

# General relativity and the cuprates

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**Gary T. Horowitz and Jorge E. Santos**

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**ABSTRACT:** We add a periodic potential to the simplest gravitational model of a superconductor and compute the optical conductivity. In addition to a superfluid component, we find a normal component that has Drude behavior at low frequency followed by a power law fall-off. Both the exponent and coefficient of the power law are temperature independent and agree with earlier results computed above  $T_c$ . These results are in striking agreement with measurements on some cuprates. We also find a gap  $\Delta = 4.0 T_c$ , a rapidly decreasing scattering rate, and “missing spectral weight” at low frequency, all of which also agree with experiments.

**KEYWORDS:** Holography and condensed matter physics (AdS/CMT), Gauge-gravity correspondence, AdS-CFT Correspondence

**ARXIV EPRINT:** [1302.6586](https://arxiv.org/abs/1302.6586)



# Transport in strange metals

GaryFest  
University of California, Santa Barbara  
May 1, 2015

Subir Sachdev

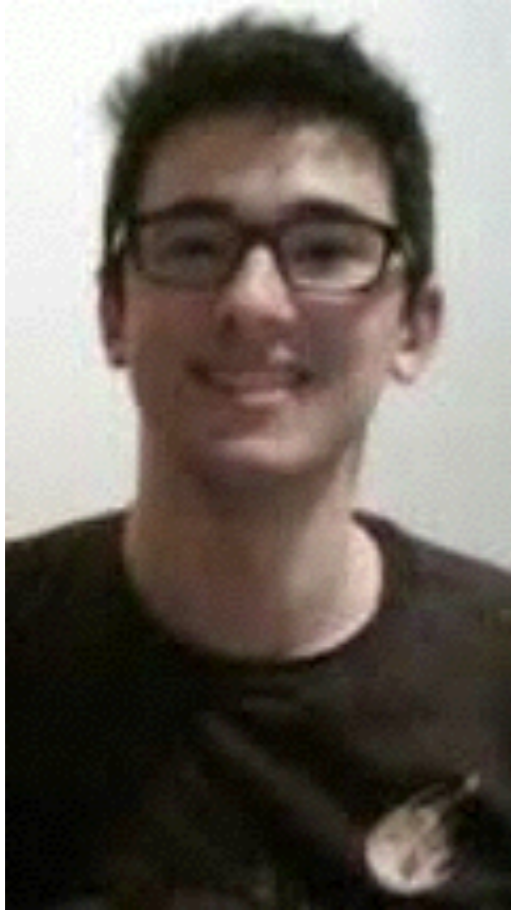


Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)



JOHN TEMPLETON  
FOUNDATION





Andrew Lucas  
Harvard



Aavishkar Patel  
Harvard



Philipp Strack  
Cologne

# Outline

## 1. Quasiparticle transport in ordinary metals

*Bloch vs. Peierls*

## 2. Transport without quasiparticles in strange metals

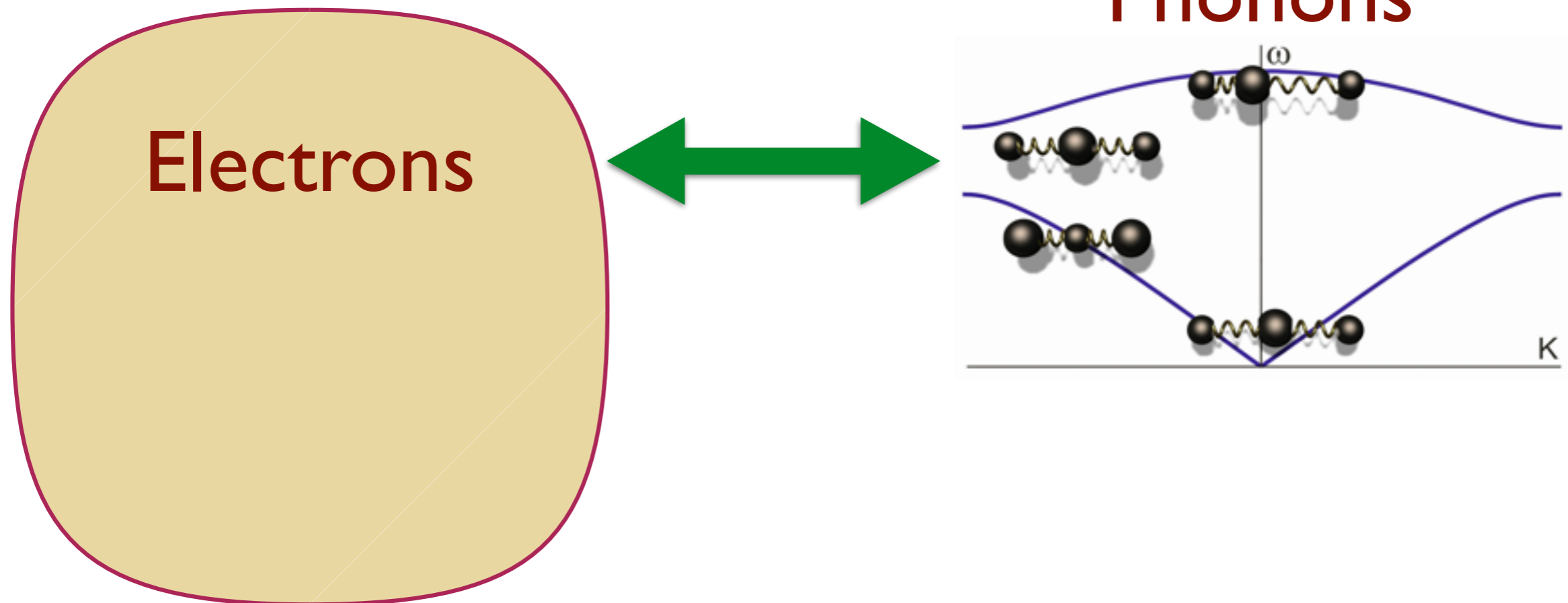
*Memory functions, holography, and hydrodynamics*

## 3. The spin density wave quantum critical point

*Transport with hyperscaling violation*

# Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch's law (1930) : a resistivity  $\rho(T) \sim T^5$ .



# Quasiparticle transport in metals:

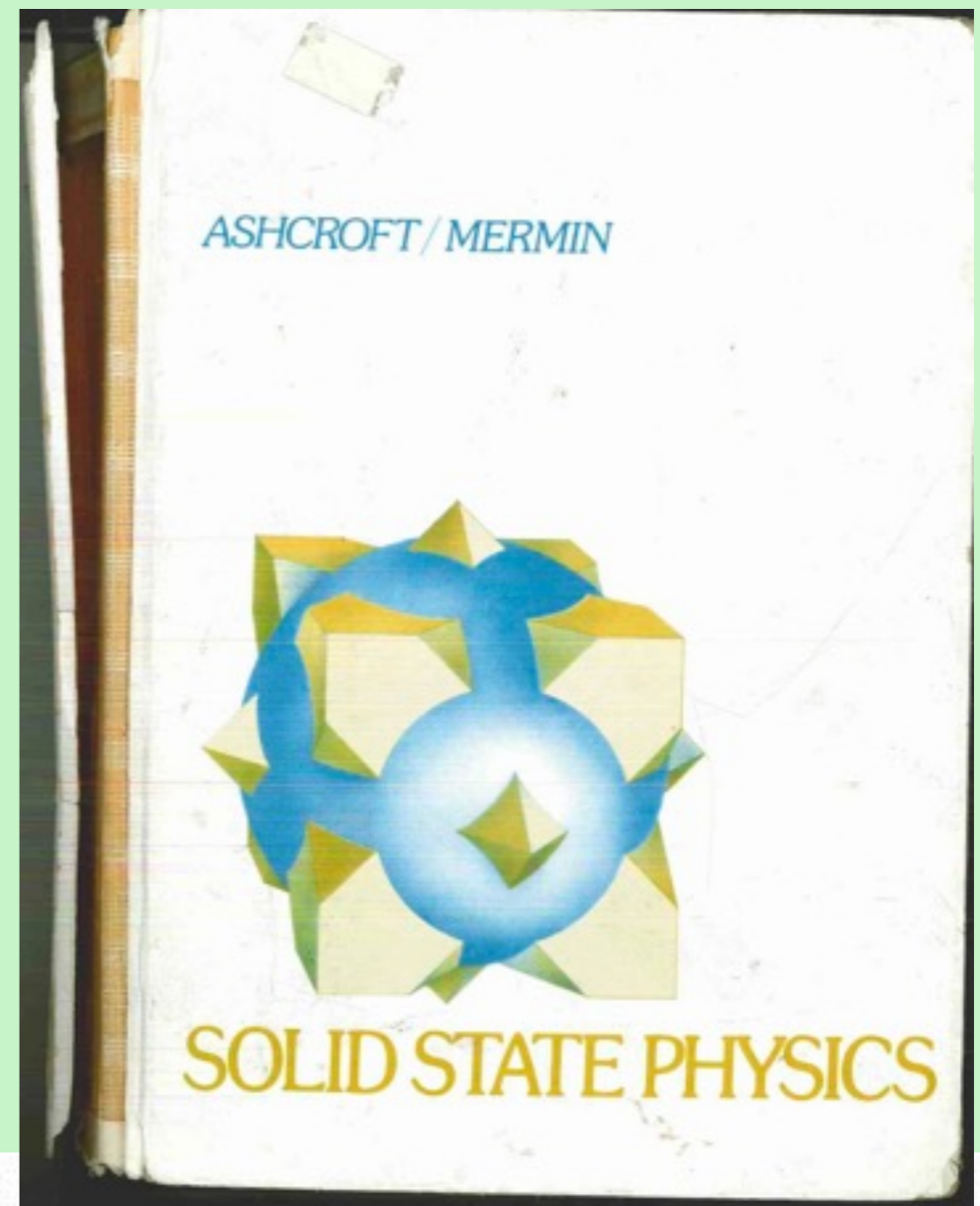
- Compute the scattering rates off phonons: this leads to resistivity  $\rho(T) \sim T^5$ .

However, this ignores “phonon drag”

## PHONON DRAG

Peierls<sup>28</sup> pointed out a way in which the low temperature resistivity might decline more rapidly than  $T^5$ .

<sup>28</sup> R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).



# Quasiparticle transport in metals:

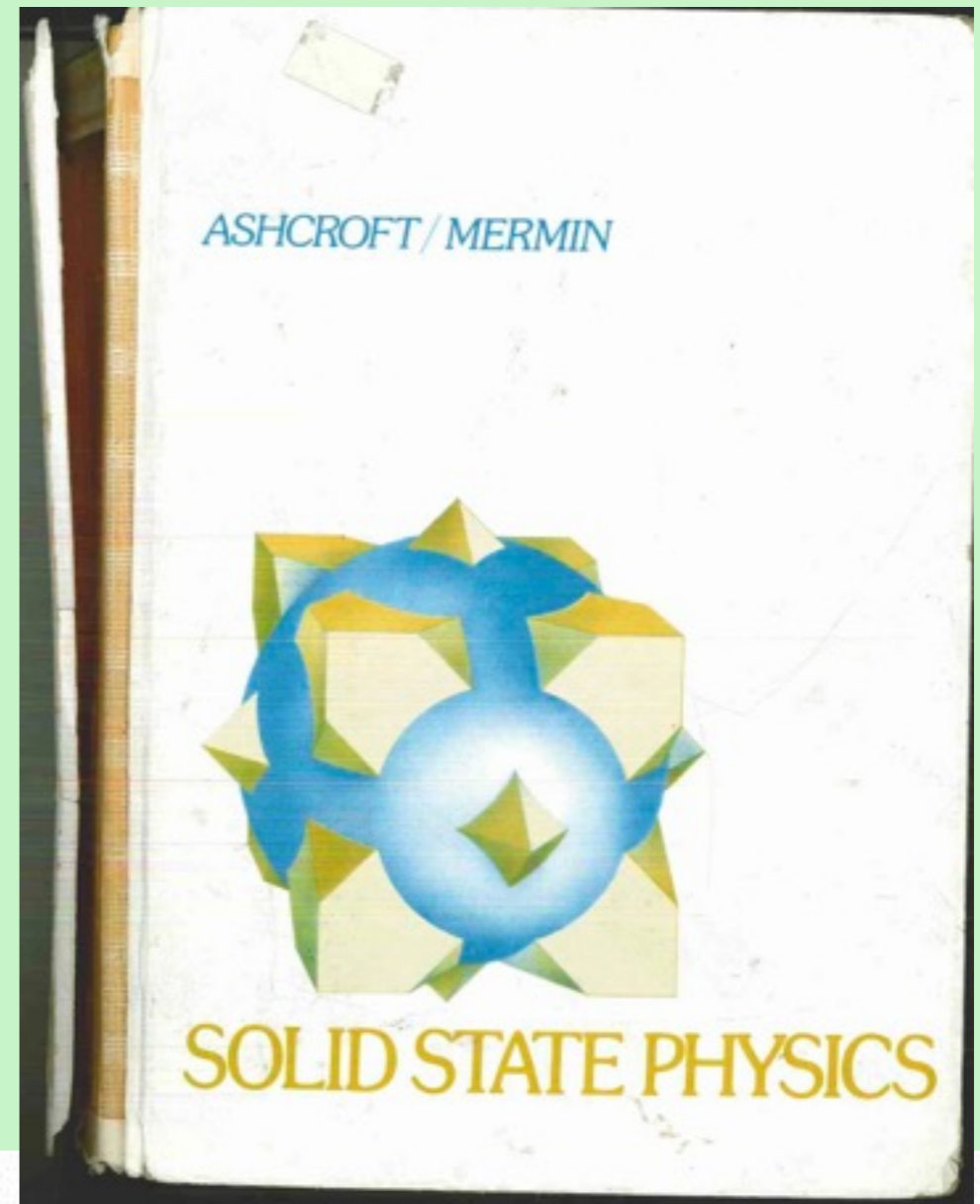
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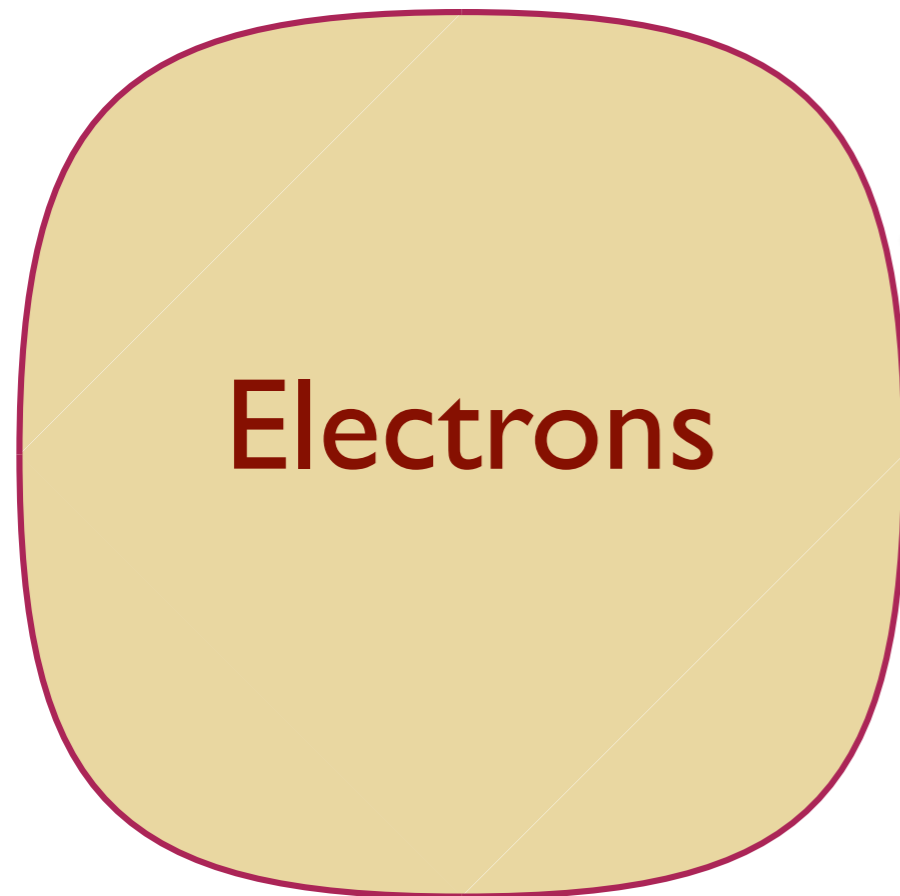
## PHONON DRAG

Peierls<sup>28</sup> pointed out a way in which the low temperature resistivity might decline more rapidly than  $T^5$ . This behavior has yet to be observed,

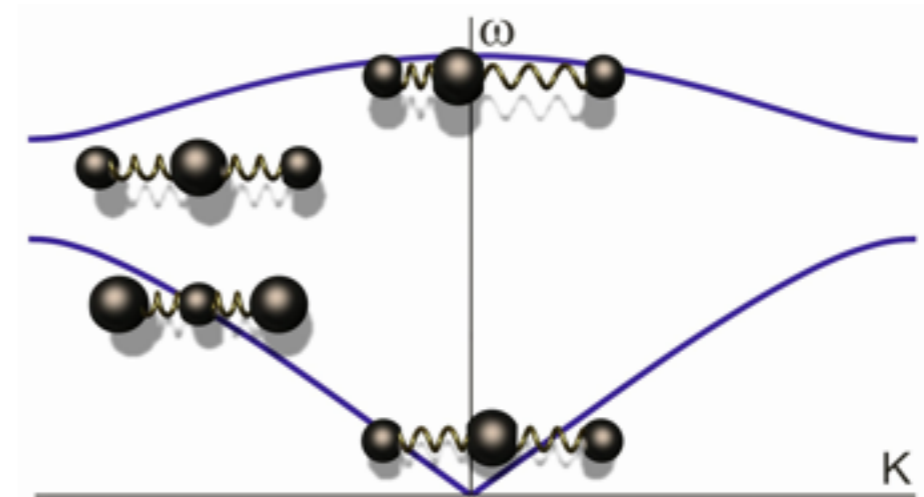
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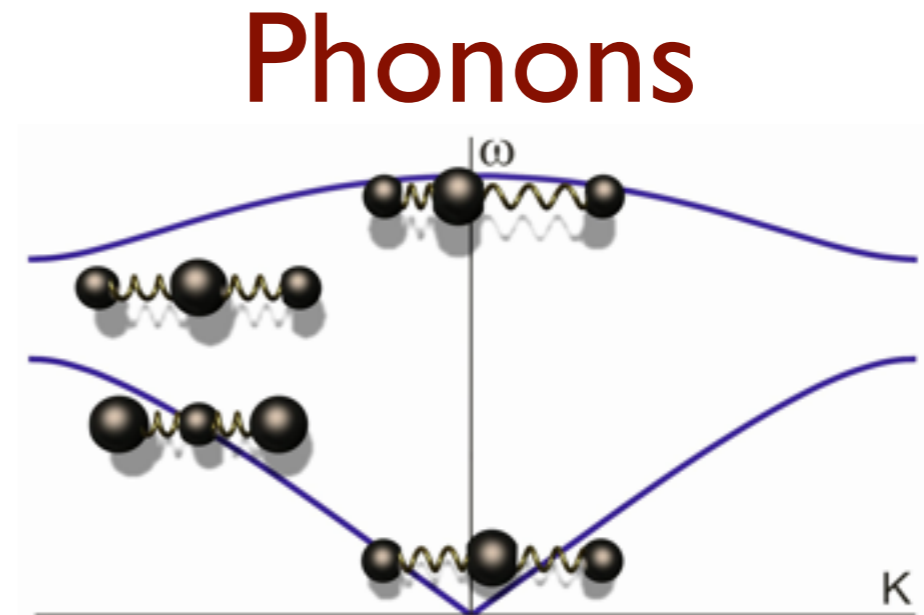
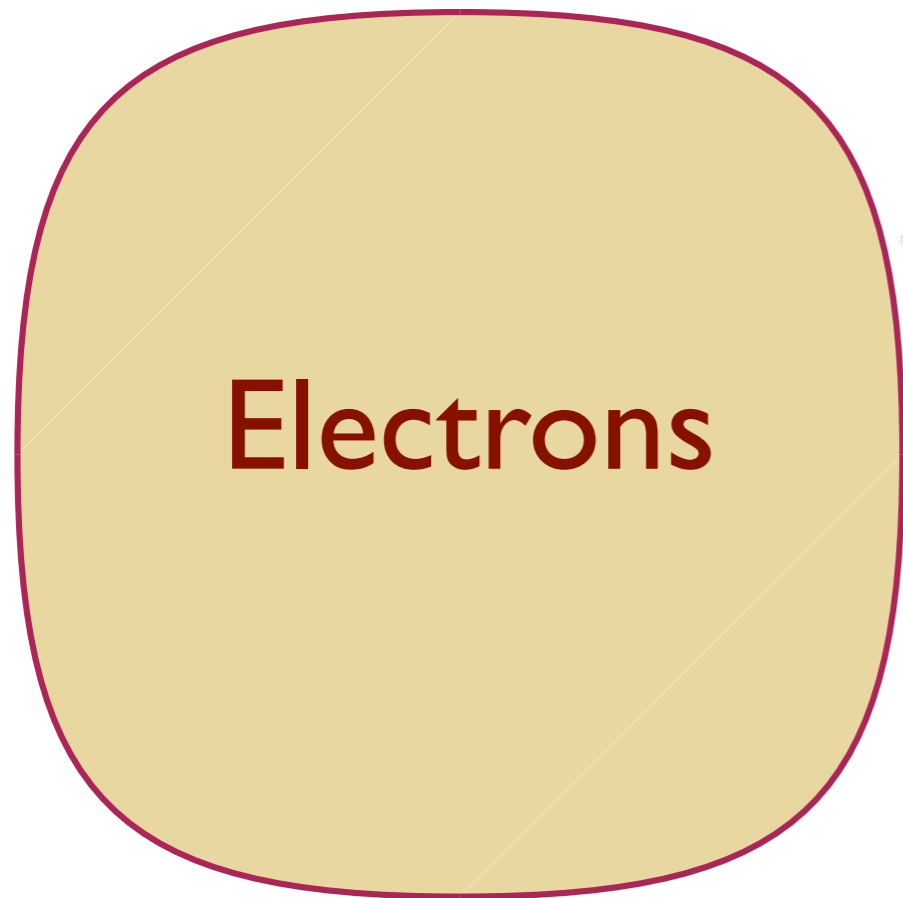
# Rates of Momentum Flow



## Phonons

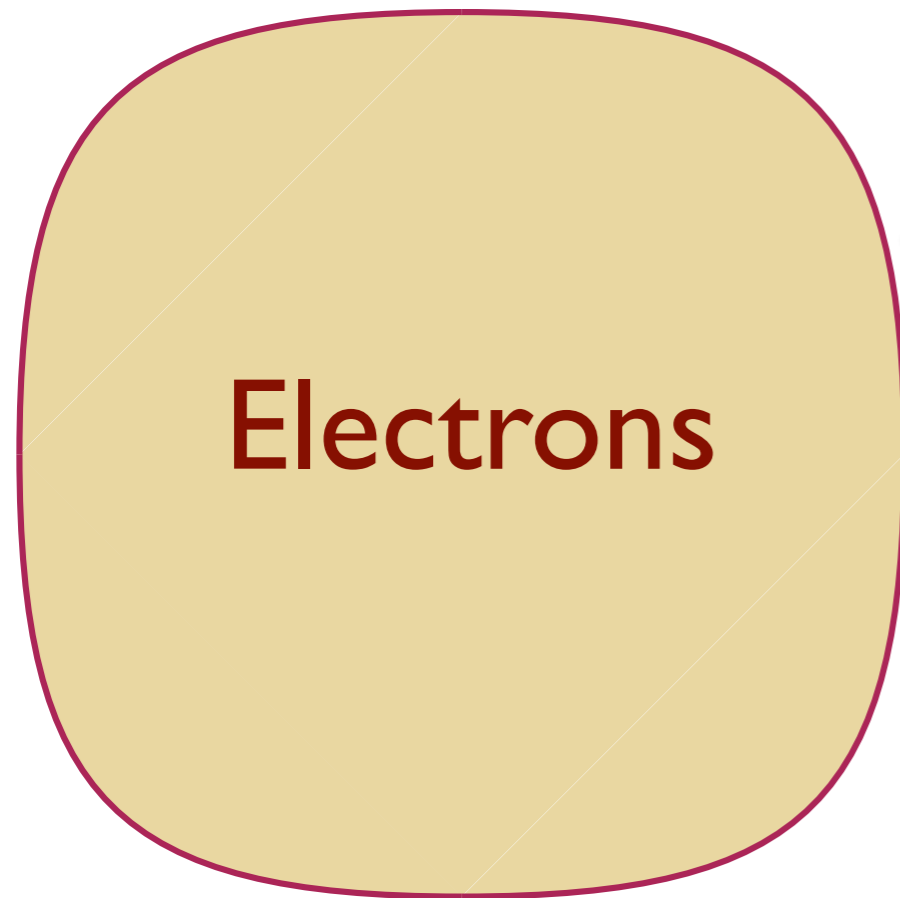


# Rates of Momentum Flow



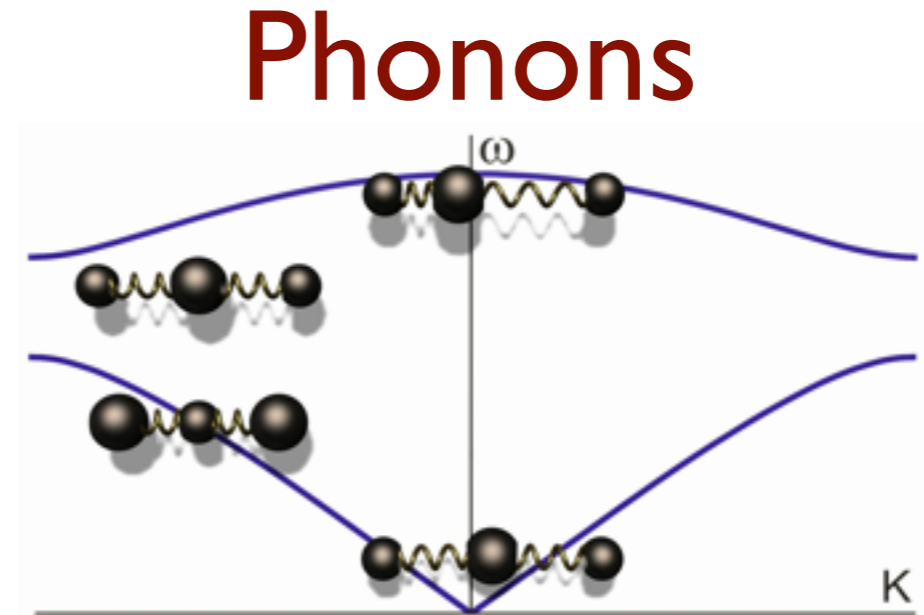
Defects

# Rates of Momentum Flow



**SLOW**

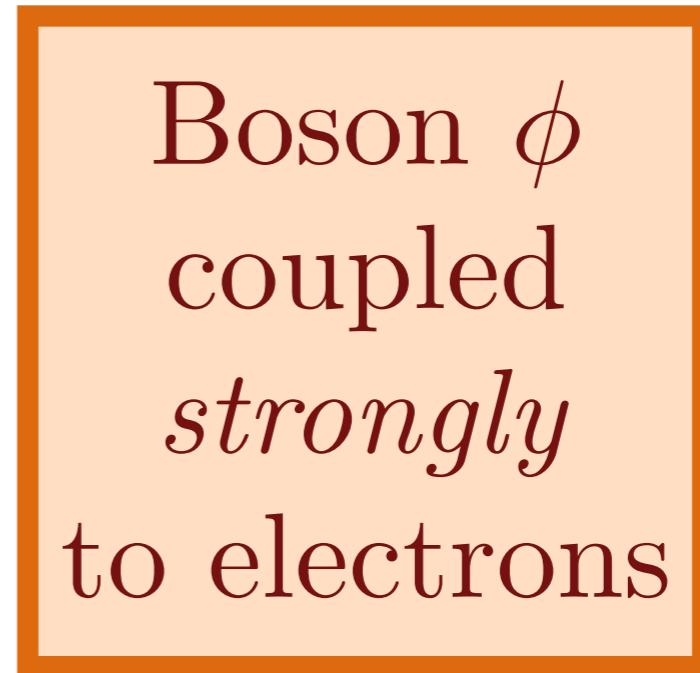
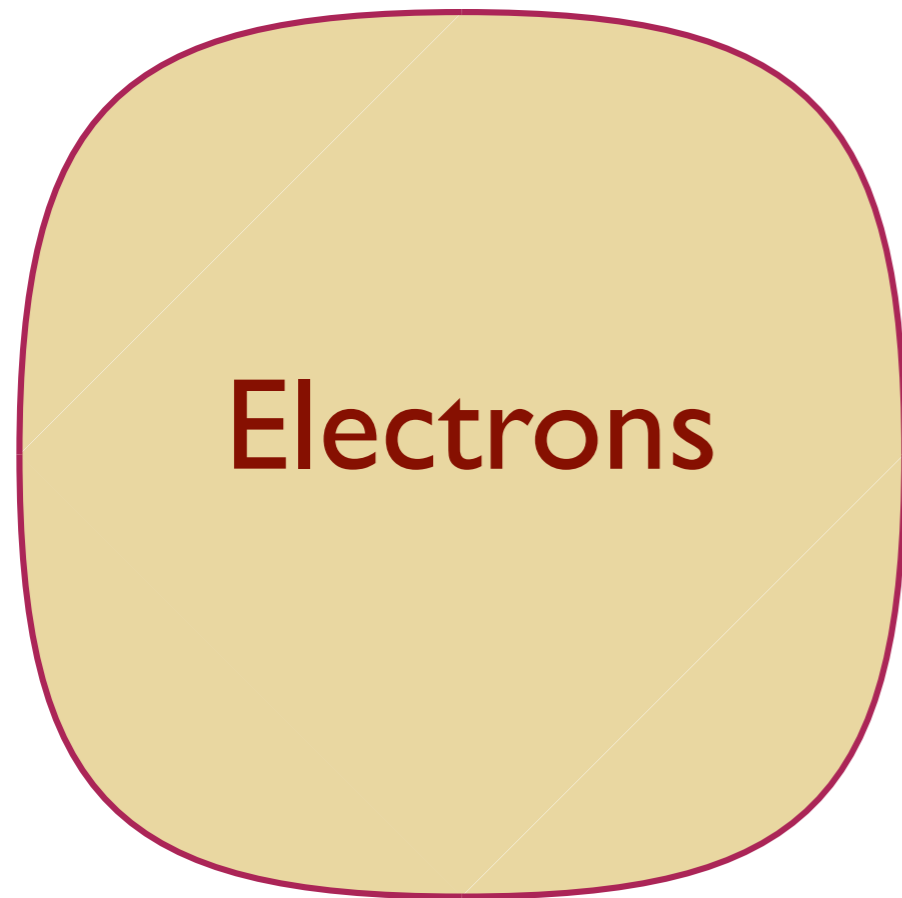
Process  
controlling  
resistivity  
(Bloch)



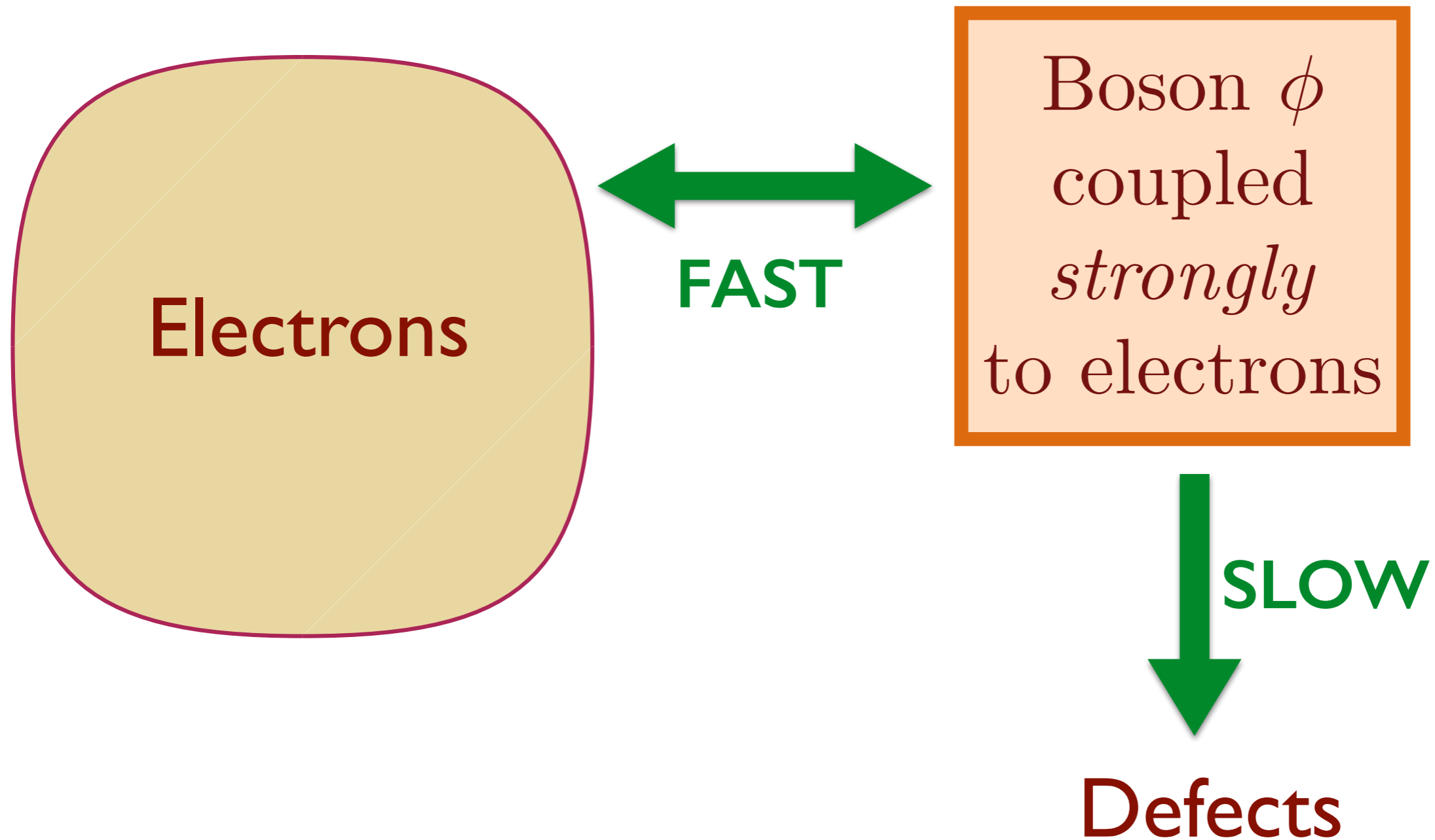
**FAST**

Defects

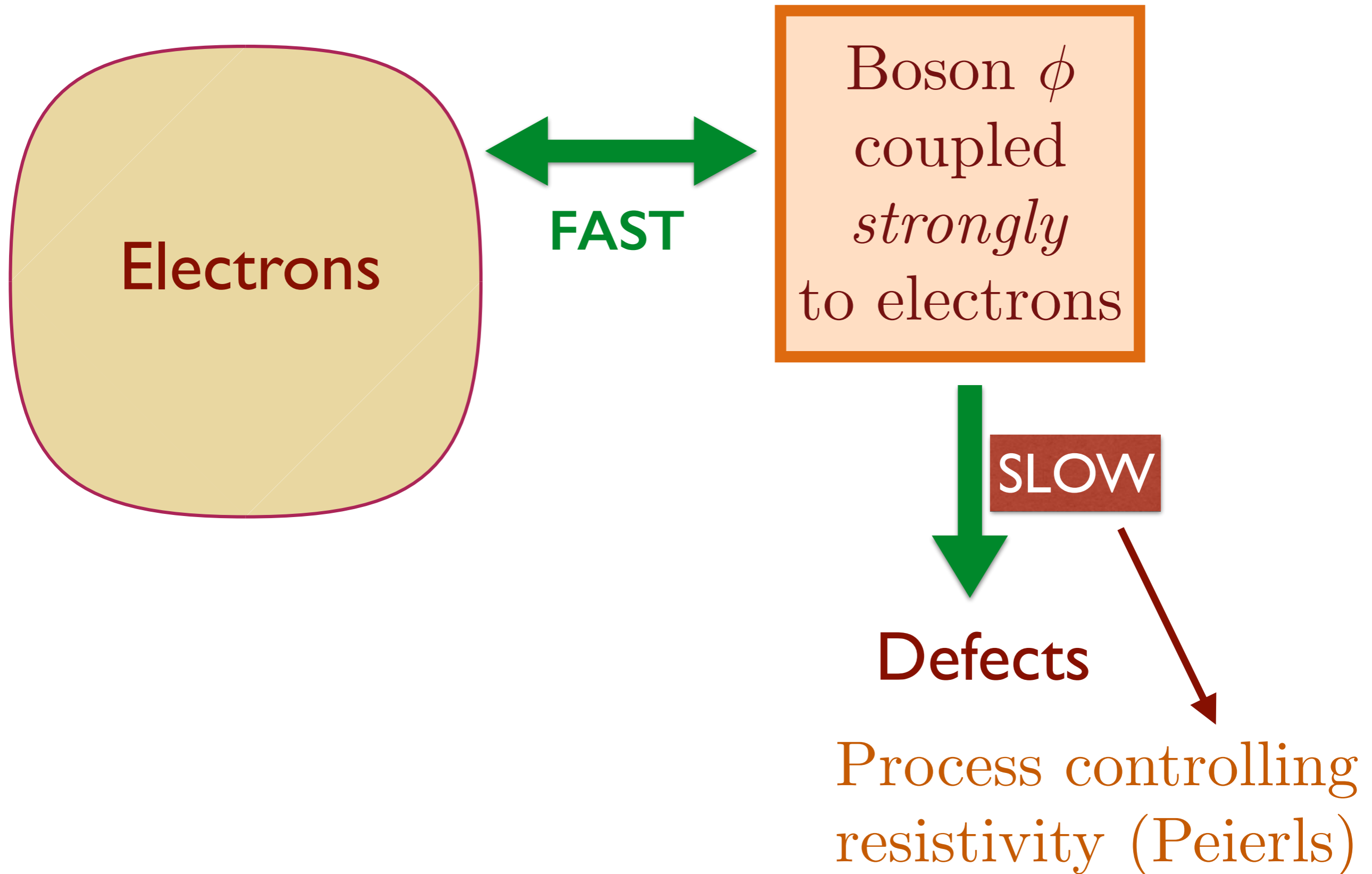
# Rates of Momentum Flow



# Rates of Momentum Flow



# Rates of Momentum Flow



# Outline

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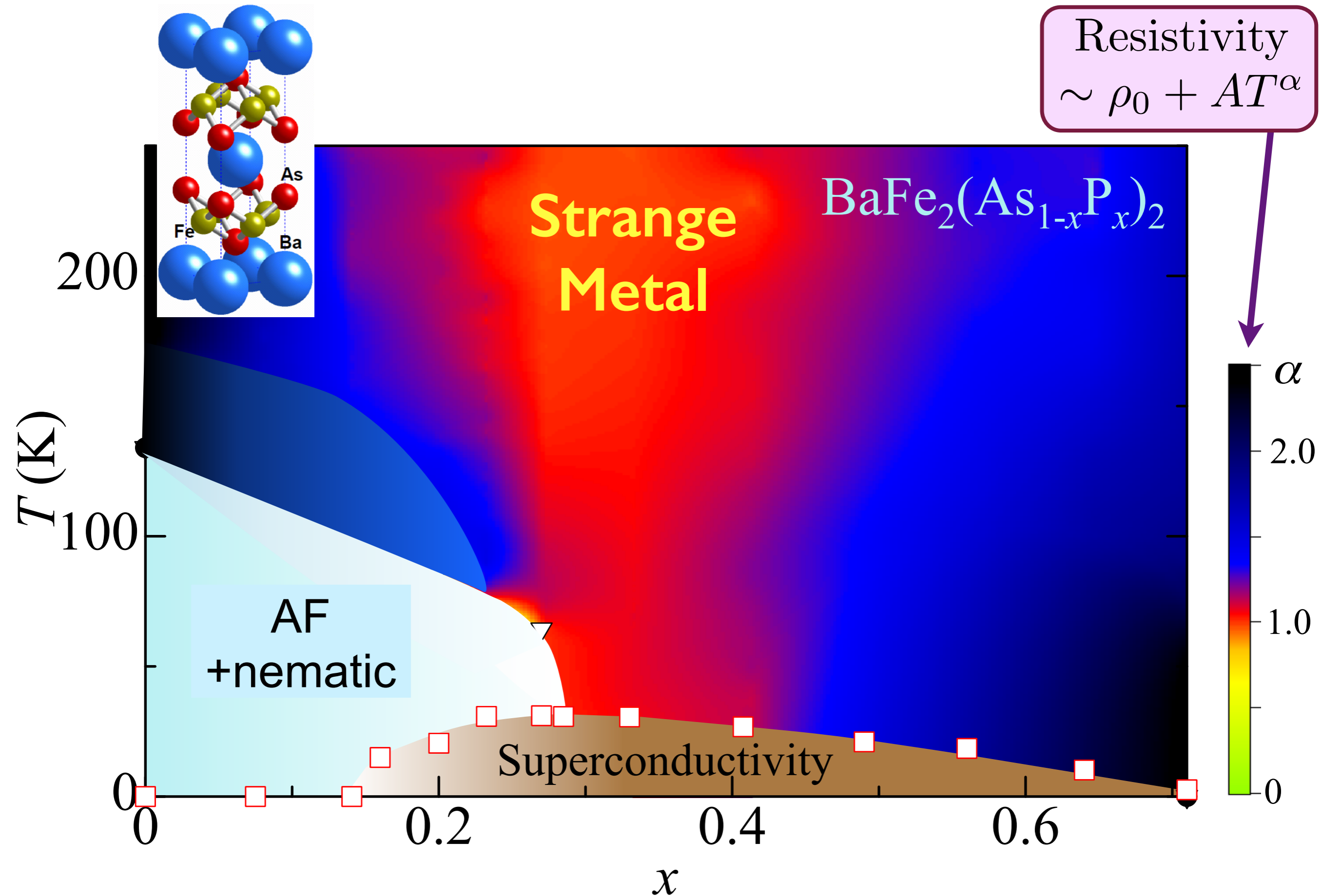
*Bloch vs. Peierls*

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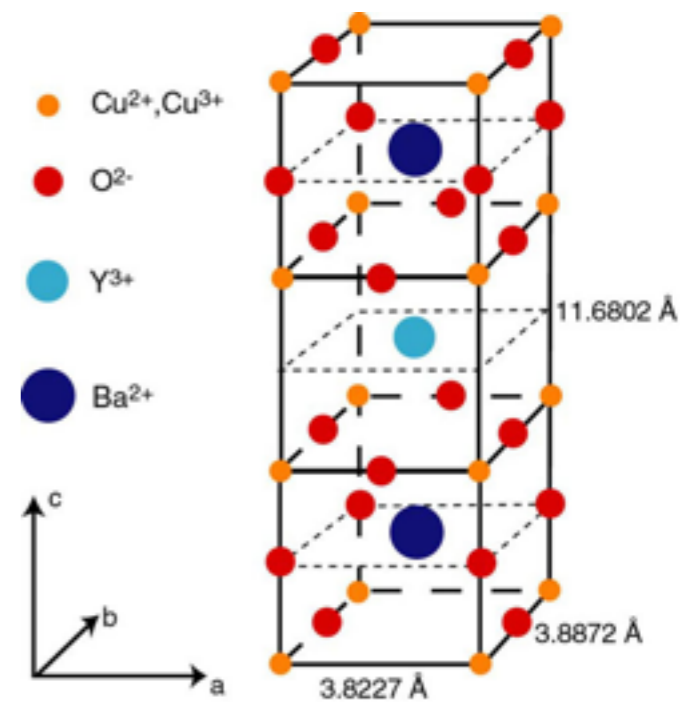
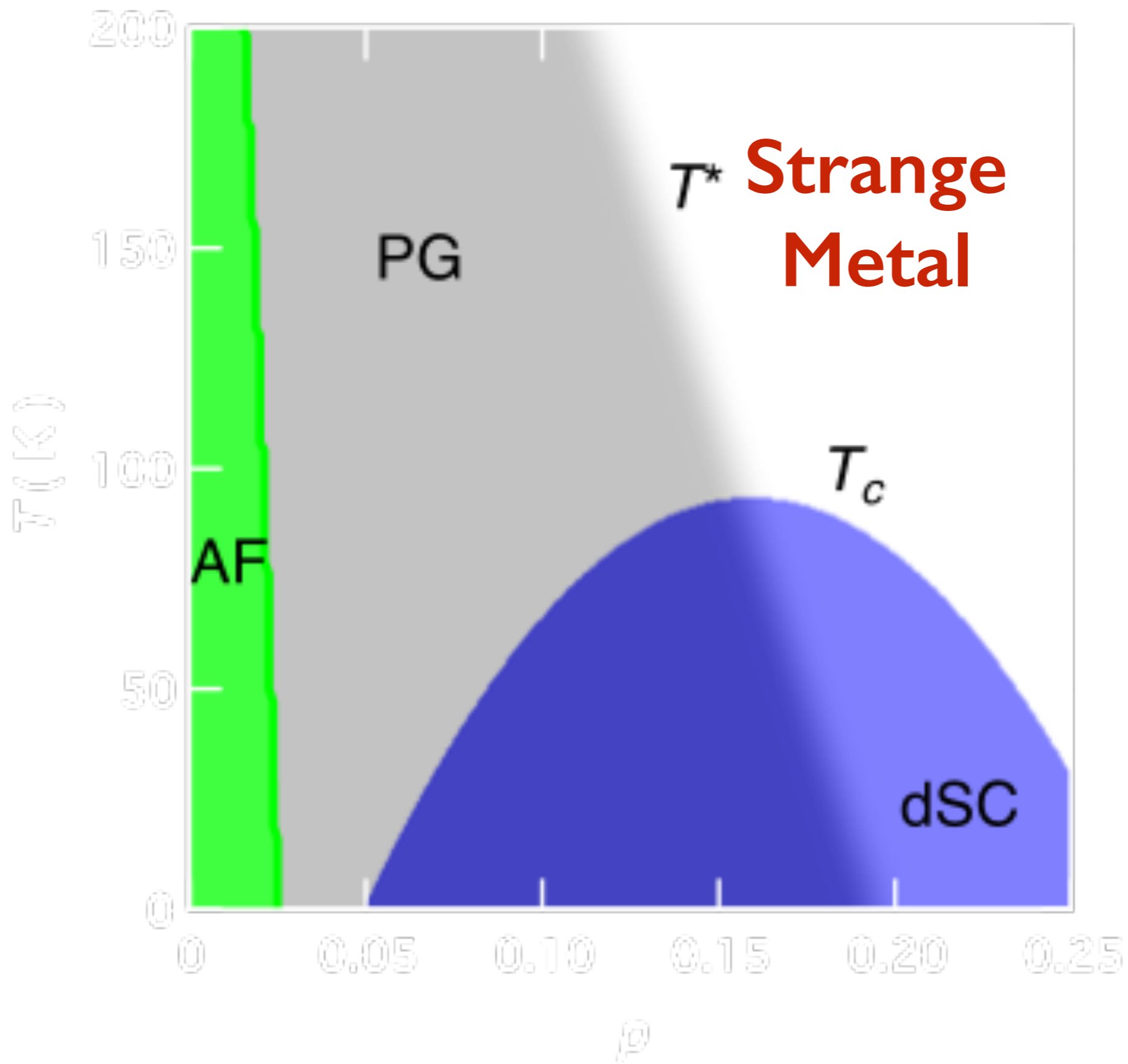
*Memory functions, holography, and hydrodynamics*

## 3. The spin density wave quantum critical point

*Transport with hyperscaling violation*



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,  
 H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,  
*Physical Review B* **81**, 184519 (2010)



universal constraints on transport

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Electrical transport at a strongly-coupled critical theory with particle-hole symmetry, obeying hyperscaling, in  $d$  spatial dimensions with dynamic critical exponent  $z$

$$\sigma = \sigma_Q \sim T^{(d-2)/z}$$

Follows from gauge invariance

( $\sigma = 1/\rho =$  conductivity)

Electrical transport at a strongly-coupled critical theory  
without particle-hole symmetry,  
with a conserved momentum  $P$

$$\sigma = \sigma_Q + \frac{Q^2}{\mathcal{M}} \pi \delta(\omega)$$

with  $Q \equiv \chi_{J_x, P_x}$  and  $\mathcal{M} \equiv \chi_{P_x, P_x}$  thermodynamic response functions

Obtained in hydrodynamics, holography, and  
by memory functions

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

A. Lucas and S. Sachdev, arXiv:1502.04704

Electrical transport at a strongly-coupled critical theory  
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$$\sigma = \sigma_Q + \frac{Q^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau_L)}$$

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Momentum relaxation by an external source  $h_L$  coupling to the operator  $\mathcal{O}$

$$H = H_0 - \int d^d x h_L(x) \mathcal{O}(x).$$

$$\frac{\mathcal{M}}{\tau_L} = \lim_{\omega \rightarrow 0} \int d^d q |h_L(q)|^2 q_x^2 \frac{\text{Im} (G_{\mathcal{O}\mathcal{O}}^R(q, \omega))_{H_0}}{\omega} + \text{higher orders in } h_L$$

**Obtained by memory functions**

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

S.A. Hartnoll and D. M. Hofman, PRL **108**, 241601 (2012)

A. Lucas, S. Sachdev and K. Schalm, PRD **89**, 066018 (2014)

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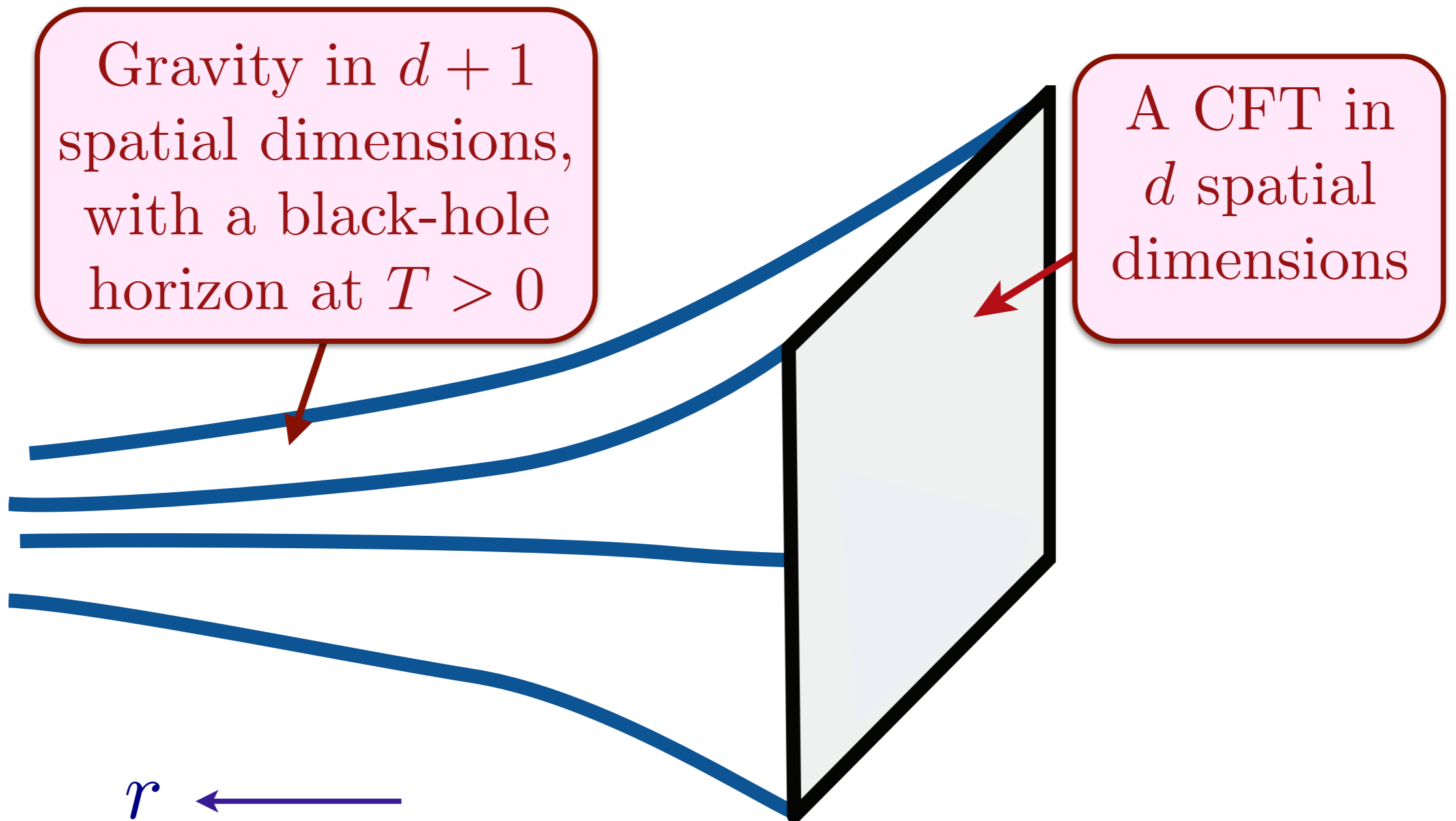
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# Holography of a conformal field theory: AdS/CFT

Gravity in  $d + 1$  spatial dimensions, with a black-hole horizon at  $T > 0$

A CFT in  $d$  spatial dimensions



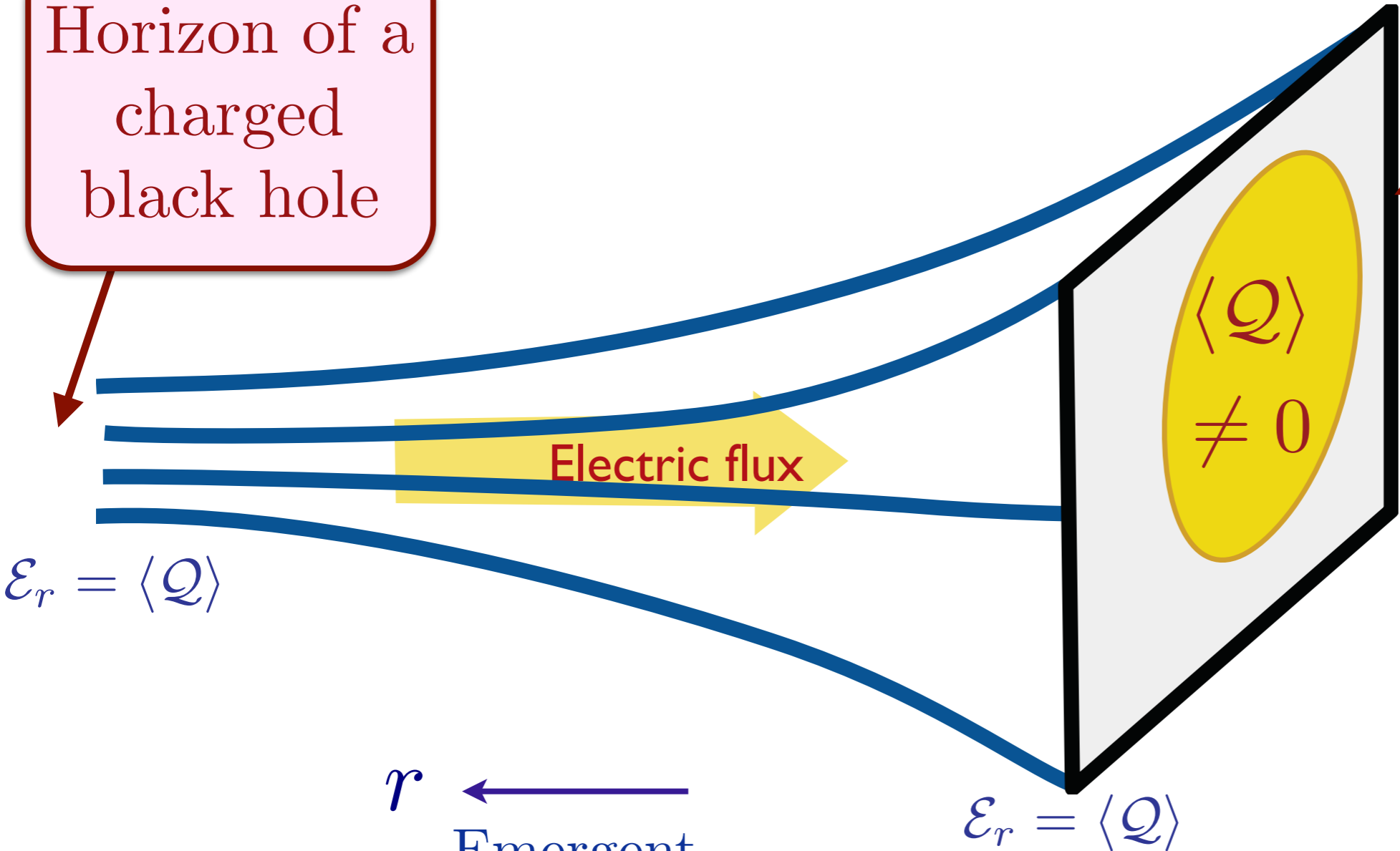
$r$  ←  
Emergent  
“holographic”  
dimension

Solvable models which have led to new insights on the transport properties of quantum matter without quasiparticles

# Holography of a strange metal: a charged black hole

Horizon of a charged black hole

A strange metal at density  $\langle Q \rangle$



Solvable models which have led to new insights on the transport properties of quantum matter without quasiparticles

# Optical Conductivity with Holographic Lattices

**JHEP 1207 (2012) 168**

Gary T. Horowitz<sup>a</sup>, Jorge E. Santos<sup>a</sup>, David Tong<sup>b</sup>

We add a gravitational background lattice to the simplest holographic model of matter at finite density and calculate the optical conductivity. With the lattice, the zero frequency delta function found in previous calculations (resulting from translation invariance) is broadened and the DC conductivity is finite. The optical conductivity exhibits a Drude peak with a cross-over to power-law behavior at higher frequencies. Surprisingly, these results bear a strong resemblance to the properties of some of the cuprates.

$$H = H_0 - \int d^d x h_L(x) \mathcal{O}(x).$$

Computed  $\sigma$  by numerical solution of Einstein equations. Found excellent agreement with memory function expression evaluated holographically for theory  $H_0$ .

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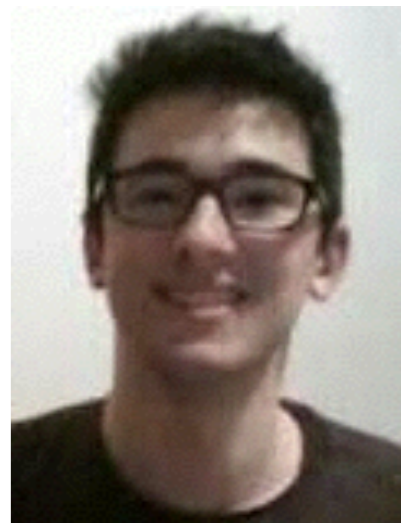
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Proof of equivalence between holography (for a large class of background metrics) and memory function formula for  $\tau_L$   
A. Lucas, JHEP 03, 071 (2015)



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$$\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} \mathcal{M} \left( \frac{1}{\tau_L} - i\omega \right),$$
$$\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2 B^2 + ((\tau_L^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2} BQ.$$

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Blake and Donos: With  $\sigma_Q \sim 1/T$  and  $\tau_L \sim 1/T^2$ , we obtain  $\sigma_{xx} \sim 1/T$  and  $\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2$ , in agreement with strange metal data on cuprates (such data cannot be explained in a quasiparticle model).

**Obtained in hydrodynamics, holography, and  
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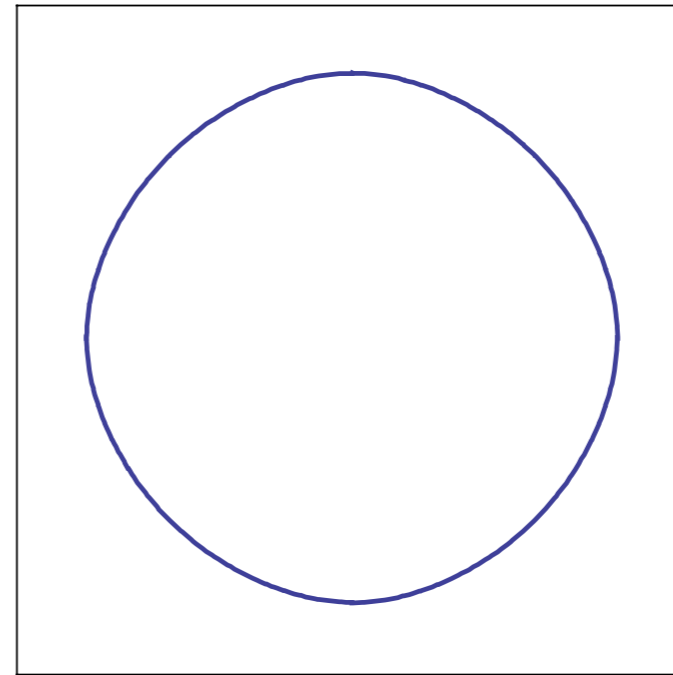
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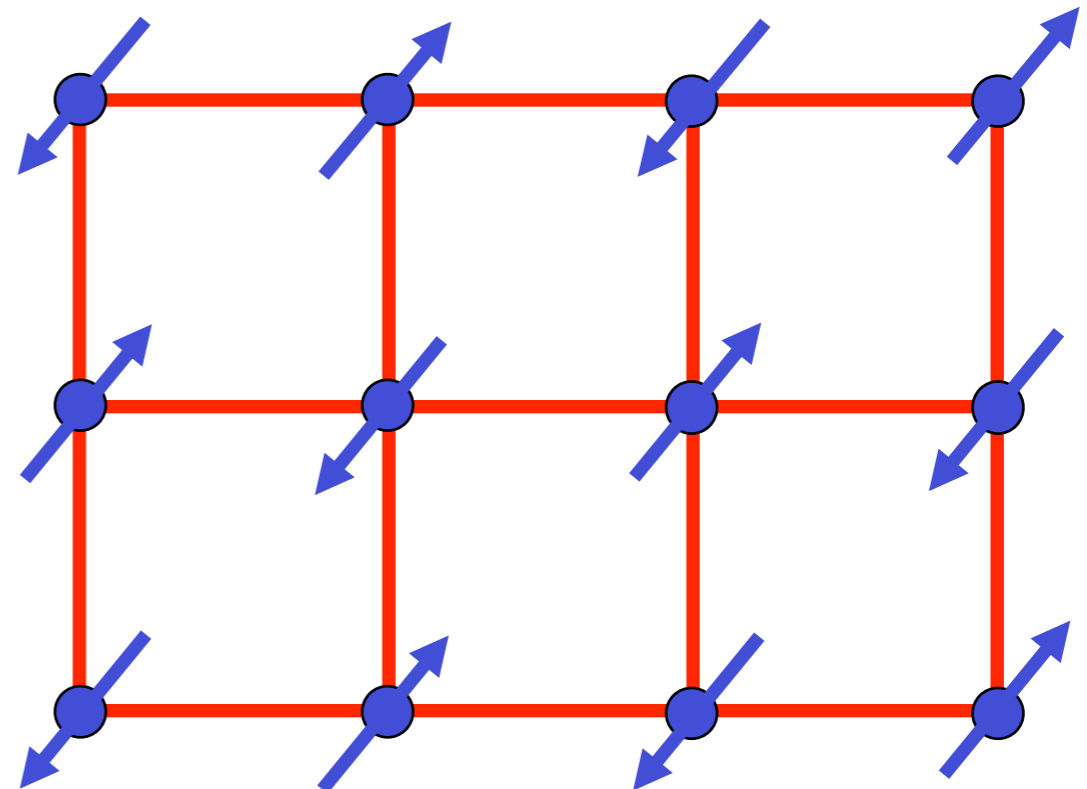
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# Fermi surface+antiferromagnetism

Metal with “large”  
Fermi surface



+

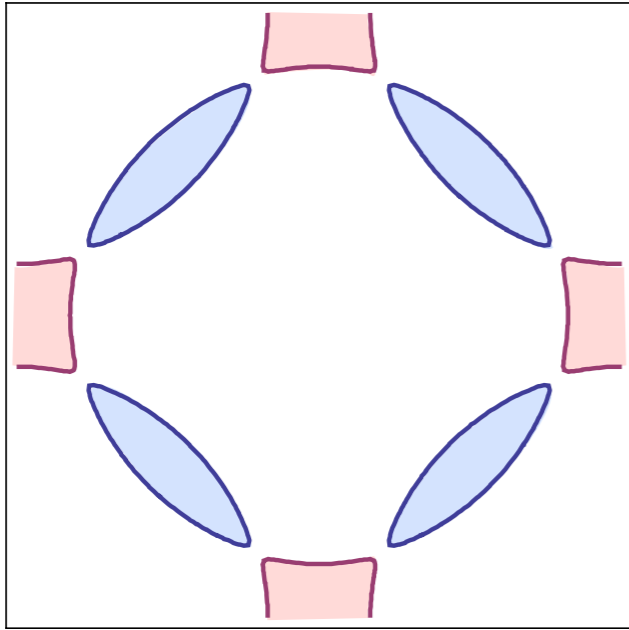


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

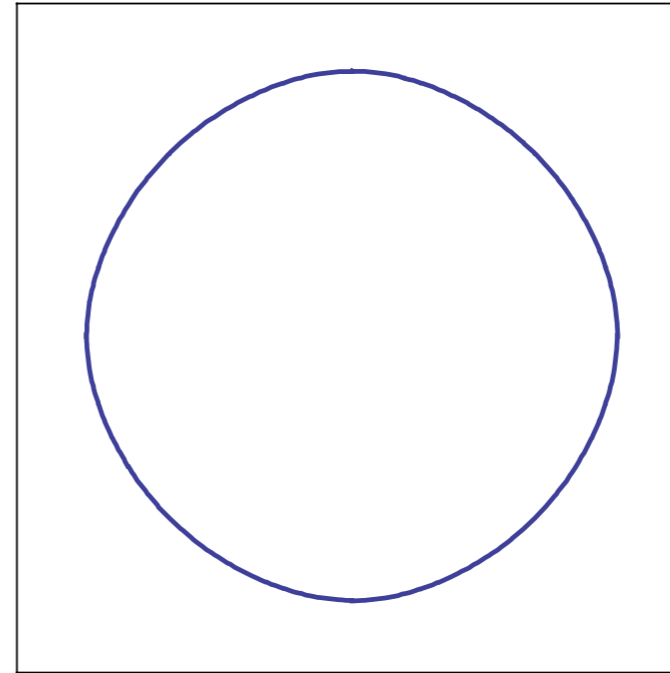
where  $\mathbf{K}$  is the ordering wavevector.

# Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

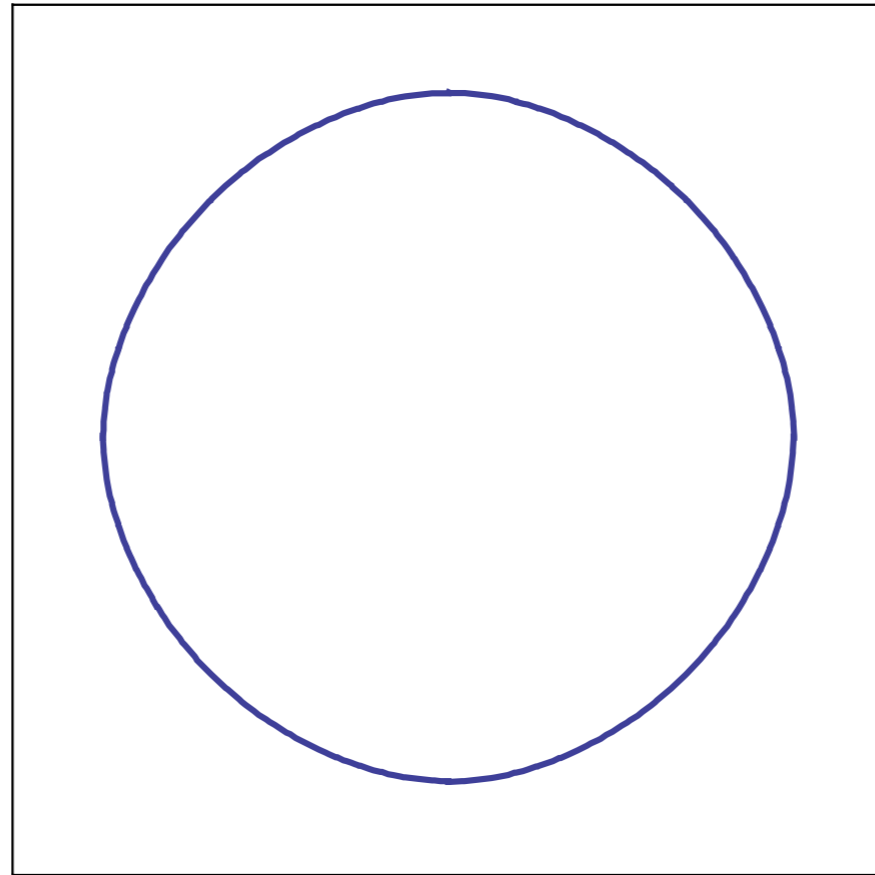
Metal with electron  
and hole pockets



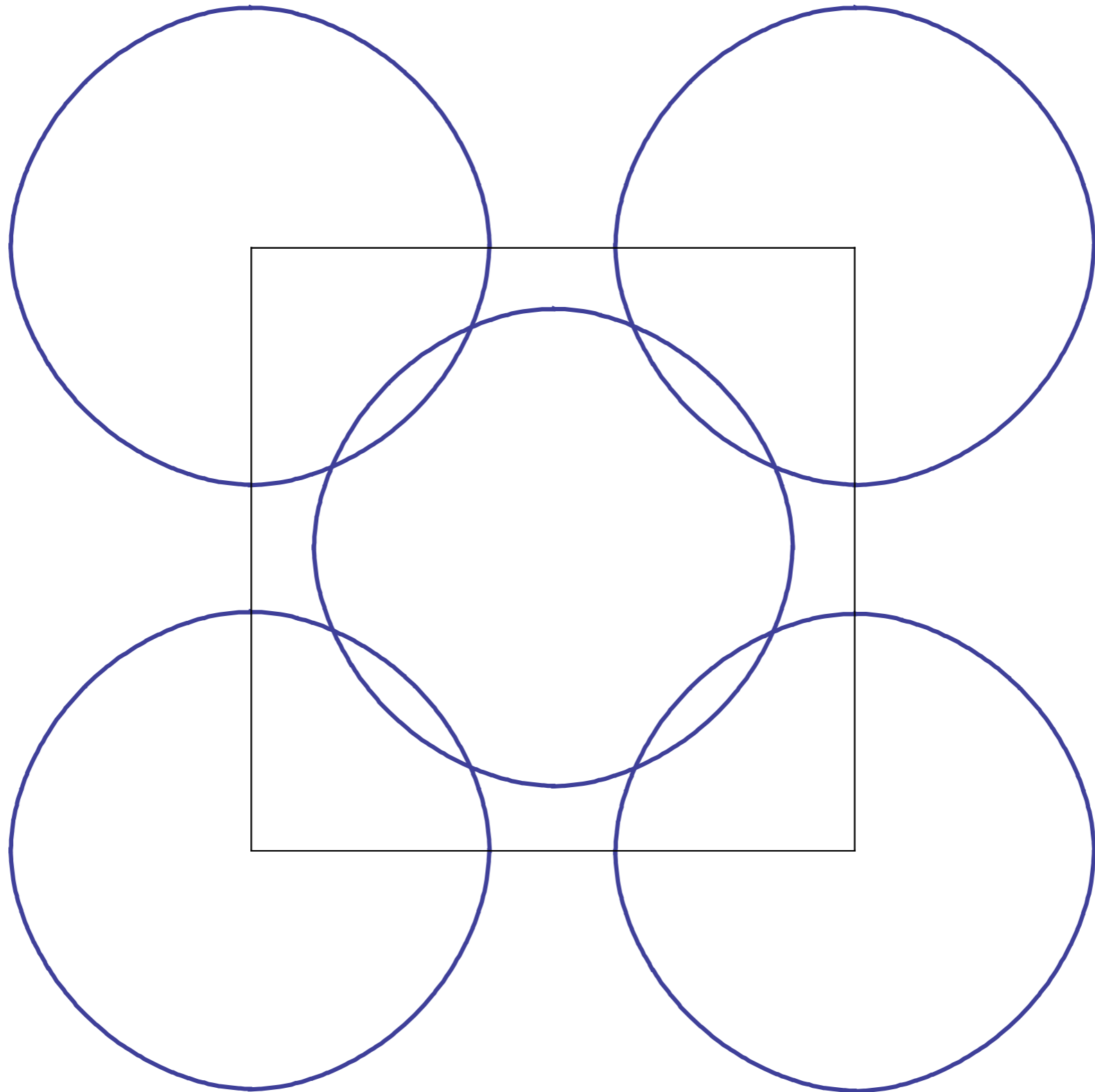
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”  
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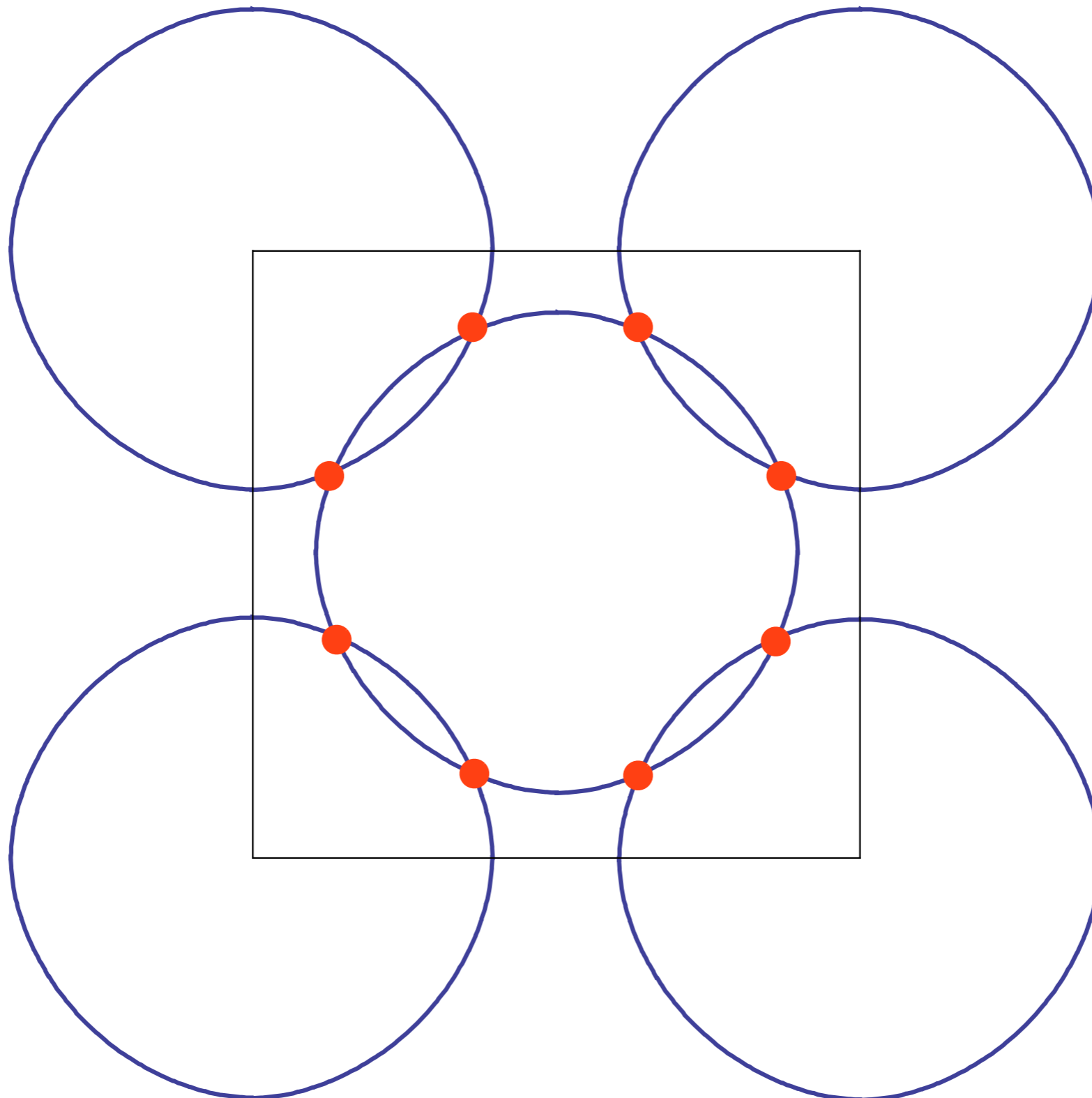
← Increasing interaction



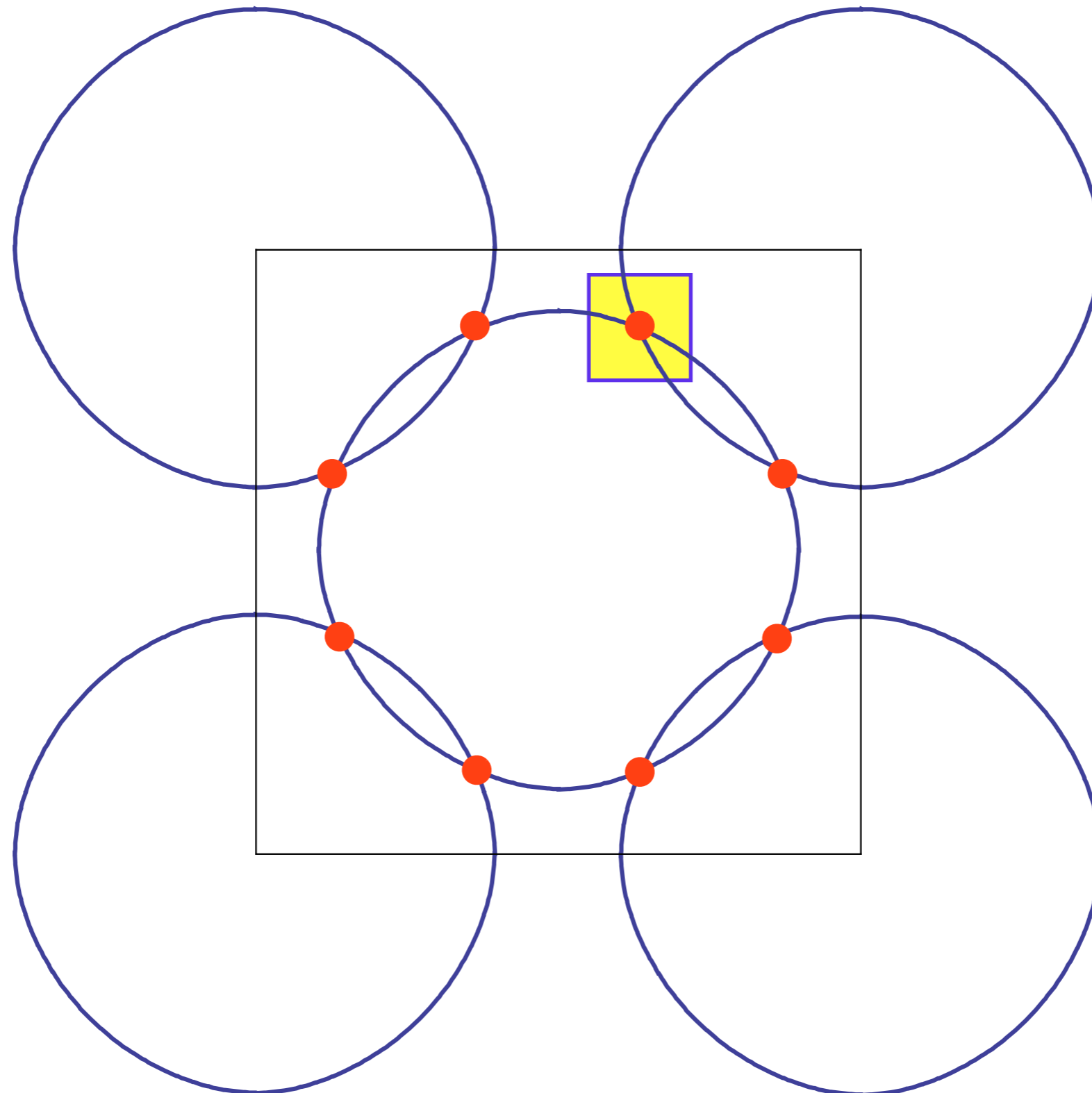
**Metal with “large” Fermi surface**



Fermi surfaces translated by  $\mathbf{K} = (\pi, \pi)$ .

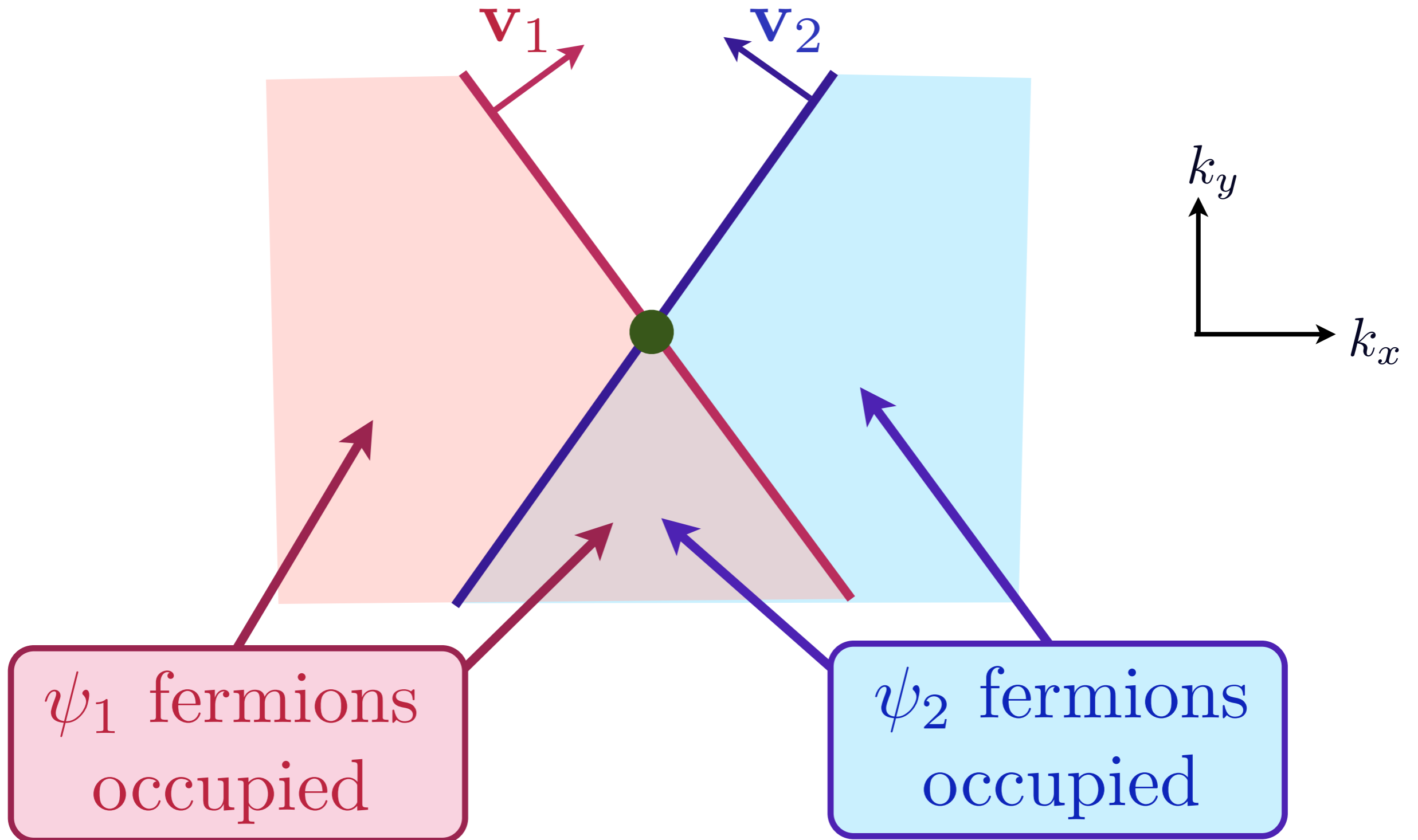


**“Hot” spots**



**Low energy theory for critical point near hot spots**

Hot-spot theory has fermions  $\psi_{1,2}$  (with Fermi velocities  $\mathbf{v}_{1,2}$ ) and boson order parameter  $\vec{\varphi}$ , and a “Yukawa” coupling  $\lambda$ . This theory is particle-hole symmetric.



$$\sigma = \sigma_Q + \frac{Q^2}{\mathcal{M}} \pi \delta(\omega)$$

There is a natural separation into the two contributions to transport:

- Particle-hole symmetric hot-spot theory yields the value of  $\sigma_Q$ .
- Remaining “cold” regions of the Fermi surface yield the contribution of the (nearly) conserved momentum mode.

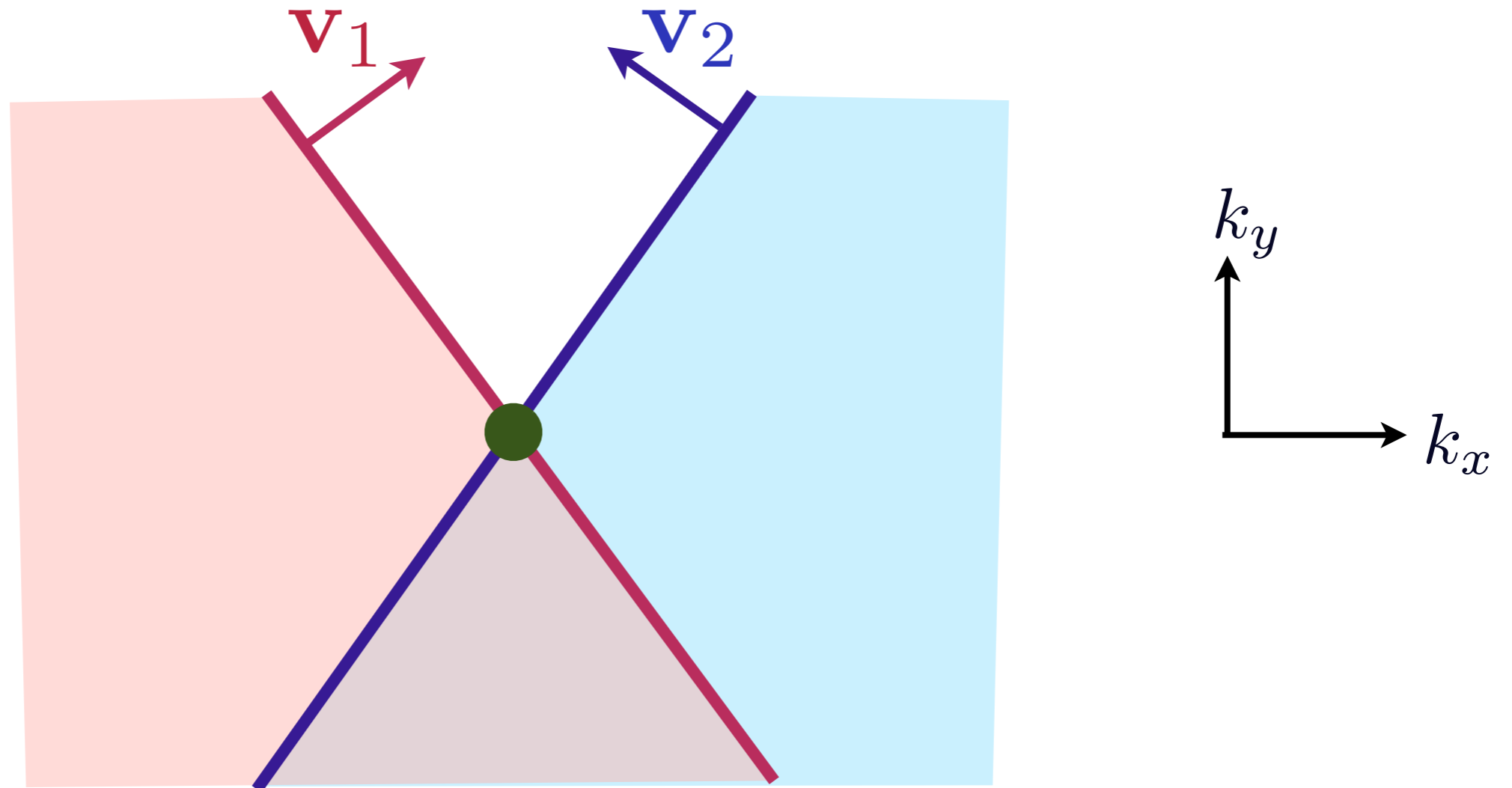
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But, all known particle-hole symmetric, strongly coupled critical theories obey hyperscaling, and so have  $\sigma_Q \sim T^0$  in  $d = 2$ .

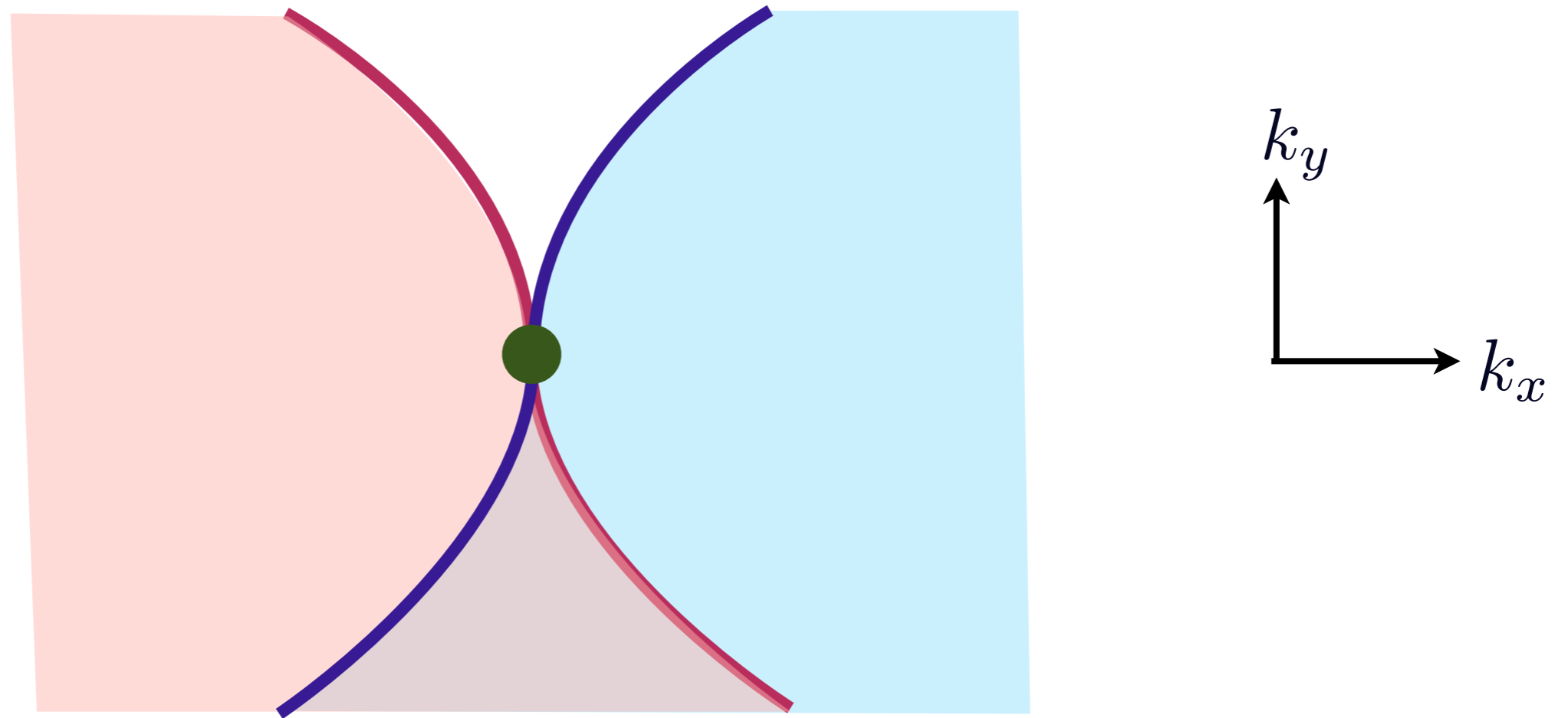
Hot-spot theory has fermions  $\psi_{1,2}$  (with Fermi velocities  $\mathbf{v}_{1,2}$ ) and boson order parameter  $\vec{\varphi}$ , and a “Yukawa” coupling  $\lambda$ .  
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This theory is particle-hole symmetric.

Renormalized Fermi surface has  $k_y \sim k_x \ln(1/k_x)$



A RG fixed point for the spin density wave critical point has recently been found by Shouvik Sur and Sung-Sik Lee (PRB **91**, 125136 (2015)) using a novel  $\epsilon$ -expansion.

- We find that the presence of gapless lines of zero energy excitations at this fixed point leads to hyperscaling violation only in the limit of vanishing boson velocity.
- Upto logarithmic corrections, the hyperscaling violation leads to the entropy density  $S \sim T^{(2-\theta)/z}$ , and  $\sigma_Q \sim T^{-\theta/z}$  with  $\theta = 1$ , and  $z = 1 + \mathcal{O}(\epsilon)$ .

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