Quantum spin liquids, and a theory of the cuprate pseudogap metal

Condensed Matter Physics in the City
University College London
June 8, 2022

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Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS
HARVARD

Yahui Zhang

Alexander Nikolaenko

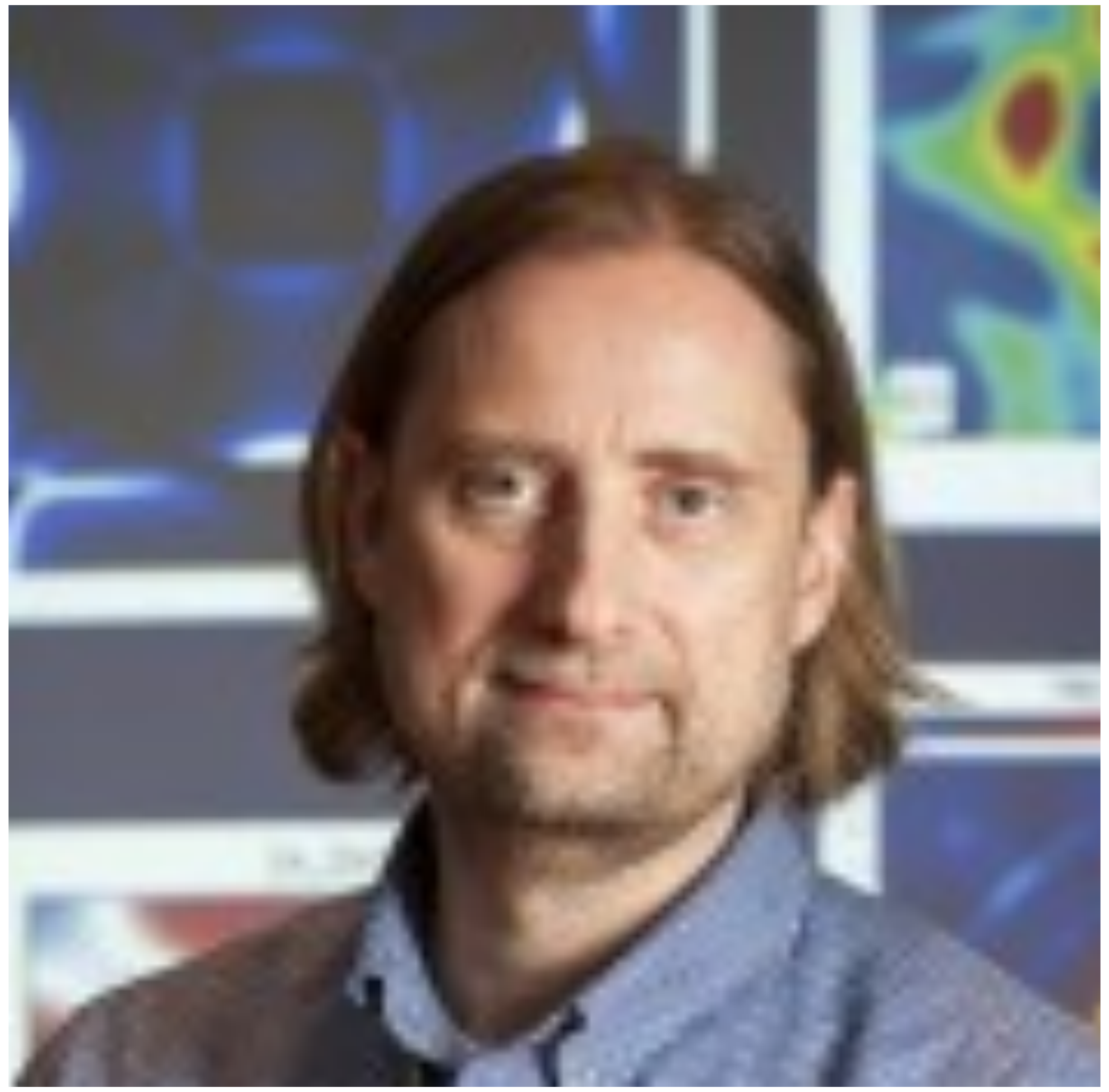
Maria Tikhanovskaya

Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

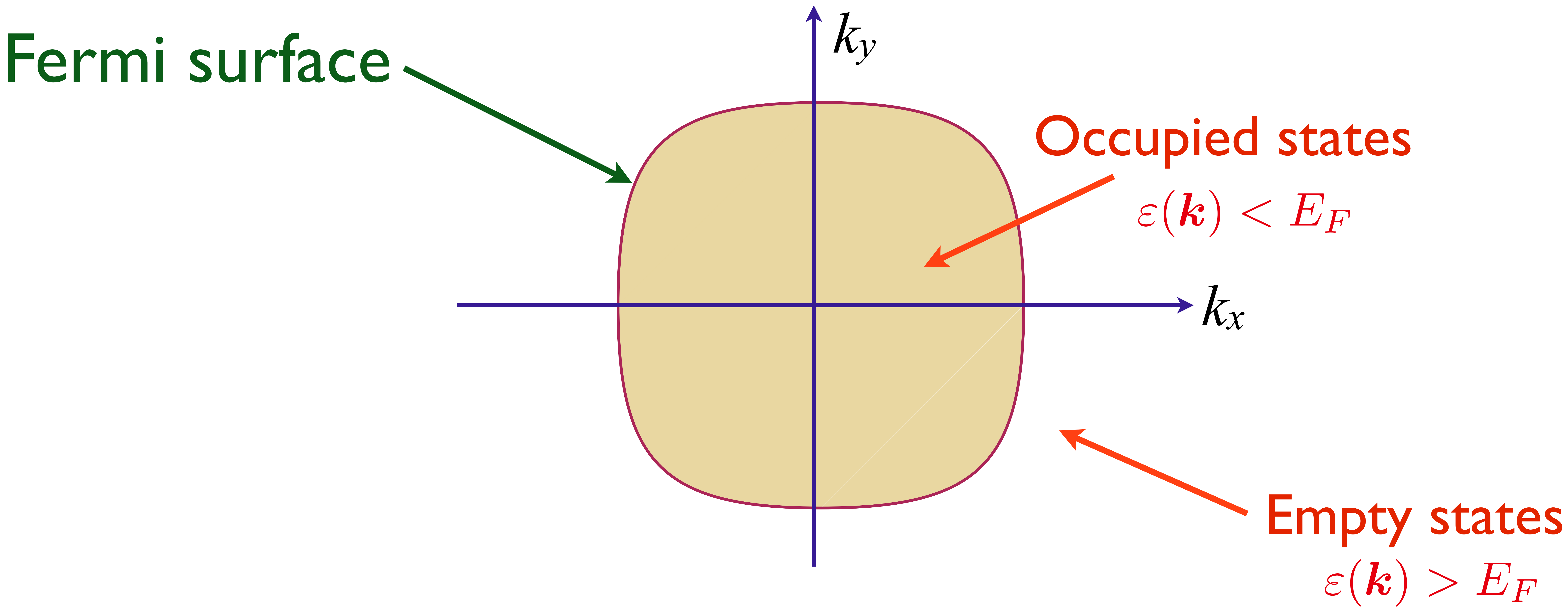
arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



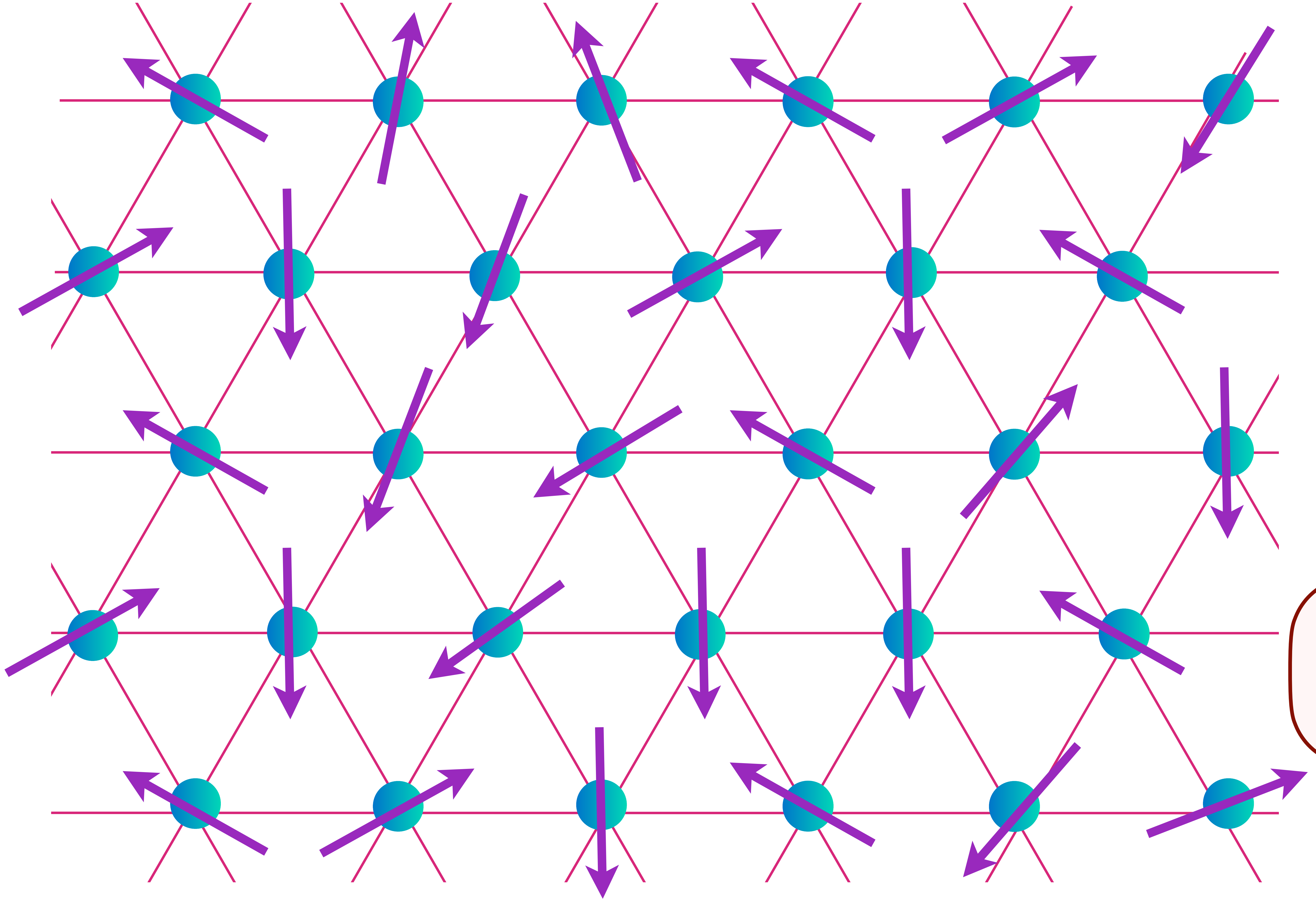
Luttinger relation

Electrons move with momentum $\mathbf{k}$ through the lattice with dispersion $\varepsilon(\boldsymbol{k})$

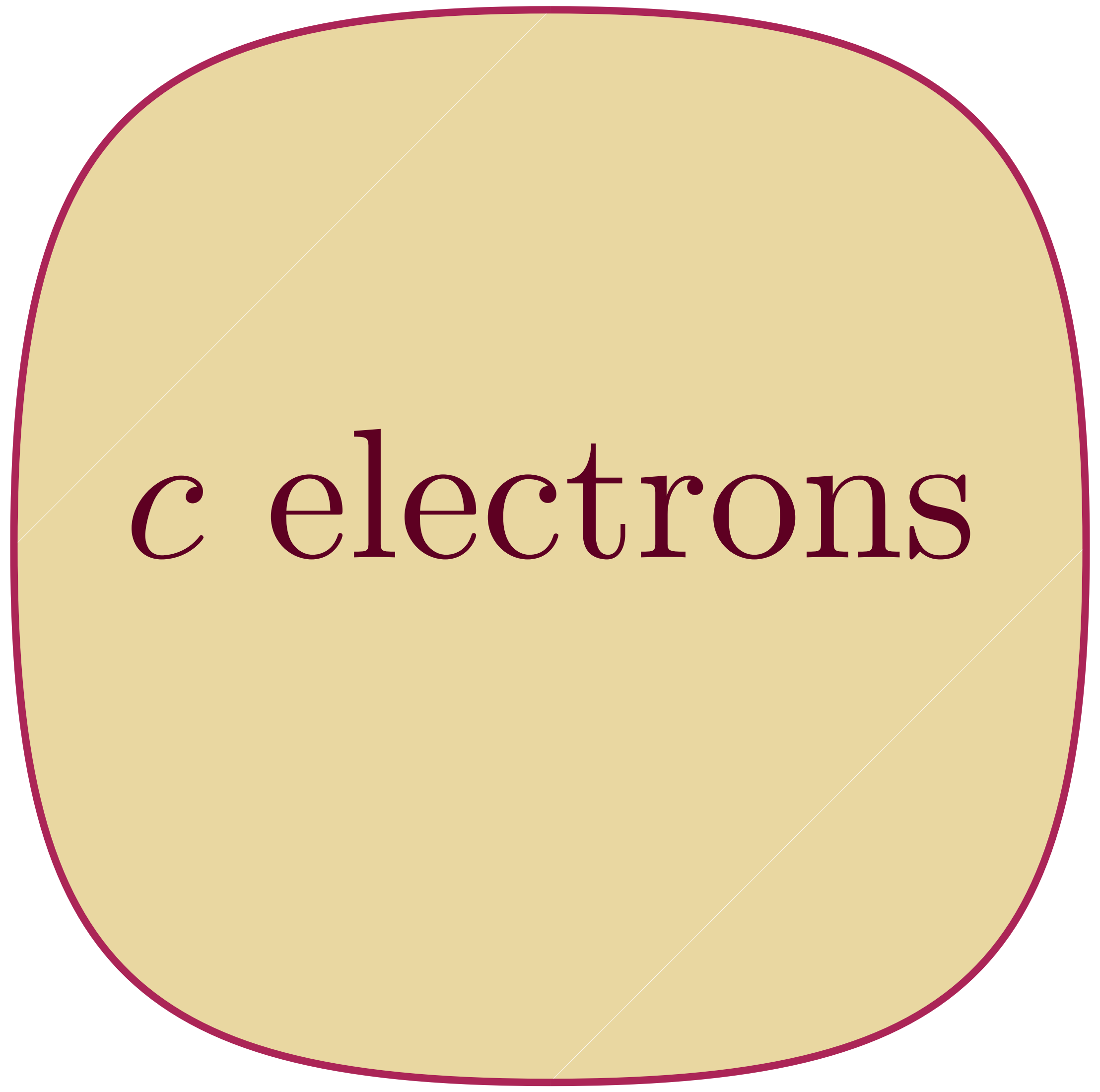


$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

Kondo lattice



Kondo exchange J_K



Density of the electrons per unit cell $= 1 + p$

f electrons

Kondo lattice: HFL phase

Luttinger relation

c and f electrons

Kondo exchange J_K

Density of the electrons per unit cell $= 1 + p$,
Fermi surface size $= 1 + p$.
Luttinger volume "large" Fermi surface.

$$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

Coleman, Read, Newns, Millis, P.A. Lee, Auerbach, Levin

Luttinger* relation

- FL*: violation of the Luttinger relation in a metal requires the presence of fractionalization and emergent gauge fields.

T. Senthil, M. Vojta, and S. S., PRL **90**, 216403 (2003); PRB **69**, 035111 (2004)

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Kondo lattice: FL* phase

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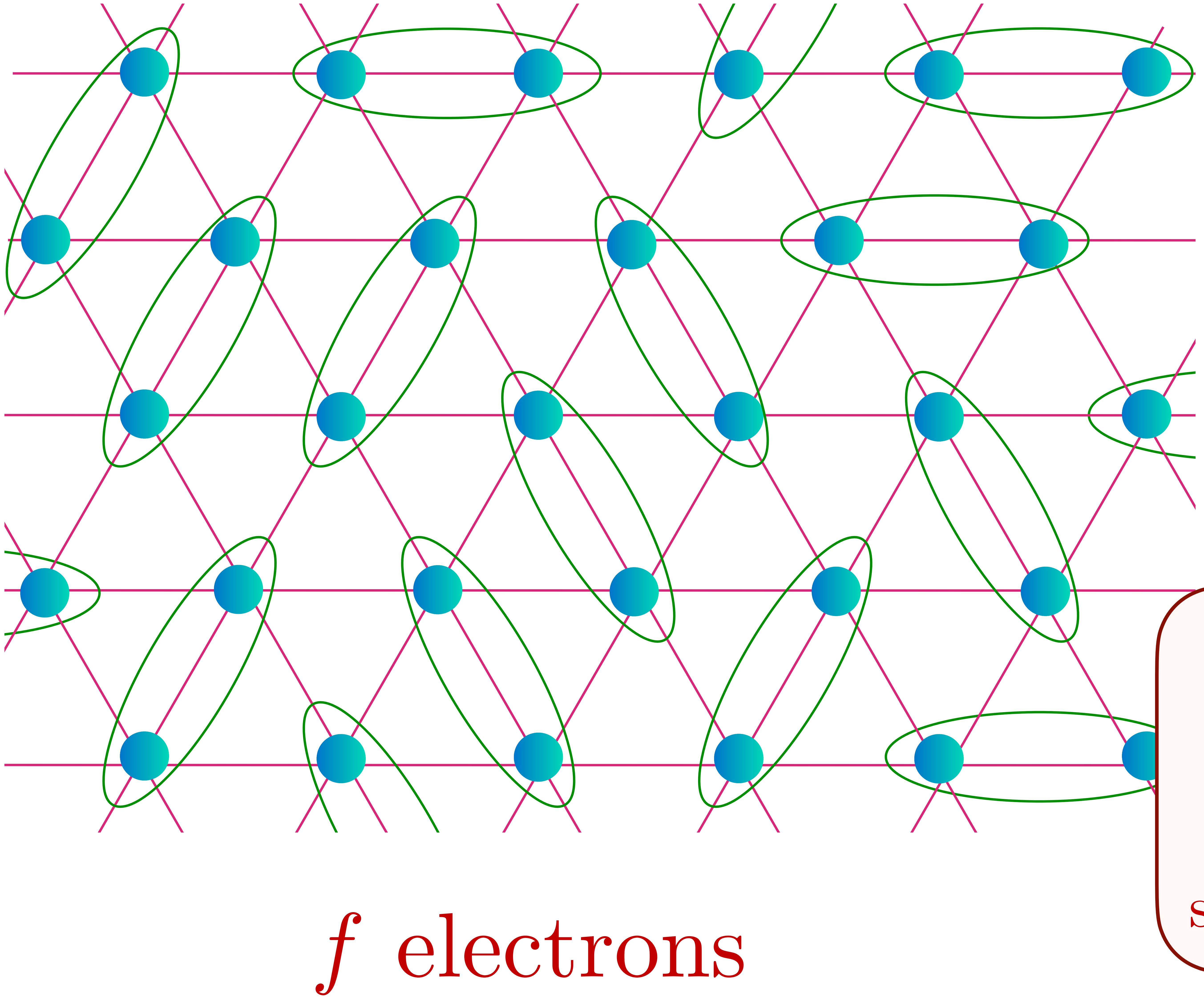

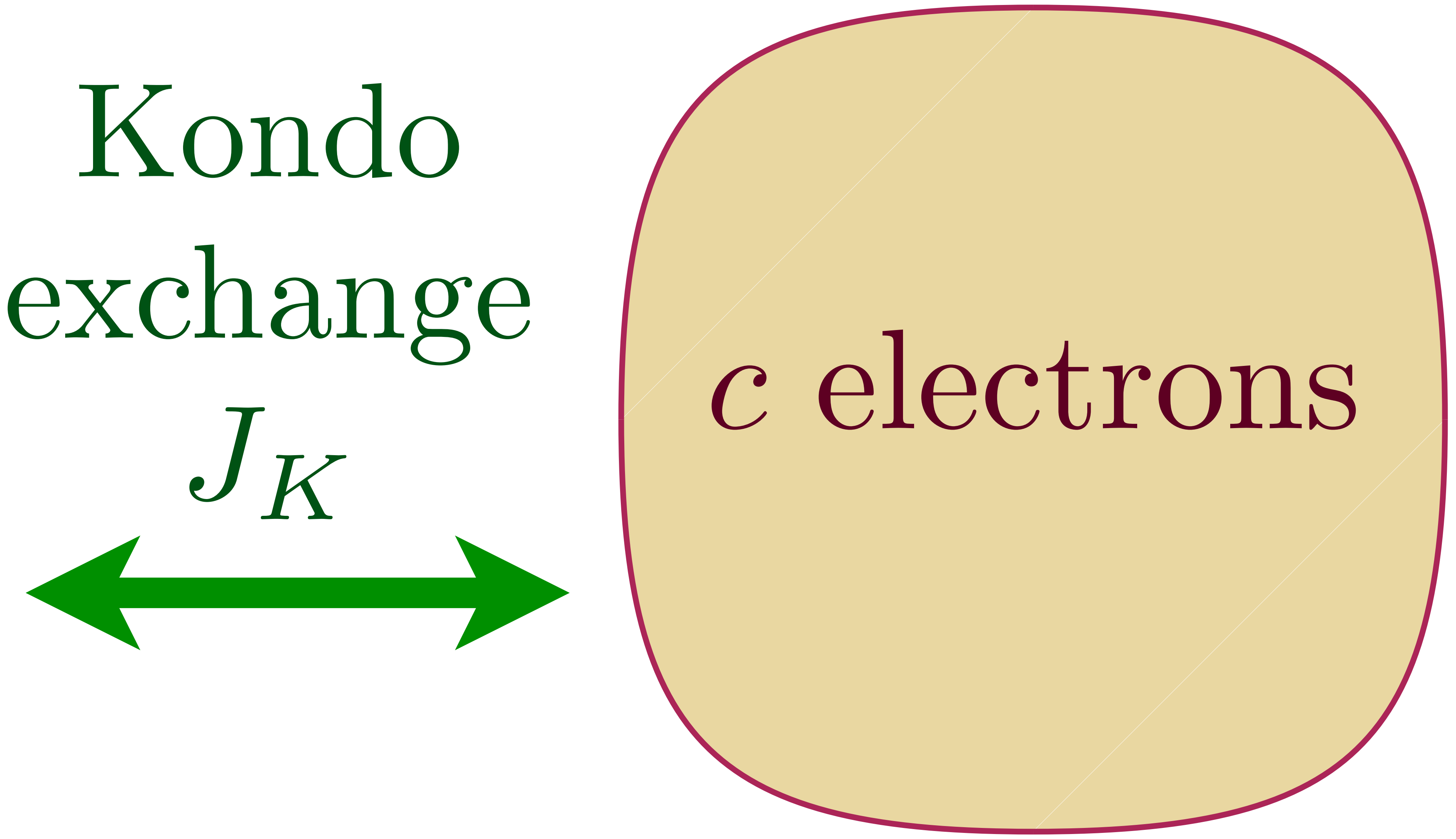

Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= p$
Non-Luttinger volume "small" Fermi
surface size is stable to all orders in J_K.

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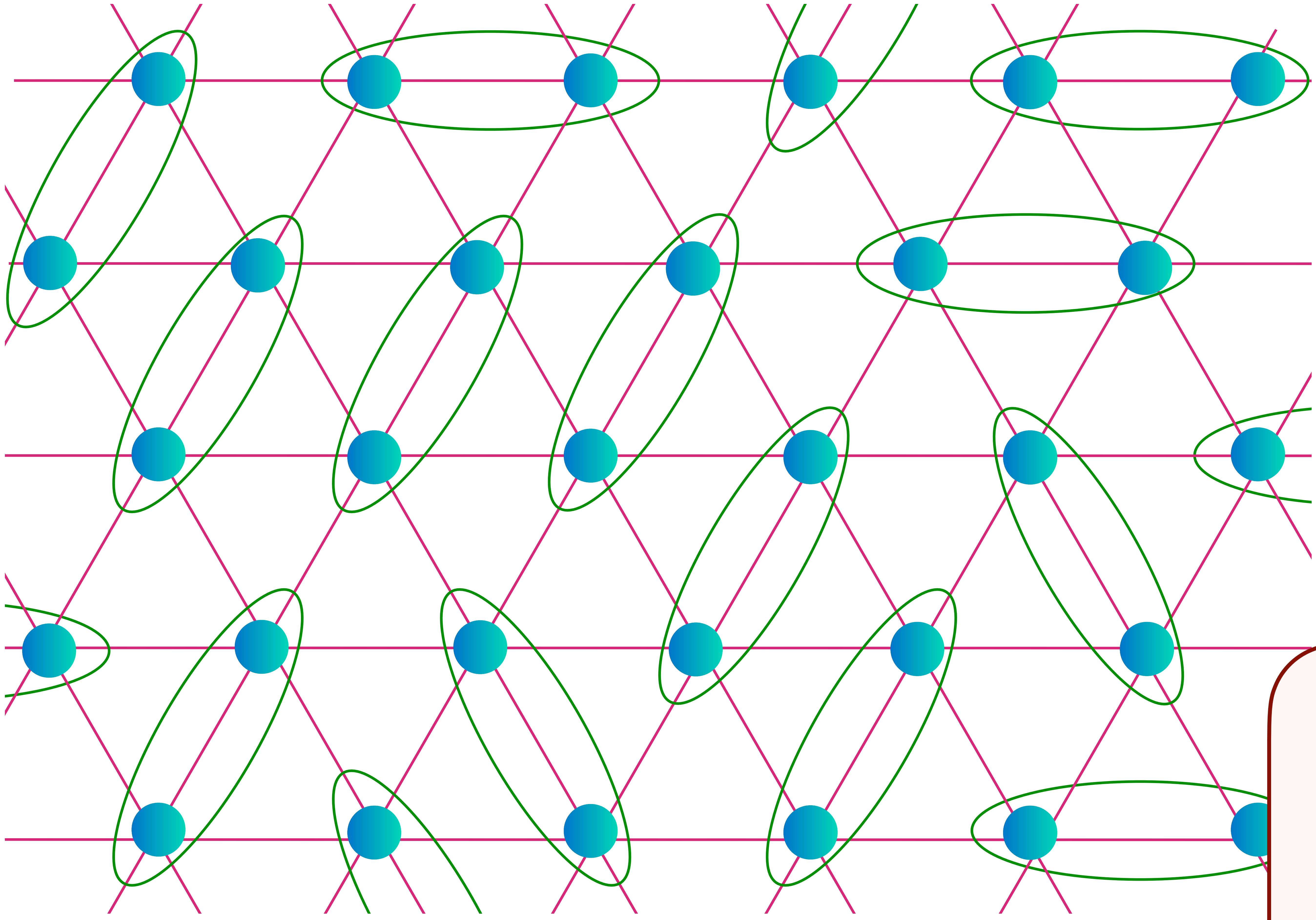

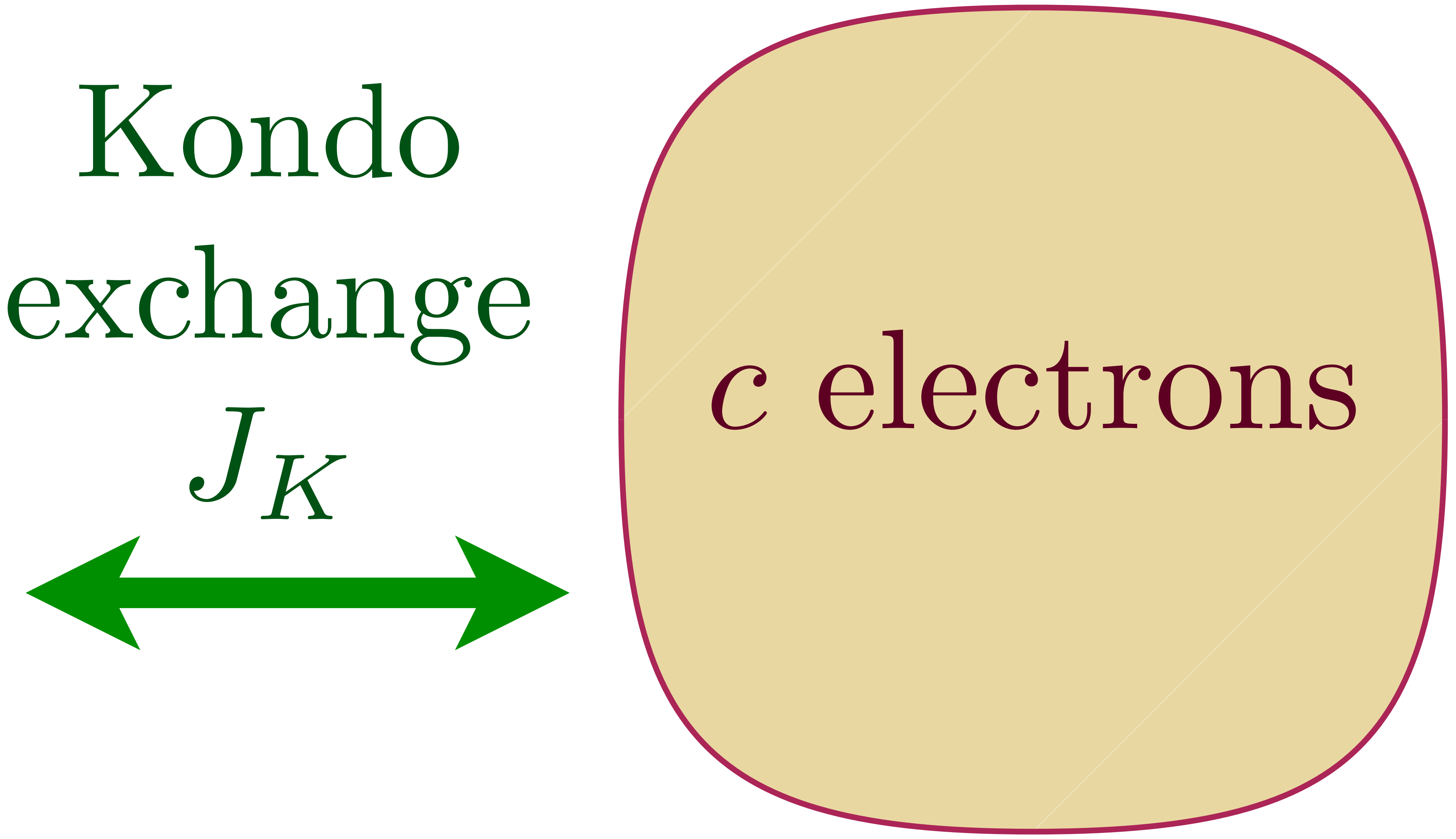

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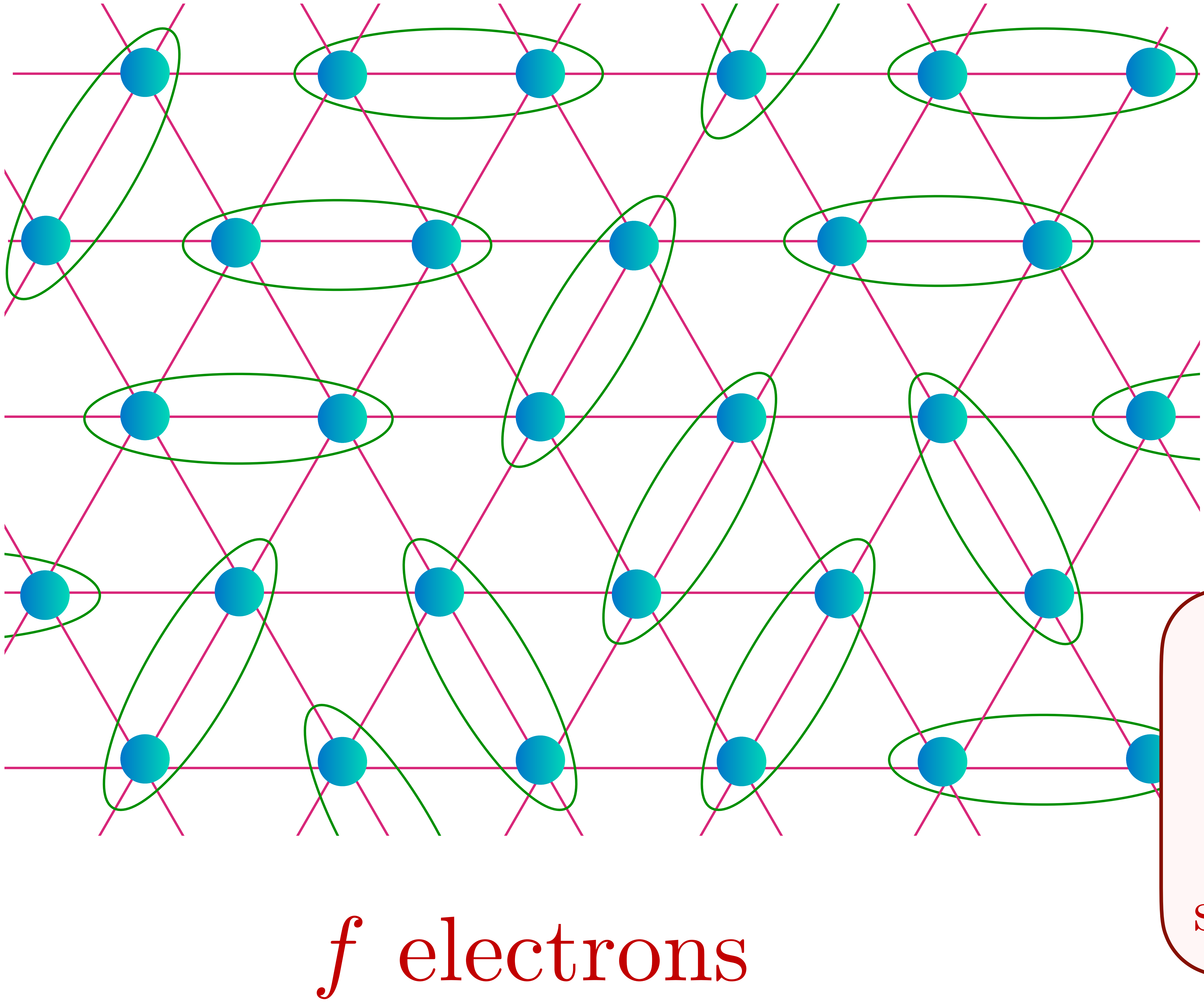

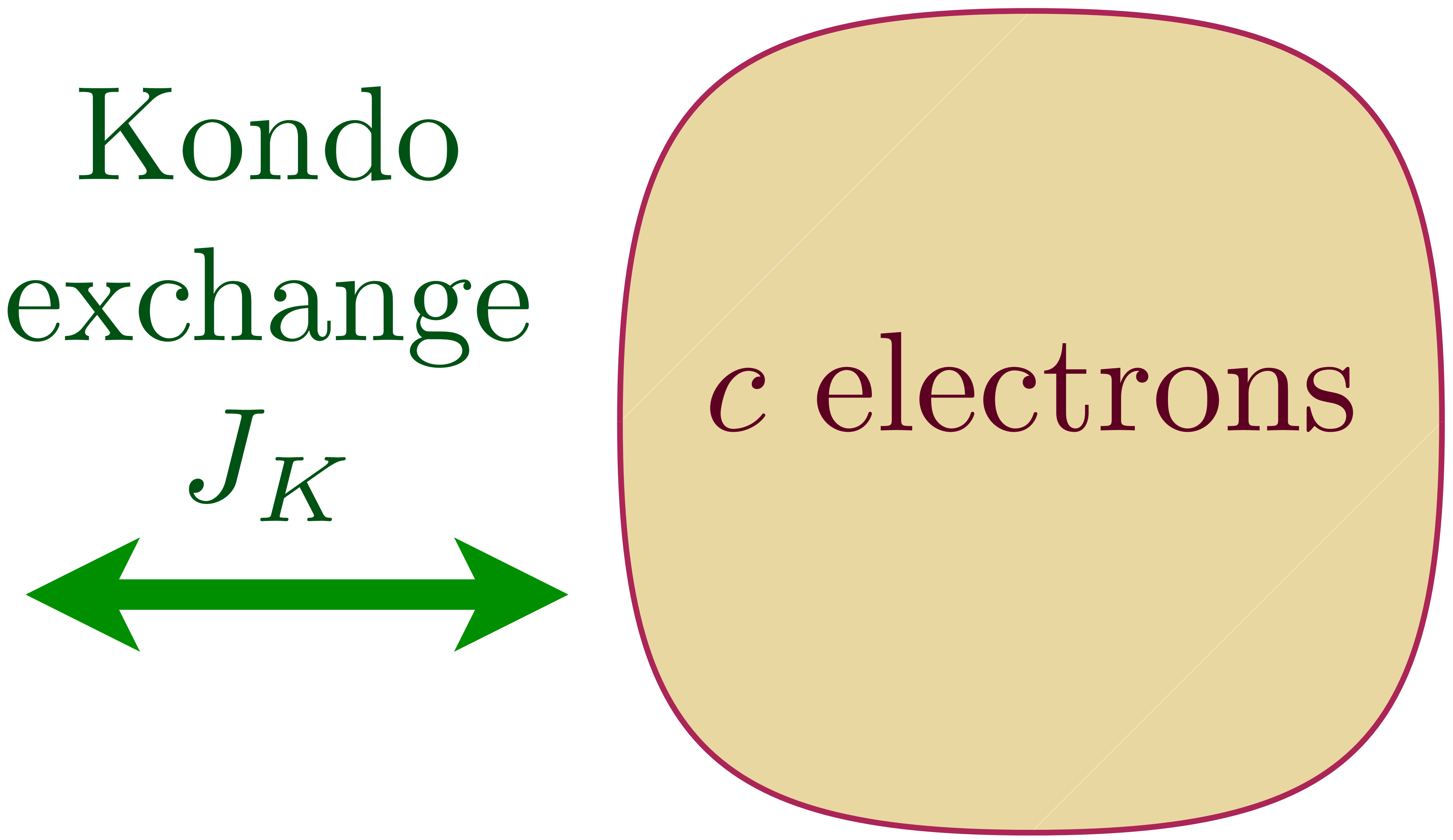

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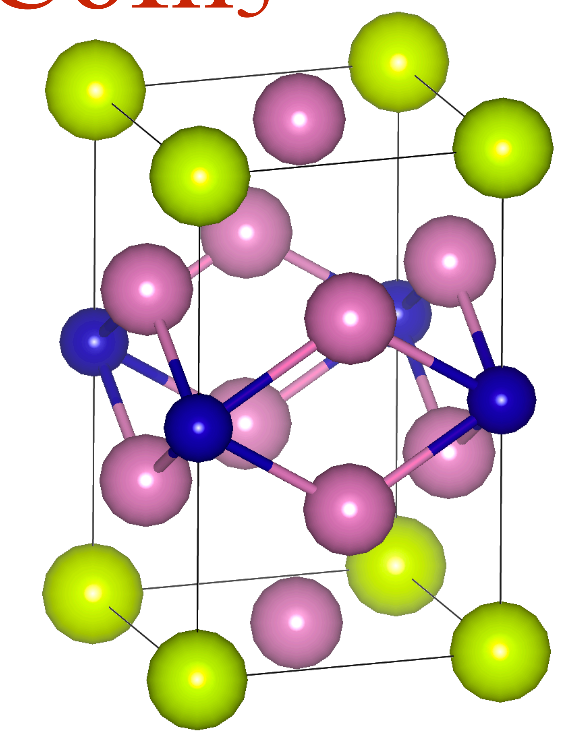
Evidence for a delocalization quantum phase transition without symmetry breaking in $CeCoIn_5$

Nikola Maksimovic, Daniel H. Eilbott, Tessa Cookmeyer, Fanghui Wan, Jan Rusz, Vikram Nagarajan, Shannon C. Haley, Eran Maniv, Amanda Gong, Stefano Faubel, Ian M. Hayes, Ali Bangura, John Singleton, Johanna C. Palmstrom, Laurel Winter, Ross McDonald, Sooyoung Jang, Ping Ai, Yi Lin, Samuel Ciocys, Jacob Gobbo, Yochai Werman, Peter M. Oppeneer, Ehud Altman, Alessandra Lanzara, James G. Analytis, Science **375**, 76 (2021).



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- Key puzzle of the cuprate pseudogap state: how do we obtain a FL* state with a Fermi surface of p holes in a *single-band model* with total hole density $1+p$? How can we democratically localize some holes and not others, when all holes are in the same band?

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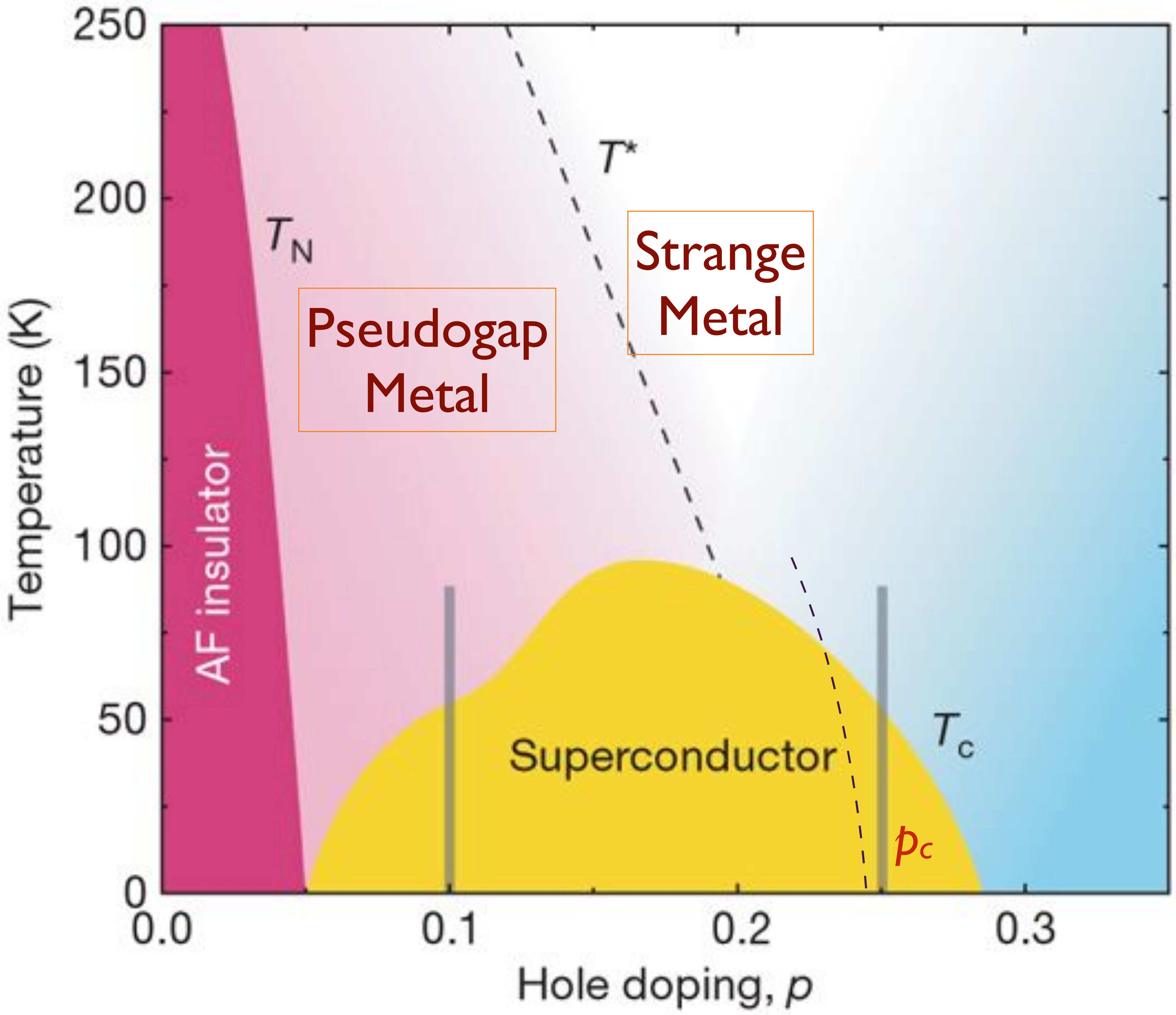
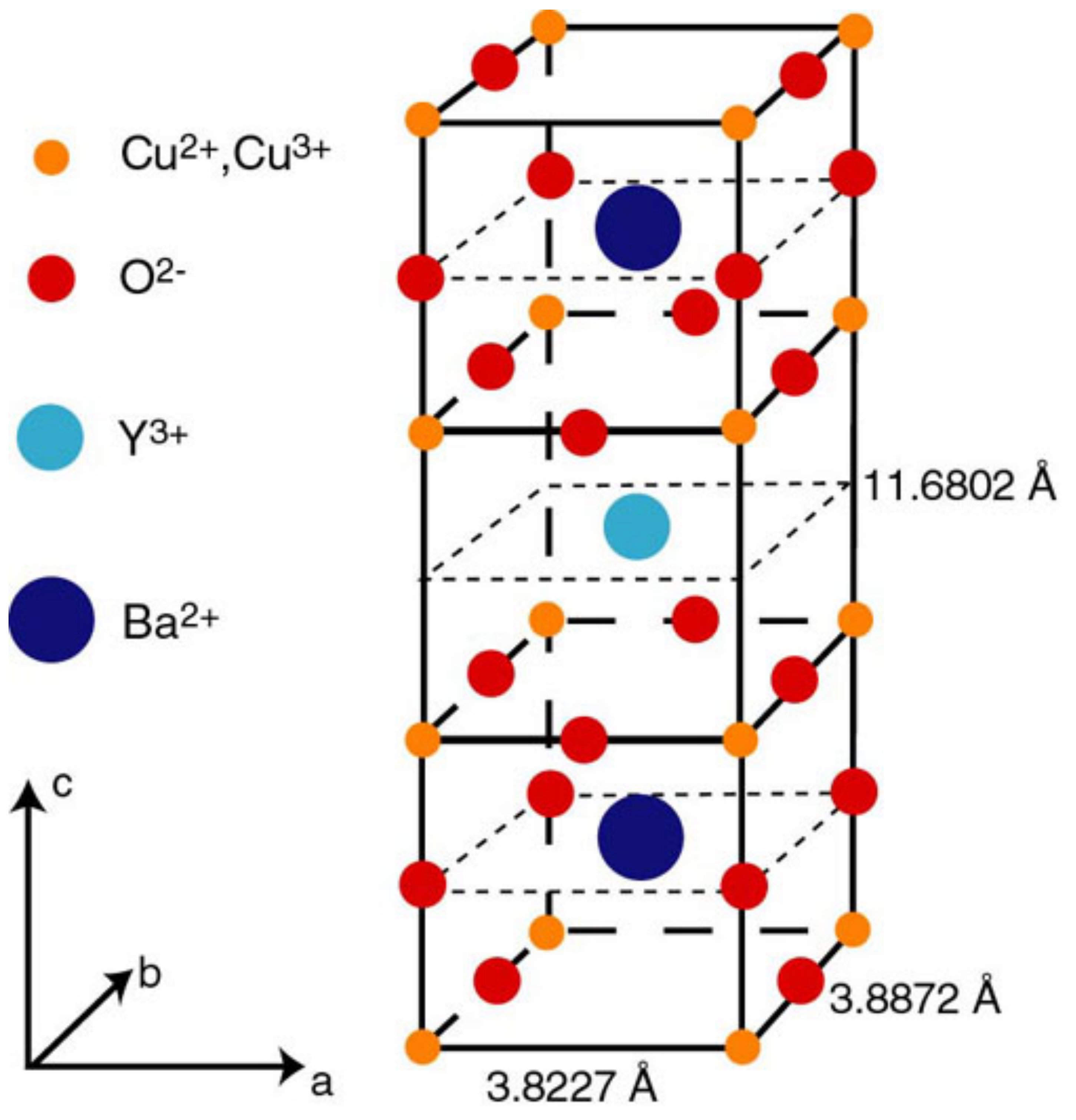
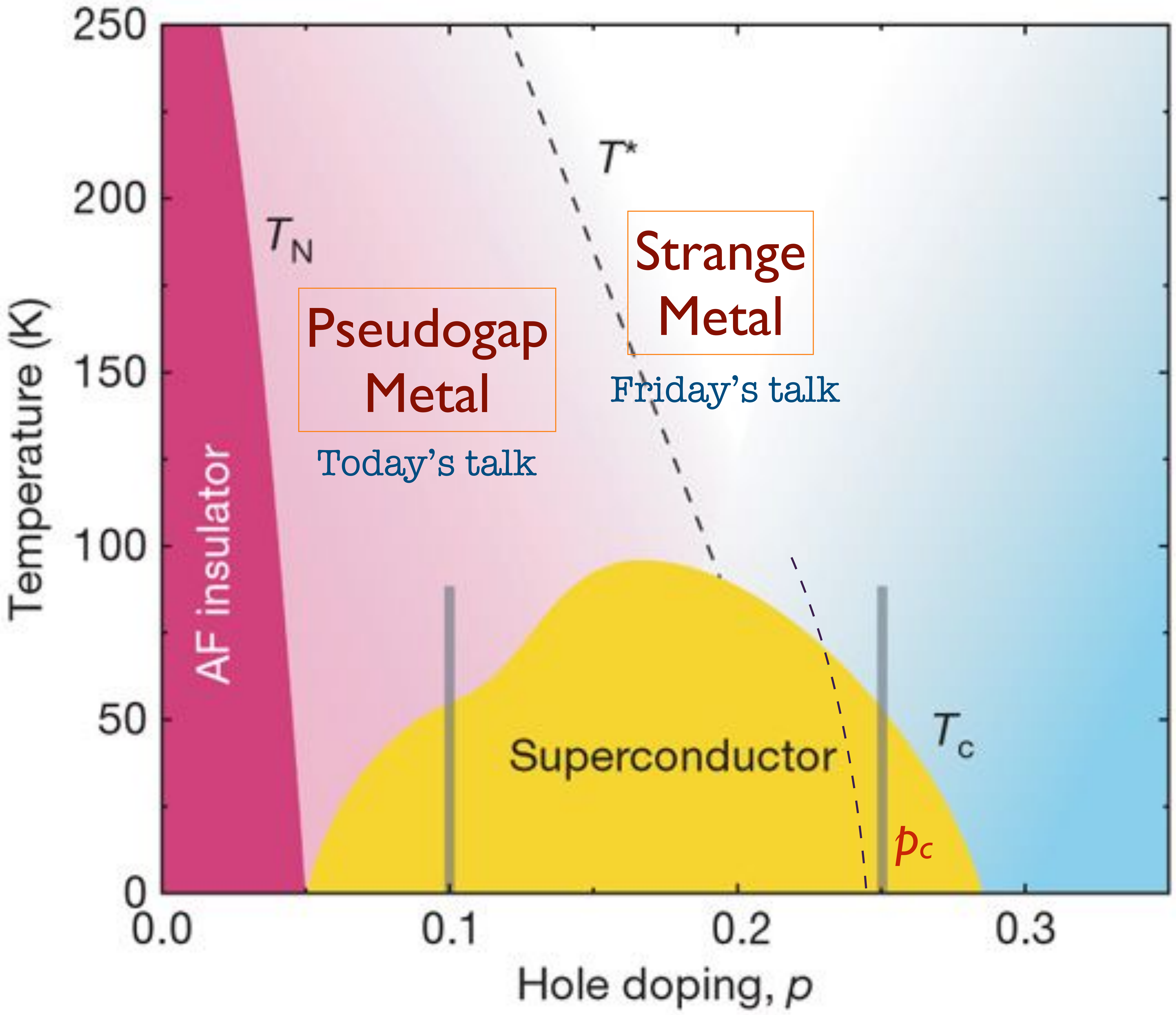
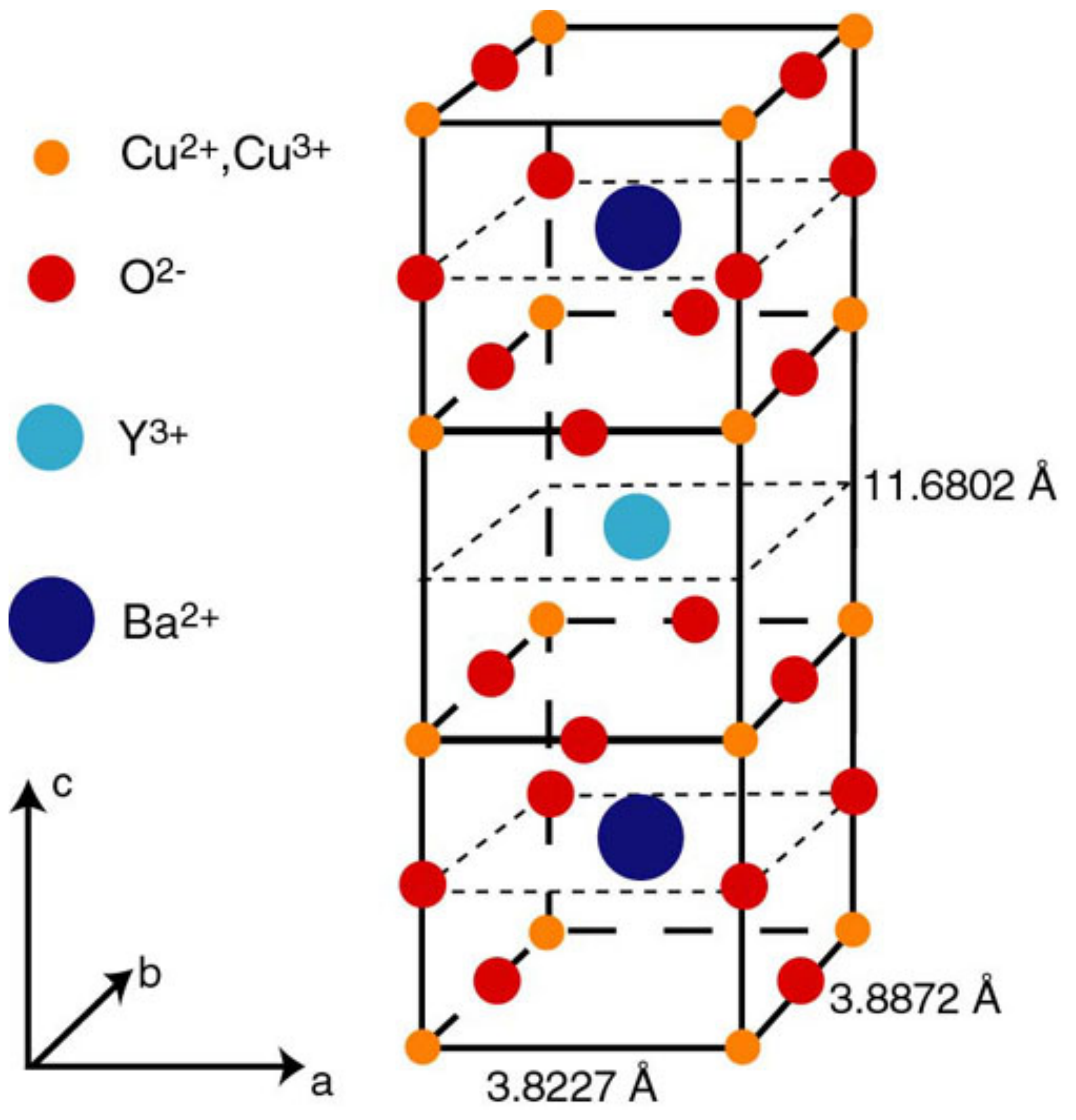
- Needed: a variational wavefunction for a FL* state in a single-band model.

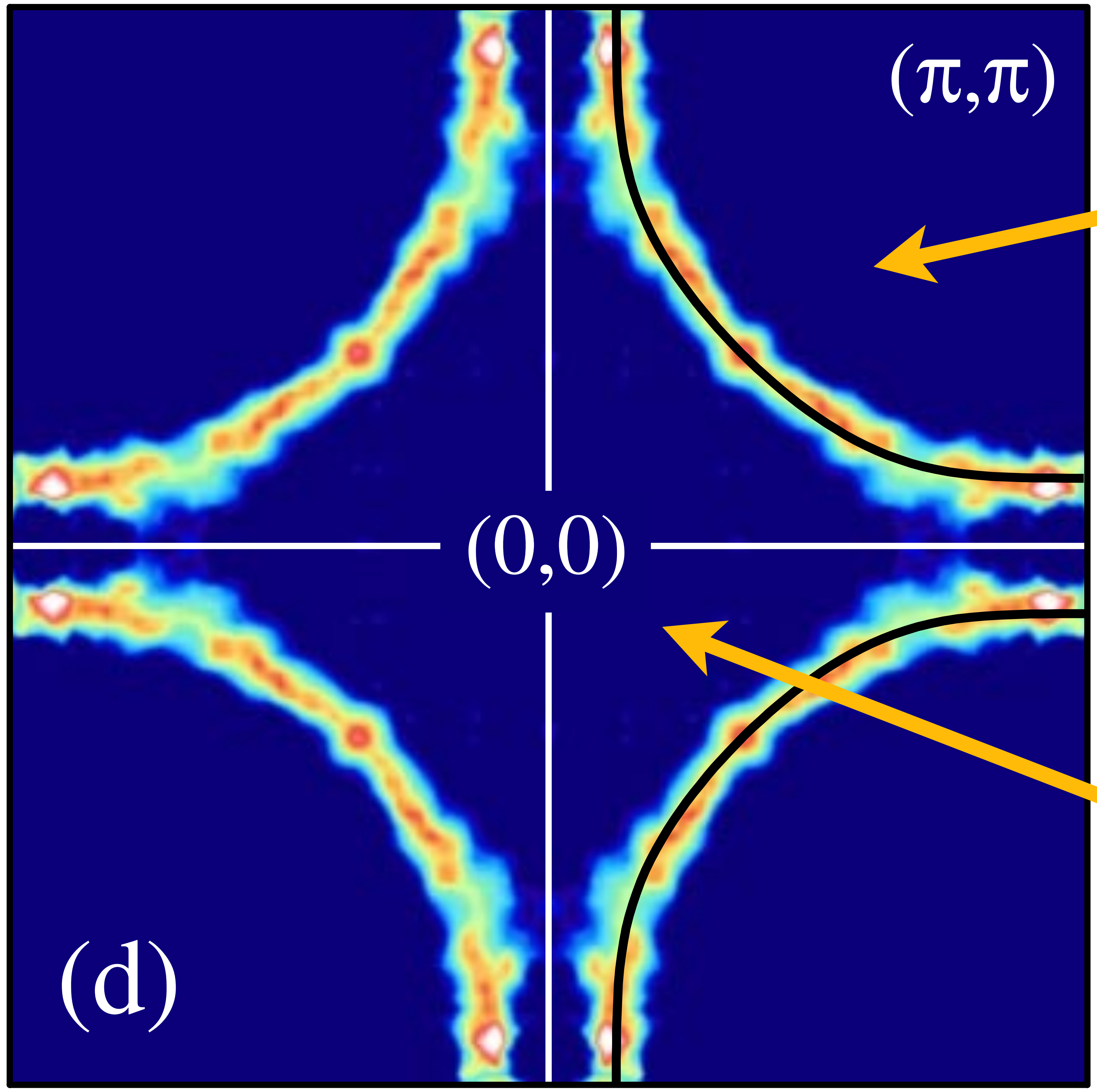

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
Pseudogap Metal
T*
Strange Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Today's talk
Strange Metal
Friday's talk
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at large p



$1+p$ holes

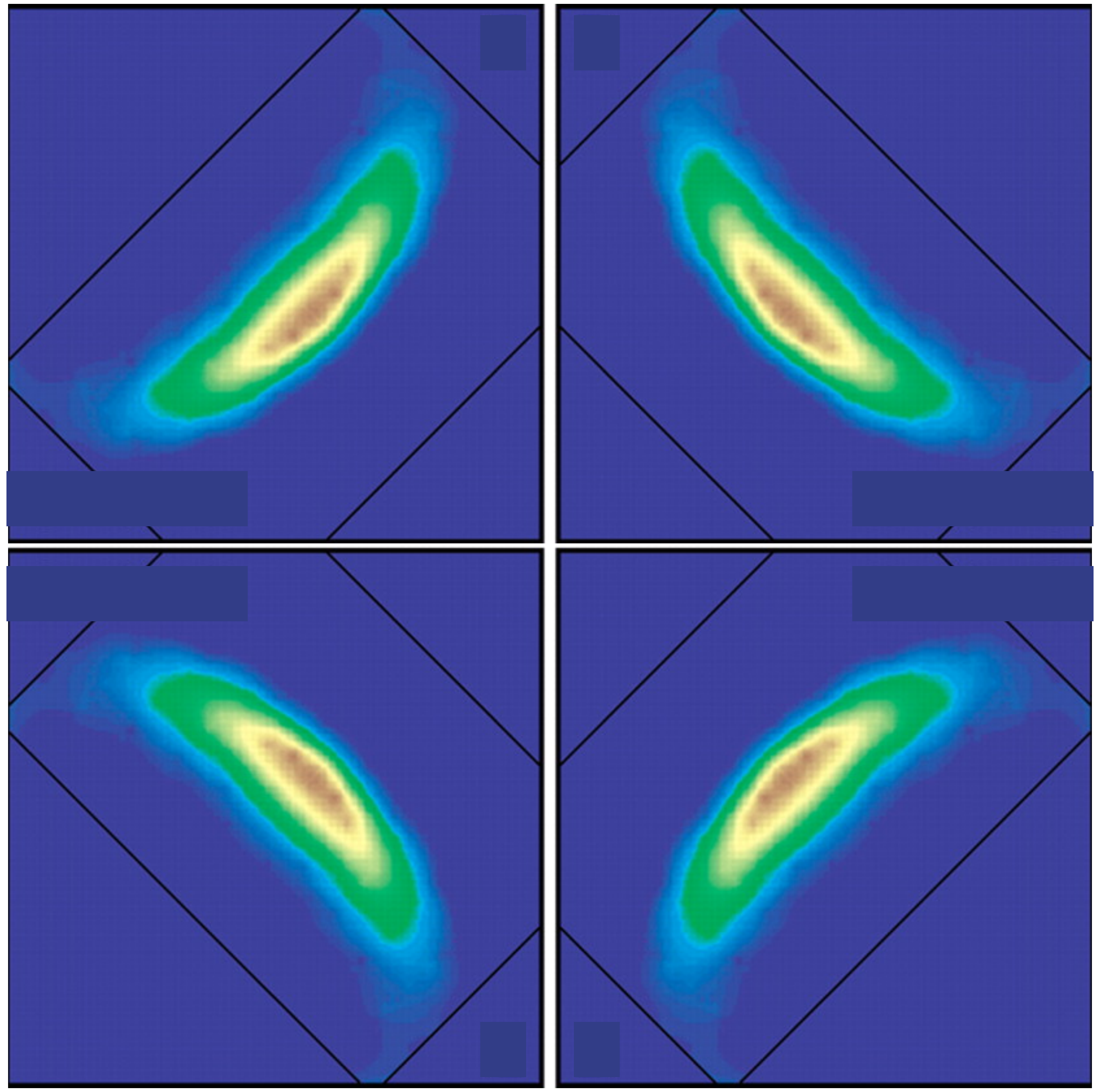
Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

1-p electrons

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Photoemission at small p

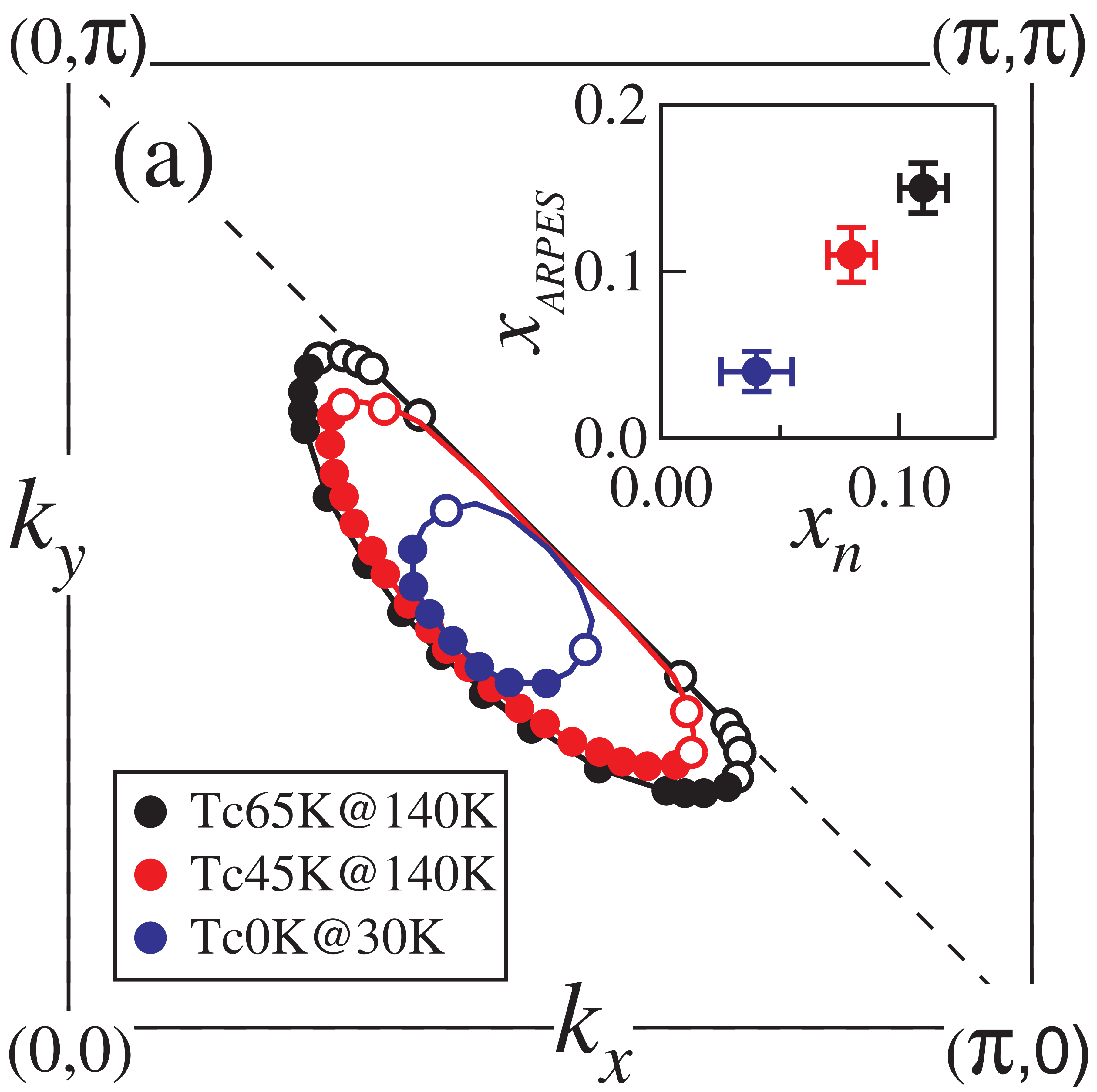


$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

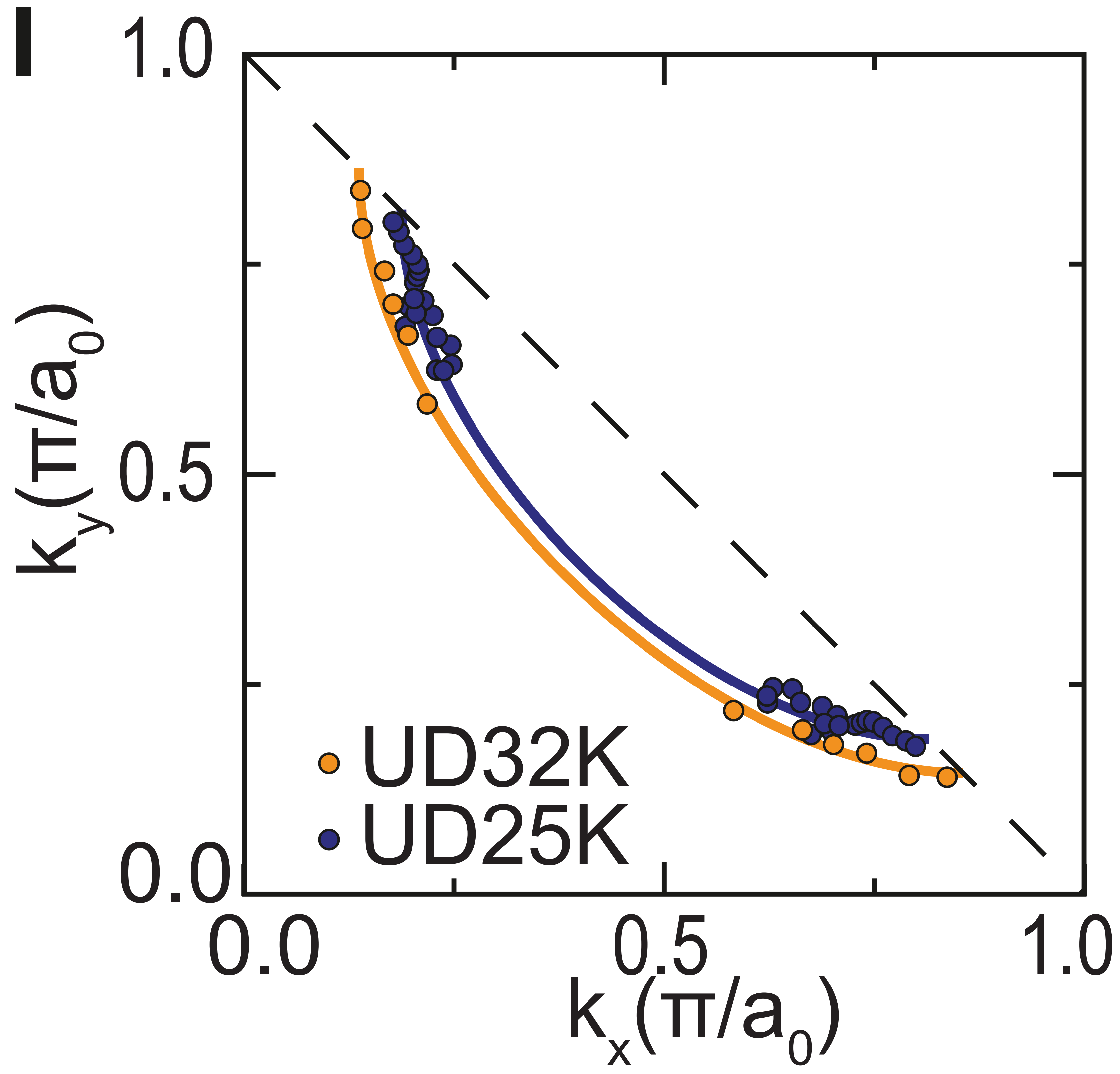
Photoemission at small p



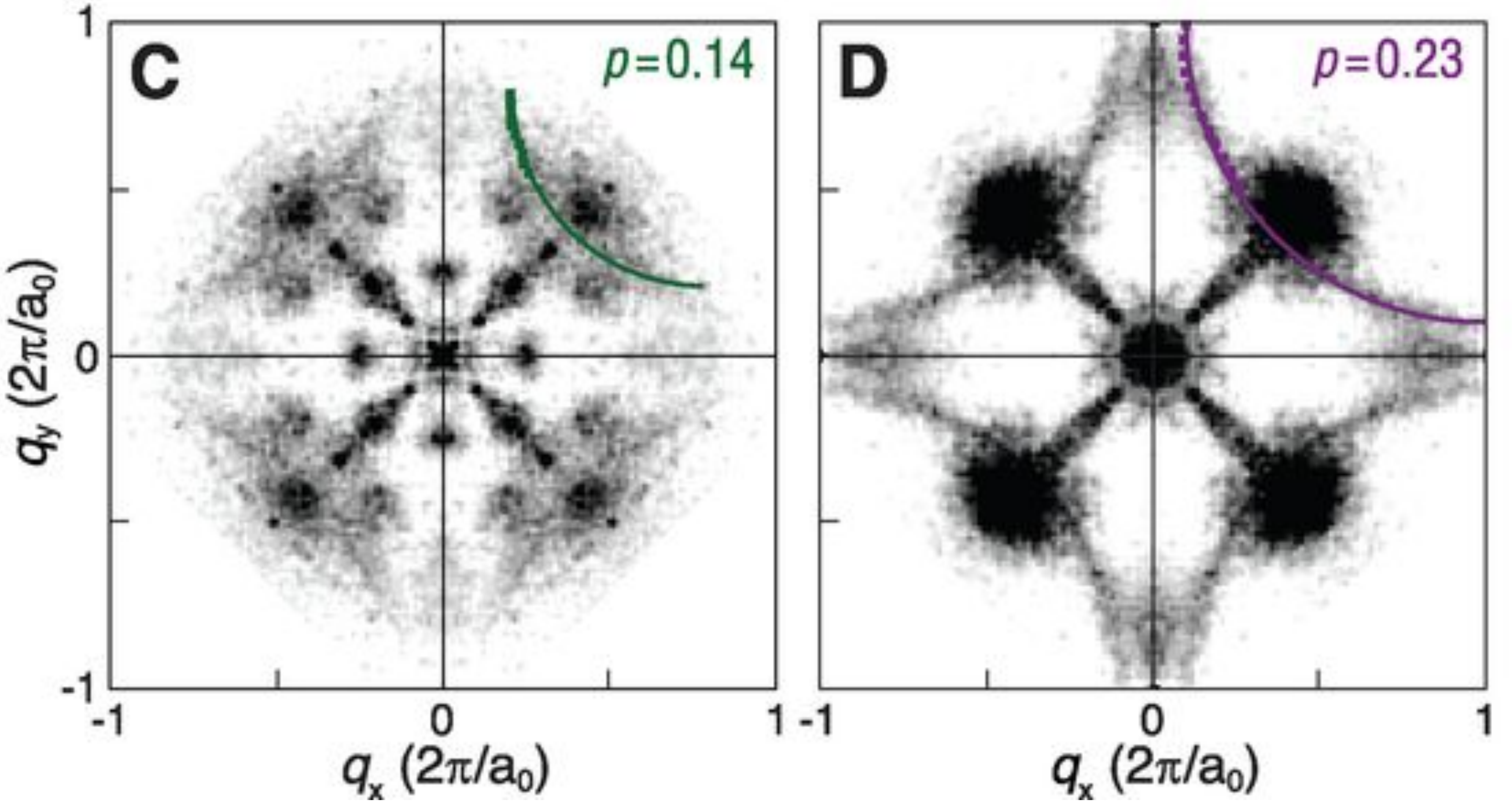
"Fermi pockets"

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

STM at small p

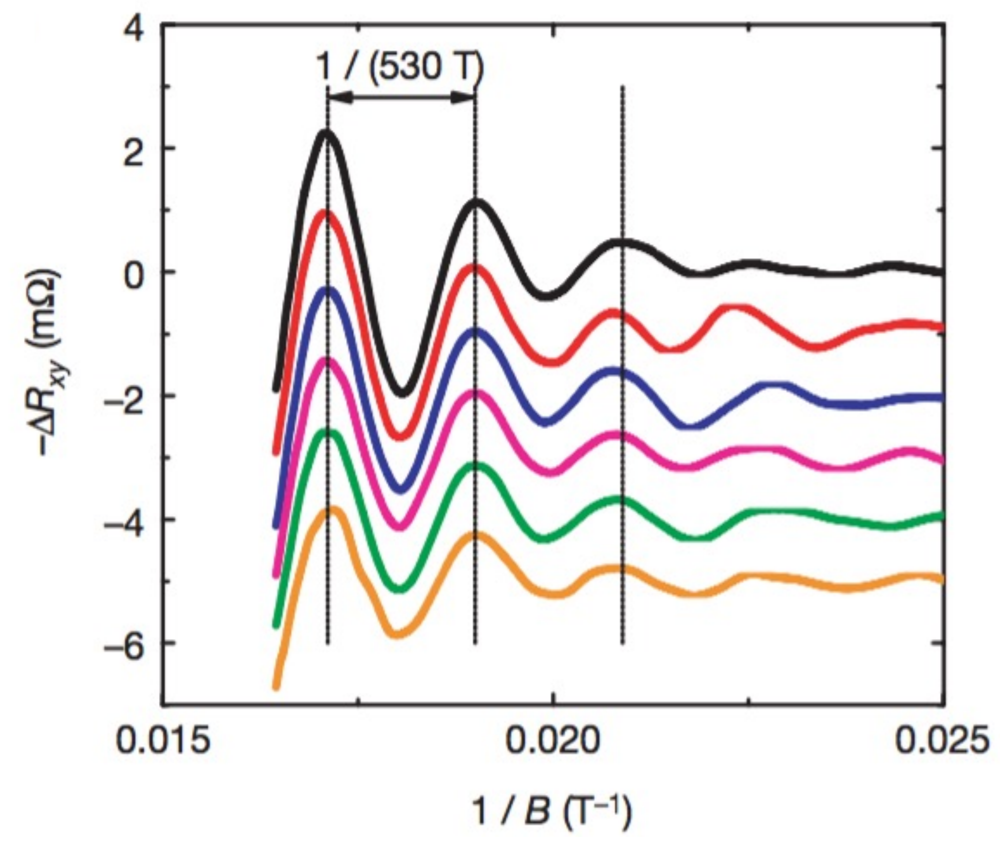


Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

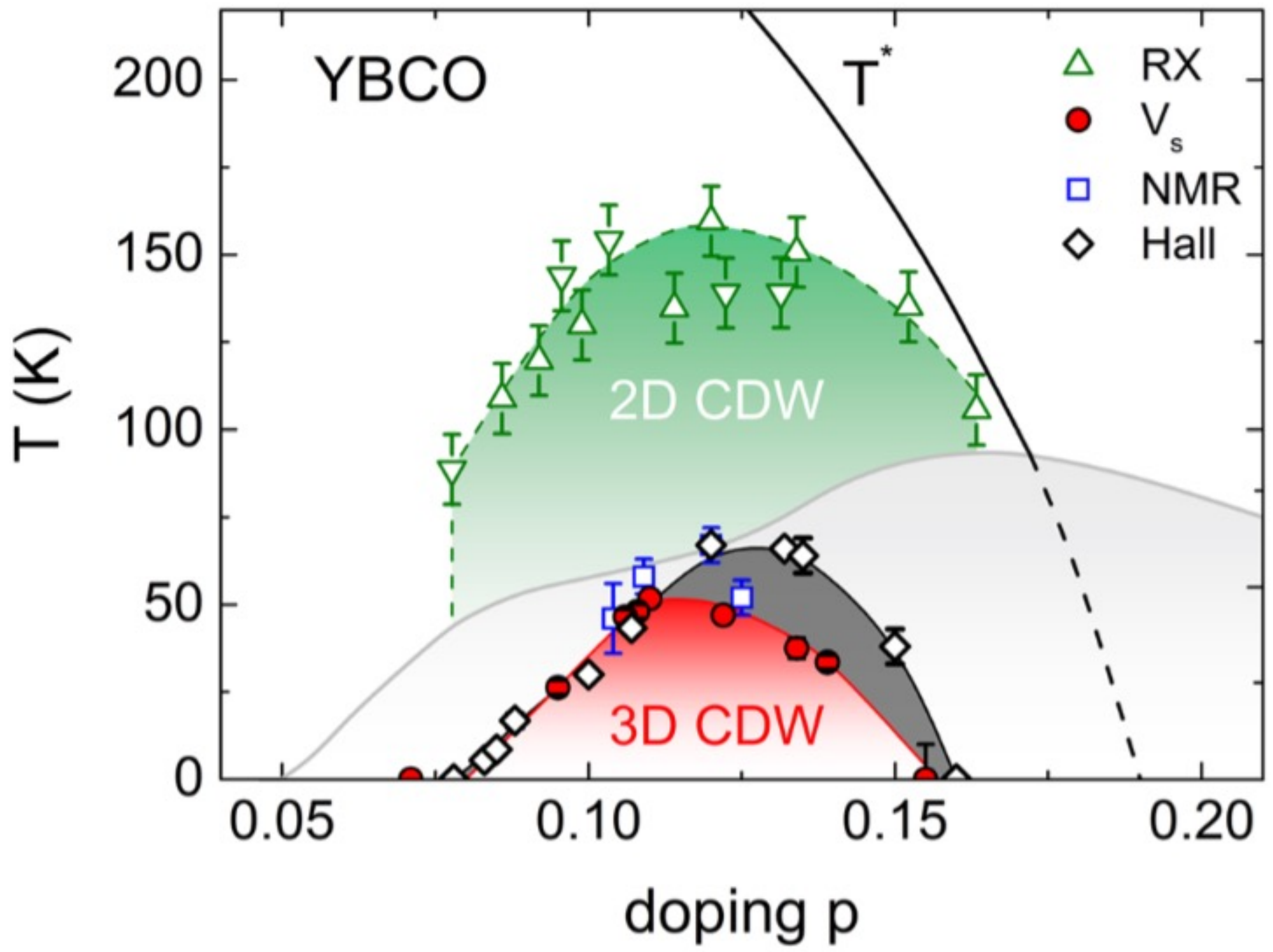


K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E. -A. Kim, J. C. Davis, Science **344**, 612 (2014)

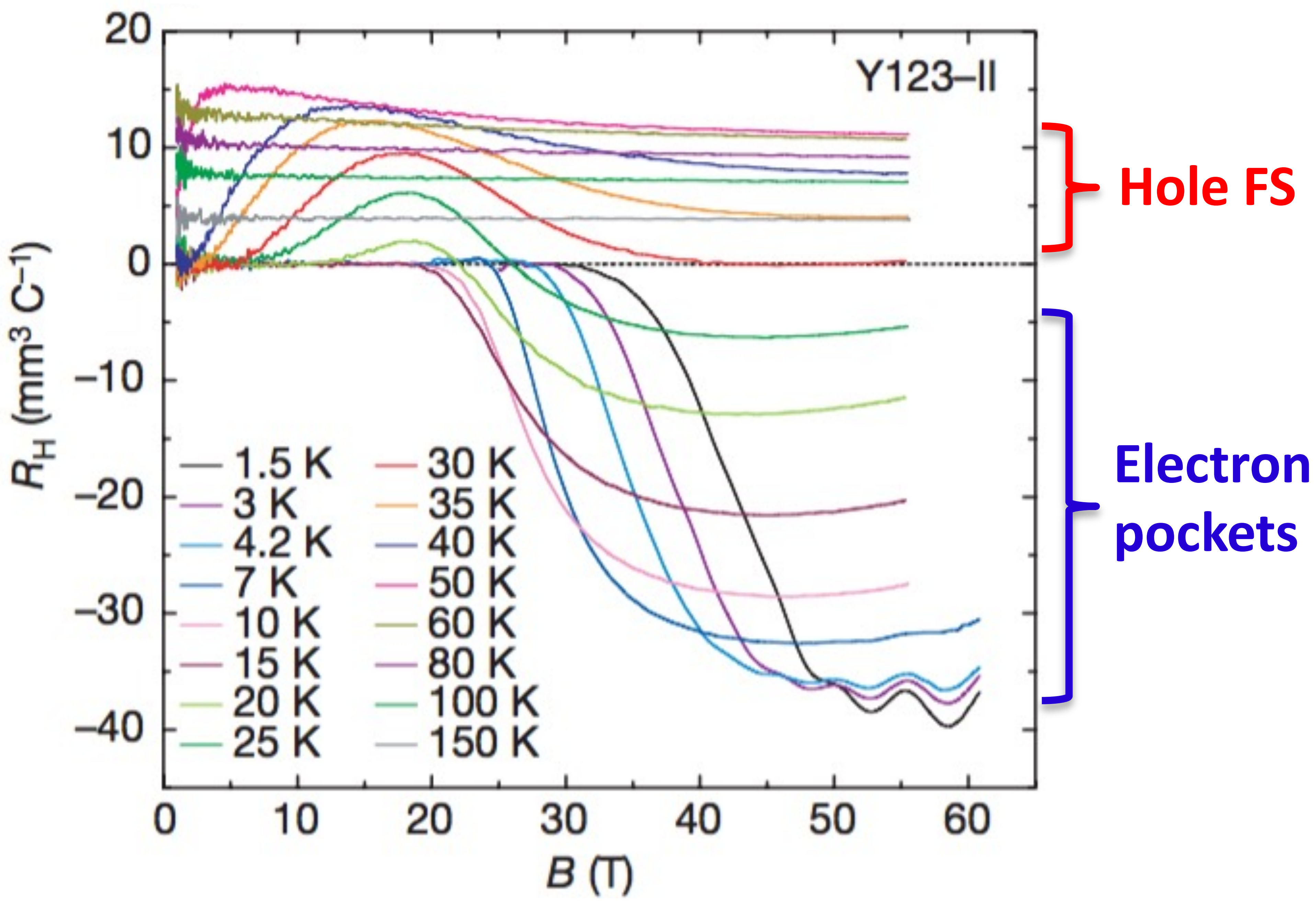
Quantum oscillation in $YBa_2Cu_3O_{6+\delta}$(YBCO)



Nicolas Doiron-Leyraud *et al.* *Nature.* **447**, 565-568 (2007)



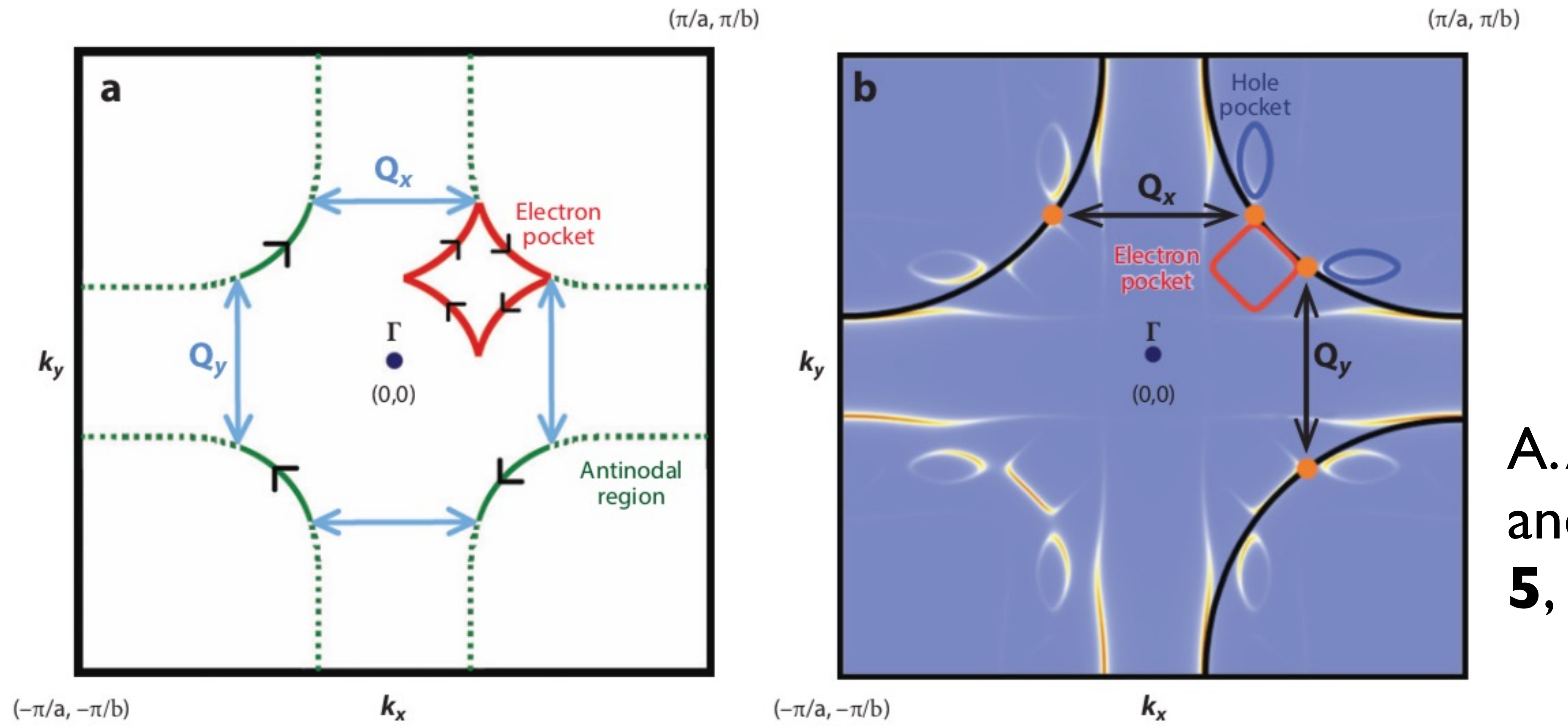
F. Laliberte, *et al.*, *npj Quantum material* **3**, 11 (2018)



Hall coefficient in YBCO

D. LeBoeuf et al., Nature 450, 533 (2007).

Electron pockets are formed due to the emergence of CDW phase.



N. Harrison and S. E. Sebastian, PRL **106**, 226402 (2011).

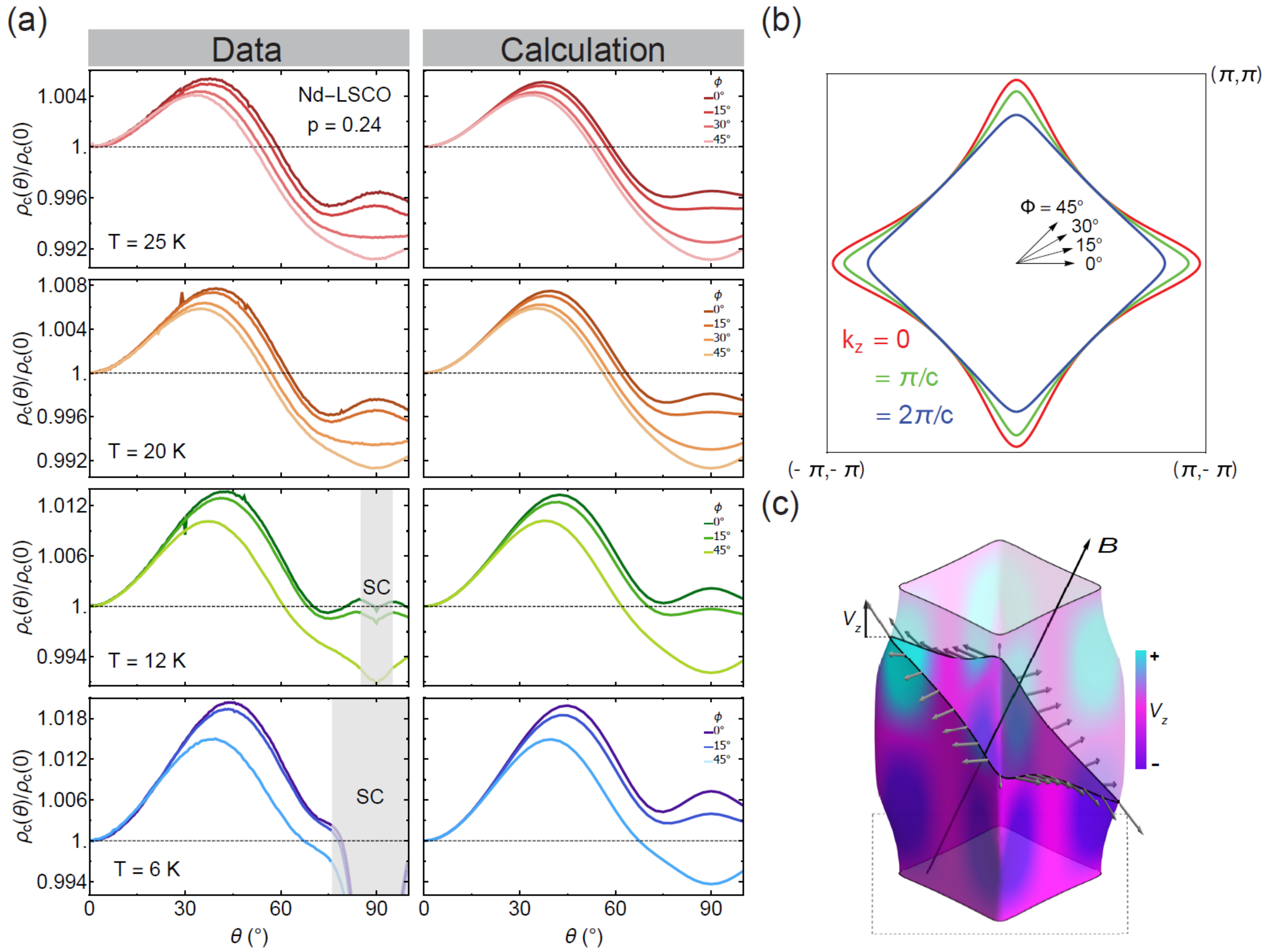
- Quantum oscillations imply a single electron pocket.
- But computations on a CDW applied to a large Fermi surface FL state lead to additional Fermi surfaces in the anti-nodal region.

A. Allais, D. Chowdhury, and S.S., Nat. Commun. **5**, 5771 (2014)

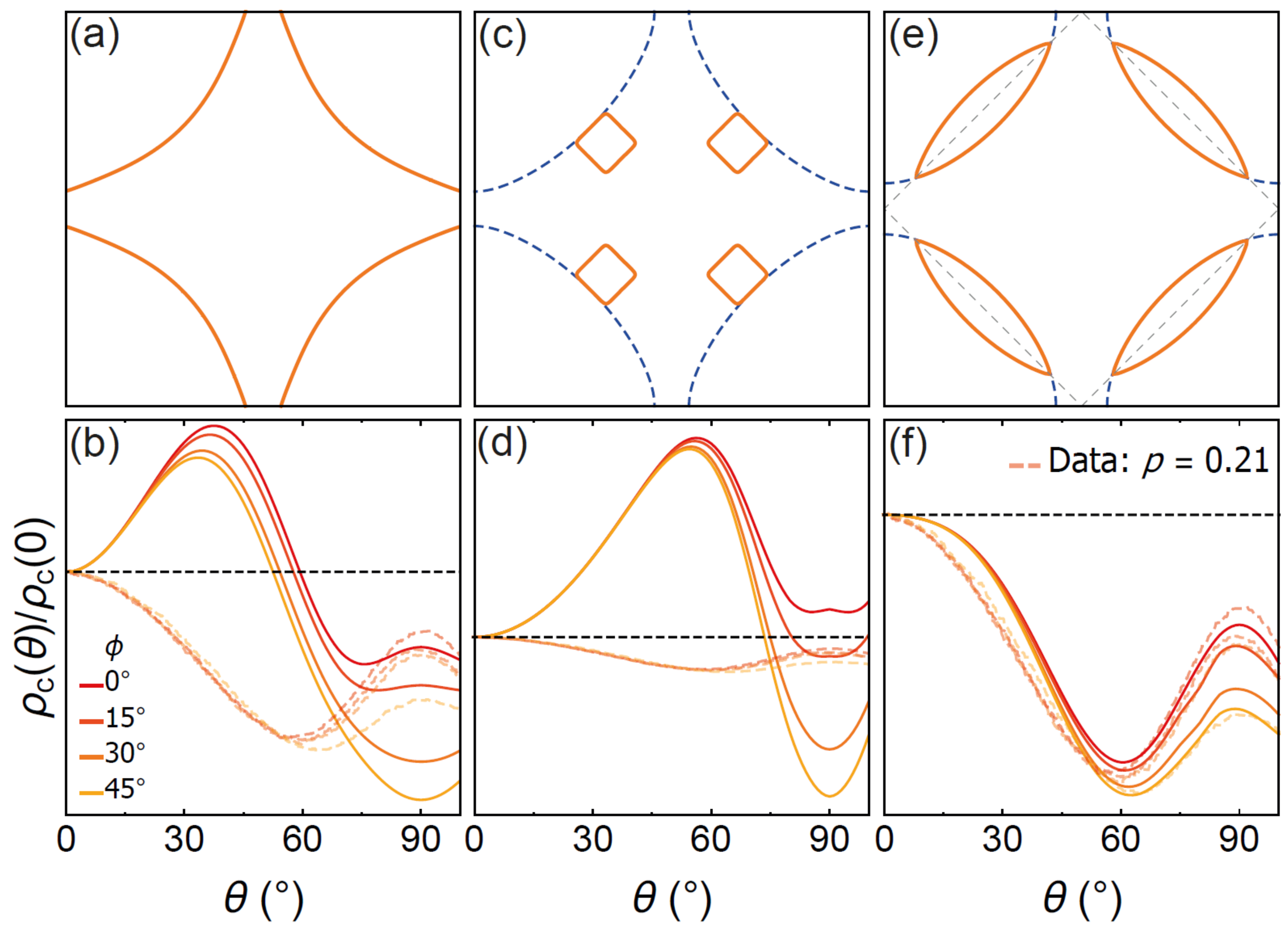
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. Above the critical doping p*—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p*, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology.



$p > p_c$ Large Fermi surface

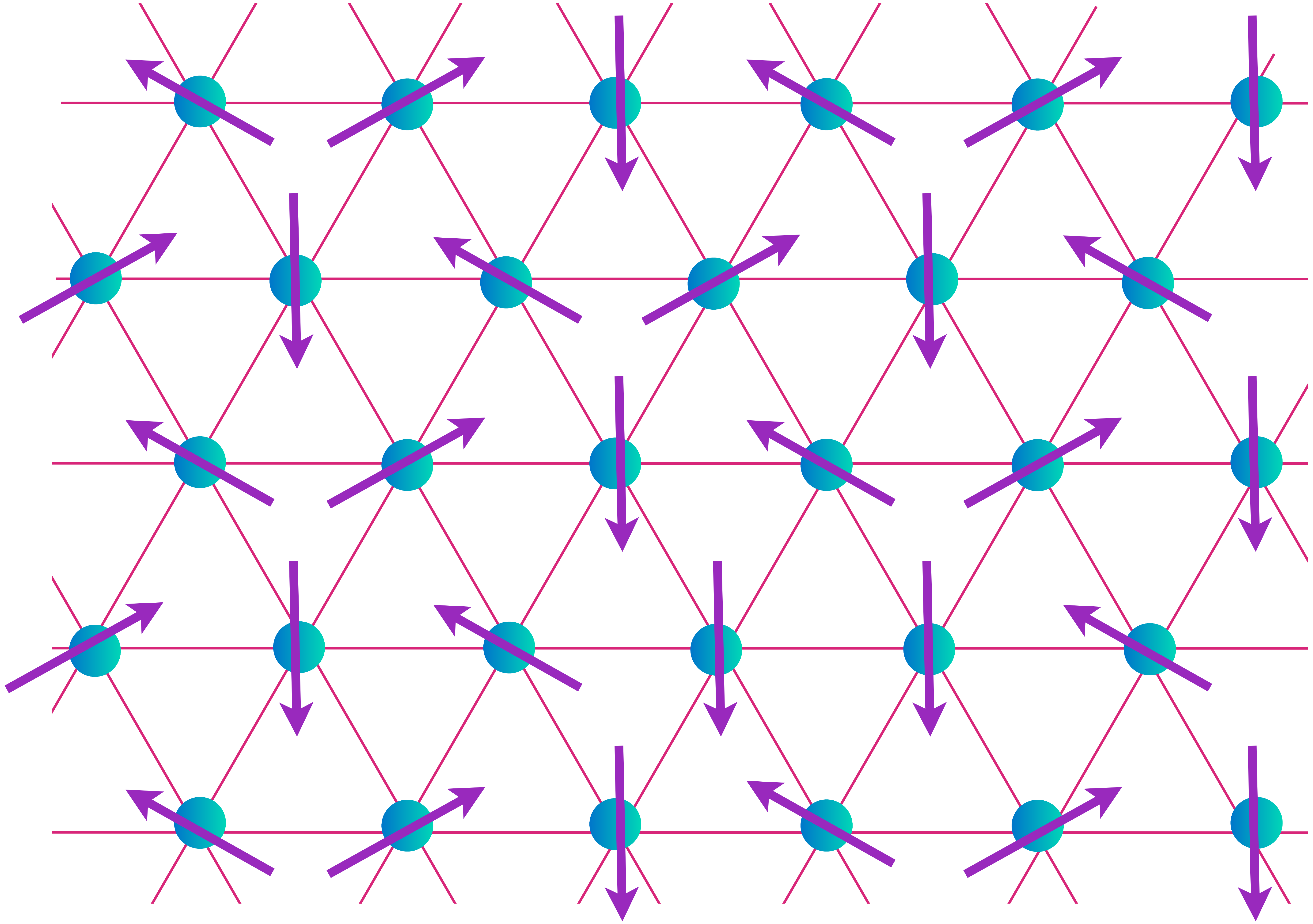


$p < p_c$ Reconstructed Fermi surface

1. Luttinger and Luttinger* relations in metals
2. Spin liquids and emergent gauge fields
3. FL* and HFL phases of the Kondo lattice
4. Paramagnon fractionalization theory of the pseudogap metal in a single band model

Triangular lattice antiferromagnet

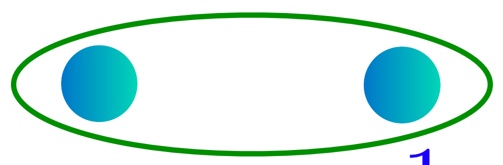
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



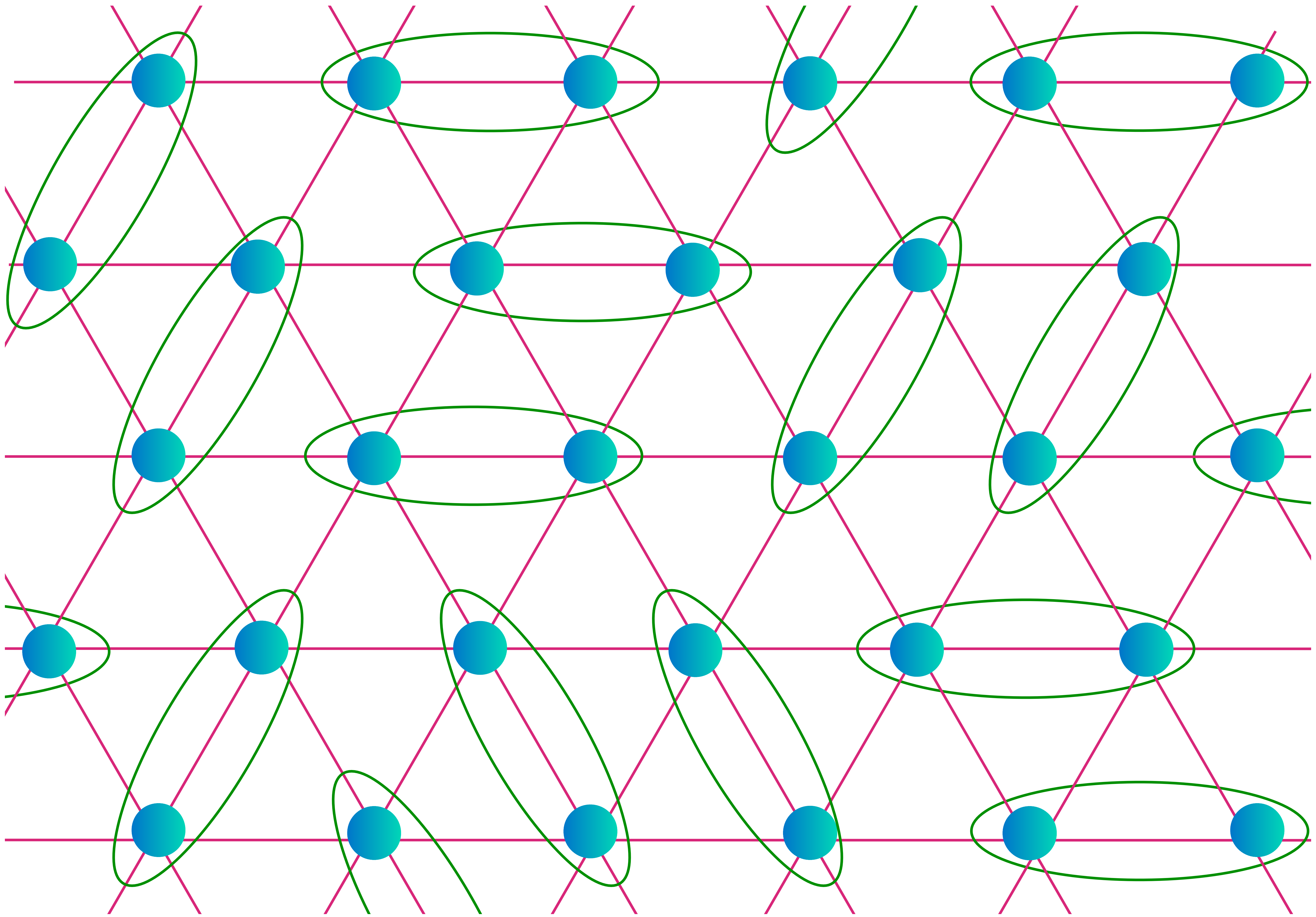
Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

Bosons at half-filling,
or a spin model with S=1/2 per unit cell



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(B_1^\dagger - B_2^\dagger\right)|0\rangle$$



$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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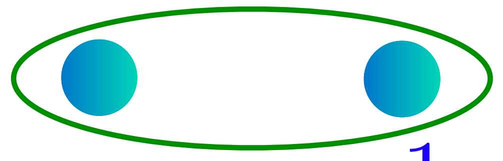

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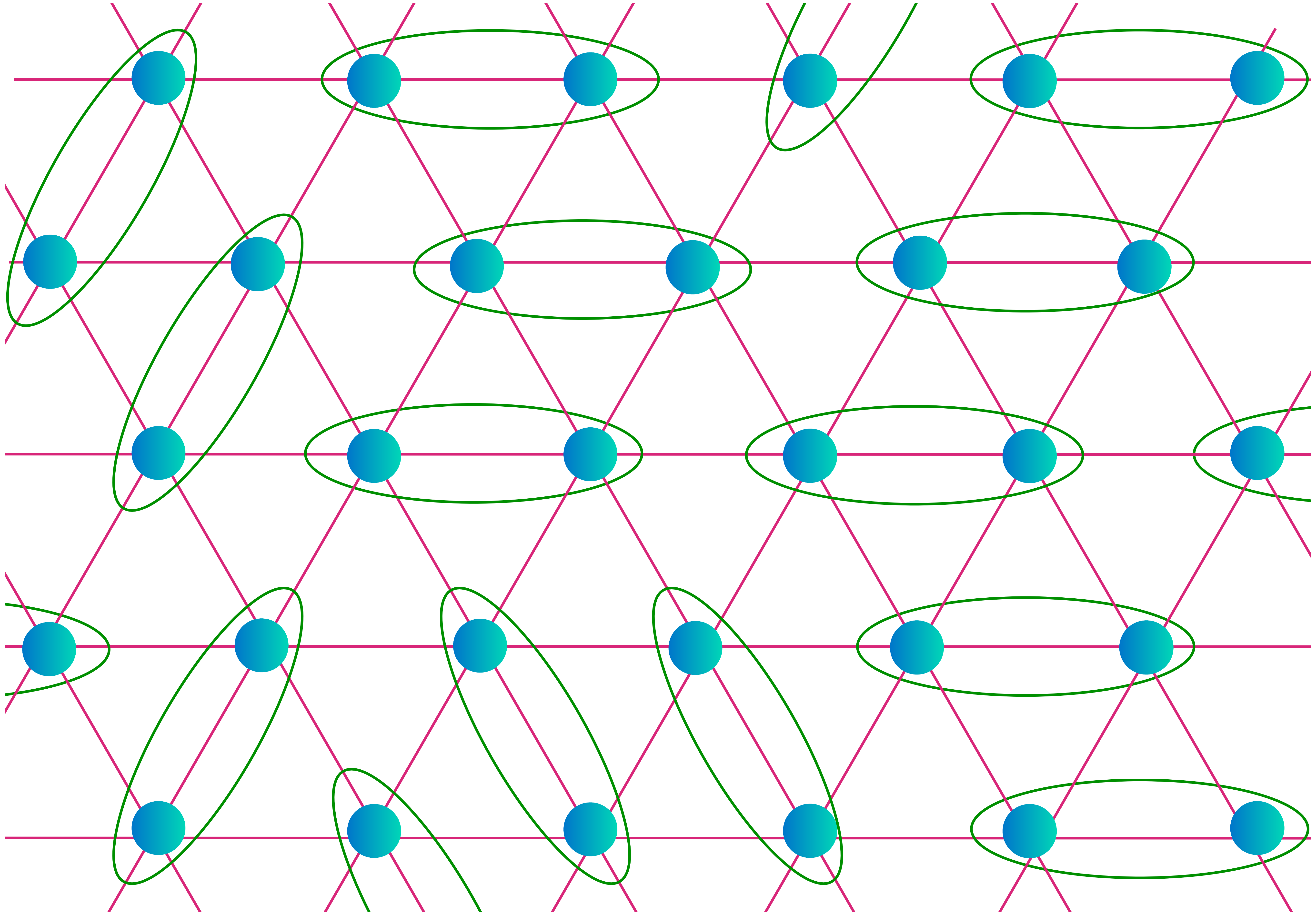

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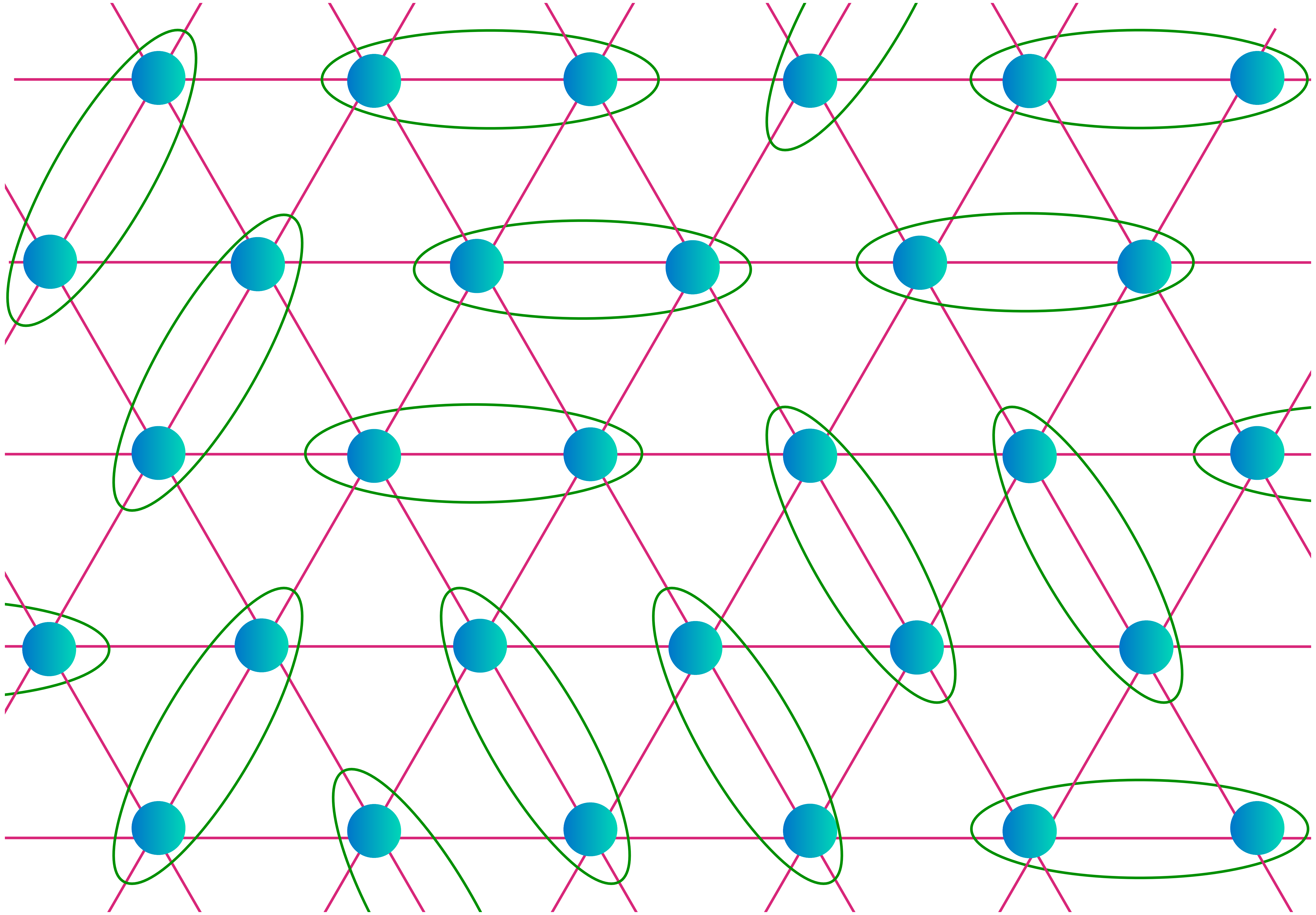

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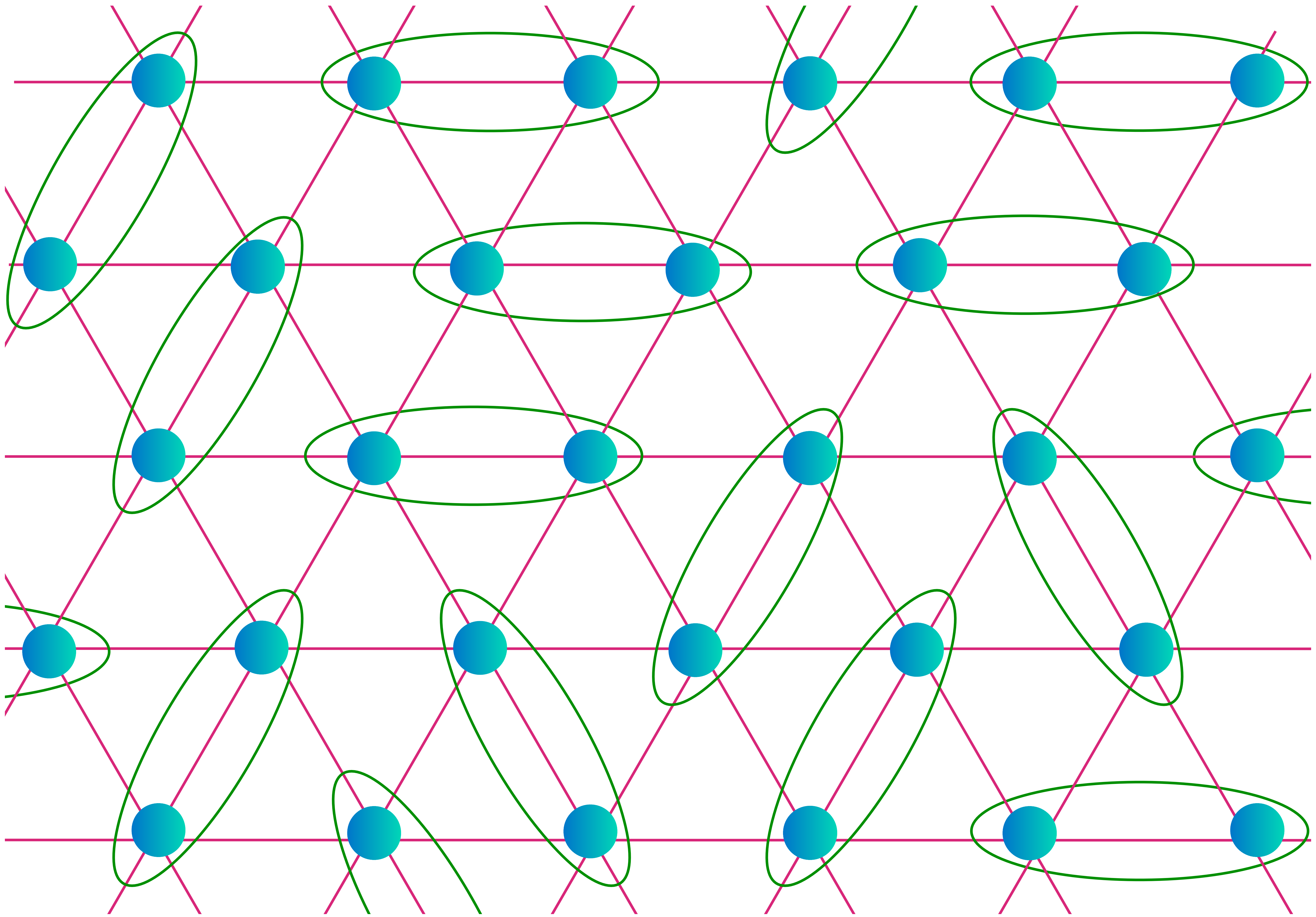

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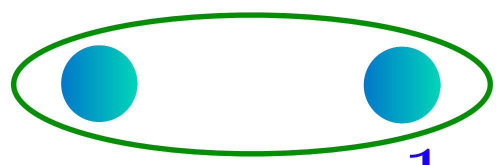

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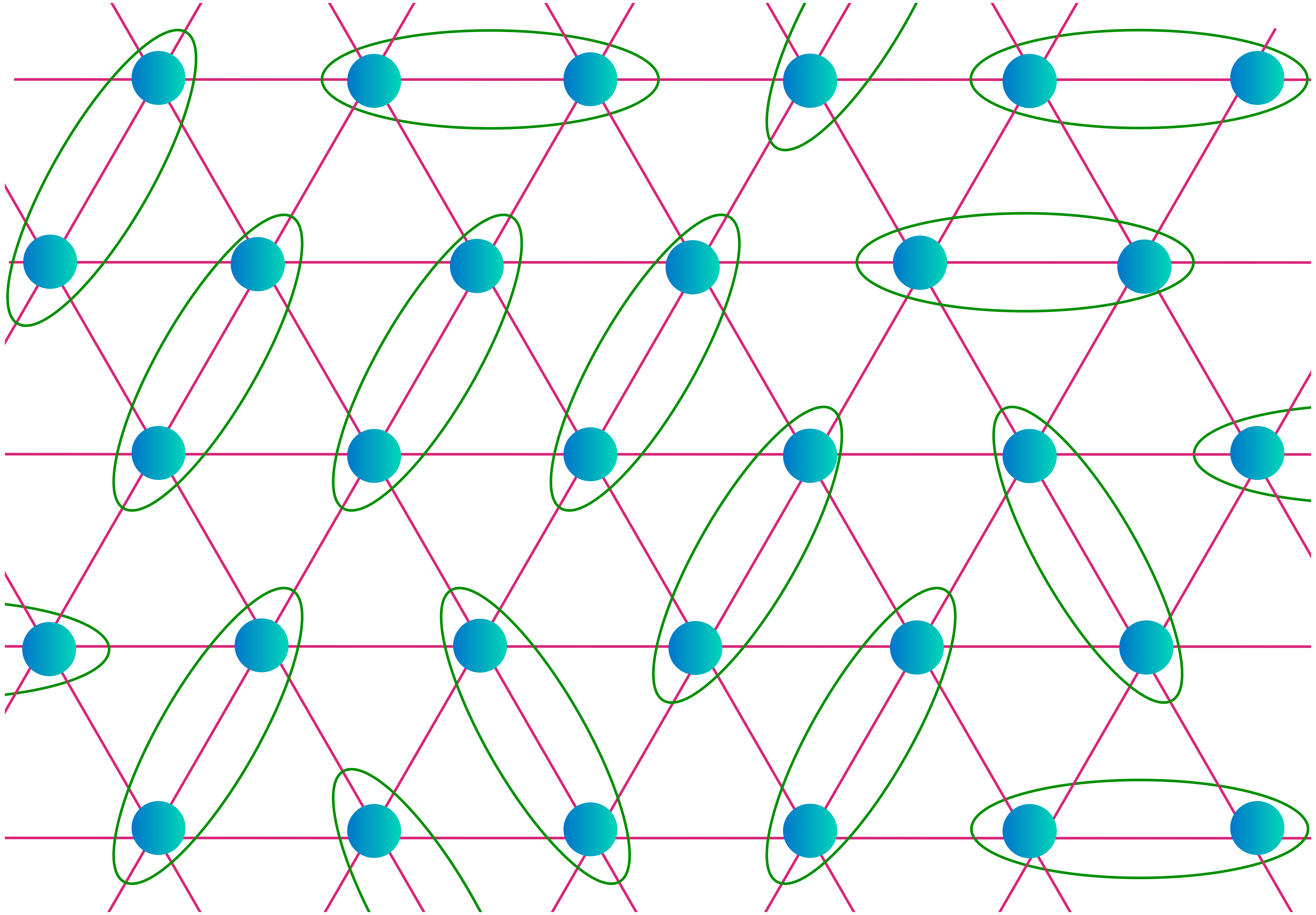

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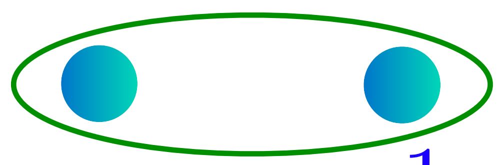

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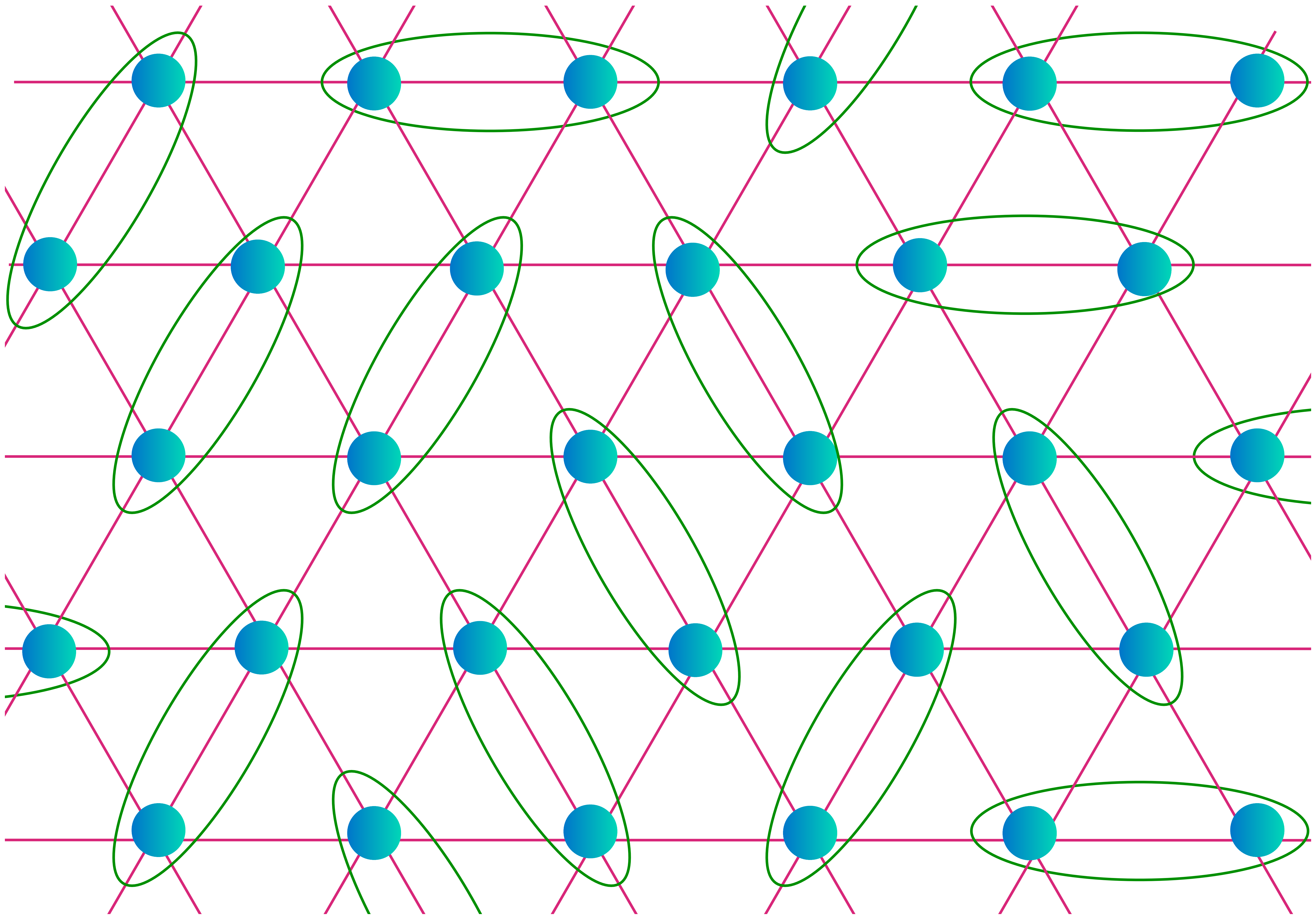

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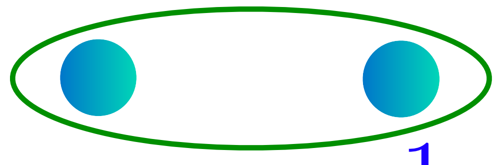
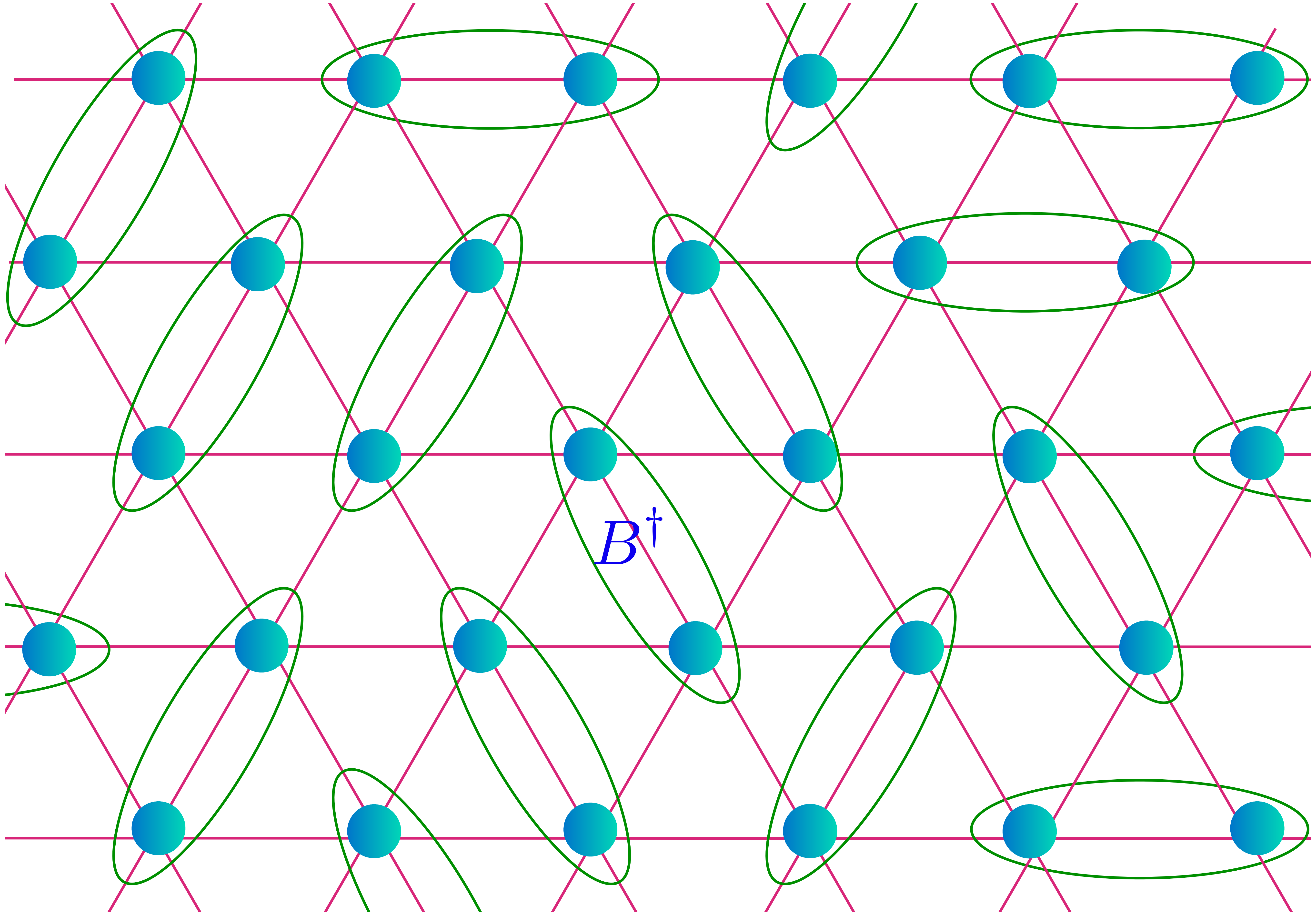
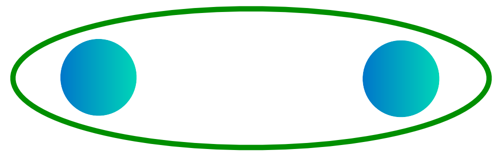
RVB: Z_2 spin liquid

Excitations with boson number 1/2



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) = \frac{1}{\sqrt{2}} \left(B_1^\dagger - B_2^\dagger \right) |0\rangle$$

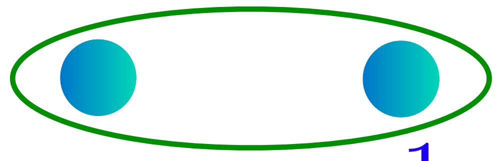


$B_2^\dagger$ 

$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} |\uparrow\uparrow\rangle$$

RVB: Z_2 spin liquid

Excitations with boson number 1/2
a "spinon"



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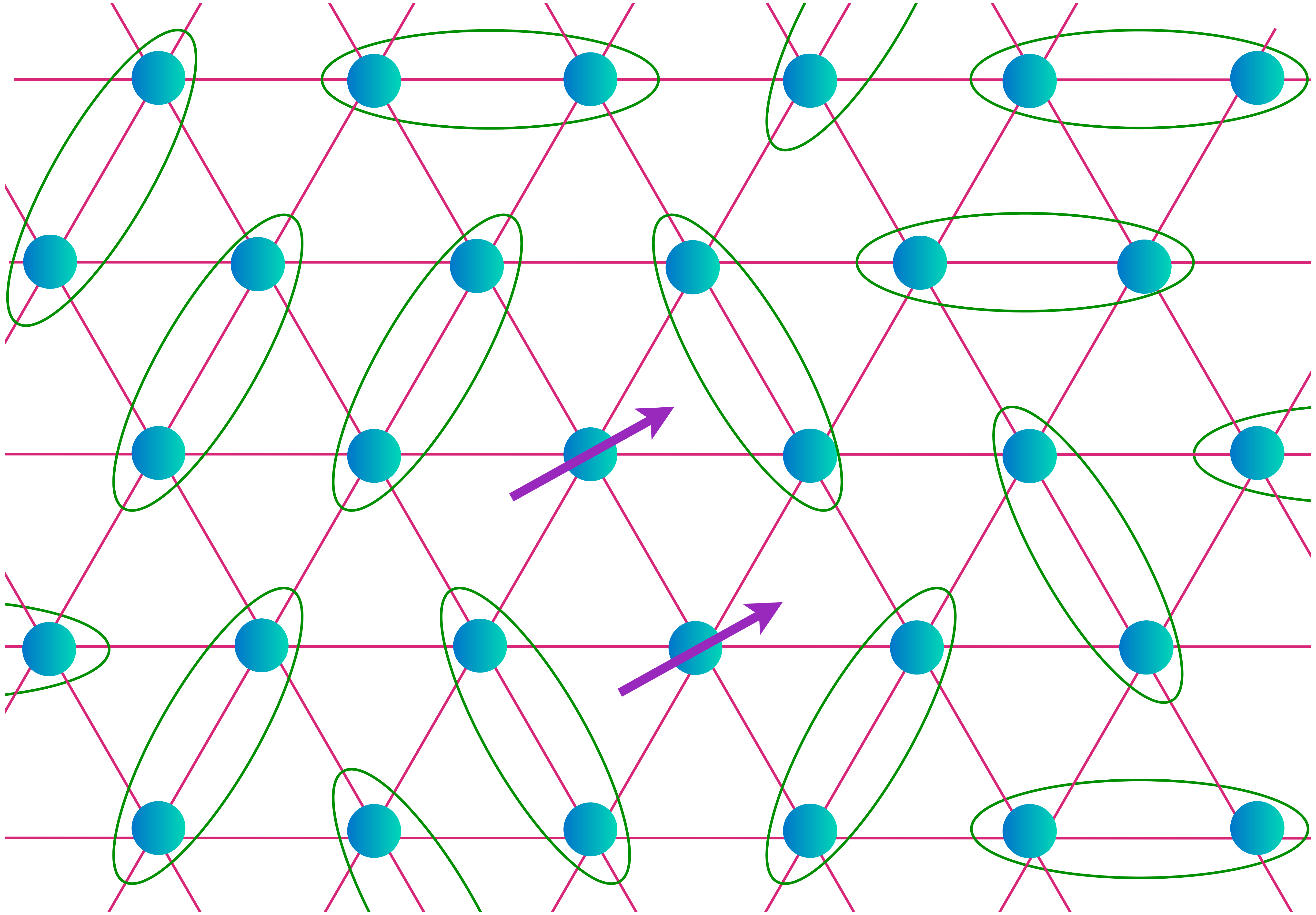
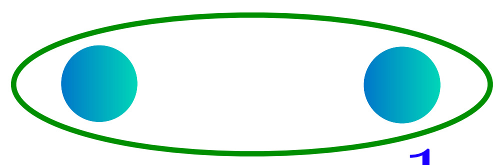


- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

RVB: Z_2 spin liquid

**Excitations with boson number 1/2
a "spinon"**



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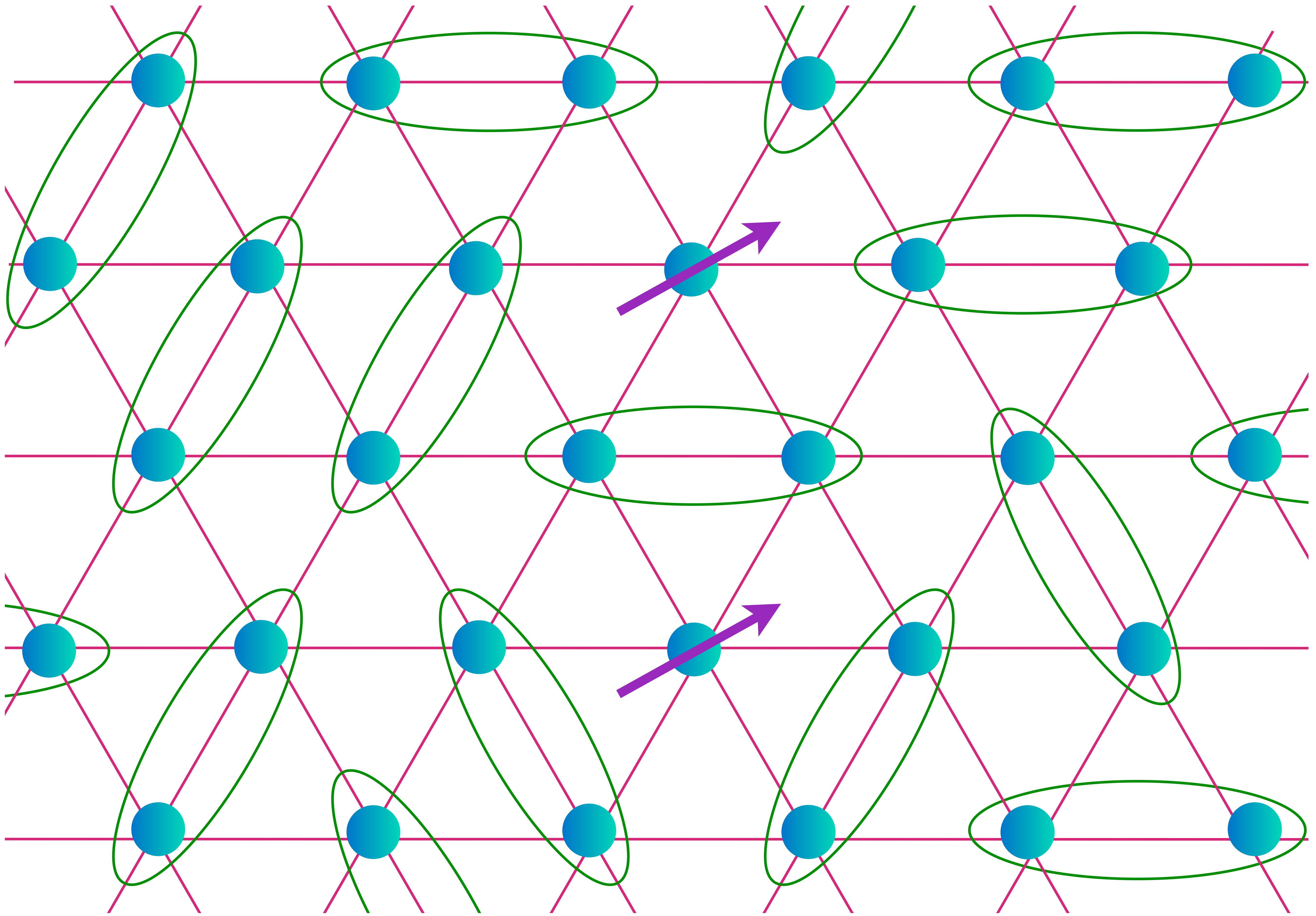

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RVB: Z_2 spin liquid

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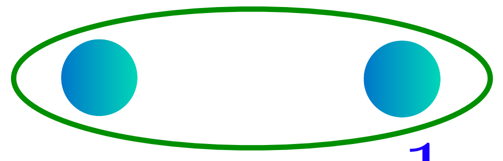

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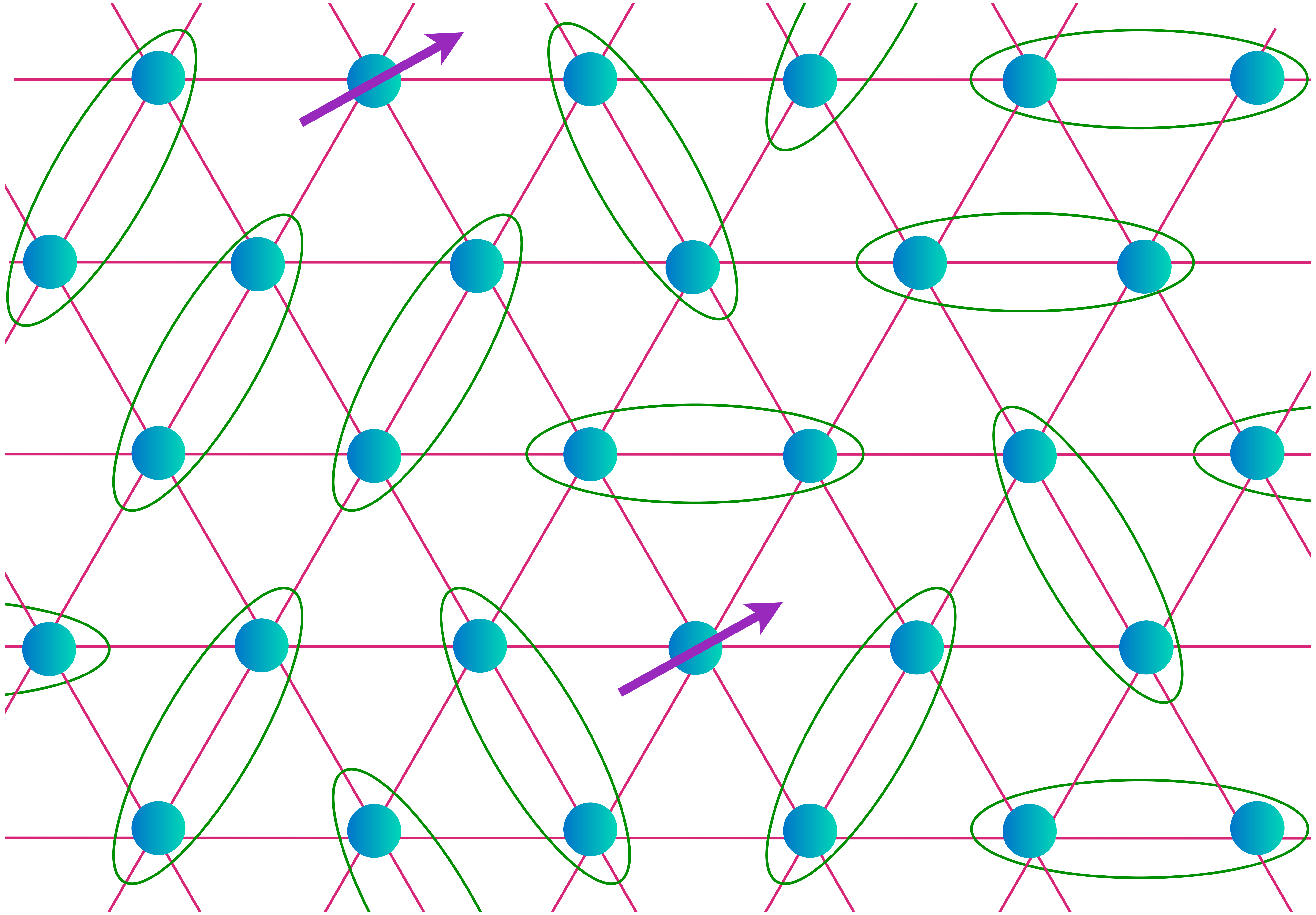

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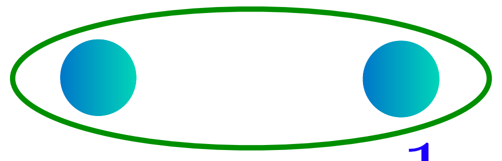

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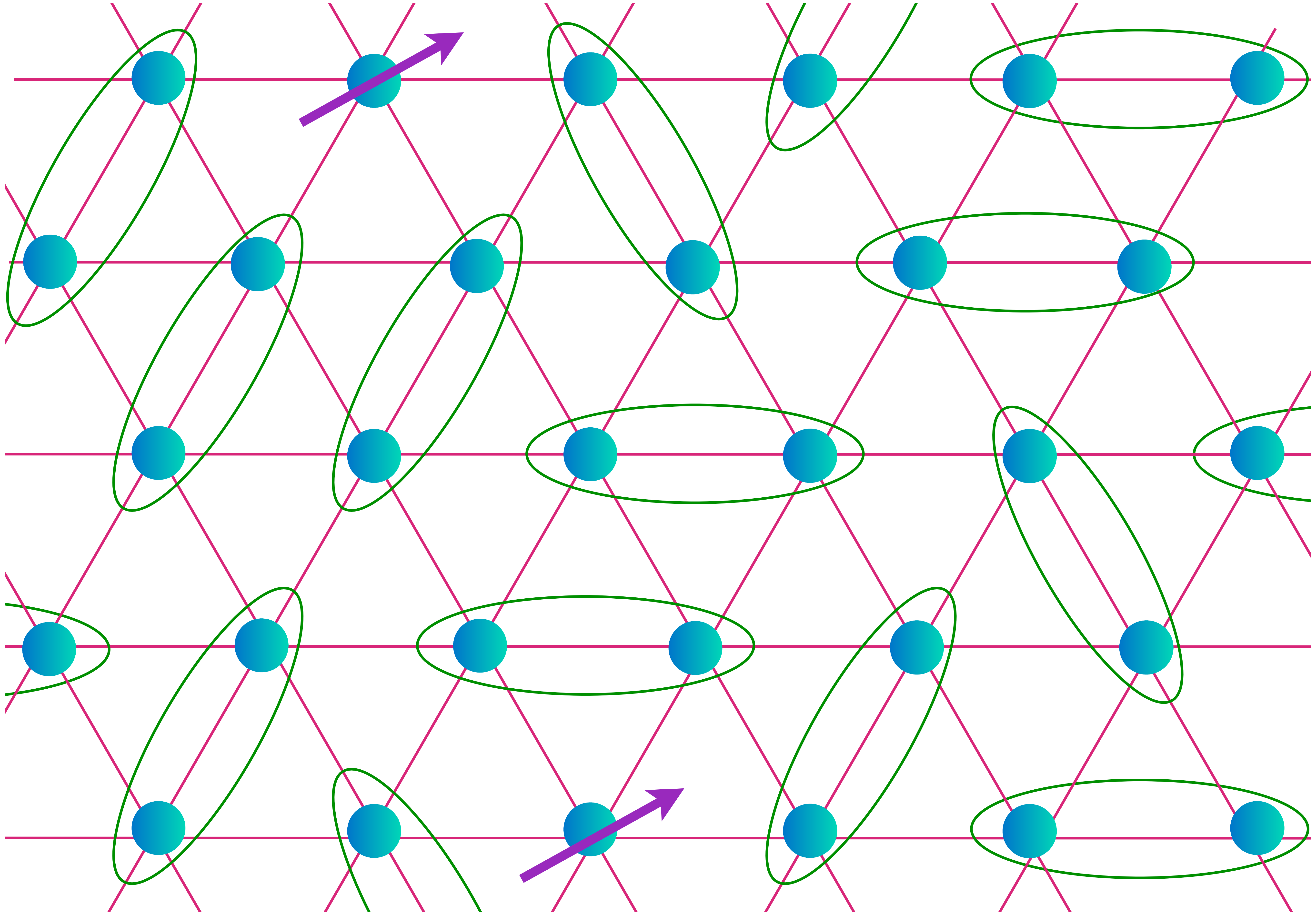

- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
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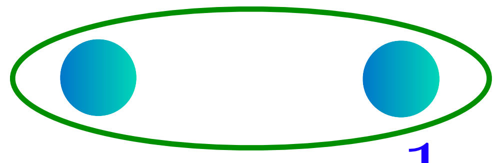
RVB: Z_2 spin liquid

Excitations with boson number 0
a vison (*m* particle)



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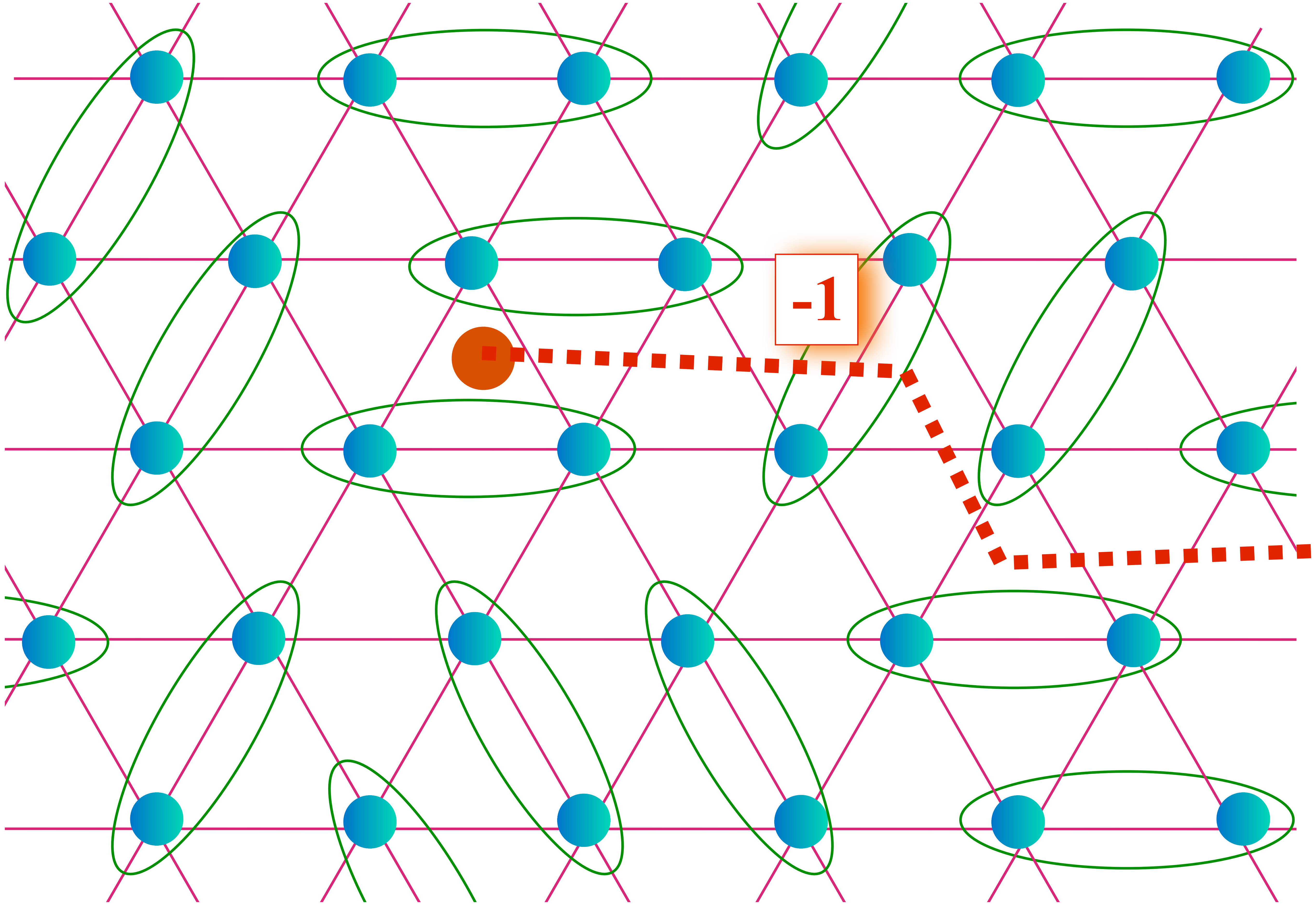

$$|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

$n_{\mathcal{D}} \rightarrow$ number of dimers crossing red line

N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

RVB: Z_2 spin liquid

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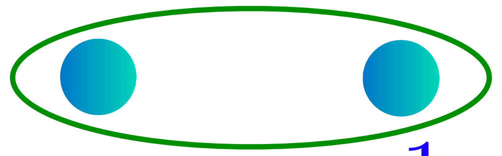

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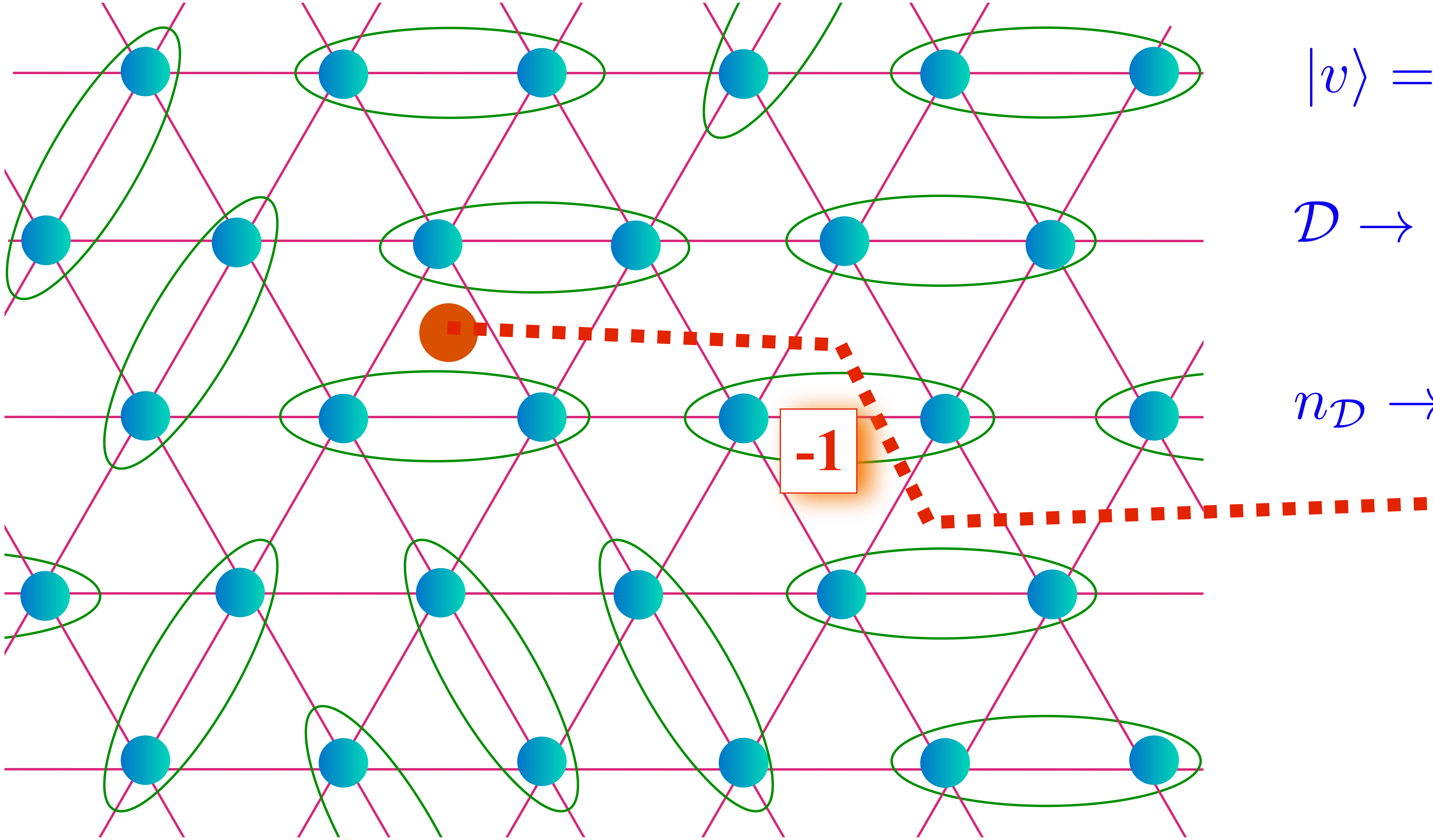

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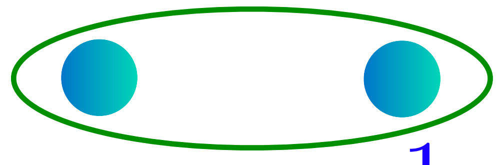

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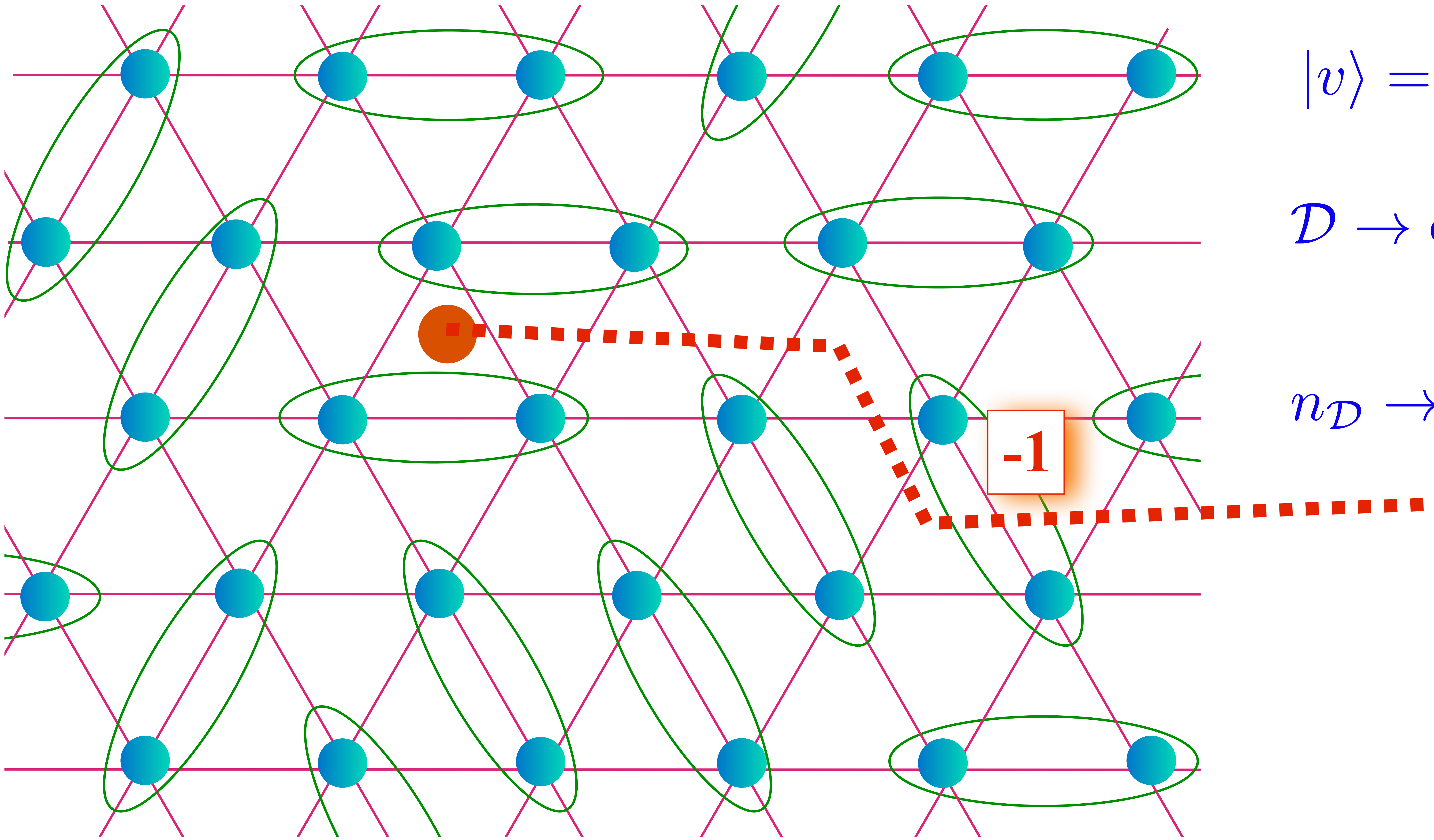

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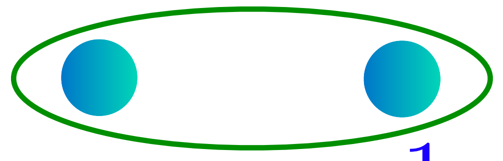

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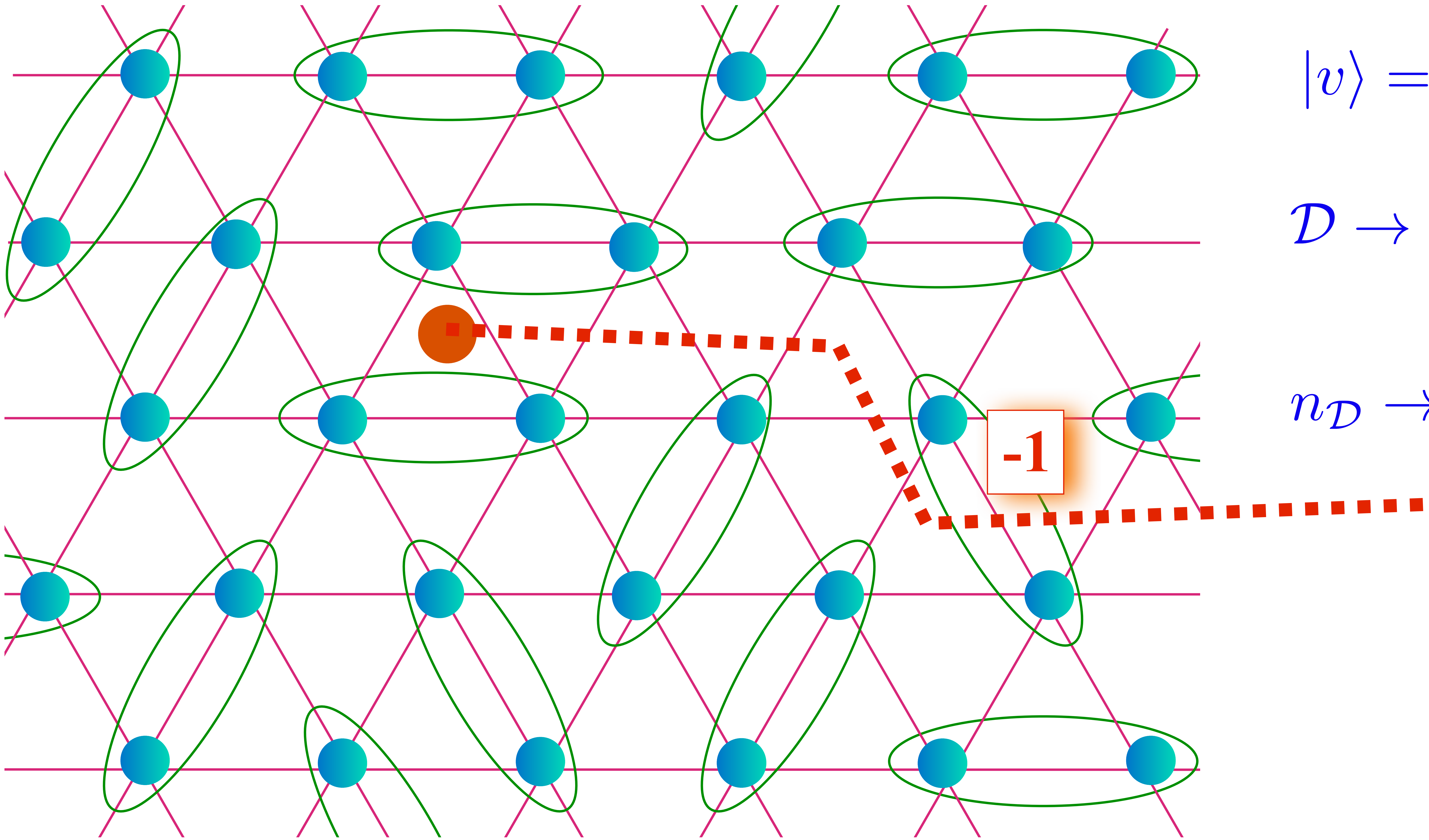

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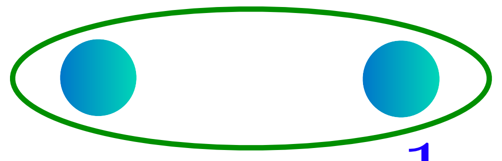

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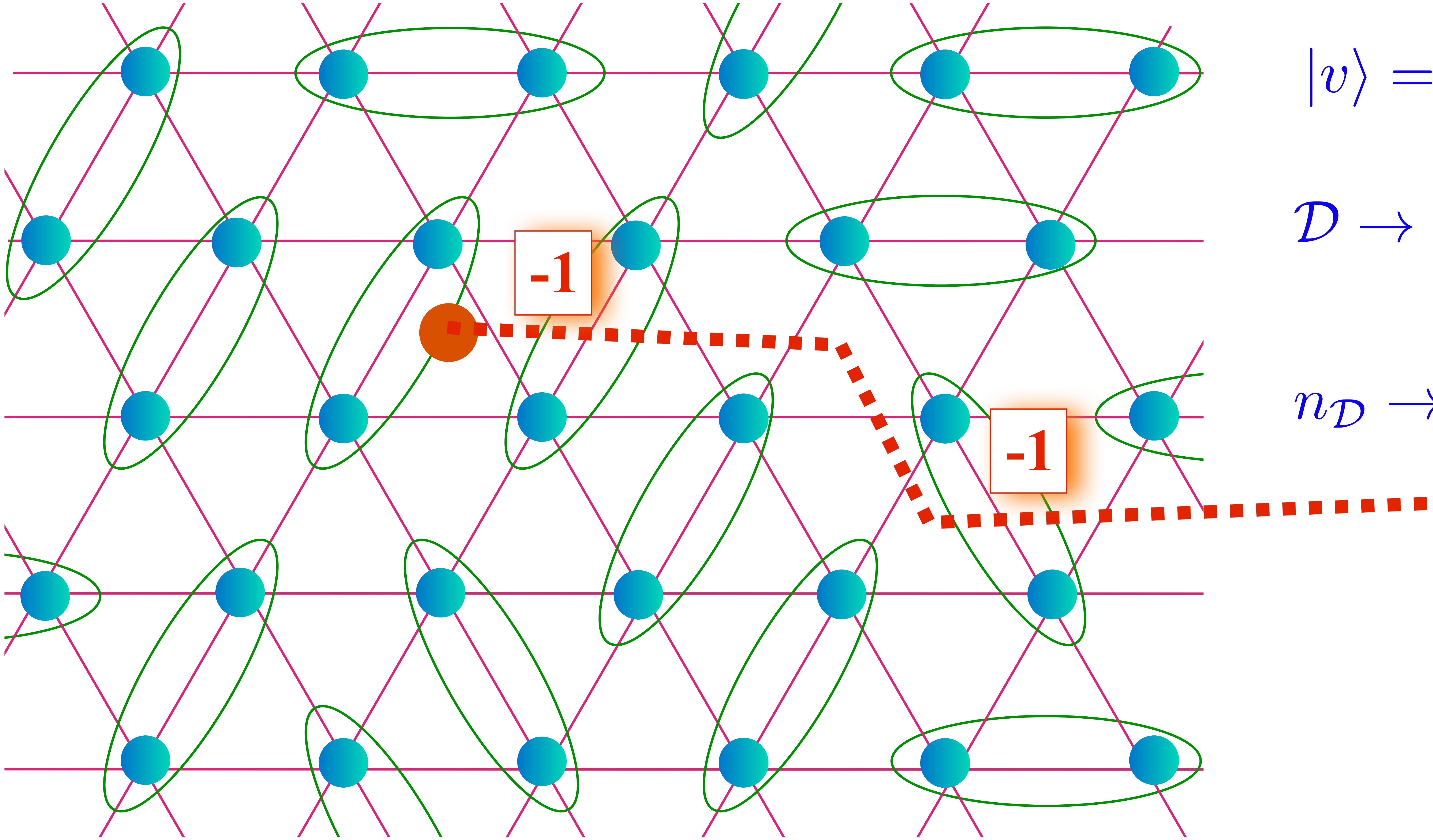

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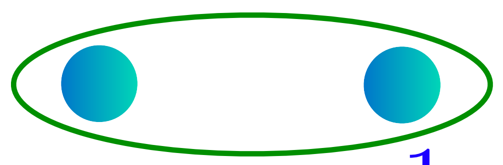

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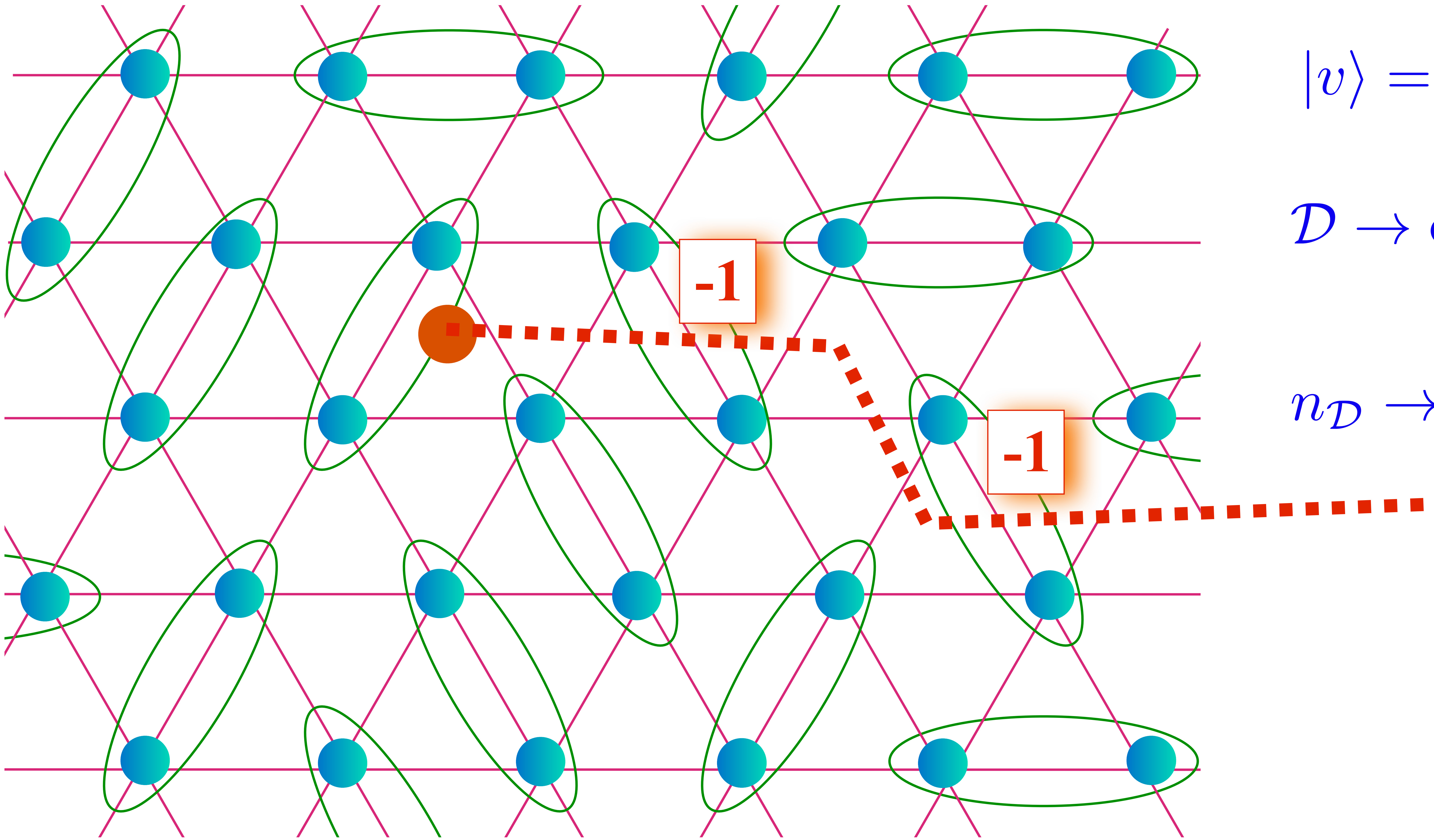

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N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

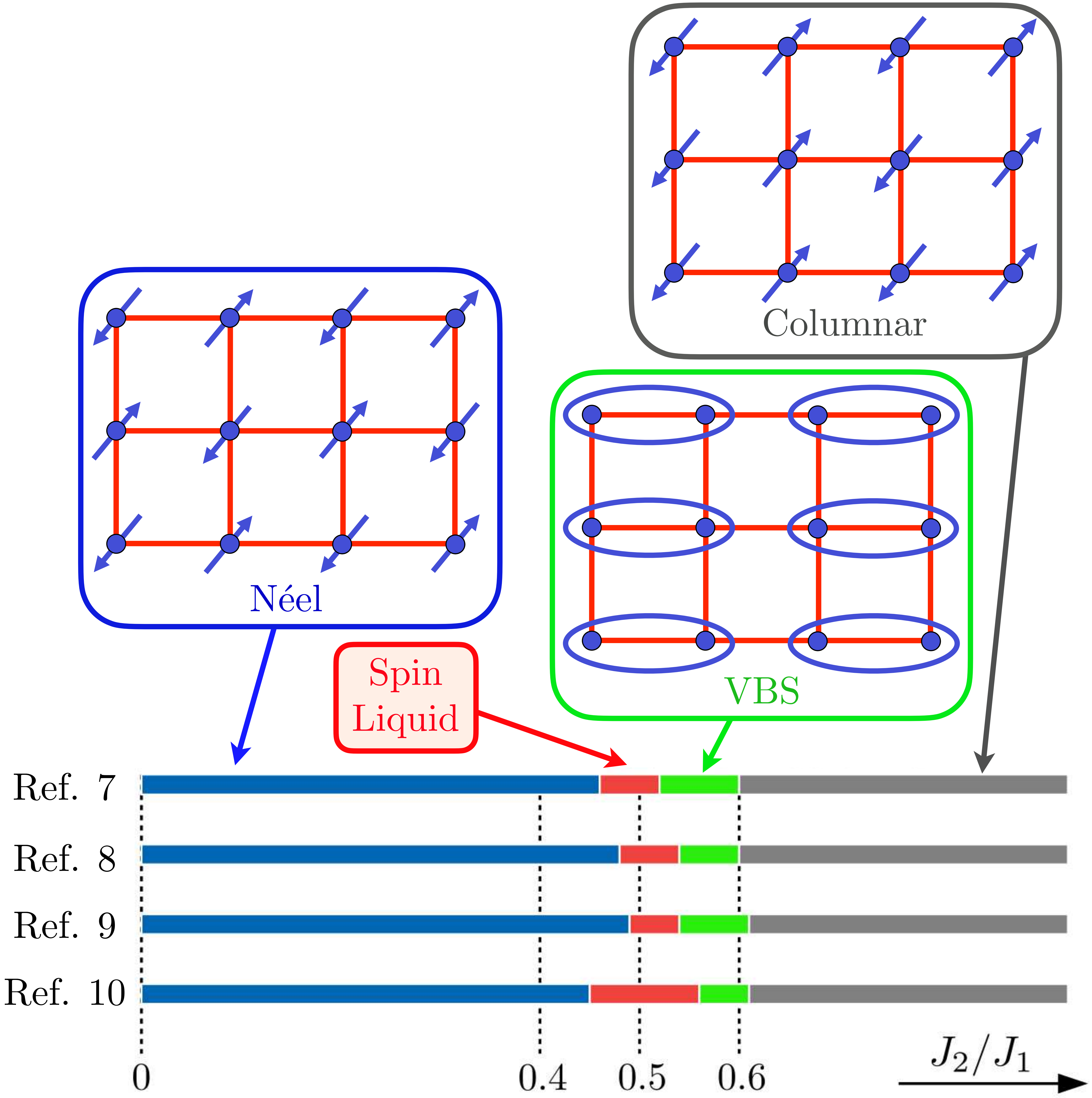
RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

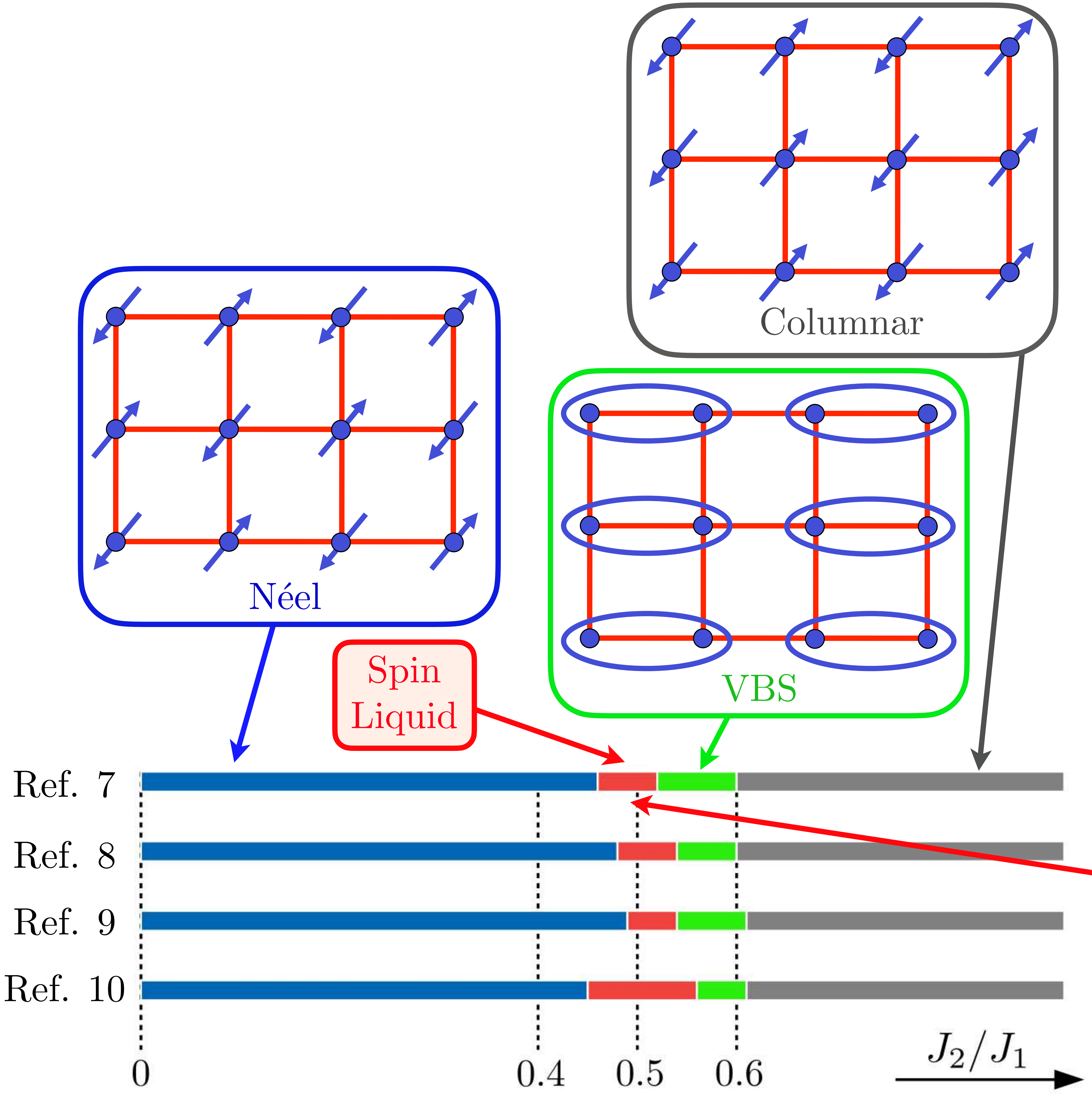
The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are 'spinon' excitations which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	$1/2$	$1/2$	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.



[7] L. Wang and A. W. Sandvik, Phys. Rev. Lett. **121**, 107202 (2018).
[8] F. Ferrari and F. Becca, Phys. Rev. B **102**, 014417 (2020).
[9] Y. Nomura and M. Imada, Phys. Rev. X **11**, 031034 (2021).
[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, arXiv:2009.01821.



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[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, arXiv:2009.01821.

Gapless Z_2 spin liquid phase !

Wen-Jun Hu,
Federico Becca,
Alberto Parola,
Sandro Sorella,
PRB **88**, 060402(R) (2013).

For the J_1-J_2 model

$$H = J_1 \sum_{ij \in n.n.} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{ij \in n.n.n.} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \,,$$

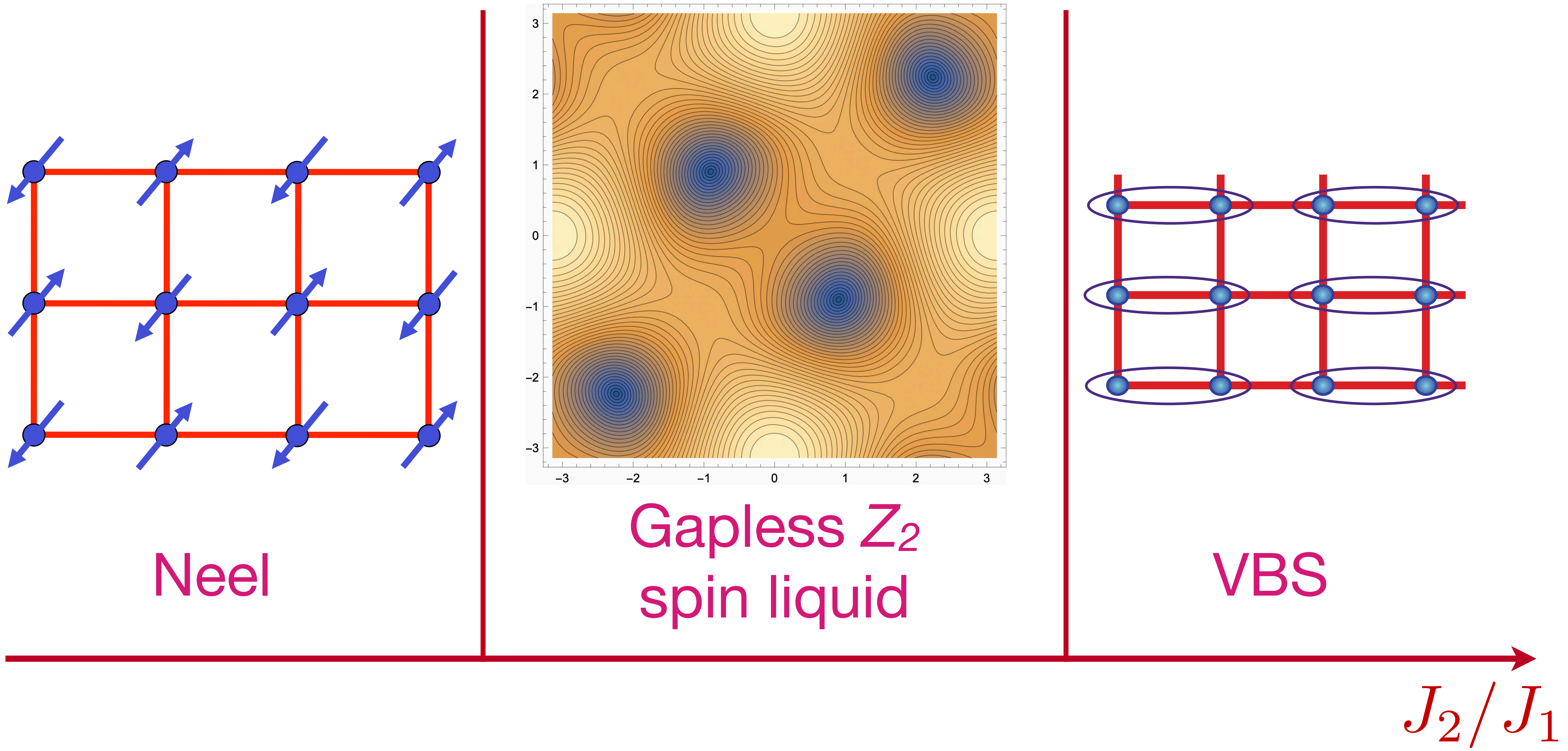
use the parton decomposition

$$\boldsymbol{S} = \frac{1}{2} f_\sigma^\dagger \boldsymbol{\tau}_{\sigma\sigma'} f_{\sigma'} \quad , \quad f_\sigma^\dagger f_\sigma = 1 \,,$$

and Hubbard-Stratonovich transformations to obtain the quadratic fermion Hamiltonian

$$\mathcal{H}_f = \sum_{i<j} \Big[P_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \text{H.c.} + Q_{ij} \varepsilon_{\sigma\sigma'} f_{i\sigma} f_{j\sigma'} + \text{H.c.} \Big] + \sum_i \lambda_i f_{i\sigma}^\dagger f_{i\sigma} \,,$$

with decoupling fields P_{ij}, Q_{ij}, λ_i. The gapless $\mathbb{Z}_2$ spin liquid state is obtained as a saddle point with $Q_{ij} = \overline{Q}_{ij}$, $P_{ij} = \overline{P}_{ij}$, $\lambda_i = \overline{\lambda}_i$ so that H_f describes a 'superconductor' with nearest-neighbor hopping, nearest-neighbor $d_{x^2-y^2}$ pairing, and second-neighbor d_{xy} pairing.



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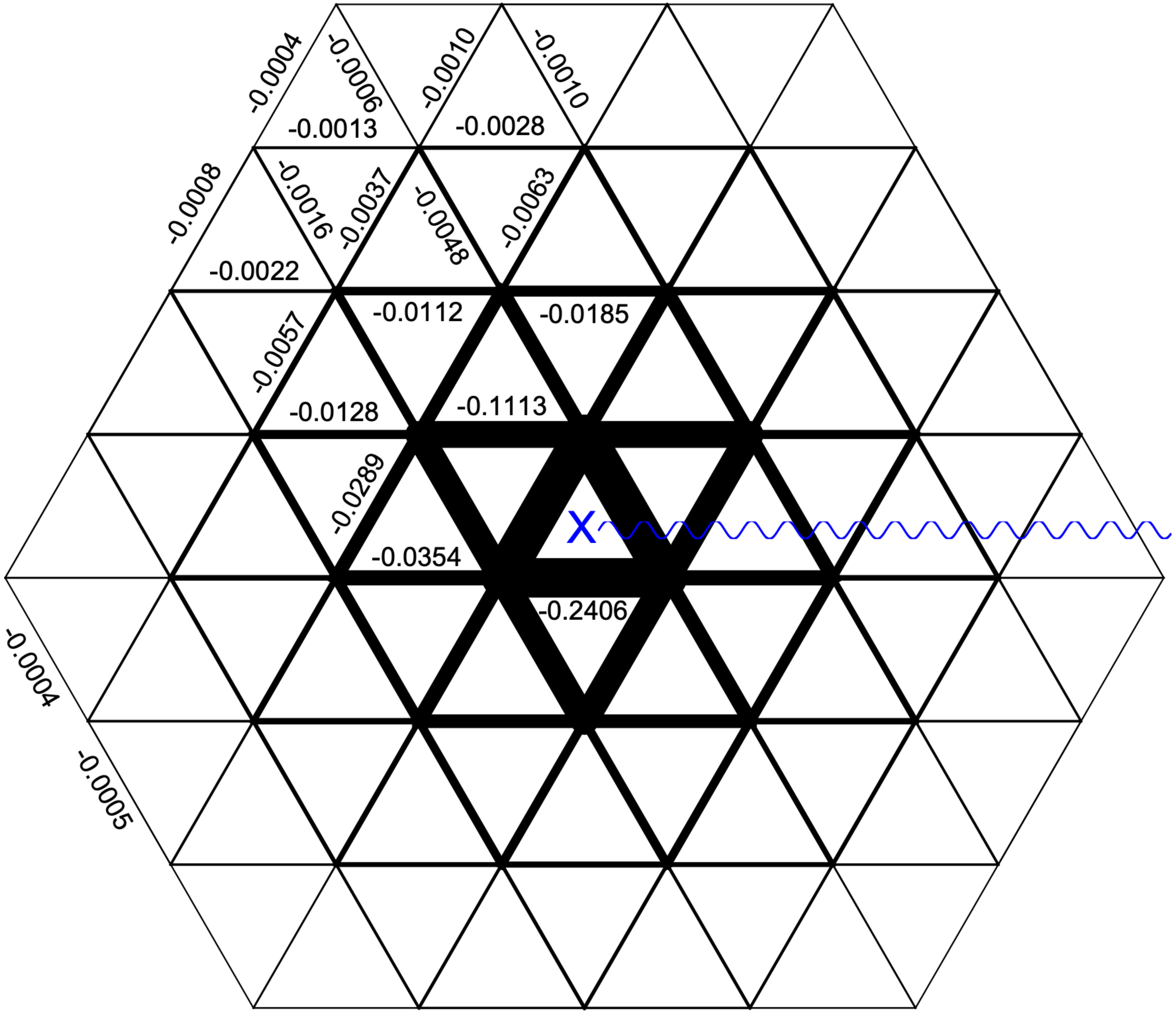
Deconfined criticality and a gapless Z_2 spin liquid in the square lattice antiferromagnet,

H. Shackleton, A. Thomson, and S. S., PRB **104**, 045110 (2021)

Henry Shackleton

Alex Thomson

There are also vison saddle points $\overline{Q}^v_{ij}$, $\overline{P}^v_{ij}$ so that $\text{sgn}(\overline{Q}^v_{ij}/\overline{Q}_{ij})$ and $\text{sgn}(\overline{P}^v_{ij}/\overline{P}_{ij})$ flip across the branch cut.



Yejin Huh, M. Punk and SS, PRB **84**, 094419 (2011)

Z_2 gauge theory with matter: describes visons and spinons

To include the visons, we restrict fluctuations of Q_{ij} and P_{ij} to

$$Q_{ij} = \overline{Q}_{ij} Z_{ij} \quad , \quad P_{ij} = \overline{P}_{ij} Z_{ij} \quad , \quad Z_{ij} = \pm 1 \, .$$

Then the system is described by the effective Hamiltonian

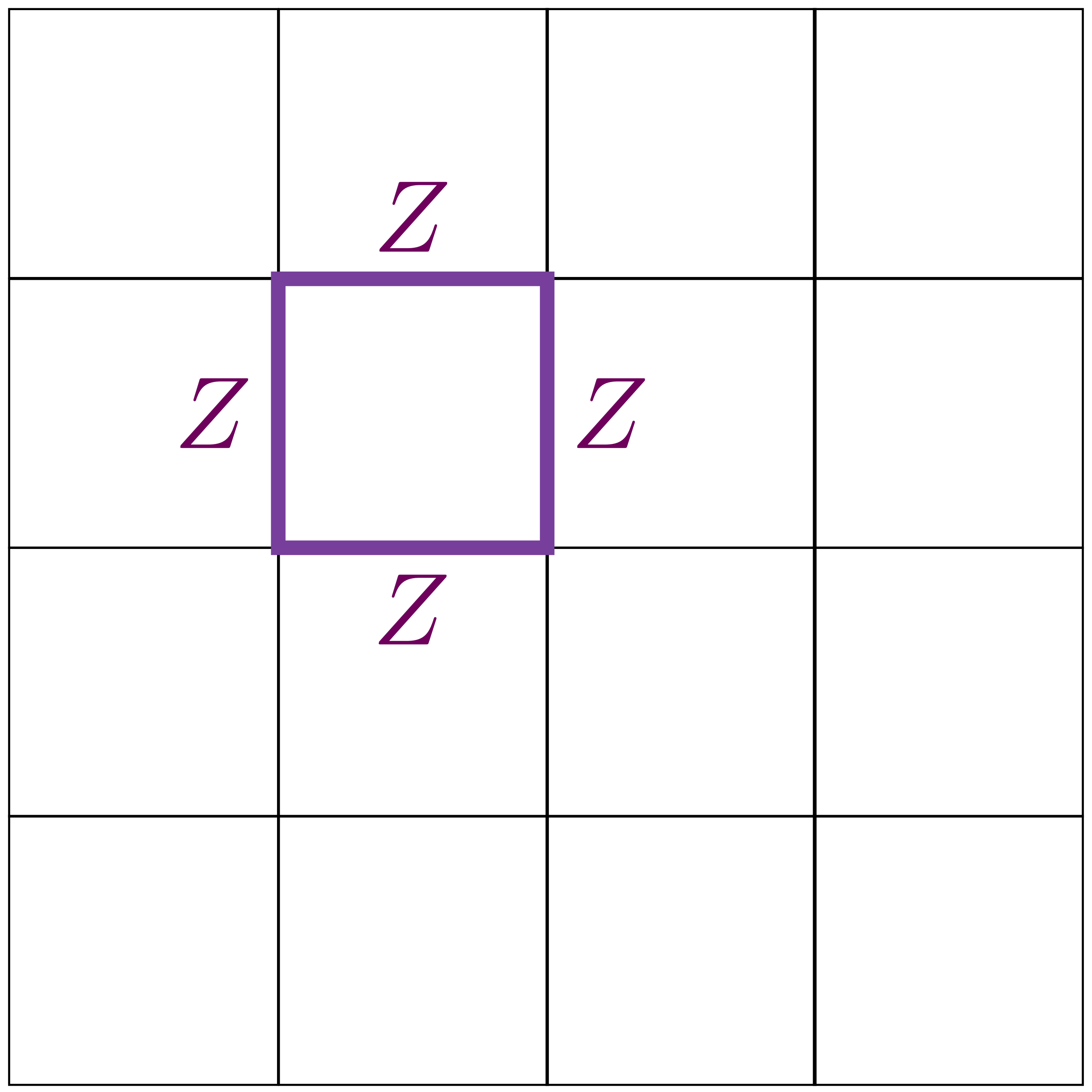
$$\begin{aligned}
\mathcal{H} =& \mathcal{H}_f + \mathcal{H}_{\mathbb{Z}_2} \\
\mathcal{H}_f =& \sum_{i<j} Z_{ij} \left[\overline{P}_{ij} f_{i\sigma}^\dagger f_{j\sigma} + \text{H.c.} + \overline{Q}_{ij} \varepsilon_{\sigma\sigma'} f_{i\sigma} f_{j\sigma'} + \text{H.c.} \right] + \sum_i \lambda_i \left(f_{i\sigma}^\dagger f_{i\sigma} - 1 \right) \\
\mathcal{H}_{\mathbb{Z}_2} =& - K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_\ell X_\ell
\end{aligned}$$

All terms are invariant under the $\mathbb{Z}_2$ gauge transformation

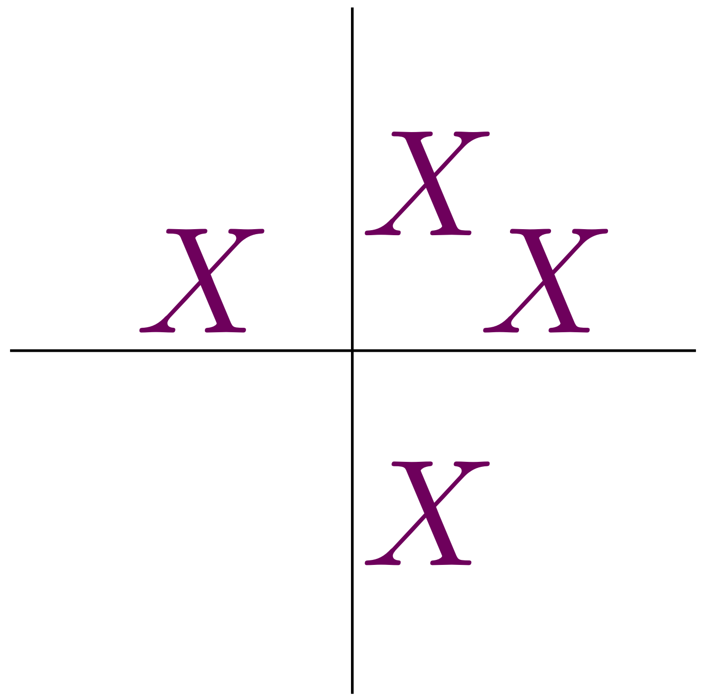
$$f_{i\sigma} \to \varrho_i f_i \quad , \quad Z_{ij} \to \varrho_i Z_{ij} \varrho_j \quad , \quad X_{ij} \to X_{ij} \quad , \quad \varrho_i = \pm 1 \, .$$

Pure Z_2 gauge theory: only describes vison sector

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$



$G_i =$



$$[\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

$G_i = (-1)^{2S}$ for spin S antiferromagnets

For $S = 1/2$ we have an 'odd' spin liquid.

R. Jalabert and S. Sachdev, Physical Review B **44**, 686 (1991)

S. Sachdev and M. Vojta, Journal of the Physical Society of Japan **69,** Suppl. B, 1 (2000); cond-mat/9910231

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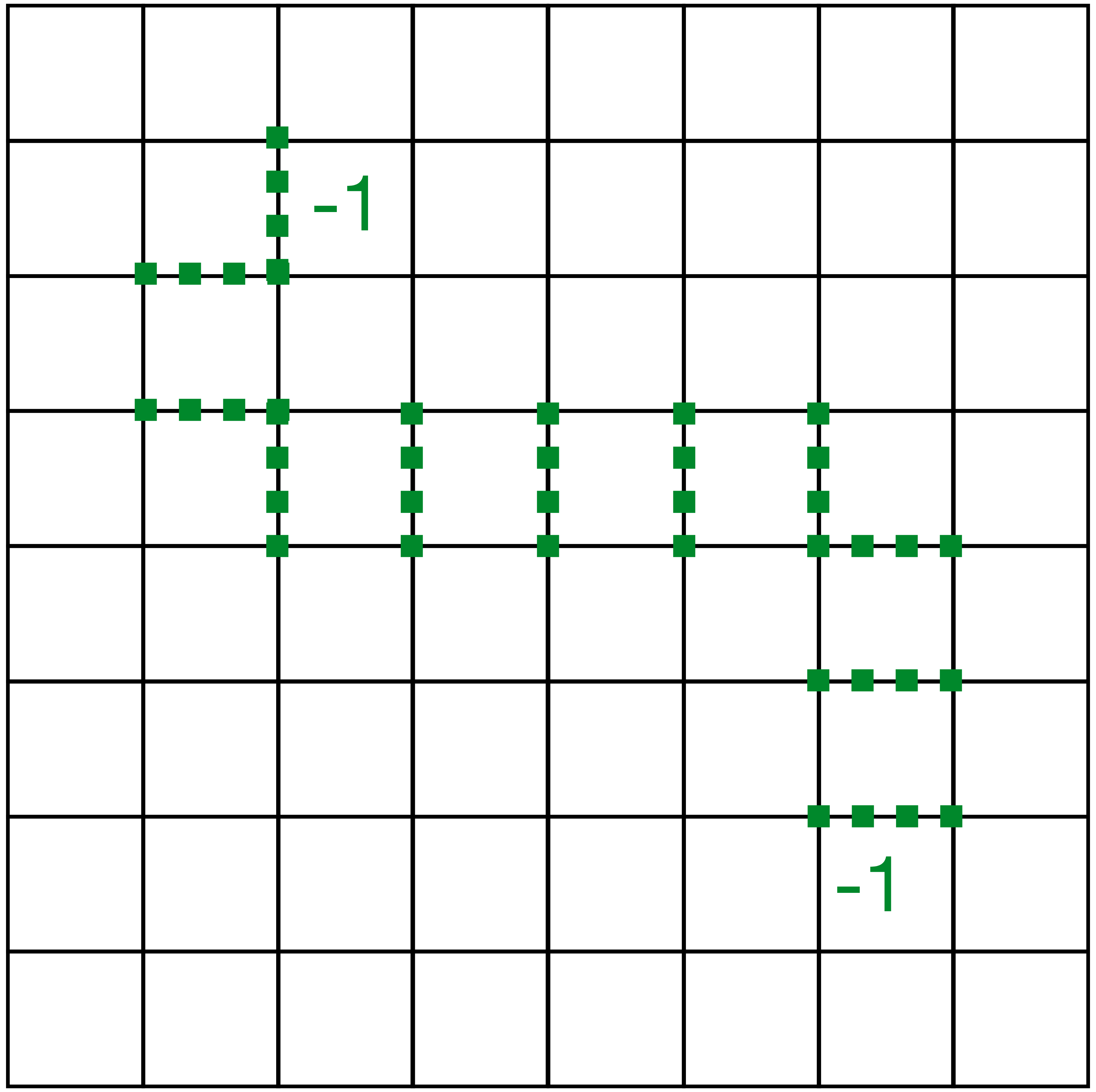


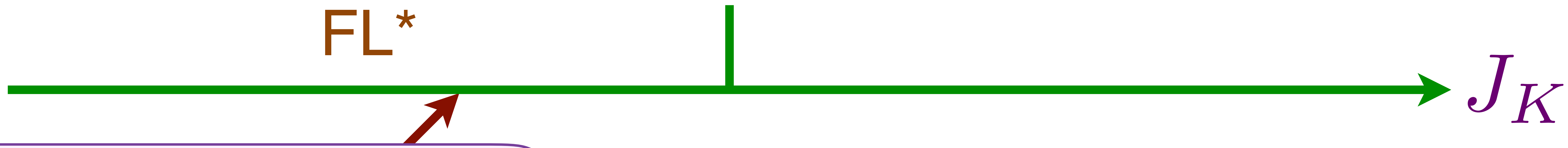
Two visons (indicated by the -1's in the plaquettes) connected by an invisible string. The dashed lines indicate the links, ℓ, on which $Z_\ell = -1$. The plaquettes with an even number of dashed lines on their edges carry no $\mathbb{Z}_2$ fluxes, and so are 'invisible'.

1. Luttinger and Luttinger* relations in metals
2. Spin liquids and emergent gauge fields
3. FL* and HFL phases of the Kondo lattice
4. Paramagnon fractionalization theory of the pseudogap metal in a single band model

Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K \, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\mathcal{H}_{\mathrm{FL}*} = -\sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{i,j} P_{ij} f^\dagger_{i\sigma} f_{j\sigma}$$



Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } f\rangle \otimes |\text{Slater determinant of } c\rangle$$

N. Andrei and P. Coleman, PRL **62**, 595 (1989)
S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)
T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

Kondo lattice

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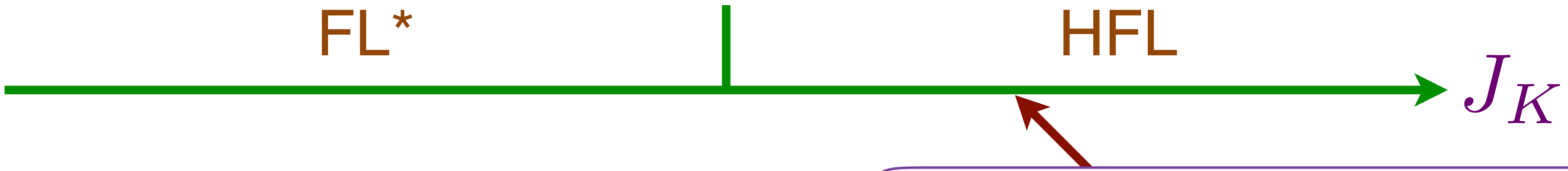
$$\mathcal{H}_{\rm HFL} = - \sum_{i,j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{i,j} P_{ij} f^\dagger_{i\sigma} f_{j\sigma} + \sum_i B \left(f^\dagger_{i\sigma} c_{i\sigma} + \text{H.c.} \right)$$



Large Fermi surface of size $1 + p$

$$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

Coleman, Read, Newns, Millis, P.A. Lee, Auerbach, Levin

1. Luttinger and Luttinger* relations in metals
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4. Paramagnon fractionalization theory of the pseudogap metal in a single band model

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^{\dagger} c_{i\sigma}$$

We use the operator equation (valid on each site i):

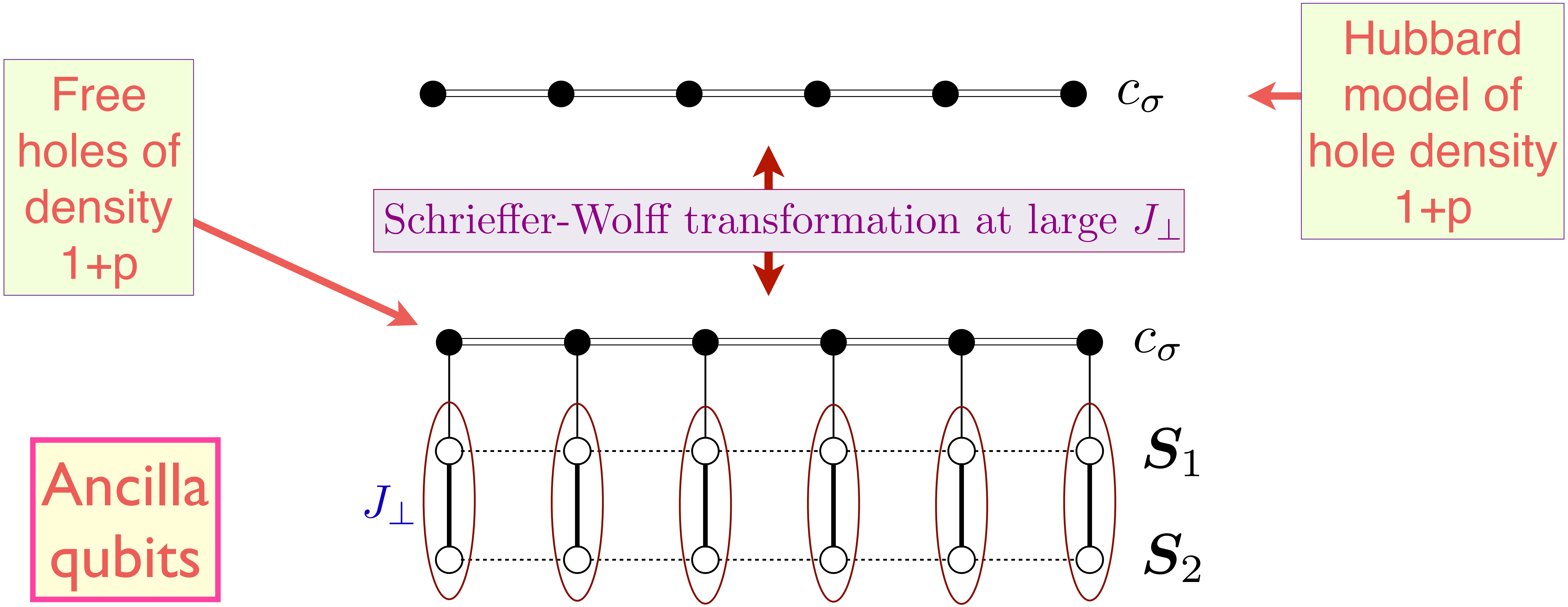
$$U\left(n_{\uparrow} - \frac{1}{2}\right)\left(n_{\downarrow} - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^{\dagger}\frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model

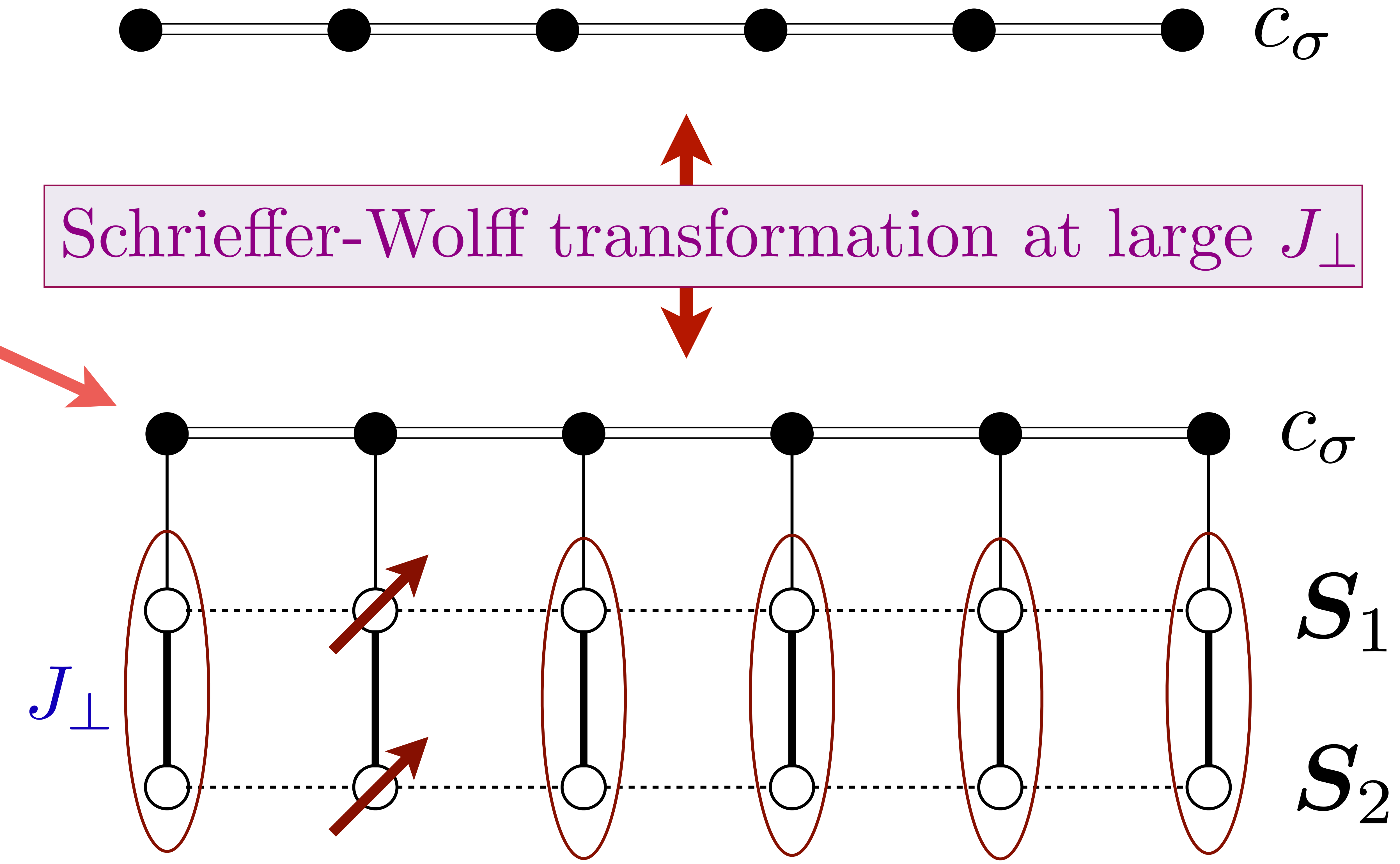


$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model

Free holes of density 1+p

Hubbard model of hole density 1+p



Ancilla qubits

$\boldsymbol{\Phi}$ paramagnon

$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

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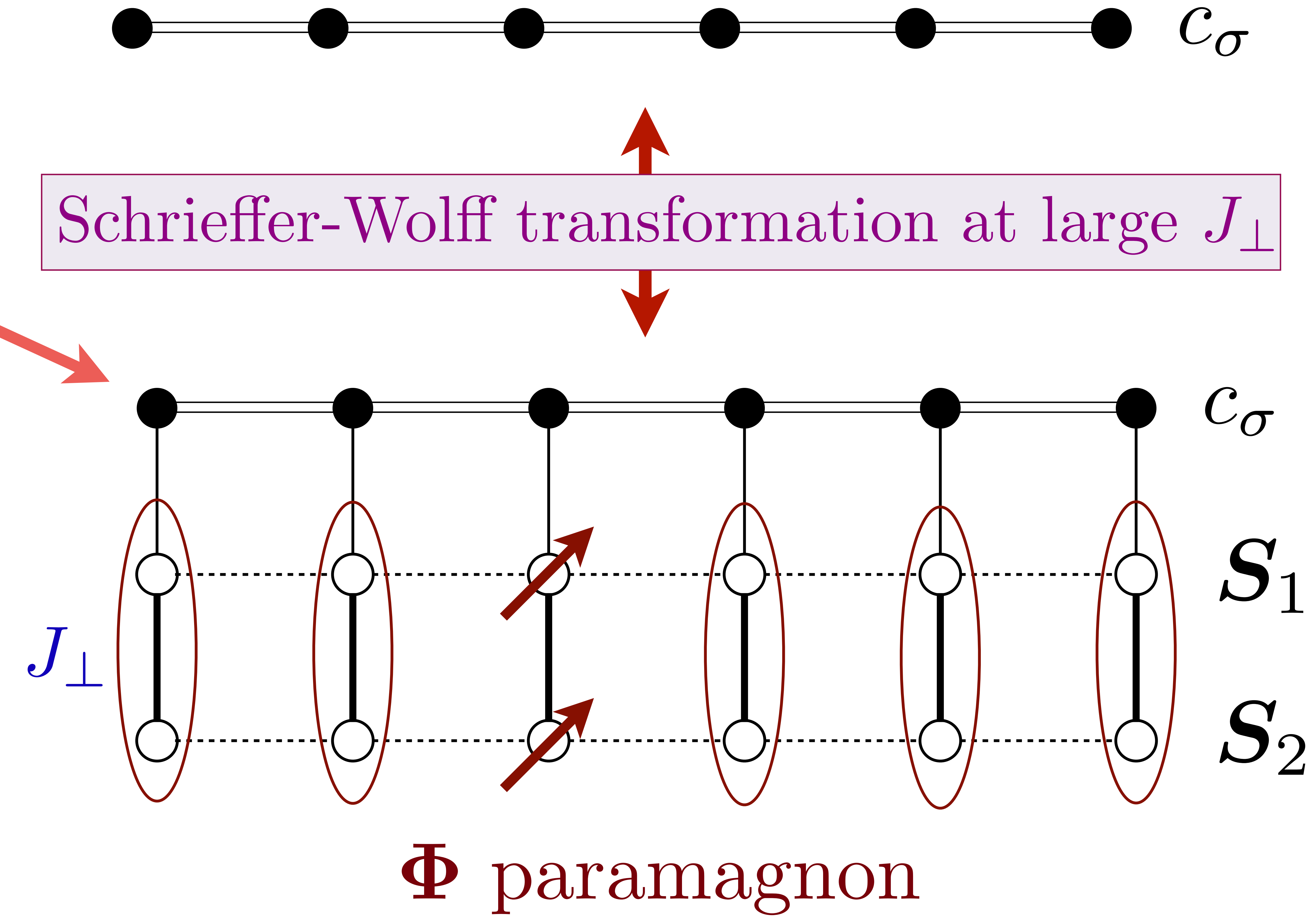

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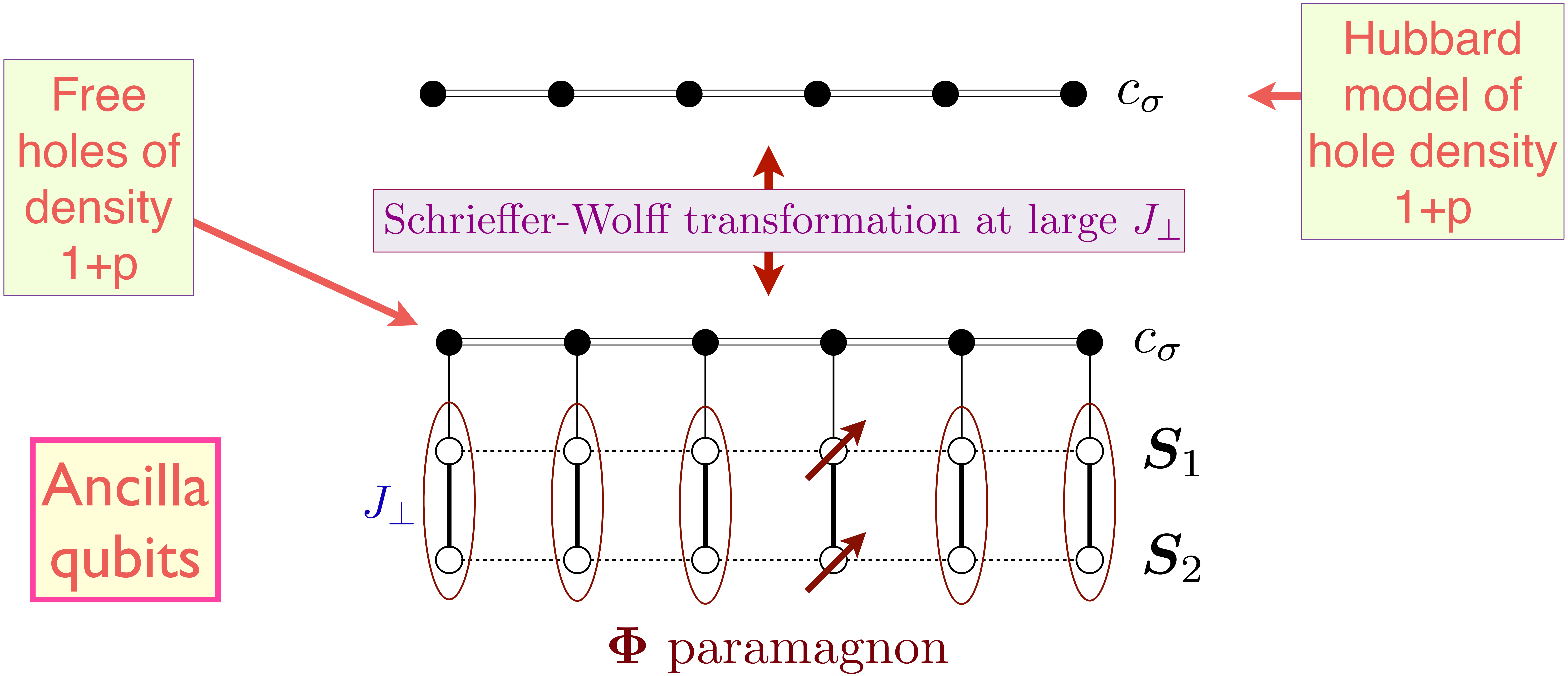

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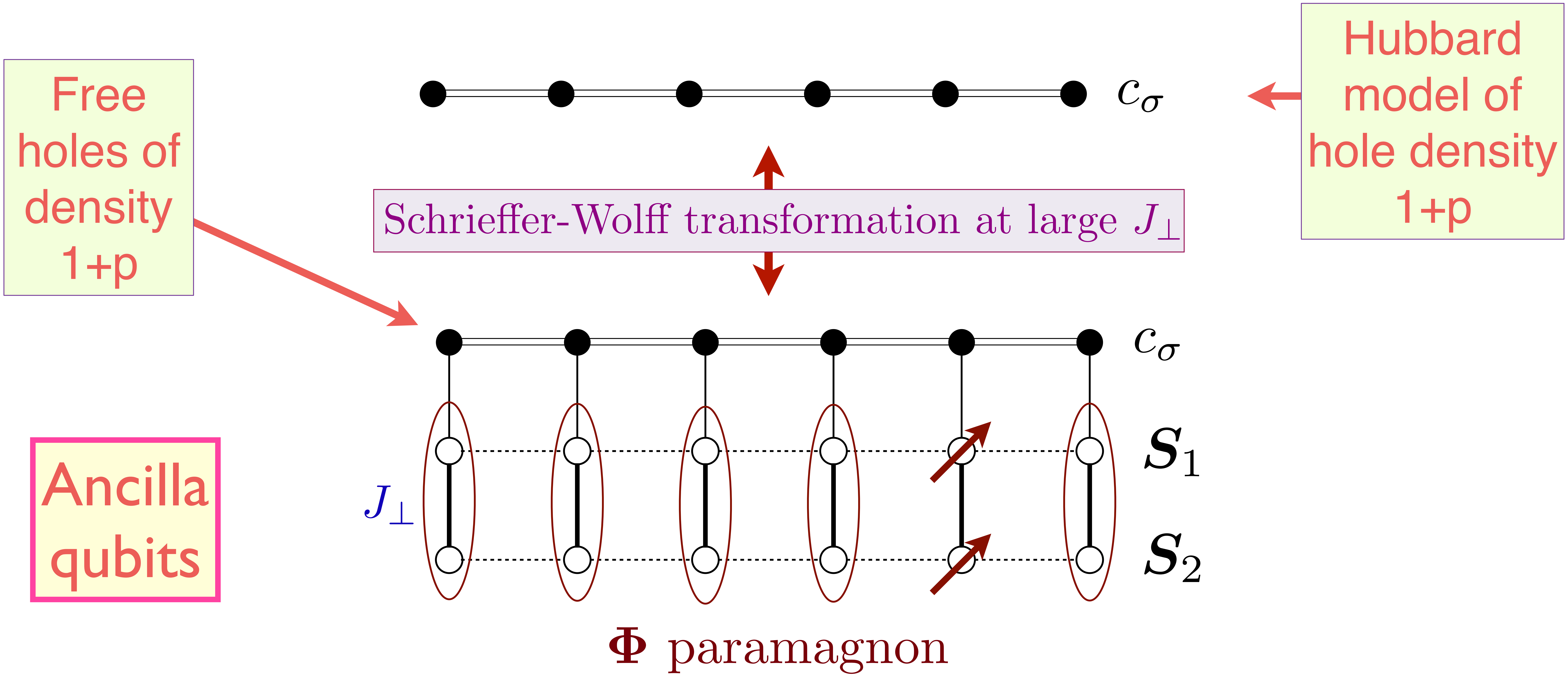

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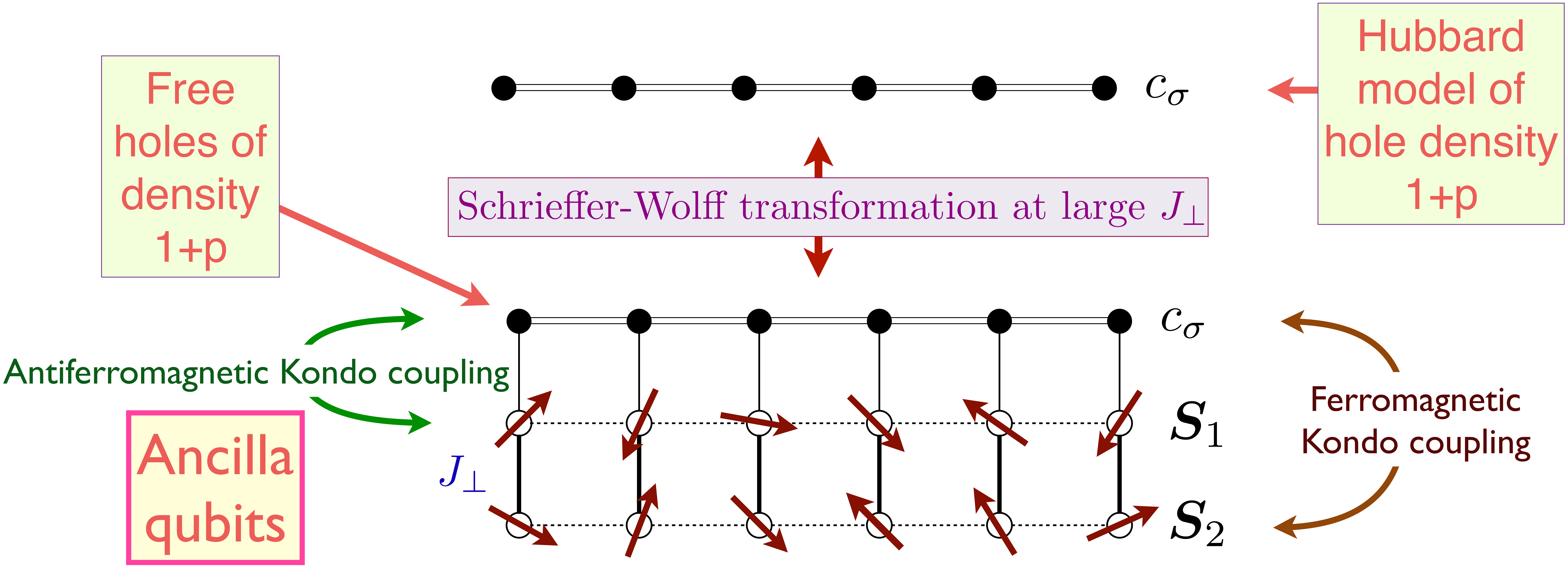
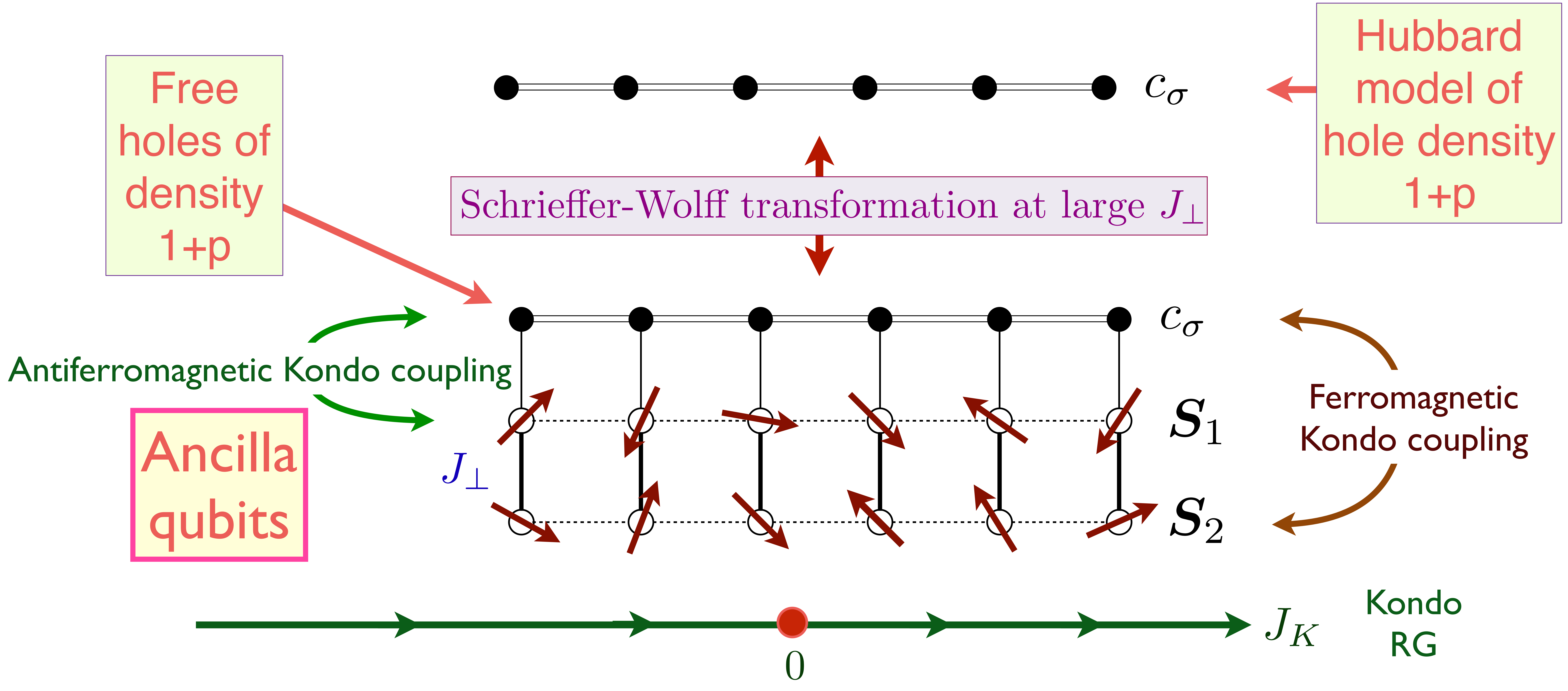
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

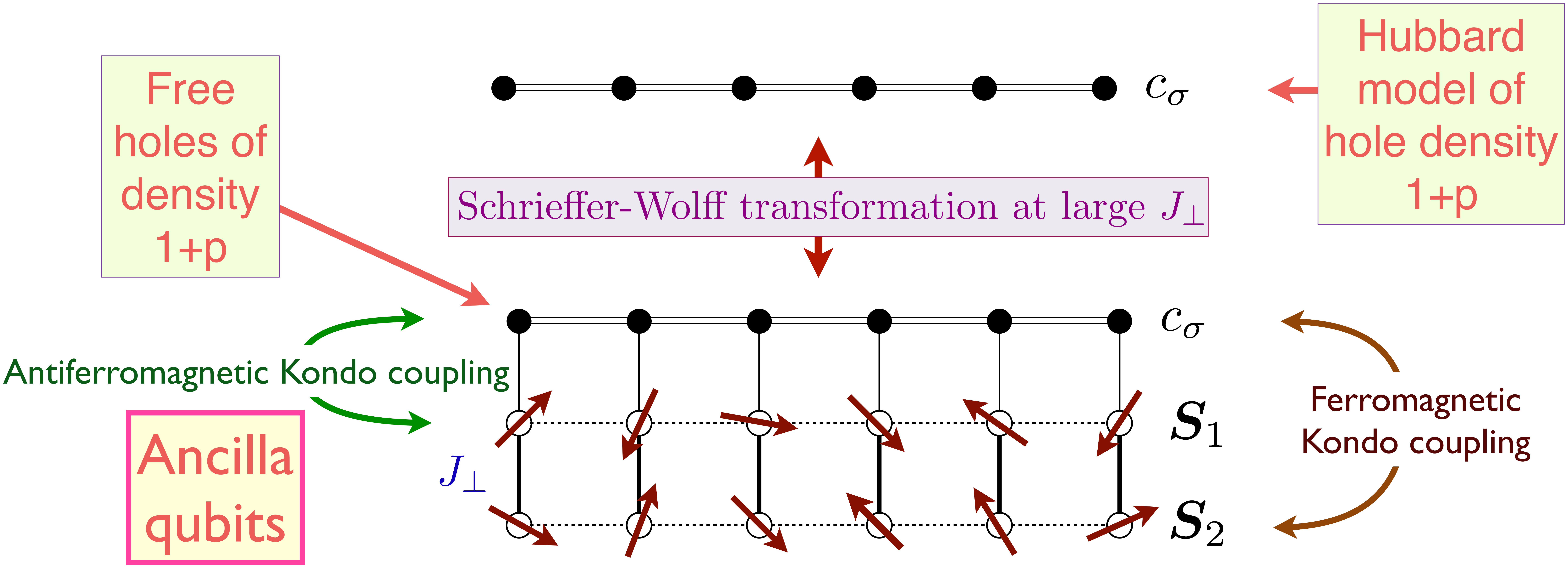
$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + J_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + -\widetilde{J}_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{2i} + \ldots$$

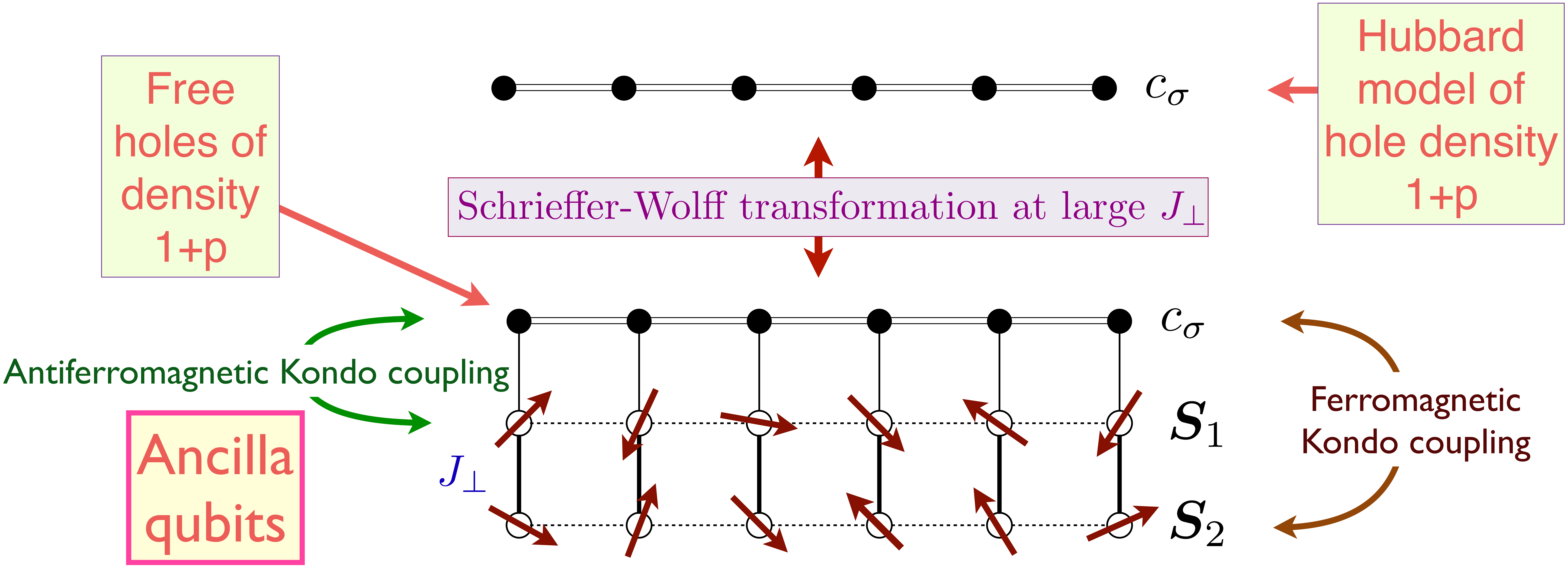
Paramagnon fractionalization theory of the Hubbard model



A FL* state is realized when the antiferromagnetic Kondo coupling dominates over $J_\perp$, and the c_σ and $\boldsymbol{S}_1$ form a "large" Fermi surface of hole density

$$(1+p)+1 = 2+p = p \bmod 2!$$

Paramagnon fractionalization theory of the Hubbard model



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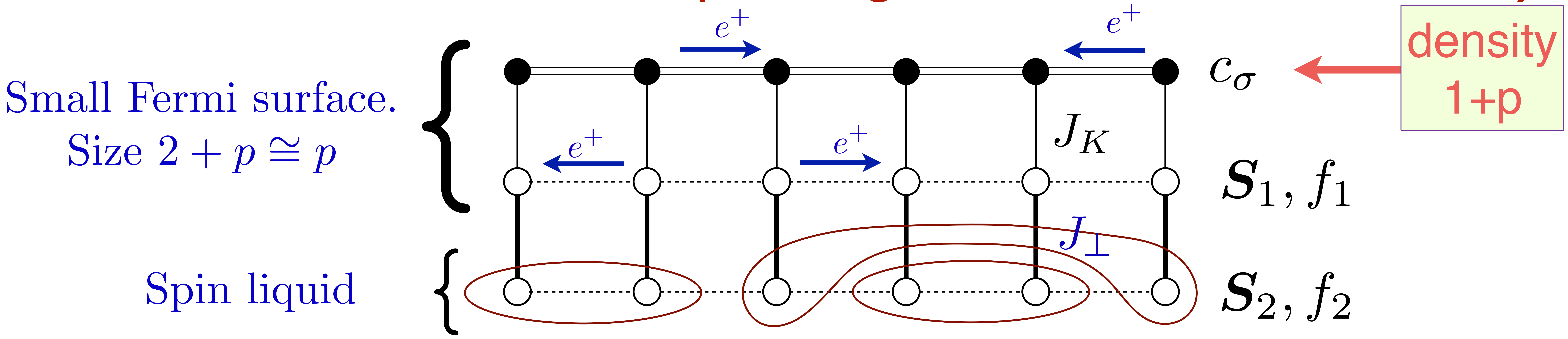
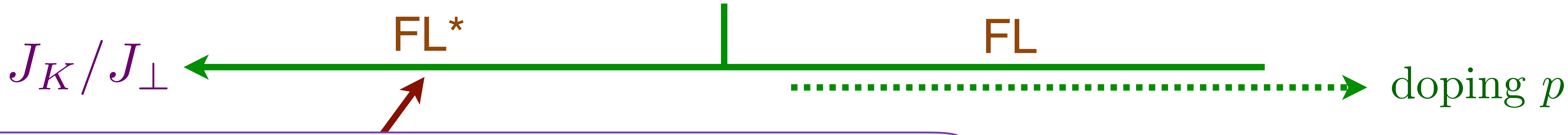
$$(1+p)+1 = 2+p = p \bmod 2!$$

The $\boldsymbol{S}_2$ must form an 'odd' spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

Trial wavefunctions in the paramagnon fractionalization theory





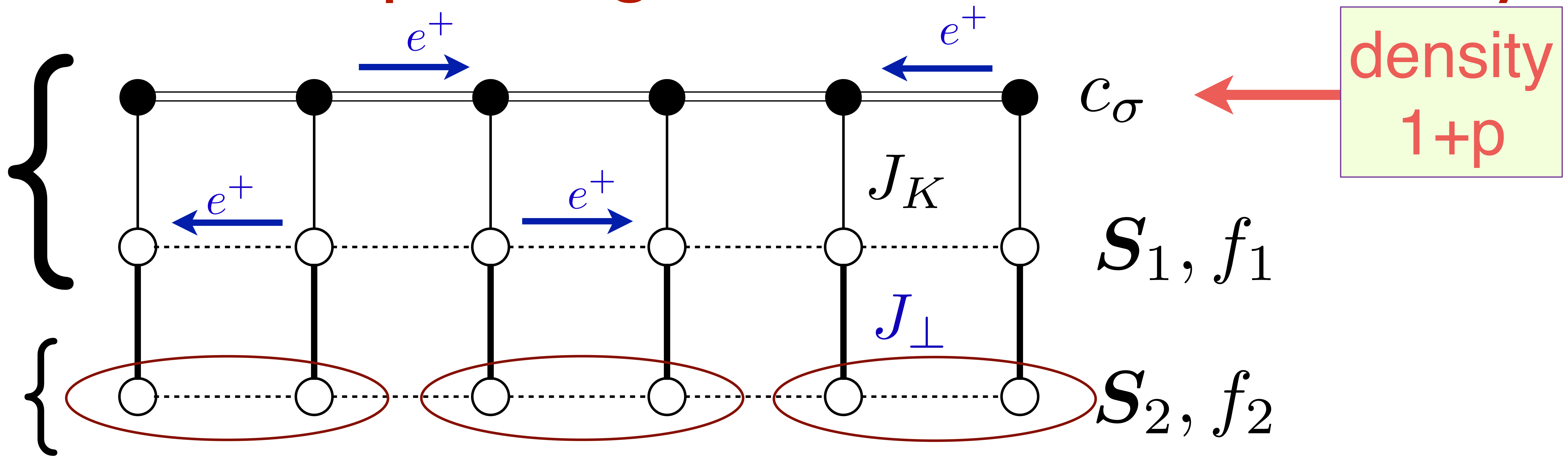
Small Fermi surface of size p

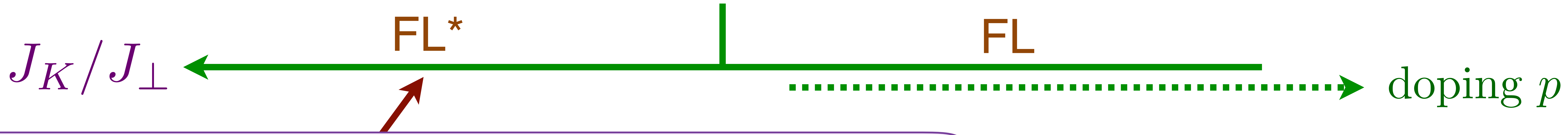
$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } f_1, f_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2+p \cong p$

Spin liquid





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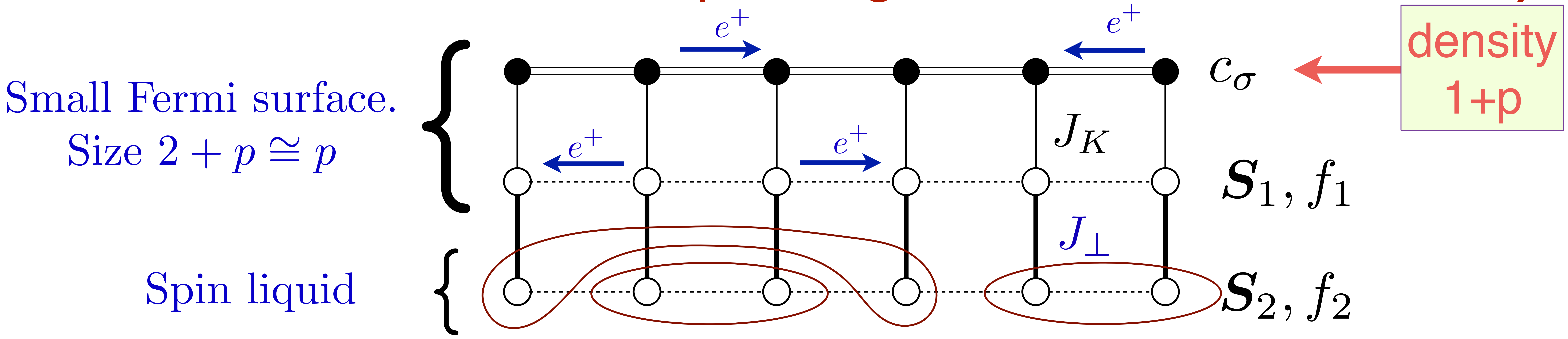
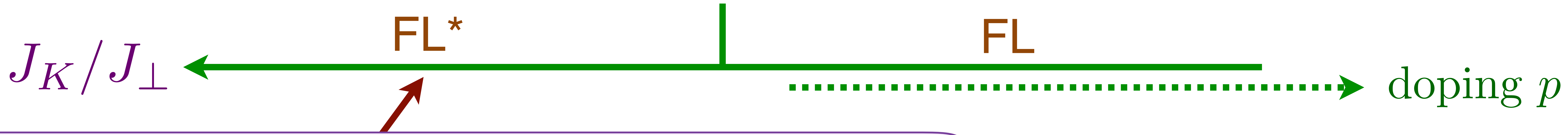
Trial wavefunctions in the paramagnon fractionalization theory

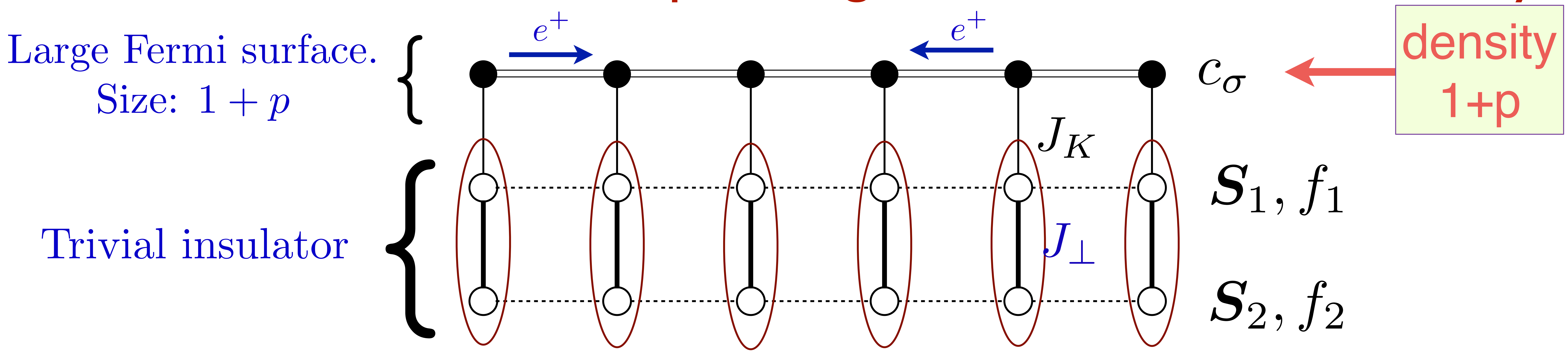


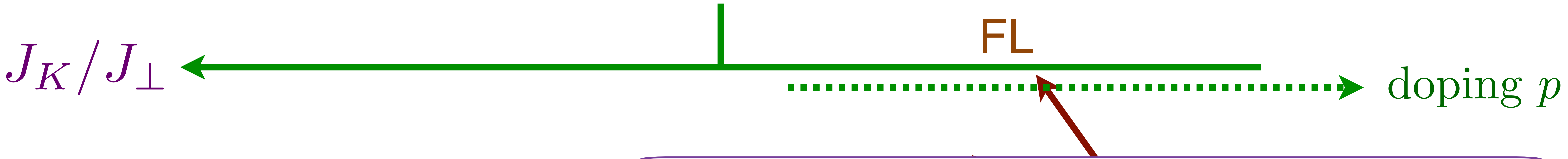


Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } f_1, f_2]$$
$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
$$\otimes |\text{Slater determinant of } f_2\rangle$$

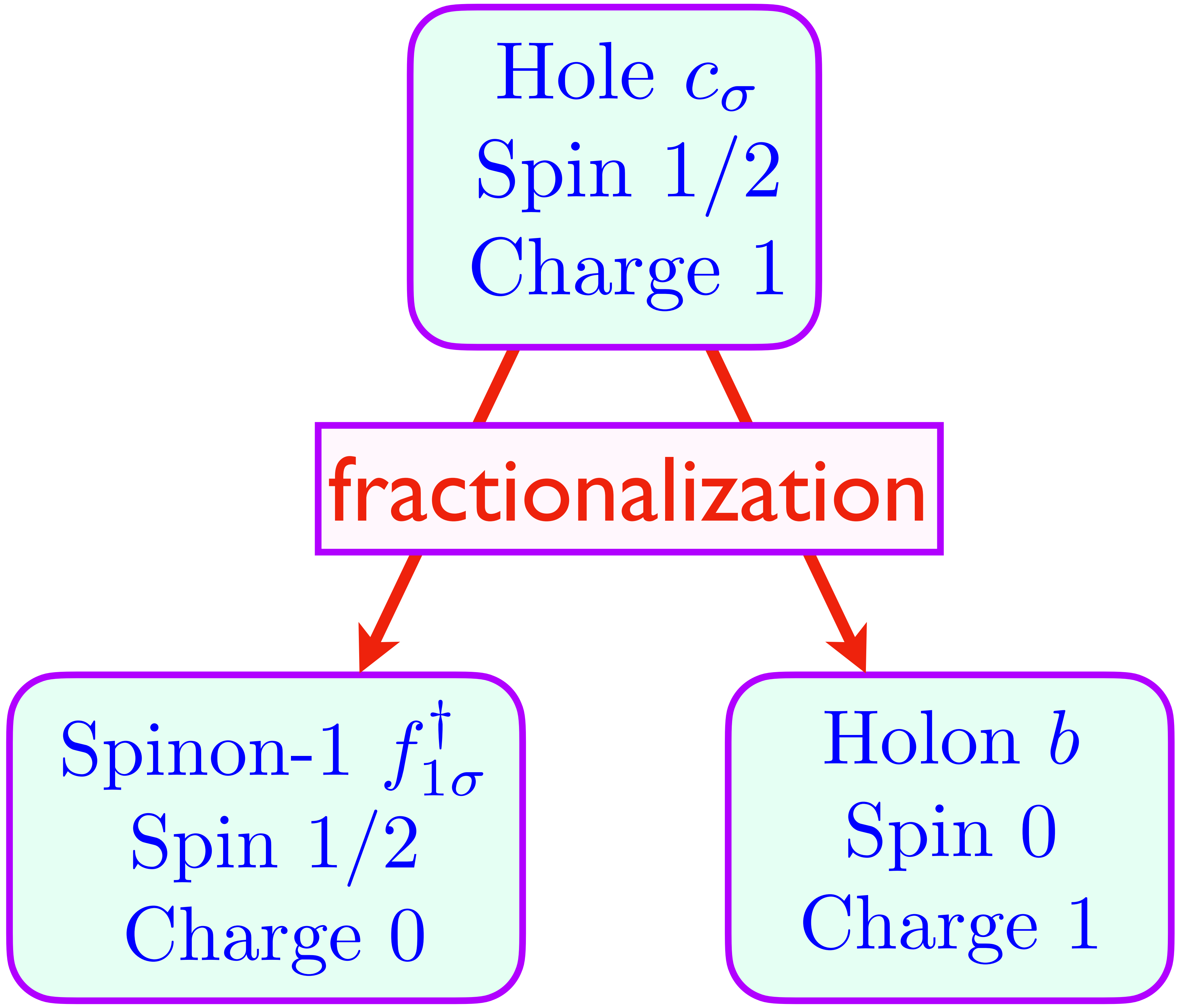
Trial wavefunctions in the paramagnon fractionalization theory



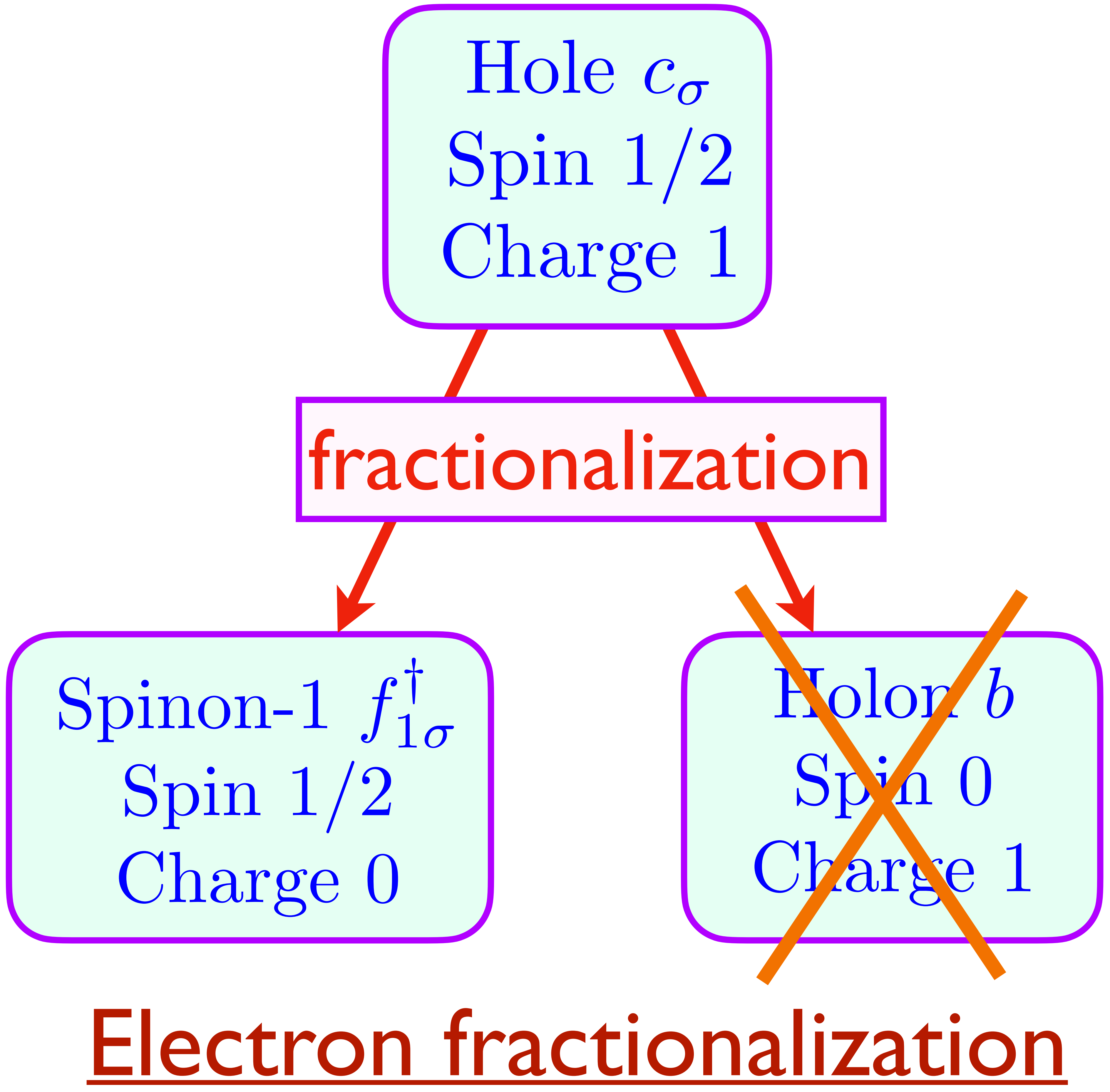


Large Fermi surface of size $1+p$

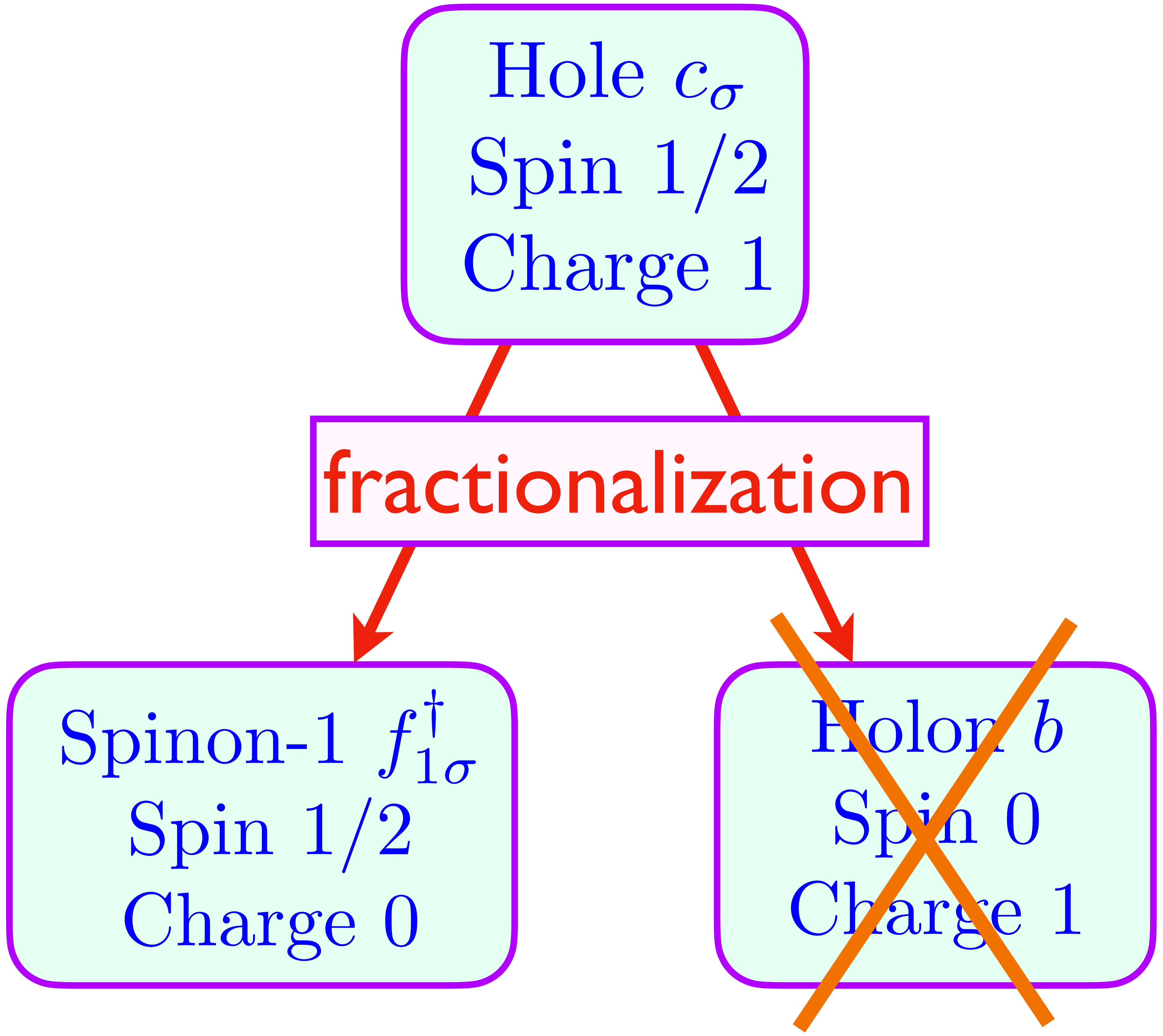
$$|\mathrm{FL}\rangle = |\text{Rung singlets of } f_1, f_2\rangle \otimes |\text{Slater determinant of } c\rangle$$



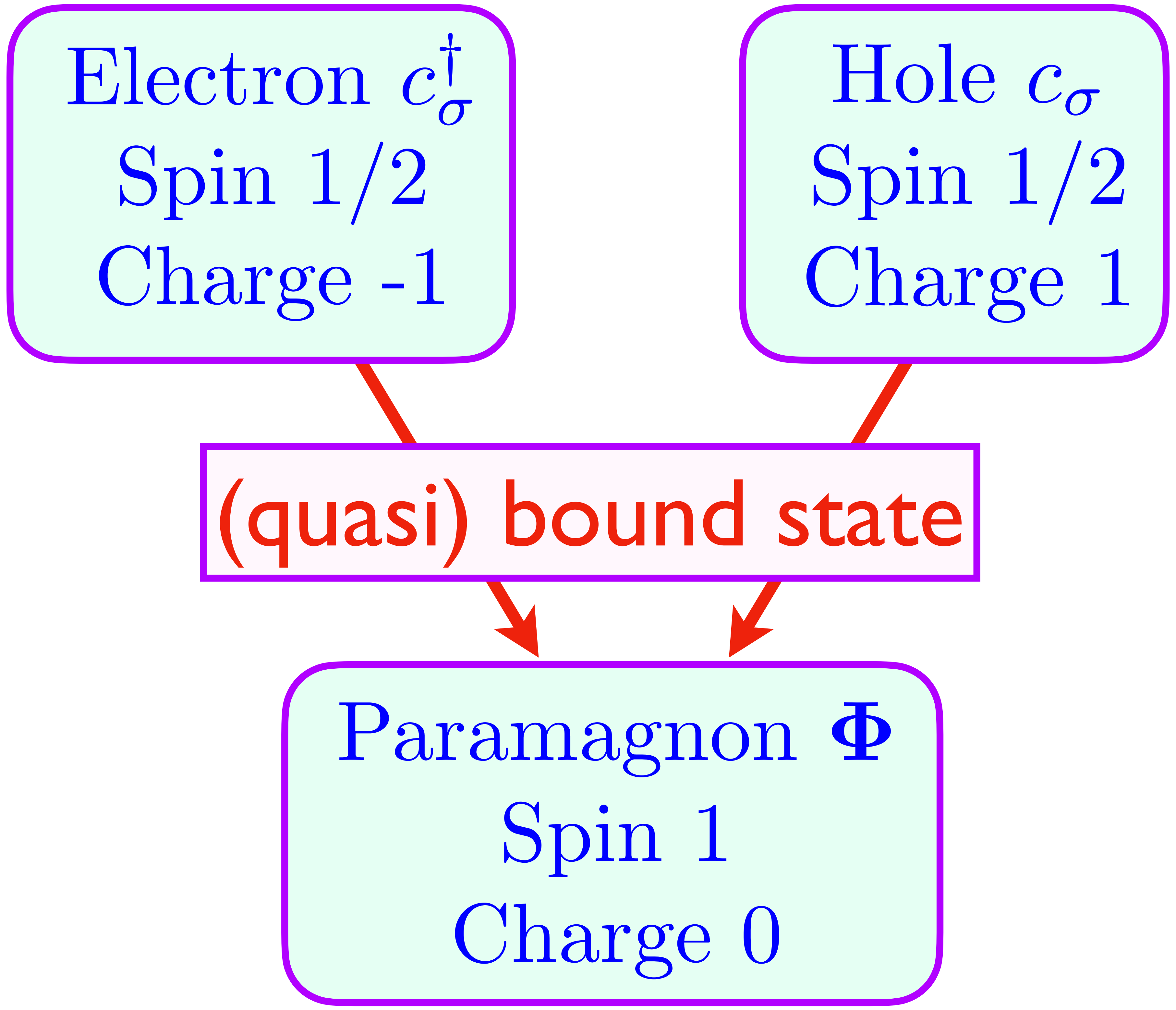
Electron fractionalization



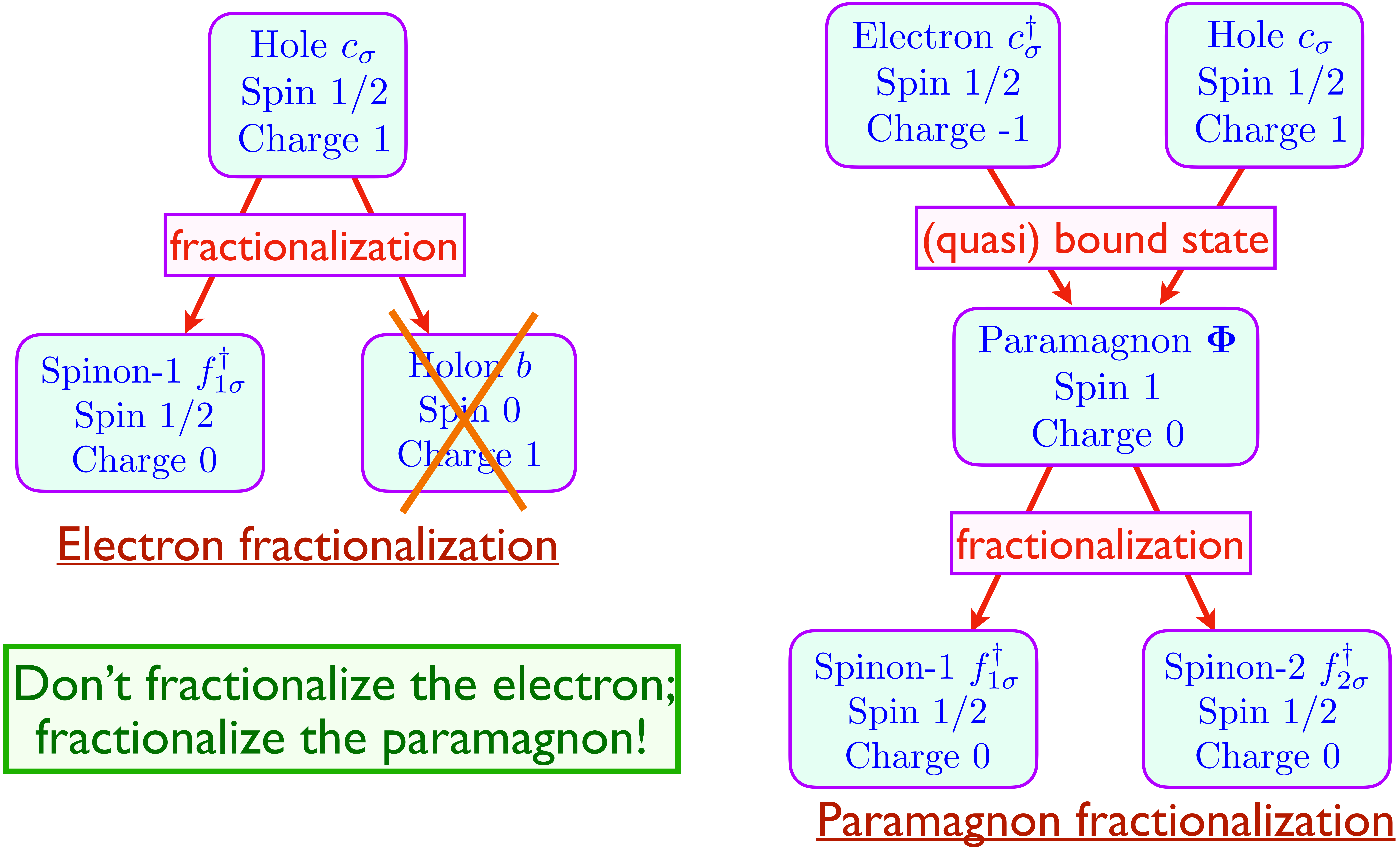
Don't fractionalize the electron;
fractionalize the paramagnon!



Electron fractionalization

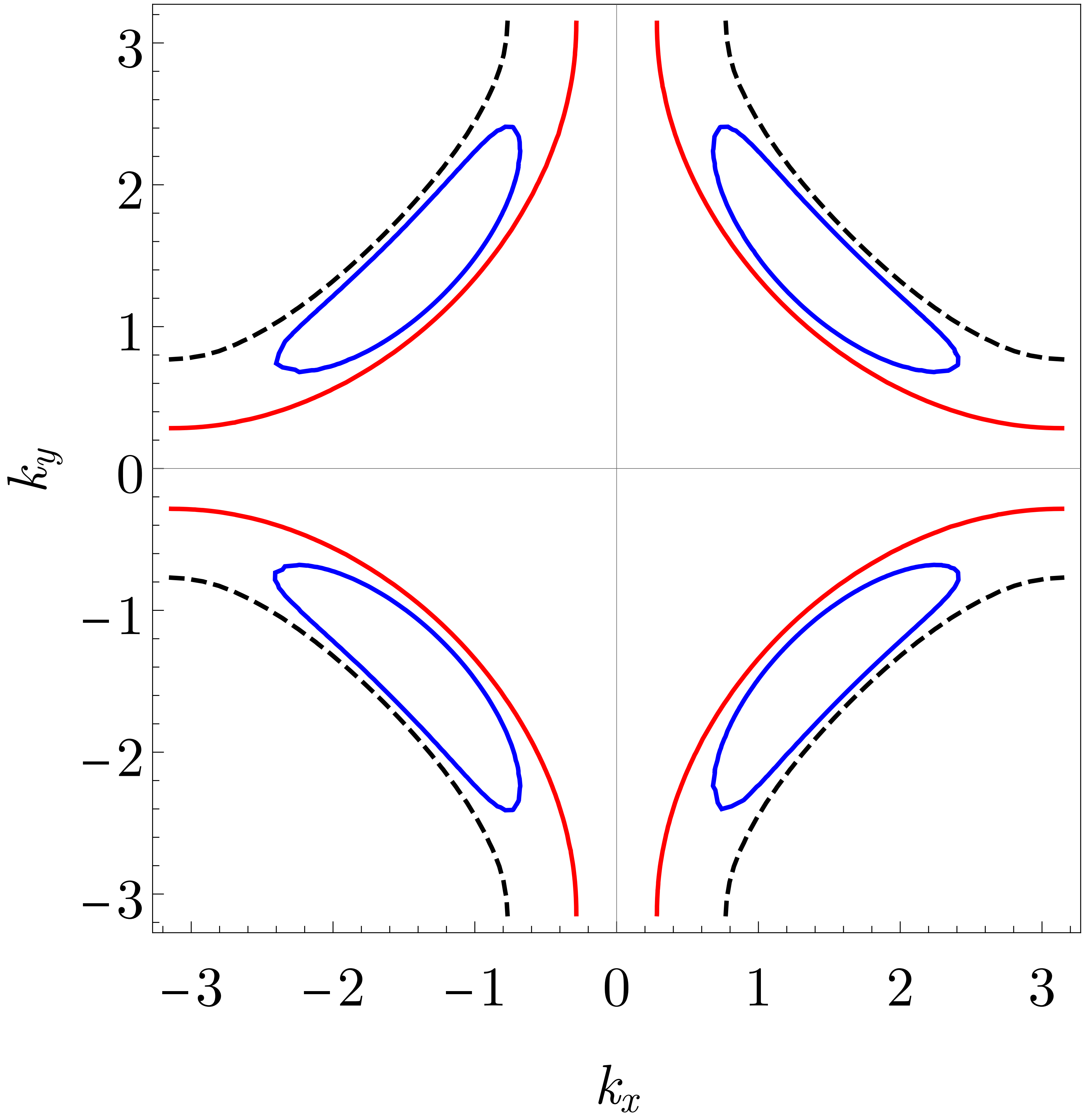


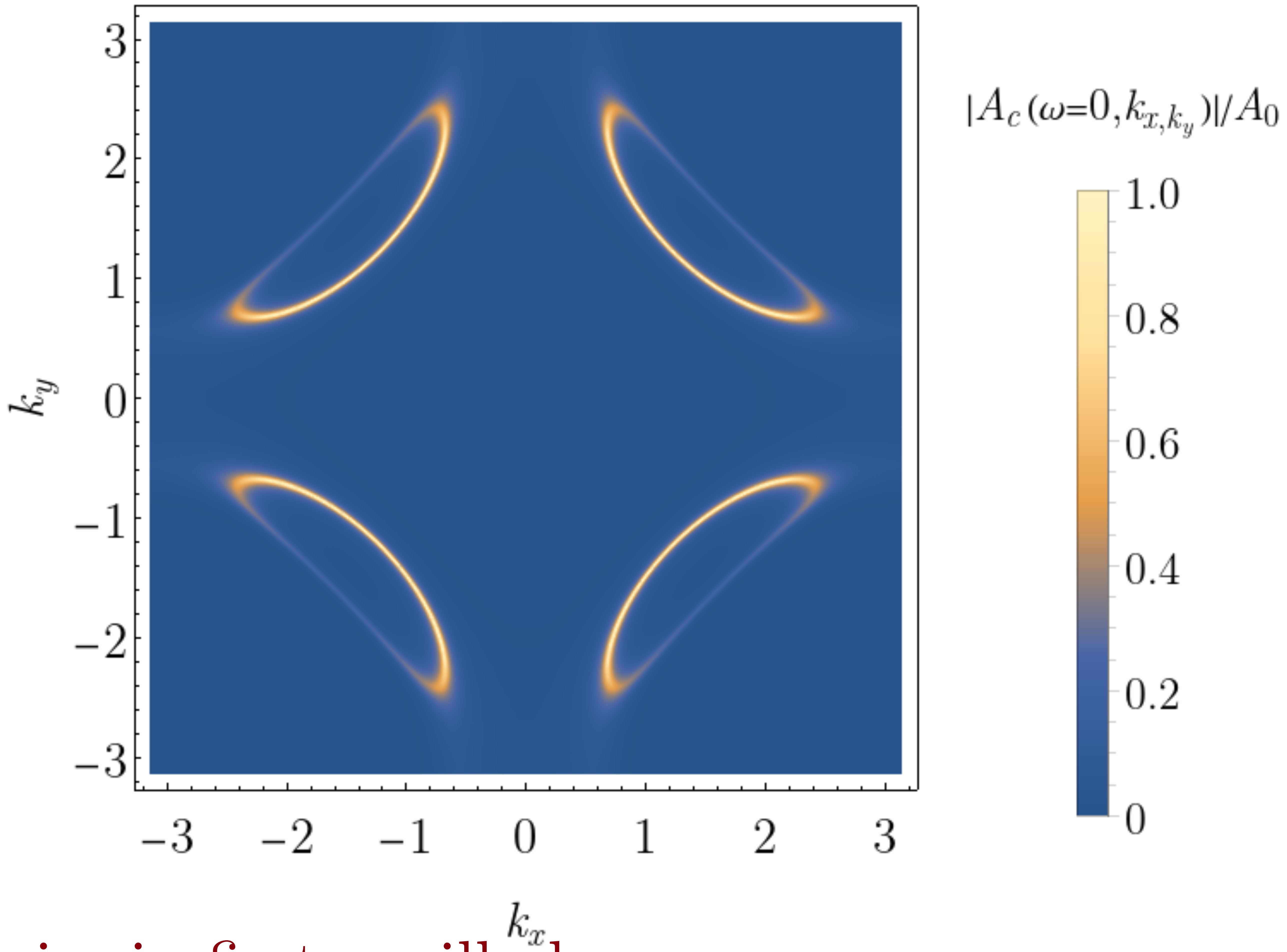
Don't fractionalize the electron; fractionalize the paramagnon!


Hole c_σ
Spin 1/2
Charge 1
fractionalization
Spinon-1 $f_{1\sigma}^\dagger$
Spin 1/2
Charge 0
Holon b
Spin 0
Charge 1
Electron fractionalization
Don't fractionalize the electron;
fractionalize the paramagnon!
Electron $c_\sigma^\dagger$
Spin 1/2
Charge -1
Hole c_σ
Spin 1/2
Charge 1
(quasi) bound state
Paramagnon $\mathbf{\Phi}$
Spin 1
Charge 0
fractionalization
Spinon-1 $f_{1\sigma}^\dagger$
Spin 1/2
Charge 0
Spinon-2 $f_{2\sigma}^\dagger$
Spin 1/2
Charge 0
Paramagnon fractionalization

FL* in a ***one-band*** model

"Fermi arc" spectral functions



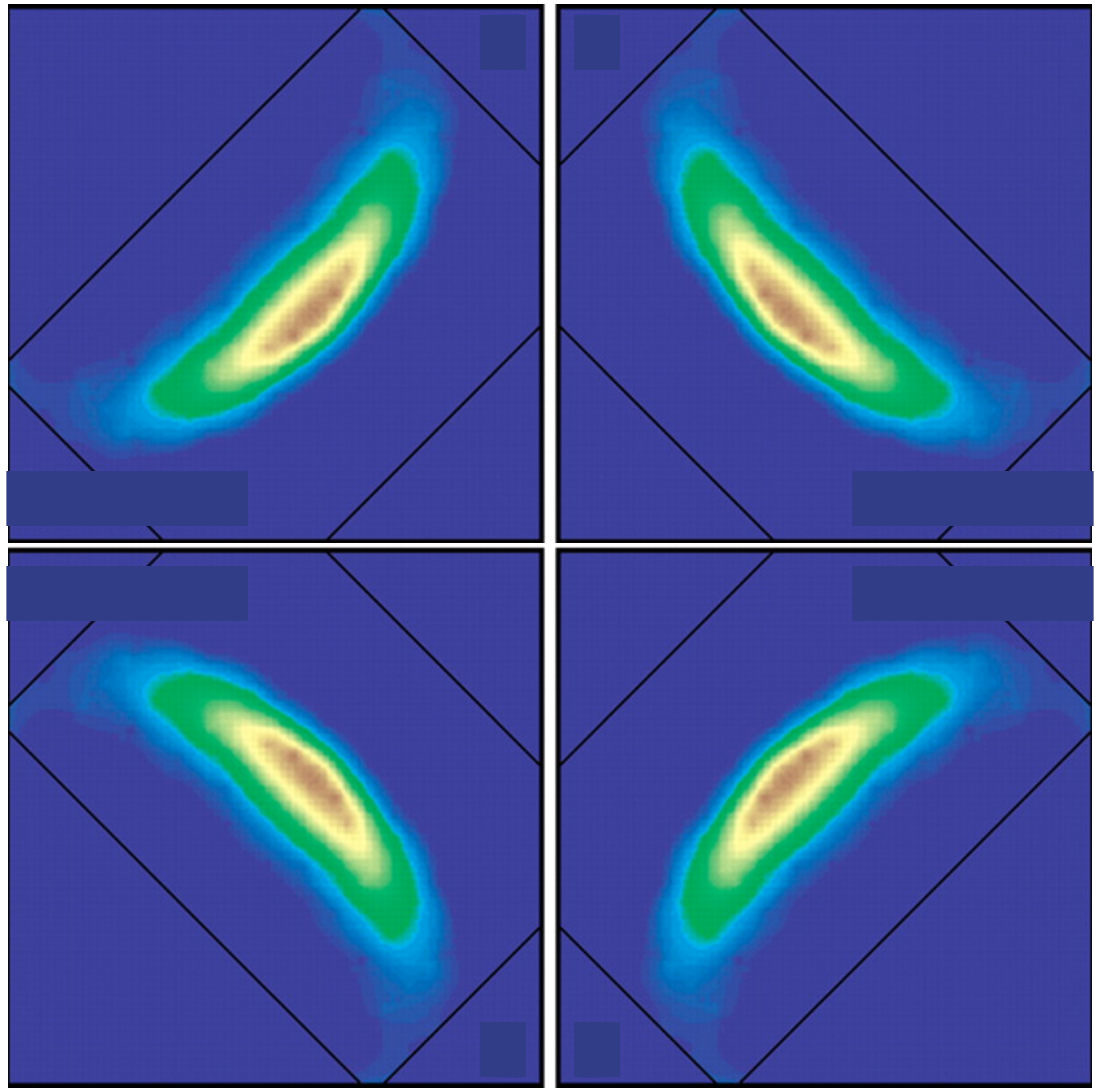


FL*: Condensate B breaks gauge symmetries in first ancilla layer.

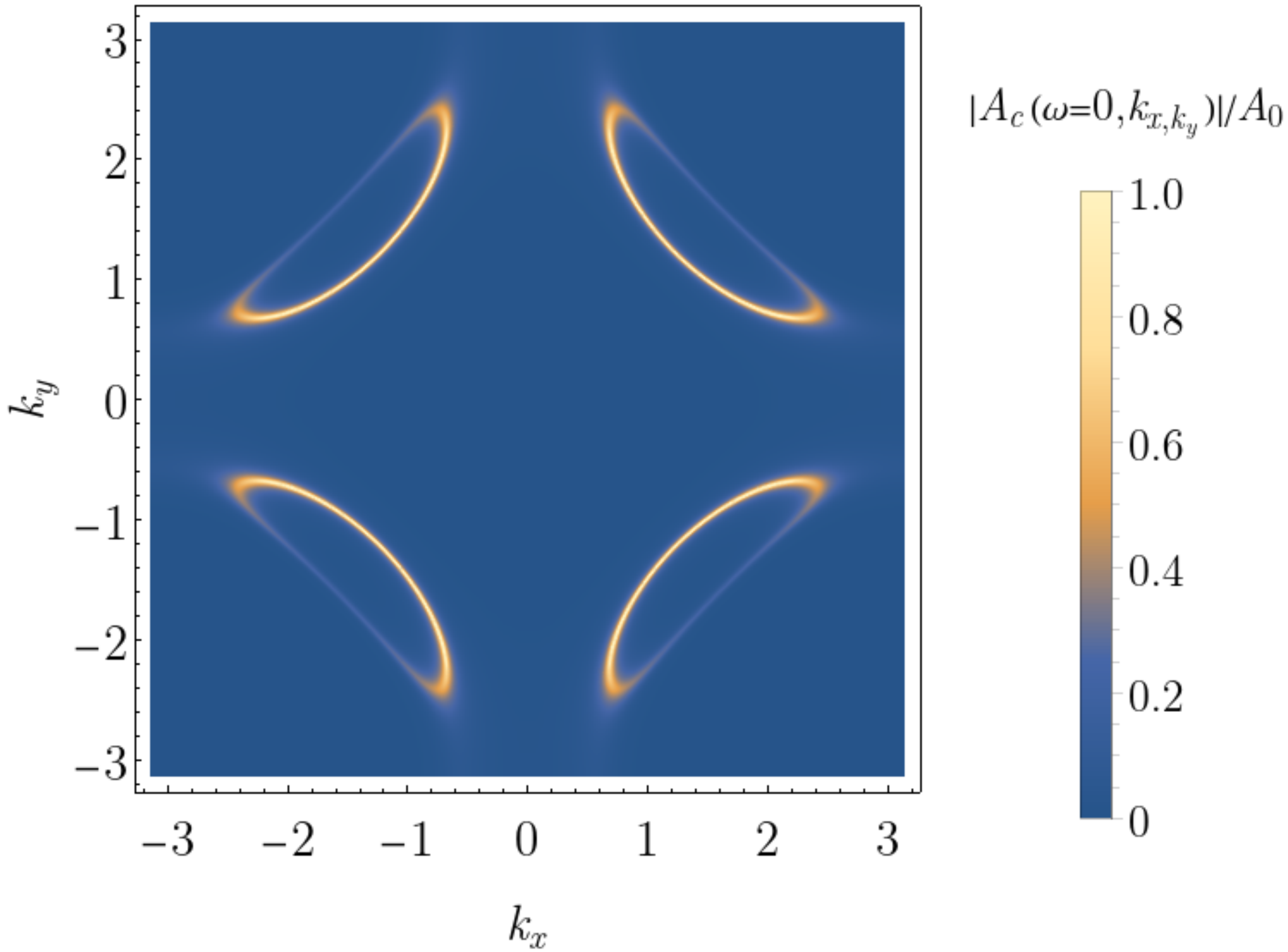
$$H = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} + \sum_i B\left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_\sigma\right)$$

Precursors:
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
Yang Qi, SS, PRB **81**, 115129 (2010)
Eun-Gook Moon, SS, PRB **83**, 224508 (2011)

Photoemission at small p



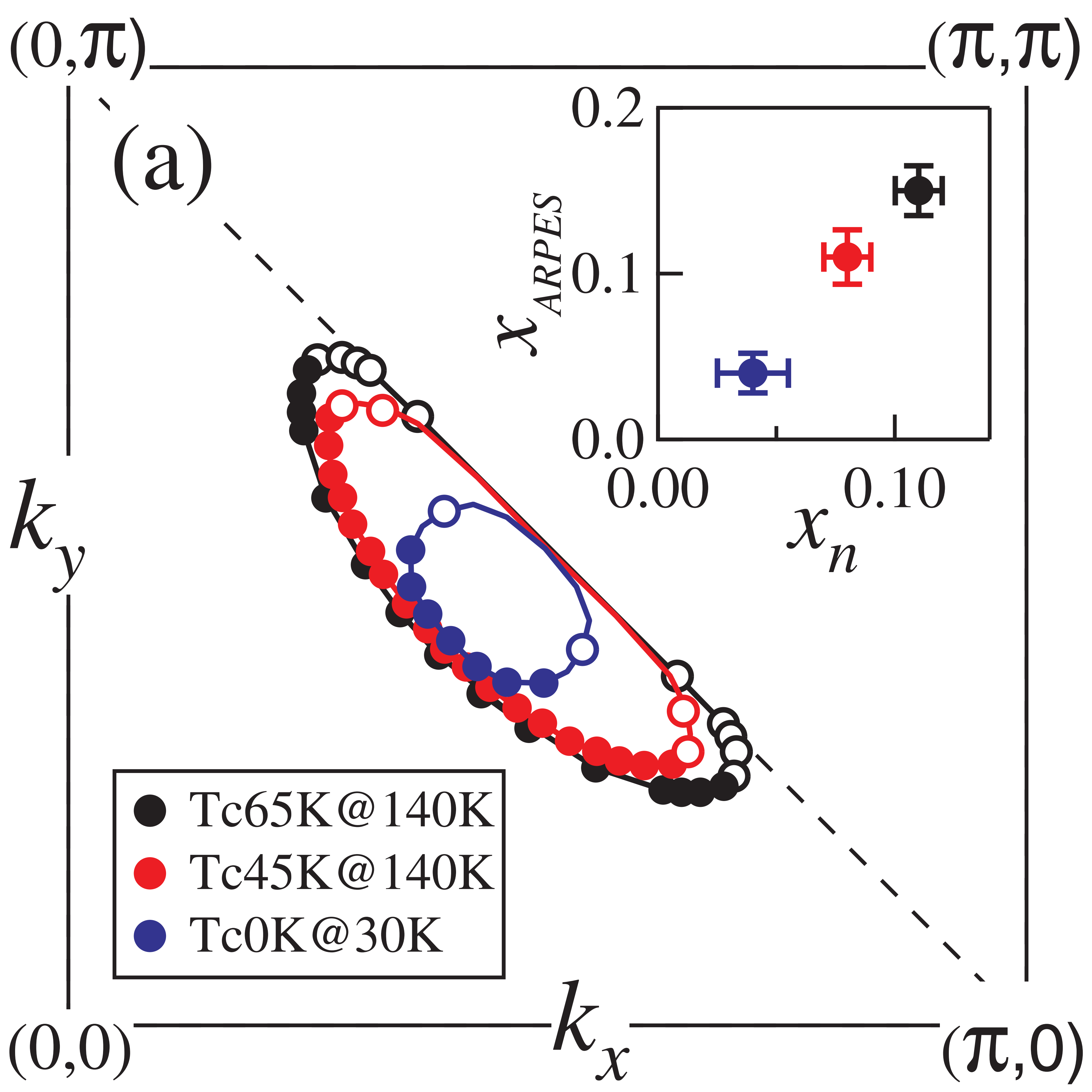
$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

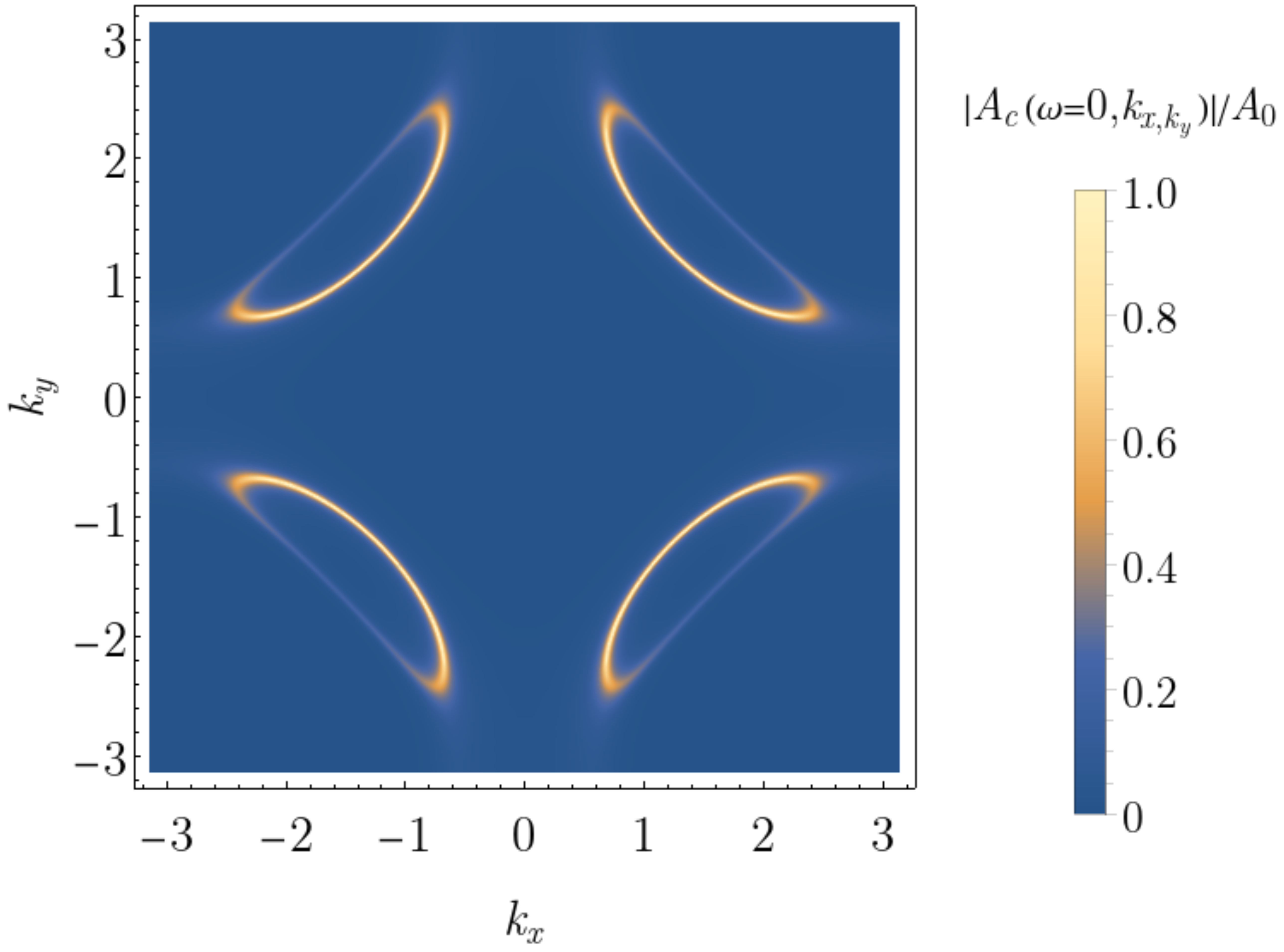


"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small p

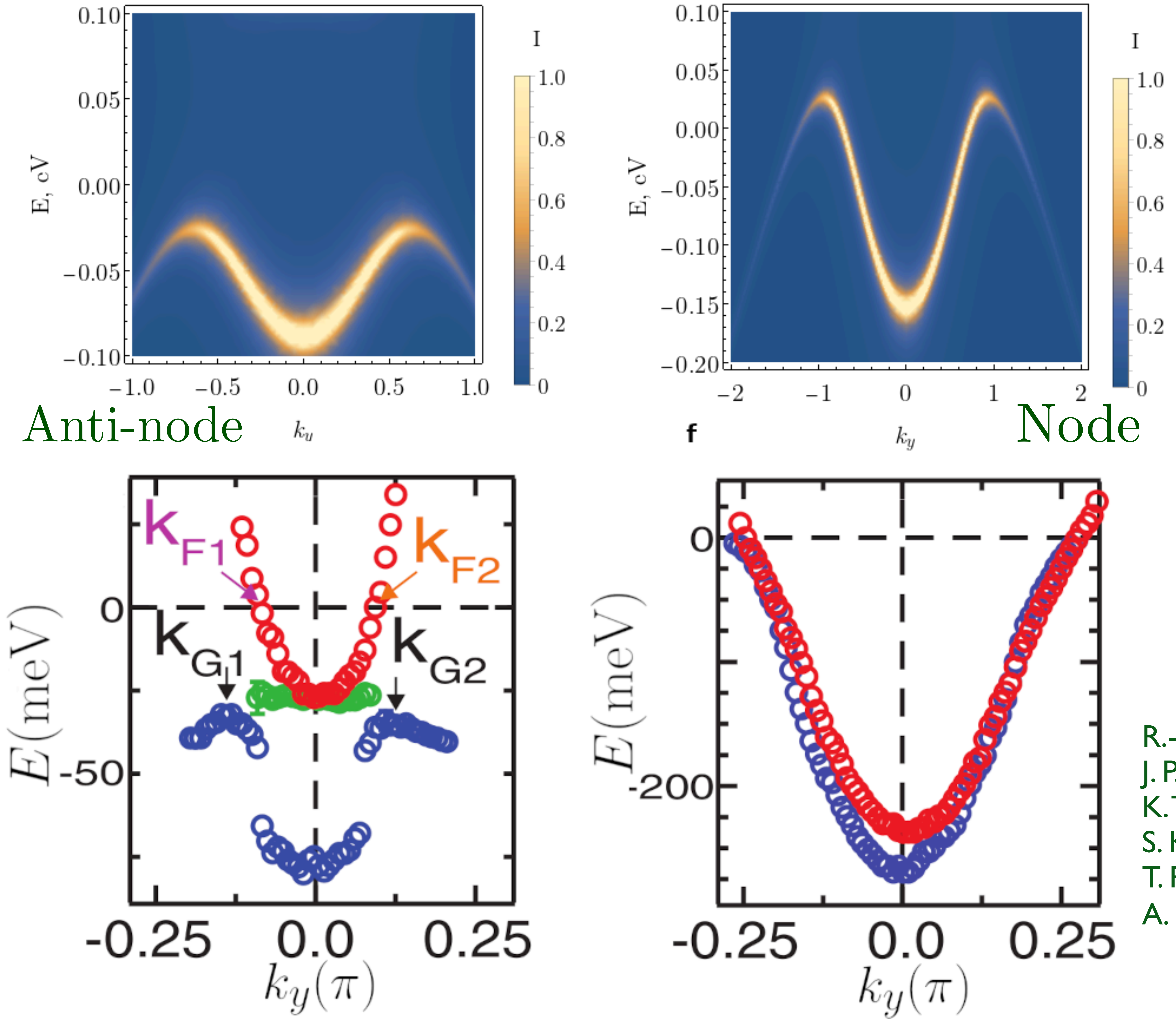




"Fermi pockets"

Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

FL* in a *one-band* model

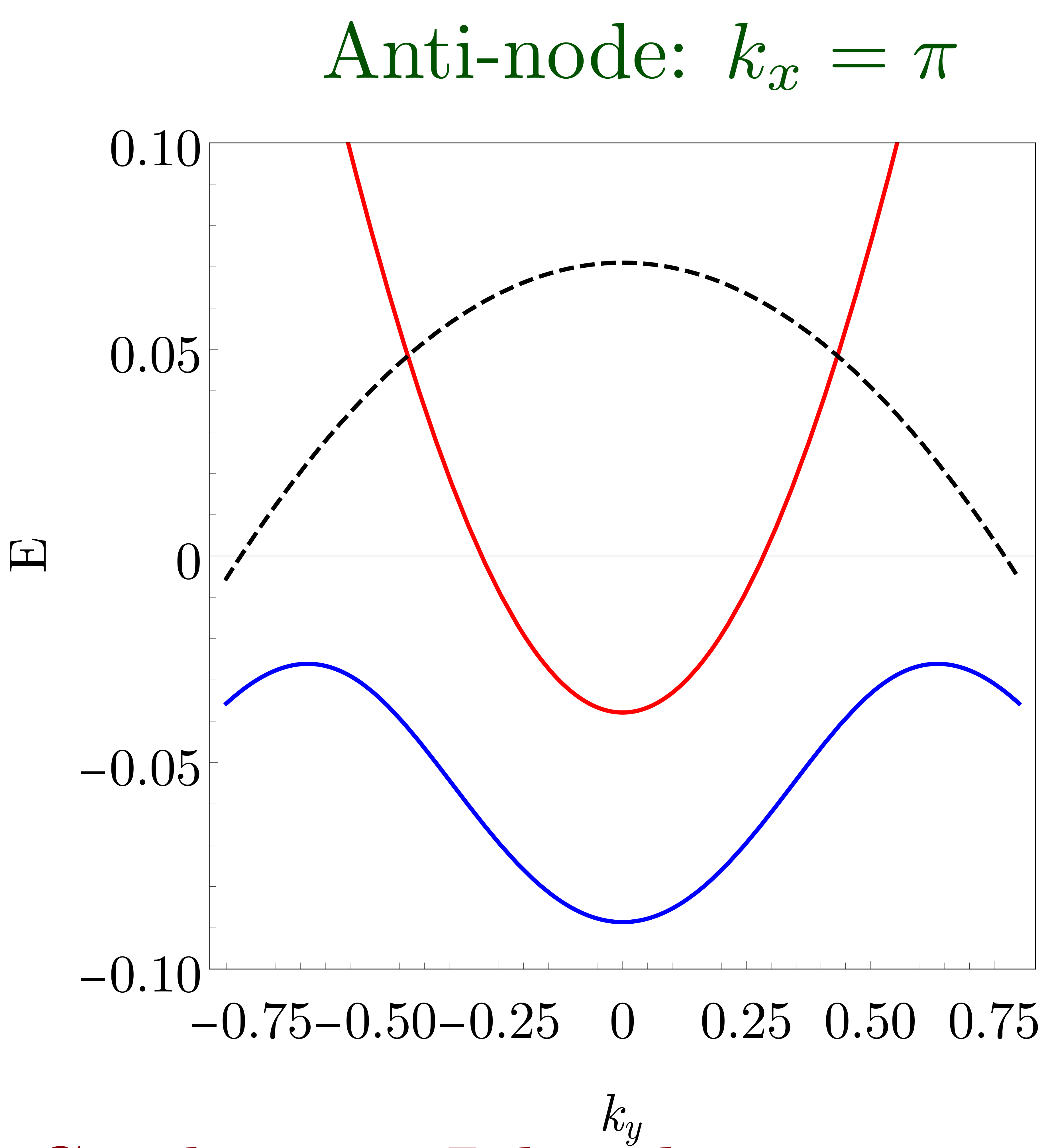


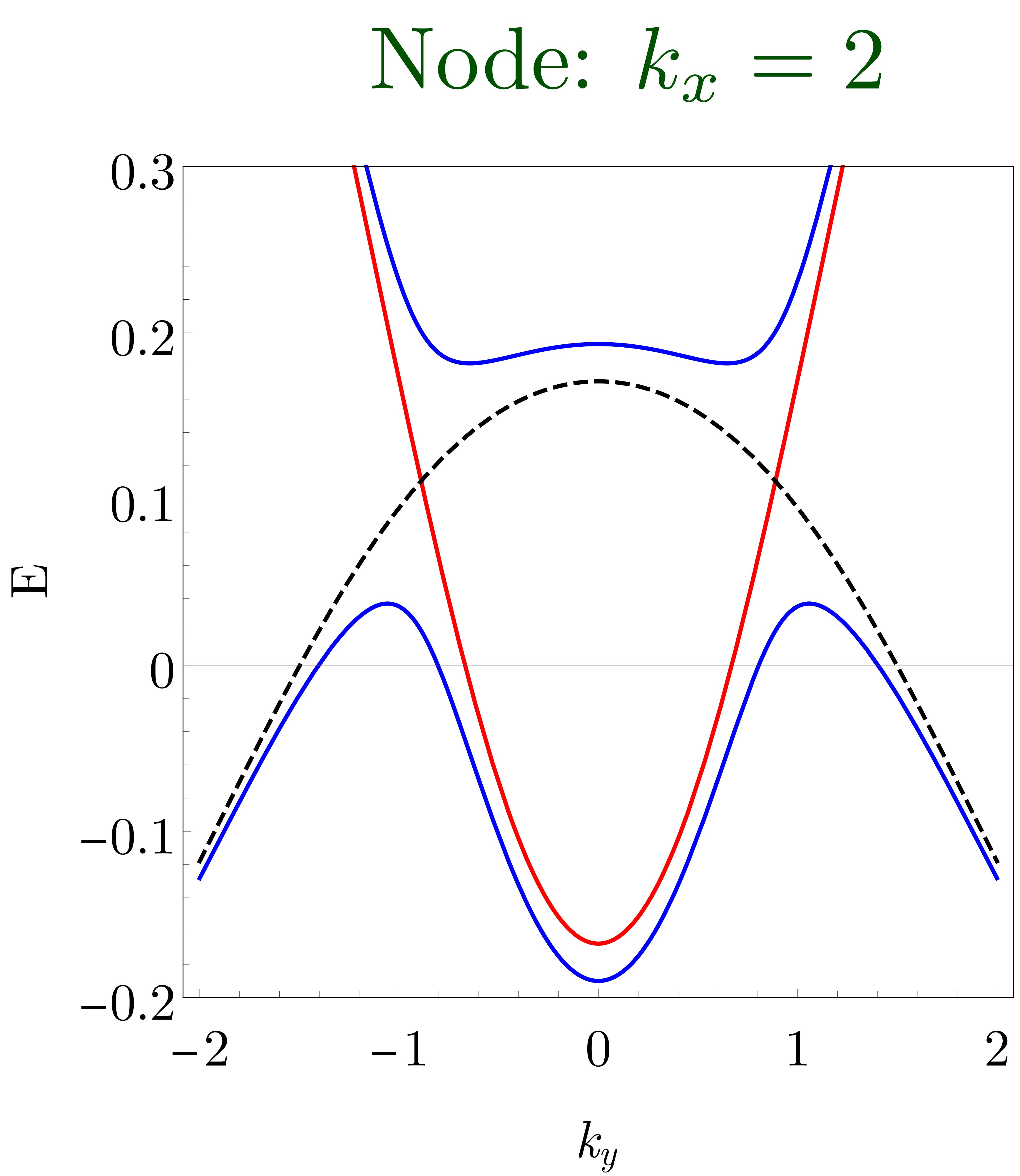
ARPES on Bi2201

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

FL* in a ***one-band*** model

Electronic dispersion





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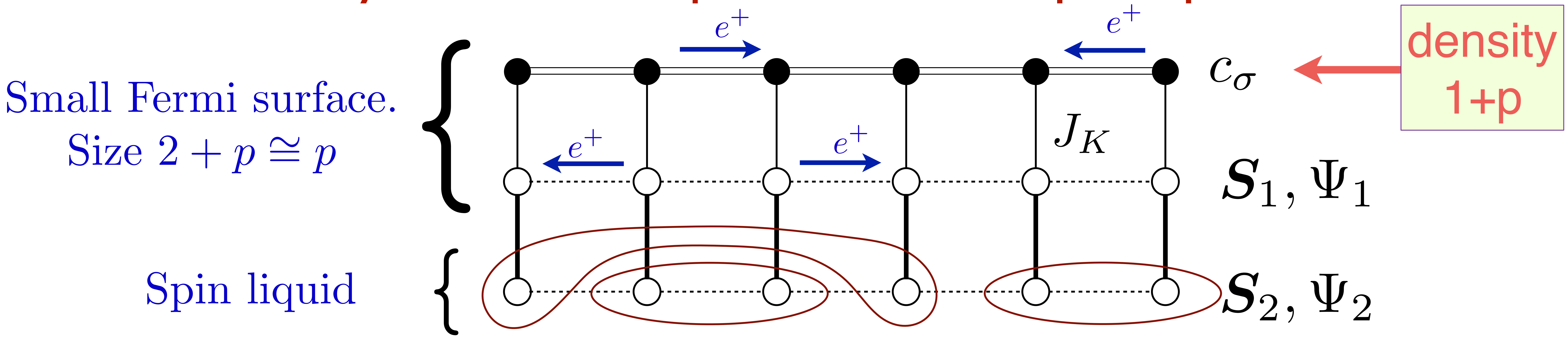
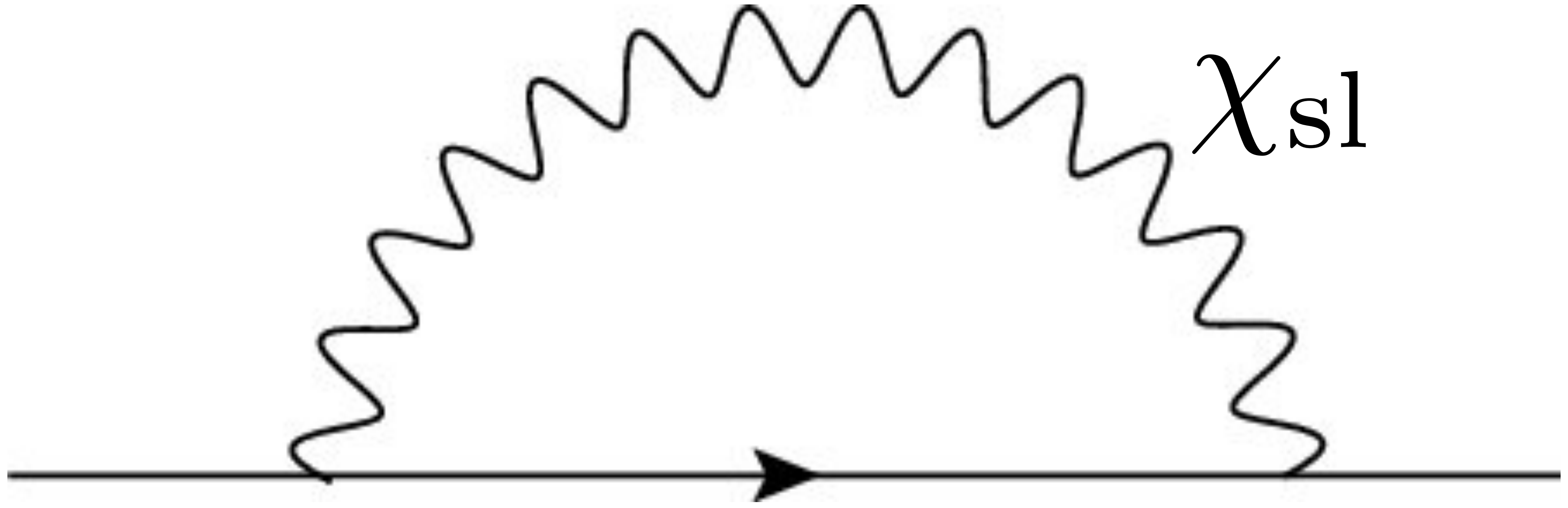
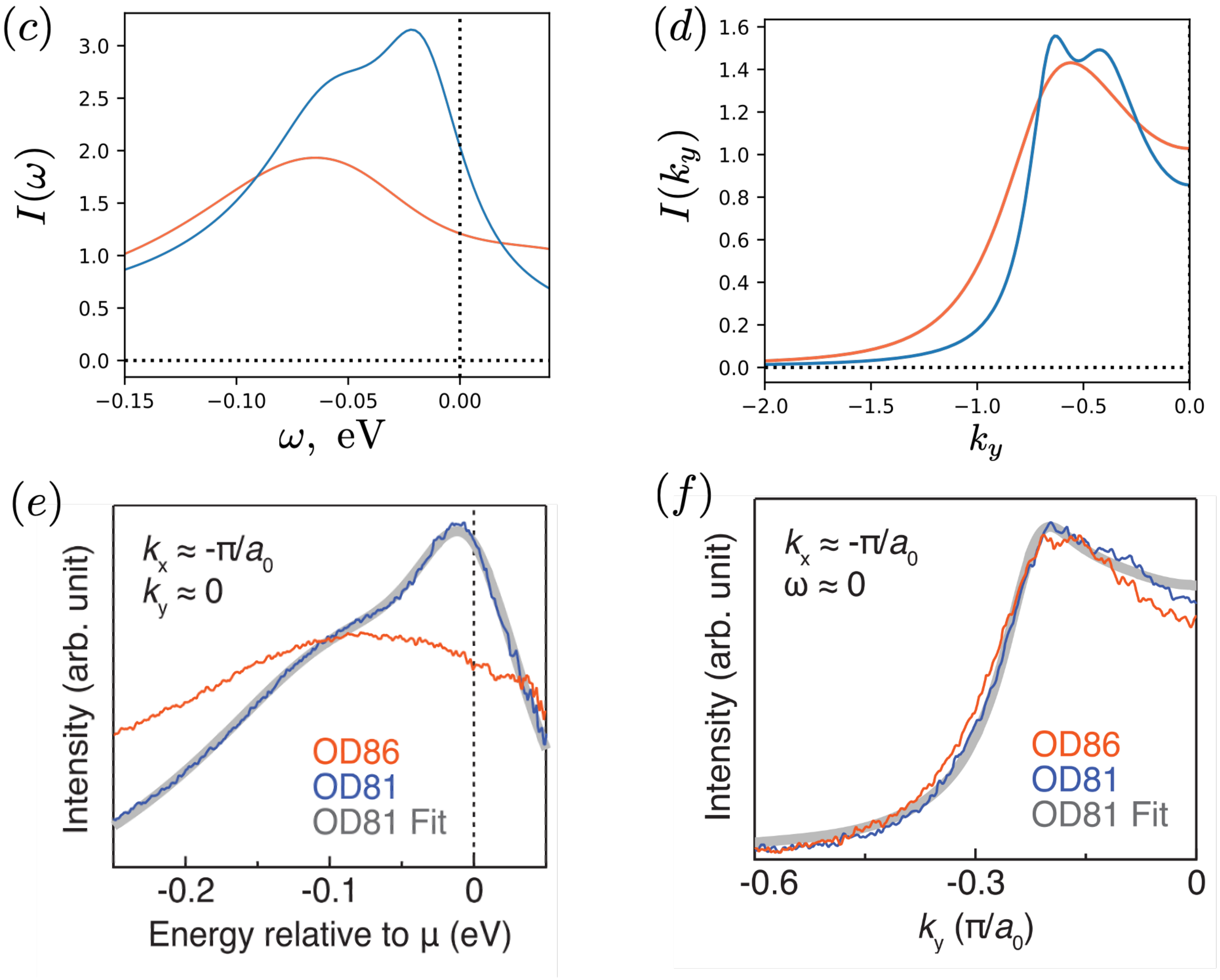
Eun-Gook Moon, SS, PRB **83**, 224508 (2011)

Dynamic consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2. We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility $\chi_{\rm sl}$.





Antinodal EDC and MDC

(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless $\mathbb{Z}_2$ spin liquid

(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

Summary

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  Don't fractionalize the mobile electron, but fractionalize the paramagnon into 'ancilla qubits'.
  Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

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  Don't fractionalize the mobile electron, but fractionalize the paramagnon into 'ancilla qubits'.
  Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.

- Outlook:

  – 'Back side' of hole pockets may be observable in cleaner samples.

  – Theory for single electron pocket in confining CDW ordered phase, as observed in quantum oscillations.

  – Neutron scattering observation of excitations of spin liquid in second ancilla layer.

  – Theory for FL*-FL transition with ghost Fermi surfaces—strange metal?