Many-fermion quantum entanglement in strange metals and in black holes

University of California, Irvine

May 21, 2026

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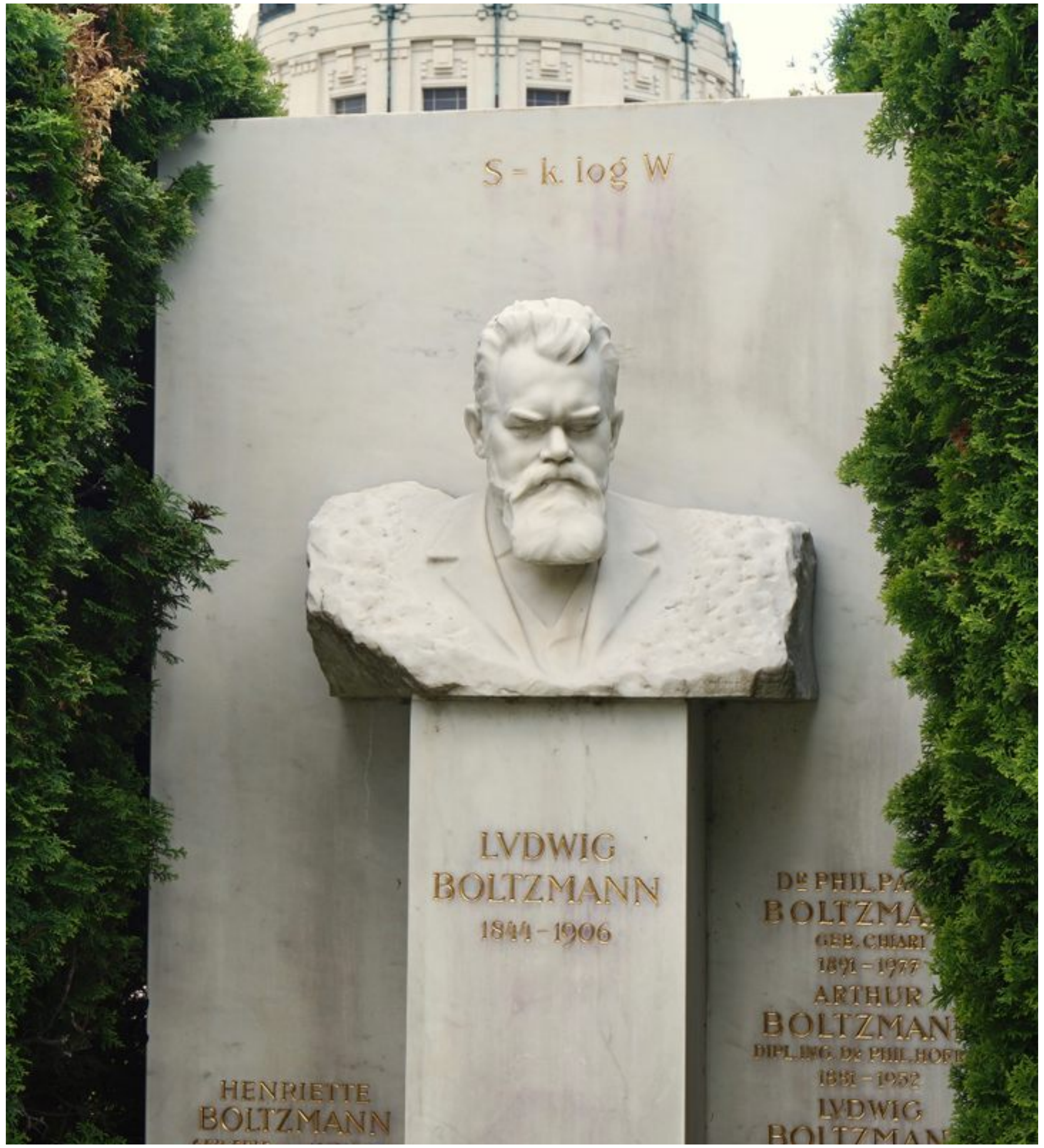
PHYSICS

HARVARD

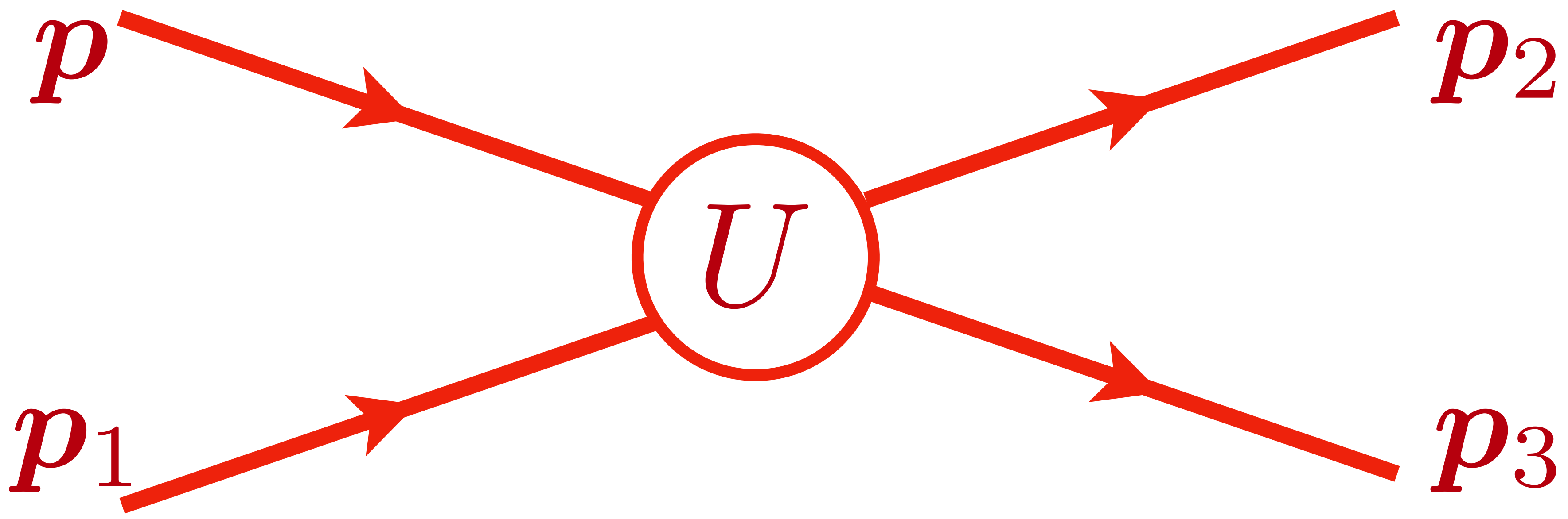
Boltzmann-Landau theory of ordinary metals: Cu, Ag

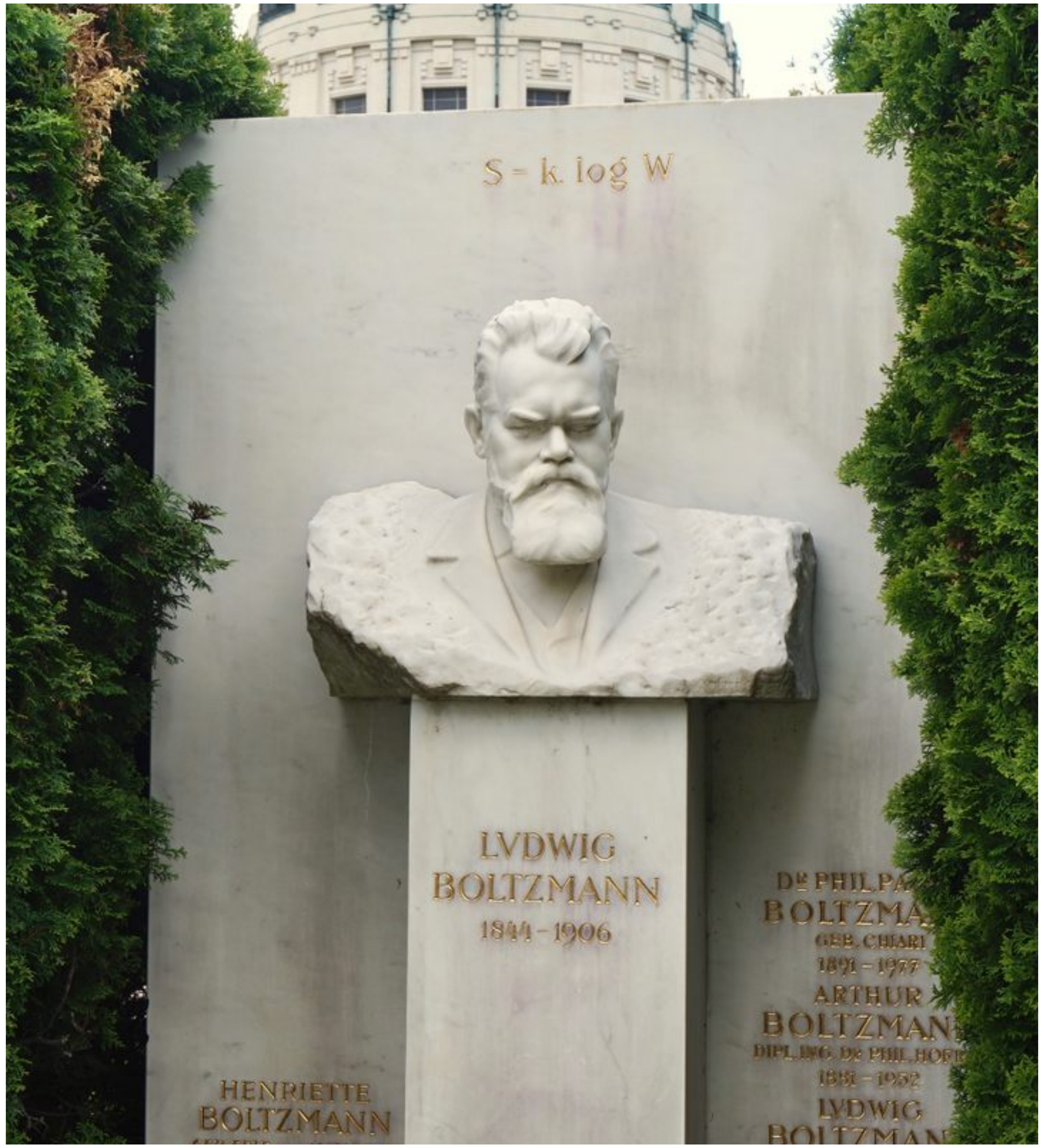
Boltzmann equation (1872)
Dilute classical gas



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Molecular chaos:
successive collisions are rare and statistically independent





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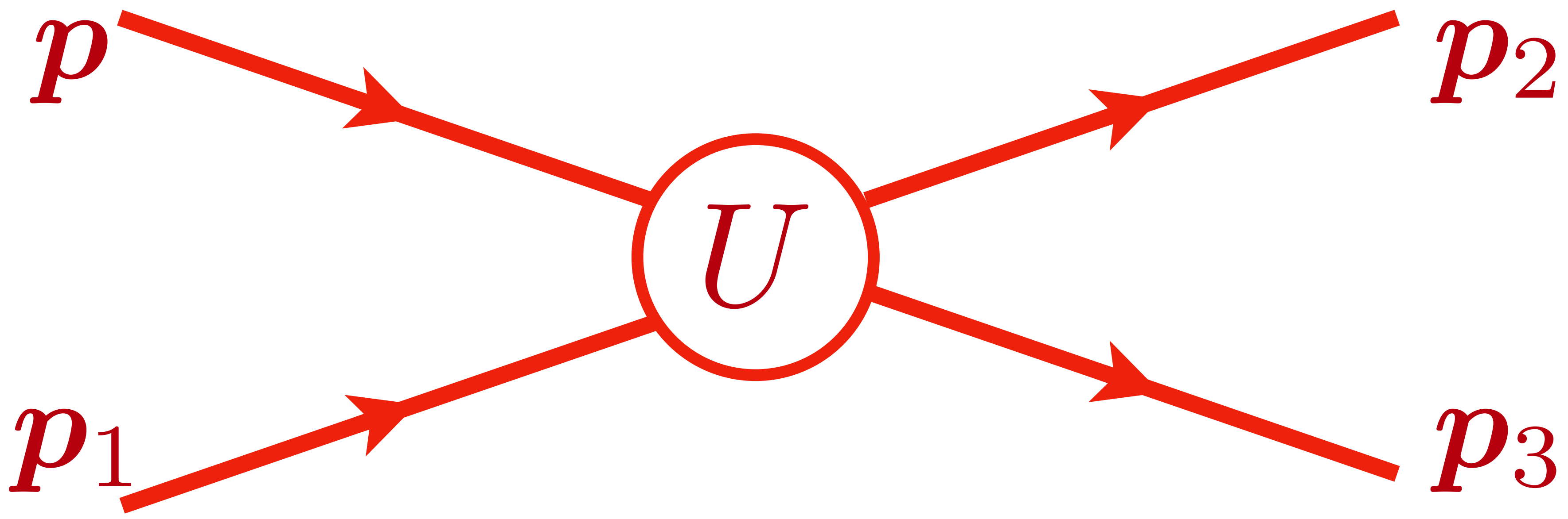
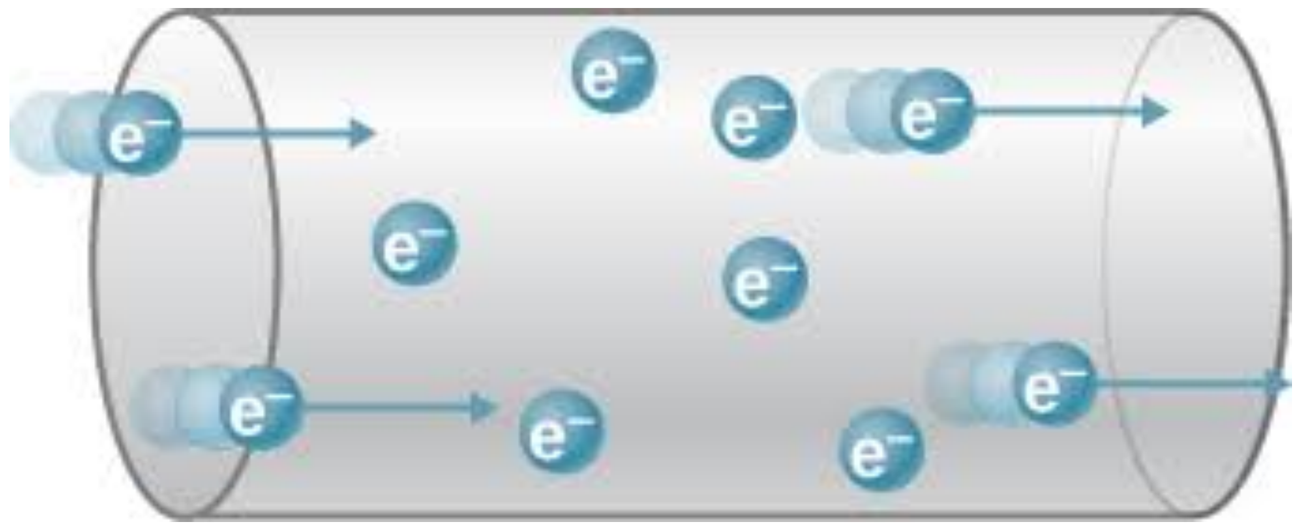
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Collisions are also rare in a dense quantum gas at low temperatures because of the Pauli exclusion principle.

Neglect quantum interference (entanglement) between successive collisions

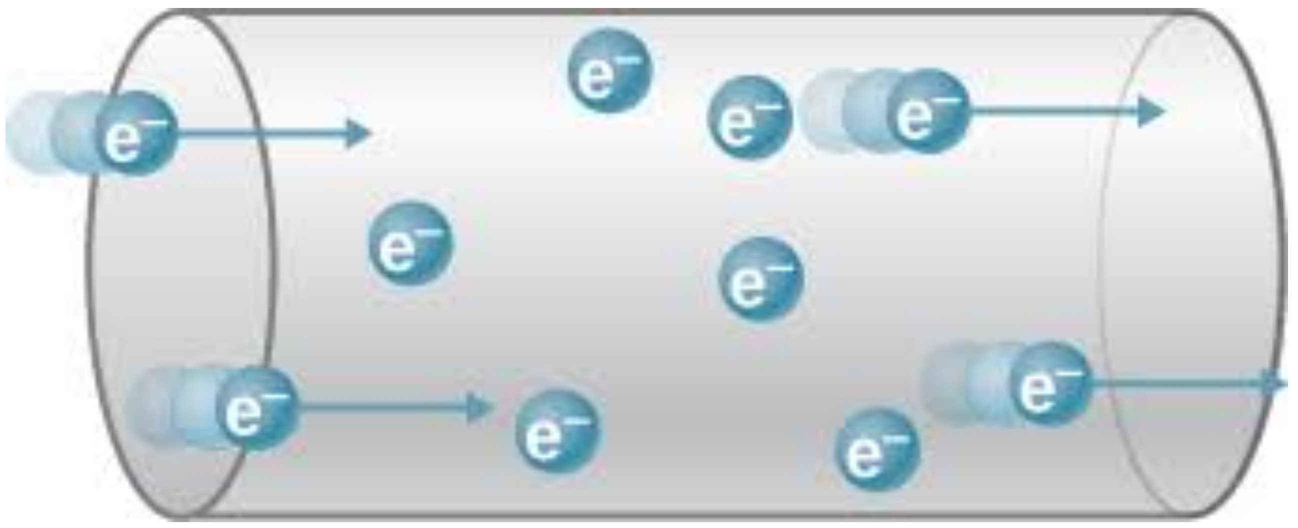


Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

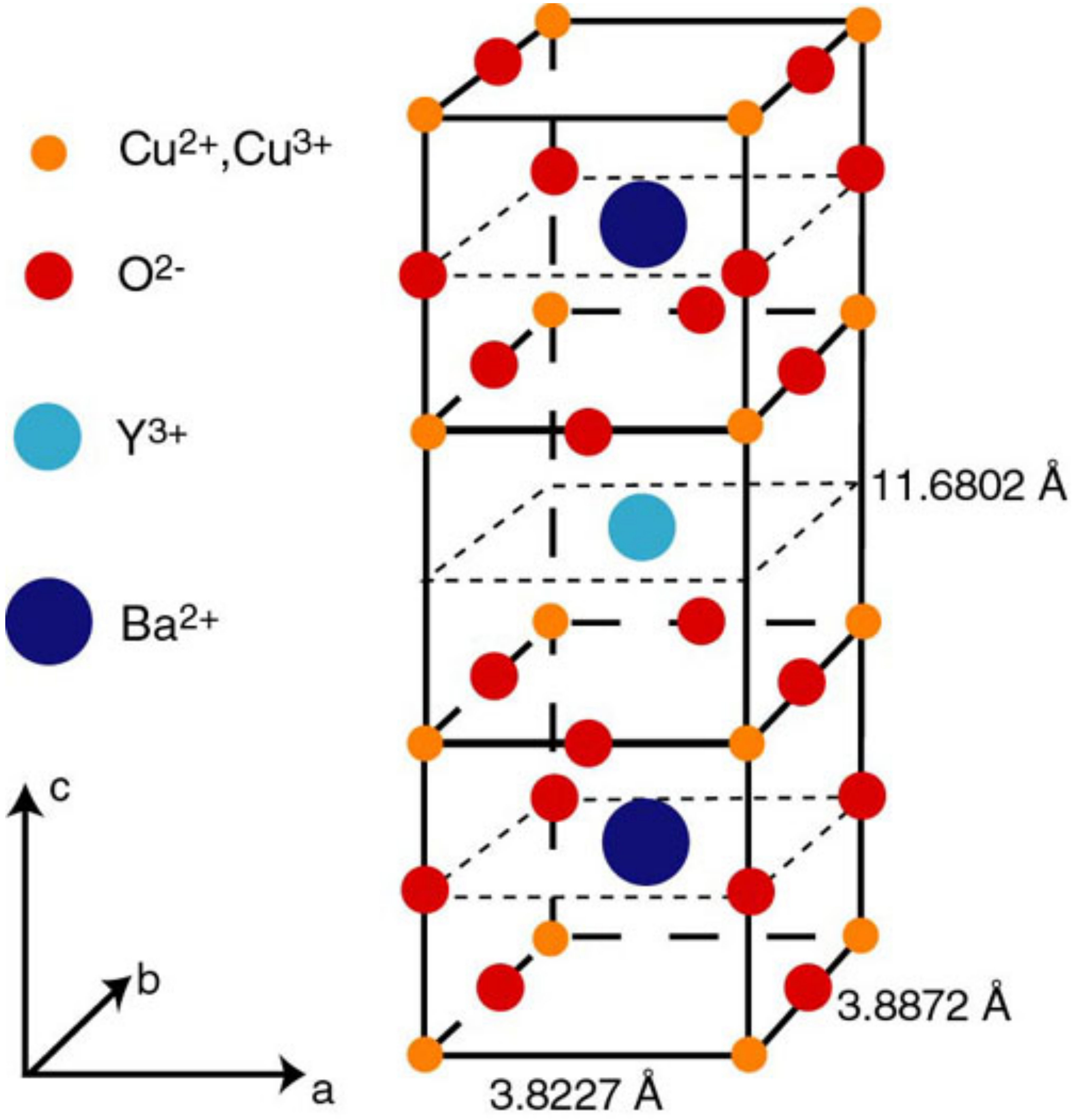
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

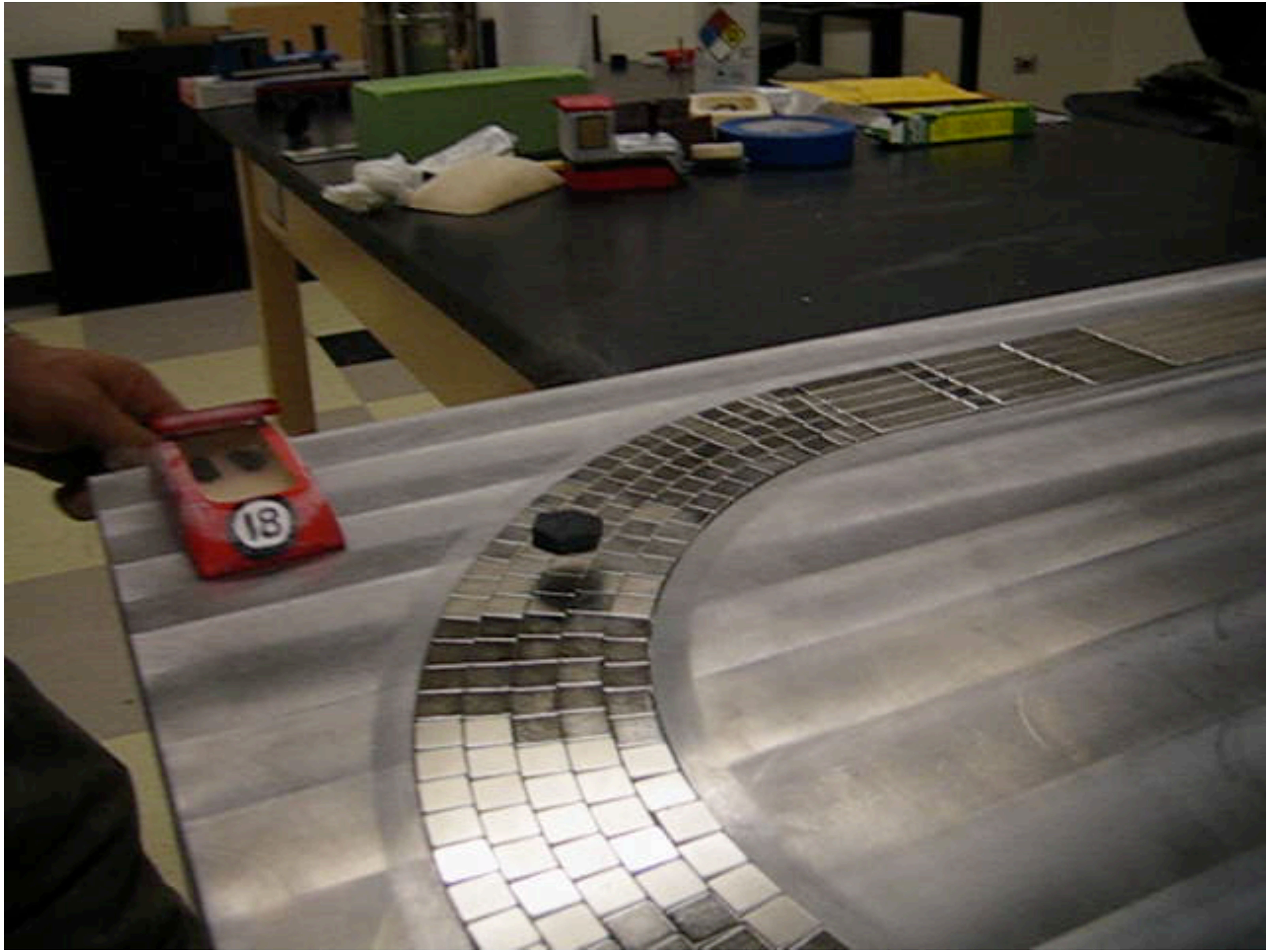
S. Sachdev, Quantum Phase Transitions (1999)

Strange metals:
the cuprates

Cuprate high temperature superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

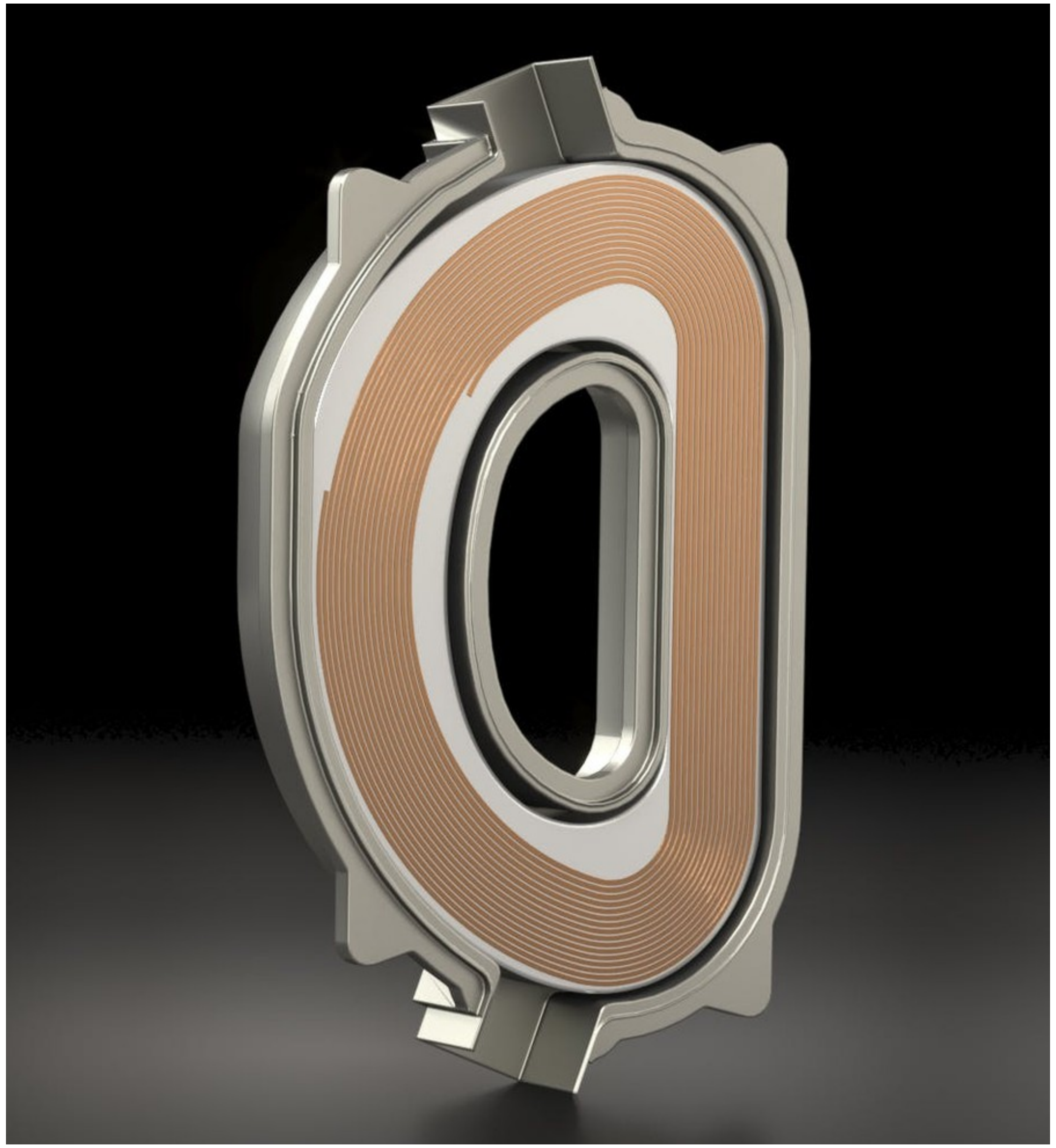
Julian Hetel and Nandini Trivedi, Ohio State University

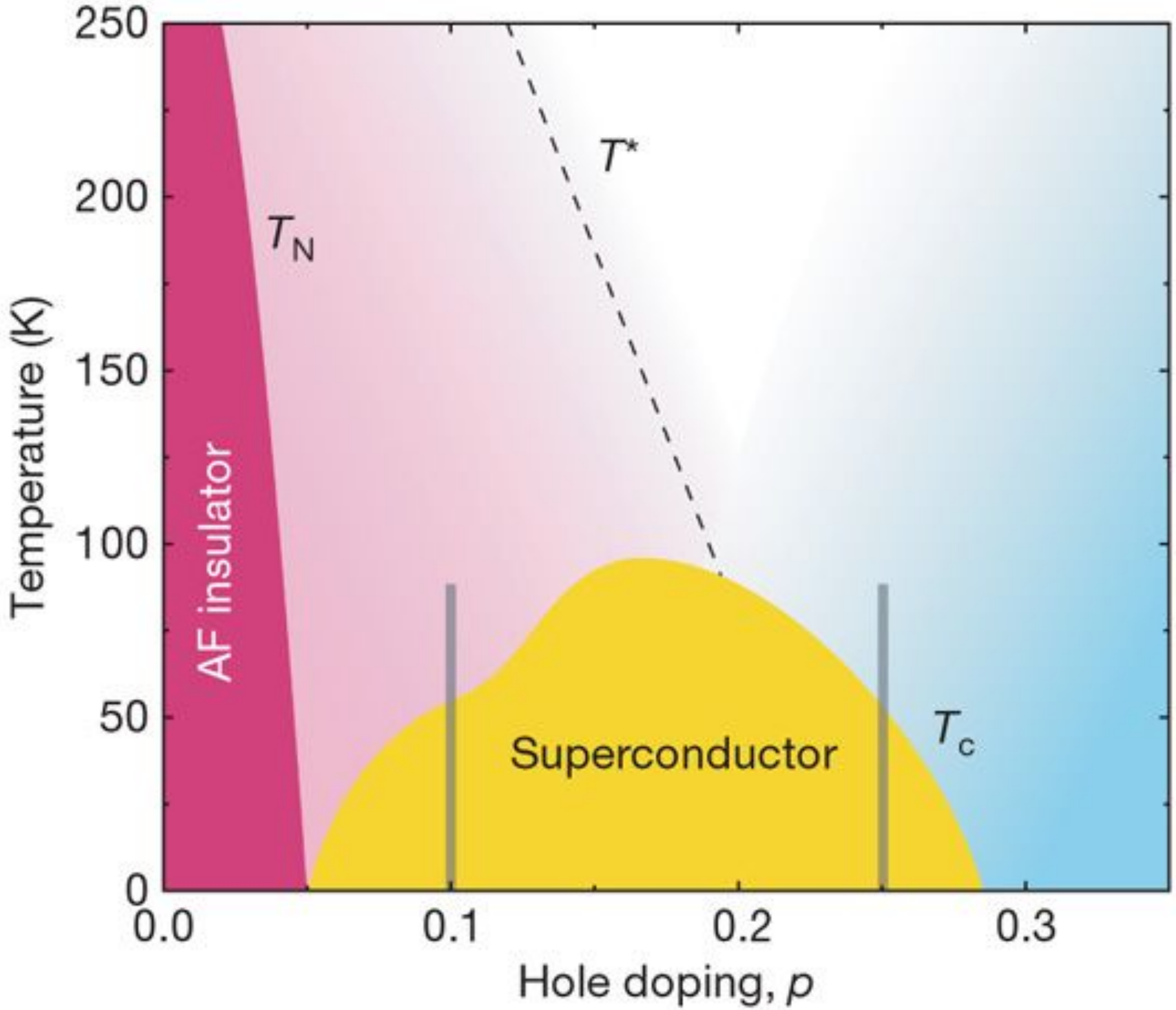
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





250
200
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0.0
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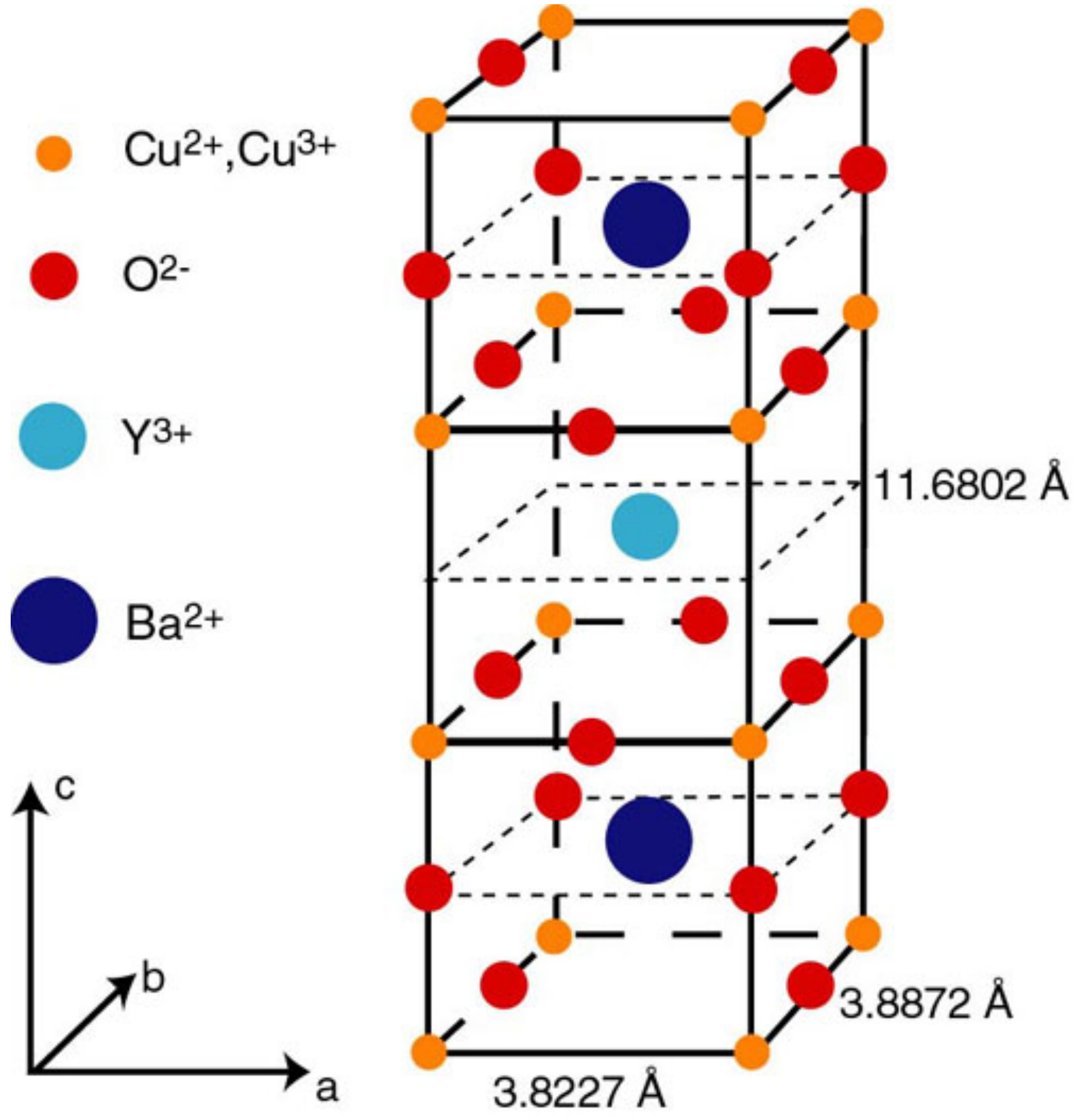
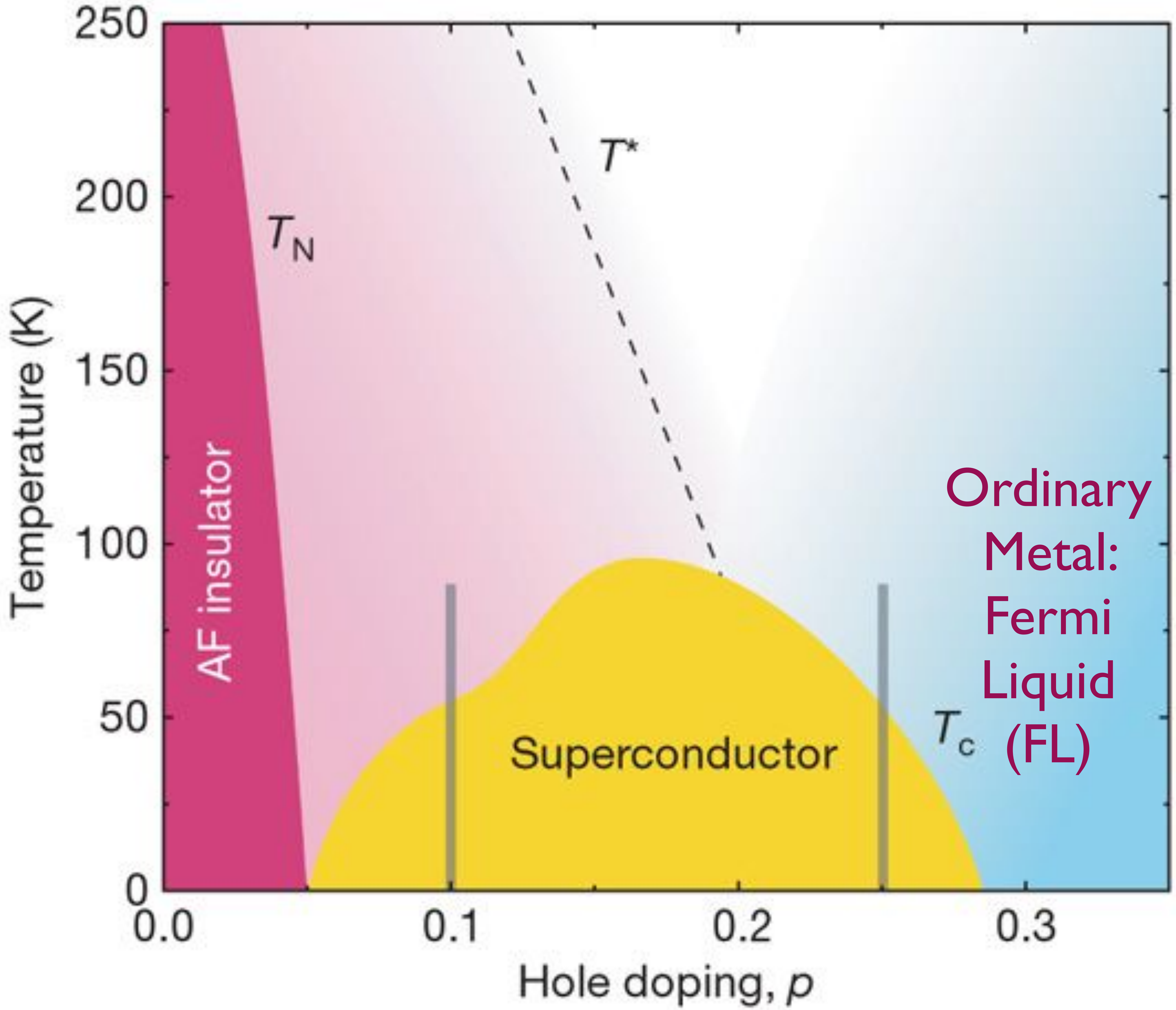
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



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Temperature (K)
Hole doping, p
AF insulator
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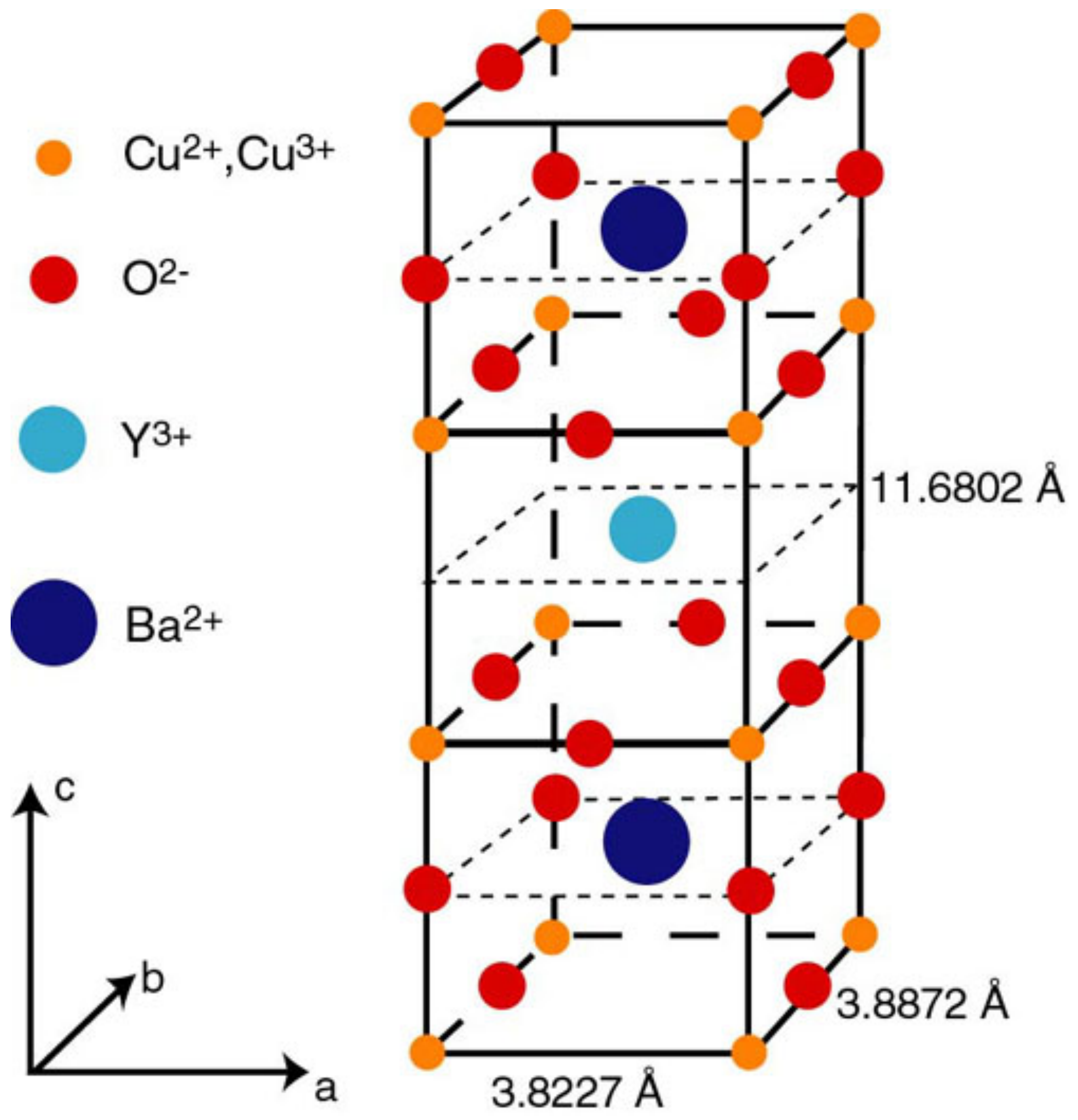
Ordinary Metal: Fermi Liquid (FL)



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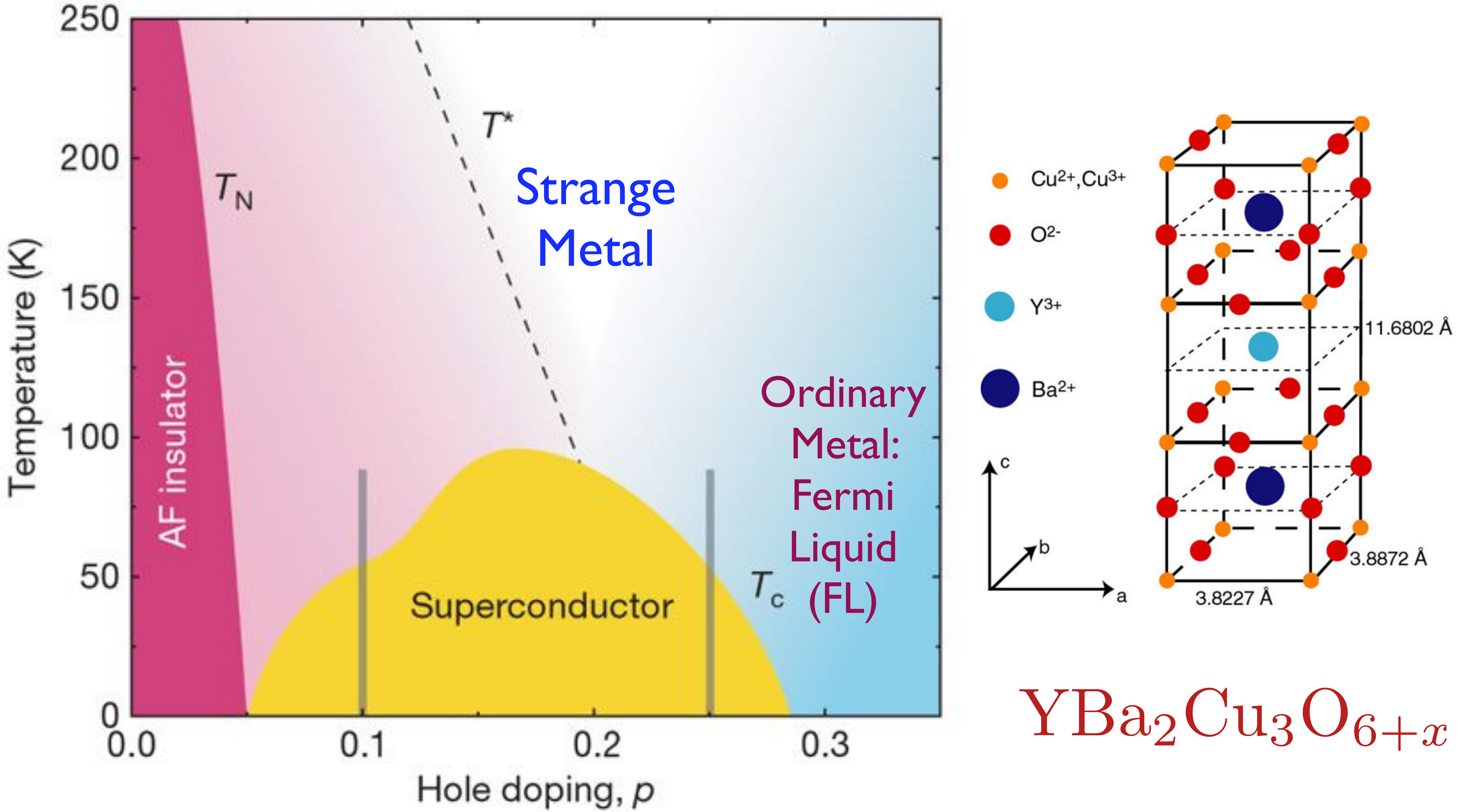
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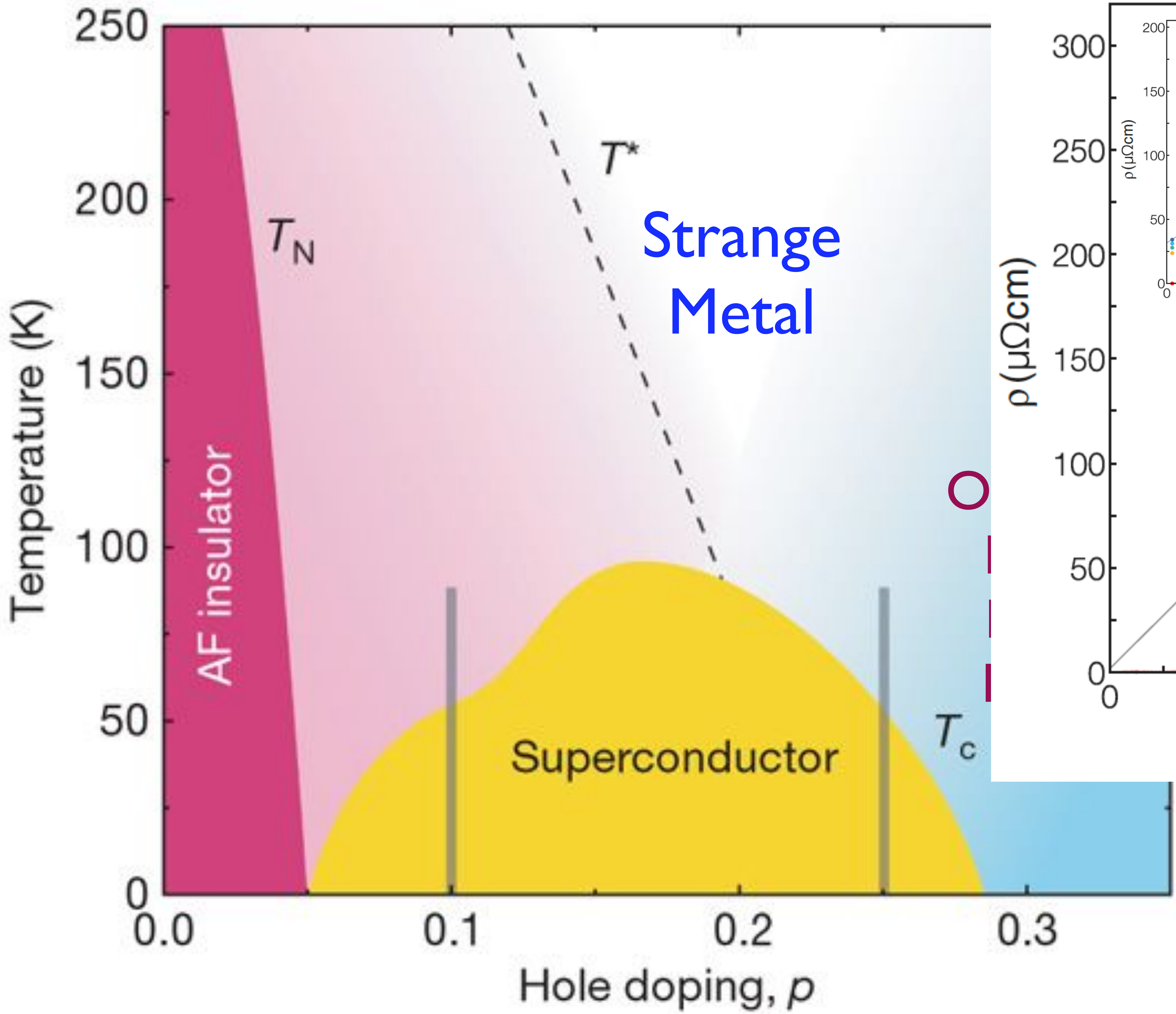
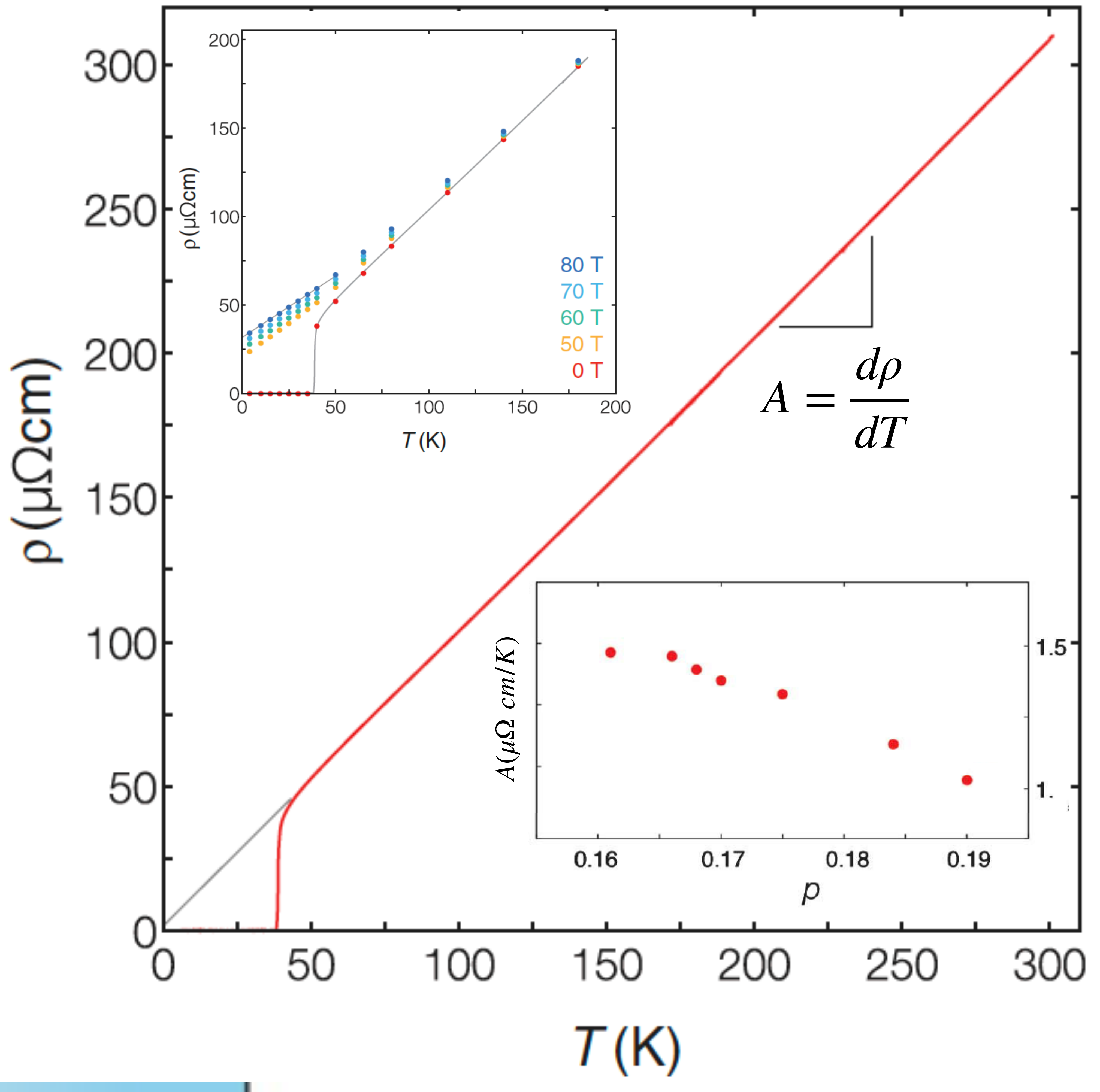




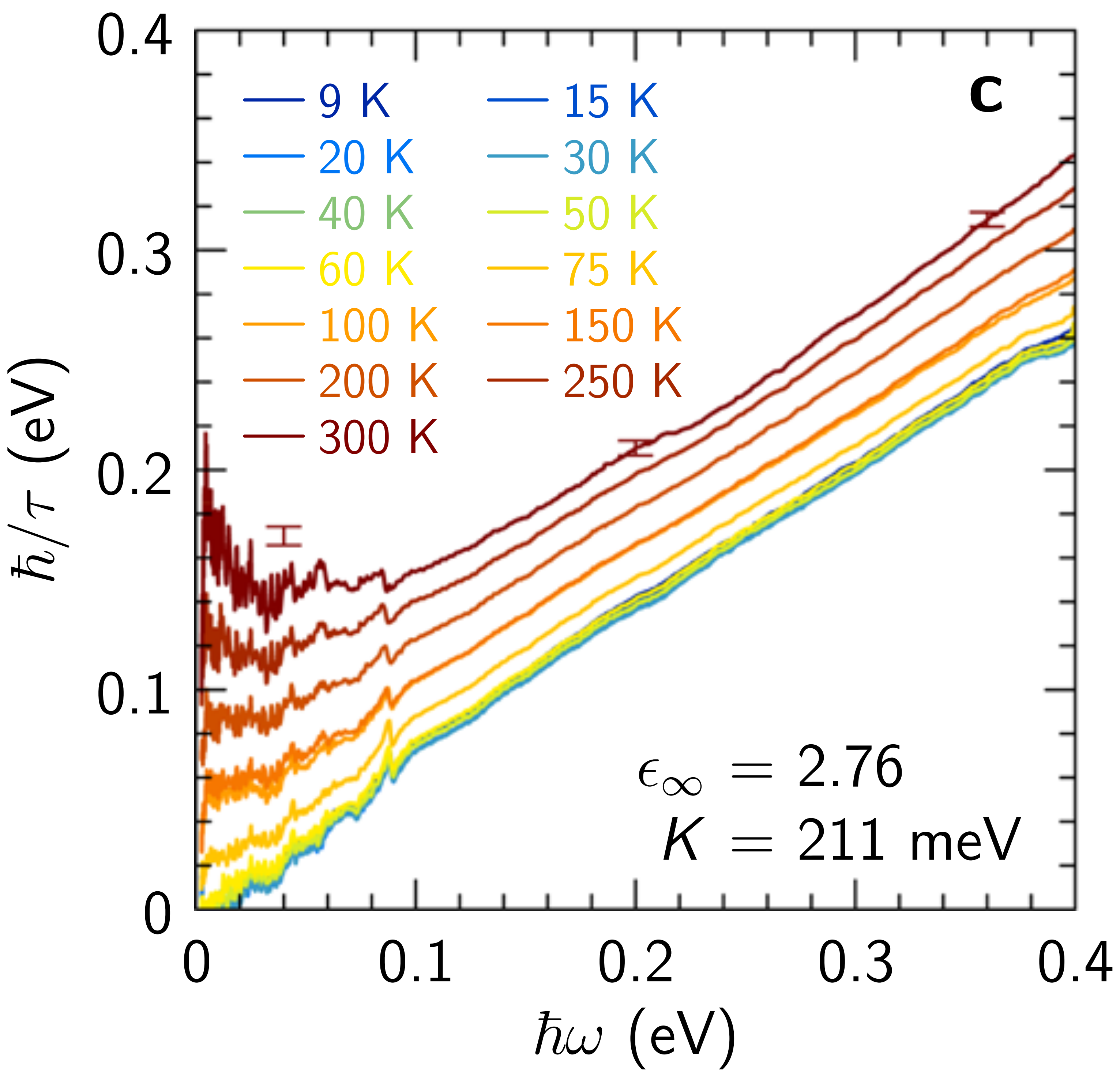
LSCO: Giraldo-Gallo et al. 2018

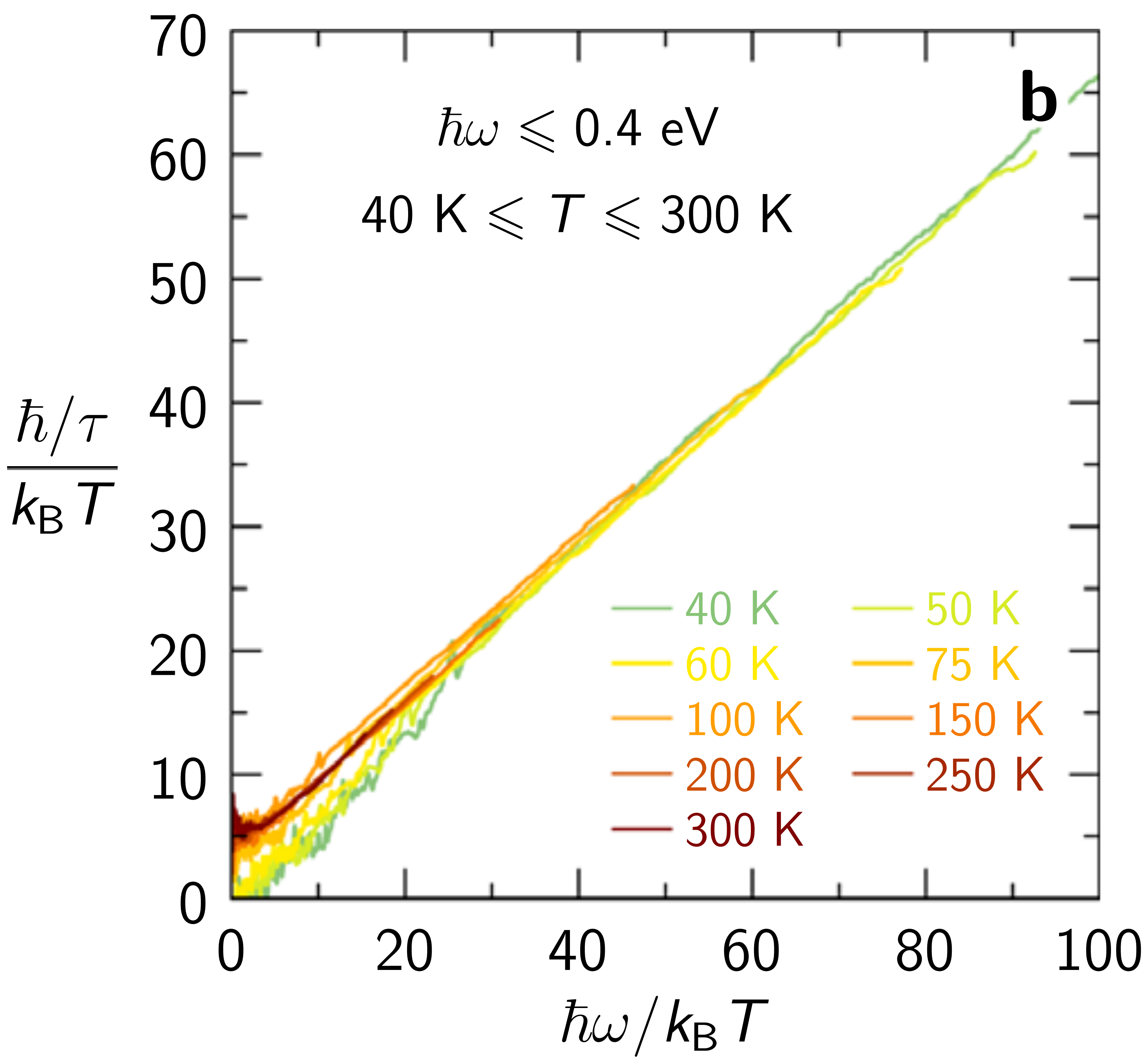
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

The time τ appears to be *independent* of interaction strength, contrary to Boltzmann.

Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

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What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

What is special about the strange metal in the hole-doped cuprates that leads to the highest temperature superconductivity?

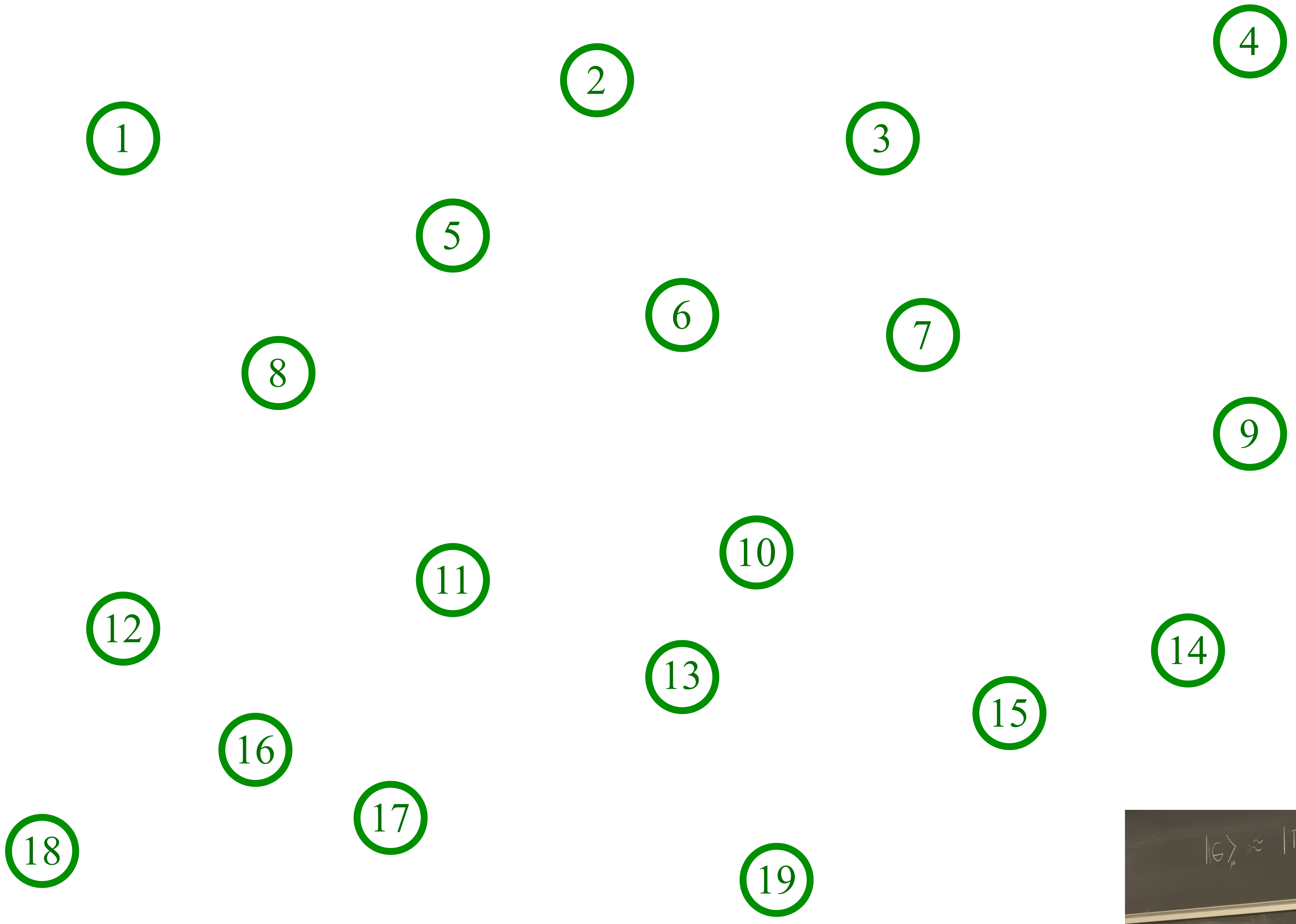
Much progress has been made in addressing these questions in last 3 decades.

But an unexpected bi-product has been a much deeper understanding of the quantum theory of many particles, which has impacted numerous other fields of physics, including the quantum theory of black holes, and quantum error correction

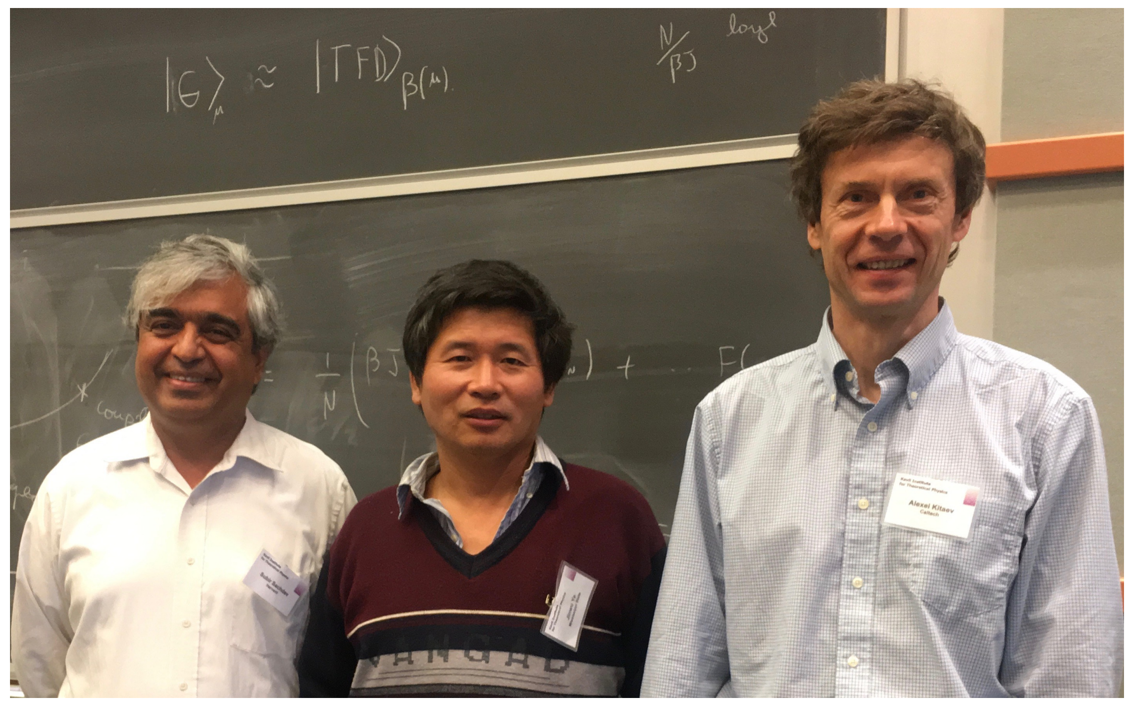
The Sachdev-Ye-Kitaev model: solvable Planckian dynamics

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

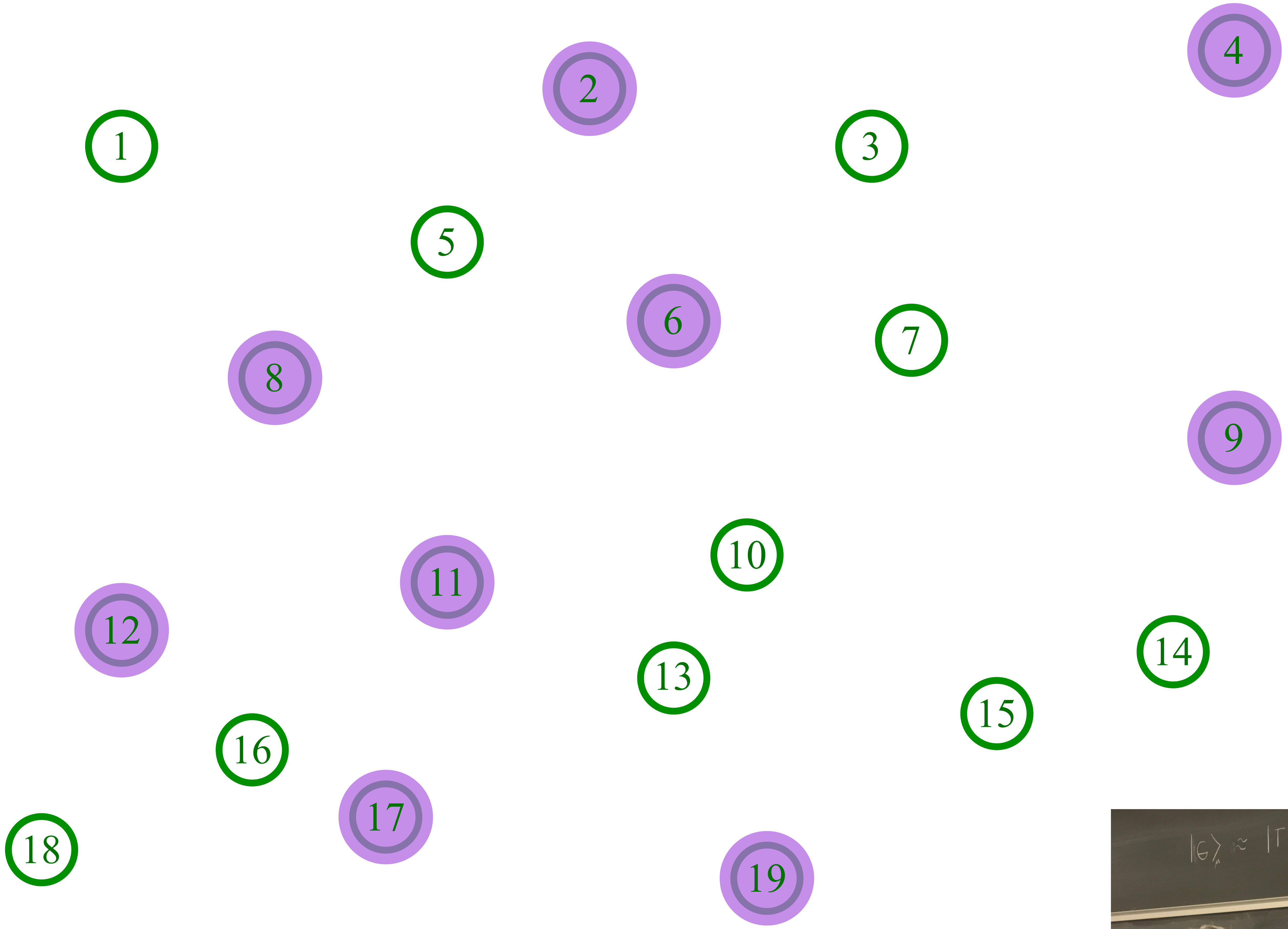


Pick a set of random positions

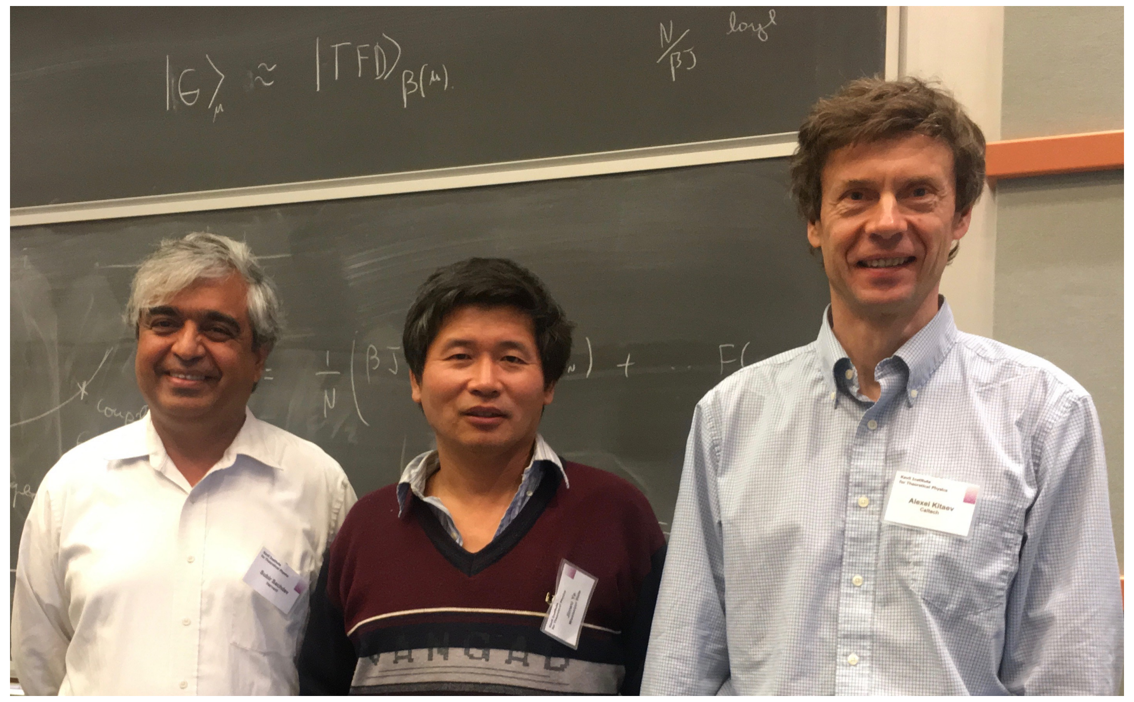


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Place electrons randomly on some sites



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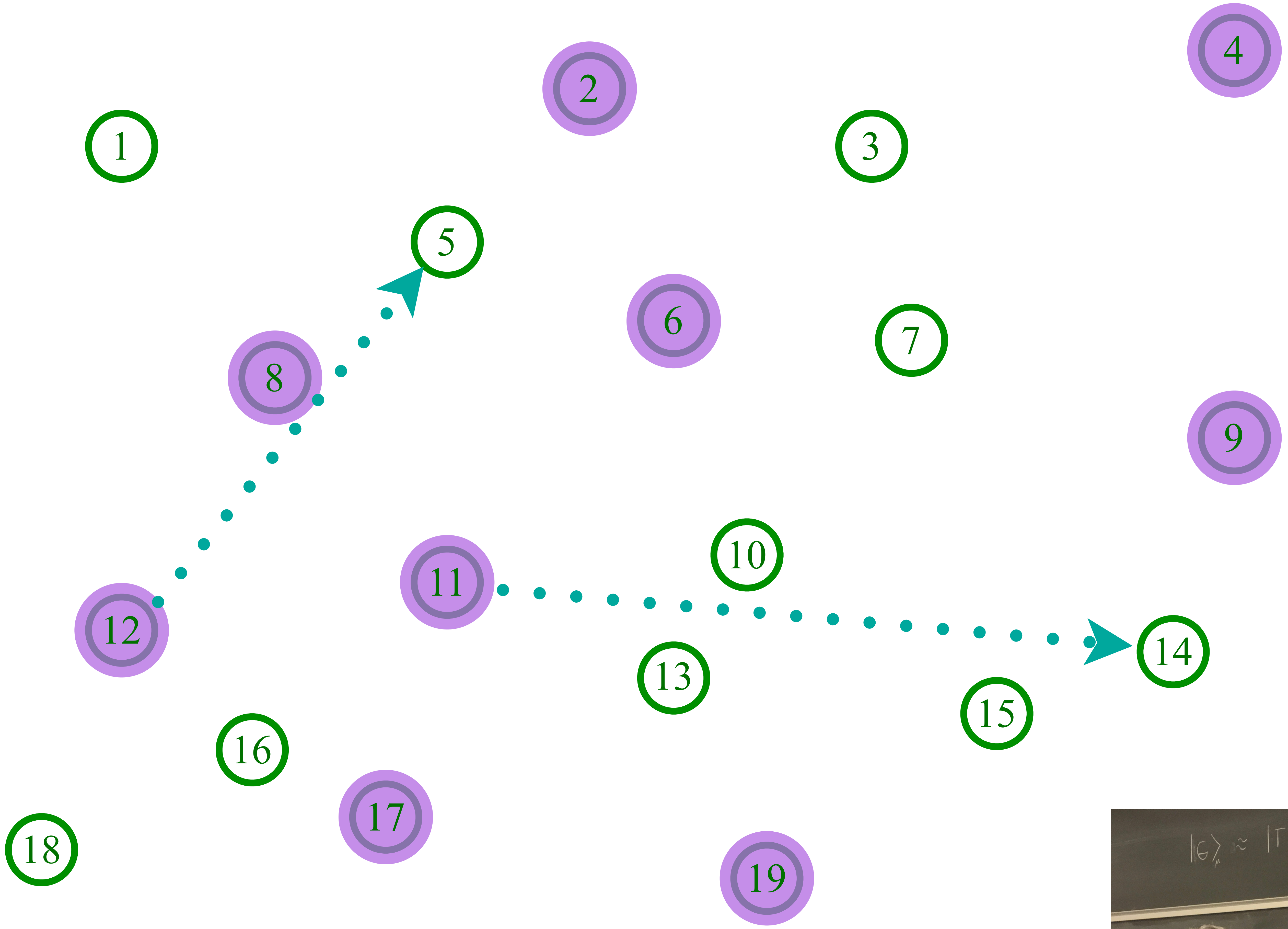
Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



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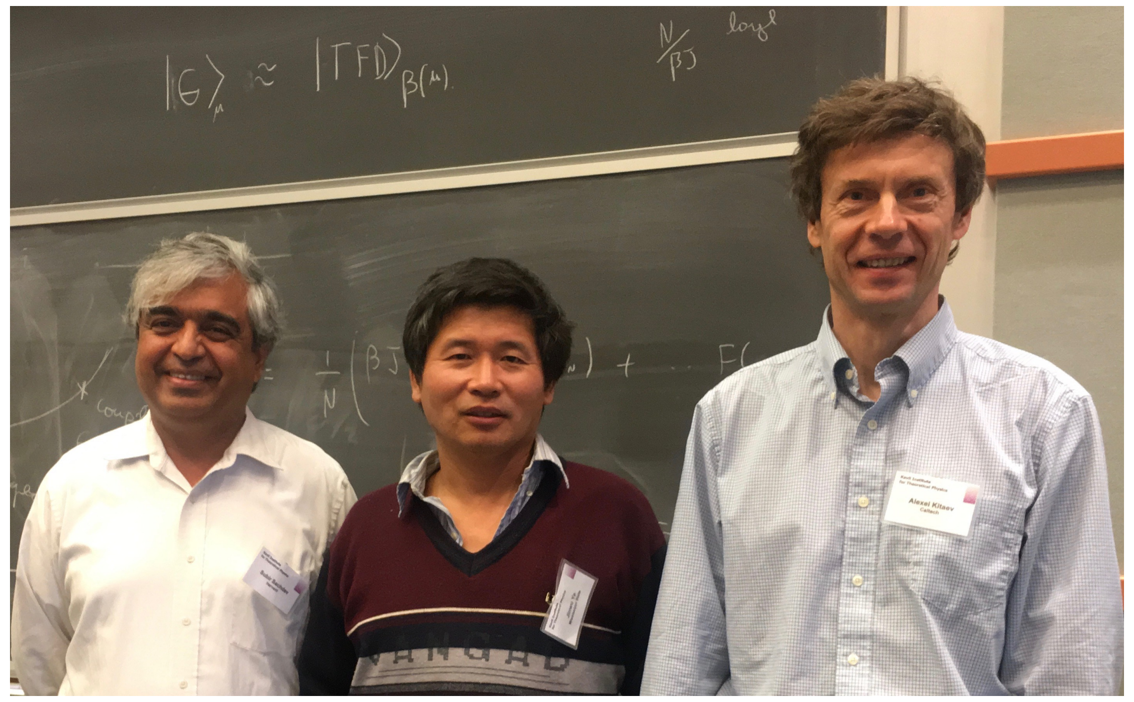

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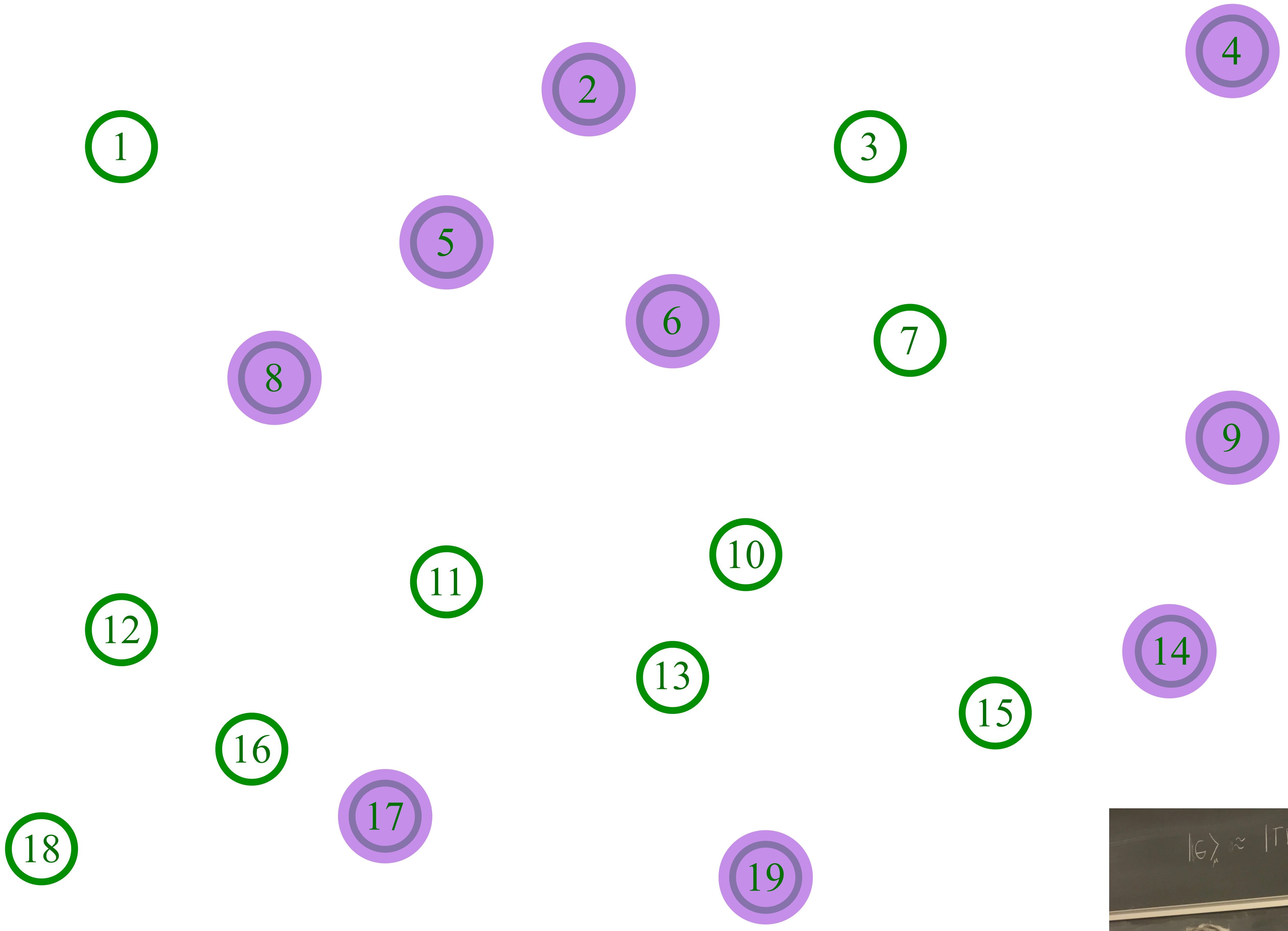
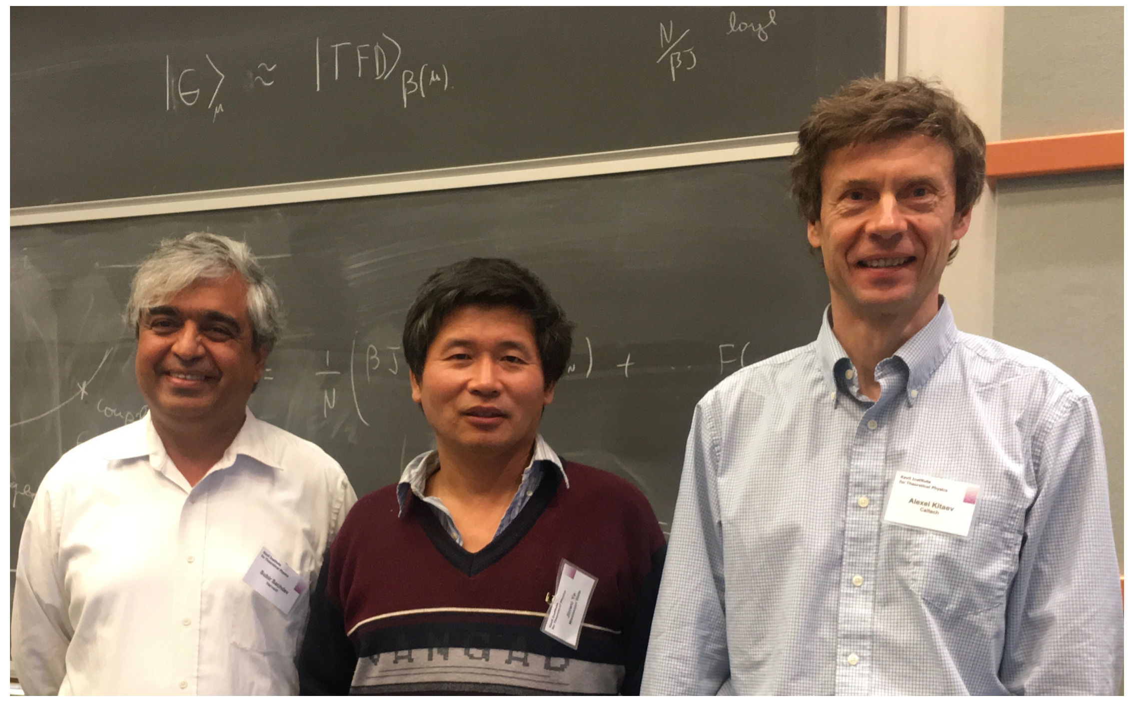


Entangle electrons pairwise randomly



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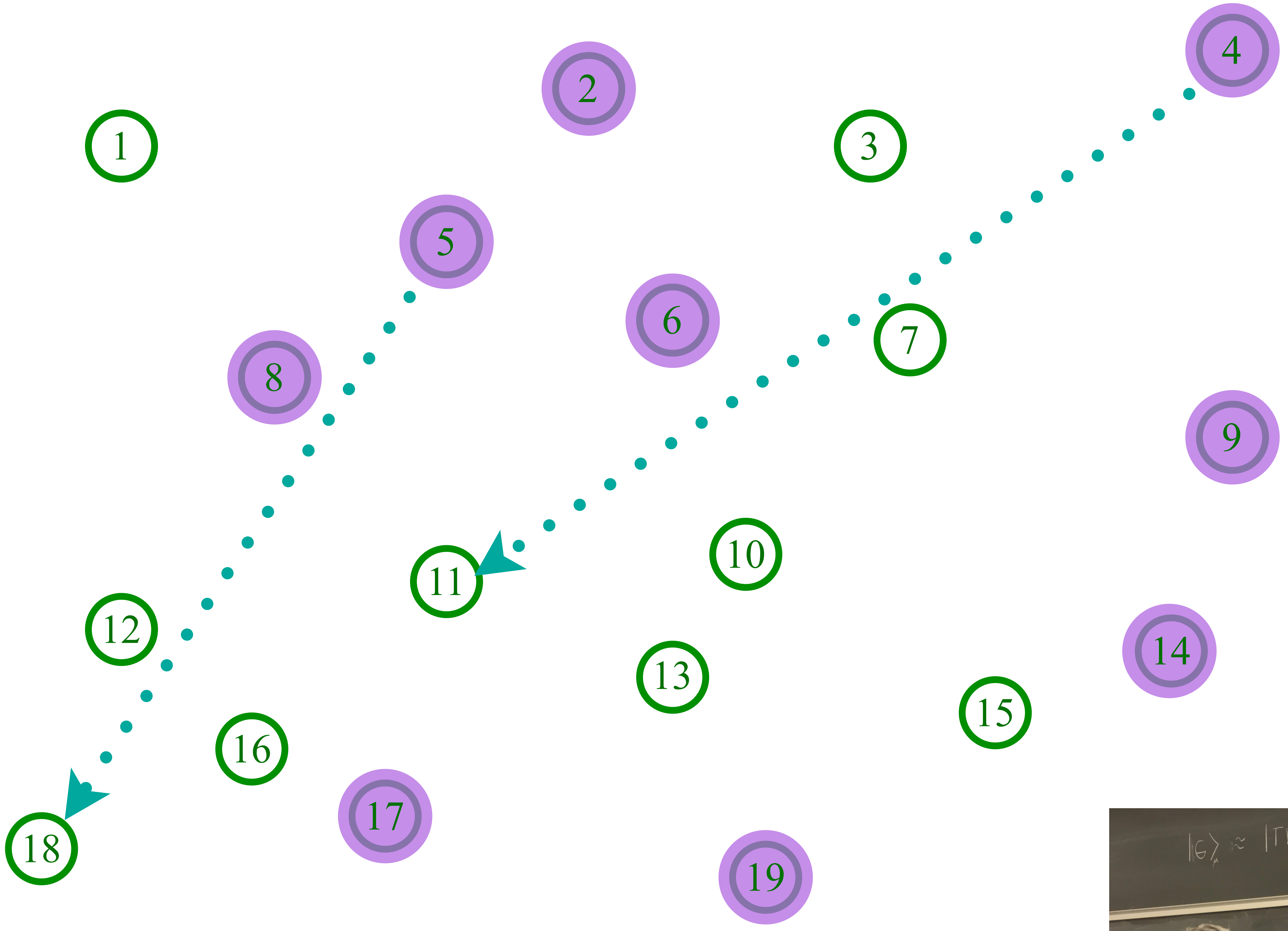
Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



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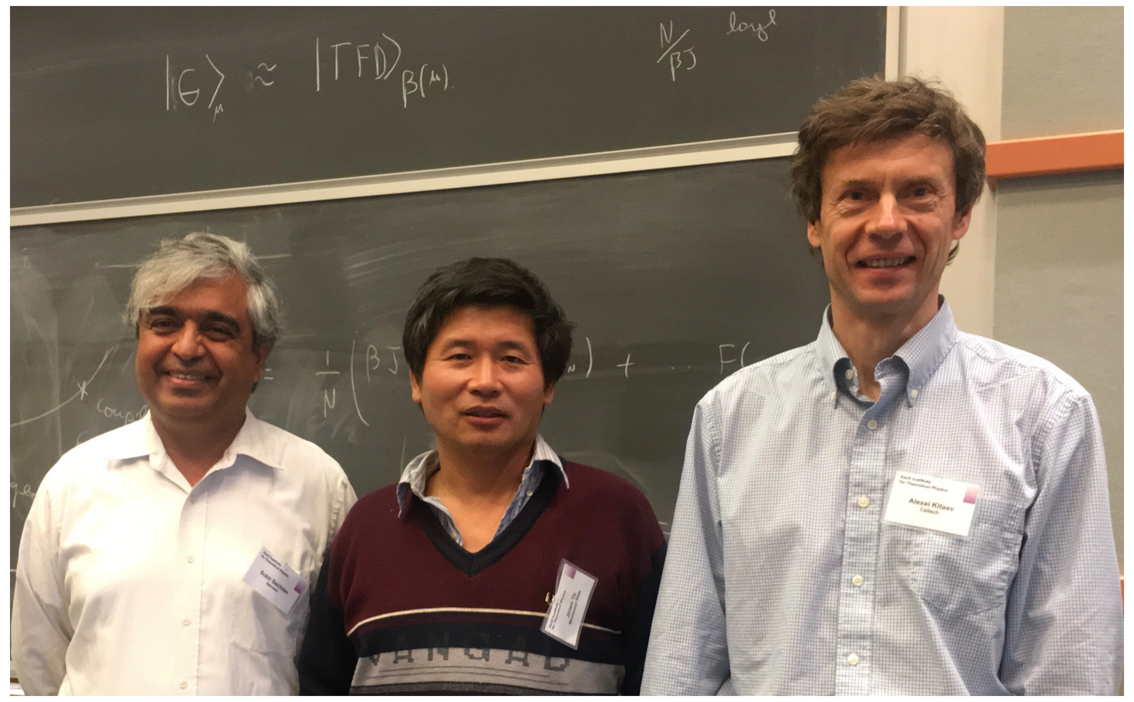

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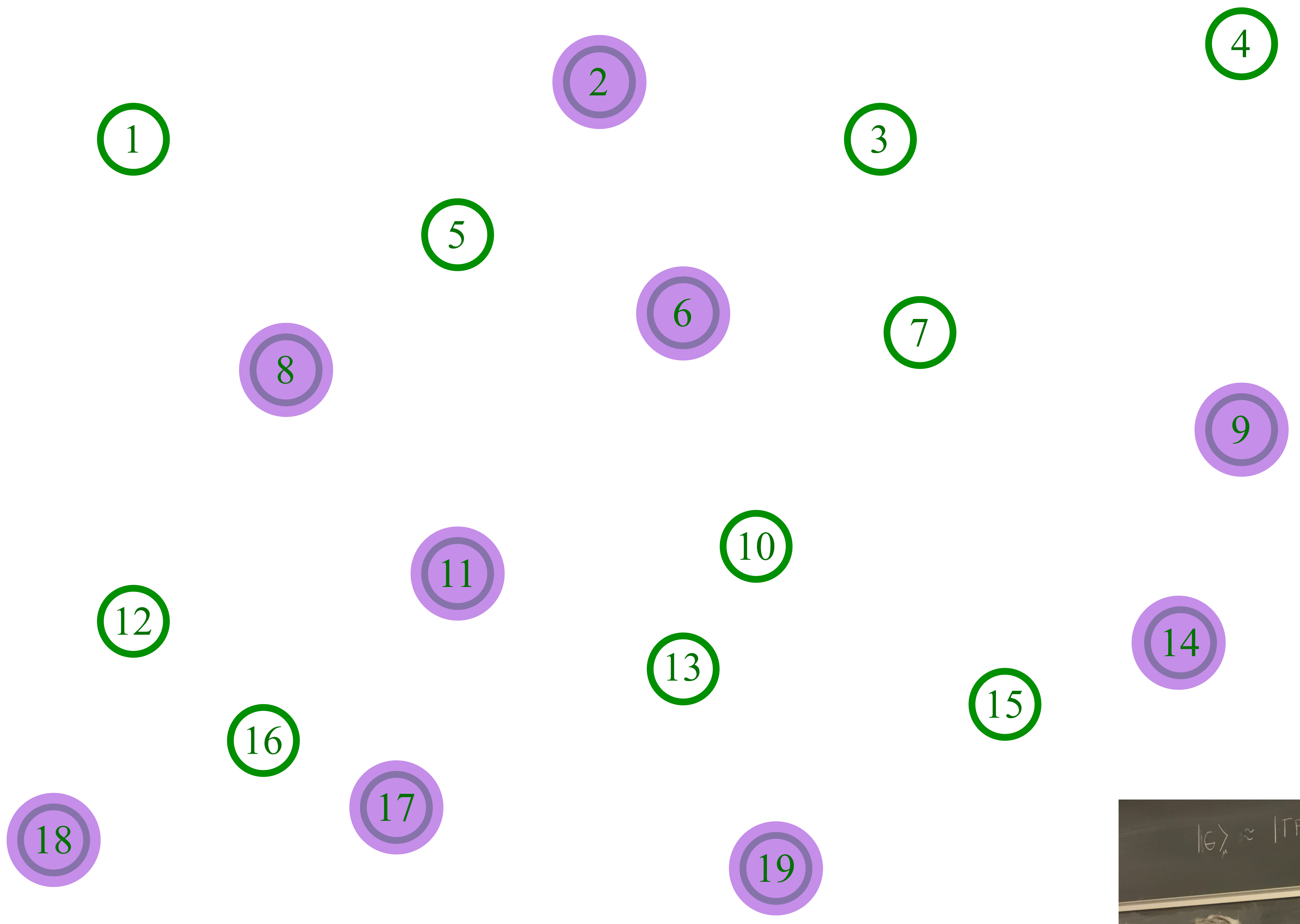

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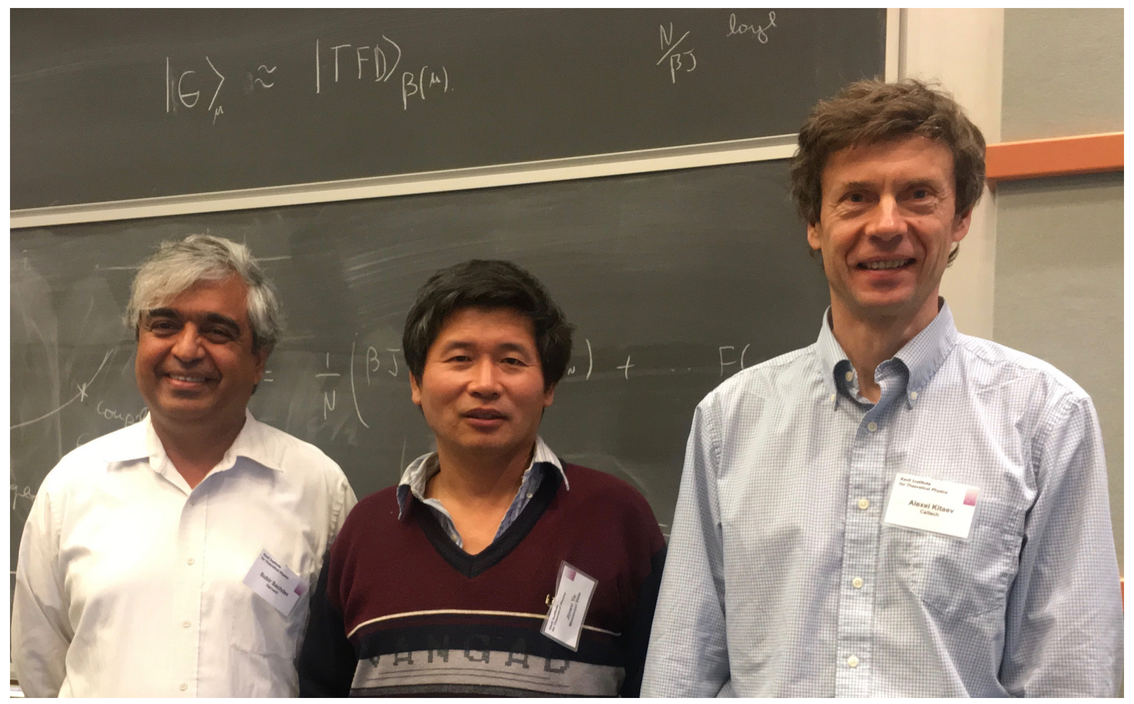

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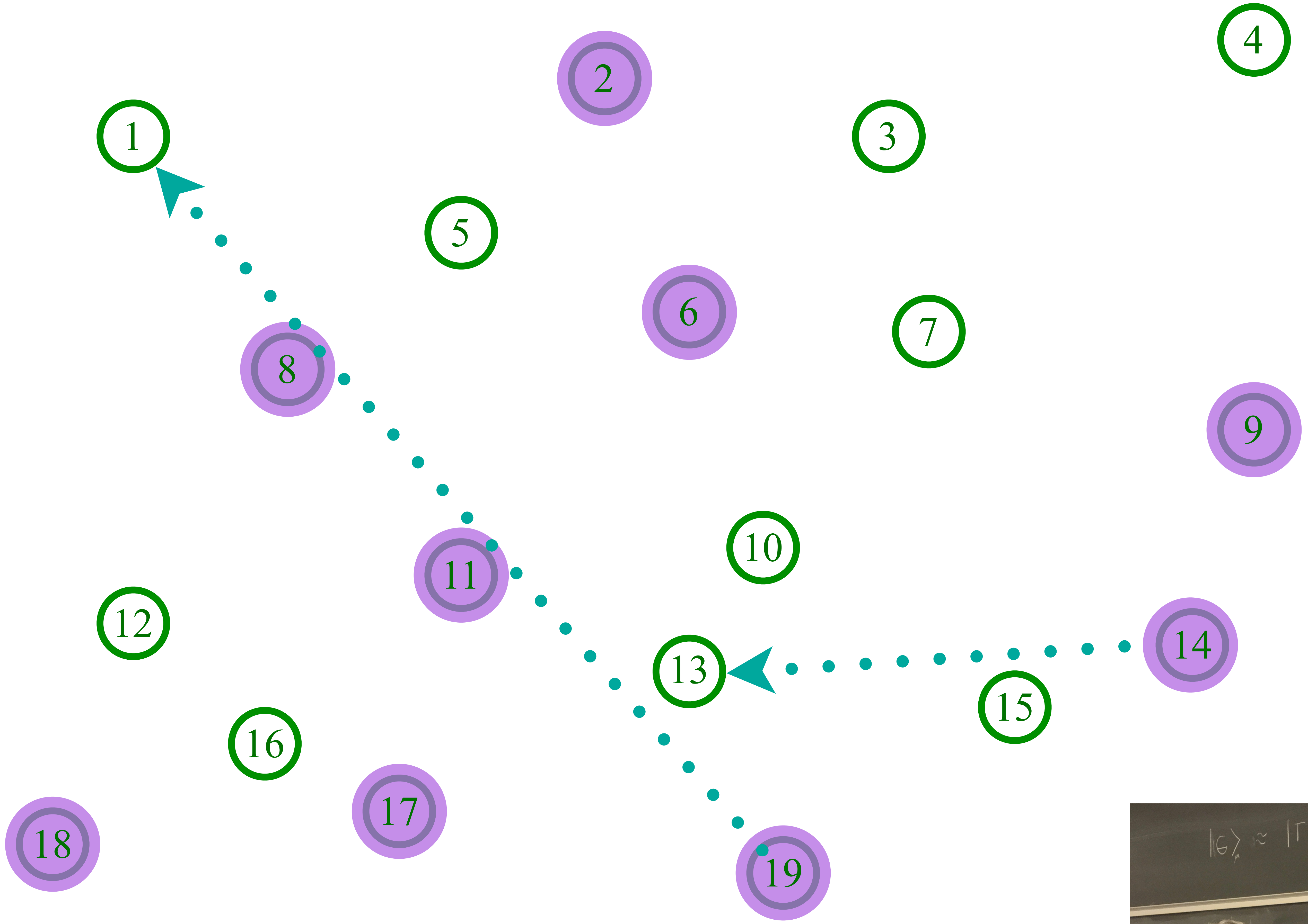
$$U_{14,19;1,13}$$



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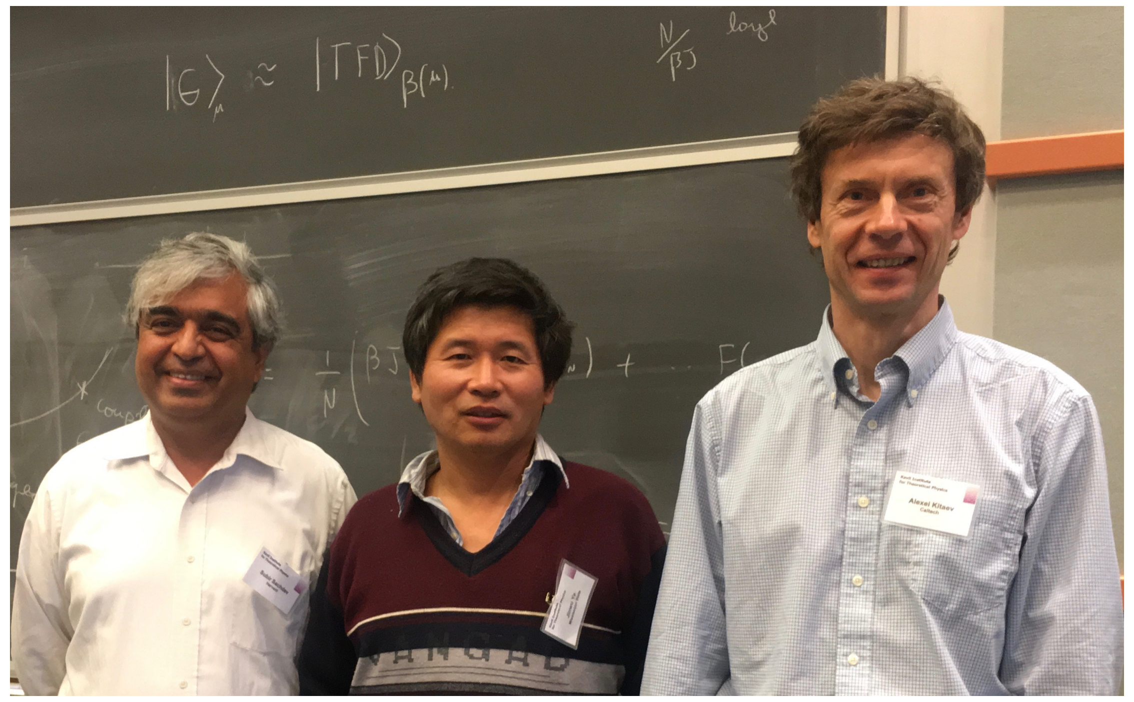

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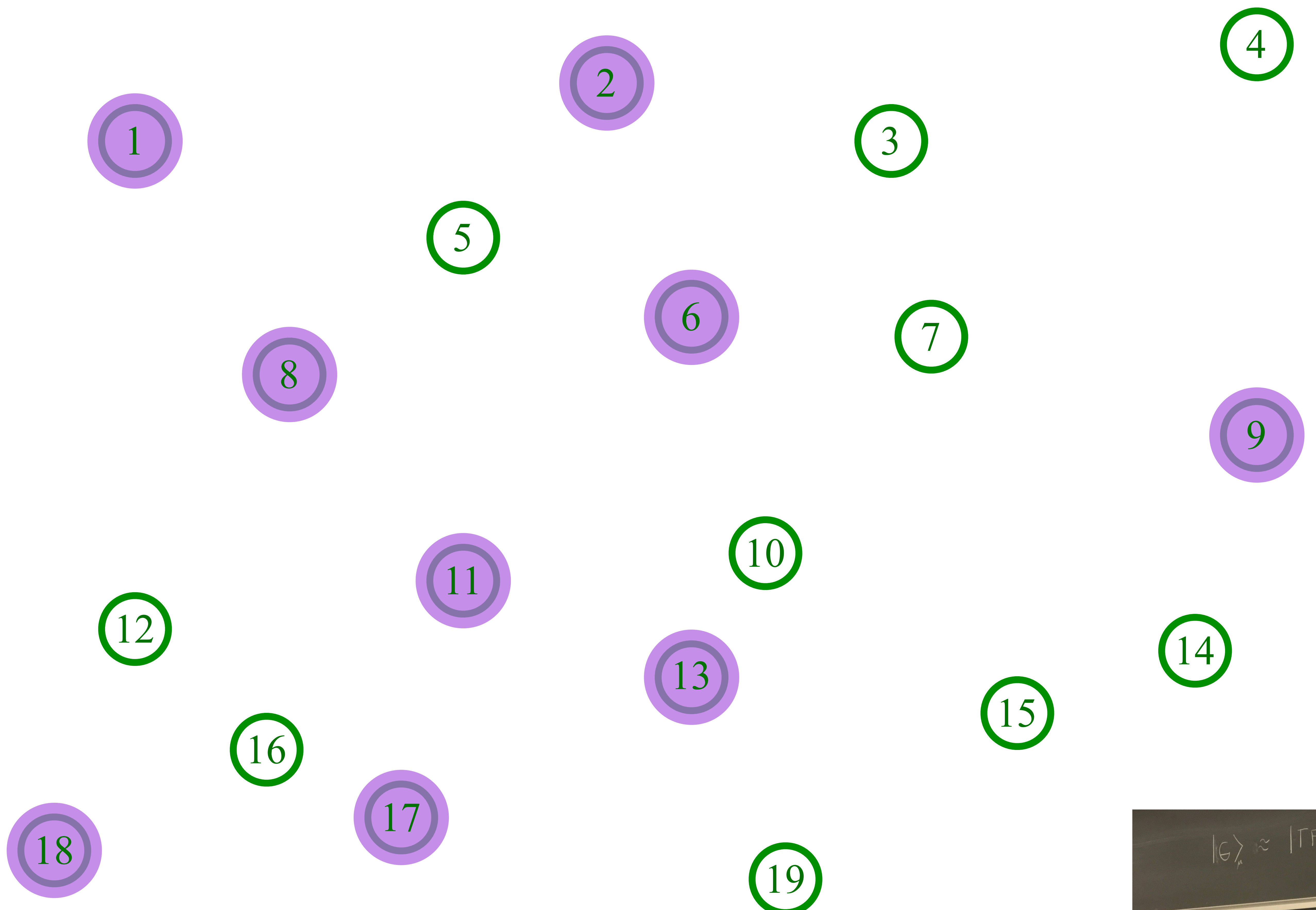

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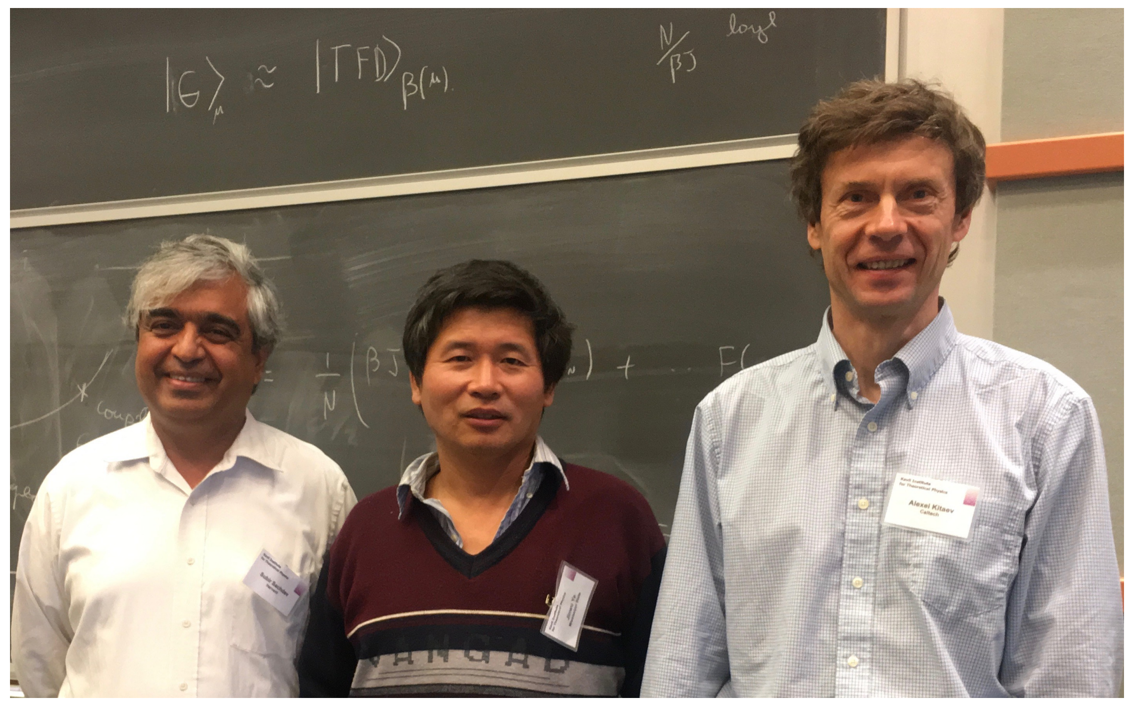

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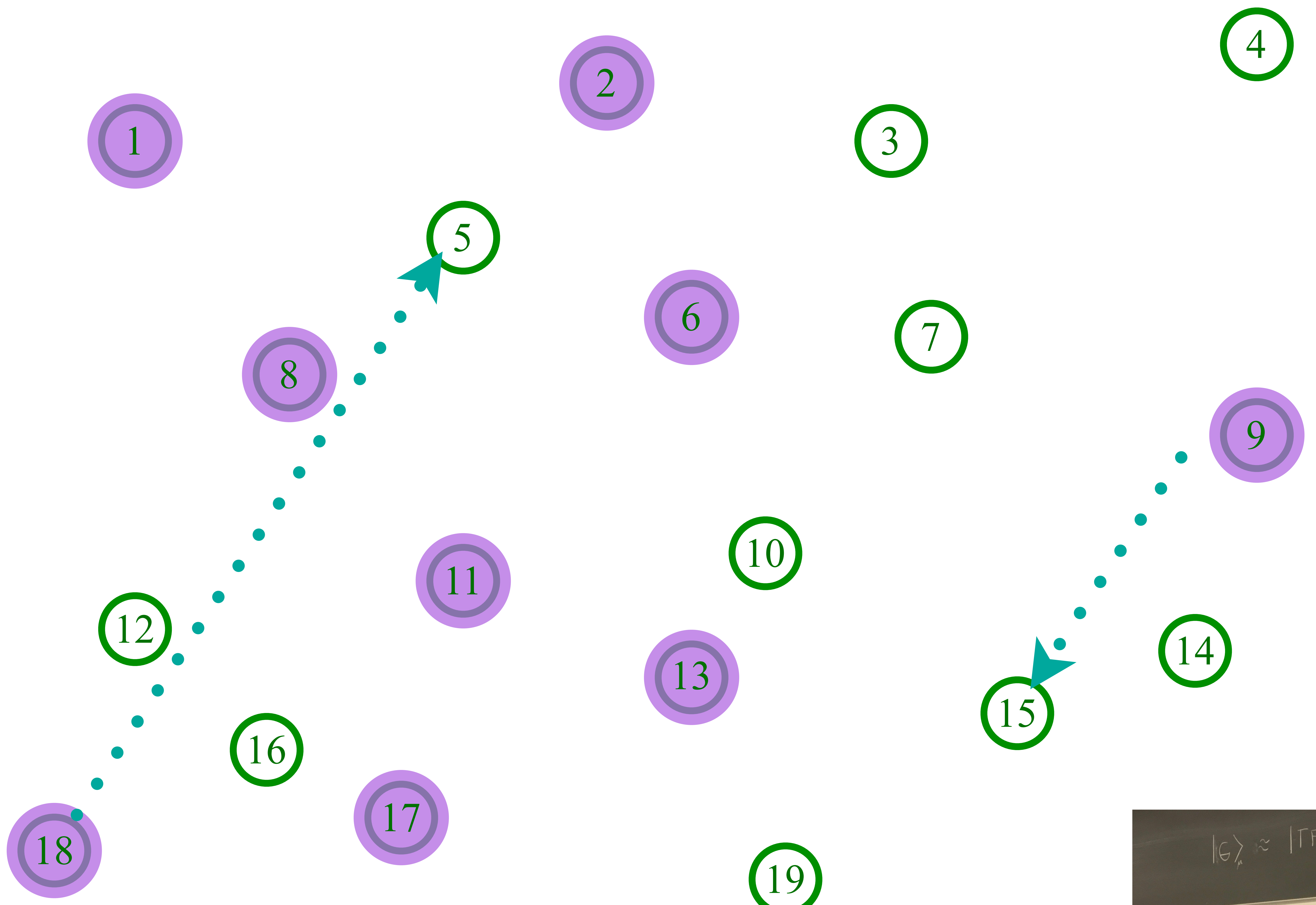
$$U_{9,18;5,15}$$



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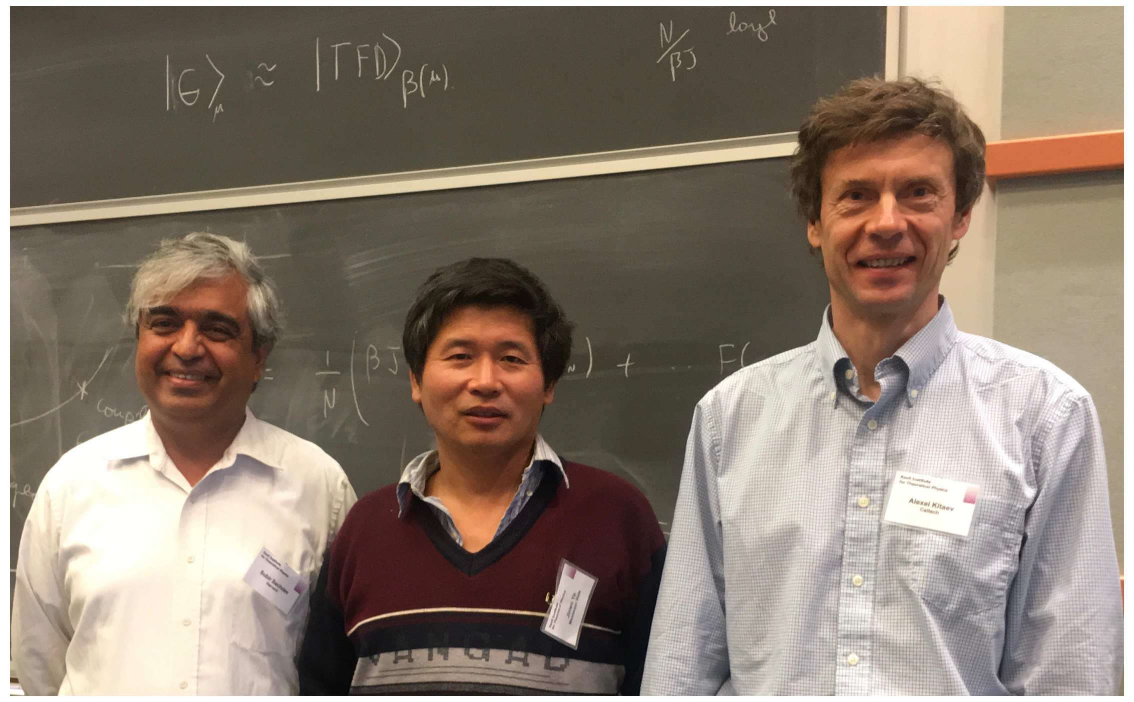

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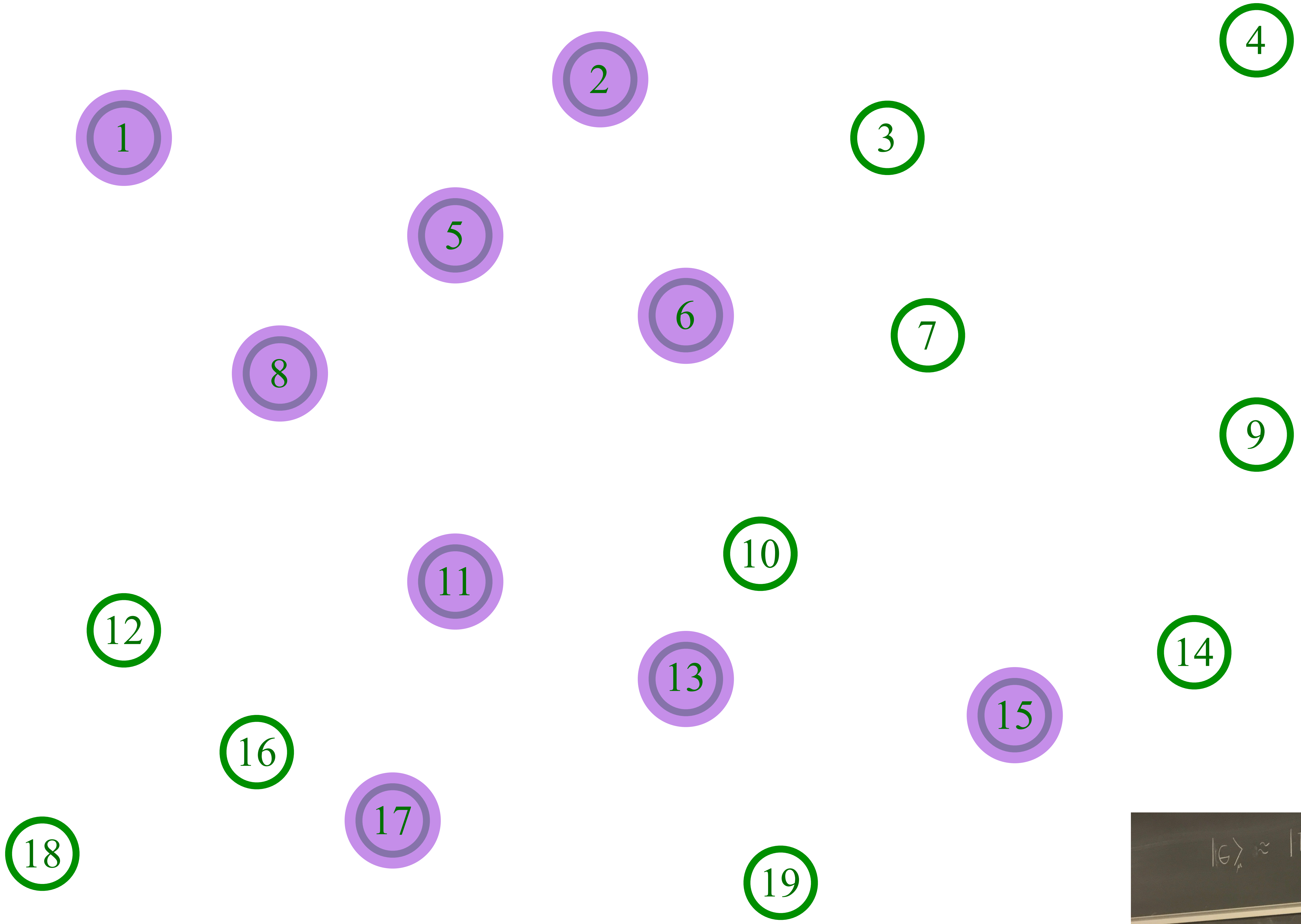

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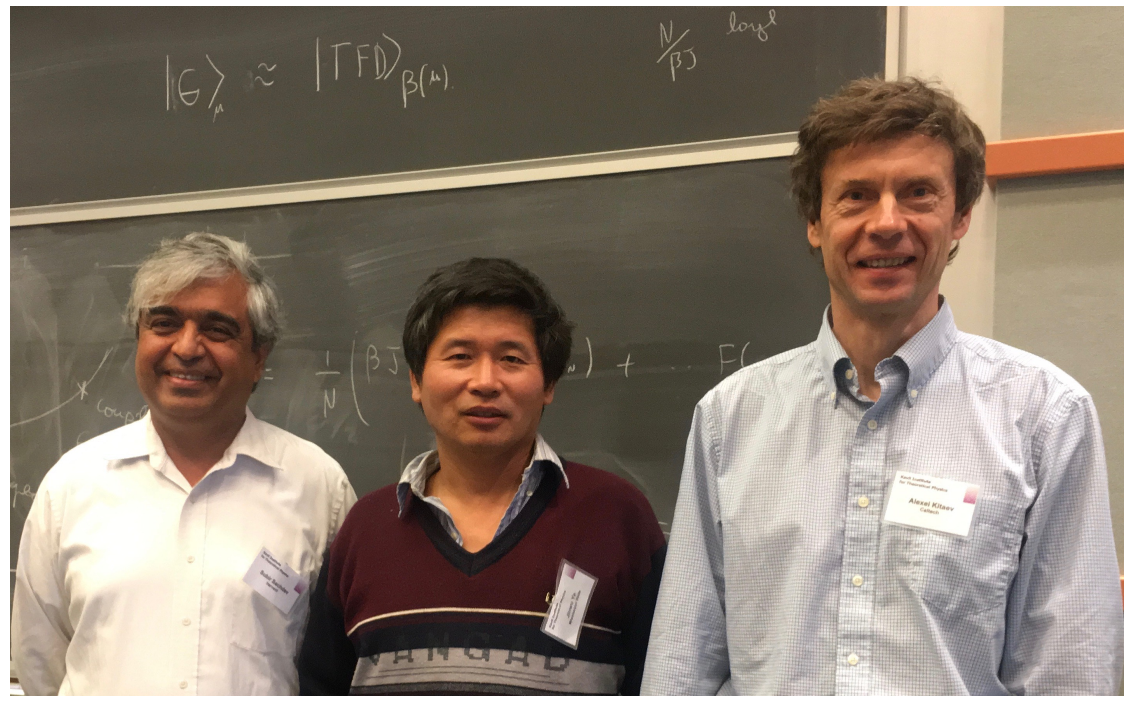
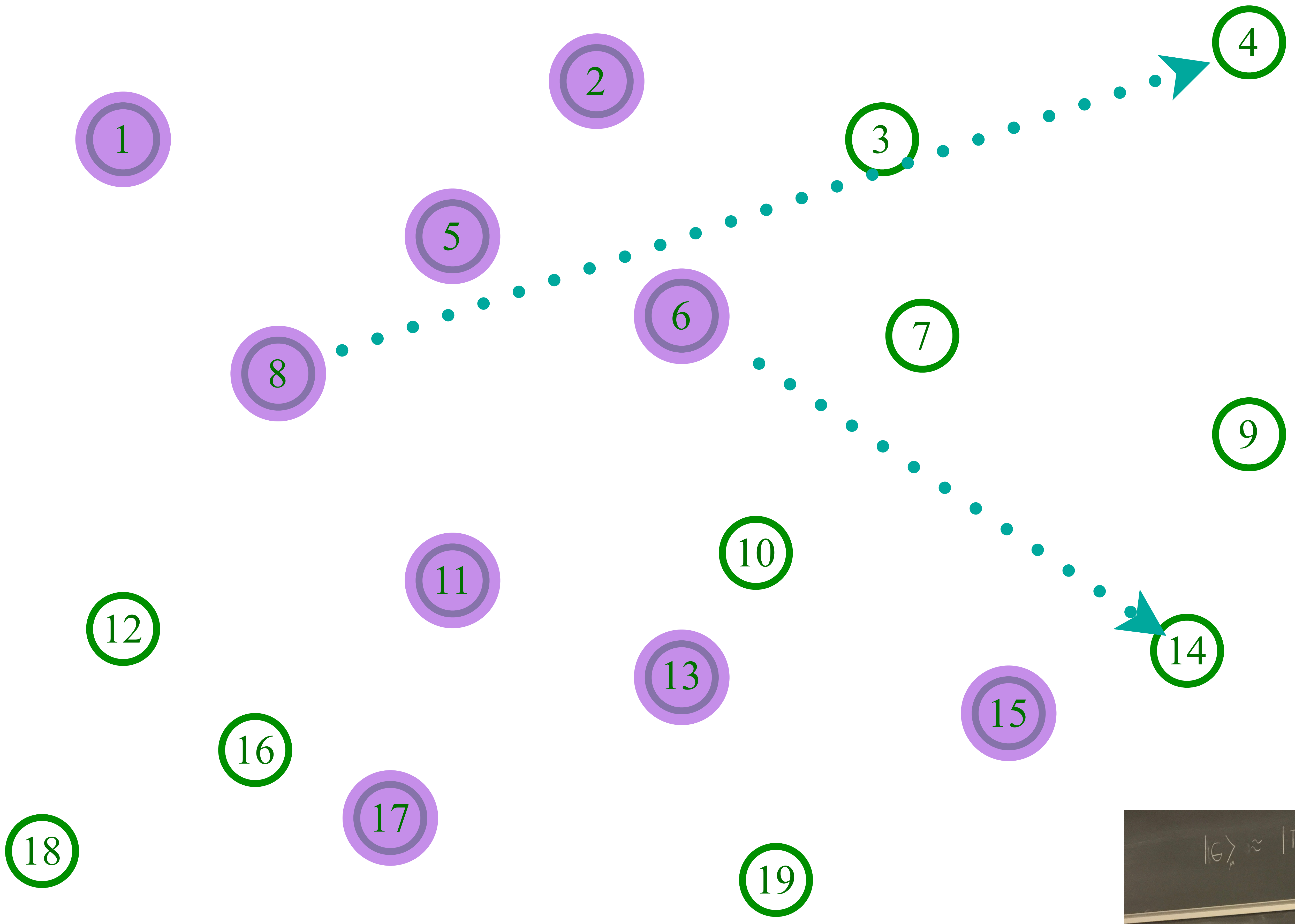


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



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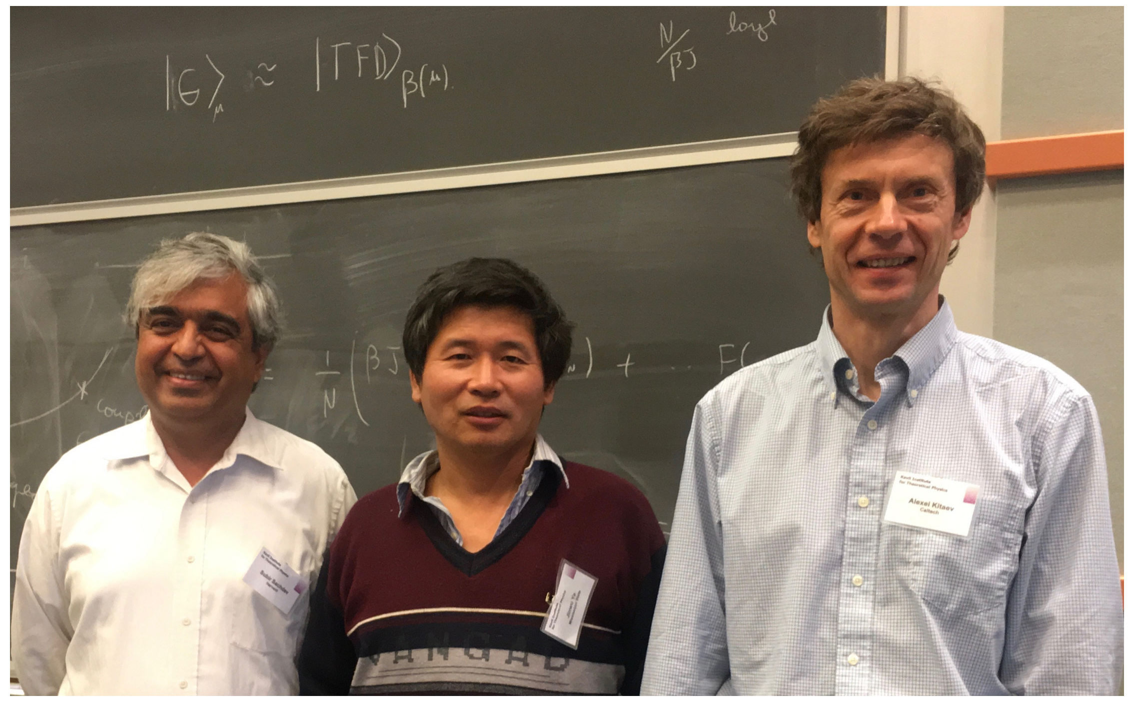

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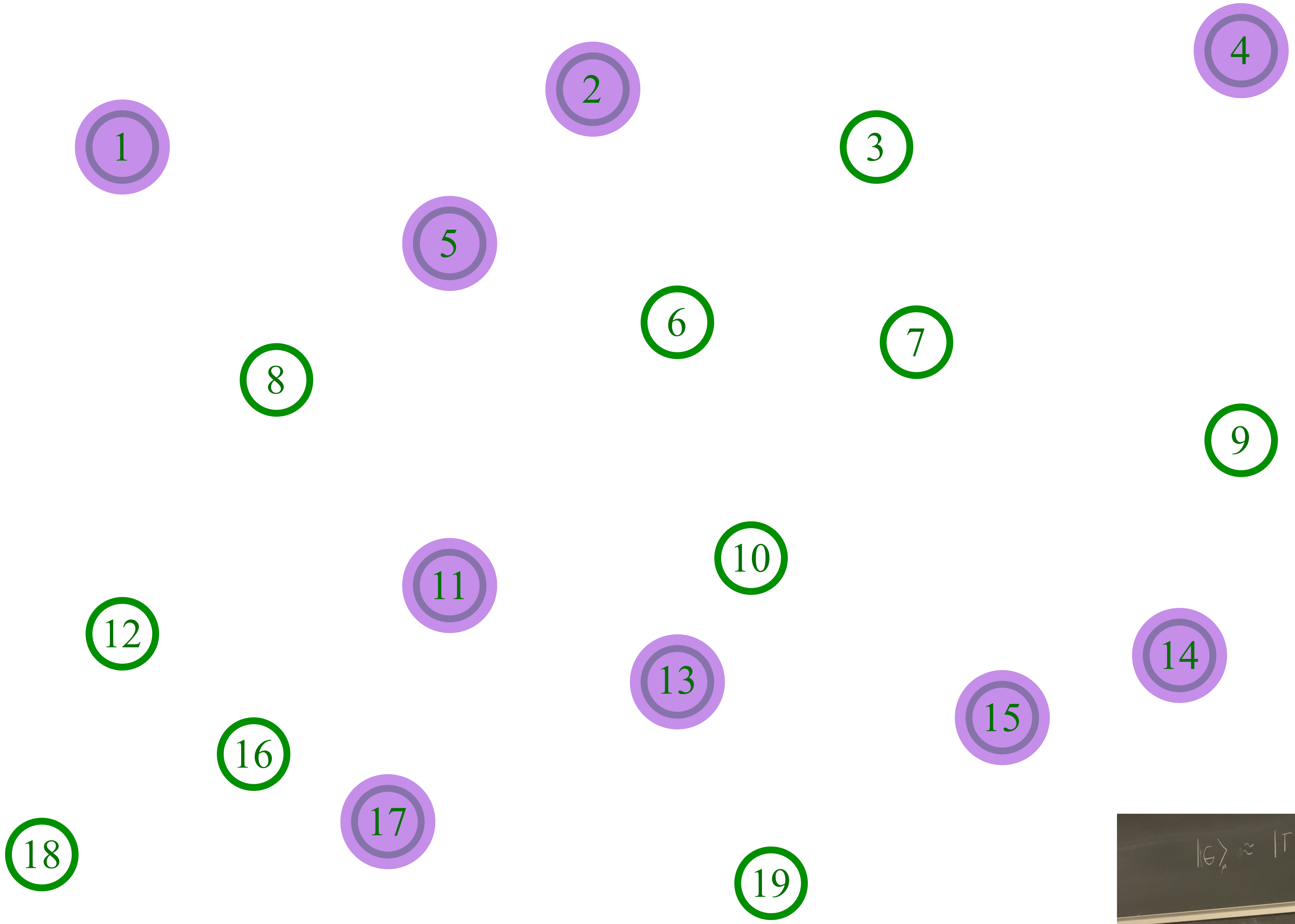

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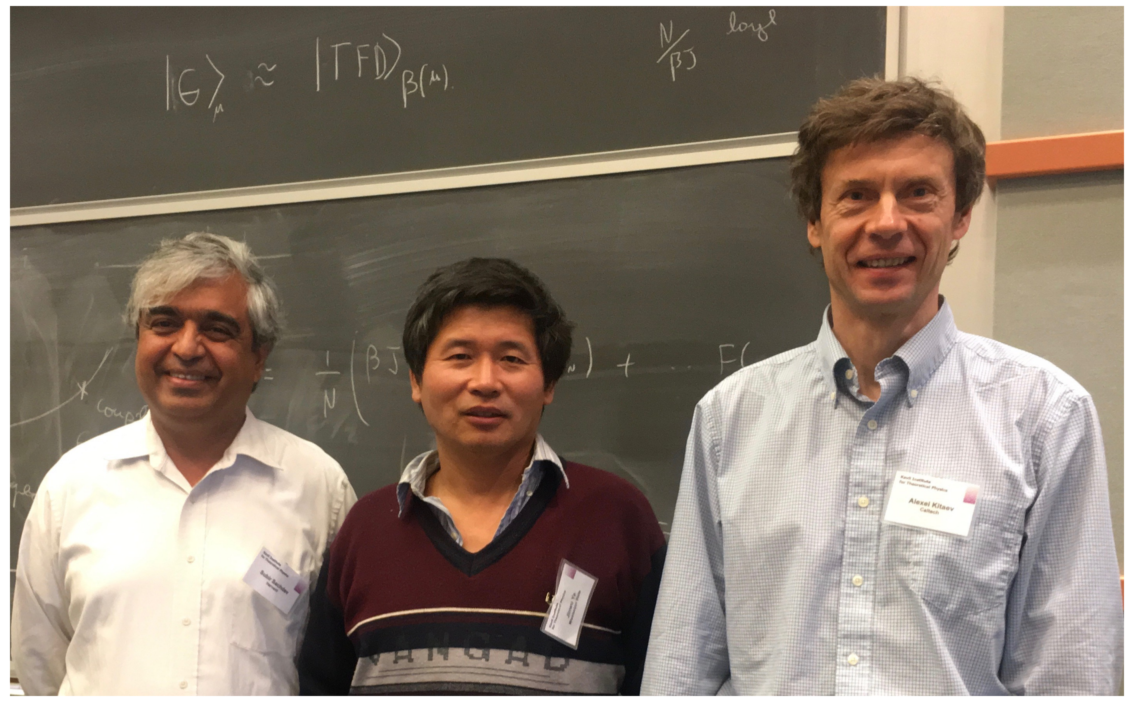


The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

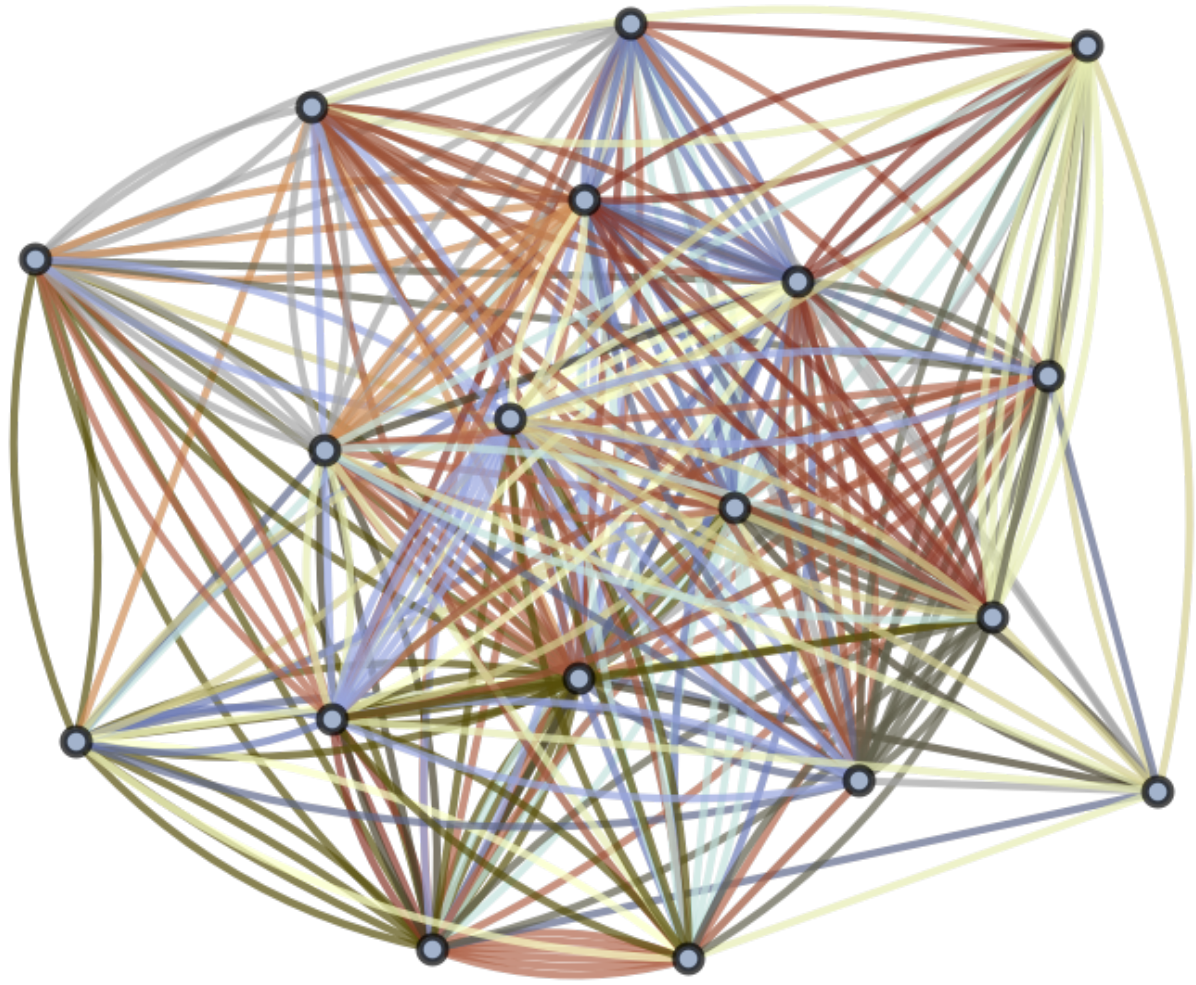
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

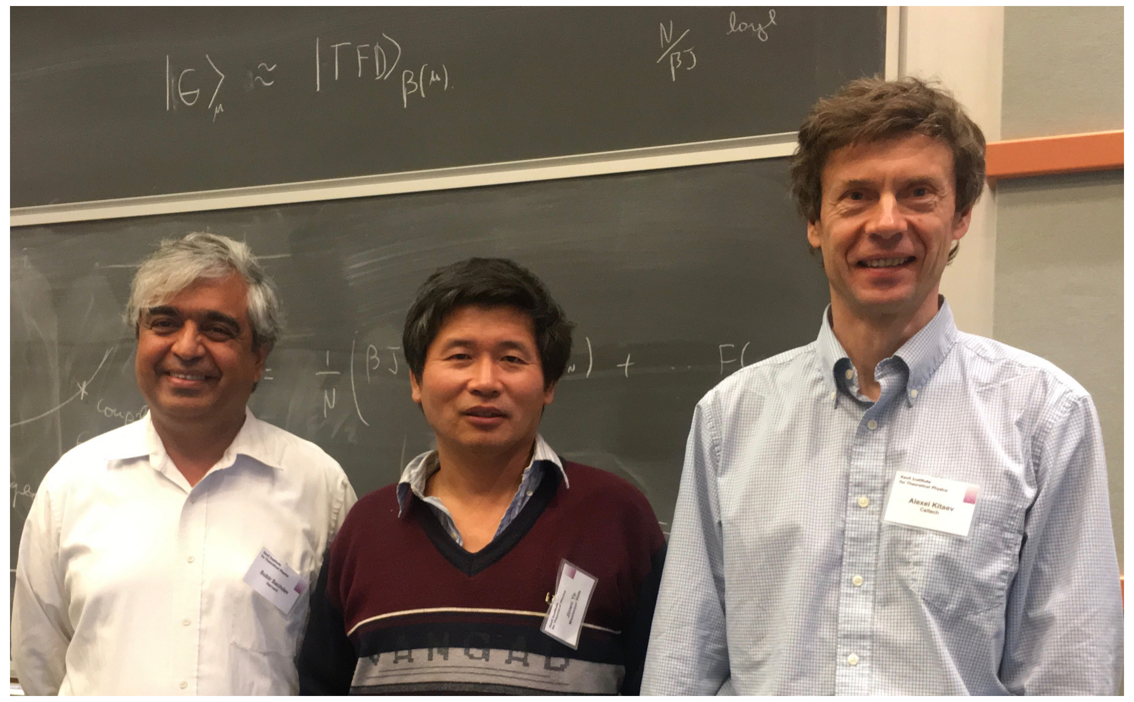
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

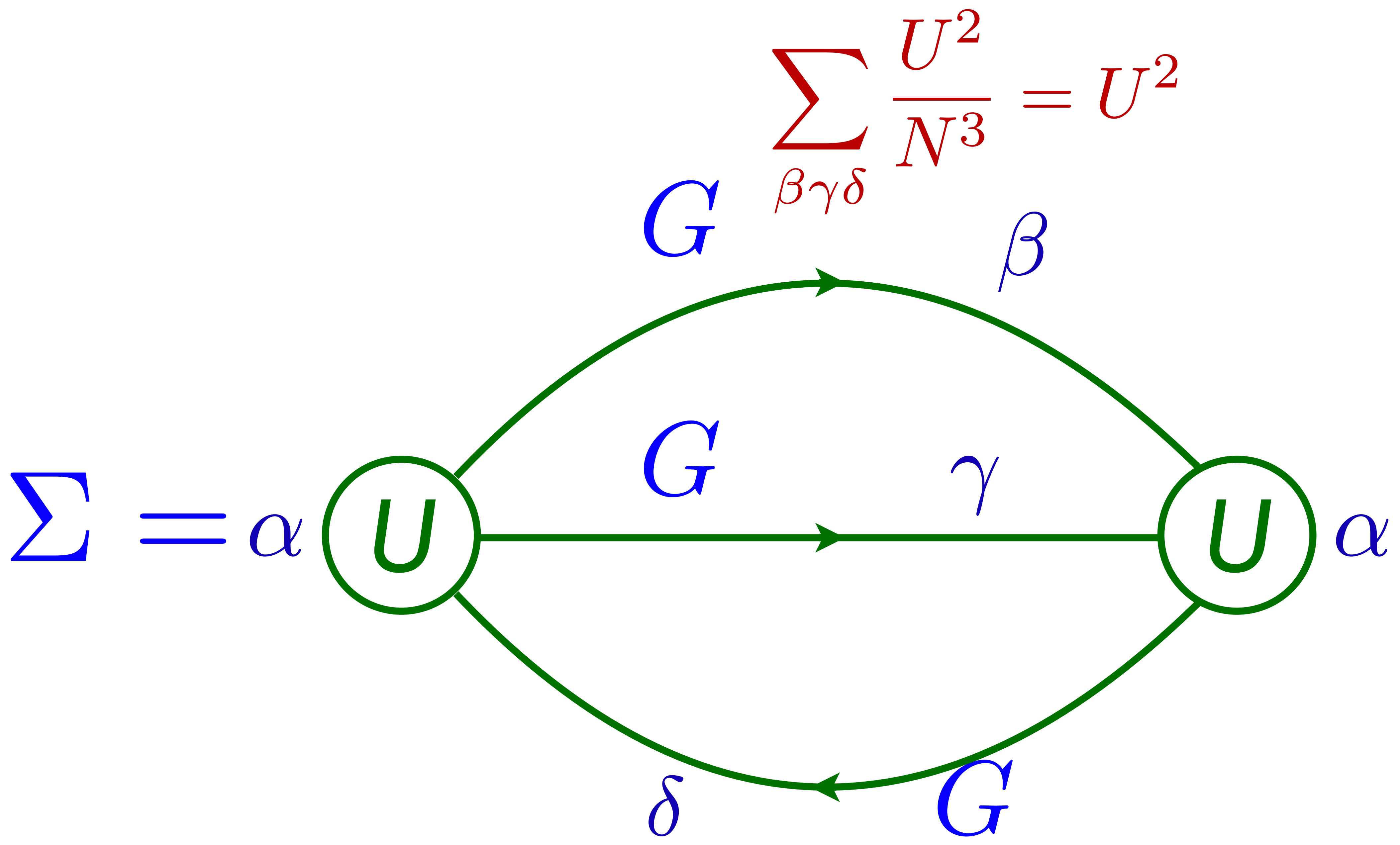


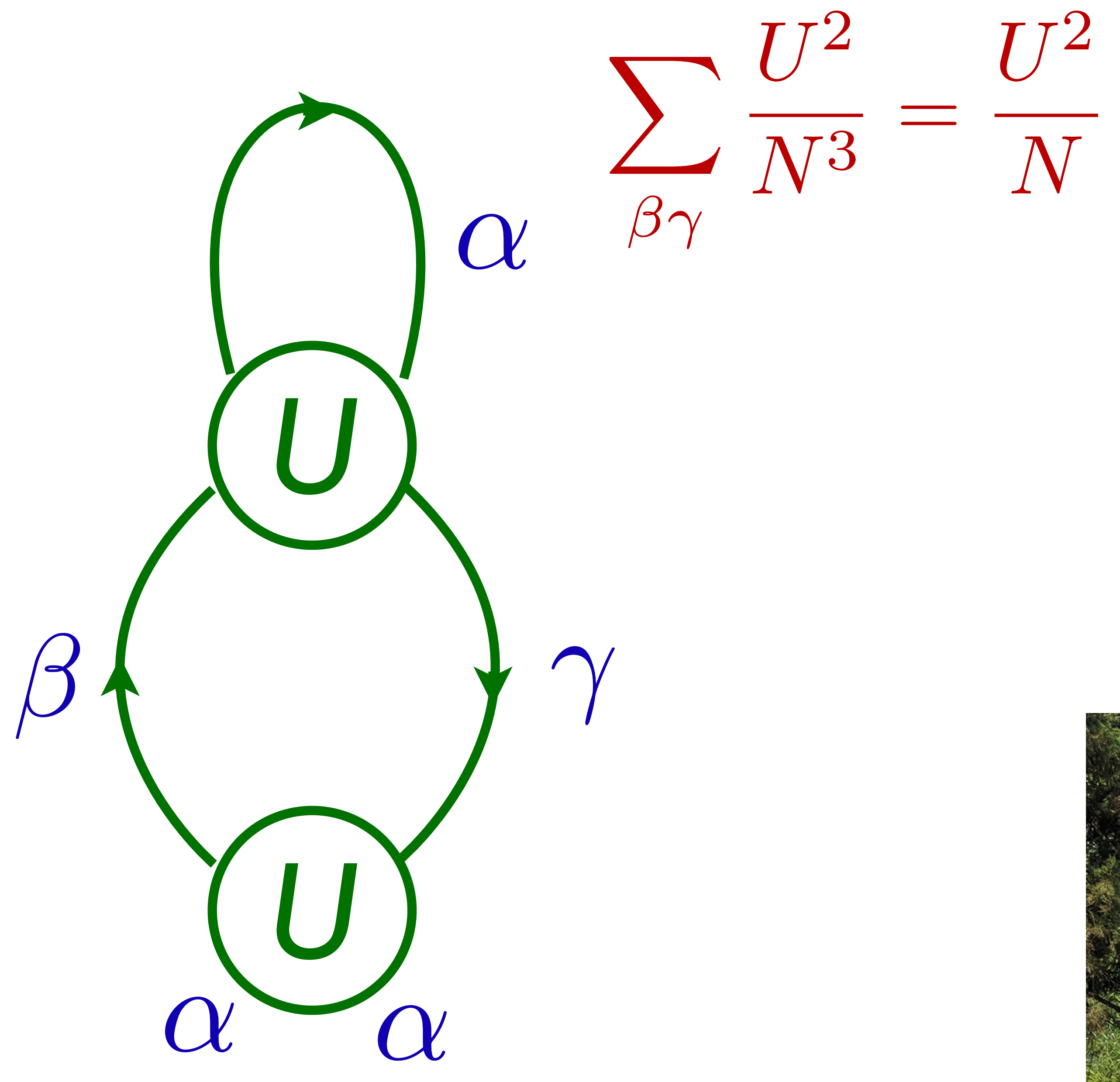


The Sachdev-Ye-Kitaev (SYK) model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$





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$$G(\tau = 0^-) = \mathcal{Q}.$$

Solution: At $T = 0$, $G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-1/2}$

For $T > 0$, $G(\omega) \sim T^{-1/2} F(\hbar\omega / k_B T)$

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

Current is carried by an "entangled quantum soup"

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

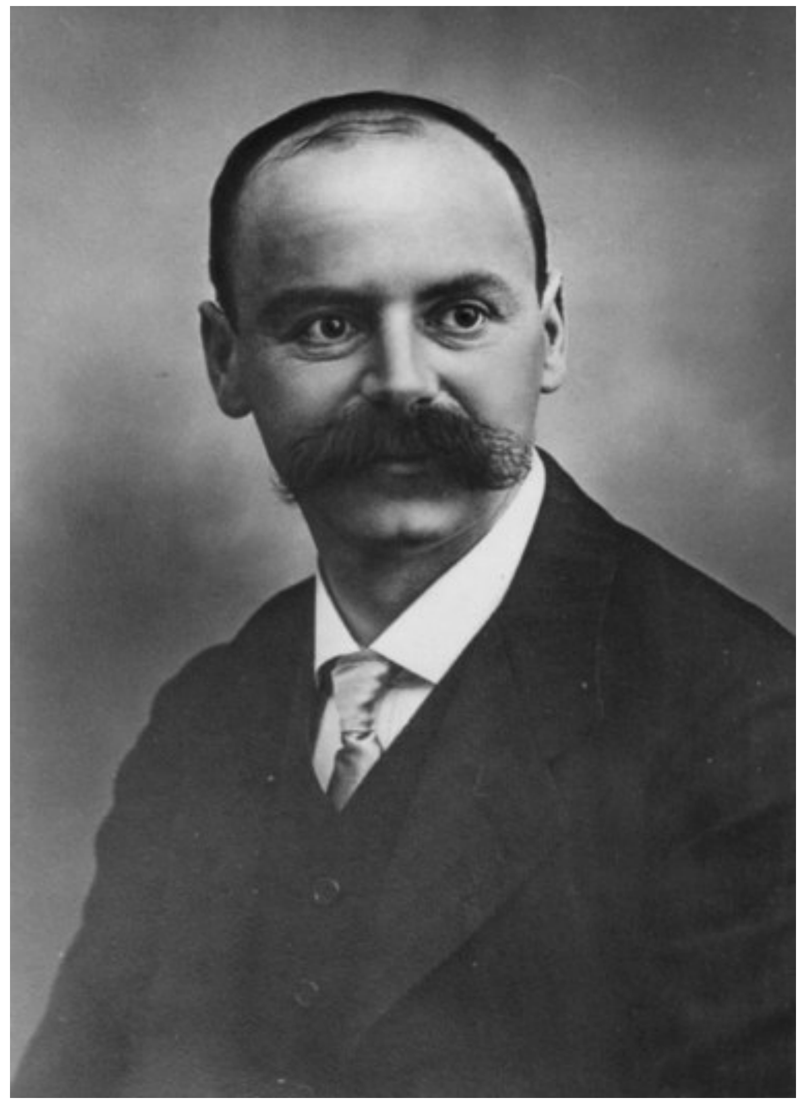
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

Planckian dynamics of black holes and the SYK model

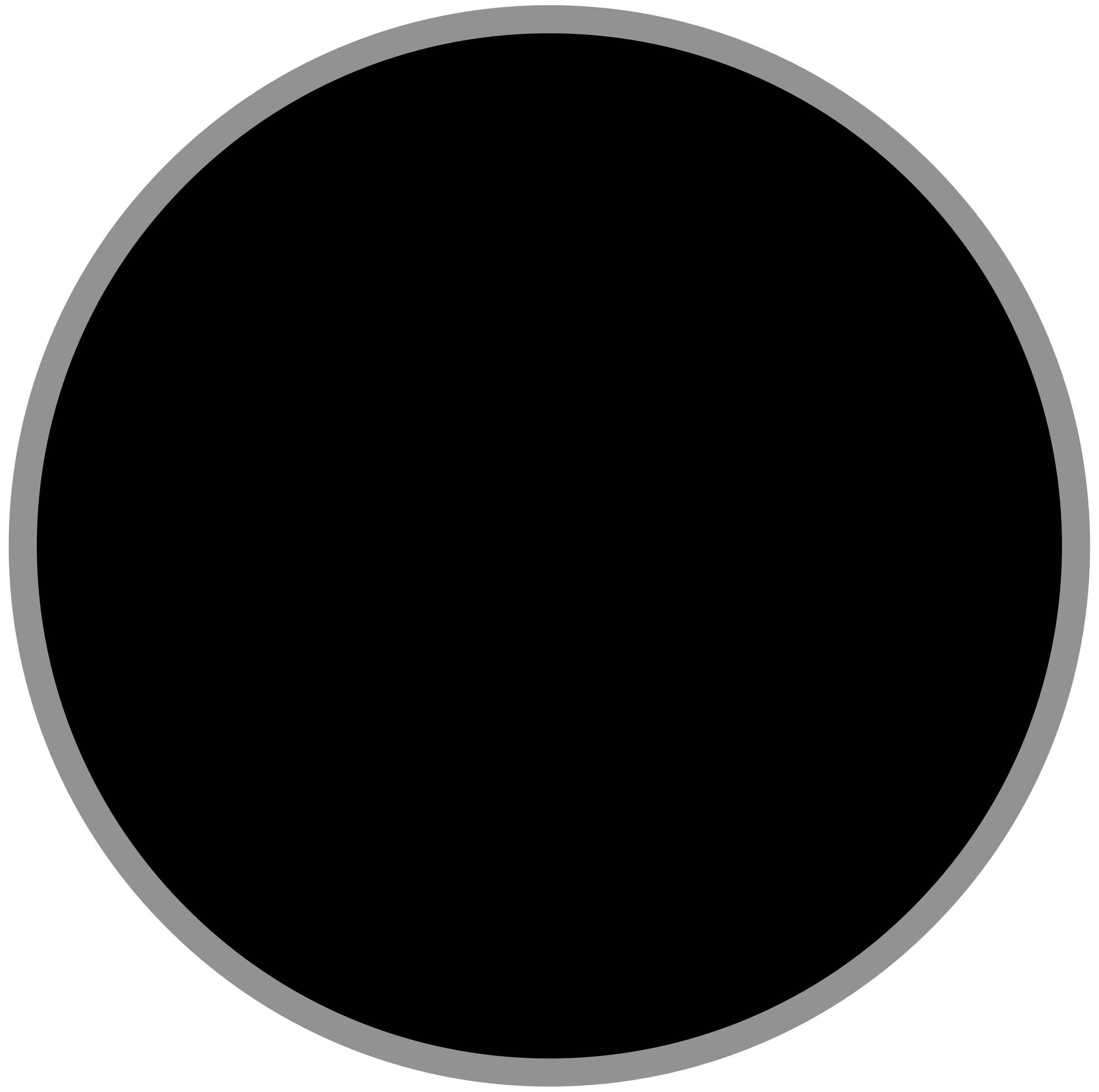
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

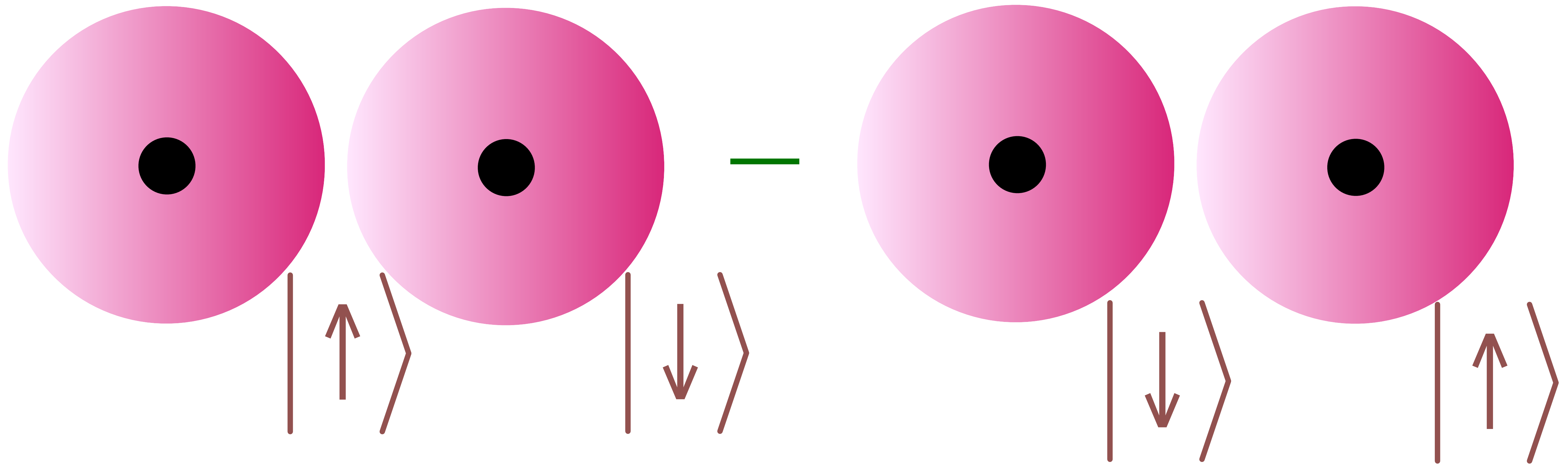
Horizon radius $R = \dfrac{2GM}{c^2}$



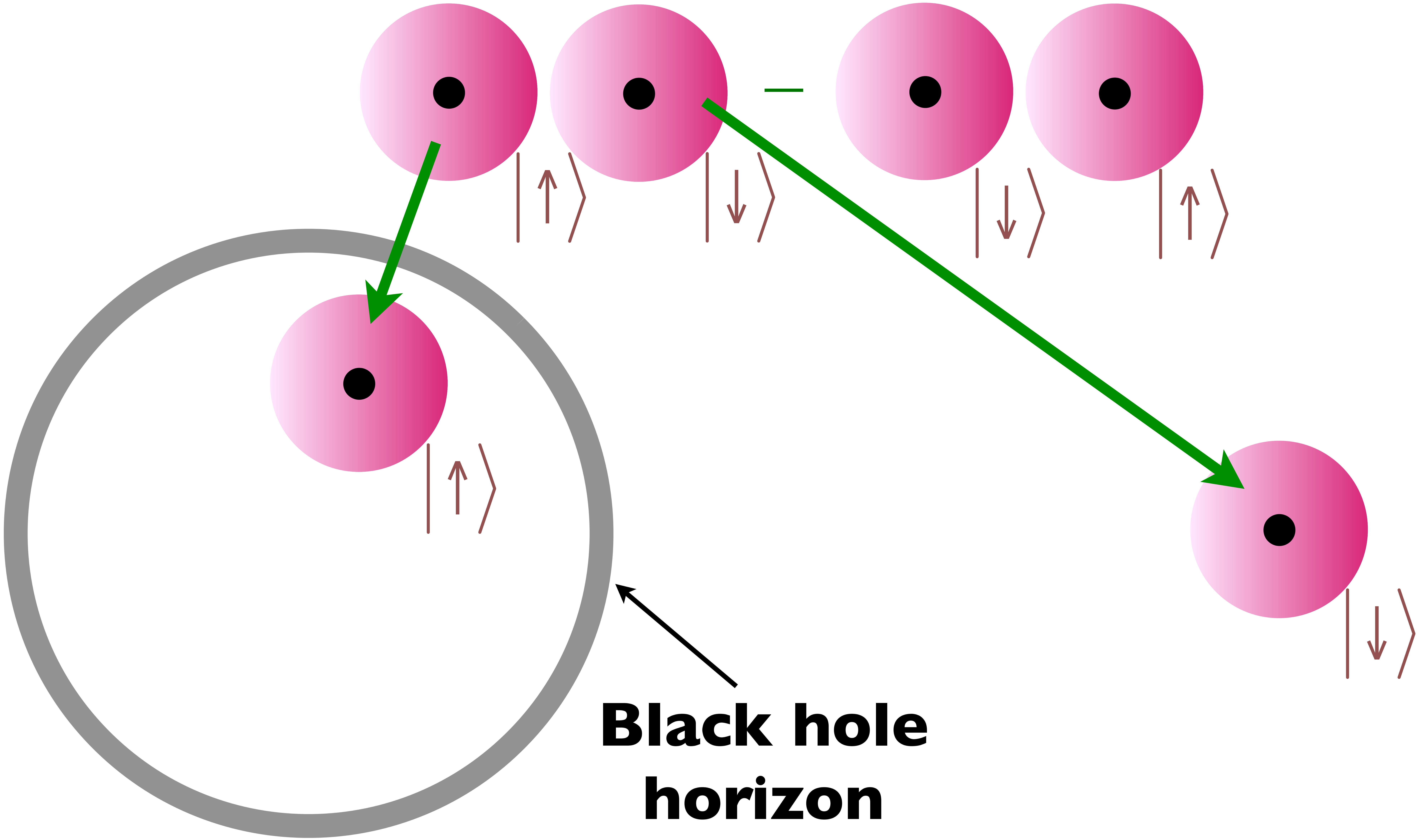
G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

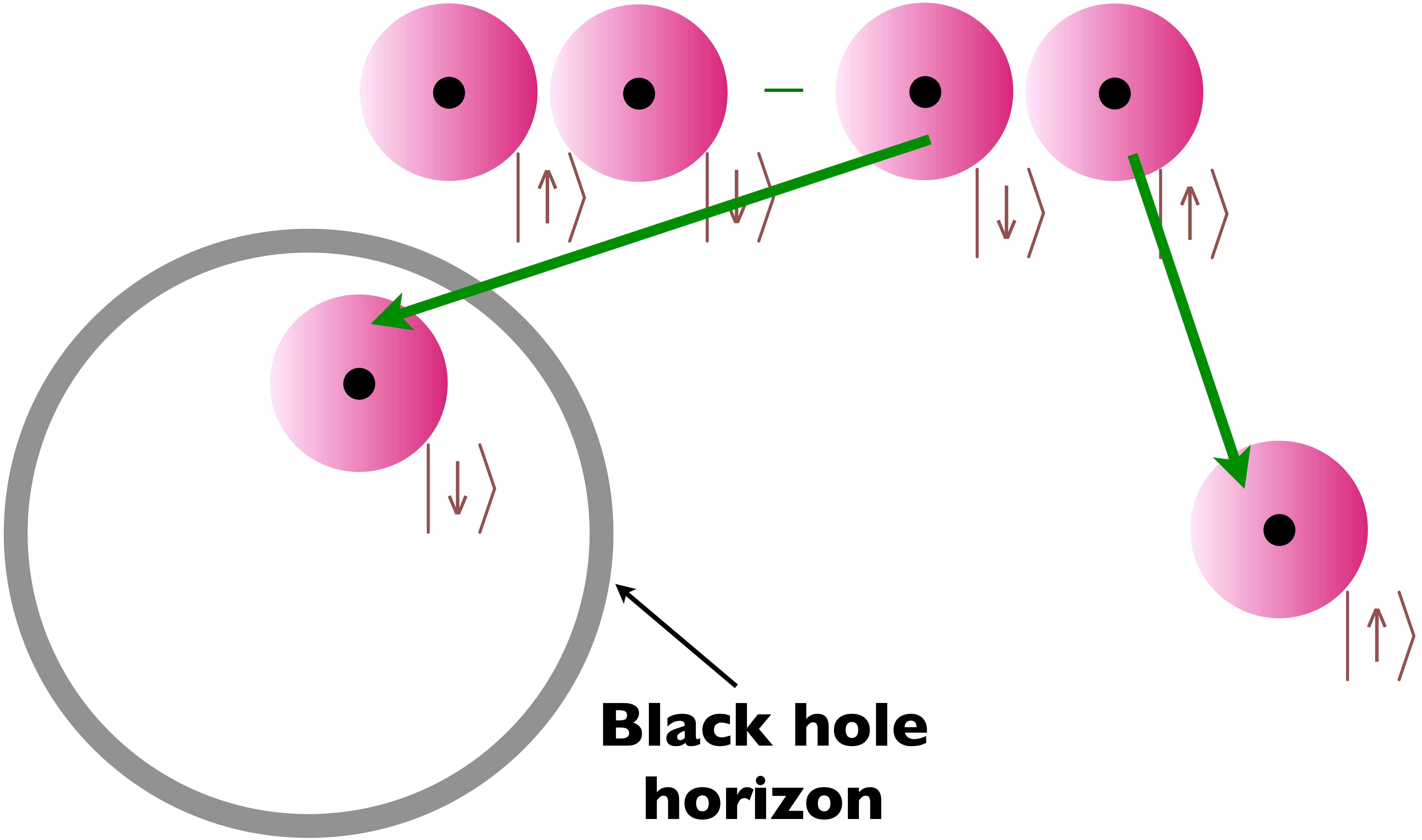
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

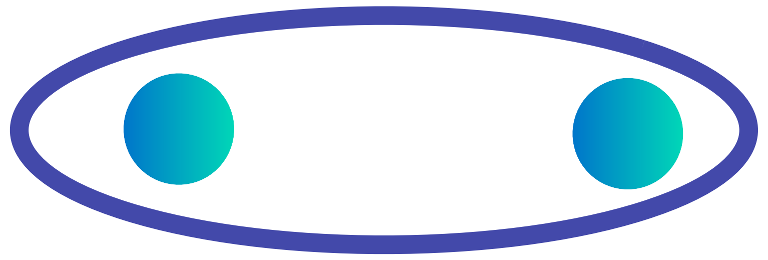


Quantum Entanglement across a black hole horizon

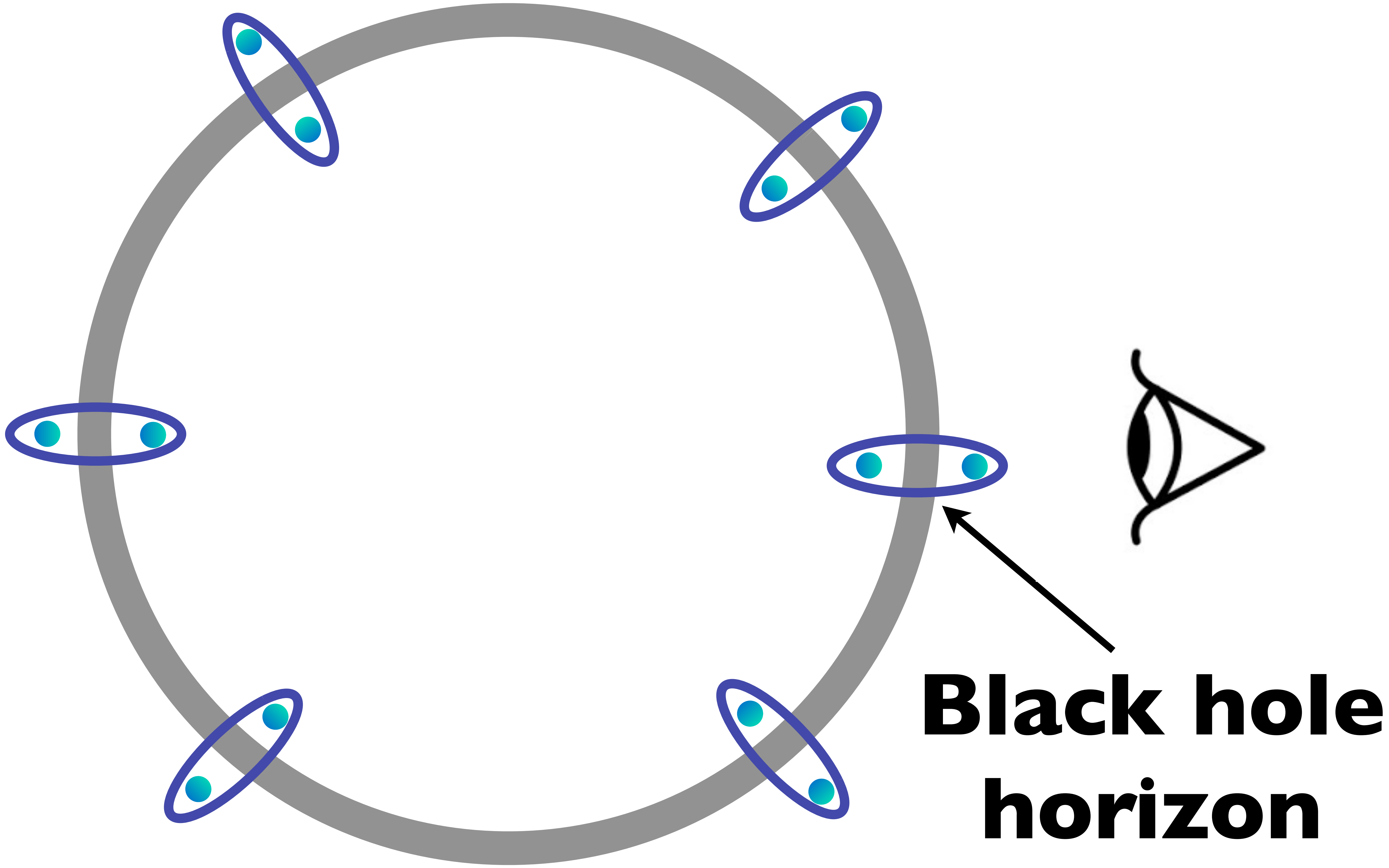


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



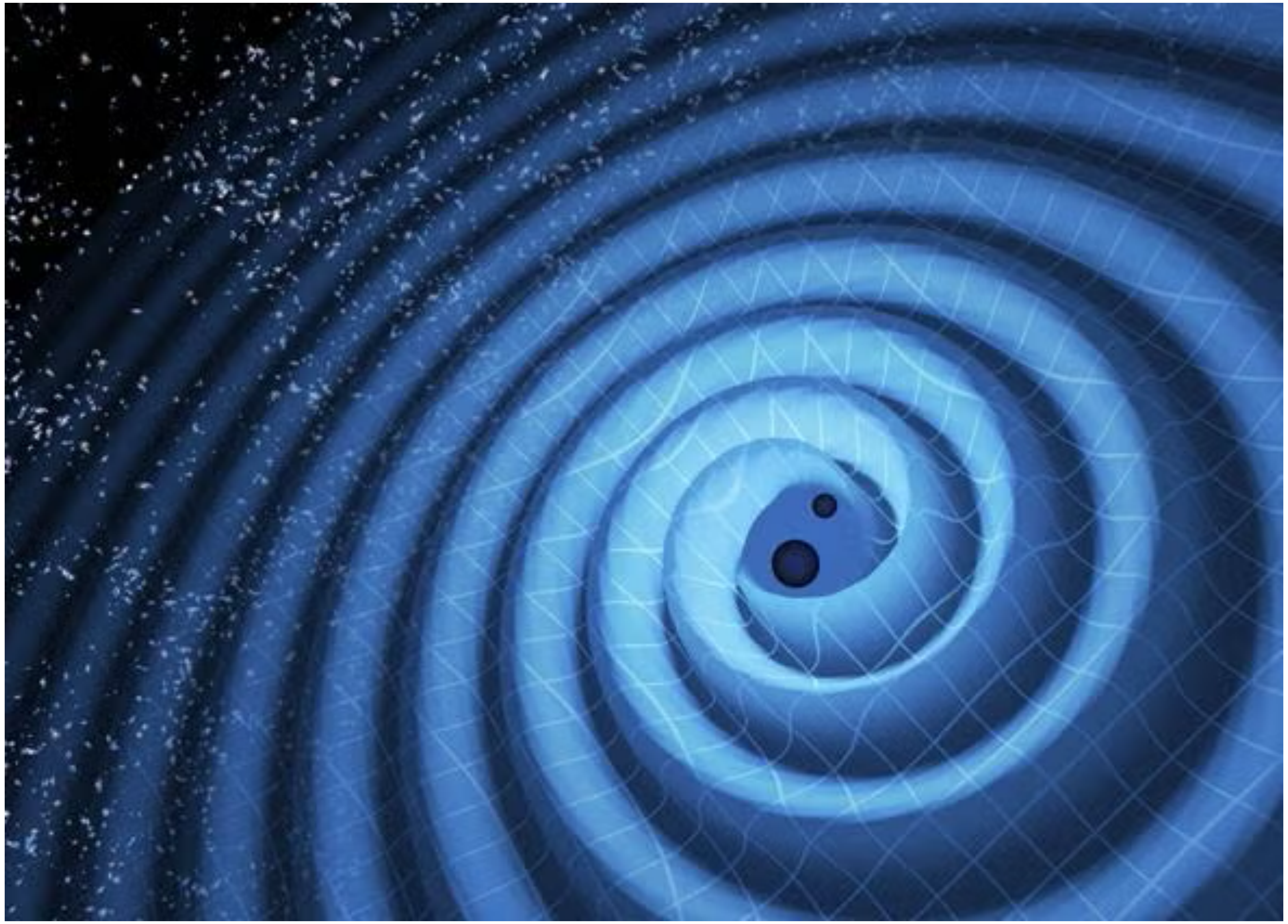
By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

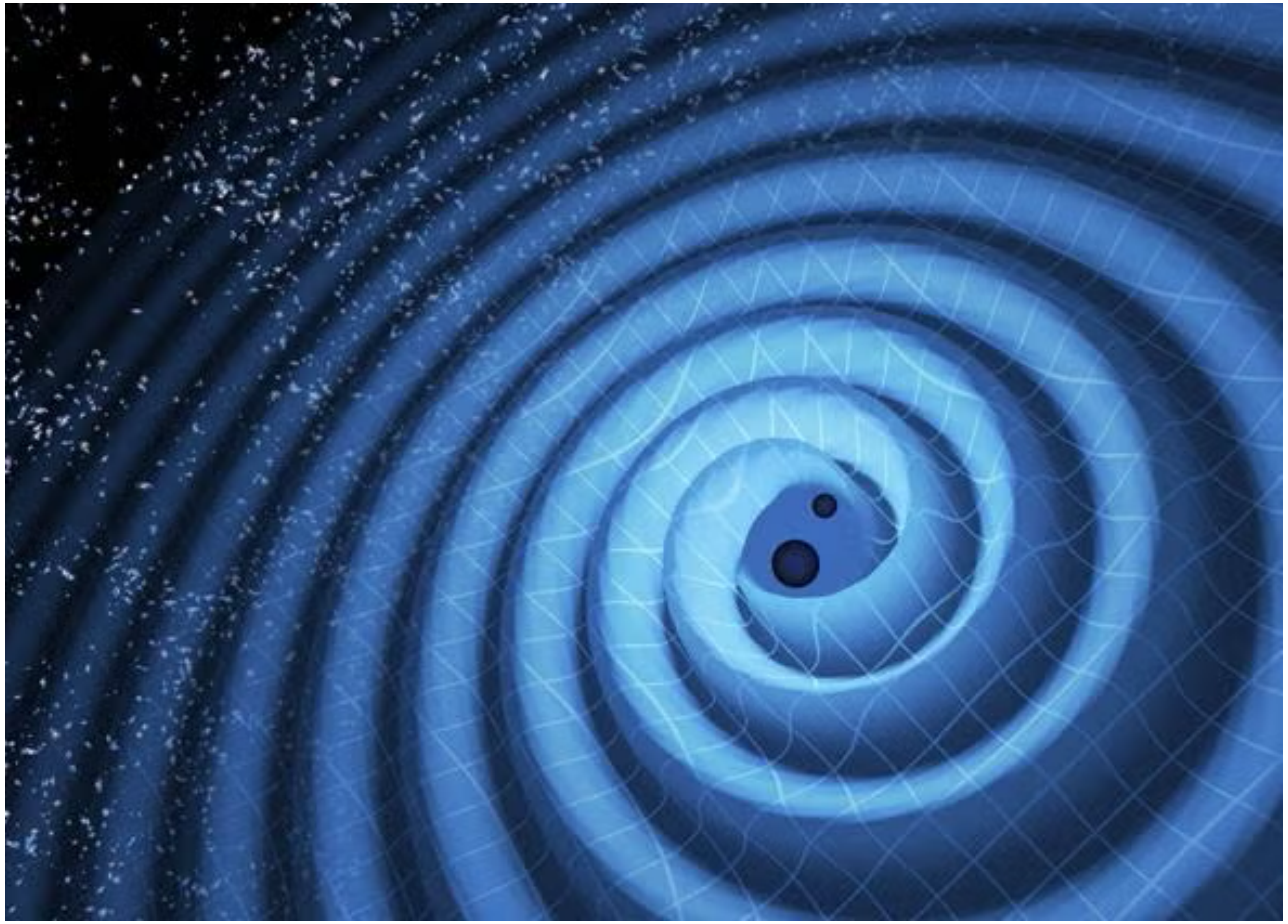


Artwork depicting gravitational waves emanating from two black holes coalescing.
LIGO/T. Pyle

$$\tau_{\text{ring-down}} \sim \frac{8\pi GM}{c^3}$$

C.V. Vishveshwara, Nature **227**, 936 (1970)

Quantum Entanglement across a black hole horizon



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C.V. Vishveshwara, Nature **227**, 936 (1970)

Using the Hawking temperature of the black hole

$$T_H = \frac{\hbar c^3}{8\pi G M k_B},$$

$$\tau_{\text{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics of quasi-normal modes!

S. Sachdev, PRL **105**, 151602 (2010)

Quantum Entanglement across a black hole horizon

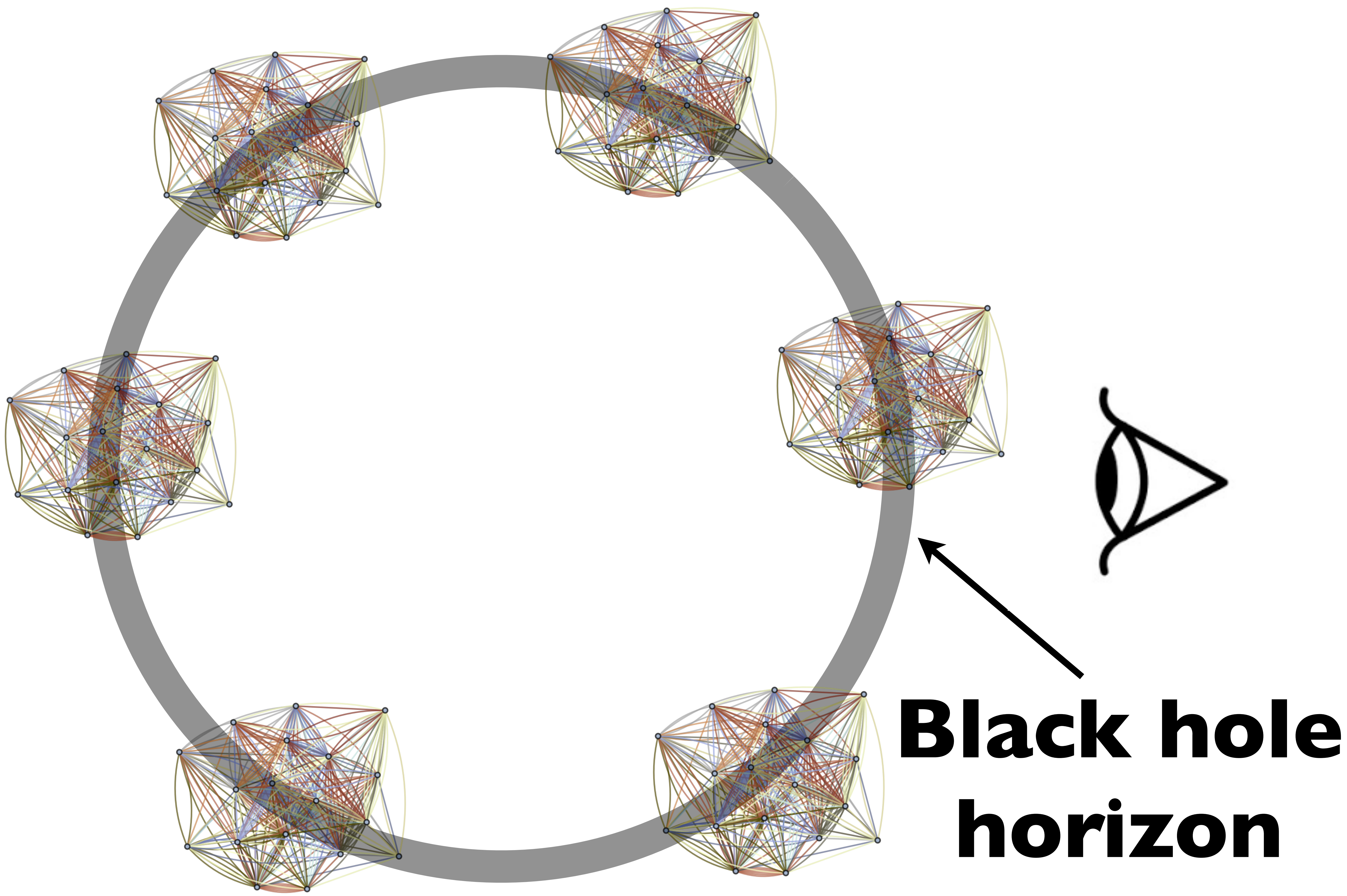
Quantum entanglement on the surface

S. Sachdev, PRL **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

"... This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R_2$ physics of Reissner- Nordström black holes."



Quantum Entanglement across a black hole horizon

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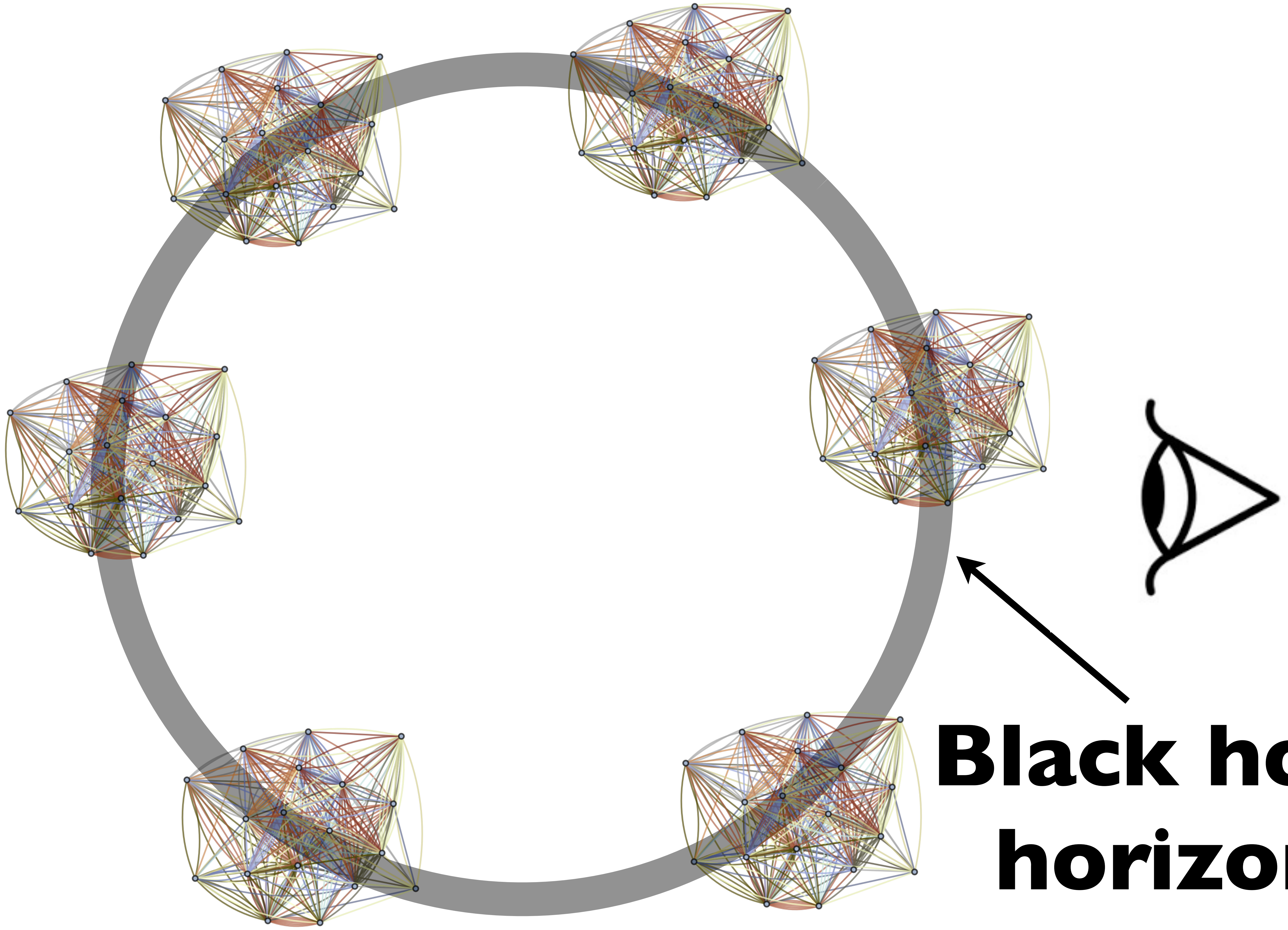
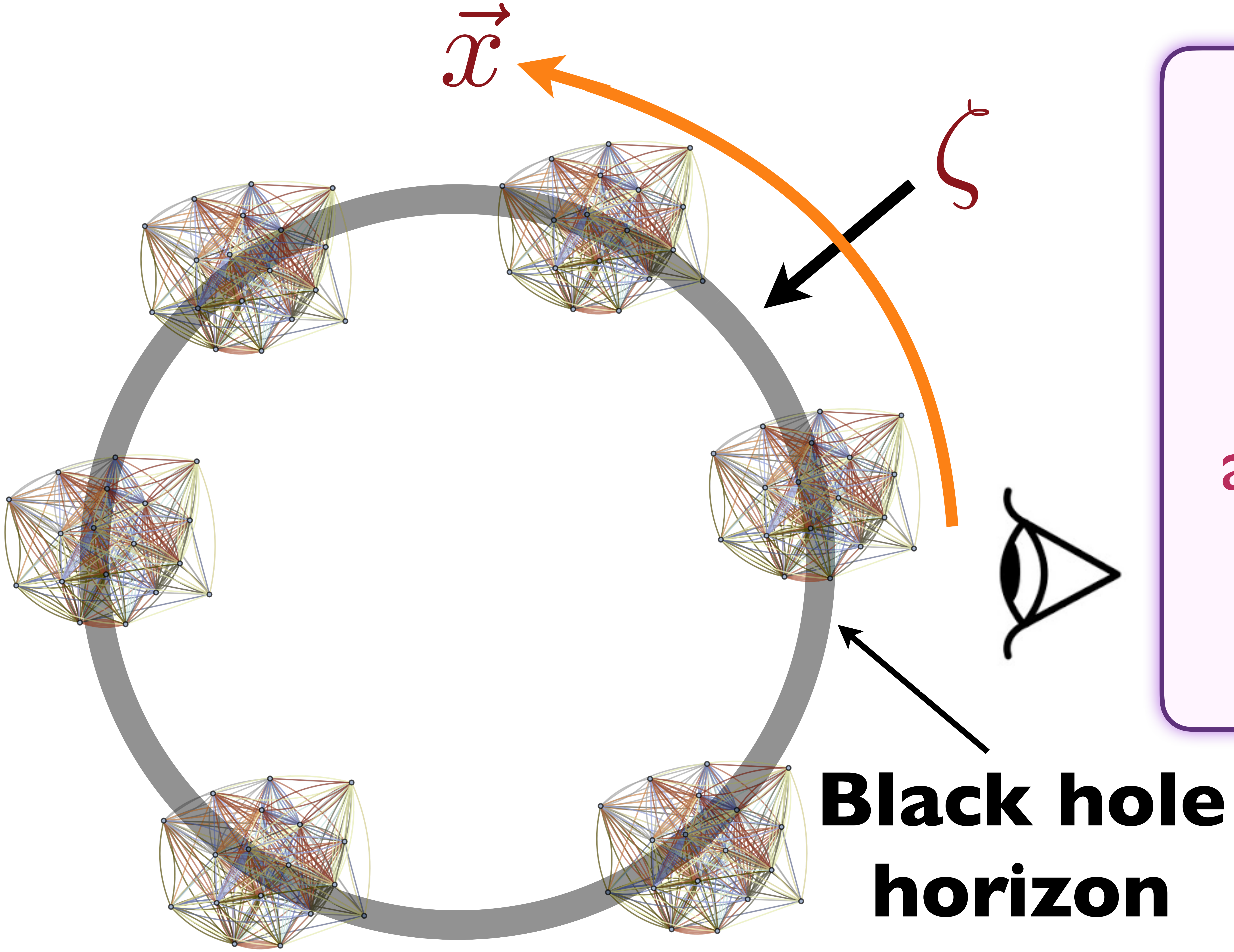


i.e. SYK models (after 2015)

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

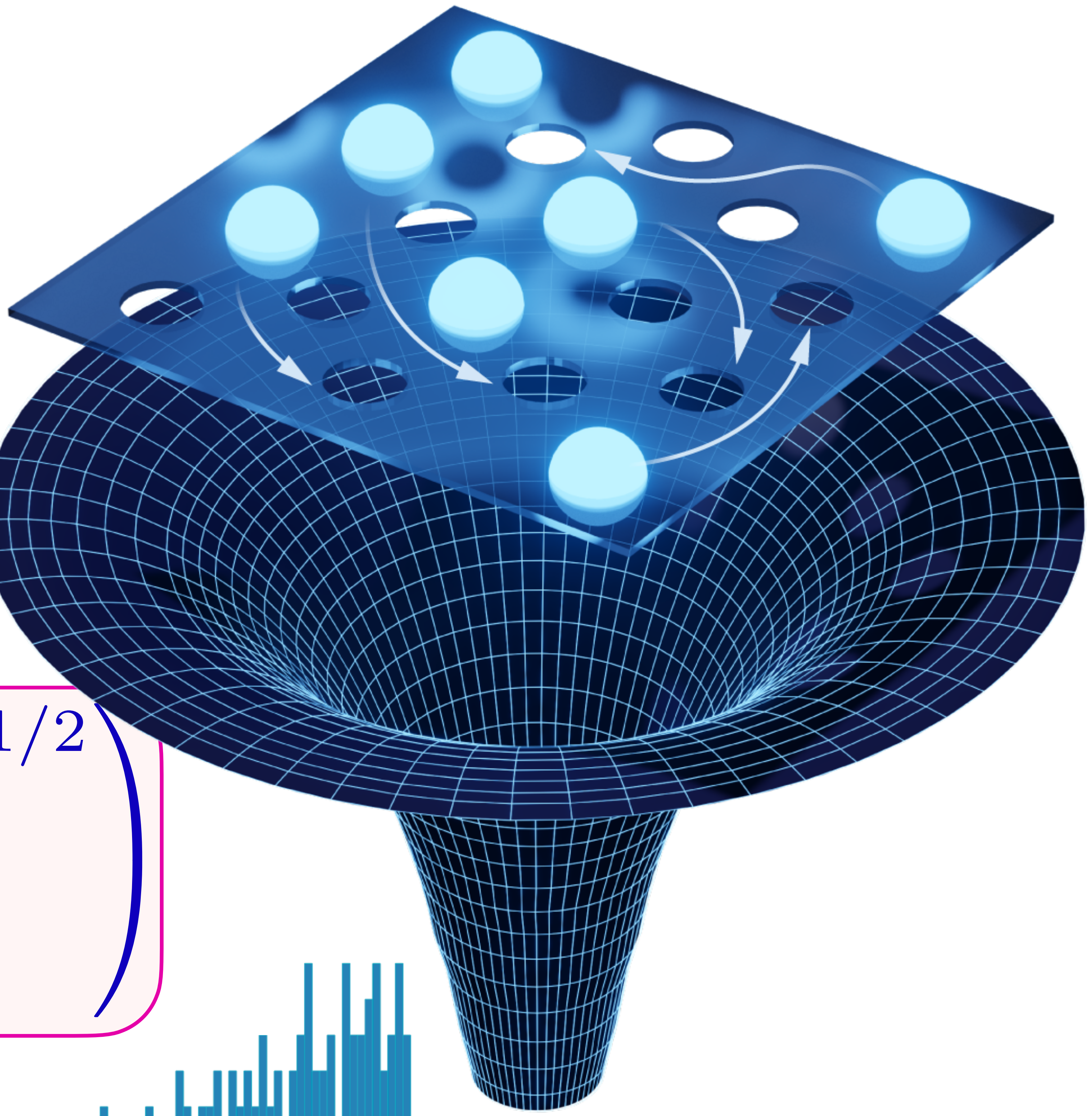


The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

Kitaev (2015), Maldacena Stanford (2015)

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

D(E) of charged black holes from the SYK model



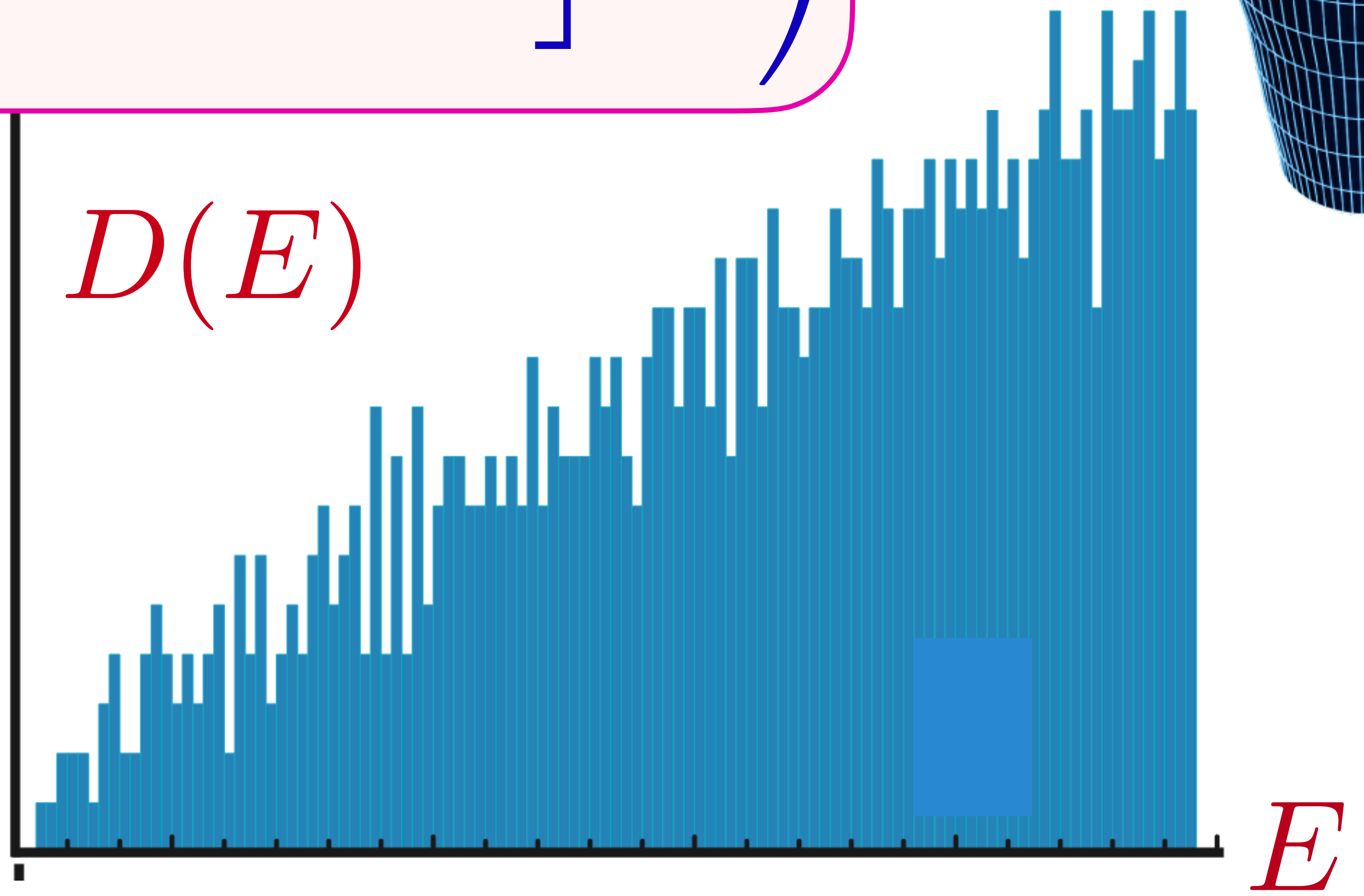
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}{A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

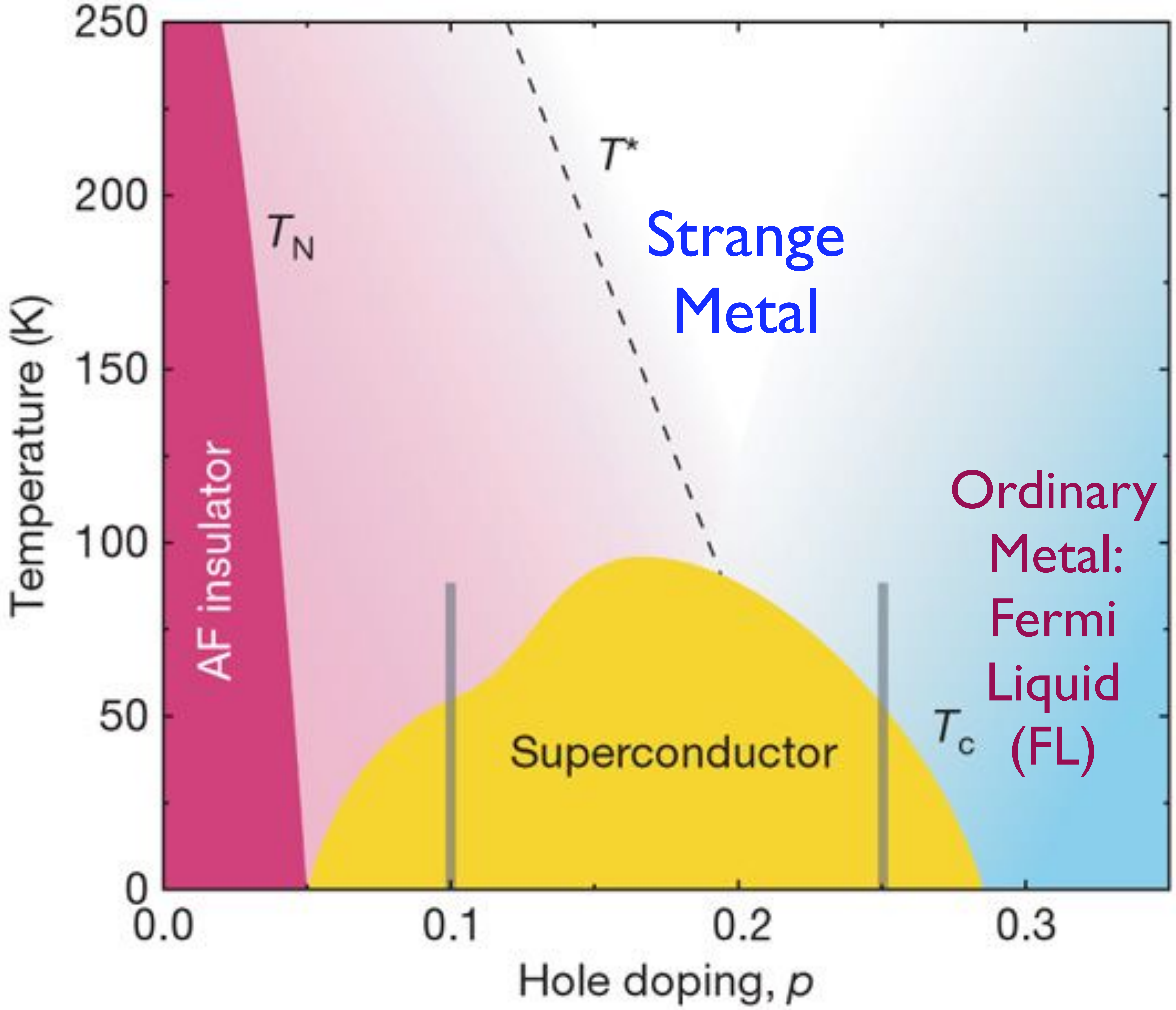
Developments from the SYK model

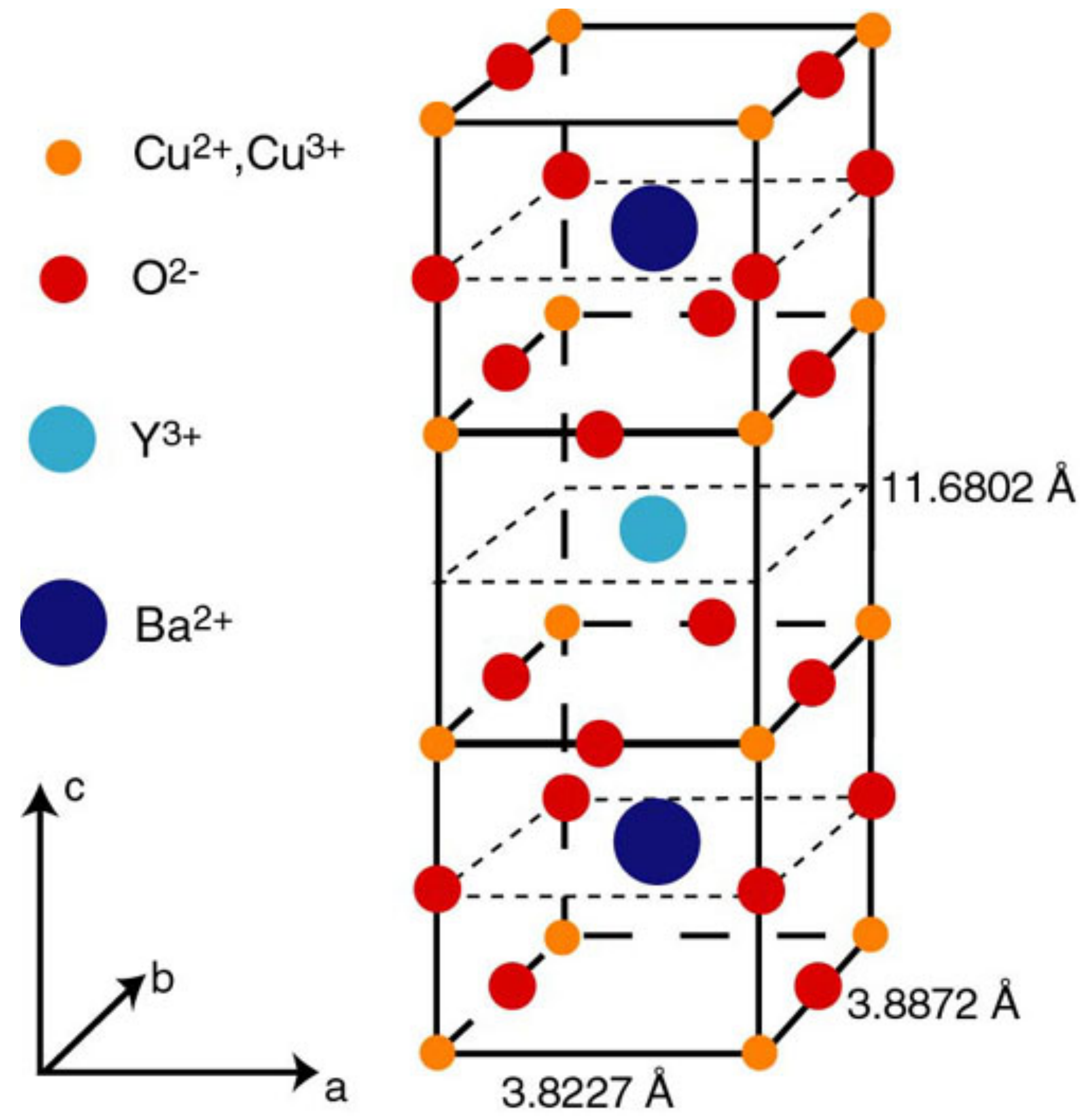


Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

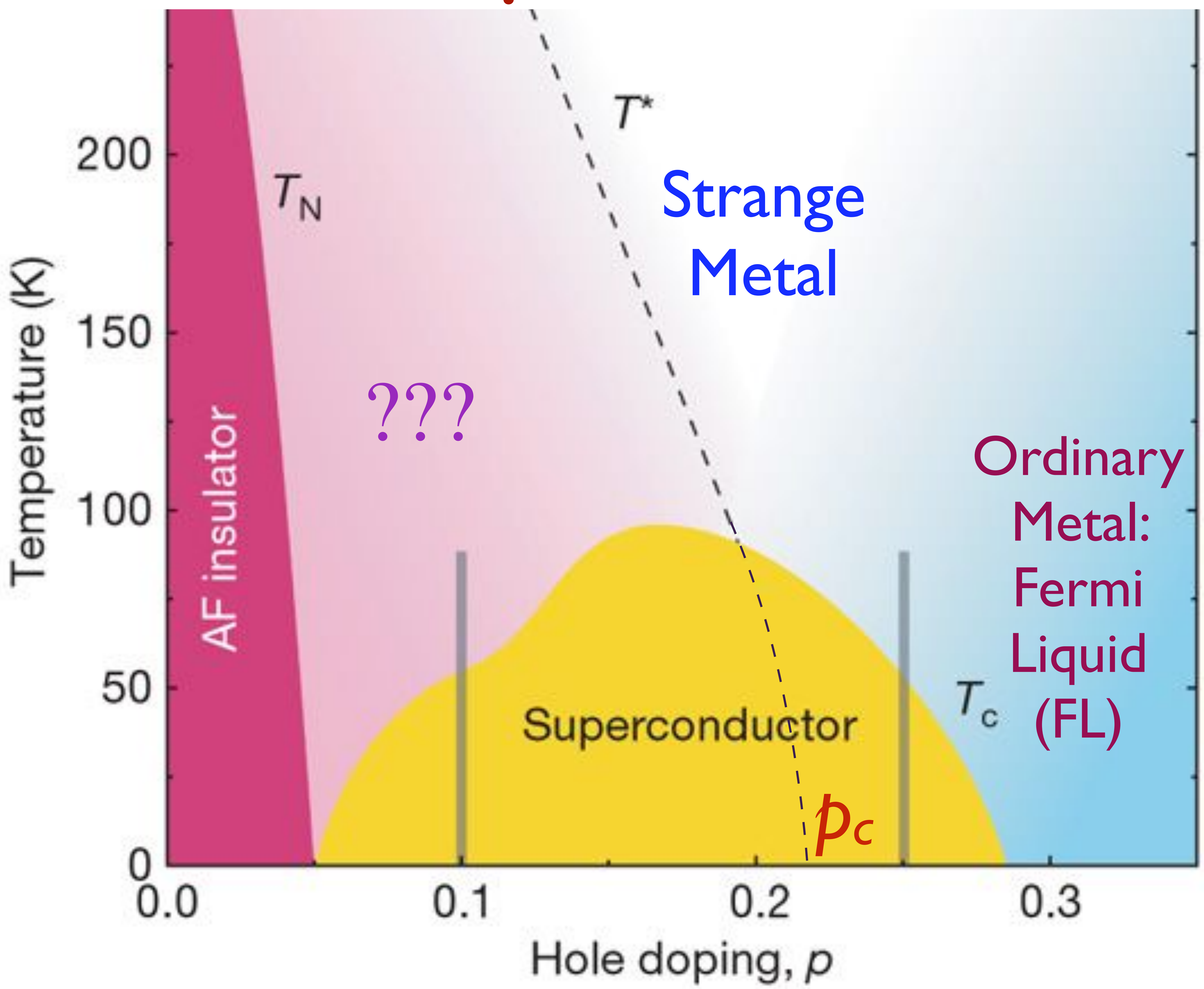
Quantum phase transitions in metals and Planckian dynamics

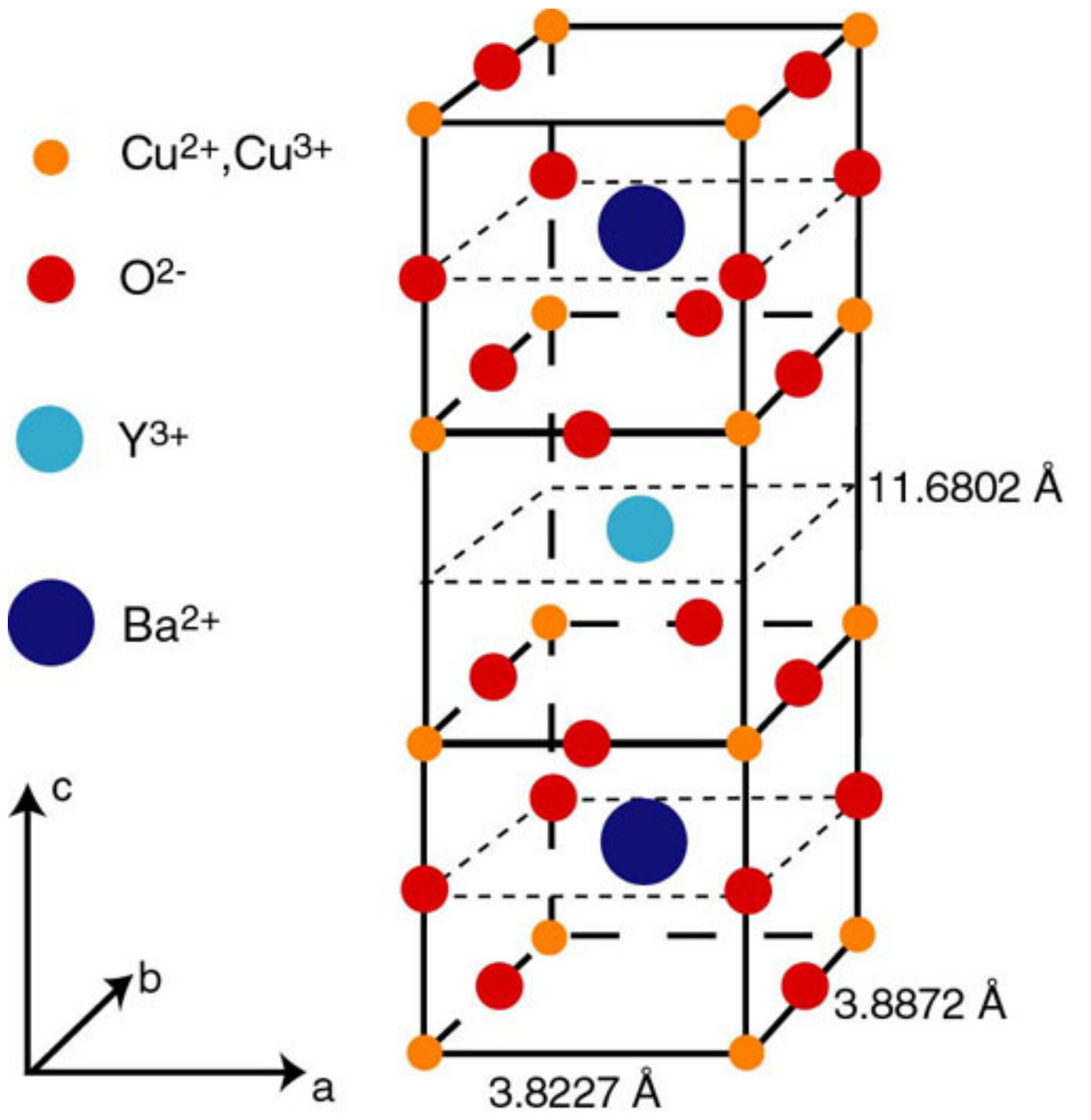

250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
T_N
T*
Strange Metal
AF insulator
Superconductor
T_c
Ordinary Metal: Fermi Liquid (FL)


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

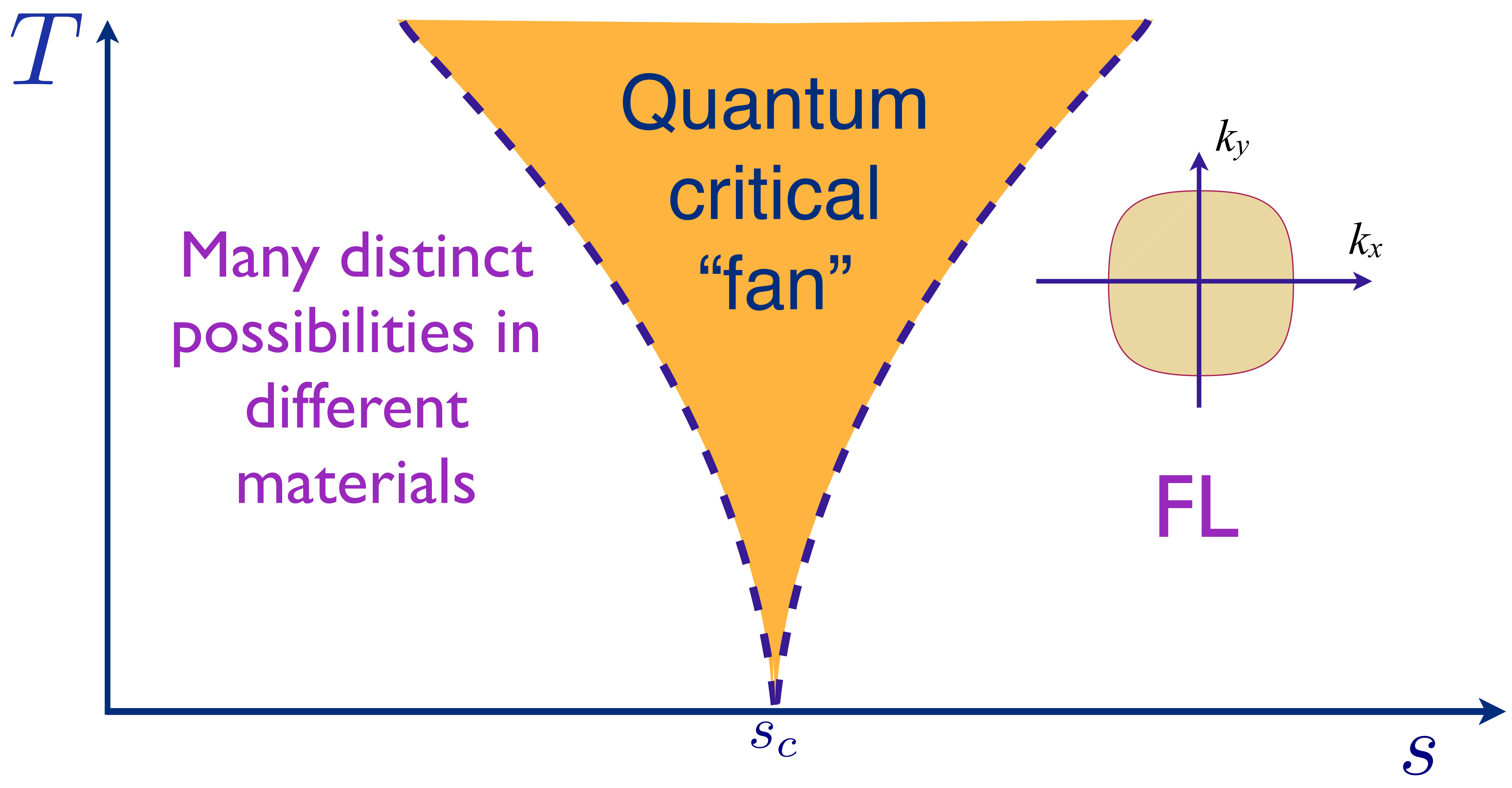
Quantum phase transition between two metals





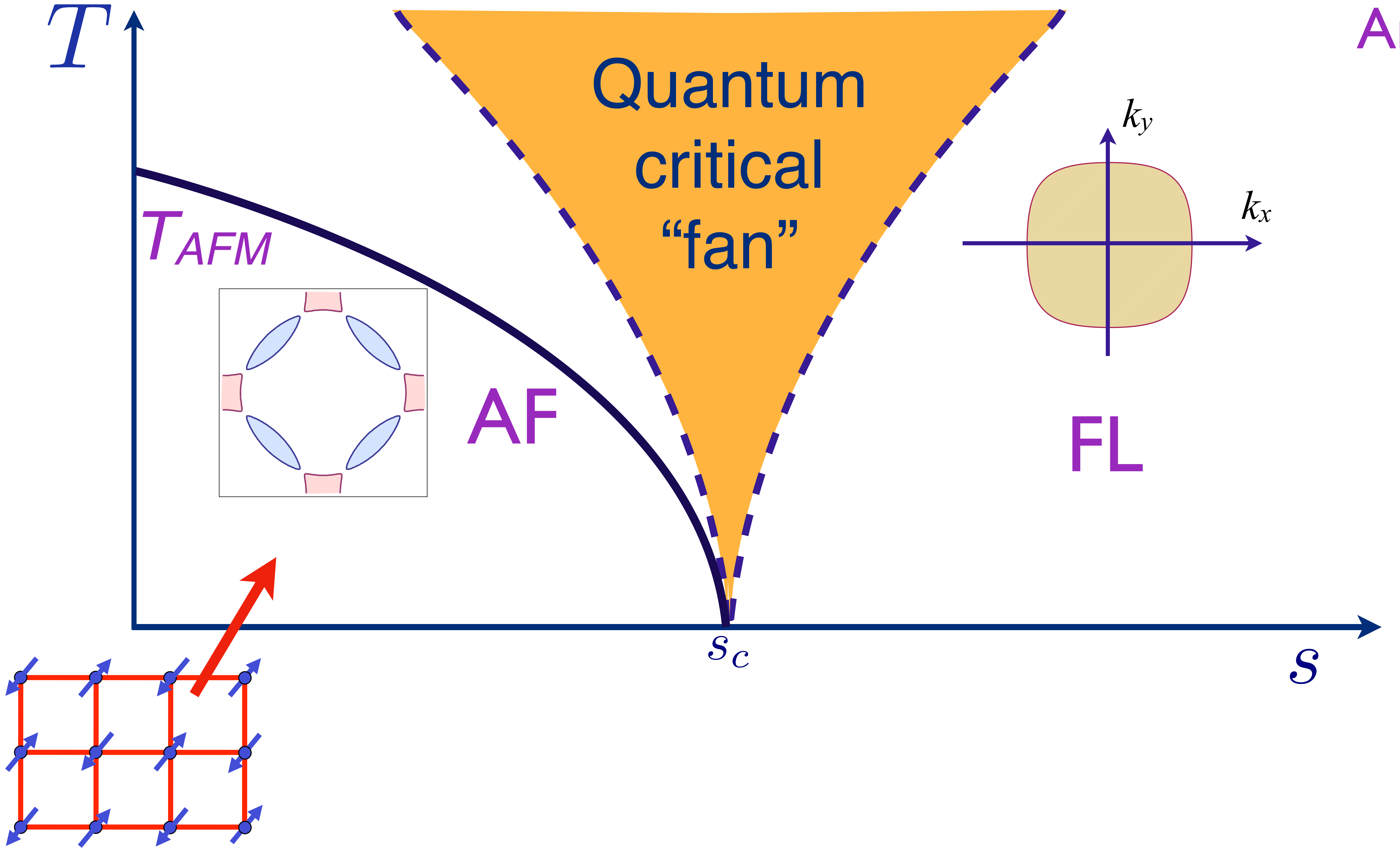
$YBa_2Cu_3O_{6+x}$

Quantum phase transition between two metals



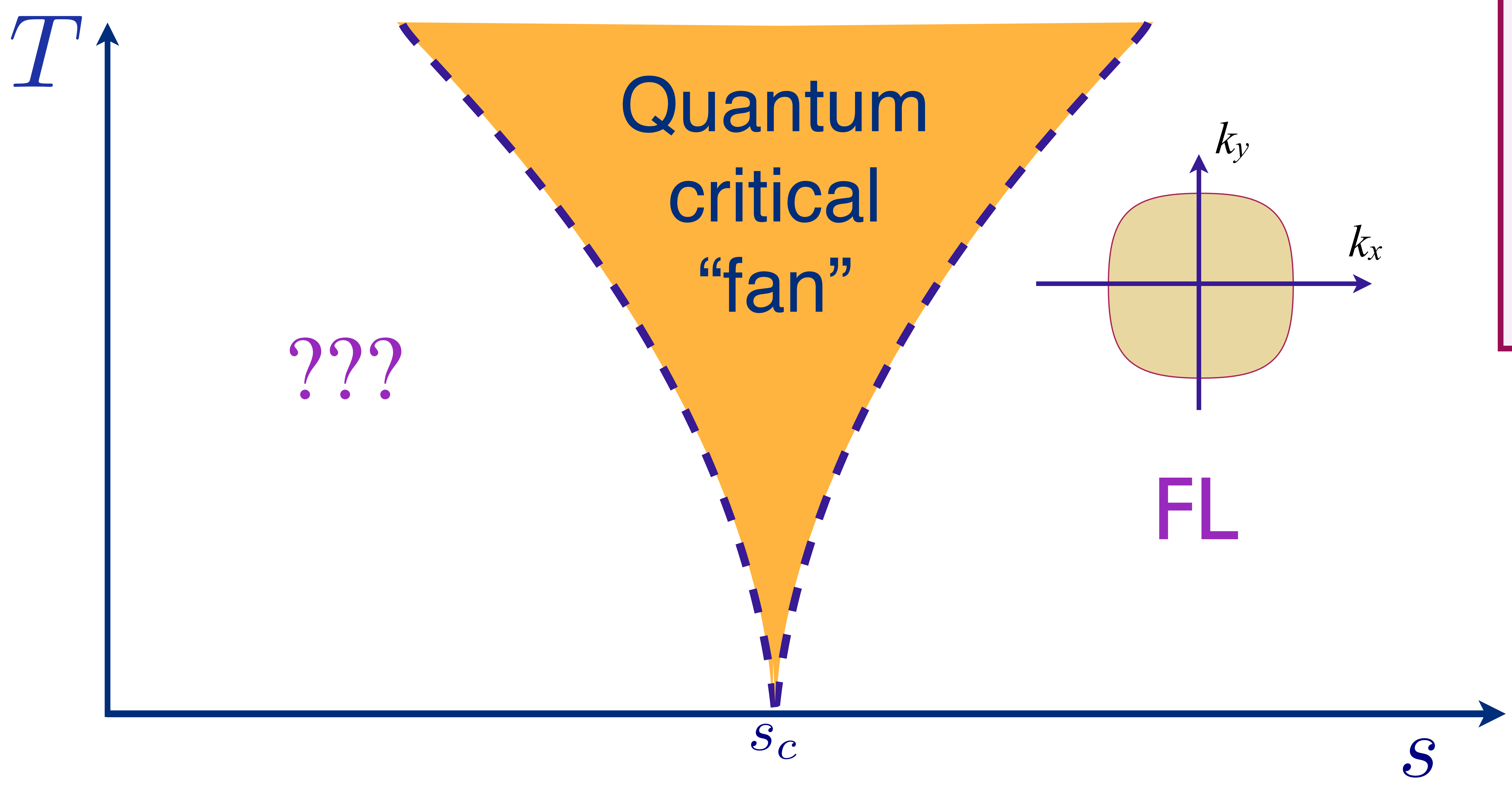
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Quantum phase transition between two metals



Antiferromagnetism
in the pnictides
and
electron-doped
cuprates

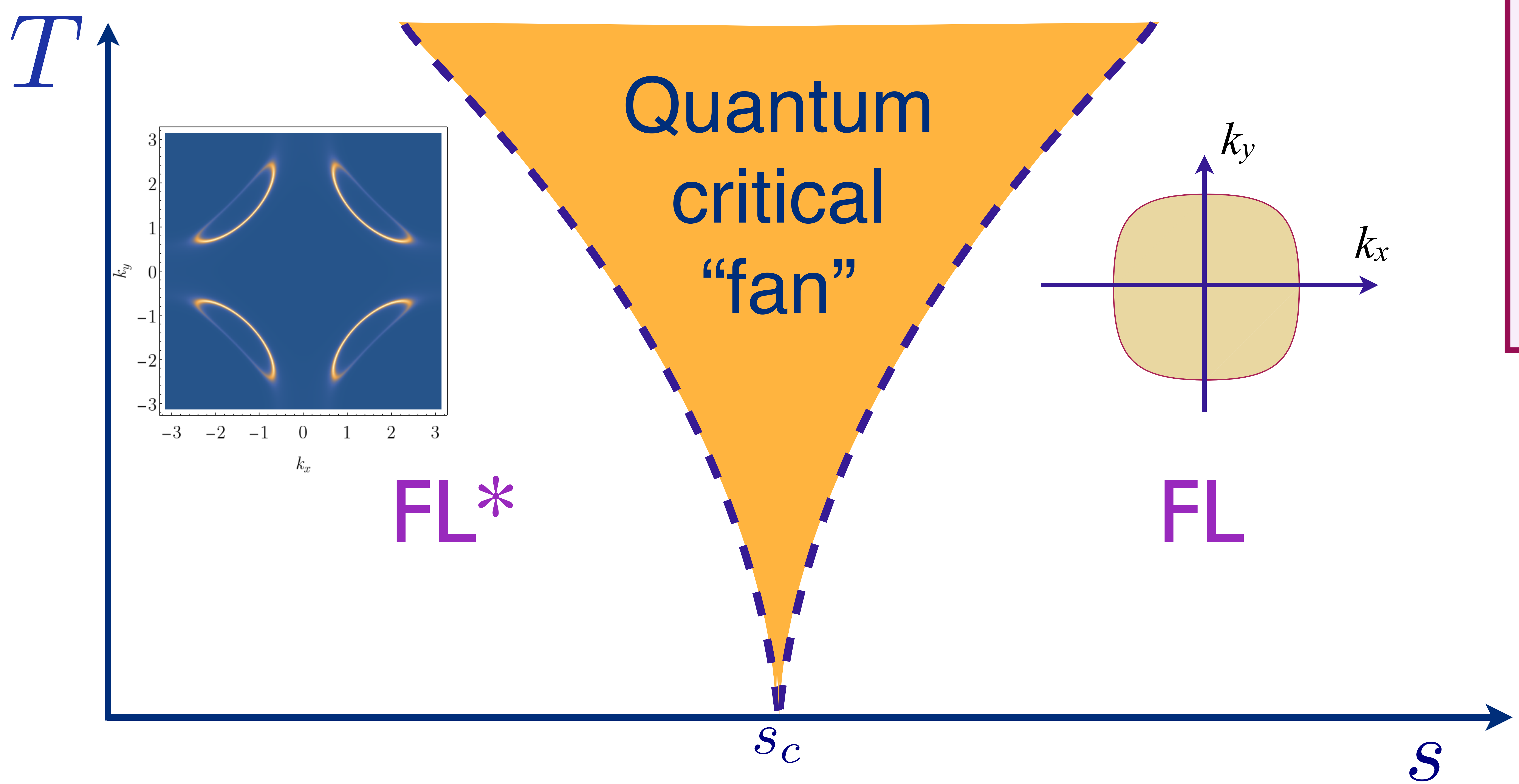
Quantum phase transition between two metals

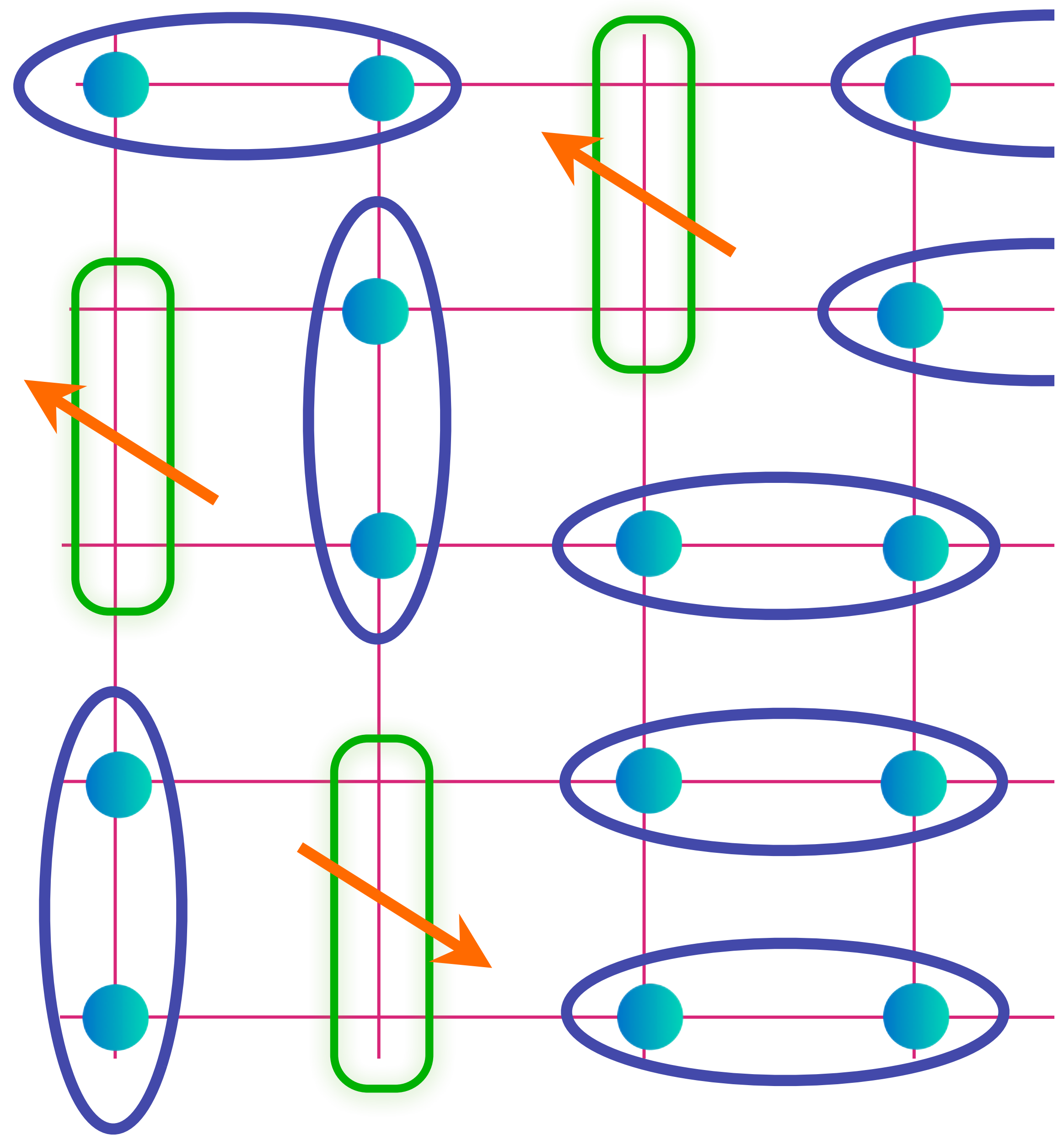


No obvious candidate with broken symmetry in hole-doped cuprates

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004); S. S., M. A. Metlitski, and M. Punk, J. Phys.: CM **24**, 294205 (2012)

Quantum phase transition between two metals

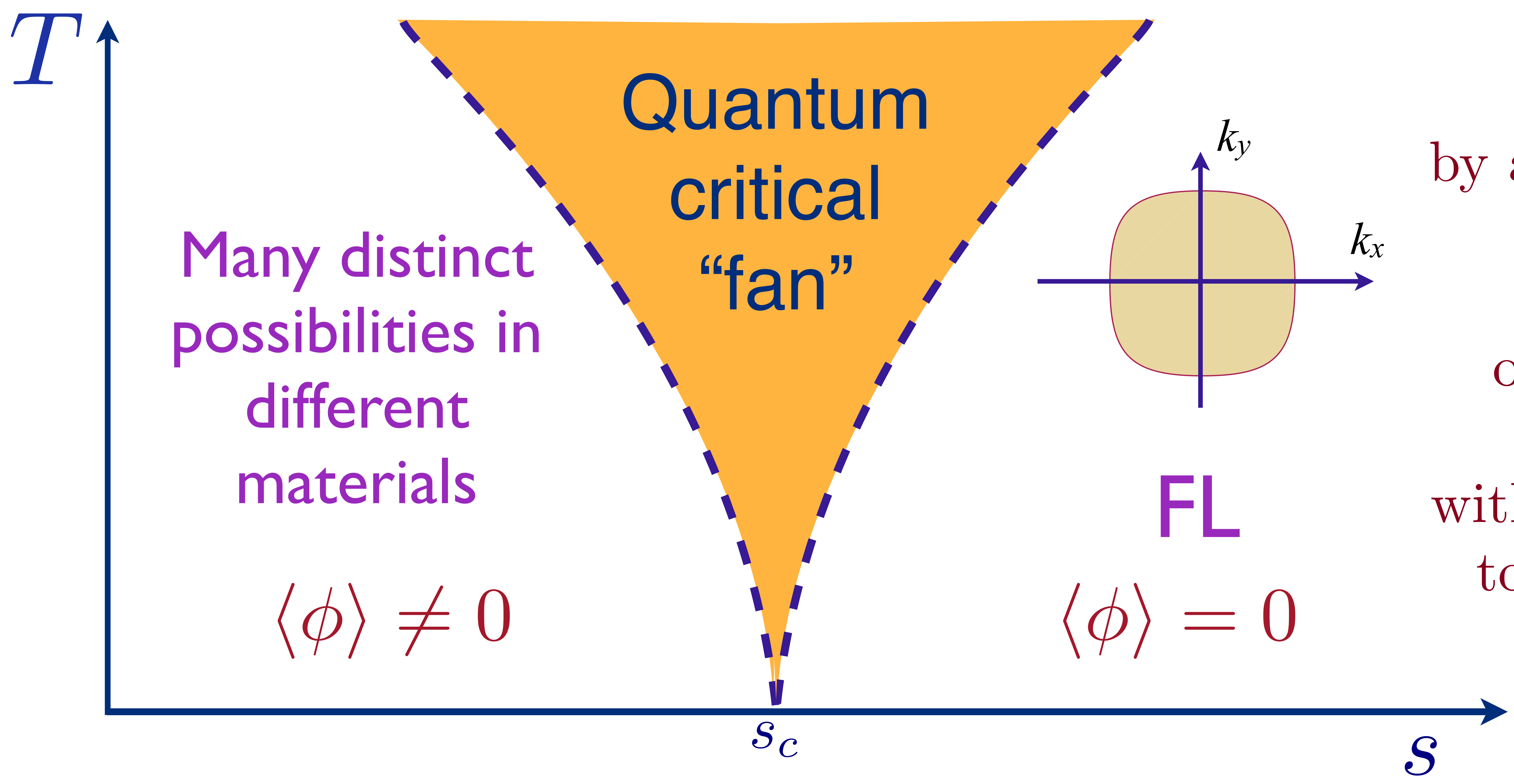




Fractionalized Fermi Liquid (FL*) in hole-doped cuprates

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004); S. S., M.A. Metlitski, and M. Punk, J. Phys.: CM **24**, 294205 (2012)

Quantum phase transition between two metals



All possibilities are described (with impurities) by a *universal* theory of a scalar field ϕ (representing an order parameter or a Higgs field) with a Yukawa coupling to fermions near the Fermi surface.

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK model

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

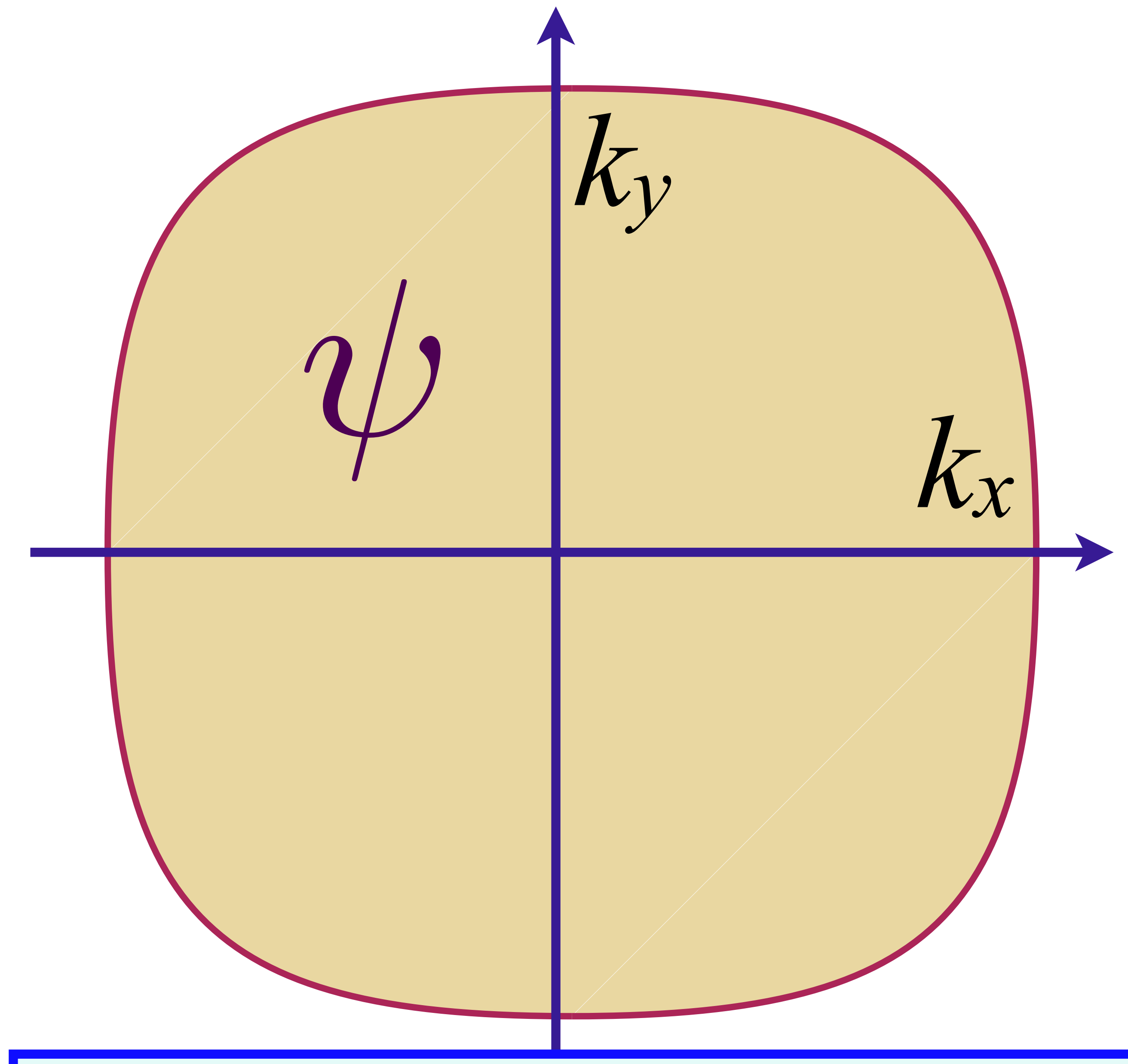
A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

2d-YSYK model: Fermi surface + critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$+s\,[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

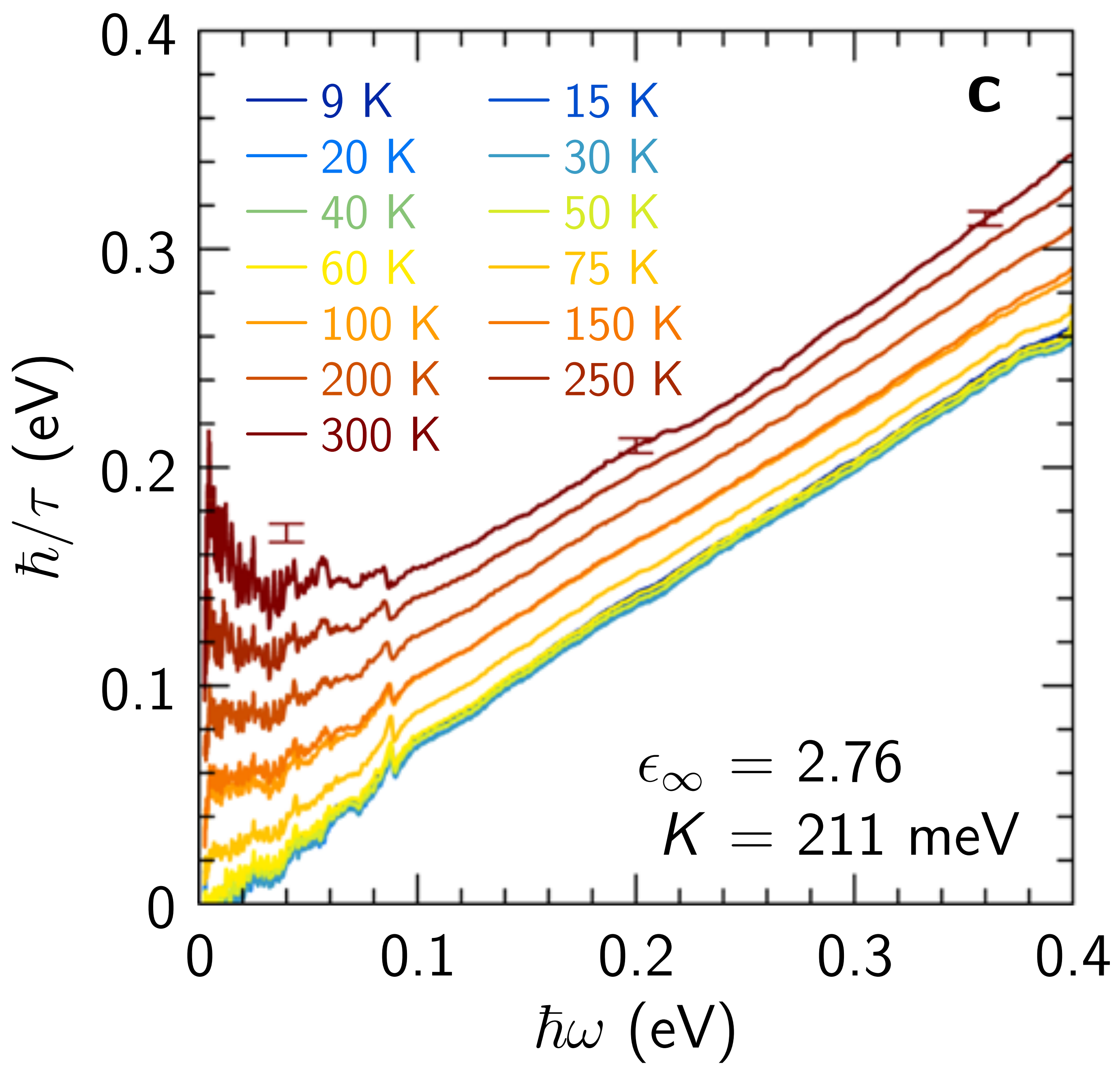
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2 + u\,[\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

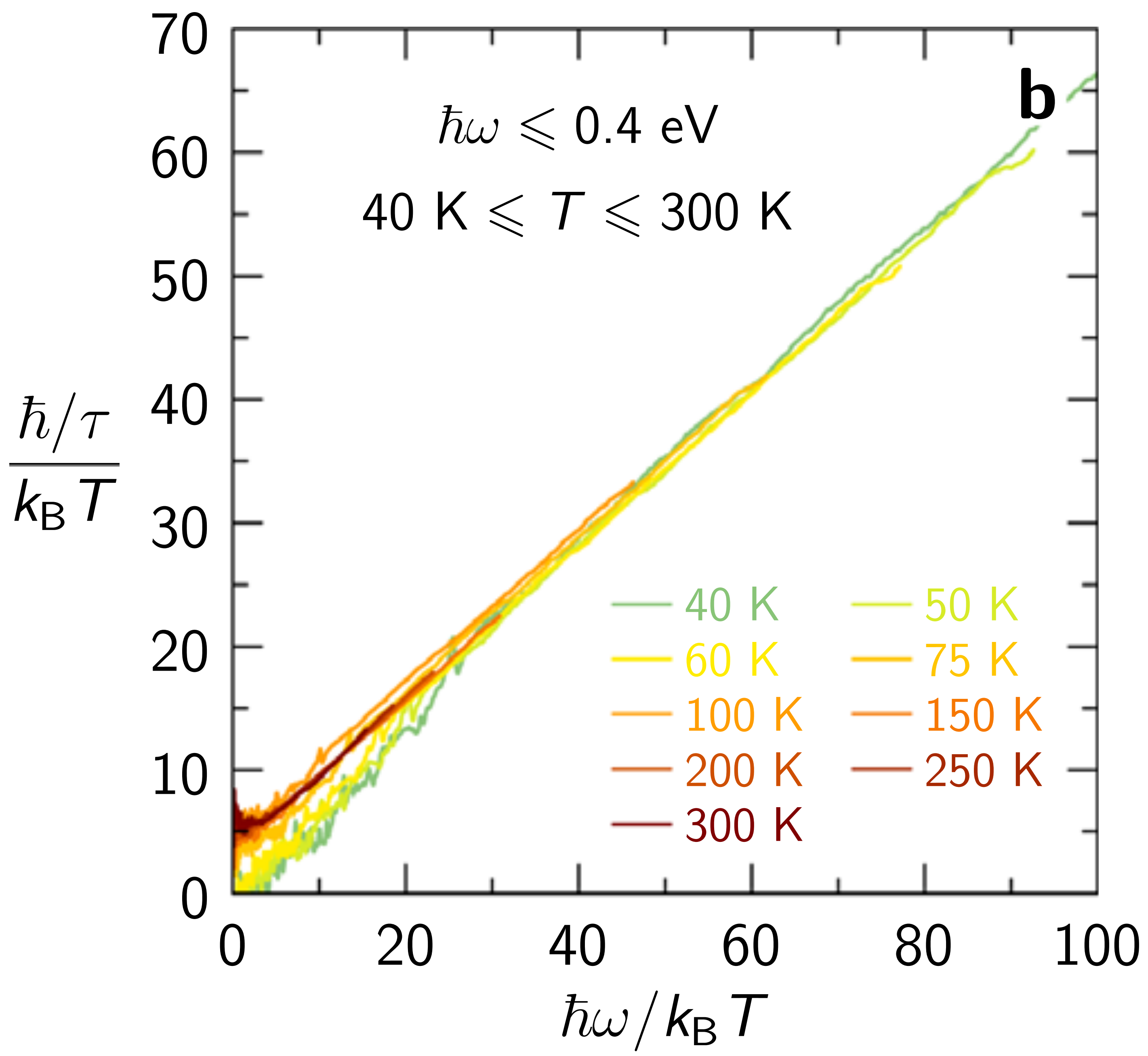
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder): the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

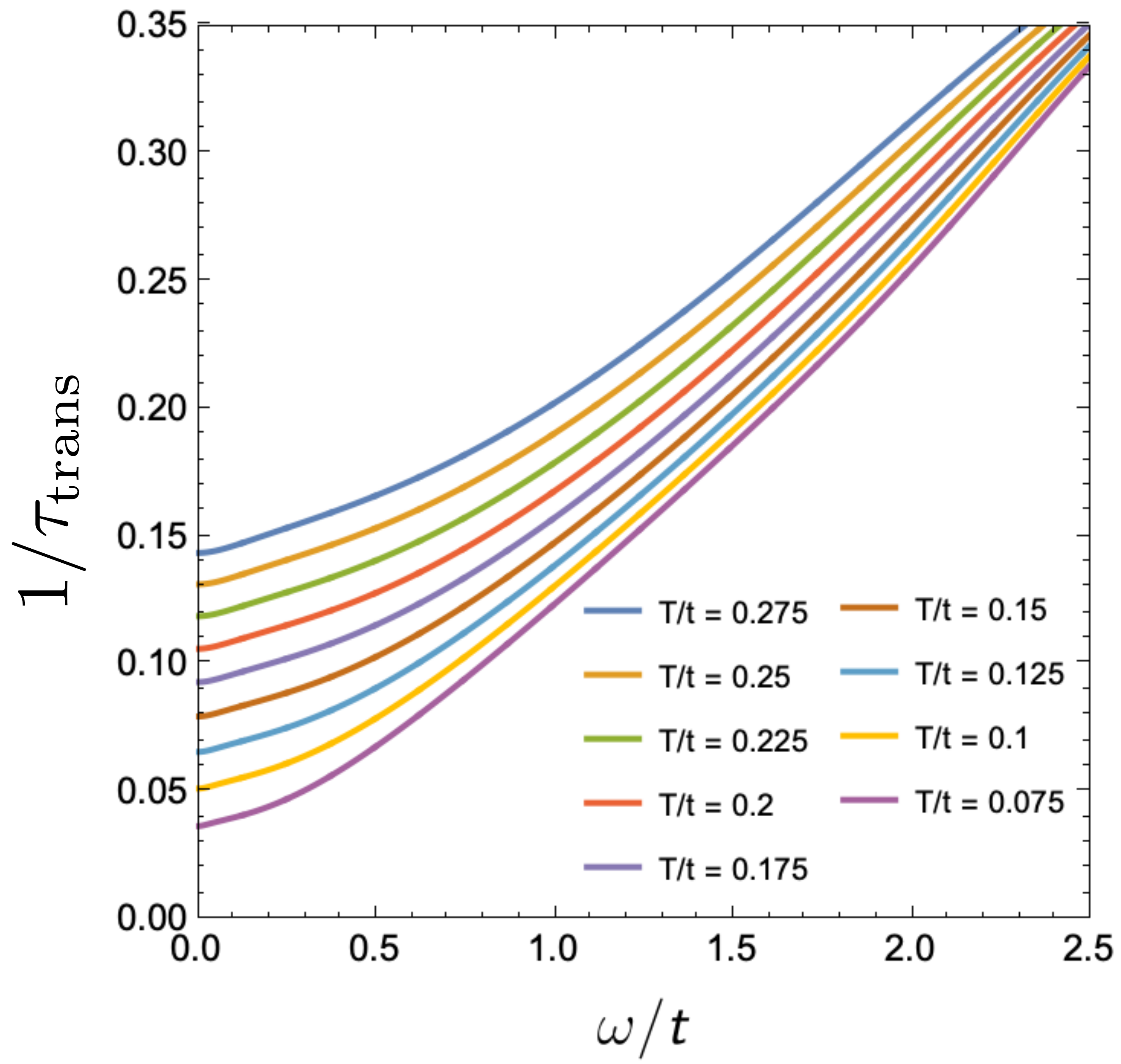


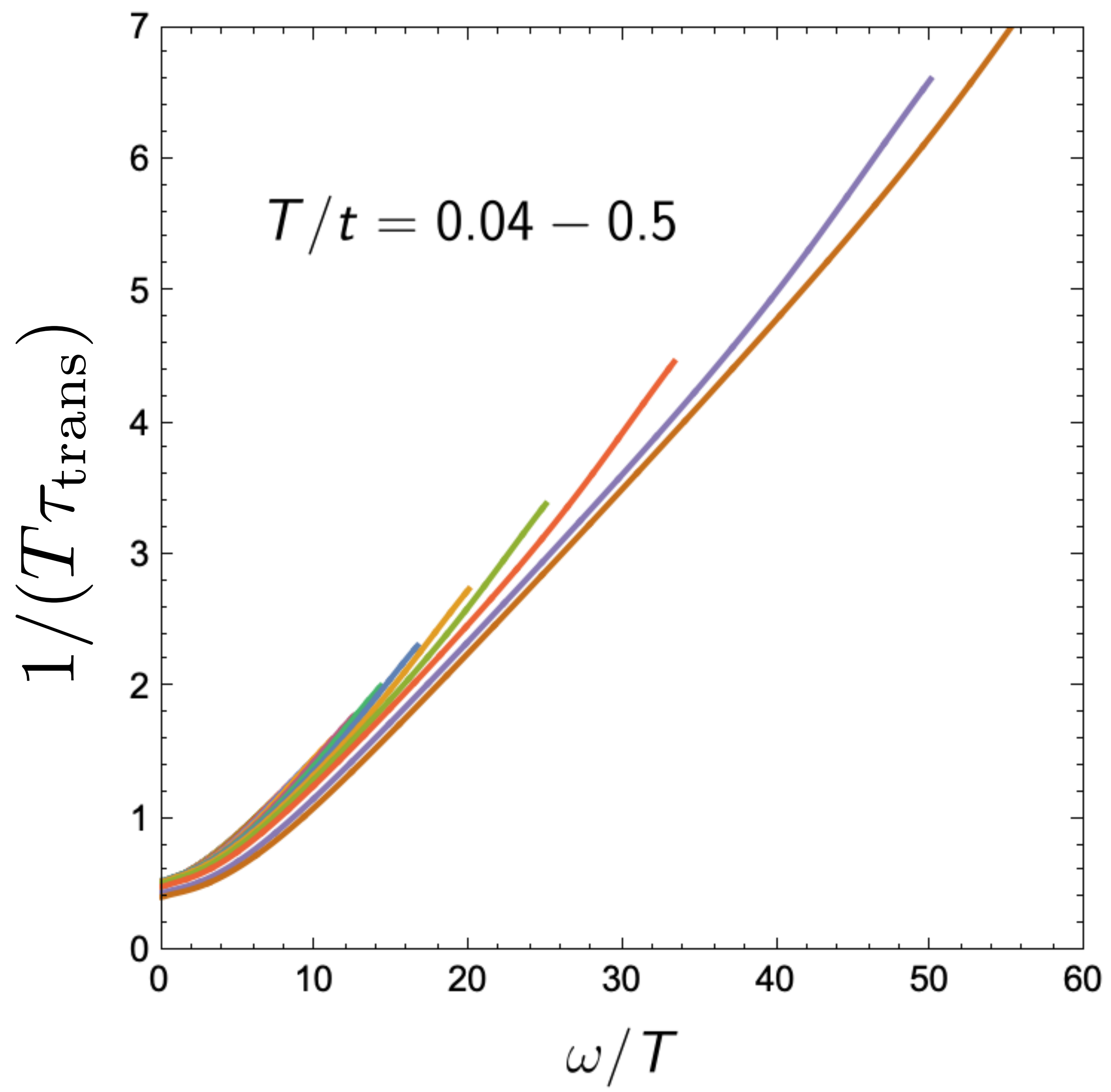


$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$





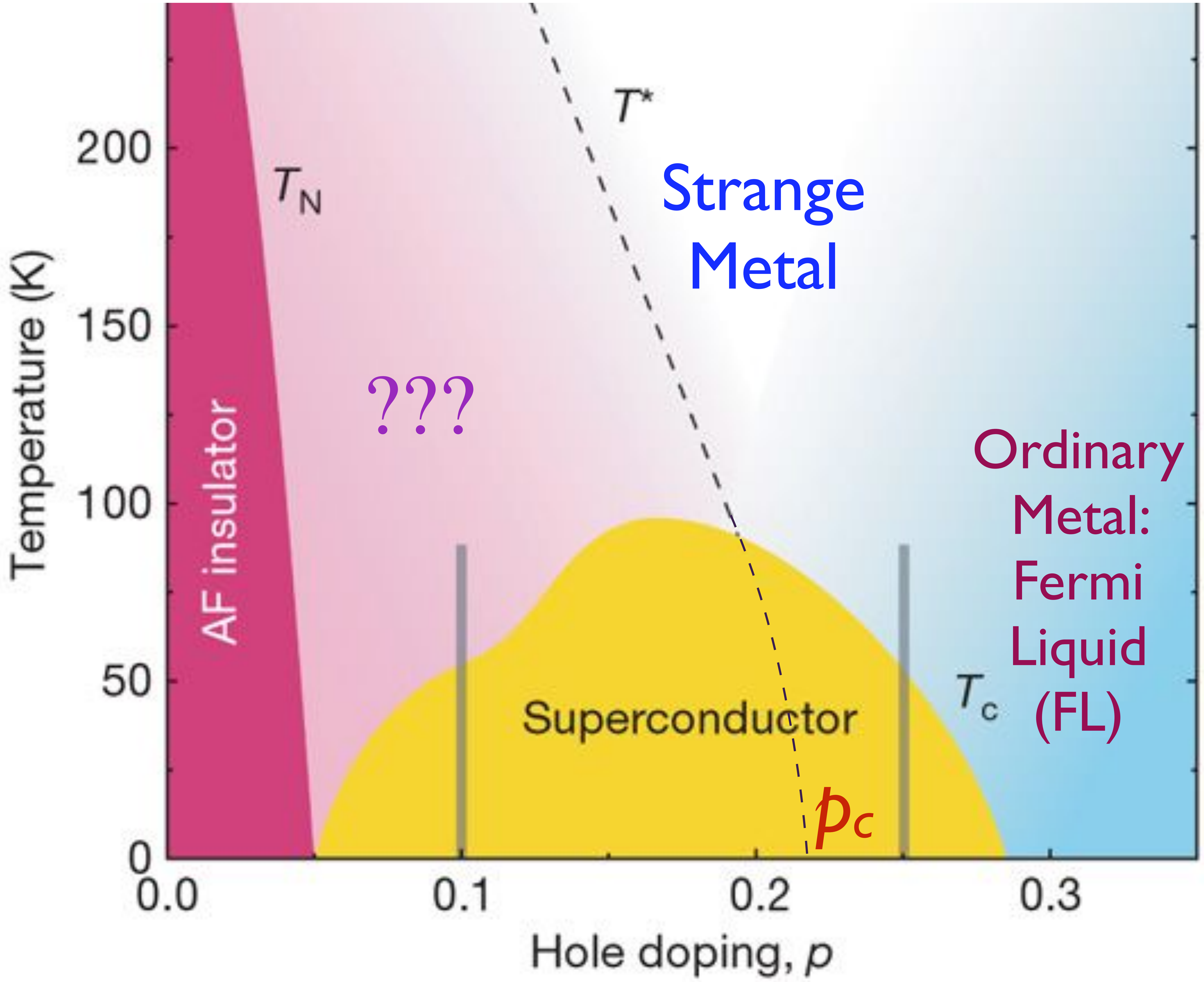
2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

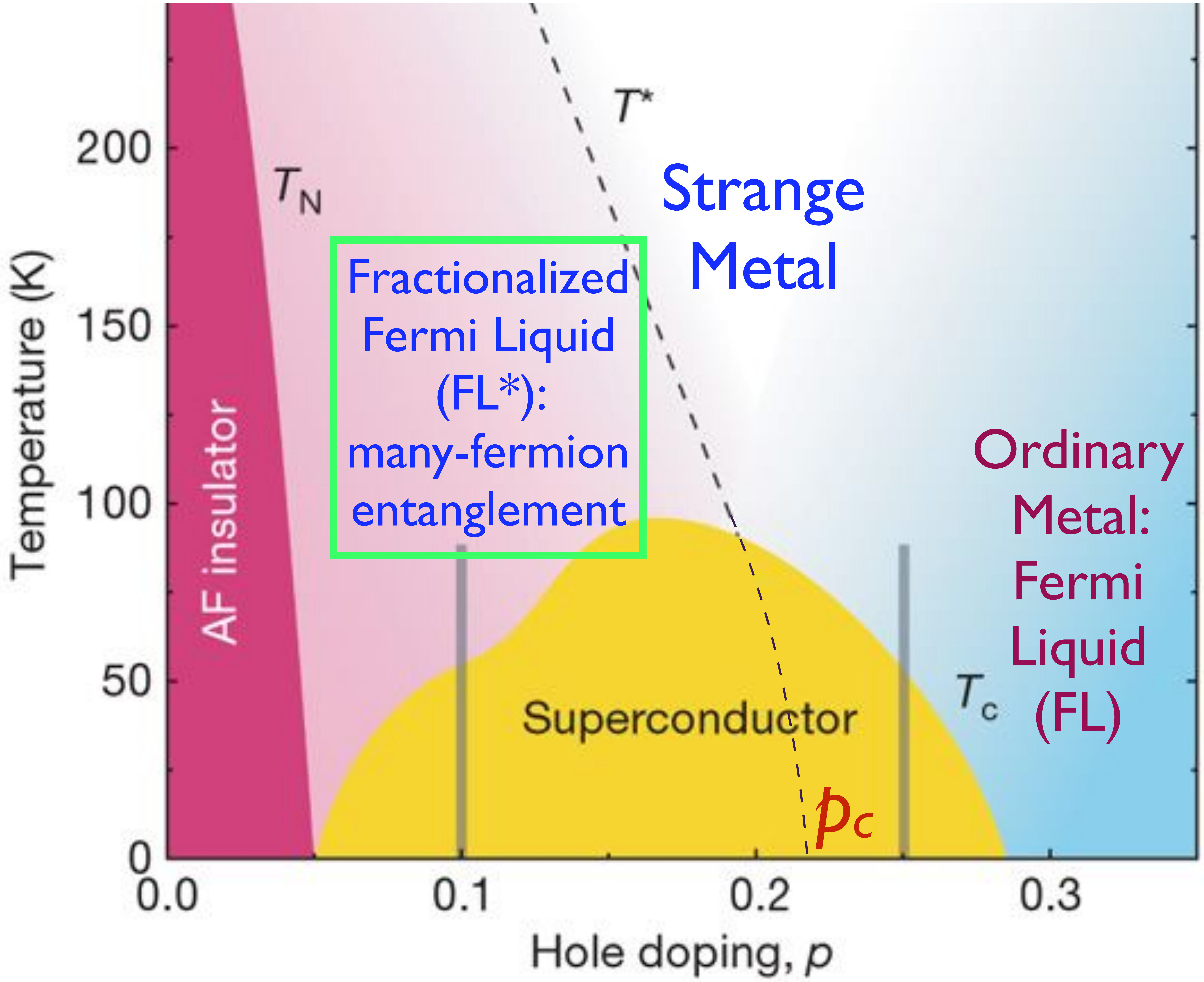
Fractionalized
Fermi liquid (FL*)
in the hole-doped cuprates

Quantum phase transitions in two-dimensional metals



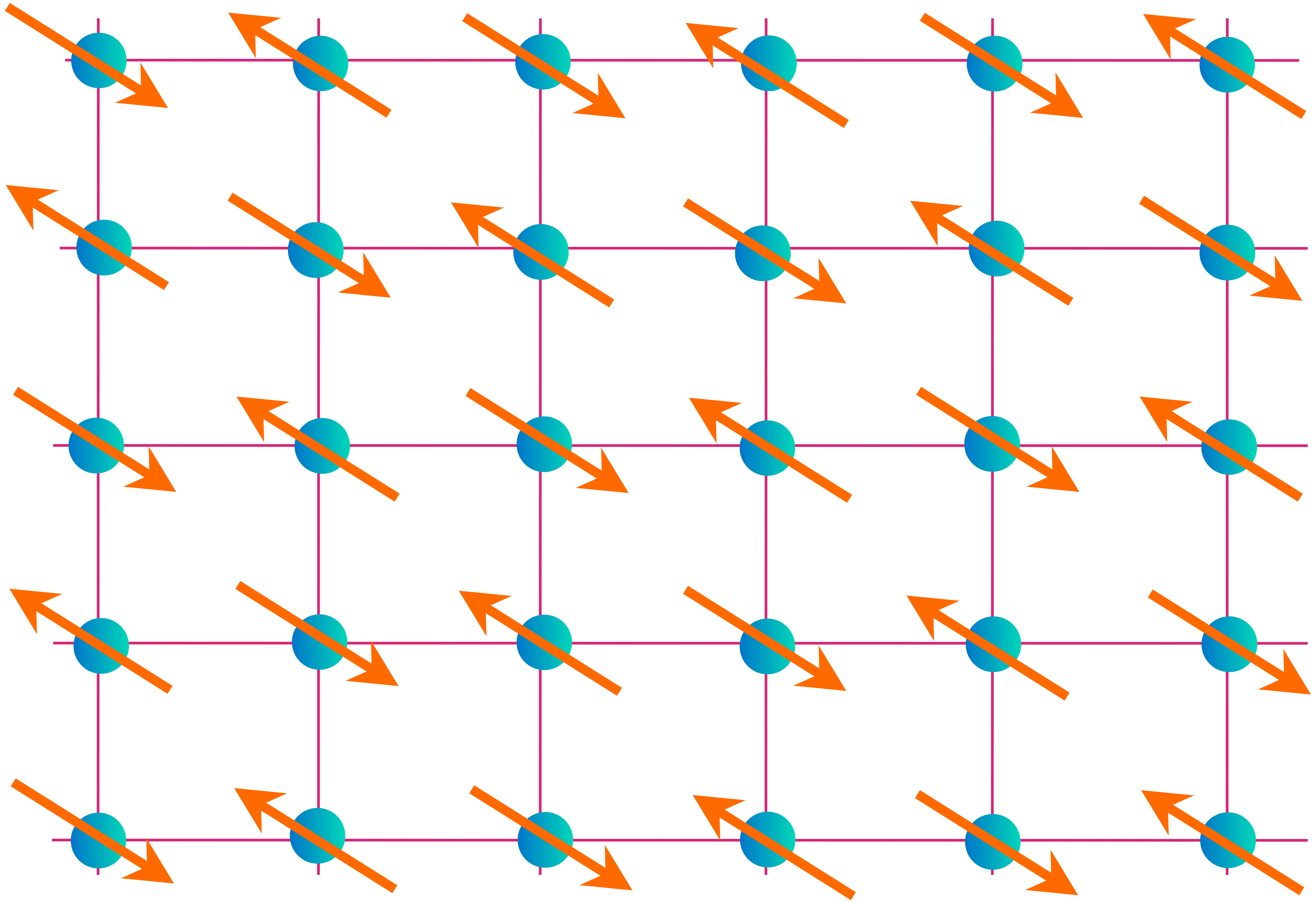
How can there be a quantum phase transition at p_c if there is no clear ordering at small p ?

Quantum phase transitions in two-dimensional metals

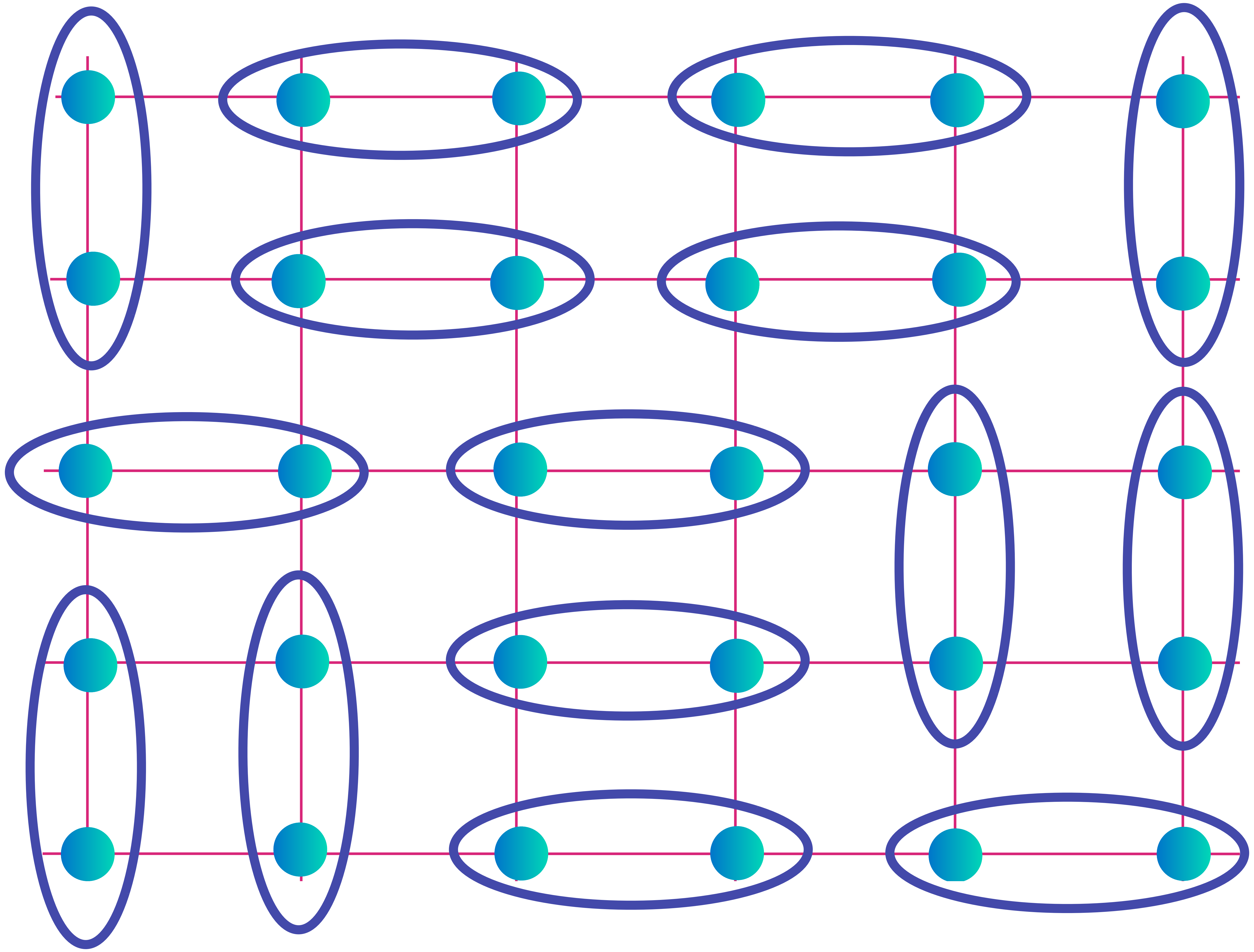


FL*:
Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

Antiferromagnet

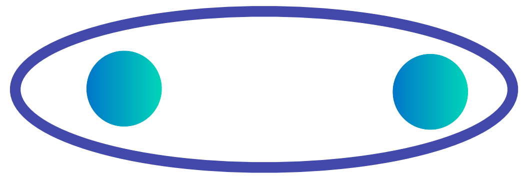


Anderson's Resonating Valence Bond (1972,1987)

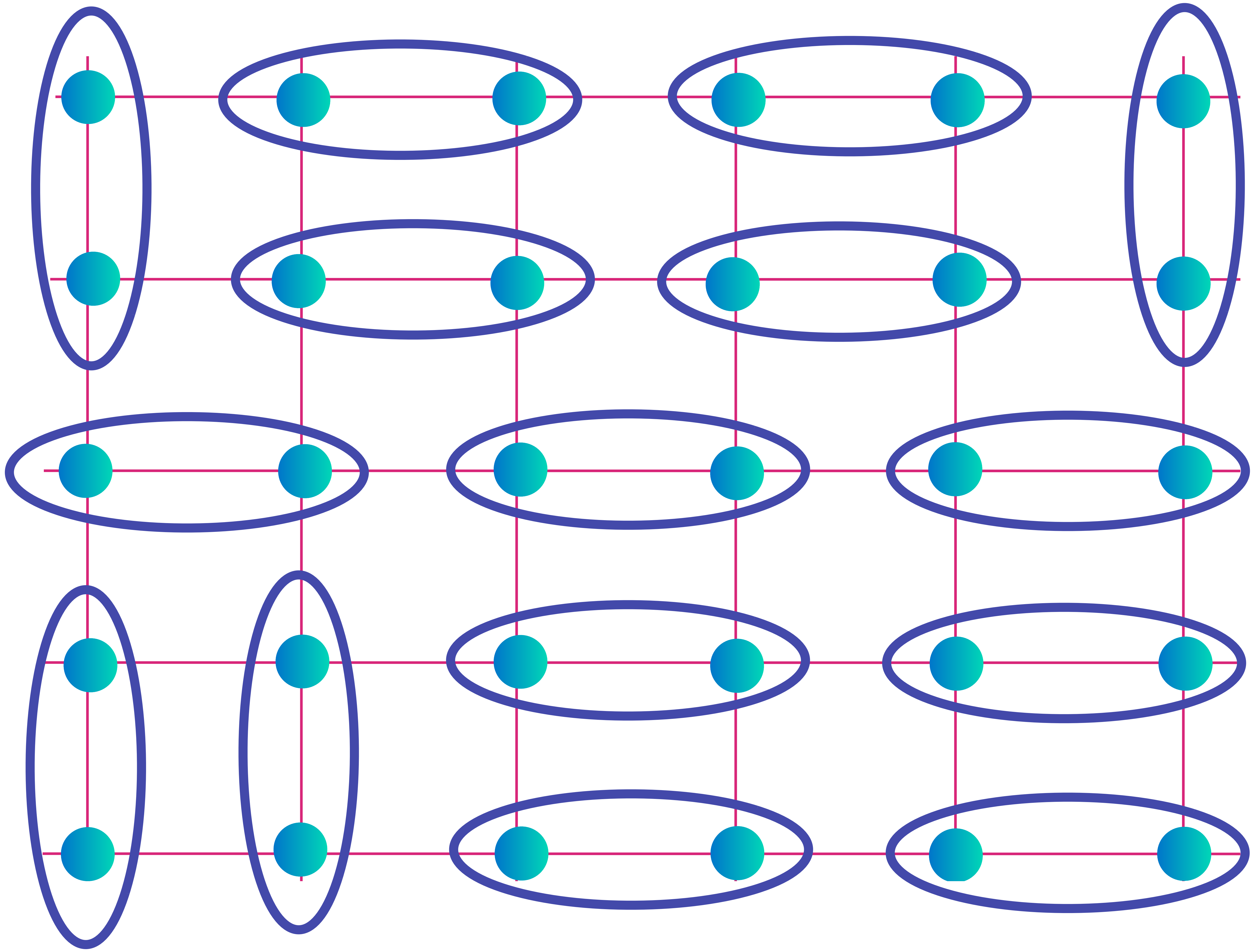


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \to$ dimer covering of lattice

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Anderson's Resonating Valence Bond (1972,1987)



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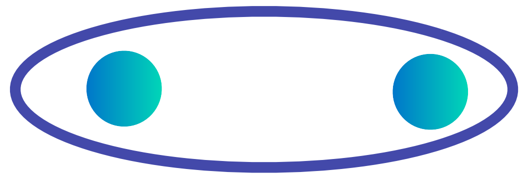 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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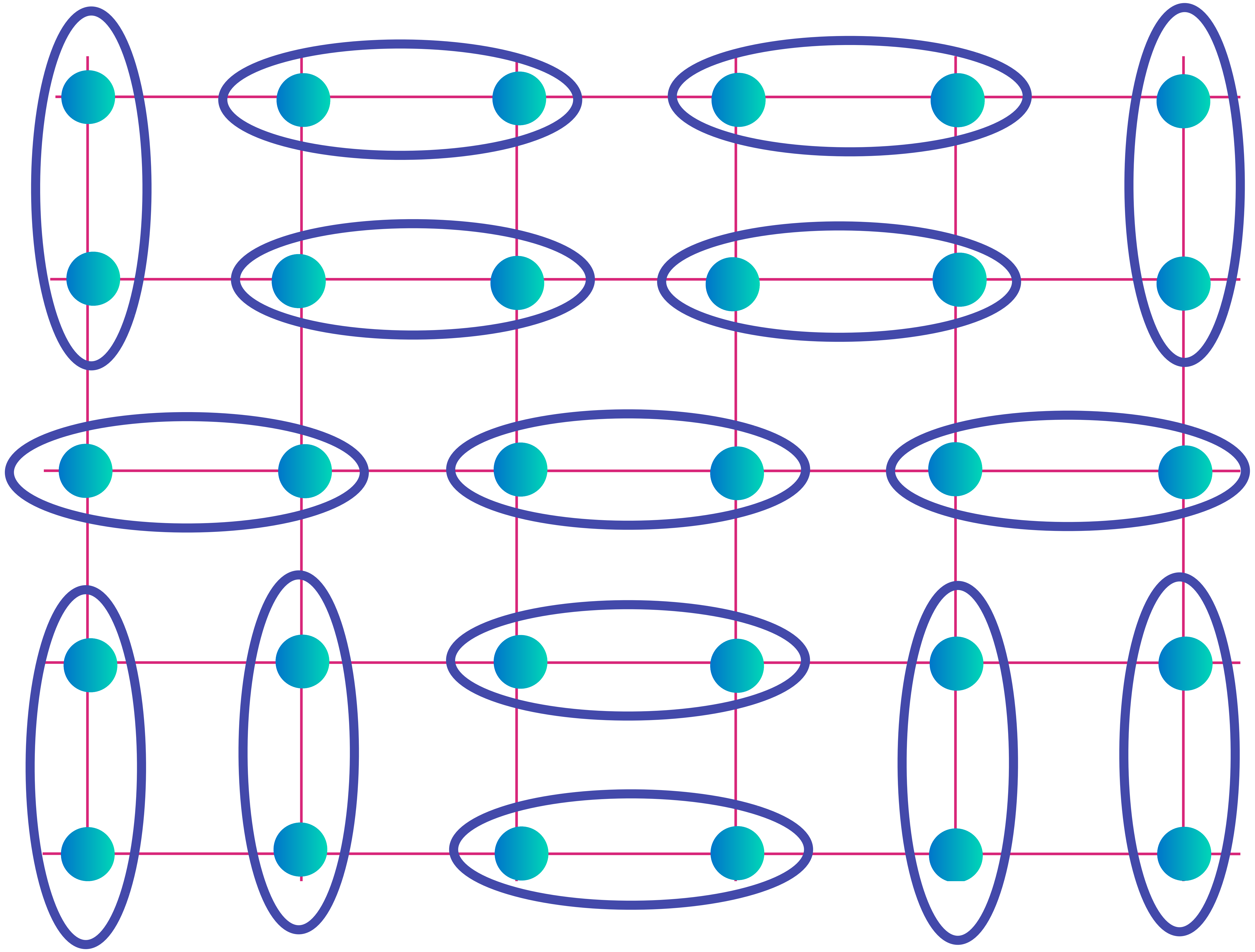

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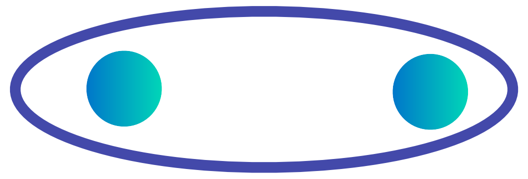 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

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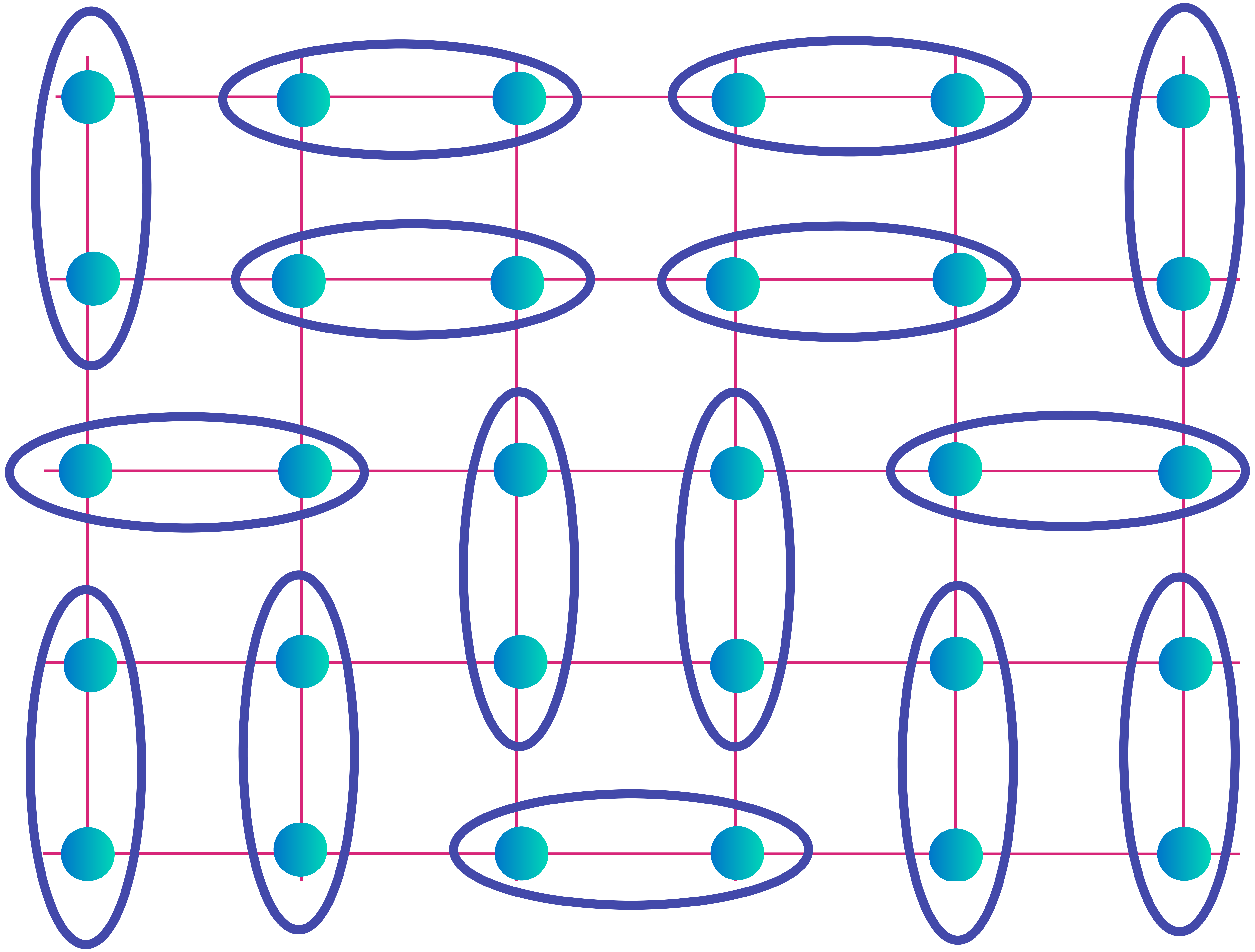

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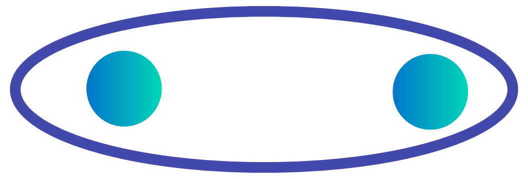 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

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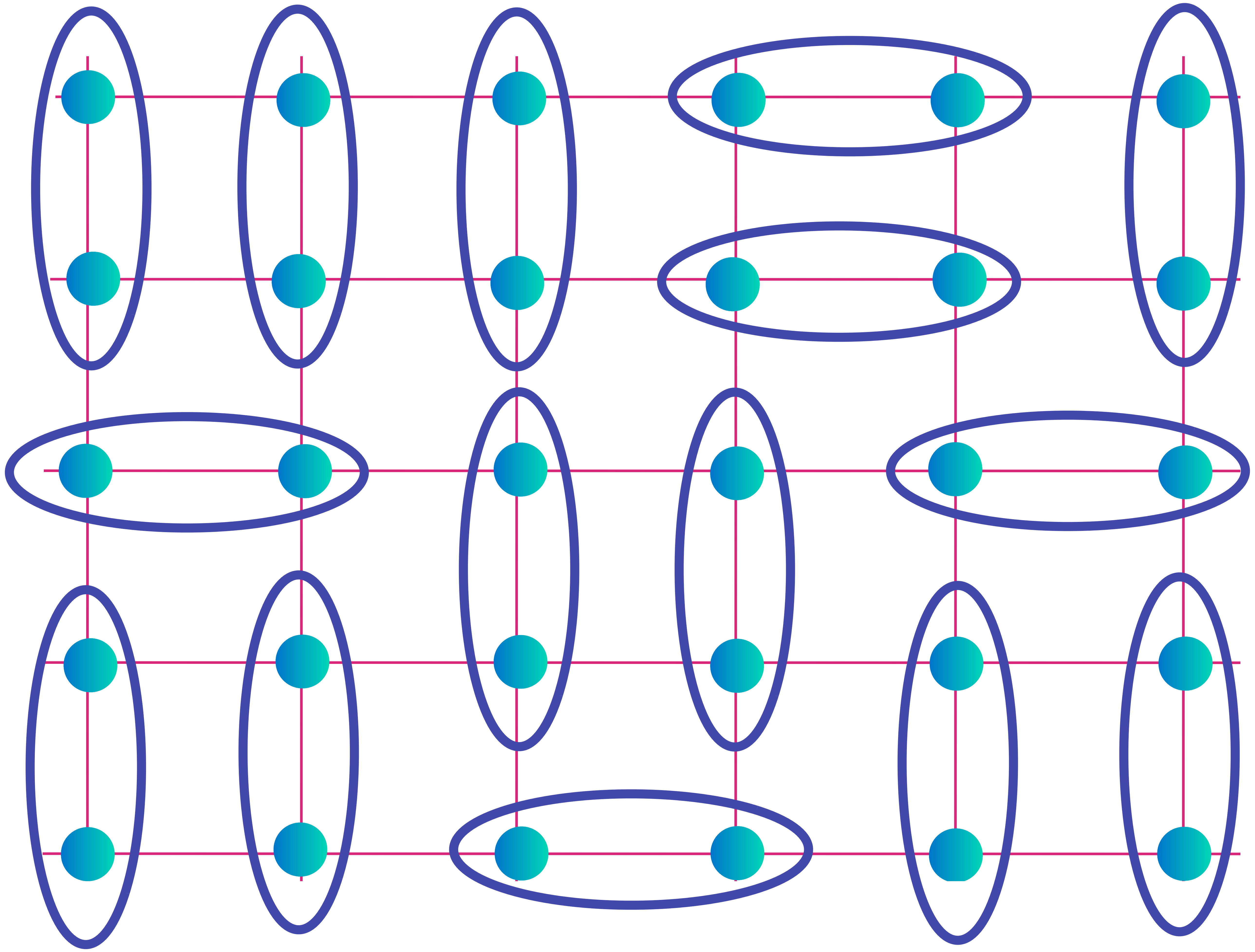

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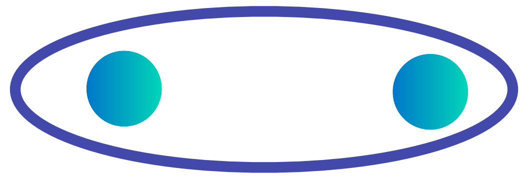 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

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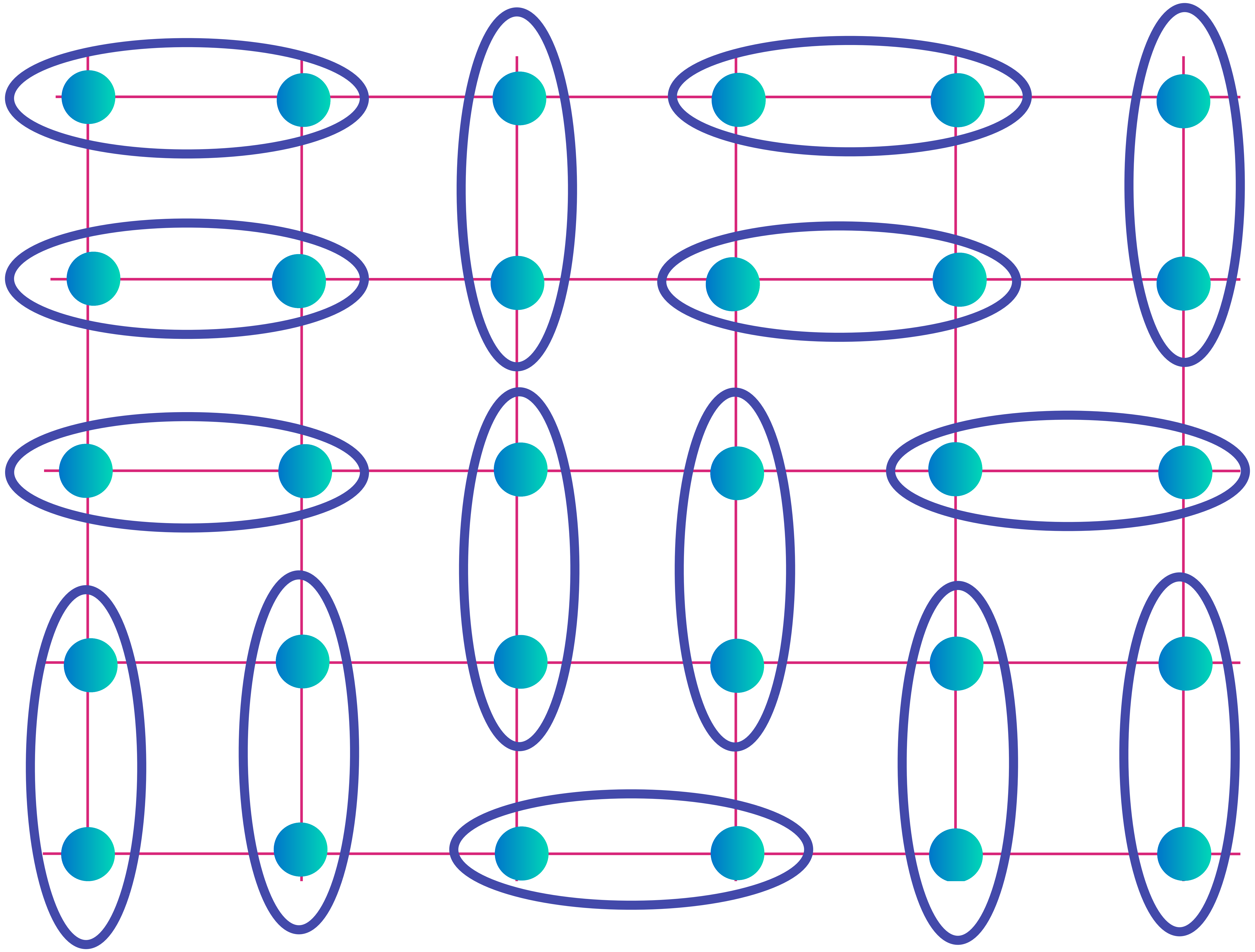

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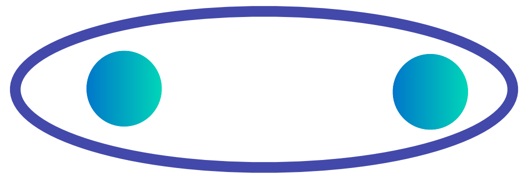 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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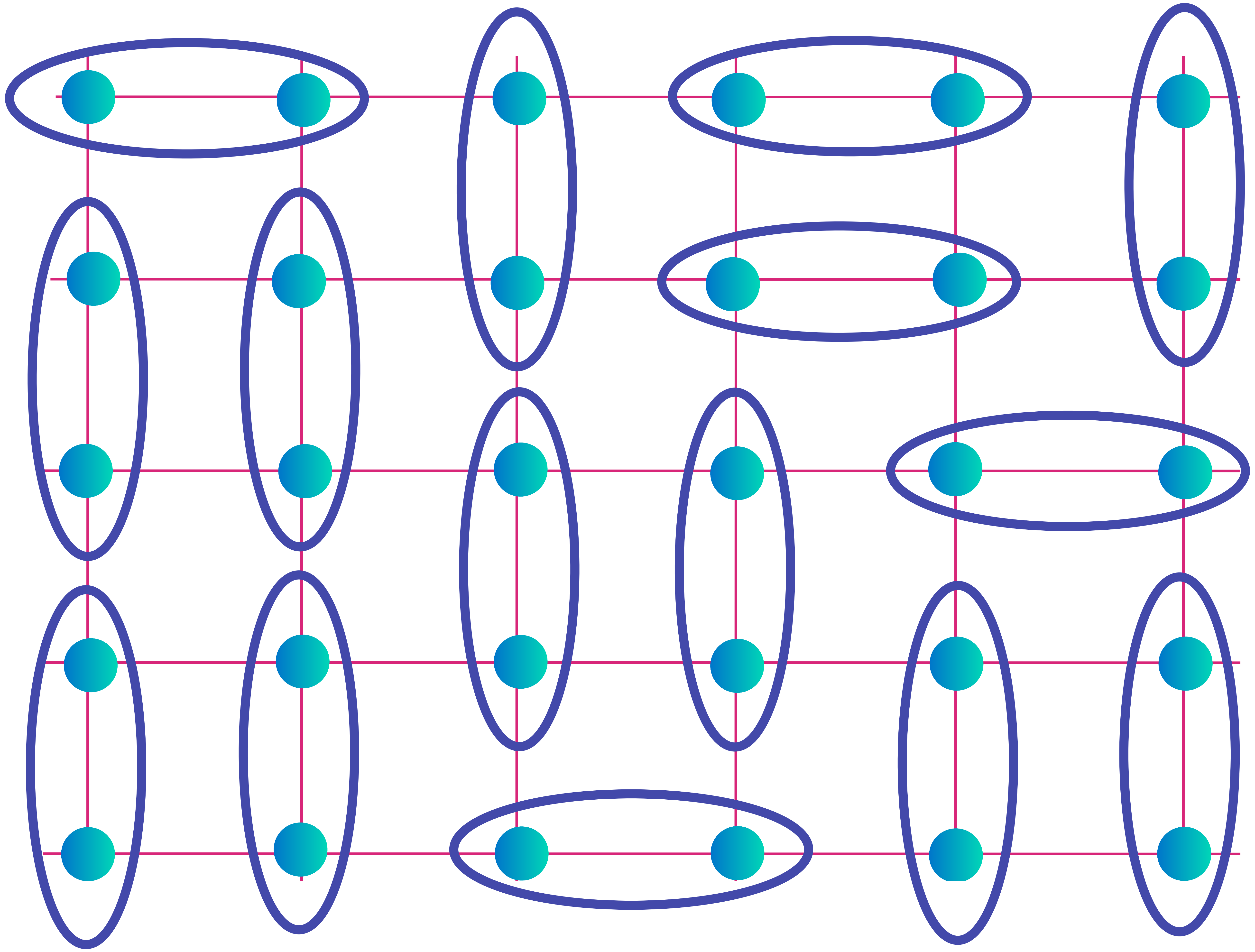

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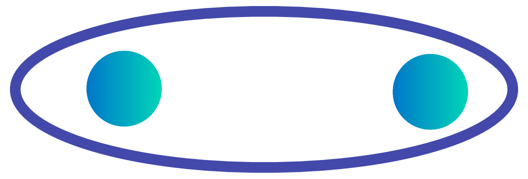
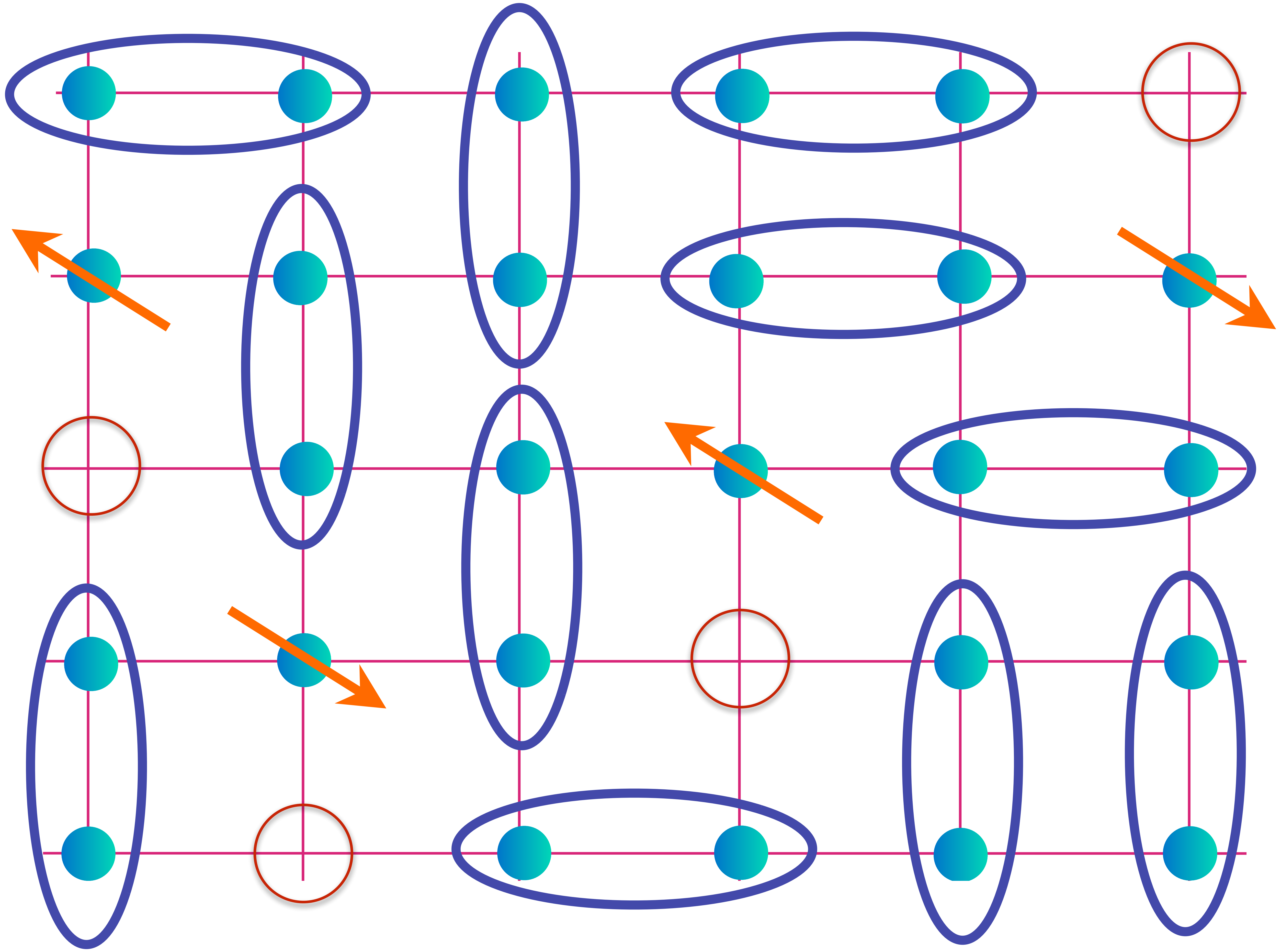
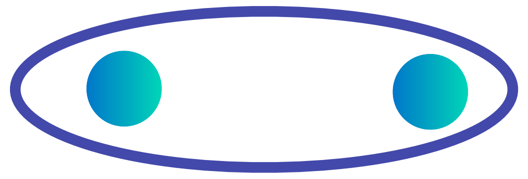
$\mathcal{D} \to$ dimer covering of lattice

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \, / \sqrt{2}$

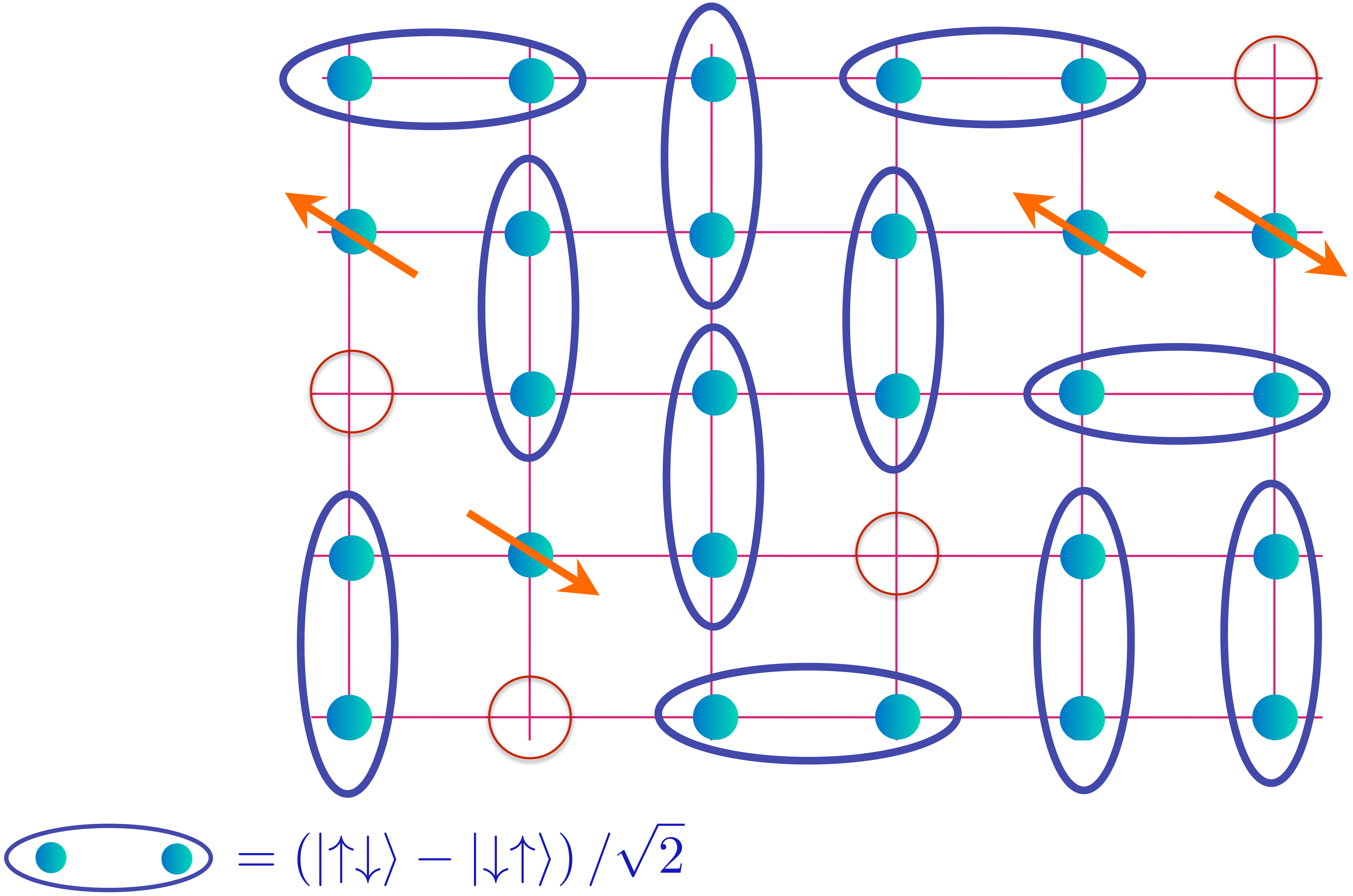
To obtain a (super)conductor we have to remove a density p of electrons



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Fractionalized particles: Spinons (charge 0, spin 1/2)

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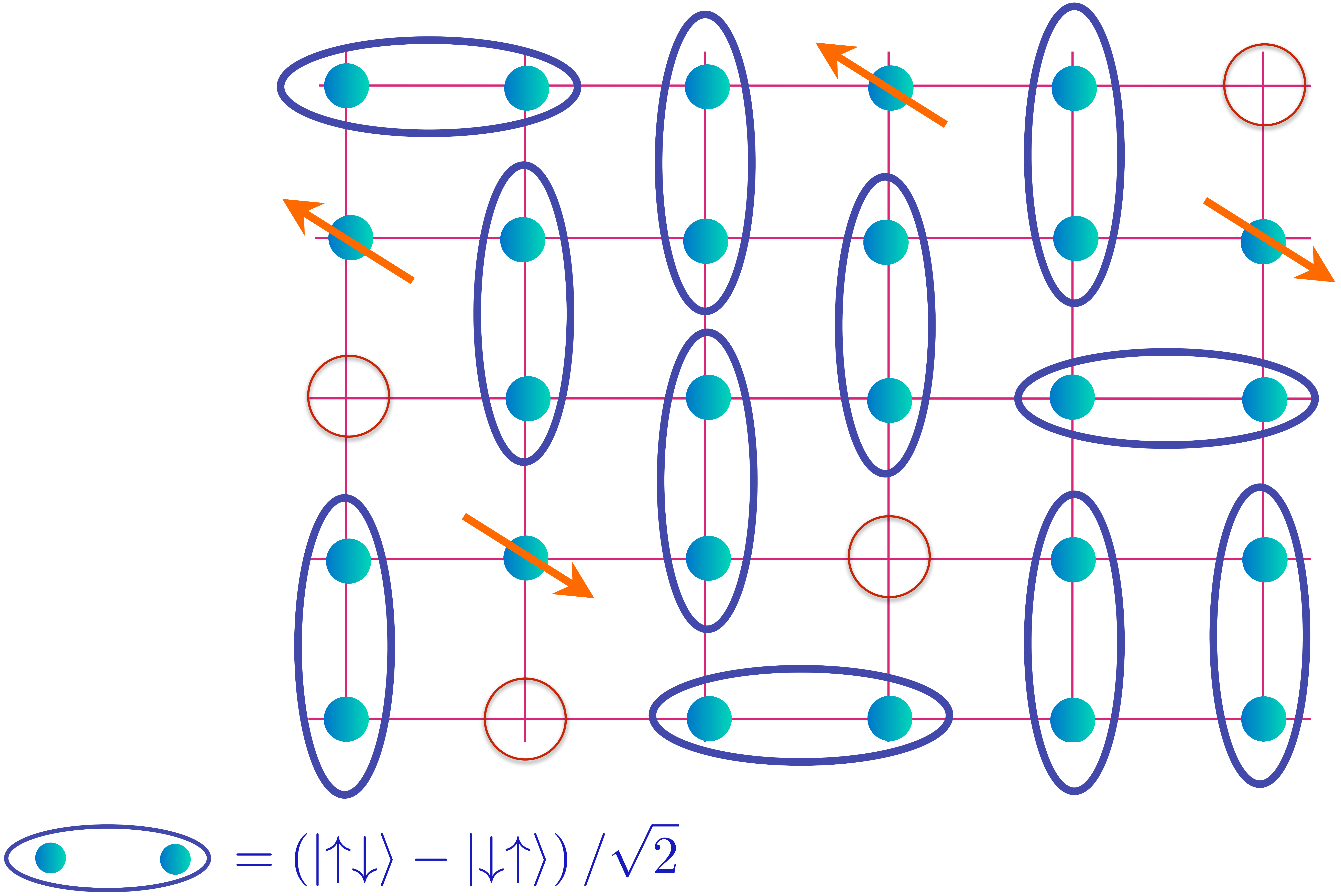

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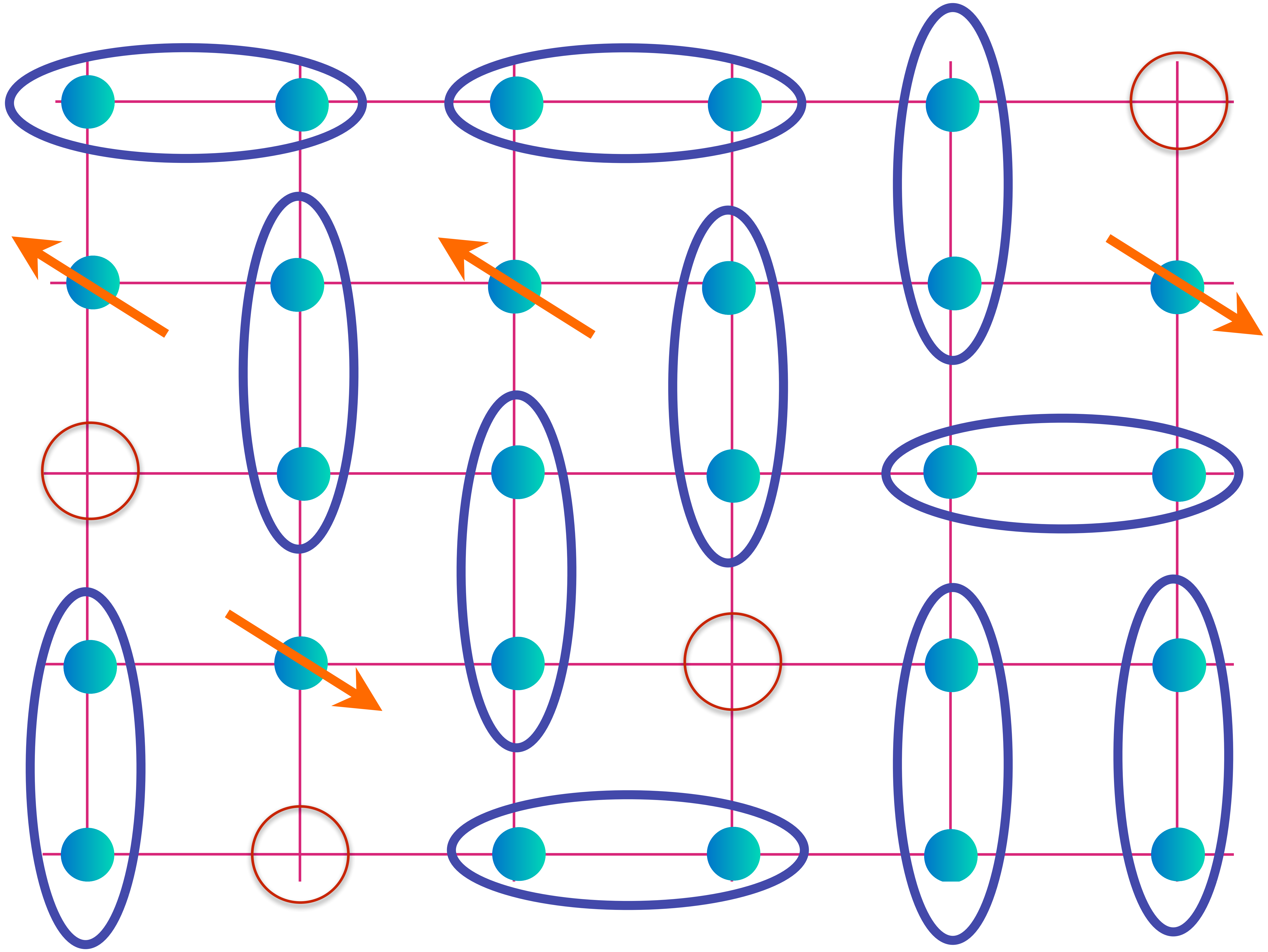

Fractionalized particles:
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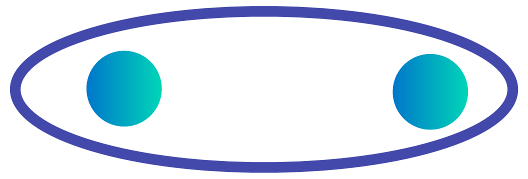 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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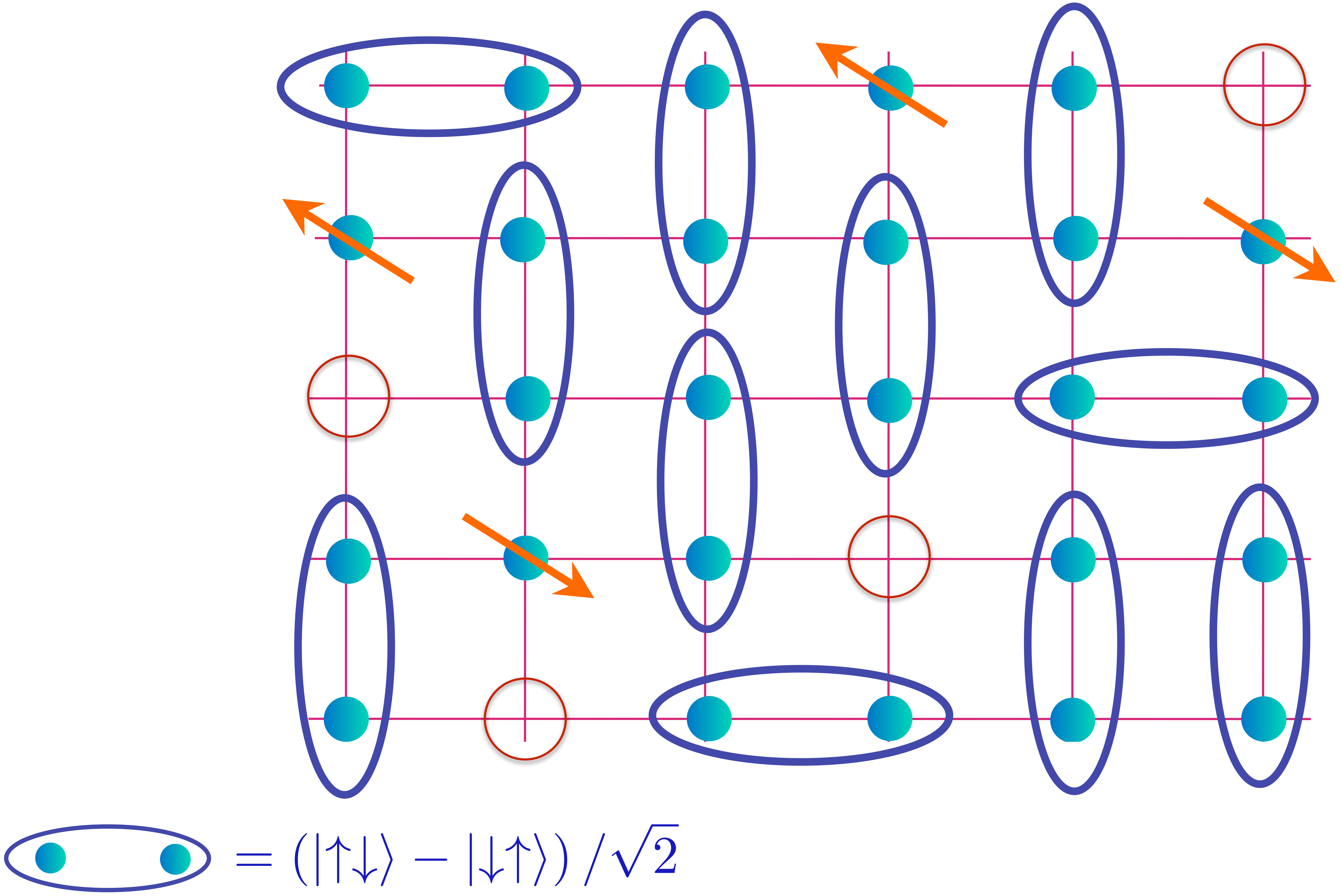

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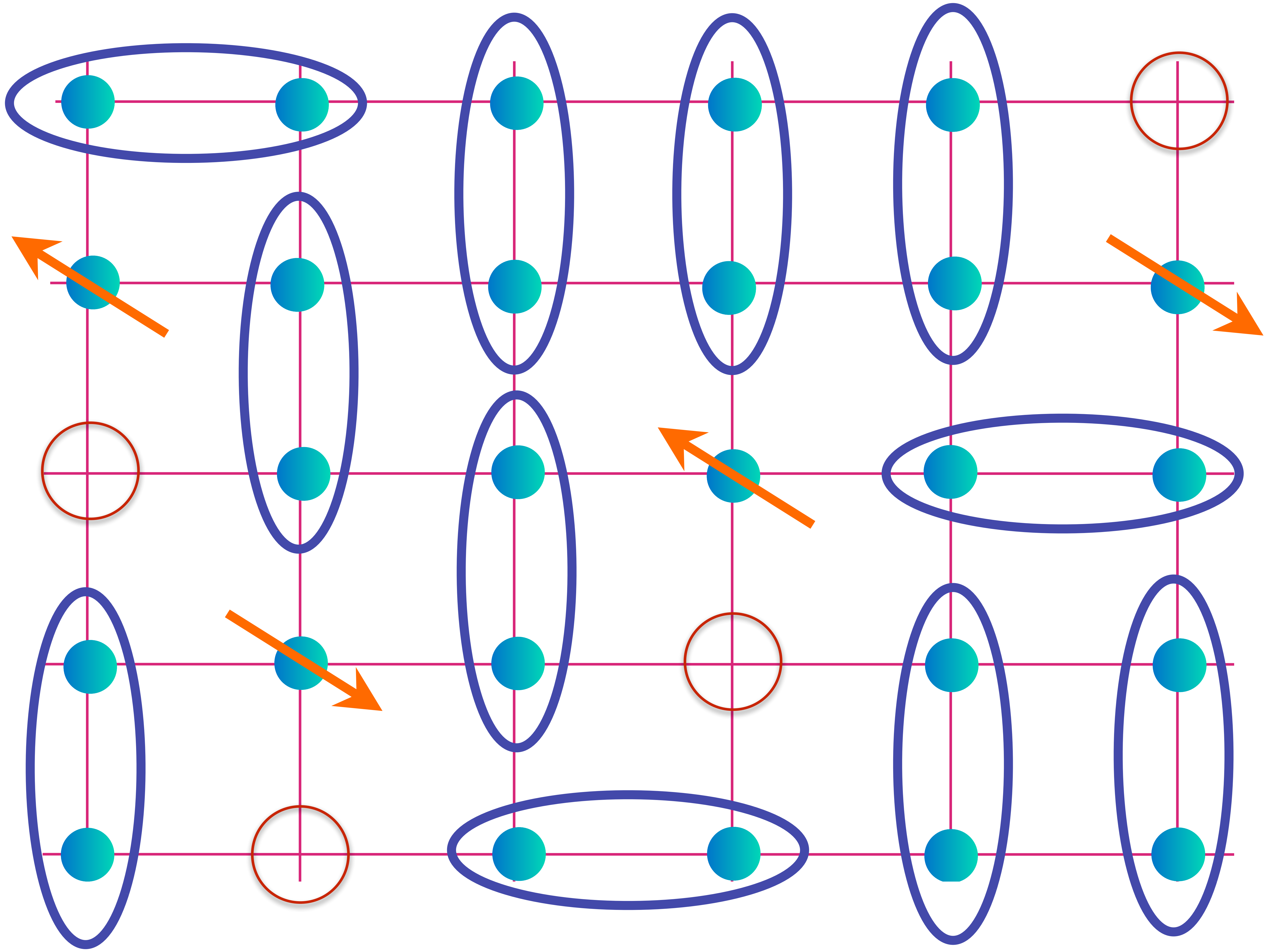

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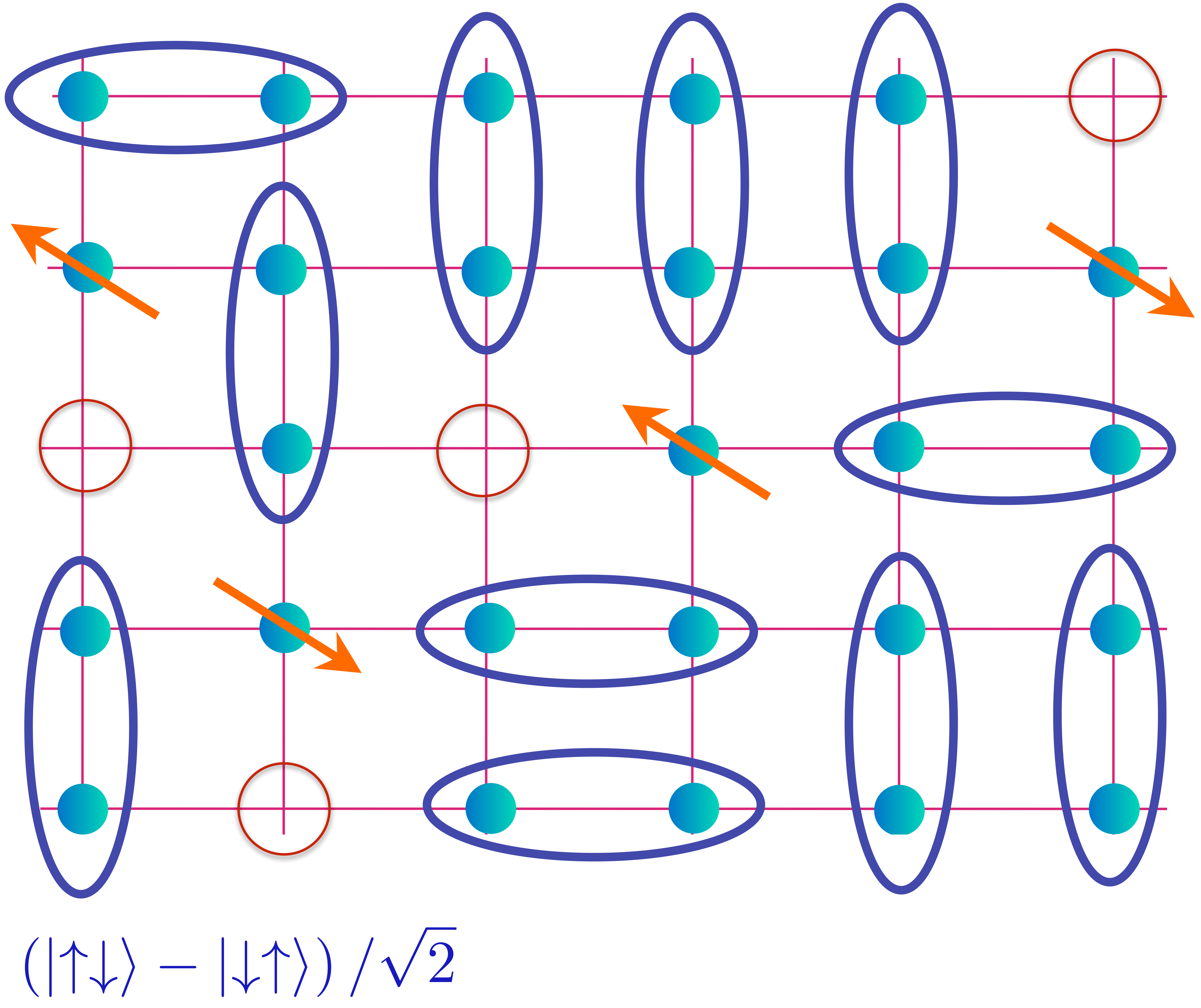
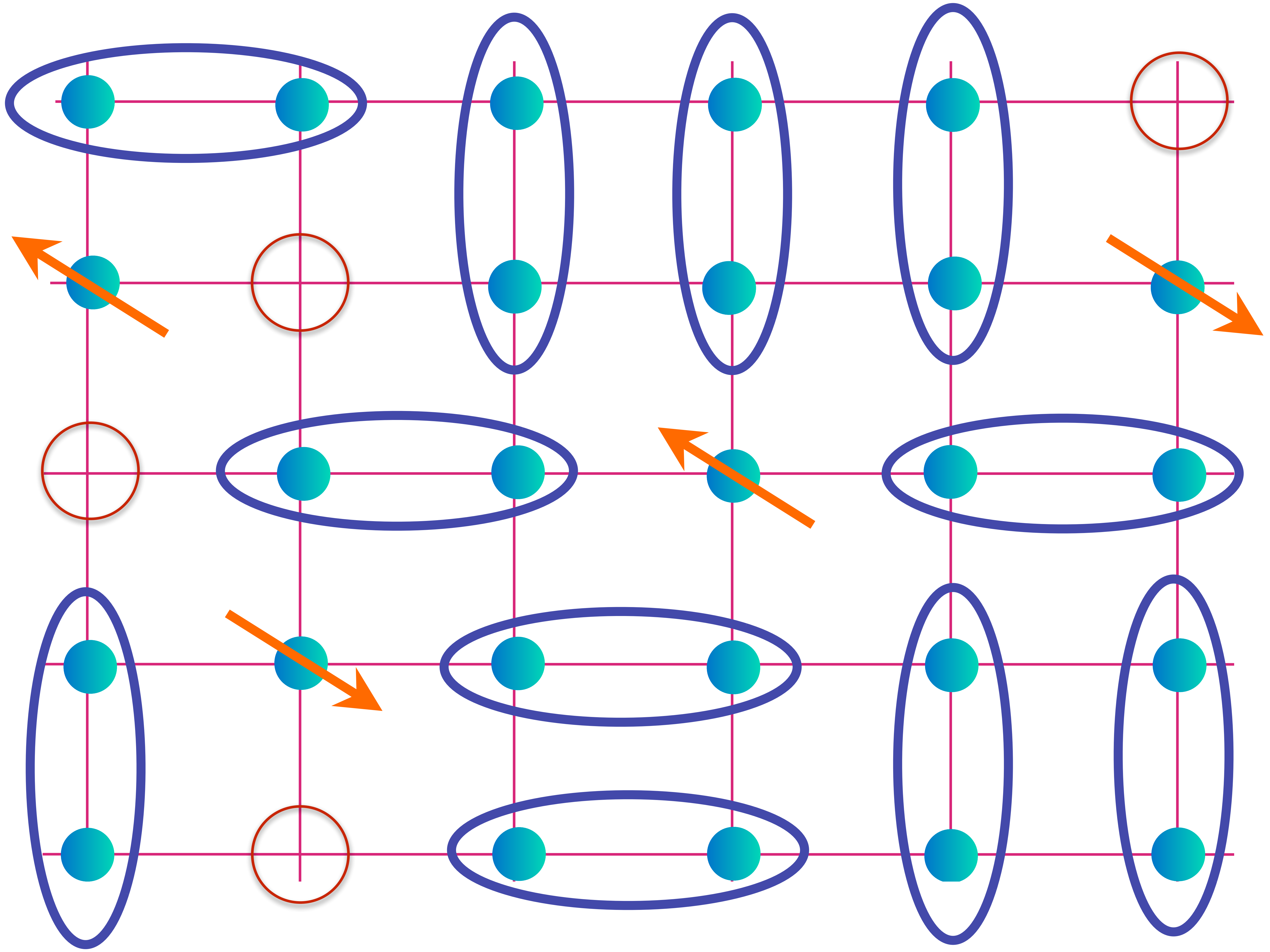


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Fractionalized particles:
Spinons
(charge 0, spin 1/2)
and
holons
(charge e, spin 0)

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Fractionalized particles:
Spinons
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and
holons
(charge e, spin 0)

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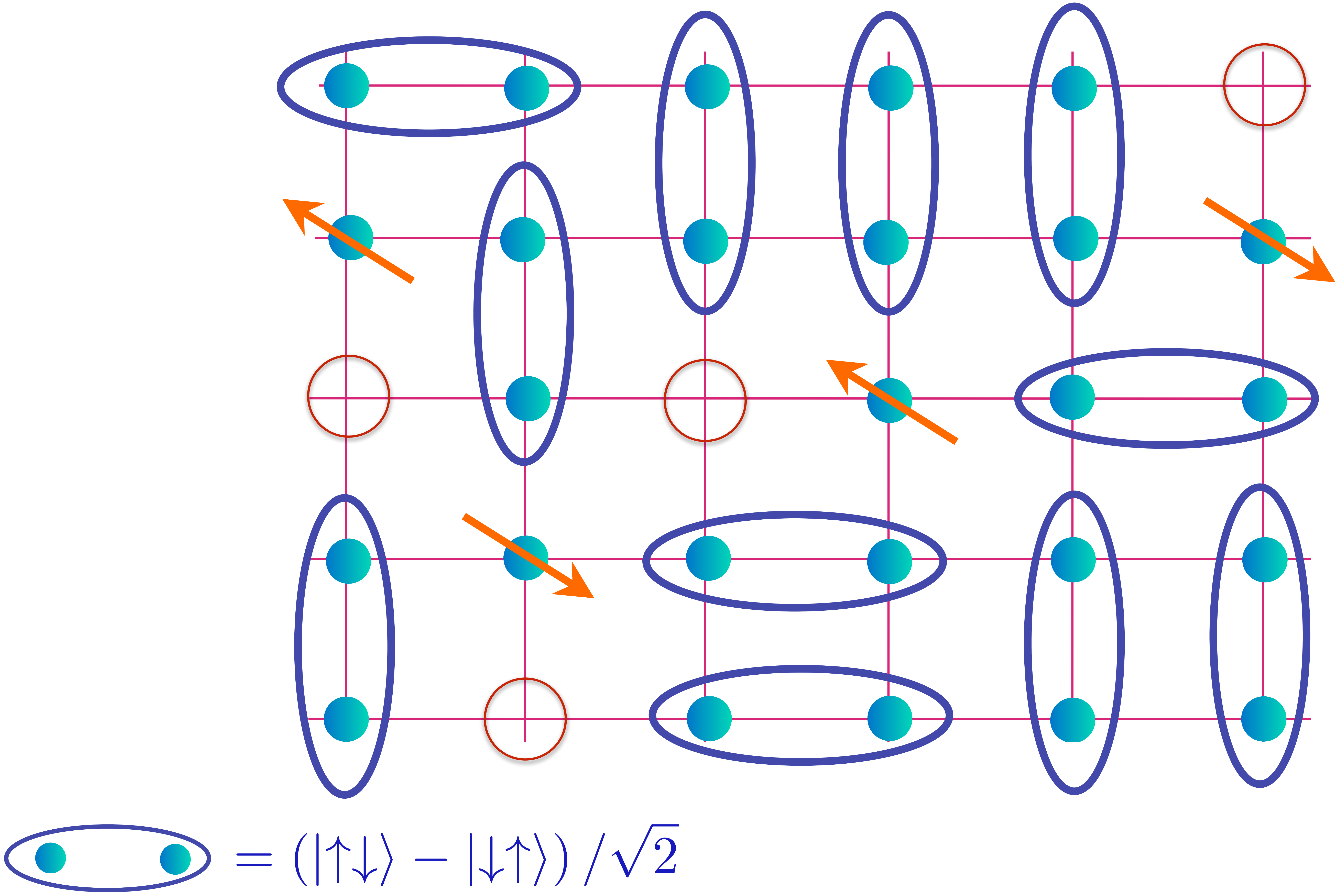

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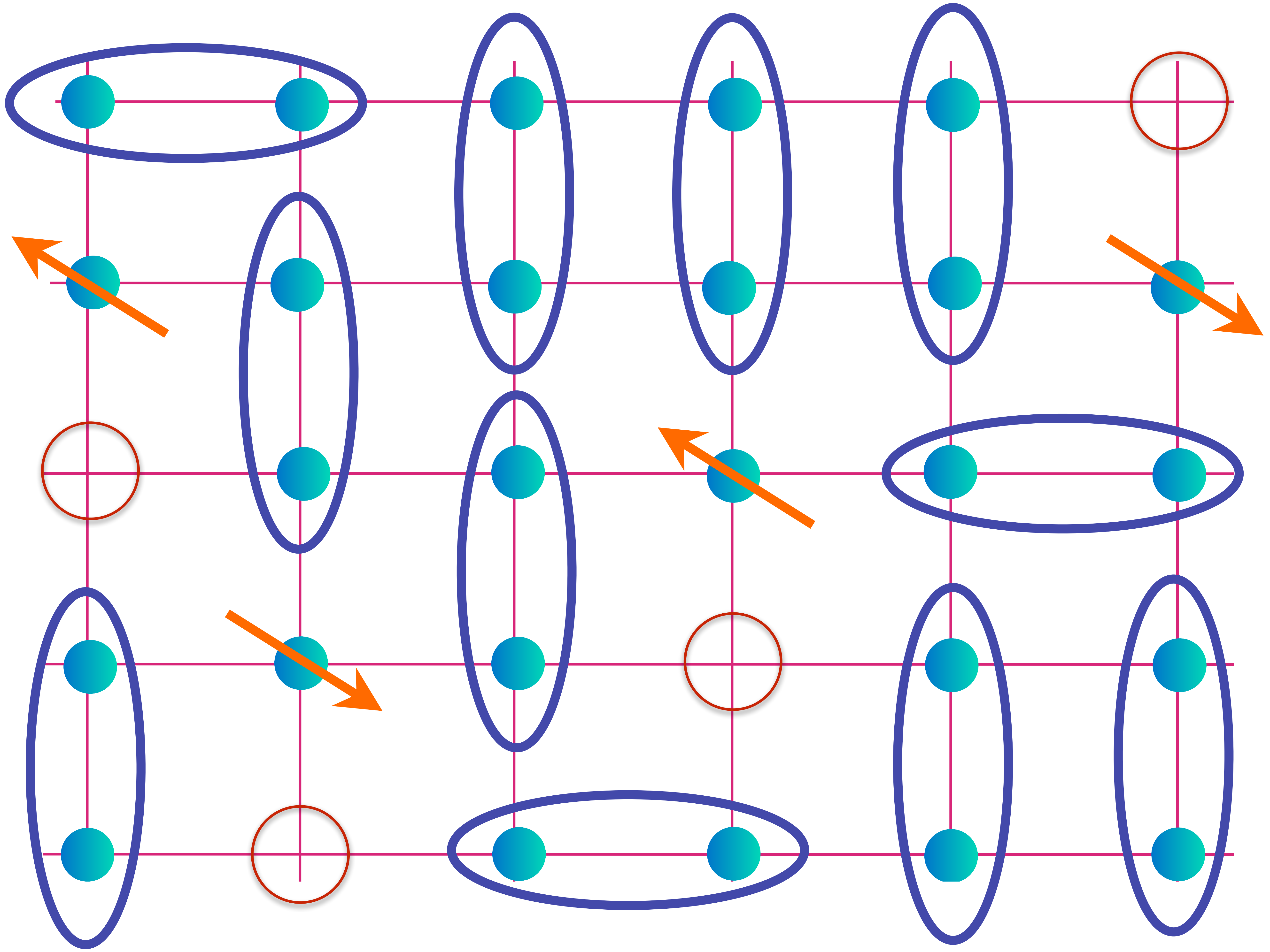

Fractionalized particles:
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and
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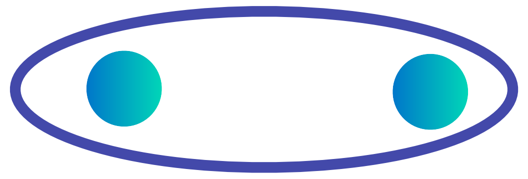
(charge e, spin 0)



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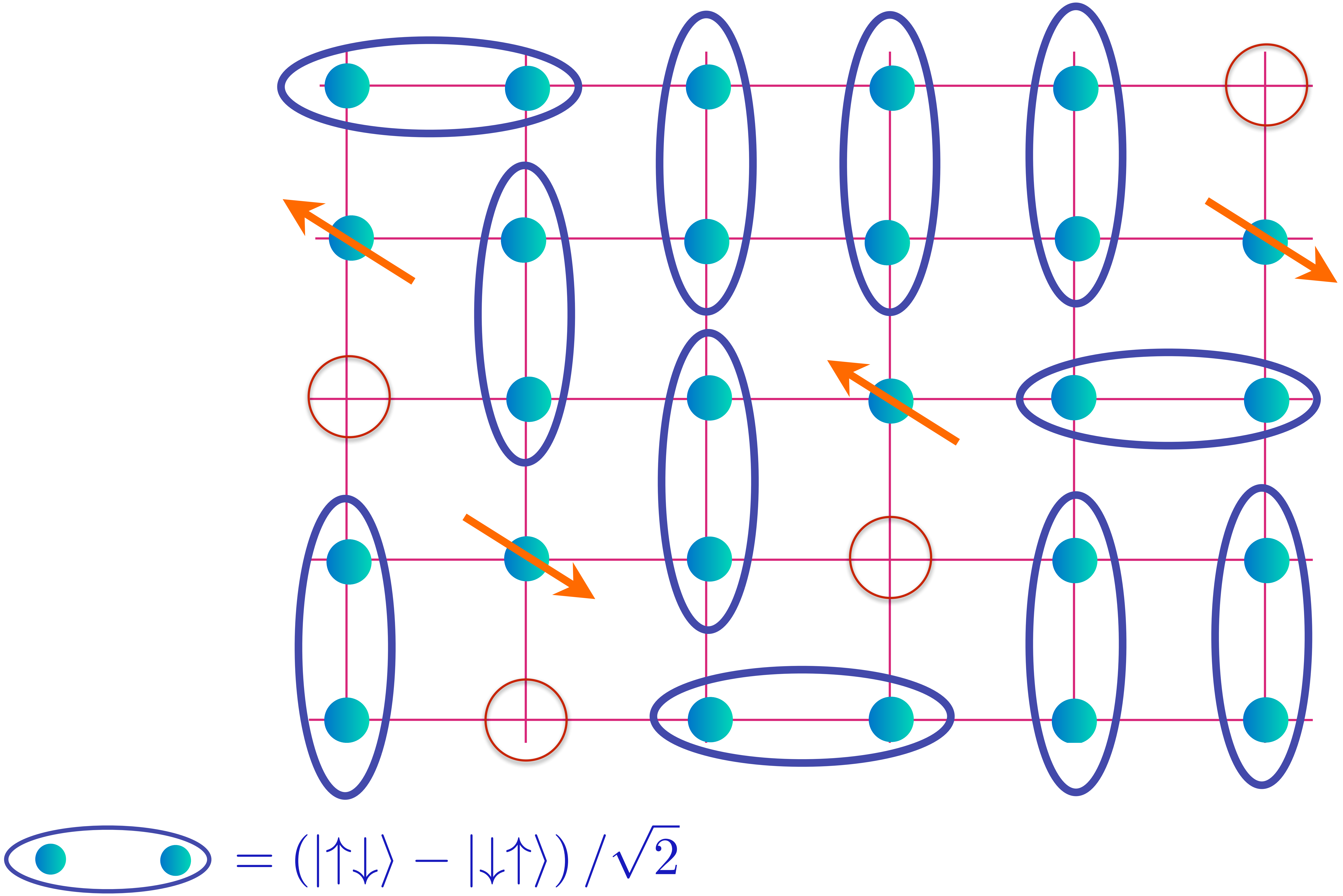


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

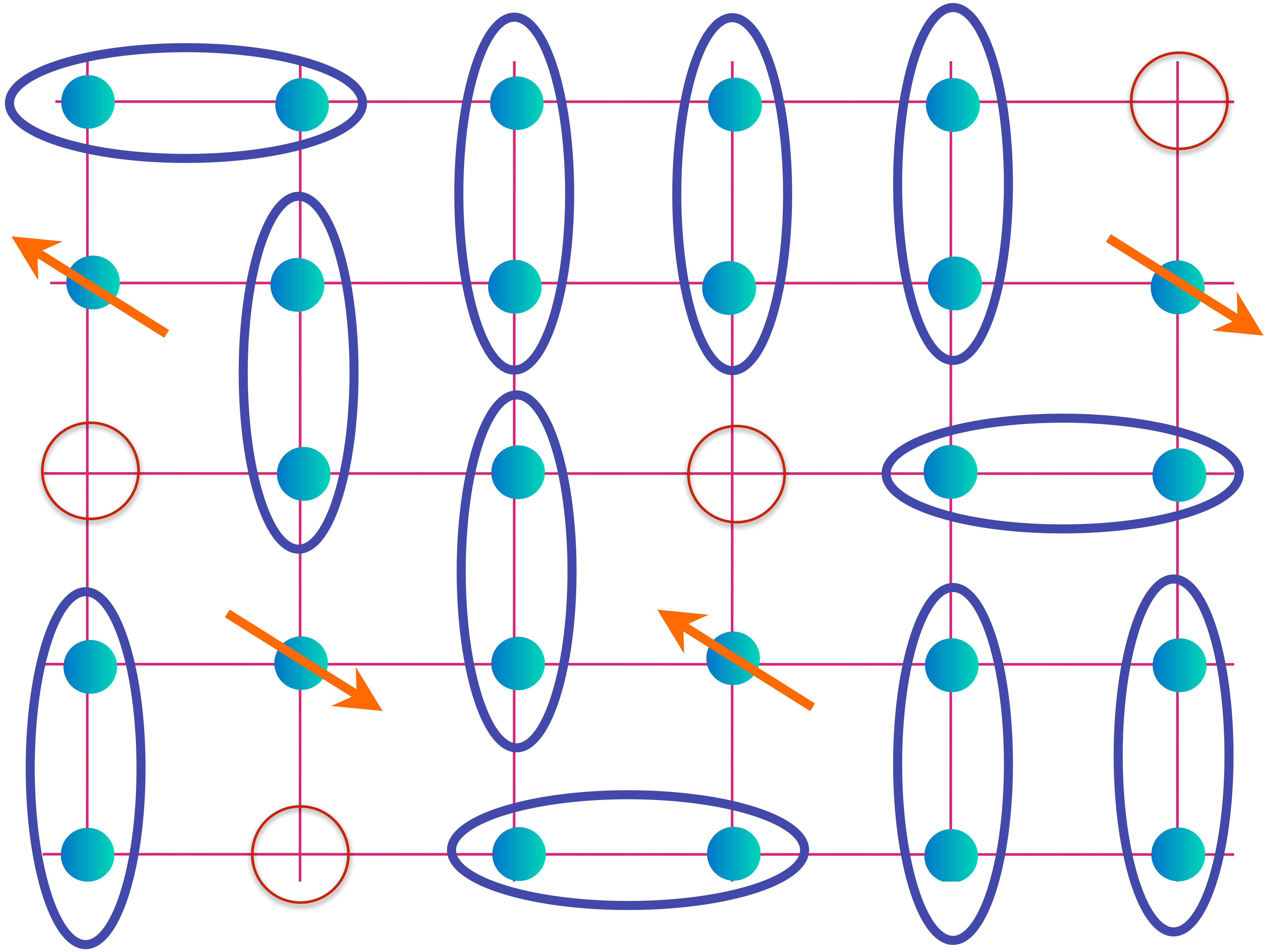
Fractionalized particles:
Spinons
(charge 0, spin $1/2$)
and
holons
(charge e, spin 0)

<u>Holons cannot tunnel between layers</u>

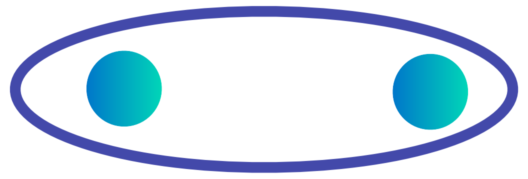
To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



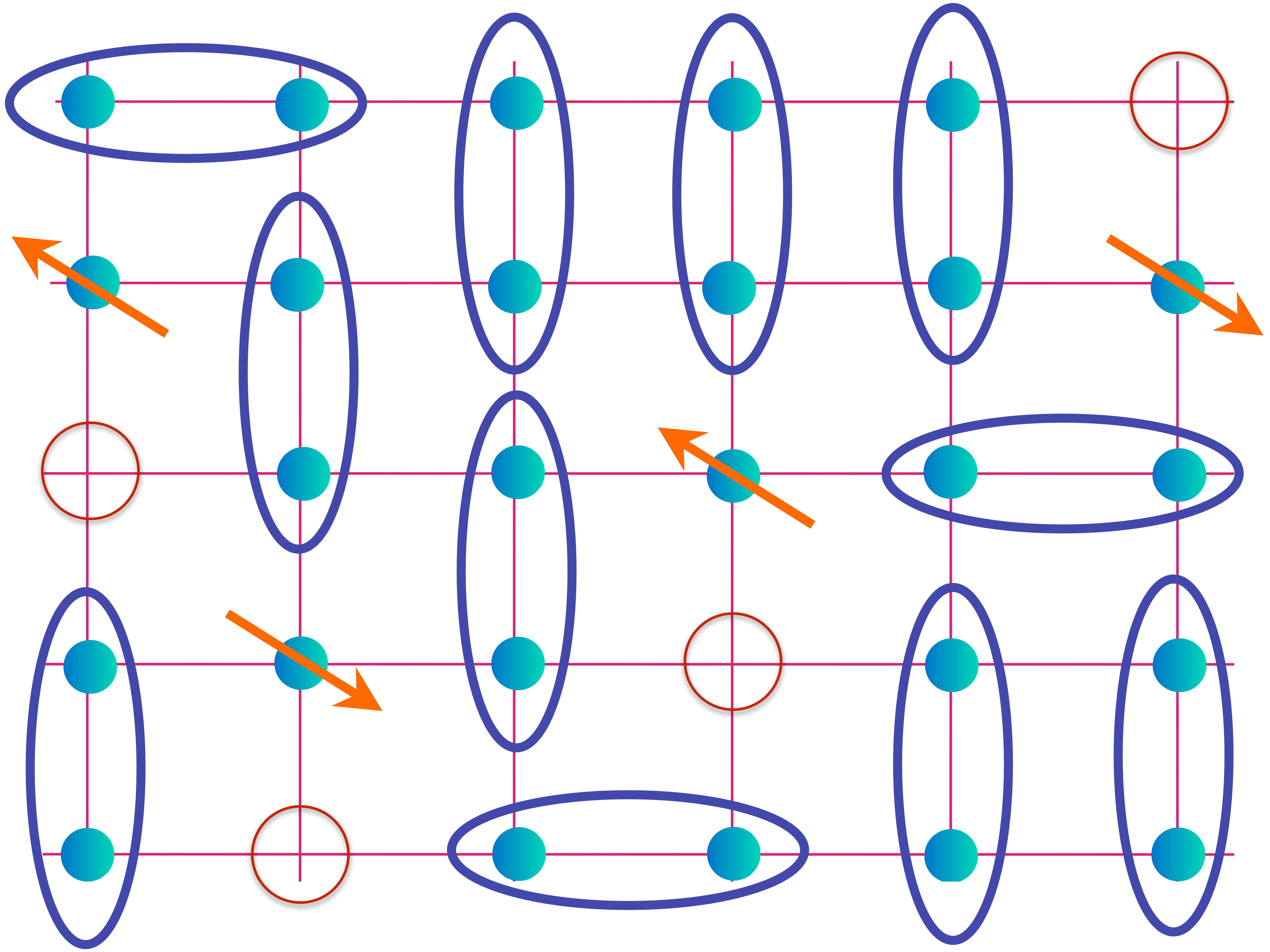
But the holons and spinons can gain energy by resonating with each other

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

To obtain a (super)conductor we have to remove a density p of electrons

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



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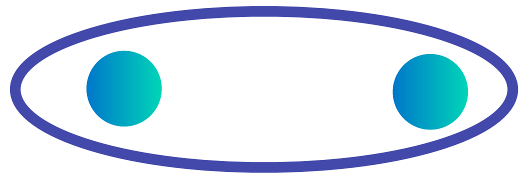

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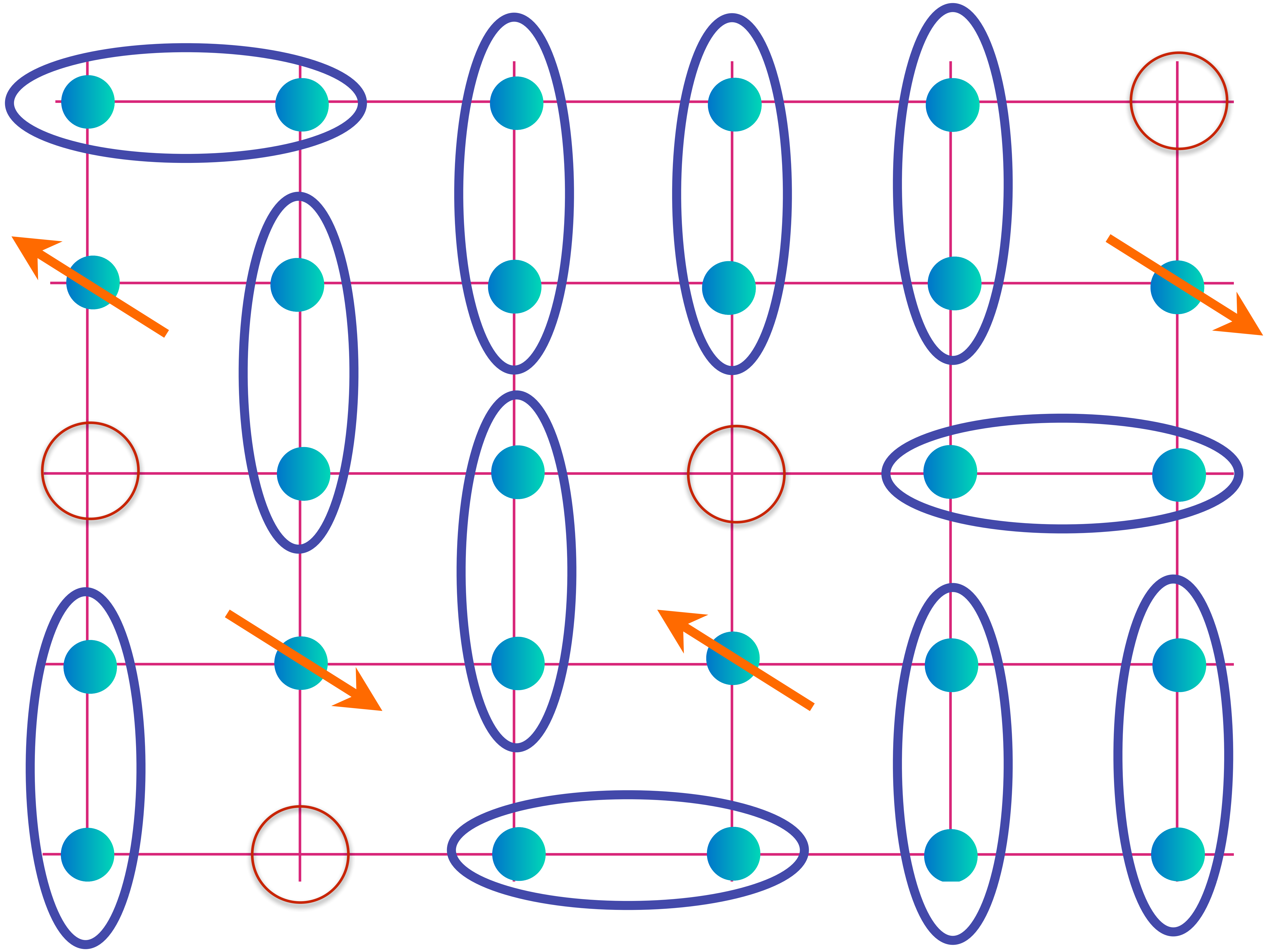

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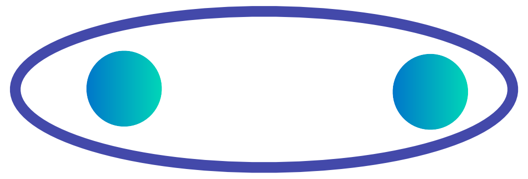
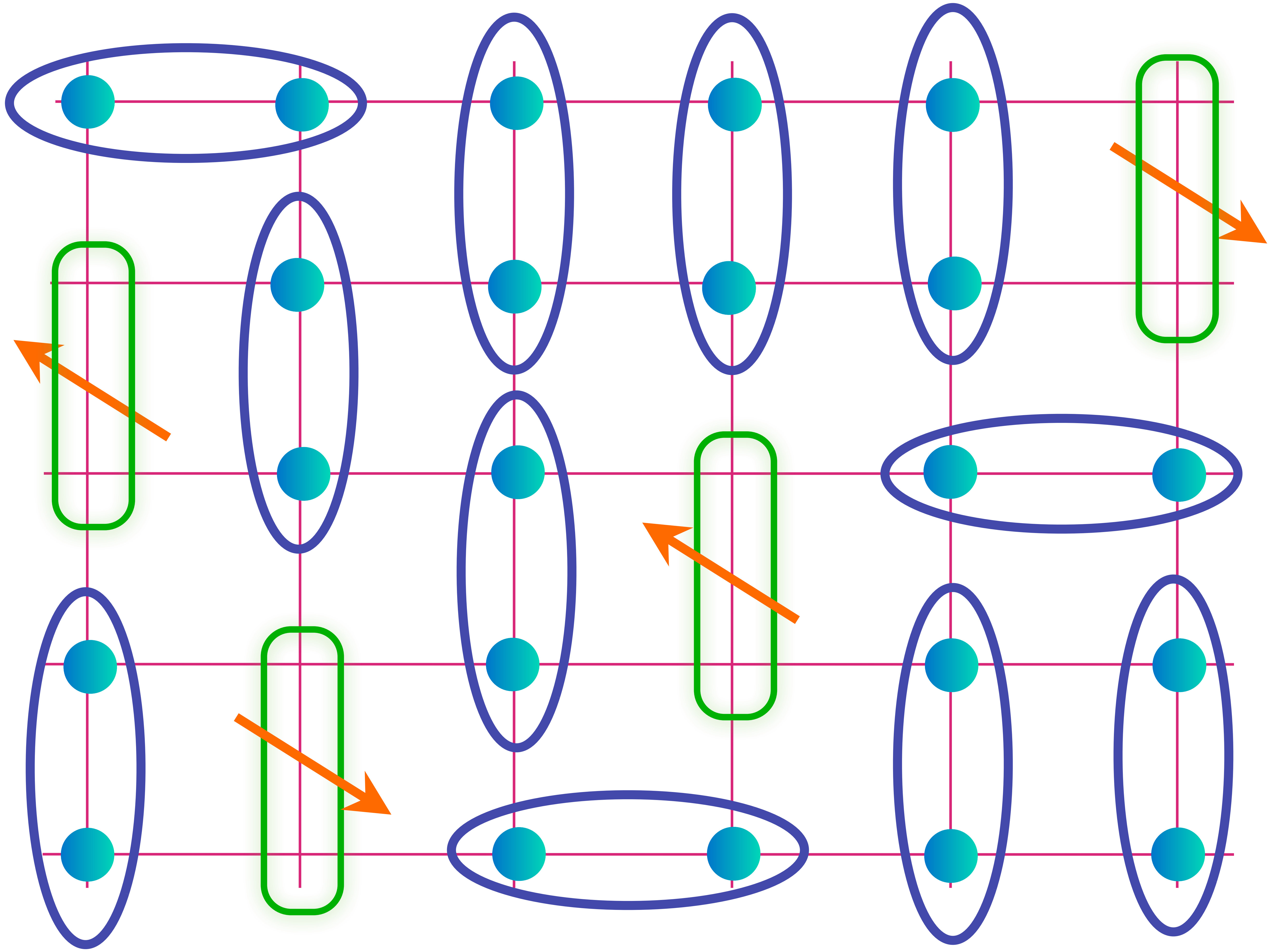


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

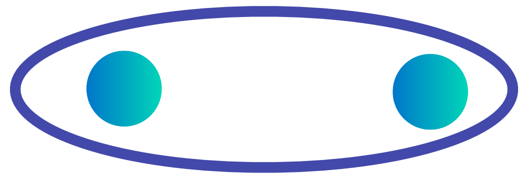
To obtain a (super)conductor we have to remove a density p of electrons

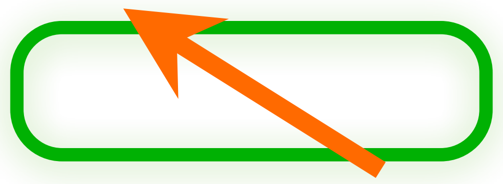
Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.



FL*:
Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007);
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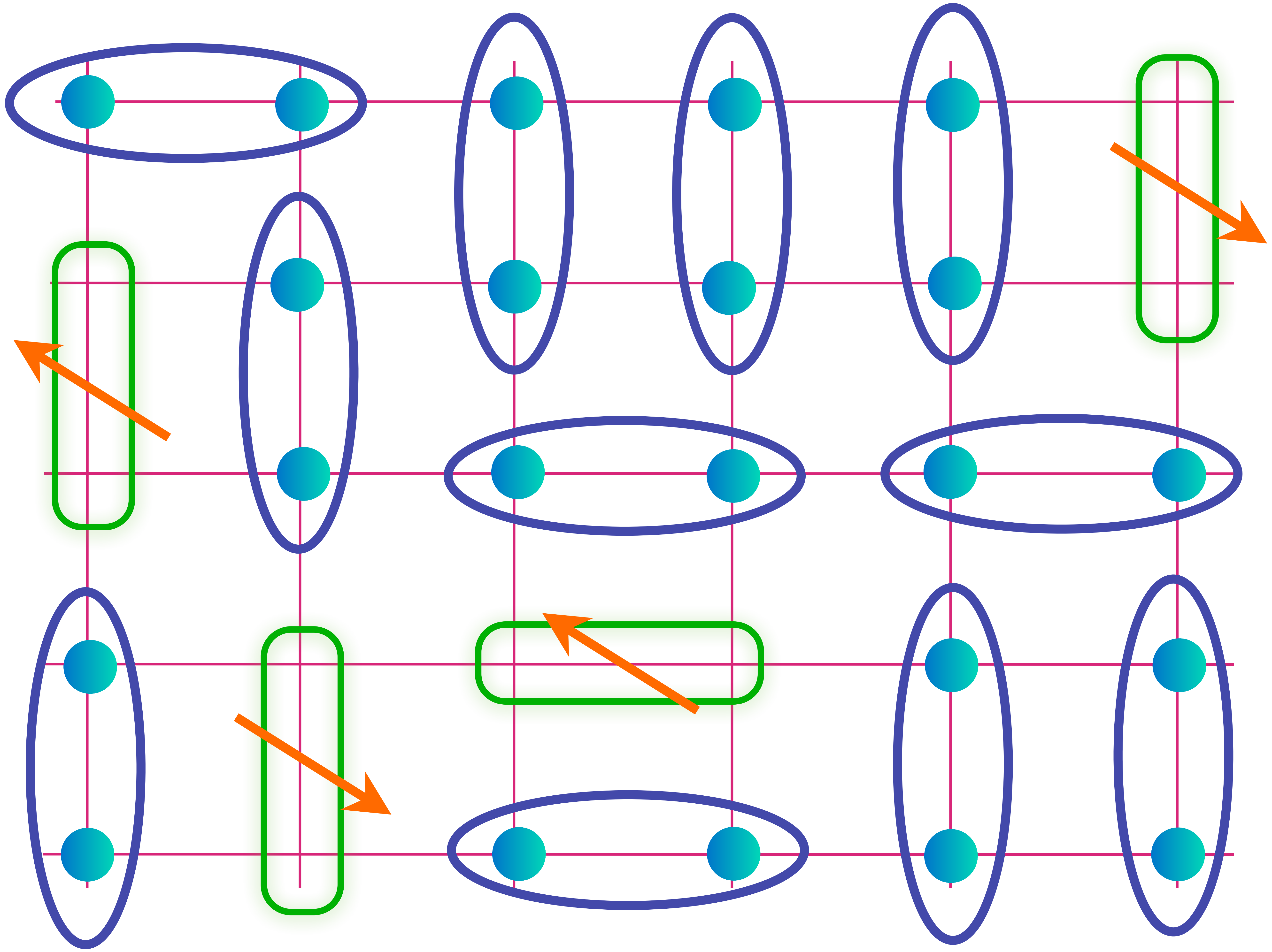

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Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

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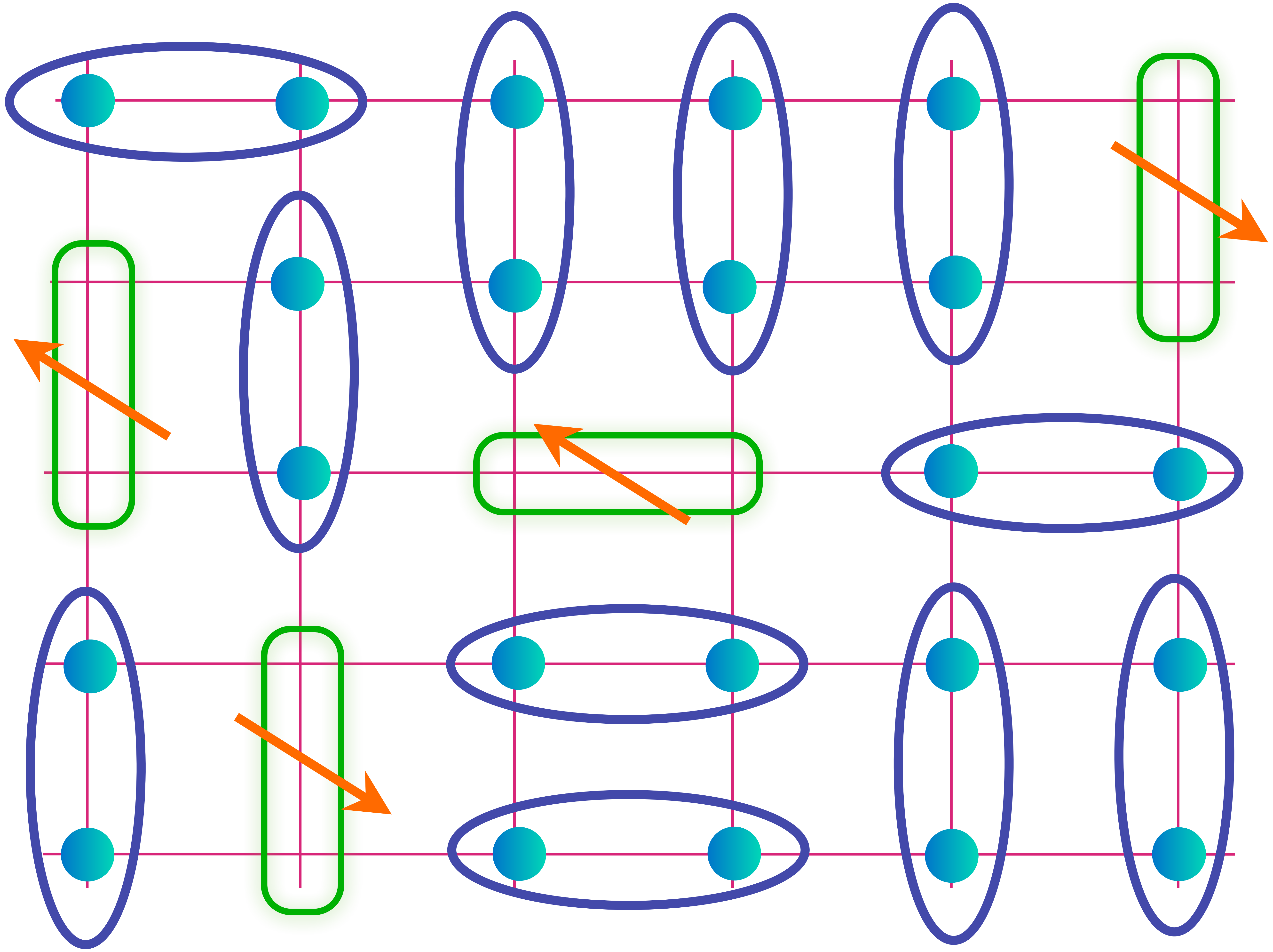

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.

FL*:
Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2} \qquad = \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$$

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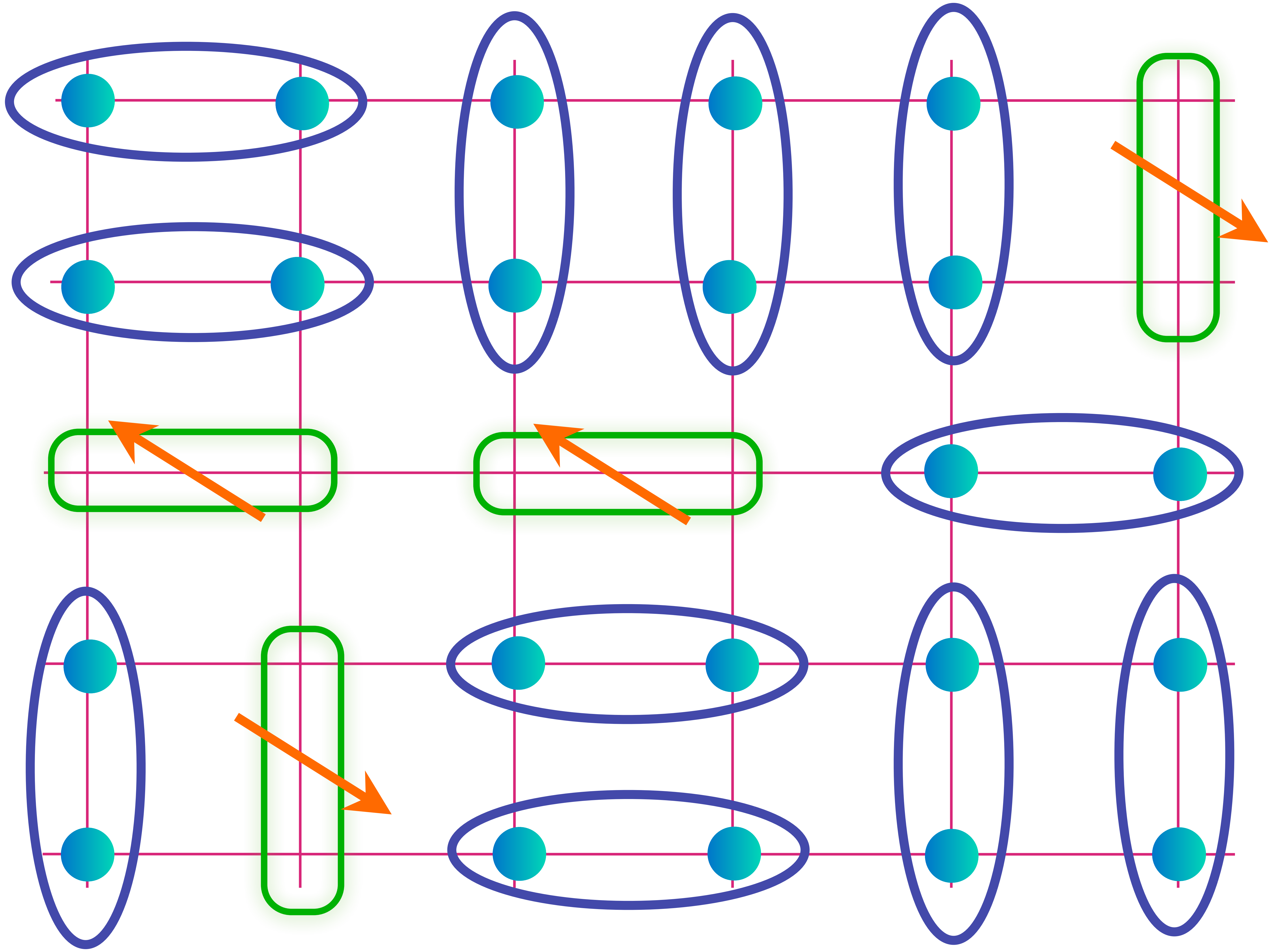

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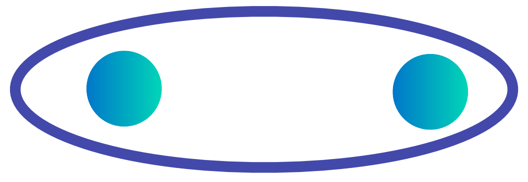
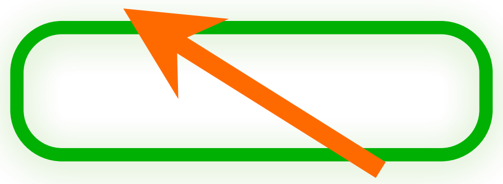
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$  $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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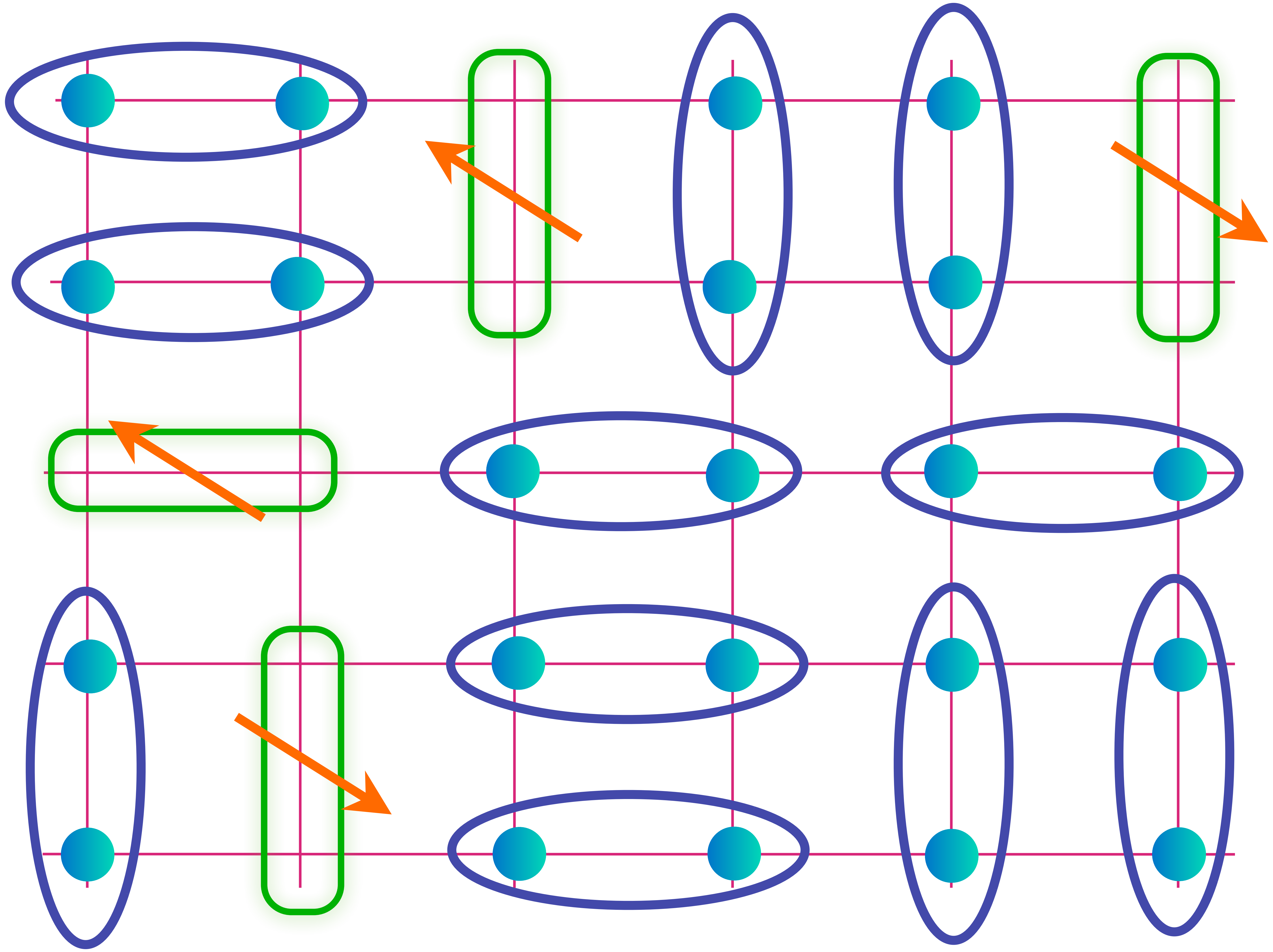

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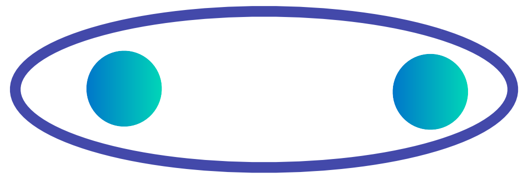
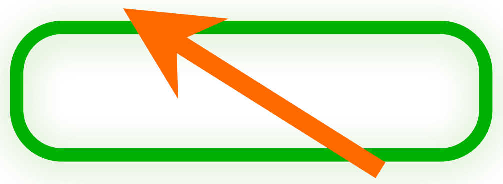
Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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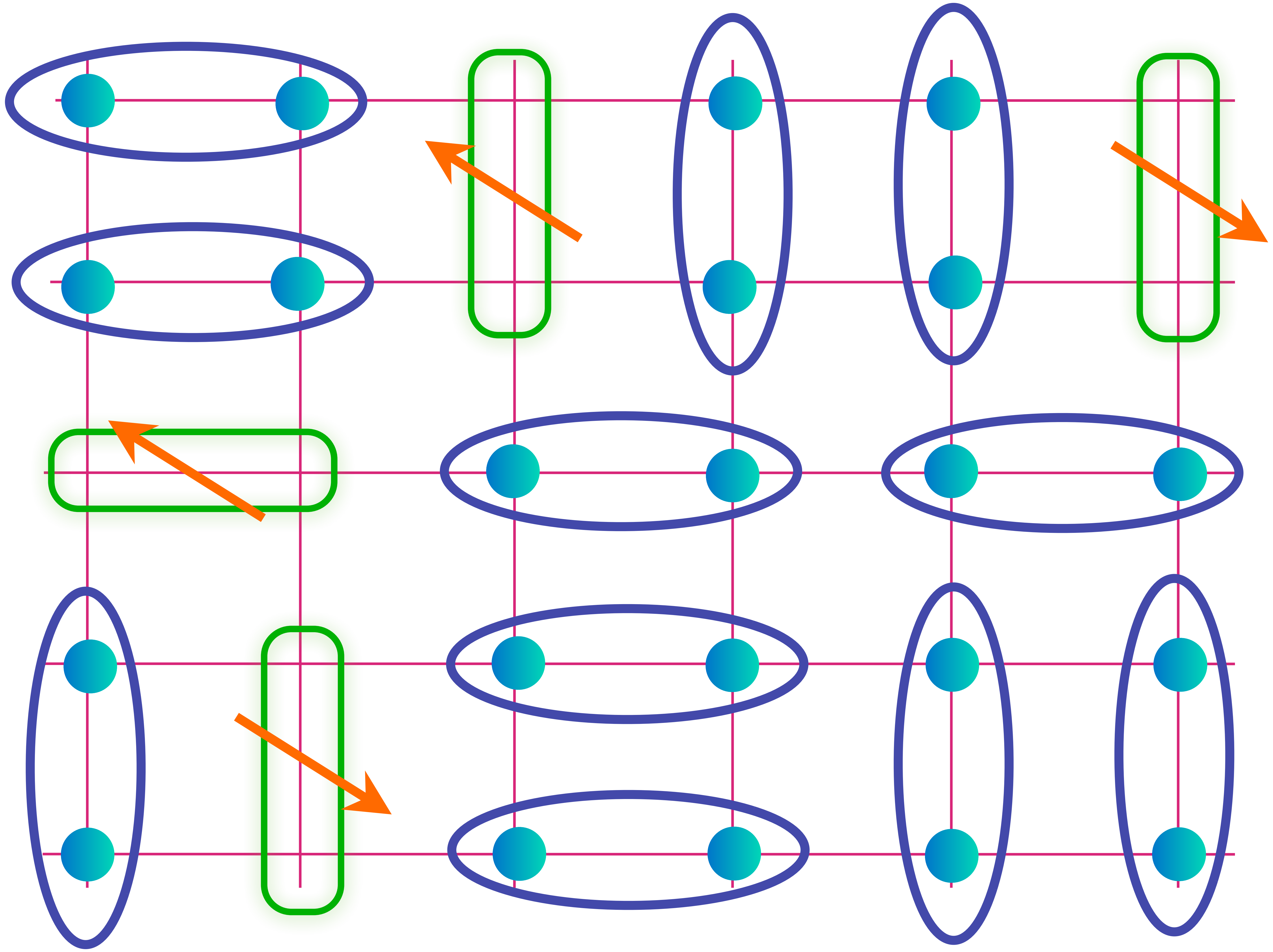

Energy cost to create spinon $\sim J$

Energy gained by bound state $\sim t$.

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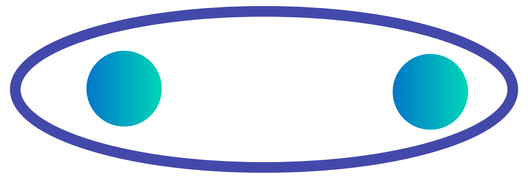
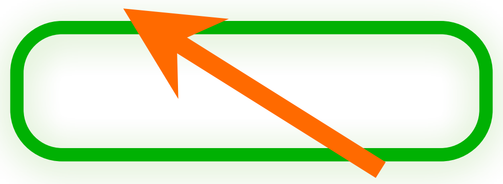
Fermi gas of holon-spinon bound states (magnetic polarons) with a "background" spin liquid

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

Green dimers can tunnel between layers

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007);
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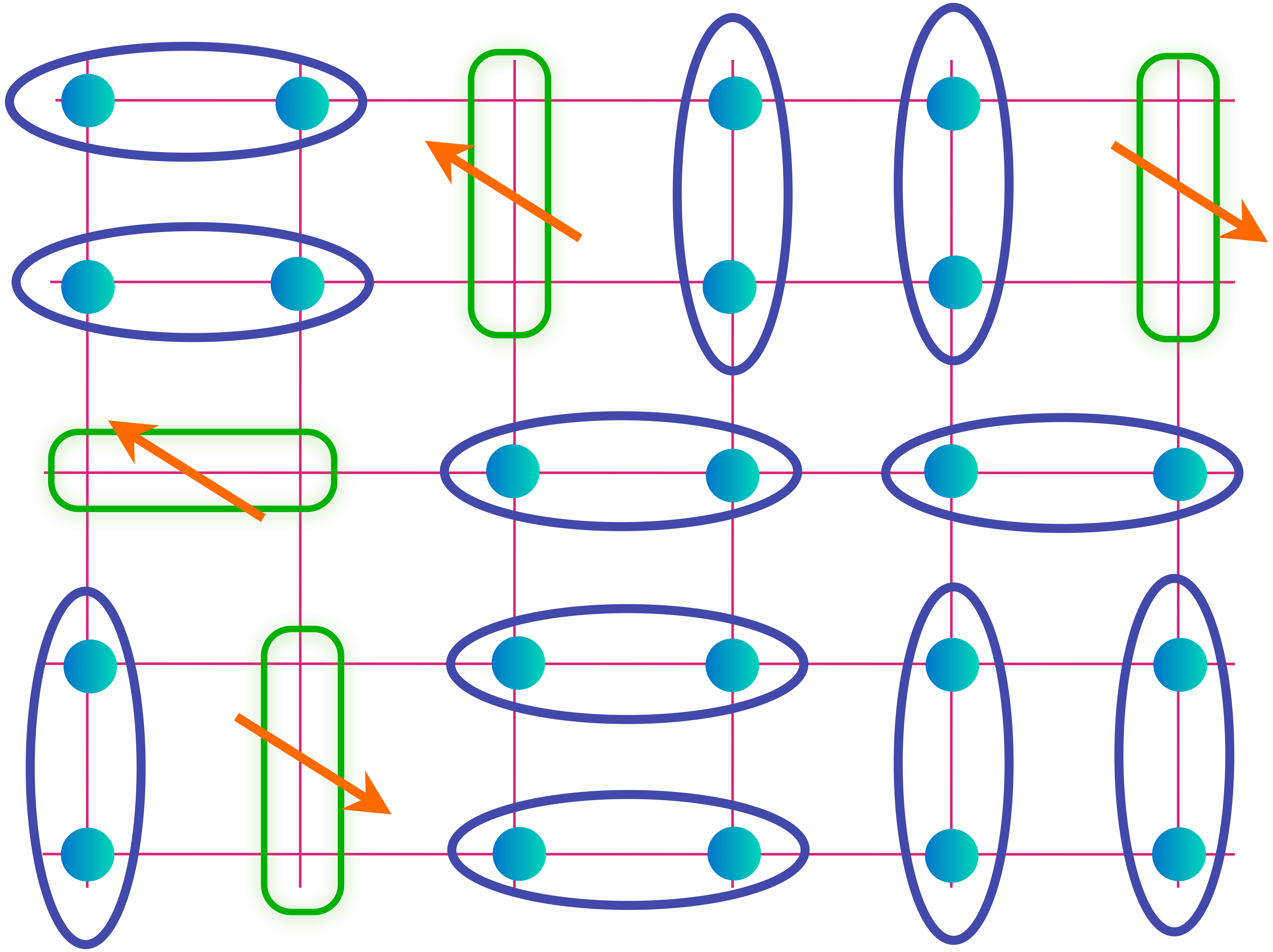
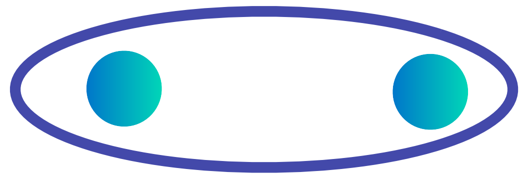
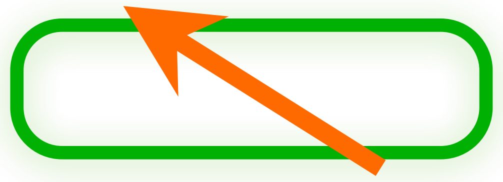


Fermi surface?

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007); R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4,** 28 (2008); Y. Qi and S. S., PRB 81, 115129 (2010); M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

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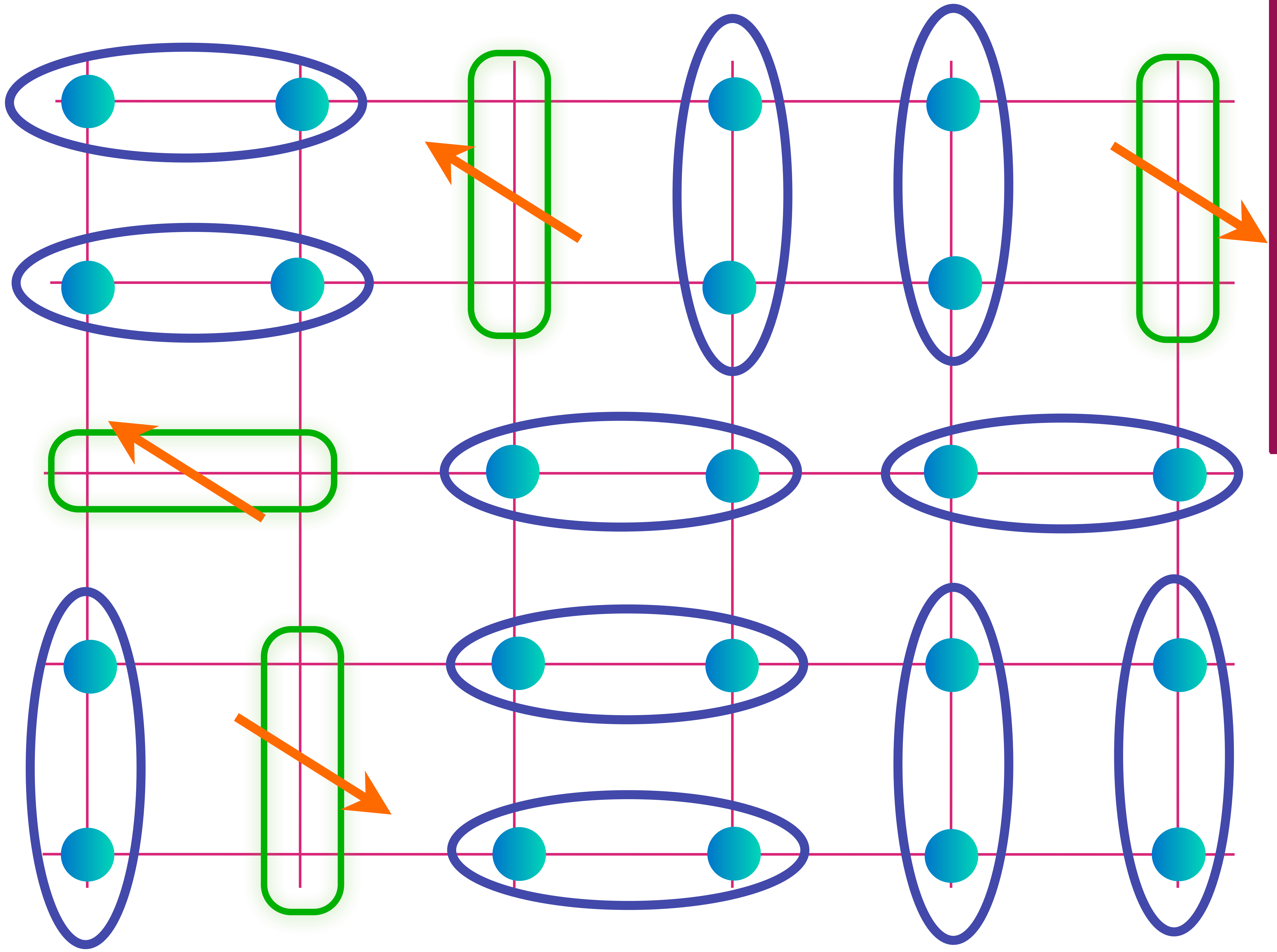
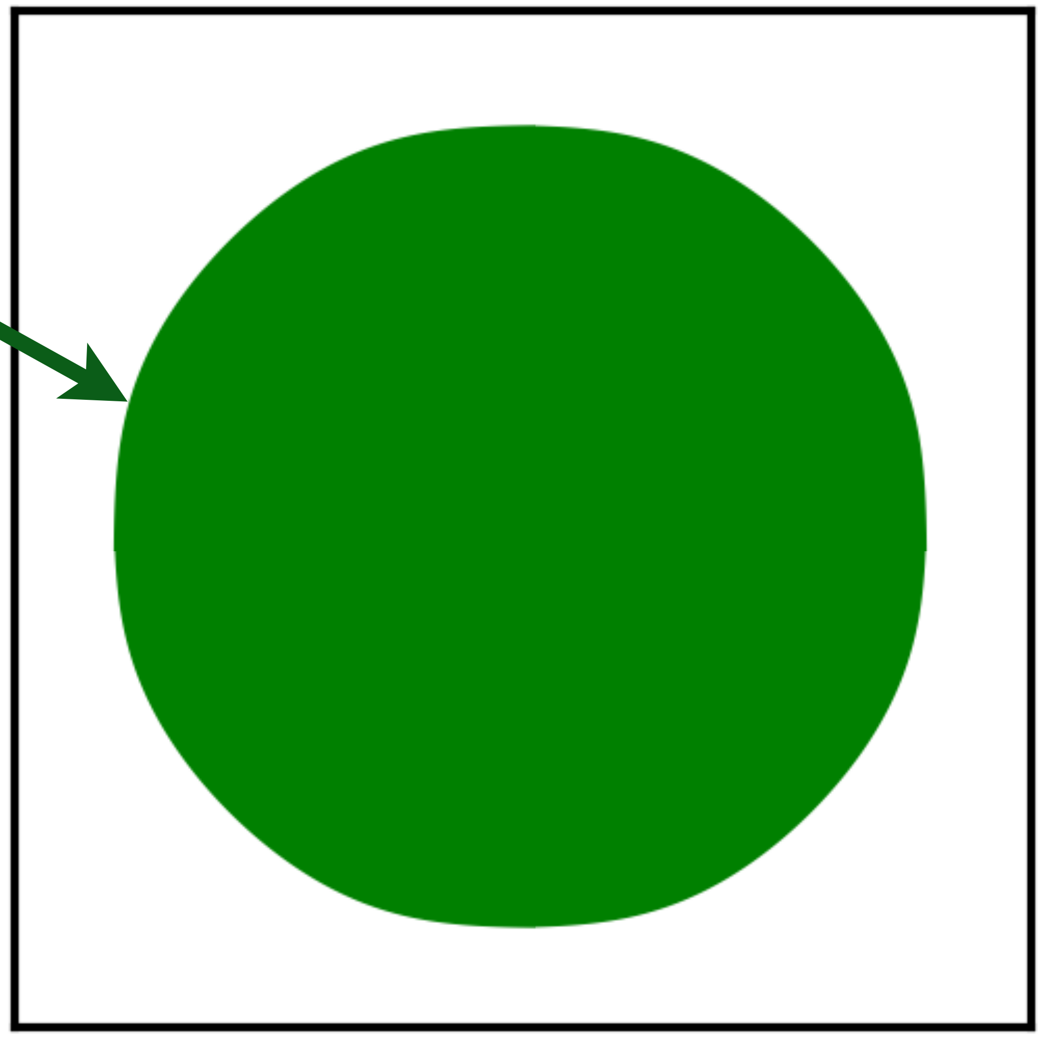


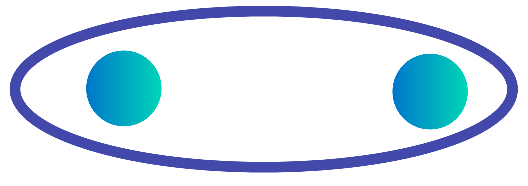
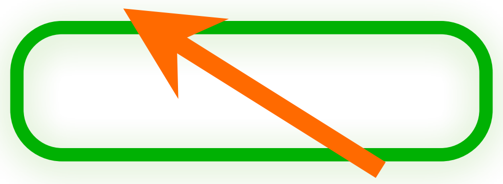
Luttinger area $(1+p)/2$



Count *all* electrons $= 1 - p$.
Holes in a filled band $= 1 + p$.

FL

Fermi surface?

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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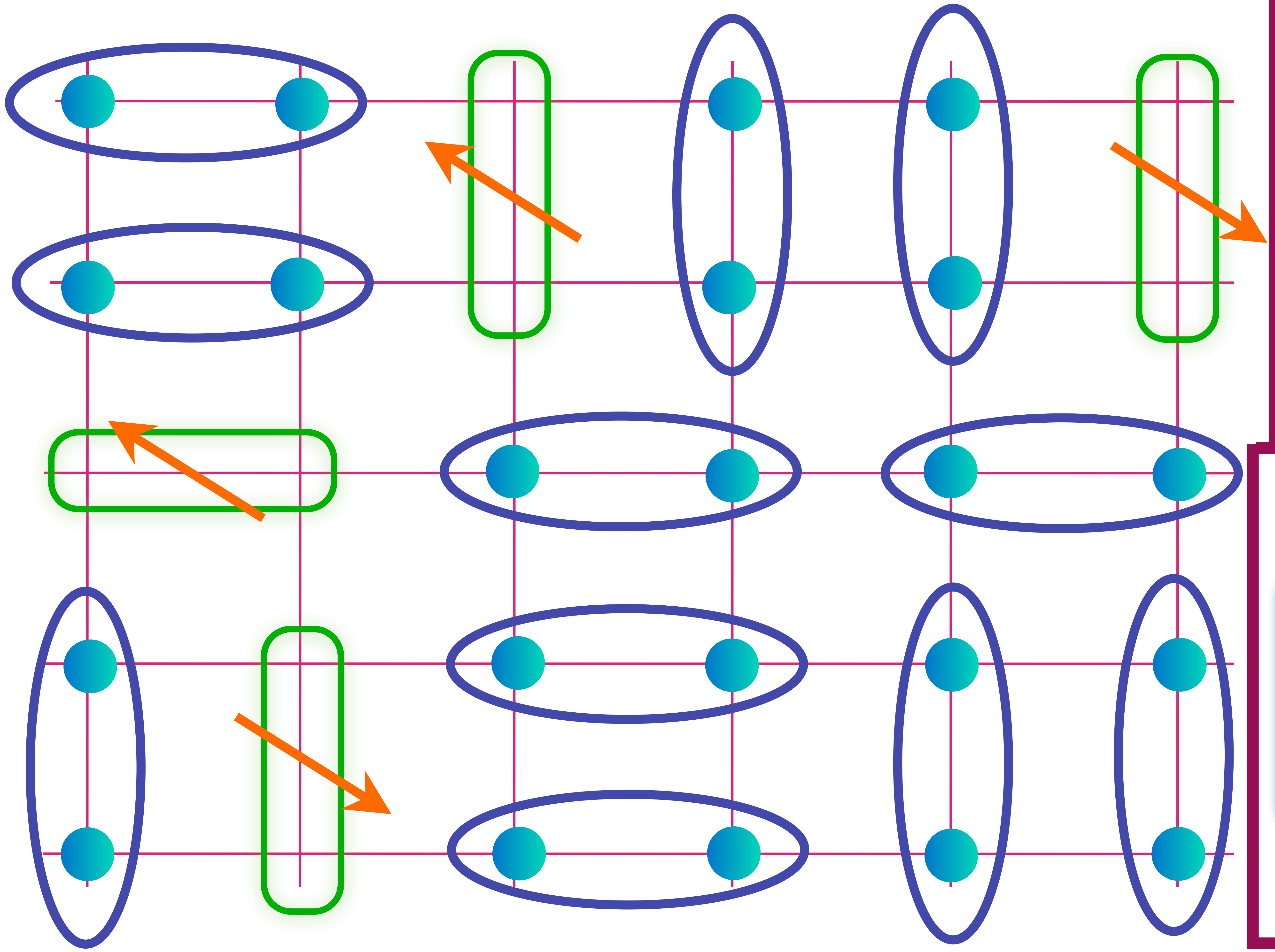
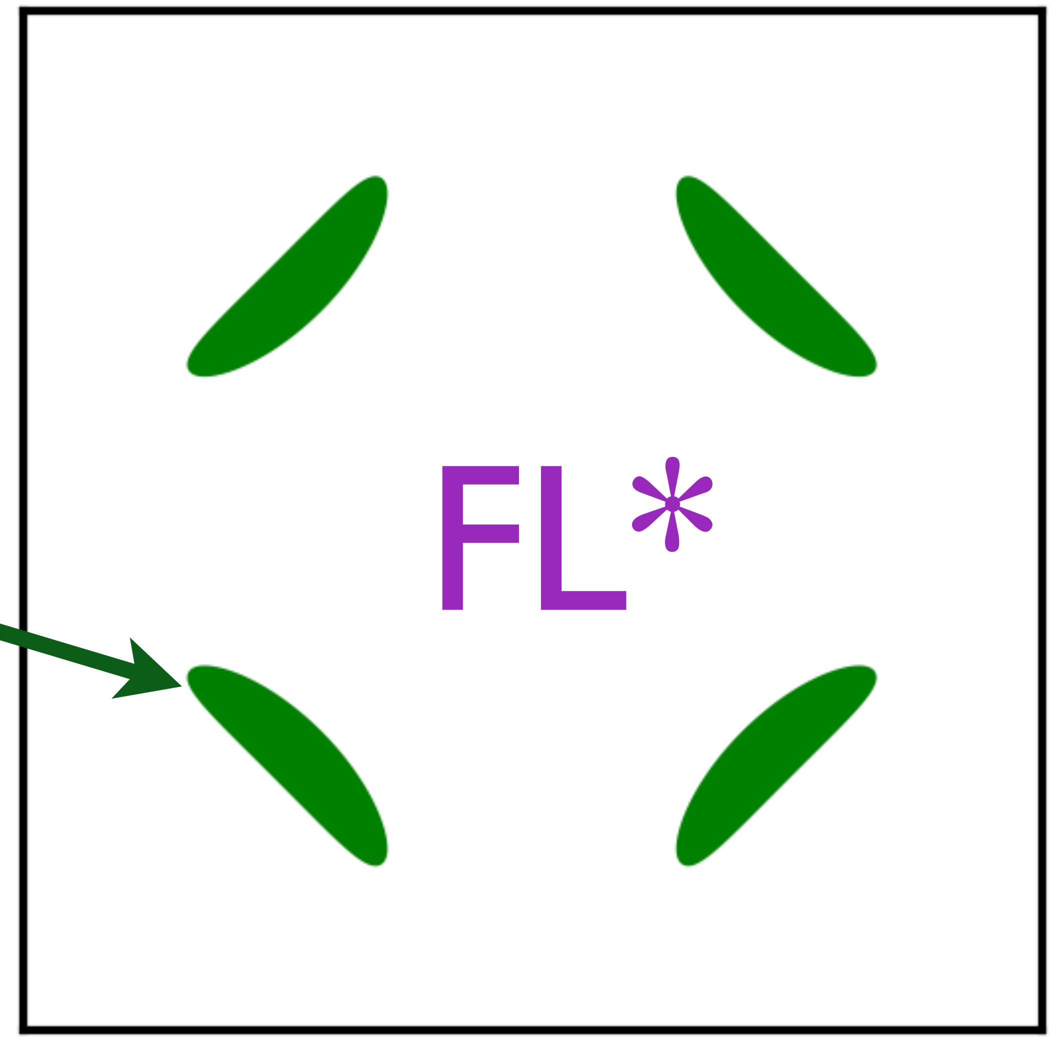
To obtain a (super)conductor we have to remove a density p of electrons



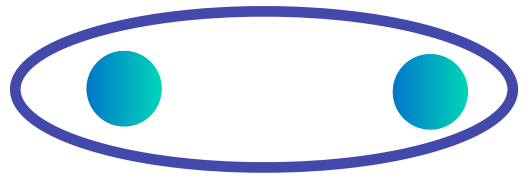
Luttinger area $(1+p)/2$

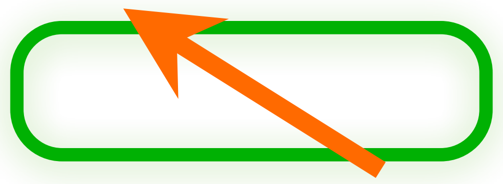
Count *all* electrons $= 1 - p$. Holes in a filled band $= 1 + p$.

Non-Luttinger area $p/8$



Count only green dimers

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

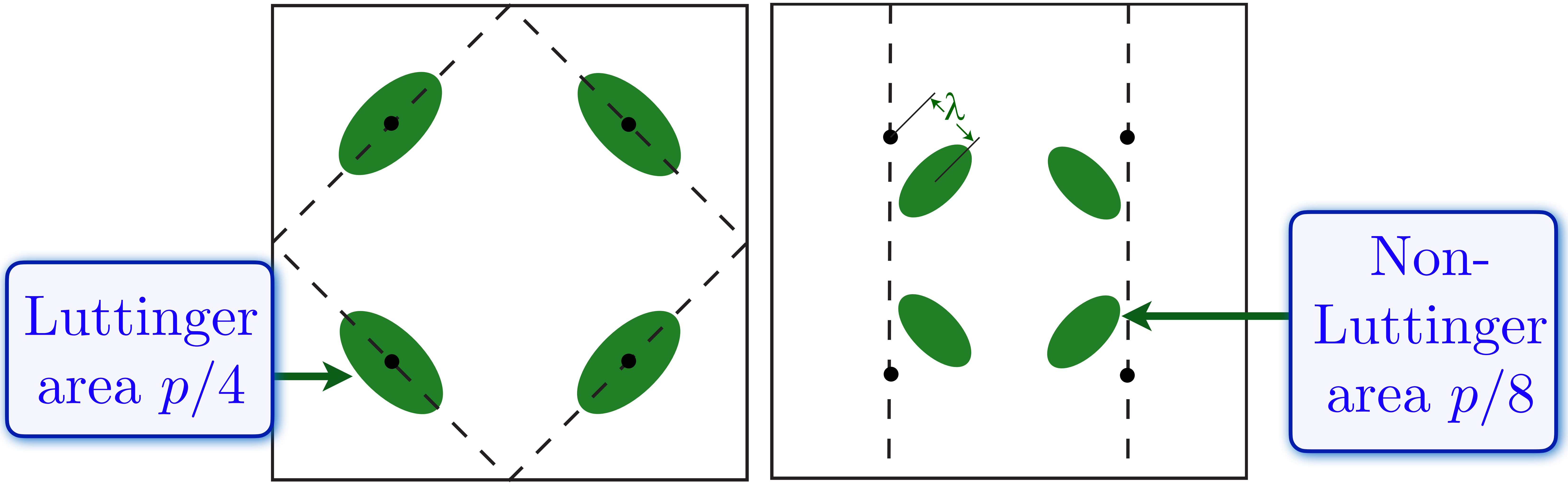
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007);
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4,** 28 (2008); Y. Qi and S. S., PRB 81, 115129 (2010); M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



the area of each ellipse is $\mathcal{A}_F = (2\pi)^2\delta/4$

AF

the area of each ellipse is $\mathcal{A}_F = (2\pi)^2\delta/8$

*FL**

Coherent inter-layer transport requires inter-layer spin correlations

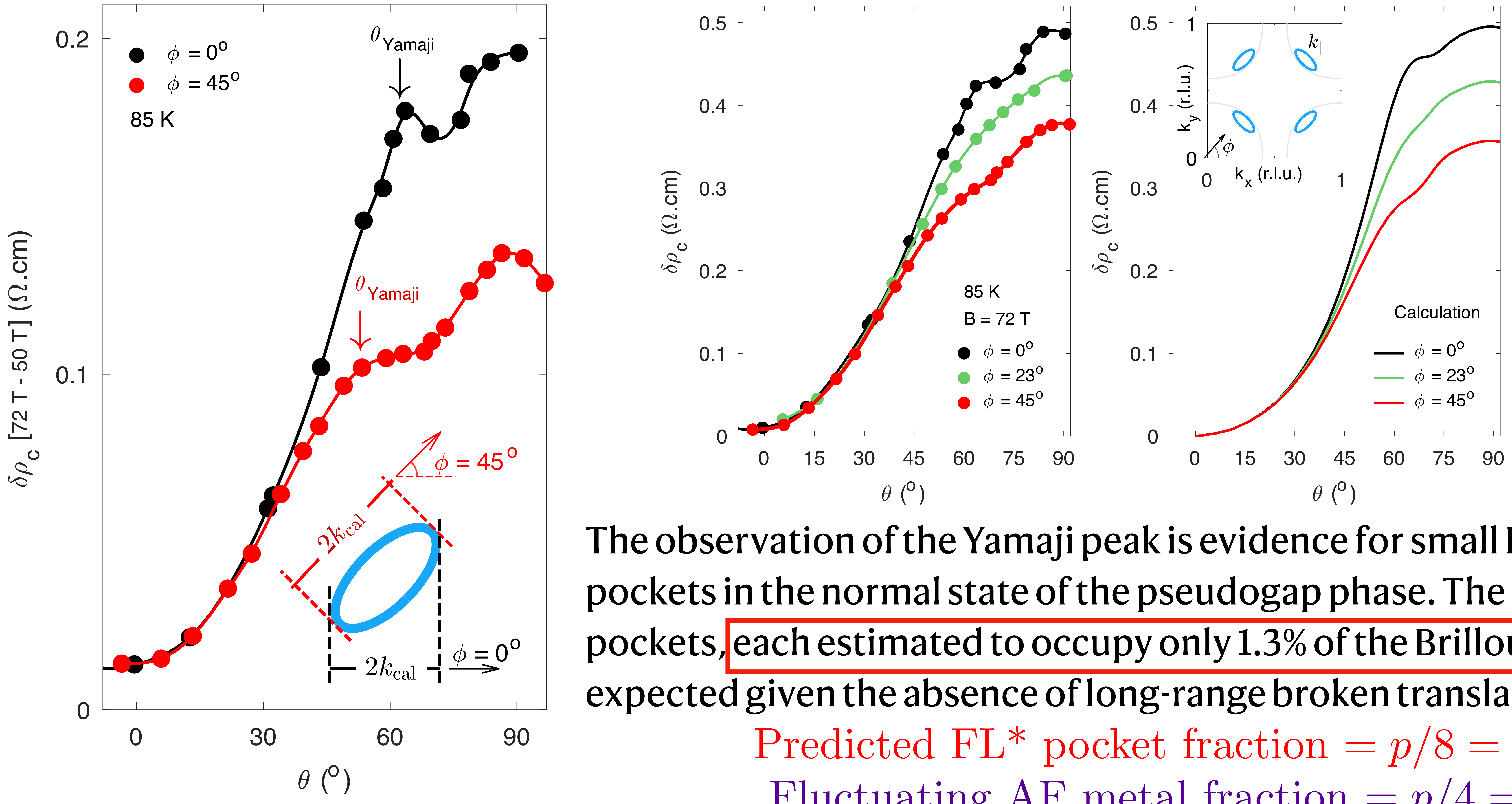
Coherent inter-layer transport

Note factor of 2 between thermally disordered antiferromagnetism and "quantum disordered" antiferromagnetism in FL*.

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan [1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela [1], Eric D. Bauer [2], Arkady Shekhter [1] & Neil Harrison [1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025



Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

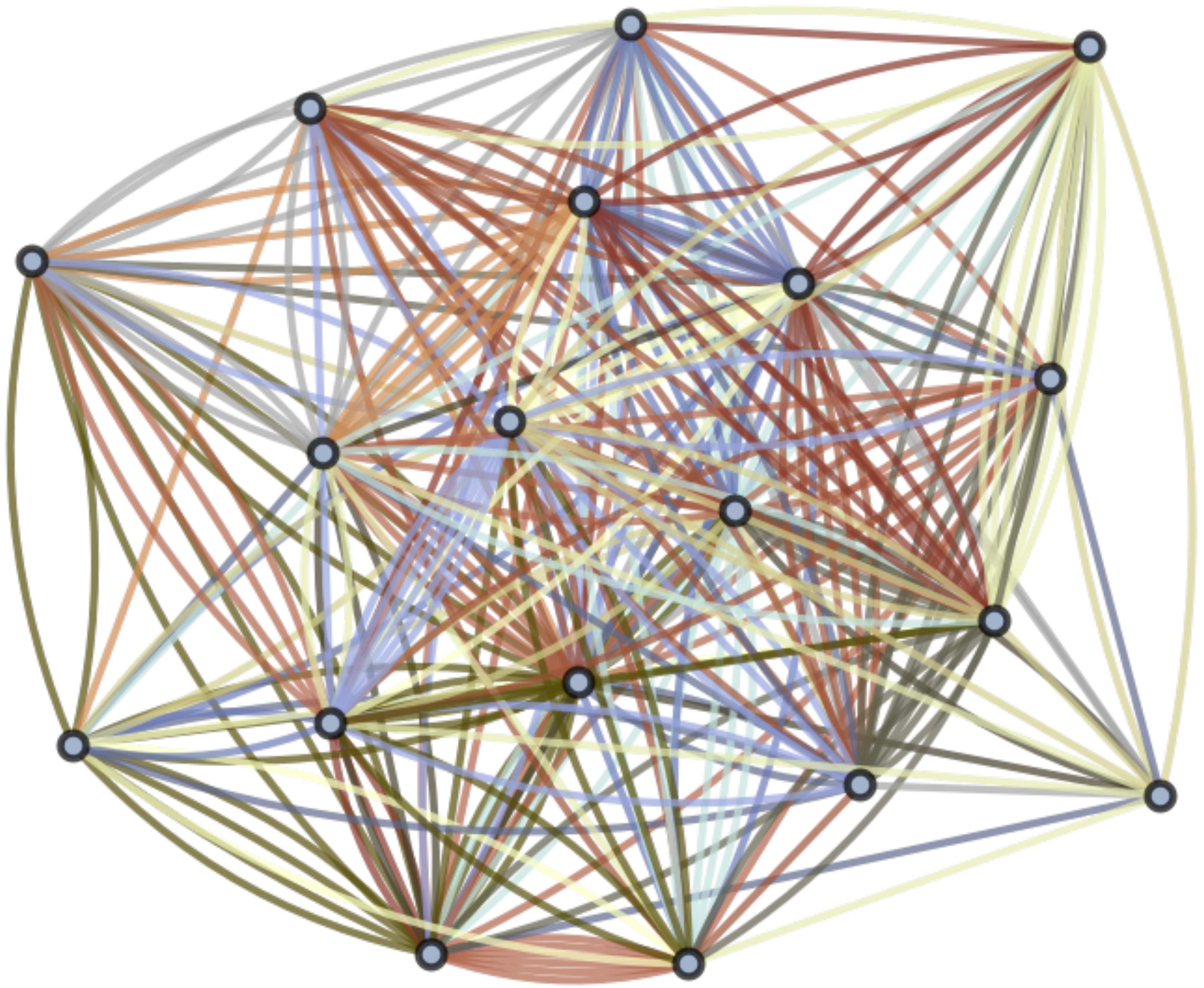
Fluctuating AF metal fraction $= p/4 = 2.5\%$.

Jing-Yu Zhao, S. Chatterjee, S. S, Ya-Hui Zhang, arXiv:2510.13943

Recap

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in Planckian dynamics and the loss of identity of the particles



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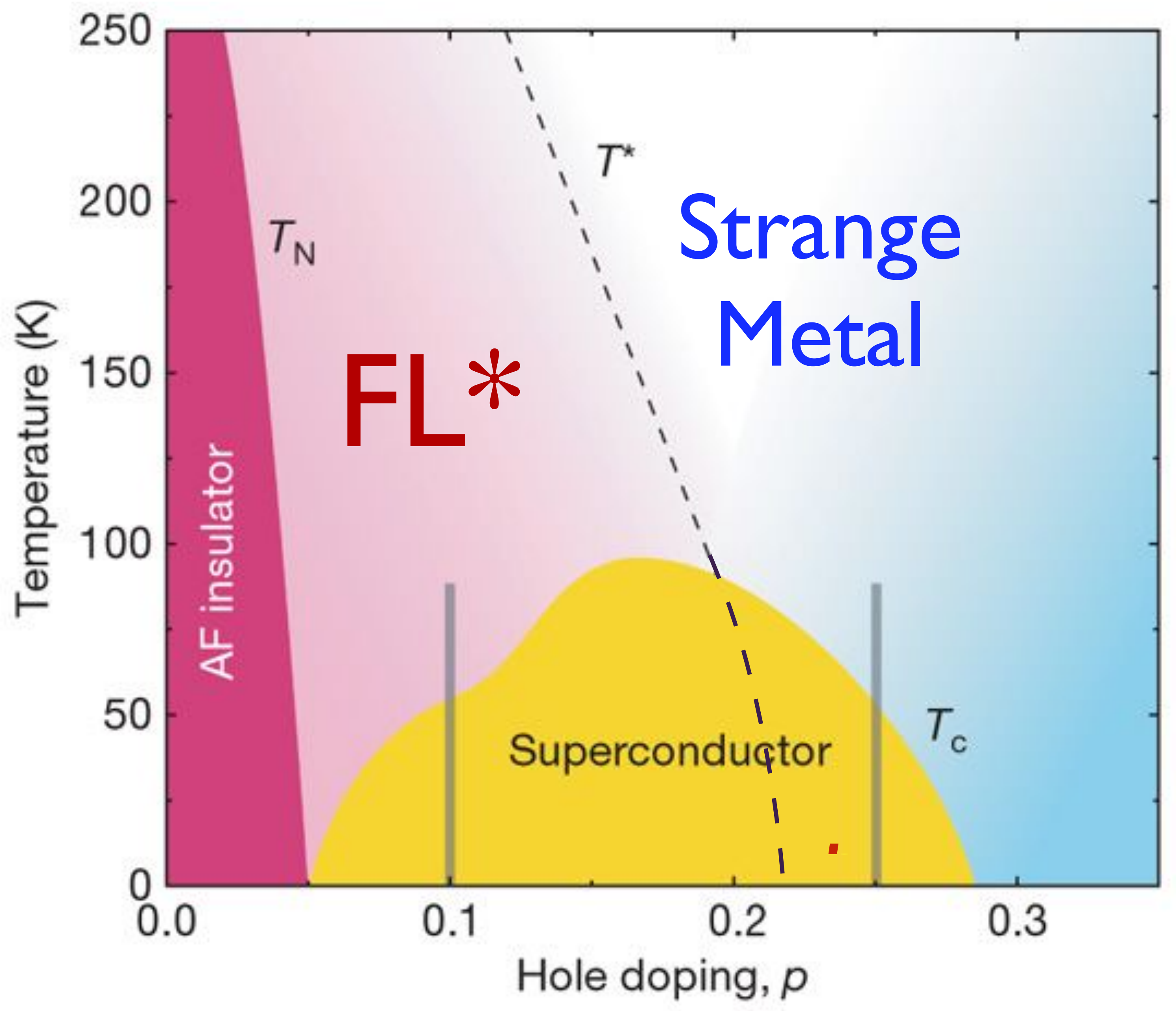
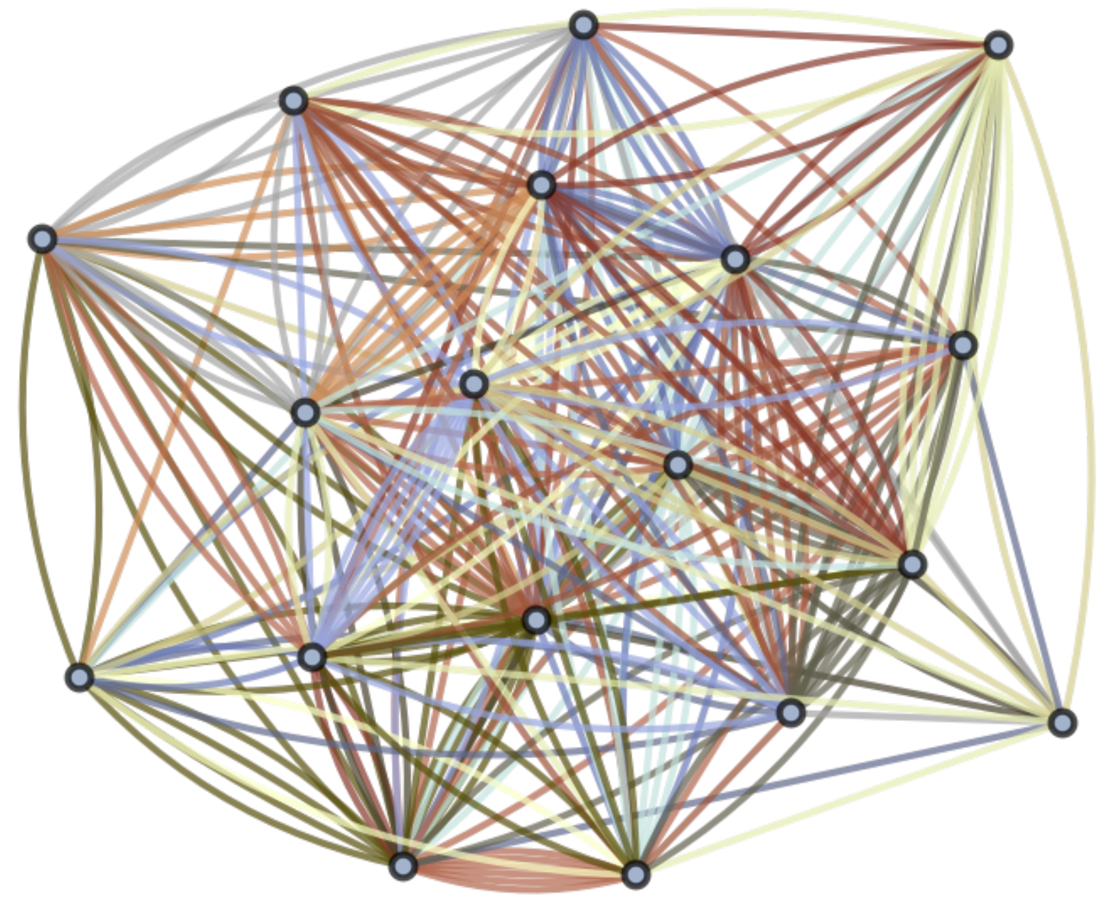
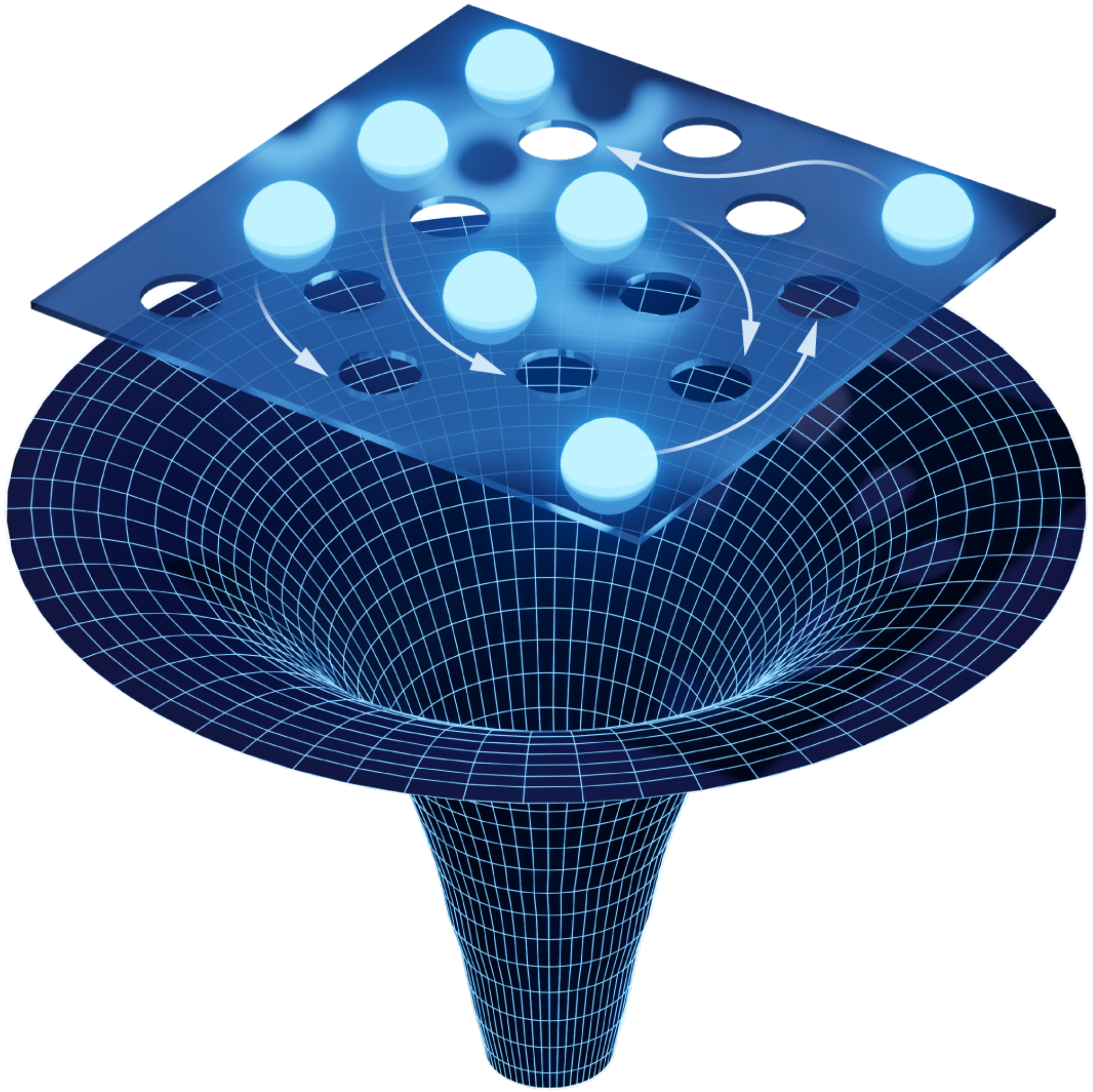


In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



The Sachdev-Ye-Kitaev (SYK) model



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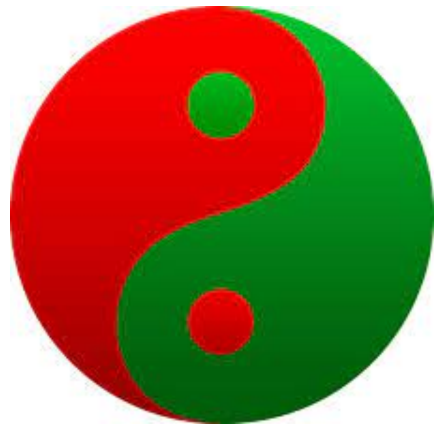
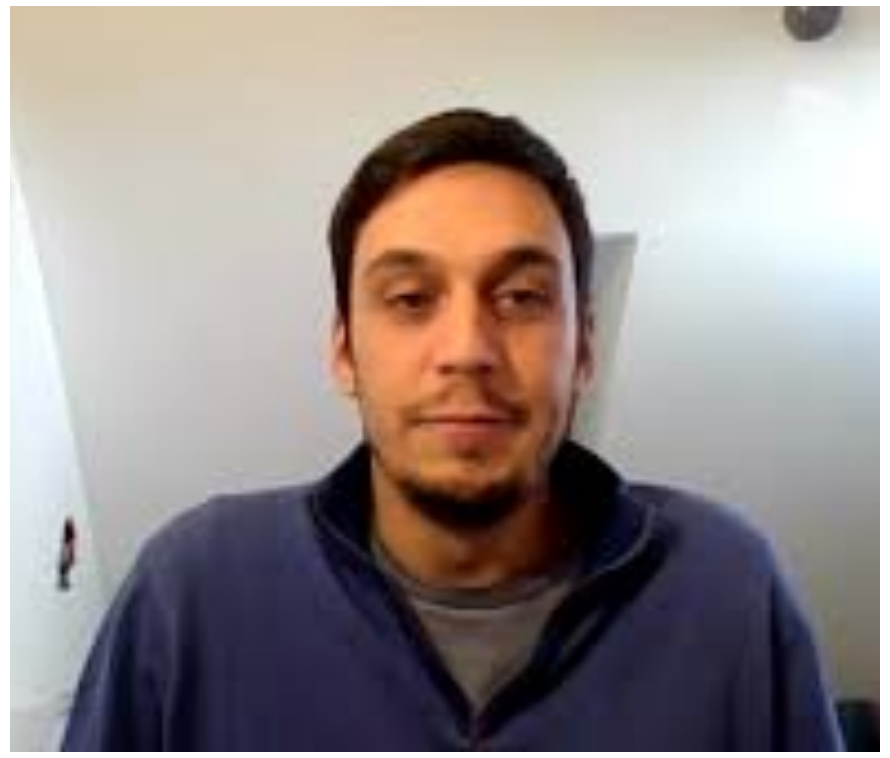


In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Maine Christos
Caltech



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