

Planckian metals

Workshop on Ultra-Quantum Matter
Institute for Advanced Study, Princeton
October 26, 2021

Subir Sachdev



INSTITUTE FOR
ADVANCED STUDY

PHYSICS



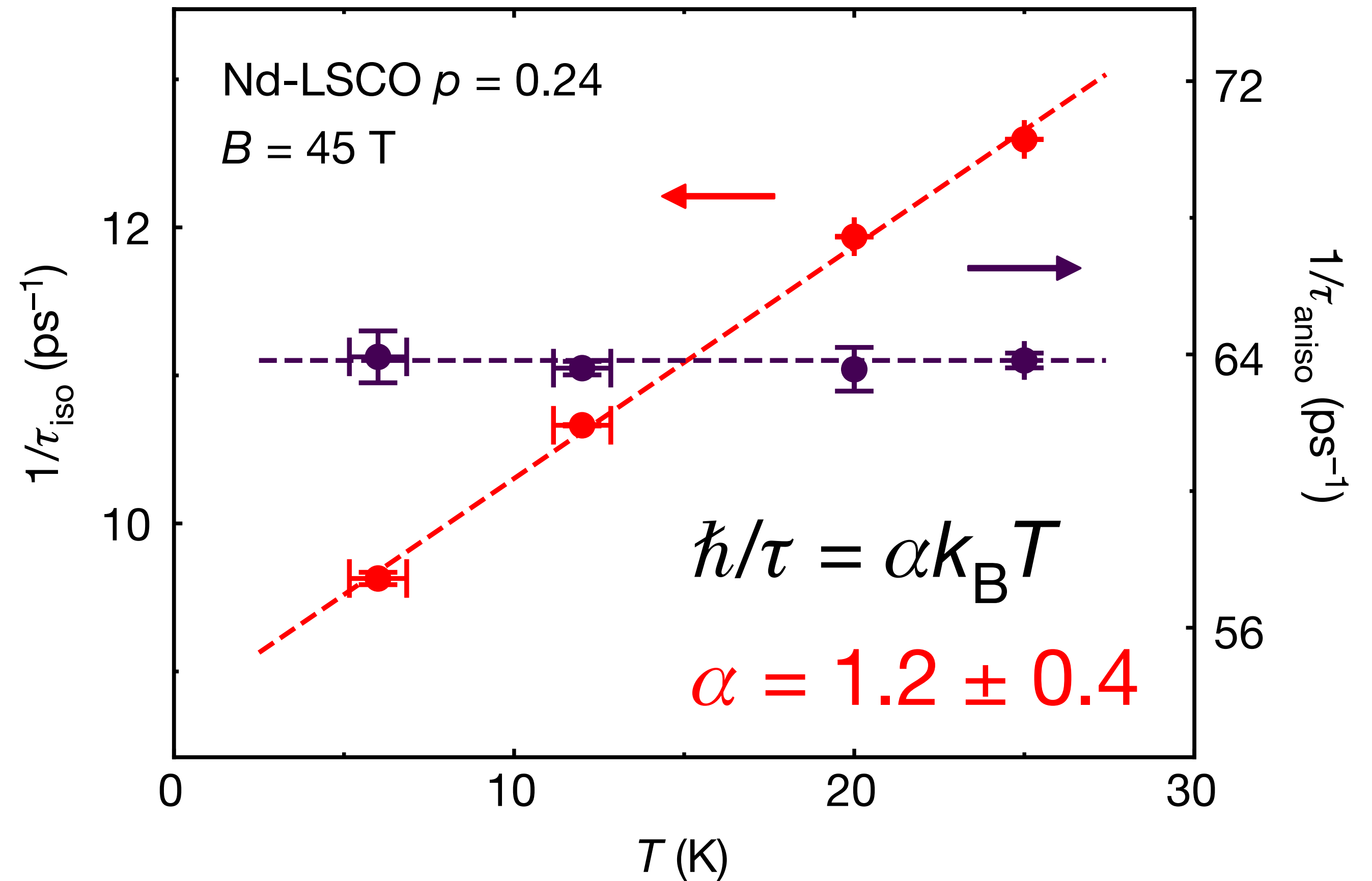
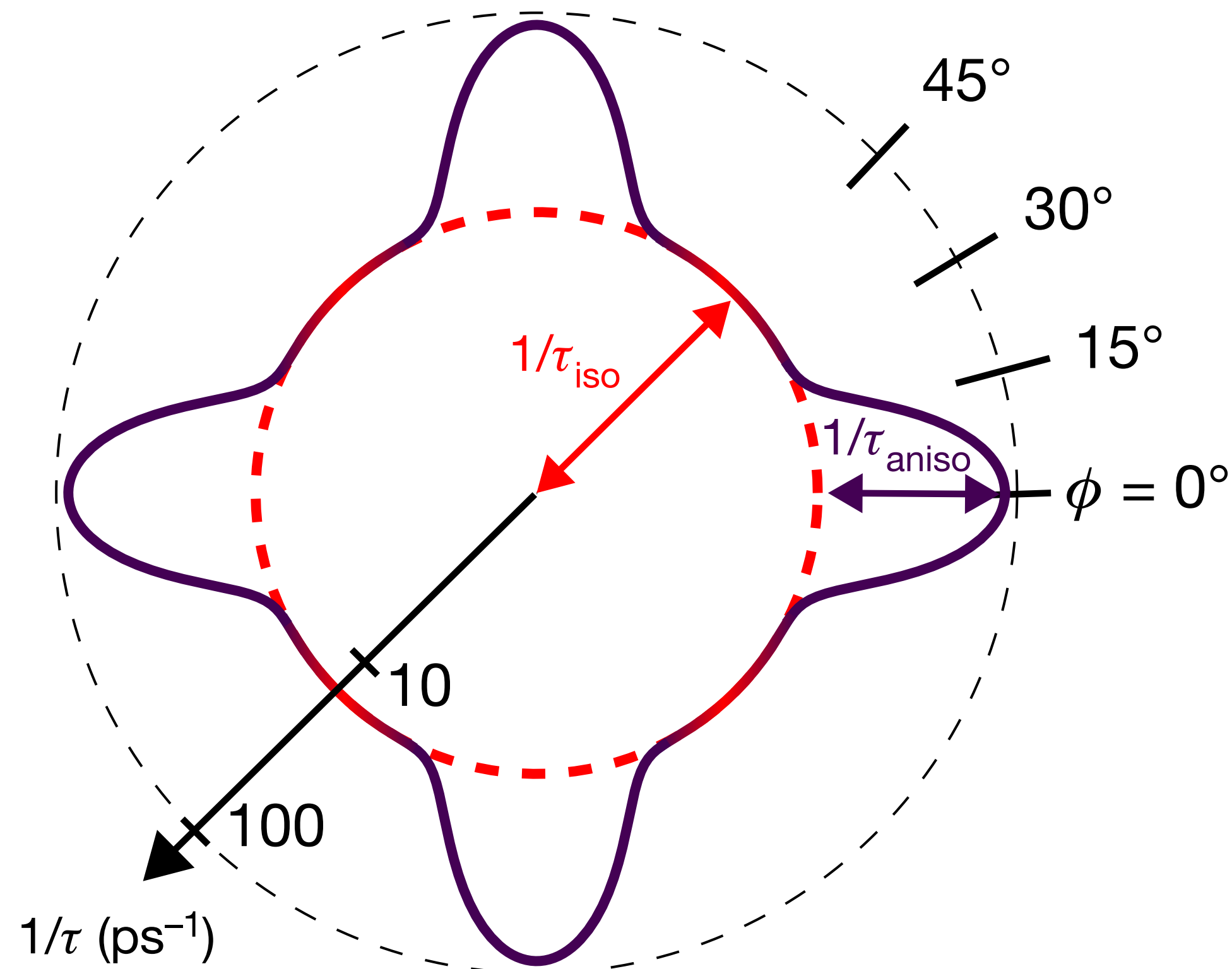
HARVARD

Talk online: sachdev.physics.harvard.edu

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

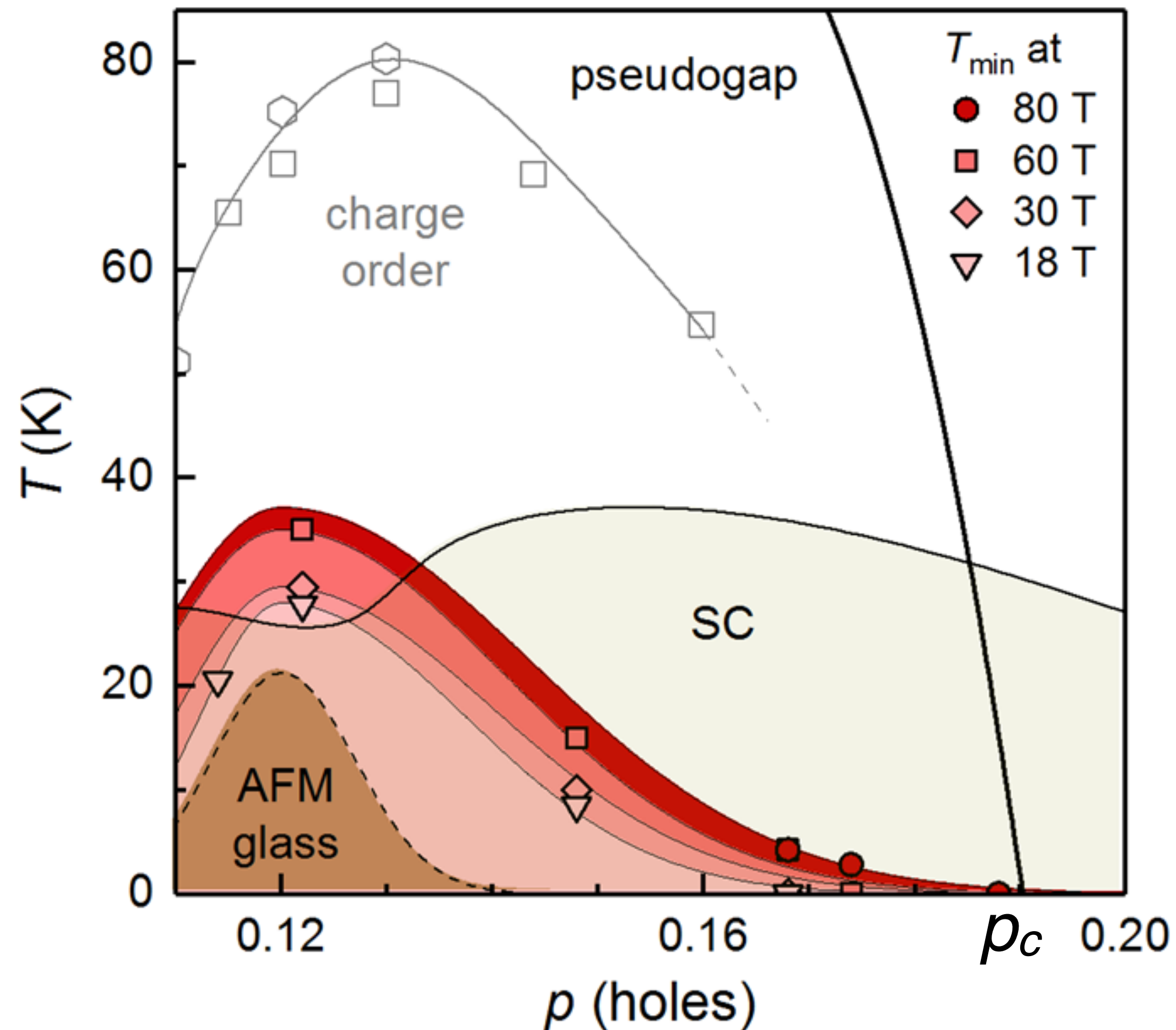
G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics **16**, 1064 (2020)

Mehdi Frachet^{1†}, Igor Vinograd^{1†}, Rui Zhou^{1,2}, Siham Benhabib¹, Shangfei Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Sanath K. Ramakrishna³, Arneil P. Reyes³, Jérôme Debray⁴, Tohru Kurosawa⁵, Naoki Momono⁶, Migaku Oda⁵, Seiki Komiya⁷, Shimpei Ono⁷, Masafumi Horio⁸, Johan Chang⁸, Cyril Proust¹, David LeBoeuf^{1*}, Marc-Henri Julien^{1*}



1. Random t-J model

2. Maximal chaos for a Fermi surface
coupled to gauge fields in 2 dimensions

3. Unification?

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Microscopically unrealistic,
describes observations



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Random t - J model doped with hole density p

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} \mathcal{P}_d c_{i\alpha}^\dagger c_{j\alpha} \mathcal{P}_d + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma} c_{i\alpha}$$

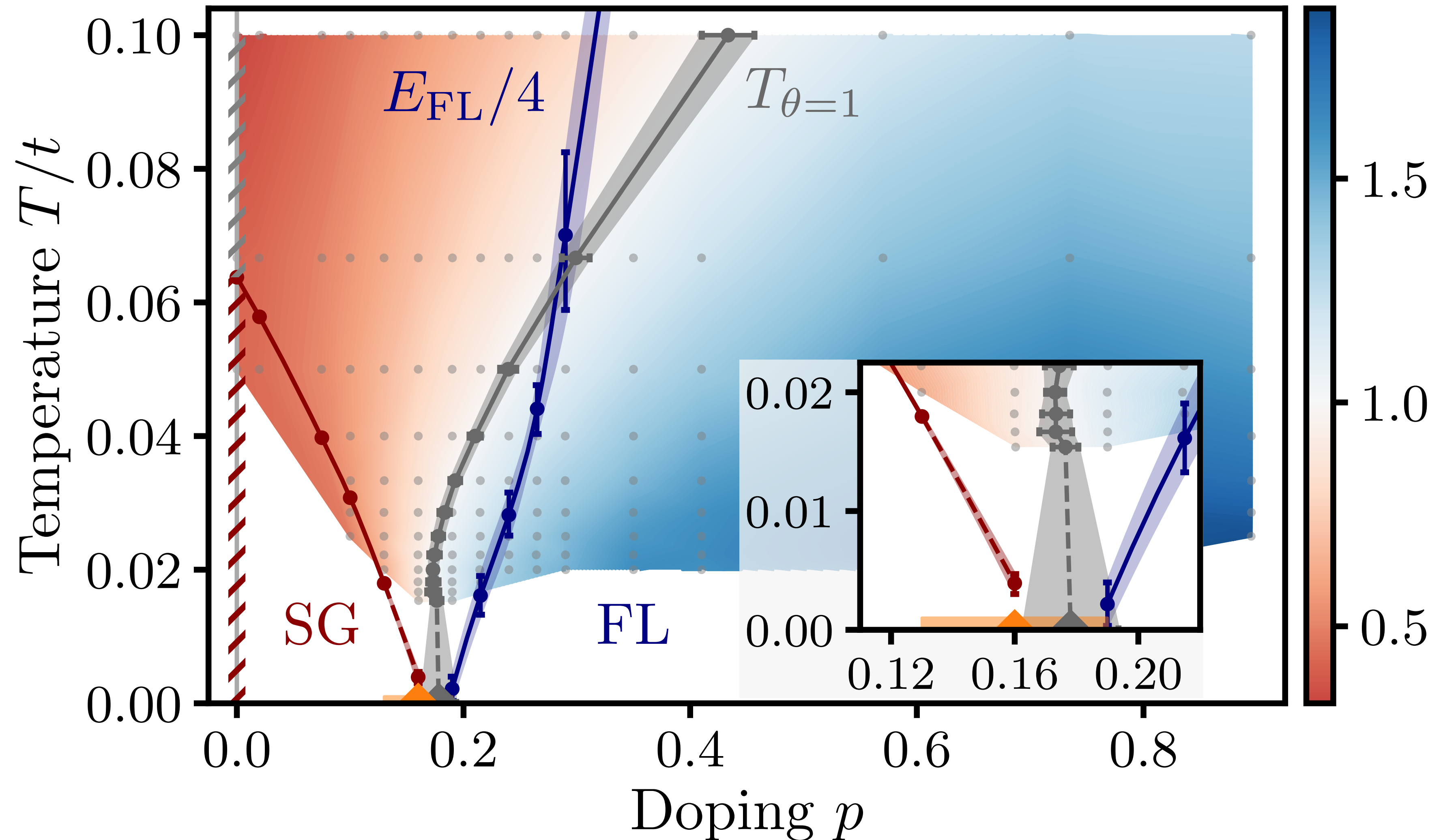
\mathcal{P}_d projects out doubly-occupied sites.

$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

$J \Rightarrow$ two-particle interaction, as in SYK
 $t \Rightarrow$ one-particle hopping, as in random matrices

Numerical solution of t - J model on a fully-connected cluster
with all-to-all and random t_{ij} and J_{ij}



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Maine Christos



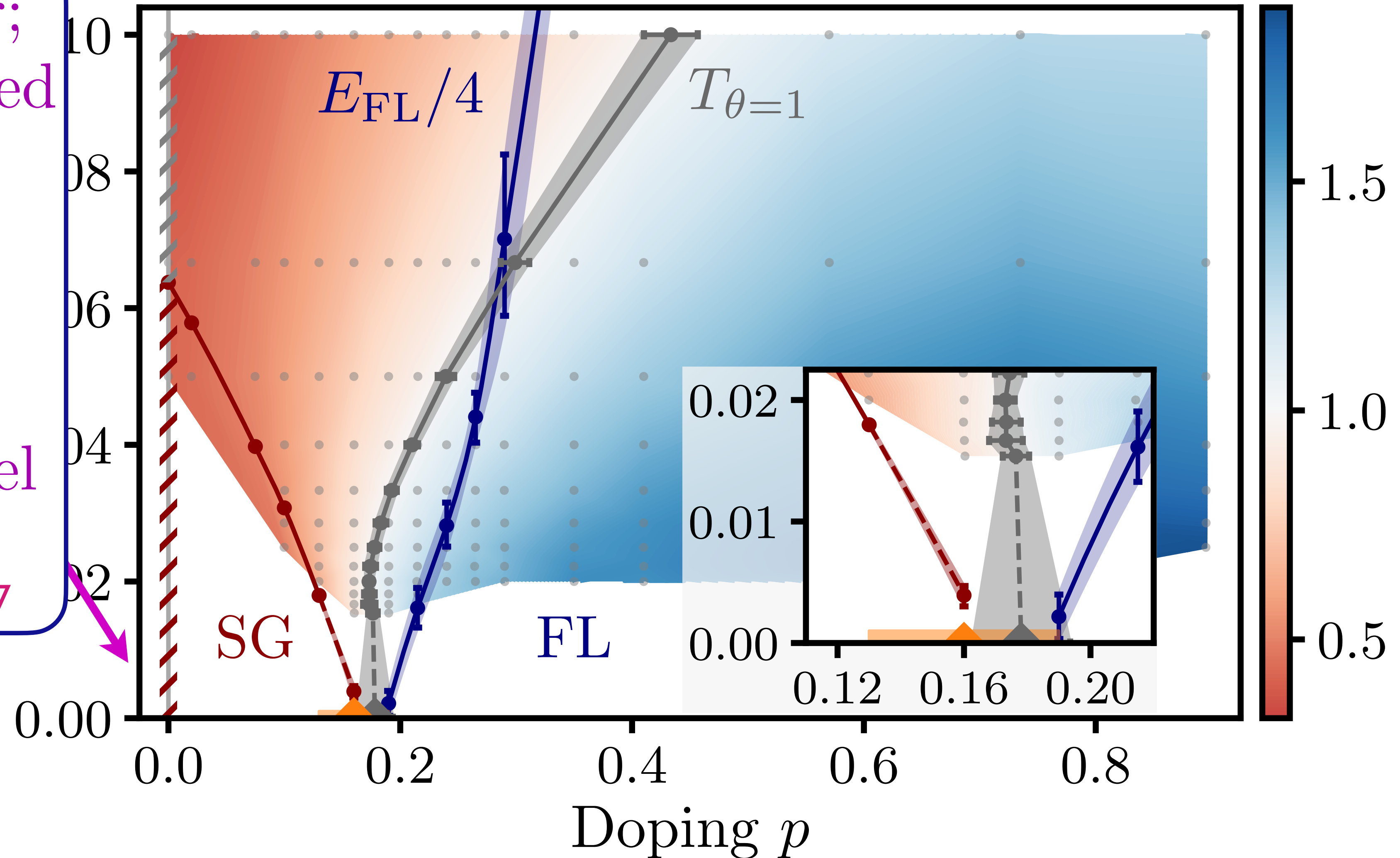
Felix Haehl

Spin glass order;
SYK fractionalized
spin liquid
for $\omega > JqEA$.

$$T_c \sim J e^{-\sqrt{\pi M}}$$

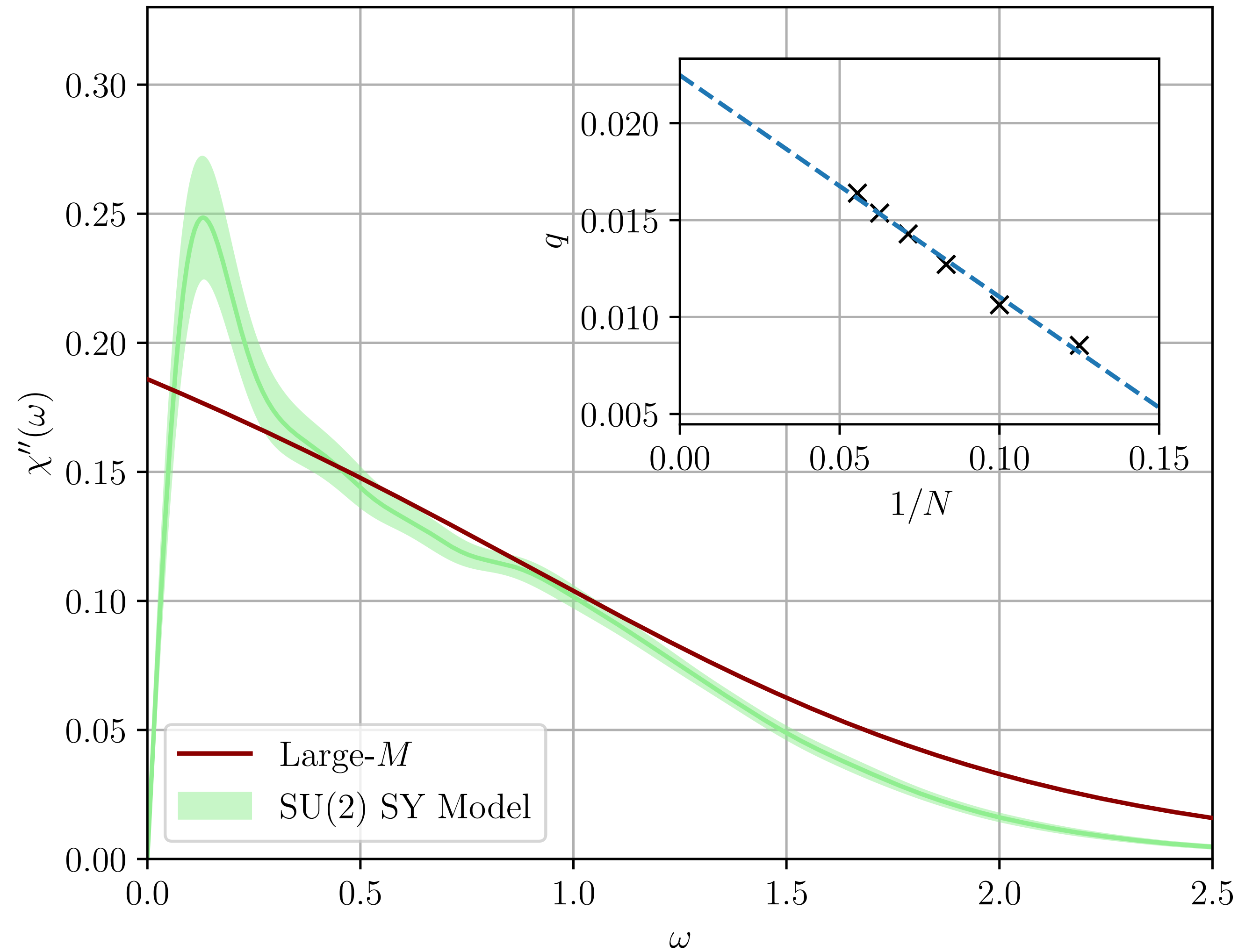
for $SU(M)$ model

M. Christos, F. M. Haehl, and
S. Sachdev, arXiv:2110.00007



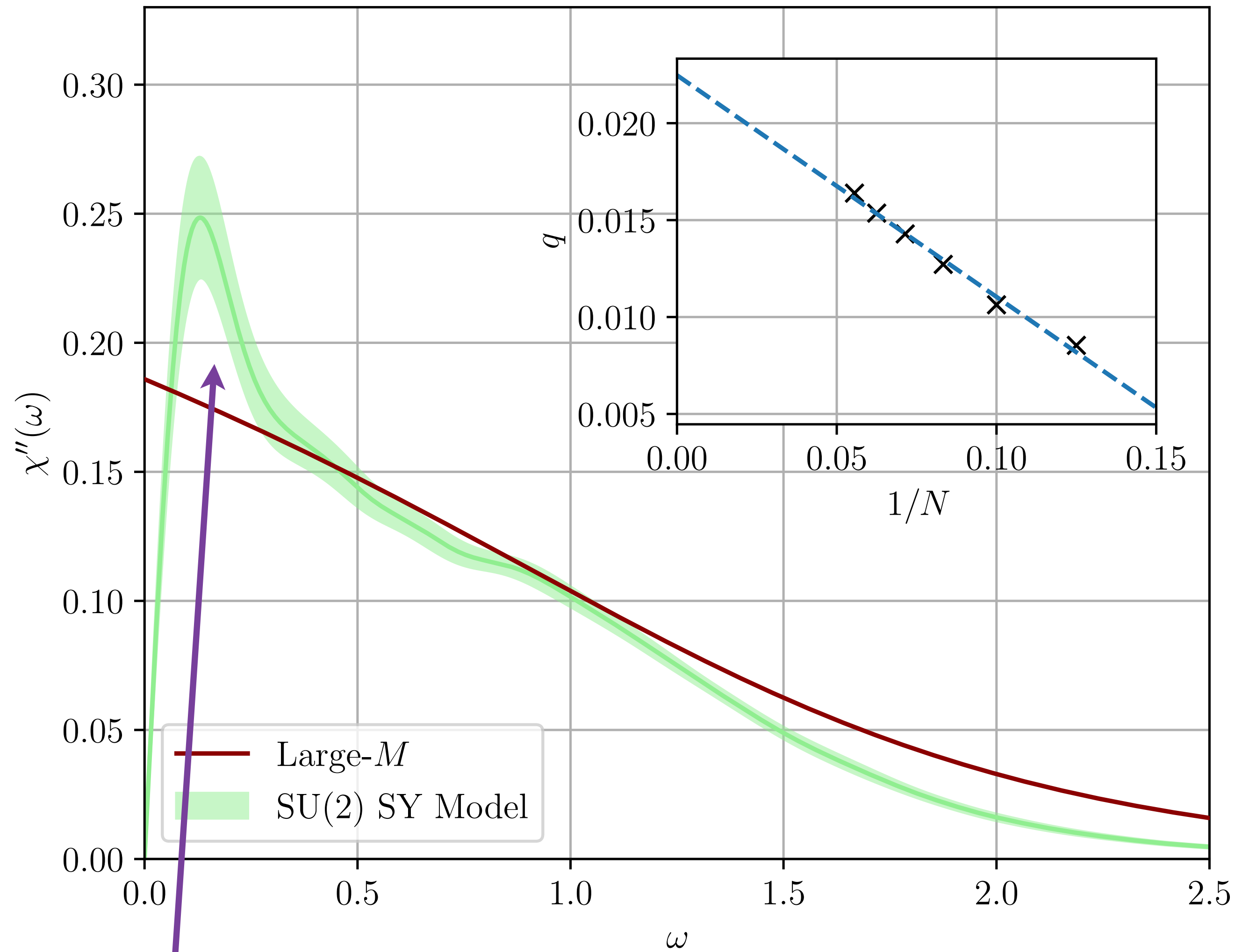
P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet, arXiv:2103.08607
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Exact diagonalization of clusters of SU(2) spins



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Exact diagonalization of clusters of SU(2) spins

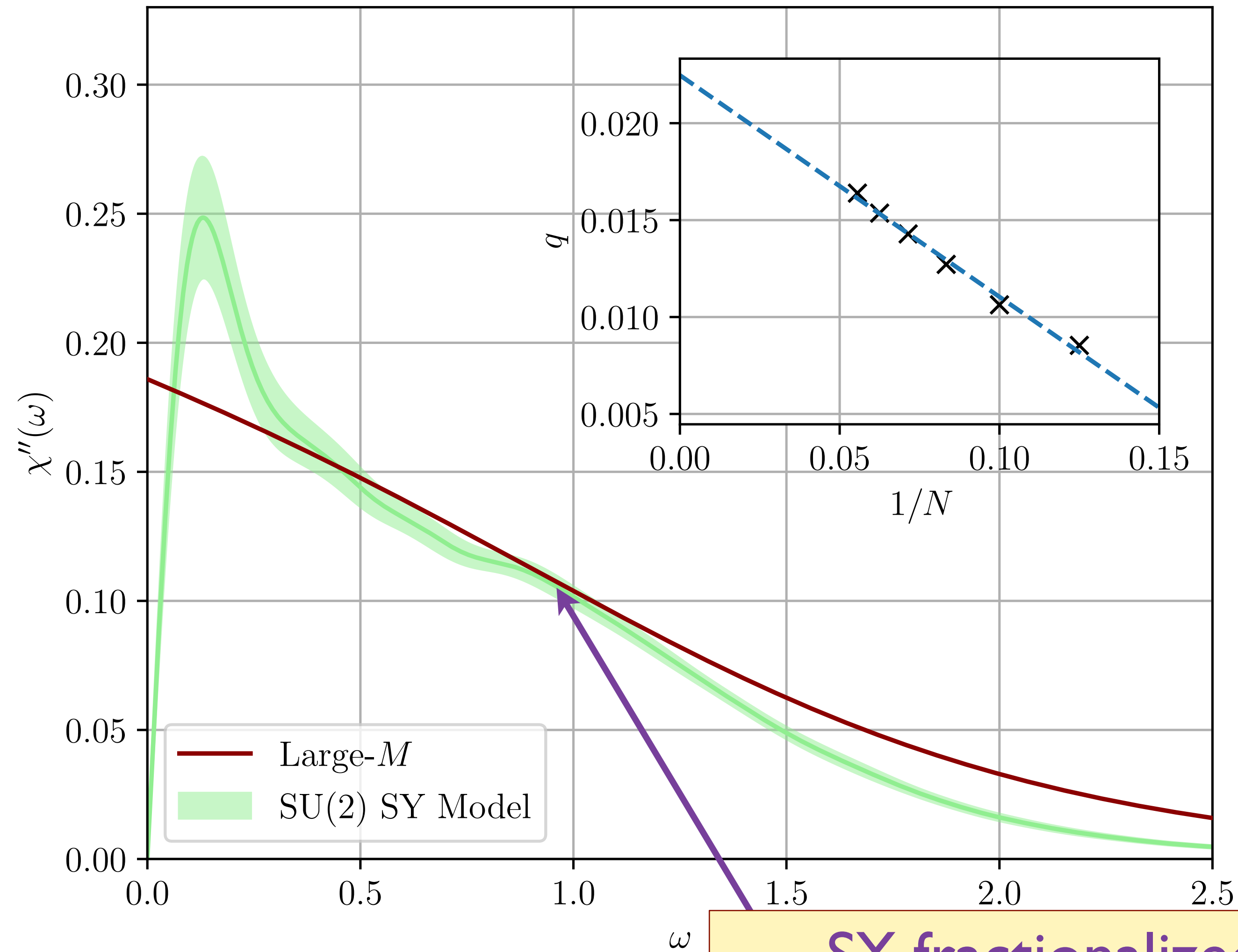


Spin glass



H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

Exact diagonalization of clusters of SU(2) spins

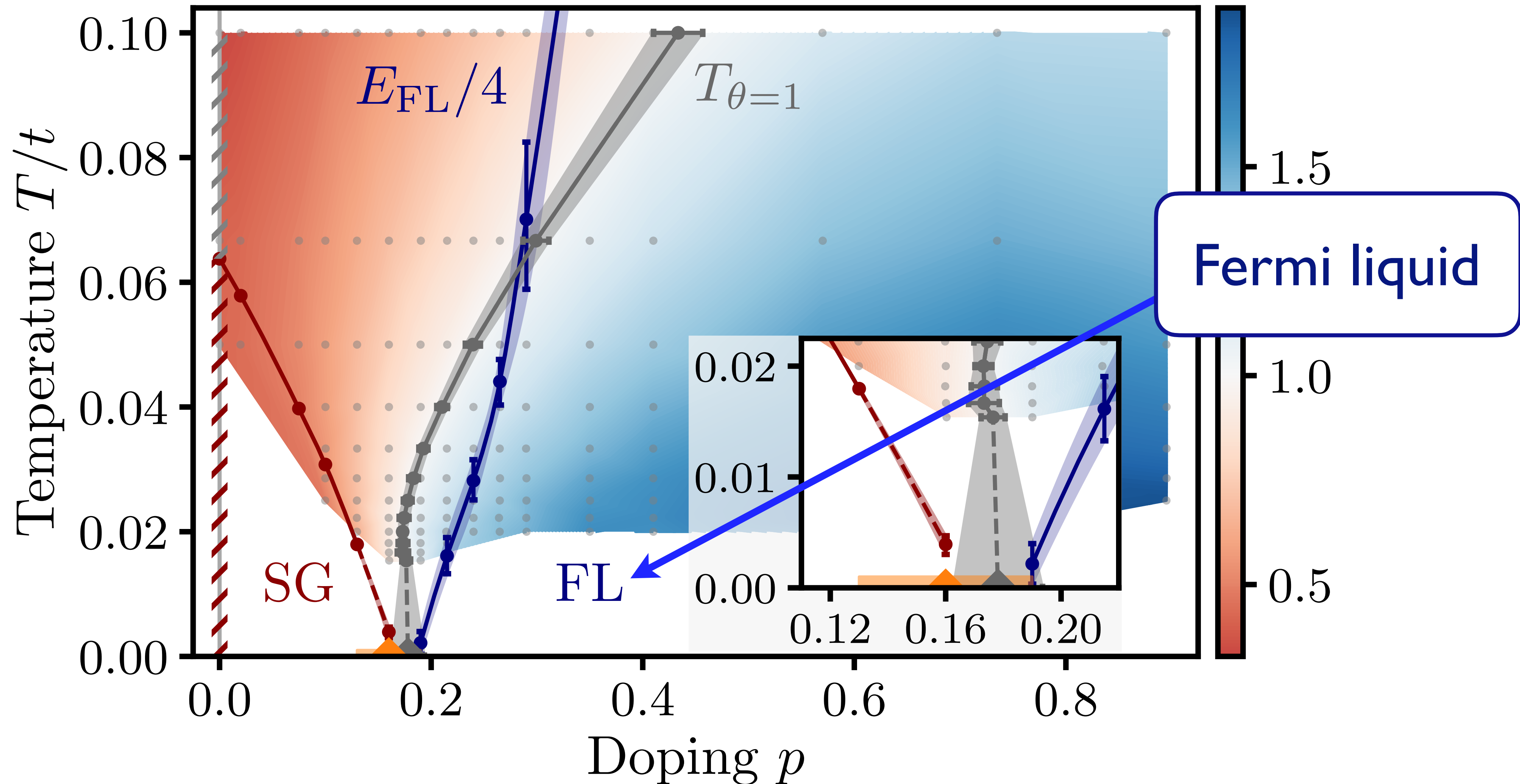


SU(2) fractionalized spin liquid
(boundary graviton of Schwarzian theory)

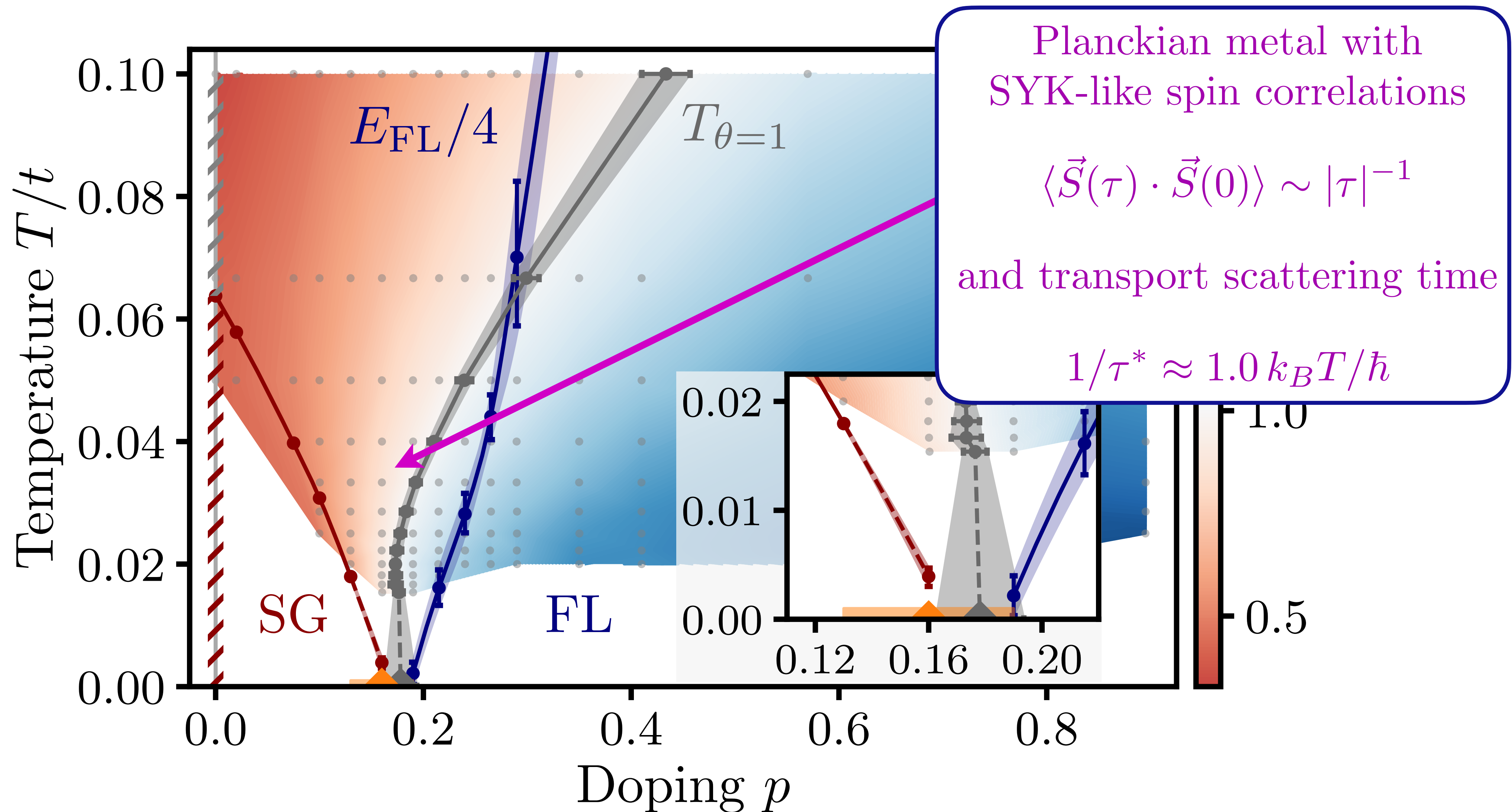
H. Shackleton, A. Wietek, A. Georges, and S. Sachdev, PRL **126**, 136602 (2021)

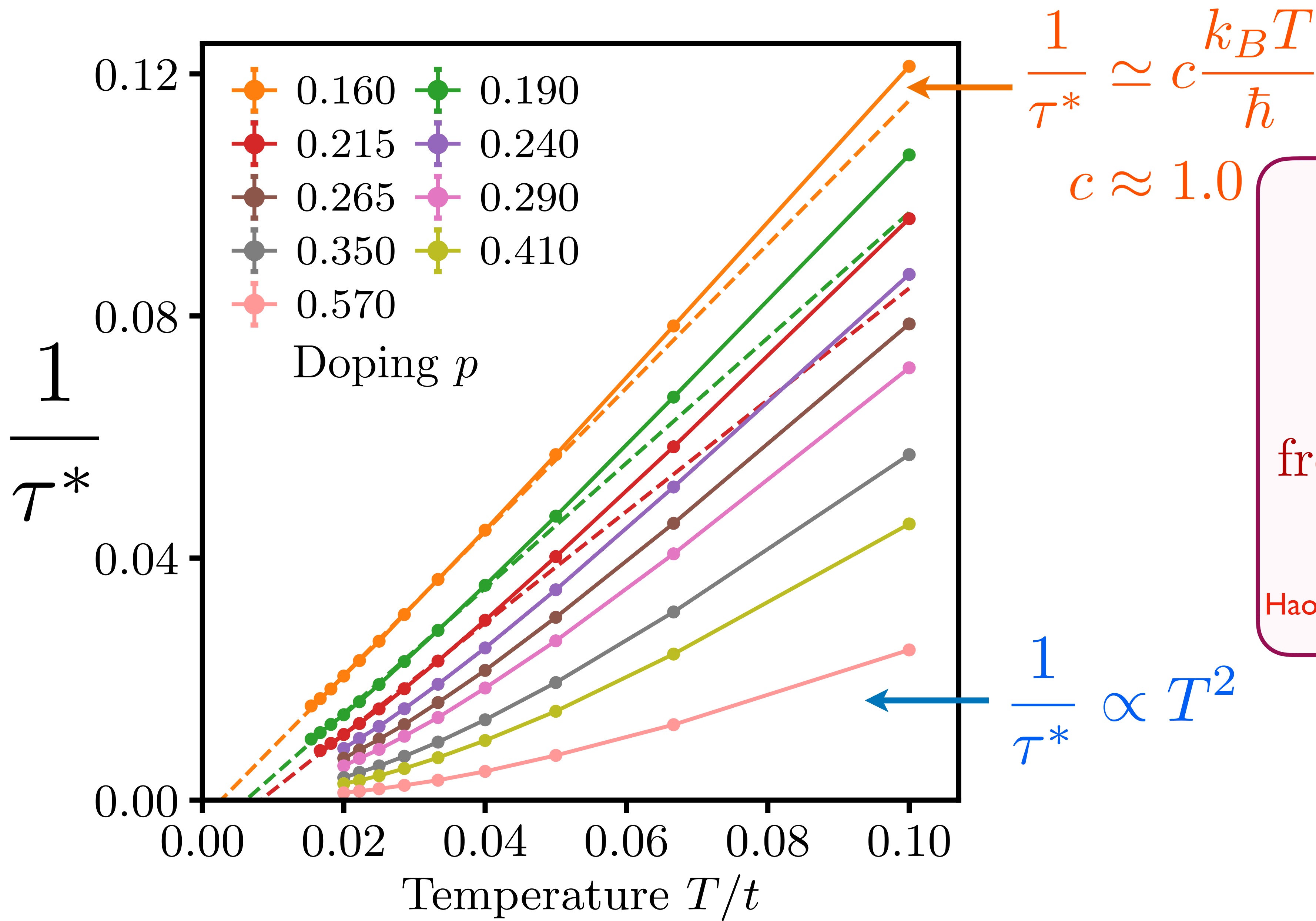


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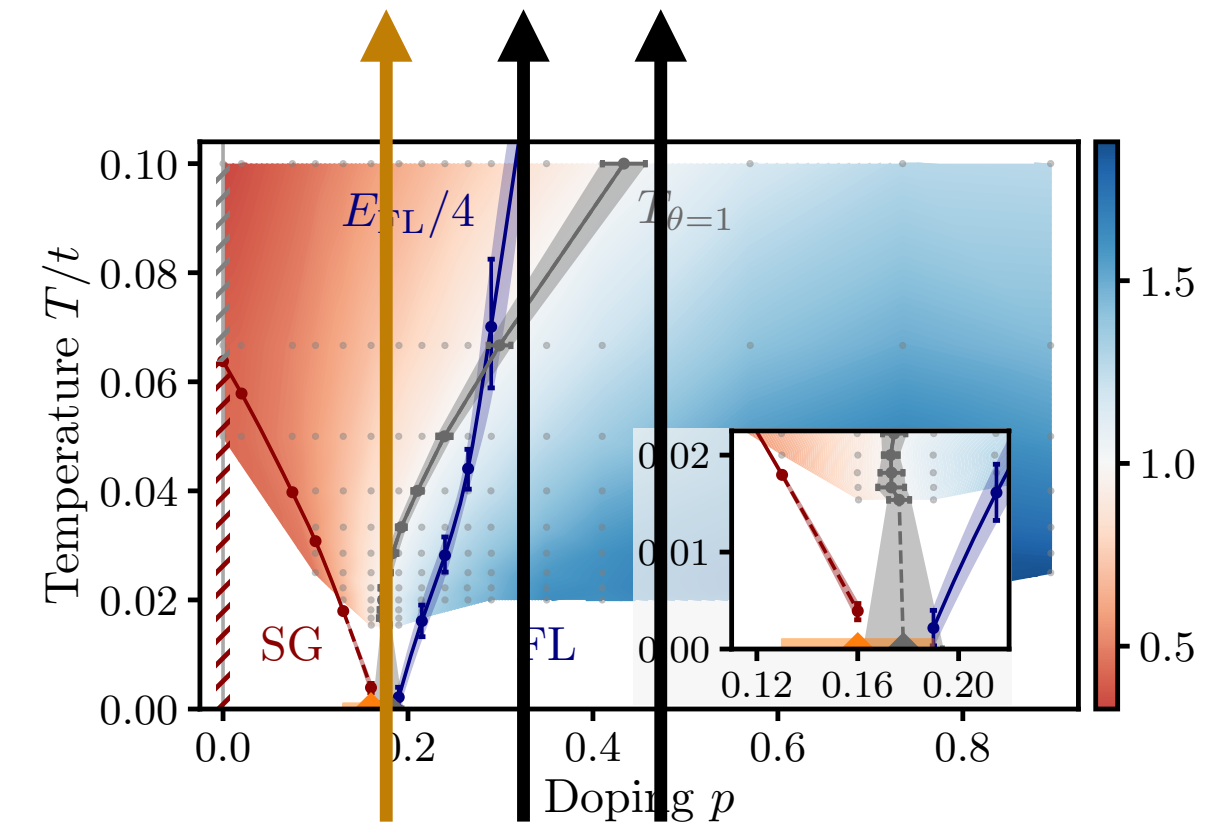




Planckian metal
for $p \approx p_c$

Linear T resistivity
from Schwarzian mode
in large M theory

Haoyu Guo, Yingfei Gu, and S. Sachdev 2020



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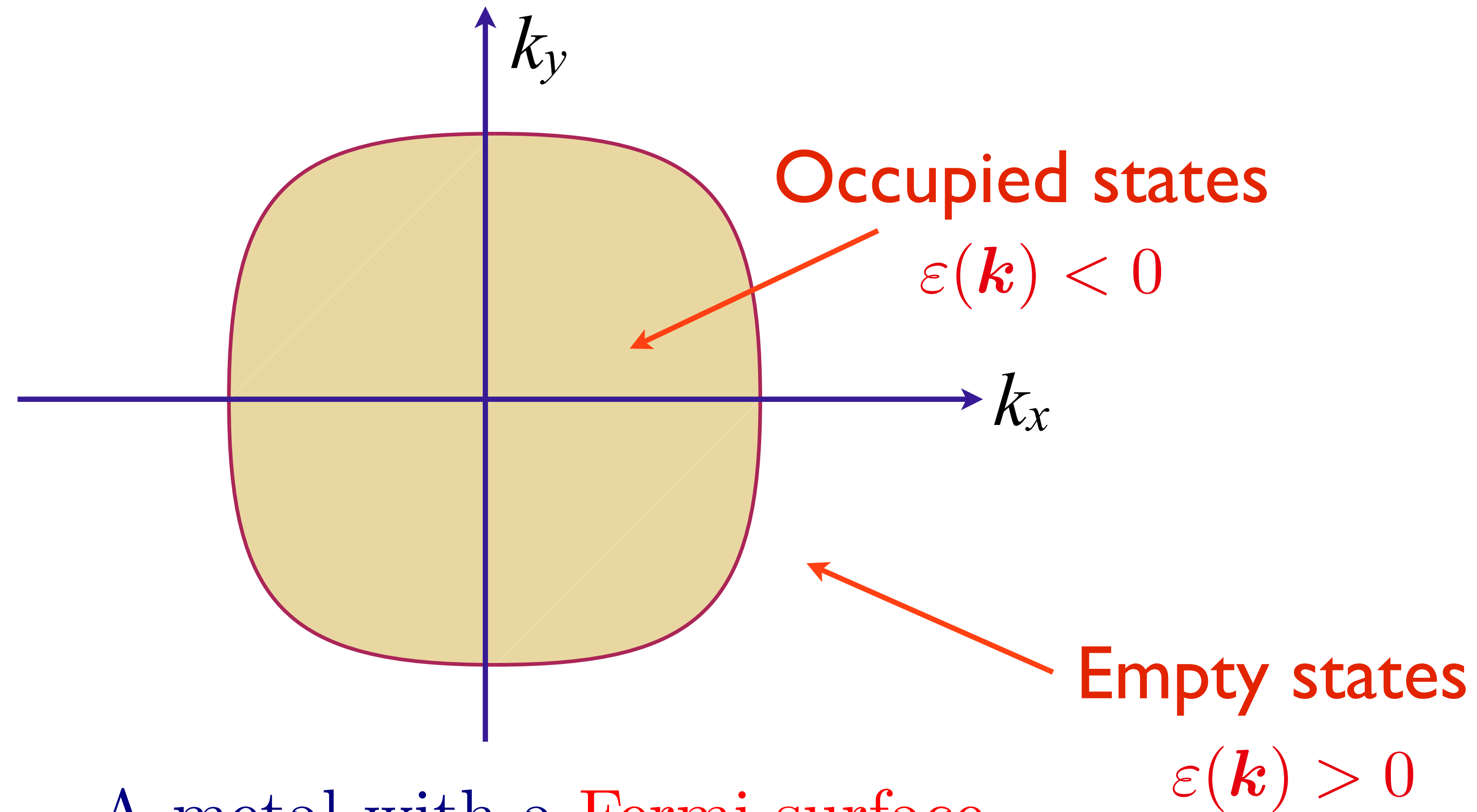
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Fermi surface coupled to a gauge field



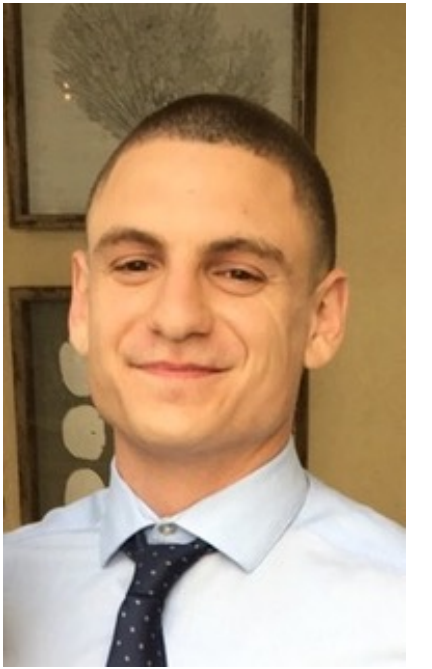
A metal with a Fermi surface minimally coupled to a gauge field \mathbf{A}

$$\mathcal{L} = c_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \epsilon(-i\nabla - g\mathbf{A}) - \mu \right) c_{\mathbf{k}} + \frac{1}{2} (\nabla \times \mathbf{A})^2$$

Large N theory of a critical Fermi surface

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.



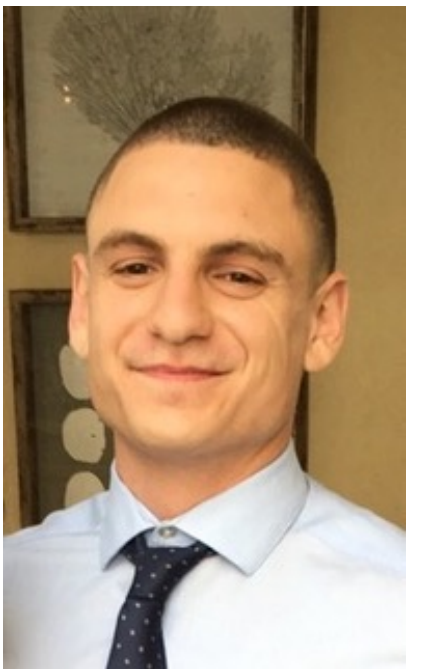
Large N theory of a critical Fermi surface

N flavors of fermions $\psi_{\pm\alpha}$,
 M flavors of a boson a_α , and
a “Yukawa coupling” $g_{\alpha\beta\gamma}$ which is a random function in
flavor space. Note: there is *no spatial randomness*. Take
the large N limit with M/N fixed.

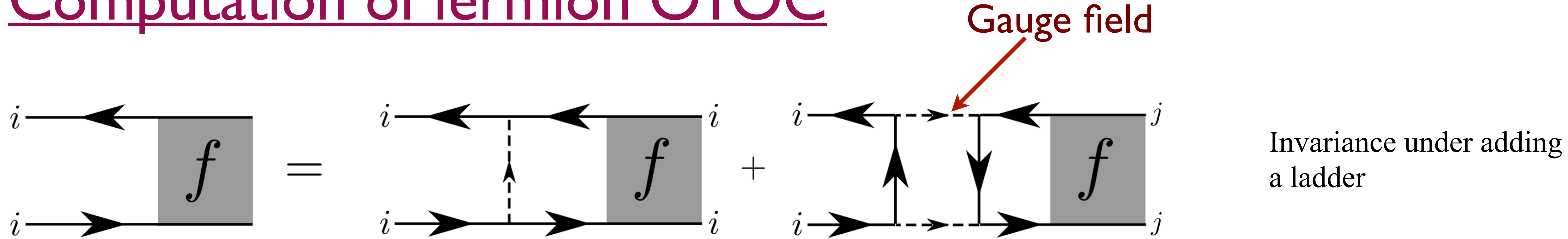
$$\mathcal{L} = \psi_{+\alpha}^\dagger (\partial_\tau - i\partial_x - \partial_y^2) \psi_{+\alpha} + \psi_{-\alpha}^\dagger (\partial_\tau + i\partial_x - \partial_y^2) \psi_{-\alpha} \\ - \frac{g_{\alpha\beta\gamma}}{N} a_\alpha \left(\eta_{+\alpha} \psi_{+\beta}^\dagger \psi_{+\gamma} + \eta_{-\alpha} \psi_{-\beta}^\dagger \psi_{-\gamma} \right) + \frac{1}{2} (\partial_y a_\alpha)^2$$

$\eta_{\pm\alpha} = \pm 1$ depending upon nature of a_α : gauge field, Higgs
field, order parameter

$$\overline{g_{\alpha\beta\gamma}} = 0 \quad , \quad \overline{|g_{\alpha\beta\gamma}|^2} = g^2$$



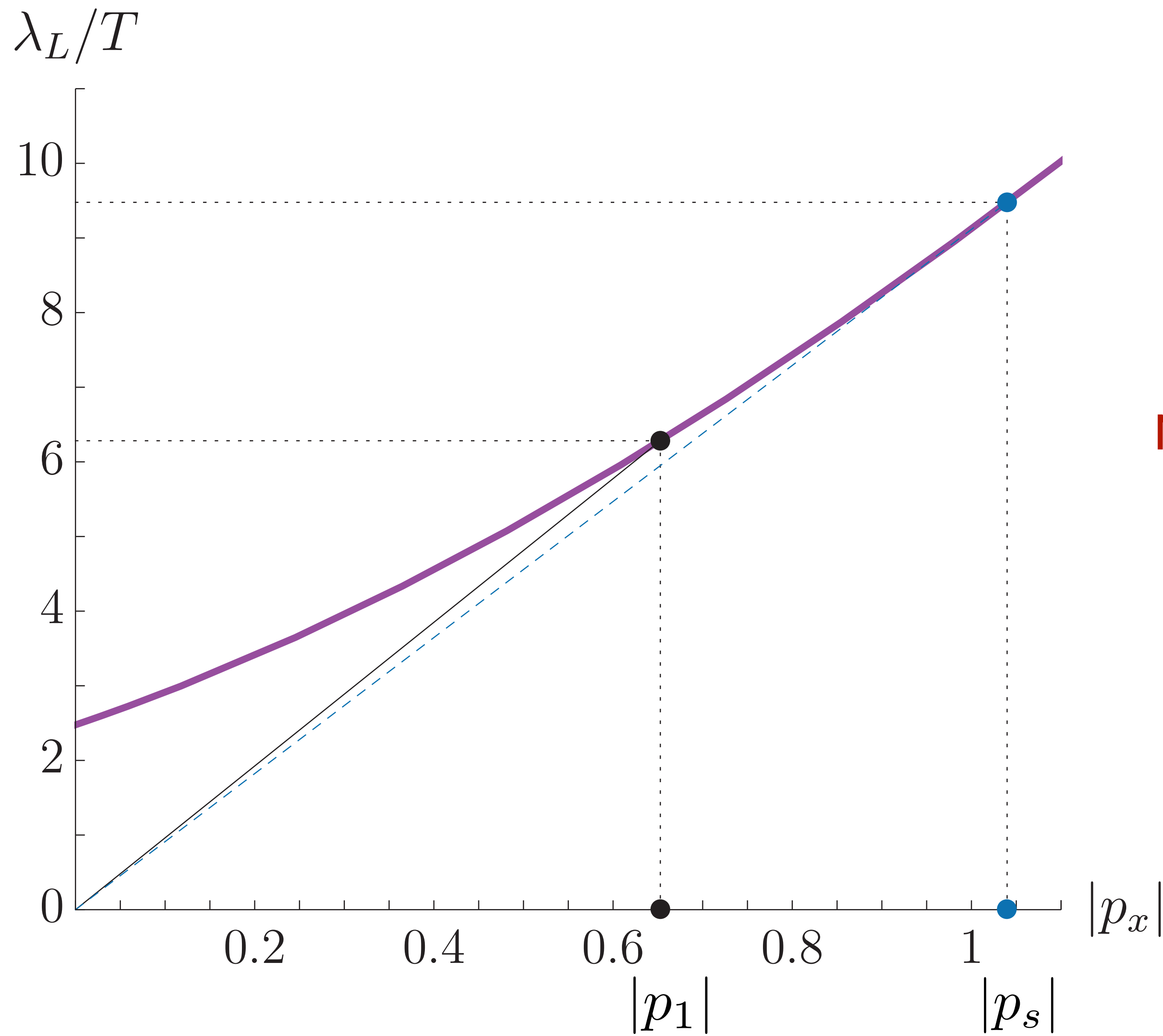
Computation of fermion OTOC



$$f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^N \int d^2x \operatorname{Tr} \left[e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \} e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \}^\dagger \right] = \int d^2x f(t, x)$$

$$\begin{aligned} & \left[T^{2/z} c_{f,z} \left(H_{1-2/z} \left(\frac{-ik_0 - \pi T}{2\pi T} \right) + H_{1-2/z} \left(\frac{-i(\omega - k_0) - \pi T}{2\pi T} \right) \right) + 2\mu(T) - p_x \right] \tilde{f}(k_0, \omega) \\ &= g^2 \int \frac{dk'_0 dk'_y}{(2\pi)^2} \frac{c_b(k_0 - k'_0) |k'_y|}{(|k'_y|^z + m^2)^2 + c_b^2(k_0 - k'_0)^2} \frac{\tilde{f}(k'_0, \omega)}{\sinh \frac{k_0 - k'_0}{2T}} \\ & - \frac{ig^4}{8z \sin(\frac{2\pi}{z}) c_b^{2-\frac{2}{z}}} \int \frac{dk'_0 dk_{01}}{(2\pi)^2} \frac{(-ik_{01})^{\frac{2}{z}-1} - (i(k_{01} - \omega))^{\frac{2}{z}-1}}{2k_{01} - \omega} \frac{\tilde{f}(k'_0, \omega)}{\cosh \frac{k_0 - k_{01}}{2T} \cosh \frac{k'_0 - k_{01}}{2T}} \end{aligned}$$

As in [Patel, Sachdev 2016]



Maria Tikhanovskaya
Harvard



Aavishkar Patel
Berkeley

Gu and Kitaev (2019):
Compute λ_L for *imaginary*
momentum: *i.e.* $p_x = i|p_x|$
and include contribution of pole

We find maximal chaos with $\lambda_L = 2\pi T$,
and butterfly velocity $v_1 = 2\pi/|p_1| \approx 9.67g^{-4/3}T^{1/3}$

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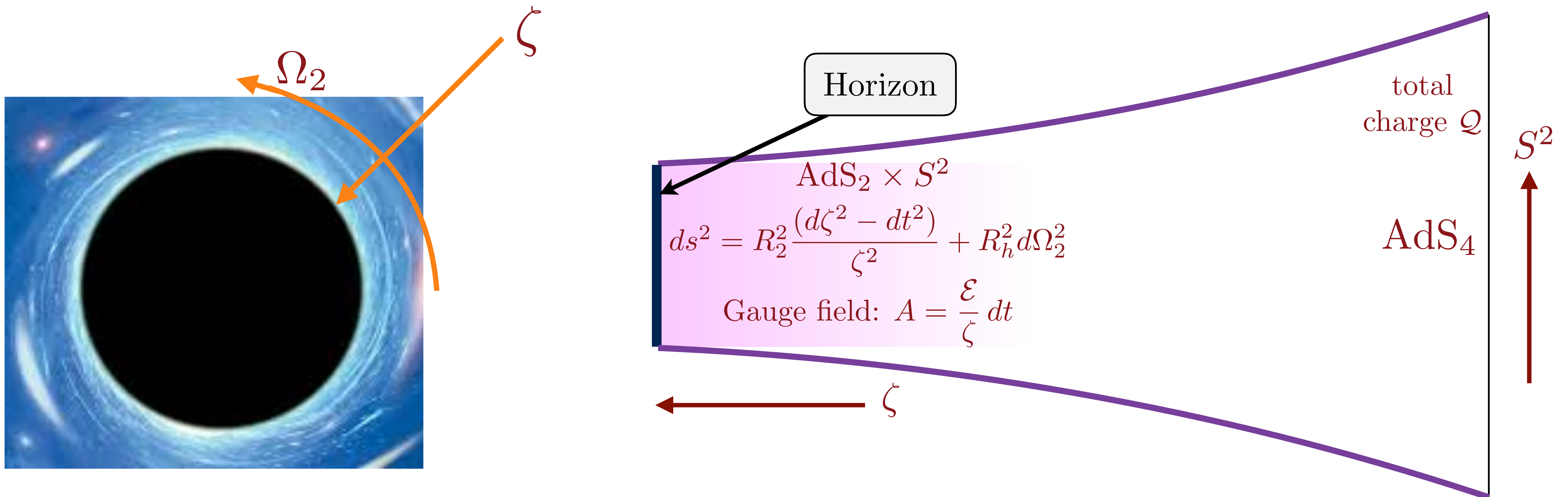
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- Why should the 2+1 dimensions t - J model be described by a 0+1 dimensional SYK-like theory over a significant temperature range ?

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- By the holographic mapping, this implies that certain models of Fermi surfaces coupled to large N gauge fields in 2+1 dimensions display a dimensional reduction to a 0+1 dimensional theory.
- The transition between the pseudogap and the Fermi liquid in the t - J model can be written as $\text{SU}(2) \times \text{U}(1)$ gauge theory.