Yukawa-SYK models
and
strange metals in two spatial dimensions

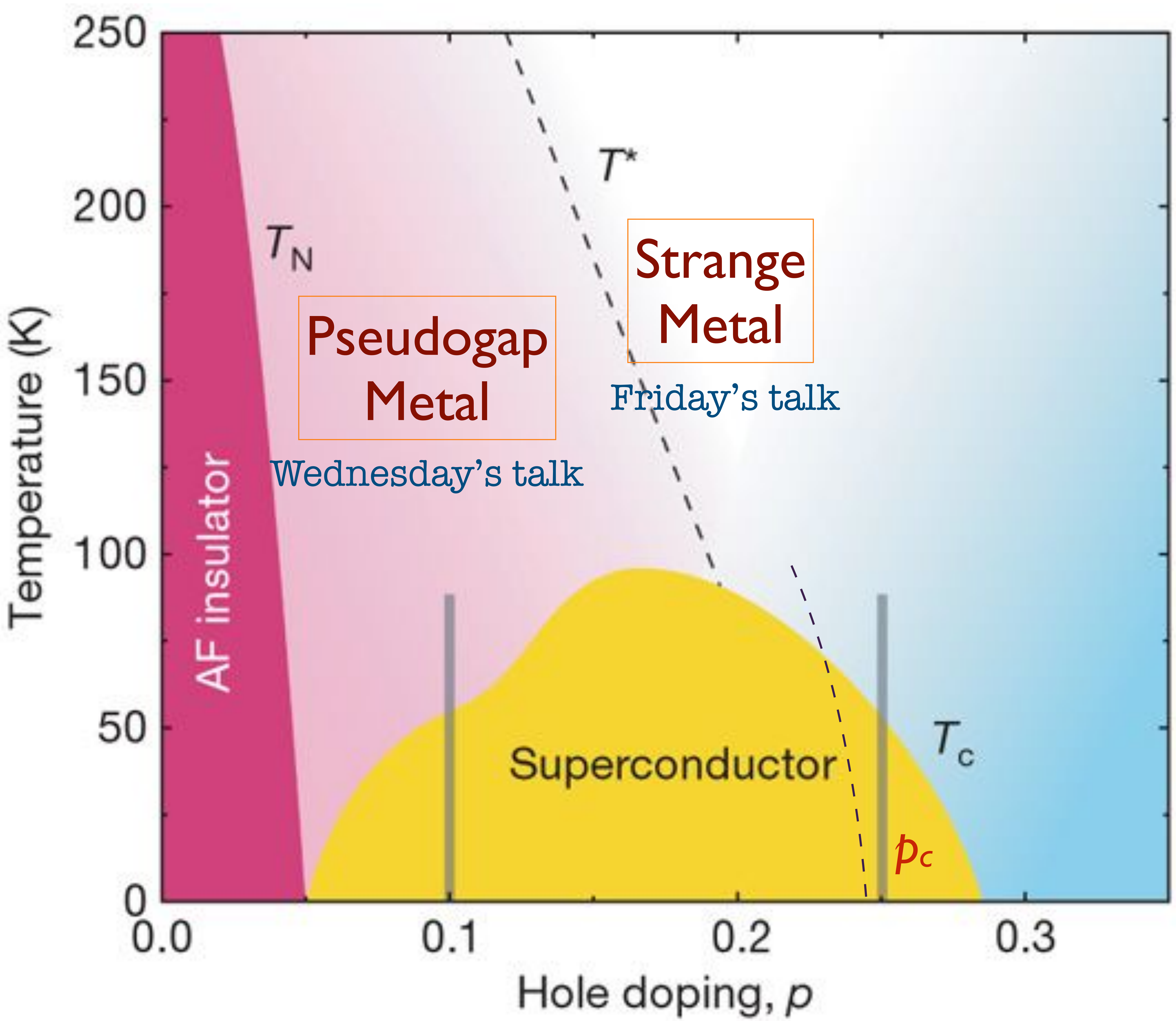
Condensed Matter Physics in the City
University College London
June 10, 2022

Subir Sachdev

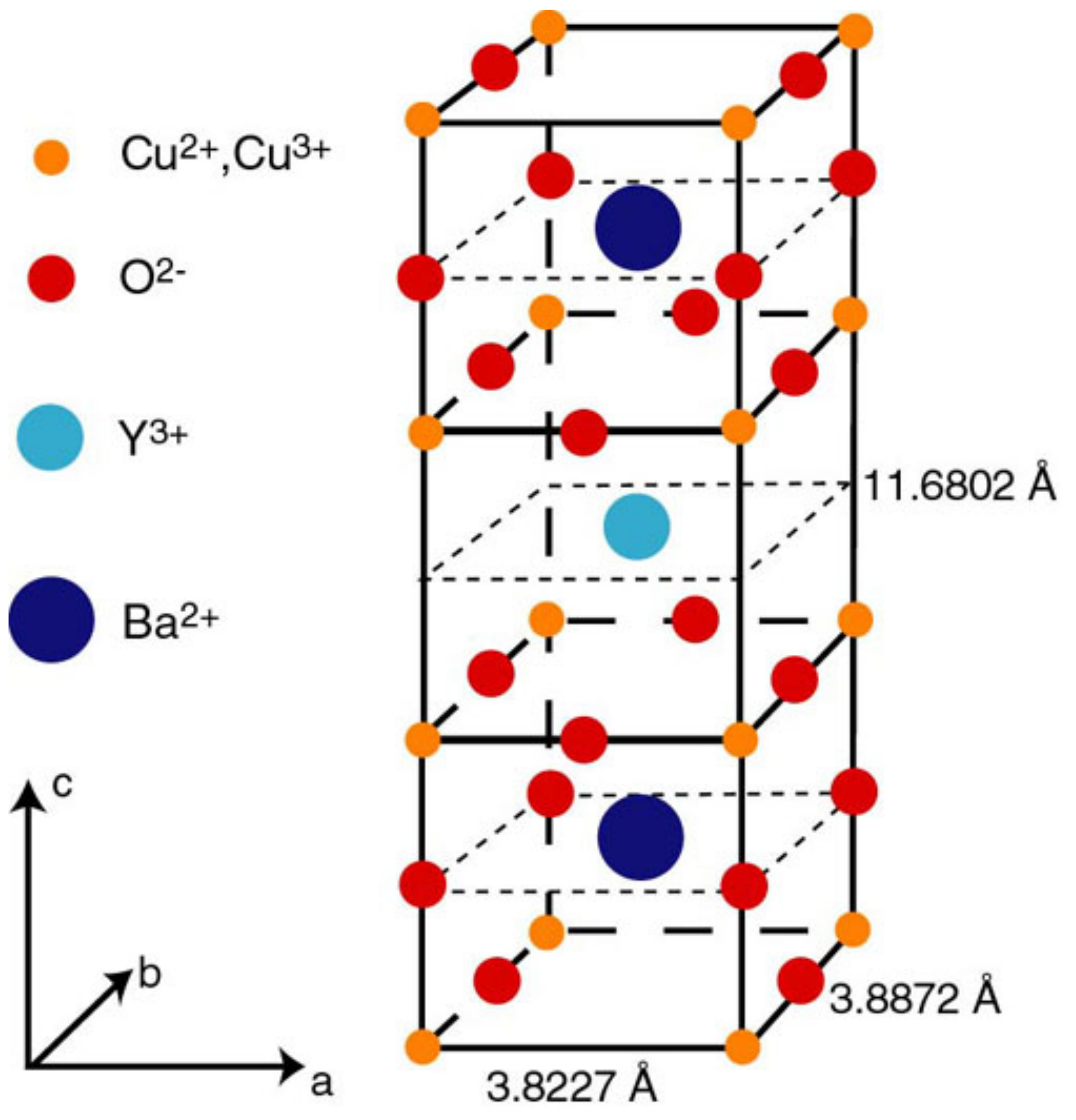
Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

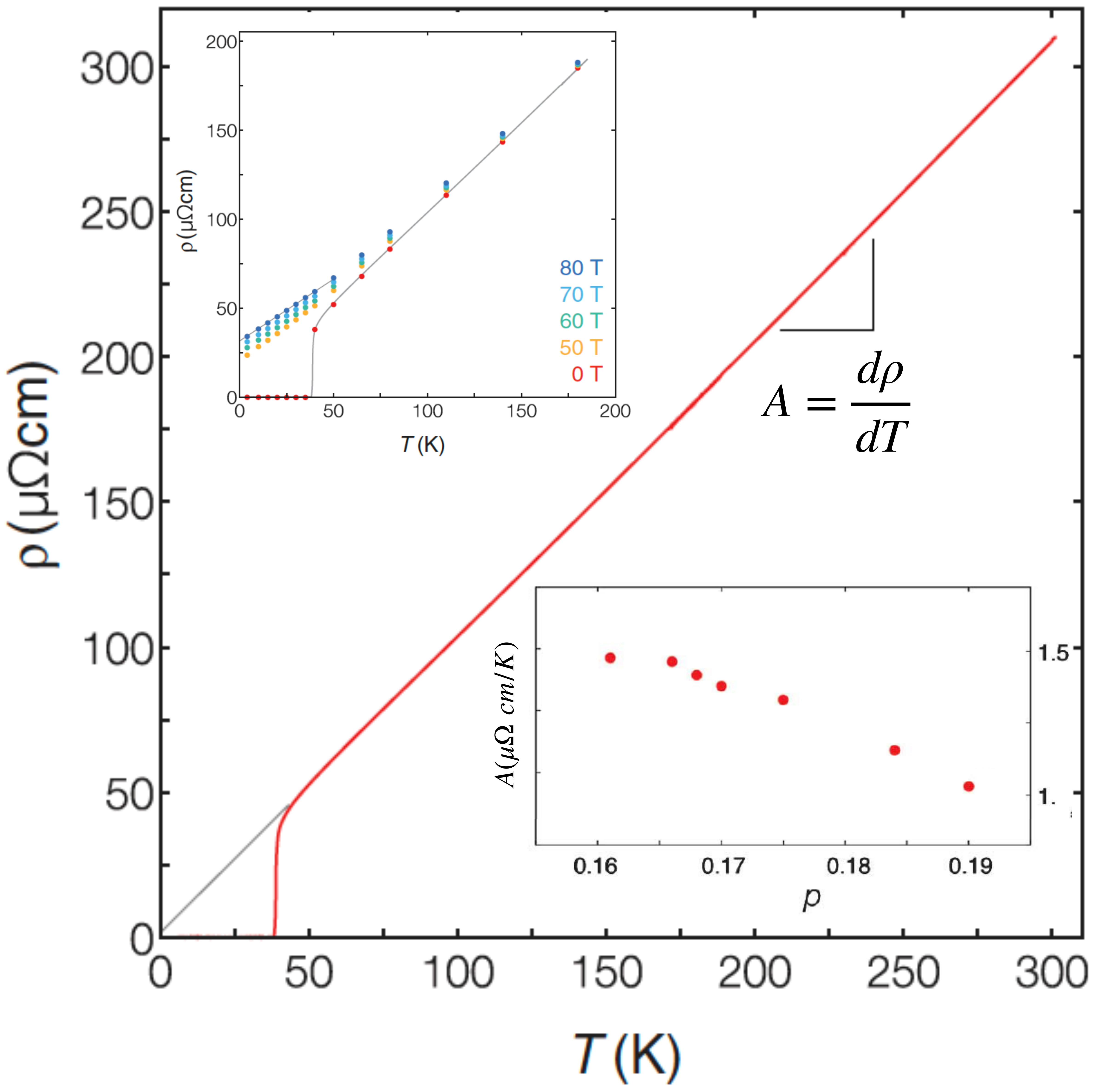
PHYSICS
HARVARD



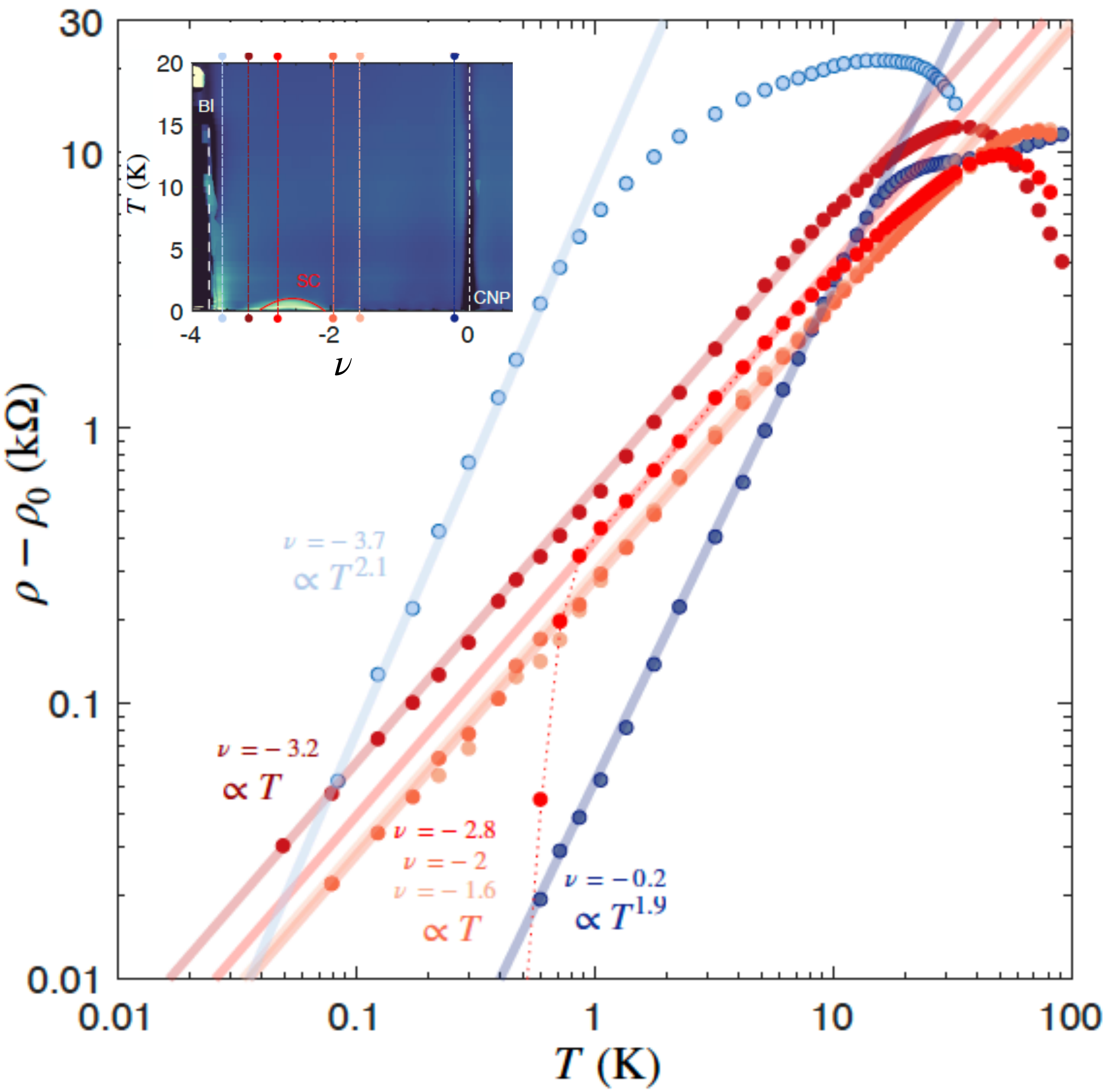
250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Pseudogap Metal
Wednesday's talk
Strange Metal
Friday's talk
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

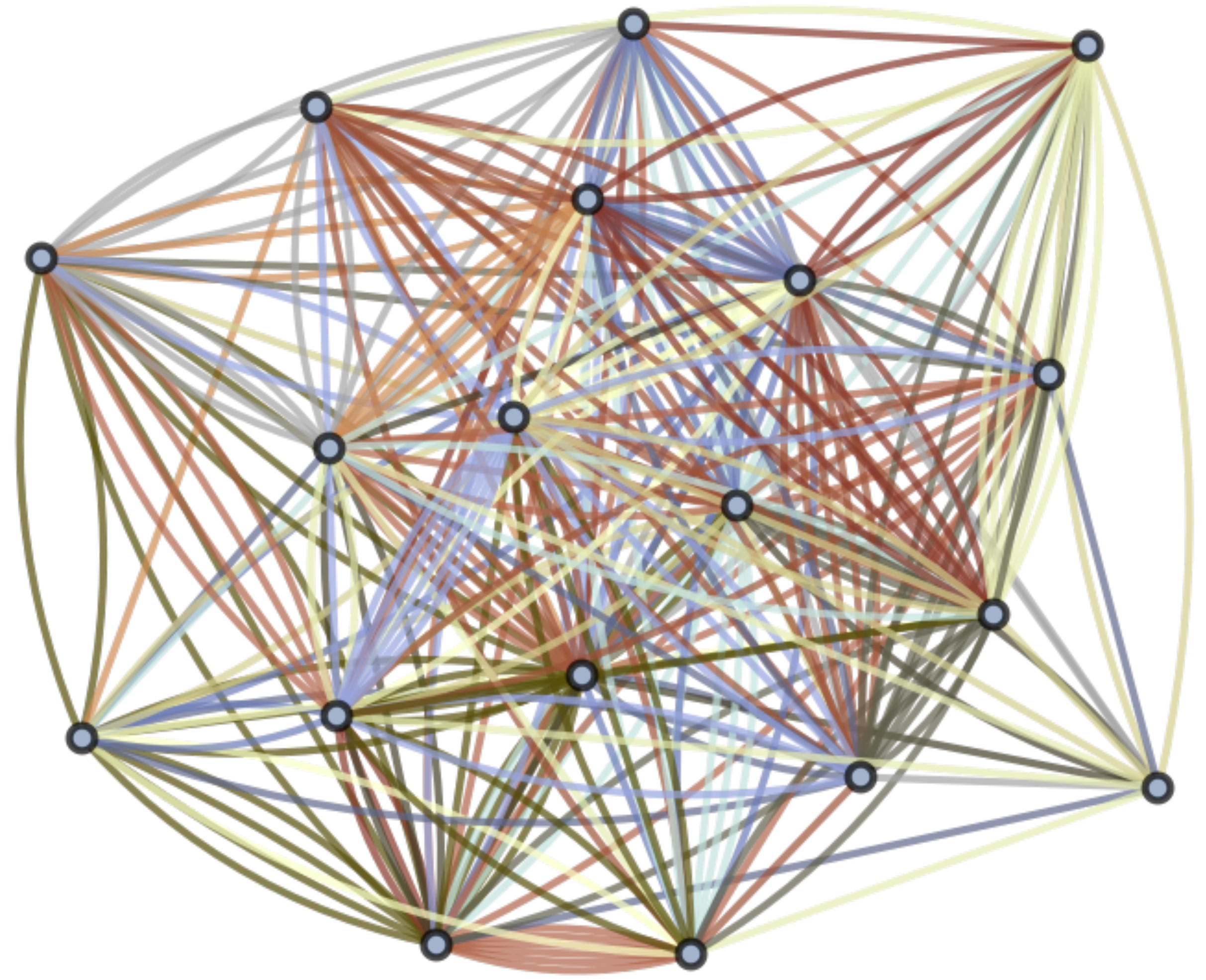
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

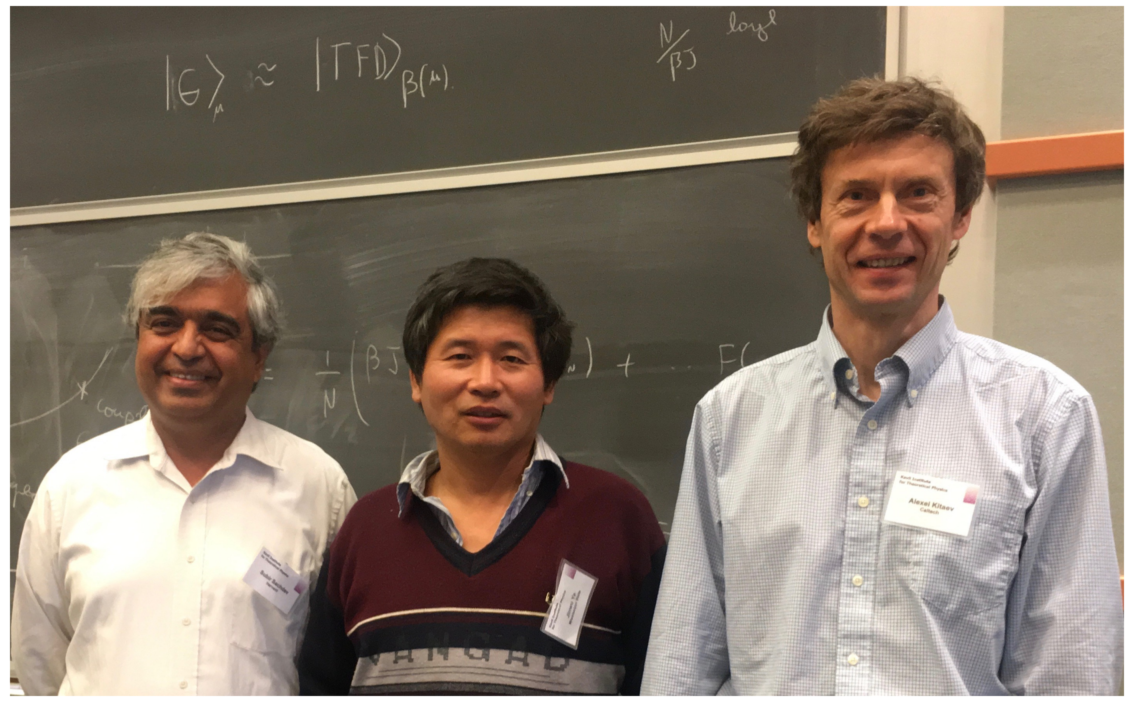
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

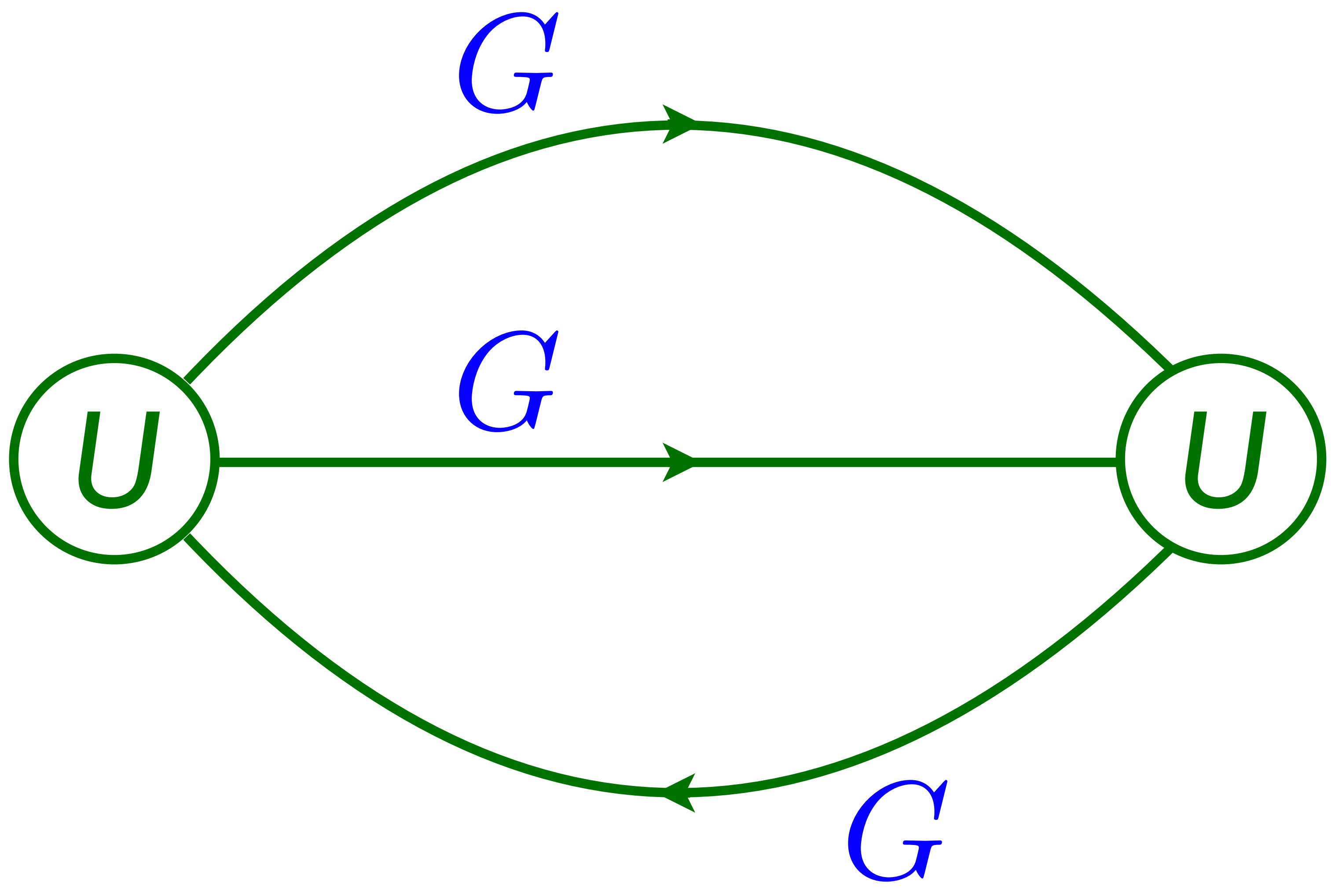




Complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

Conformal solution at $\mu = 0$, $G(\tau) \sim \dfrac{\mathrm{sgn}(\tau)}{\sqrt{|\tau|}}$.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The Sachdev-Ye-Kitaev (SYK) model

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, RMP

- Green's function has Planckian time scaling $G(\omega, T) \sim \omega^{-1/2} F(\hbar\omega/k_B T)$.

- No quasiparticle decomposition of many-body states.

- Leading (dangerously) irrelevant operator is a time reparameterization soft mode $\tau \to f(\tau)$.

- The effective action for $f(\tau)$ is the quantum theory of a charged black hole in Einstein-Maxwell theory in 3+1 spacetime dimensions at low T.

- The path integral over $f(\tau)$ determines the many-body density of states $D(E)$ at low energy E. For a generic charged black hole in 3+1 spacetime dimensions with $T = 0$ horizon area A_0, we have the new universal result

$$D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\sqrt{\pi} A_0^{3/2} \frac{c^3}{\hbar G} \frac{E}{\hbar c}\right]^{1/2}\right)$$

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

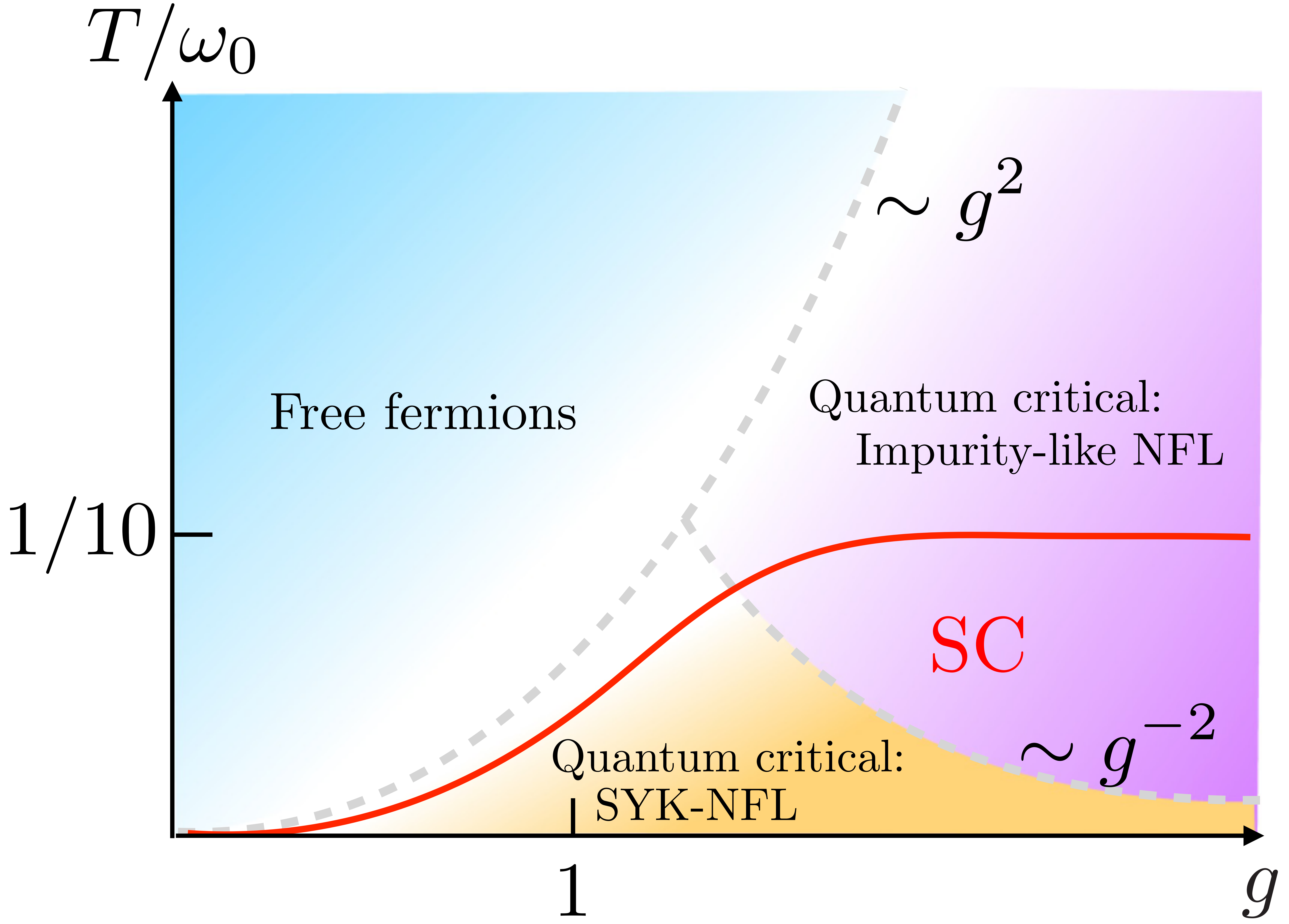
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

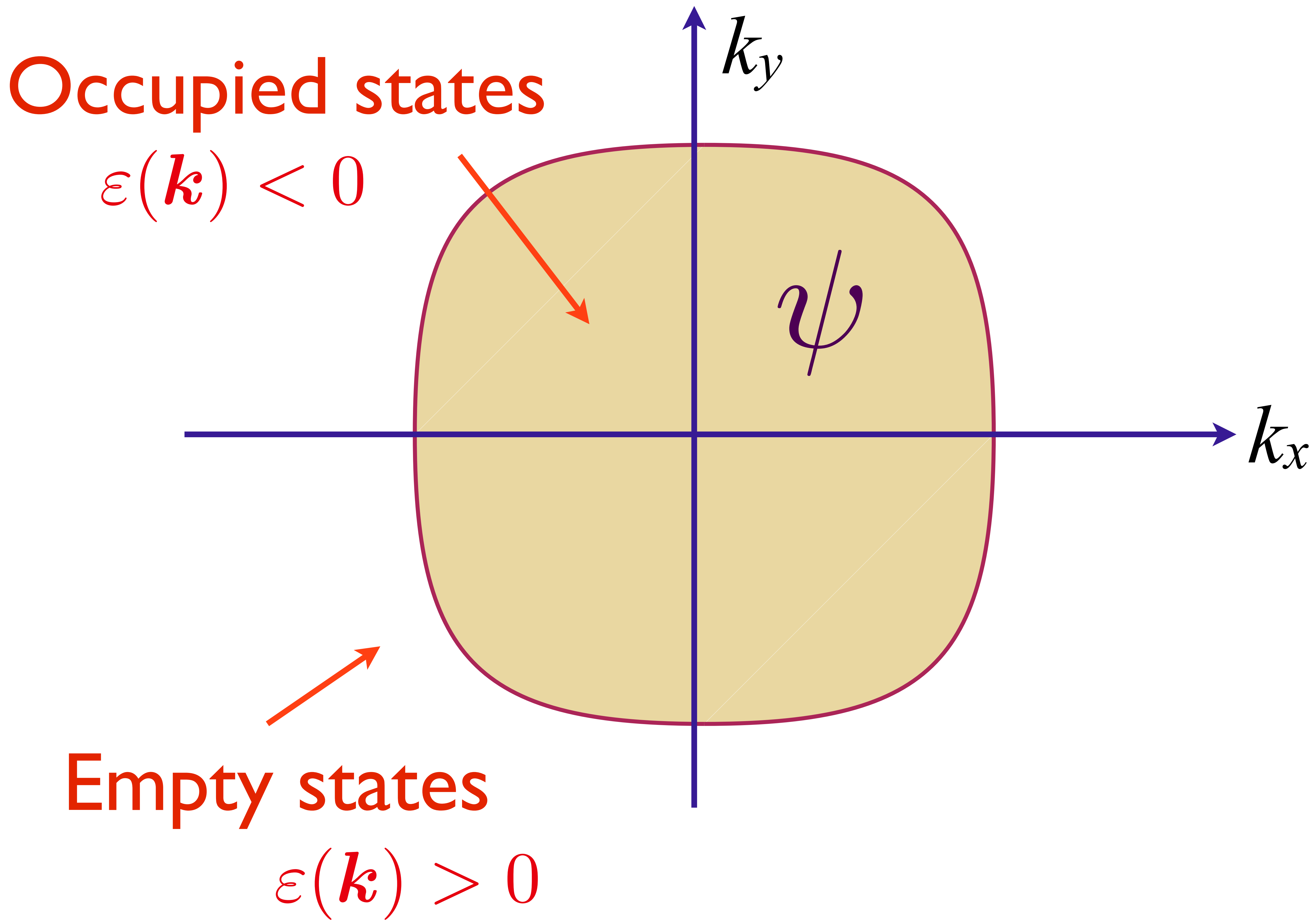
Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models and a large *N* theory of a critical Fermi surface in two spatial dimensions

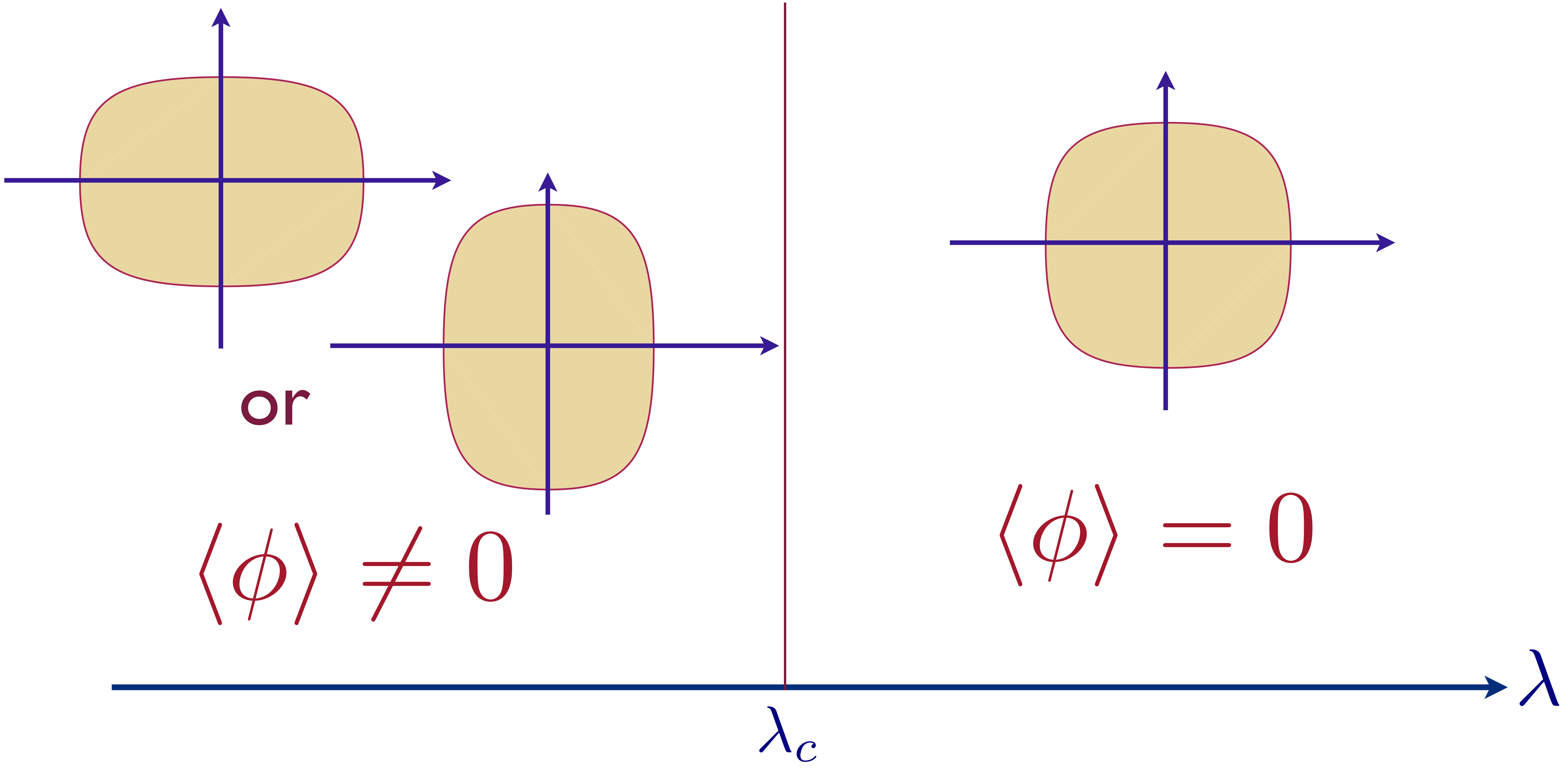
Fermi surface coupled to a critical boson



+

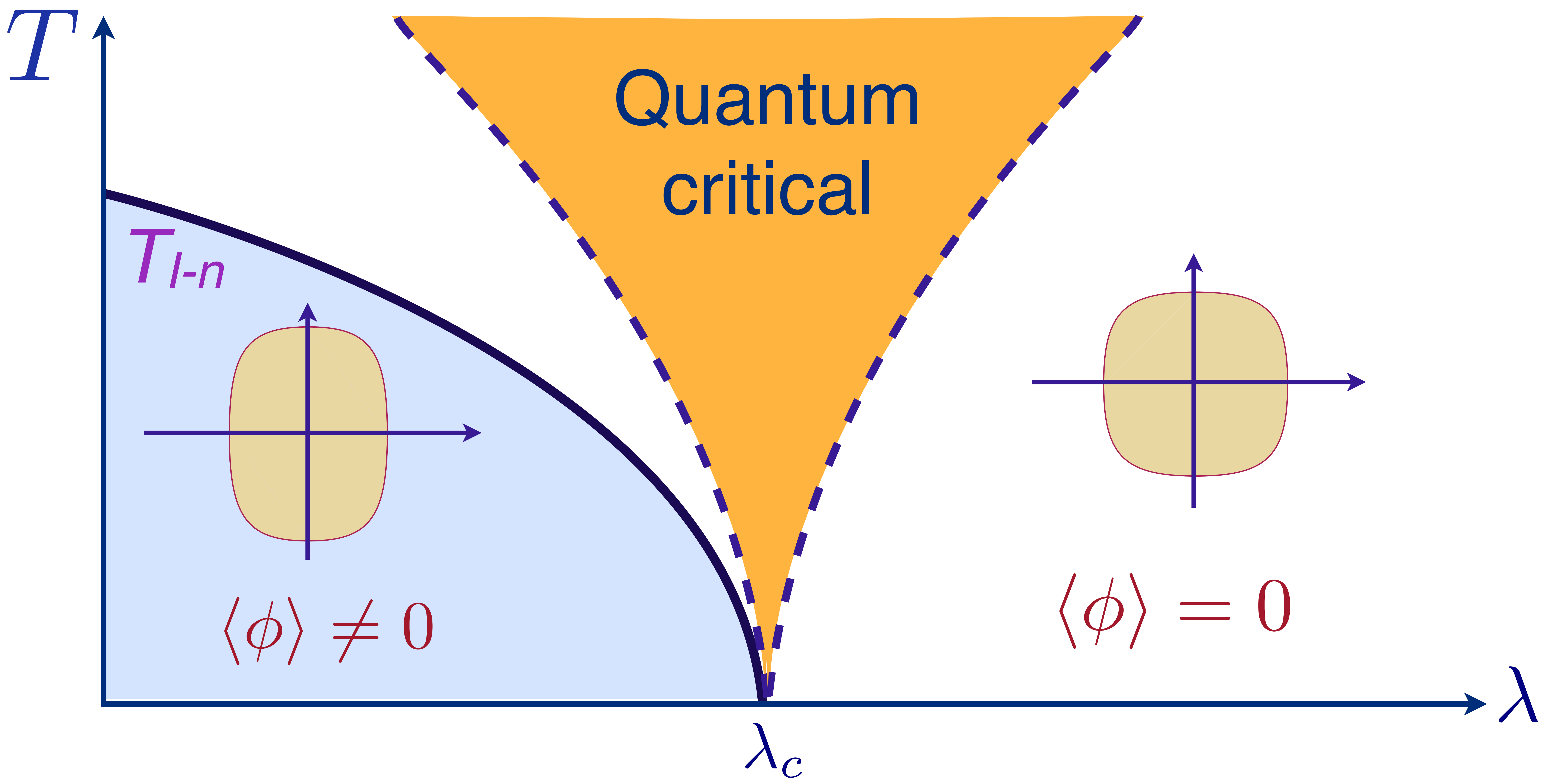
a
critical
boson
ϕ

Quantum criticality of Ising-nematic ordering in a metal



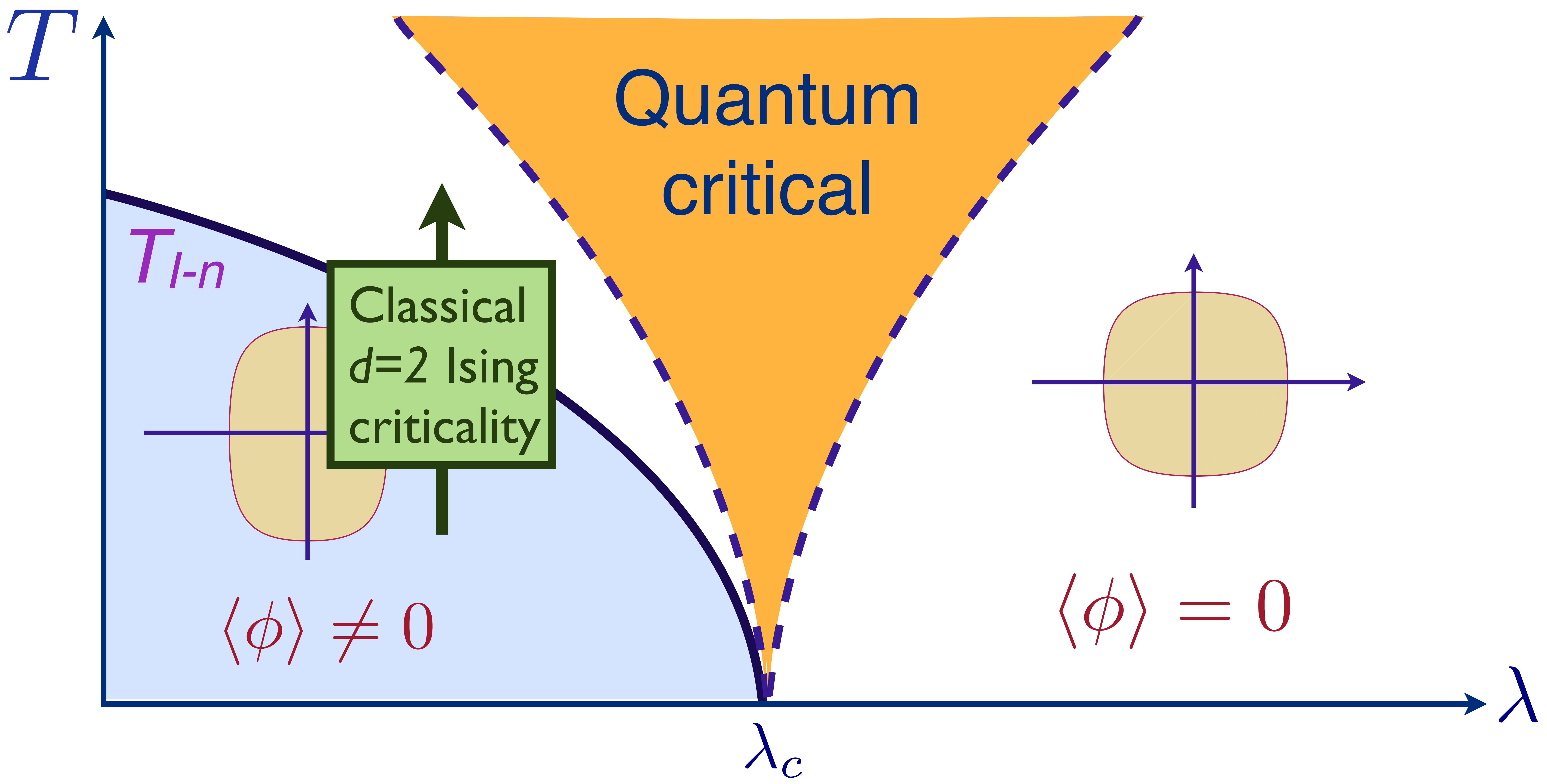
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



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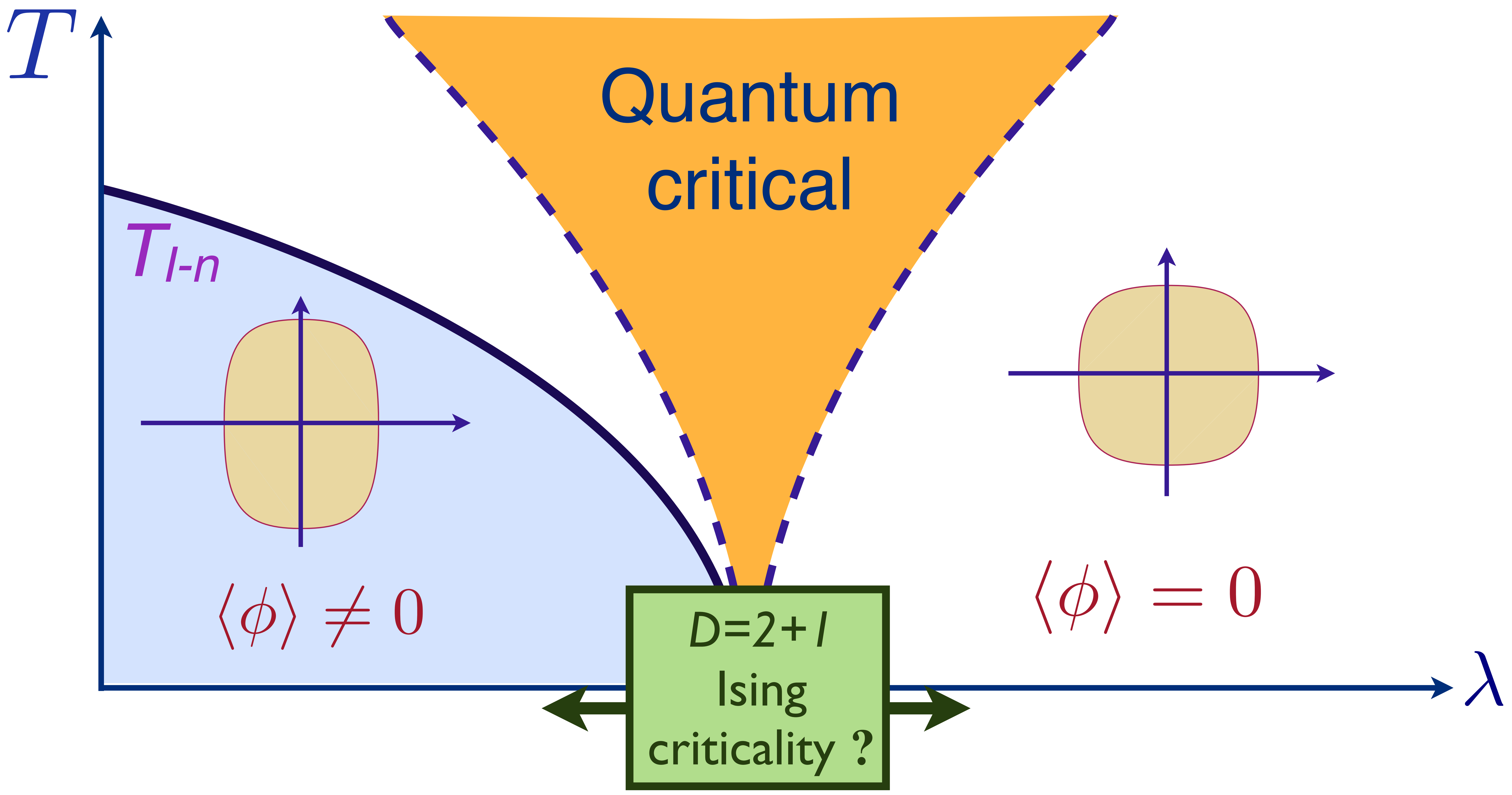

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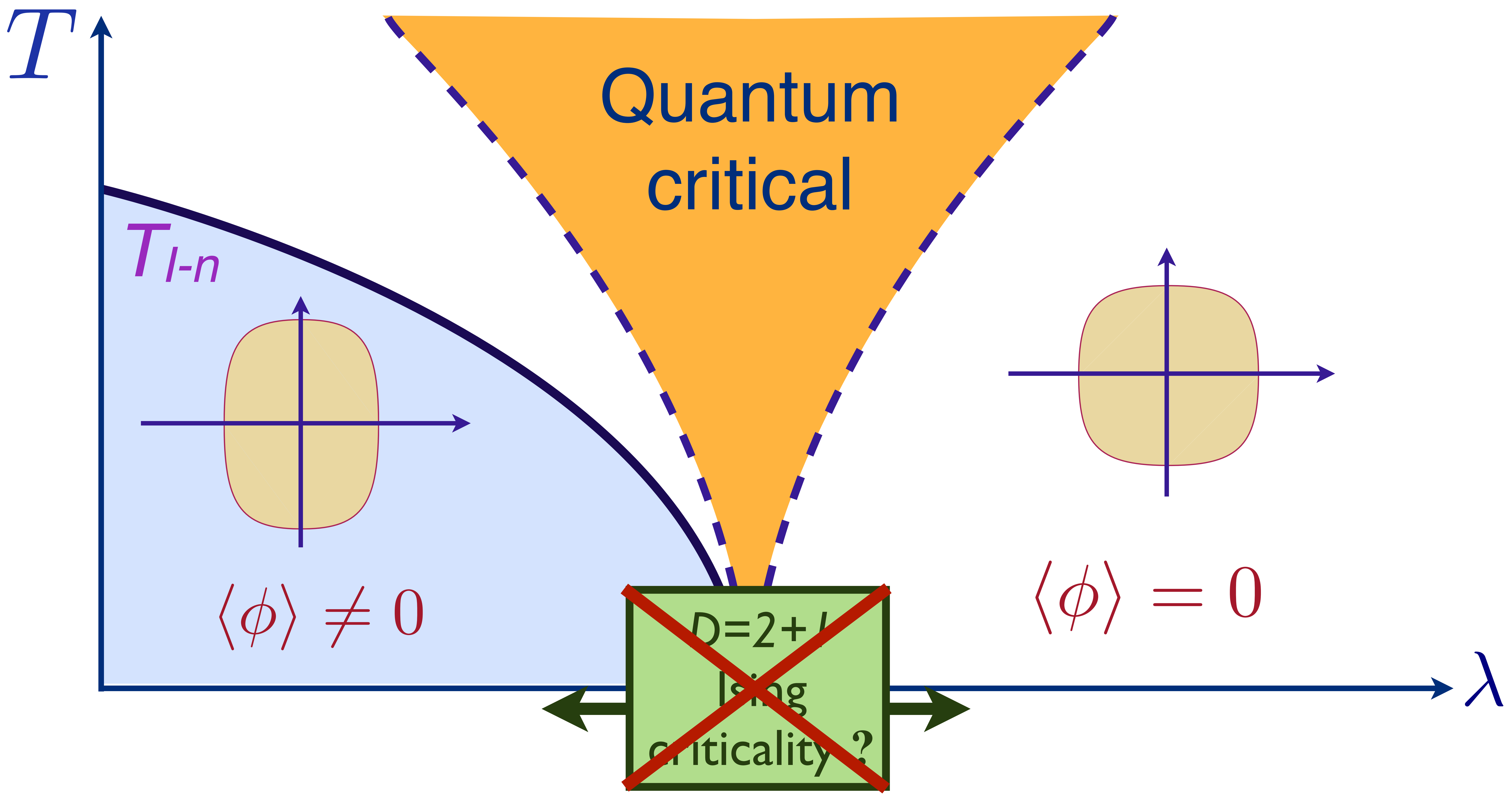

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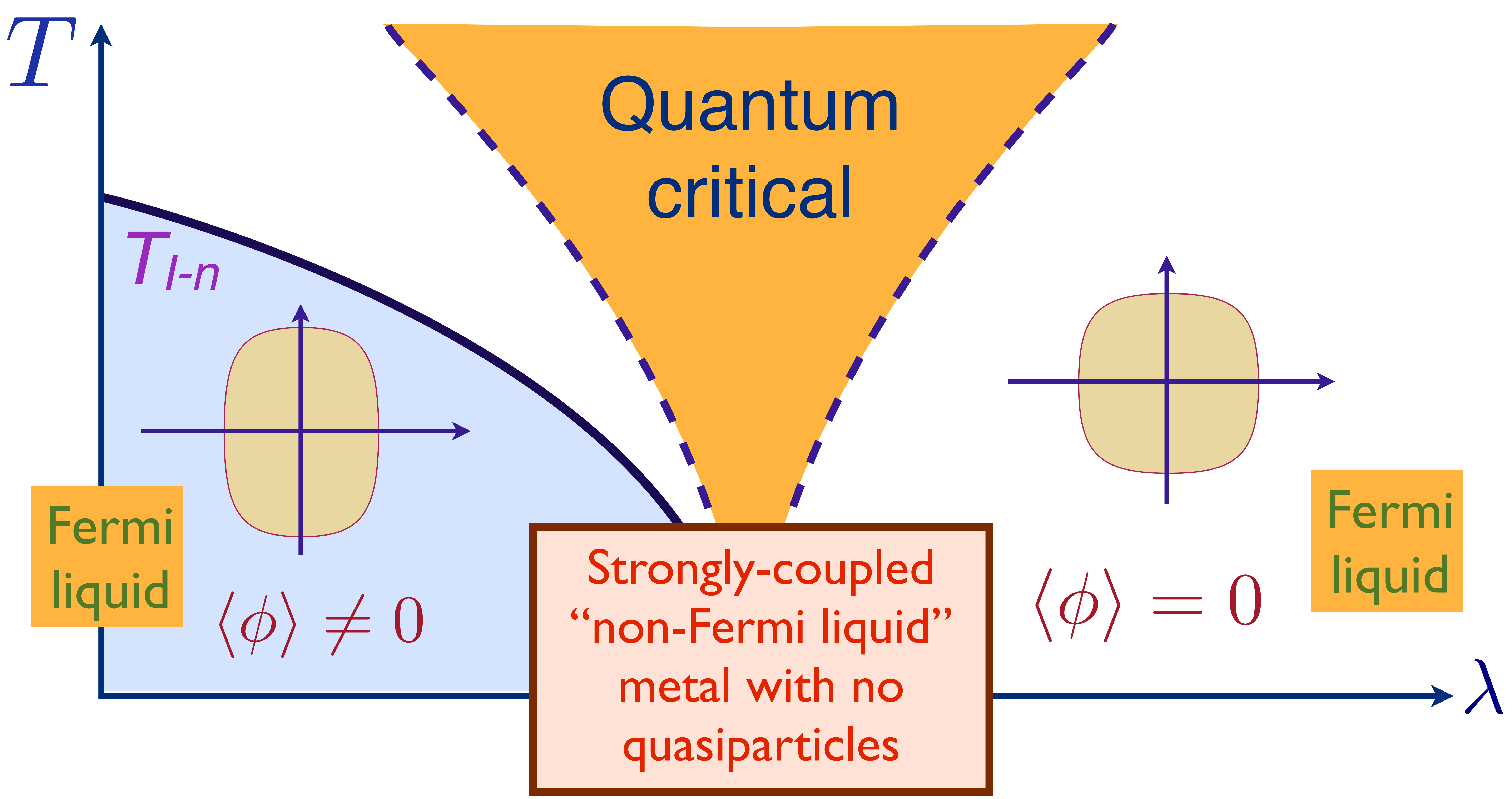

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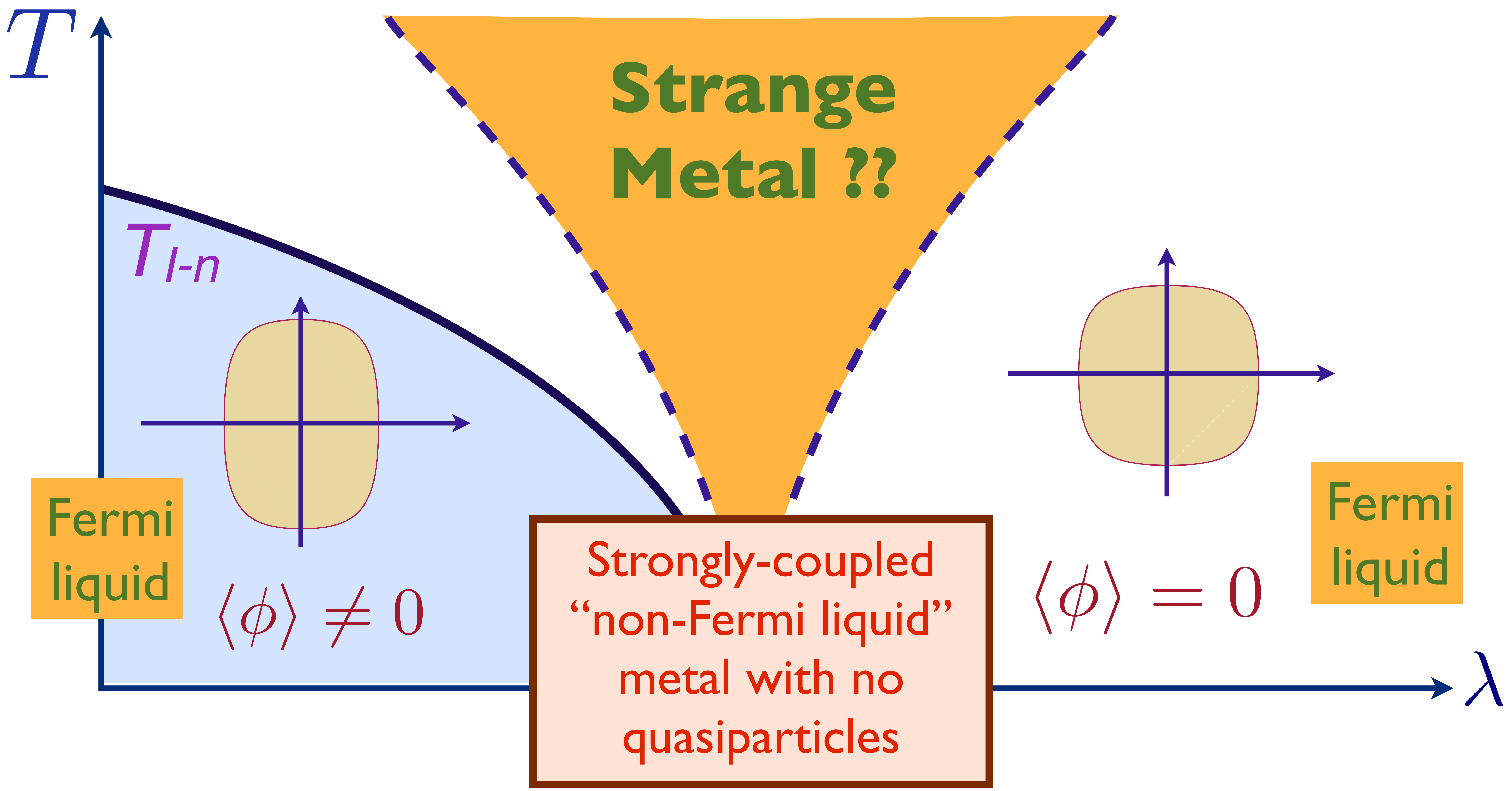

Phase diagram as a function of T and λ

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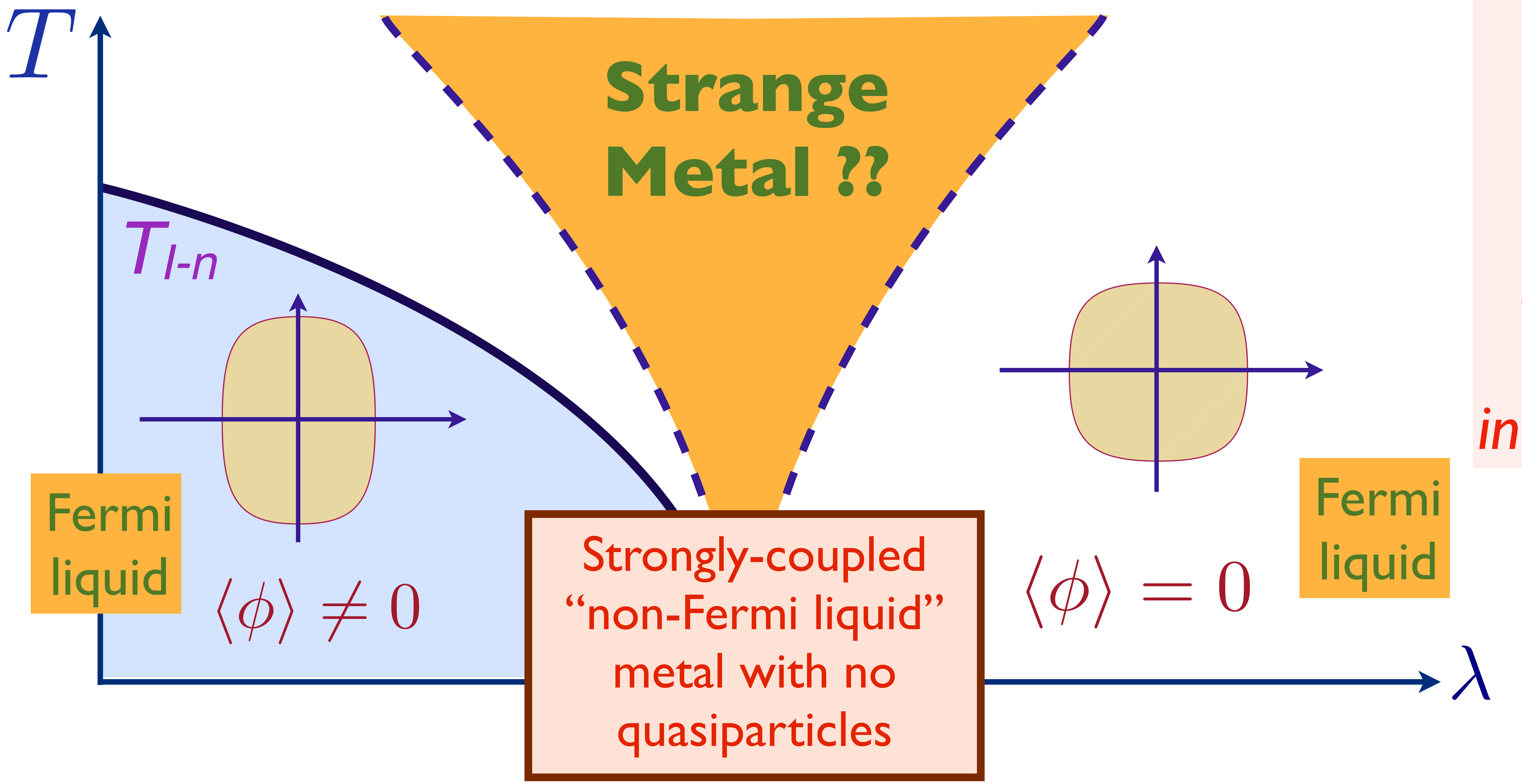
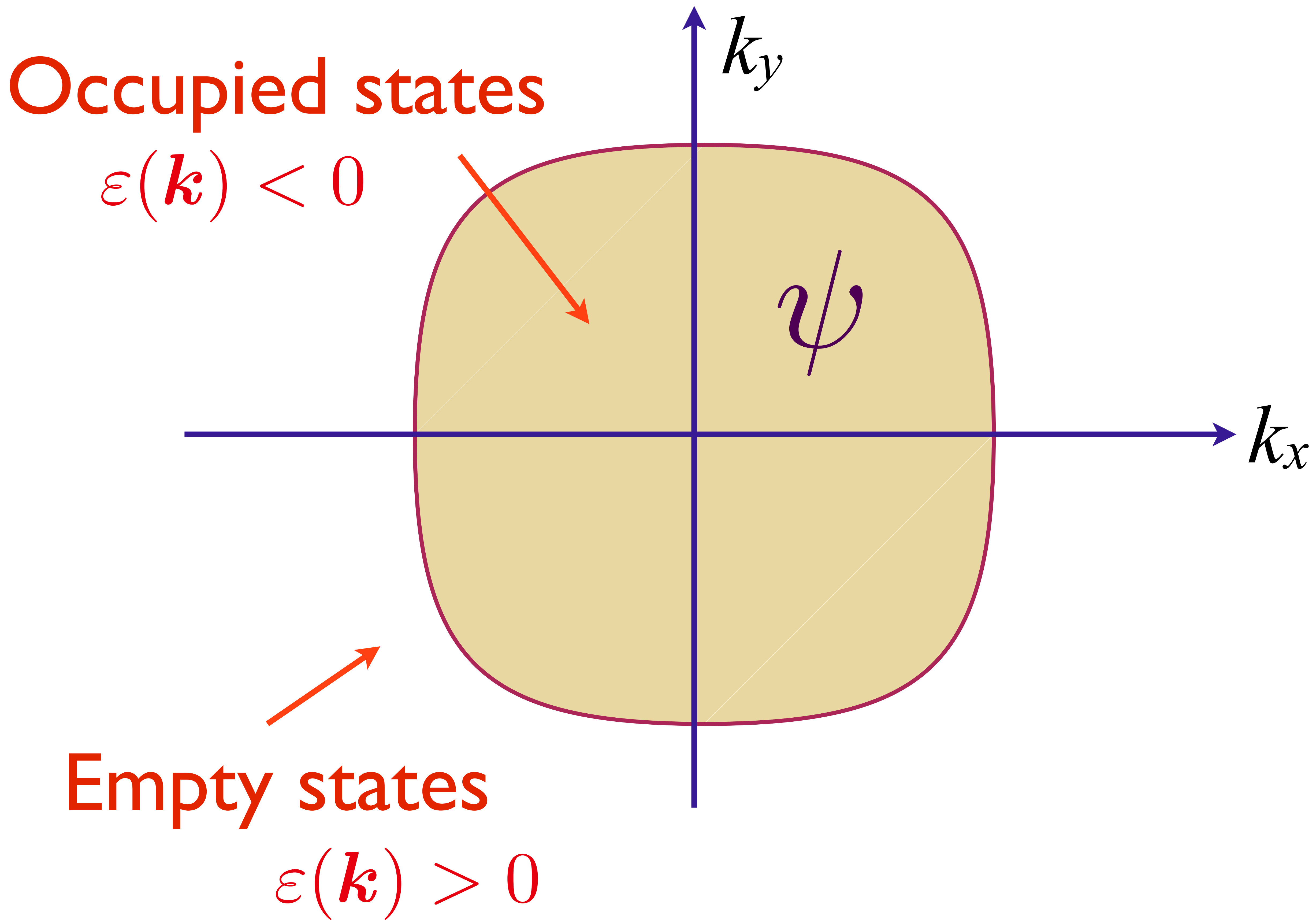


No.

Need spatial *disorder* in the *interactions*

Phase diagram as a function of T and λ

Fermi surface coupled to a critical boson



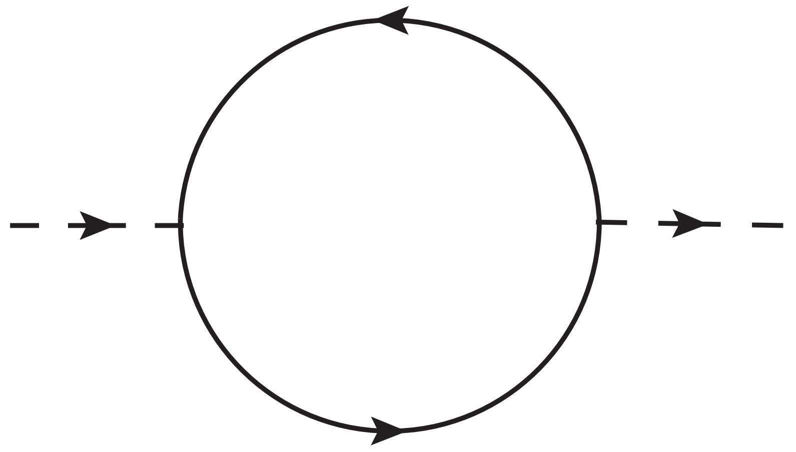
+

a critical boson ϕ

Fermi surface coupled to a critical boson

"Yukawa" coupling: $\quad g \int d^2r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

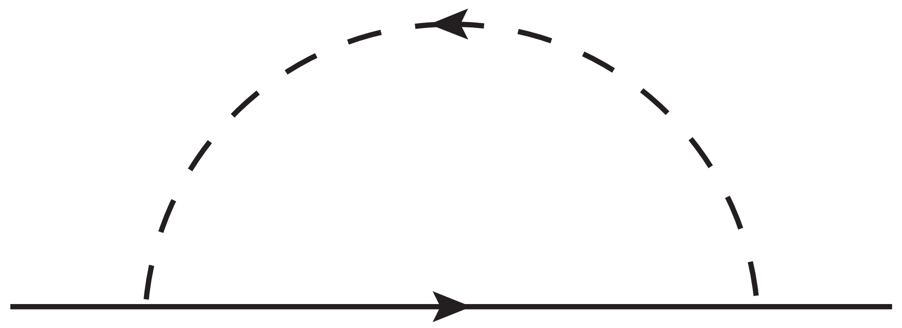
Boson self energy $\Pi(q, i\Omega) \sim -g^2 \dfrac{|\Omega|}{q}$ (Landau damping)



Boson Green's function $D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|/q}$

Fermion self energy $\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}$

Fermion Green's function $G(\boldsymbol{k}, i\omega) = \dfrac{1}{i\omega \mp k_x - k_y^2 - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$



P.A. Lee (1989)

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface coupled to a critical boson

"Yukawa" coupling: $\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

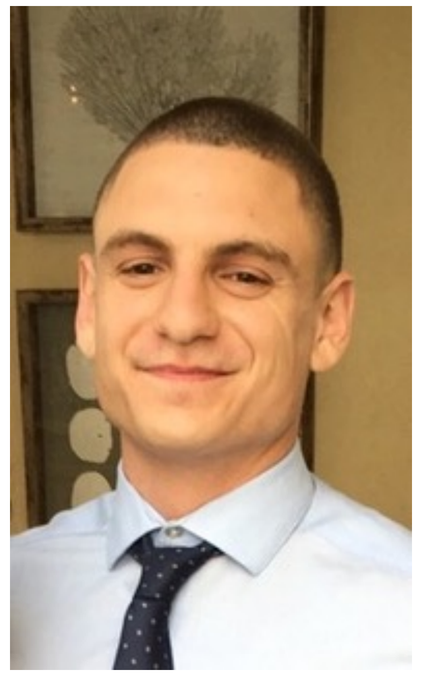
$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory



The theory self-averages, and the average partition function can be written exactly as a 'G-Σ' theory involving a path integral over *bilocal in spacetime*. We introduce the spacetime co-ordinate $X \equiv (\tau, x, y)$, and all Green's functions and self energies in the path integral are functions of two spacetime co-ordinates X_1 and X_2.

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(X_1, X_2)\mathcal{D}\Sigma(X_1, X_2)\mathcal{D}D(X_1, X_2)\mathcal{D}\Pi(X_1, X_2) \exp\left[-N I(G, \Sigma, D, \Pi)\right] .$$

The G-Σ-D-Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) &= \frac{g^2}{2}\mathrm{Tr}\left(G \cdot [GD]\right) - \mathrm{Tr}(G \cdot \Sigma) + \frac{1}{2}\mathrm{Tr}(D \cdot \Pi) \\ &- \ln\det\left[(\partial_{\tau_1} + \varepsilon(-i\boldsymbol{\nabla}_1))\,\delta(X_1 - X_2) + \Sigma(X_1, X_2)\right] \\ &\quad + \frac{1}{2}\ln\det\left[\left(-\partial_{\tau_1}^2 - \boldsymbol{\nabla}_1^2 + s\right)\delta(X_1 - X_2) - \Pi(X_1, X_2)\right] . \end{aligned}$$

where we have introduced notation

$$\mathrm{Tr}\left(f \cdot g\right) \equiv \int dX_1 dX_2\, f(X_2, X_1) g(X_1, X_2) .$$

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

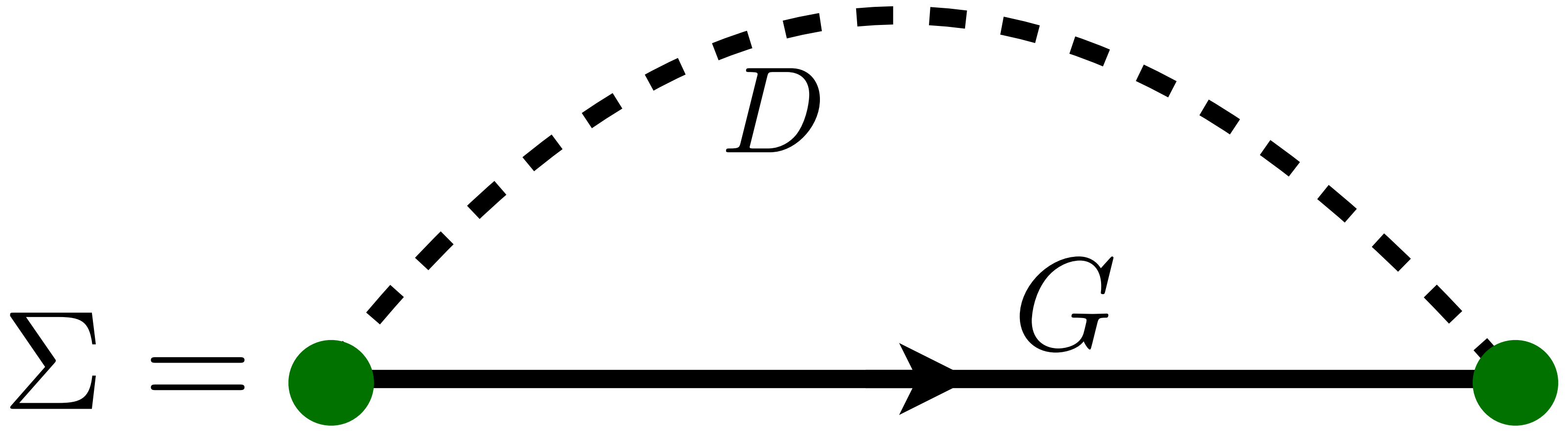
The saddle point equations are

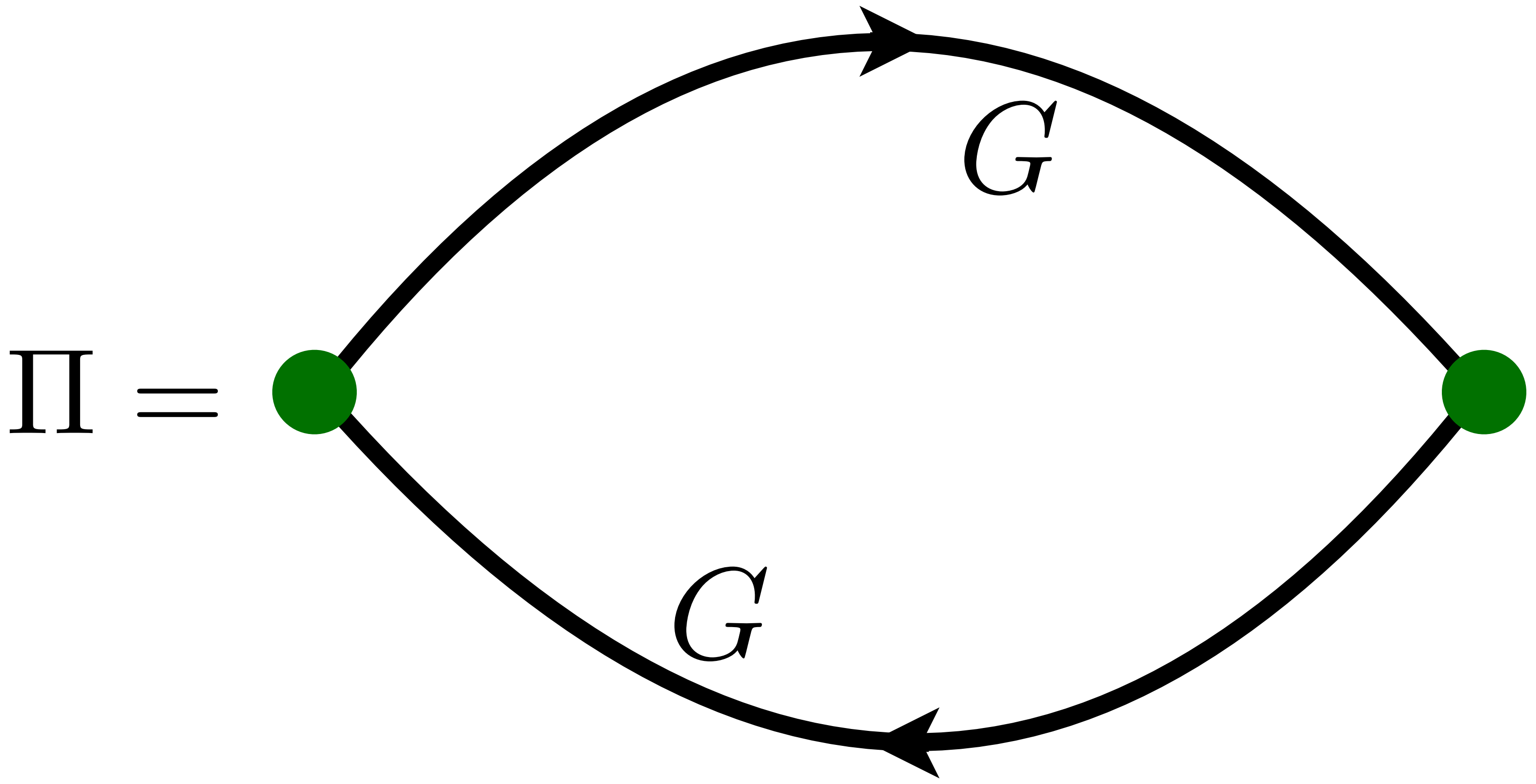
$$\Sigma(\boldsymbol{r},\tau) = g^2 \lambda D(\boldsymbol{r},\tau) G(\boldsymbol{r},\tau),$$

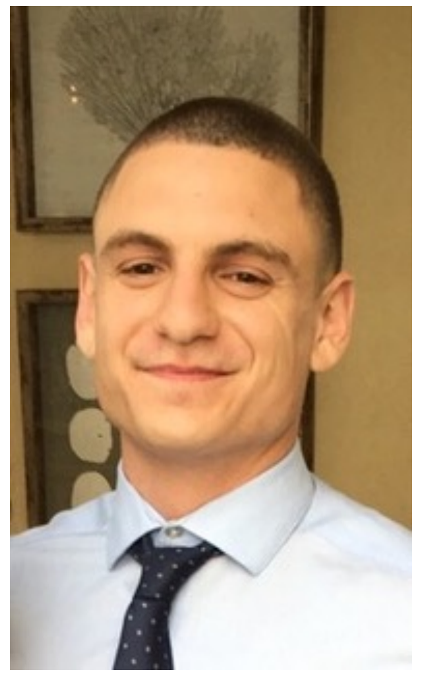
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau) G(\boldsymbol{r},\tau),$$

$$G(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n)},$$

$$D(\boldsymbol{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q}, i\Omega_m)}.$$







Exact Solution at small ω:

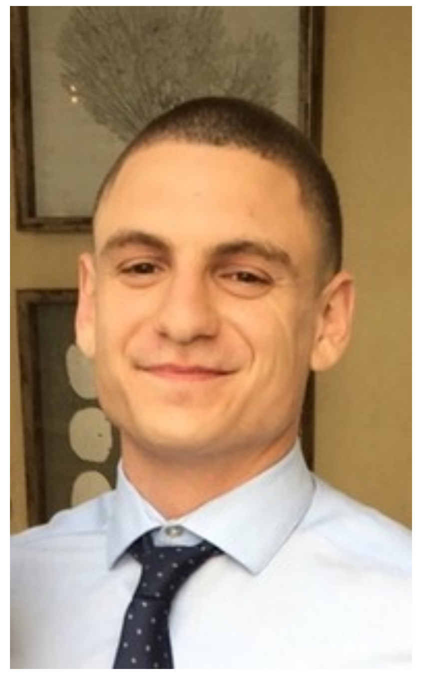
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}}, i\omega)}$$

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

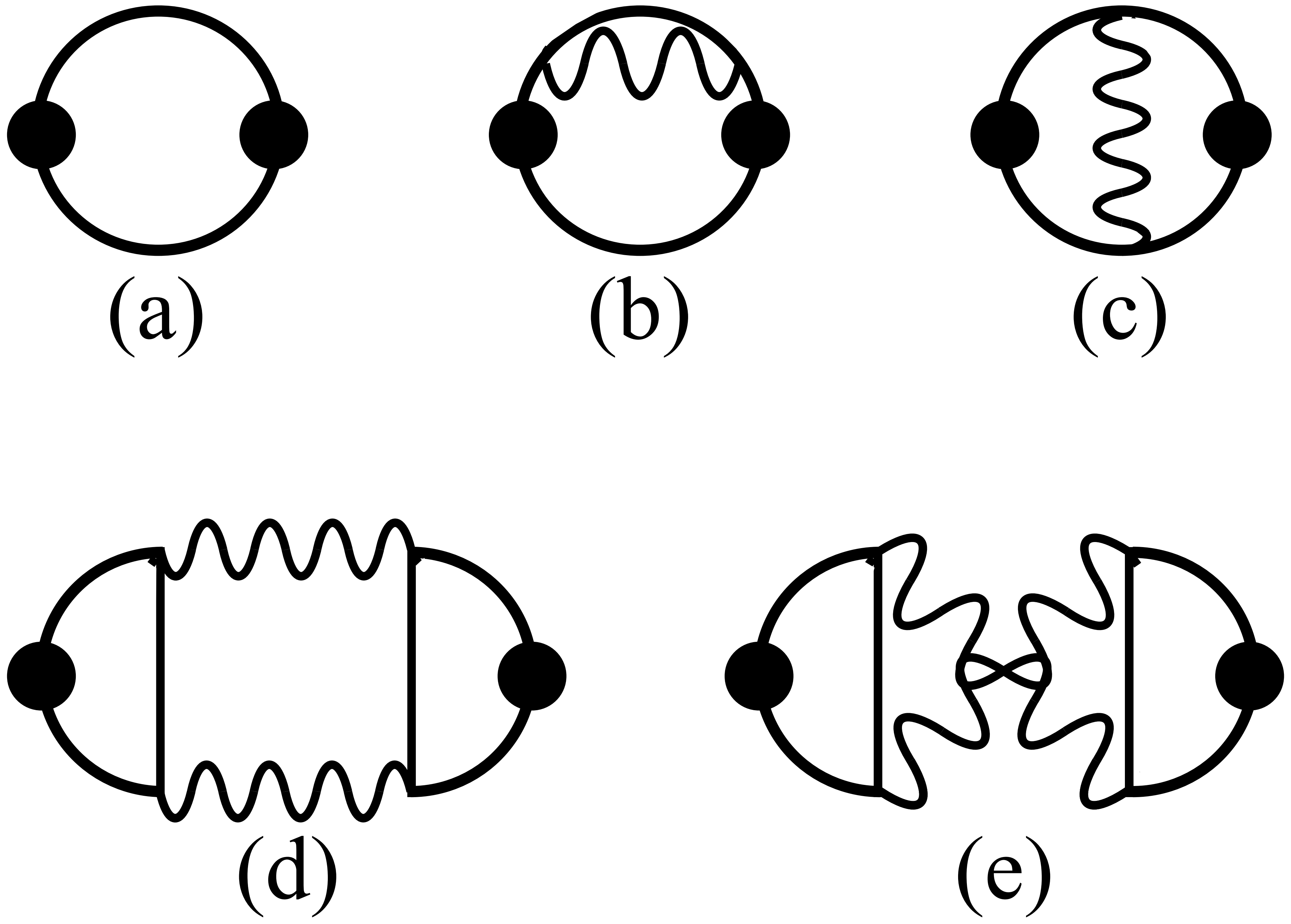
Strange metal in two spatial dimensions from spatially random interactions

Fermi surface coupled to a critical boson


"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$



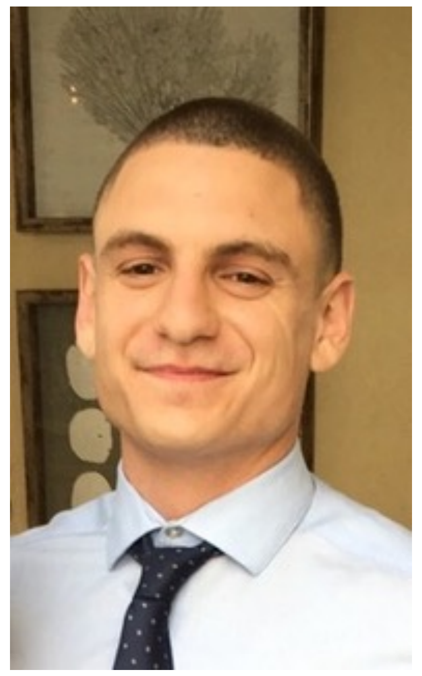
+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

These results do not account for conservation of total momentum *i.e.* 'boson drag'.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson



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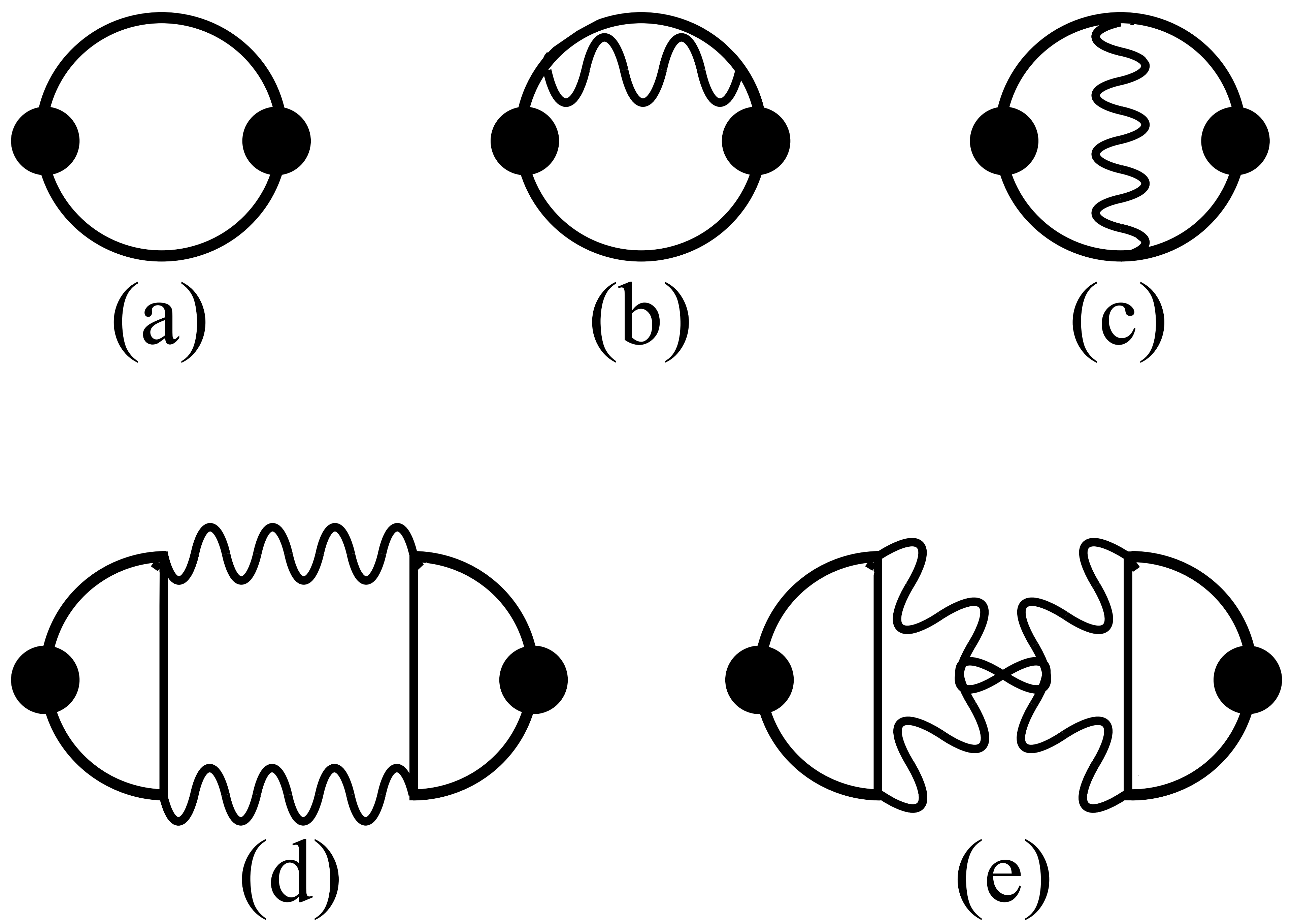


+ all ladders and bubbles…..

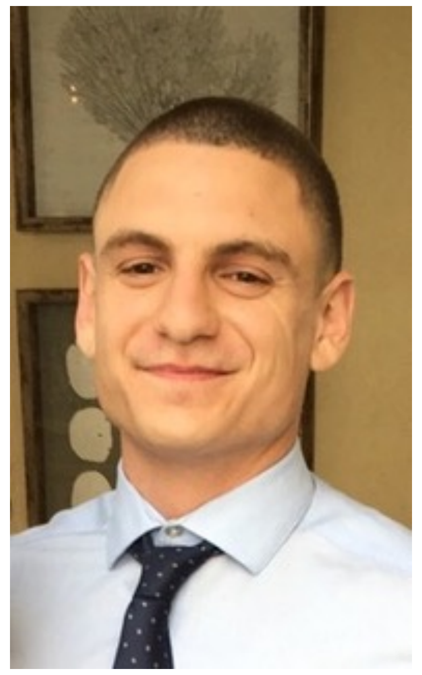
Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson


"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

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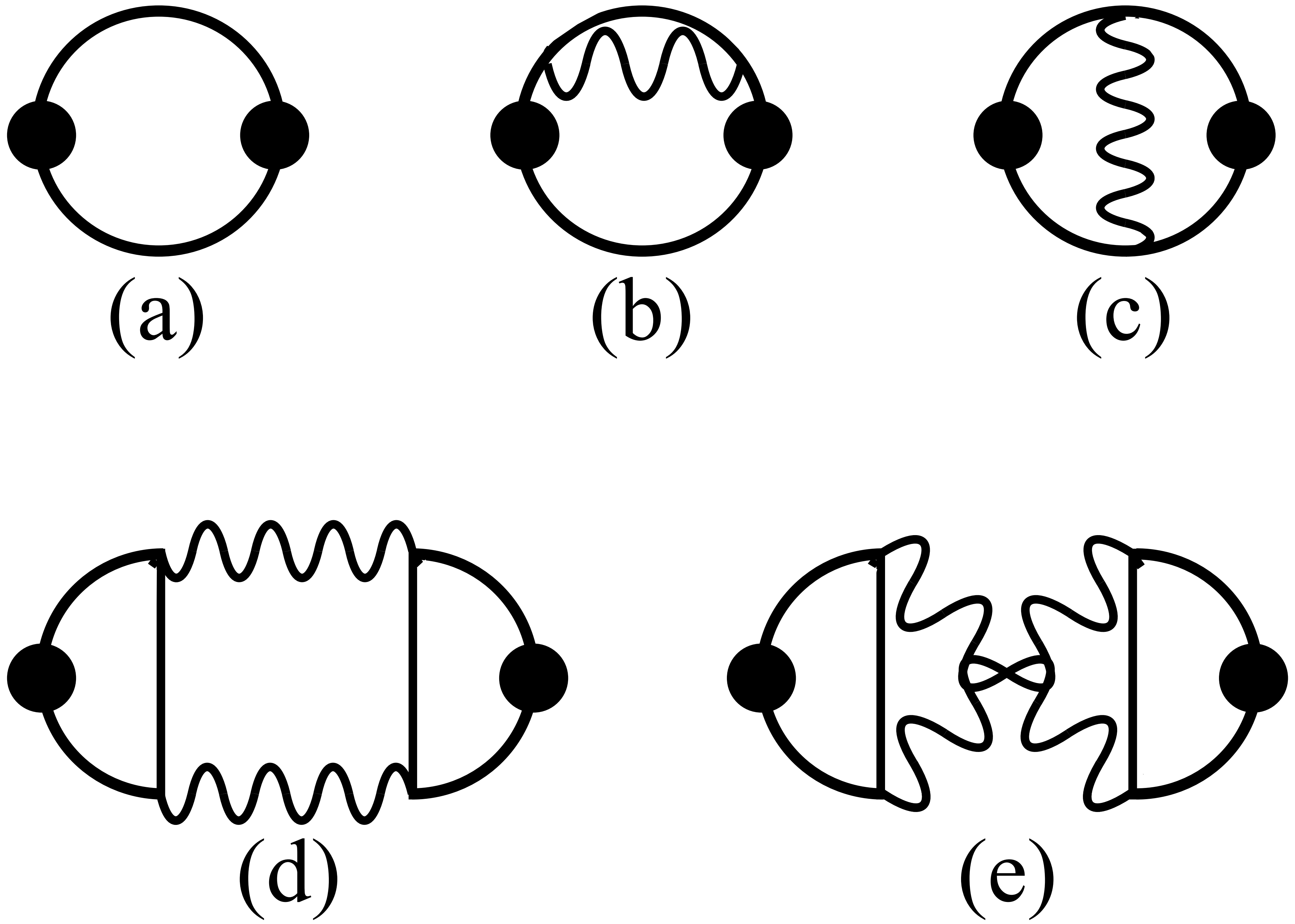

+ all ladders and bubbles.....

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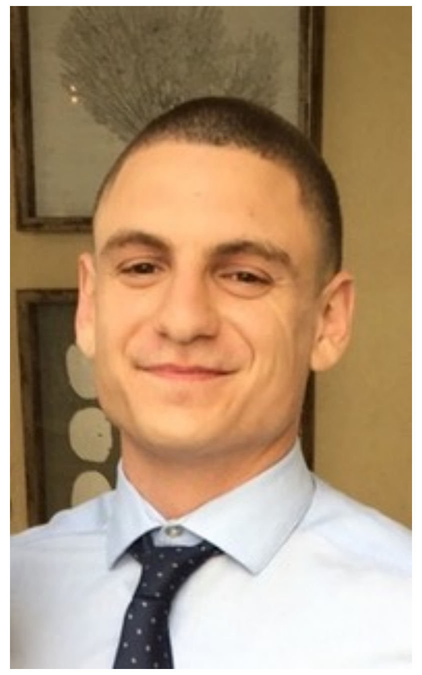
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \ldots$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder



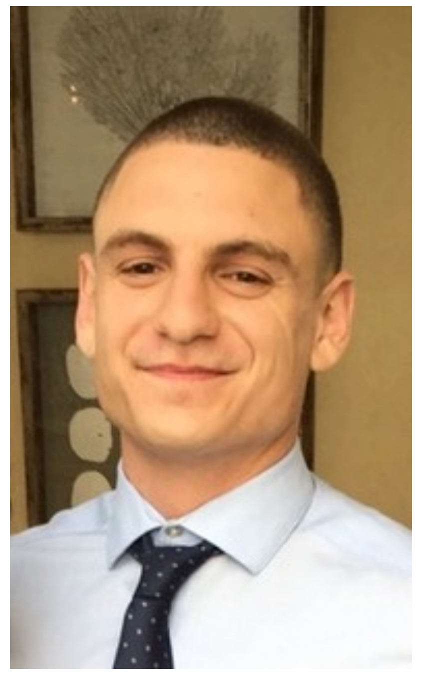
"Yukawa" coupling: $$\frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random potential: $$+\frac{1}{\sqrt{N}}\int d^2r d\tau\, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl} g_{abc}} = g^2\,\delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r) v_{lm}(r')} = v^2\,\delta(r-r')\delta_{il}\delta_{jm}$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder



"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N} \displaystyle\int d^2r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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Marginal Fermi liquid self energy and *T log T* specific heat

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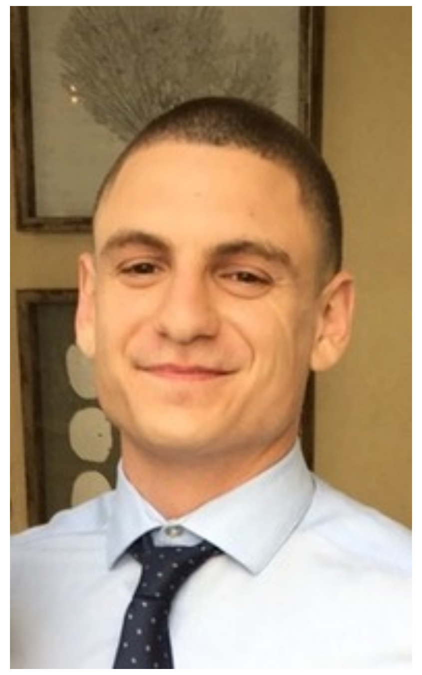

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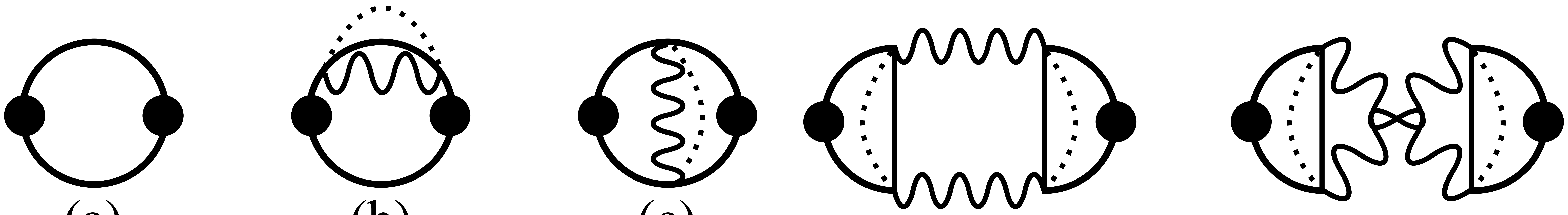

(a) σ_v (b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$ (c) $\sigma_{V,g}$ (d) (e)

+ all ladders and bubbles.....

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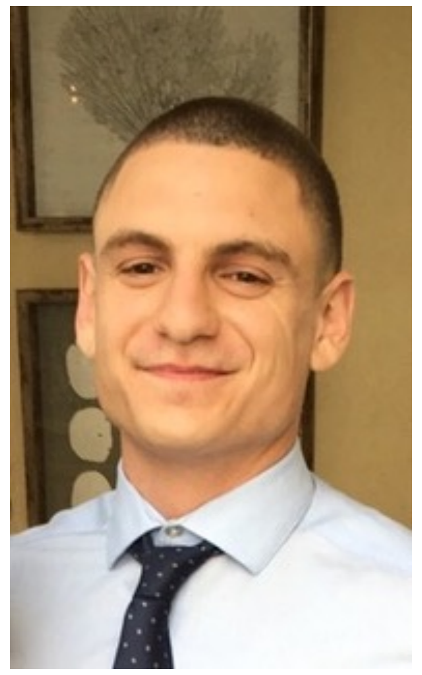


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The g^2 log term does not contribute to transport

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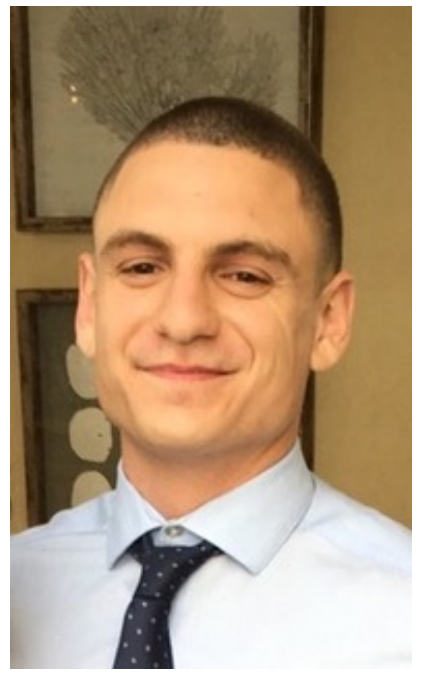


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With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots.$$

with $1/\rho(0) \sim 1/\tau_{\rm trans} \sim v^2$.

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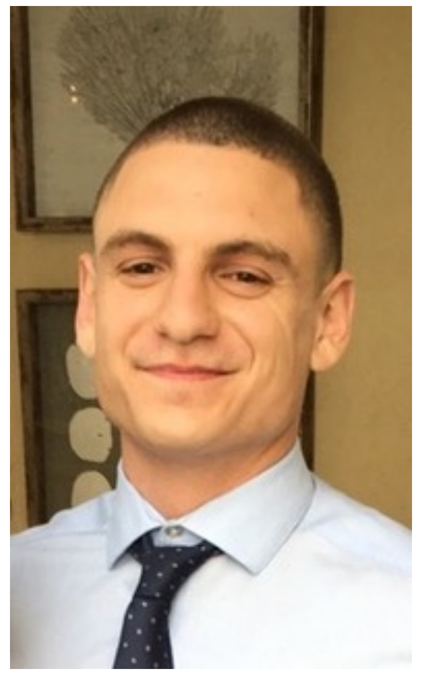


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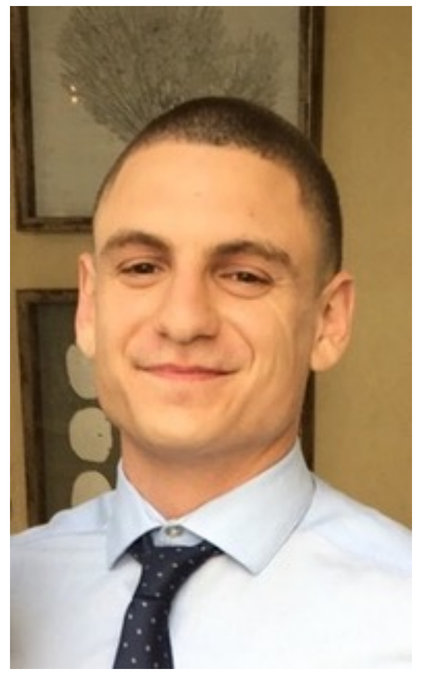

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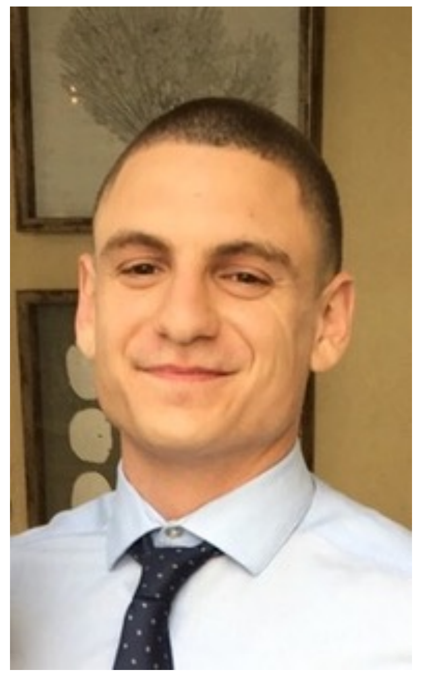


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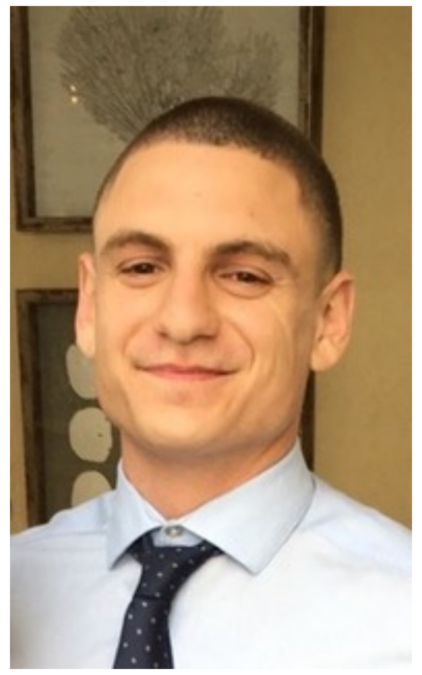

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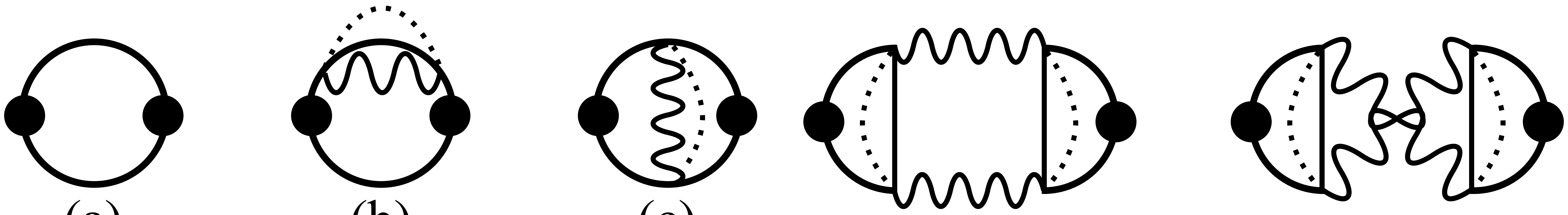

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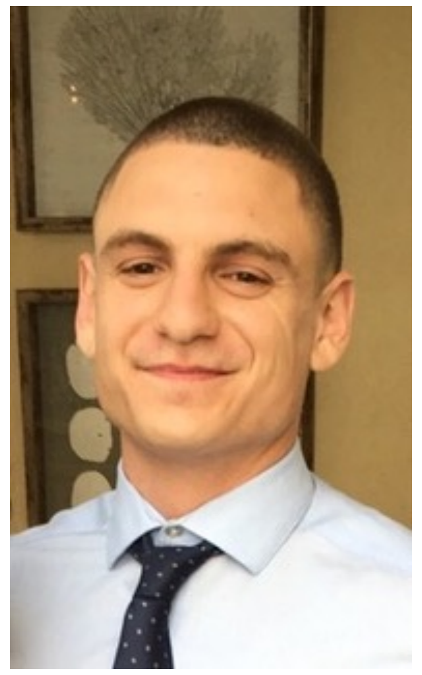


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Conductivity:

The g^2 log term does not contribute to transport
but the g'^2 log term does!

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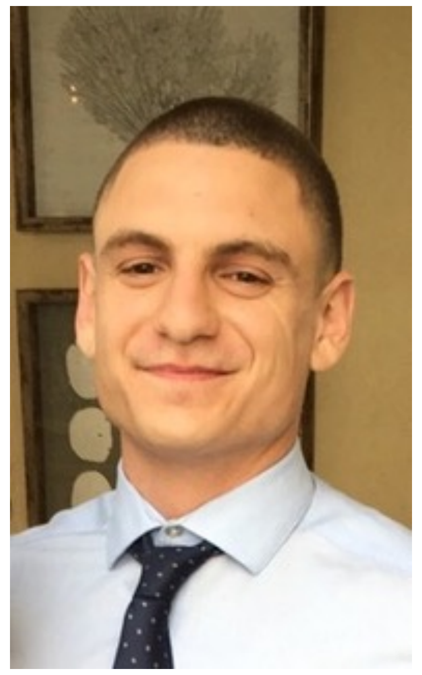

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Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega \, m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

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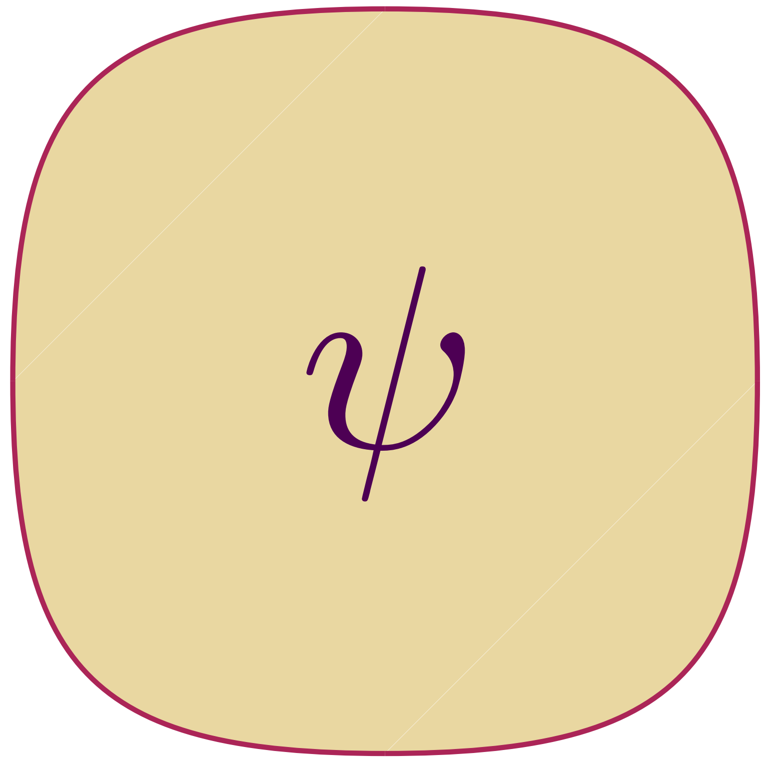
Strange metal from a Yukawa-SYK model



+

a critical boson

ϕ

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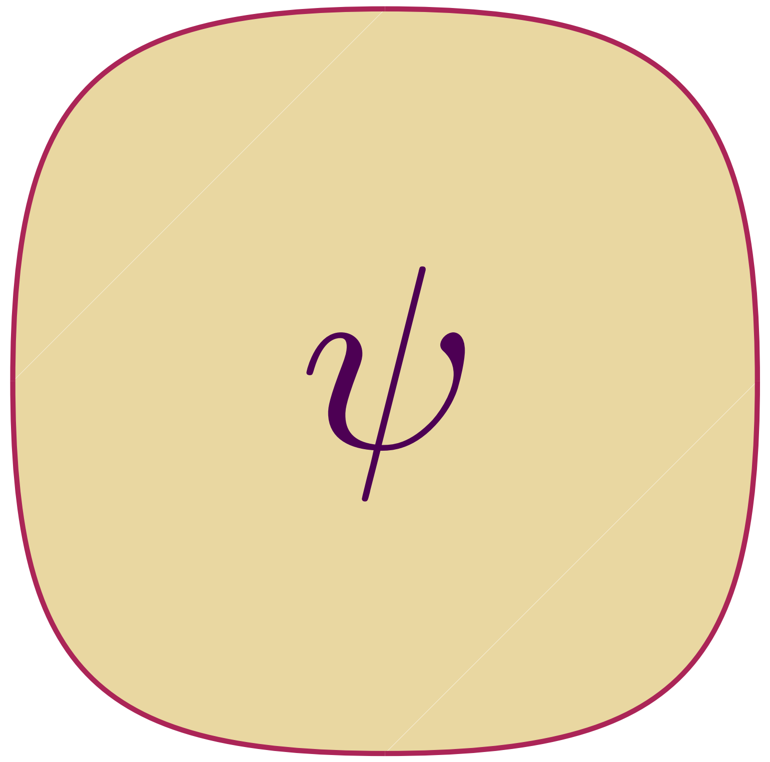

+ a critical boson

ϕ

Non-Fermi liquid with $T^{2/3}$ specific heat,
but conductivity $\sigma(\omega) \sim \delta(\omega)$

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Strange metal from a Yukawa-SYK model

ψ + a critical boson ϕ

MFL self-energy, $T\ln(1/T)$ specific heat,
but T-independent 'residual' resistivity,
and negligible optical conductivity

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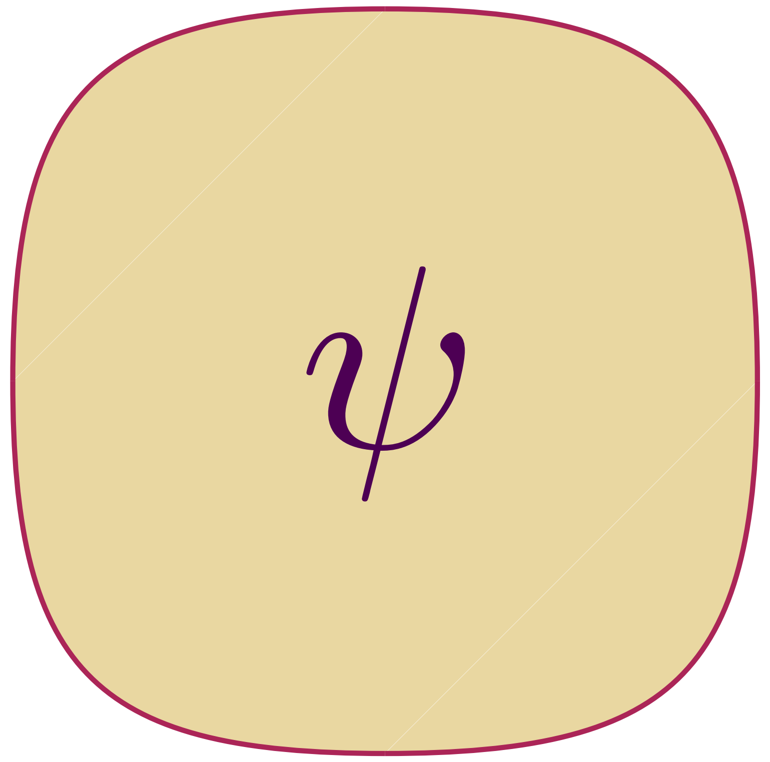

$+$ a critical boson ϕ

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