

The quantum criticality of metals in two dimensions: antiferromagnetism, d-wave superconductivity and bond order

ICTP, Trieste
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Subir Sachdev





Max Metlitski



Erez Berg



**Rolando
La Placa**

Quantum phase transitions with loss of antiferromagnetic order

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I. Dimerized insulators:

Conformal field theory

Quantum phase transitions with loss of antiferromagnetic order

1. Dimerized insulators:

Conformal field theory

2. Metals (at weaker coupling):

The iron-based superconductors

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*The hole-doped cuprate
superconductors*

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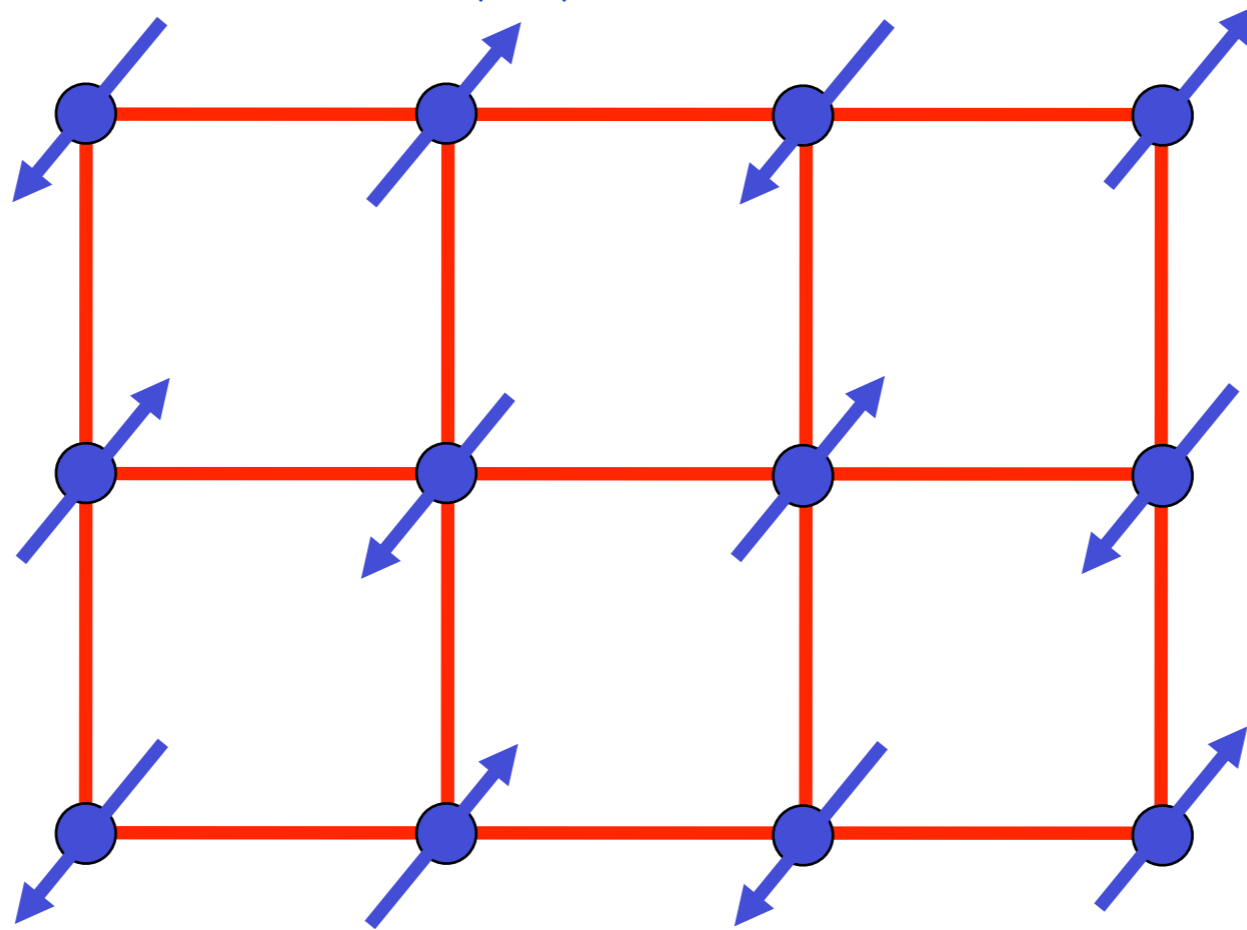
The iron-based superconductors

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Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

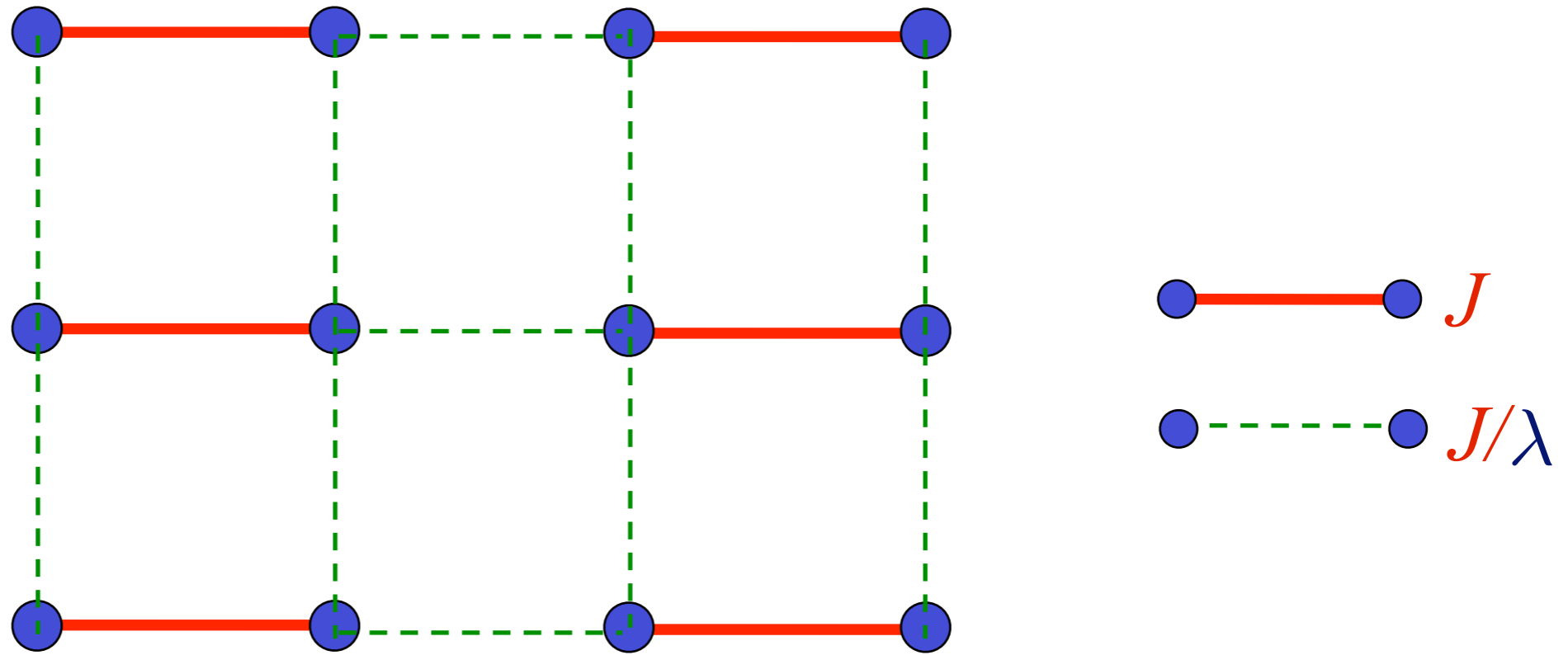
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

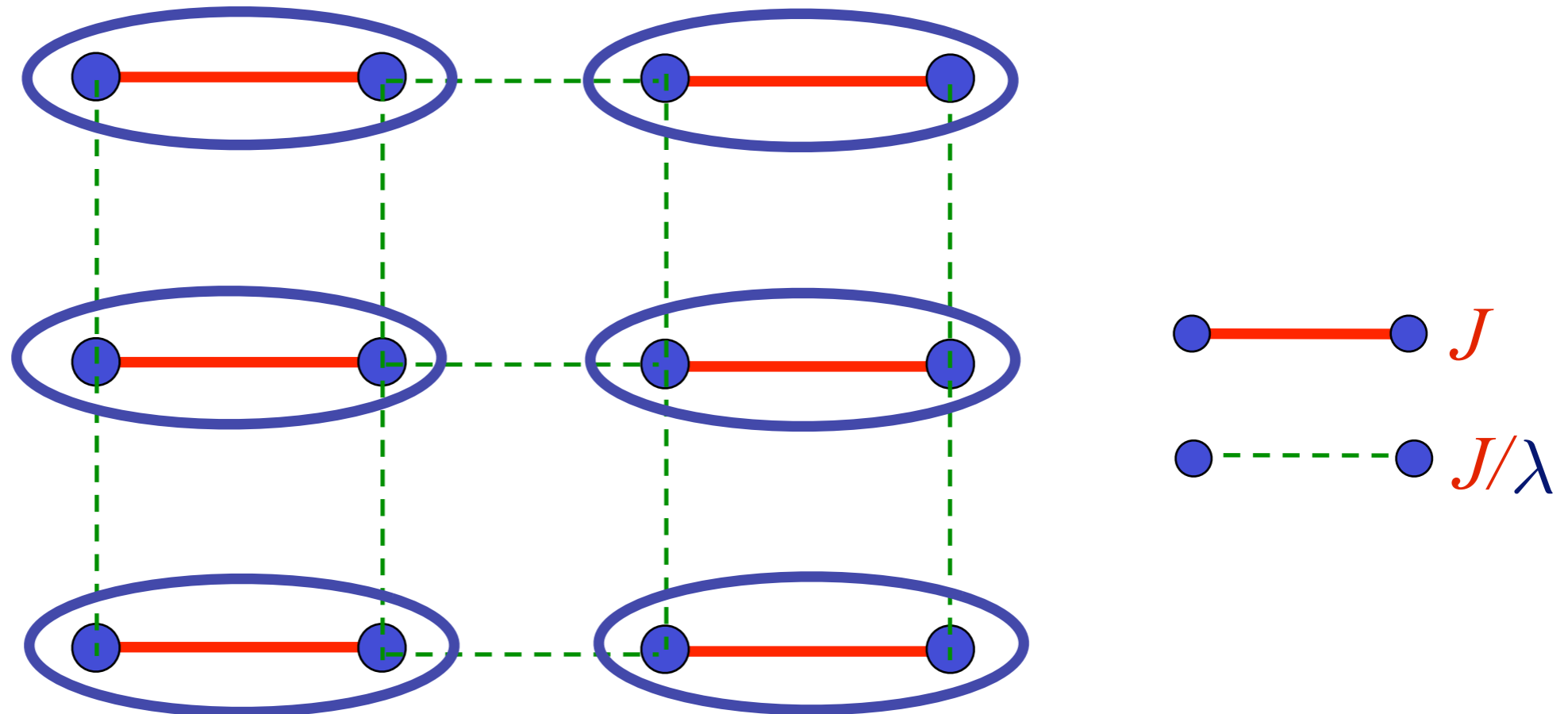
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

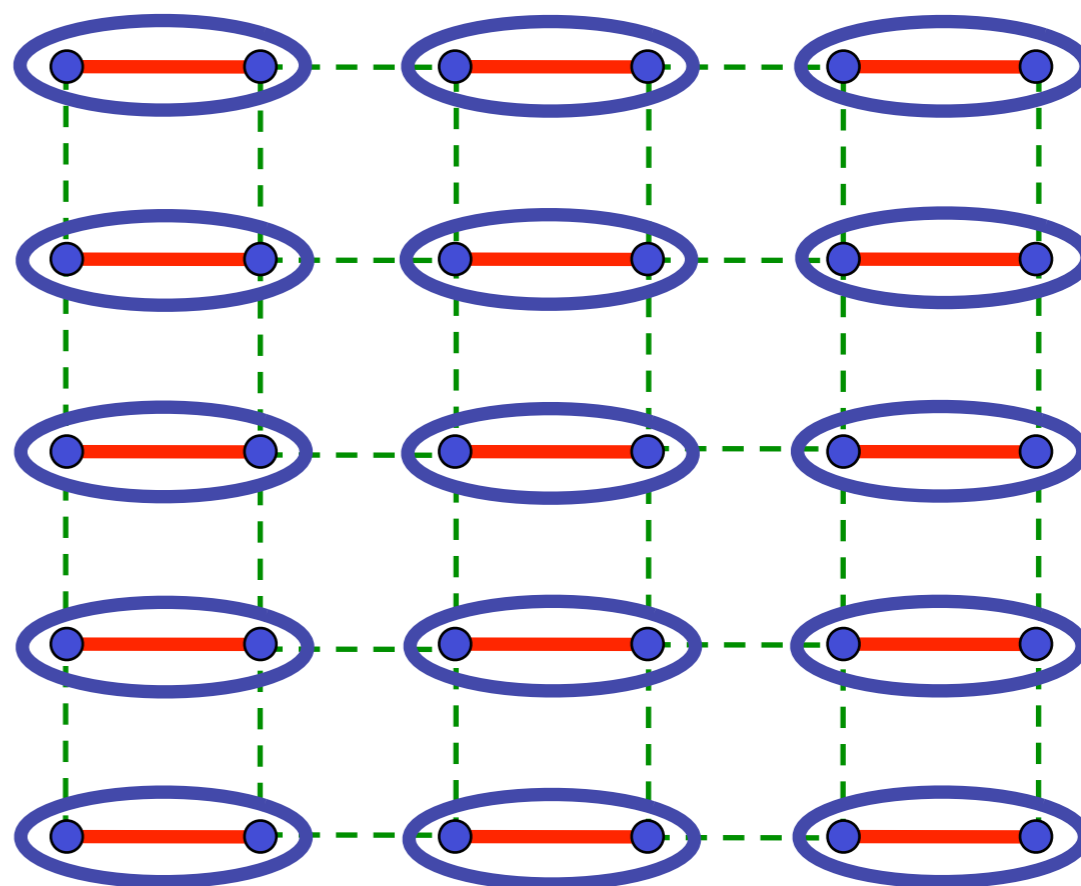
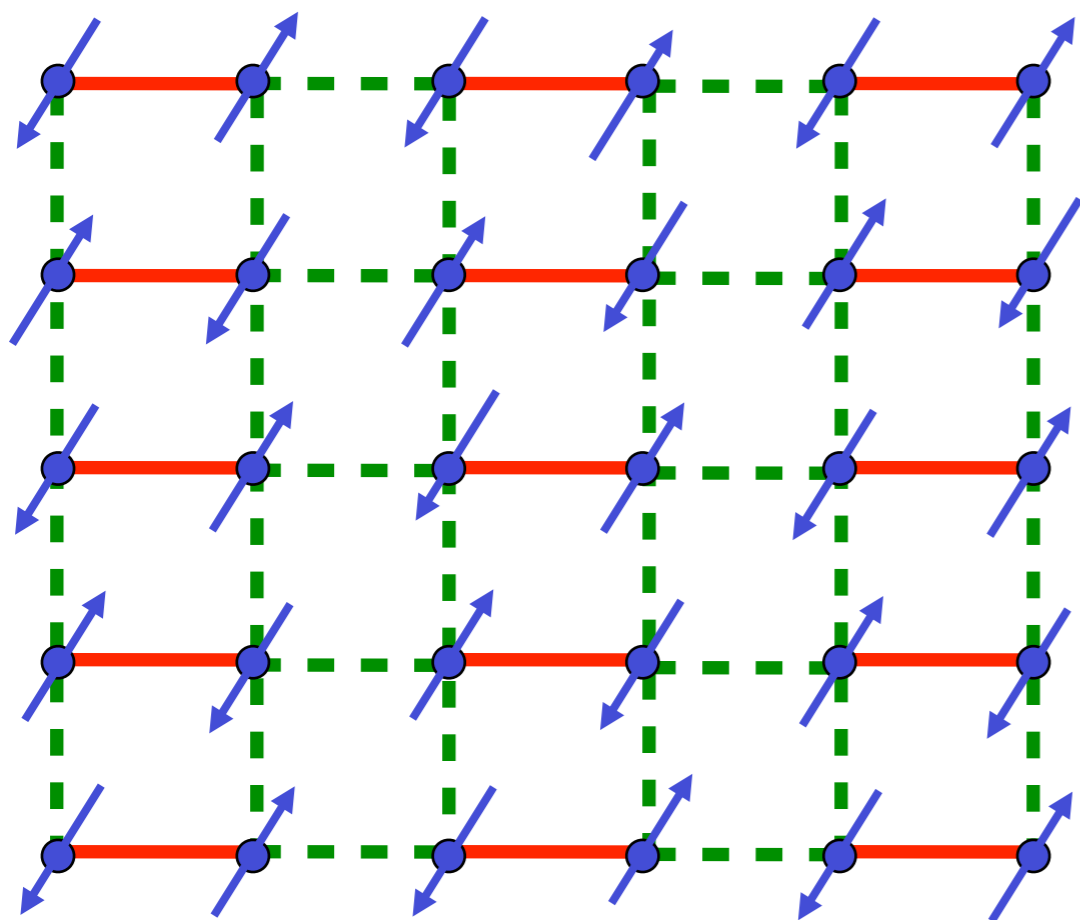
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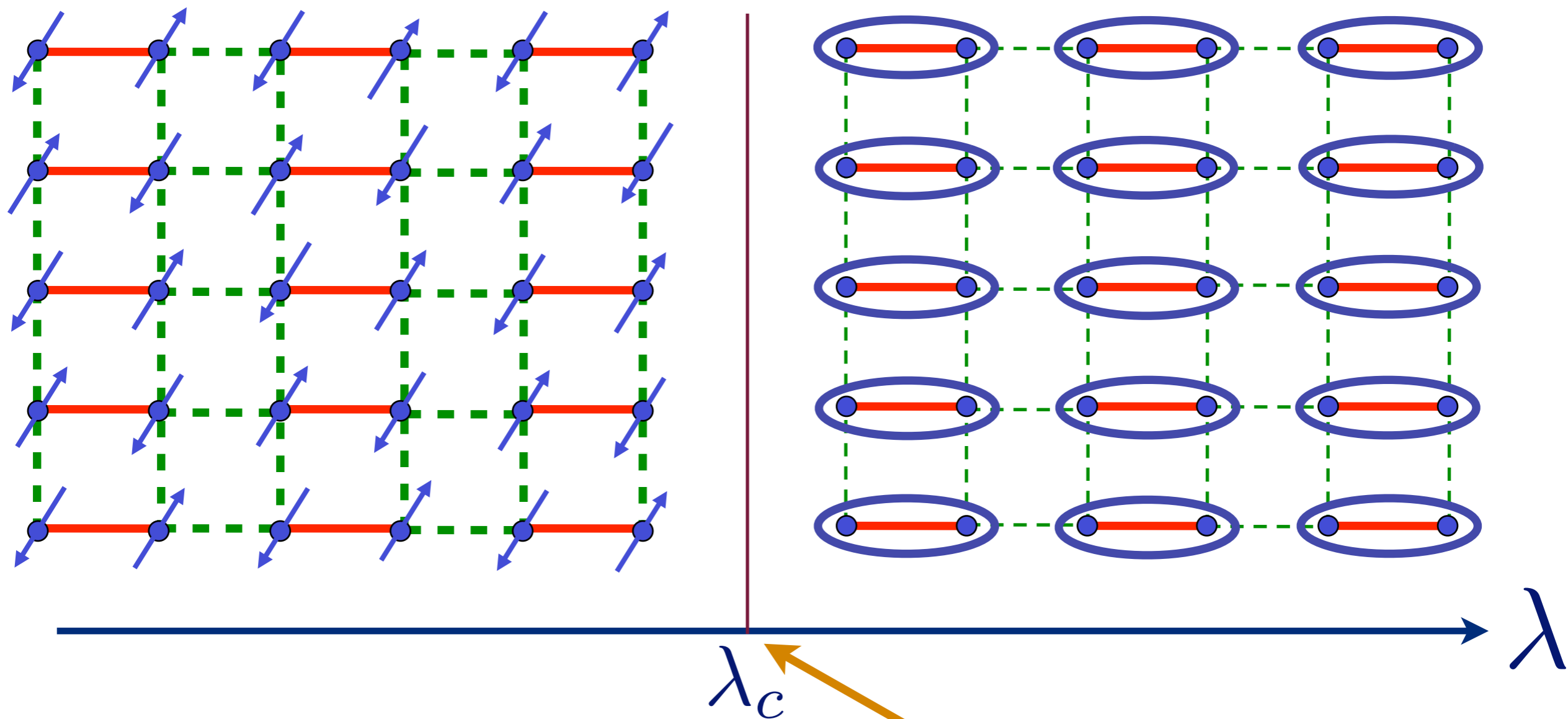
Ground state is a “quantum paramagnet”
with spins locked in valence bond singlets

$$\text{Valence bond singlet} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$\text{Diagram of two blue spheres connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

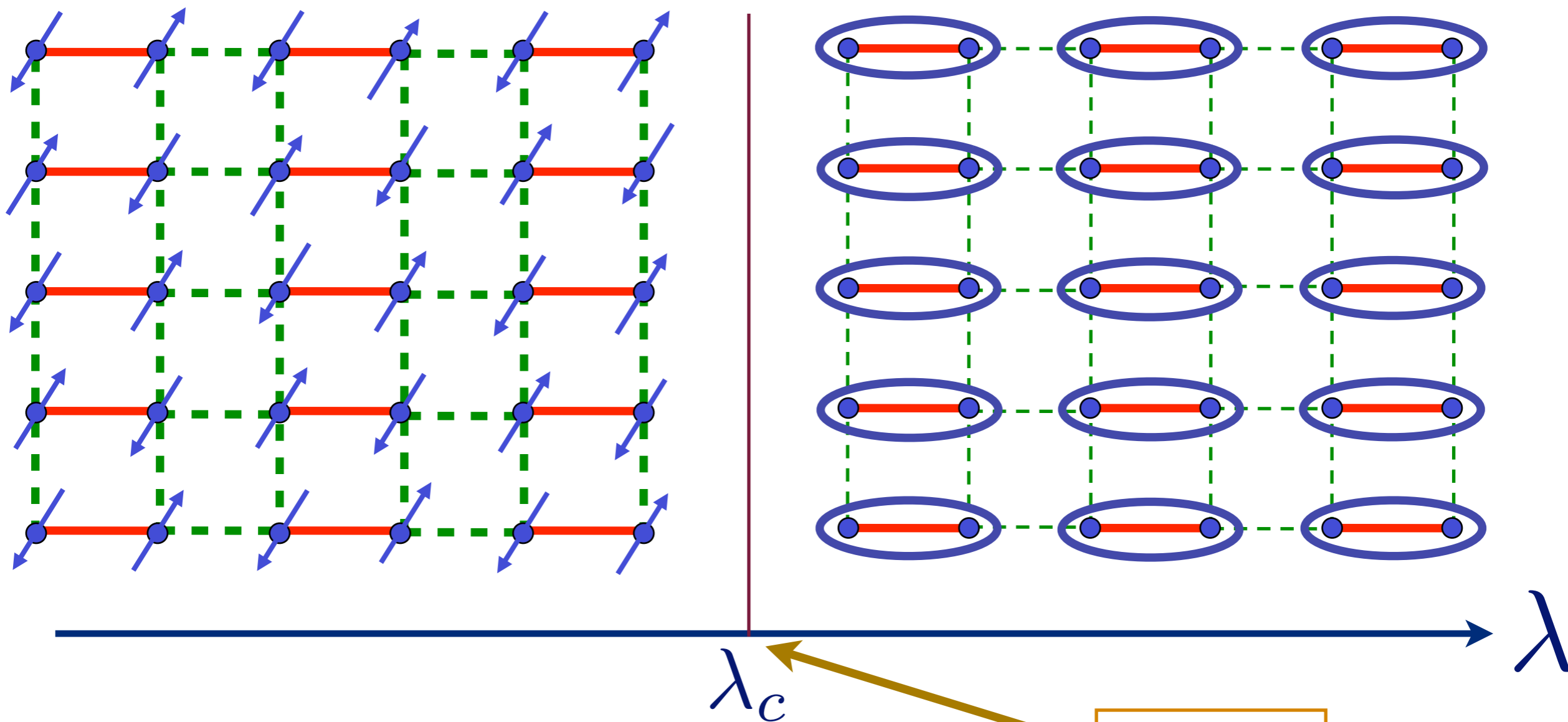


$$\text{Diagram of two blue dots connected by a red line, enclosed in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



Quantum critical point with non-local entanglement in spin wavefunction

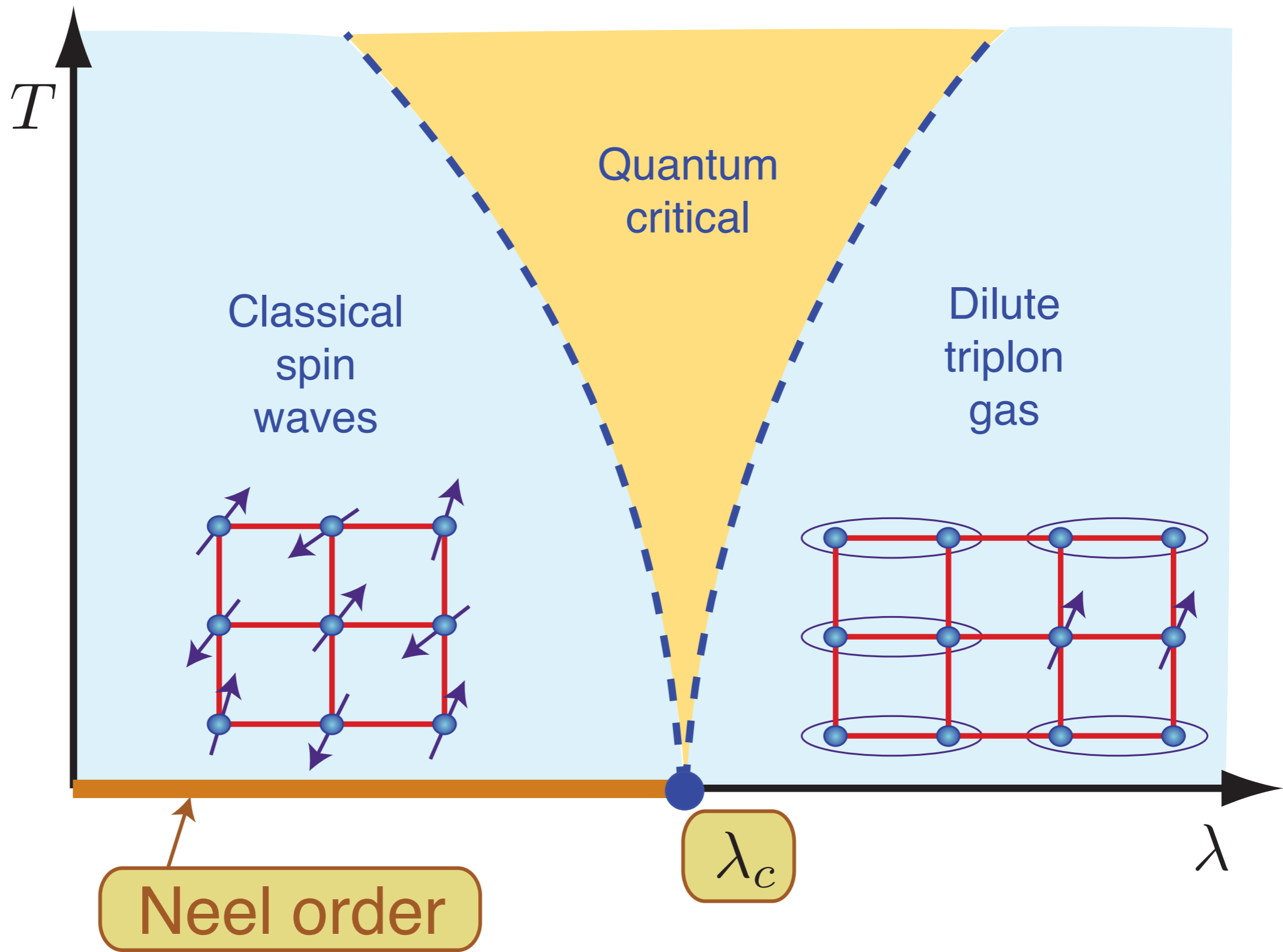
$$\text{Diagram of two sites in a blue oval} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



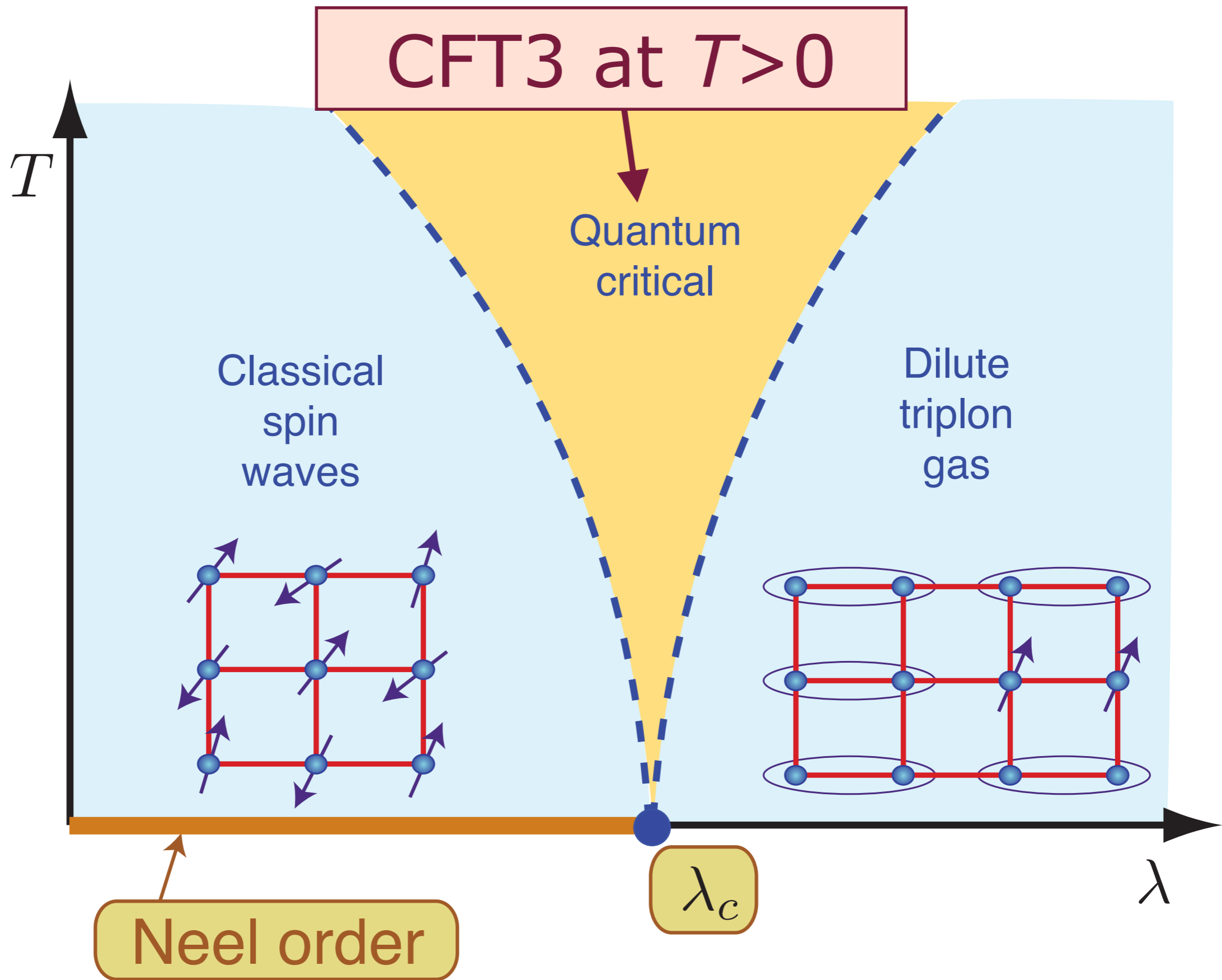
$O(3)$ order parameter $\vec{\varphi}$

CFT3

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right]$$



S. Sachdev and
 J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).



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 J. Ye, *Phys. Rev. Lett.*
69, 2411 (1992).

Quantum phase transitions with loss of antiferromagnetic order

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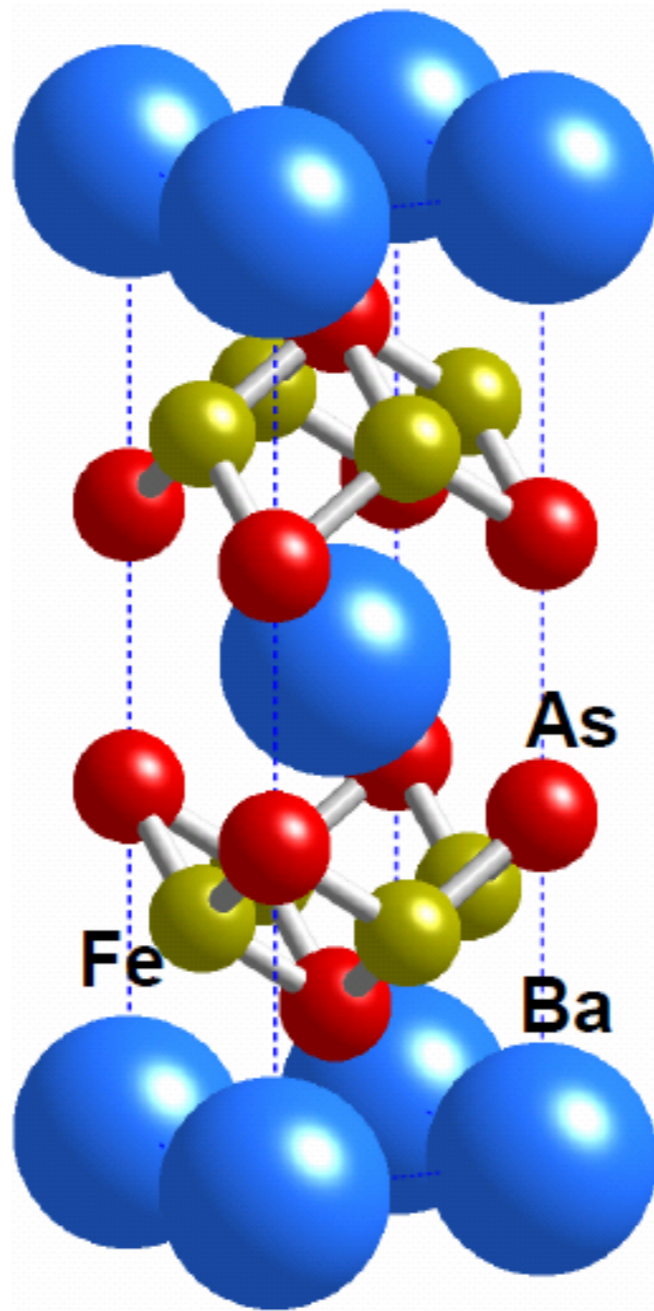
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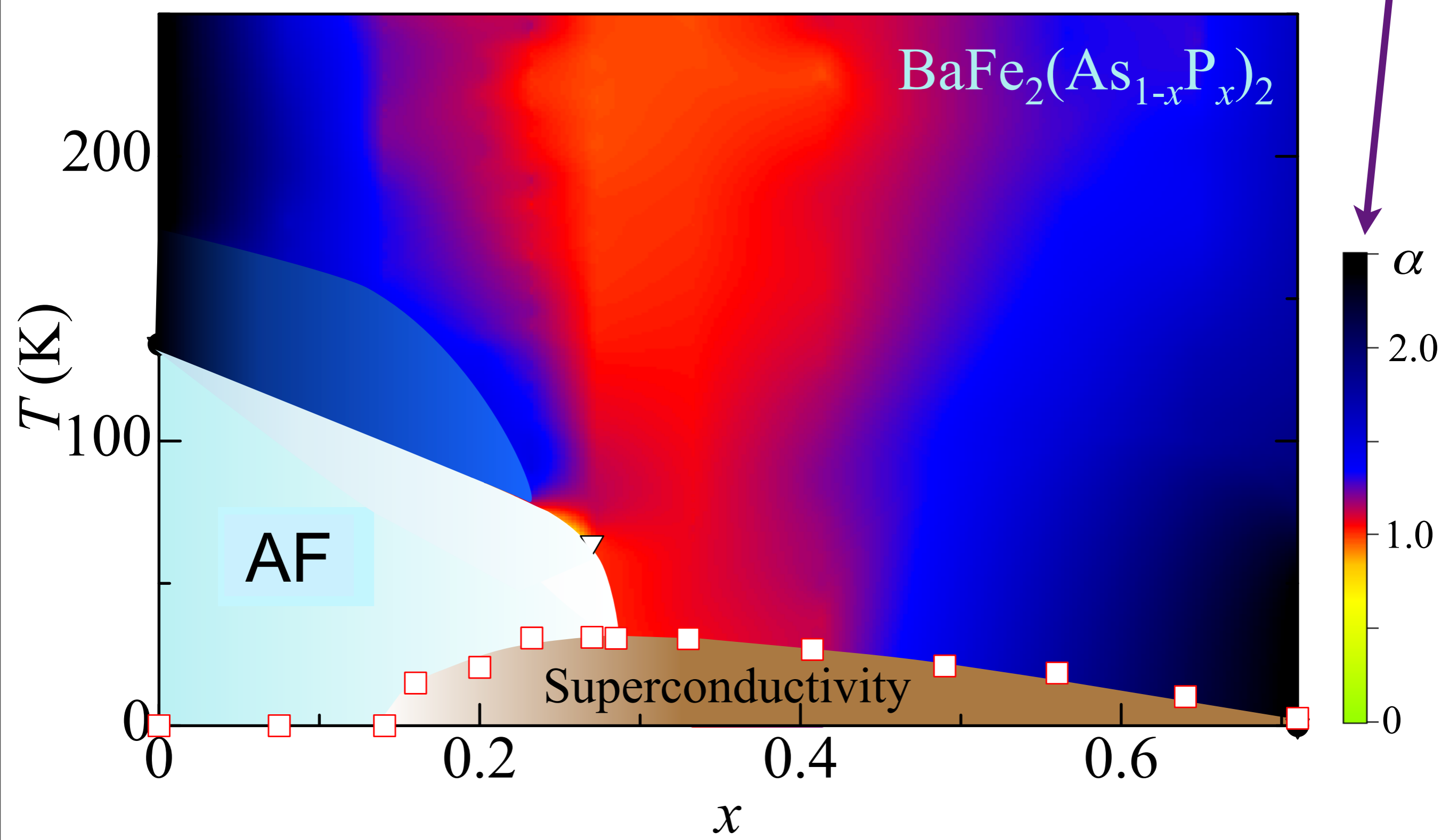
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Iron pnictides:

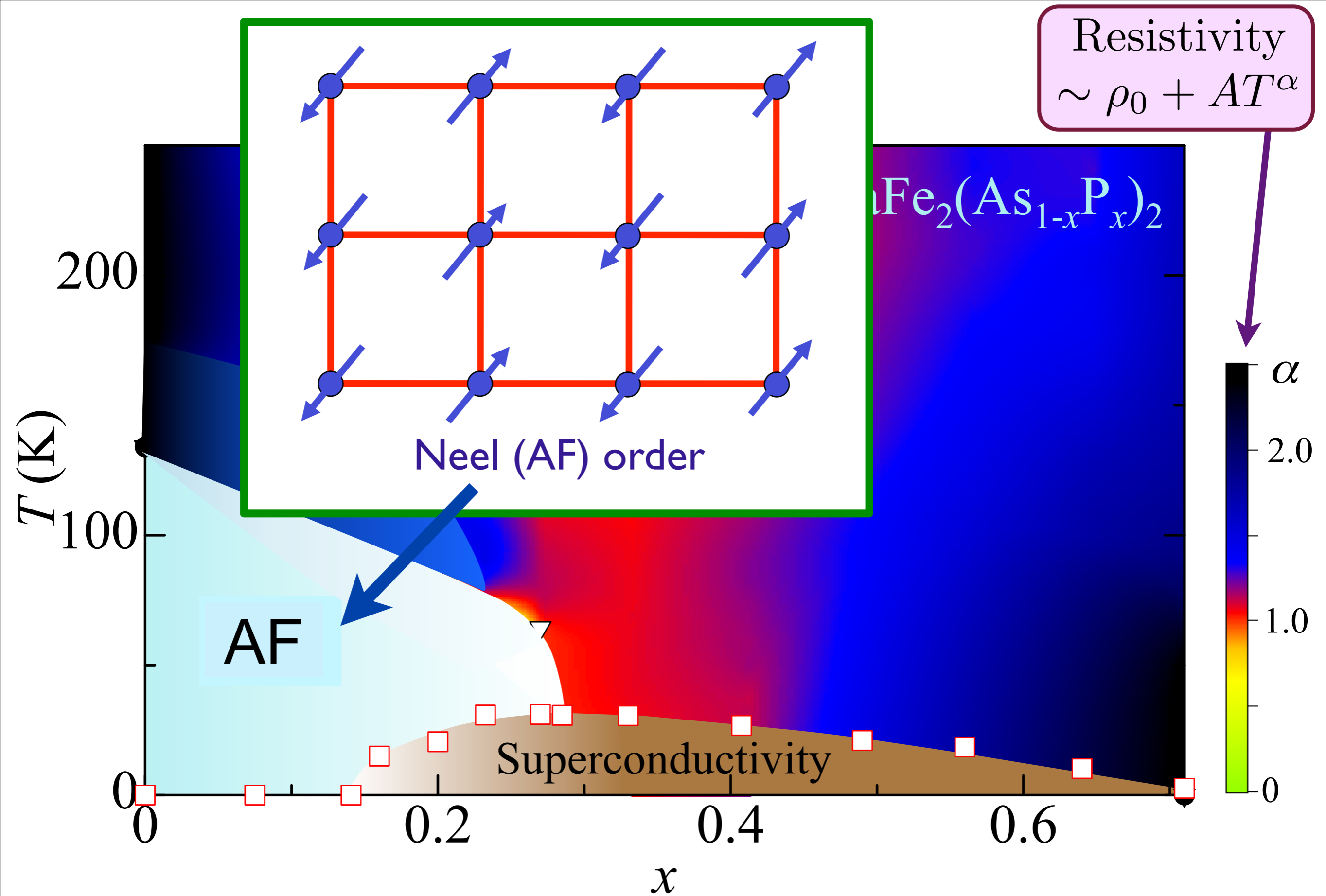
a new class of high temperature superconductors



Resistivity
 $\sim \rho_0 + AT^\alpha$

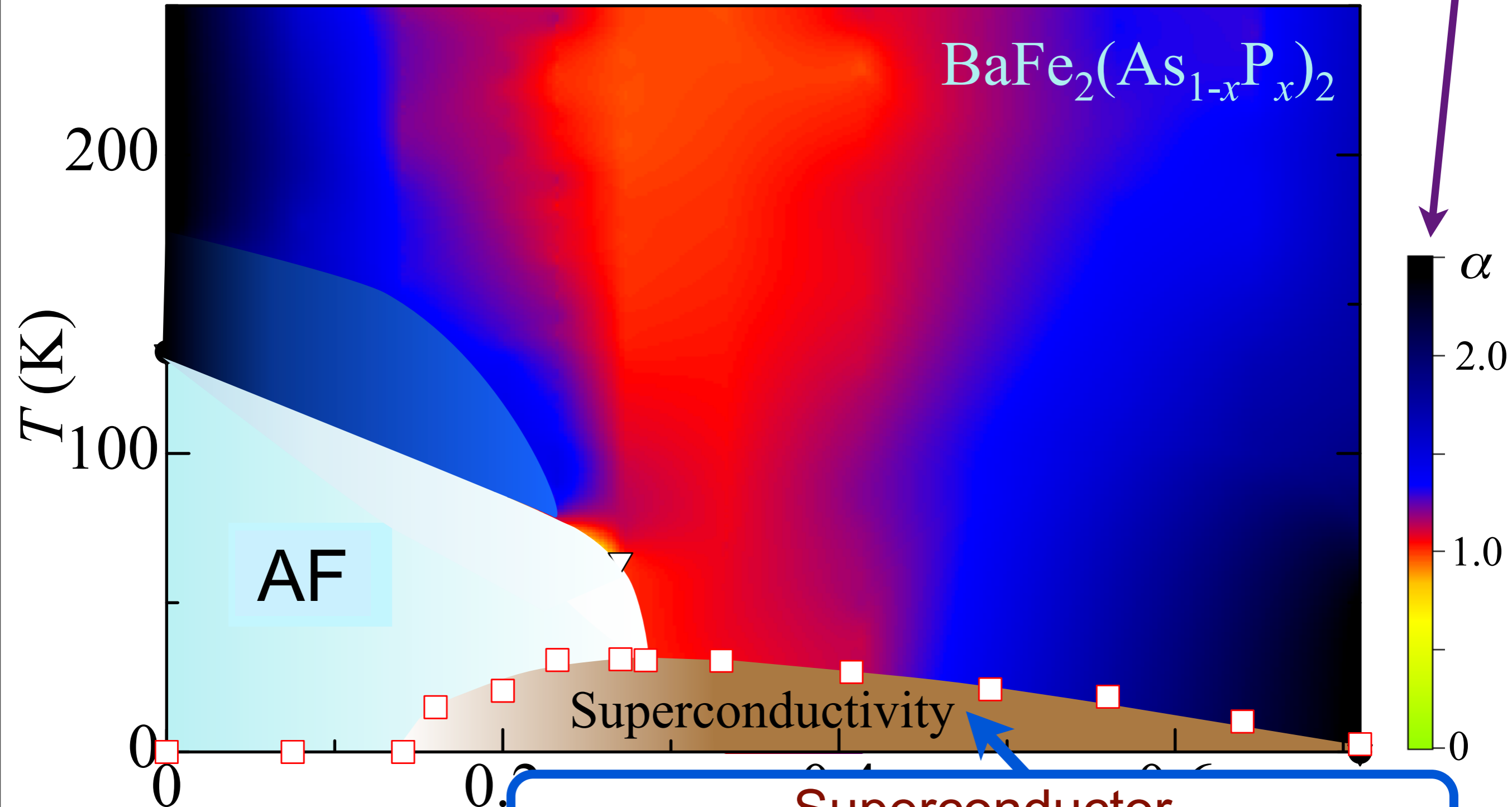


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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 H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
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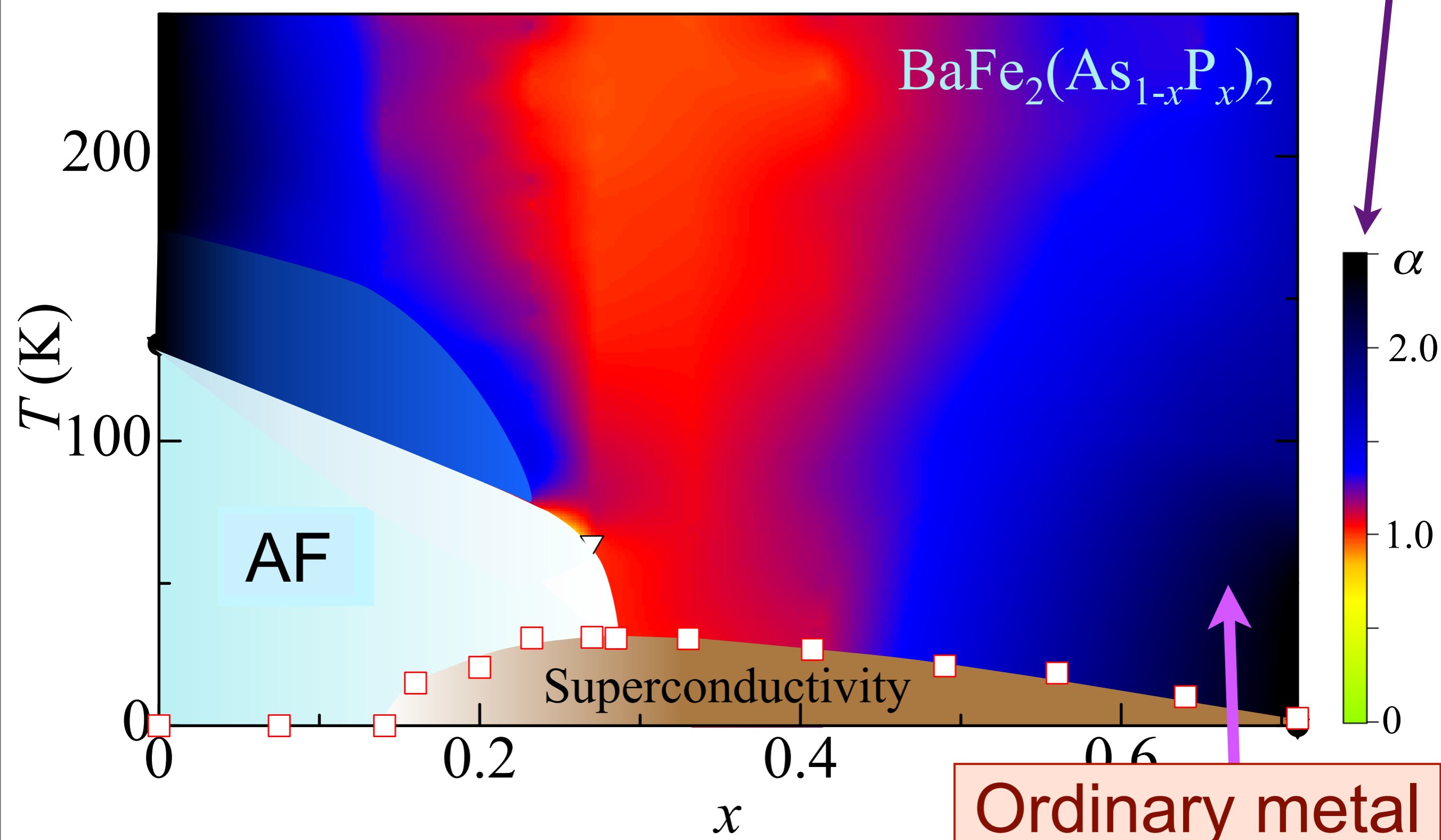
Resistivity
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Superconductor
Bose condensate of pairs of electrons

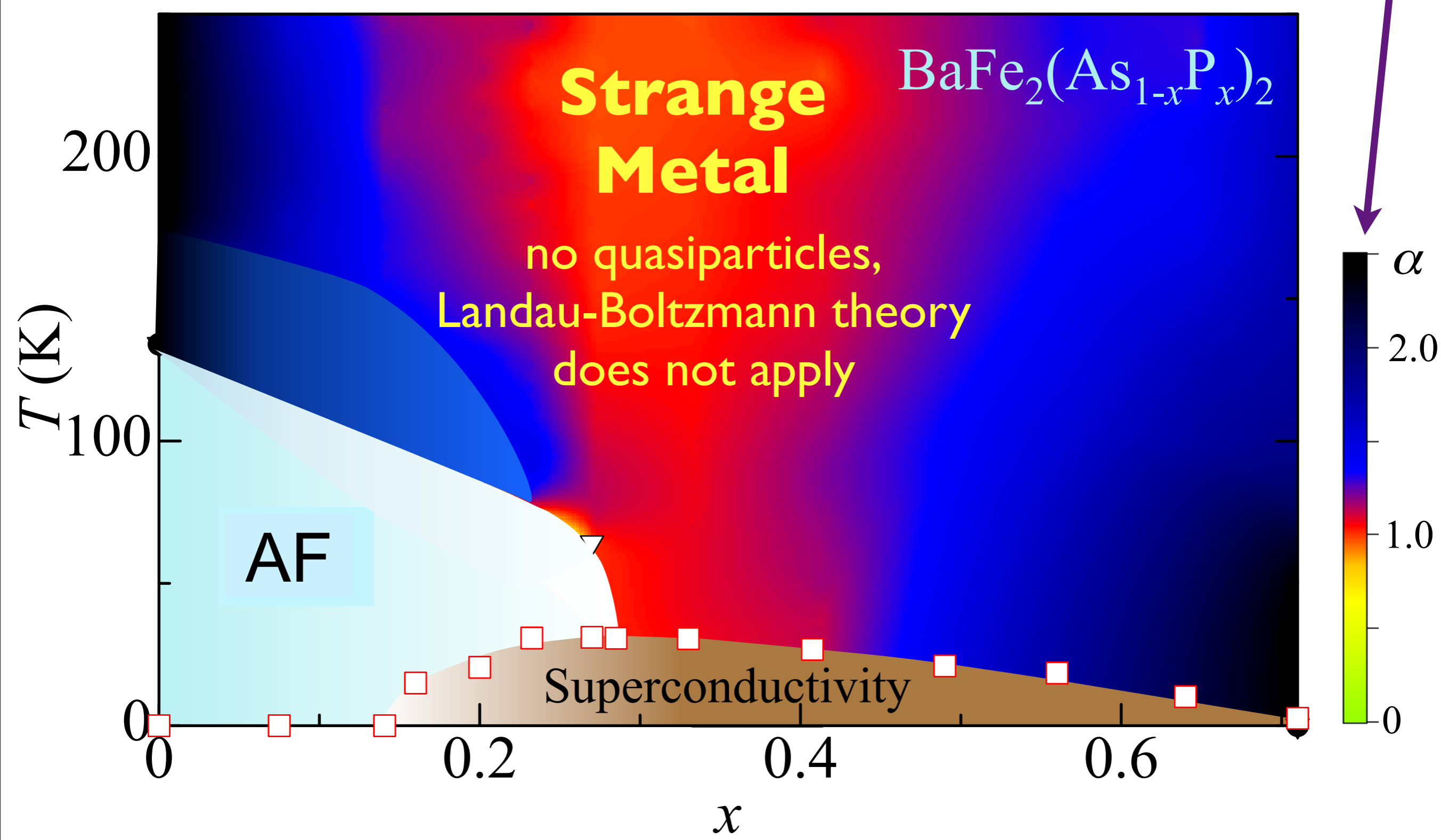
S. Kasahara, T. Shiba
H. Ikeda, H. Iakeya, K. Hirata, I. Terashima, and Y. Matsuda,
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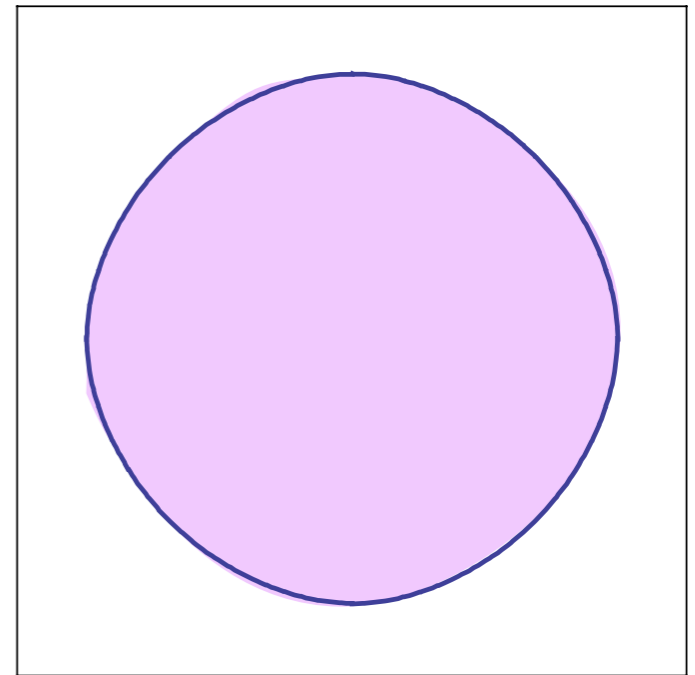
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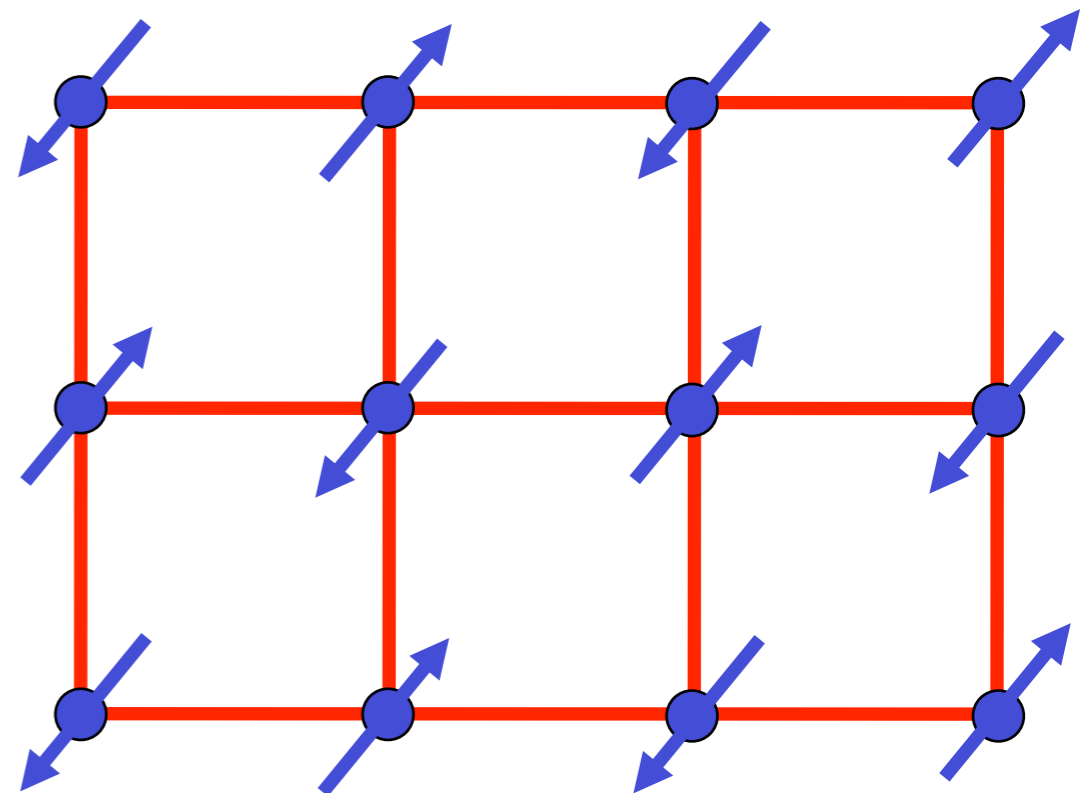
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Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



+



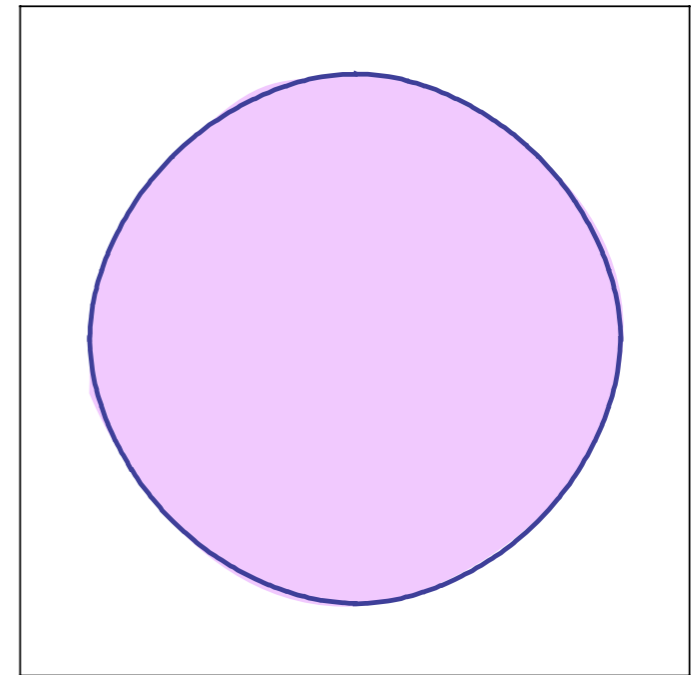
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

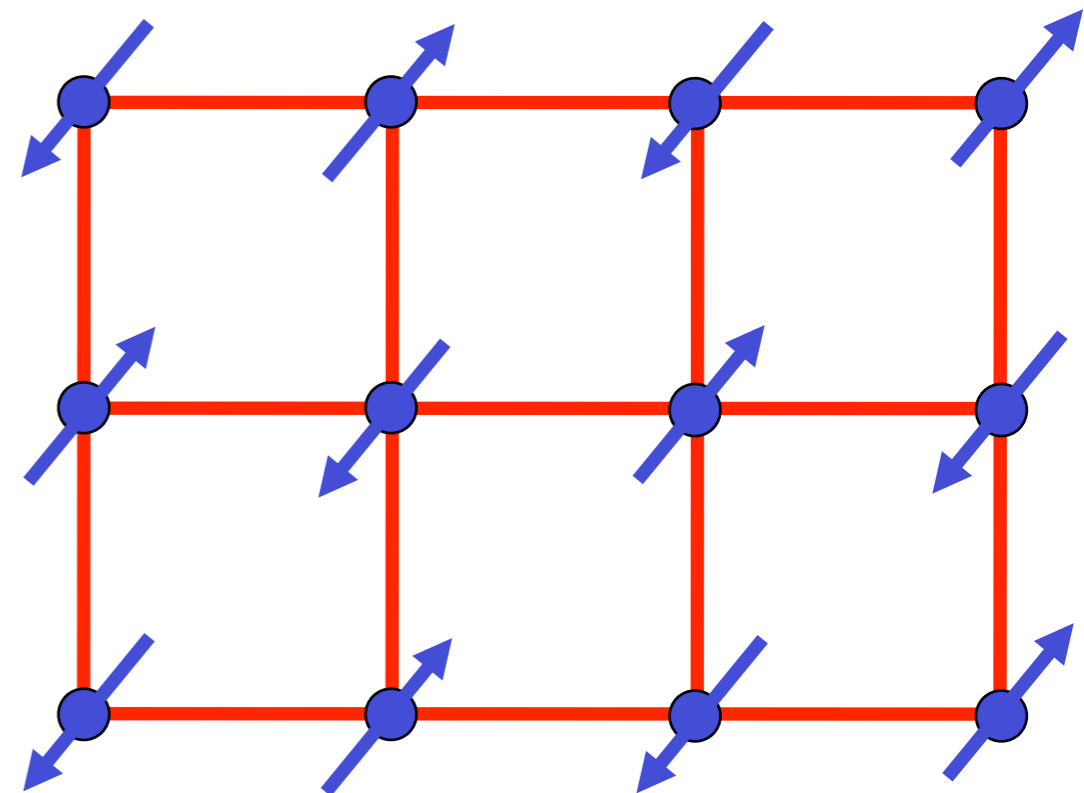
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism

$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$



+



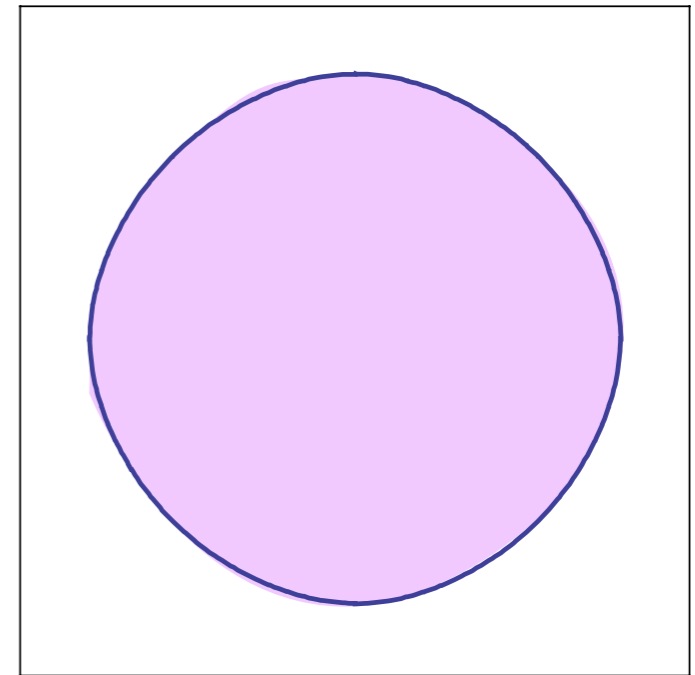
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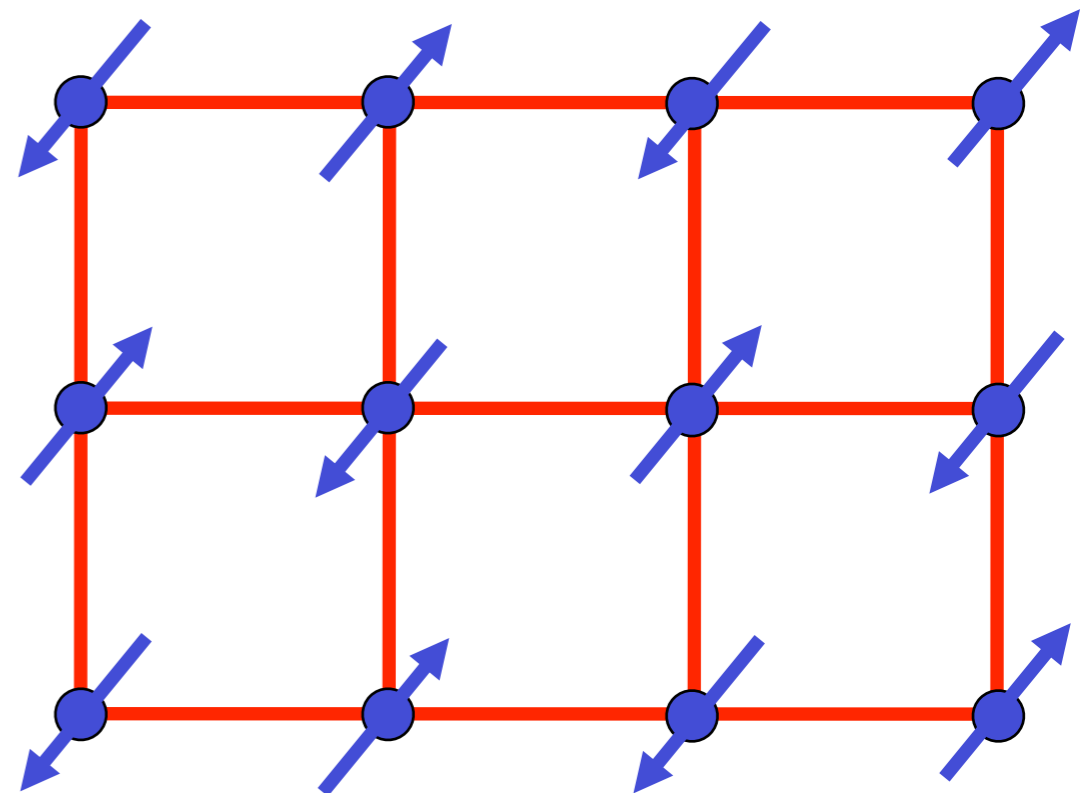
$$H_0 = - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$



+

$$H_{\text{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}, \beta}$$

where $\vec{\sigma}$ are the Pauli matrices.



Fermi surface+antiferromagnetism

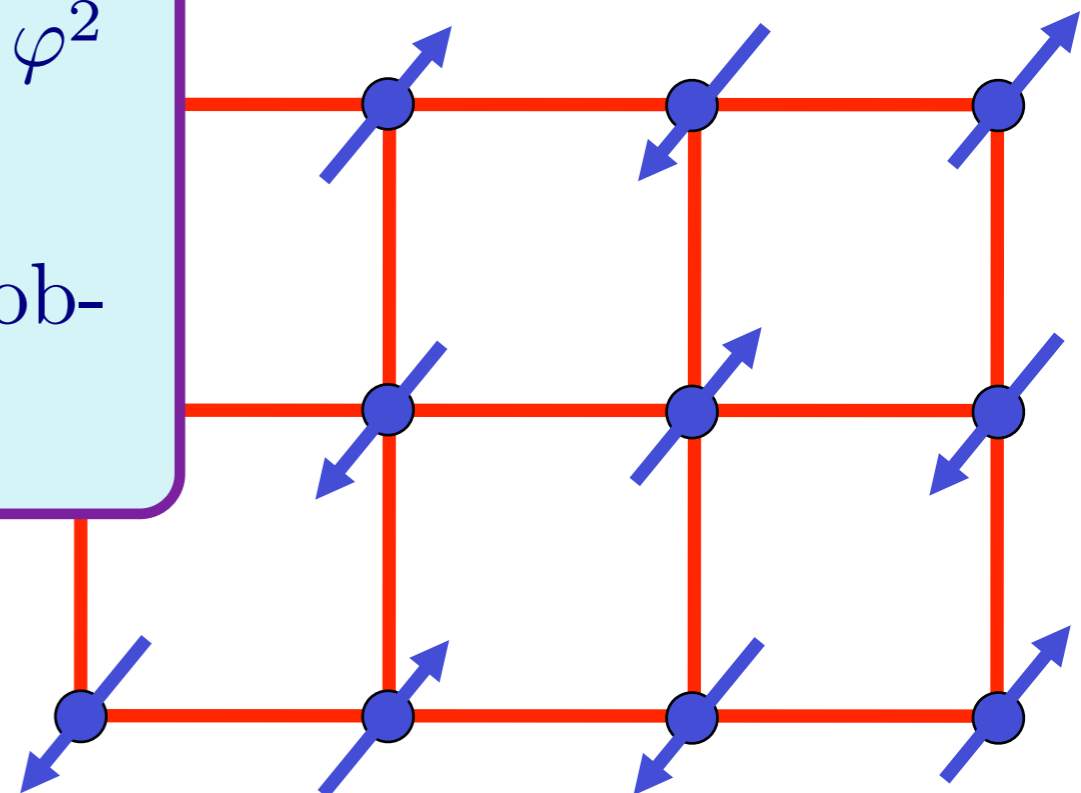
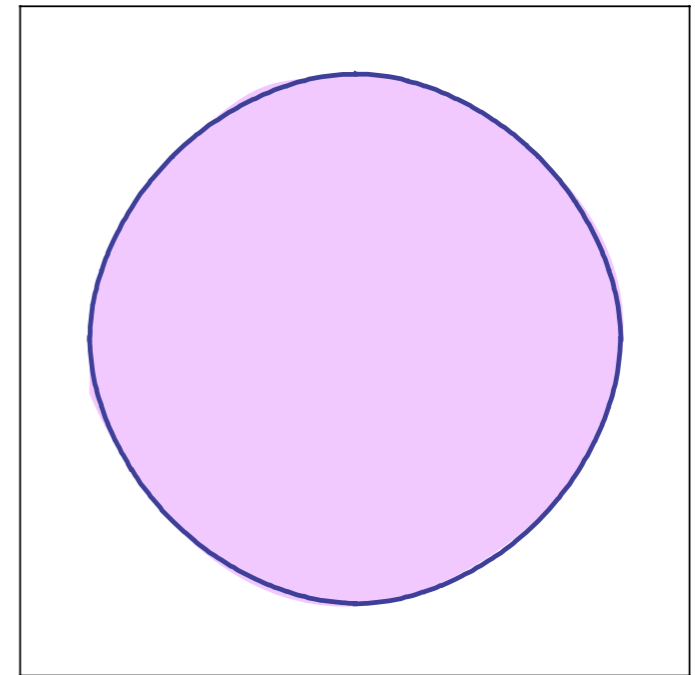
$$H_0 = - \sum_{i < j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The electron dispersions obtained by diagonalizing $H_0 + H_{\text{sdw}}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

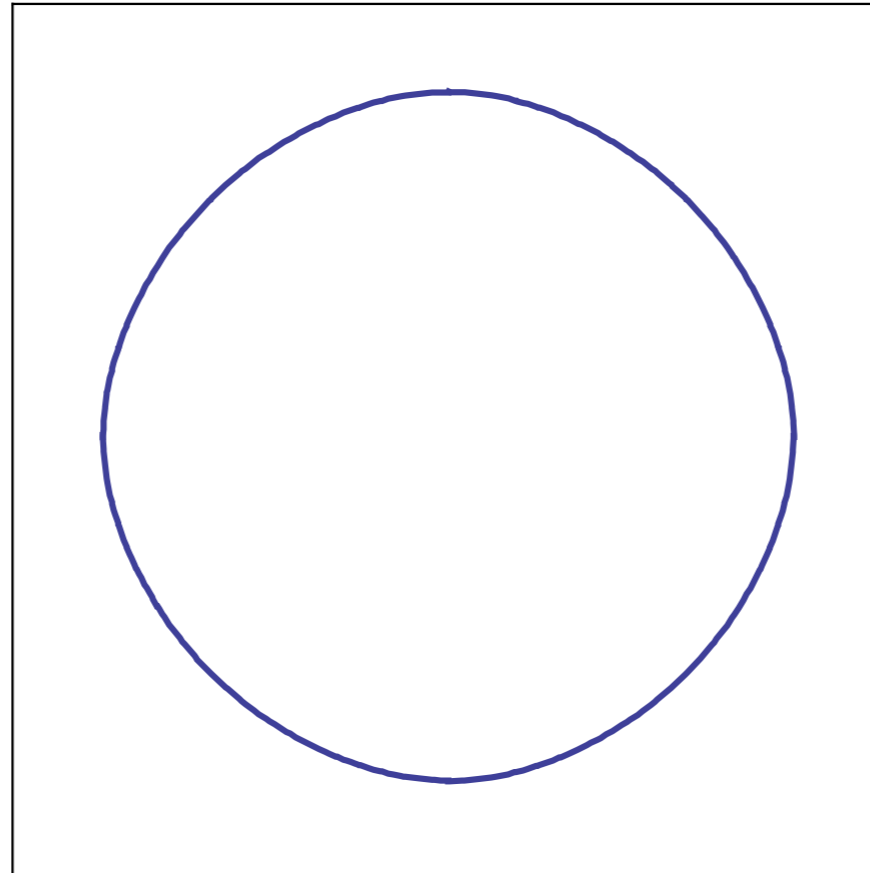
$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right)^2 + \varphi^2}$$

We now fill the lowest energy states to obtain the new Fermi surface

where $\vec{\sigma}$ are the Pauli matrices.

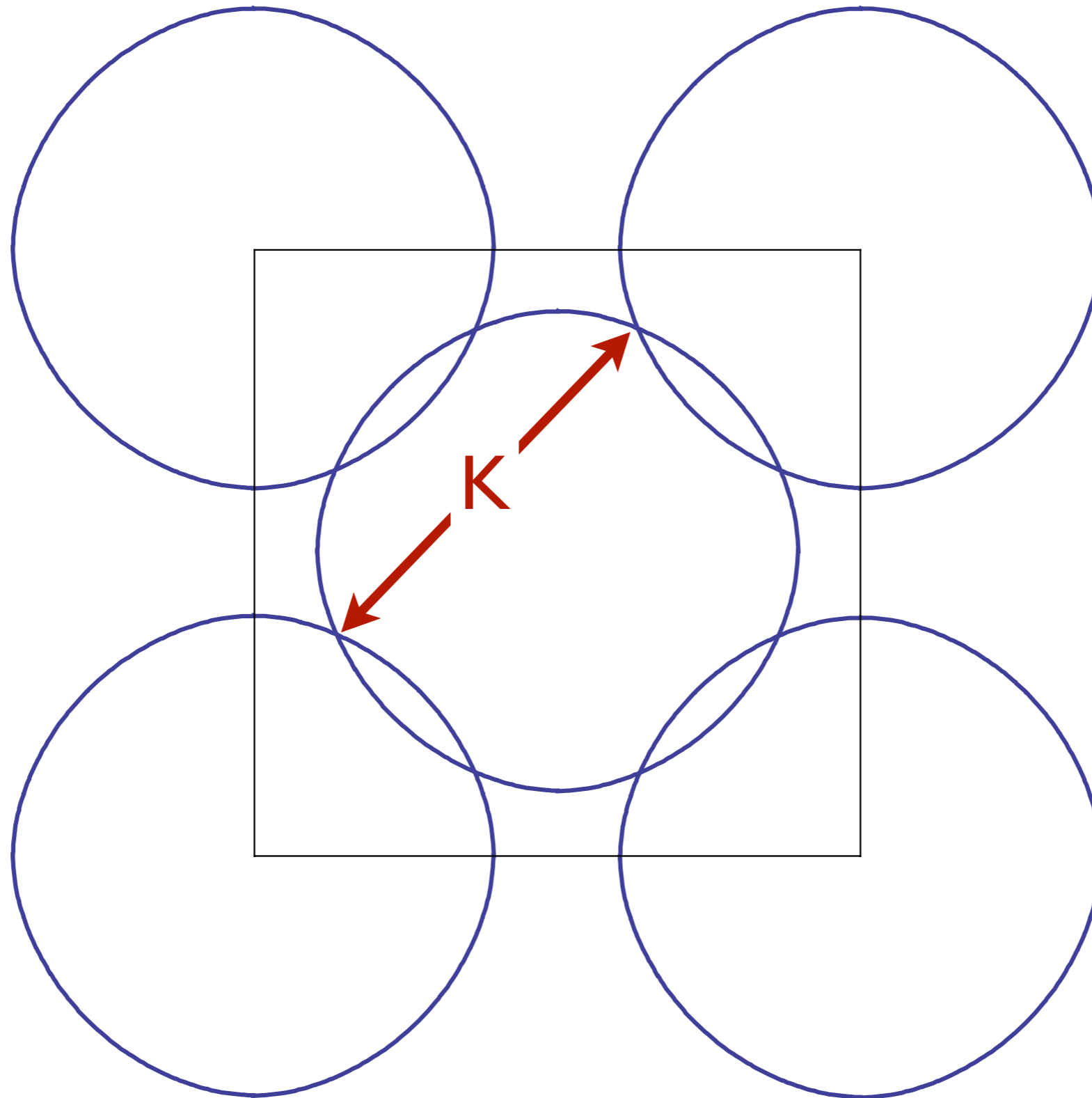


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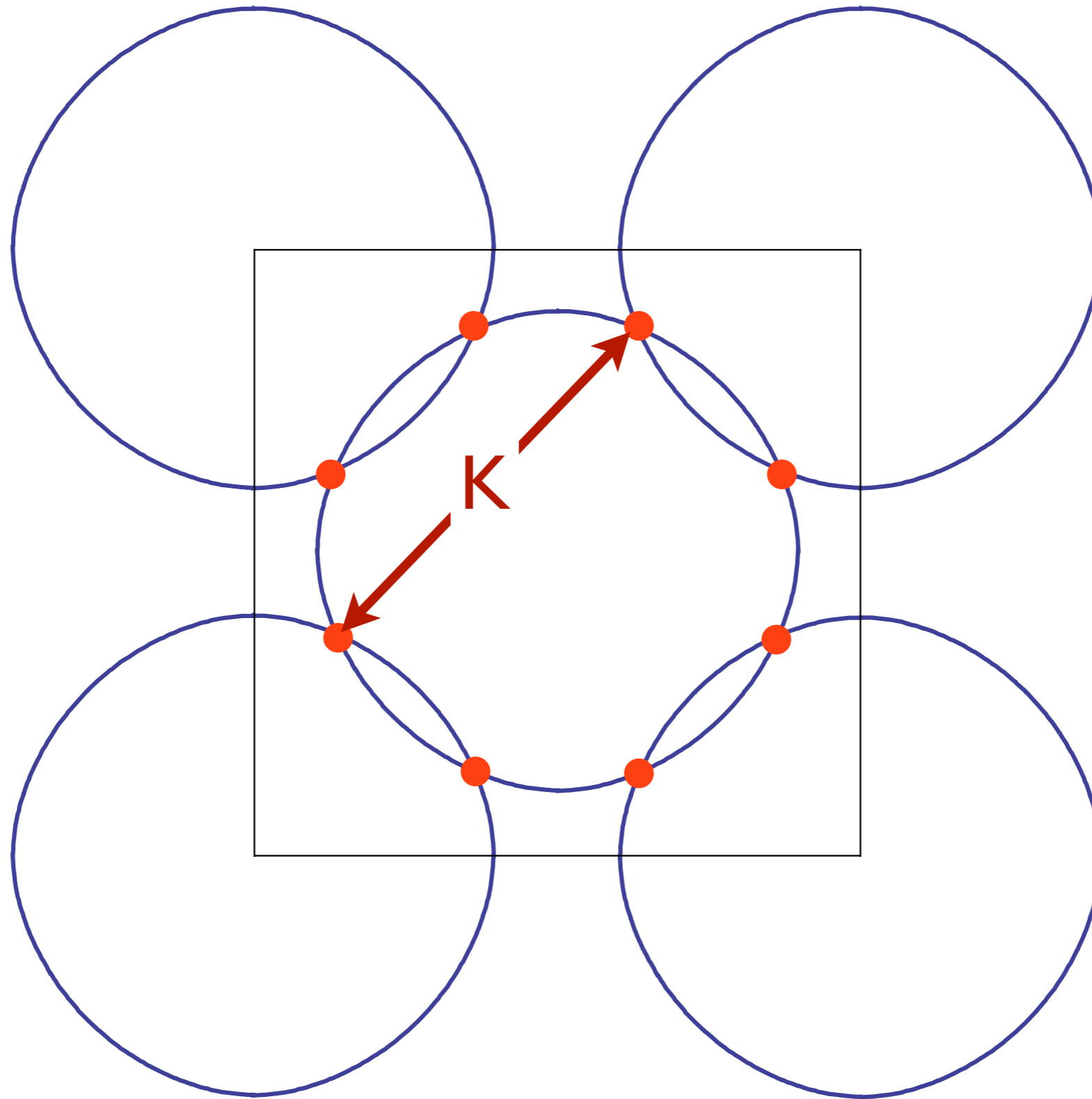
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



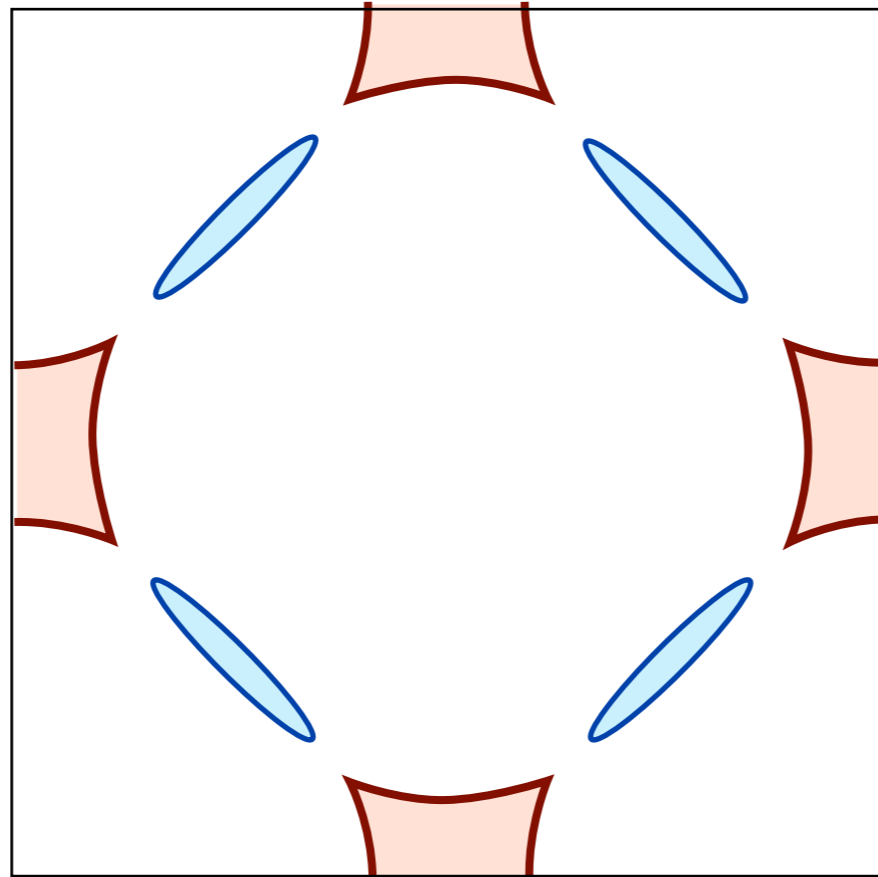
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



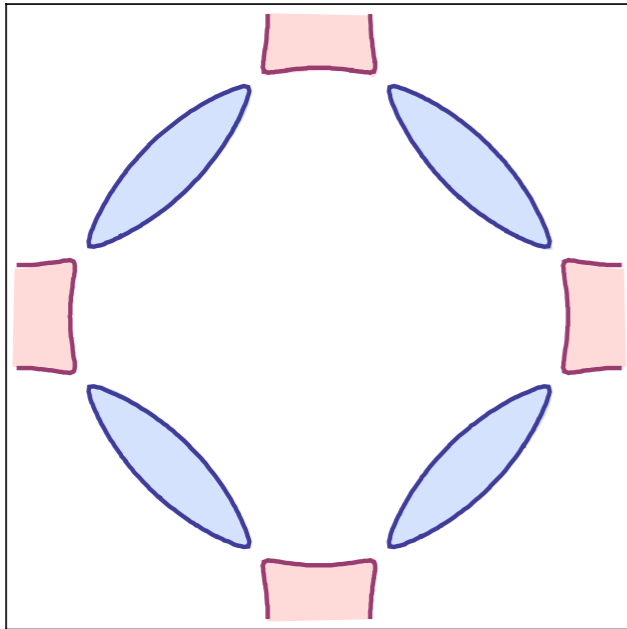
“Hot” spots

Fermi surface+antiferromagnetism



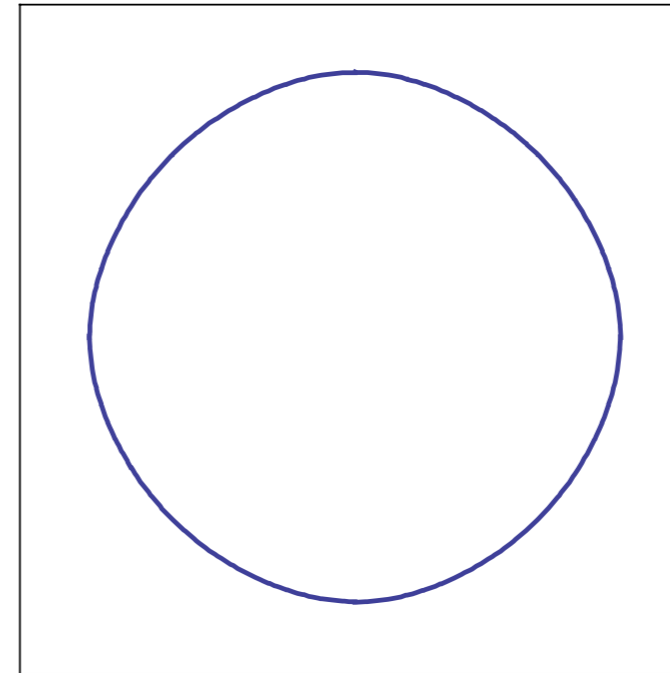
Electron and hole pockets in
antiferromagnetic phase
with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

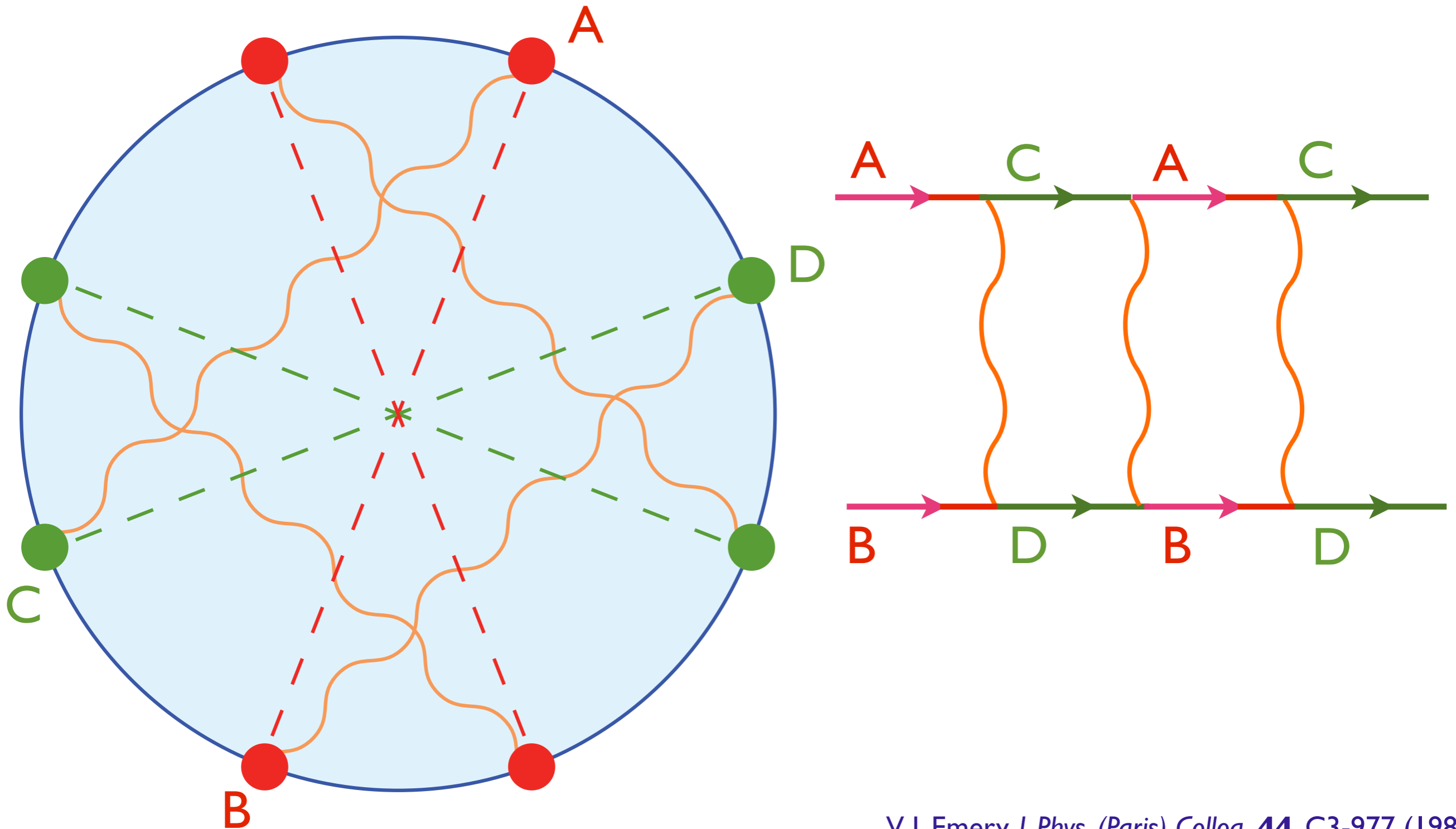


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

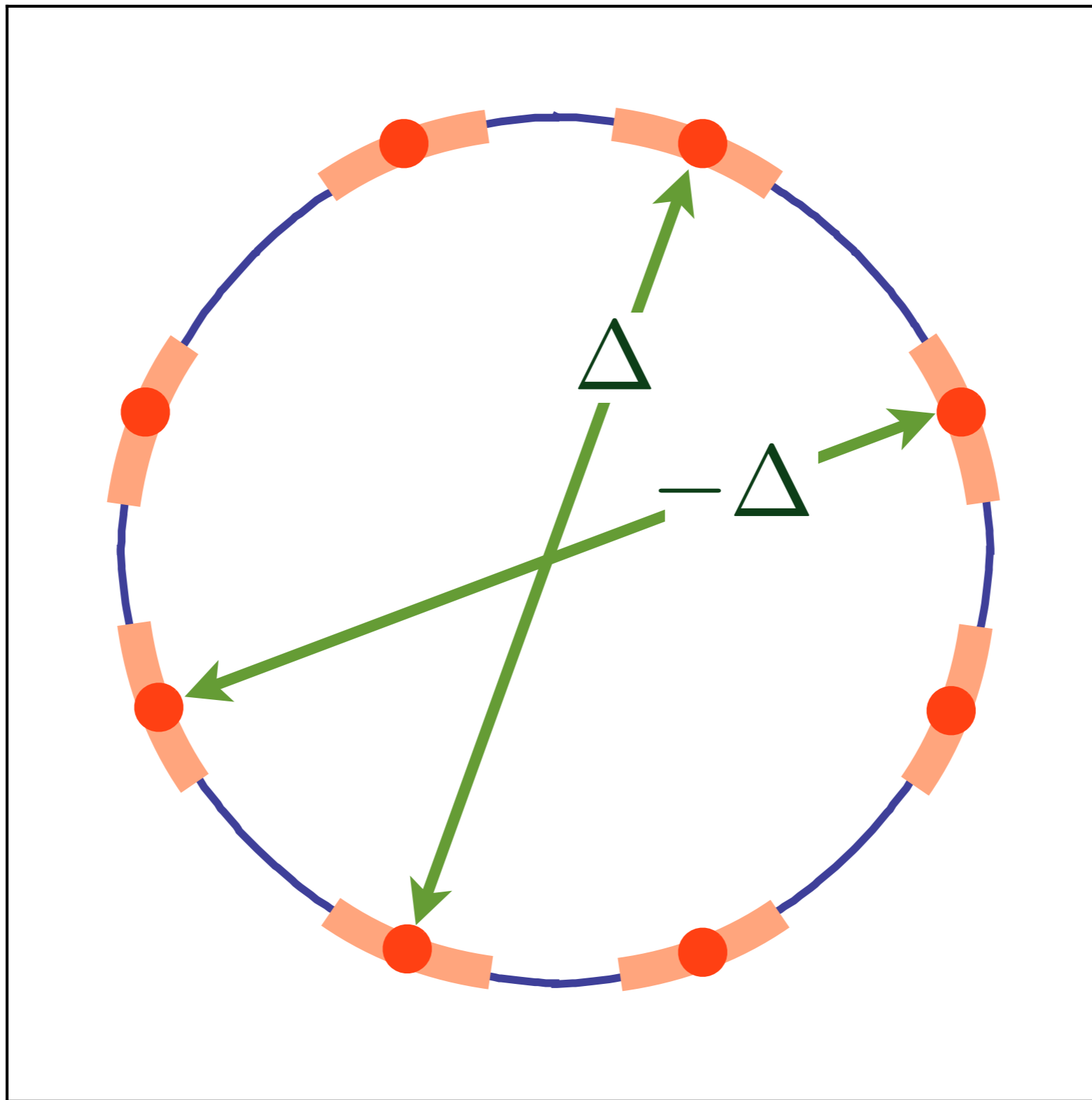
r

Pairing “glue” from antiferromagnetic fluctuations



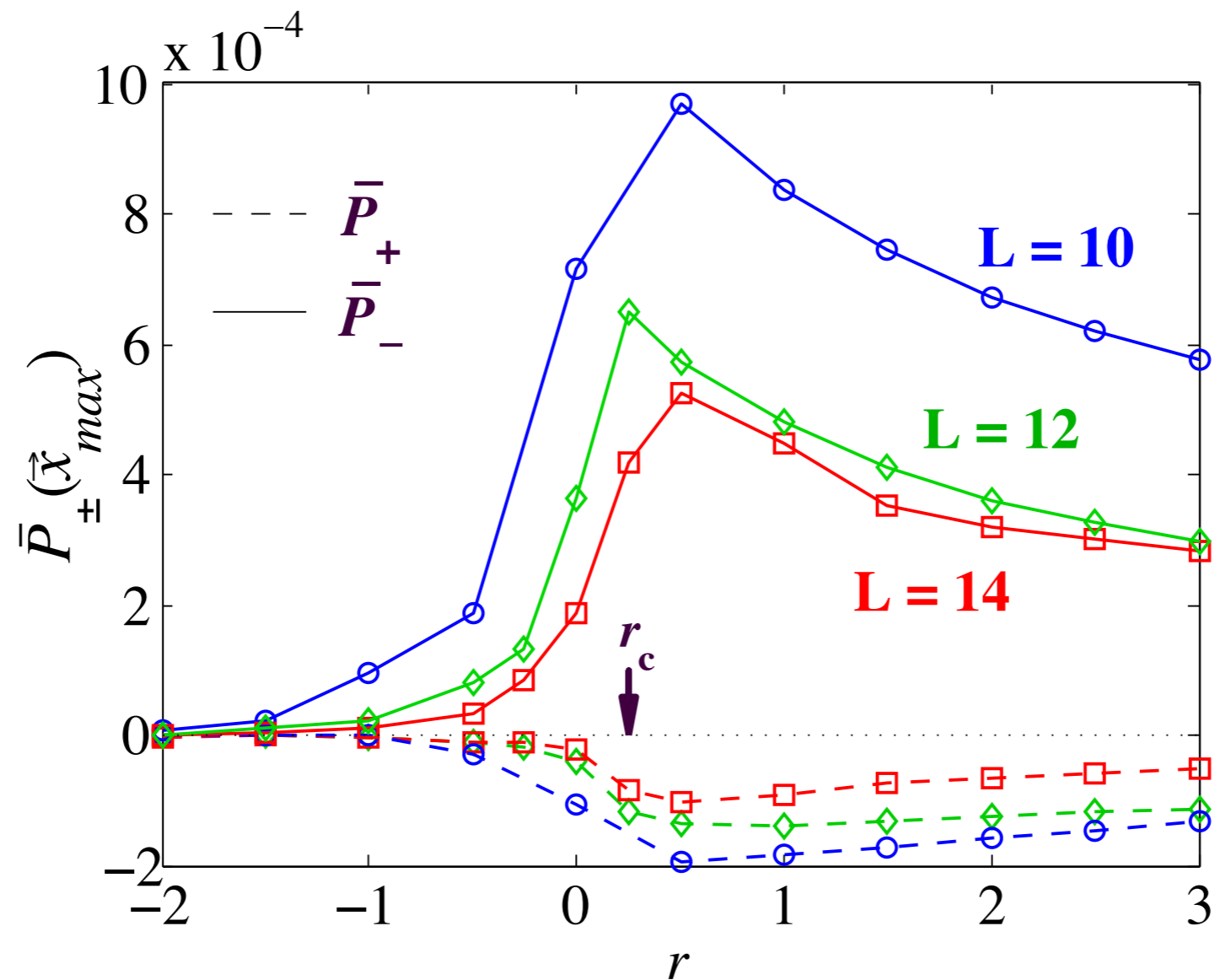
V.J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at and near hot spots

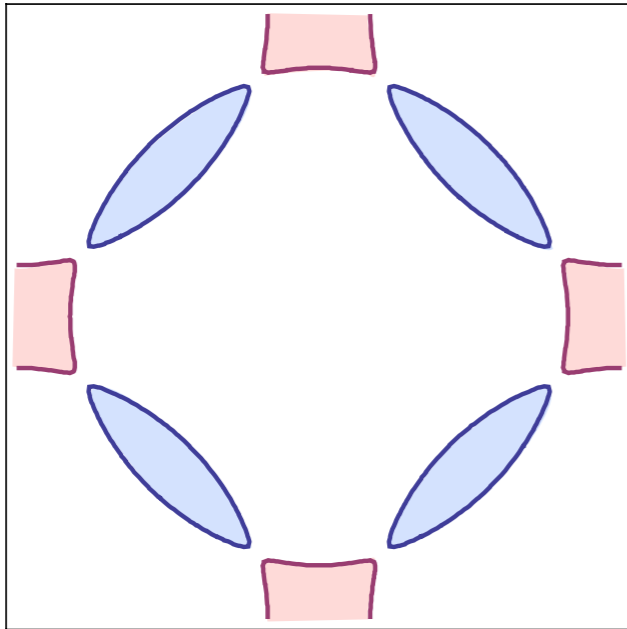
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s/d pairing amplitudes P_{+}/P_{-}
as a function of the tuning parameter r

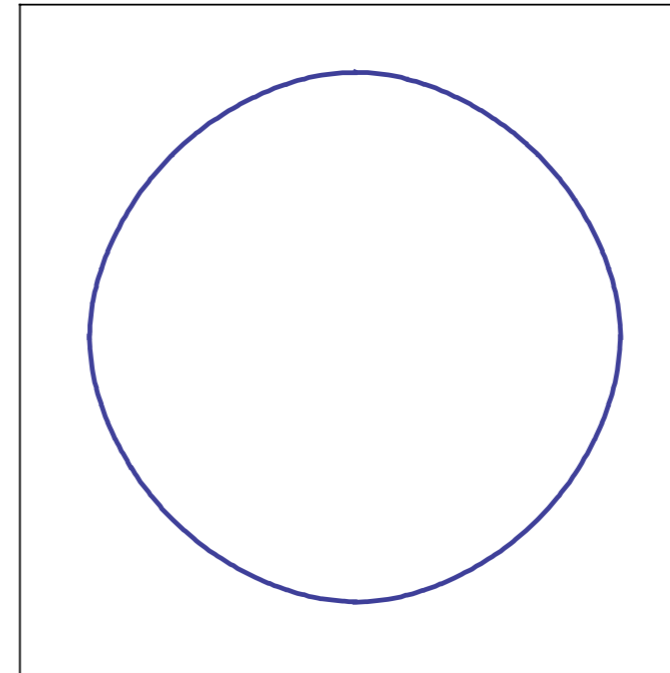
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

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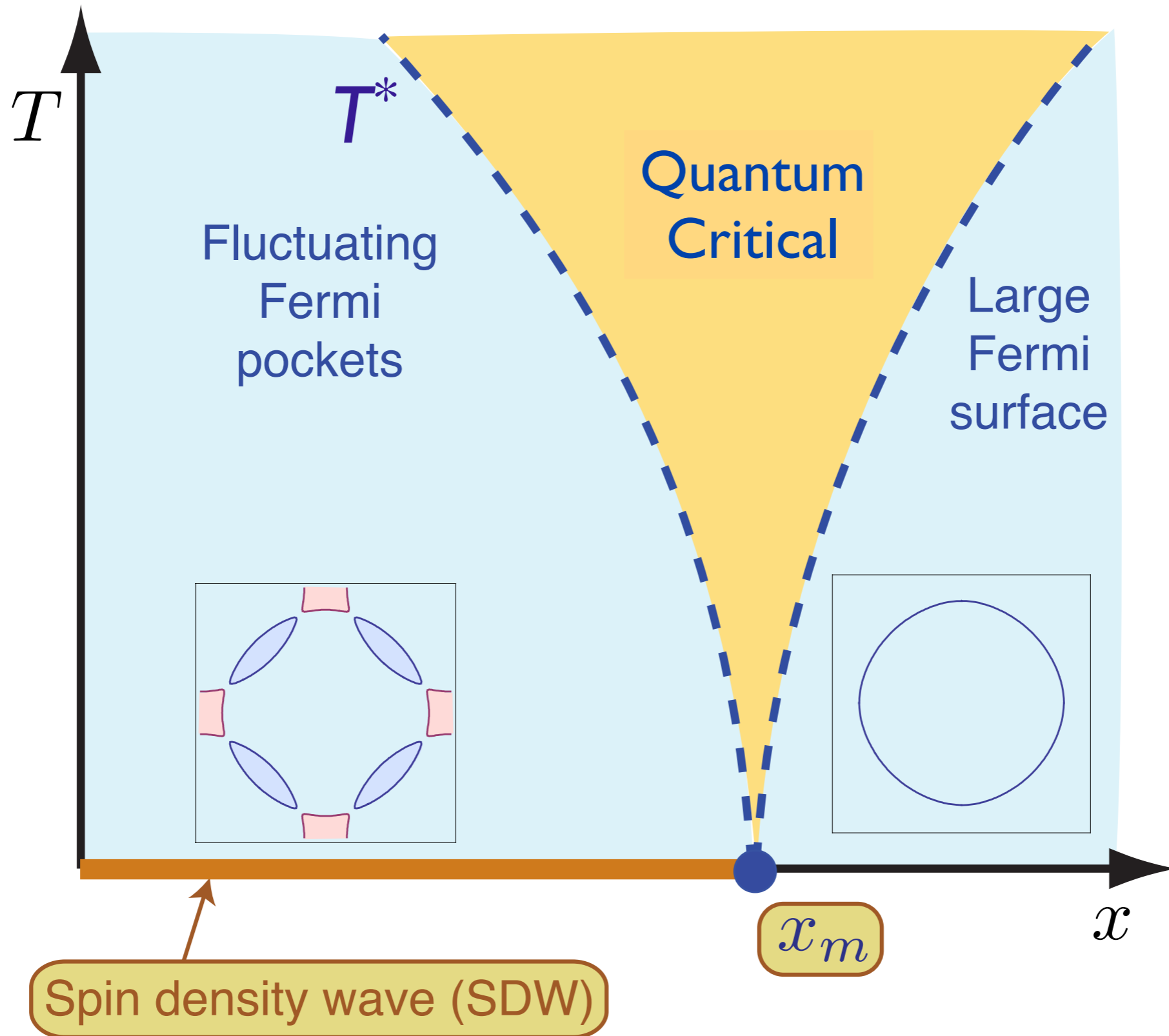


$$\langle \vec{\varphi} \rangle = 0$$

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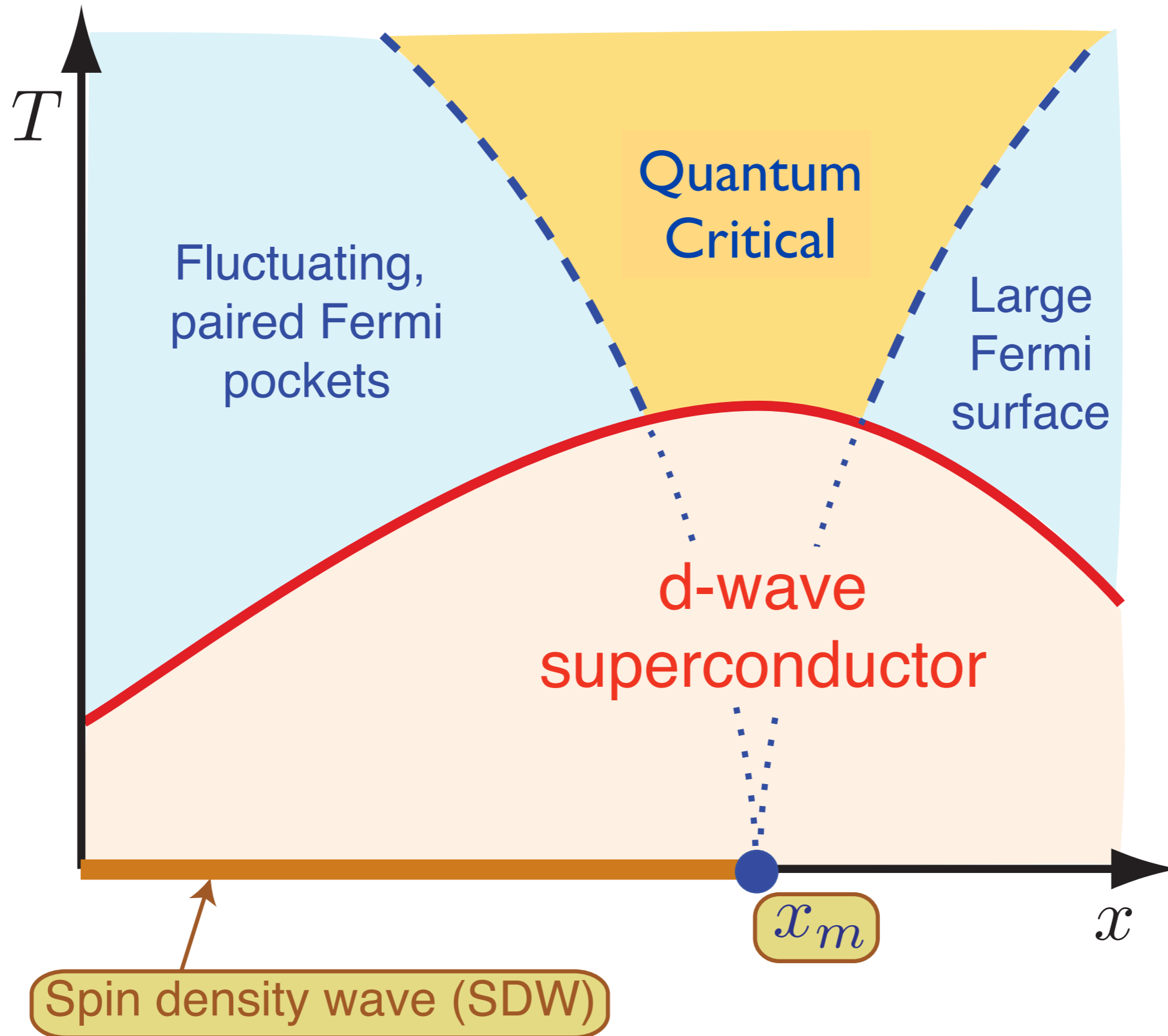
r

Fermi surface+antiferromagnetism



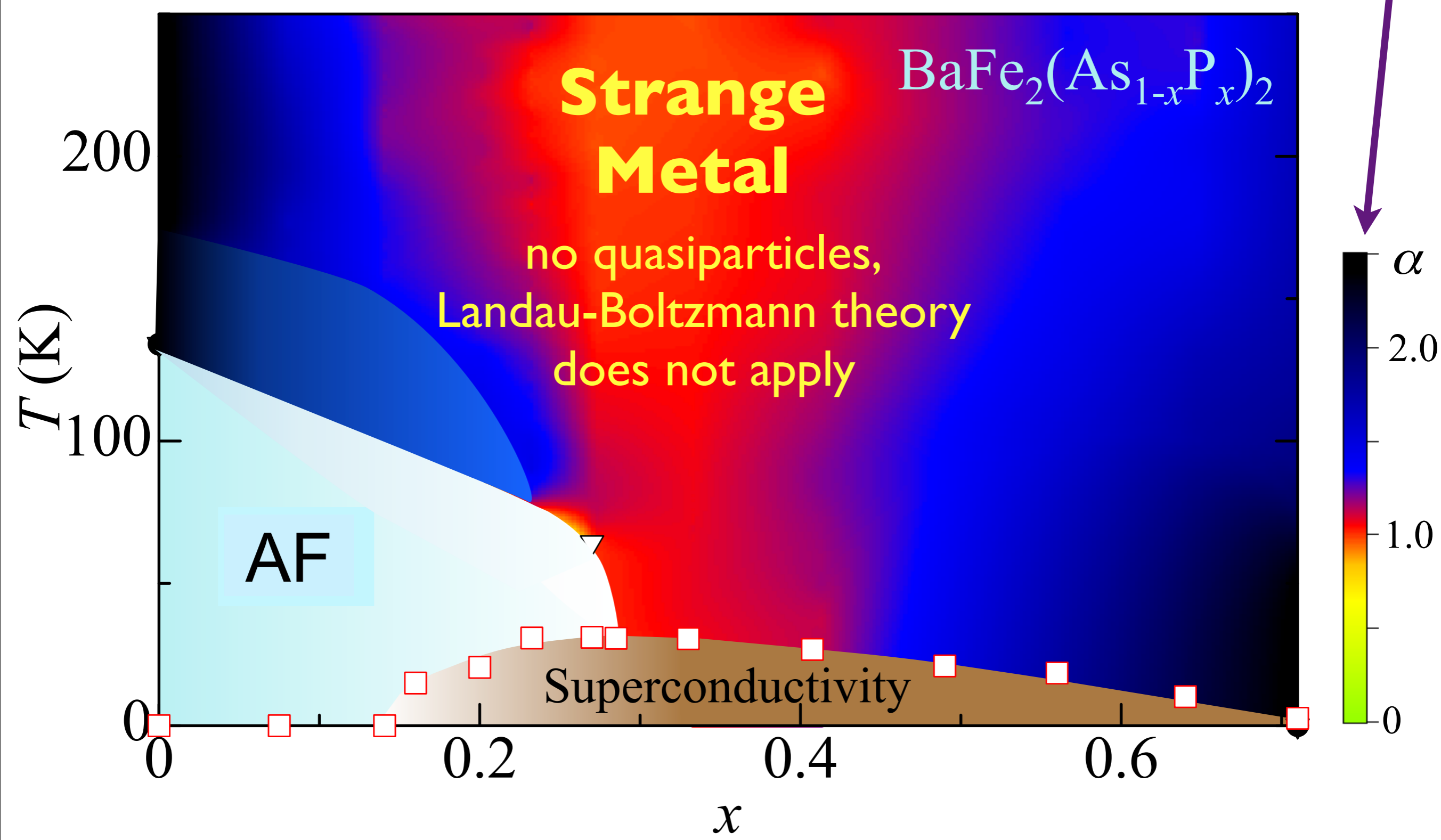
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
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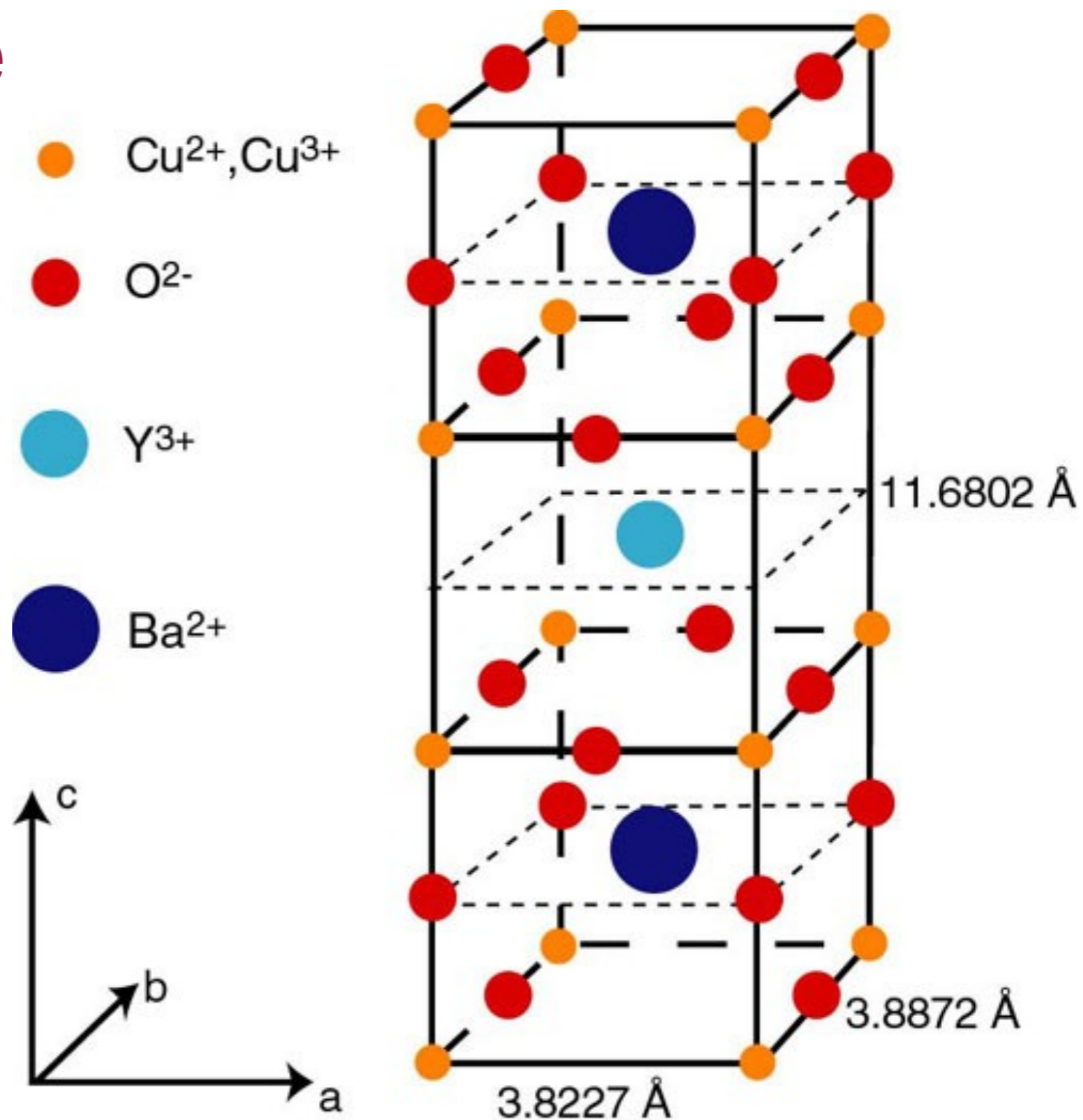
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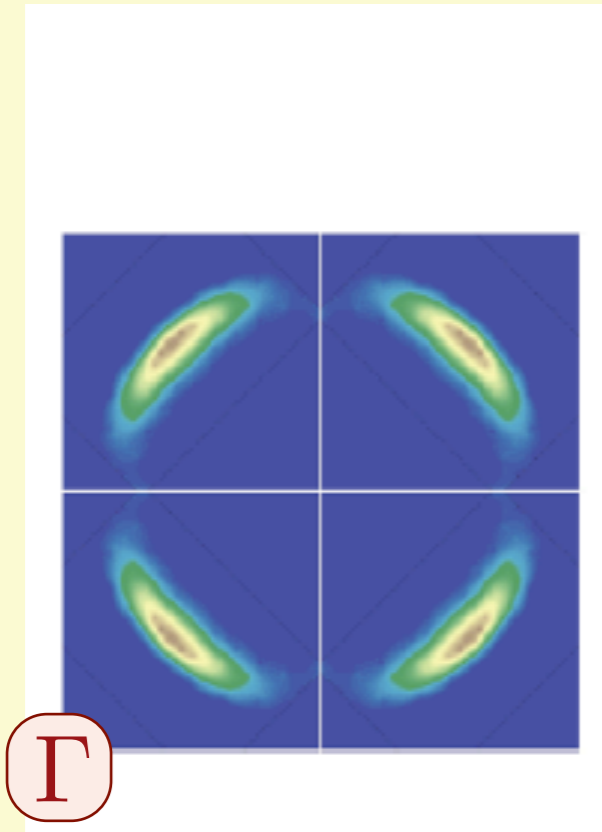
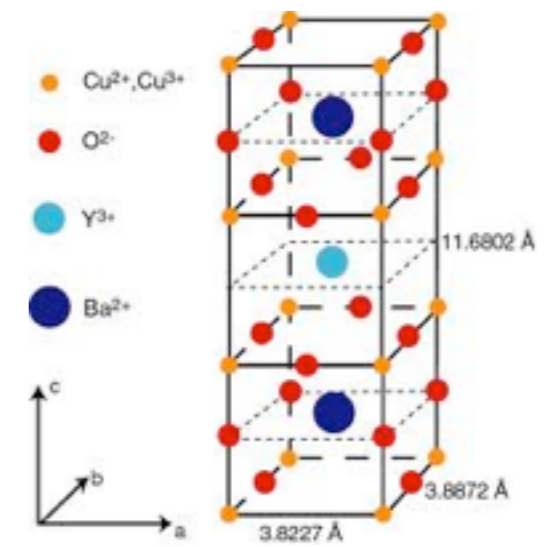
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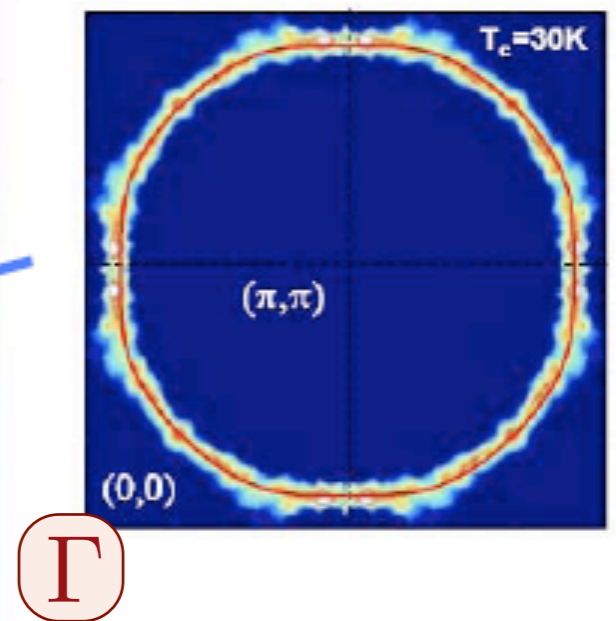
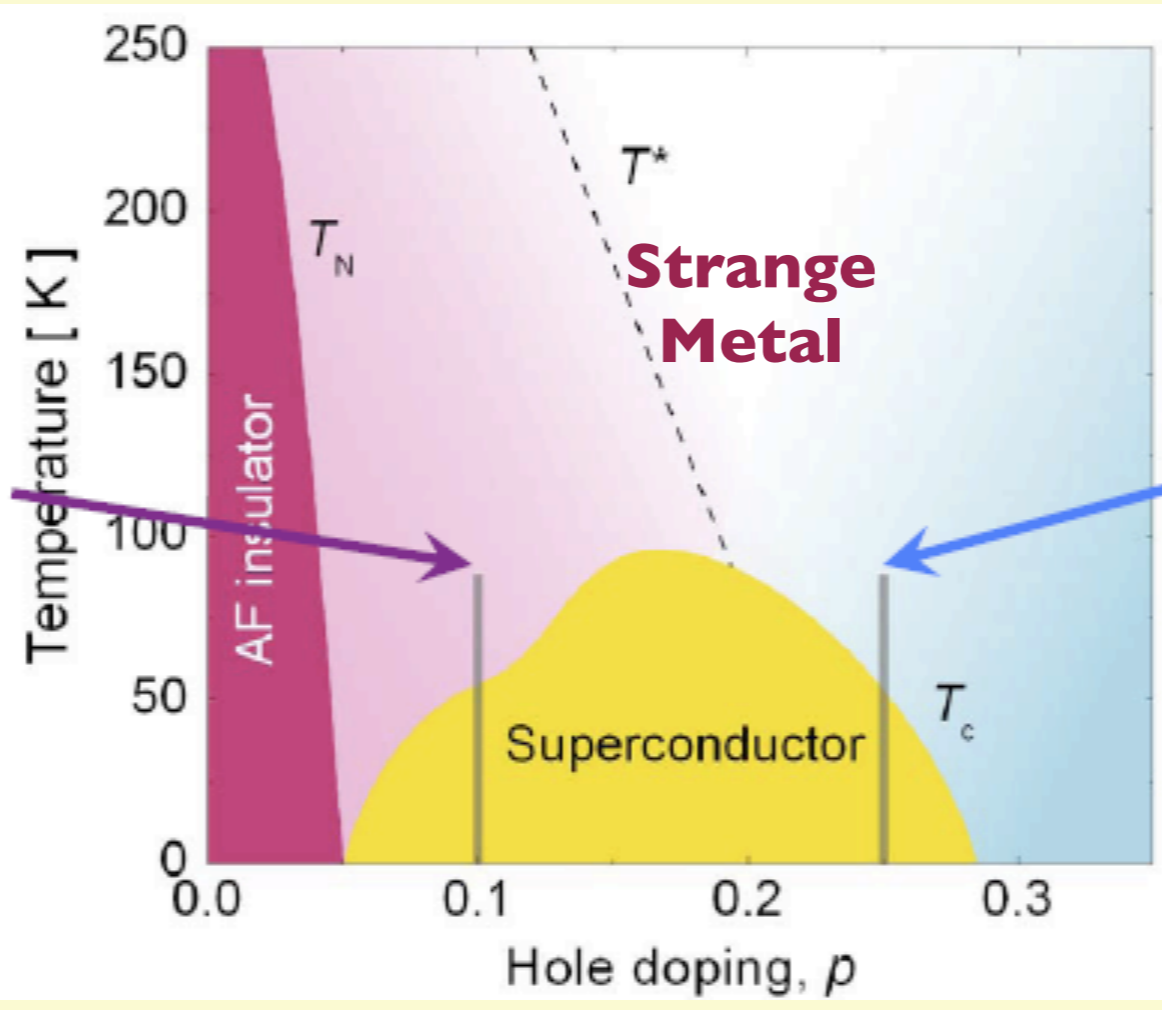
High temperature superconductors





K.M. Shen et al., Science 2005

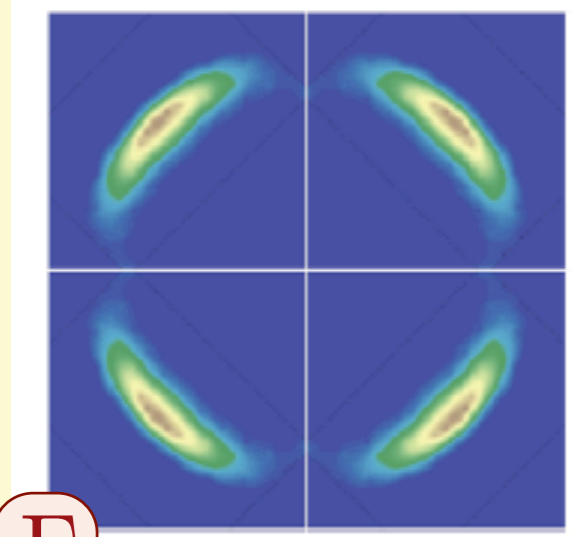
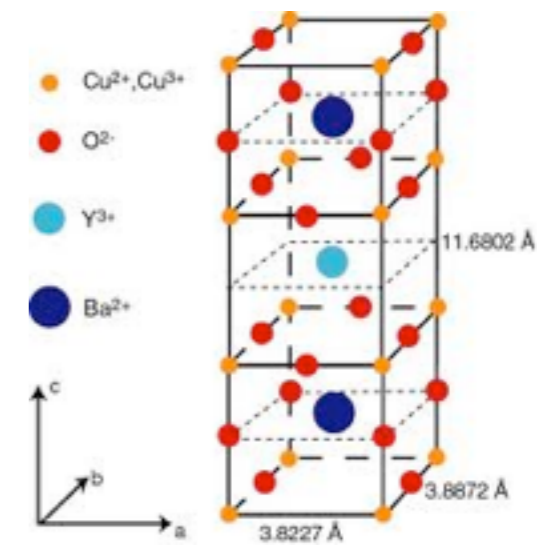
Smaller hole Fermi-pockets



M. Platé et al., PRL 2005

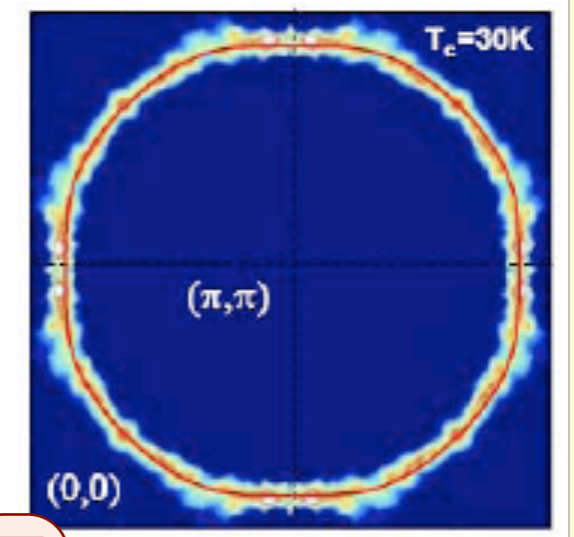
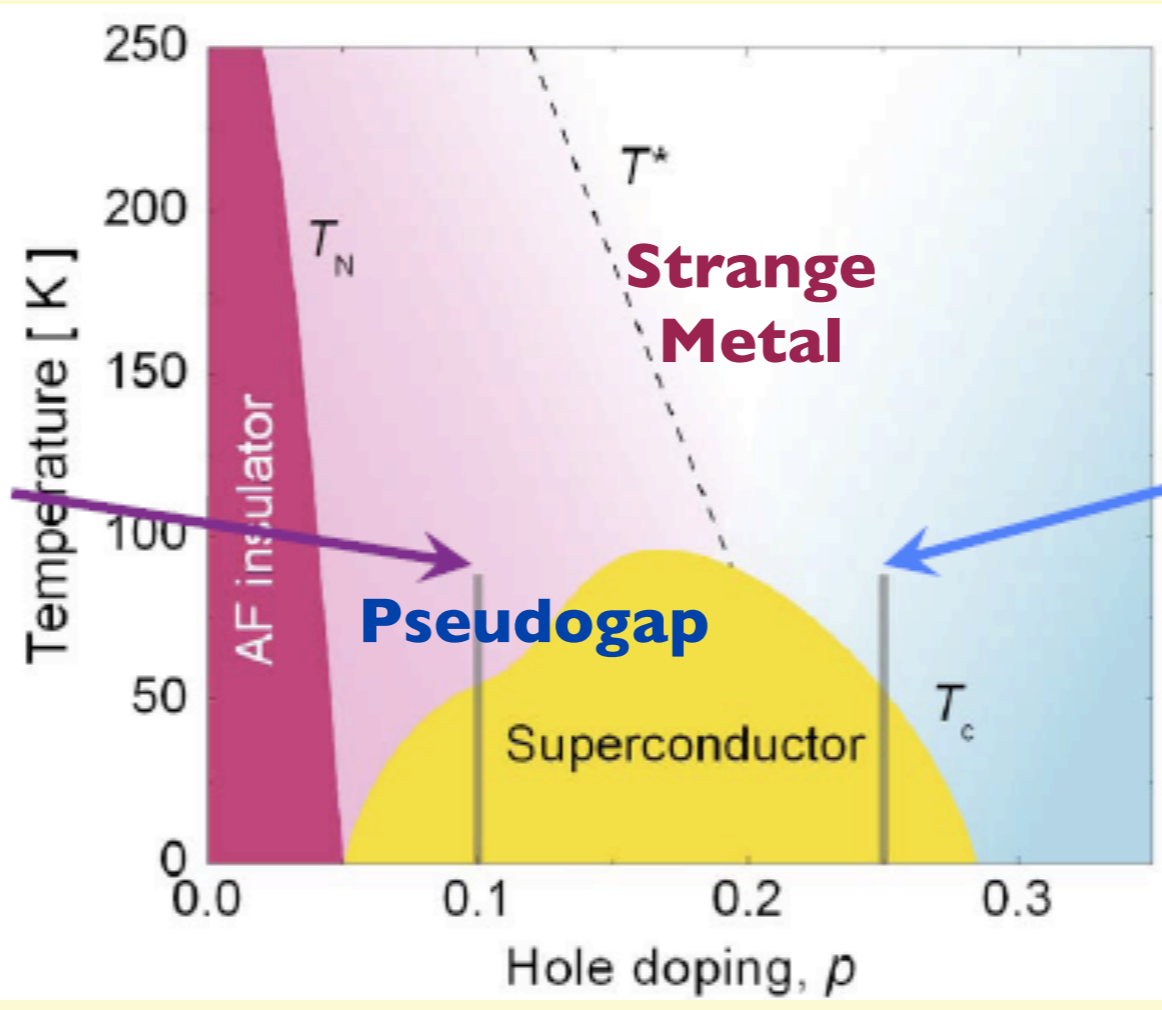
Large hole Fermi surface

What about the pseudogap ?



K.M. Shen et al., Science 2005

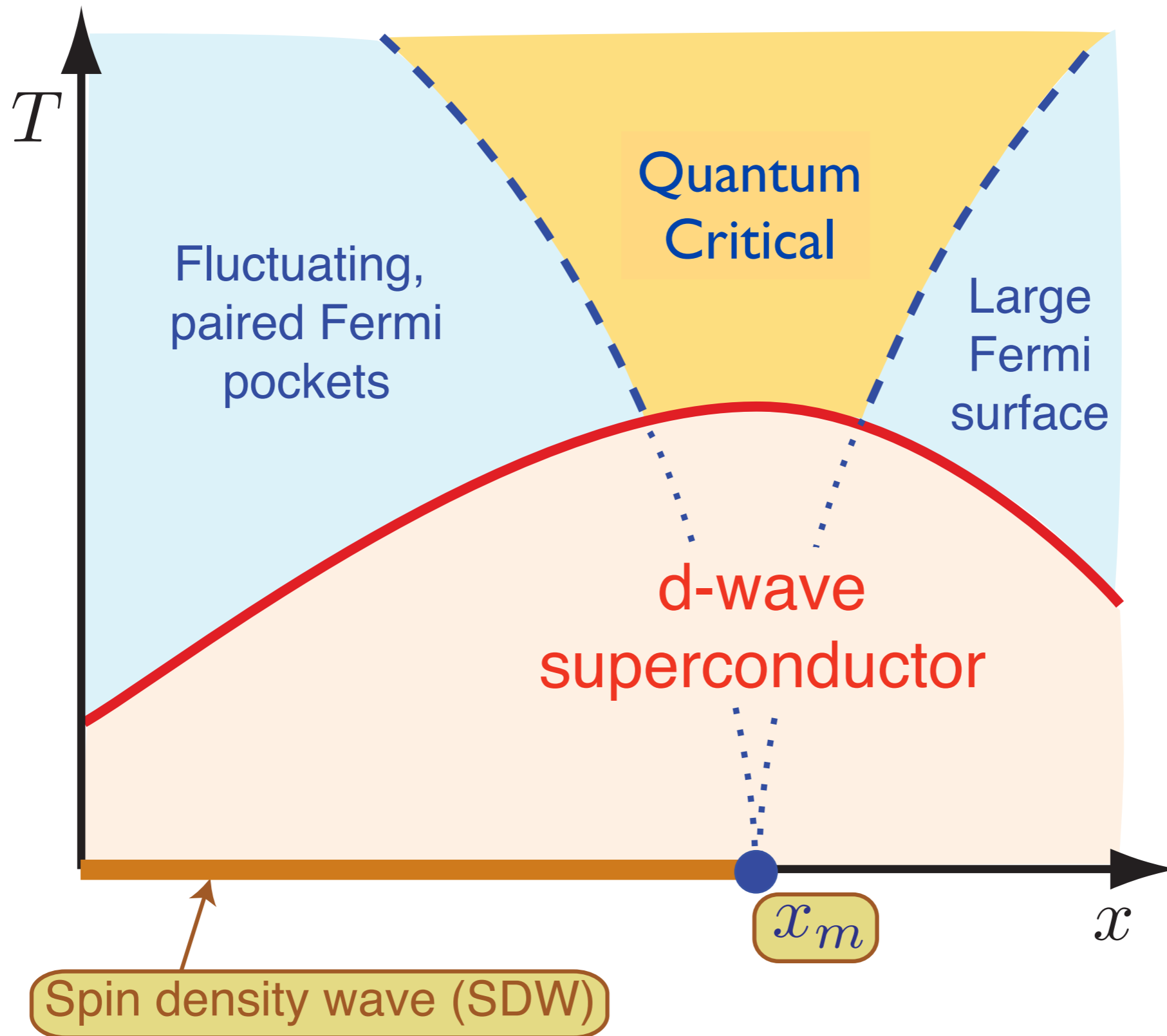
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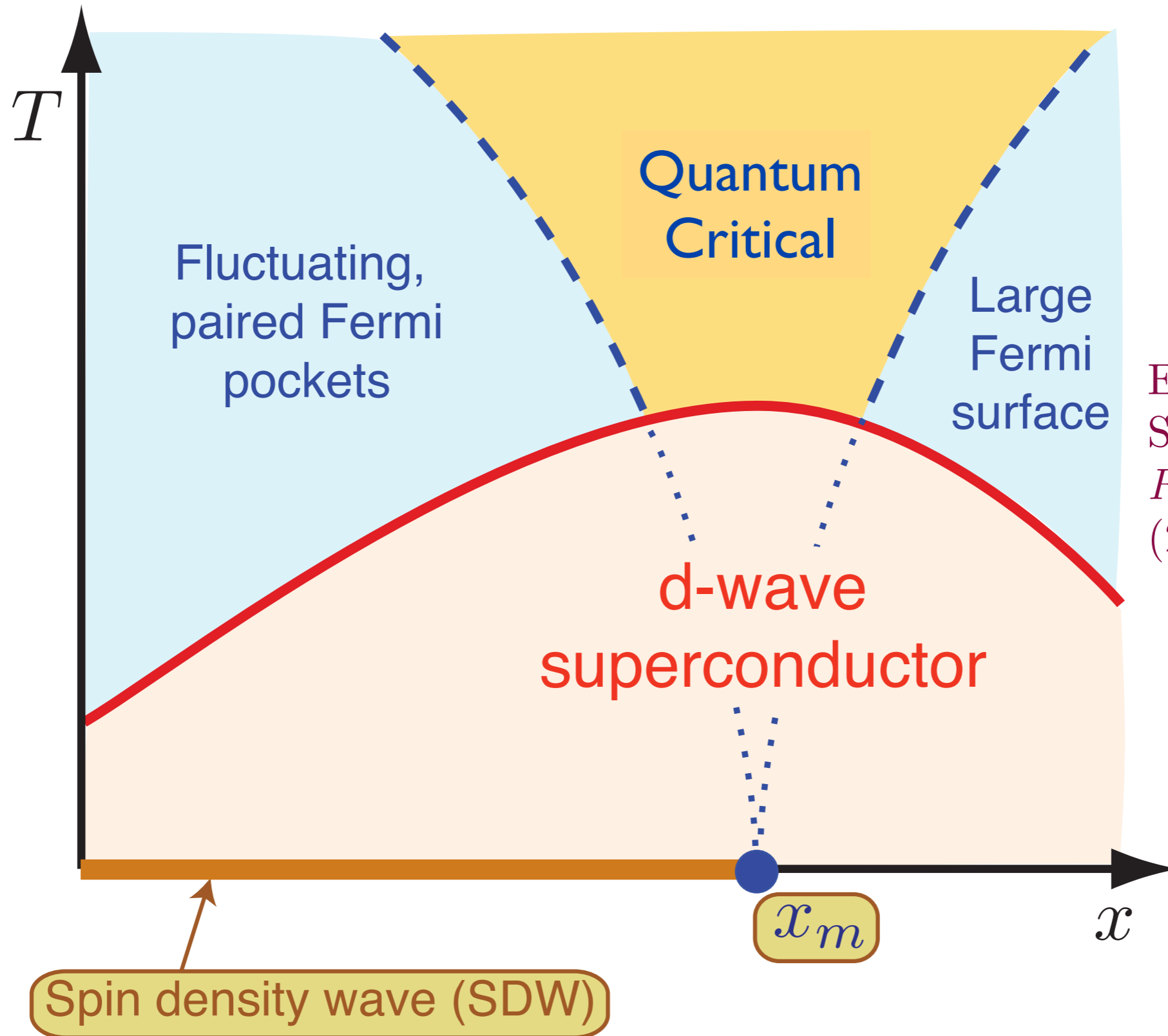
Large hole Fermi surface

Fermi surface+antiferromagnetism



QCP for the onset of SDW order is actually within a superconductor

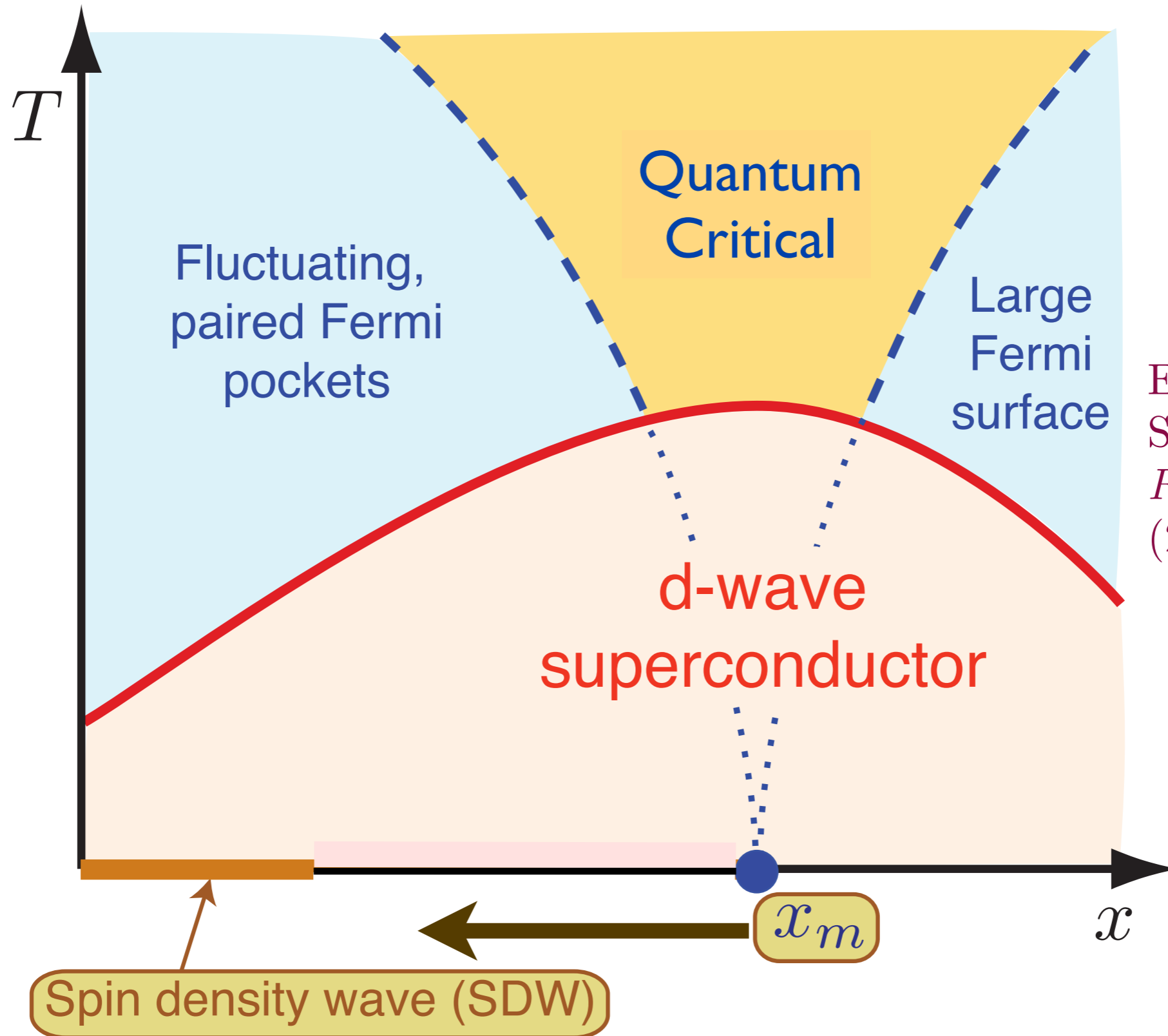
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

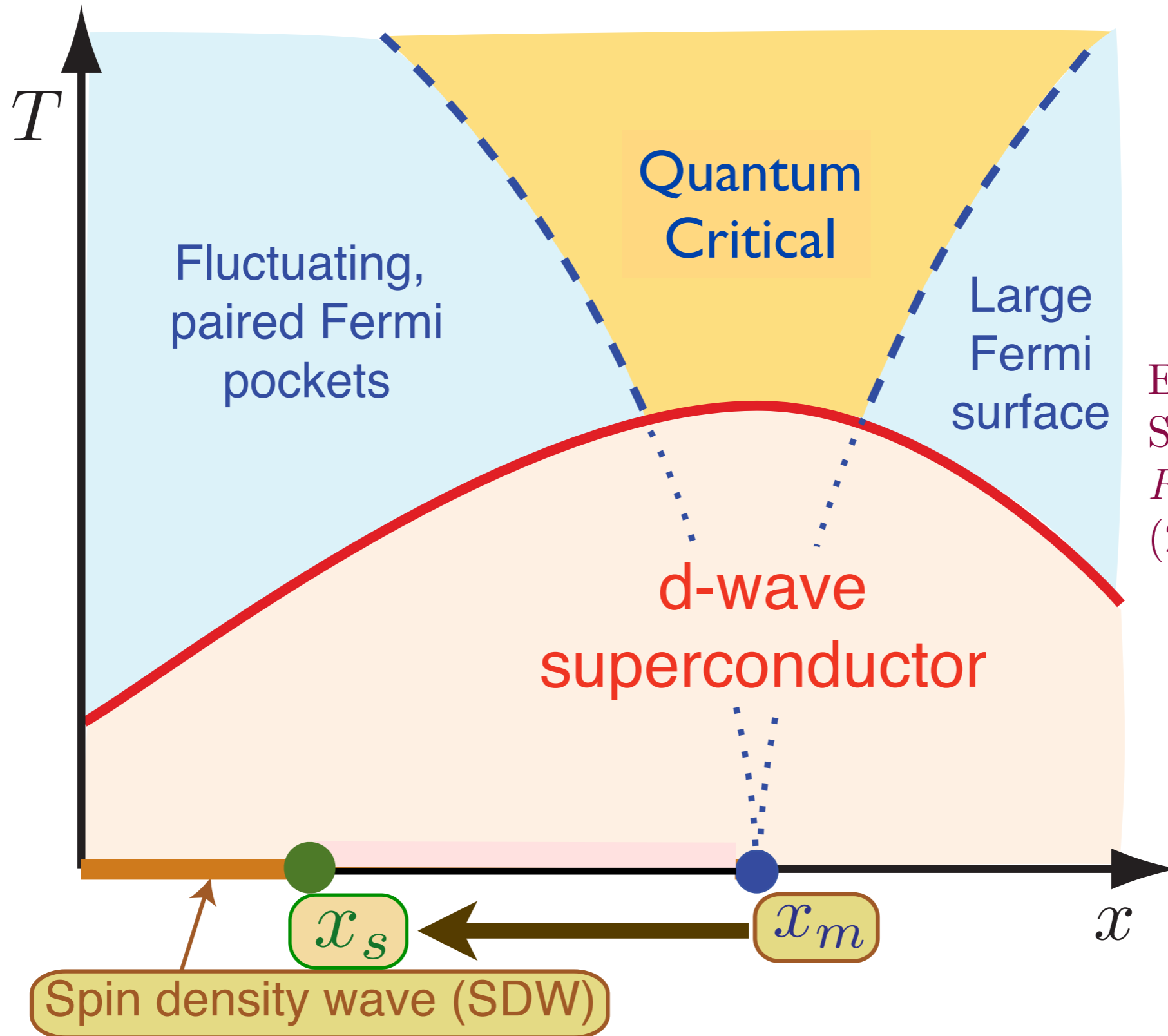
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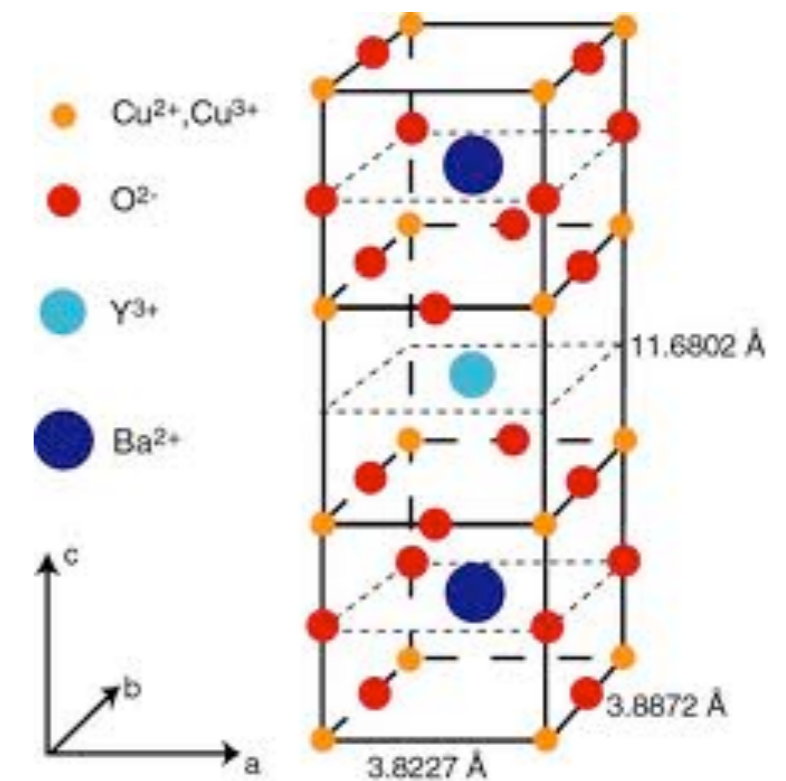
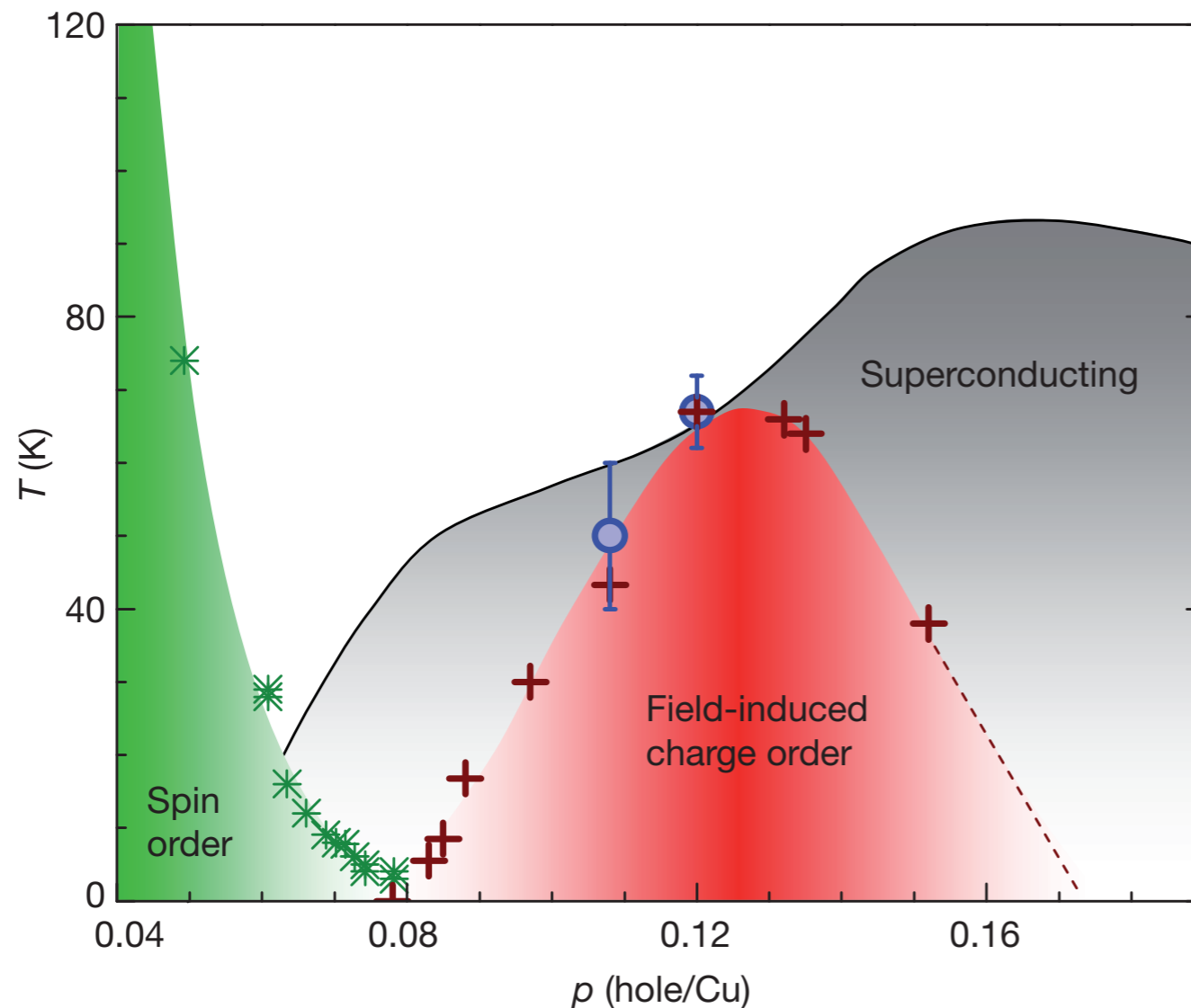
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Magnetic-field-induced charge-stripe order in the high-temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Mladen Horvatić¹, Claude Berthier¹, W. N. Hardy^{2,3}, Ruixing Liang^{2,3}, D. A. Bonn^{2,3} & Marc-Henri Julien¹

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.
- The pseudospin partner of d -wave superconductivity is an incommensurate d -wave bond order
- These orders form a pseudospin doublet, which is responsible for the “pseudogap” phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)

C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)

M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)

K. B. Efetov, H. Meier, and C. Pépin, Nature Physics **9**, 442 (2013).

S. Sachdev and R. La Placa, Phys. Rev. Lett. in press arXiv:1303.2114

1. Pseudospin symmetry between d -wave superconductivity and bond order

Continuum field theory with exact pseudospin symmetry

2. Approximate pseudospin symmetry on the lattice t - J model

Pseudogap and bond order in the underdoped cuprates

1. Pseudospin symmetry between d -wave superconductivity and bond order

Continuum field theory with exact pseudospin symmetry

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Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction.
Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i < j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta} \right)$$

which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \rightarrow U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J .

- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
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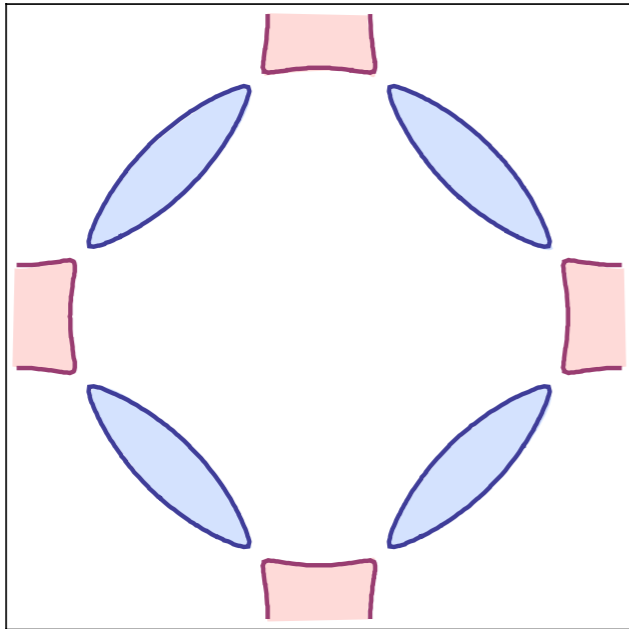
$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta} \right)$$

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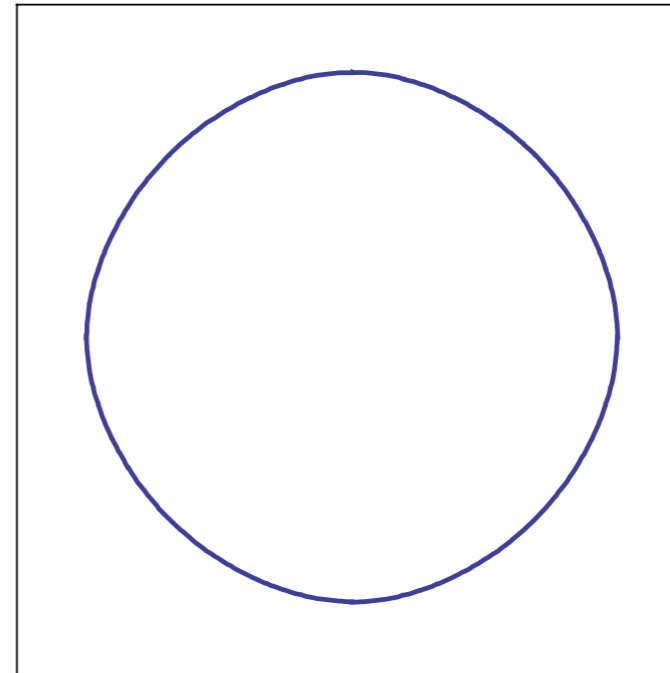
We will find important consequences of the pseudospin symmetry in ordinary metals with antiferromagnetic correlations.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

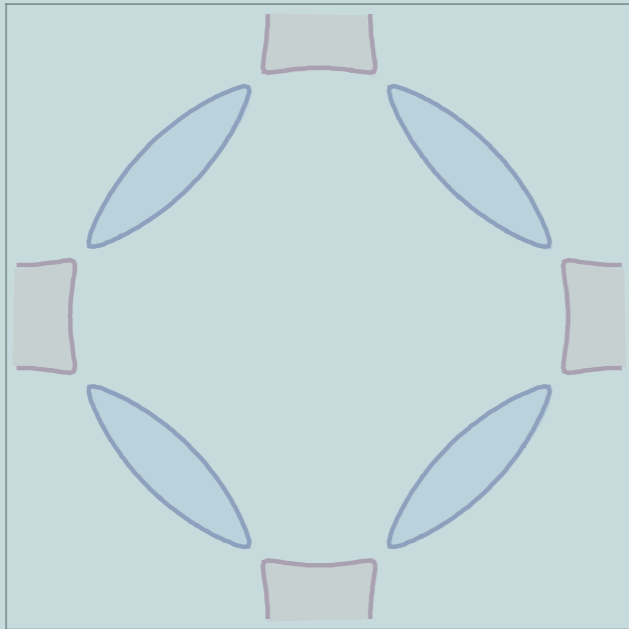


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

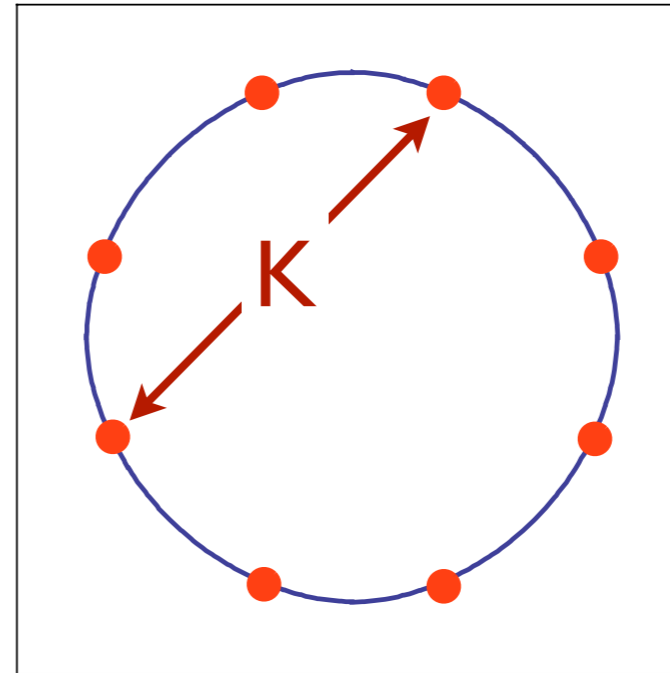
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Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
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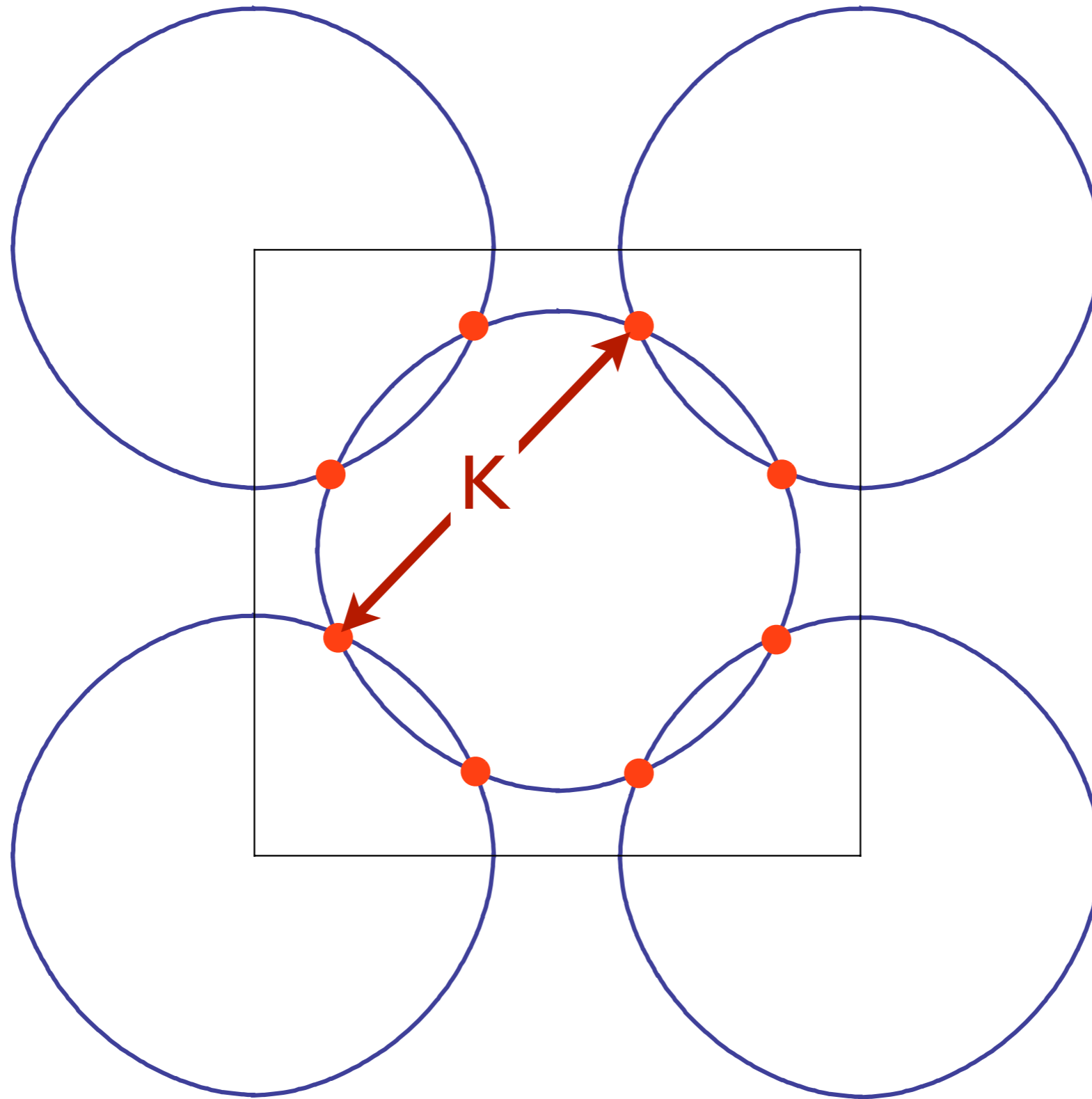
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

Focus on
this
region

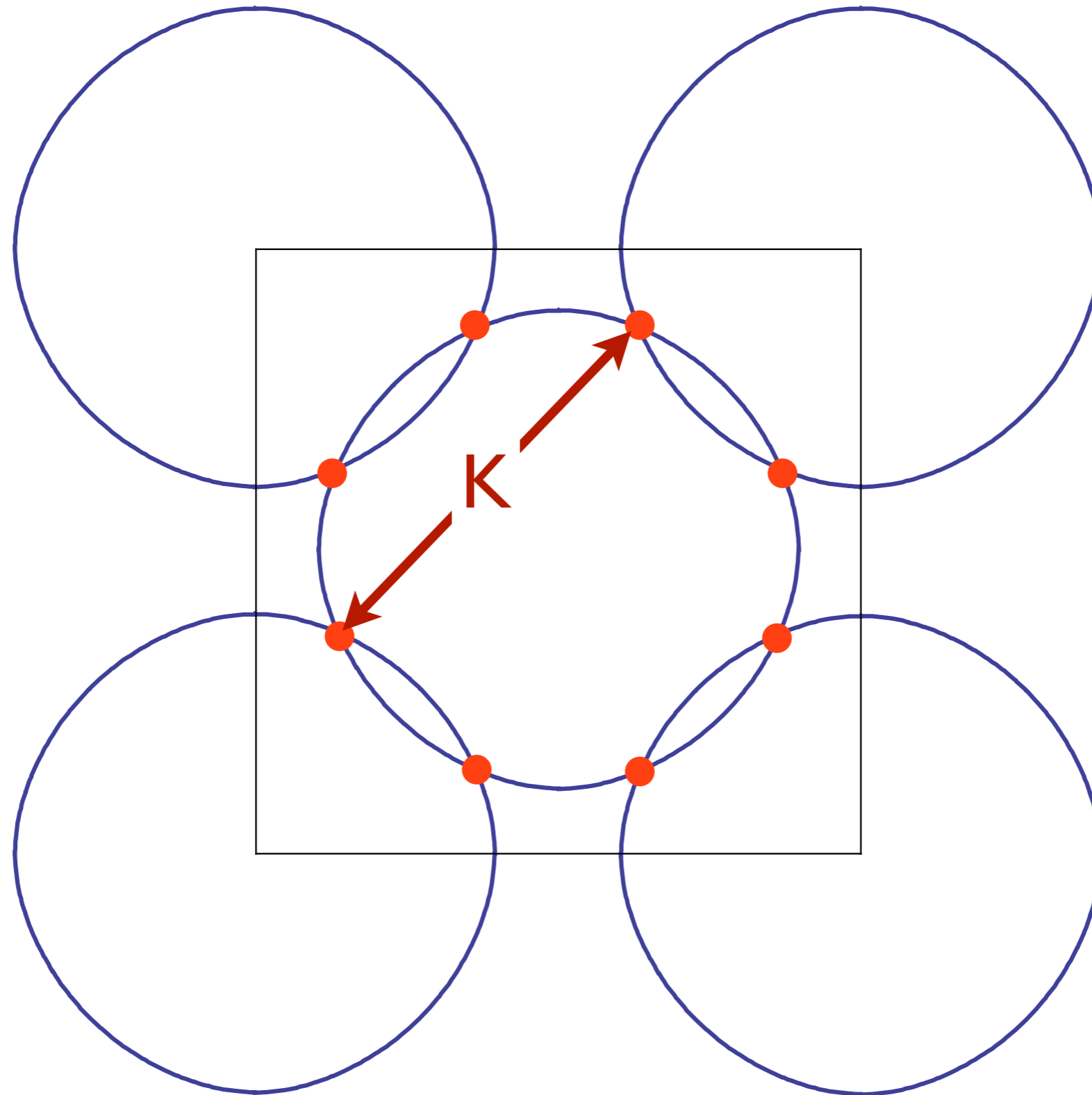
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Fermi surface+antiferromagnetism



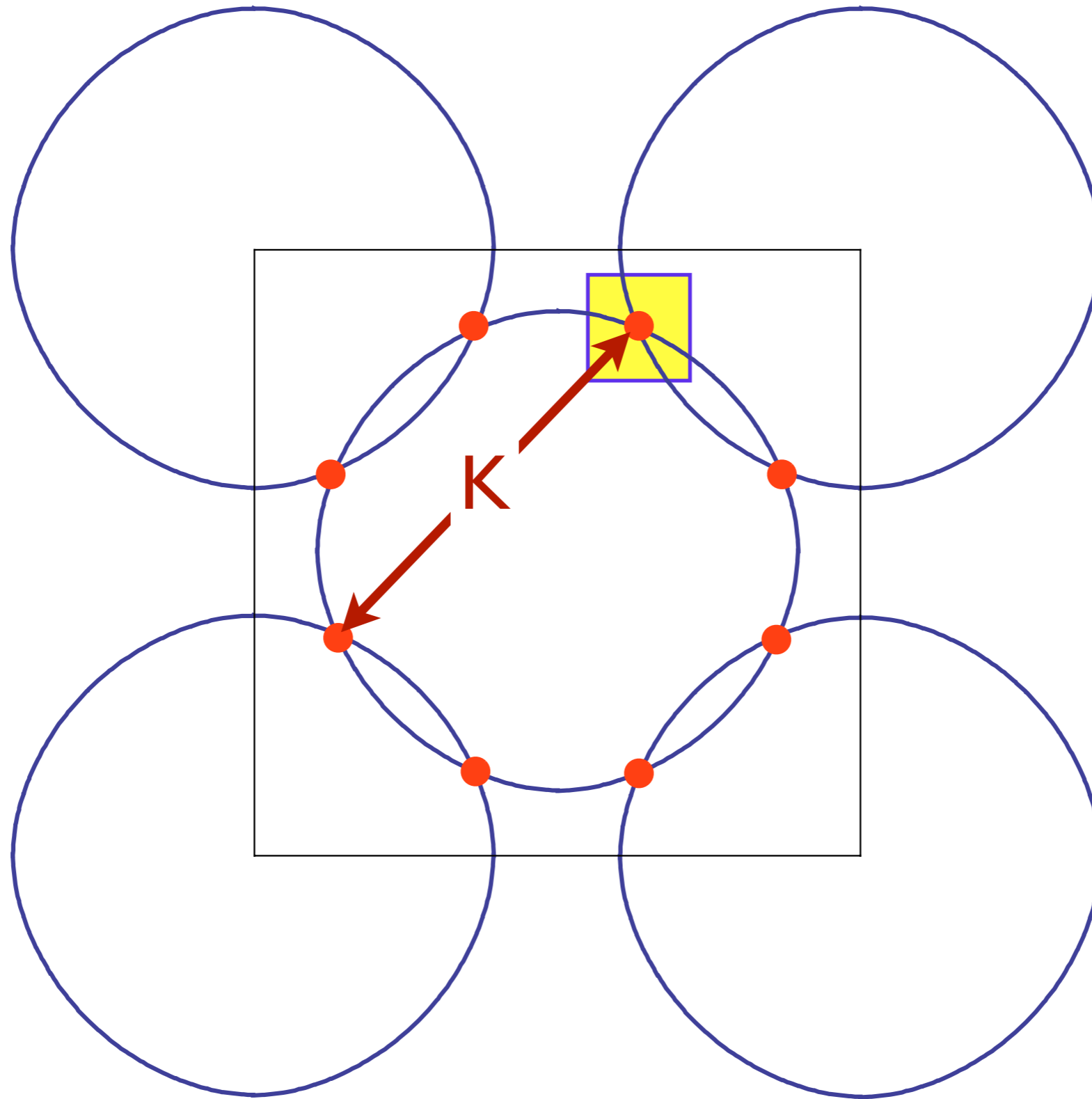
“Hot” spots

Fermi surface+antiferromagnetism



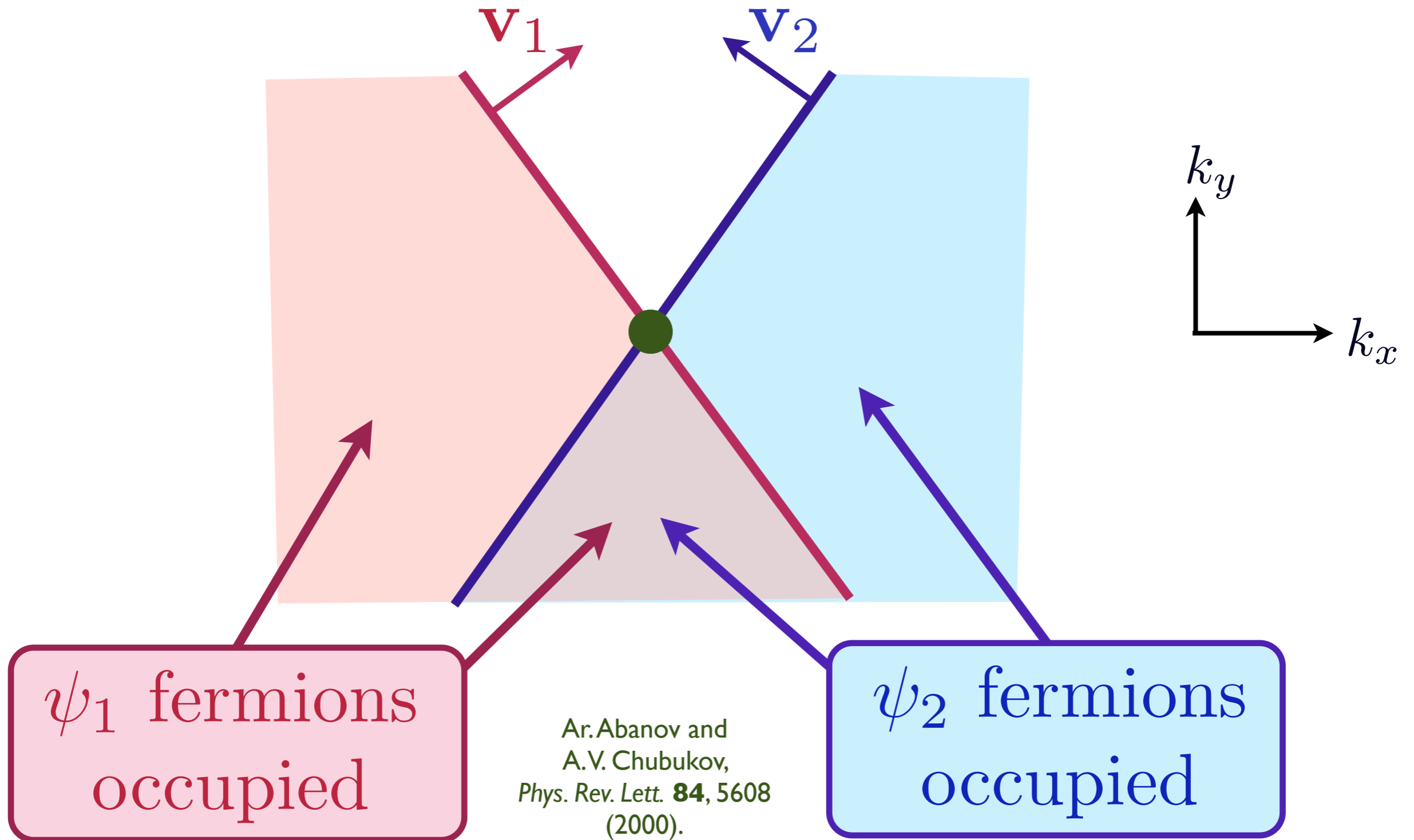
Low energy theory for critical point near hot spots

Fermi surface+antiferromagnetism

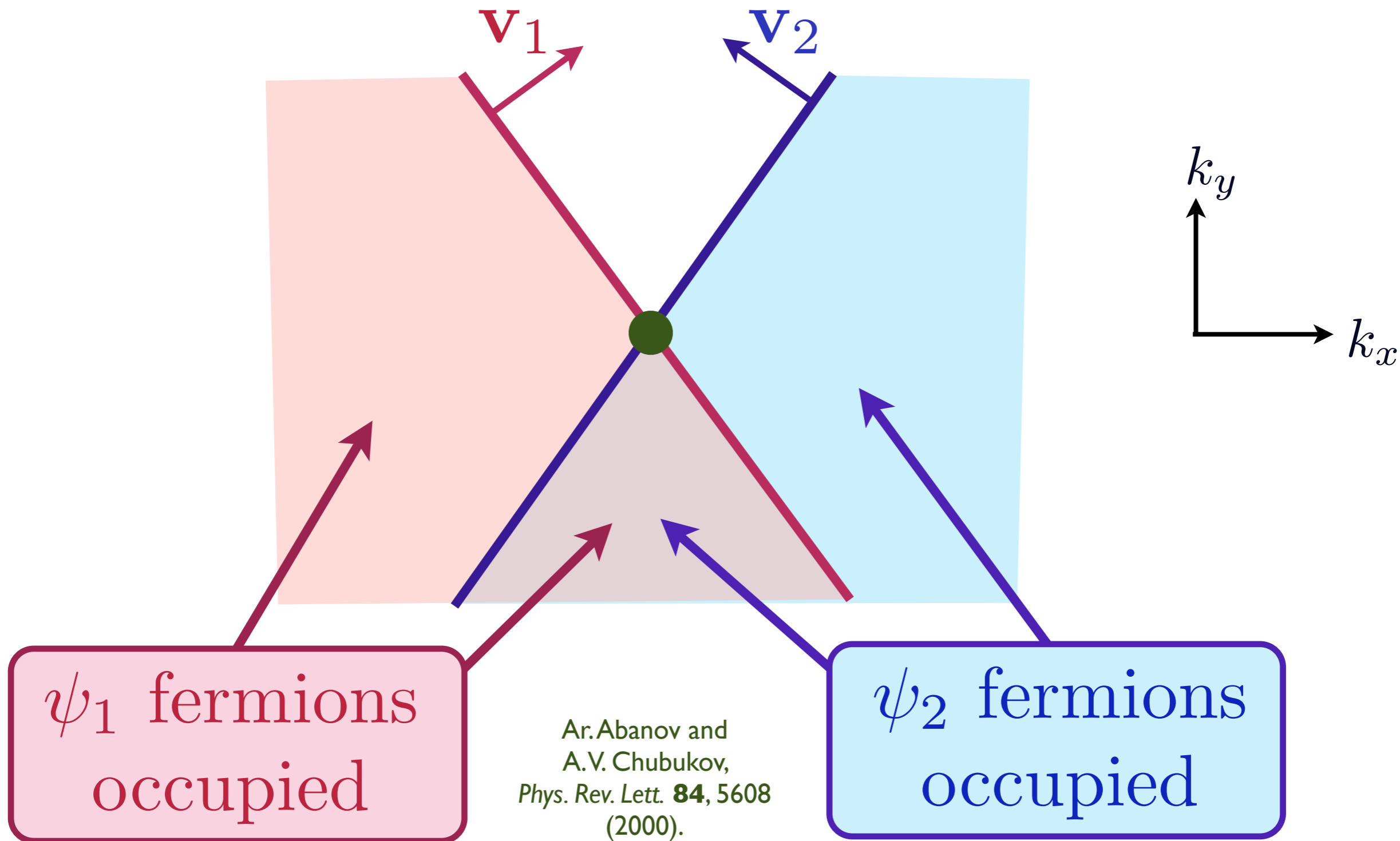


Low energy theory for critical point near hot spots

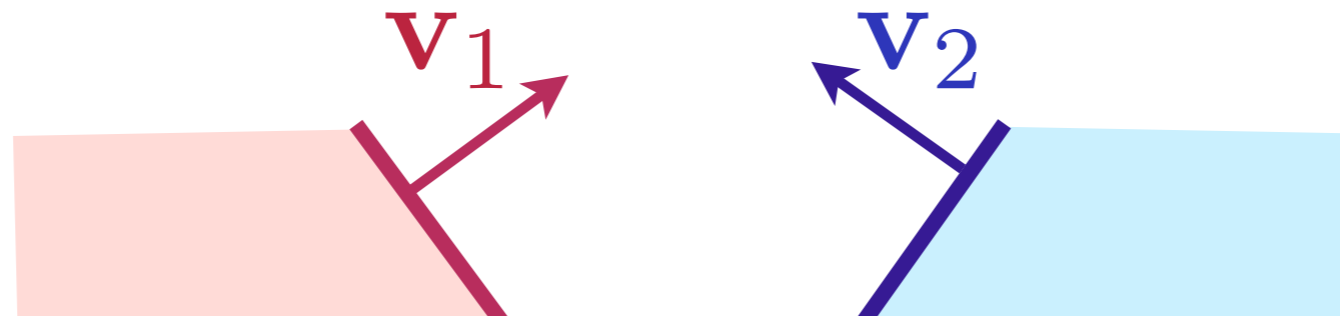
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



$$\mathcal{S} = \int d^2r d\tau \left[\psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha} \right. \\
 \left. + \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \right]$$



$$\mathcal{S} = \int d^2r d\tau \left[\psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha} + \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \right]$$



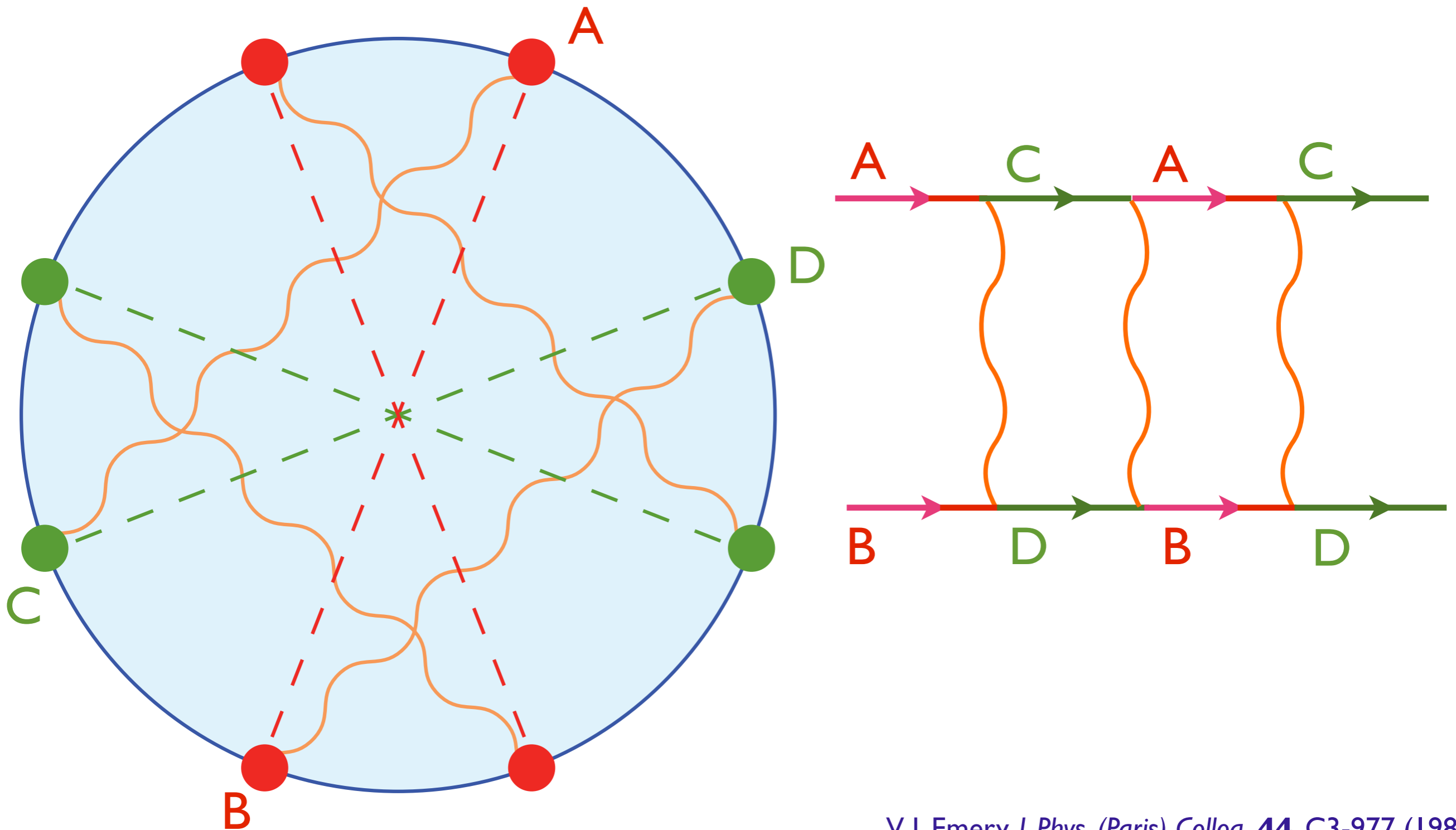
This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots.

ψ_1 fermions occupied

ψ_2 fermions occupied

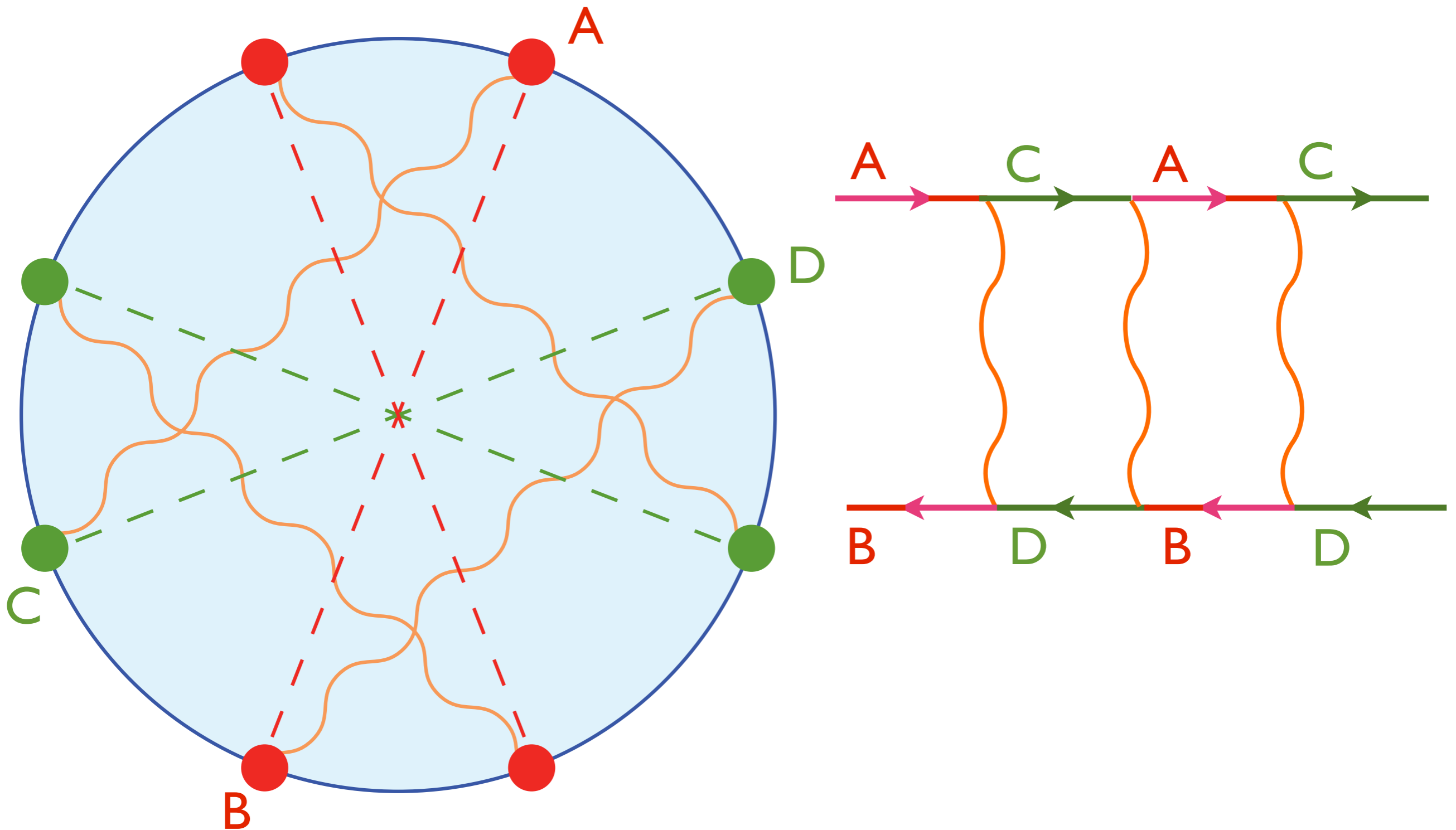
M.A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

Pairing “glue” from antiferromagnetic fluctuations



V.J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Same “glue” leads to particle-hole pairing



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

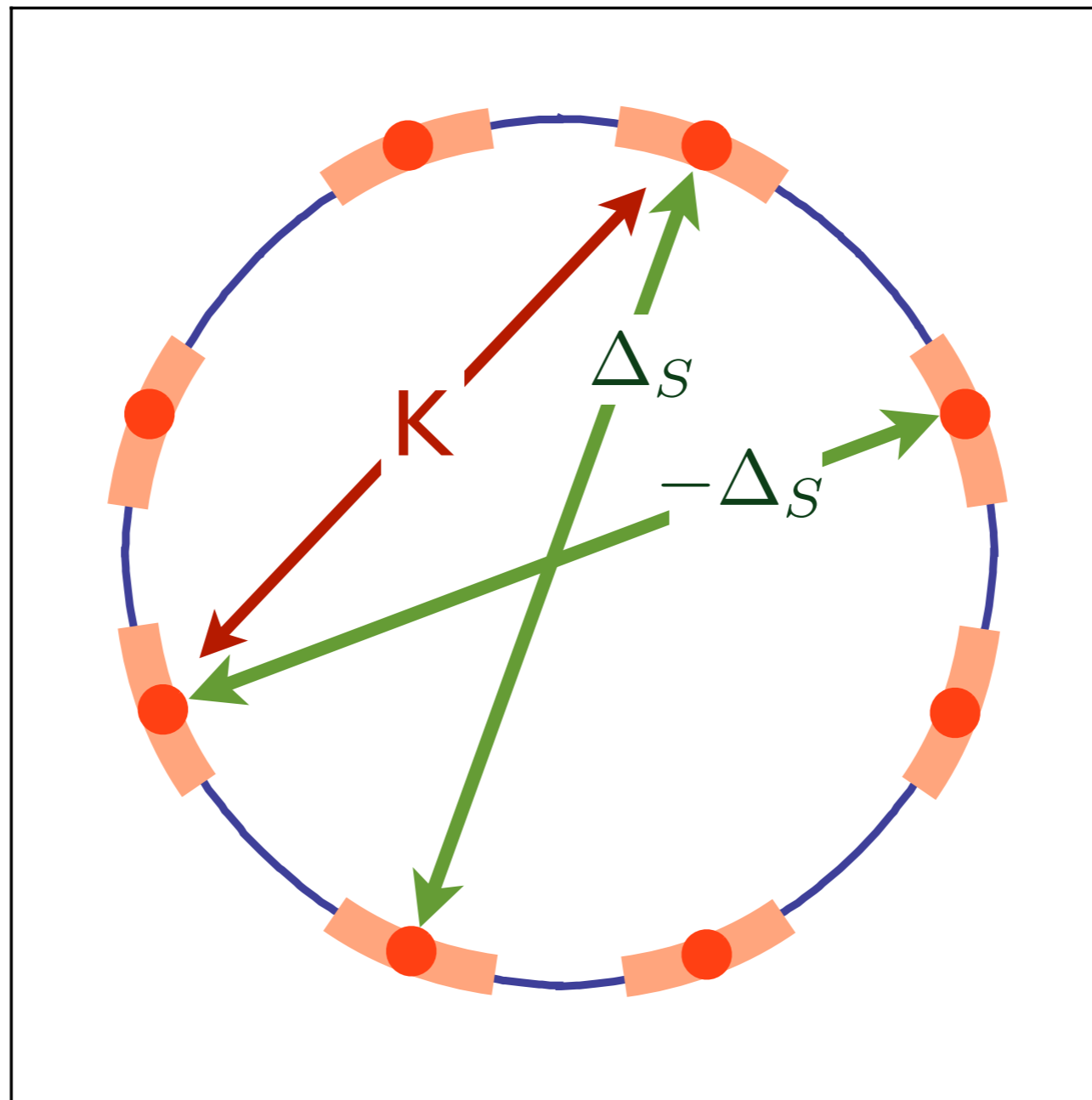
$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

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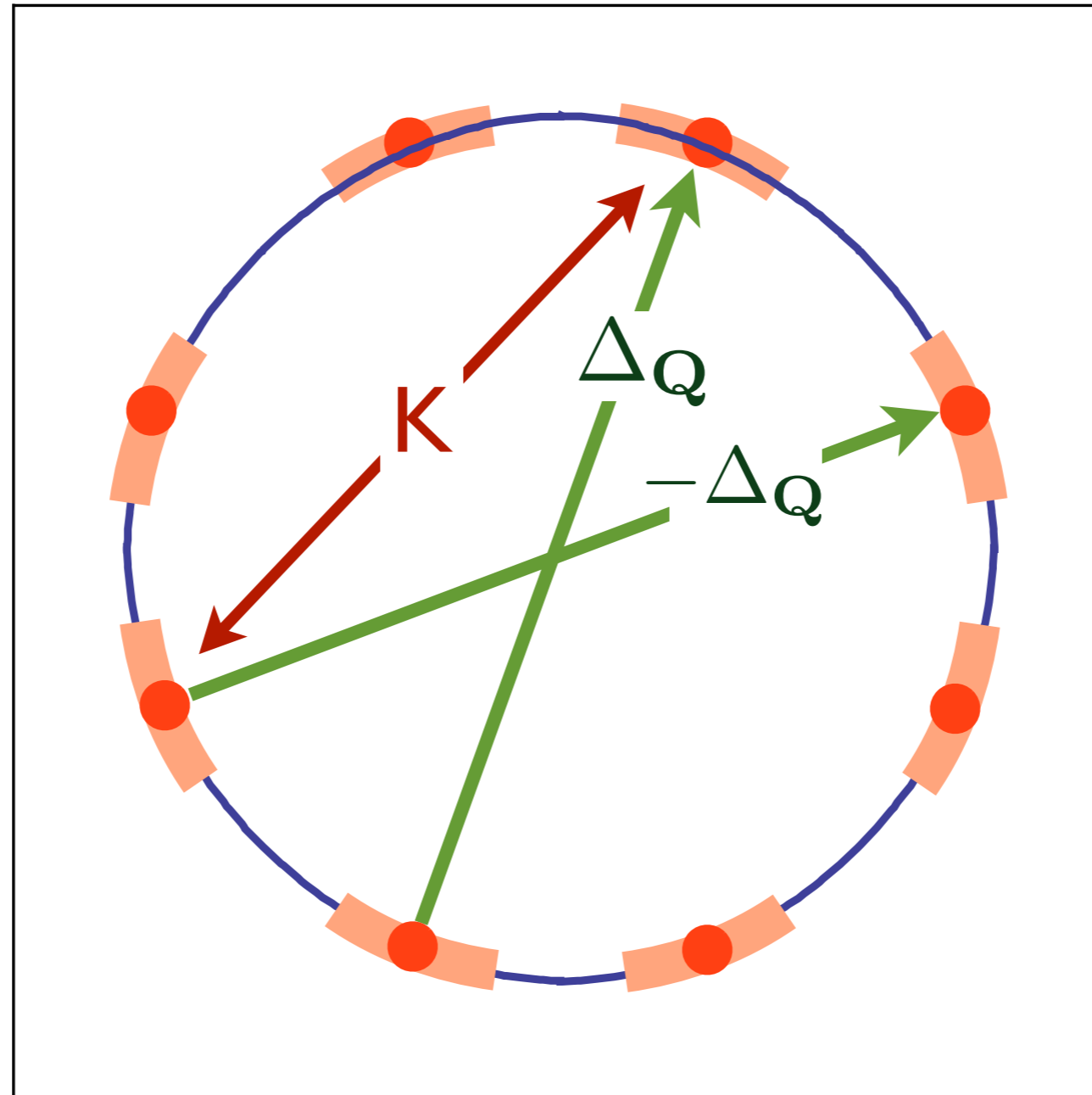


d-wave superconductor: particle-particle pairing at and near hot spots, with sign-changing pairing amplitude

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After
pseudospin
rotation on
half the
hot-spots

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)

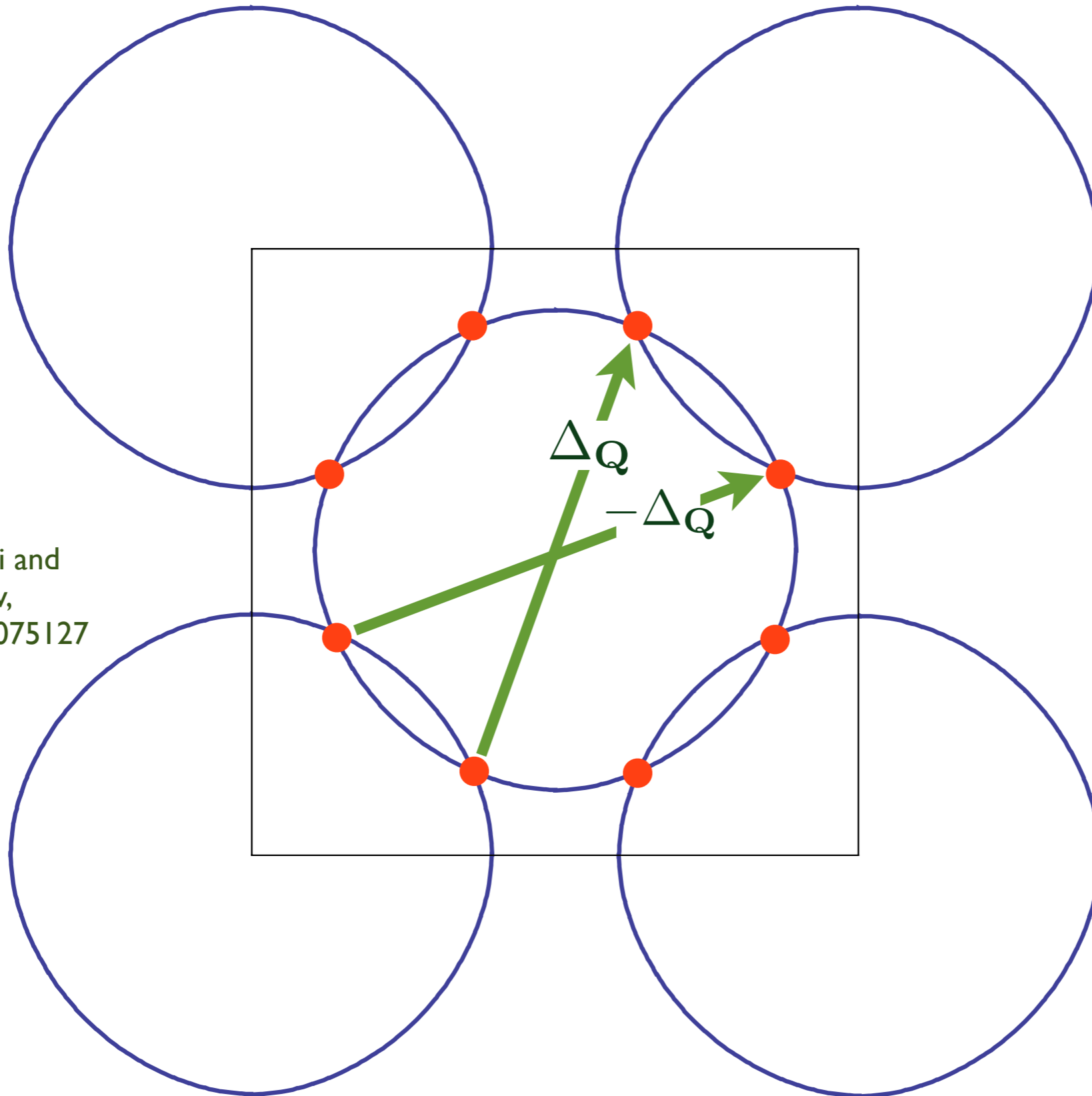


\mathbf{Q} is ' $2k_F$ '
wavevector

Incommensurate d-wave bond order:
particle-hole pairing at and near hot spots, with
sign-changing pairing amplitude

Incommensurate d -wave bond order

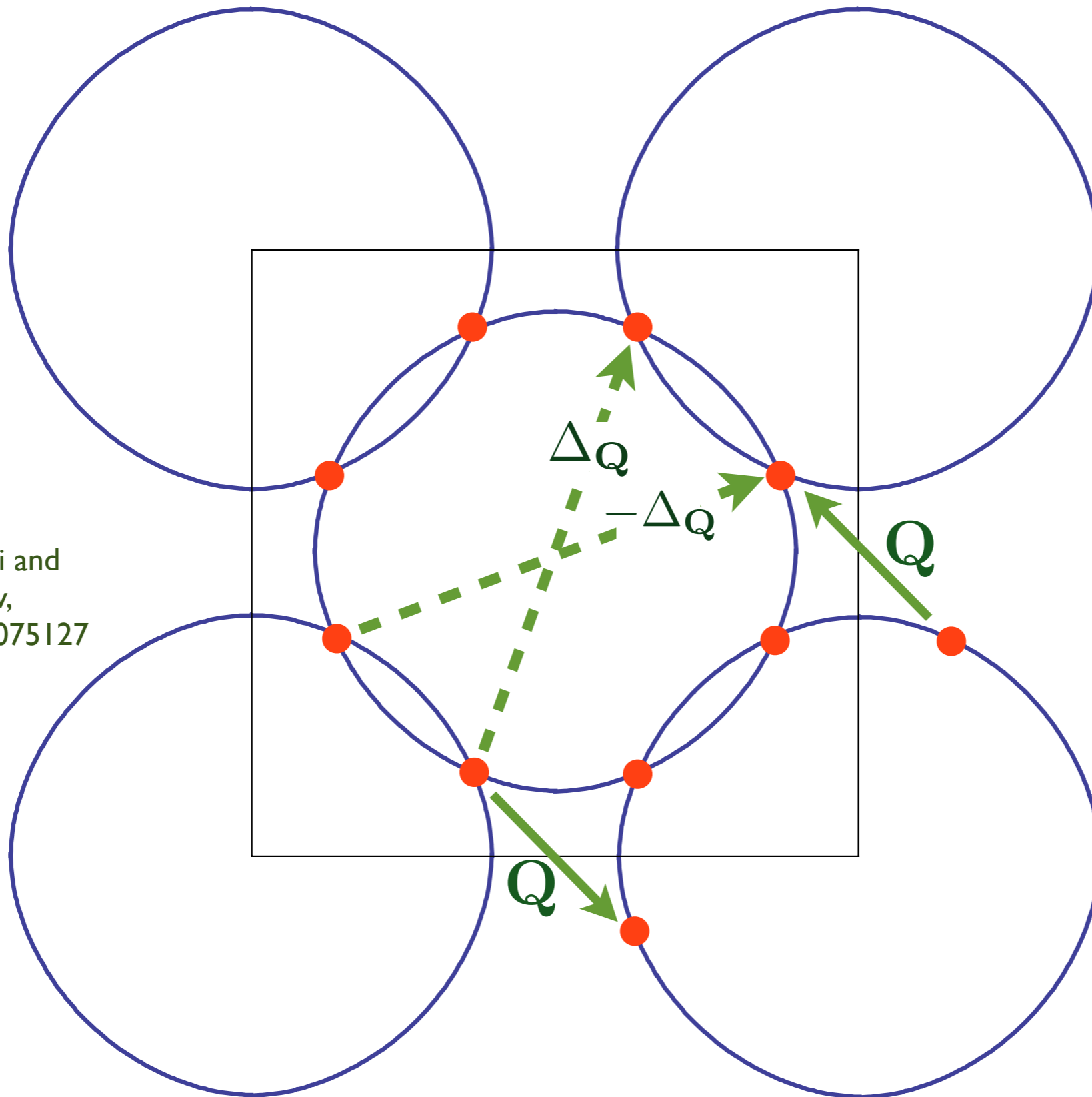
M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)



$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta\mathbf{Q}(\cos k_x - \cos k_y)$$

Incommensurate d -wave bond order

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)



$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_Q (\cos k_x - \cos k_y)$$

Incommensurate d -wave bond order

Consider modulation in an off-site “density” like variable at sites \mathbf{r}_i and \mathbf{r}_j

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle \sim \left[\sum_{\mathbf{k}} \Delta_{\mathbf{Q}}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

relative co-ord. average co-ord.

The wavevector \mathbf{Q} is associated with a modulation in the *average* co-ordinate $(\mathbf{r}_i - \mathbf{r}_j)/2$: this determines the wavevector of the neutron/X-ray scattering peak.

The interesting part is the dependence on the *relative* co-ordinate $\mathbf{r}_i - \mathbf{r}_j$. Assuming time-reversal, the order parameter $\Delta_{\mathbf{Q}}(\mathbf{k})$ can always be expanded as

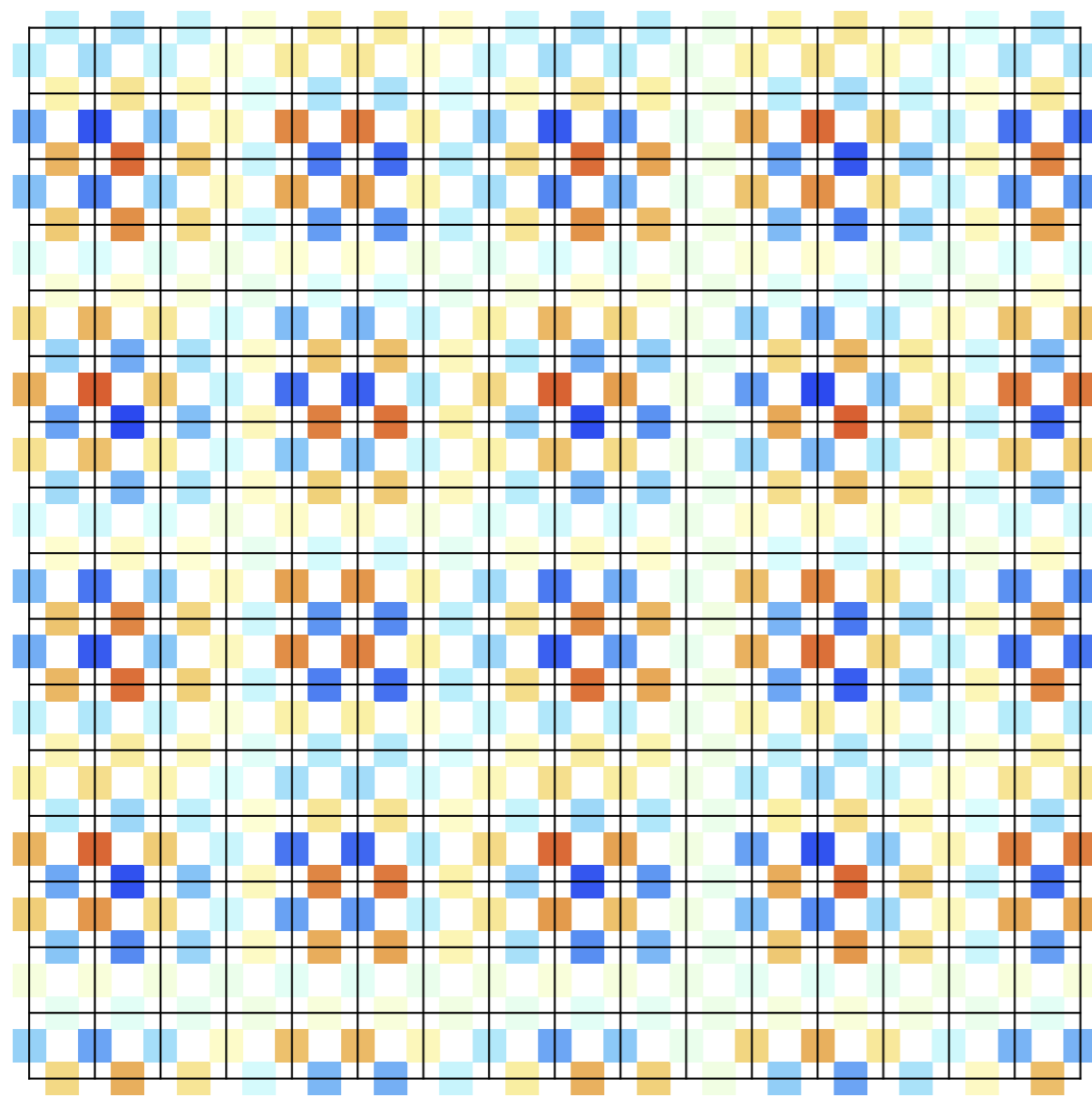
$$\Delta_{\mathbf{Q}}(\mathbf{k}) = c_s + c_{s'}(\cos k_x + \cos k_y) + c_d(\cos k_x - \cos k_y) + \dots$$

The usual charge-density-wave has only $c_s \neq 0$.

The bond-ordered state we find has

$$|c_d| \gg c_s, c_{s'}, \dots$$

Incommensurate d -wave bond order



“Bond density”
measures amplitude
for electrons to be
in spin-singlet
valence bond.

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle$$

where \mathbf{Q} extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$ is non-zero *only* when \mathbf{r}, \mathbf{s} are nearest neighbors.

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
 $\text{Re}[\Delta_S], \text{Im}[\Delta_S]$



d-wave bond order
 $\text{Re}[\Delta_Q], \text{Im}[\Delta_Q]$

- $O(4)$ symmetry is exact in the continuum hot-spot theory for the onset of antiferromagnetism

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
 $\text{Re}[\Delta_S], \text{Im}[\Delta_S]$



d-wave bond order
 $\text{Re}[\Delta_Q], \text{Im}[\Delta_Q]$

- Symmetry is “anomalous” because it requires independent rotation of left- and right-movers: it is broken by any lattice regularization of the theory.

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
 $\text{Re}[\Delta_S], \text{Im}[\Delta_S]$



d-wave bond order
 $\text{Re}[\Delta_Q], \text{Im}[\Delta_Q]$

- Symmetry is broken by Fermi surface curvature (which prefers Δ_S) and density-density Coulomb repulsion (which prefers Δ_Q).

Order parameter for the pseudogap

4-component order parameter

d-wave pairing
 $\text{Re}[\Delta_S], \text{Im}[\Delta_S]$



d-wave bond order
 $\text{Re}[\Delta_Q], \text{Im}[\Delta_Q]$

- Thermal fluctuations restore symmetry, and the pseudogap is a regime of a fluctuating $O(4)$ order parameter (which cannot have a phase transition with $O(4)$ symmetry).

K. B. Efetov, H. Meier, and C. Pepin, *Nature Physics* **9**, 442 (2013)

1. Pseudospin symmetry between d -wave superconductivity and bond order

Continuum field theory with exact pseudospin symmetry

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Pseudogap and bond order in the underdoped cuprates

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superconductivity and bond order
*Continuum field theory with
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*Pseudogap and bond order
in the underdoped cuprates*

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Optimize the free energy w.r.t. a mean field Hamiltonian which allows for spin-singlet charge order ($\Delta_{\mathbf{Q}}(\mathbf{k})$):

$$H_{MF} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{\mathbf{k}, \mathbf{Q}} \Delta_{\mathbf{Q}}(\mathbf{k}) c_{\mathbf{k}+\mathbf{Q}/2, \alpha}^\dagger c_{\mathbf{k}-\mathbf{Q}/2, \alpha}$$

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

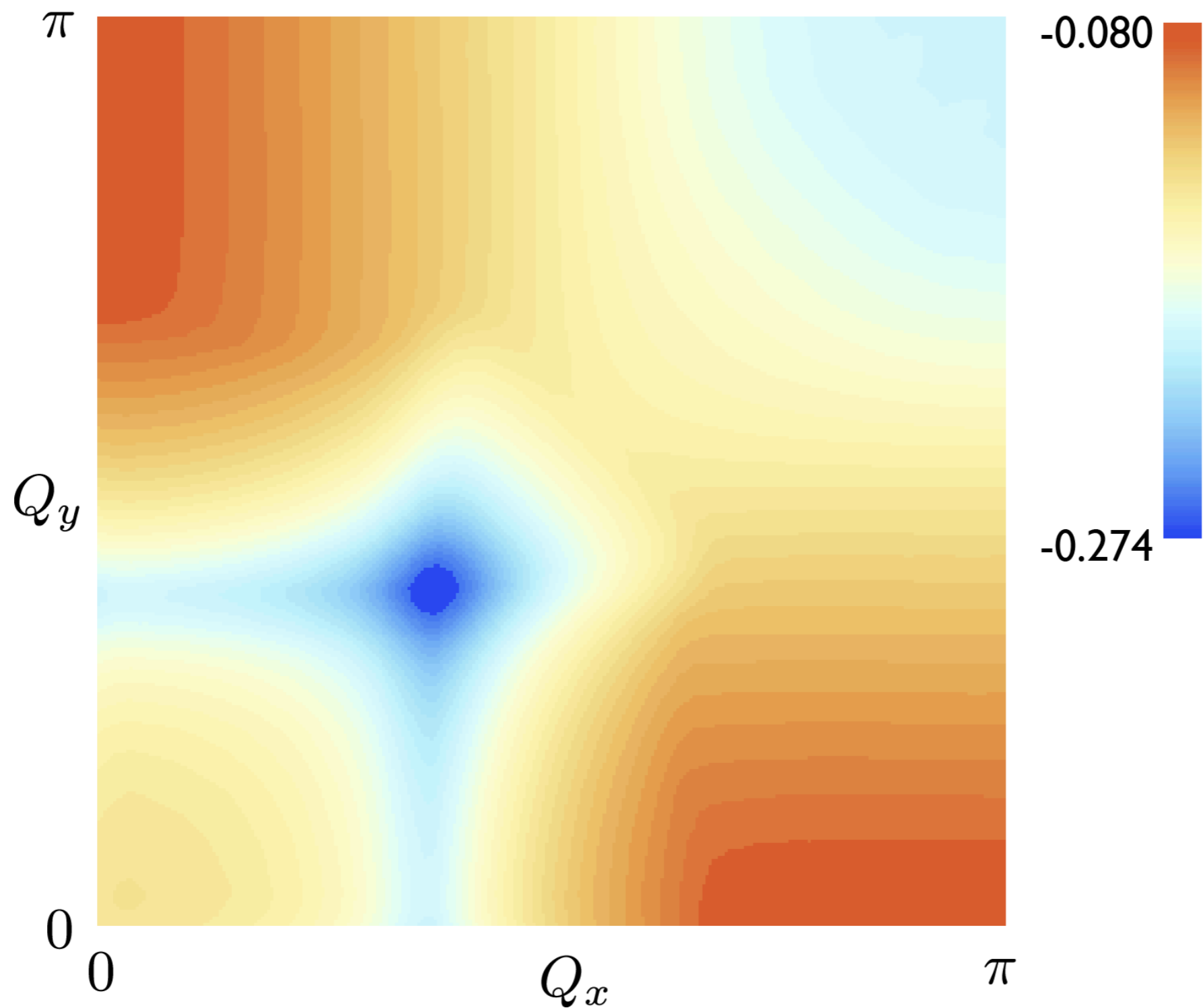
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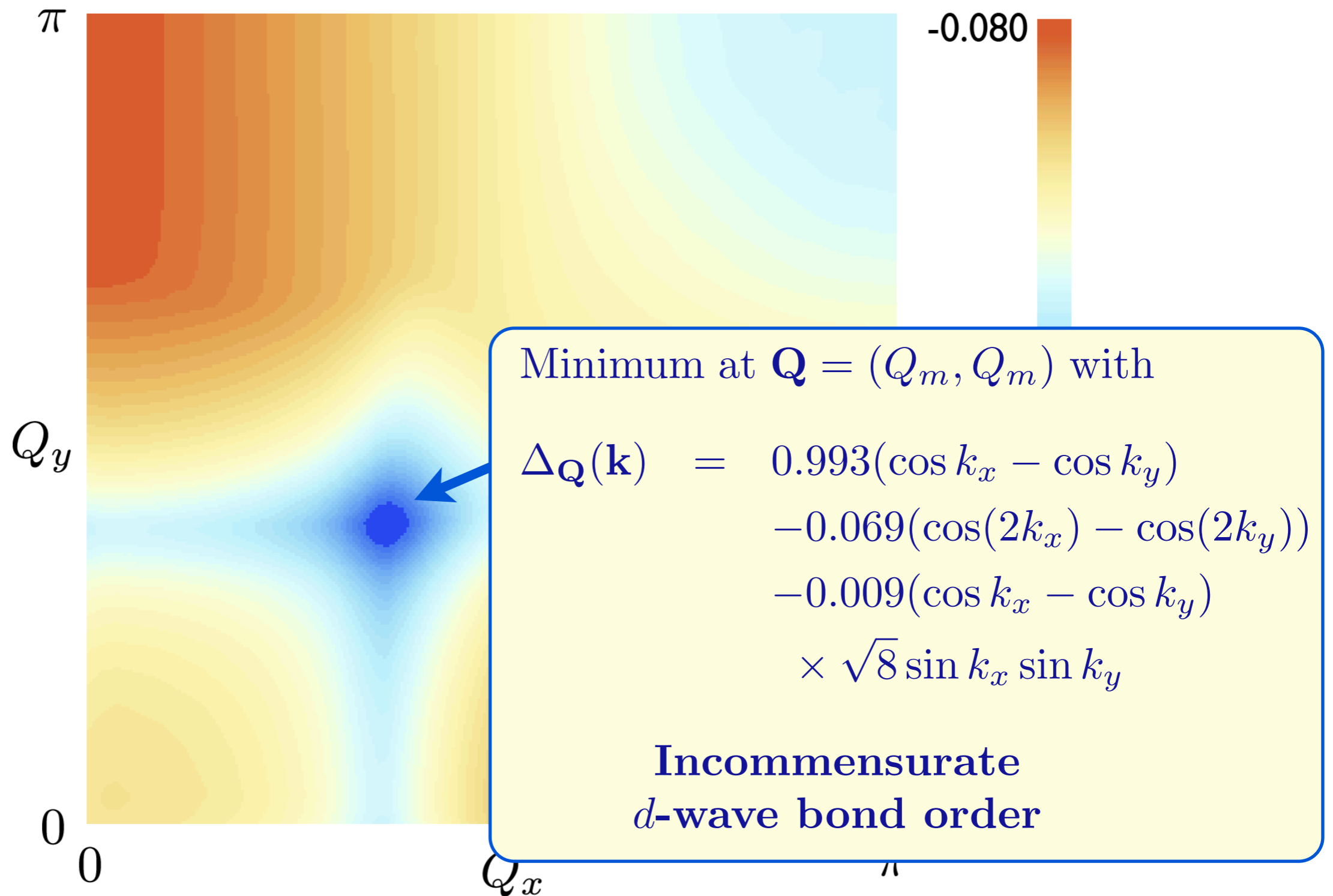
Expanding the free energy in powers of the order parameters we obtain

$$F = F_0 + \sum_{\mathbf{k}, \mathbf{Q}} \Delta_{\mathbf{Q}}^*(\mathbf{k}) \mathcal{M}_{\mathbf{Q}}(\mathbf{k}, \mathbf{k}') \Delta_{\mathbf{Q}}(\mathbf{k}')$$

We compute the eigenvalues, $1 + \lambda_{\mathbf{Q}}$, and eigenfunctions, $\Delta_{\mathbf{Q}}(\mathbf{k})$ of the kernel $\mathcal{M}_{\mathbf{Q}}(\mathbf{k}, \mathbf{k}')$

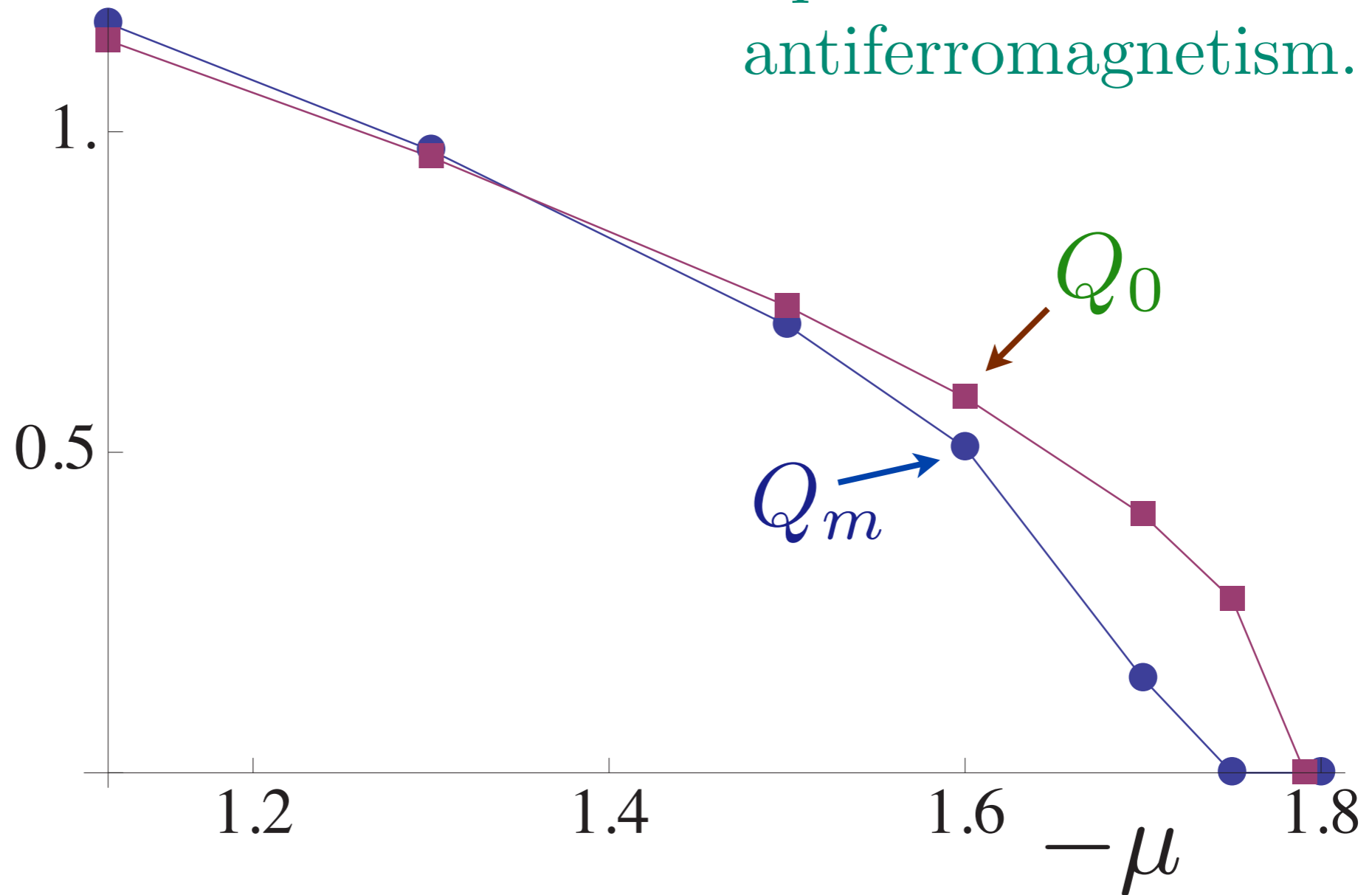
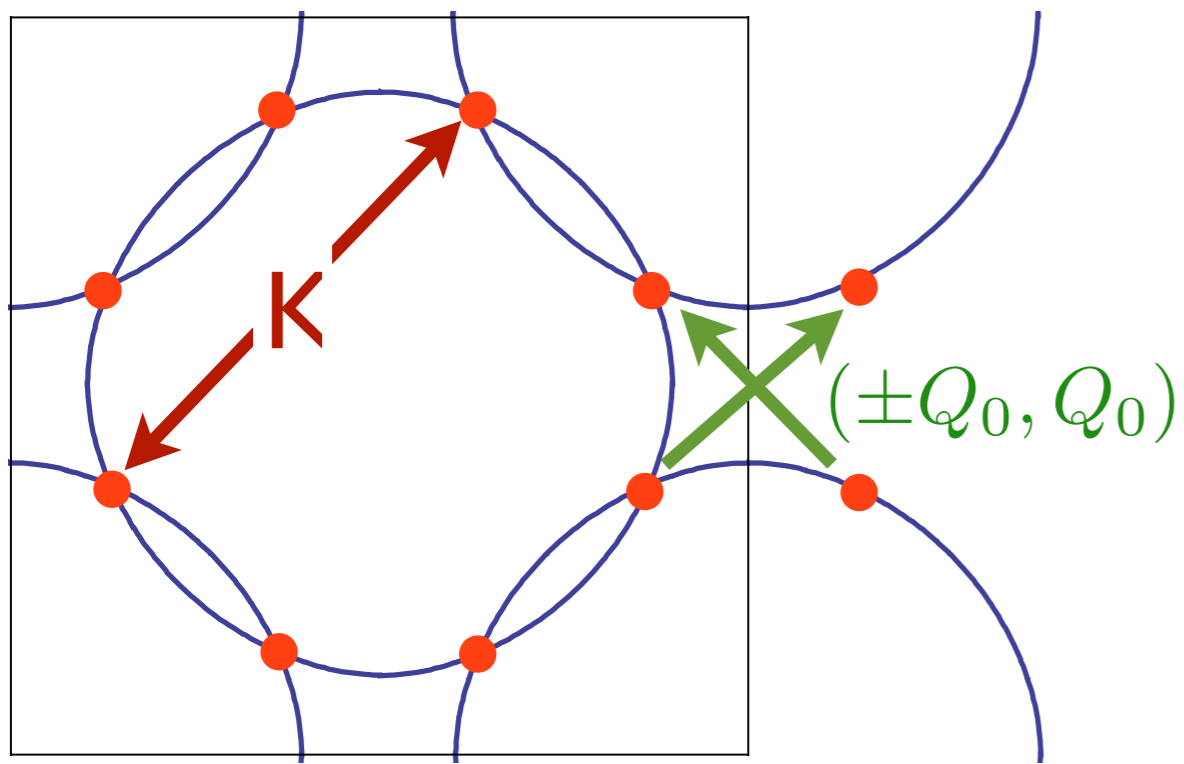


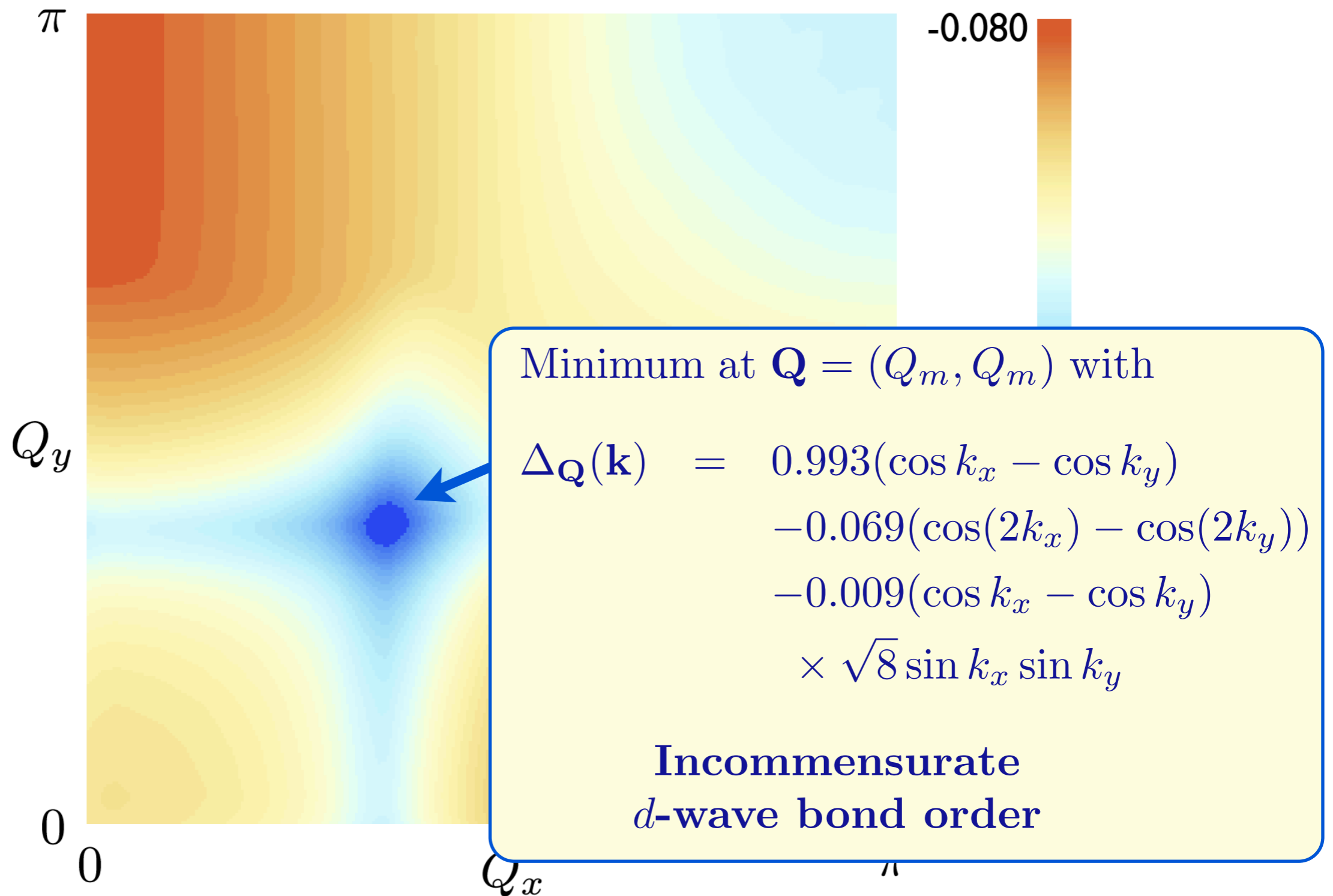
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.



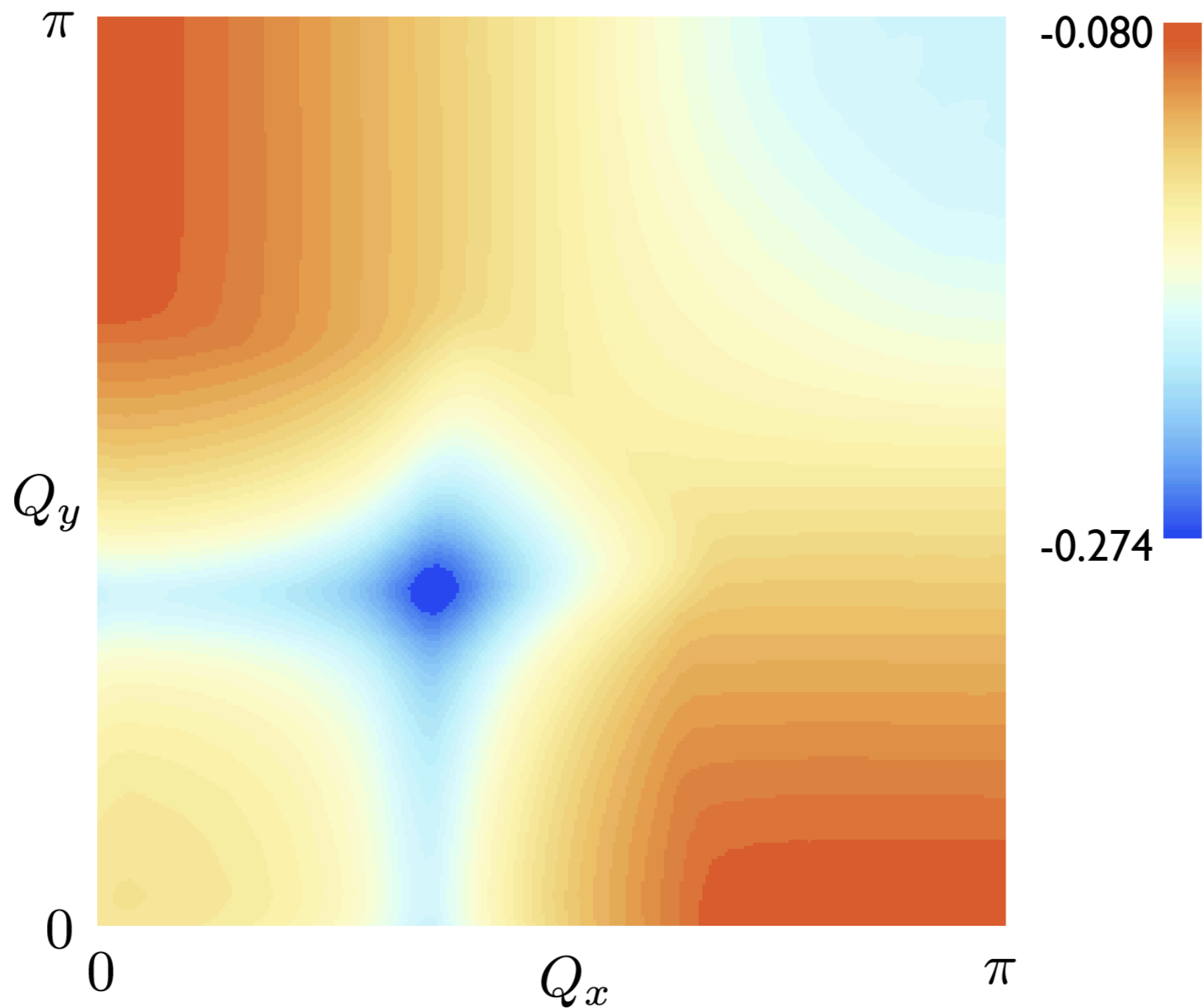
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.

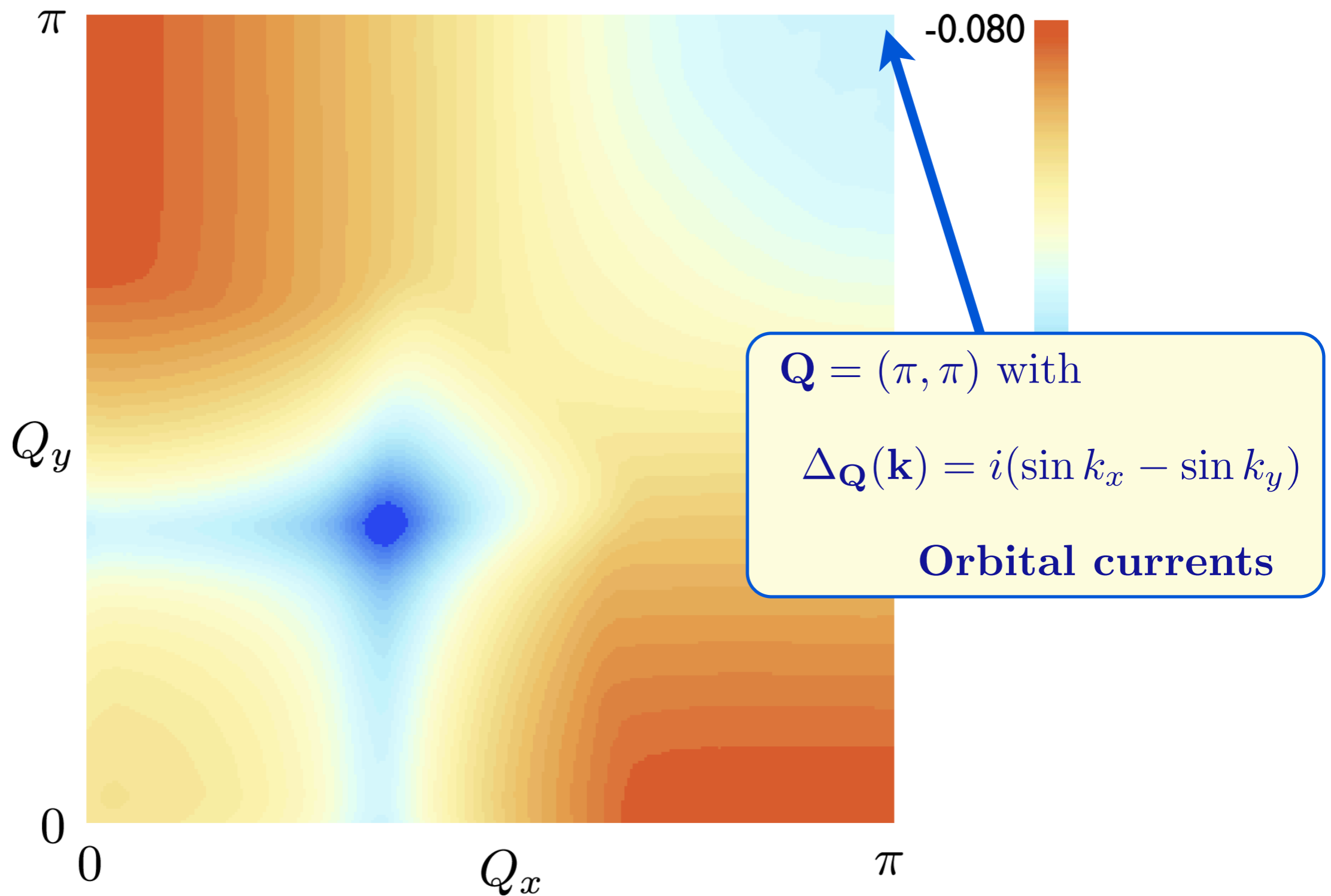




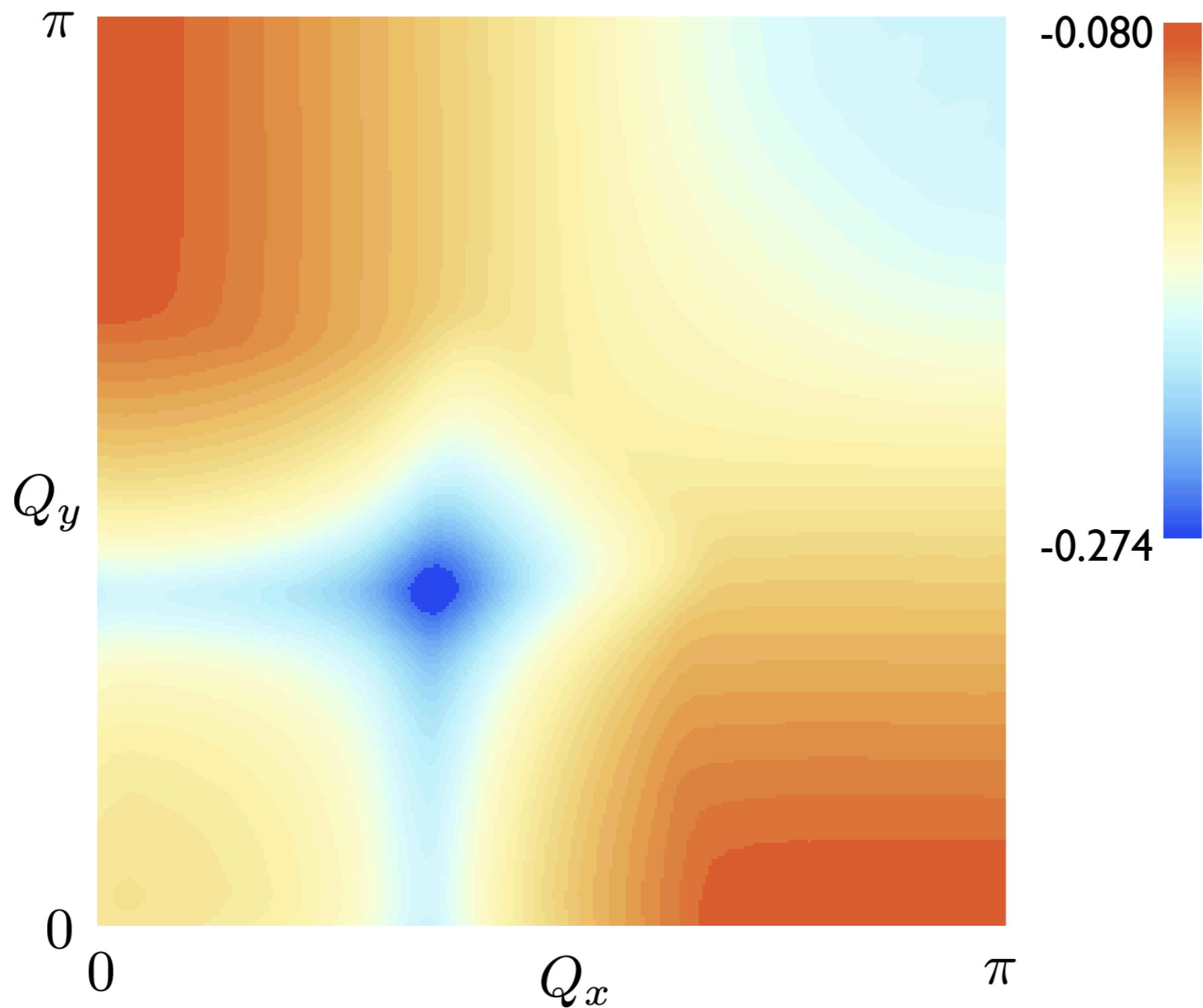
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.



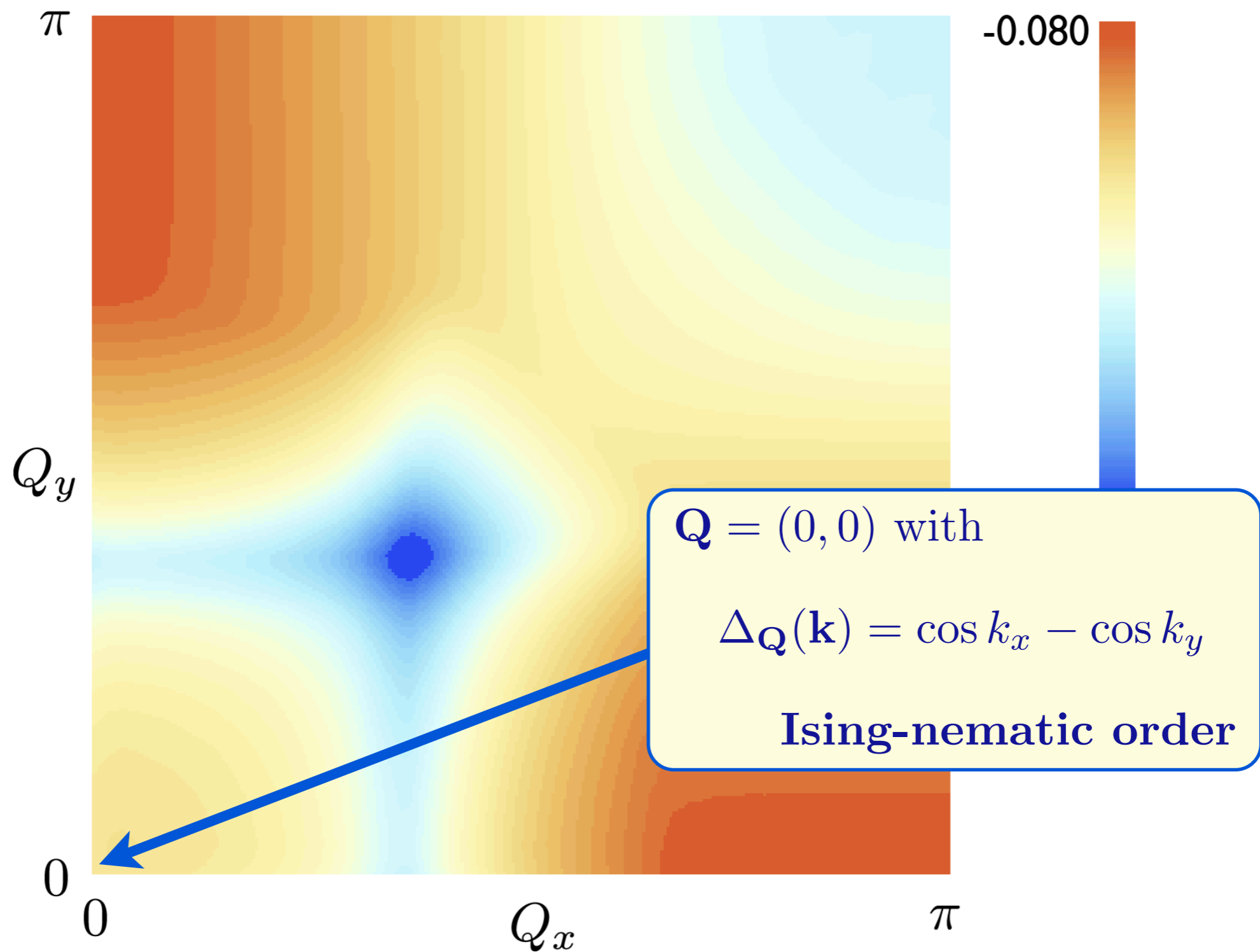
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.



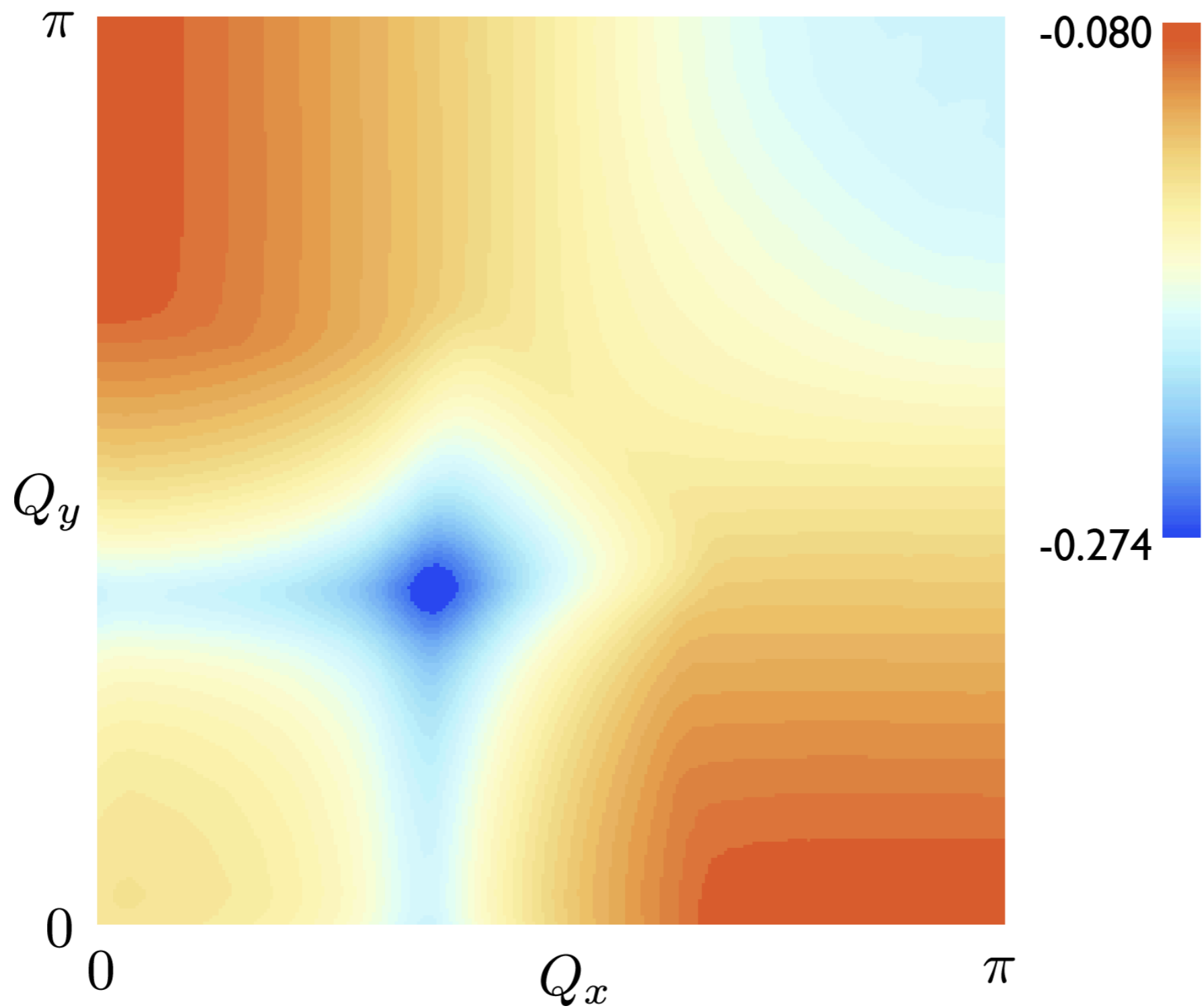
Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

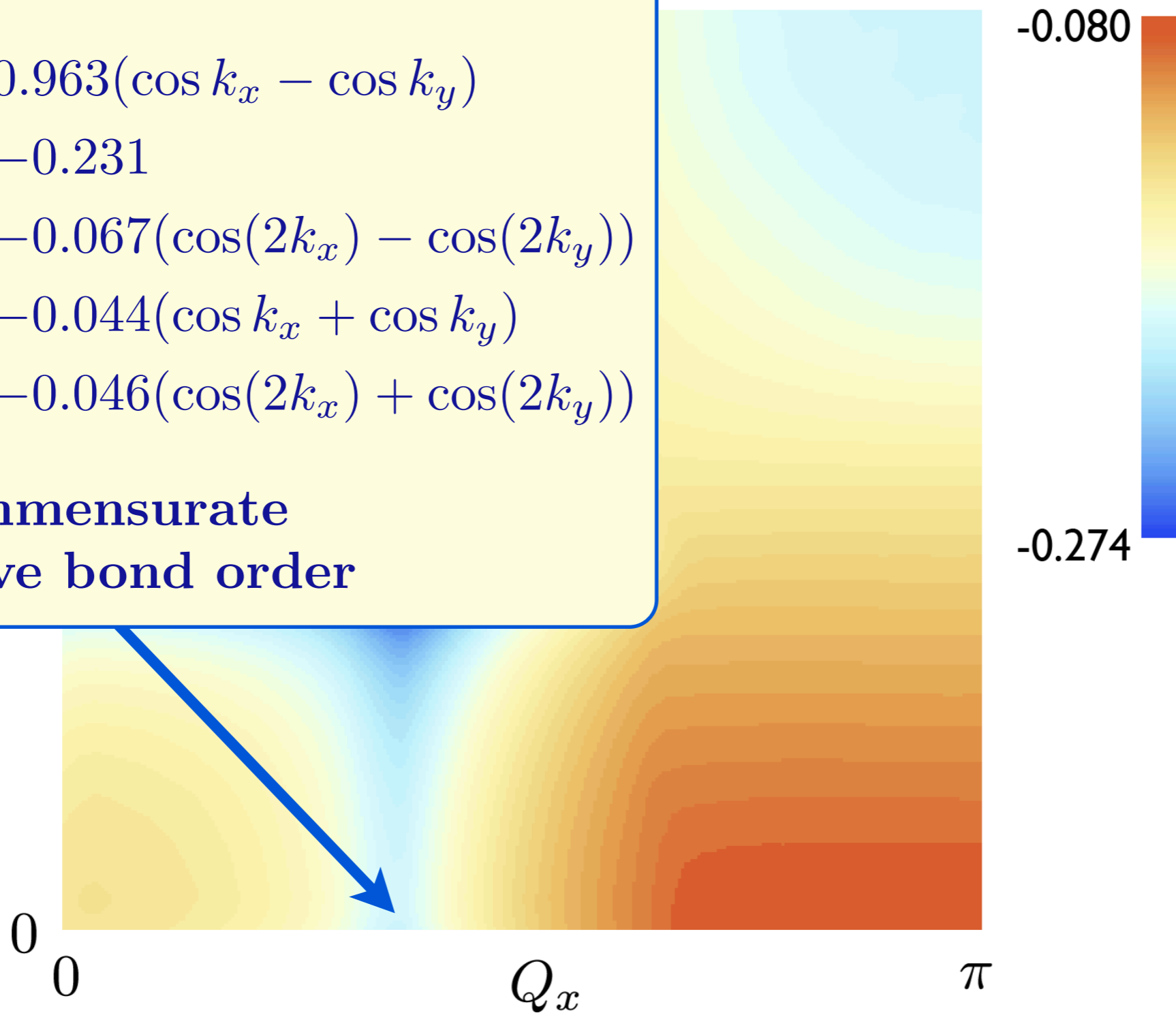


Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

$\mathbf{Q} = (Q_m, 0)$ with

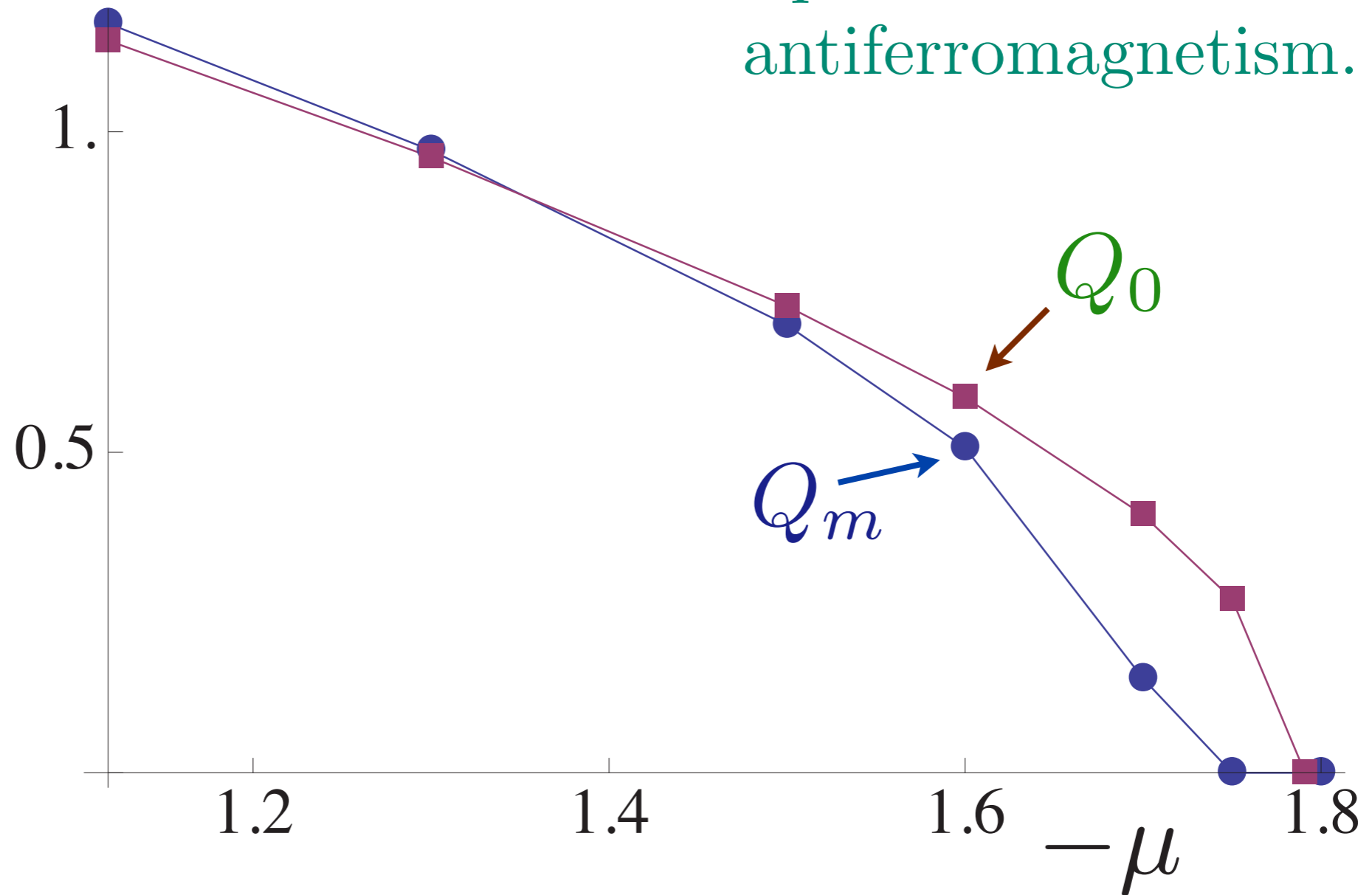
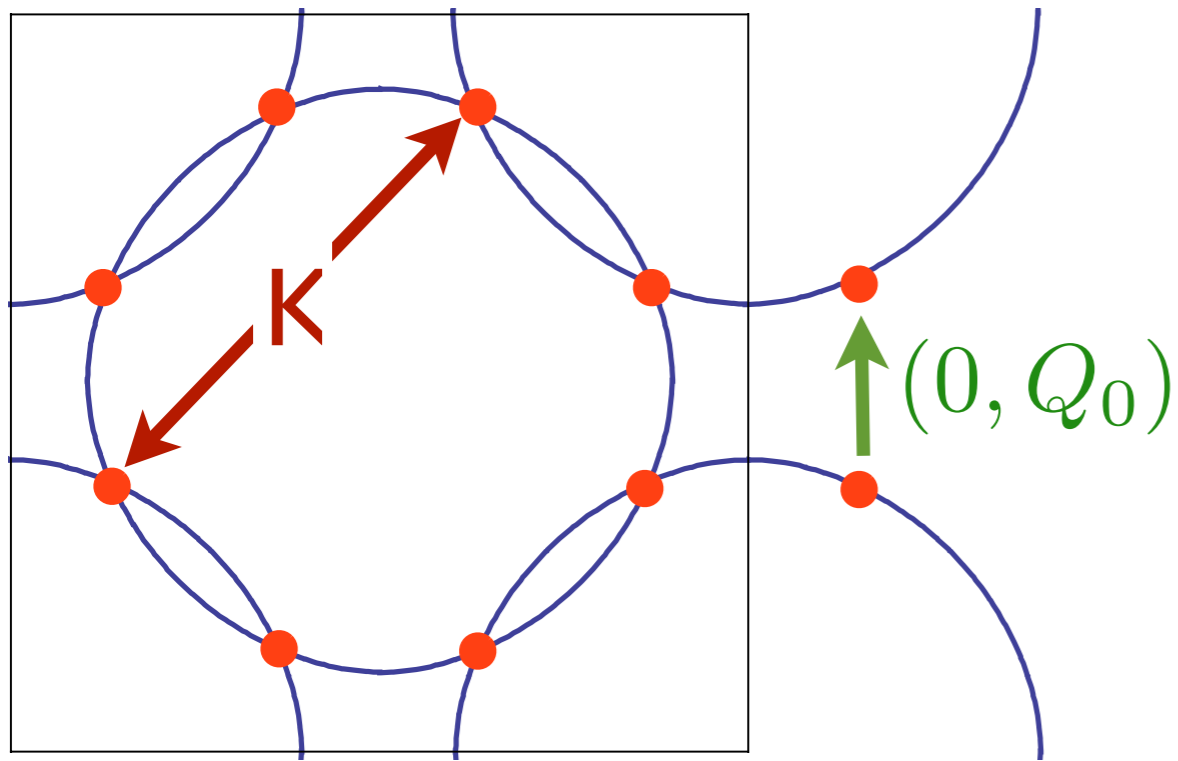
$$\Delta_{\mathbf{Q}}(\mathbf{k}) = \begin{aligned} &0.963(\cos k_x - \cos k_y) \\ &-0.231 \\ &-0.067(\cos(2k_x) - \cos(2k_y)) \\ &-0.044(\cos k_x + \cos k_y) \\ &-0.046(\cos(2k_x) + \cos(2k_y)) \end{aligned}$$

Incommensurate
 d_{+s} -wave bond order

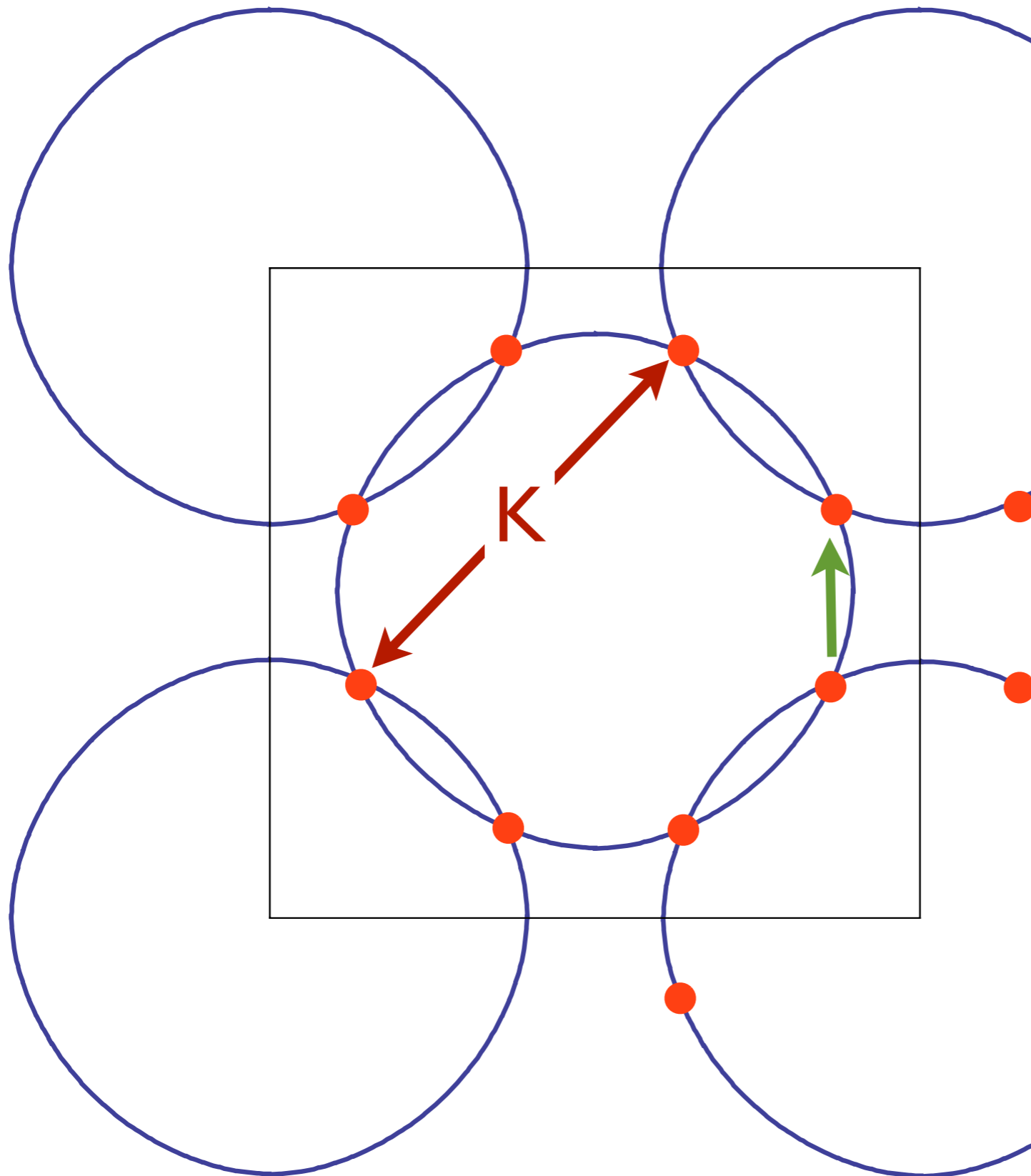


Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}/J_0$.

Remarkable agreement between the value of Q_m from Hartree-Fock in a metal with short-range *incommensurate* spin correlations, and the value of Q_0 from hot spots of *commensurate* antiferromagnetism.



Incommensurate d -wave bond order



Observed low T
ordering.

Our computations show
that the charge order is
predominantly d -wave
also at this Q .

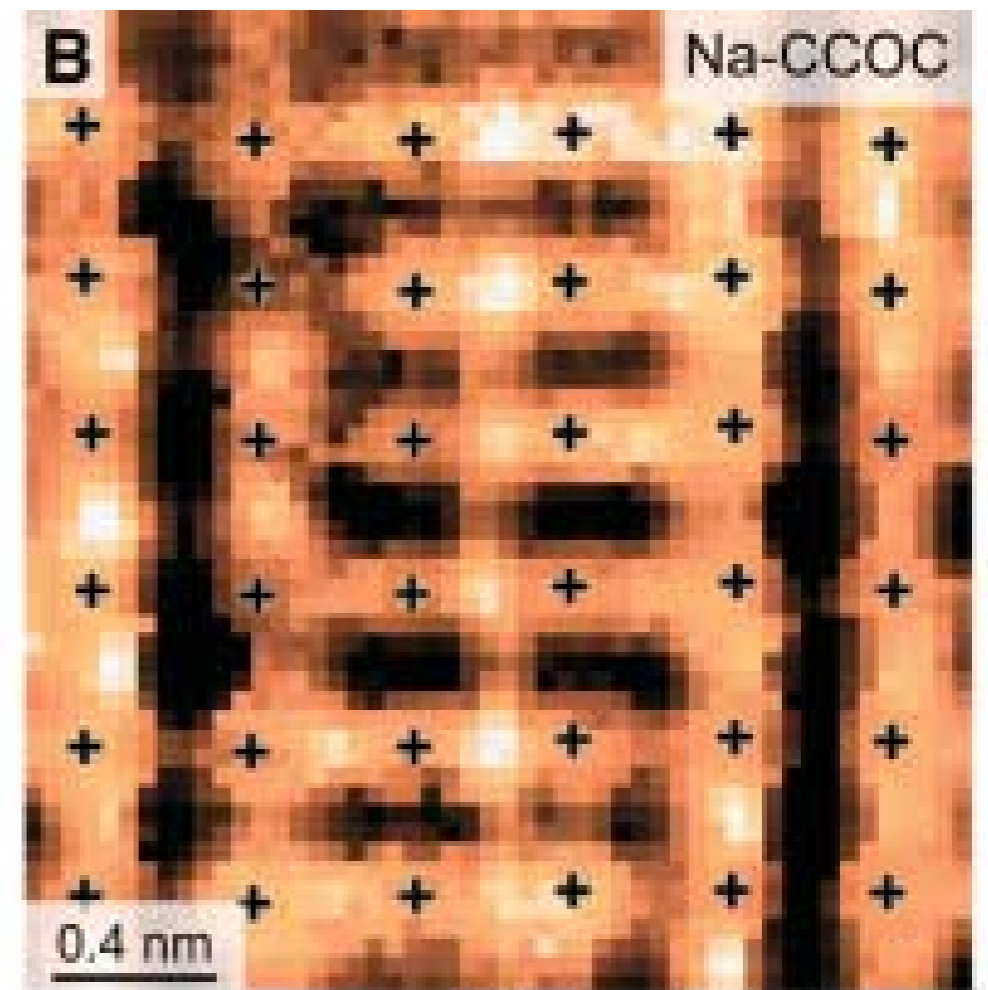
This Q is preferred in
computations of bond
order within the
superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters in press; arXiv:1303.2114
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)

An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,¹ C. Taylor,¹ K. Fujita,^{1,2} A. Schmidt,¹ C. Lupien,³ T. Hanaguri,⁴ M. Azuma,⁵ M. Takano,⁵ H. Eisaki,⁶ H. Takagi,^{2,4} S. Uchida,^{2,7} J. C. Davis^{1,8*}

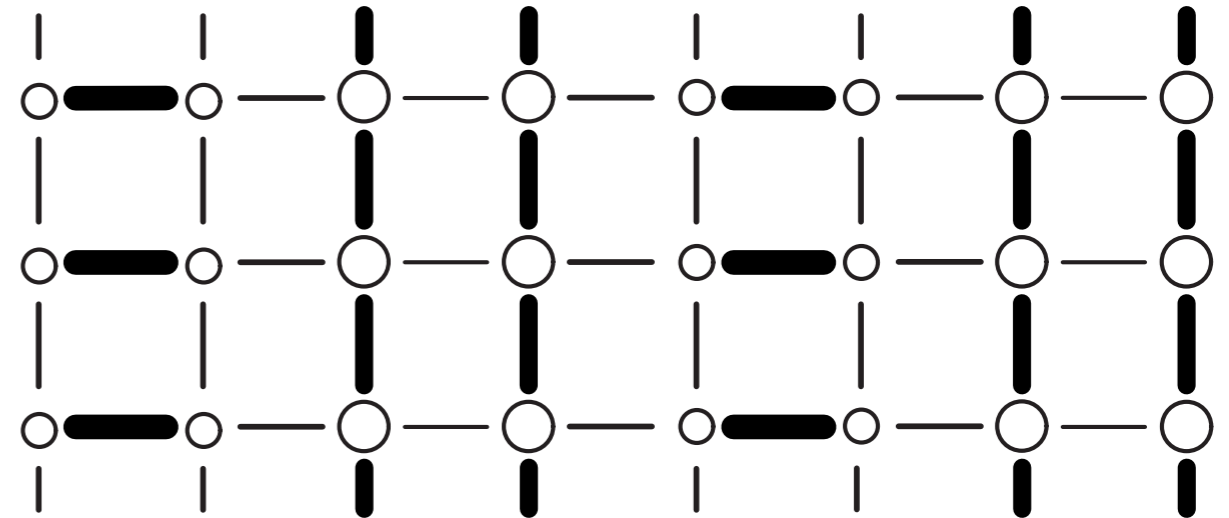
9 MARCH 2007 VOL 315 SCIENCE



PHYSICAL REVIEW B 77, 094504 (2008)

Superconducting *d*-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

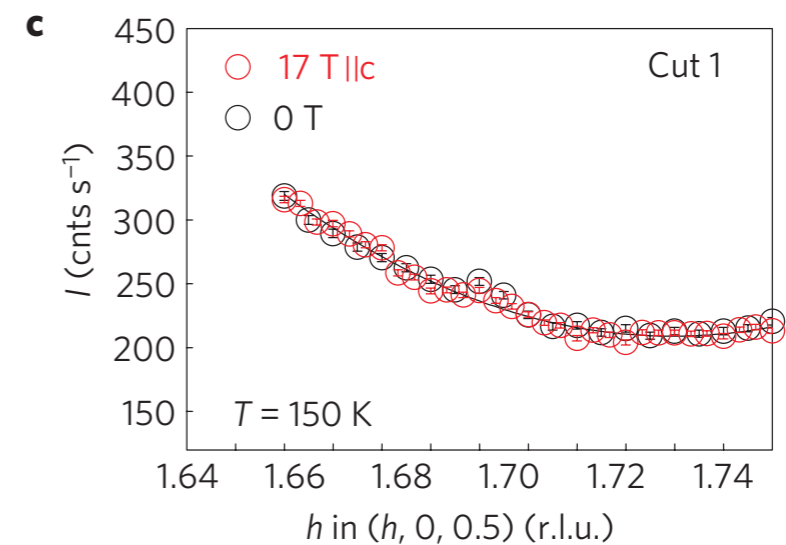
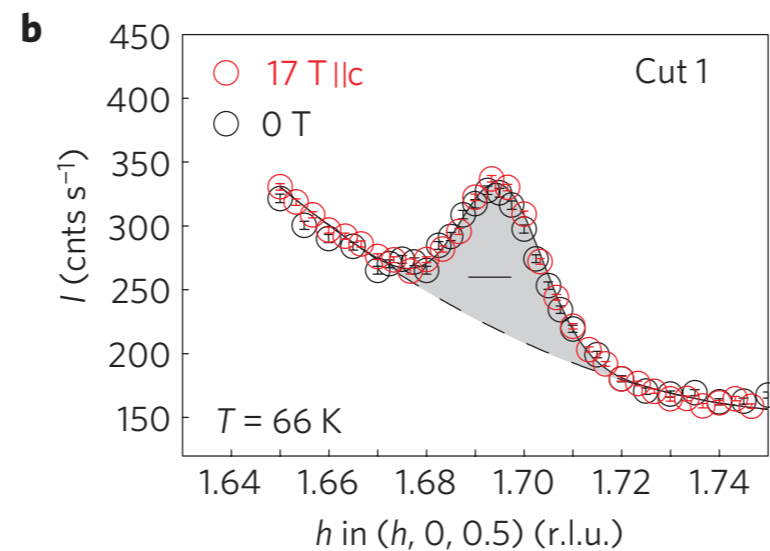
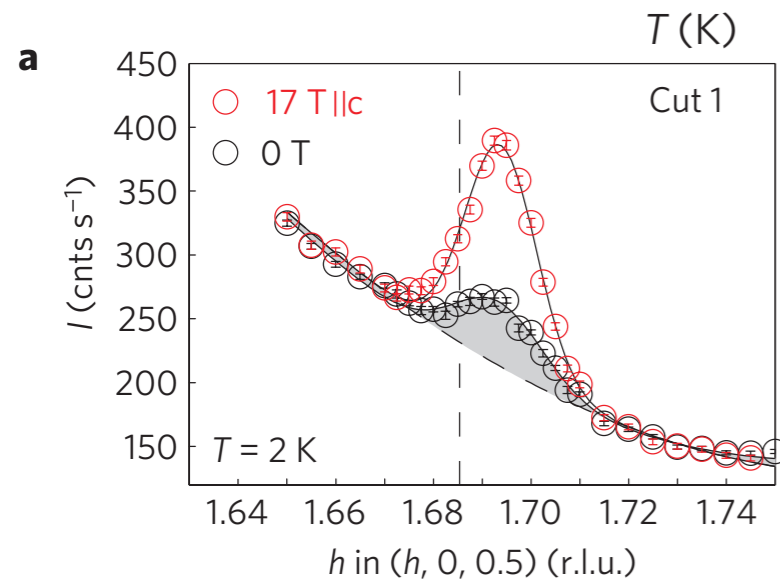
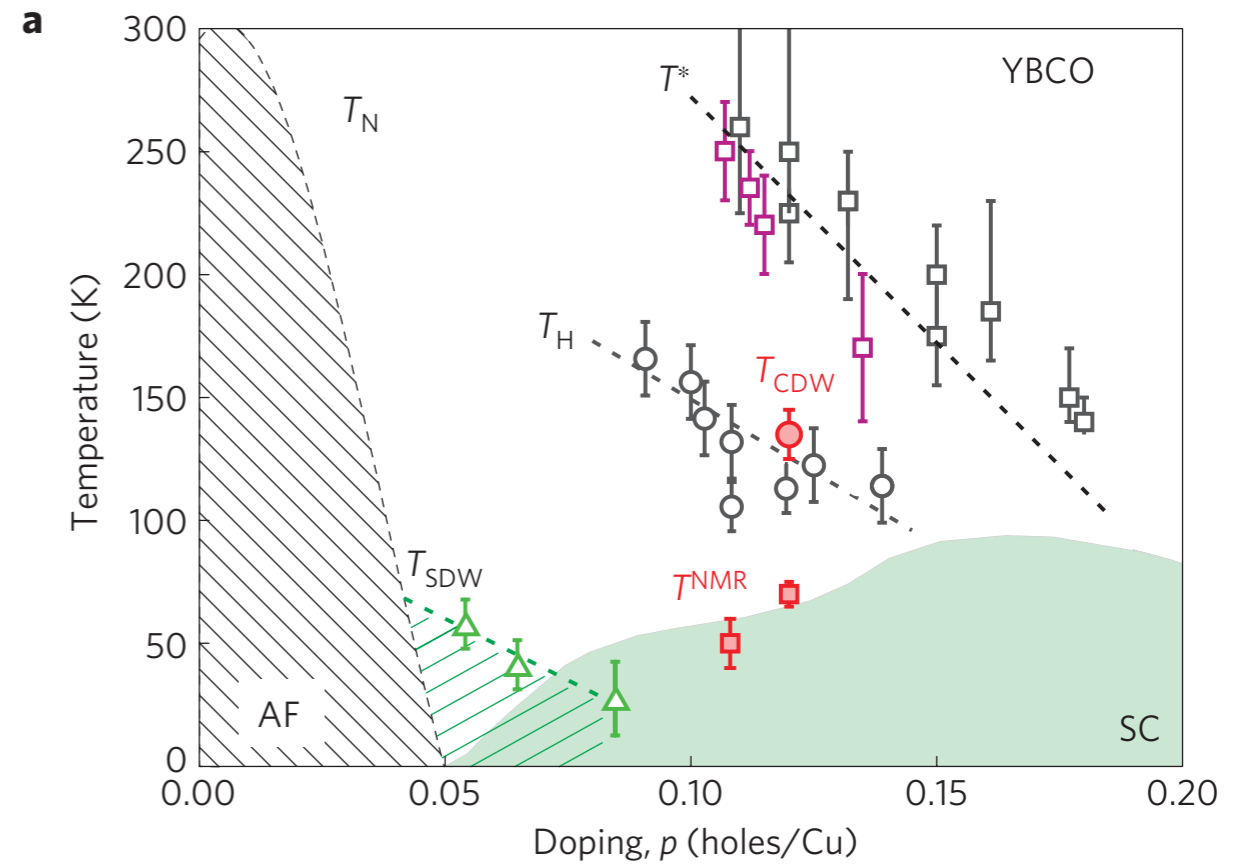
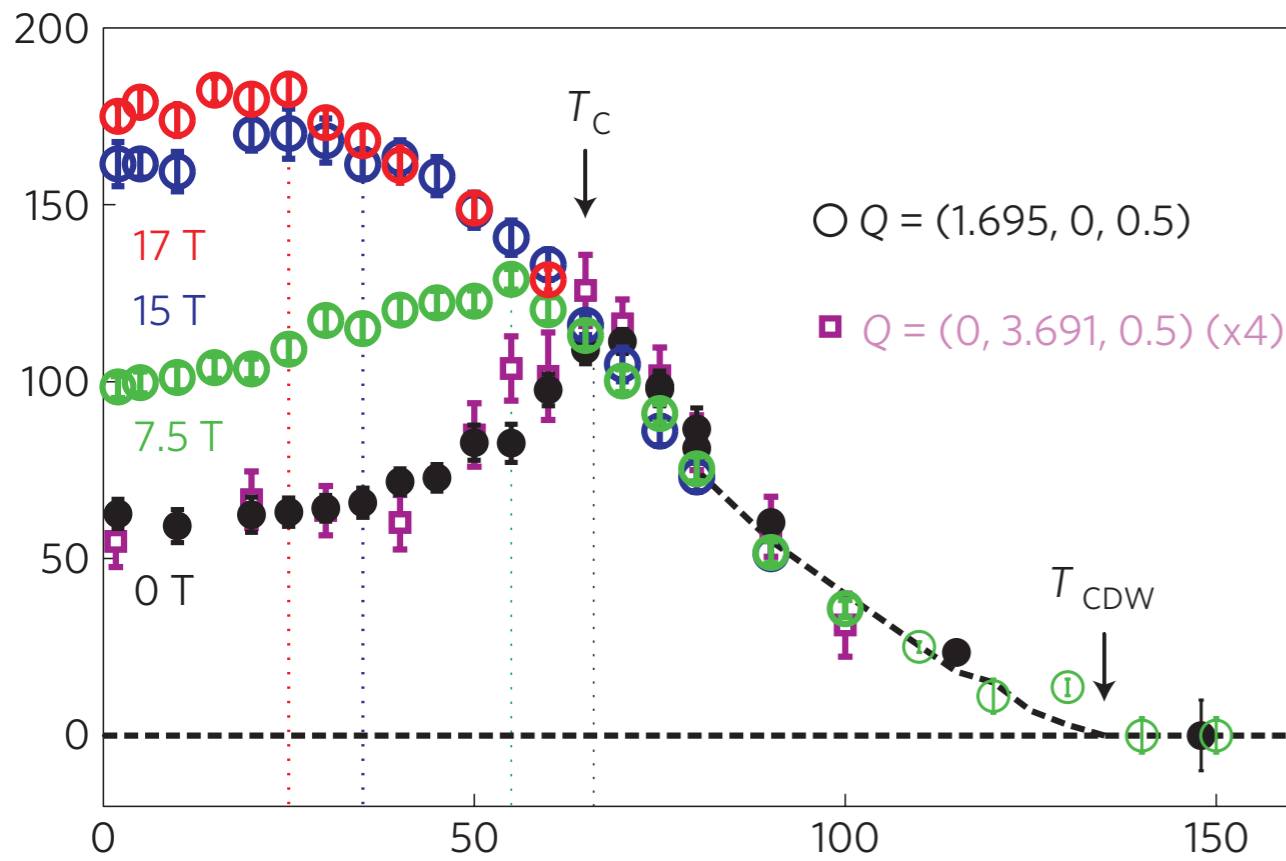
Matthias Vojta and Oliver Rösch



We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both *s*- and *d*-wave channels, with nonlocal Coulomb repulsion suppressing the *s*-wave component.

Direct observation of competition between superconductivity and charge density wave order in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

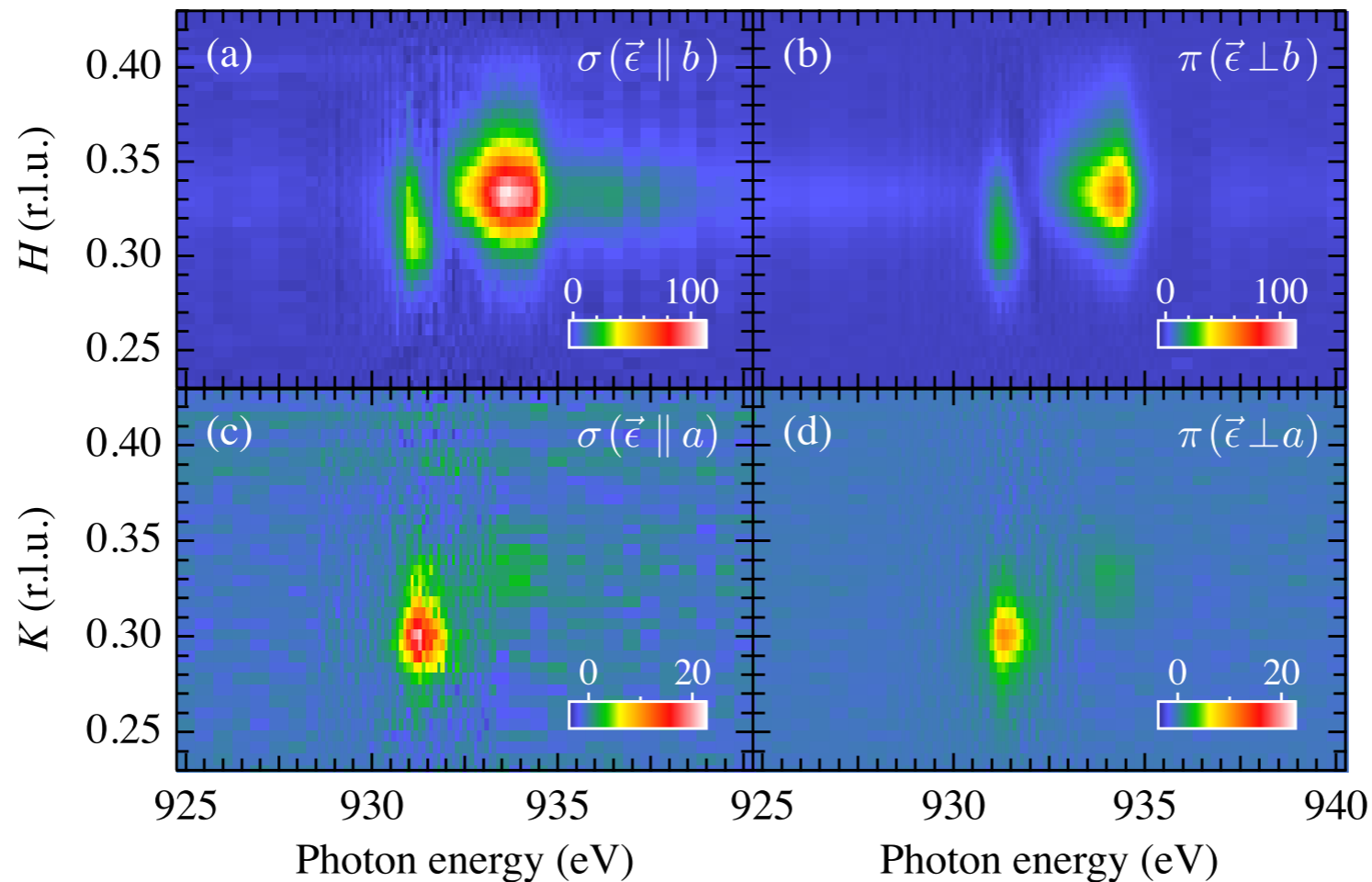
J. Chang^{1,2*}, E. Blackburn³, A. T. Holmes³, N. B. Christensen⁴, J. Larsen^{4,5}, J. Mesot^{1,2}, Ruixing Liang^{6,7}, D. A. Bonn^{6,7}, W. N. Hardy^{6,7}, A. Watenphul⁸, M. v. Zimmermann⁸, E. M. Forgan³ and S. M. Hayden⁹



Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $\text{YBa}_2\text{Cu}_3\text{O}_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

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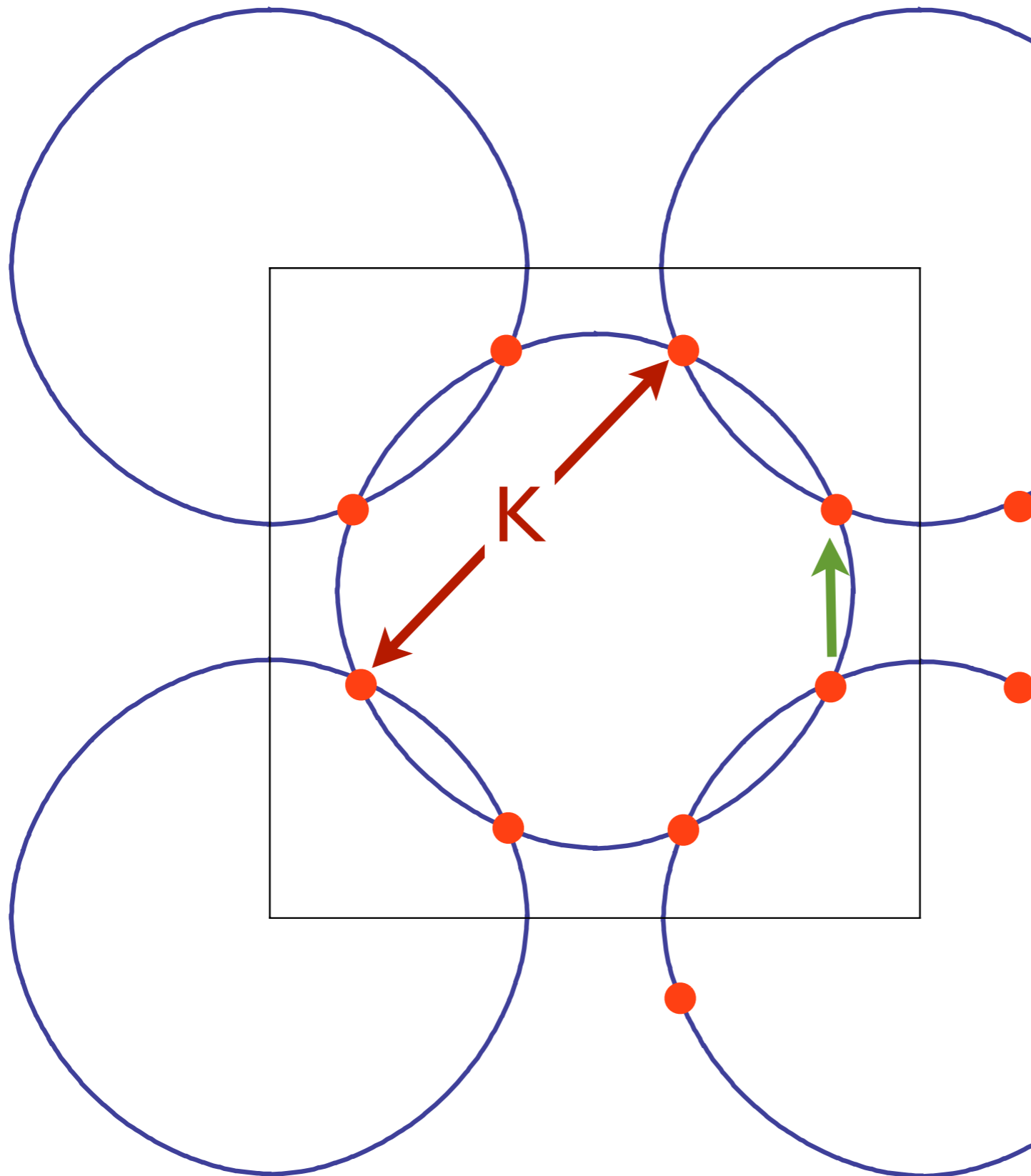
PRL **109**, 167001 (2012)



Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu $2p$ to $3d_{x^2-y^2}$ transition, similar to stripe-ordered 214 cuprates.

These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.

Incommensurate d -wave bond order



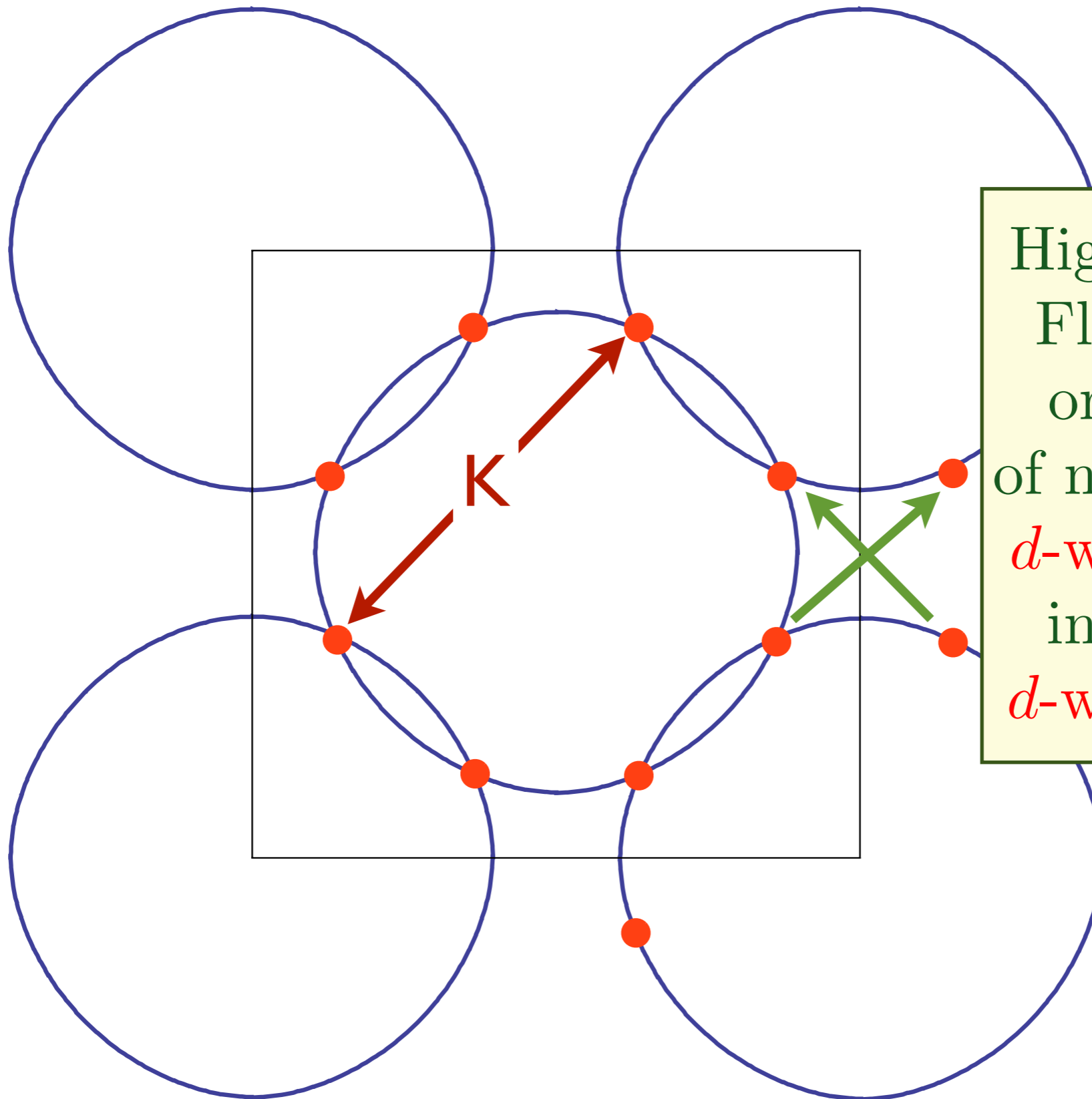
Observed low T
ordering.

Our computations show
that the charge order is
predominantly d -wave
also at this Q .

This Q is preferred in
computations of bond
order within the
superconducting phase.

S. Sachdev and R. La Placa, Physical Review Letters in press; arXiv:1303.2114
M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)

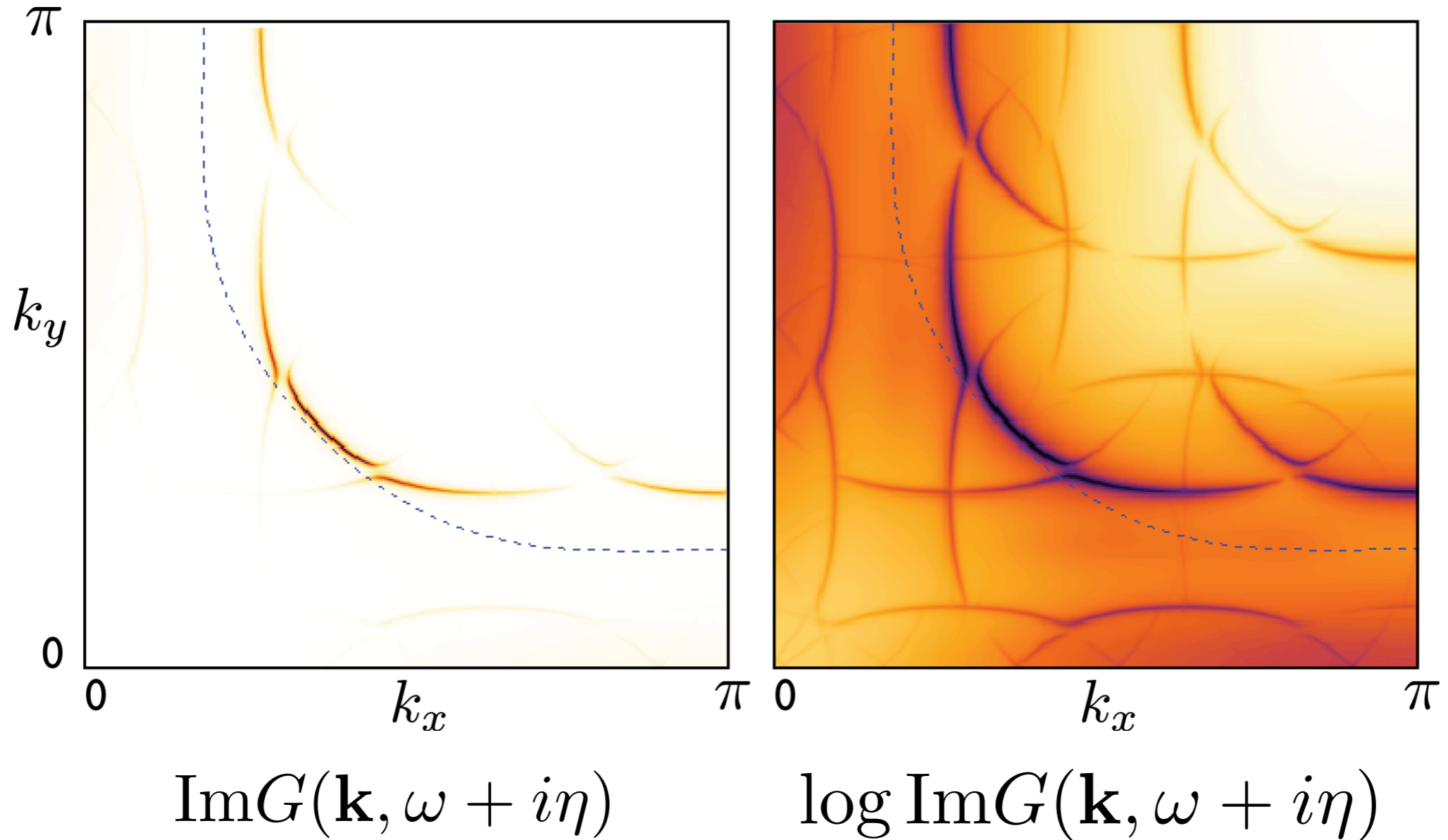
Incommensurate d -wave bond order



High T pseudogap:
Fluctuating $O(4)$
order parameter
of nearly degenerate
 d -wave pairing and
incommensurate
 d -wave bond order.

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Electron spectral function



$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle \propto \Delta_{\mathbf{Q}}(\mathbf{k}) = \begin{cases} \Delta_s + \Delta_d(\cos k_x - \cos k_y) & , \quad \mathbf{Q} = (\pm Q_0, 0) \\ \Delta_s - \Delta_d(\cos k_x - \cos k_y) & , \quad \mathbf{Q} = (0, \pm Q_0) \end{cases}$$

$$\text{with } \Delta_s/\Delta_d = -0.234.$$

Summary

Antiferromagnetism in metals and the high temperature superconductors

- Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)

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- Antiferromagnetic quantum criticality leads to d -wave superconductivity (supported by sign-problem-free Monte Carlo simulations)
- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to d -wave superconductivity, and to a charge density wave with a d -wave form factor. This is a promising explanation of the pseudogap regime.