

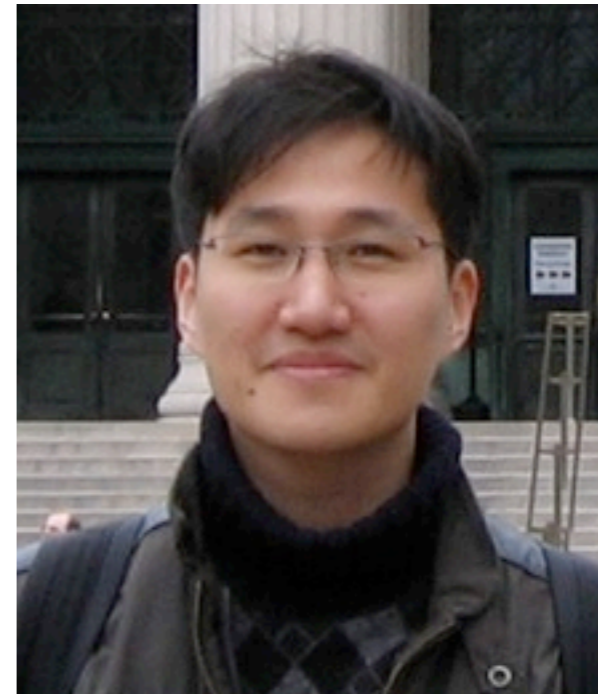
The phase diagrams of the high temperature superconductors

Talk online: sachdev.physics.harvard.edu





Max Metlitski, Harvard



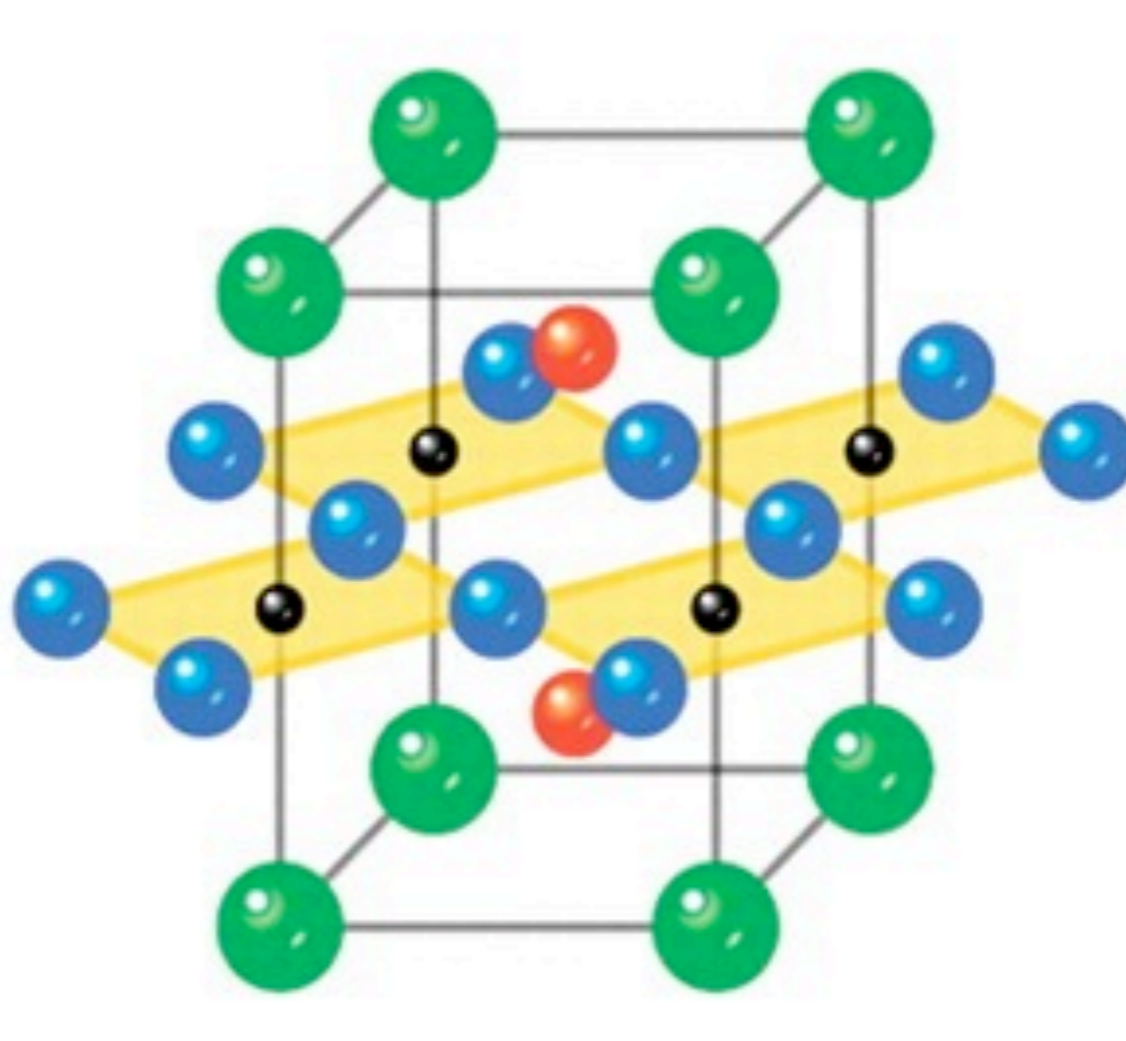
Eun Gook Moon, Harvard



The cuprate superconductors

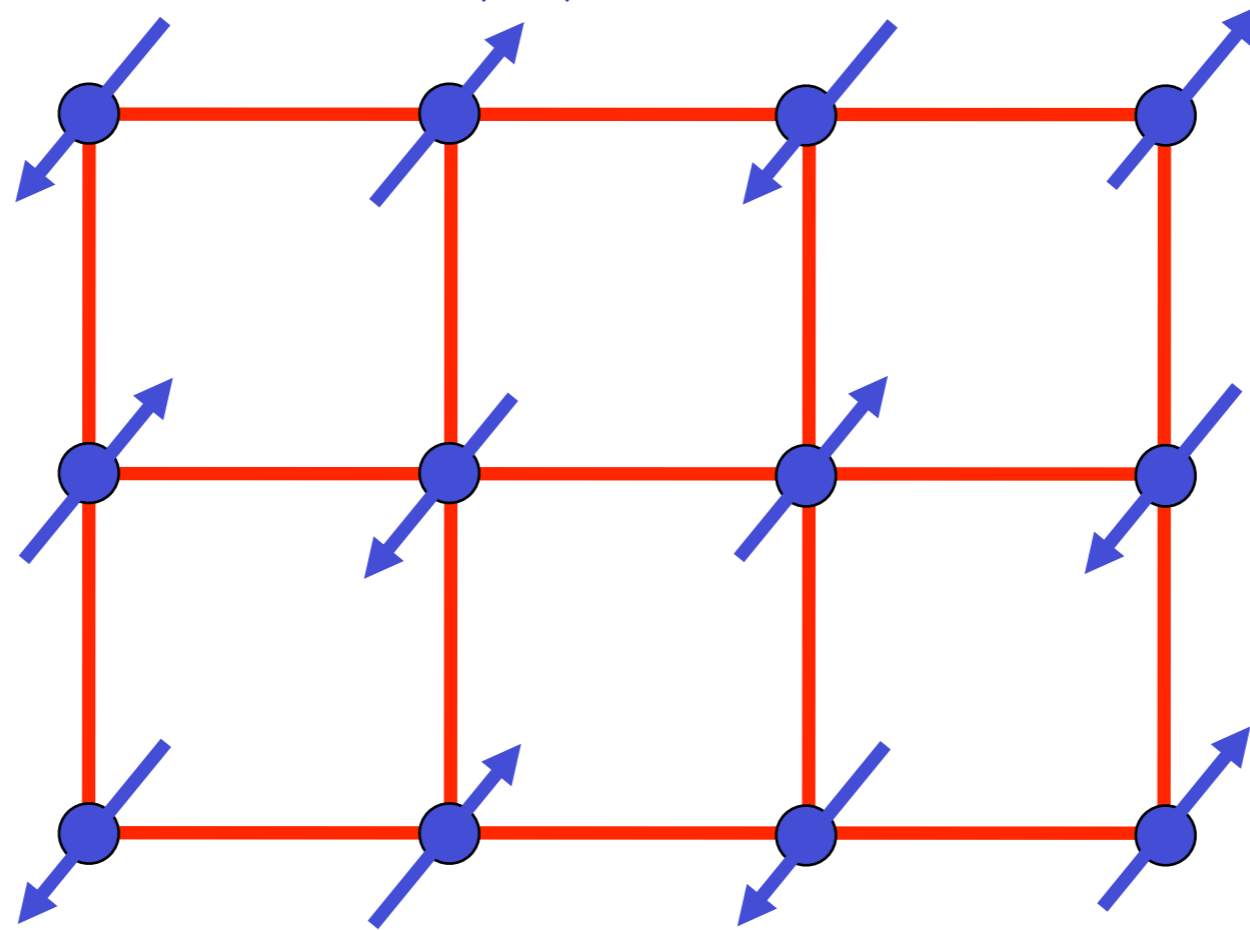
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

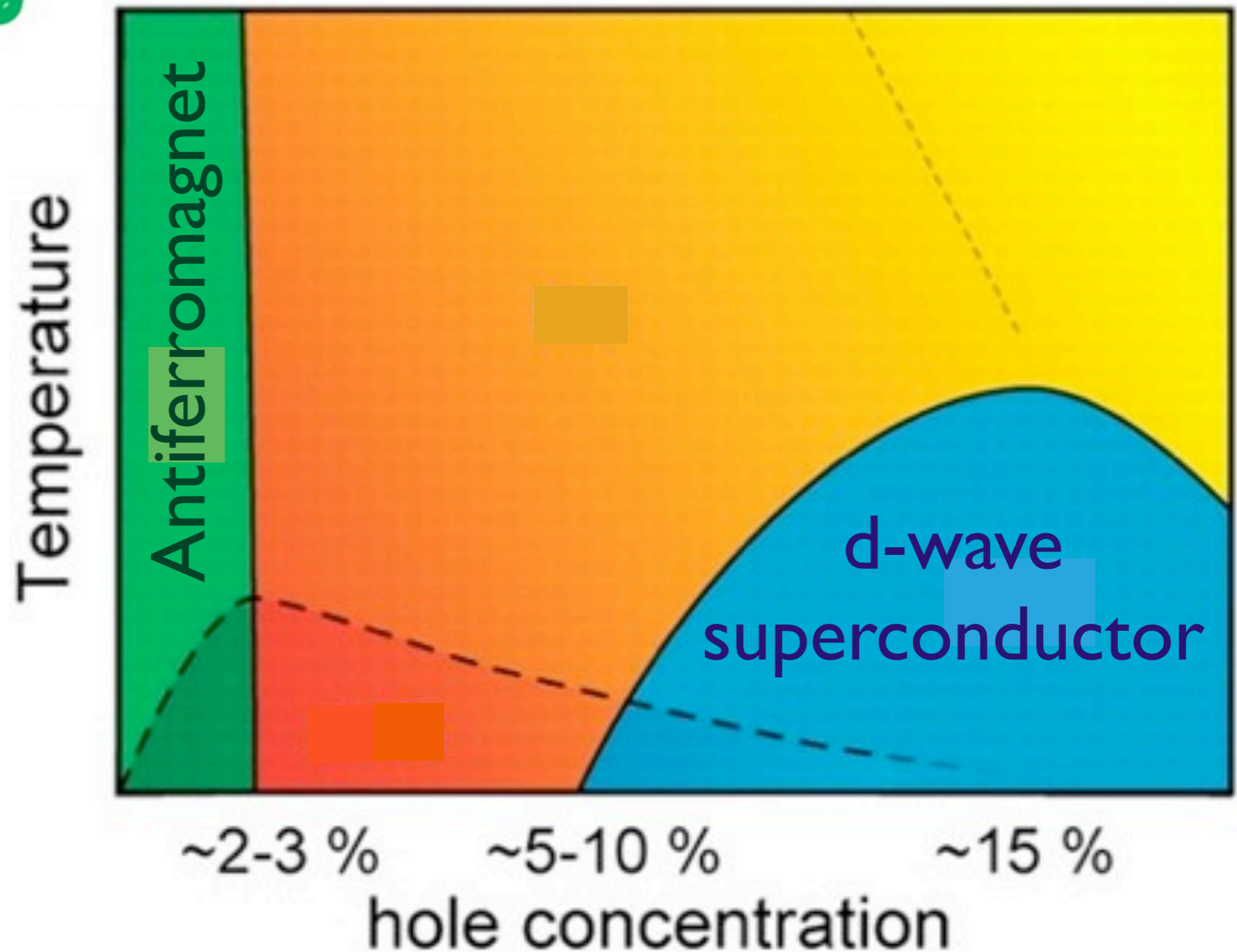
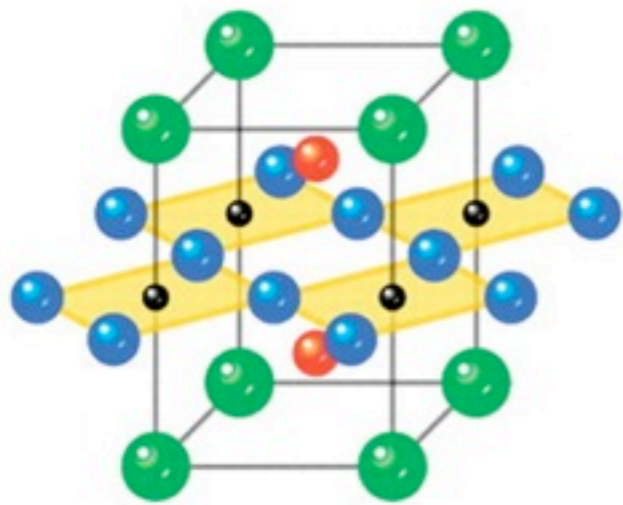
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

The cuprate superconductors

Na-CCOC

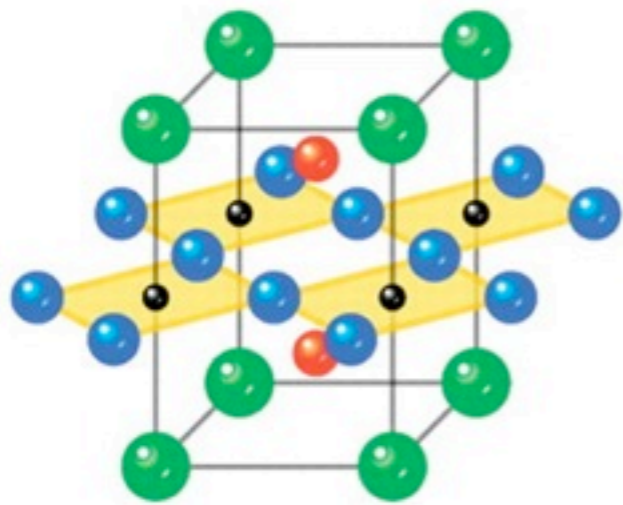
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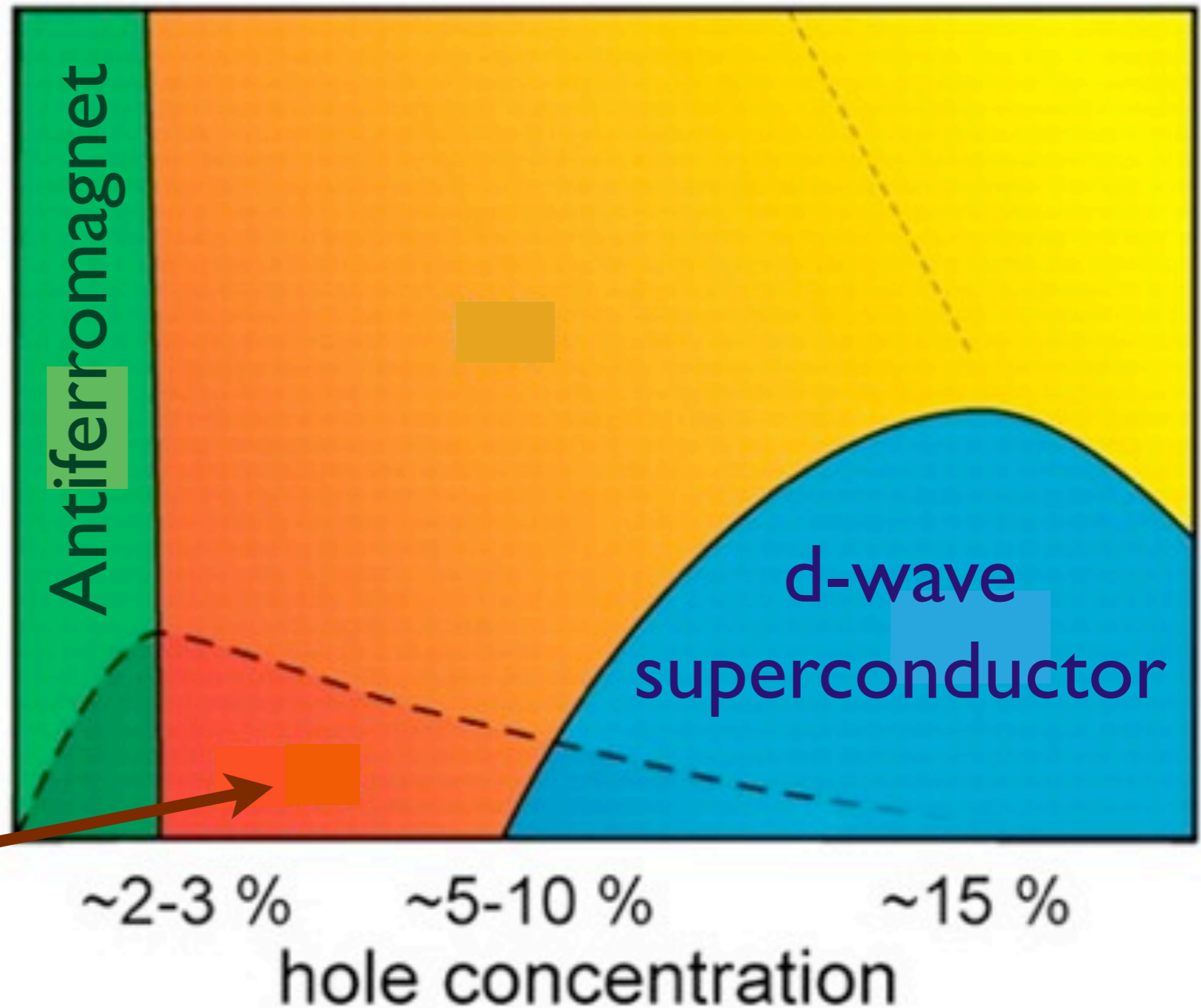
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Temperature

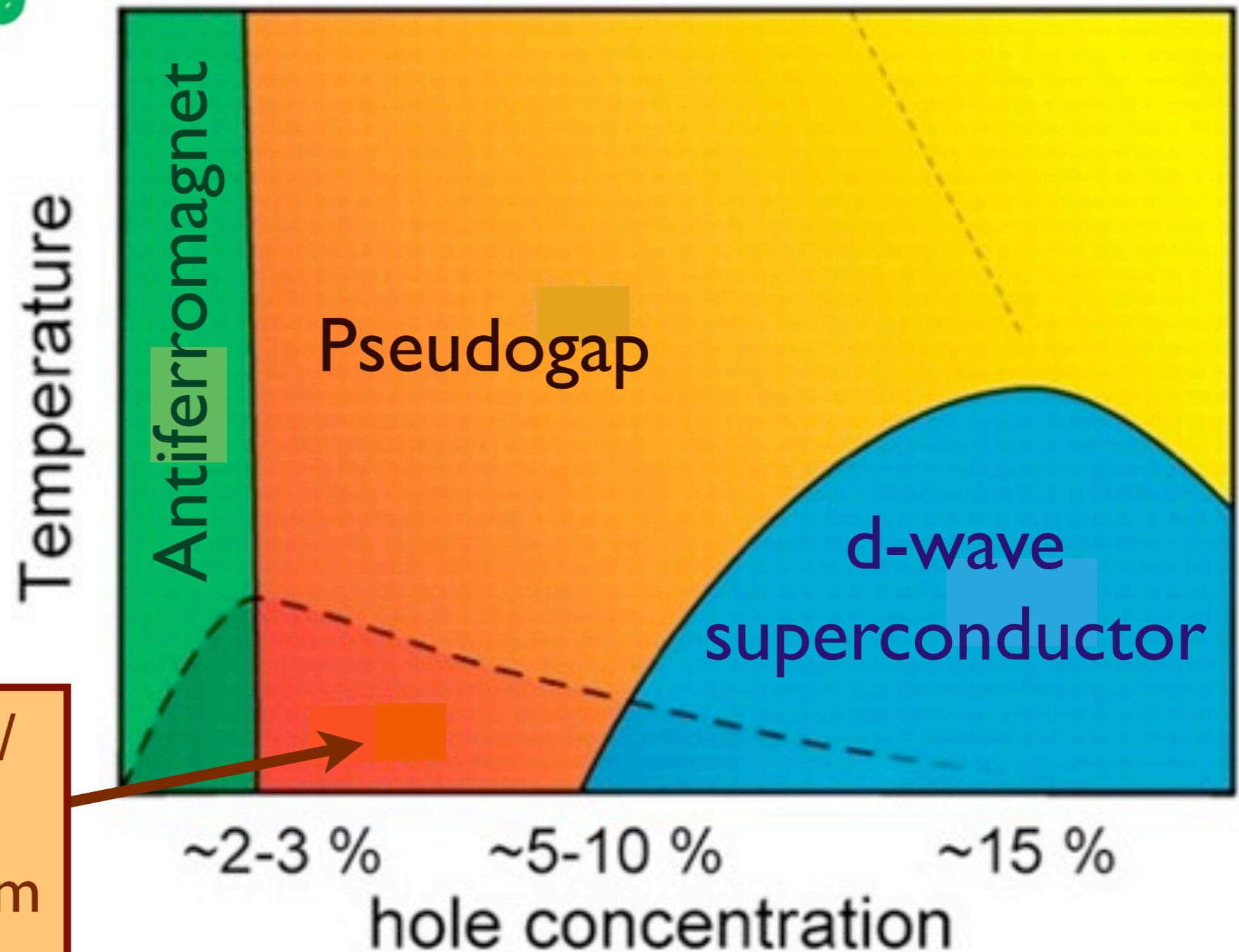
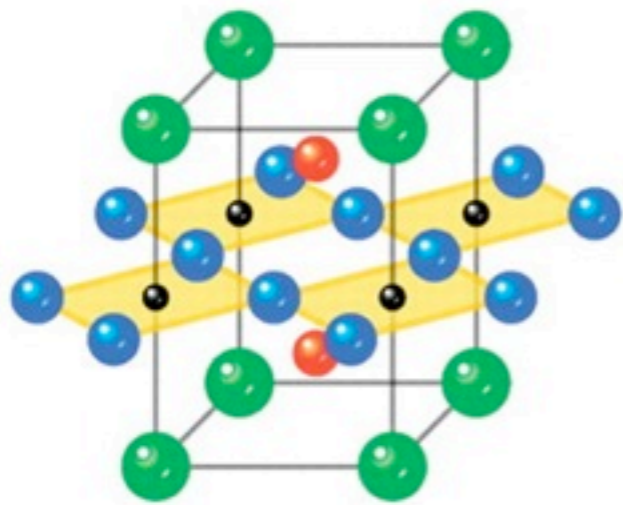


Incommensurate/
disordered
antiferromagnetism
and charge order

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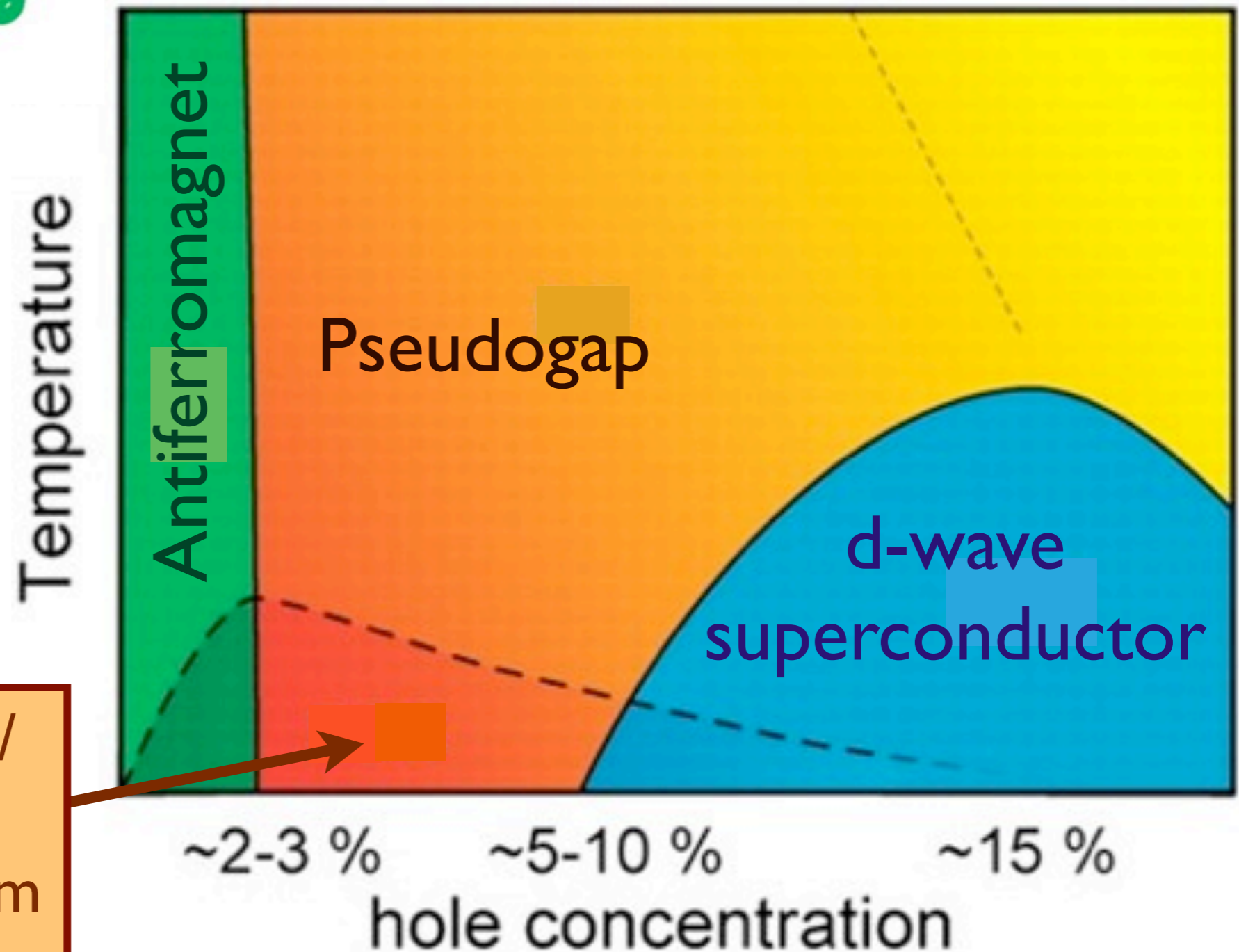
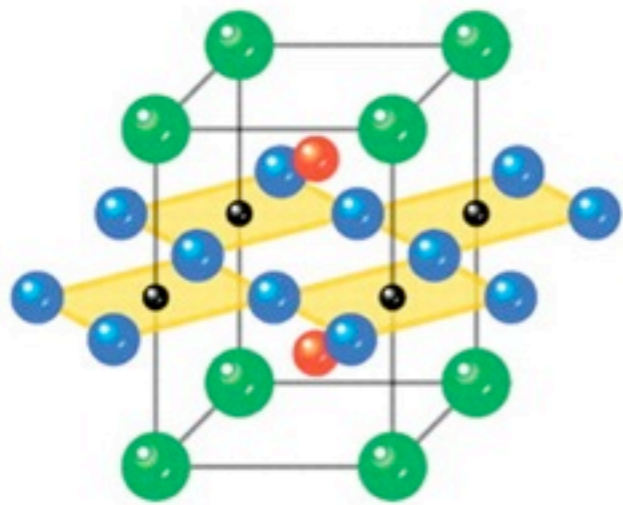


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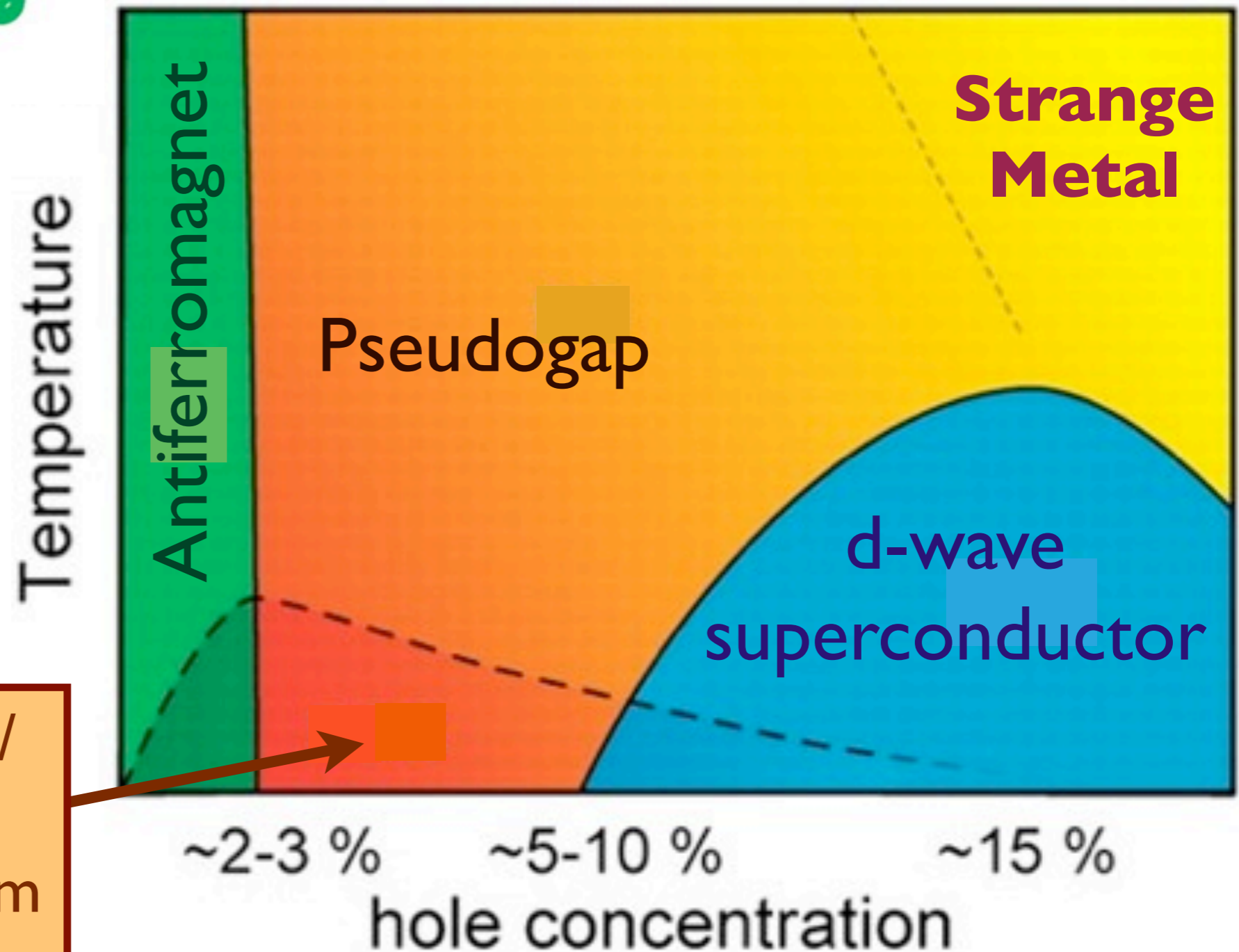
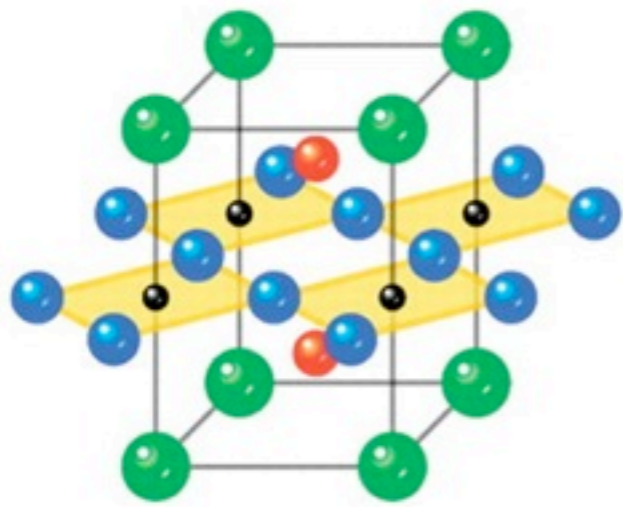


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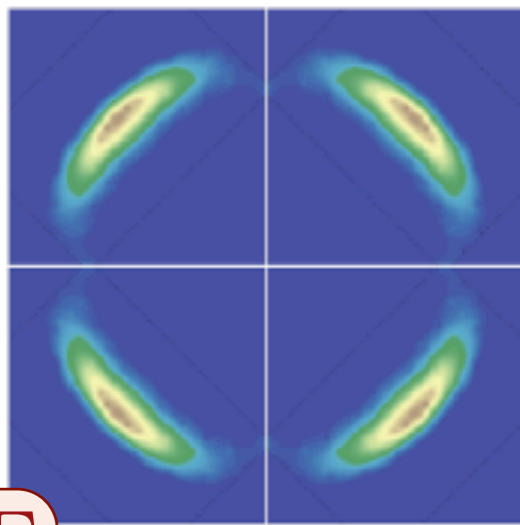
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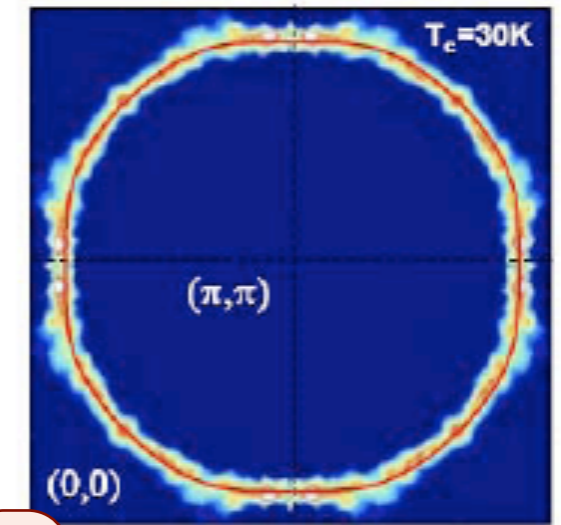
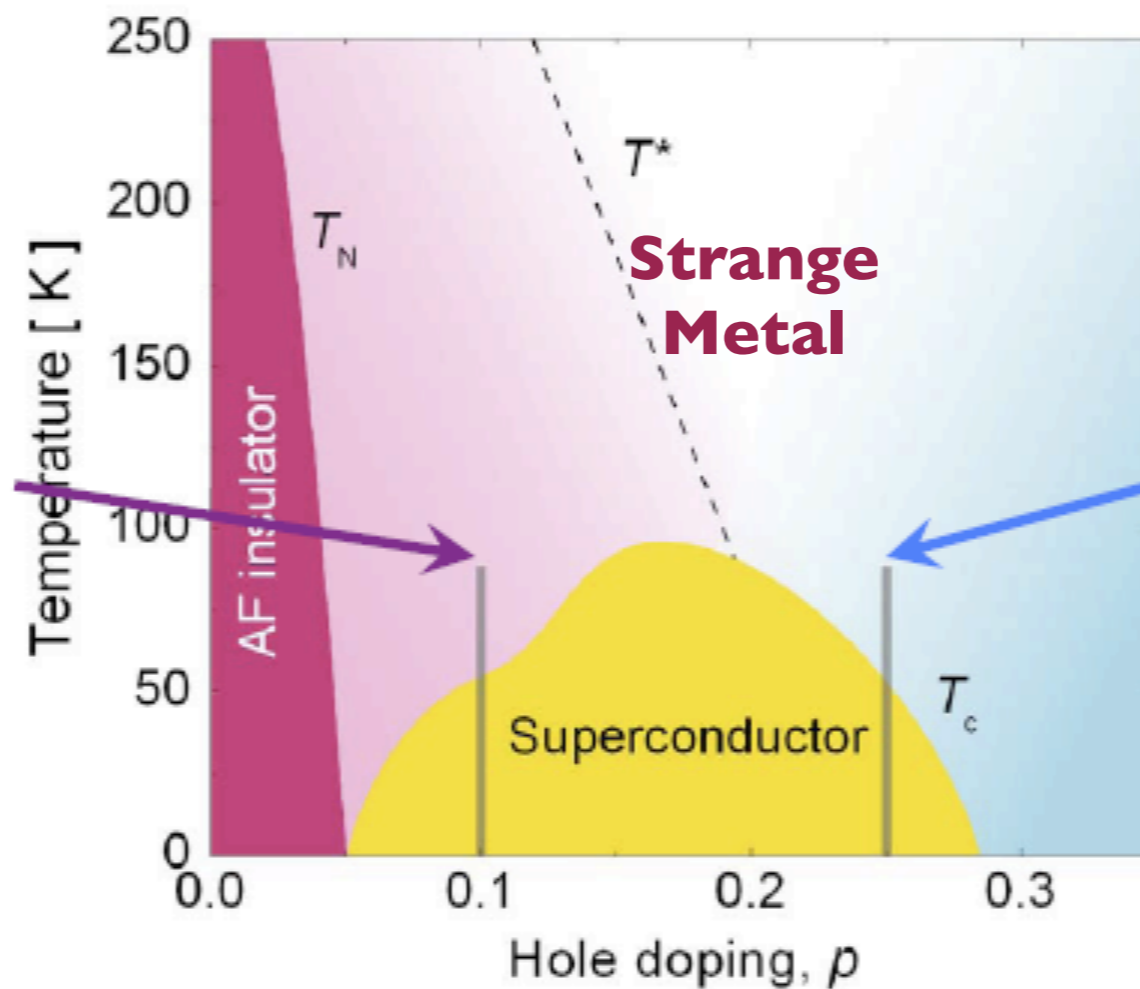


Incommensurate/
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Γ

K.M. Shen et al., Science 2005



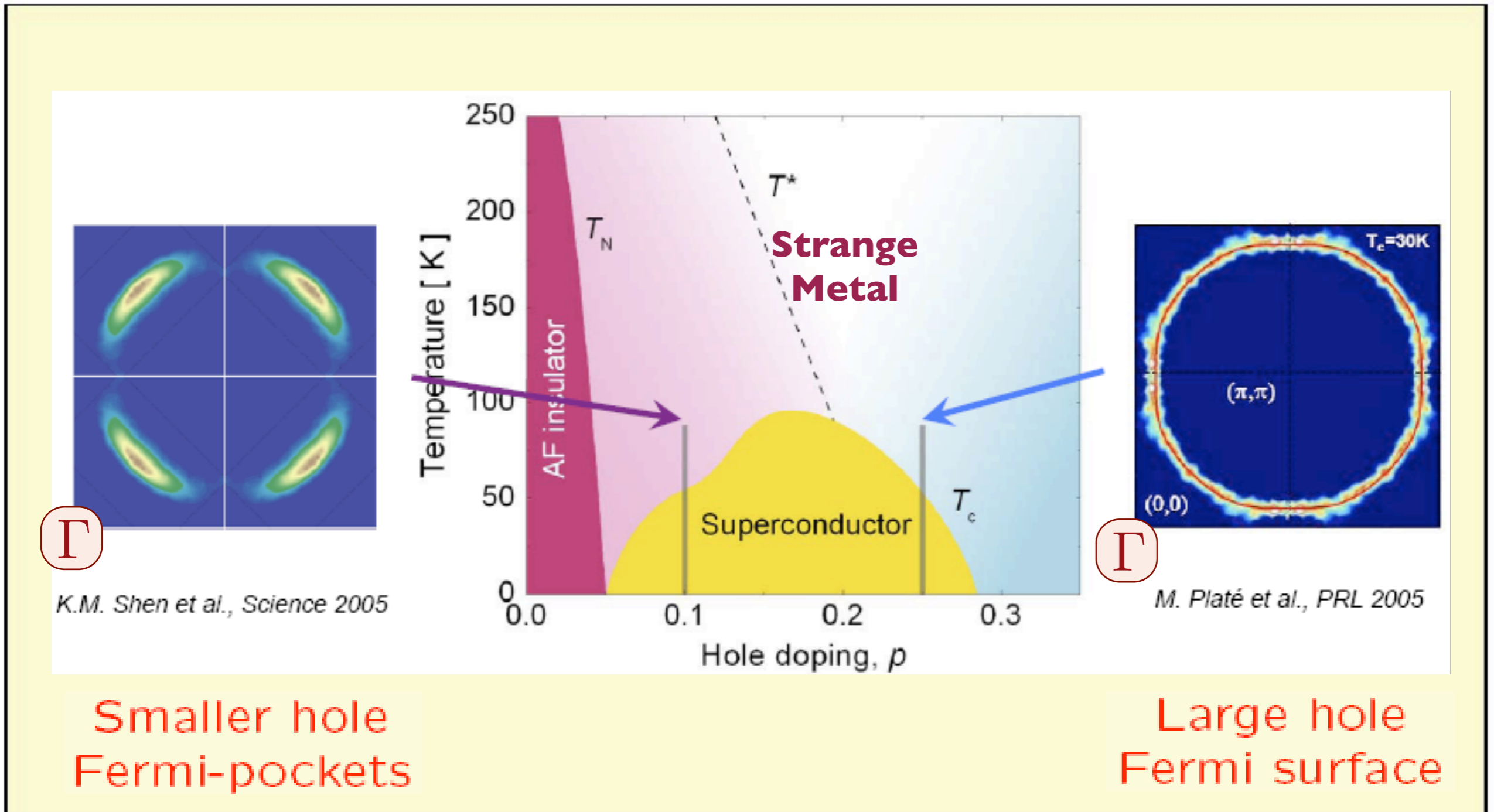
Γ

M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

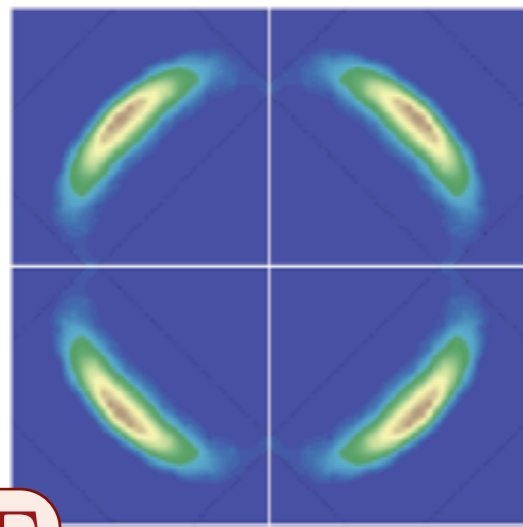
Large hole
Fermi surface

Key Ingredients:



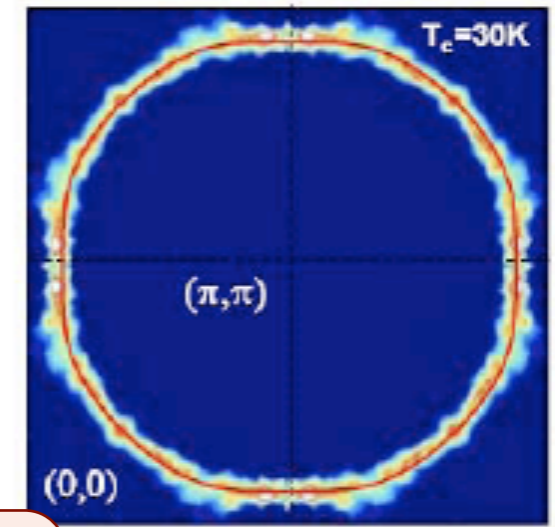
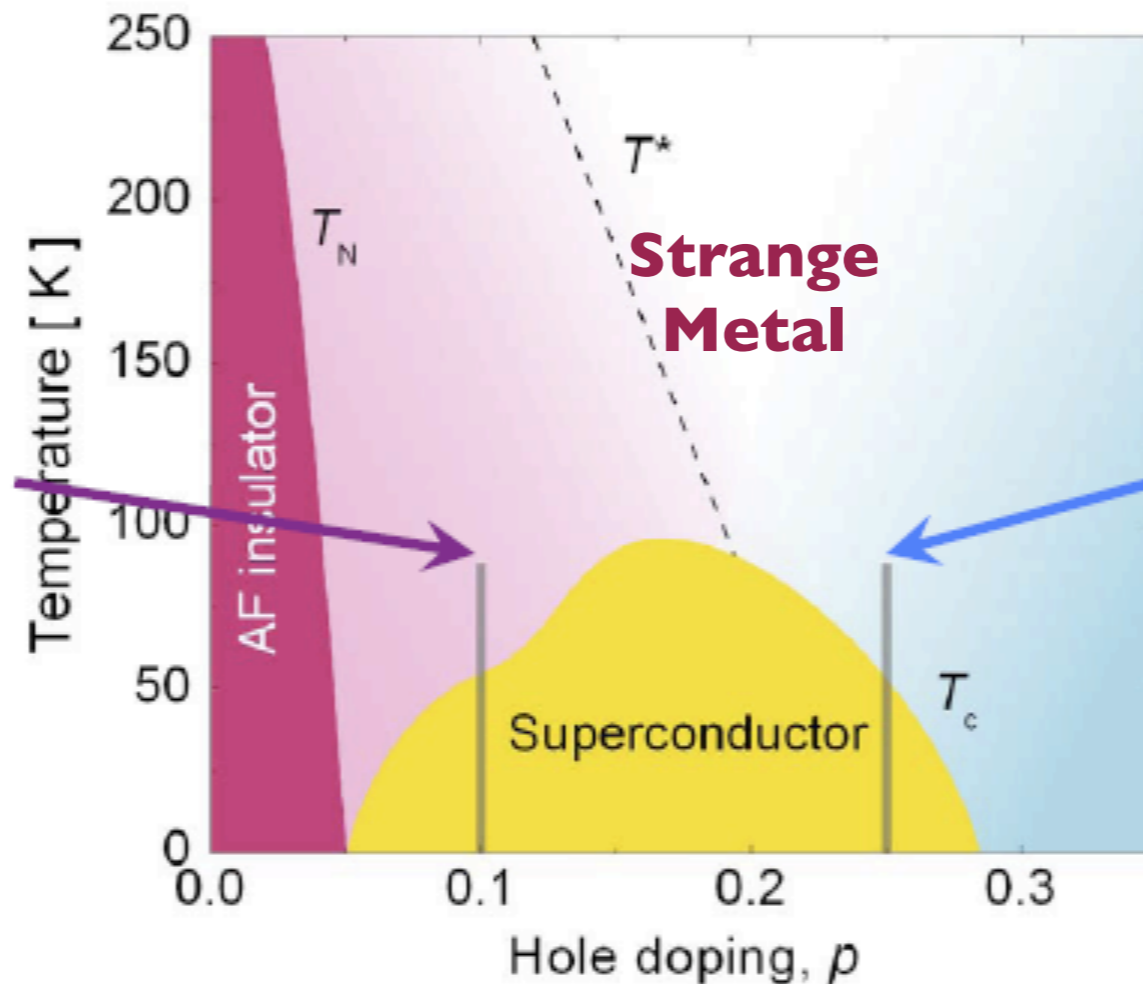
Key Ingredients:

Antiferromagnetism (AF)
Spin density wave (SDW)



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K.M. Shen et al., Science 2005



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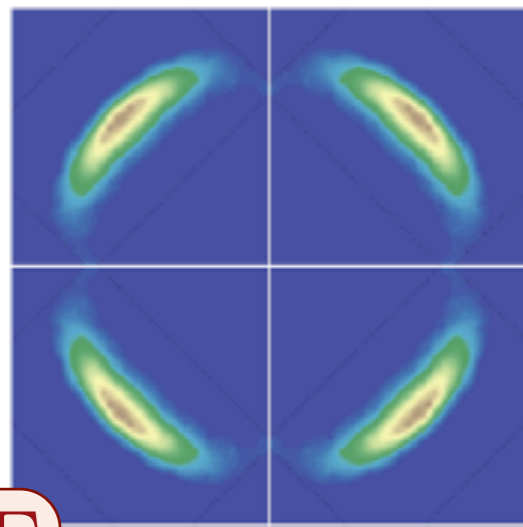
Smaller hole
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Key Ingredients:

d-wave superconductivity

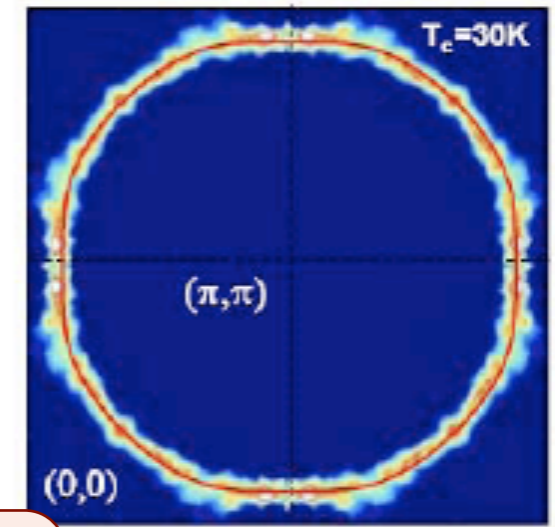
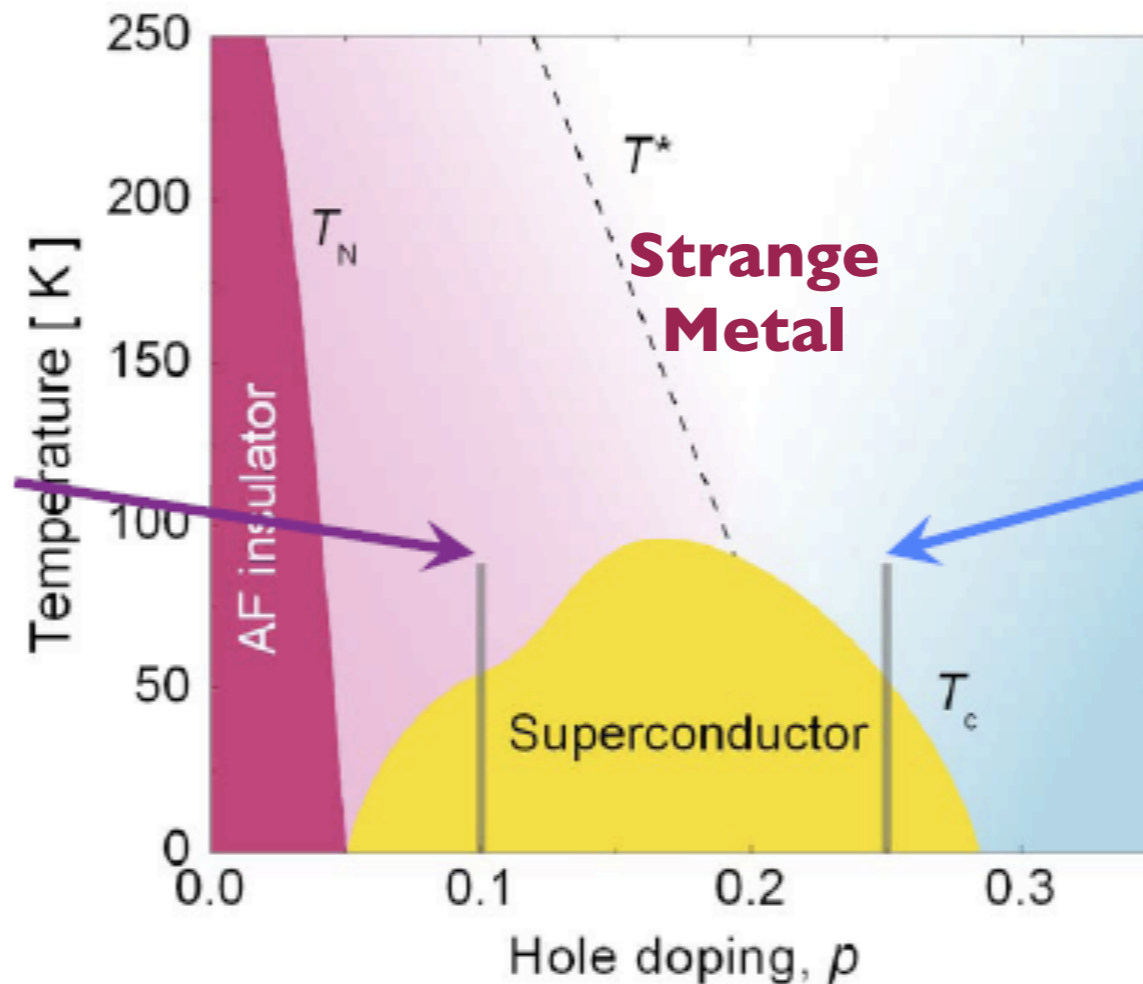
Antiferromagnetism (AF)
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Smaller hole
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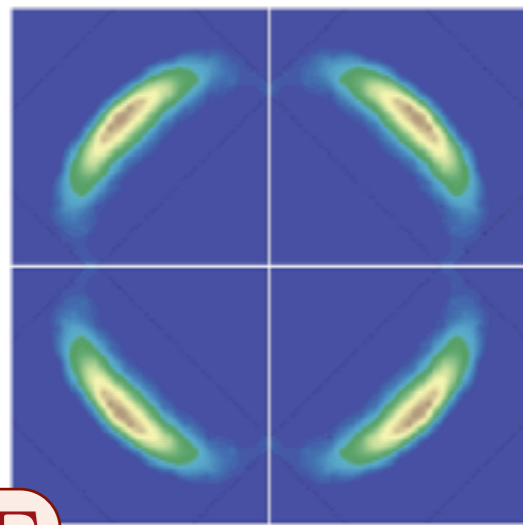
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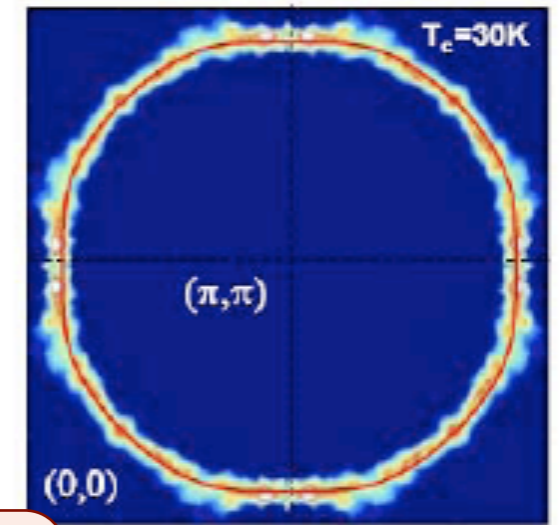
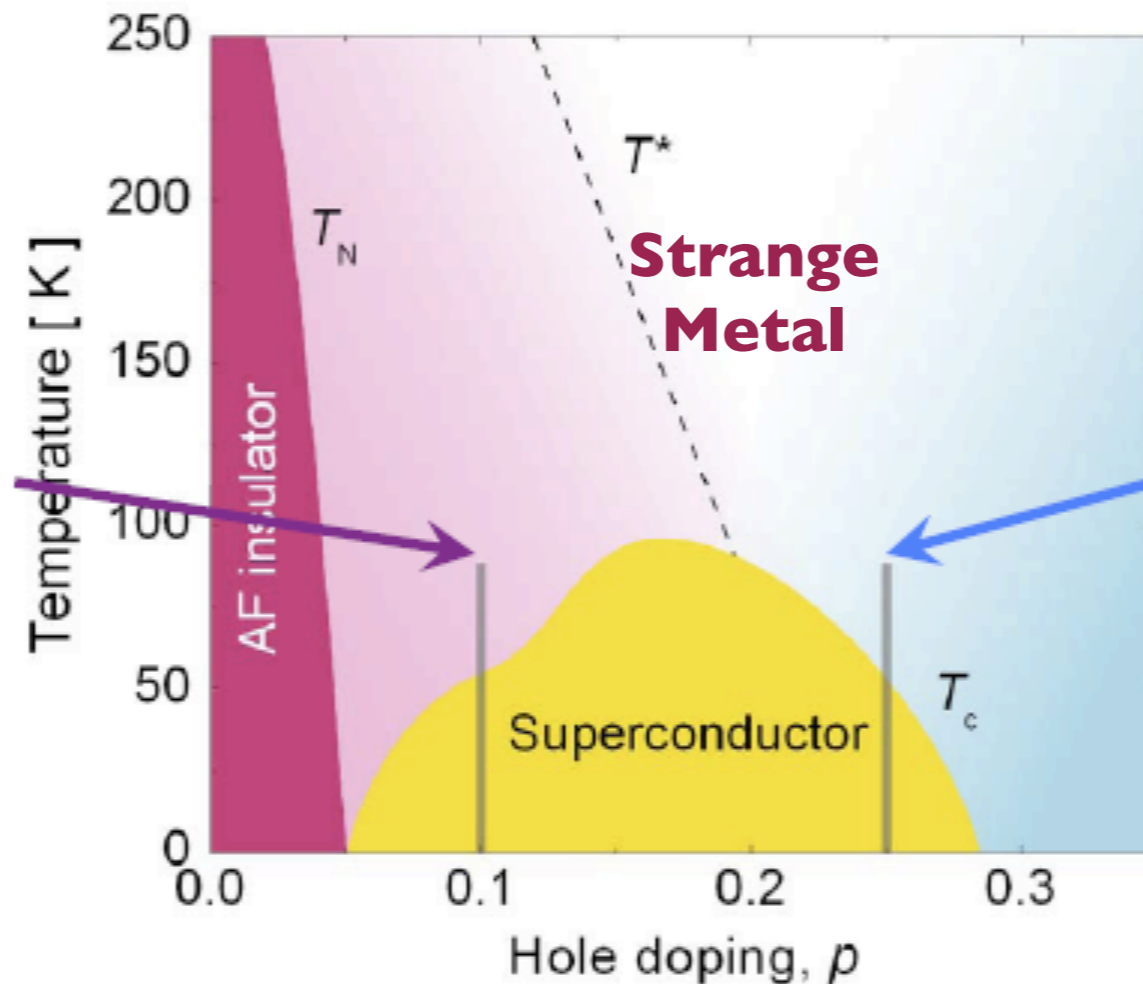
Antiferromagnetism (AF)
Spin density wave (SDW)

Fermi surface change



Γ

K.M. Shen et al., Science 2005

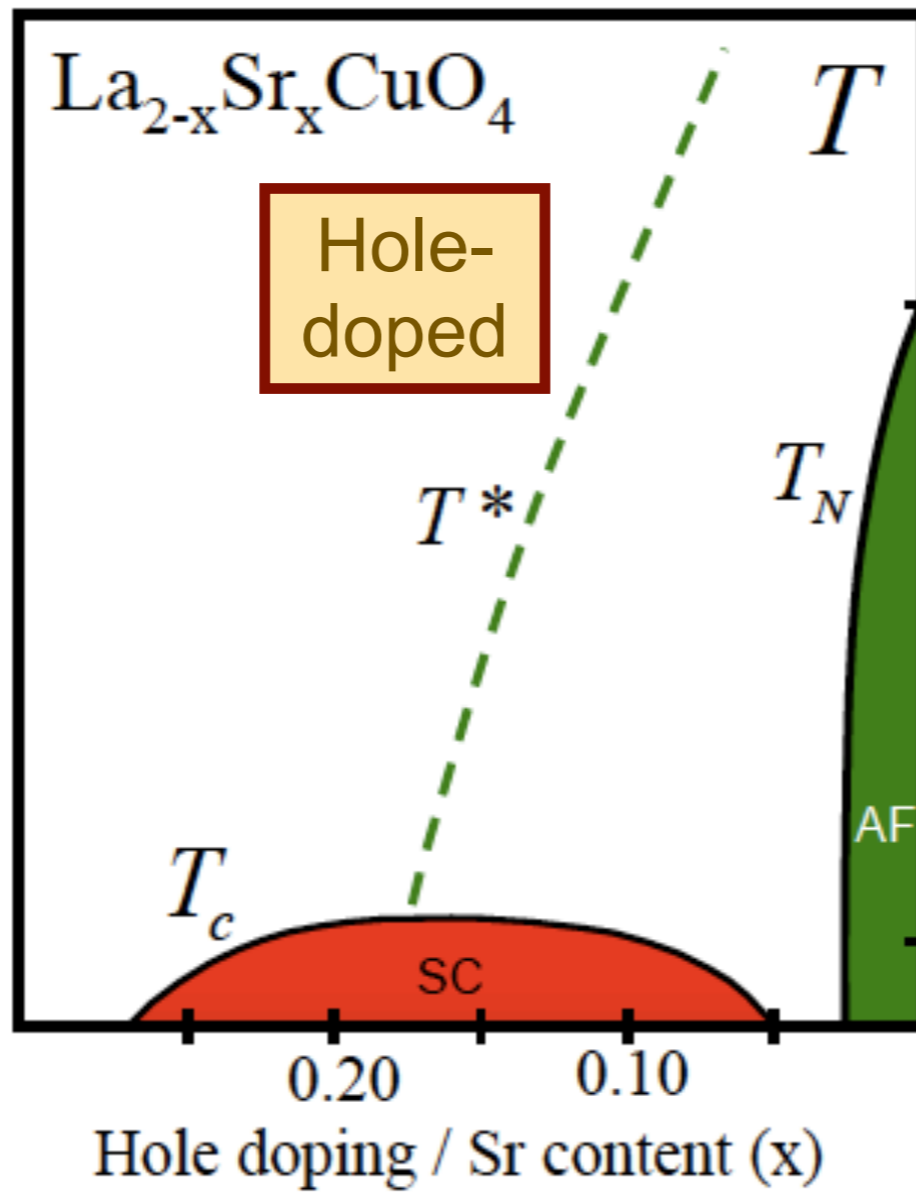


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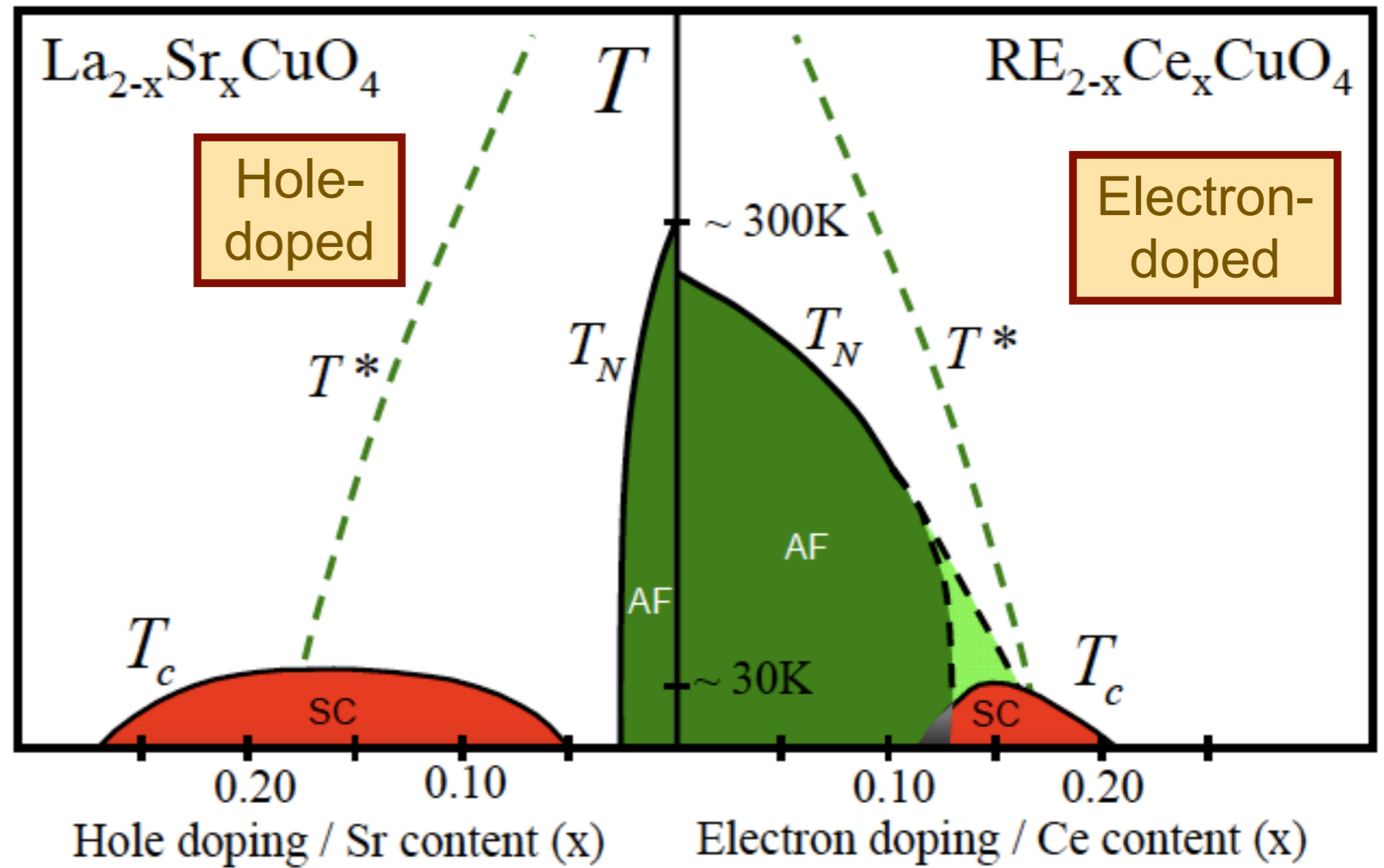
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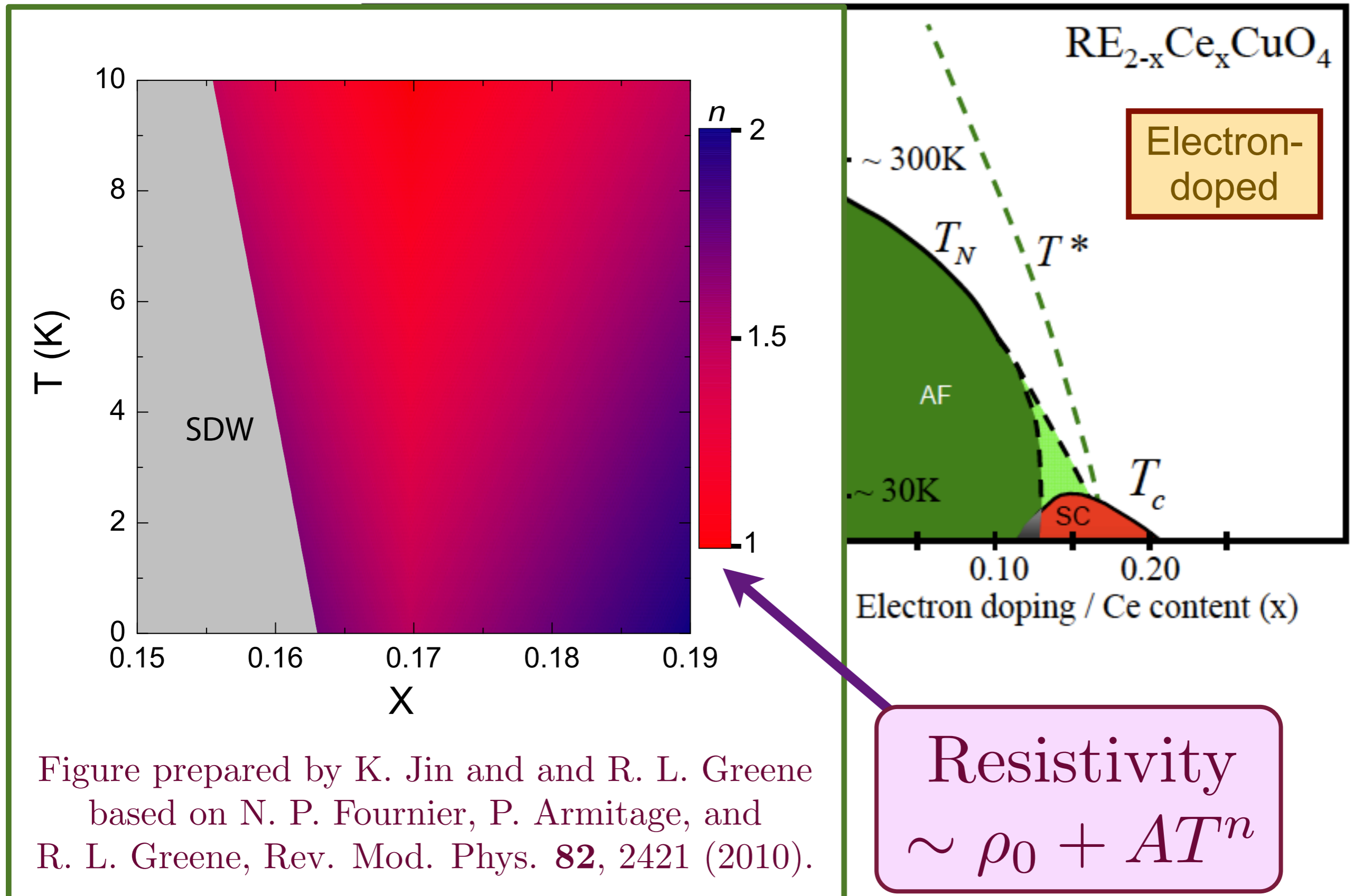
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Electron-doped cuprate superconductors



Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

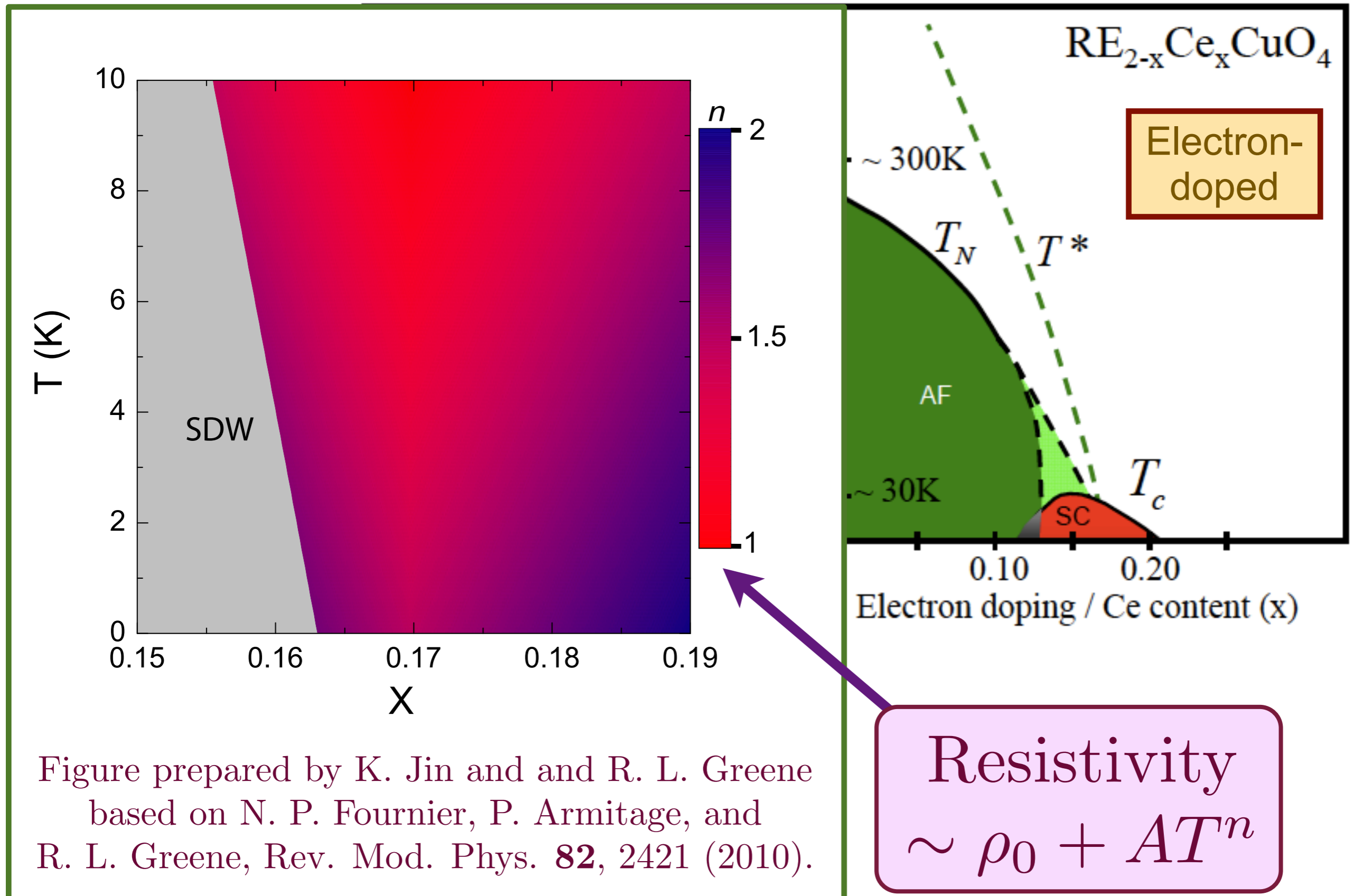
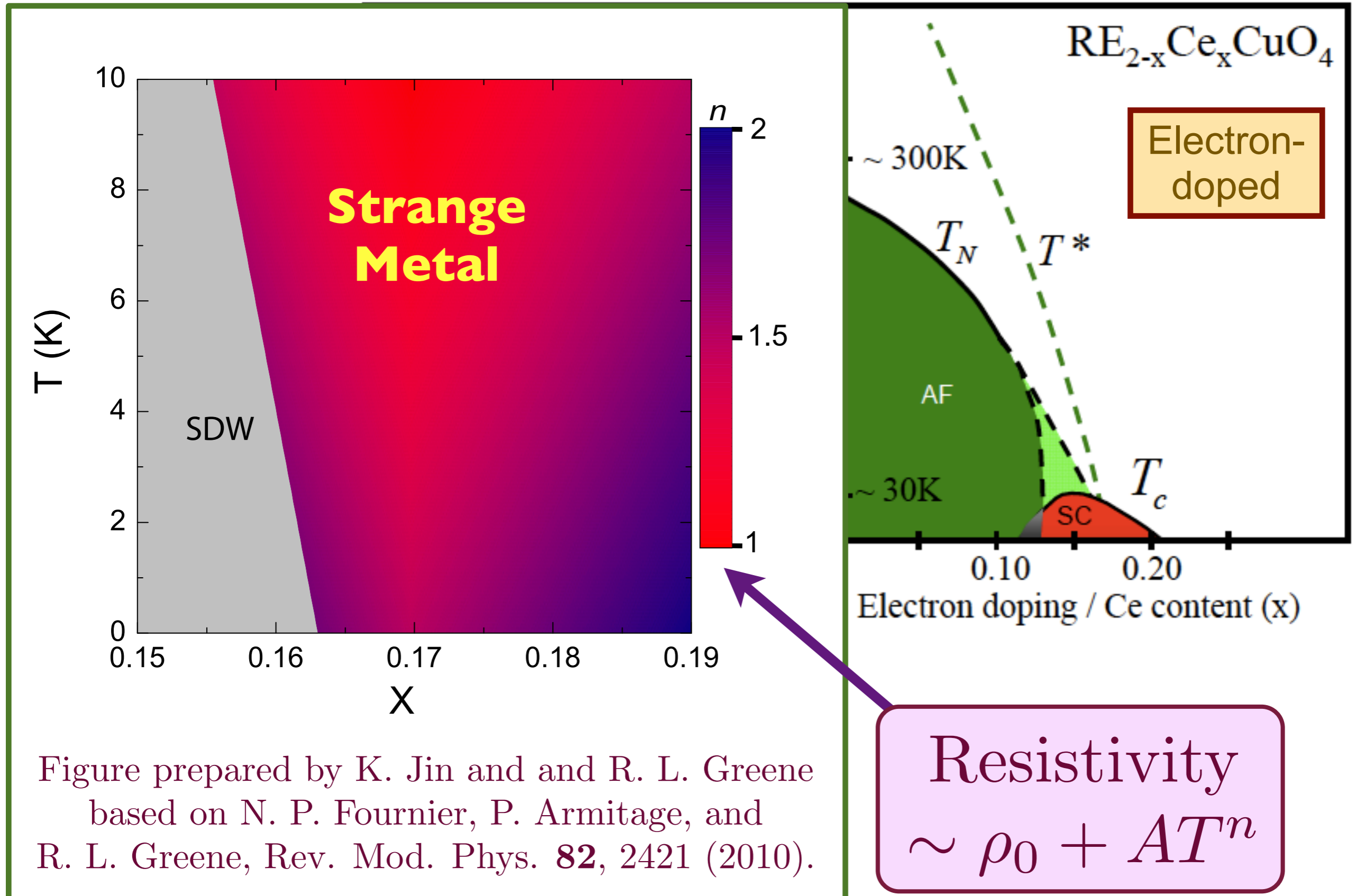


Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

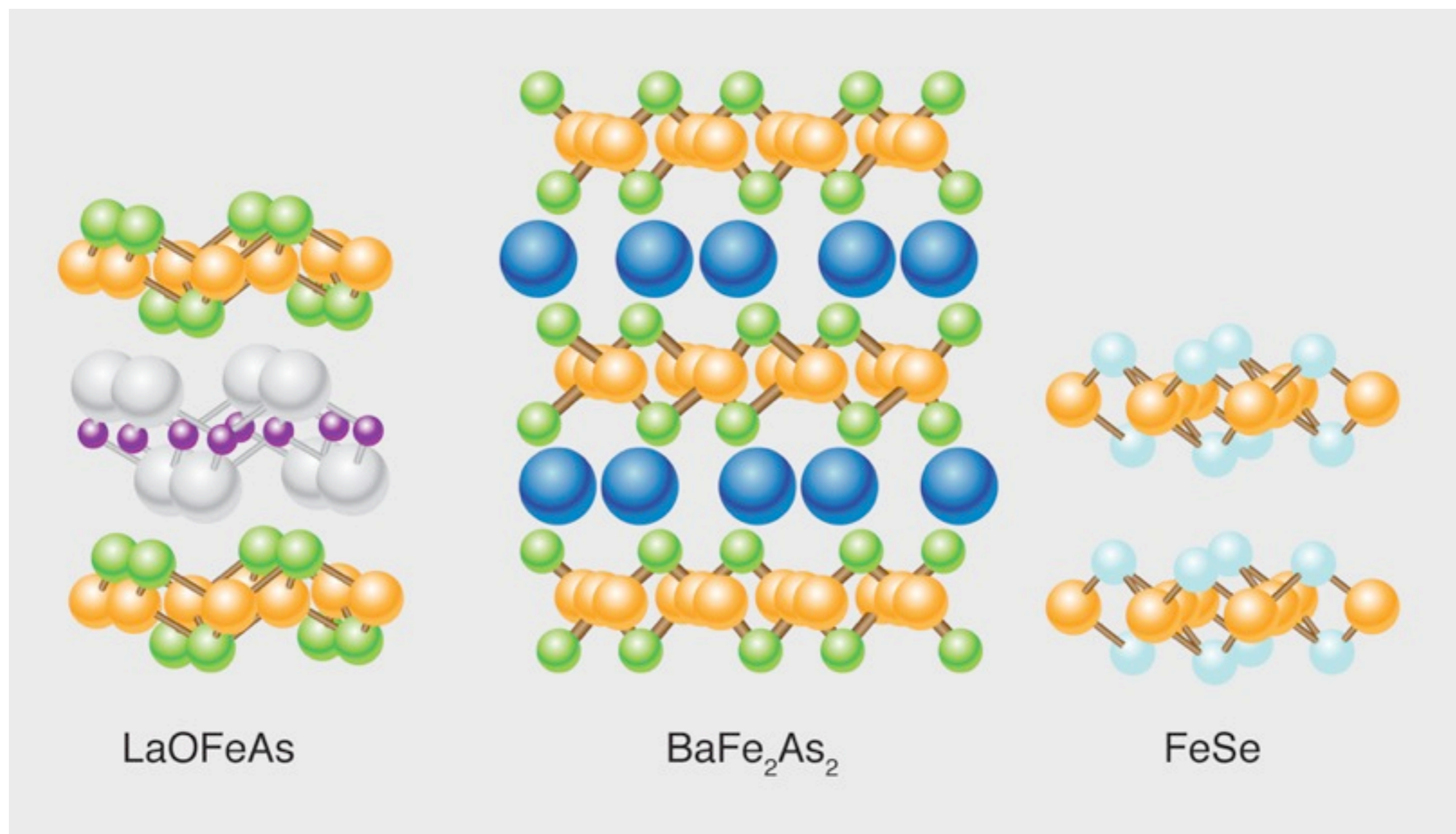
Resistivity
 $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors



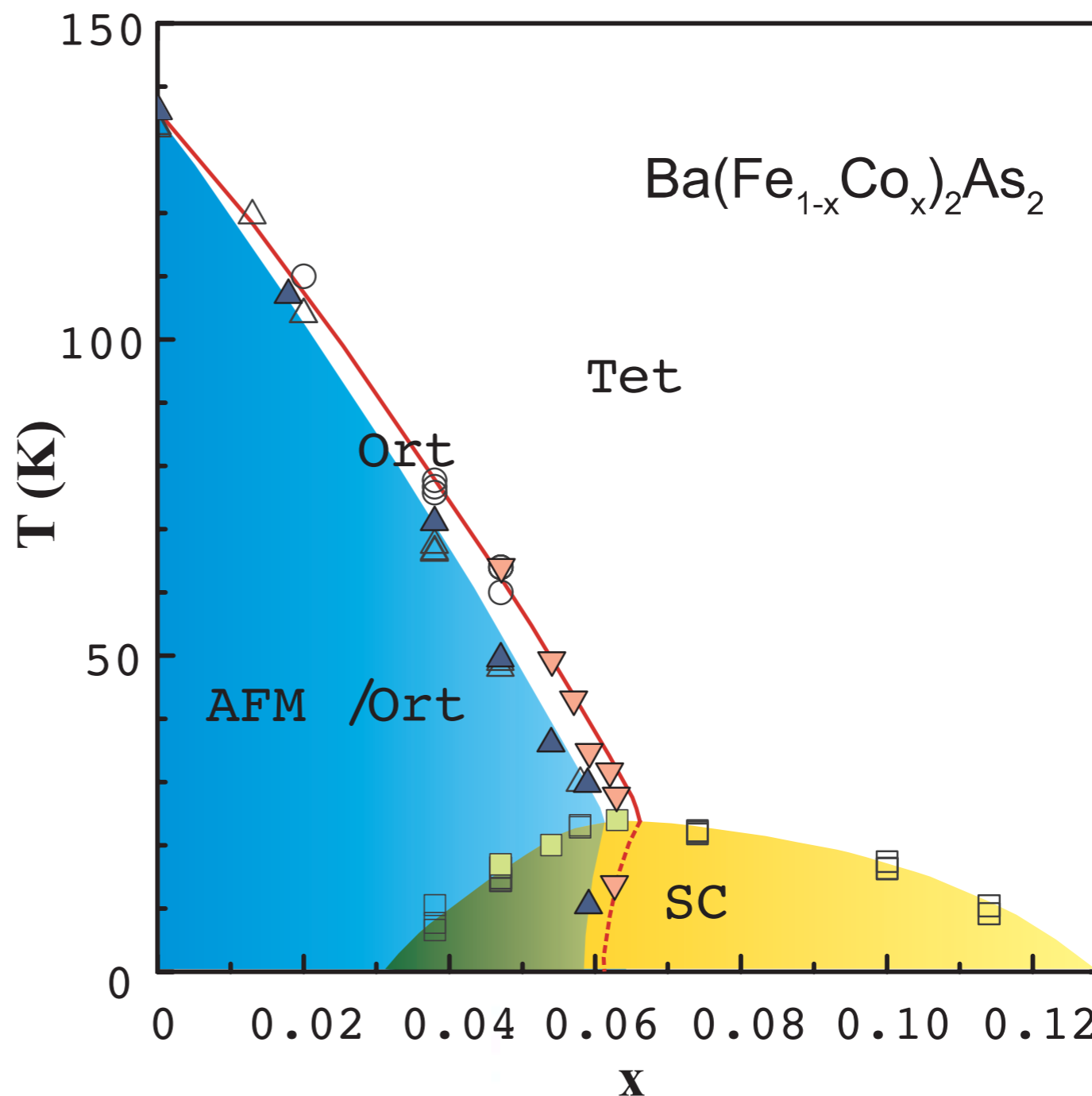
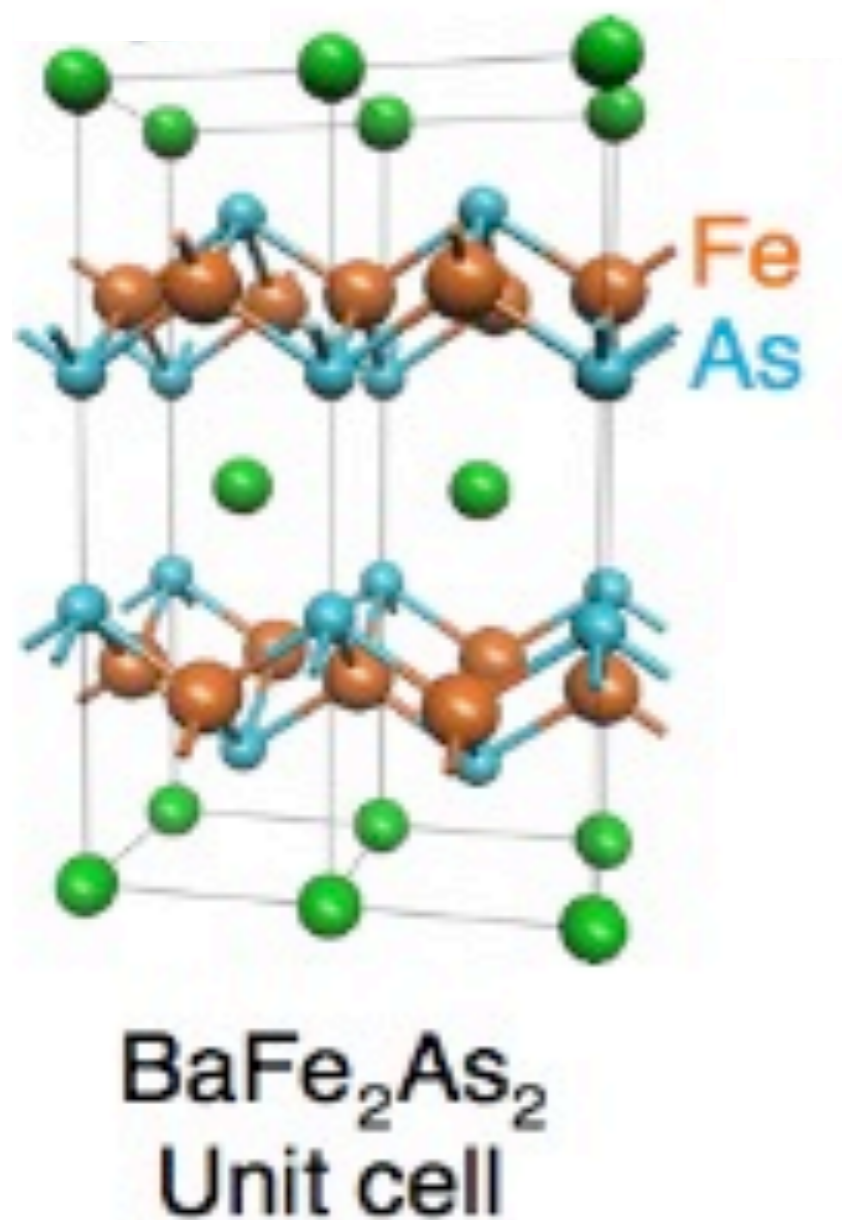
Iron pnictides:

a new class of high temperature superconductors



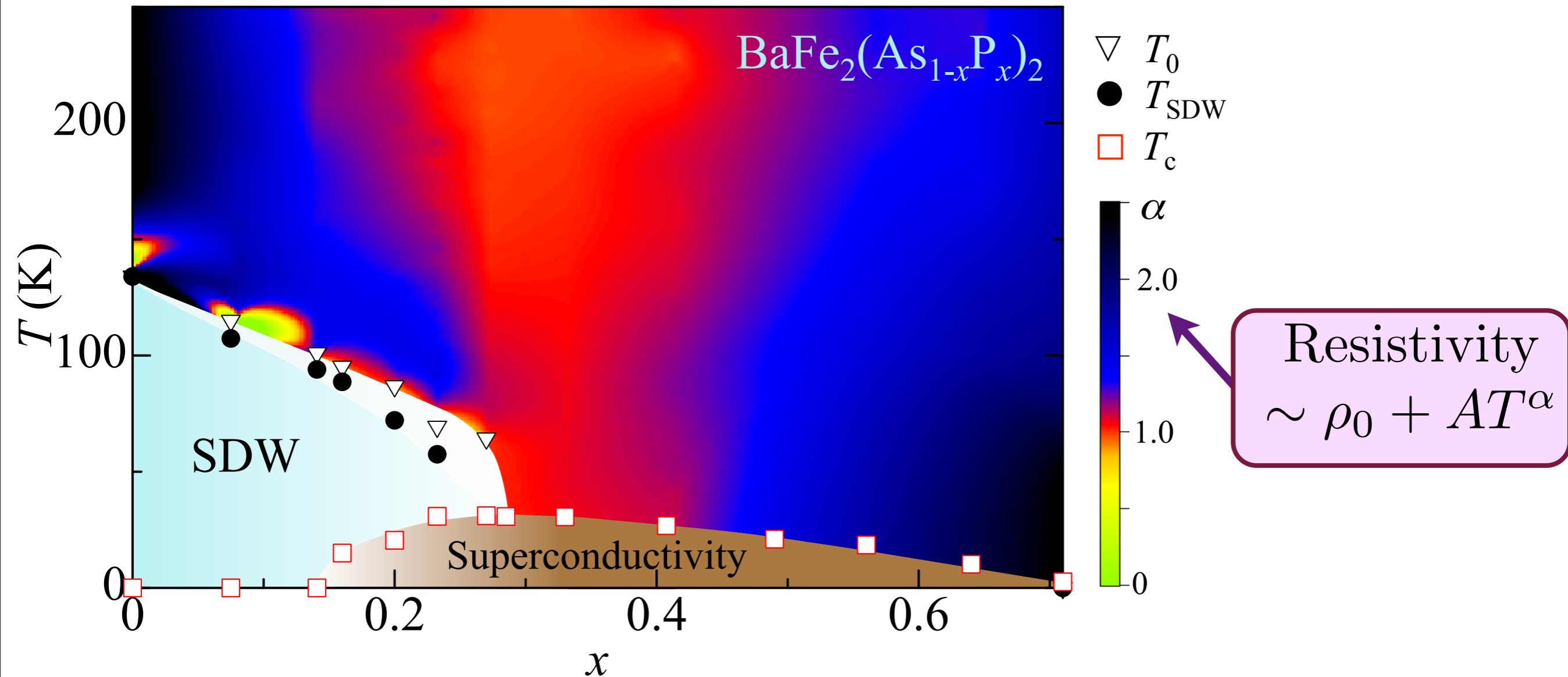
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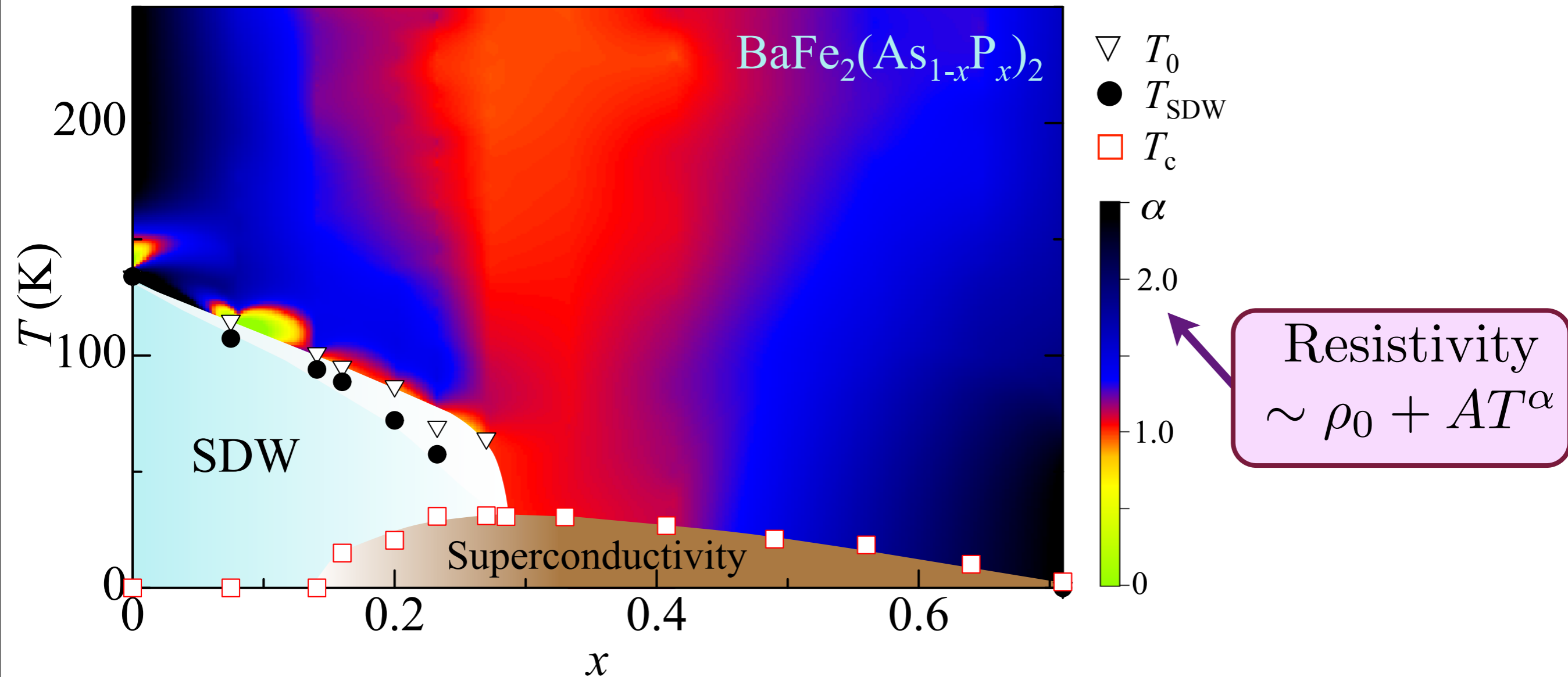
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



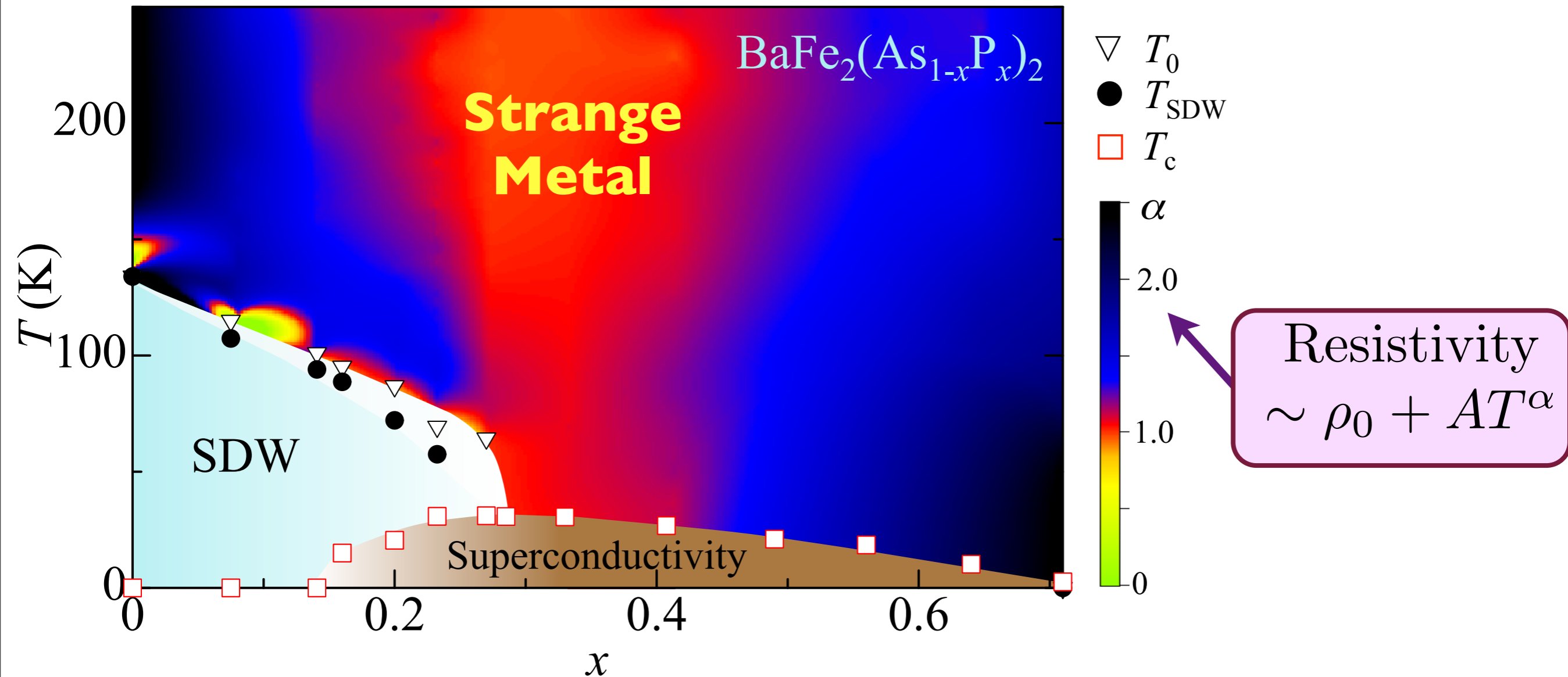
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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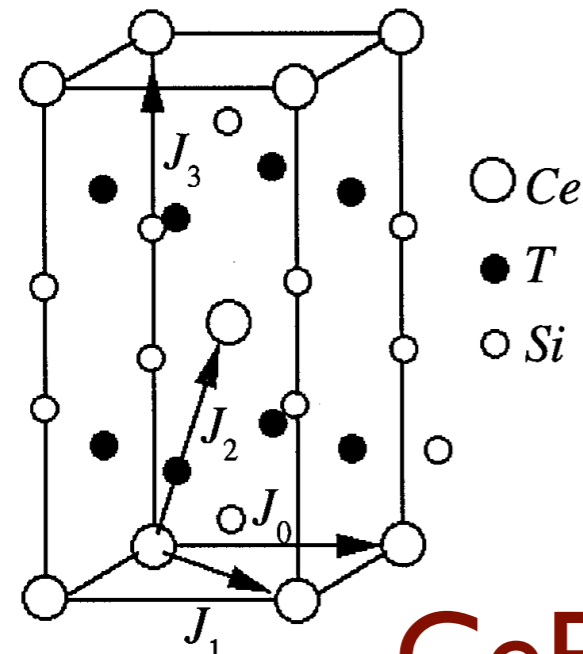
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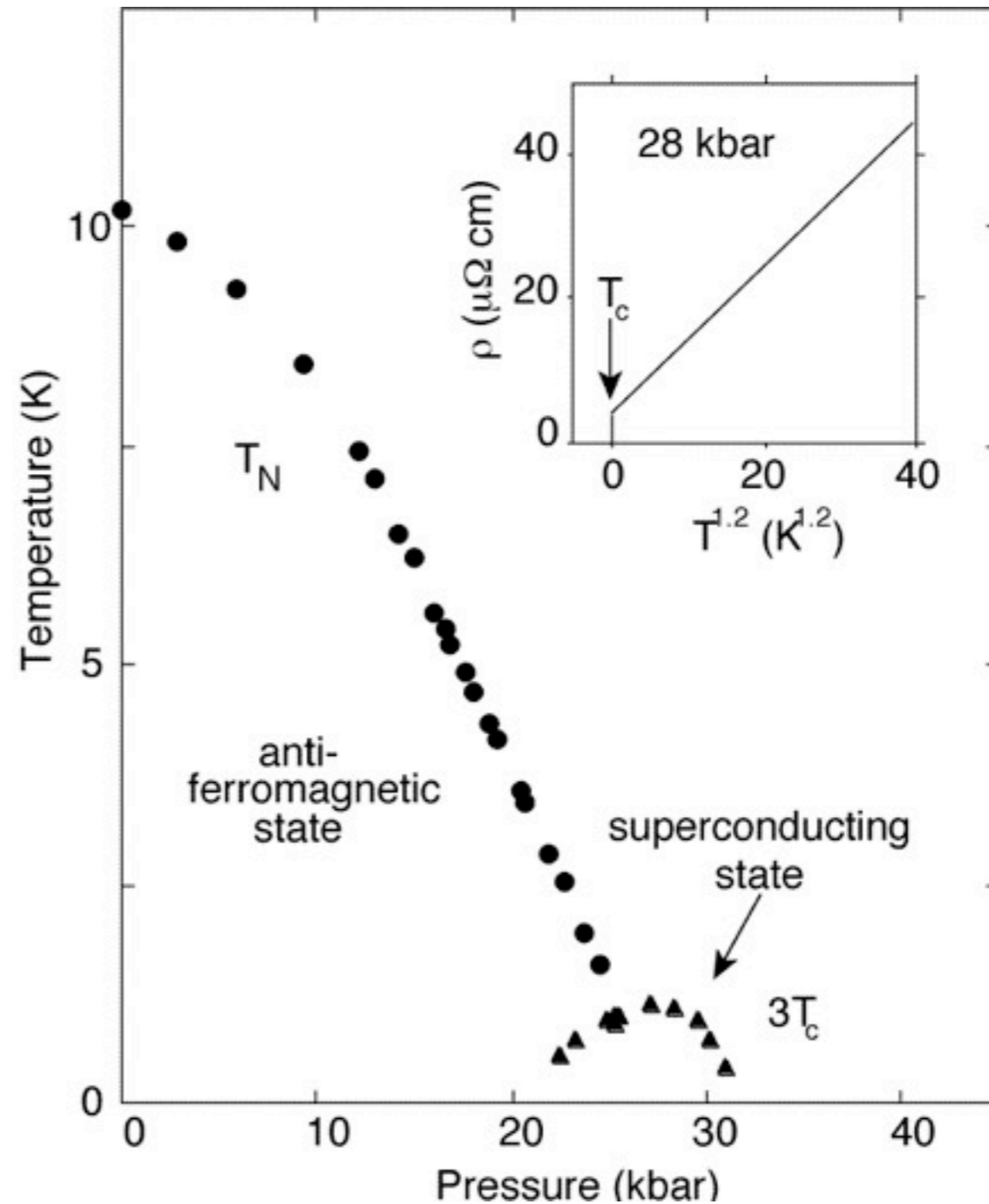
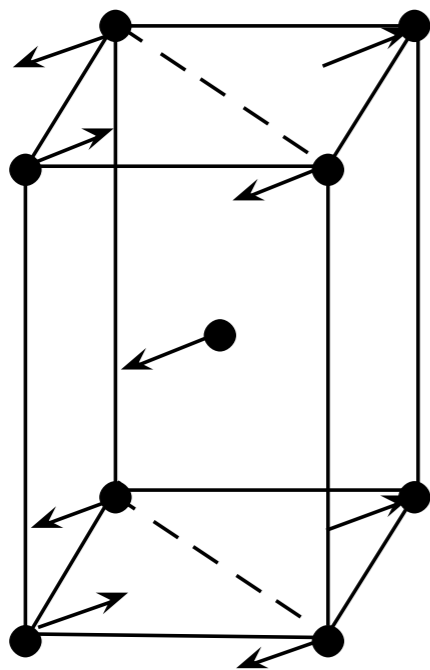


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Lower T_c superconductivity in the heavy fermion compounds

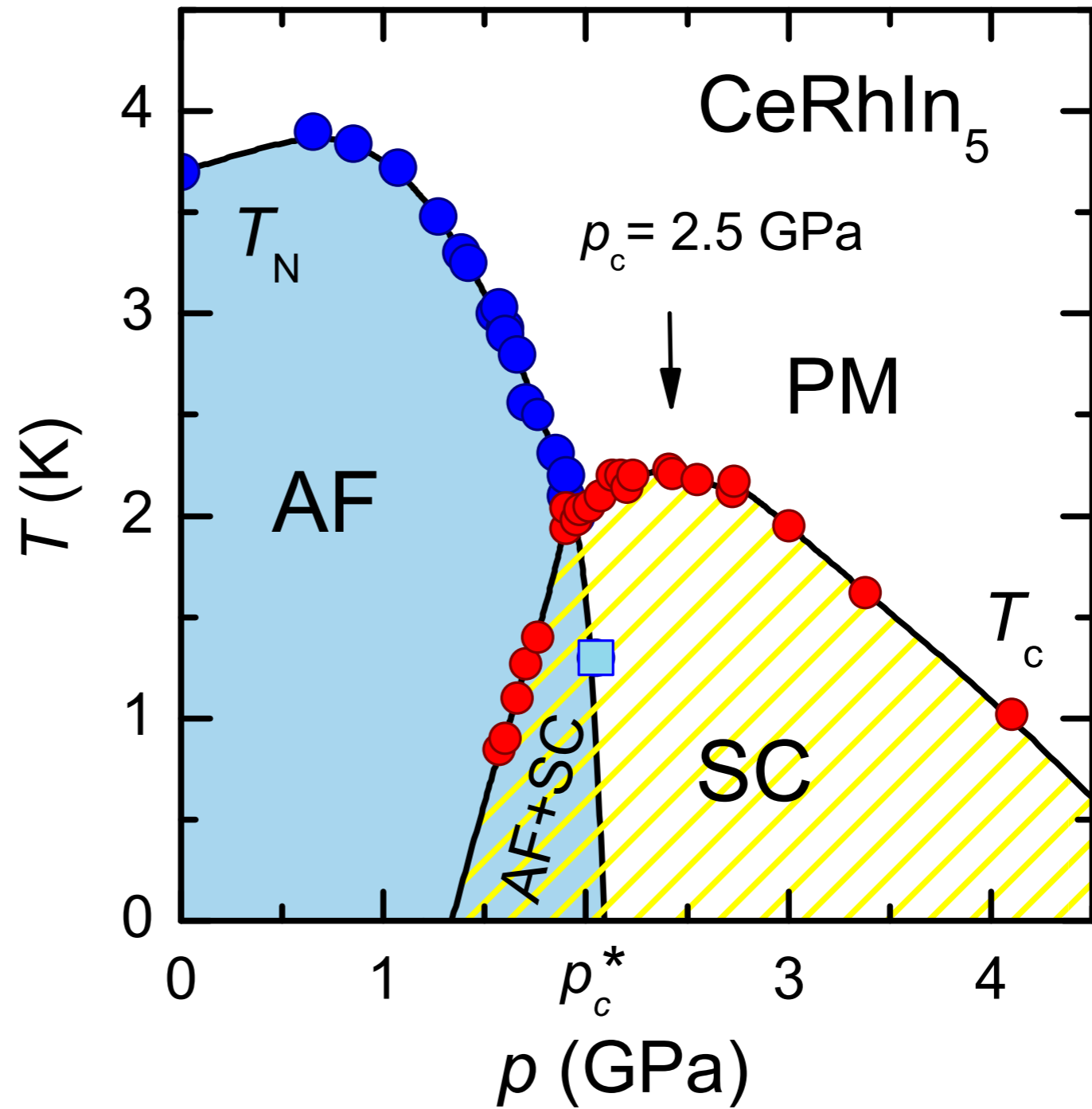
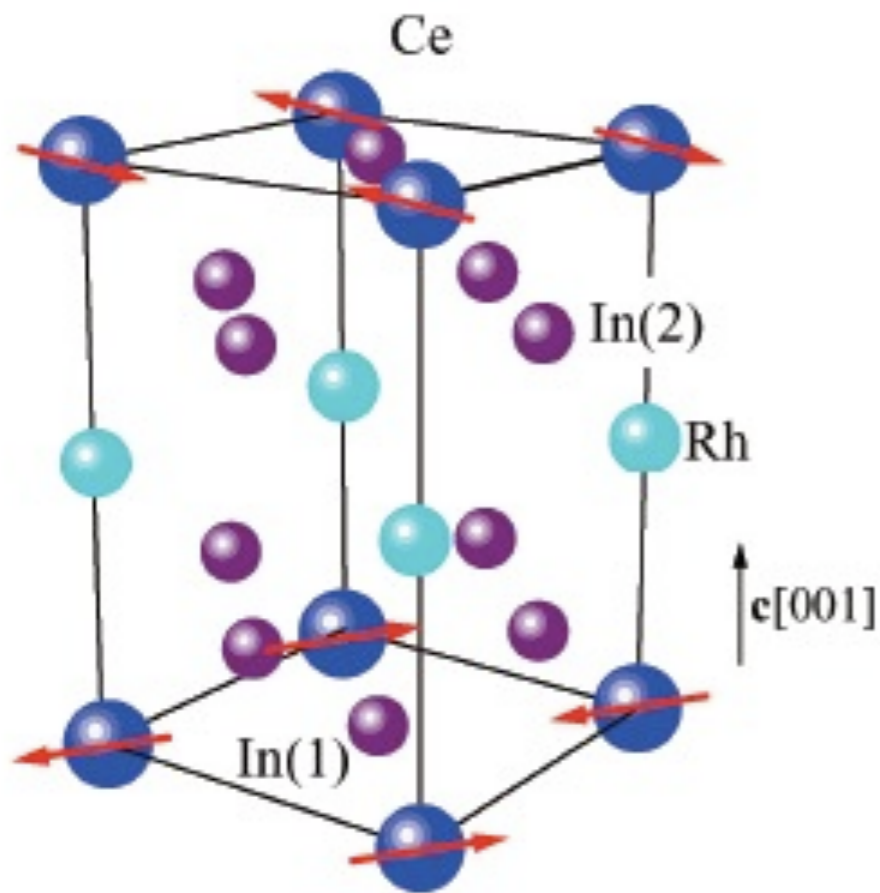


a



N. D. Mathur, F. M. Grosche, S. R. Julian, I. R. Walker, D. M. Freye, R. K. W. Haselwimmer, and G. G. Lonzarich, *Nature* **394**, 39 (1998)

Lower T_c superconductivity in the heavy fermion compounds



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

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- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Outline

1. Loss of antiferromagnetism in an insulator

Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal

From large Fermi surfaces to Fermi pockets, d-wave superconductivity, and competing orders

3. Strongly-coupled quantum criticality in metals

The AdS/CFT correspondence

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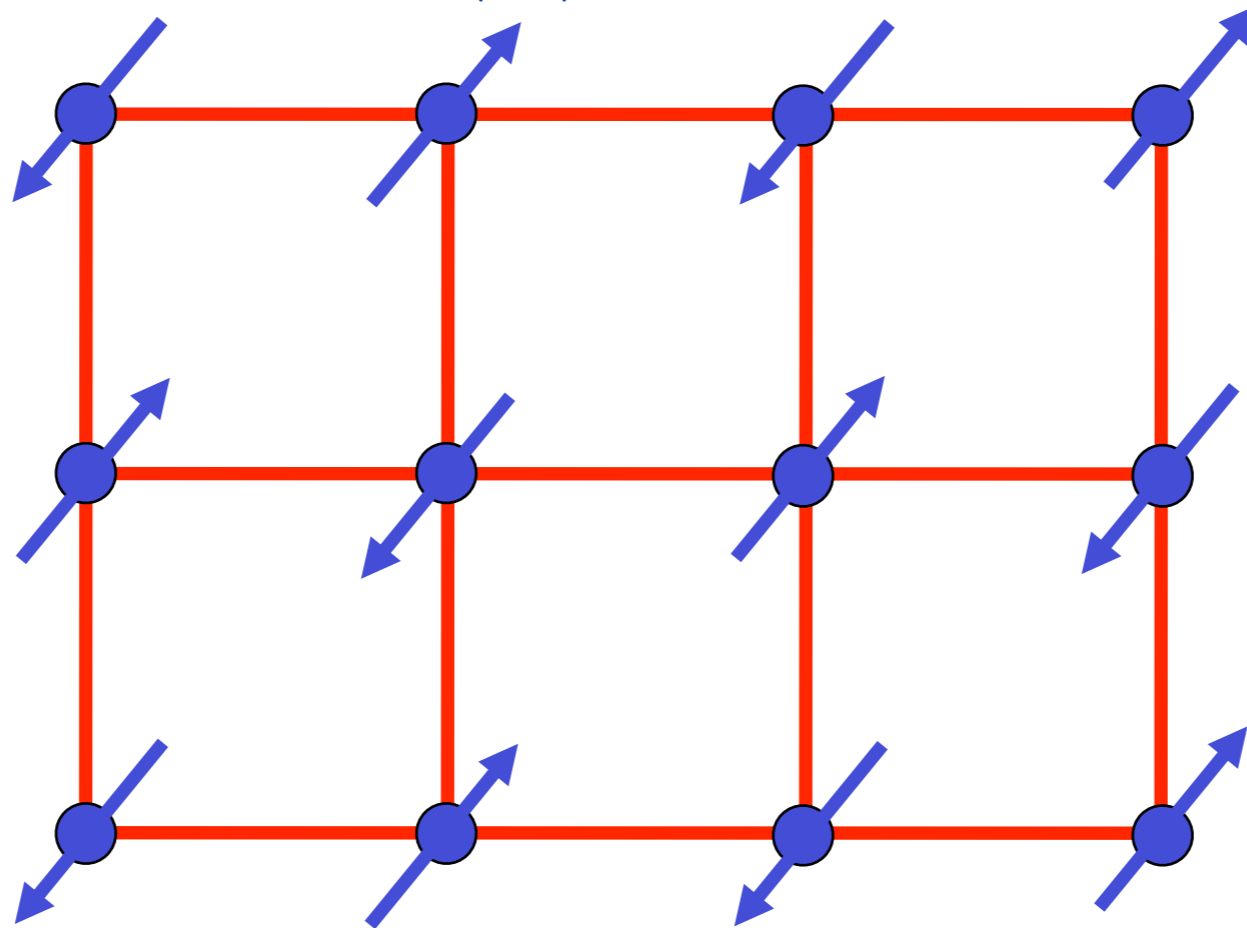
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Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

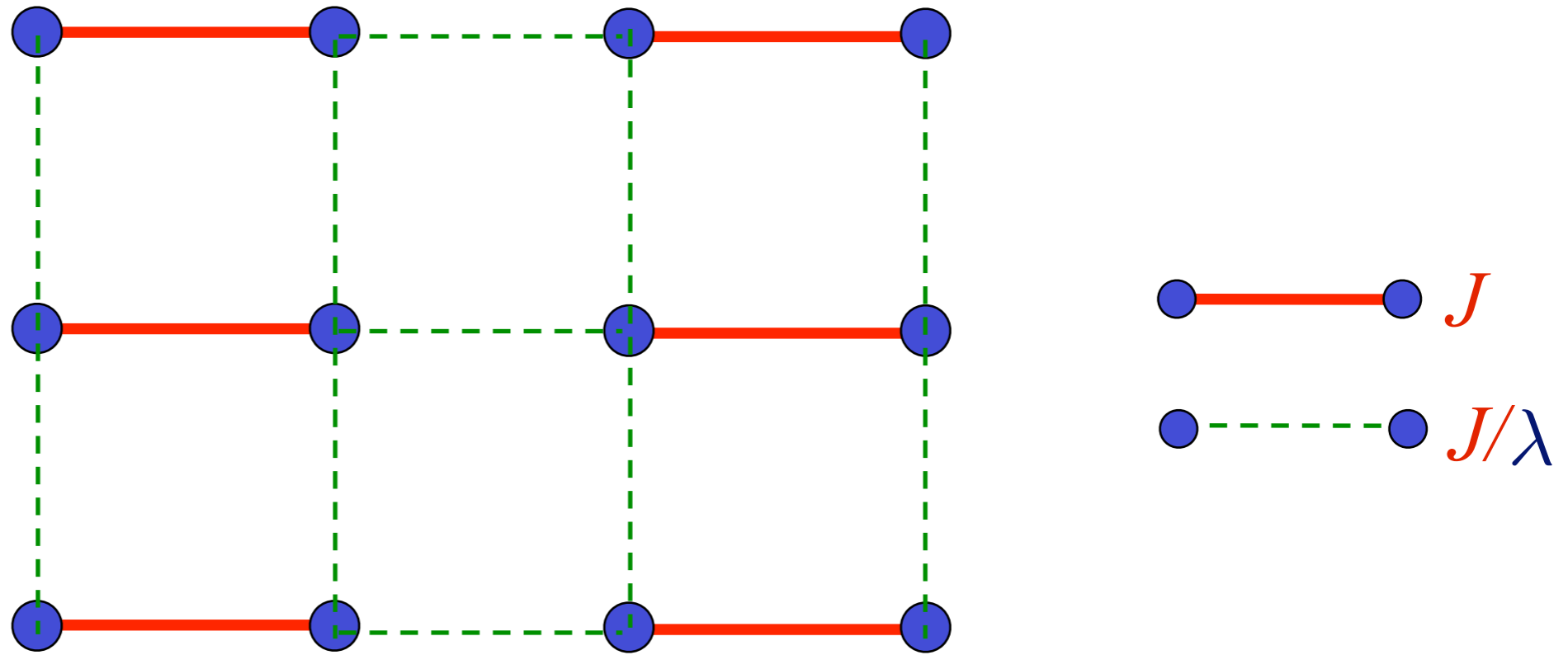
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

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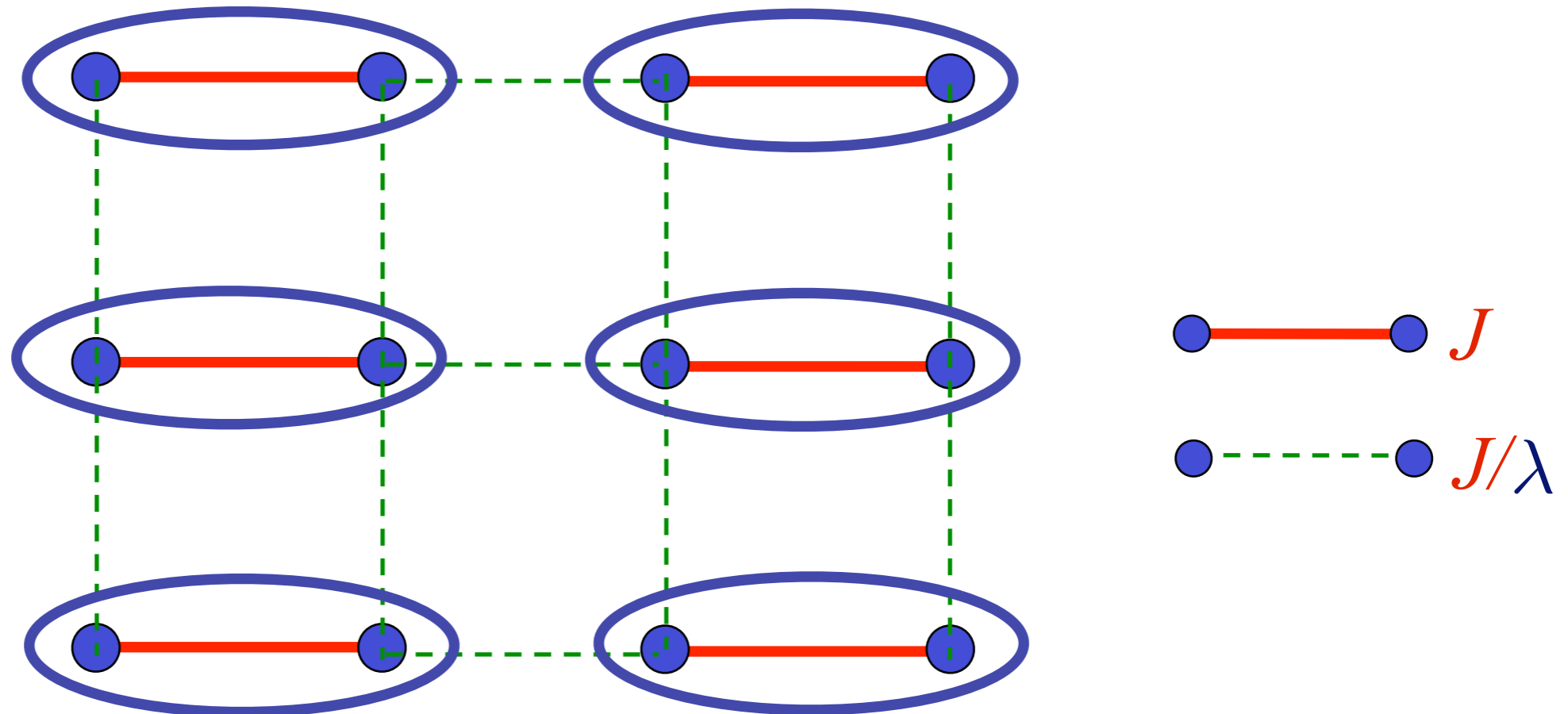
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Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

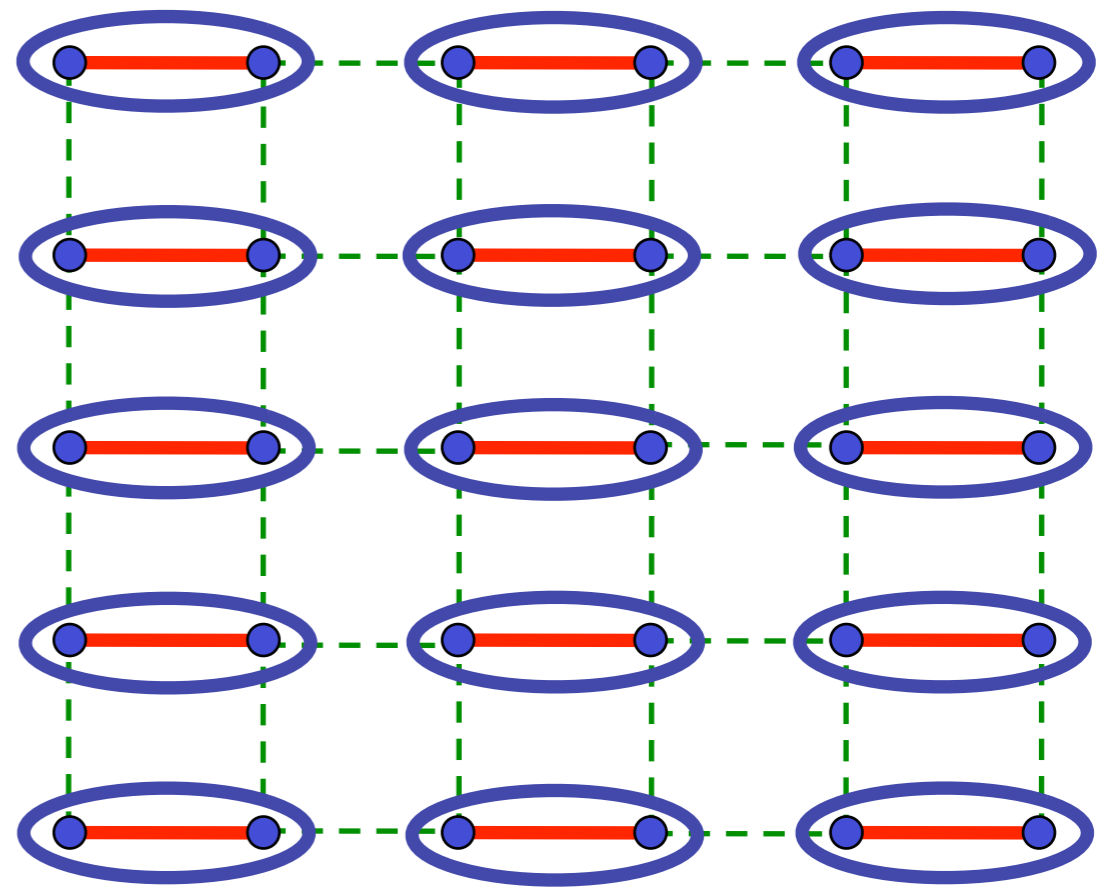
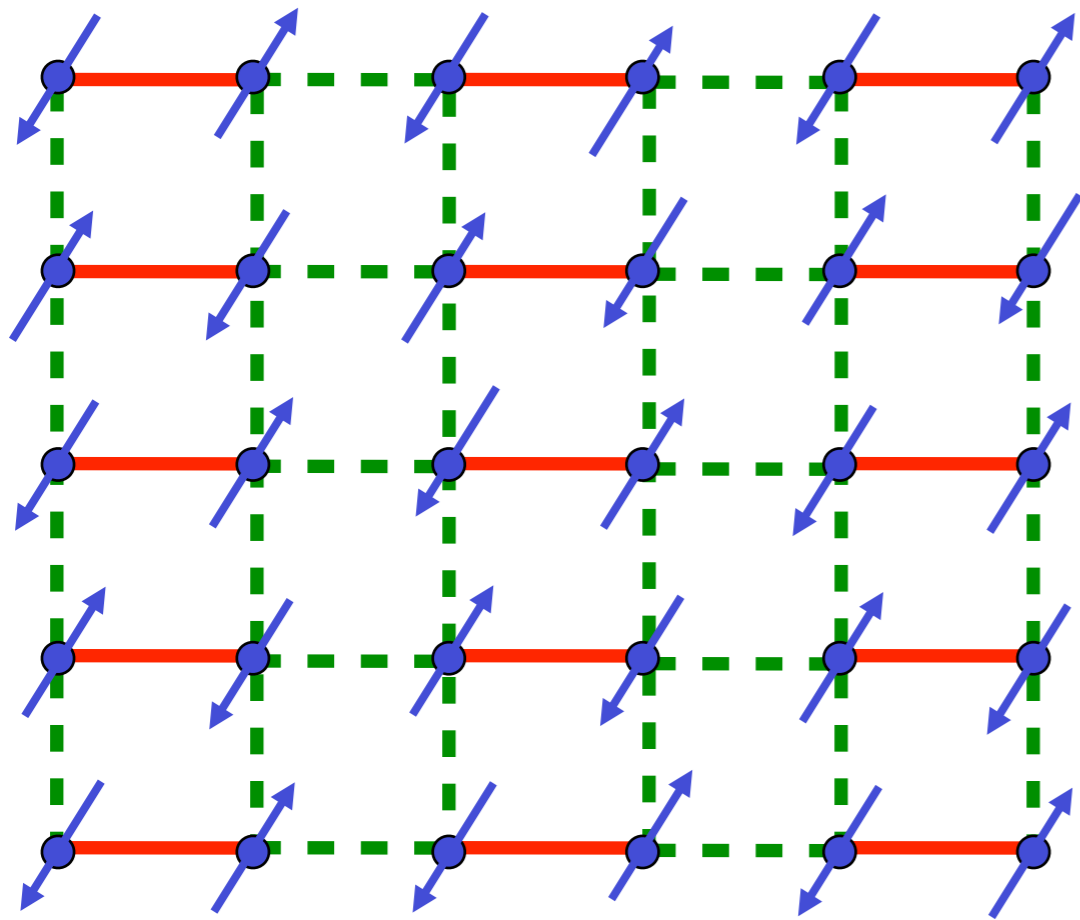
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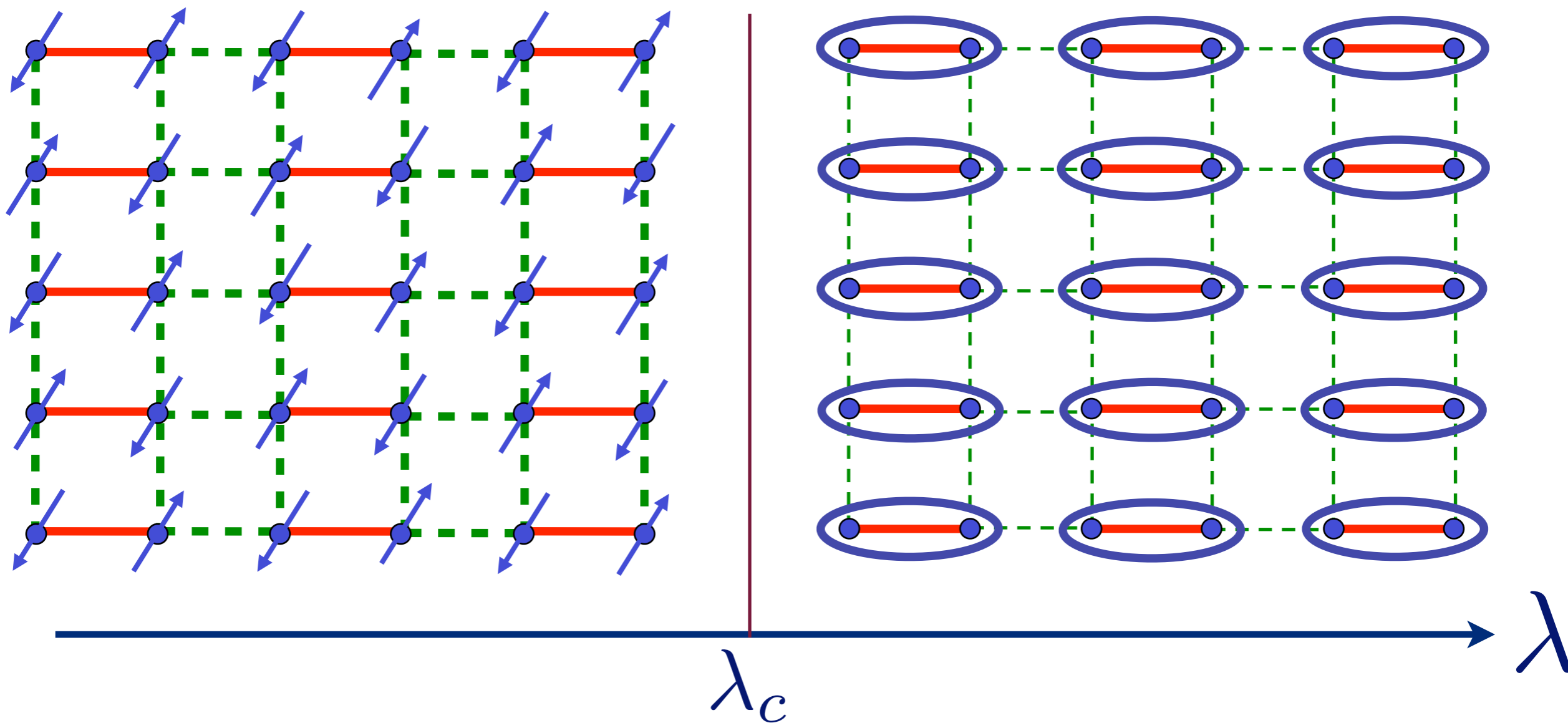
Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$\text{[Diagram of two blue dots connected by a red line, enclosed in a blue oval]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



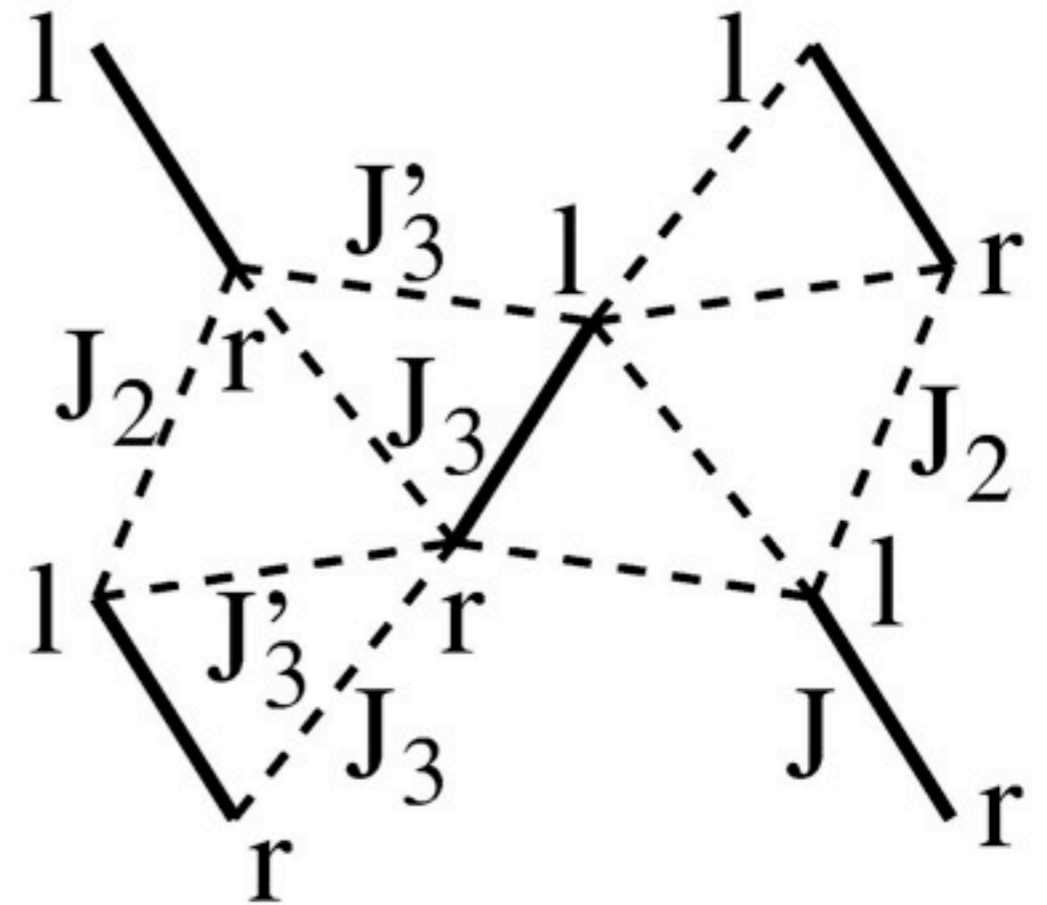
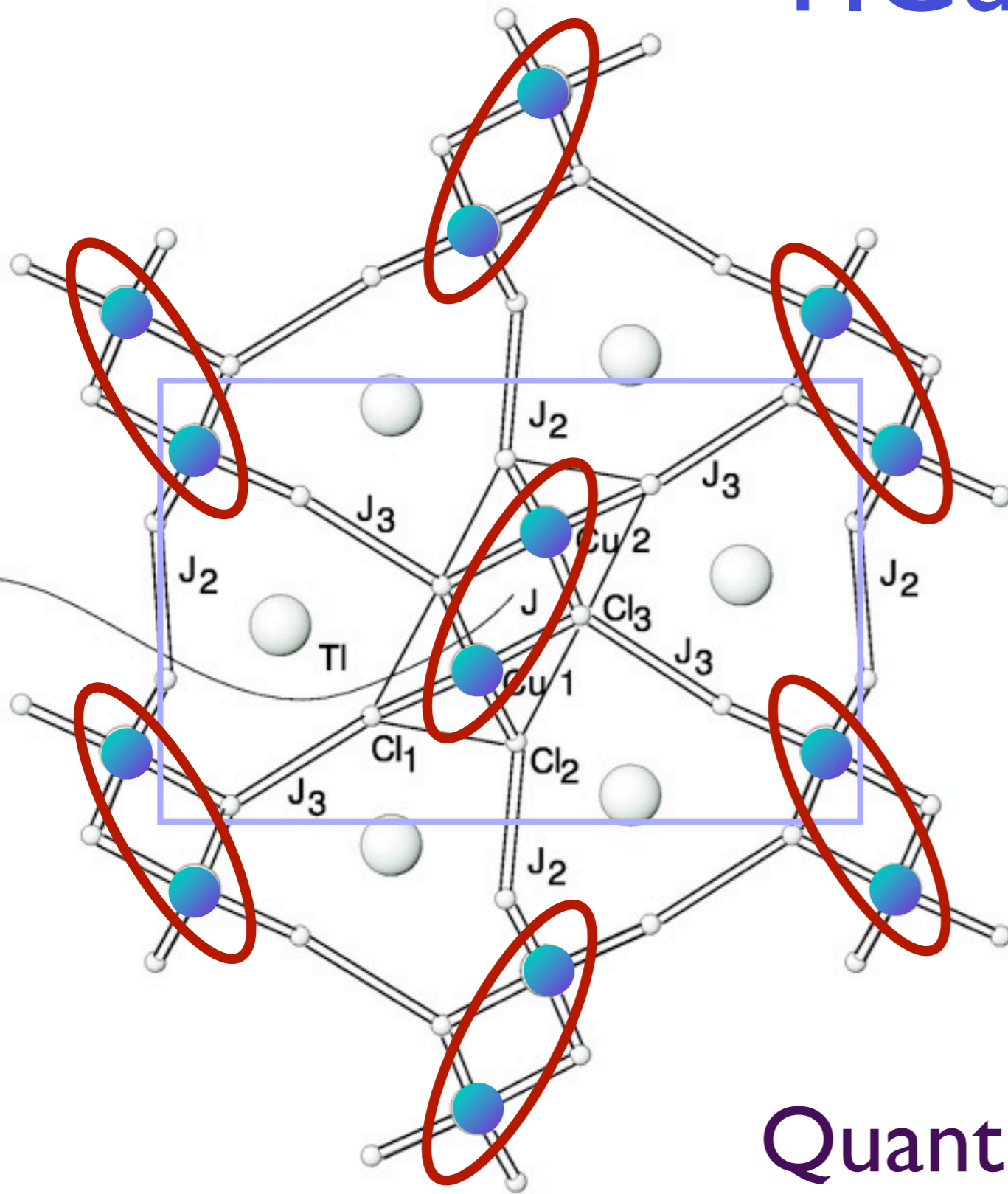
$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Pressure in TlCuCl_3

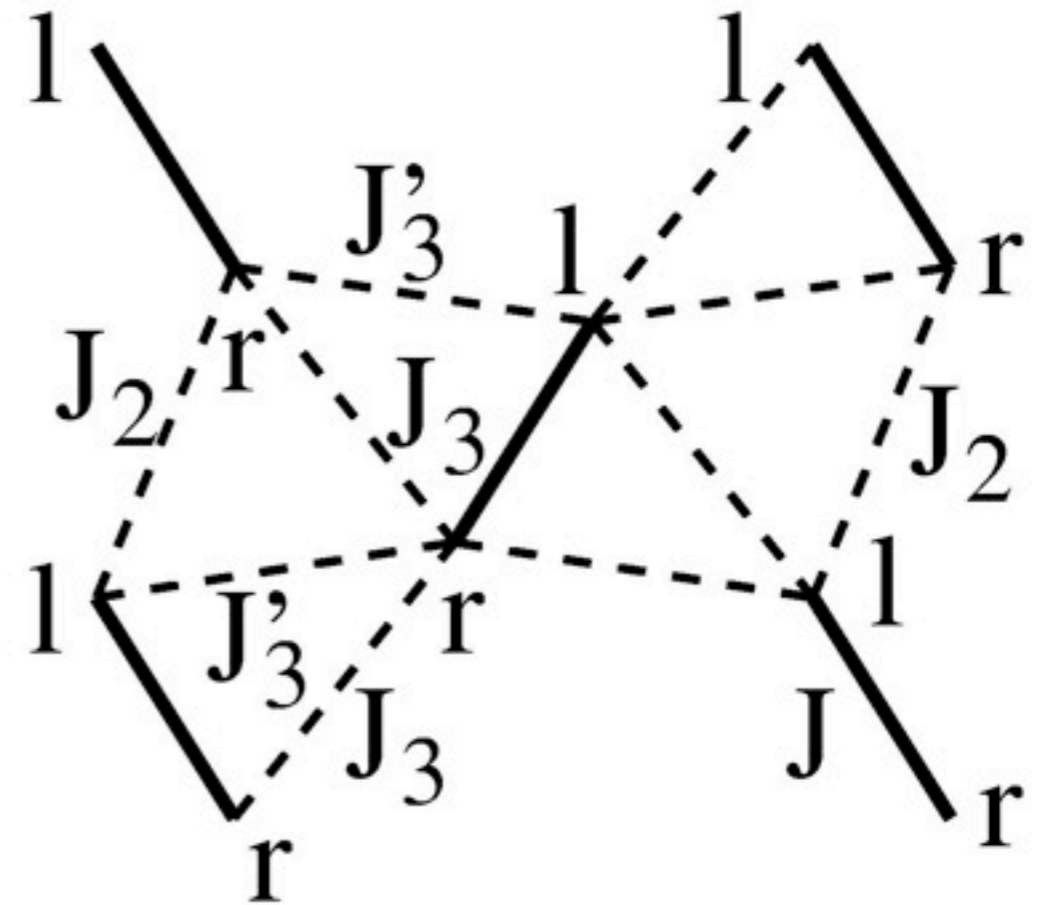
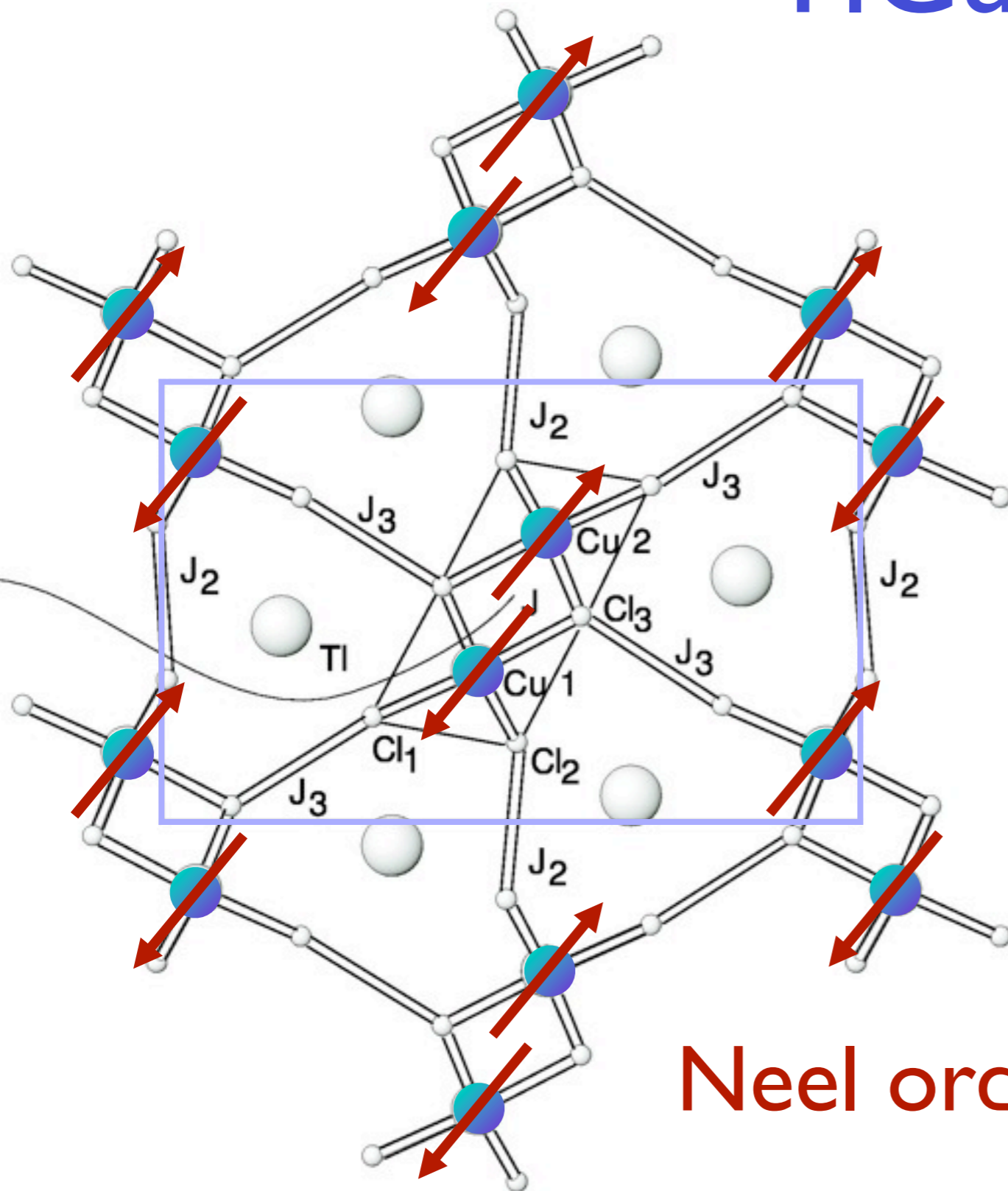
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka,
Journal of the Physical Society of Japan, **73**, 1446 (2004).

TlCuCl₃



Quantum paramagnet at
ambient pressure

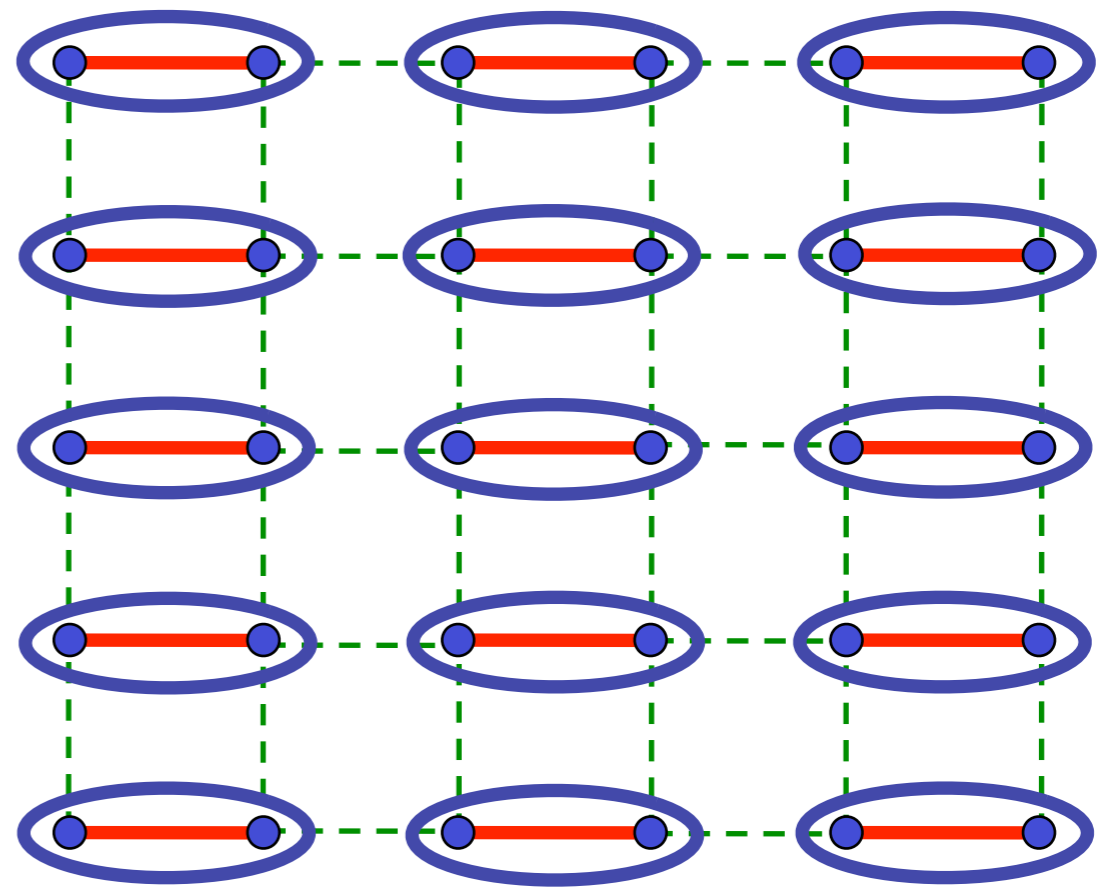
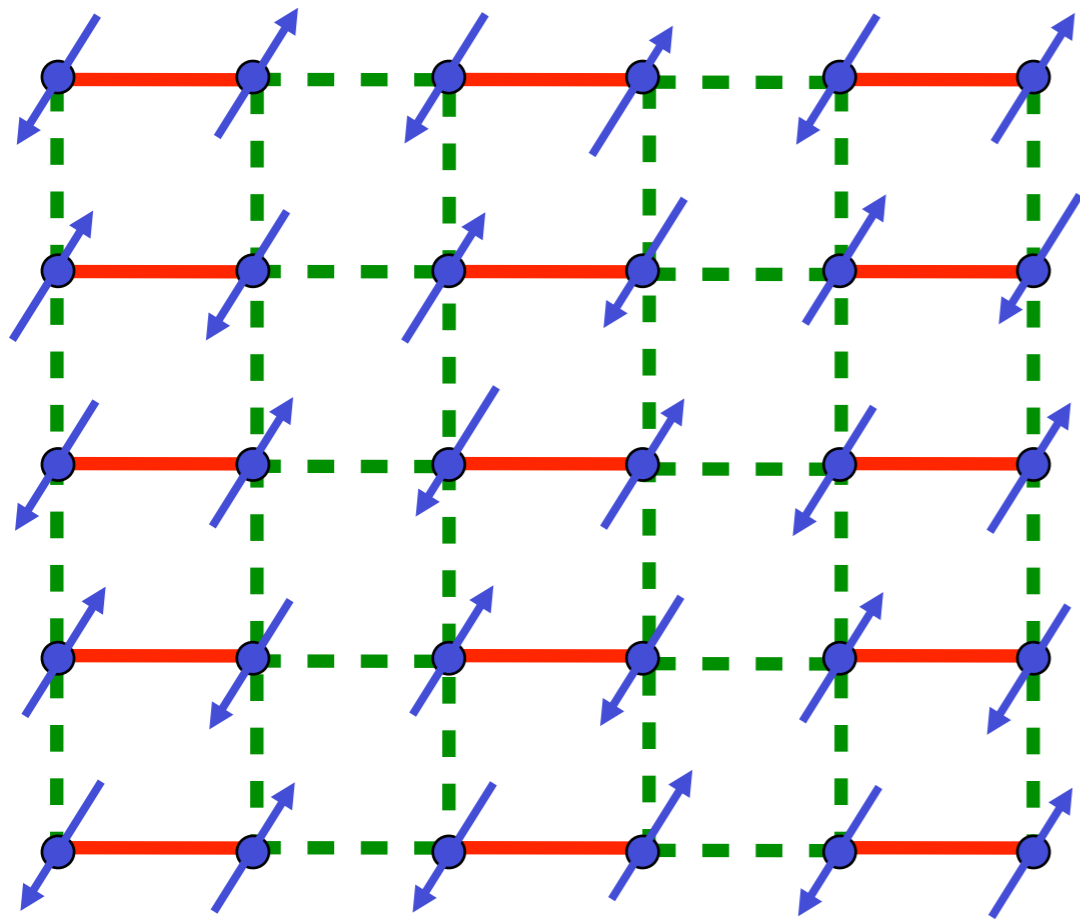
TlCuCl₃



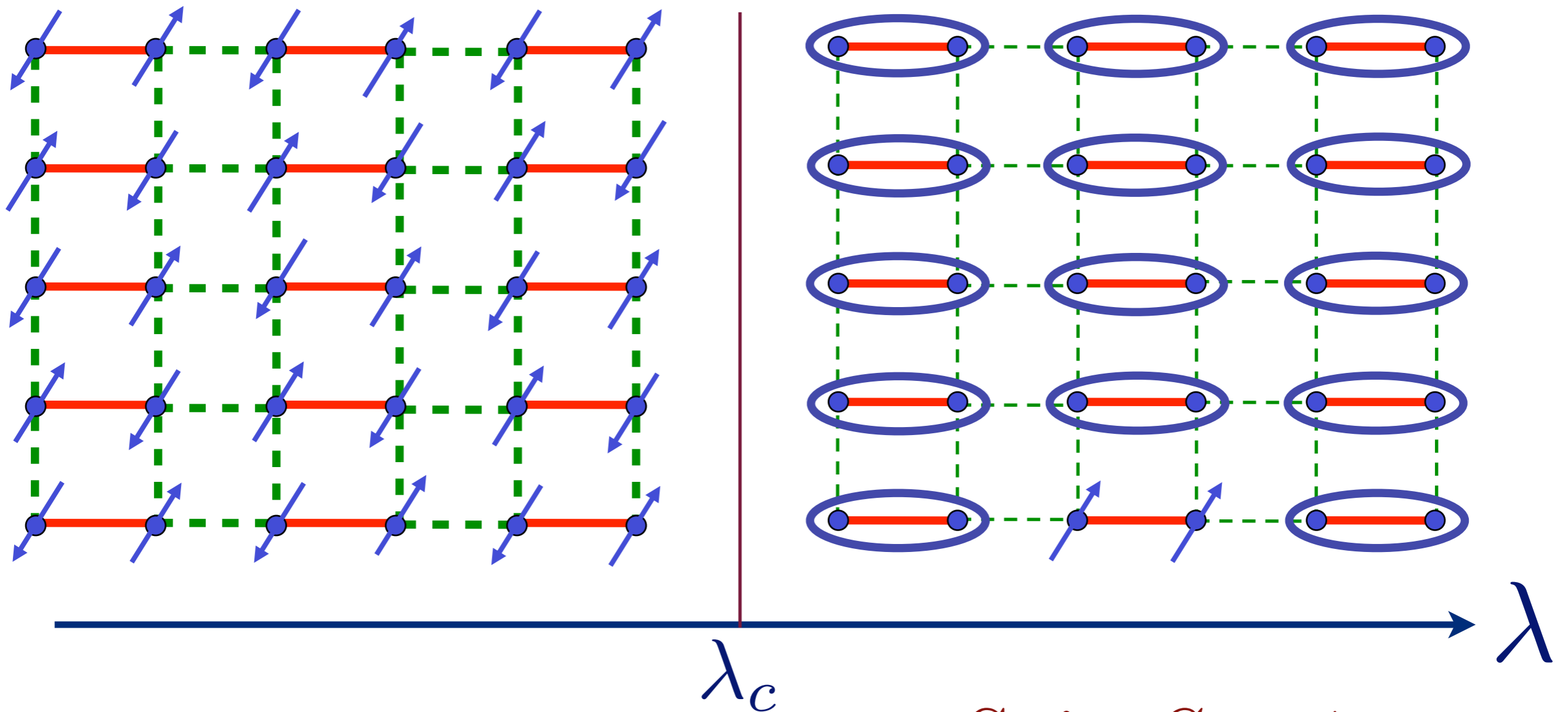
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

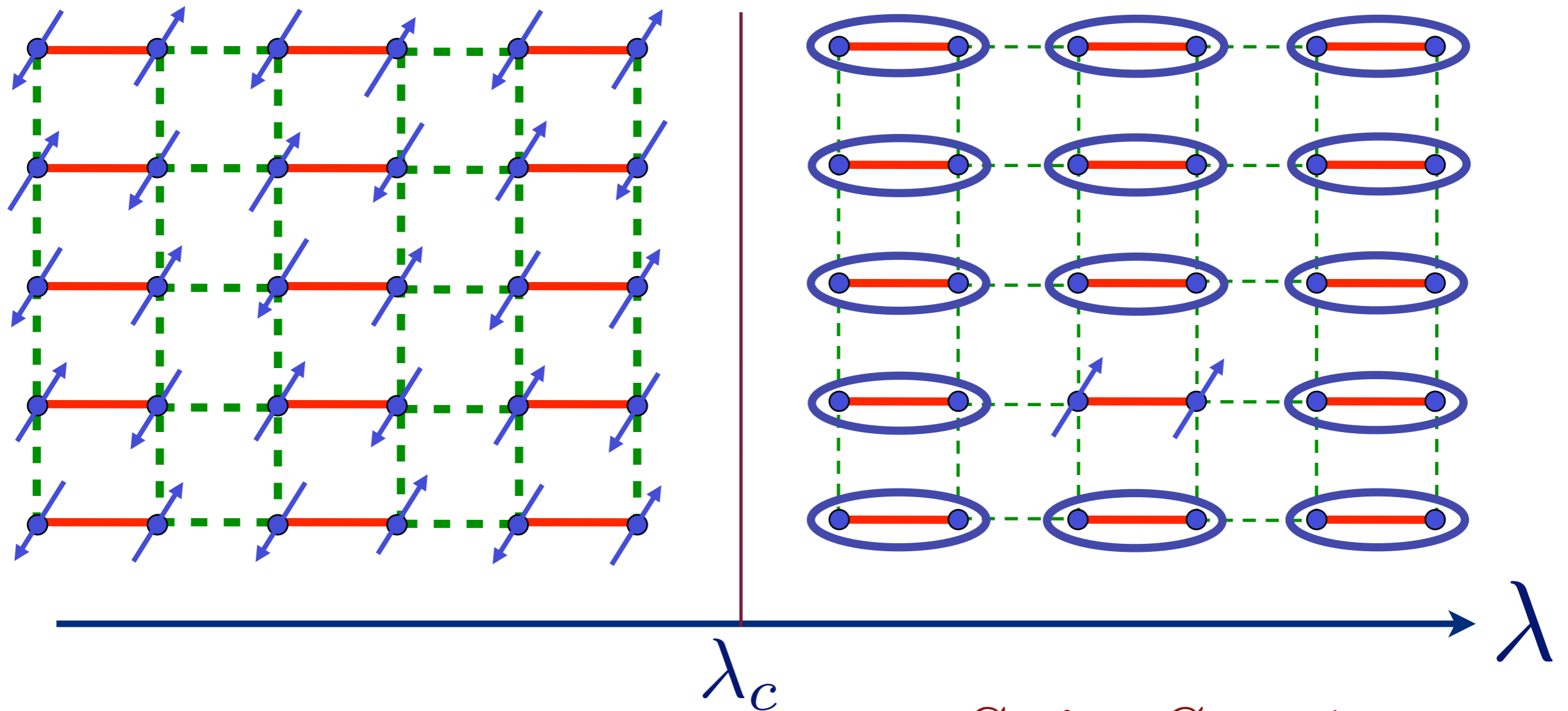


Excitation spectrum in the paramagnetic phase



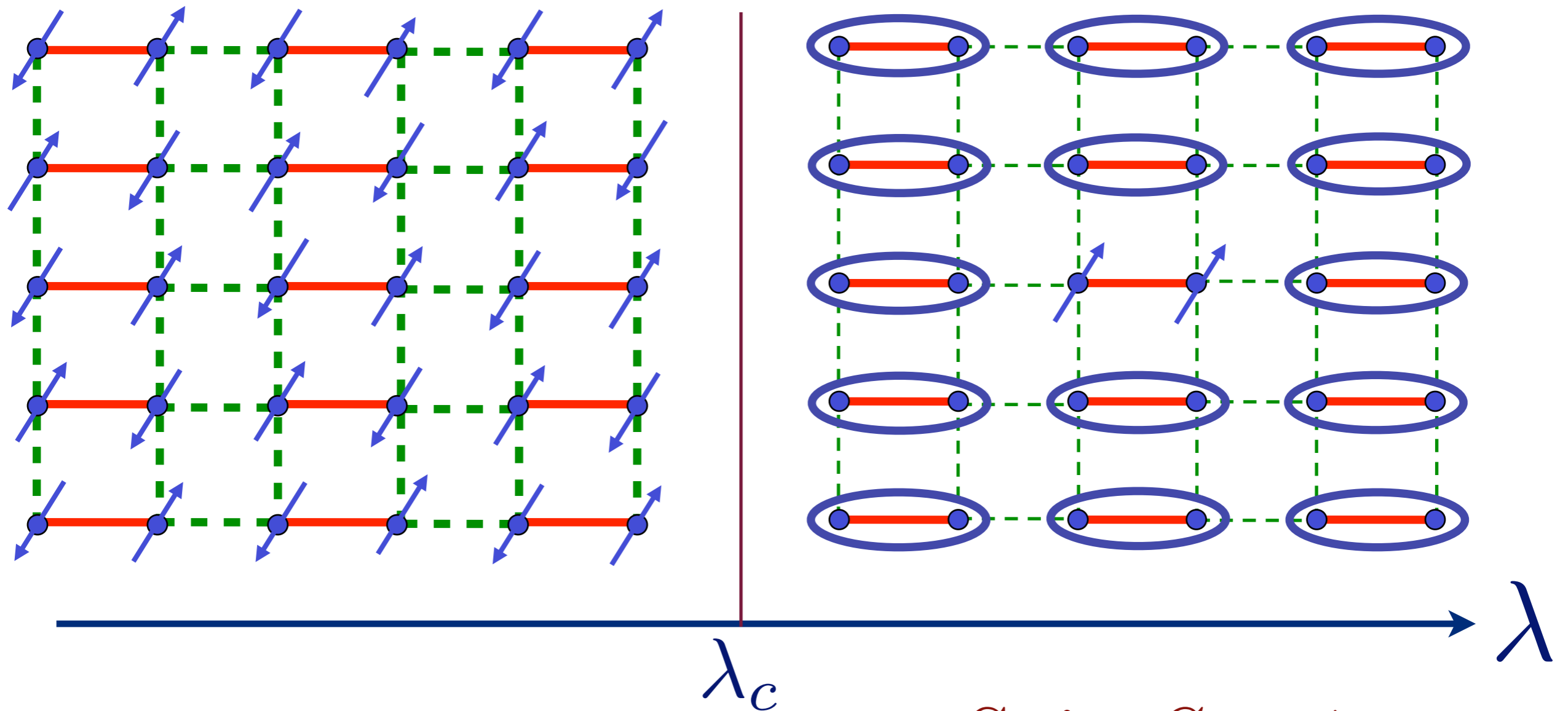
Spin $S = 1$
“triplon”

Excitation spectrum in the paramagnetic phase



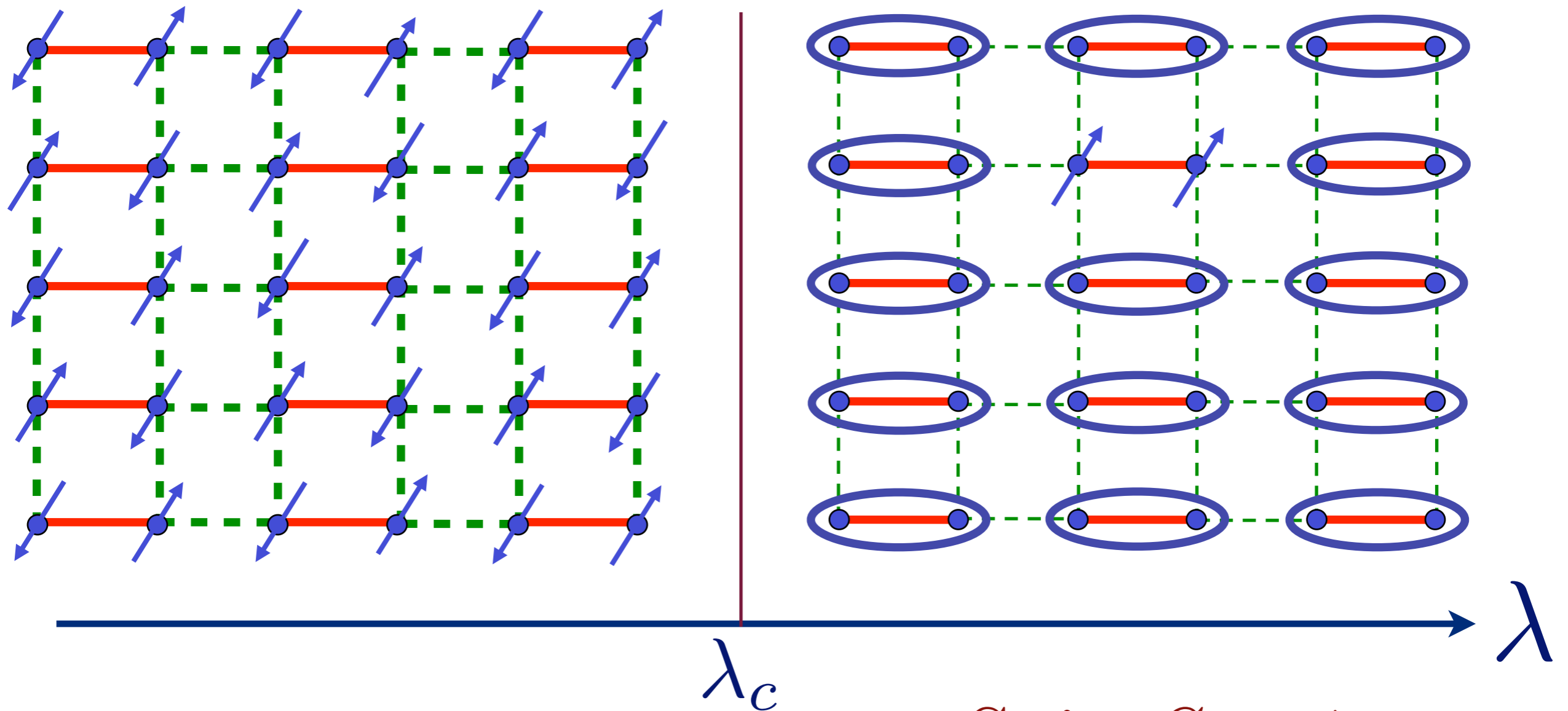
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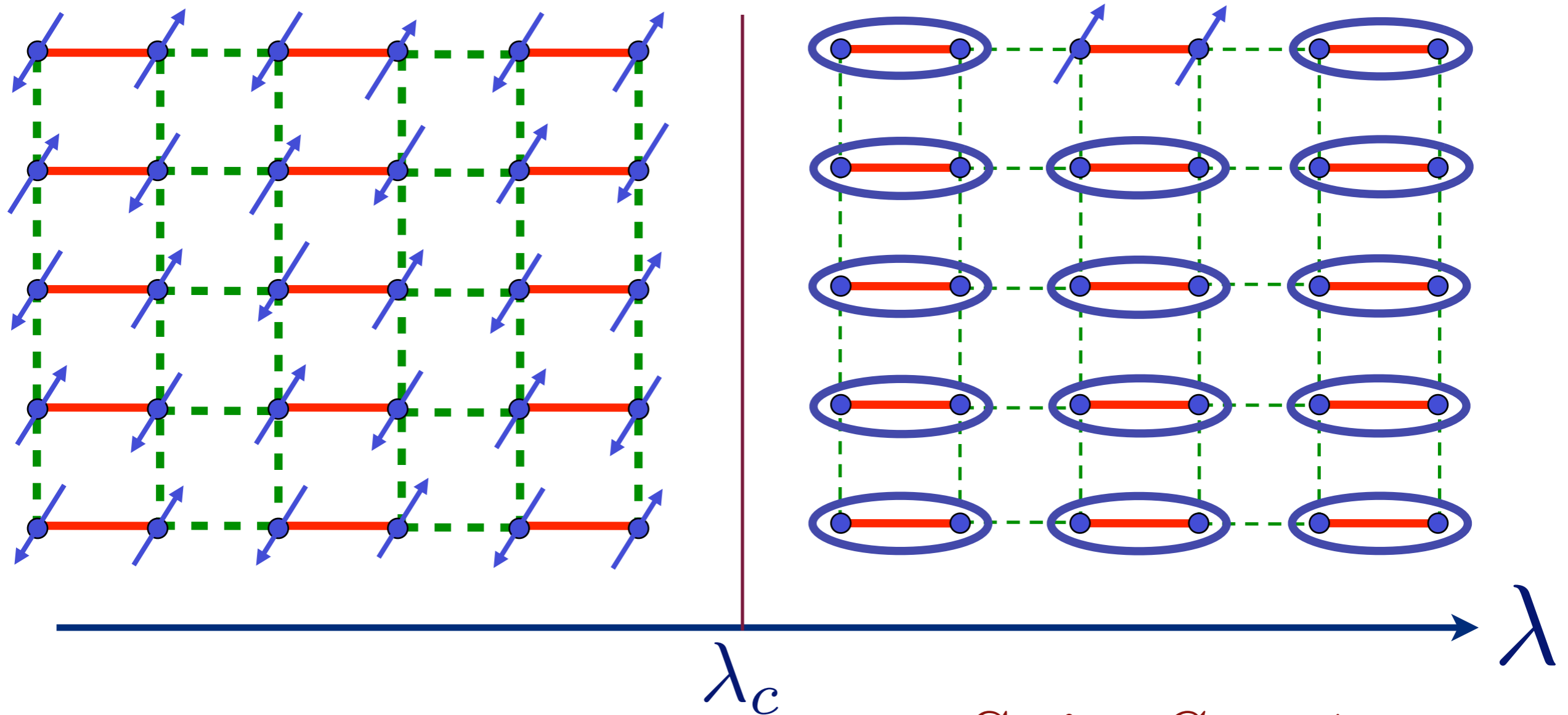
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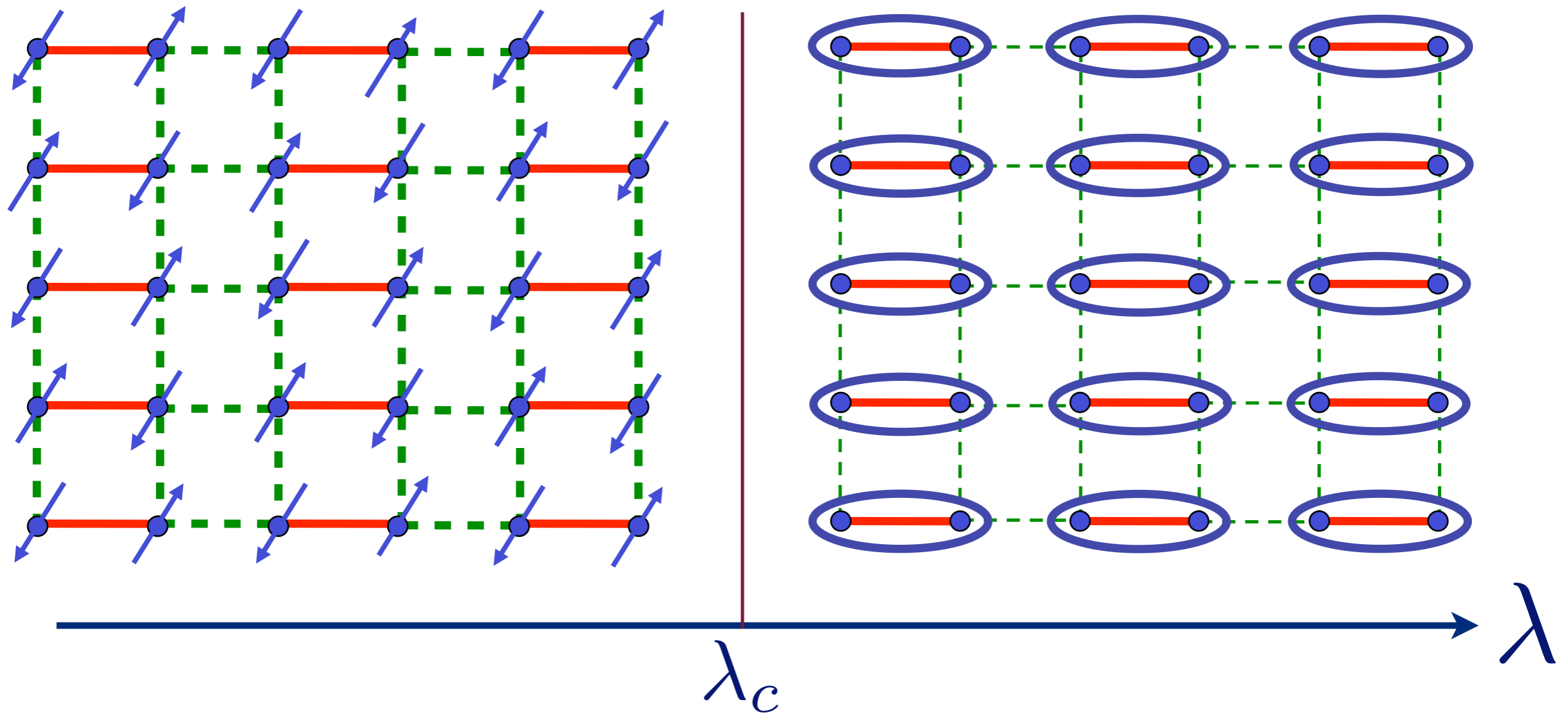
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Excitation spectrum in the paramagnetic phase



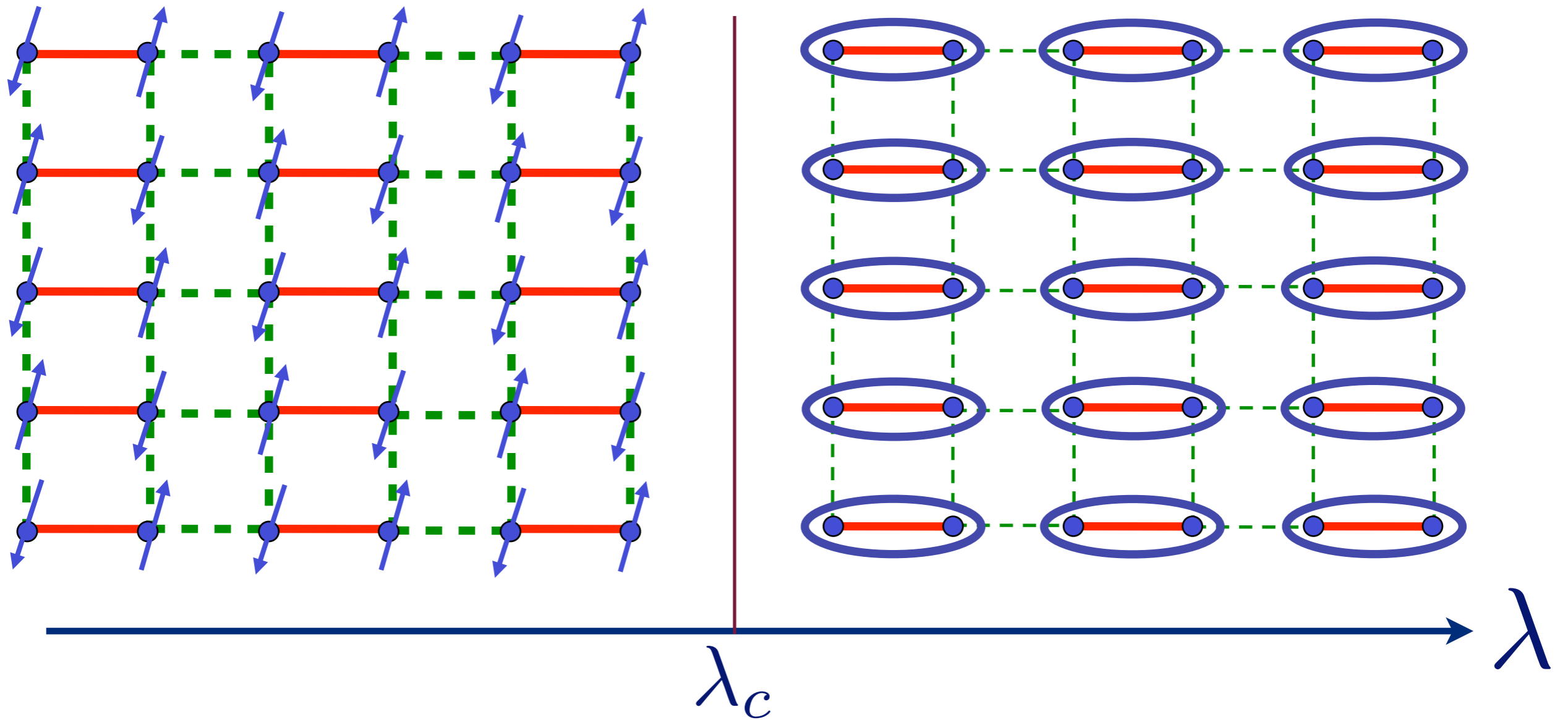
Spin $S = 1$
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Excitation spectrum in the Néel phase



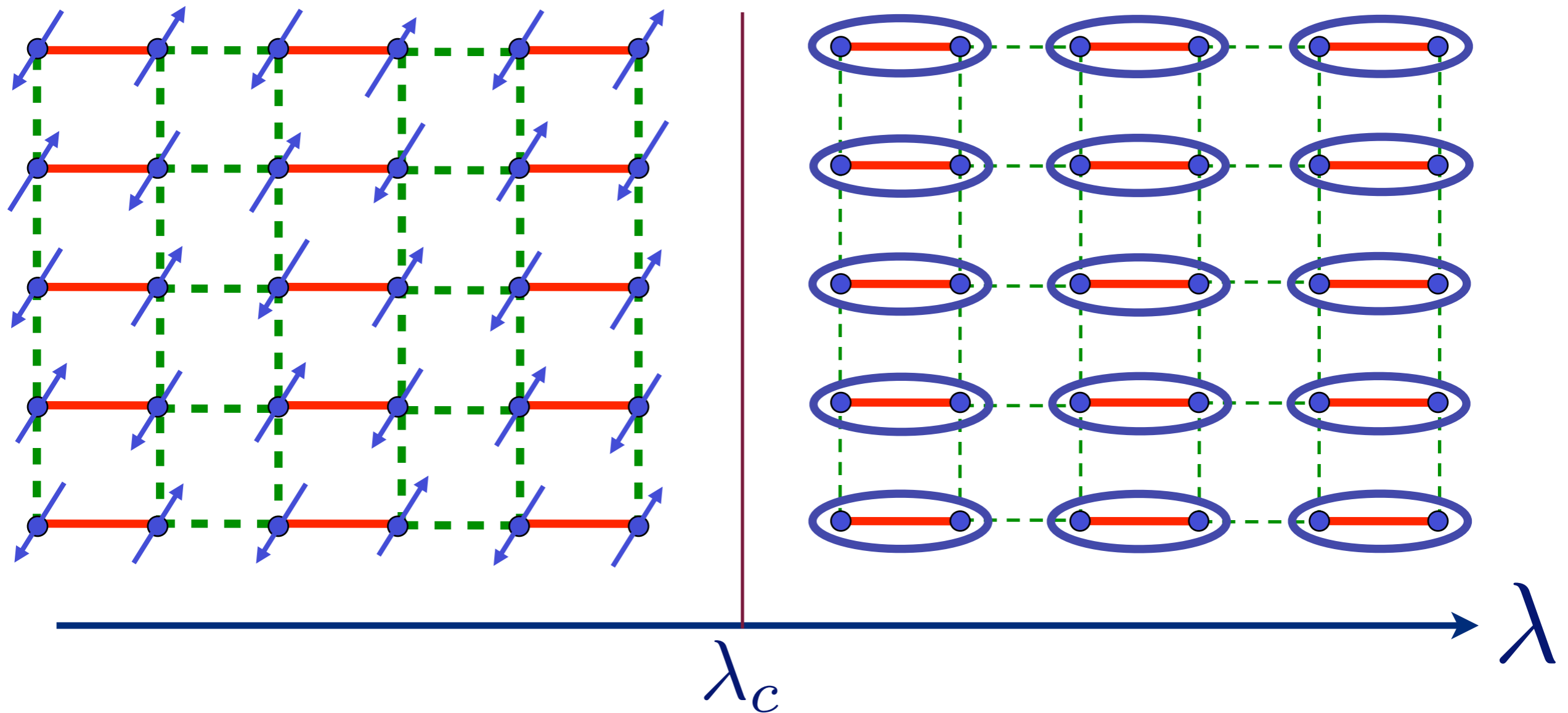
Spin waves

Excitation spectrum in the Néel phase



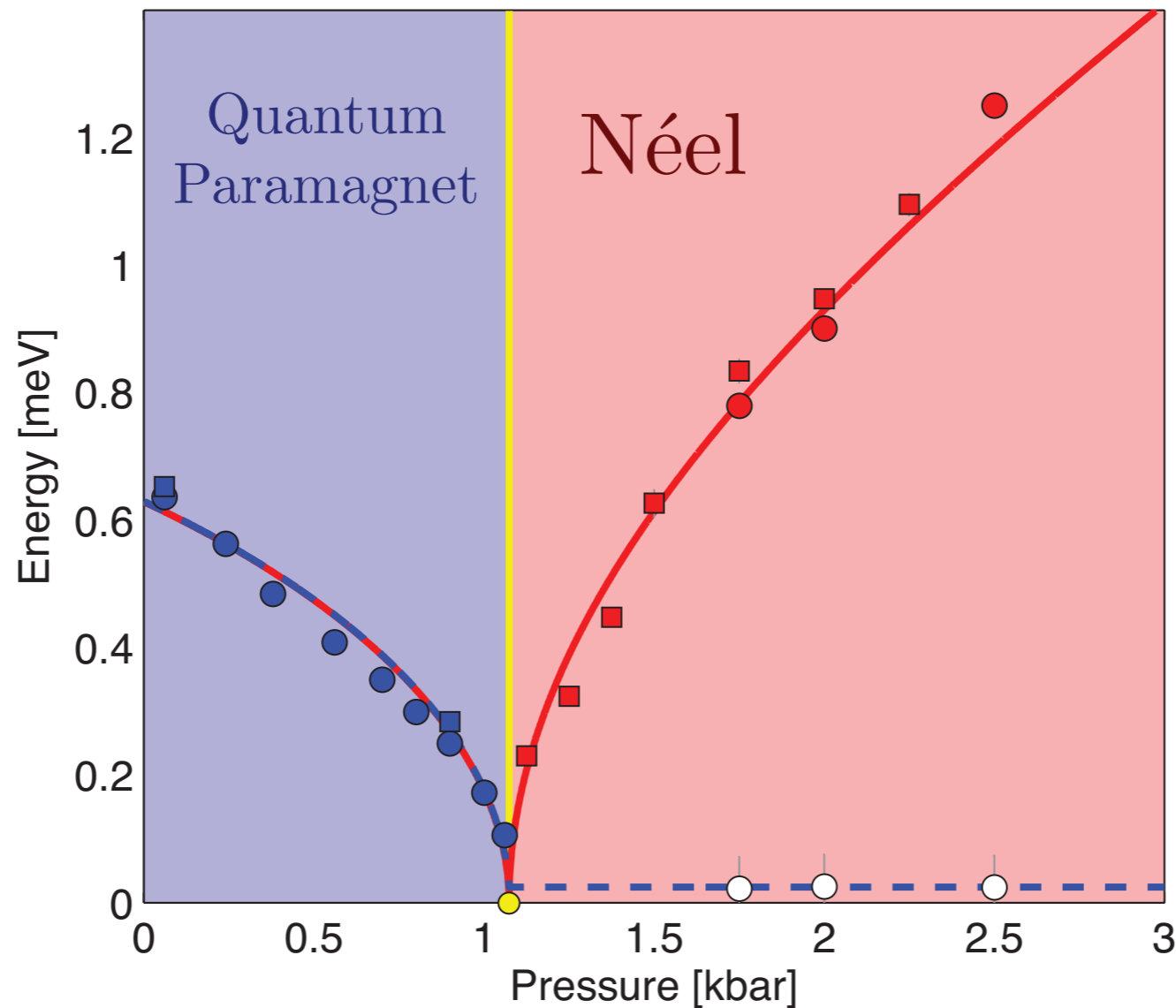
Spin waves

Excitation spectrum in the Néel phase



Spin waves

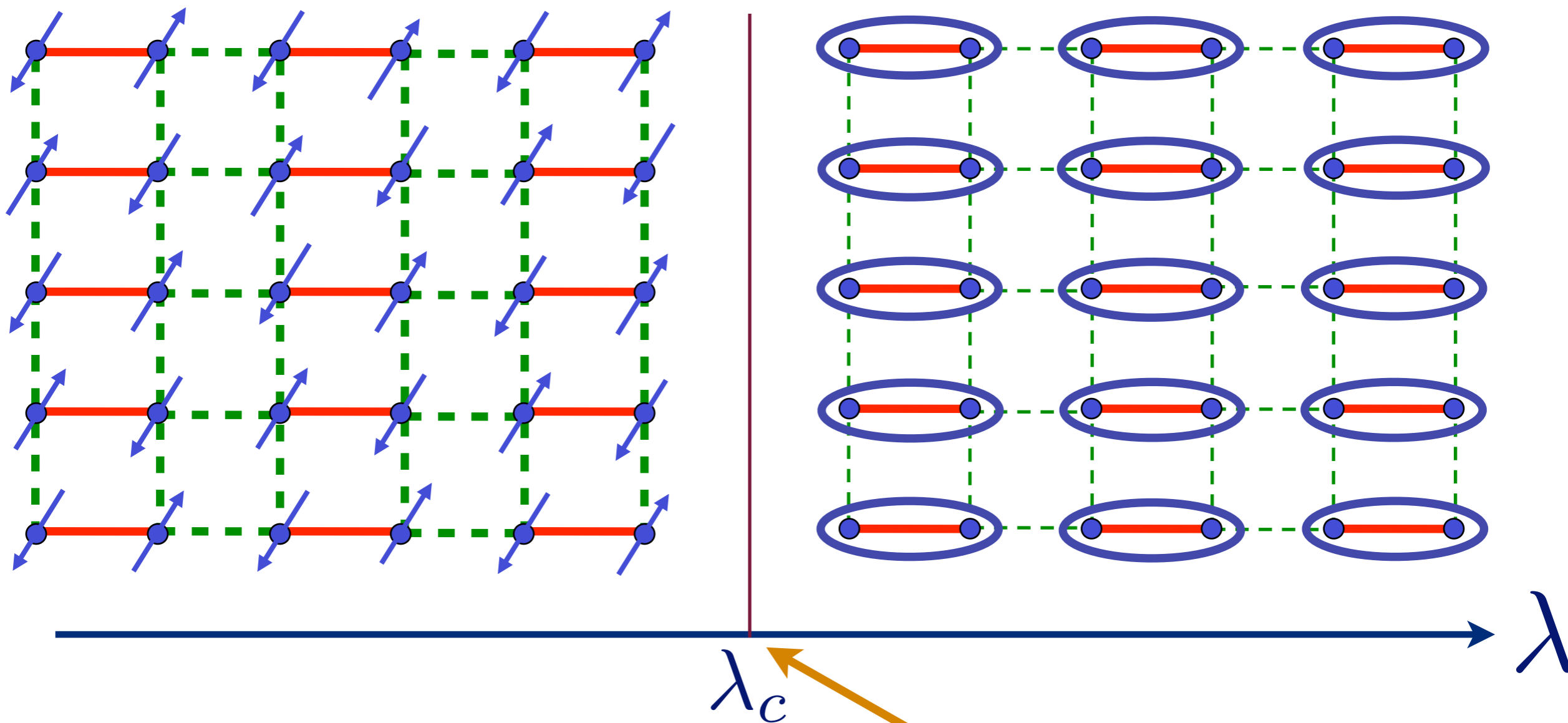
TlCuCl₃ with varying pressure



Observation of $3 \rightarrow 2$ low energy modes,
emergence of new Higgs particle in the Néel phase.

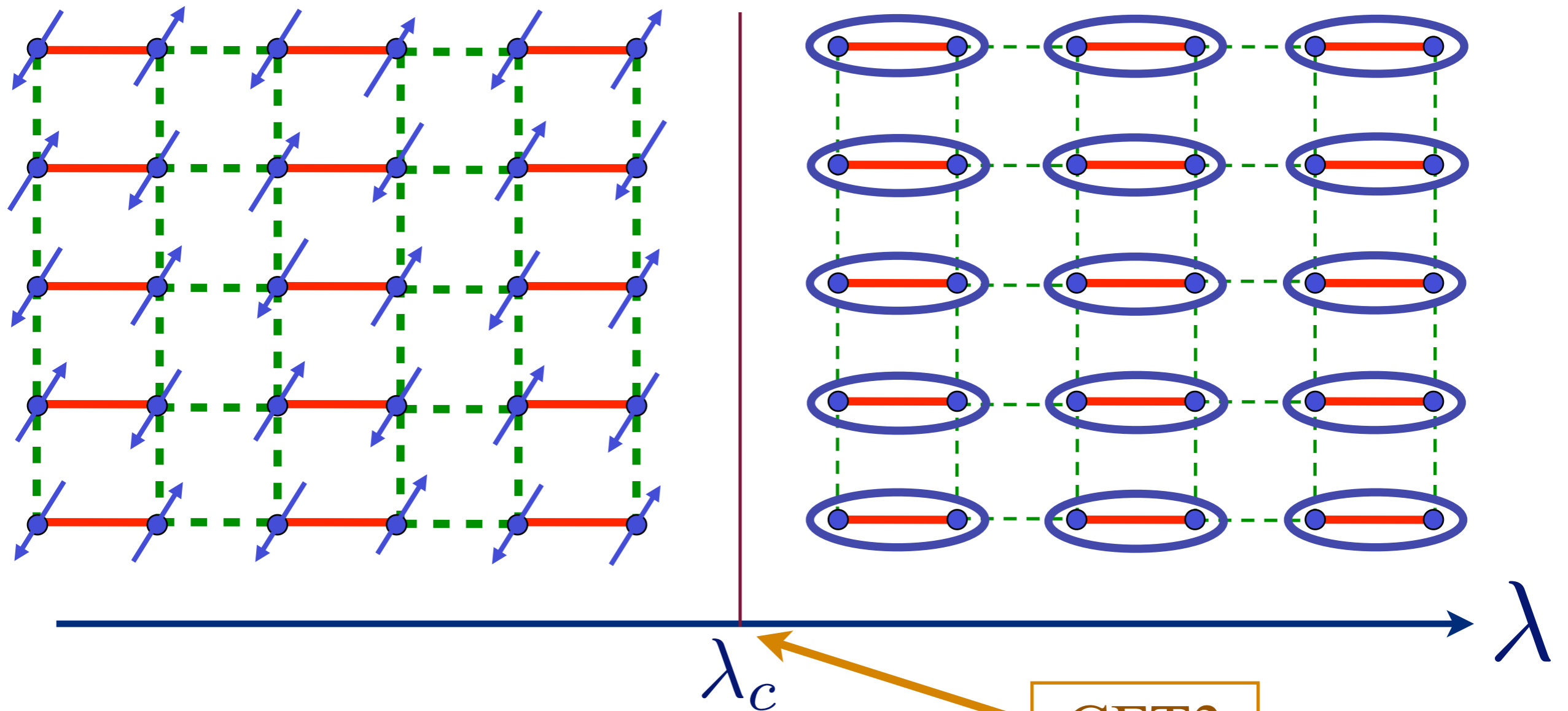
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

$$\text{[Diagram: Two blue dots connected by a red line, enclosed in a blue oval]} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Quantum critical point with non-local entanglement in spin wavefunction

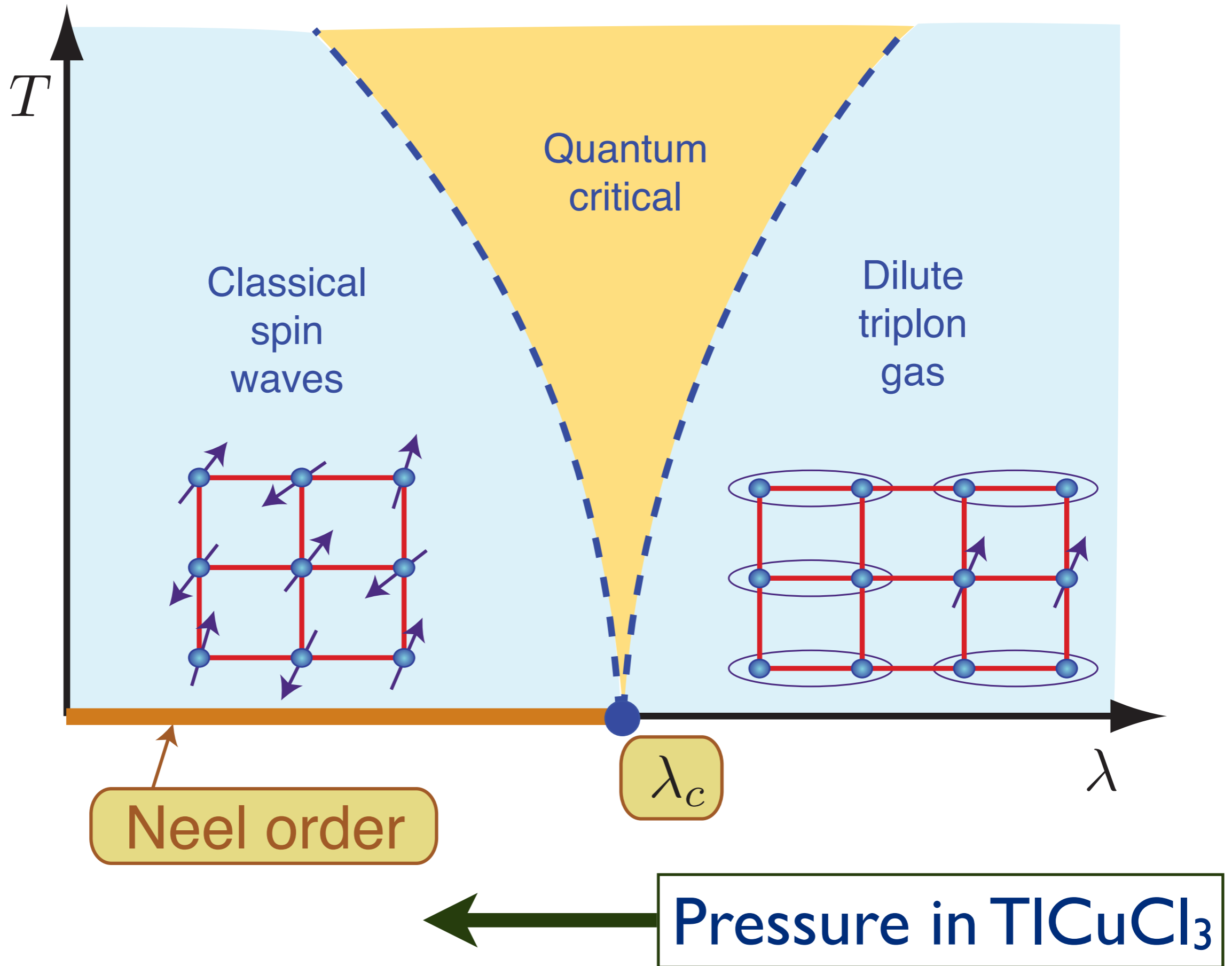
Description using Landau-Ginzburg field theory



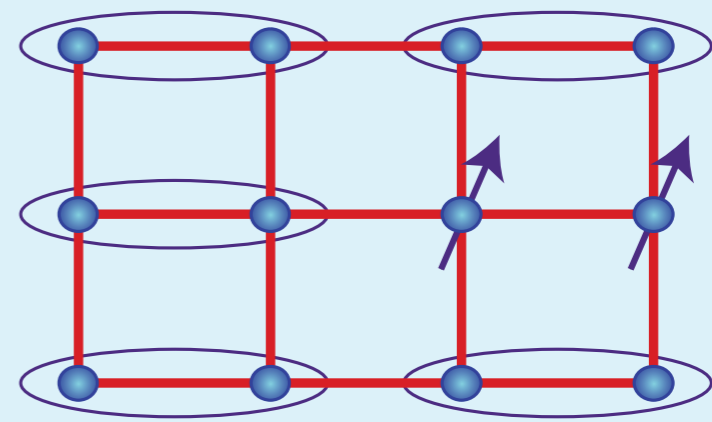
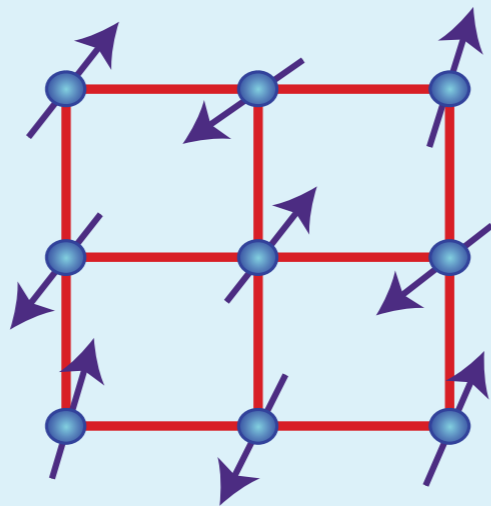
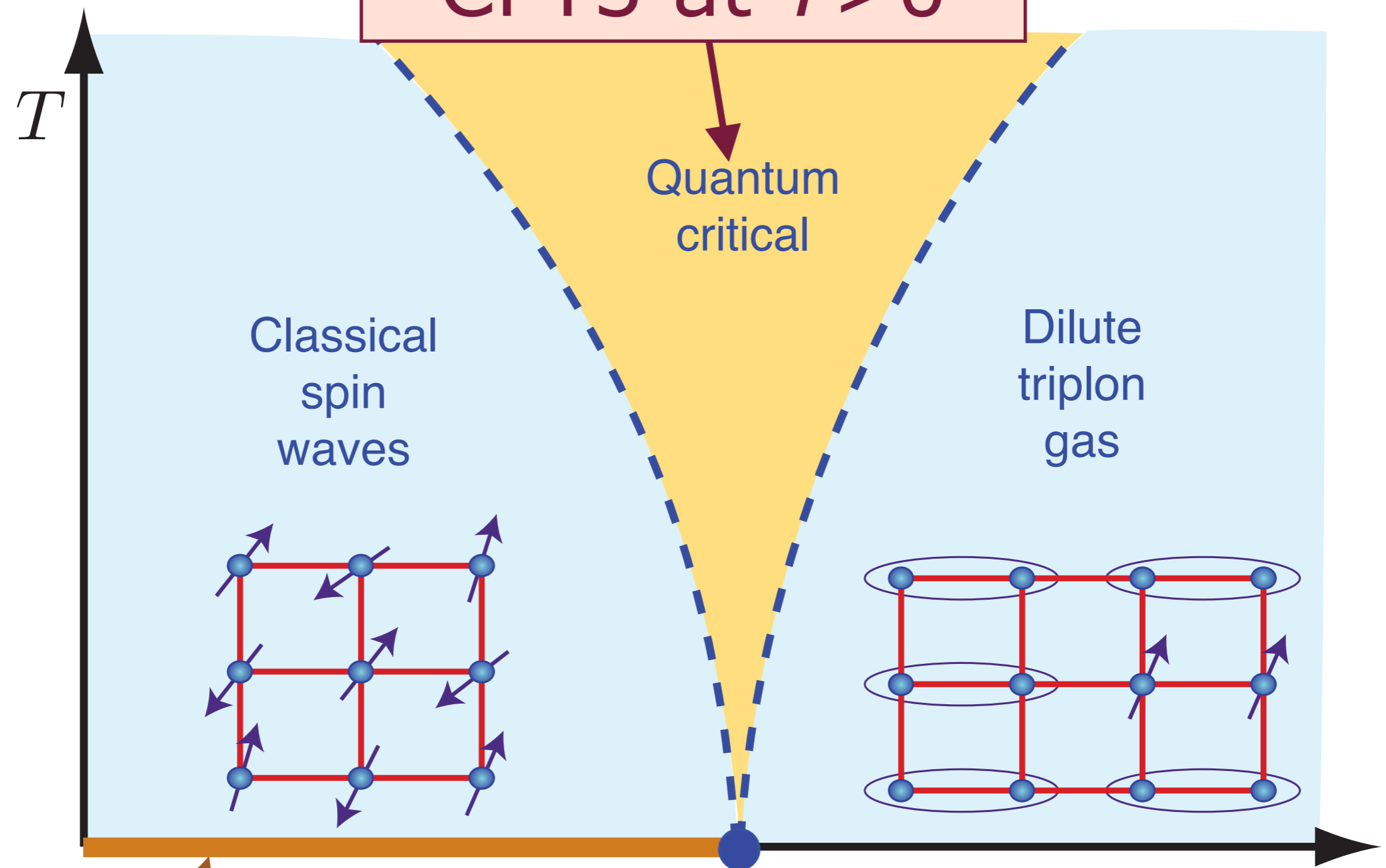
$O(3)$ order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



CFT3 at $T > 0$

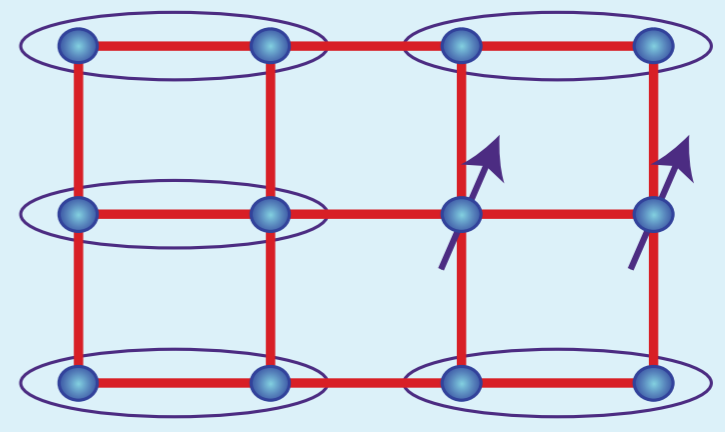
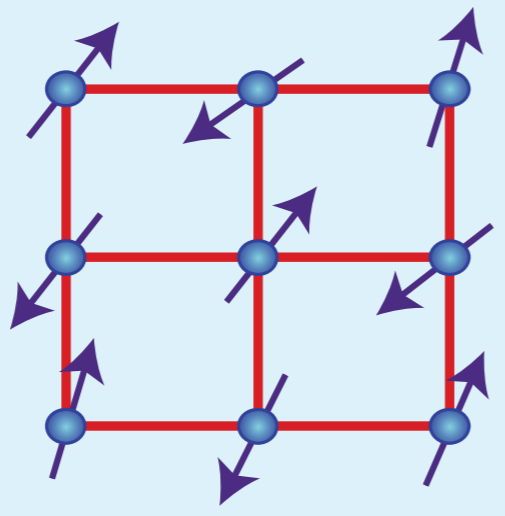
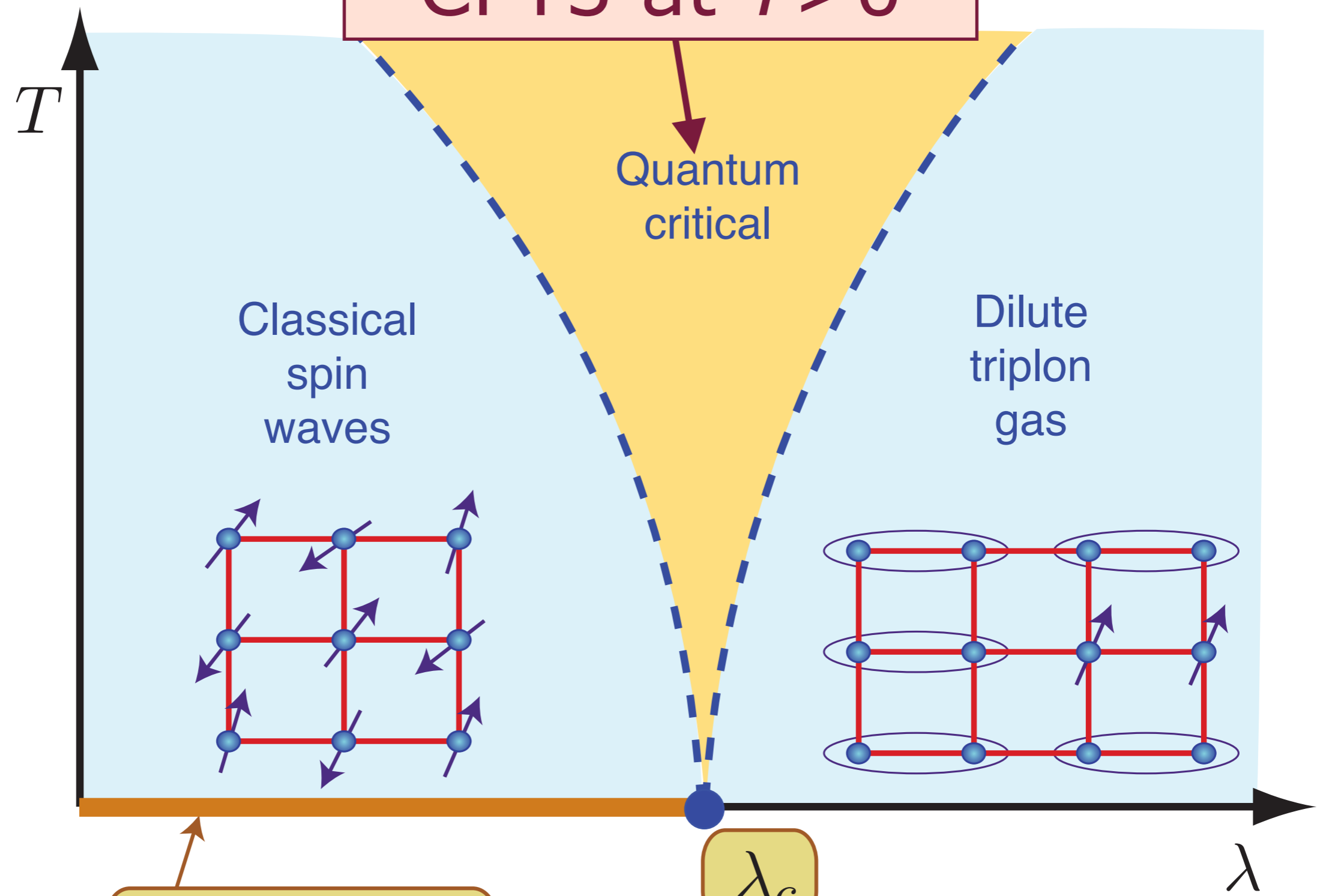


Neel order

λ_c

Pressure in $TlCuCl_3$

CFT3 at $T > 0$



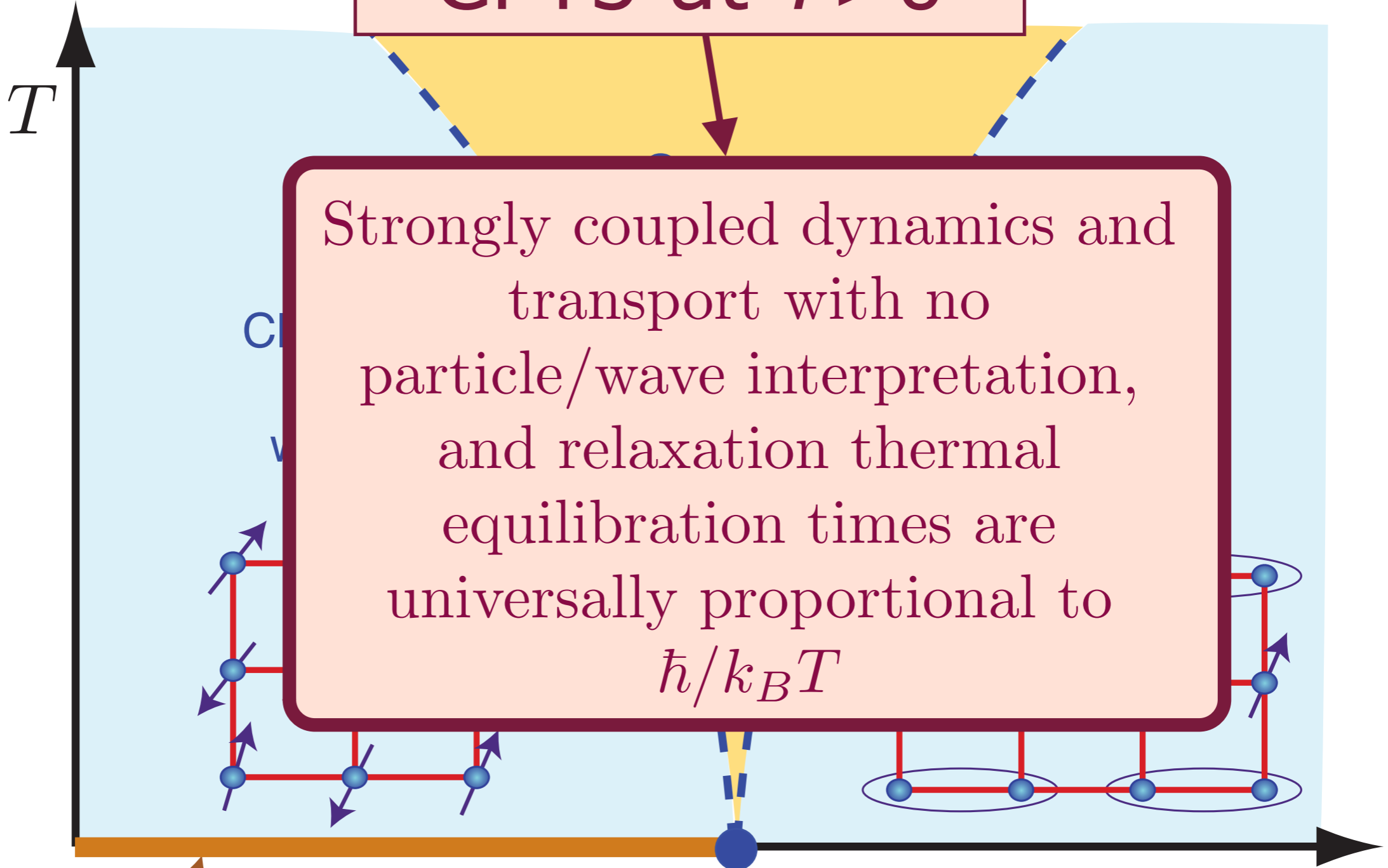
Neel order

λ_c

Pressure in $TlCuCl_3$



CFT3 at $T > 0$



Neel order

λ_c

Pressure in $TlCuCl_3$

Outline

1. Loss of antiferromagnetism in an insulator

Coupled-dimer antiferromagnets and quantum criticality

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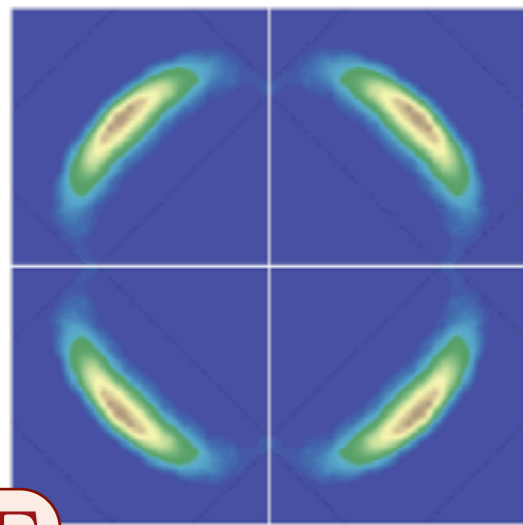
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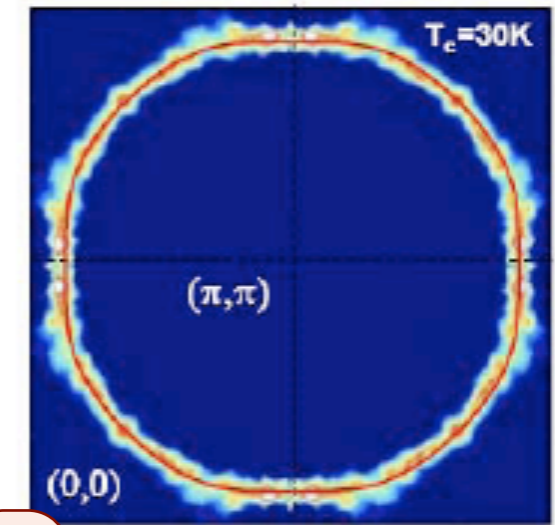
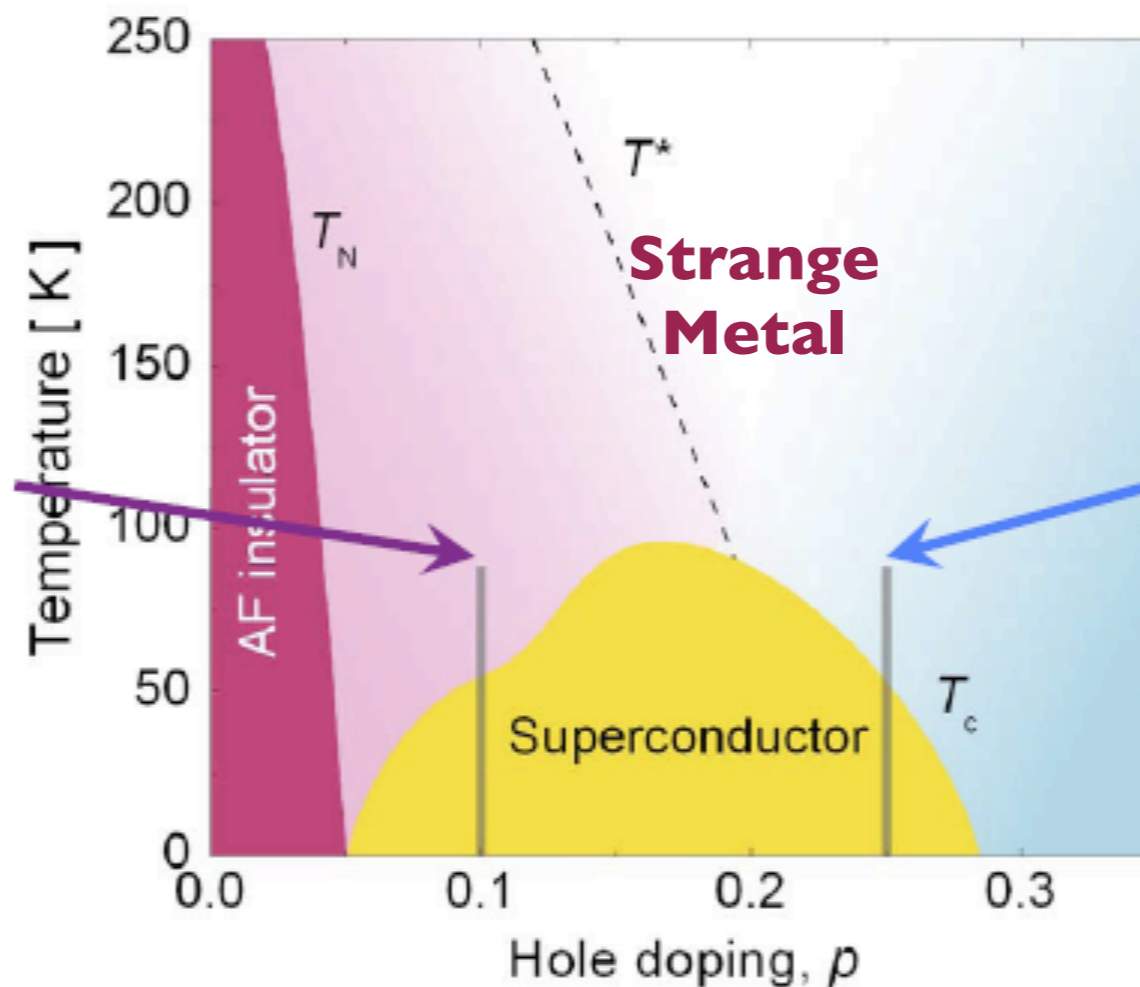
The AdS/CFT correspondence

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Γ

K.M. Shen et al., Science 2005



Γ

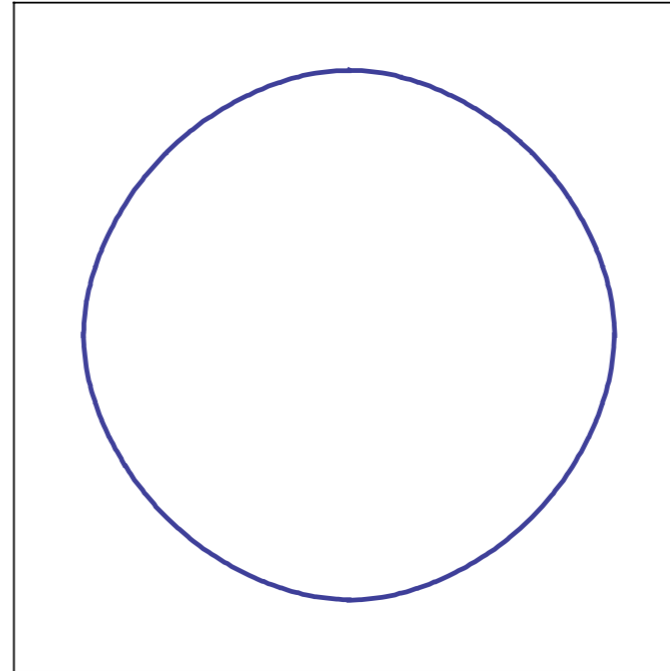
M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

Large hole
Fermi surface

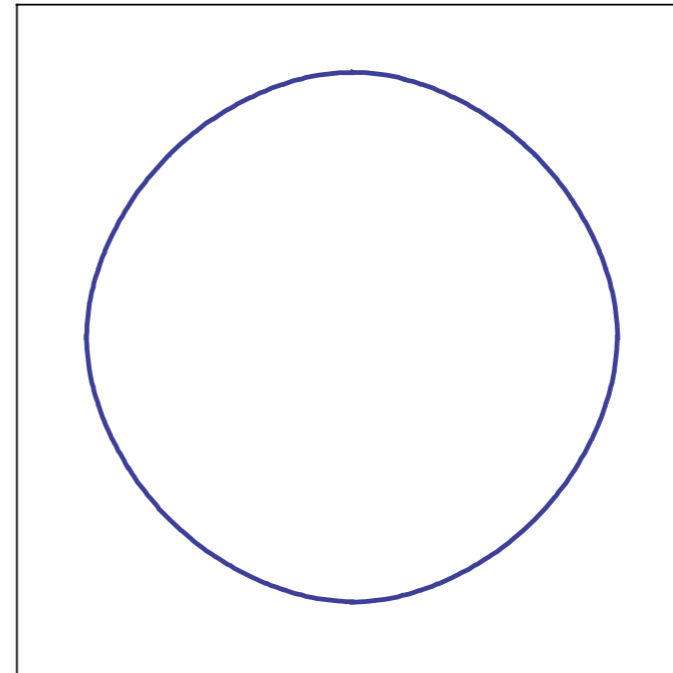
Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

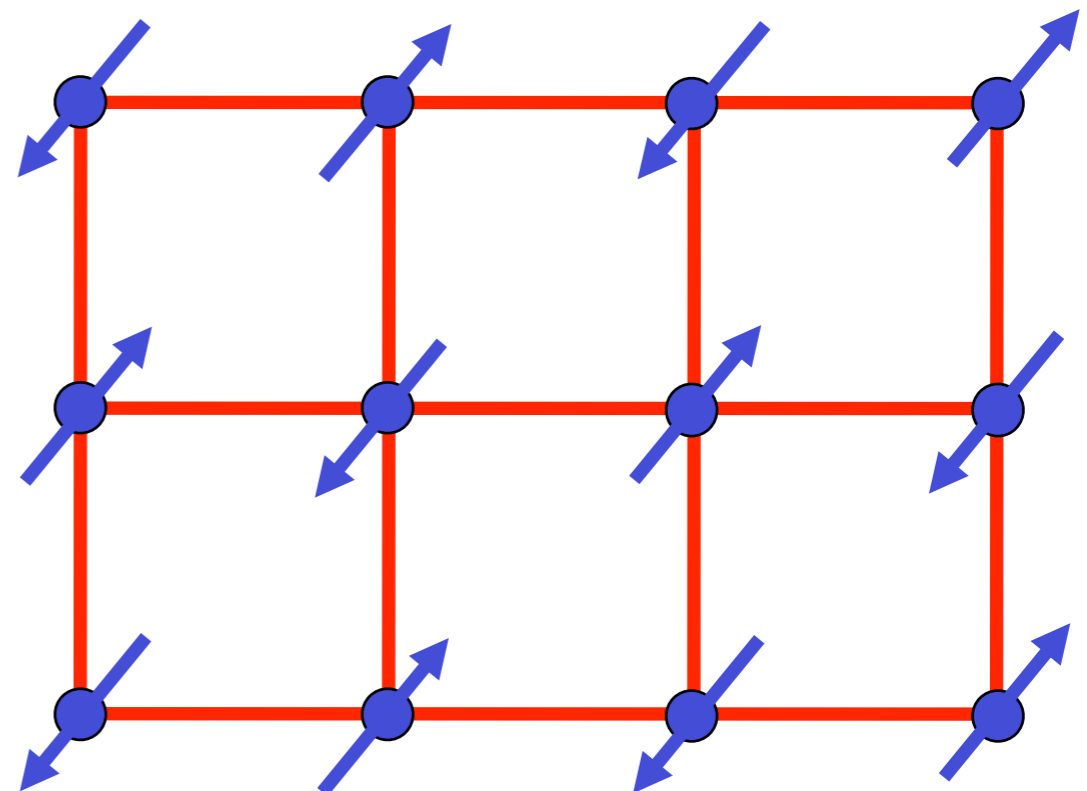


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+

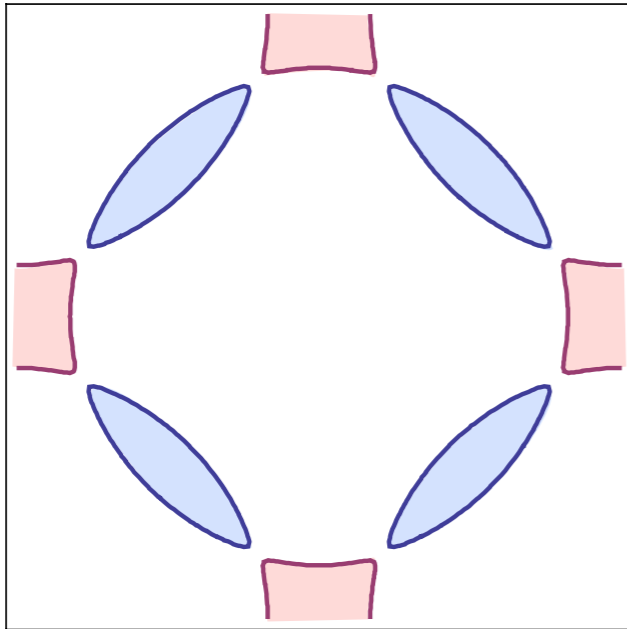


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

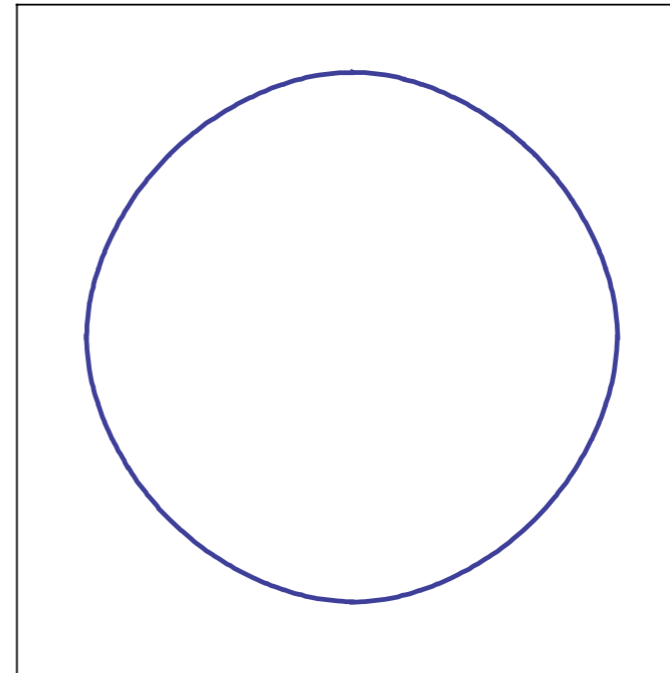
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



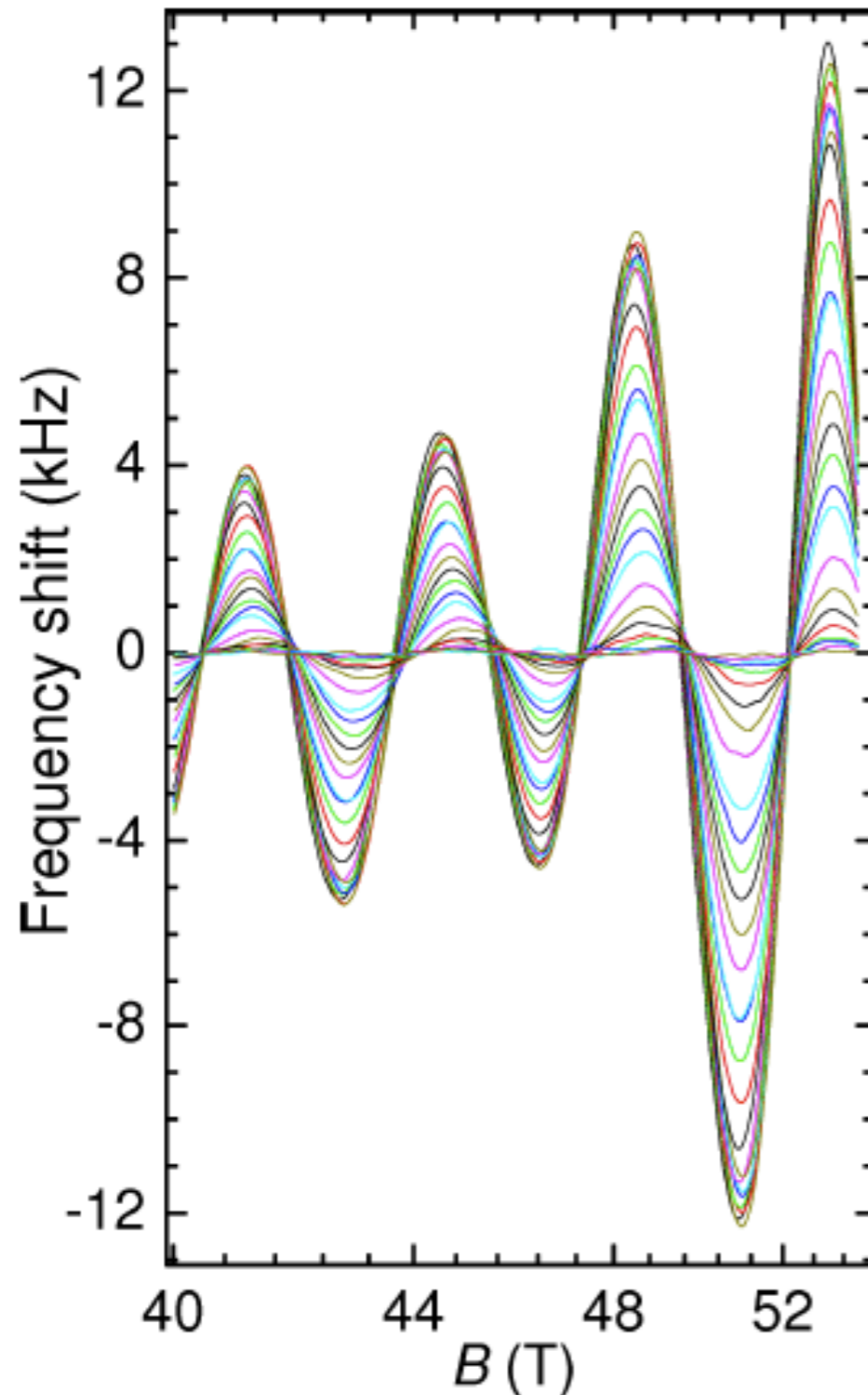
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

S

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Evidence for small Fermi pockets

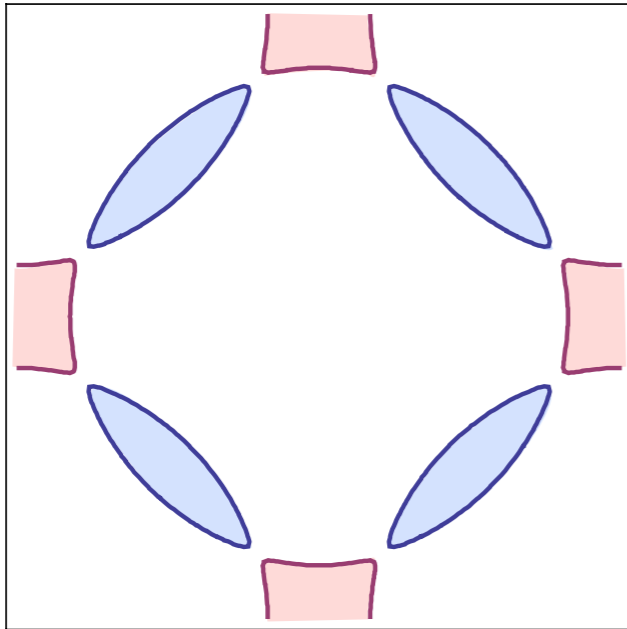


Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D.A. Bonn,
W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

Original observation:
N. Doiron-Leyraud, C. Proust,
D. LeBoeuf, J. Levallois,
J.-B. Bonnemaïson, R. Liang,
D.A. Bonn, W. N. Hardy,
and L. Taillefer,
Nature **447**, 565 (2007)

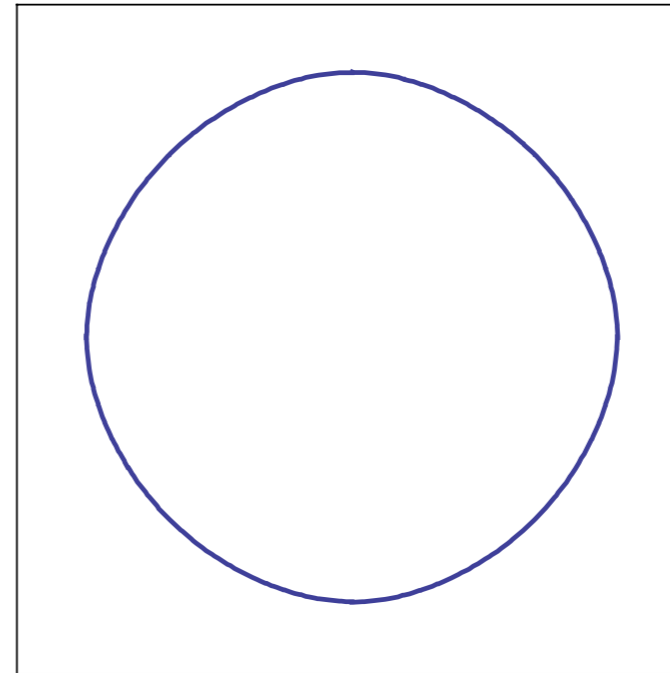
FIG. 2: Magnetic quantum oscillations measured in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



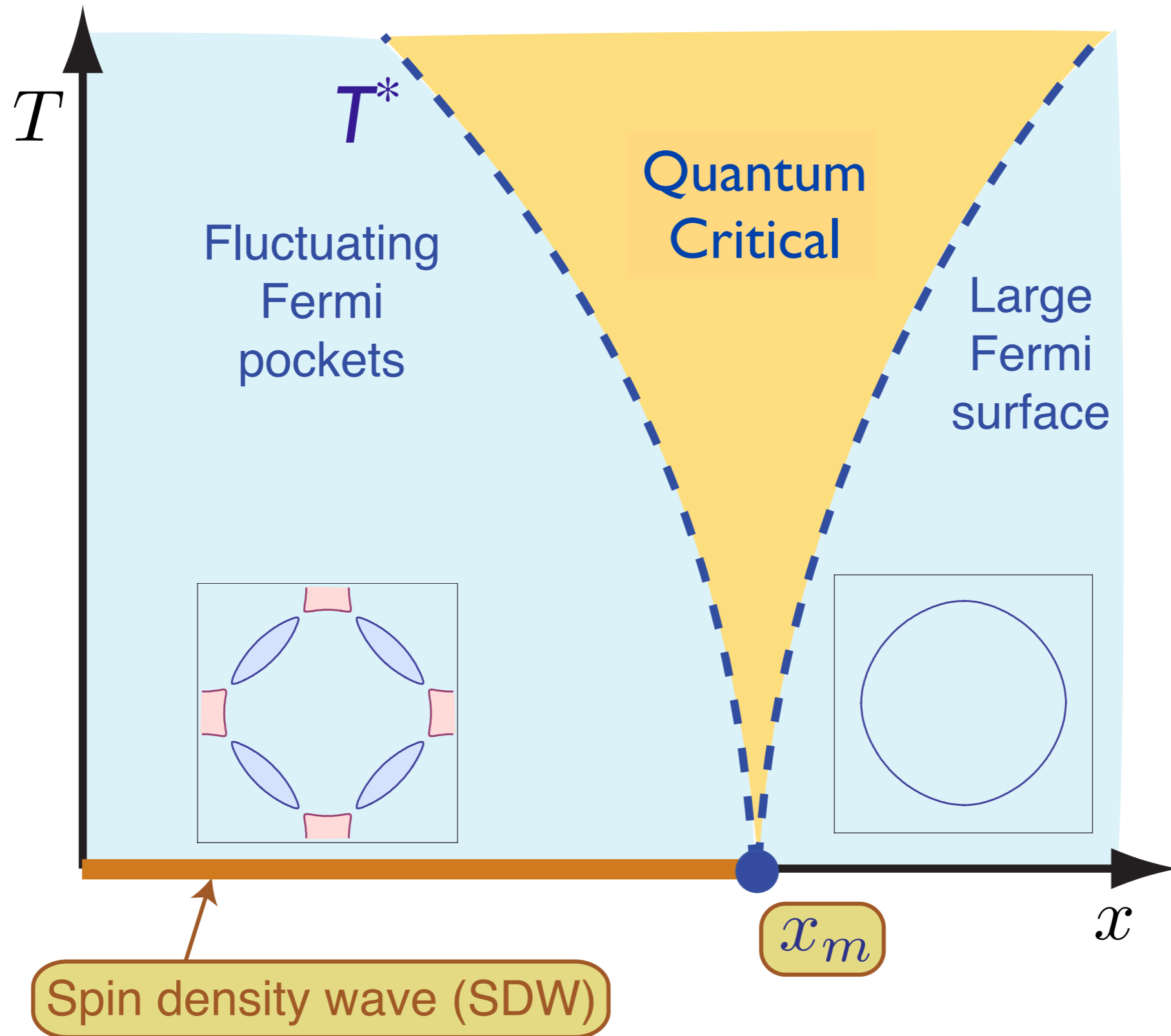
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Metal with “large”
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S

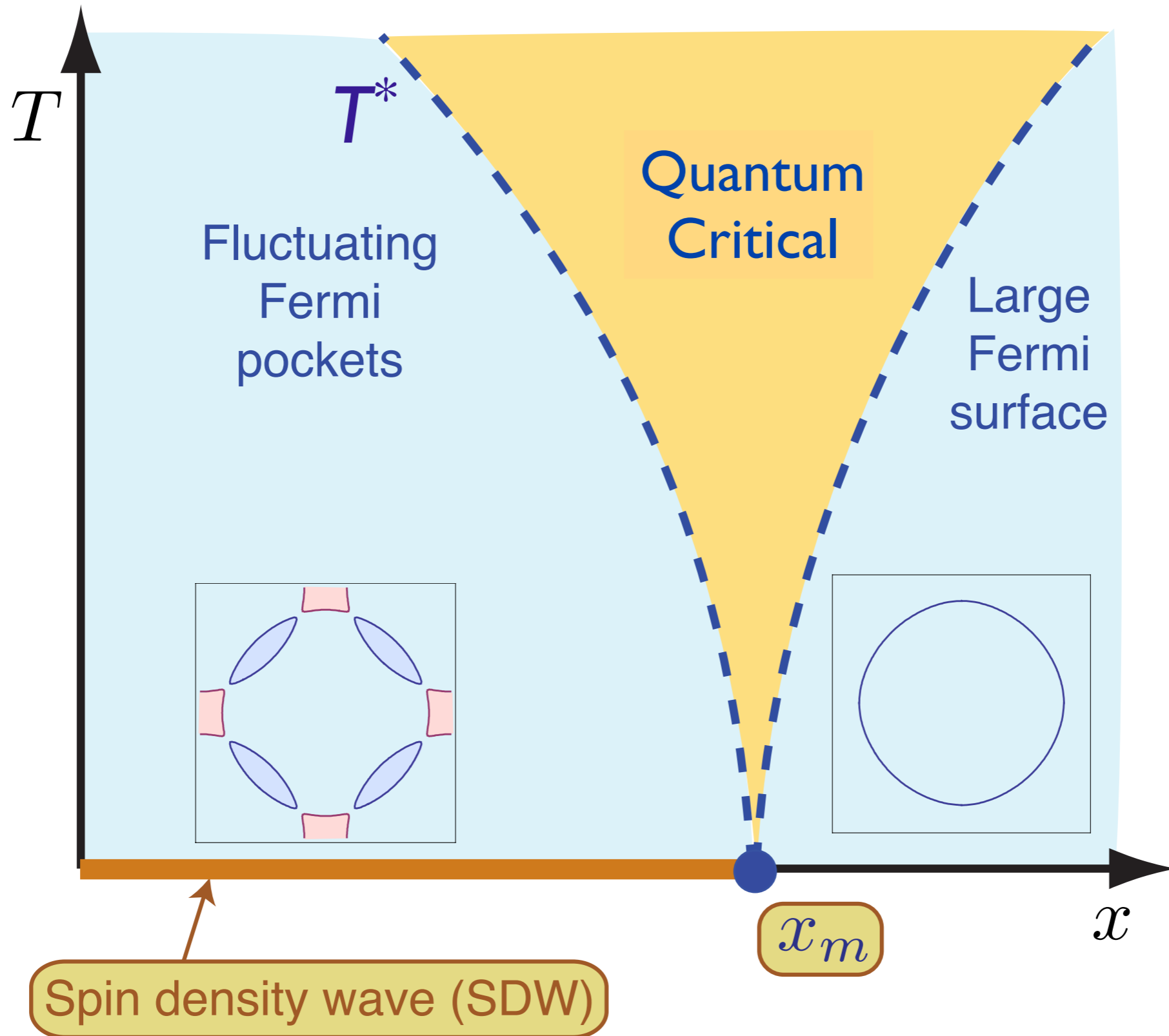
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



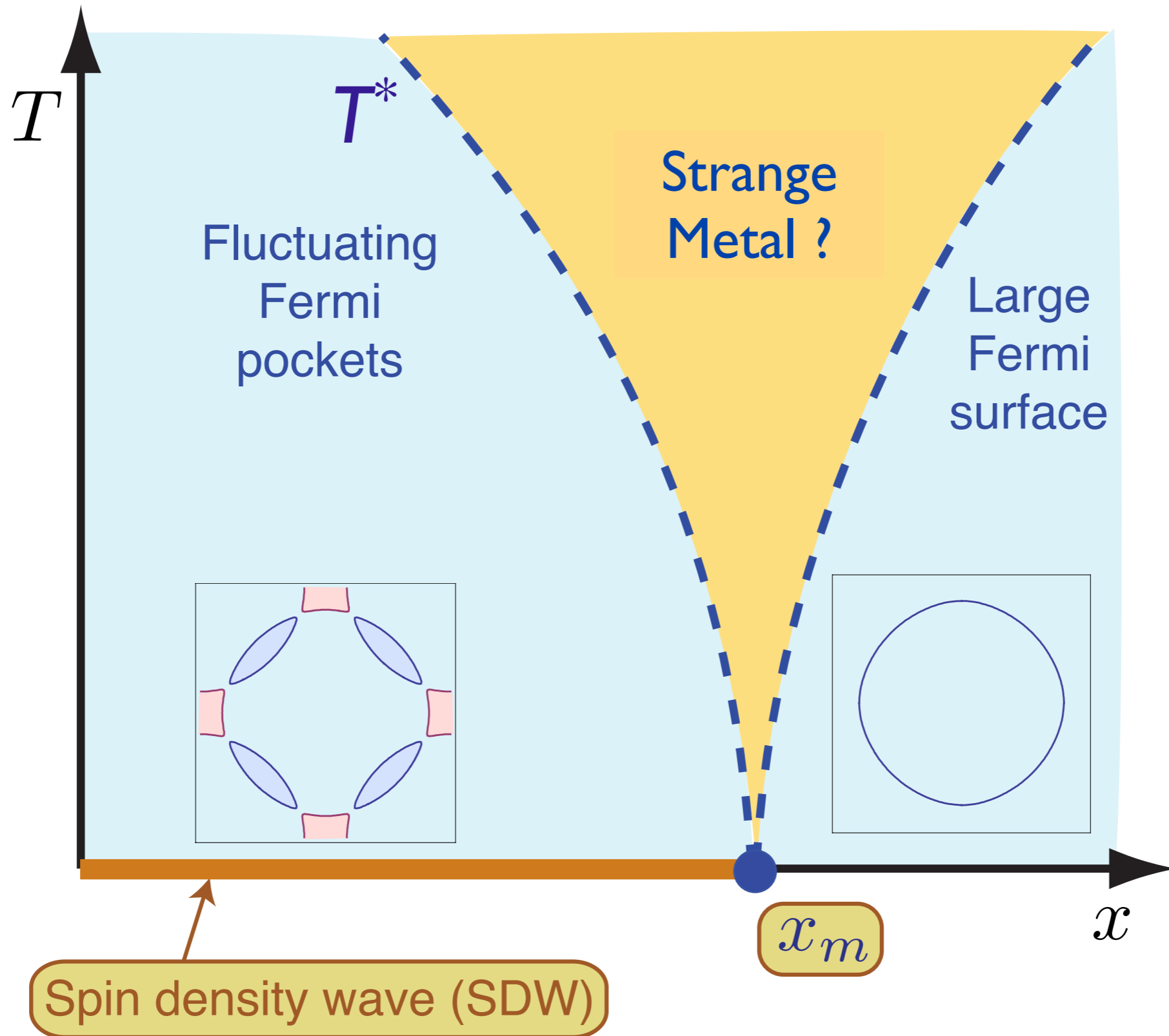
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

***d*-wave pairing near a spin-density-wave instability**

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†

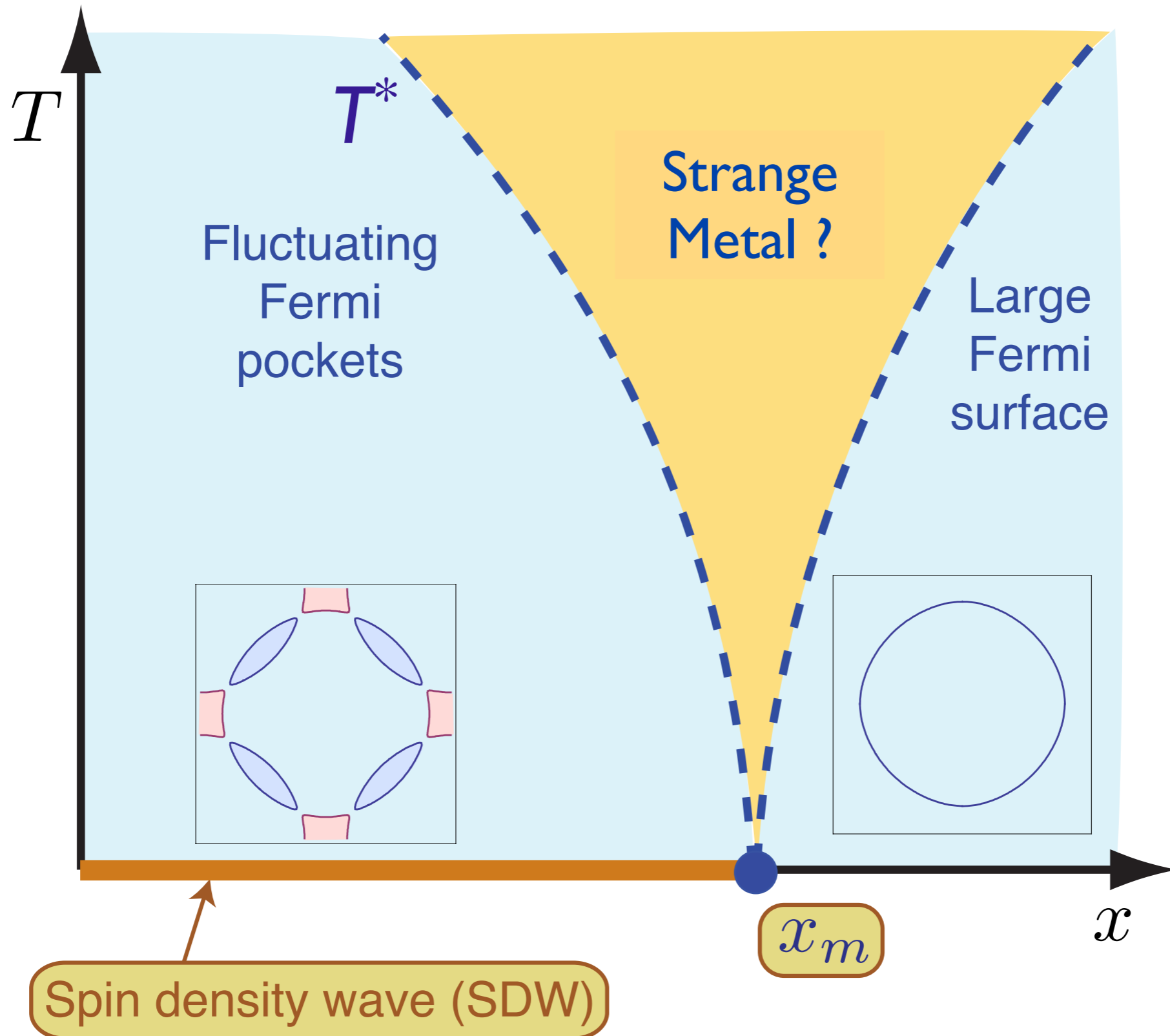
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 23 June 1986)

We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet *d*-wave pairing interaction. For a cubic band the singlet ($d_{x^2-y^2}$ and $d_{3z^2-r^2}$) channels are enhanced while the singlet (d_{xy}, d_{xz}, d_{yz}) and triplet *p*-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

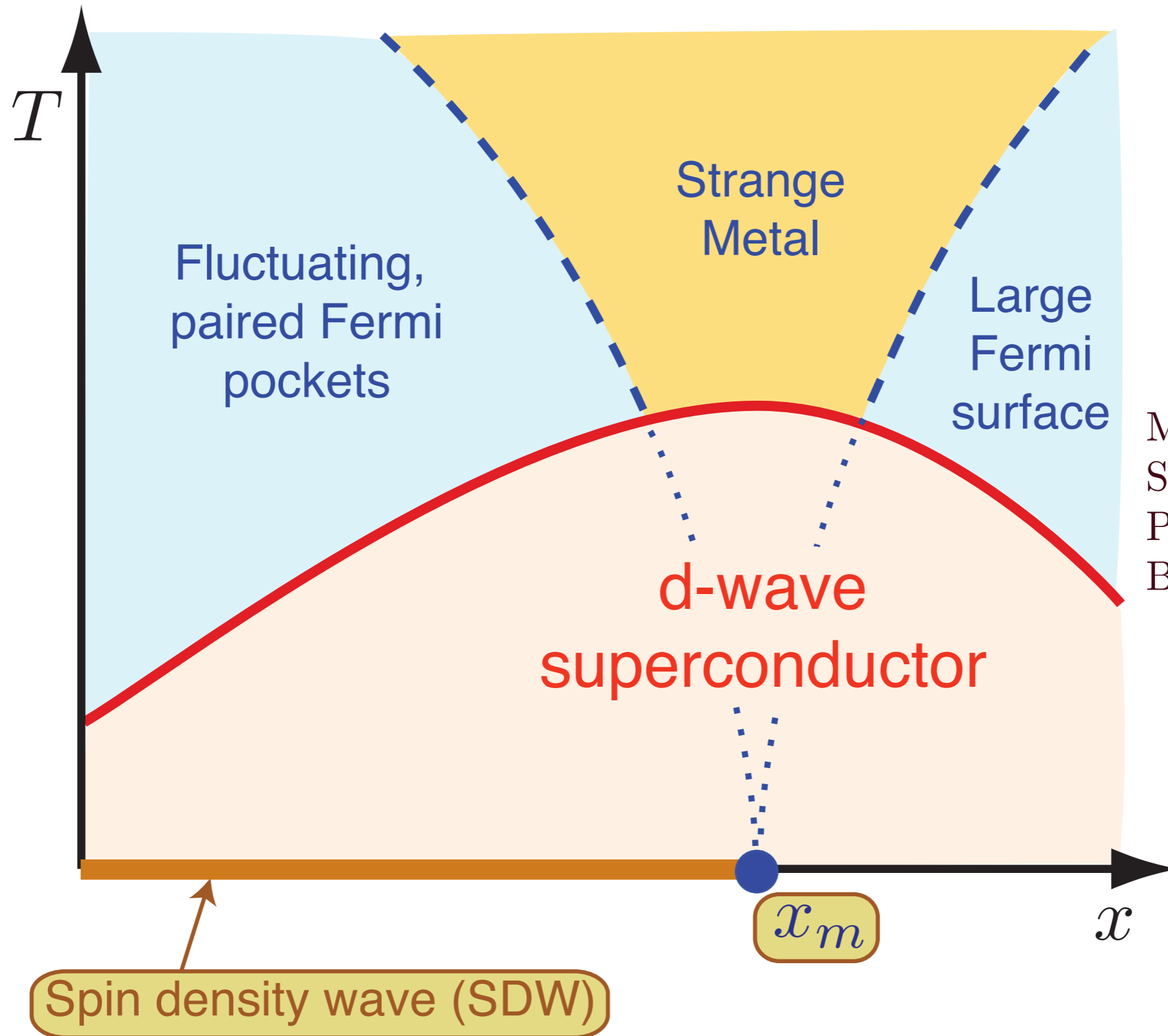
Physical Review B **34**, 8190 (1986)

Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

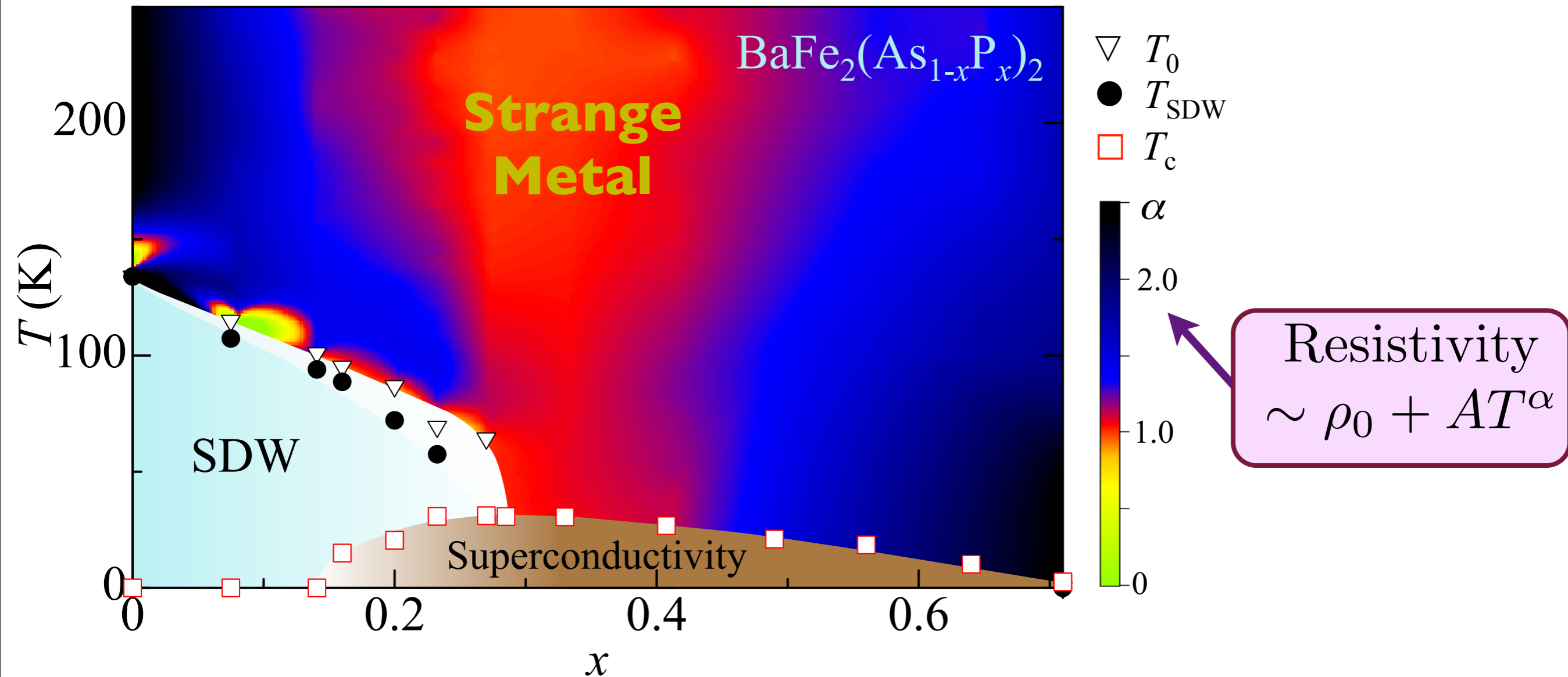
Theory of quantum criticality in the cuprates



M. A. Metlitski and
S. Sachdev,
Physical Review
B **82**, 075128 (2010)

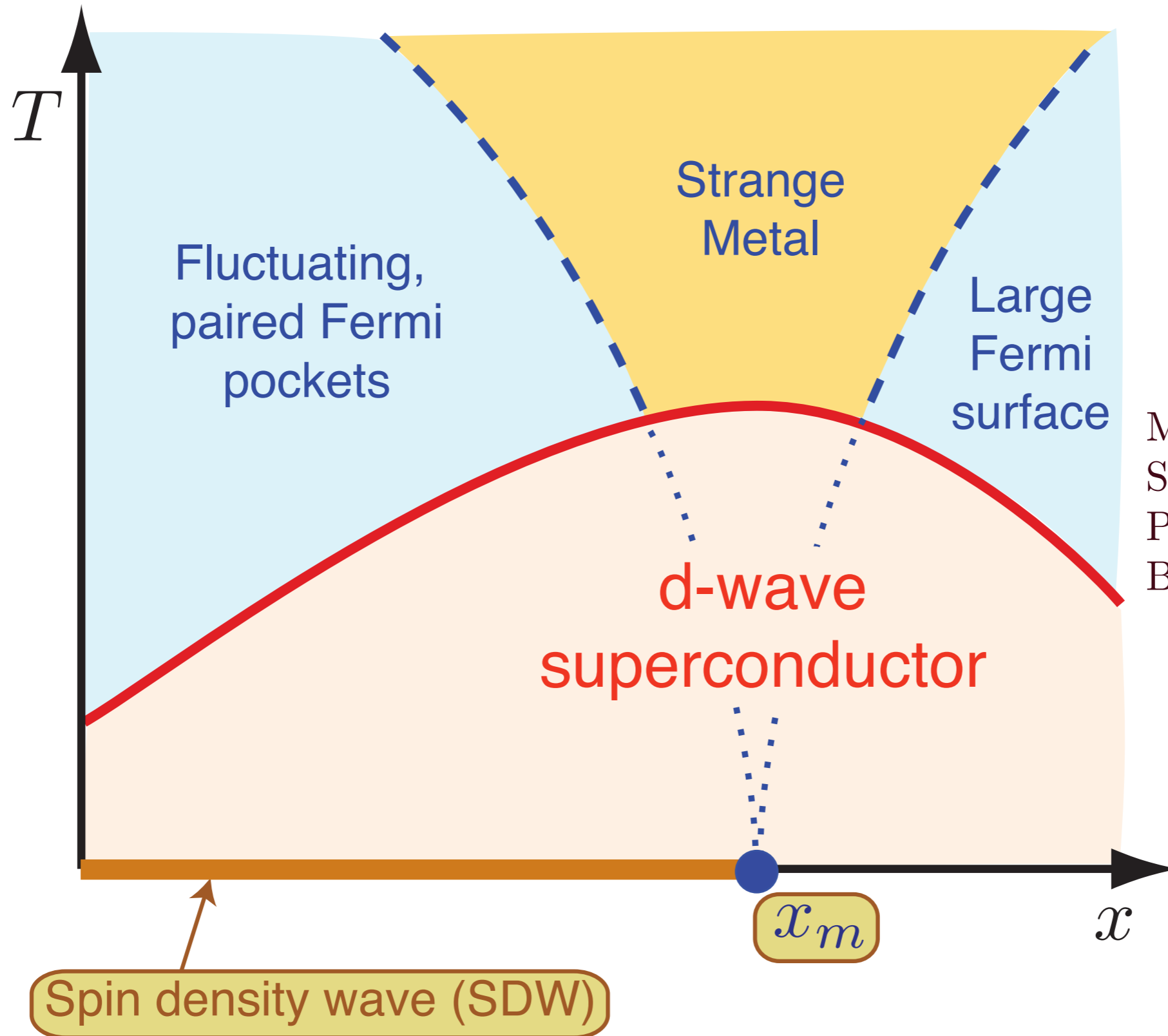
SDW quantum critical point is unstable to d -wave superconductivity
This instability is stronger than that in the BCS theory

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

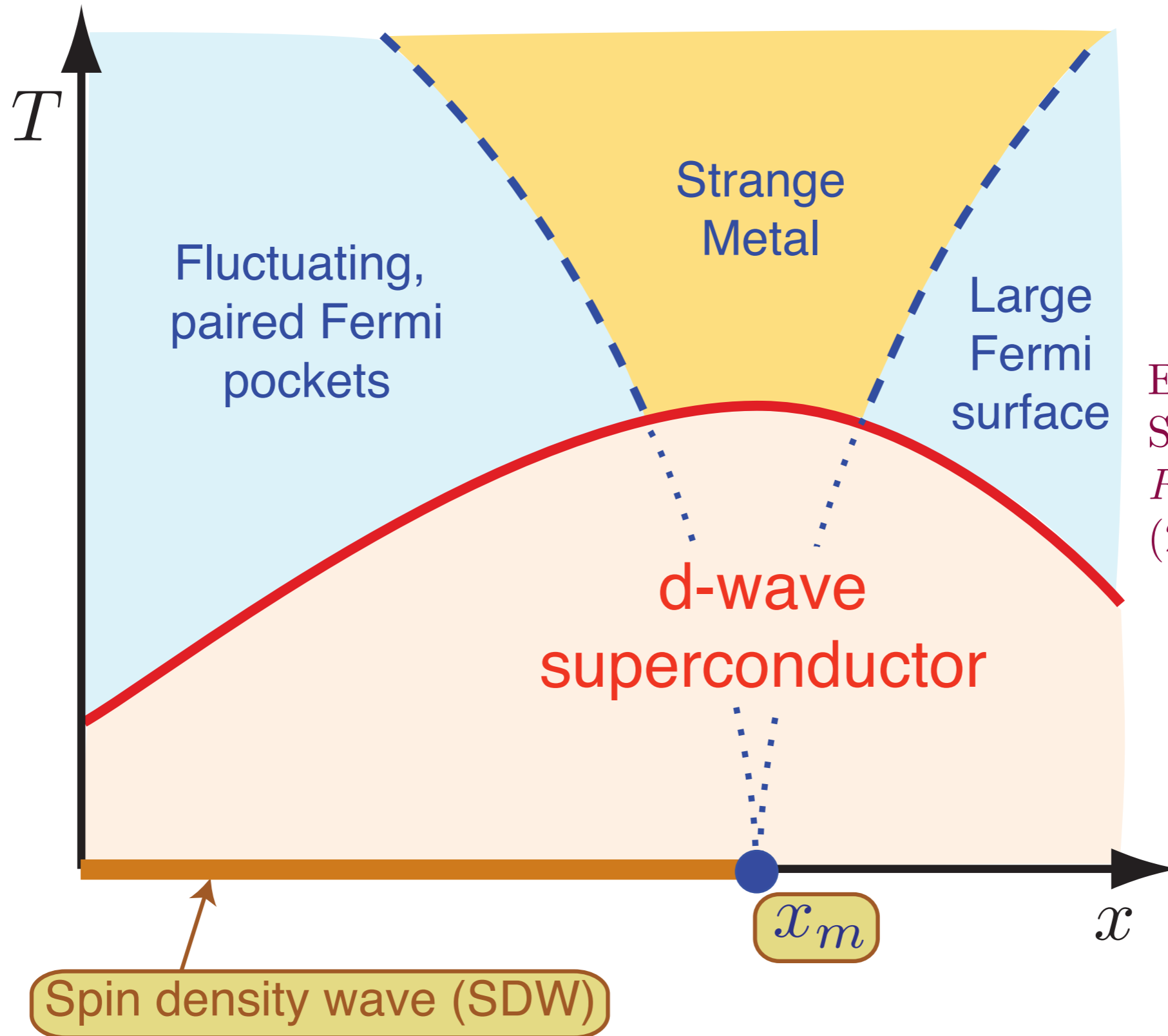
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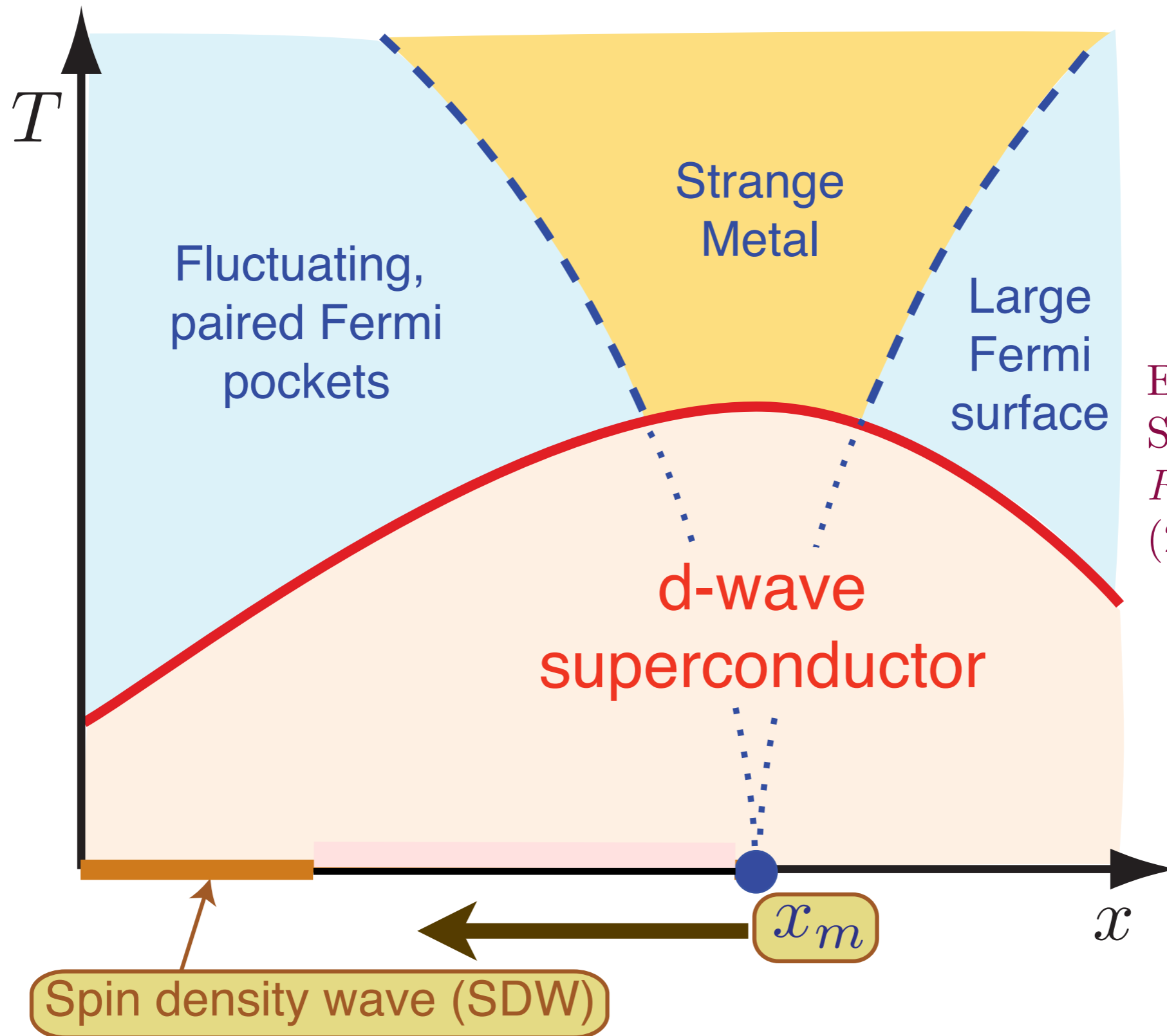
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

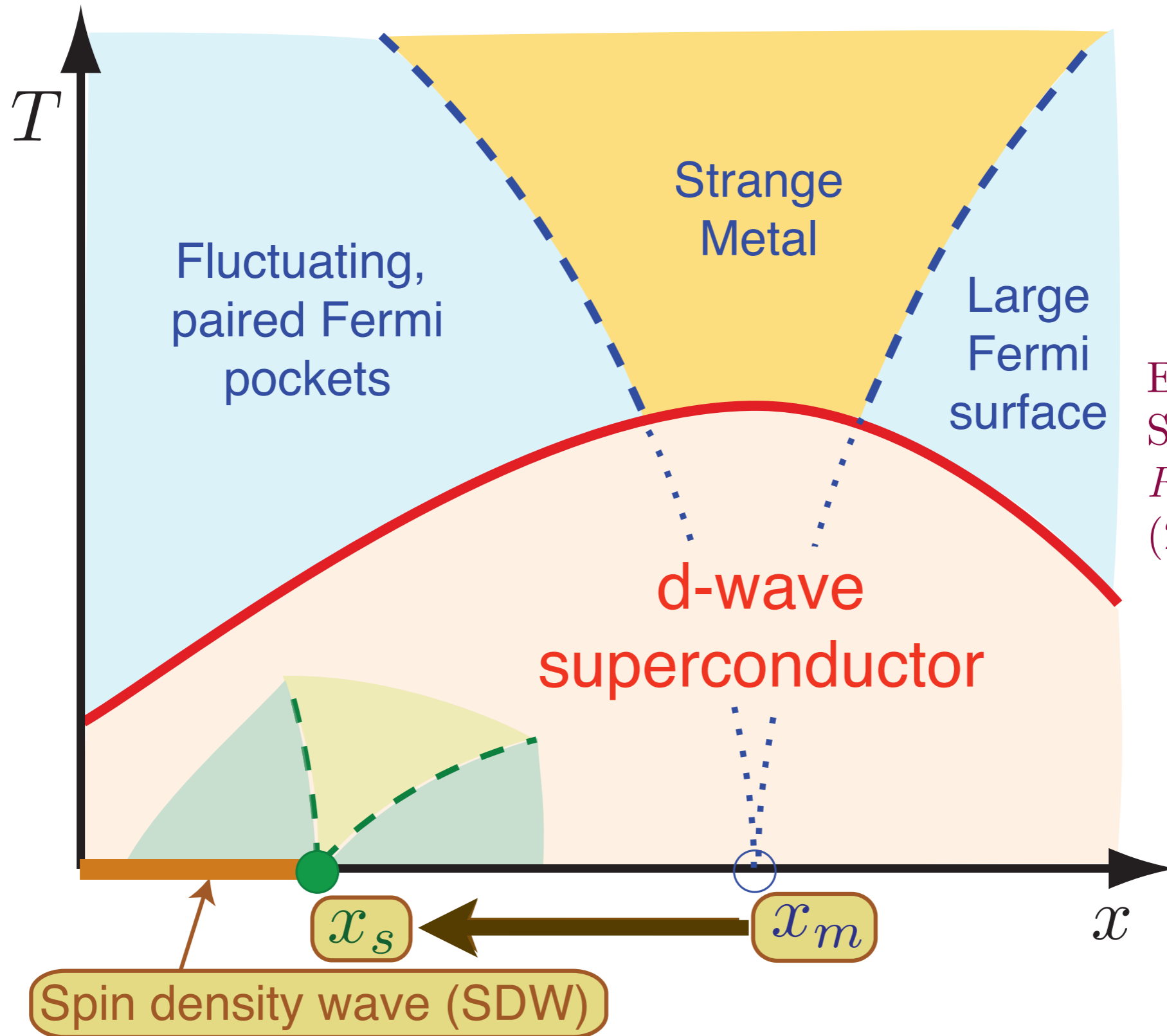
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

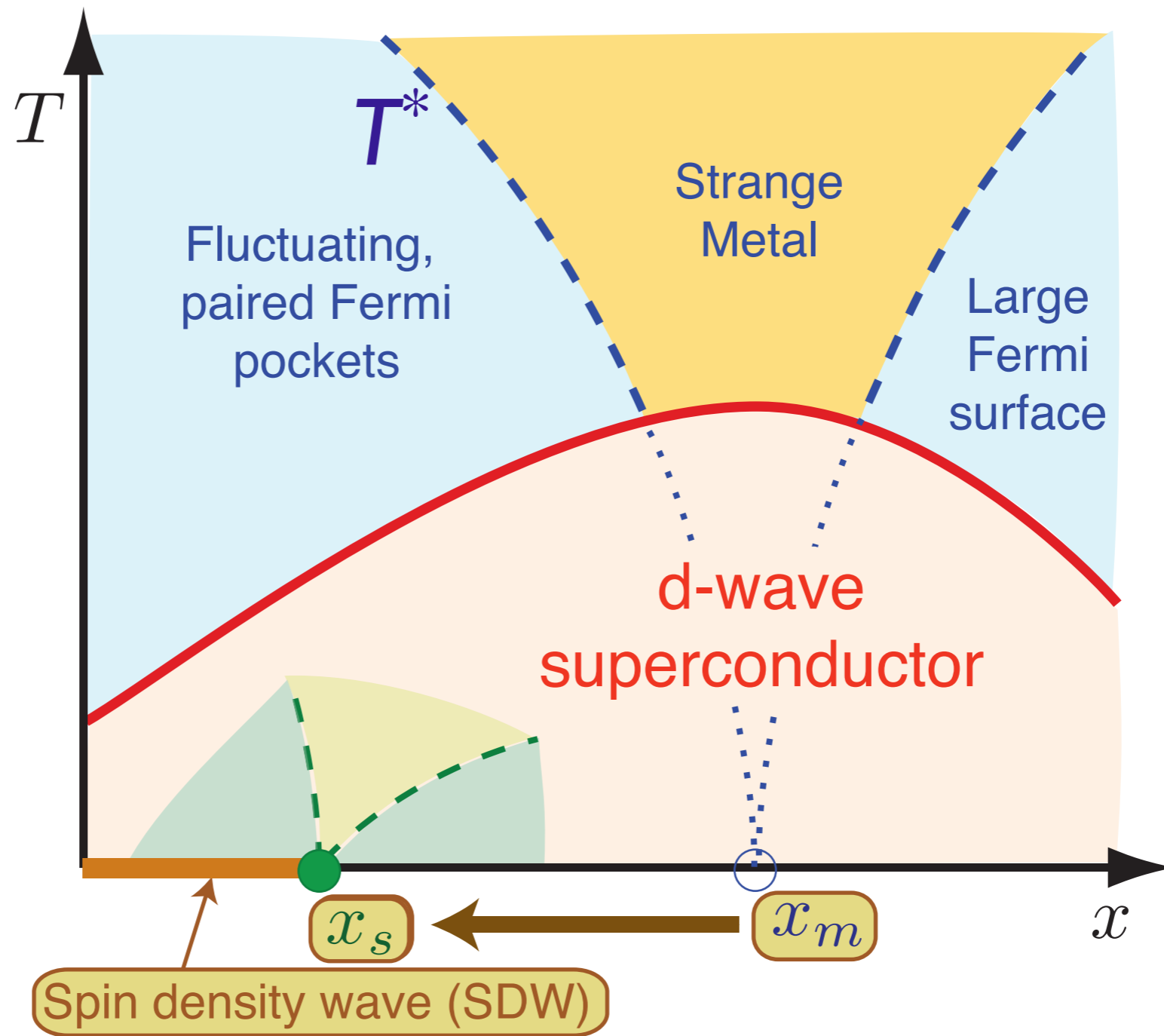
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Theory of quantum criticality in the cuprates

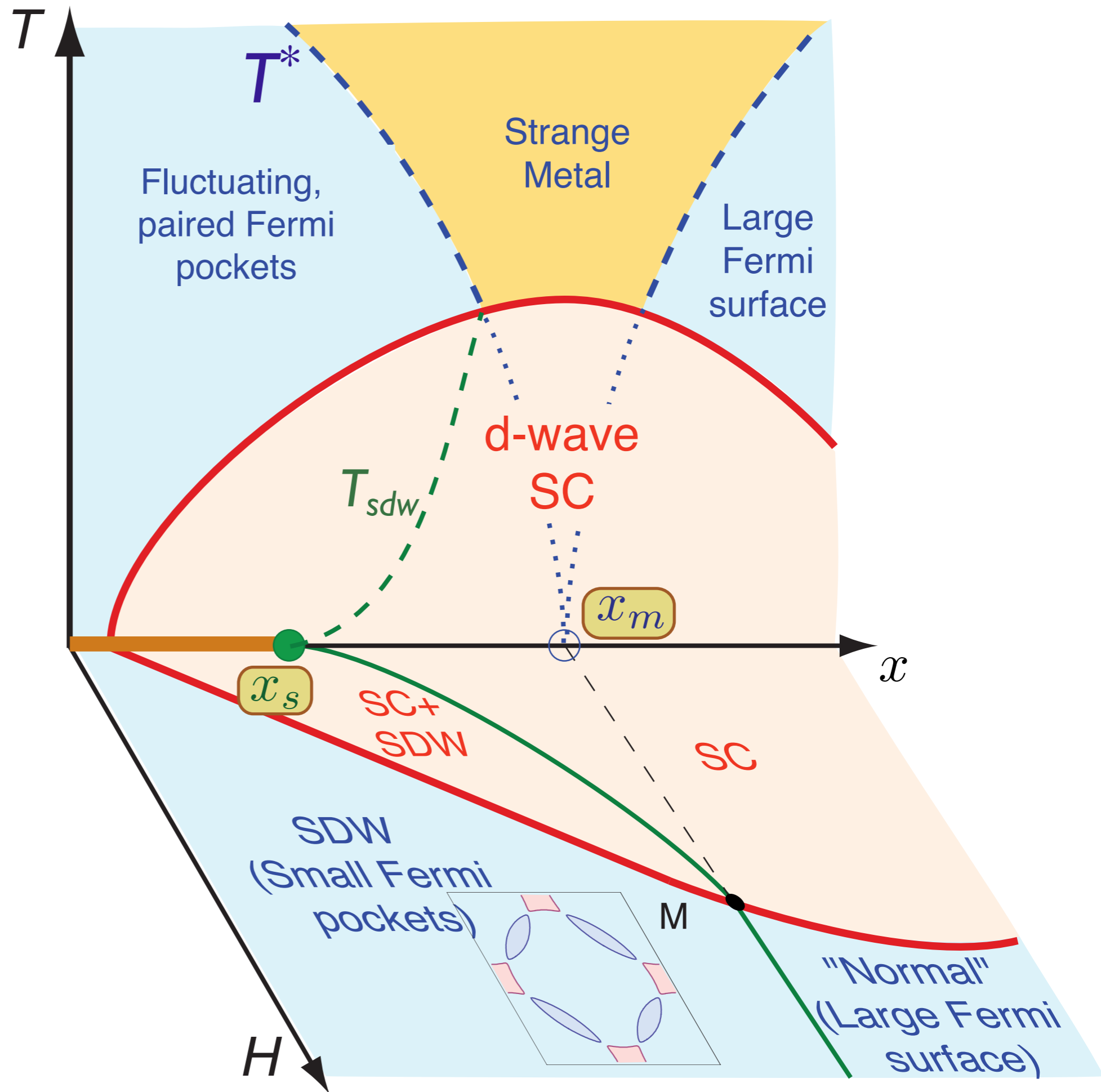


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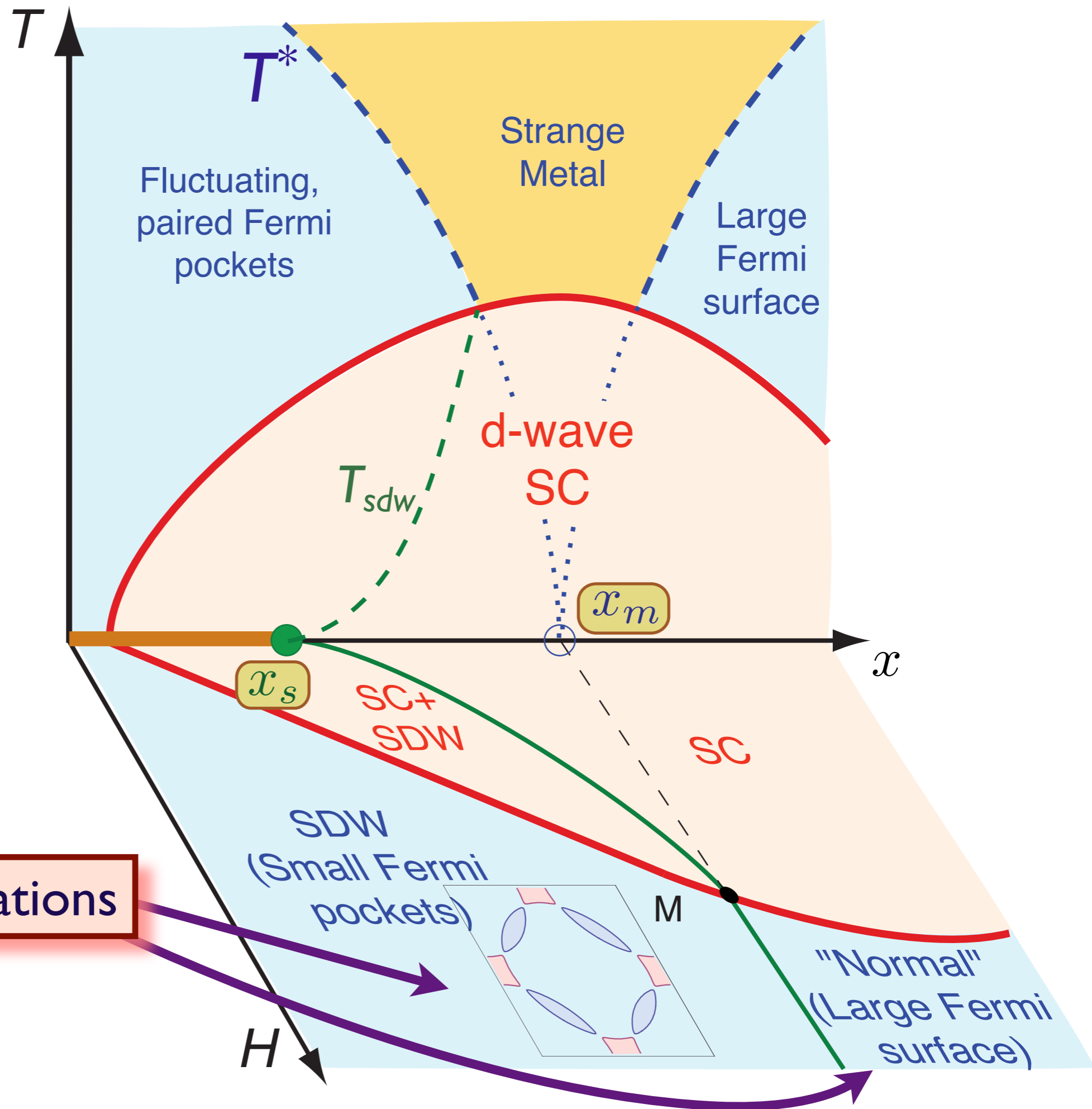
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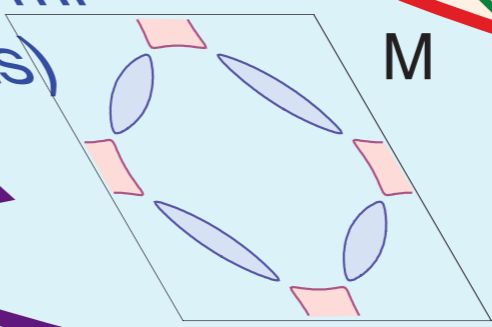
E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).



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and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).

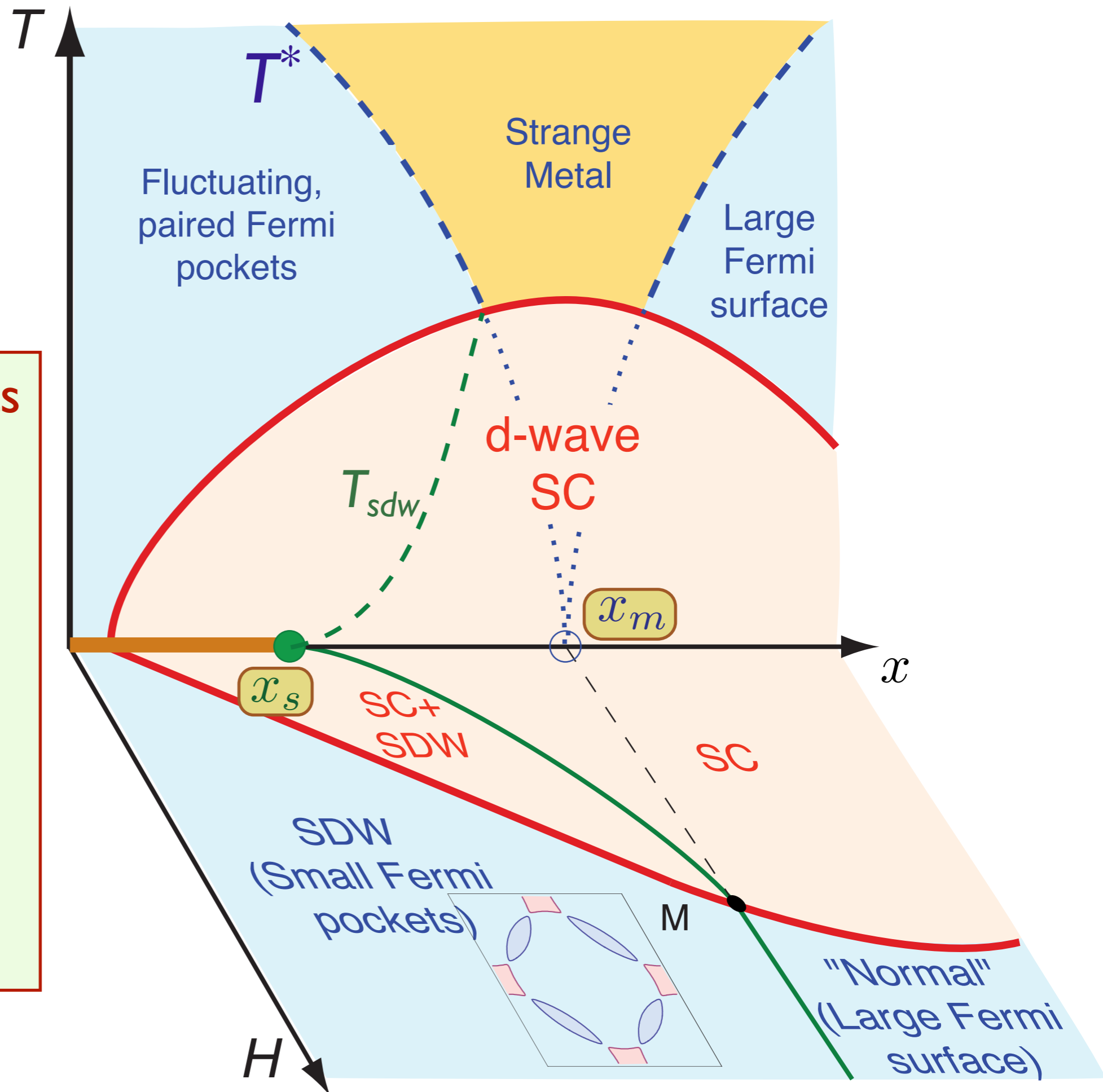


Quantum oscillations

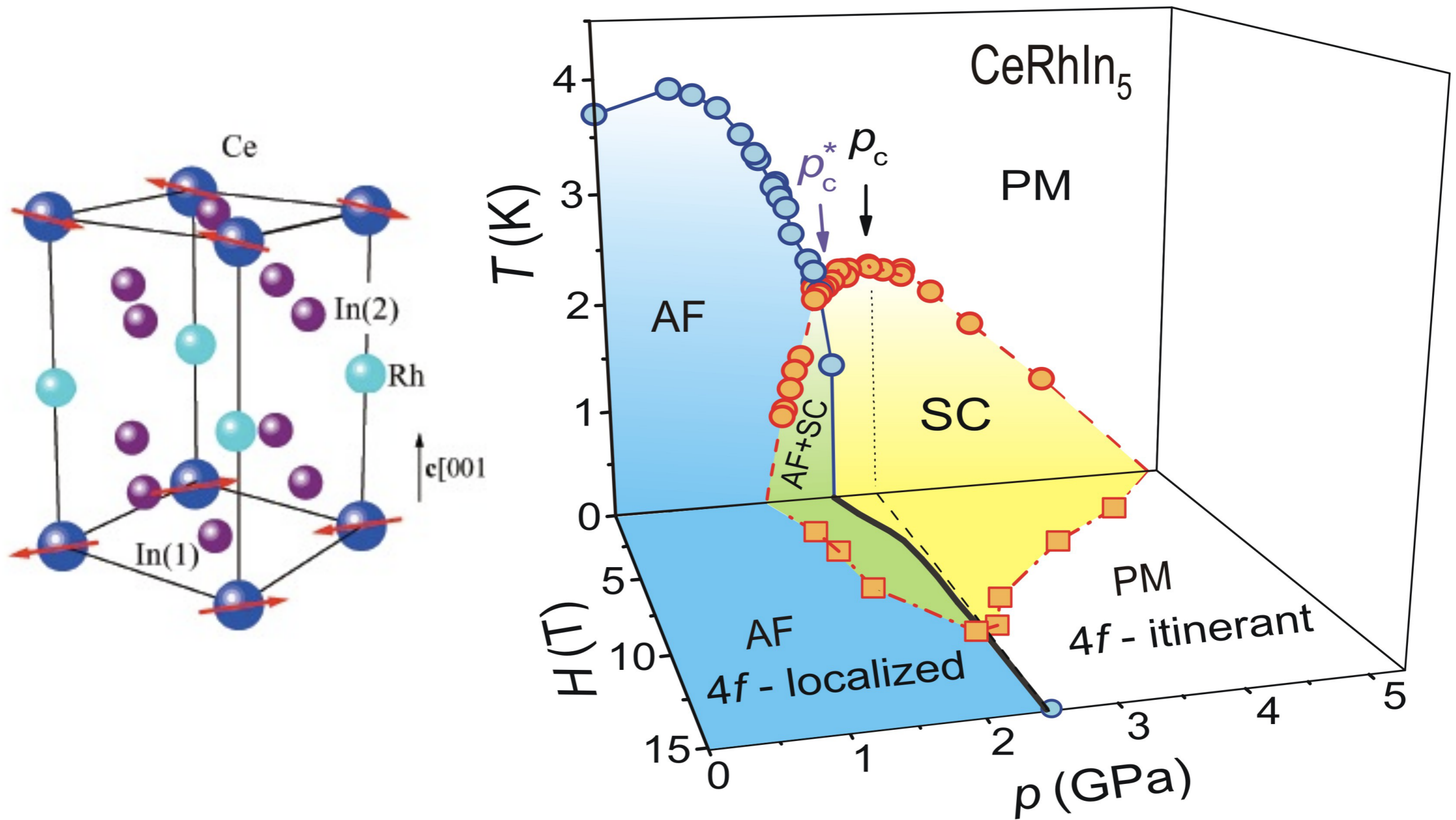


E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).

Many experiments
have presented
evidence for the
predicted
green quantum
phase transition
line
from SC to SC
+SDW in a
magnetic field



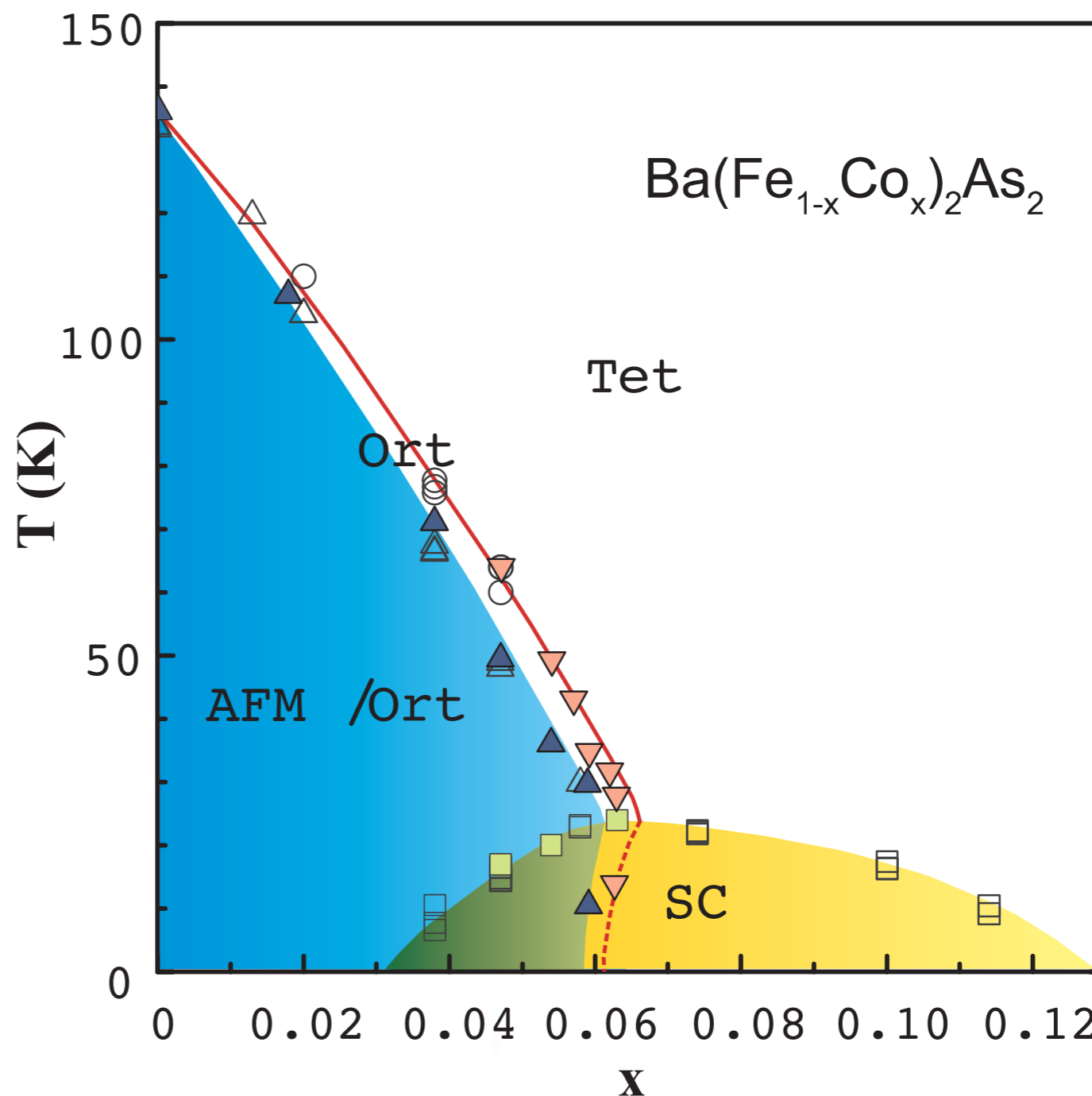
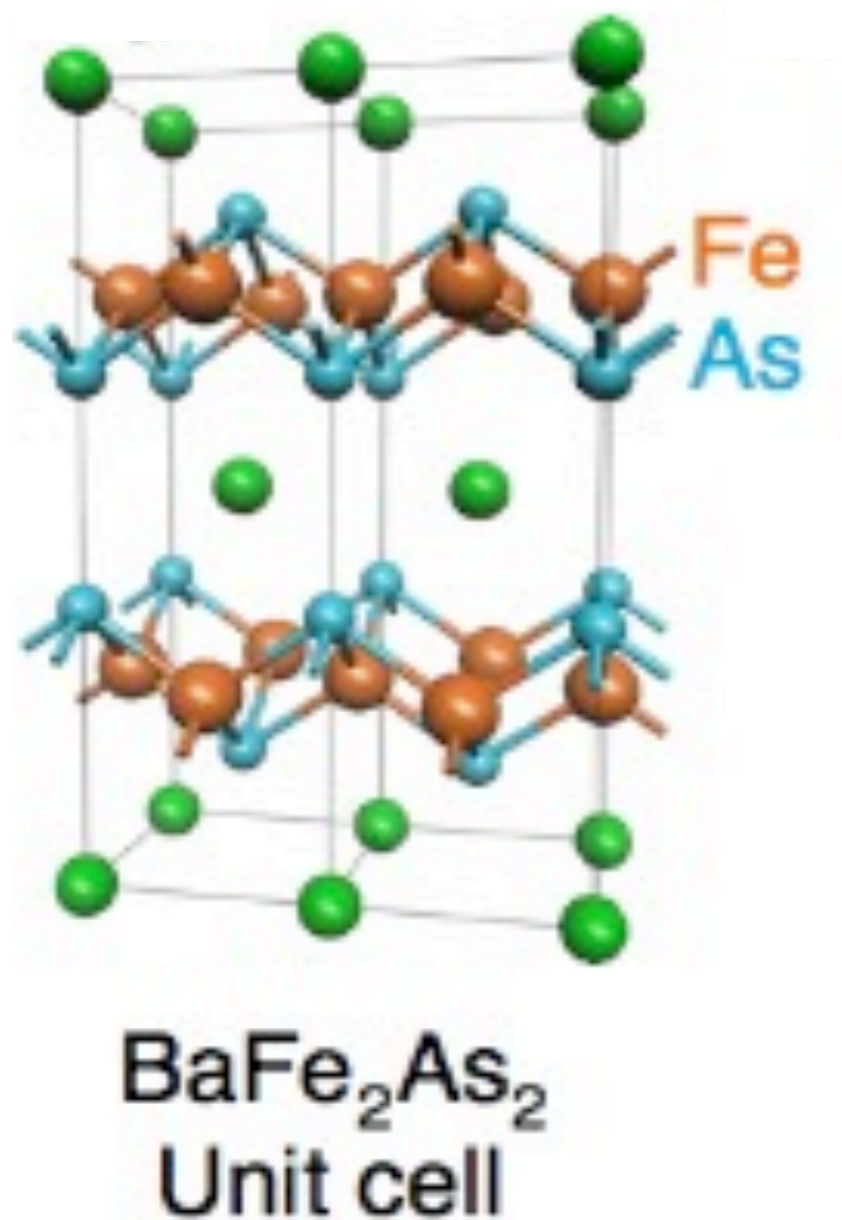
Similar phase diagram for CeRhIn₅



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Iron pnictides:

a new class of high temperature superconductors



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

Outline

1. Loss of antiferromagnetism in an insulator

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2. Onset of antiferromagnetism in a metal

From large Fermi surfaces to Fermi pockets, d-wave superconductivity, and competing orders

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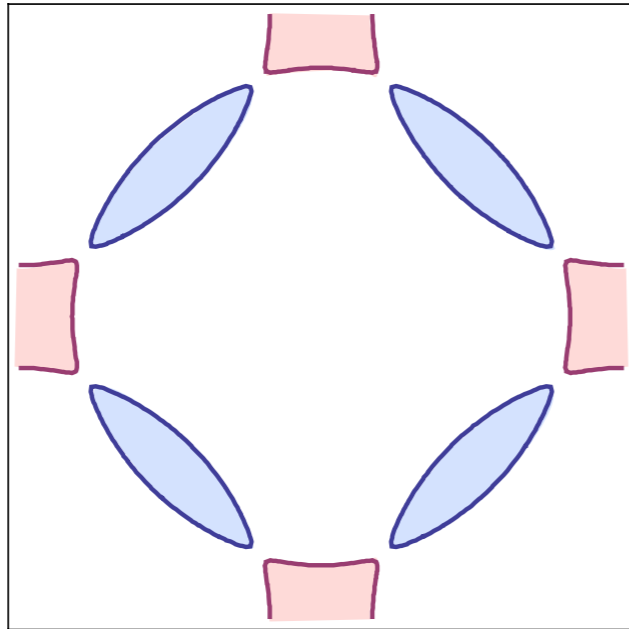
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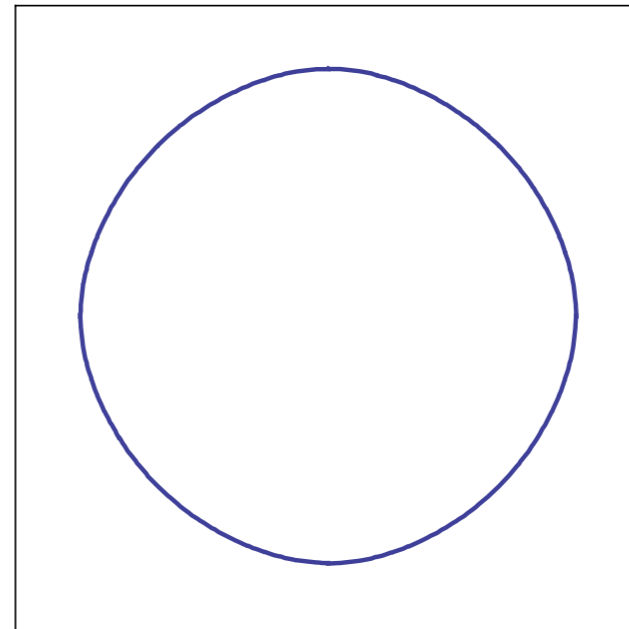
Quantum criticality of the onset of antiferromagnetism in a metal

$$\langle \vec{\varphi} \rangle \neq 0$$



Metal with electron
and hole pockets

$$\langle \vec{\varphi} \rangle = 0$$

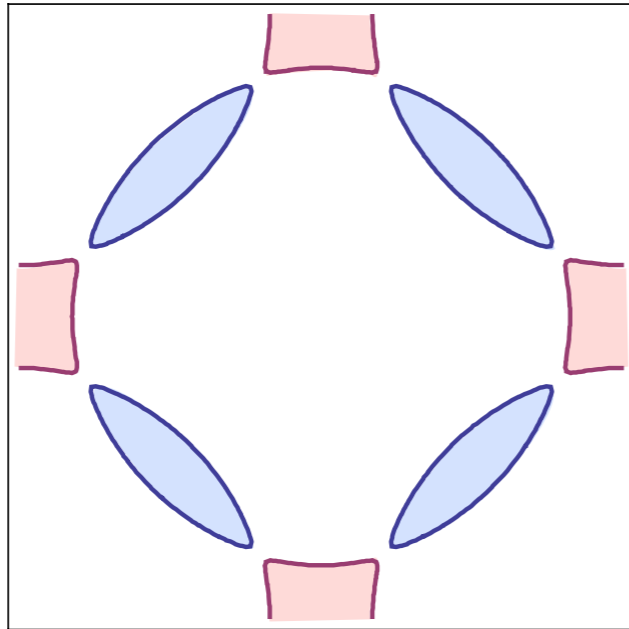


Metal with “large”
Fermi surface

$\rightarrow S$

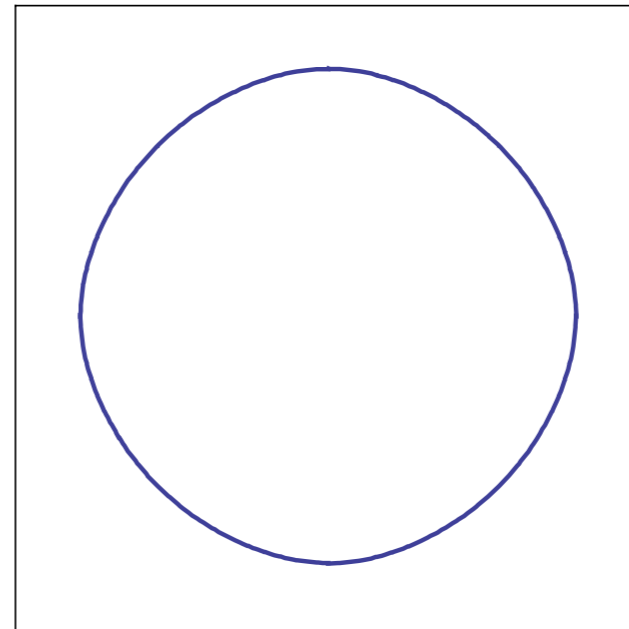
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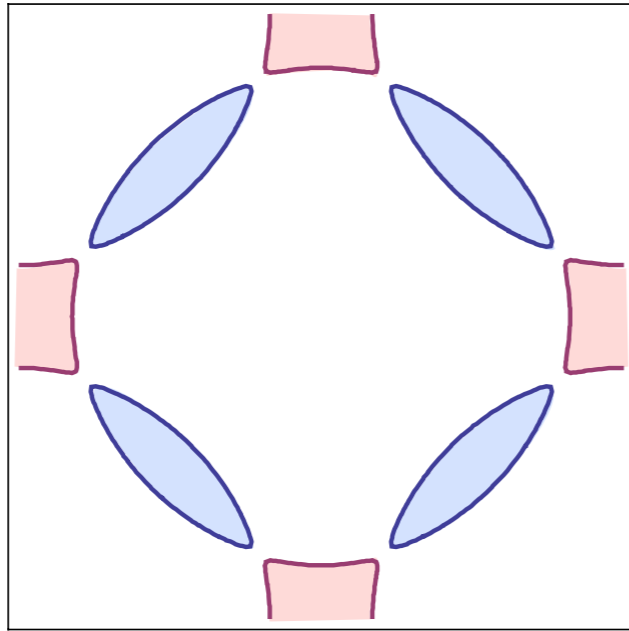


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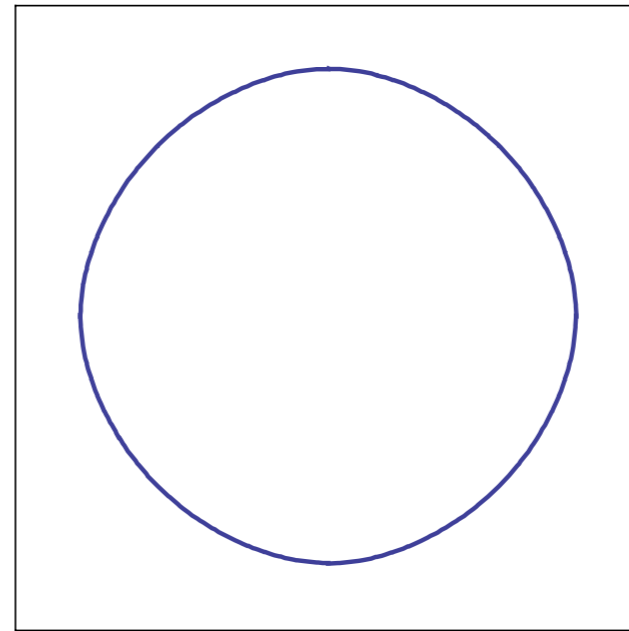
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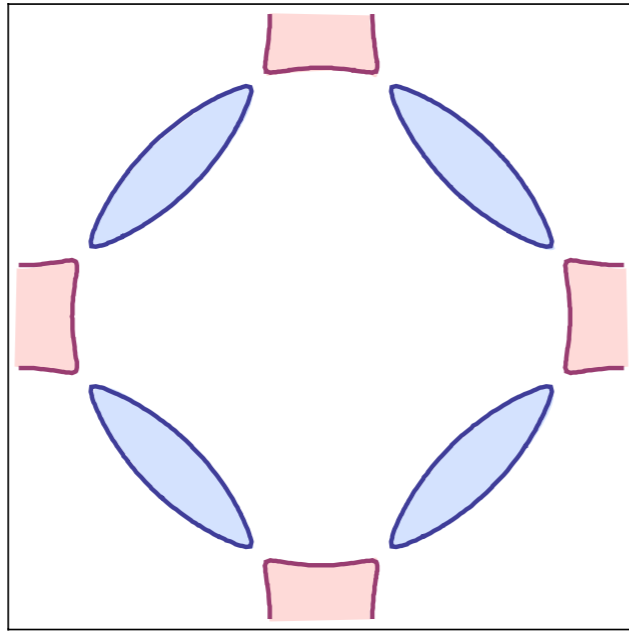


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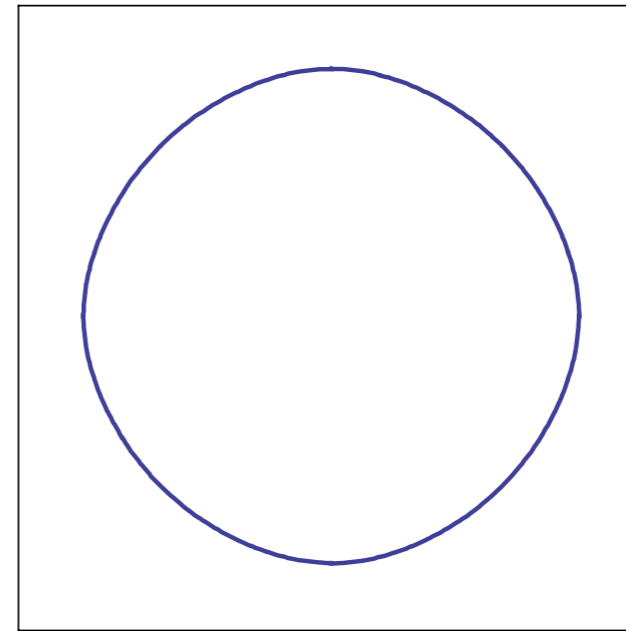
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$\rightarrow S$

- Quantum critical theory is strongly-coupled in two (but not higher) spatial dimensions (but not a CFT).

M.A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075128 (2010)

Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

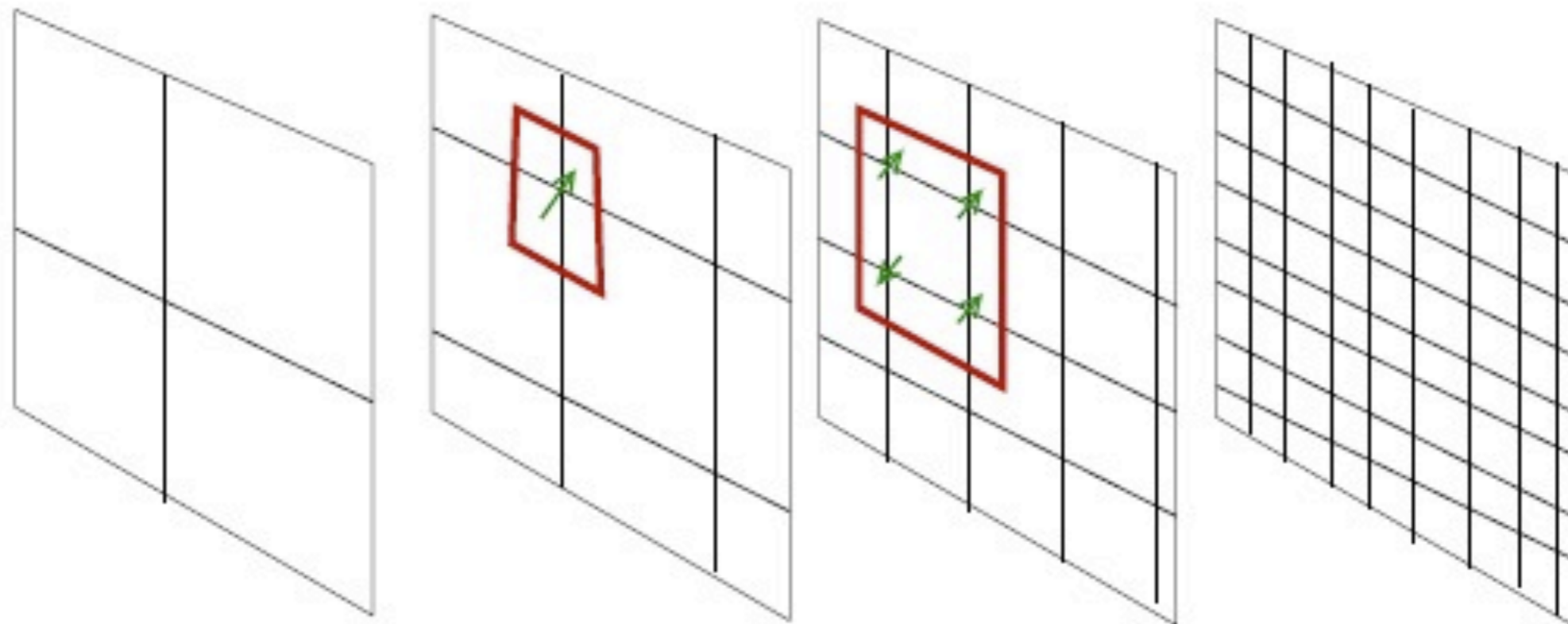
where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .

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$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u .

Key idea: \Rightarrow Implement u as an extra dimension, and map to a local theory in $D + 1$ dimensions.



u

J. McGreevy, arXiv0909.0518

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b u$$

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$$x^\mu \rightarrow x^\mu / b \quad , \quad u \rightarrow b u$$

This is an invariance of the *metric* of the theory in $D + 1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2 / u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS_{D+1} , and L is the AdS radius.

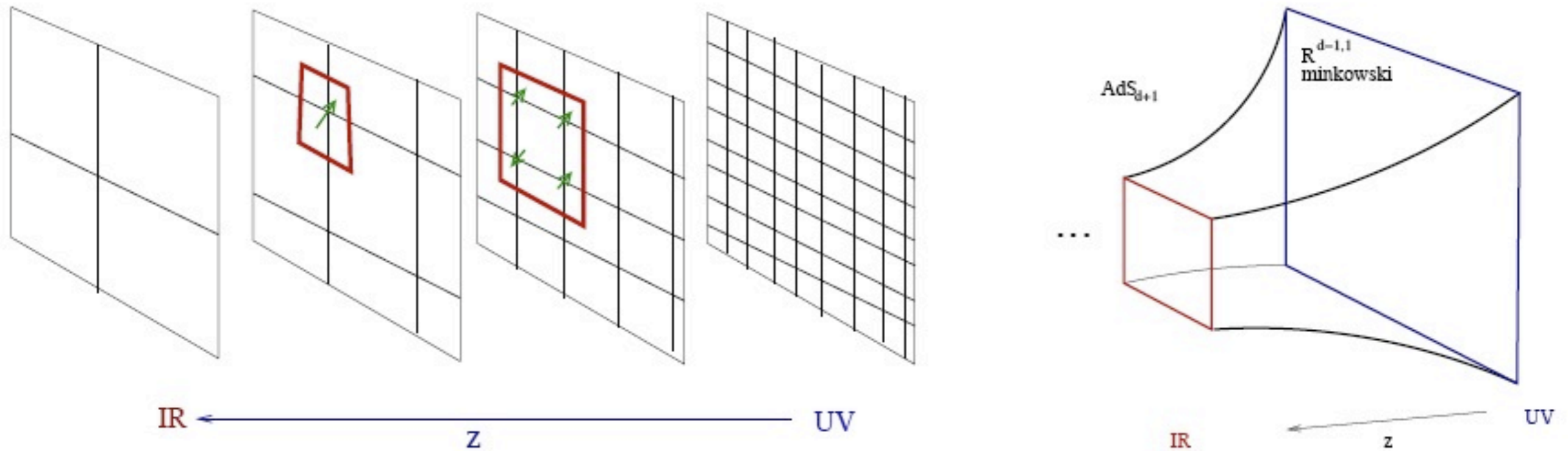
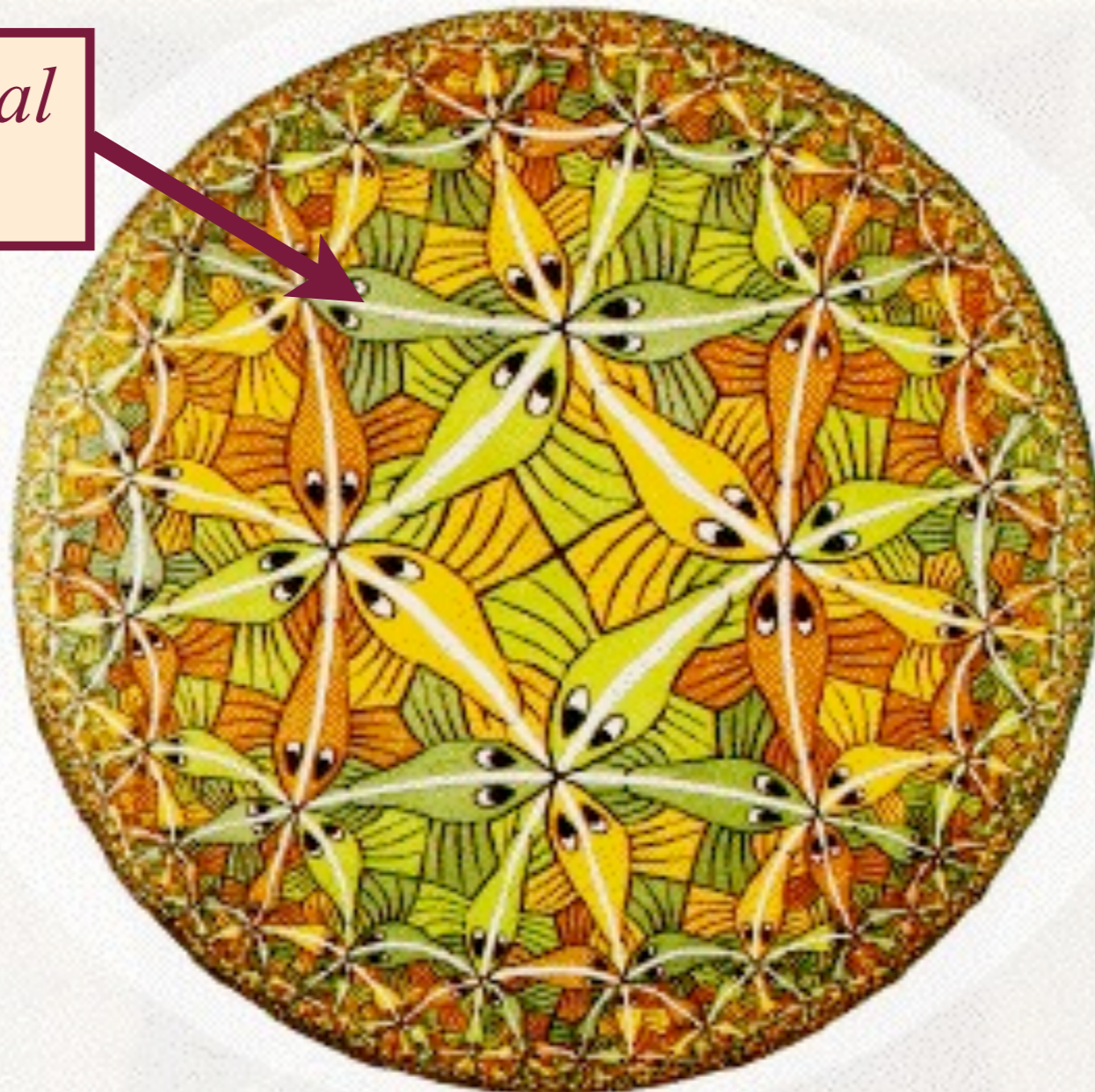


Figure 1: The extra (‘radial’) dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z . The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions

*3+1 dimensional
AdS space*

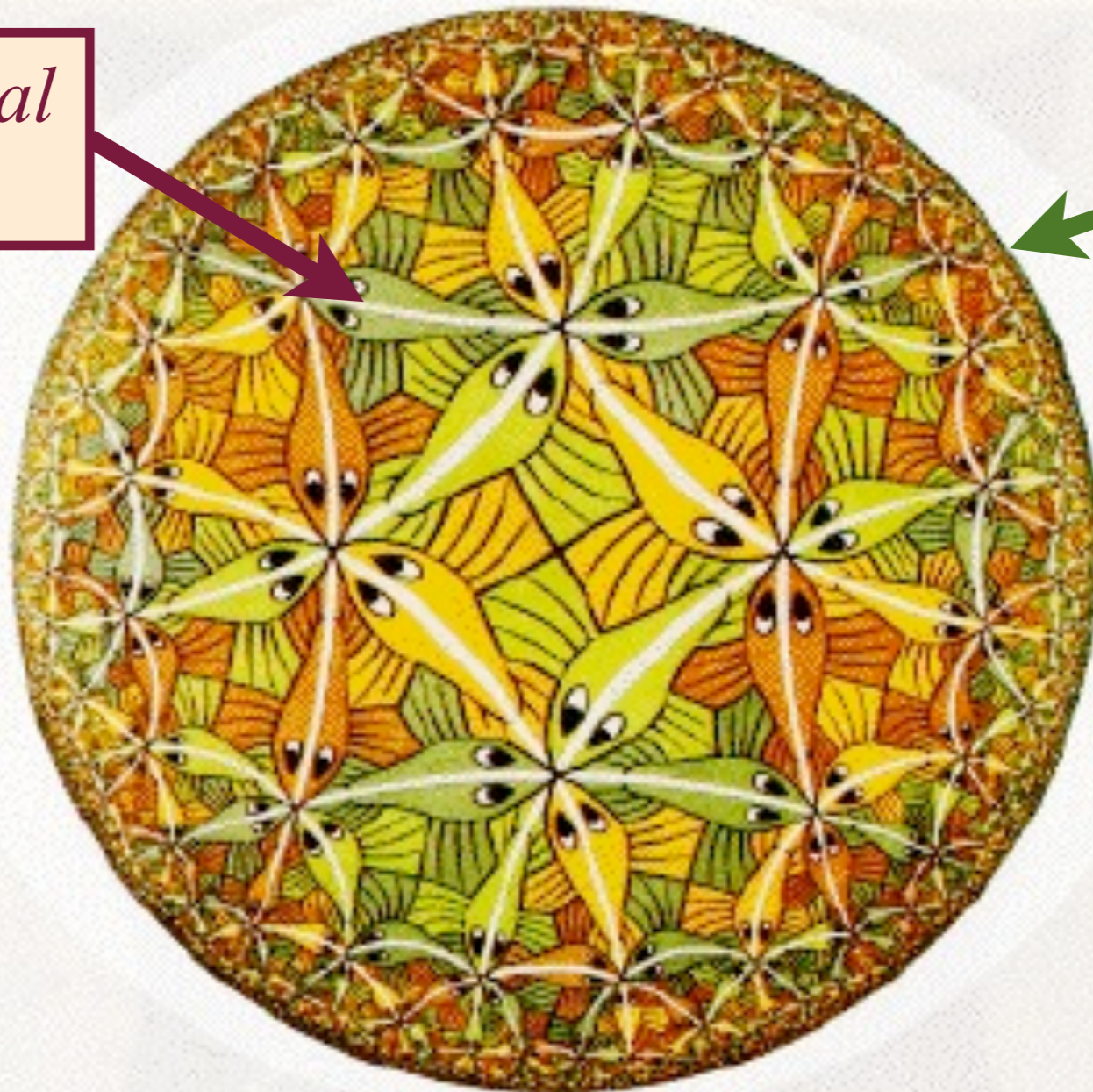


Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence

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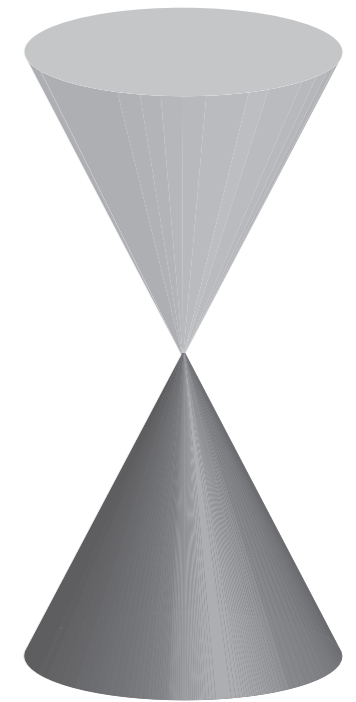
*3+1 dimensional
AdS space*



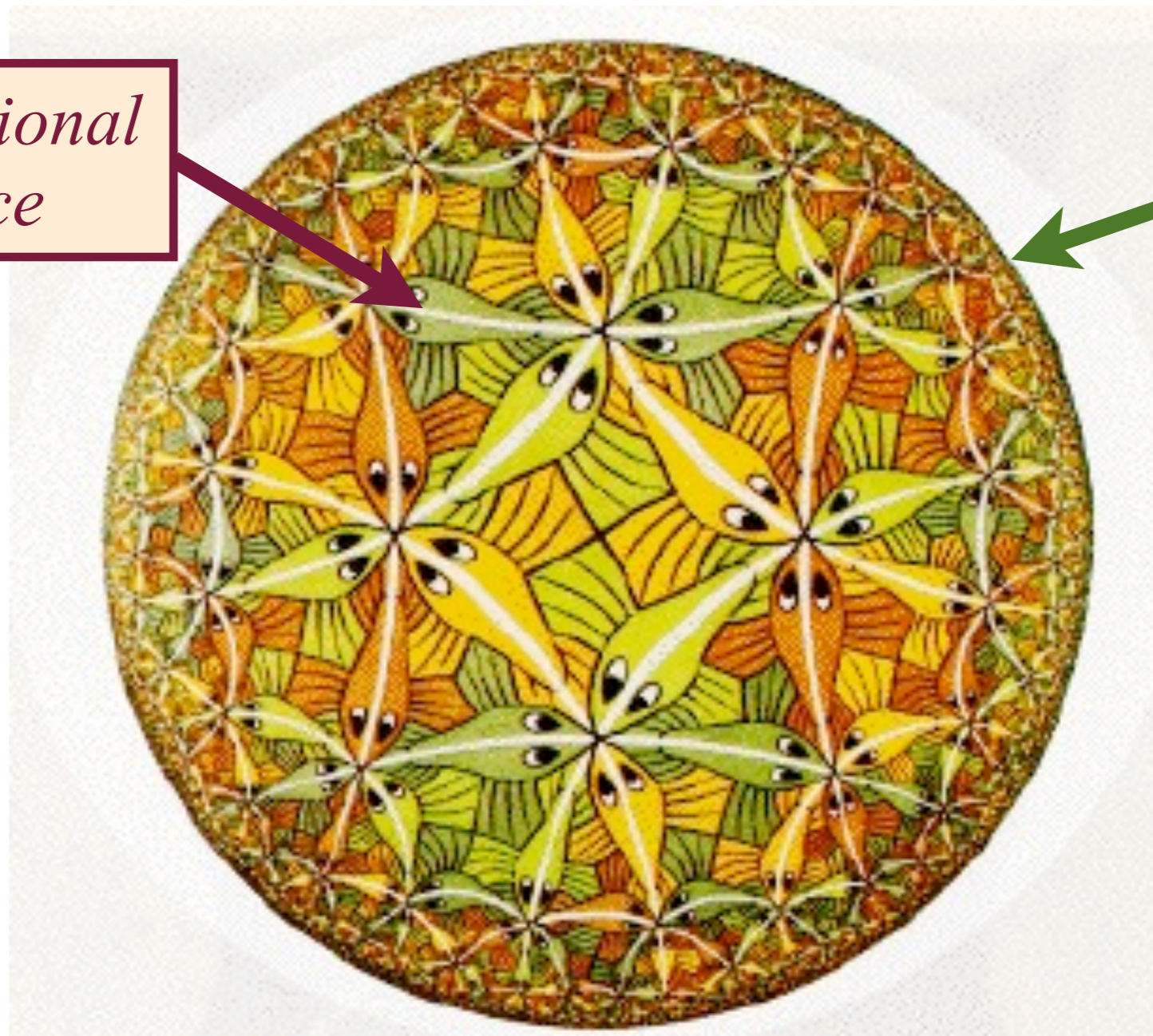
A 2+1
dimensional
system at its
quantum
critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence



*3+1 dimensional
AdS space*

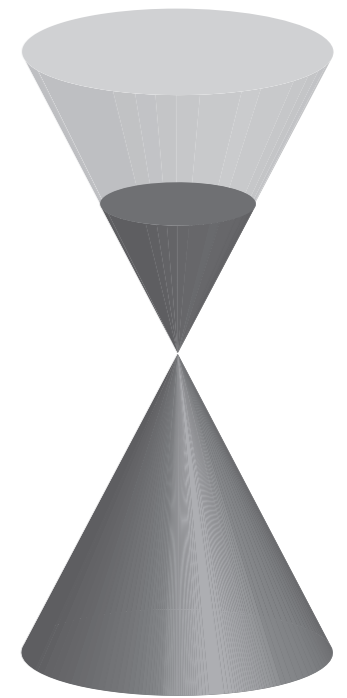


A 2+1
dimensional
system at its
quantum
critical point

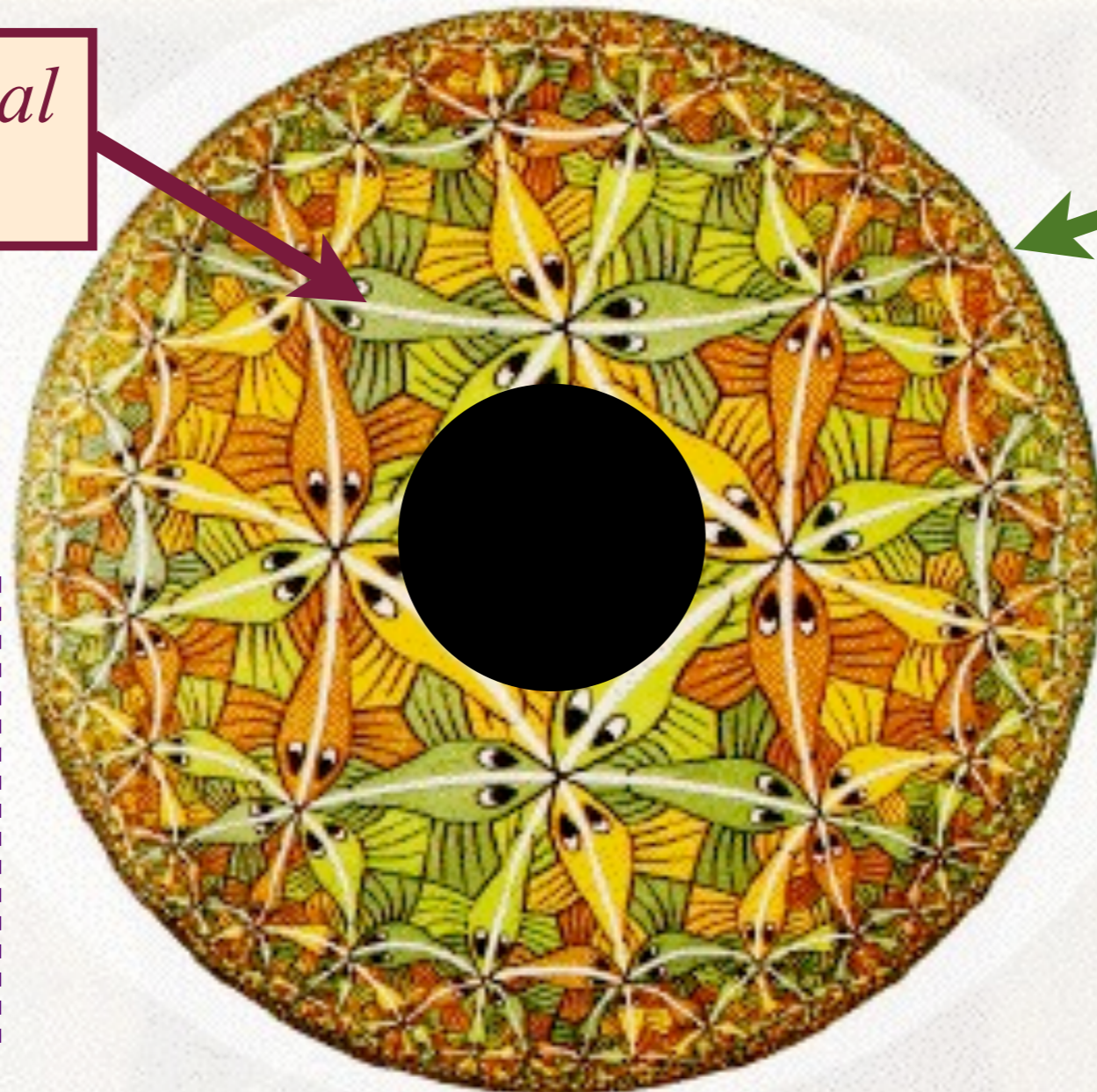
Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence

Move away from the quantum critical point to a system of matter at non-zero density: equivalent to adding an electrical charge to the black hole.



*3+1 dimensional
AdS space*



Finite
density
matter in
2+1
dimensions

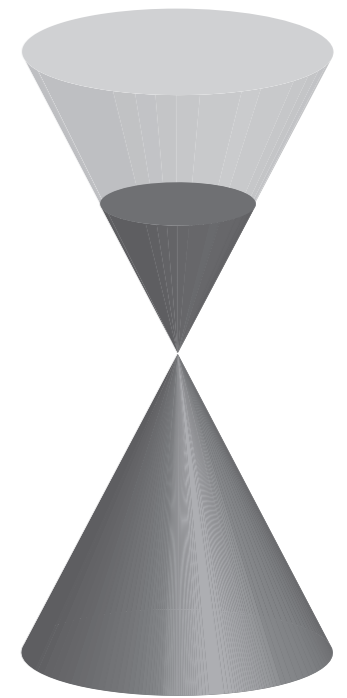
Extremal
Reissner-
Nordstrom
black hole

AdS/CFT correspondence

Examine the free energy and Green's function of a probe particle

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

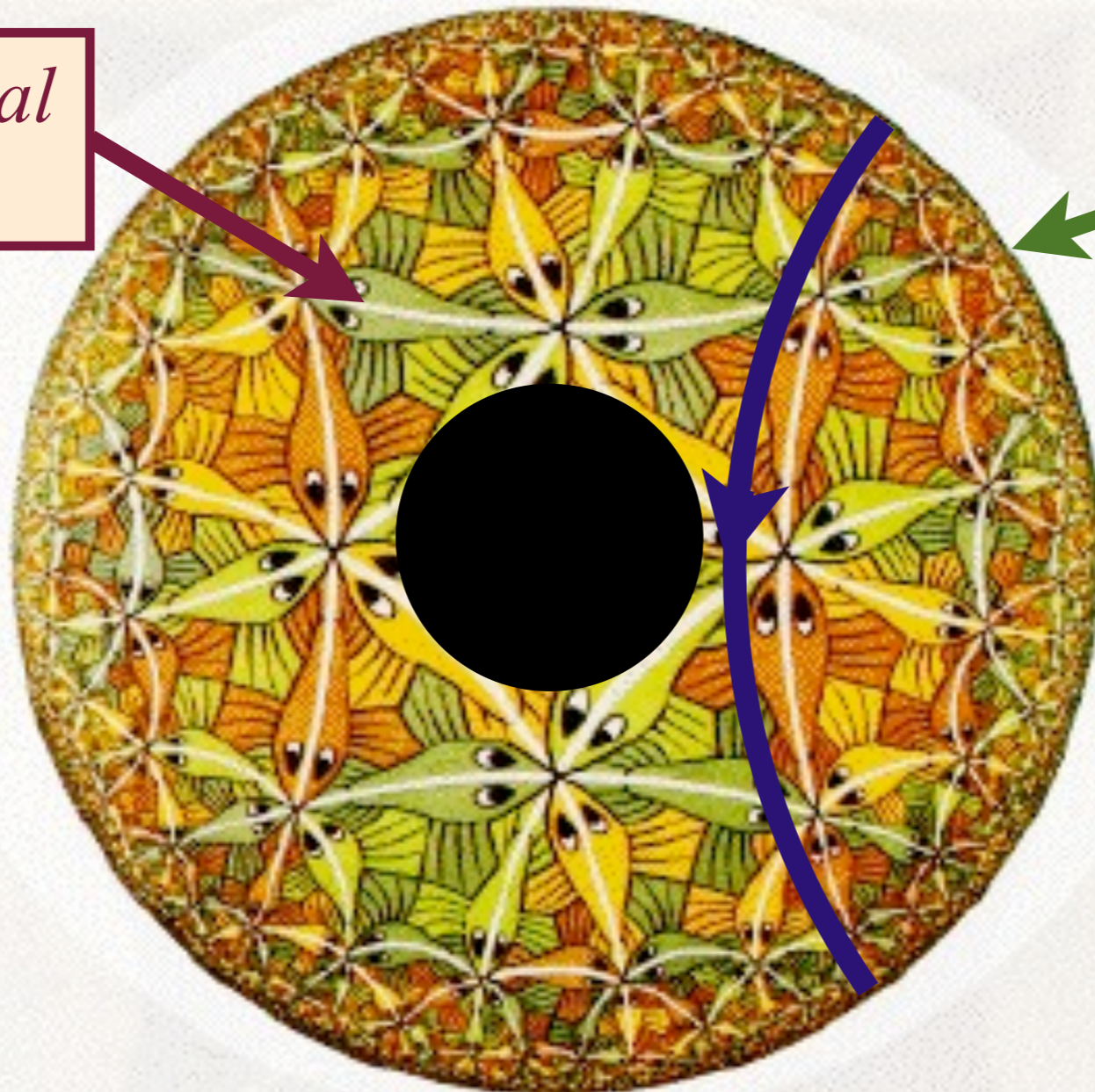
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788



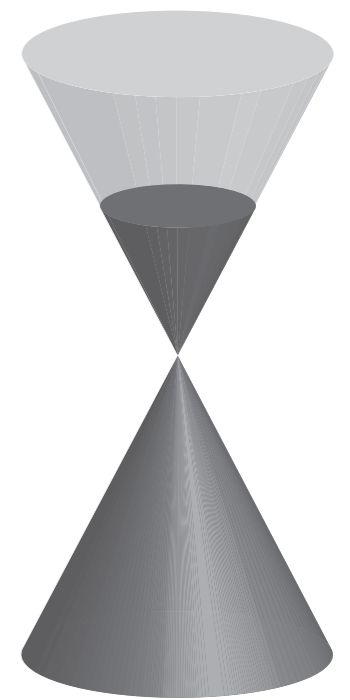
*3+1 dimensional
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Finite
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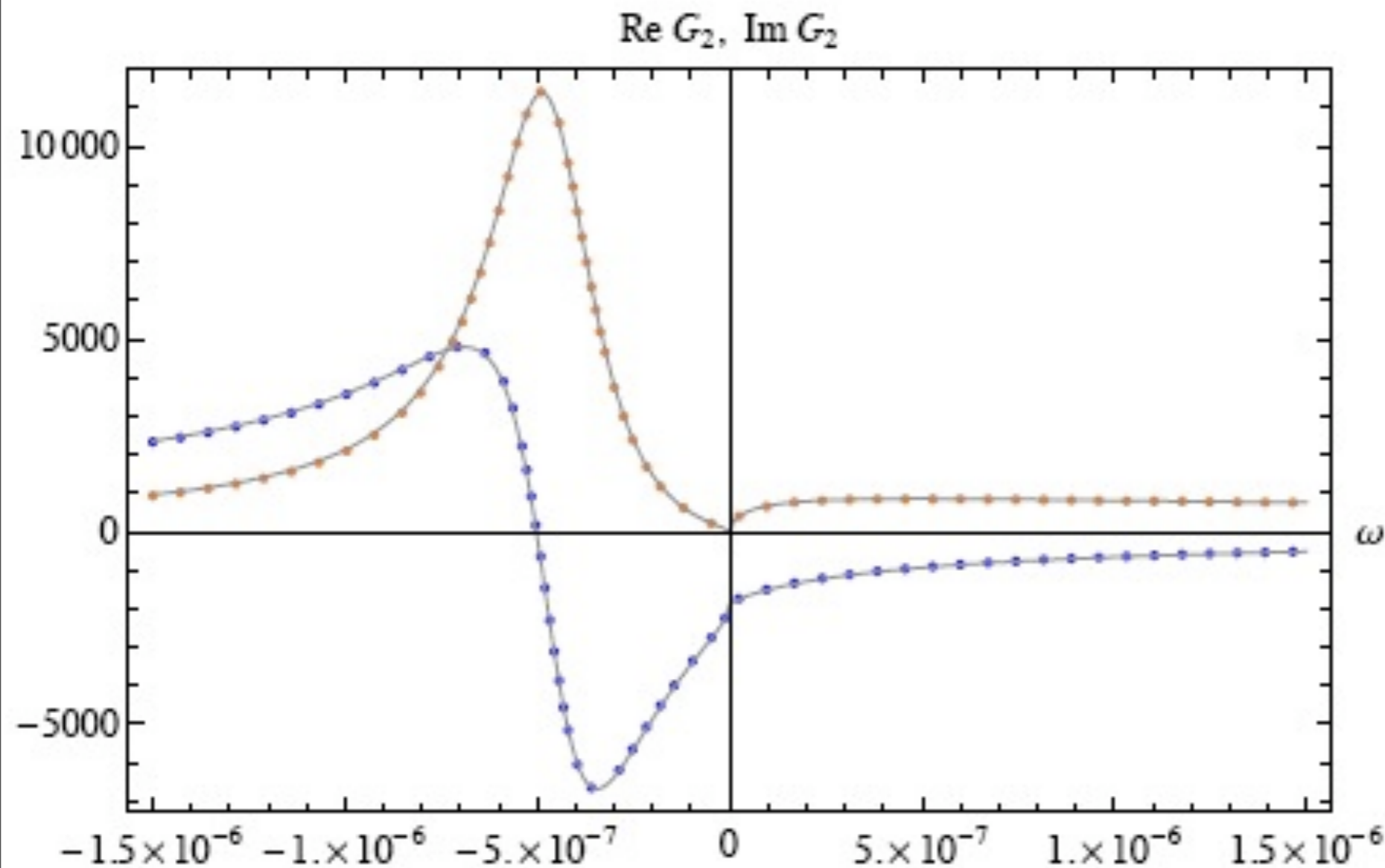
Extremal
Reissner-
Nordstrom
black hole



Green's function of a fermion



T. Faulkner, H. Liu,
J. McGreevy, and
D. Vegh,
arXiv:0907.2694



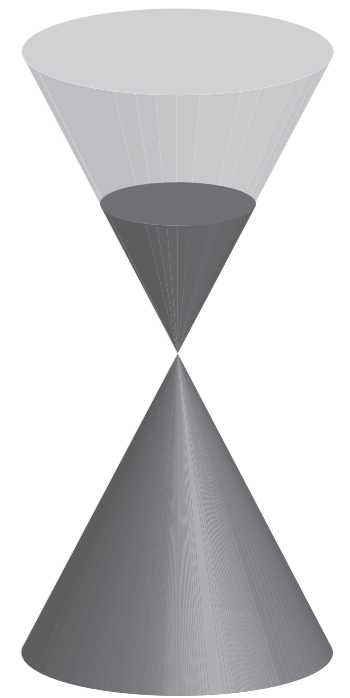
$$G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);

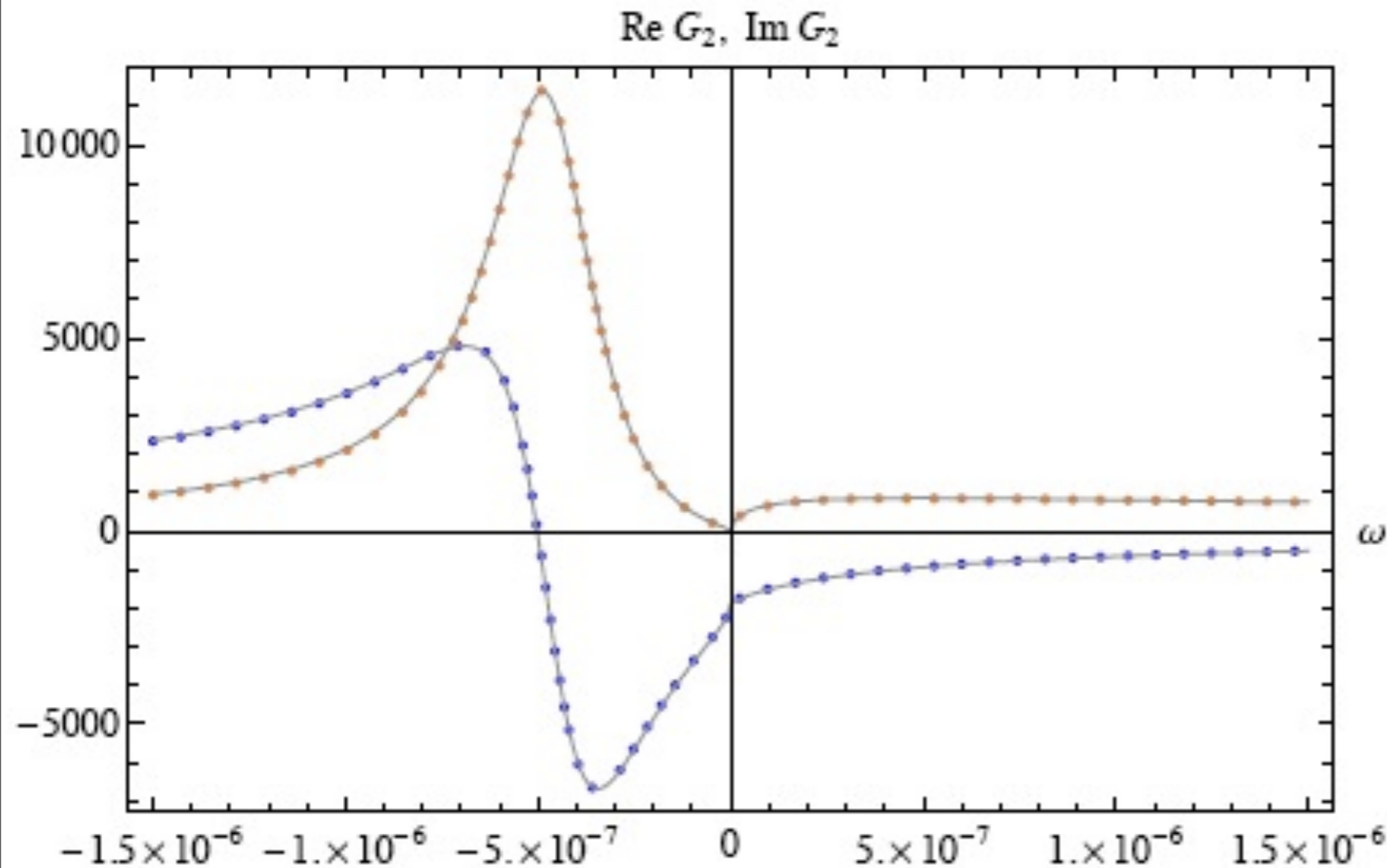
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);

F. Denef, S.A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

Green's function of a fermion



T. Faulkner, H. Liu,
J. McGreevy, and
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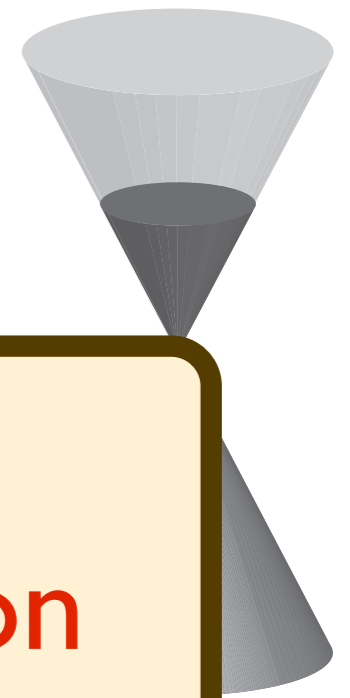
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Green's function of a fermion



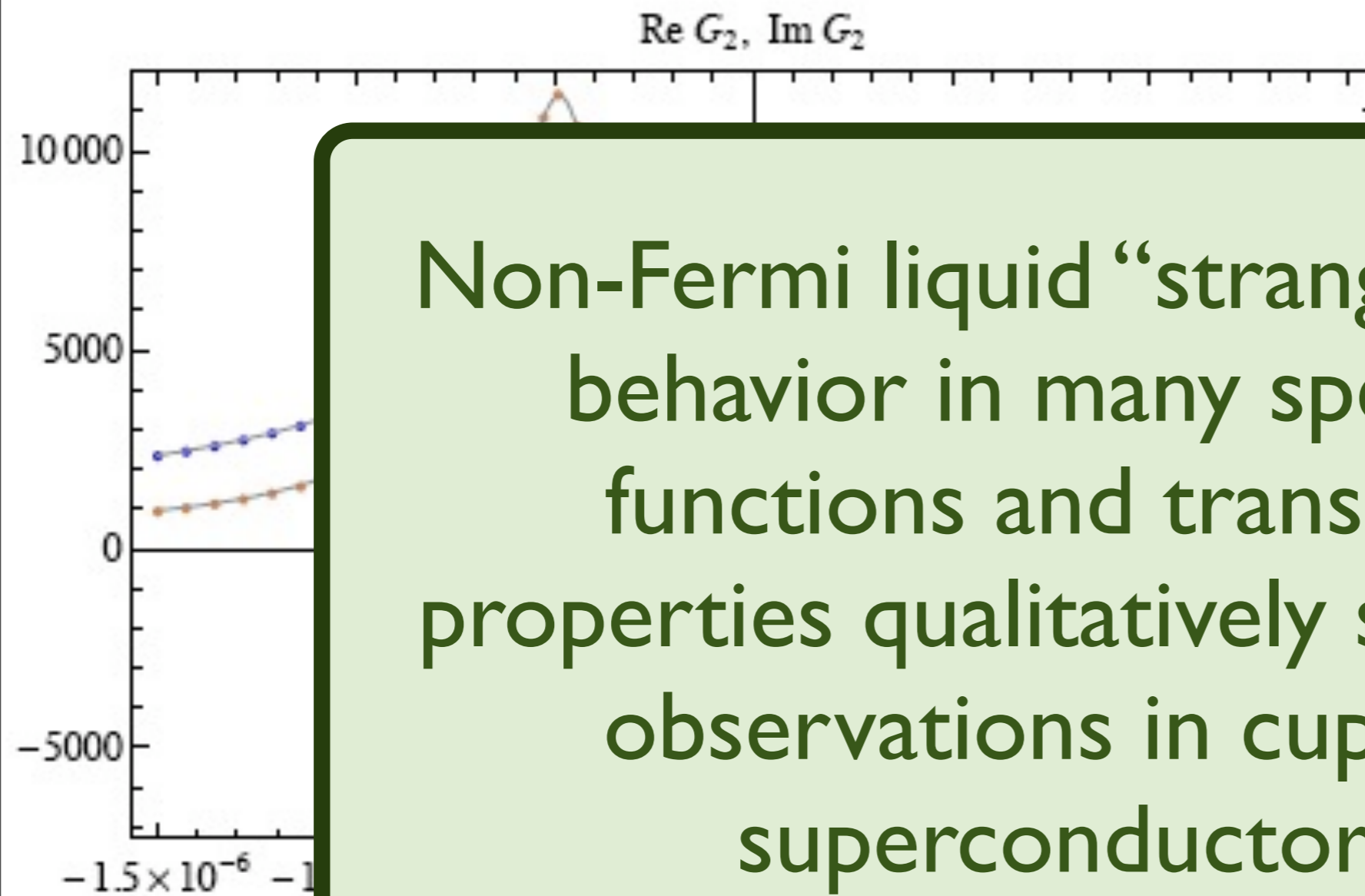
H. Liu,
y, and
n,
.2694

A Fermi surface: Green's function is singular on a spherical surface at a non-zero $k=k_F$. But the nature of the singularity differs from that in Landau's Fermi liquid theory.

$$G(k, \omega) \sim \frac{1}{\omega - v_F(k - k_F) - i\omega\theta(k)}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
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F. Denef, S.A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

Green's function of a fermion



Non-Fermi liquid “strange metal”
behavior in many spectral
functions and transport
properties qualitatively similar to
observations in cuprate
superconductors

$$G(k, \omega) \sim \frac{1}{\omega - v_F(k - k_F) - i\omega^\theta(k)}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
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.2694

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*
- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Questions and answers

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Yes

● *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*

● *What is the physics of the strange metal ?*

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Proposal: strongly-coupled quantum criticality of Fermi surface change in a metal