

# Quantum entanglement and the phases of matter

University of North Carolina, Chapel Hill  
April 18, 2016

Subir Sachdev

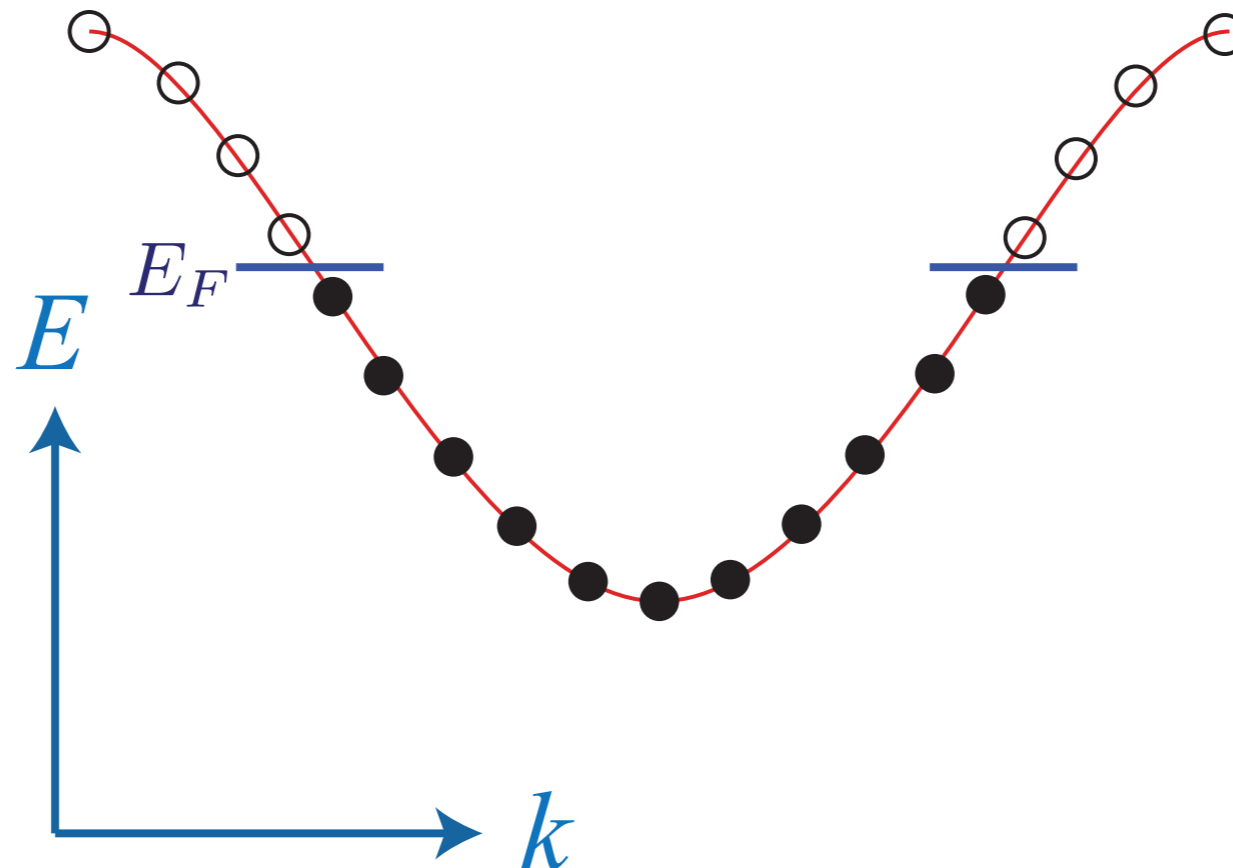
Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



# Foundations of quantum many body theory:

## I. Ground states connected adiabatically to independent electron states

### Metals

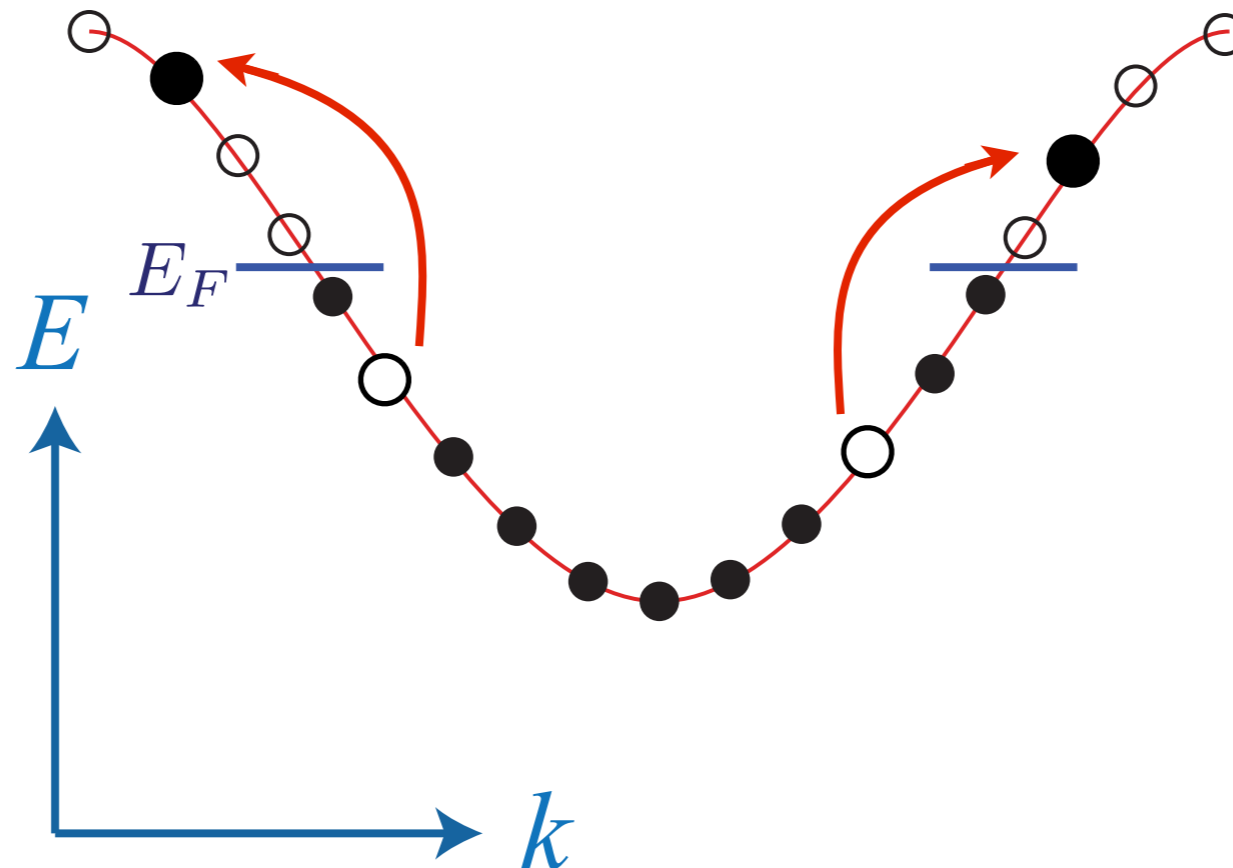


# Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

## Metals

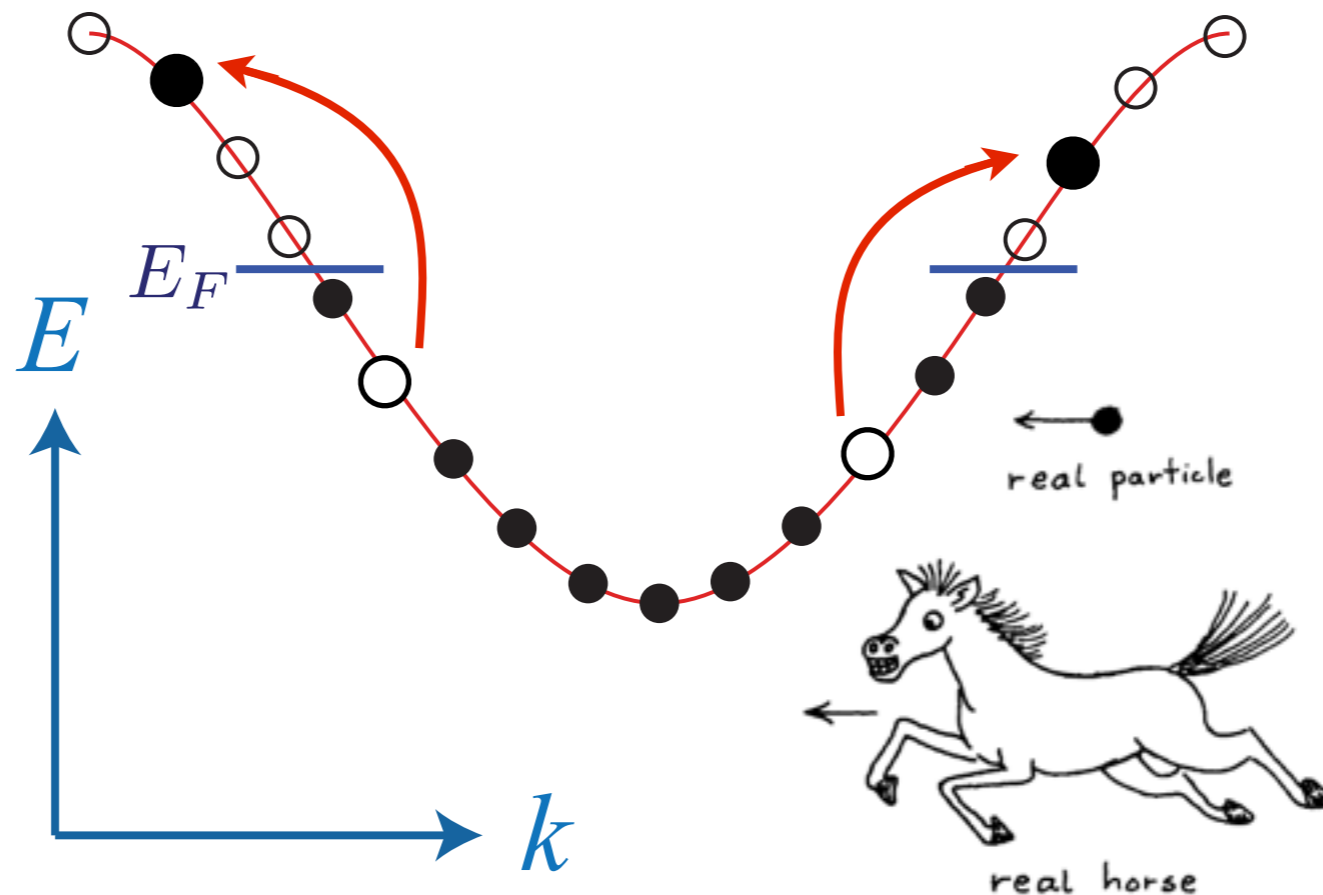


# Foundations of quantum many body theory:

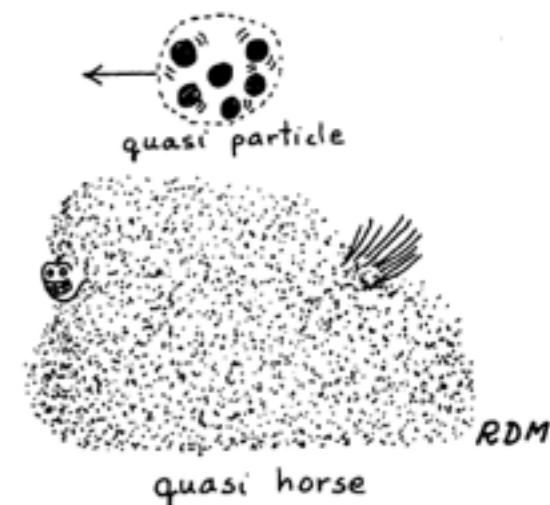
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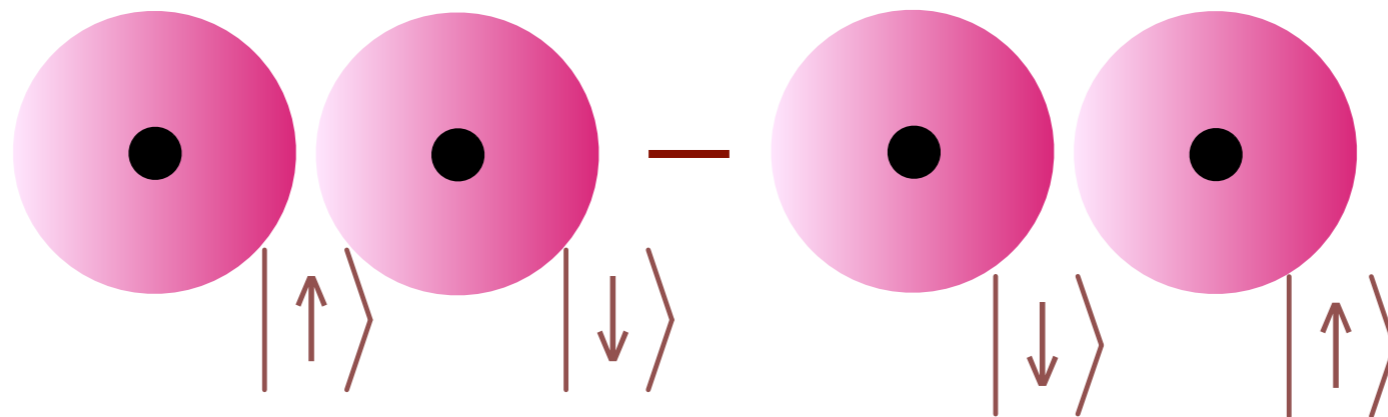


A Guide to Feynman Diagrams in the Many-body Problem, R.D. Mattuck, Dover (1992)



## Quantum entanglement:

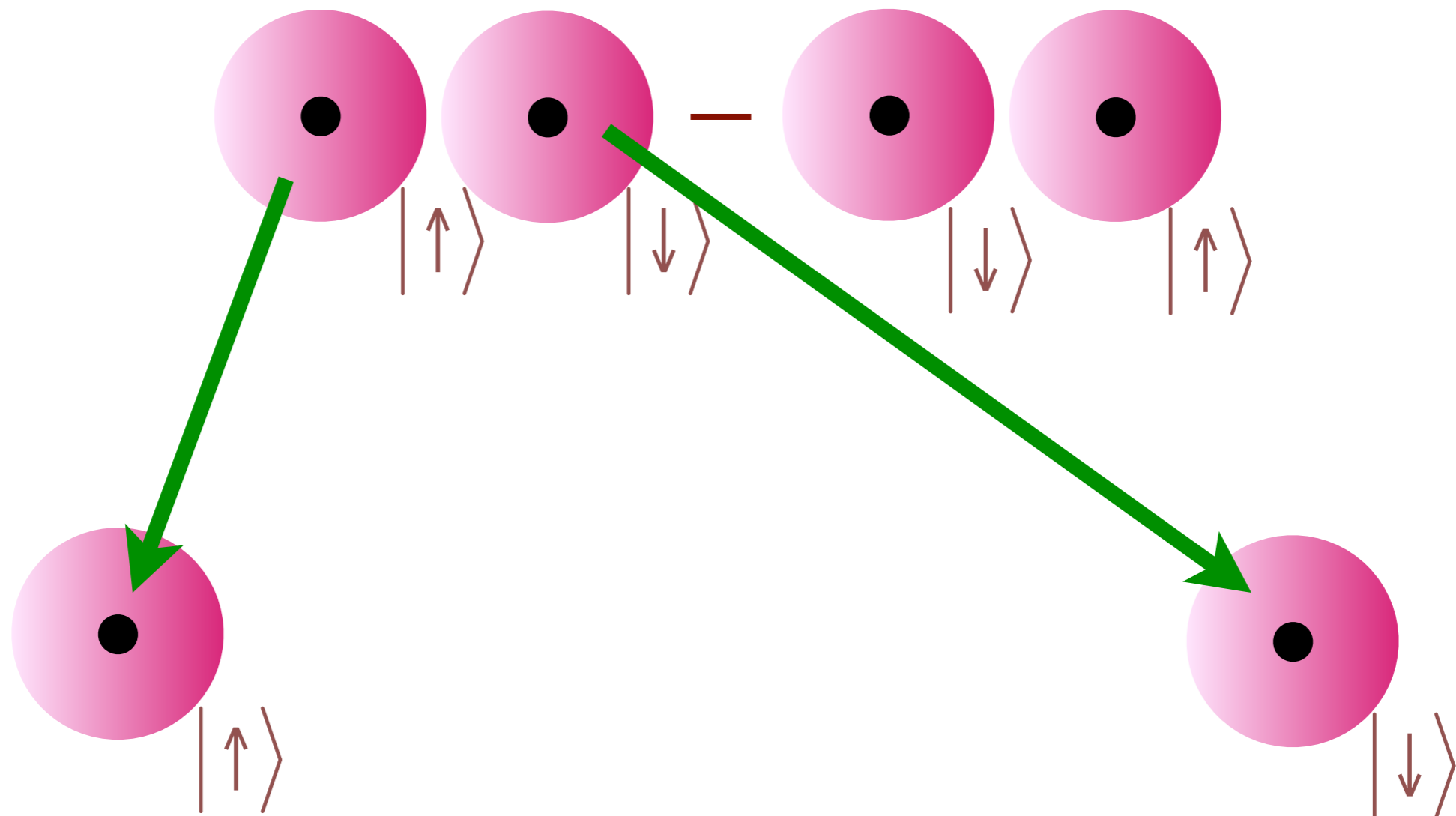
*EPR pair: Non-local correlations between quantum measurements due to superposition between many-electron states*



Hydrogen molecule

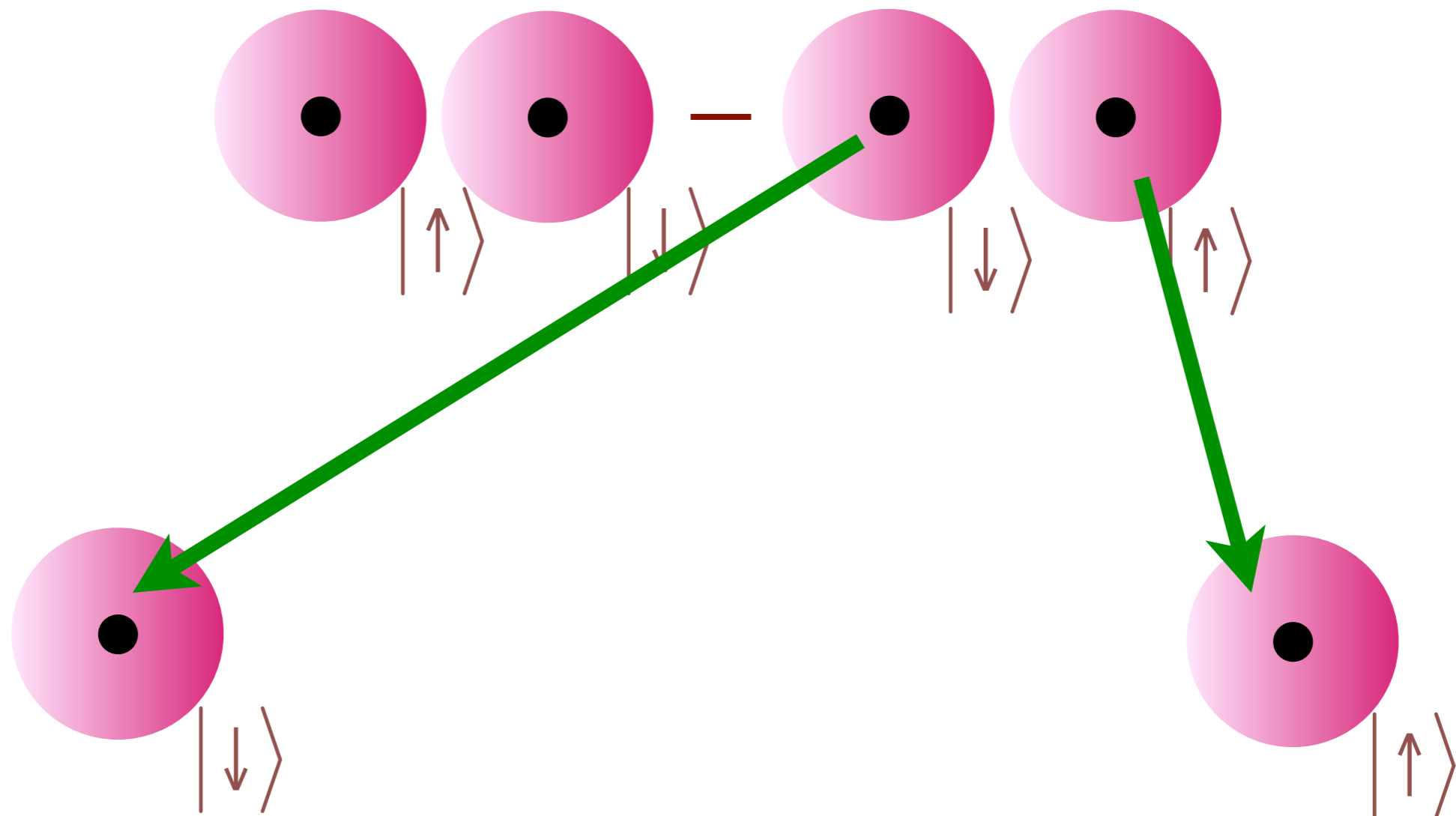
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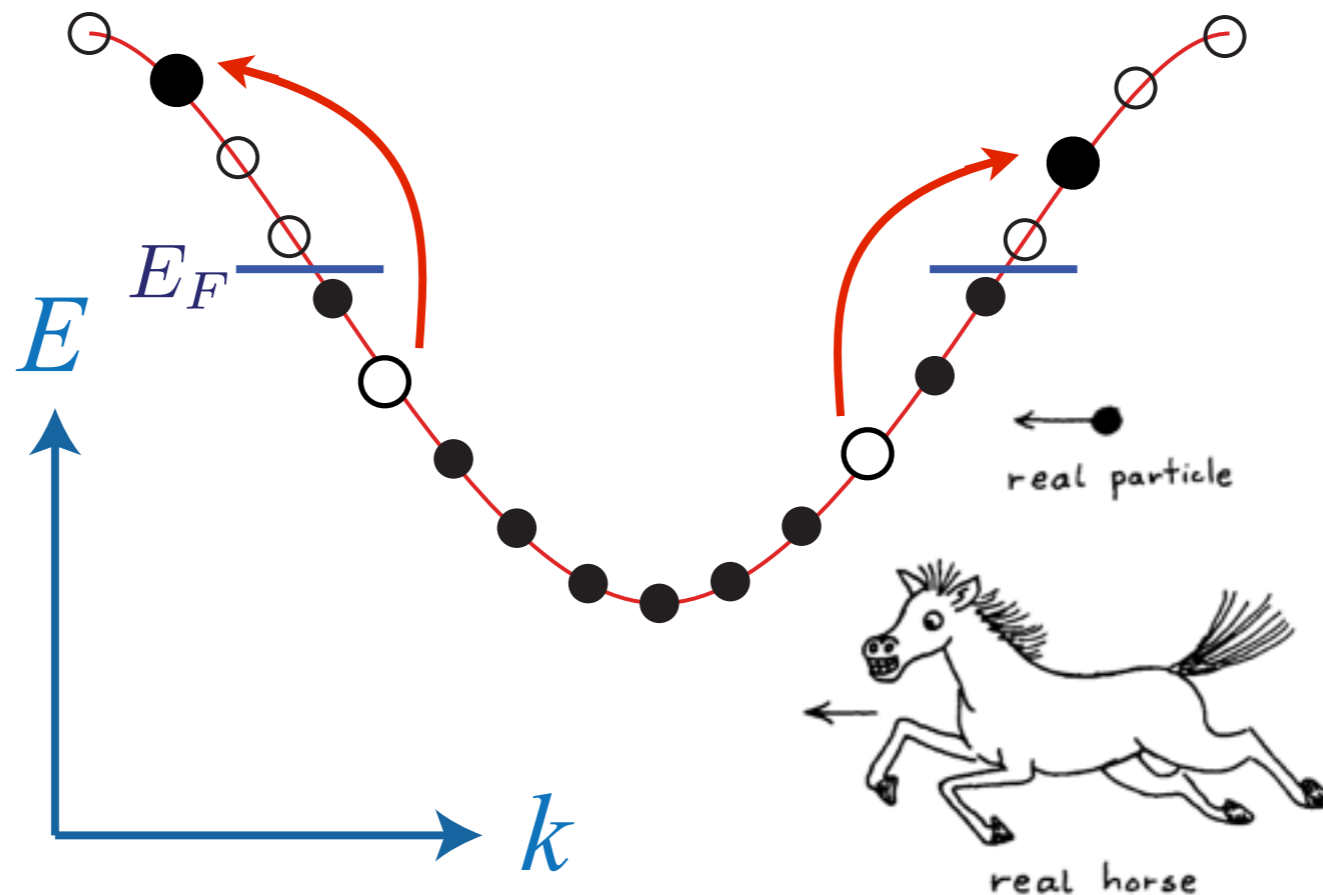


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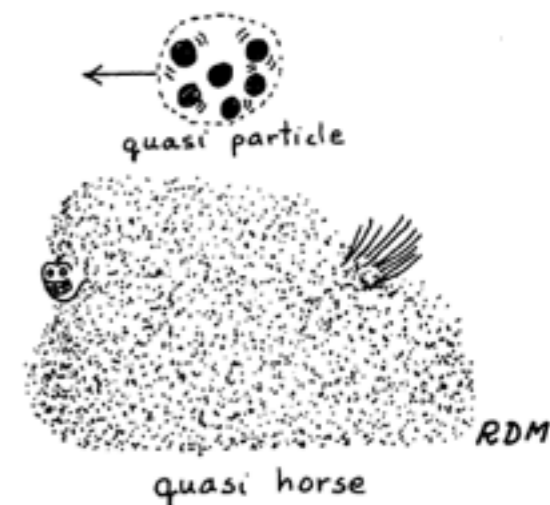
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## Metals



A Guide to Feynman Diagrams in the Many-body Problem, R.D. Mattuck, Dover (1992)



## Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. Boltzmann-Landau theory of quasiparticles

### Famous example:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. No quasiparticles

## Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

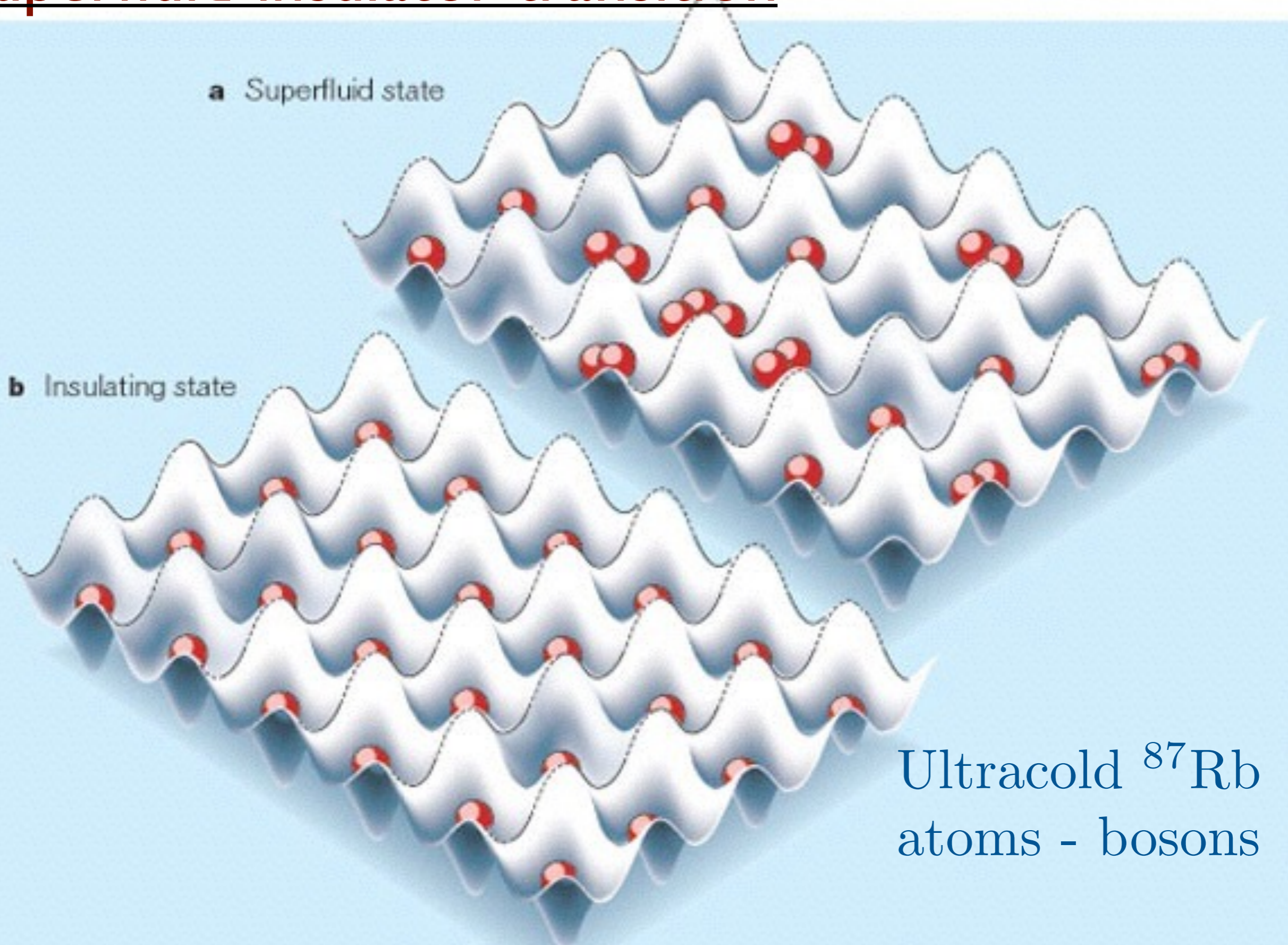
### 2. No quasiparticles

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- *Charged black hole horizons in anti-de Sitter space*

## Quantum matter without quasiparticles:

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# Superfluid-insulator transition



Ultracold  $^{87}\text{Rb}$   
atoms - bosons

$$\underline{U \gg t}$$



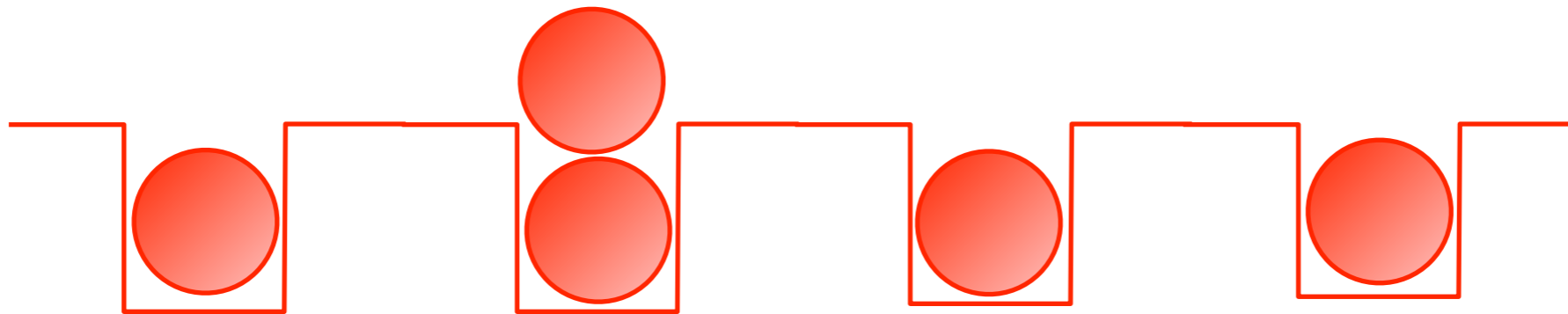
Insulator (the vacuum)  
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

On-site repulsion between bosons =  $U$   
Tunneling amplitude between sites =  $t$

$$\underline{U \gg t}$$

Excitations of the insulator:

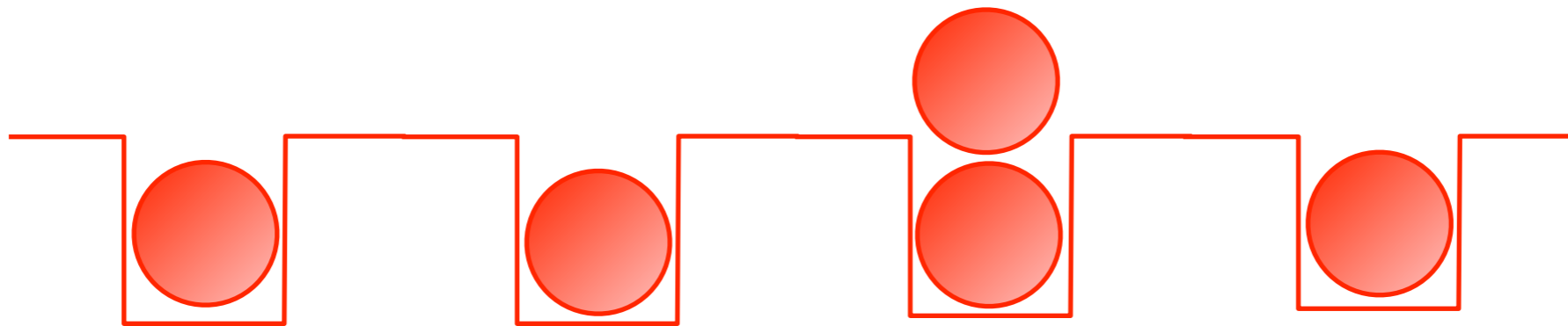


Particles  $\sim \psi^\dagger$

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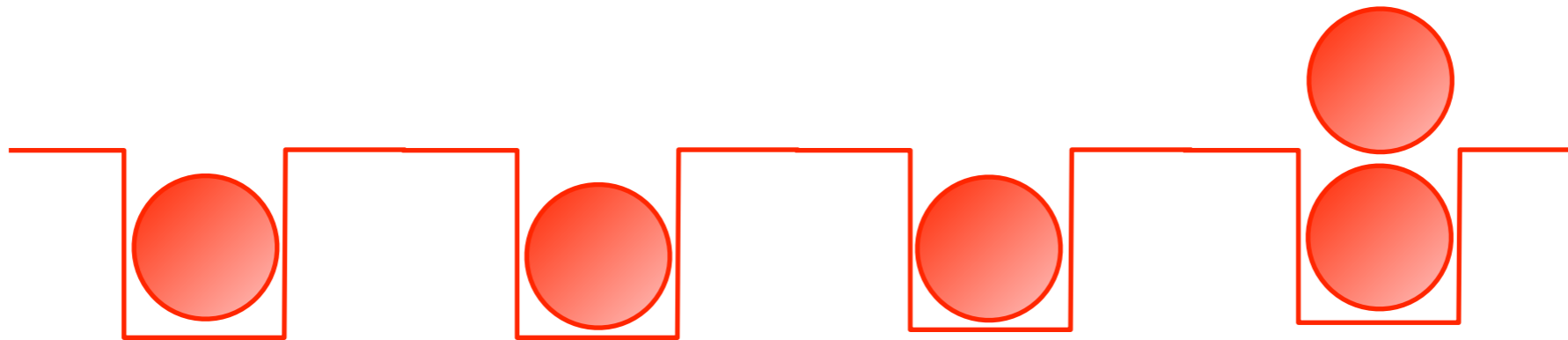


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Excitations of the insulator:



Holes  $\sim \psi$

On-site repulsion between bosons =  $U$   
Tunneling amplitude between sites =  $t$

$$\underline{U \gg t}$$

Excitations of the insulator:



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On-site repulsion between bosons =  $U$   
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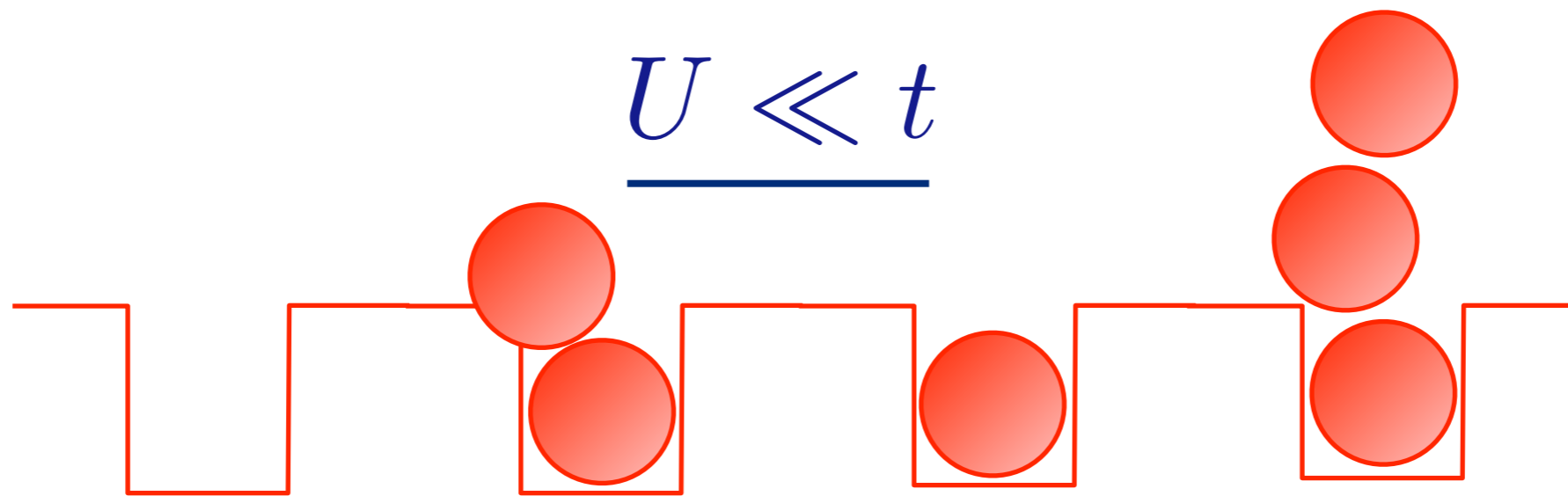
$$\underline{U \gg t}$$

Excitations of the insulator:



Holes  $\sim \psi$

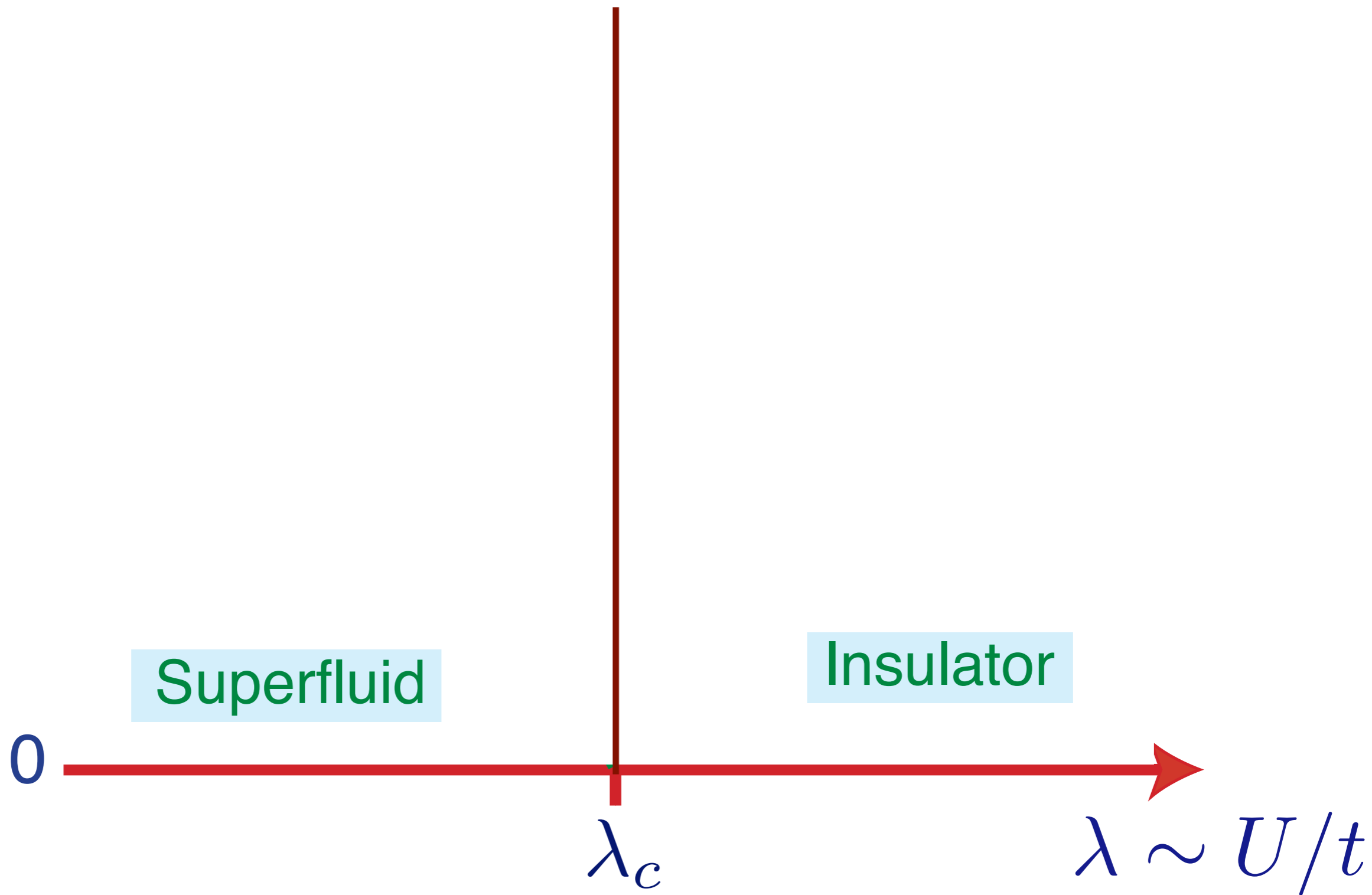
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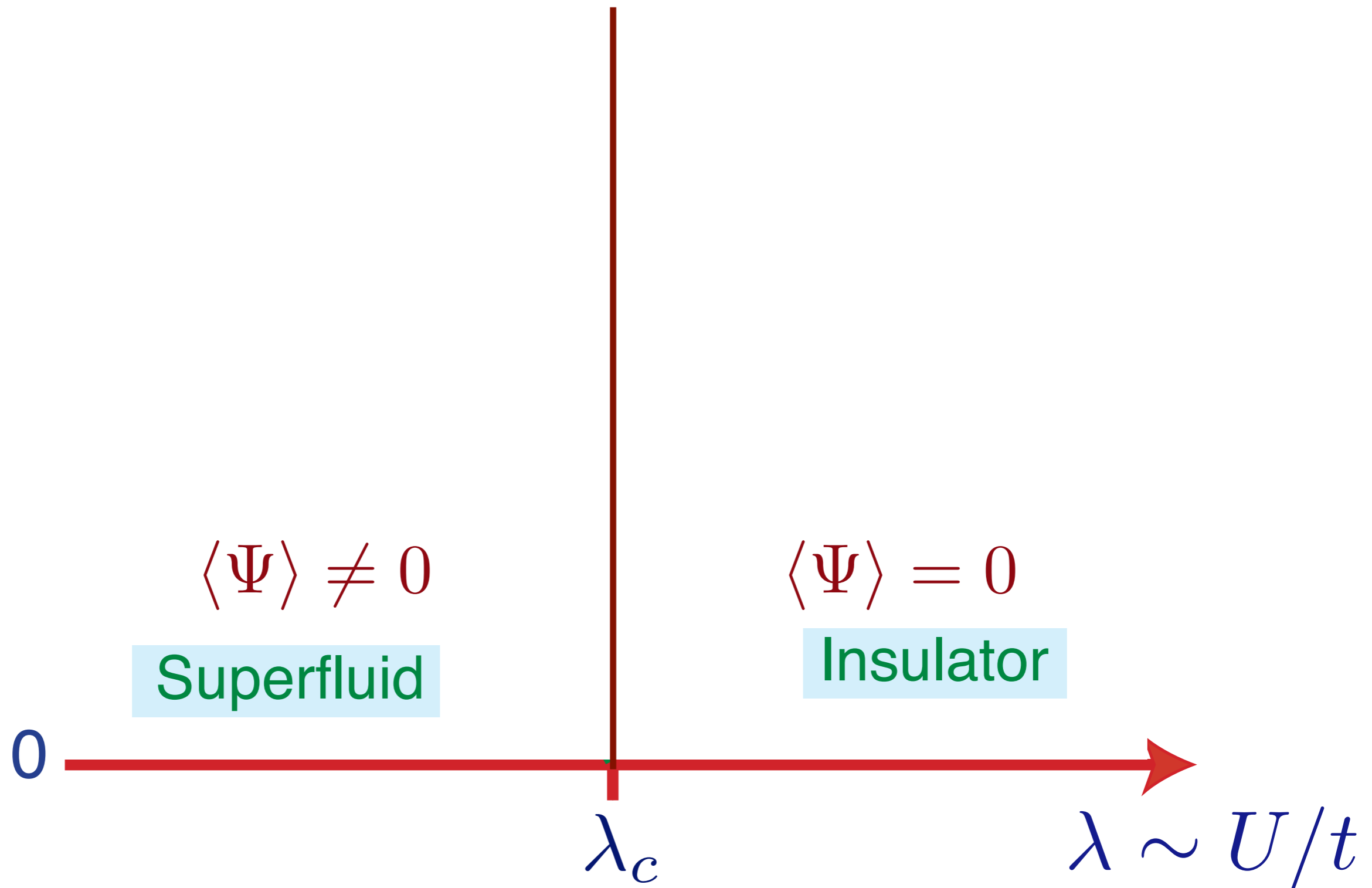
Superfluid  
at small repulsion between bosons

$$|\text{Ground state}\rangle = \left[ \sum_i b_i^\dagger \right]^N |0\rangle$$

On-site repulsion between bosons =  $U$   
Tunneling amplitude between sites =  $t$



$\Psi \rightarrow$  a complex field representing the Bose-Einstein condensate of the superfluid

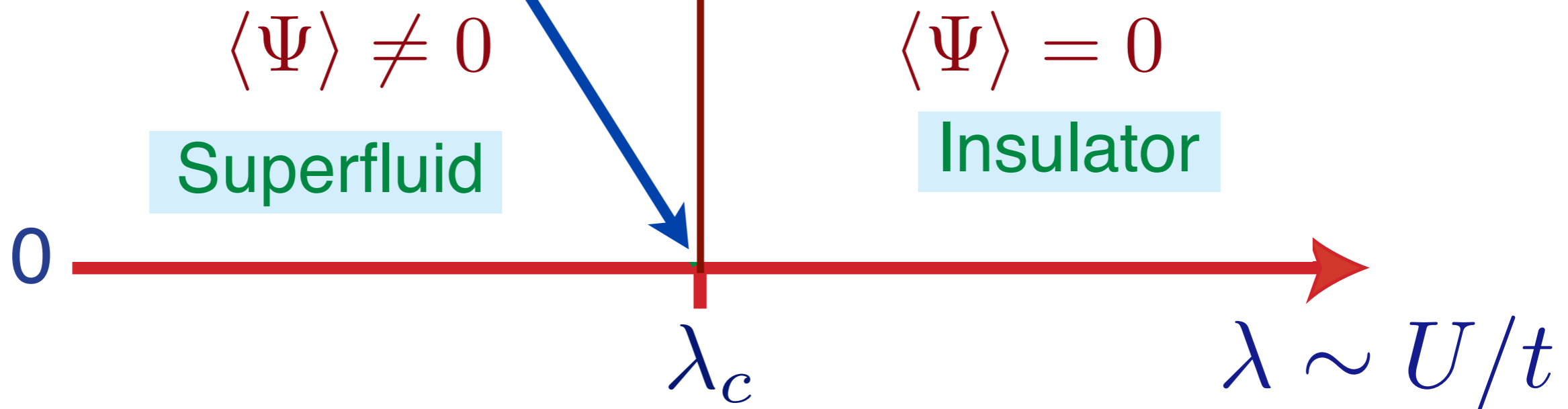


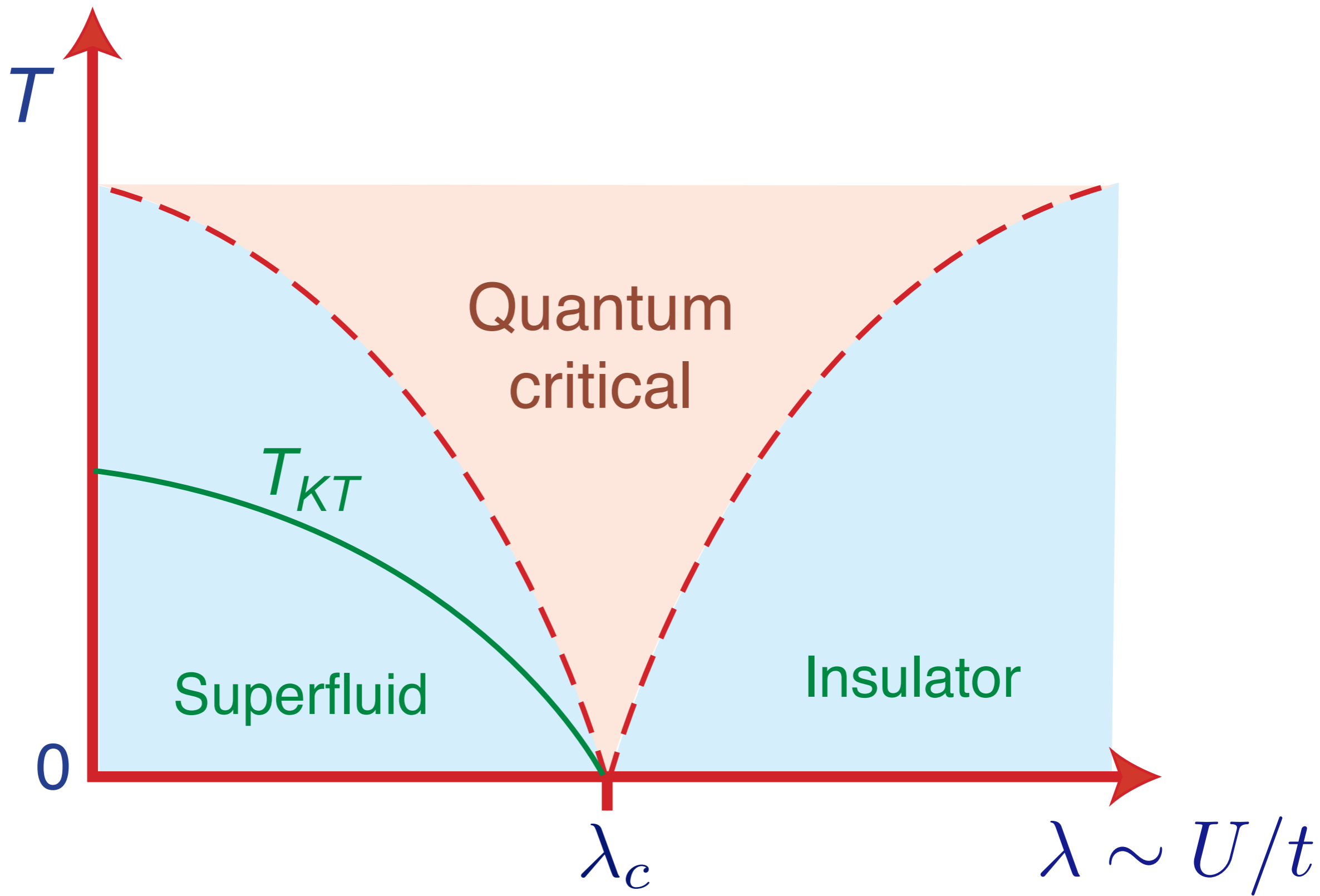
$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

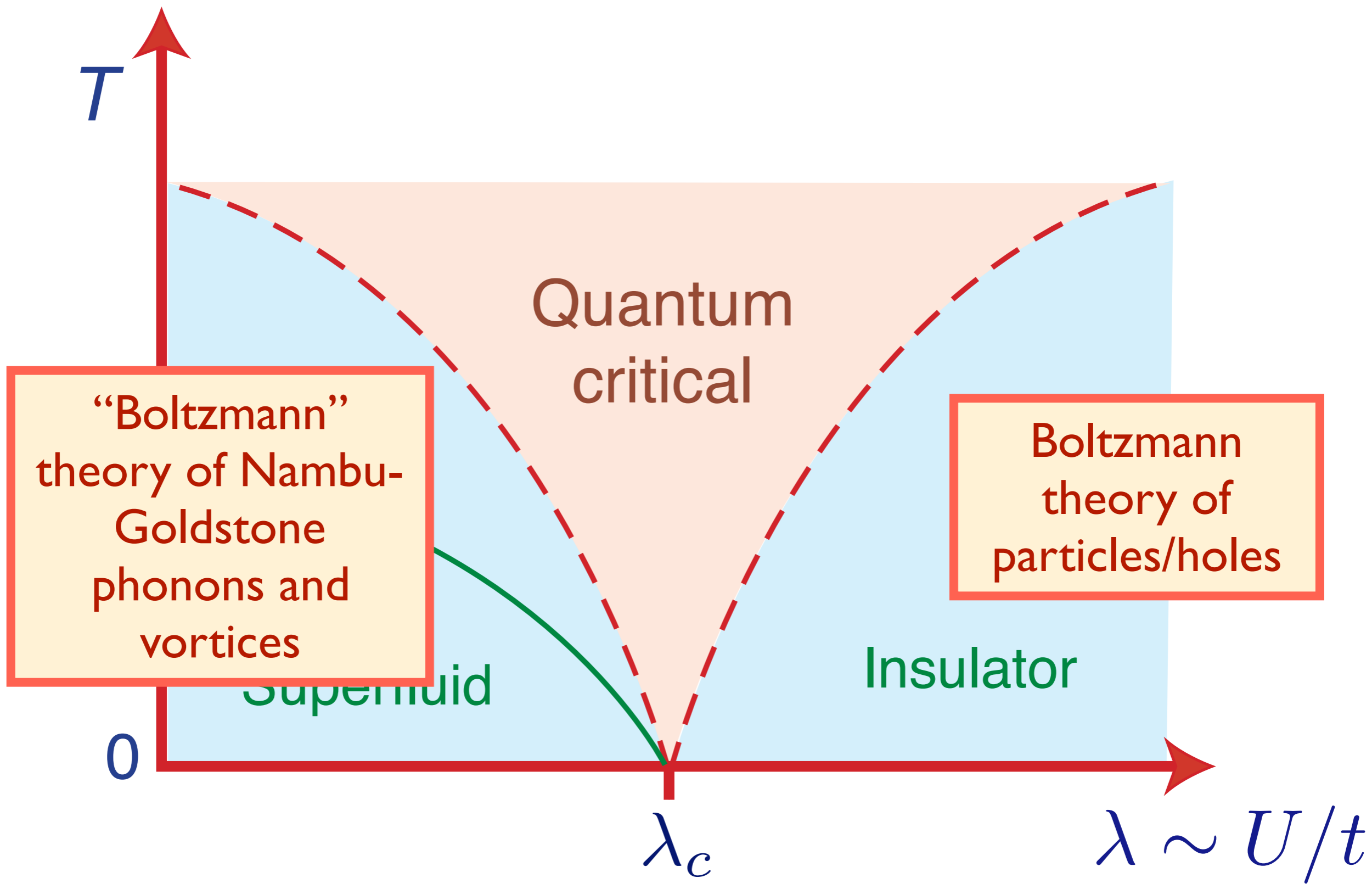
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

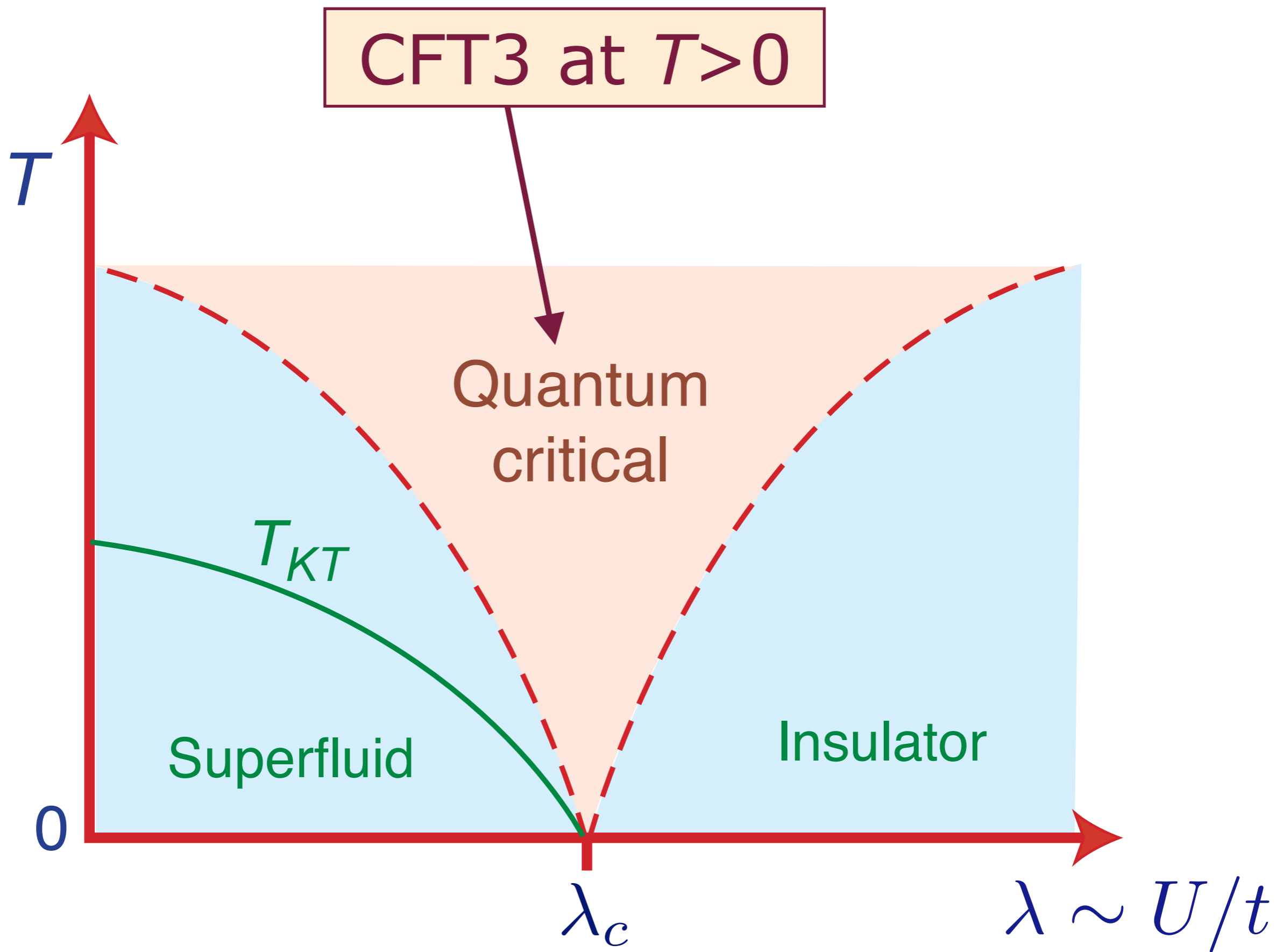
*Long-range quantum entanglement  
and no quasiparticles:*

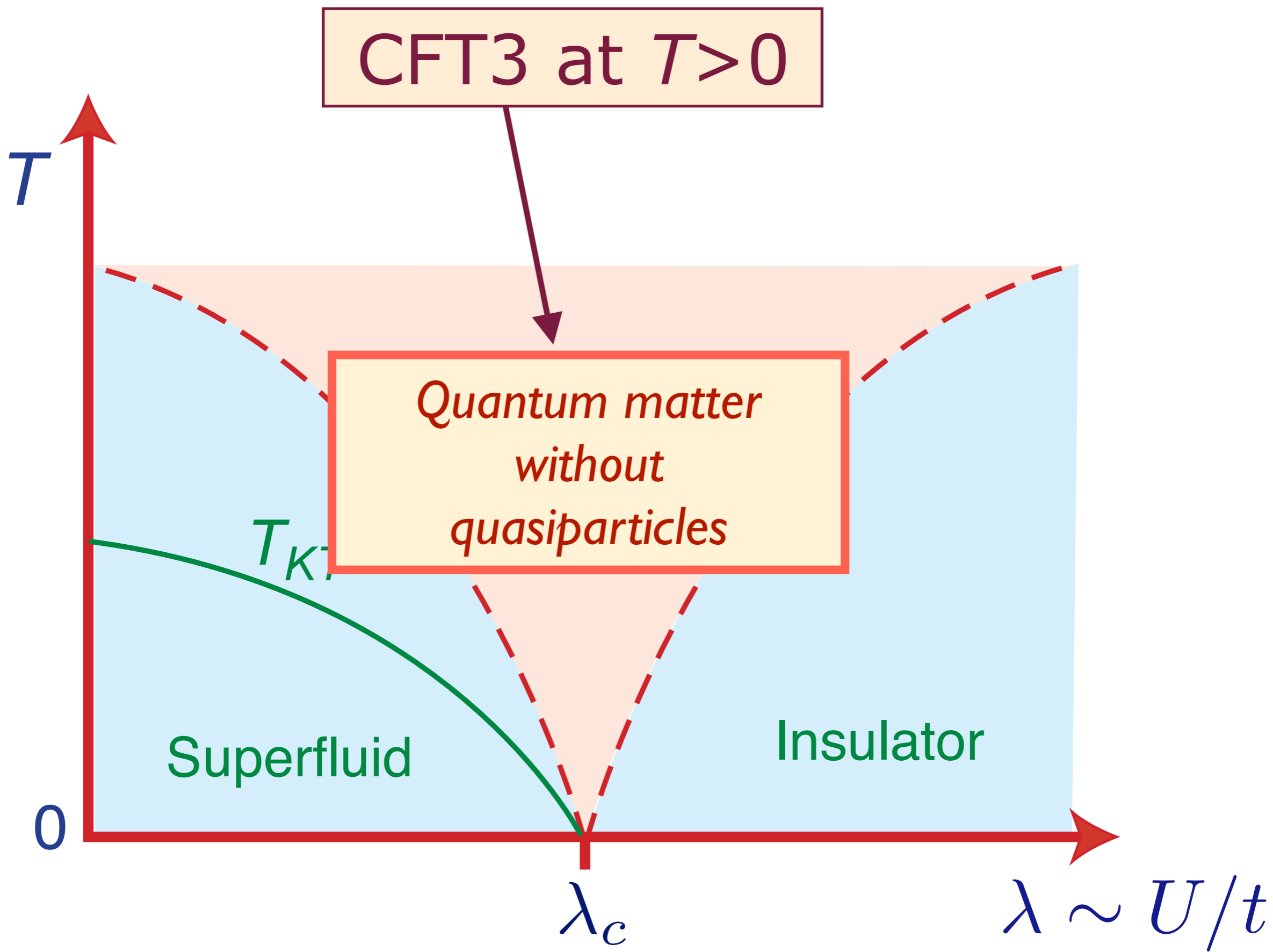
A conformal field theory  
in 2+1 spacetime dimensions:  
a CFT3

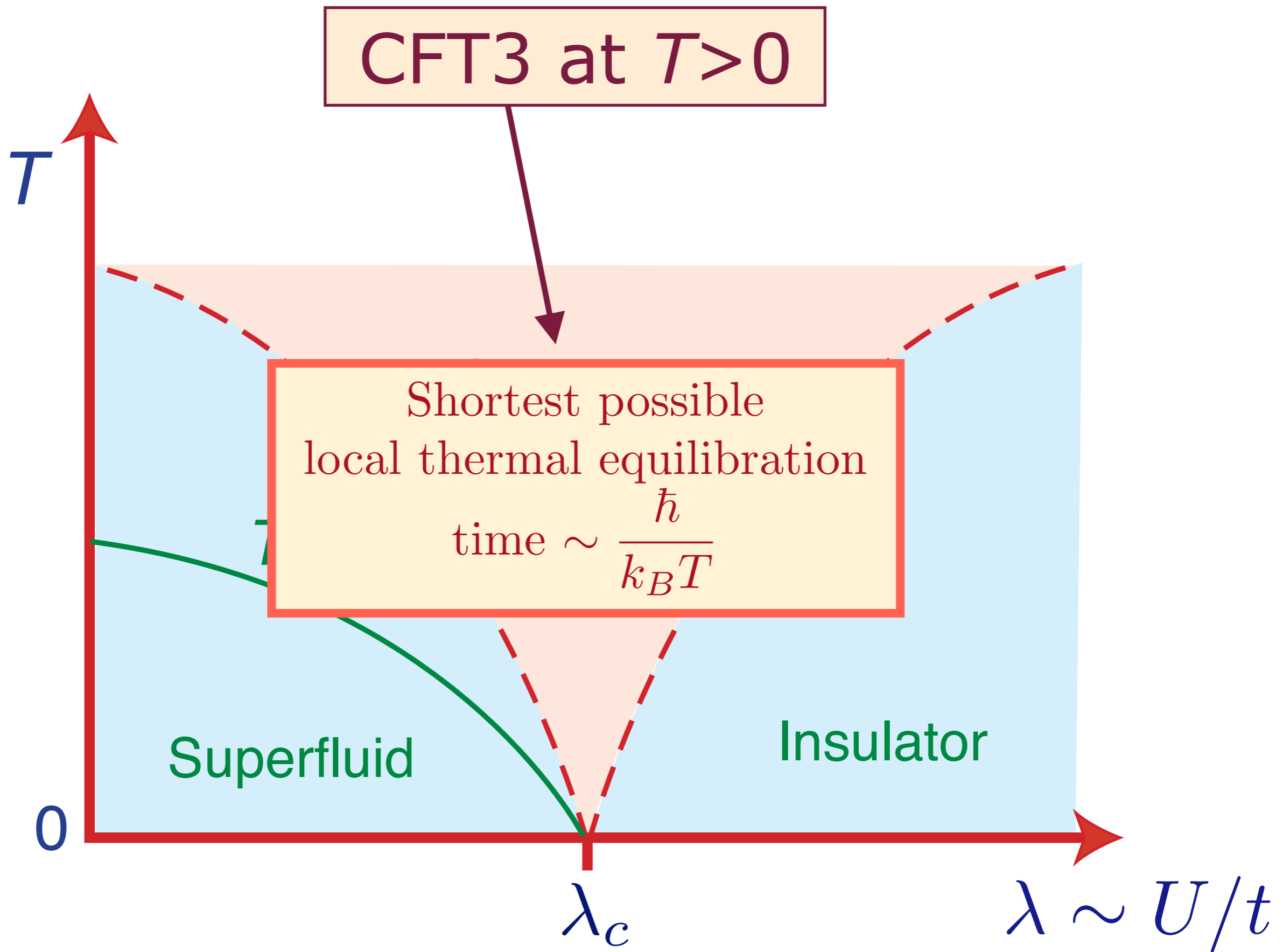






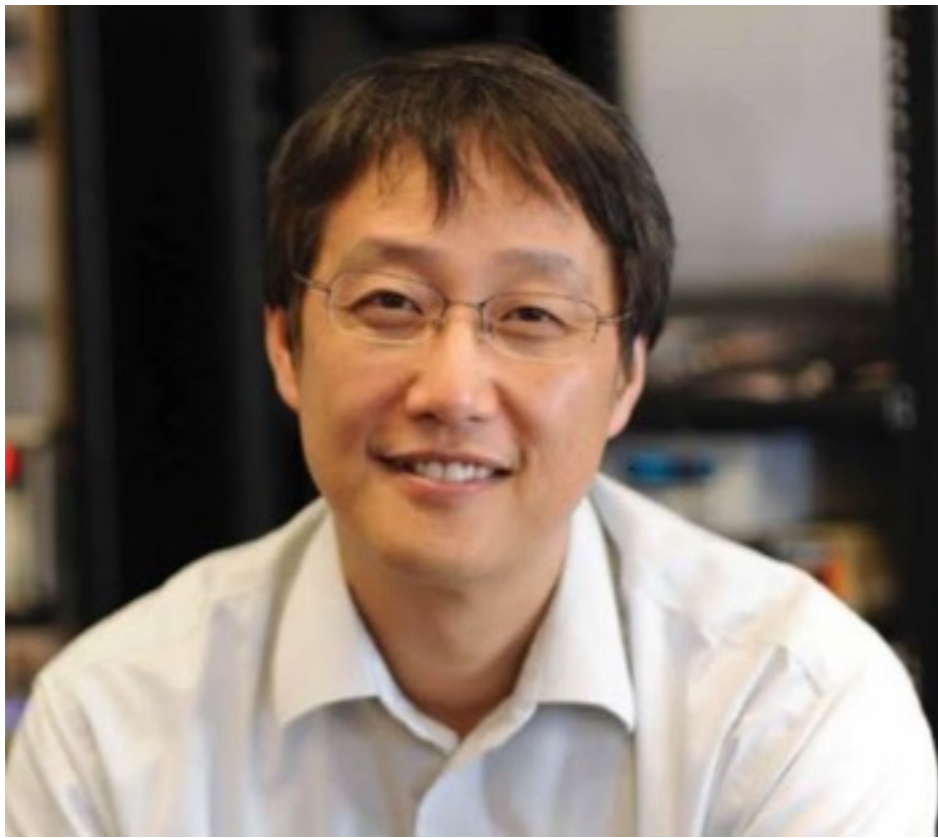






## Quantum matter without quasiparticles:

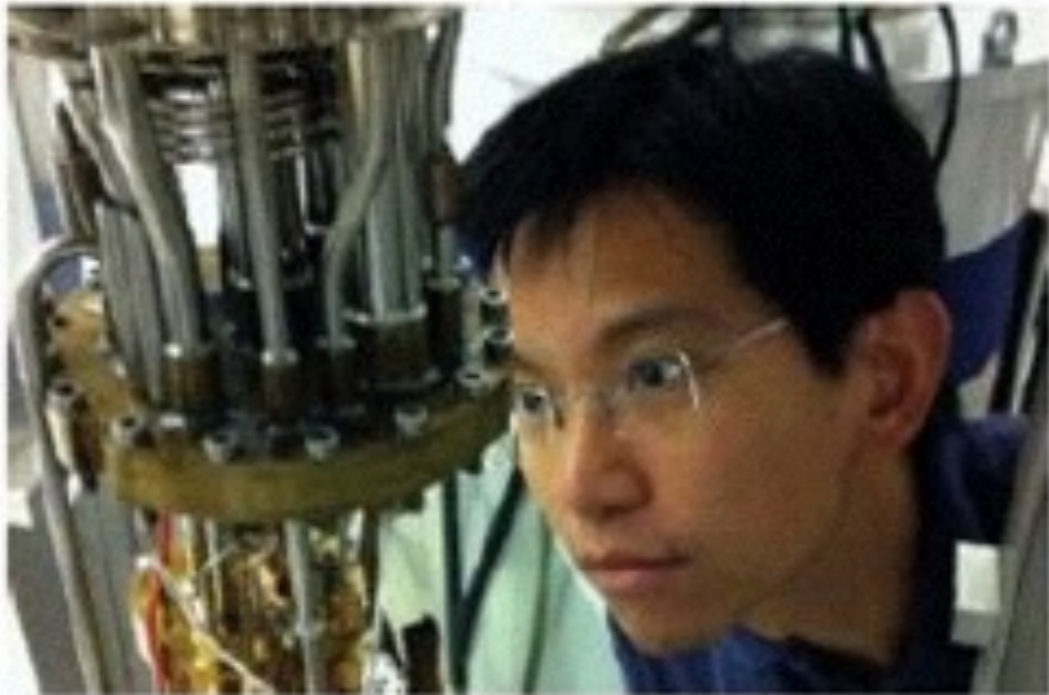
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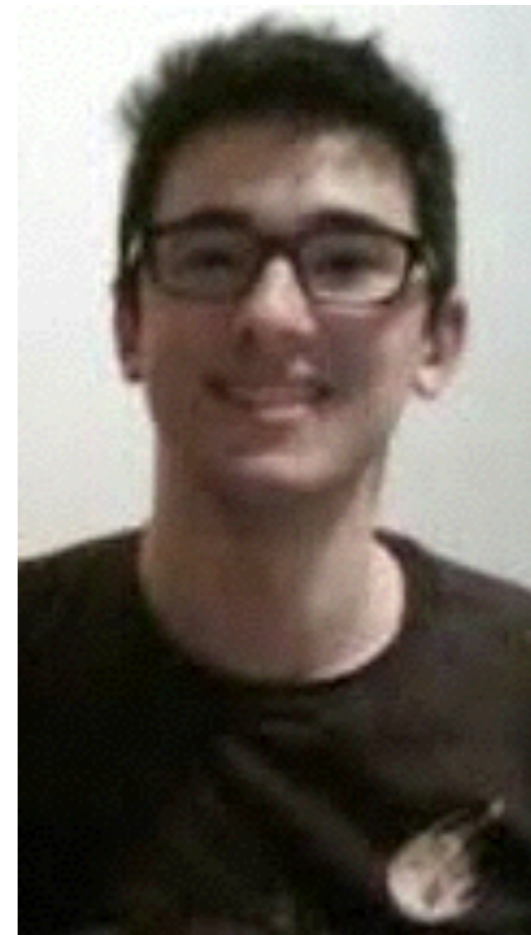
Philip Kim



Jesse Crossno

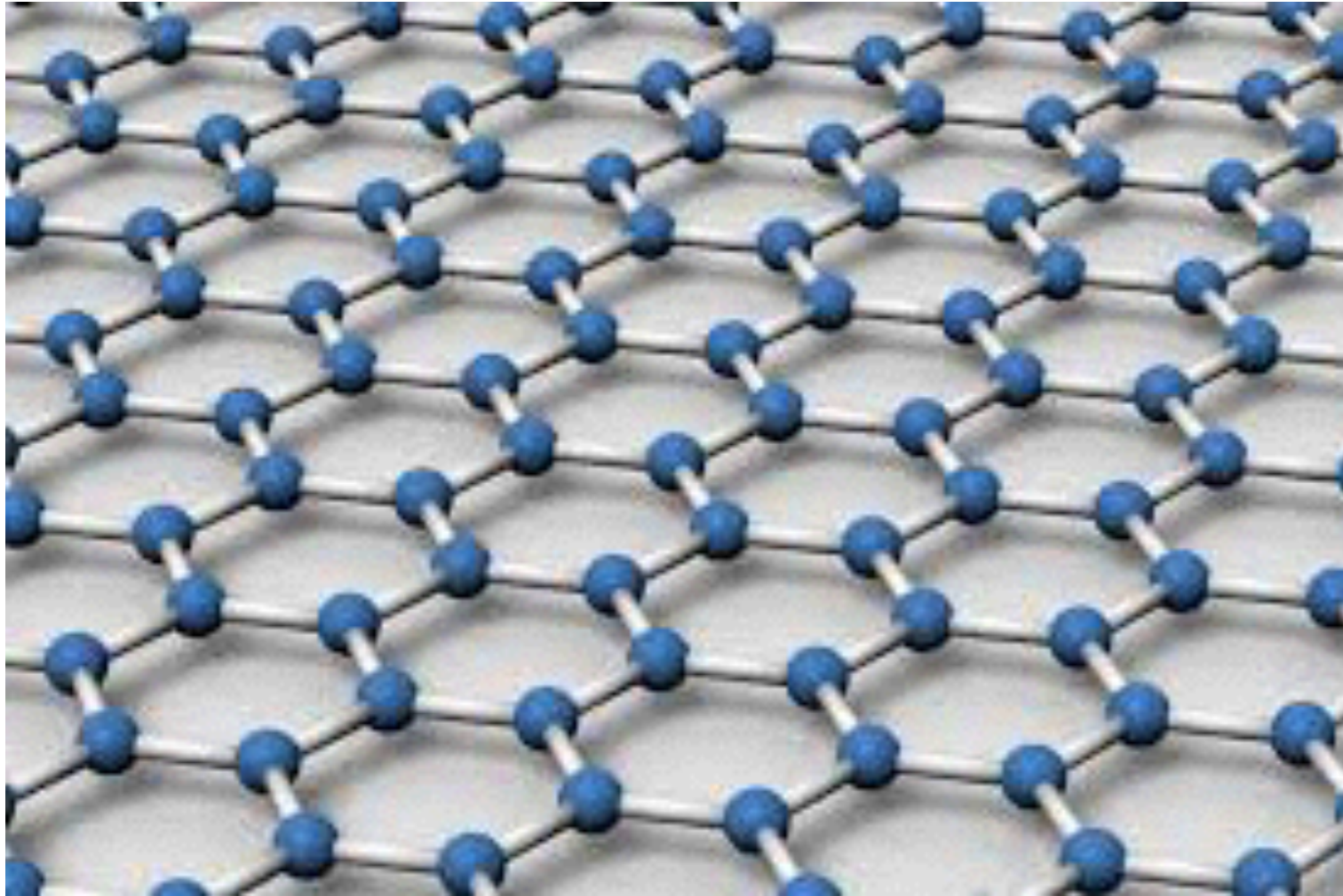


Kin Chung Fong

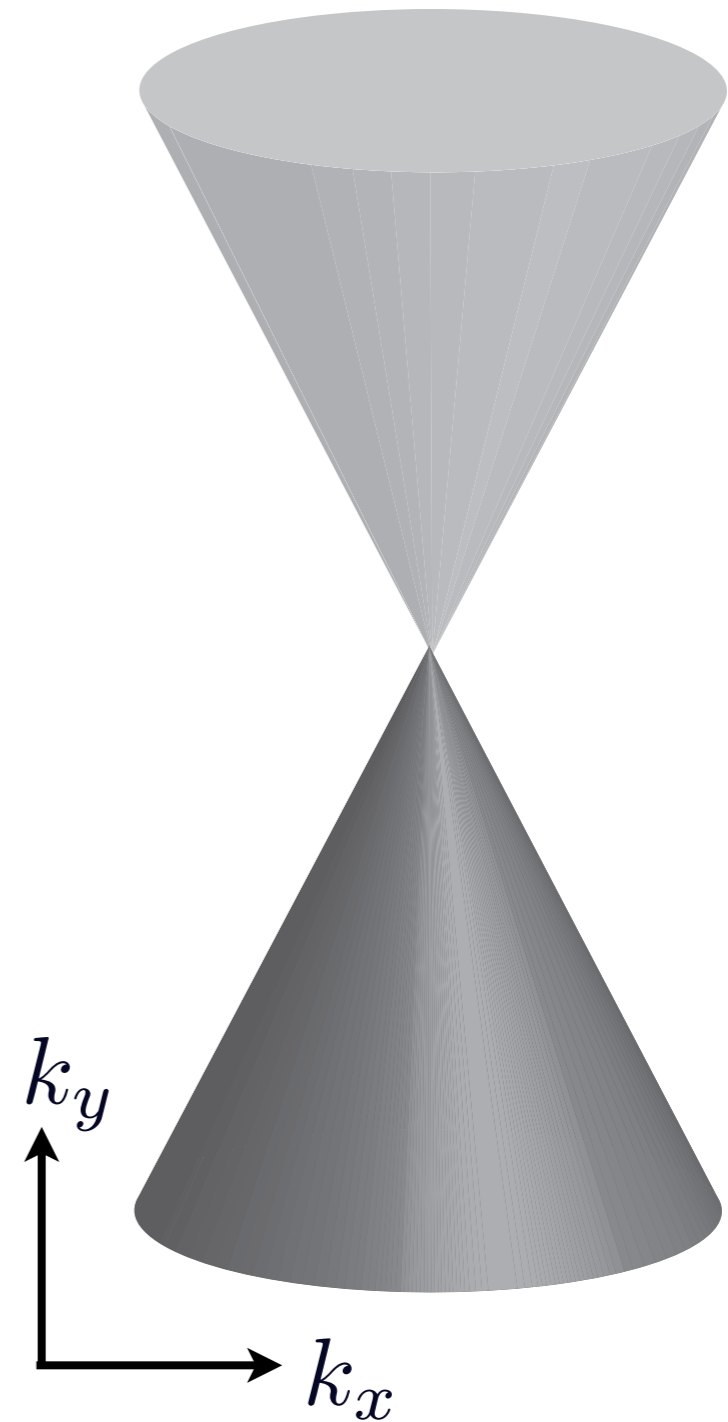


Andrew Lucas

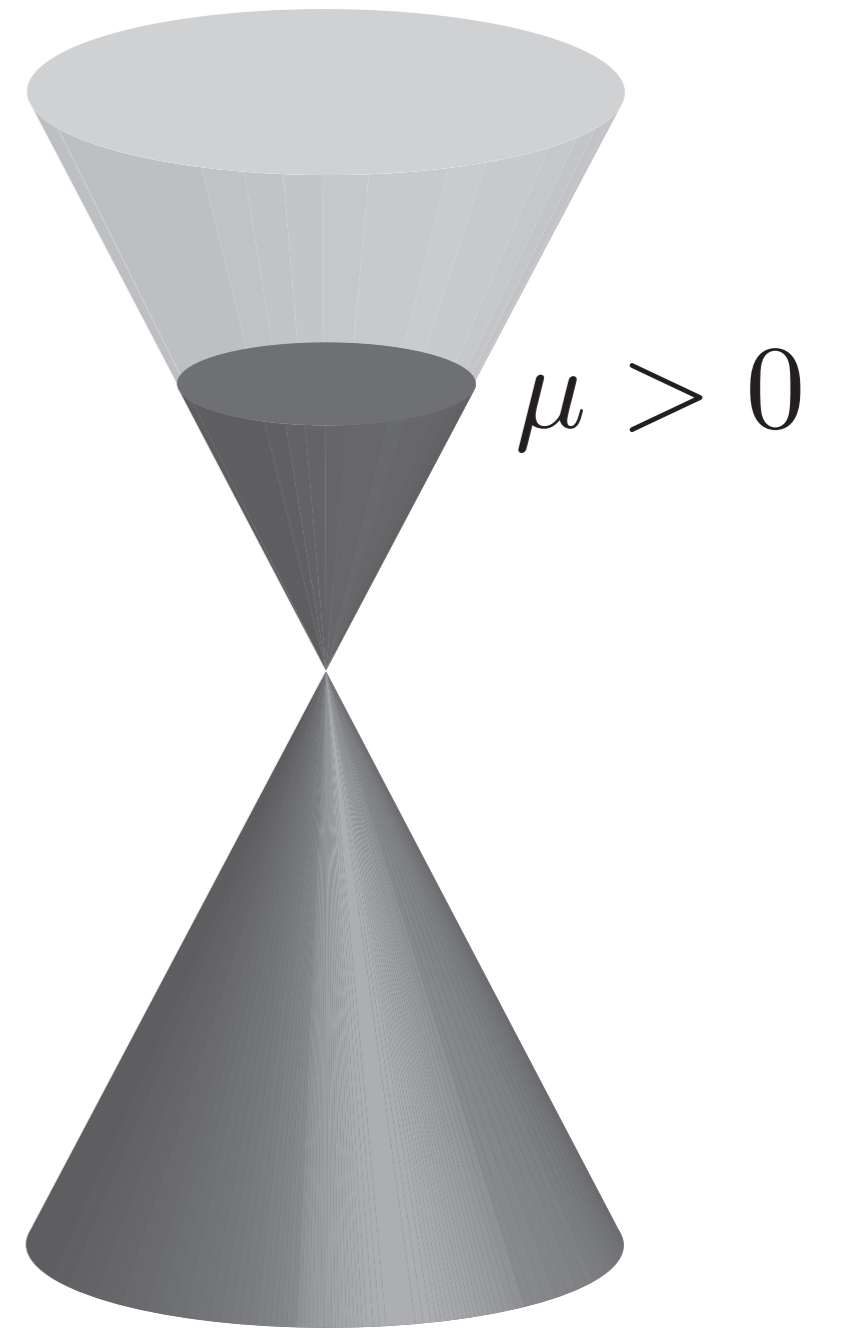
# Graphene



Same “Hubbard” model as for ultracold atoms, but for electrons on the honeycomb lattice

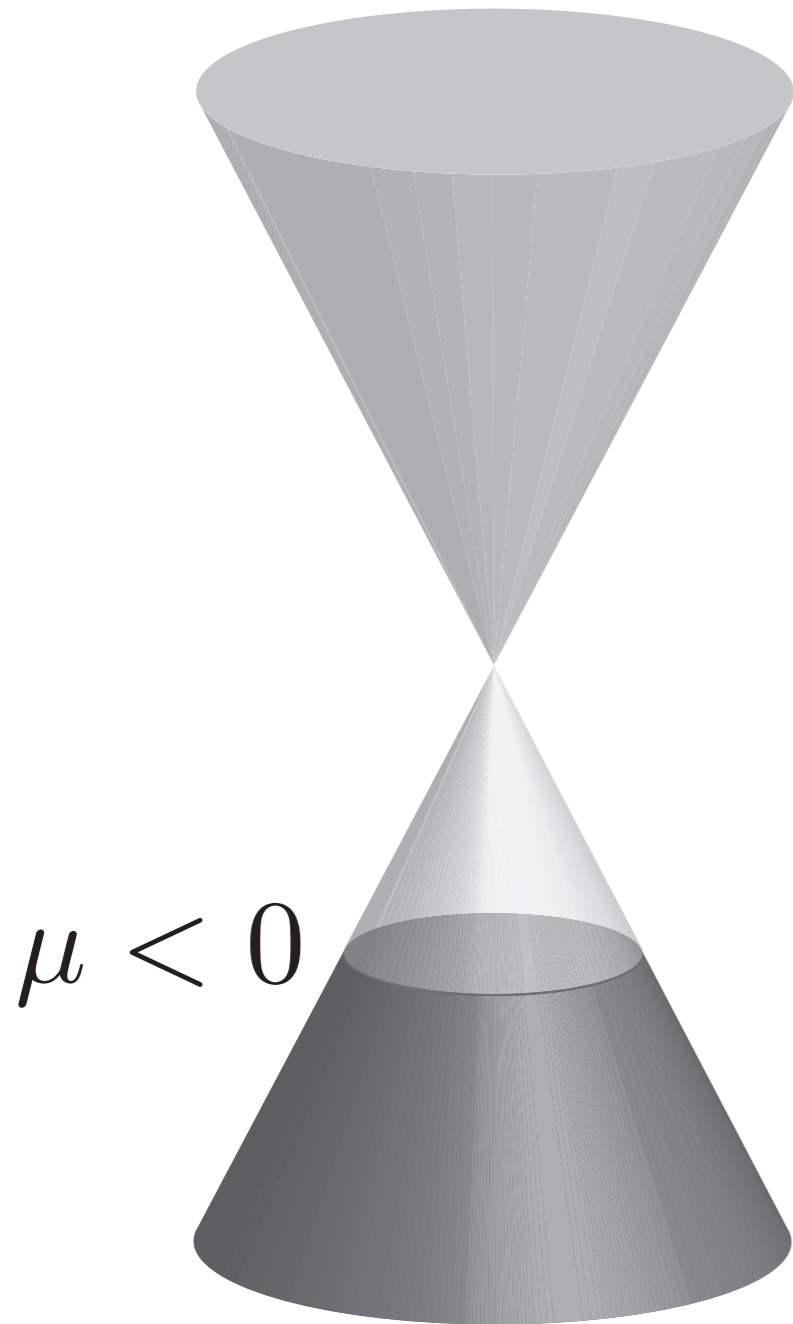


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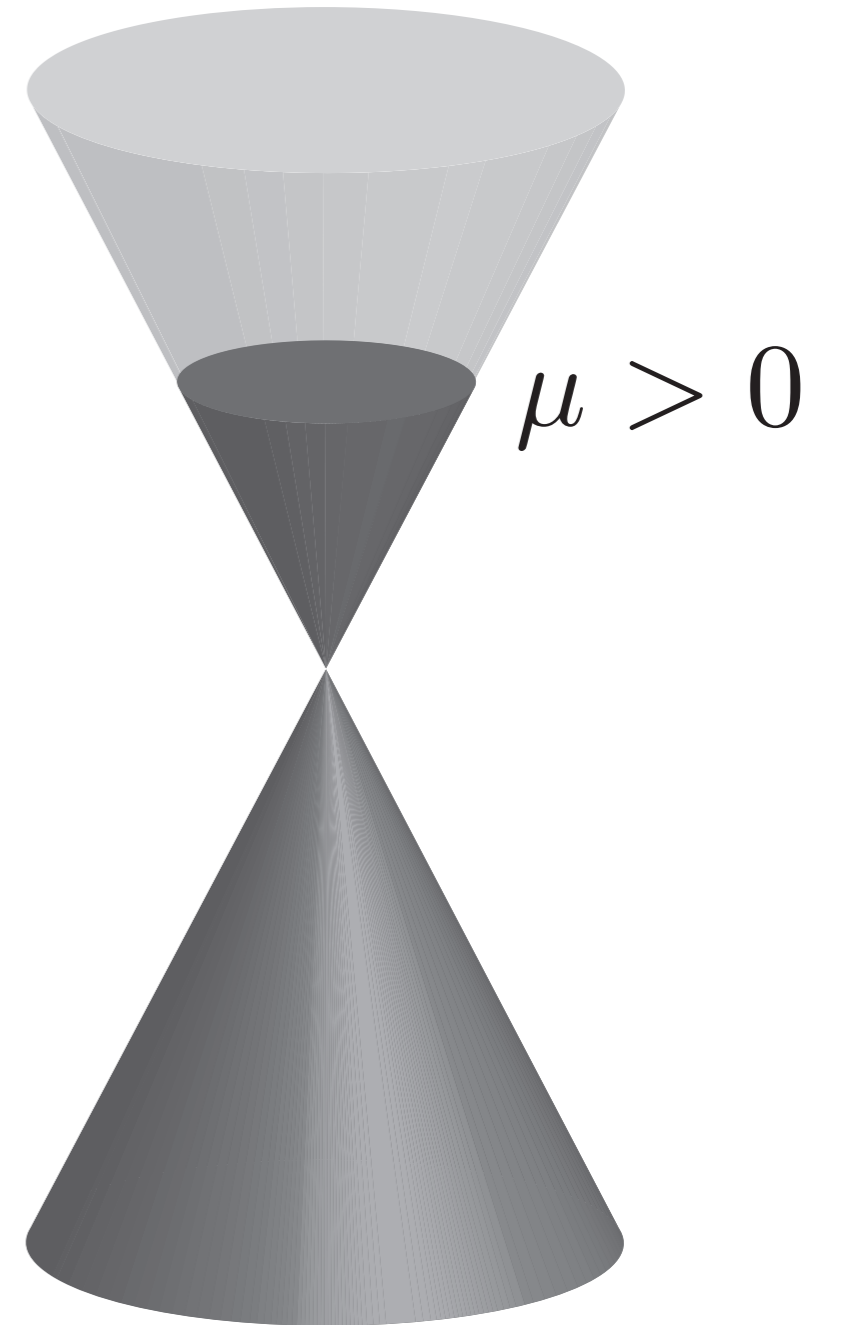


**Electron  
Fermi surface**

# Graphene

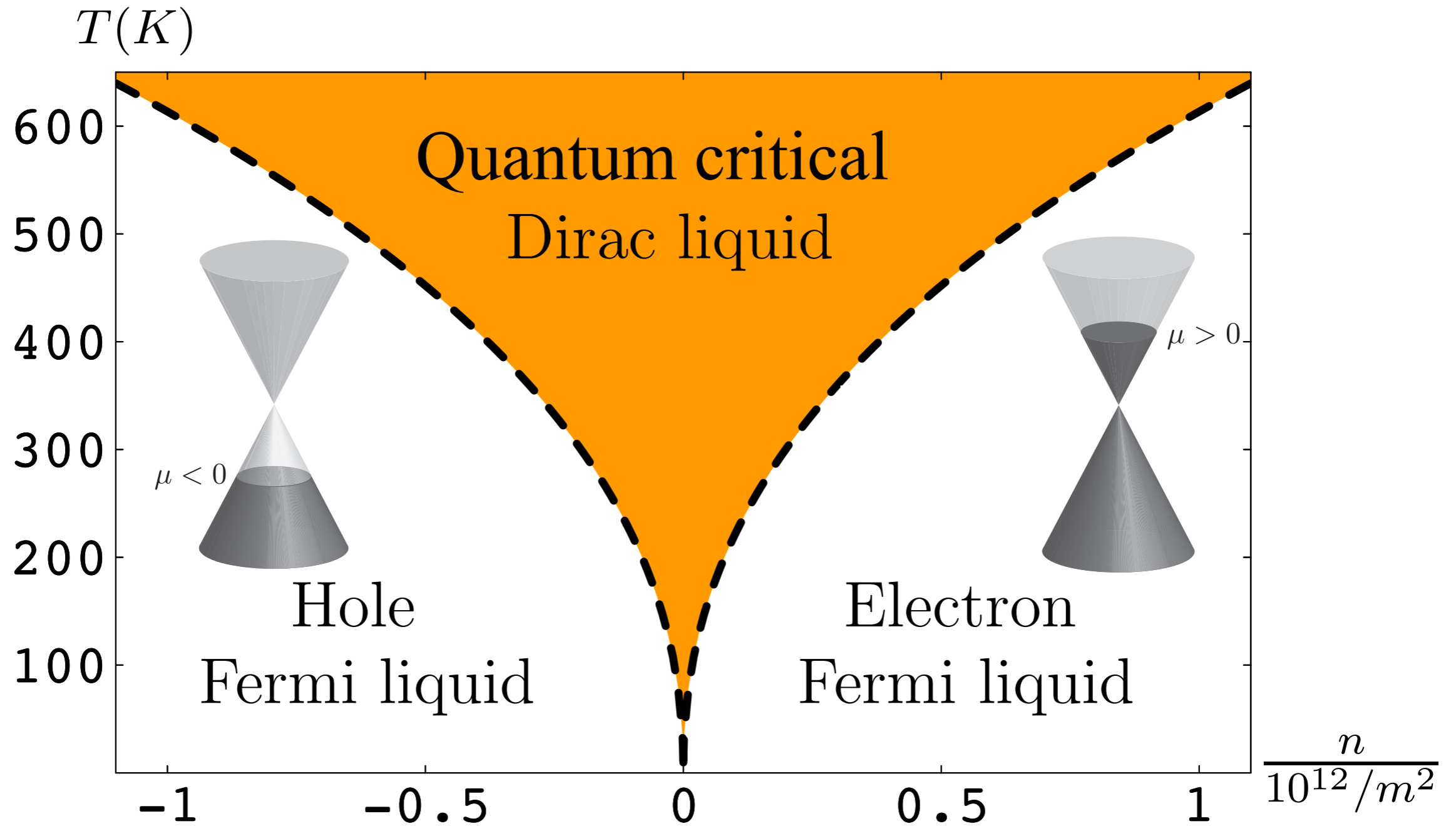


**Hole  
Fermi surface**



**Electron  
Fermi surface**

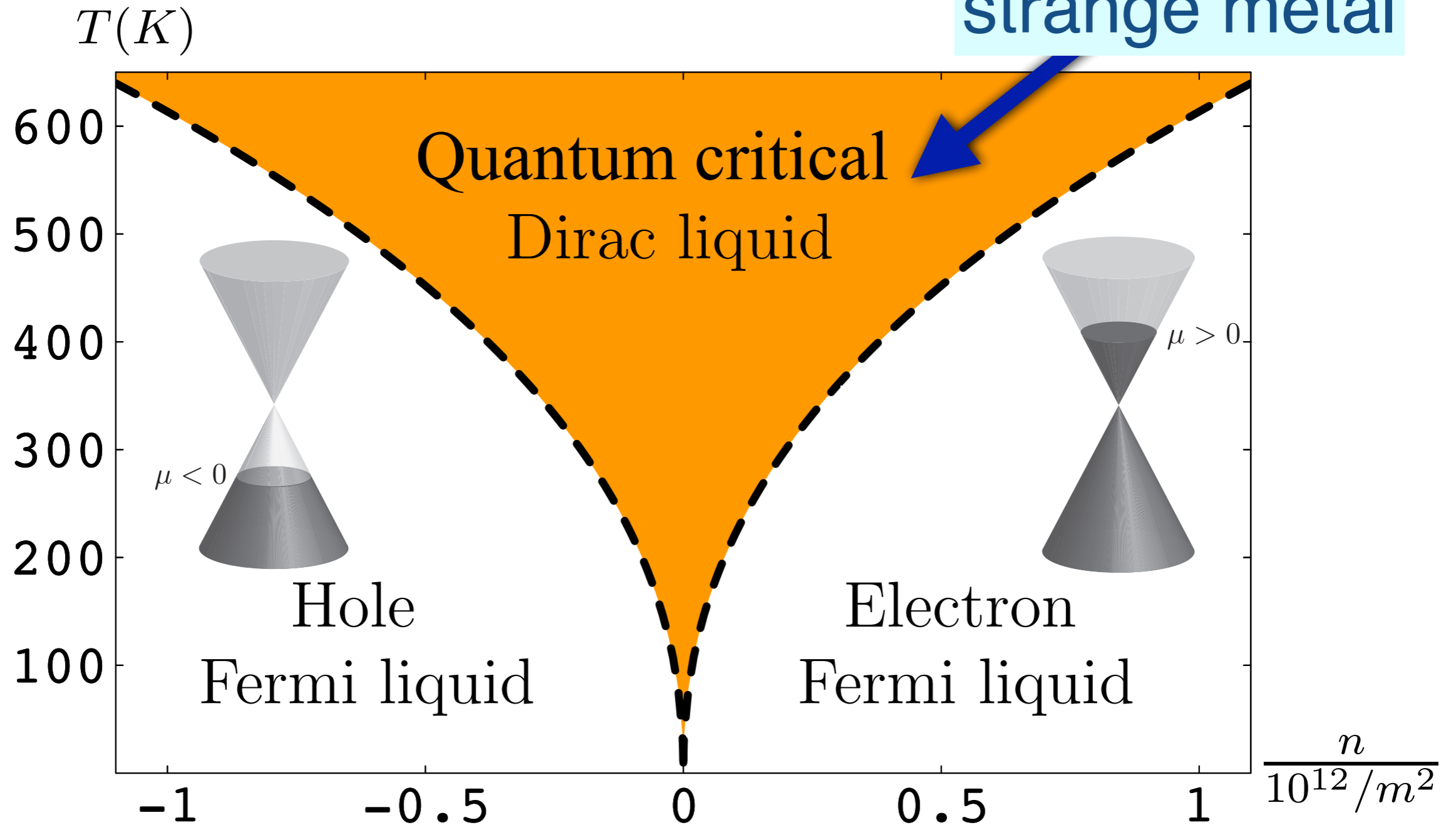
# Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)  
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)  
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

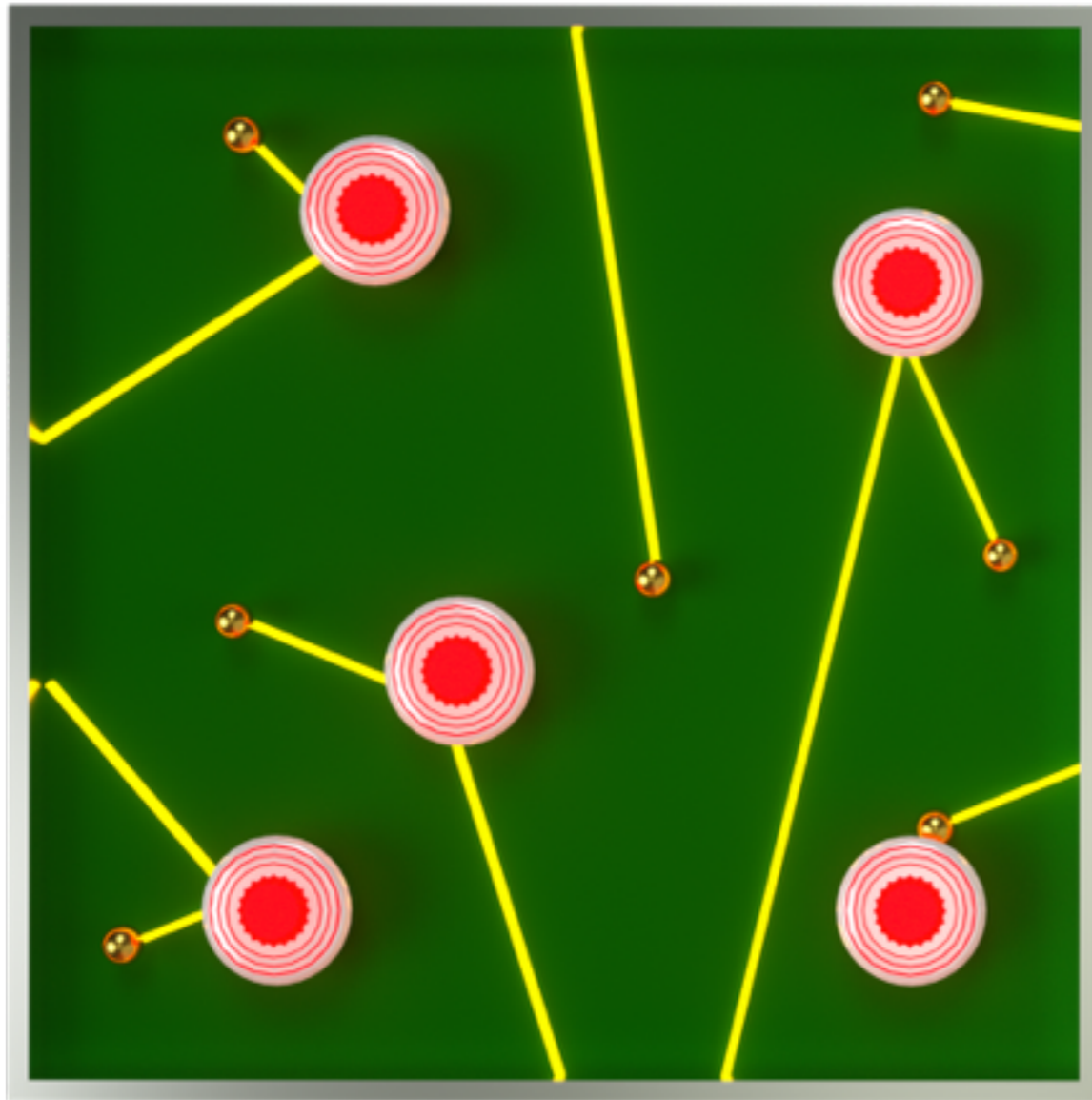
# Graphene

Predicted  
strange metal

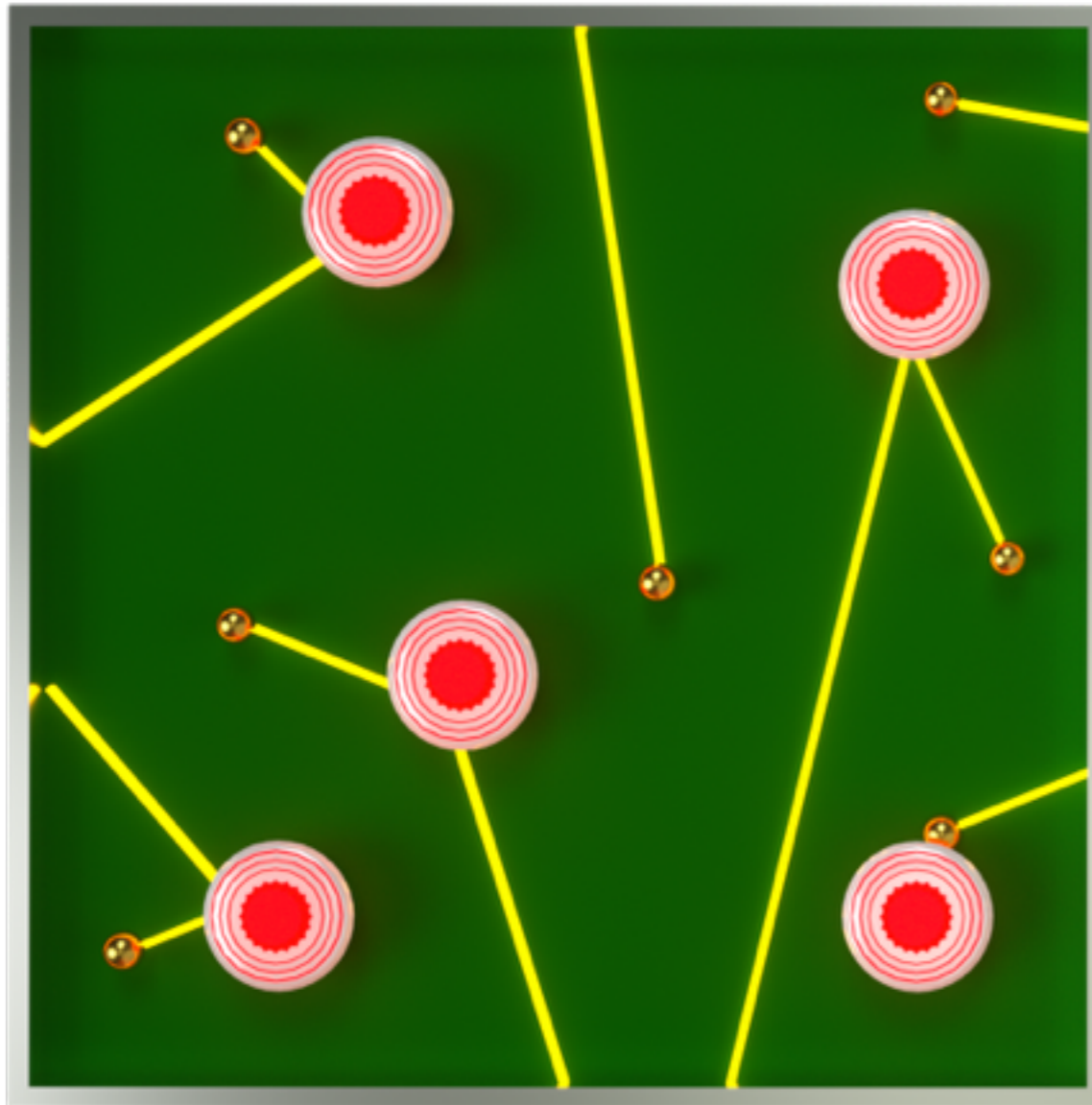


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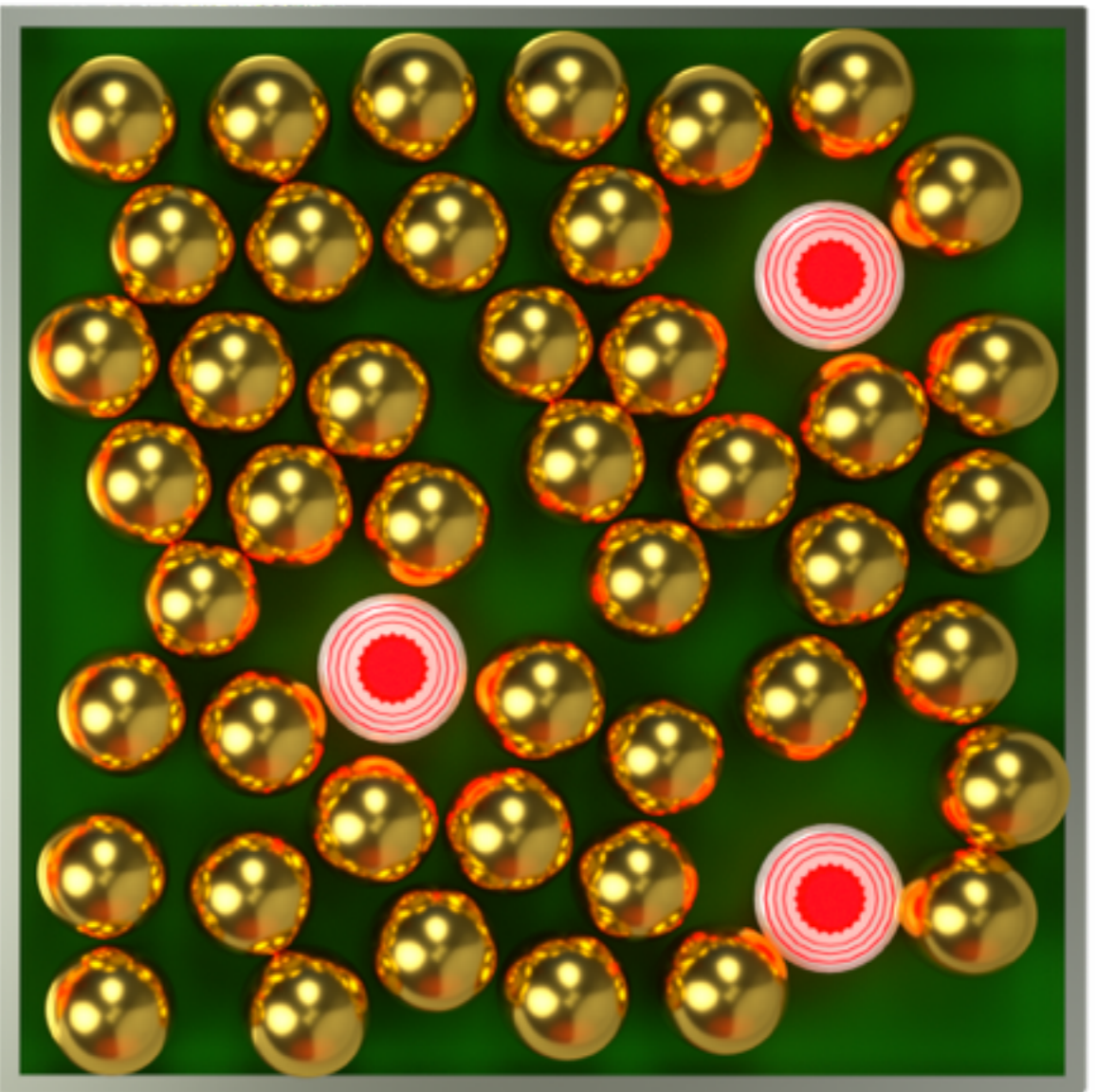
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)



Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

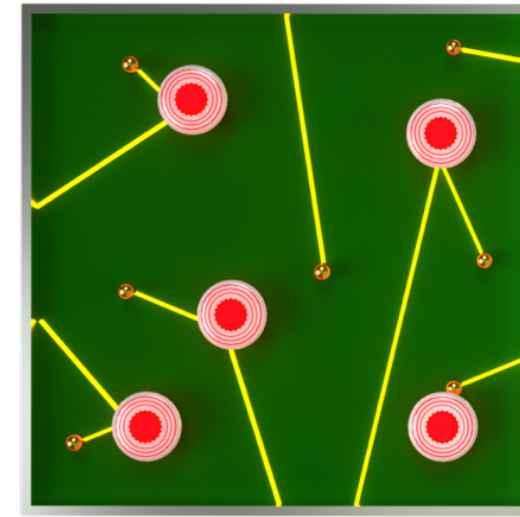


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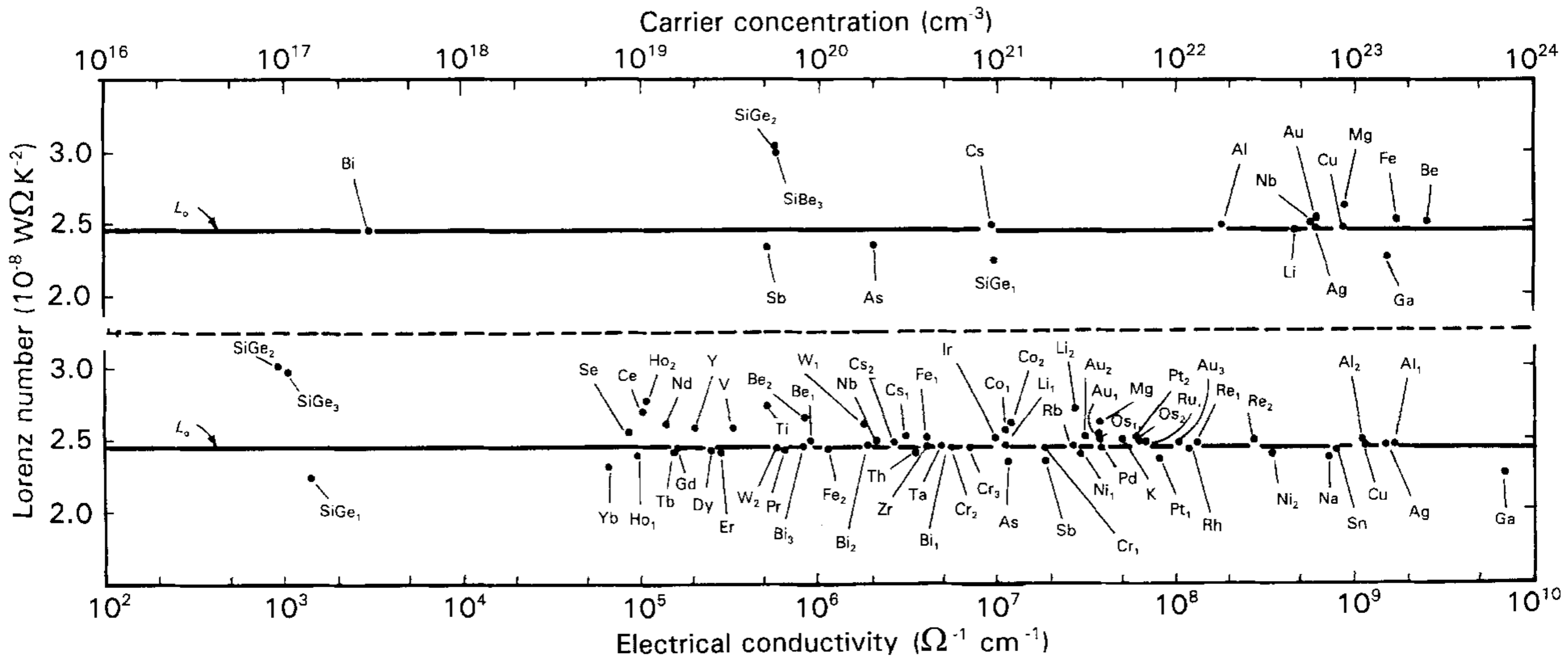
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities

# Thermal and electrical conductivity with quasiparticles

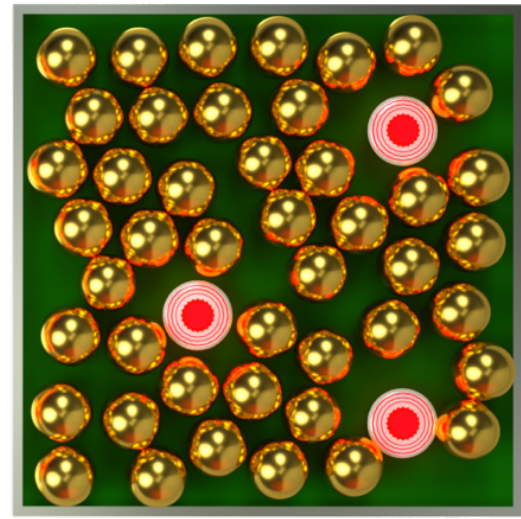


- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{\text{W} \cdot \Omega}{\text{K}^2}.$$



# Transport in Strange Metals



For a strange metal  
with a “relativistic” Hamiltonian,  
hydrodynamic, holographic,  
and memory function methods yield

$$\text{Lorentz ratio } L = \kappa / (T\sigma) \\ = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q)\right)^2}$$

$Q \rightarrow$  electron density;  $\mathcal{H} \rightarrow$  enthalpy density

$\sigma_Q \rightarrow$  quantum critical conductivity

$\tau_{\text{imp}} \rightarrow$  momentum relaxation time from impurities.

Note that for a clean system ( $\tau_{\text{imp}} \rightarrow \infty$  first),

the Lorentz ratio diverges  $L \sim 1/Q^4$ ,

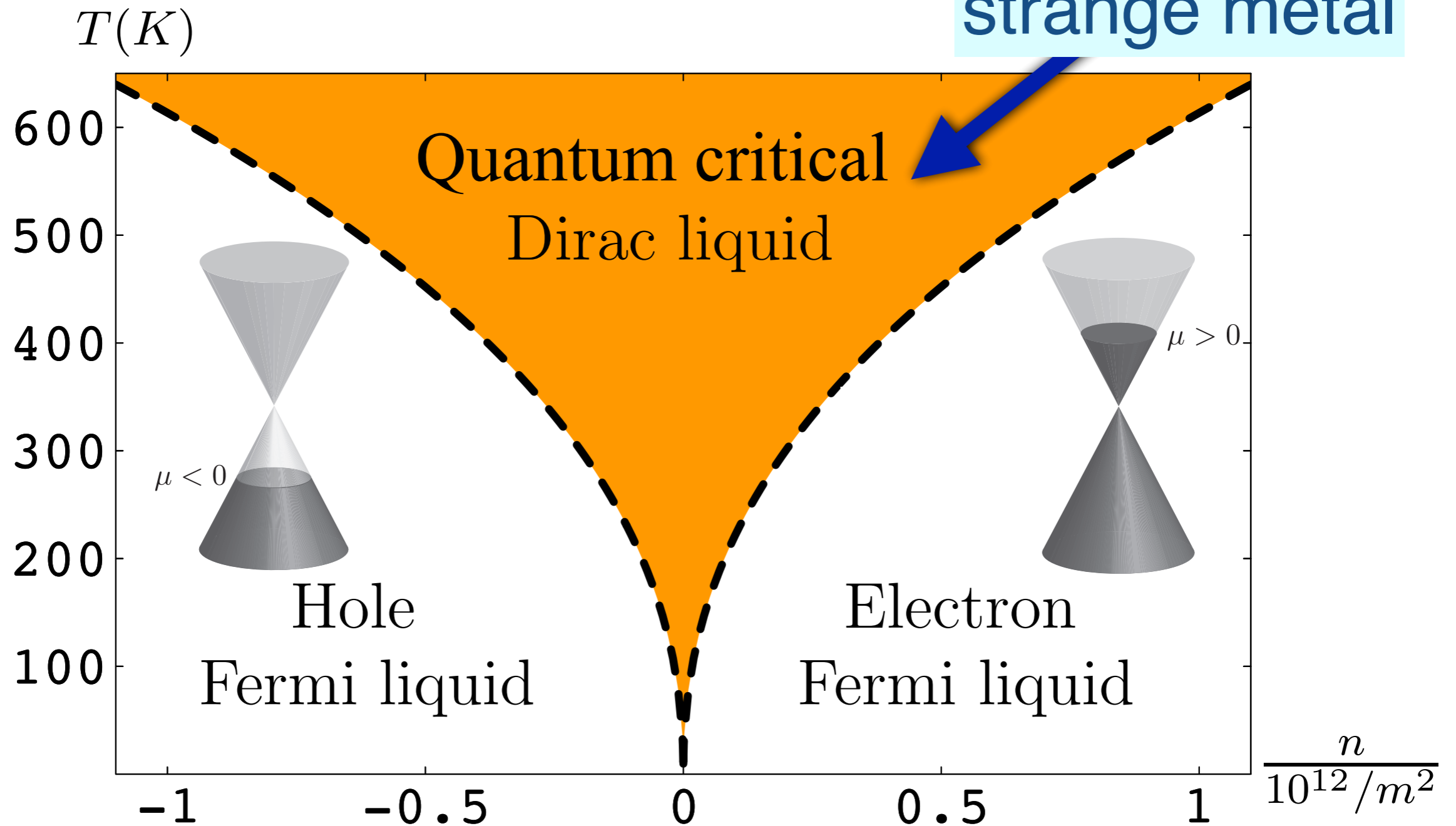
as we approach “zero” electron density at the Dirac point.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

# Graphene

Predicted  
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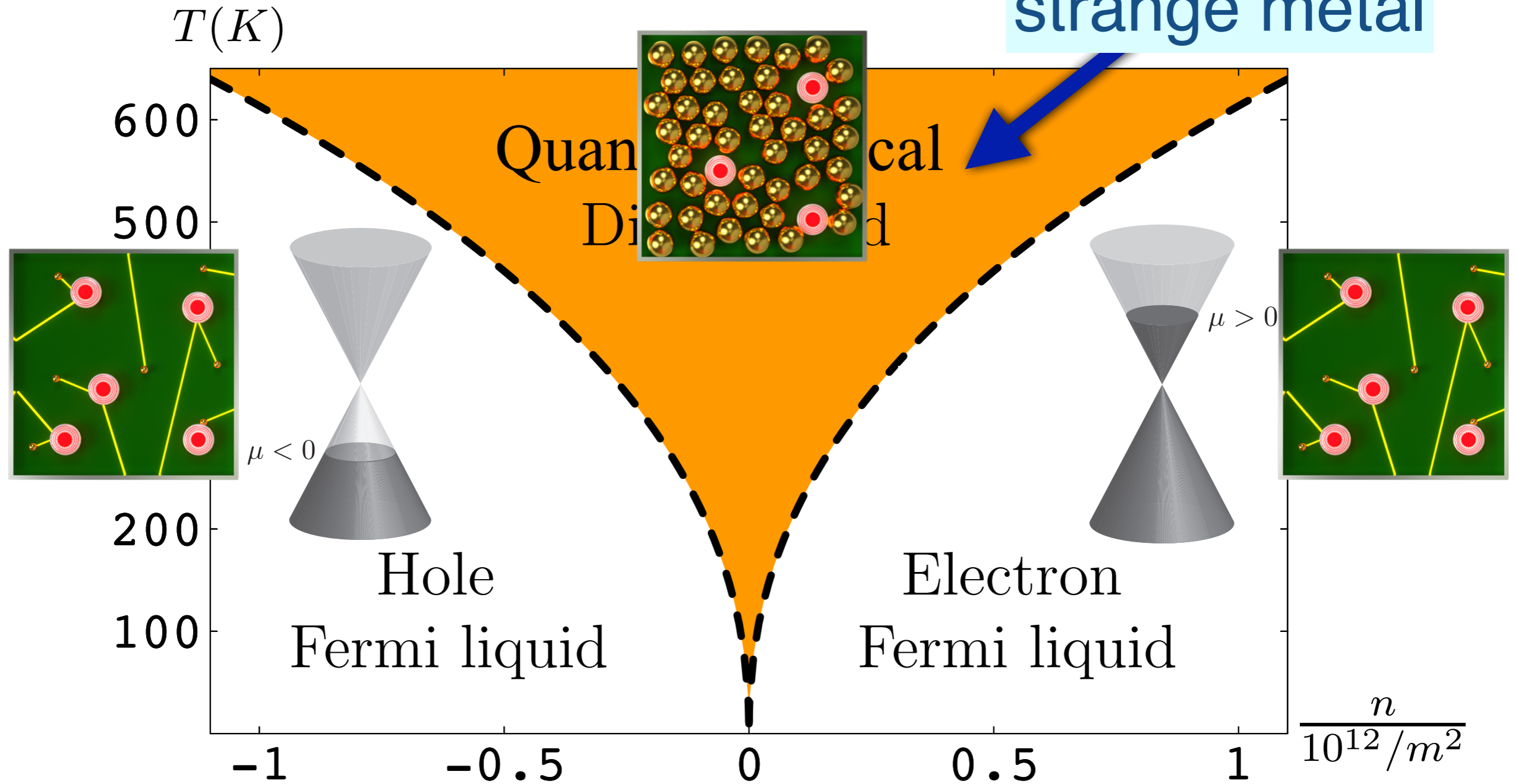


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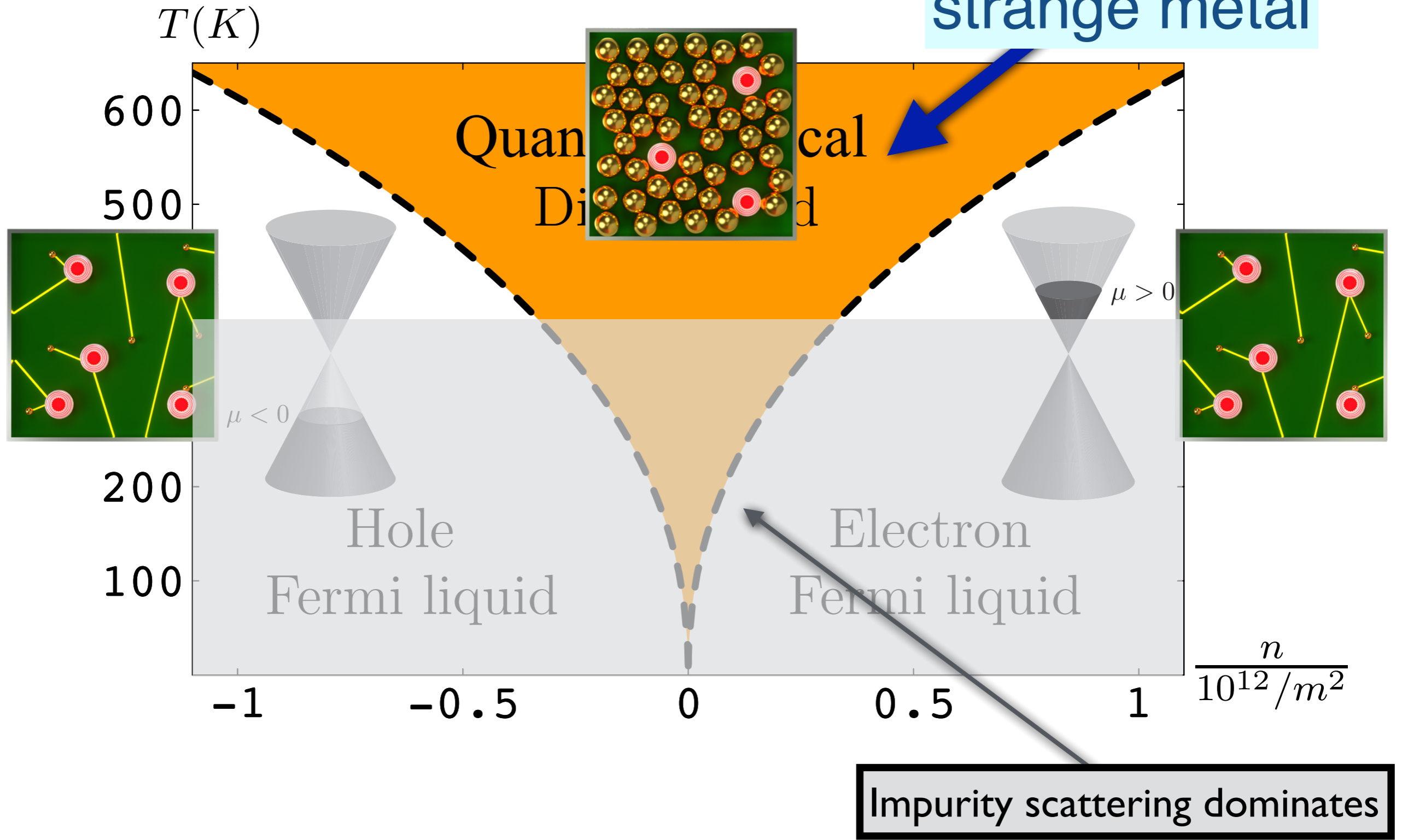


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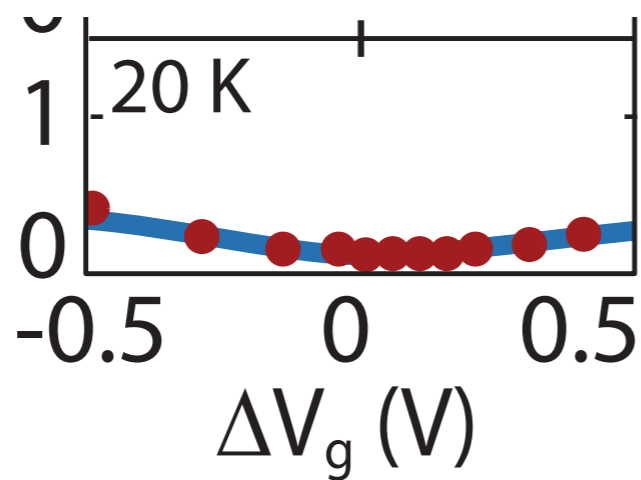
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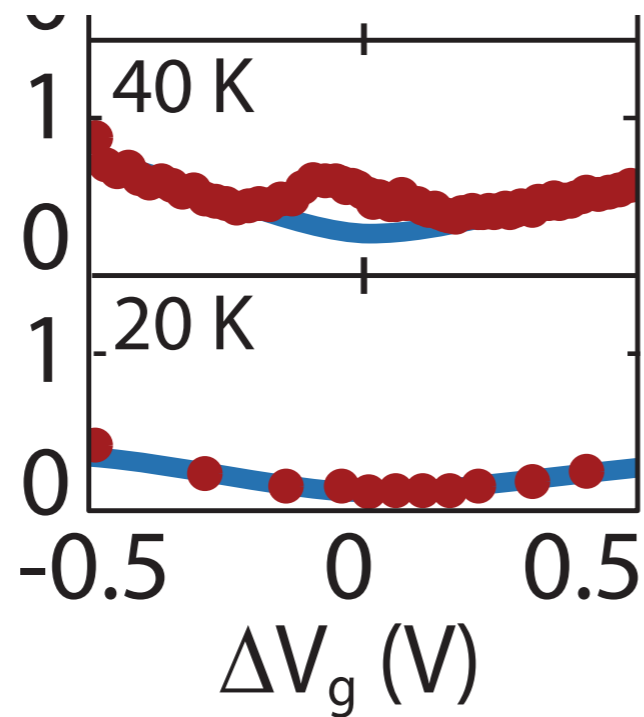
Thermal Conductivity (nW/K)



Red dots: data

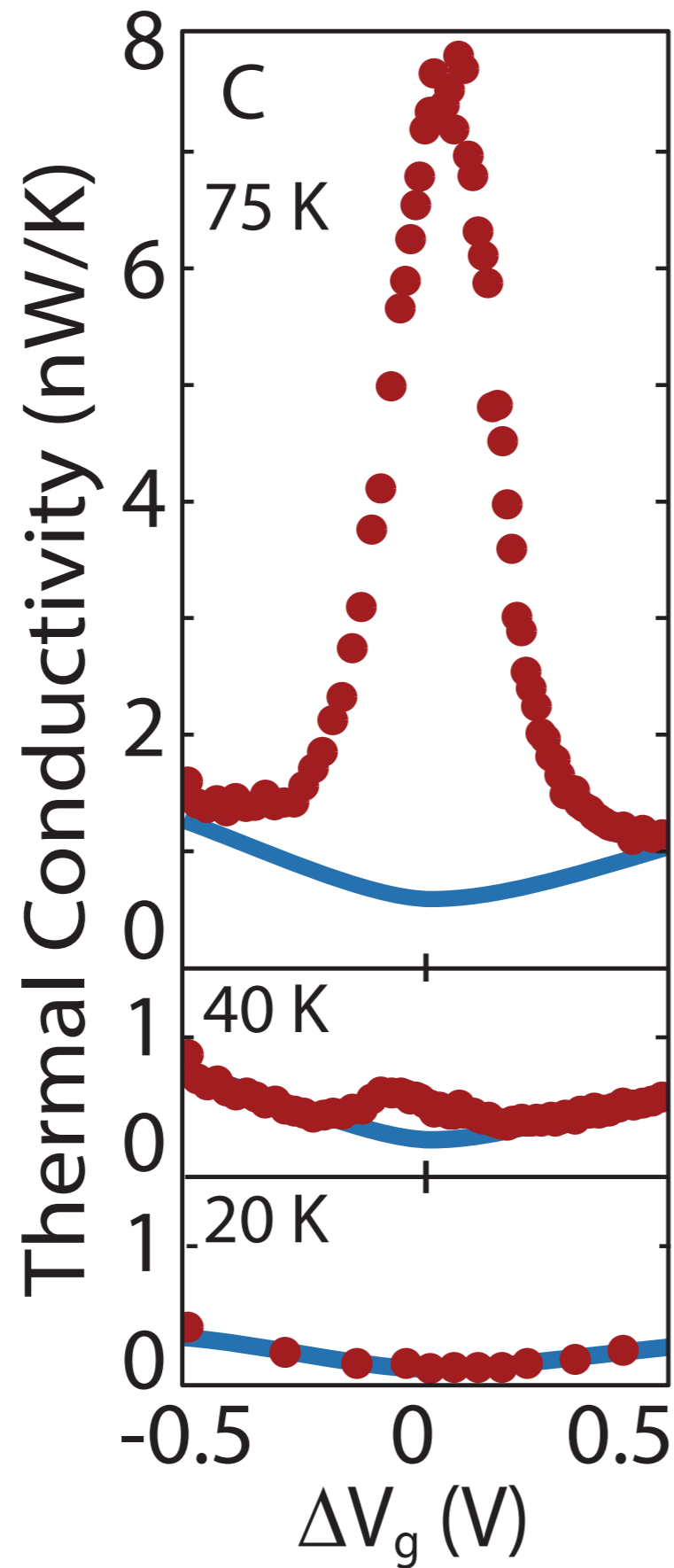
Blue line: value for  $L = L_0$

Thermal Conductivity (nW/K)



Red dots: data

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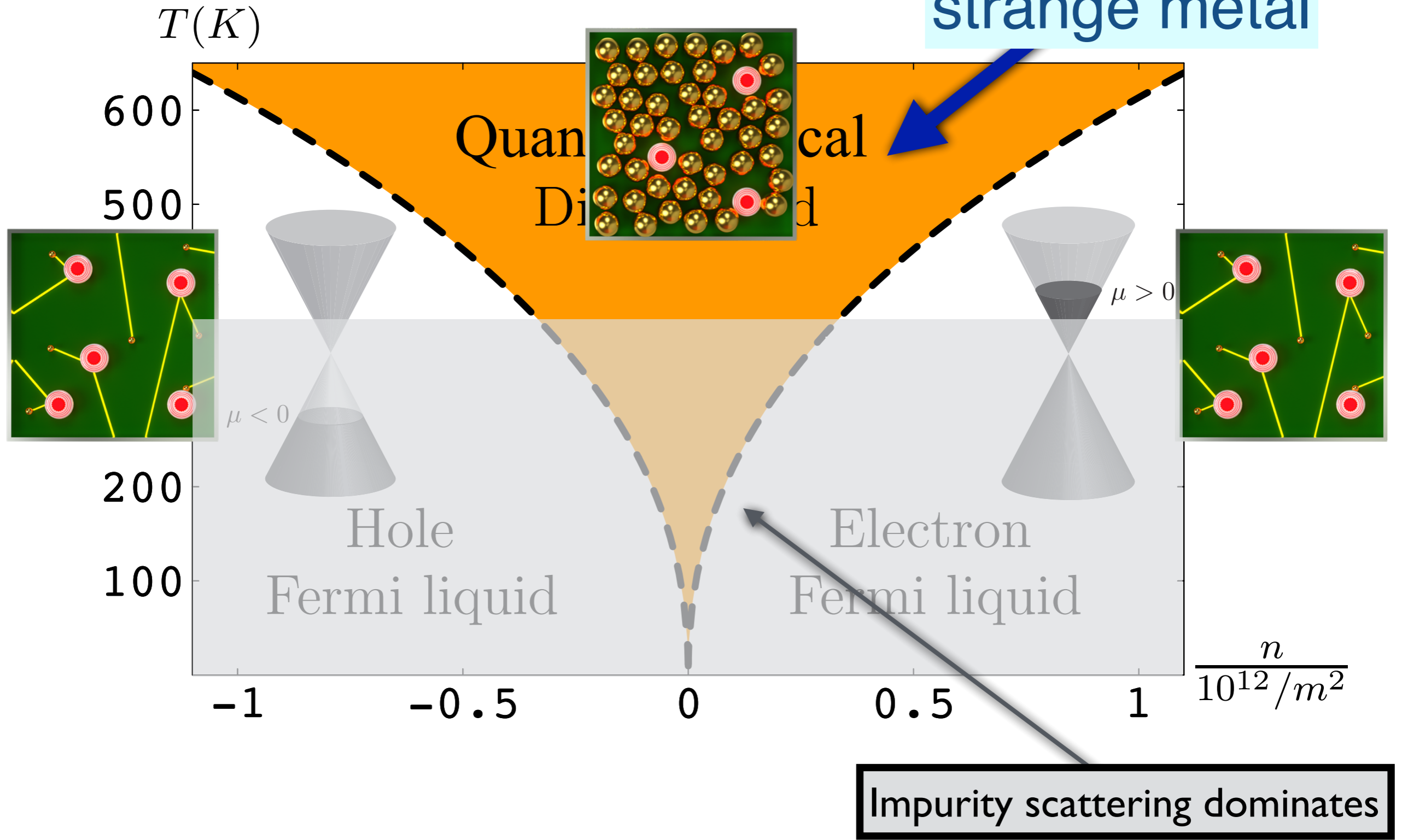


Red dots: data

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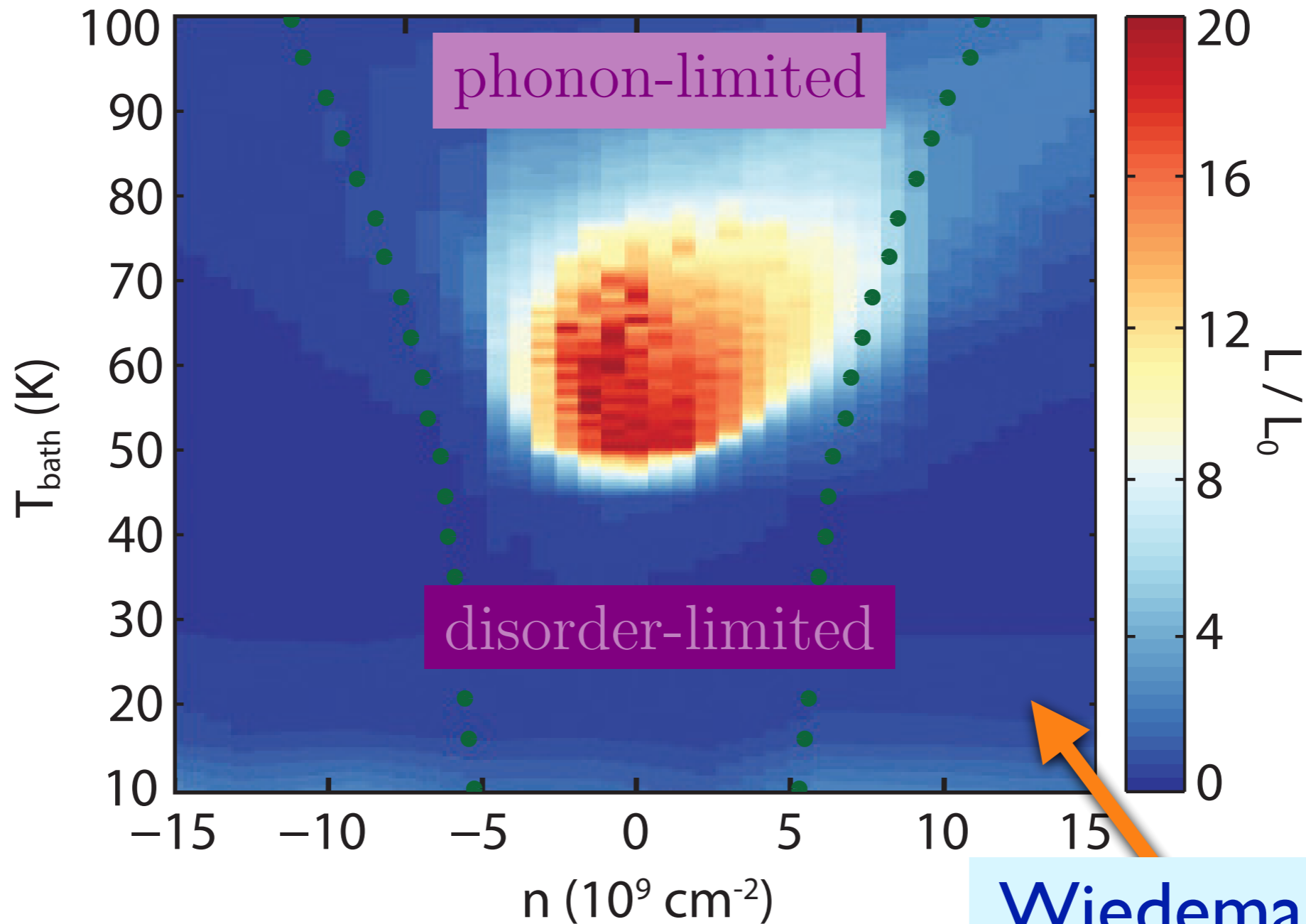
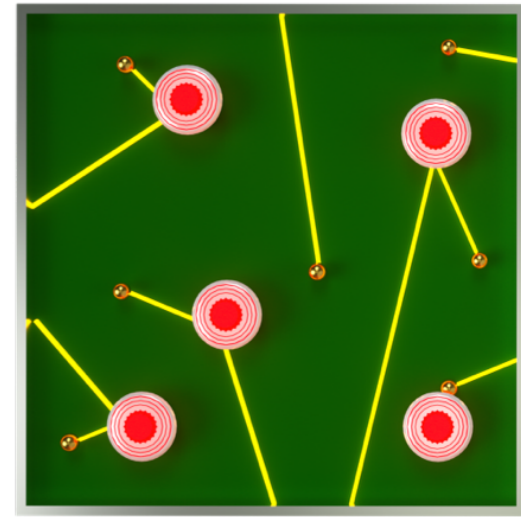


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J. Crossno et al., Science **351**, 1058 (2016)

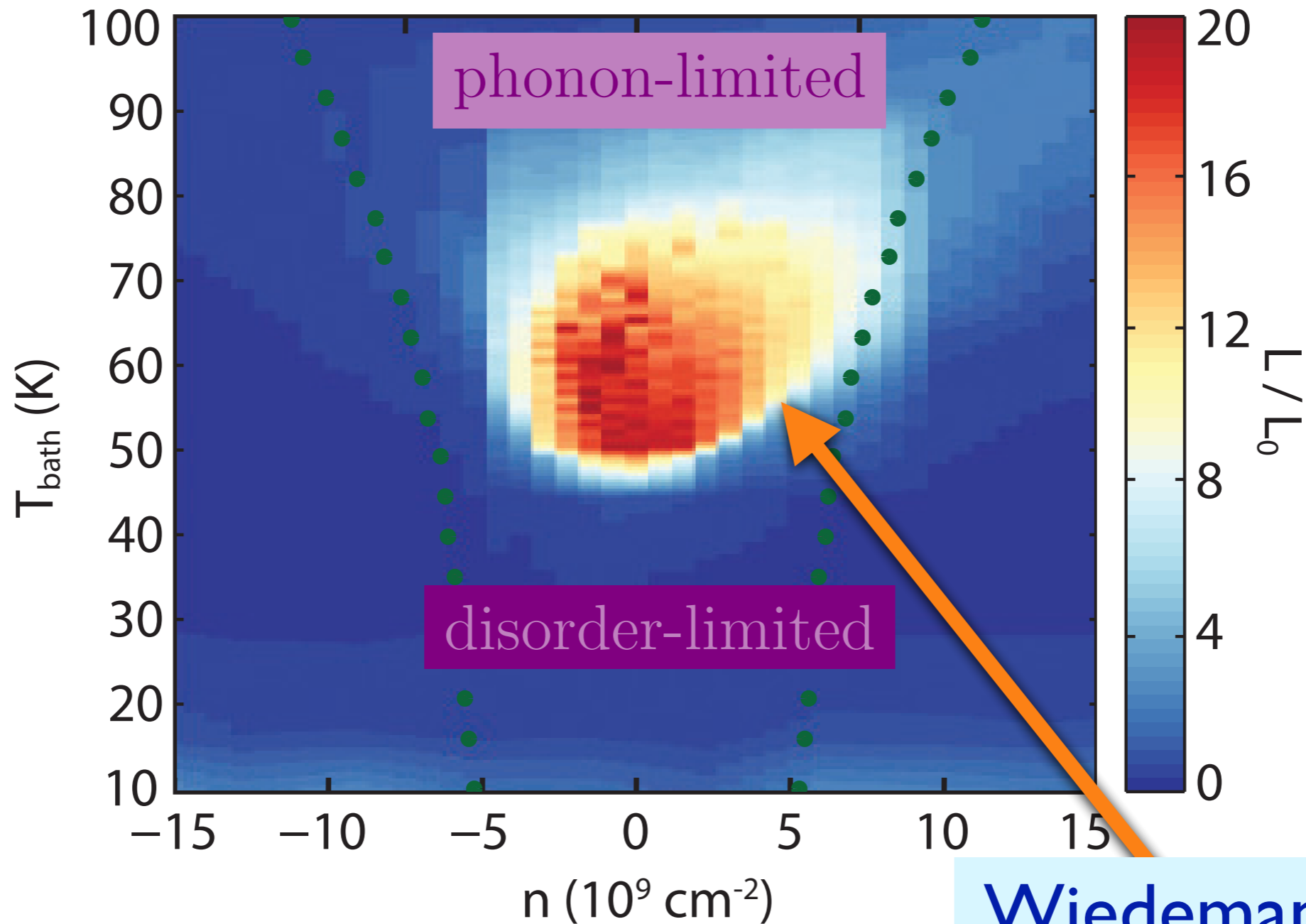
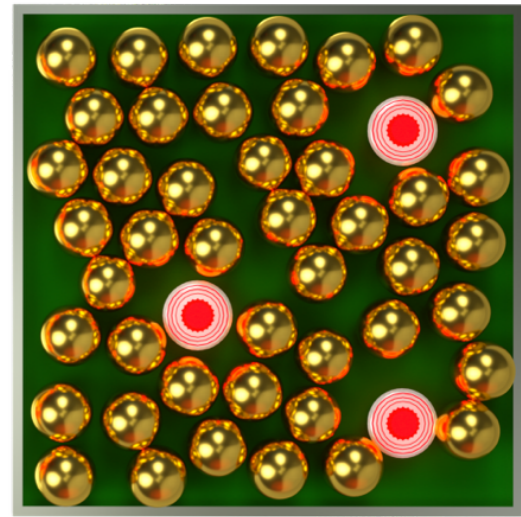
# Strange metal in graphene



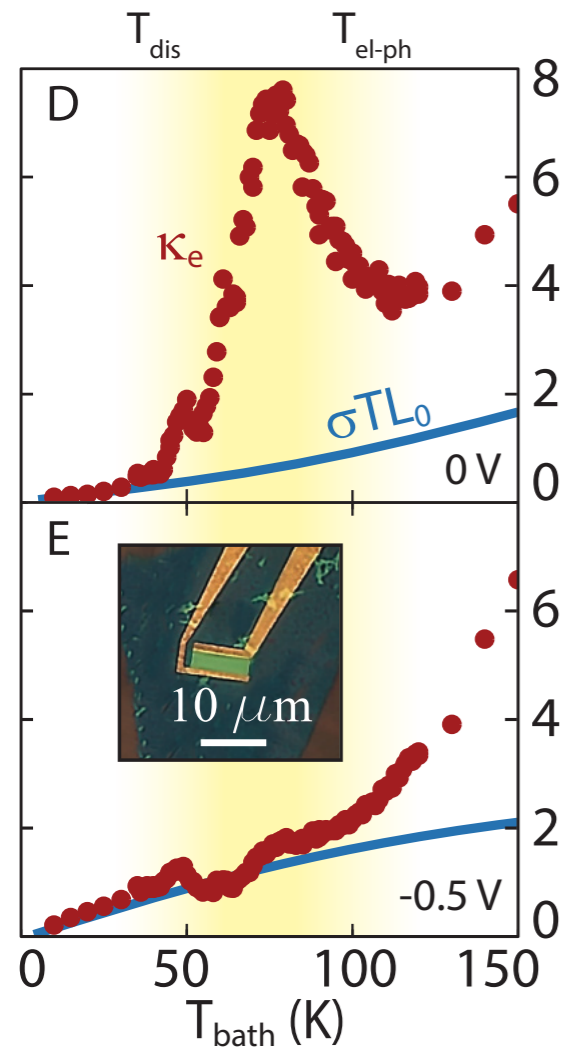
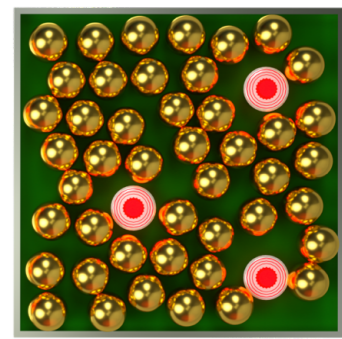
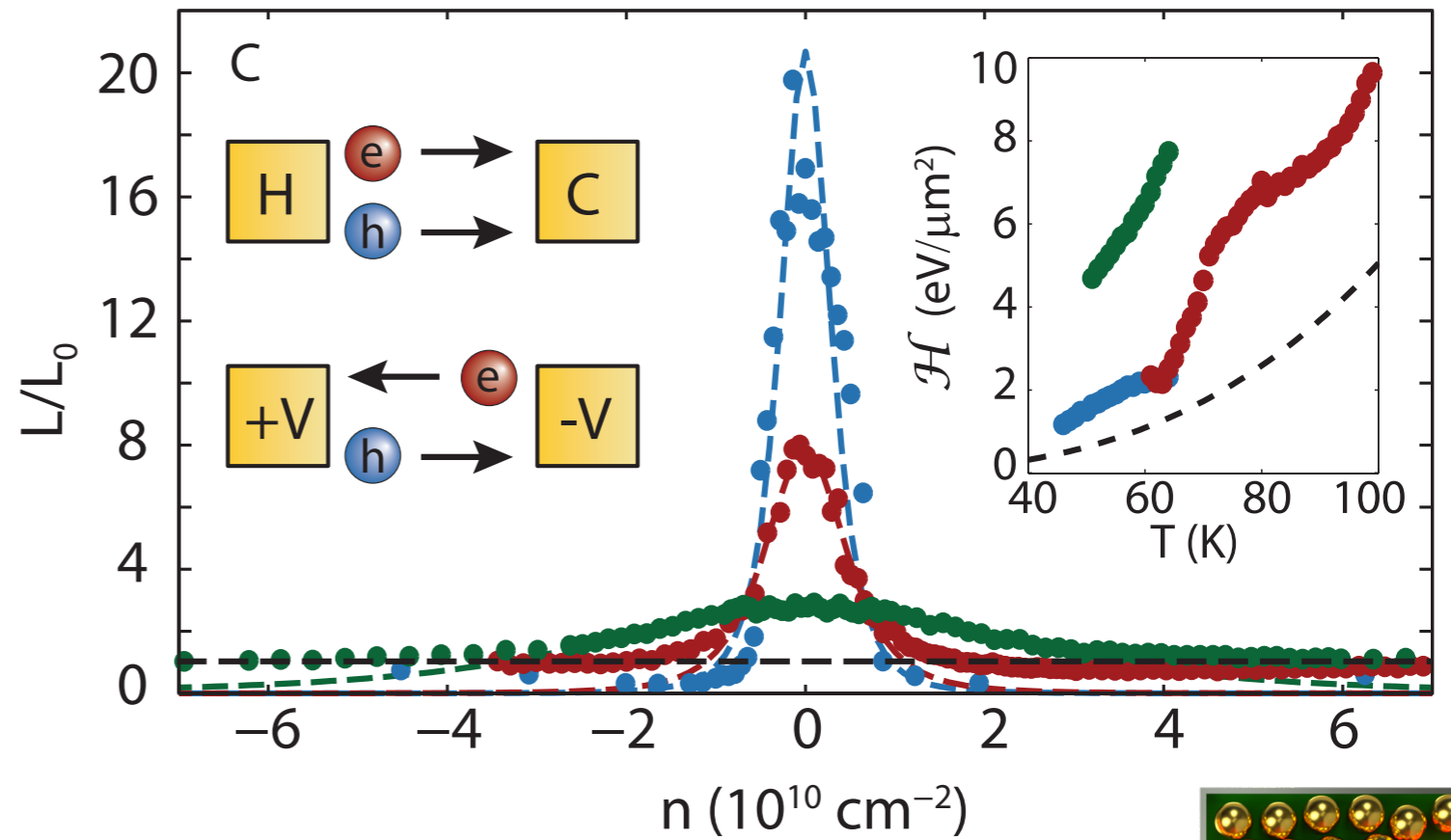
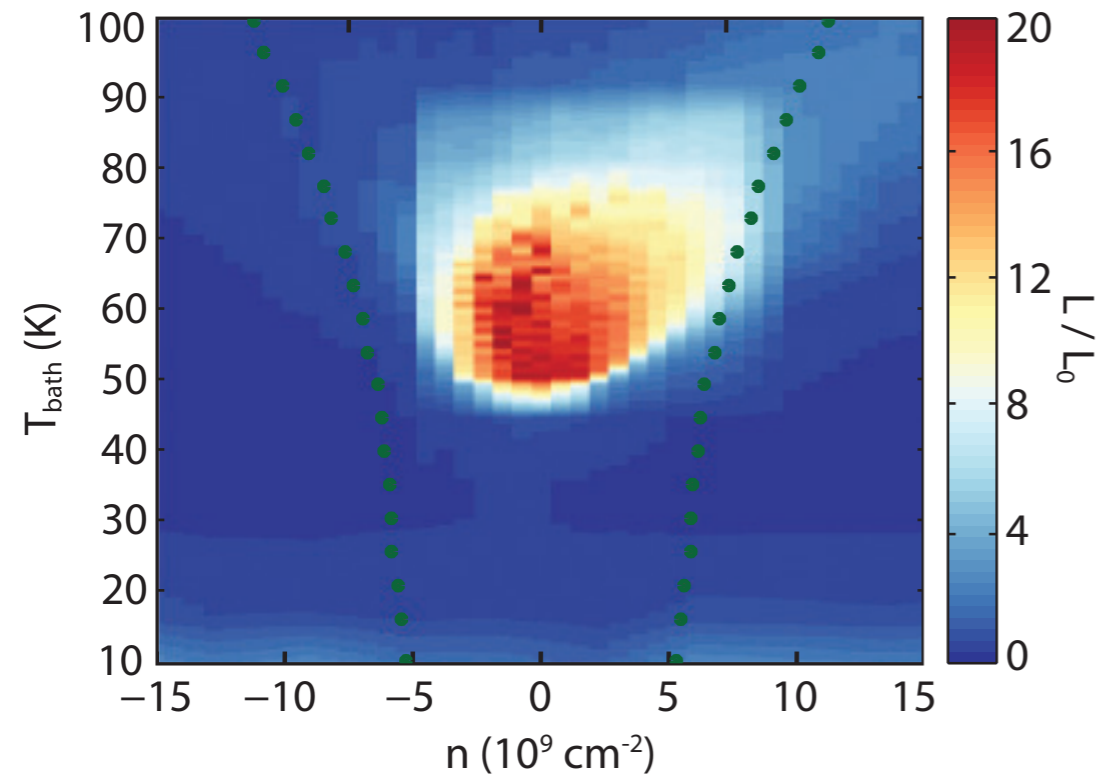
Wiedemann-Franz  
obeyed

J. Crossno et al., Science **351**, 1058 (2016)

# Strange metal in graphene



**Wiedemann-Franz  
violated !**



Lorentz ratio  $L = \kappa / (T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}$$

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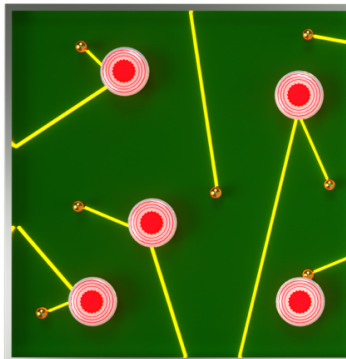
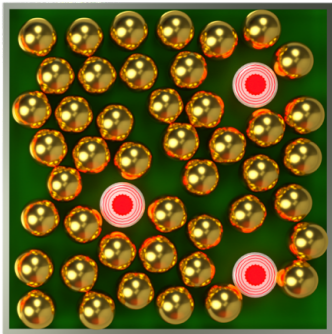
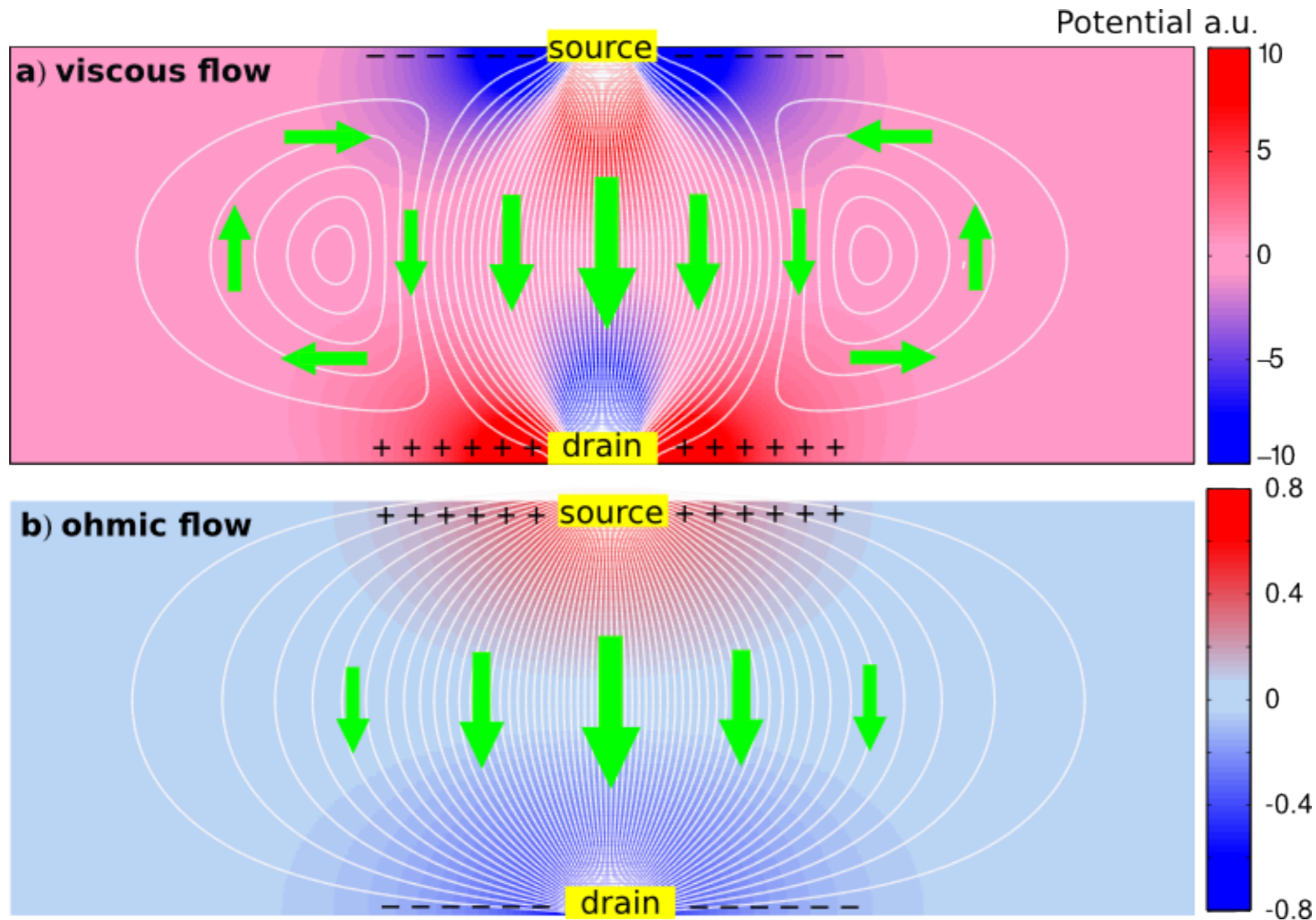
$\tau_{\text{imp}} \rightarrow$  momentum relaxation time from impurities

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J. Crossno et al., Science **351**, 1058 (2016)

# Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene



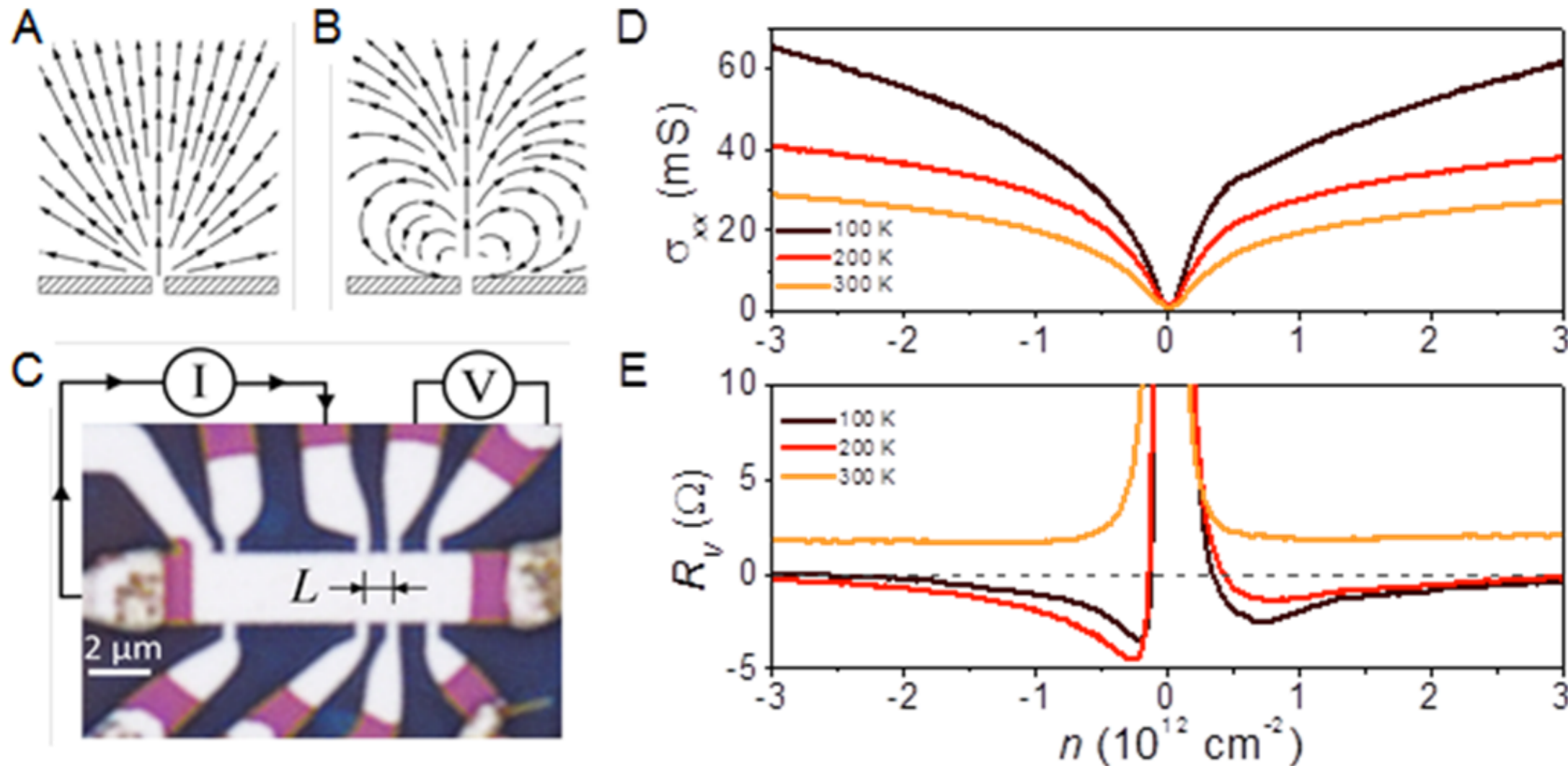
L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

# Strange metal in graphene

Science 351, 1055 (2016)

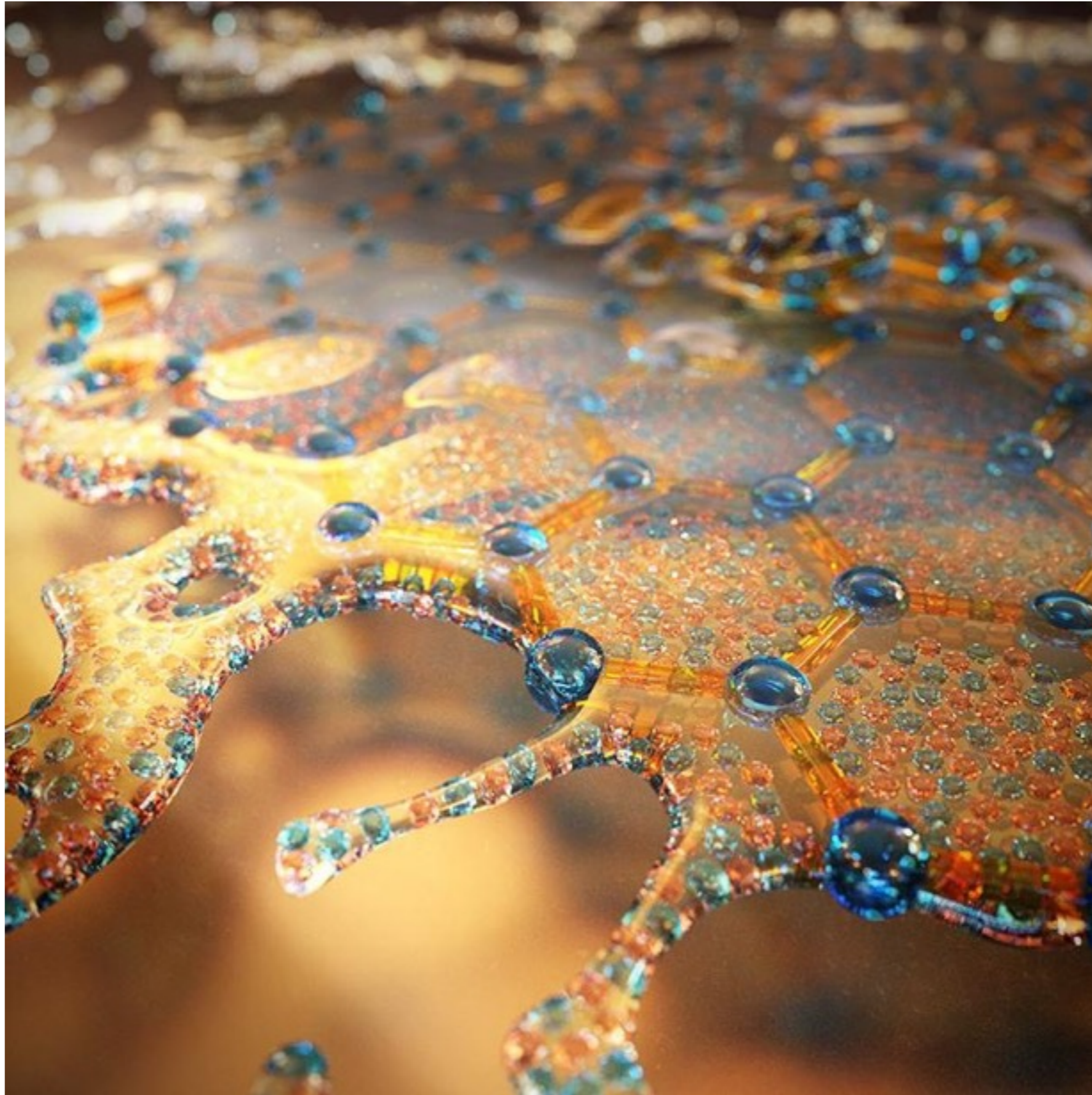
## Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin<sup>1</sup>, I. Torre<sup>2,3</sup>, R. Krishna Kumar<sup>1,4</sup>, M. Ben Shalom<sup>1,5</sup>, A. Tomadin<sup>6</sup>, A. Principi<sup>7</sup>, G. H. Auton<sup>5</sup>, E. Khestanova<sup>1,5</sup>, K. S. Novoselov<sup>5</sup>, I. V. Grigorieva<sup>1</sup>, L. A. Ponomarenko<sup>1,4</sup>, A. K. Geim<sup>1</sup>, M. Polini<sup>3,6</sup>



**Figure 1.** Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero  $\nu$  (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity  $\sigma_{xx}$  and  $R_V$  for this device as a function of  $n$  induced by applying gate voltage.  $I = 0.3 \mu\text{A}$ ;  $L = 1 \mu\text{m}$ . For more detail, see Supplementary Information.

# Graphene: “a metal that behaves like water”



## Quantum matter without quasiparticles:

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- *Charged black hole horizons in anti-de Sitter space*

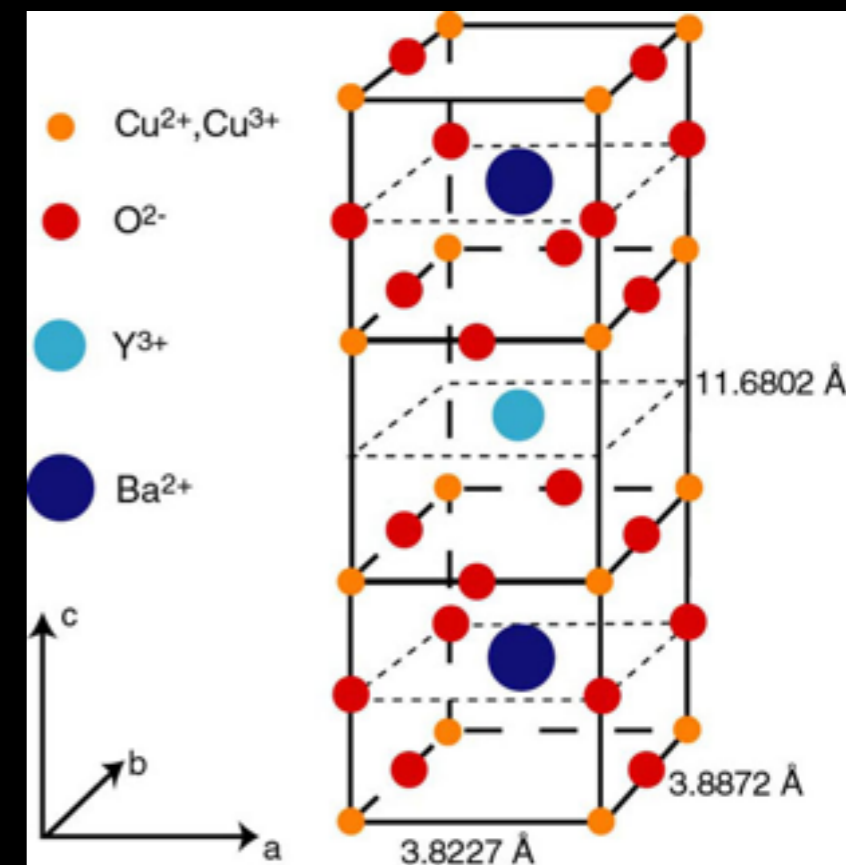
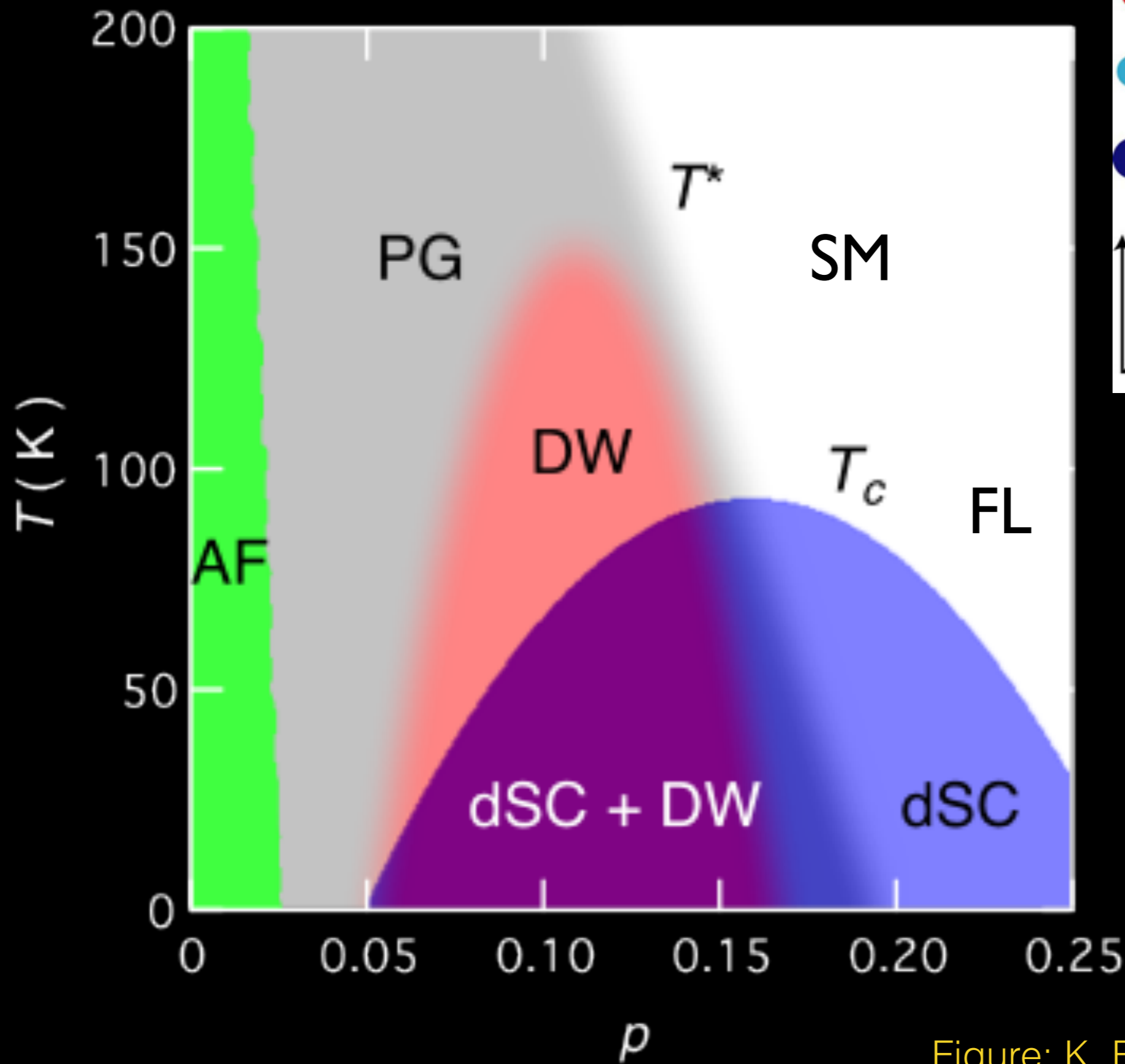
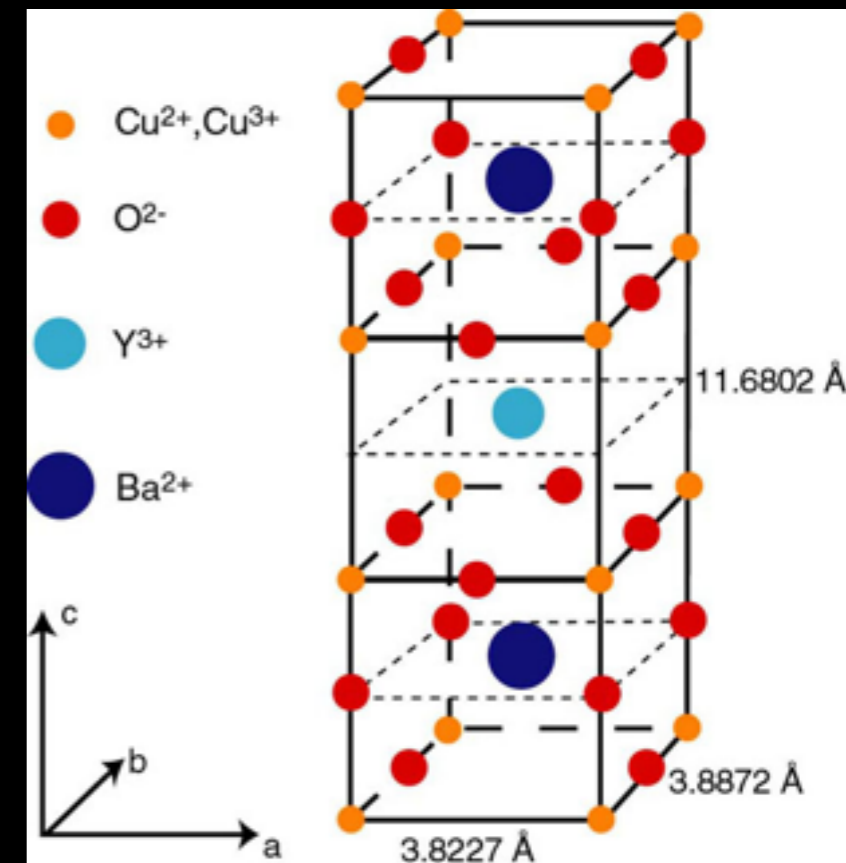
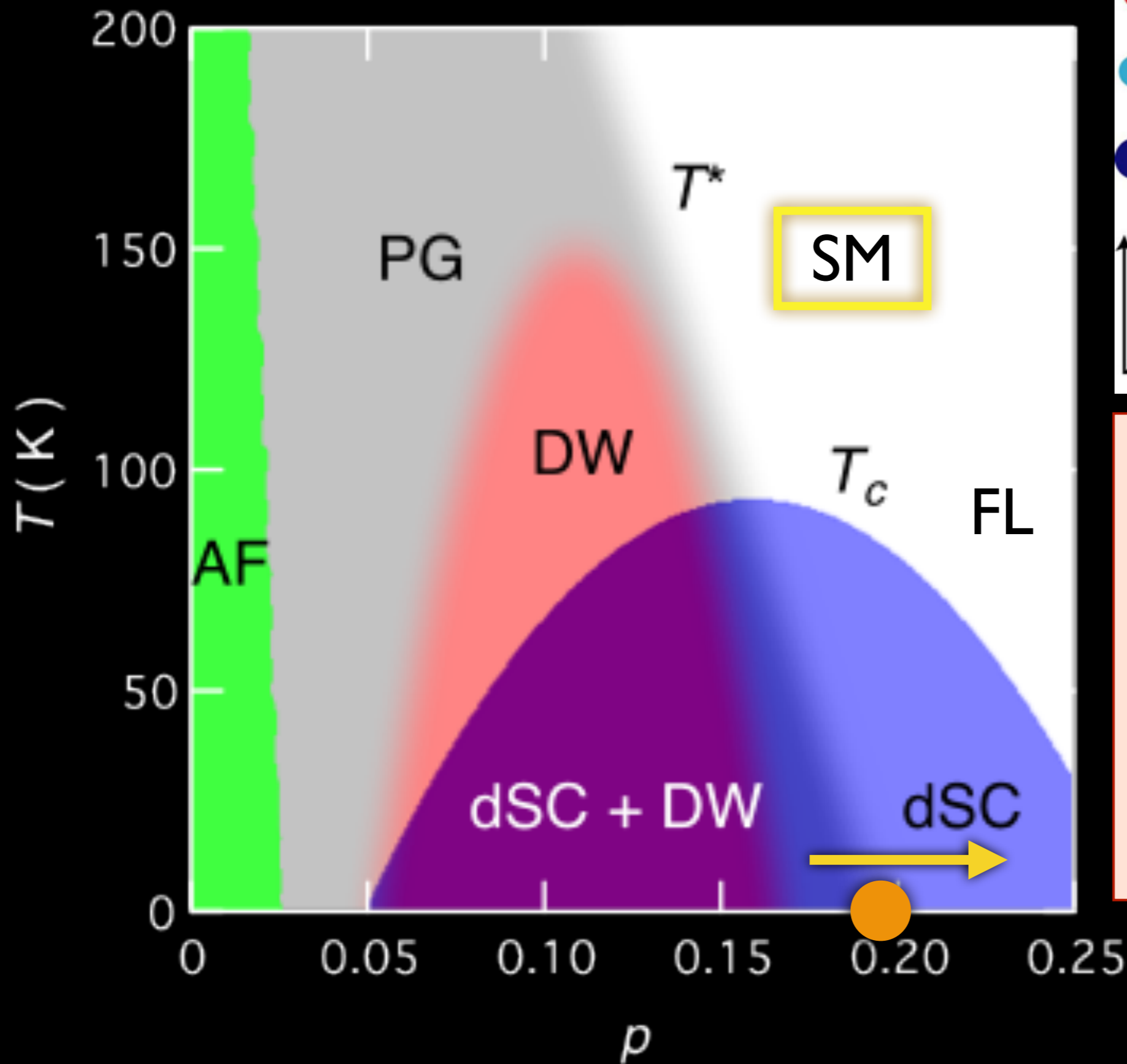
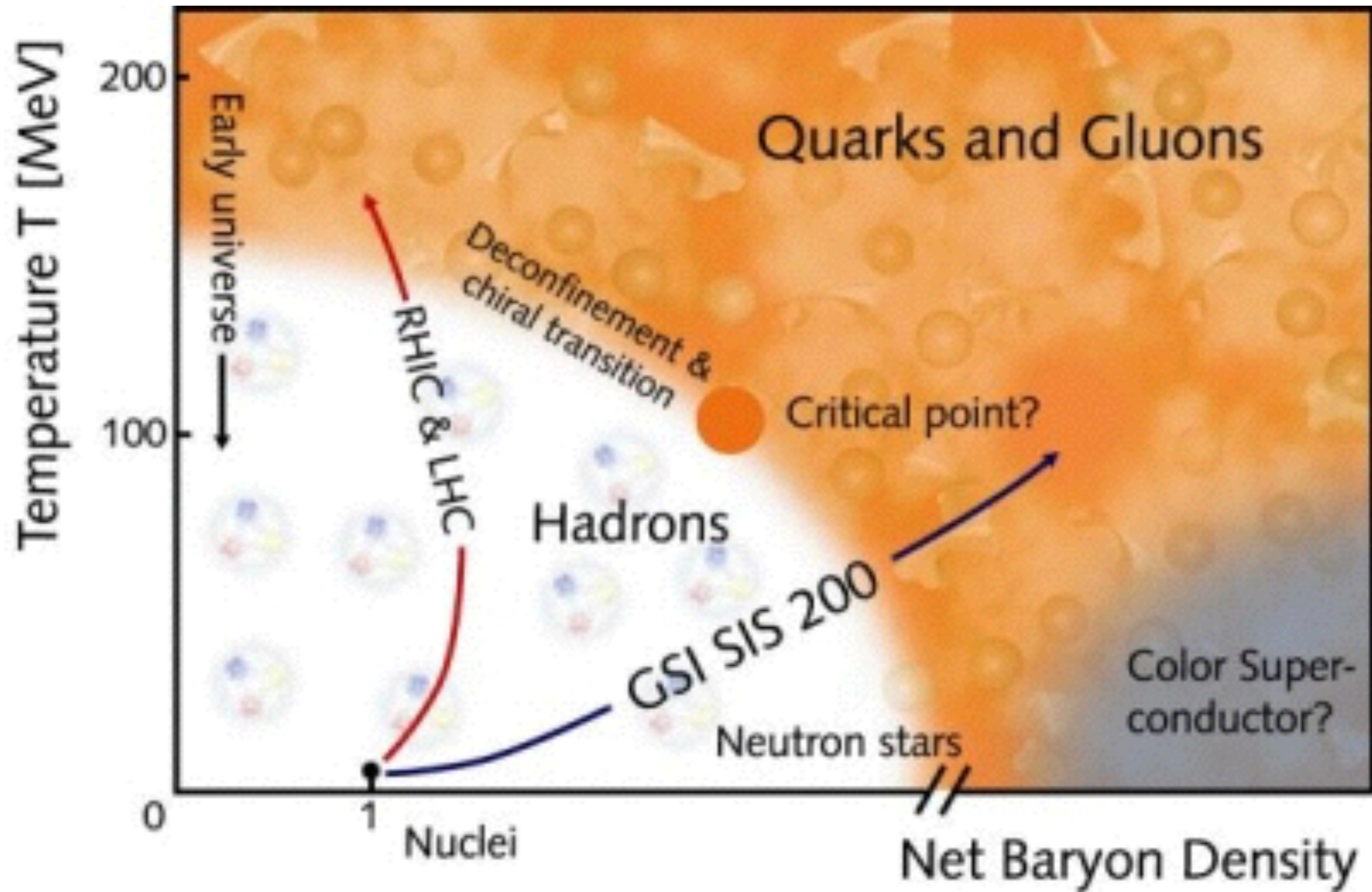


Figure: K. Fujita and J. C. Seamus Davis



Proposed a  $\text{SU}(2)$  gauge theory for transition for quantum-criticality at optimal doping, as the origin of strange metal (SM) behavior at higher  $T$

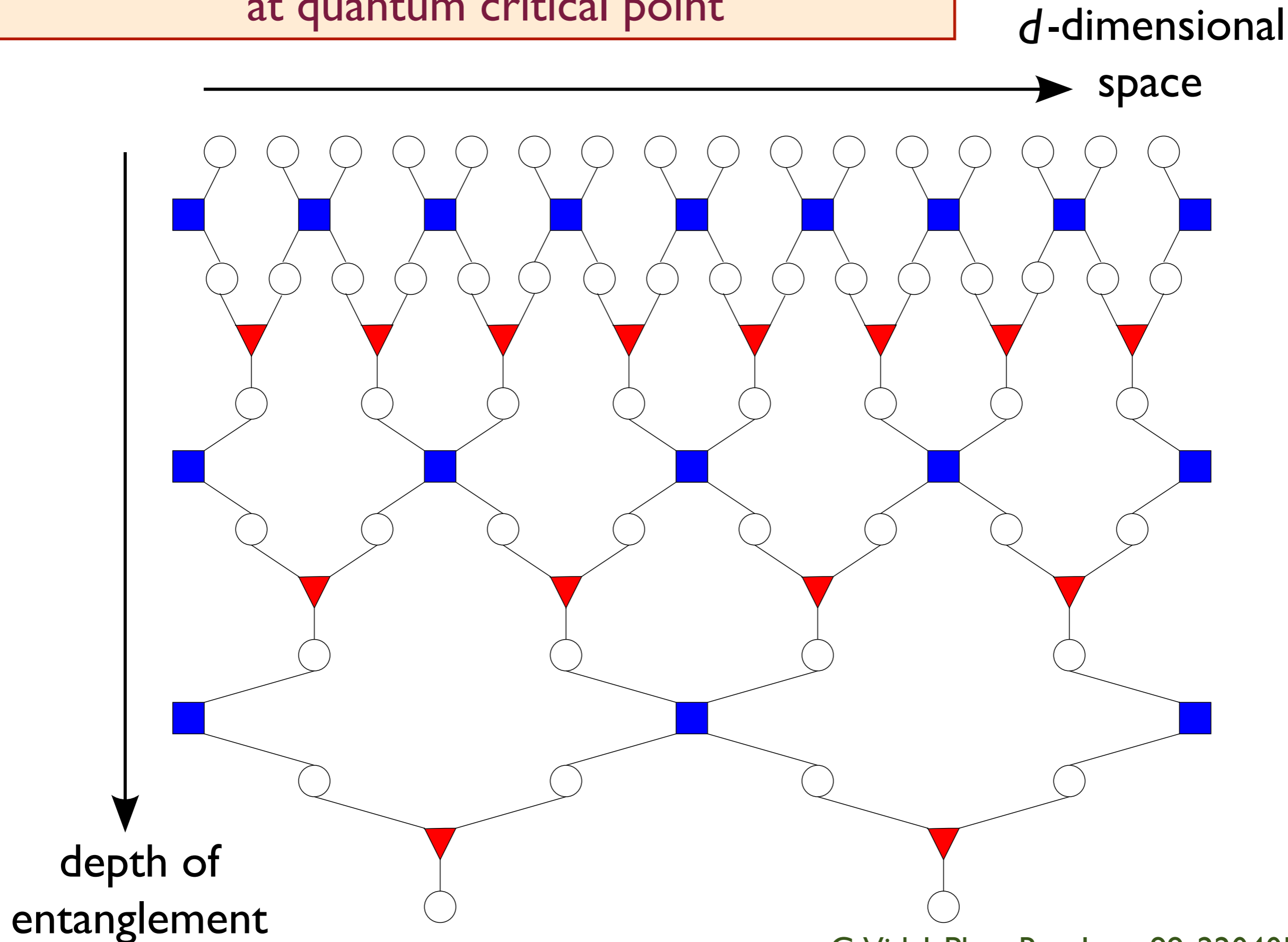
# Quark-gluon plasma



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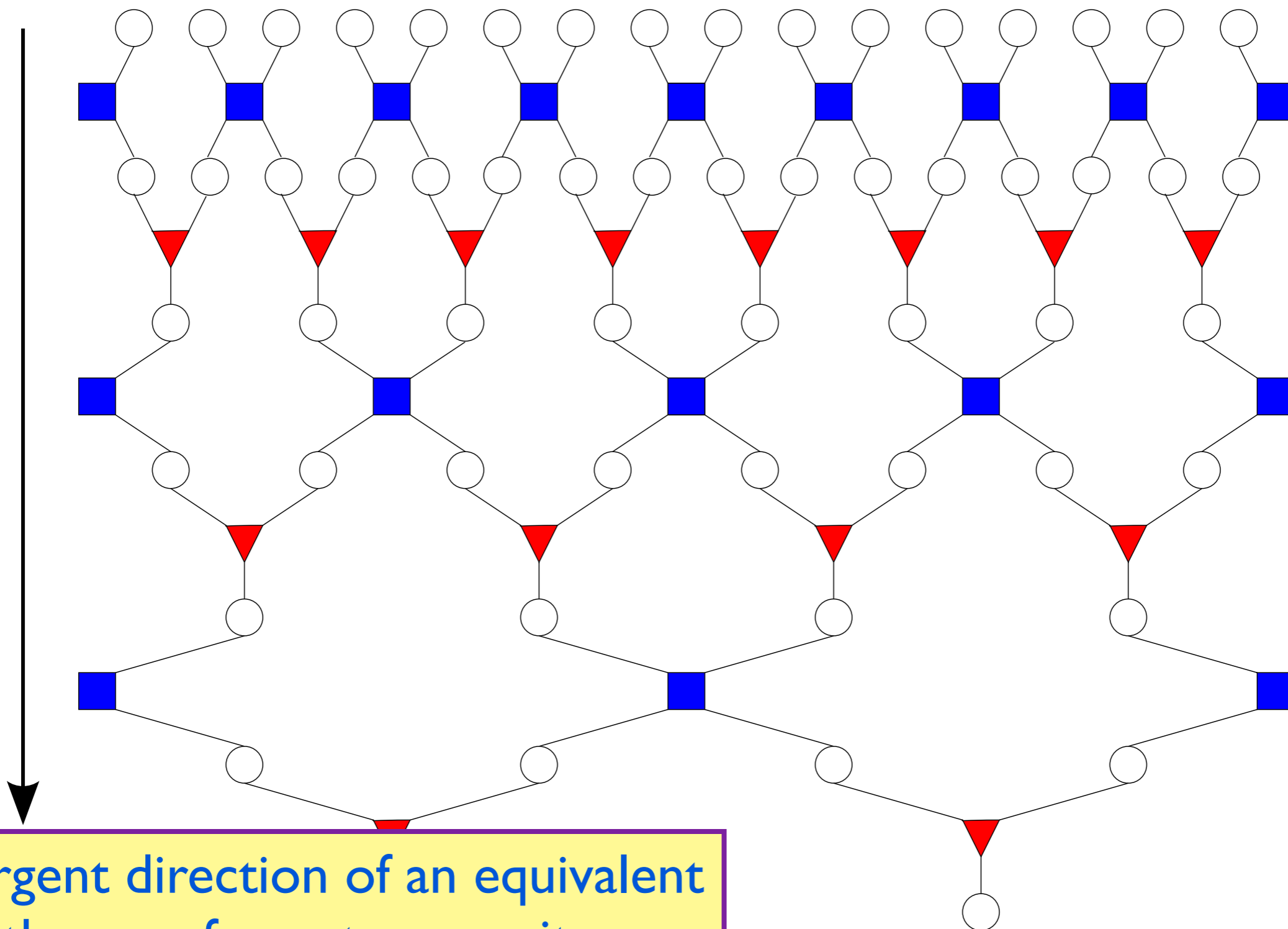
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# Tensor network representation of entanglement at quantum critical point



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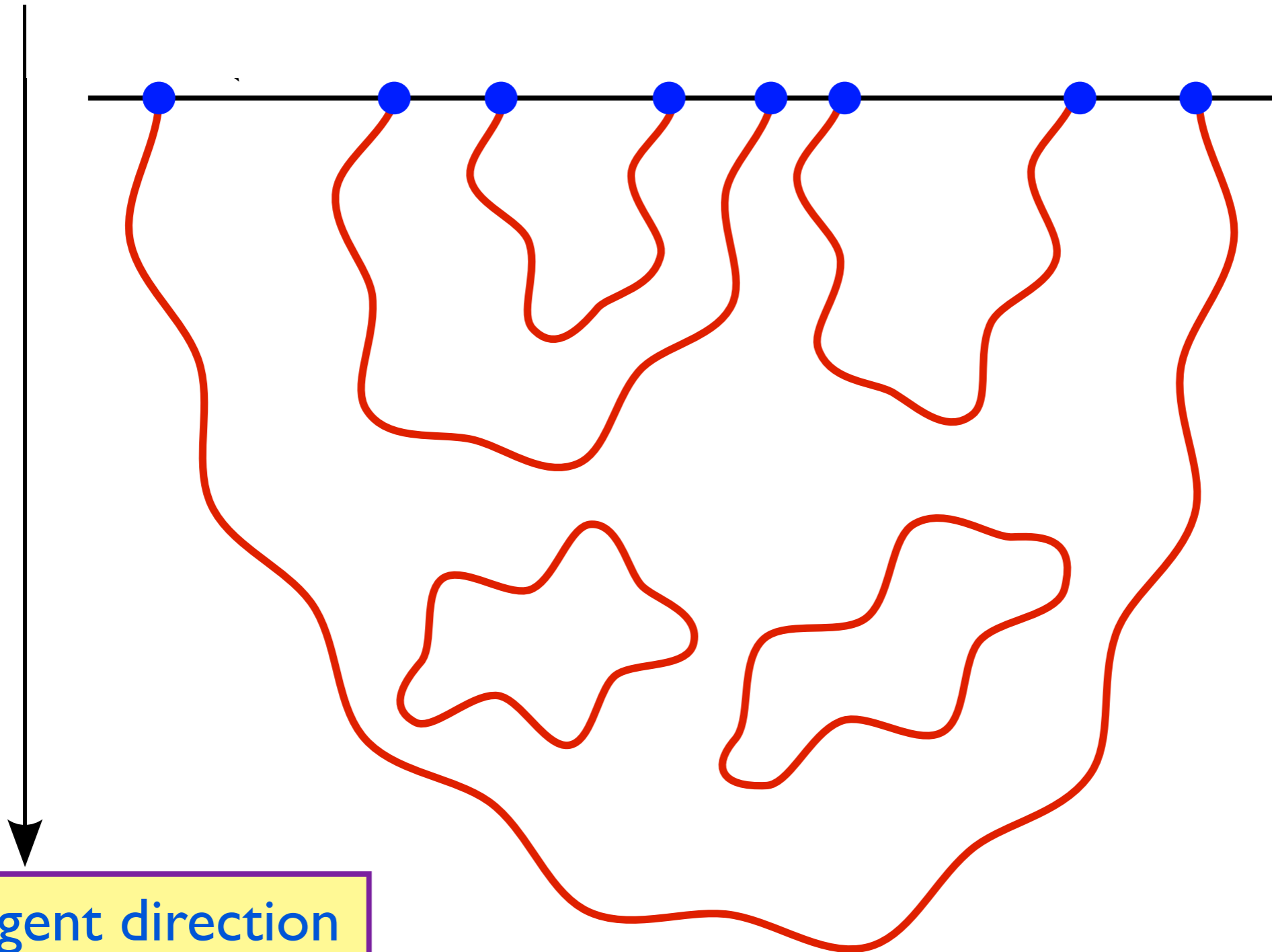
$d$ -dimensional  
space



Emergent direction of an equivalent  
theory of quantum gravity

String theory near  
a D-brane

$d$ -dimensional  
space



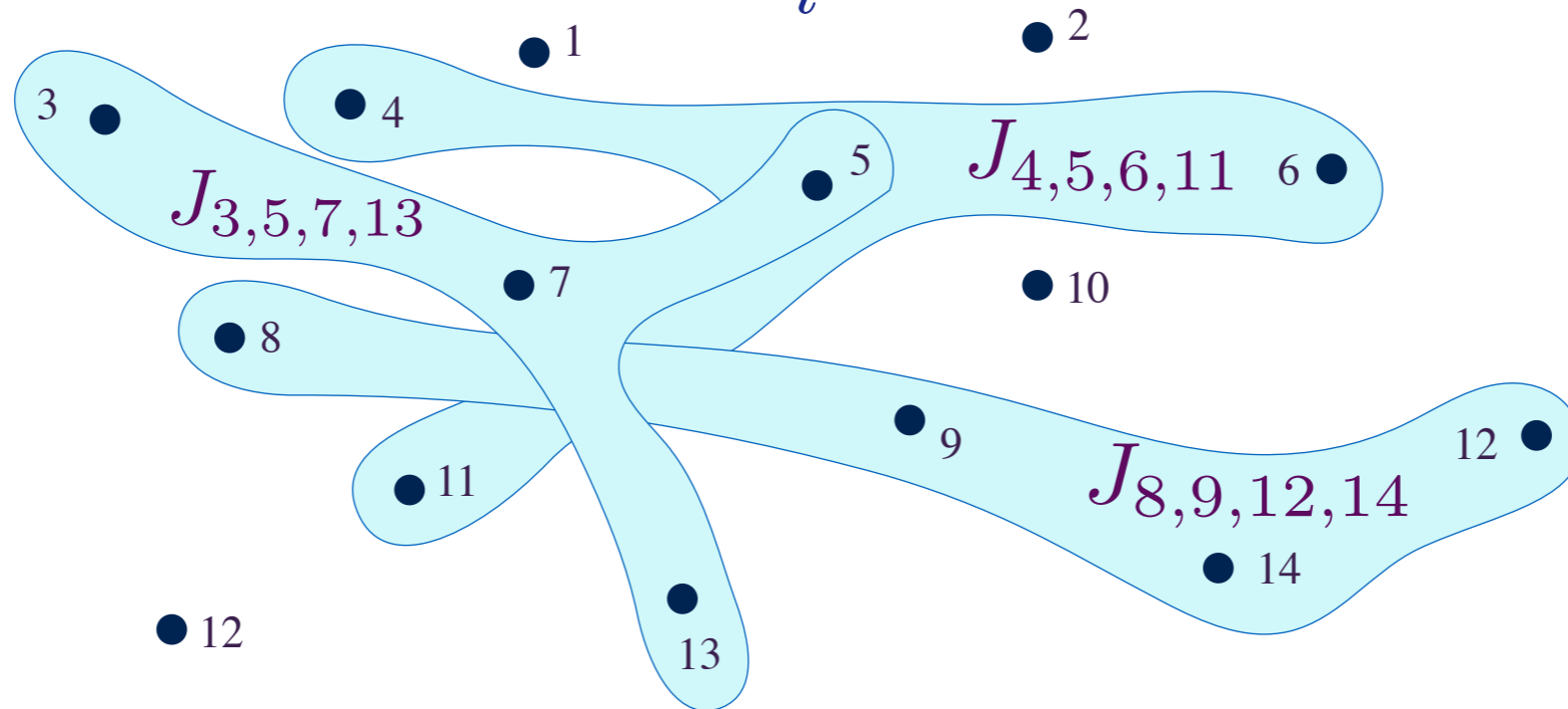
Emergent direction  
of AdS<sub>4</sub>

# Infinite-range (SYK) model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;kl}$  are independent random variables with  $\overline{J_{ij;kl}} = 0$  and  $\overline{|J_{ij;kl}|^2} = J^2$   
 $N \rightarrow \infty$  yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

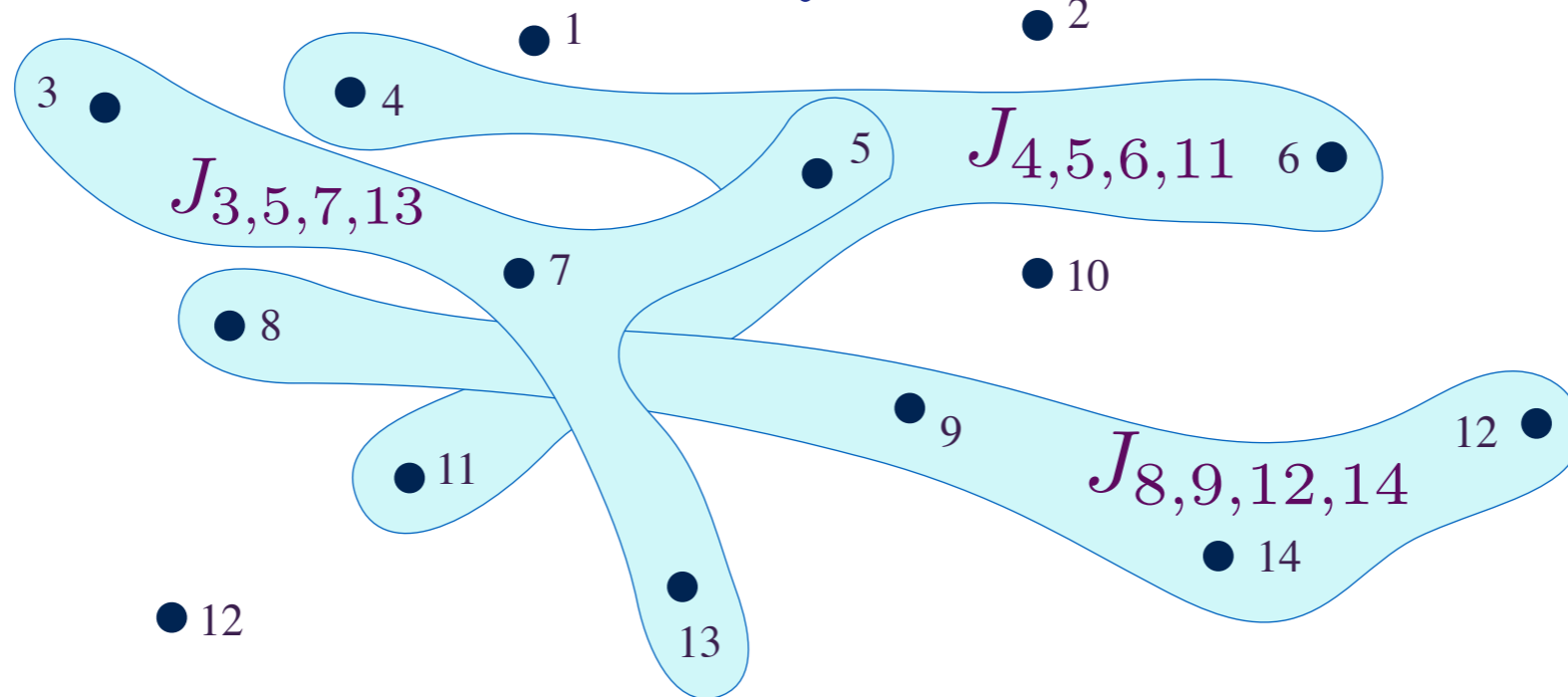
A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

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A fermion can move only by entangling with another fermion:  
the Hamiltonian has “nothing but entanglement”.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

## Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\text{Im } G(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$  encodes the particle-hole asymmetry

While  $\mathcal{E}$  determines the *low* energy spectrum, it is determined by the *total* fermion density  $\mathcal{Q}$ :

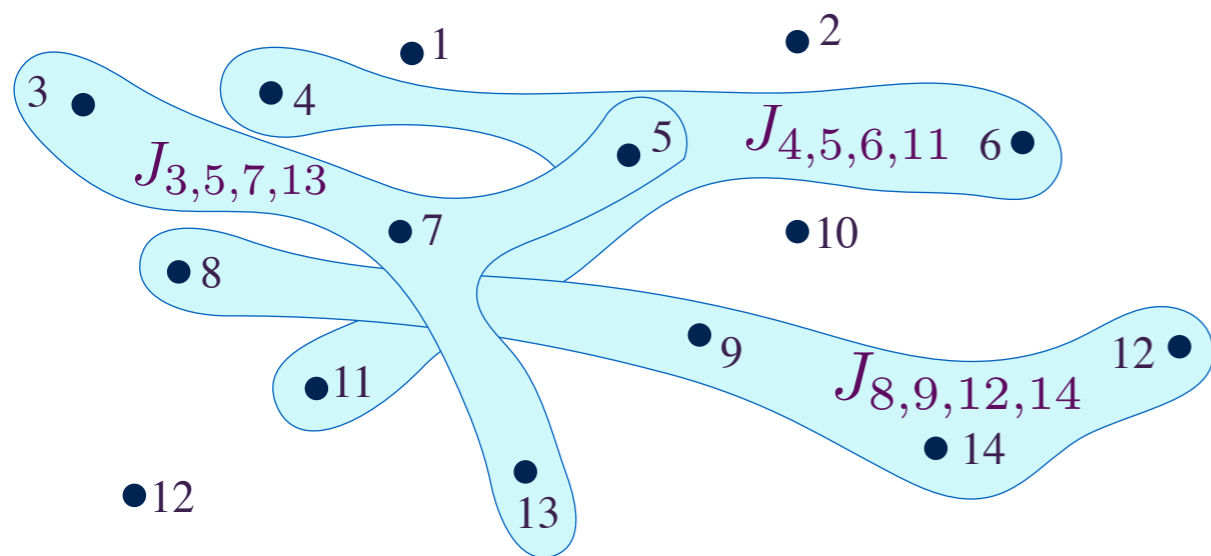
$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1}(e^{2\pi\mathcal{E}}).$$

Analog of the relationship between  $\mathcal{Q}$  and  $k_F$  in a Fermi liquid.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

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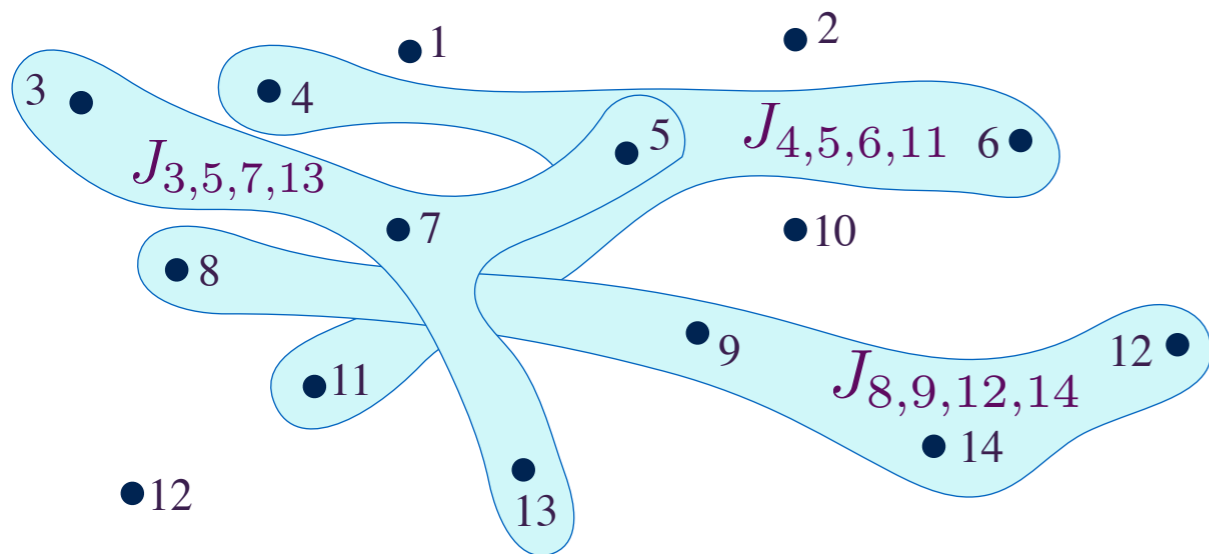
$$Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle.$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state'  
determines  $\mathcal{E}$  as a function of  $Q$

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Microscopic zero temperature  
entropy density,  $\mathcal{S}$ , obeys

$$\frac{\partial \mathcal{S}}{\partial Q} = 2\pi\mathcal{E}$$

# Holographic gravity theory

Start with simplest theory of Einstein gravity and Maxwell electromagnetism

$$\mathcal{S}_{EM} = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right]$$

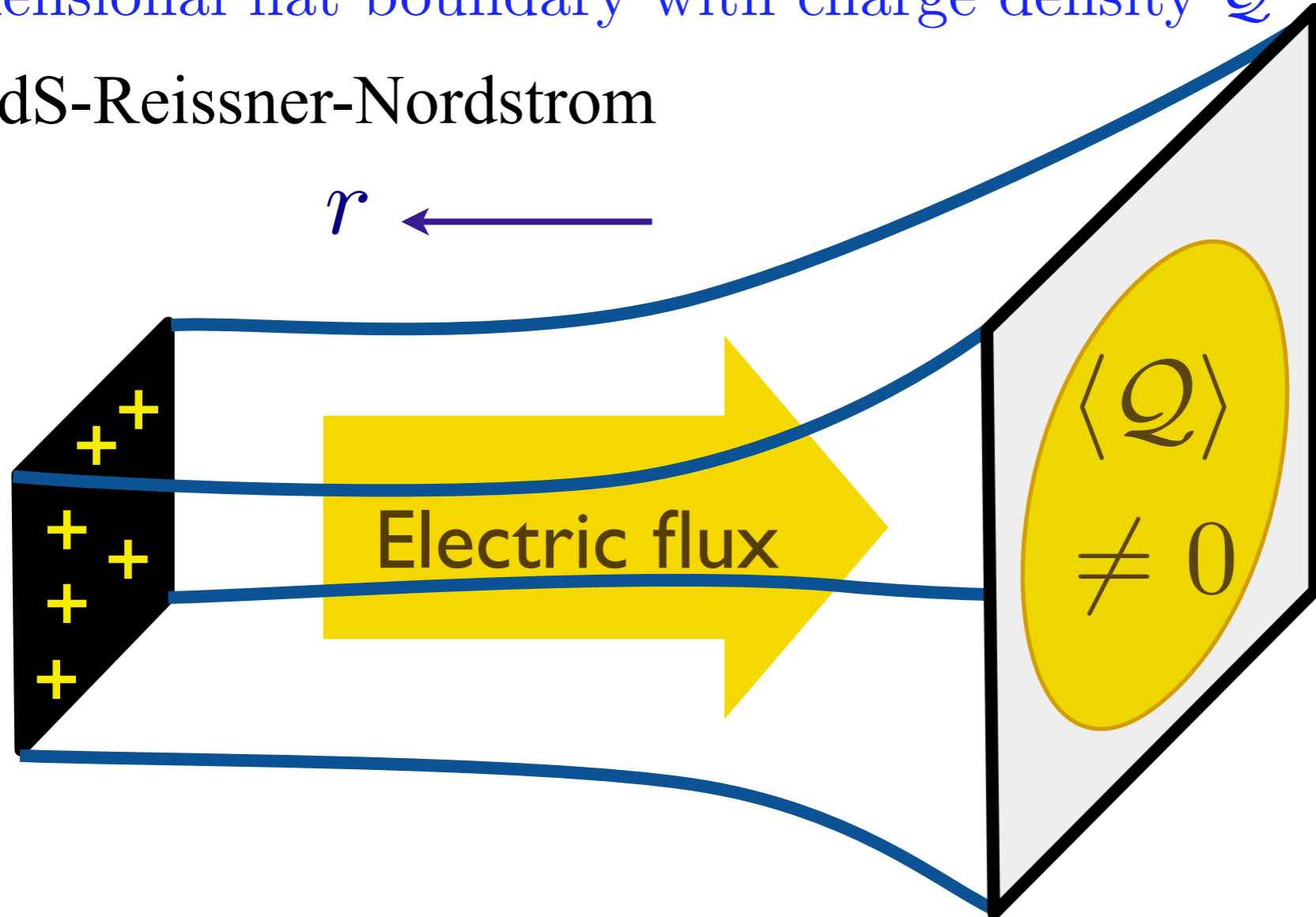
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Solve equations of motion in the presence of a  $d$ -dimensional flat boundary with charge density  $\mathcal{Q}$

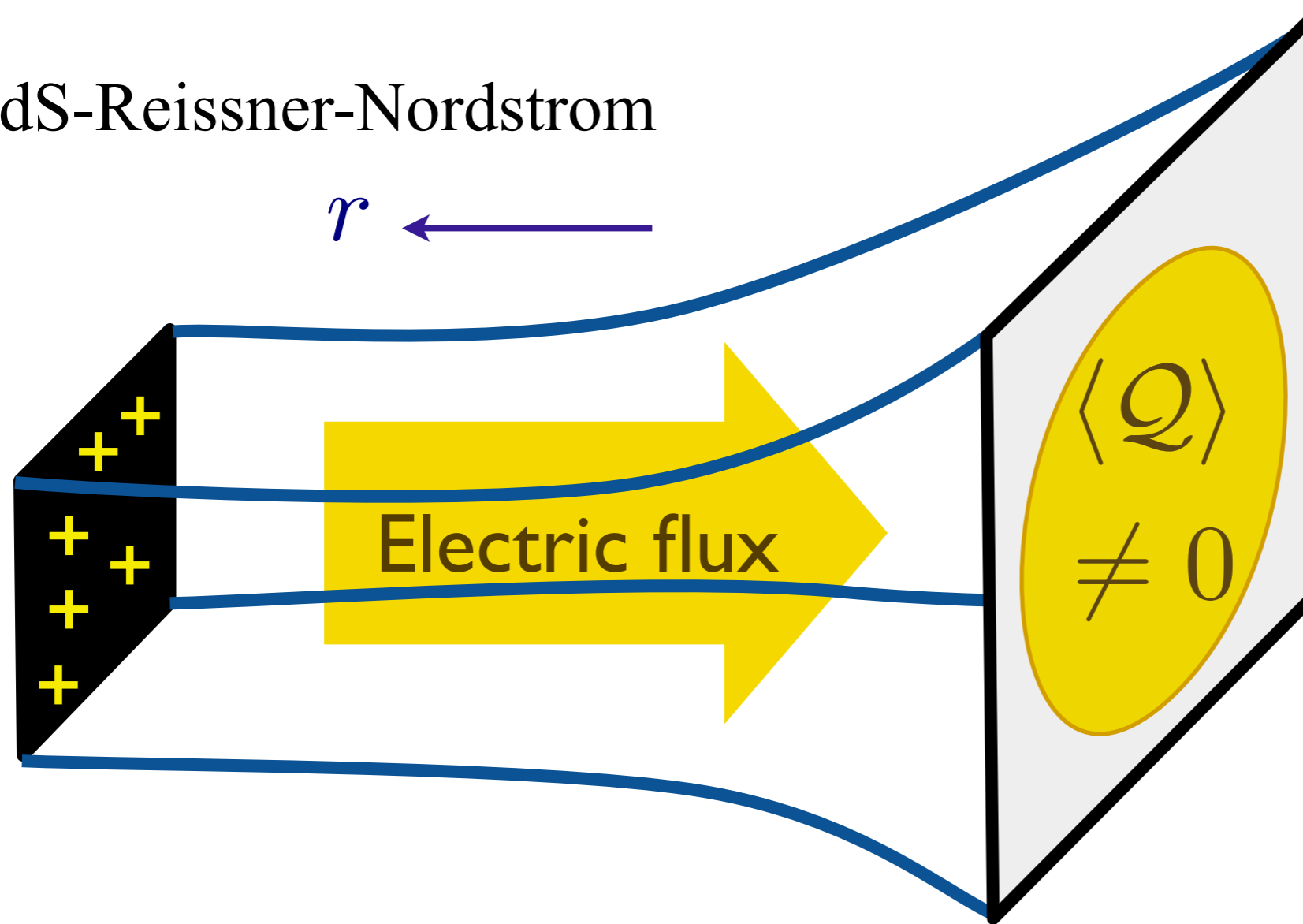
AdS-Reissner-Nordstrom



Quantum matter on the boundary with a variable charge density  $\mathcal{Q}$  of a global U(1) symmetry.

# Charged black branes

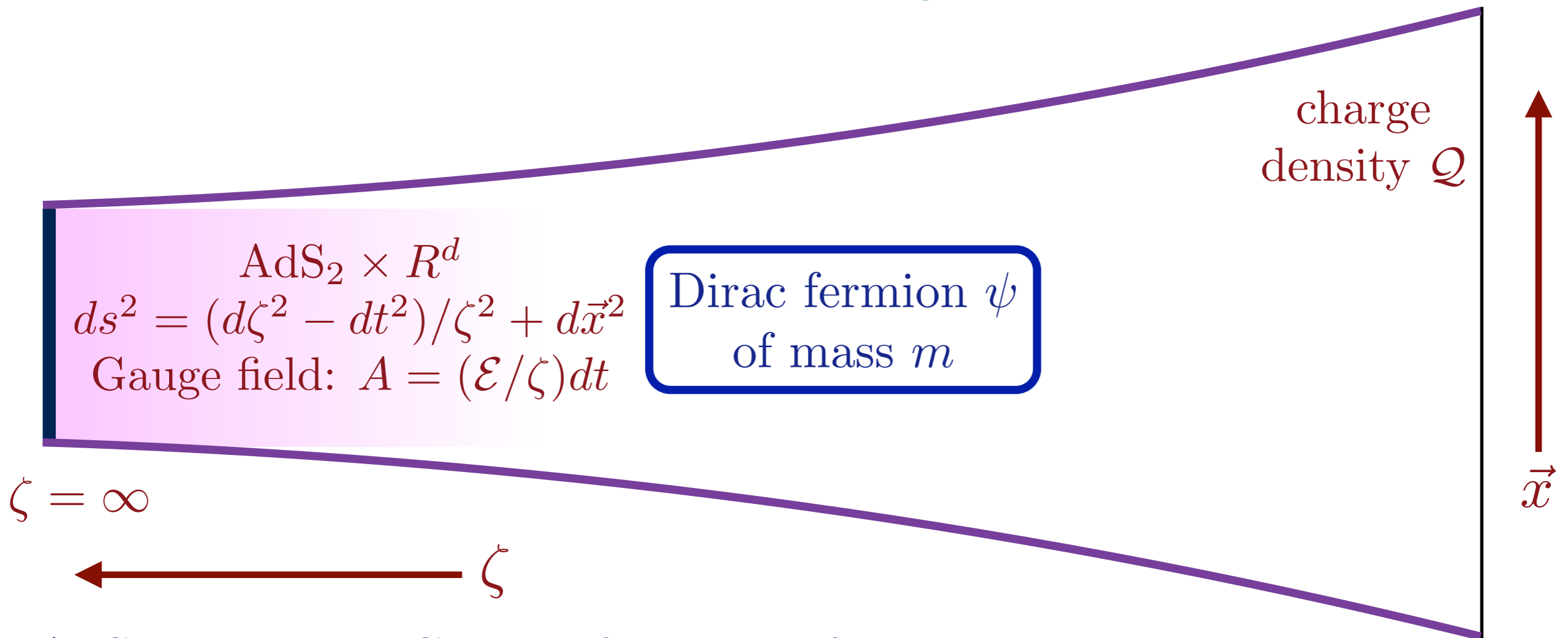
AdS-Reissner-Nordstrom



Quantum matter on the boundary with a variable charge density  $Q$  of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density,  $Q$ , at  $T = 0$  which does not have any quasiparticle excitations.

# Quantum fields on charged black branes



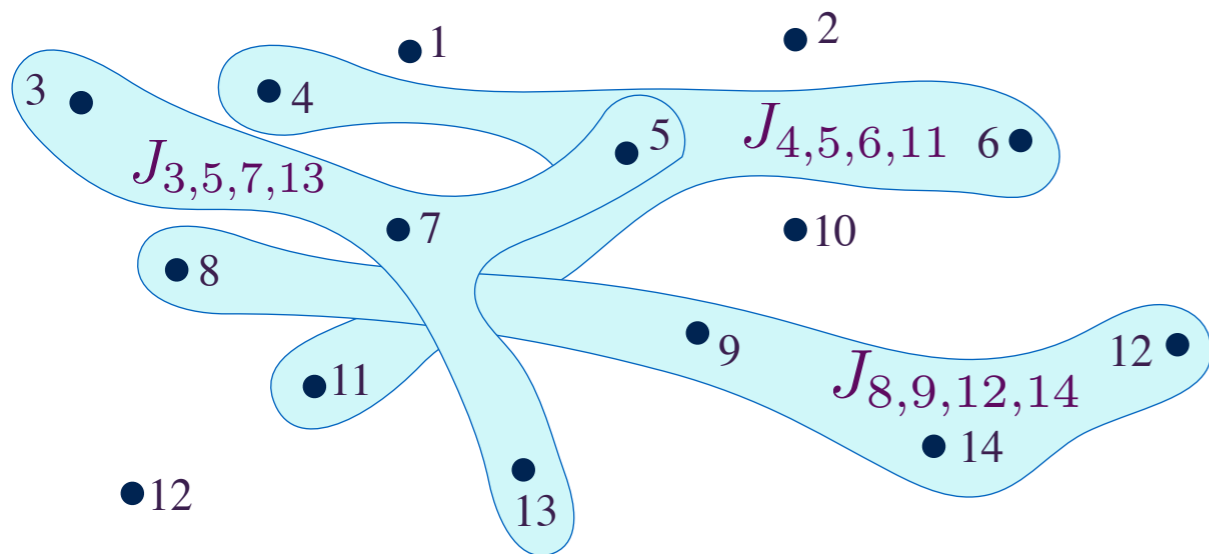
AdS<sub>2</sub> boundary Green's function of  $\psi$  at  $T = 0$

$$\text{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-(1-2\Delta)}, & \omega < 0. \end{cases}$$

where the fermion scaling dimension  $\Delta$  is a function of  $m$

$\mathcal{E}$  encodes the particle-hole asymmetry

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Local fermion density of states

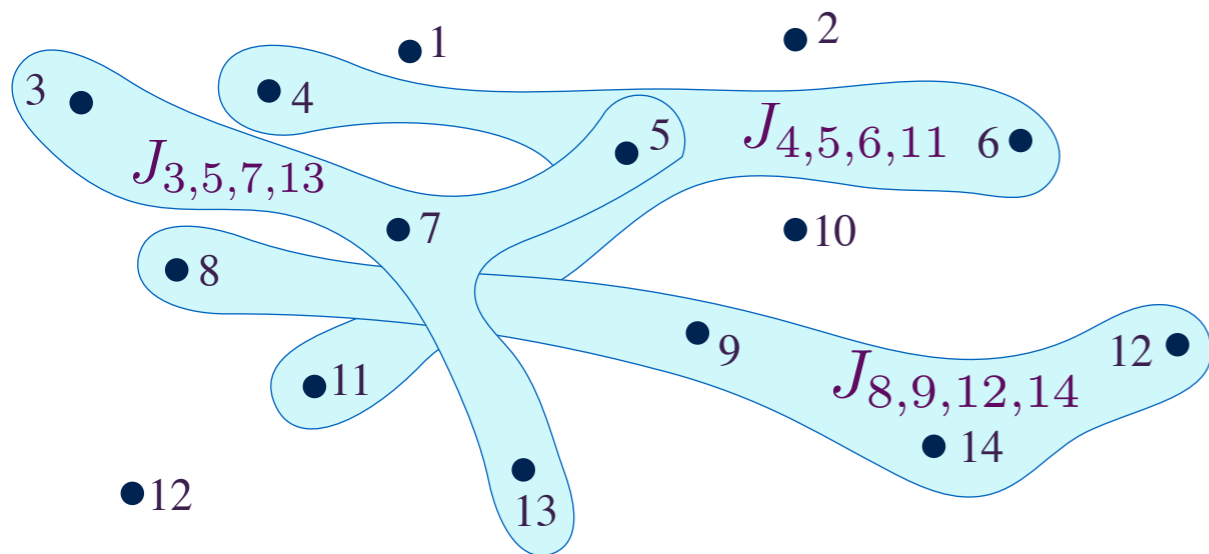
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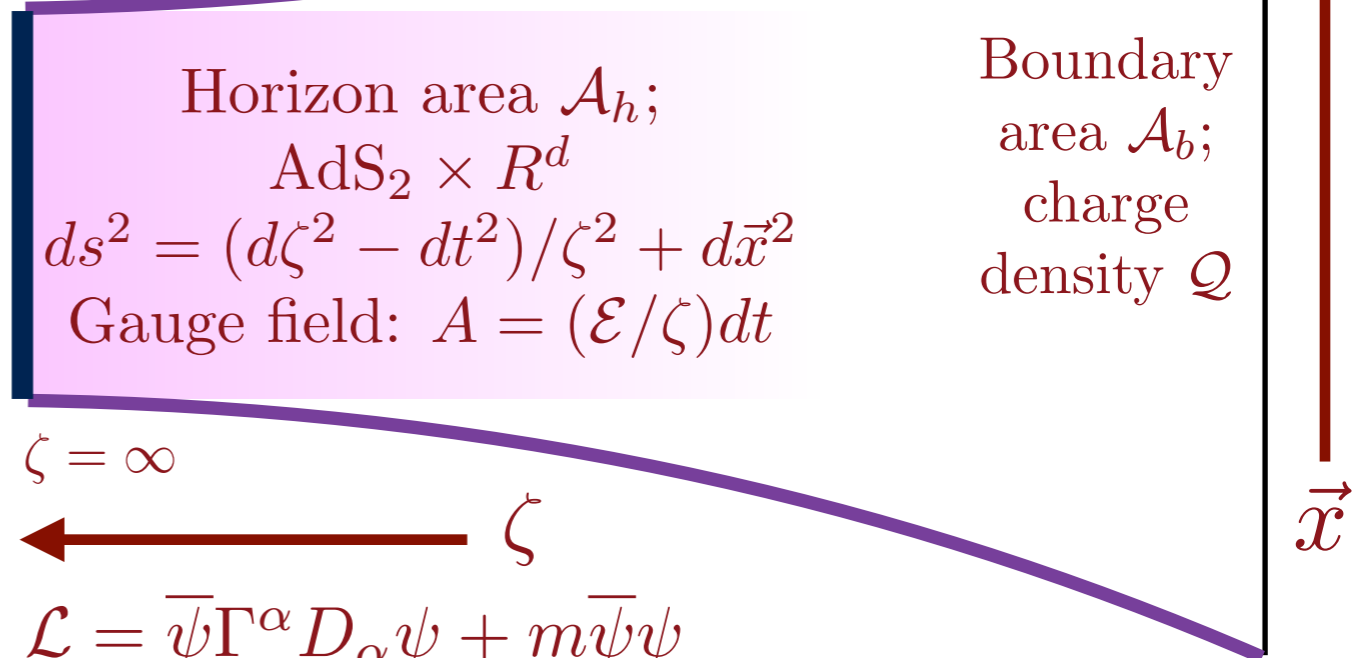
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Einstein-Maxwell theory  
+ cosmological constant



Horizon area  $\mathcal{A}_h$ ;  
 $\text{AdS}_2 \times R^d$   
 $ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$   
Gauge field:  $A = (\mathcal{E}/\zeta)dt$

Boundary  
area  $\mathcal{A}_b$ ;  
charge  
density  $\mathcal{Q}$

$\zeta = \infty$

$\zeta$

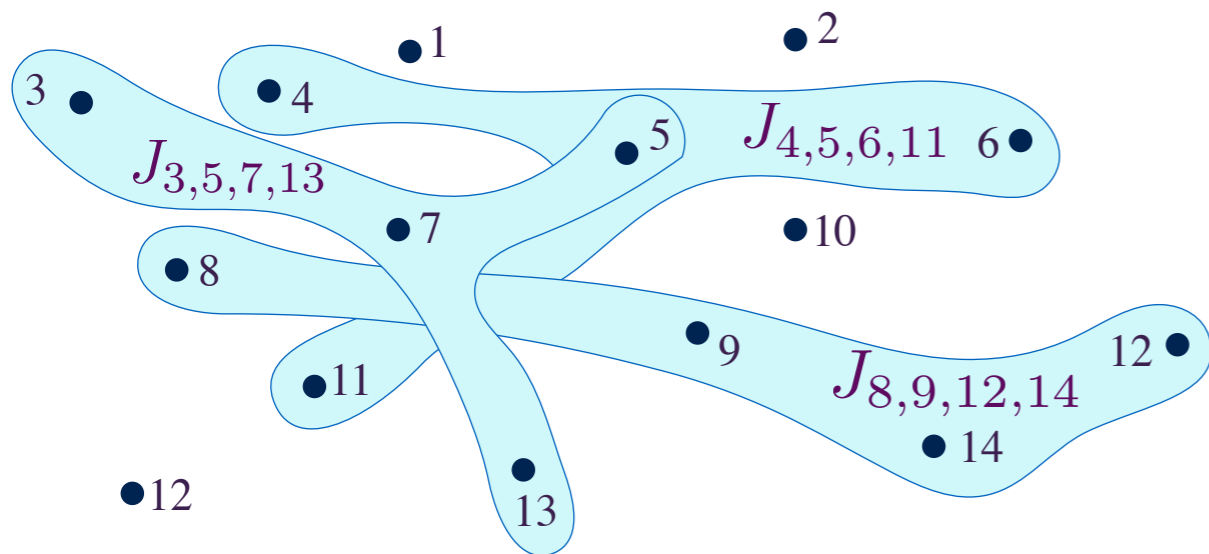
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‘Equation of state’ relating  $\mathcal{E}$   
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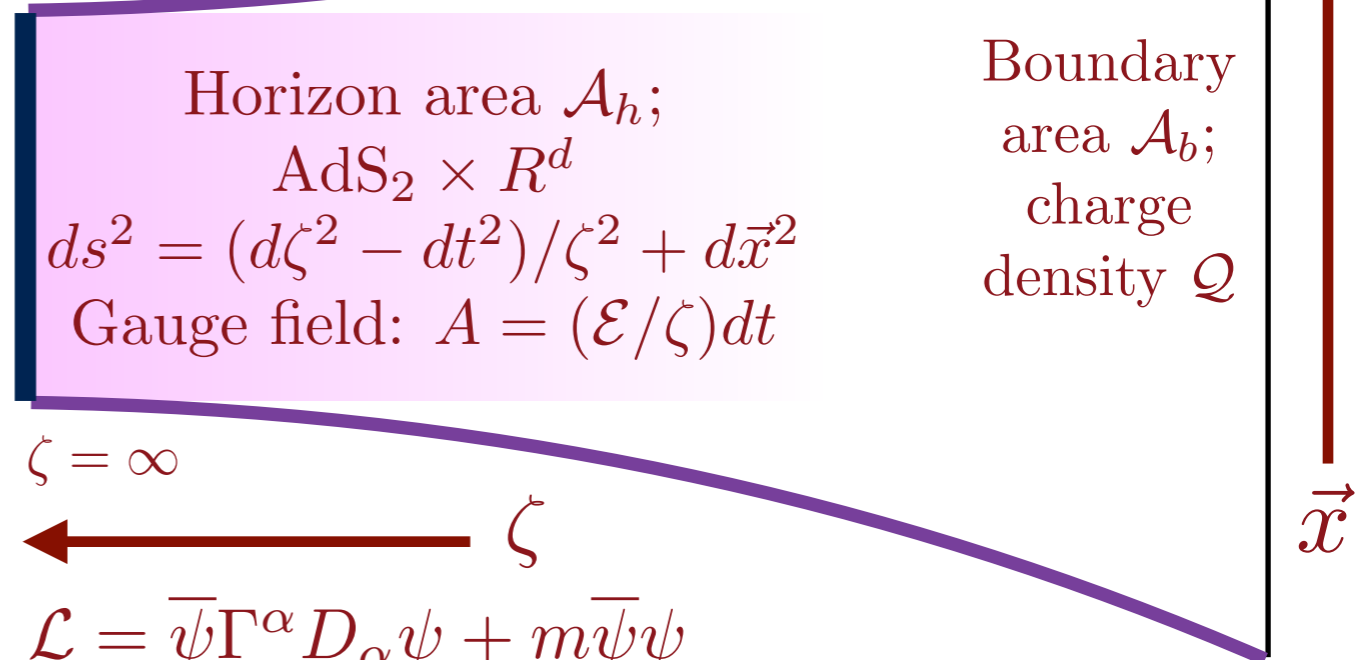
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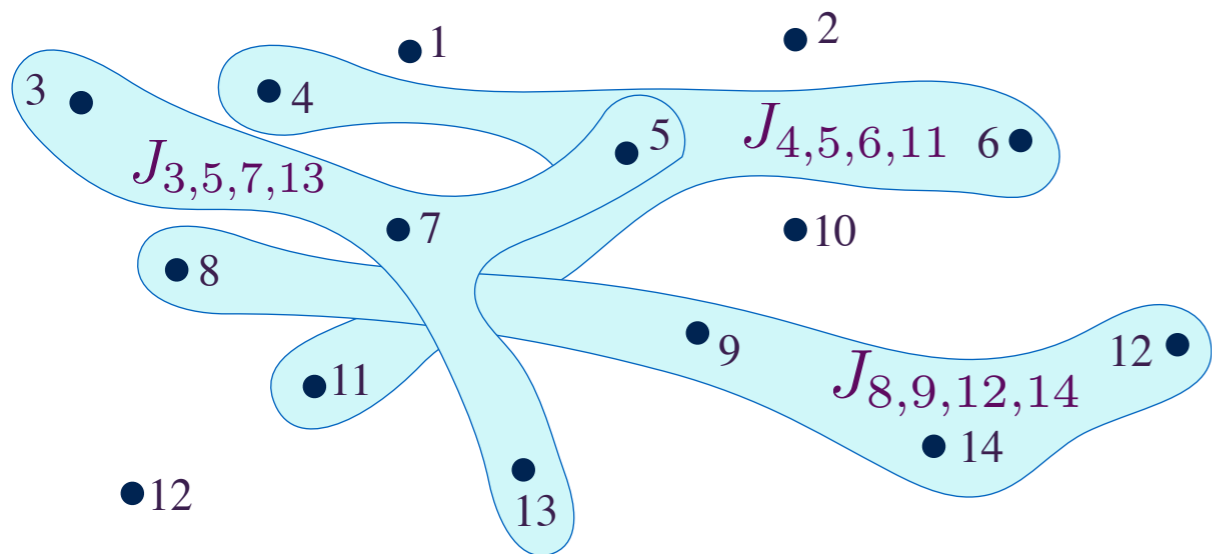
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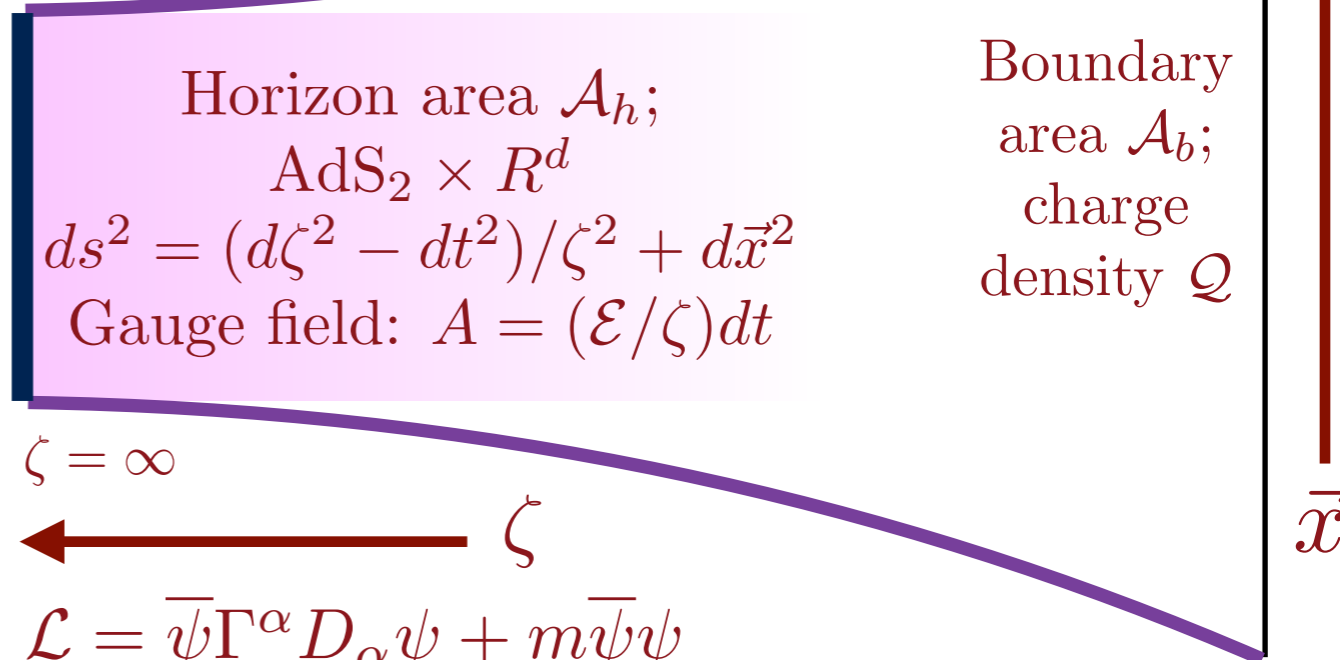
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Evidence for AdS<sub>2</sub> gravity dual of  $H$

Einstein-Maxwell theory + cosmological constant



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## A bound on quantum chaos:

- The “Lyapunov exponent” for chaos,  $\lambda_L$ , is given by out-of-time-order correlators, and for quantum systems near equilibrium, it obeys the bound  $\lambda_L \leq 2\pi k_B T / \hbar$ .

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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S. H. Shenker and D. Stanford, arXiv:1306.0622

- The bound is also saturated by the SYK model

A. Kitaev, unpublished

J. Polchinski and V. Rosenhaus, arXiv:1601.06768

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# Entangled quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time  $\sim \frac{\hbar}{k_B T}$
- Continuously variable density,  $\mathcal{Q}$   
(conformal field theories are usually at fixed density,  $\mathcal{Q} = 0$ )
- Theory built from hydrodynamics/holography  
/memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.
- Future work: detection of hydrodynamic flow in other strange metals . . . . .