A simple model of entangled qubits, describing black holes and superconductors

Trinity Mathematical Society
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Subir Sachdev

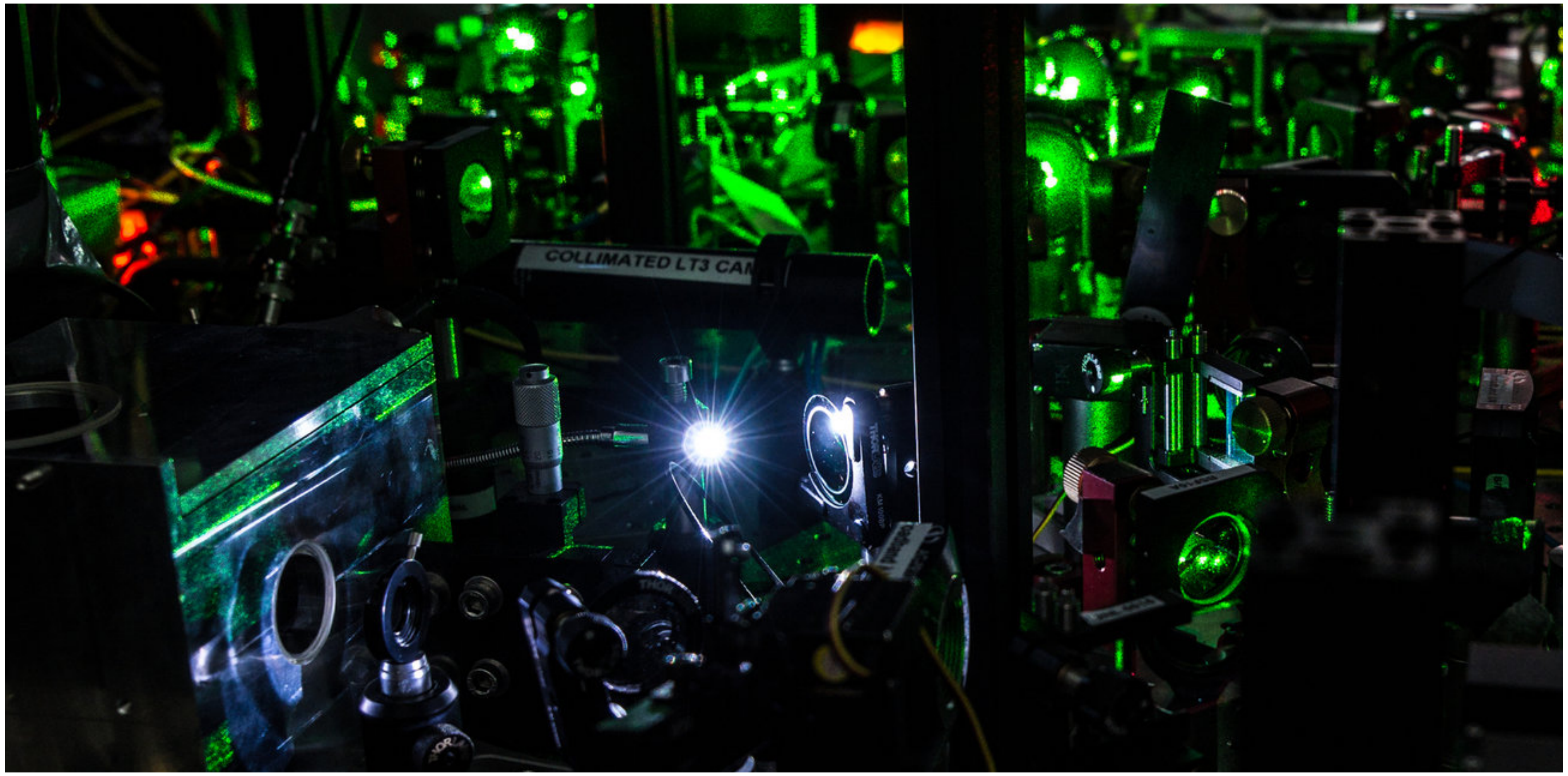
Talk online: sachdev.physics.harvard.edu

The New York Times

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

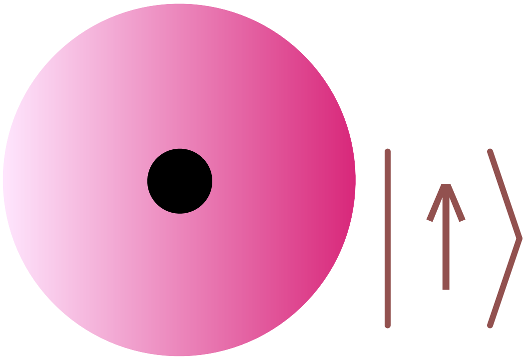


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

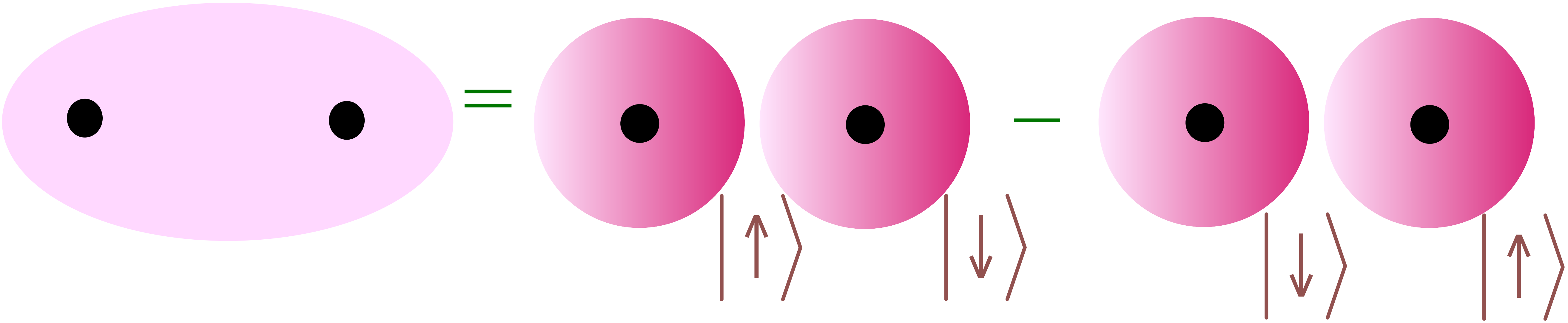
Quantum entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

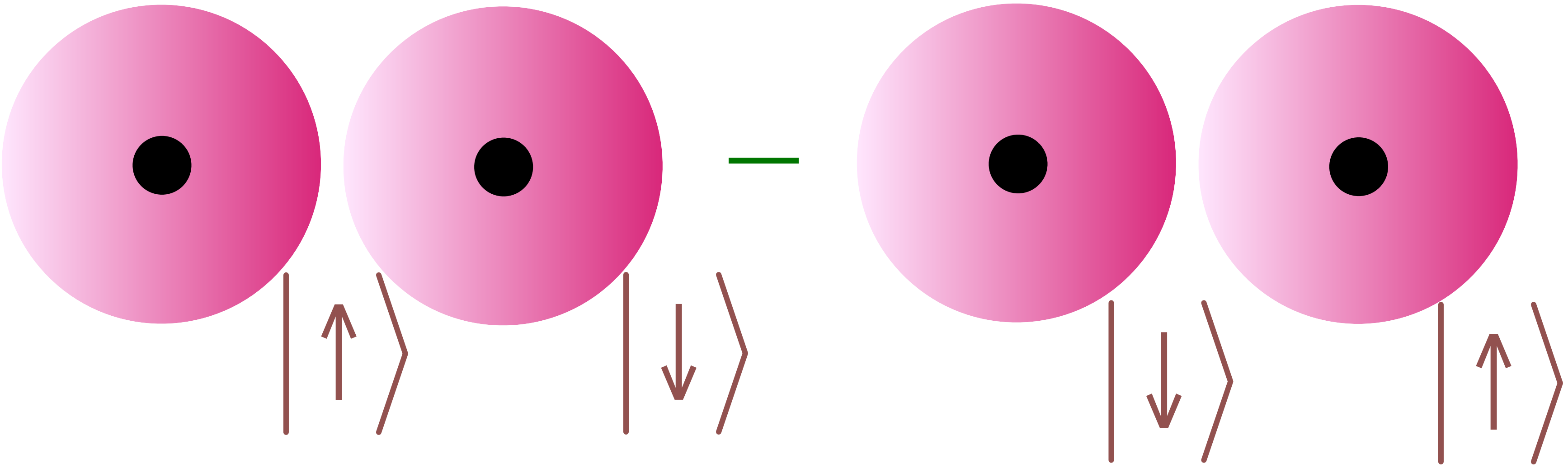


Hydrogen molecule:



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Quantum Entanglement: quantum superposition with more than one particle



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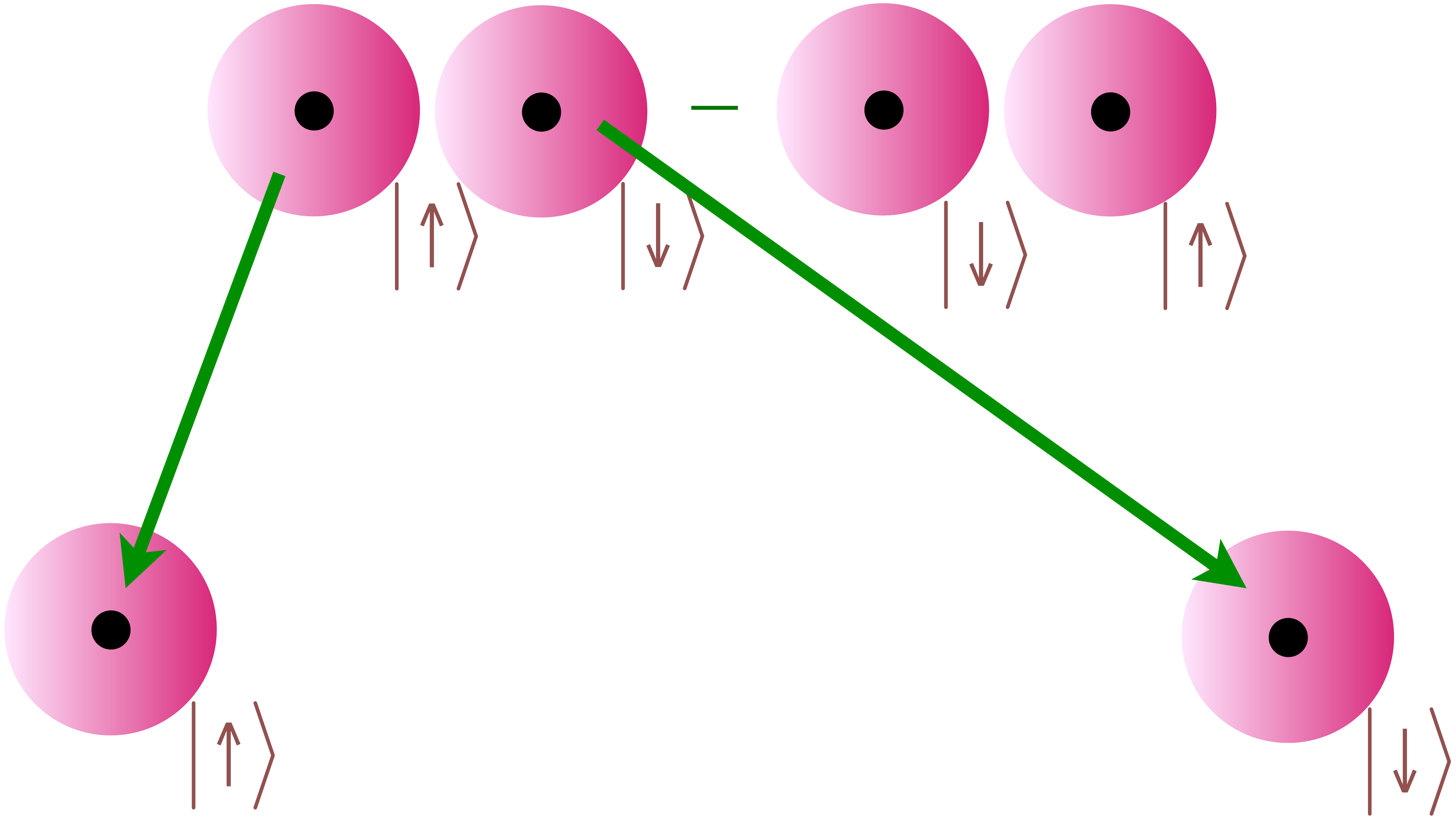

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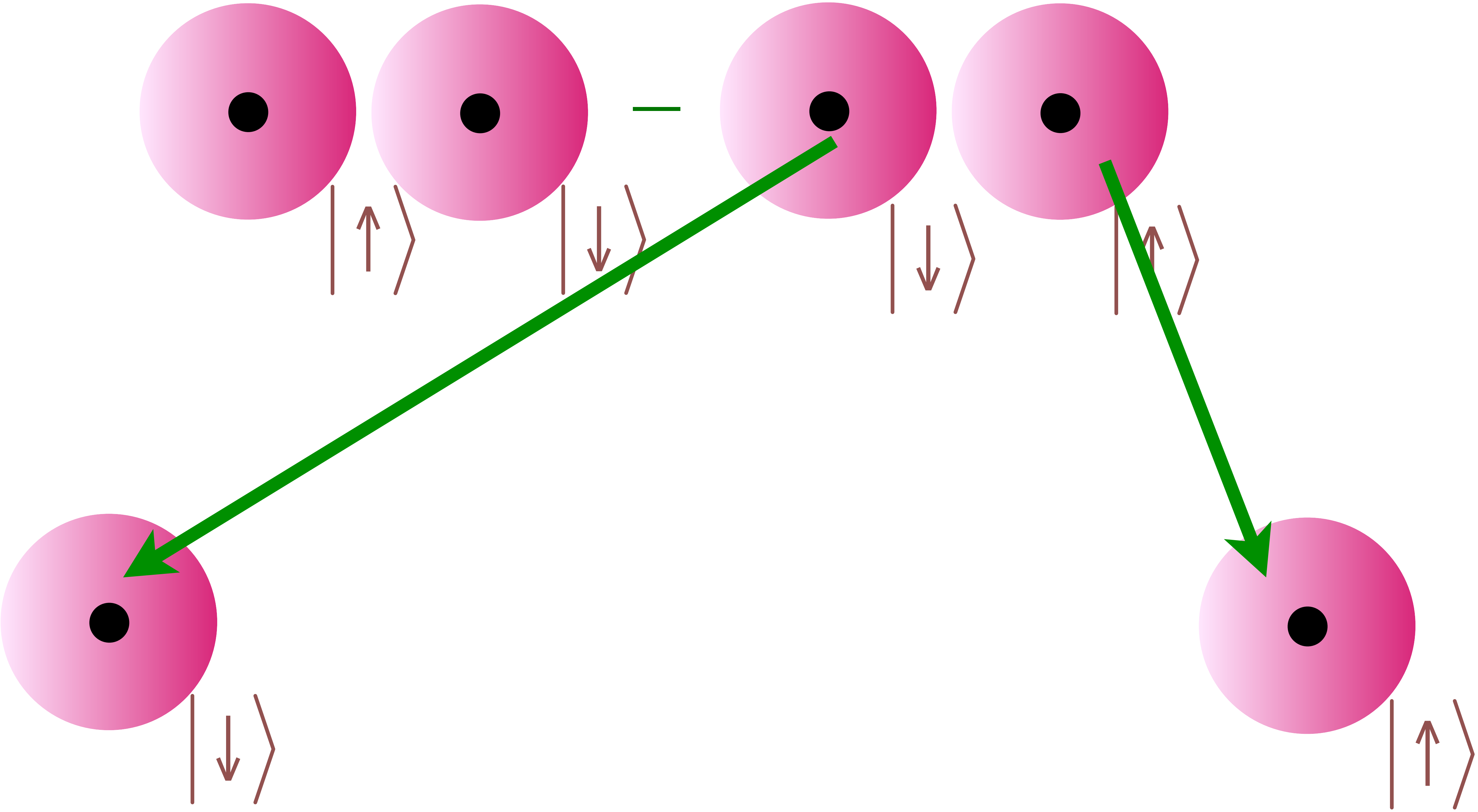

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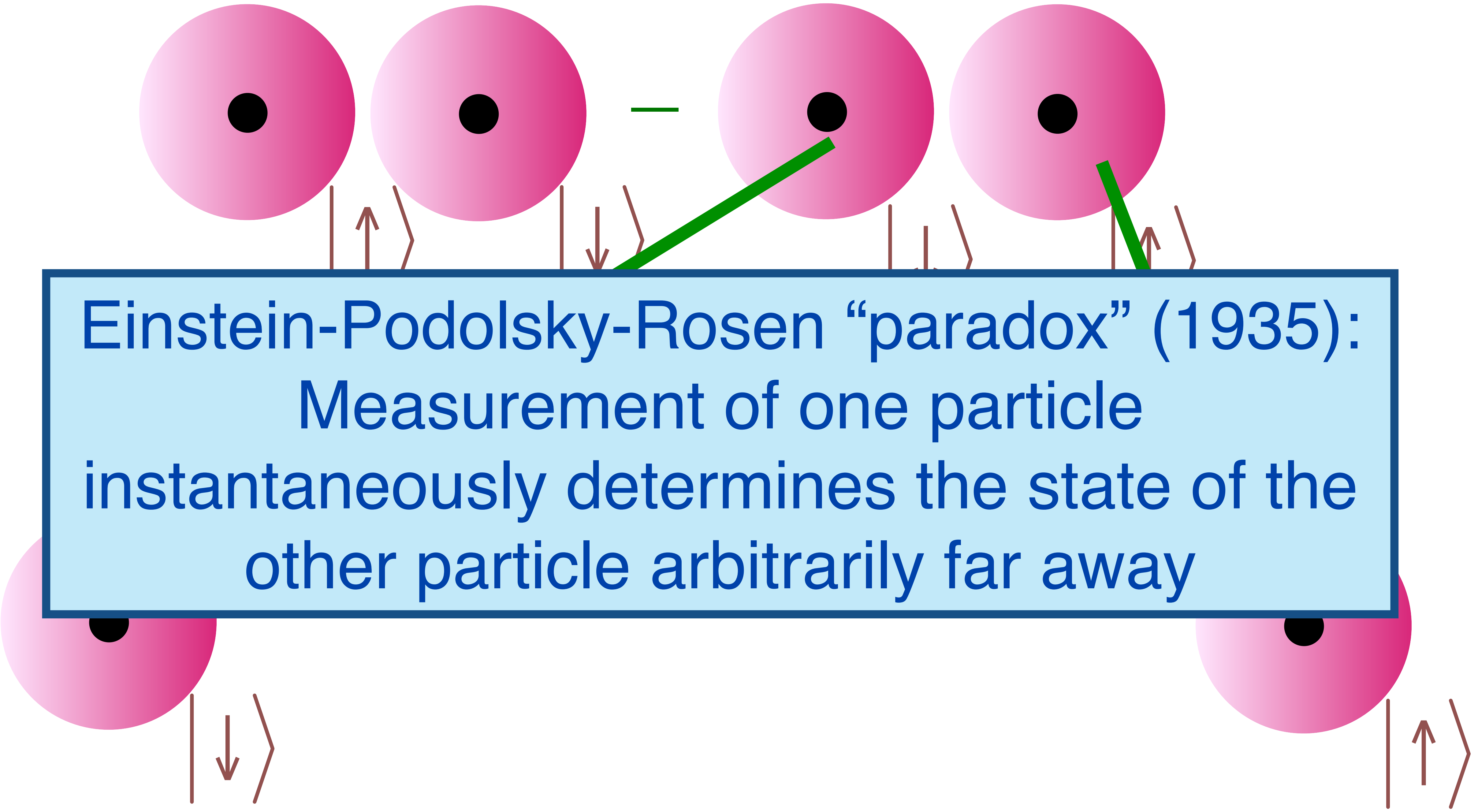

Einstein-Podolsky-Rosen "paradox" (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

Quantum entanglement

Quantum entanglement

A simple qubit model

A qubit: 2 states $|\uparrow\rangle$, $|\downarrow\rangle$.

Pauli gates:

$$X\,|\uparrow\rangle = |\downarrow\rangle \quad , \quad X\,|\downarrow\rangle = |\uparrow\rangle$$

$$Y\,|\uparrow\rangle = i\,|\downarrow\rangle \quad , \quad Y\,|\downarrow\rangle = -i\,|\uparrow\rangle$$

$$Z\,|\uparrow\rangle = |\uparrow\rangle \quad , \quad Z\,|\downarrow\rangle = -\,|\downarrow\rangle$$

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A 2-qubit Hamiltonian: $\mathcal{H} = J\left(X_1X_2 + Y_1Y_2 + Z_1Z_2\right)$

Ground state: $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1\,|\downarrow\rangle_2 - |\downarrow\rangle_1\,|\uparrow\rangle_2\right)$

Energy $= -3J$

Excited states: $|\uparrow\rangle_1\,|\uparrow\rangle_2, \quad |\downarrow\rangle_1\,|\downarrow\rangle_2, \quad \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1\,|\downarrow\rangle_2 + |\downarrow\rangle_1\,|\uparrow\rangle_2\right)$

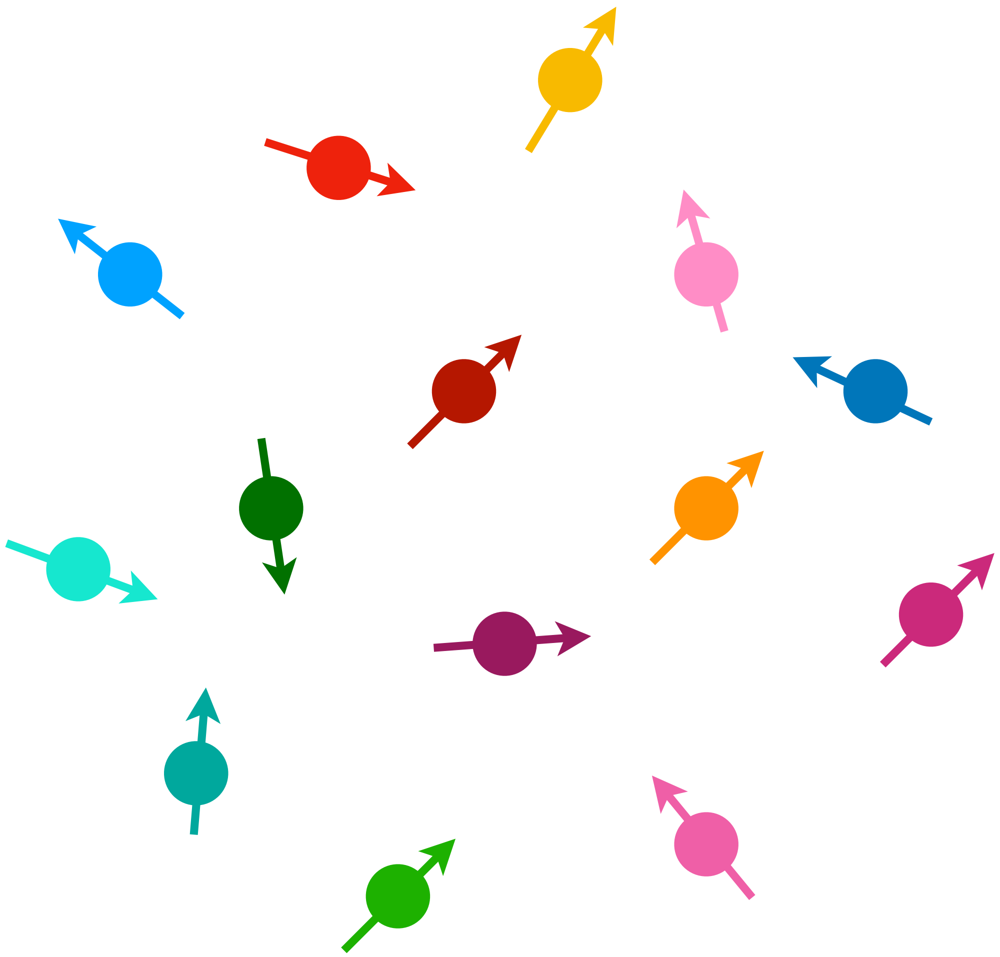
Energy $= J$

The simple model

$$\mathcal{H} = \sum_{i<j=1}^{N} J_{ij} \left(X_i X_j + Y_i Y_j + Z_i Z_j \right)$$

J_{ij} are random numbers with zero mean, and variance $\sim J^2/N$

(Technical comment: the solvable model has states in the self-conjugate representation of SU(M), and we take the limit $N \to \infty$ followed by the limit $M \to \infty$)



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

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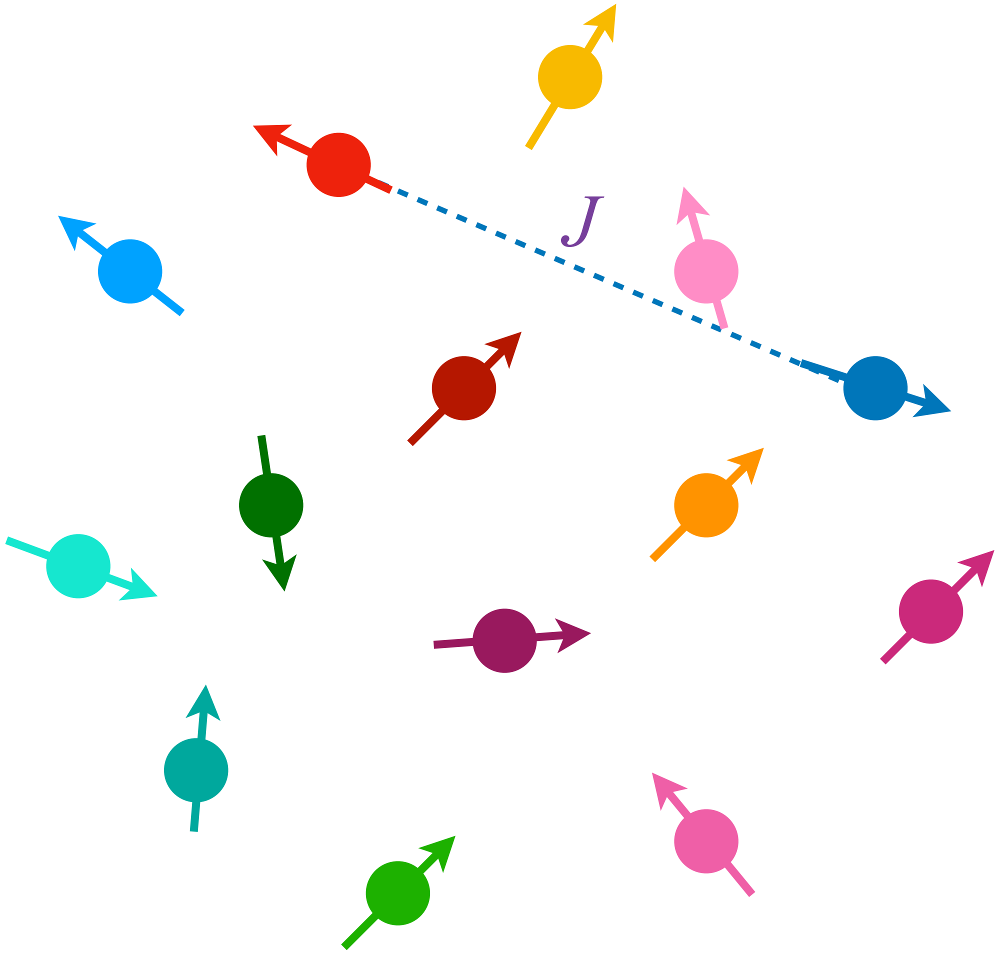

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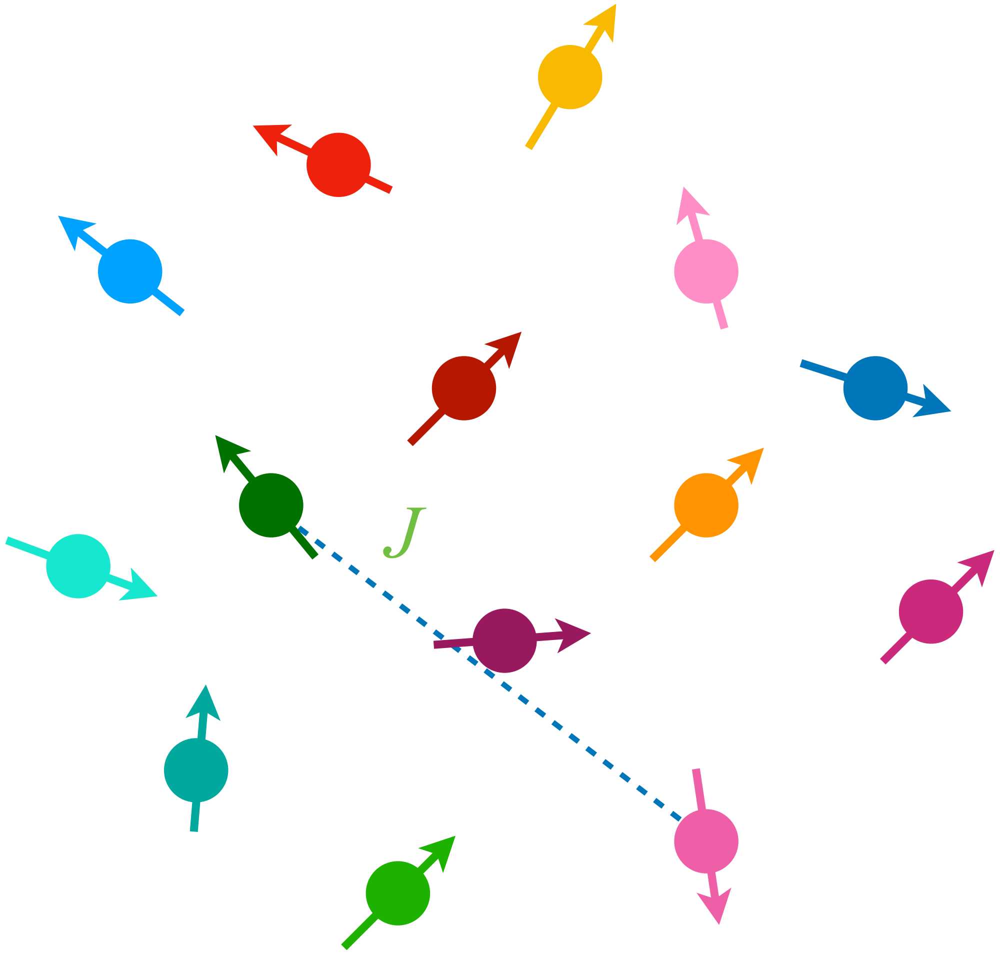

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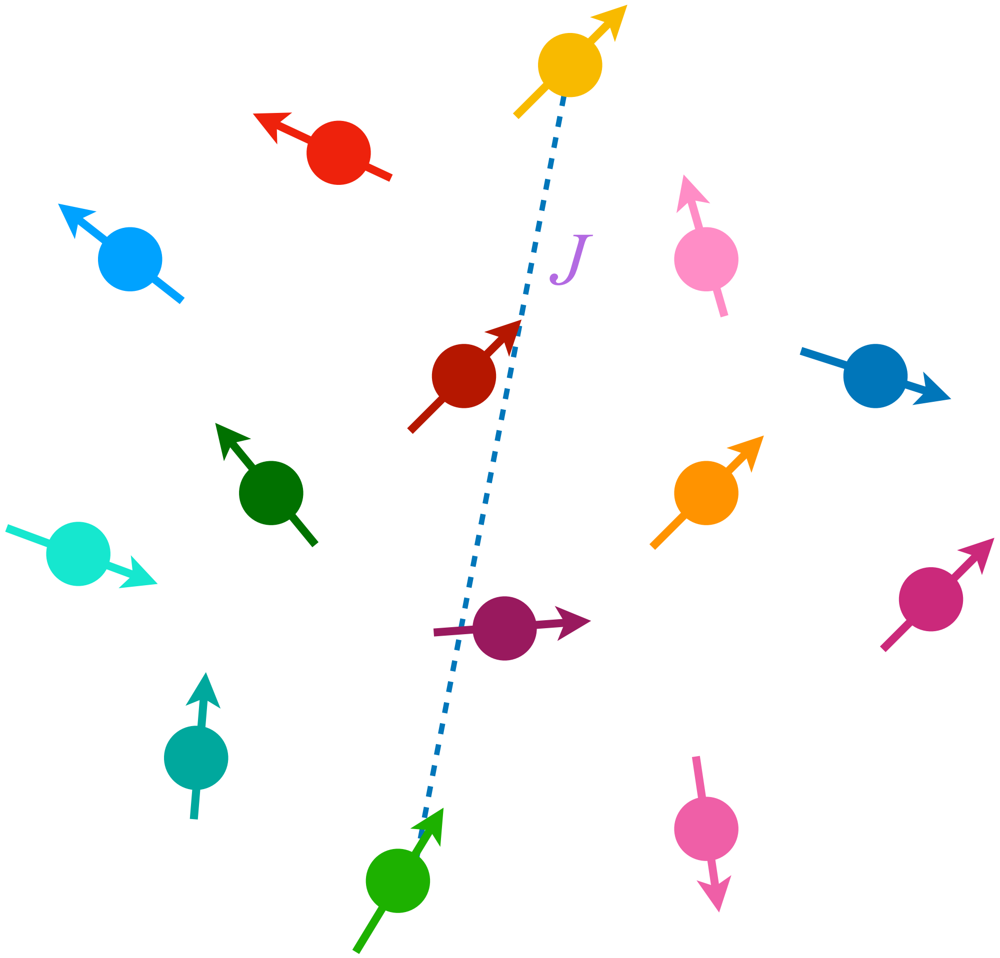

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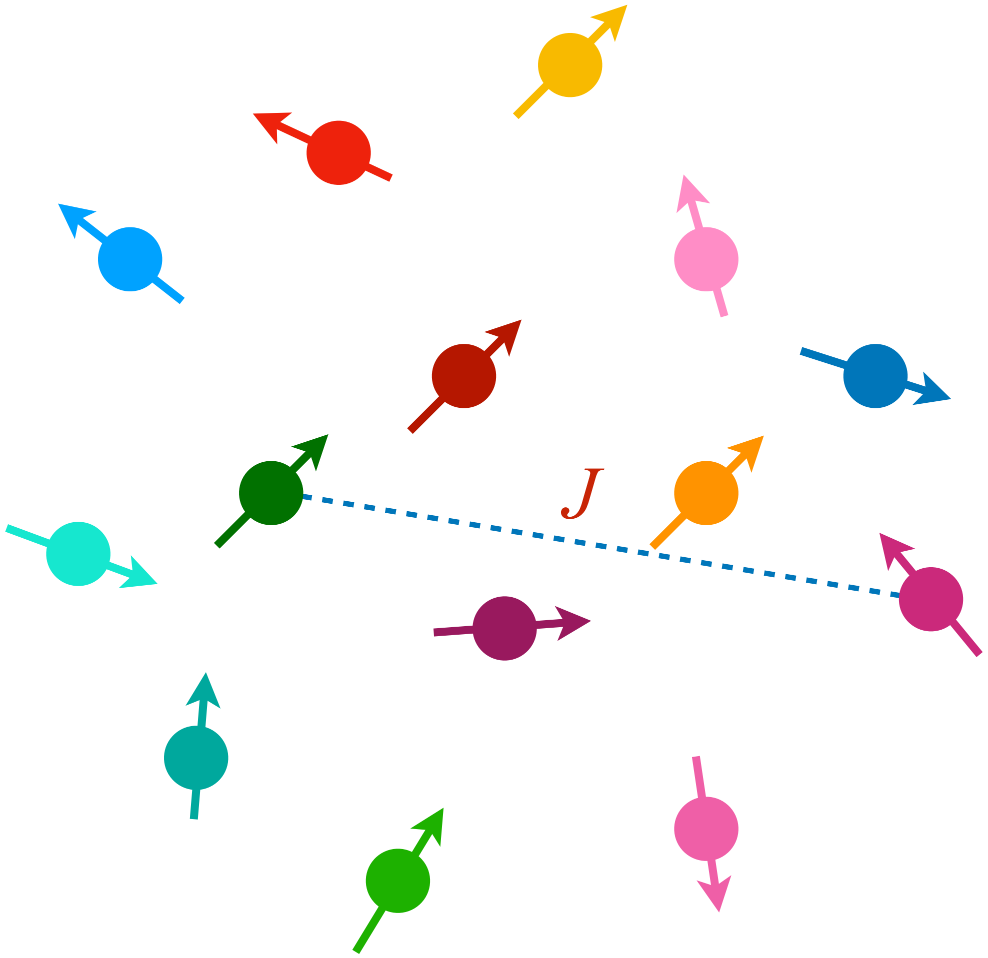

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The main result

For $k_B T \ll J$

$$
\begin{aligned}
\mathcal{Z} &= \operatorname{Tr} \exp\left(-\frac{\mathcal{H}}{k_B T}\right) \\
&= \exp\left(N \frac{S_0}{k_B}\right) \int \mathcal{D} f(\tau) \exp\left(-\frac{1}{\hbar} \mathcal{S}_{\rm 2D-gravity}\left[f(\tau)\right]\right)
\end{aligned}
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S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
A. Kitaev (2015)
J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

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S_0 is the $T \to 0$ entropy of the qubit model.
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

It maps on to the Bekenstein-Hawking
entropy of charged black holes

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
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- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: τ on a circle of circumference $\hbar/(k_B T)$.
- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a ‘boundary graviton’.
- The action of 2D-gravity, $\mathcal{S}_{2\mathrm{D-gravity}}$, is constrained by an emergent time reparameterization symmetry which is broken down to a conformal symmetry (SL(2,R)).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)
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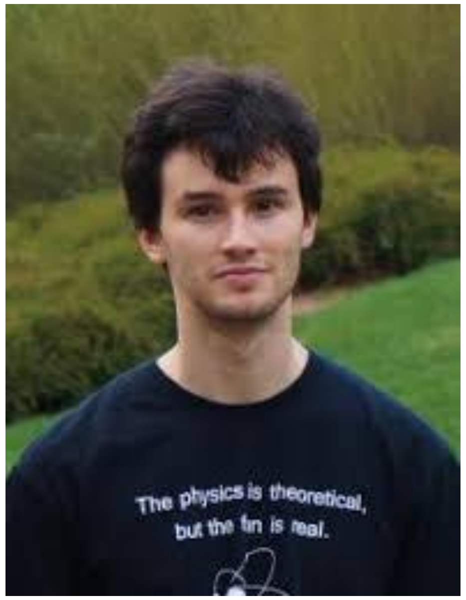
Maria Tikhanovskaya

Haoyu Guo



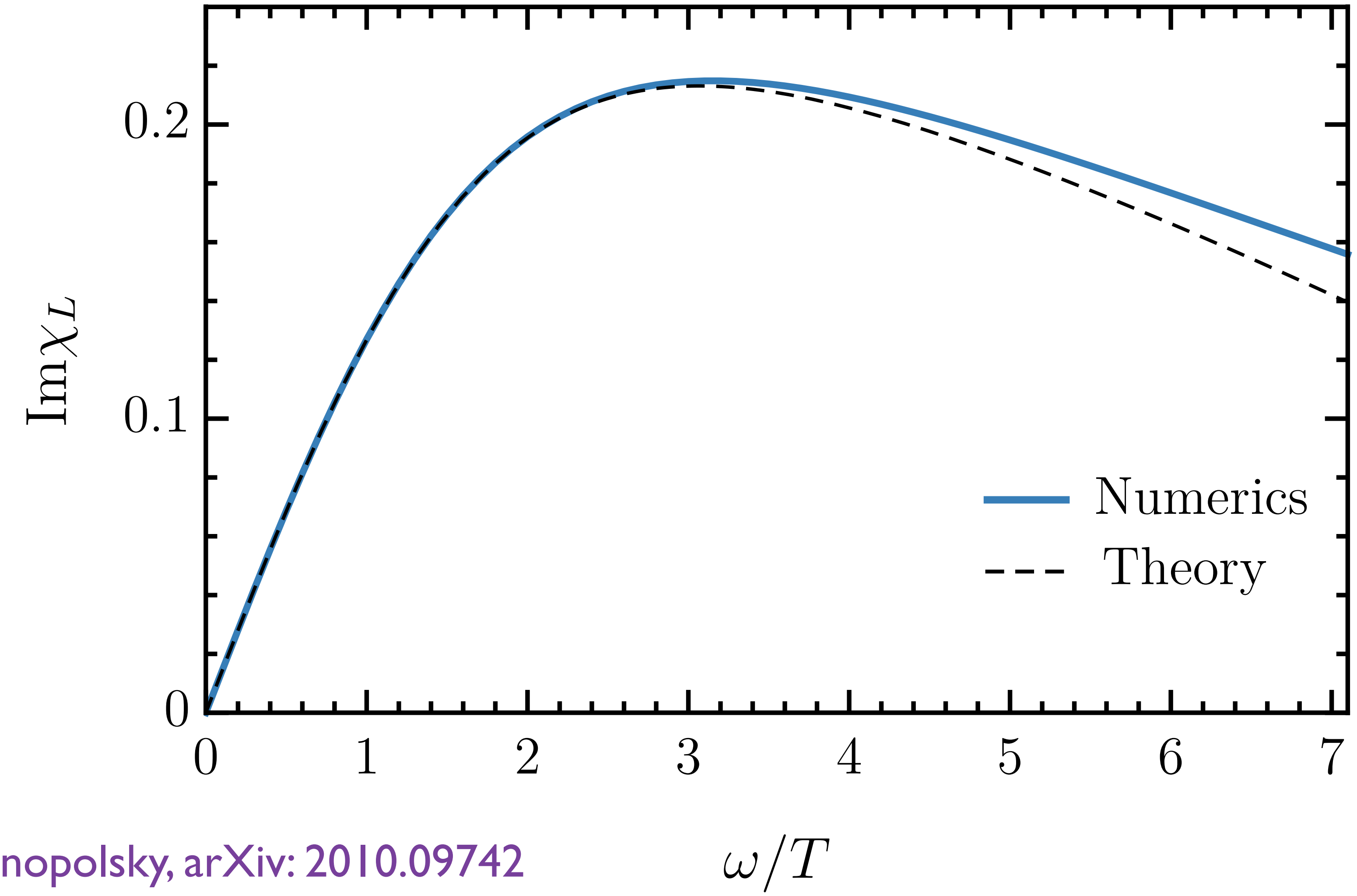
Grigory Tarnopolsky

arXiv:2010.09742

Consequences of 2D-gravity for the dynamic spin susceptibility

$$\chi_L(\omega) = \sum_n |\langle 0| X_i |n\rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\chi_L(\omega) \sim \tanh\left(\frac{\hbar\omega}{2k_B T}\right)\left[1 - \mathcal{C}\gamma\,\omega\tanh\left(\frac{\hbar\omega}{2k_B T}\right) - \ldots\right]$$



Maria Tikhanovskaya, Haoyu Guo, S. Sachdev, G. Tarnopolsky, arXiv: 2010.09742

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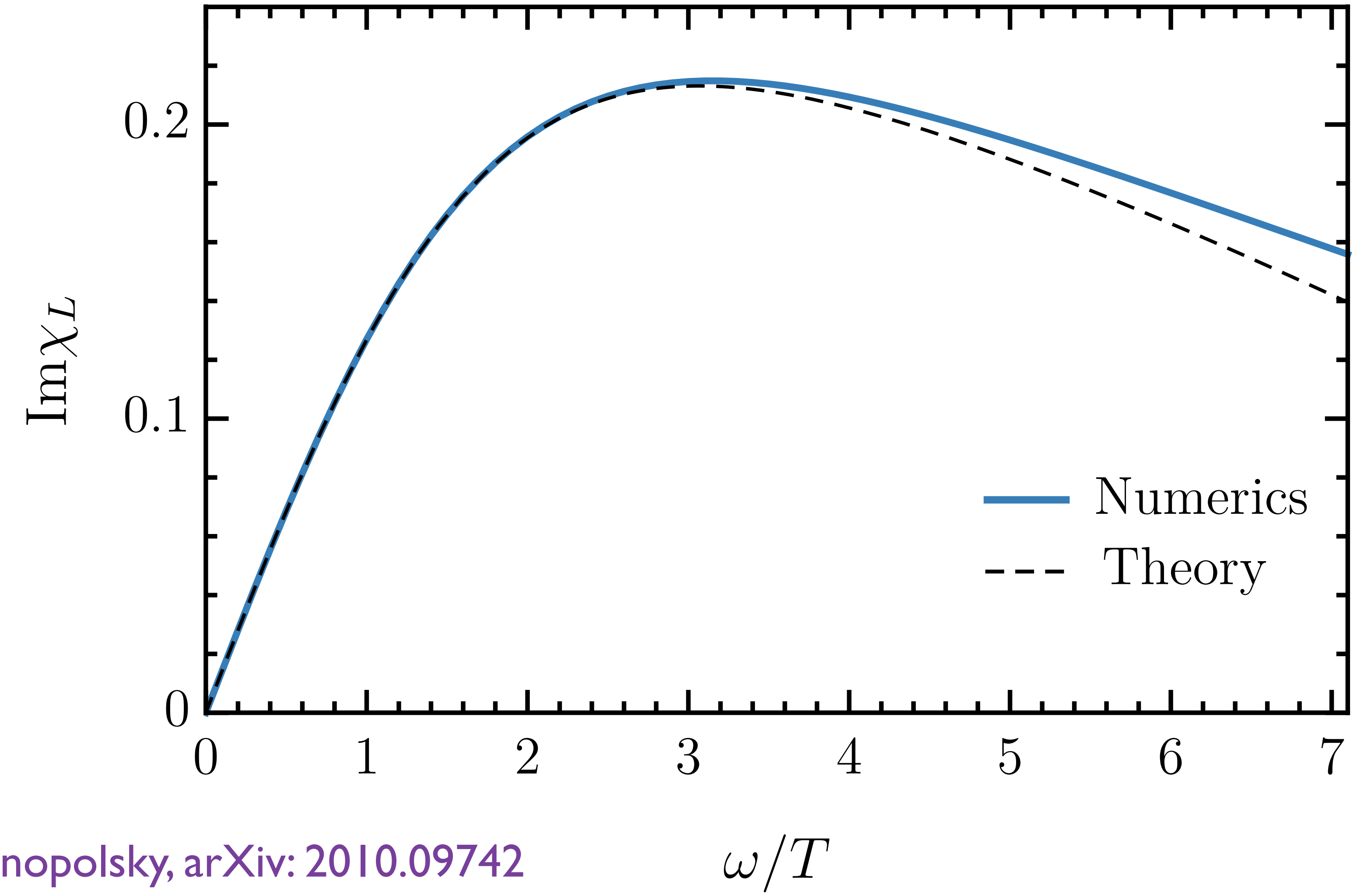
Conformally (SL(2,R)) invariant result with characteristic dissipative time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)



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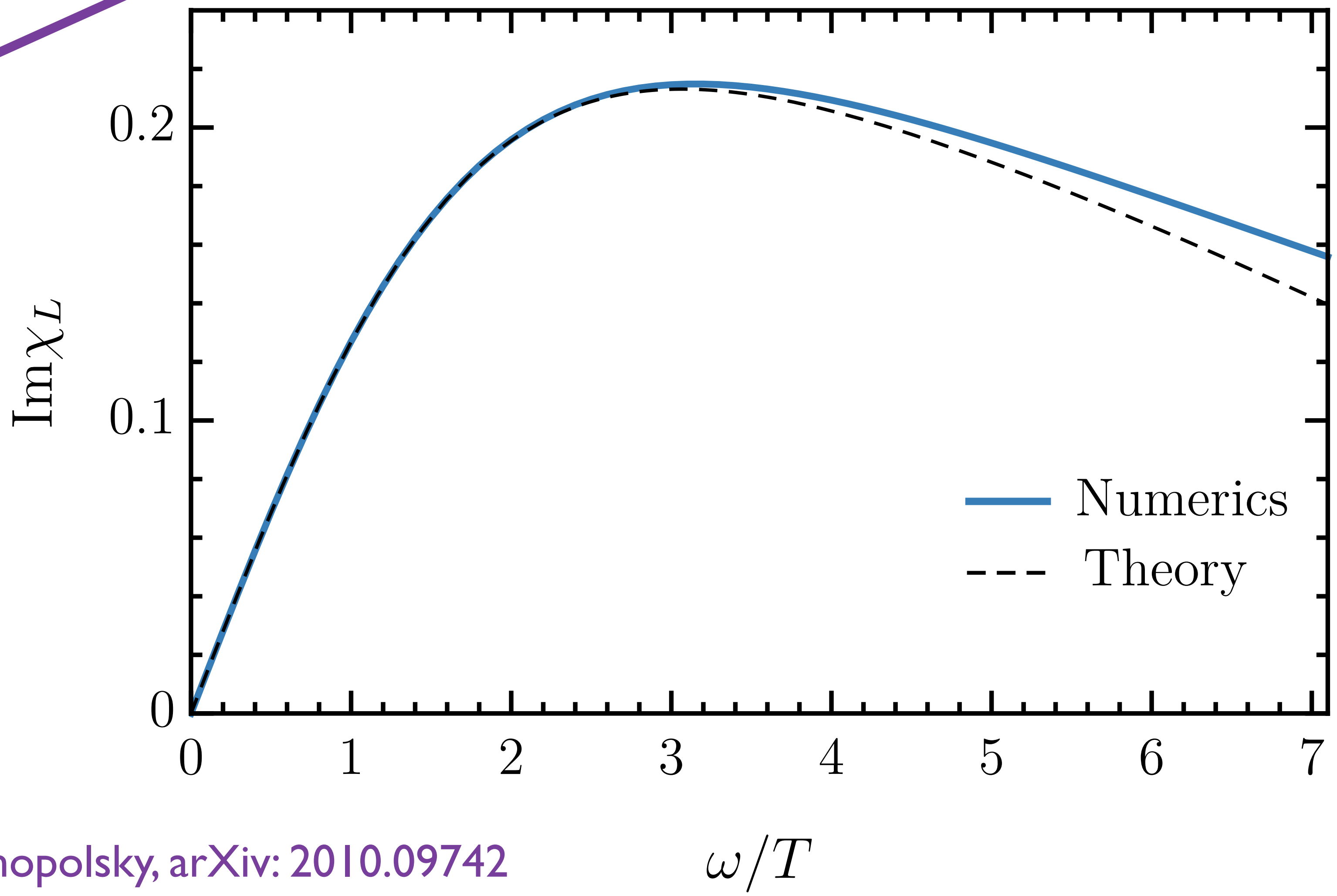
Correction from the boundary graviton



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Quantum entanglement

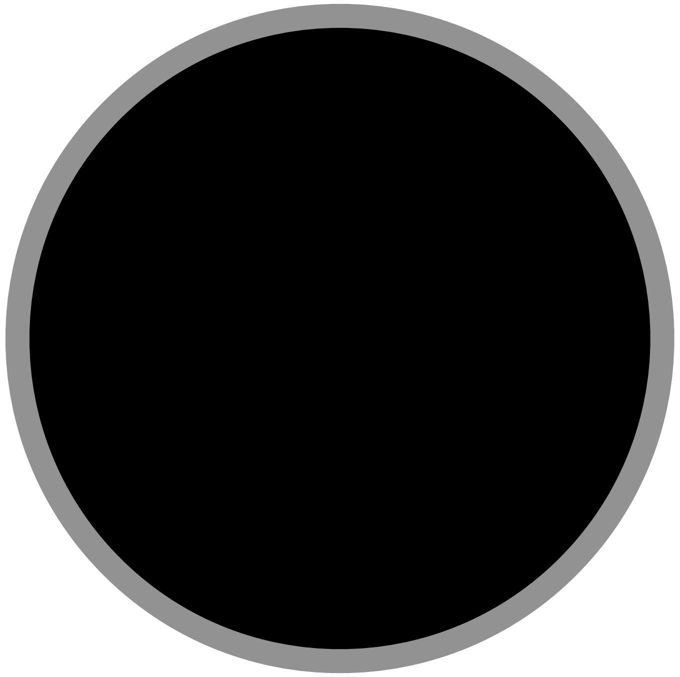
Black holes

A simple qubit model

Black Holes

Objects so dense that light is gravitationally bound to them.

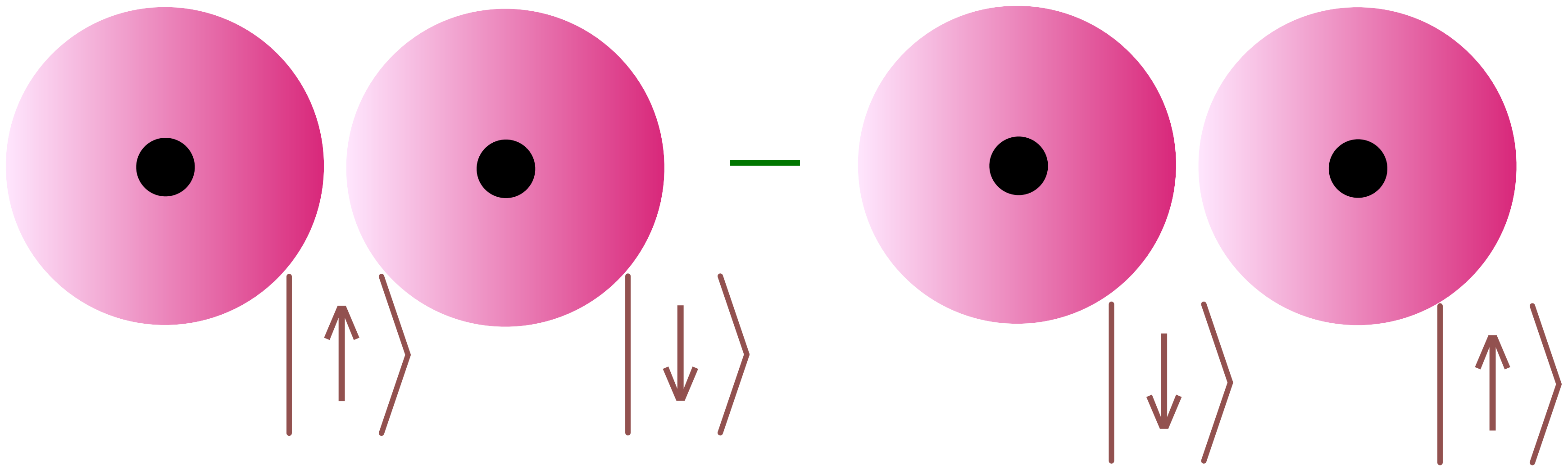
In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

Quantum Entanglement across a black hole horizon



$$|\uparrow\rangle \; |\downarrow\rangle \; - \; |\downarrow\rangle \; |\uparrow\rangle$$

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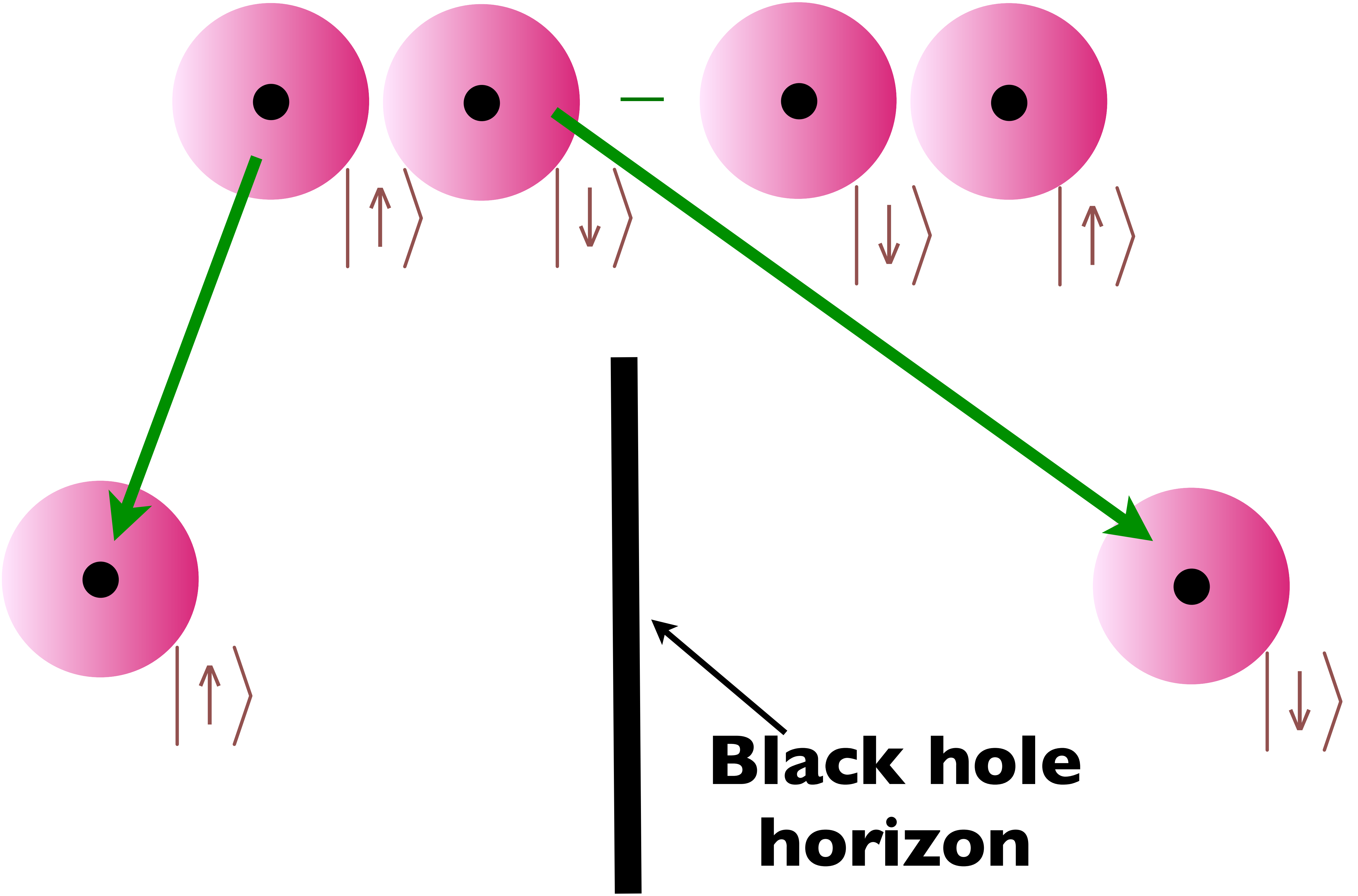
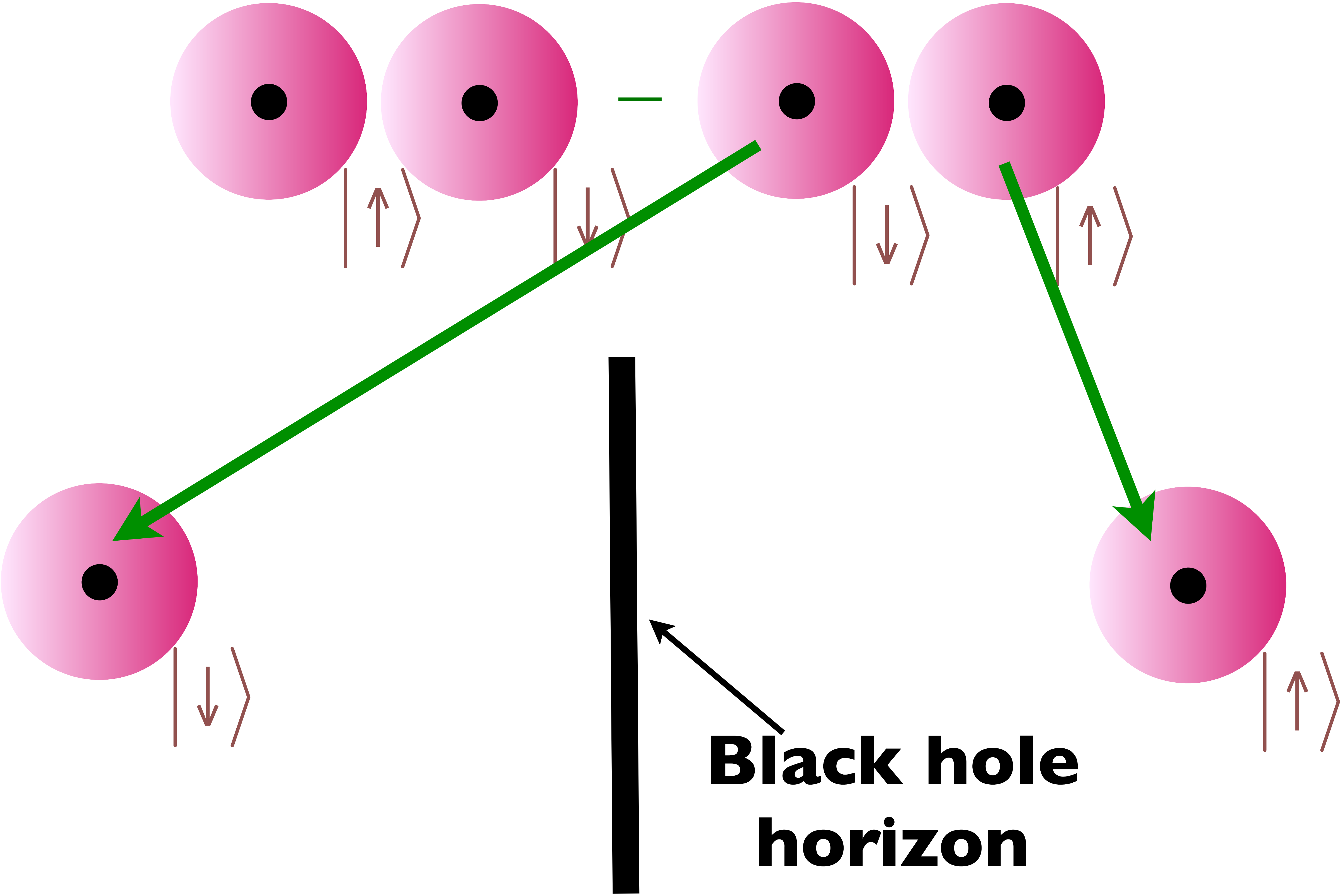


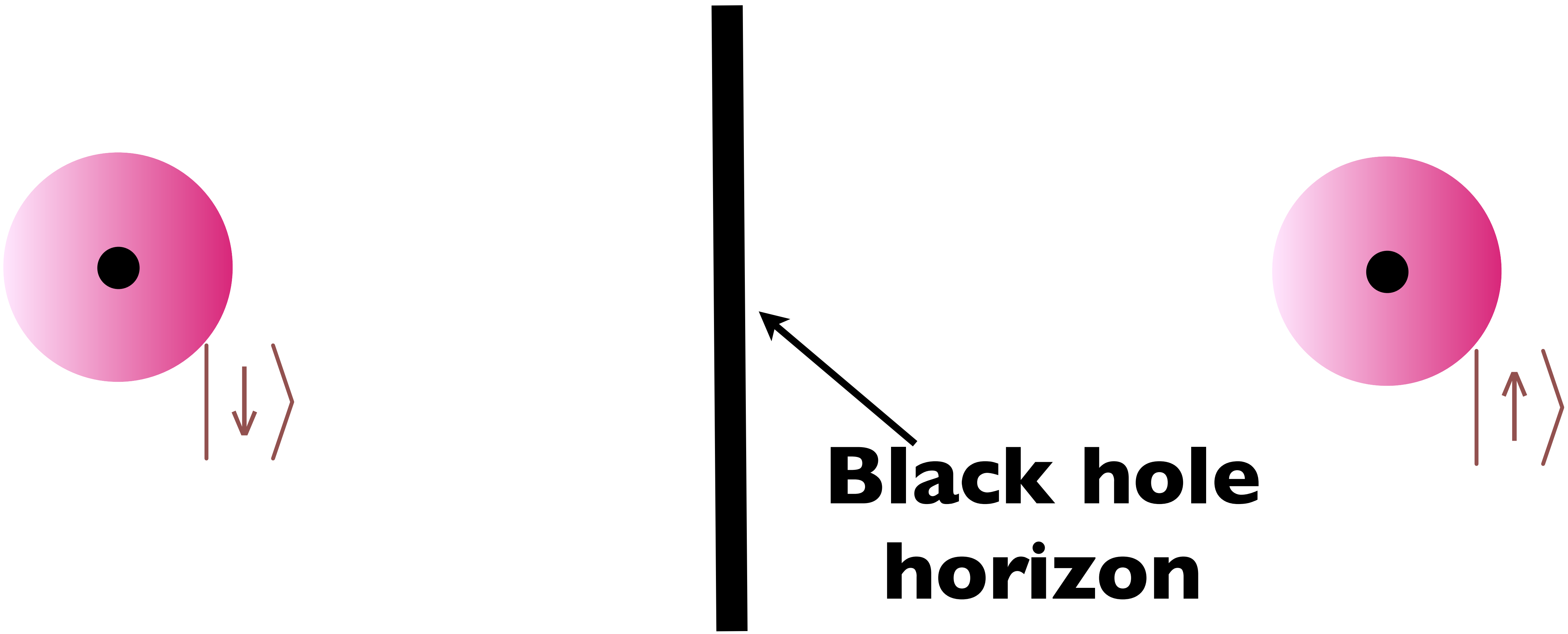
Quantum Entanglement across a black hole horizon



Black hole horizon

Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



$|\downarrow\rangle$

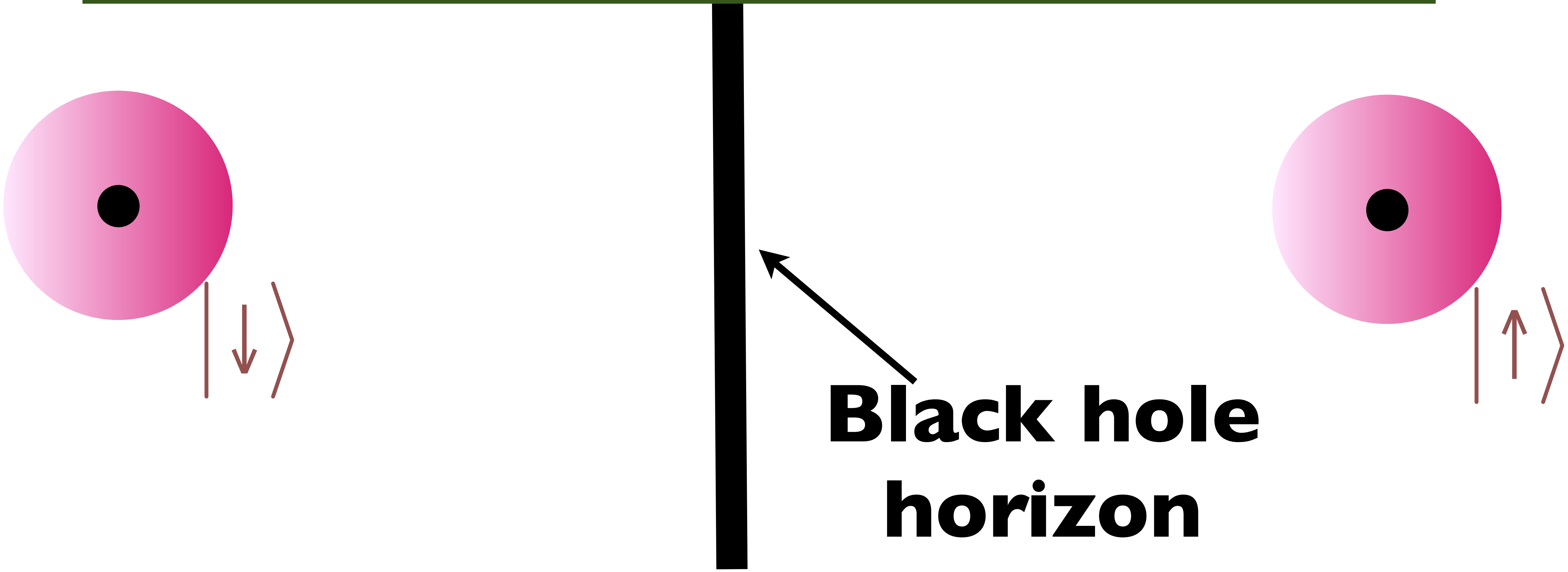
Black hole horizon

$|\uparrow\rangle$

Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

(because to an outside observer, the state of the electron inside the black hole is an unknown)



$|\downarrow\rangle$

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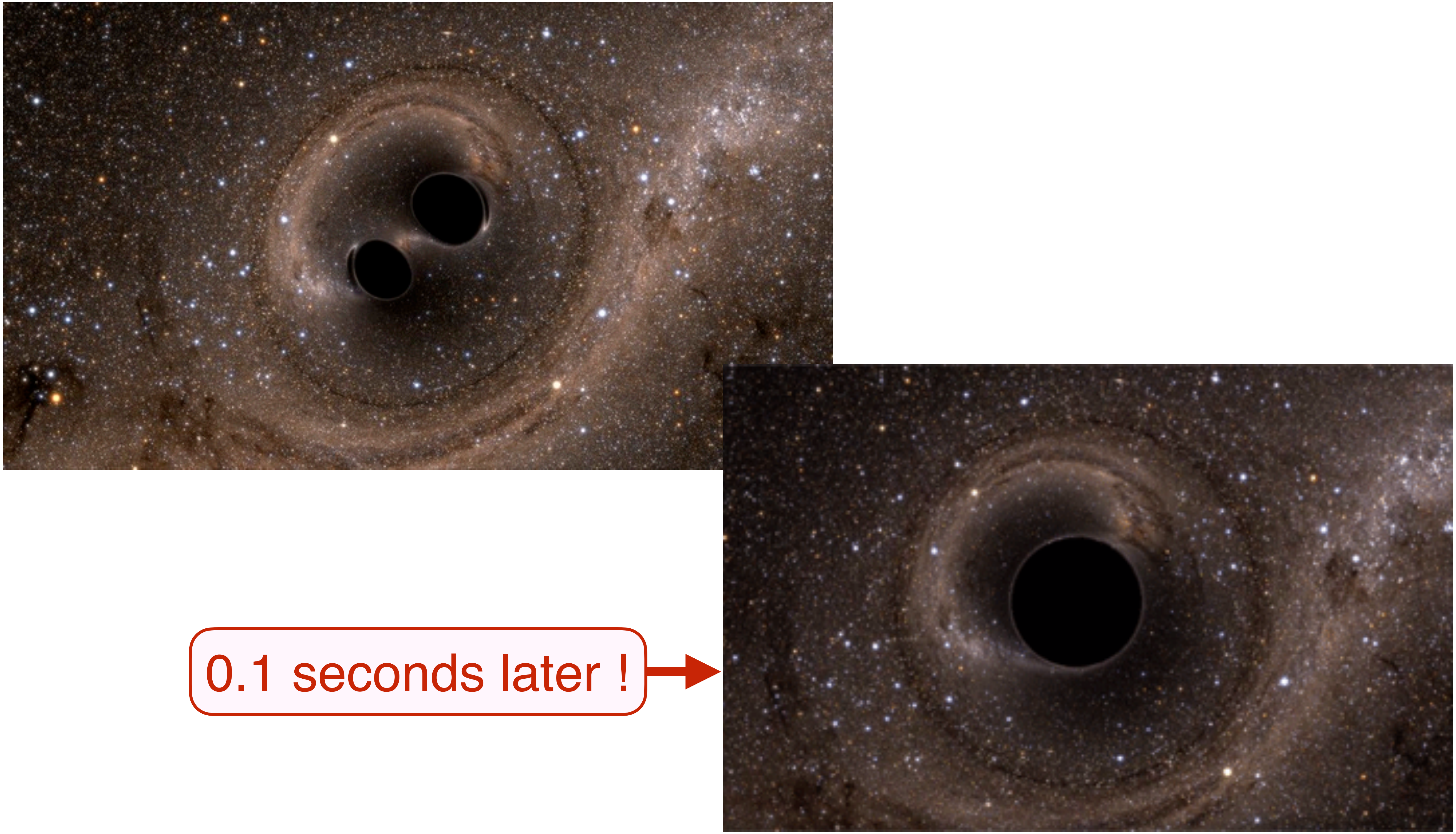
Black hole horizon

Quantum Black holes

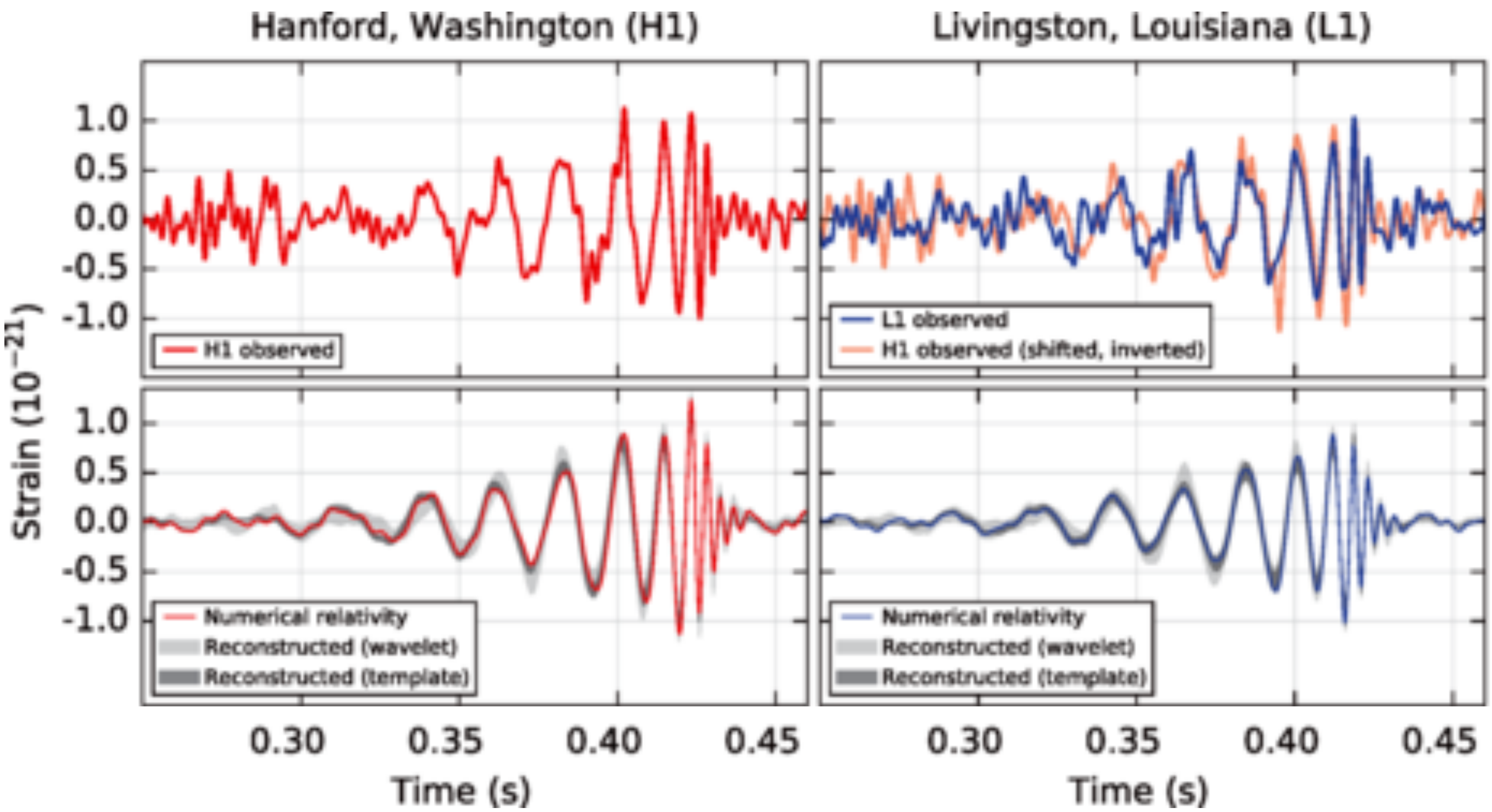
- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

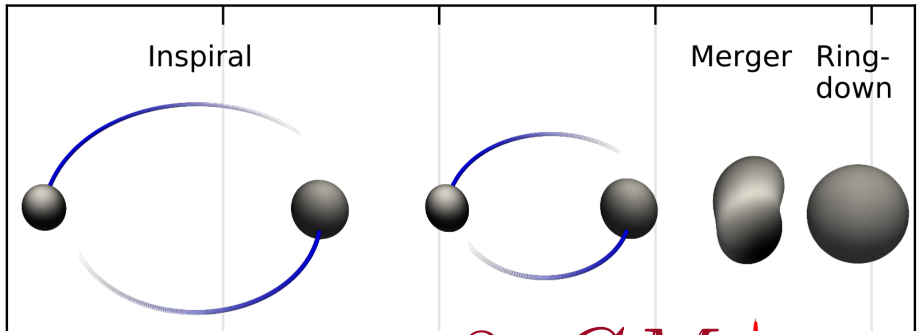
J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away



0.1 seconds later !





LIGO
September 14, 2015

- The ring-down time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals

$$\boxed{\frac{\hbar}{k_B T_H},}$$

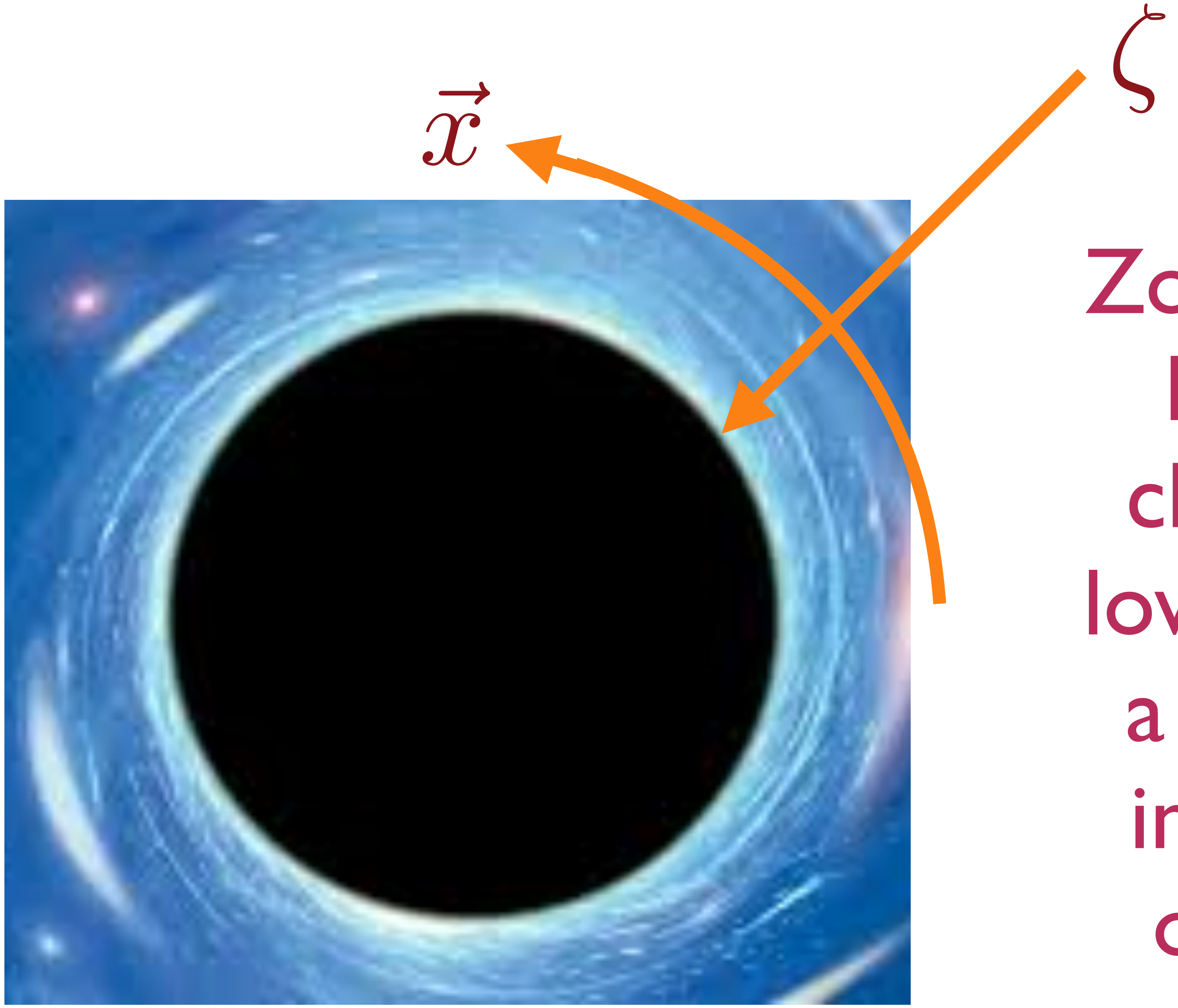
$\hbar$ Planck's constant, k_B Boltzmann's constant

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

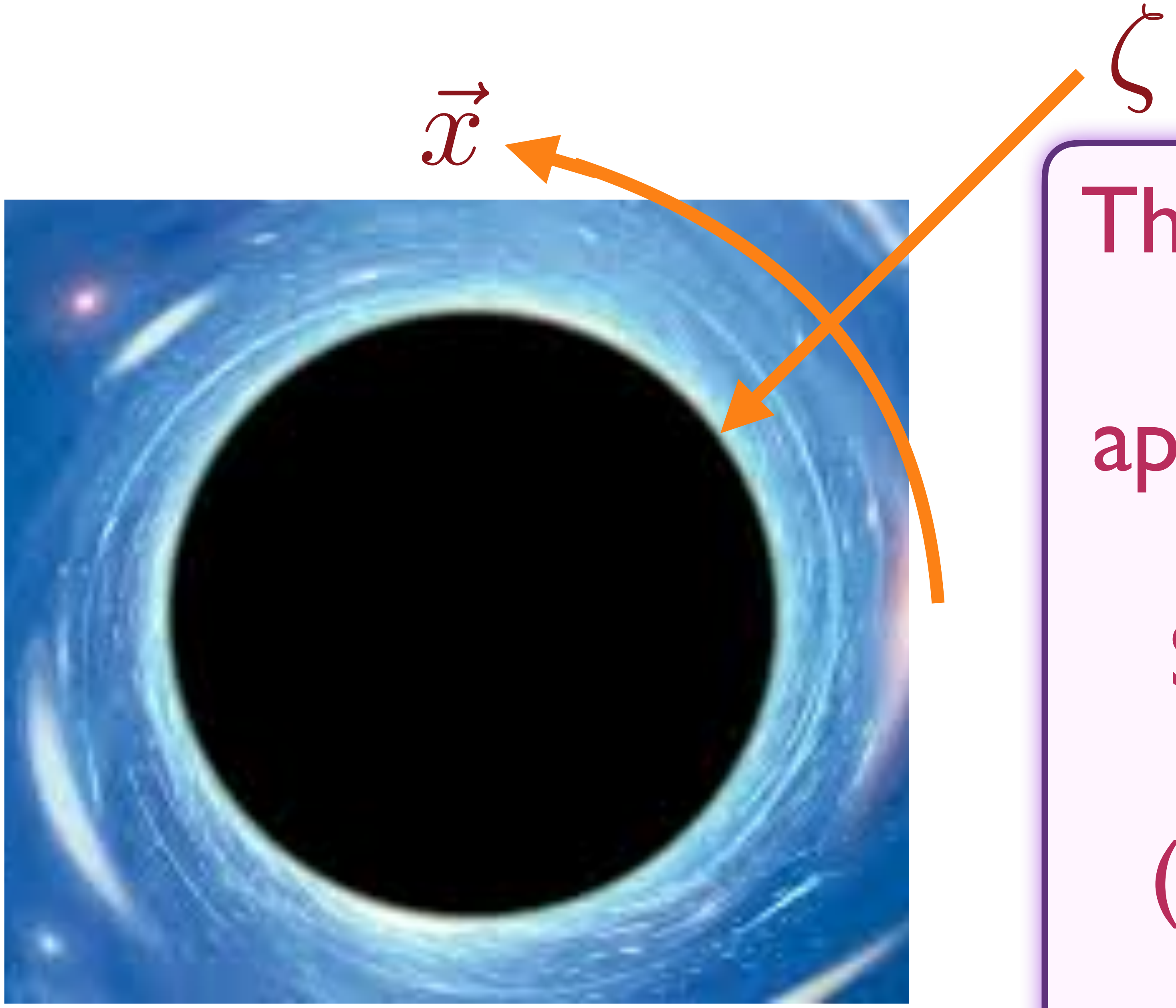
Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space (ζ) and one time dimension

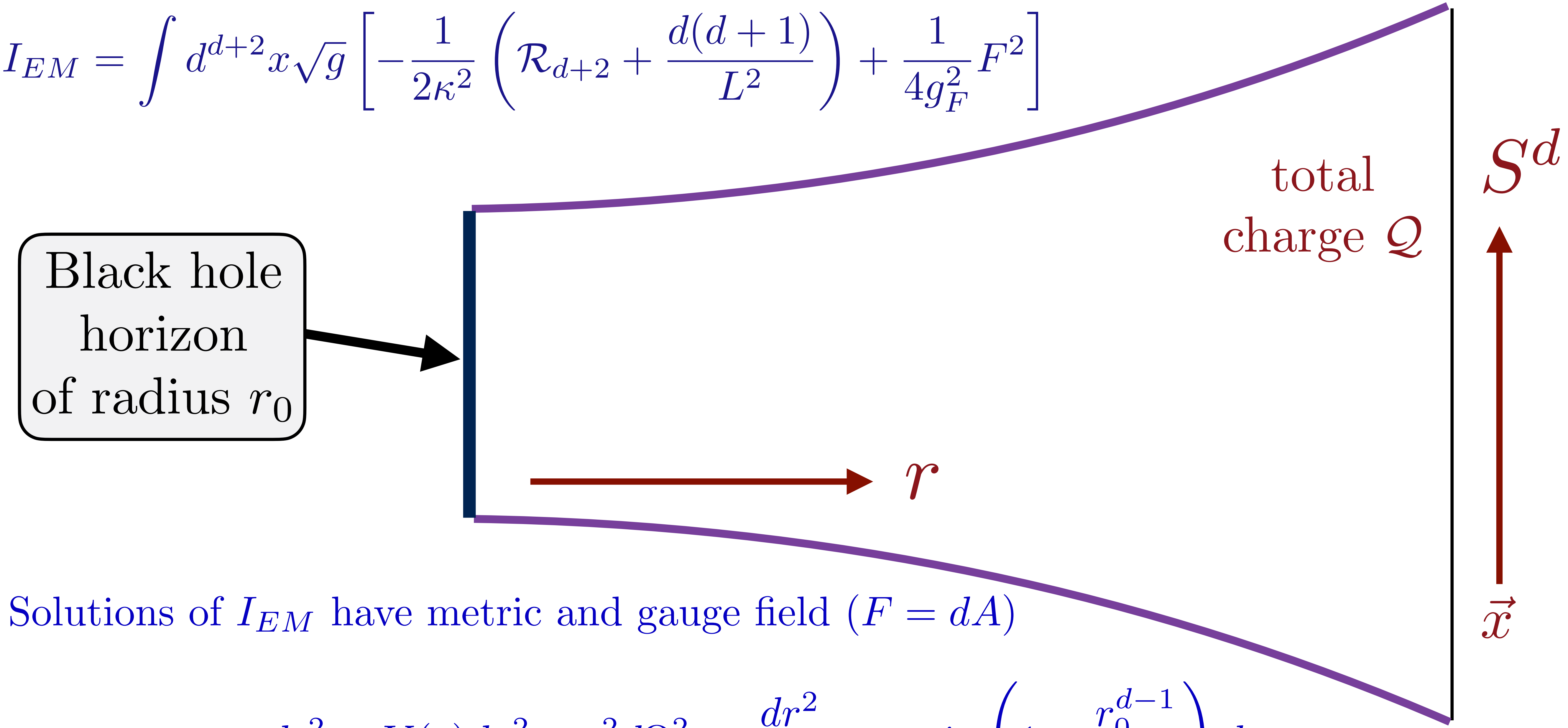
Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



This 2D-gravity theory is precisely that appearing in the low T limit of the Sachdev-Ye-Kitaev (SYK) models (including the qubit model)!

Charged black holes

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right]$$



Solutions of I_{EM} have metric and gauge field $(F = dA)$

$$ds^2 = V(r)d\tau^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad i\mu\left(1 - \frac{r_0^{d-1}}{r^{d-1}}\right)d\tau$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}.$$

where $d\Omega_d^2$ is the metric of the d-sphere. All parameters of the solution are determined in terms of the chemical potential μ, and the Hawking temperature of horizon, T.

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{L R_h}{\sqrt{d(d+1)R_h^2 + (d-1)^2 L^2}} .$$

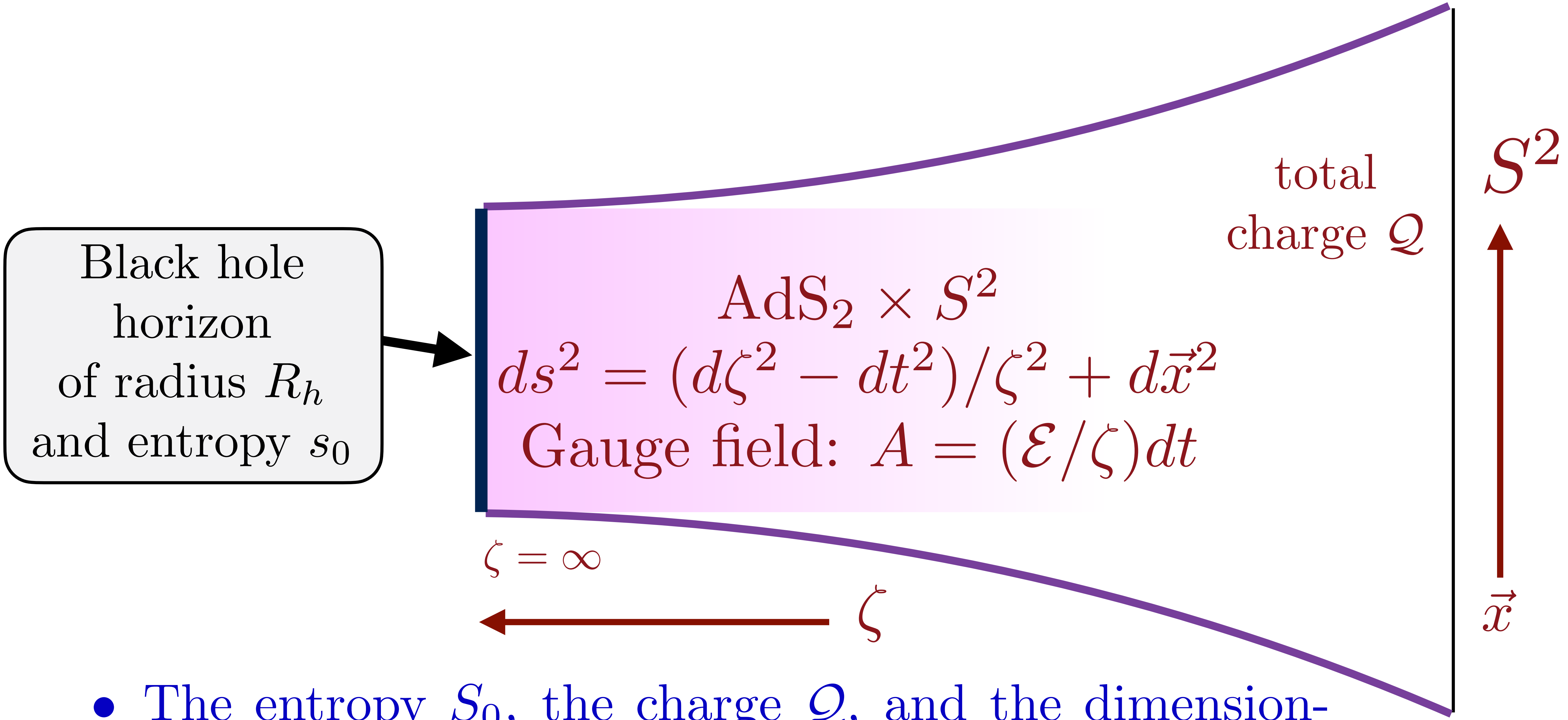
Then the near-horizon metric becomes AdS$_2\times$ S$_d$, with

$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_d^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt \, .$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{2\left[d(d+1)R_h^2 + (d-1)^2 L^2\right]} .$$

Charged black holes



Black hole horizon of radius R_h and entropy s_0

$$\mathrm{AdS}_2 \times S^2$$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

total charge $\mathcal{Q}$

S^2

$\vec{x}$

$\zeta = \infty$

ζ

- The entropy $\mathcal{S}_0$, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey the same thermodynamic relation as the SYK model

$$\boxed{\frac{d\mathcal{S}_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

2D gravity and black holes

- Reparameterization invariance is a defining property of Einstein's theory of gravity
- In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

A. Kitaev, 2015
J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

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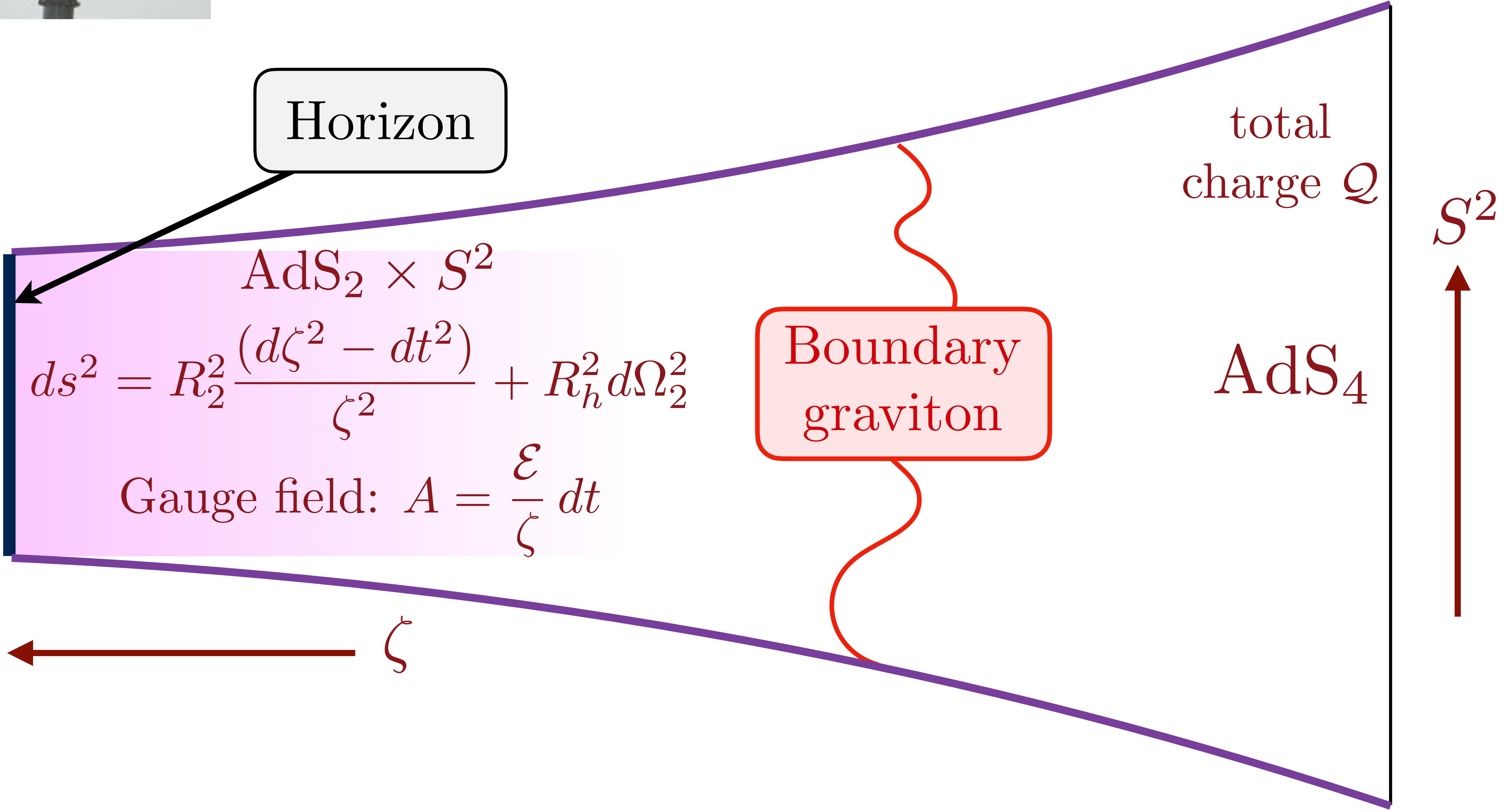
- Reparameterization invariance is a defining property of Einstein's theory of gravity
- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

Low T quantum fluctuations about the Einstein-Maxwell theory of charged black holes in $d \geq 2$ spatial dimensions leads to the same 2D gravity theory as the SYK models.

P. Nayak, A. Shukla, R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547
U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239
P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

SYK model and charged black holes



Horizon

$\mathrm{AdS}_2 \times S^2$

$$ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2$$

Gauge field: $A = \dfrac{\mathcal{E}}{\zeta} dt$

ζ

Boundary graviton

total charge $\mathcal{Q}$

AdS_4

S^2

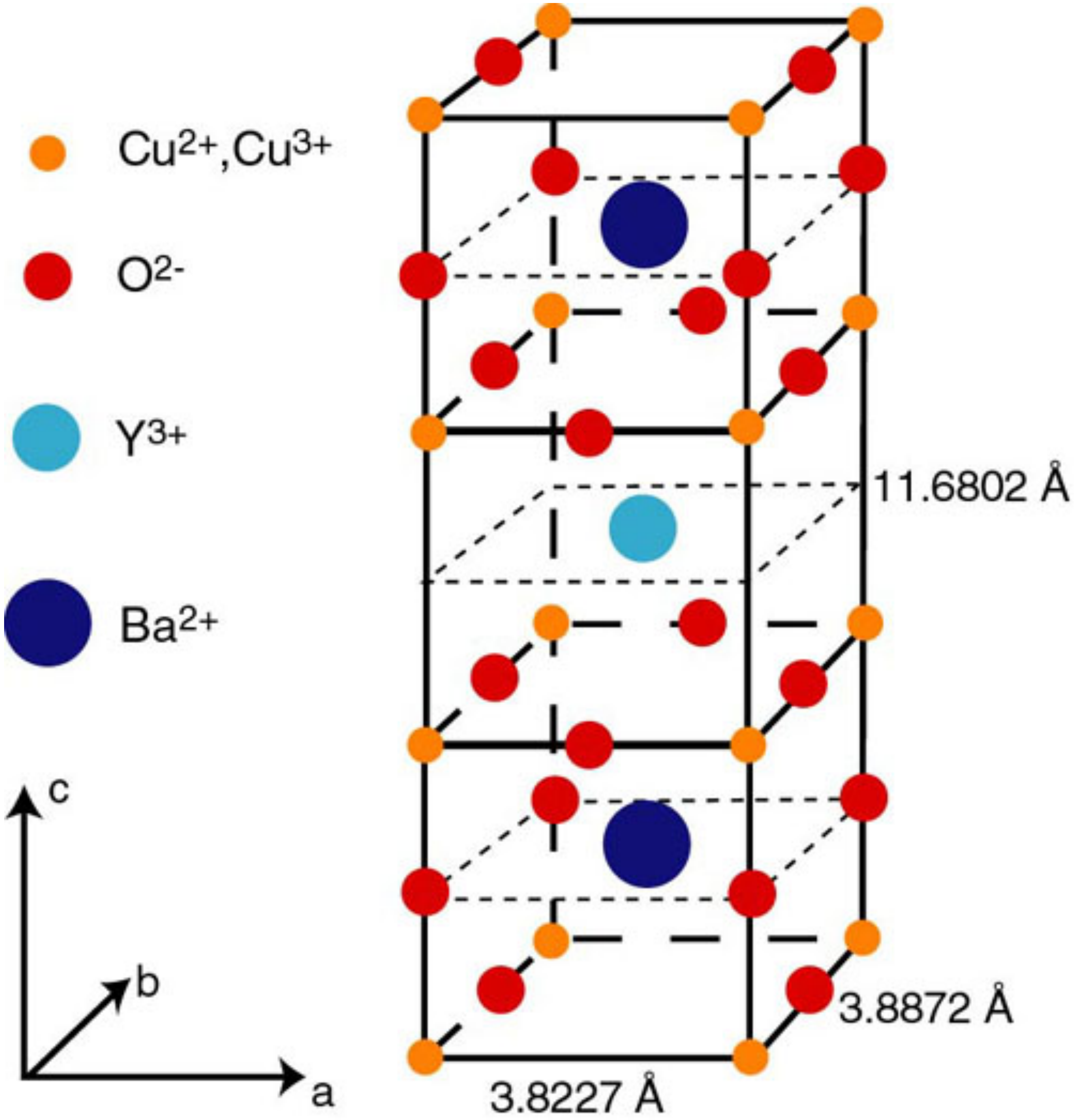
Quantum entanglement

A simple qubit model

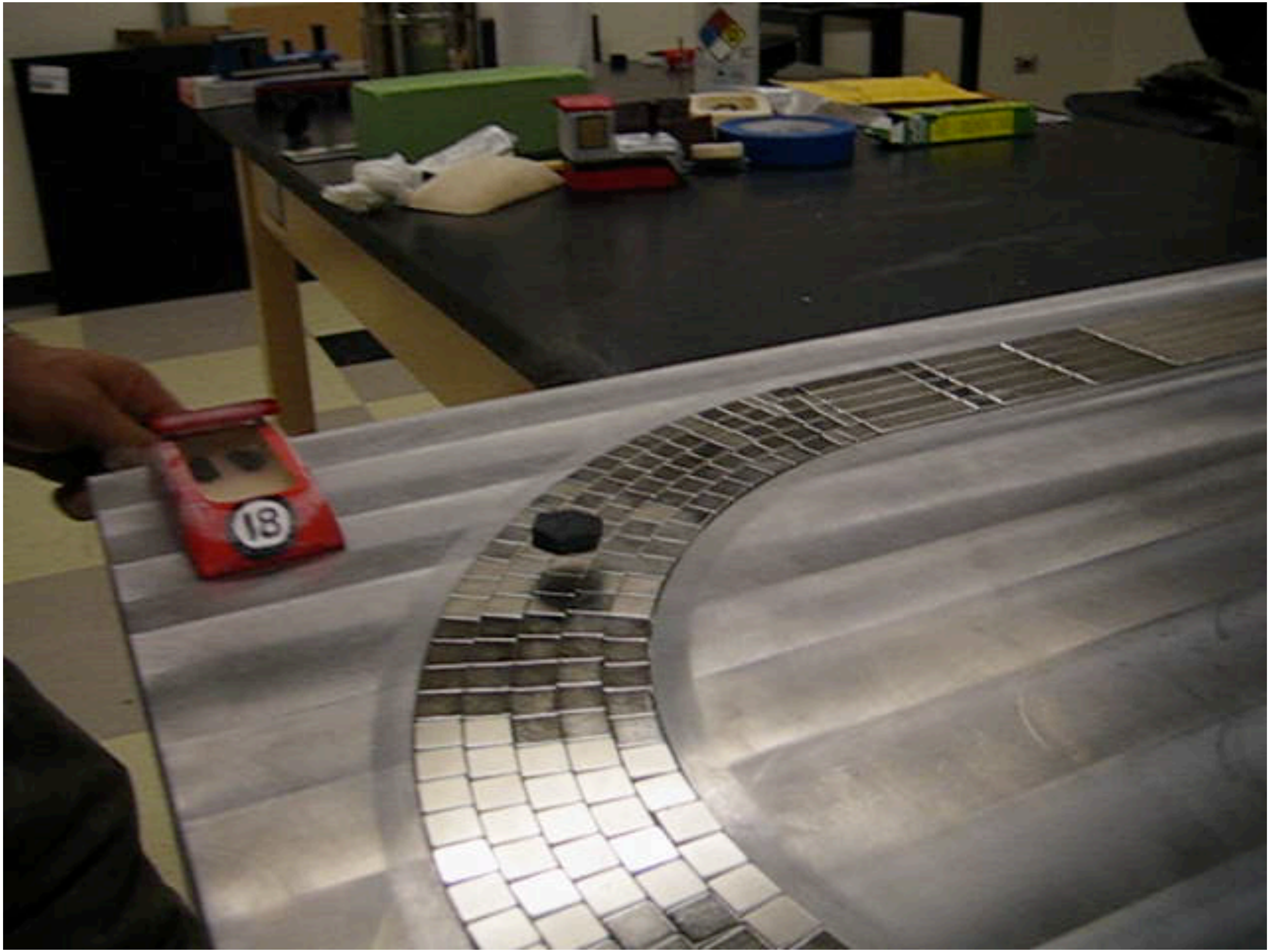
Black holes

Copper-based superconductors

Cuprate superconductors



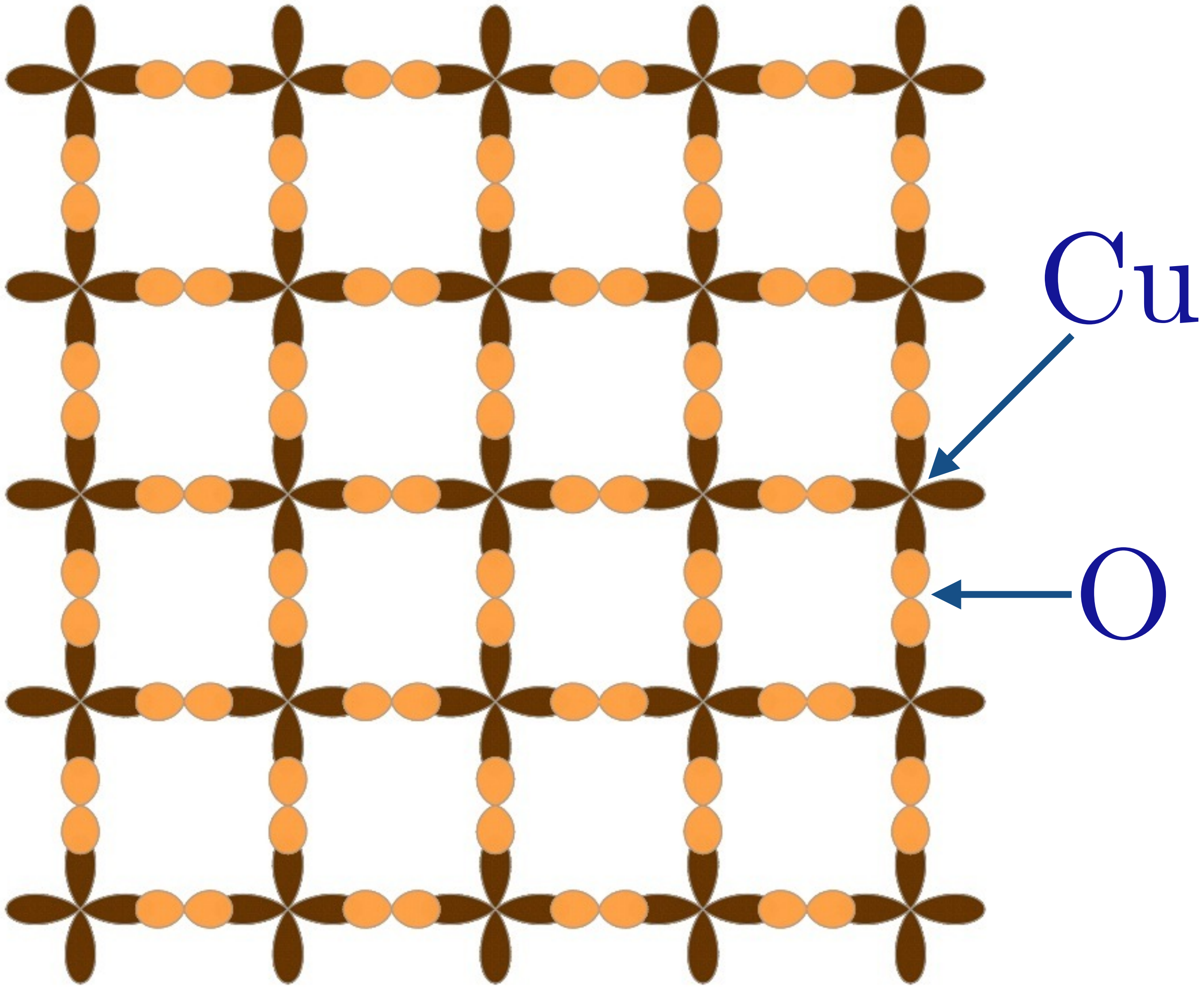
$YBa_2Cu_3O_{6+x}$



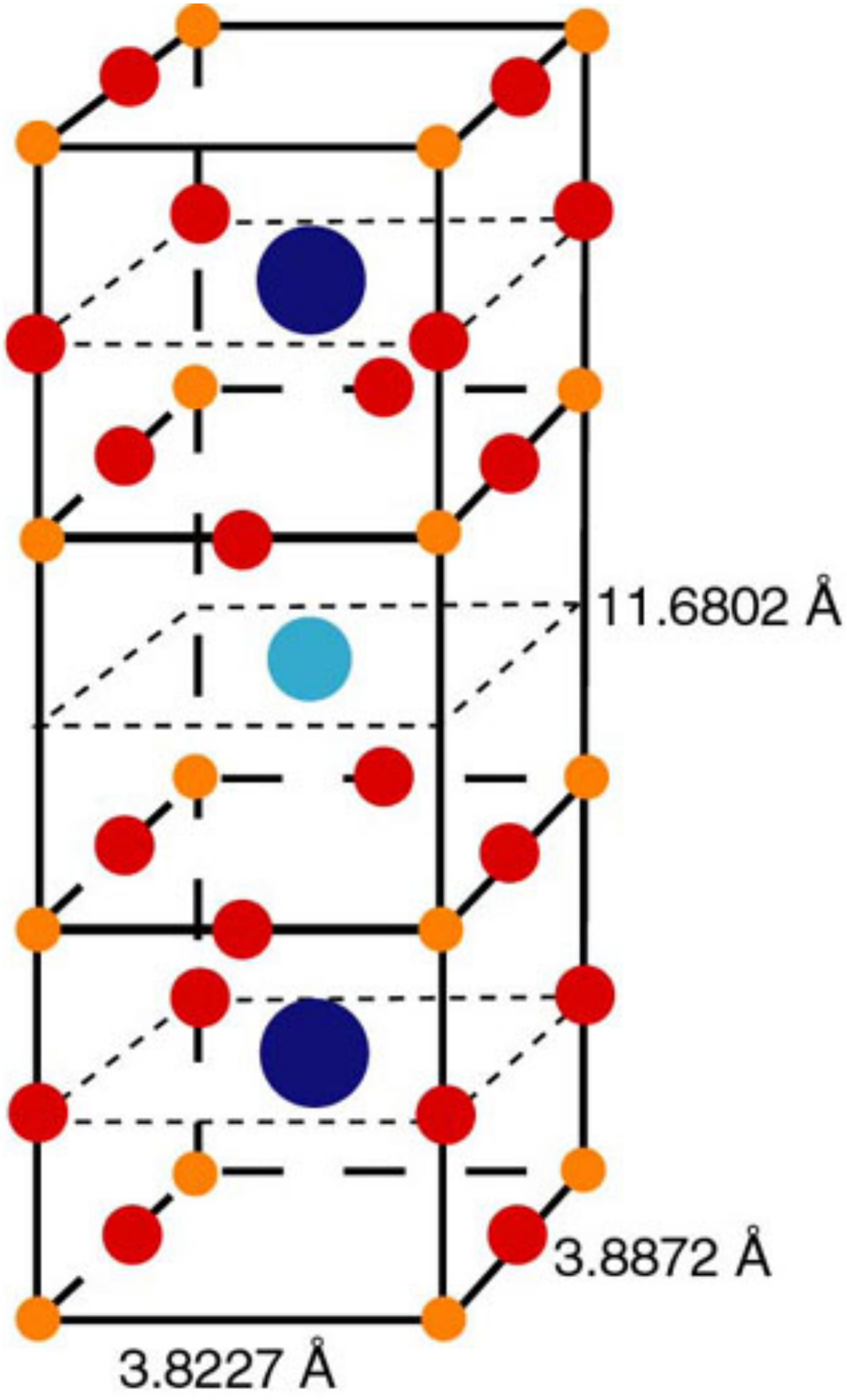
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

High temperature superconductors

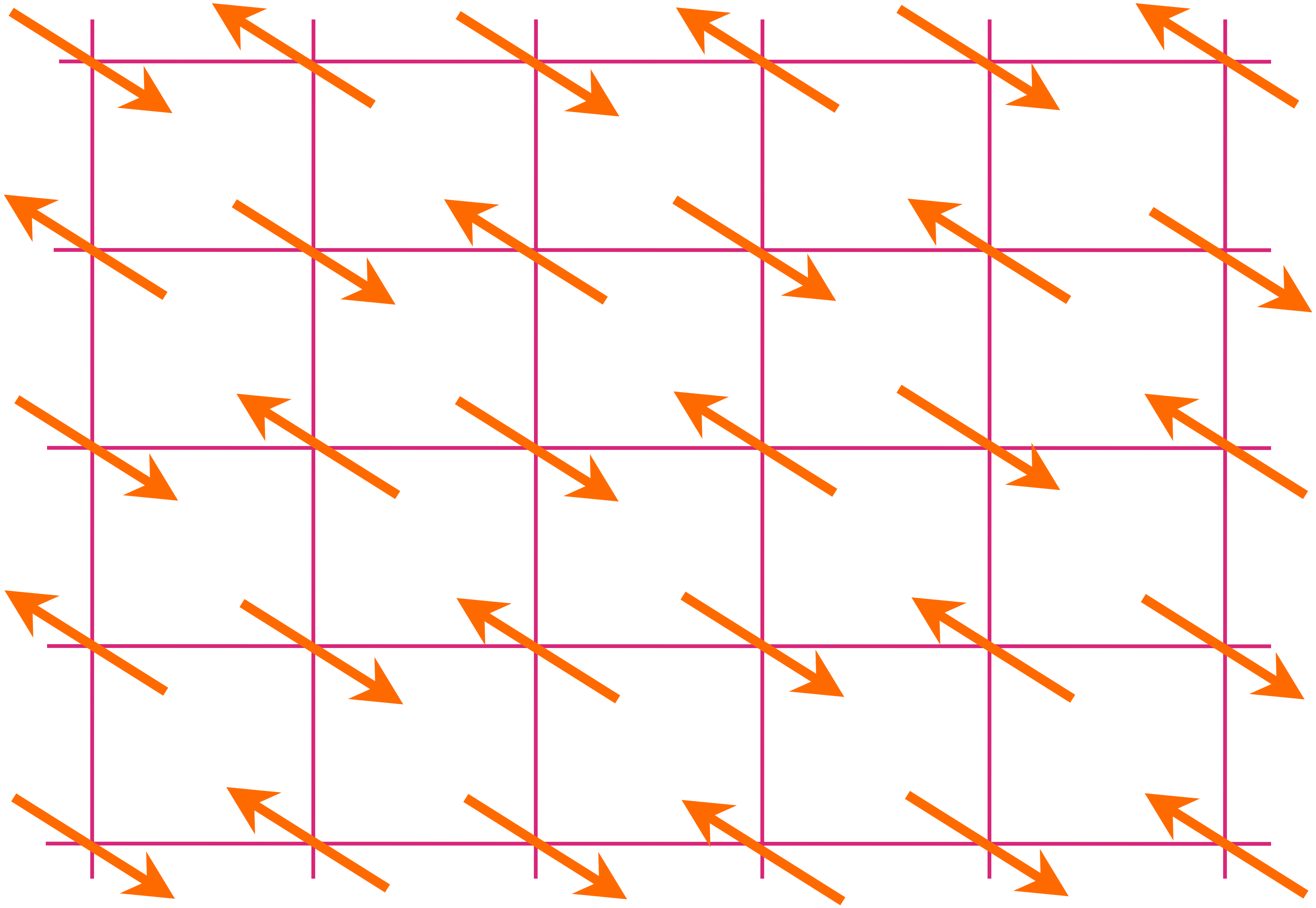


CuO$_2$ plane

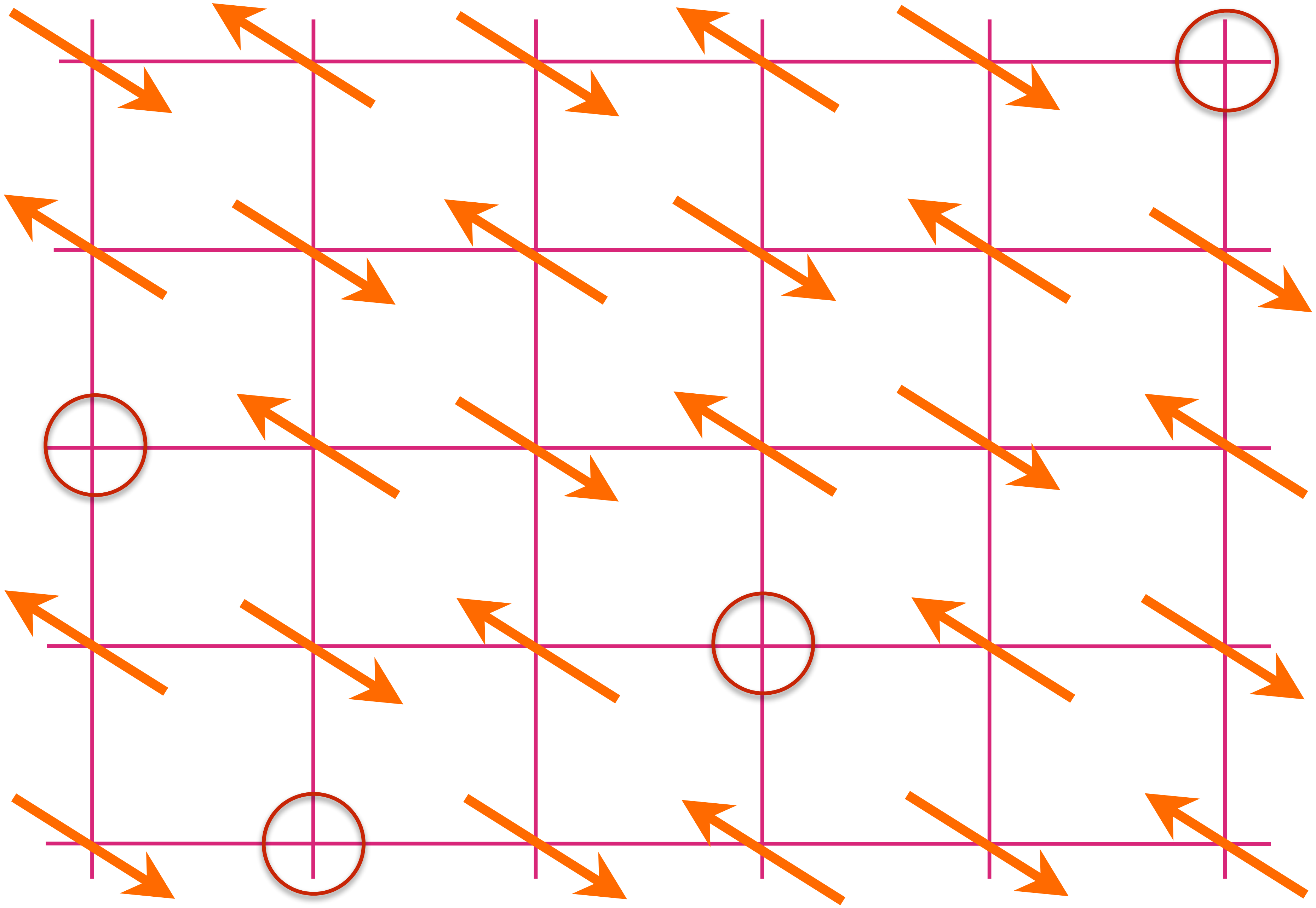


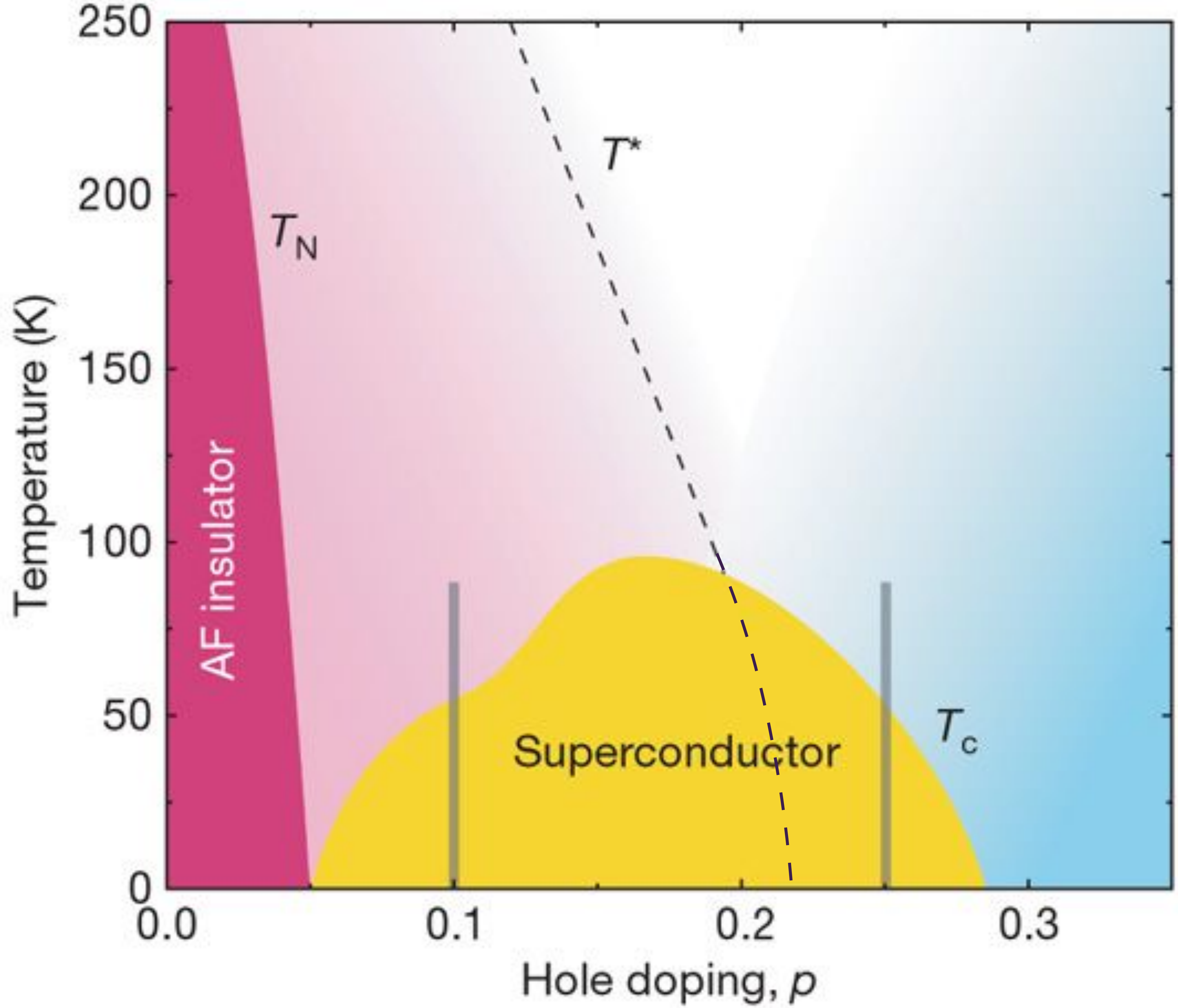
$YBa_2Cu_3O_{6+x}$

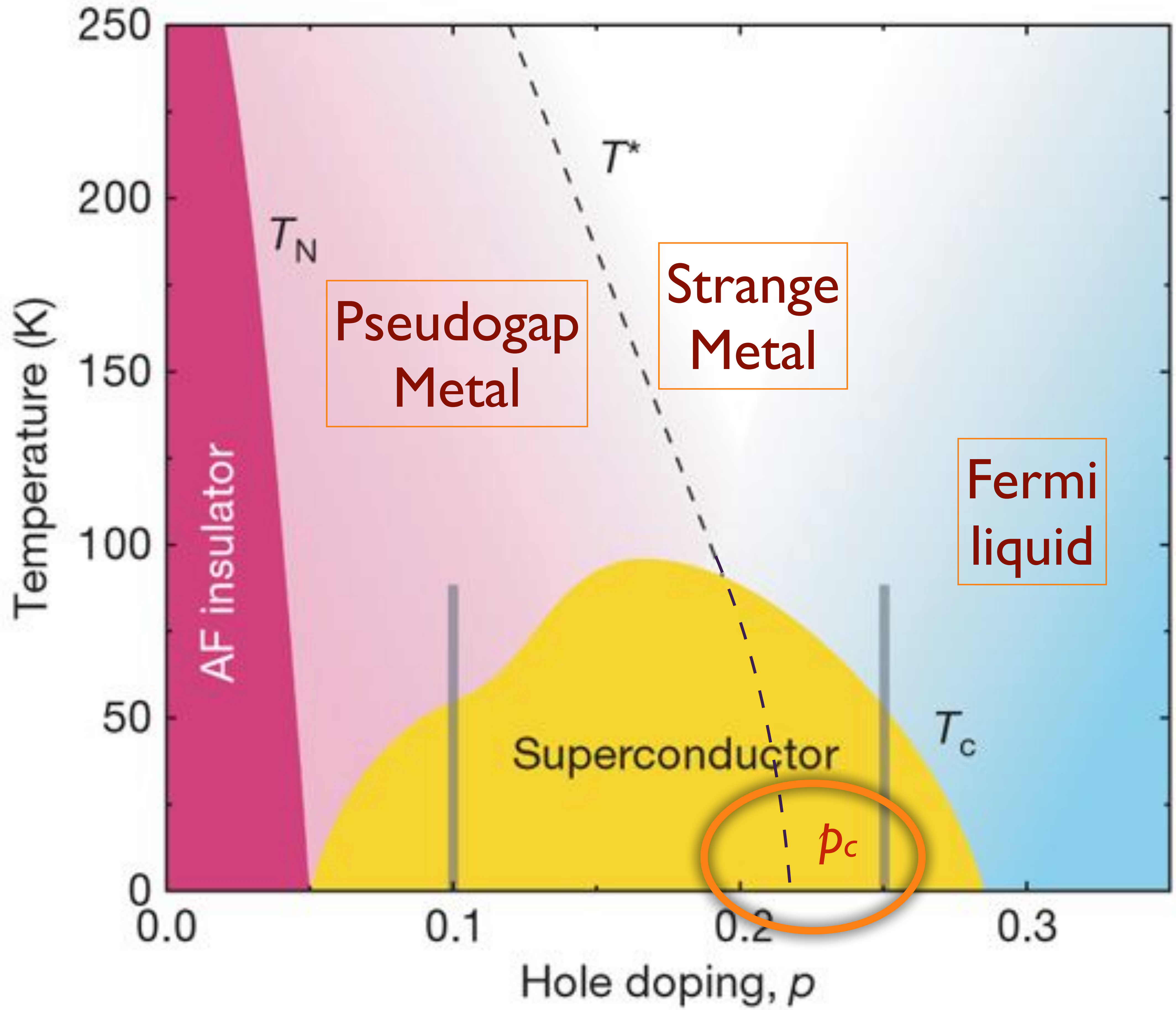
Insulating antiferromagnet



Antiferromagnet doped with hole density p




250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c



Pseudogap Metal

Strange Metal

Fermi liquid

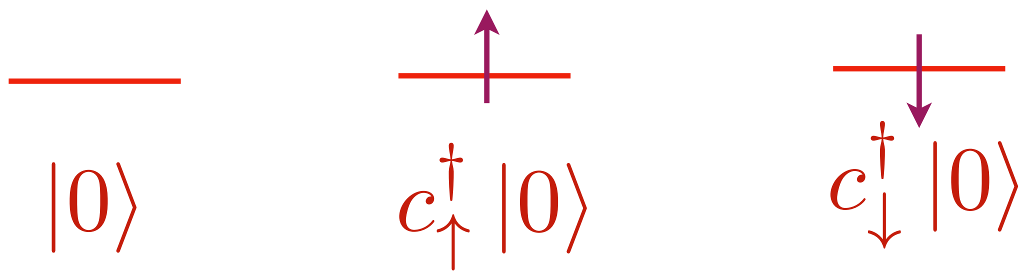
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

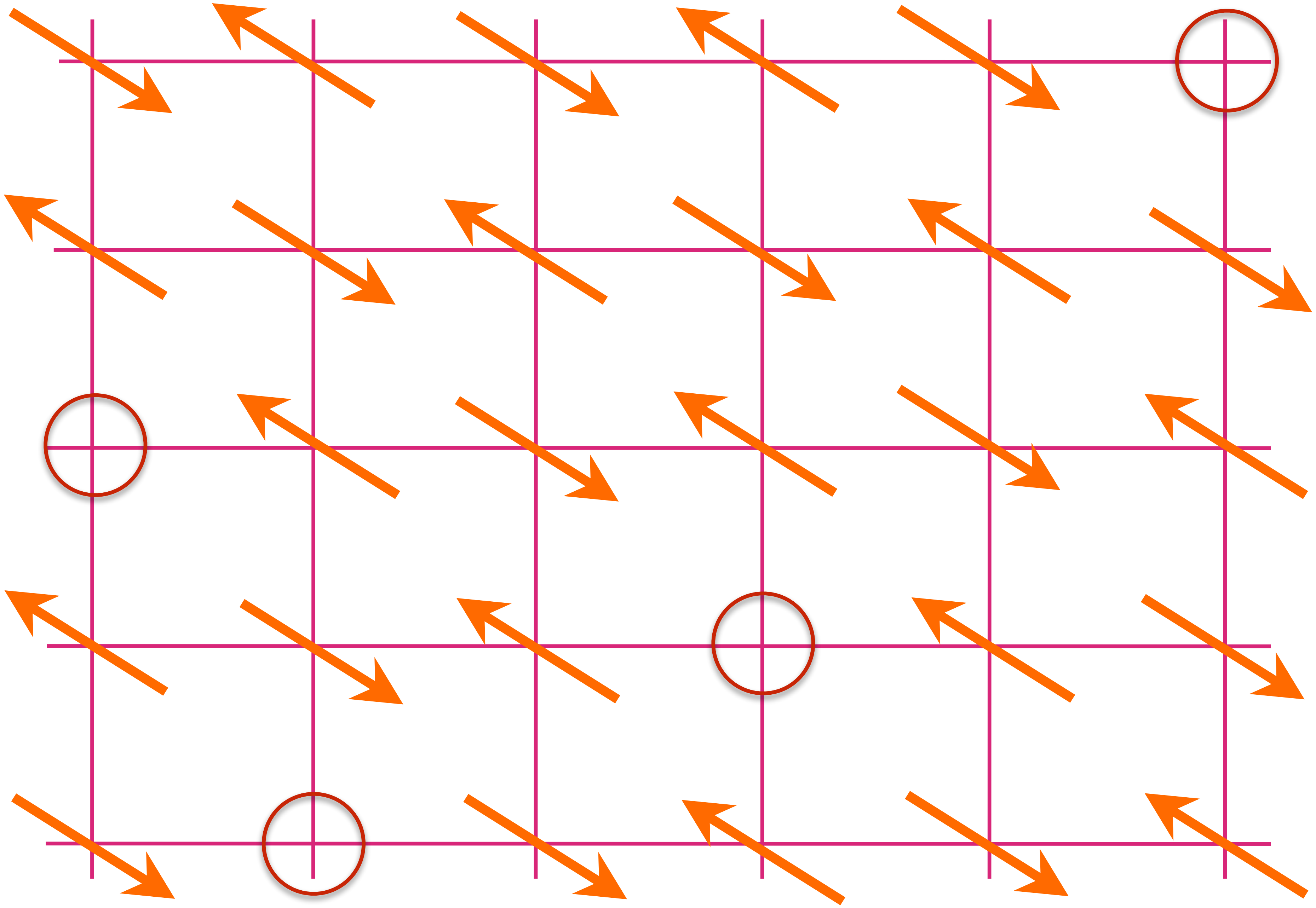
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$

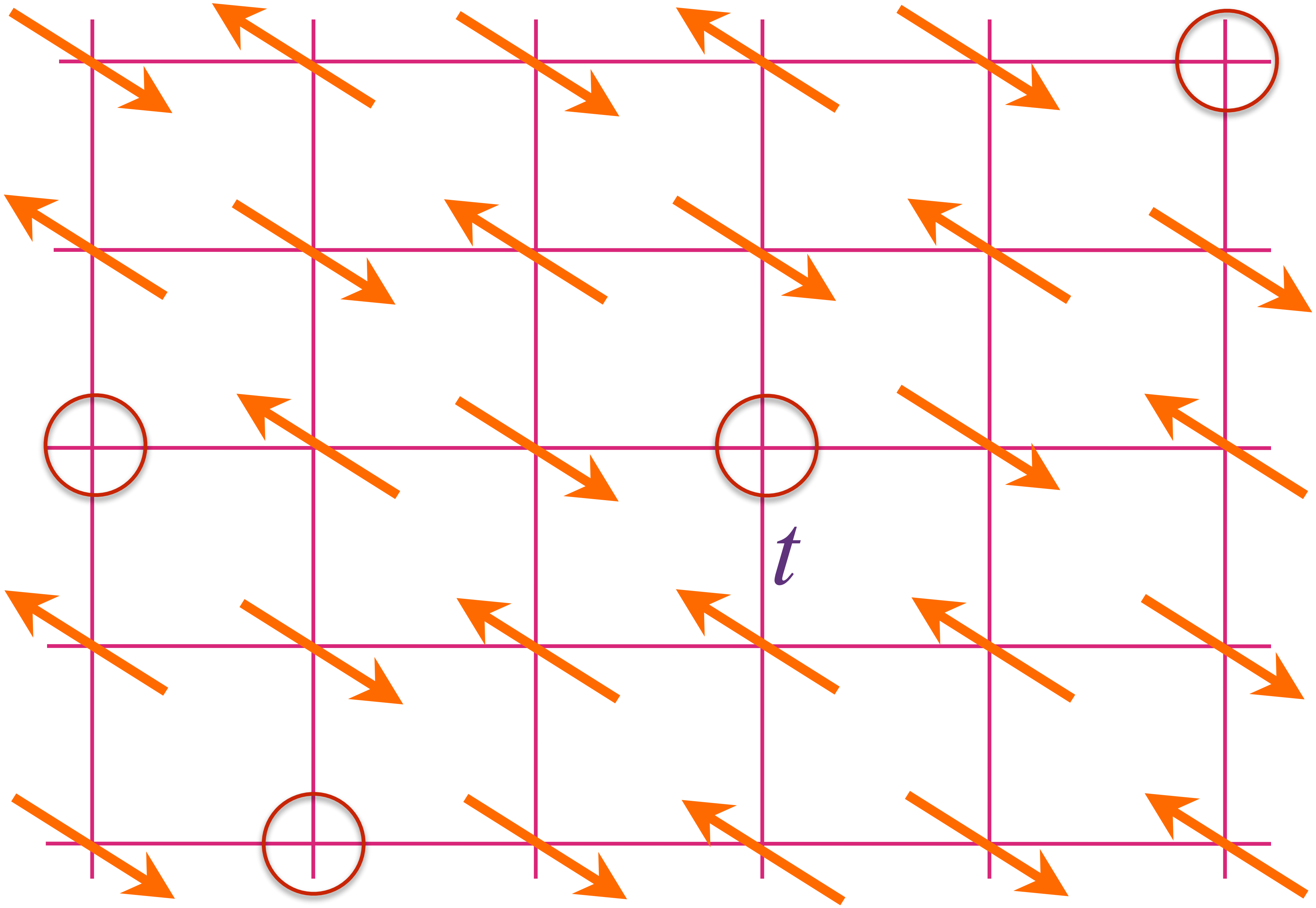


Real-space view



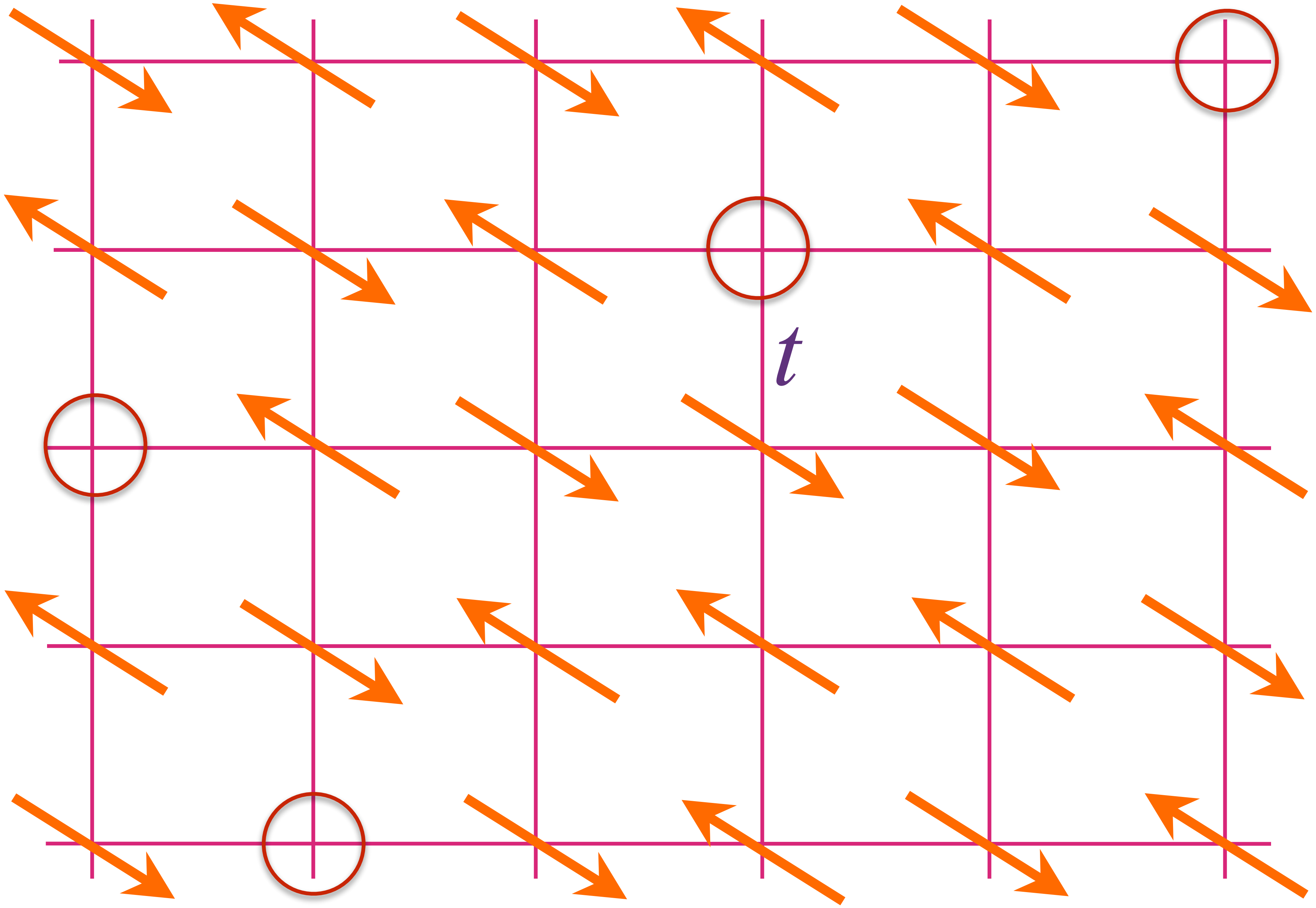
p mobile holes in a background of fluctuating spins

Real-space view



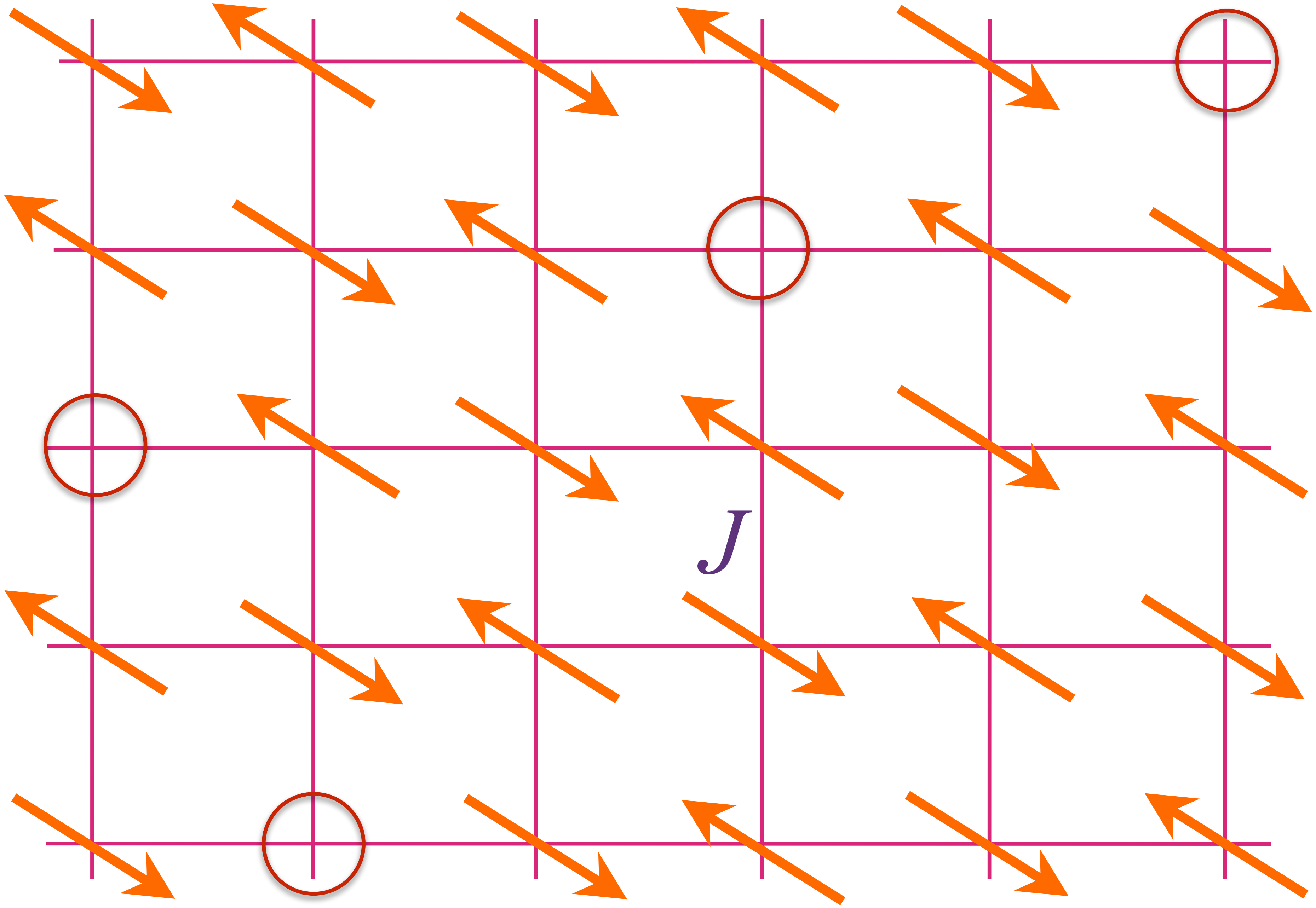
p mobile holes in a background of fluctuating spins

Real-space view



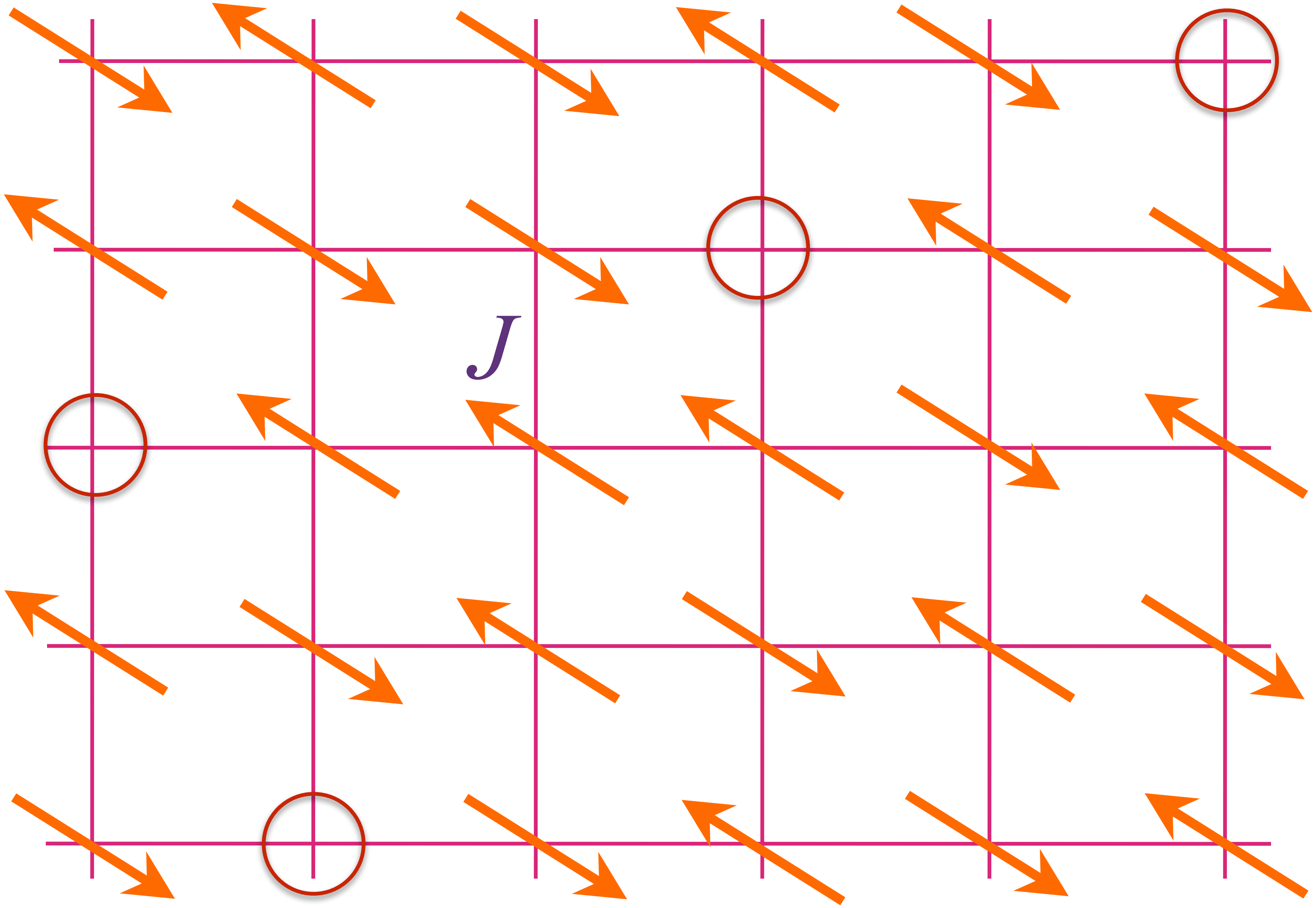
p mobile holes in a background of fluctuating spins

Real-space view



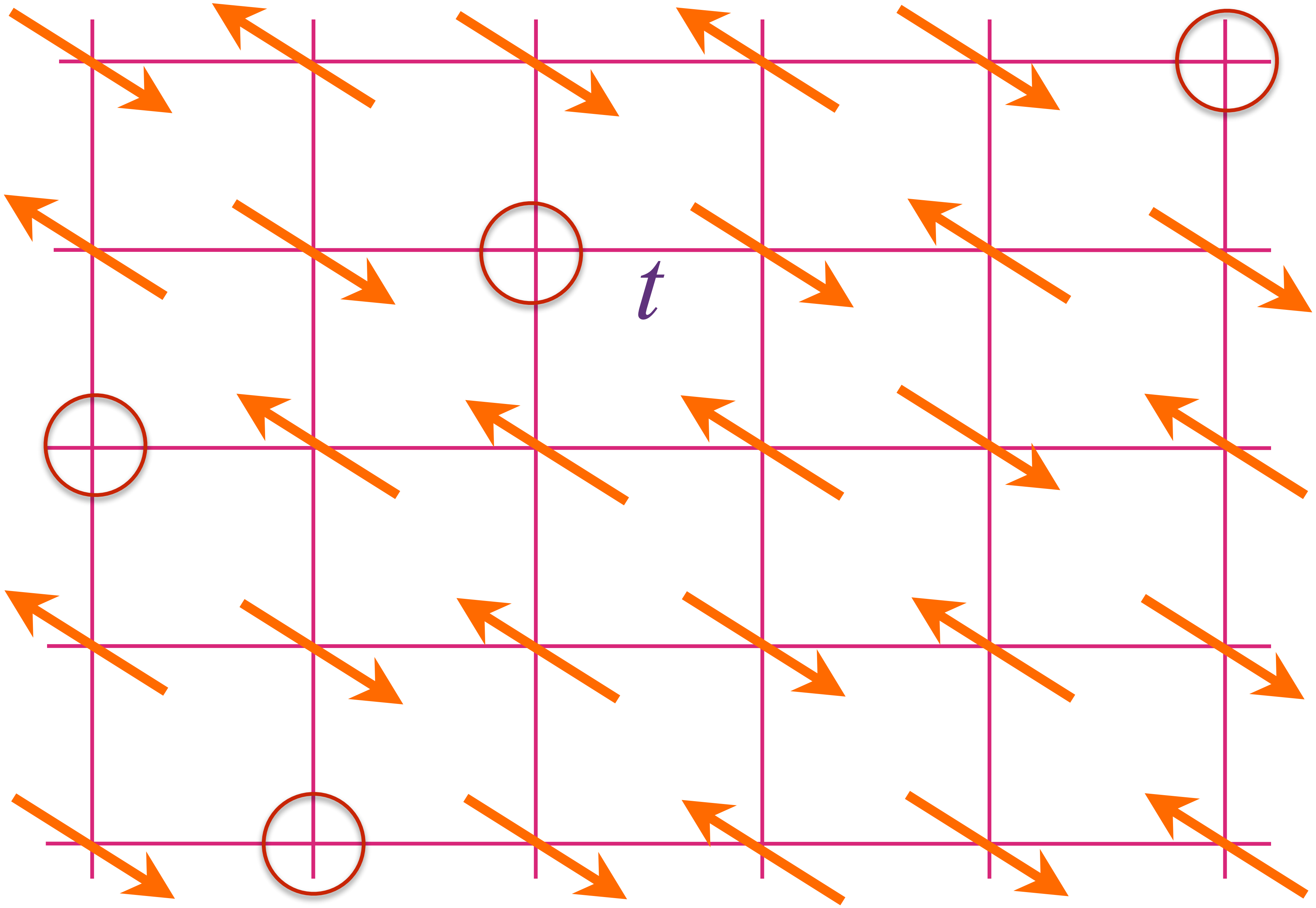
p mobile holes in a background of fluctuating spins

Real-space view



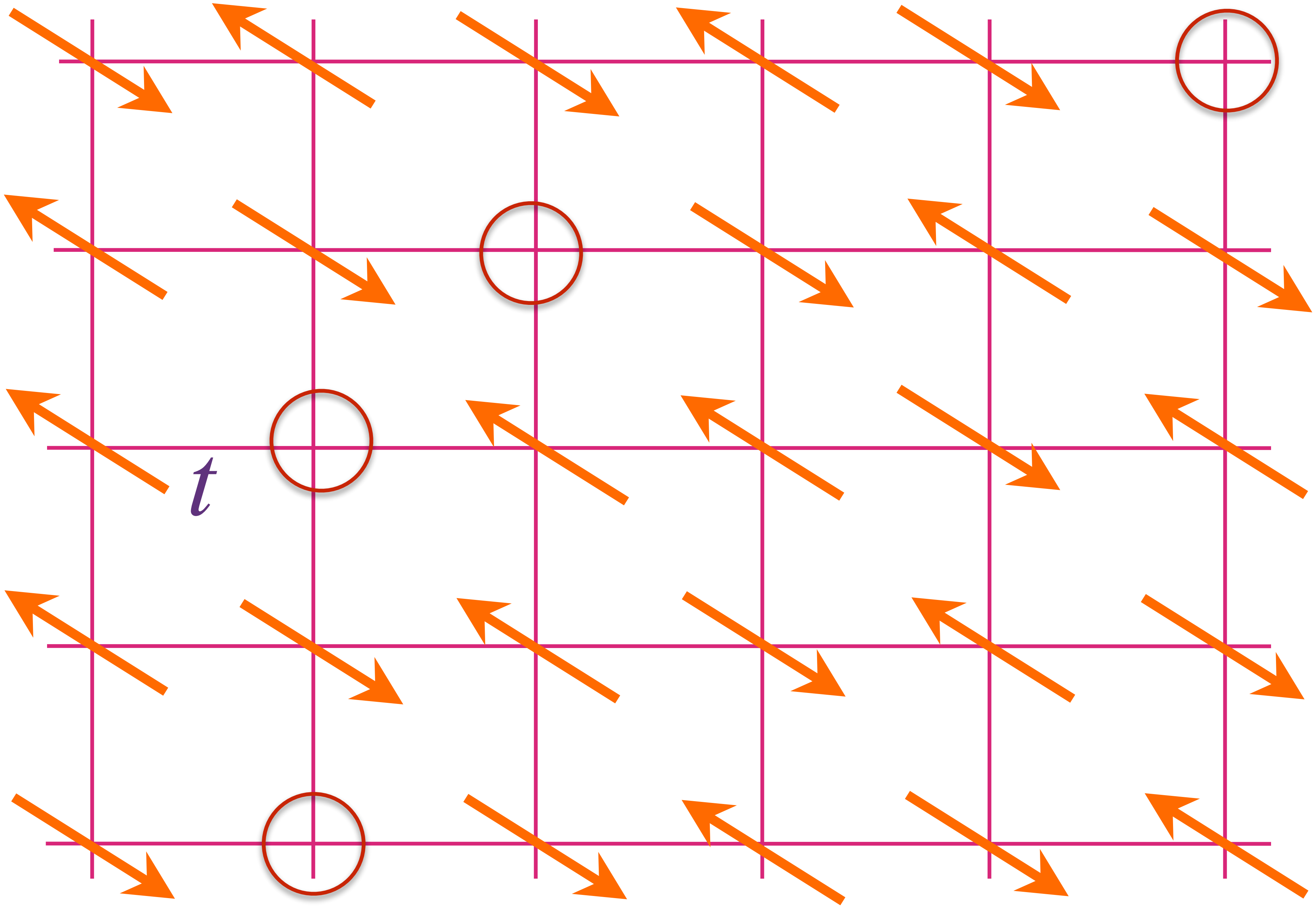
p mobile holes in a background of fluctuating spins

Real-space view



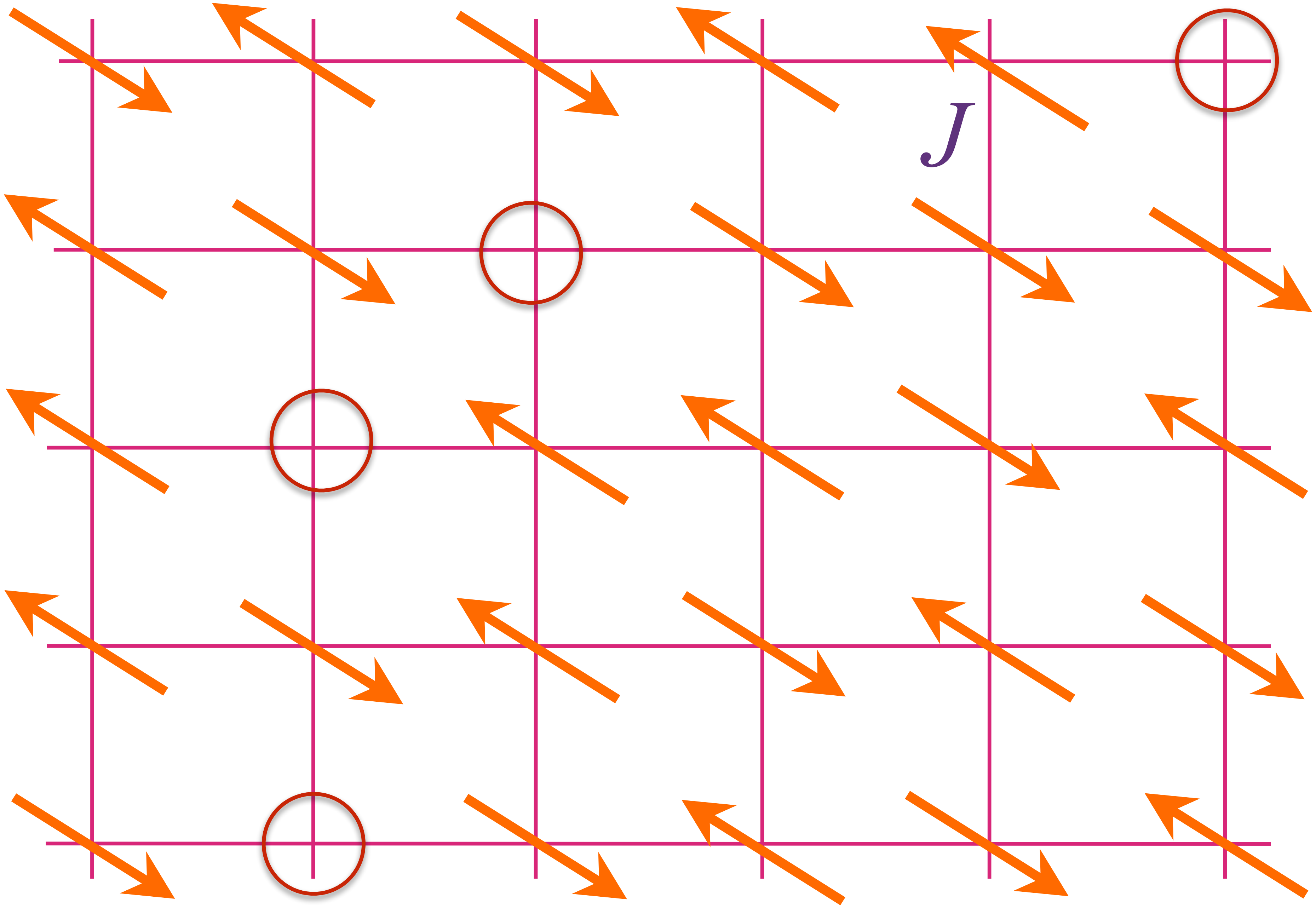
p mobile holes in a background of fluctuating spins

Real-space view



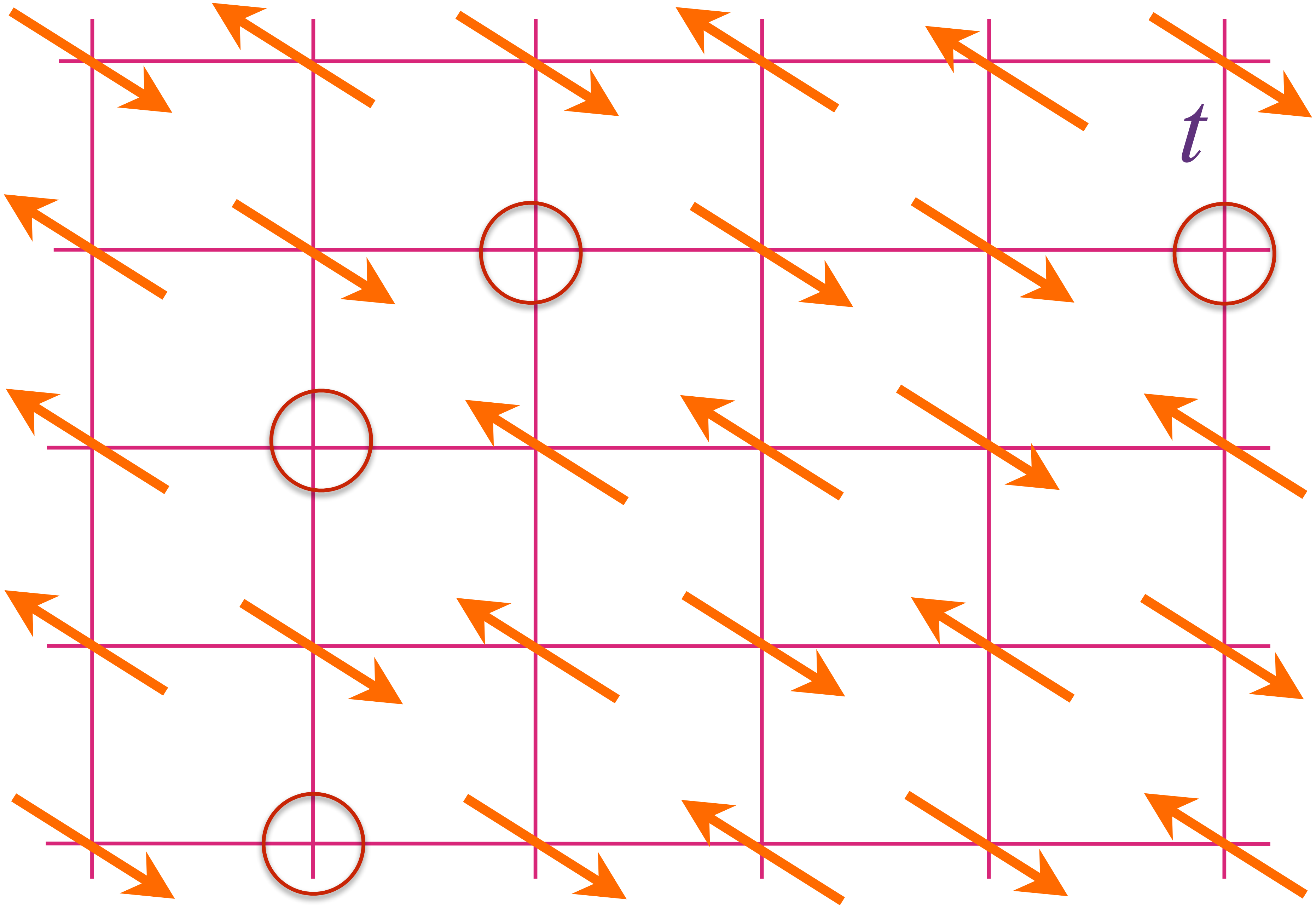
p mobile holes in a background of fluctuating spins

Real-space view



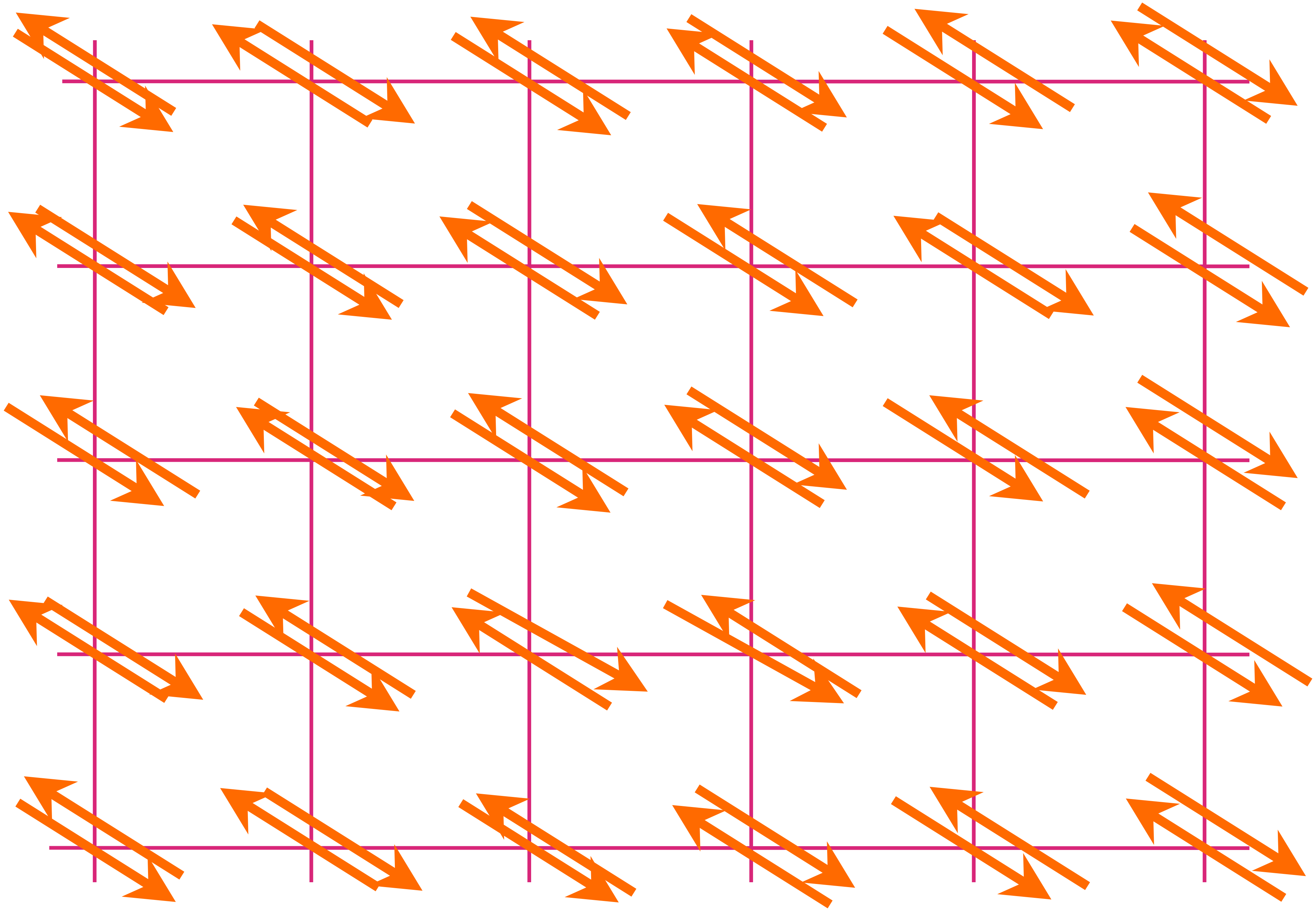
p mobile holes in a background of fluctuating spins

Real-space view



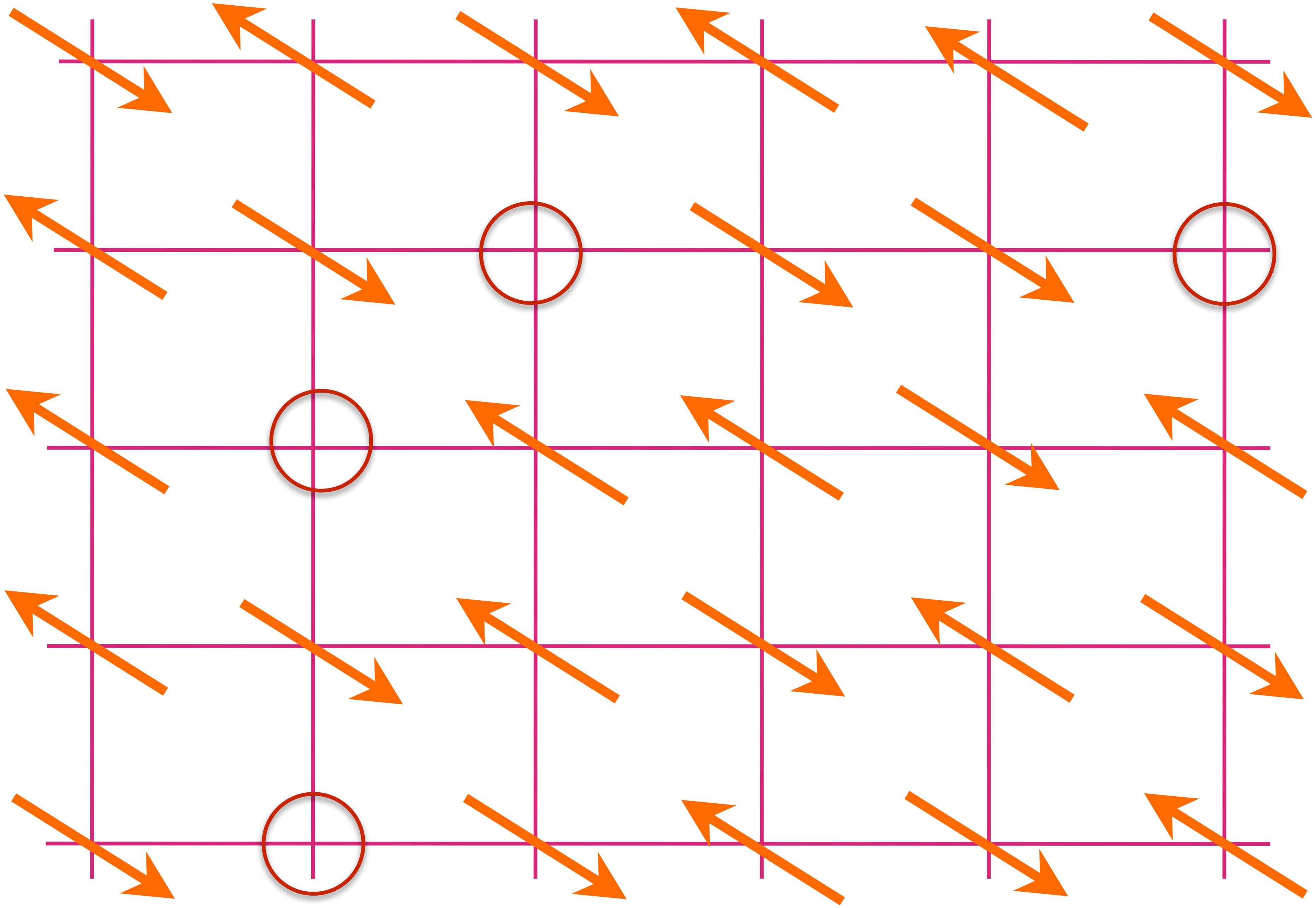
p mobile holes in a background of fluctuating spins

Momentum-space view



Filled Band

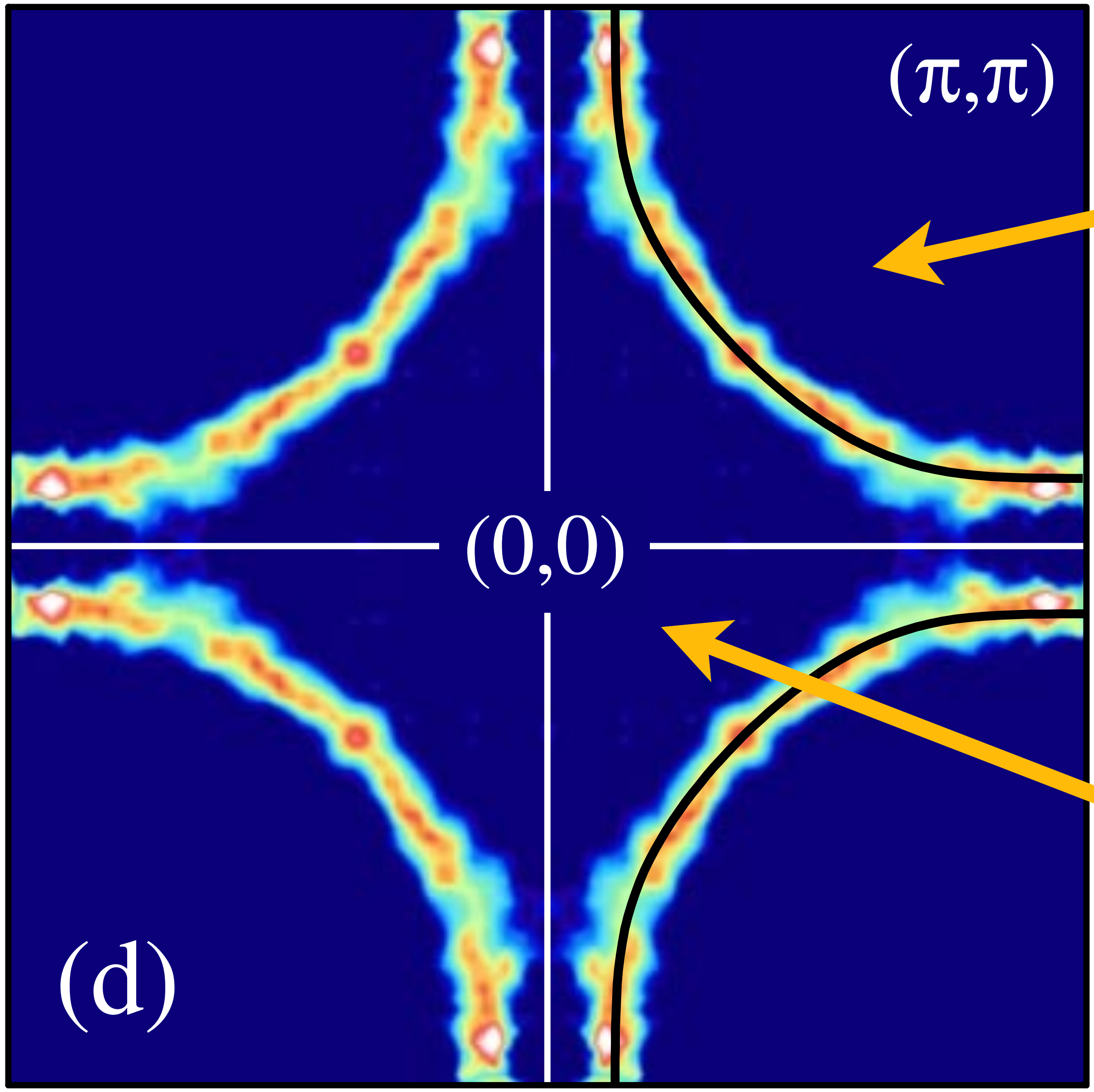
Momentum-space view



1-p mobile electrons =

1+p mobile holes in a filled band

Momentum-space view at large p



$1+p$ holes

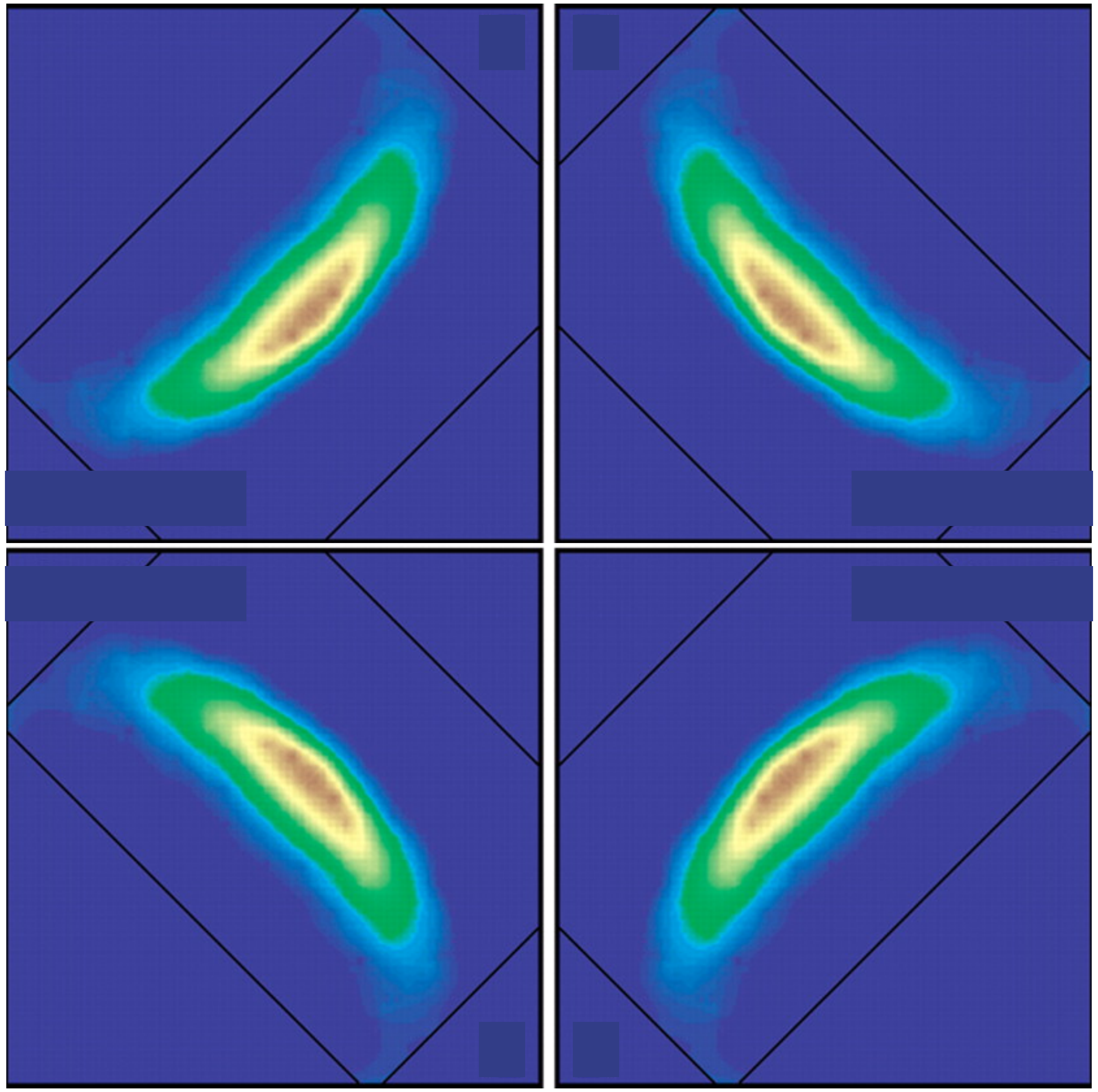
1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Momentum-space view at small *p*



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

“Fermi arcs”

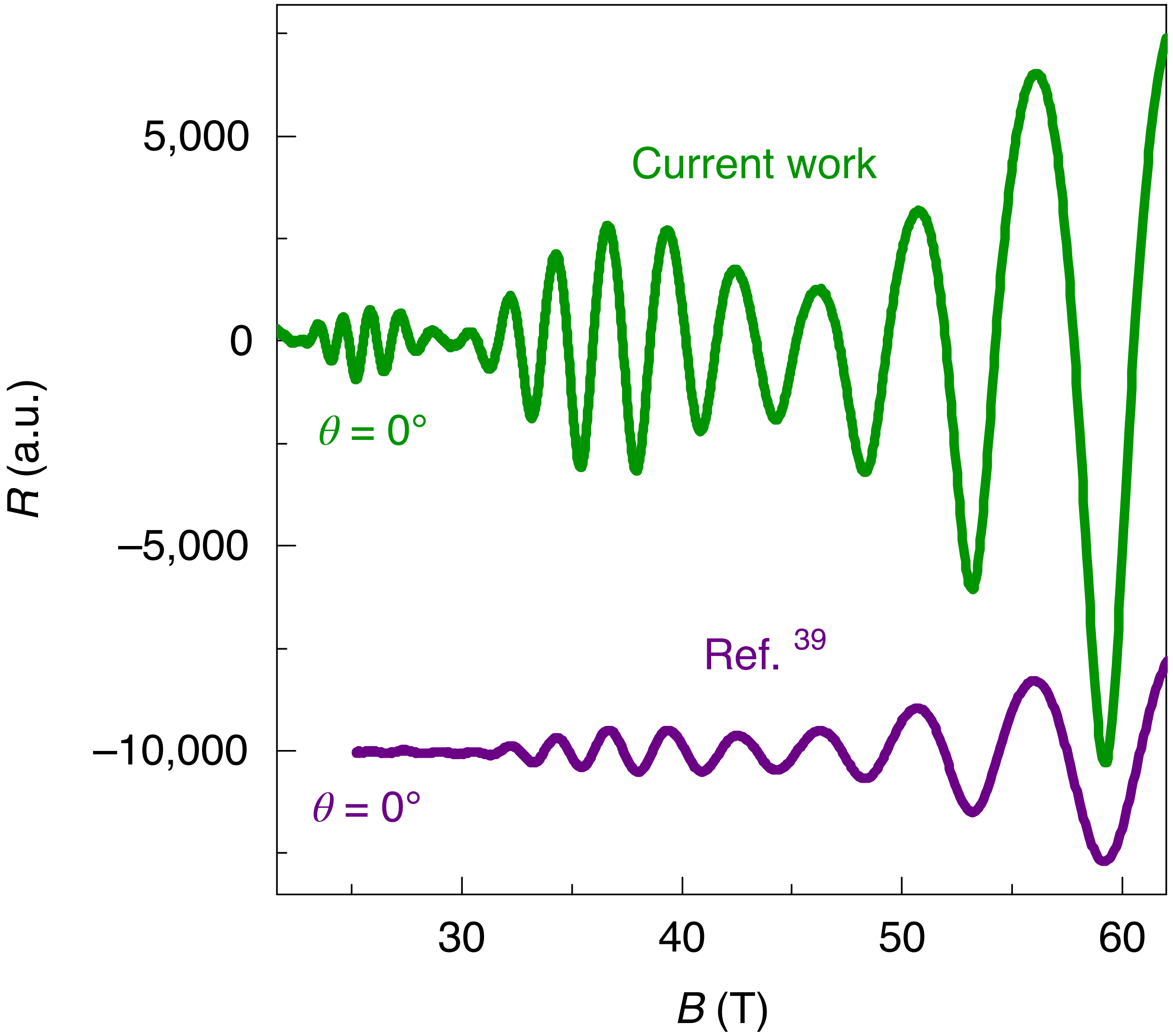
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

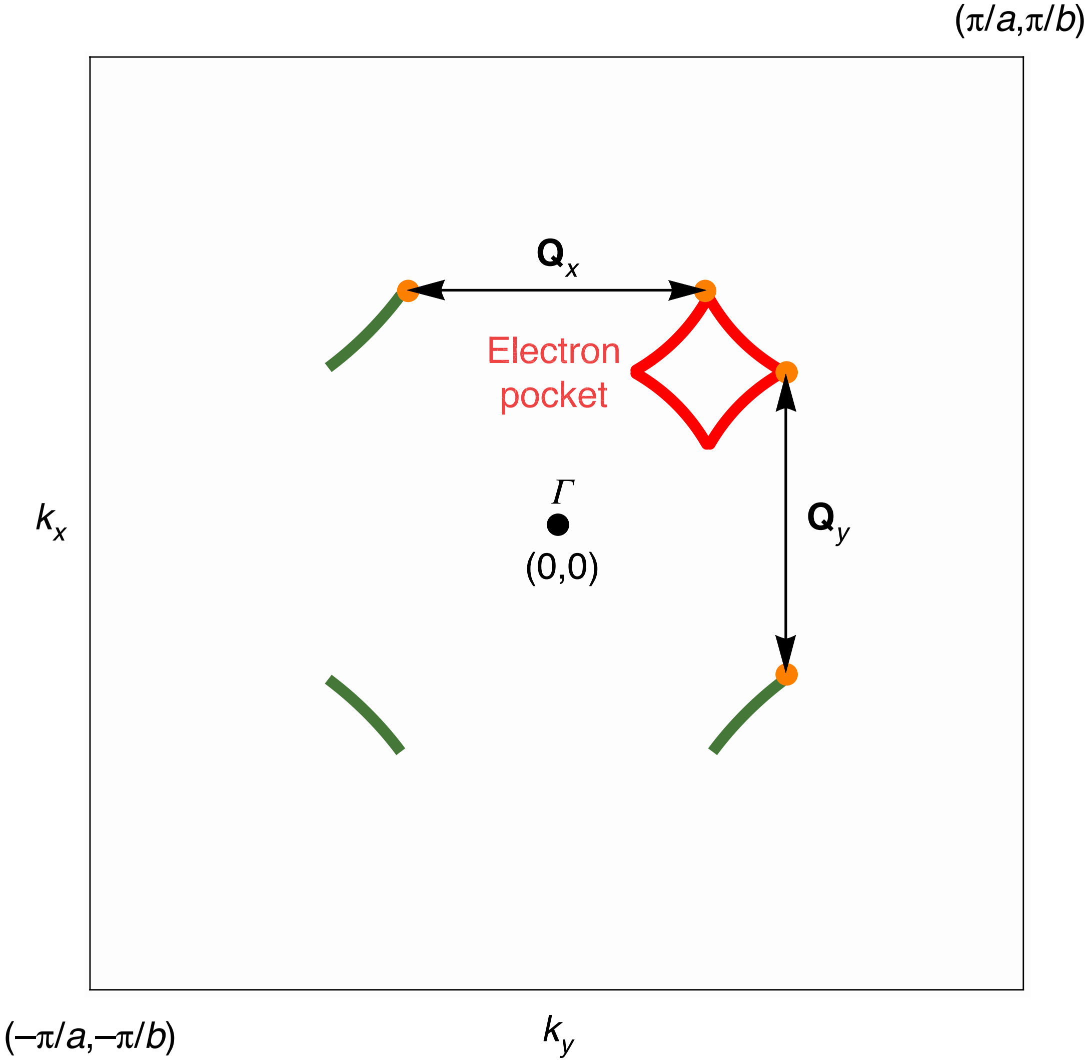
NATURE PHYSICS | VOL 16 | AUGUST 2020 | 841–847

Hard antinodal gap revealed by quantum oscillations in the pseudogap regime of underdoped high-T_c superconductors

Máté Hartstein, Yu-Te Hsu, Kimberly A. Modic, Juan Porras, Toshinao Loew, Matthieu Le Tacon, Huakun Zuo, Jinhua Wang, Zengwei Zhu, Mun K. Chan, Ross D. McDonald, Gilbert G. Lonzarich, Bernhard Keimer, Suchitra E. Sebastian and Neil Harrison

Suchitra Sebastian, Cavendish



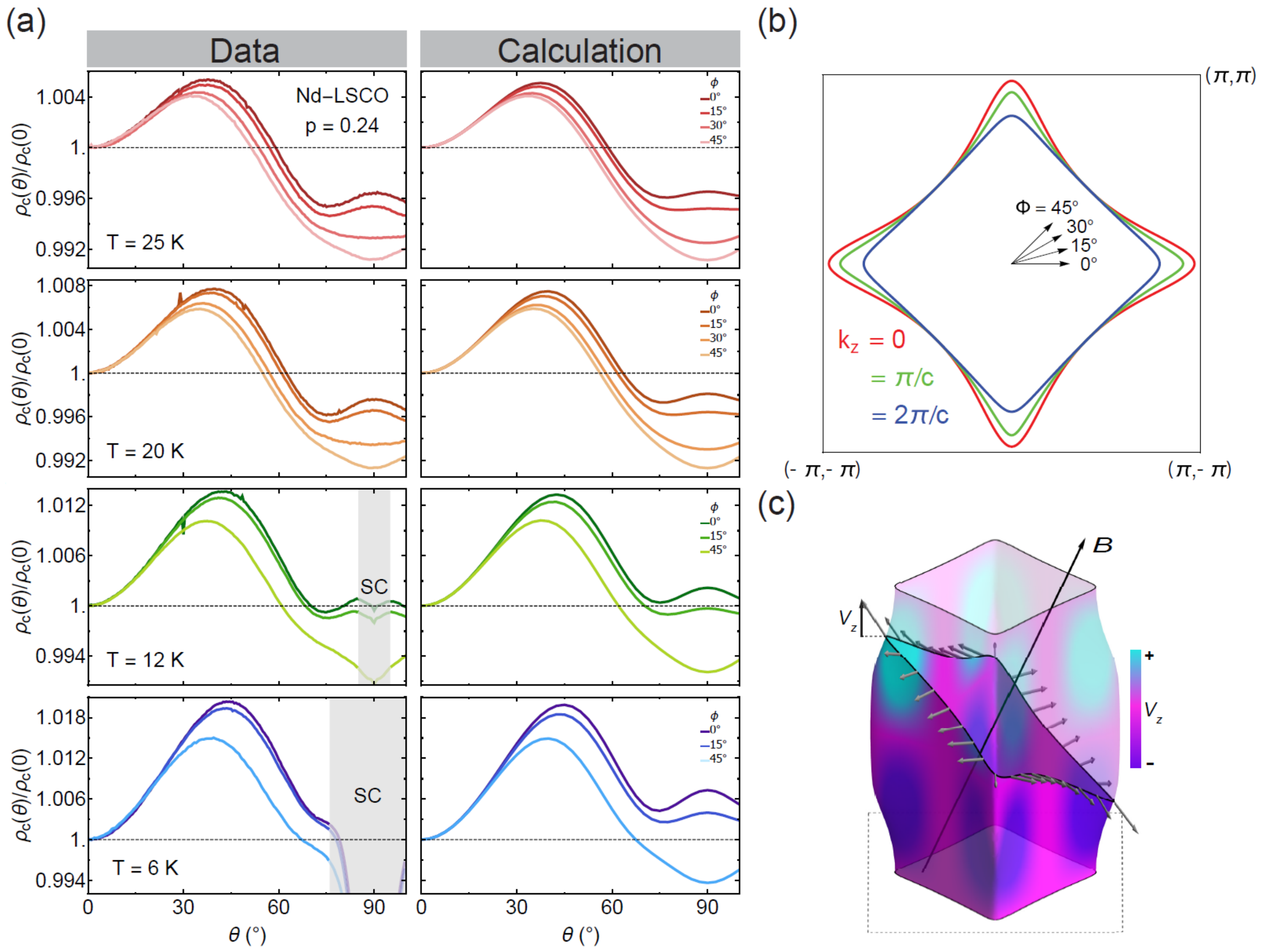


"Fermi arcs" reconstructed by charge density wave order at high magnetic fields in $YBa_2Cu_3O_{6.55}$

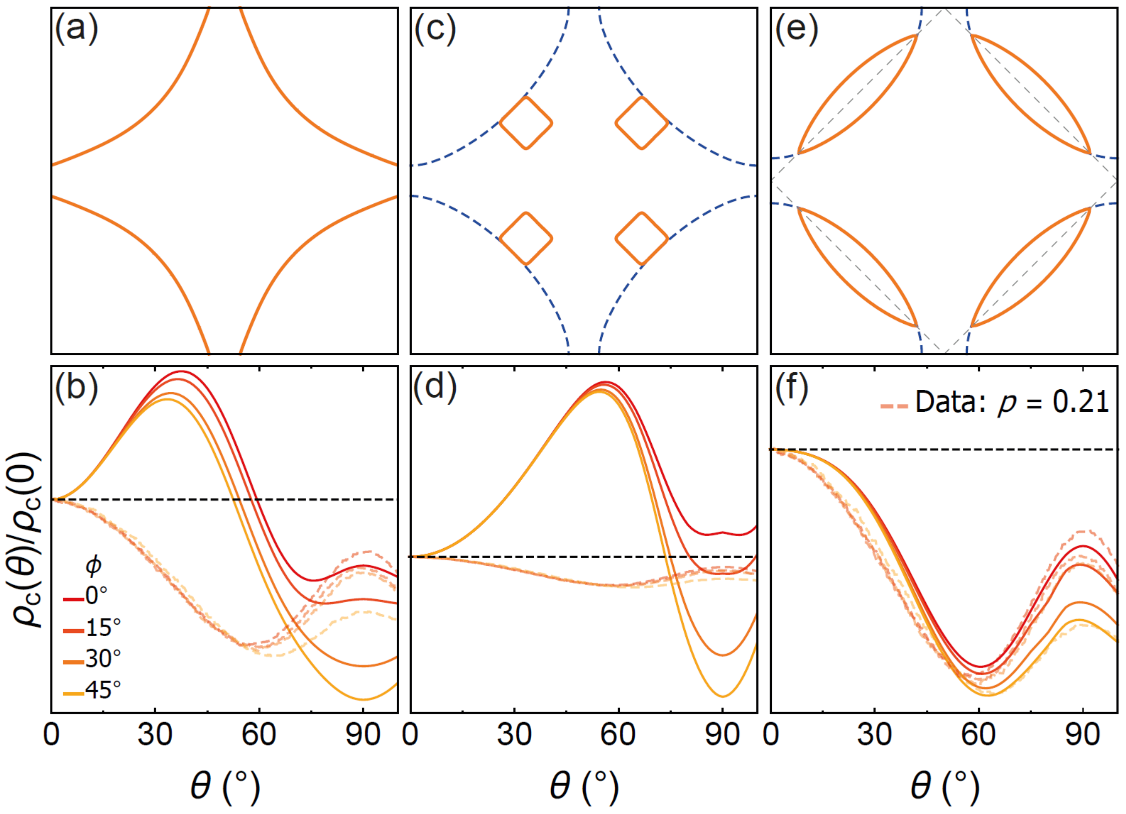
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. Above the critical doping p*—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p*, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a Q = (π,π) wavevector. While static Q = (π,π) antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

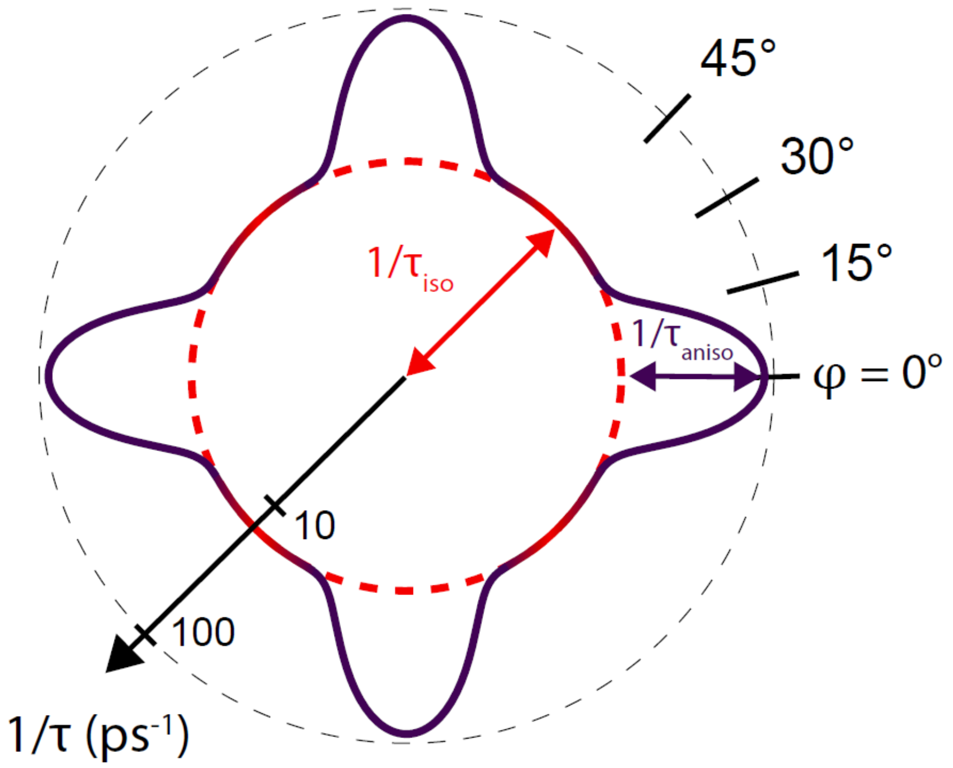


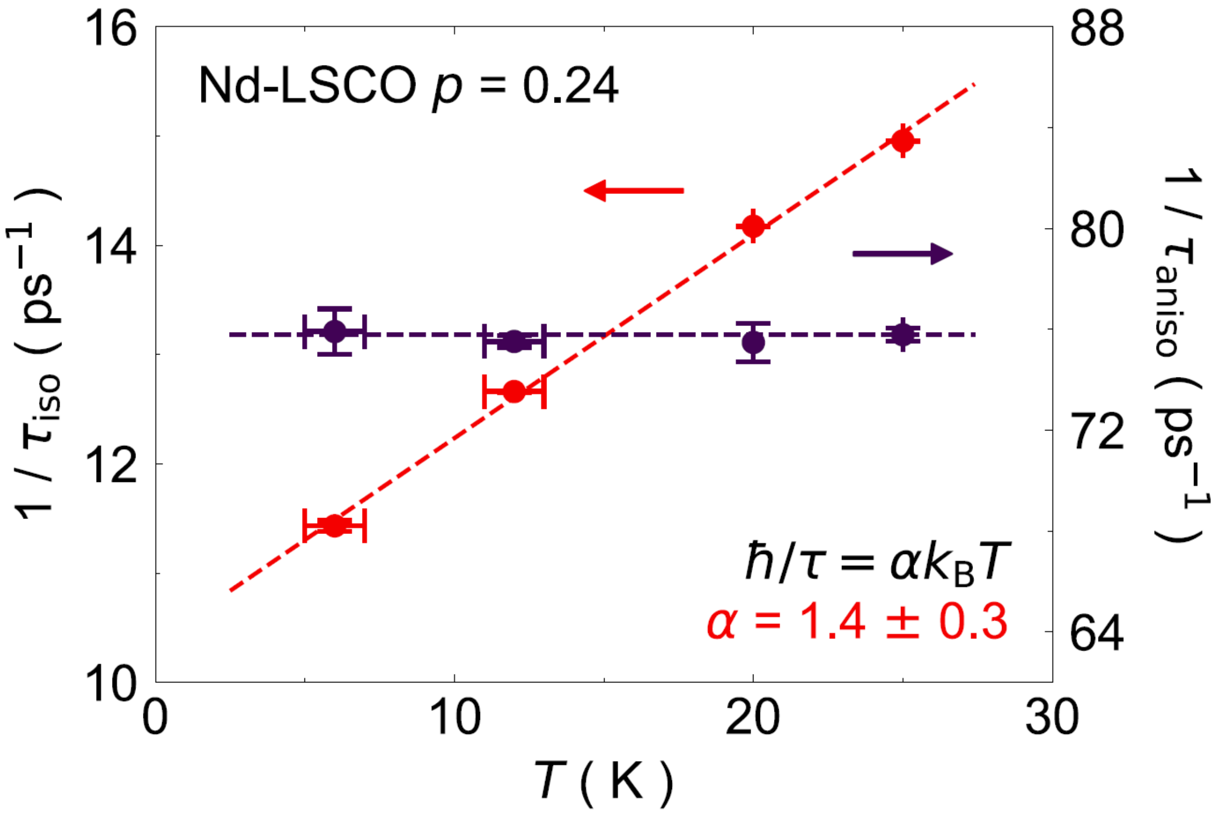
$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

An Isotropic, *T*-linear Scattering Rate





$$\frac{1}{\tau} = \frac{1}{\tau_{\text{aniso}}\left(\vec{k}\right)} + \frac{\alpha}{\hbar} k_B T$$

G. Grissonanche, Y. Fang *et al.*, 2020

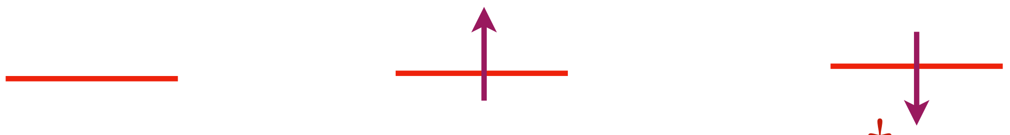
t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



$|0\rangle$ $\quad$ $c_{\uparrow}^{\dagger} |0\rangle$ $\quad$ $c_{\downarrow}^{\dagger} |0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

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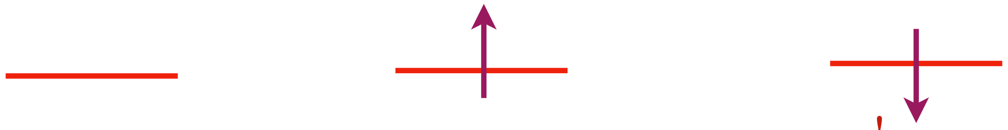
$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$$



$|0\rangle$ $\quad$ $c_\uparrow^\dagger |0\rangle$ $\quad$ $c_\downarrow^\dagger |0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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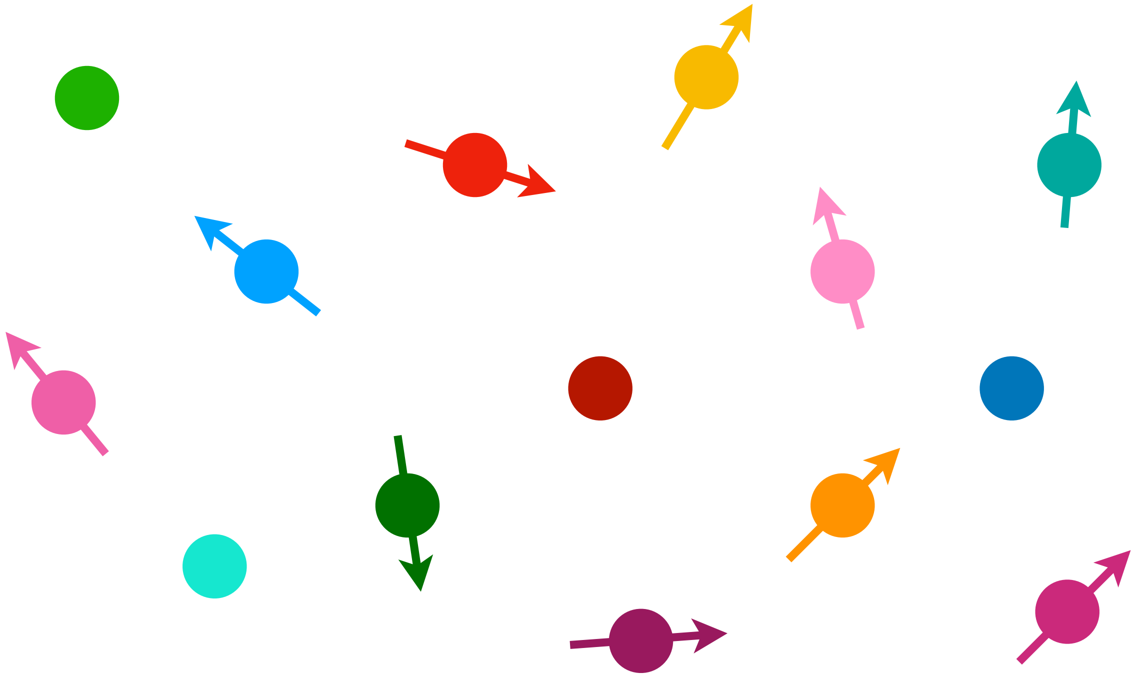

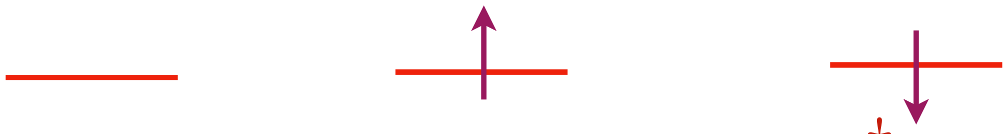

$|0\rangle \qquad c_{\uparrow}^{\dagger} |0\rangle \qquad c_{\downarrow}^{\dagger} |0\rangle$

Random *t*-*J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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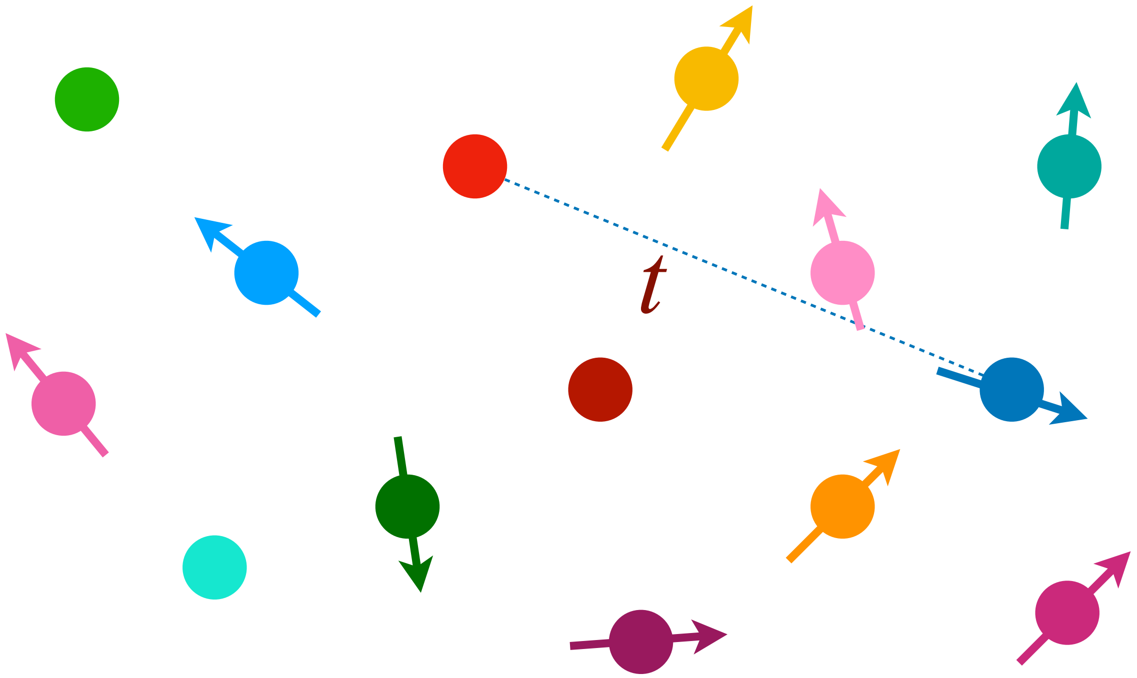

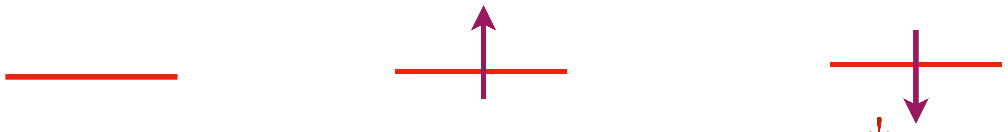

$|0\rangle$ $\qquad$ $c^\dagger_\uparrow |0\rangle$ $\qquad$ $c^\dagger_\downarrow |0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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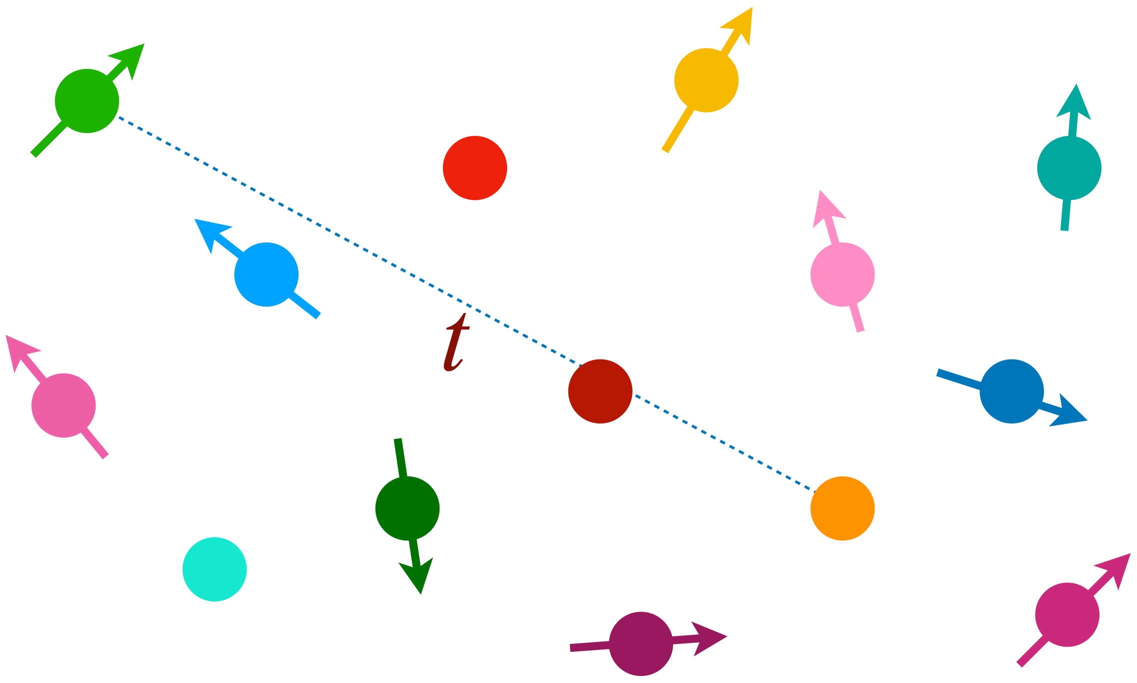

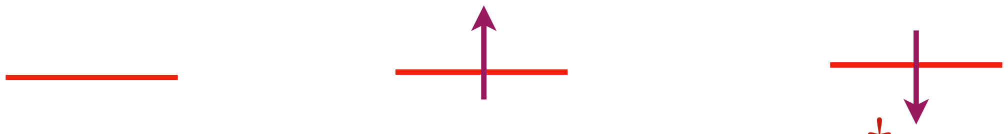

$|0\rangle$ $\quad c_{\uparrow}^{\dagger}\,|0\rangle$ $\quad c_{\downarrow}^{\dagger}\,|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}}\sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}}\sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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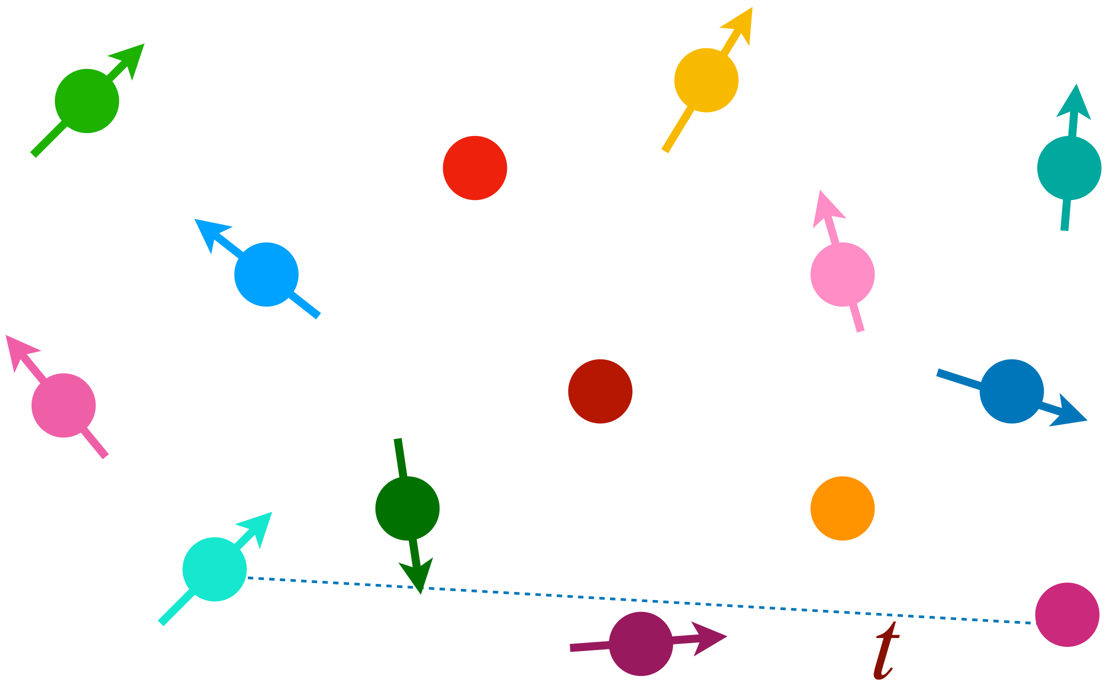
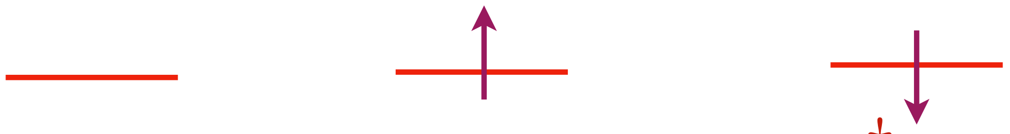


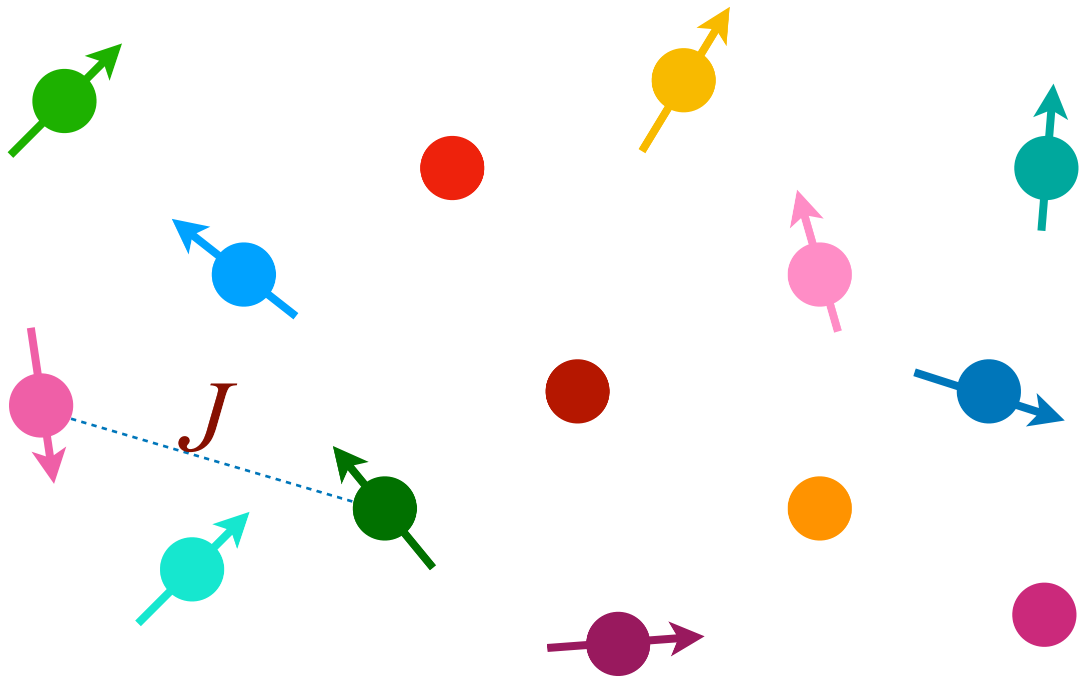


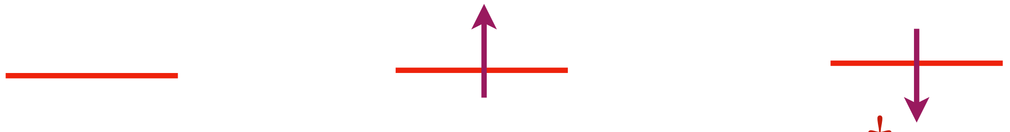
$|0\rangle$ $c_{\uparrow}^{\dagger}|0\rangle$ $c_{\downarrow}^{\dagger}|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





$|0\rangle$ $\qquad c_{\uparrow}^{\dagger}\,|0\rangle$ $\qquad c_{\downarrow}^{\dagger}\,|0\rangle$

Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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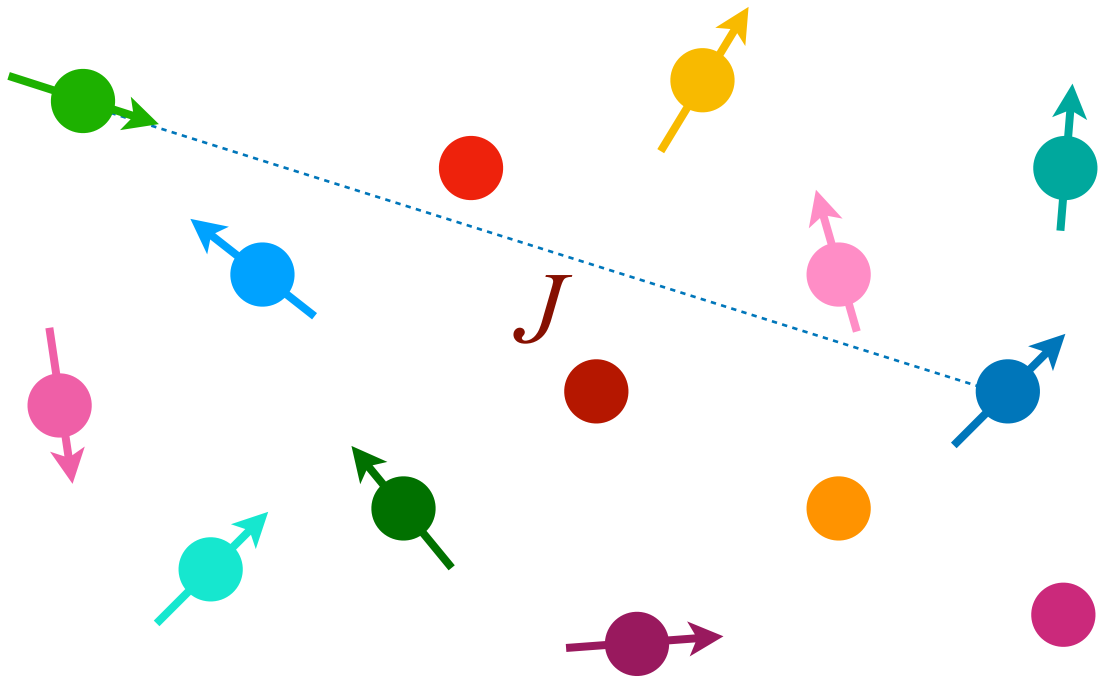

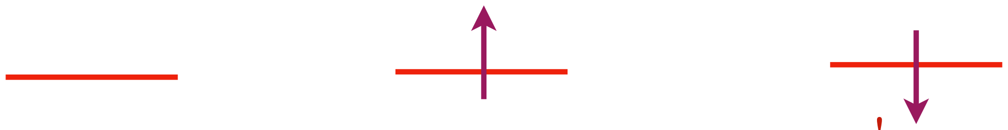

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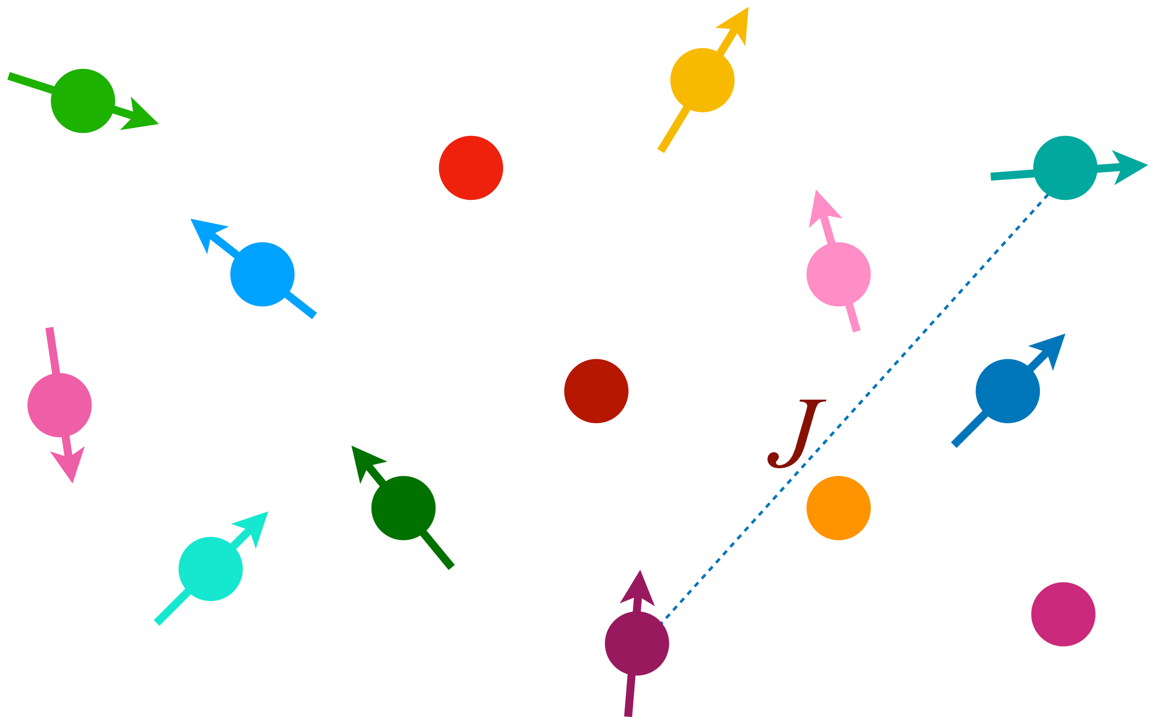
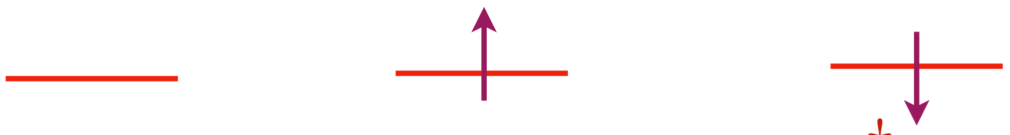
Random *t-J* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.





$|0\rangle$ $\quad$ $c_{\uparrow}^{\dagger}\,|0\rangle$ $\quad$ $c_{\downarrow}^{\dagger}\,|0\rangle$

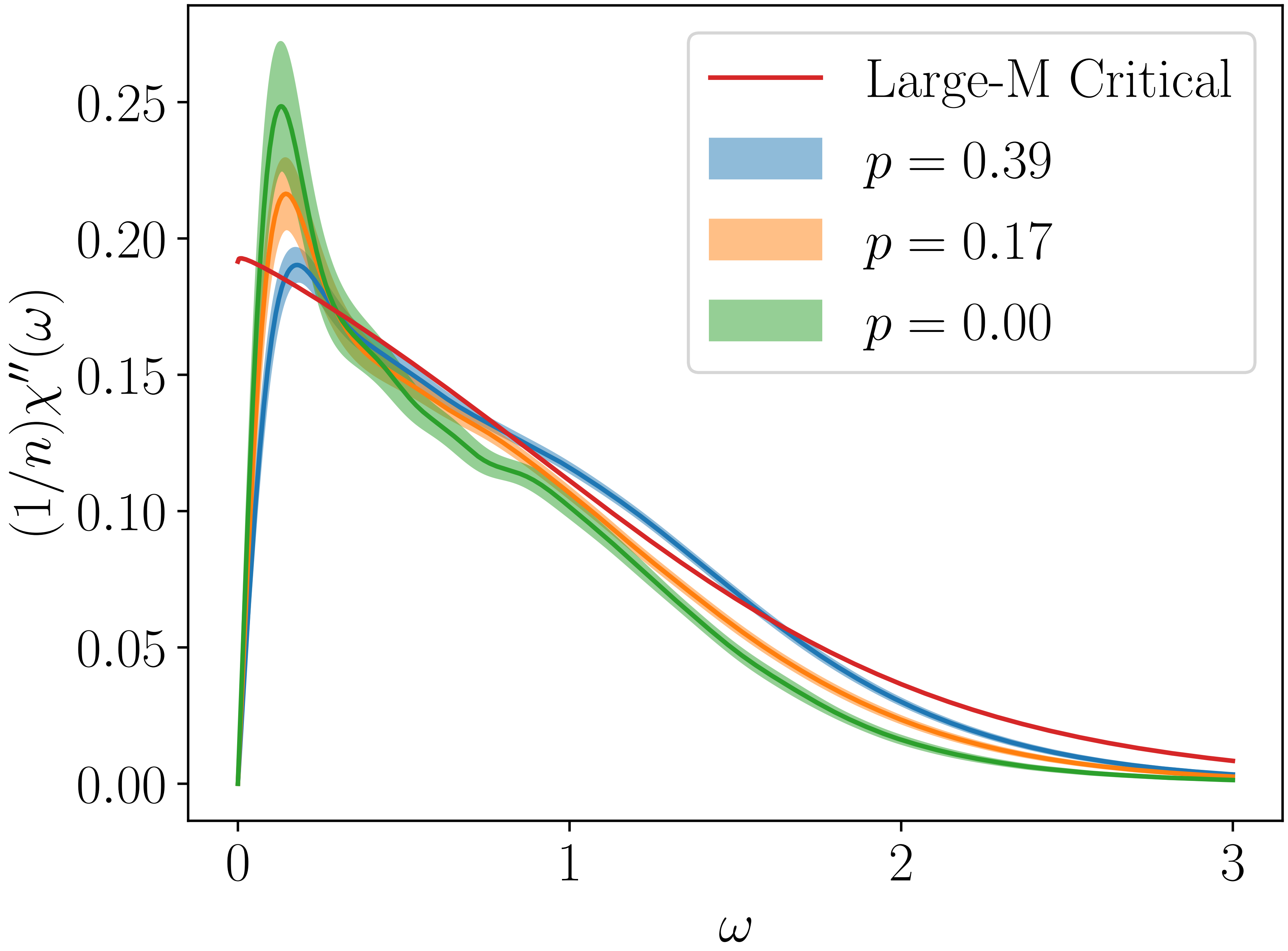
Henry Shackleton

Alexander Wietek

Antoine Georges

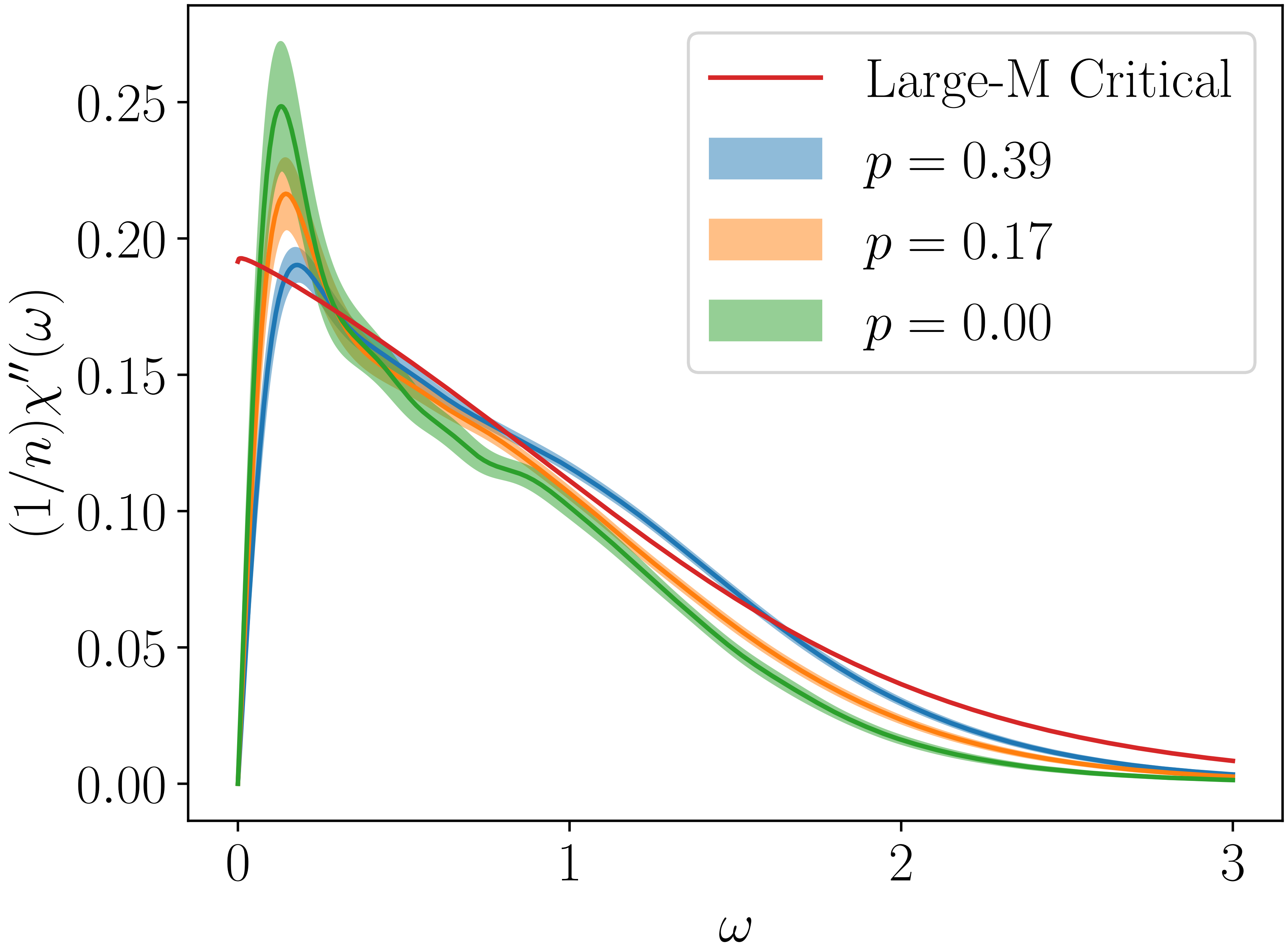
to appear….

Numerical exact diagonalization and SYK theory



Evidence for a quantum critical point at $p = p_c \approx 0.3$
with SYK criticality.
Spin glass order for $p < p_c$

Numerical exact diagonalization and SYK theory



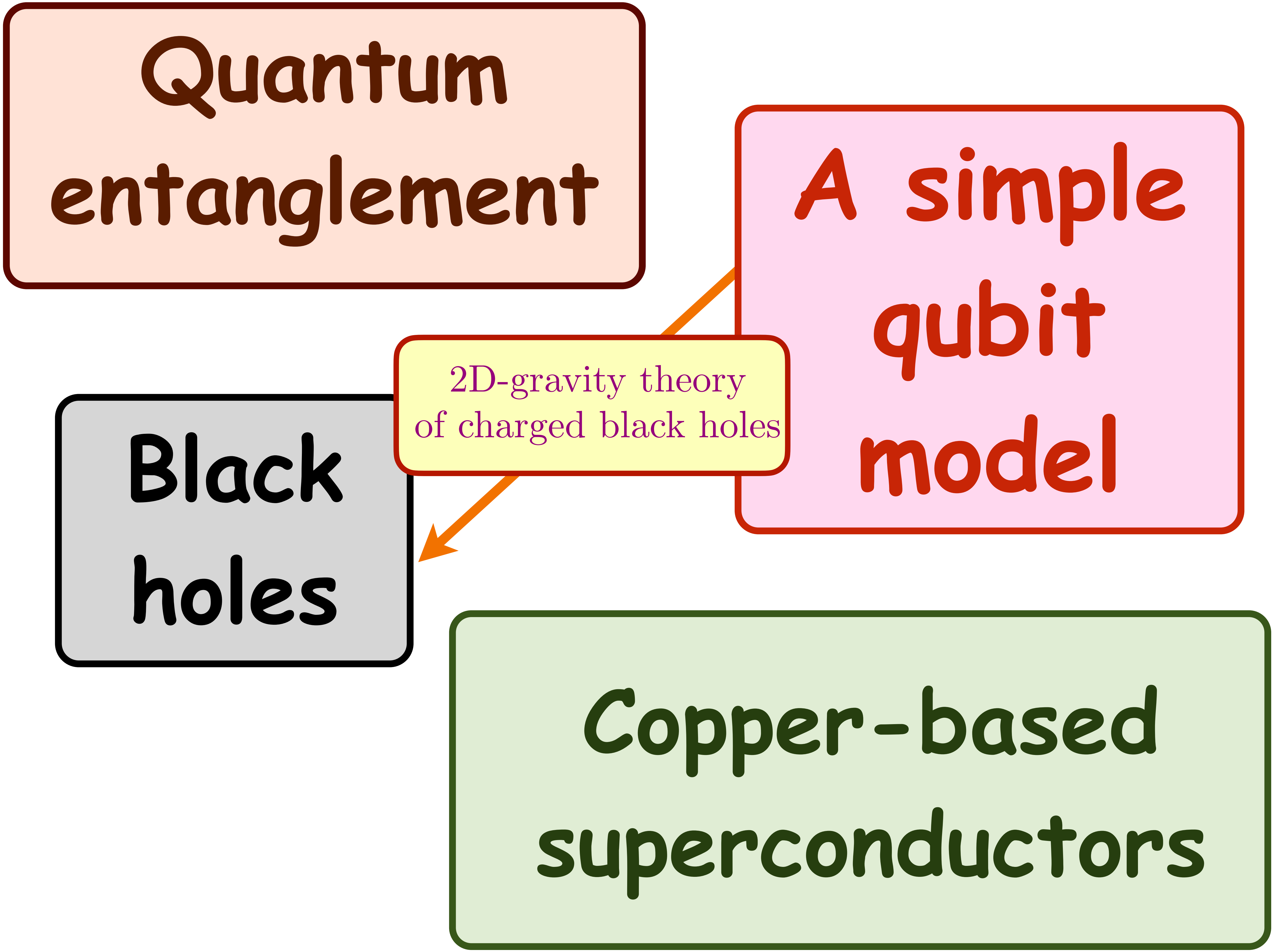
Numerics matches many other observations, including the breakdown of the Luttinger-volume Fermi surface for $p < p_c$, and Planckian dissipation at scale $\hbar/(k_B T)$.

Quantum entanglement

A simple qubit model

Black holes

Copper-based superconductors



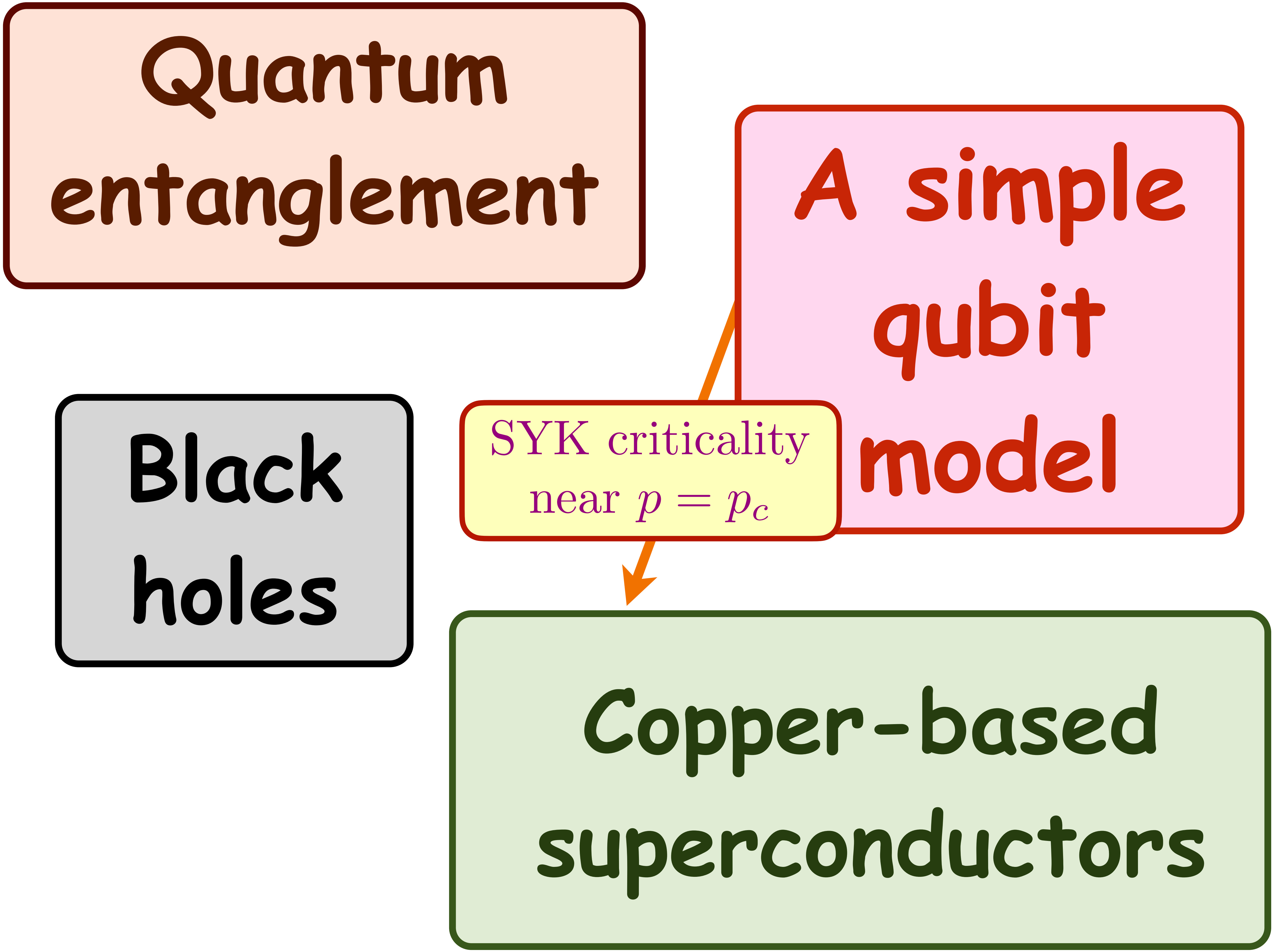
Quantum entanglement

A simple qubit model

2D-gravity theory of charged black holes

Black holes

Copper-based superconductors



Quantum entanglement

A simple qubit model

SYK criticality near $p = p_c$

Black holes

Copper-based superconductors