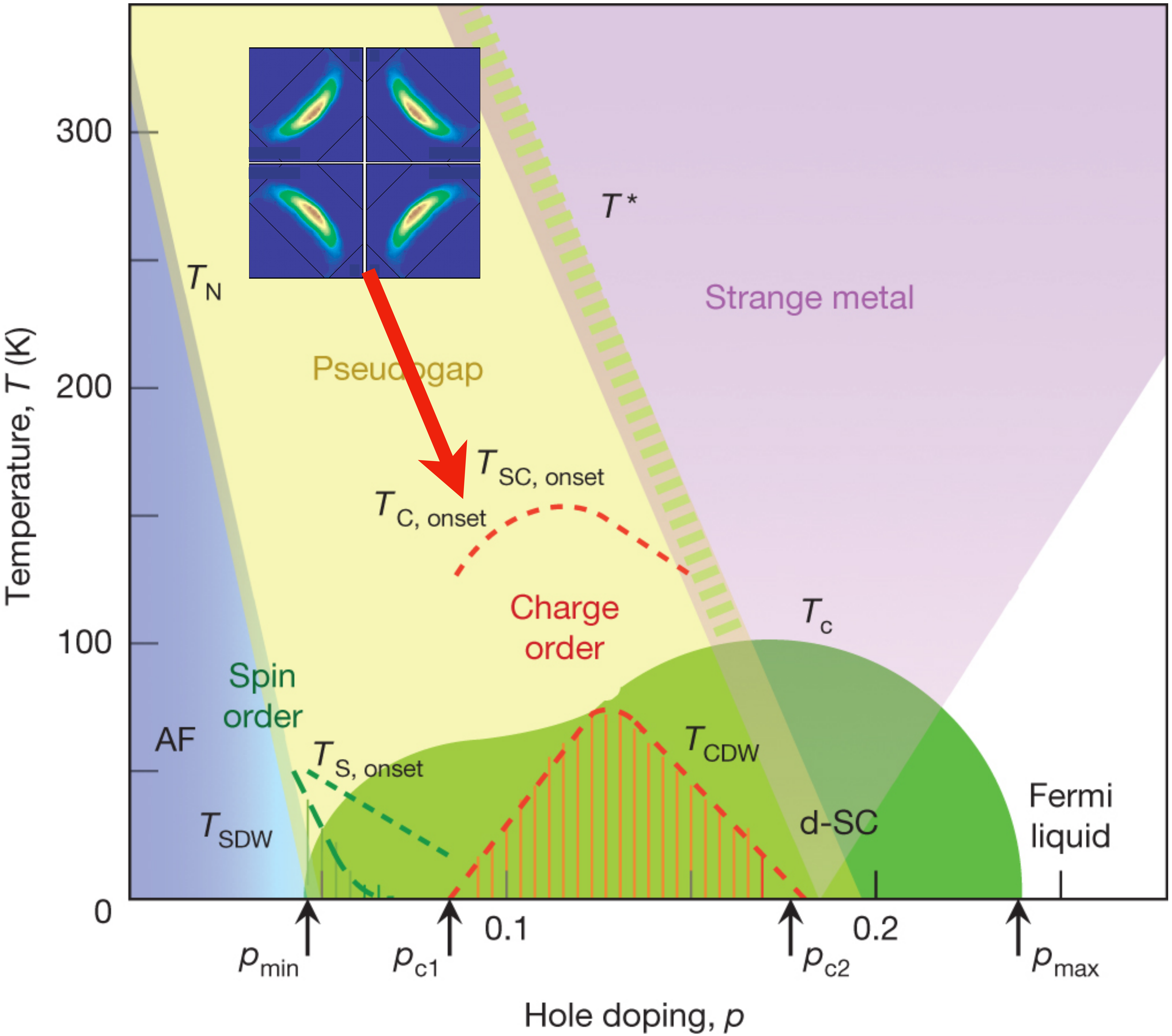
FL
p

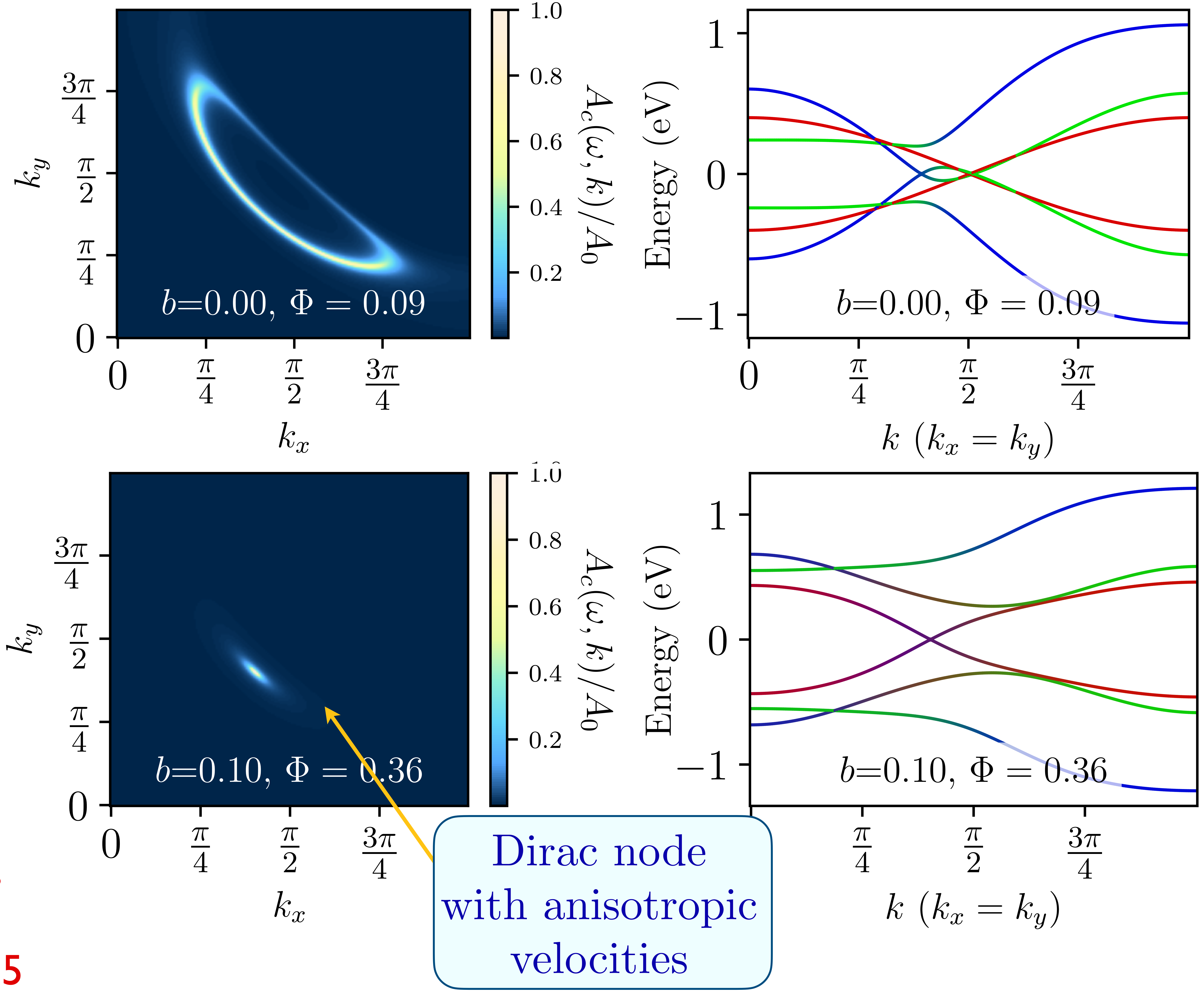
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Spinons move in π flux $\Rightarrow B$ moves in π flux $\Rightarrow$ Near-degenerate instabilities to dSC with 4 nodal points and to charge order.

Predictions

Electron spectral density in hole-doped cuprates

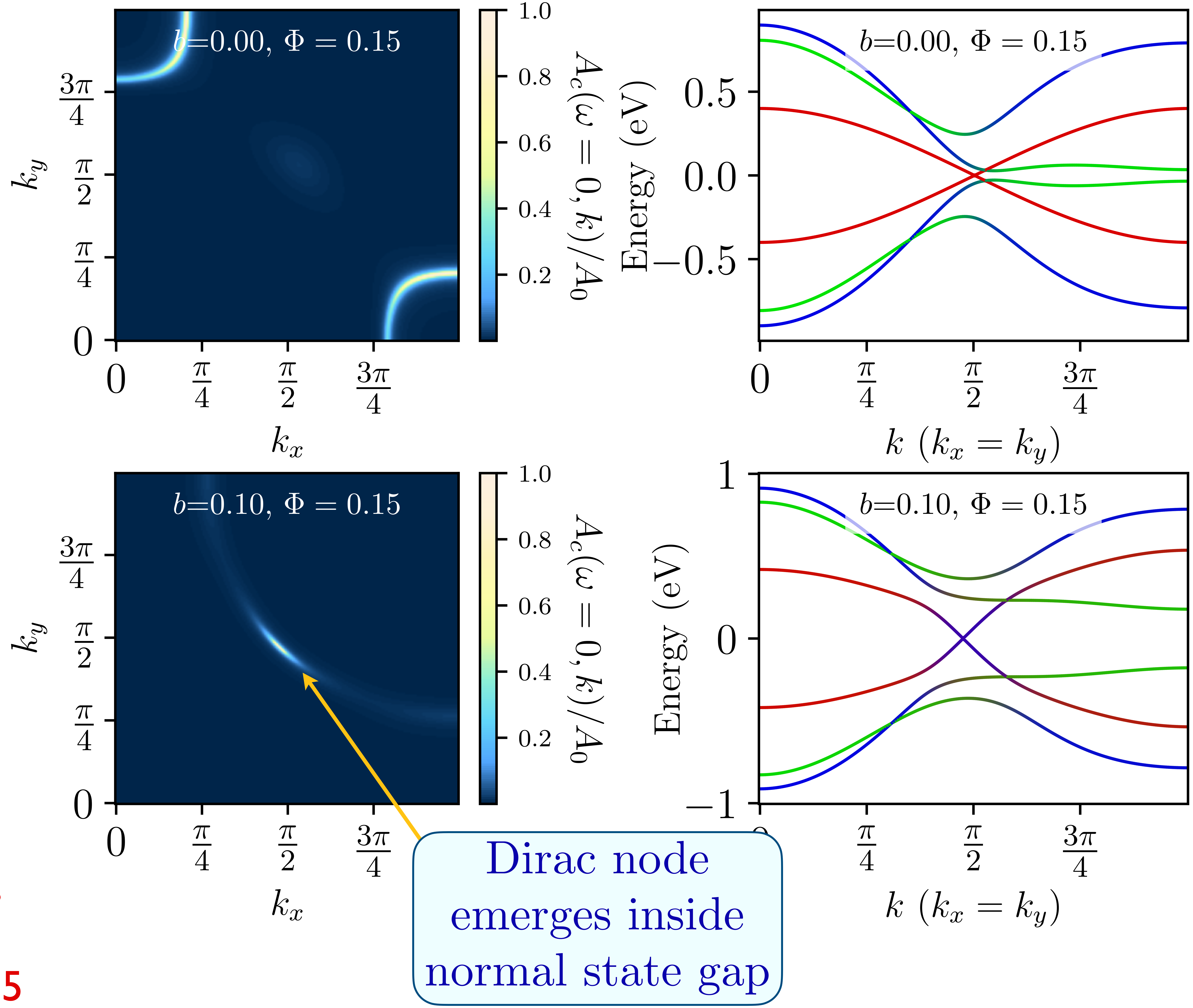


FL*

dSC

Maine Christos and S.Sachdev, arXiv:2308.03835

Electron spectral density in electron-doped cuprates



FL*

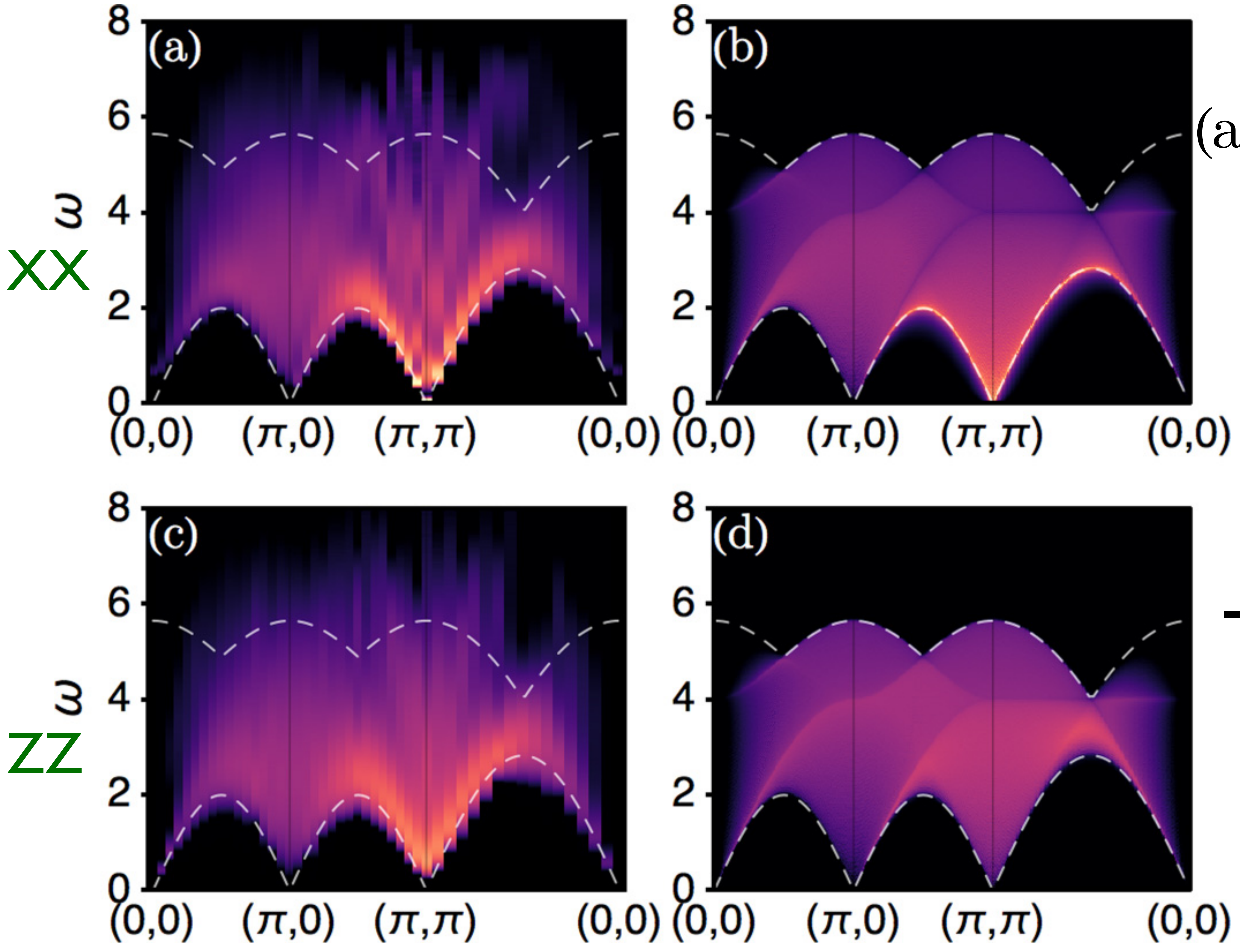
dSC

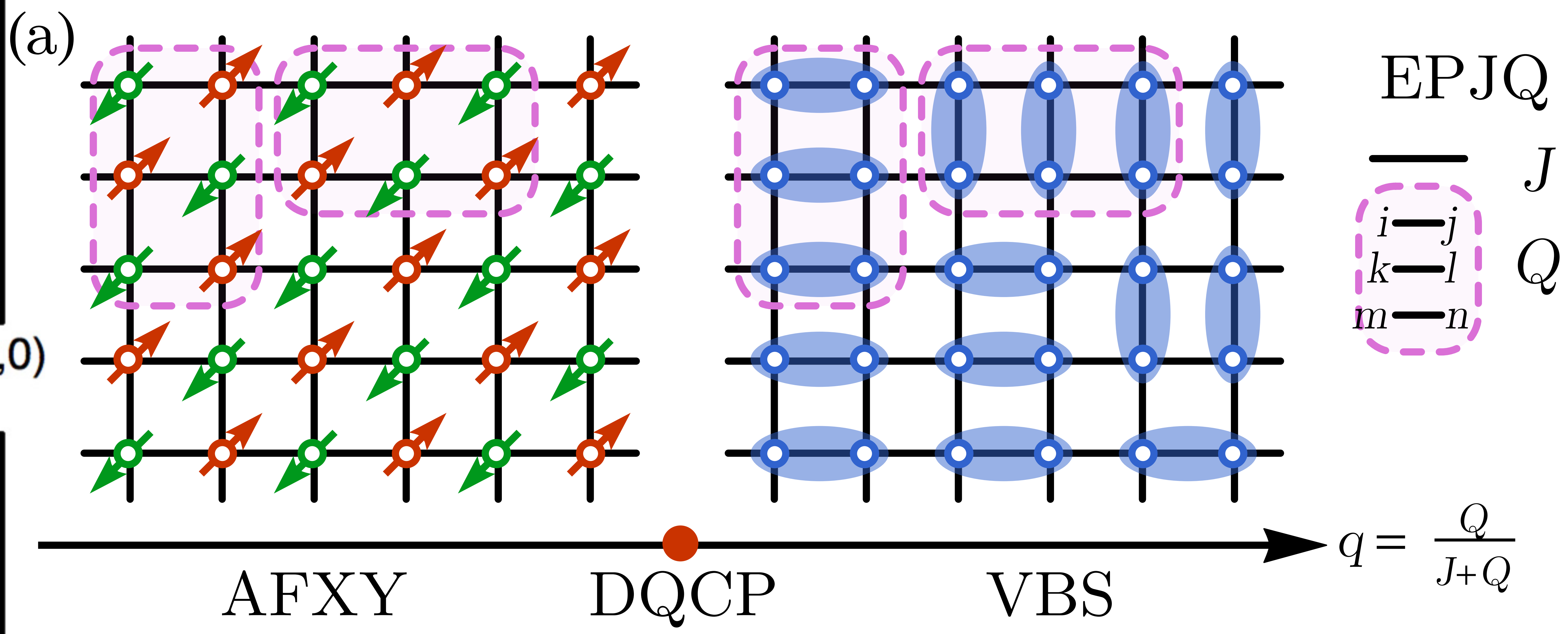
Maine Christos and S.Sachdev, arXiv:2308.03835

Observable by neutron scattering in pseudogap ?

QMC

Free fermion spinons in π-flux

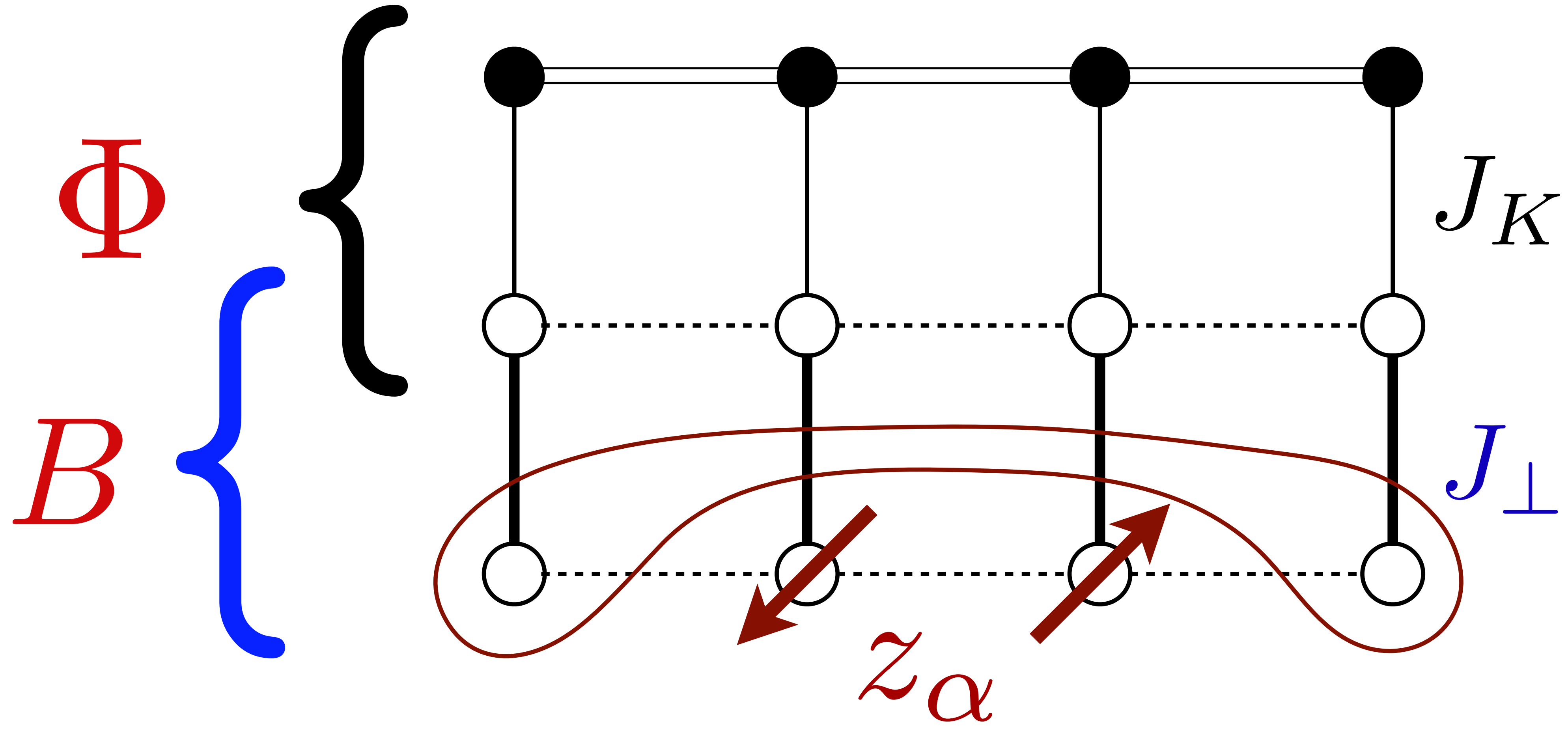




Nvsen Ma, Guang-Yu Sun, Yi-Zhuang You, Cenke Xu, Ashvin Vishwanath, Anders W. Sandvik, and Zi Yang Meng, *Phys. Rev. B* **98**, 174421 (2018)

Summary

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620; C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)



p

T
p
Φ
B
J_K
$J_\perp$
z_α
FL*
Hole pockets
and spin liquid
Strange
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d-wave superconductivity.
Charge order.
AFM
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Charge order.
(π,π)
(0,0)
(d)
FL

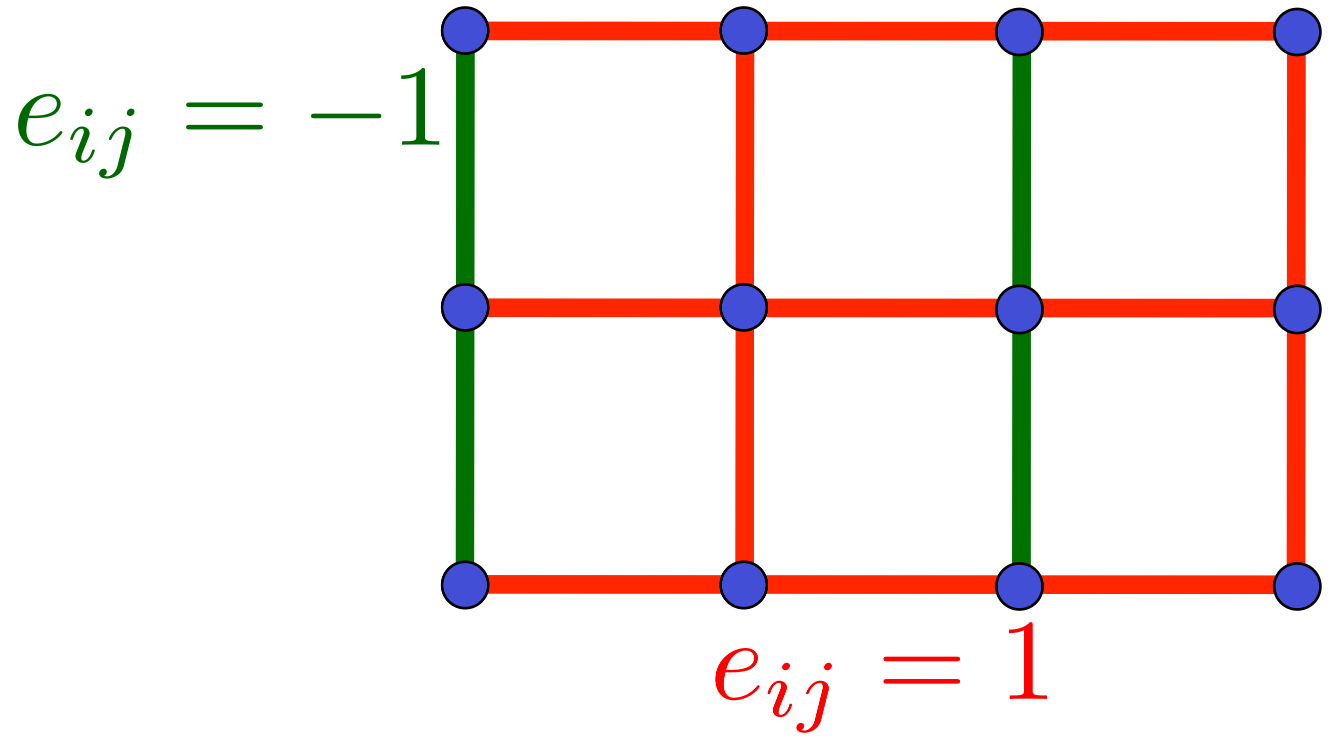
Extra slides

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



- The nearest-neighbor effective Hamiltonian for charge e, SU(2)$_N$ fundamental boson B_i is constrained by the fact that the composite of B_i and Ψ_i is an electron:

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \ldots$$

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} |\Delta_{\boldsymbol{i}\boldsymbol{j}}|^2$$
$$+ J_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} Q_{\boldsymbol{i}\boldsymbol{j}}^2 + K_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} J_{\boldsymbol{i}\boldsymbol{j}}^2 .$$

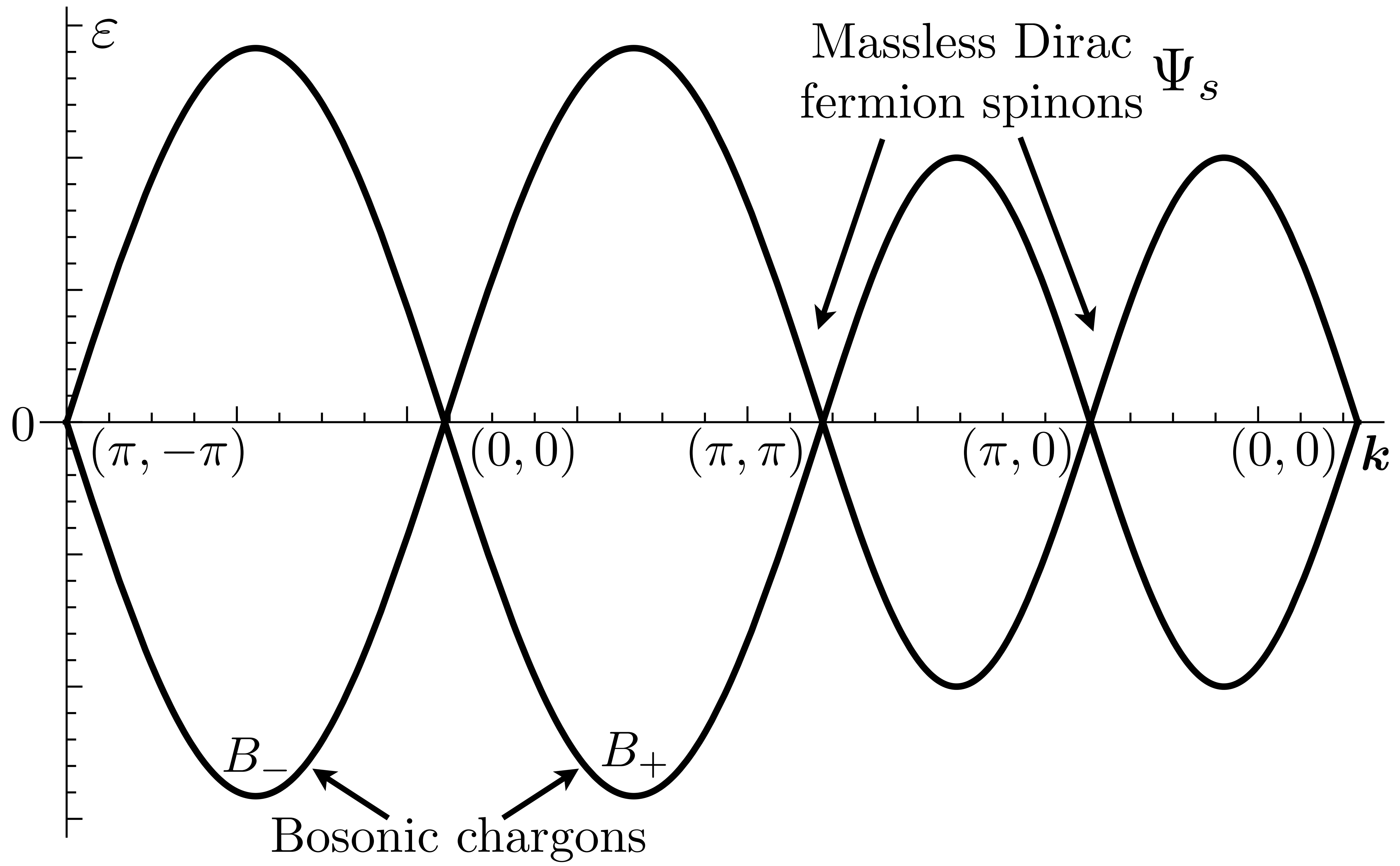
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

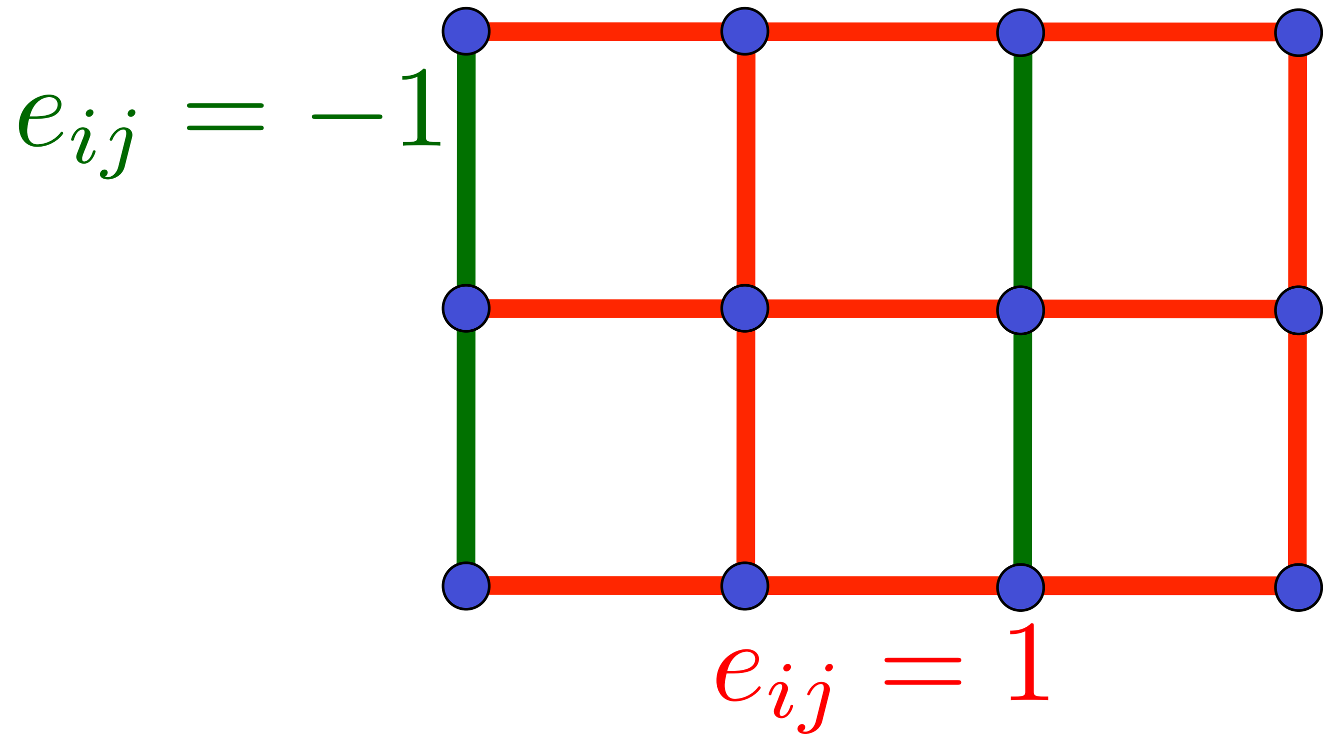
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{i}\boldsymbol{j}} = Q_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{i}\boldsymbol{j}} = -J_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

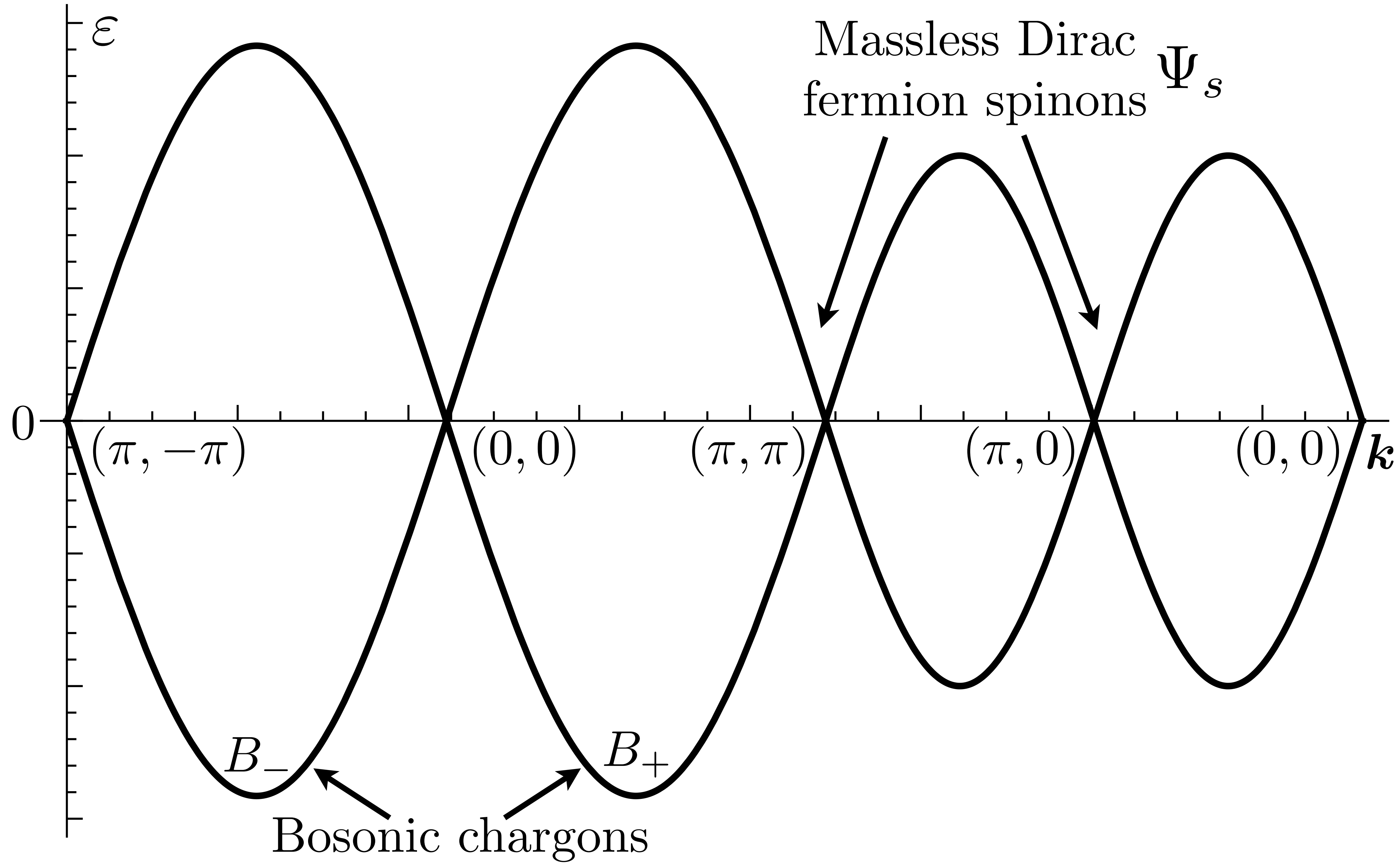
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{i}\boldsymbol{j}} = \Delta_{\boldsymbol{j}\boldsymbol{i}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{b\boldsymbol{j}}$.

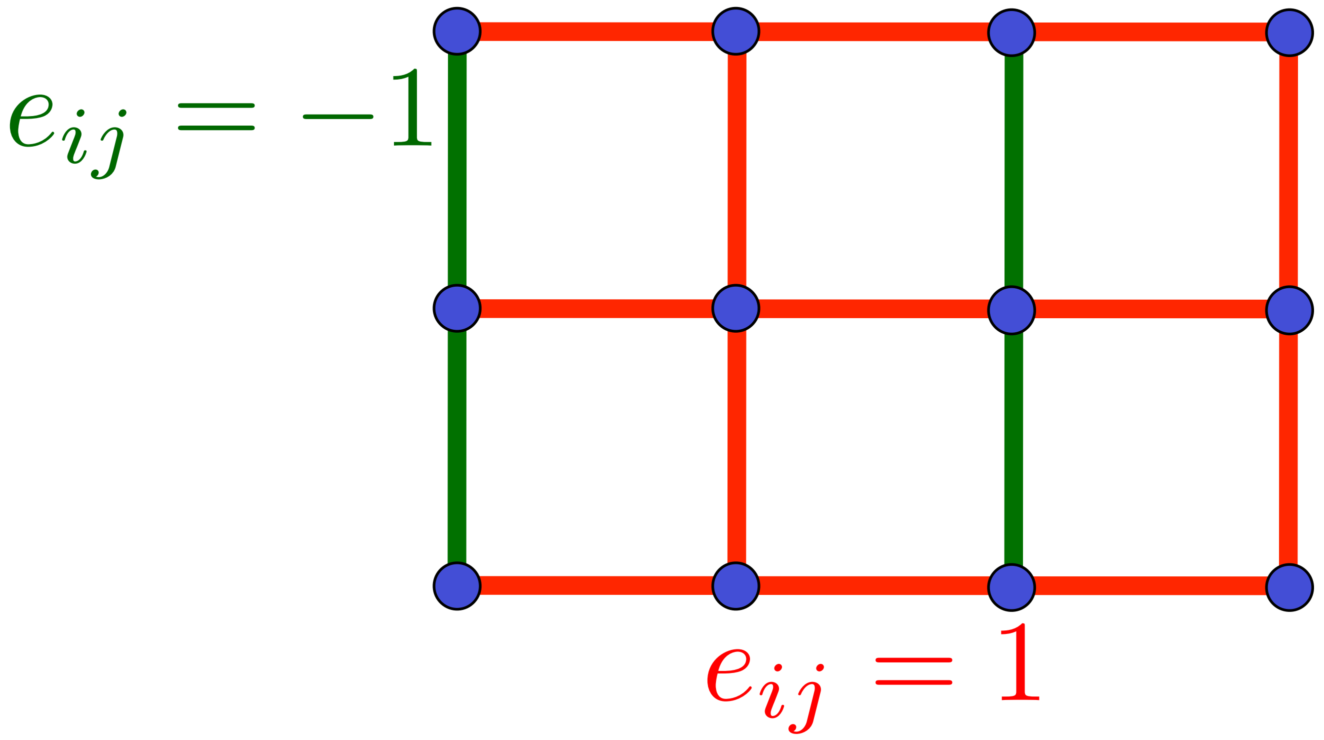
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





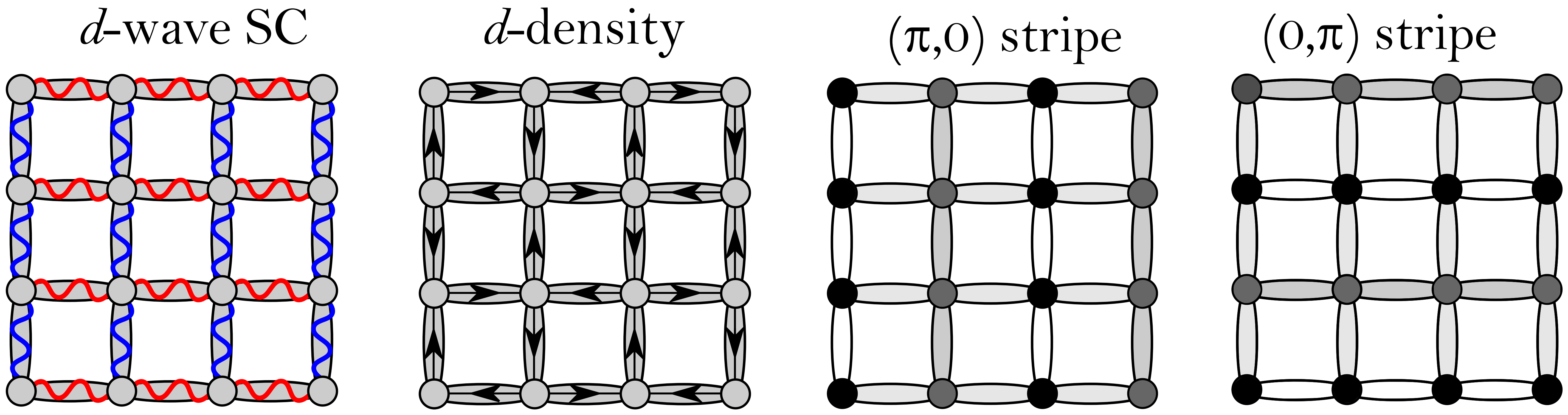
Confinement of $SU(2)_N$ gauge theory by charge fluctuations





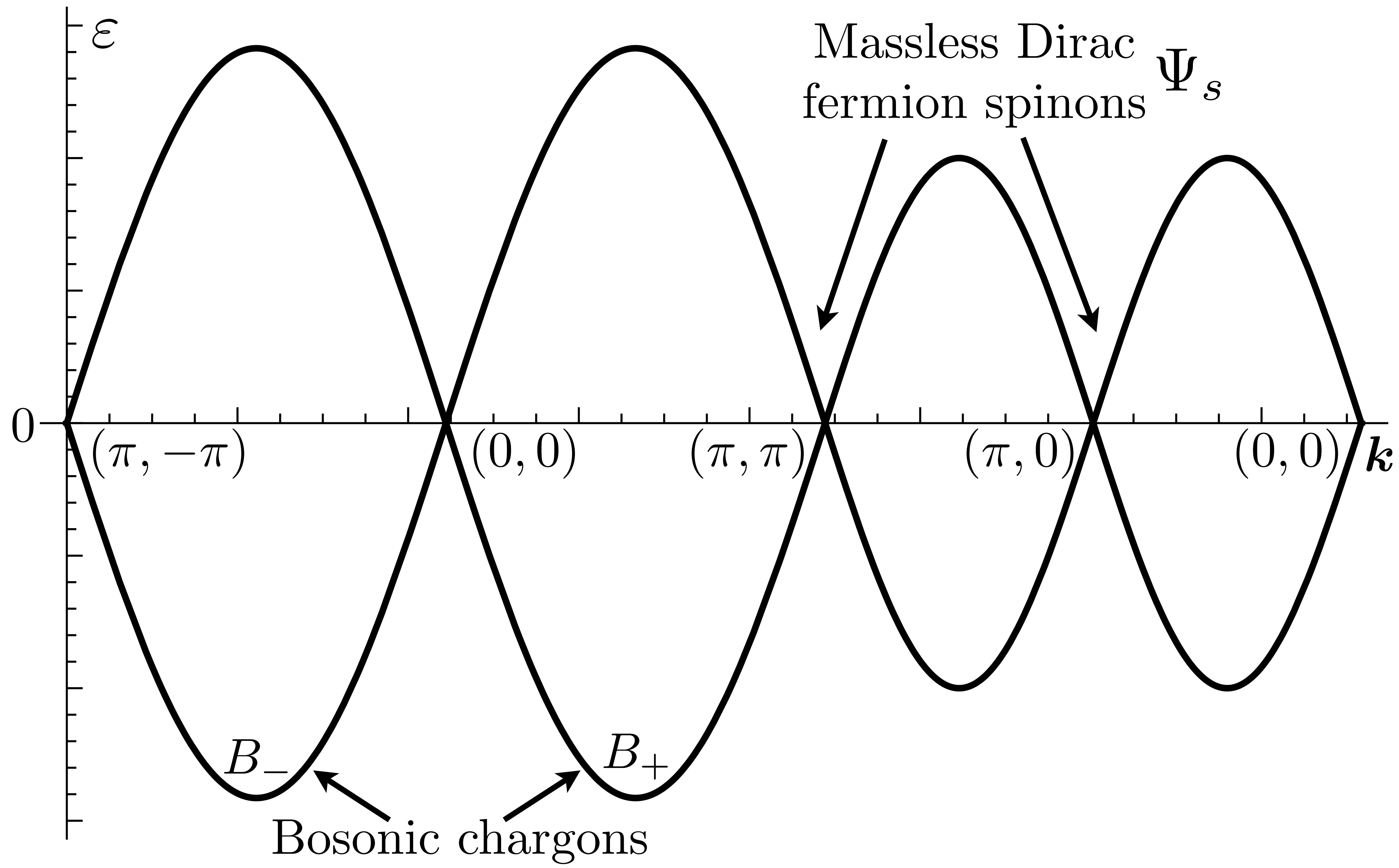
$SU(2)_N$ gauge-invariant and $SU(2)$ spin invariant order parameters of Higgs phases:

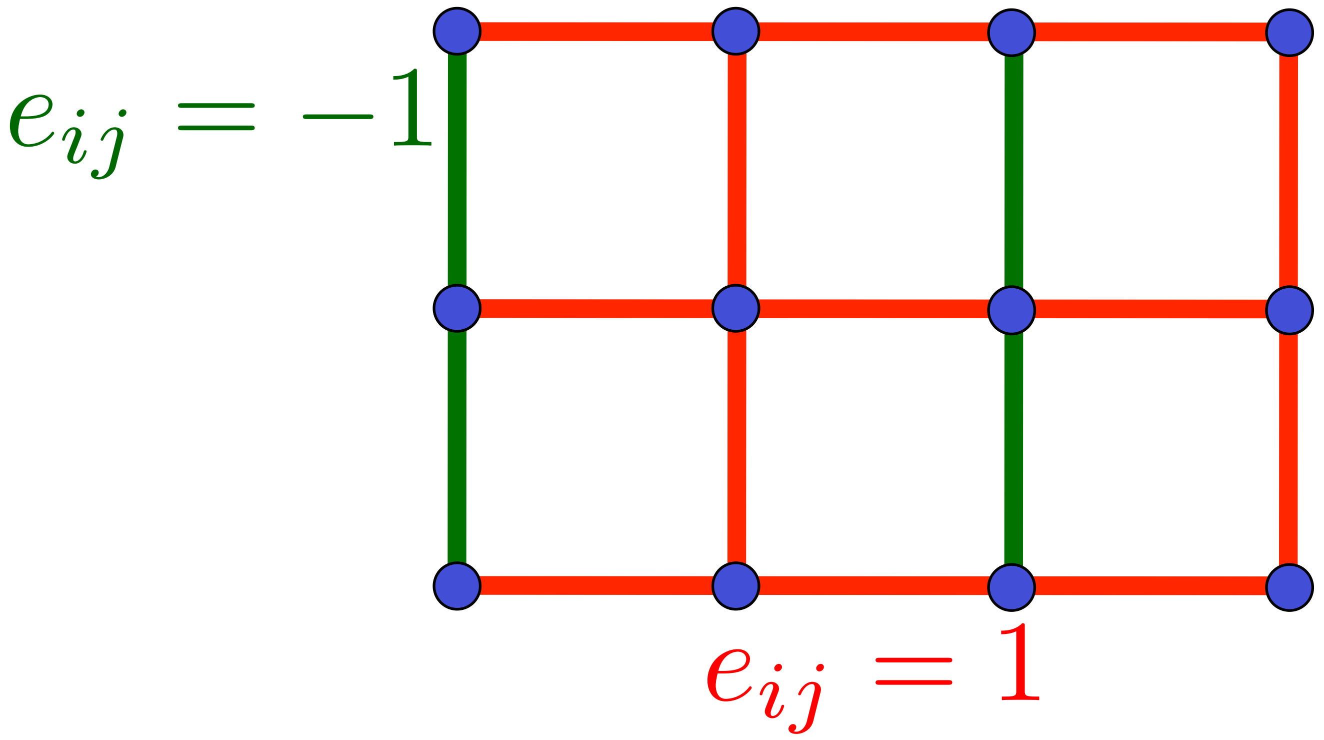
$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$

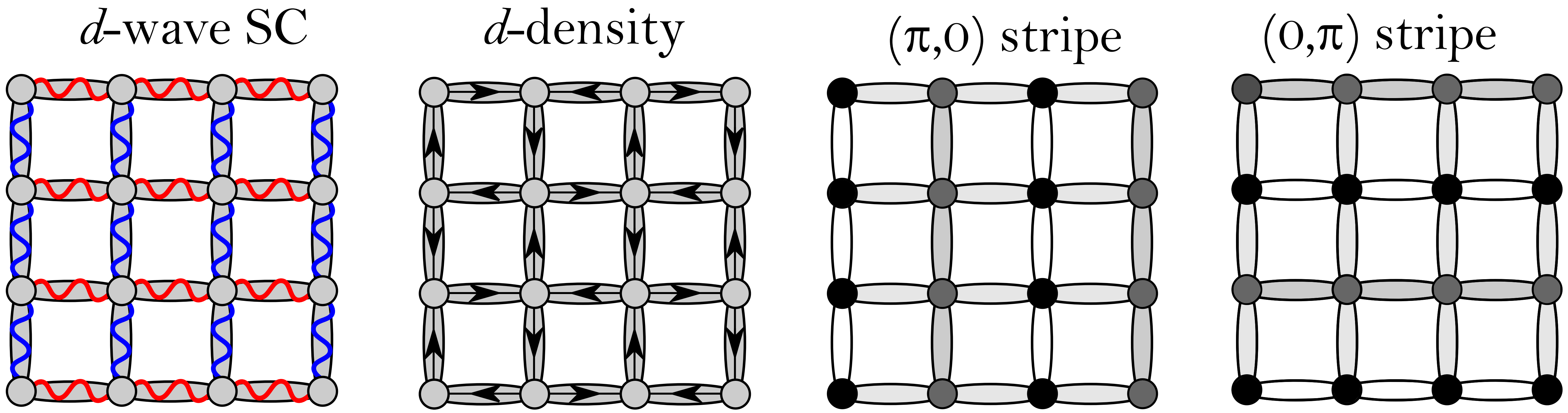


The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a $SO(5)_b$ rotation symmetry between these orders.

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

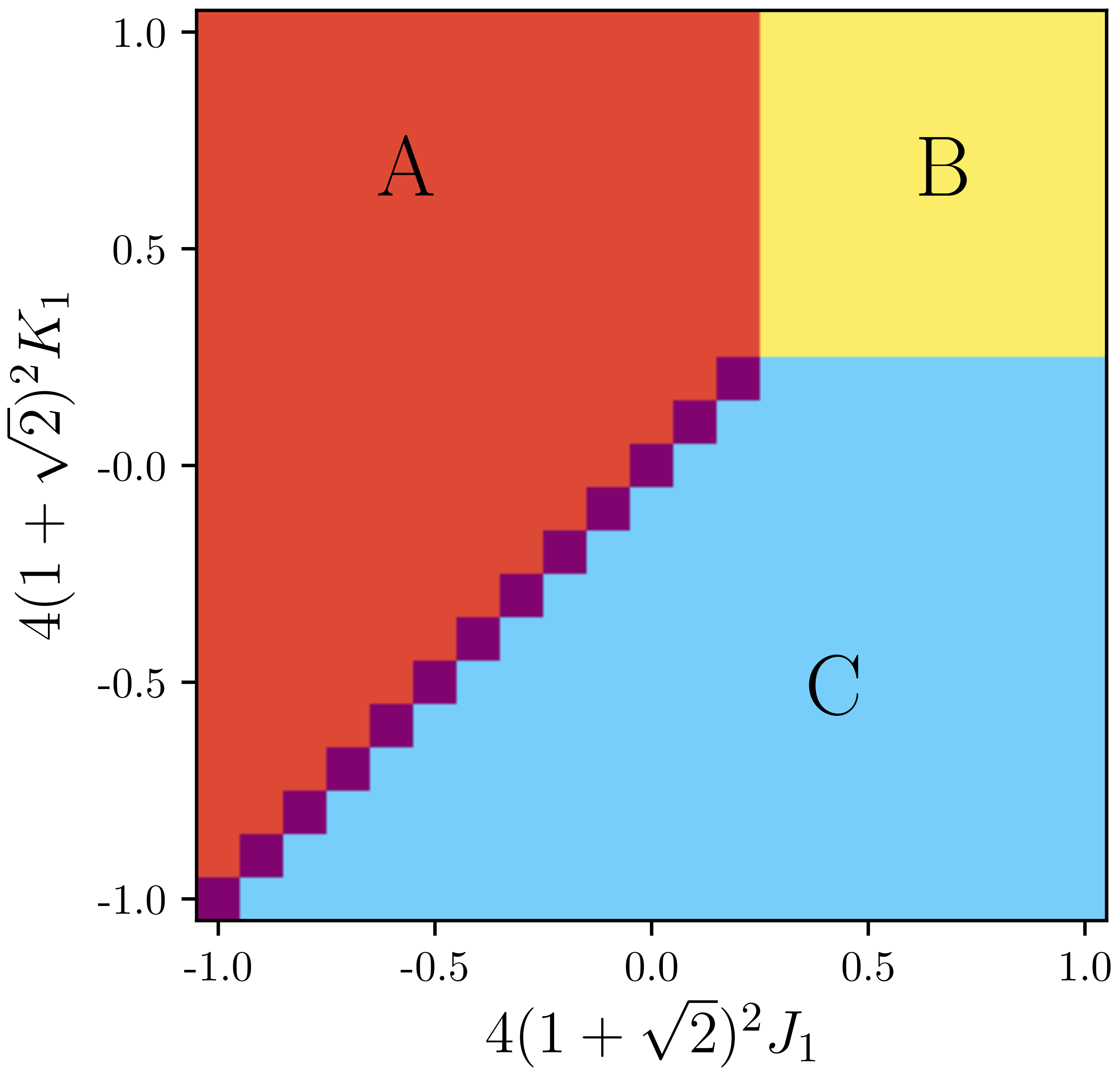






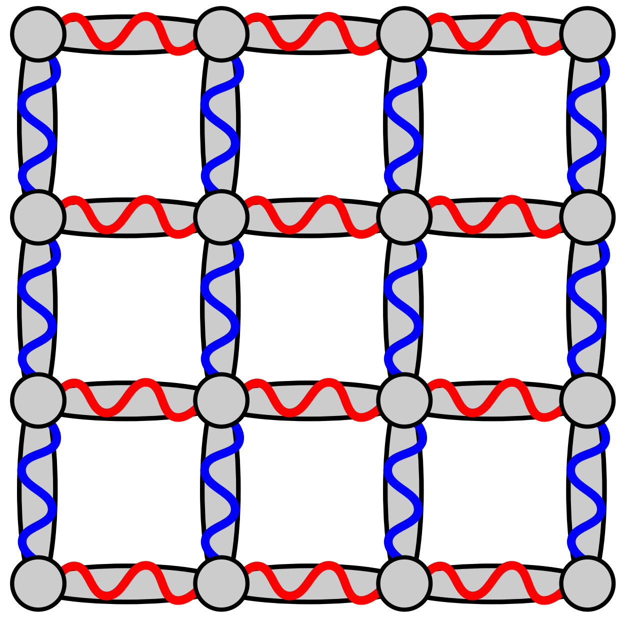
The B_{av}
($a \to SU(2)_N$ gauge, $v \to$ valley)
are the "square roots" of
conventional
d-wave superconductor,
charge density wave,
pair density wave
…

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

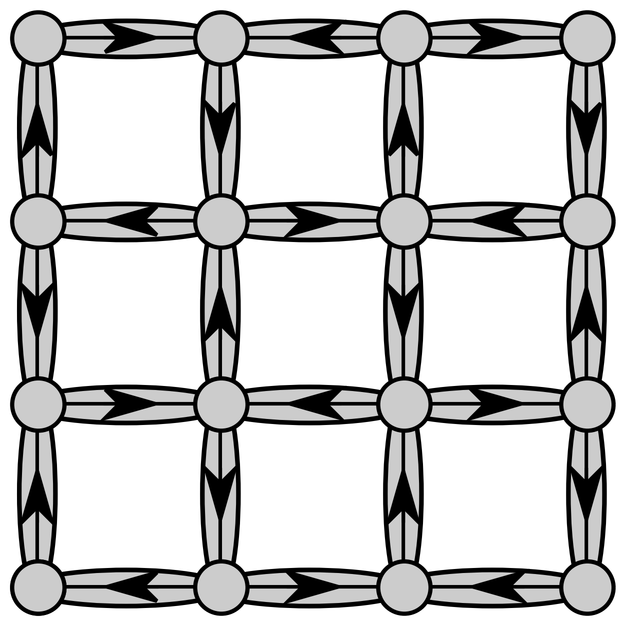


$$\langle B \rangle \neq 0$$

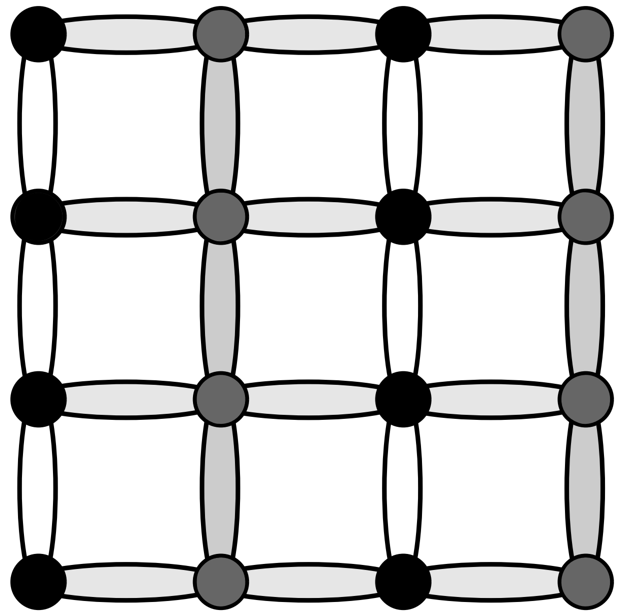
Phase B
d-wave SC



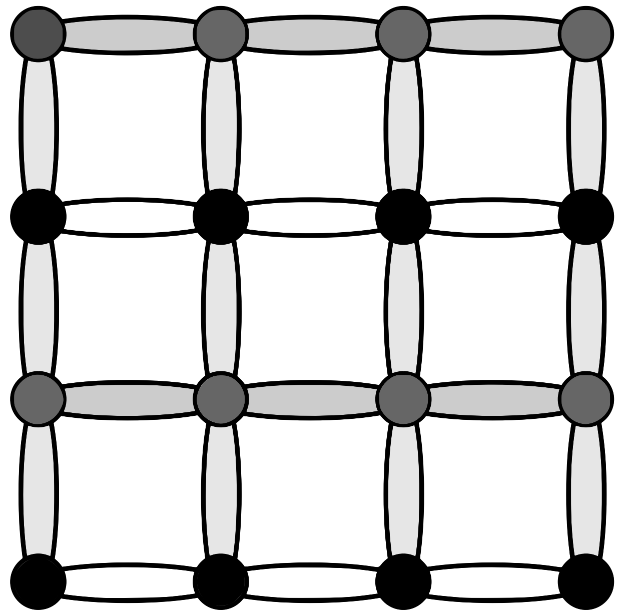
Phase C
d-density



Phase A
(π,0) stripe



Phase A
(0,π) stripe



Global phase diagram of $SU(2)_N$ gauge theory

Global phase diagram of SU(2)$_N$ gauge theory

Unified SU(2)xU(1) gauge theory of spinons, electrons and Higgs bosons:
uncanny similarities to the Salam-Weinberg-Glashow theory of weak interactions

- The electromagnetic U(1) is effectively global, because $\alpha \ll 1$.

- The fermionic spinons transform as a fundamental of gauge SU(2), with a massless Dirac spectrum

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) \,,$$

  where U_{ij} is the (lattice) SU(2) gauge field. The spinons are the analog of the neutrinos

- The Higgs sector has a boson B_i which is fundamental of SU(2)

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \mathcal{O}(B_i^4)$$

- The hole pockets in the nodal region of the Brillouin zone are described by electron $\bar{c}_{i\alpha}$ which have a Yukawa coupling to the spinons and the Higgs field $B_i = (B_{1i}, B_{2i})$:

$$H_Y = \sum_{ij} \bar{t}_{ij} \bar{c}_{i\alpha}^\dagger \bar{c}_{j\alpha} + i \sum_i \left(B_{1i} f_{i\alpha}^\dagger \bar{c}_{i\alpha} - B_{2i} \varepsilon_{\alpha\beta} f_{i\alpha} \bar{c}_{i\beta} \right) + \text{H.c.} \,.$$