# The time reparameterization soft mode

Simons Collaboration on Ultra-Quantum Matter
Virtual Meeting
June 2-4, 2020

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS

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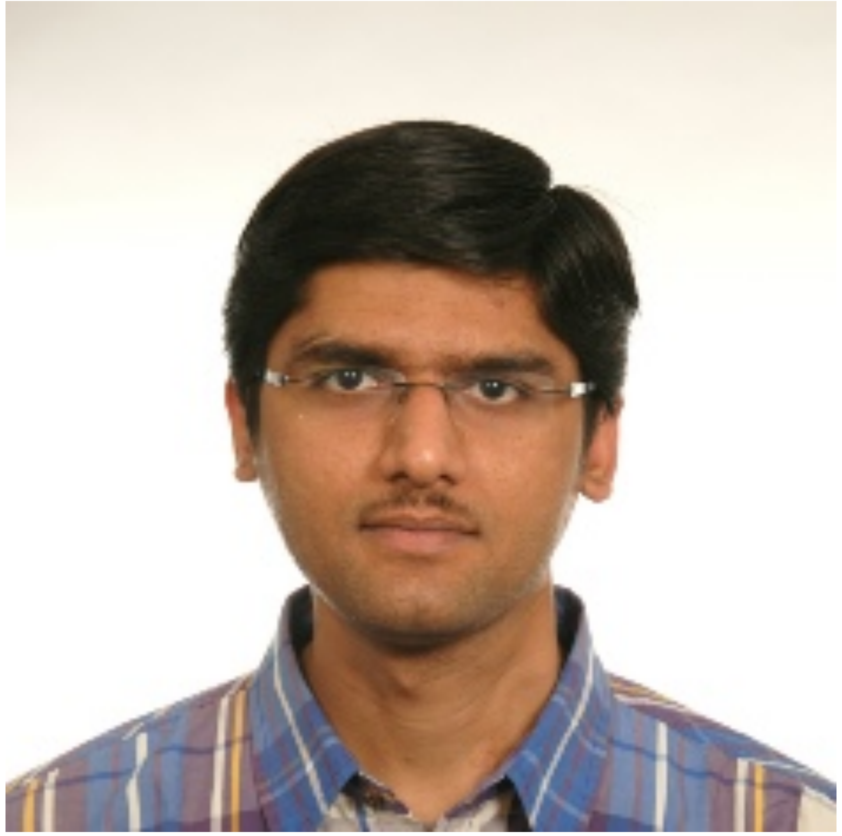
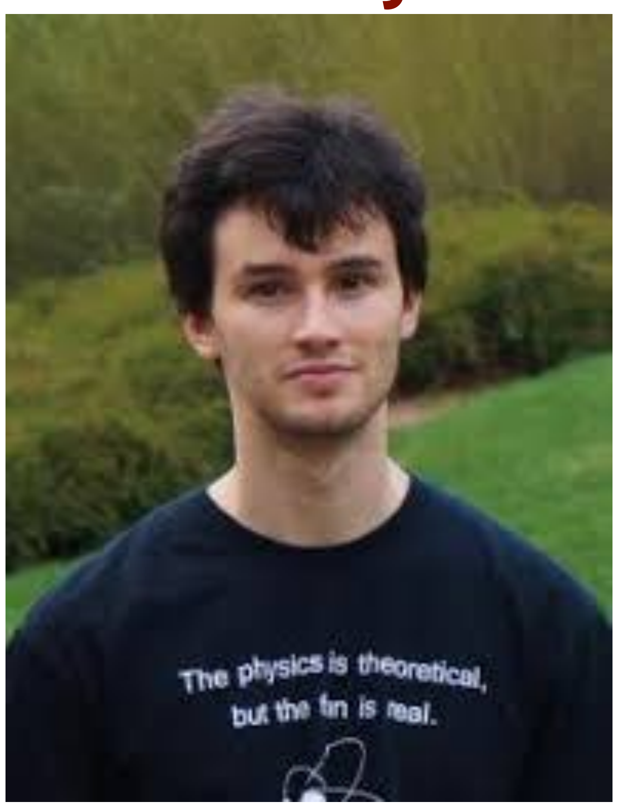


Darshan Joshi

Chenyuan Li

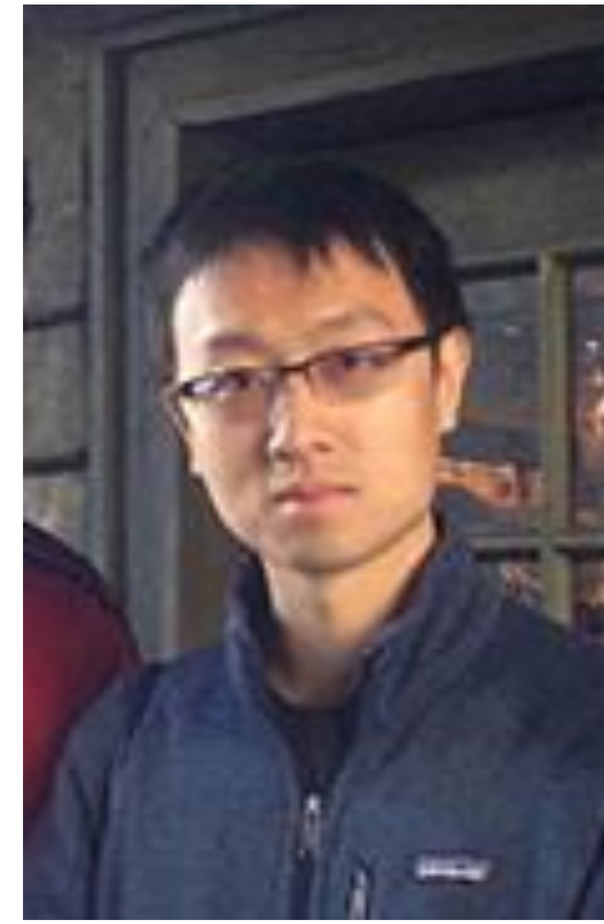



Grigory Tarnopolsky

Physical Review X
**10**, 021033 (2020)

Antoine Georges

Annals of Physics, **418**, 168202 (2020)



Haoyu Guo

Yingfeu Gu

# Challenge for theory:

A model of a metal in which the resistivity, $\rho$, obeys

$$\lim_{T \to 0} \frac{d\rho}{dT} \neq 0$$

$$\rho(T) = \rho(0) + AT + \ldots, \quad T \to 0\,.$$

# 2 key ingredients

1. Emergent gauge symmetry and fractionalization

$$c_\alpha = f_\alpha b^\dagger$$
$$f_\alpha \to e^{i\phi} f_\alpha \ , \ b \to b\, e^{i\phi}$$

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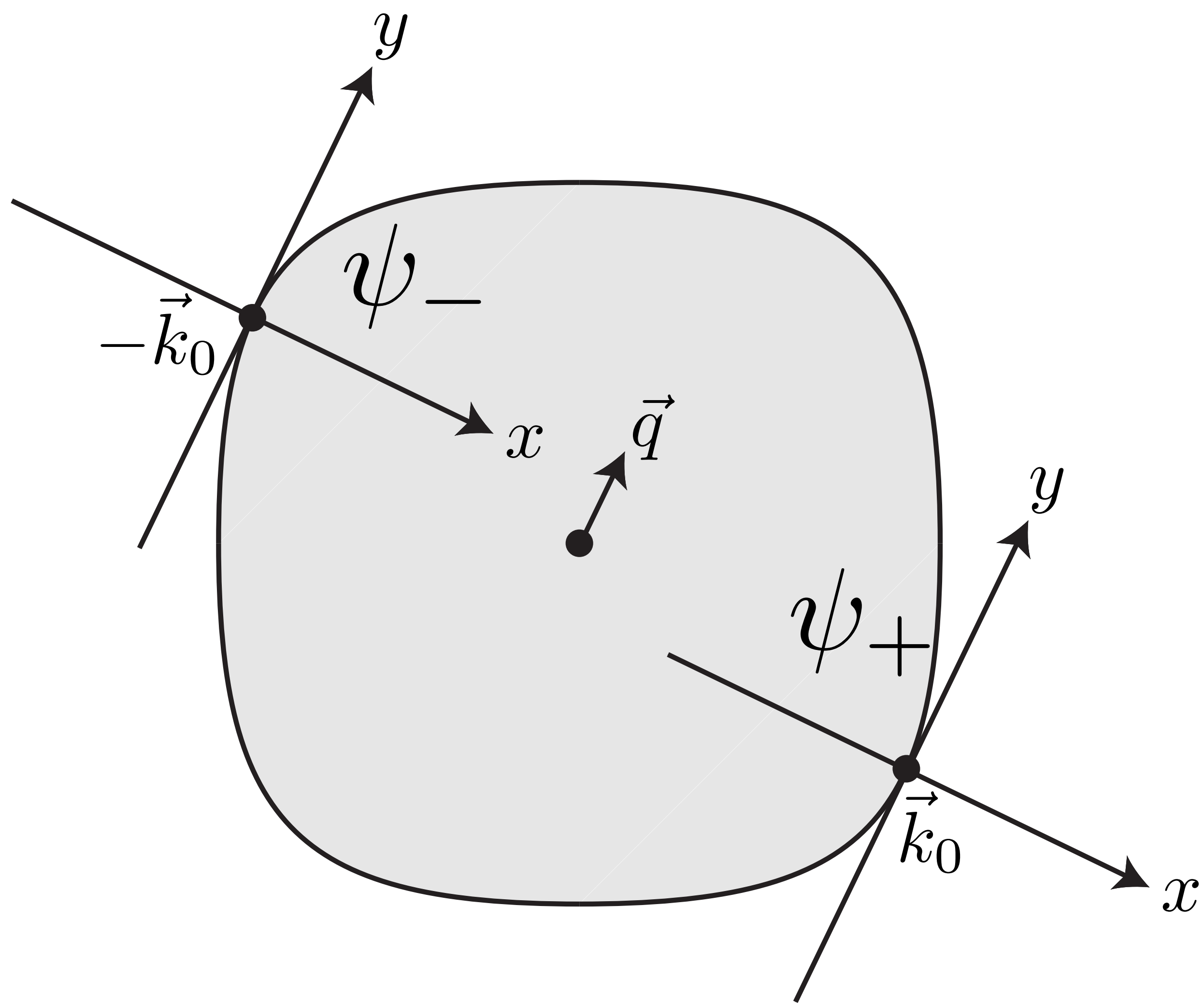
2. Time reparameterization symmetry

$$\tau \to f(\tau)$$

Present in models of non-Fermi liquids:
quantum matter at variable density
without quasiparticle excitations

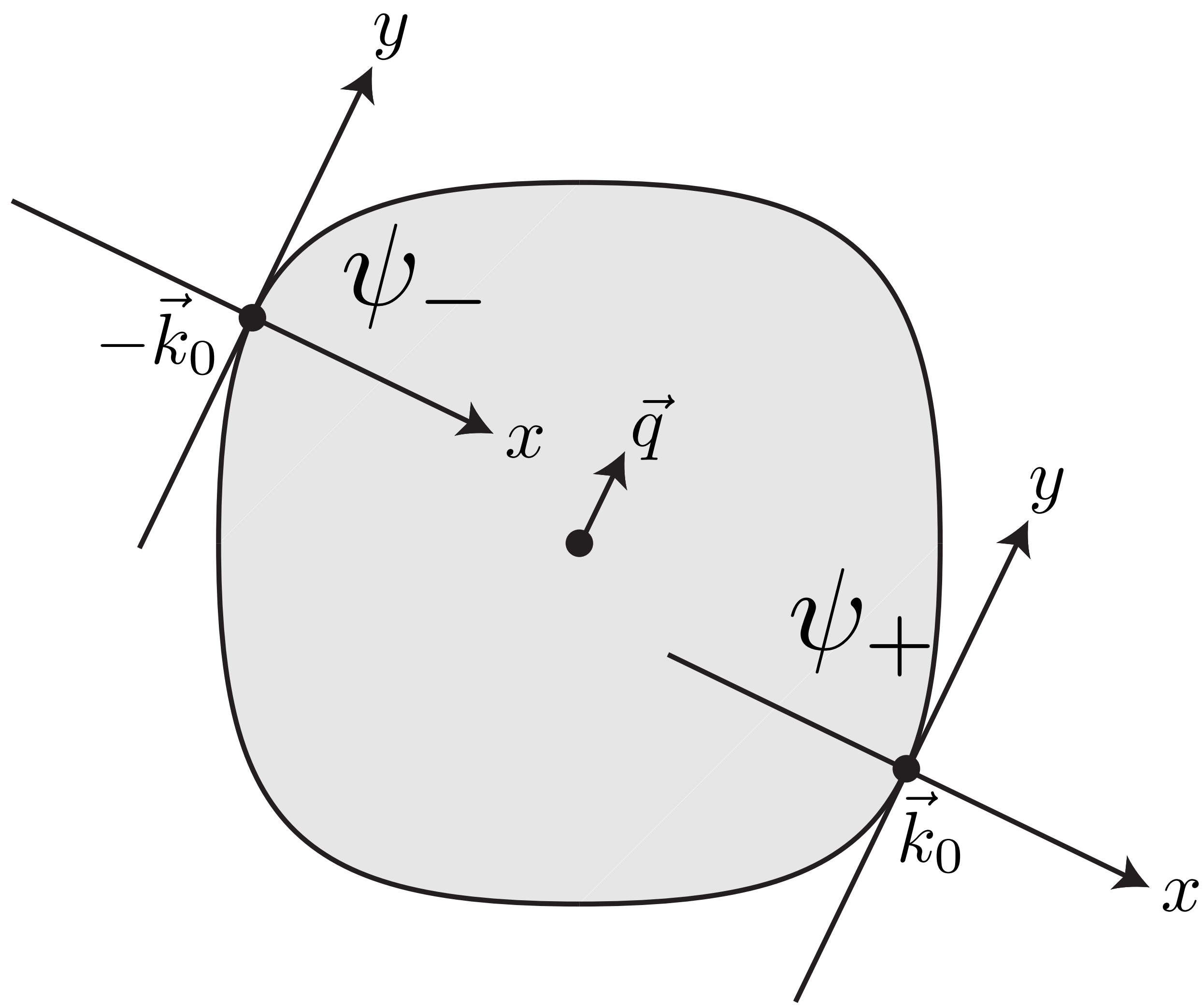
# Fermi surface coupled to a gauge field




- Gauge fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$. In Landau gauge $\vec{A} = (a, 0)$.

# Fermi surface coupled to a gauge field




$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

# Fermi surface coupled to a gauge field

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

## Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\, s \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

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&\quad - a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y a \right)^2
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Because the bare time derivatives are irrelevant, the critical theory has an emergent time reparameterization symmetry:

$$\begin{aligned}
\tau &\rightarrow f(\tau) \\
d\tau &\rightarrow f'(\tau)\, d\tau \\
x &\rightarrow [f'(\tau)]^{1/z}\, x \\
y &\rightarrow [f'(\tau)]^{1/(2z)}\, y \\
A &\rightarrow [f'(\tau)]^{1/z}\, A \\
\psi &\rightarrow [f'(\tau)]^{-(2z+1)/(4z)}\, \psi
\end{aligned}$$

Work in progress with Haoyu Guo and Aavishkar Patel

1. SYK criticality +
   *time reparameterization soft mode*

2. SYK lattice models

3. *Fractionalization* and SYK criticality
   in $t$-$J$ models with random exchange

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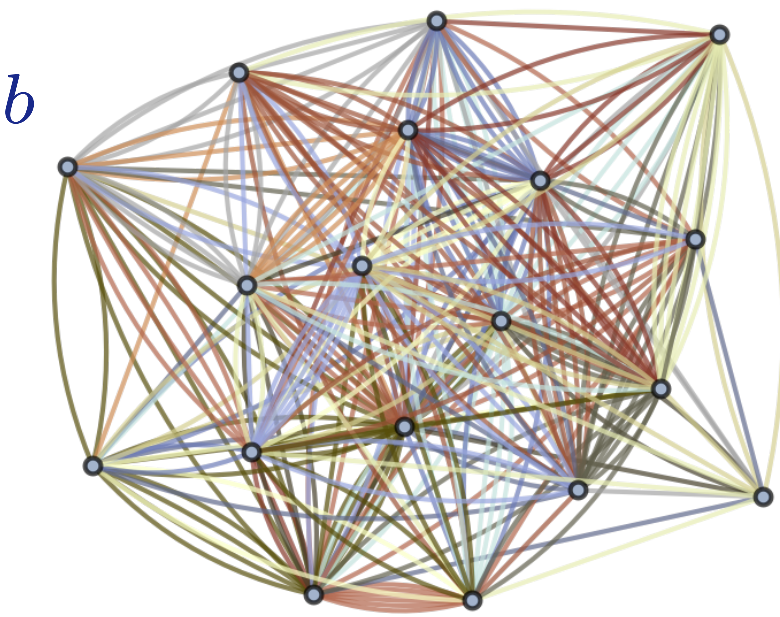
# The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{a,b,c,d=1}^{N} U_{ab;cd}\, c_a^\dagger c_b^\dagger c_c c_d - \mu \sum_a c_a^\dagger c_a$$

$$c_a c_b + c_b c_a = 0 \quad , \quad c_a c_b^\dagger + c_b^\dagger c_a = \delta_{ab}$$

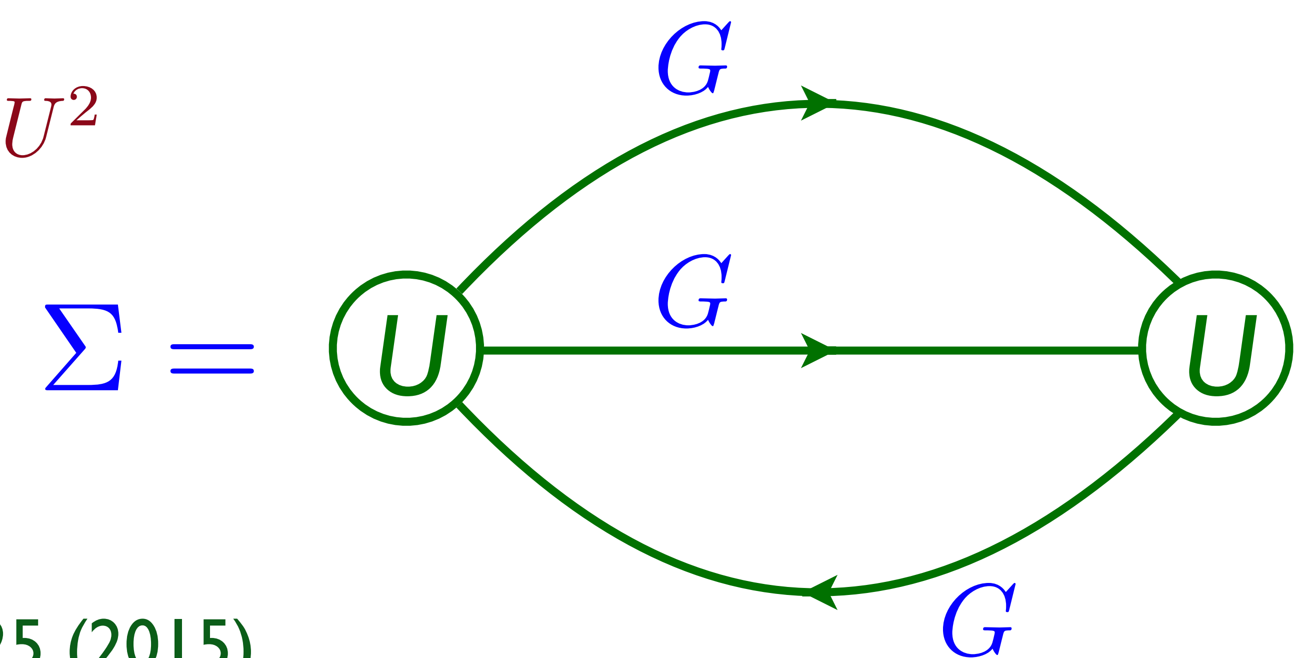
$$\mathcal{Q} = \frac{1}{N} \sum_a c_a^\dagger c_a$$



$U_{ab;cd}$ are independent random variables

with $\overline{U_{ab;cd}} = 0$ and $\overline{|U_{ab;cd}|^2} = U^2$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)}$$




S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# The complex SYK model

**Key properties**

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $\mathcal{Q} = 1/2$.

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PRL **70**, 3339 (1993)

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2. There is a non-zero extensive entropy as $T \to 0$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = \mathcal{S}_0(\mathcal{Q}) \neq 0$$

This entropy is *not* due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-N\mathcal{S}_0}$ down to the ground state.

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3. Thermal equilibration in a 'Planckian time' $\sim \hbar/(k_B T)$

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

# The complex SYK model

## Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. A Schwarzian action for this soft mode is implied by an emergent SL(2,R) symmetry. Specifically, the entropy is $S(T)/N = \mathcal{S}_0(\mathcal{Q}) + \gamma T$, where $\gamma$ propotional to the co-efficient of the Schwarzian.

A. Kitaev, KITP talk (2015)

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)

A. Kitaev and J. Suh, JHEP 183 (2018)

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$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

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$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll U$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations. The singular part of the self-energy and the Green's function obey

$$\int_0^\beta d\tau_2\, \Sigma_{\rm sing}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\Sigma_{\rm sing}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)$$

A. Kitaev, 2015

S. Sachdev, PRX **5,** 041025 (2015)

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5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\left\langle c_a^\dagger(t) c_b(0) c_a(t) c_b^\dagger(0) \right\rangle = C_0 + C_1 \left( \frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

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with $\lambda = 2\pi k_B T/\hbar$.

6. For spinful fermions, spin correlations decay as
$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim 1/|\tau|$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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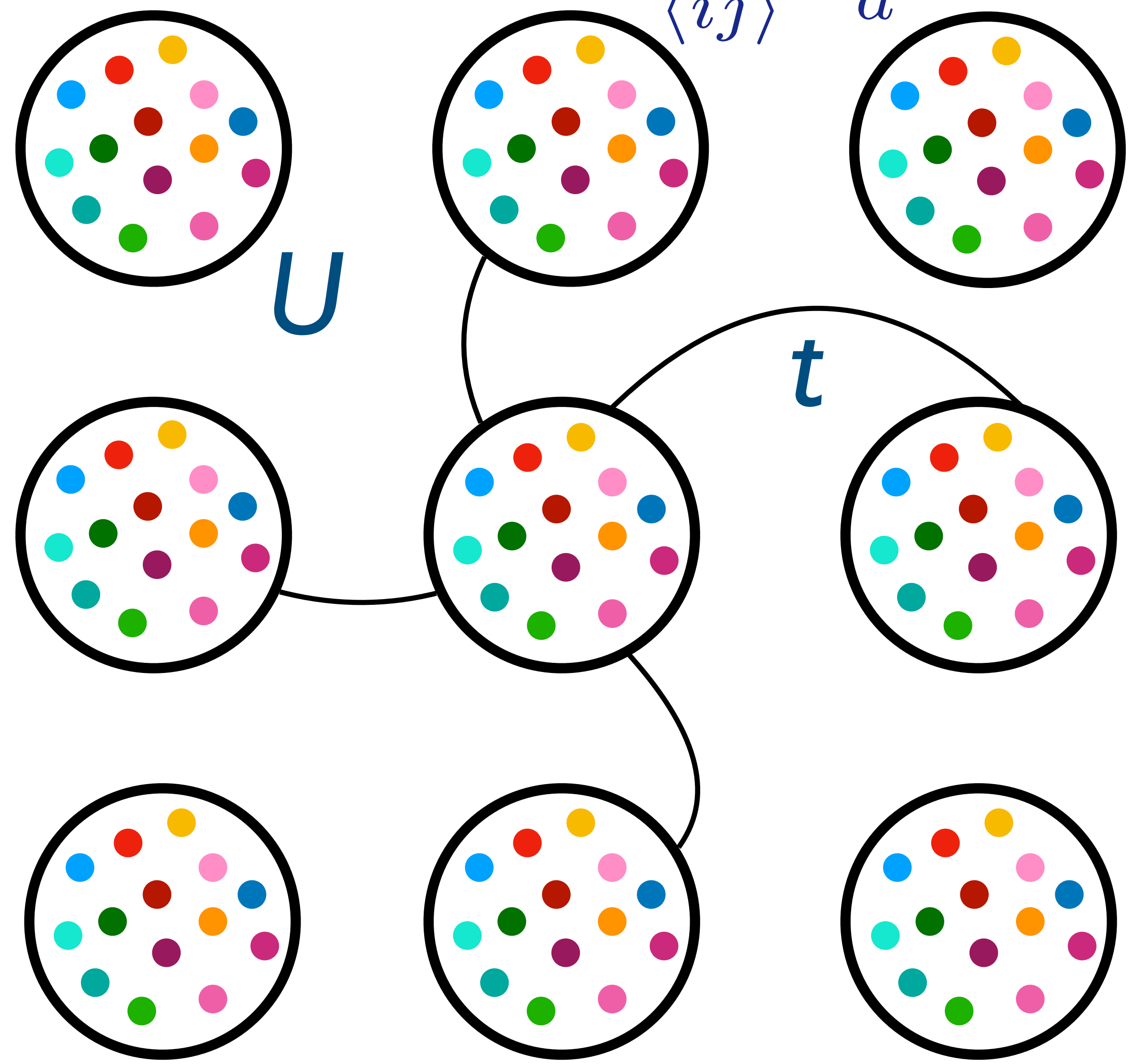
# A strange metal: lattice of SYK islands

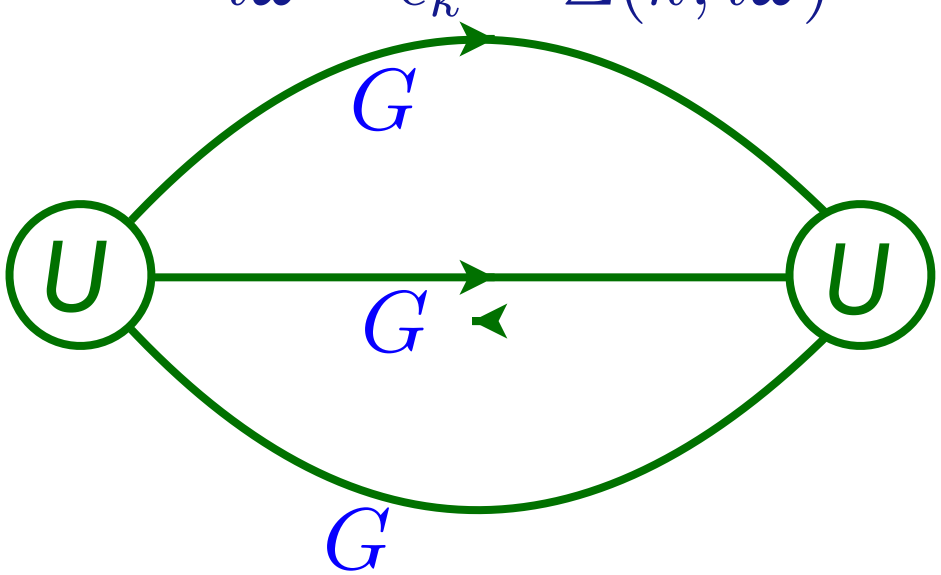
$$H = \frac{1}{(2N)^{3/2}} \sum_{i} \sum_{a,b,c,d=1}^{N} U_{i,ab;cd}\, c_{ia}^\dagger c_{ib}^\dagger c_{ic} c_{id} - t \sum_{\langle ij \rangle} \sum_{a} c_{ia}^\dagger c_{ja}$$

Random interaction within each island $U$.

Amplitude to hop between islands $t$.




$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$\Sigma =$ 


Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

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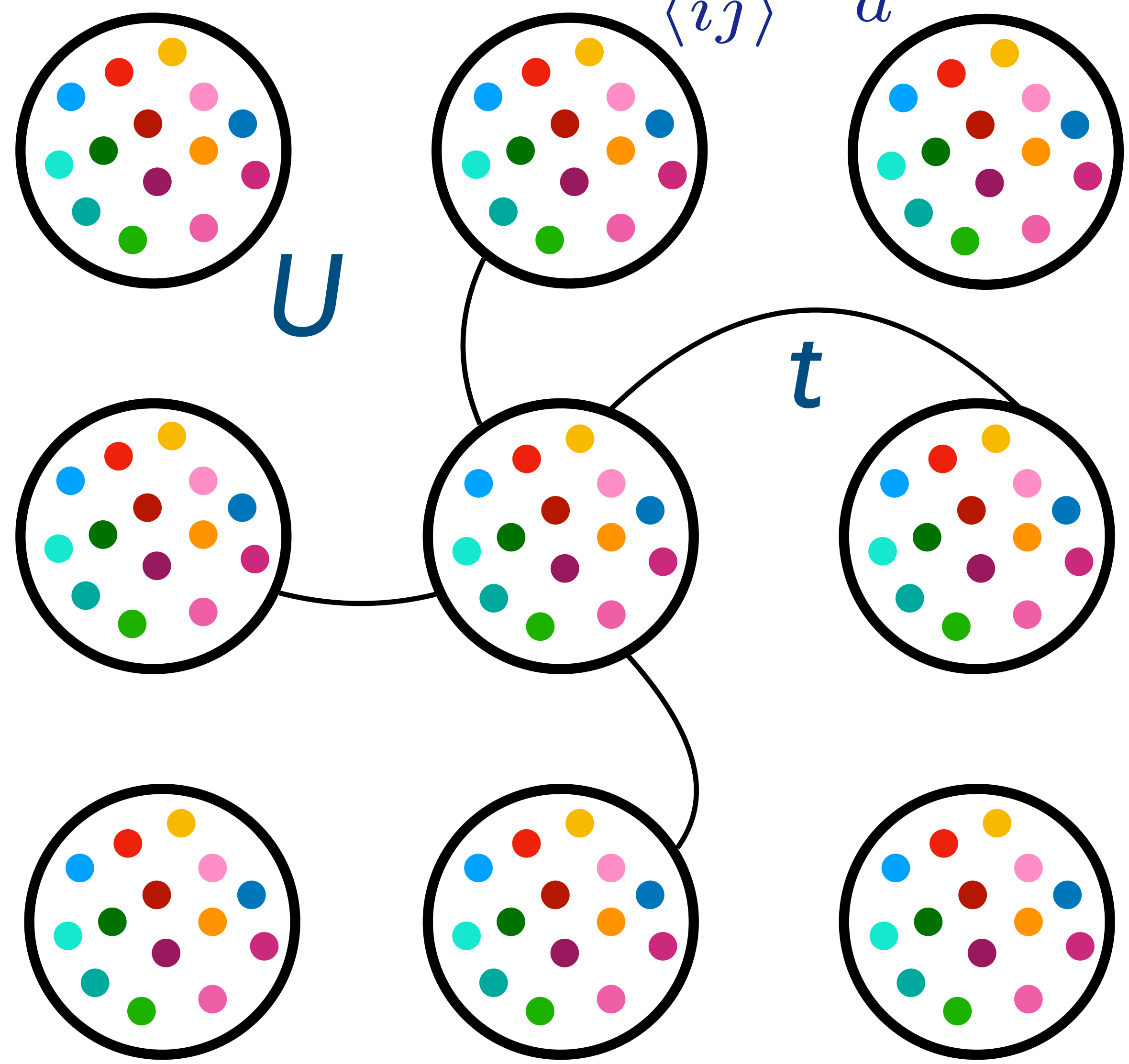
Model yields SYK criticality and resistivity

$$\rho \sim T$$

for $t^2/U \lesssim T \lesssim U$

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# The $t$-$J$ model

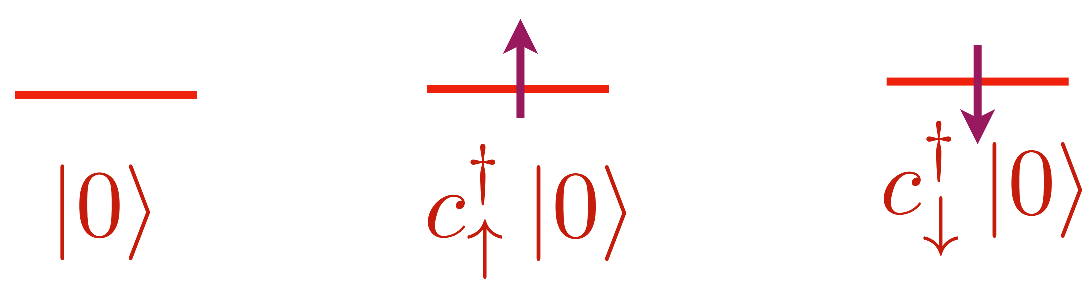
$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c^\dagger_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \boxed{\sum_\alpha c^\dagger_{i\alpha} c_{i\alpha} \leq 1,} \quad \sum_\alpha \left\langle c^\dagger_{i\alpha} c_{i\alpha} \right\rangle = 1 - p$$

We take the large $z$ limit in a lattice with co-ordination number $z$ and $J_{ij}$ random, $\overline{J_{ij}} = 0$, $\overline{J^2_{ij}} = J^2$



# Fractionalization in the *t*-*J* model

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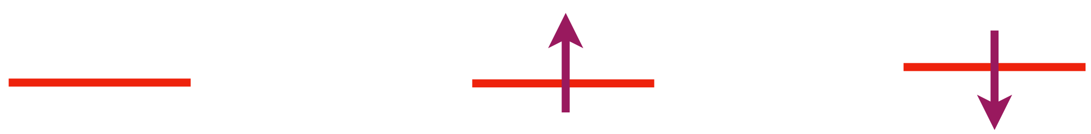
Each site has 3 states which we map to the '*superspin*' space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):



$b^\dagger \left| v \right\rangle$ $\quad$ $f_\uparrow^\dagger \left| v \right\rangle$ $\quad$ $f_\downarrow^\dagger \left| v \right\rangle$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \rightarrow b e^{i\phi} \, , \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$

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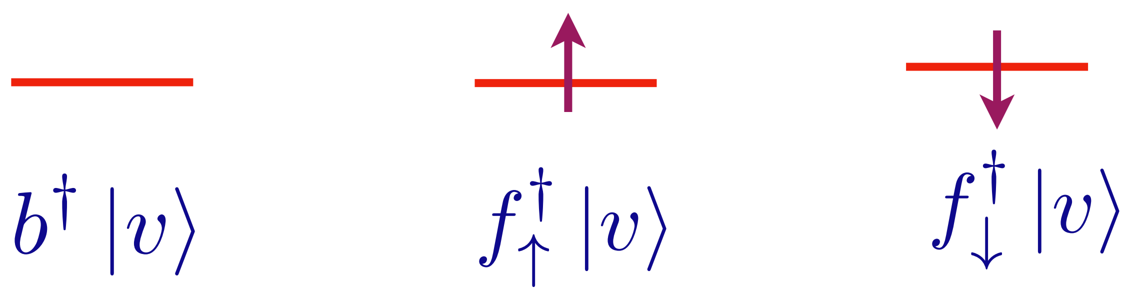


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U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(1|2) super-spin space; both $t$ and $J$ terms in $H$ are *quartic* in terms of fractionalized particles.

# Fractionalization in the $t$-$J$ model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} \mathfrak{f}_i \mathfrak{f}_j^\dagger \mathfrak{b}_{i\alpha}^\dagger \mathfrak{b}_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} \mathfrak{b}_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \mathfrak{b}_{i\beta} \cdot \mathfrak{b}_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \mathfrak{b}_{j\delta}$$

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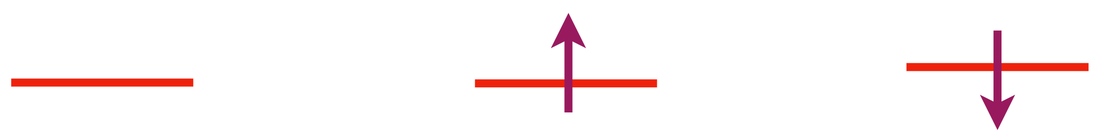

$\mathfrak{f}^\dagger |v\rangle$ $\quad$ $\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\quad$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(2|1) super-spin space; both $t$ and $J$ terms in $H$ are *quartic* in terms of fractionalized particles.

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

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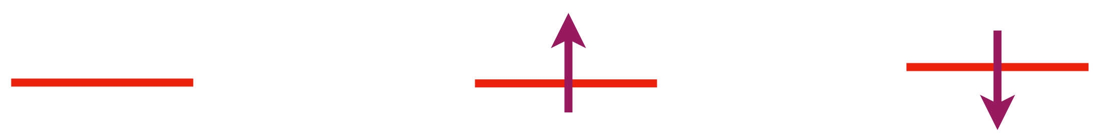

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SU(1|2) ≡ SU(2|1)

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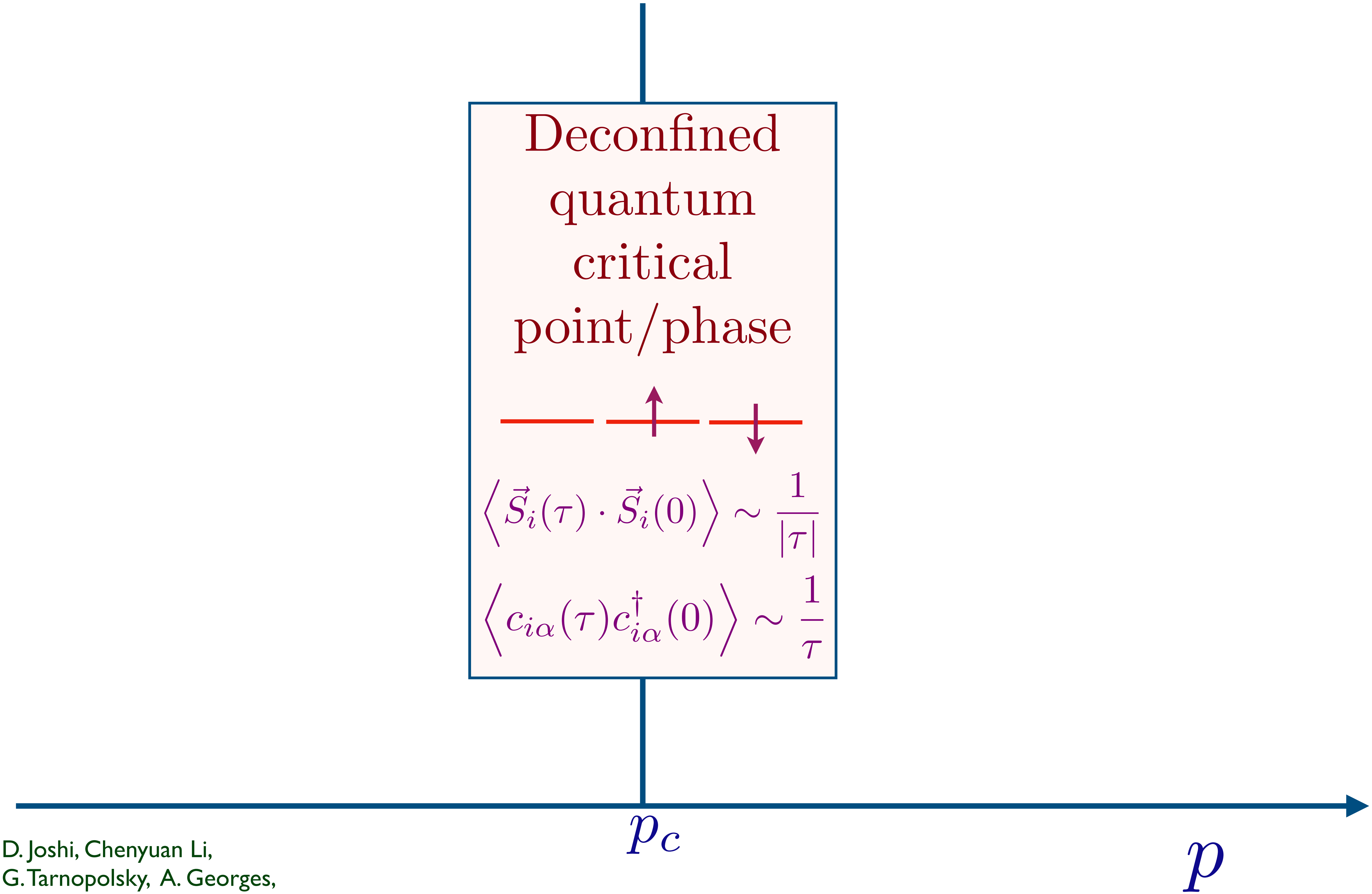
# $t$-$J$ phase diagram: RG using *either* SU(2|1) or SU(1|2)




D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

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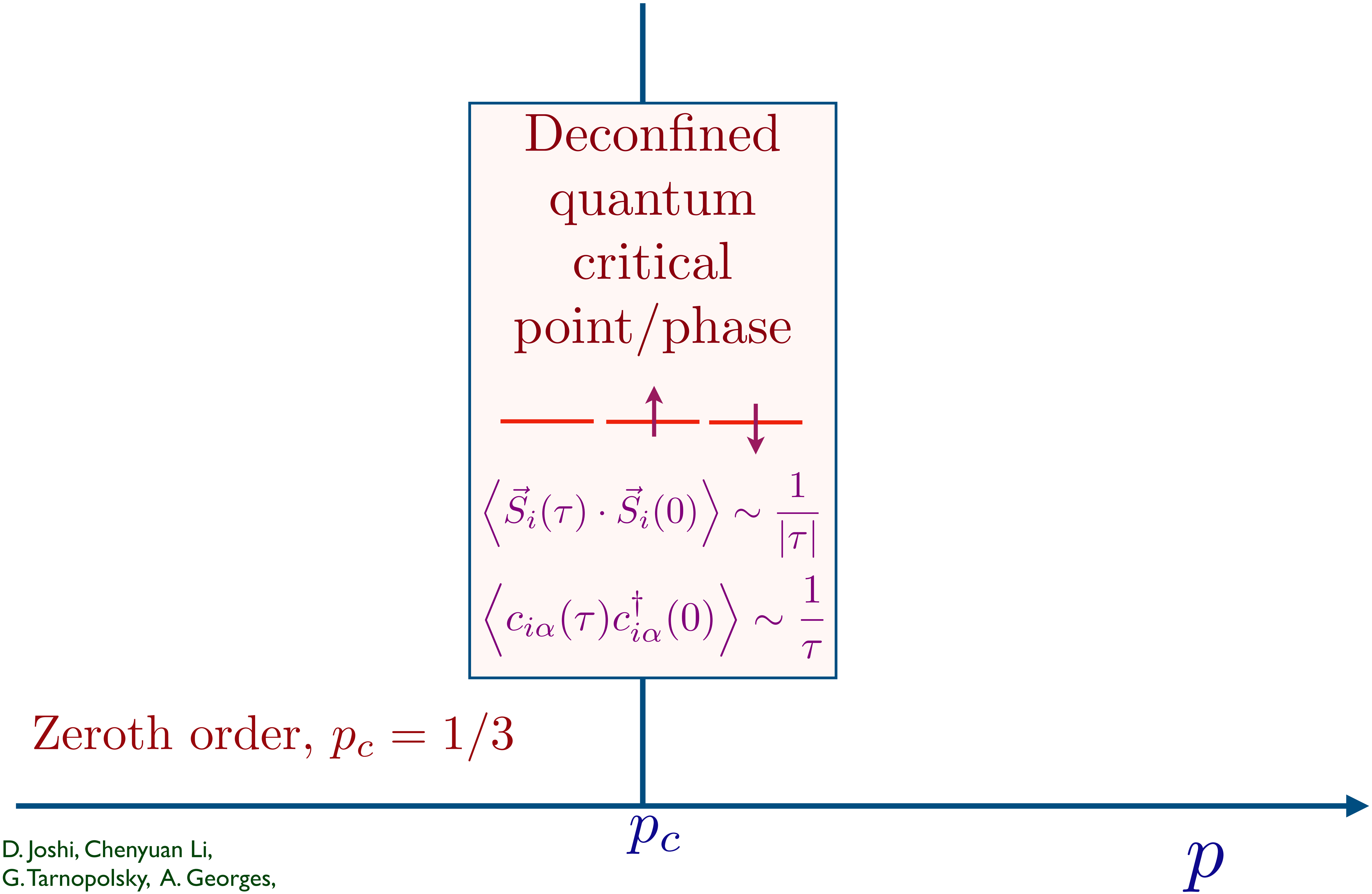


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PRX **10**, 021033 (2020)

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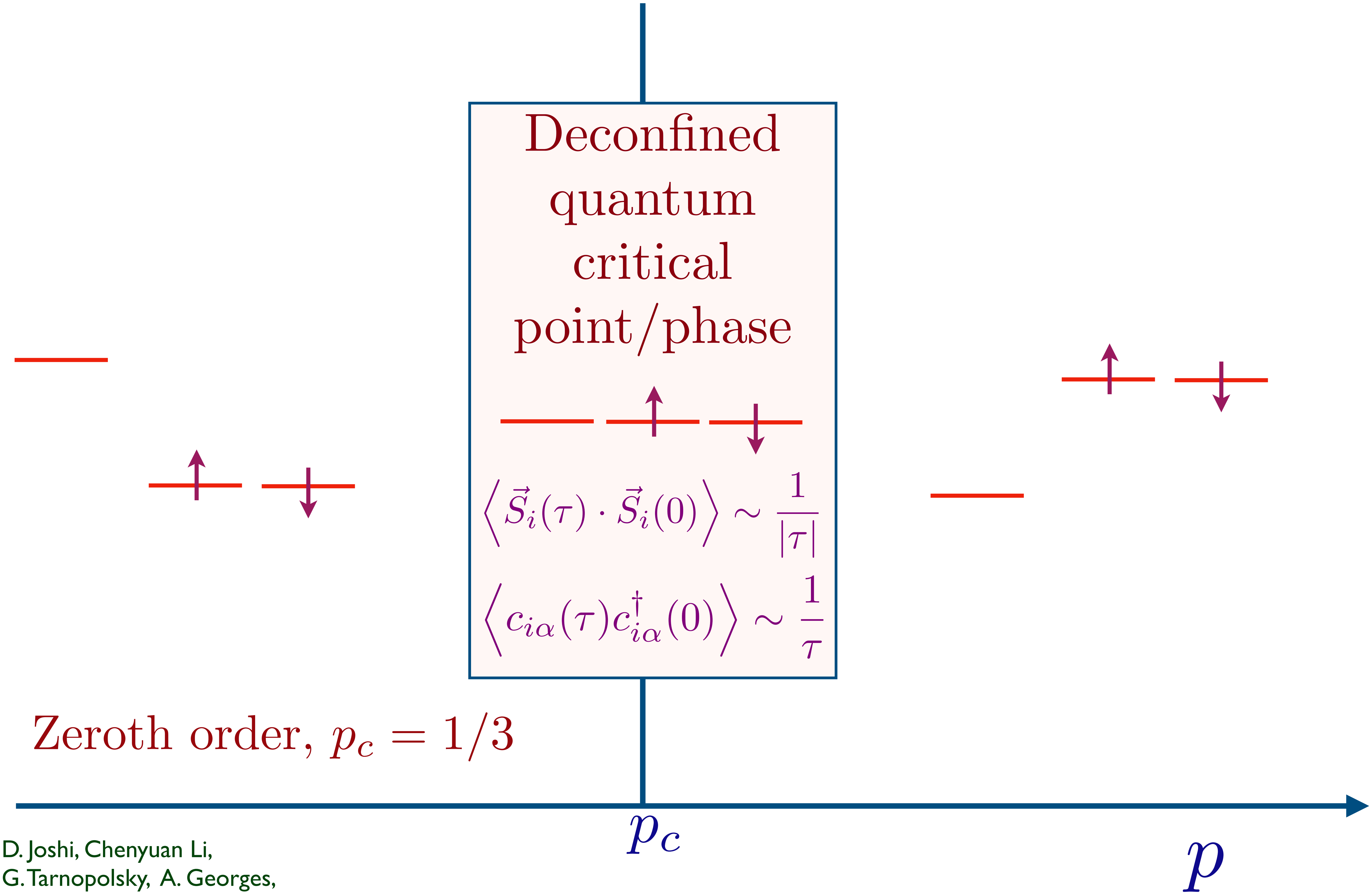


D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

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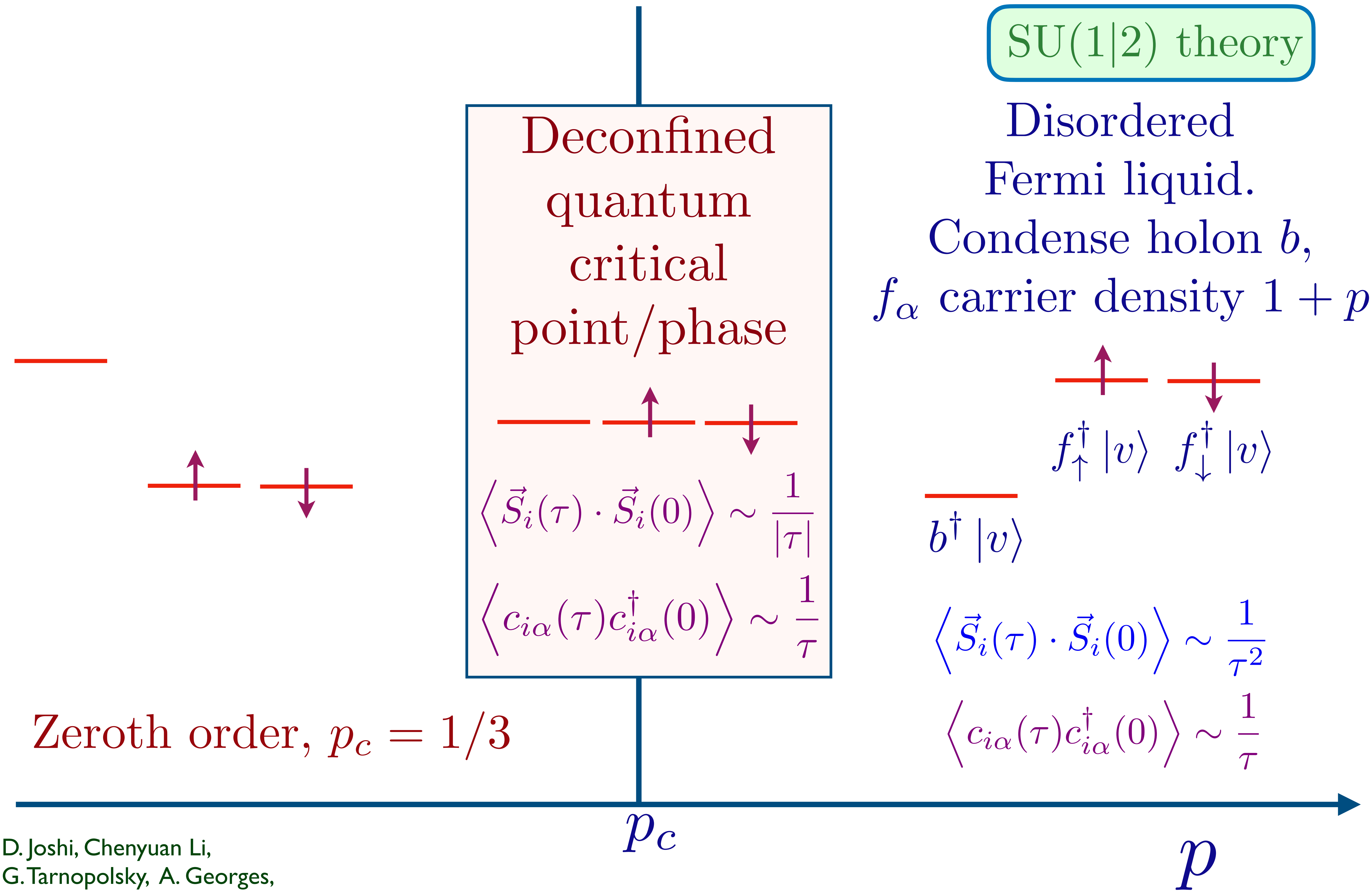


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# $t$-$J$ phase diagram: RG using *either* SU(2|1) or SU(1|2)

**SU(2|1) theory**

Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density $p$

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

**Deconfined quantum critical point/phase**

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

**SU(1|2) theory**

Disordered Fermi liquid.
Condense holon $b$,
$f_\alpha$ carrier density $1 + p$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

$p_c$ $p$

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# Fractionalization in the $t$-$J$ model

## Large $M$ limit of $\text{SU}(M'|M)$ theory

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

Here $\mu_f$ and $\mu_b$ are chemical potentials chosen to satisfy

$$
\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p \,.
$$

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# Fractionalization in the $t$-$J$ model

## Large $M$ limit of SU$(M'|M)$ theory

The critical solution which is self-consistent in both the $t$ and $J$ terms has $\Delta_b = \Delta_f = 1/4$, implying

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \begin{cases} \dfrac{A_+}{|\tau|} & , \quad \tau > 0 \\ -\dfrac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} .$$

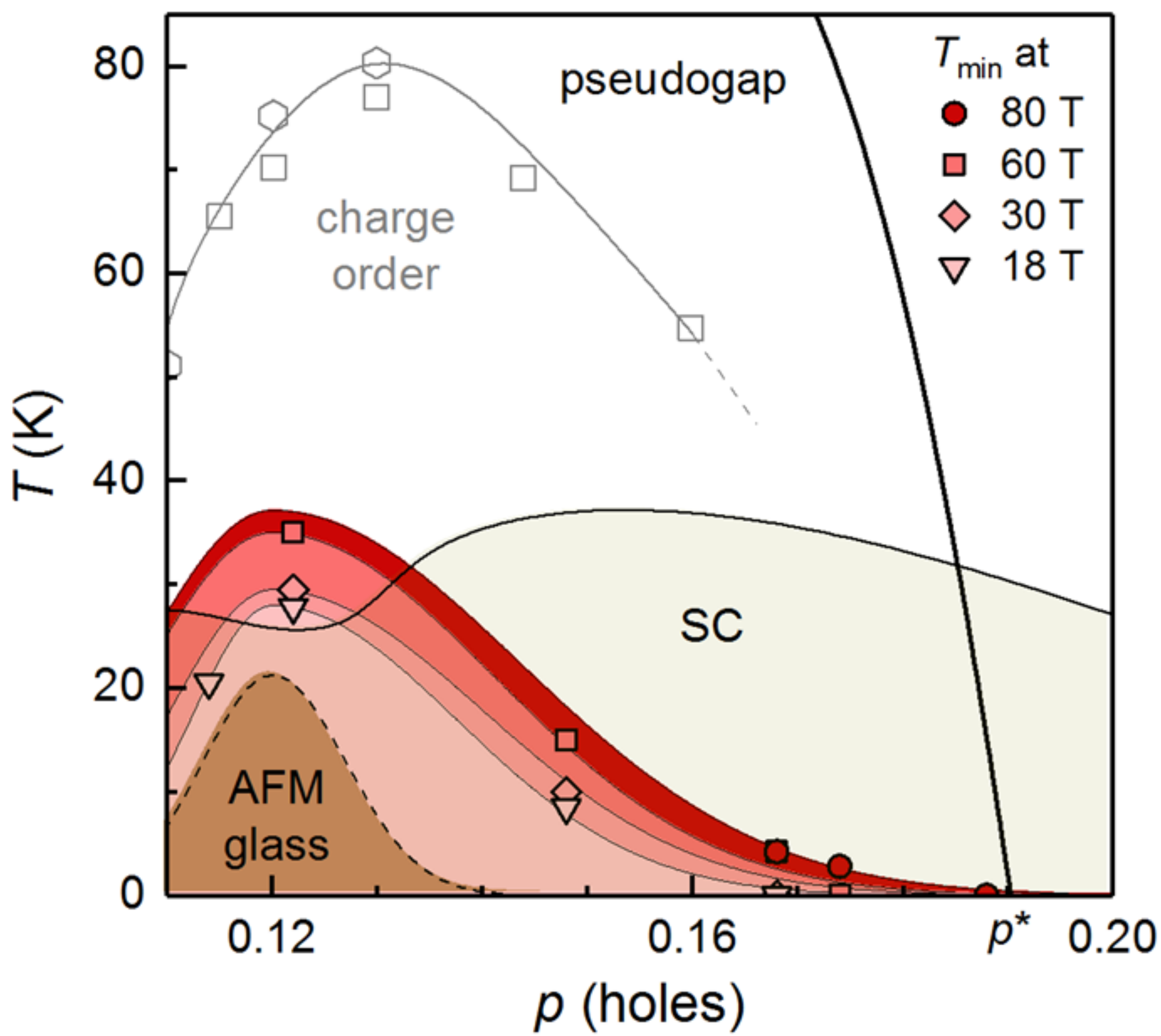
RG computations show that these results for the exponents of gauge-invariant operators are expected to be exact beyond the large $M$ limit.

D. Joshi, Chenyuan Li,
G. Tarnopolsky, A. Georges,
S. Sachdev,
PRX **10**, 021033 (2020)

# Hidden magnetism at the pseudogap critical point of a high temperature superconductor

arXiv:1909.10258

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]




**Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$.** Temperature – doping phase diagram representing $T_{min}$, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of $T_{min}$ in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature $T_f$ (see Fig. 2), $T_{min}$ is closely tied to $T_f$ and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

At the critical point/phase of the $t$-$J$ model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

At the critical point/phase of the $t$-$J$ model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c^{\dagger}_{i\alpha}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.

# Time reparameterization soft mode

The leading corrections to the SL(2,R) invariant critical Green's function arise from the time reparameterization soft mode, and these take the form

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{\pi T}{\sin(\pi T \tau)} \left( 1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large $M$ limit) scaling function, and $\alpha_G$ is universally proportional to the co-efficient $\alpha_S$ of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

# Time reparameterization soft mode

Finally, computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + 8\alpha_G \frac{T}{J} + \ldots\right)$$

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

# Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

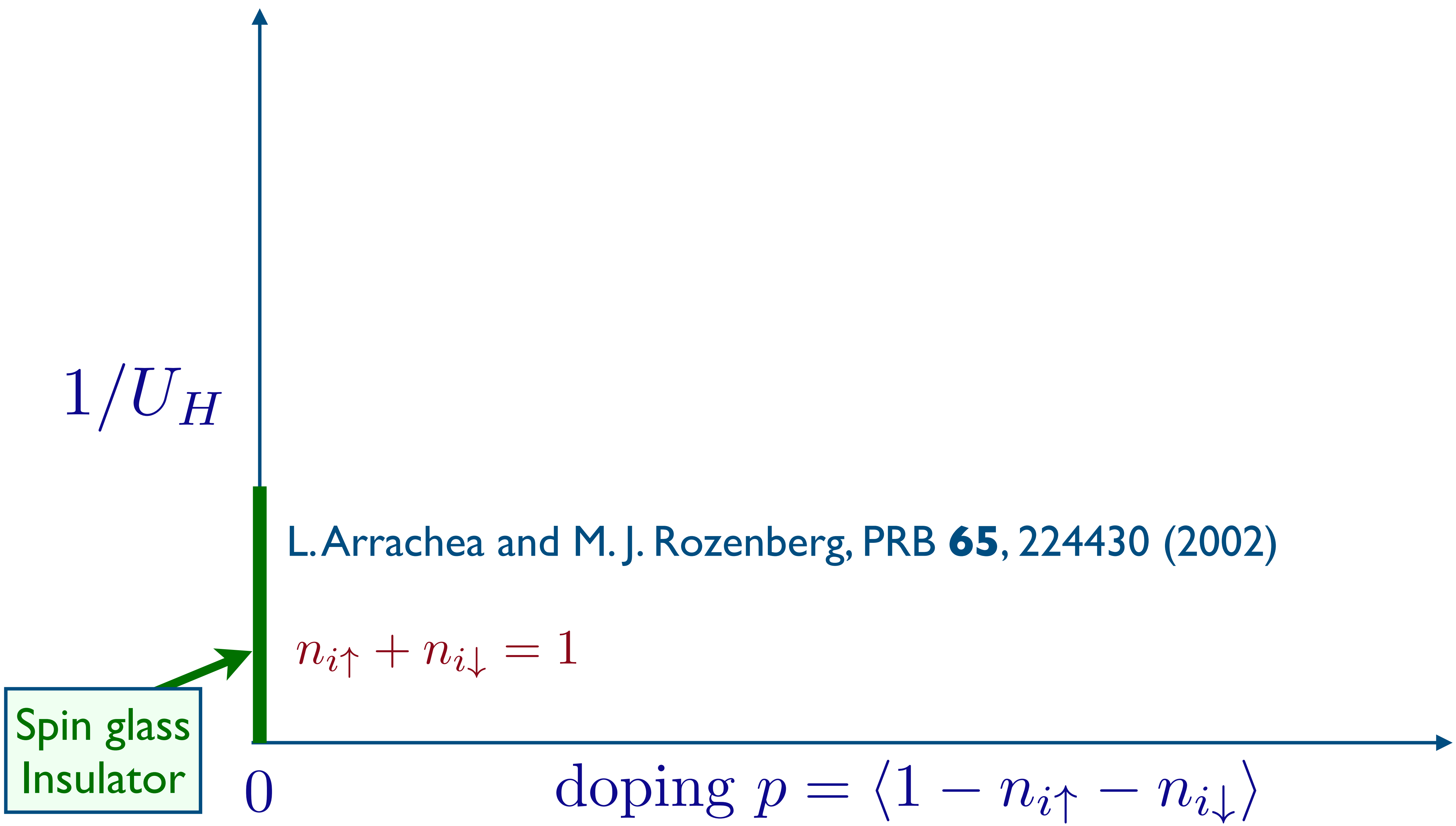
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$

$$U_H > 0 \text{ non-random}$$

# Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



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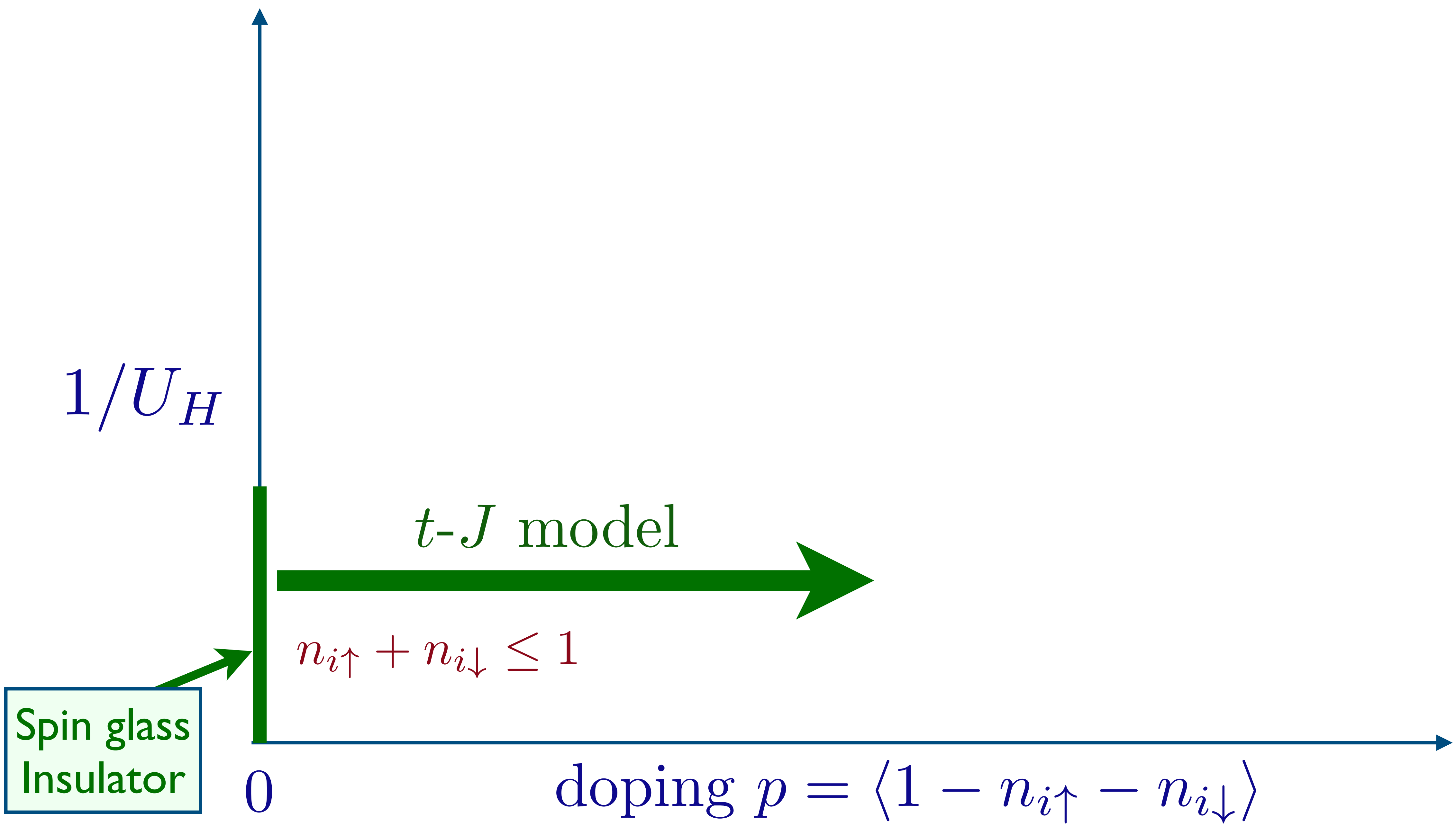

# Random $t$-$J$-$U_H$ model

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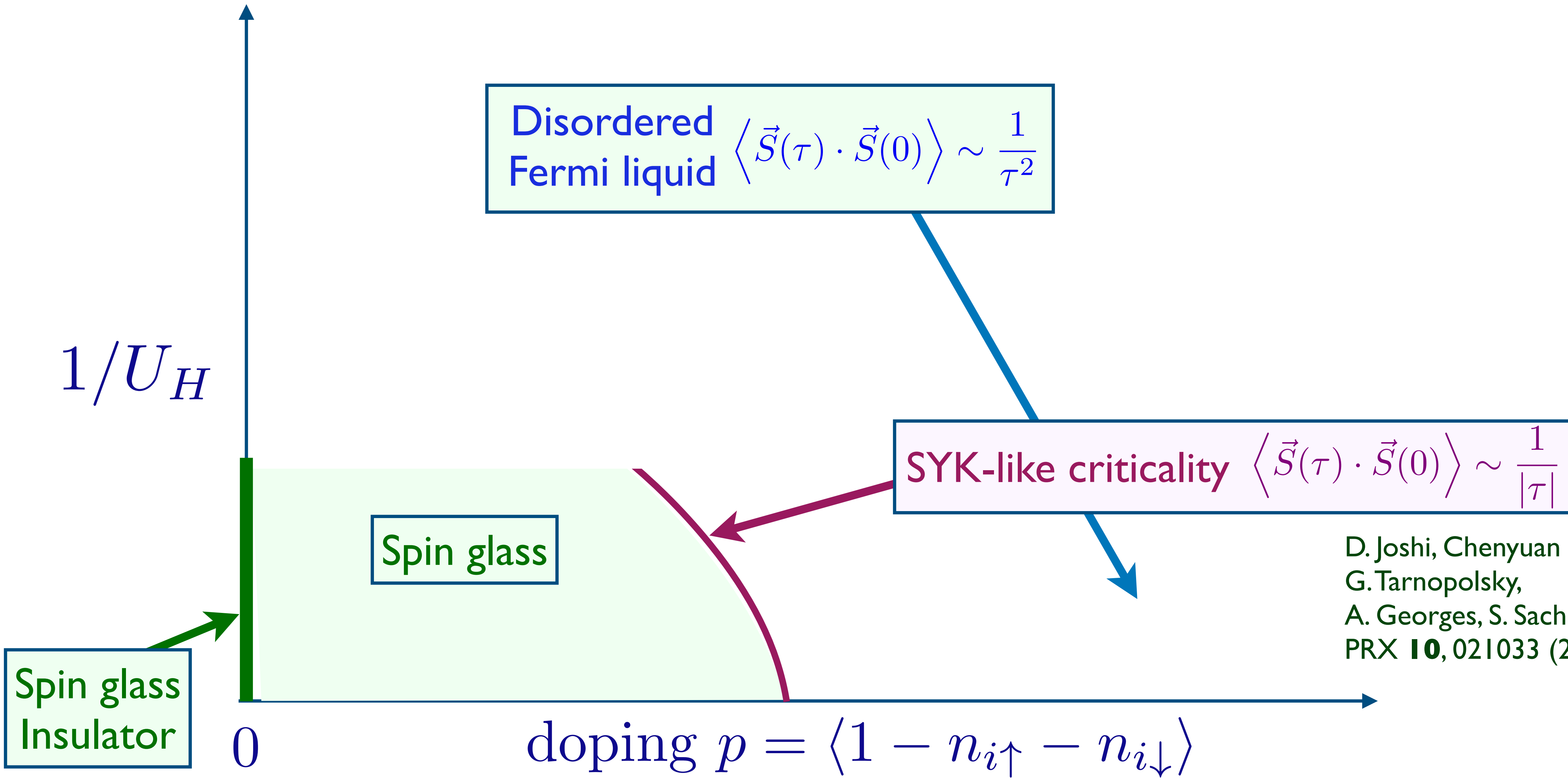


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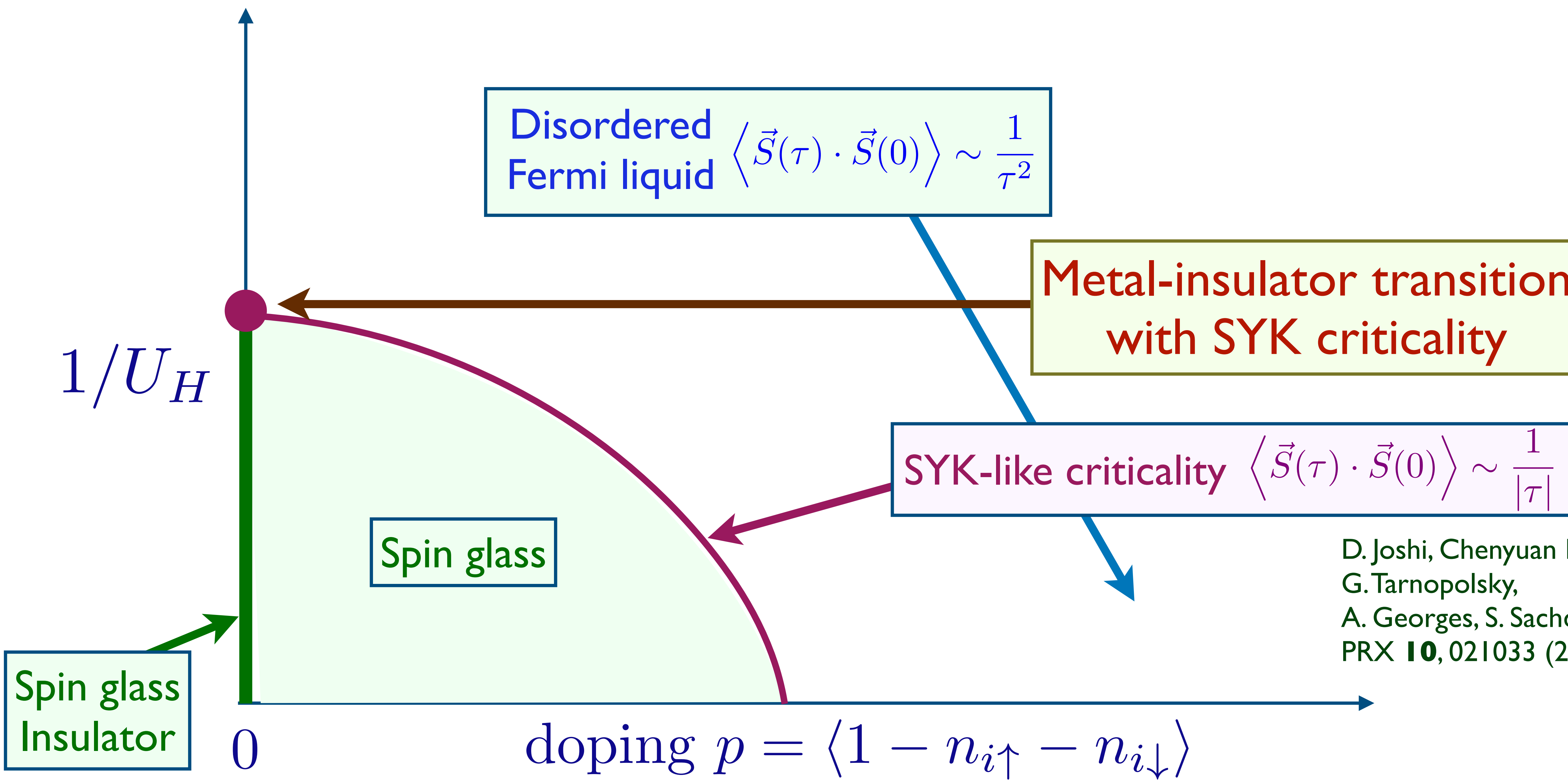
# Random *t*-*J*-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



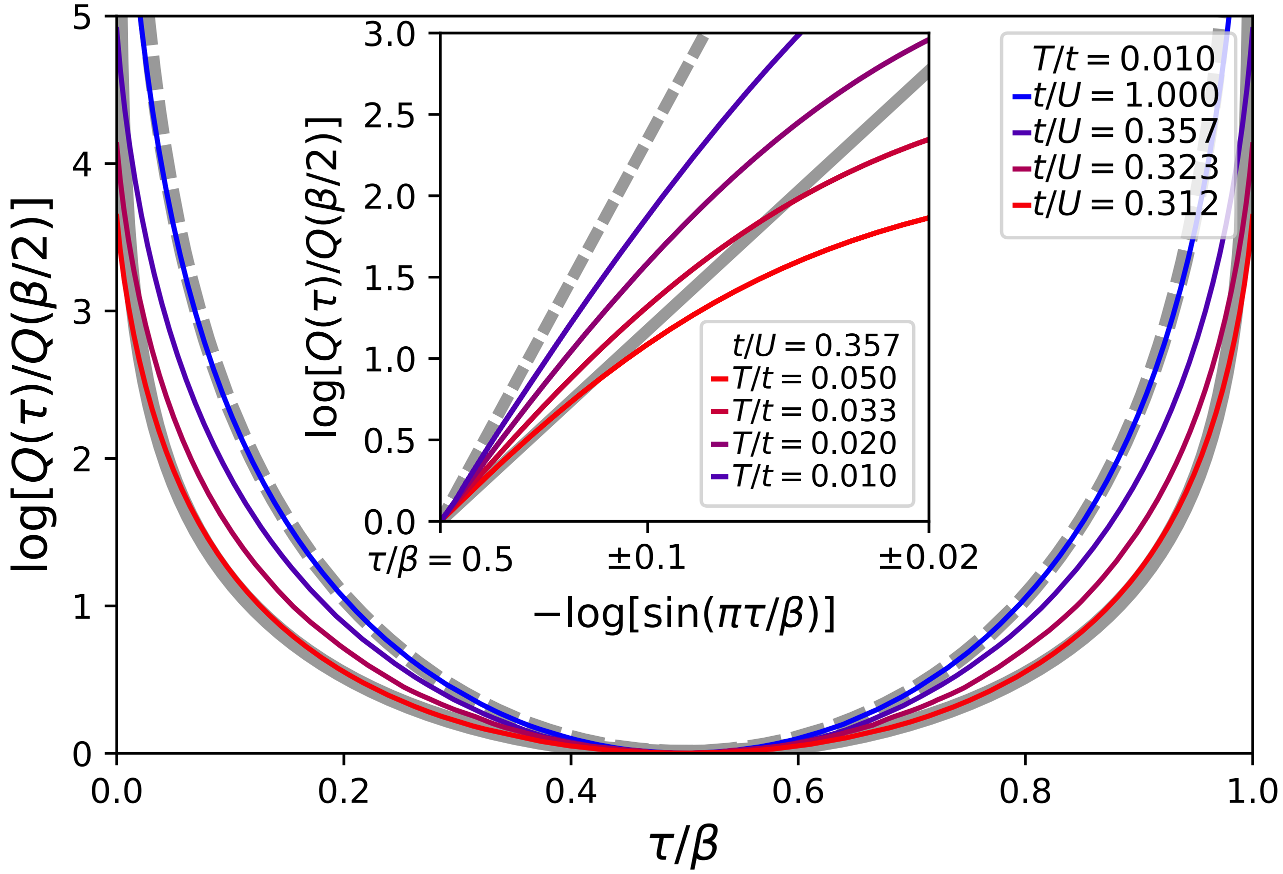

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

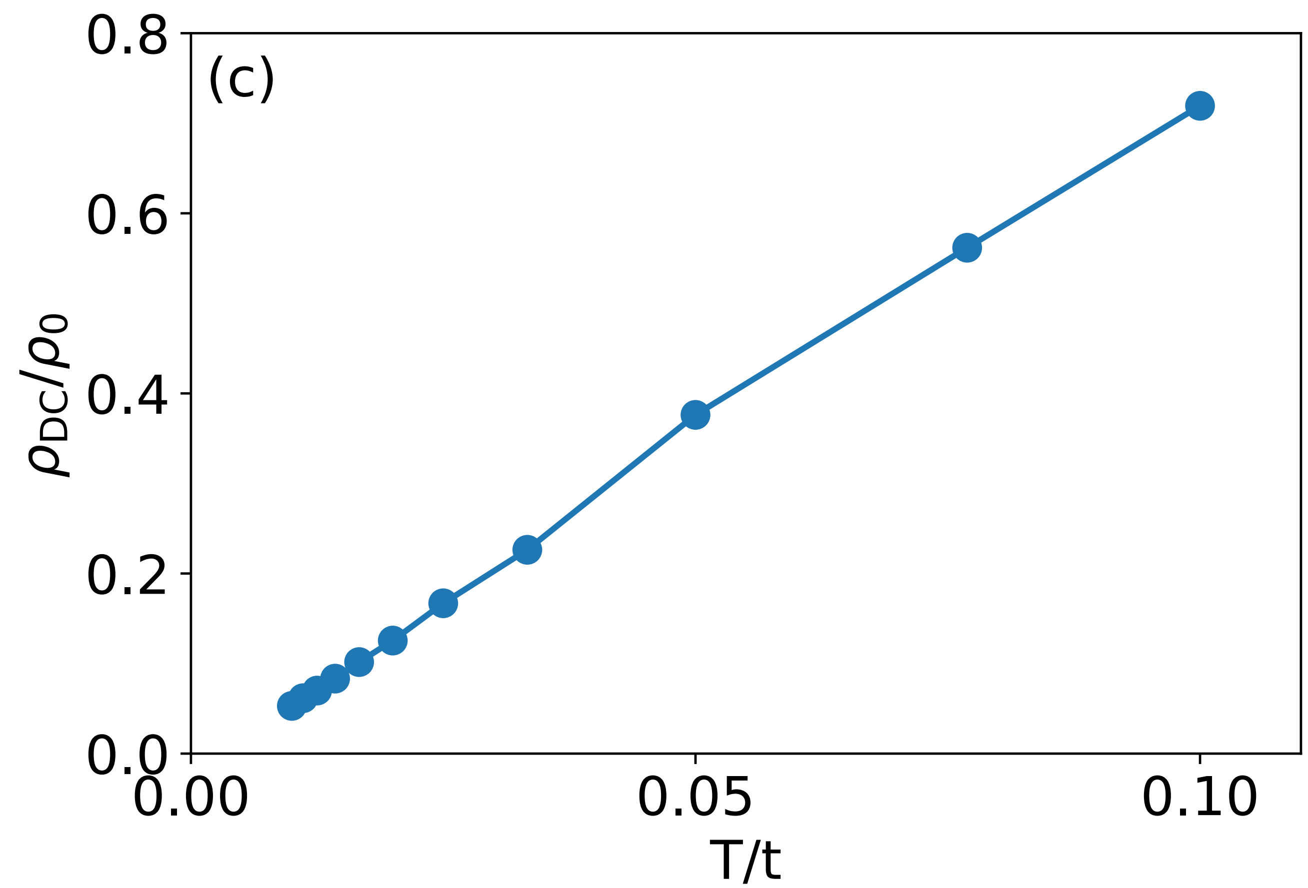
arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

# Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim




Critical spin correlations:

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$$

Resisitivity $\rho \sim T$ to the lowest $T$ at the critical point in a large-dimension model

# Challenge for theory:

A model of a metal in which the resistivity, $\rho$, obeys

$$\lim_{T \to 0} \frac{d\rho}{dT} \neq 0$$

$$\rho(T) = \rho(0) + AT + \ldots, \quad T \to 0\,.$$

## 2 key ingredients

1. Emergent gauge symmetry and fractionalization

$$c_\alpha = f_\alpha b^\dagger$$
$$f_\alpha \to e^{i\phi} f_\alpha \;,\; b \to b\, e^{i\phi}$$

2. Time reparameterization symmetry

$$\tau \to f(\tau)$$

Present in models of non-Fermi liquids:
quantum matter at variable density
without quasiparticle excitations