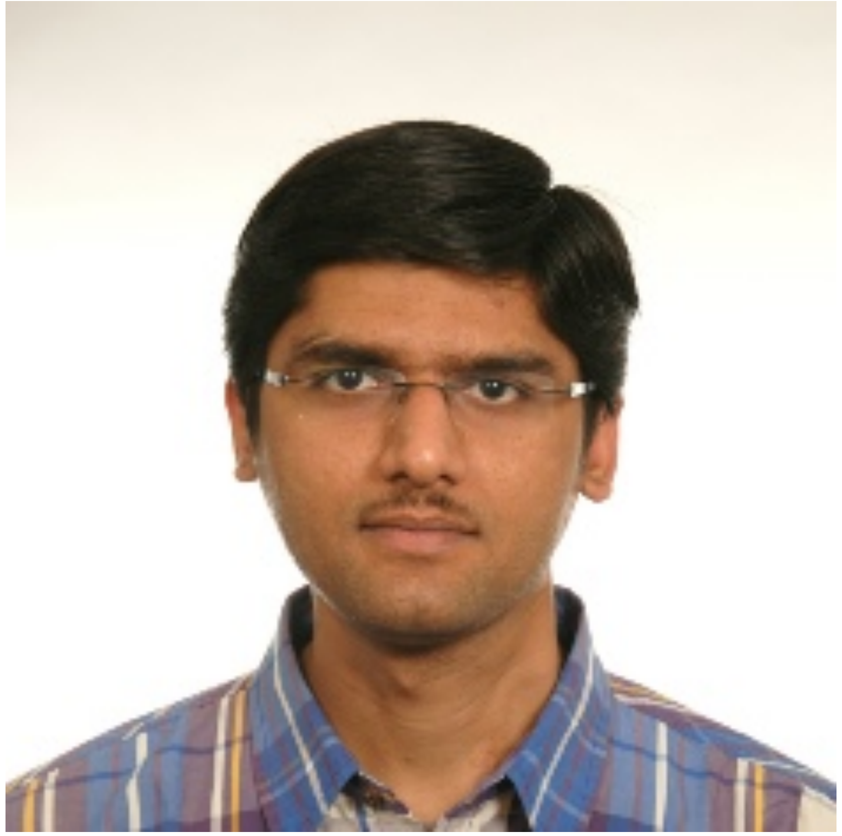
The strange quantum physics of the high temperature superconductors

University of Massachusetts, Amherst
March 11, 2020

Subir Sachdev

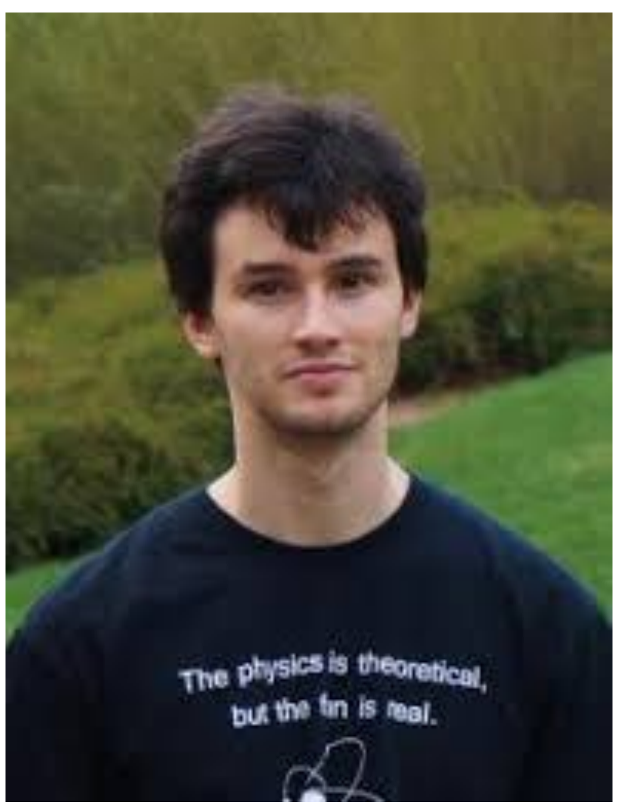
Talk online: sachdev.physics.harvard.edu



Darshan Joshi

Chenyuan Li

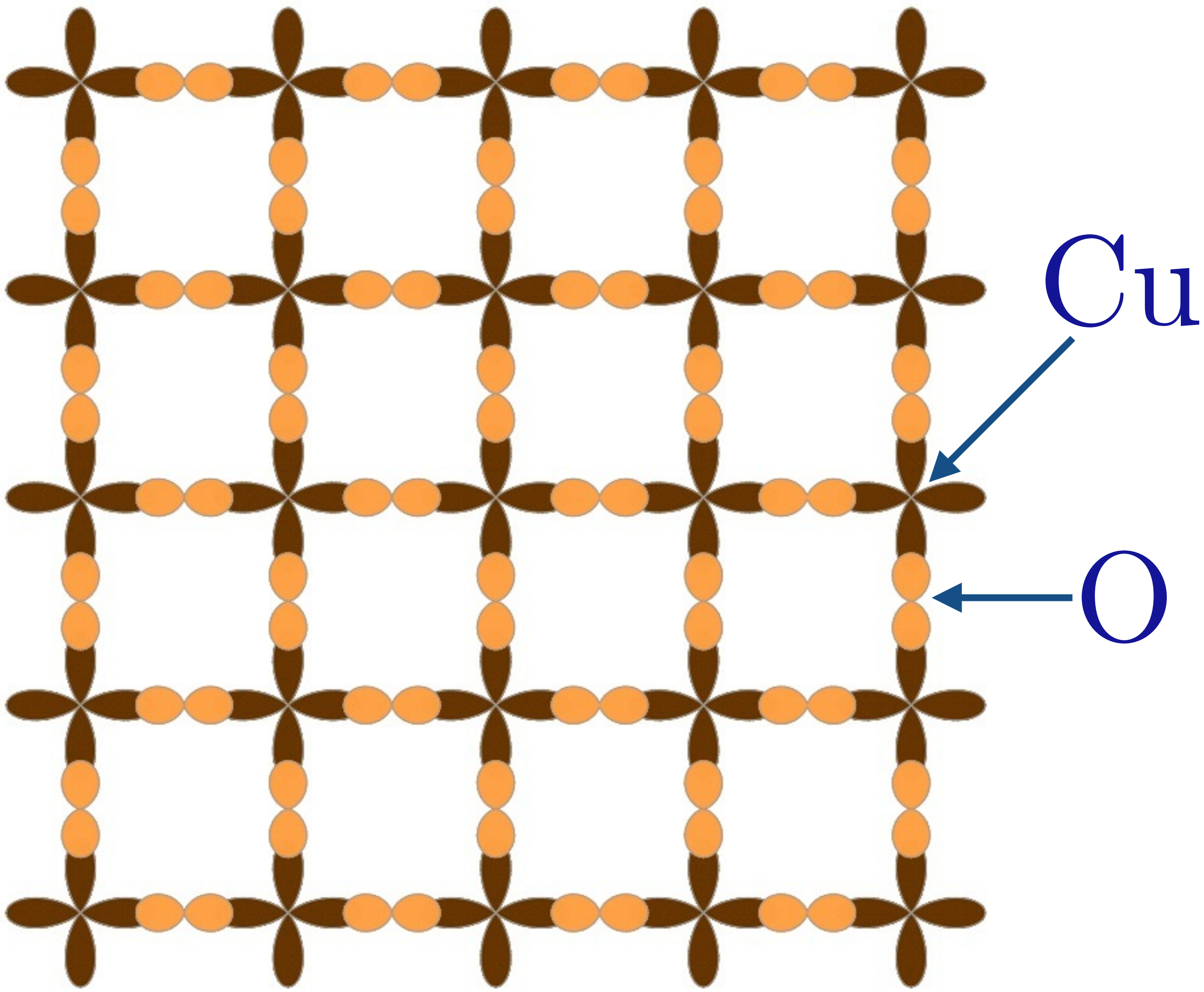
arXiv:1912.08822



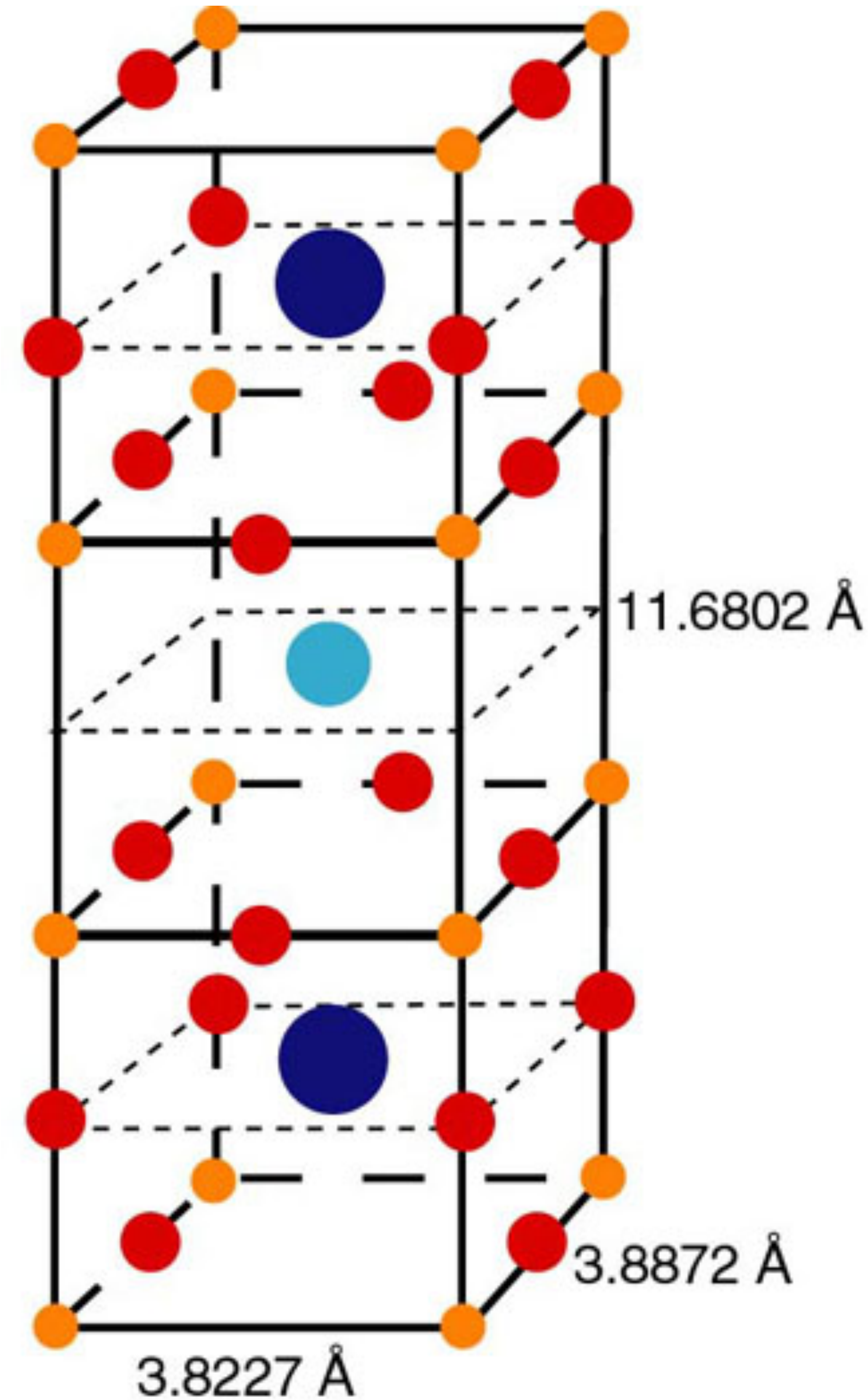
Grigory Tarnopolsky

Antoine Georges

High temperature superconductors

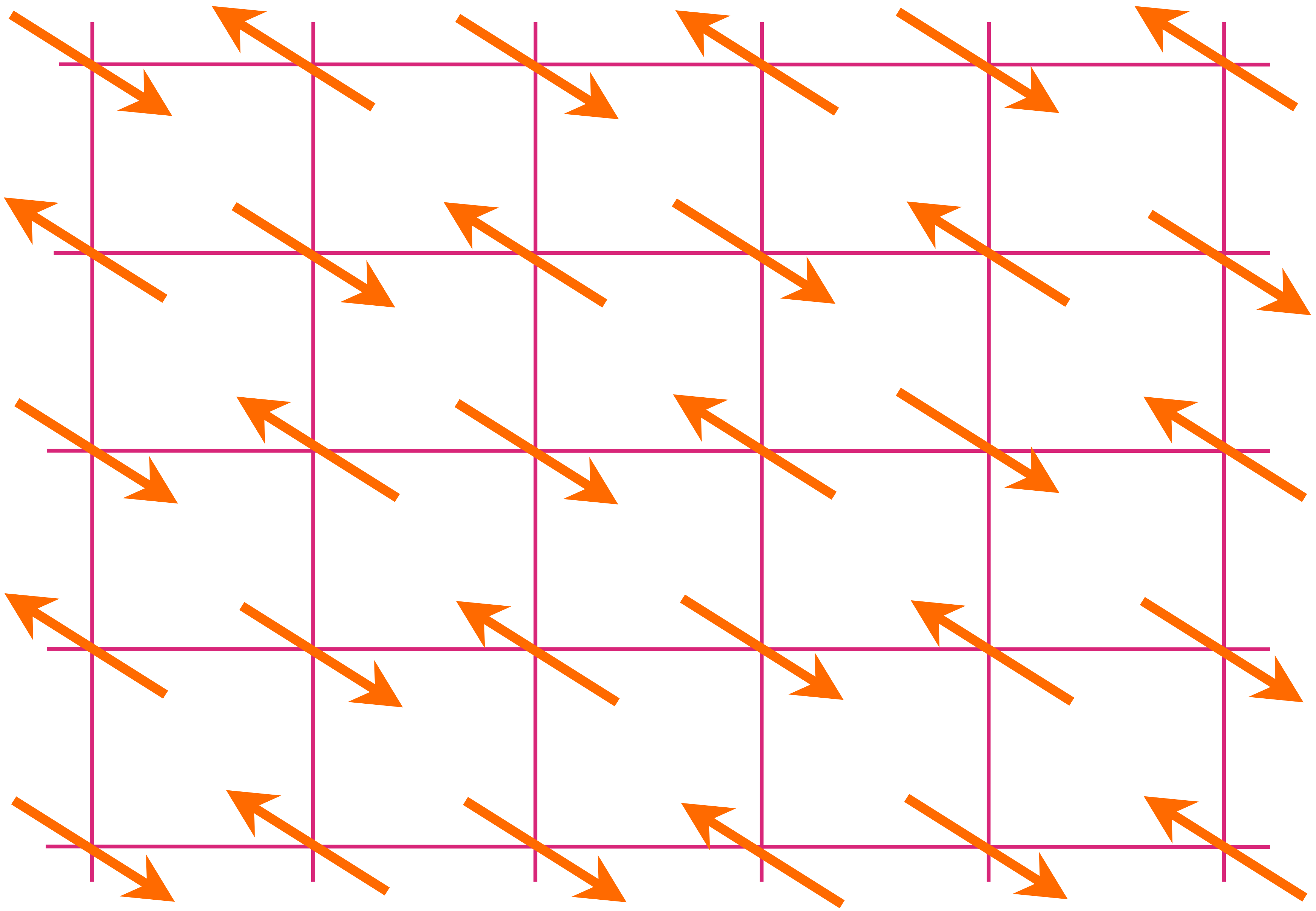


CuO_2 plane

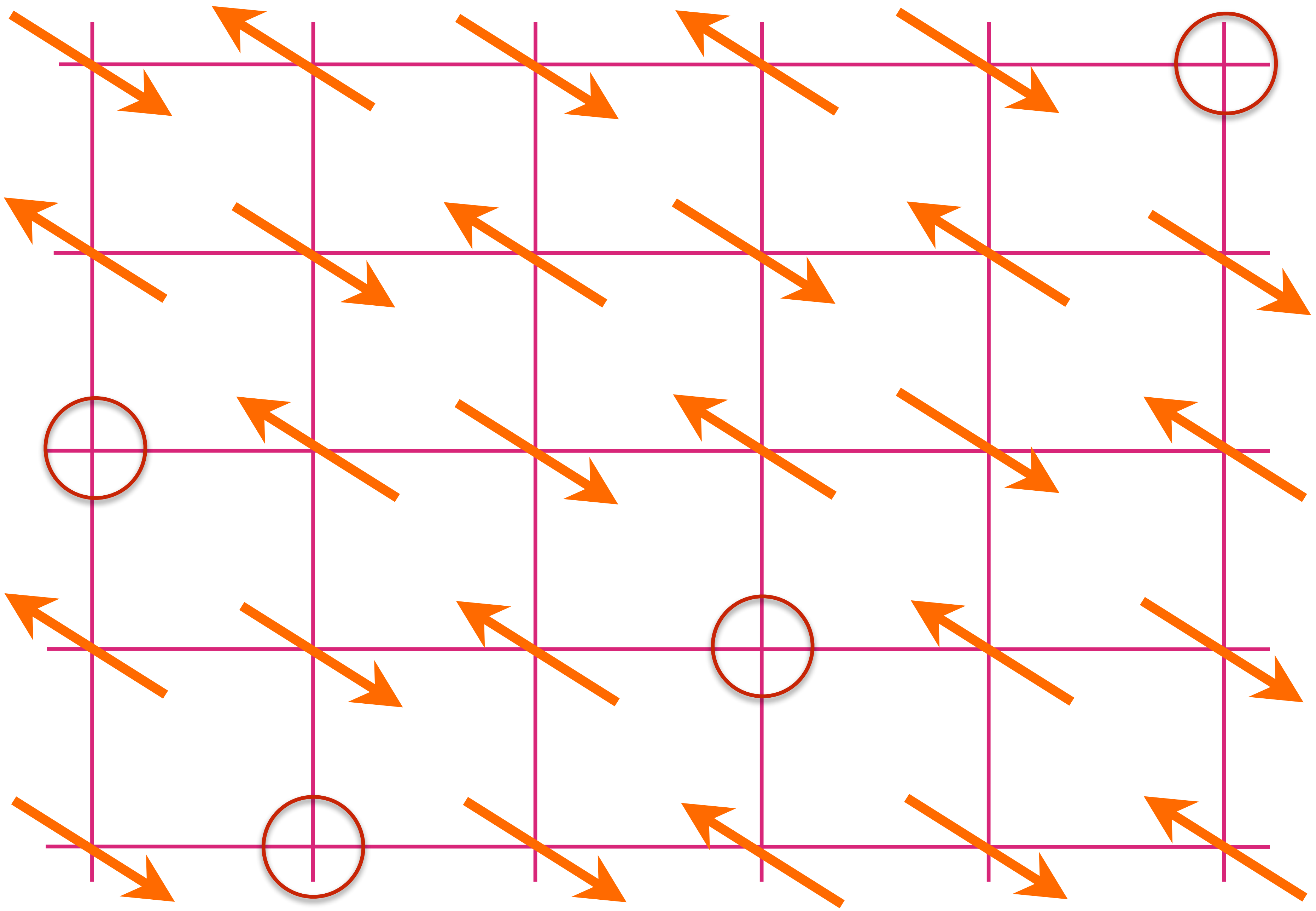


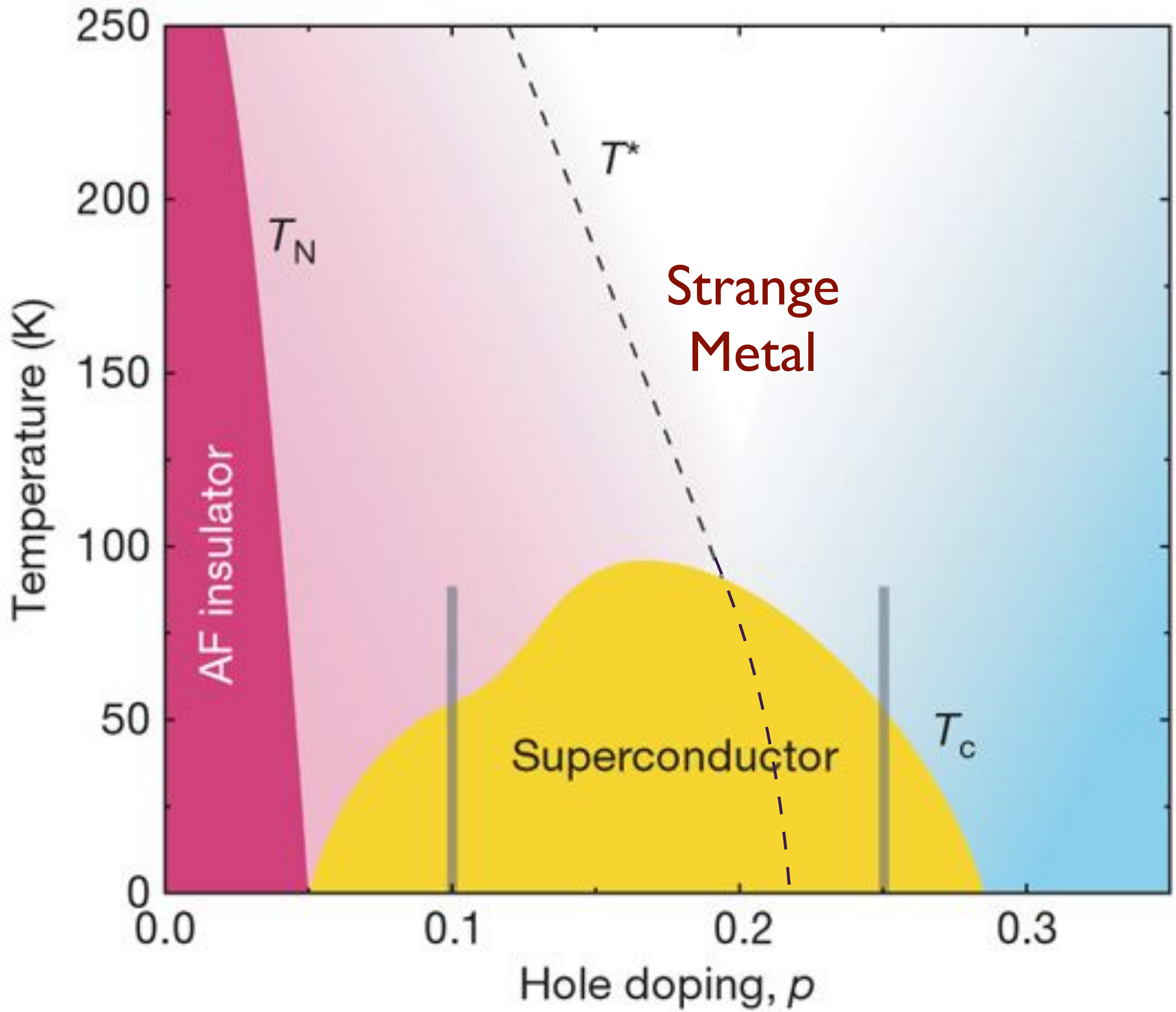
$YBa_2Cu_3O_{6+x}$

Insulating antiferromagnet



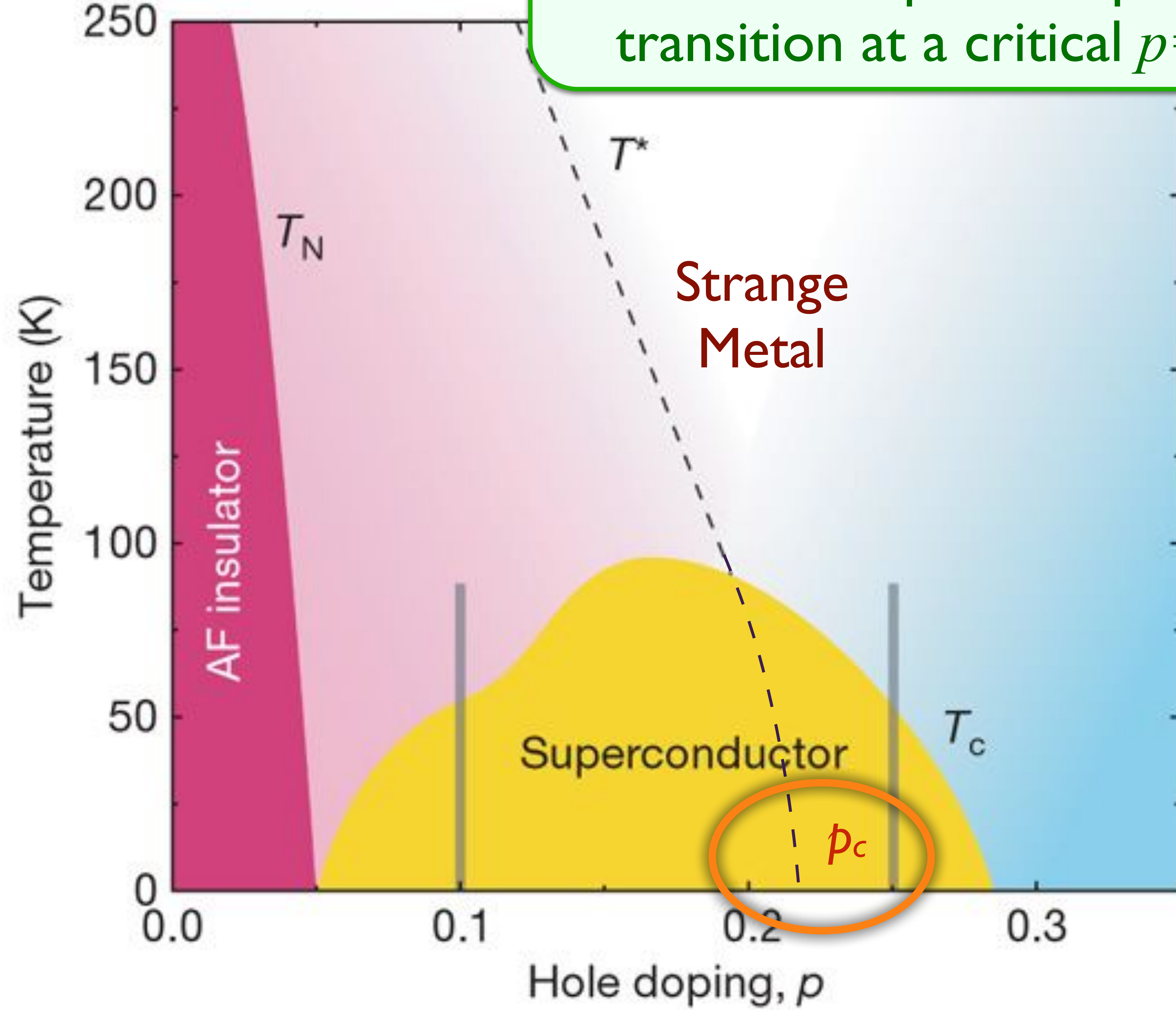
Antiferromagnet doped with hole density p



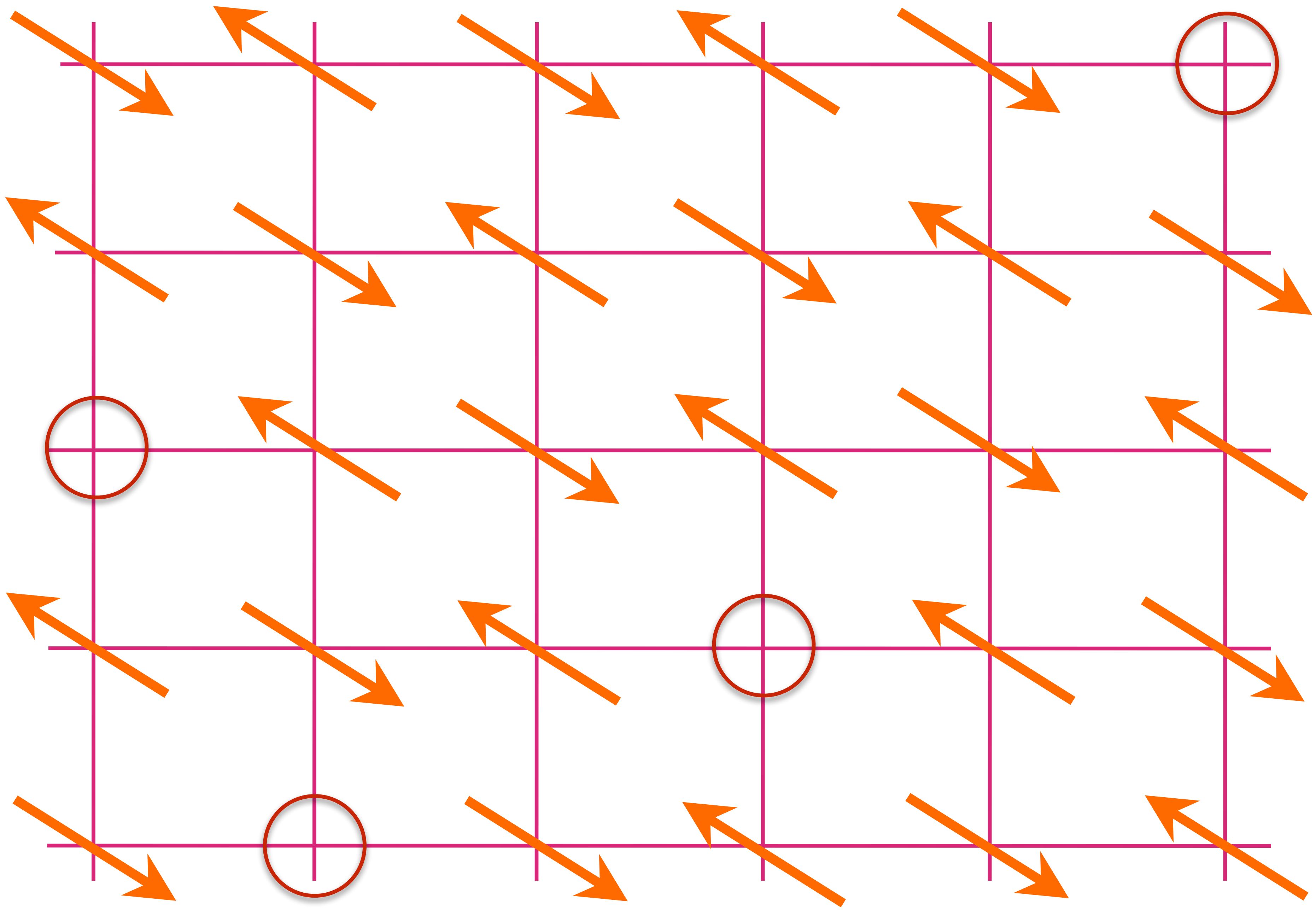


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c

Is there a quantum phase transition at a critical $p = p_c$?

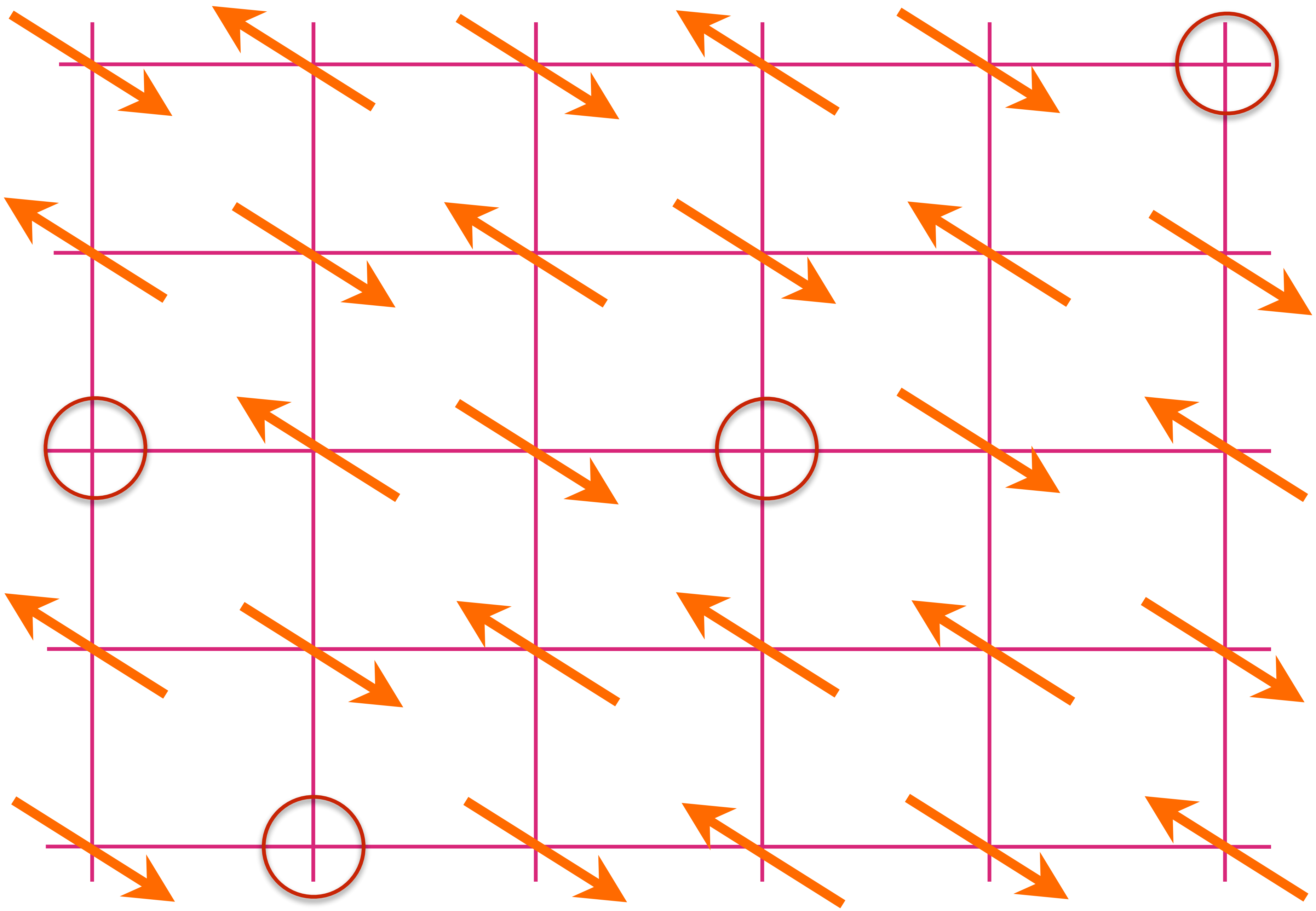


Real-space view at small p



p mobile holes in a background of fluctuating spins

Real-space view at small p



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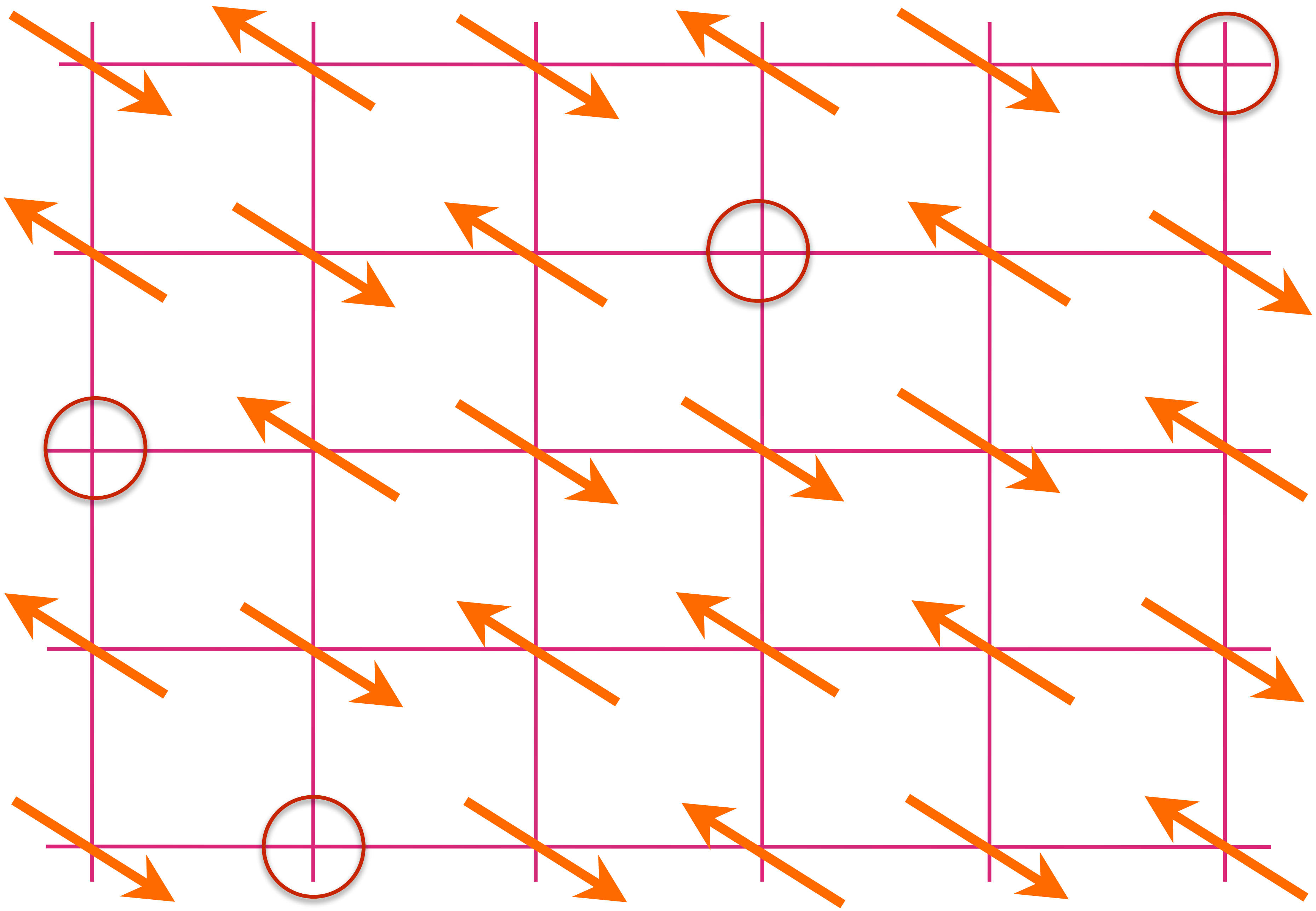

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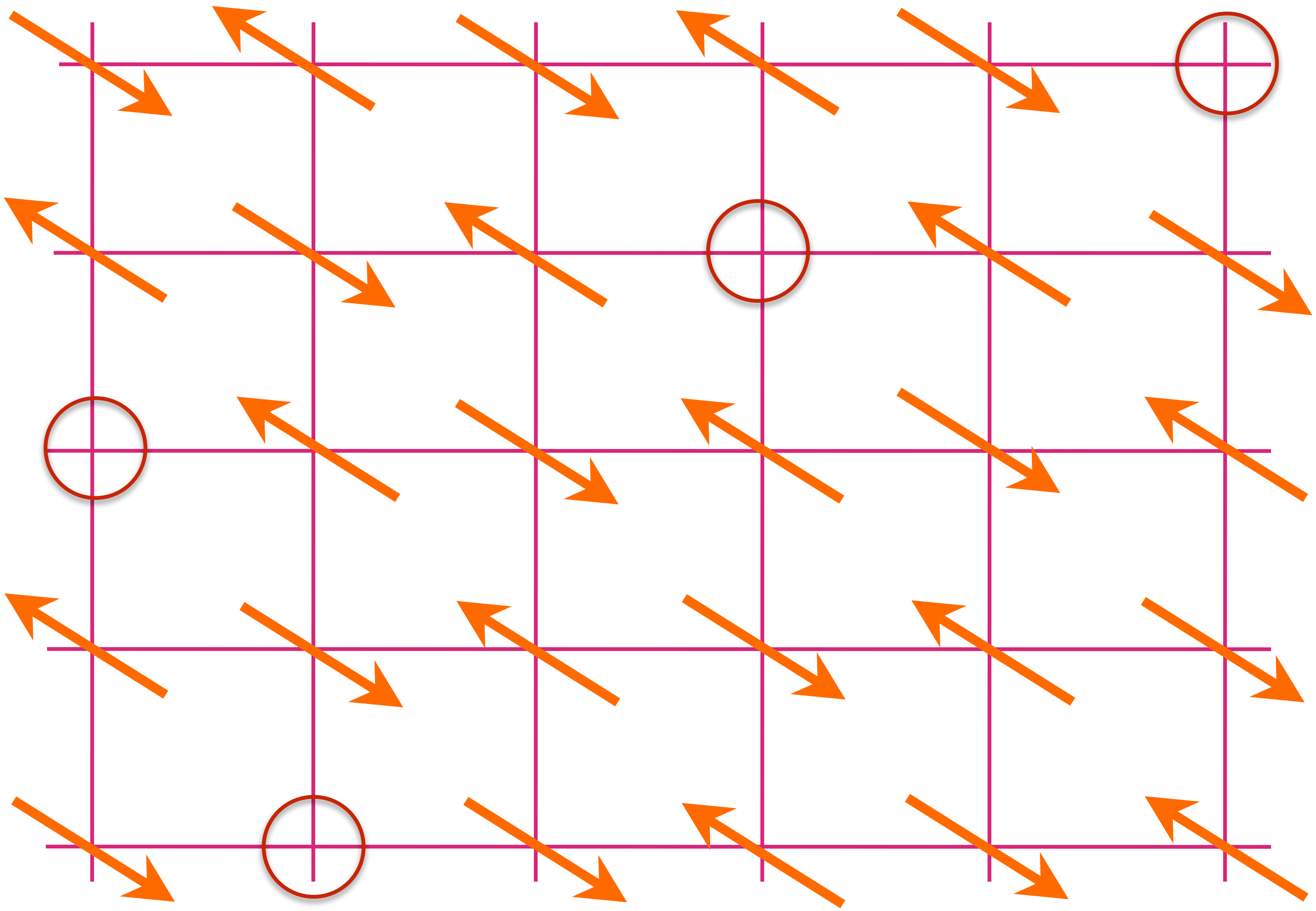

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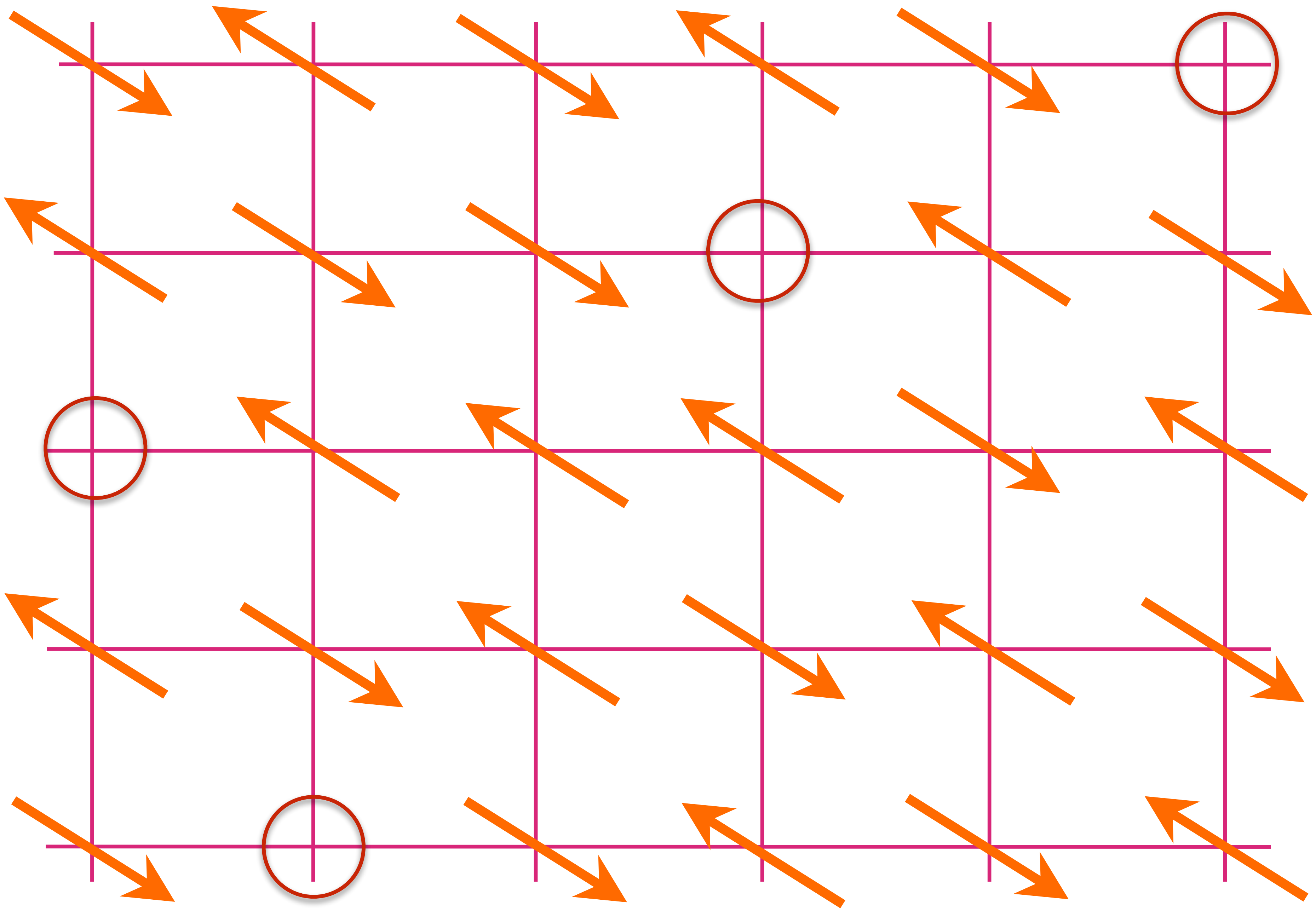

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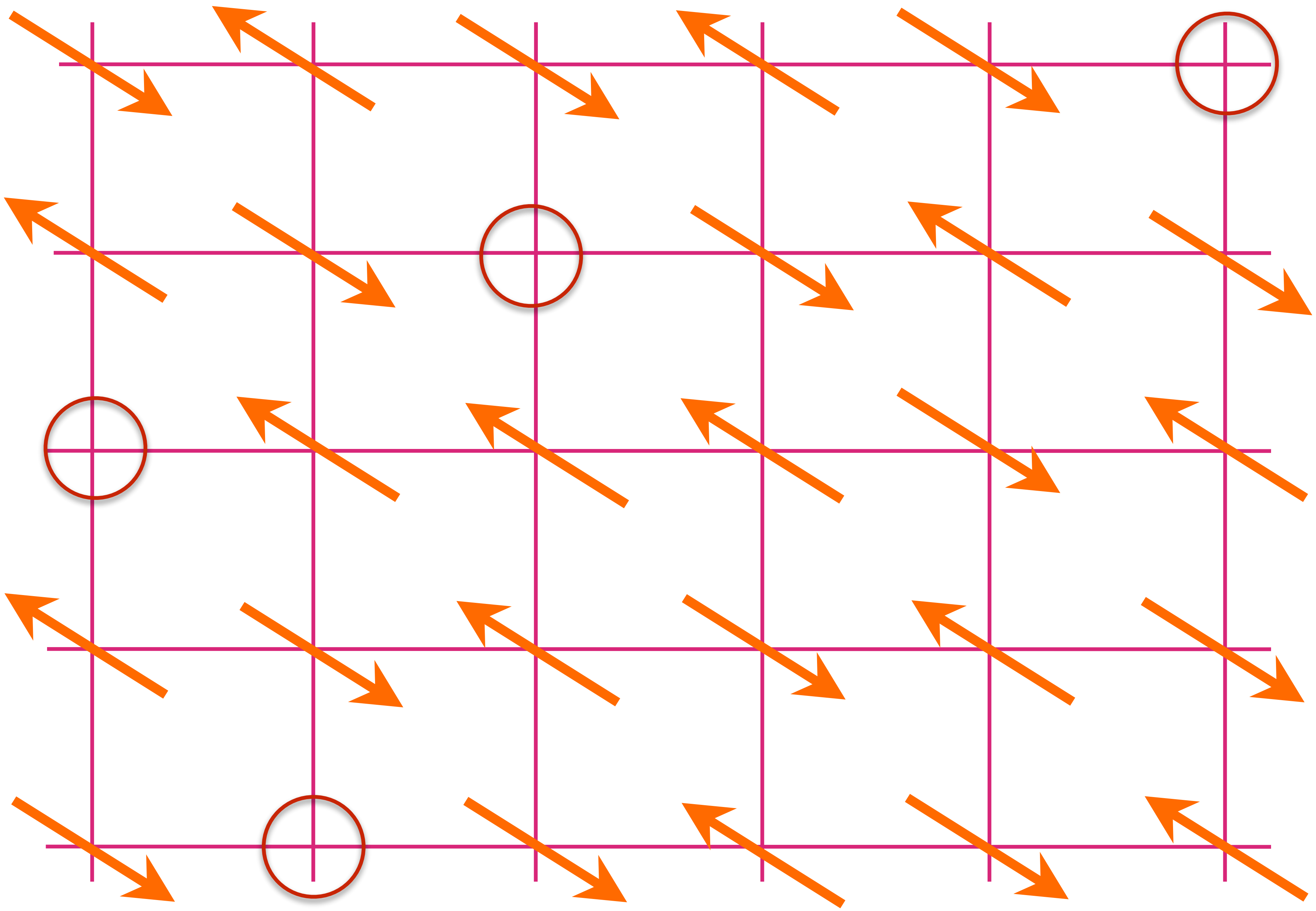

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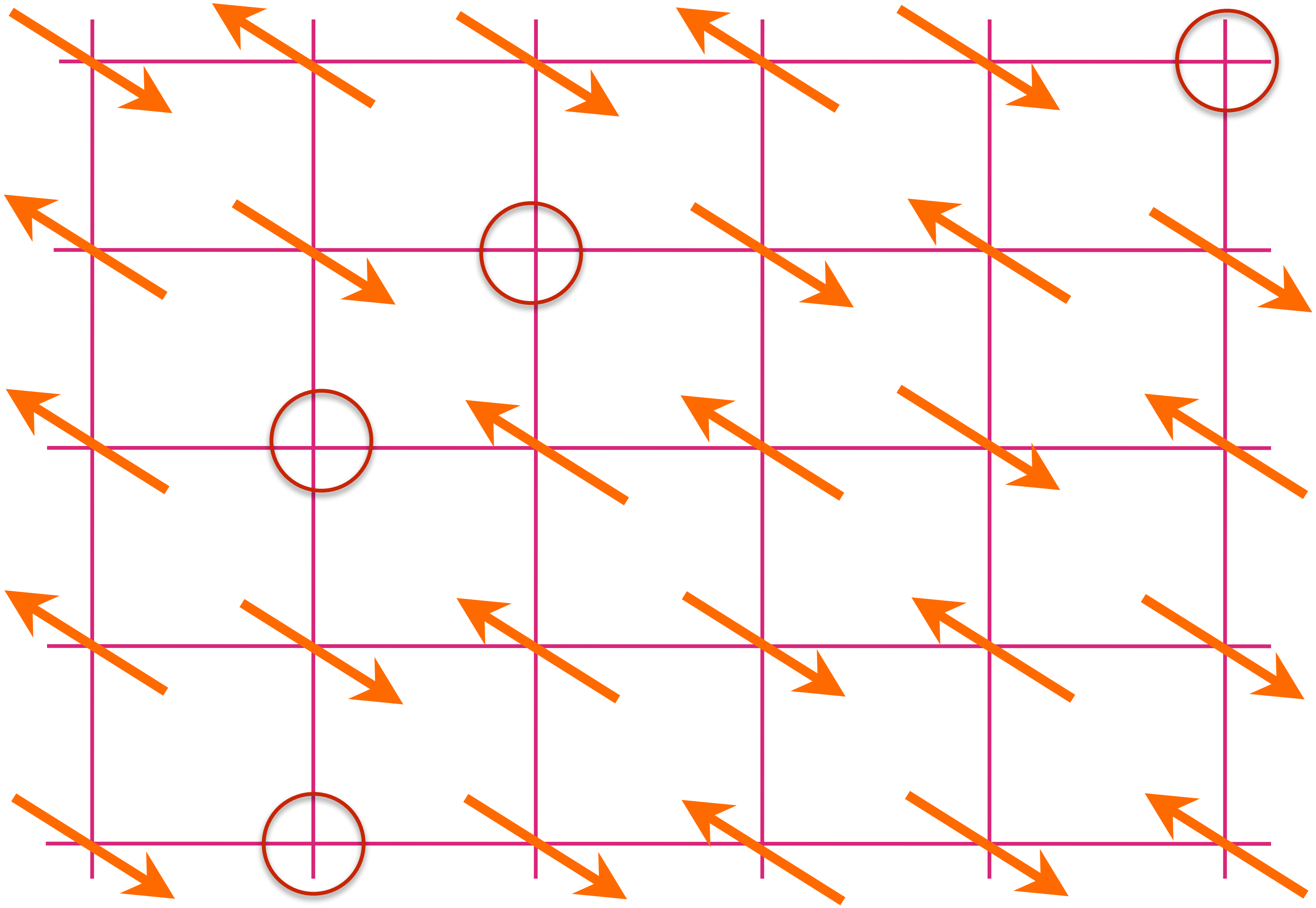

p mobile holes in a background of fluctuating spins

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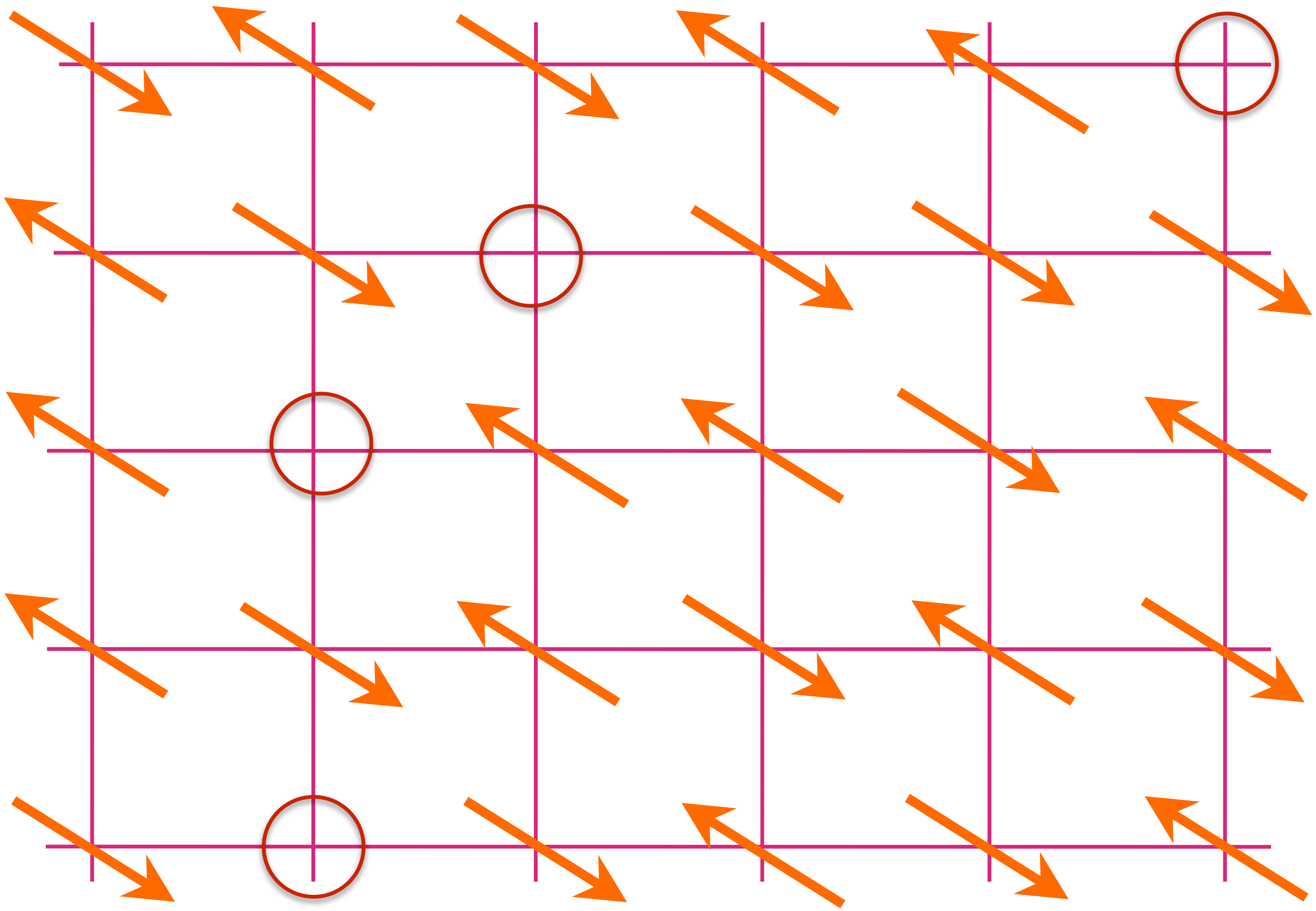
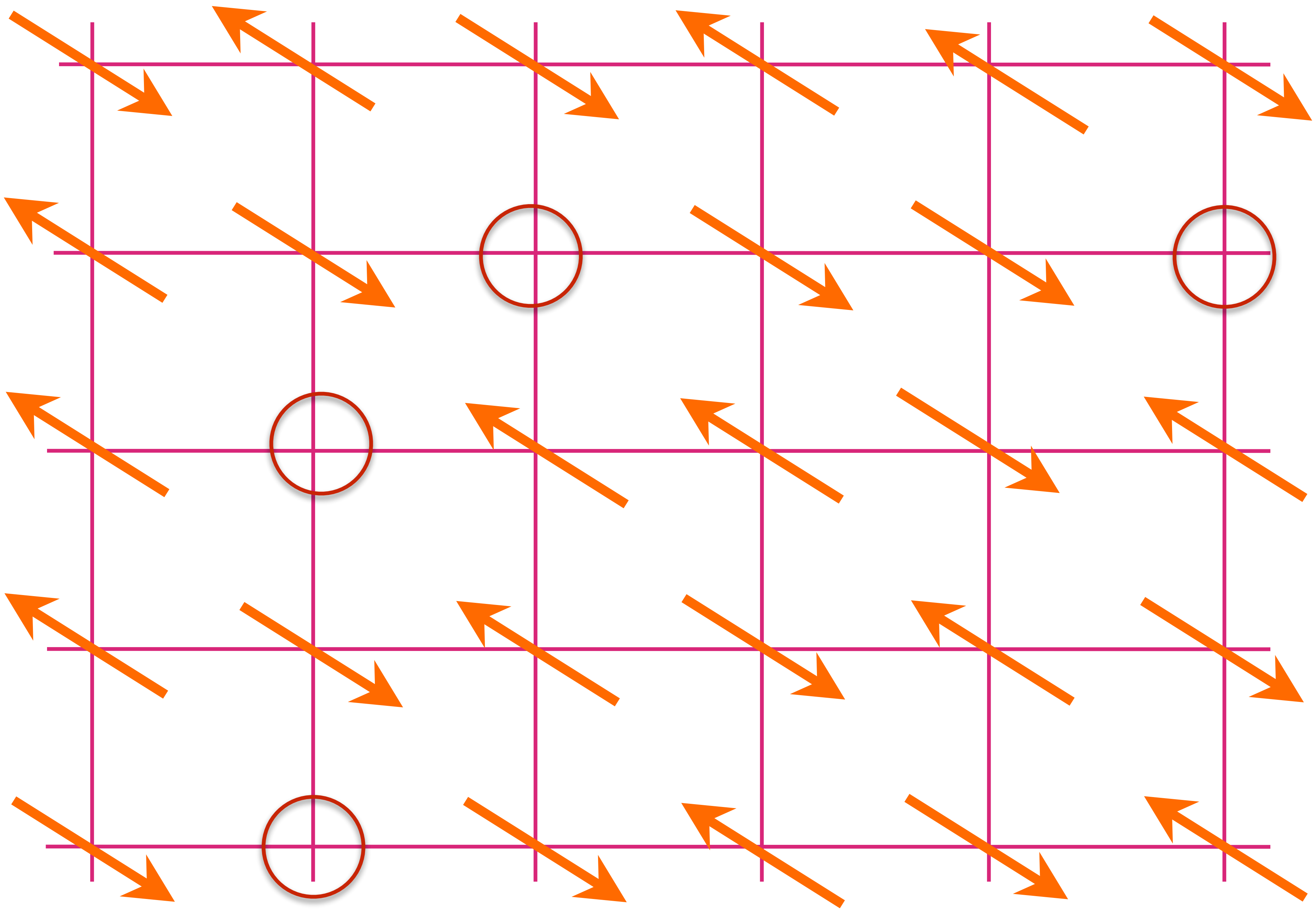


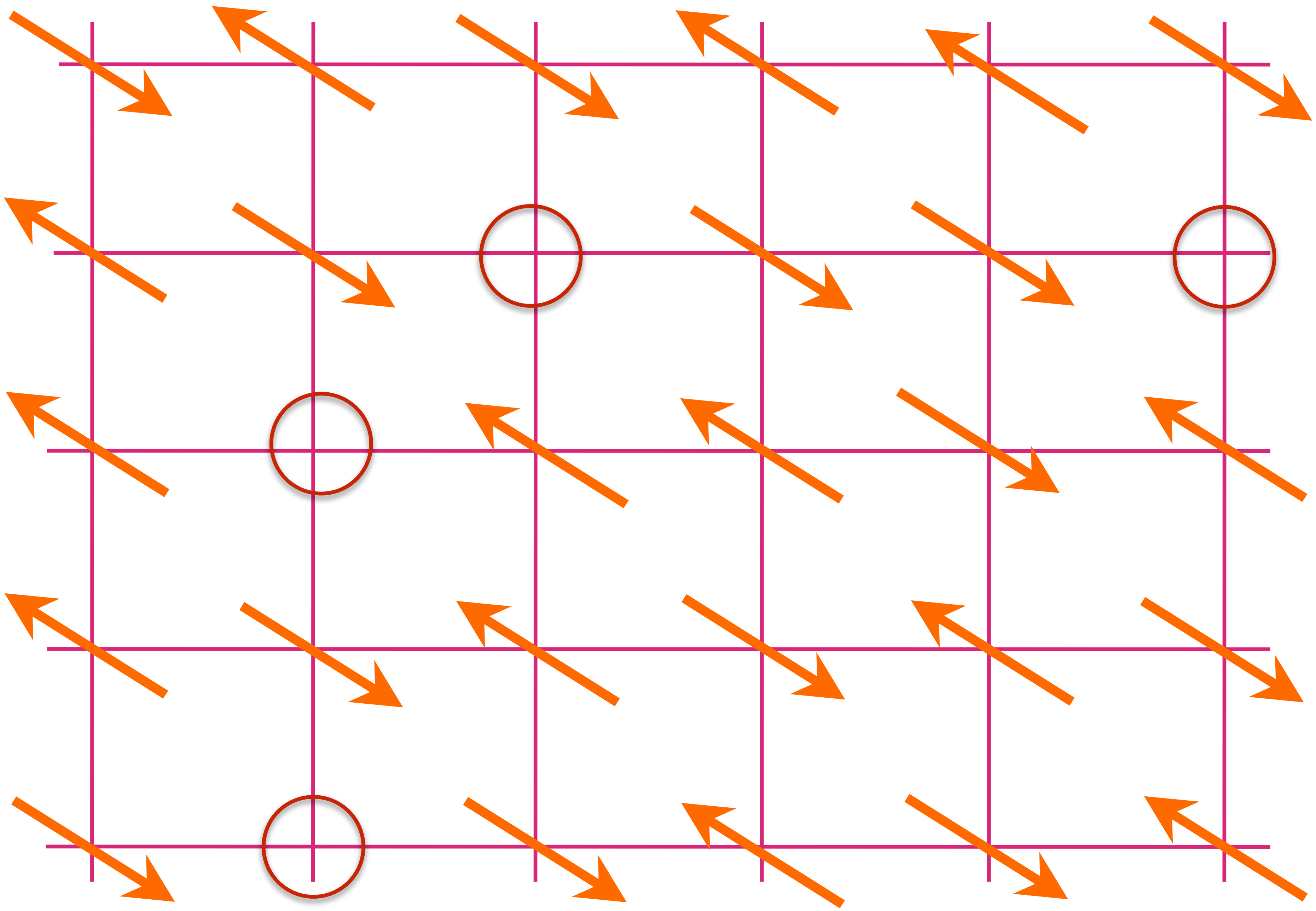
p mobile holes in a background of fluctuating spins

Real-space view at small p



p mobile holes in a background of fluctuating spins

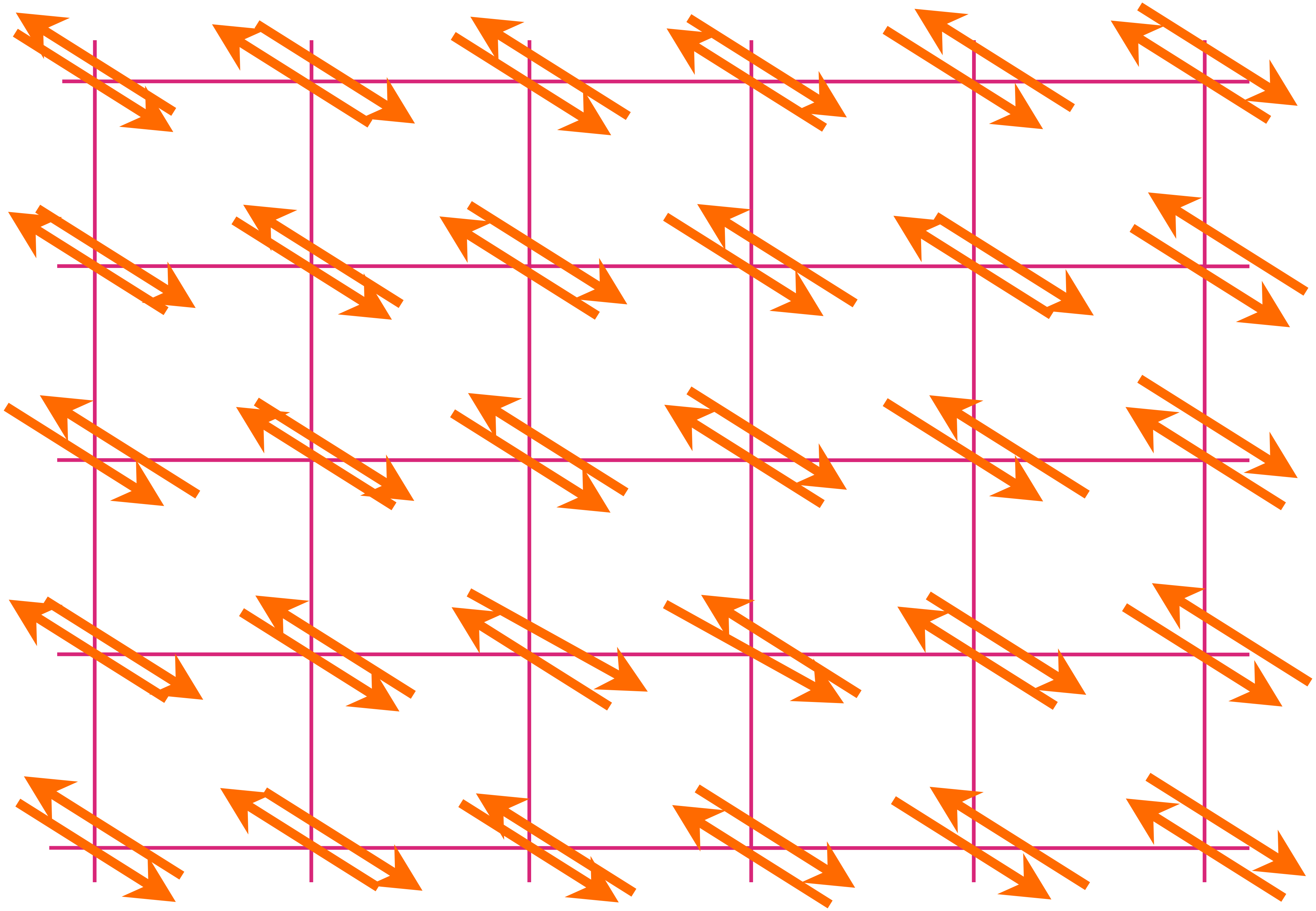
Momentum-space view at large p



$1\text{-}p$ mobile electrons =

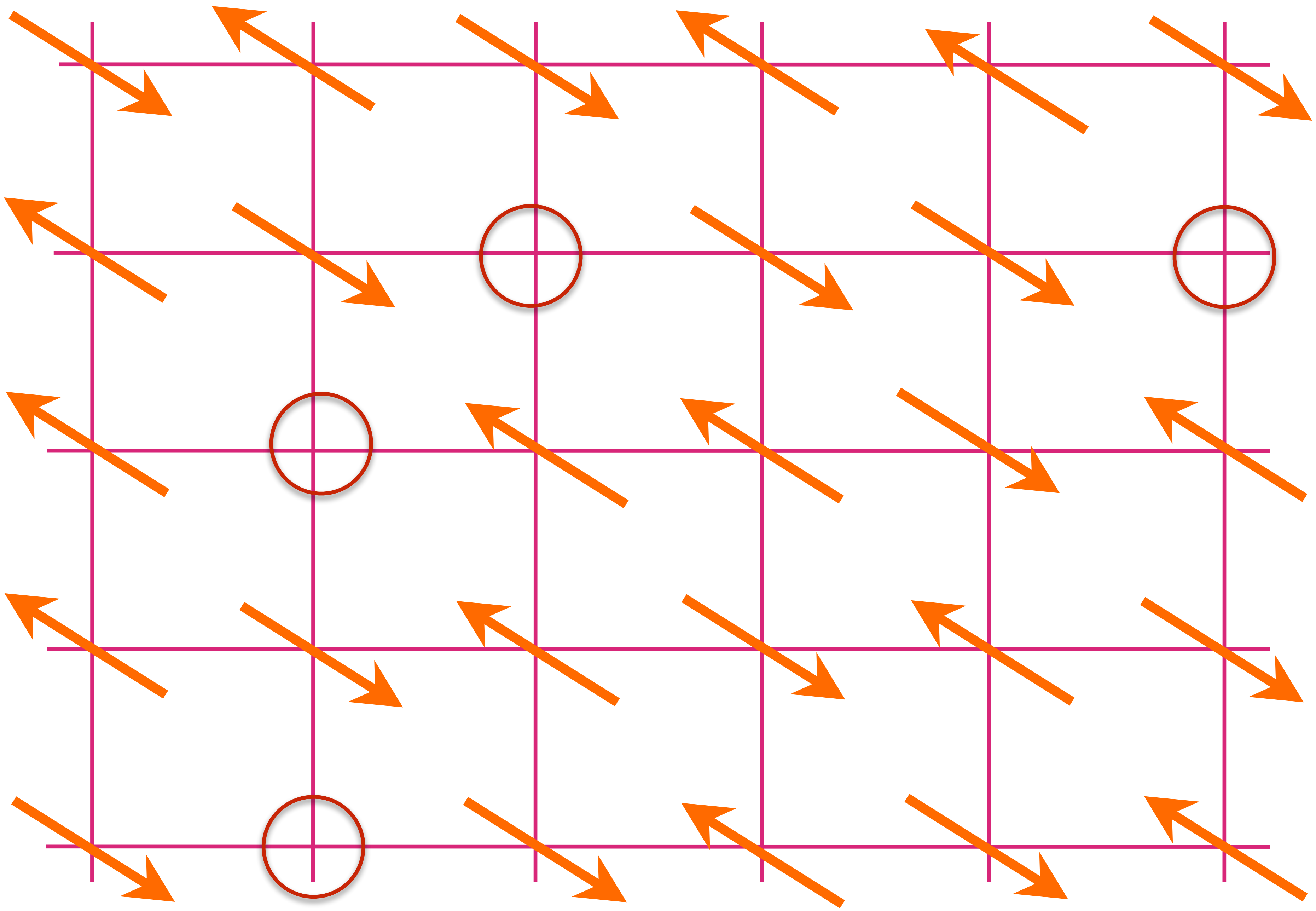
$1+p$ mobile holes in a filled band

Momentum-space view at large p



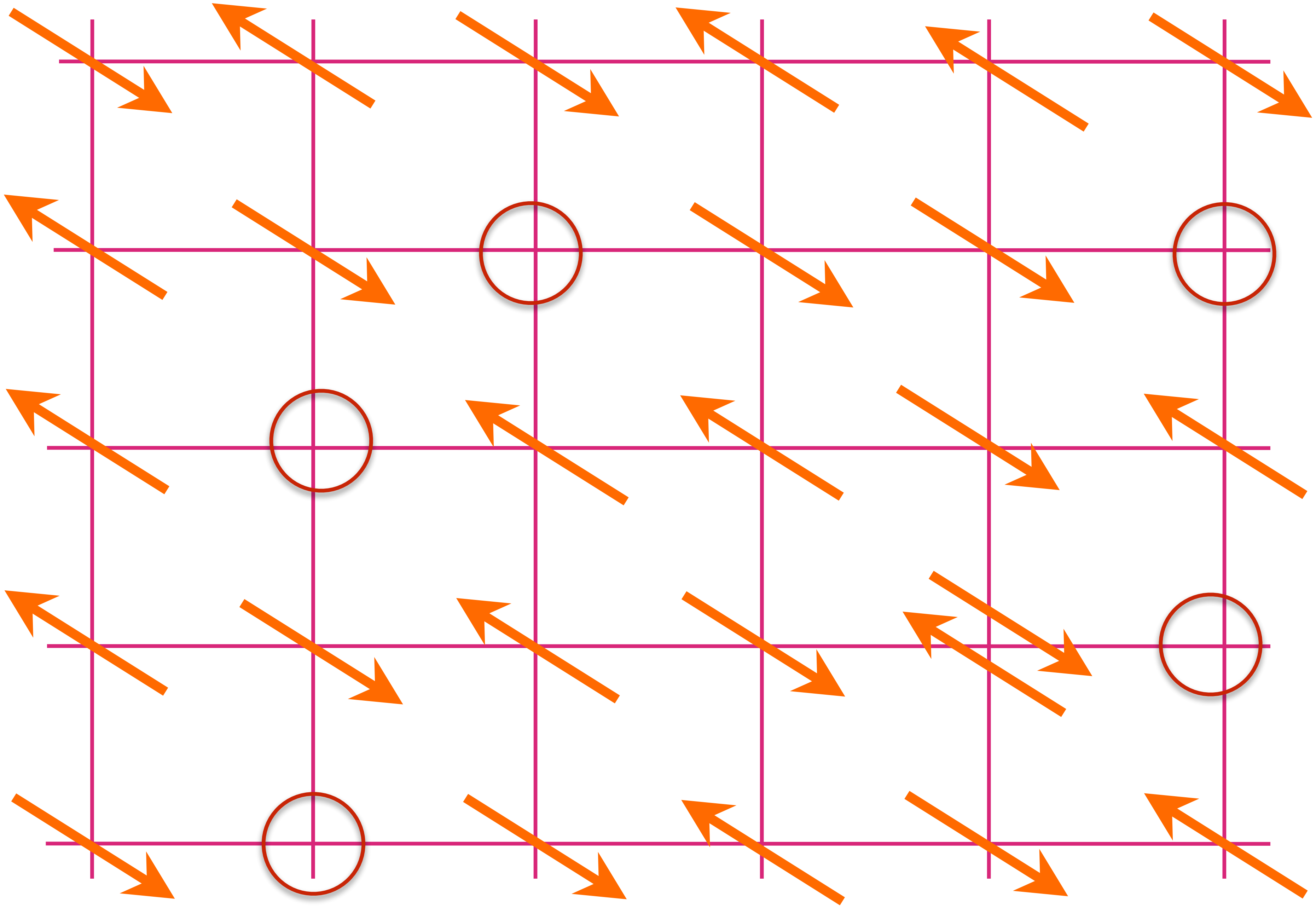
Filled Band

Momentum-space view at large p



1-p mobile electrons =
1+p mobile holes in a filled band

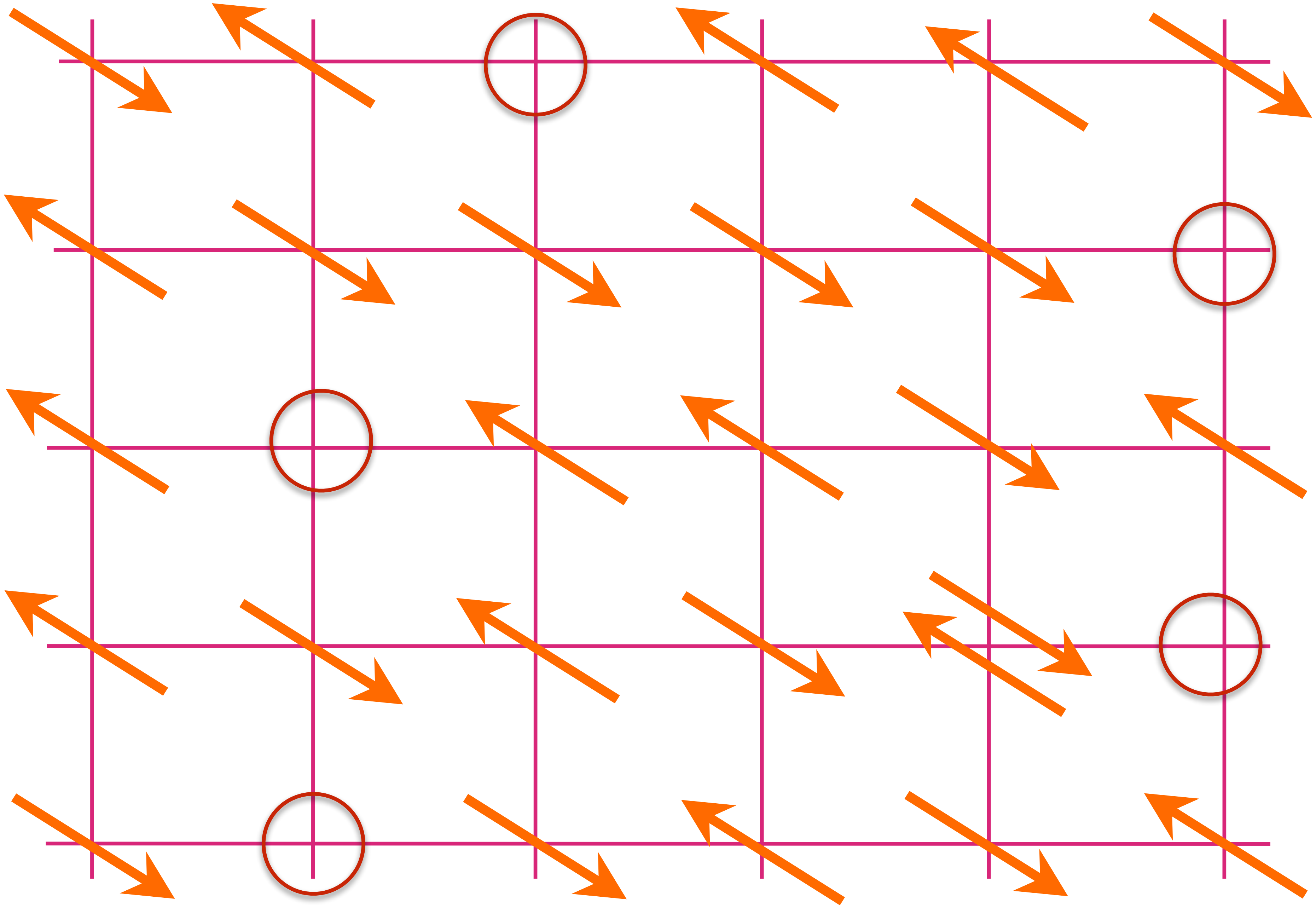
Momentum-space view at large p



1-p mobile electrons =

1+p mobile holes in a filled band

Momentum-space view at large p



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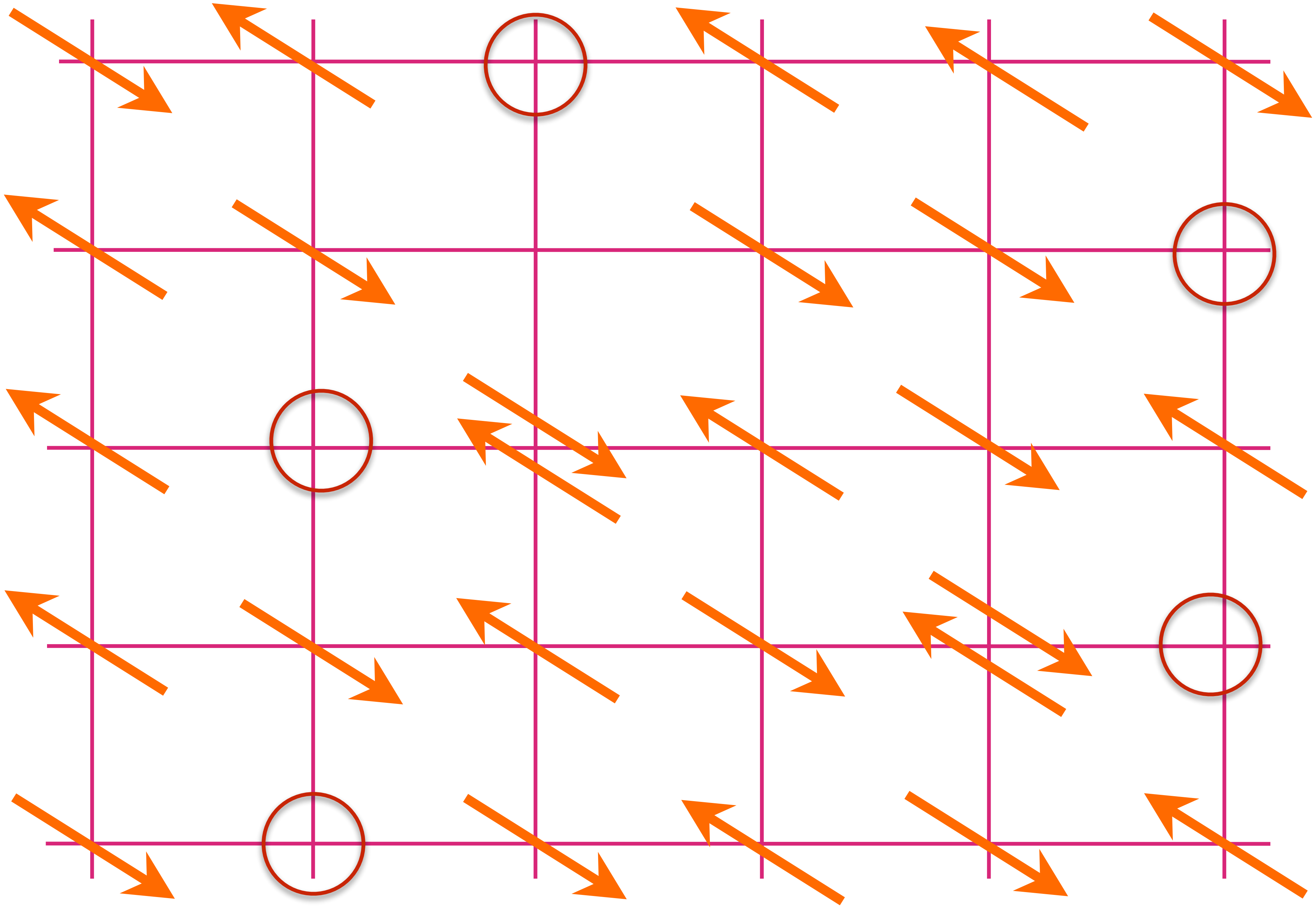

$1\text{-}p$ mobile electrons =
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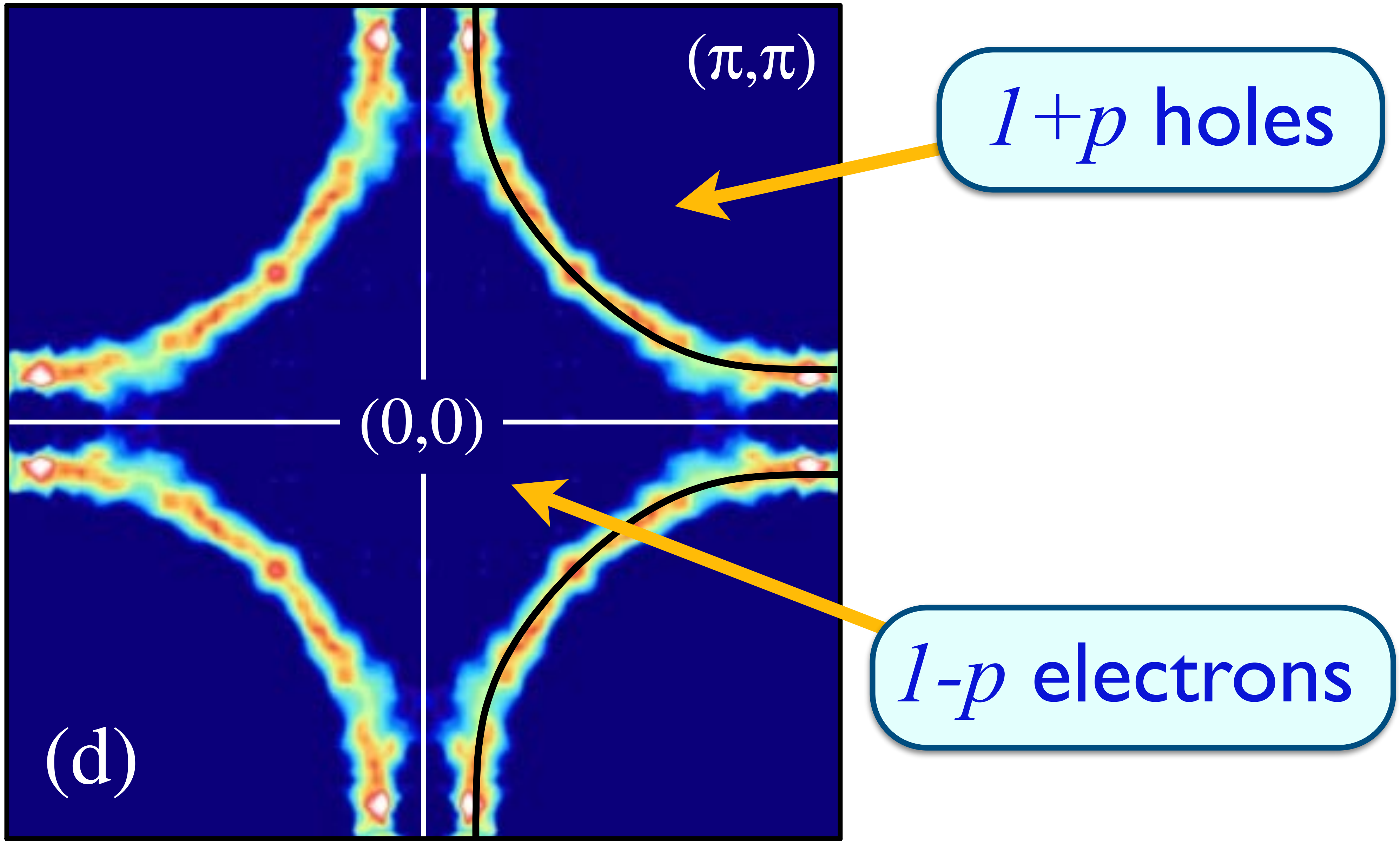
Momentum-space view at large p

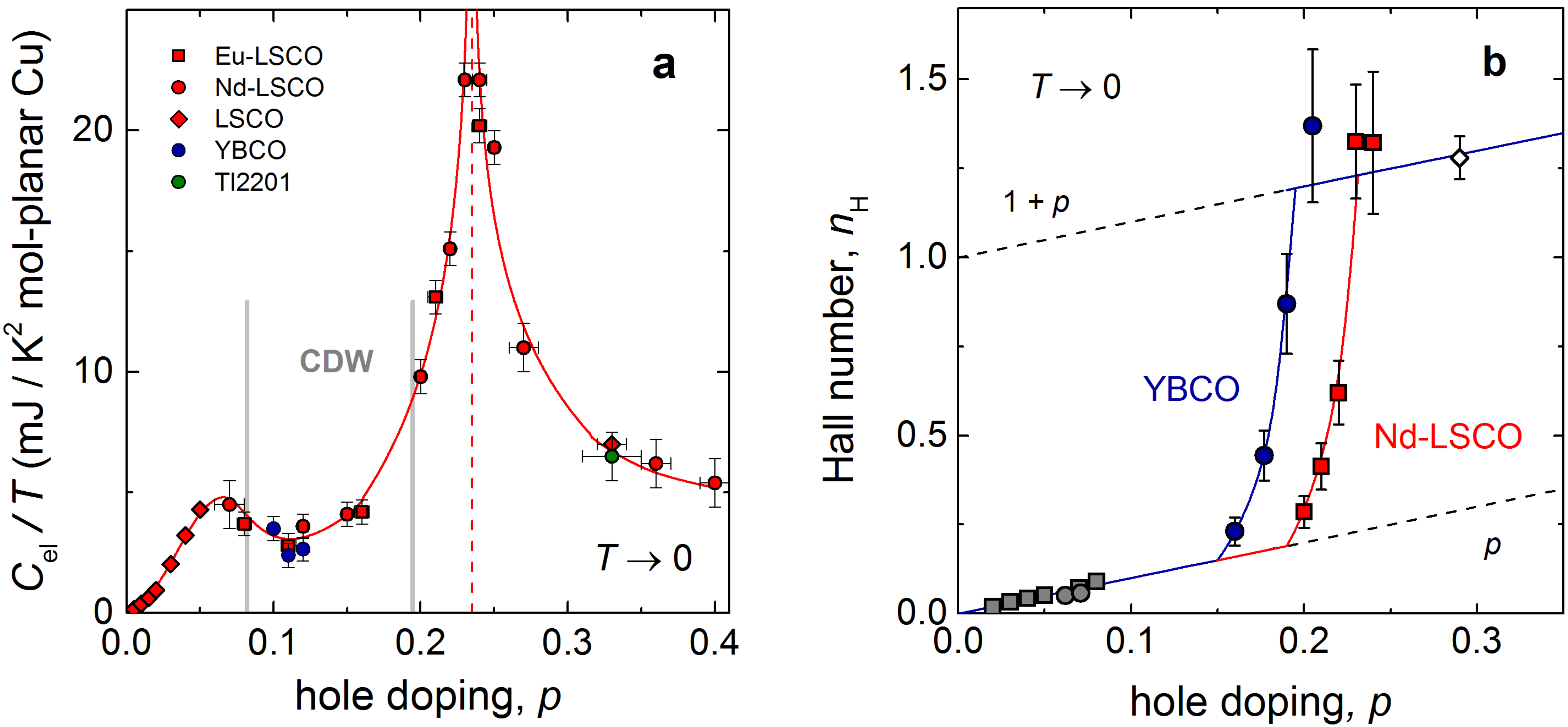


$1+p$ mobile holes in a filled band

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

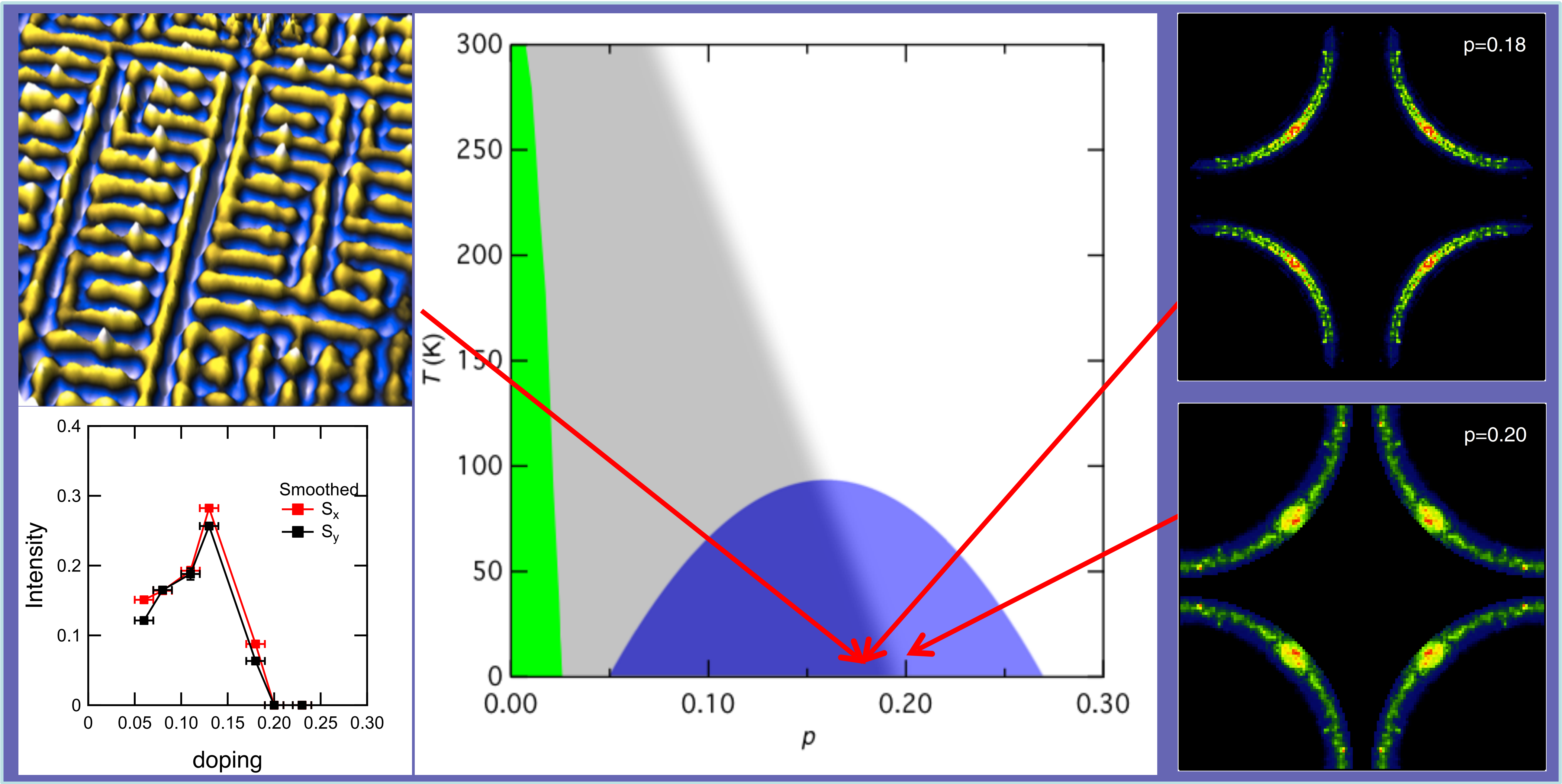
Cyril Proust and Louis Taillefer, arXiv:1807.0507



Hole doped cuprates

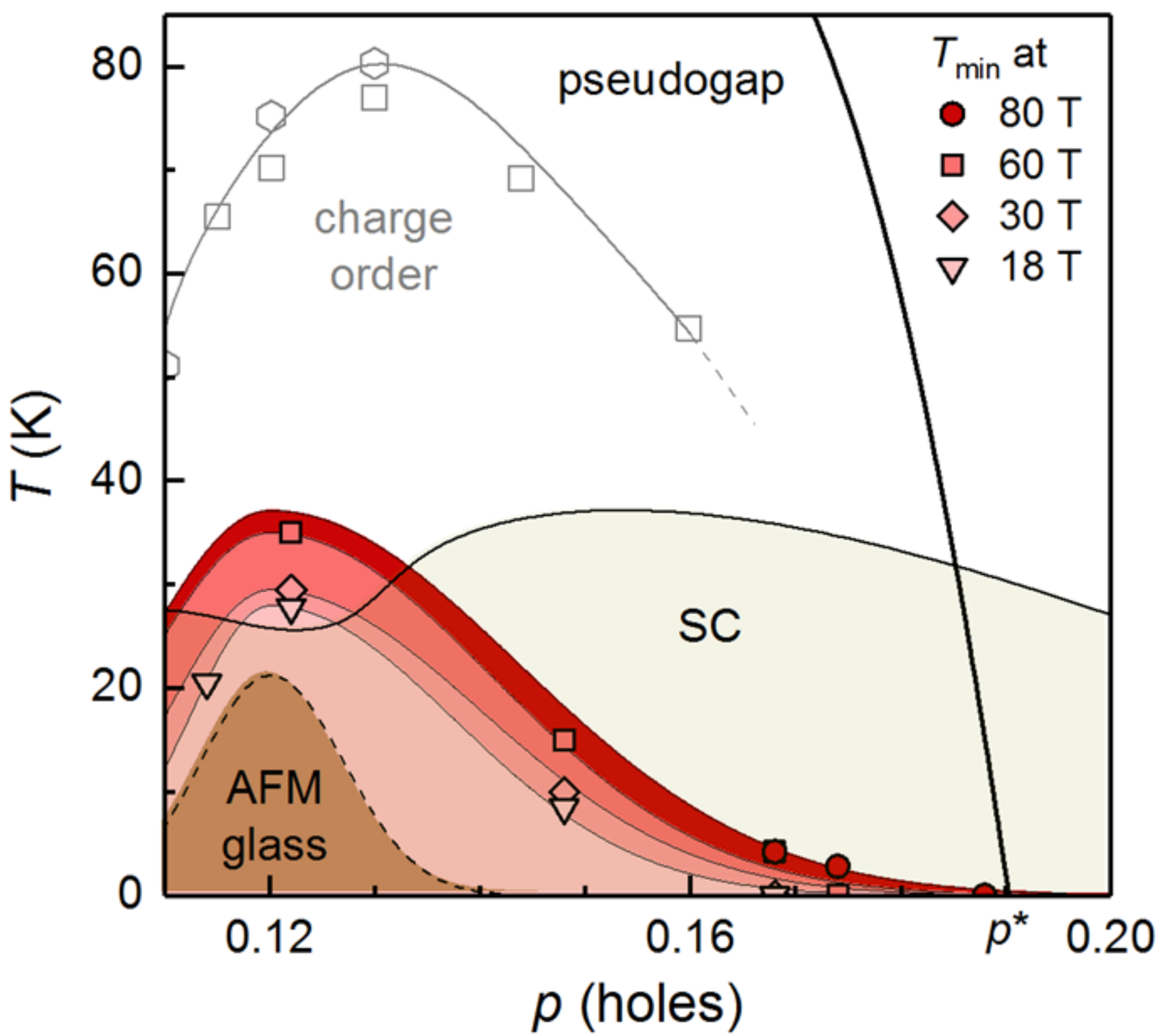
Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

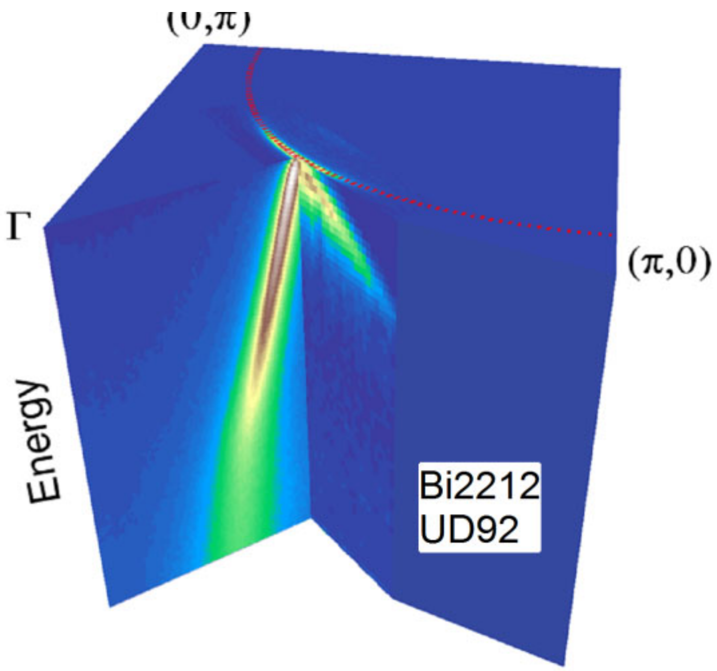
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]

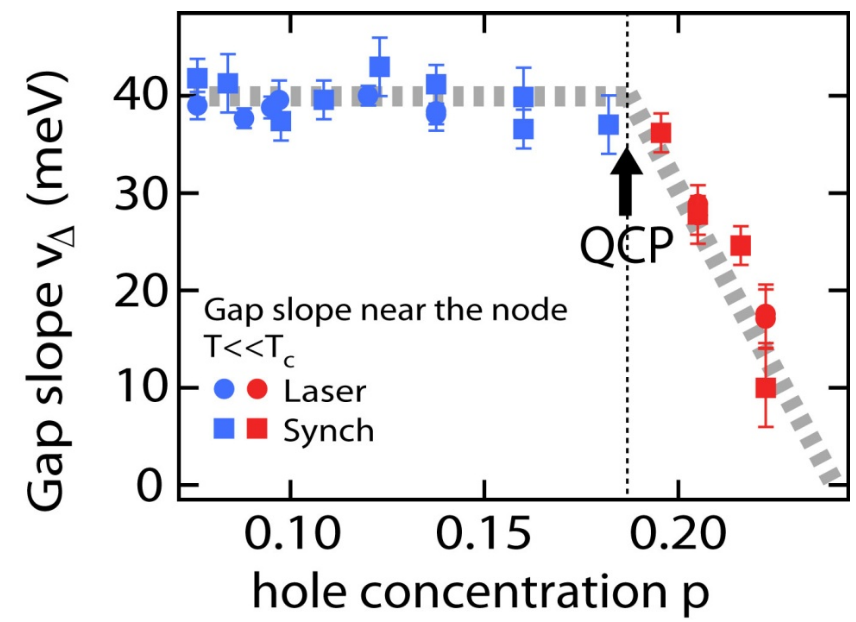


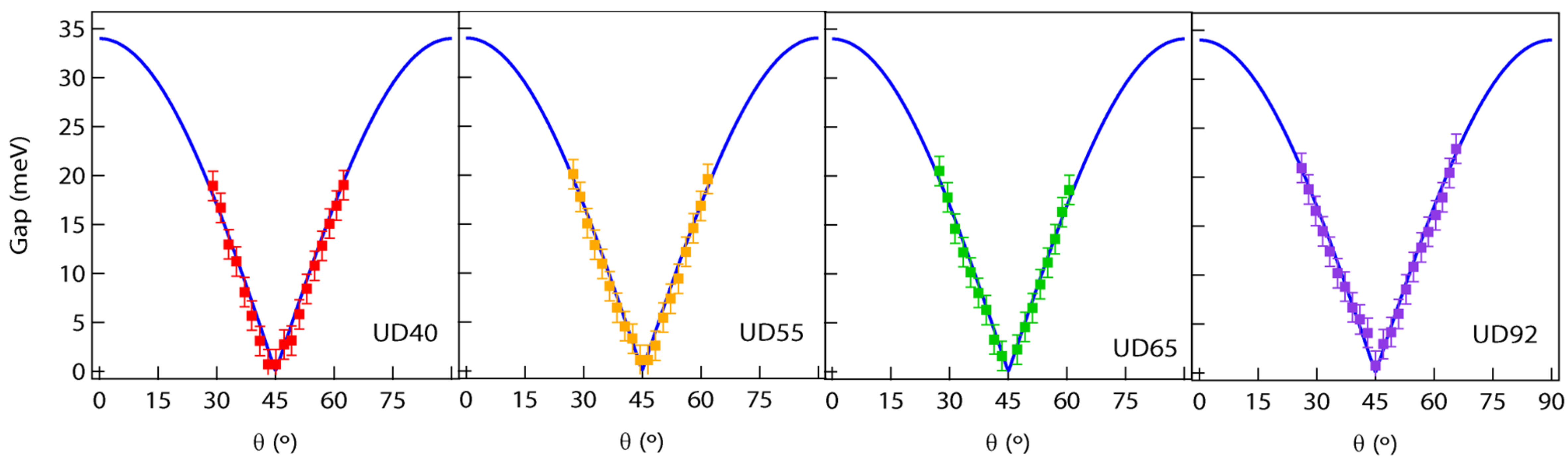
arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Precision Measurement of the Node

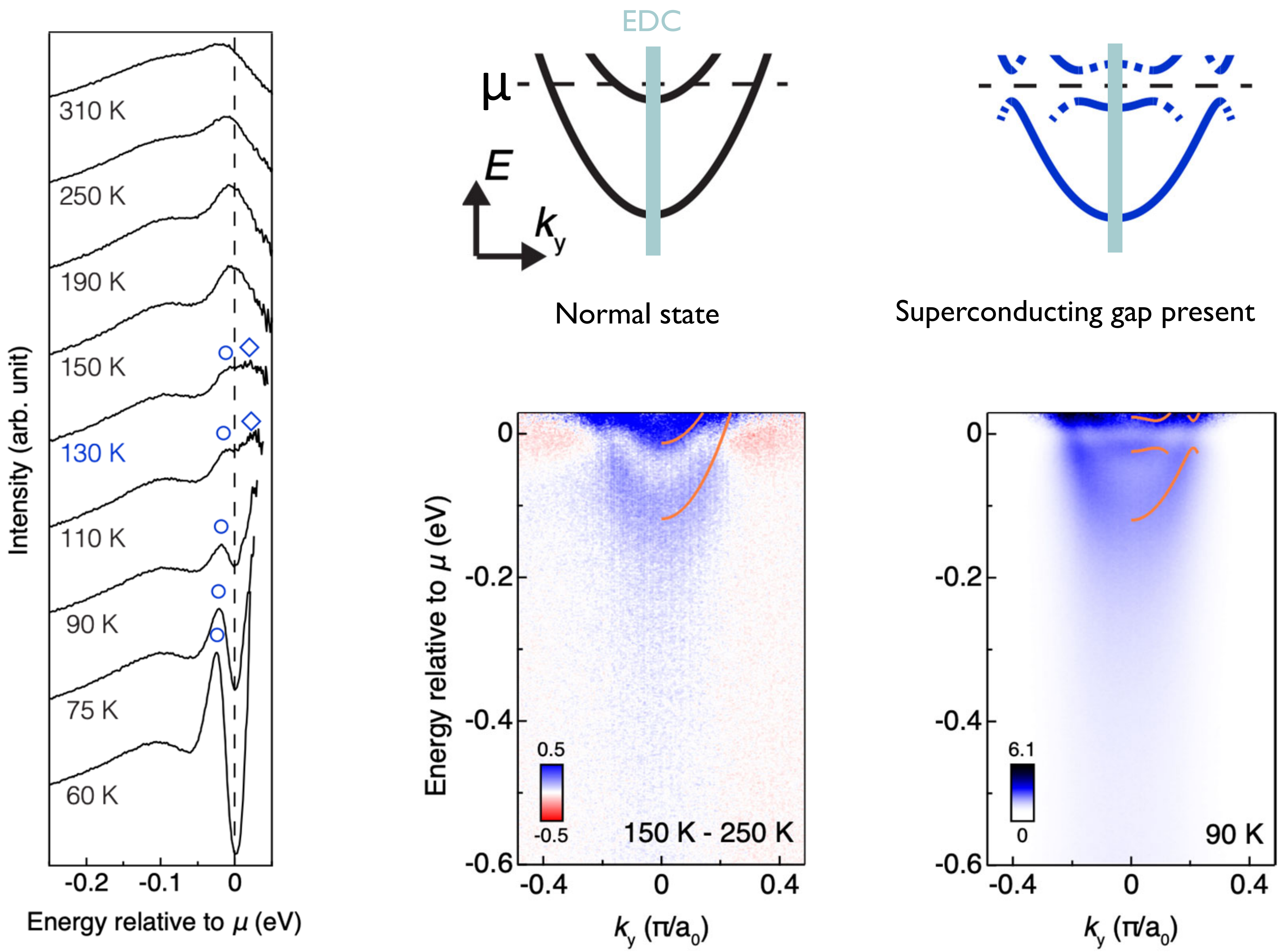






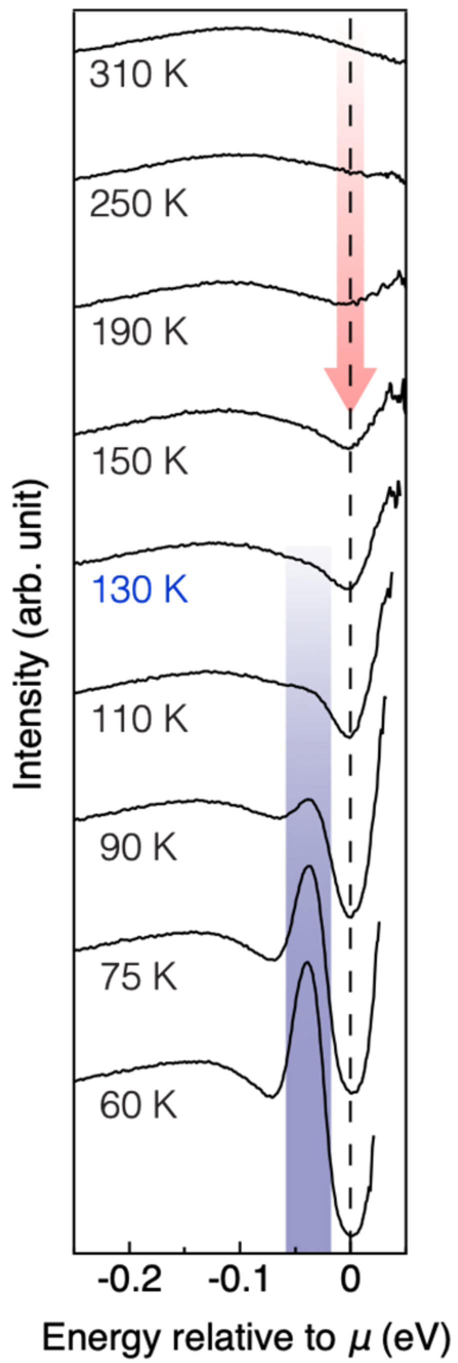
I. M. Vishik, M. Hashimoto, Rui-Hua He, Wei-Sheng Lee, Felix Schmitt, Donghui Lu, R. G. Moore, C. Zhang, W. Meevasana, T. Sasagawa, S. Uchida, Kazuhiro Fujita, S. Ishida, M. Ishikado, Yoshiyuki Yoshida, Hiroshi Eisaki, Zahid Hussain, Thomas P. Devereaux, and Zhi-Xun Shen, PNAS **109**, 18332 (2012)

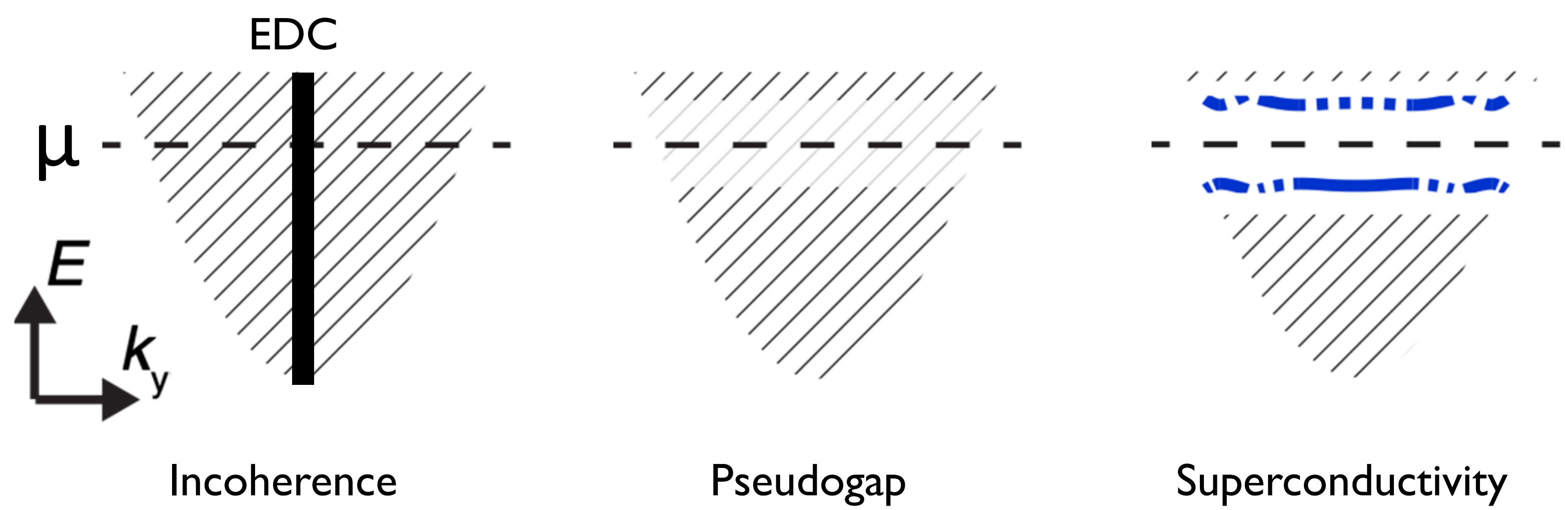
One gap for p > 0.19 (T_c ~ 81 K)

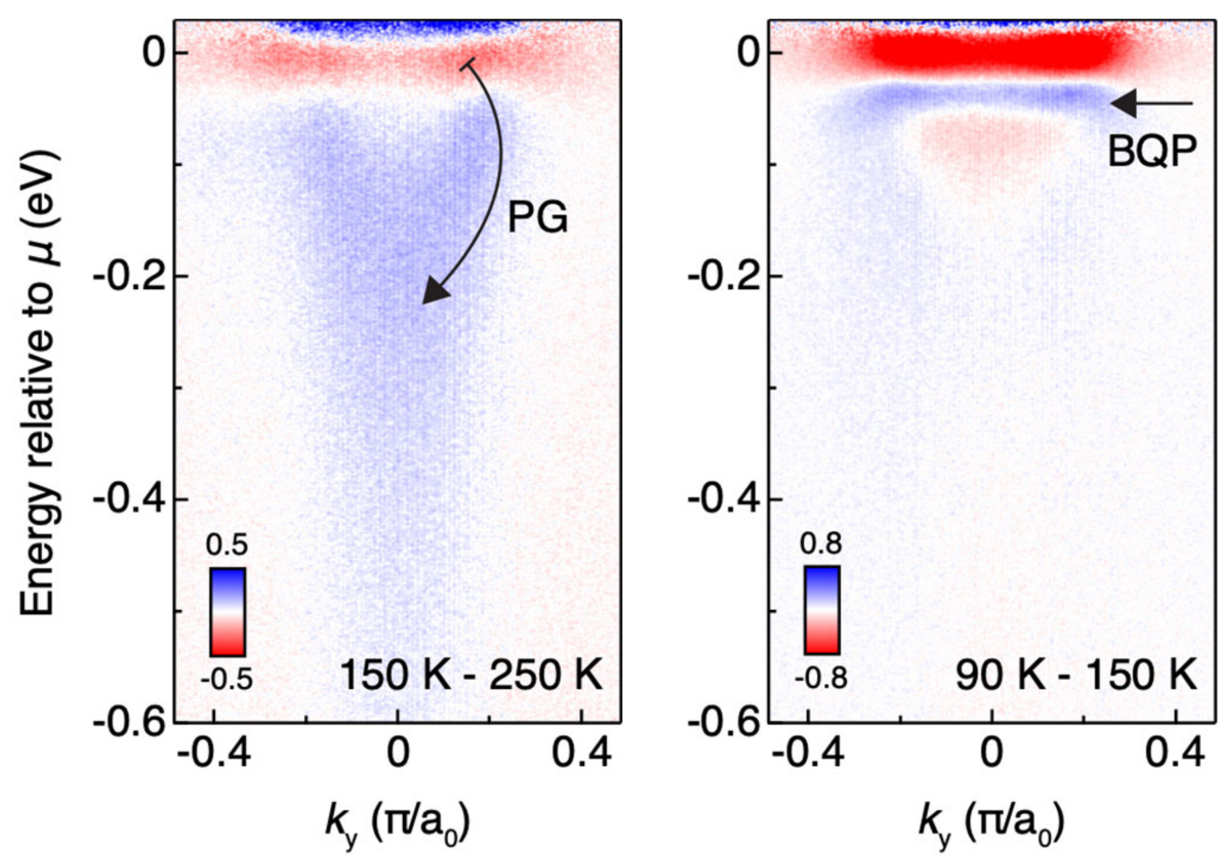


Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

Two “gaps” for p < 0.19 (T_c~ 86 K)







Su-Di Chen, Makoto Hashimoto, Yu He, Dongjoon Song, Ke-Jun Xu, Jun-Feng He, T. P. Devereaux, Hiroshi Eisaki, Dong-Hui Lu, J. Zaanen, Zhi-Xun Shen, *Science* **366**, 6469 (2019)

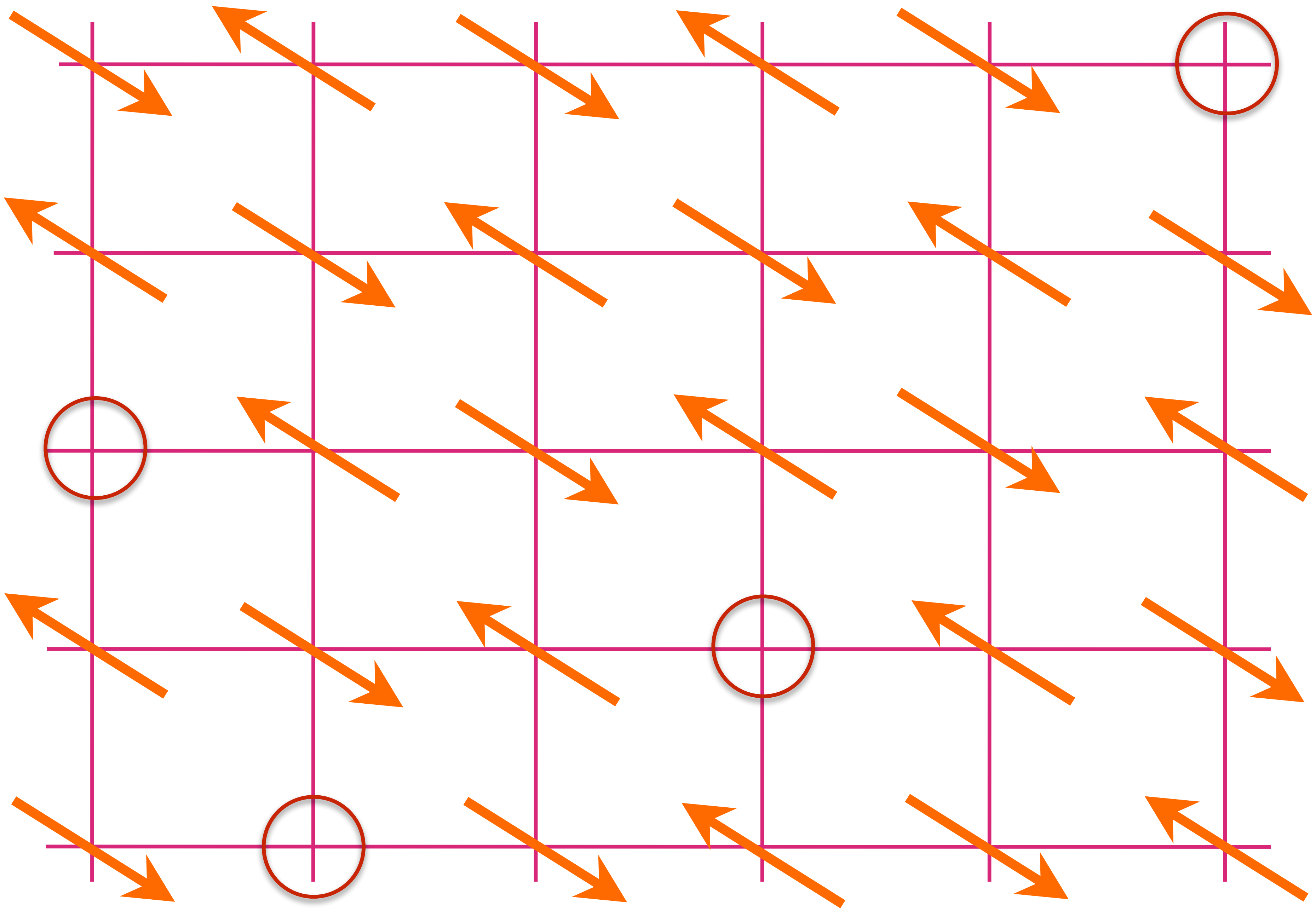
1. Deconfined quantum criticality of random *t*-*J* models

2. Linear-T resistivity and SYK criticality

1. Deconfined quantum criticality of random *t*-*J* models

2. Linear-T resistivity and SYK criticality

Real-space view at small p



p mobile holes in a background of fluctuating spins

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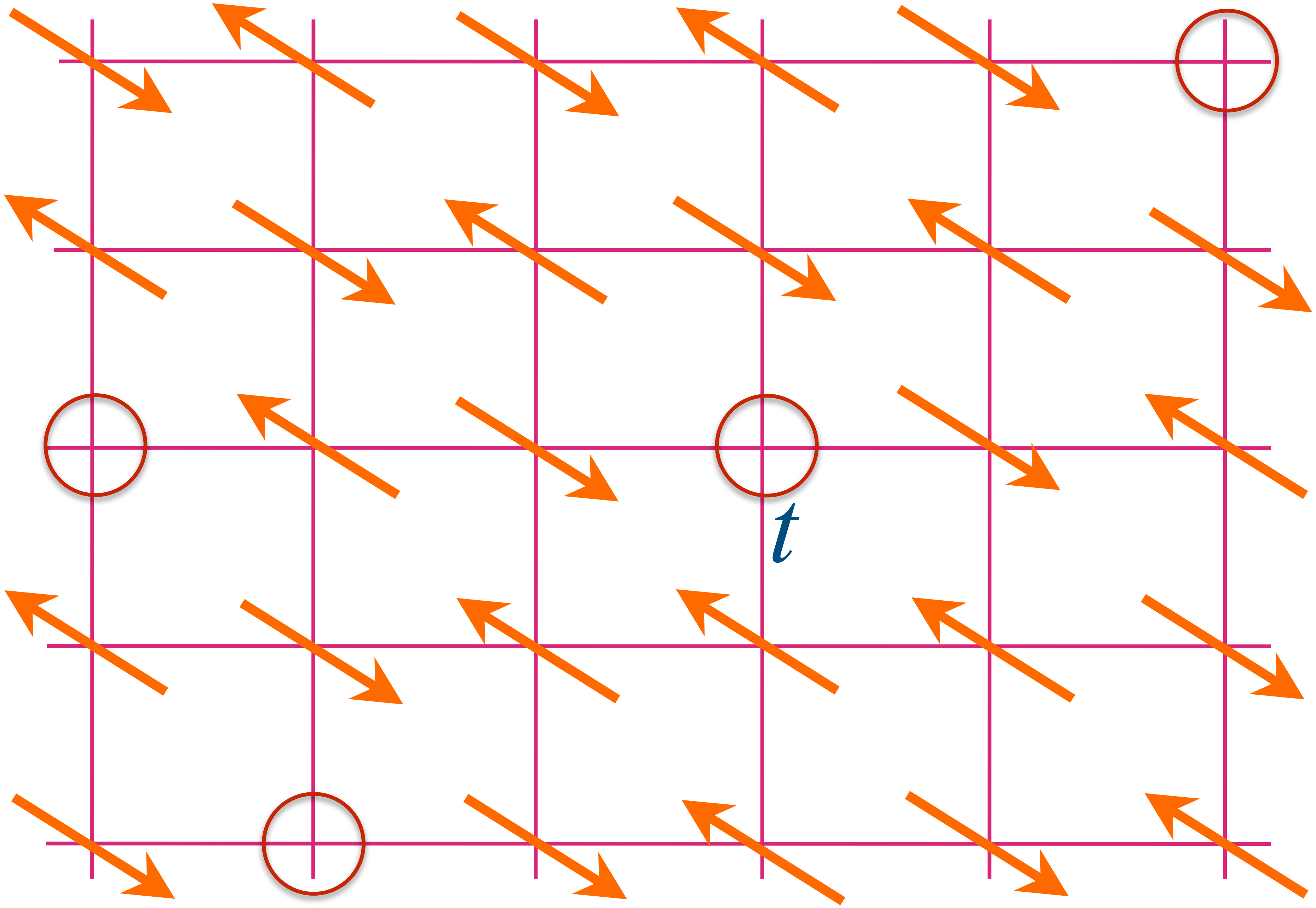

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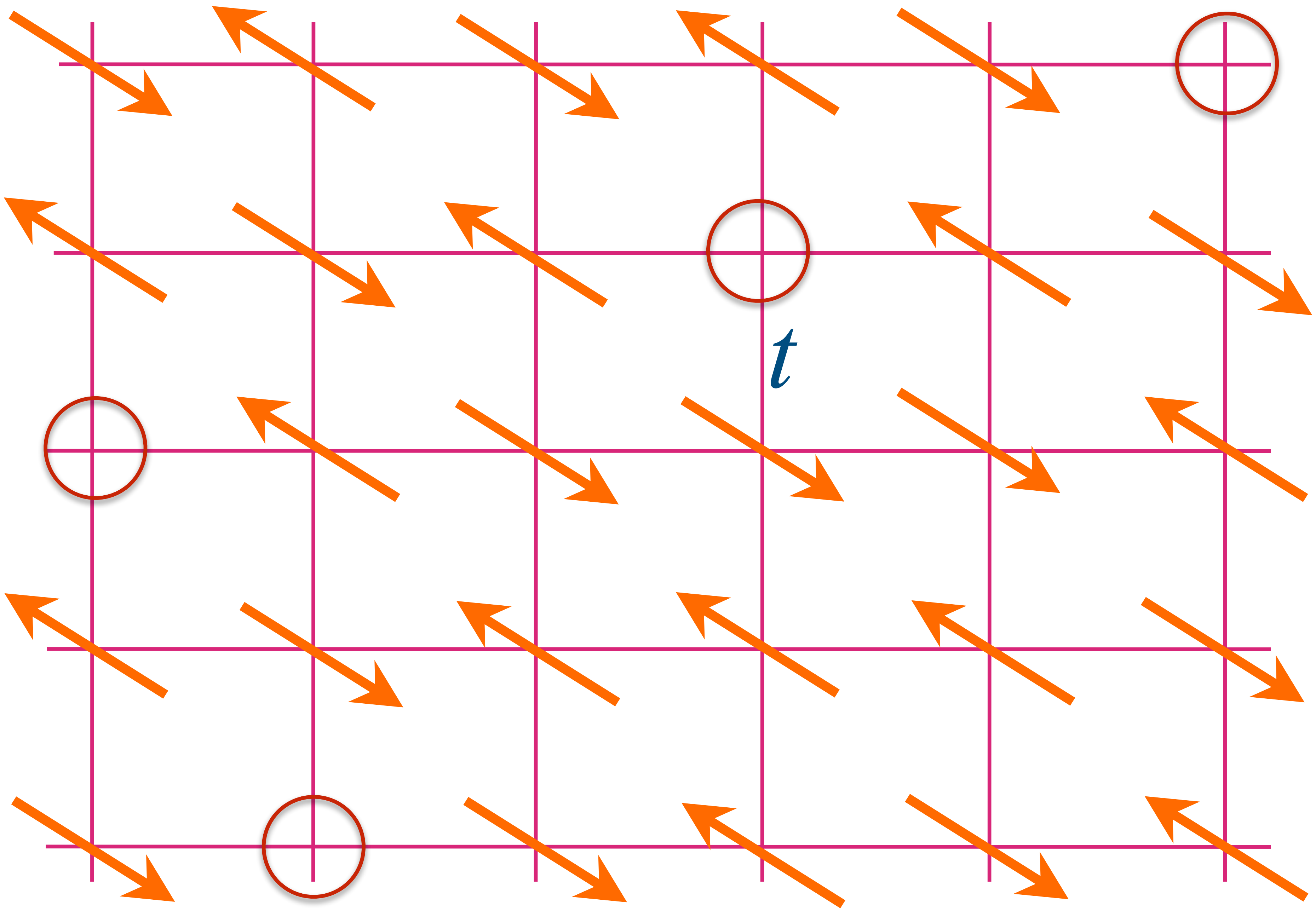

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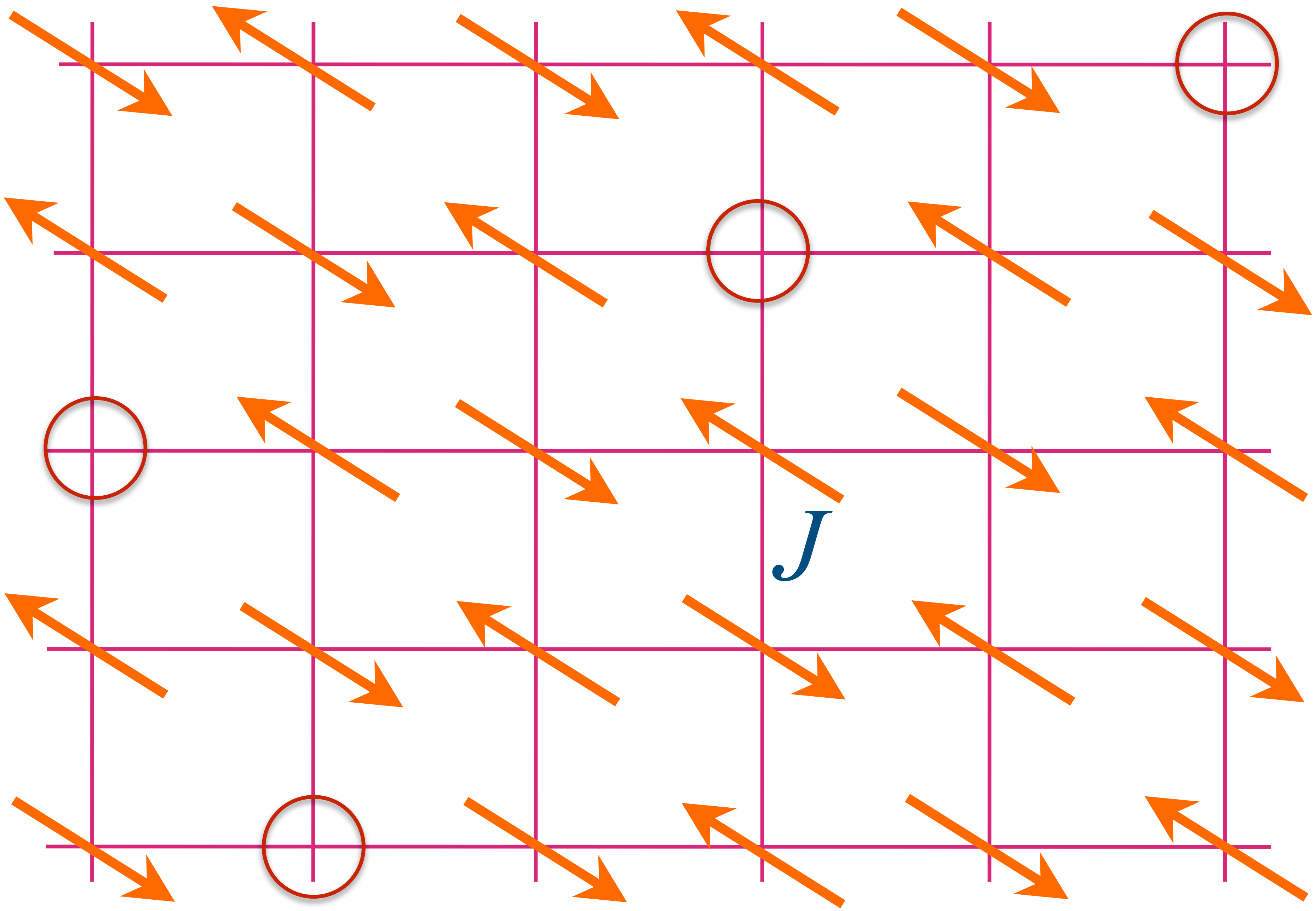


p mobile holes in a background of fluctuating spins

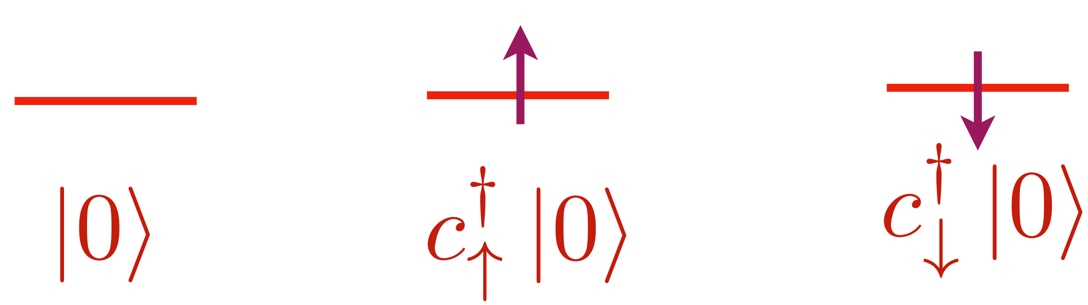
t-J model

$$H = \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

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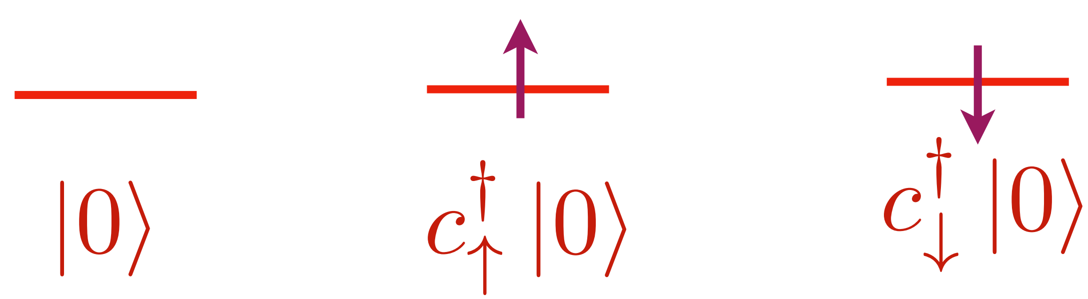
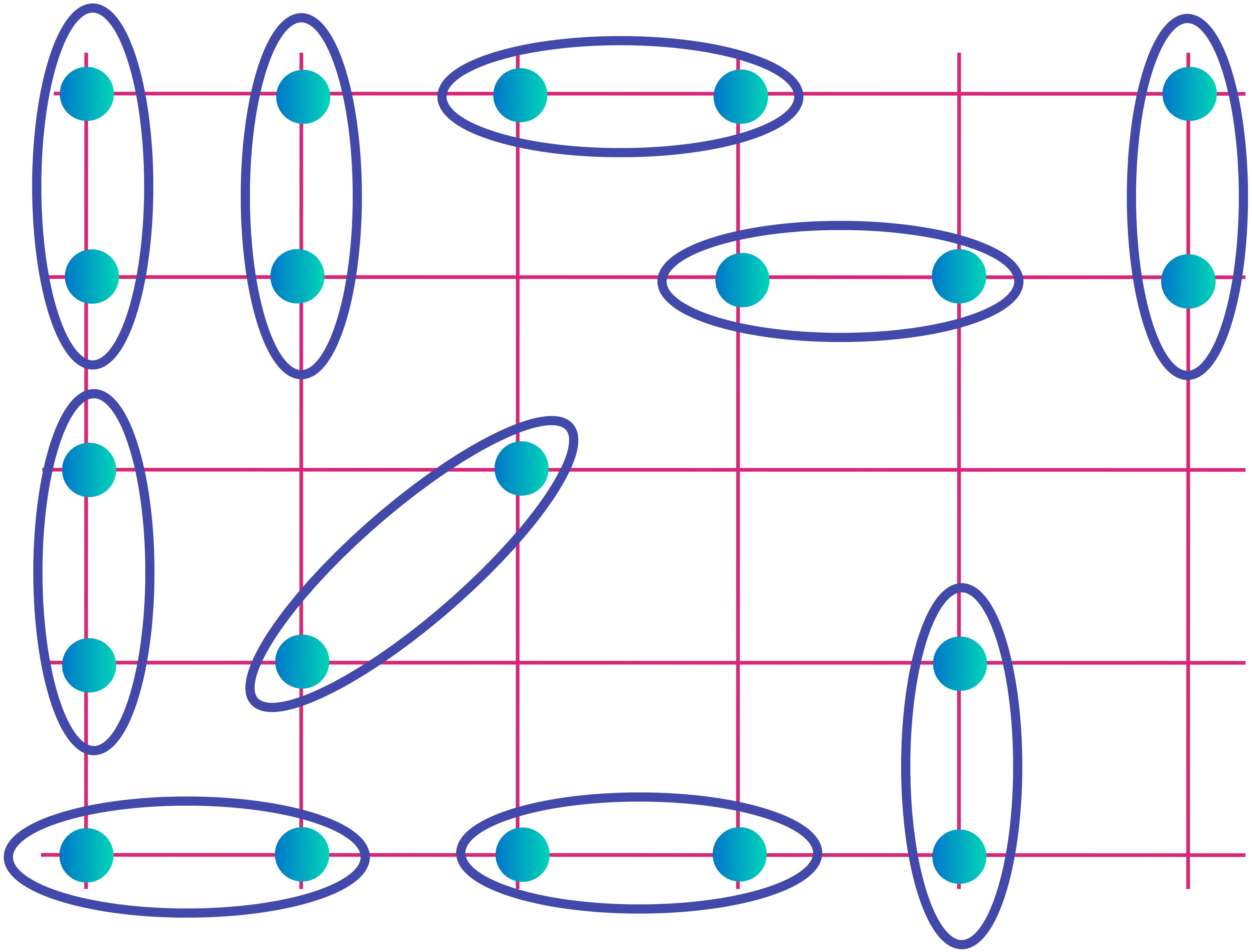
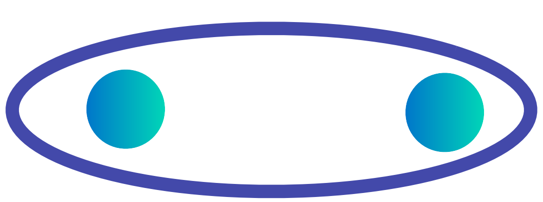
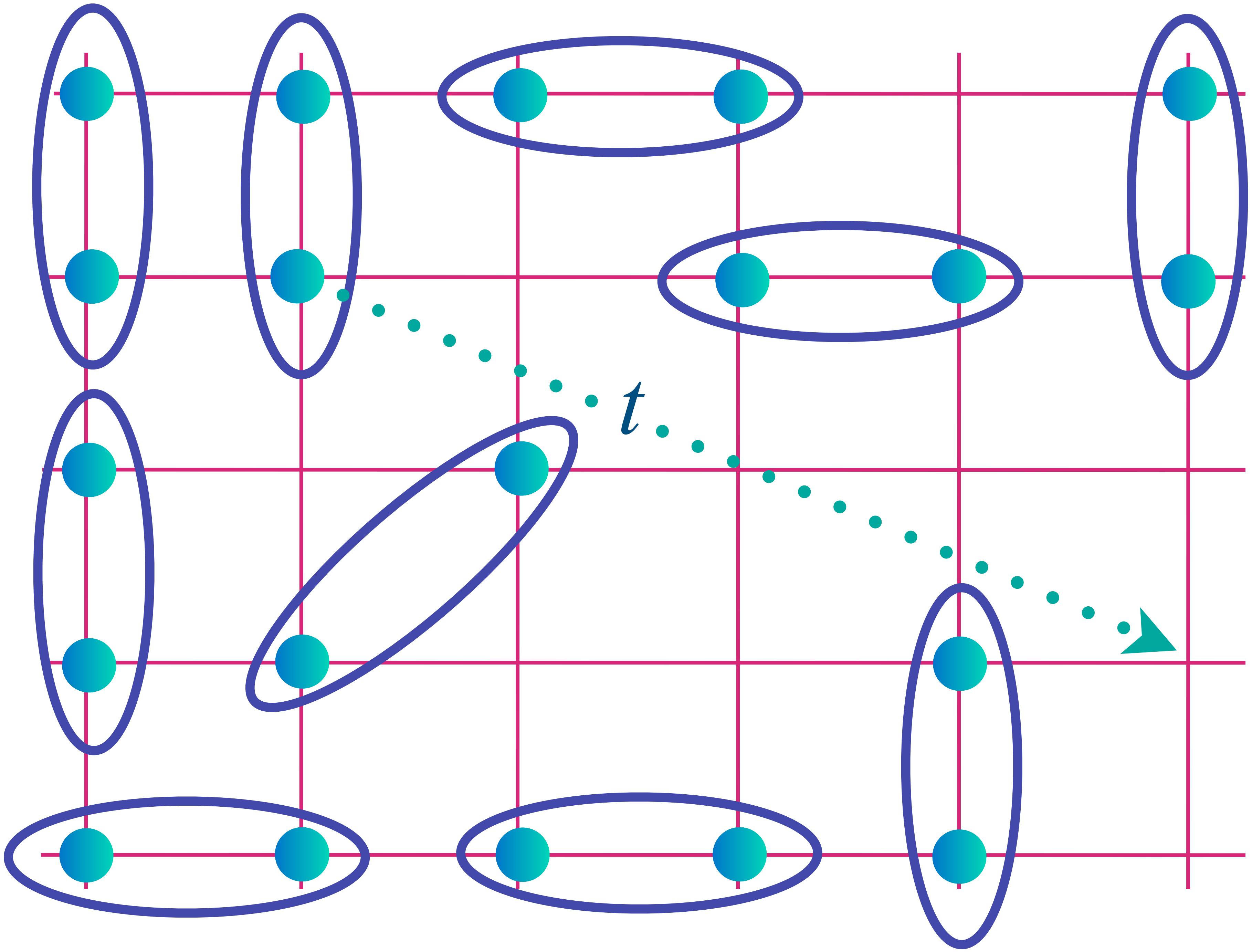
$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \ \overline{J^2_{ij}} = J^2$$
$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \ \overline{t^2_{ij}} = t^2$$





Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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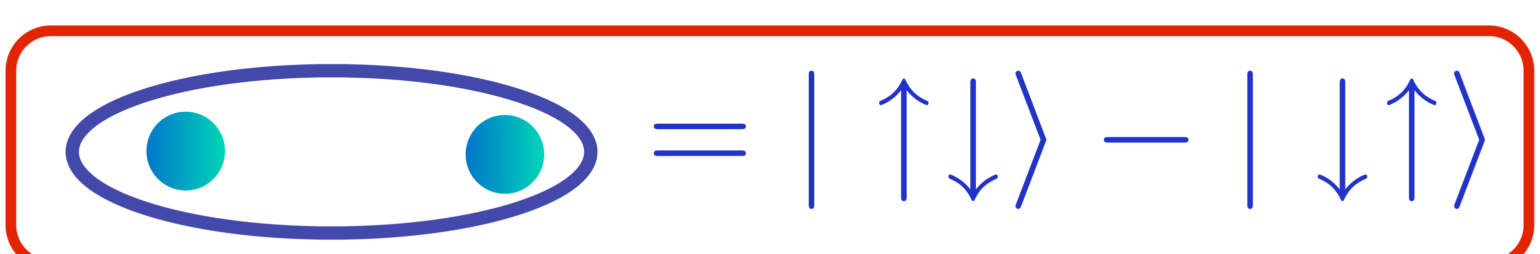

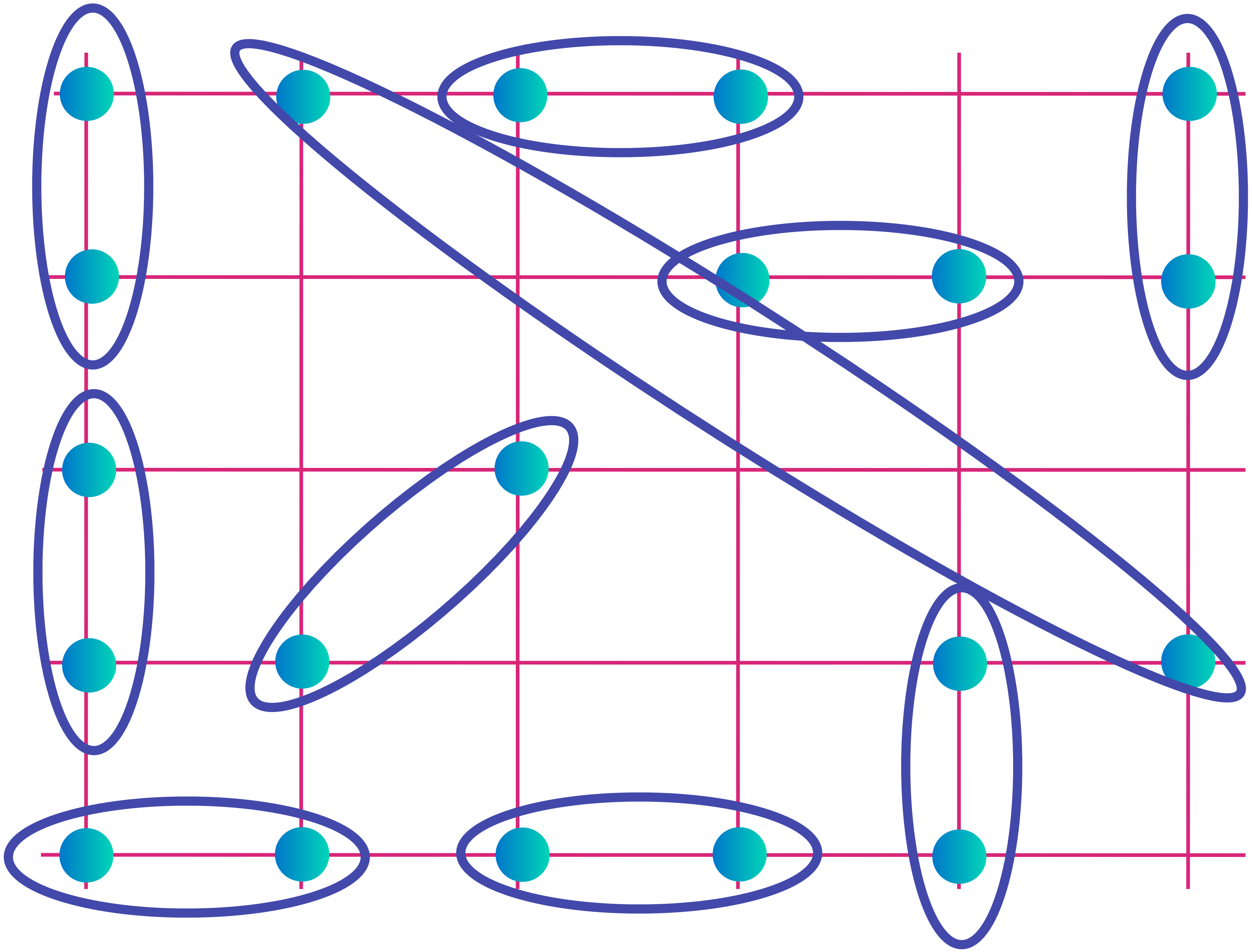

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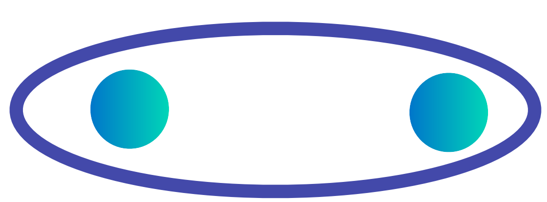
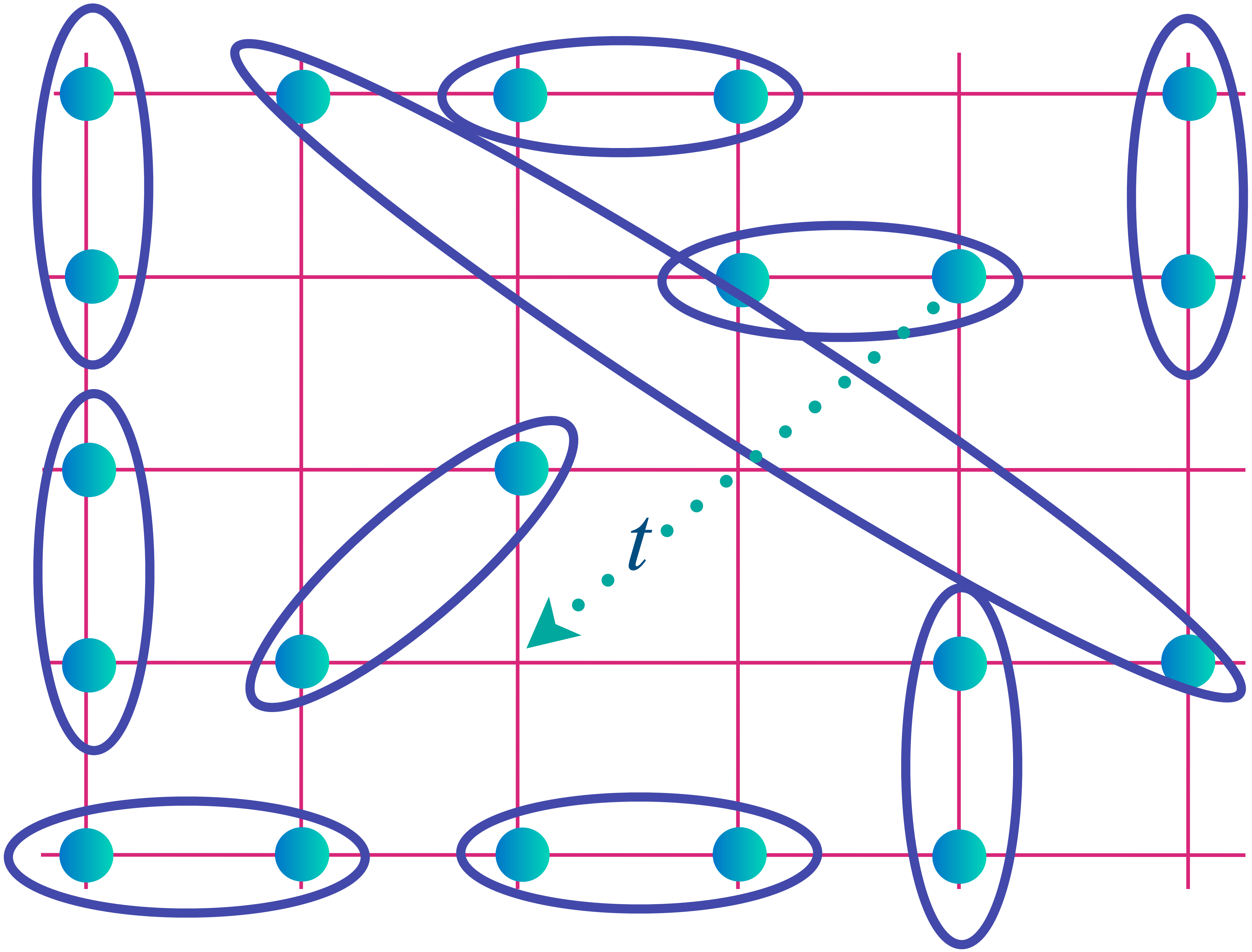
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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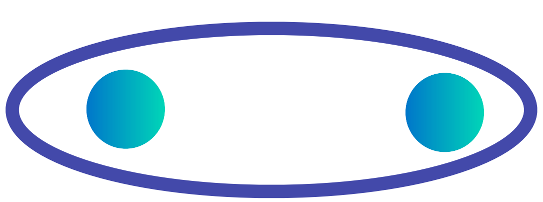
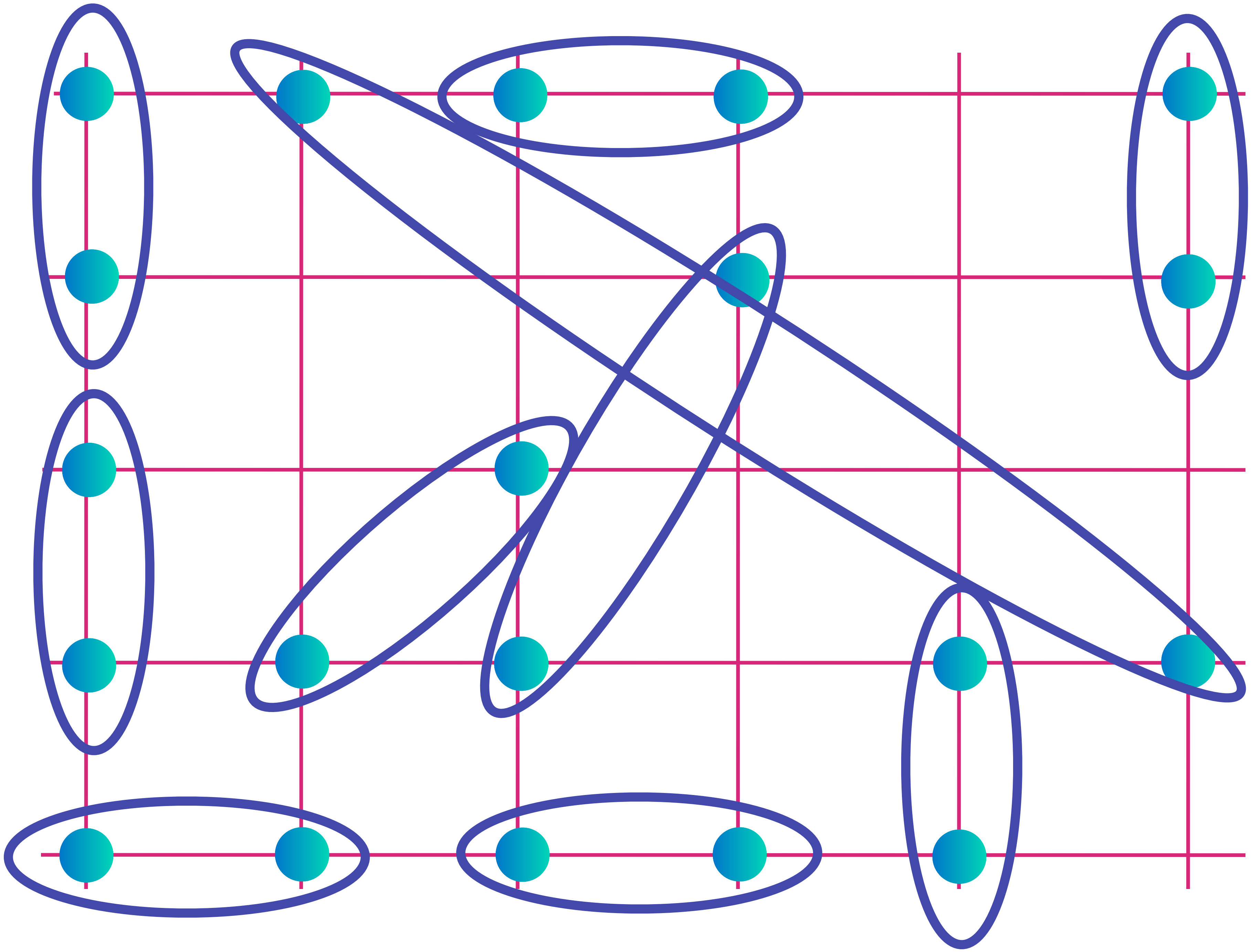
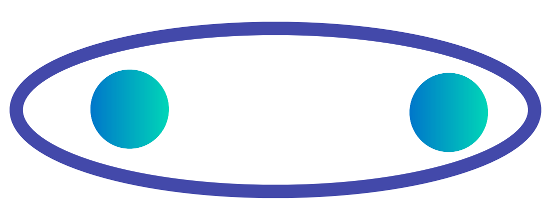


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude



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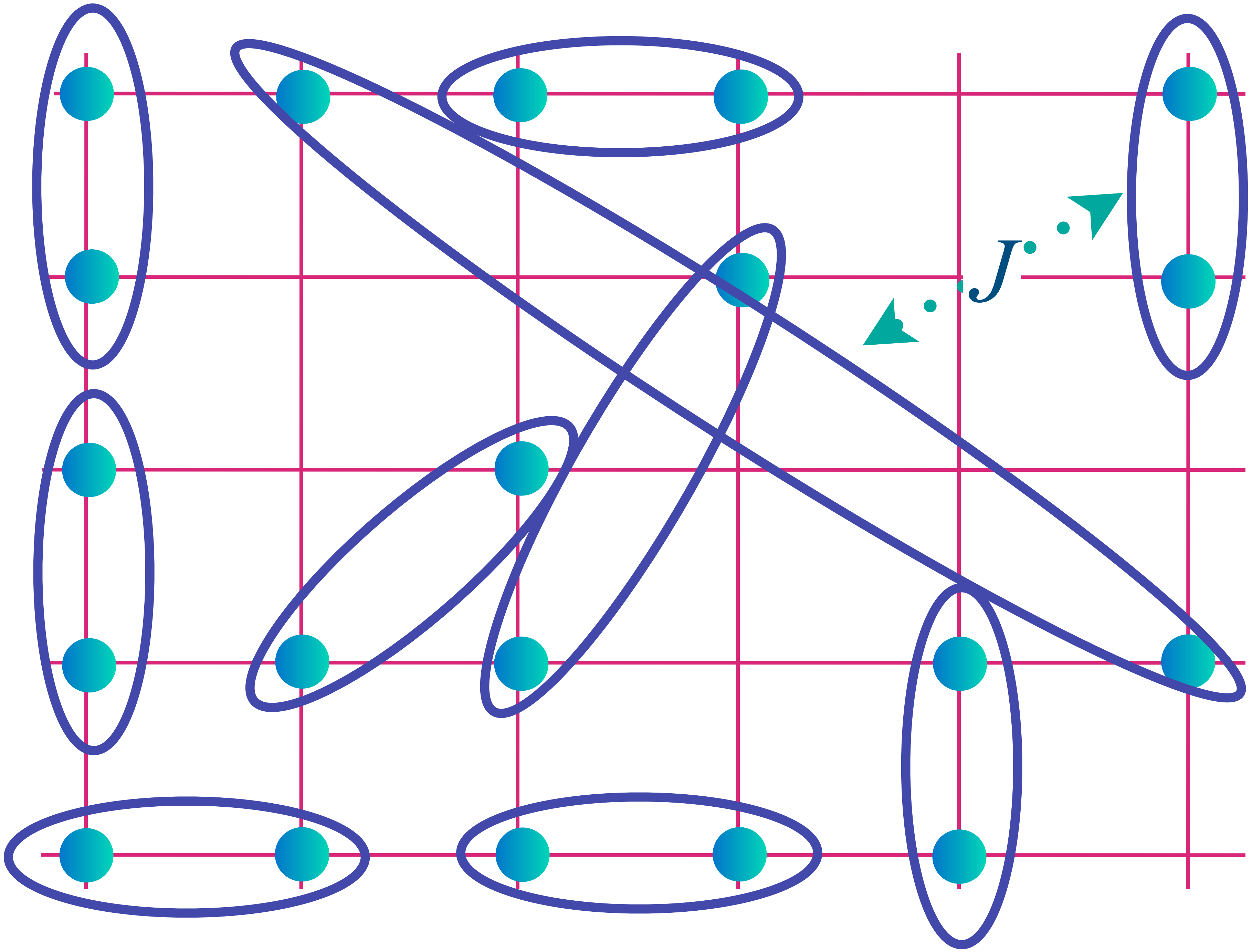

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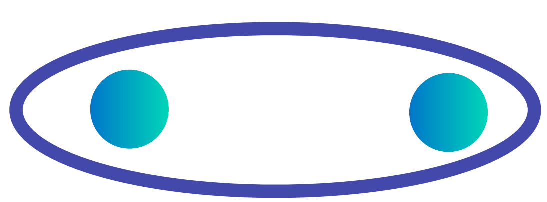
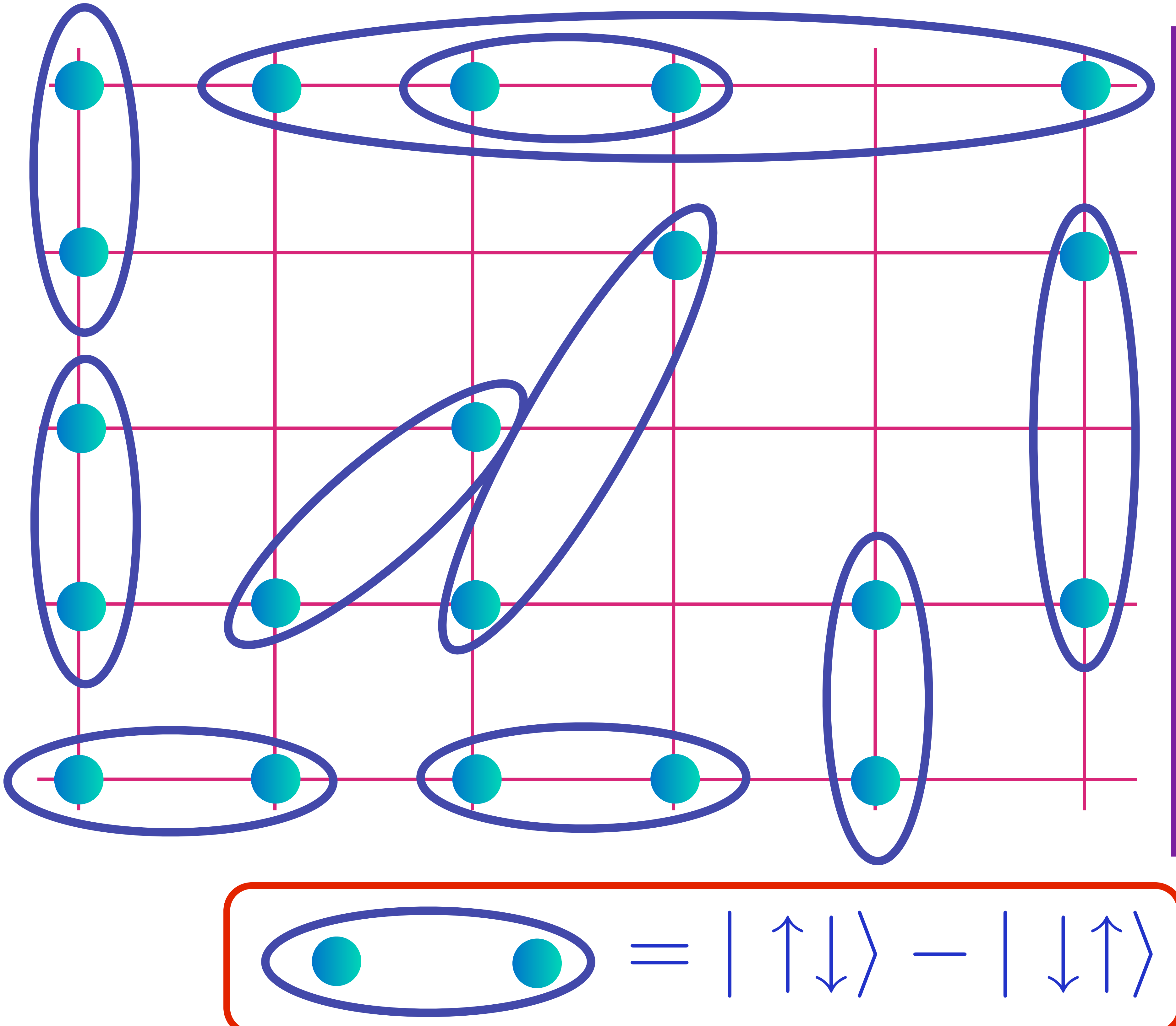
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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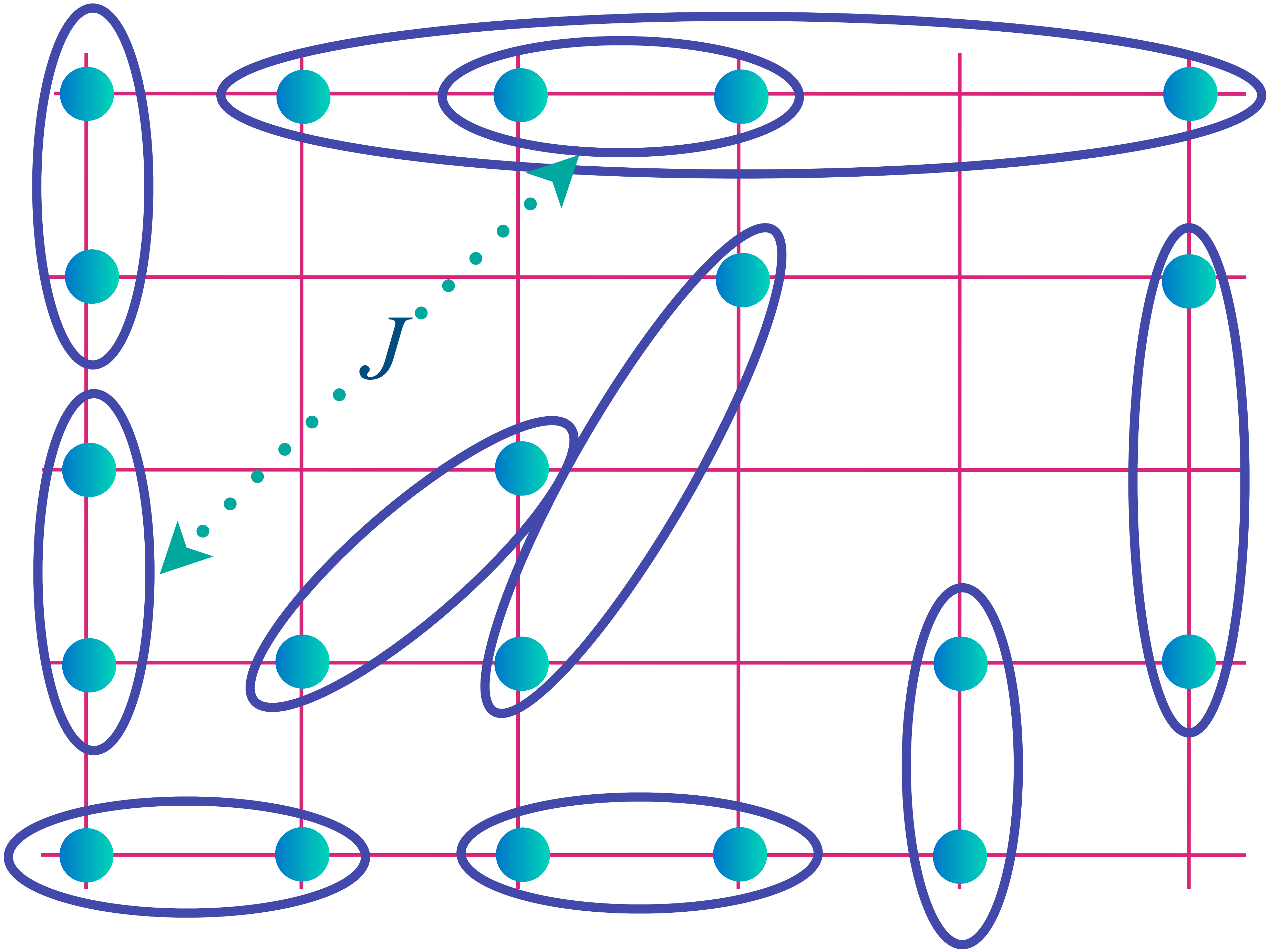
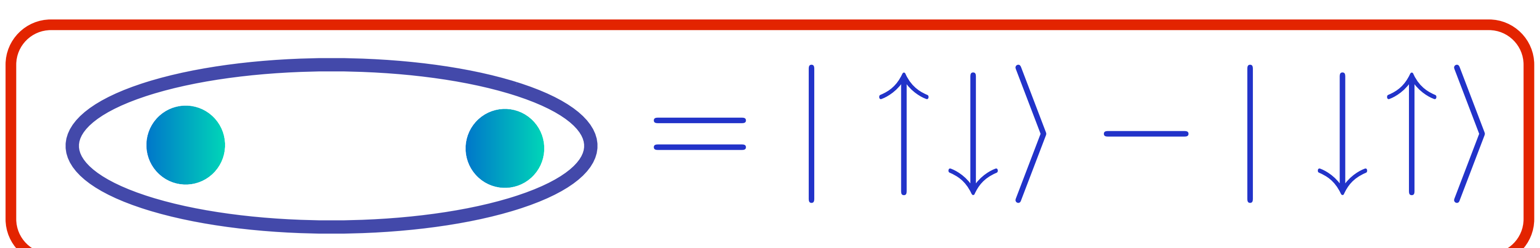
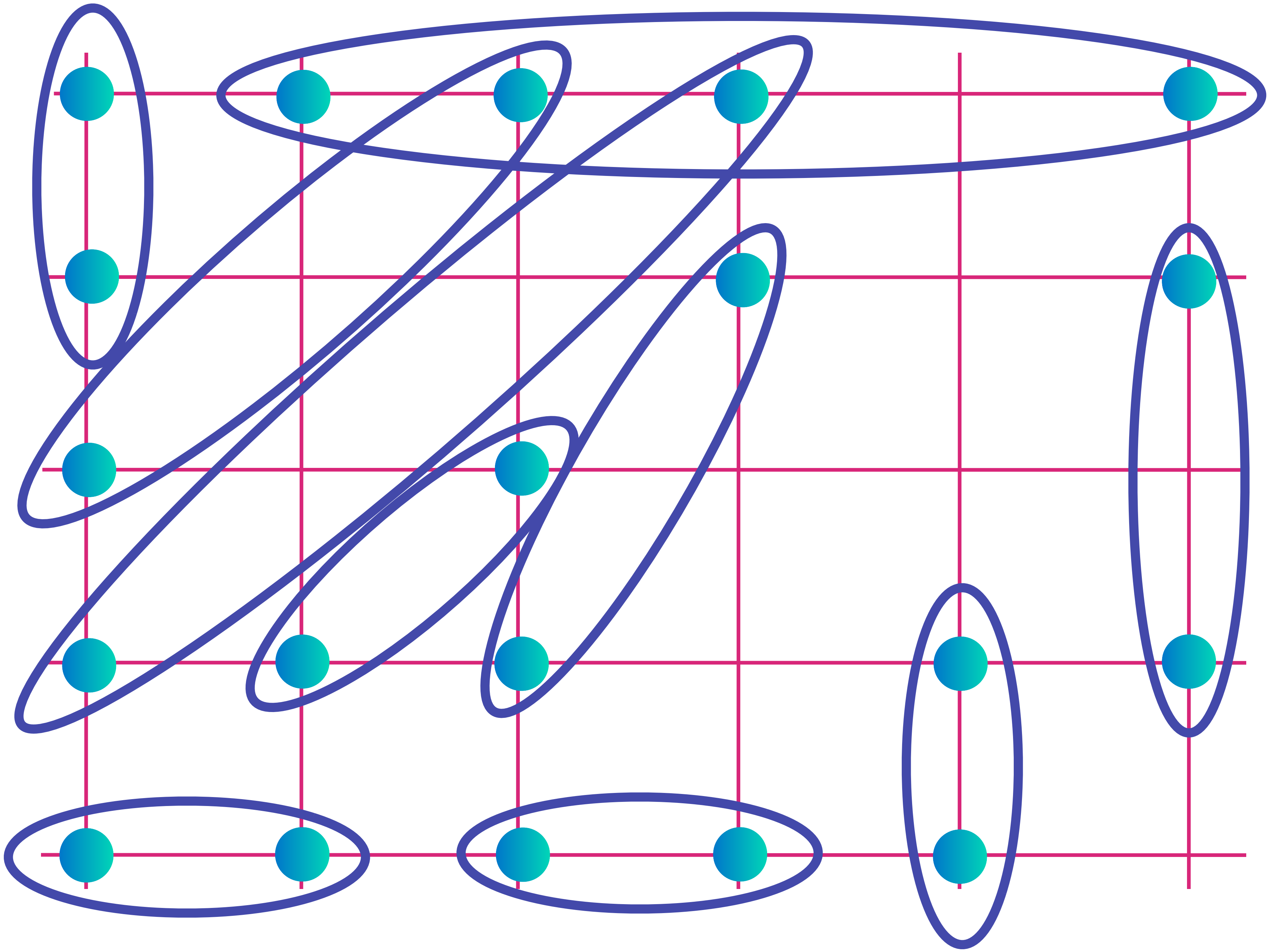
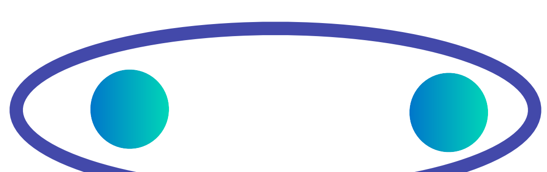


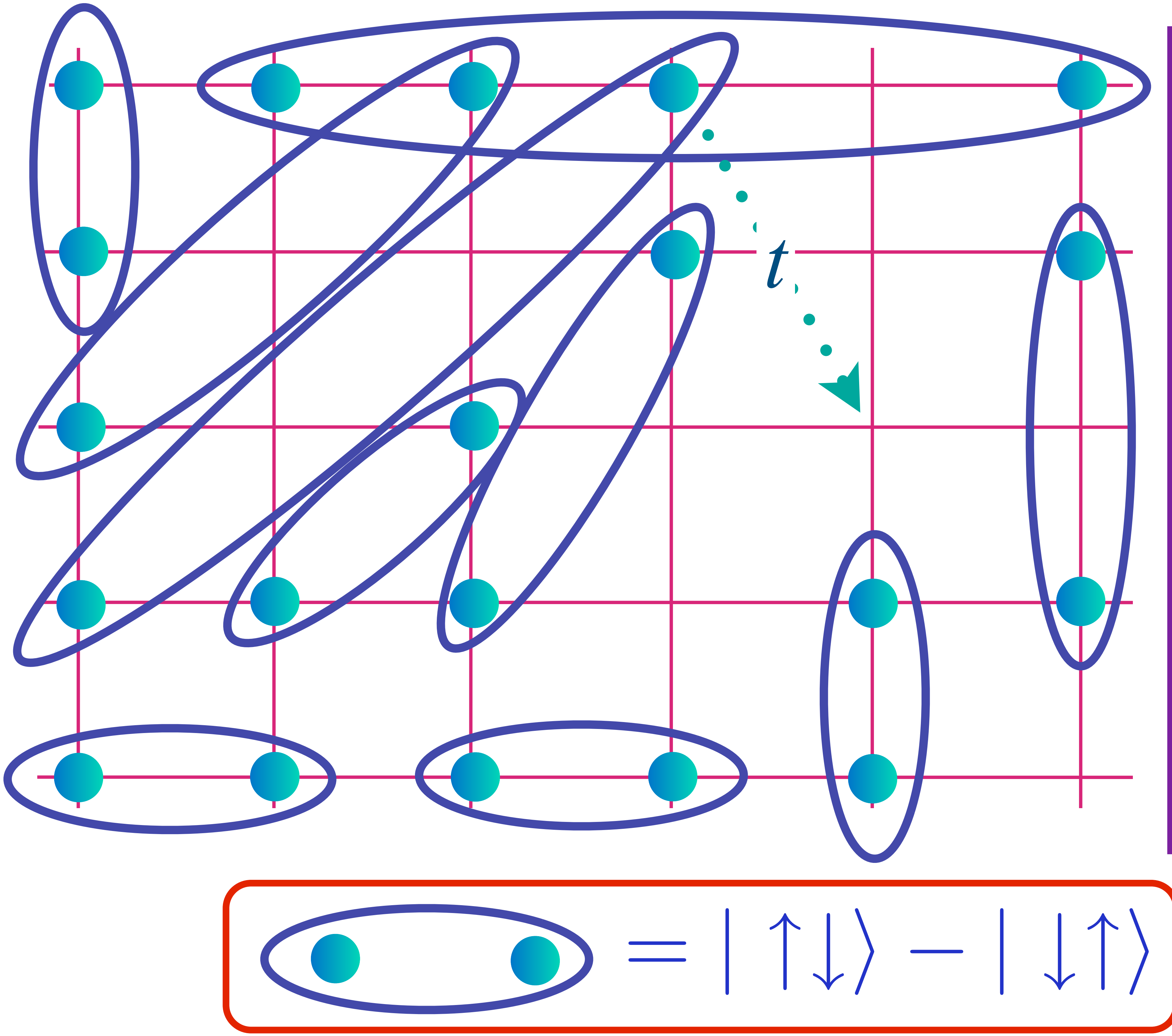
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude



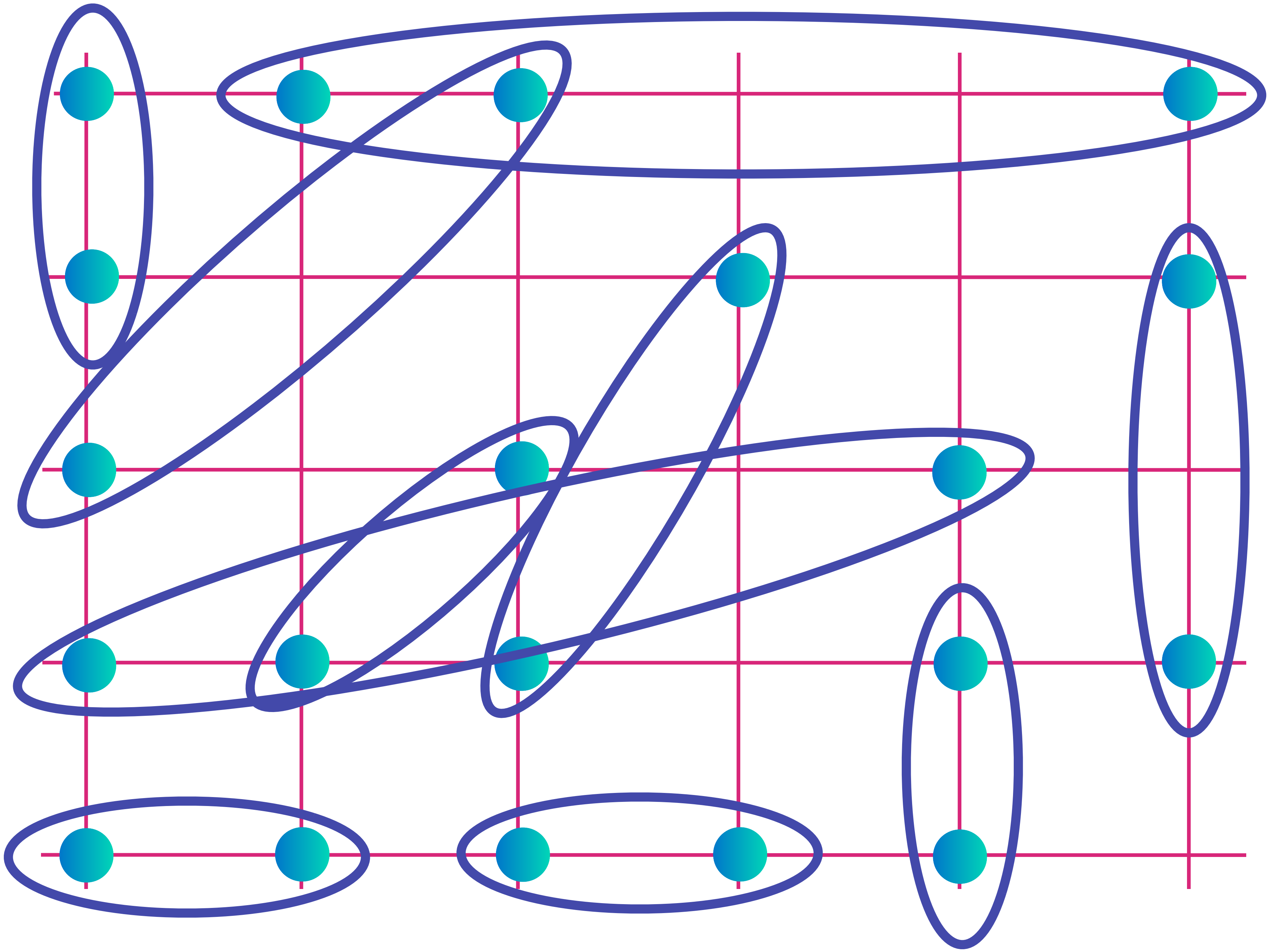


Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

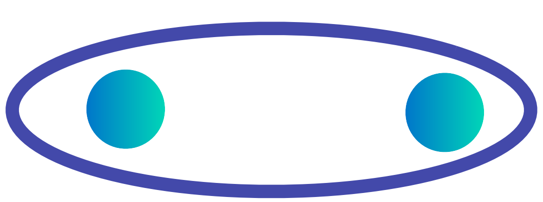
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



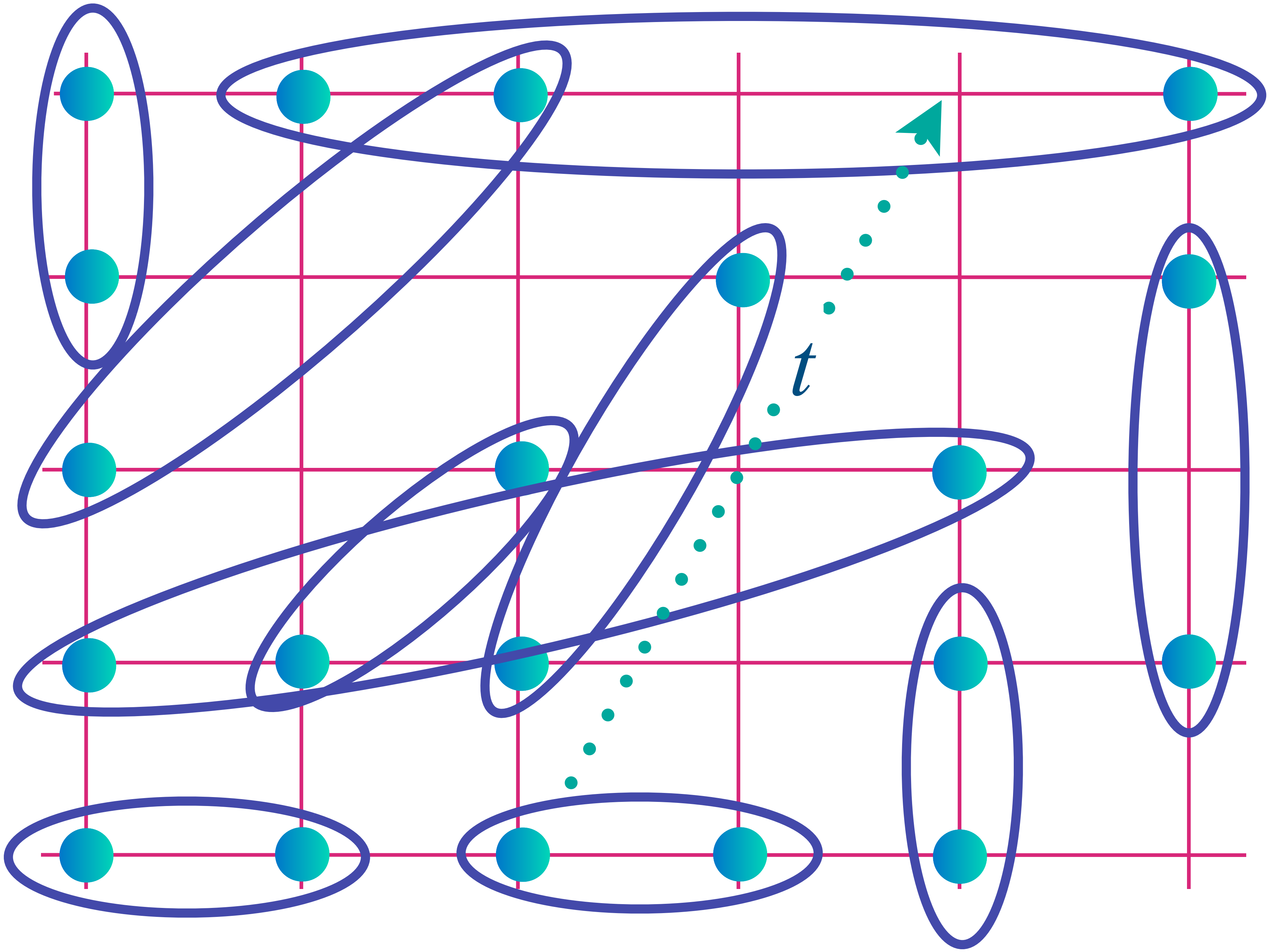
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude



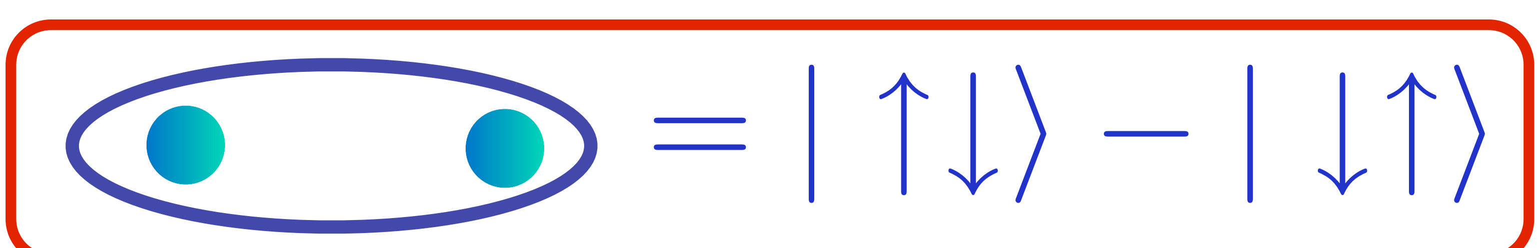
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

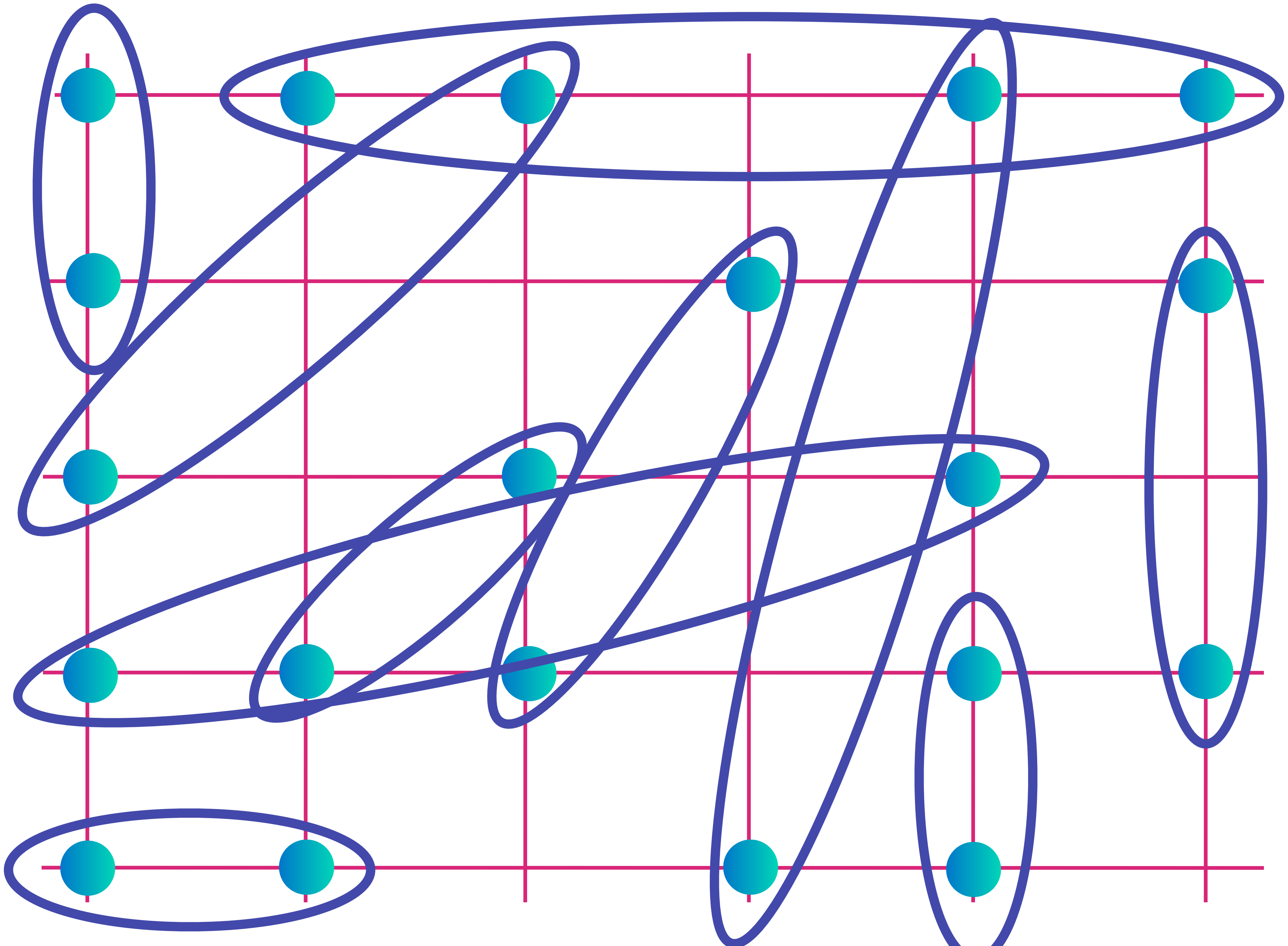


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude





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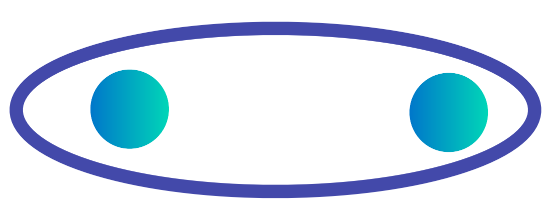
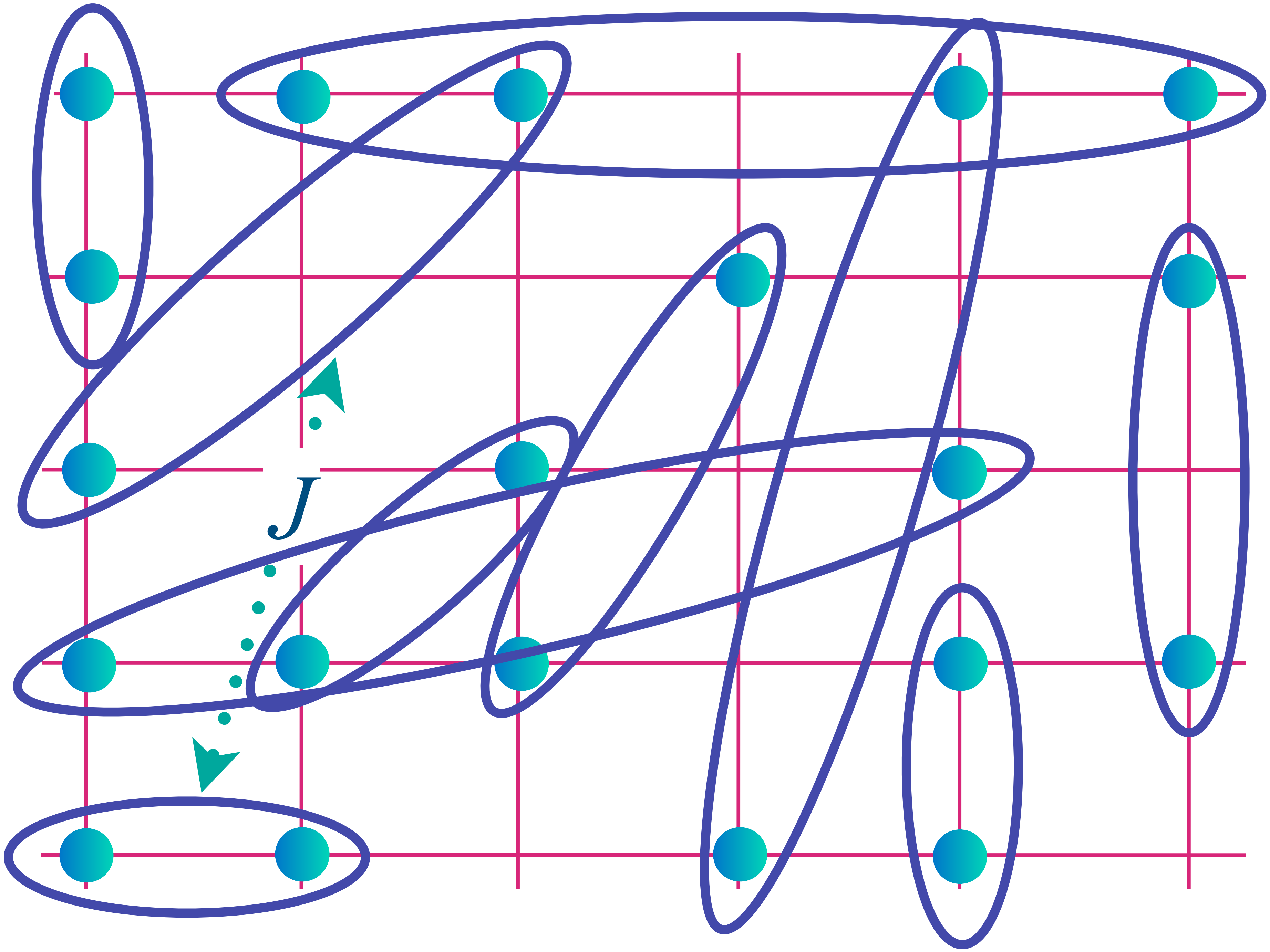
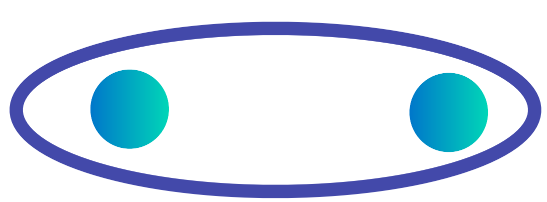
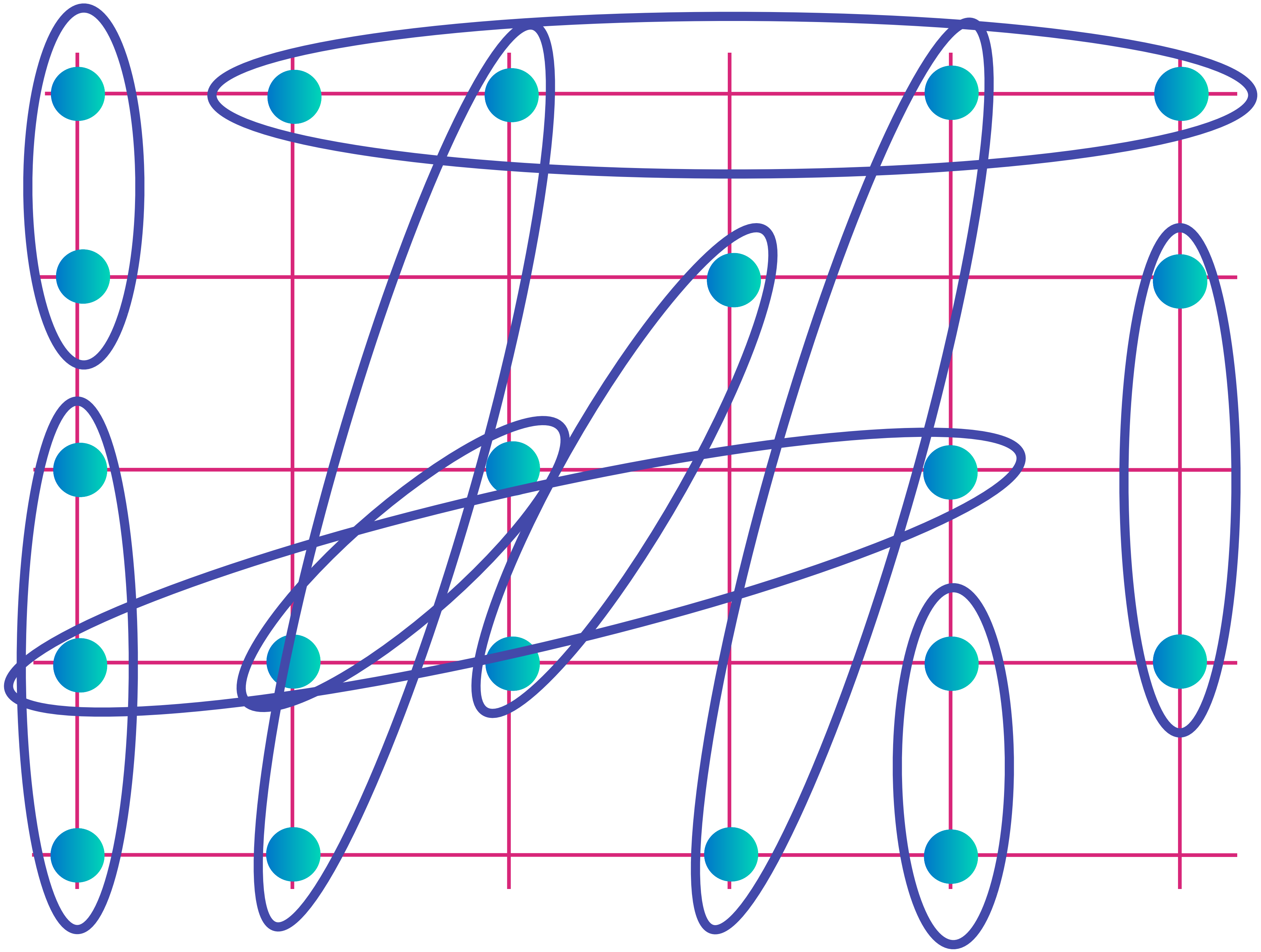
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



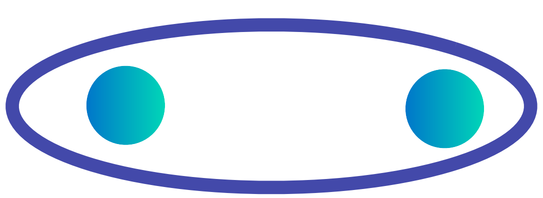
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



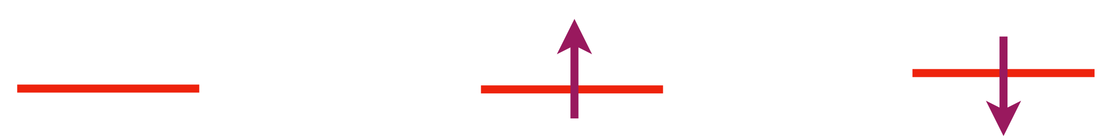
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

<u>*t-J* model</u>

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$b^\dagger \left|v\right\rangle \qquad f_\uparrow^\dagger \left|v\right\rangle \qquad f_\downarrow^\dagger \left|v\right\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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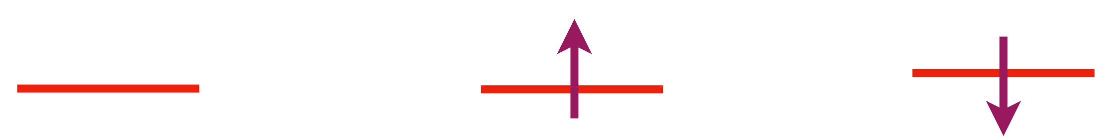


$\mathfrak{f}^\dagger \left|v\right\rangle$ $\quad$ $\mathfrak{b}_\uparrow^\dagger \left|v\right\rangle$ $\quad$ $\mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

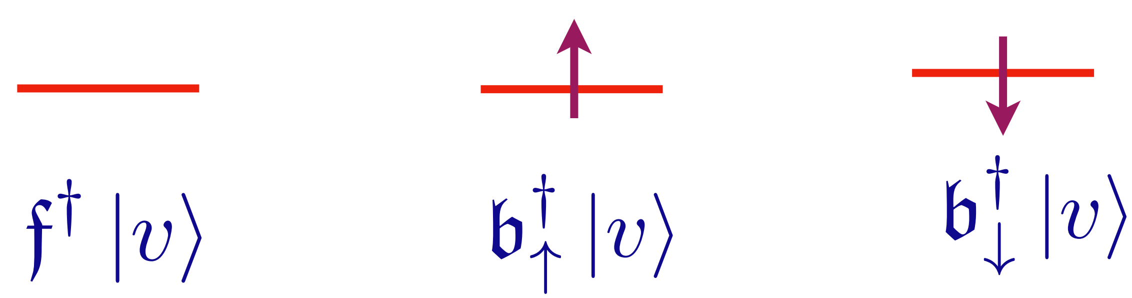
U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



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$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

U(1) gauge invariance, $\quad \mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} = 1$$

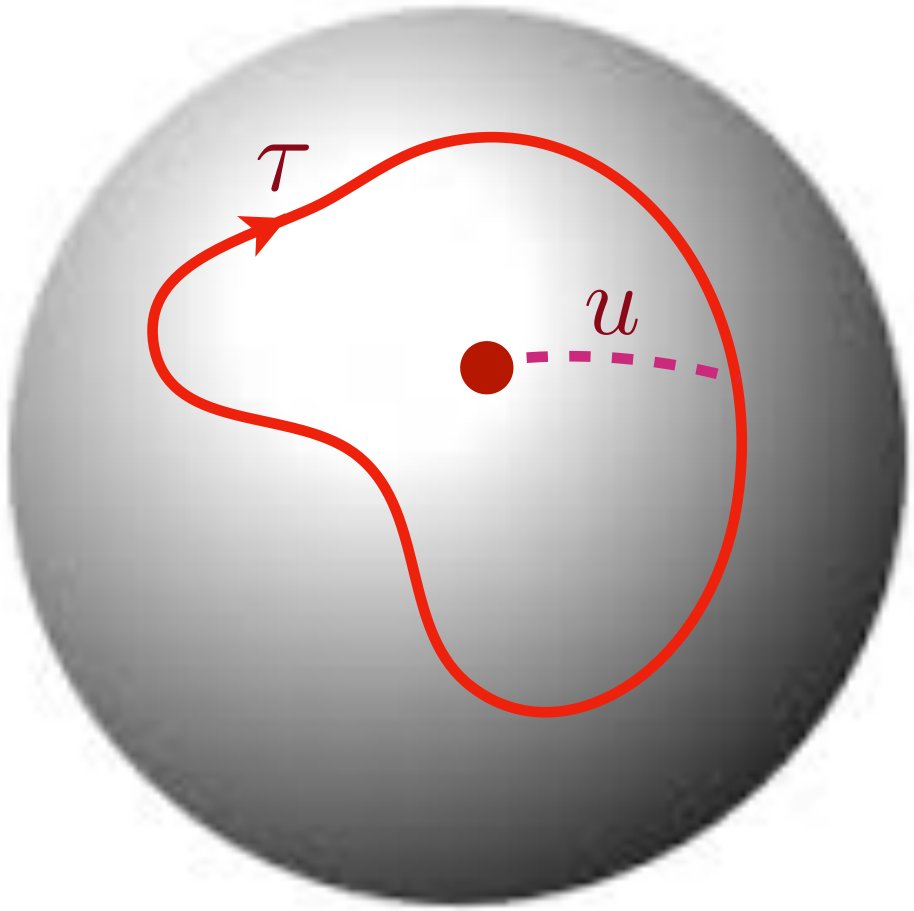
$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

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$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau').$$

From this action we compute

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

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$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.$$

For a model with $\mathrm{SU}(M \to \infty)$ symmetry: critical 'spin liquid' state with $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim 1/|\tau|$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Numerical solution for $\mathrm{SU}(2)$ symmetry: spin glass order: $\lim_{\tau\to\infty} \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \neq 0$

L. Arrachea and M. J. Rozenberg, PRB **65**, 224430 (2002)

t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

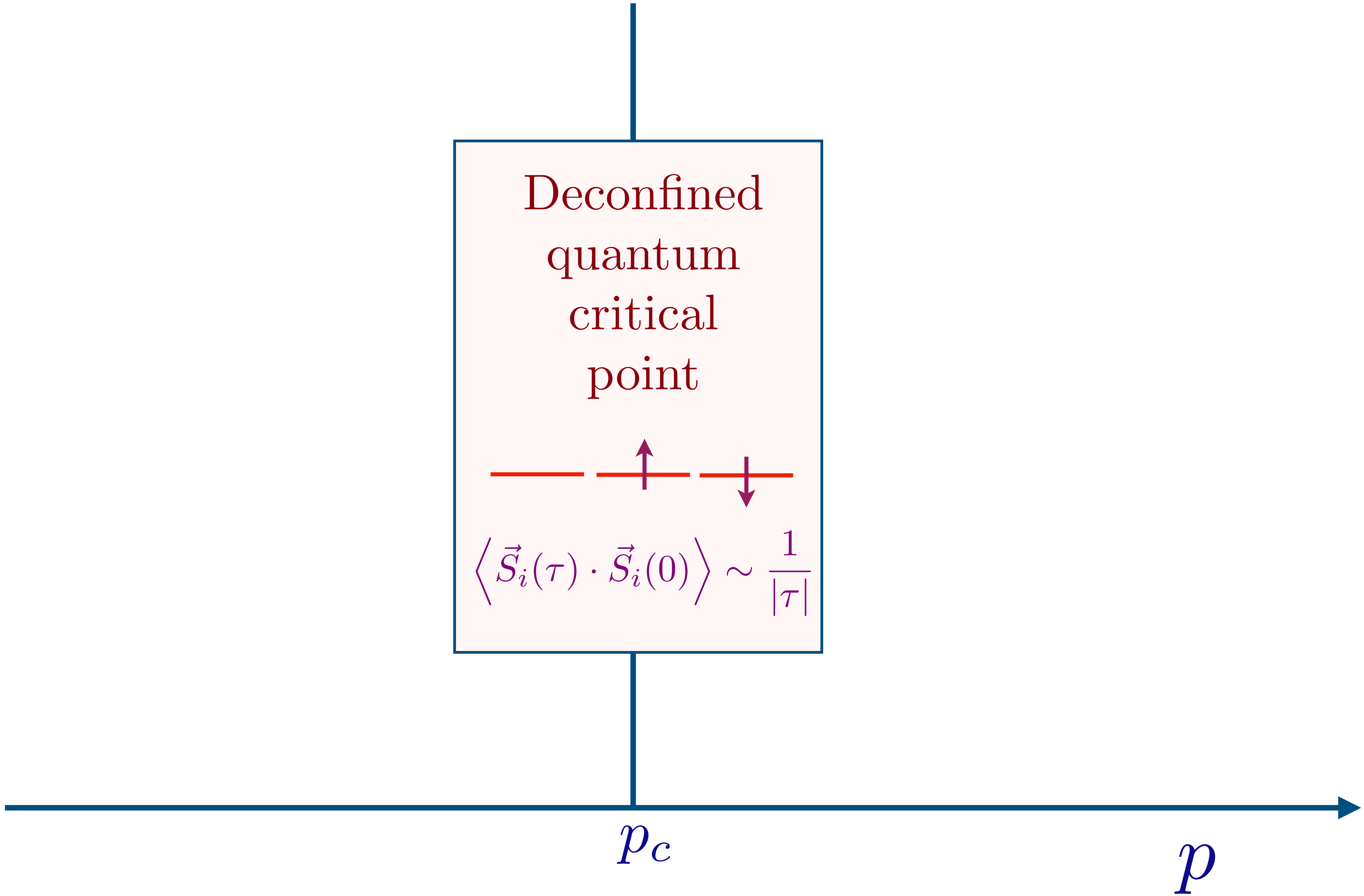
$$\mathcal{S}_B = i \int_0^1 du \int d\tau \, \mathrm{Tr}\left(\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P} \right)$$

$$\mathcal{S}_{tJ} = \int d\tau d\tau' \, \mathrm{Tr}\left(\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau') \right) + \int d\tau \mathrm{Tr}\left(s_0 \, \mathcal{P}(\tau) \right) .$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a 'Zeeman superfield' s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

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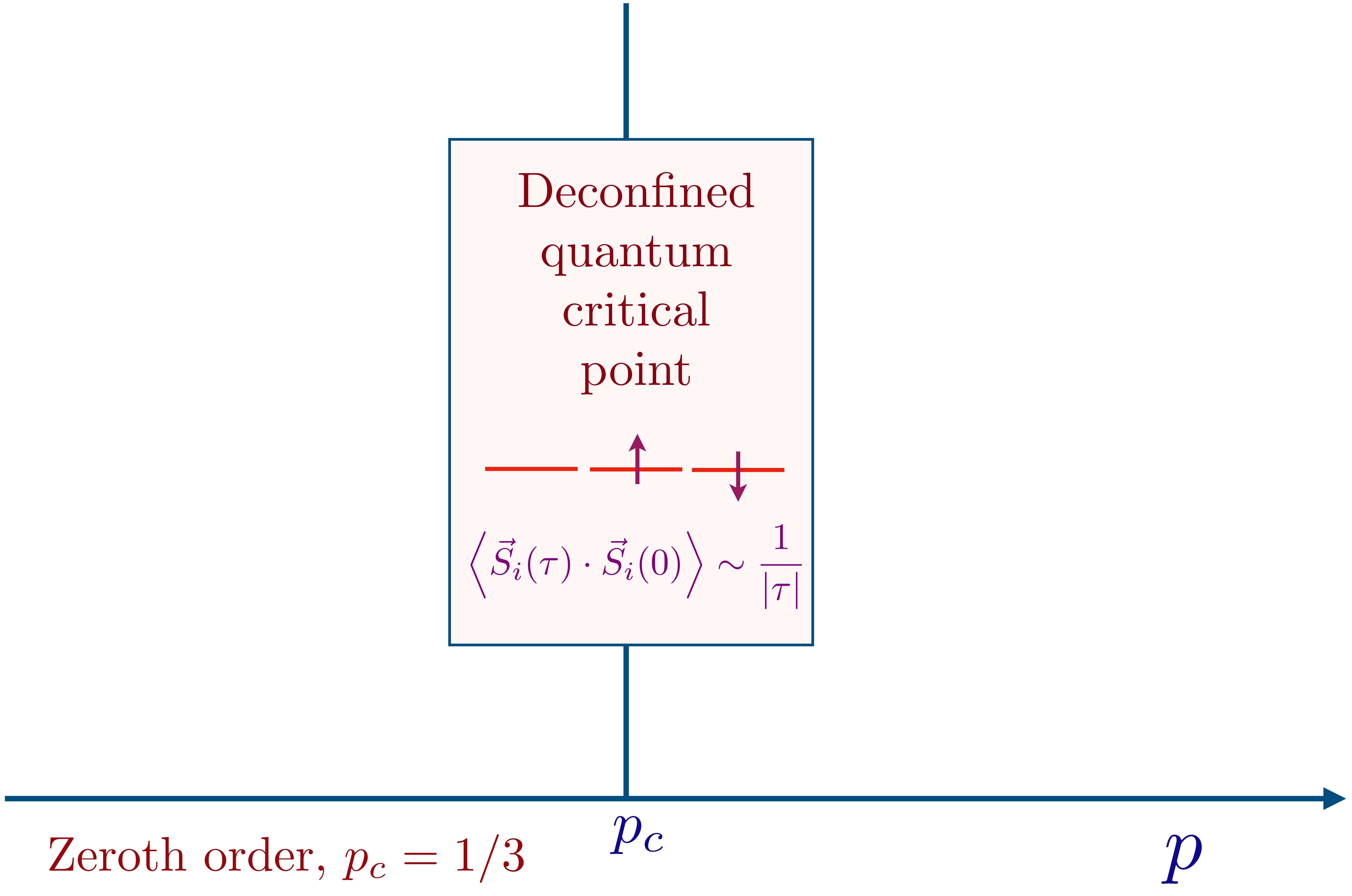
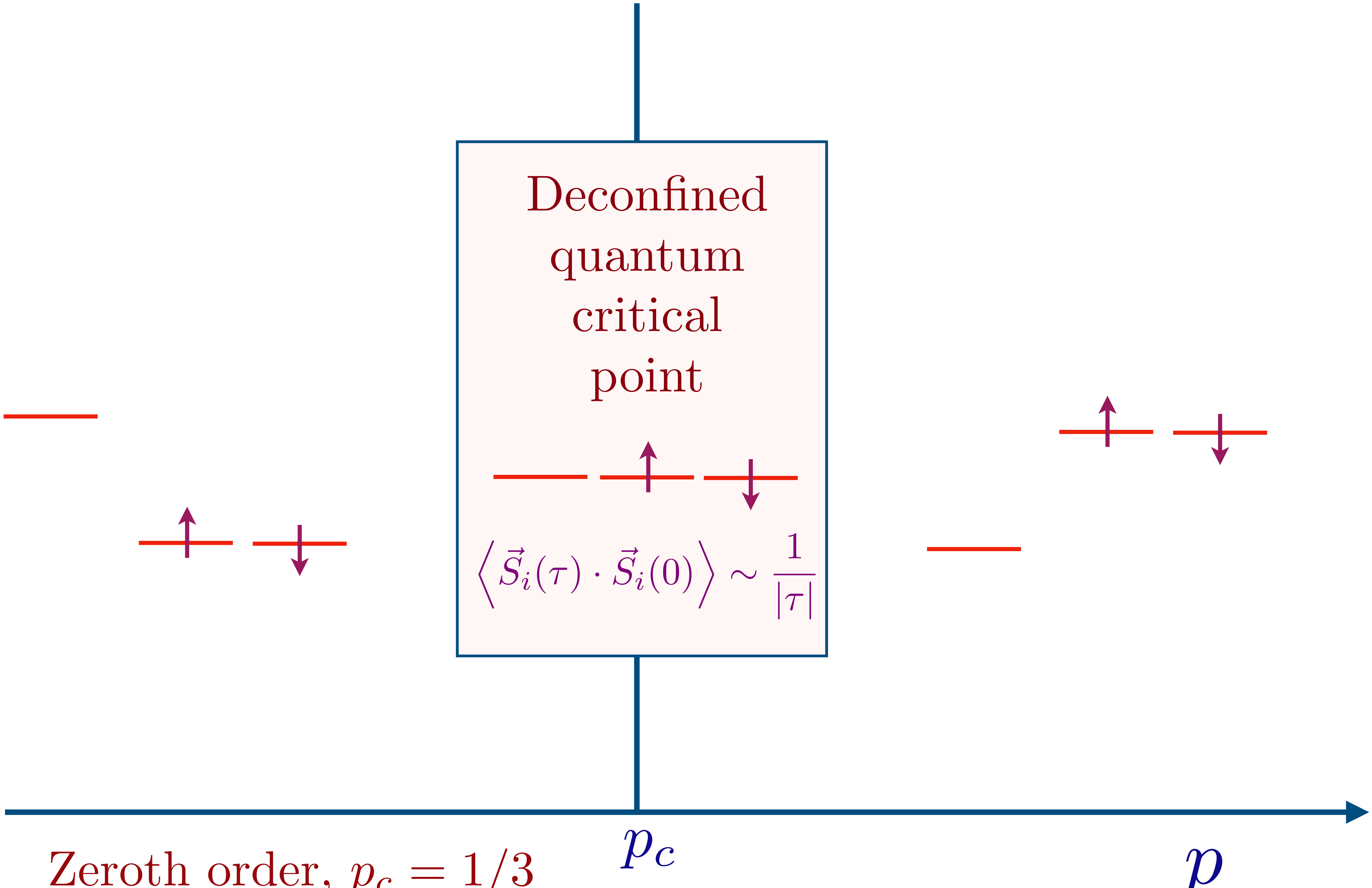


D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

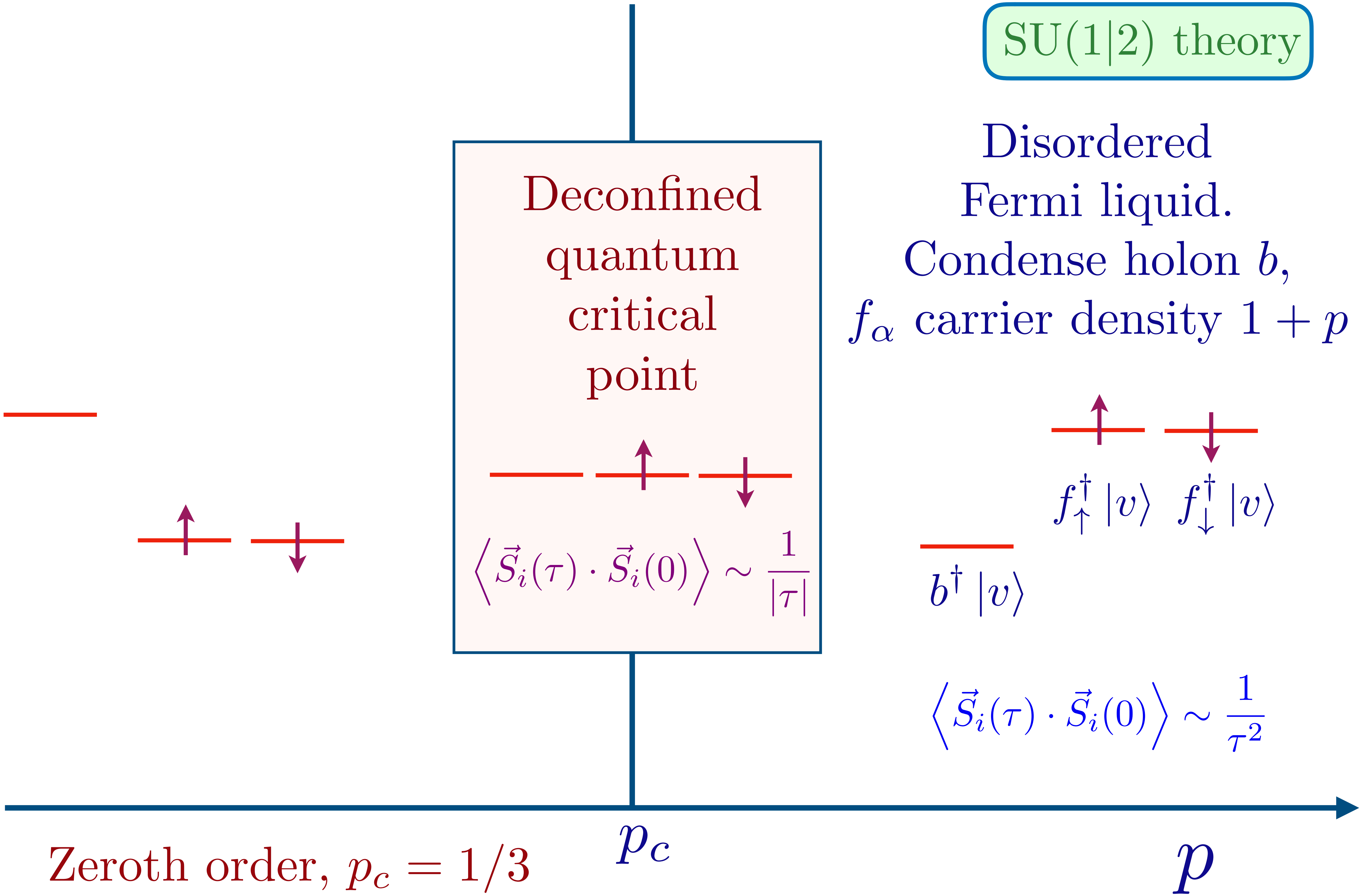
t-J model phase diagram



Zeroth order, $p_c = 1/3$

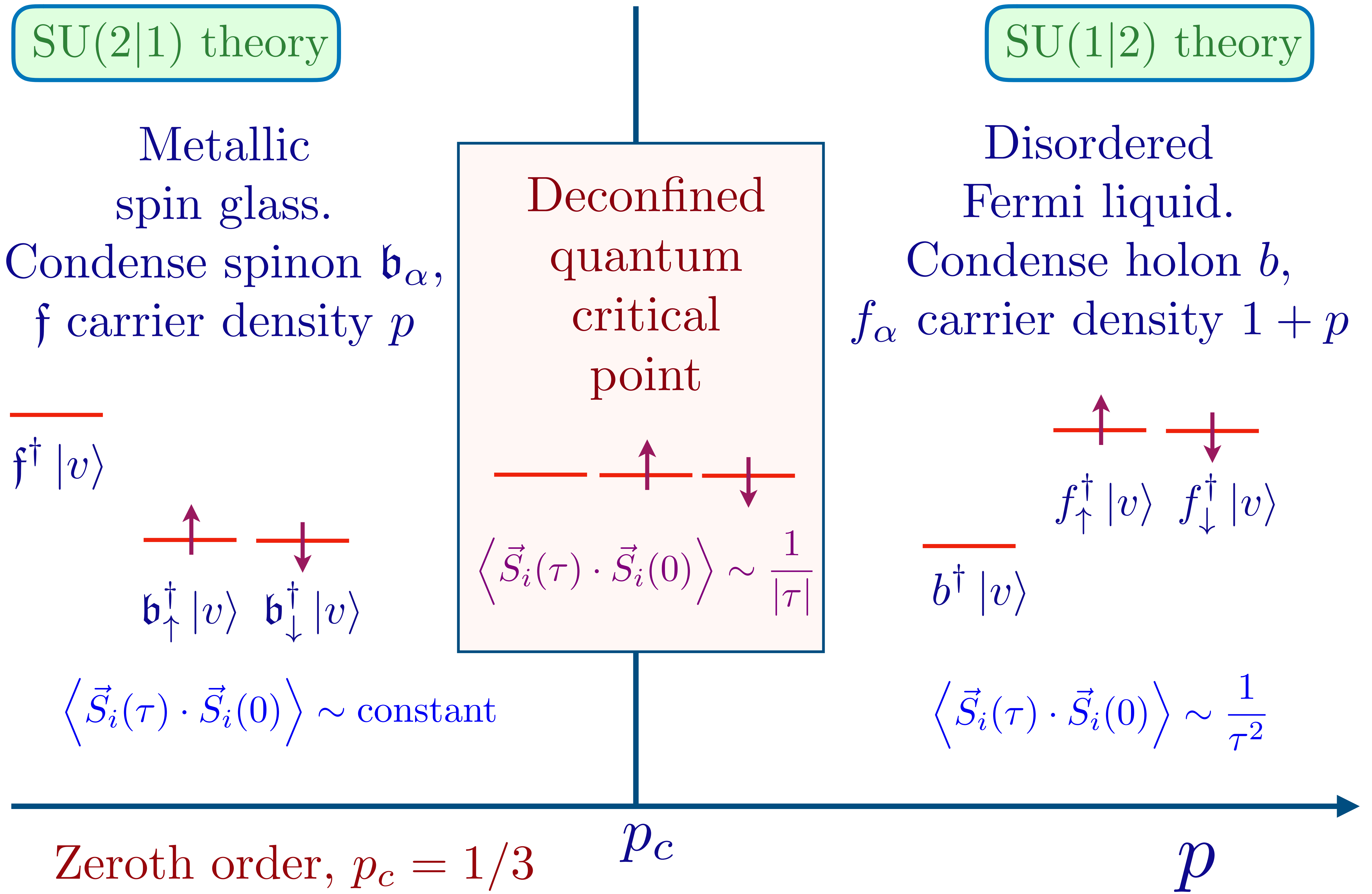
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

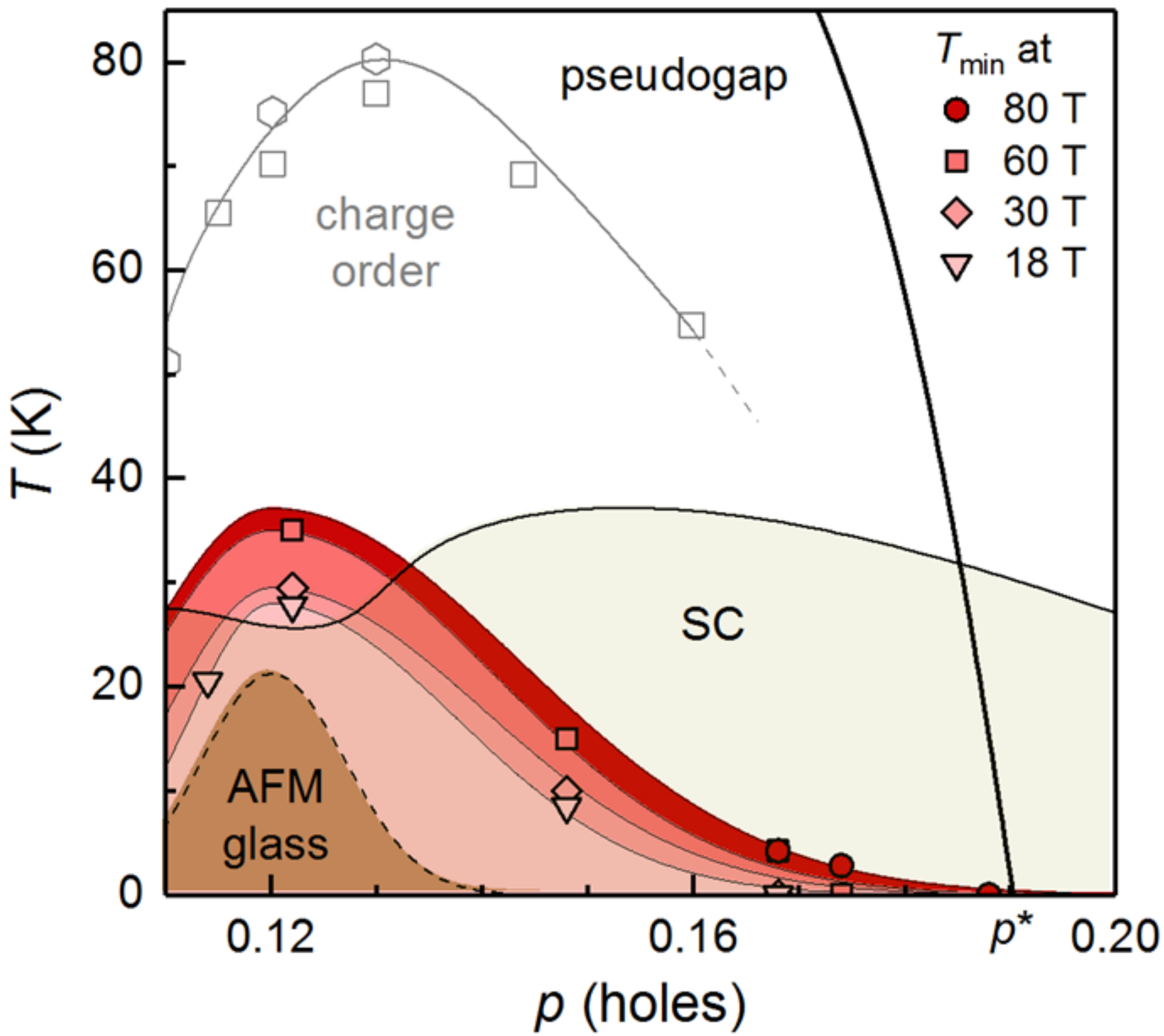
t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

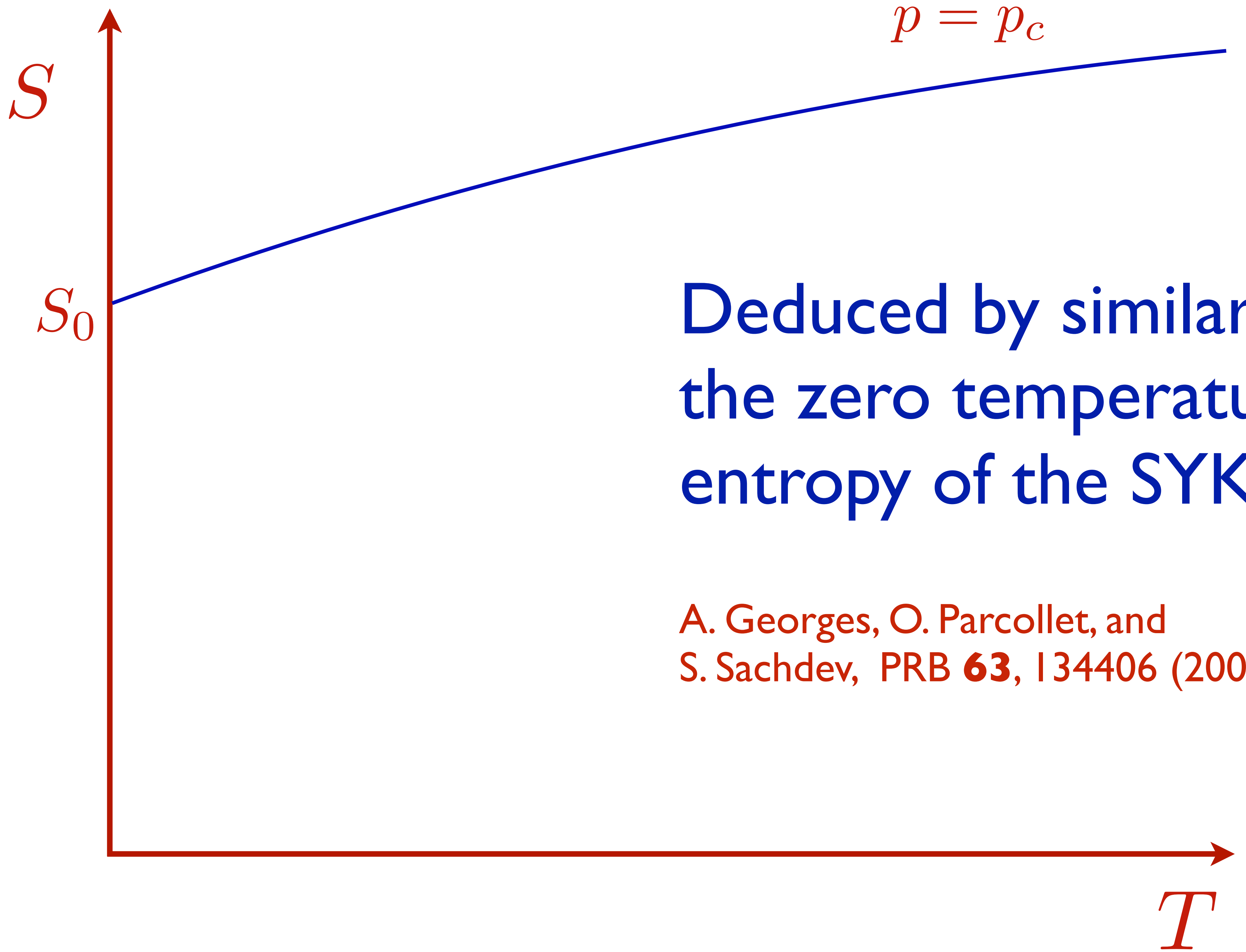
Mehdi Frachet[1†], Igor Vinograd[1†], Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

t-J model entropy

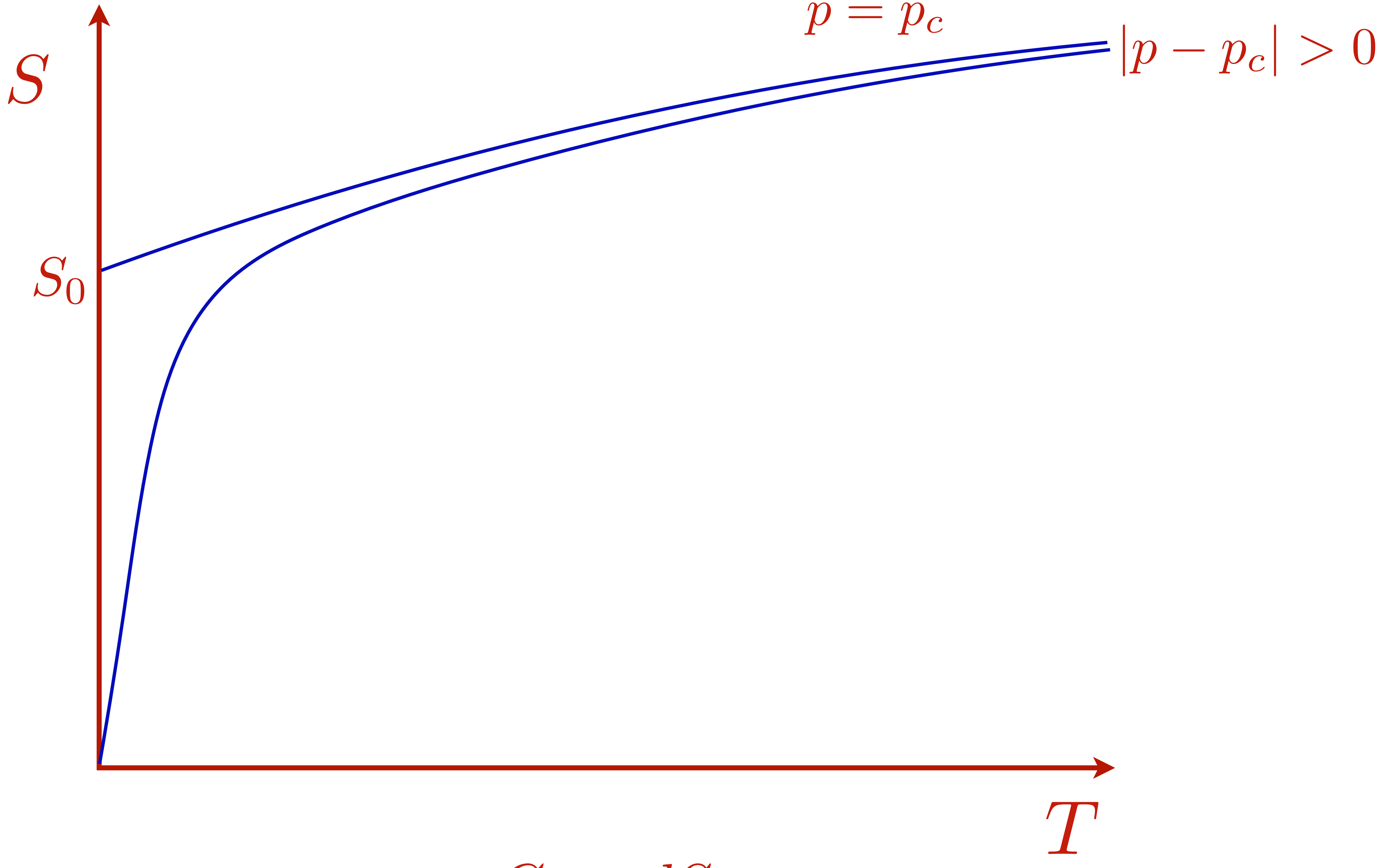


Deduced by similarity to the zero temperature entropy of the SYK model

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

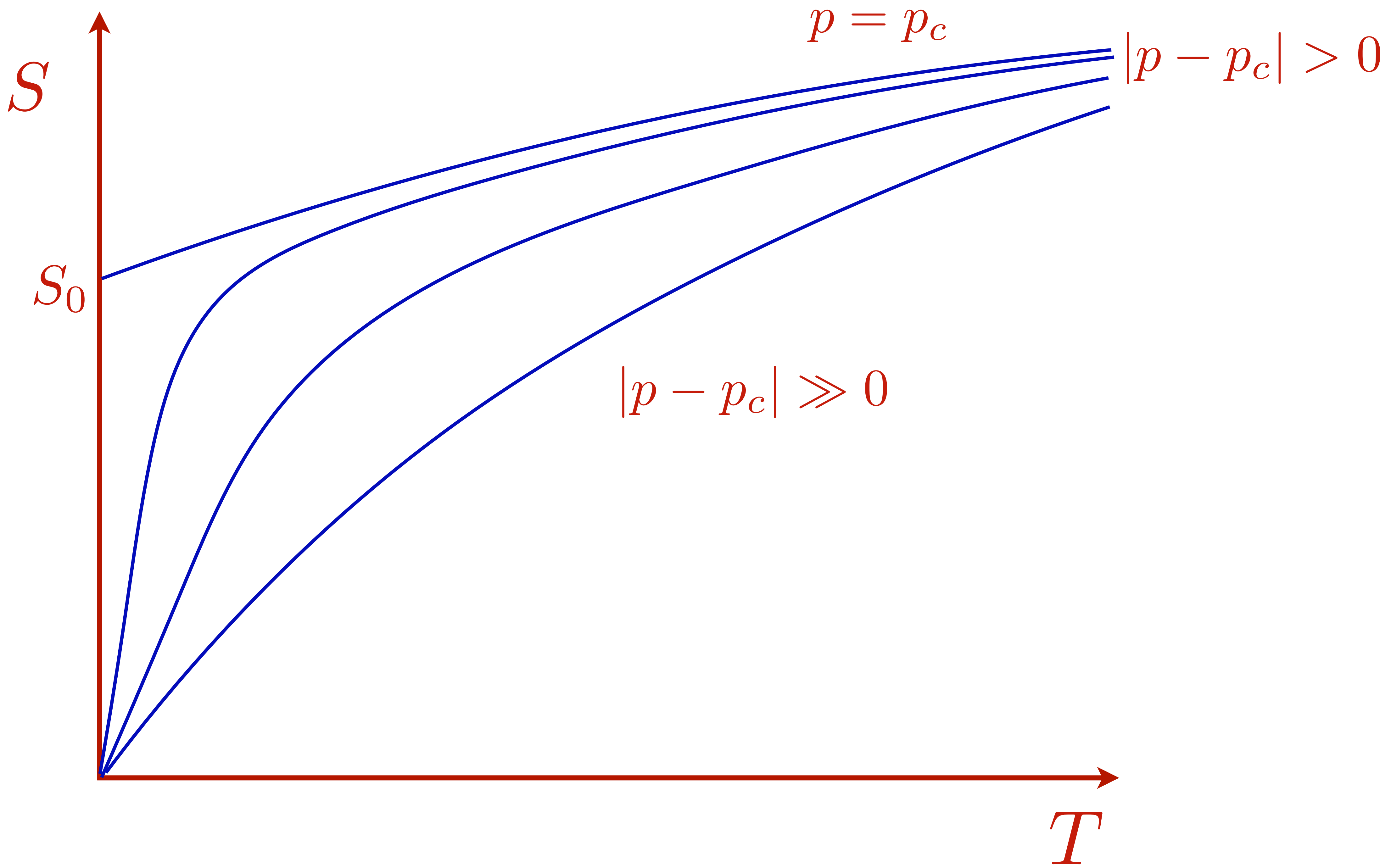
$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy



$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy

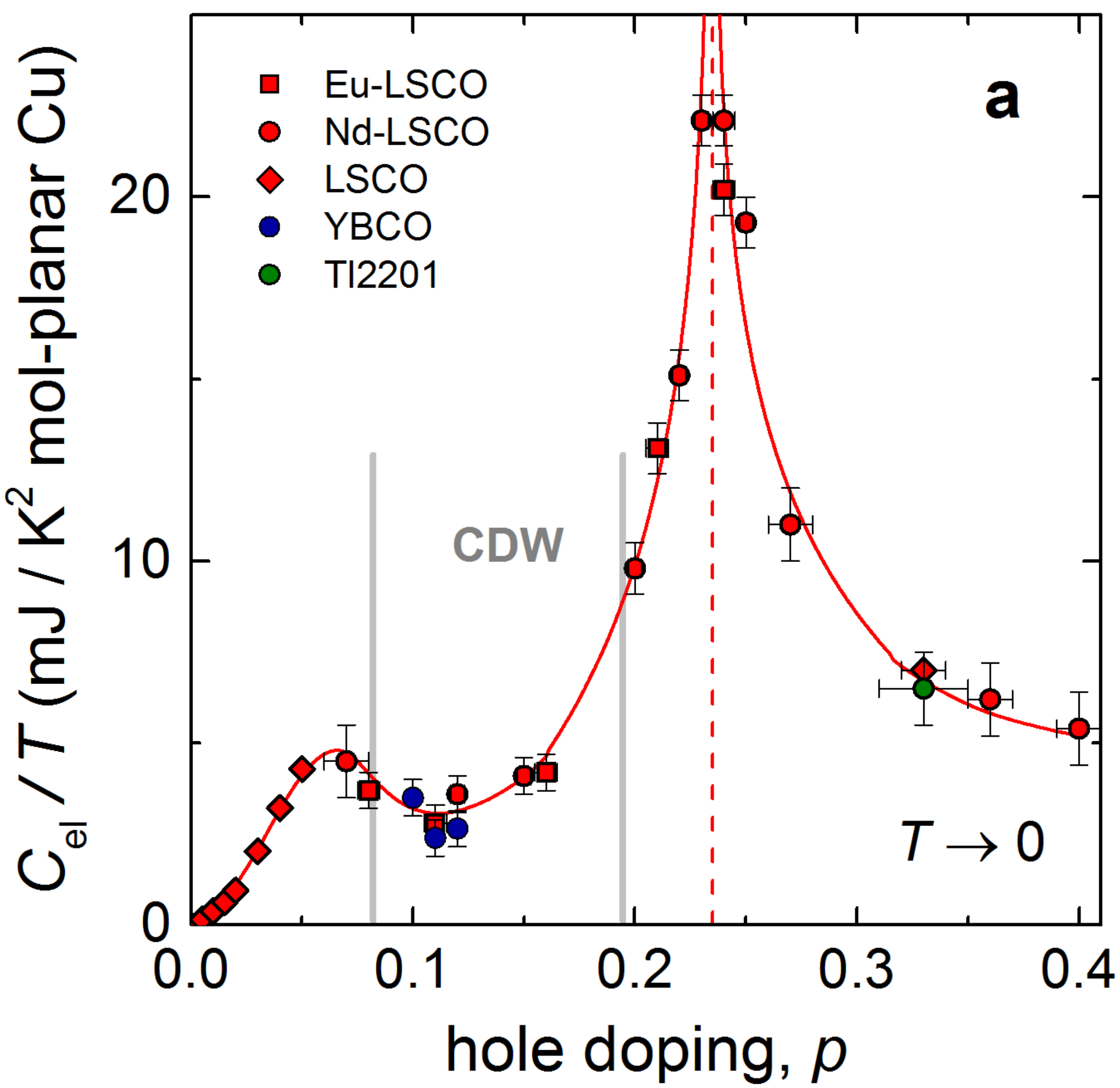


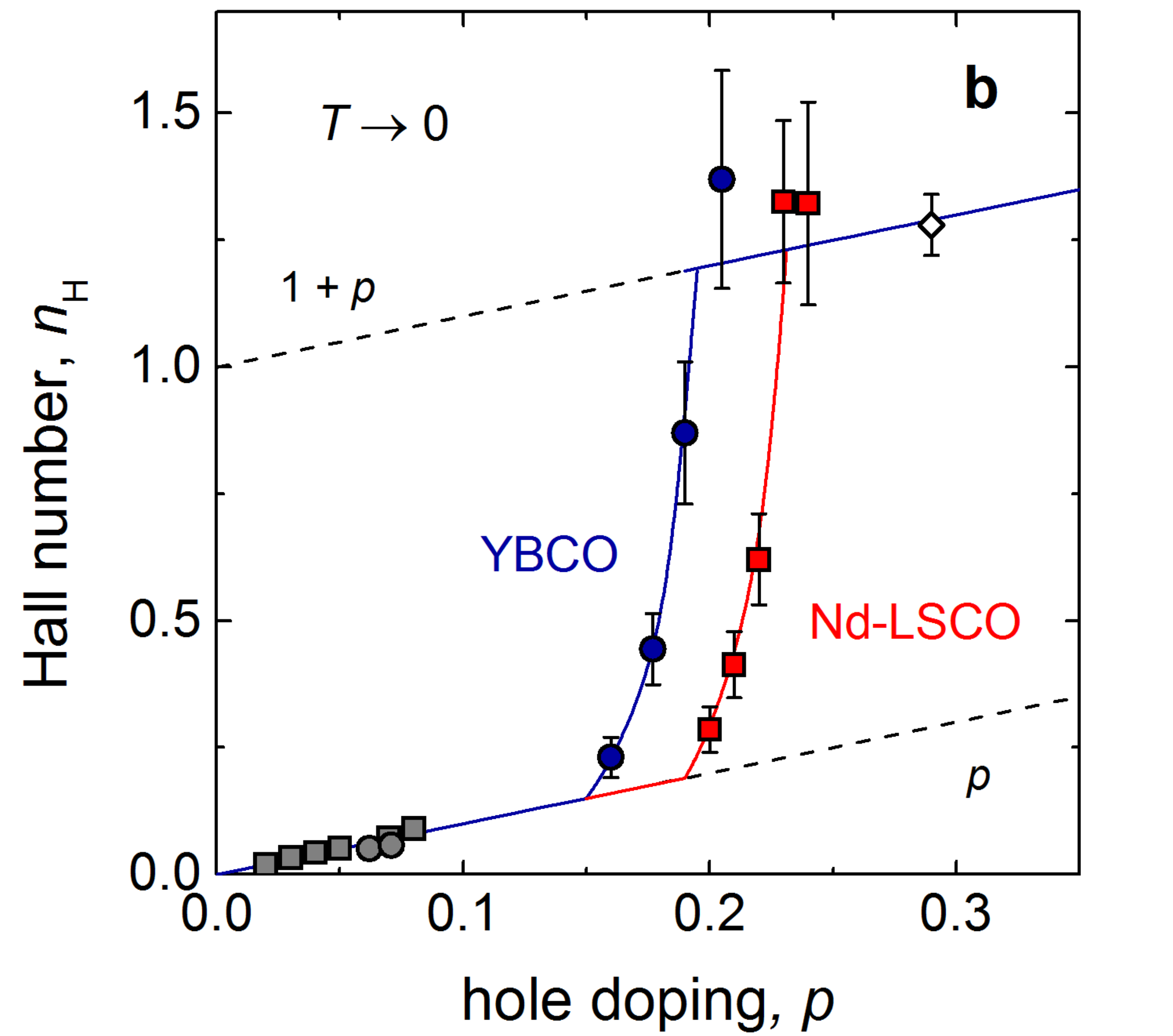
$$\frac{C}{T} = \frac{dS}{dT}$$

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

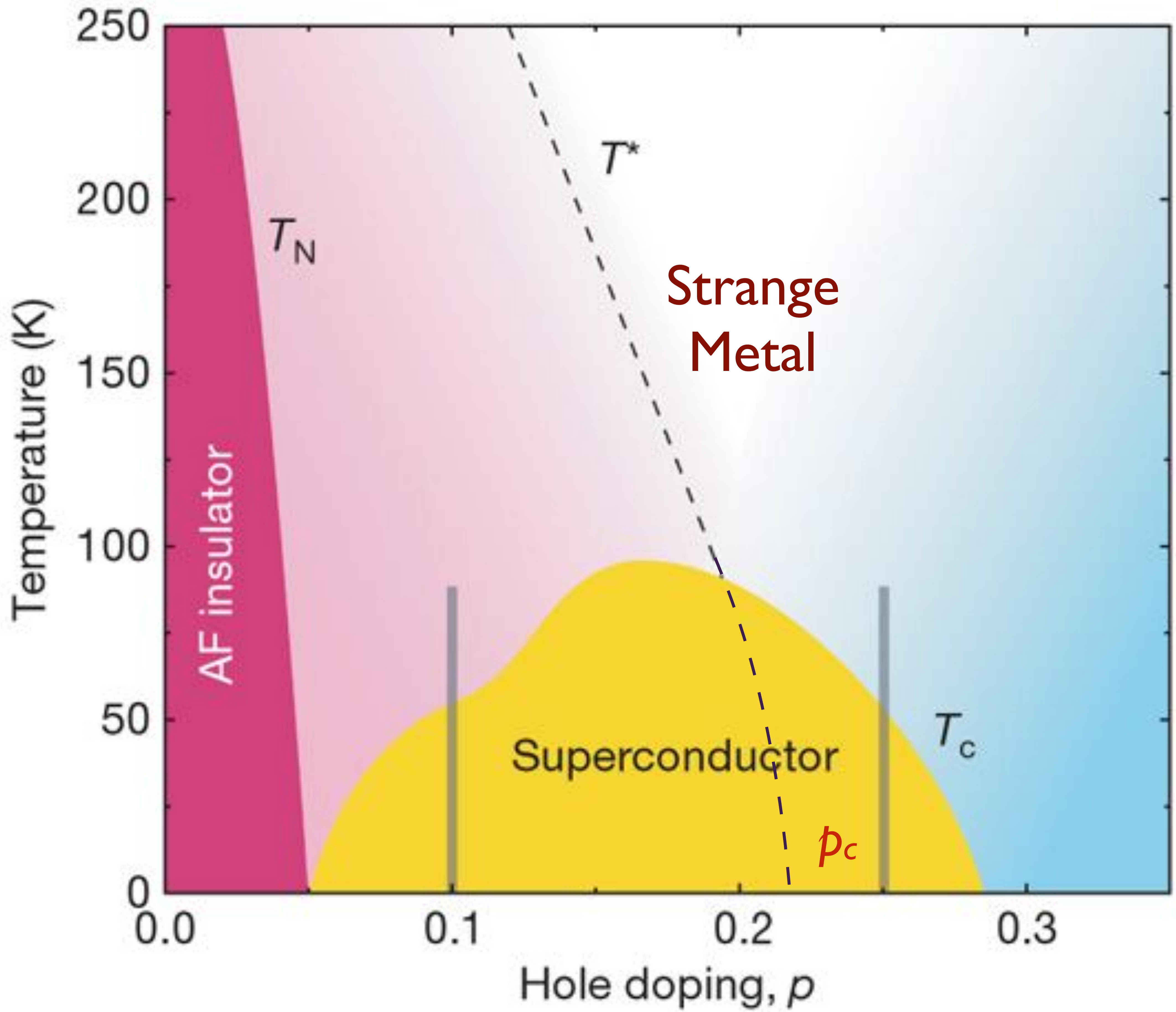
Cyril Proust and Louis Taillefer, arXiv:1807.0507



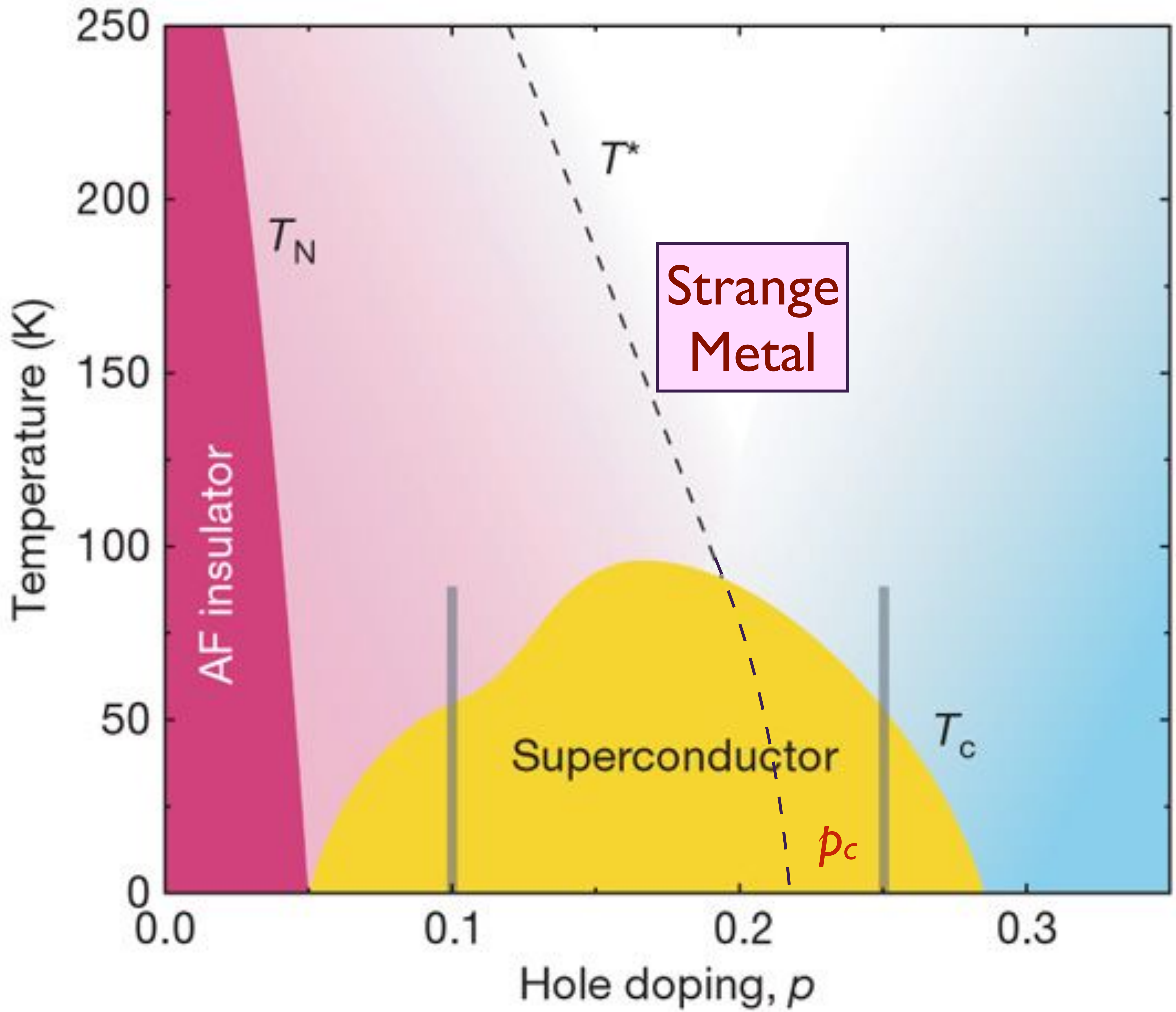


1. Deconfined quantum criticality of random *t*-*J* models

2. Linear-T resistivity and SYK criticality



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
p_c


250
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Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange
Metal
Superconductor
T_c
p_c

Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau}$$

with a universal scattering rate

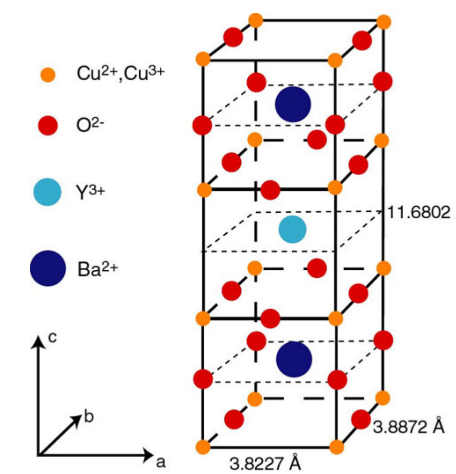
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar} ,$$

independent of the strength of interactions!

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

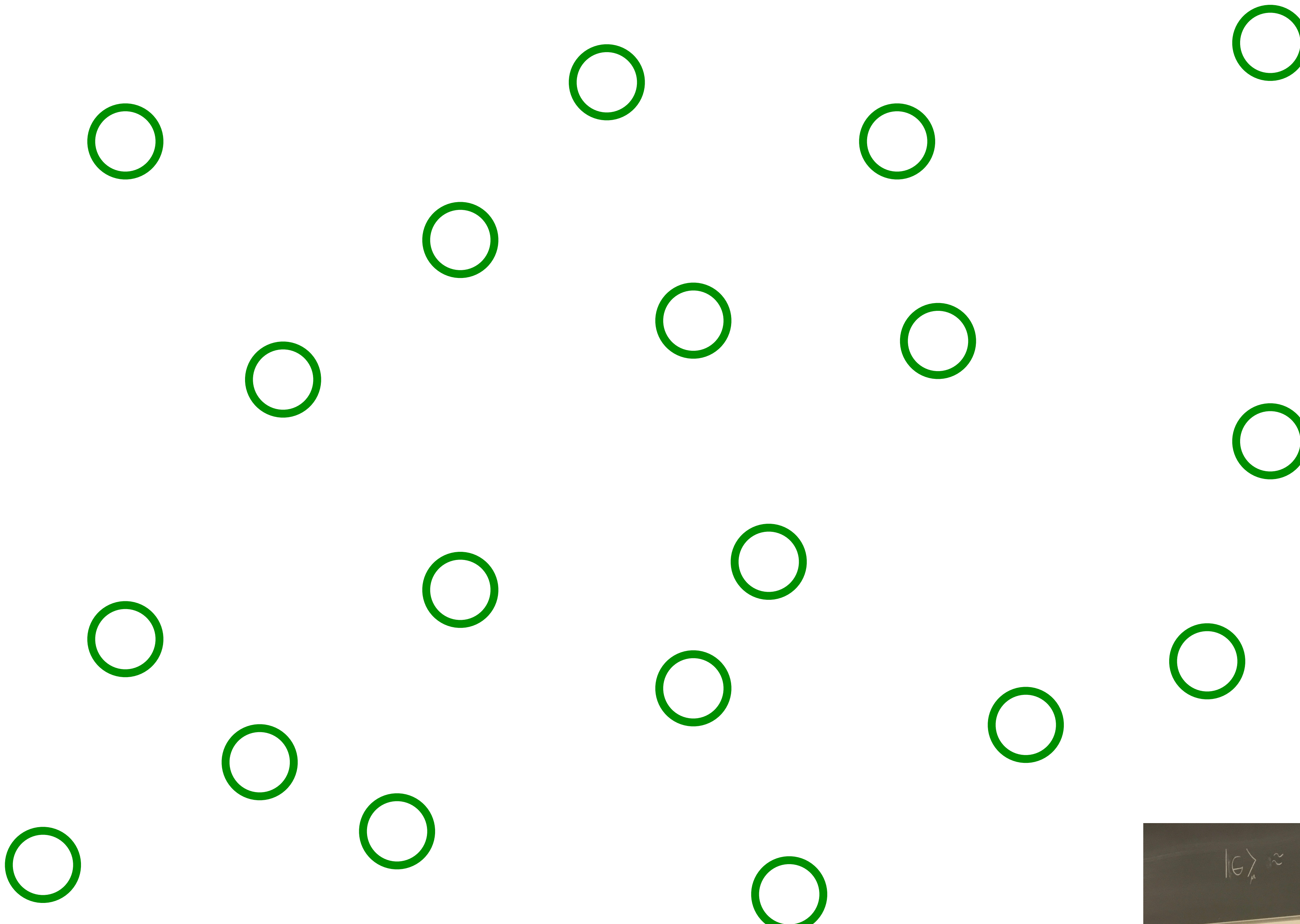
Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

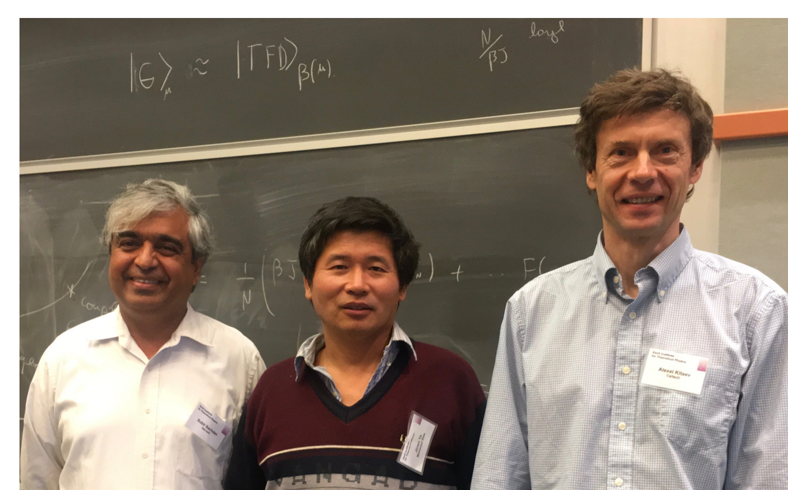


A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

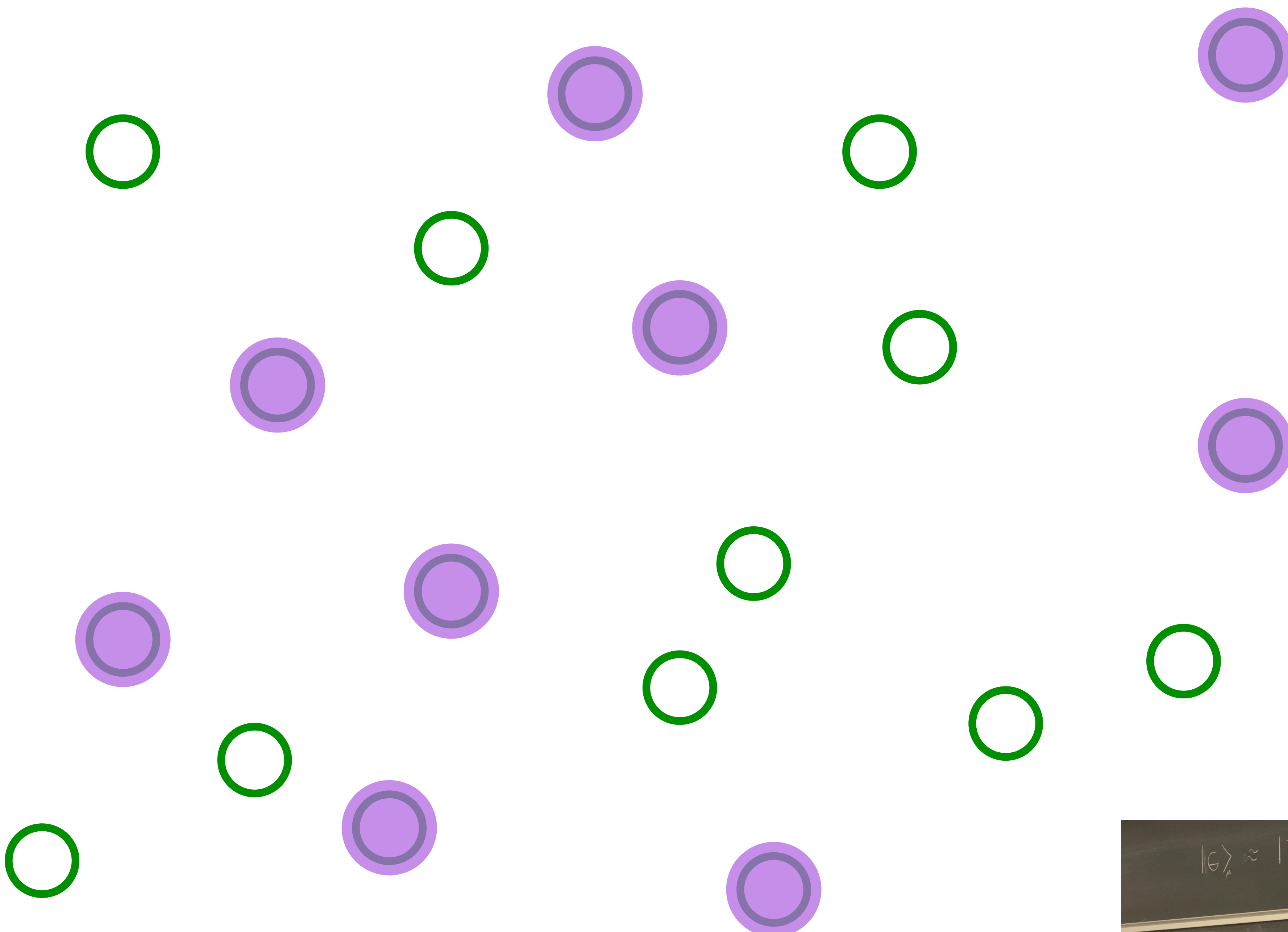
The Sachdev-Ye-Kitaev (SYK) model



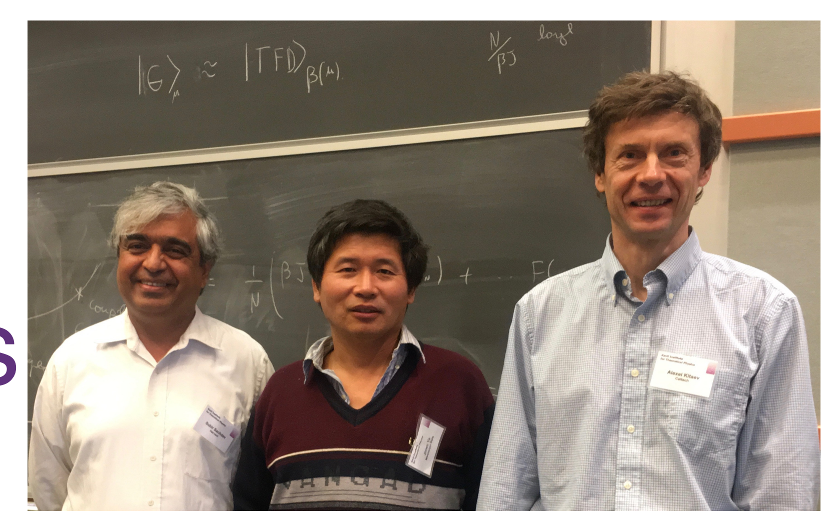
Pick a set of random positions



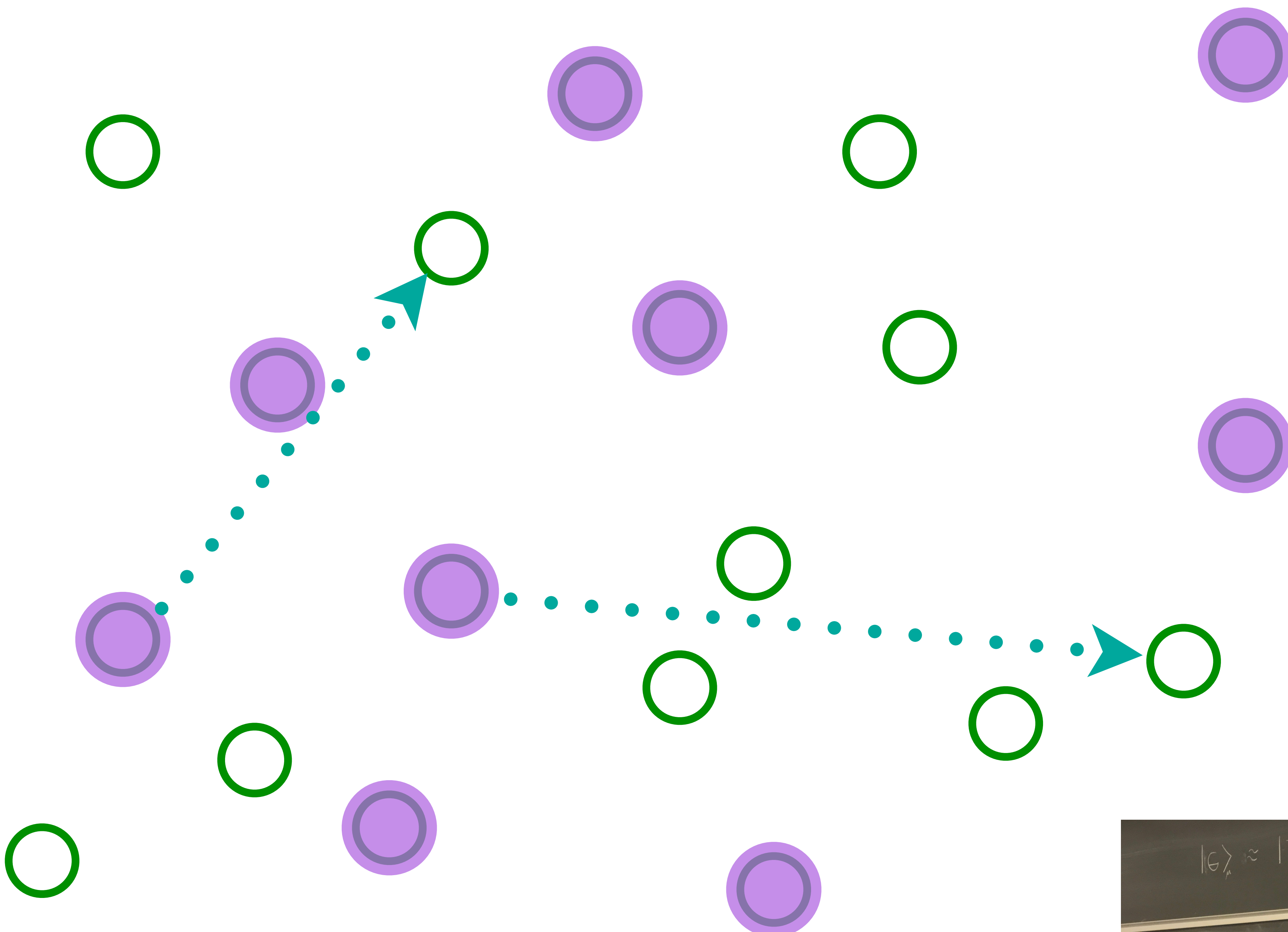
The SYK model



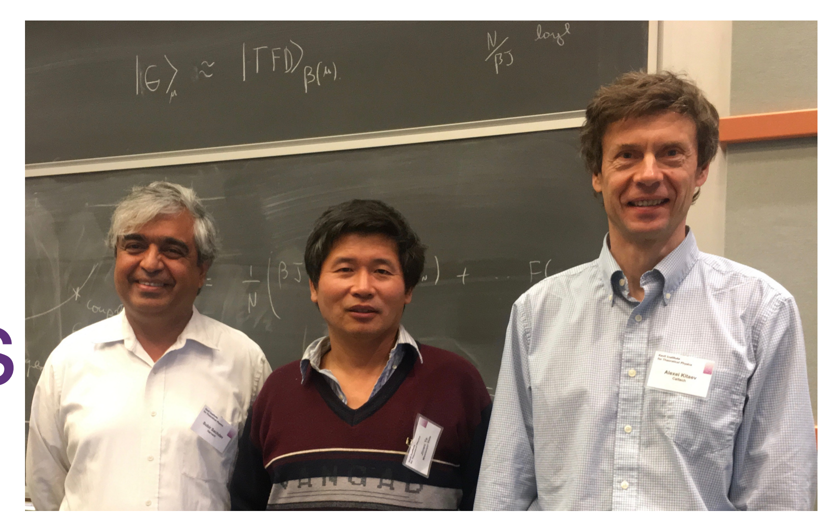
Place electrons randomly on some sites



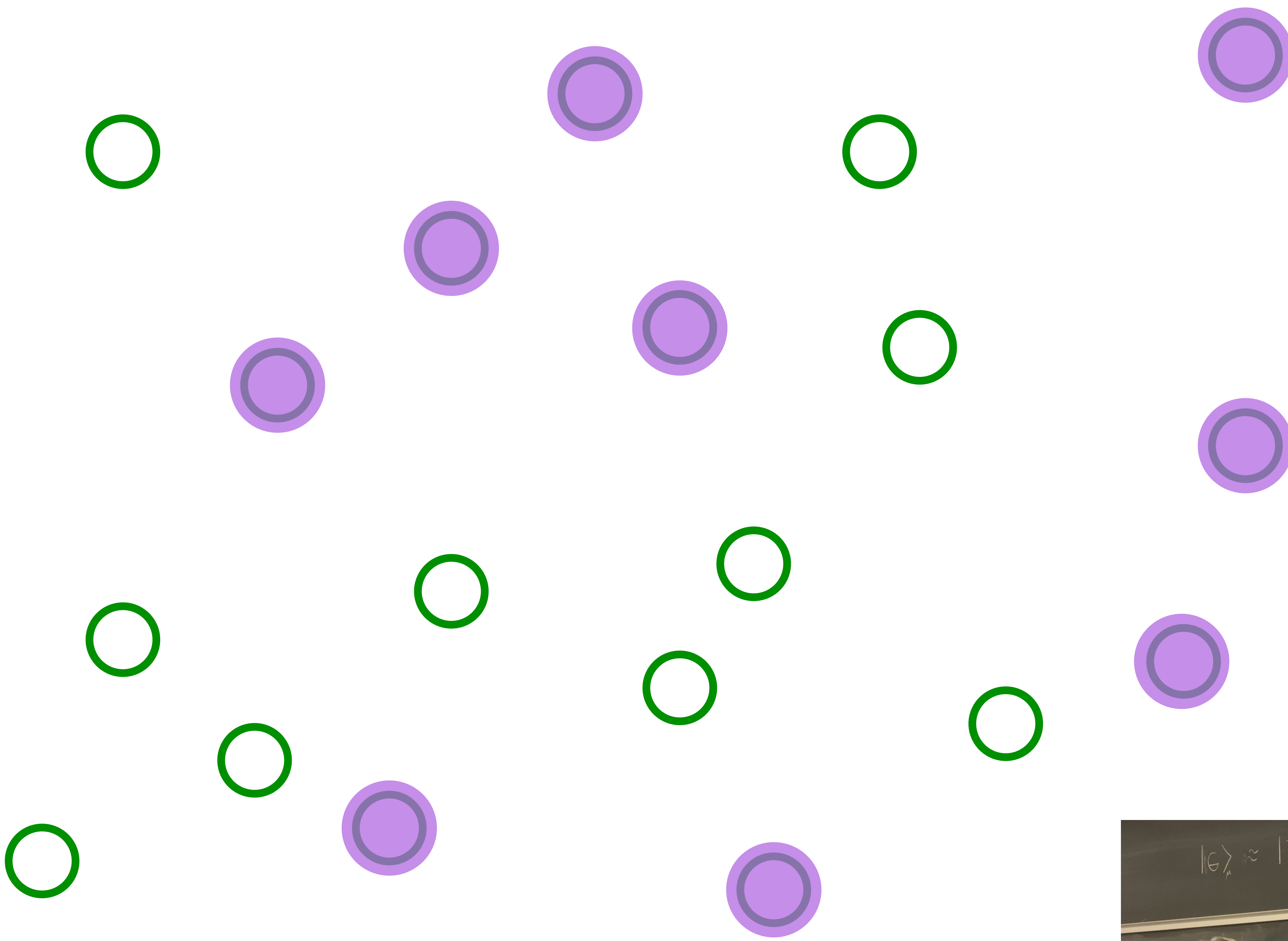
The SYK model



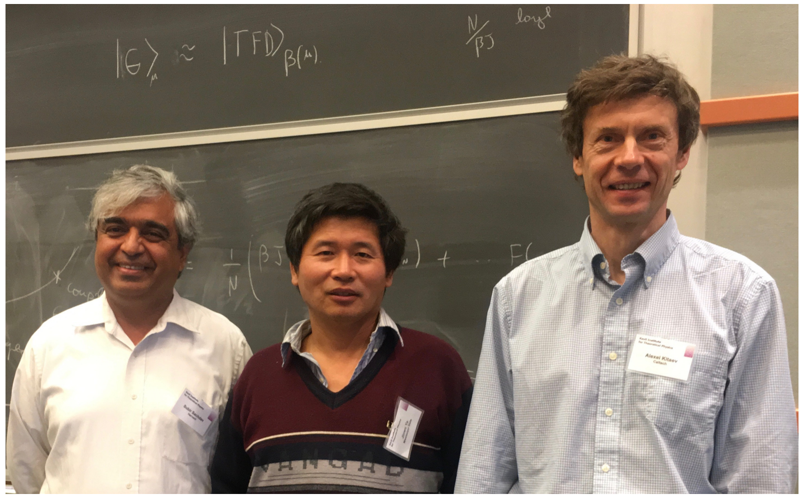
Place electrons randomly on some sites



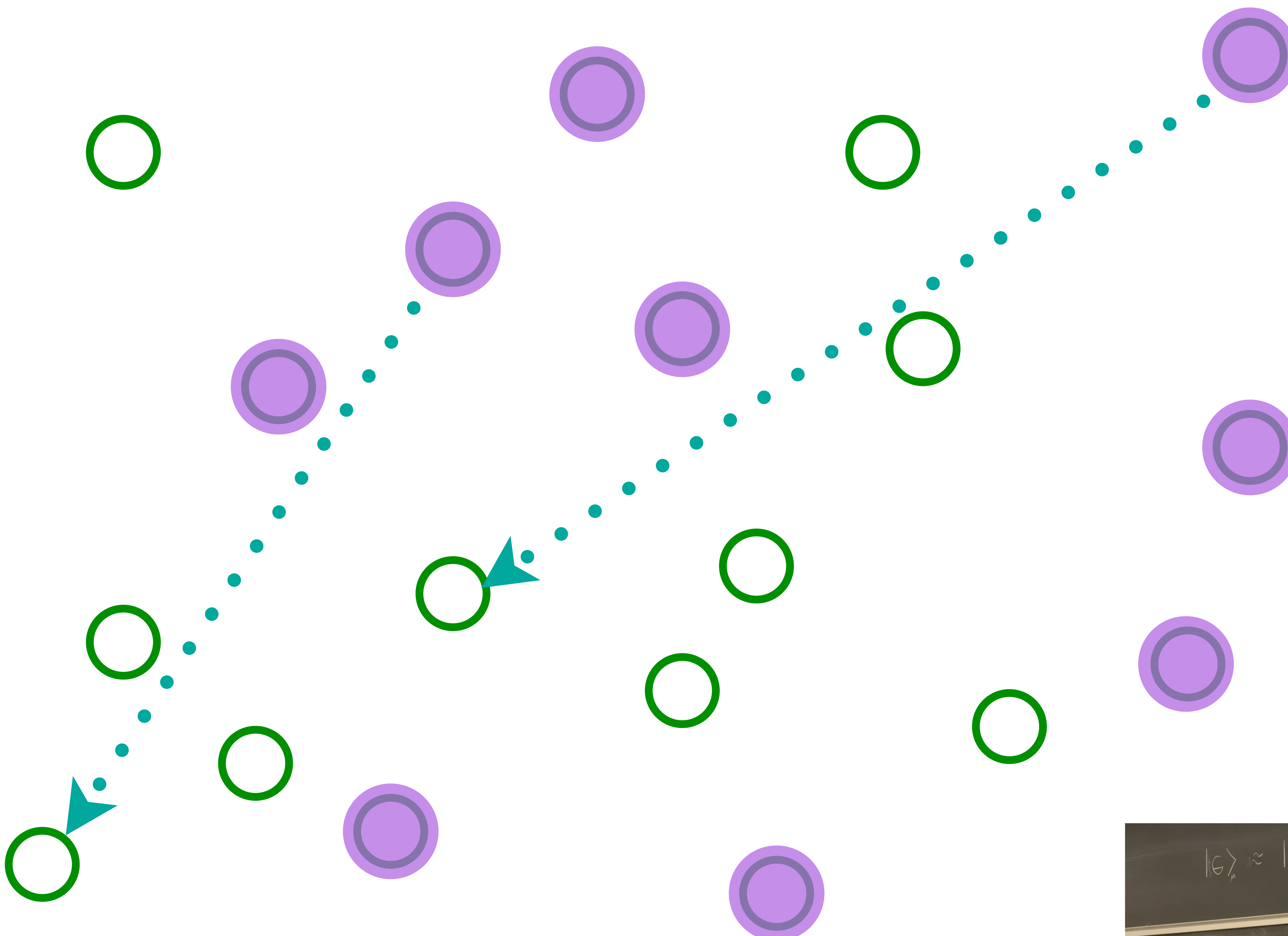
The SYK model

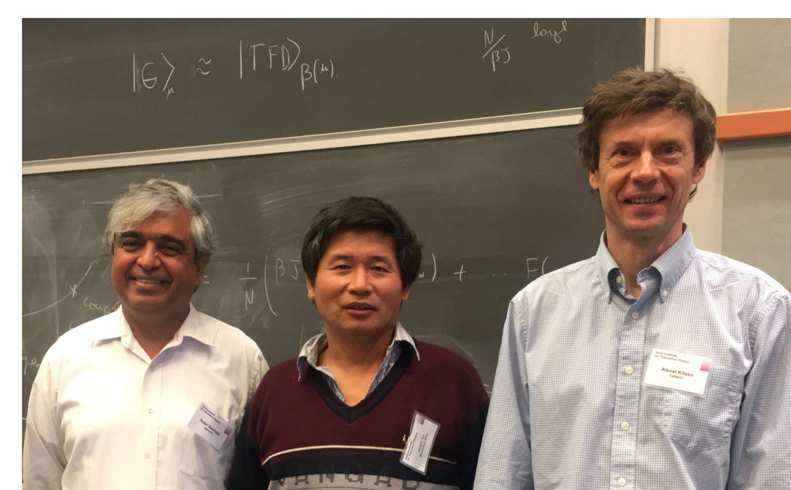


Entangle electrons pairwise randomly



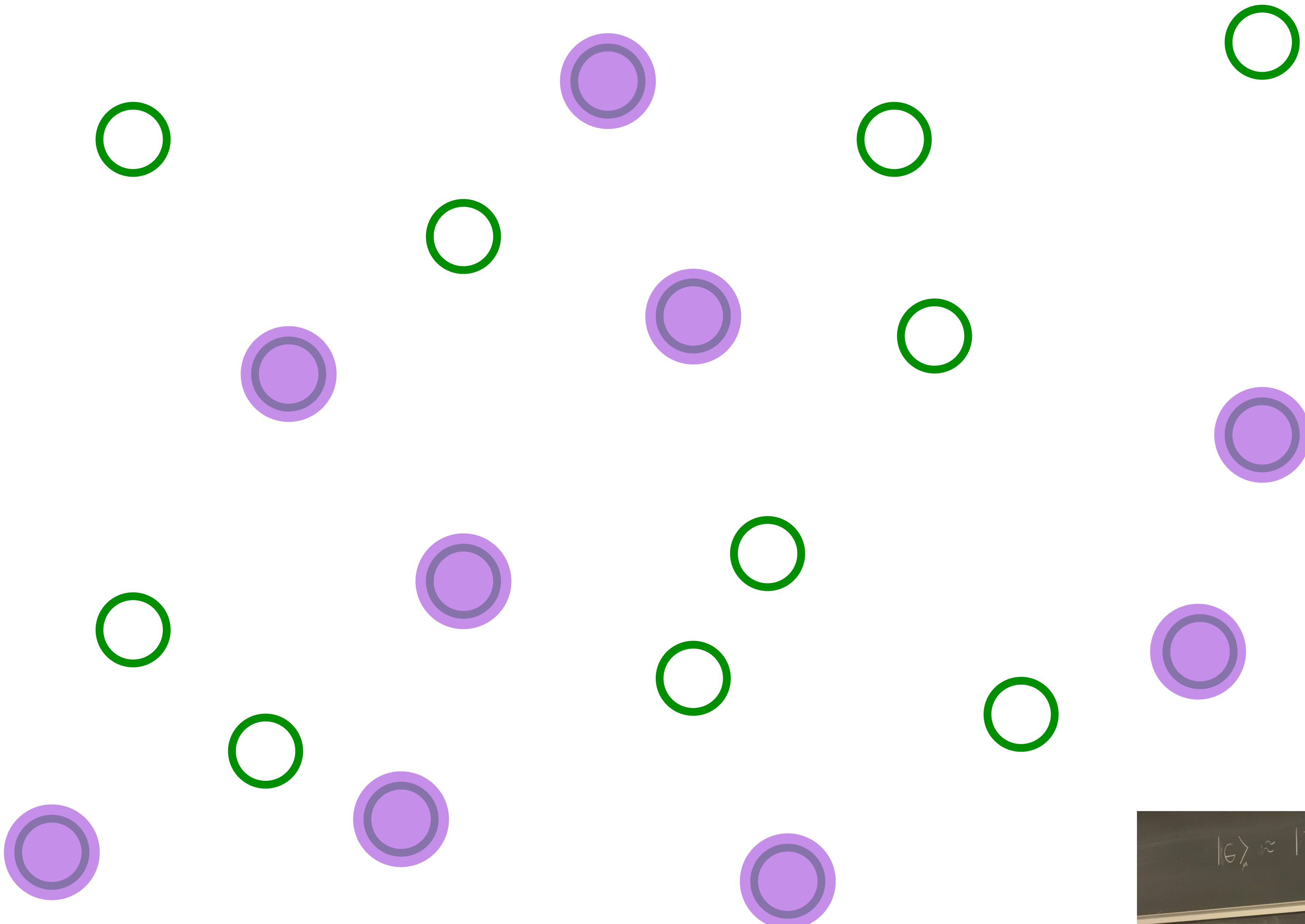
The SYK model



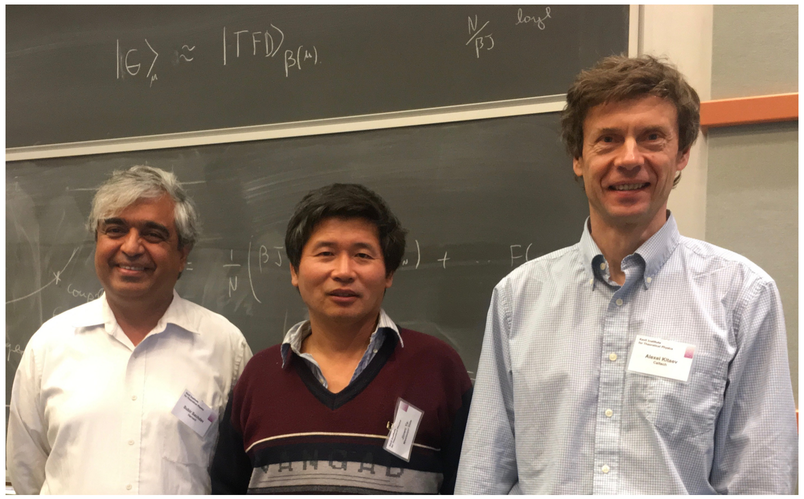


Entangle electrons pairwise randomly

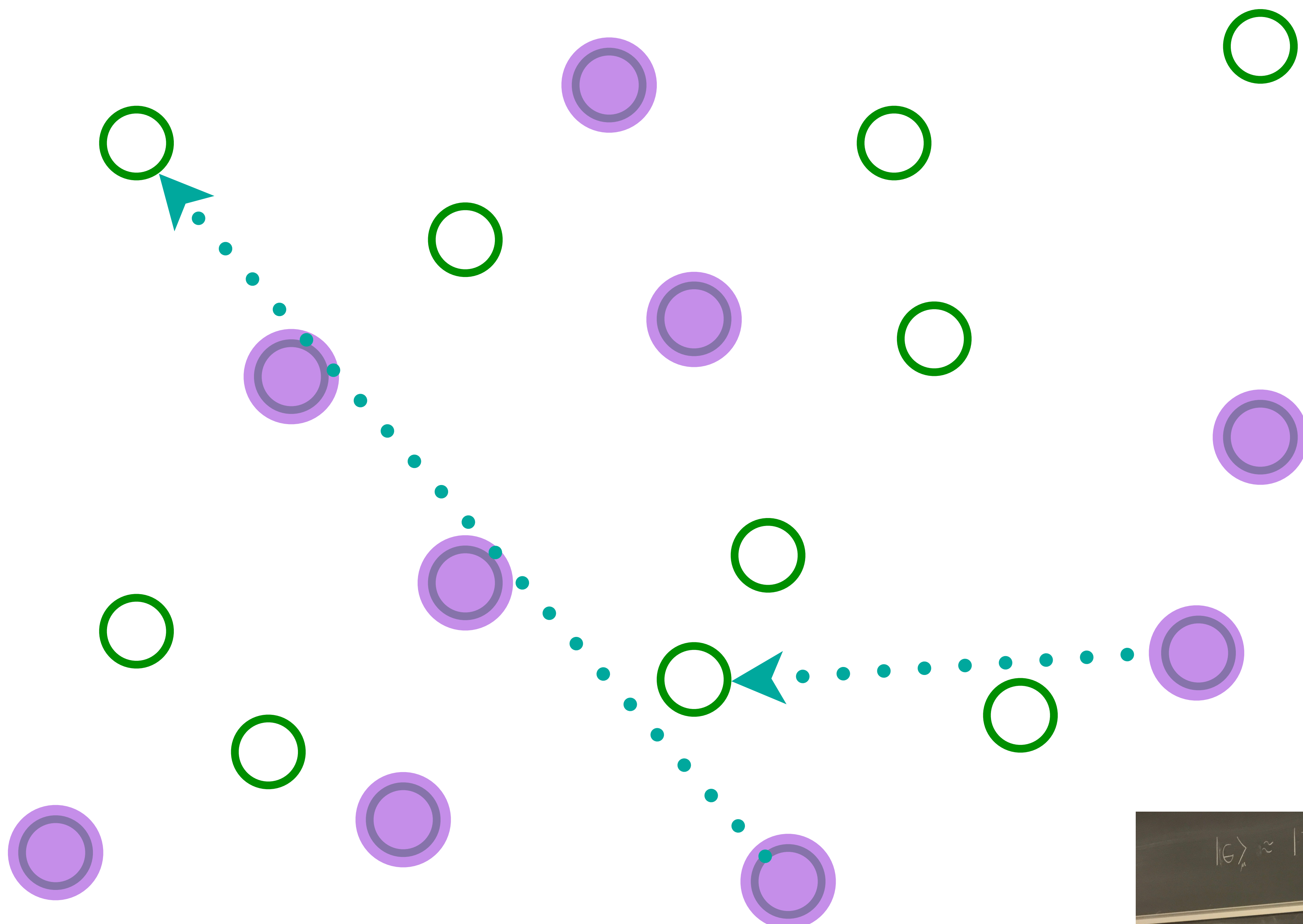
The SYK model



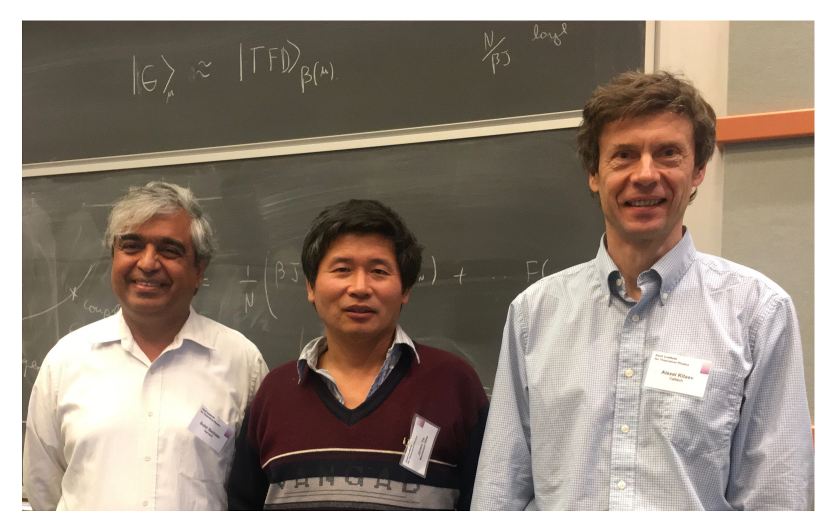
Entangle electrons pairwise randomly



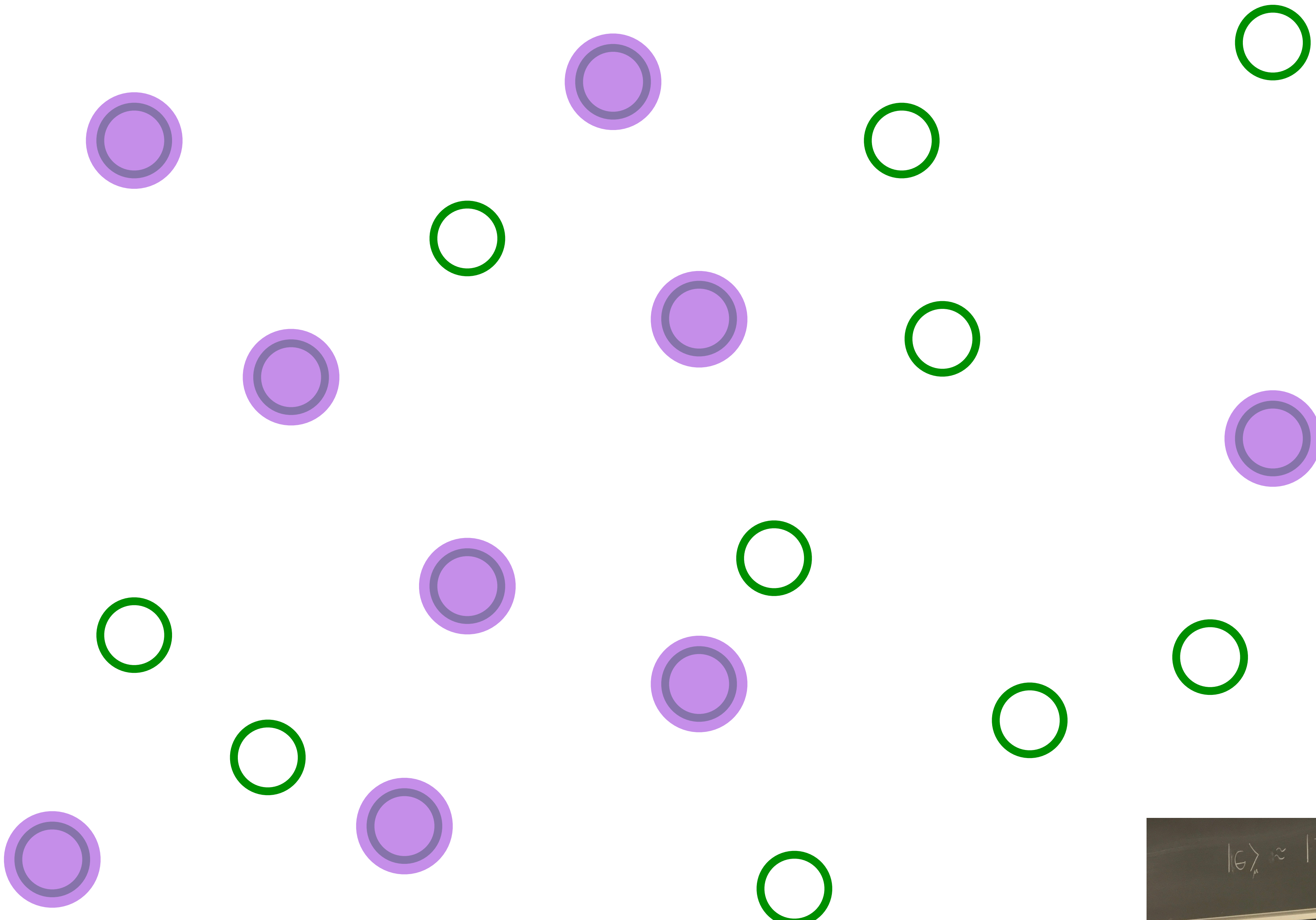
The SYK model



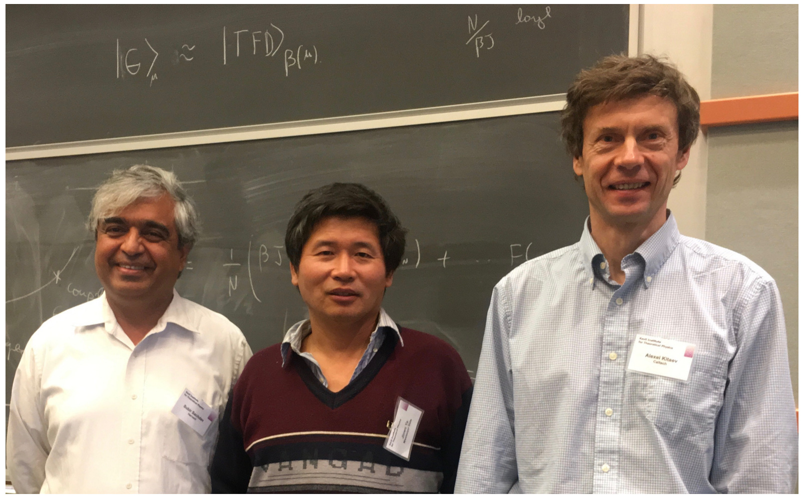
Entangle electrons pairwise randomly



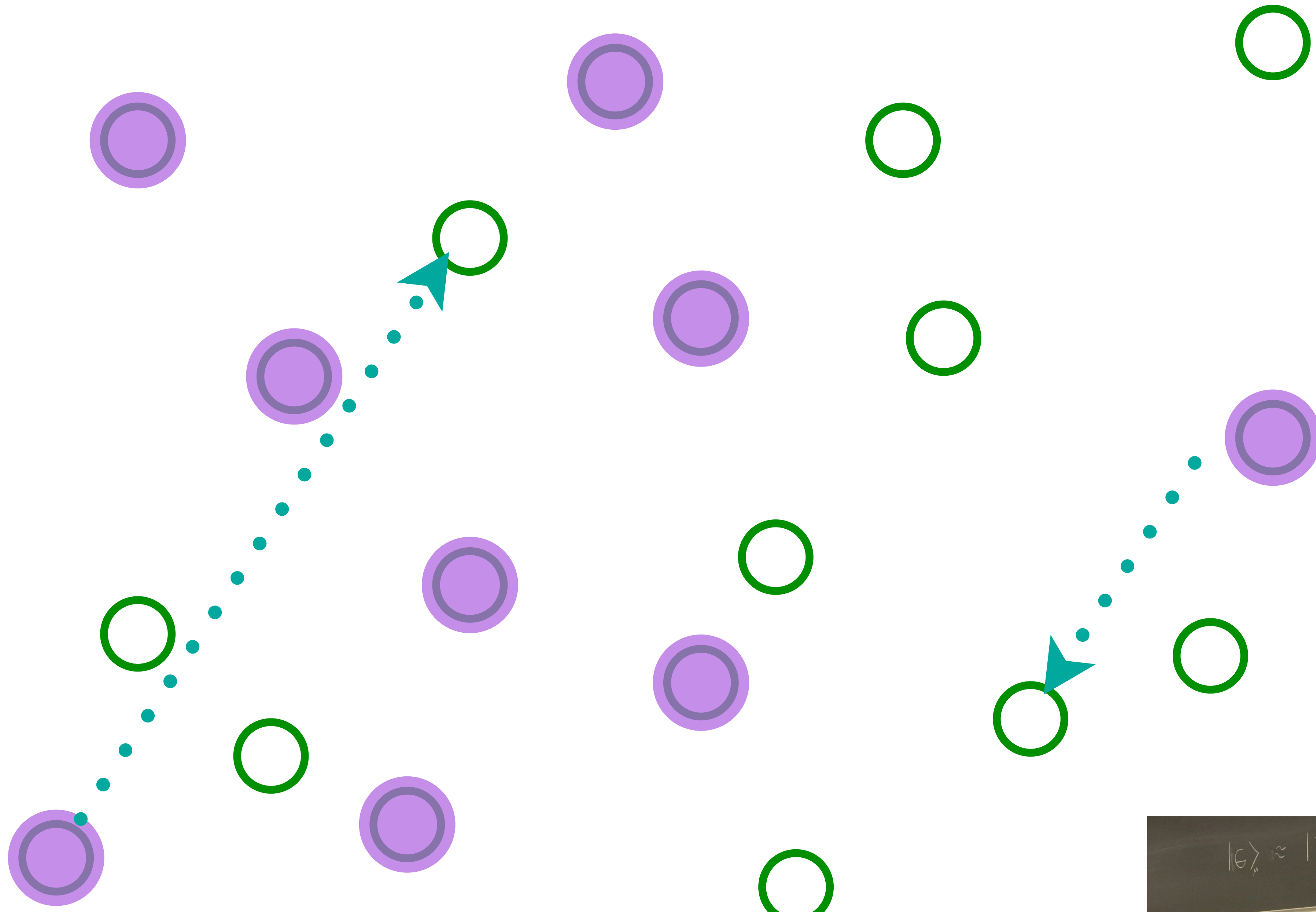
The SYK model



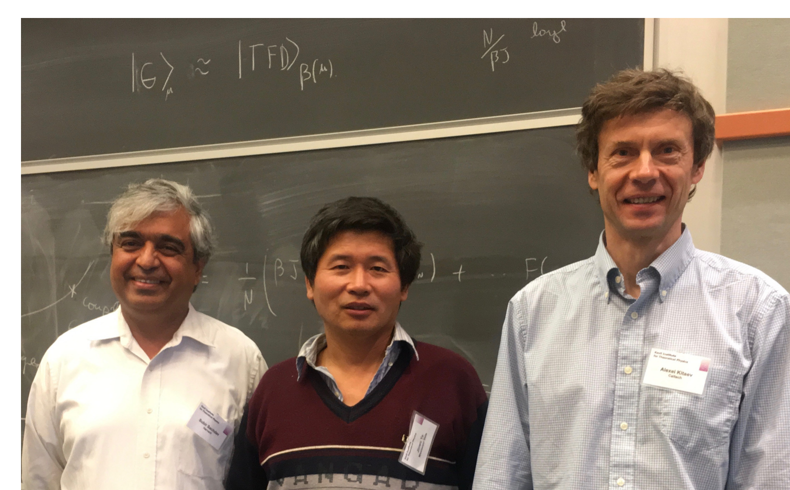
Entangle electrons pairwise randomly



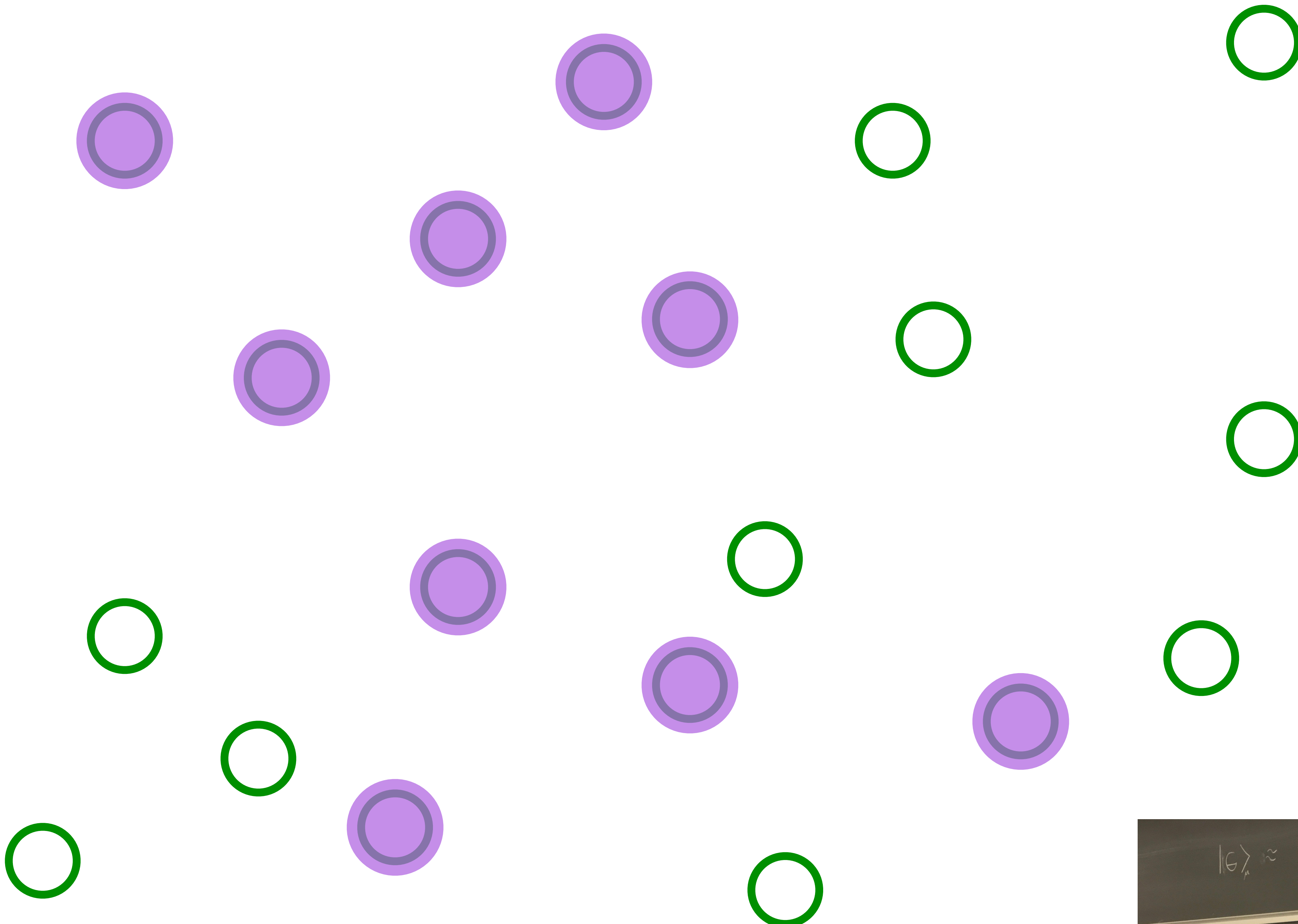
The SYK model



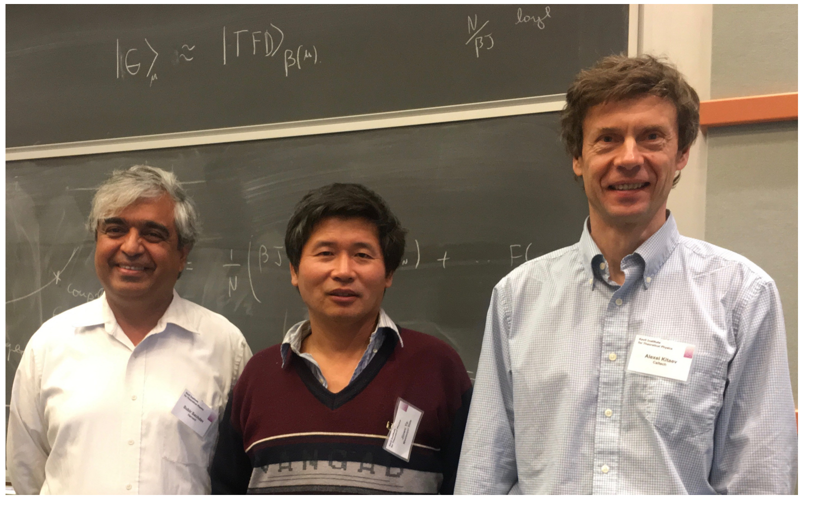
Entangle electrons pairwise randomly



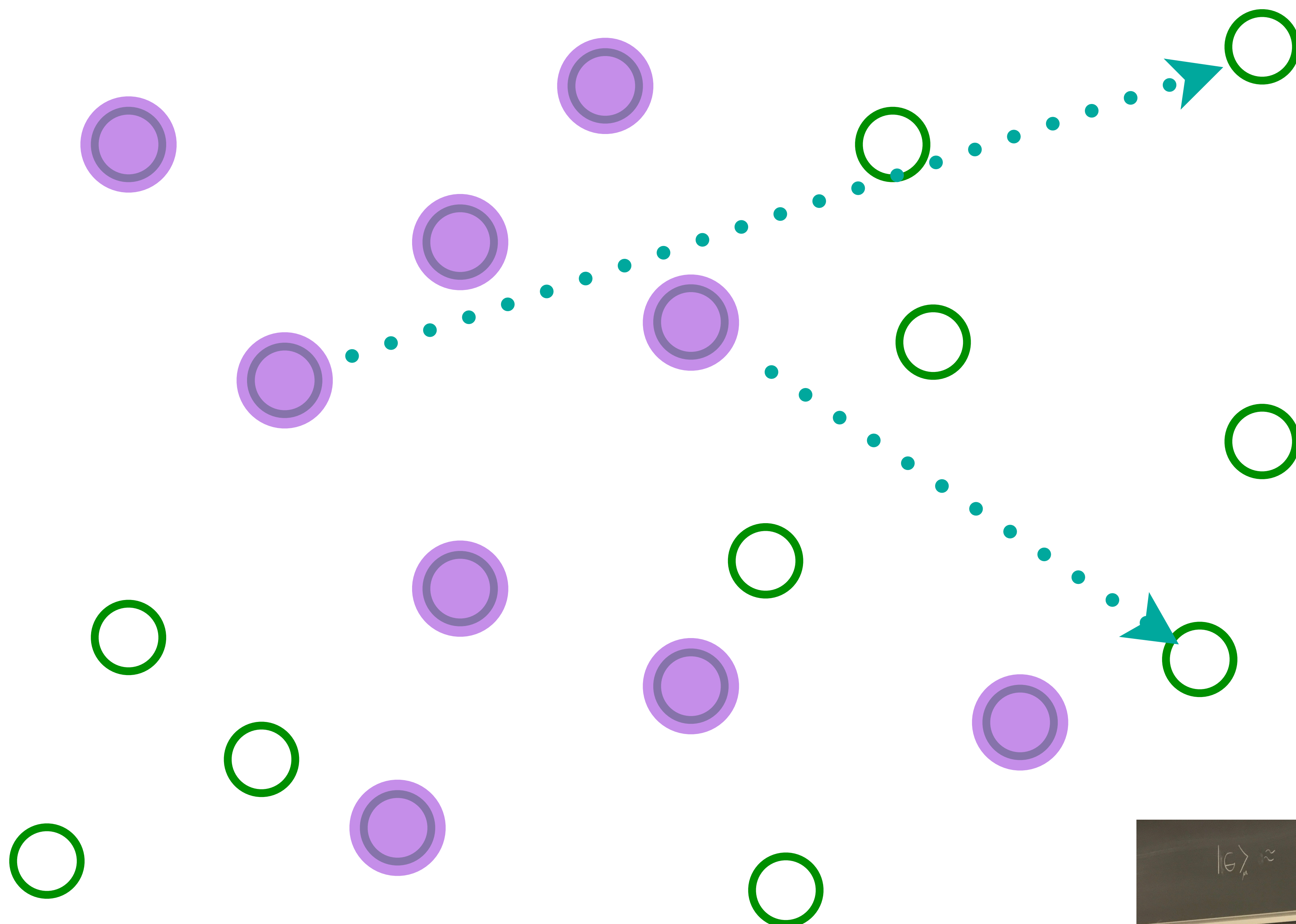
The SYK model



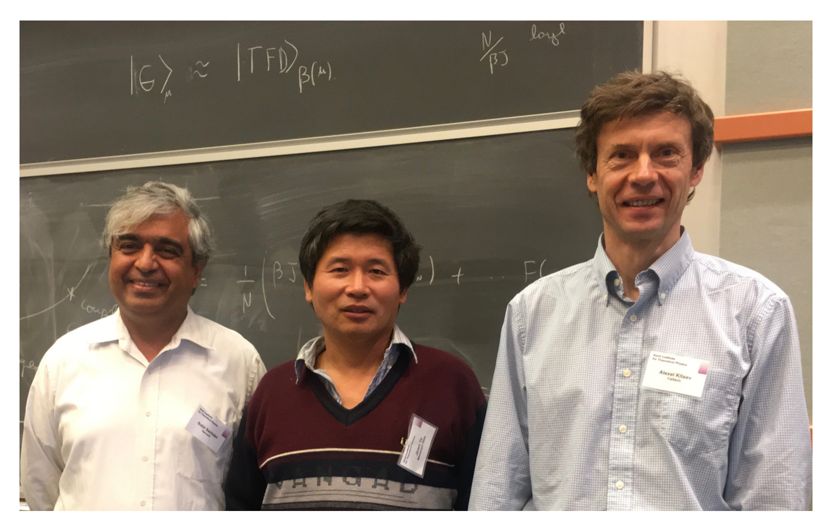
Entangle electrons pairwise randomly



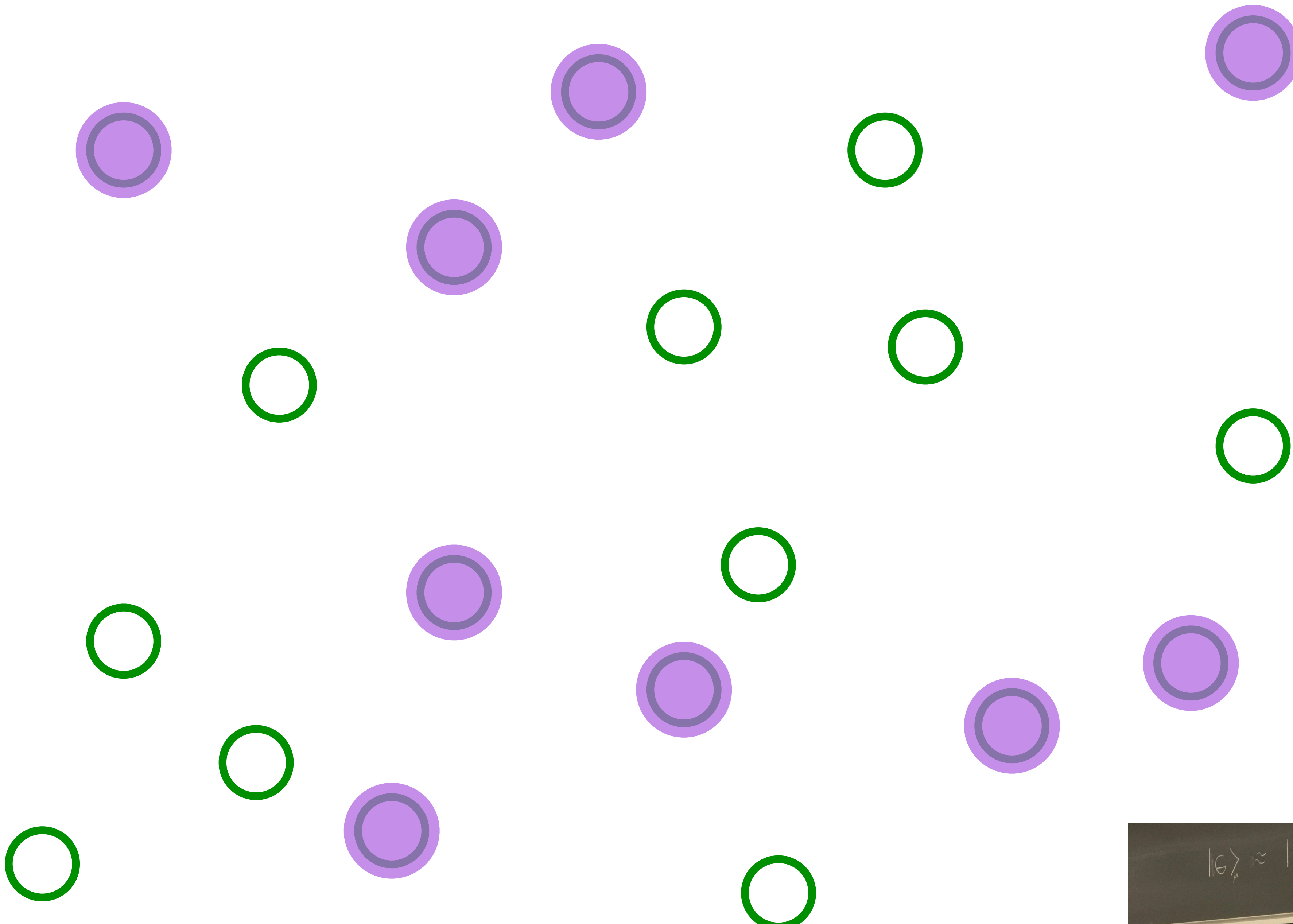
The SYK model



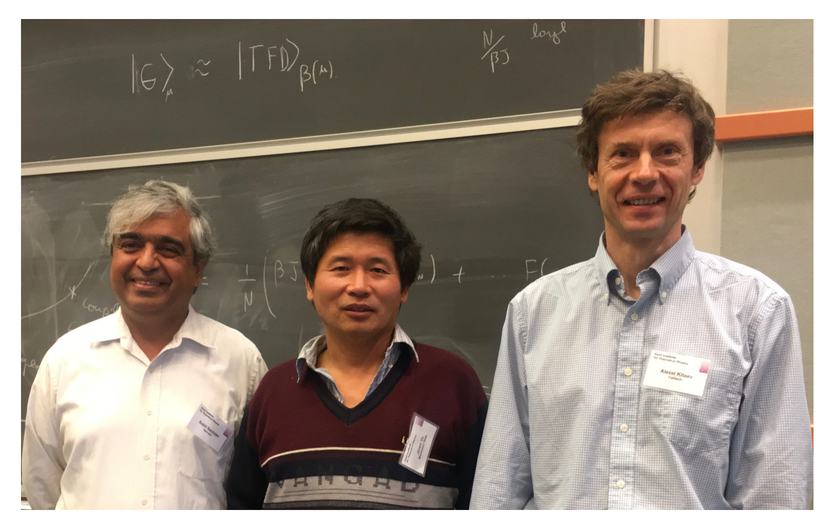
Entangle electrons pairwise randomly



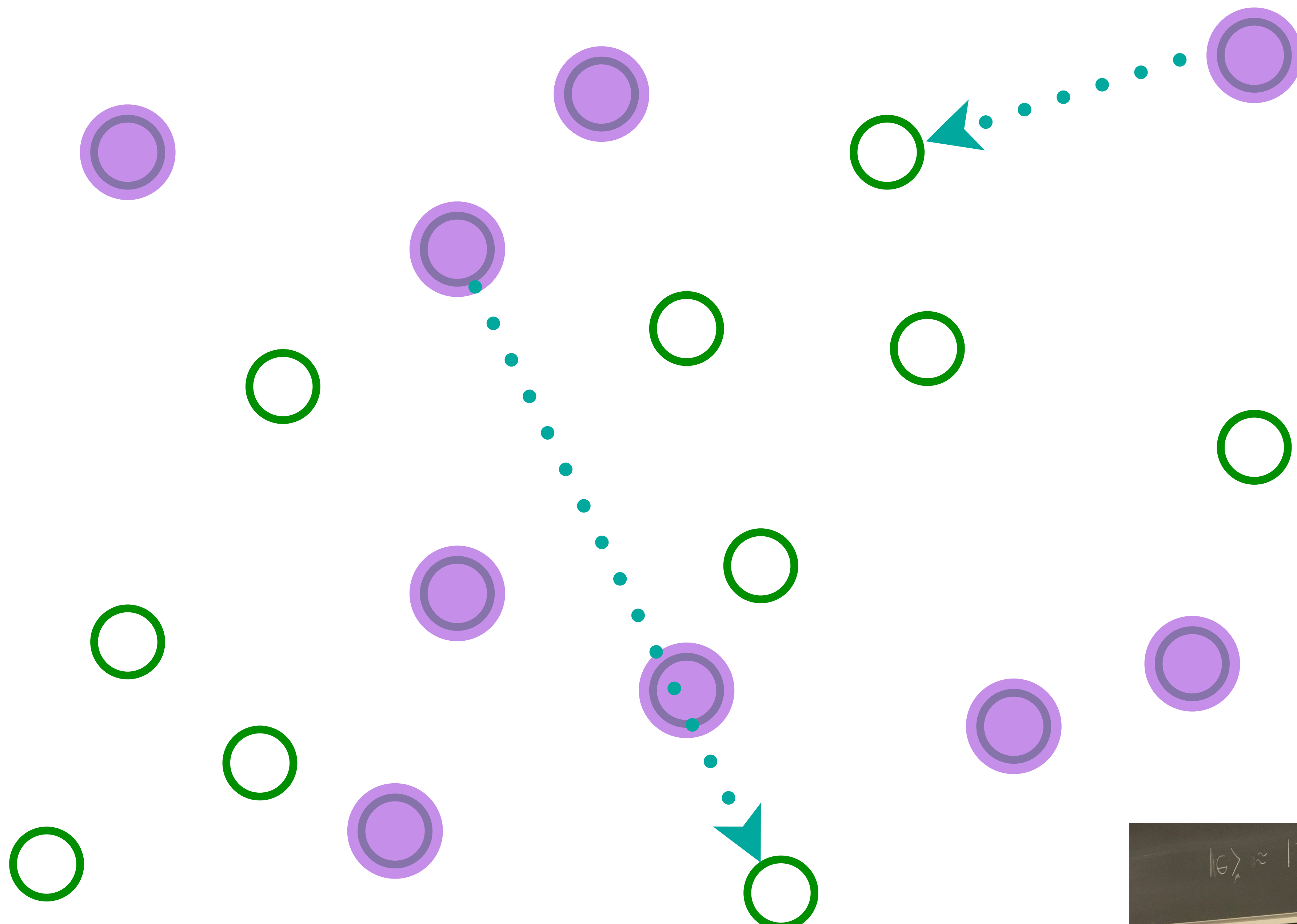
The SYK model



Entangle electrons pairwise randomly



The SYK model



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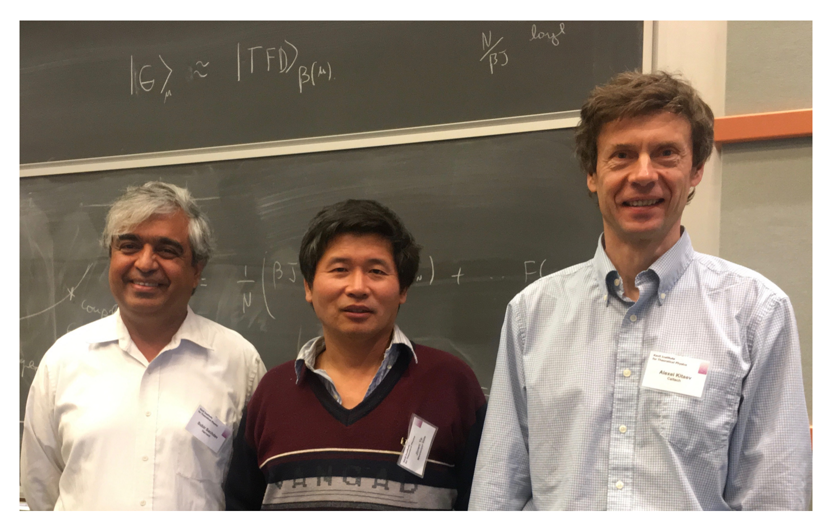
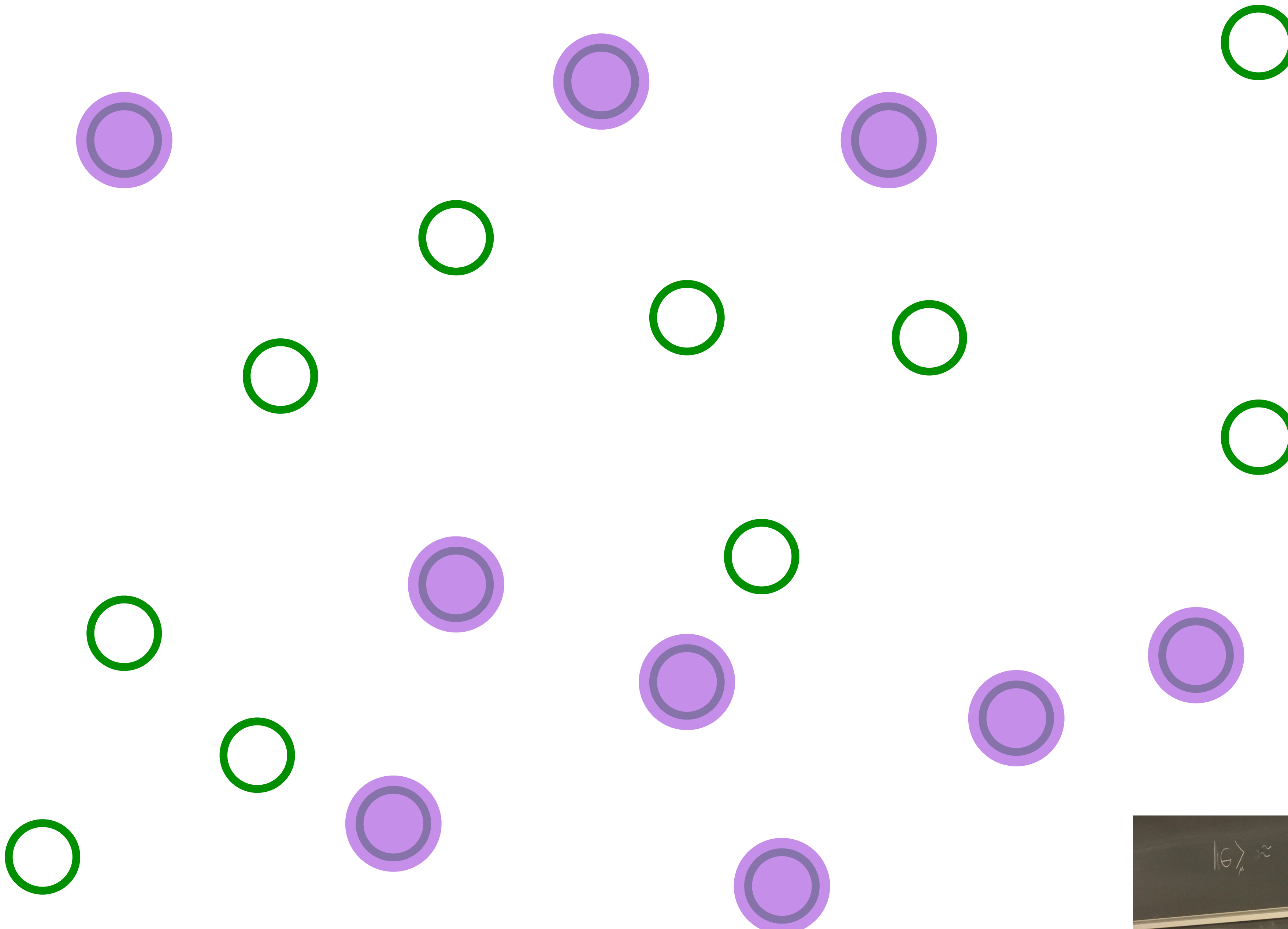
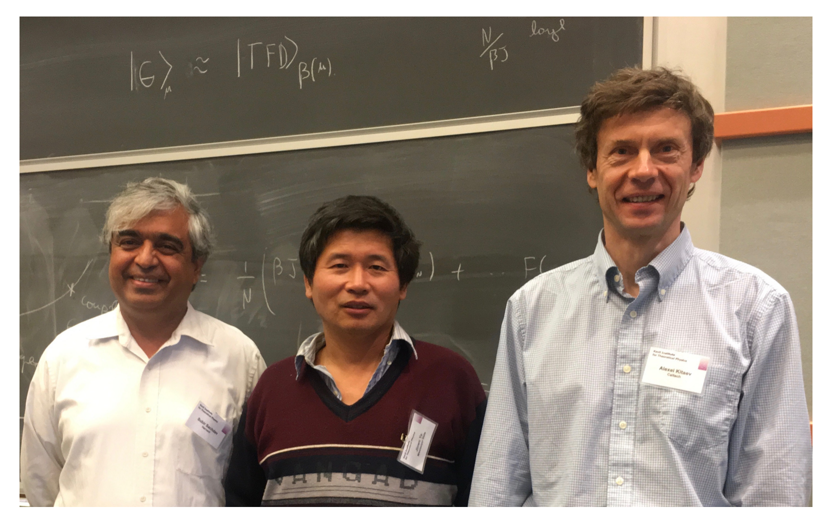


The SYK model



Entangle electrons pairwise randomly



The SYK model

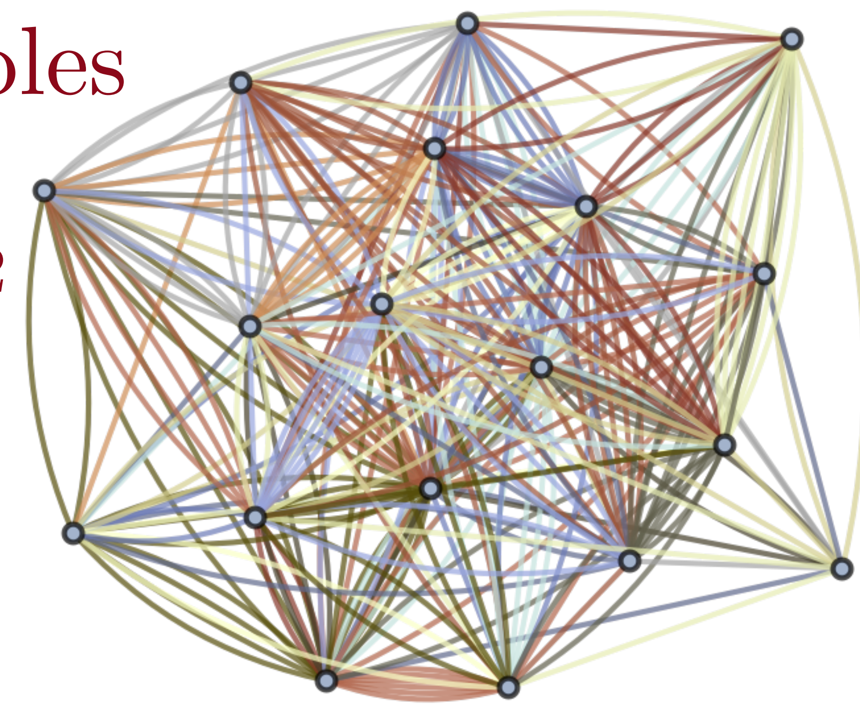
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

A strange metal: lattice of SYK islands

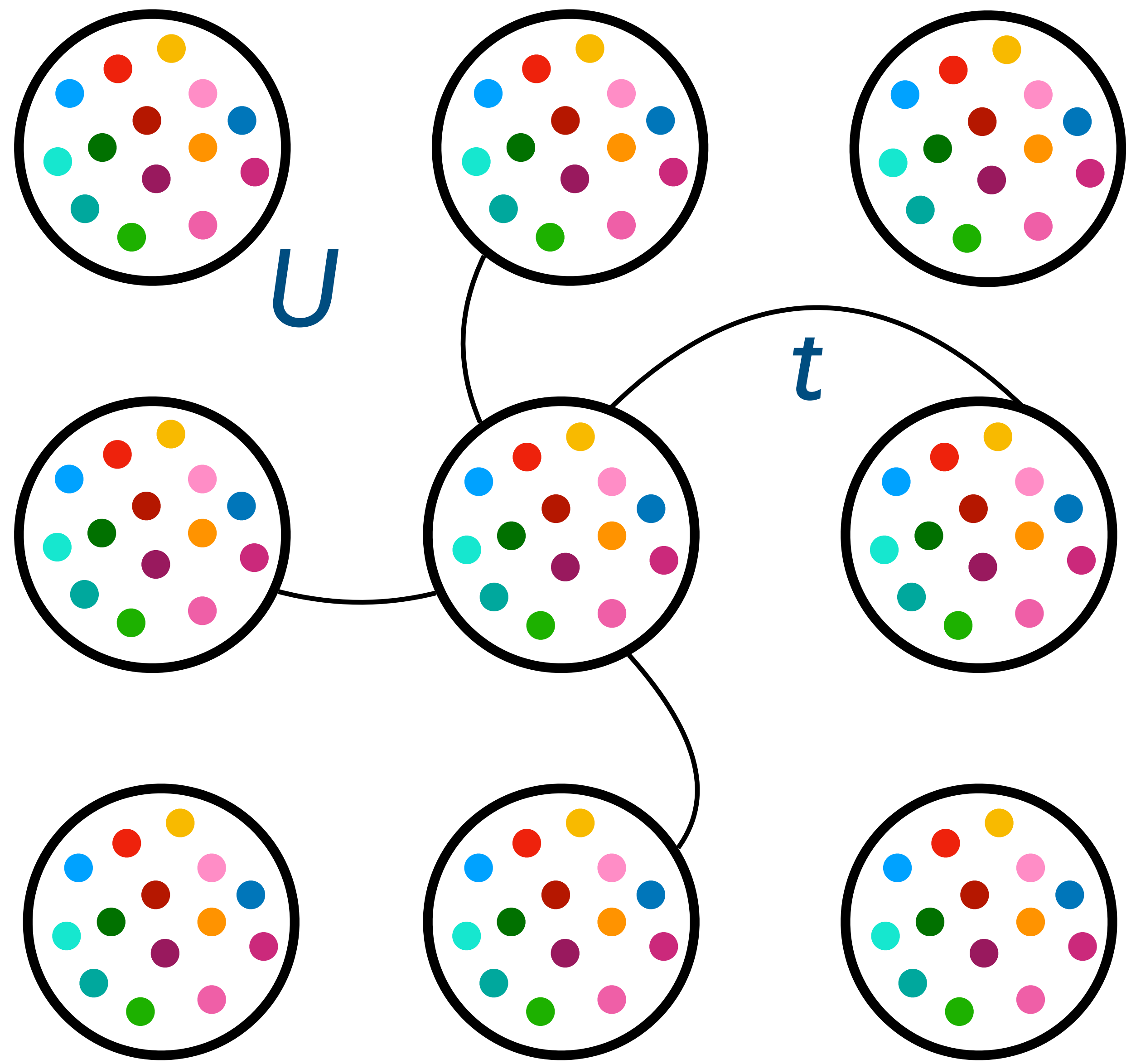
Random interaction within each island U.

Amplitude to hop between islands t.

Model yields

$$\rho \sim T \text{ and } \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$$

for $t^2/U \lesssim T \lesssim U$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

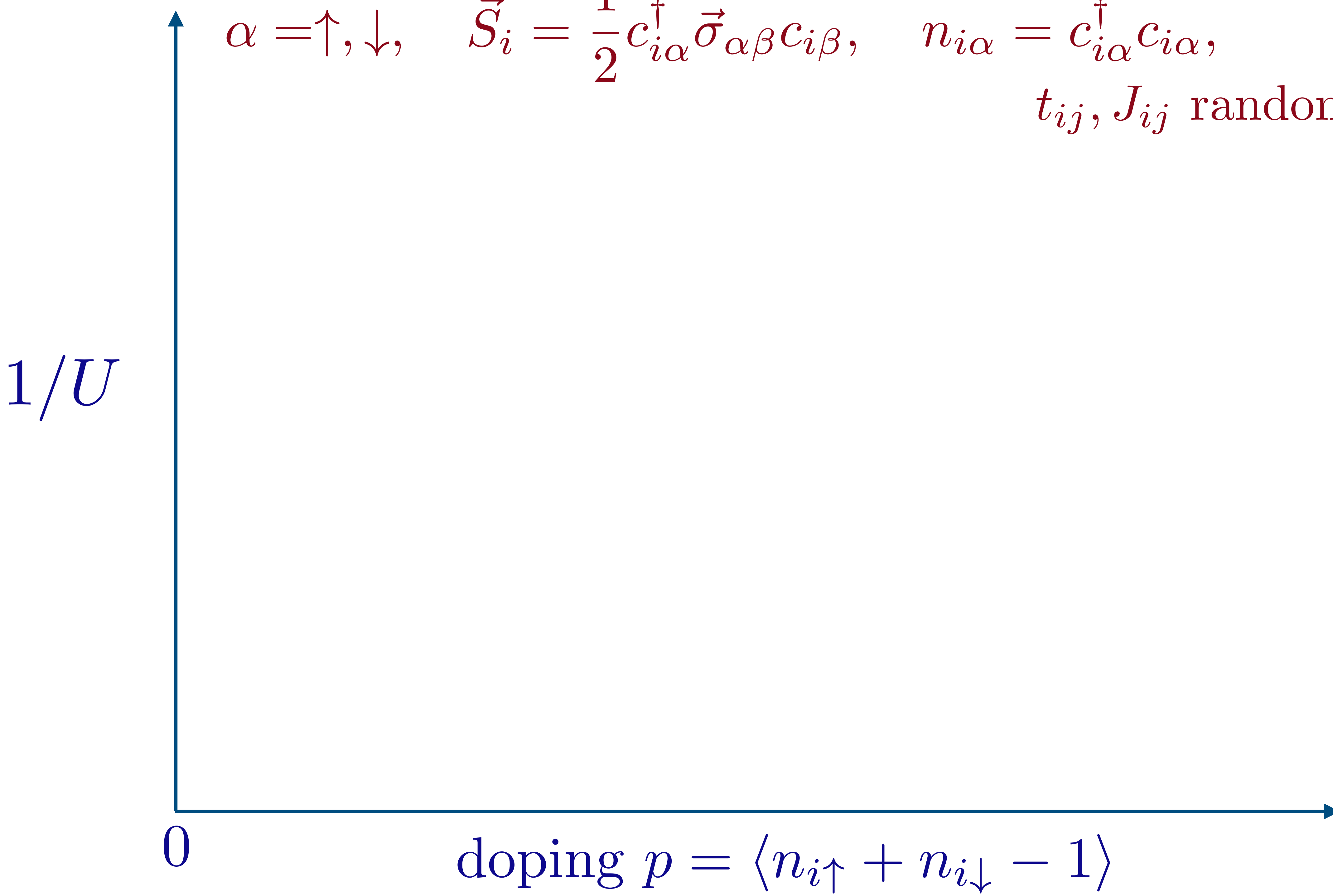
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

Random t-J-U model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$

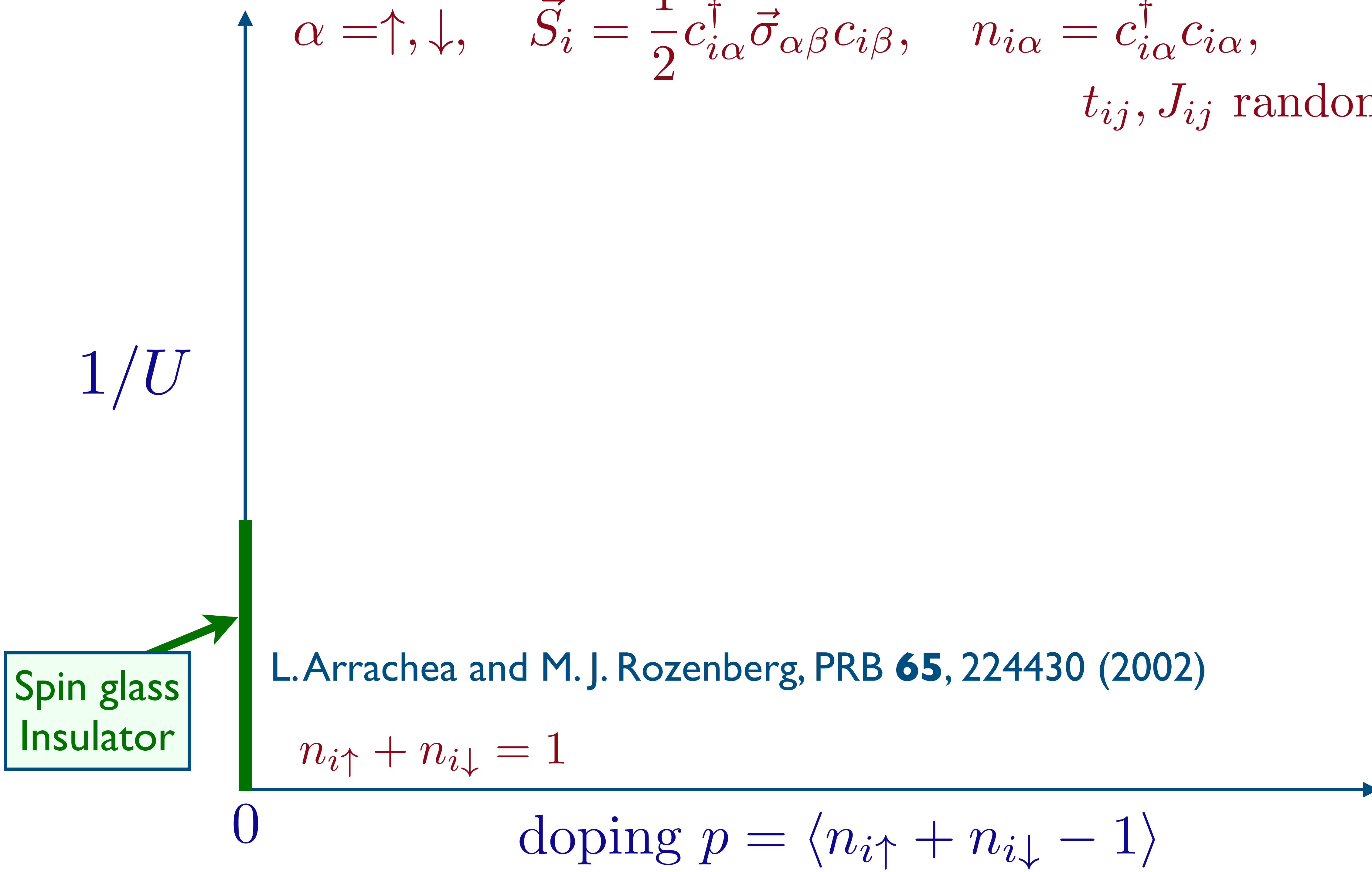


Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \boxed{\frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j} + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$



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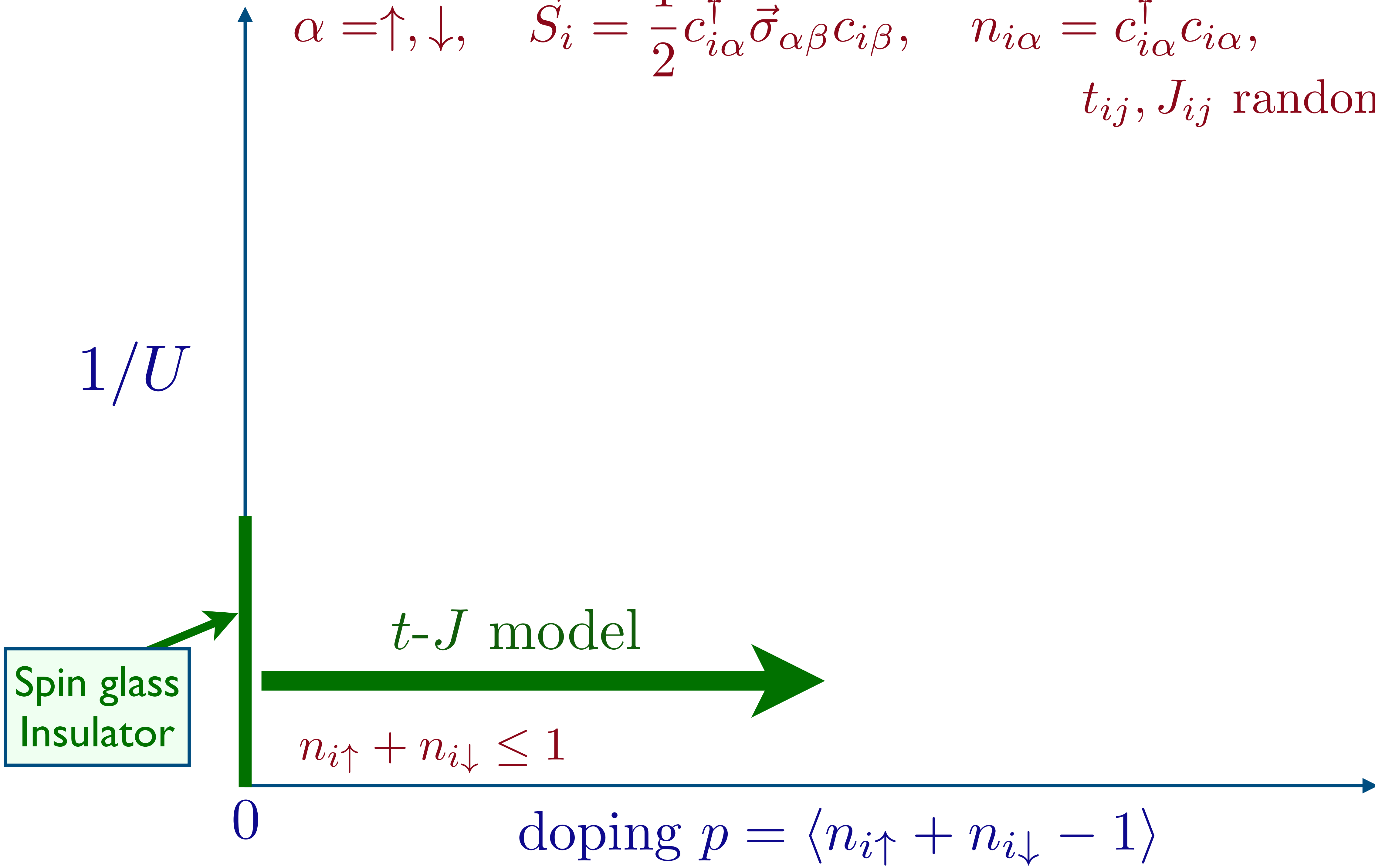

Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

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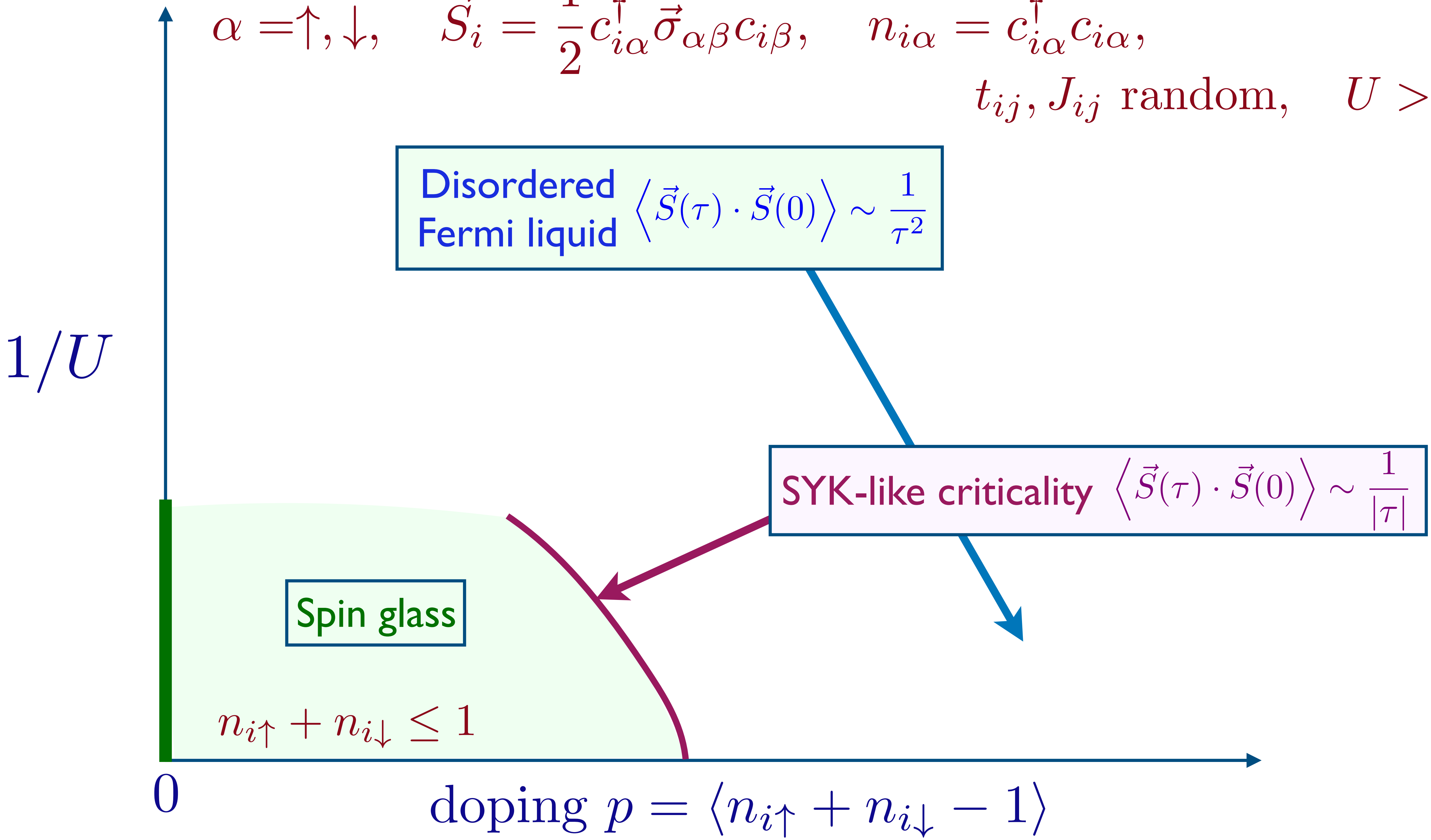

Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}}\sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}}\sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U\sum_{i=1}^{N} n_{i\uparrow}n_{i\downarrow}$$

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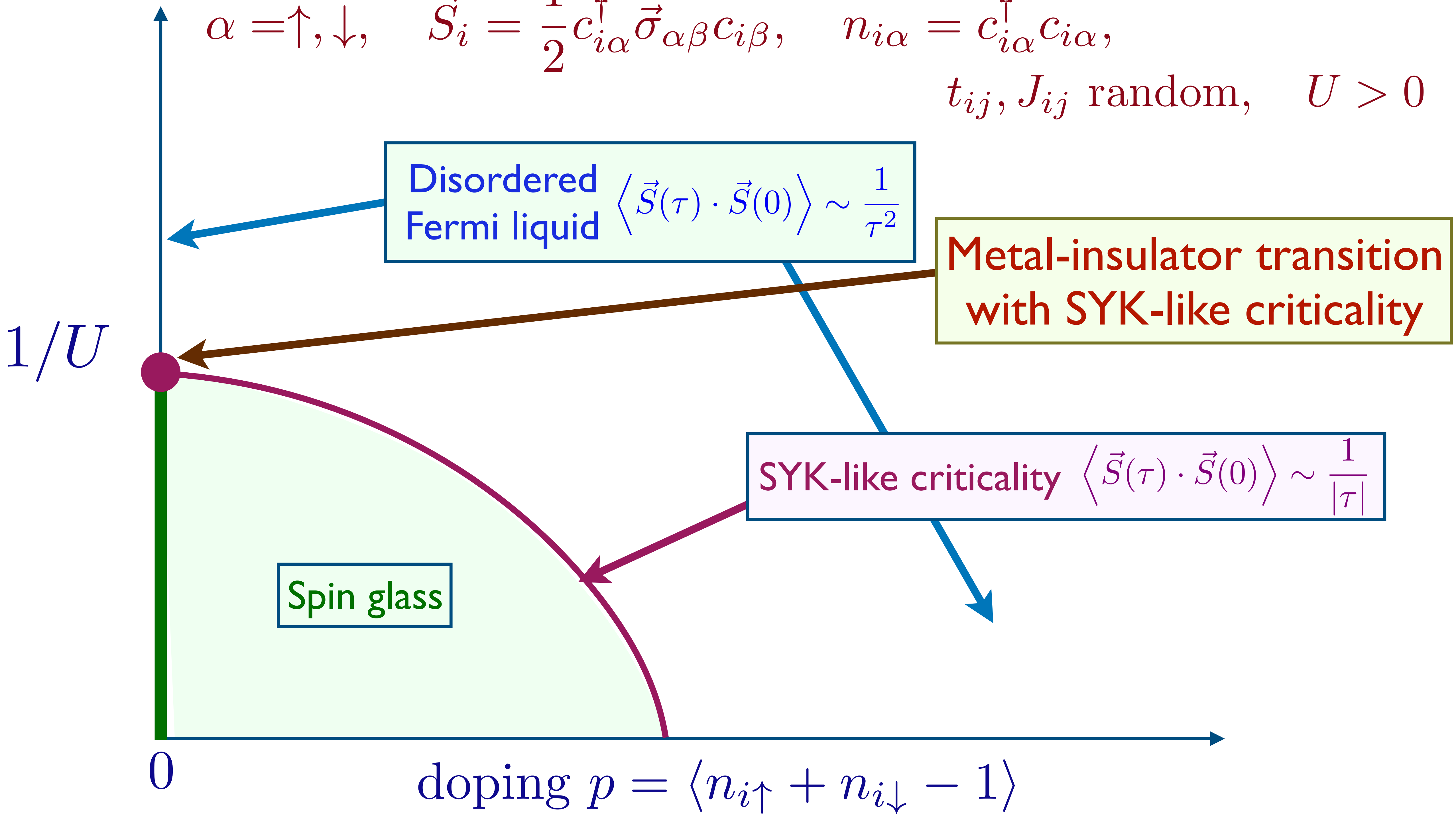


arXiv.org > cond-mat > arXiv:2002.07181

Condensed Matter > Strongly Correlated Electrons

Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-$1/2$ fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-Ah Kim

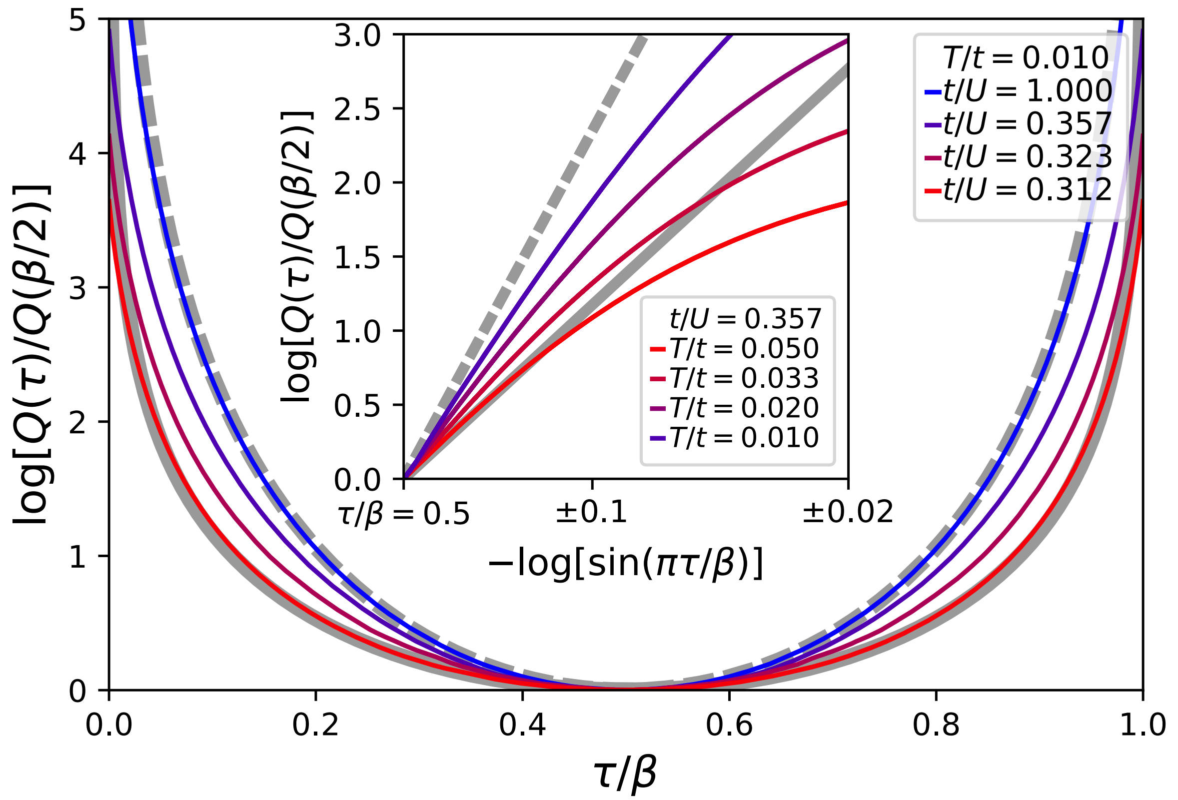
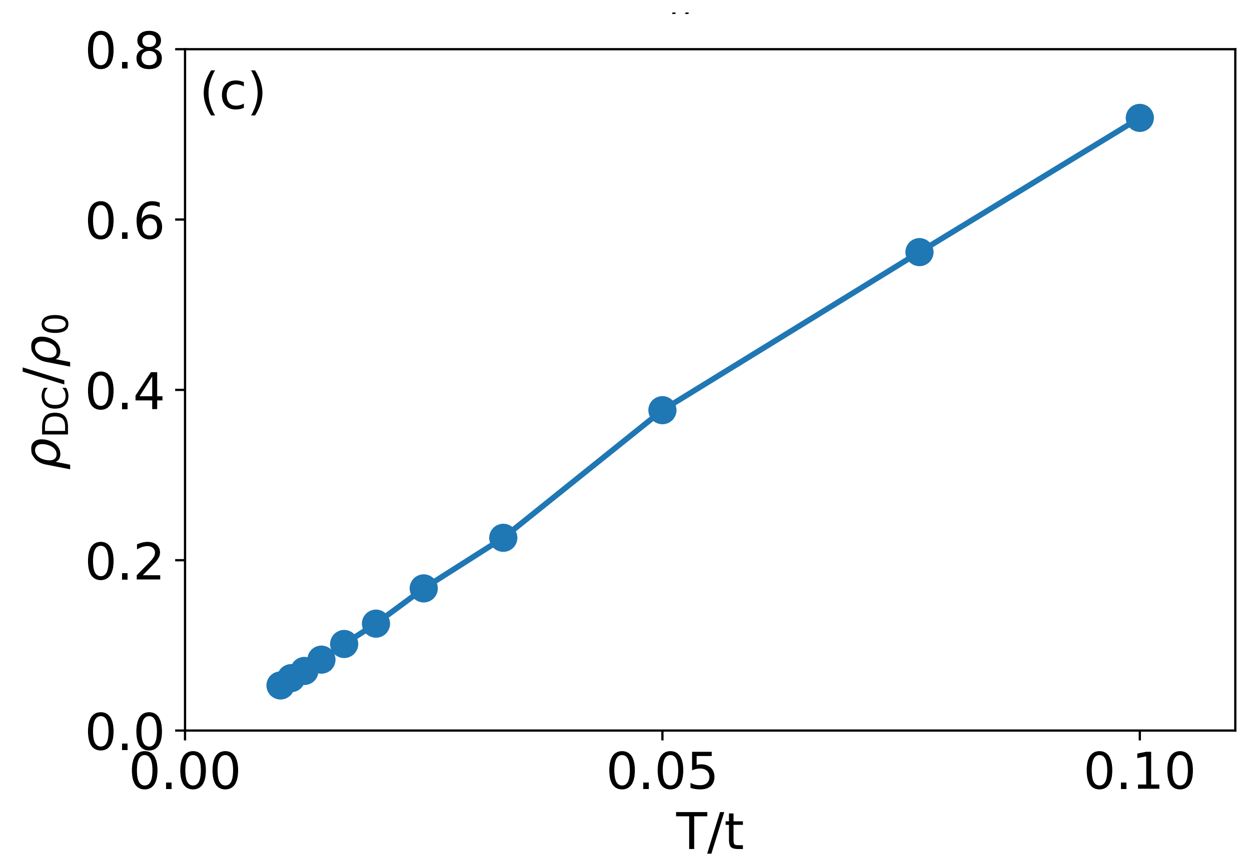


FIG. 2. Main: Spin susceptibility $\log[Q(\tau)/Q(\beta/2)]$ vs τ/β for $J/t = 0.5$ and $T/t = 0.01$, across several t/U. Grey curves show $(1/\sin\pi\tau/\beta)^{\alpha}$ with $\alpha = 1$ (solid) and $\alpha = 2$ (dashed). Color scheme follows the blue (FL) and red (QSL) gradient of Fig. 1. Inset: Spin susceptibility $\log[Q(\tau)/Q(\beta/2)]$ vs $-\log[\sin(\pi\tau/\beta)]$, for $J/t = 0.5$ and $t/U = 0.357$, across a range of T, demonstrating scaling behavior of $Q(\tau)$ near $\tau = \beta/2$. Grey curves show $\alpha = 1, 2$ (solid, dashed).



- The t-J model with random and all-to-all hopping and exchange displays a phase diagram which captures many of the key characteristics of the cuprate phase diagram.

- Its optimal doping criticality displays SYK-like criticality, including a (likely) linear-in-T resitivity down to $T = 0$.