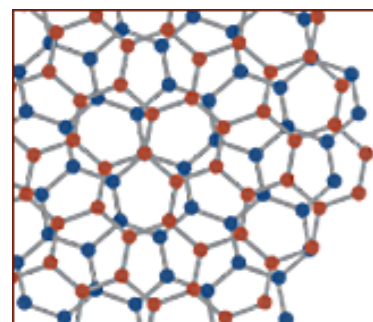


Planckian metals

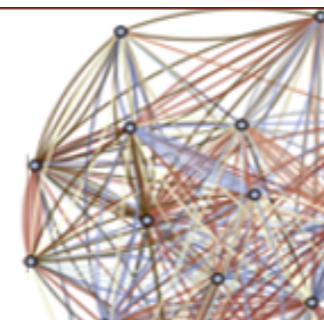
Aavishkar Patel and Subir Sachdev
PRL **123**, 066601 (2019)

Harvard University, September 12, 2019

Talk online: sachdev.physics.harvard.edu



Simons Collaboration on
Ultra-Quantum Matter



Universal T -linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142-147

A. Legros^{1,2}, S. Benhabib³, W. Tabis^{3,4}, F. Laliberté¹, M. Dion¹, M. Lizaire¹, B. Vignolle³, D. Vignolles³, H. Raffy⁵, Z. Z. Li⁵, P. Auban-Senzier⁵, N. Doiron-Leyraud¹, P. Fournier^{1,6}, D. Colson², L. Taillefer^{1,6*} and C. Proust^{3,6*}

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,^{1,2} Tristin Metz,² Christopher Eckberg,² Kevin Kirshenbaum,² Alex Hughes,² Renxiong Wang,² Limin Wang,² Shanta R. Saha,² I-Lin Liu,^{2,3,4} Nicholas P. Butch,^{2,4} Zhonghao Liu,^{5,6} Sergey V. Borisenko,⁵ Peter Y. Zavalij,⁷ and Johnpierre Paglione^{2,8}

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,^{1,*} Debanjan Chowdhury,^{1,*} Daniel Rodan-Legrain,¹ Oriol Rubies-Bigordà,¹ Kenji Watanabe,² Takashi Taniguchi,² T. Senthil,^{1,†} and Pablo Jarillo-Herrero^{1,†}

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown¹, Debayan Mitra¹, Elmer Guardado-Sanchez¹, Reza Nourafkan², Alexis Reymbaut², Charles-David Hébert², Simon Bergeron², A.-M. S. Tremblay^{2,3}, Jure Kokalj^{4,5}, David A. Huse¹, Peter Schauf^{1*}, Waseem S. Bakr^{1†}

Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ , is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

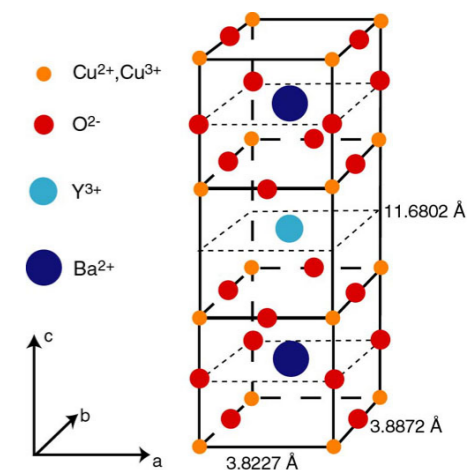
independent of the strength of interactions!



Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11 \text{ kbar}$	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of T -linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, *Nature Physics* **15**, 142 (2019)

1. SYK lattice models
2. Planckian metal ansatz
3. Resonant SYK models
4. Random t - J - U models

1. SYK lattice models

2. Planckian metal ansatz

3. Resonant SYK models

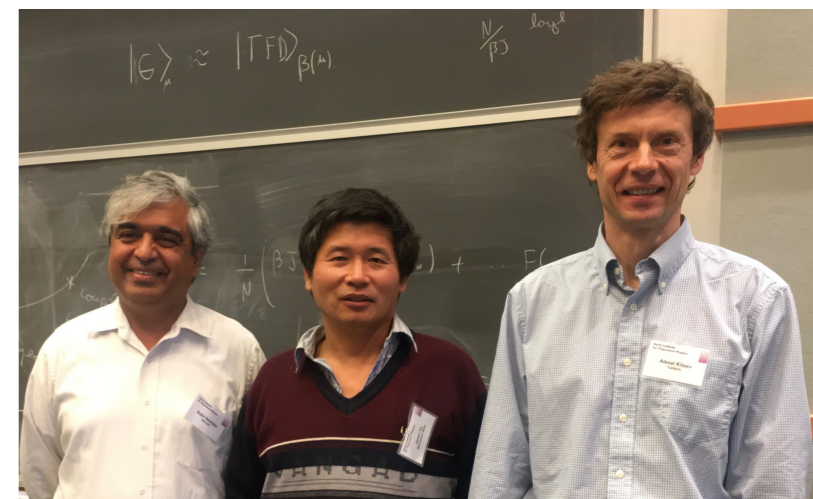
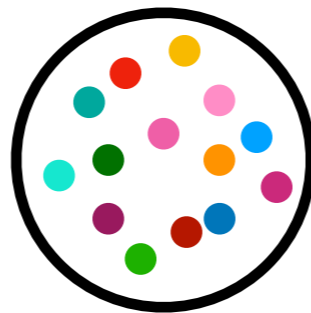
4. Random t - J - U models

The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} + \epsilon \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

There is a one-parameter family of critical solutions with varying Q , characterized by a dimensionless parameter \mathcal{E} .

For long times $\tau > 0$

$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = e^{\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = e^{-\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

\mathcal{E} determines the particle-hole asymmetry, and $A(\mathcal{E})$ is a known function.

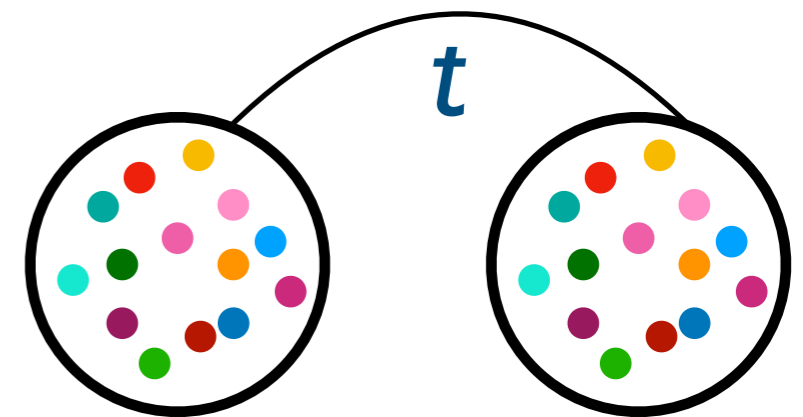
\mathcal{E} is determined by ϵ/U .

In a Fermi liquid,

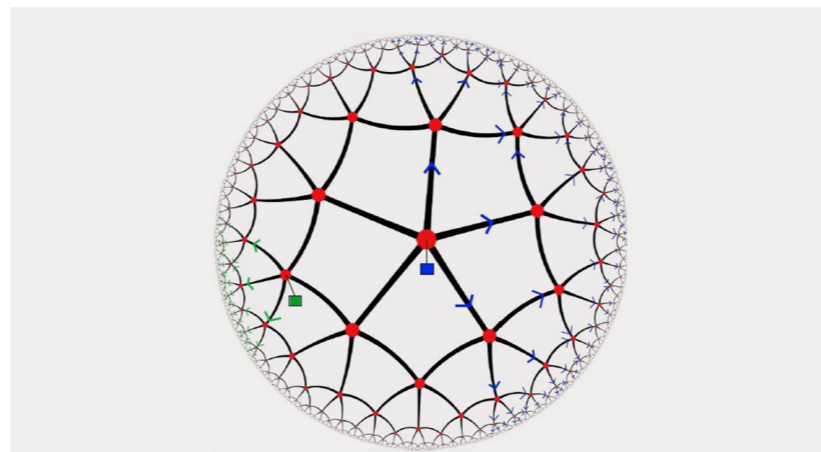
$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle = \langle c_\alpha^\dagger(\tau) c_\alpha(0) \rangle = \tilde{A}/\tau$$

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

Equivalent to an
 “eternal traversable wormhole” U
 between two black holes with
 AdS₂ horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491



A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

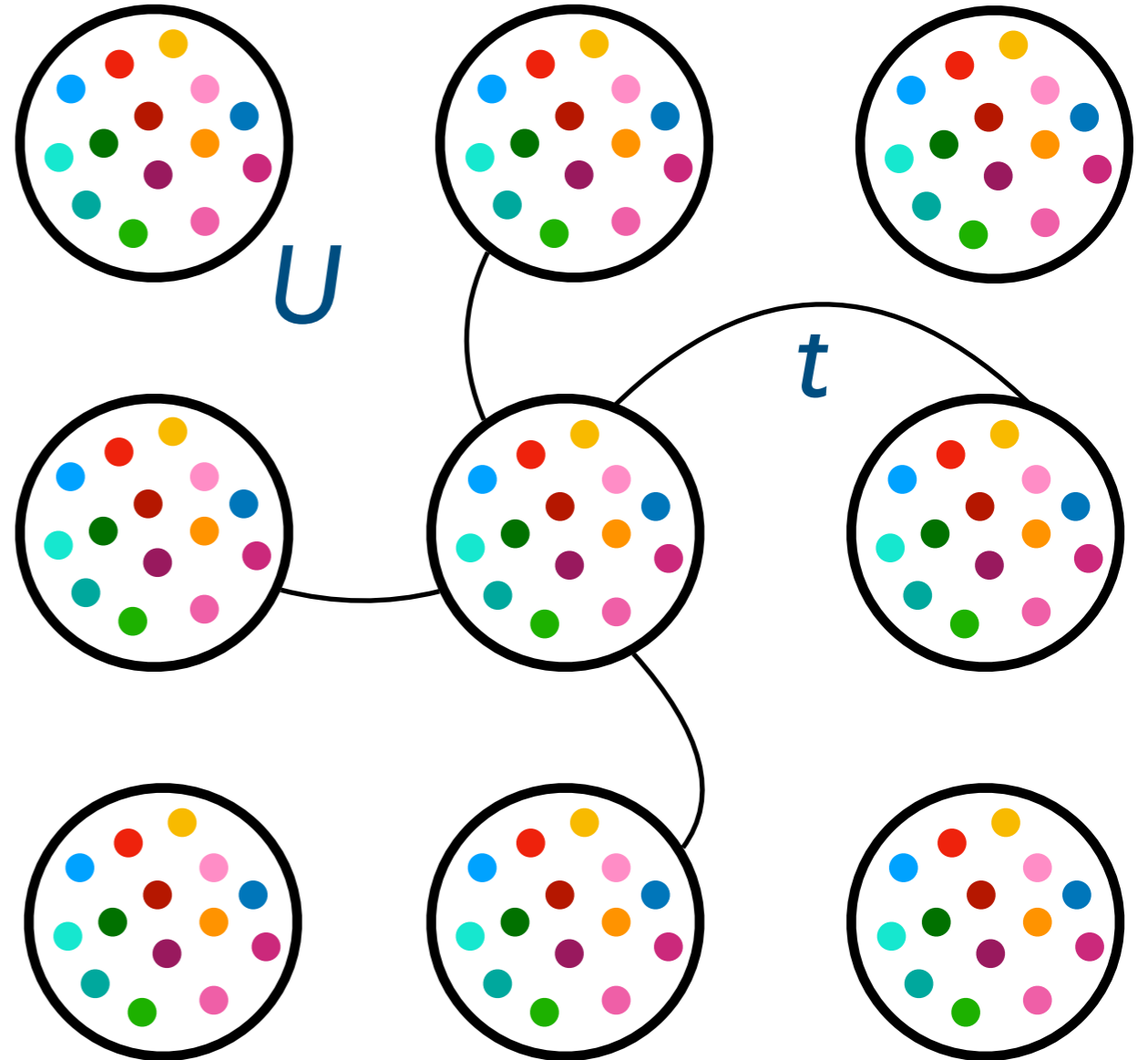
Choose $U \gg t$ on-site,
and the same on all sites;
yields ‘incoherent metal’
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\text{SYK}}(\epsilon, \hbar\omega / (k_B T))$$

independent of \mathbf{k} .

There is linear-in- T resistivity
but only with bad metal
behavior with $\rho > h/e^2$, and
co-efficient dependent upon U :

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);
Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman,
Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy,
Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999);
Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{i, \alpha\beta; \gamma\delta} c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c_{i\alpha}^\dagger c_{j\alpha}$$

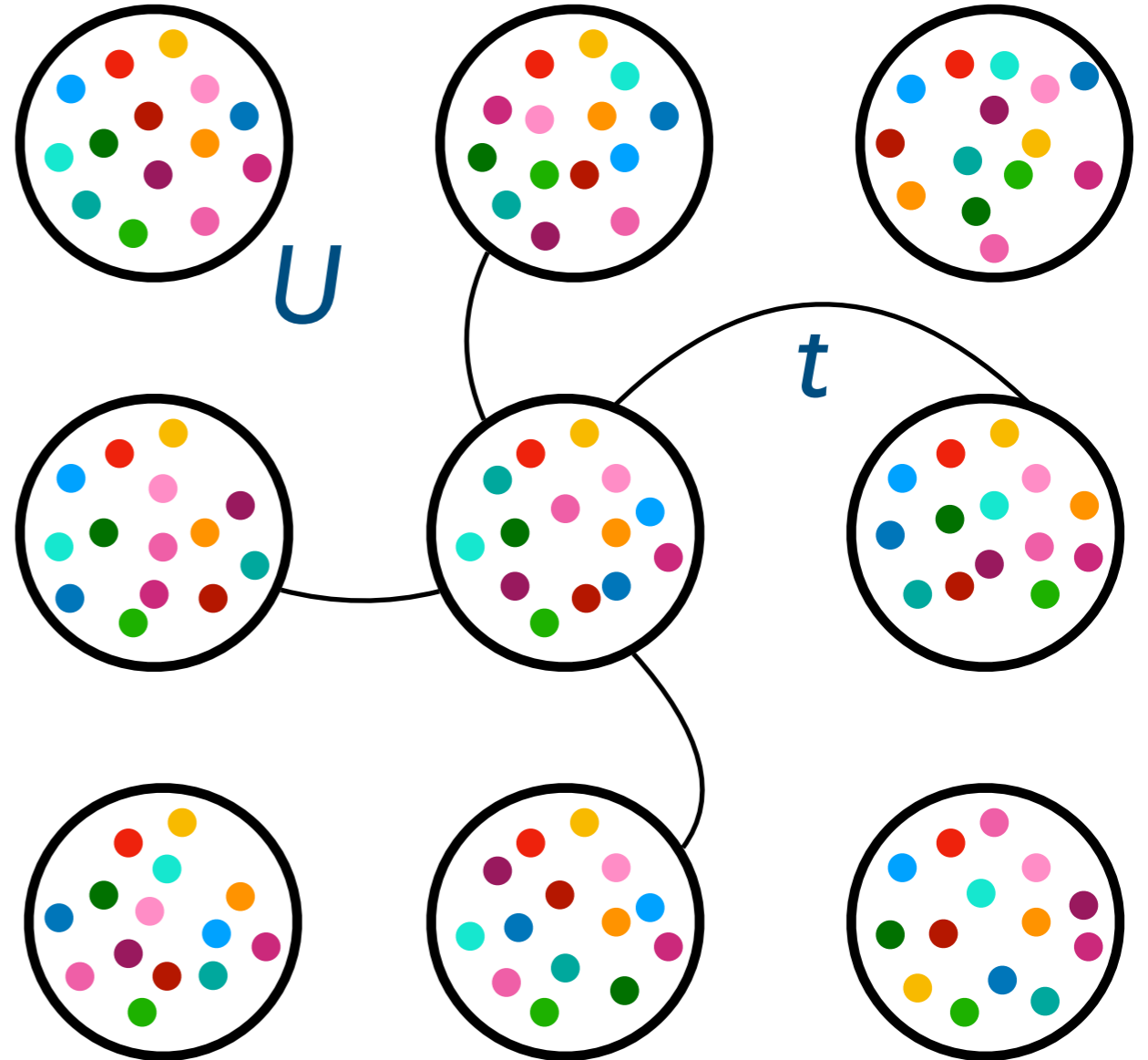
Choose $U \gg t$ on-site,
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1. SYK lattice models

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Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi\mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

The parameter \mathcal{E} is independent of k ,
and determined by the total density

Planckian metal ansatz with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}_k} \frac{A(\mathcal{E}_k)}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}_k} \frac{A(\mathcal{E}_k)}{\sqrt{U\tau}}$$



We choose a linear function $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$:
the particle-hole asymmetry changes as
we cross the Fermi surface.

$\mathbb{C} = 0.41$ for the SYK model.



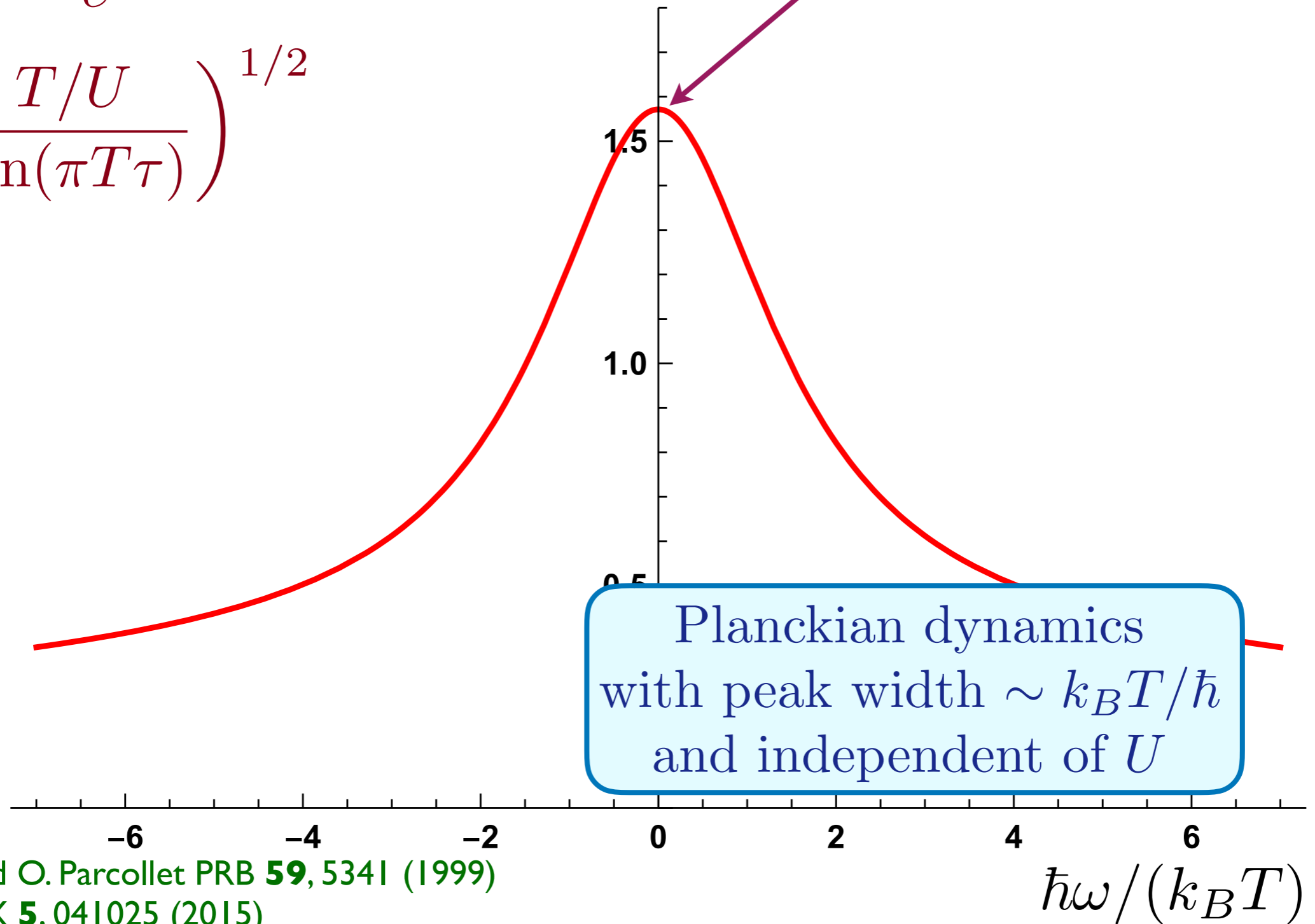
$G^R(\epsilon_k, \omega)$ is the
Fourier transform of

$$G(\epsilon_k, \tau) \sim e^{-2\pi\mathcal{E}_k T \tau}$$

$$\times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$



$G^R(\epsilon_k, \omega)$ is the Fourier transform of

$$G(\epsilon_k, \tau) \sim e^{-2\pi\mathcal{E}_k T \tau}$$

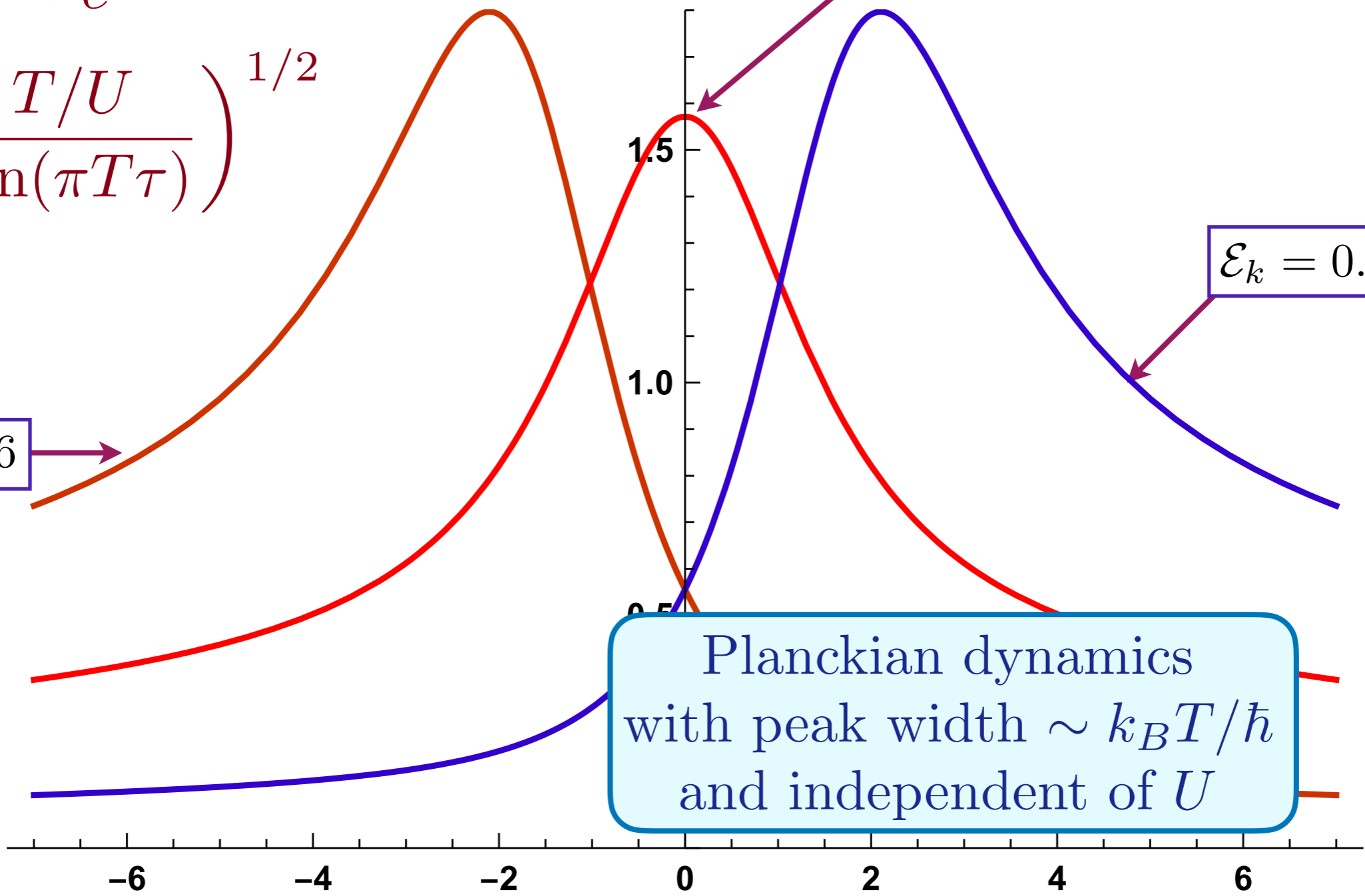
$$\times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$

$$\mathcal{E}_k = 0.26$$

$$\mathcal{E}_k = -0.26$$



Planckian dynamics with peak width $\sim k_B T / \hbar$ and independent of U

$G^R(\epsilon_k, \omega)$ is the Fourier transform of

$$G(\epsilon_k, \tau) \sim e^{-2\pi\mathcal{E}_k T \tau}$$

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$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$

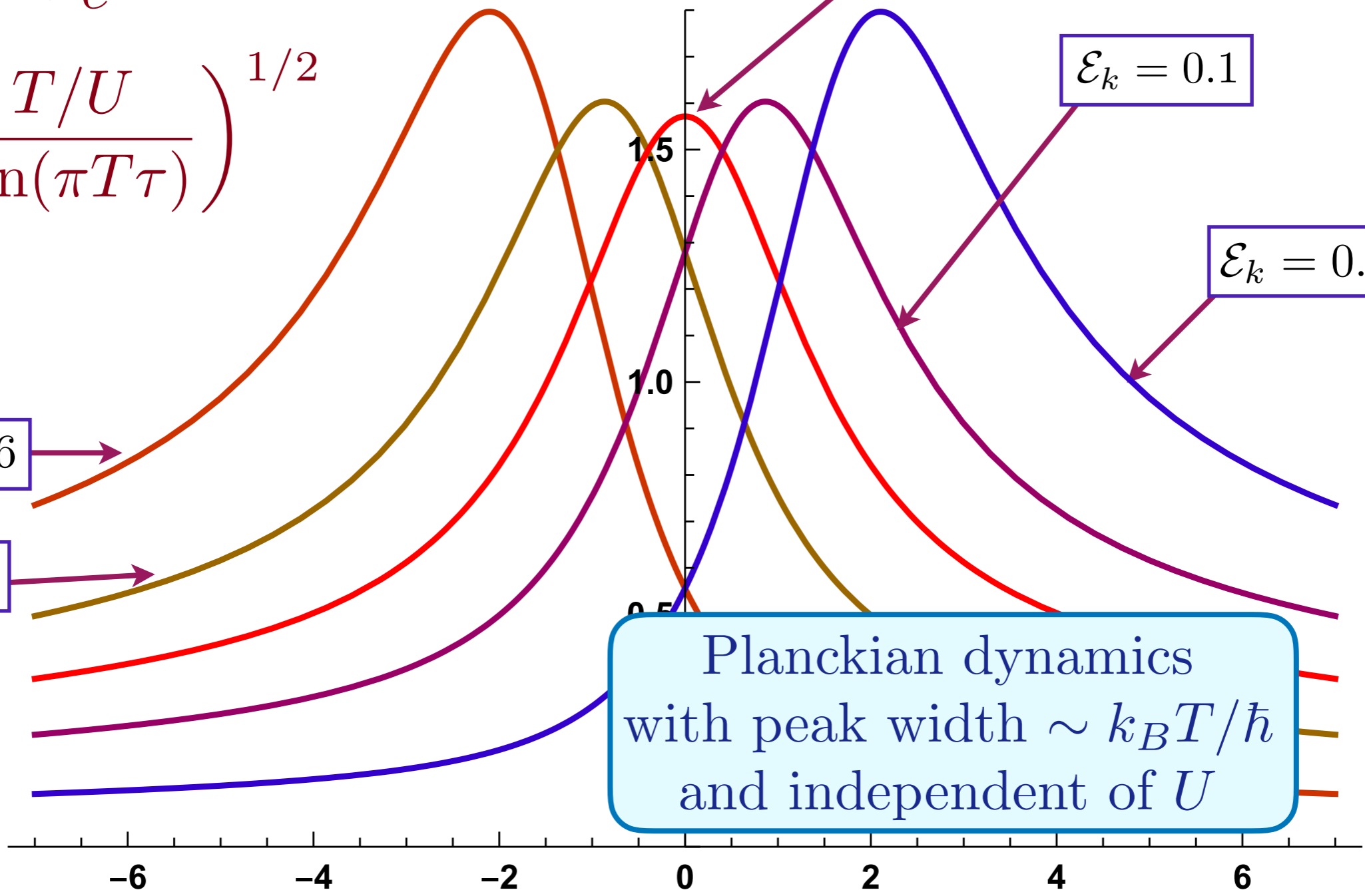
$$\mathcal{E}_k = 0.1$$

$$\mathcal{E}_k = 0.26$$

$$\mathcal{E}_k = -0.26$$

$$\mathcal{E}_k = -0.1$$

Planckian dynamics
with peak width $\sim k_B T / \hbar$
and independent of U



A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

$$\hbar\omega / (k_B T)$$

$G^R(\epsilon_k, \omega)$ is the Fourier transform of

$$G(\epsilon_k, \tau) \sim e^{-2\pi\mathcal{E}_k T \tau}$$

$$\times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E}_k = \mathbb{C} \frac{\epsilon_k}{U}$$

$$-\text{Im}G^R(\omega) \quad \mathcal{E}_k = 0$$

$$\mathcal{E}_k = 0.1$$

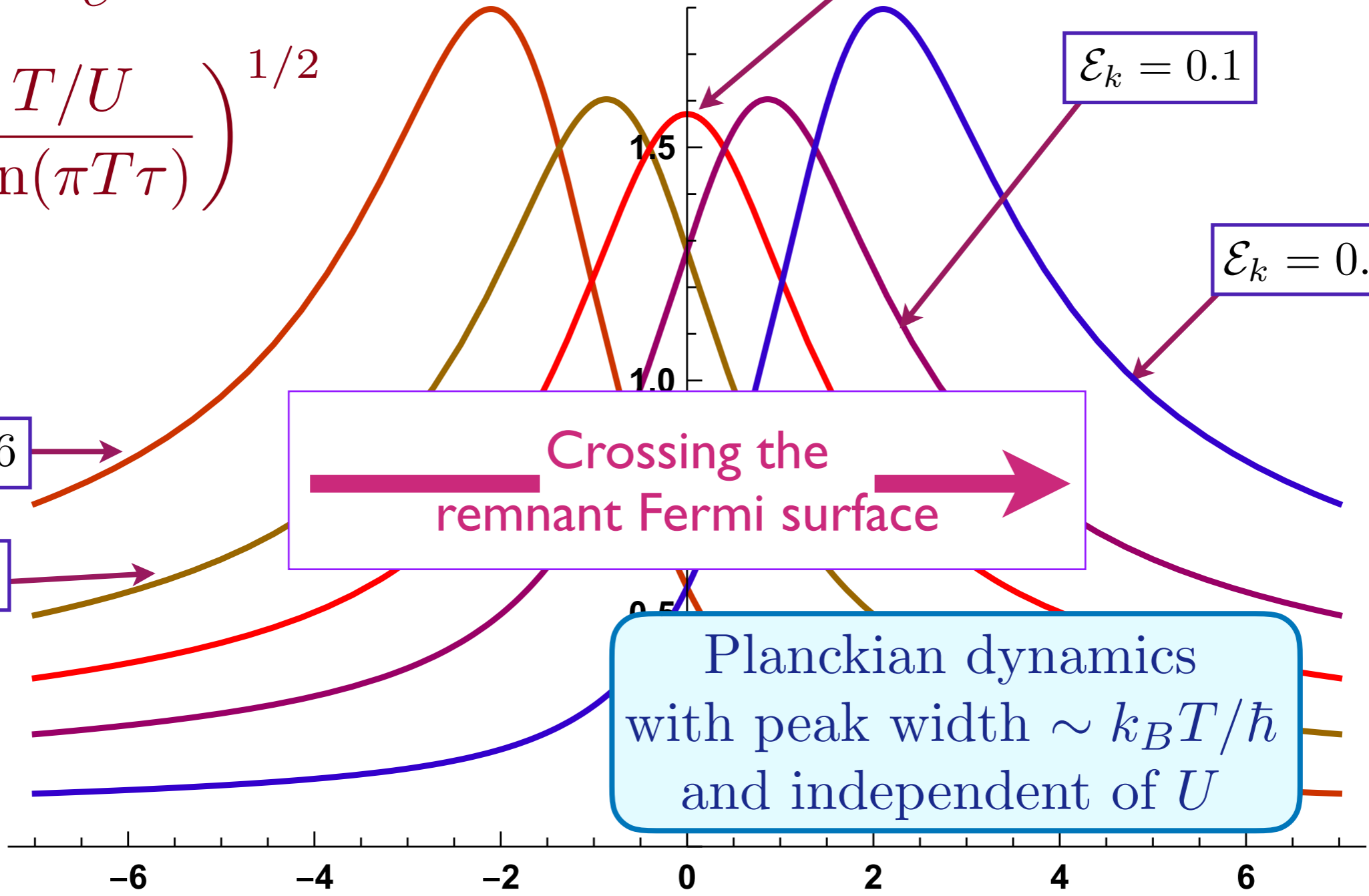
$$\mathcal{E}_k = 0.26$$

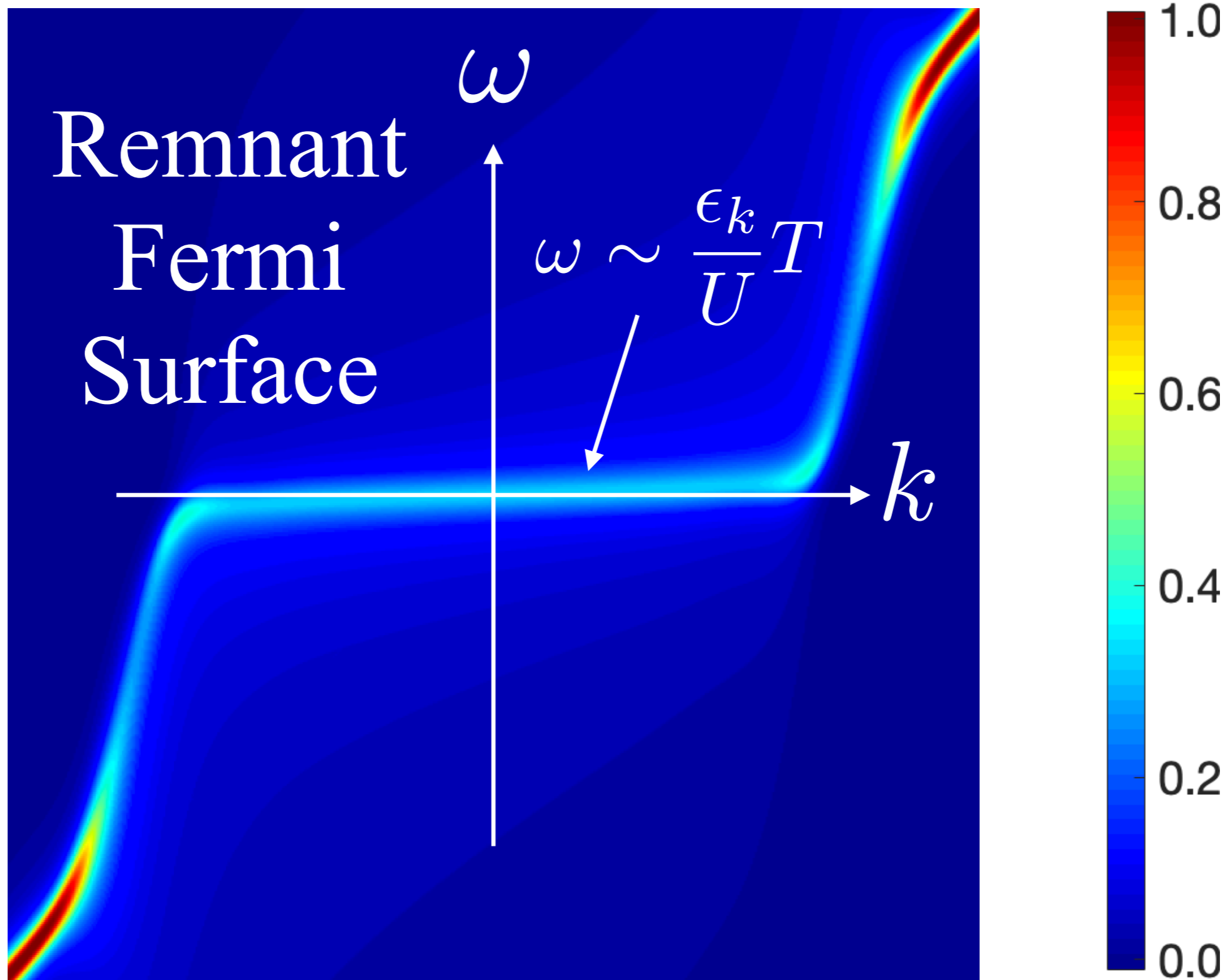
$$\mathcal{E}_k = -0.26$$

$$\mathcal{E}_k = -0.1$$

Crossing the remnant Fermi surface

Planckian dynamics with peak width $\sim k_B T / \hbar$ and independent of U





Resistivity of a Planckian metal as $T \rightarrow 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{Ne^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\text{Im} G_{\text{SYK}}^R \left(\epsilon, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left(\frac{\omega}{2T} \right)$$

Resistivity of a Planckian metal as $T \rightarrow 0$

From the Kubo formula, in the large N limit

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$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}, \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U,$$

where

$$m^* = \frac{d V_{FS}}{\oint_{FS} |\mathbf{v}_F|},$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^* .

Resistivity of a Planckian metal as $T \rightarrow 0$

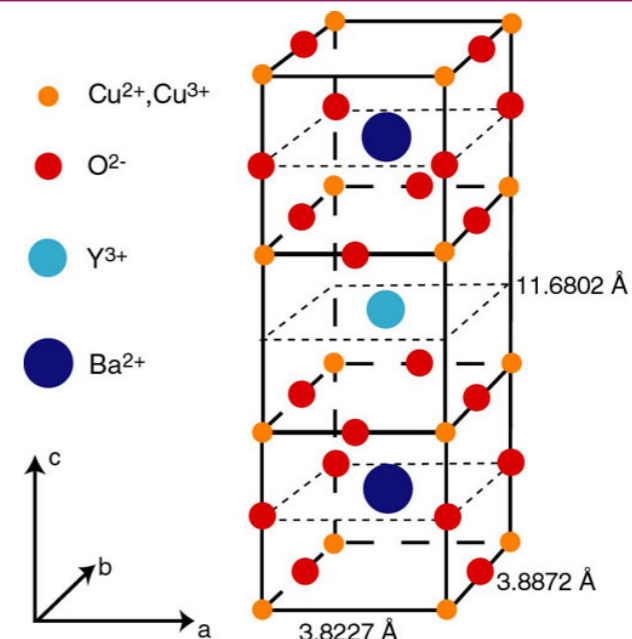
$$\rho = \frac{m^*}{ne^2} 2.71\mathbb{C} \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

The number \mathbb{C} is defined by $\mathcal{E}_k = \mathbb{C} \epsilon_k / U$ as $|\epsilon_k| \rightarrow 0$. This is determined by UV physics, and was found to be very weakly dependent upon the ratio of the energy width of the interactions, W_U , to U . Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.



Aavishkar Patel



A.A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

1. SYK lattice models

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Generalized SYK models

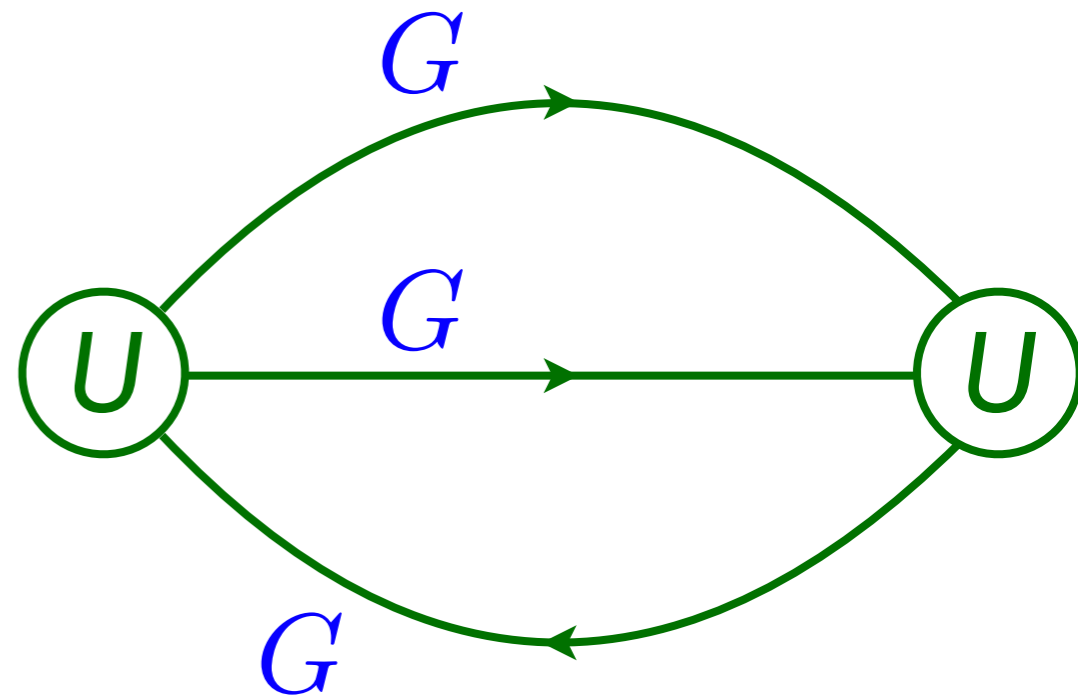
$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

The large N limit is still given by the sum of “melon” diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$$\Sigma =$$



Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

The large N limit is still given by the sum of “melon” diagrams.

For many generic models in this class with $U \gg W$,
 $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$,
and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);
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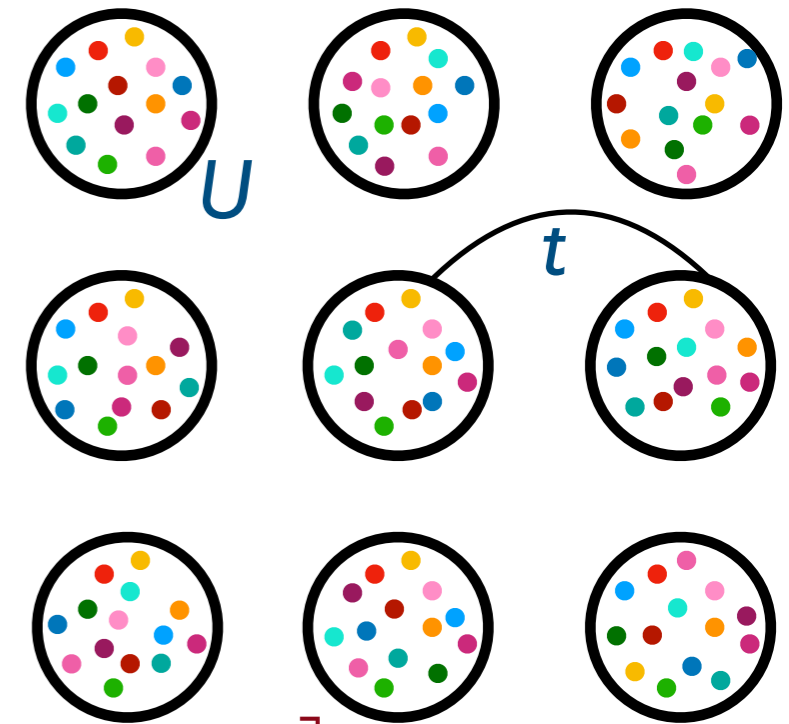
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

Rewriting of lattice model of incoherent and bad metal in momentum space



$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right]$$

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta}(k_a) c_{k_1\alpha}^\dagger c_{k_2\beta}^\dagger c_{k_3\gamma} c_{k_4\delta} \\ + \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$
 ϵ_k has a bandwidth W .

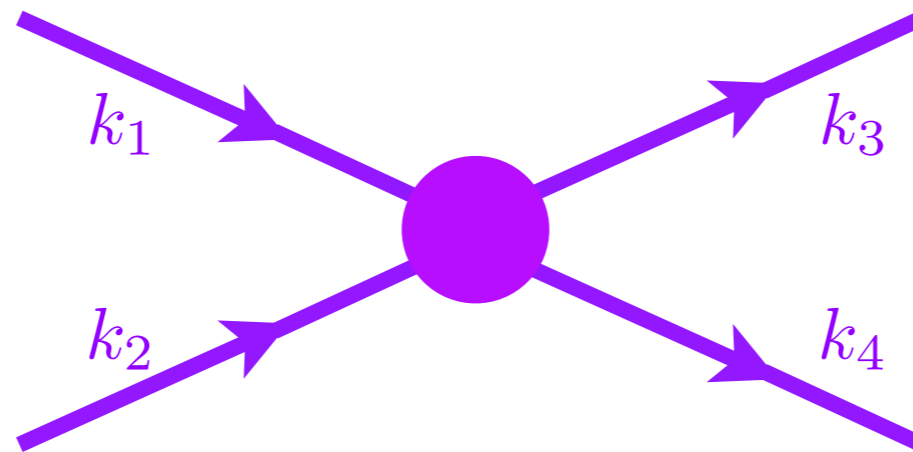
We examine a model with weaker $W \lesssim U$, but impose a **resonance condition**.

This leads to a solution which obeys the Planckian ansatz as $T \rightarrow 0$.

$$\overline{U(k_1, k_2, k_3, k_4) U^*(k_5, k_6, k_7, k_8)} = \\ U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8) \right] \\ \times \left[\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8}) \right]$$

This implies off-site interactions with correlations which decay with a power-law in space.

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we “integrate these out” in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k , which we have already accounted for.

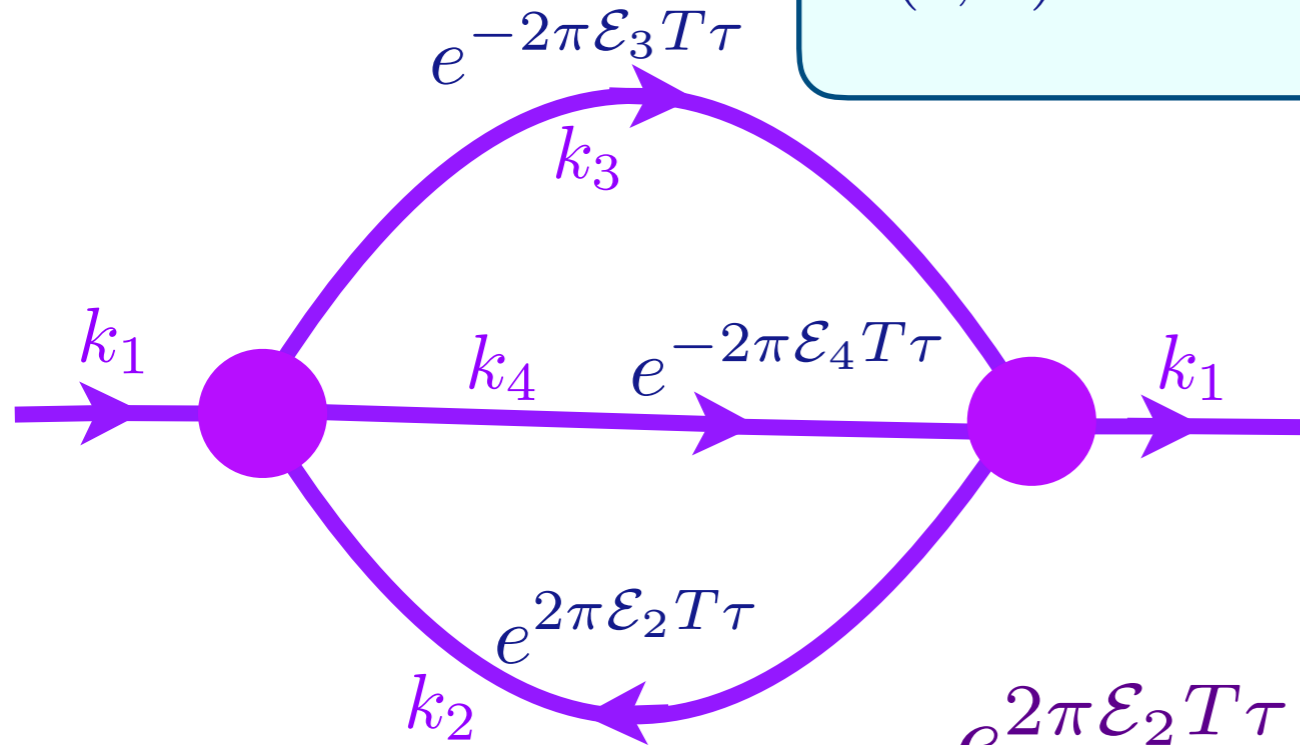
Keep only the interactions resonant in the bare quasiparticle energy with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi\mathcal{E}T\tau} \left(\frac{T/U}{\sin(\pi T\tau)} \right)^{1/2}, \quad 0 < \tau < 1/T.$$



$$e^{2\pi\mathcal{E}_2 T\tau} e^{-2\pi\mathcal{E}_3 T\tau} e^{-2\pi\mathcal{E}_4 T\tau} = e^{-2\pi\mathcal{E}_1 T\tau}$$

if

$$\mathcal{E}_a = \mathbb{C}\epsilon_a/U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a Planckian metal as $T \rightarrow 0$ with a remnant Fermi surface:
 $G(k, \omega) = G_{\text{SYK}}(\epsilon_k, \hbar\omega/(k_B T)),$
 with $\mathcal{E}_k = \mathbb{C}\epsilon_k/U$

Planckian metals with a remnant Fermi surface

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \rightarrow 0$.
- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasi-particle dispersion ϵ_k , while resonant interactions have to be treated self-consistently.
- The resonance is a single ‘fine-tuning’ condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \rightarrow 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $\epsilon_k = 0$, and an effective mass m^* defined by the dispersion of ϵ_k , with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.



Aavishkar Patel (Harvard → Miller Fellow at Berkeley)



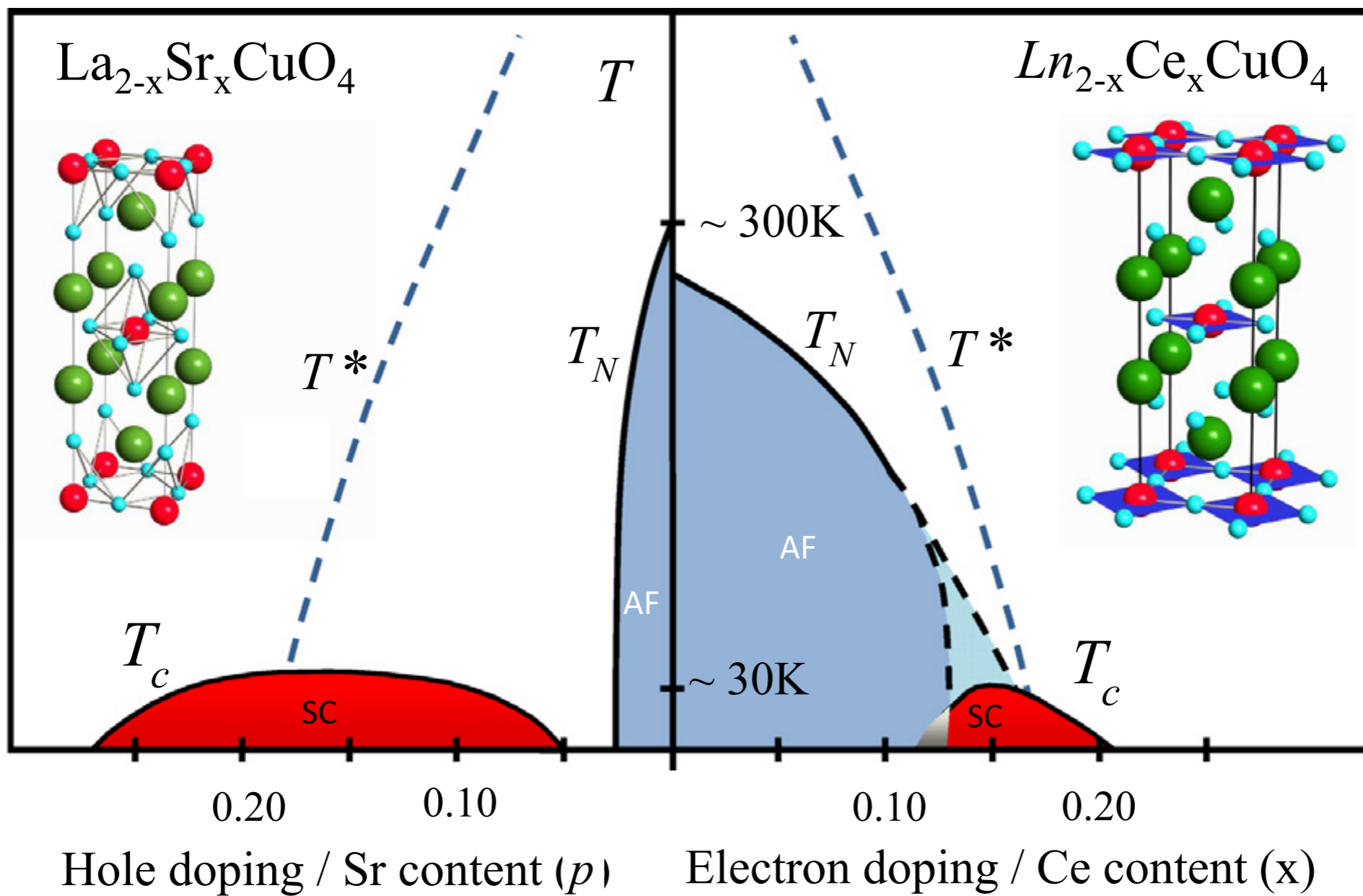
1. SYK lattice models

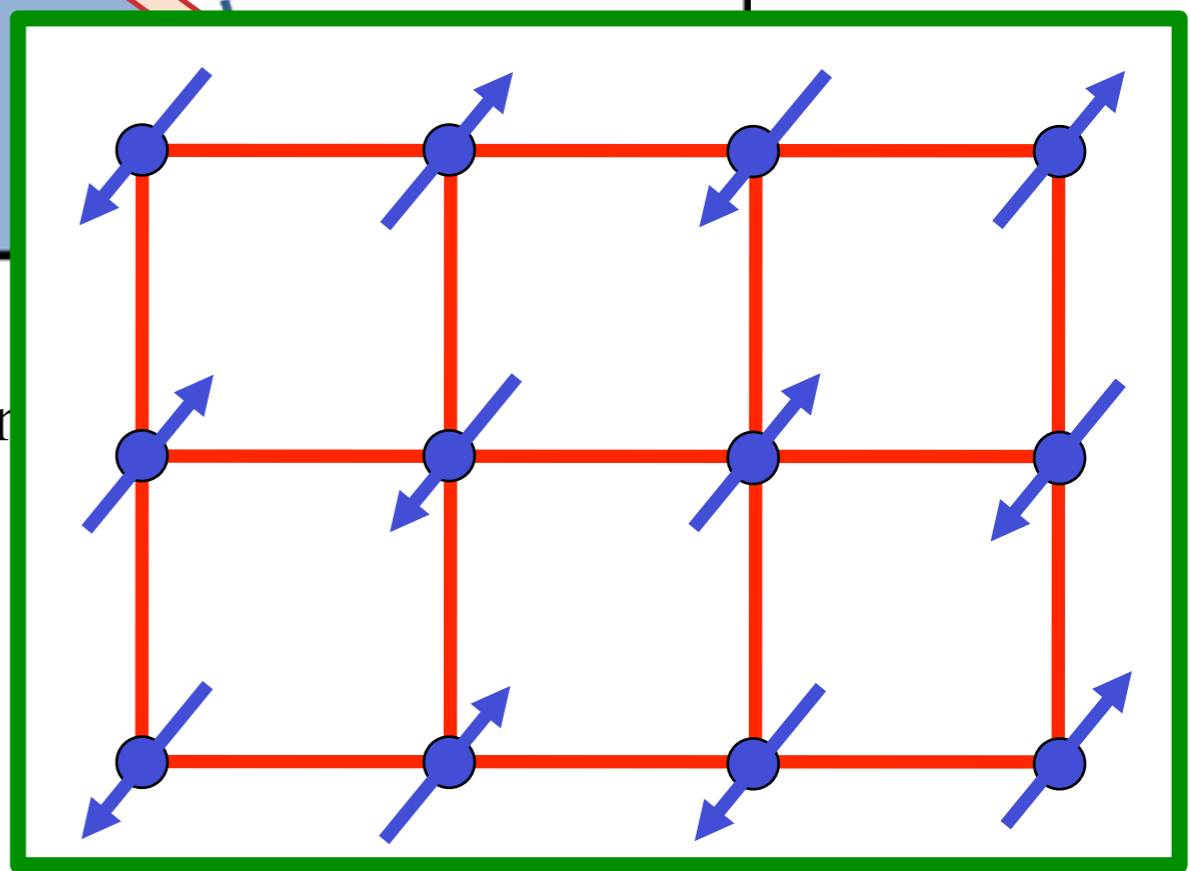
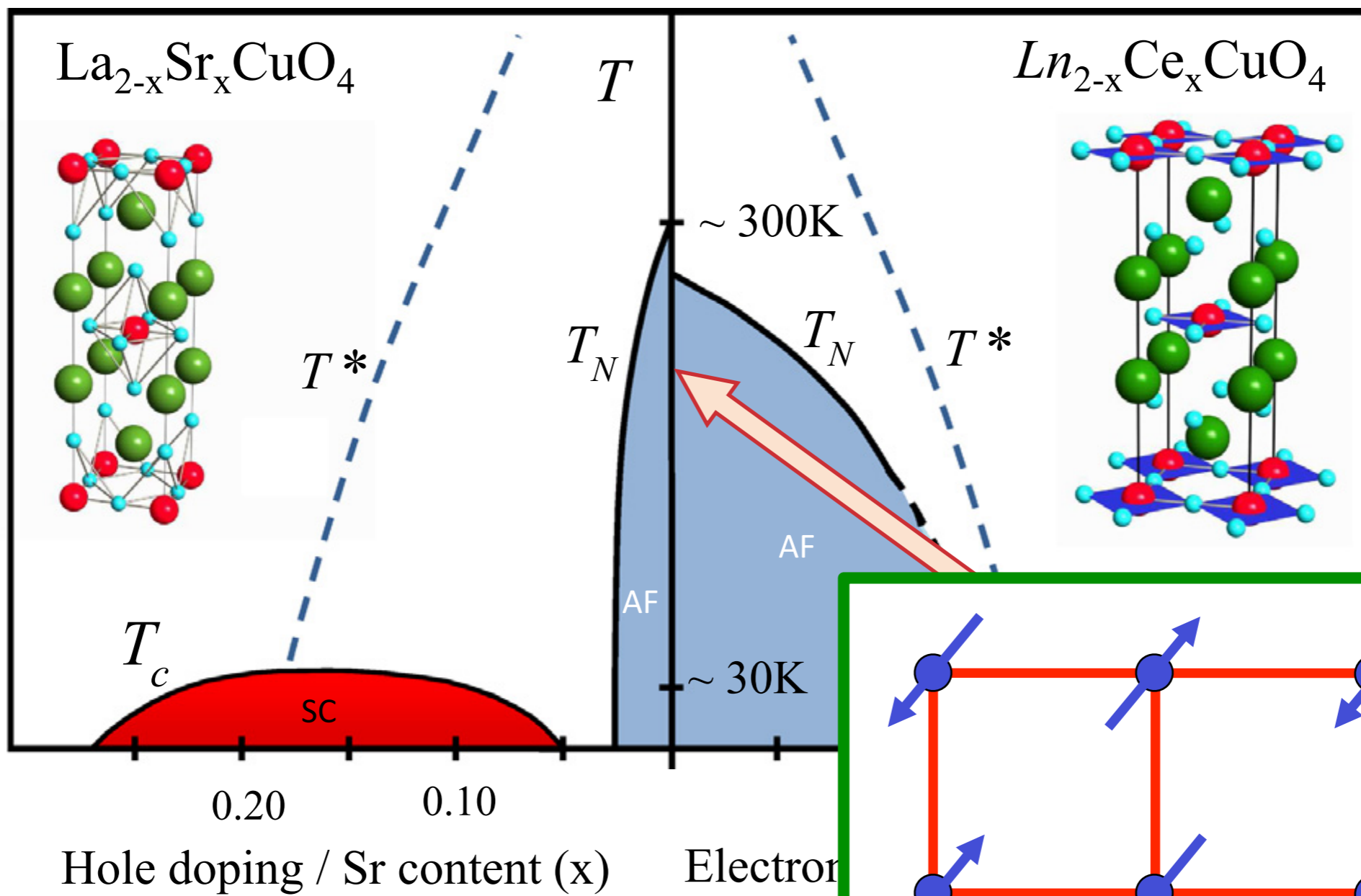
2. Planckian metal ansatz

3. Resonant SYK models

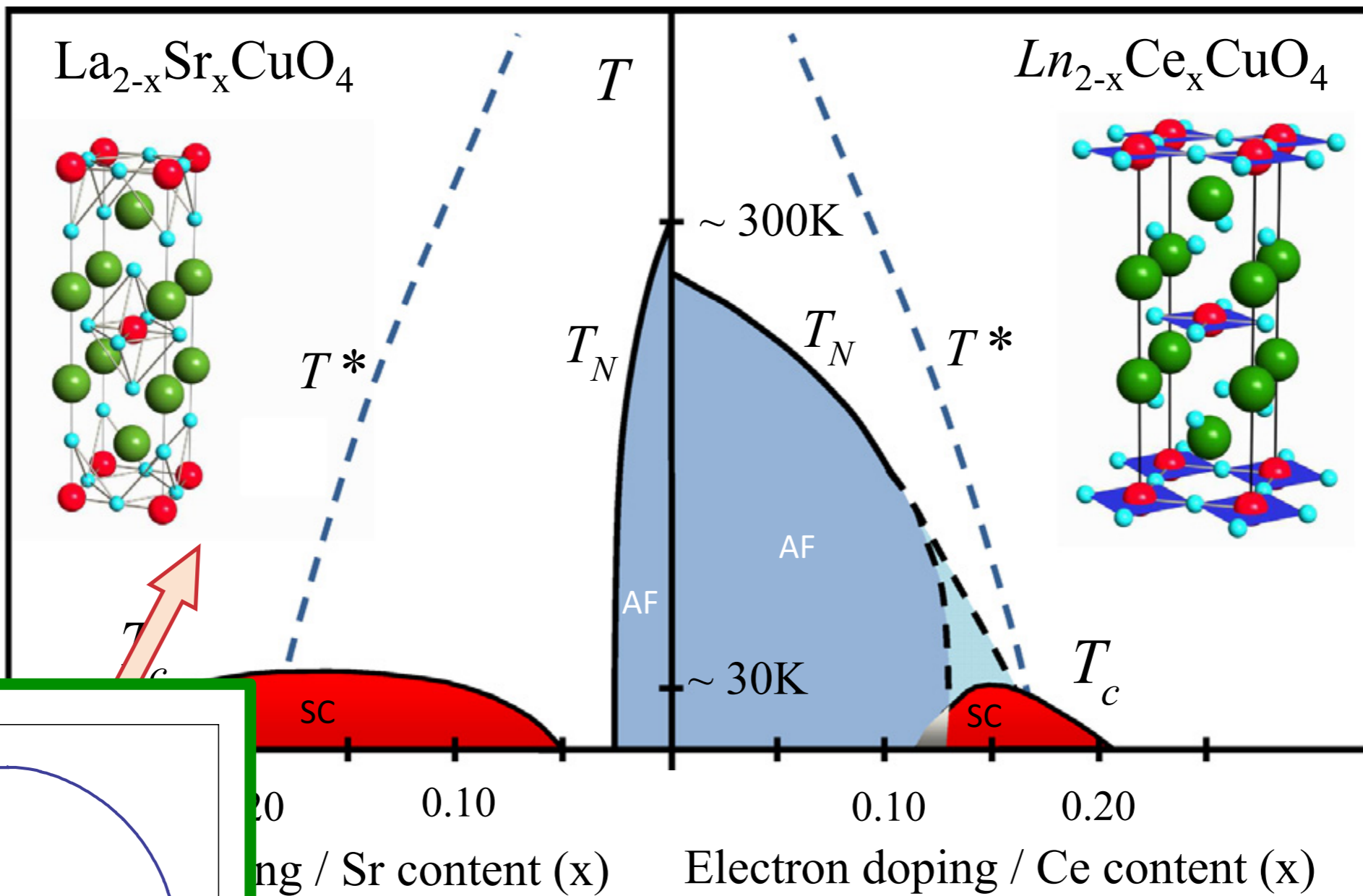
4. Random t - J - U models

Ongoing work with Antoine Georges, Yingfei Gu,
Darshan Joshi, Chenyuan Li, Olivier Parcollet,
Gregory Tarnopolsky



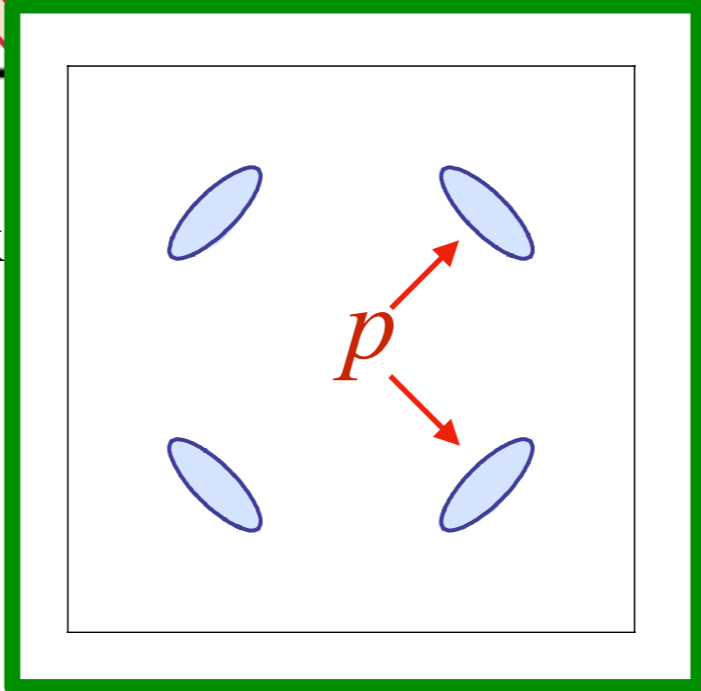
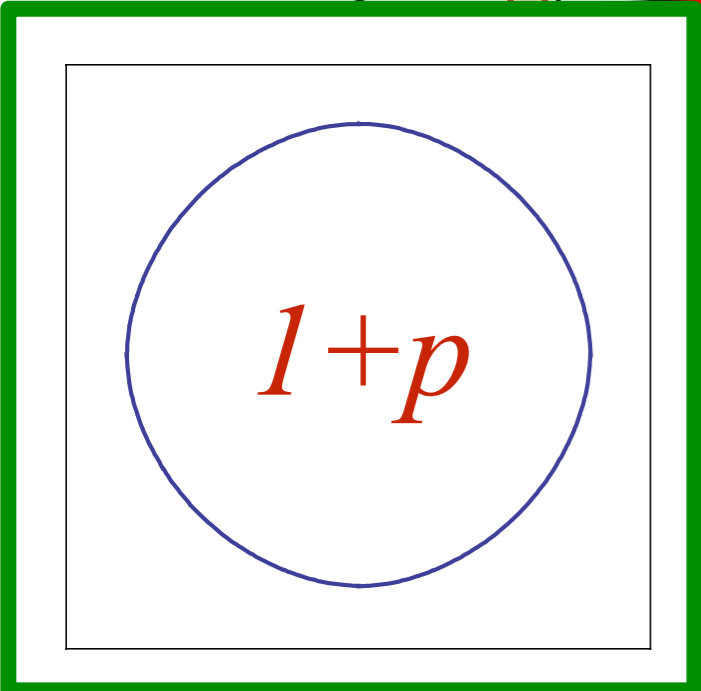
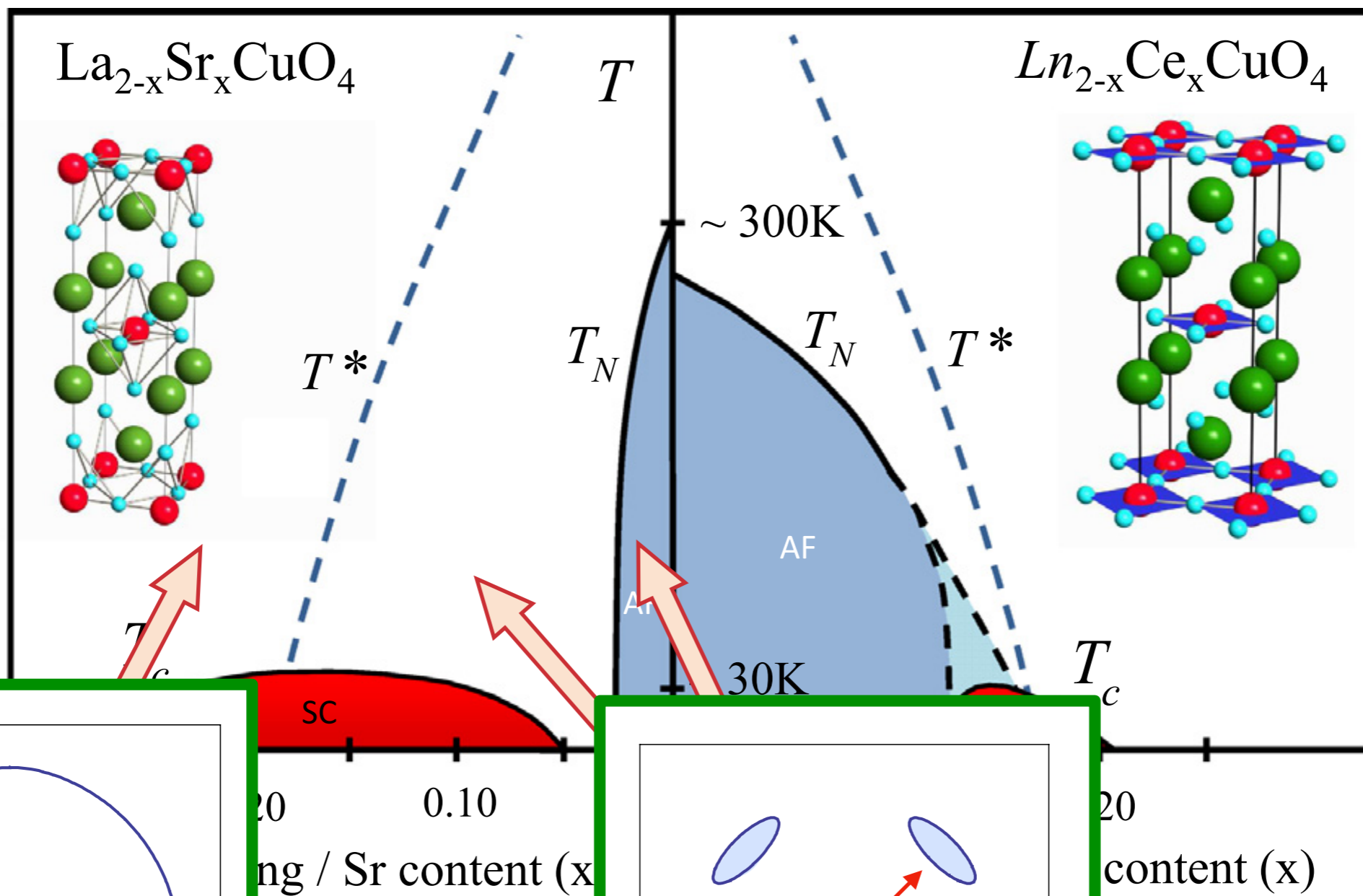


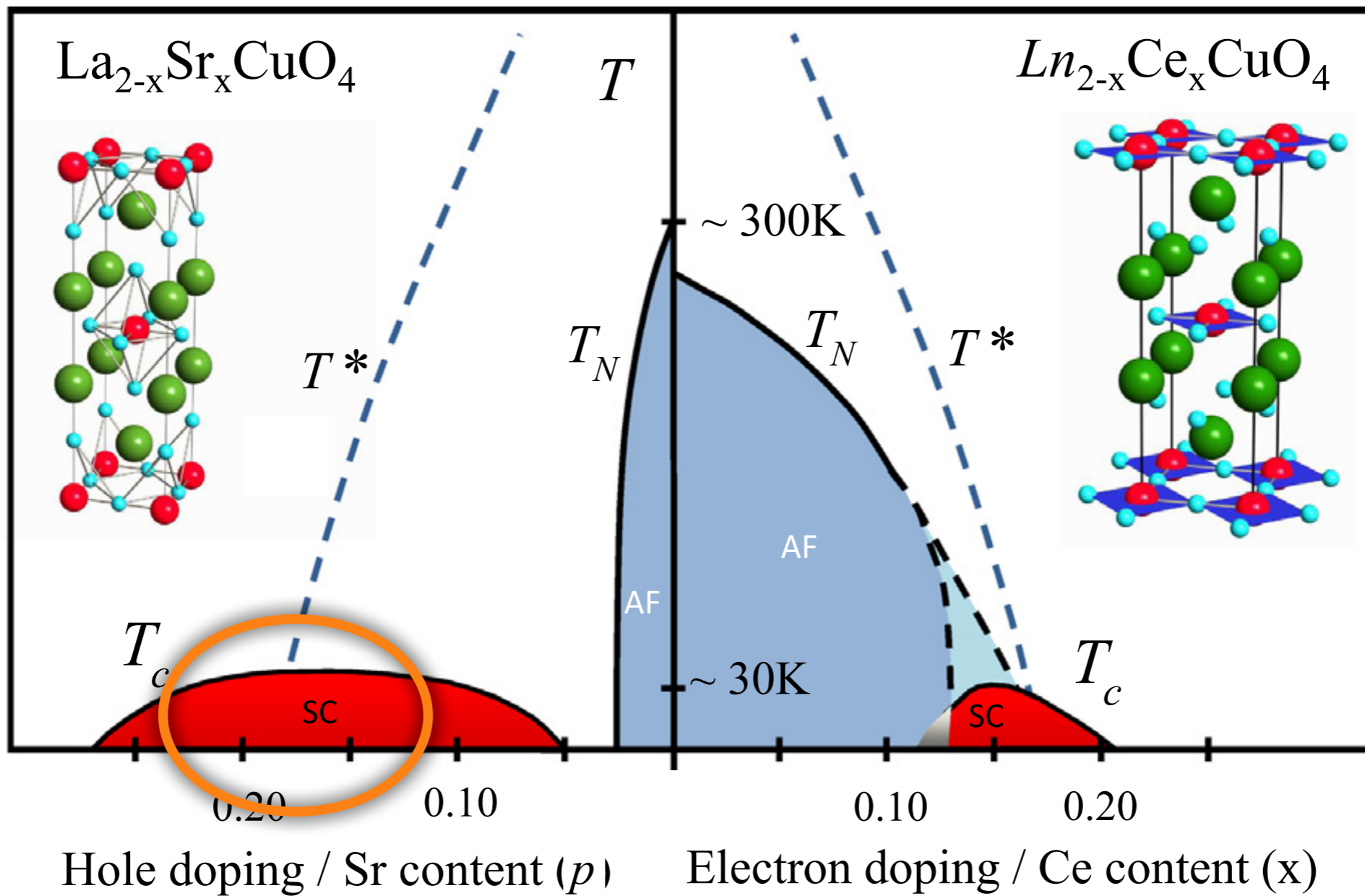
Insulating Antiferromagnet



$1+p$

Conventional Fermi liquid

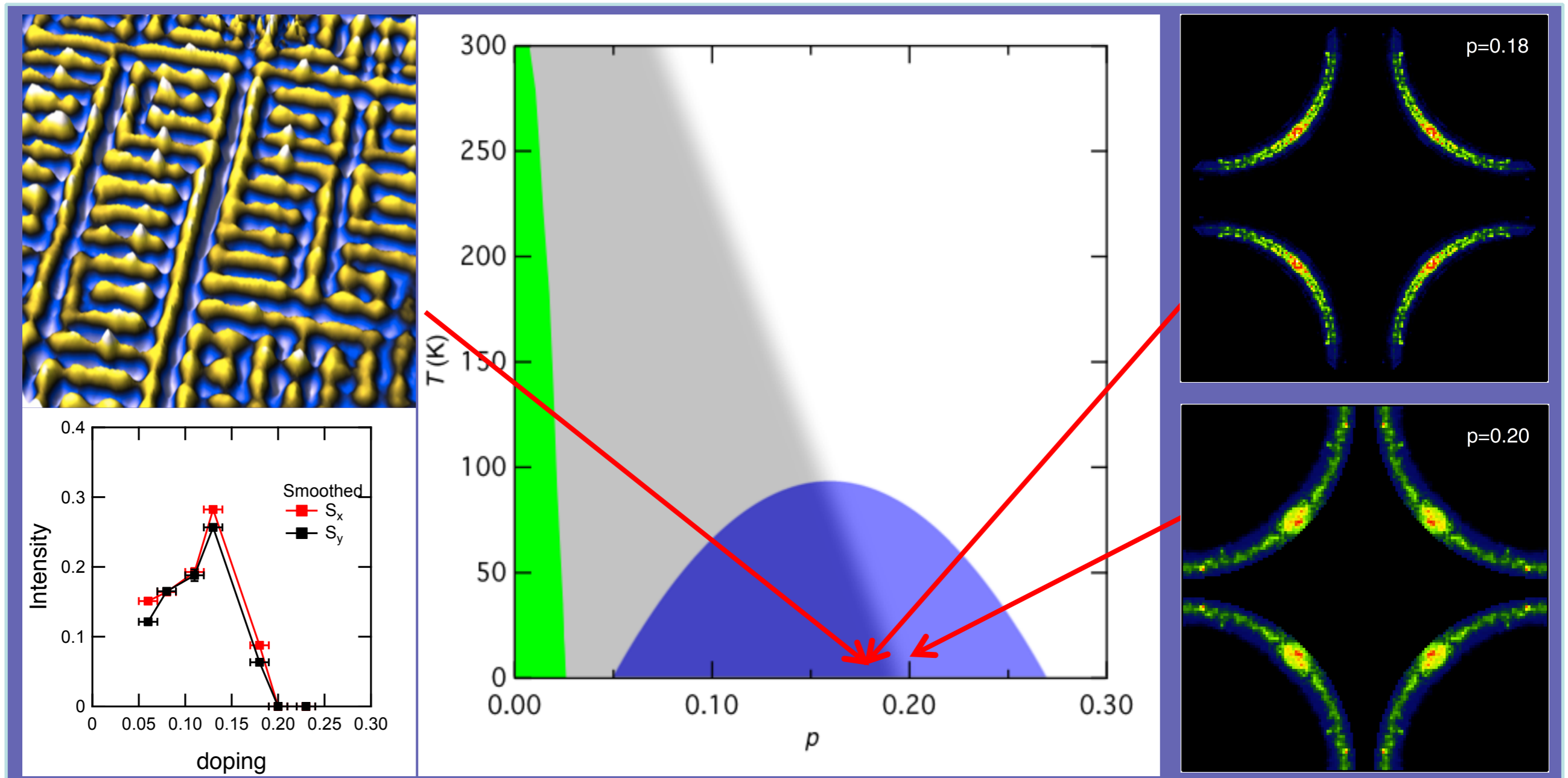




Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, *Science* **344**, 608 (2014)

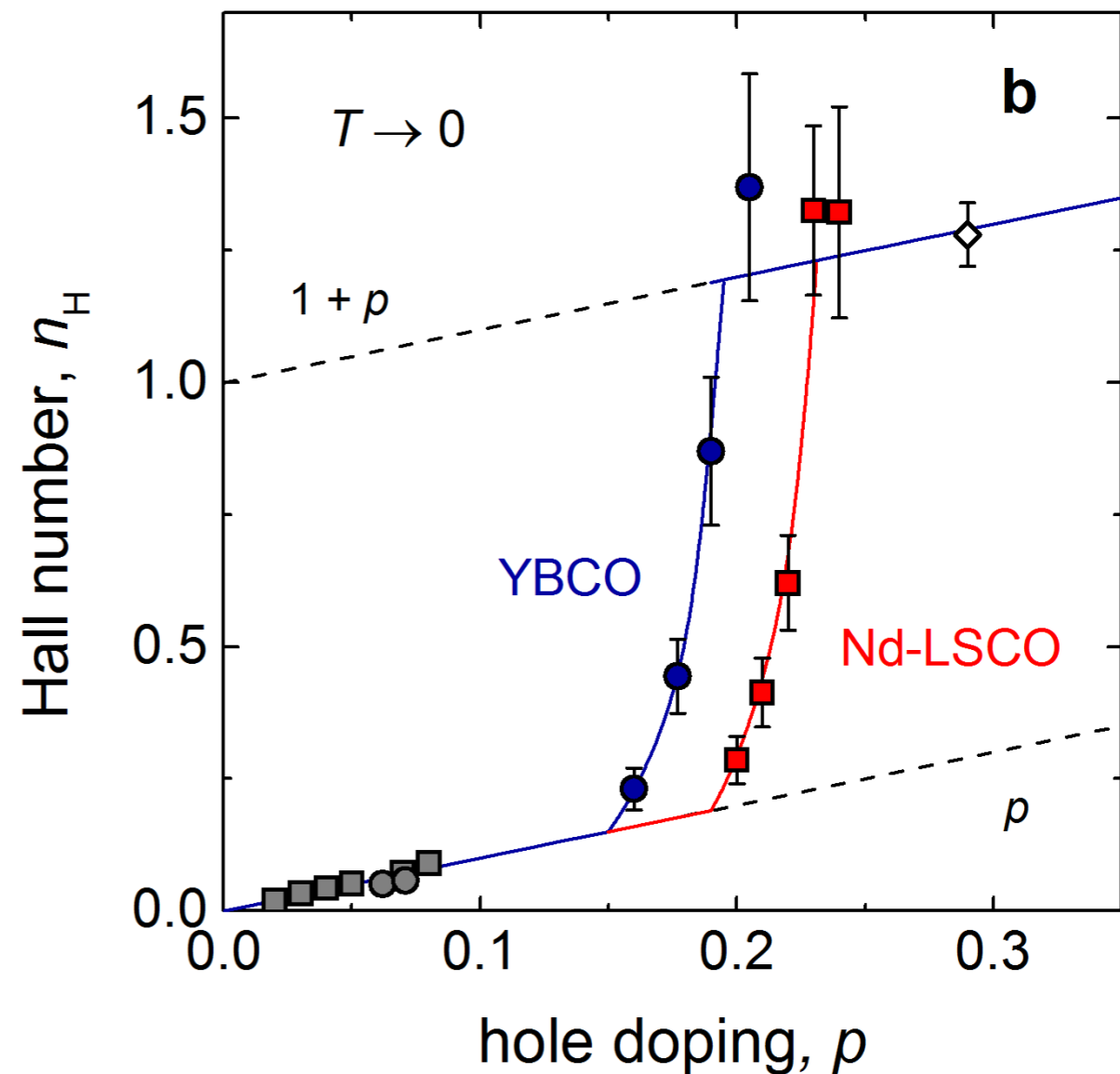
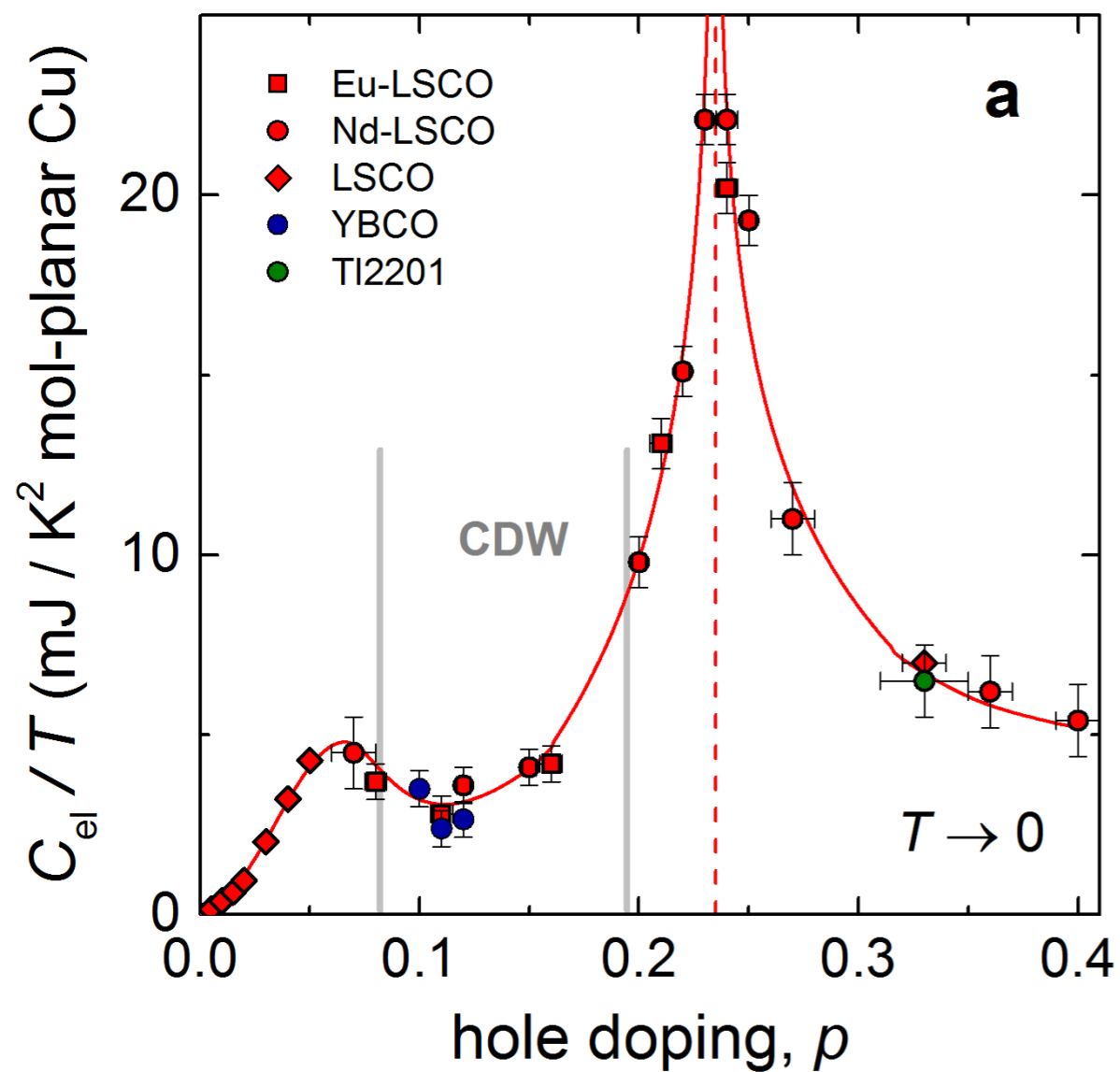
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, *Science* **344**, 612 (2014)

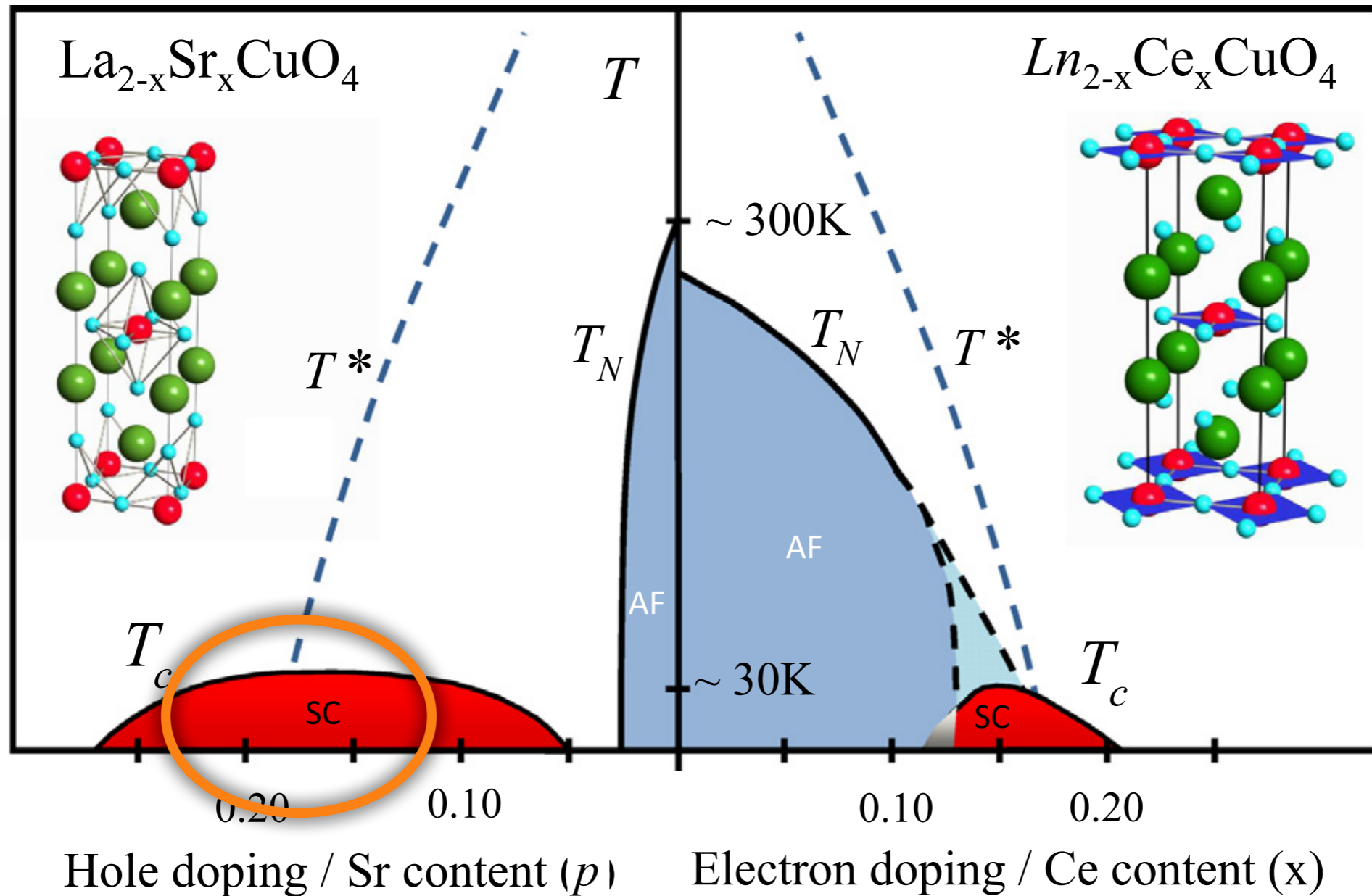


Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507



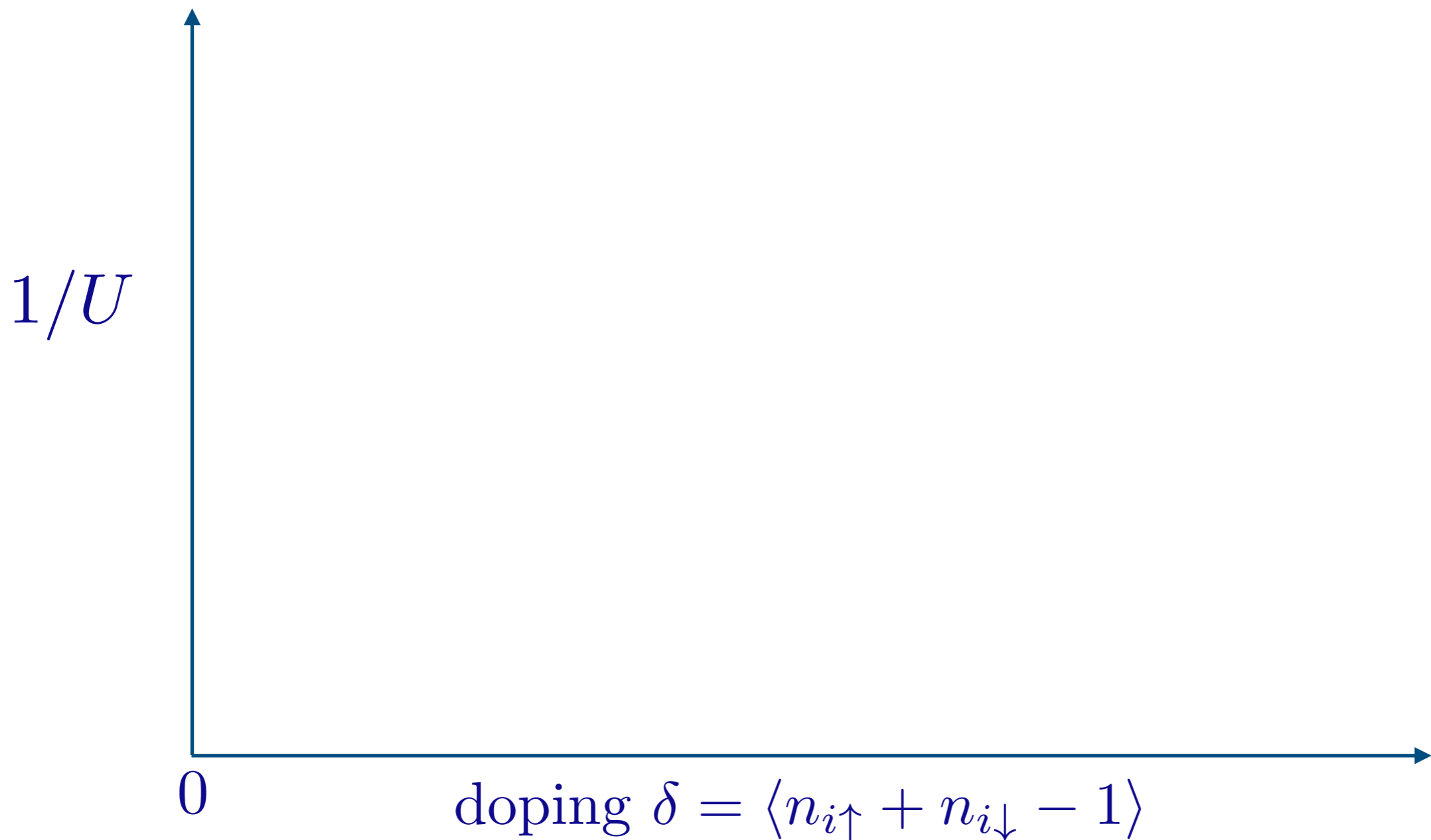


Is there a quantum critical point near optimal doping, not associated with the onset of antiferromagnetism?

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^N n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha}, \quad t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

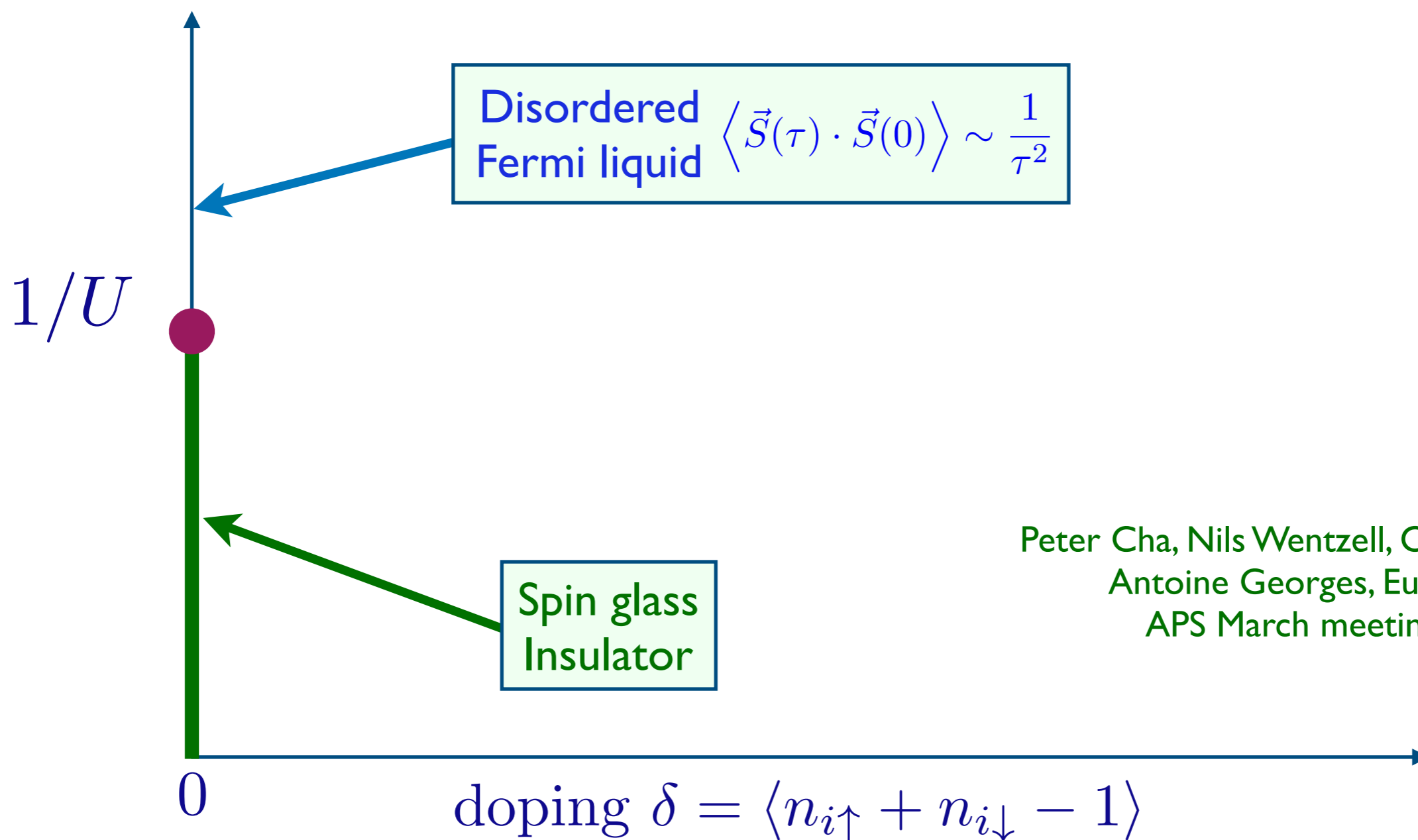
Solve as $N \rightarrow \infty$: numerical, large M ($SU(2) \rightarrow SU(M)$), ϵ expansions.



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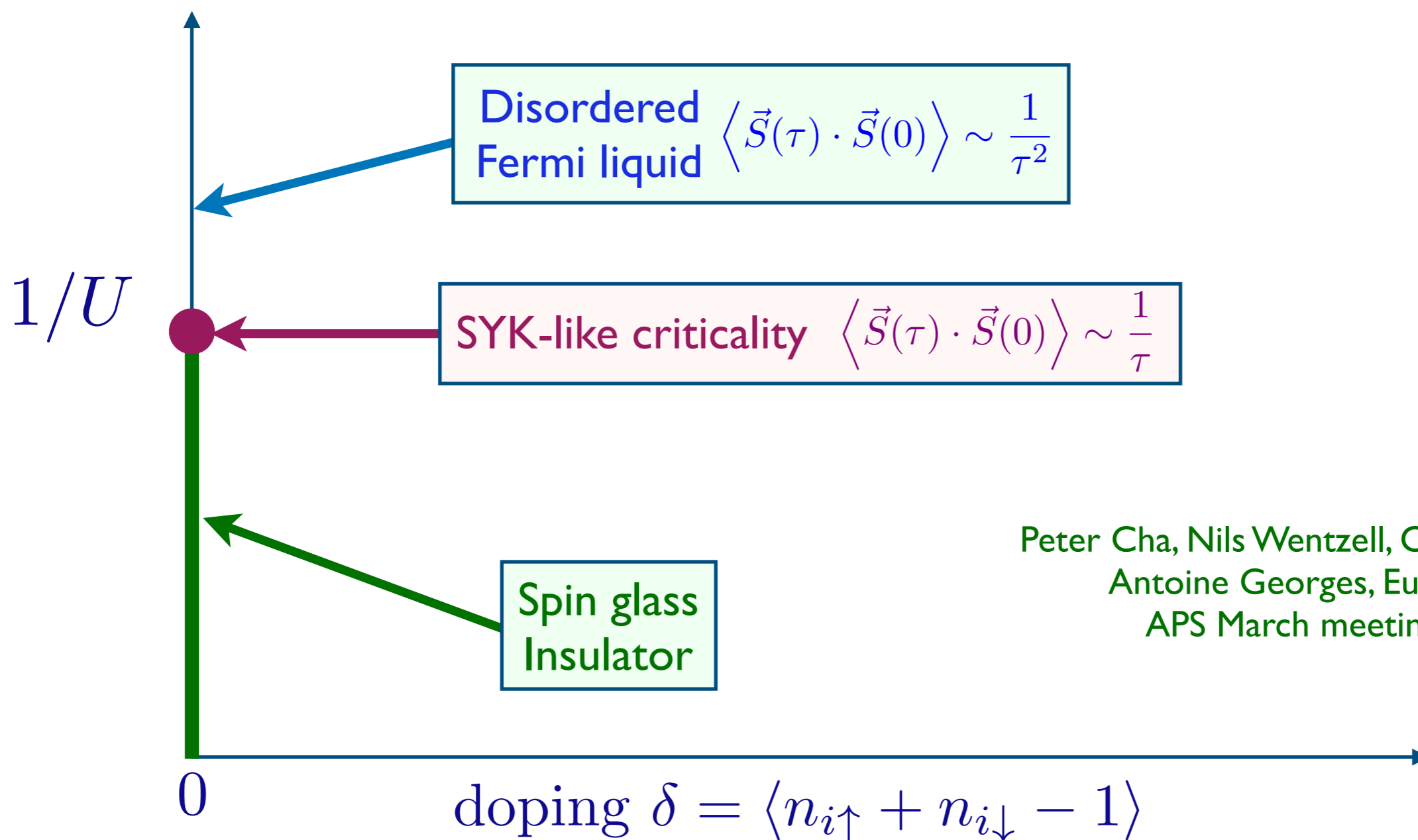


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Antoine Georges, Eun-ah Kim,
APS March meeting 2019

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