



AdS/CFT and condensed matter

Talk online: sachdev.physics.harvard.edu



Particle theorists

Sean Hartnoll, KITP

Christopher Herzog, Princeton

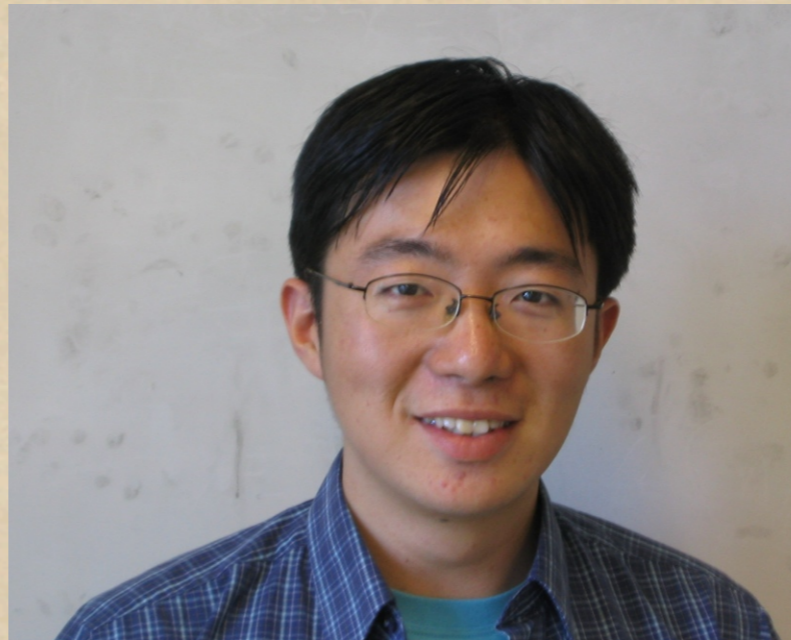
Pavel Kovtun, Victoria

Dam Son, Washington

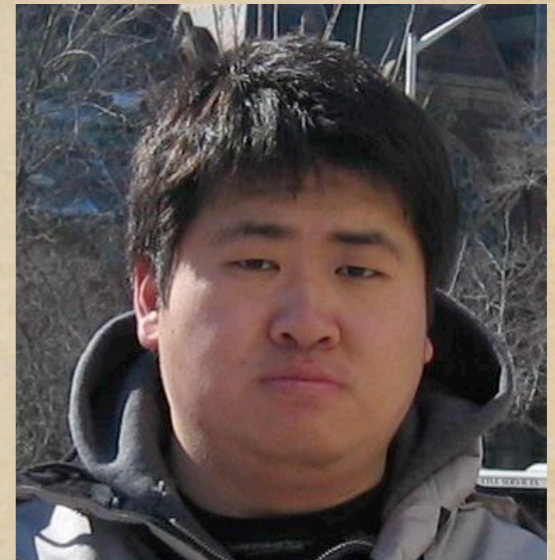
Condensed matter
theorists



Markus Mueller
Geneva



Cenke Xu
Harvard



Yang Qi
Harvard

Outline

1. CFT3s in condensed matter physics

(a) Coupled-dimer antiferromagnets

(b) Triangular lattice antiferromagnets

(c) The superfluid-insulator transition

2. Black holes and quantum criticality

3. Applications

Nernst effect in the cuprate superconductors

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(b) Triangular lattice antiferromagnets

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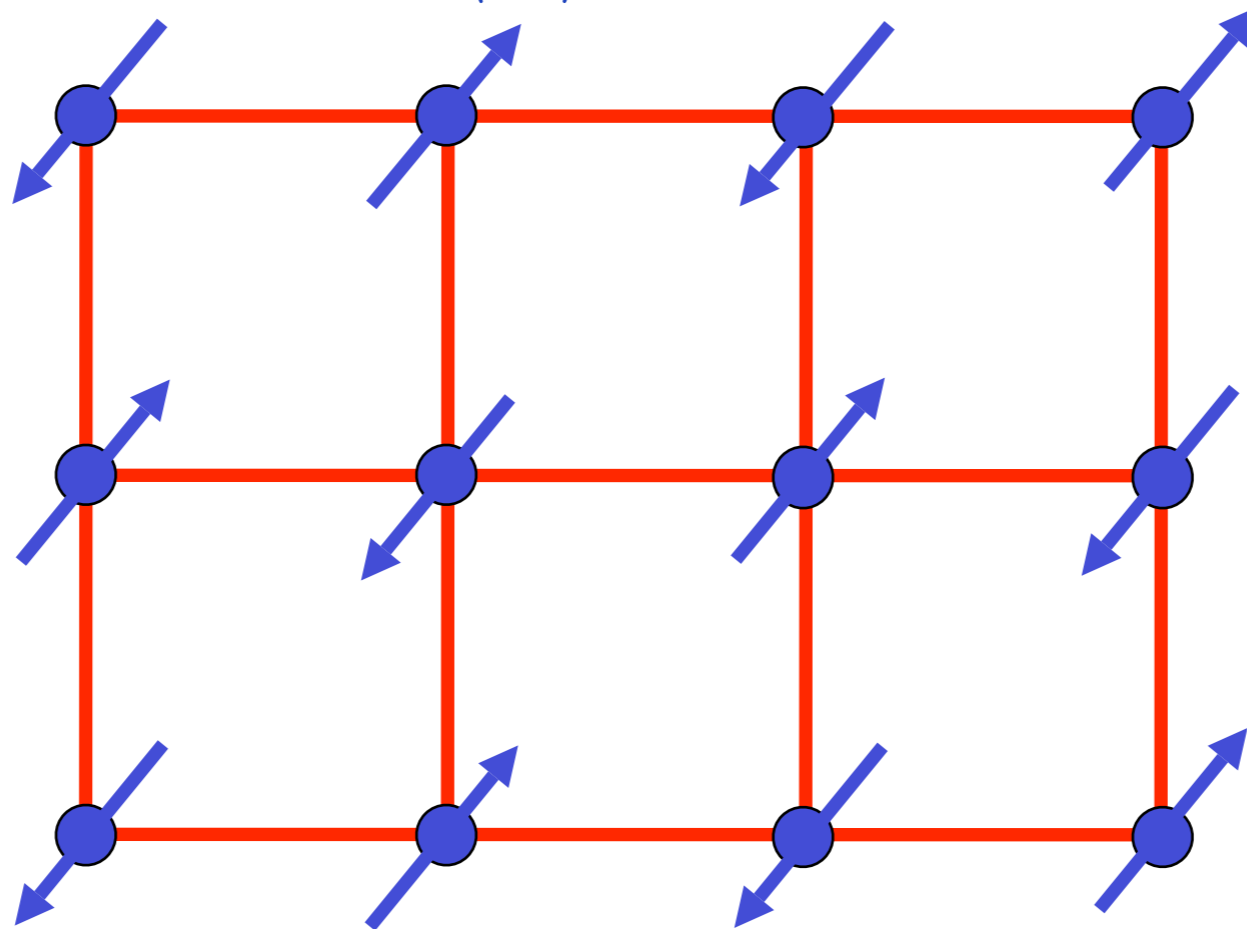
2. Black holes and quantum criticality

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Nernst effect in the cuprate superconductors

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

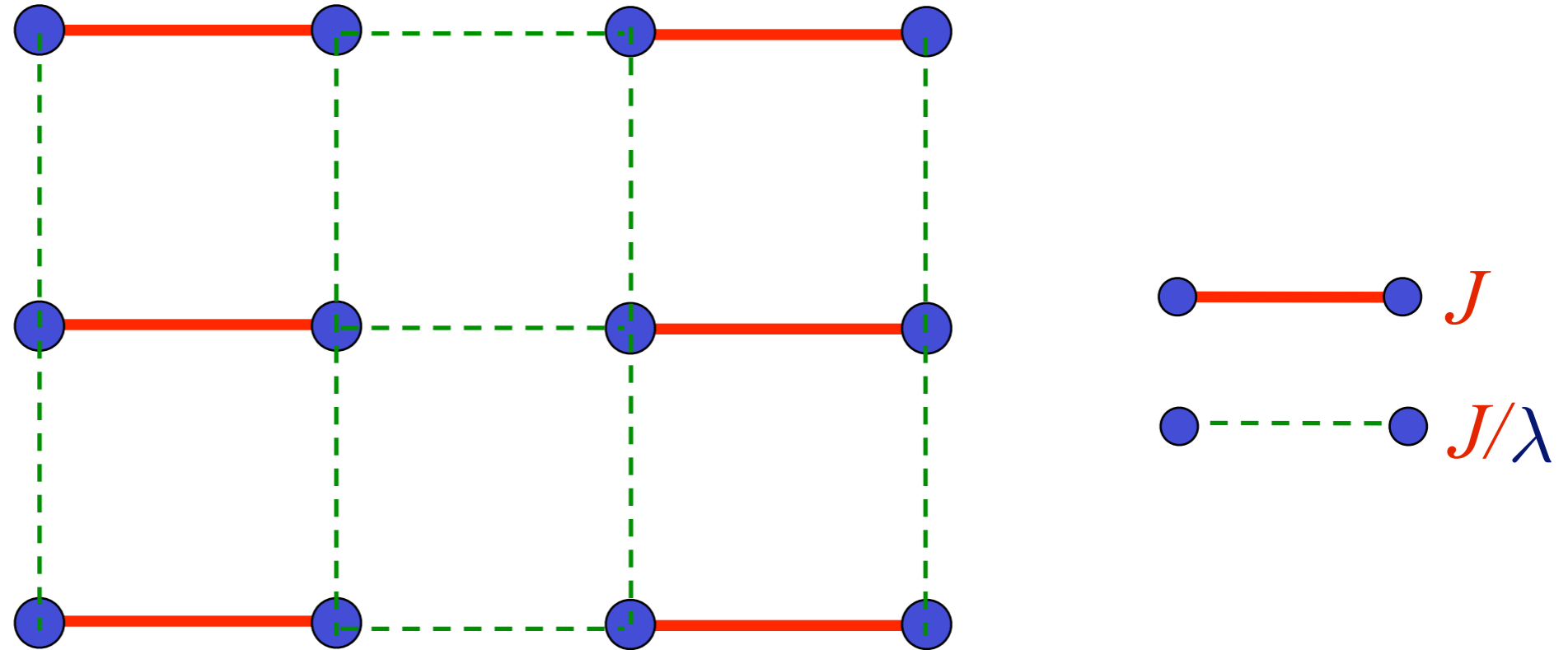
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

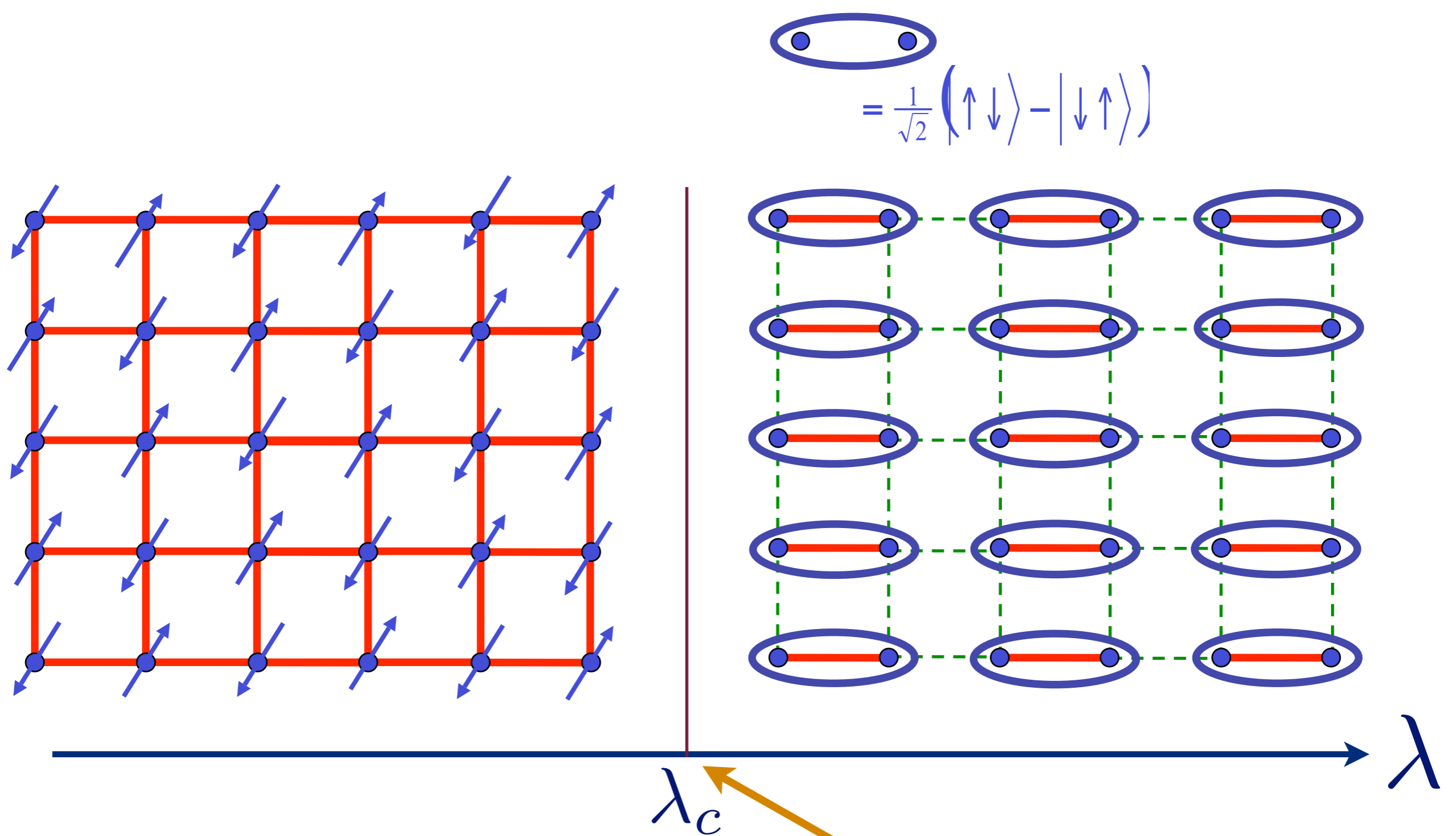
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

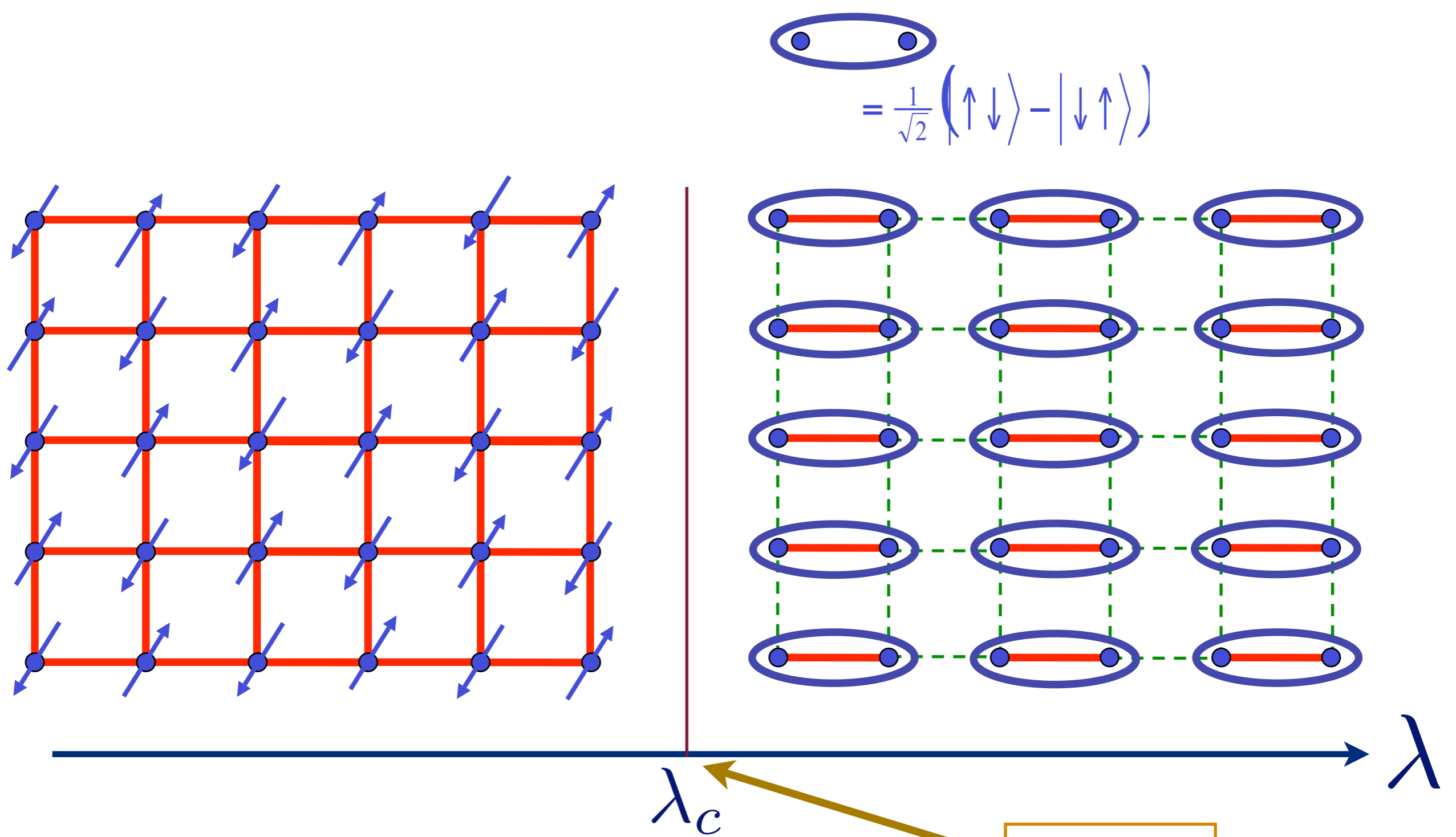
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase



Quantum critical point with non-local entanglement in spin wavefunction



$O(3)$ order parameter $\vec{\varphi}$

CFT3

$$\mathcal{S} = \int d^2r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν , β/ν , and η . We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c . The bottom group are results for the plaquette model. Numbers in [...] brackets denote the $\chi^2/\text{d.o.f.}$ For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
1.9096 $-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
1.9096 $+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107 ^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
1.8230 $-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
1.8230 $+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

^a $L > 12$.

^b $L > 16$.

^c $L > 20$.

^dPrevious best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418

M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic
RG of CFT3
E.Vicari *et al.*

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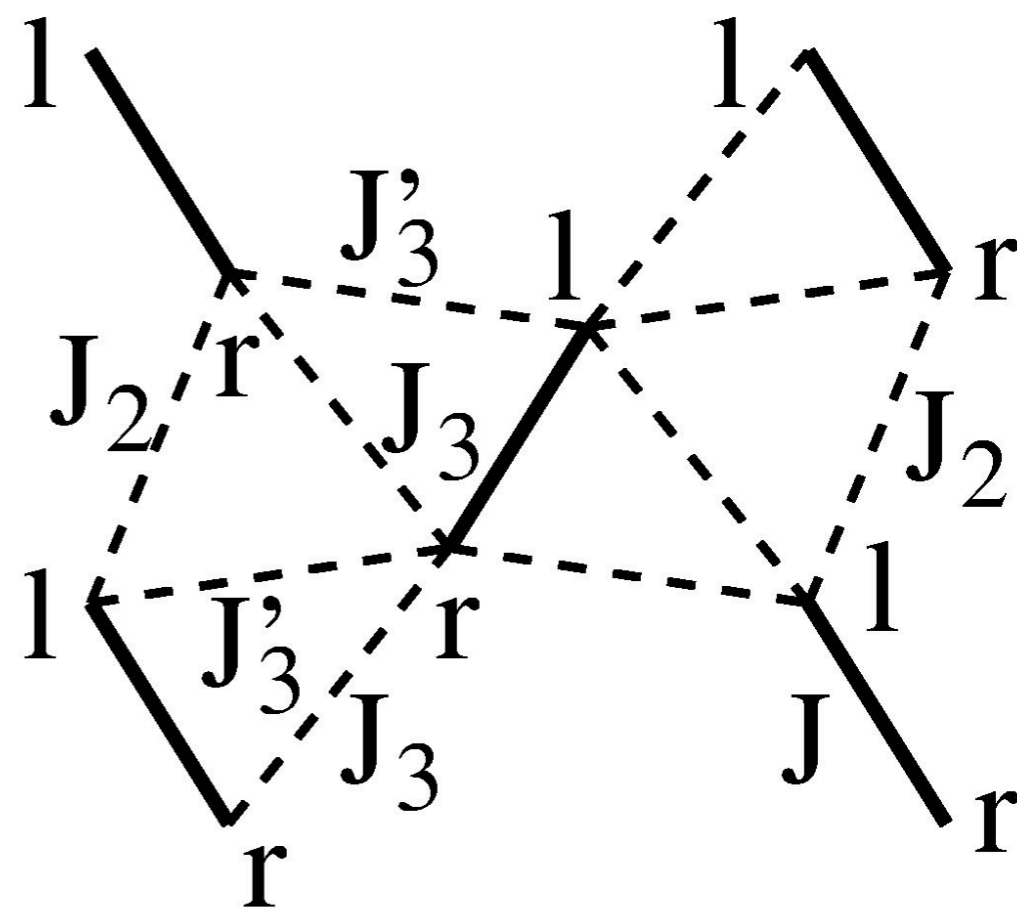
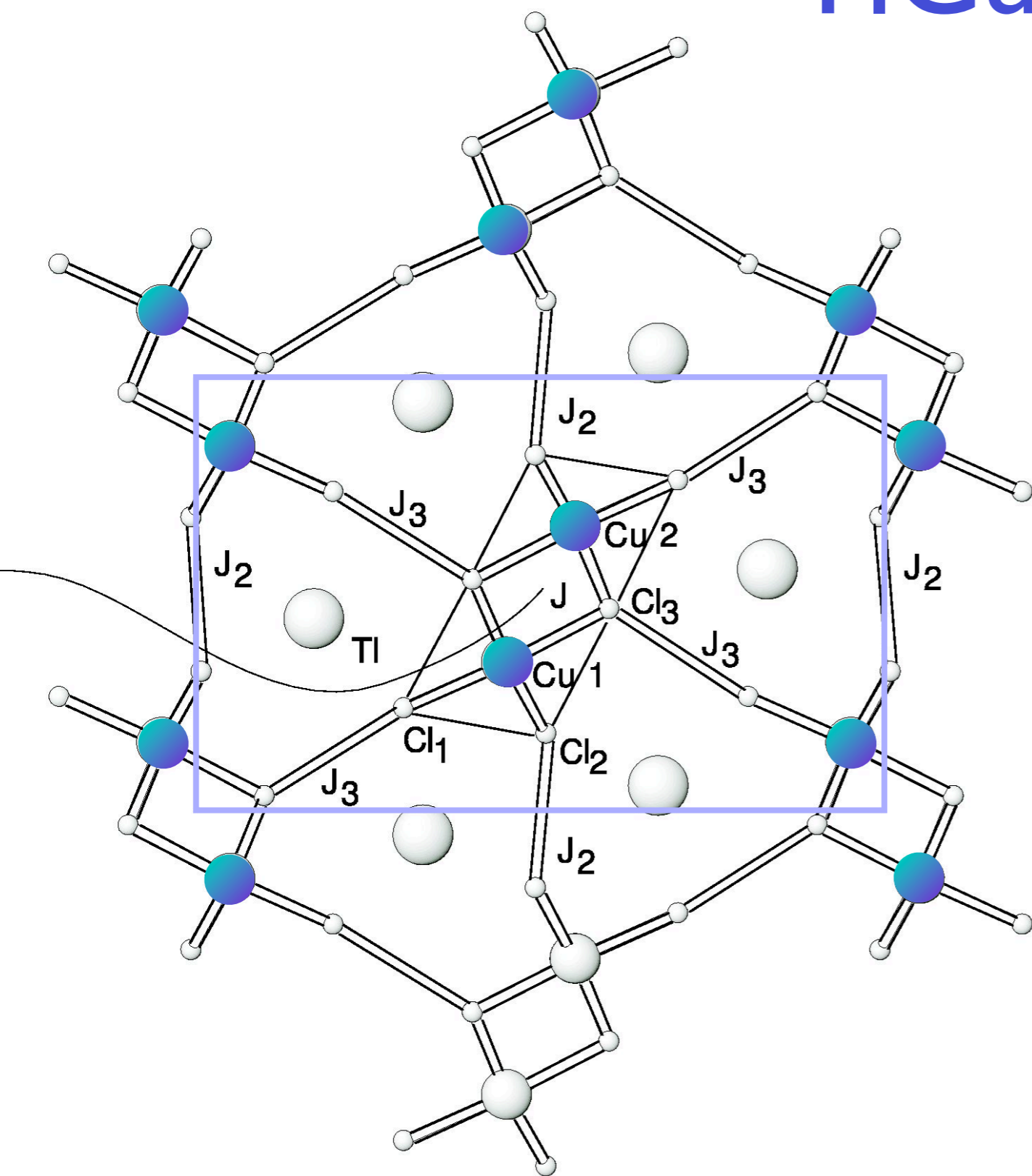
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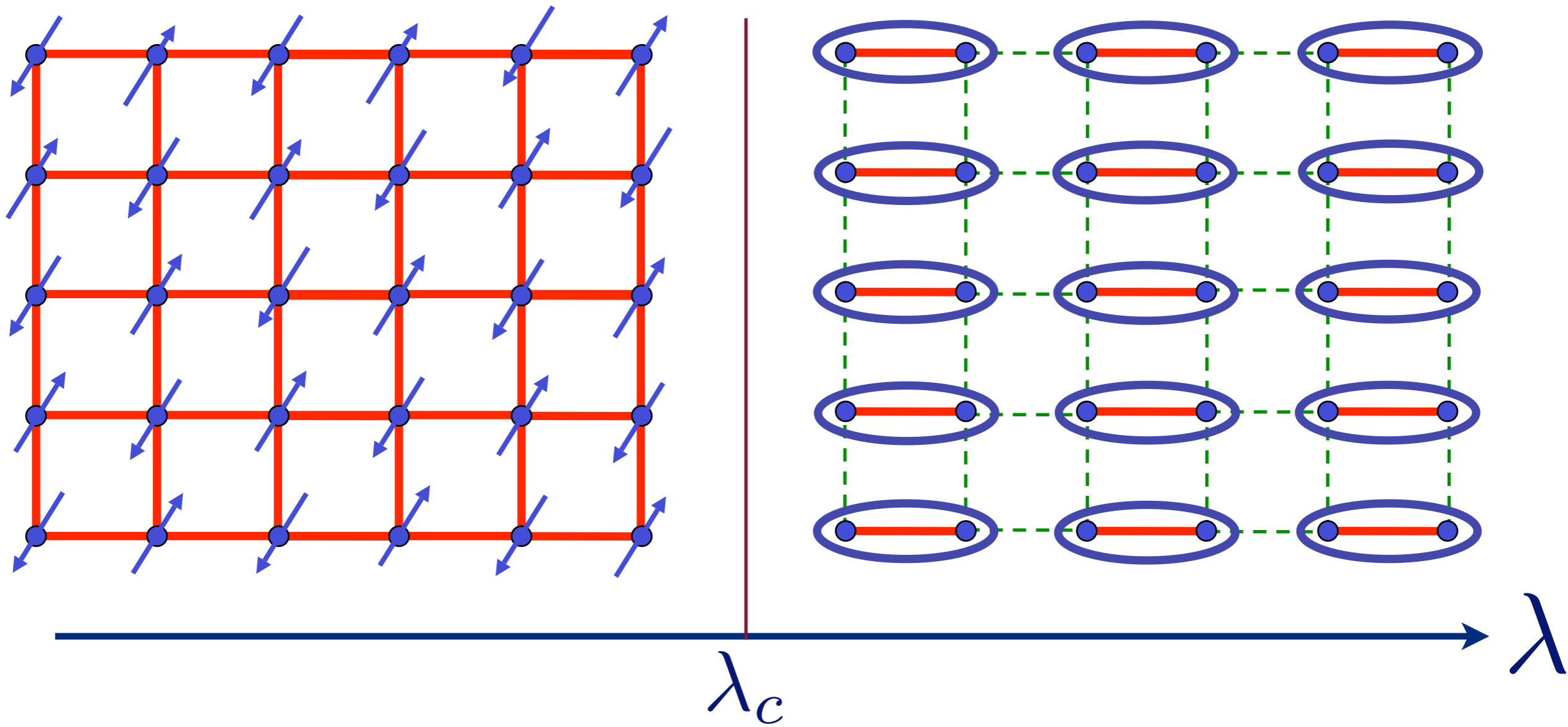
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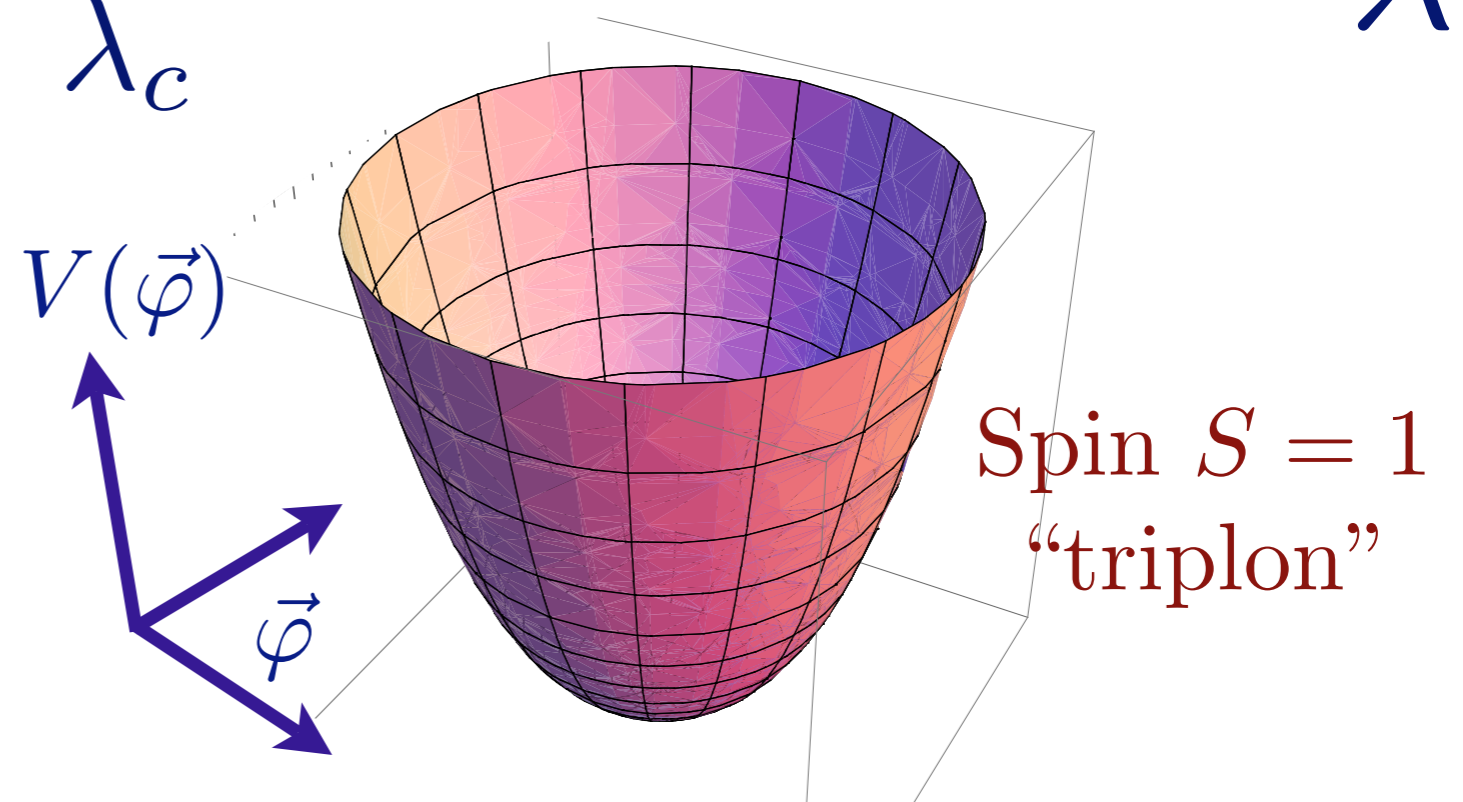
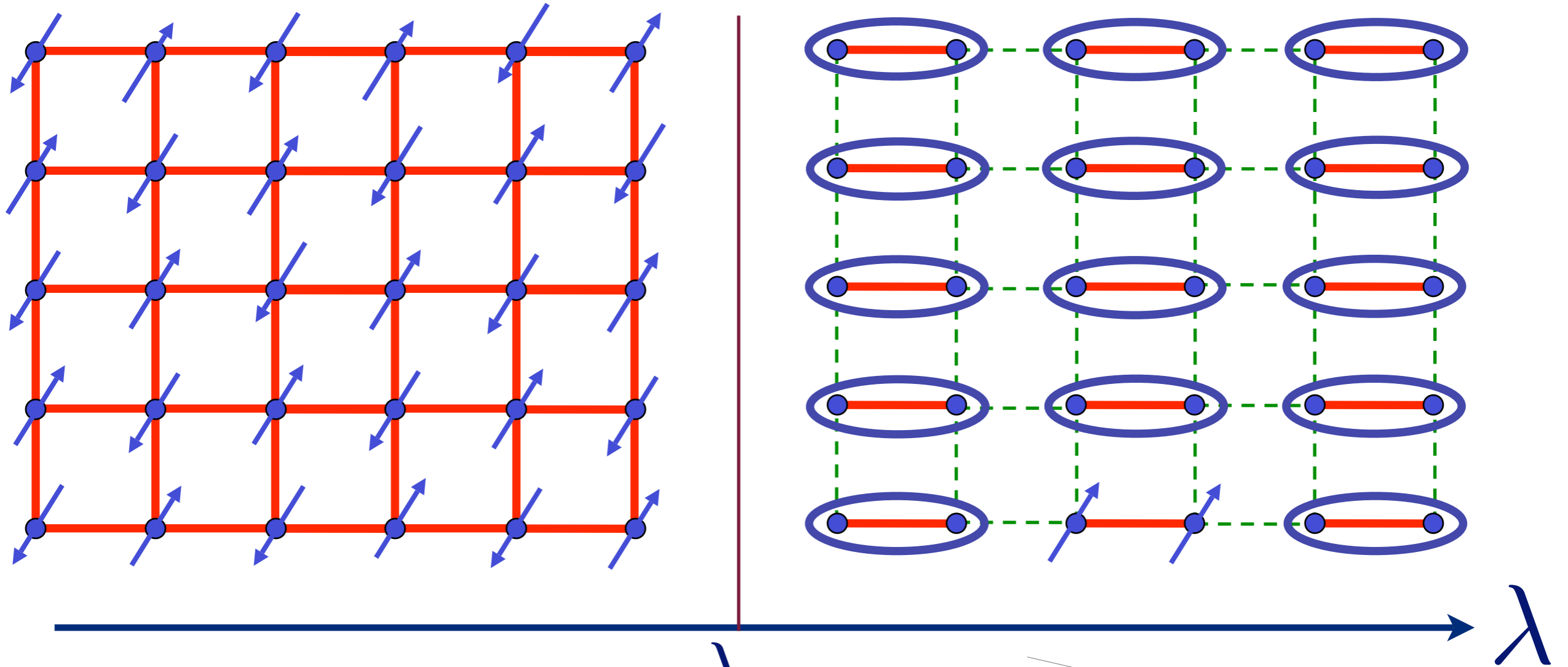
TlCuCl₃



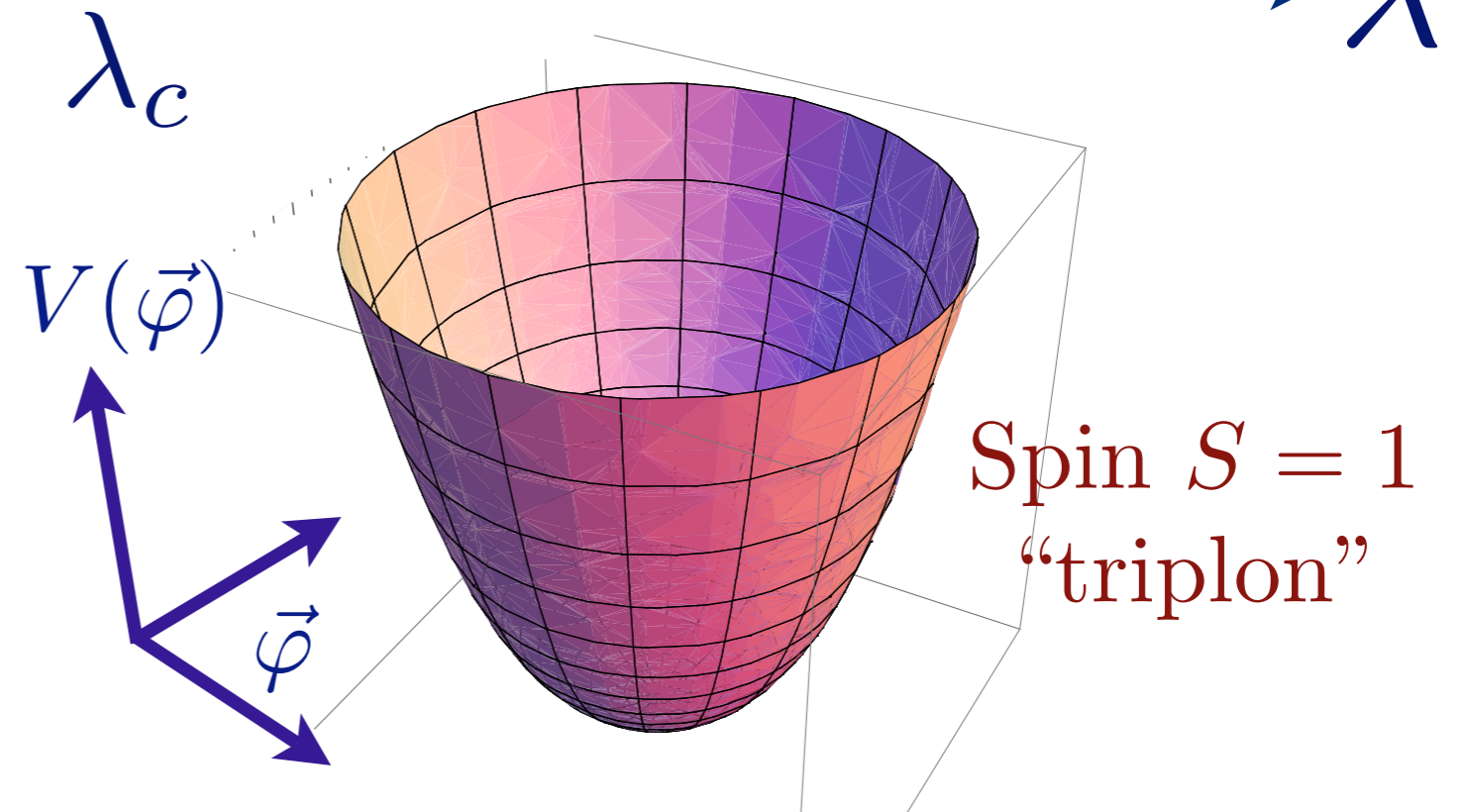
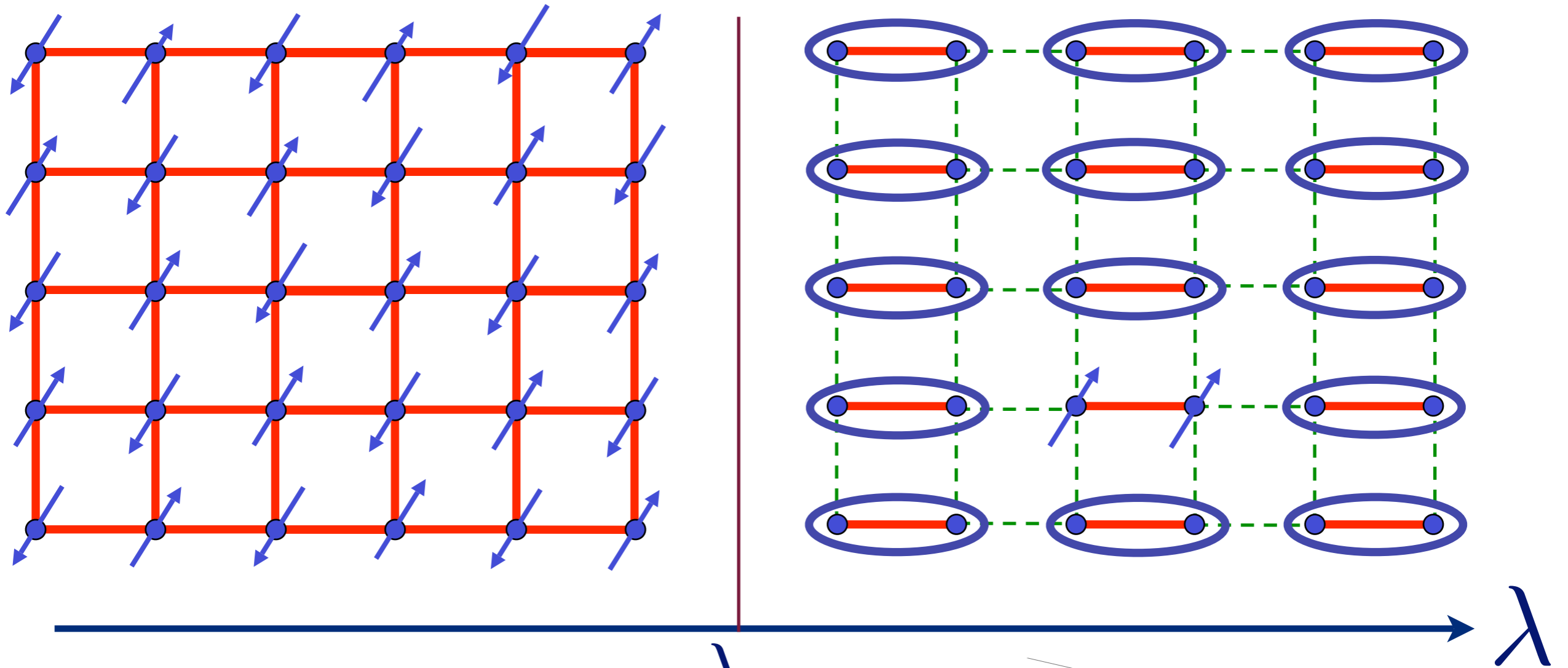


← Pressure in TlCuCl_3

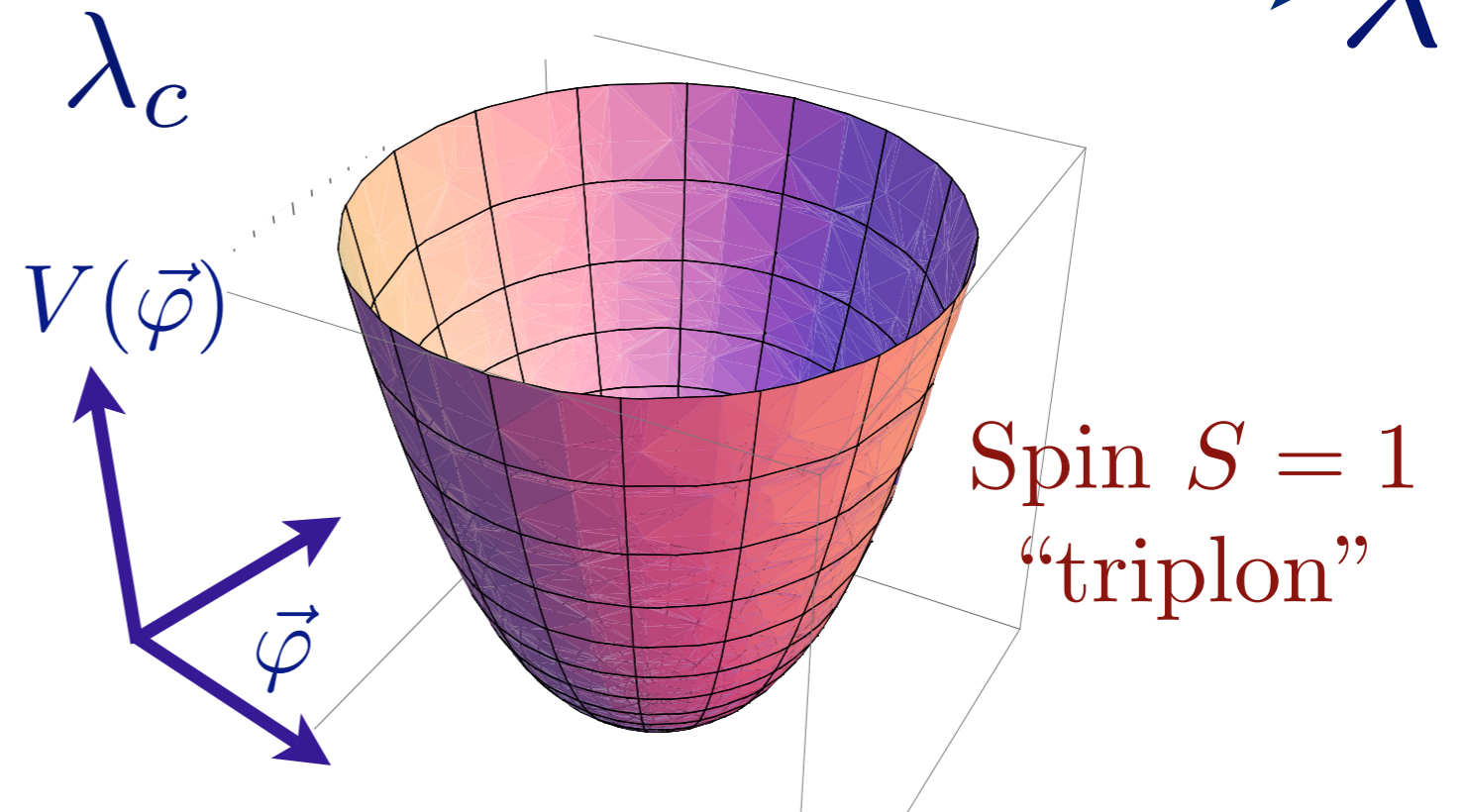
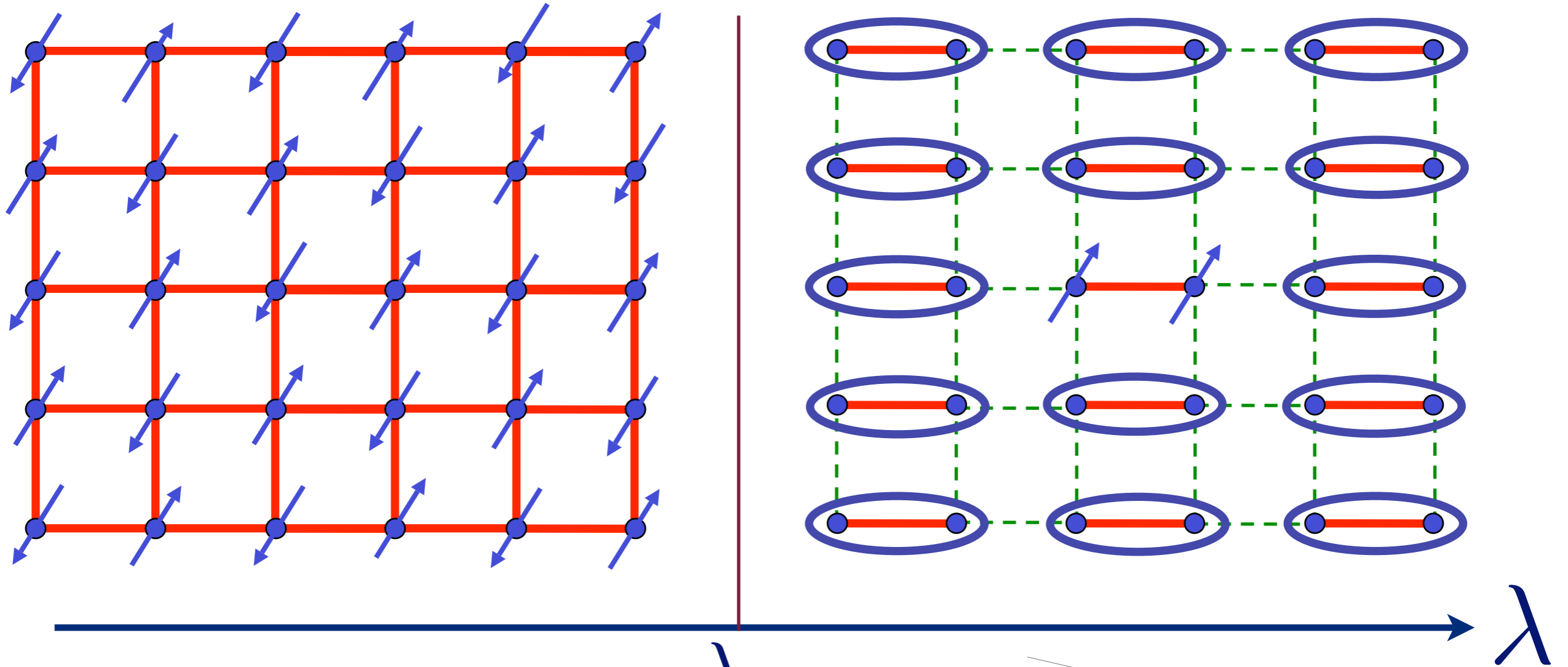
Excitation spectrum in the paramagnetic phase



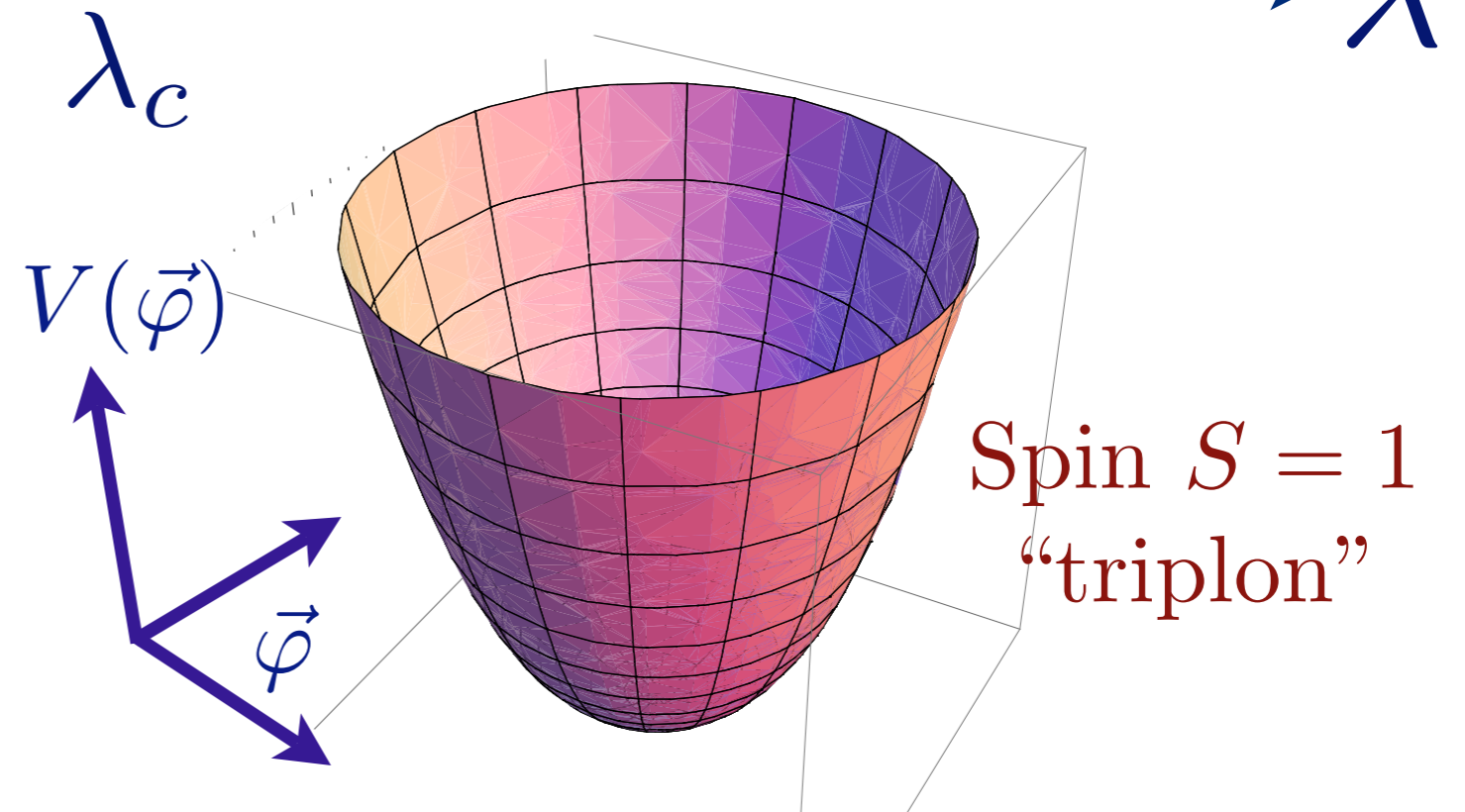
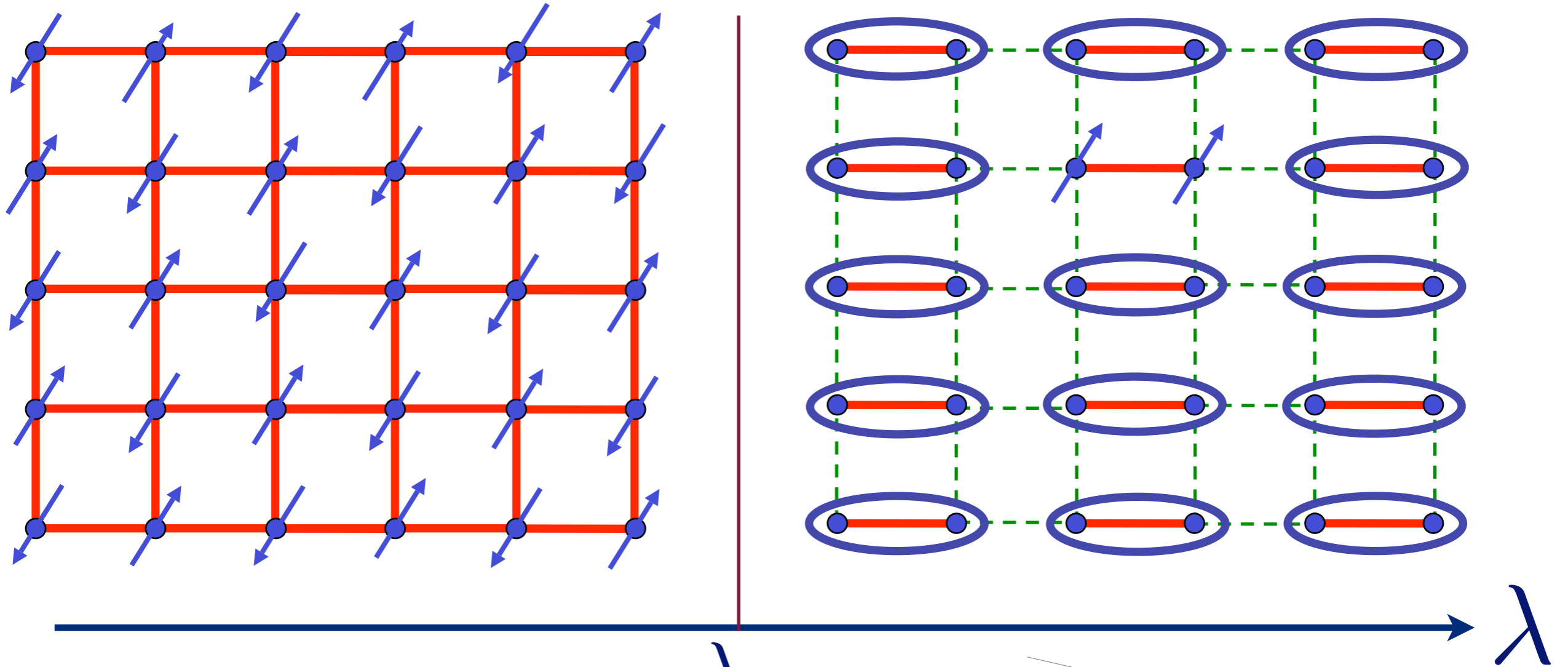
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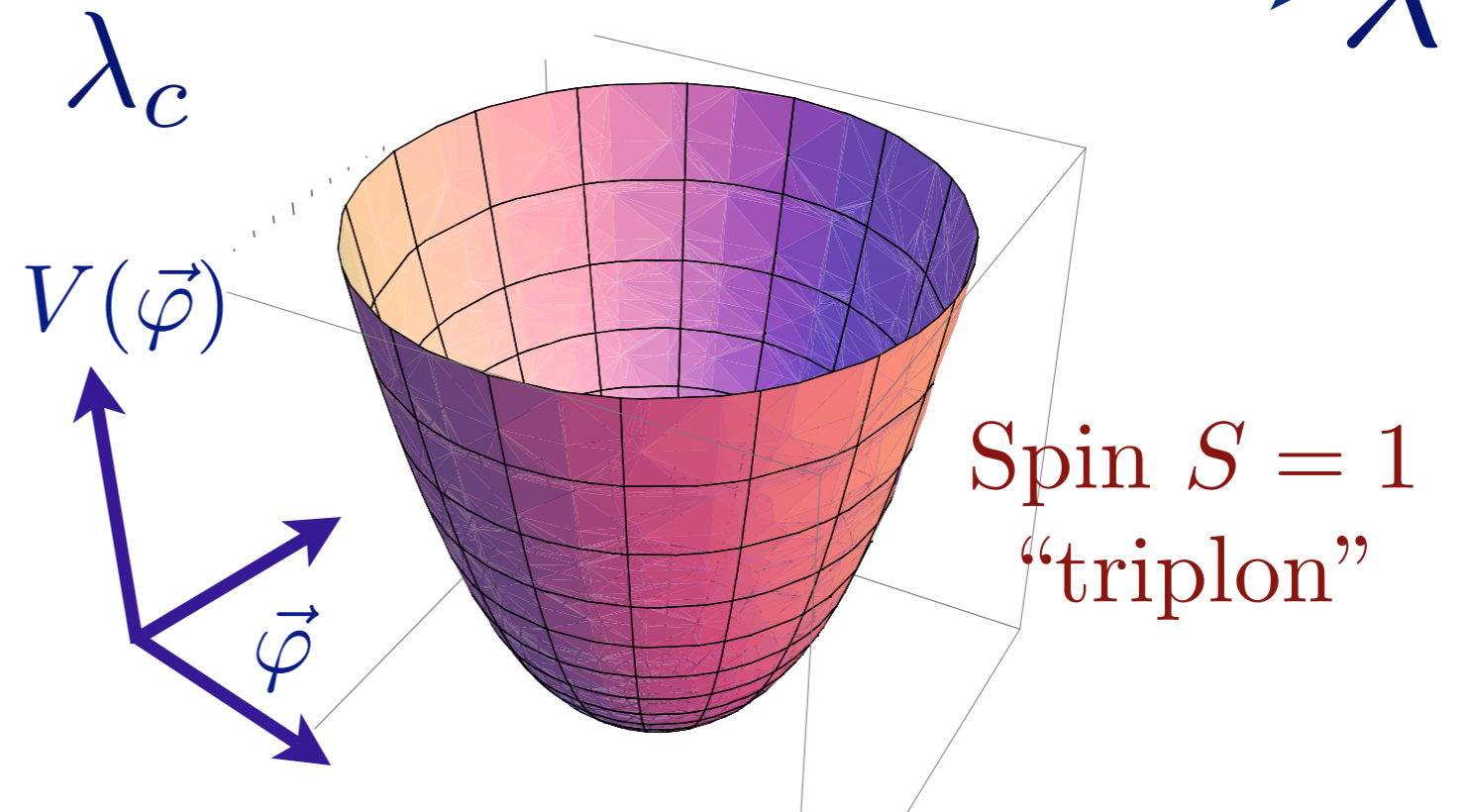
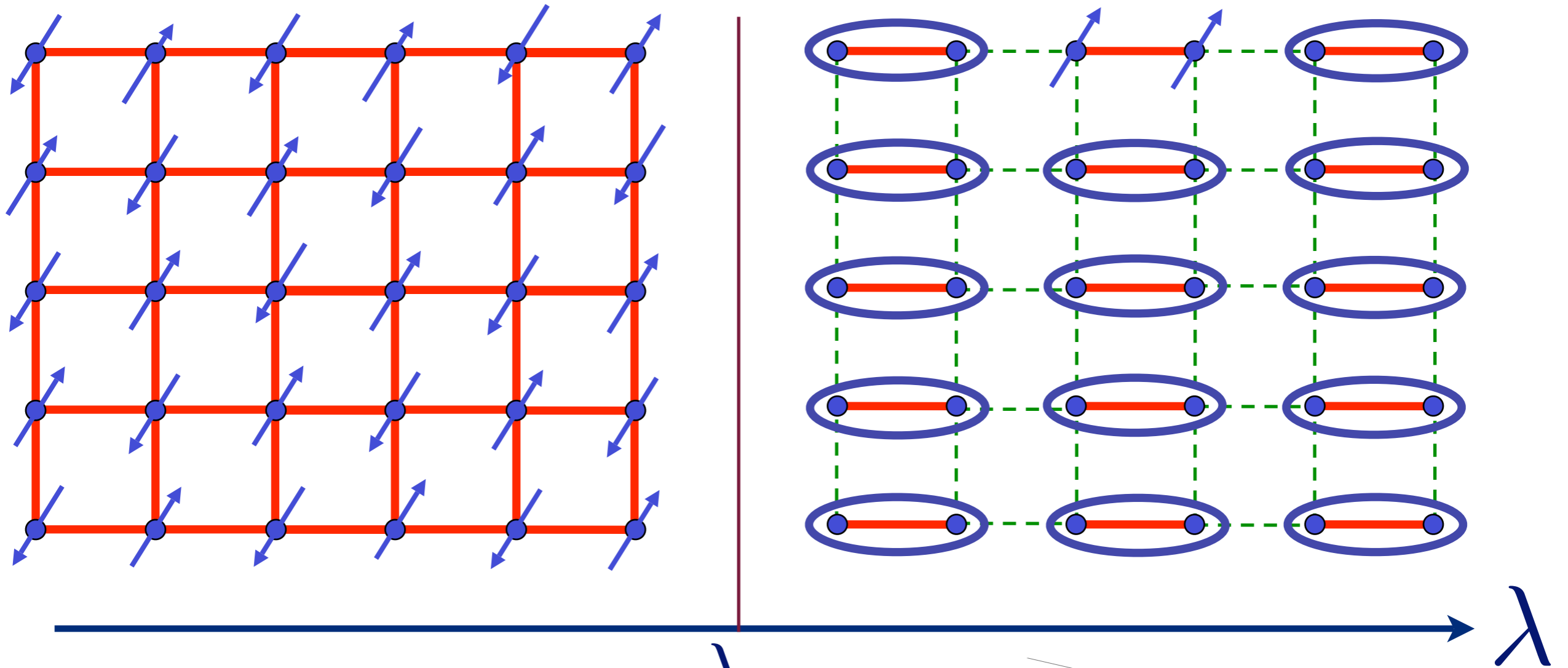
Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



TlCuCl₃ at ambient pressure

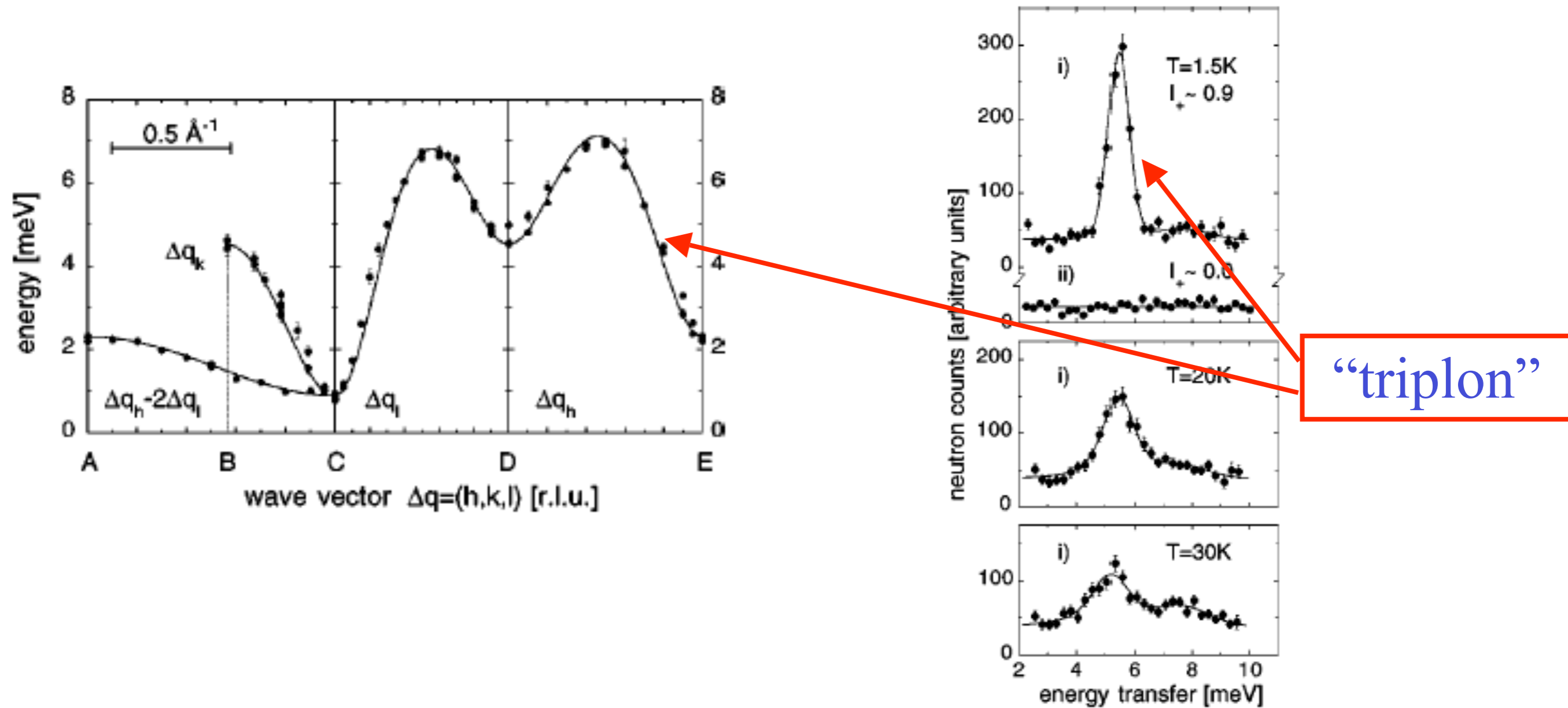
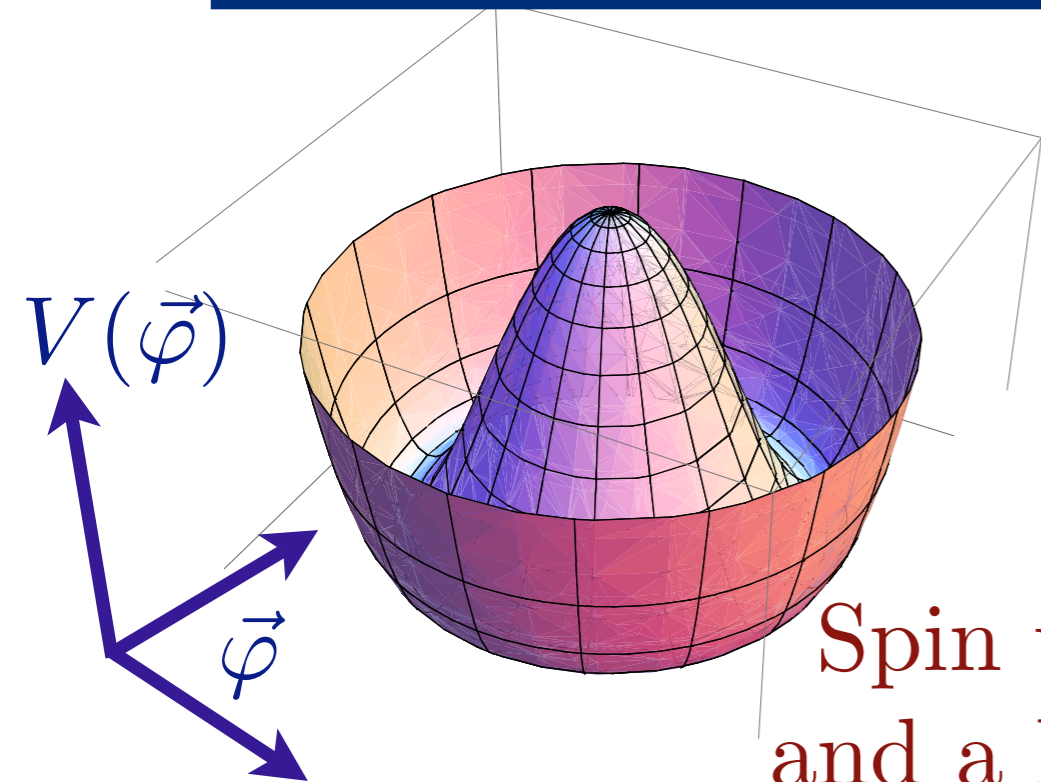
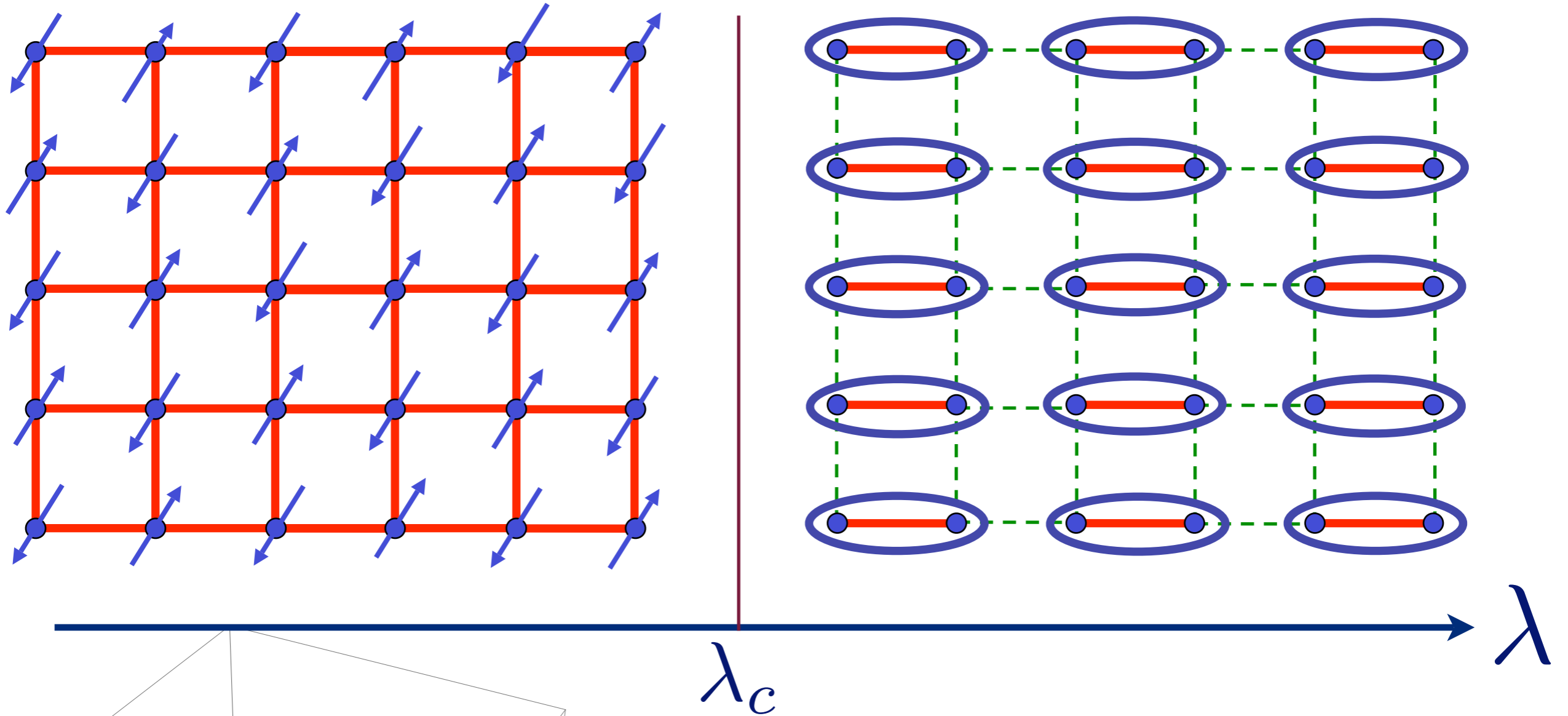


FIG. 1. Measured neutron profiles in the a^*c^* plane of TlCuCl₃ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u.]. The spectrum at $T=1.5\text{K}$

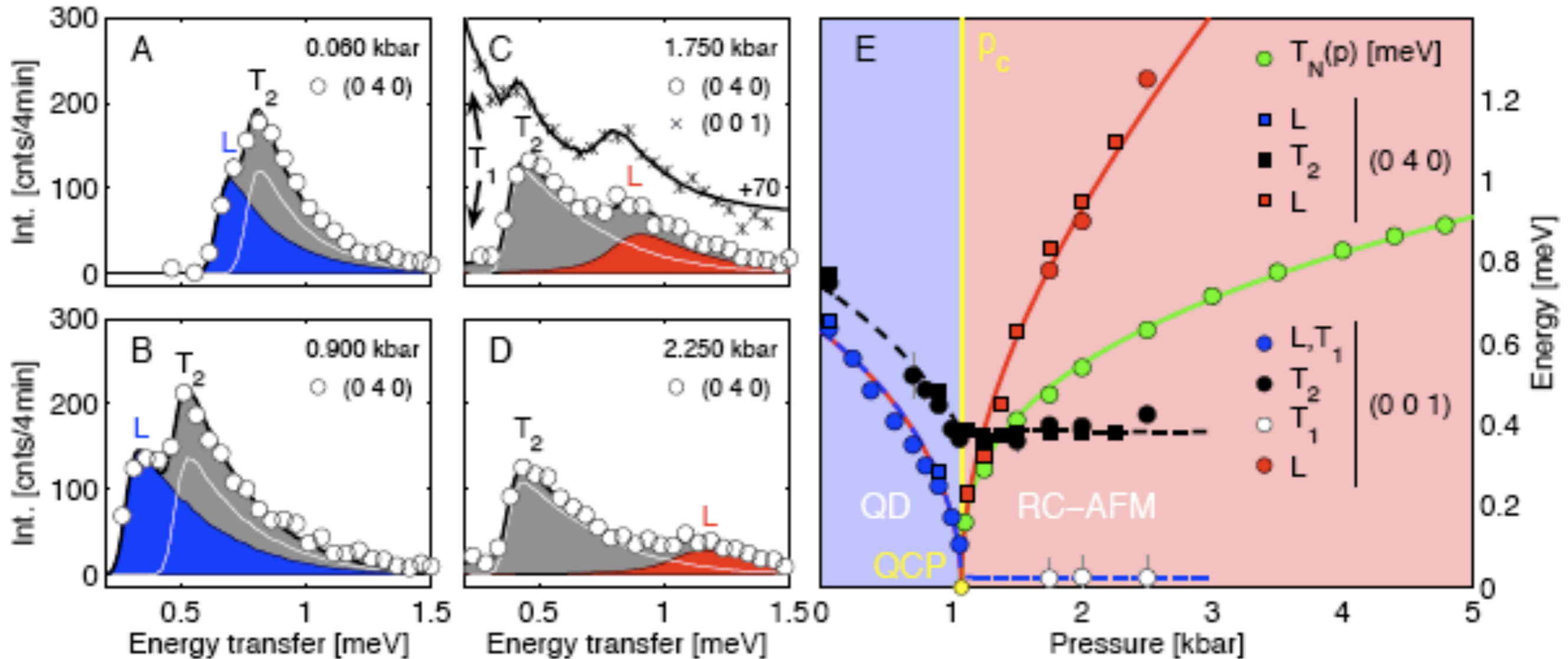
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev. B* 63 172414 (2001).

Excitation spectrum in the Néel phase



Spin waves (“Goldstone” modes)
and a longitudinal “Higgs” particle

TiCuCl₃ with varying pressure

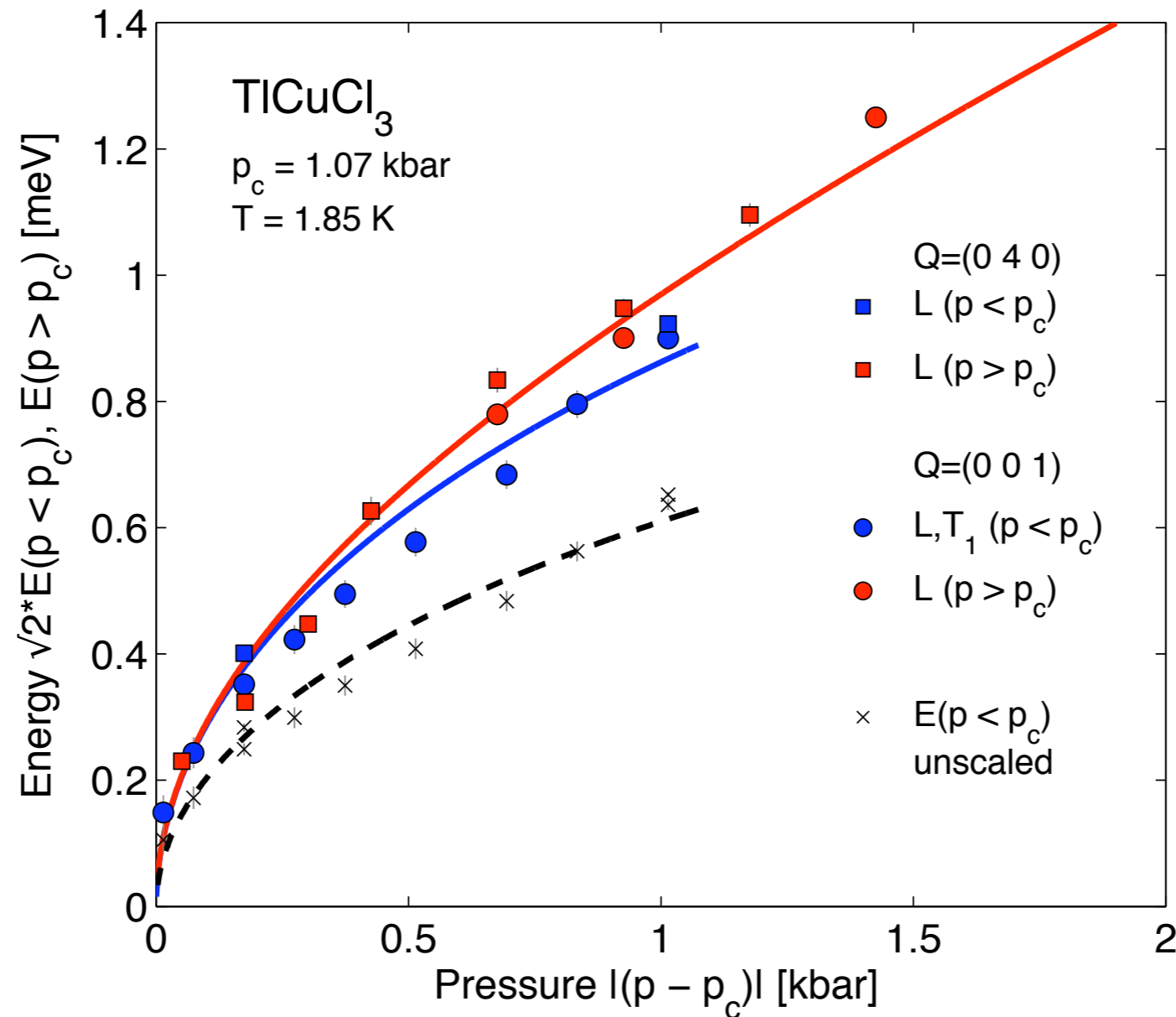


Observation of 3 \rightarrow 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorro, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

$$\frac{\text{Energy of "Higgs" particle}}{\text{Energy of triplon}} = \sqrt{2}$$



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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(b) Triangular lattice antiferromagnets

(c) The superfluid-insulator transition

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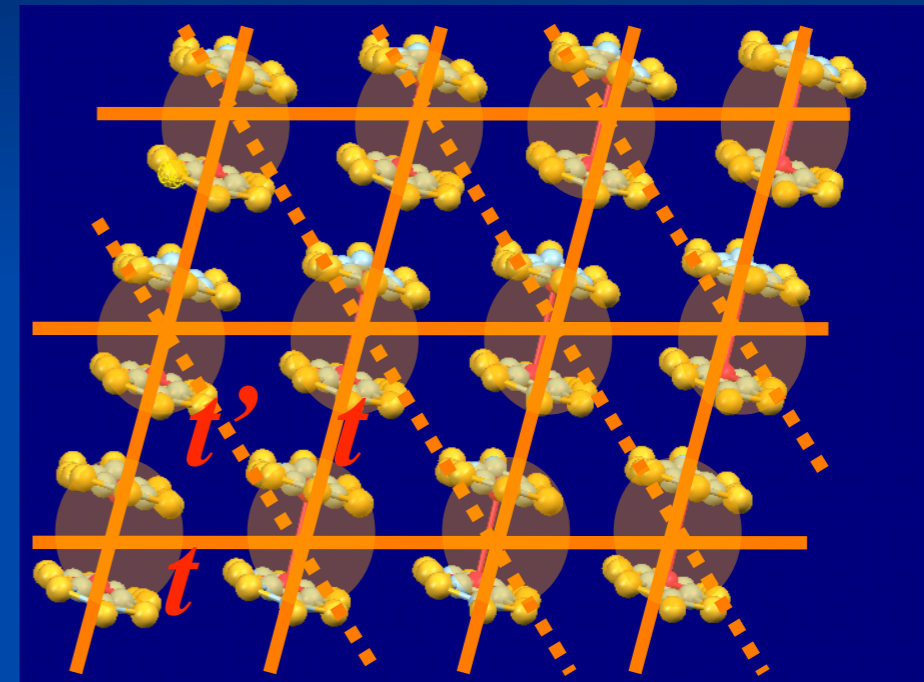
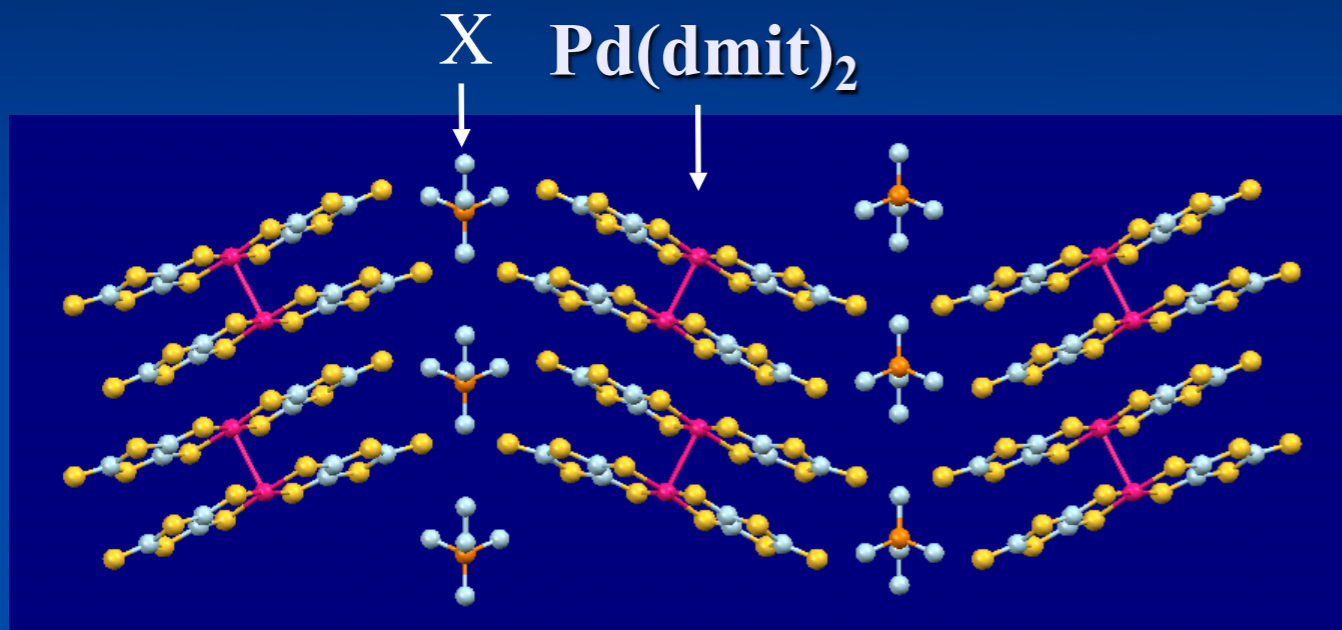
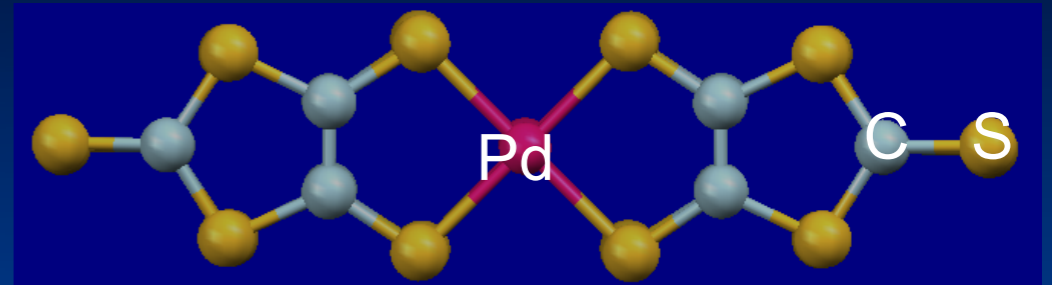
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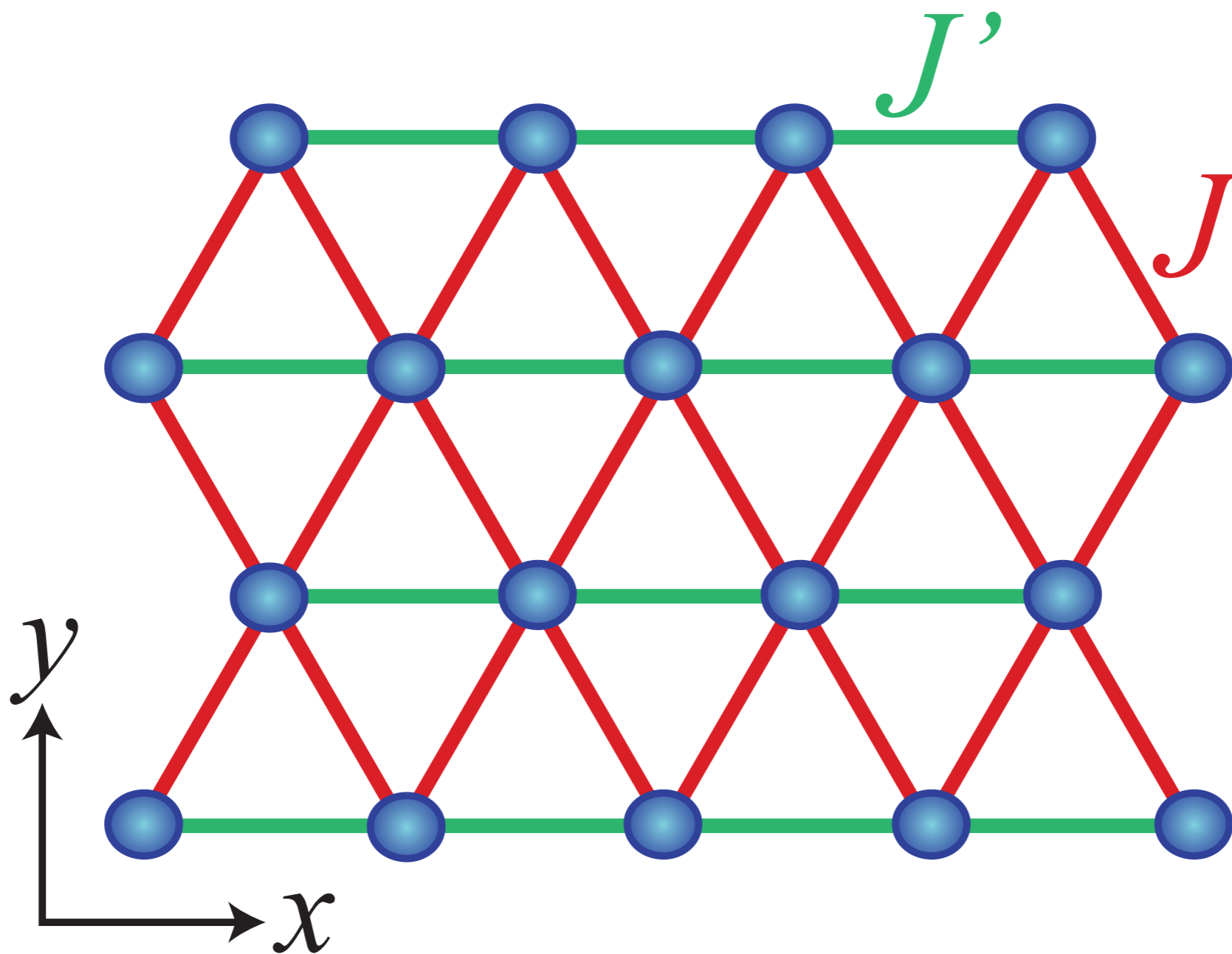
Half-filled band \rightarrow Mott insulator with spin $S = 1/2$

Triangular lattice of $[\text{Pd}(\text{dmit})_2]_2$

\rightarrow frustrated quantum spin system

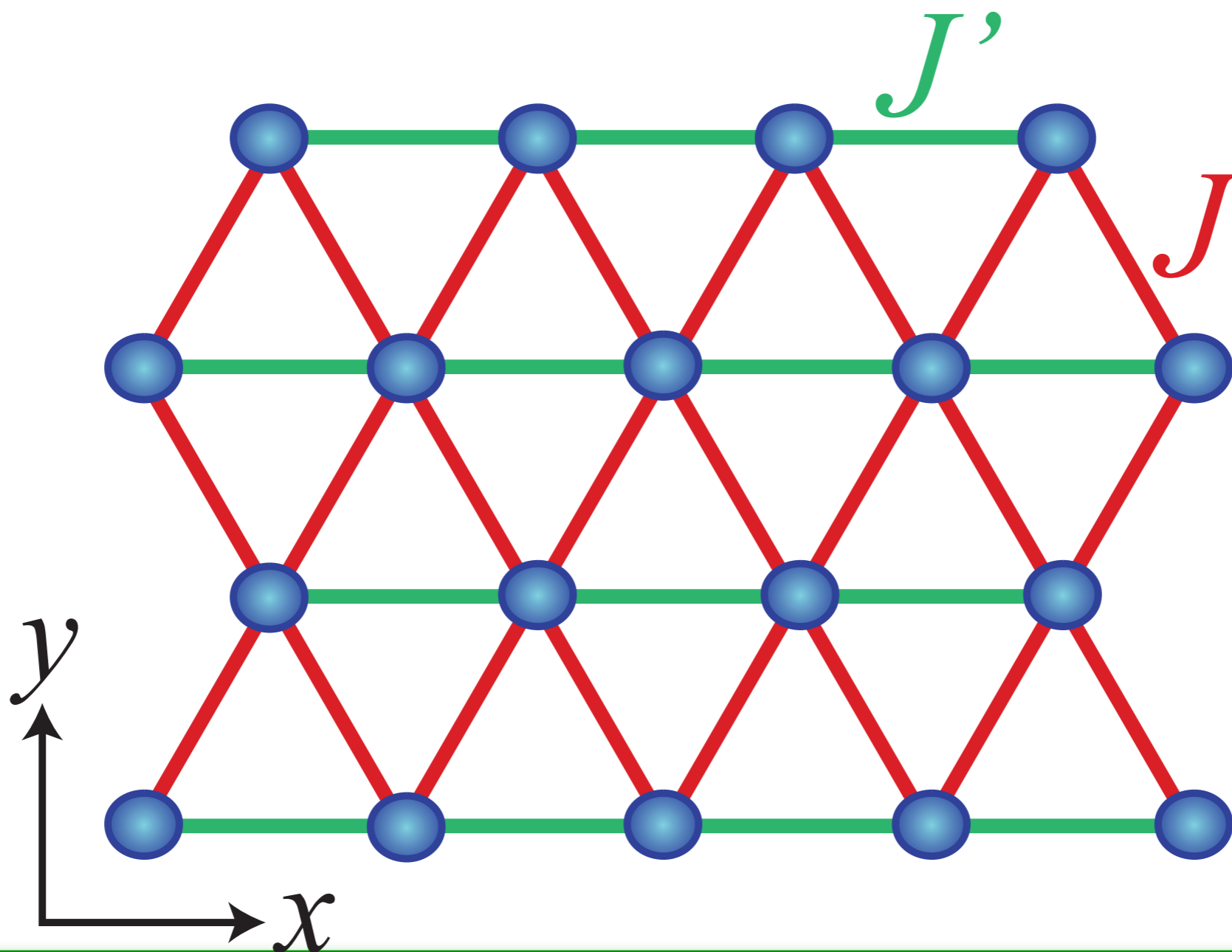
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



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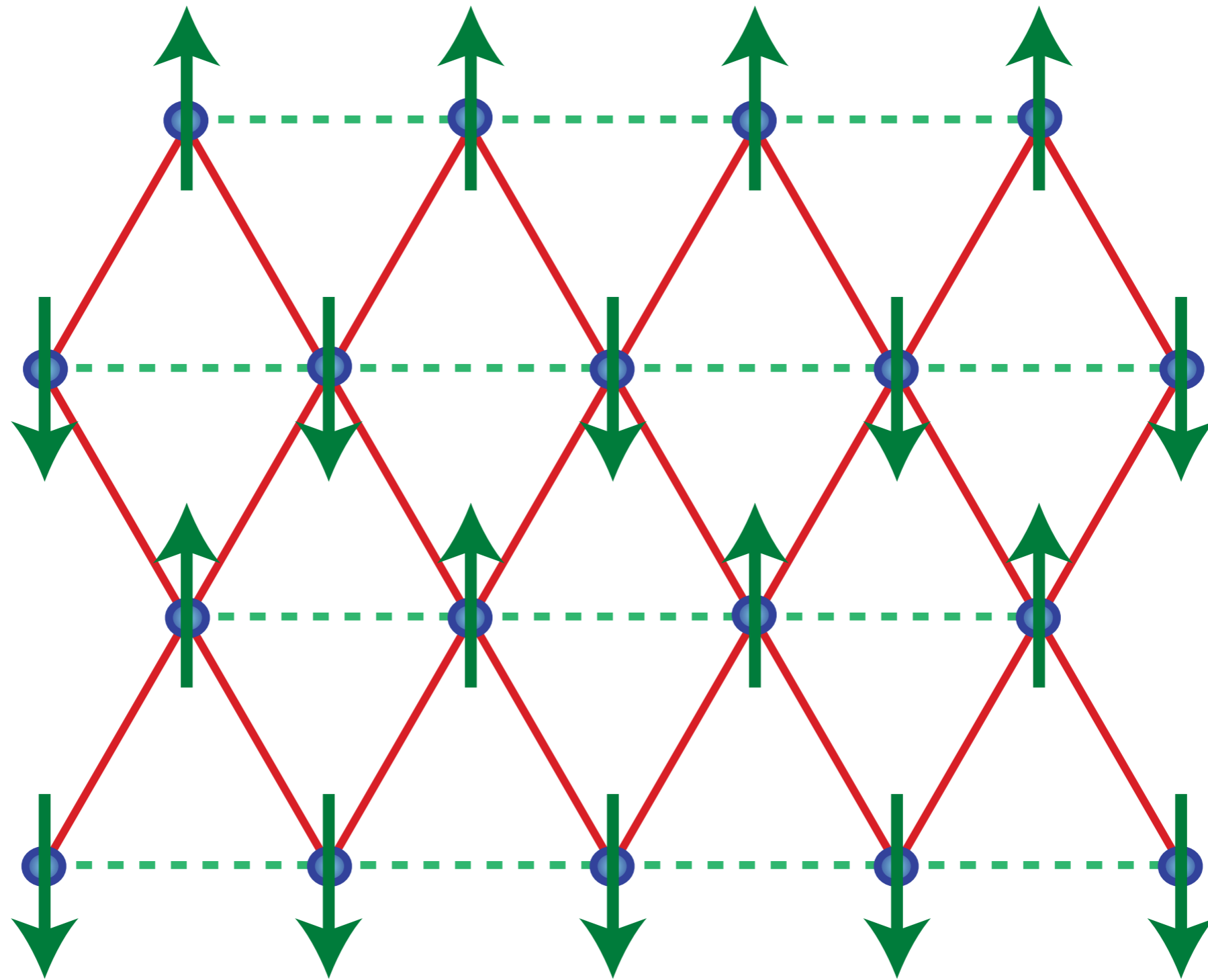
$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



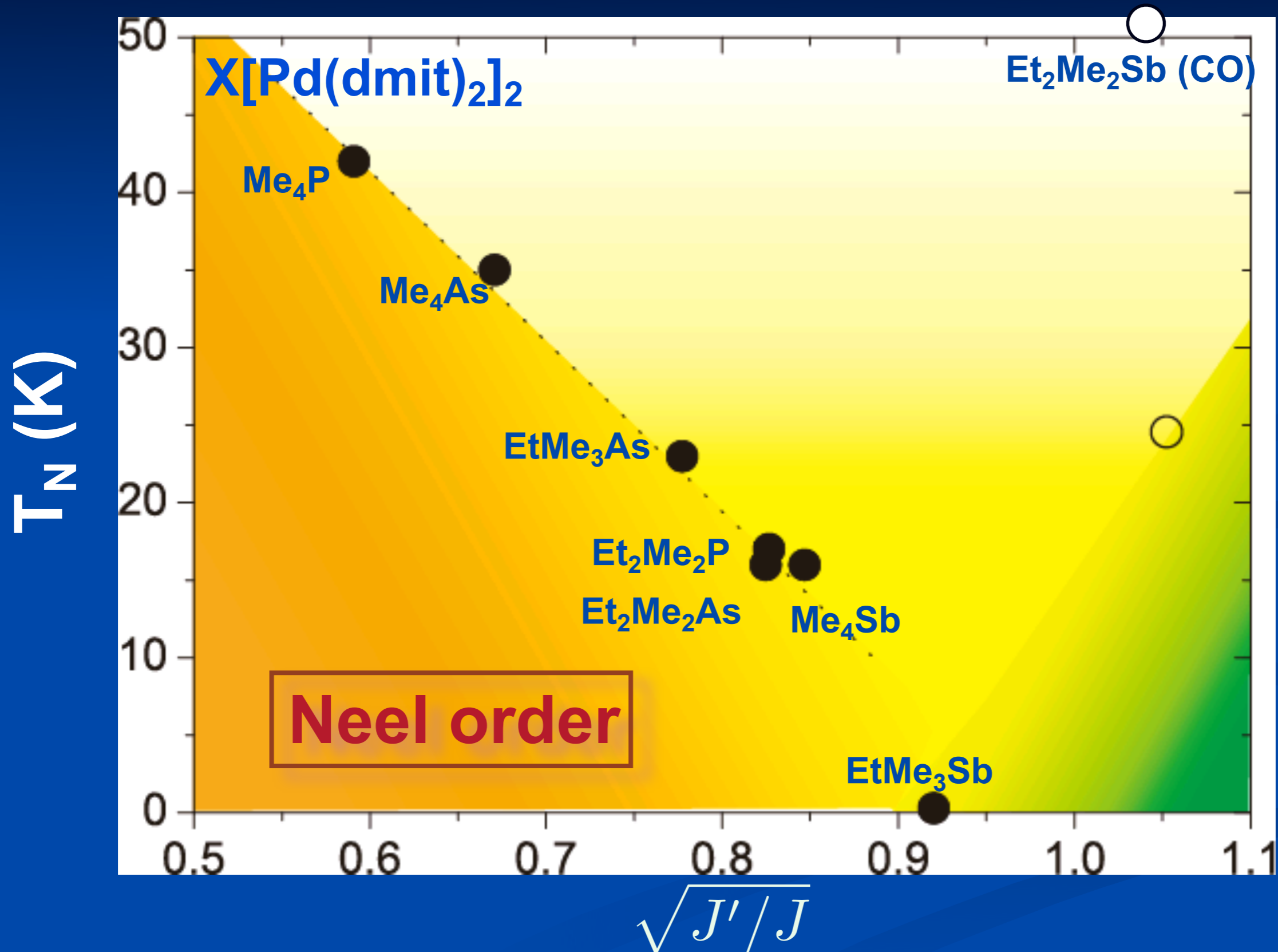
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

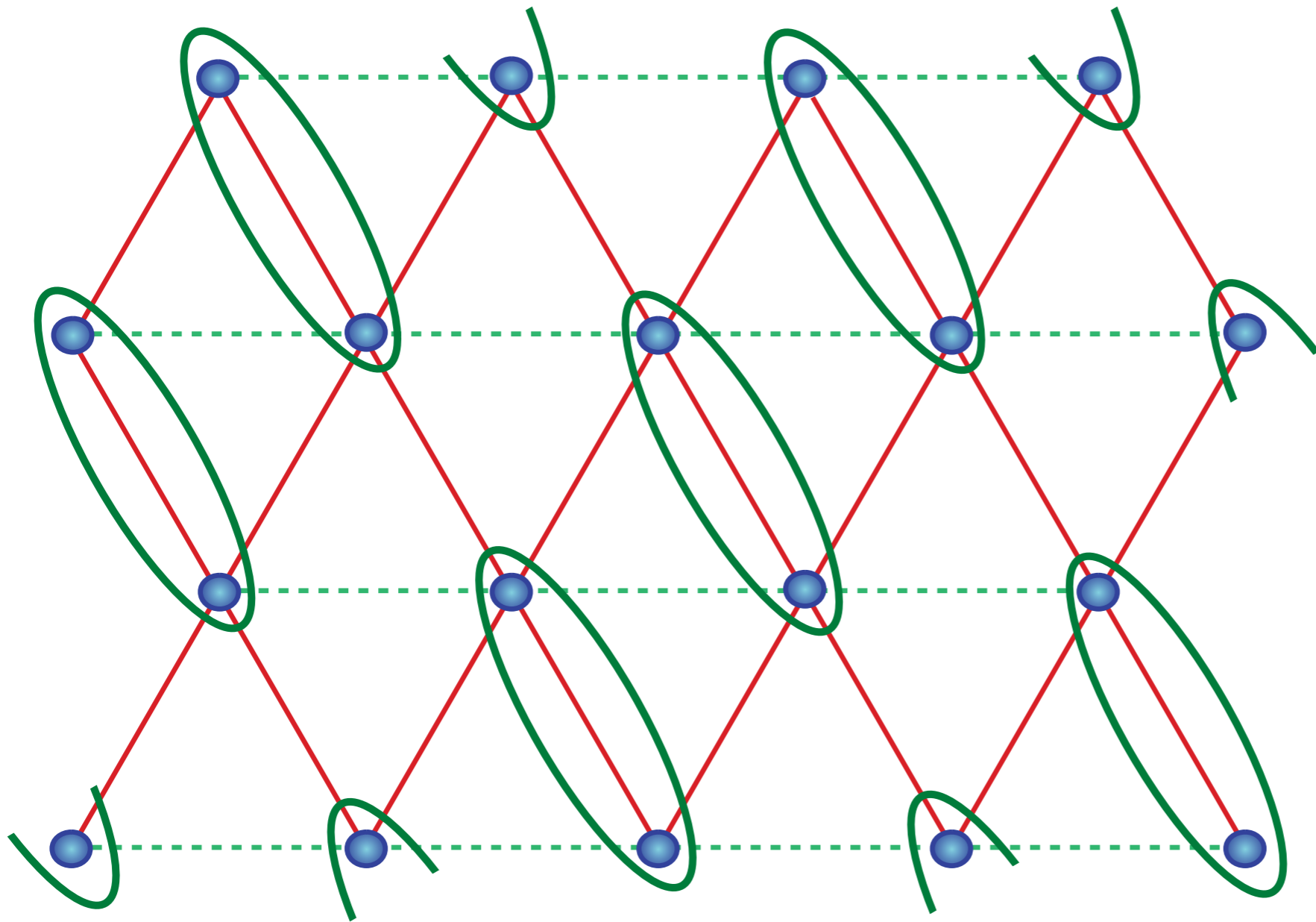
Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO

Magnetic Criticality



Anisotropic triangular lattice antiferromagnet

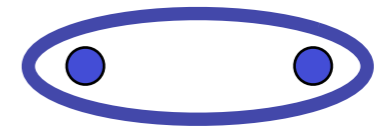
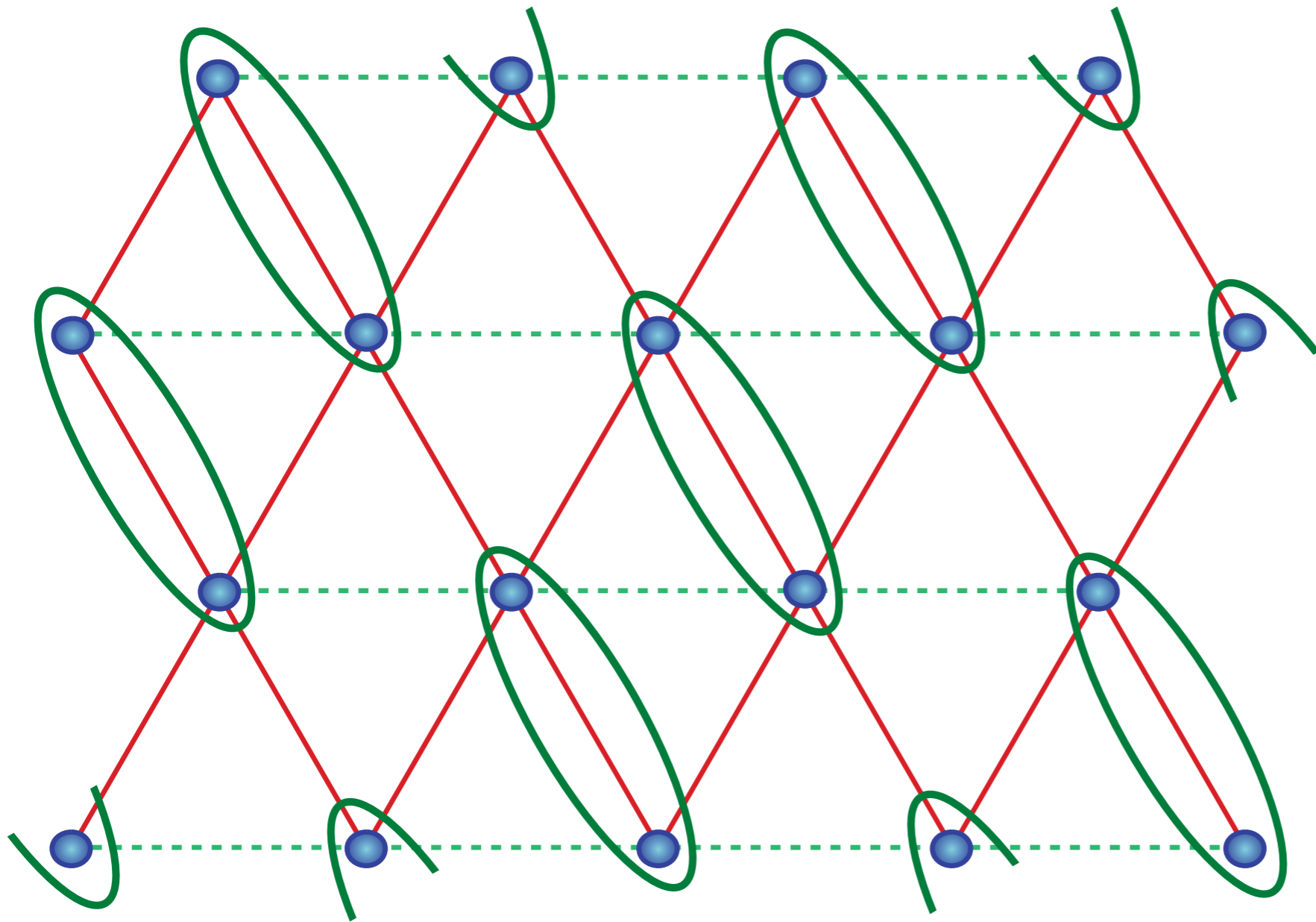


$$\begin{array}{c} \text{Diagram of two spheres in an oval} \\ = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}} \end{array}$$

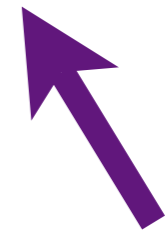
Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry



$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

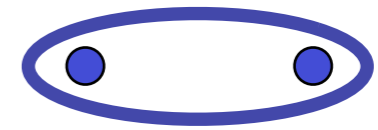
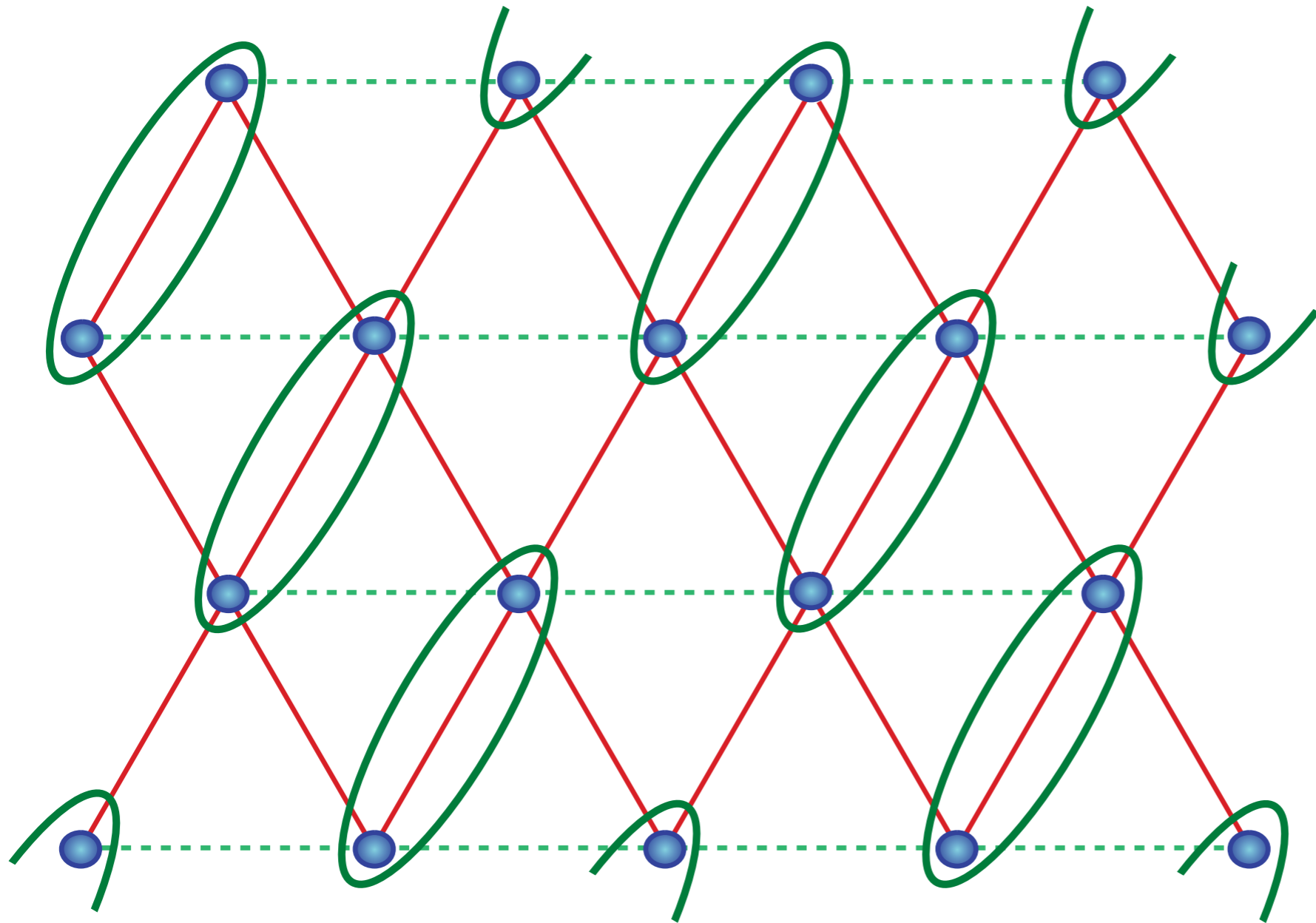


Valence bond solid (VBS)

Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry



$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

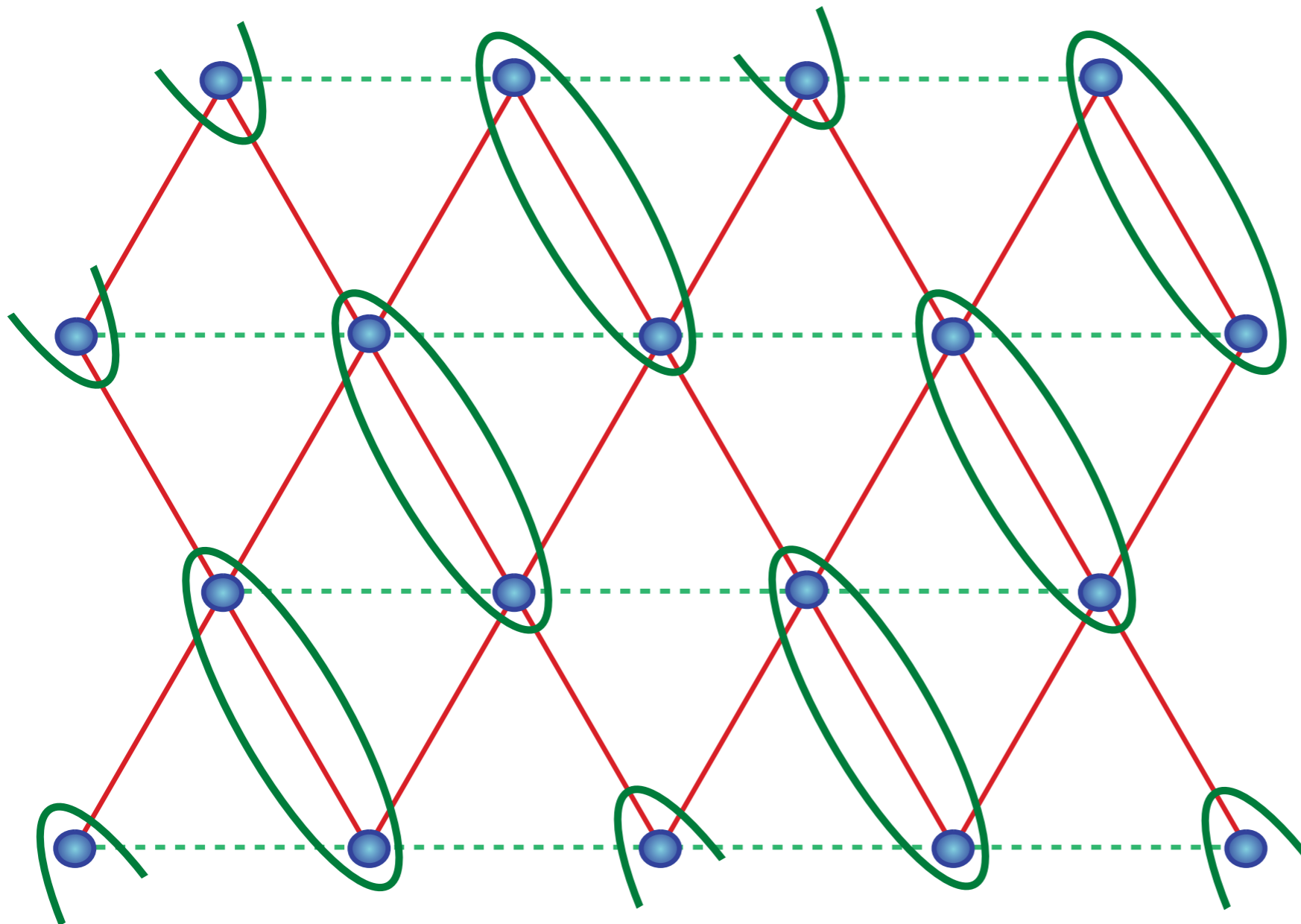


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Anisotropic triangular lattice antiferromagnet

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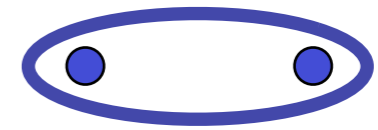
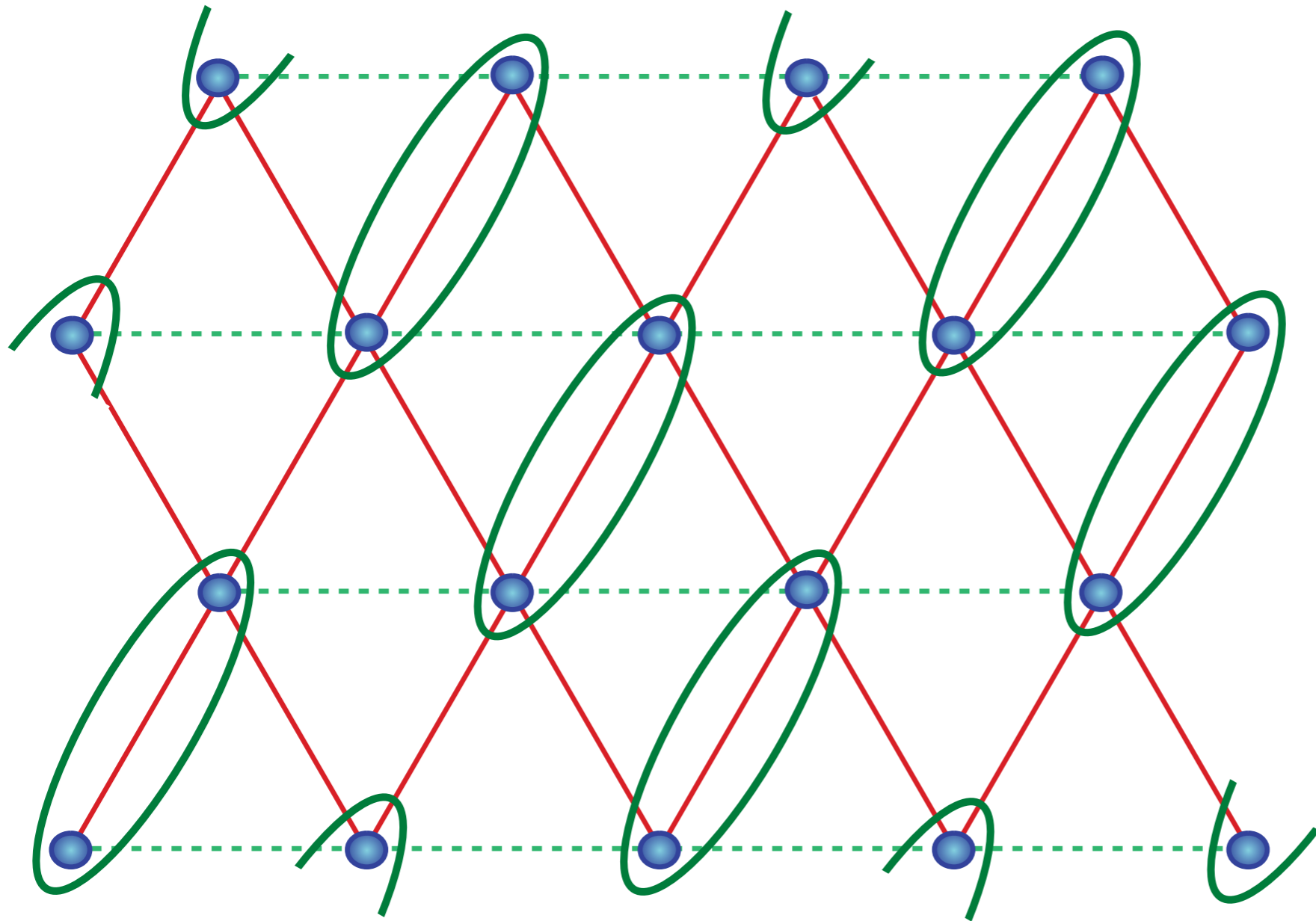


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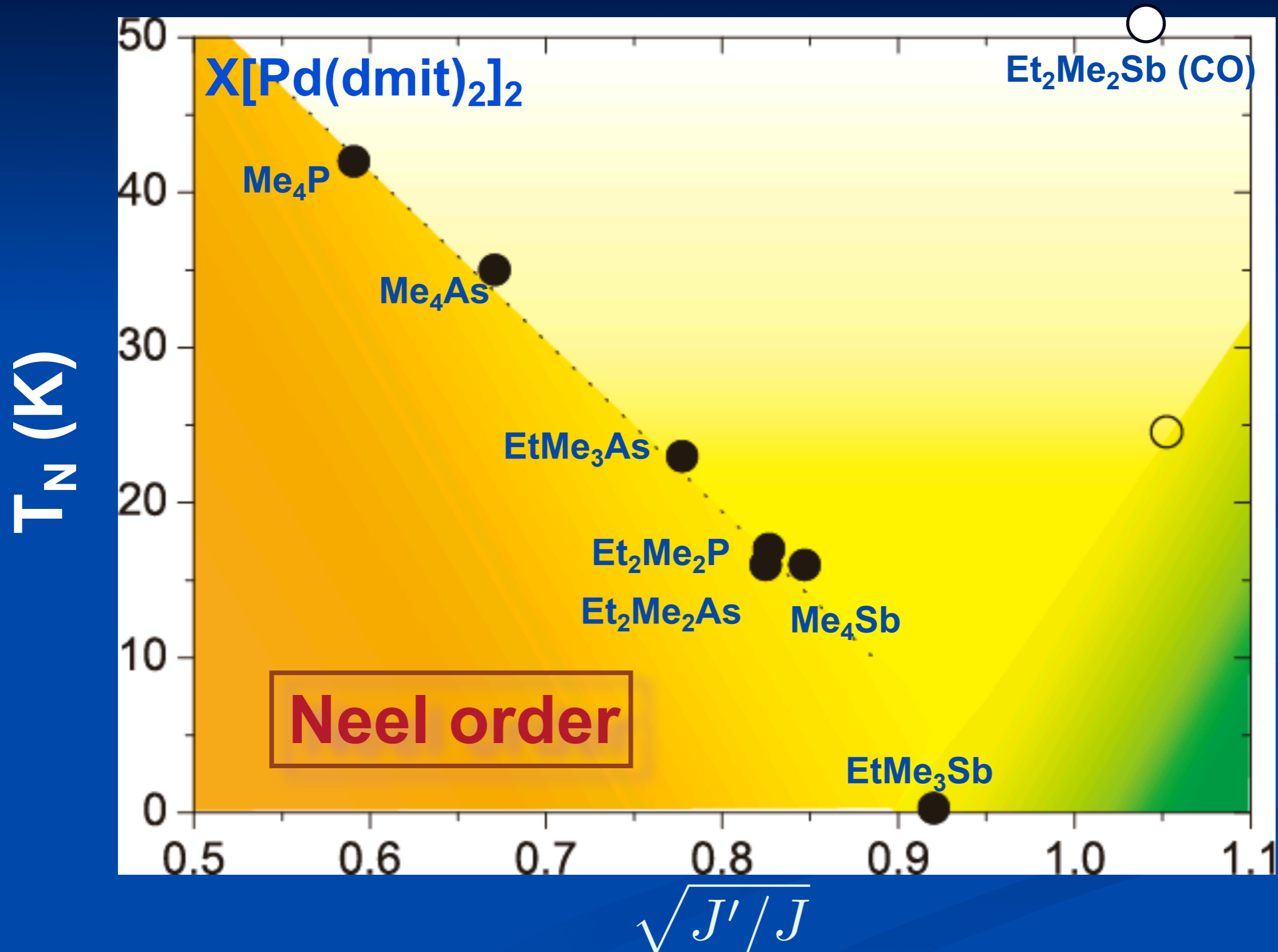
Possible ground state for intermediate J'/J

Anisotropic triangular lattice antiferromagnet

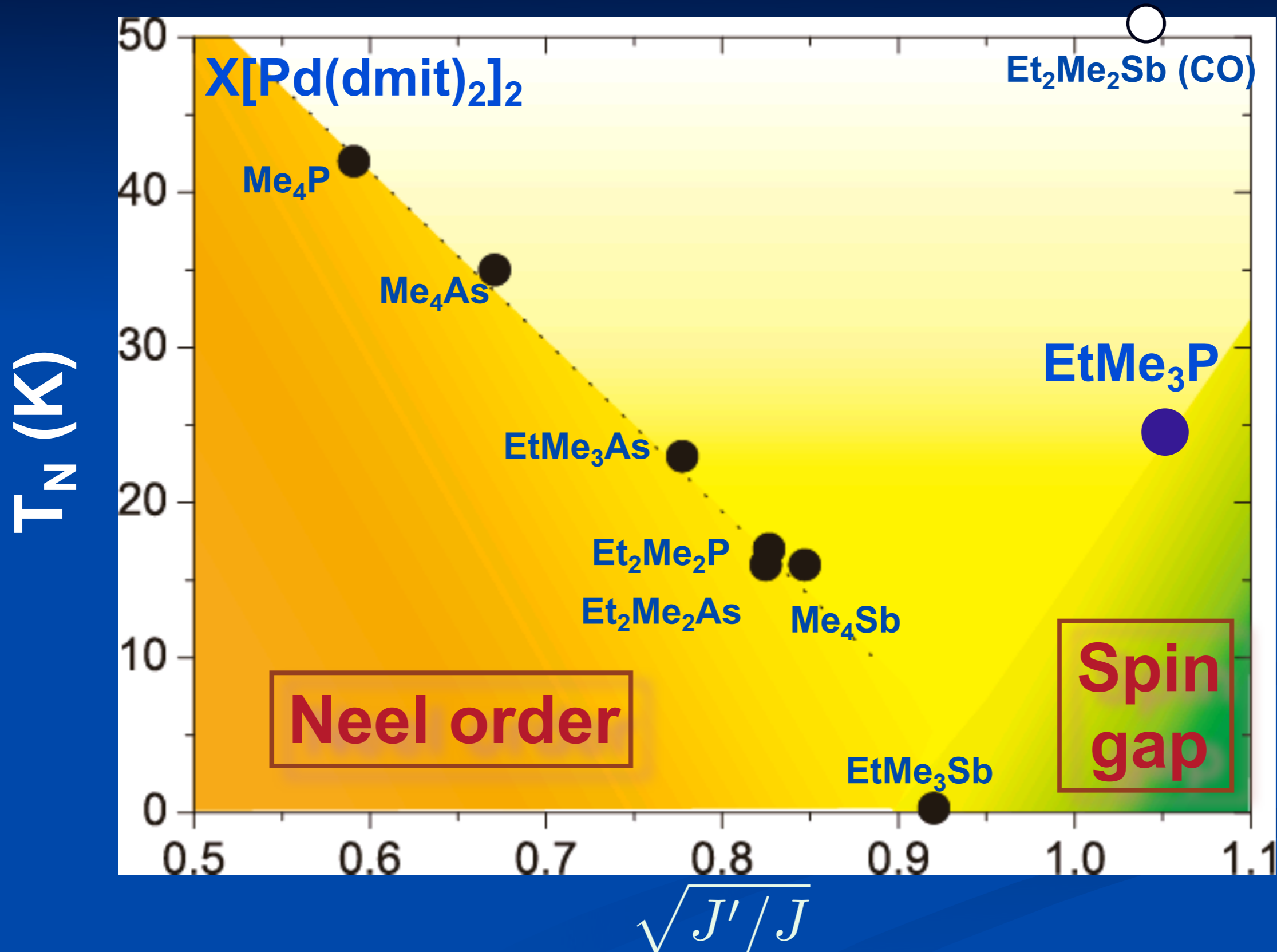
Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

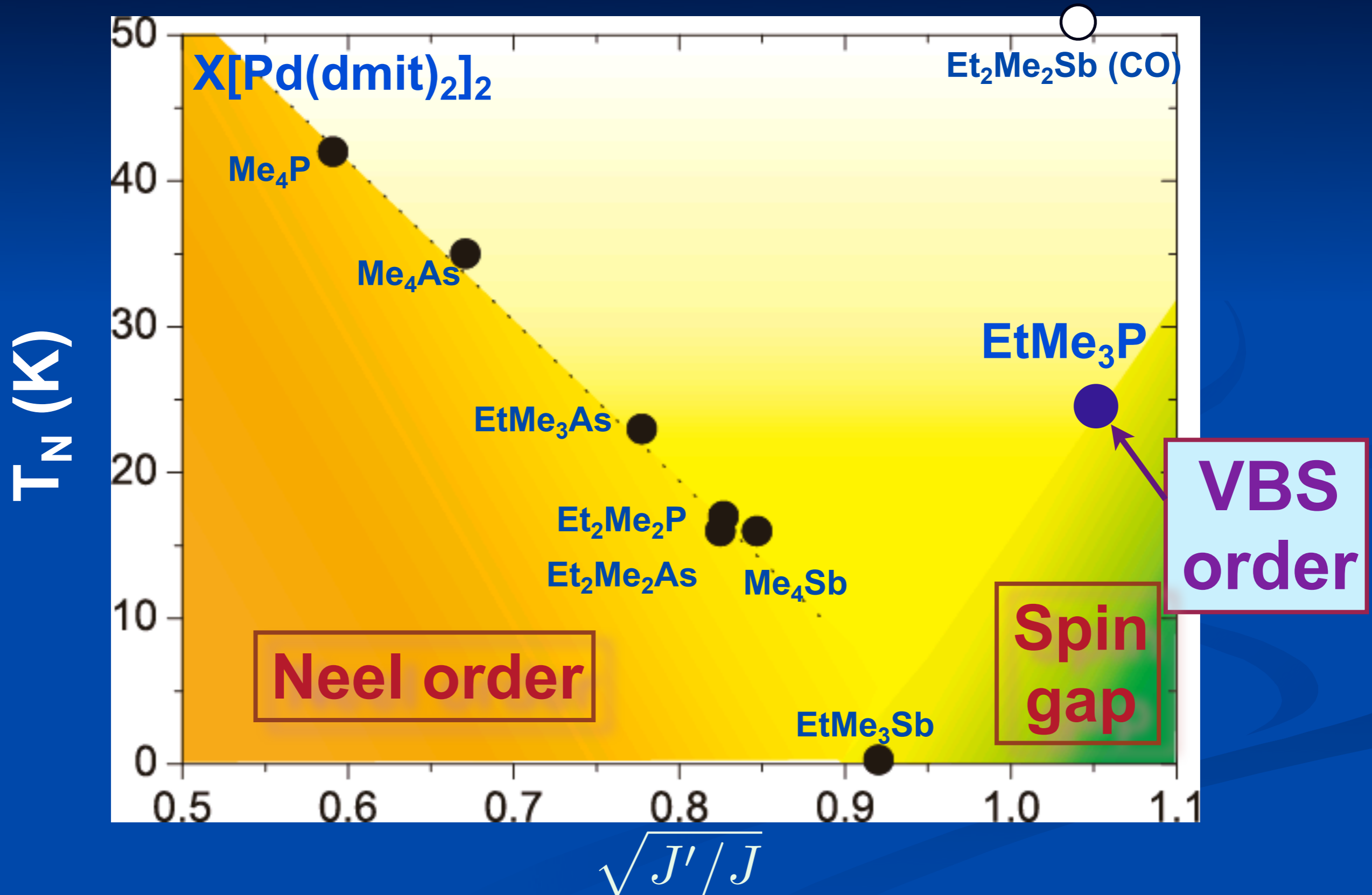
Magnetic Criticality



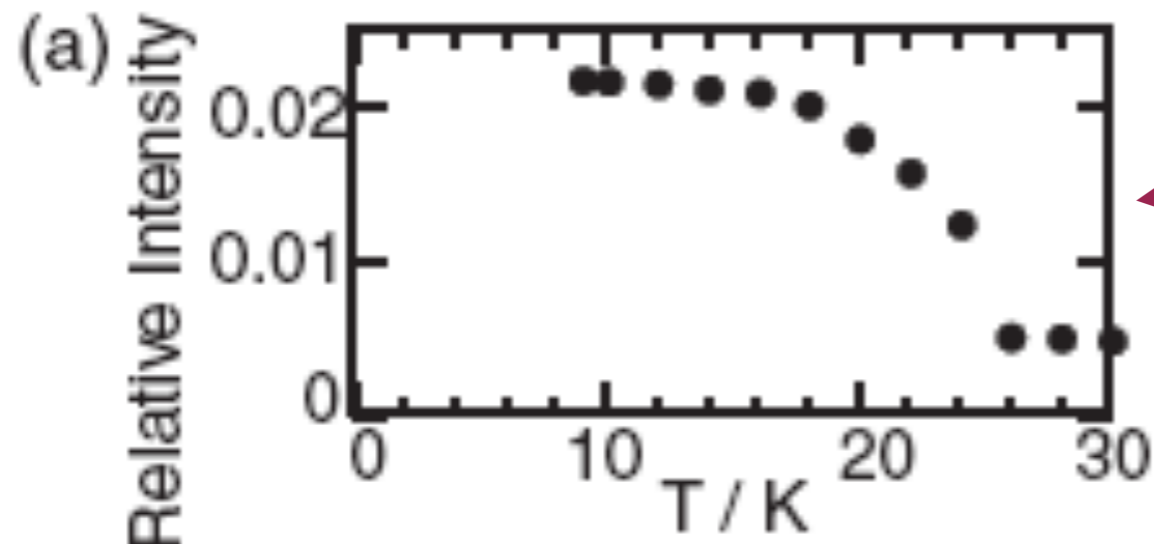
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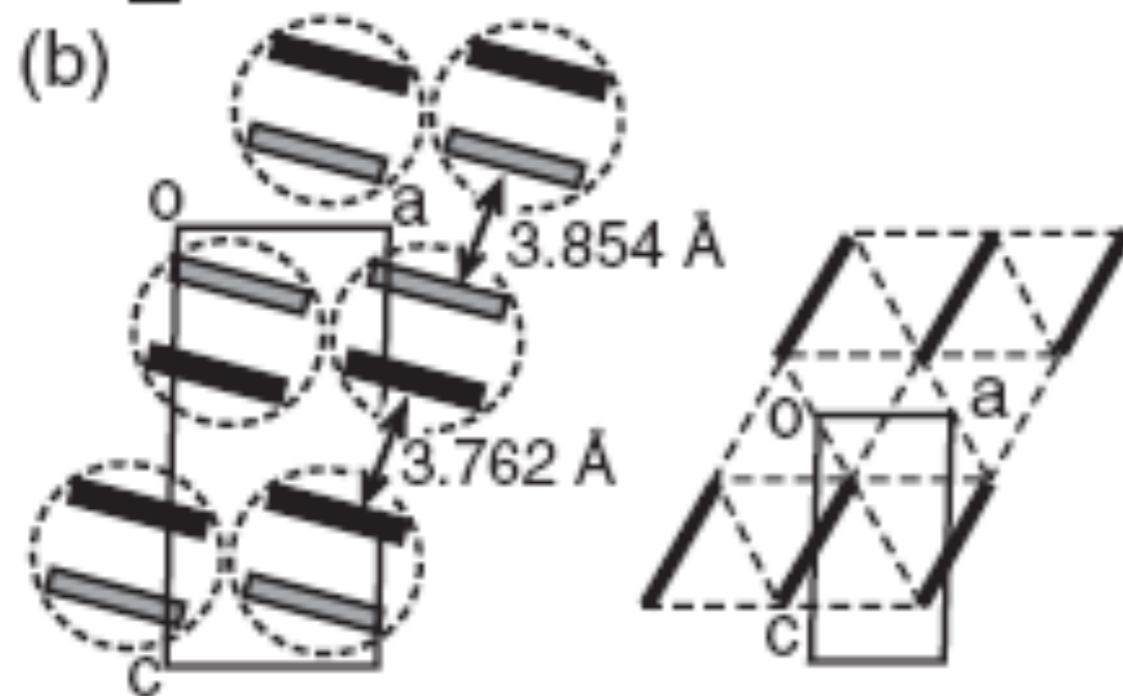
Magnetic Criticality



Observation of a valence bond solid (VBS) in $\text{ETMe}_3\text{P}[\text{Pd}(\text{dmit})_2]_2$



X-ray scattering

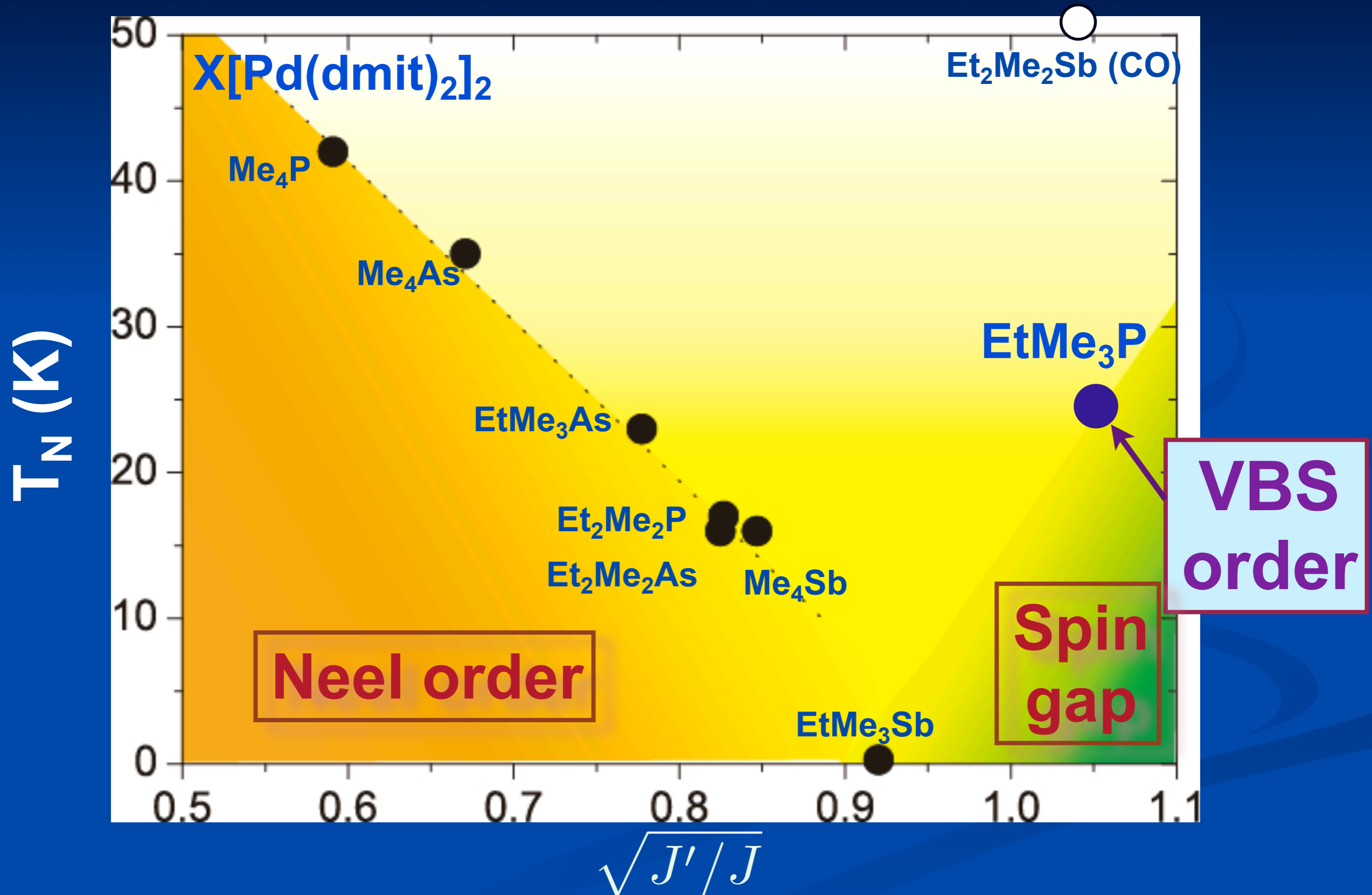


Spin gap ~ 40 K
 $J \sim 250$ K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality

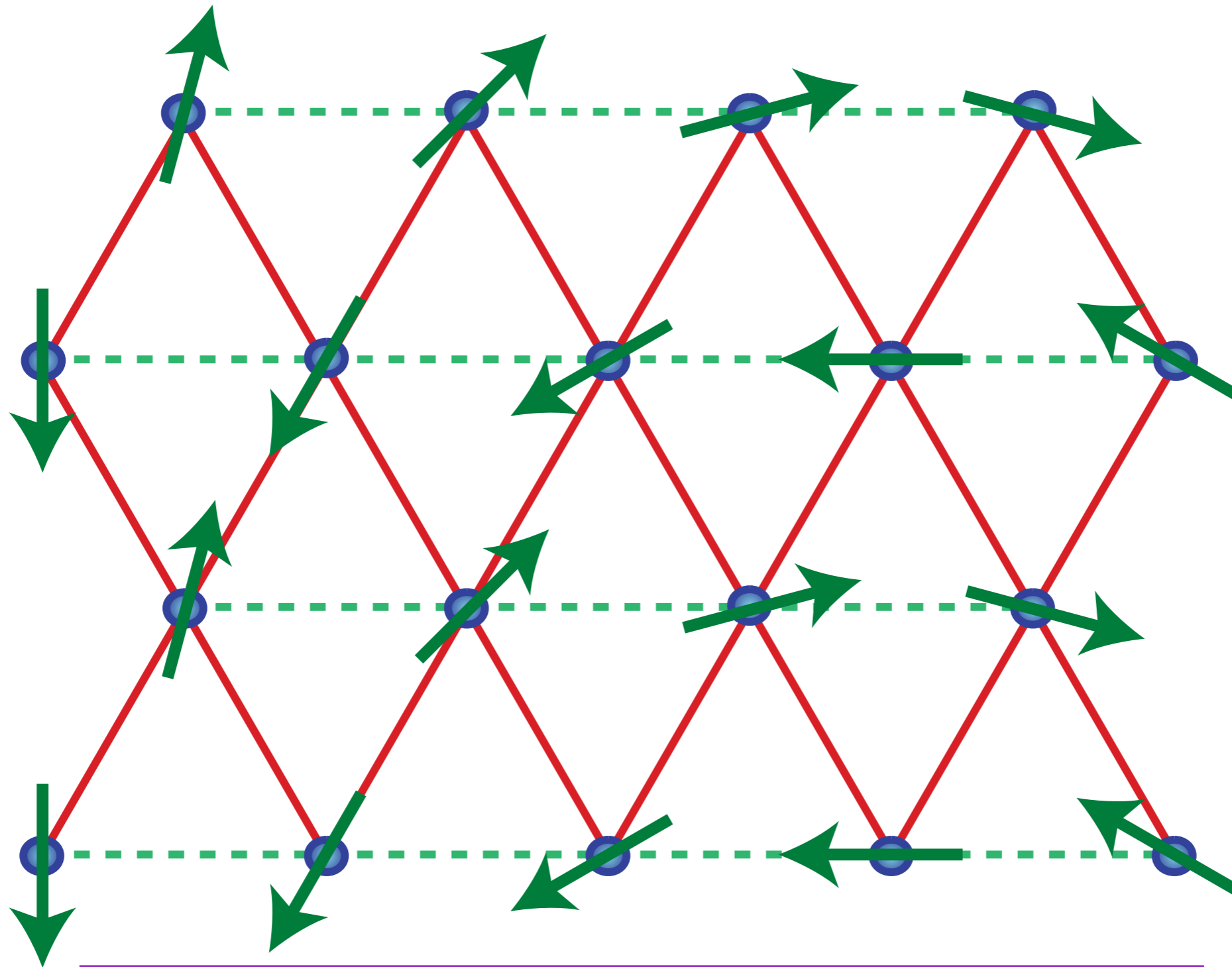


Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO

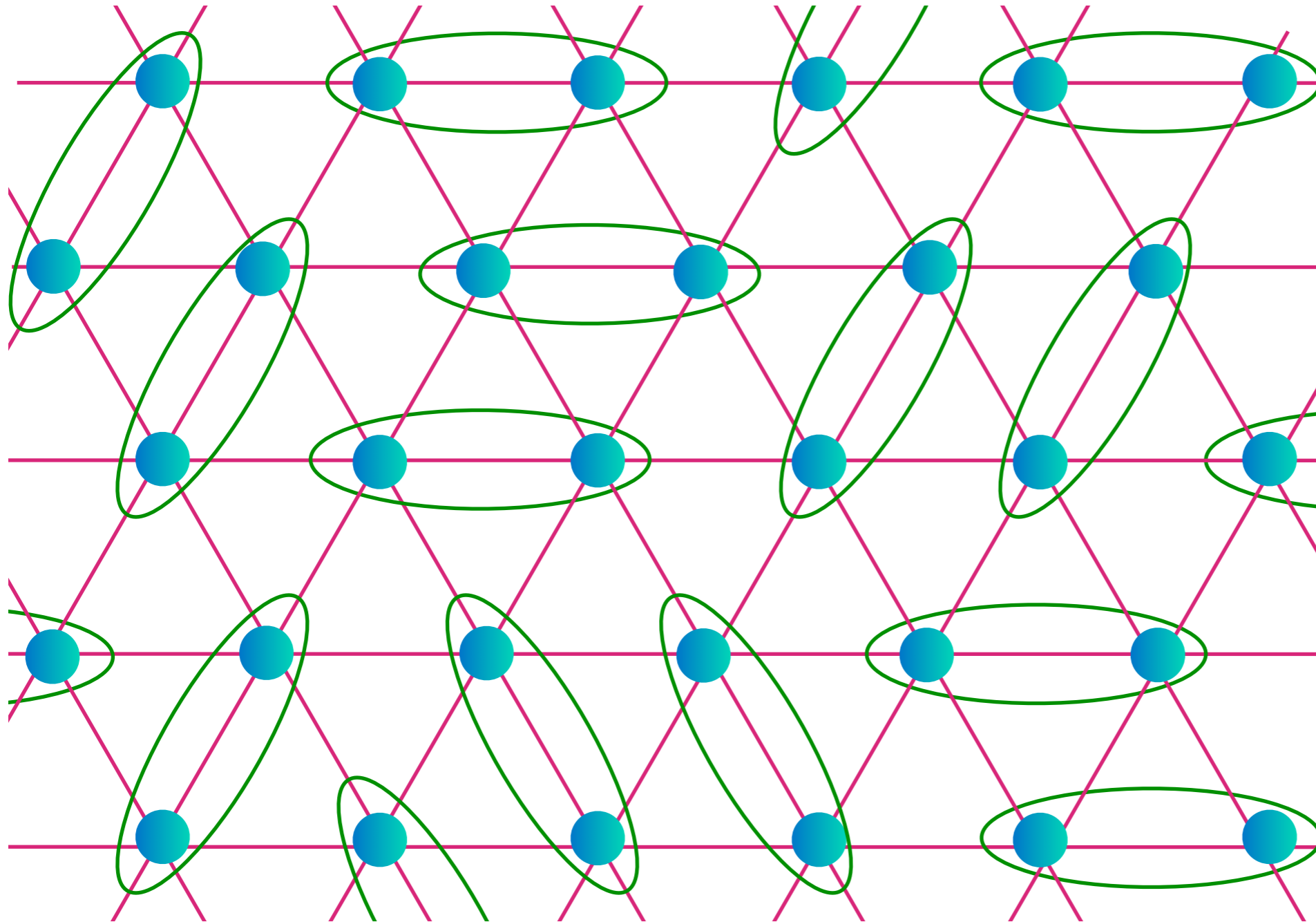
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO
- Z_2 spin liquid: preserves all symmetries of Hamiltonian

Triangular lattice antiferromagnet

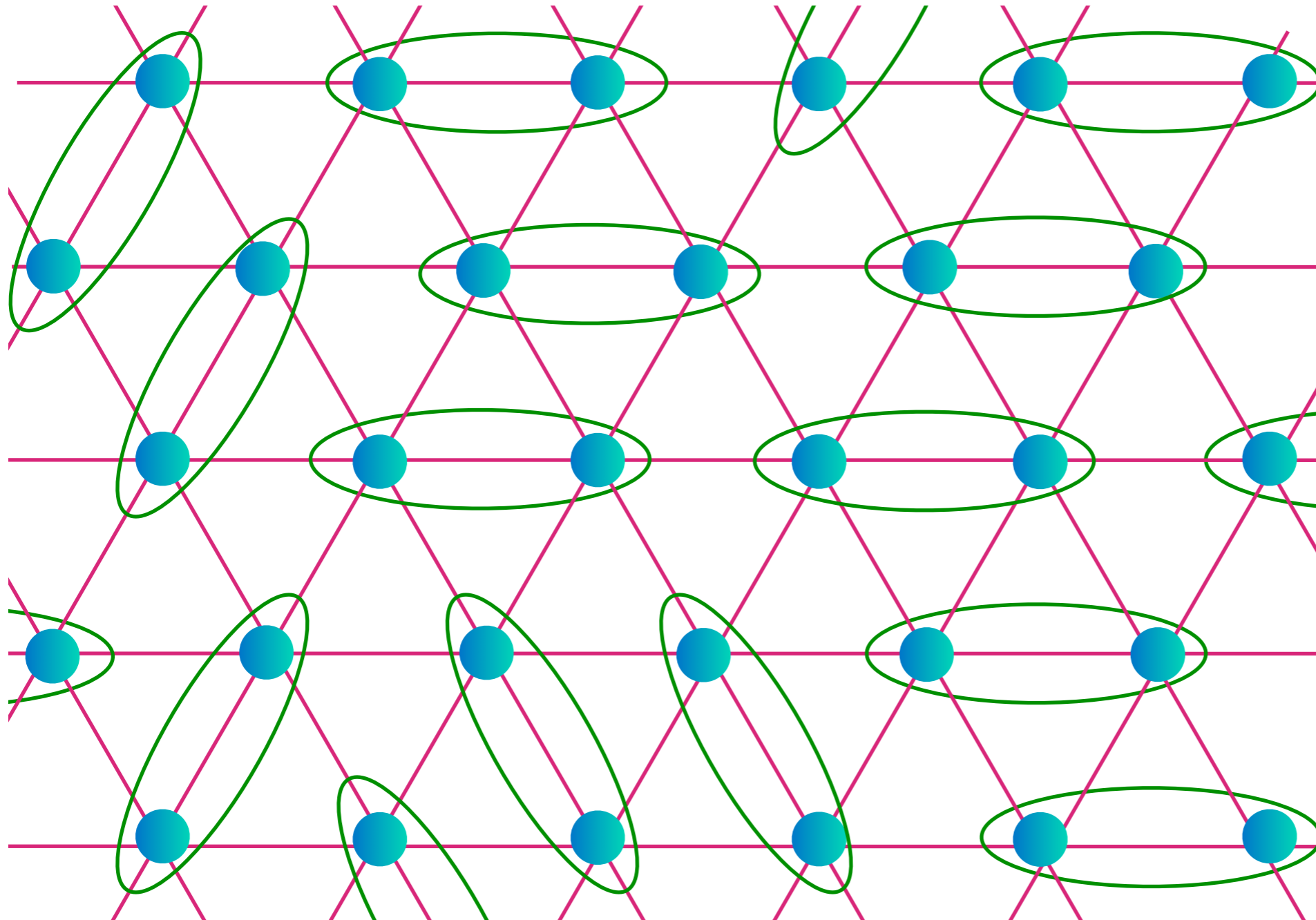
Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell

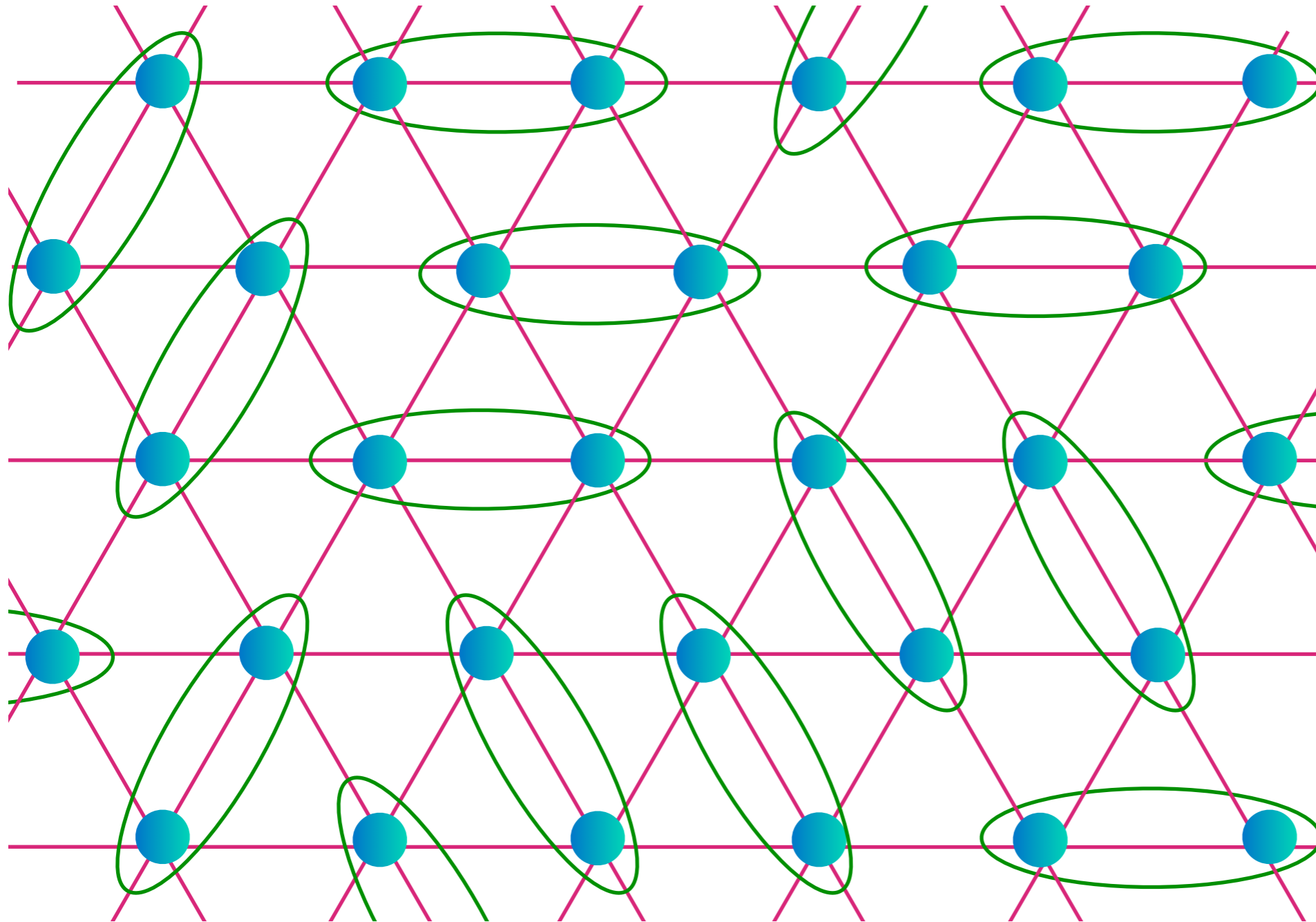


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Triangular lattice antiferromagnet

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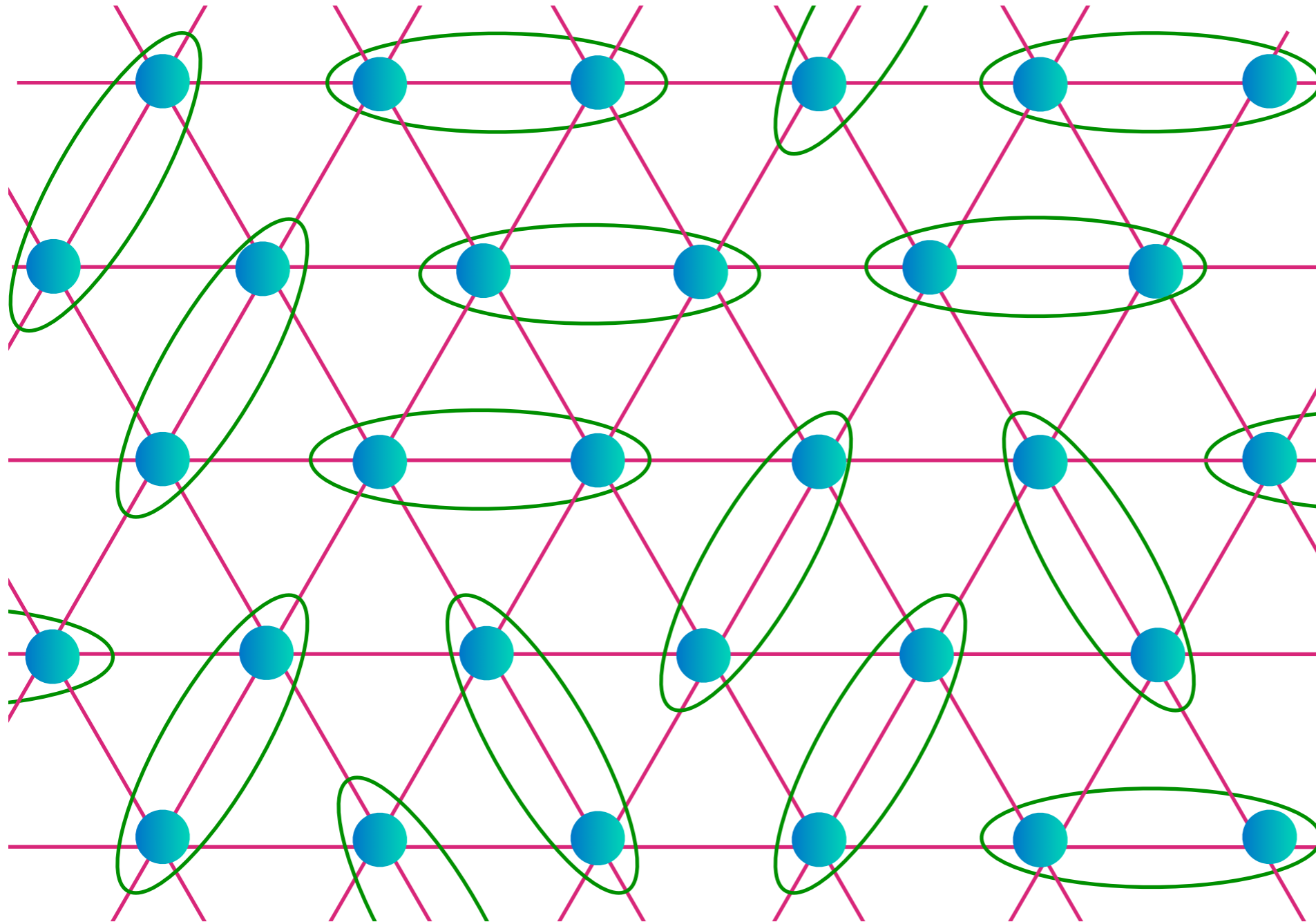
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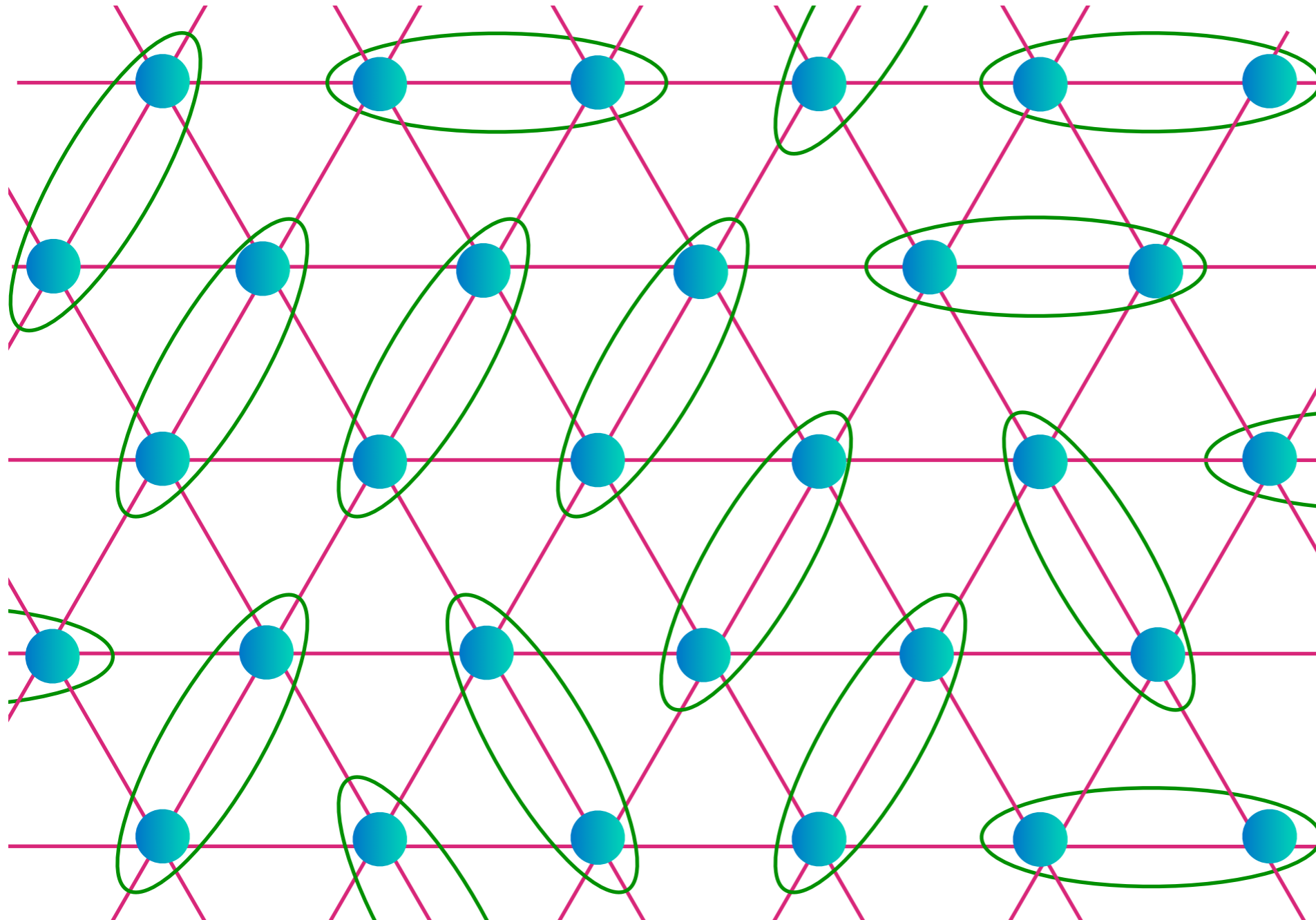
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Triangular lattice antiferromagnet

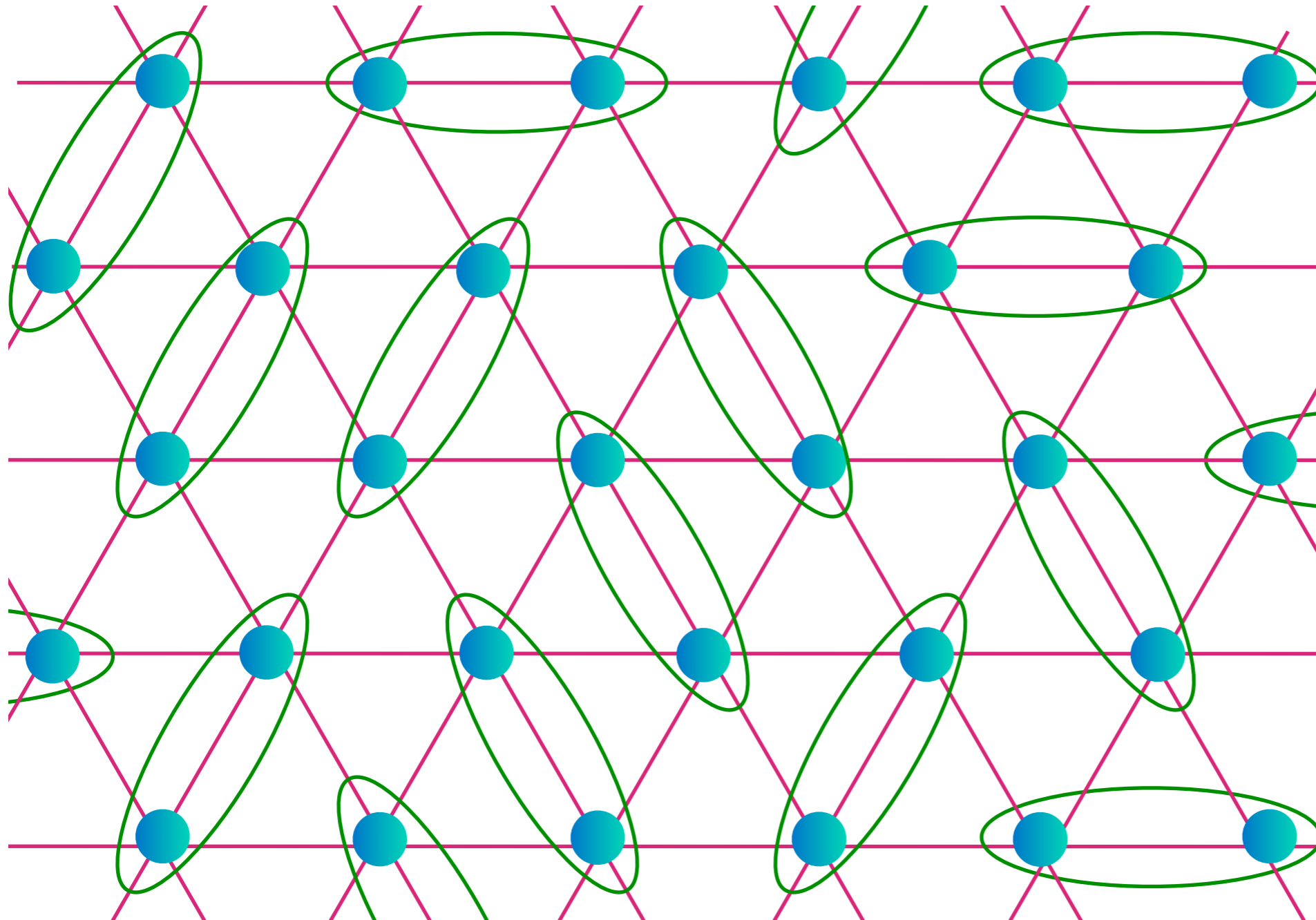
Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



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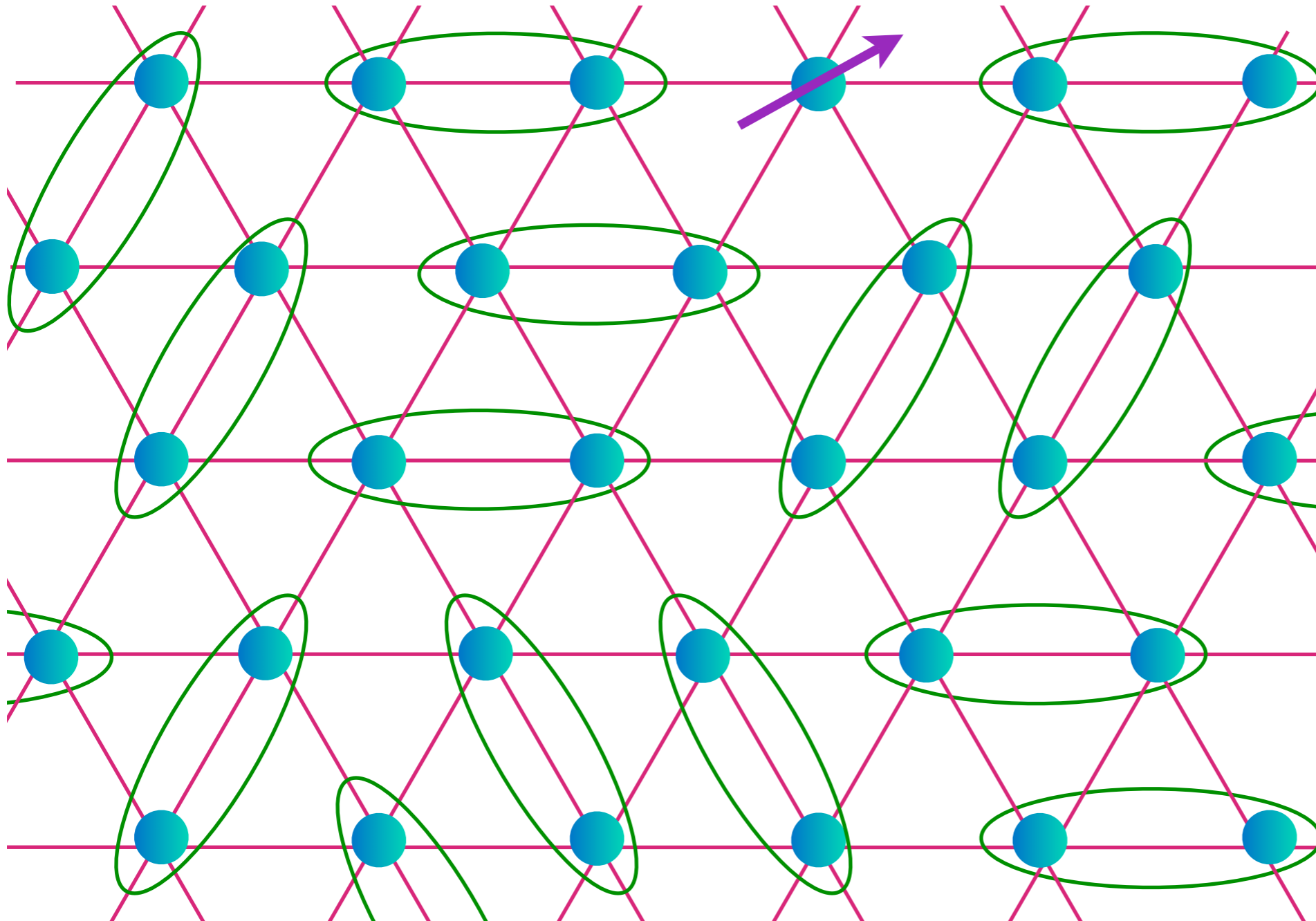


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Excitations of the Z_2 Spin liquid

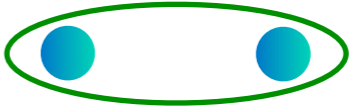
A spinon

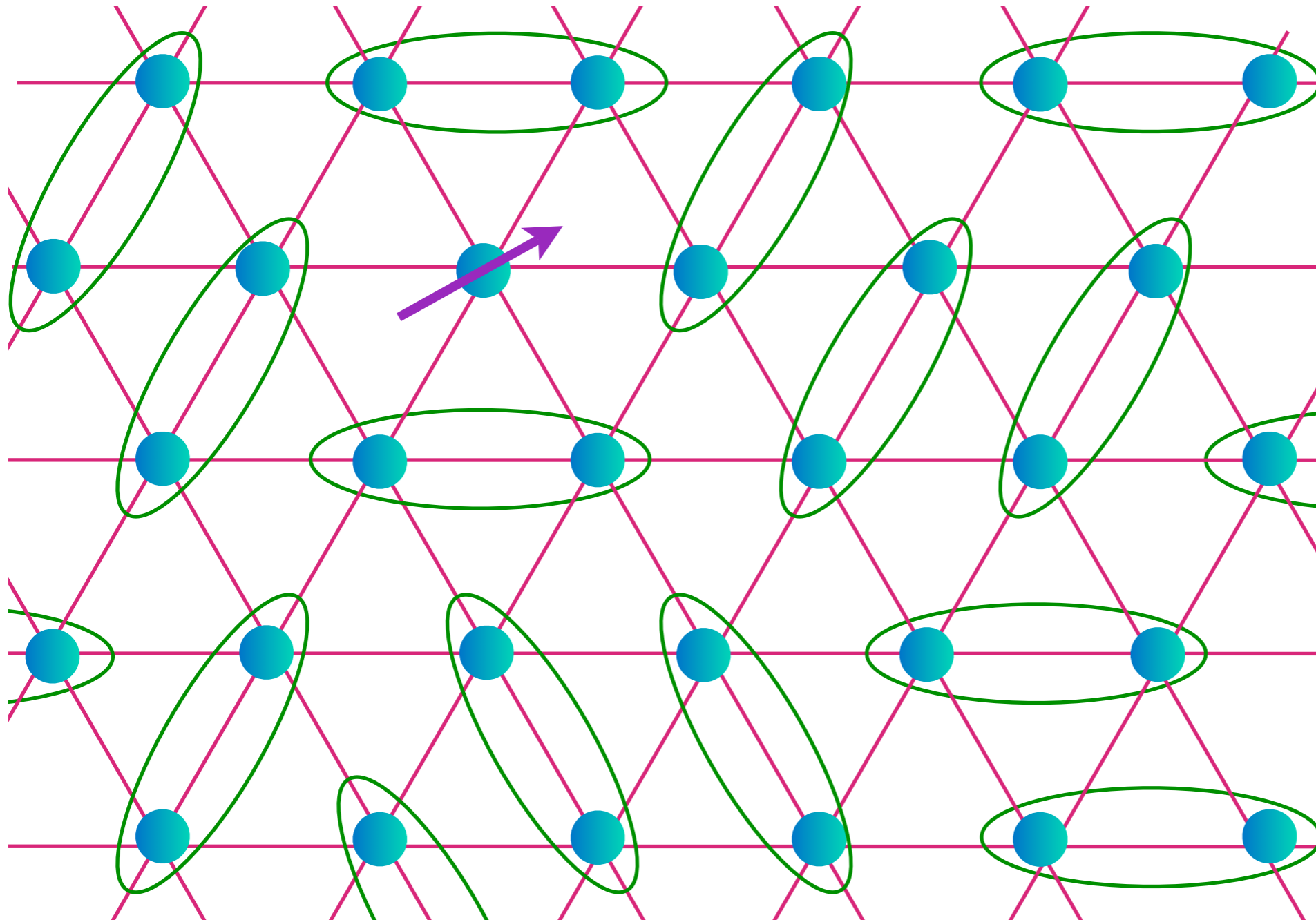

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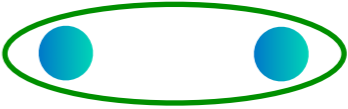
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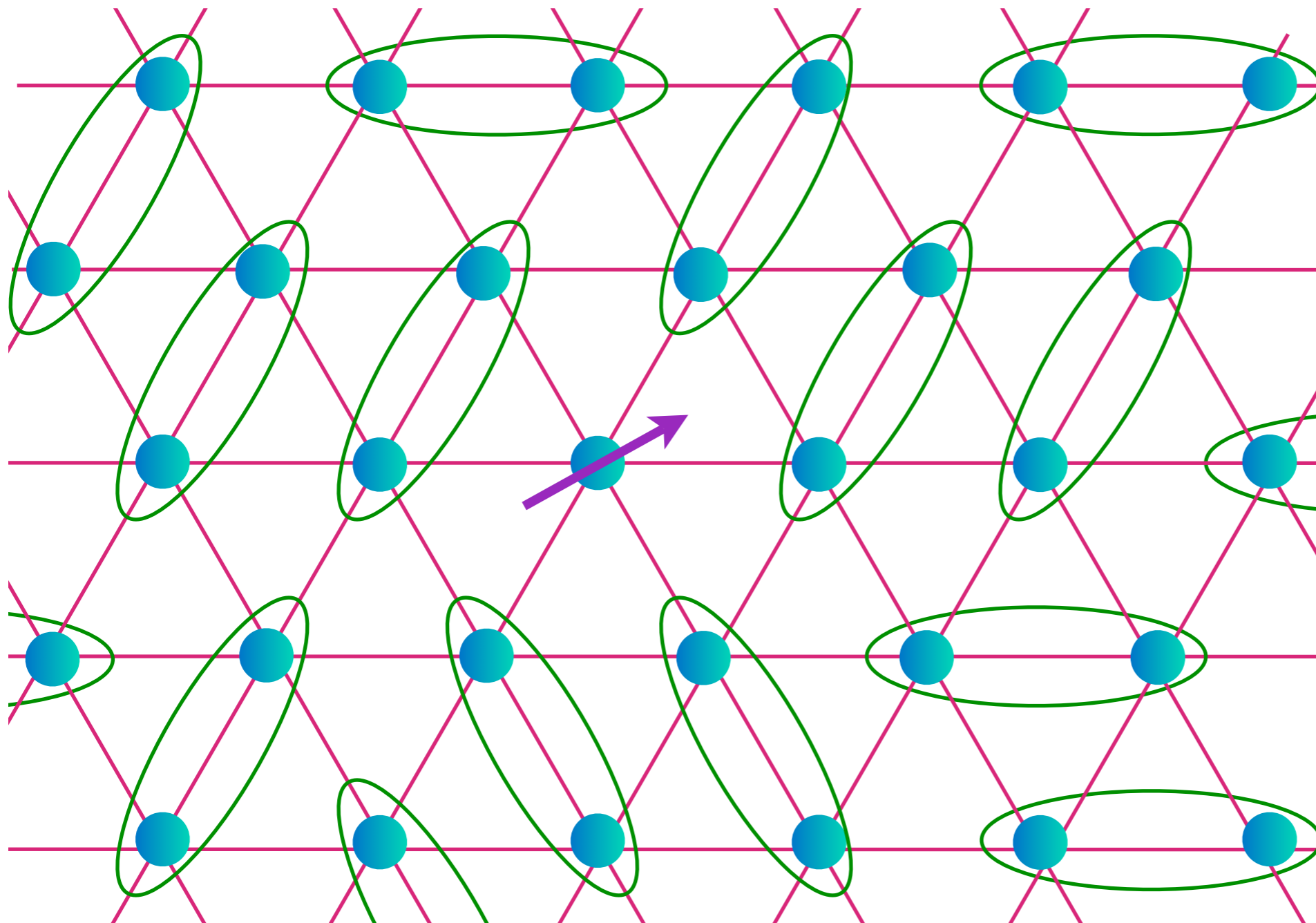

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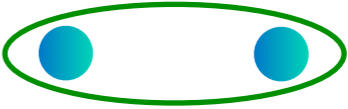
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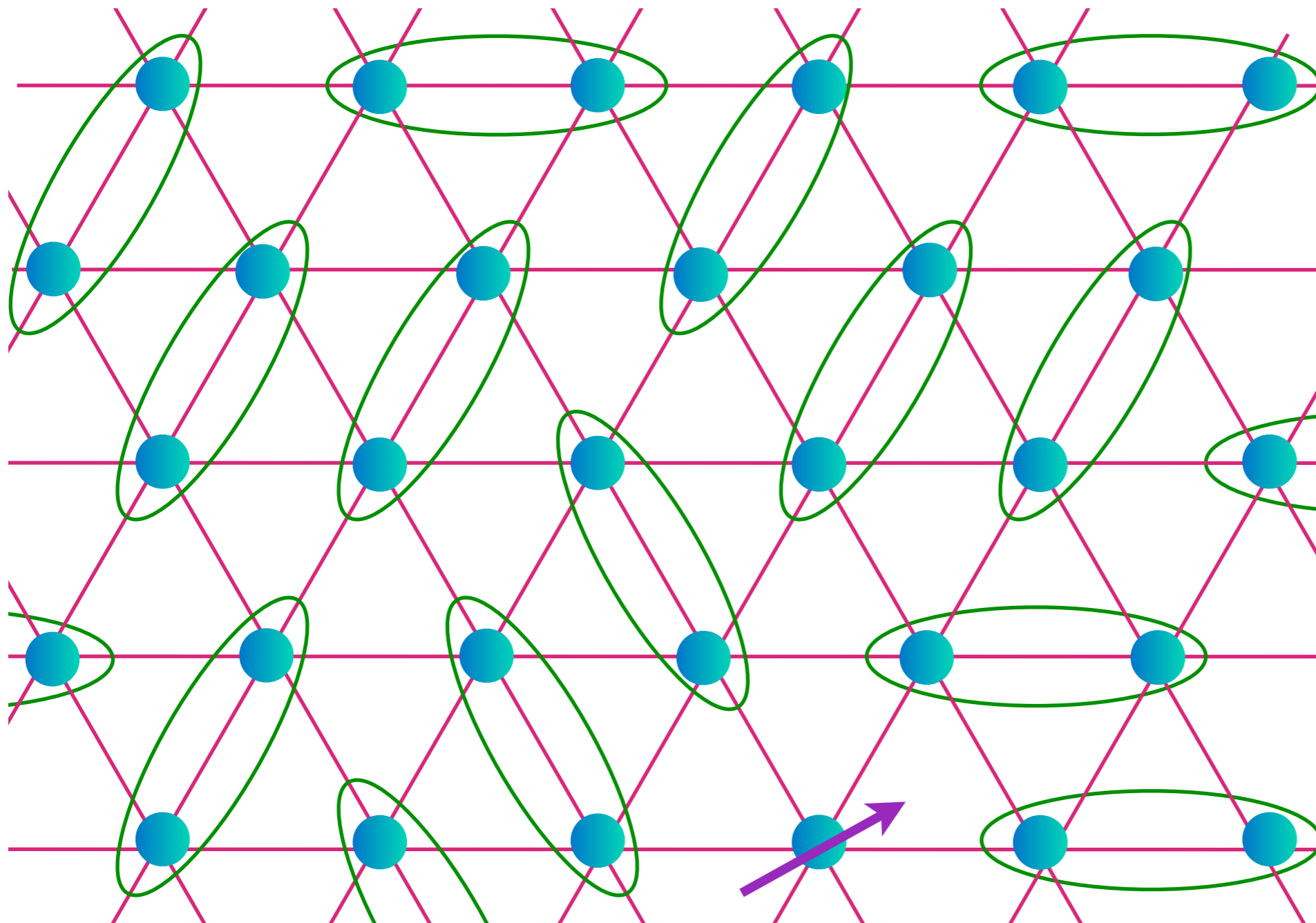

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Excitations of the Z_2 Spin liquid

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The theory for quantum phase transitions is expressed in terms of fluctuations of z_α , and *not* the order parameter $\vec{\varphi}$.

Effective theory for z_α must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Excitations of the Z_2 Spin liquid

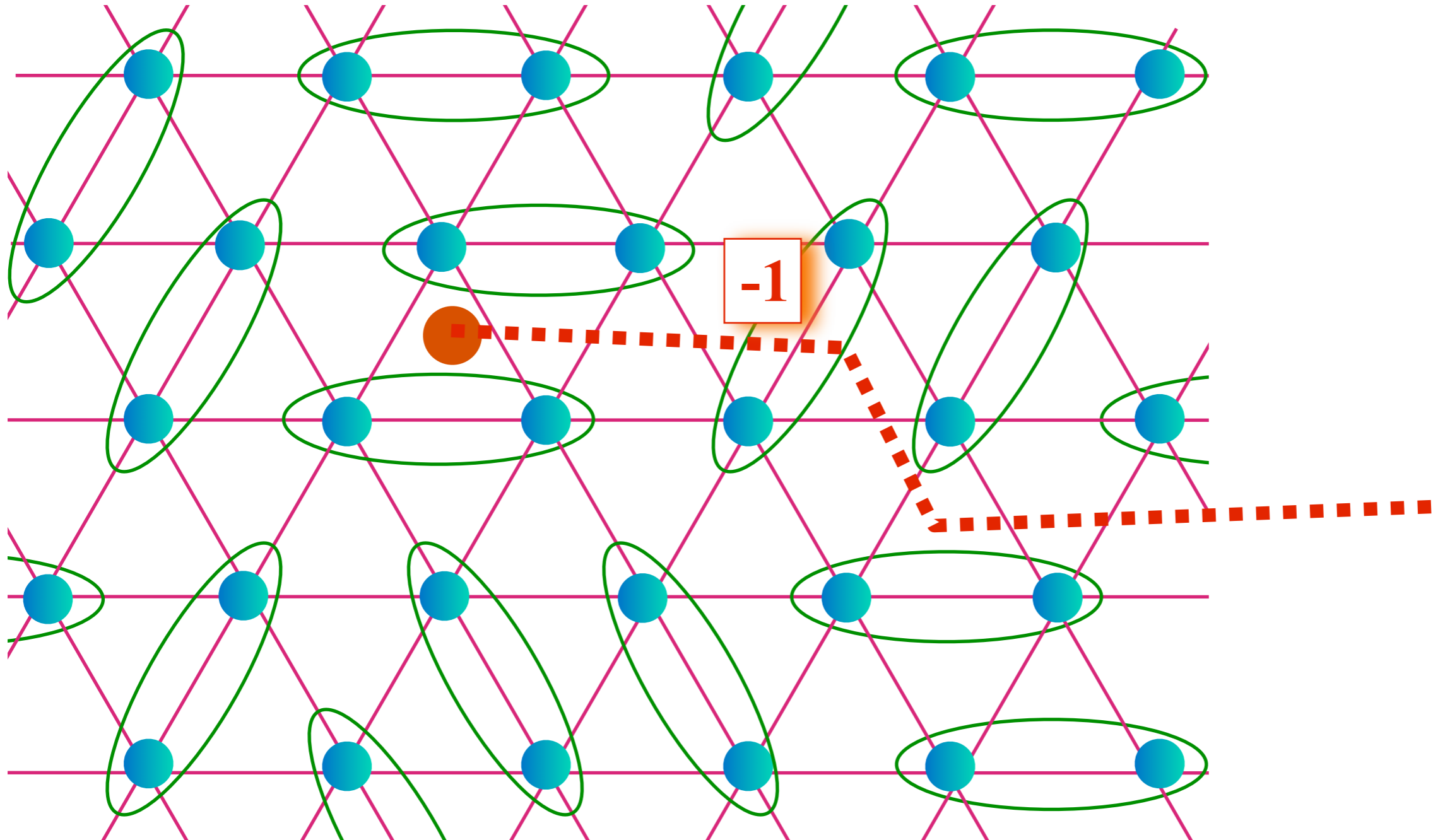
A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate
- A vison is an Abrikosov vortex in the pair condensate of spinons
- Visions are the dark matter of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

Excitations of the Z_2 Spin liquid

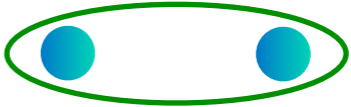
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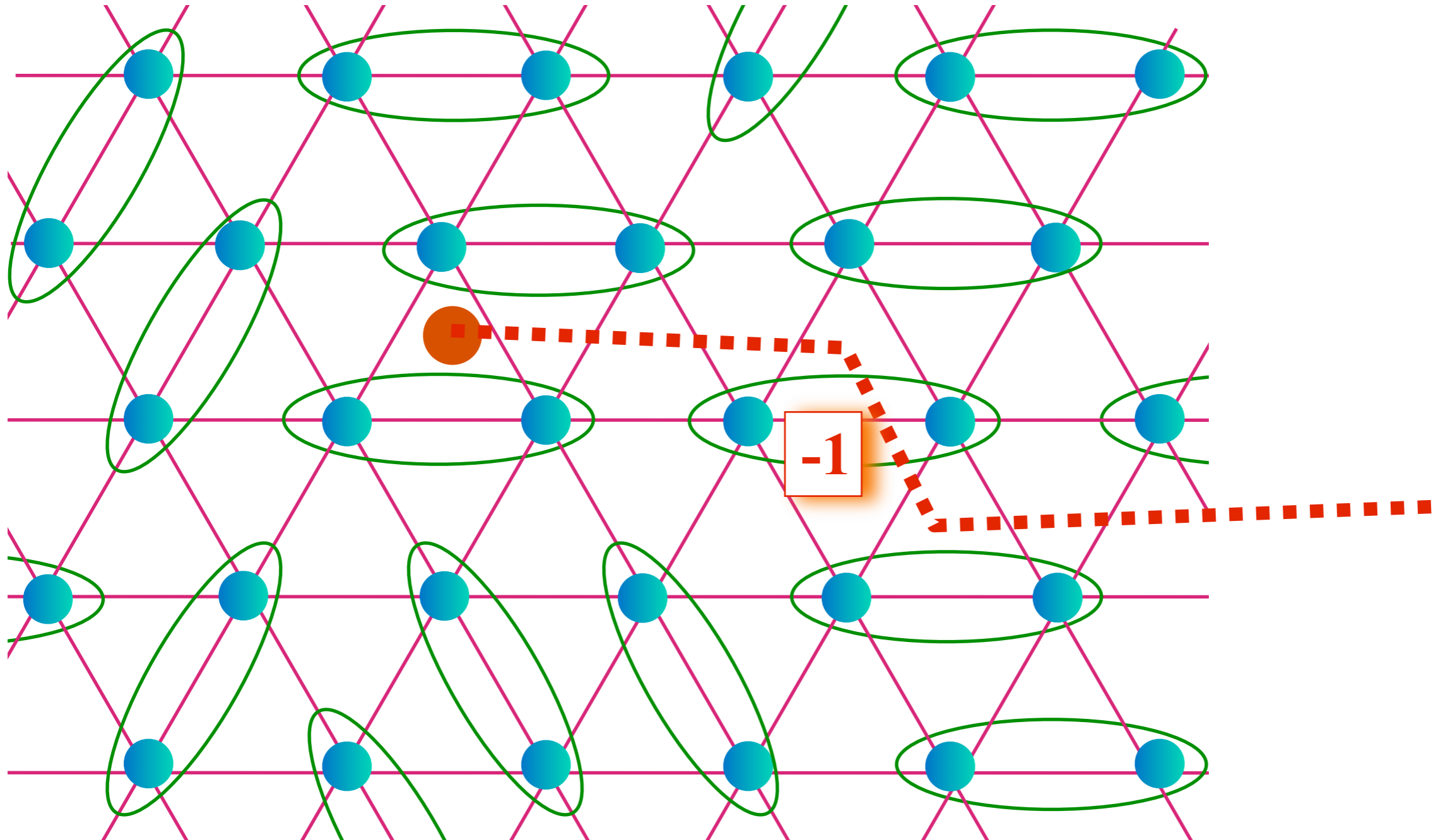
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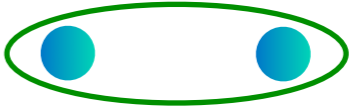
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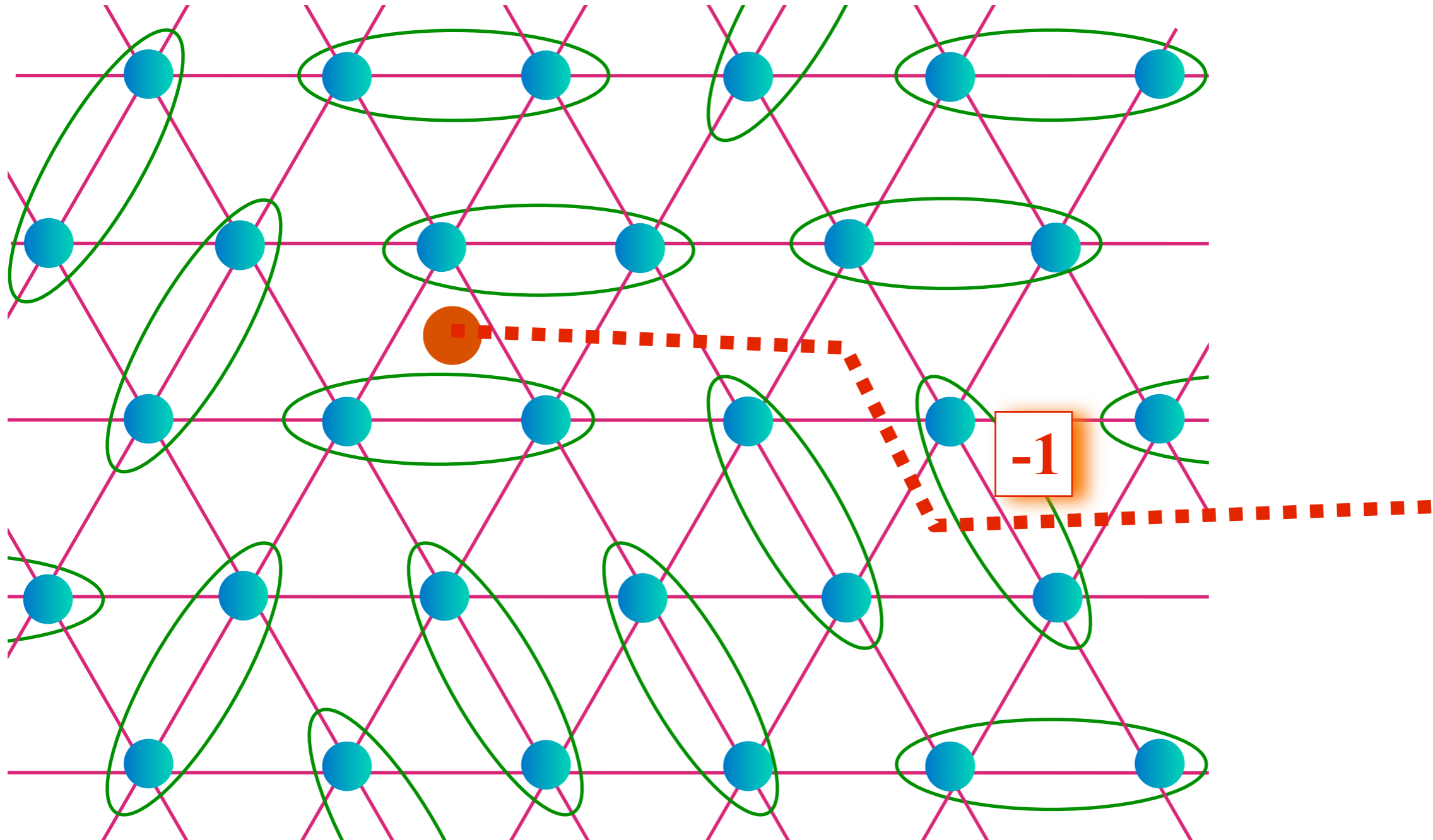

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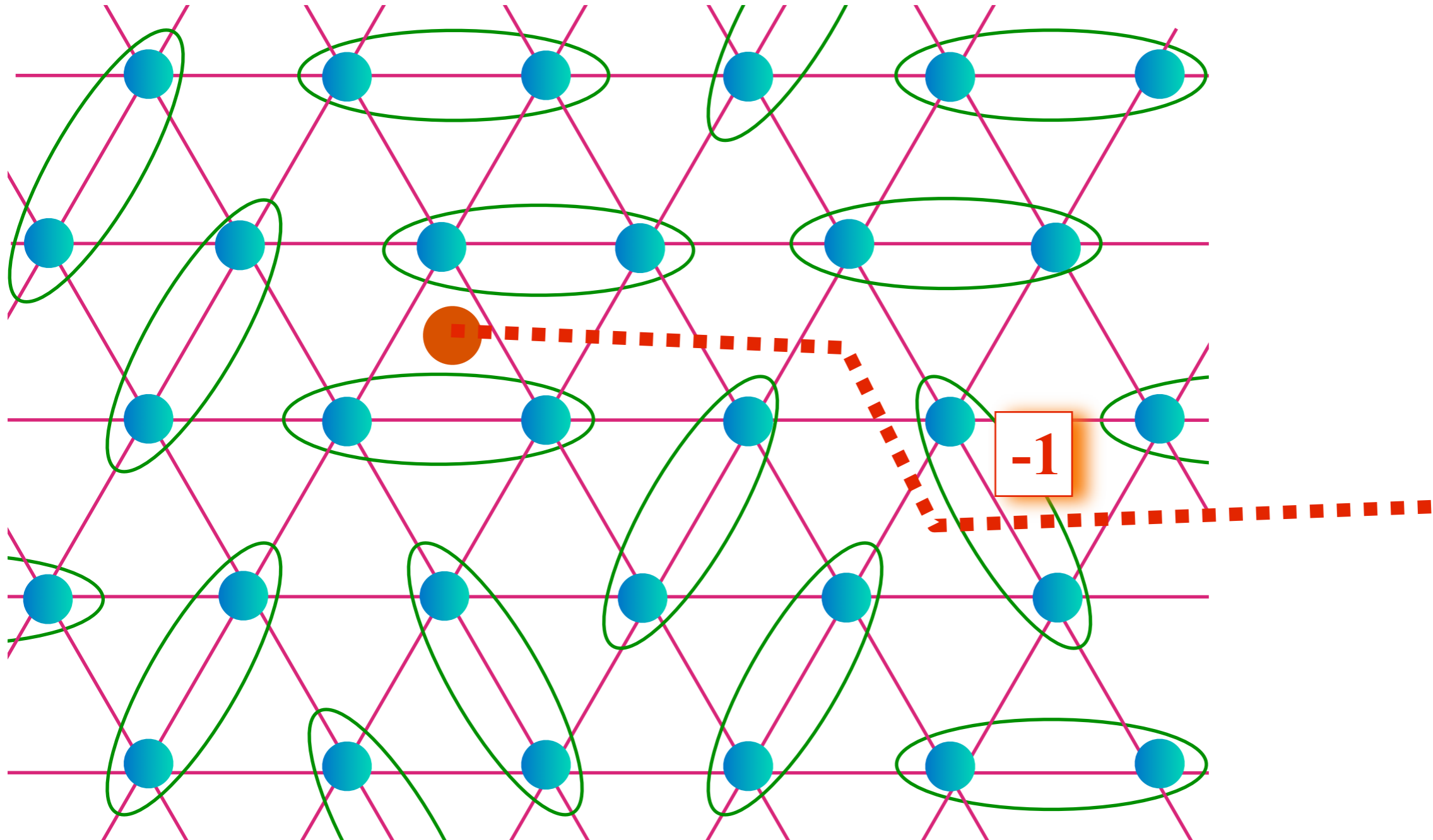

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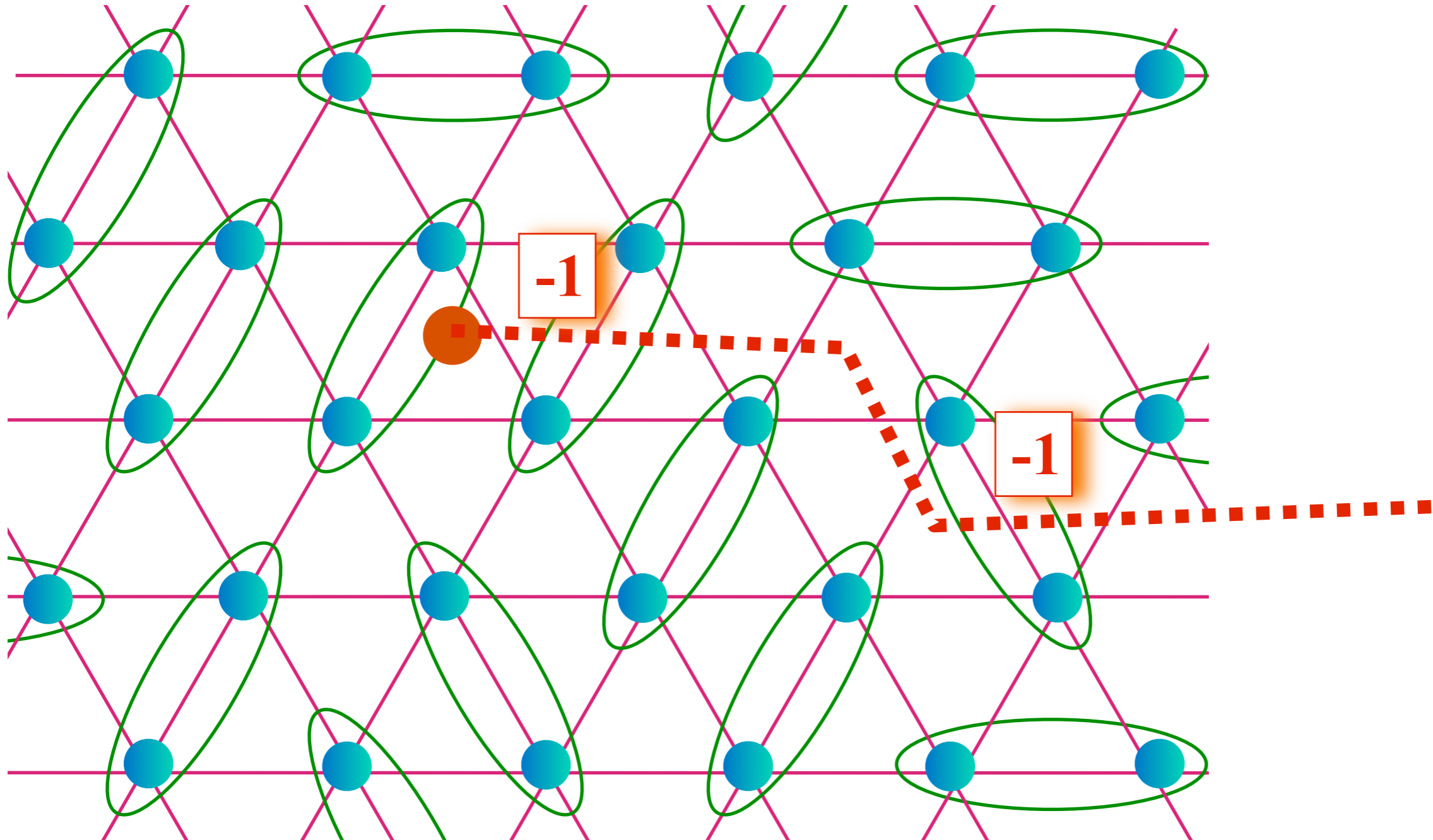
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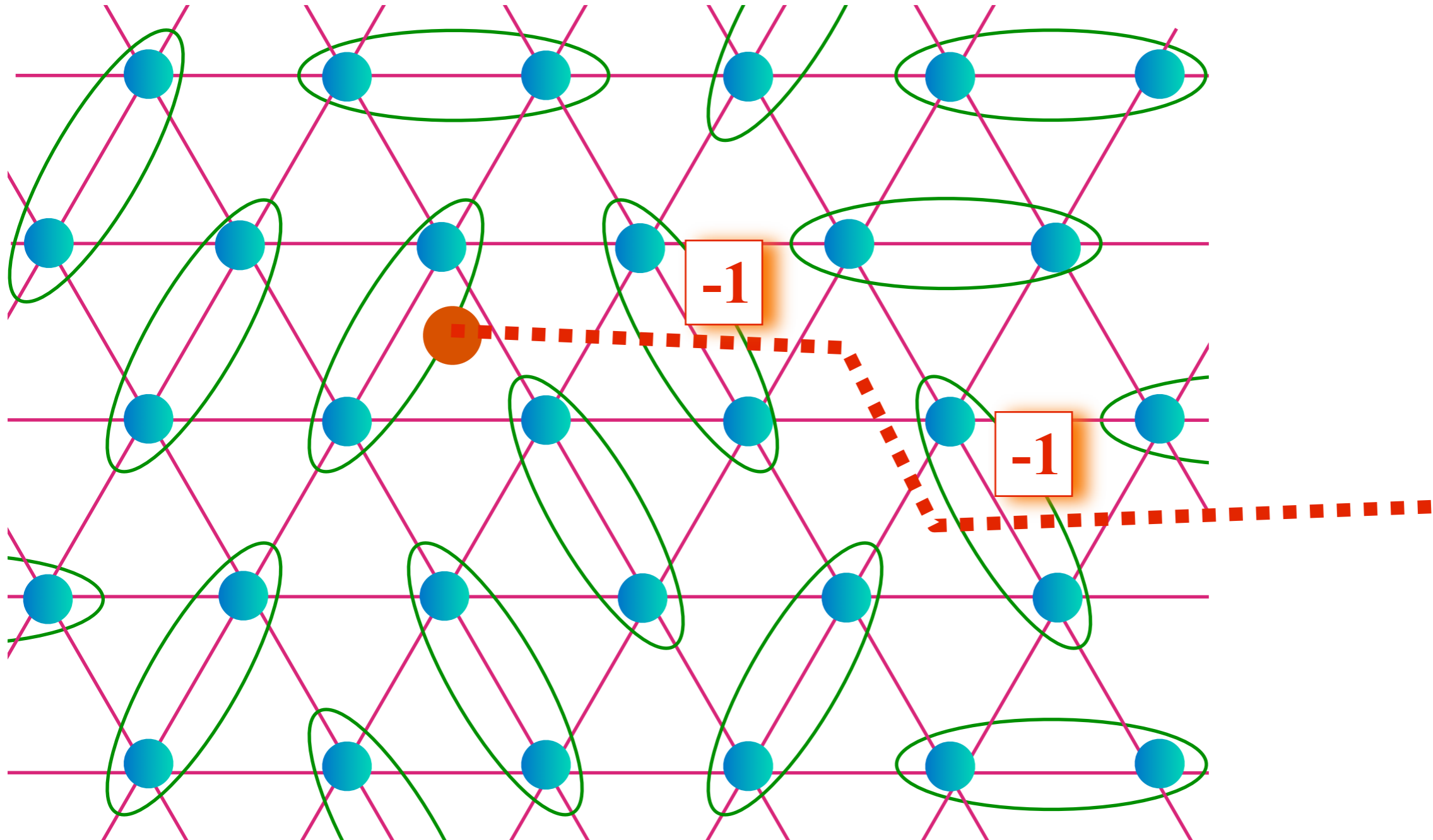
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A Simple Toy Model (A. Kitaev, 1997)

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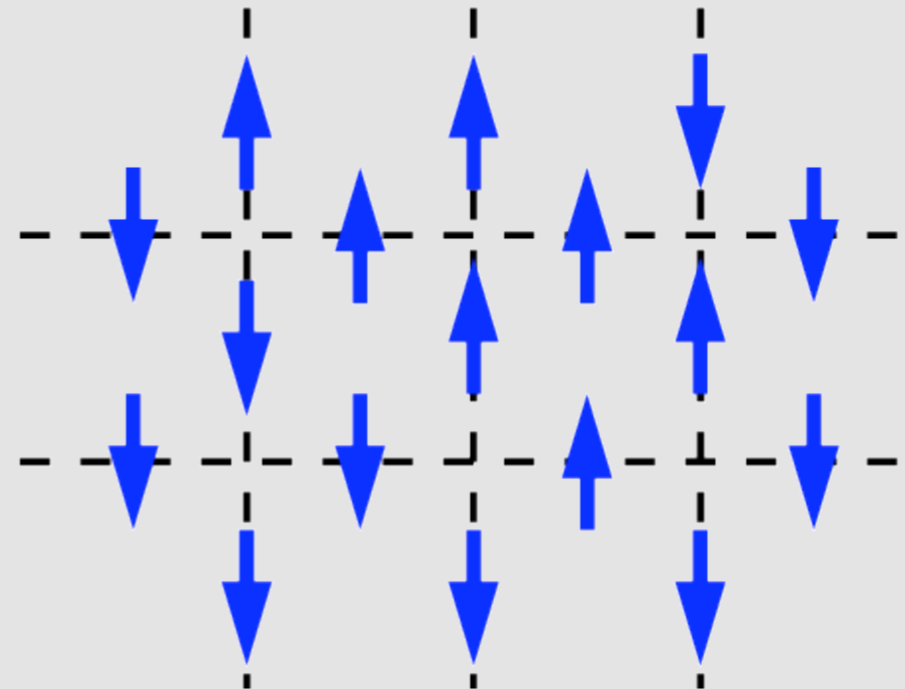
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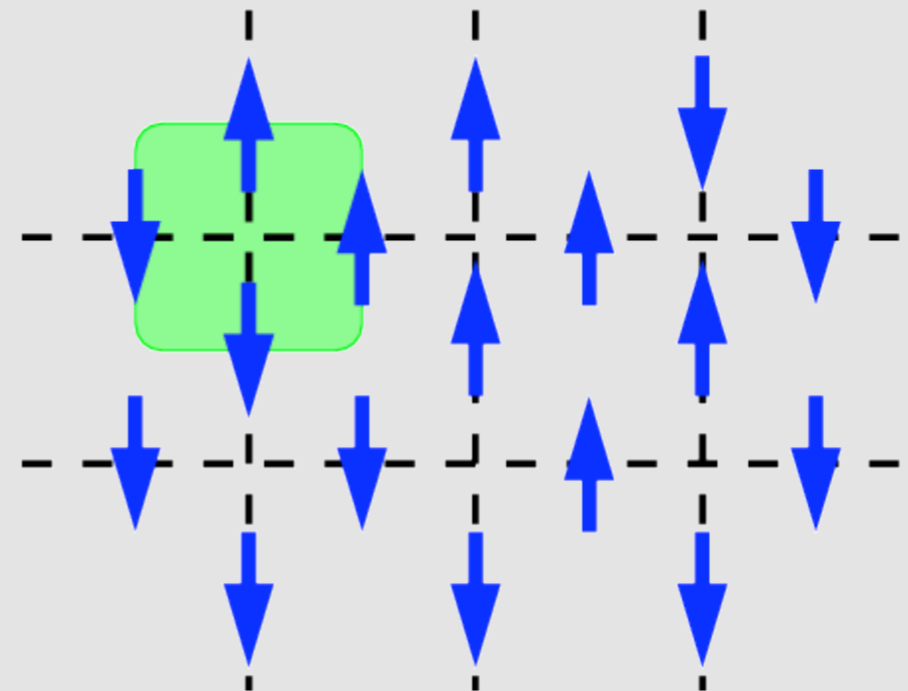
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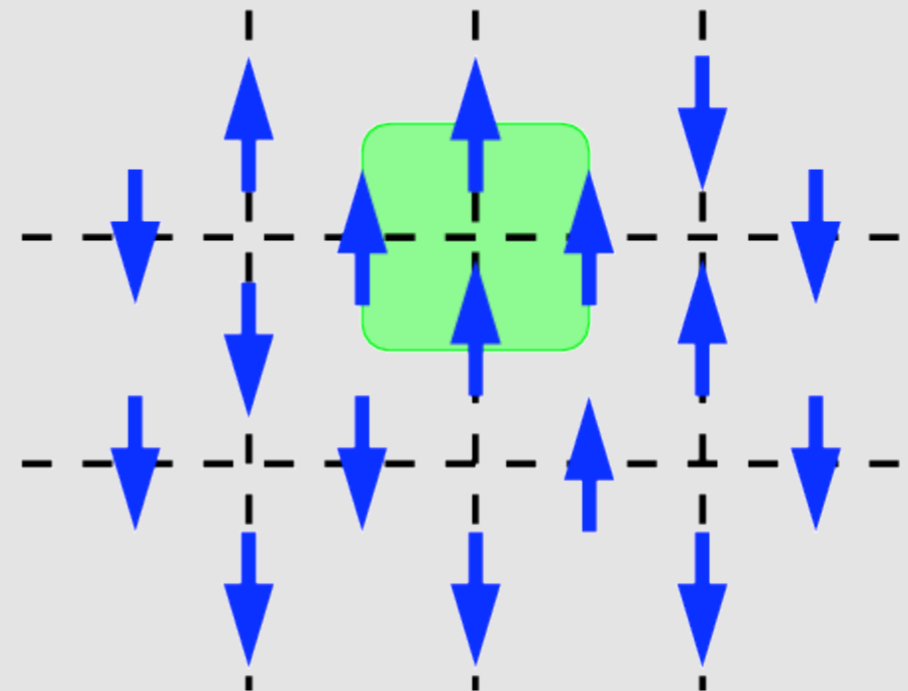
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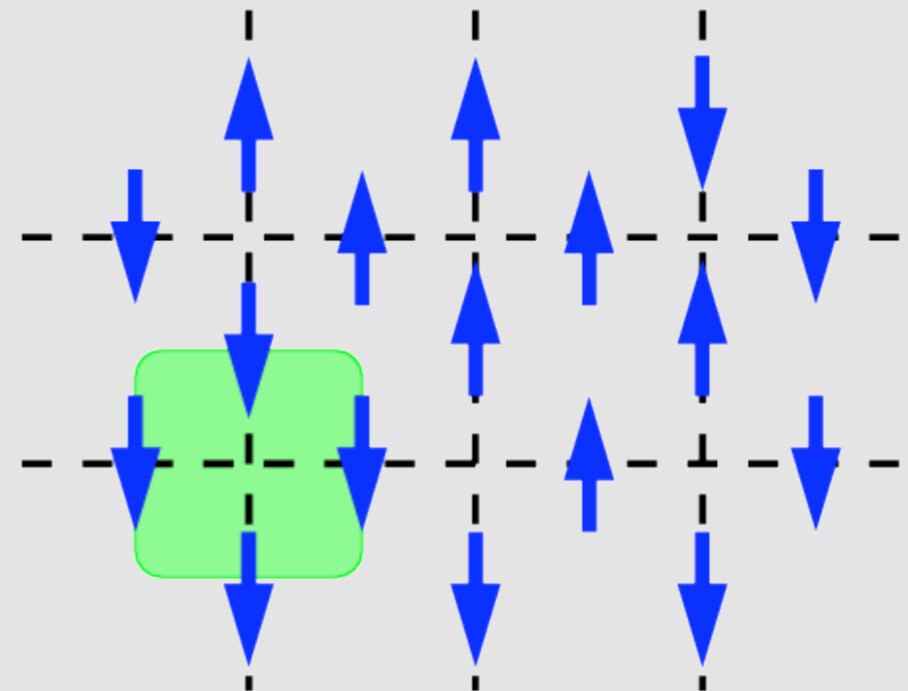
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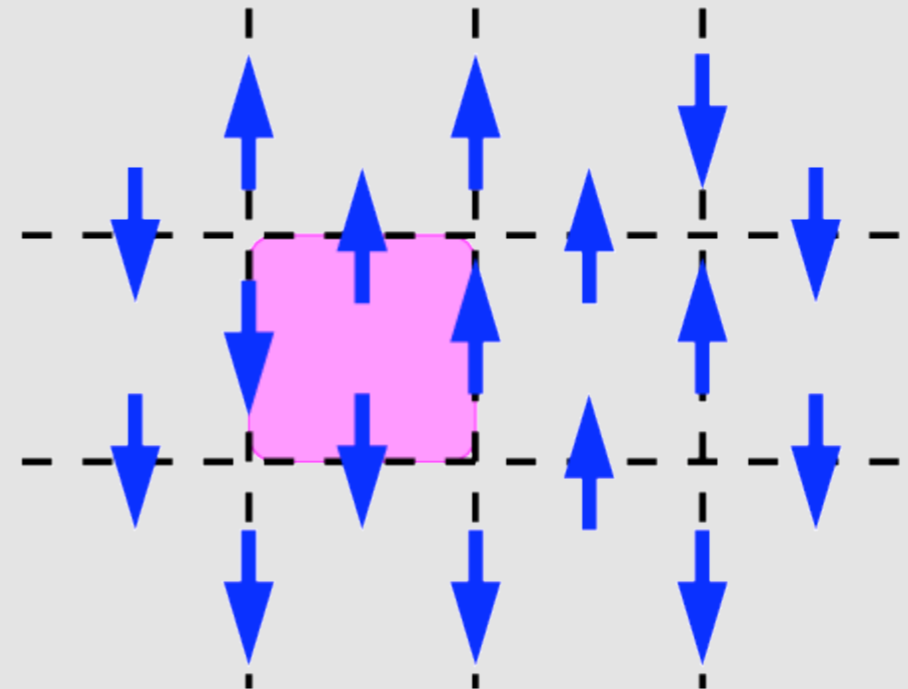
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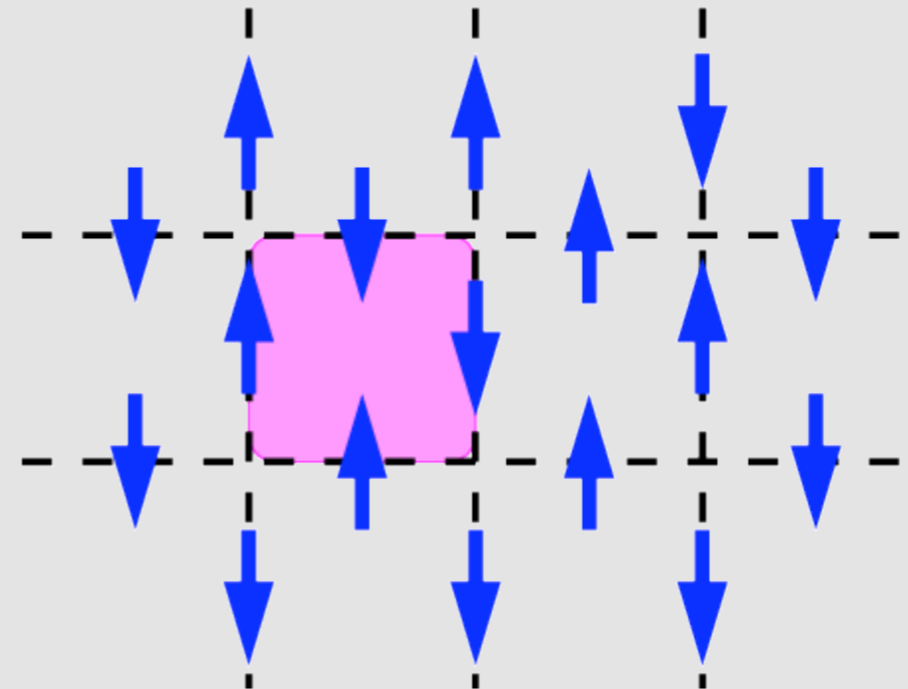
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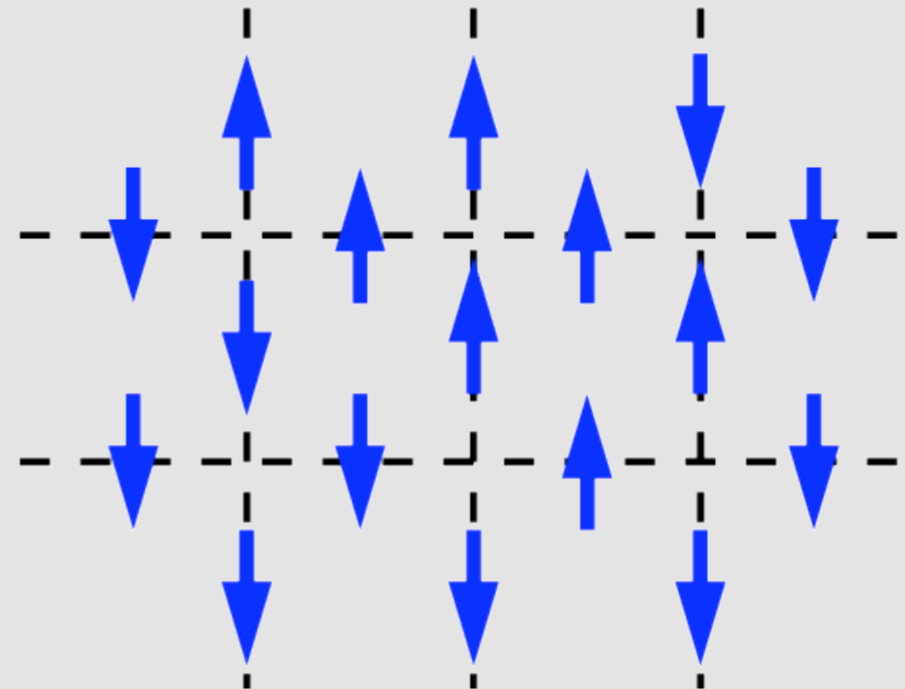
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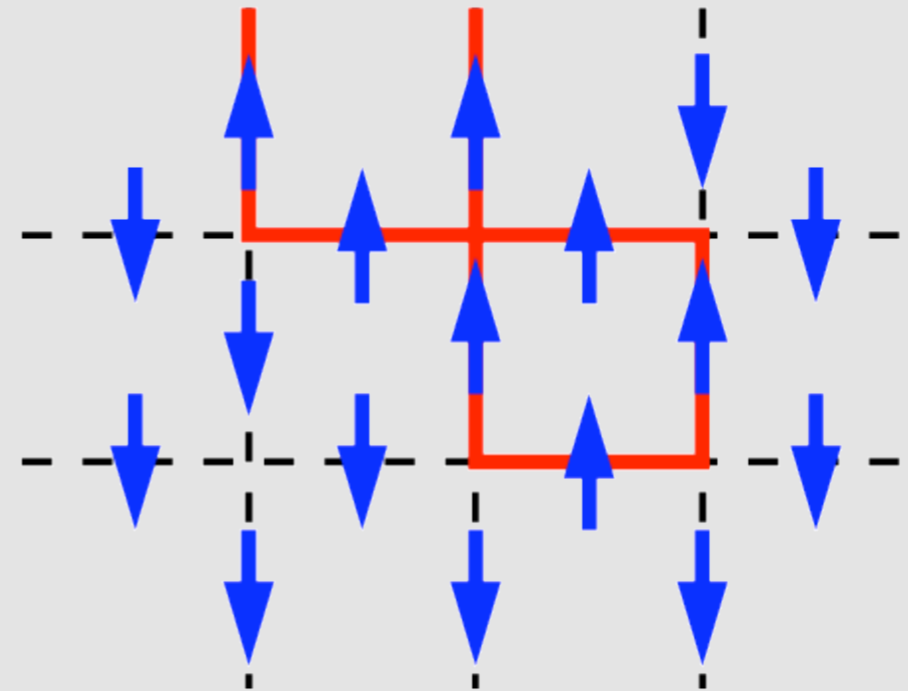
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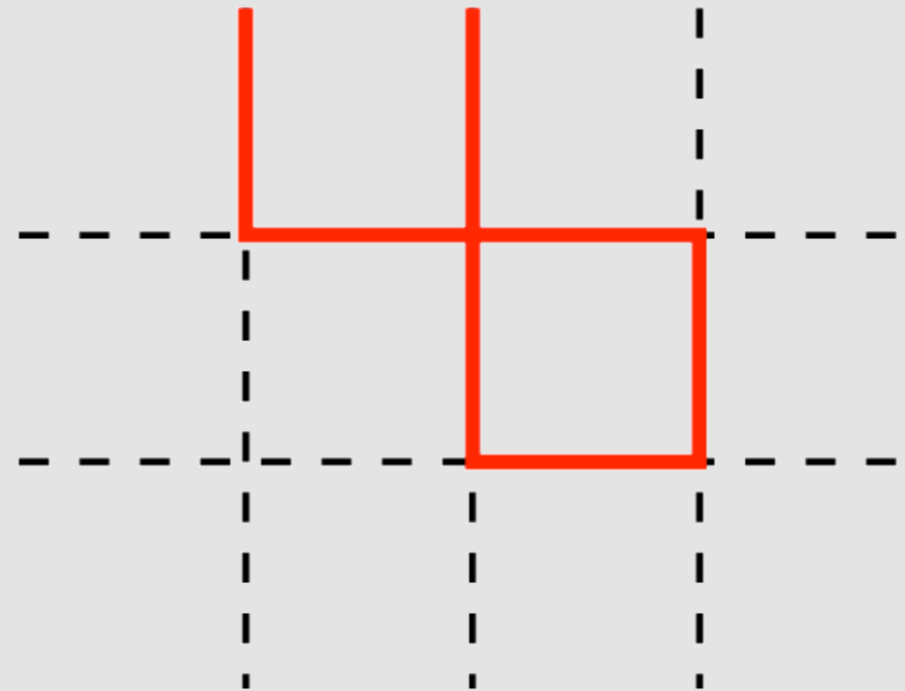
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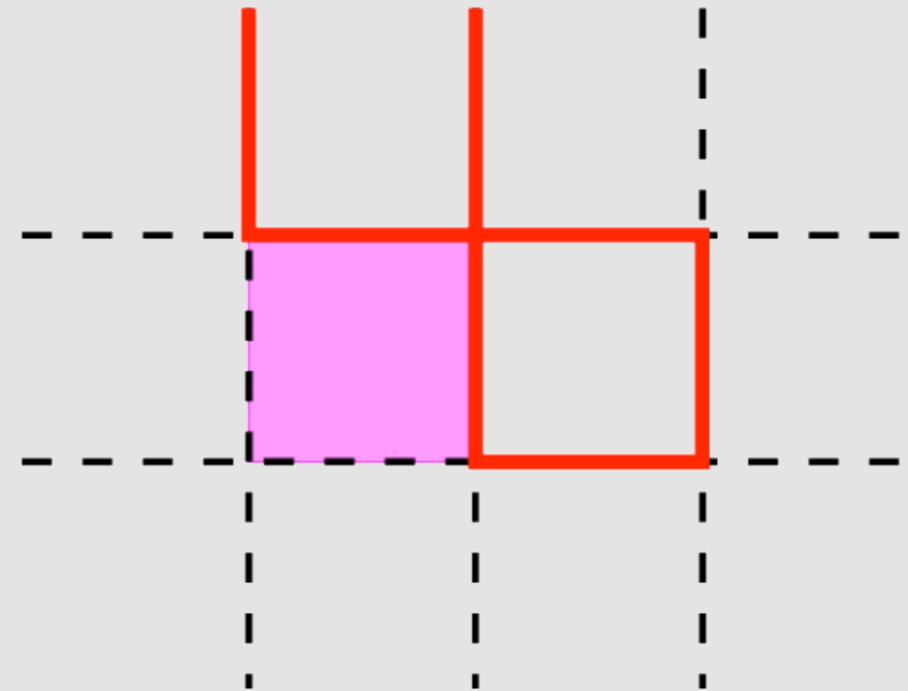
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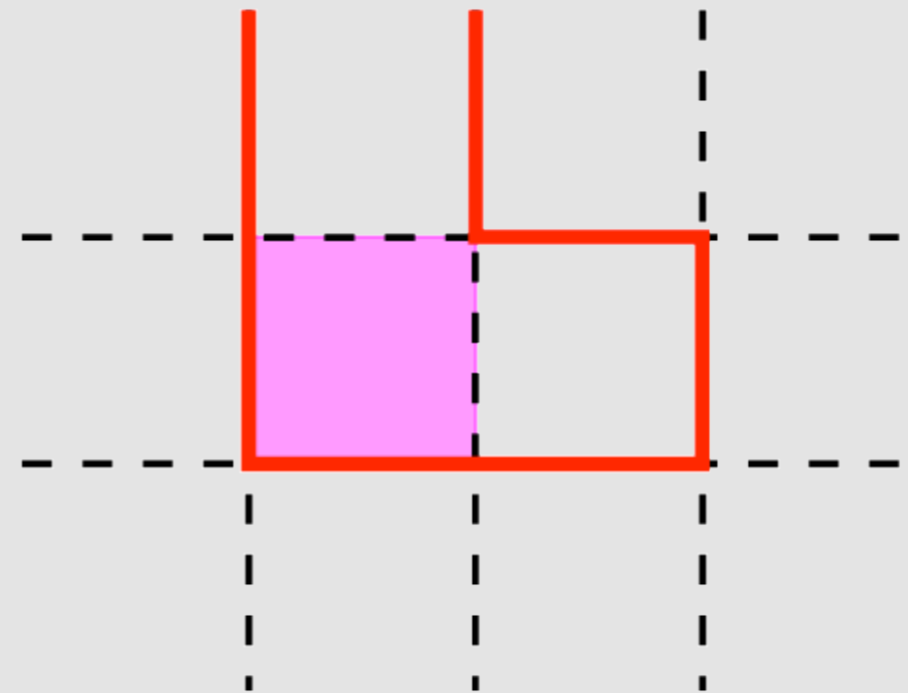
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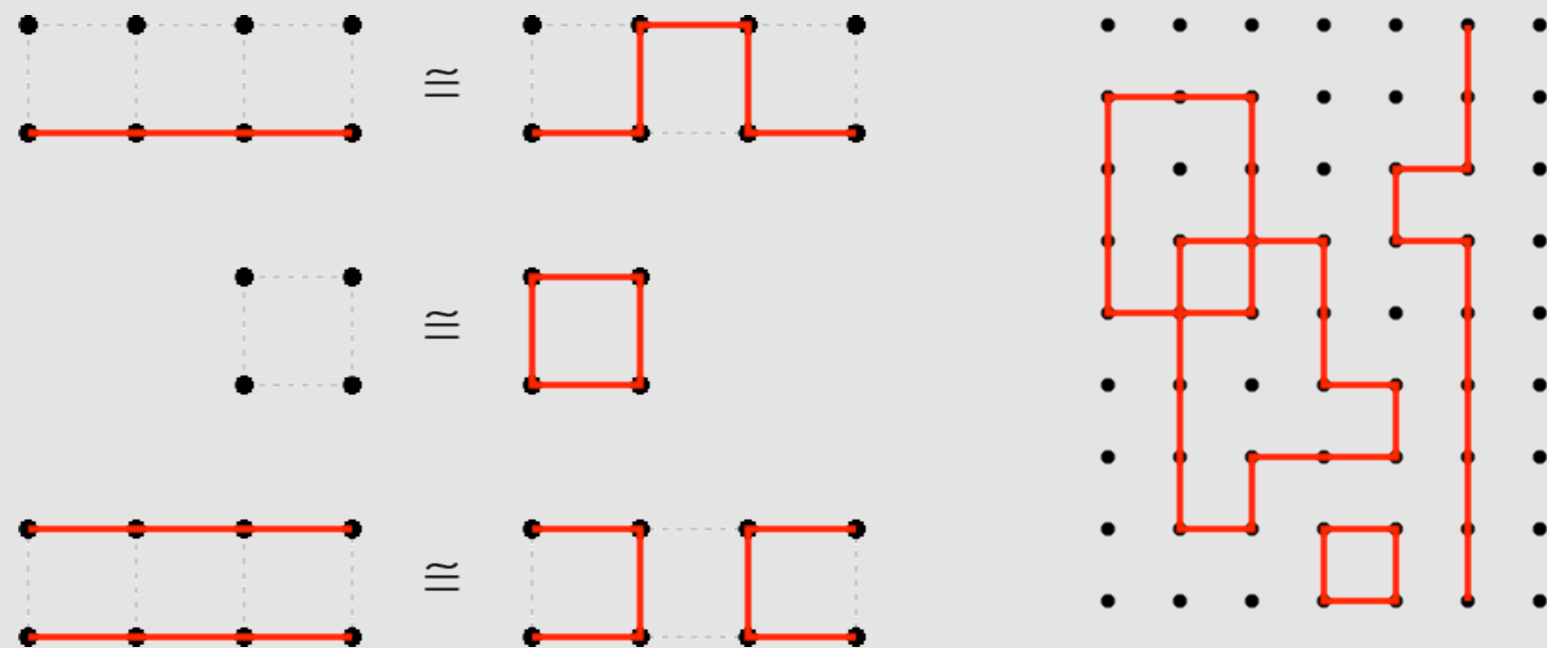
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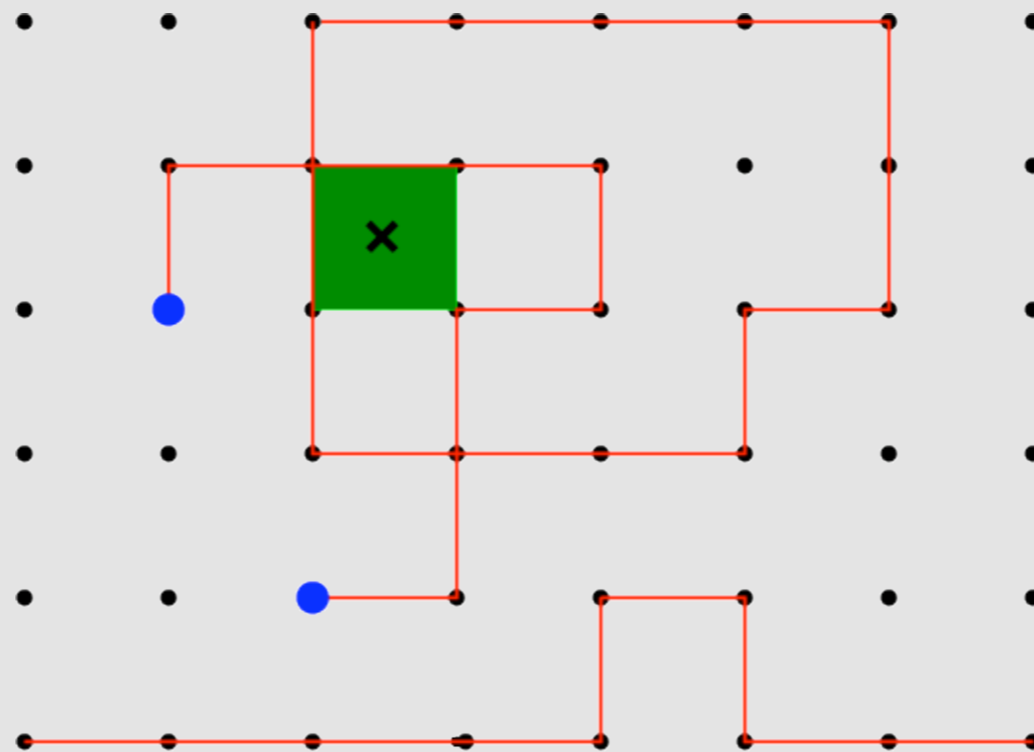
- Ground state: all $A_i=1$, $F_p=1$
- Pictorial representation: color each link with an up-spin.
- $A_i=1$: closed loops.
- $F_p=1$: every plaquette is an equal-amplitude superposition of inverse images.



The GS wavefunction takes the same value on configurations connected by these operations. It does not depend on the geometry of the configurations, only on their topology.

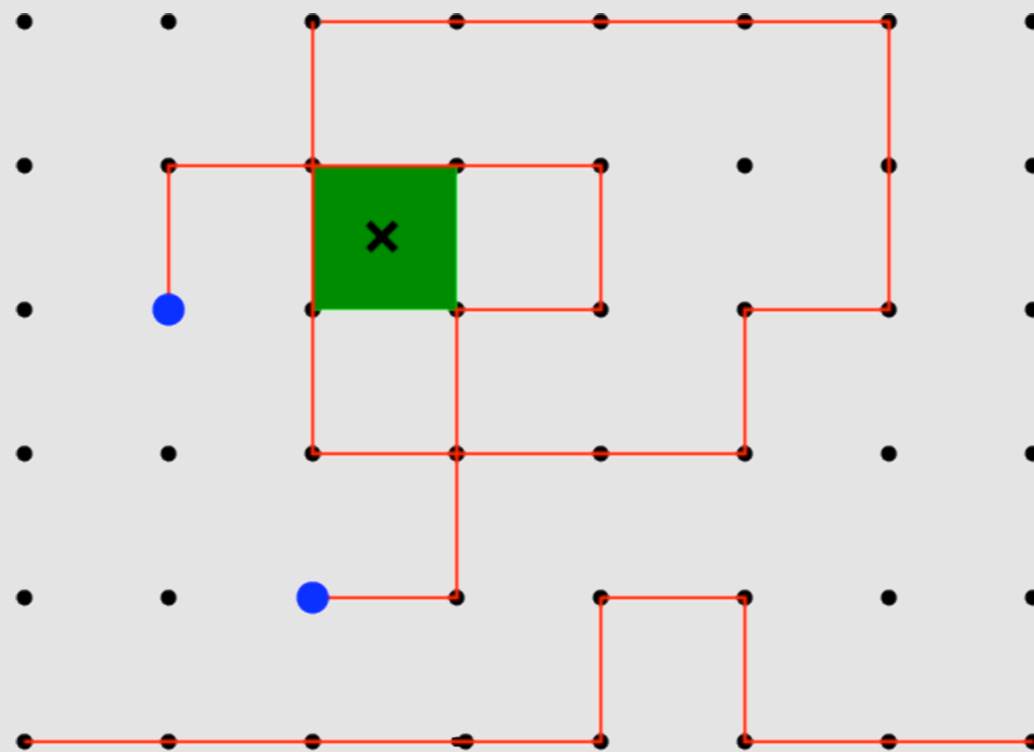
Properties of Excitations

- “Electric” particle, or $A_i = -1$ – endpoint of a line (*a “spinon”*)
- “Magnetic particle”, or *vortex*: $F_p = -1$ – a “flip” of this plaquette changes the sign of a given term in the superposition.
- Charges and vortices interact via topological Aharonov-Bohm interactions.



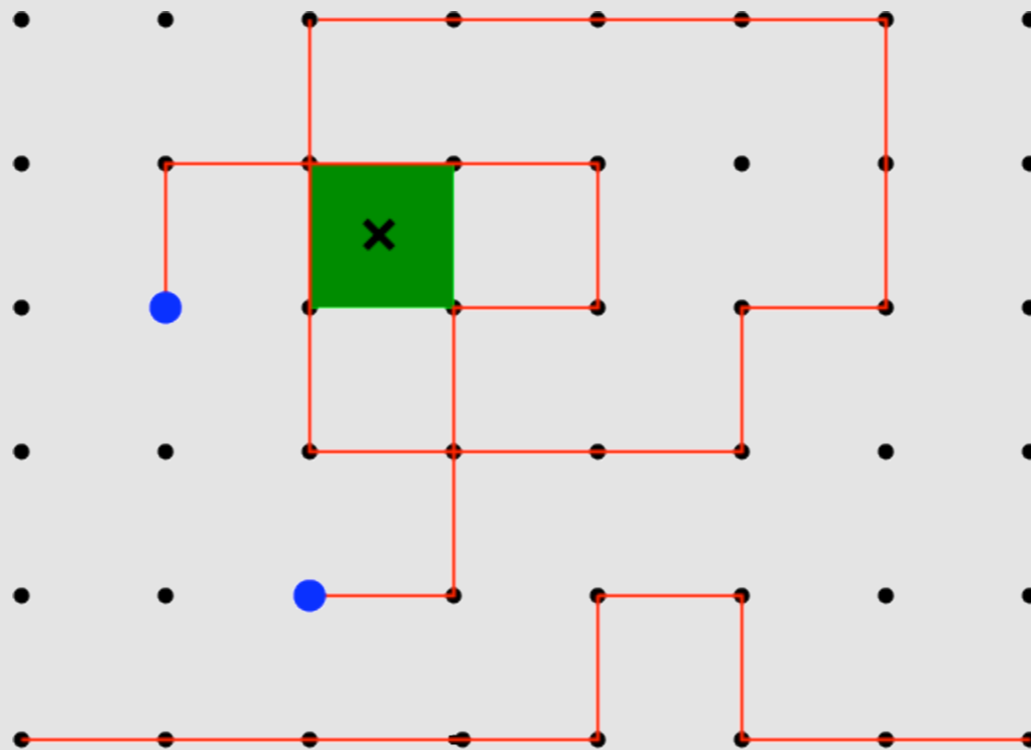
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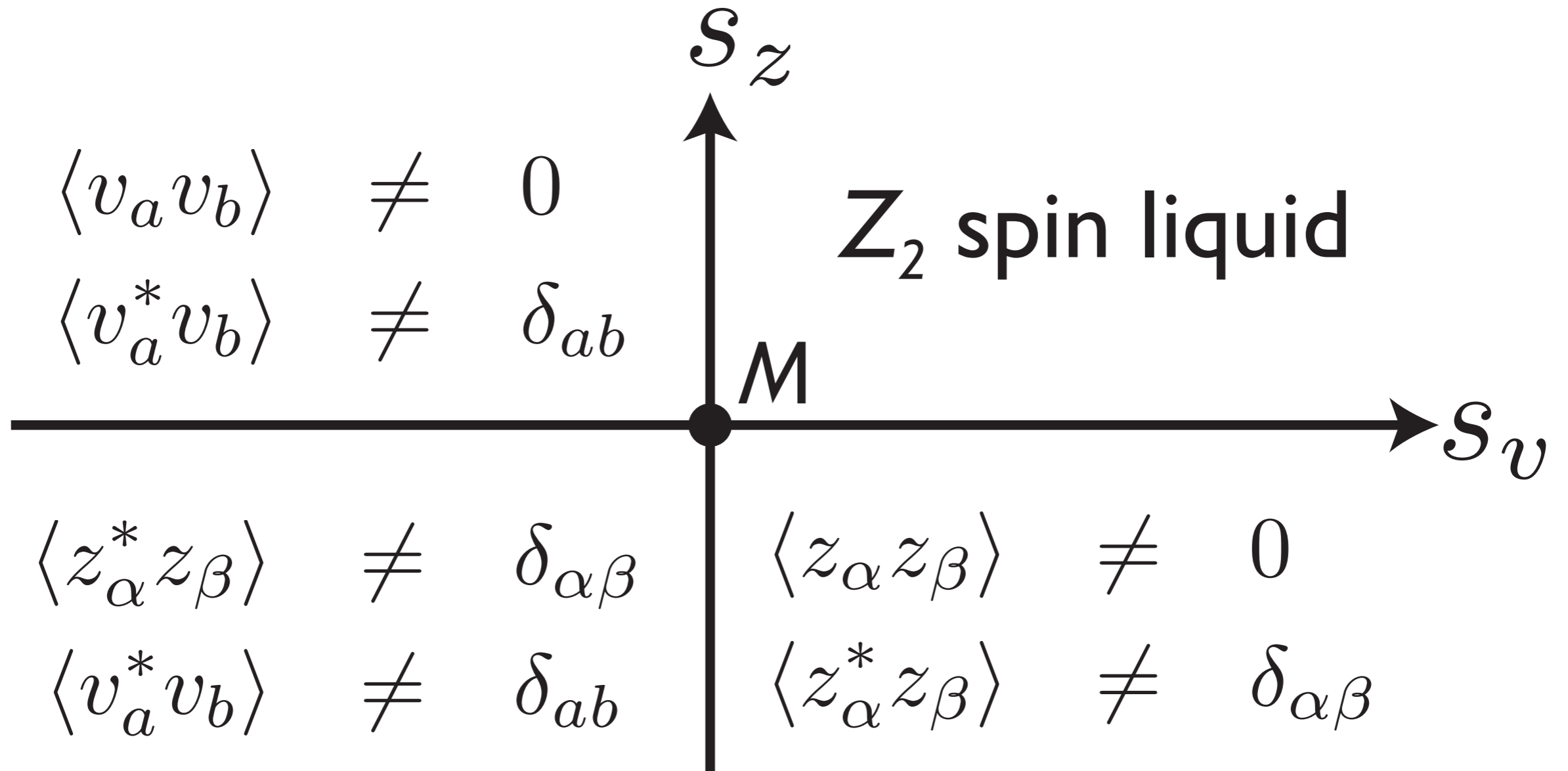
- *Spinons and visons are mutual semions*

Mutual Chern-Simons Theory

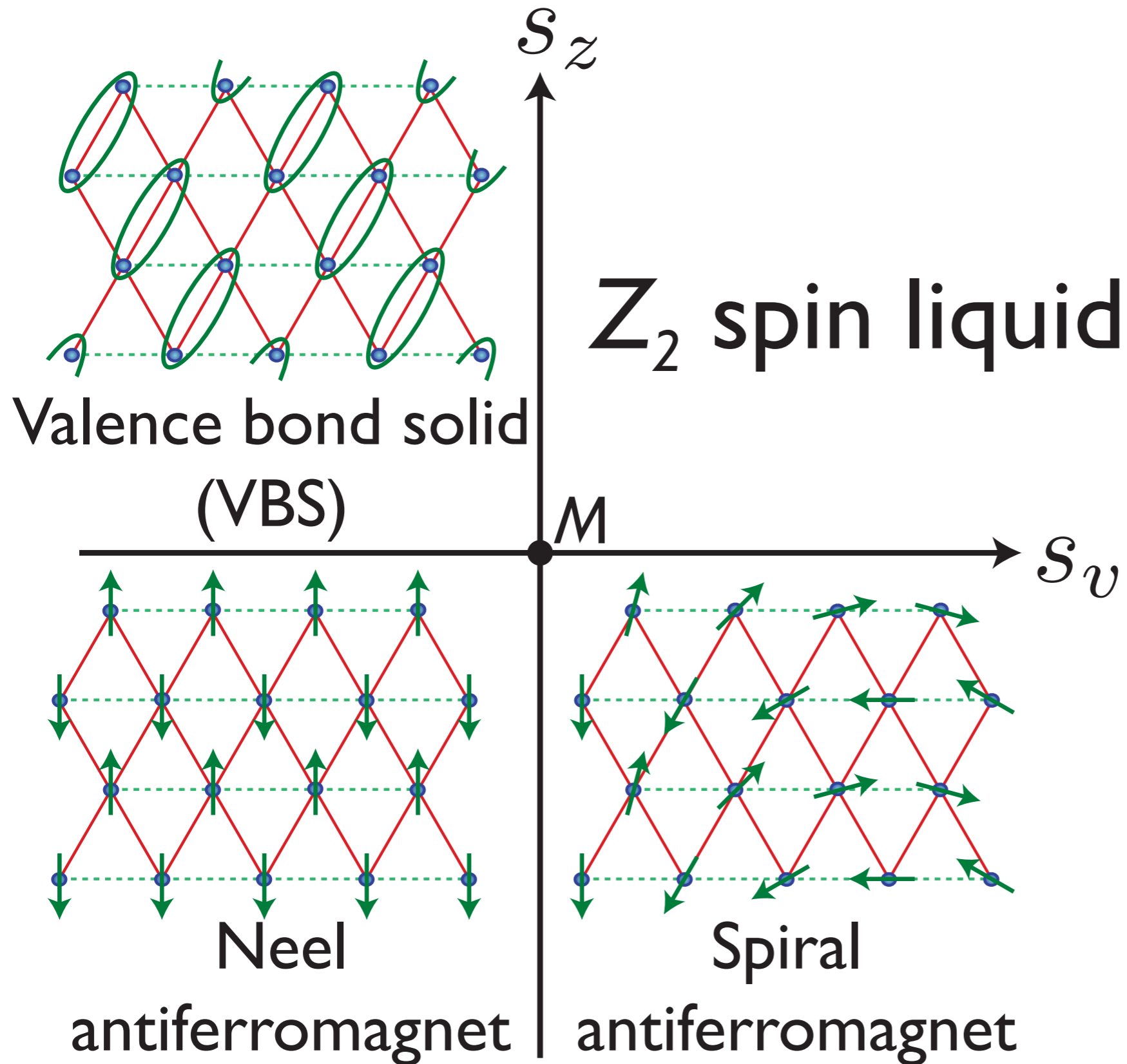
Express theory in terms of the physical excitations of the Z_2 spin liquid: the spinons, z_α , and the visons. After accounting for Berry phase effects, the visons can be described by complex fields v_a , which transforms non-trivially under the square lattice space group operations.

A related Berry phase is the phase of -1 acquired by a spinon encircling a vortex. This is implemented in the following “mutual Chern-Simons” theory at $k = 2$:

$$\begin{aligned} \mathcal{L} &= \sum_{\alpha=1}^2 \left\{ |(\partial_\mu - ia_\mu)z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\ &+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - ib_\mu)v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\ &+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \dots \end{aligned}$$



Theoretical global phase diagram

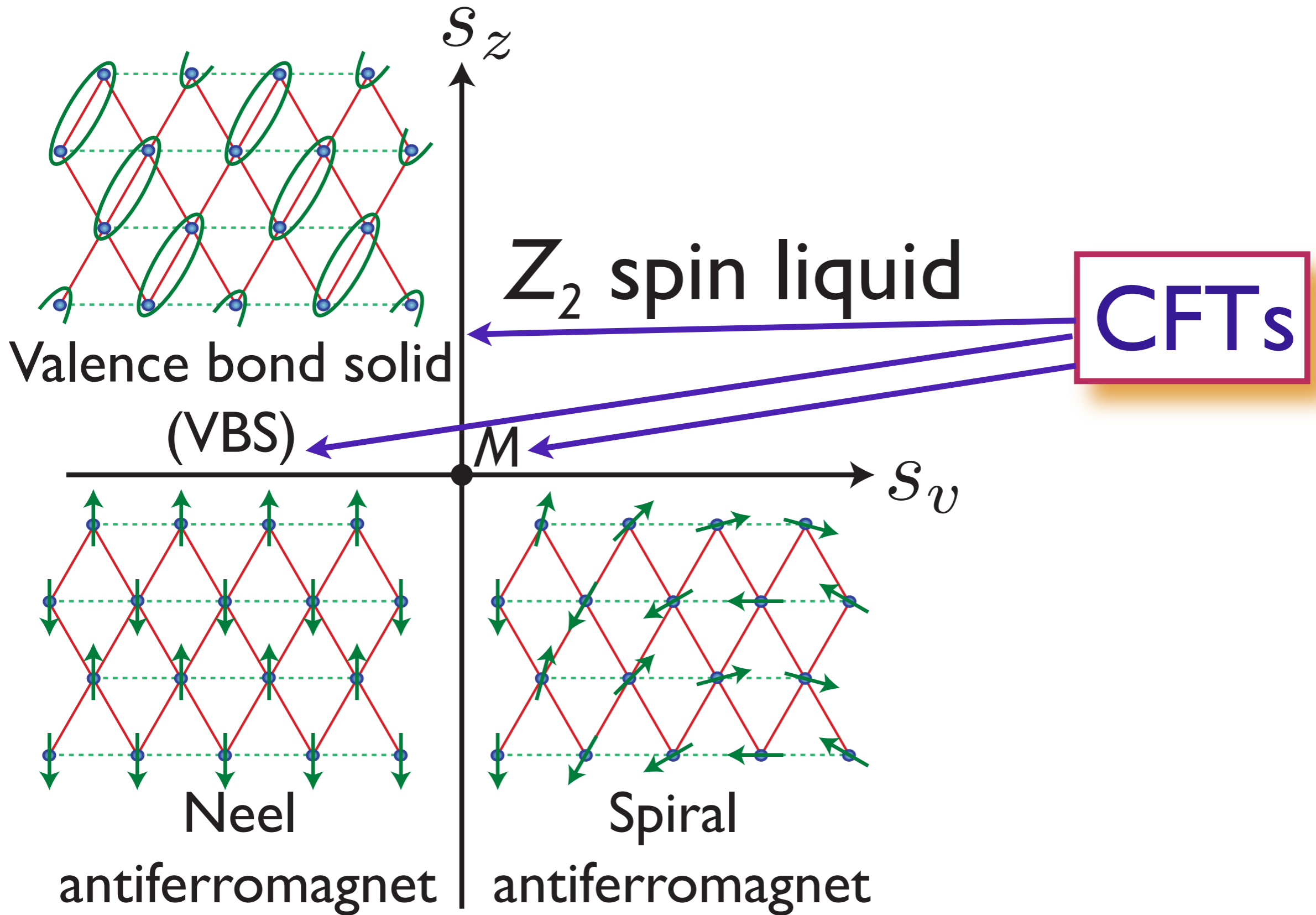


N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

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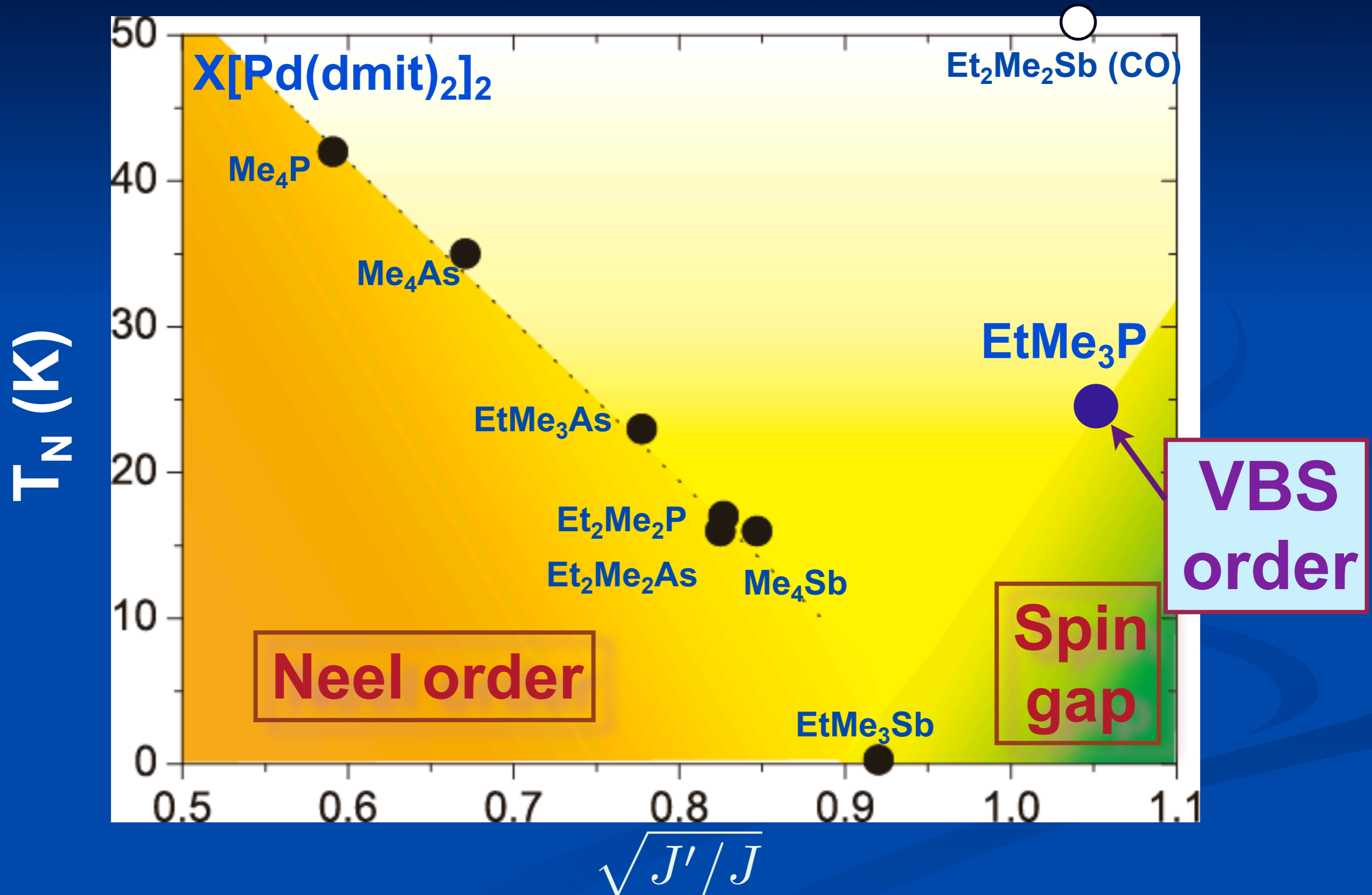


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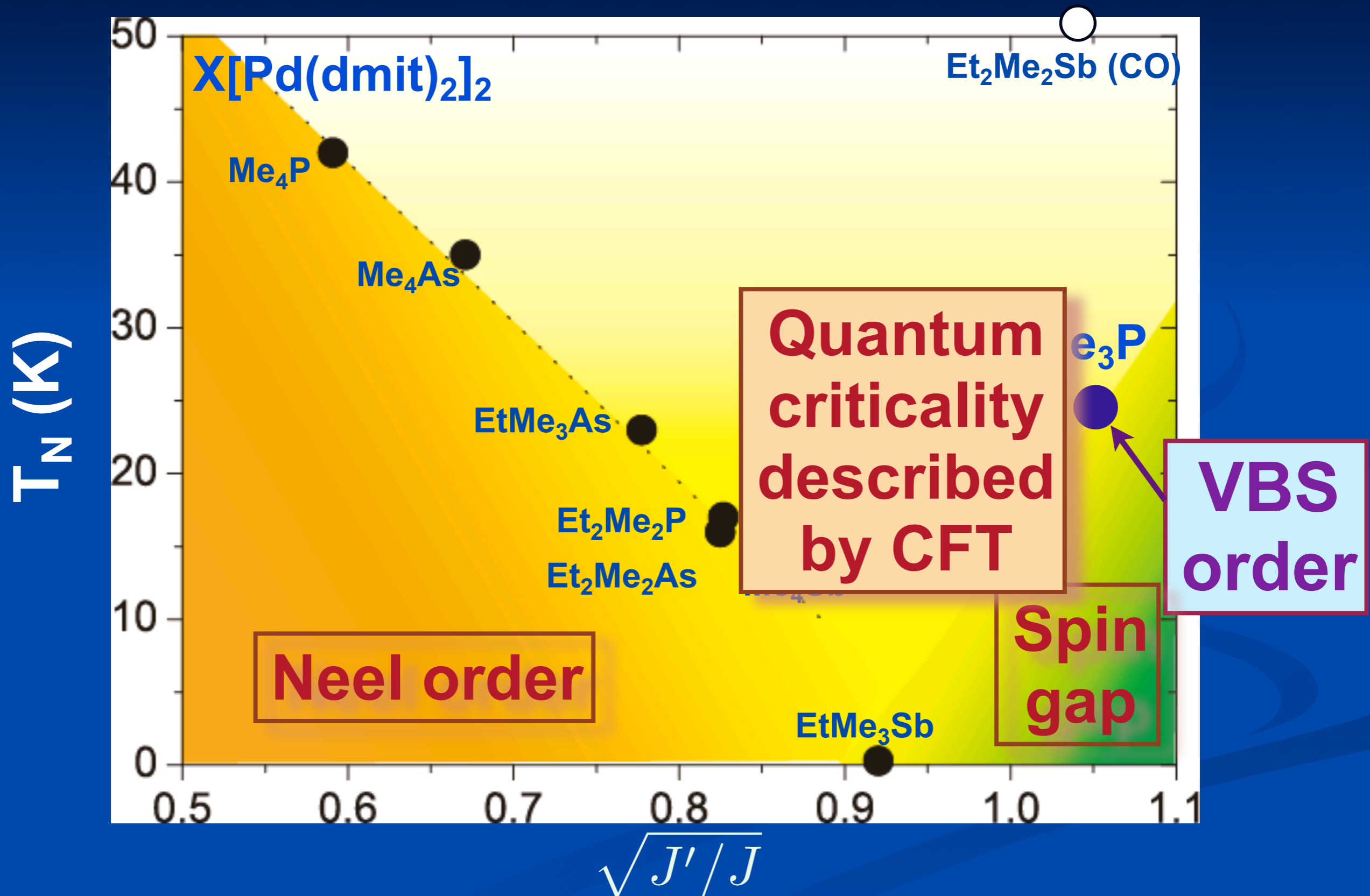
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Magnetic Criticality



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From quantum antiferromagnets to string theory

A direct generalization of the CFT of the multicritical point M ($s_z = s_v = 0$) to $\mathcal{N} = 4$ supersymmetry and the $U(N)$ gauge group was shown by O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP **0810**, 091 (2008) to be dual to a theory of quantum gravity (M theory) on $AdS_4 \times S^7/Z_k$.

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(a) Coupled-dimer antiferromagnets

(b) Triangular lattice antiferromagnets

(c) The superfluid-insulator transition

2. Black holes and quantum criticality

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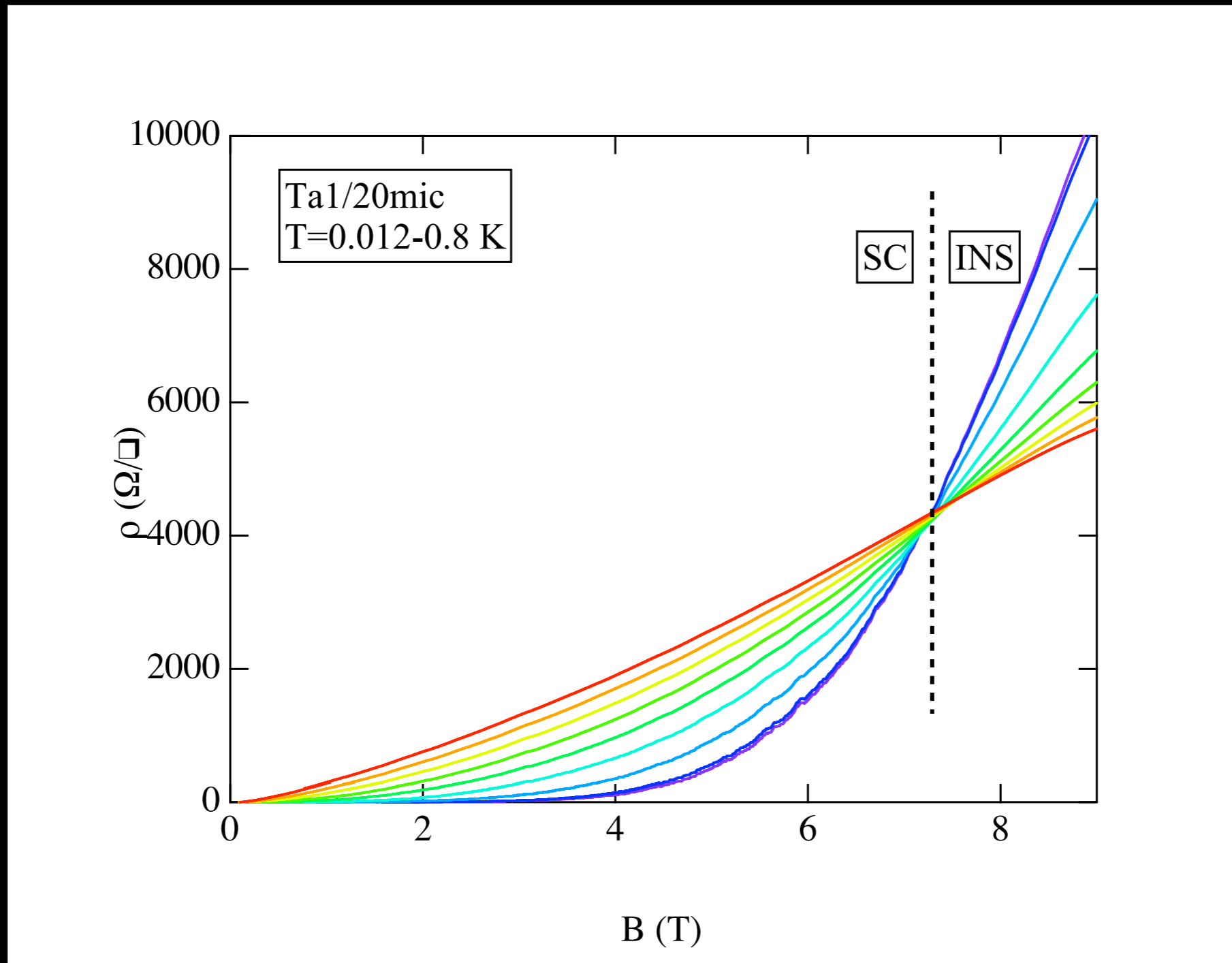
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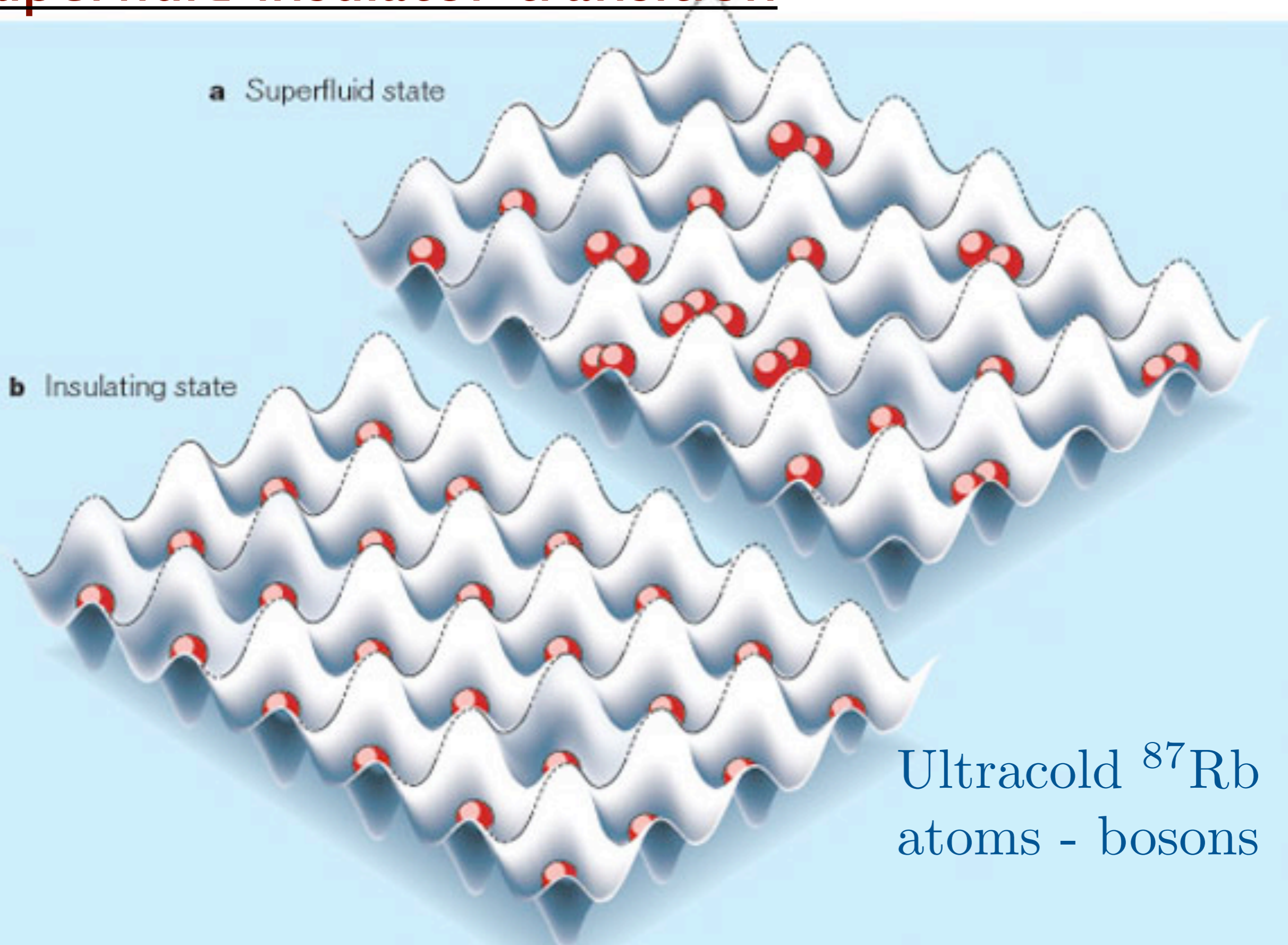
Superfluid-insulator transition

Indium Oxide films

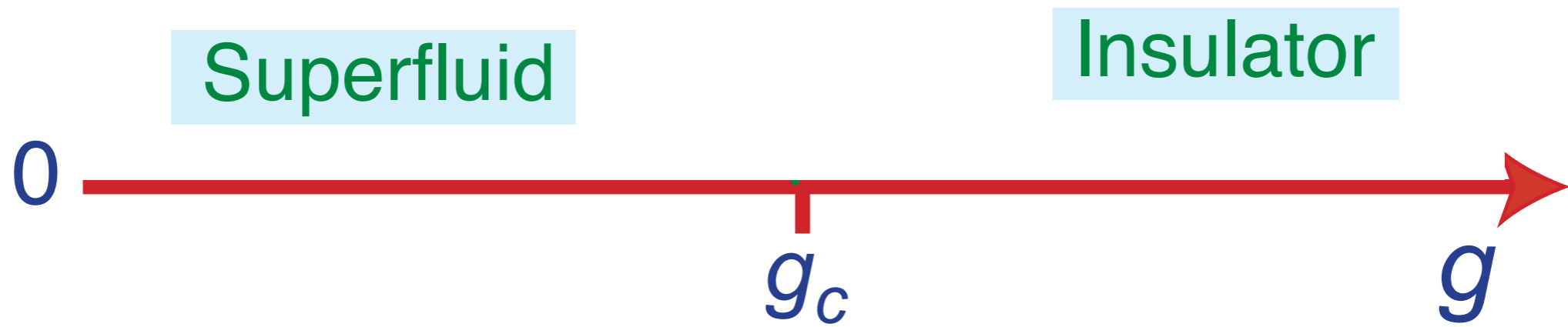


G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K.A. Moler, *Europhys. Lett.* **75**, 611 (2006).

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons



$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c) |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

$$\langle \psi \rangle \neq 0$$

Superfluid

$$\langle \psi \rangle = 0$$

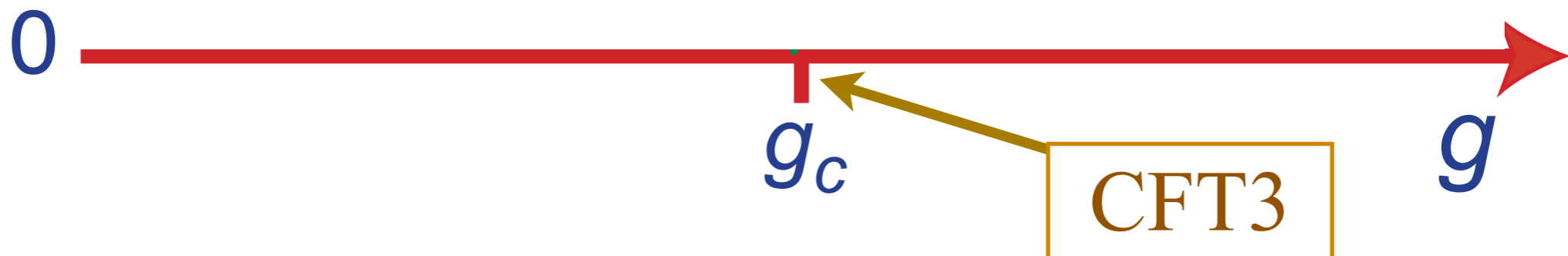
Insulator

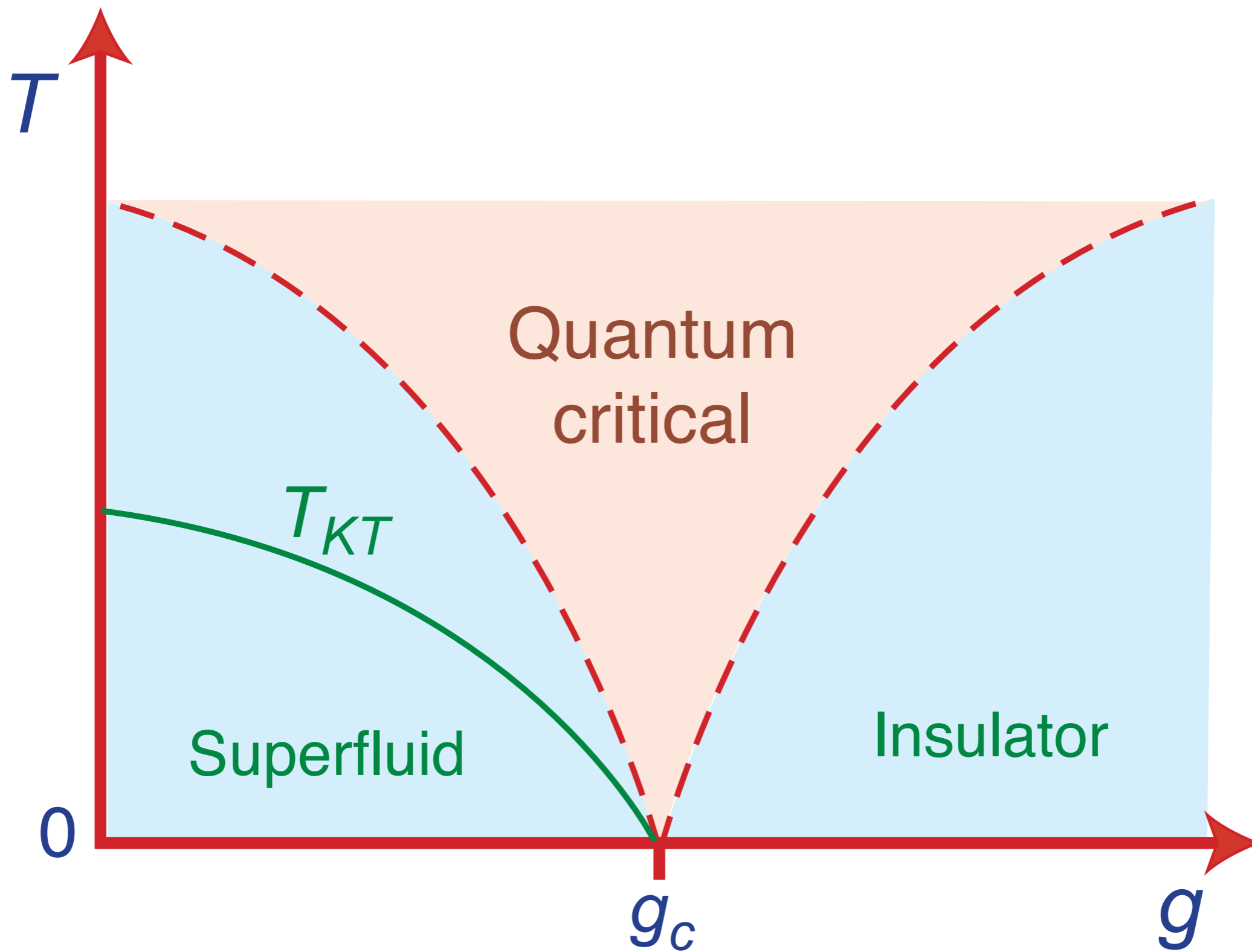
0

g_c

CFT3

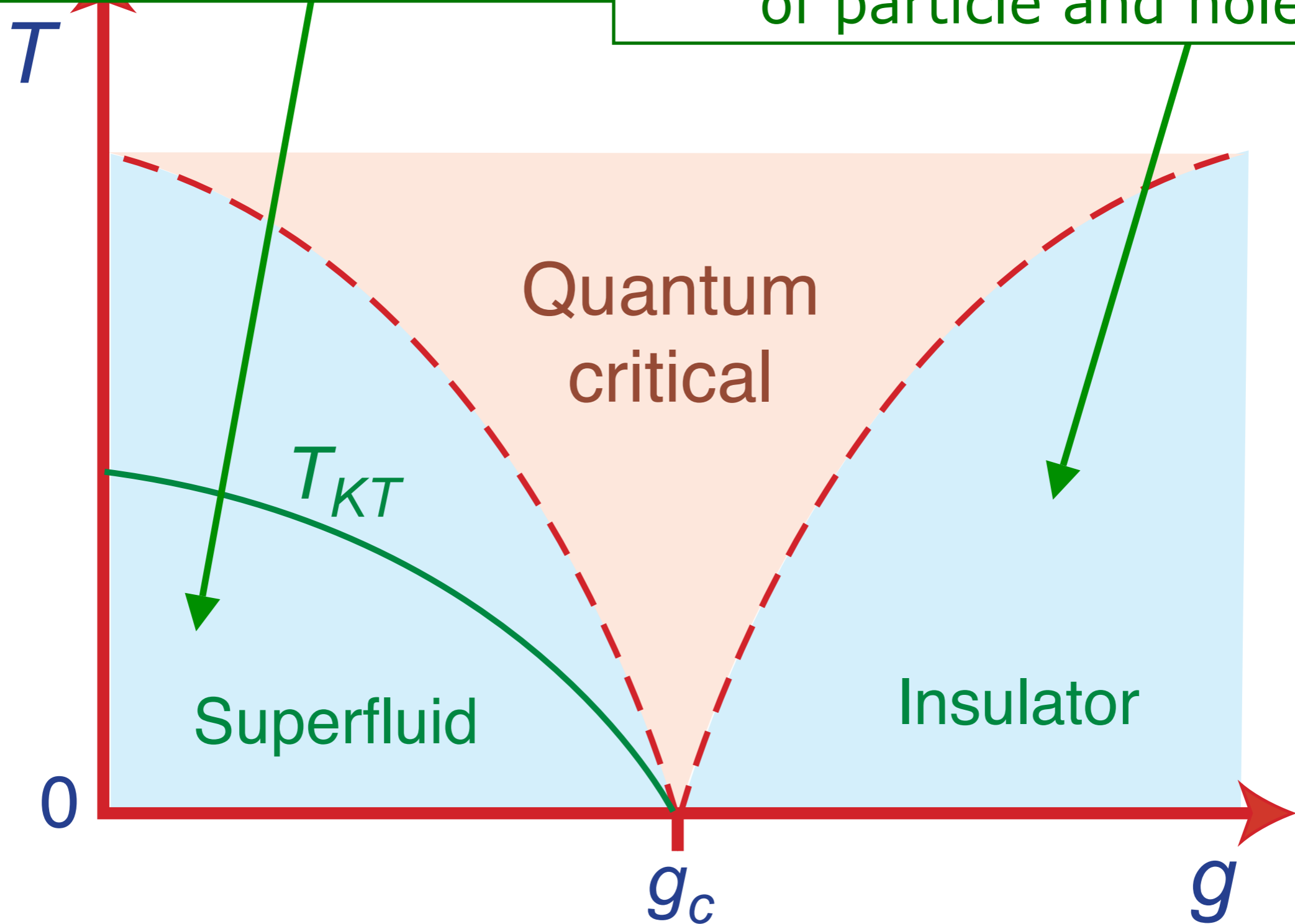
g

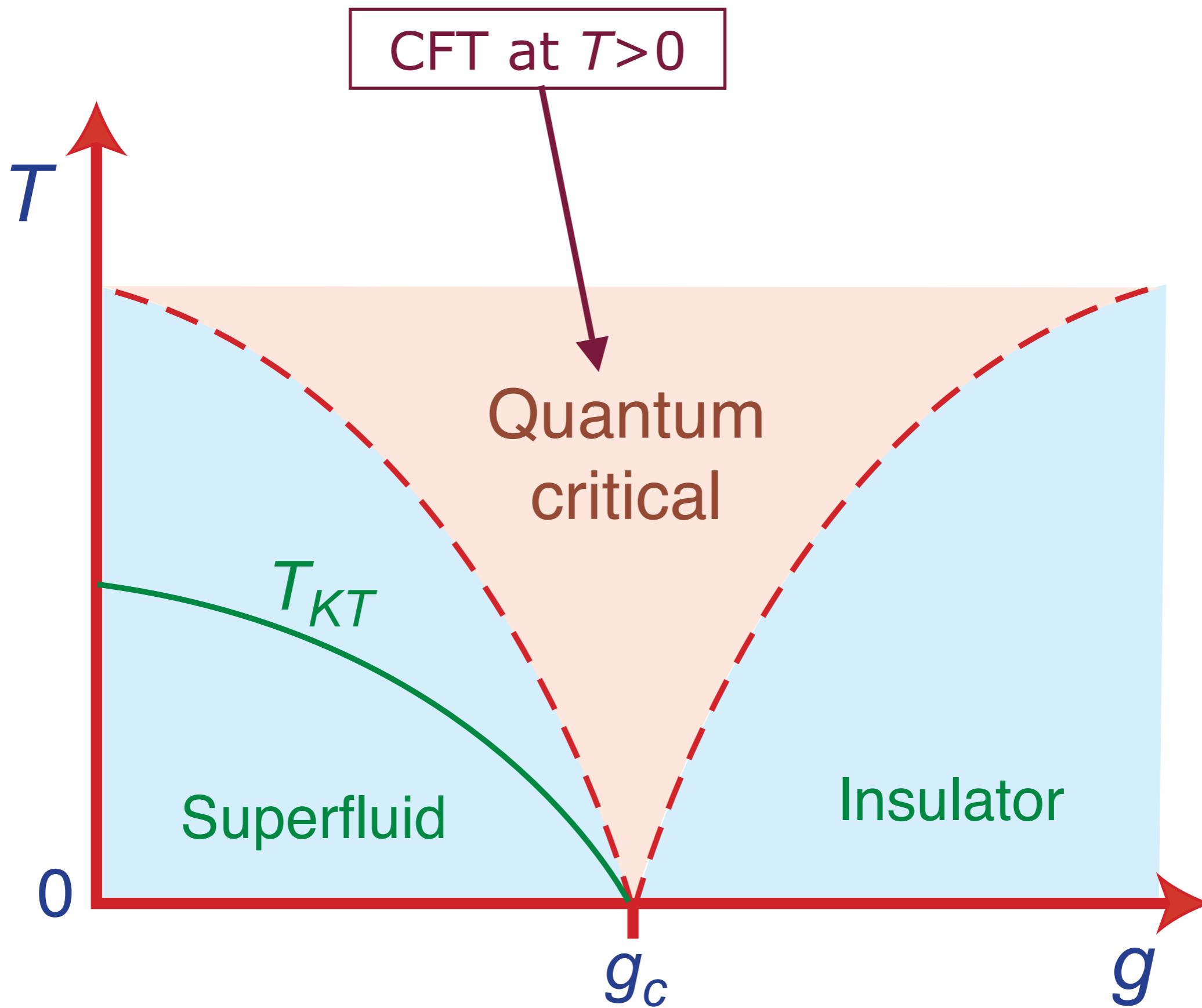




Classical vortices and wave oscillations of the condensate

Dilute Boltzmann/Landau gas of particle and holes





Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of “light”.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \rightarrow 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

However, for *all* CFT3s, at $\hbar\omega \ll k_B T$, we have the Einstein relation

$$\chi(k, \omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D \chi_c = \frac{4e^2}{h} \Theta_1 \Theta_2$$

where the **compressibility**, χ_c , and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(h\nu)^2} \Theta_1 \quad ; \quad D = \frac{h\nu^2}{k_B T} \Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

Density correlations in CFTs at $T > 0$

In CFTs collisions are “phase” randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \frac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \frac{4e^2}{h} \Theta_1 \Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

Quantum critical transport

Quantum “*perfect fluid*”
with shortest possible
relaxation time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

Quantum critical transport

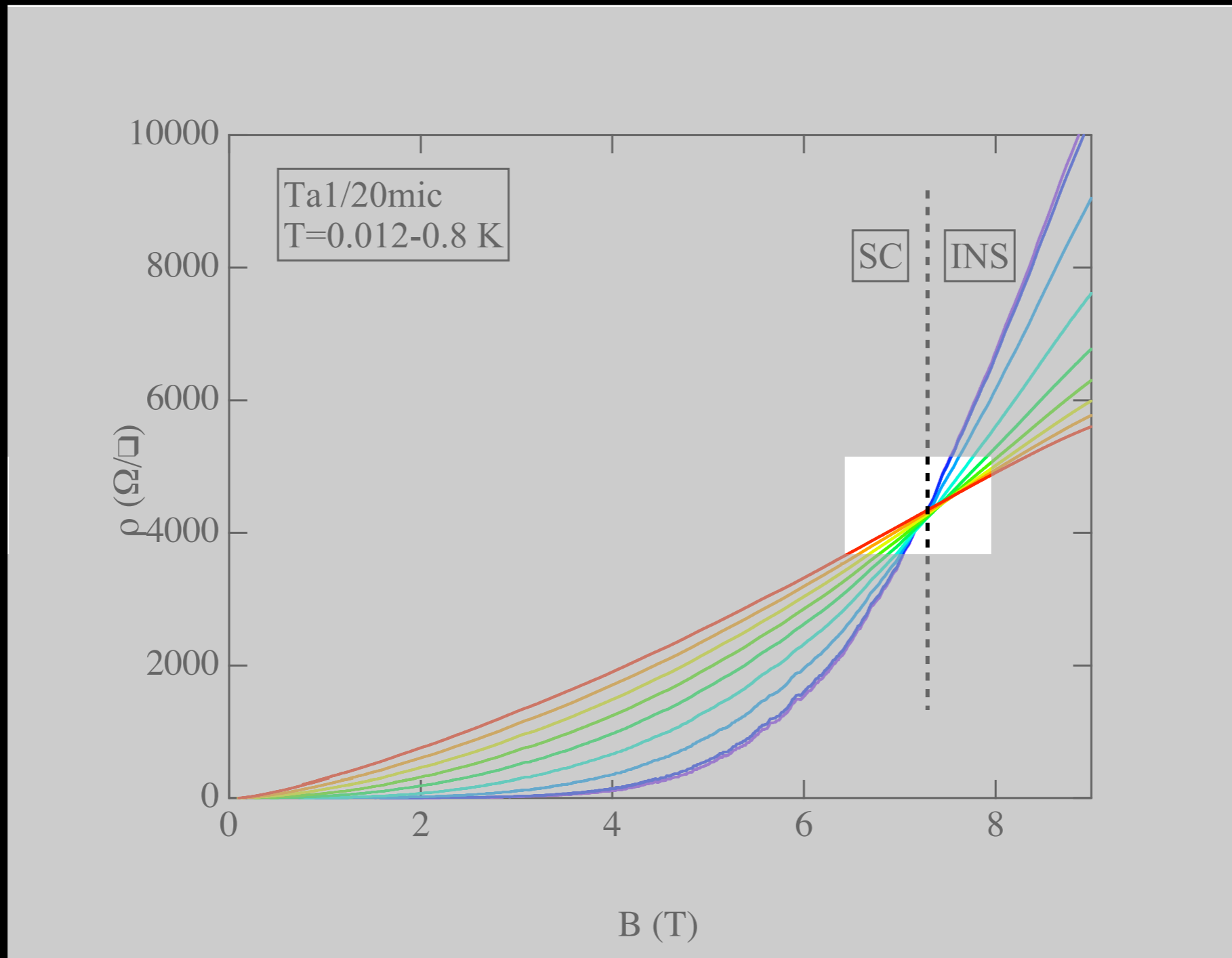
Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

Superfluid-insulator transition

Indium Oxide films



G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K.A. Moler, *Europhys. Lett.* **75**, 611 (2006).

Quantum critical transport

Transport co-efficients not determined
by collision rate, but by
universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)]$$

Outline

1. CFT3s in condensed matter physics

(a) Coupled-dimer antiferromagnets

(b) Triangular lattice antiferromagnets

(c) The superfluid-insulator transition

2. Black holes and quantum criticality

3. Applications

Nernst effect in the cuprate superconductors

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Black Holes

Objects so massive that light is gravitationally bound to them.

Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole **horizon** is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \frac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

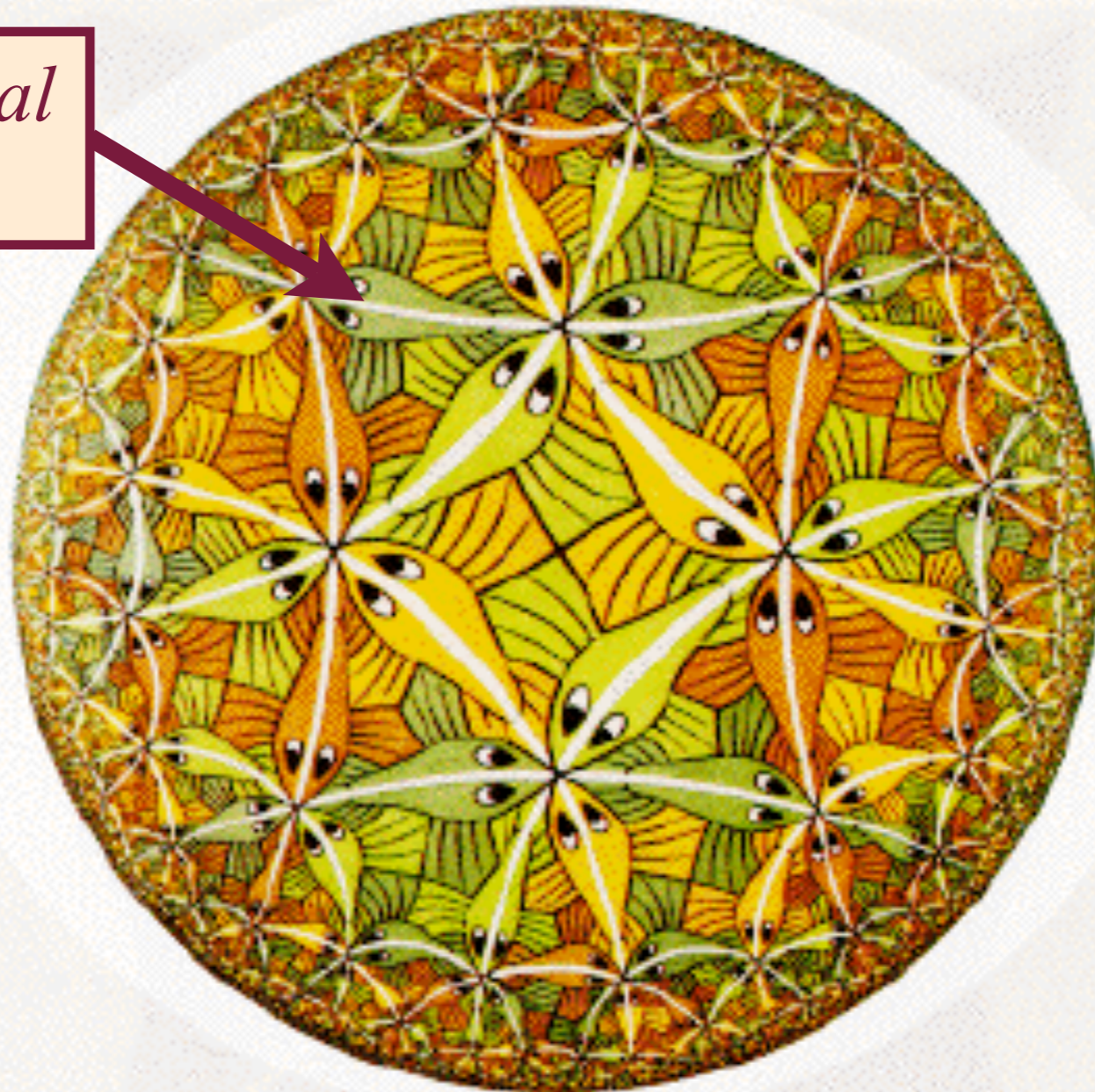
Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Horizon temperature: $4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

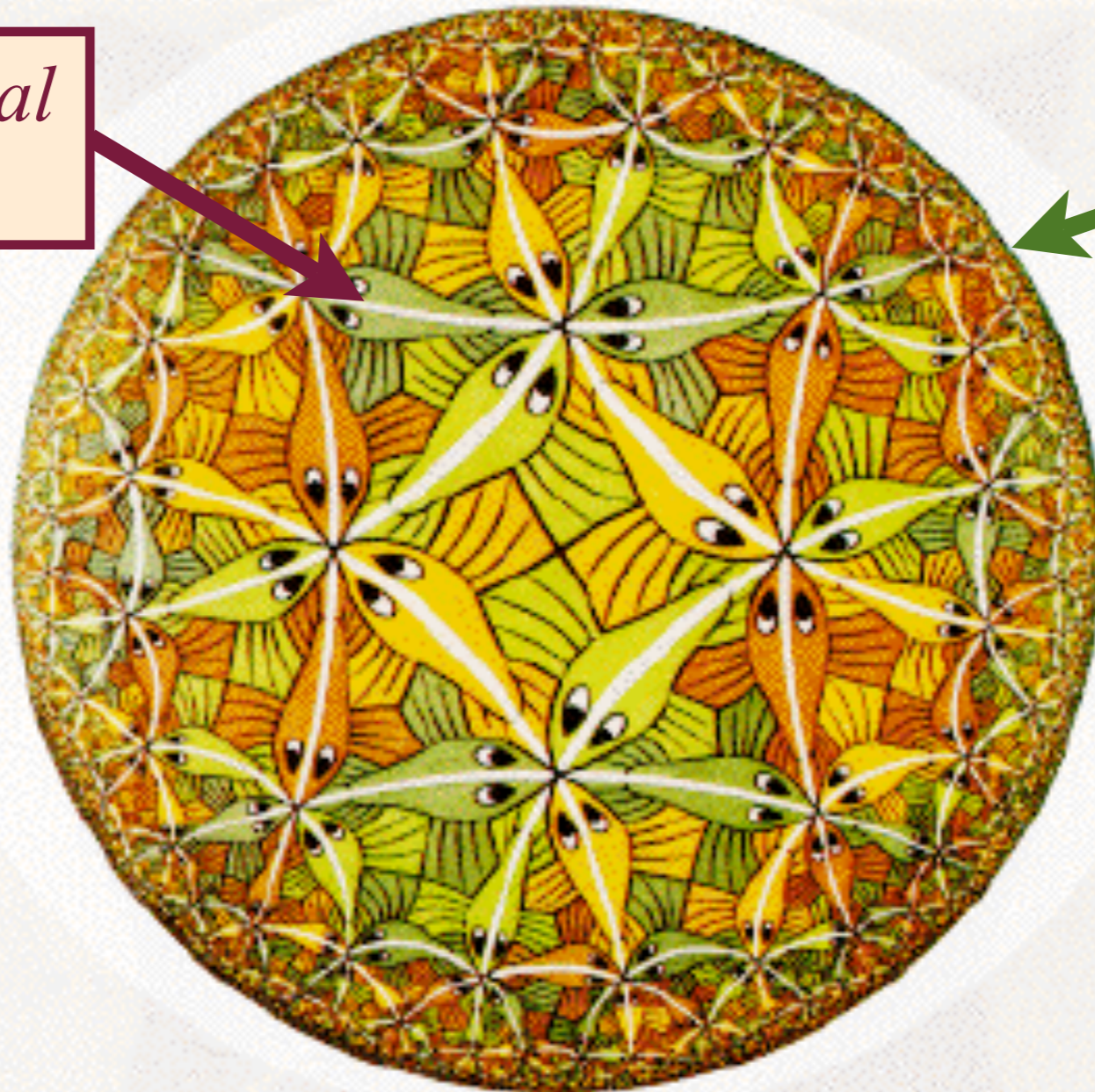
*3+1 dimensional
AdS space*



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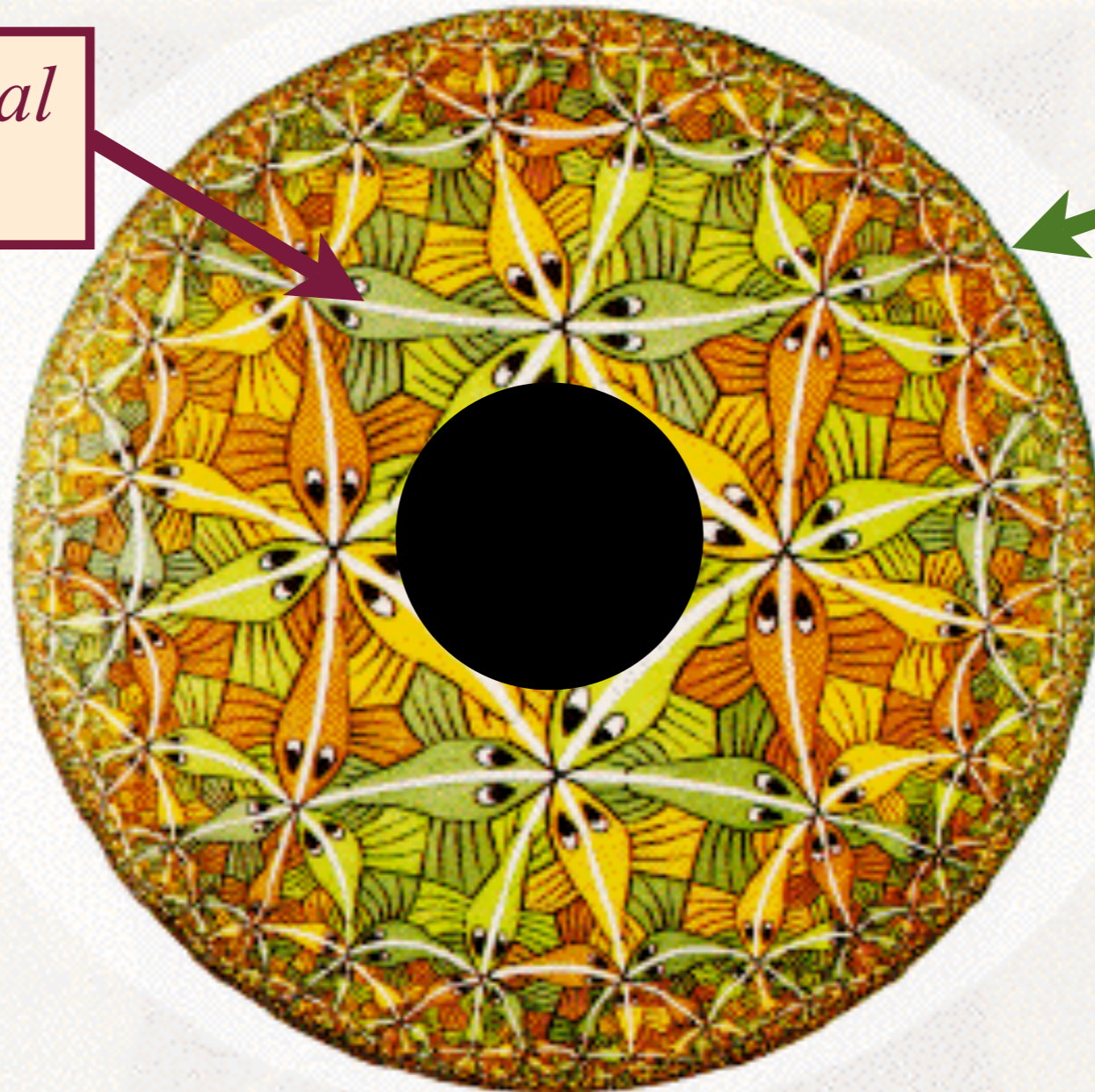
A 2+1
dimensional
system at its
quantum
critical point

AdS/CFT correspondence

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*3+1 dimensional
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Quantum
criticality in
2+1
dimensions



Black hole
temperature
=
temperature
of quantum
criticality

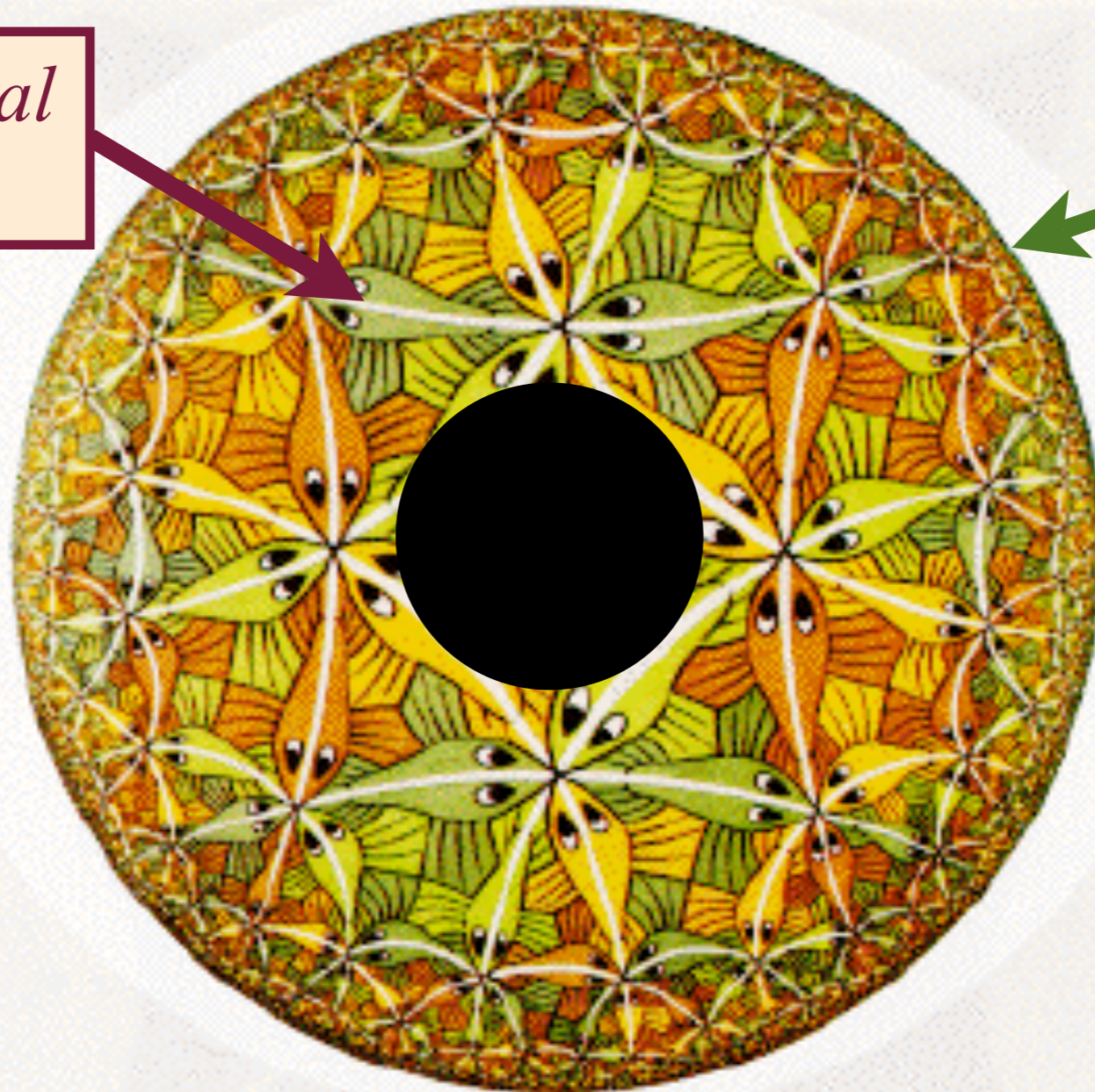
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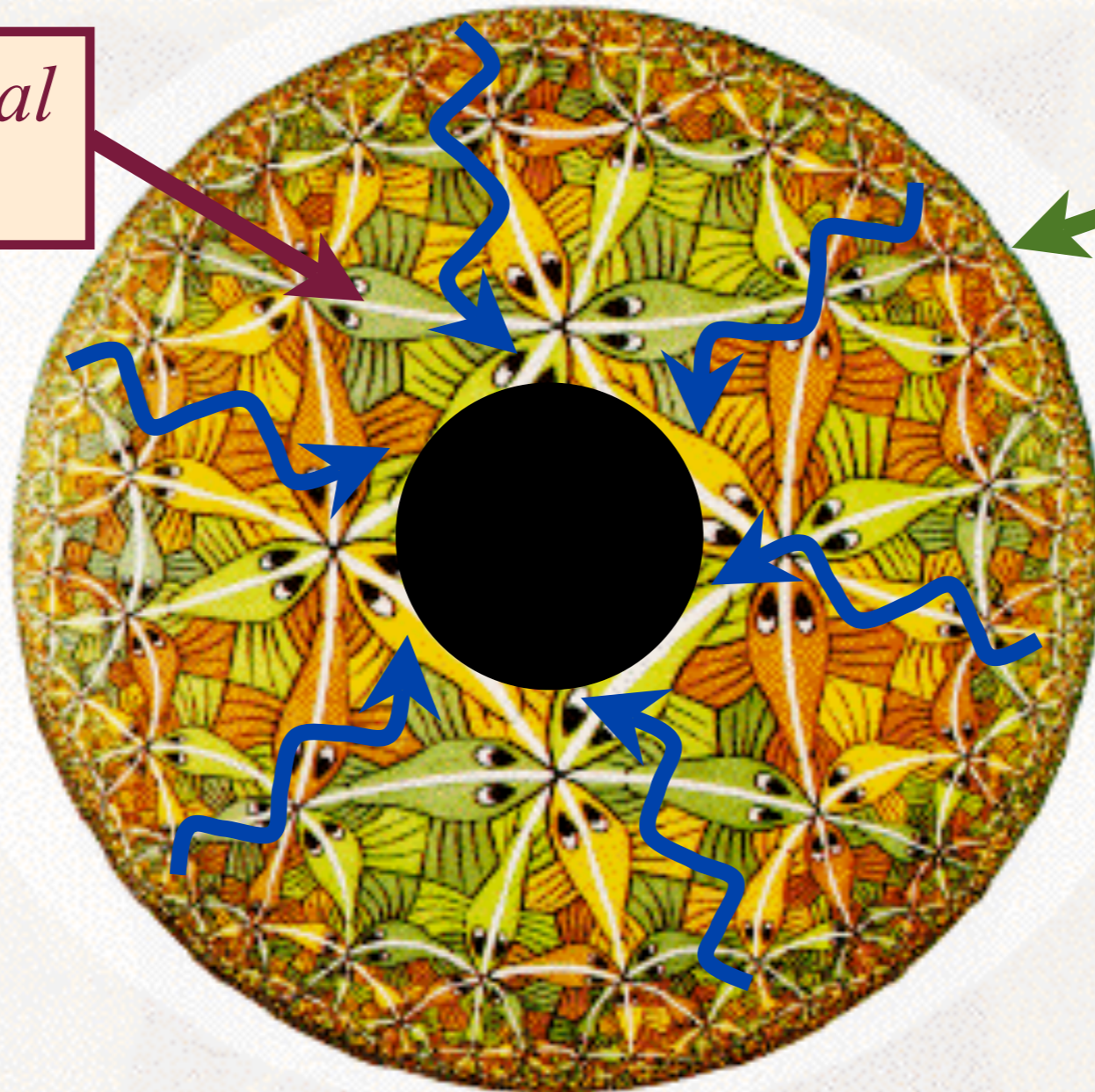
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AdS space*

Quantum
criticality in
2+1
dimensions

Quantum
critical
dynamics =
waves in
curved
space



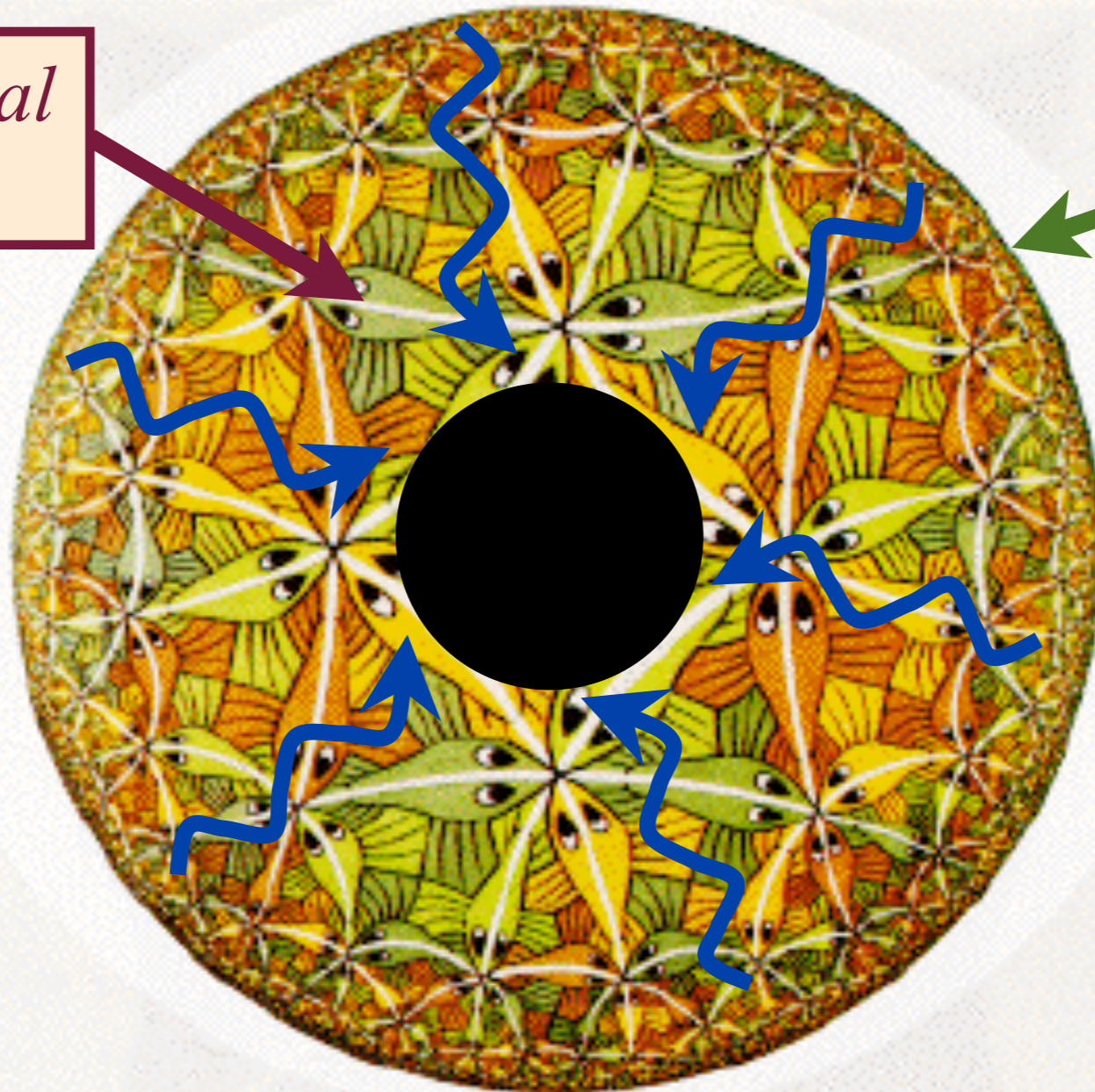
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Quantum
criticality in
2+1
dimensions

Friction of
quantum
criticality =
waves
falling into
black hole



Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics
Uses physical model but strong-coupling makes explicit solution difficult
2. Solve Einstein-Maxwell equations in the background of a black hole in AdS space
*Yields hydrodynamic relations which apply to general classes of quantum critical systems.
First exact numerical results for transport co-efficients (for supersymmetric systems).*

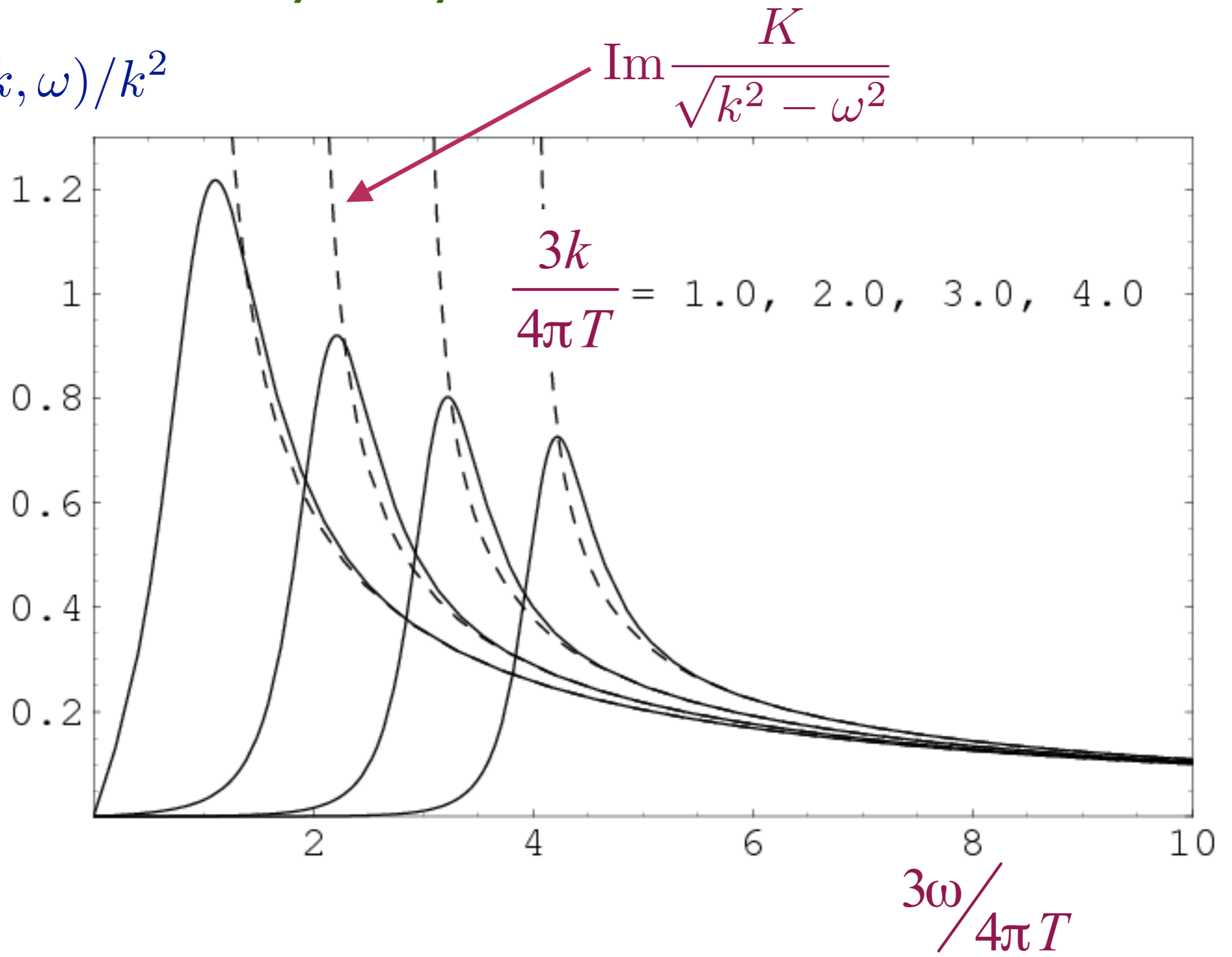
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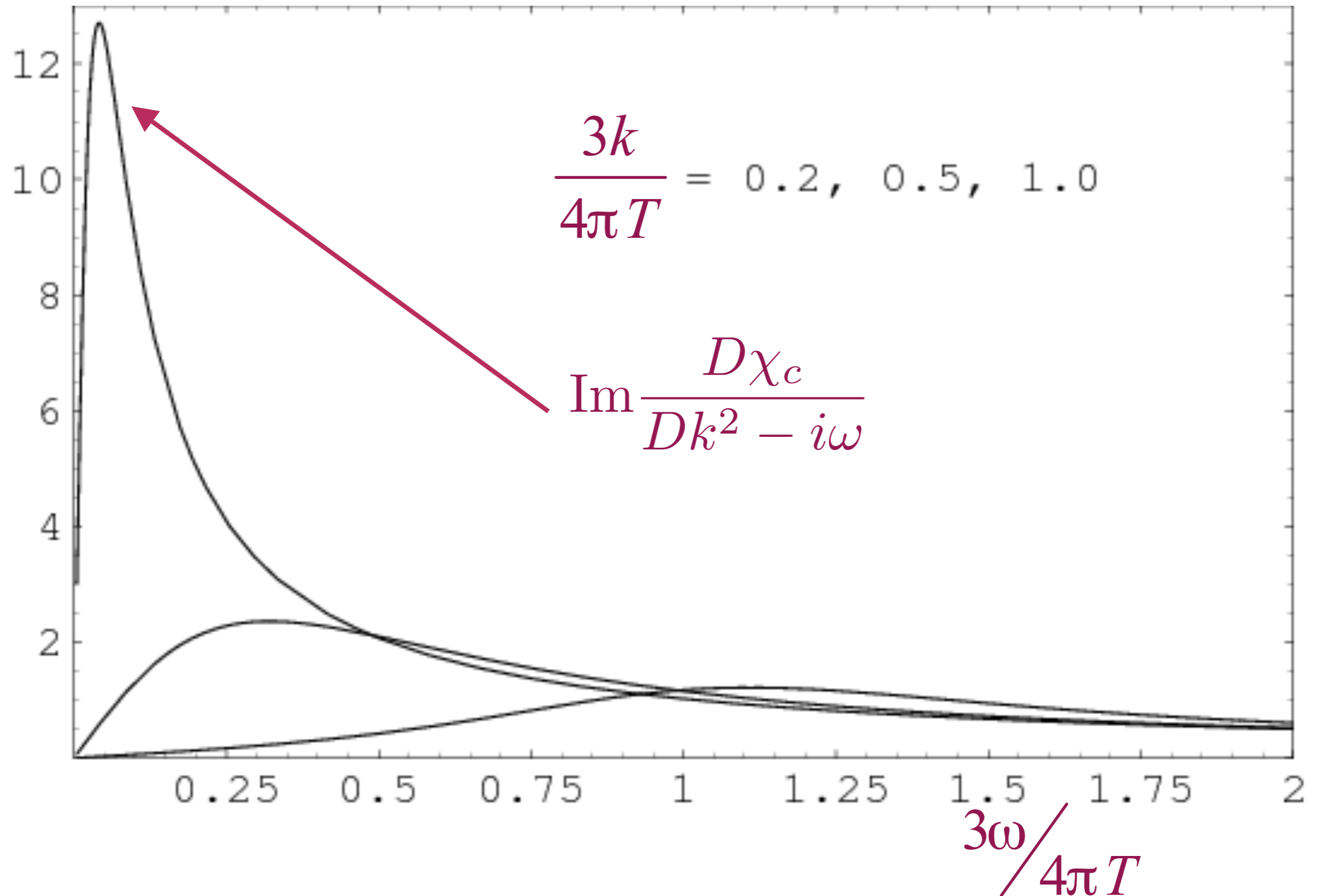
Collisionless to hydrodynamic crossover of SYM3

$$\text{Im}\chi(k, \omega)/k^2$$



Collisionless to hydrodynamic crossover of SYM3

$\text{Im}\chi(k, \omega)/k^2$



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Nernst effect

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Low viscosity fluid

3. Fermi gas at unitarity

Non-relativistic AdS/CFT

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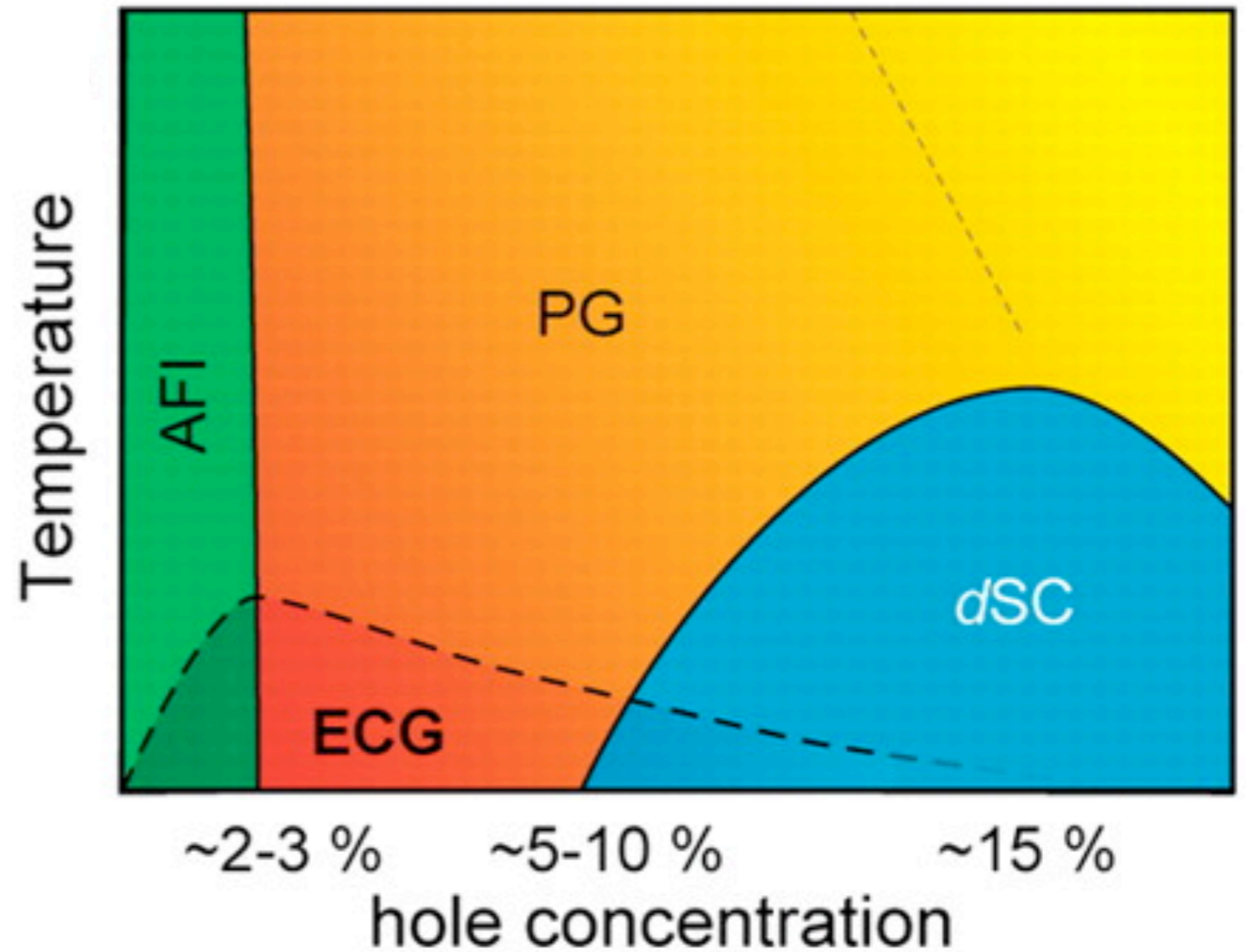
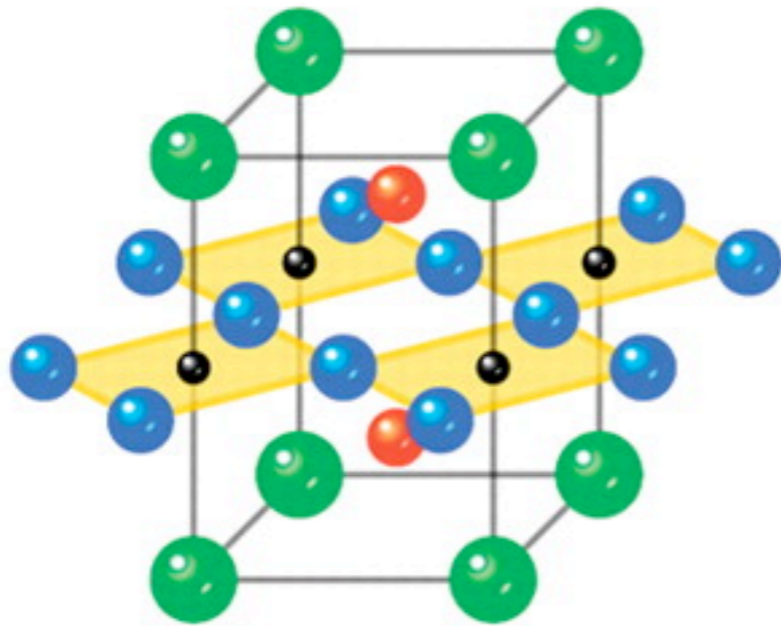
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The cuprate superconductors

Na-CCOC

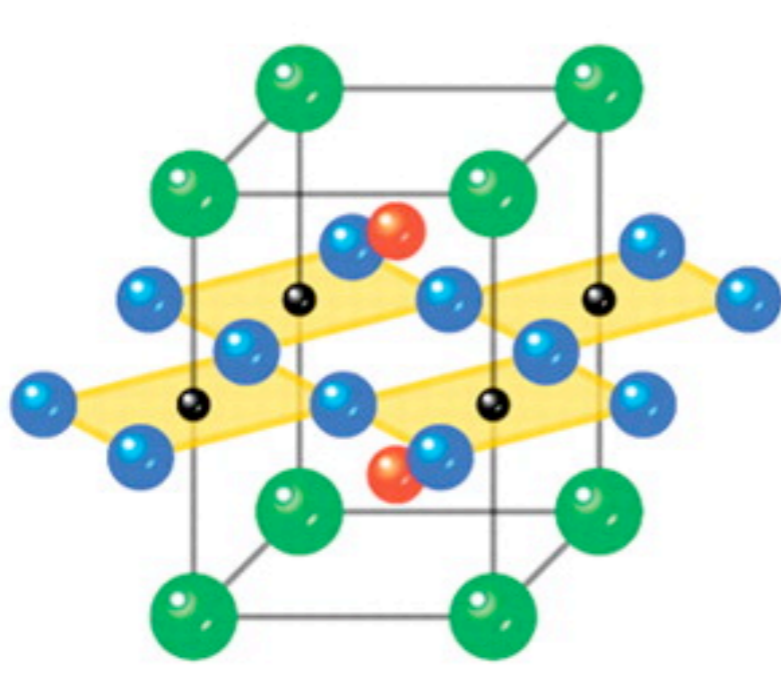
- Cu
- Ca/Na
- O
- Cl



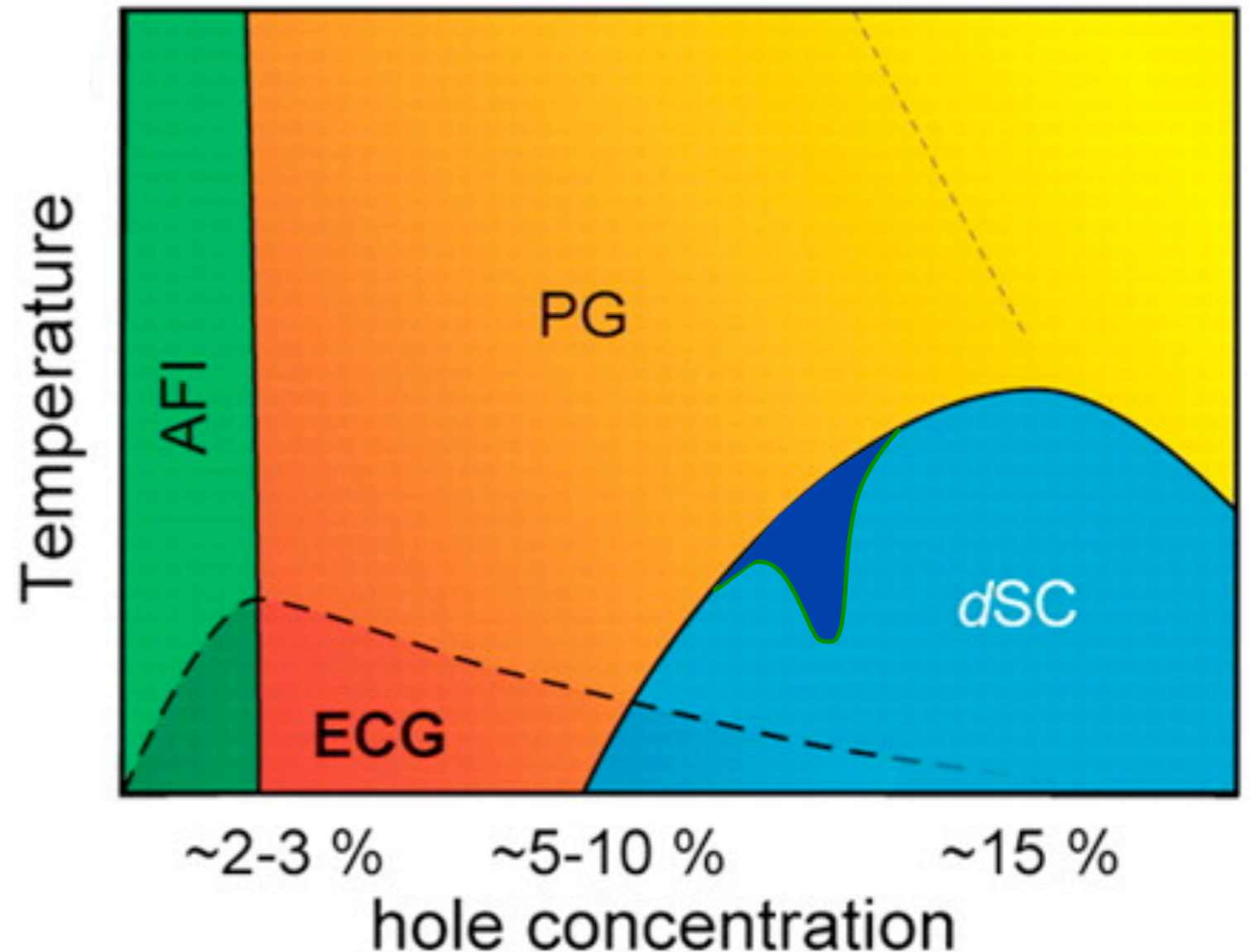
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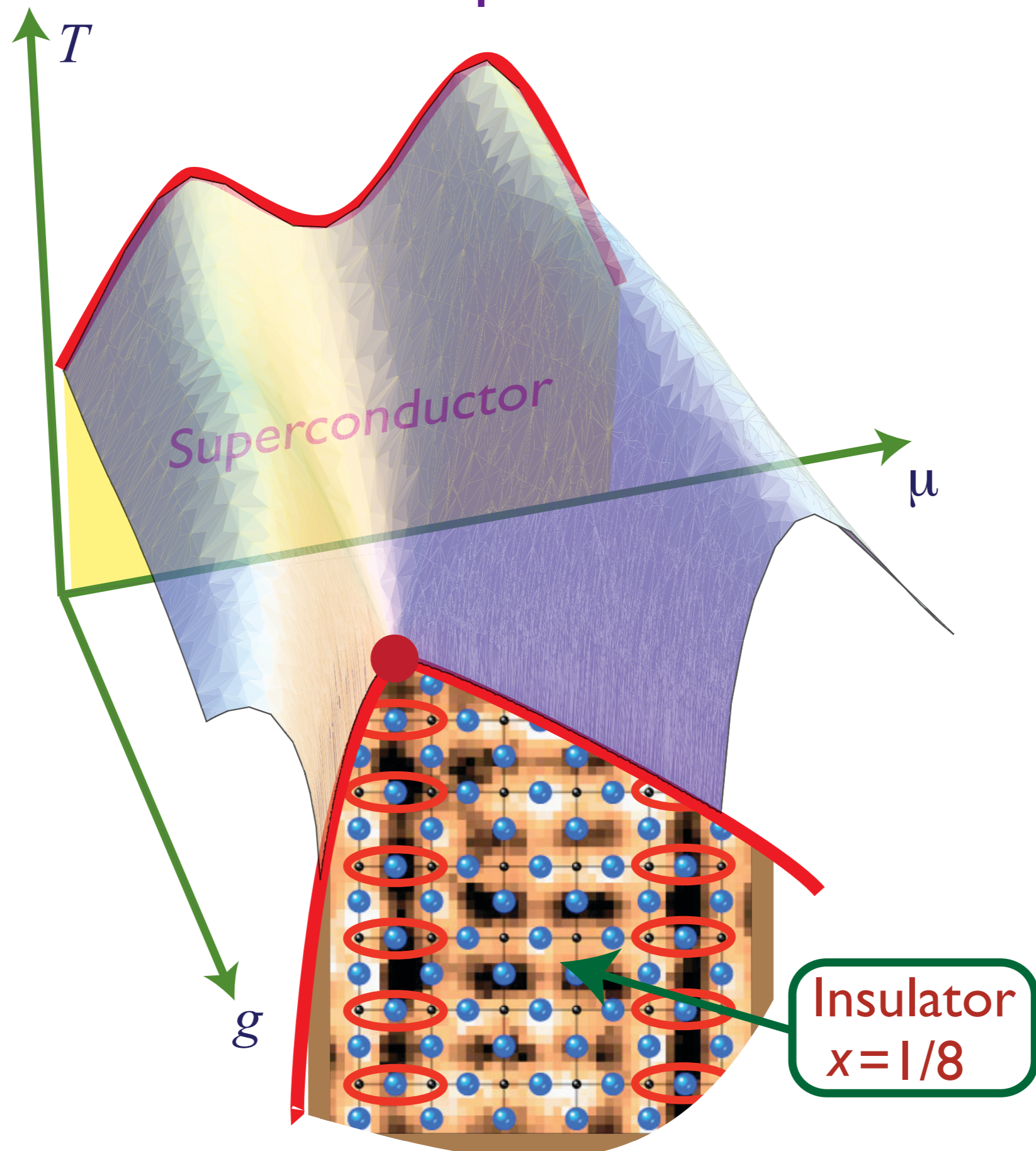
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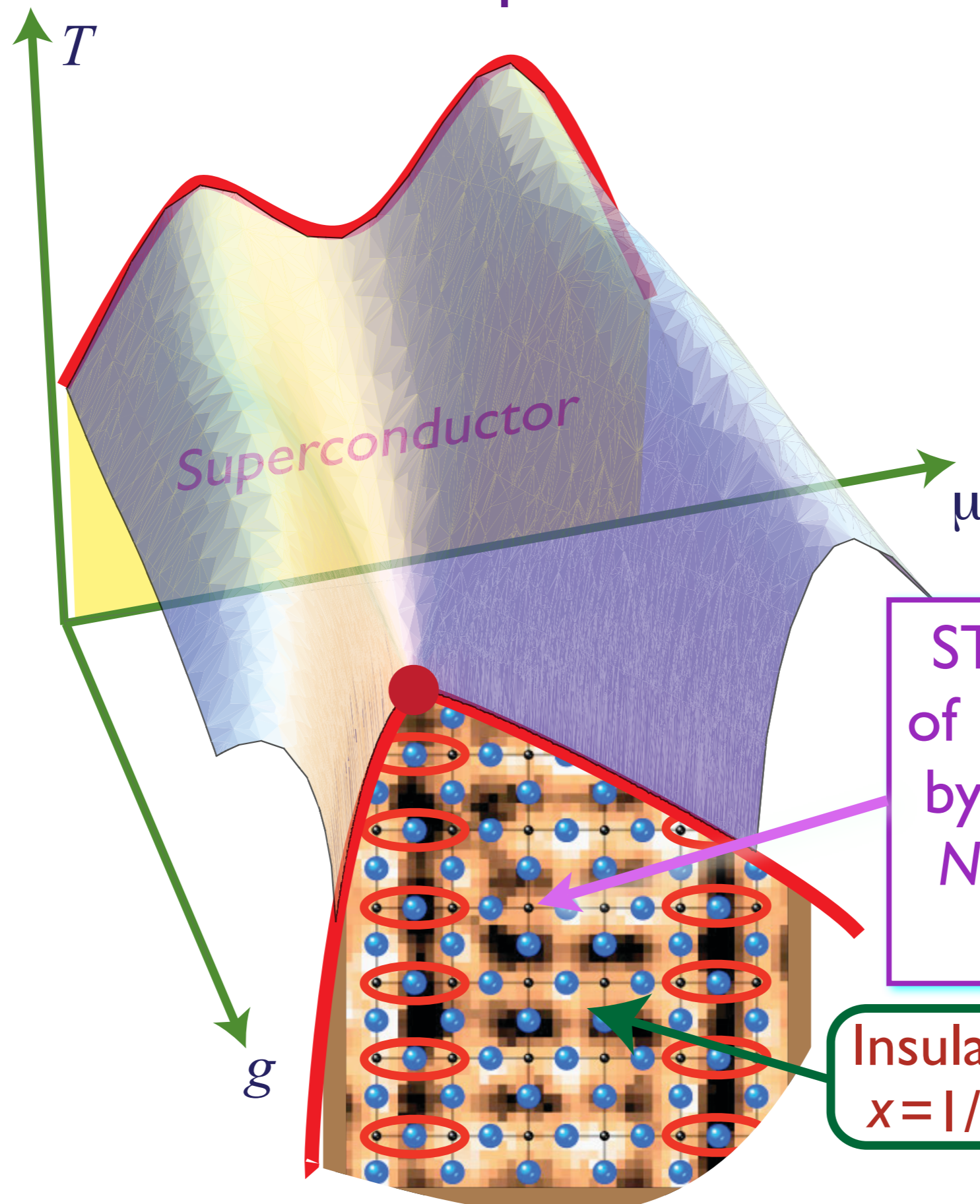
Proximity to an insulator at 12.5% hole concentration



Cuprates



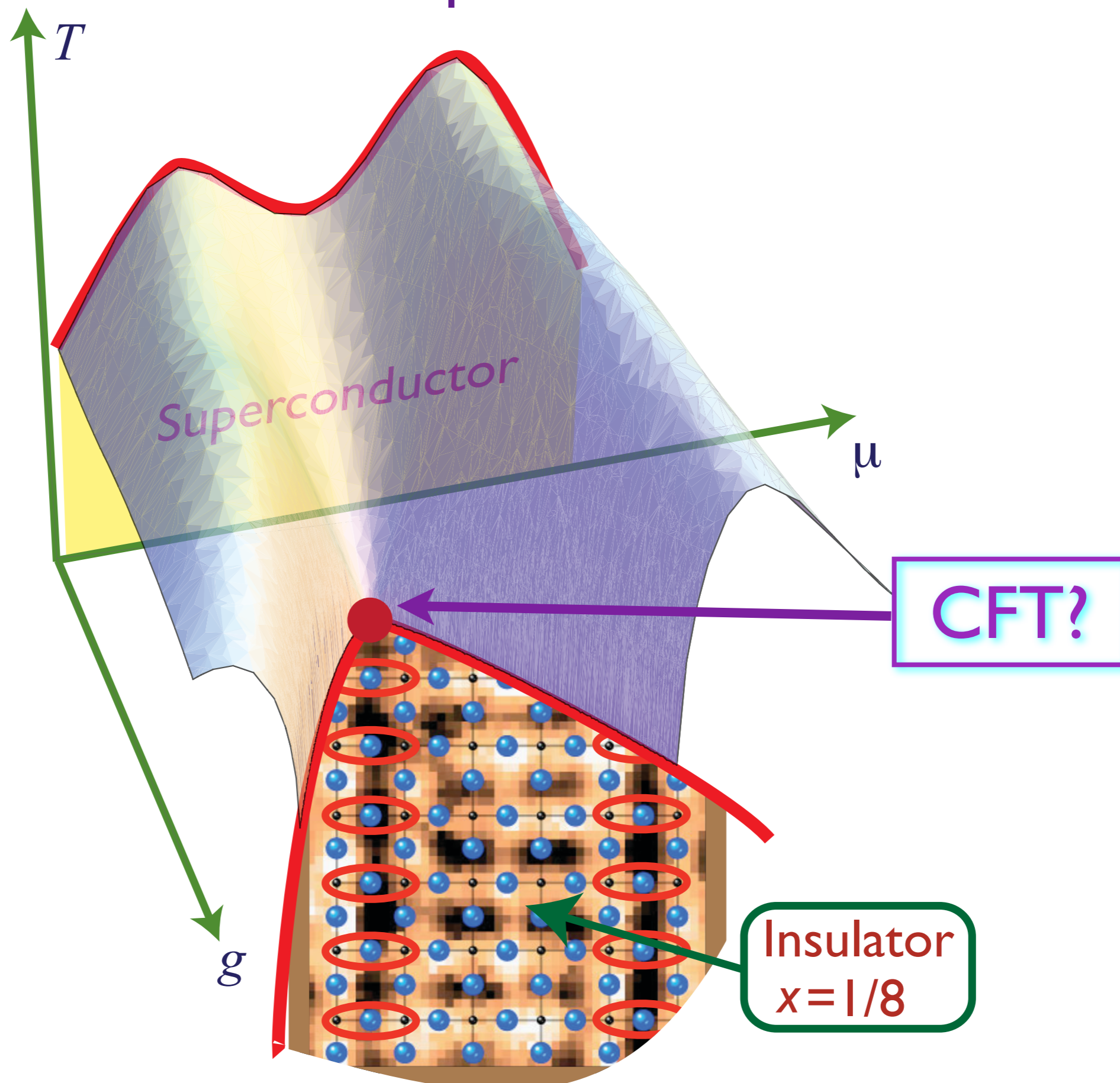
Cuprates



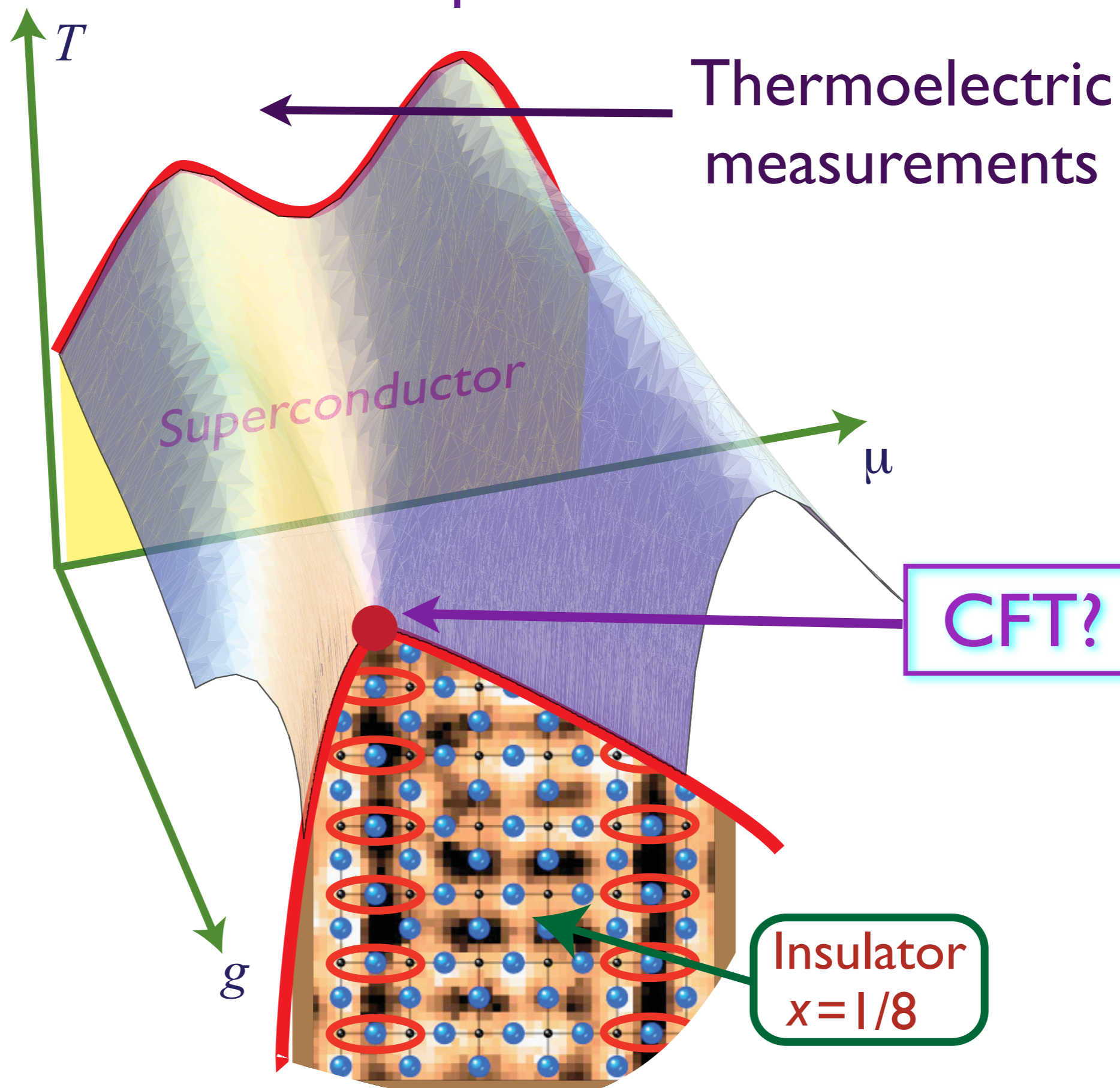
STM observations
of VBS modulations
by Y. Kohsaka *et al.*,
Nature **454**, 1072
(2008)

Insulator
 $x = 1/8$

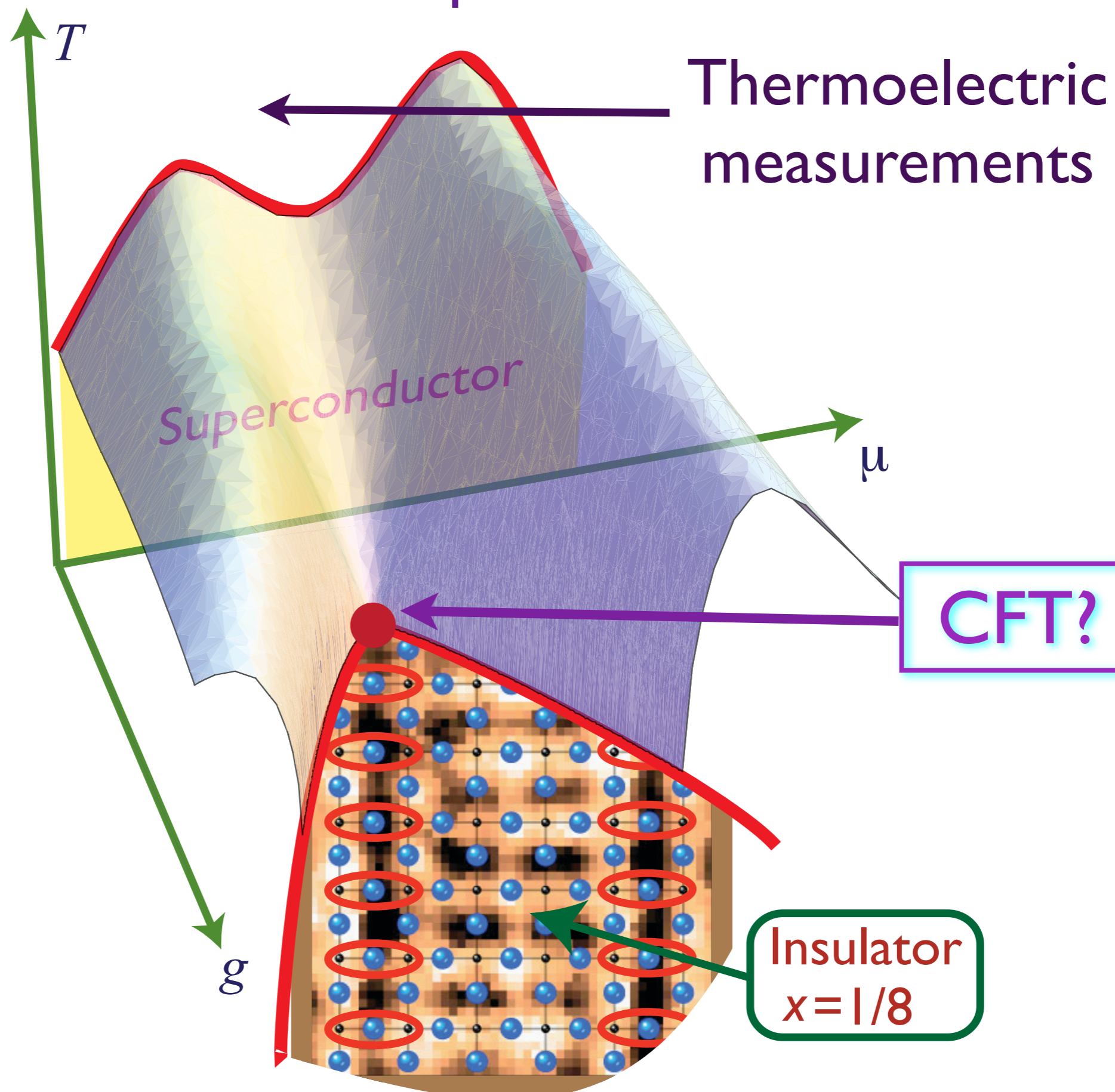
Cuprates



Cuprates



Cuprates



Hydrodynamic cyclotron resonance at a frequency

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and with width

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

where B = magnetic field, ρ = charge density away from density of CFT, ε = energy density, P = pressure, v = velocity of “light” in CFT, and $\sigma_Q e^2/h$ is the universal conductivity of the CFT.

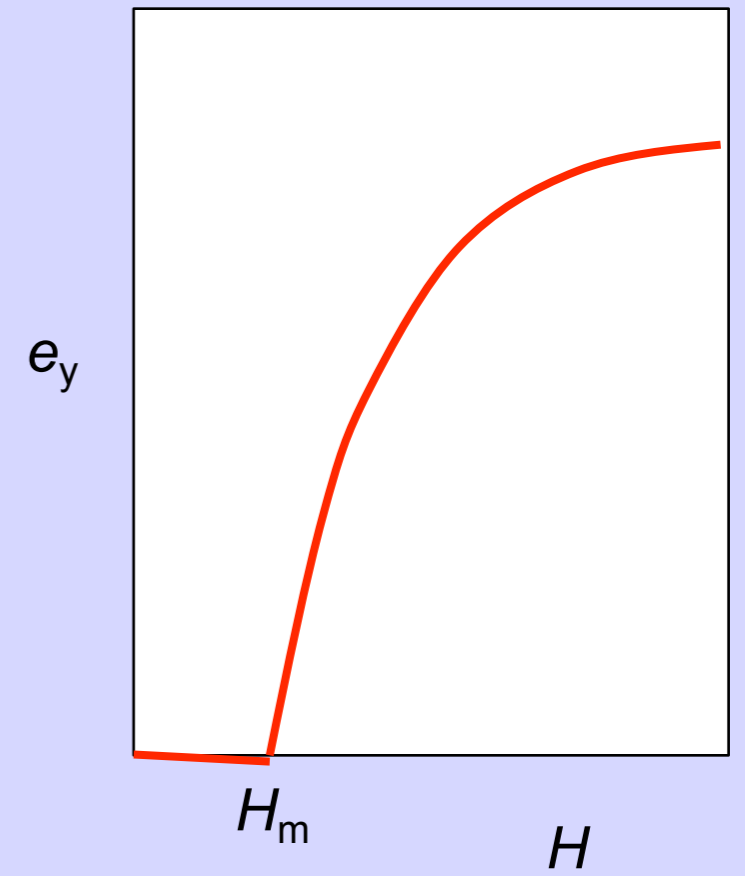
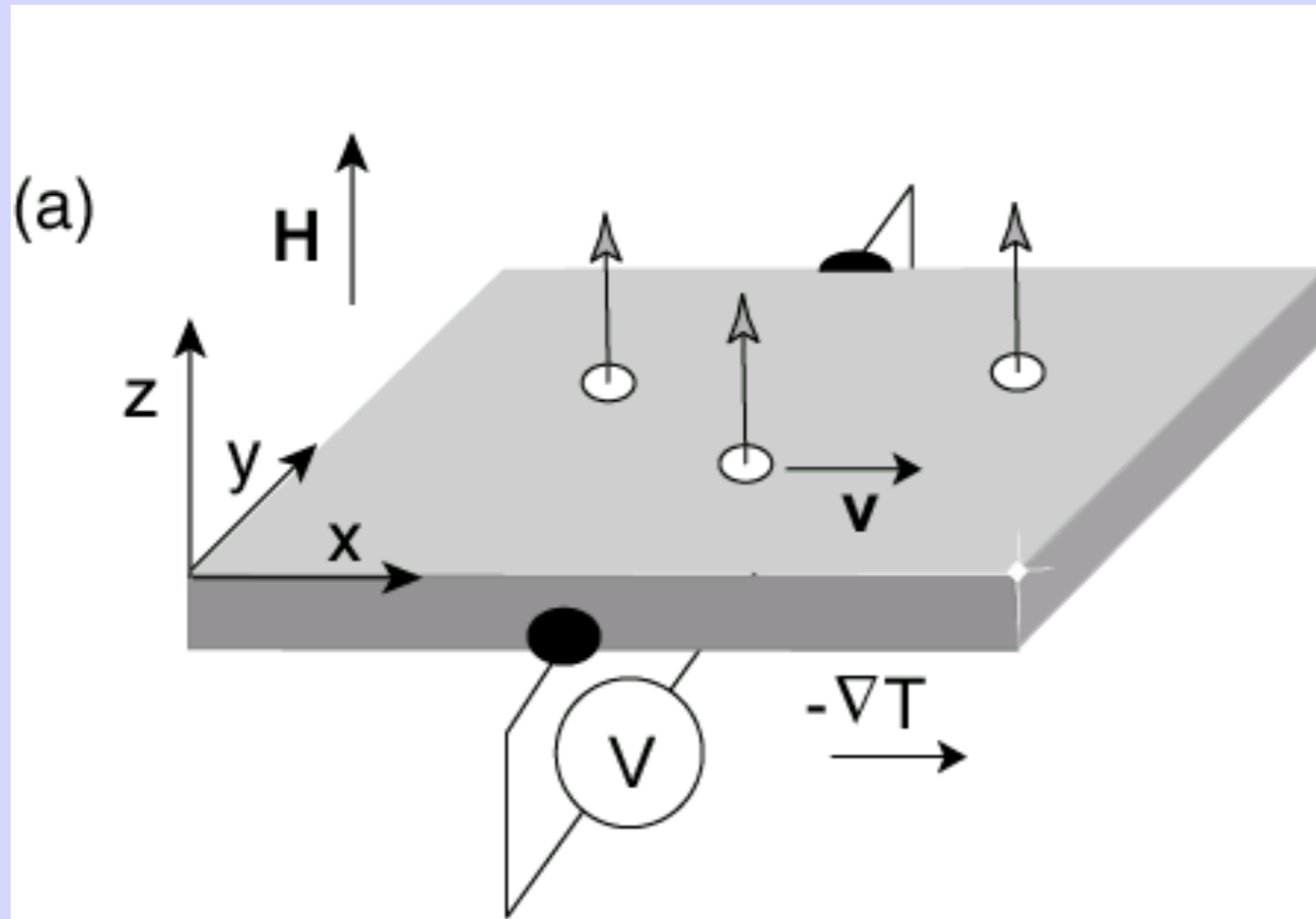
“Wiedemann-Franz”-like relation for thermal conductivity, κ at $B = 0$

$$\kappa = \sigma_Q \left(\frac{k_B^2 T}{e^{*2}} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a “Wiedemann-Franz” relation for “vortices”

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left(\frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

Nernst experiment



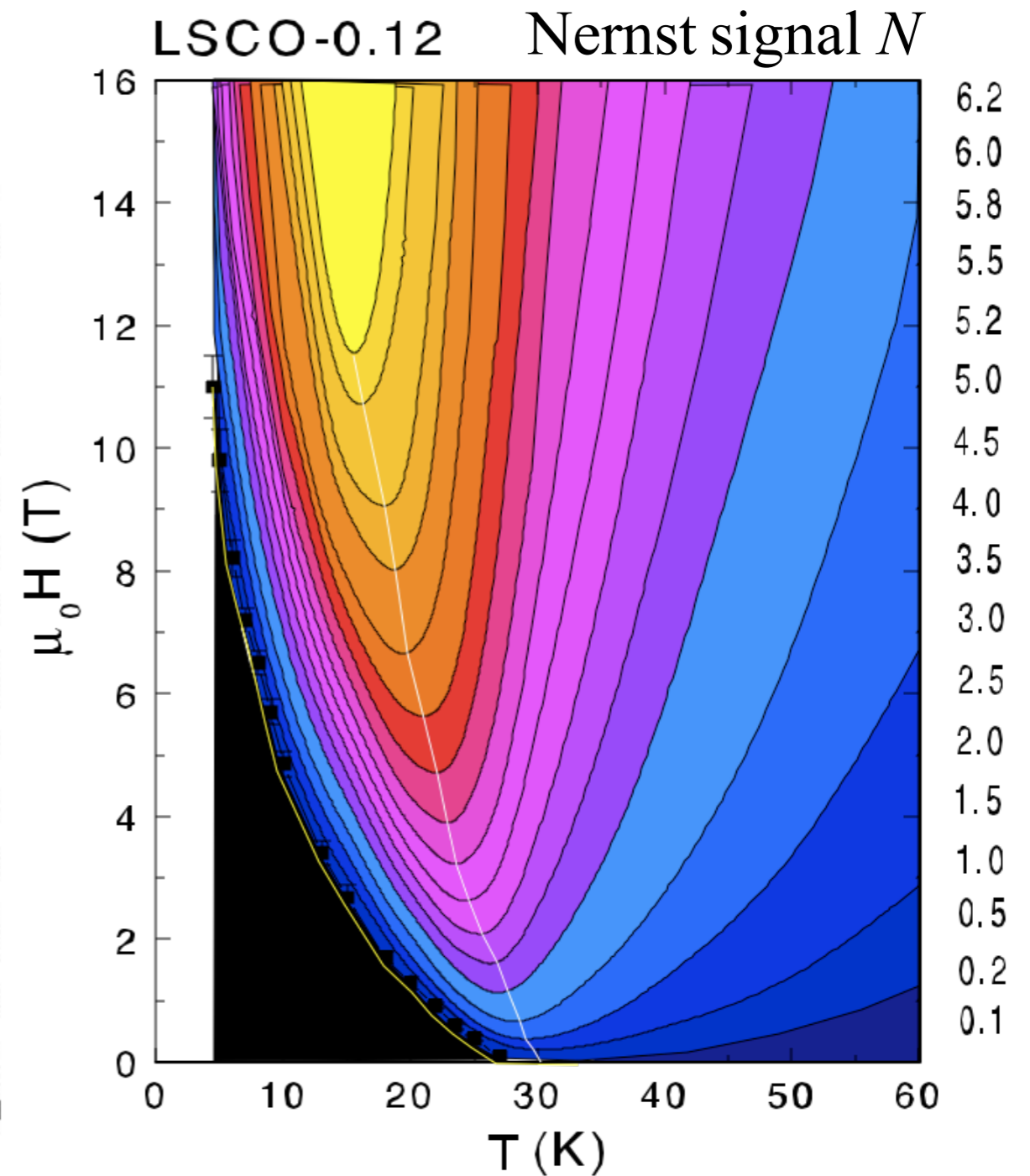
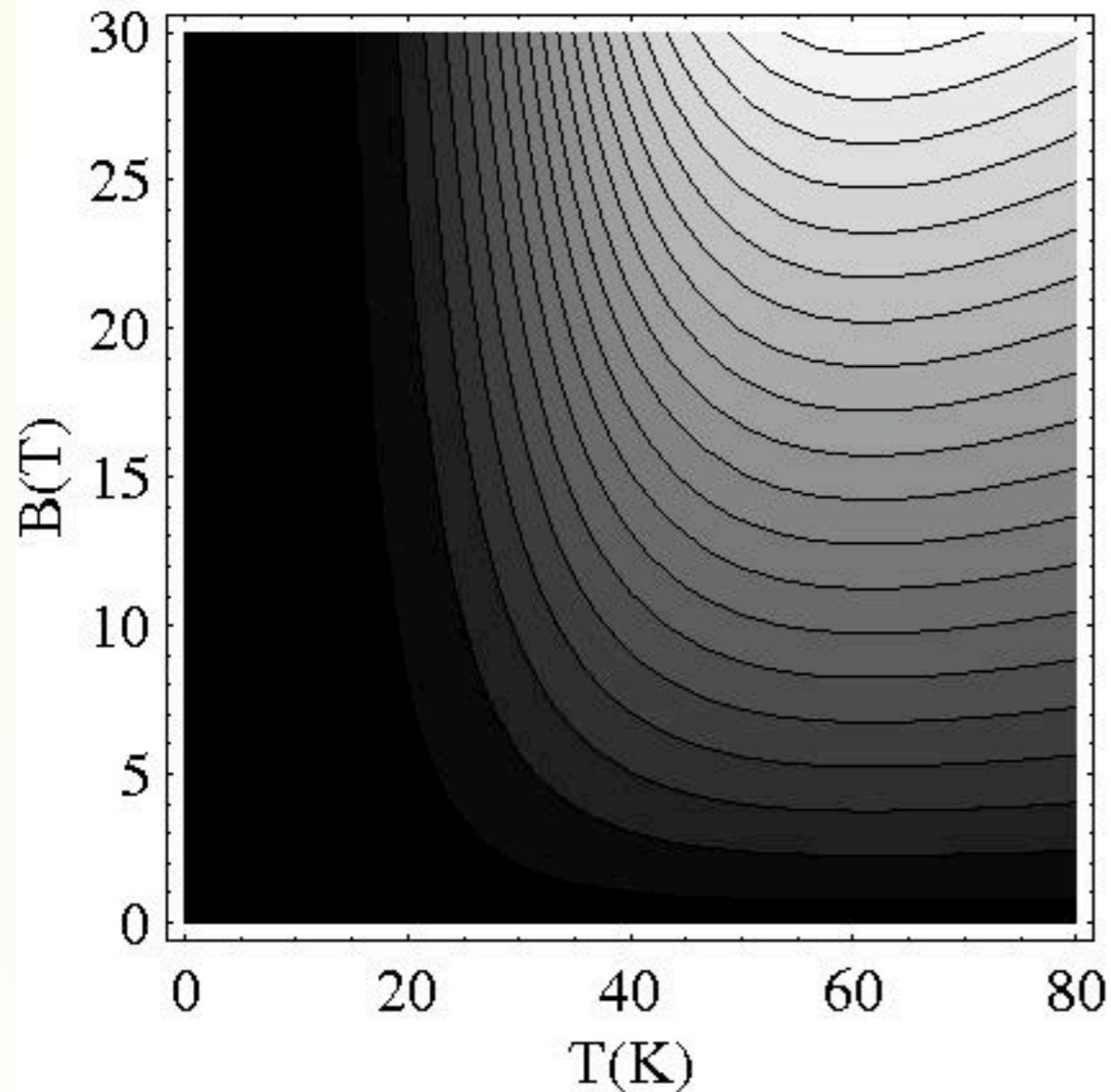
Nernst signal (transverse thermoelectric response)

$$e_N = \left(\frac{k_B}{e^*} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right) \left[\frac{\omega_c / \tau_{\text{imp}}}{(\omega_c^2 / \gamma + 1 / \tau_{\text{imp}})^2 + \omega_c^2} \right]$$

where τ_{imp} is the momentum relaxation time due to impurities or umklapp scattering.

LSCO Experiments

Theory for N



Y. Wang, L. Li, and N. P. Ong, *Phys. Rev. B* **73**, 024510 (2006).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev. B* **76** 144502 (2007)

B and T dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, τ_{imp} and ν .

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Similar results apply to electronic transport in graphene, where the relativistic Dirac spectrum of the electrons leads to analogies with the hydrodynamics of CFTs. We have made specific quantitative predictions for THz experiments on graphene at room temperature in the presence of a modest applied magnetic field.

Applications:

1. Magneto-thermo-electric transport near the superconductor-insulator transition and in graphene

Hydrodynamic cyclotron resonance
Nernst effect

2. Quark-gluon plasma

Low viscosity fluid

3. Fermi gas at unitarity

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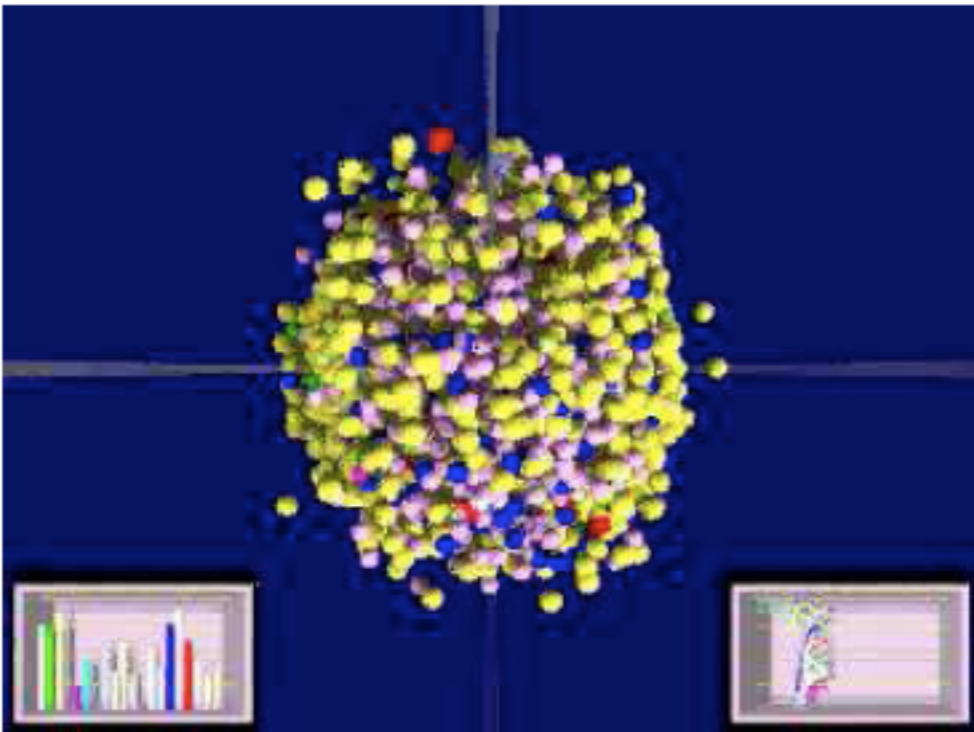
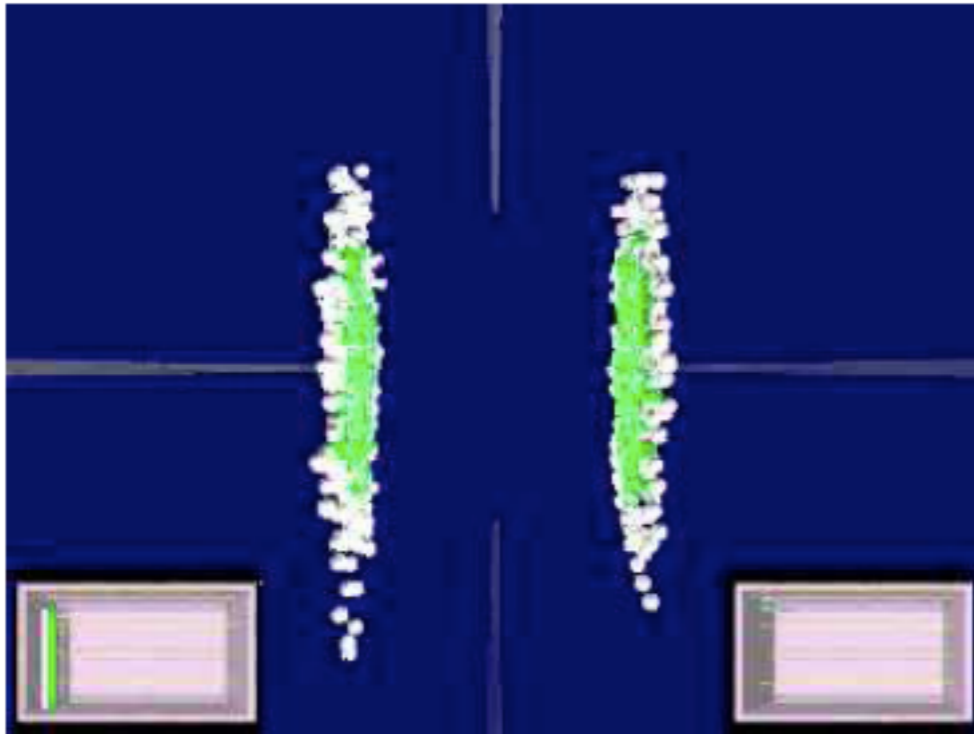
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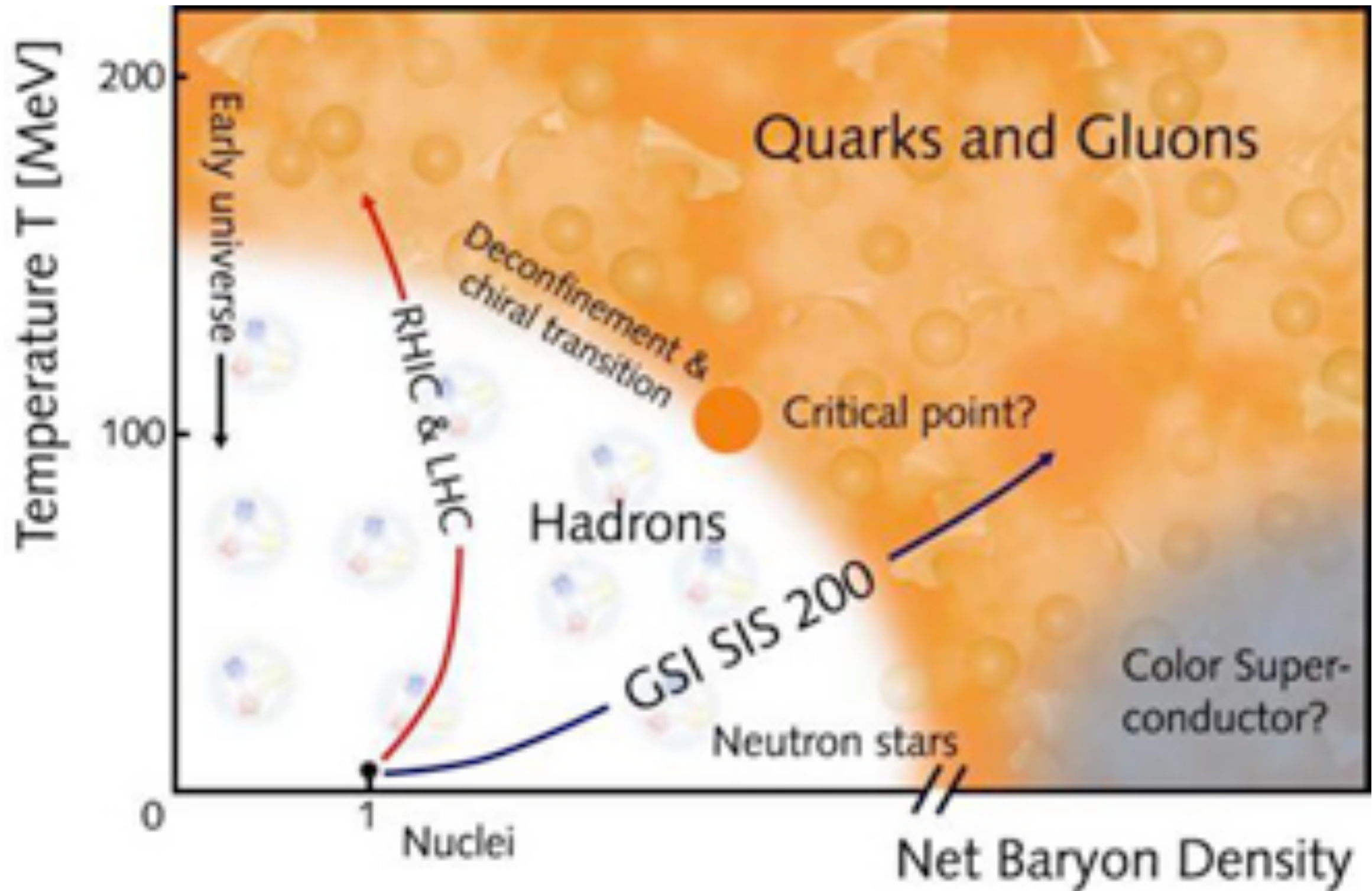
Non-relativistic AdS/CFT

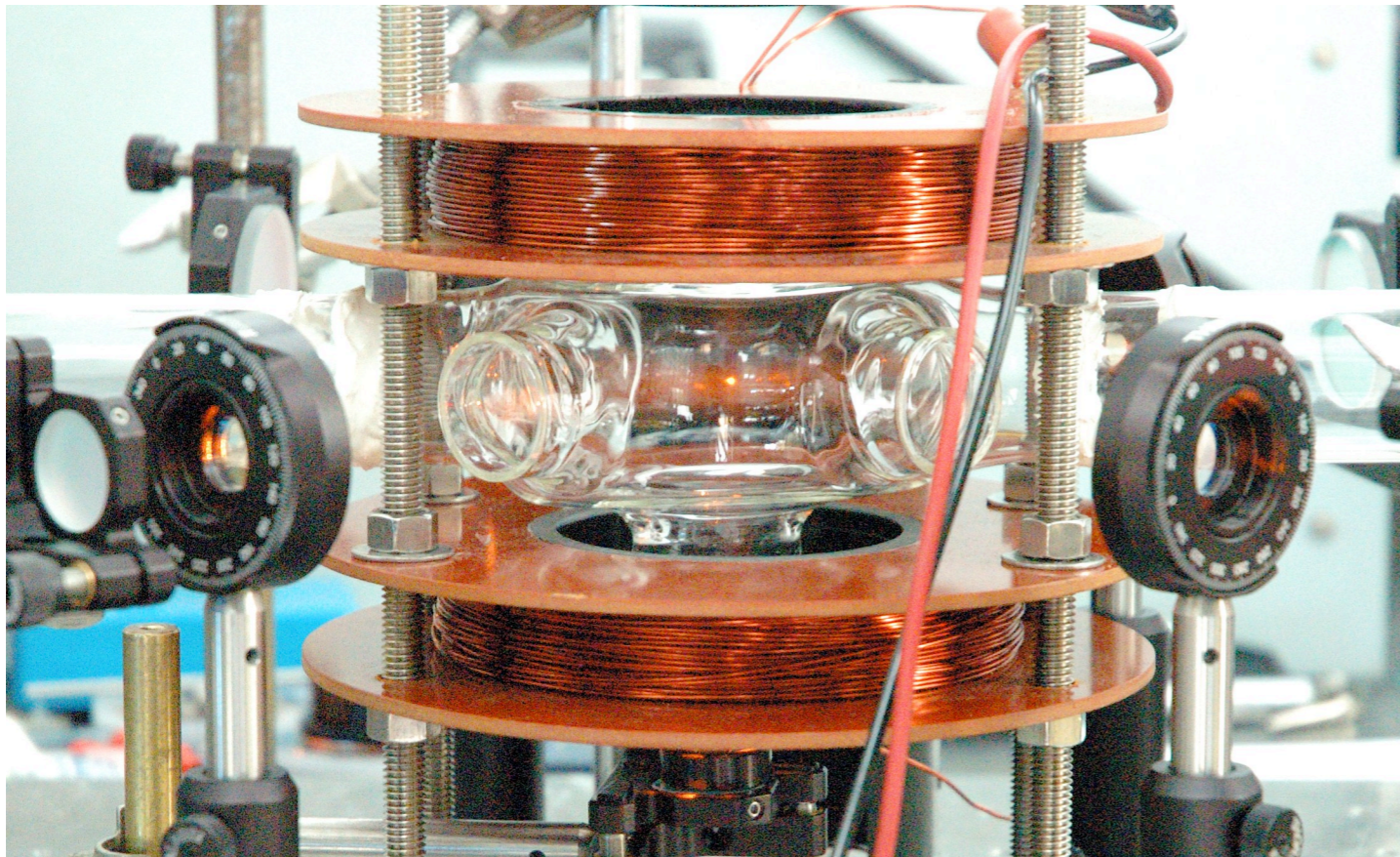
Au+Au collisions at RHIC



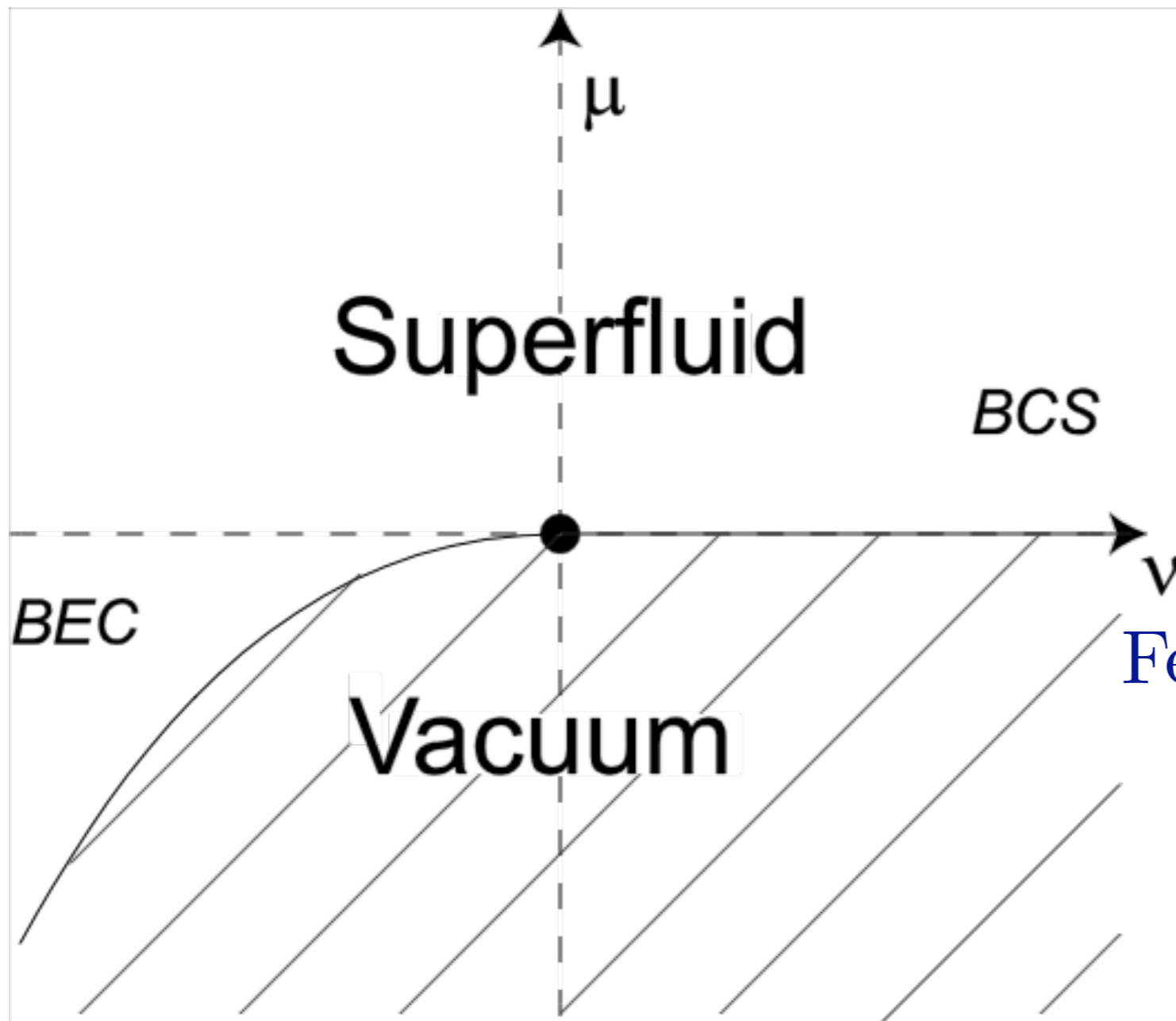
Quark-gluon plasma can be described as “quantum critical QCD”

Phases of nuclear matter

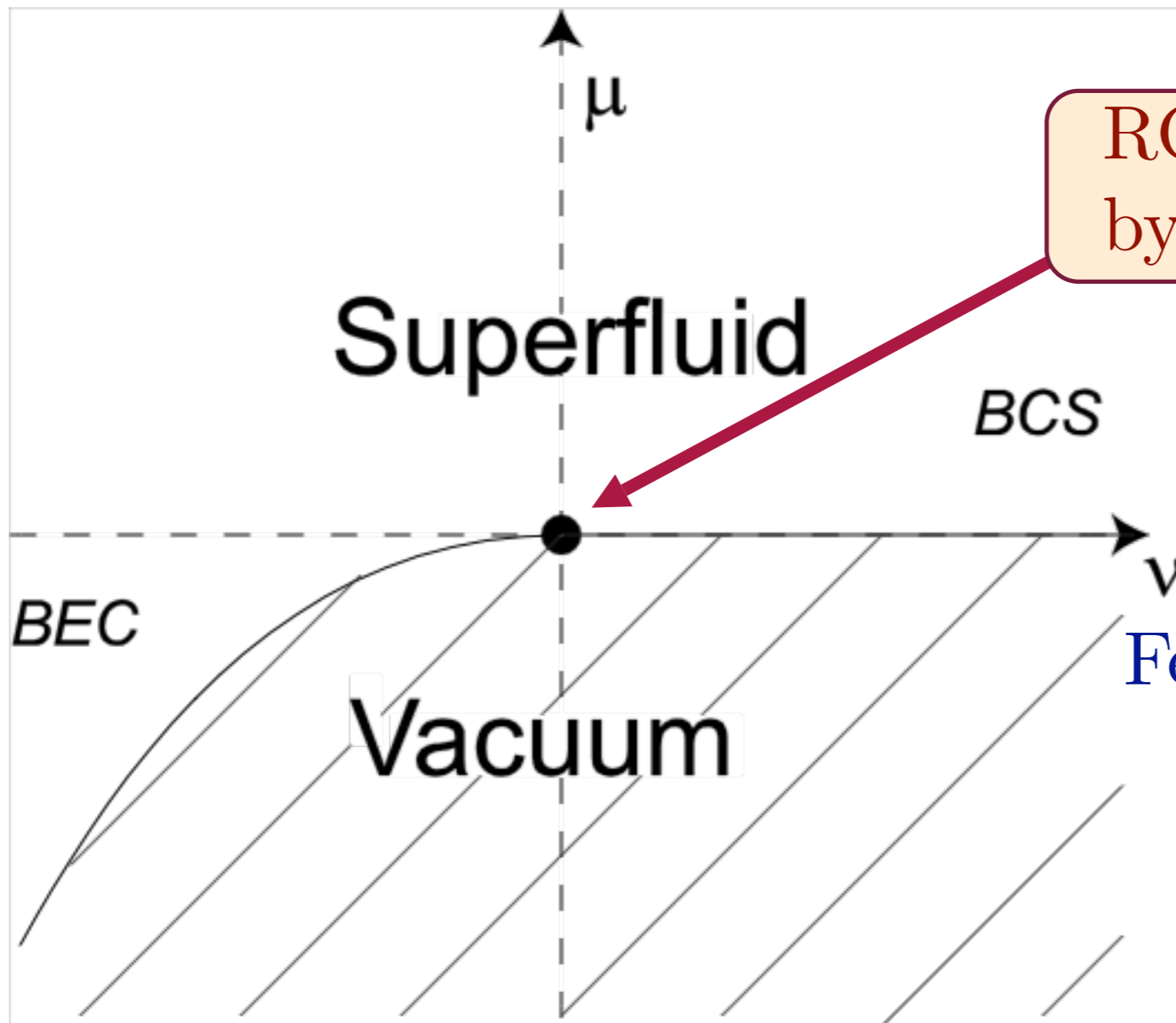




**$S=1/2$ Fermi gas
at a Feshbach
resonance**

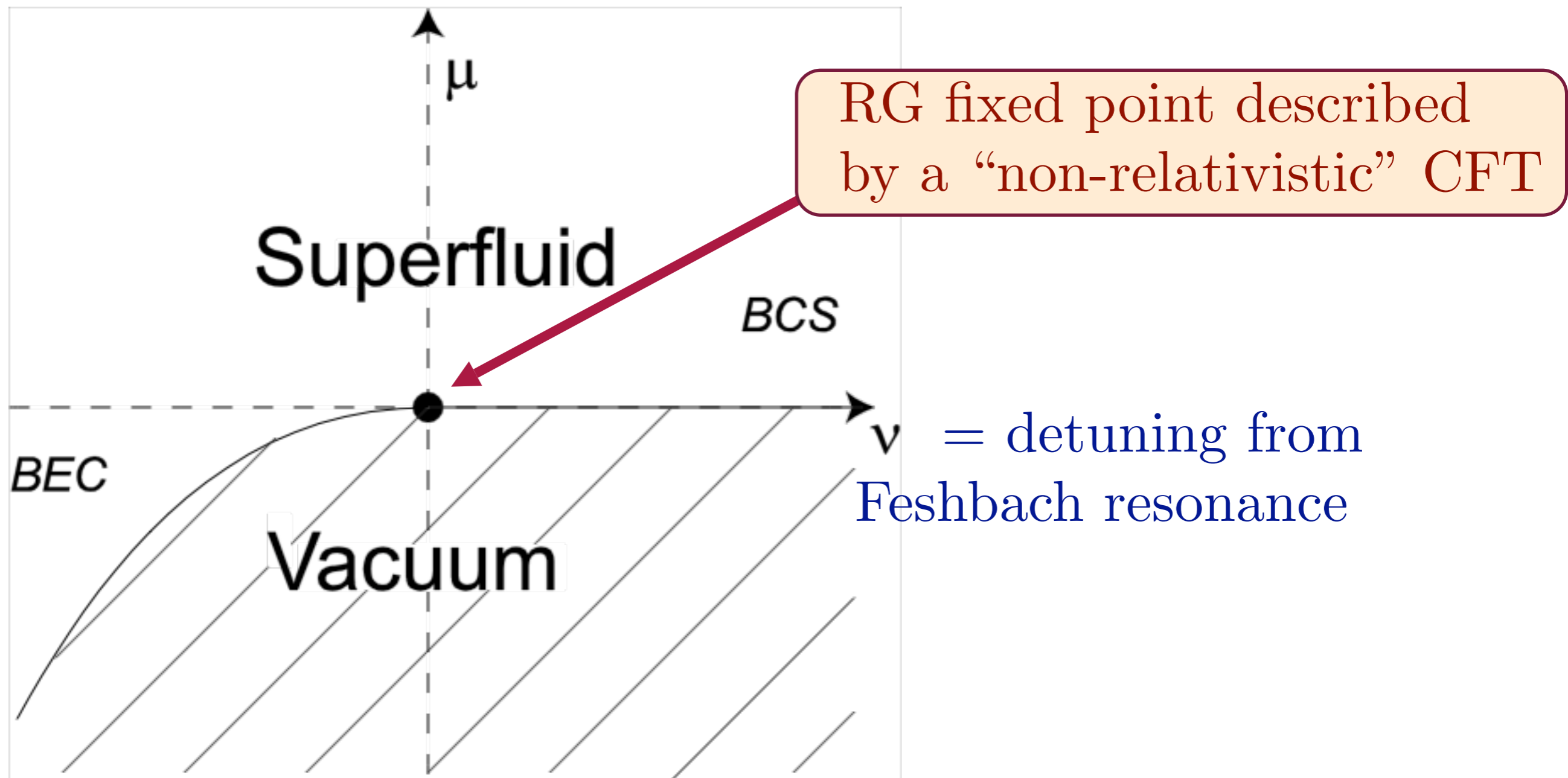


ν = detuning from
Feshbach resonance



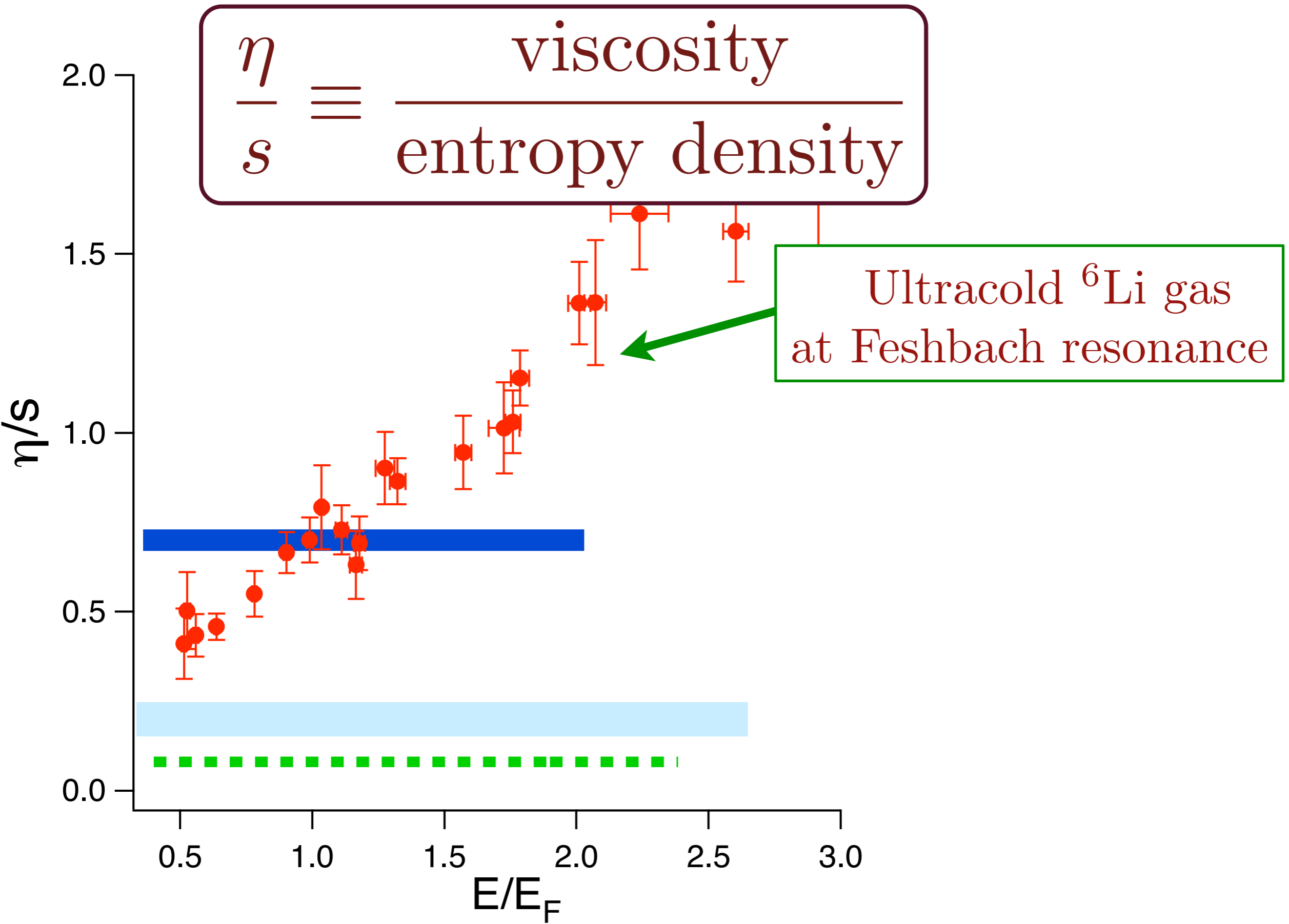
RG fixed point described by a “non-relativistic” CFT

ν = detuning from Feshbach resonance



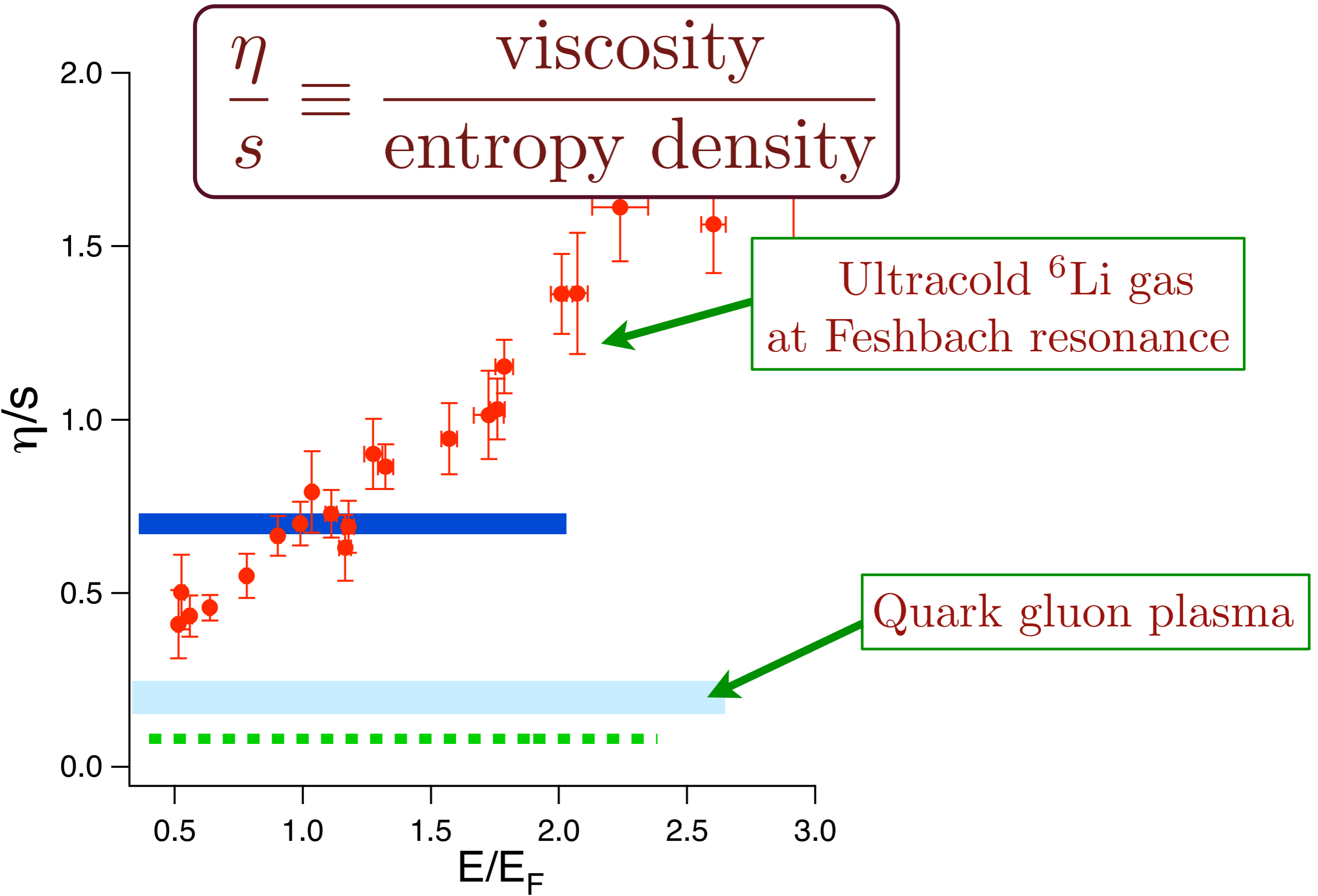
CFT is dual to quantum gravity models on AdS space. Explicit solutions of such gravity models with supersymmetry have been obtained

P. Nikolic and S. Sachdev, *Phys. Rev. A* **75**, 033608 (2007); D. T. Son, arXiv:0804.3972; K. Balasubramanian and J. McGreevy, arXiv:0804.4053; W. D. Goldberger, arXiv:0806.2867; J. L. F. Barbón and C. A. Fuertes, arXiv:0806.3244; J. Maldacena, D. Martelli, and Y. Tachikawa, arXiv:0807.1100; A. Adams, K. Balasubramanian, and J. McGreevy, arXiv:0807.1111.



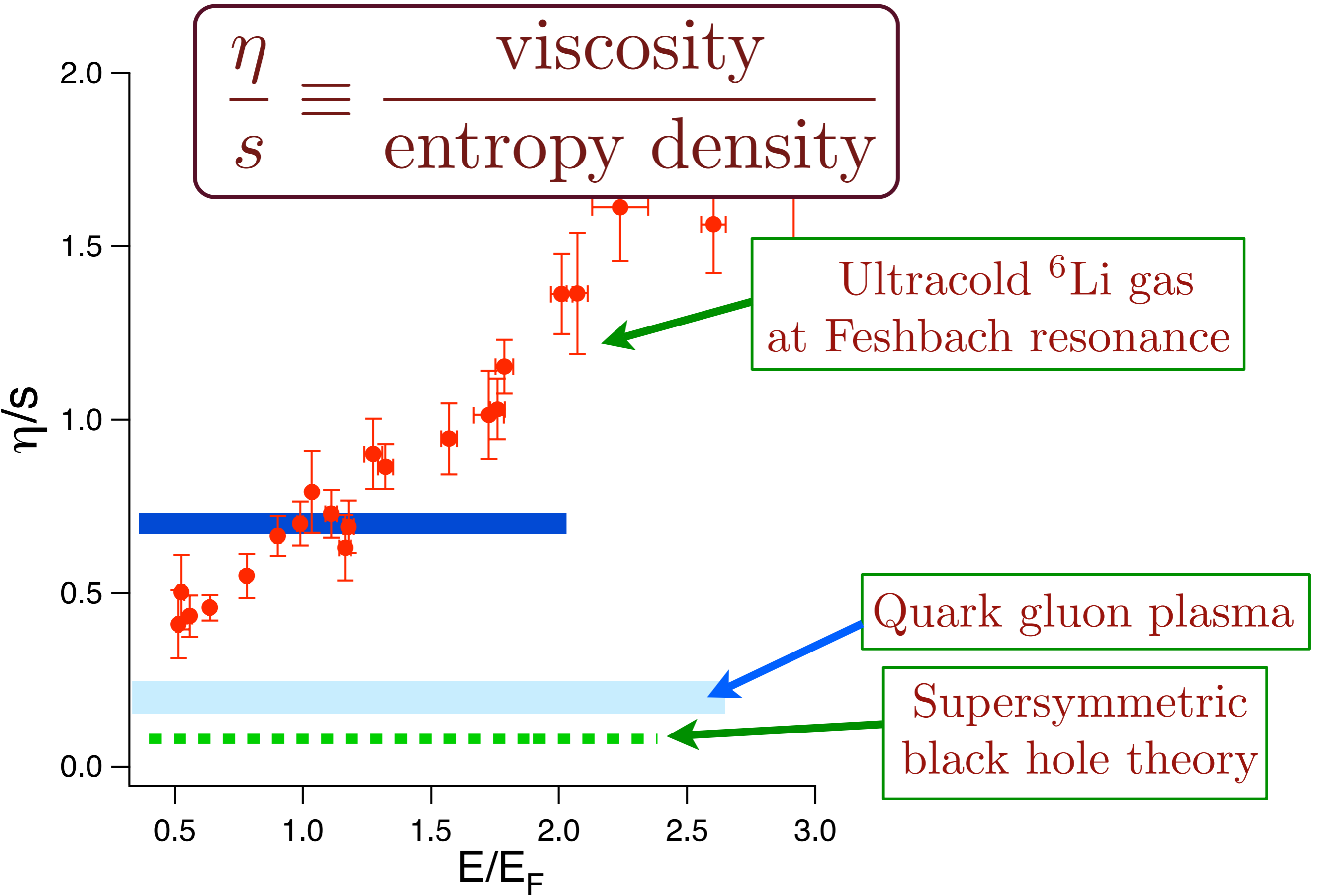
T. Schafer, *Phys. Rev.A* **76**, 063618 (2007).

A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, *J. Low Temp. Physics* **150**, 567 (2008)



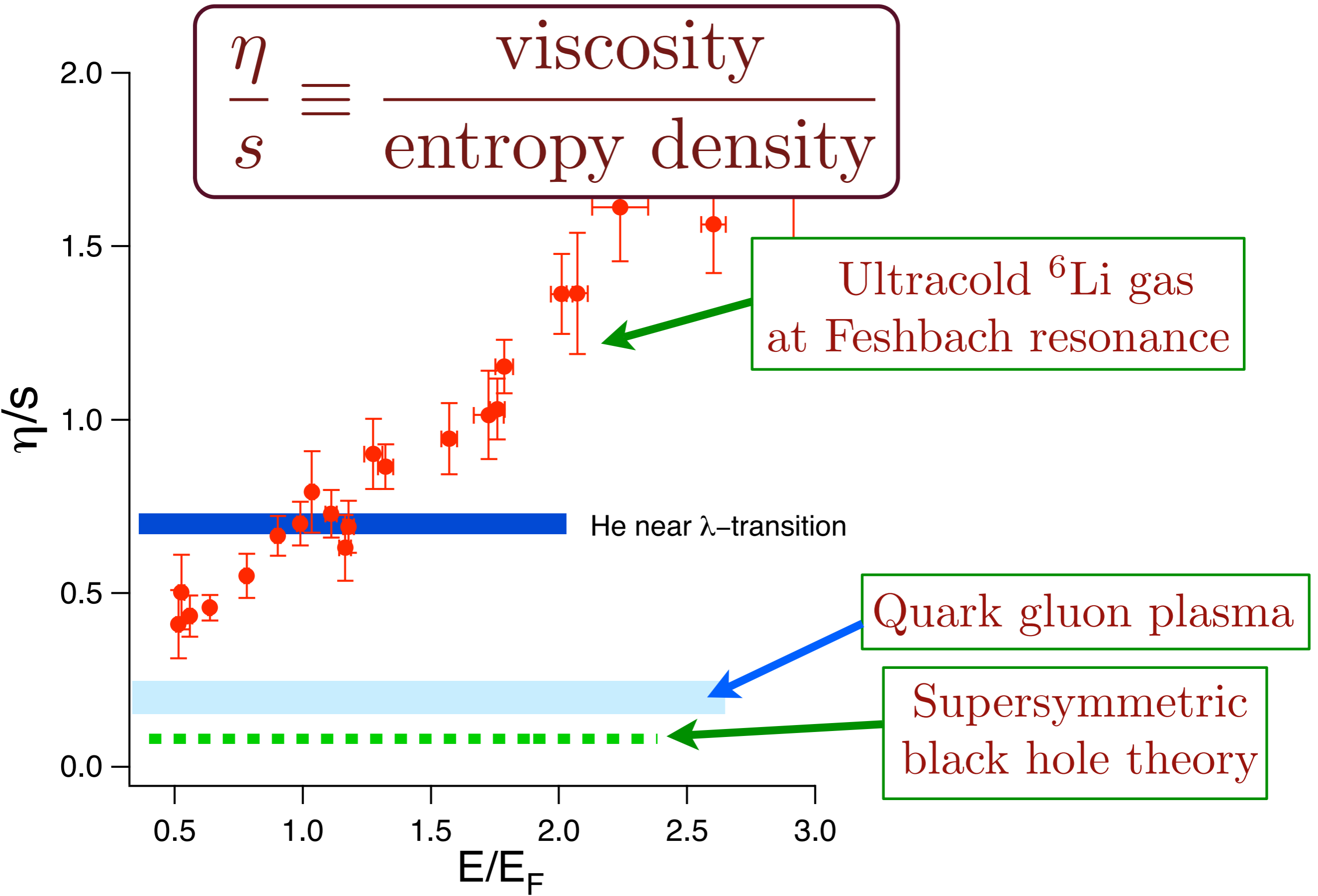
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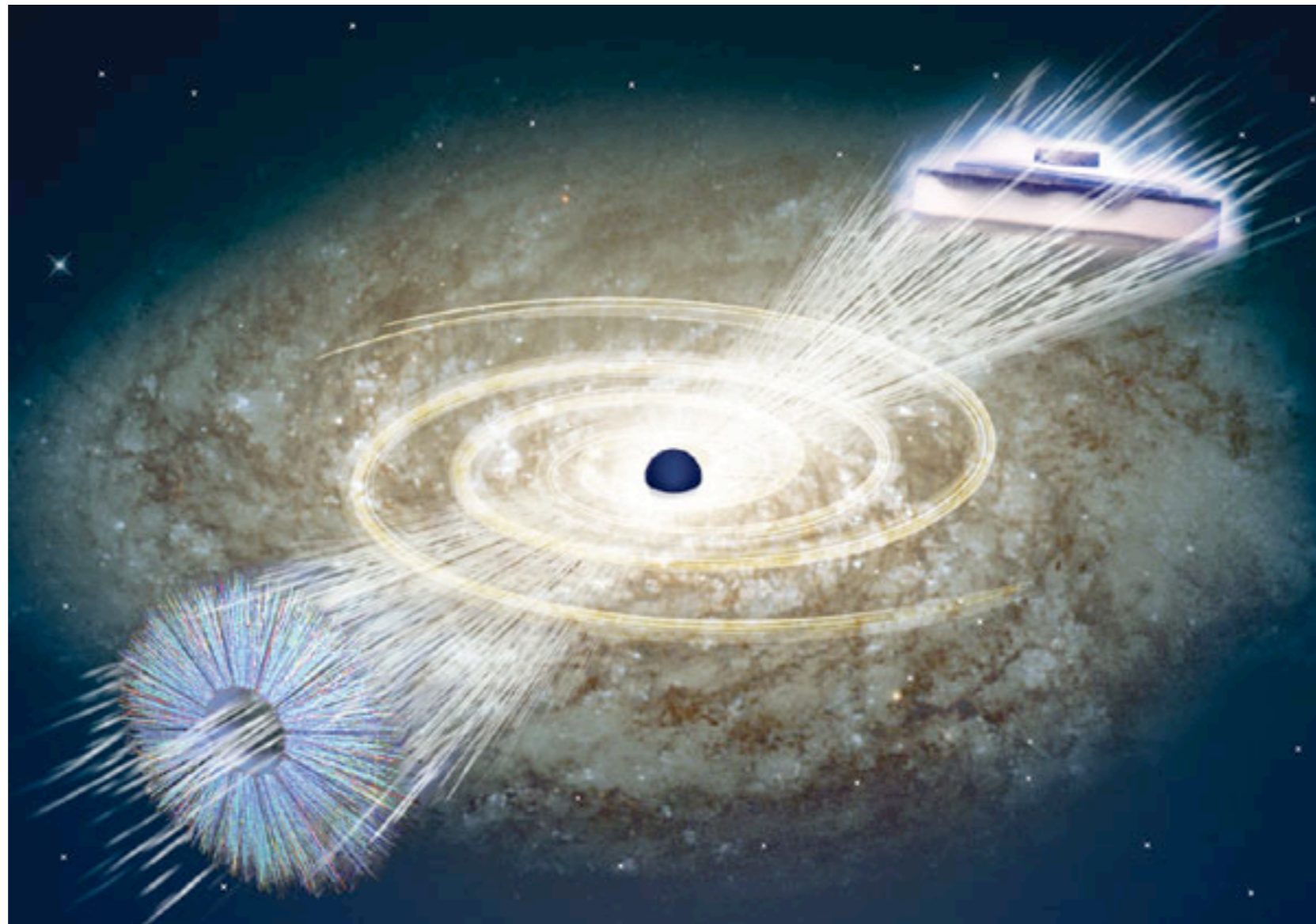
A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, *J. Low Temp. Physics* **150**, 567 (2008)

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.