

Paramagnon fractionalization theory of the cuprate pseudogap metal

Simons Collaboration on Ultra-Quantum Matter
Annual Meeting
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Subir Sachdev



Talk online: sachdev.physics.harvard.edu

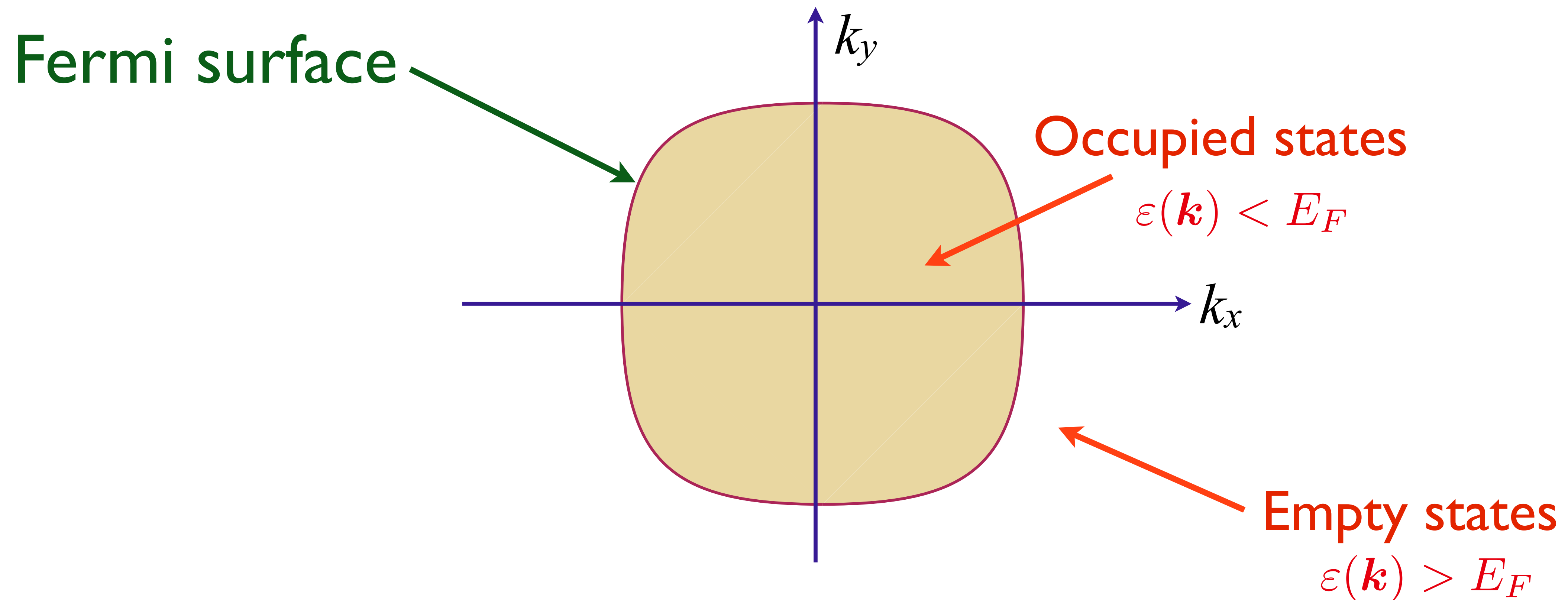


INSTITUTE FOR
ADVANCED STUDY



Luttinger relation

Electrons move with momentum \mathbf{k} through the lattice with dispersion $\varepsilon(\mathbf{k})$



$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

1. Spin liquids and violations of the Luttinger relation:

FL* and HFL phases of the Kondo lattice model.

2. Hubbard model - the vanilla FL phase

3. Hubbard model - the FL* phase:

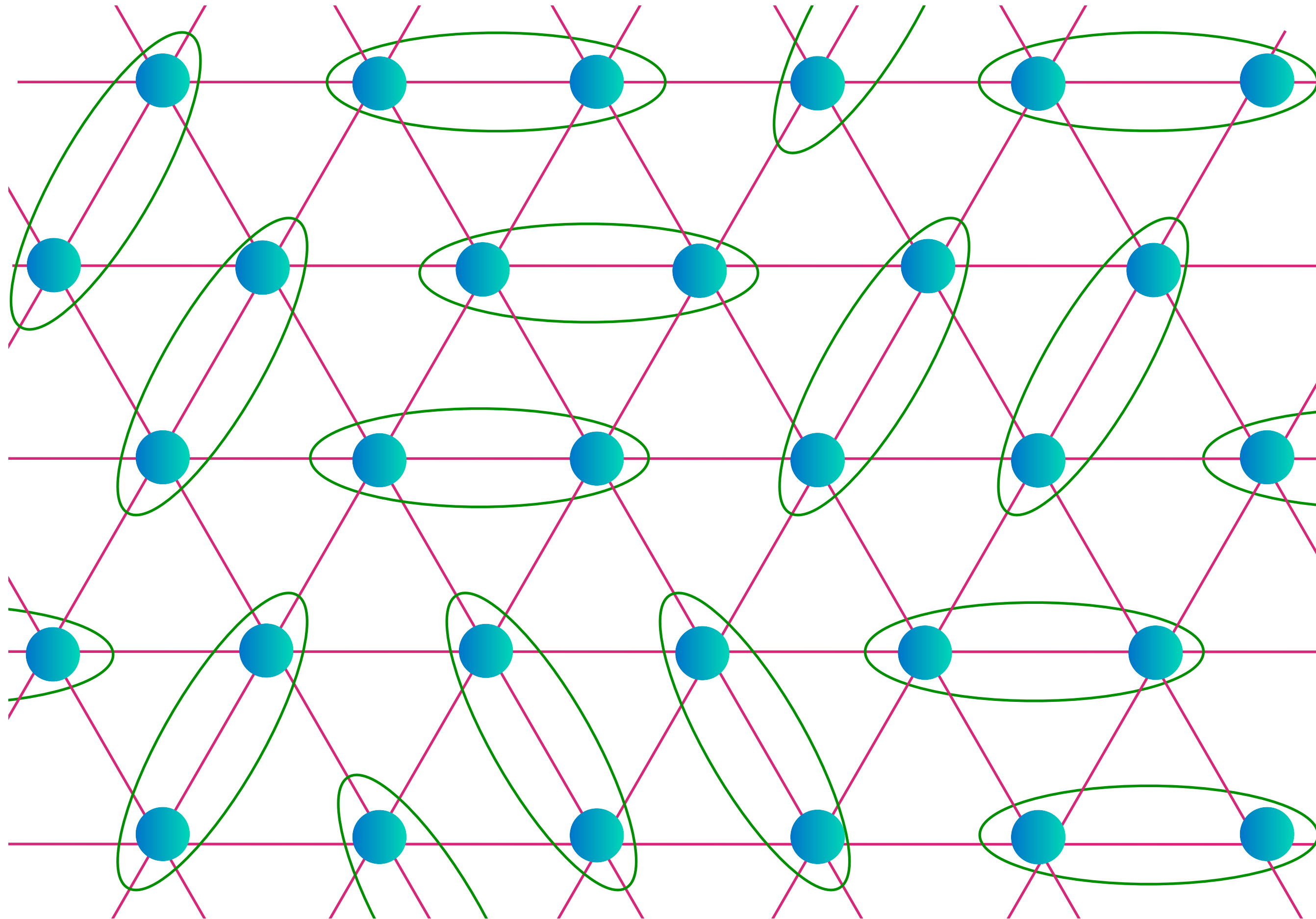
fractionalizing the paramagnon

4. Hubbard model - nature of spin liquid

5. Quantum criticality

Mott insulator: Triangular lattice antiferromagnet

Resonating valence bonds:
 Z_2 spin liquid



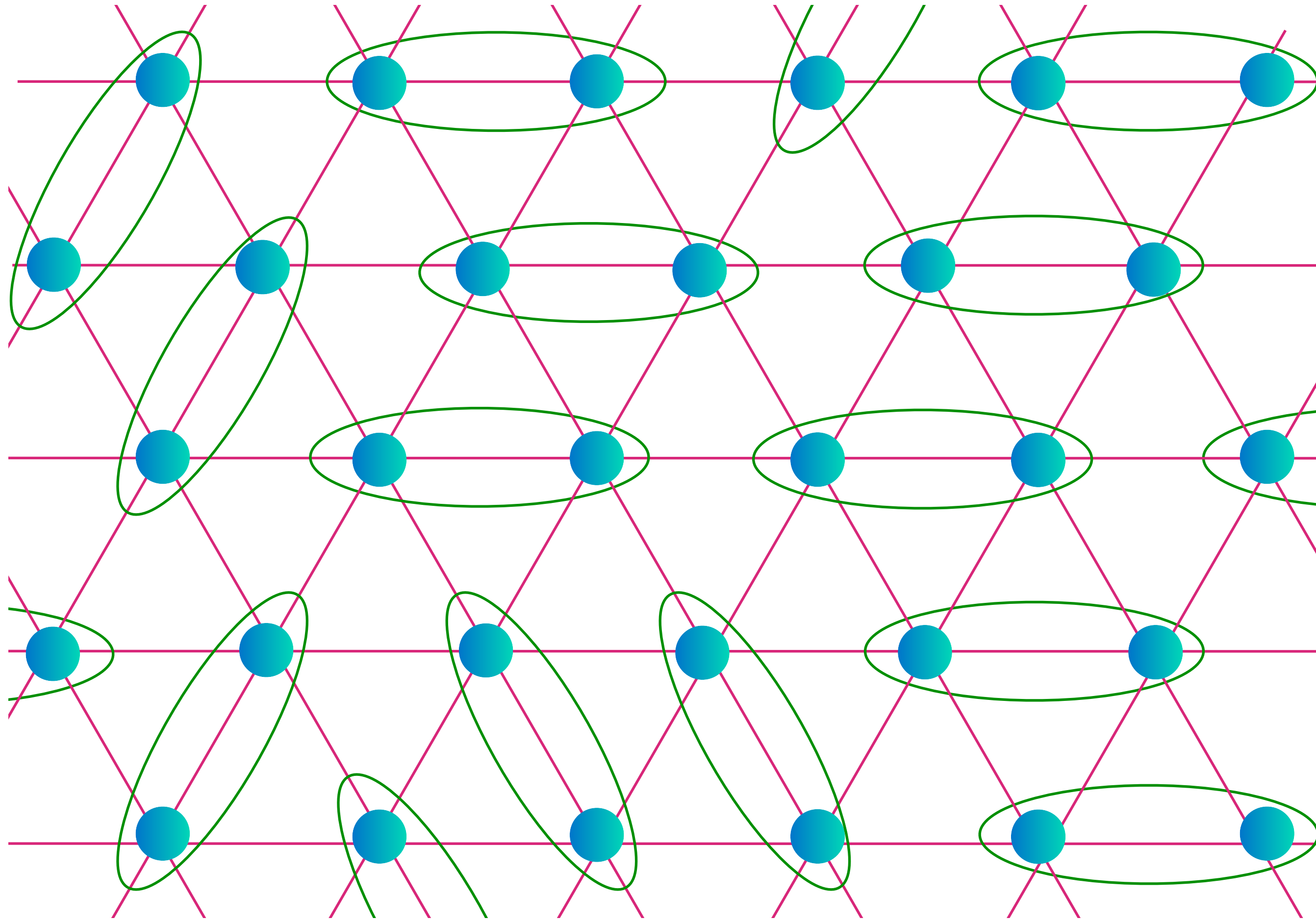
$$\begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \end{array} \\ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering
of lattice

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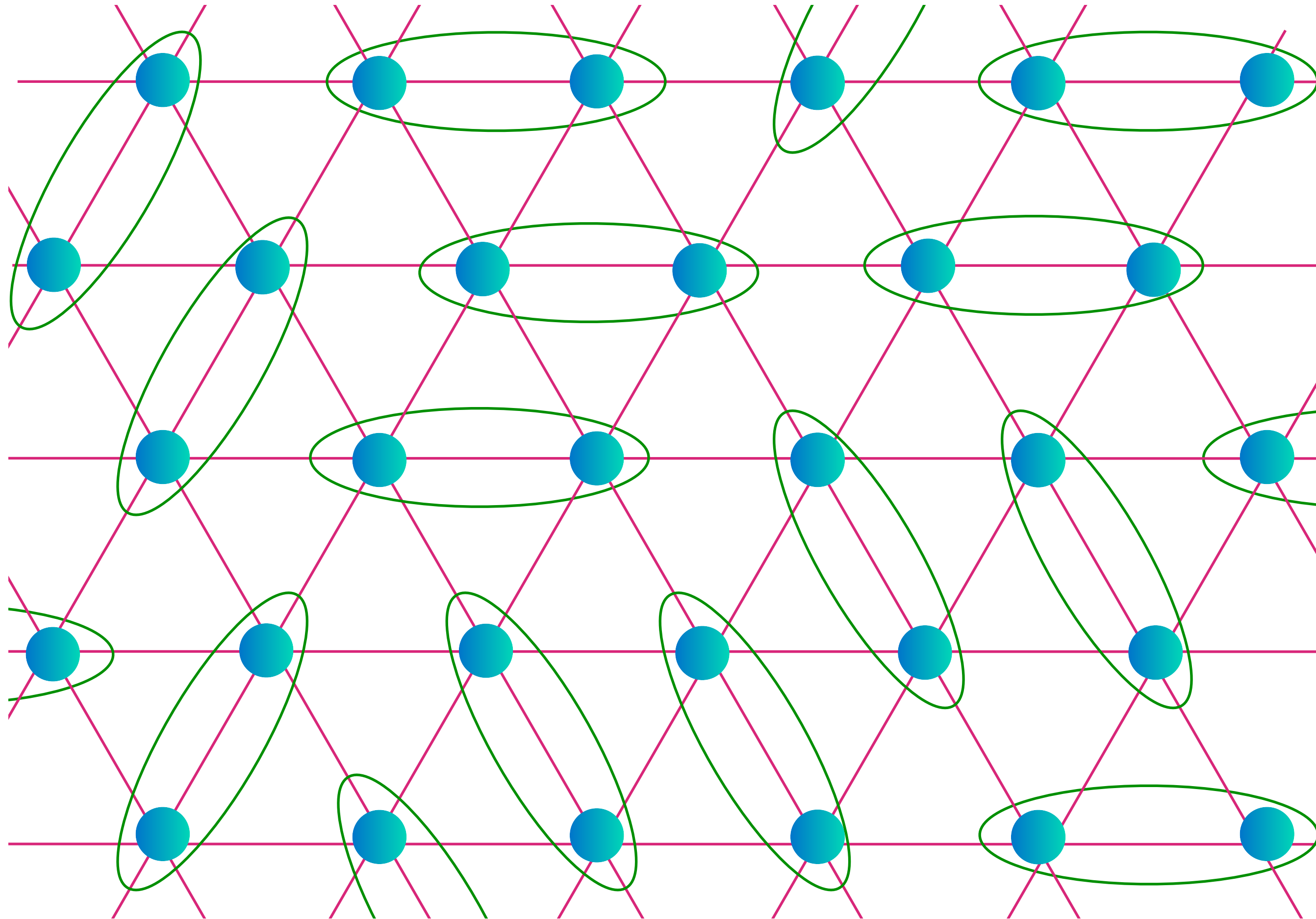
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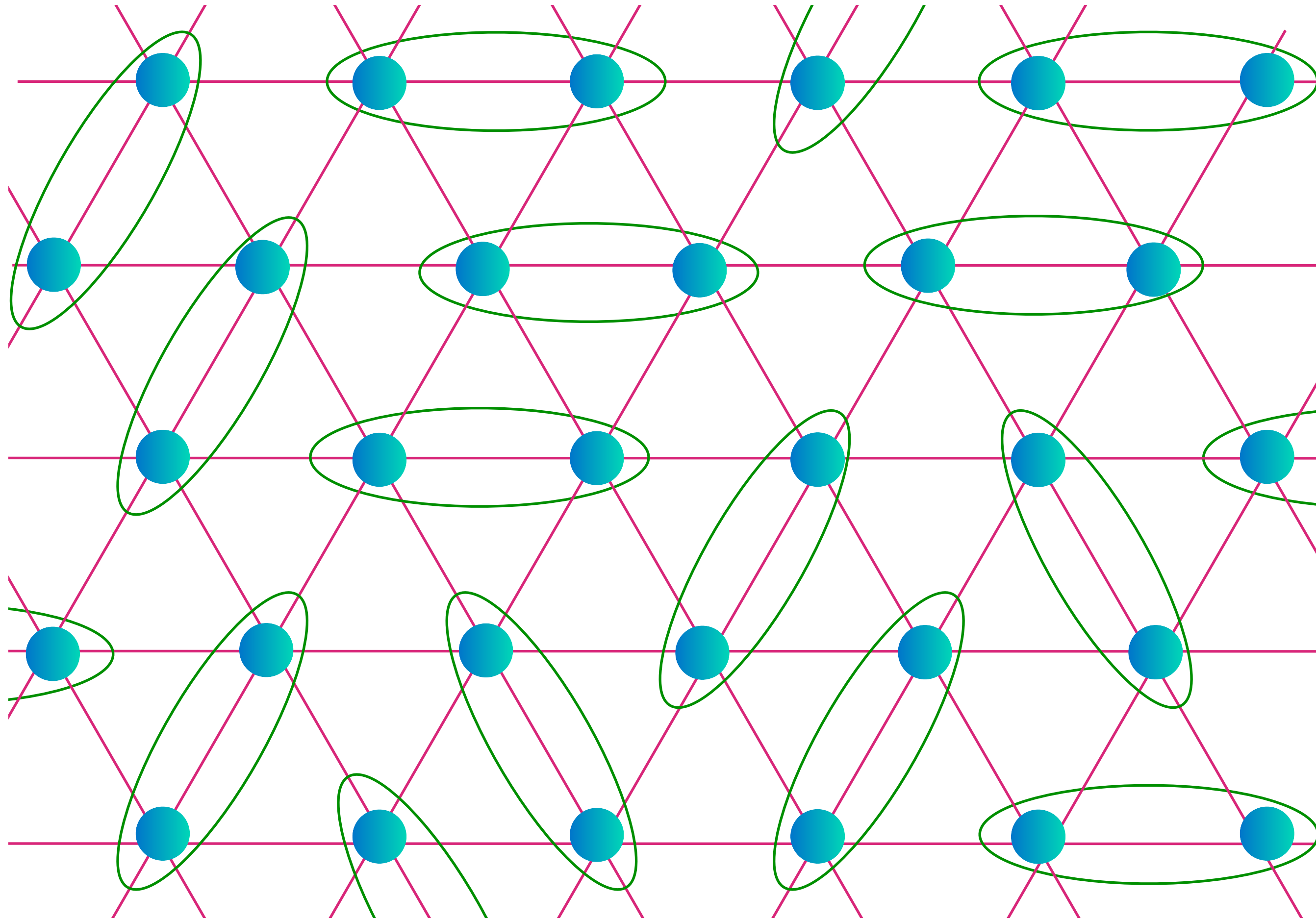
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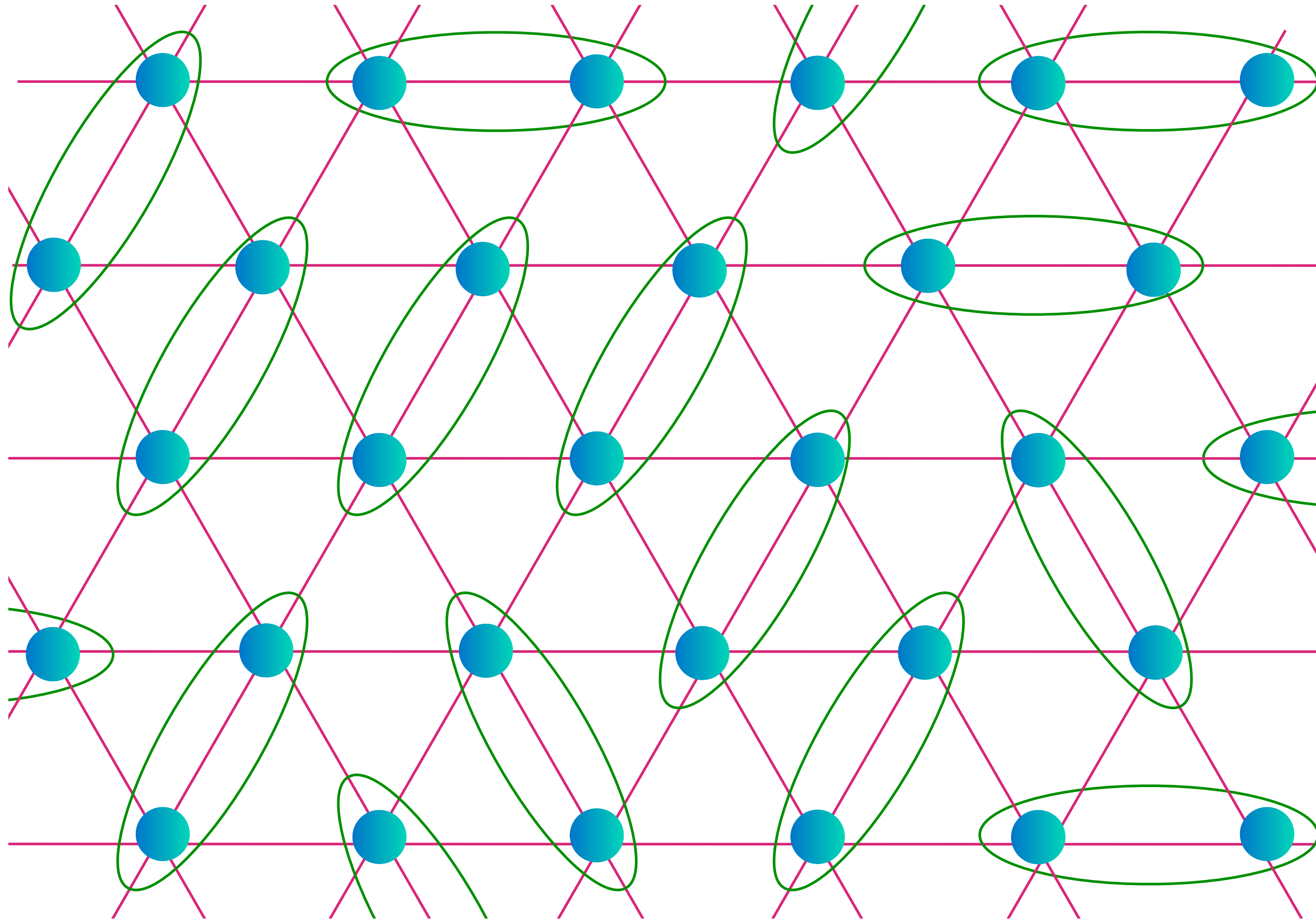
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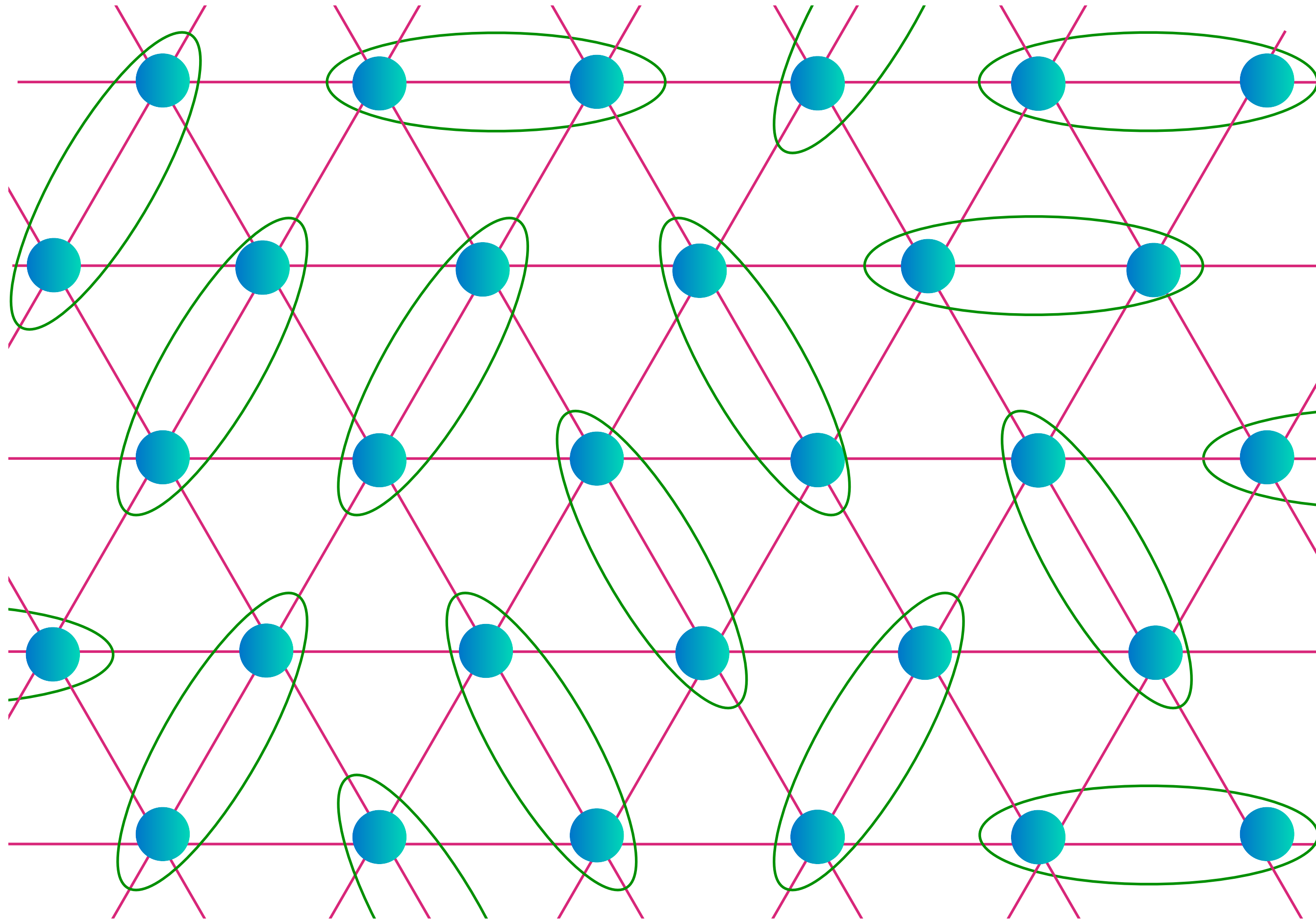
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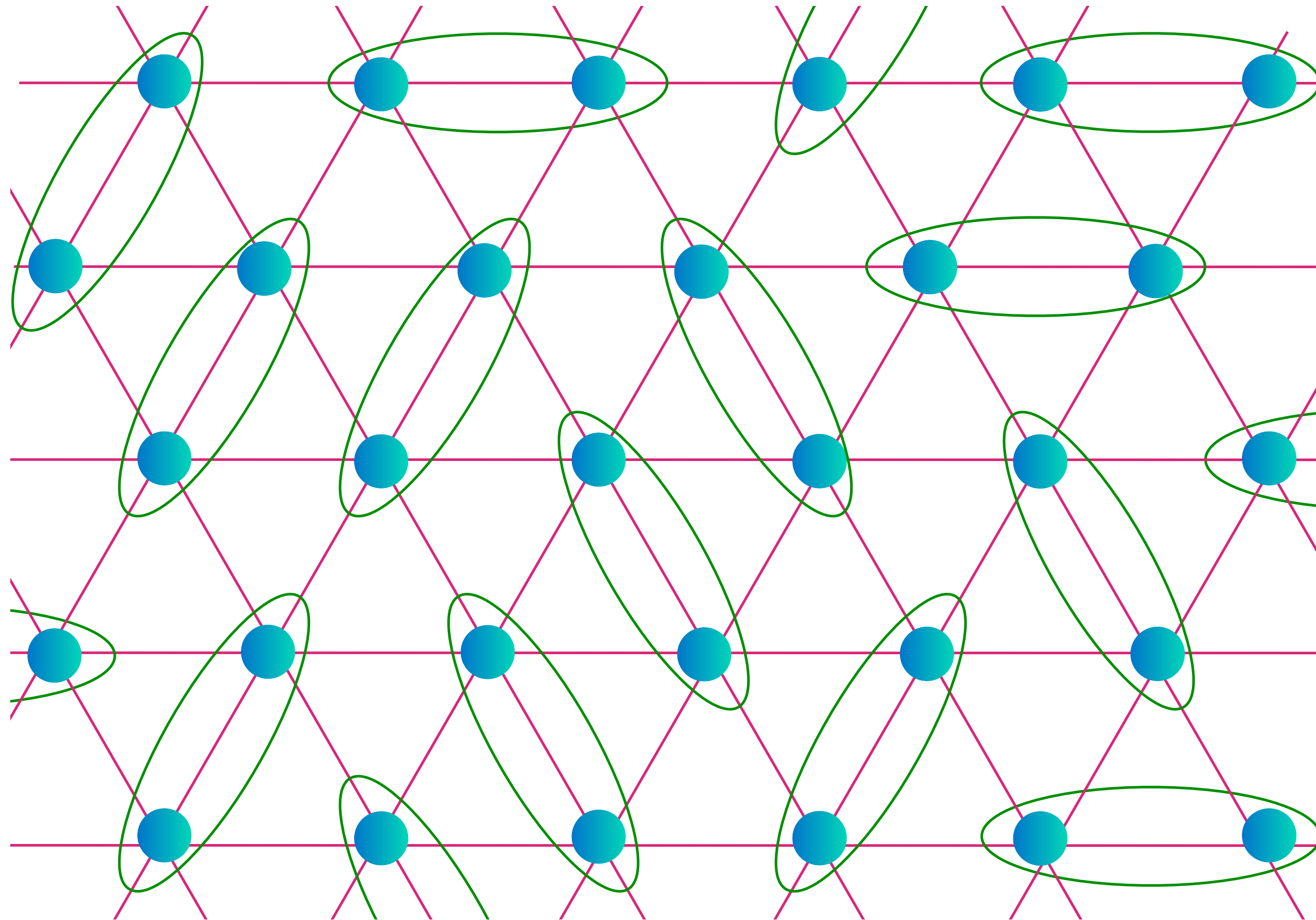
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$\mathcal{D} \rightarrow$ dimer covering
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Density of the electrons
per unit cell = 1,
but no Fermi surface
 \Rightarrow Violation of
Luttinger relation

Kondo lattice

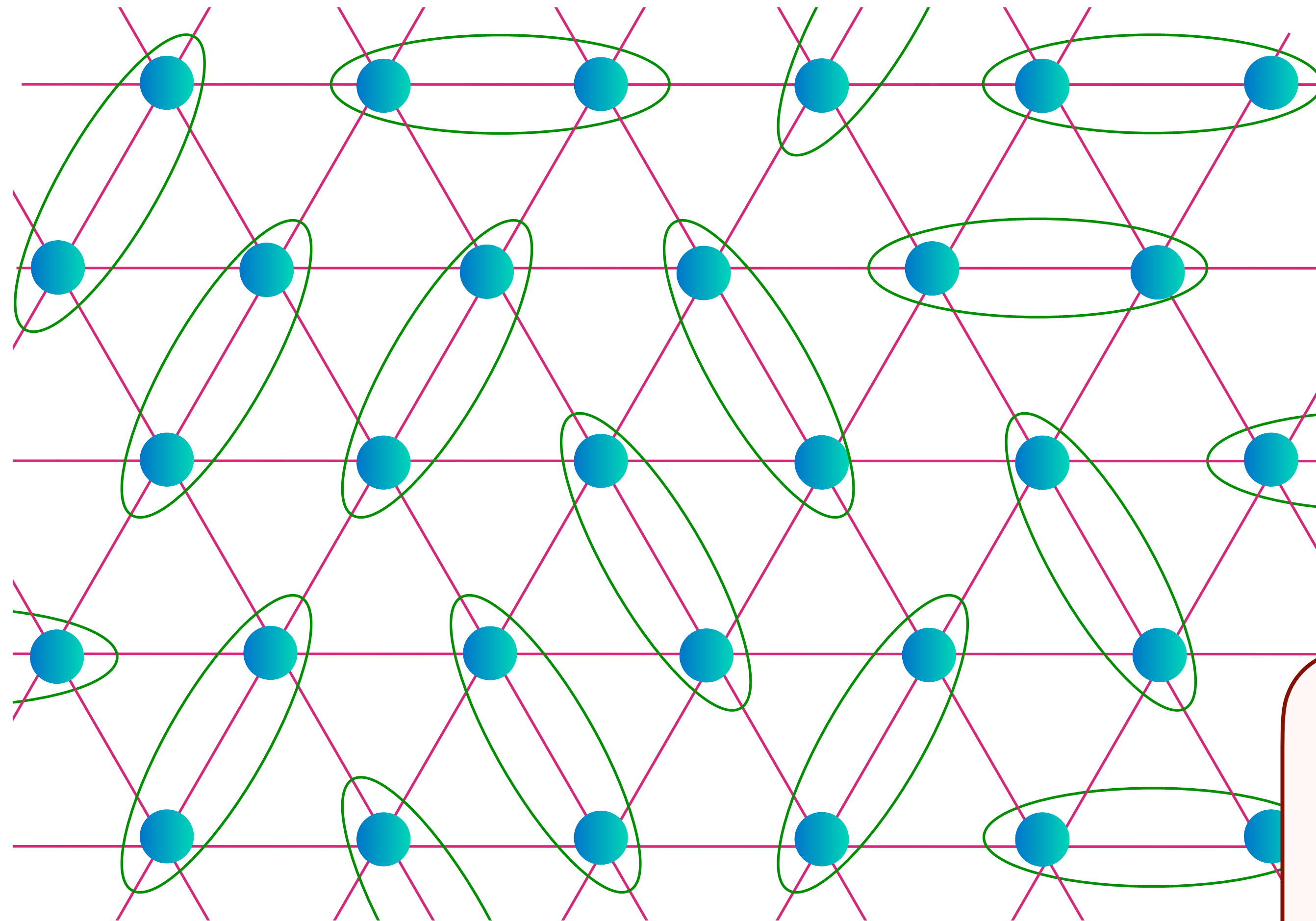


f electrons

c electrons

Density of the electrons
per unit cell = $1 + p$,
Fermi surface size = p

Kondo lattice: FL* phase



Kondo
exchange

J_K

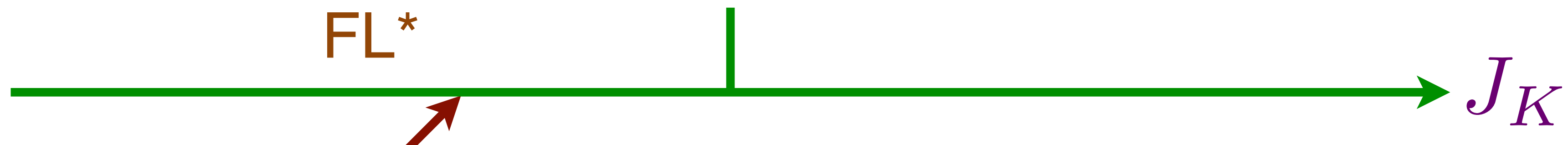
c electrons

f electrons

Density of the electrons
per unit cell = $1 + p$,
Fermi surface size = p
Non-Luttinger volume “small” Fermi
surface size is stable to all orders in J_K .

Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_i + \sum_{\langle ij \rangle} J_H \mathbf{S}_i \cdot \mathbf{S}_j$$



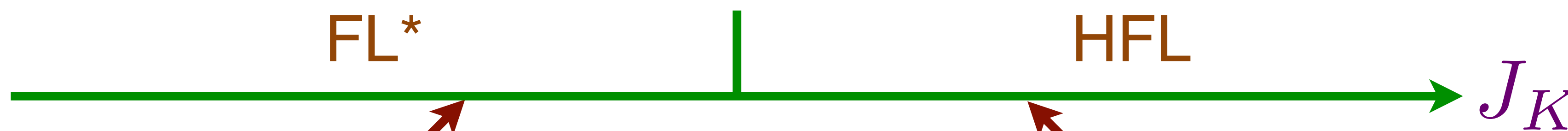
Small Fermi surface of size p

$|\text{FL*}\rangle = [\text{Projection onto one } f \text{ per site}]$
 \boxtimes |Slater determinant of f \rangle
 \otimes |Slater determinant of c \rangle

S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

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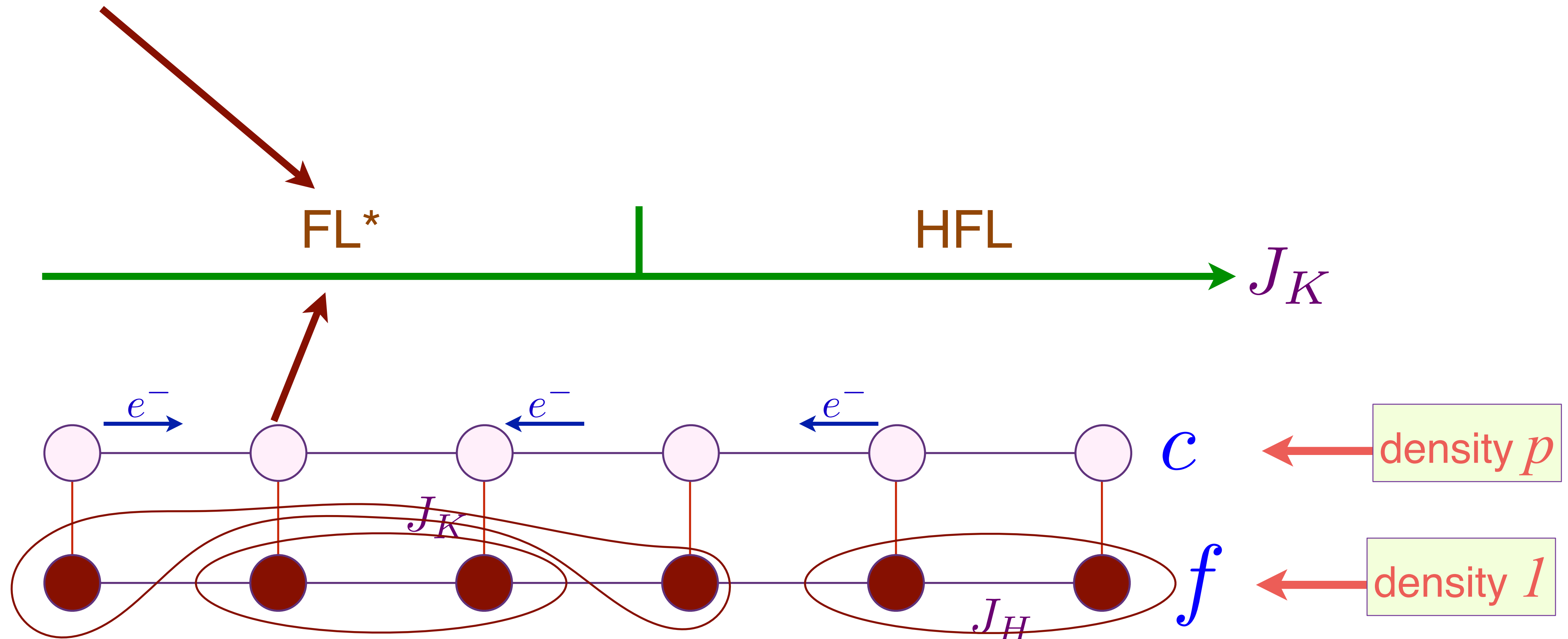
$|\text{FL}^*\rangle = [\text{Projection onto one } f \text{ per site}]$
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Large Fermi surface of size $1 + p$

$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}]$
 $\boxtimes |\text{Slater determinant of } (c, f)\rangle$

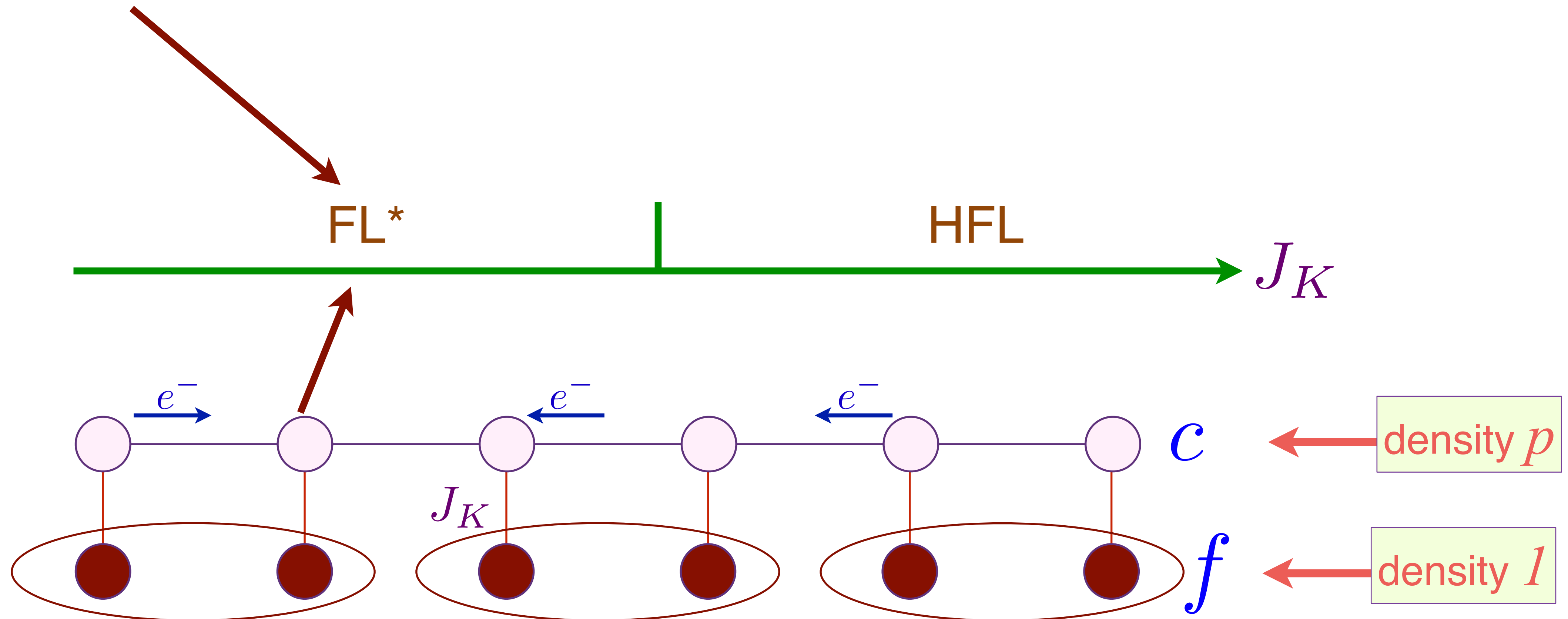
Kondo lattice

- Use fermionic spinons $\mathcal{S}_i = \frac{1}{2} f_{i\sigma}^\dagger \boldsymbol{\tau}_{\sigma\sigma'} f_{i\sigma'}$, $\sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} = 1$. This introduces a $U(1)_{\text{gauge}}$ gauge symmetry $f_{i\sigma} \rightarrow f_{i\sigma} e^{-i\vartheta_i}$ and the total symmetry is $U(1)_{\text{gauge}} \times U(1)_{\text{em}}$.
- In the FL* state, the $U(1)_{\text{gauge}}$ symmetry is at least partially unbroken, and the Luttinger relation applies only to the c fermions carrying $U(1)_{\text{em}}$ charges.



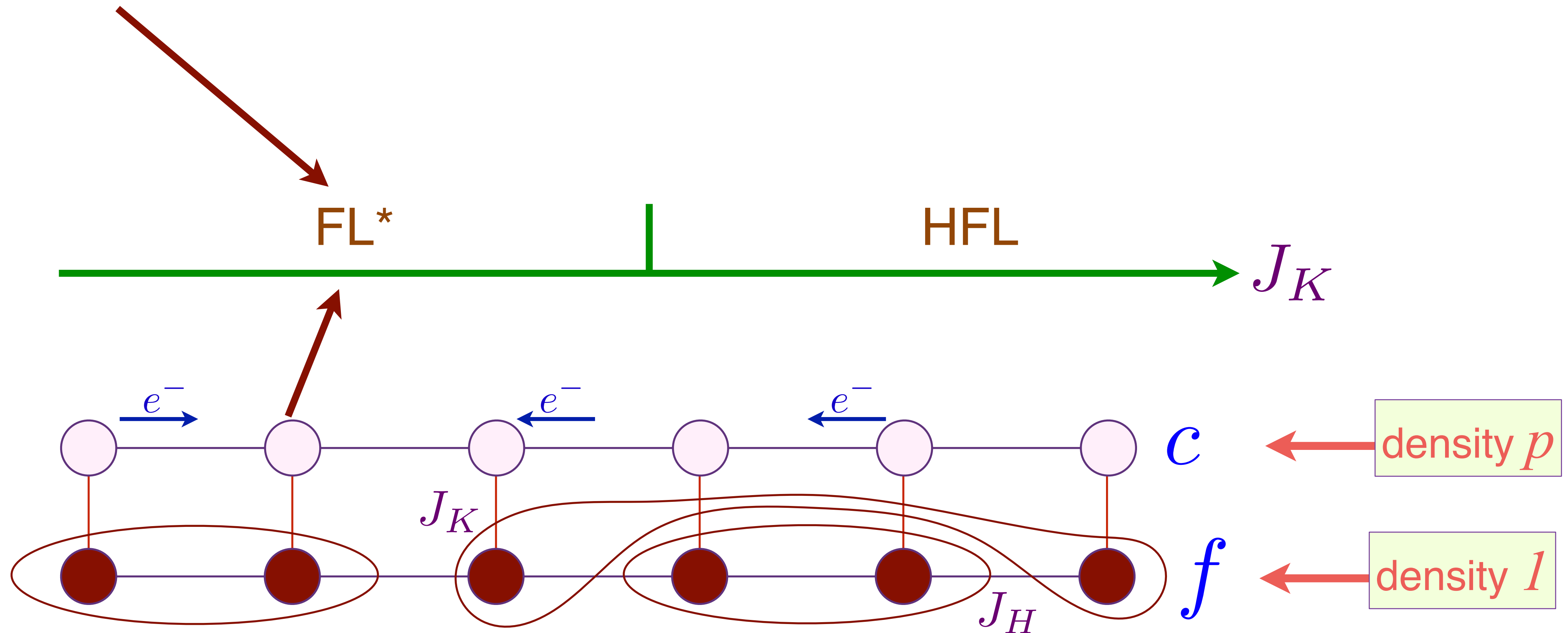
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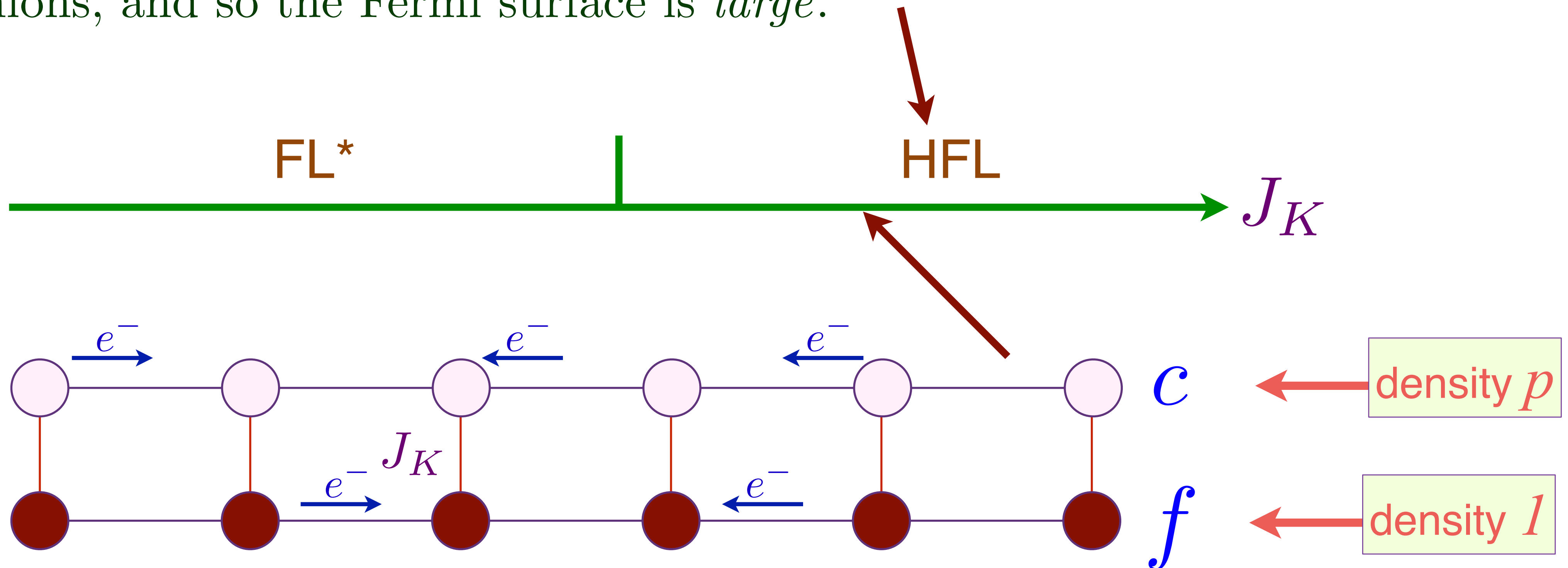
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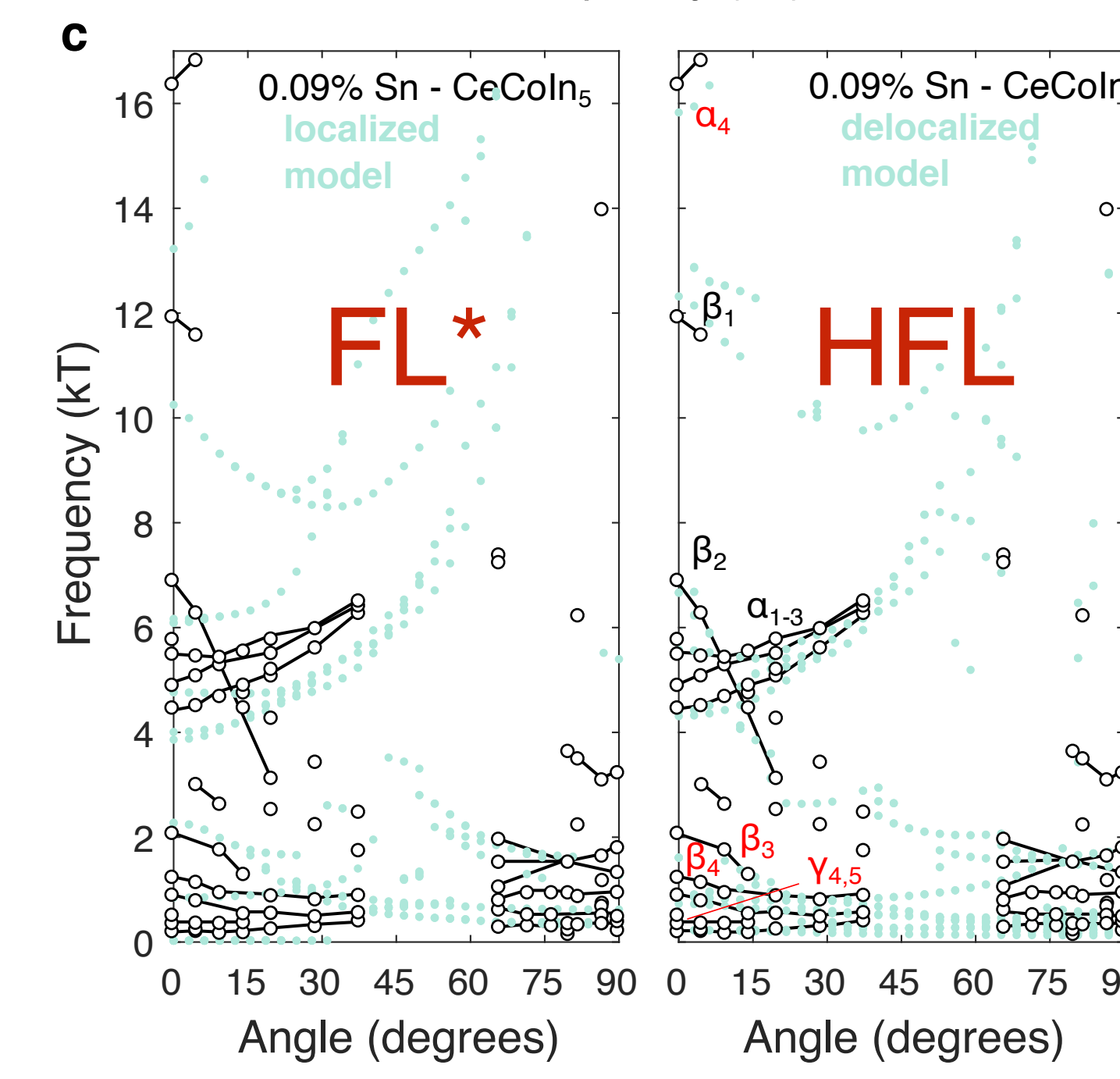
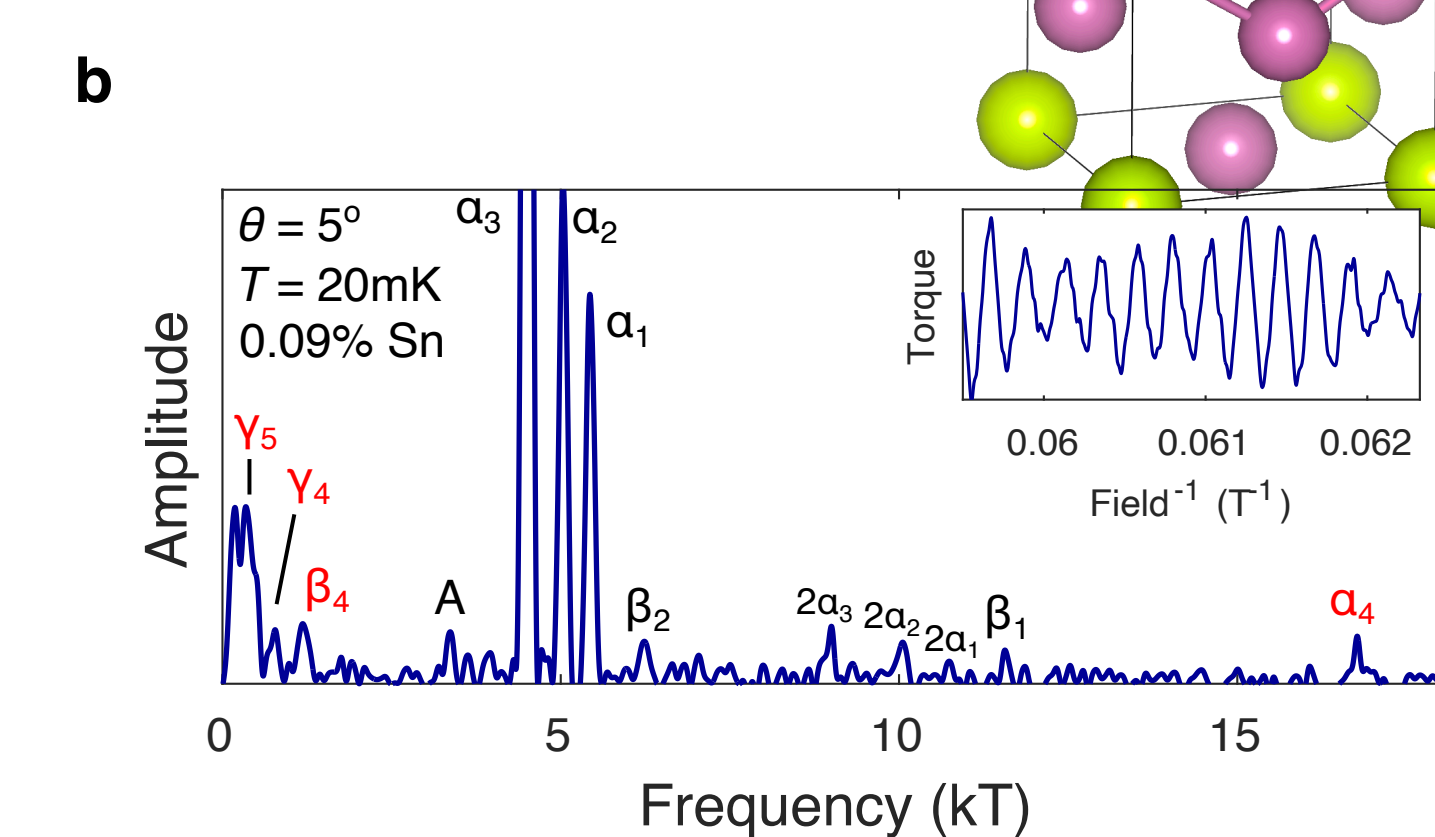
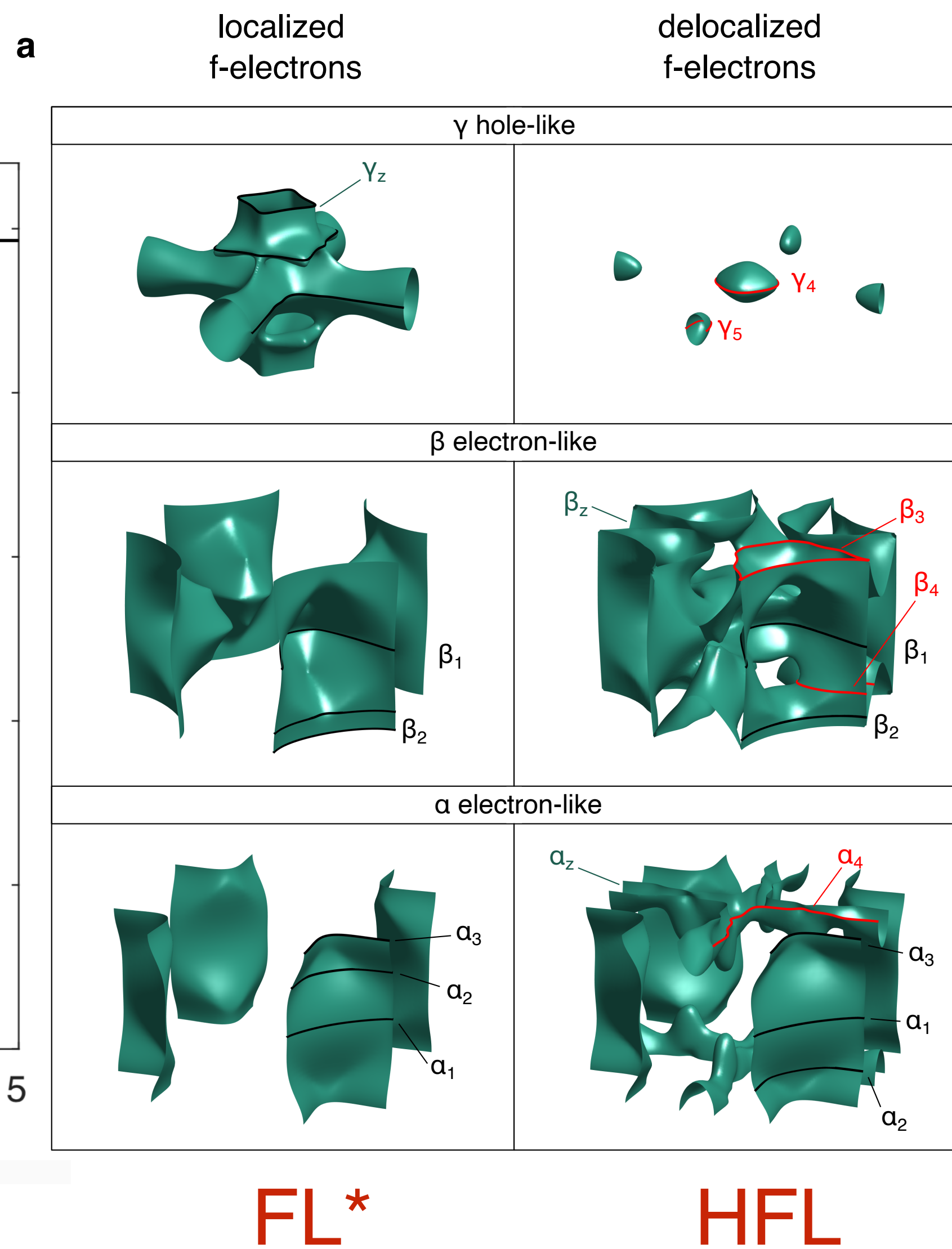
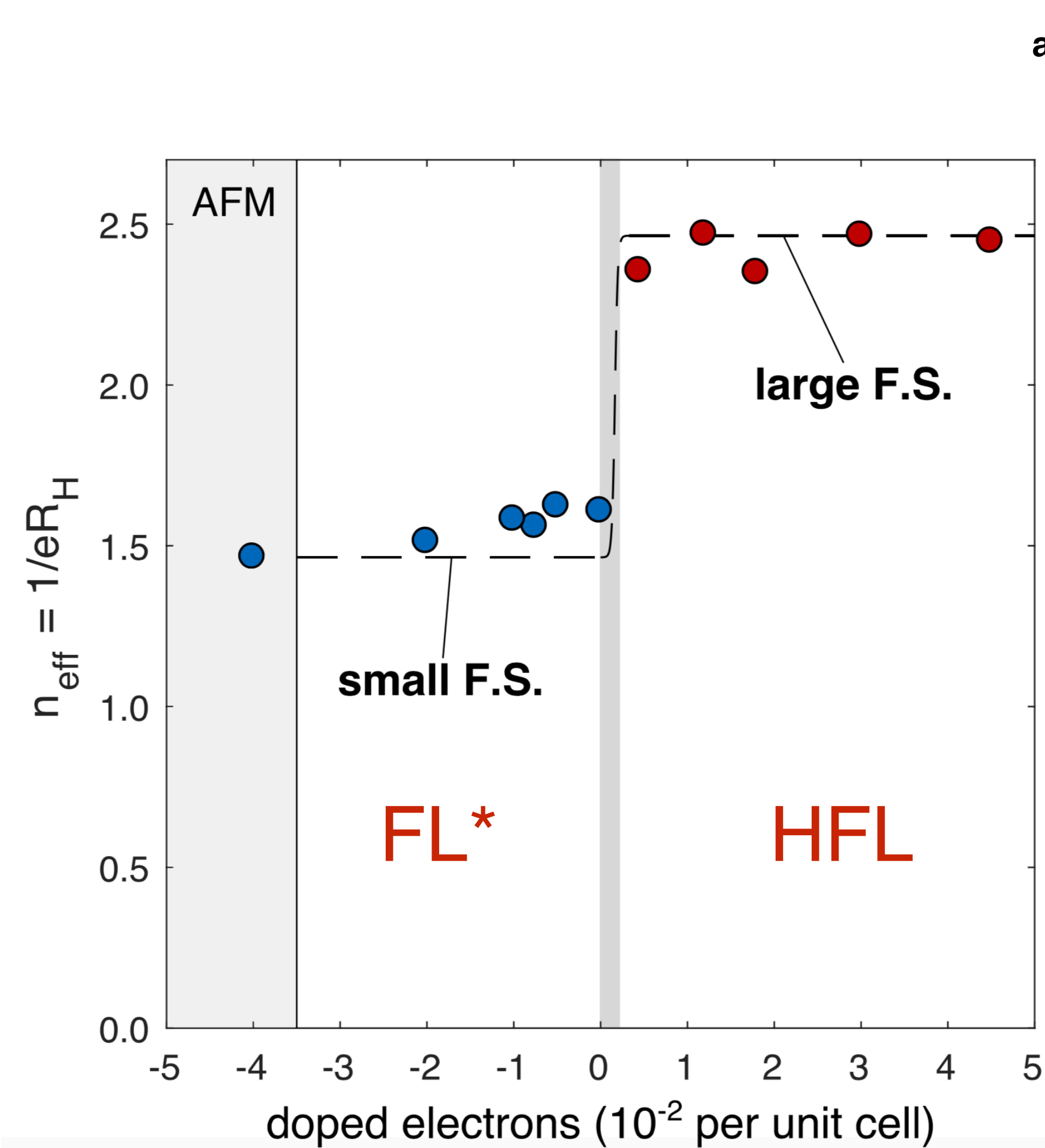
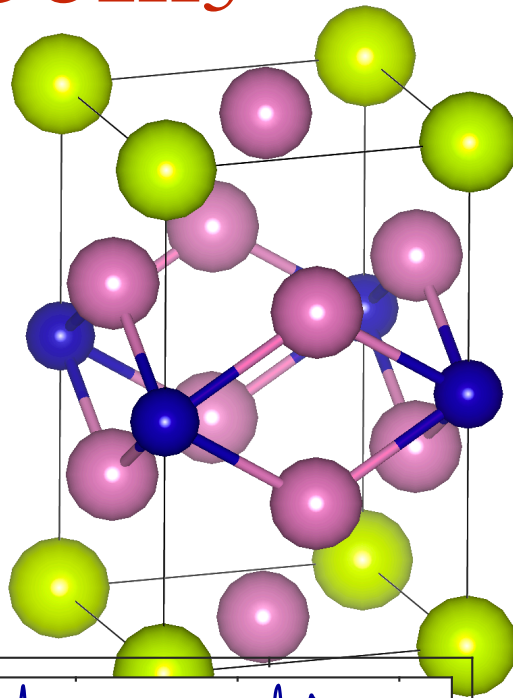
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- In the HFL state, the $U(1)_{\text{gauge}} \times U(1)_{\text{em}}$ symmetry is ‘Higgsed’ by the condensation of the hybridization boson $B_i \sim f_{i\sigma}^\dagger c_{i\sigma}$ to a diagonal $U(1)_{\text{diag}}$ symmetry. The Luttinger arguments can only be applied to the unbroken $U(1)_{\text{diag}}$ symmetry, which counts *both* c and f fermions, and so the Fermi surface is *large*.



Evidence for a delocalization quantum phase transition without symmetry breaking in CeCoIn_5

Nikola Maksimovic, Daniel H. Eilbott, Tessa Cookmeyer, Fanghui Wan, Jan Ruzs, Vikram Nagarajan, Shannon C. Haley, Eran Maniv, Amanda Gong, Stefano Faubel, Ian M. Hayes, Ali Bangura, John Singleton, Johanna C. Palmstrom, Laurel Winter, Ross McDonald, Sooyoung Jang, Ping Ai, Yi Lin, Samuel Ciocys, Jacob Gobbo, Yochai Werman, Peter M. Oppeneer, Ehud Altman, Alessandra Lanzara, James G. Analytis, *Science* **375**, 76 (2021).



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FL* and HFL phases of the Kondo lattice model.

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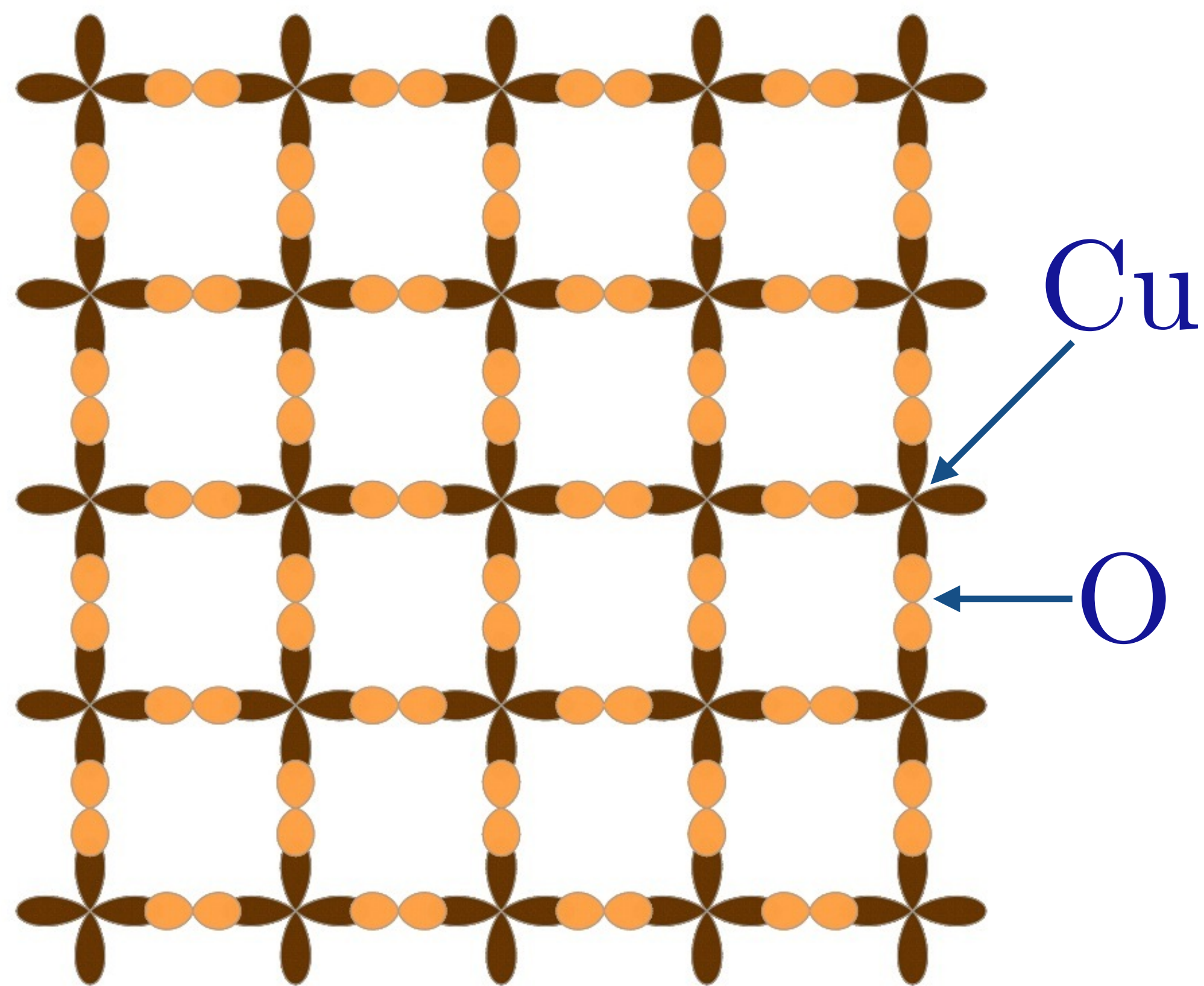
fractionalizing the paramagnon

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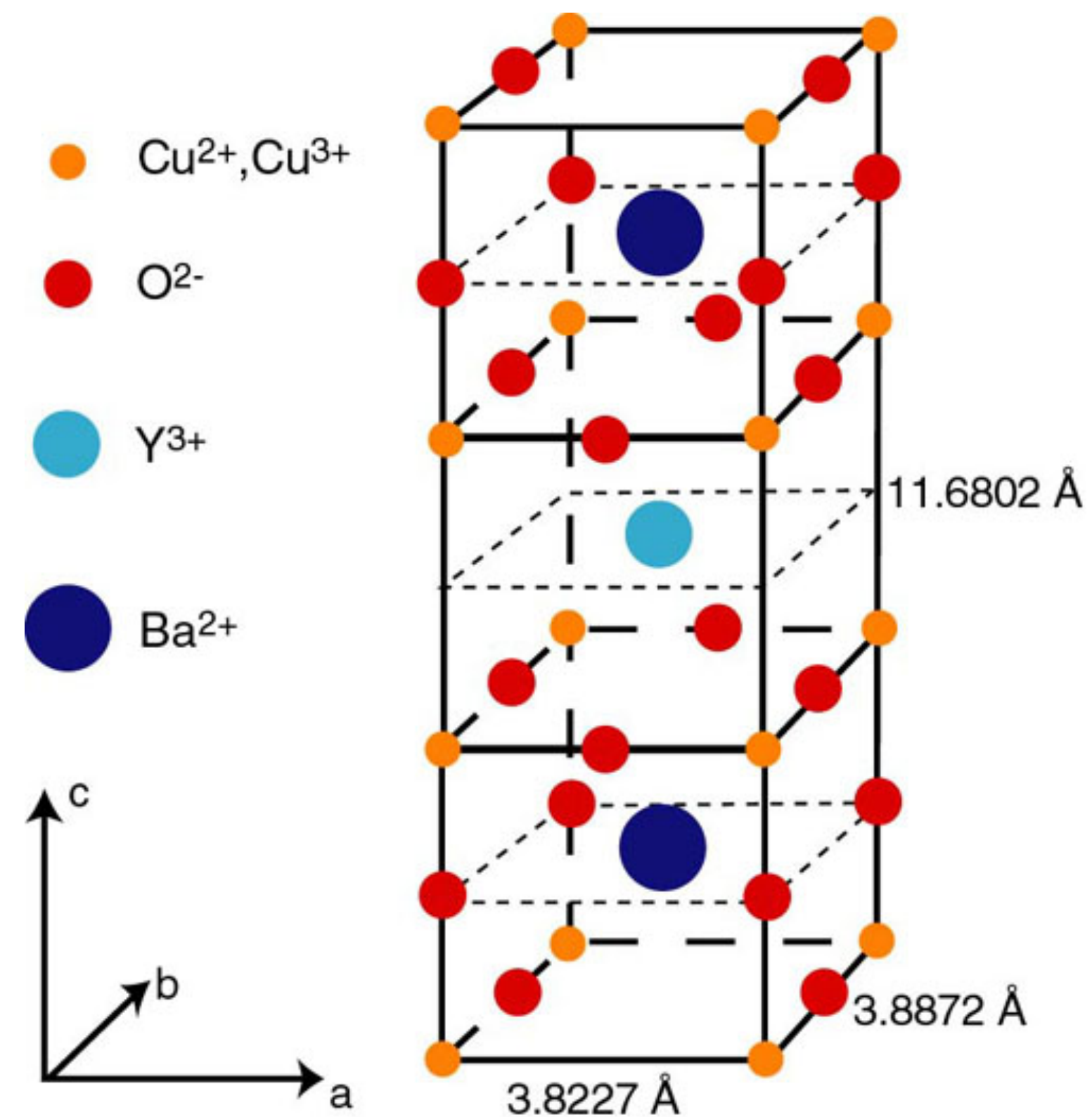
5. Quantum criticality

$$\mathcal{H}_H = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

High
temperature
superconductors

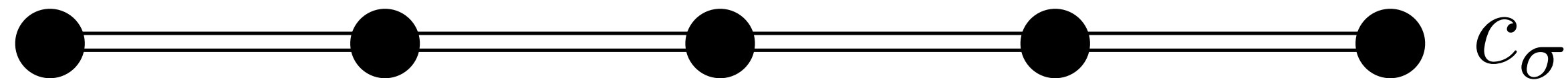


CuO_2 plane



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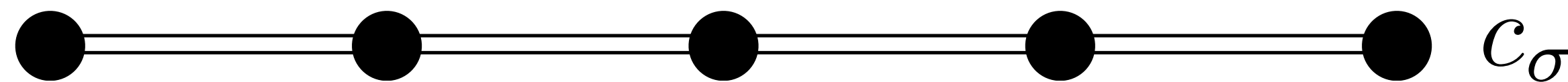
- The Luttinger theorem implies a FL phase with ‘large’ Fermi surface of size $1 + p$ holes (or $1 - p$ electrons) for all U and all p .



density
 $1+p$

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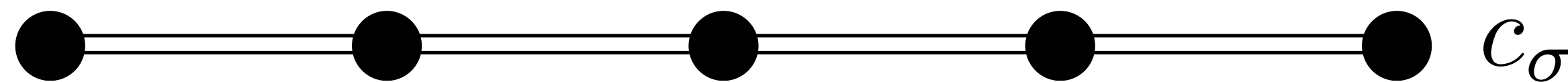
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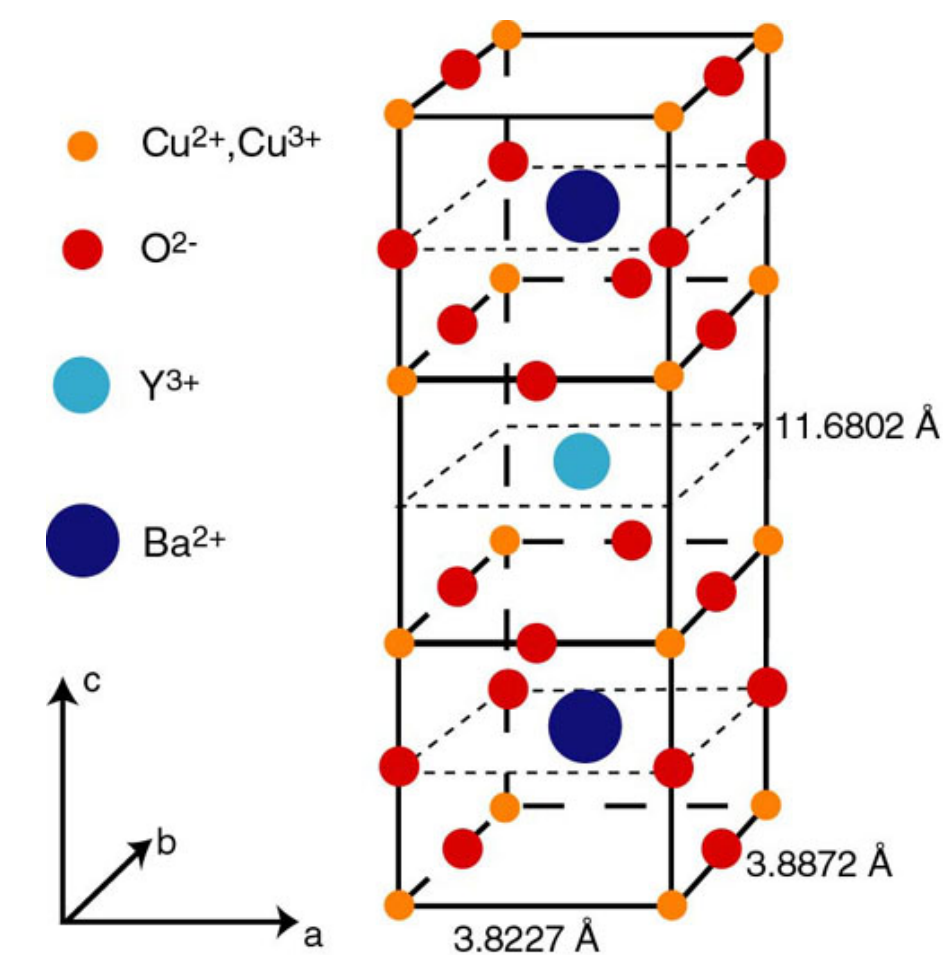
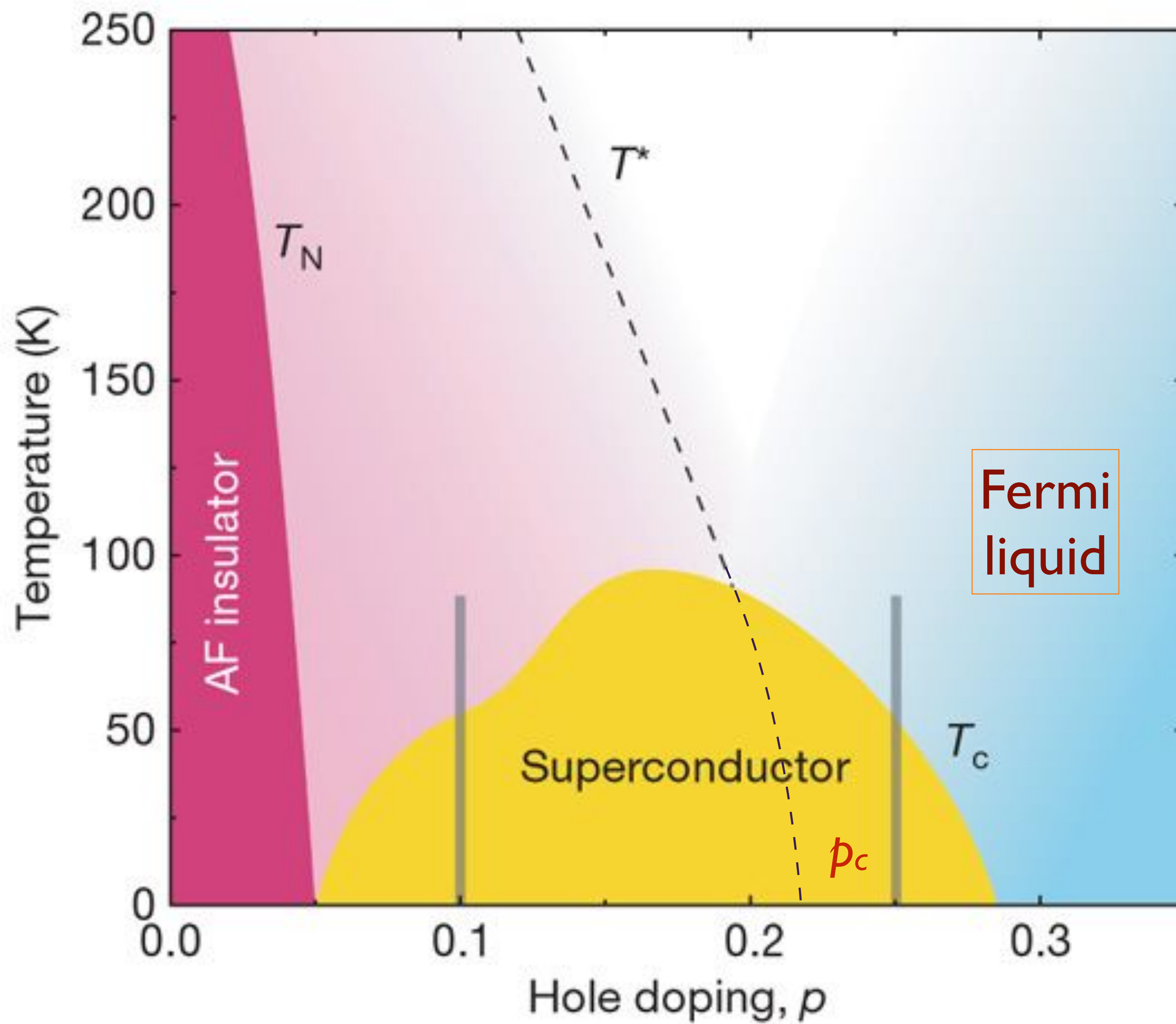
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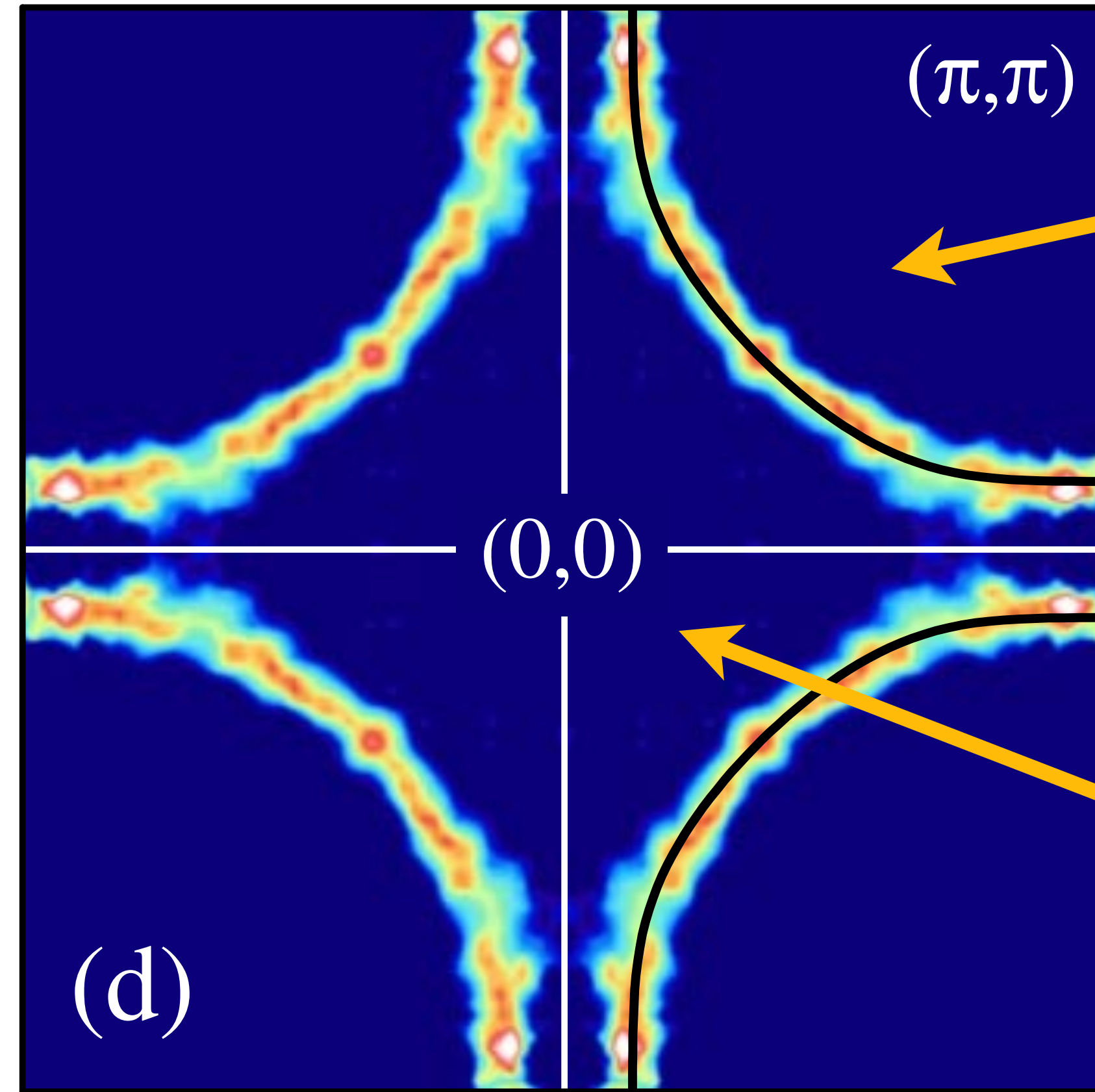
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- The main effect of the projection is a ‘Brinkman-Rice’ enhancement of the quasiparticle mass as $p \rightarrow 0$, with $m^*/m \sim 1/p$.



density
 $1+p$



Photoemission at large p



$l+p$ holes

Overdoped $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$
 $T_c = 30\text{K}$

$l-p$ electrons

$l+p$ mobile holes in a filled band

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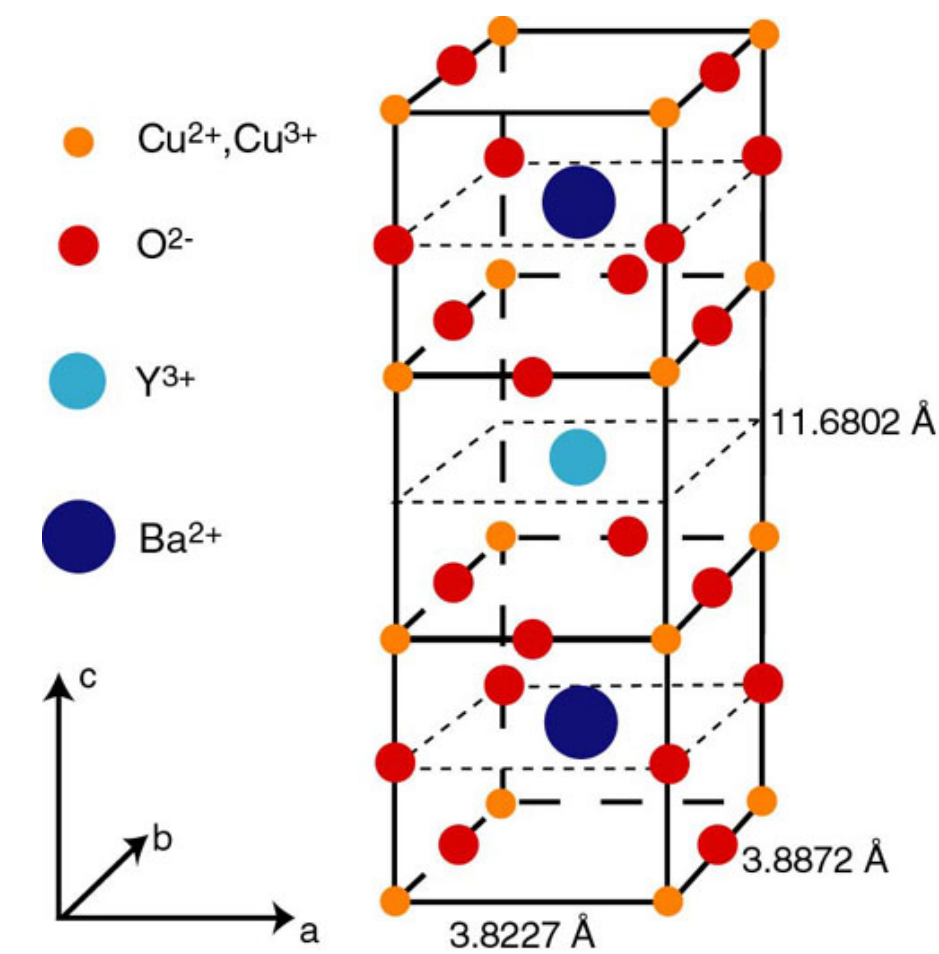
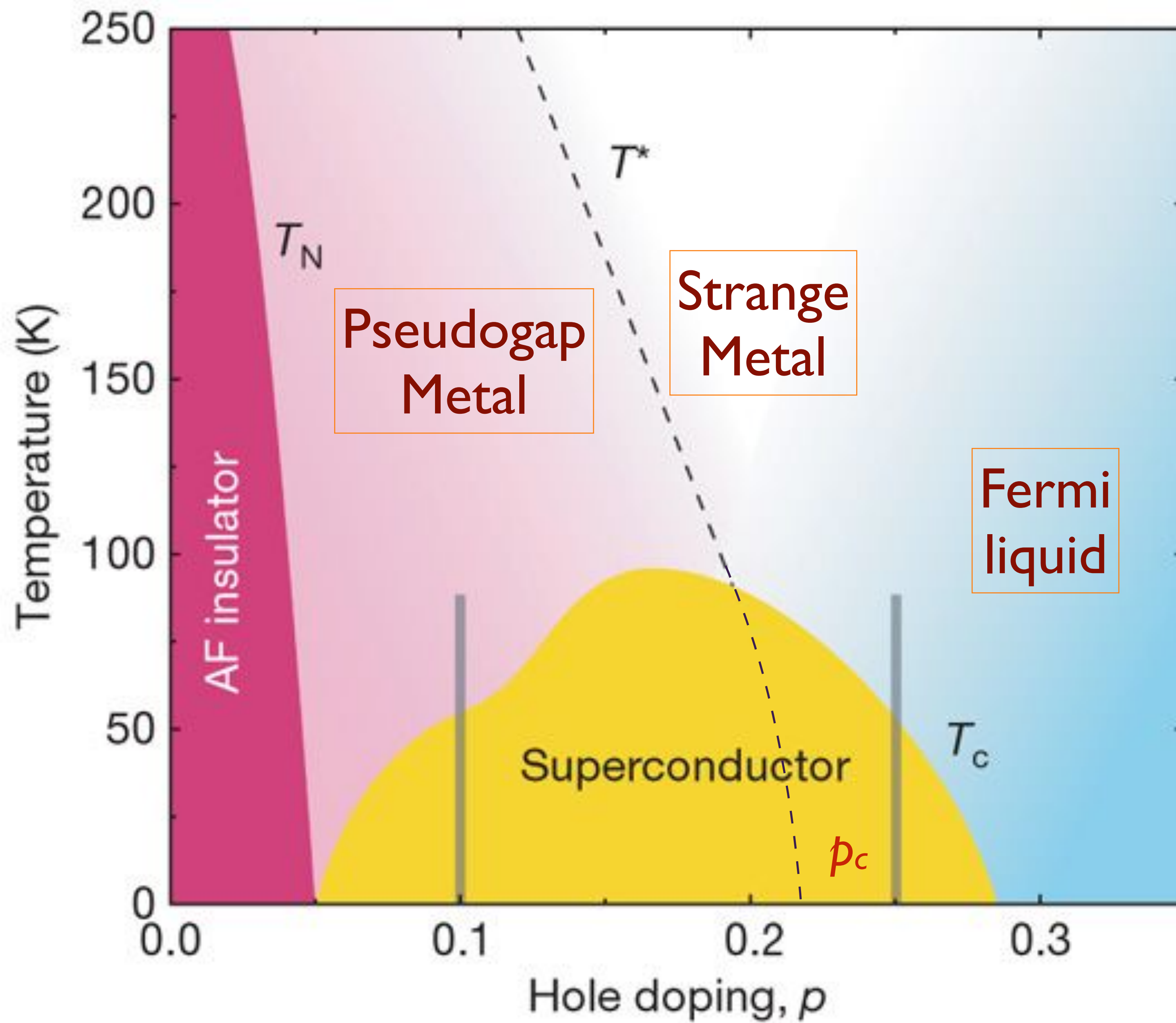
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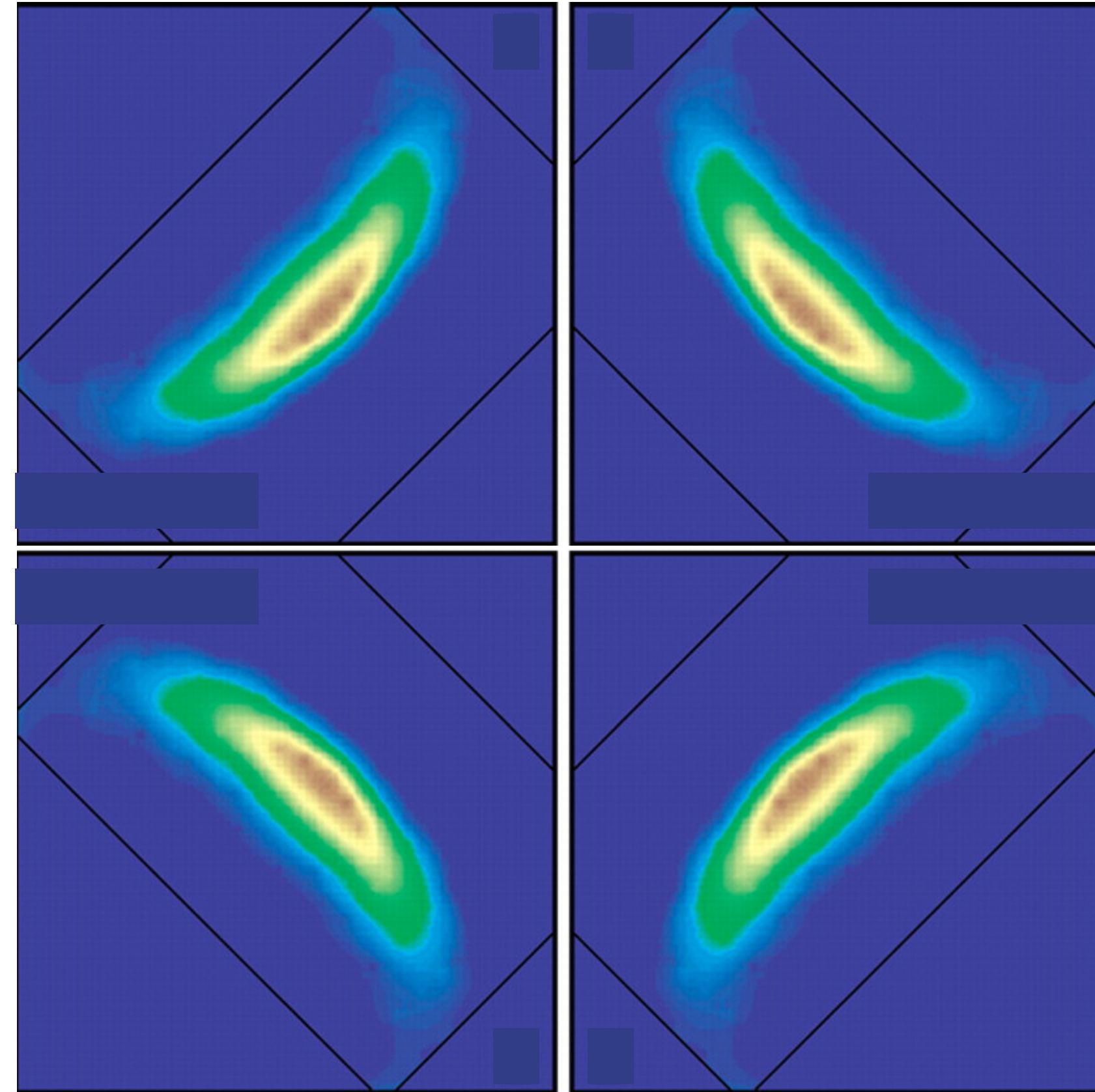
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Photoemission at small p



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$
at $x = 0.10$

“Fermi arcs”

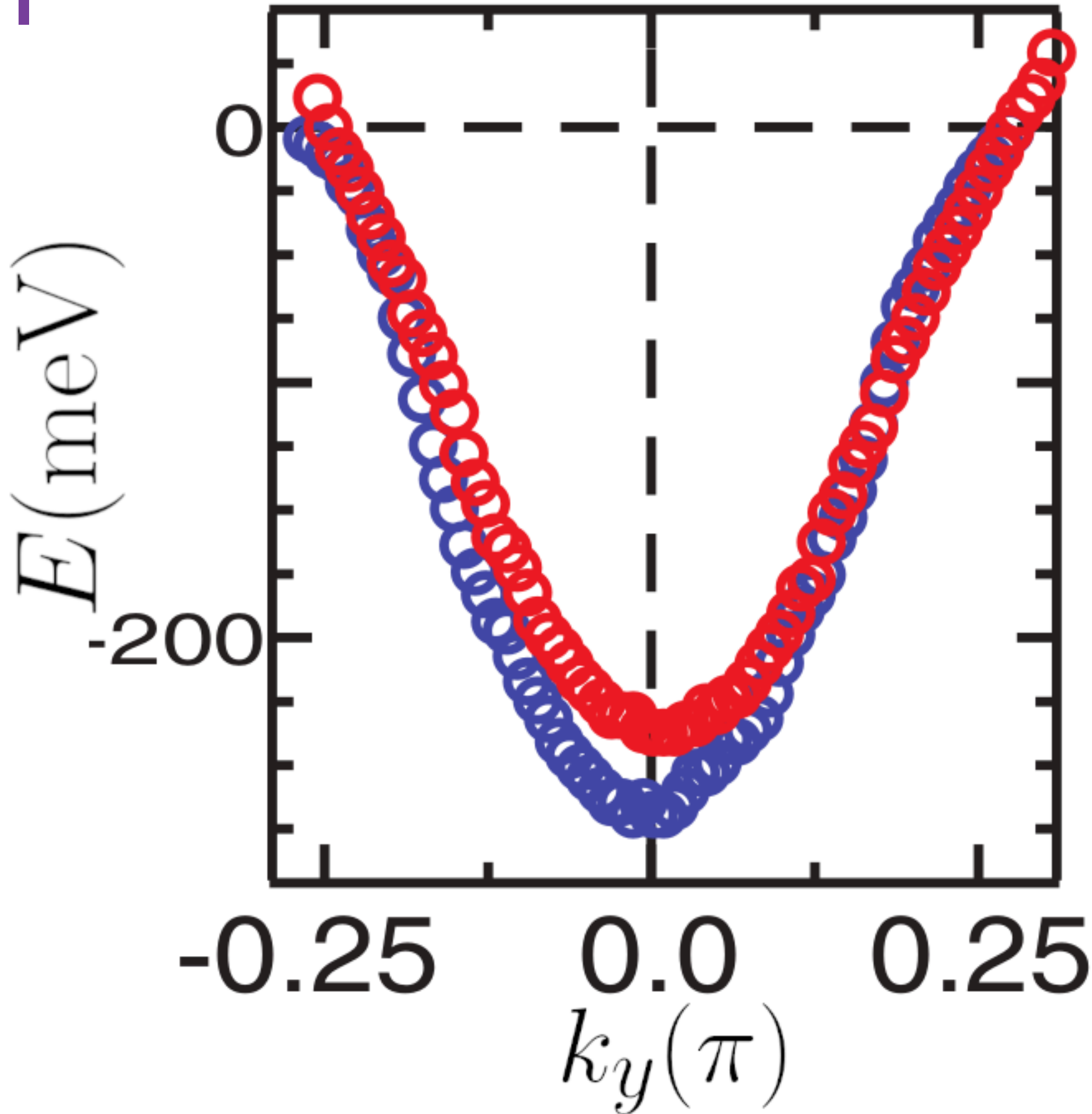
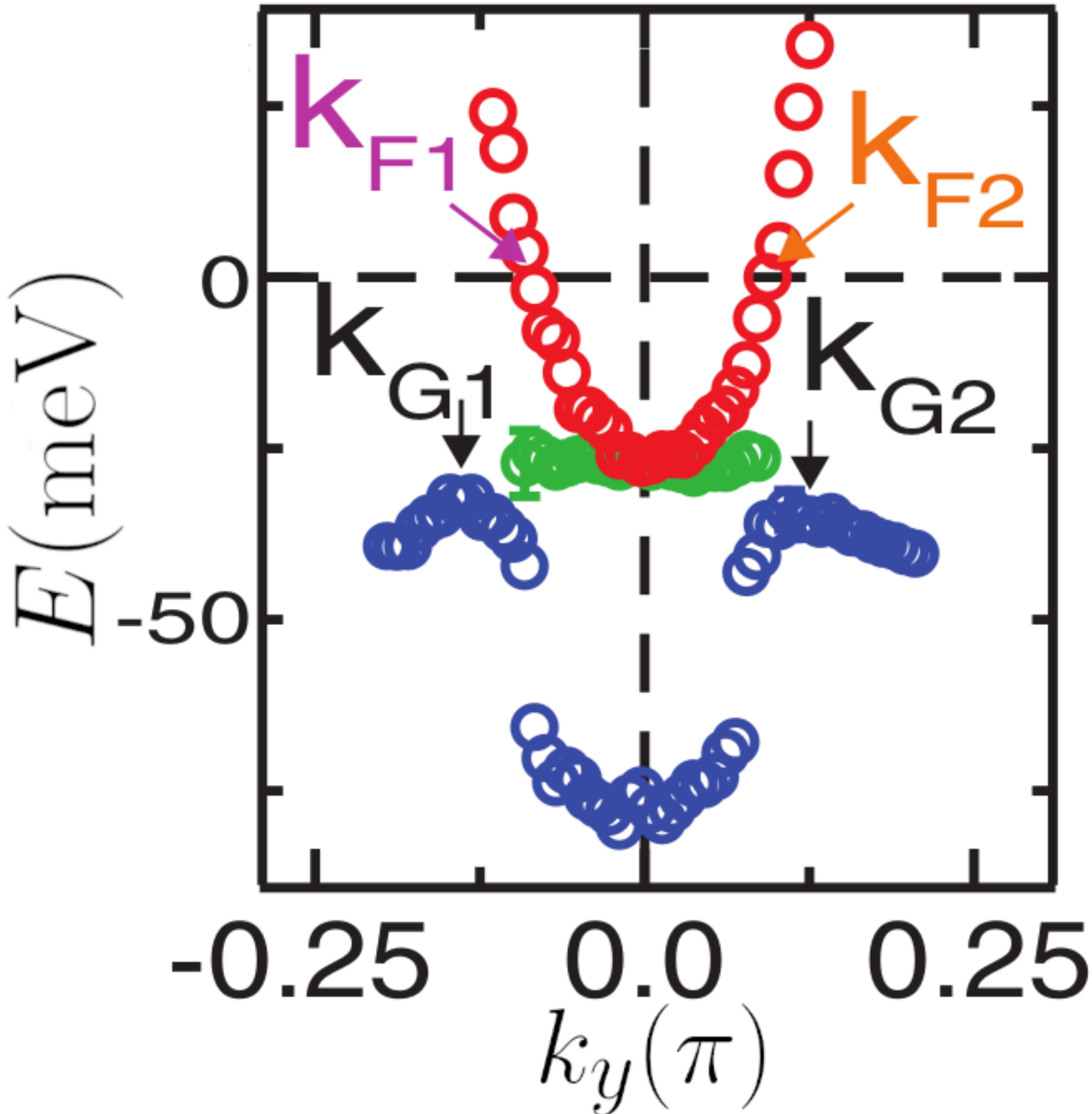
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

Photoemission at small and large p

Anti-node: $k_x = \pi$

Node: $k_x = 2$

Bi2201

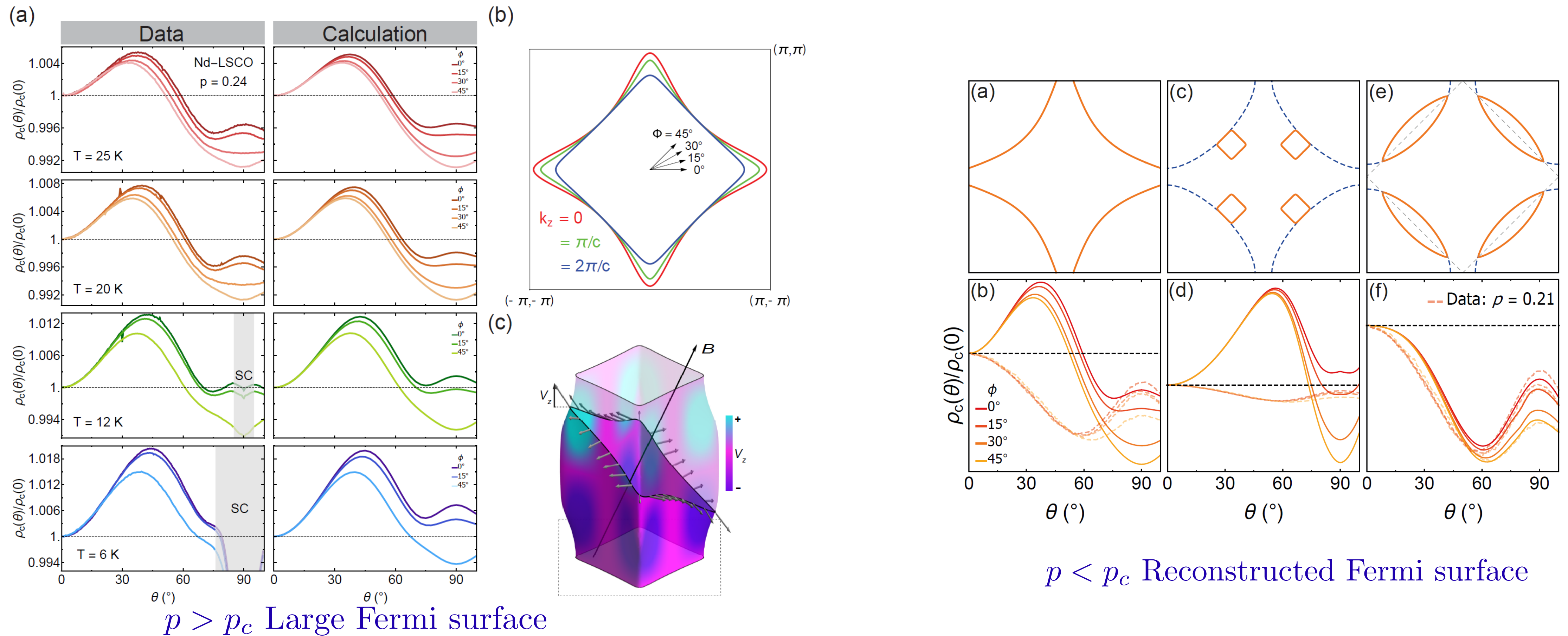


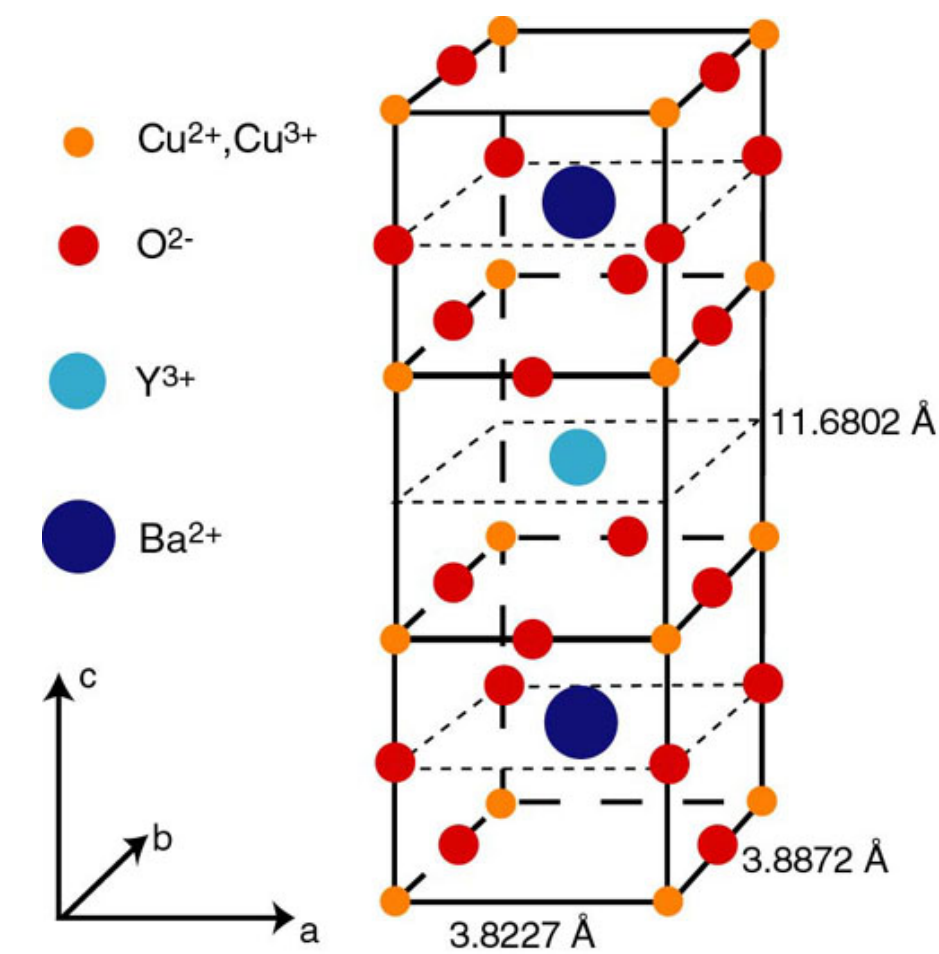
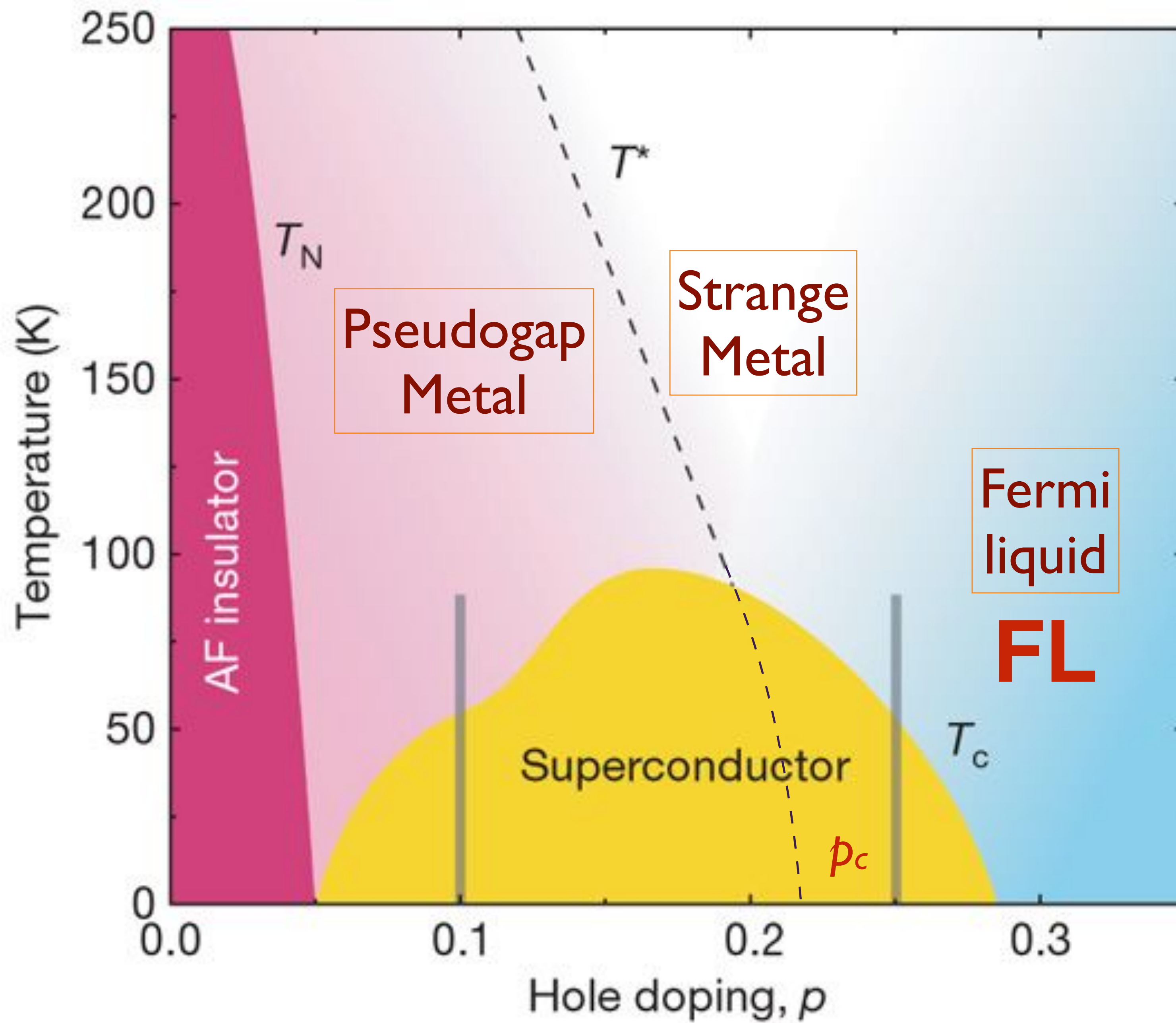
R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

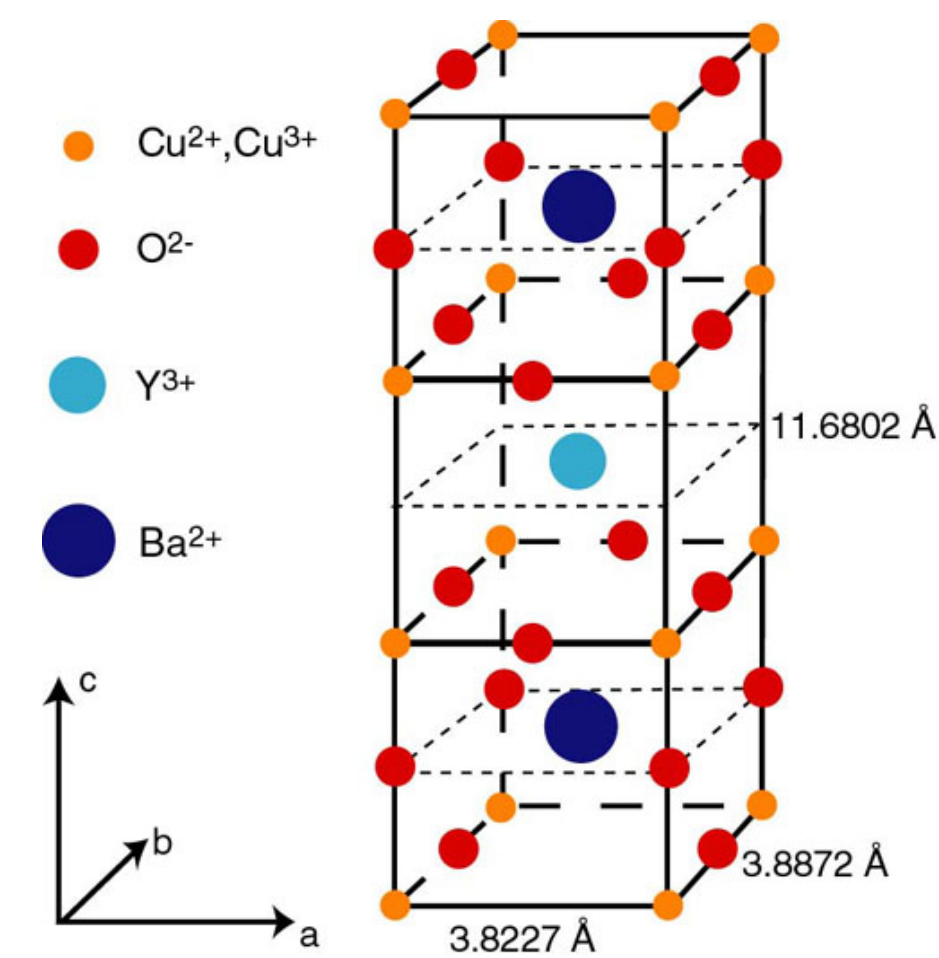
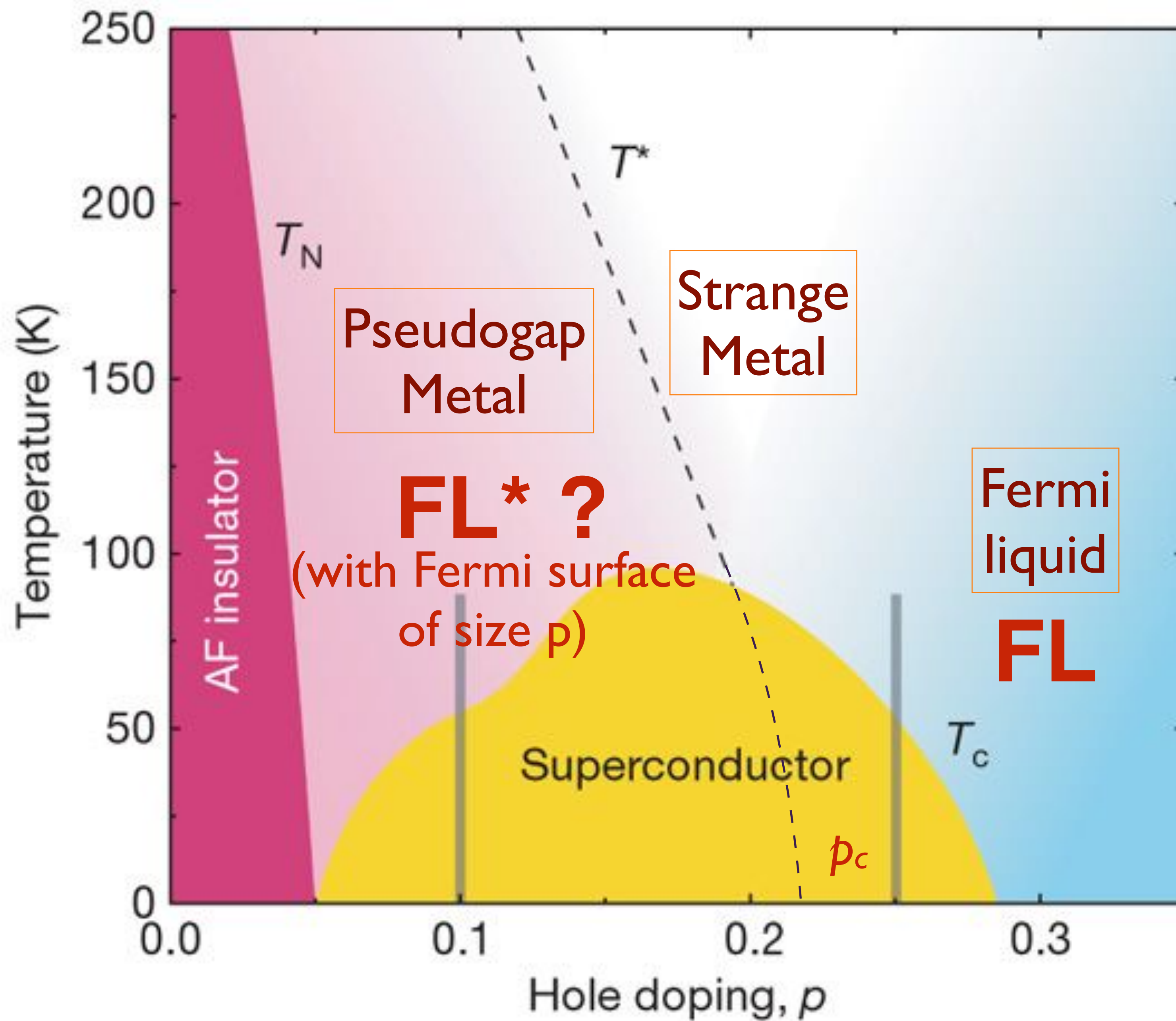
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics, in press

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate Nd-LSCO. Above the critical doping p^* we extract a Fermi surface geometry that is in quantitative agreement with angle-resolved photoemission. Below p^* the ADMR is qualitatively different, revealing a clear transformation of the Fermi surface. Changes in the quasiparticle lifetime across p^* are ruled out as the cause of this transformation. Instead we find that our data are most consistent with a reconstruction of the Fermi surface by a $Q = (\pi, \pi)$ wavevector.







The pseudogap metal \approx FL* (these papers fractionalize the mobile electron)

X.-G. Wen and P. A. Lee, “Theory of Underdoped Cuprates,” *Phys. Rev. Lett.* **76**, 503 (1996), [arXiv:cond-mat/9506065](#) [cond-mat].

J.-W. Mei, S. Kawasaki, G.-Q. Zheng, Z.-Y. Weng, and X.-G. Wen, “Luttinger-volume violating Fermi liquid in the pseudogap phase of the cuprate superconductors,” *Phys. Rev. B* **85**, 134519 (2012), [arXiv:1109.0406](#) [cond-mat.supr-con].

K.-Y. Yang, T. M. Rice, and F.-C. Zhang, “Phenomenological theory of the pseudogap state,” *Phys. Rev. B* **73**, 174501 (2006), [arXiv:cond-mat/0602164](#) [cond-mat.supr-con].

N. J. Robinson, P. D. Johnson, T. M. Rice, and A. M. Tsvelik, “Anomalies in the pseudogap phase of the cuprates: competing ground states and the role of umklapp scattering,” *Reports on Progress in Physics* **82**, 126501 (2019), [arXiv:1906.09005](#) [cond-mat.supr-con].

J. Feldmeier, S. Huber, and M. Punk, “Exact solution of a two-species quantum dimer model for pseudogap metals,” *Phys. Rev. Lett.* **120**, 187001 (2018), [arXiv:1712.01854](#) [cond-mat.str-el].

B. Verheijden, Y. Zhao, and M. Punk, “Solvable lattice models for metals with Z_2 topological order,” *SciPost Physics* **7**, 074 (2019), [arXiv:1908.00103](#) [cond-mat.str-el].

J. Brunkert and M. Punk, “Slave-boson description of pseudogap metals in t - J models,” *Physical Review Research* **2**, 043019 (2020), [arXiv:2002.04041](#) [cond-mat.str-el].

The pseudogap metal = FL* (these papers fractionalize the mobile electron)

Quantum phases of the Shraiman-Siggia model by S. Sachdev *Physical Review B* **49**, 6770 (1994).

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, “Algebraic charge liquids,” *Nature Physics* **4**, 28 (2008), [arXiv:0706.2187 \[cond-mat.str-el\]](#).

Y. Qi and S. Sachdev, “Effective theory of Fermi pockets in fluctuating antiferromagnets,” *Phys. Rev. B* **81**, 115129 (2010), [arXiv:0912.0943 \[cond-mat.str-el\]](#).

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M. Punk and S. Sachdev, “Fermi surface reconstruction in hole-doped t - J models without long-range antiferromagnetic order,” *Phys. Rev. B* **85**, 195123 (2012), [arXiv:1202.4023 \[cond-mat.str-el\]](#).

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S. Sachdev, H. D. Scammell, M. S. Scheurer, and G. Tarnopolsky, “Gauge theory for the cuprates near optimal doping,” *Phys. Rev. B* **99**, 054516 (2019), [arXiv:1811.04930 \[cond-mat.str-el\]](#).

S. Sachdev, “Topological order, emergent gauge fields, and Fermi surface reconstruction,” *Rep. Prog. Phys.* **82**, 014001 (2019), [arXiv:1801.01125 \[cond-mat.str-el\]](#).

The pseudogap metal = FL*

Main lesson from the Kondo lattice

Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b_i^\dagger$ or $f_i b_{i\sigma}^\dagger$.

We avoid this particular fractionalization because the excitations around the small Fermi surface carry spin-1/2 and charge e , just like the bare electron: so it is cumbersome to fractionalize all the electrons (which occurs on energy scale J), and then undo the fractionalization for a small density of them by forming bound states of spinons and holons (which occurs on an energy scale t). In particular, no complete theory of this bound state formation has yet been presented.

The pseudogap metal = FL*

Main lesson from the Kondo lattice

Do not fractionalize the mobile electron, $c_{i\sigma} \neq f_{i\sigma} b_i^\dagger$ or $f_i b_{i\sigma}^\dagger$.

Fractionalize the paramagnon instead!

We avoid this particular fractionalization because the excitations around the small Fermi surface carry spin-1/2 and charge e , just like the bare electron: so it is cumbersome to fractionalize all the electrons (which occurs on energy scale J), and then undo the fractionalization for a small density of them by forming bound states of spinons and holons (which occurs on an energy scale t). In particular, no complete theory of this bound state formation has yet been presented.



Yahui Zhang

arXiv: 2001.09159

arXiv: 2103.05009



**Alexander
Nikolaenko**

arXiv: 2006.01140

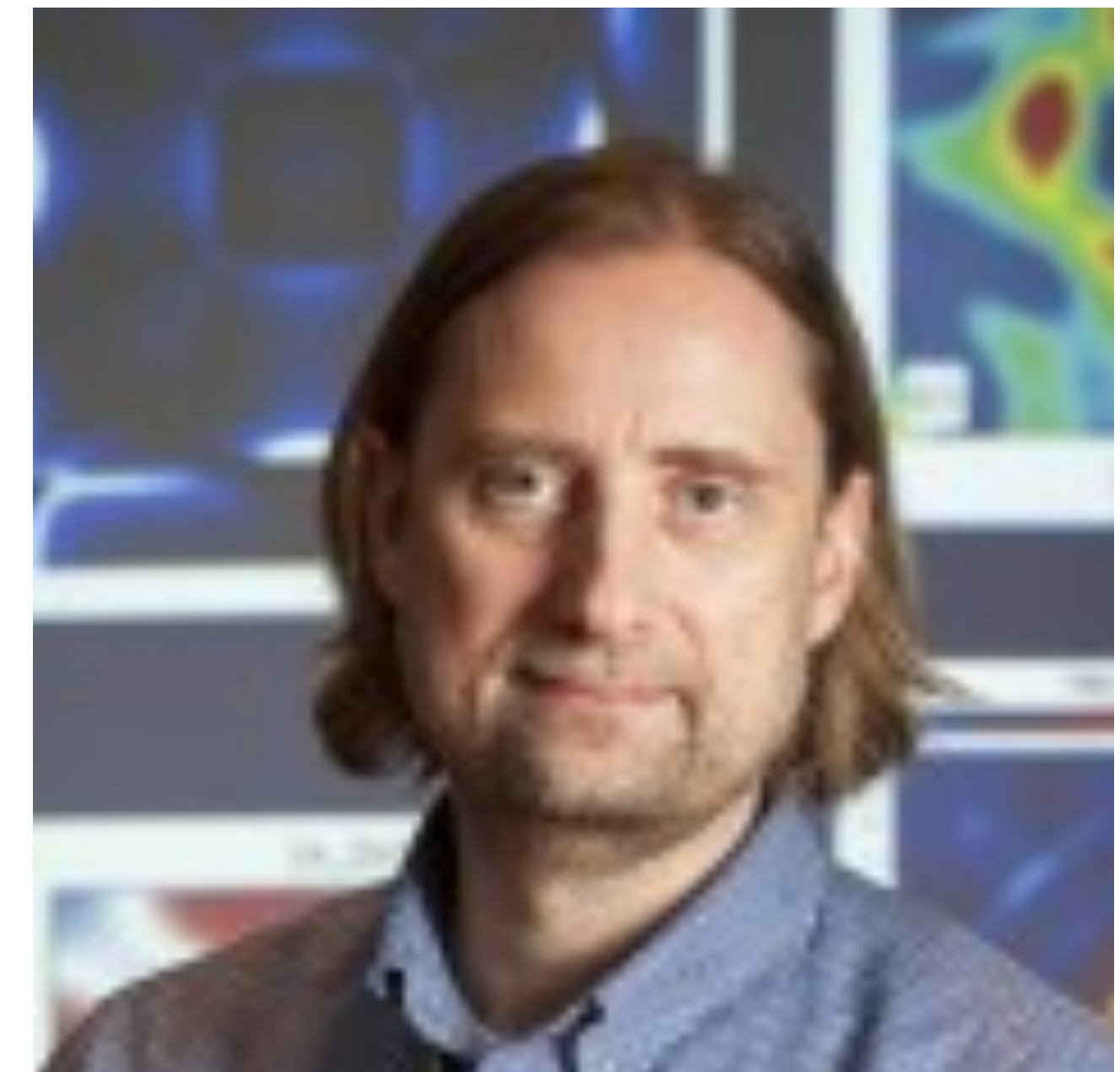
arXiv: 2111.13703



**Maria
Tikhanovskaya**



Eric Mascot



Dirk Morr

Paramagnon theory of the Hubbard model

$$H = - \sum_{i < j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U \left(n_\uparrow - \frac{1}{2} \right) \left(n_\downarrow - \frac{1}{2} \right) = -\frac{2U}{3} \mathbf{S}^2 + \frac{U}{4}$$

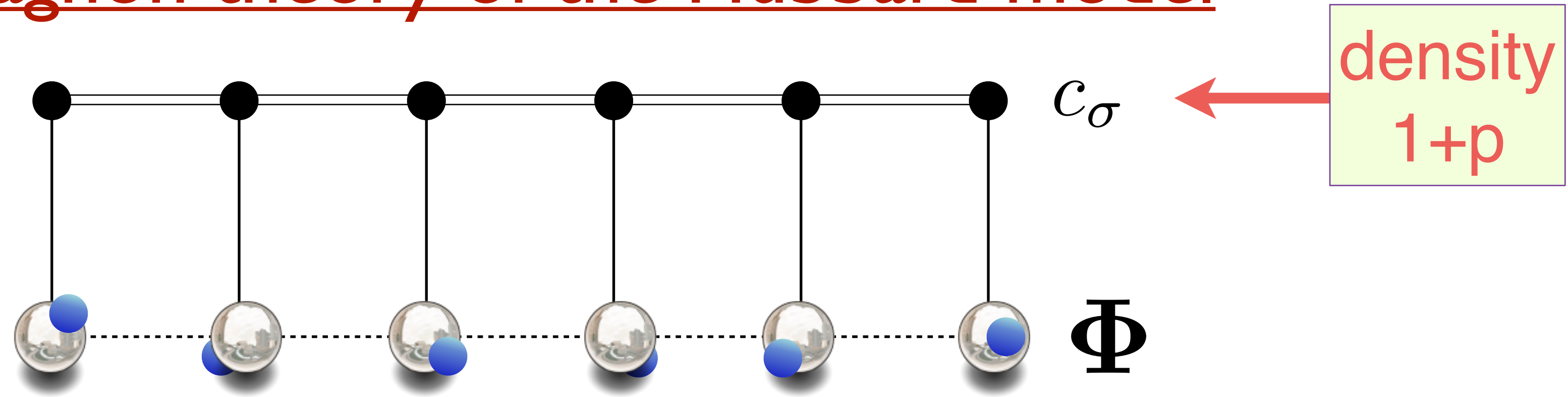
Then we decouple the interaction via

$$\exp \left(\frac{2U}{3} \sum_i \int d\tau \mathbf{S}_i^2 \right) = \int \mathcal{D}\Phi_i(\tau) \exp \left(- \sum_i \int d\tau \left[\frac{3}{8U} \Phi_i^2 - \Phi_i \cdot c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \right)$$

This yields the ‘Hertz-Millis’ theory for a ‘paramagnon quantum rotor’ Φ_i coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model

Quantum
rotors
 $|\Phi_i| = 1$

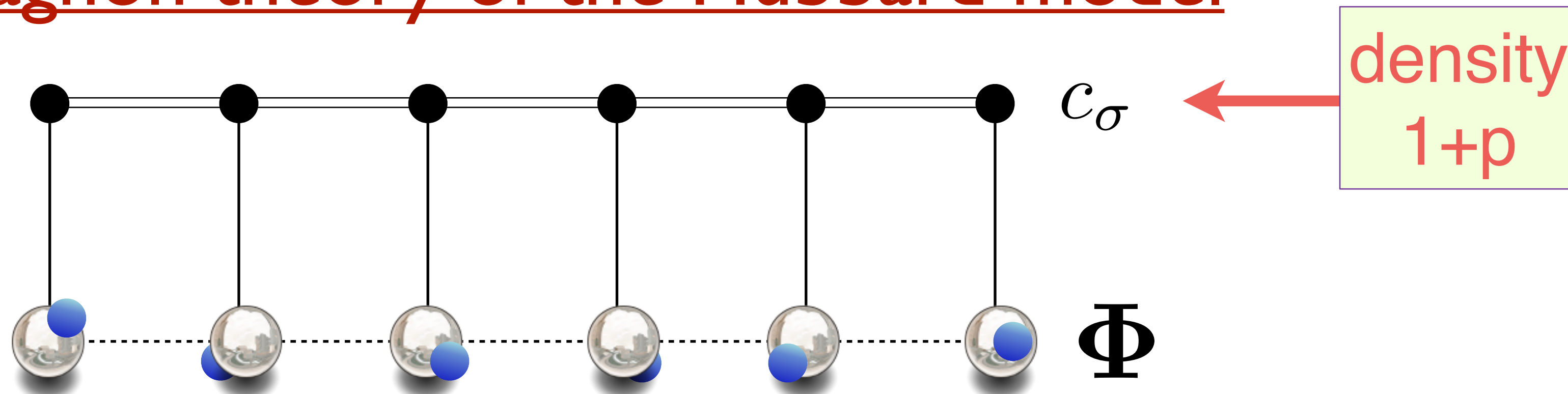


$$\mathcal{H}_{\text{rotor}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} L_i^2 - \sum_i c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \Phi_i$$

$$[L_{i\alpha}, L_{j\beta}] = i\epsilon_{\alpha\beta\gamma} \delta_{ij} L_{i\gamma}, \quad [L_{i\alpha}, \Phi_{j\beta}] = i\epsilon_{\alpha\beta\gamma} \delta_{ij} \Phi_{i\gamma}, \quad [\Phi_{i\alpha}, \Phi_{j\beta}] = 0$$

Paramagnon theory of the Hubbard model

Quantum rotors
 $|\Phi_i| = 1$



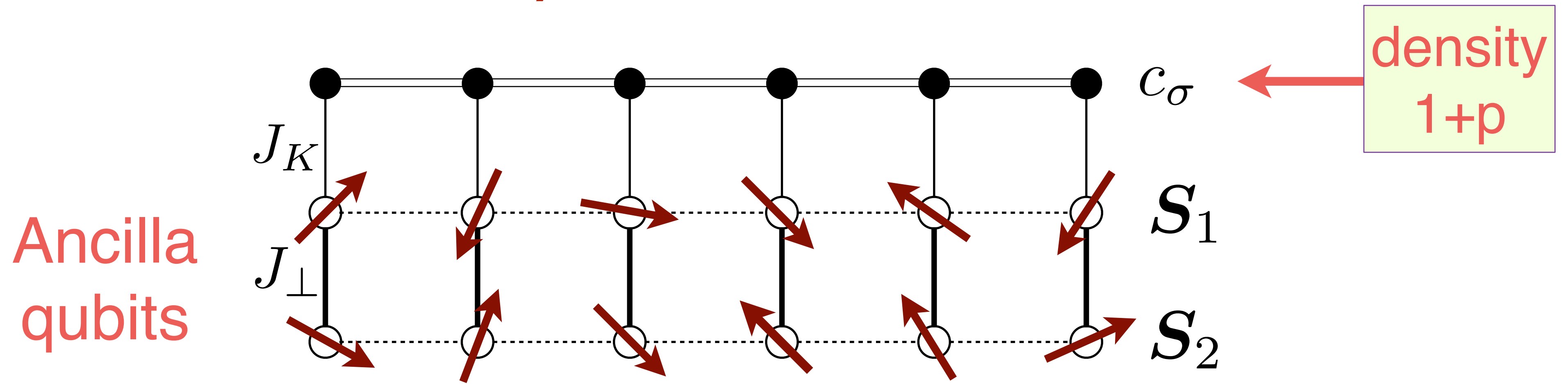
$$\mathcal{H}_{\text{rotor}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \frac{g}{2} L_i^2 - \sum_i c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \Phi_i$$

Each rotor has eigenvalues $g\ell(\ell + 1)/2$, degeneracy $2\ell + 1$, $\ell = 0, 1, 2, \dots$. Restrict to the $\ell = 0, 1$ states, and represent each rotor by 2 “ancilla qubits”, $S = 1/2$ spins \mathbf{S}_{1i} and \mathbf{S}_{2i} , with an antiferromagnetic coupling $J_\perp = g$

$$\mathbf{L}_i = \mathbf{S}_{1i} + \mathbf{S}_{2i}$$

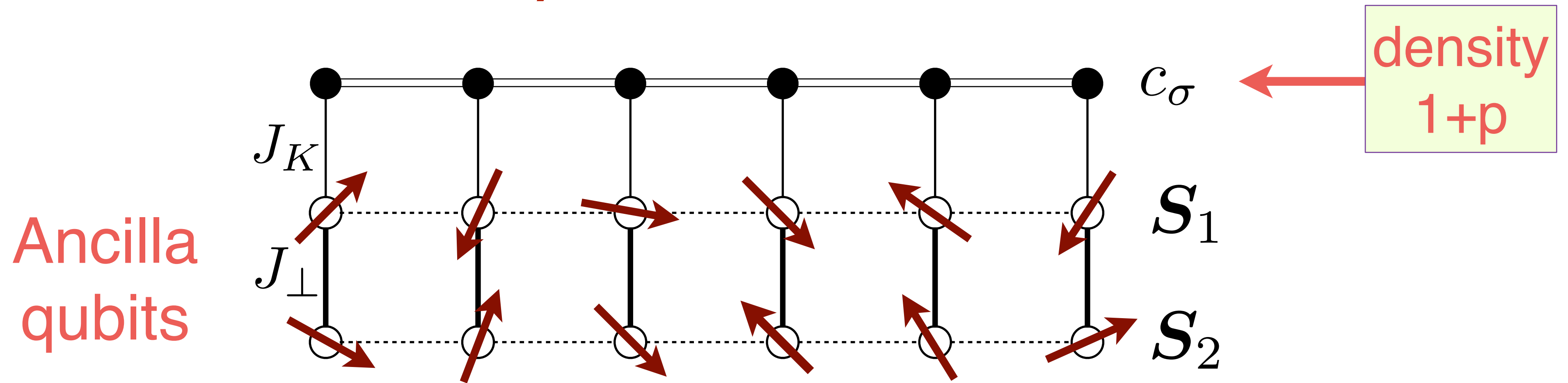
$$\Phi_i = \frac{1}{\sqrt{3}} (\mathbf{S}_{2i} - \mathbf{S}_{1i})$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \left[J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_{1i} + J_\perp \mathbf{S}_{1i} \cdot \mathbf{S}_{2i} \right] + \sum_{\langle ij \rangle} [J_1 \mathbf{S}_{1i} \cdot \mathbf{S}_{1j} + J_2 \mathbf{S}_{2i} \cdot \mathbf{S}_{2j}]$$

Ancilla theory of the Hubbard model



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + \sum_i \left[J_K c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \mathbf{S}_{1i} + J_{\perp} \mathbf{S}_{1i} \cdot \mathbf{S}_{2i} \right] + \sum_{\langle ij \rangle} [J_1 \mathbf{S}_{1i} \cdot \mathbf{S}_{1j} + J_2 \mathbf{S}_{2i} \cdot \mathbf{S}_{2j}]$$

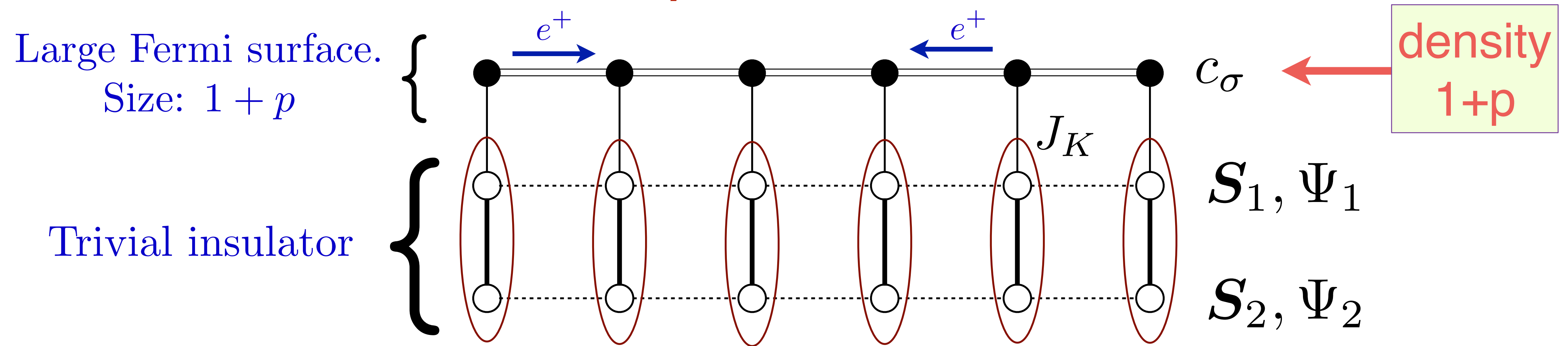
Performing a Schrieffer-Wolff transformation in powers of $1/J_{\perp}$, we obtain

$$\mathcal{H} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + U \sum_i \left[c_{i\uparrow}^\dagger c_{i\uparrow} \right] \left[c_{i\downarrow}^\dagger c_{i\downarrow} \right] + J \sum_{\langle ij \rangle} \left[c_{i\sigma}^\dagger \frac{\tau_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \cdot \left[c_{j\rho}^\dagger \frac{\tau_{\rho\rho'}}{2} c_{j\rho'} \right]$$

i.e. we recover a Hubbard-Heisenberg model with *no ancillas* and

$$U = \frac{3J_K^2}{8J_{\perp}} + \frac{3J_K^3}{16J_{\perp}^2} + \dots, \quad J = \frac{J_K^2 (J_1 + J_2)}{4J_{\perp}^2}$$

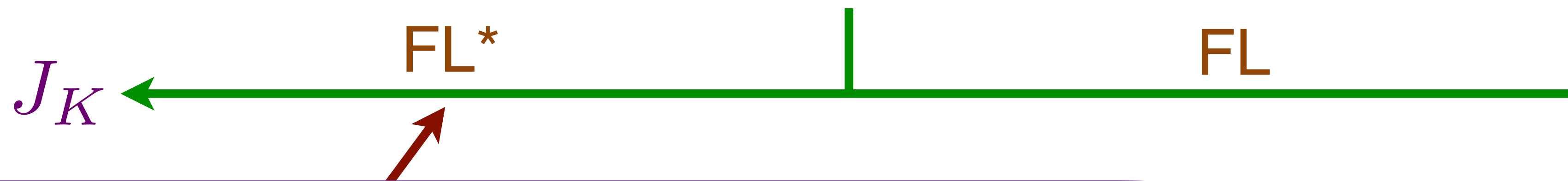
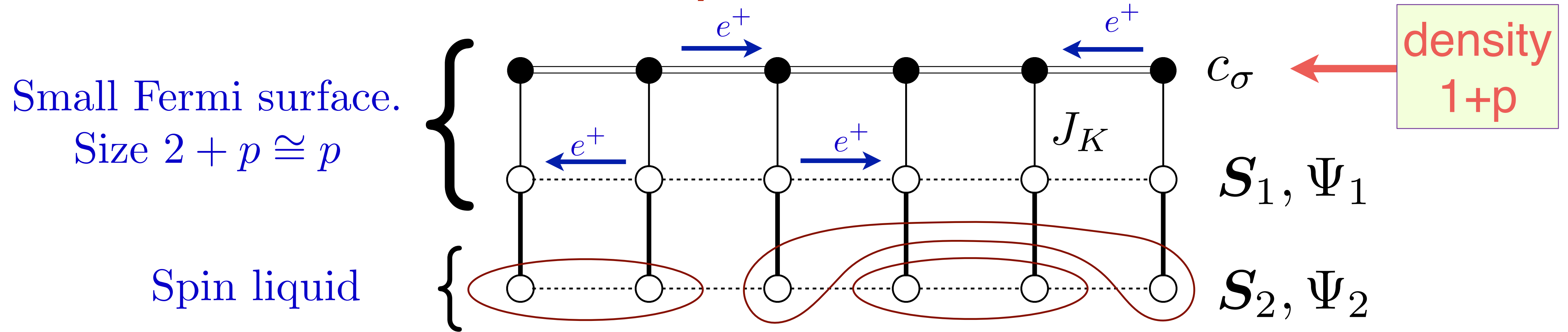
Ancilla theory of the Hubbard model



Large Fermi surface of size $1 + p$

$$|\text{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

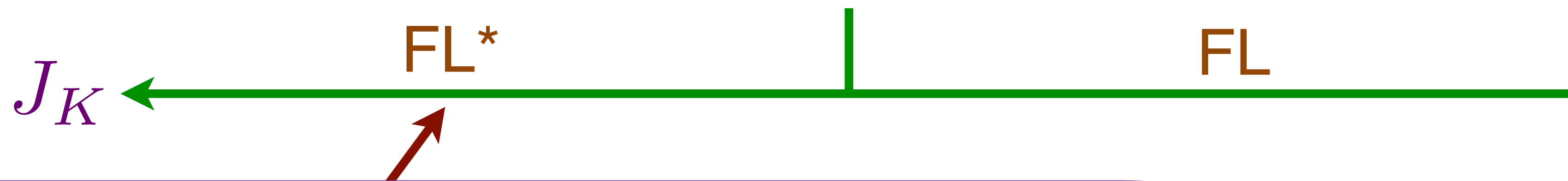
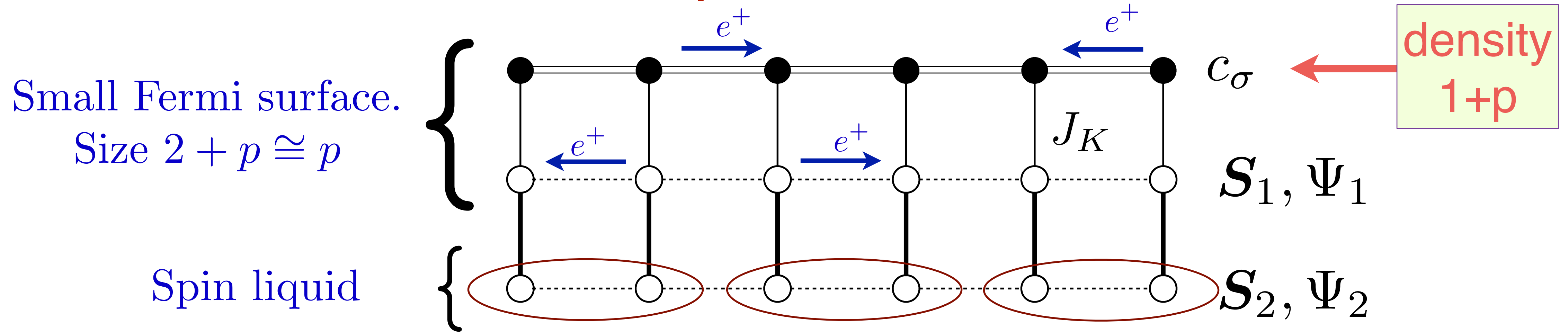
Ancilla theory of the Hubbard model



Small Fermi surface of size p

$$\begin{aligned}
 |\text{FL}^*\rangle &= [\text{Projection onto rung singlets of } \Psi_1, \Psi_2] \\
 &\quad \bowtie |\text{Slater determinant of } (c, \Psi_1)\rangle \\
 &\quad \otimes |\text{Slater determinant of } \Psi_2\rangle
 \end{aligned}$$

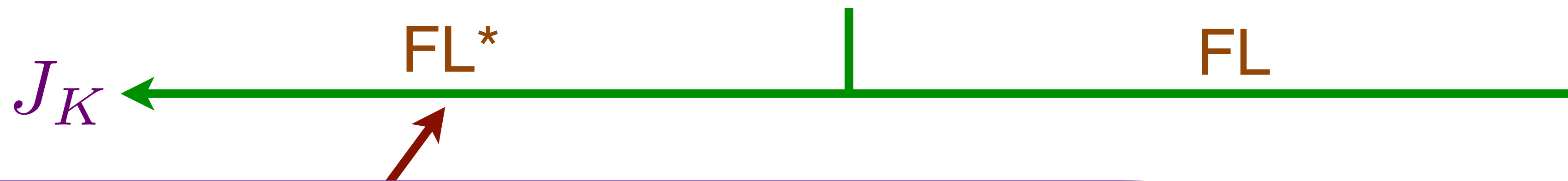
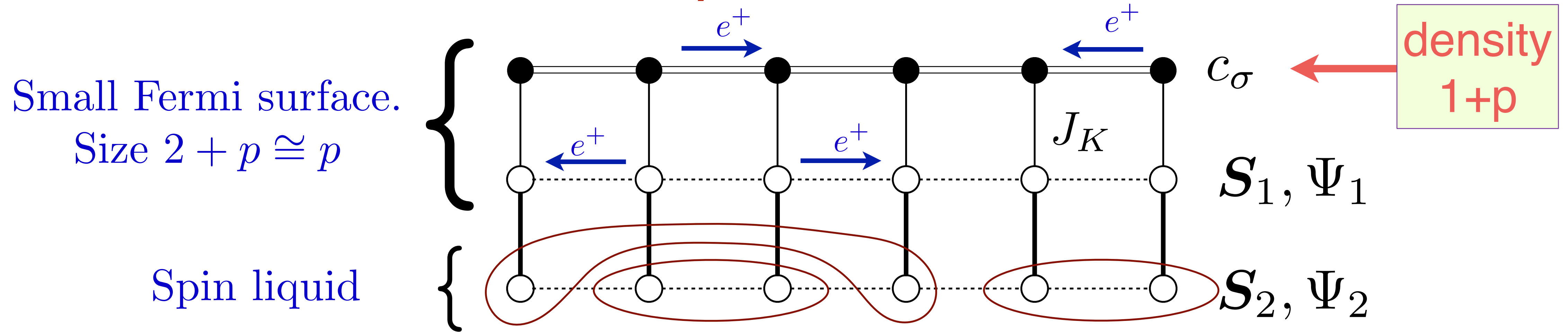
Ancilla theory of the Hubbard model



Small Fermi surface of size p

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 &\quad \otimes |\text{Slater determinant of } \Psi_2\rangle
 \end{aligned}$$

$(\text{SU}(2)_1 \times \text{SU}(2)_2 \times \text{SU}(2)_S) / \mathbb{Z}_2$ gauge theory of **one-band** model

Write fermion operators as 2×2 matrices

$$\Psi_1 = \begin{pmatrix} \Psi_{1\uparrow} & -\Psi_{1\downarrow}^\dagger \\ \Psi_{1\downarrow} & \Psi_{1\uparrow}^\dagger \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \Psi_{2\uparrow} & -\Psi_{2\downarrow}^\dagger \\ \Psi_{2\downarrow} & \Psi_{2\uparrow}^\dagger \end{pmatrix}$$

Constraints $\Psi_\alpha^\dagger \Psi_\alpha = 1$ and $\tilde{\Psi}_\alpha^\dagger \tilde{\Psi}_\alpha = 1$ lead to:

P.A. Lee, N. Nagaosa, and
X.-G. Wen, RMP **78**, 17 (2006)

$$\begin{aligned} \text{SU}(2)_1 : \quad \Psi_1 &\rightarrow \Psi_1 U_1, & \Psi_2 &\rightarrow \Psi_2 \\ \text{SU}(2)_2 : \quad \Psi_1 &\rightarrow \Psi_1, & \Psi_2 &\rightarrow \Psi_2 U_2 \end{aligned}$$

Rung singlet formation $\mathcal{S}_1 + \mathcal{S}_2 \approx 0$ leads to:

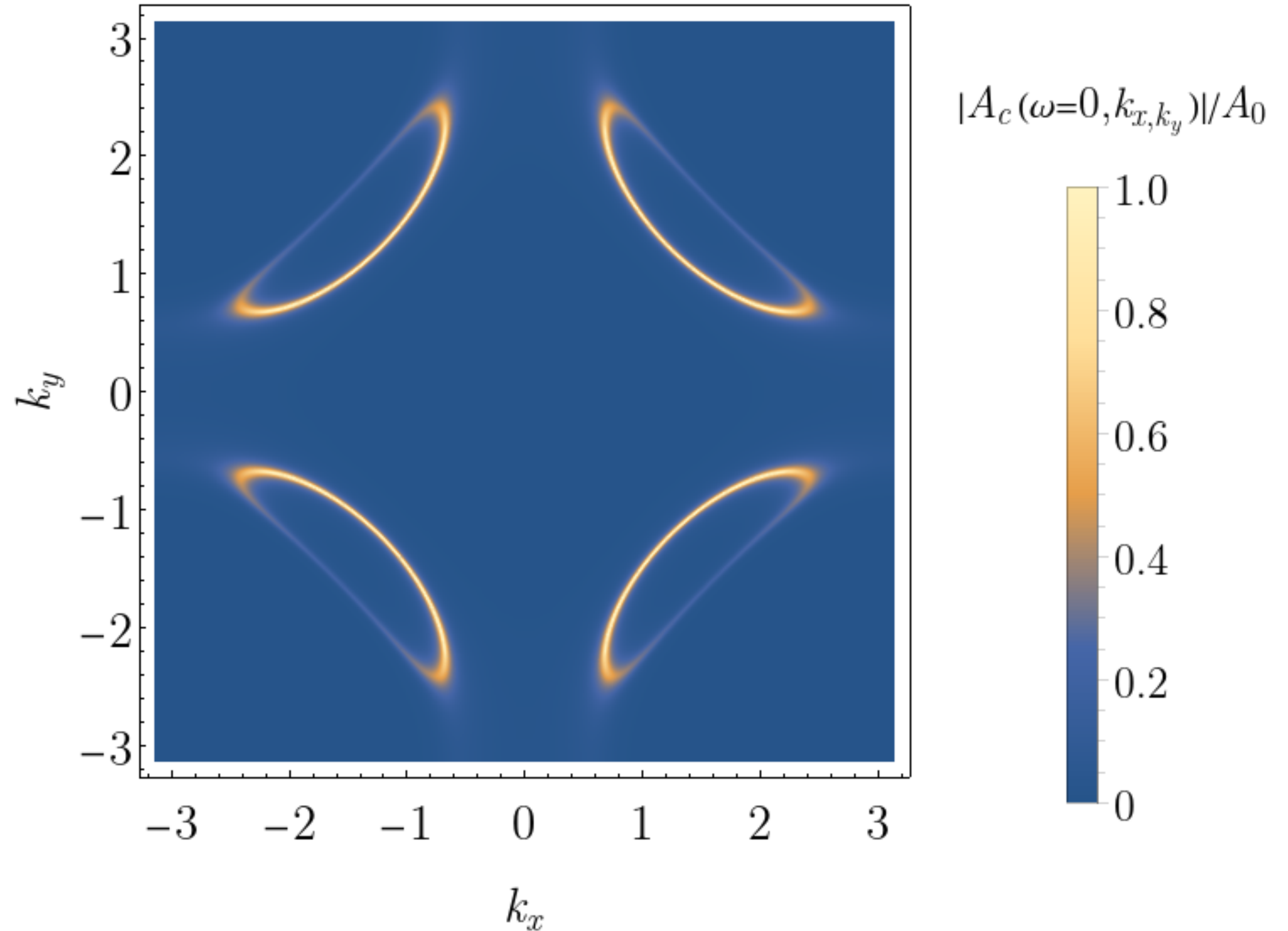
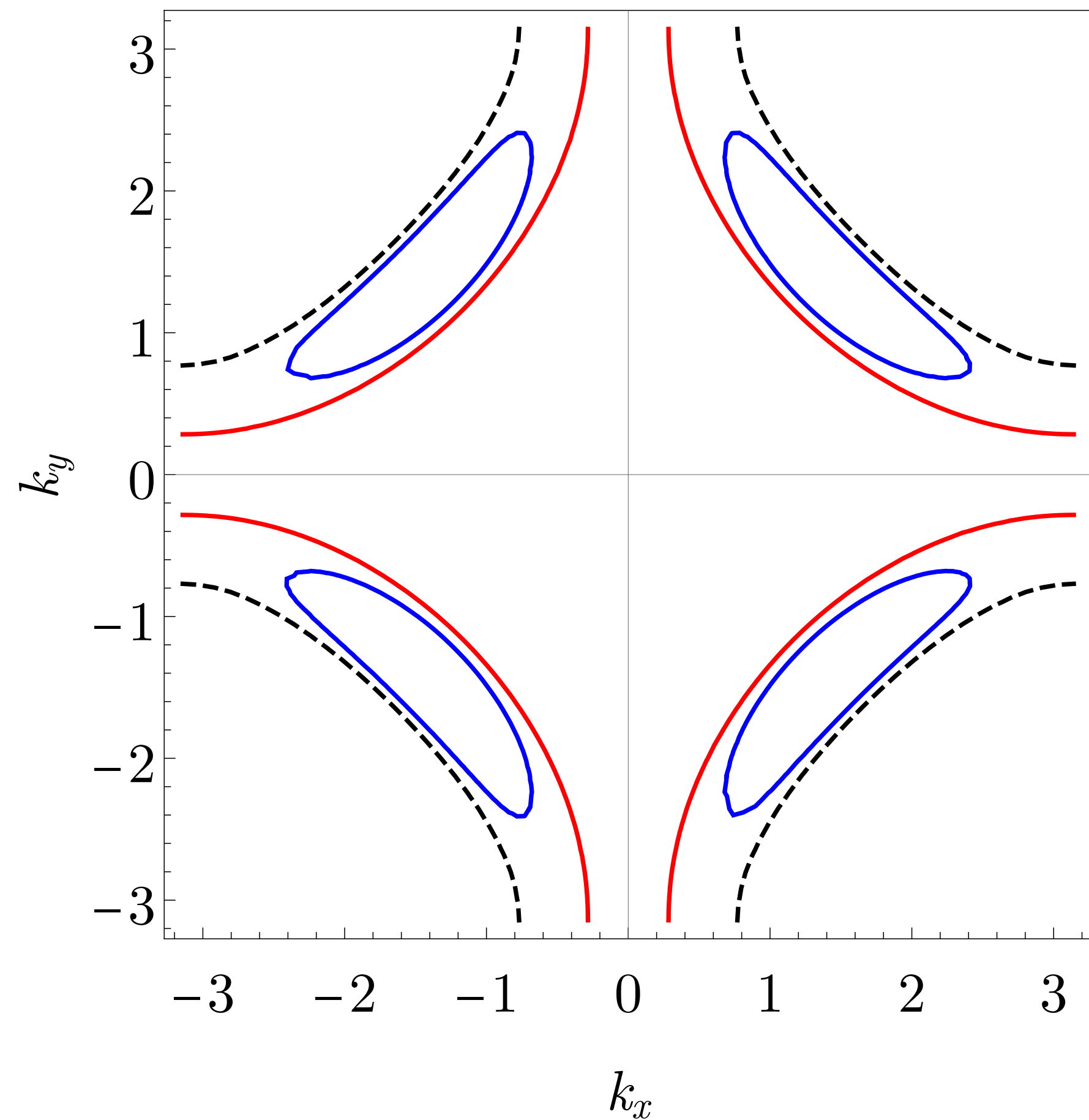
S. Sachdev, M.A. Metlitski, Yang Qi, and
Cenke Xu, PRB **80**, 155129 (2009)

S. Sachdev, H. D. Scammell, M. S. Scheurer,
and G. Tarnopolsky, PRB **99**, 054516 (2019)

$$\text{SU}(2)_S : \quad \Psi_1 \rightarrow U_S \Psi_1, \quad \Psi_2 \rightarrow U_S \Psi_2$$

FL* in a **one-band** model

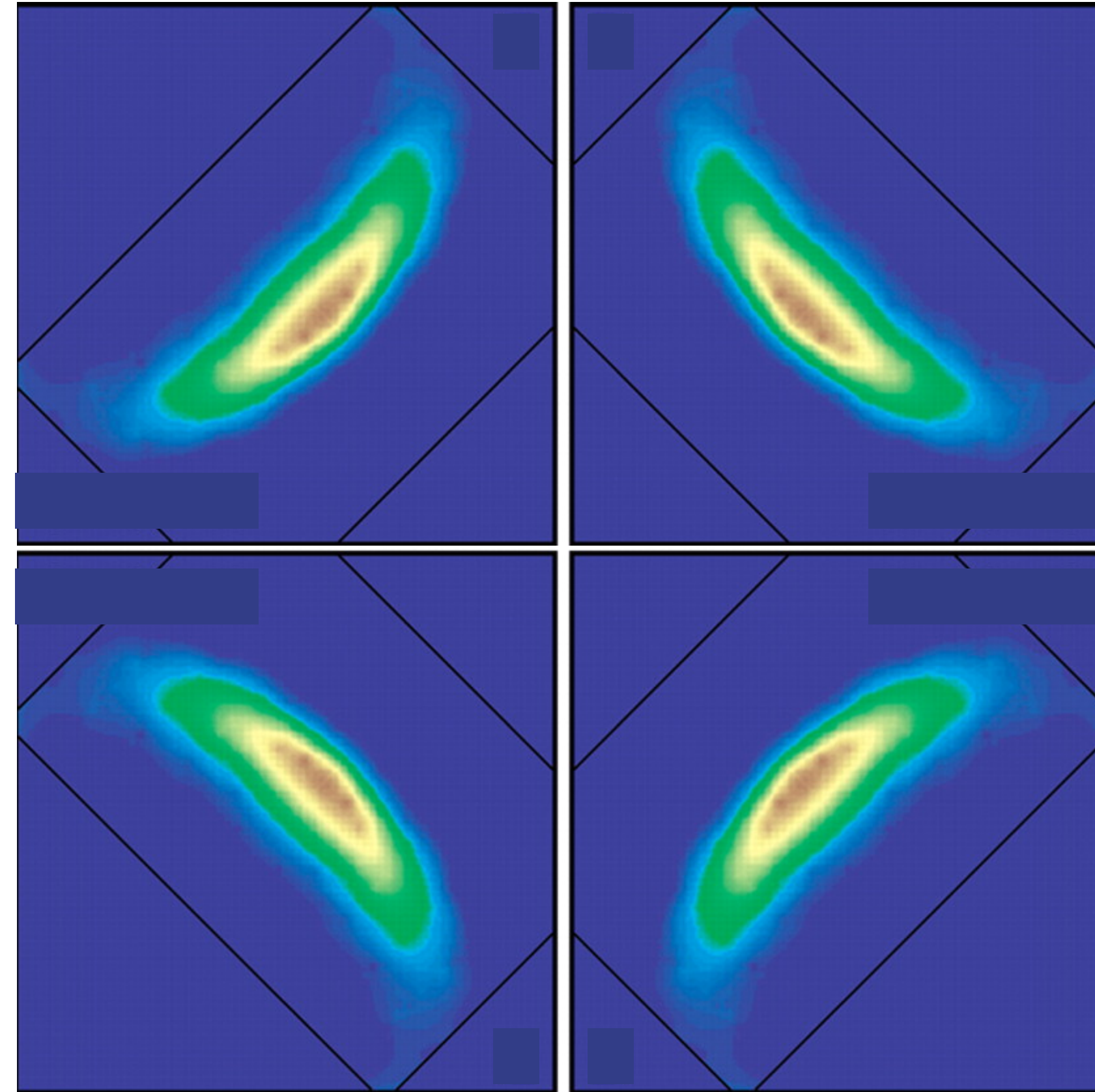
“Fermi arc” spectral functions



FL* Hamiltonian: $[(\text{SU}(2)_1 \times \text{SU}(2)_S)/\mathbb{Z}_2] \times \text{U}(1)_{\text{em}}$ is broken to $\text{U}(1)_{\text{diag}}$ by Higgs condensate Φ :

$$H = - \sum_{i,j} t_{ij} c_{i;\alpha}^\dagger c_{j;\alpha} + \sum_{i,j} t_{1,ij} \Psi_{1i;\alpha}^\dagger \Psi_{1j;\alpha} + \sum_i \Phi (c_{i;\alpha}^\dagger \Psi_{1i;\alpha} + \Psi_{1i;\alpha}^\dagger c_{i;\alpha})$$

Photoemission at small p



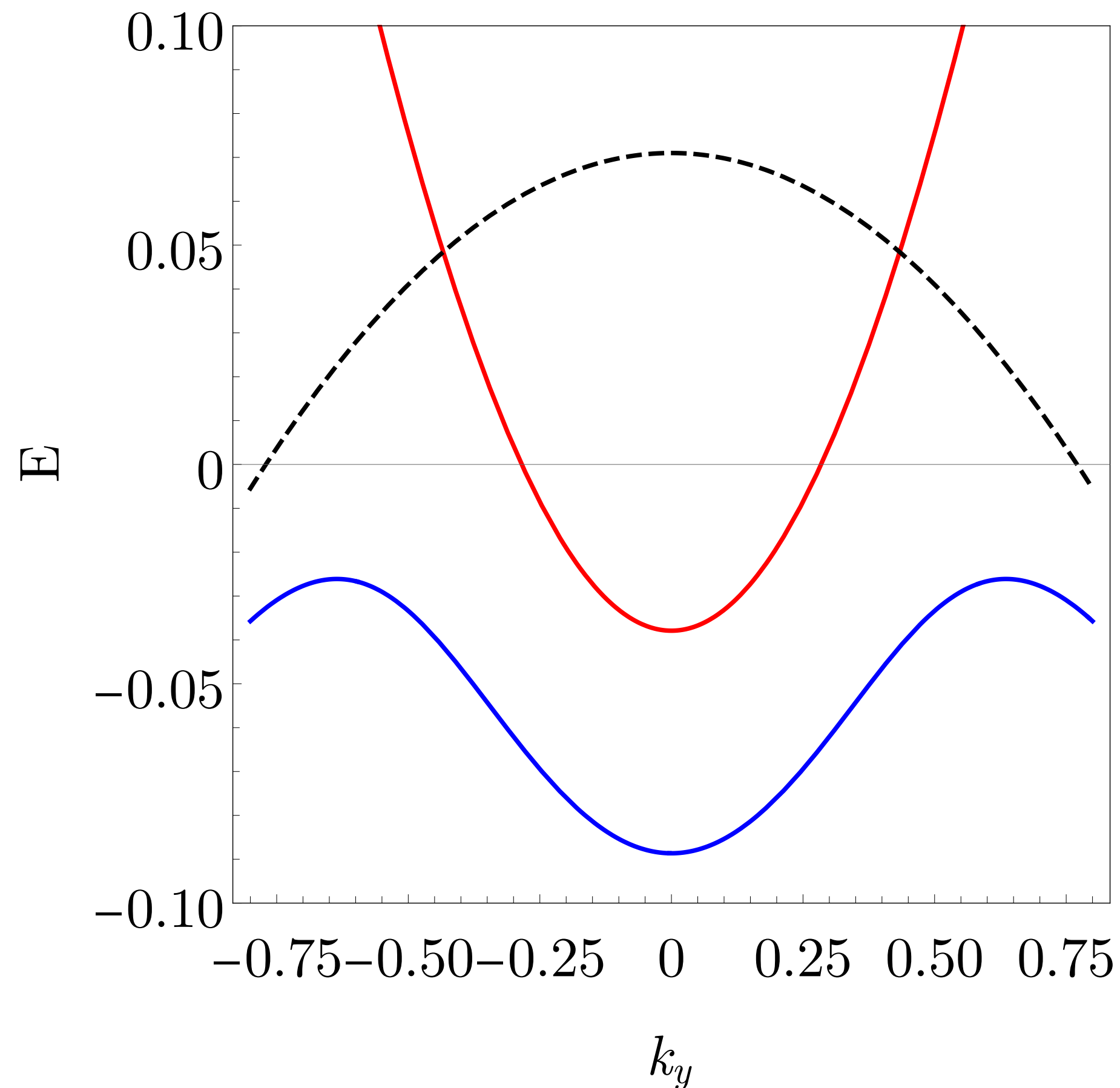
$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$
at $x = 0.10$

“Fermi arcs”

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

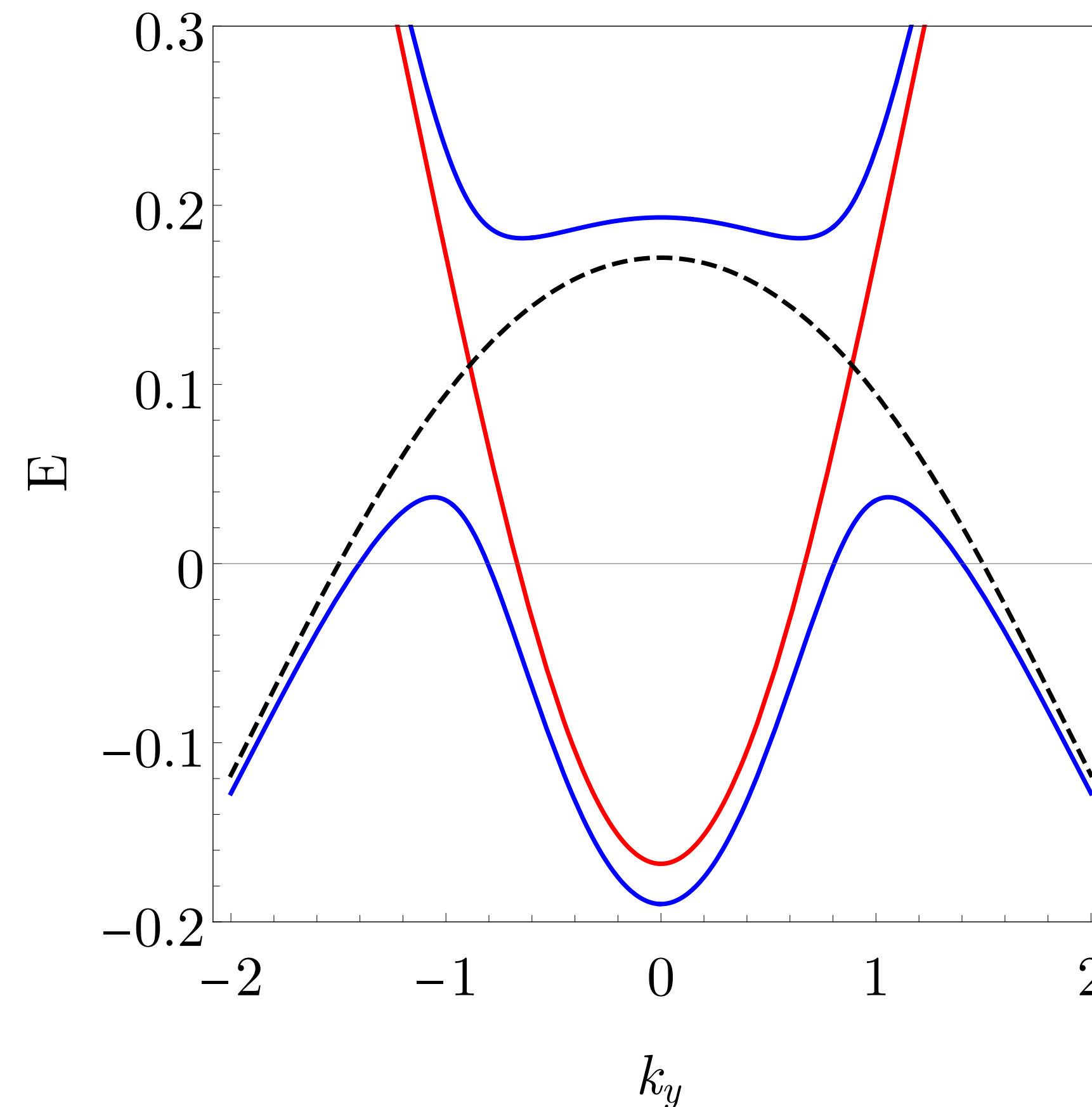
FL* in a **one-band** model

Anti-node: $k_x = \pi$



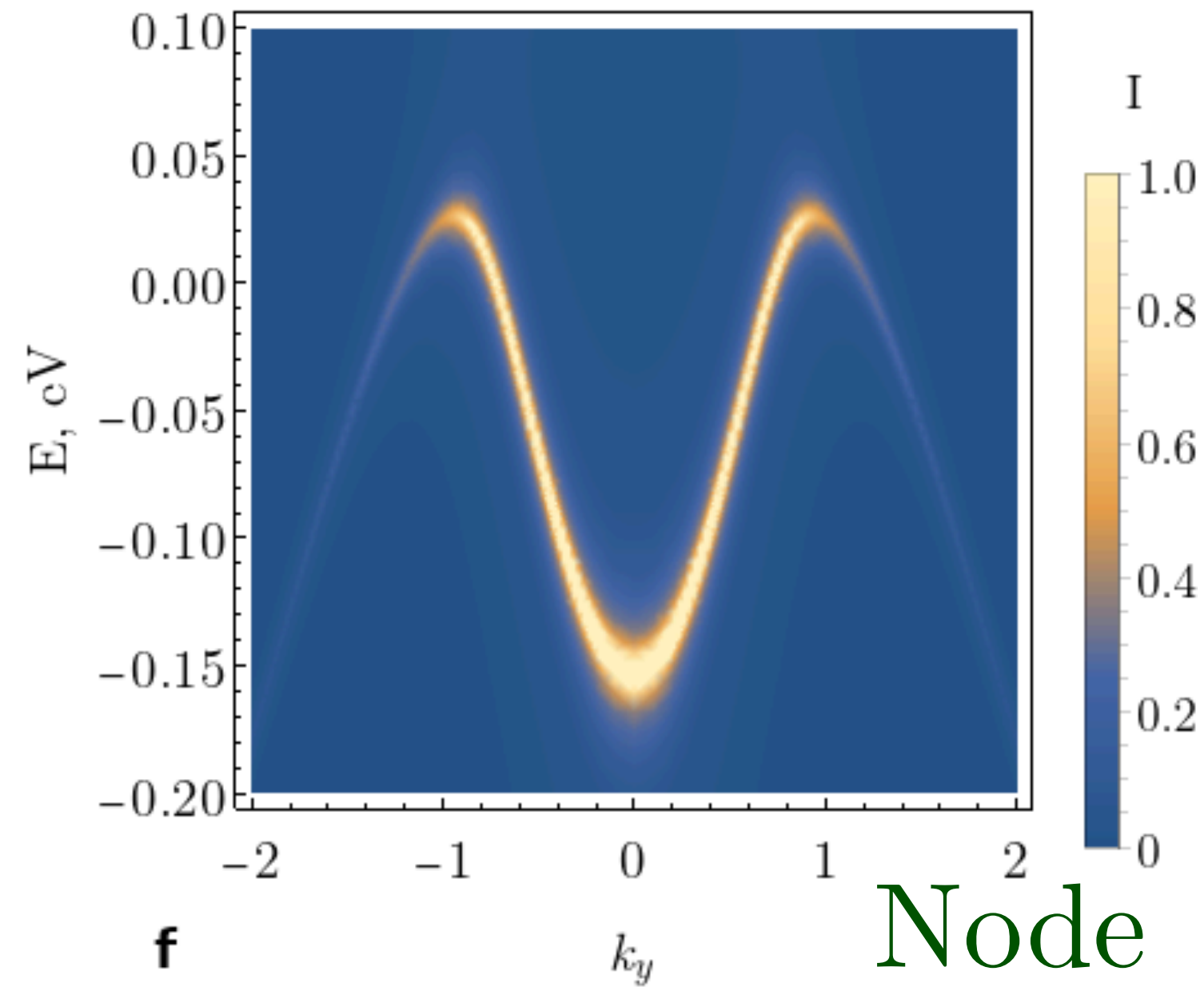
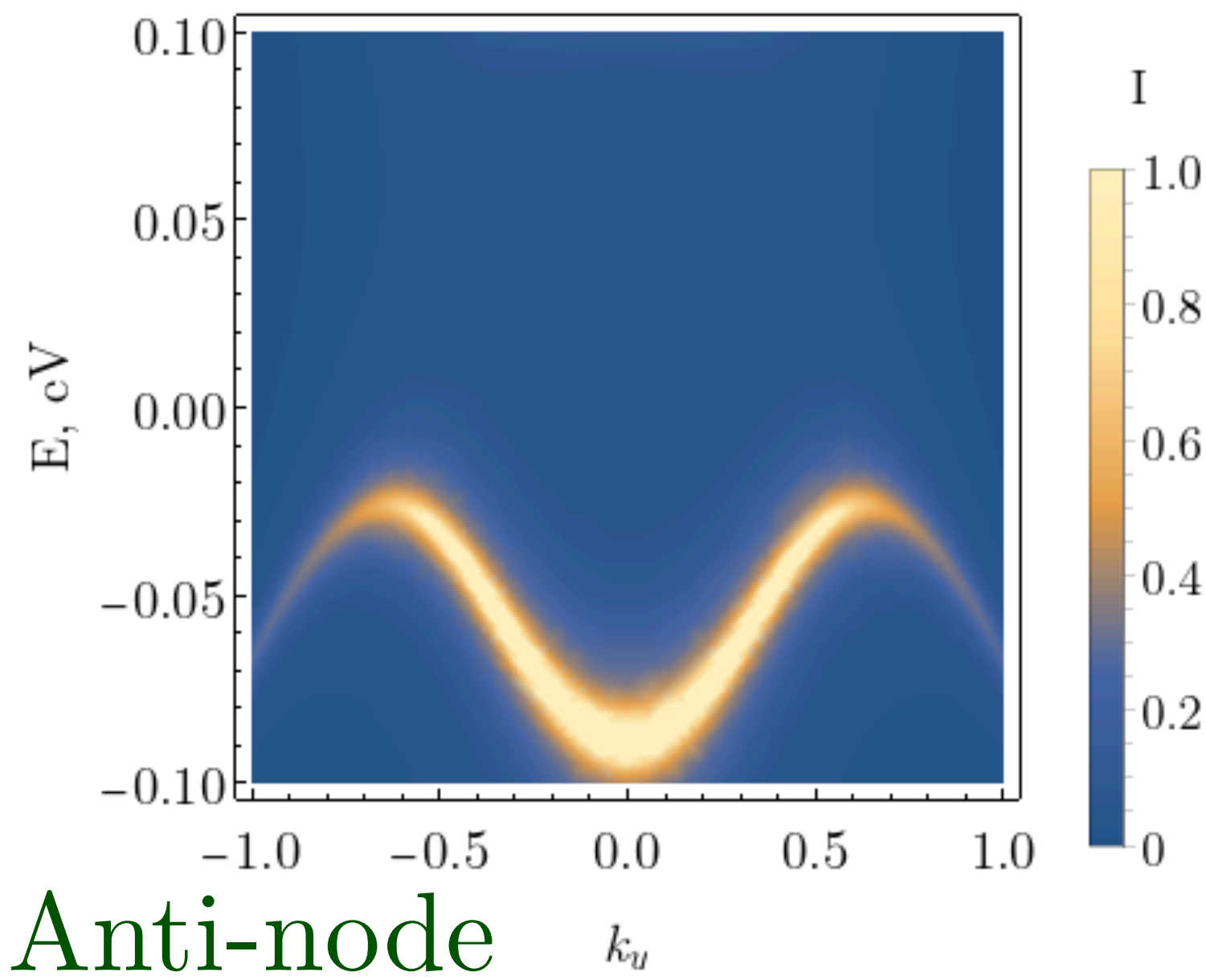
Electronic dispersion

Node: $k_x = 2$

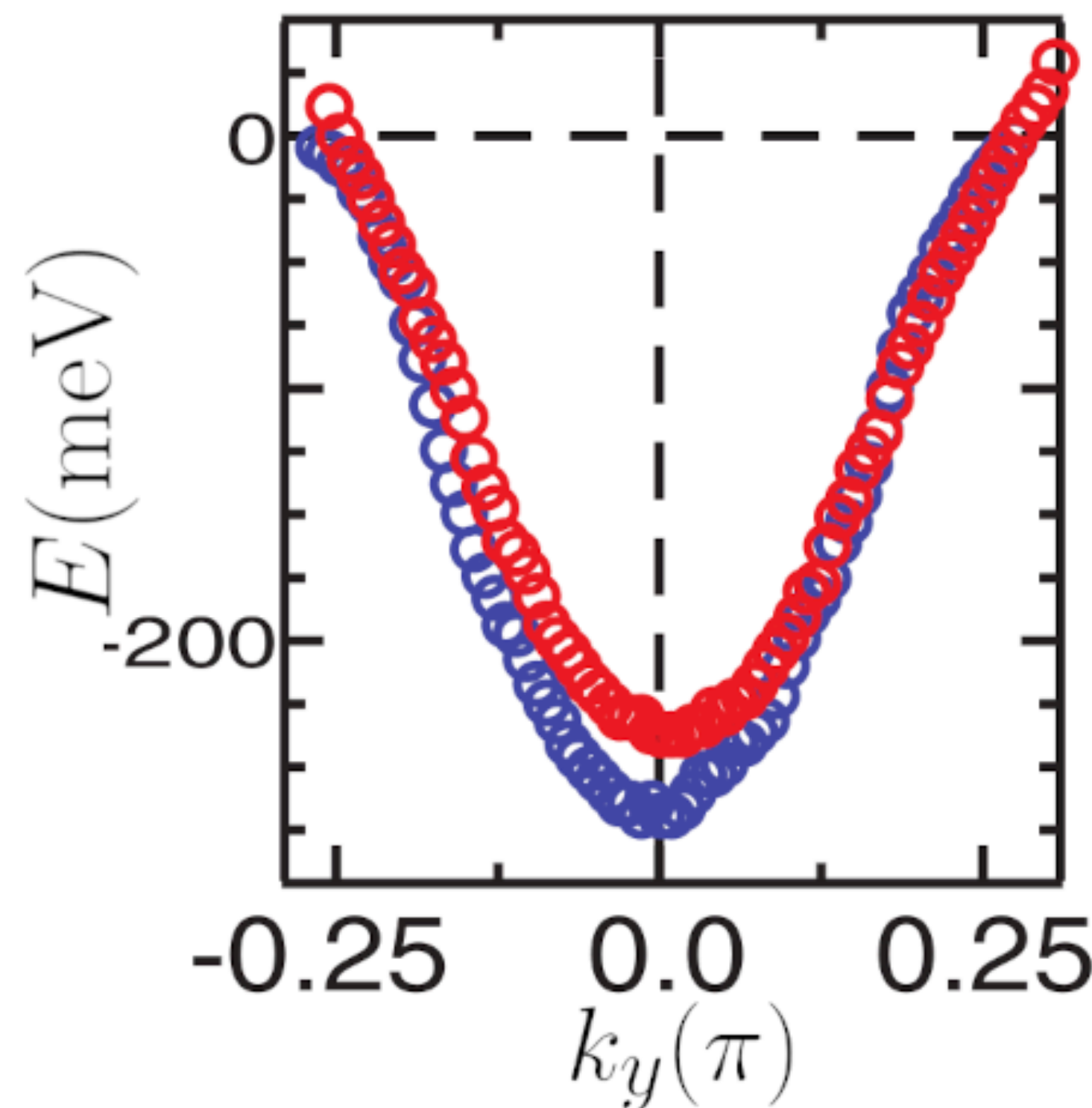
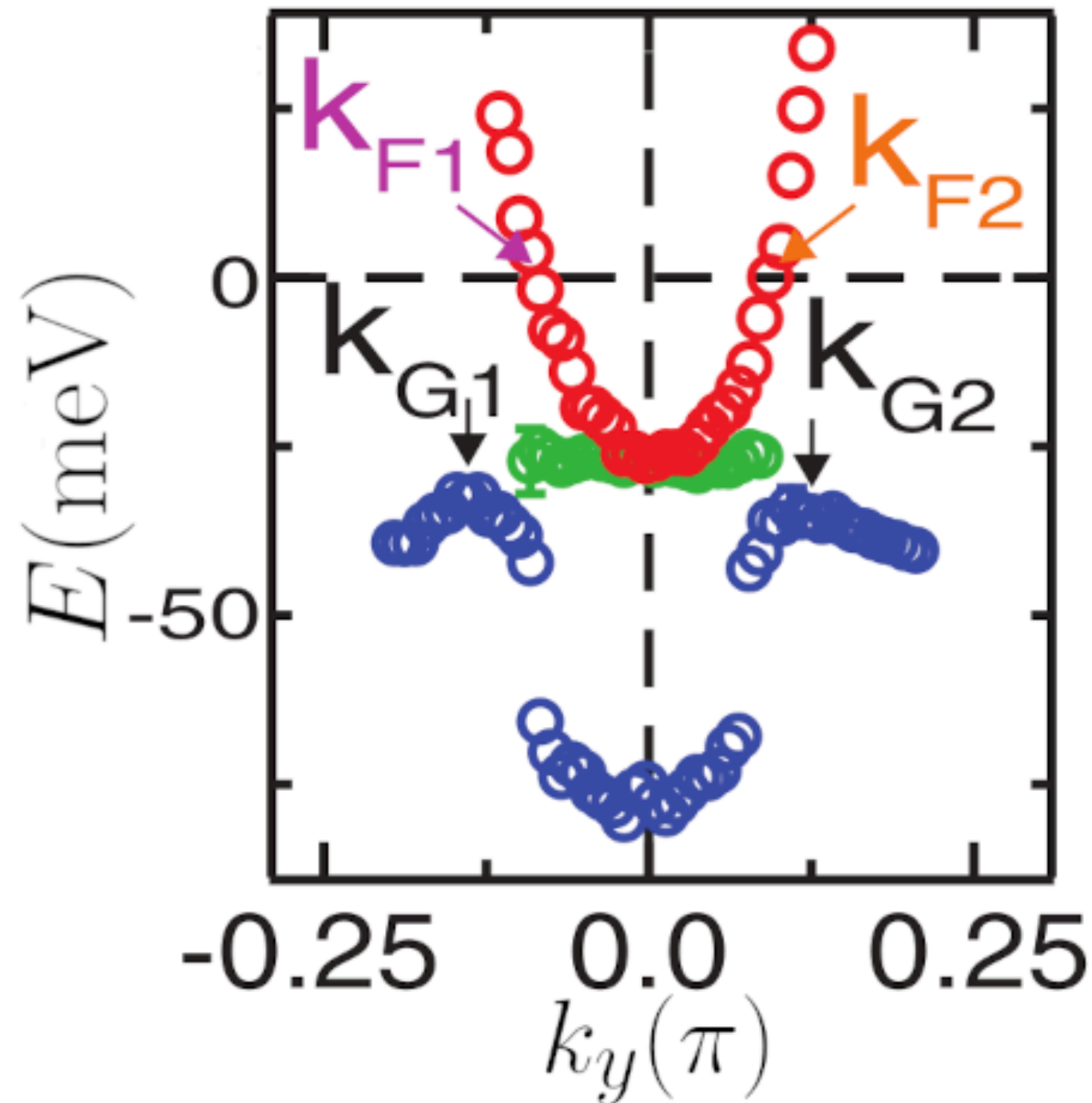


FL* Hamiltonian: $[(SU(2)_1 \times SU(2)_S) / \mathbb{Z}_2] \times U(1)_{em}$ is broken to $U(1)_{diag}$ by Higgs condensate Φ :

$$H = - \sum_{i,j} t_{ij} c_{i;\alpha}^\dagger c_{j;\alpha} + \sum_{i,j} t_{1,ij} \Psi_{1i;\alpha}^\dagger \Psi_{1j;\alpha} + \sum_i \Phi (c_{i;\alpha}^\dagger \Psi_{1i;\alpha} + \Psi_{1i;\alpha}^\dagger c_{i;\alpha})$$



FL* in a **one-band** model



ARPES on Bi2201

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

1. Spin liquids and violations of the Luttinger relation:

FL* and HFL phases of the Kondo lattice model.

2. Hubbard model - the vanilla FL phase

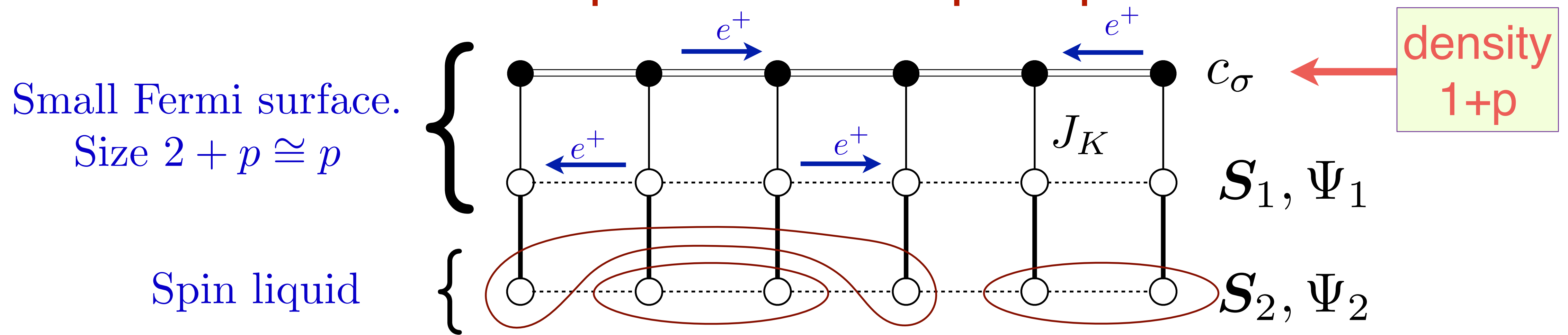
3. Hubbard model - the FL* phase:

fractionalizing the paramagnon

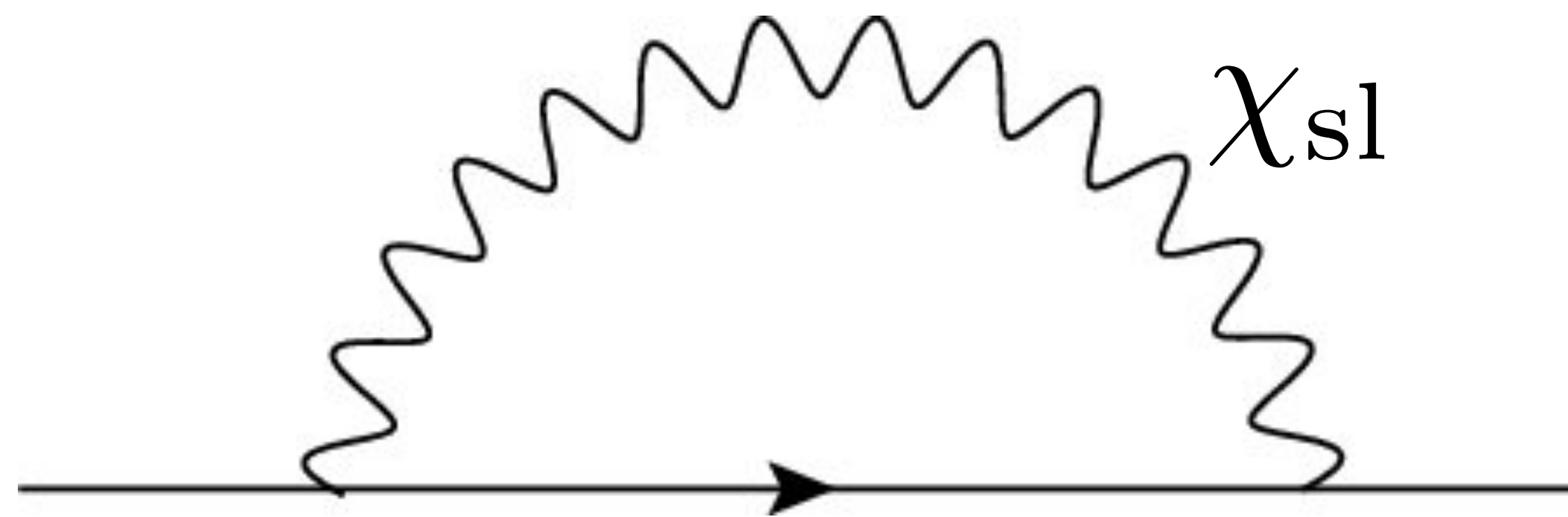
4. Hubbard model - nature of spin liquid

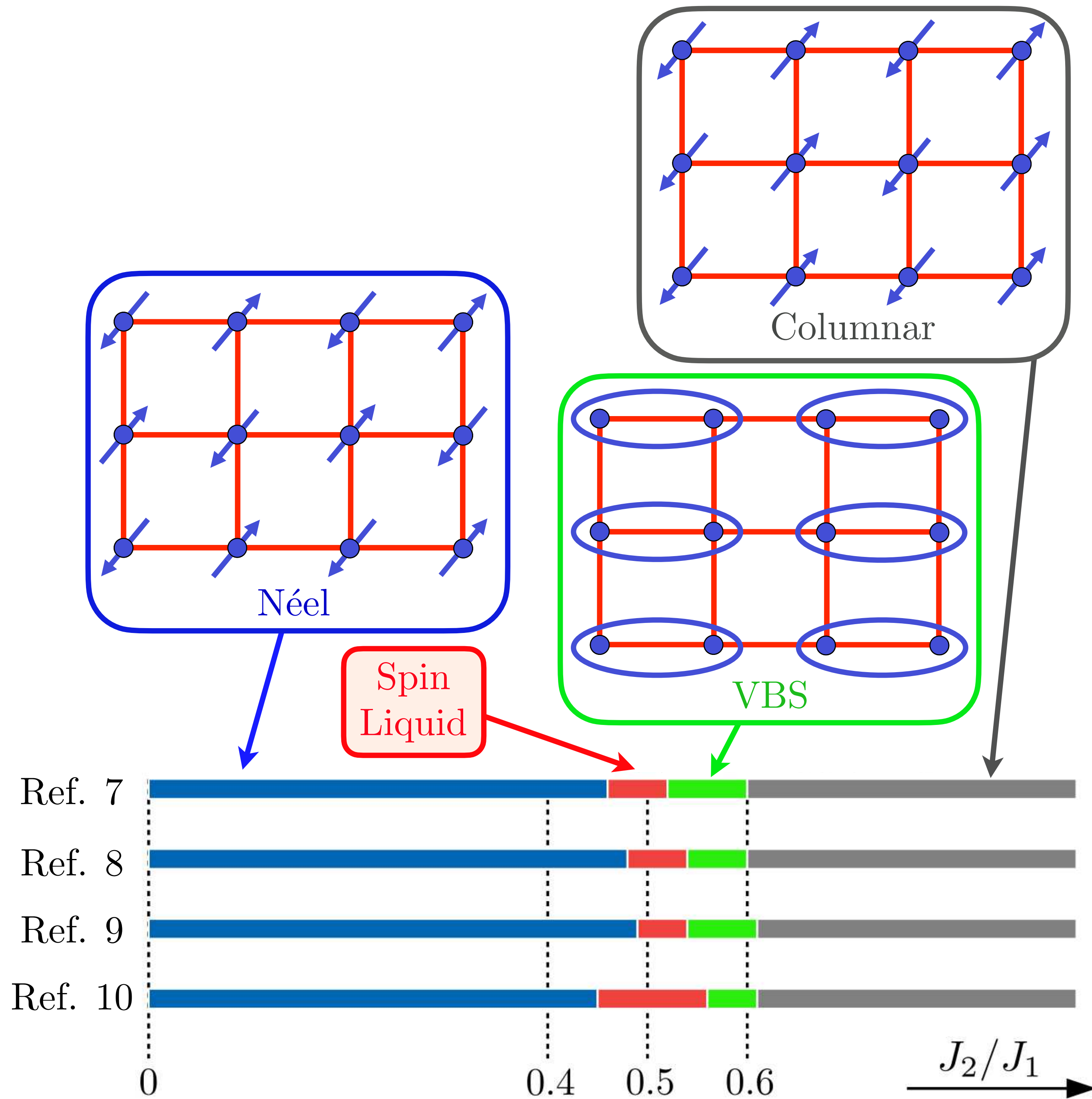
5. Quantum criticality

Consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2 . We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility χ_{sl} .



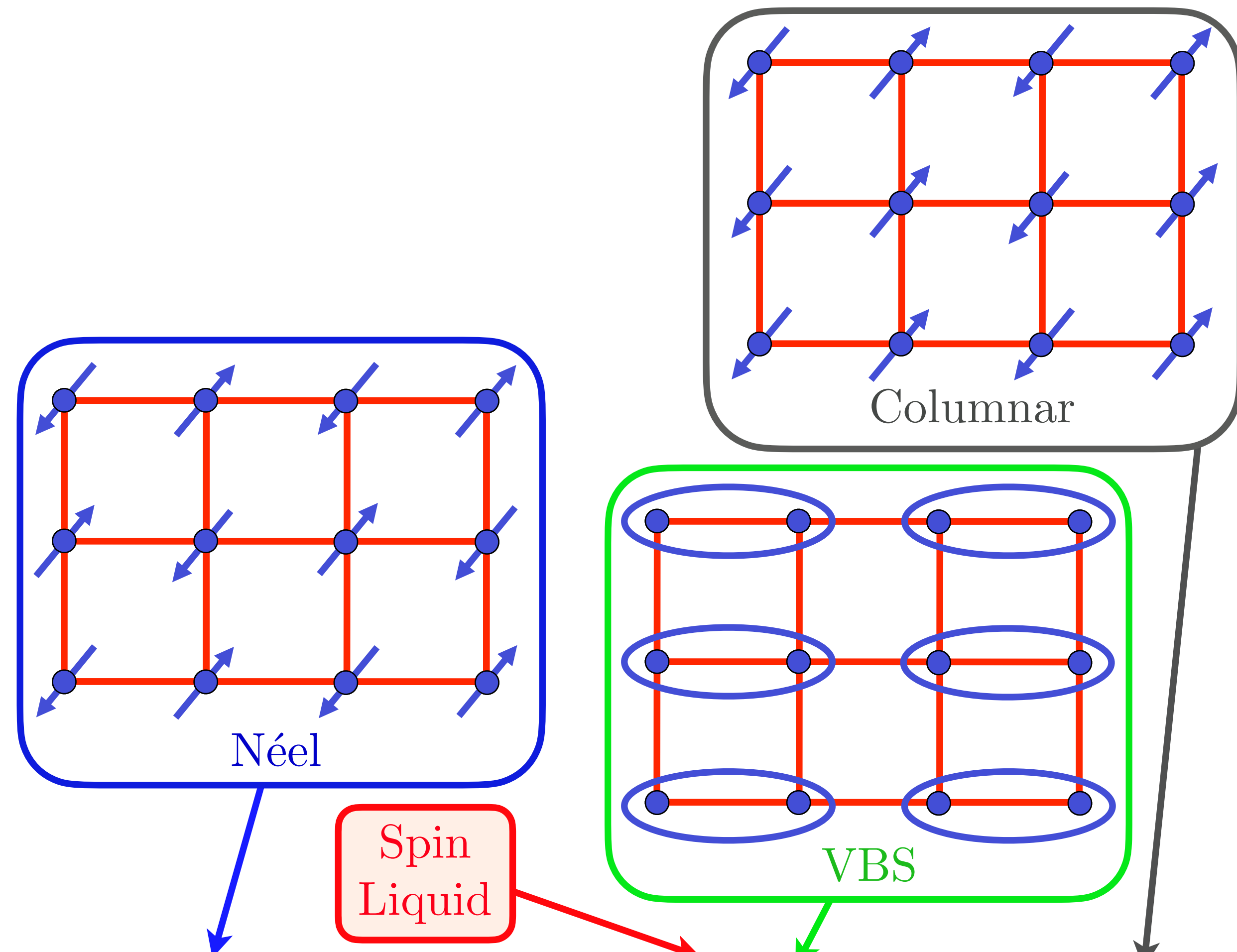


[7] L. Wang and A. W. Sandvik, Phys. Rev. Lett. **121**, 107202 (2018).

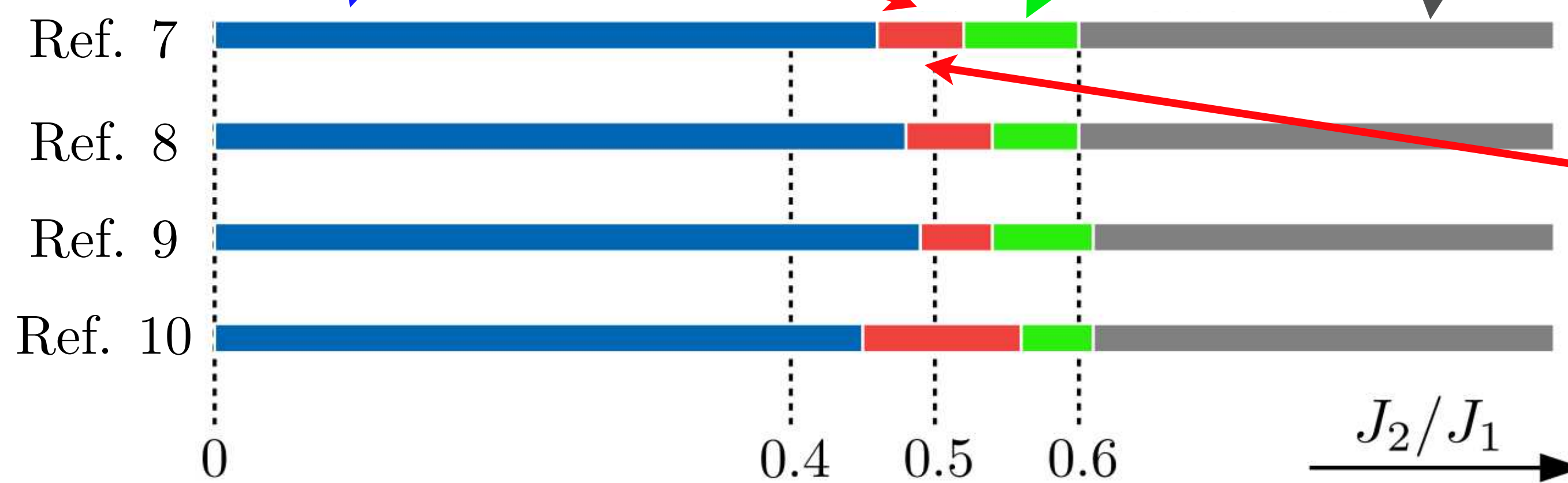
[8] F. Ferrari and F. Becca, Phys. Rev. B **102**, 014417 (2020).

[9] Y. Nomura and M. Imada, Phys. Rev. X **11**, 031034 (2021).

[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, arXiv:2009.01821.

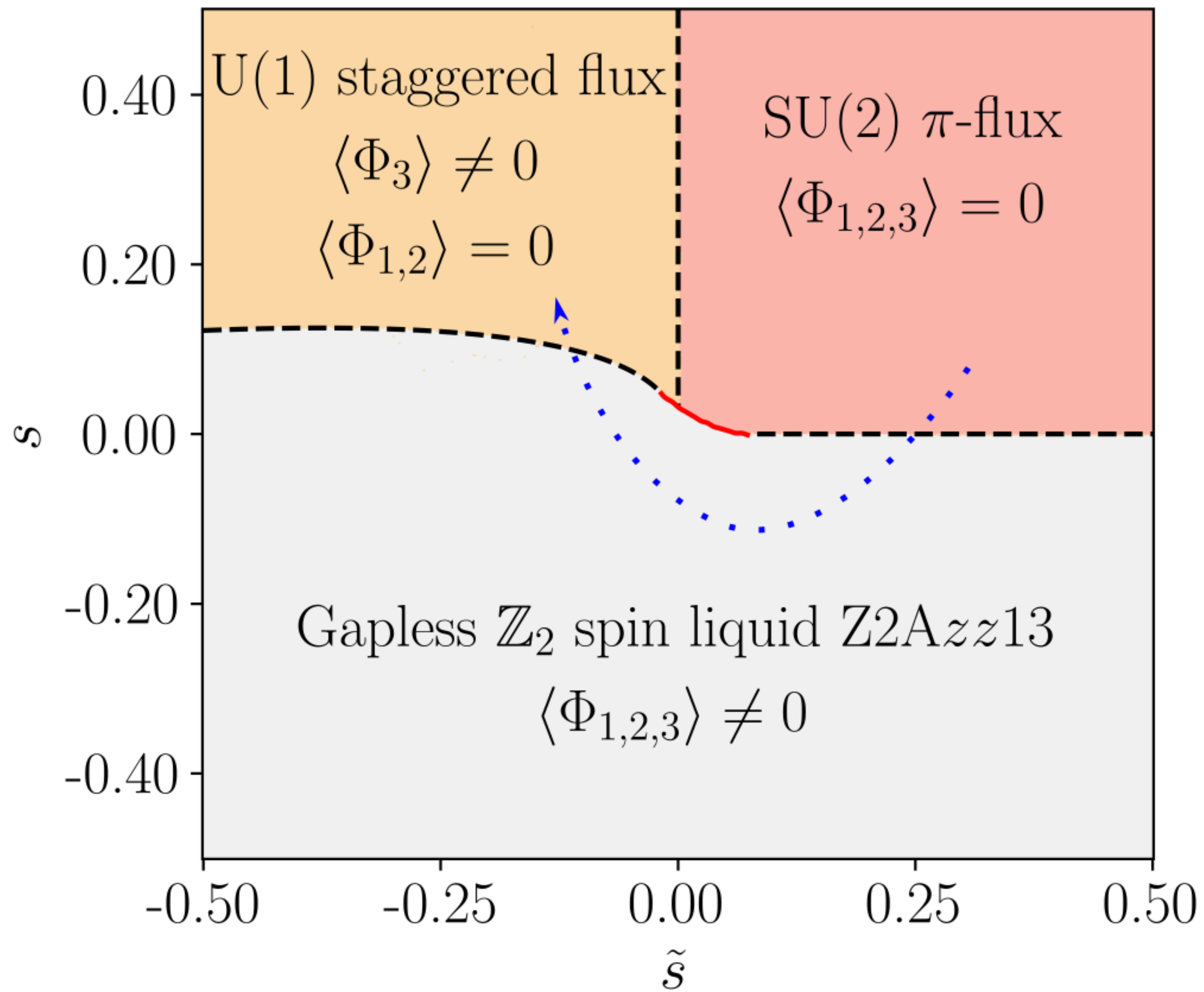


- [7] L. Wang and A. W. Sandvik, Phys. Rev. Lett. **121**, 107202 (2018).
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- [10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, arXiv:2009.01821.



Gapless
 Z_2 spin liquid
phase !

Wen-Jun Hu,
Federico Becca,
Alberto Parola,
Sandro Sorella,
PRB **88**, 060402(R) (2013).



**Henry
Shackleton**



**Alex
Thomson**
PRB **104**, 045110 (2021)

SU(2) gauge theory for transition from π -flux to gapless \mathbb{Z}_2 spin liquid:

Condense *two* adjoint Higgs scalars, Φ_1^a, Φ_2^a without breaking any symmetry.

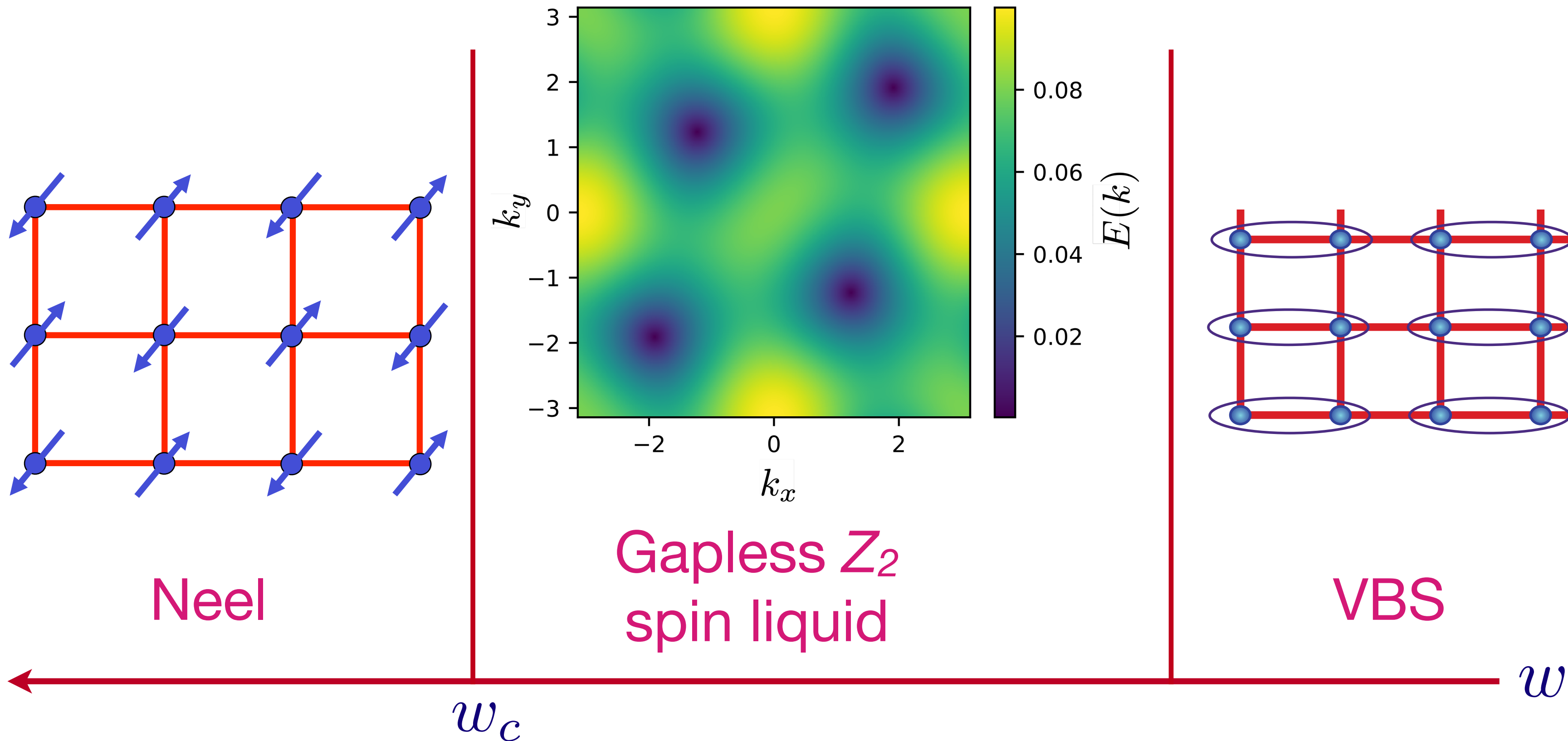
$$\mathcal{S} = \int d^2x d\tau \left[i\bar{\psi}\gamma_\mu (\partial_\mu - ia_\mu^a \sigma^a) \psi + \dots \right. \\ \left. + \lambda (\Phi_1^a \bar{\psi} \mu^z \gamma^x \sigma^a \psi + \Phi_2^a \bar{\psi} \mu^x \gamma^y \sigma^a \psi) + \frac{w}{2} (\Phi_1^a \Phi_1^a + \Phi_2^a \Phi_2^a) + \dots \right]$$



Henry
Shackleton



Alex
Thomson
PRB 104, 045110 (2021)



SU(2) gauge theory for transition from π -flux to gapless \mathbb{Z}_2 spin liquid:

Condense *two* adjoint Higgs scalars, Φ_1^a, Φ_2^a without breaking any symmetry.

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This theory has an emergent subsystem symmetries:

$$\begin{aligned} \psi &\rightarrow \exp(i f_a(x) \mu^z \sigma^a) \psi, & \Phi_{1a} &\rightarrow \Phi_{1a} + \partial_x f_a, \\ \psi &\rightarrow \exp(i g_a(y) \mu^x \sigma^a) \psi, & \Phi_{2a} &\rightarrow \Phi_{2a} + \partial_y g_a, \end{aligned}$$

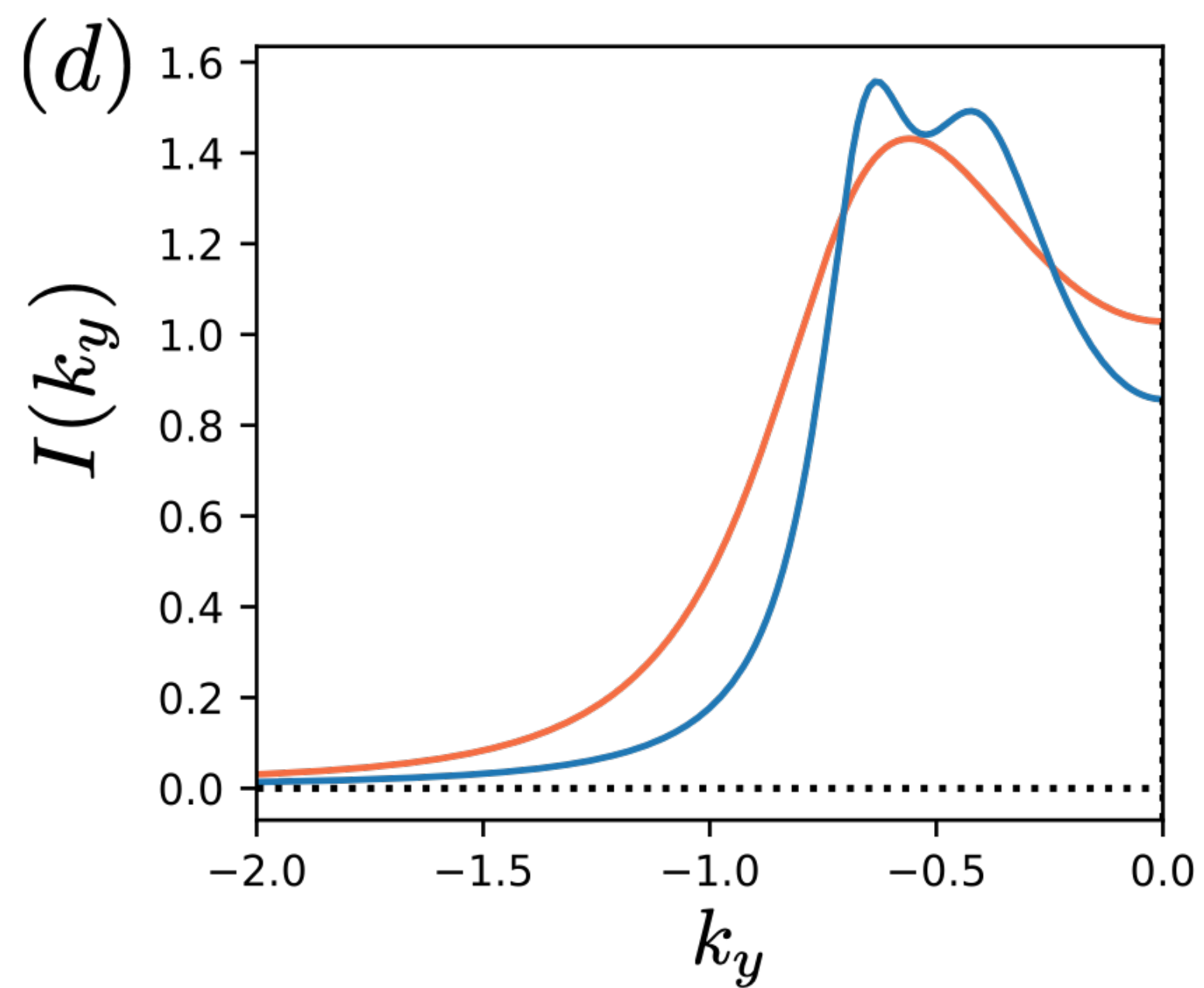
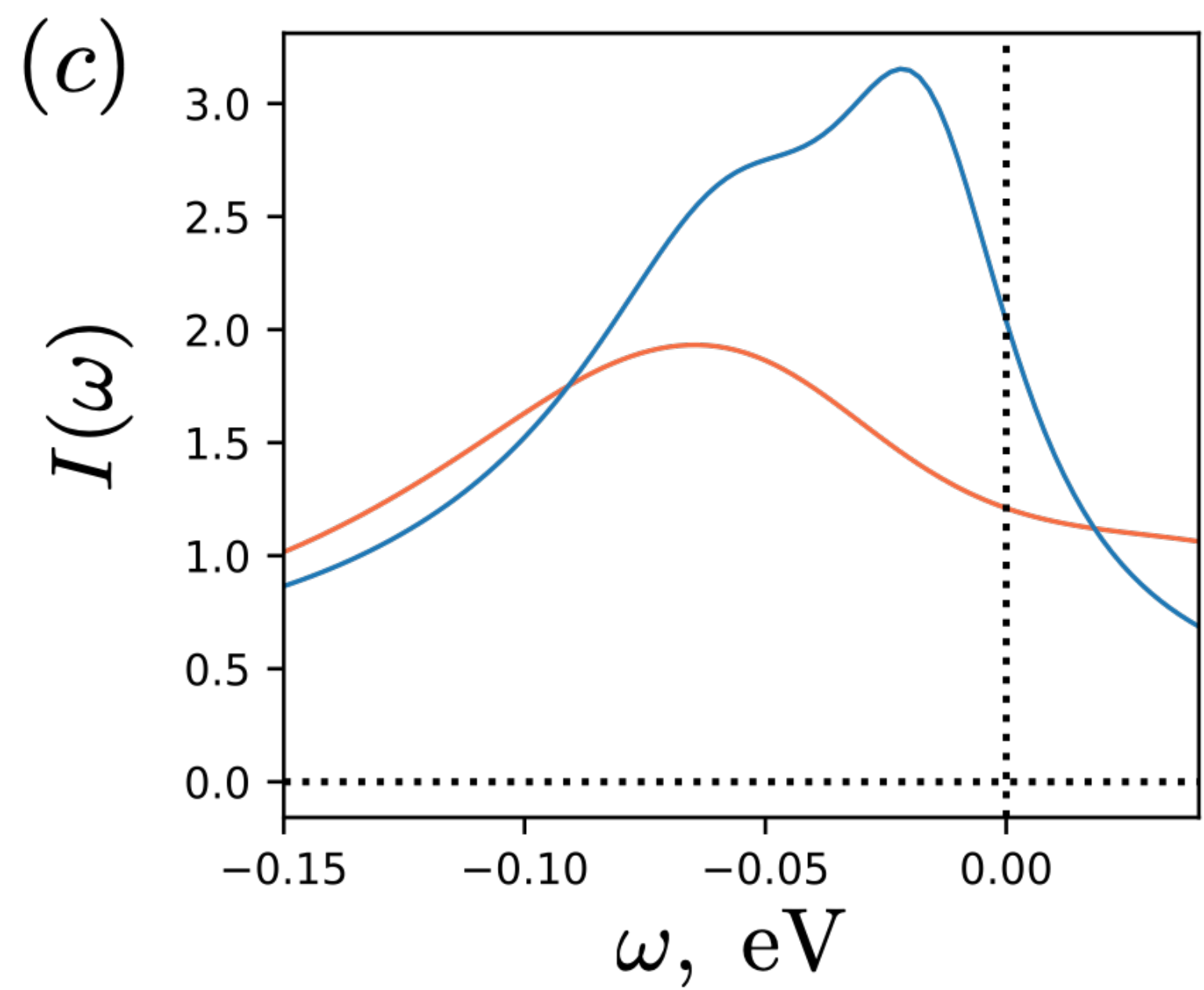
which leads to anomalous ‘log²’ scaling.



Henry
Shackleton

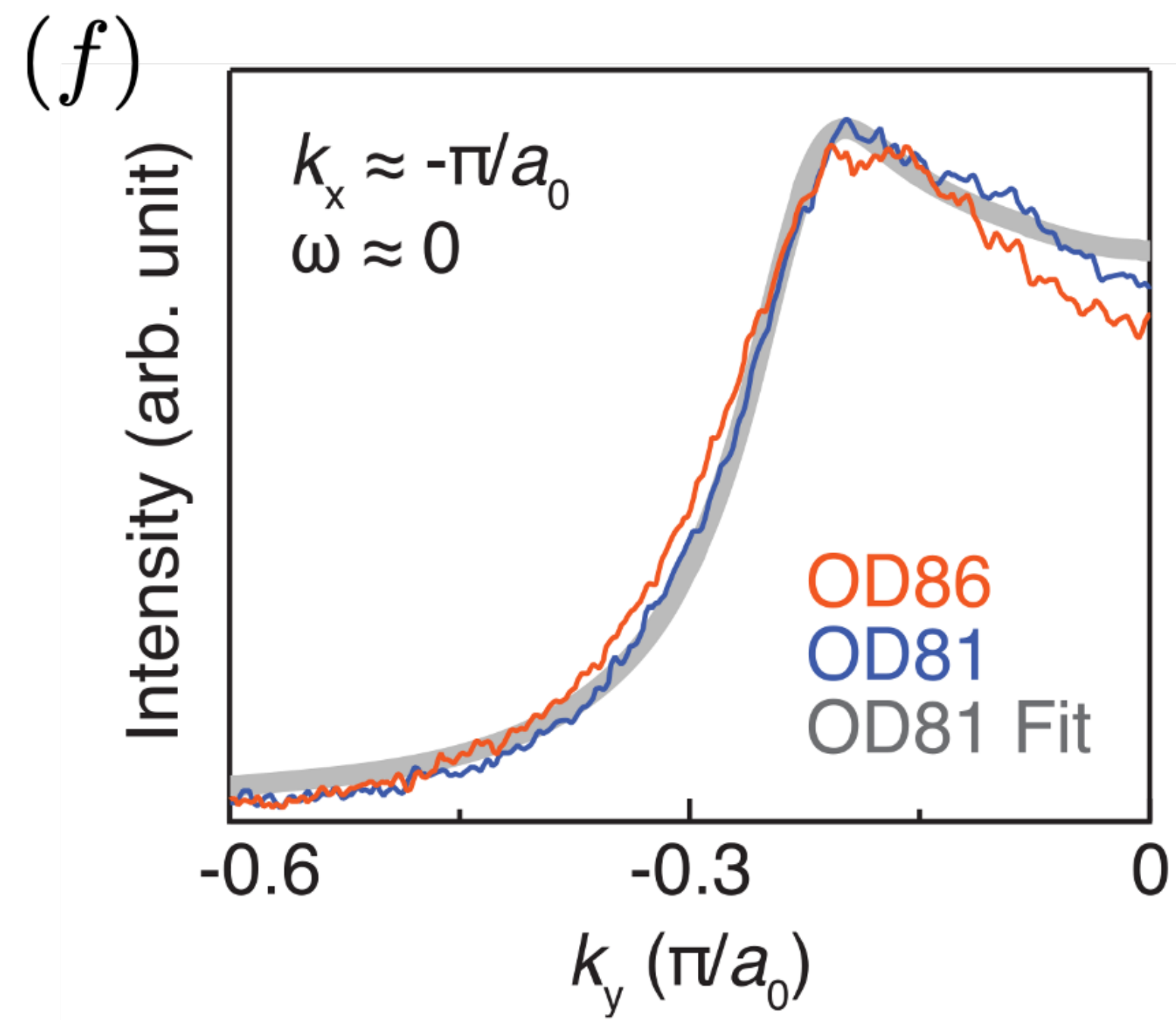
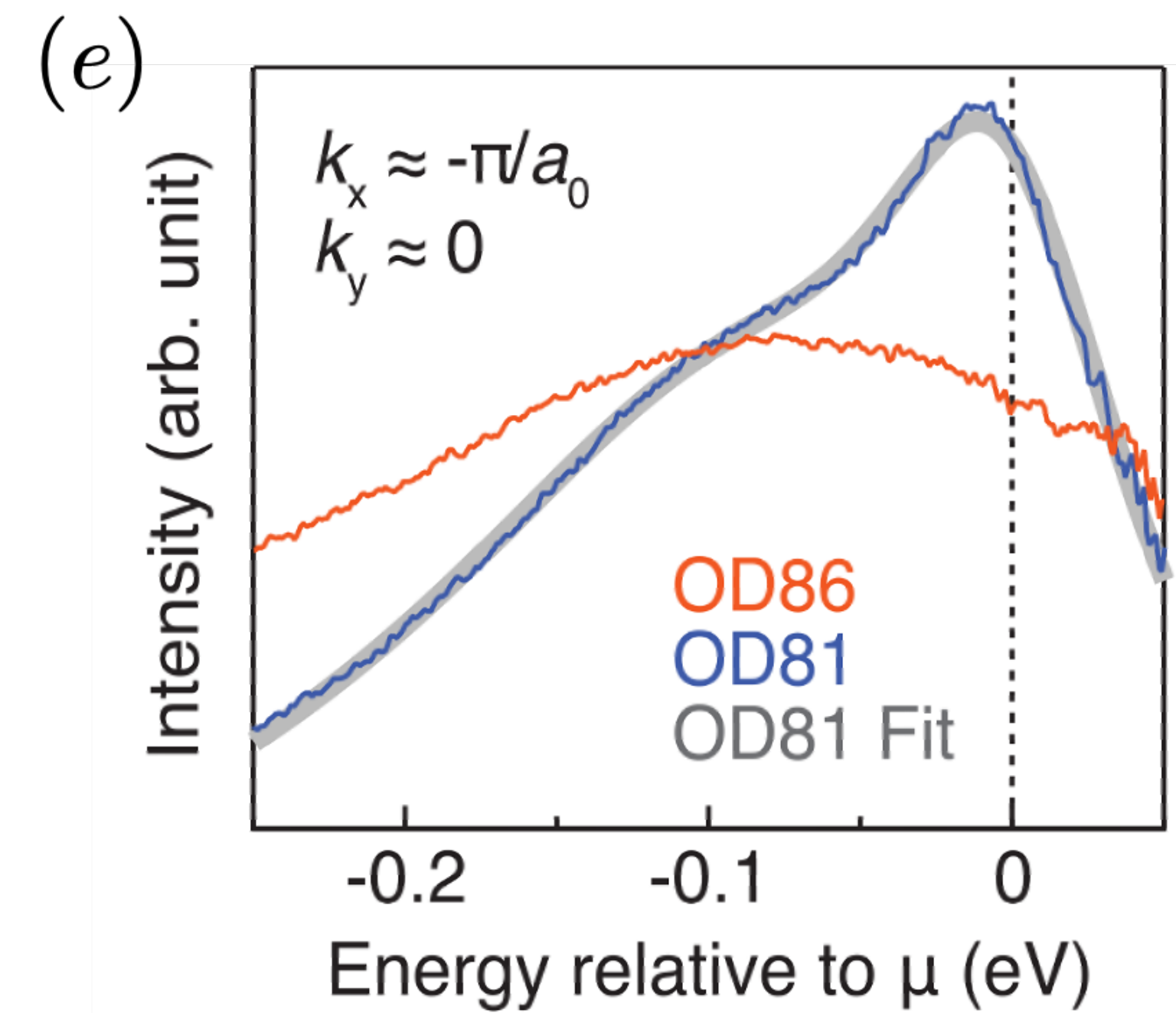


Alex
Thomson
PRB **104**, 045110 (2021)



Antinodal EDC and MDC

(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless \mathbb{Z}_2 spin liquid



(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

1. Spin liquids and violations of the Luttinger relation:

FL* and HFL phases of the Kondo lattice model.

2. Hubbard model - the vanilla FL phase

3. Hubbard model - the FL* phase:

fractionalizing the paramagnon

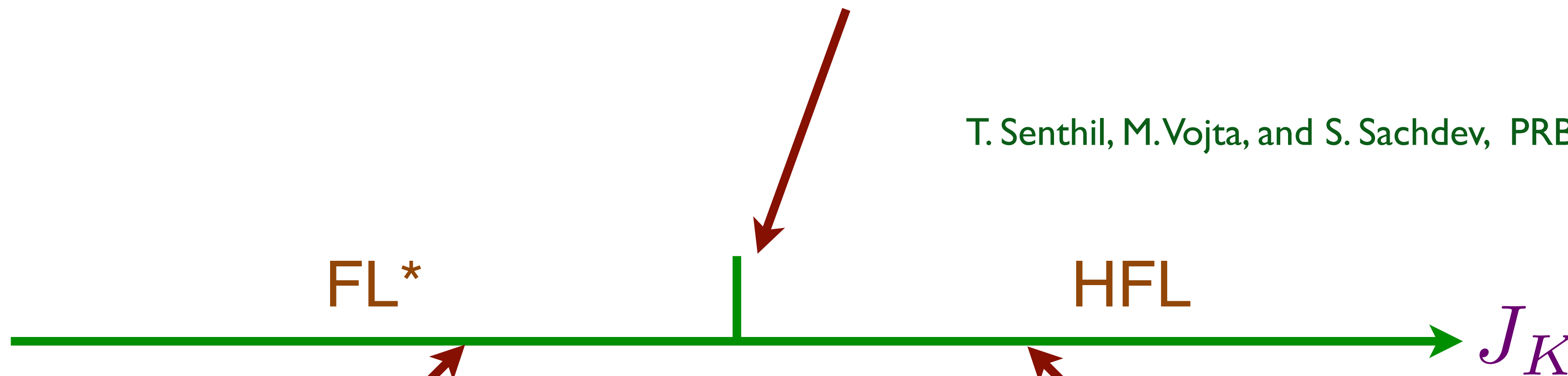
4. Hubbard model - nature of spin liquid

5. Quantum criticality

Kondo lattice

Deconfined criticality of a U(1) gauge theory with a Higgs field, spinons, and a small Fermi surface of electrons.
(FL* can be replaced by a confining phase with AFM or VBS order).

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)



Small Fermi surface of size p

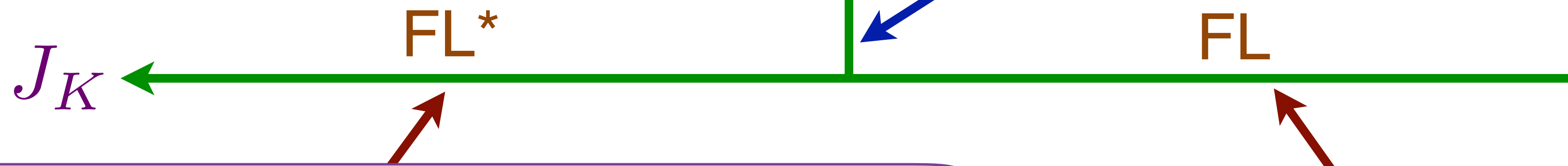
$|\text{FL}^*\rangle = [\text{Projection onto one } f \text{ per site}]$
 $\boxtimes |\text{Slater determinant of } f\rangle$
 $\otimes |\text{Slater determinant of } c\rangle$

Large Fermi surface of size $1 + p$

$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}]$
 $\boxtimes |\text{Slater determinant of } (c, f)\rangle$

Ancilla theory of the Hubbard model

- Deconfined criticality of a $(\text{SU}(2)_S \times \text{U}(1)_1)/\mathbb{Z}_2$ gauge theory.
- ‘Hybridization-Higgs’ boson $\sim C_\sigma^\dagger \Psi_a$ which condenses on the FL* side (in Kondo lattice, Higgs boson was condensed on the FL side).
- Gauge-charged ‘ghost’ Fermi surface of Ψ_1 fermions.
- Large Fermi surface of c_σ gauge-neutral electrons.



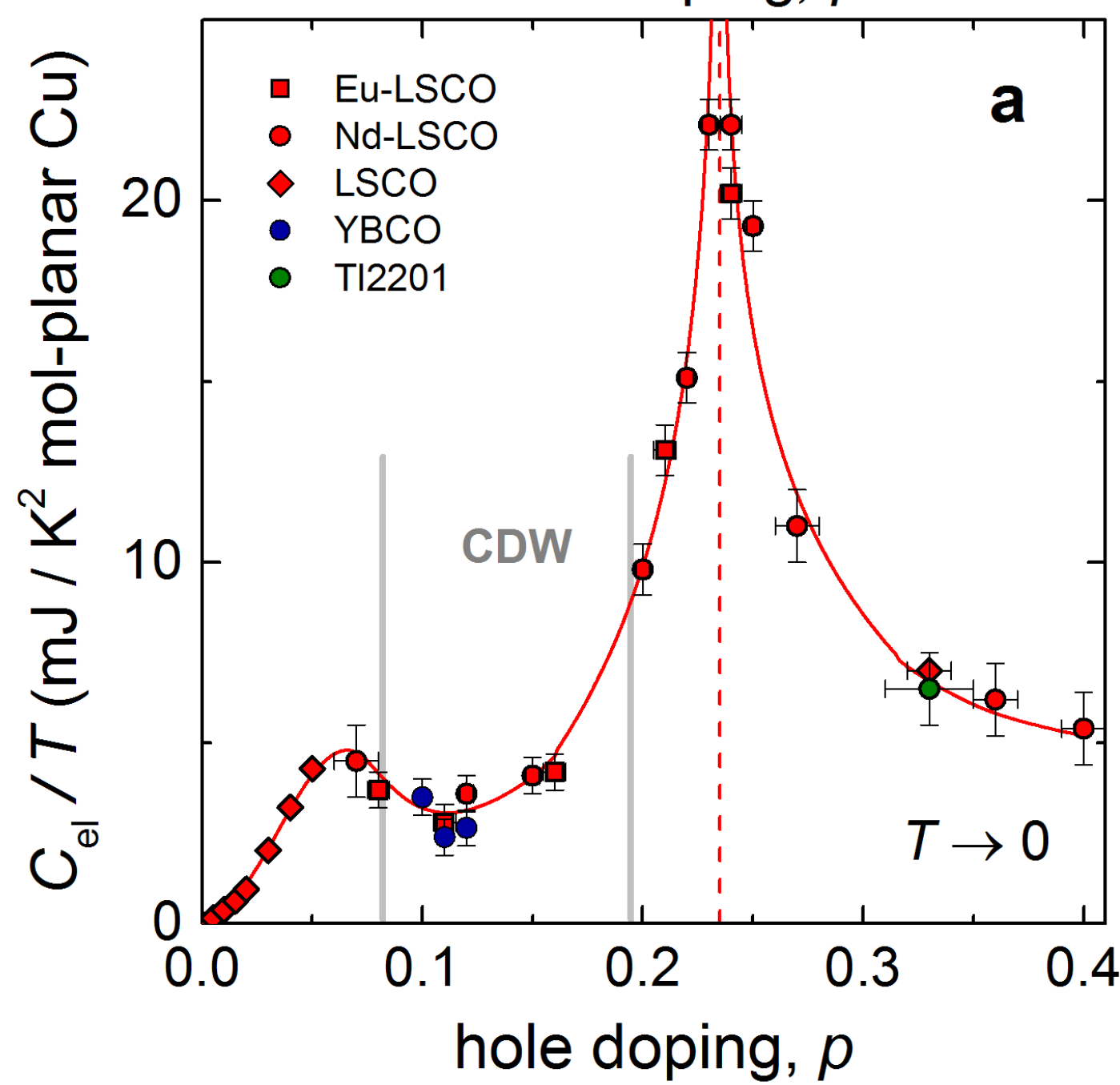
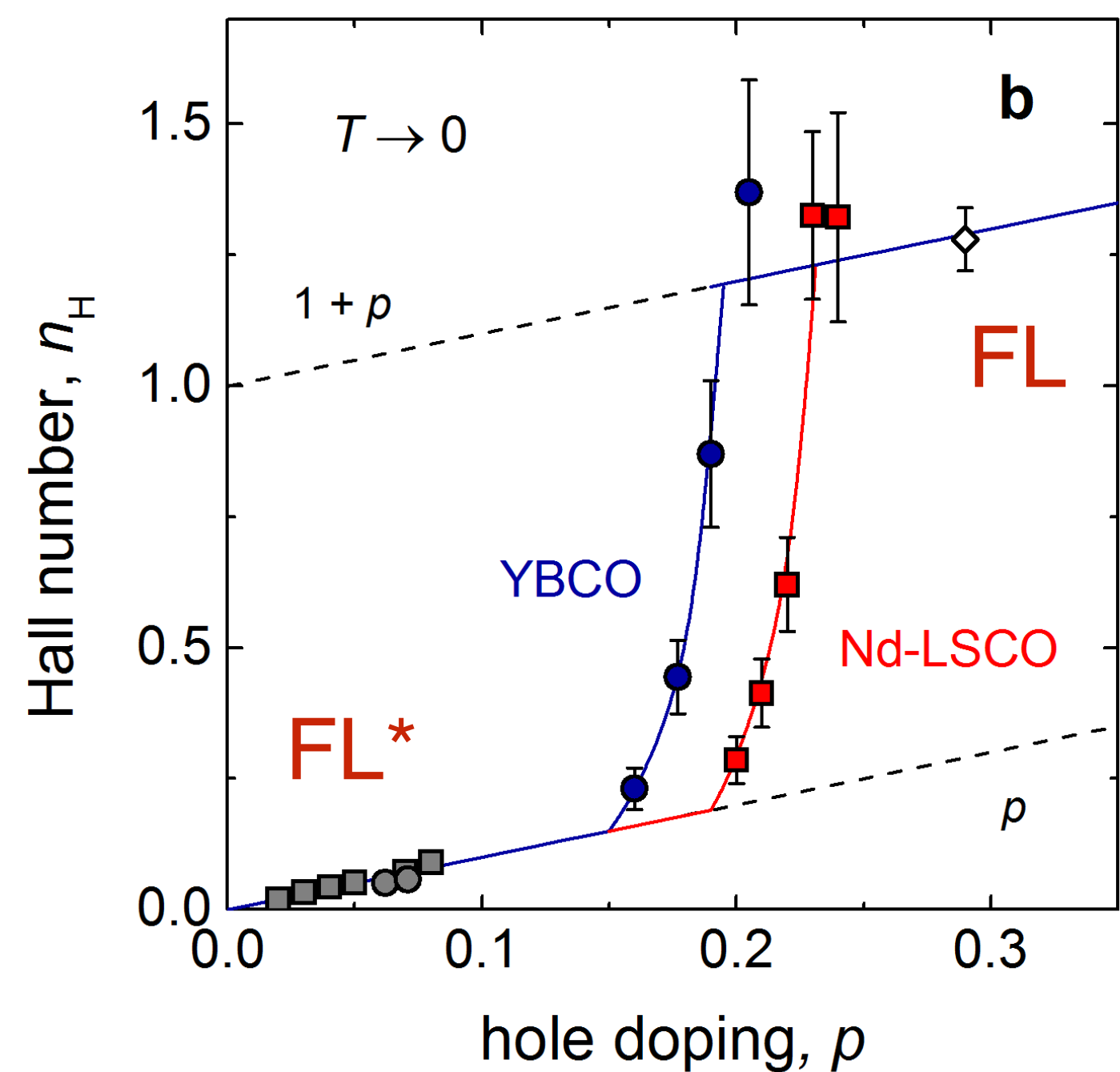
Small Fermi surface of size p

$$|\text{FL}^*\rangle = [\text{Projection onto rung singlets of } \Psi_1, \Psi_2] \\ \times |\text{Slater determinant of } (c, \Psi_1)\rangle \\ \otimes |\text{Slater determinant of } \Psi_2\rangle$$

Large Fermi surface of size $1 + p$

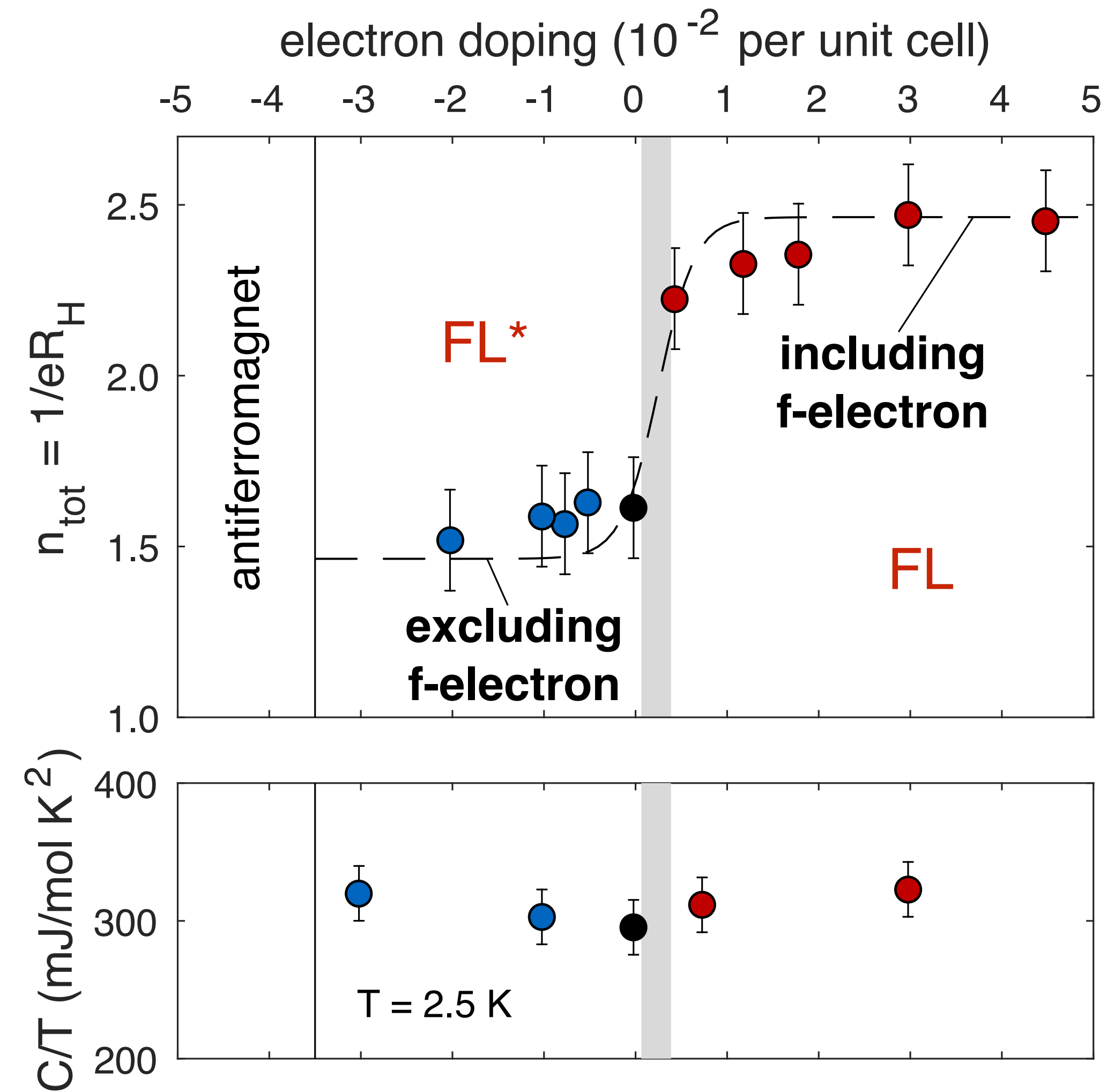
$$|\text{FL}\rangle = |\text{Rung singlets of } \Psi_1, \Psi_2\rangle \\ \otimes |\text{Slater determinant of } c\rangle$$

Cuprates



Evidence for ghost Fermi surfaces in the FL^* - FL transition in a single-band model ?

CeCoIn₅



FL*

- Recent evidence for a FL* phase in a Kondo lattice: CeCoIn₅ (Maksimovic *et al.*, Science **375**, 76 (2021), and in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019). And perhaps YbB₁₂ (Liu *et al.* arXiv:2102.09545)?

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- Ancilla theory of FL* for the pseudogap metal of the cuprates: Don't fractionalize the mobile electron, but fractionalize the 'paramagnon rotor' into 'ancilla qubits'. Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

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- Theory of FL-FL* transition on a single band Hubbard model: $(\text{SU}(2) \times \text{U}(1))/\mathbb{Z}_2$ gauge theory coupled to hybridization boson, a gauge-neutral *large* Fermi surface of electrons, and a 'ghost' Fermi surface. Prediction: critical 'ghost' Fermi surfaces near the transition.