Quantum spin liquids and the cuprate high temperature superconductors

University of Toronto
March 22, 2024
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. Sachdev, *Proc. Nat. Acad. Sci.* **120**, e2302701120 (2023)

Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.Sachdev, *Phys. Rev. B* **108**, 045123 (2023)

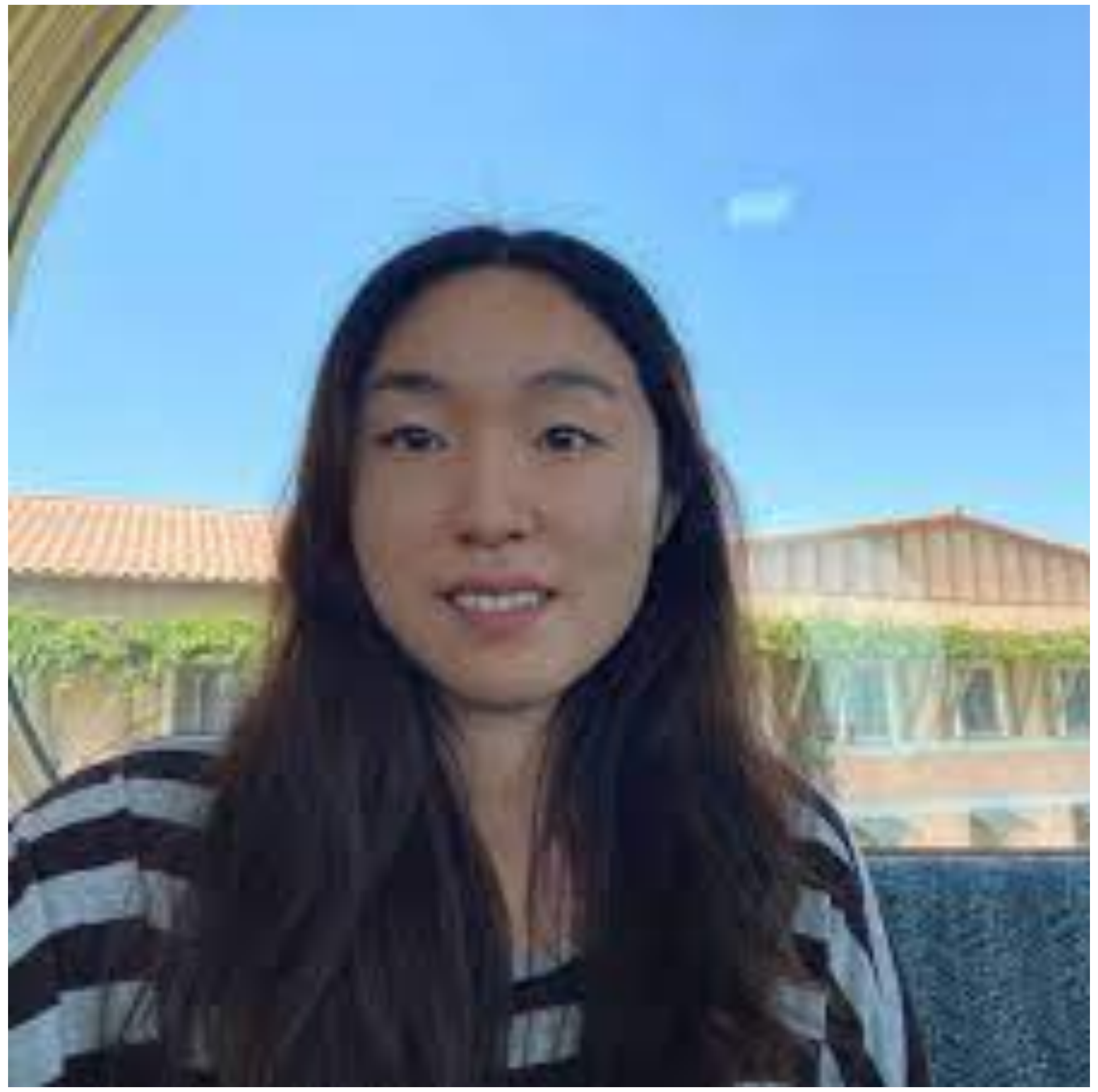
Maine Christos and S.Sachdev, *npj Quantum Materials* **9**, 4 (2024)

M. Christos, H. Shackleton, S. Sachdev, and Zhu-Xi Luo, arXiv:2402.09502

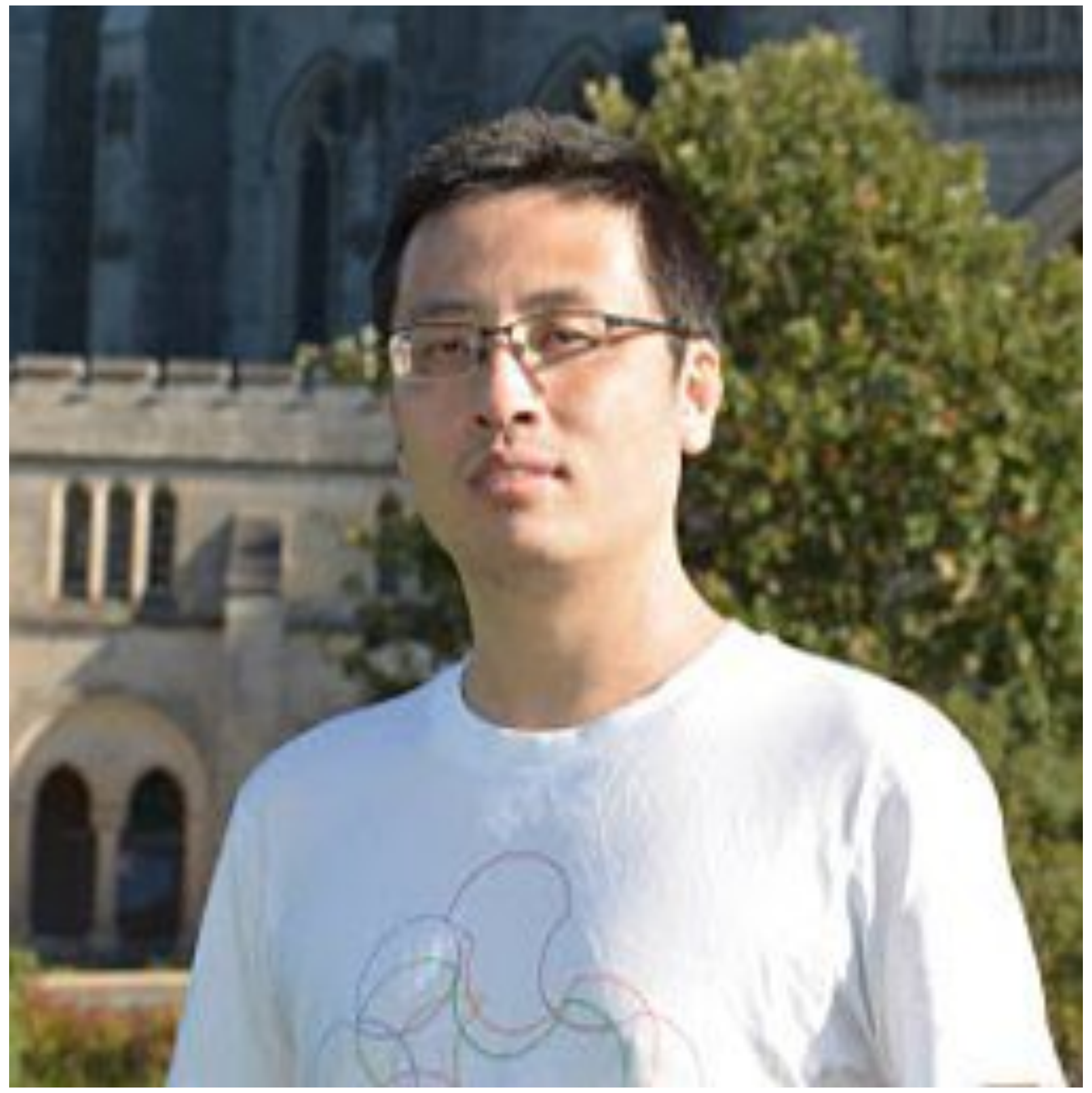
PHYSICS
VE RI TAS
HARVARD

Maine Christos

Mathias Scheurer



Zhu-Xi Luo



Ya-Hui Zhang

Henry Shackleton

S=1/2 square lattice antiferromagnet

From the pseudogap metal to
the d-wave superconductor

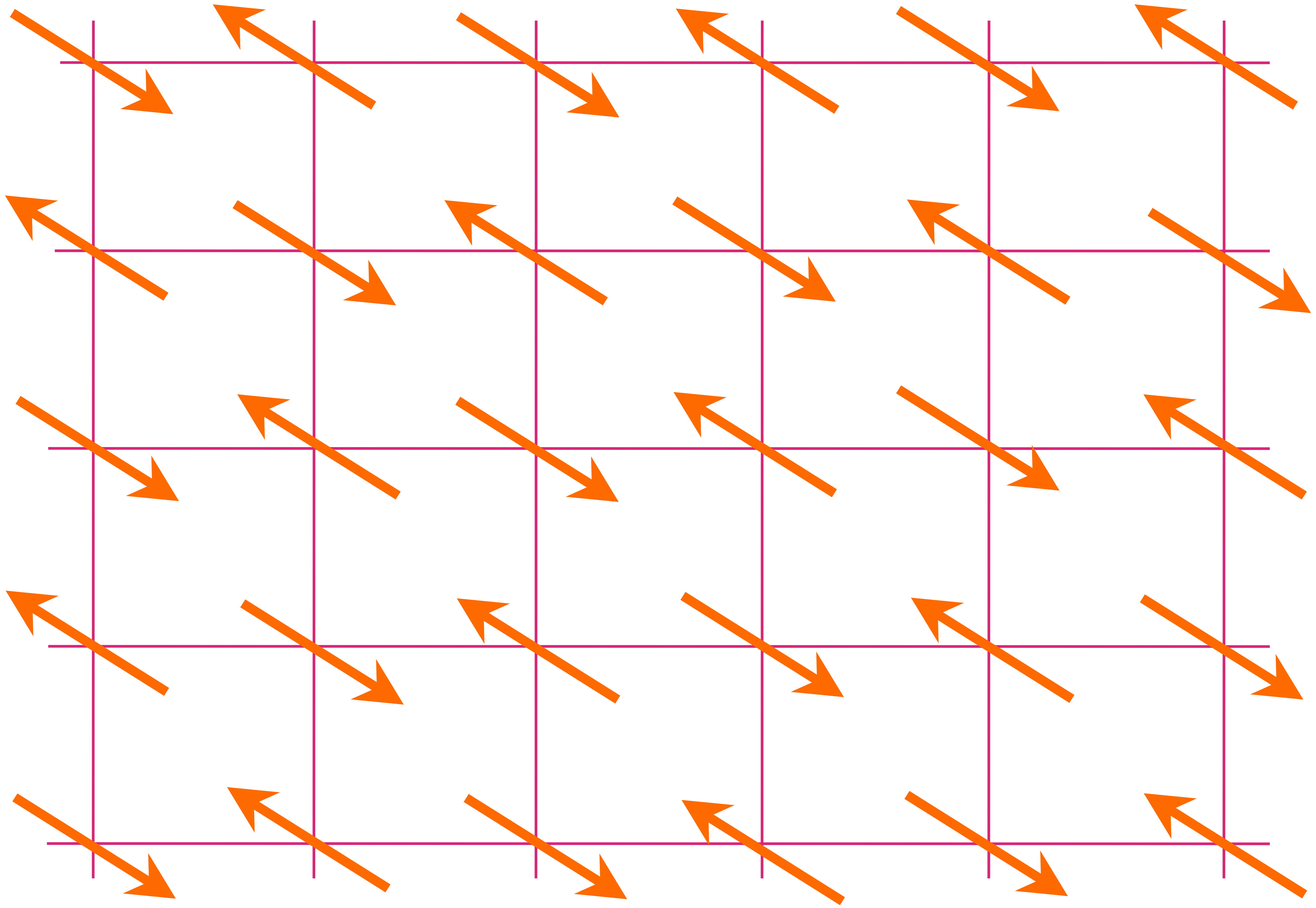
Confinement of the π-flux/CP^1 spin liquid,
d-wave superconductivity, and charge order

S=1/2 square lattice antiferromagnet

From the pseudogap metal to
the d-wave superconductor

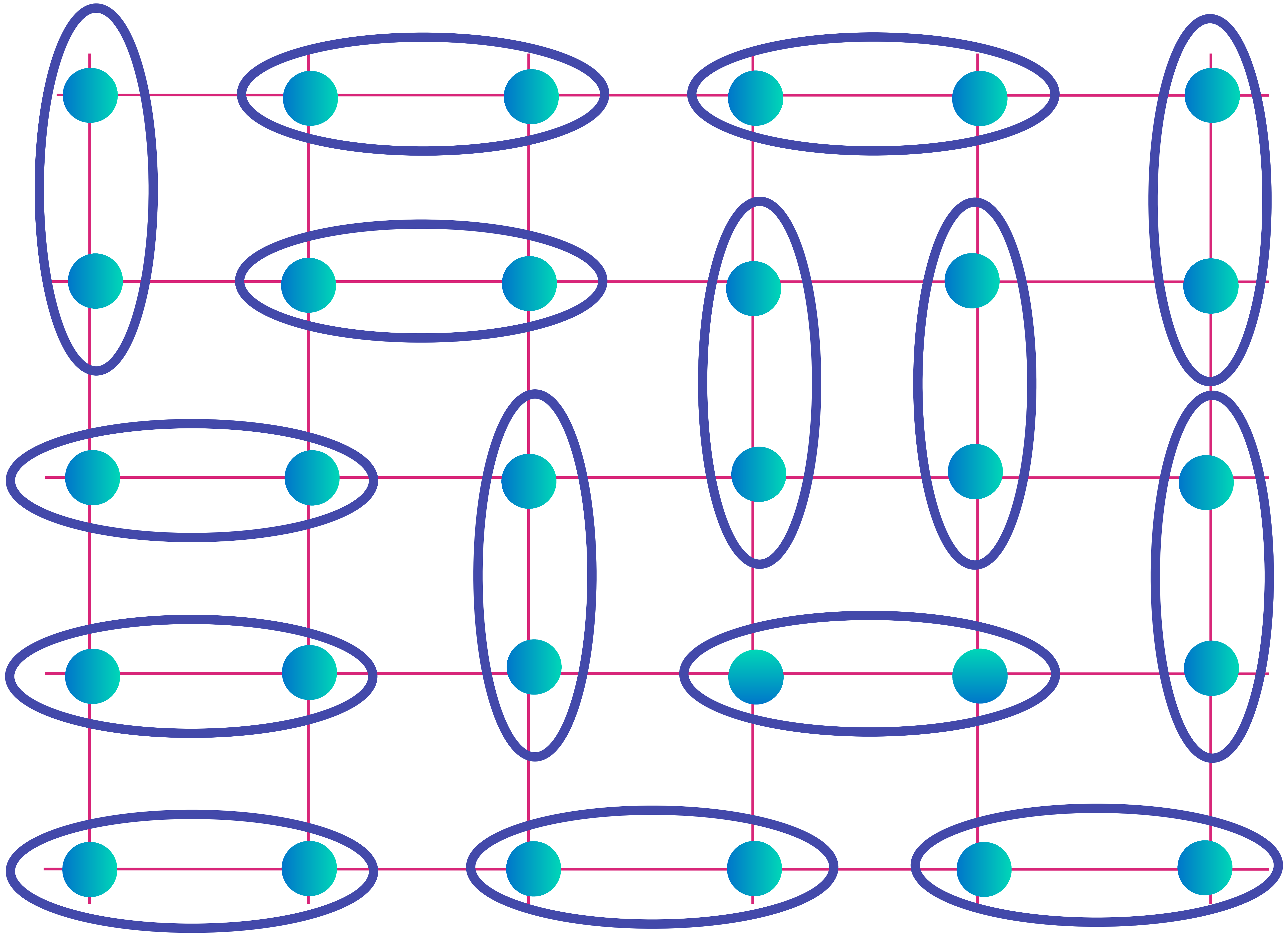
Confinement of the π-flux/CP^1 spin liquid,
d-wave superconductivity, and charge order

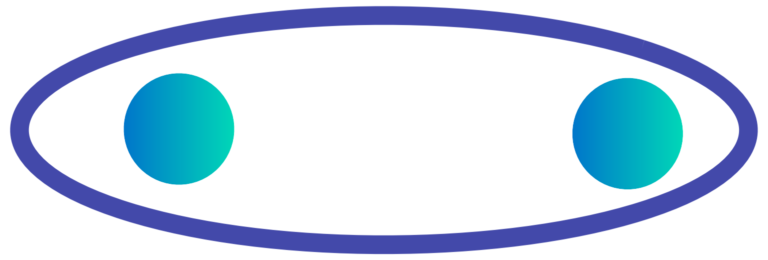
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Insulating antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

P.W. Anderson (1973)

Spin liquid

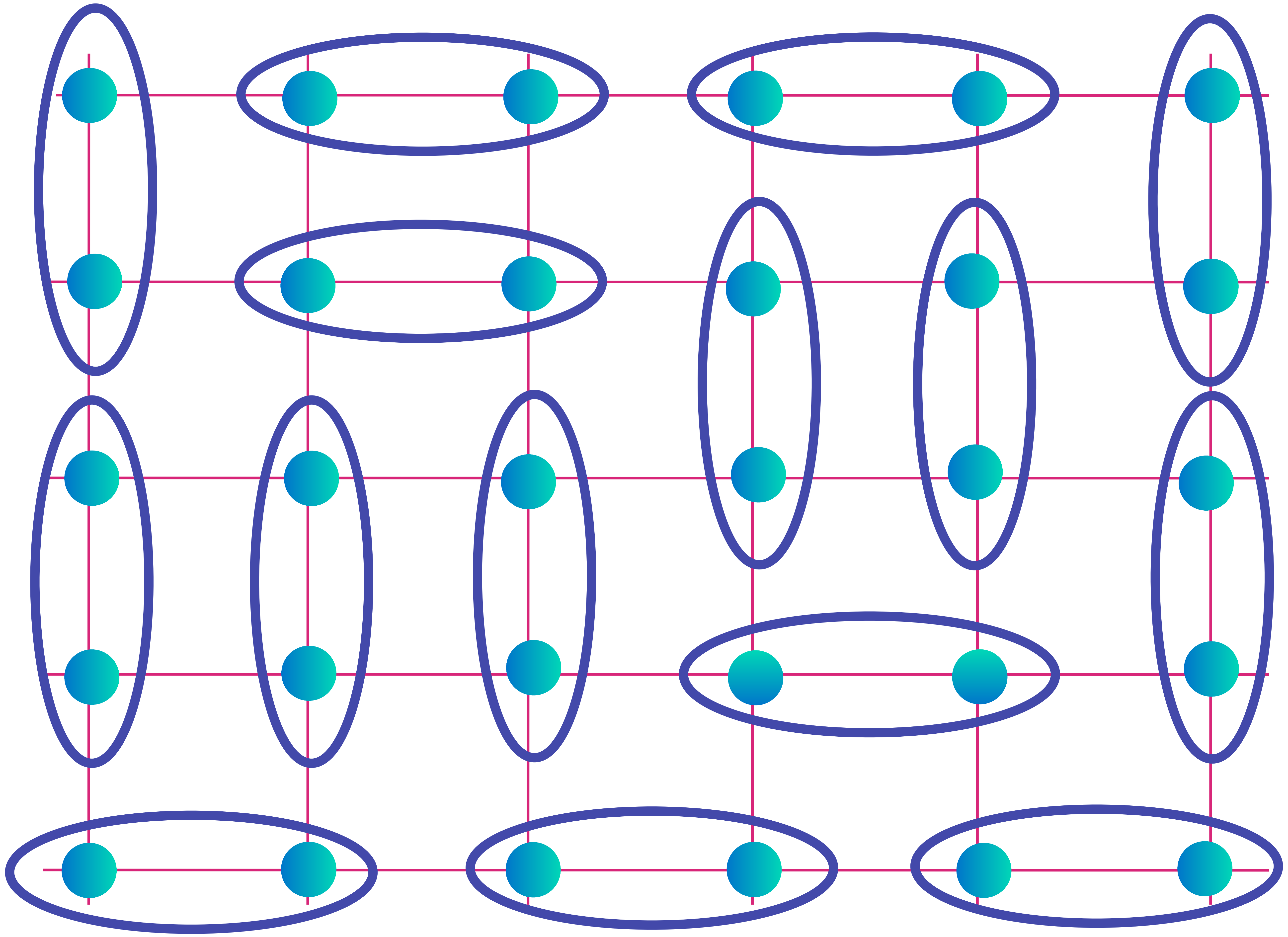
Electrons form entangled pairs, and the pairs entangle across the entire sample

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

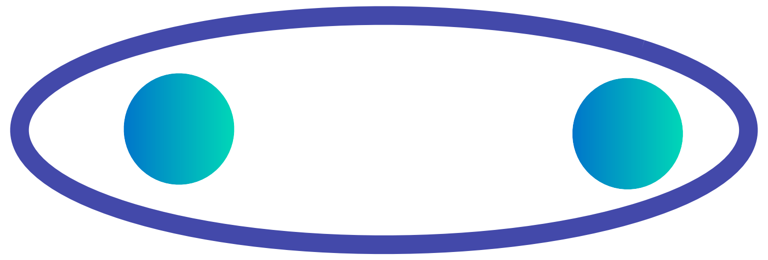
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

P.W. Anderson (1973)

Spin liquid



Electrons form entangled pairs, and the pairs entangle across the entire sample



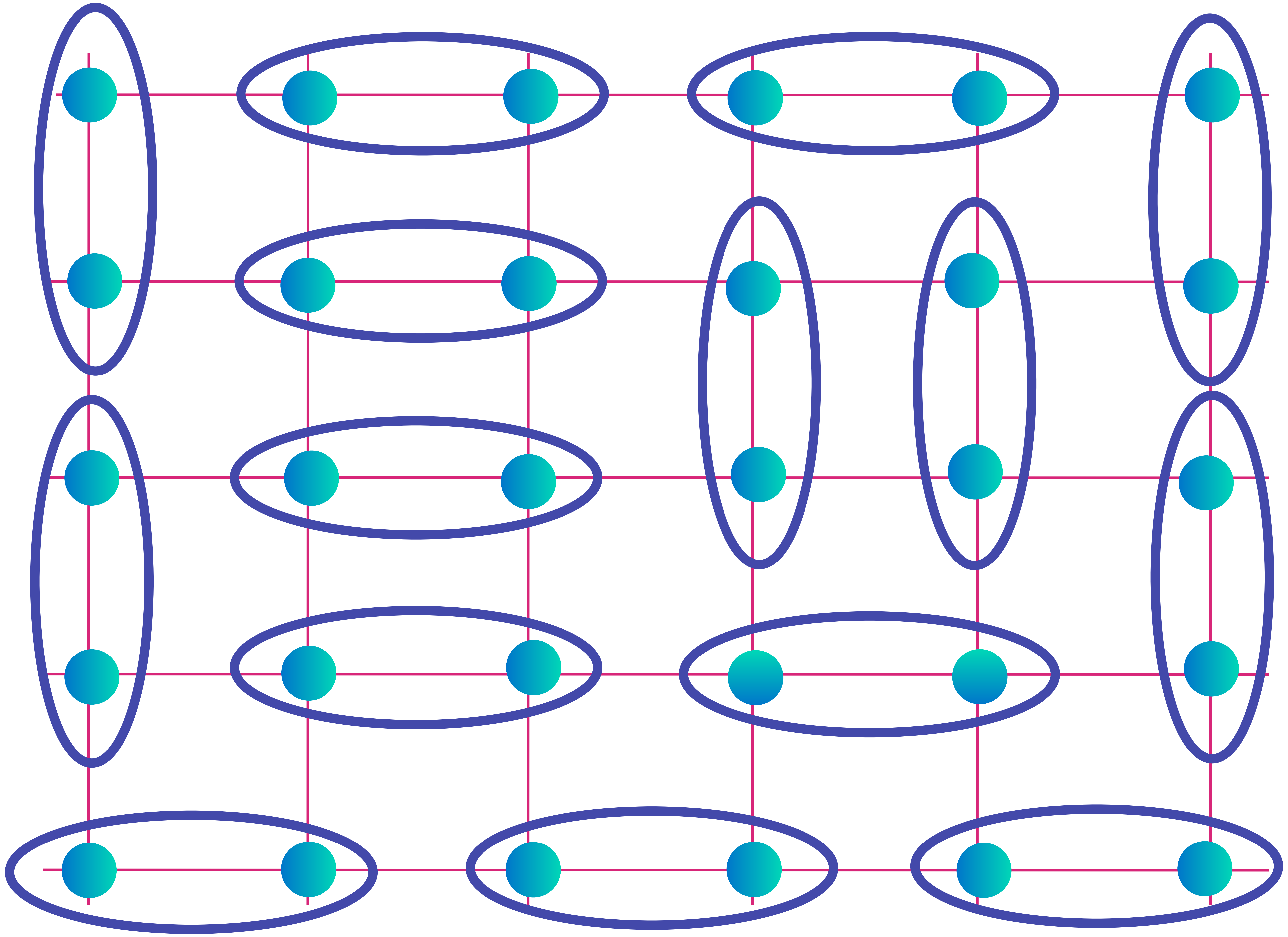
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

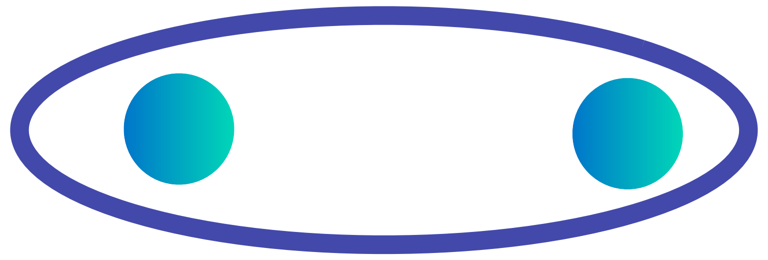
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

P.W. Anderson (1973)

Spin liquid



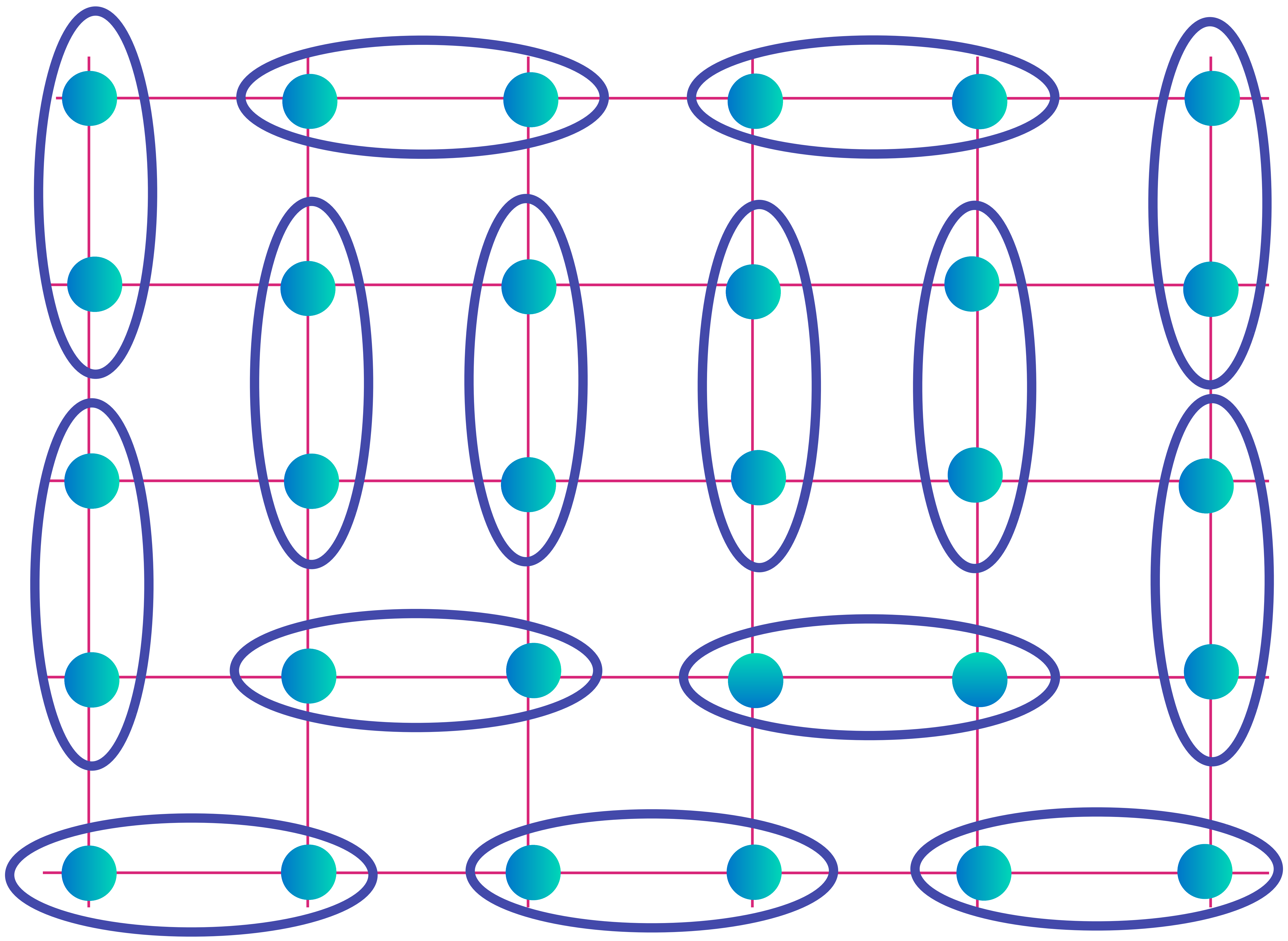
Electrons <u>f</u>orm entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

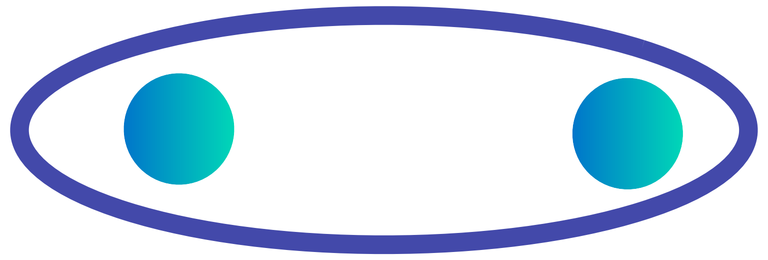
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

P.W. Anderson (1973)



Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample



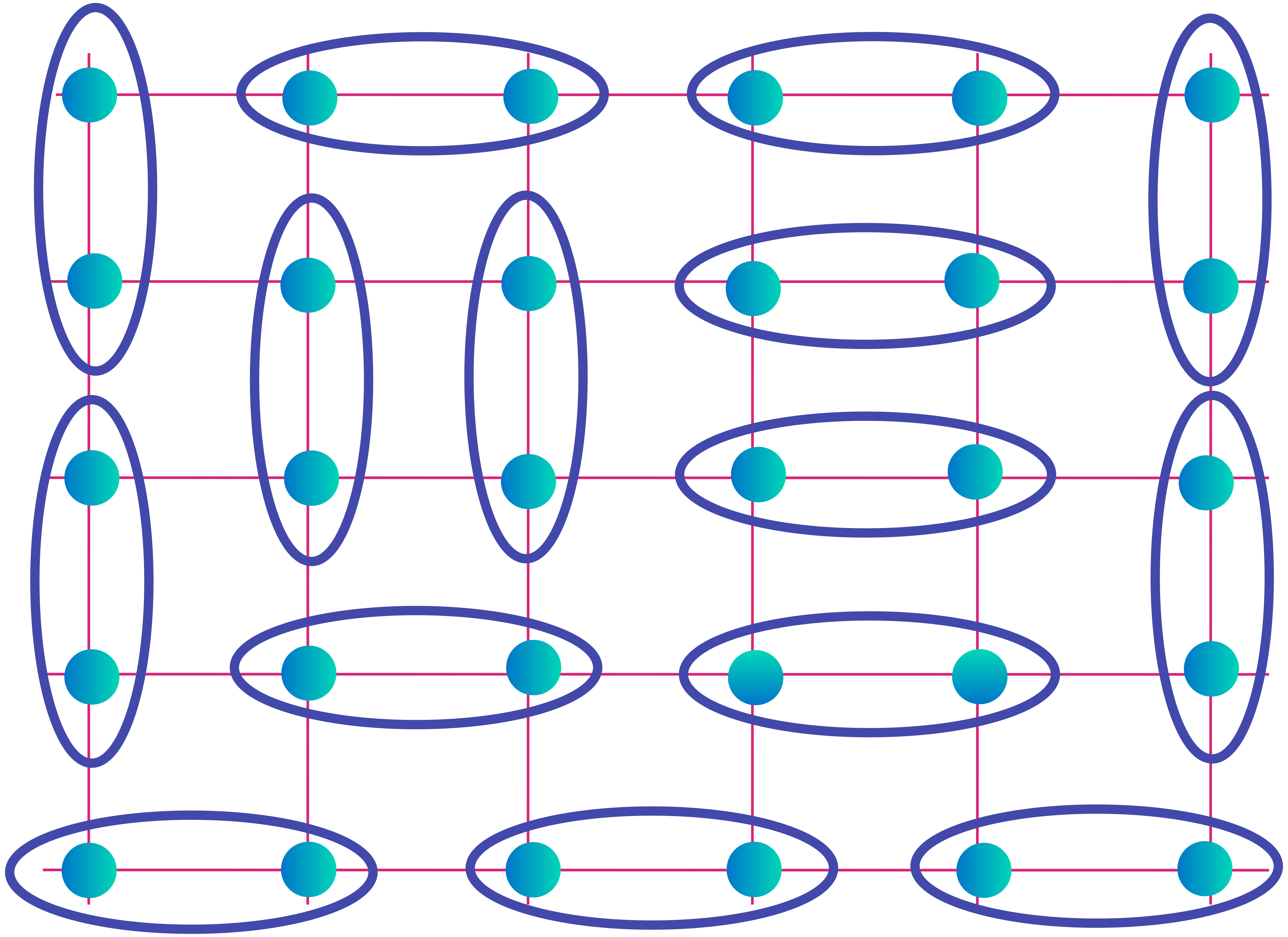
$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

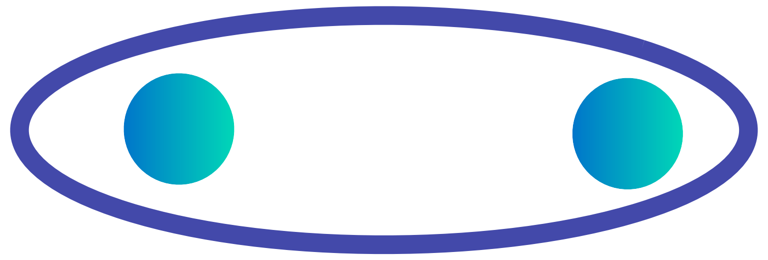
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

P.W. Anderson (1973)

Spin liquid

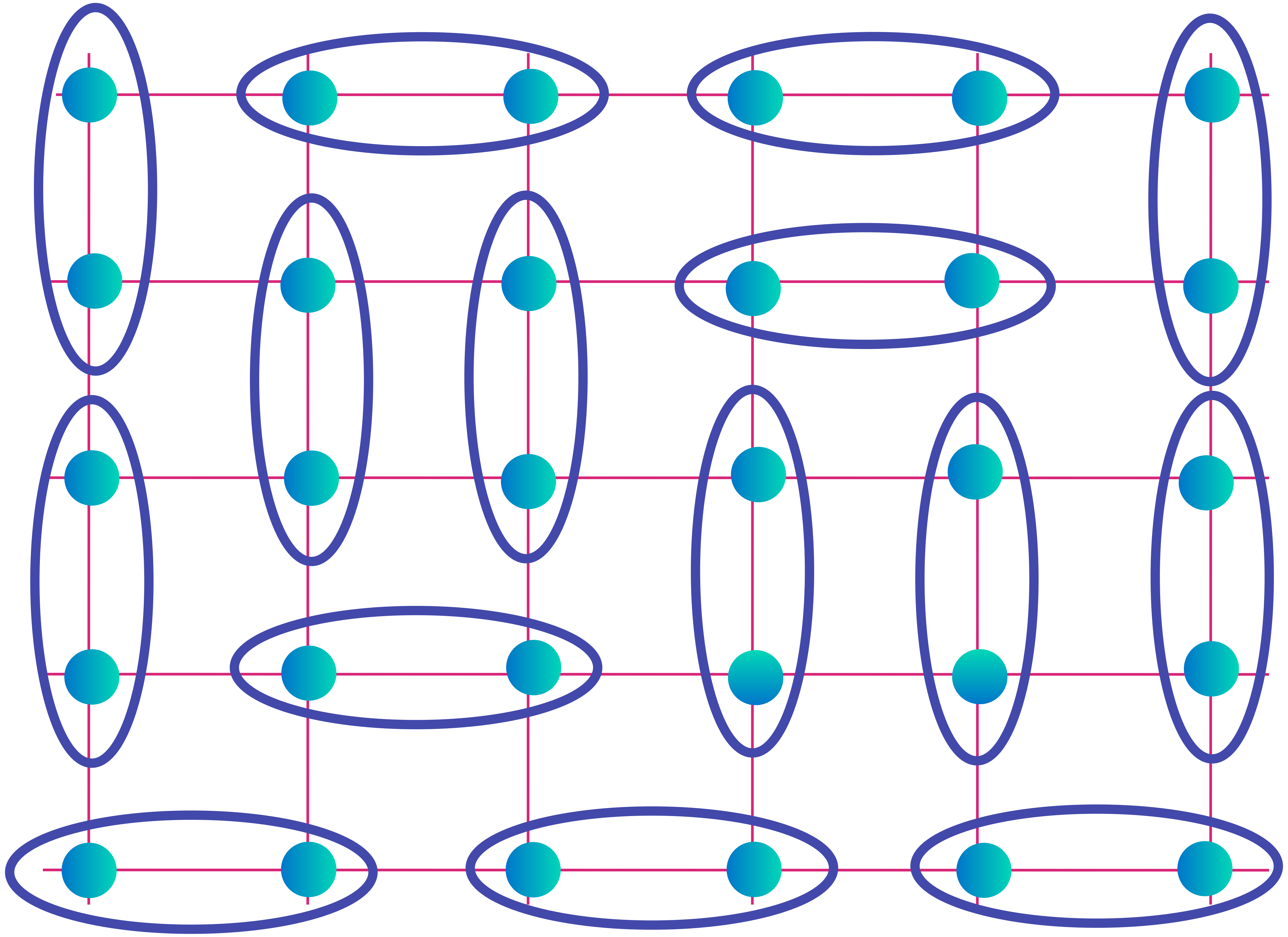


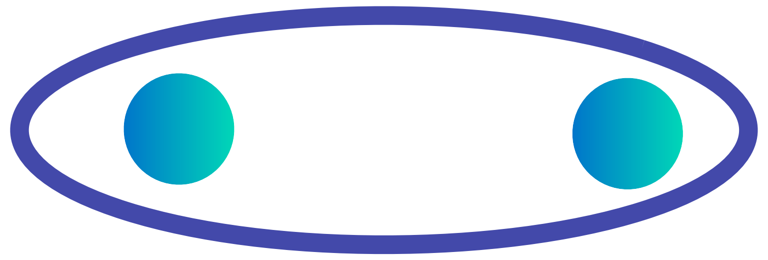
Electrons form entangled pairs, and the pairs entangle across the entire sample

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

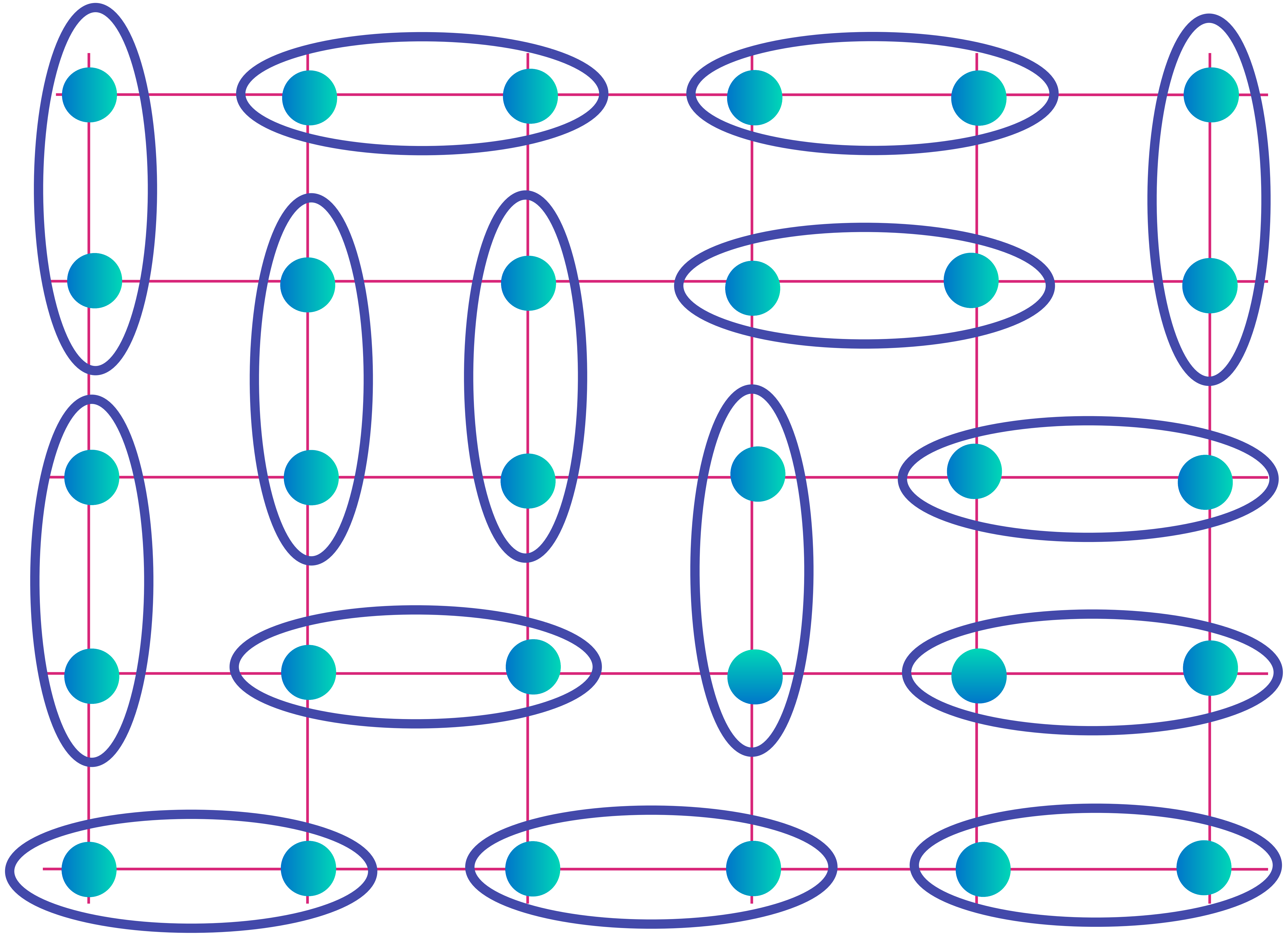
P.W. Anderson (1973)

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

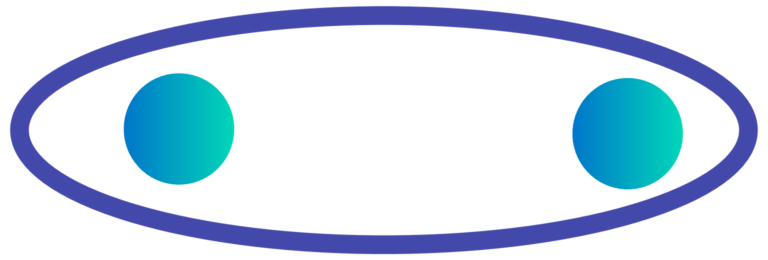
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



P.W. Anderson (1973)

Spin liquid

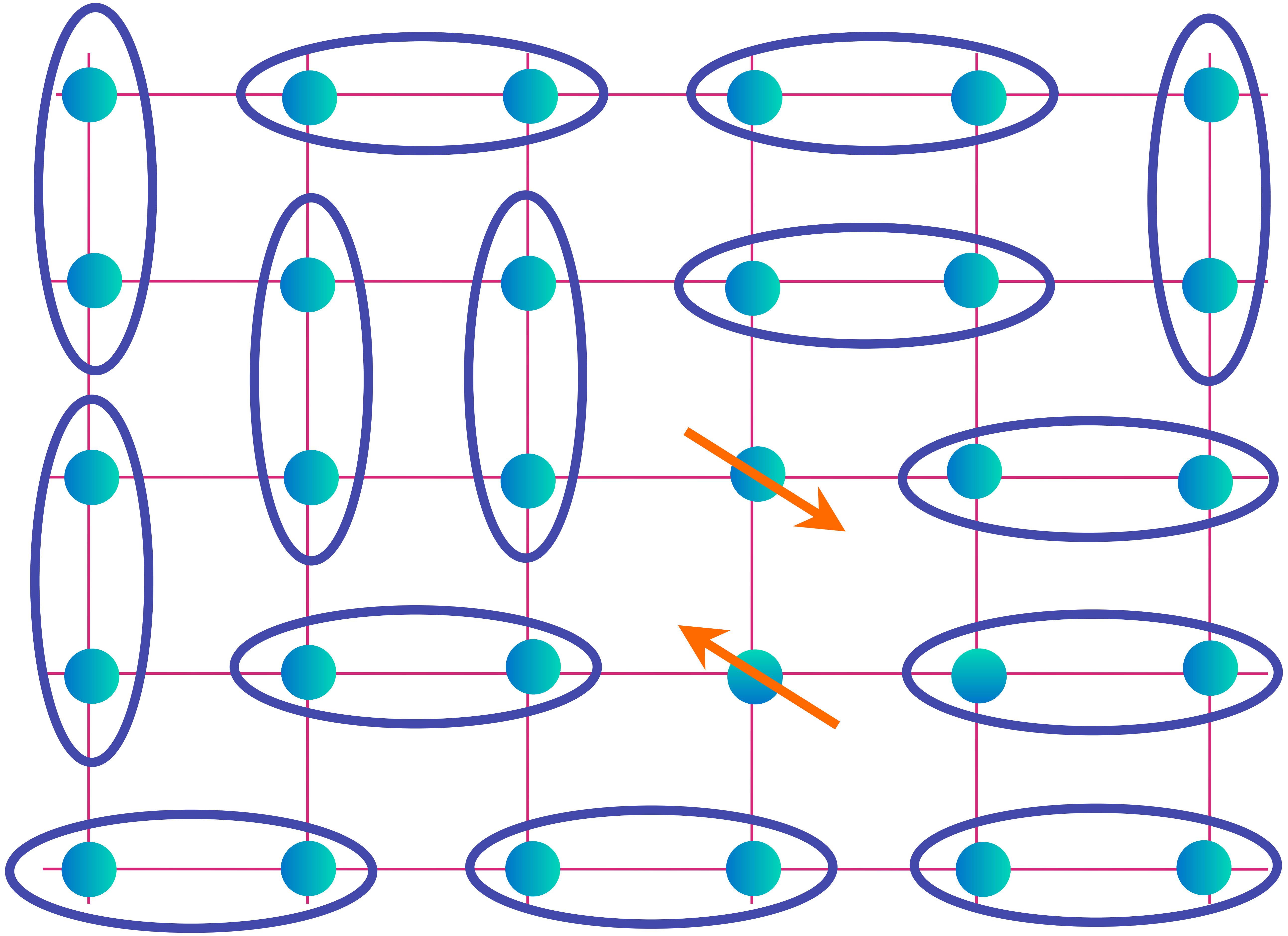
Electrons form entangled pairs, and the pairs entangle across the entire sample

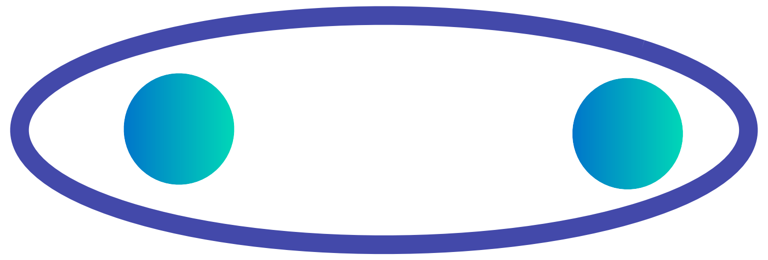


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





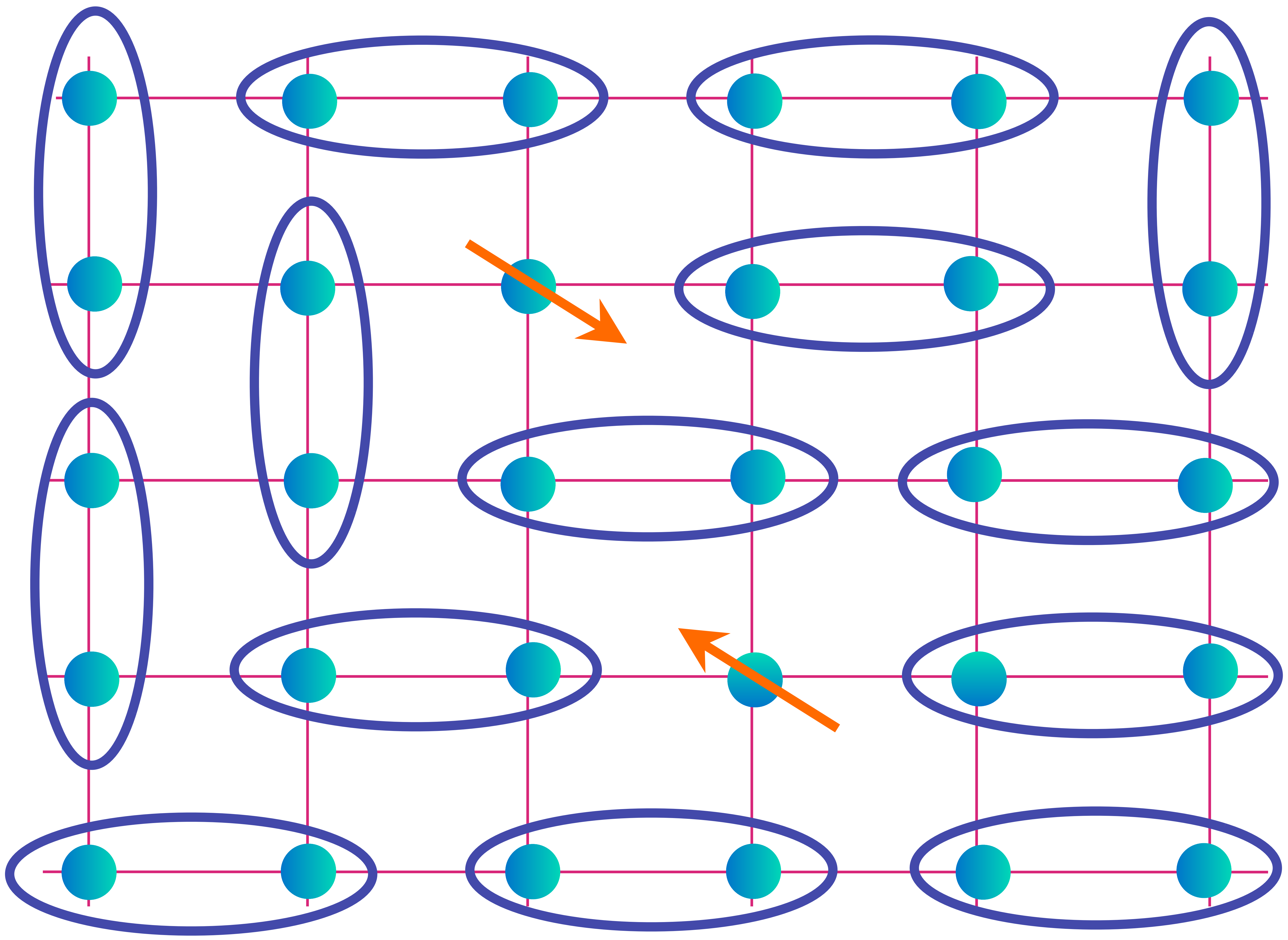
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



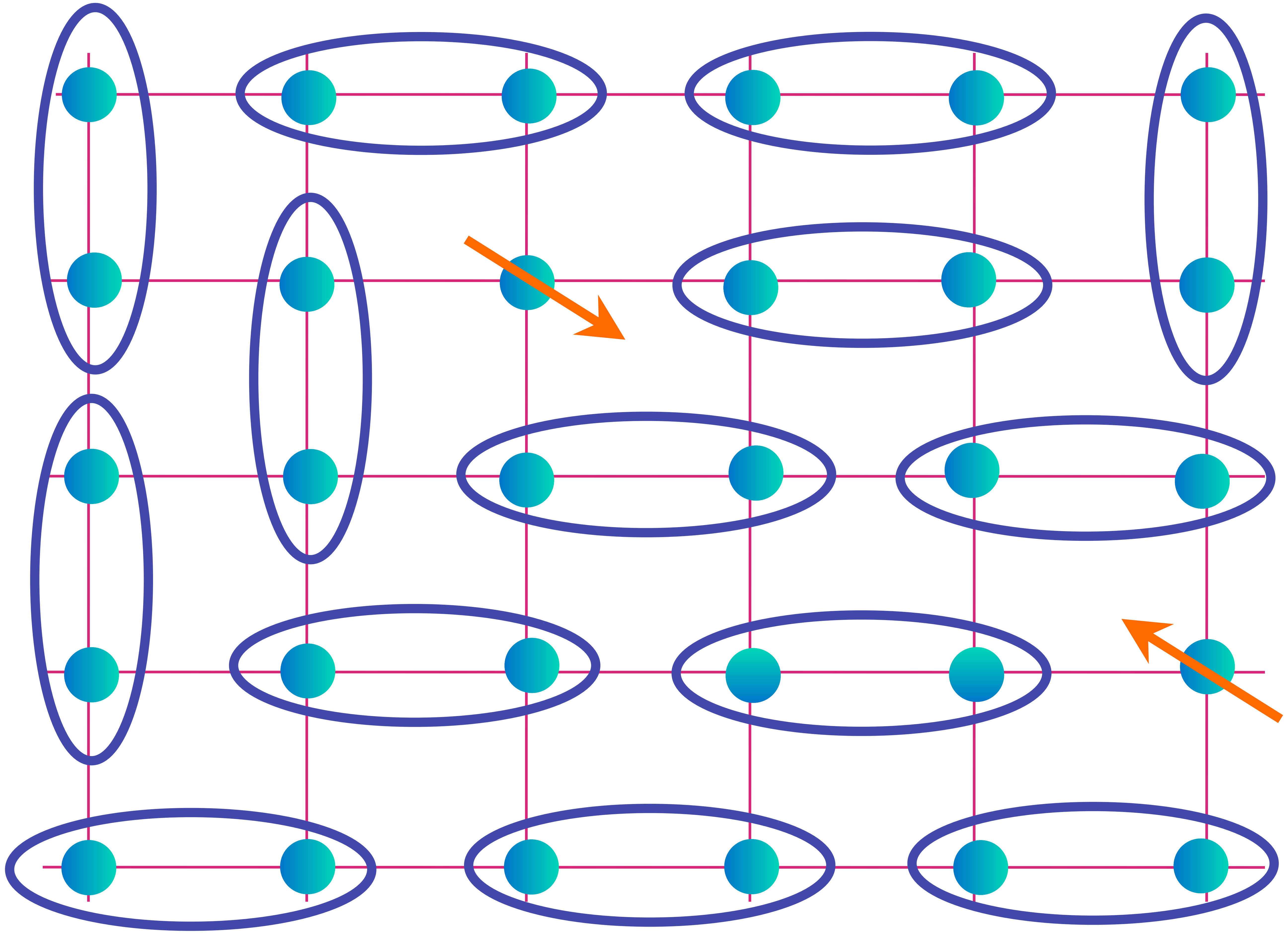
$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

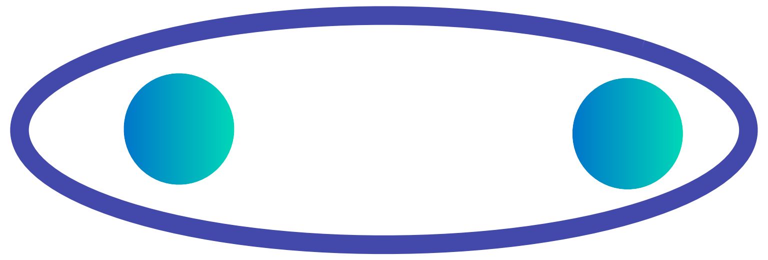
G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Spin liquid

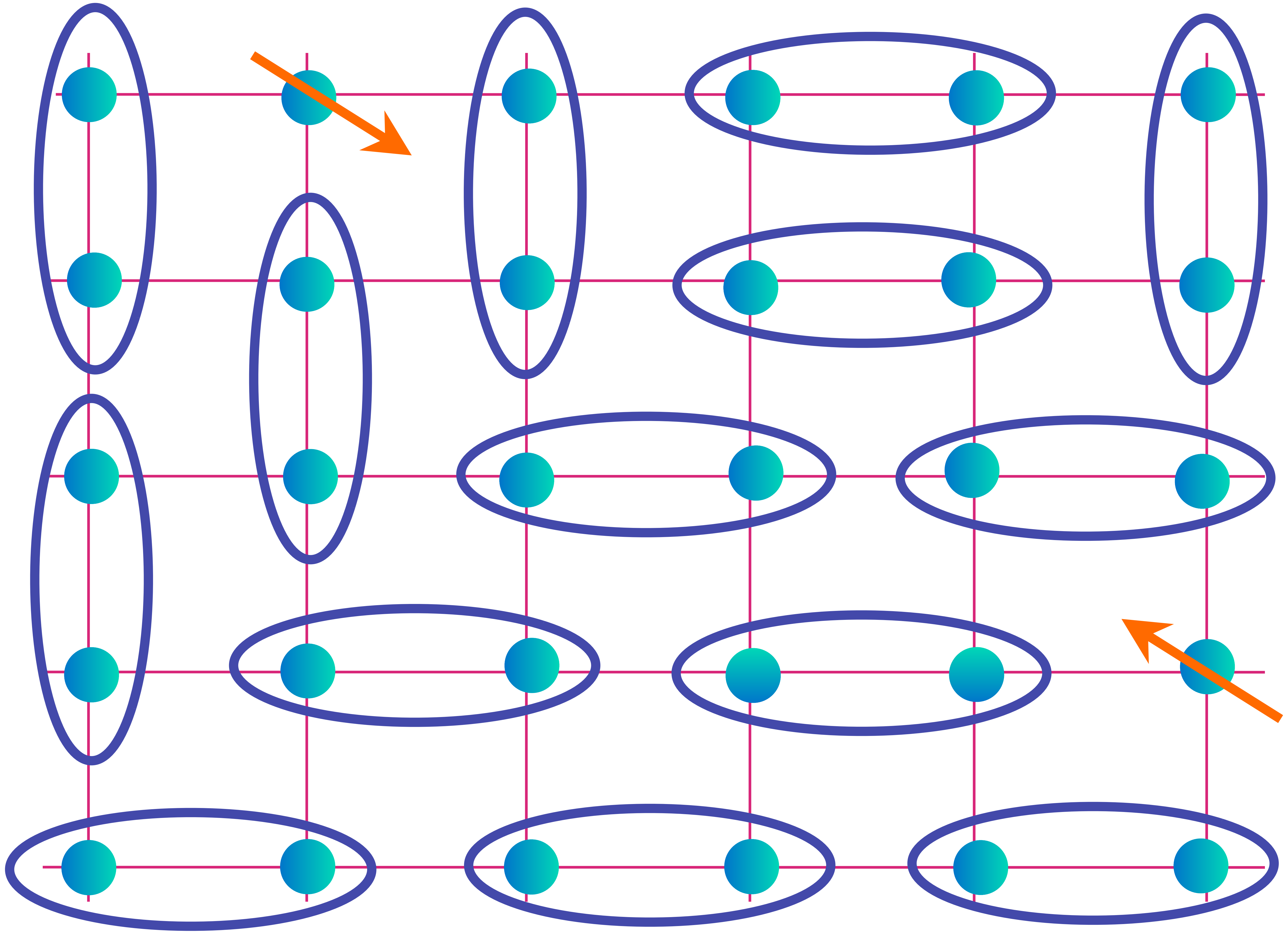
Fractionalized spinon excitations with spin S=1/2 and charge 0.

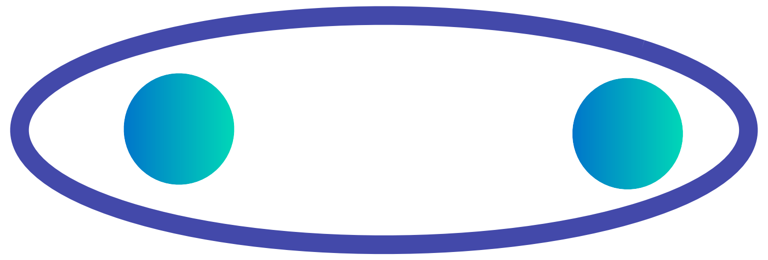


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$




$$= \,| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle$$

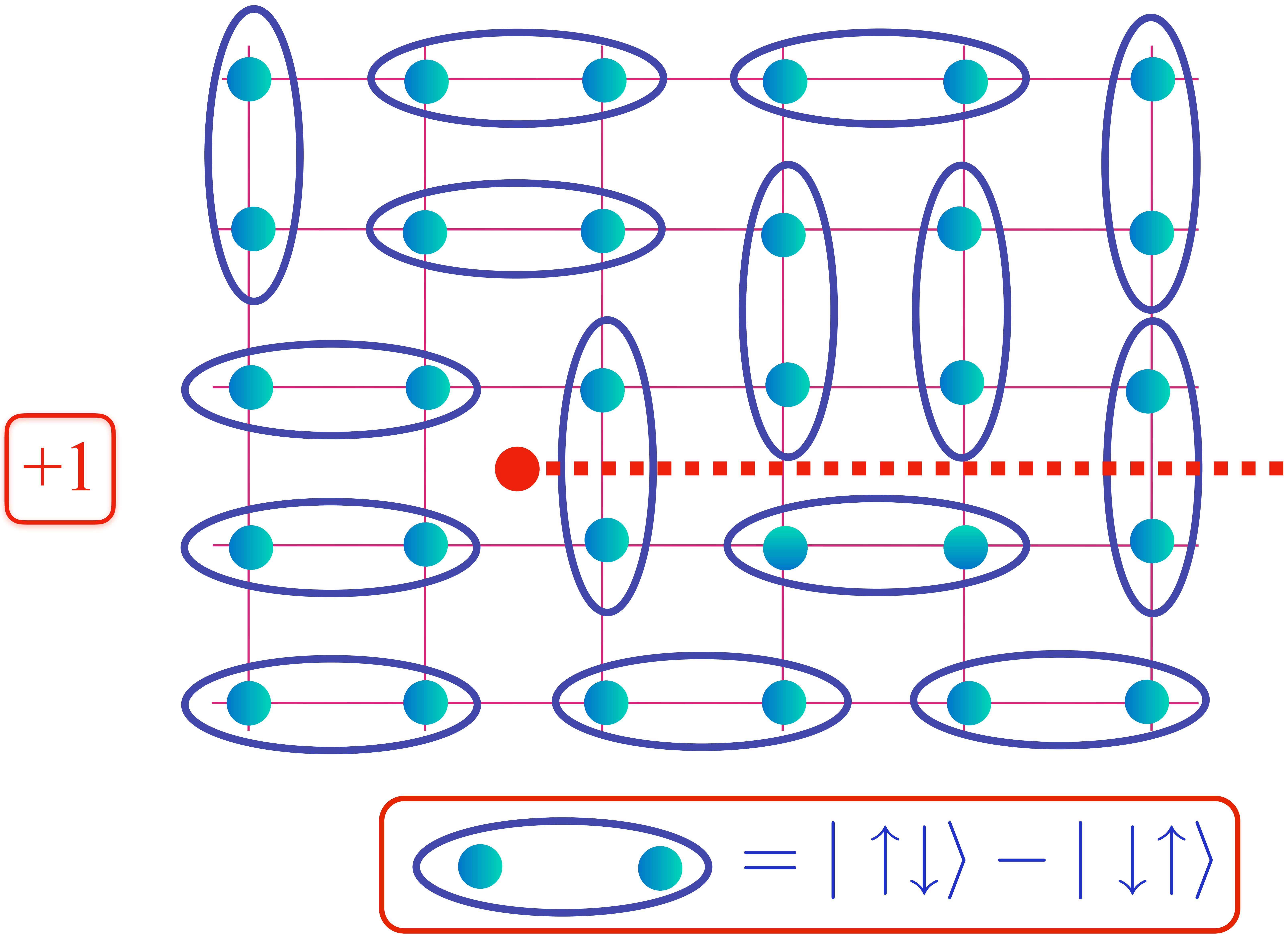
Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

G. Baskaran, Z. Zou, P.W. Anderson, *Solid State Comm.* **63**, 973 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, *Phys. Rev. B* **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$\mathbb{Z}_2$ spin liquid

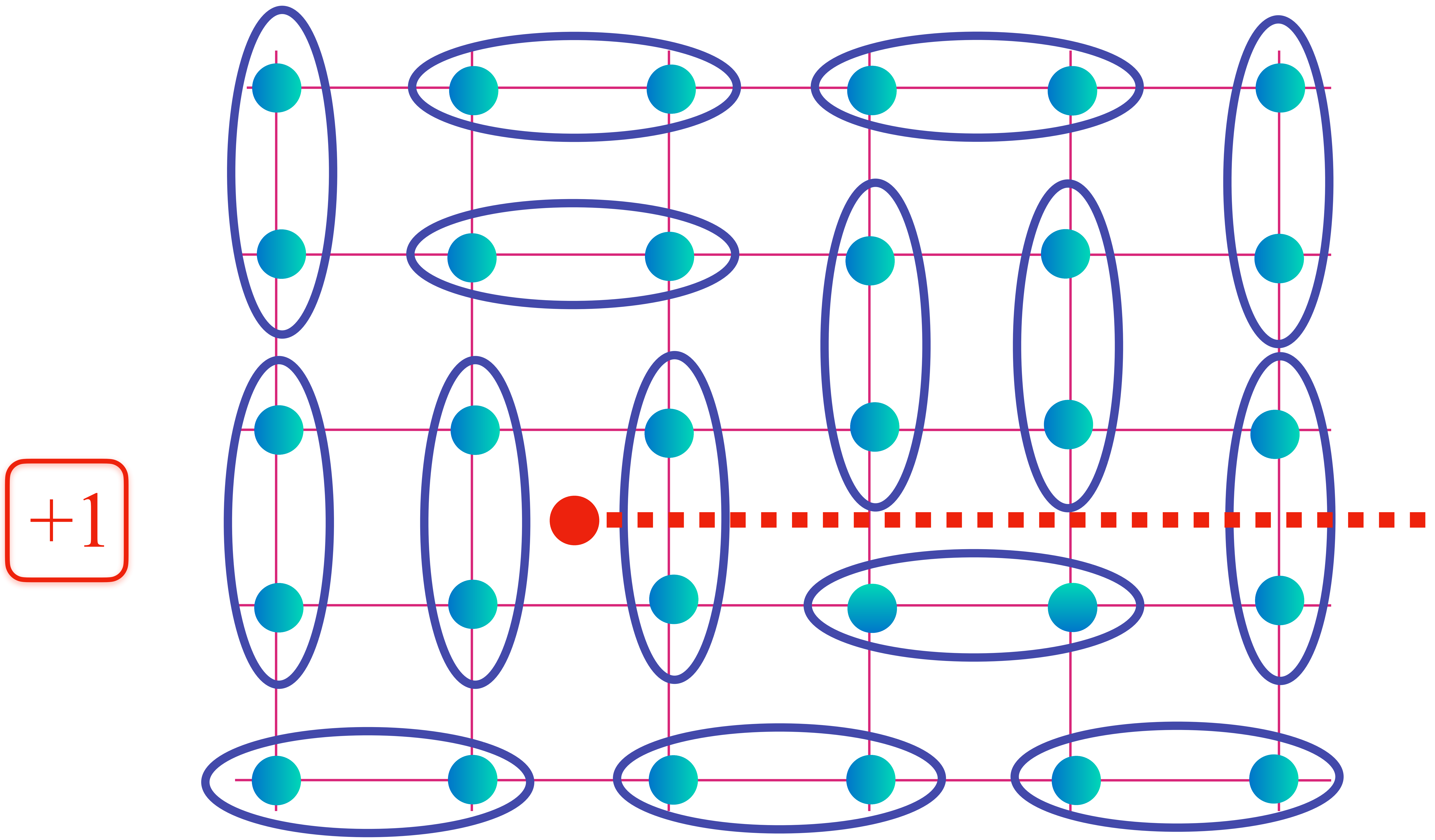
Vison excitation with $S = 0$.

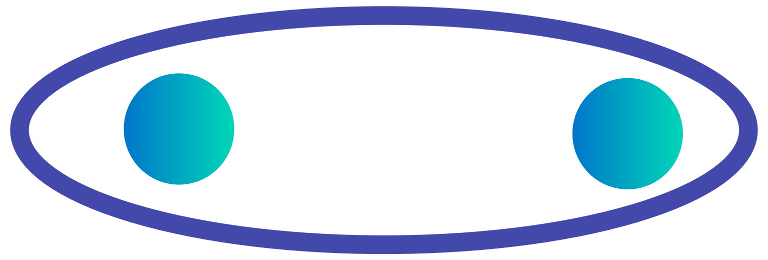
Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

$\mathbb{Z}_2$ spin liquid

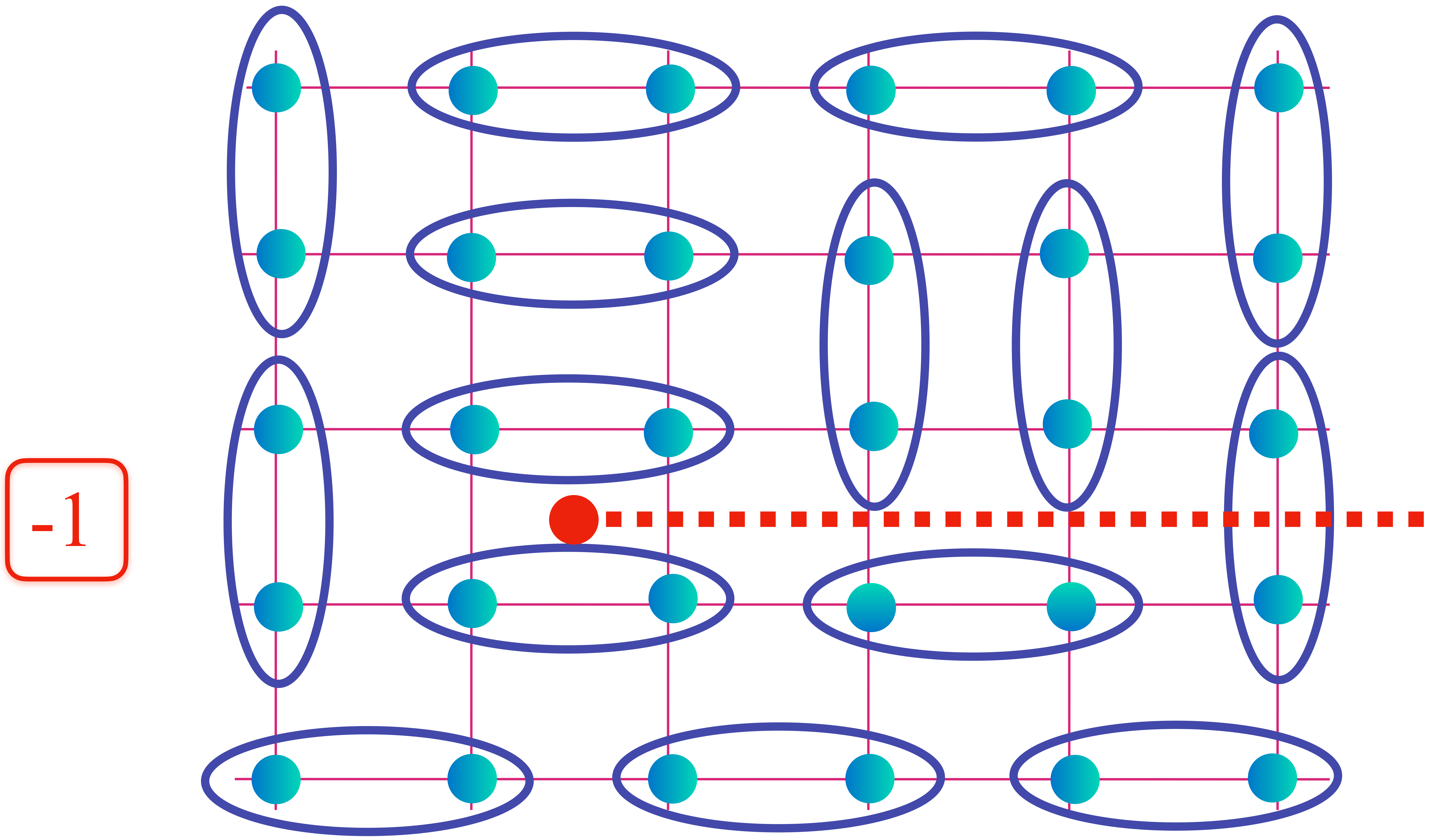
Vison excitation with $S = 0$.

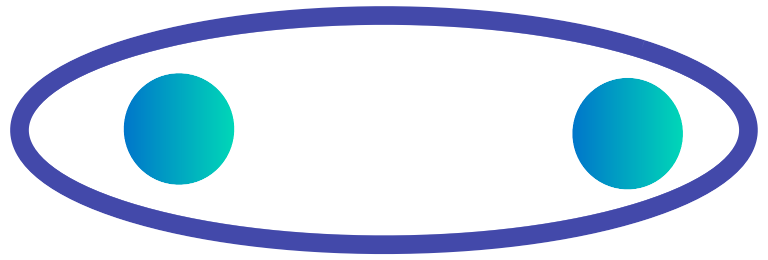
Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$\mathbb{Z}_2$ spin liquid

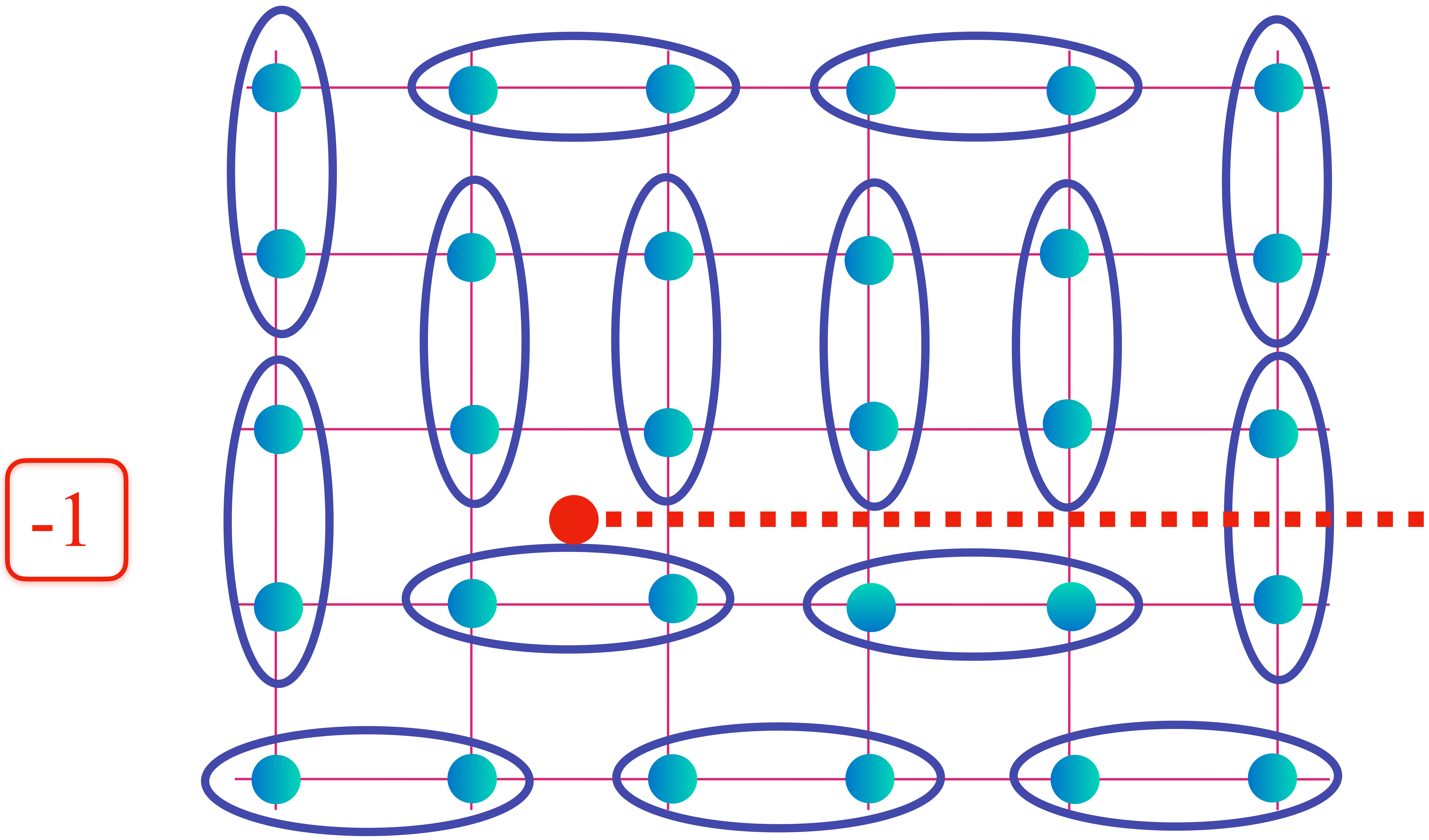
Vison excitation with $S = 0$.

Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$\mathbb{Z}_2$ spin liquid

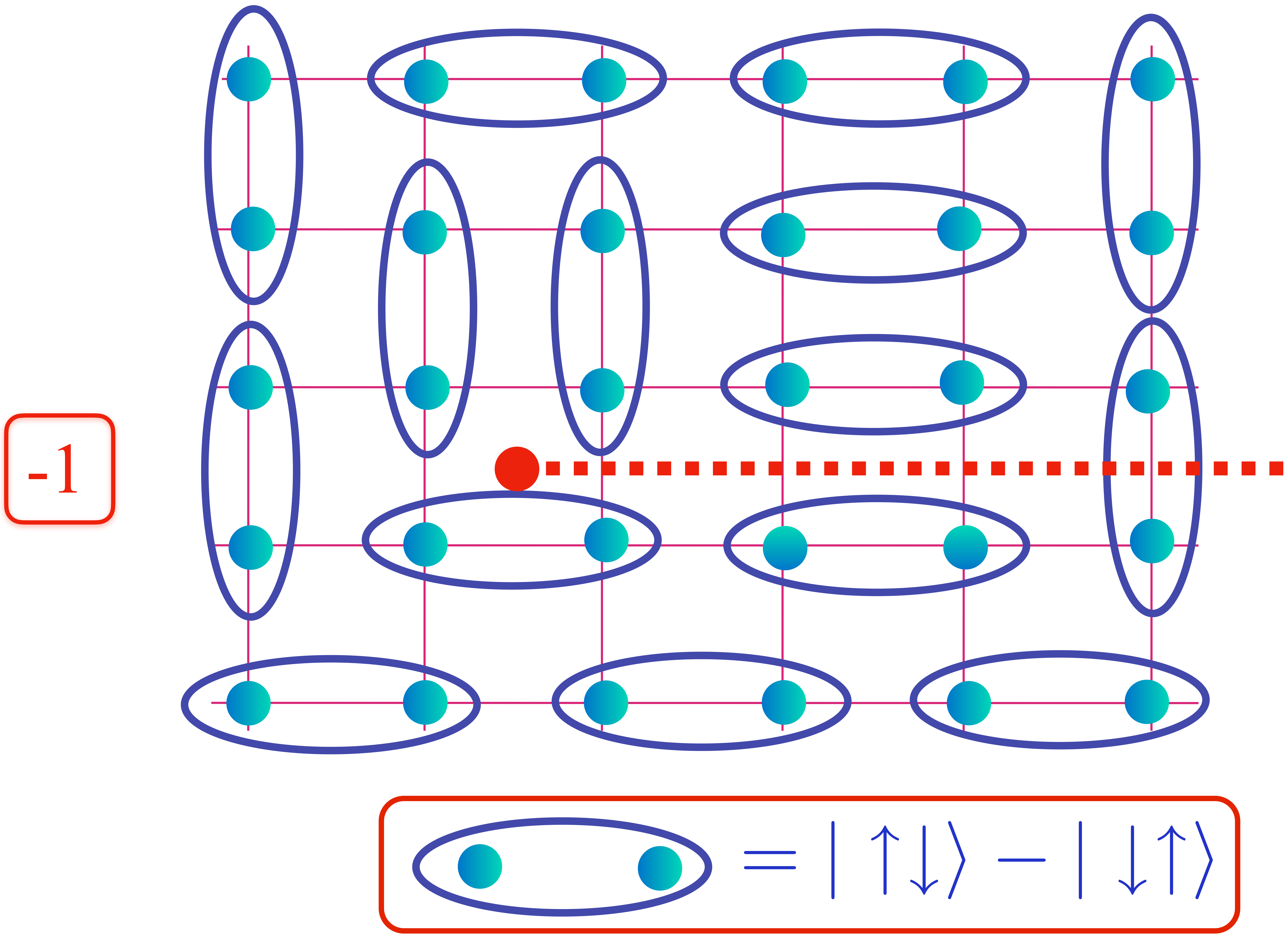
Vison excitation with $S = 0$.

Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$\mathbb{Z}_2$ spin liquid

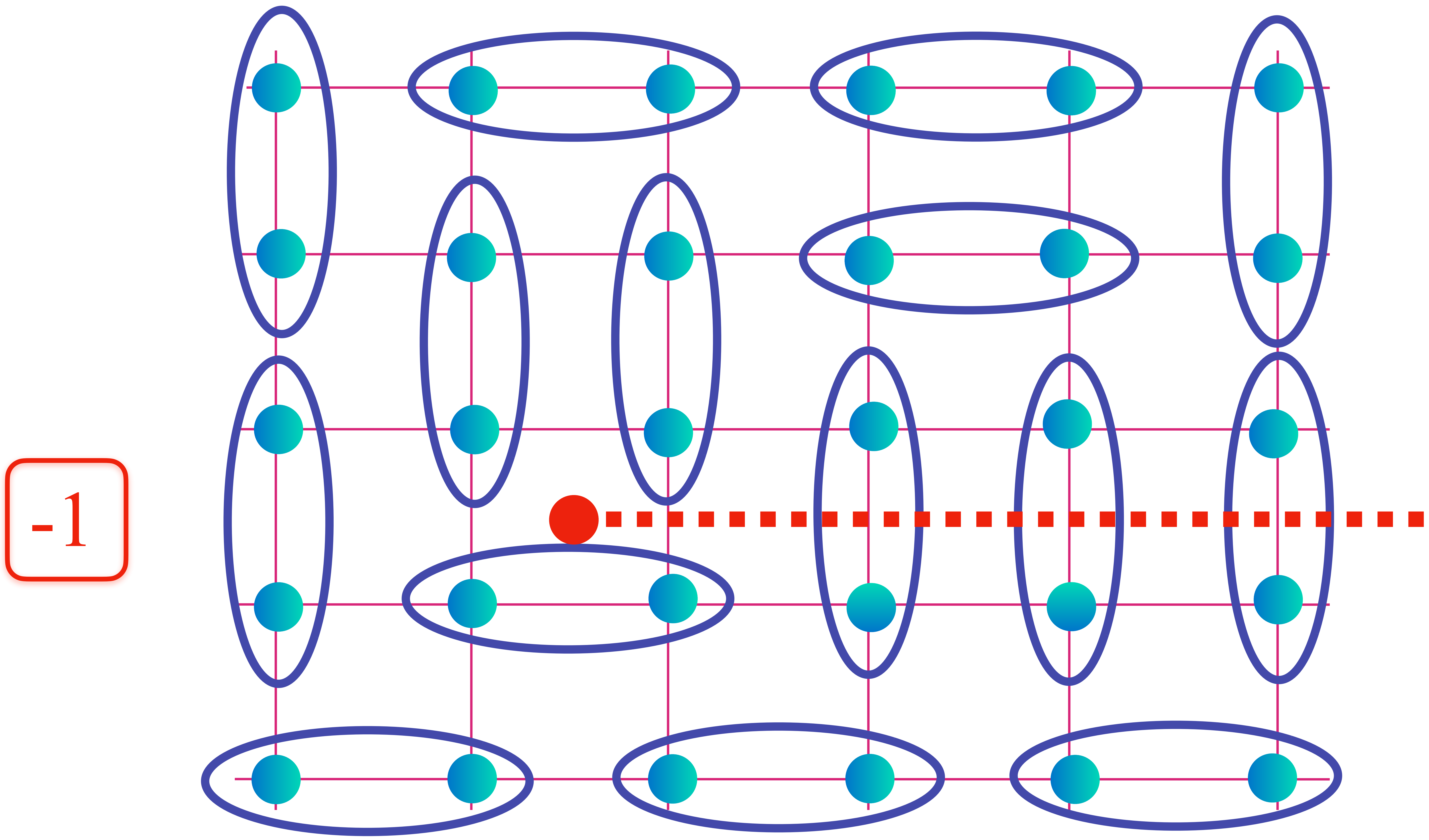
Vison excitation with $S = 0$.

Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$\mathbb{Z}_2$ spin liquid

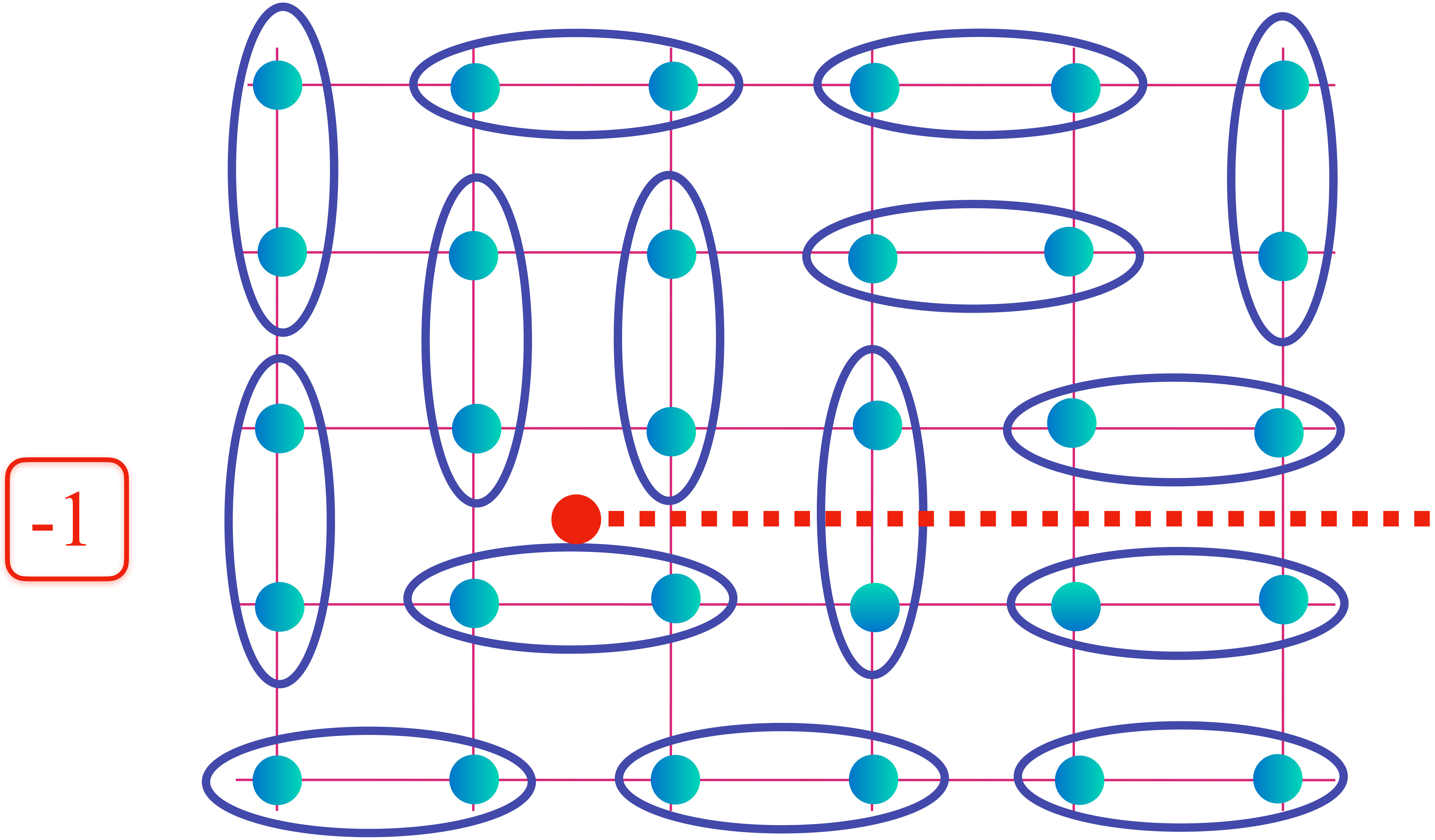
Vison excitation with $S = 0$.

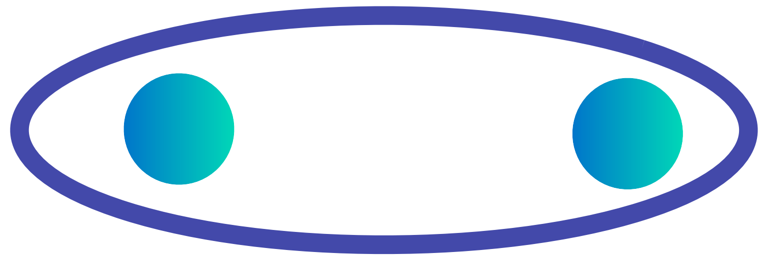
Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$




$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

$\mathbb{Z}_2$ spin liquid

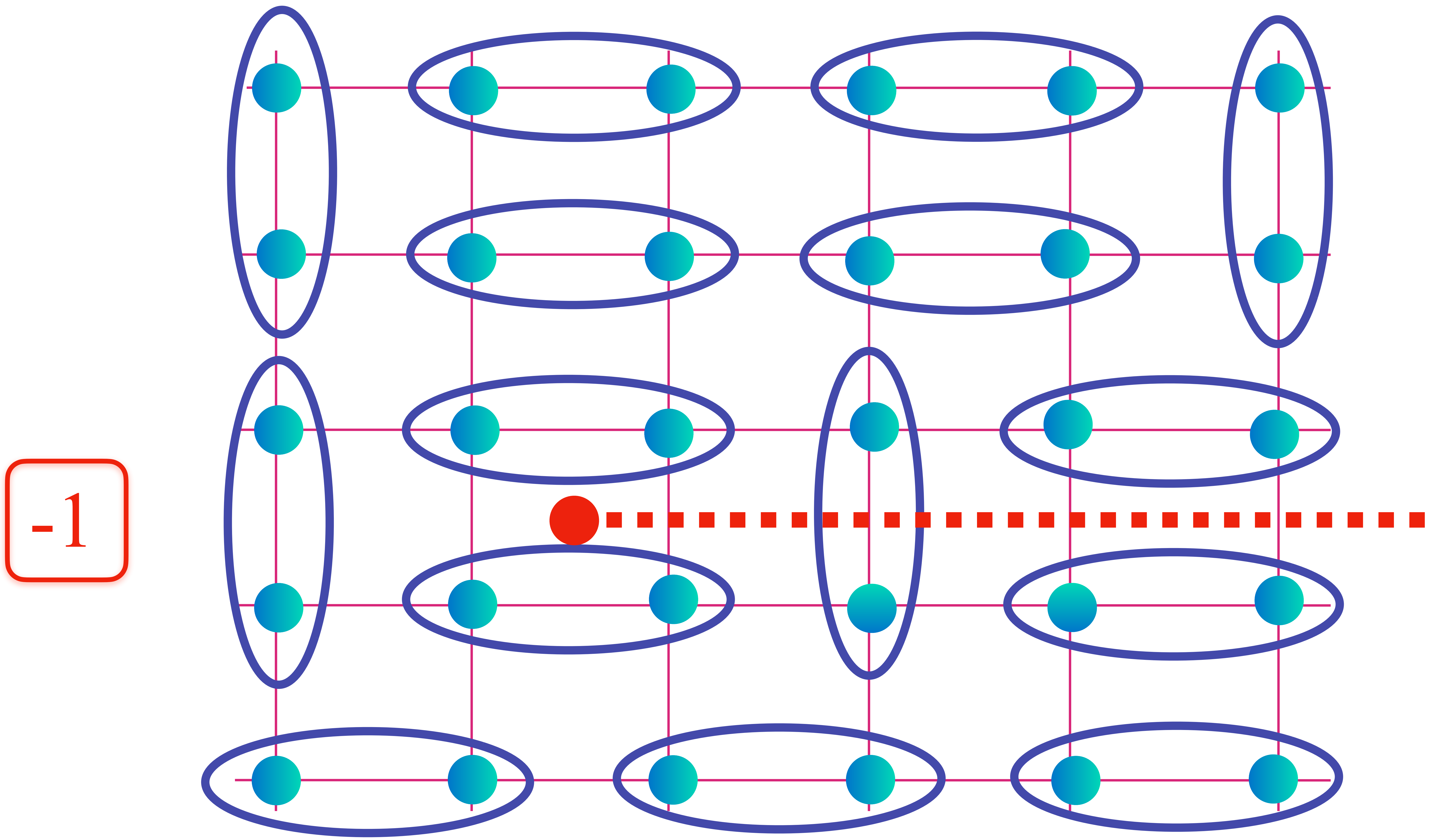
Vison excitation with $S = 0$.

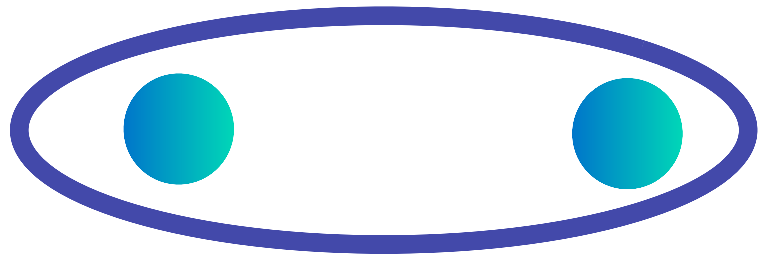
Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$\mathbb{Z}_2$ spin liquid

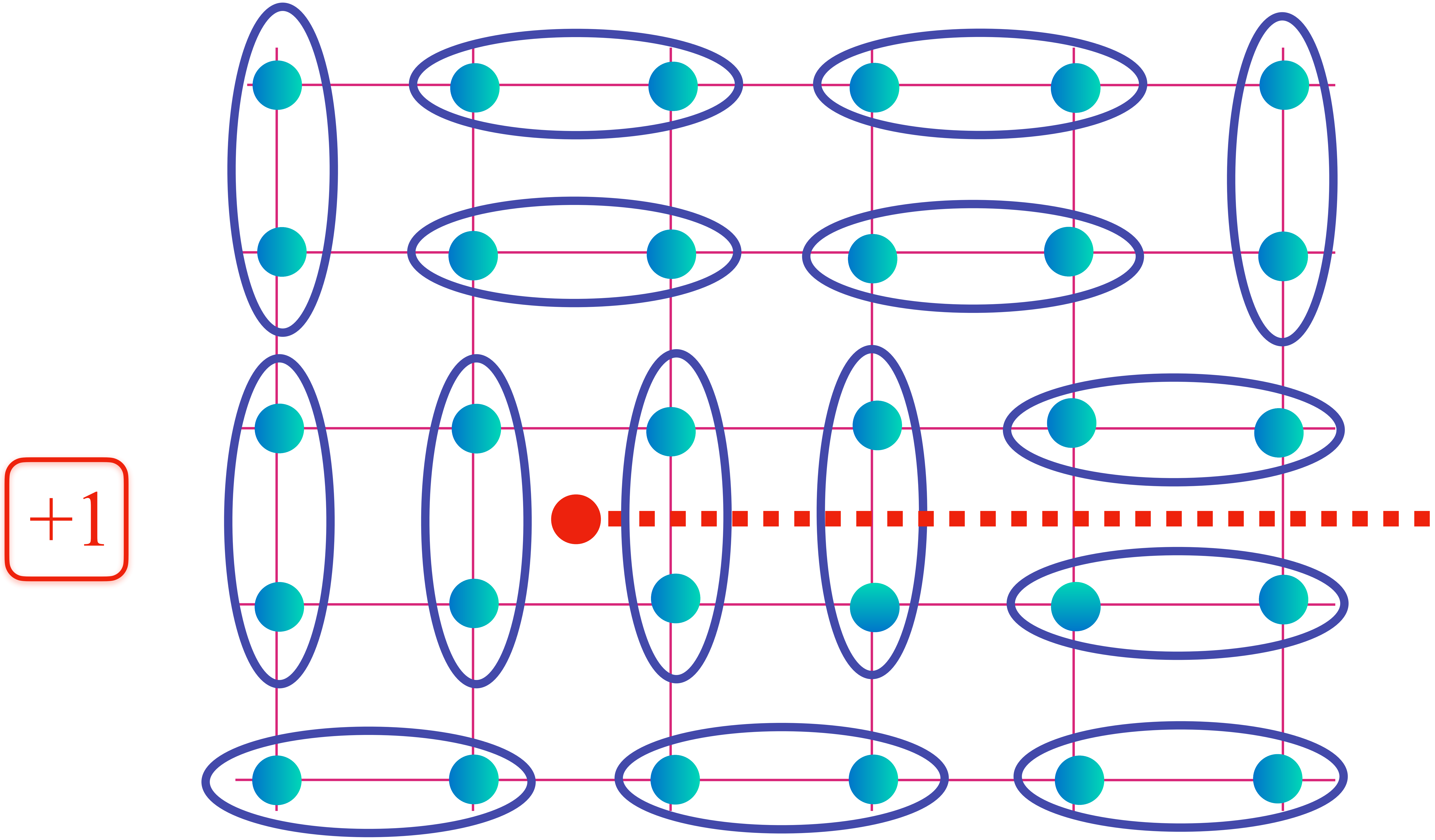
Vison excitation with $S = 0$.

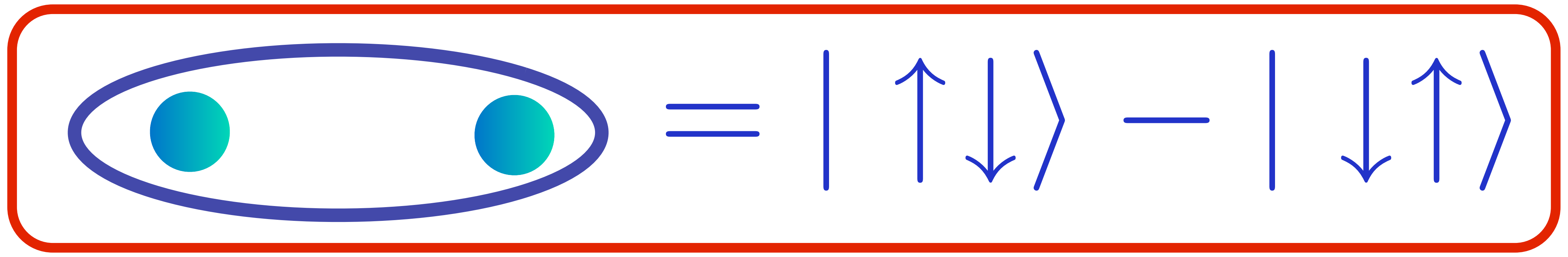
Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

N. Read and S. Sachdev, Phys. Rev. Lett. **66** (1991) 1773
X. G. Wen, Phys. Rev. B **44** (1991) 2664

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





$\mathbb{Z}_2$ spin liquid

Vison excitation with $S = 0$.

Spinons and visons are mutual semions.

The topological structure of the $\mathbb{Z}_2$ spin liquid is identical to Kitaev's toric code, discovered later (spinons and visons correspond to the e and m particles).

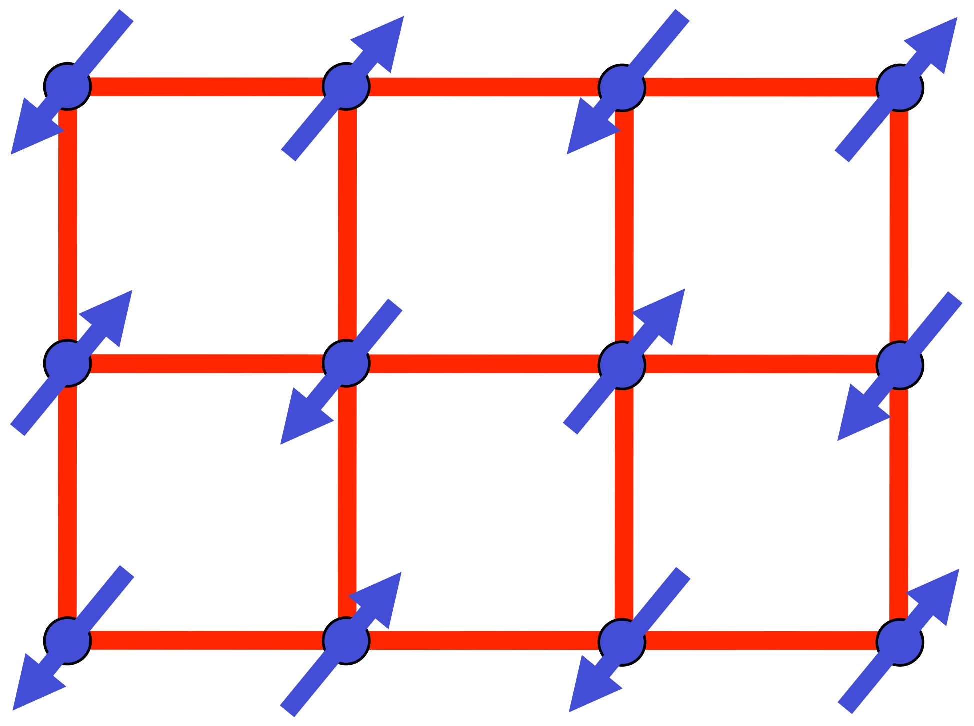
Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \quad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$

Mean-field spin liquid
with gapped bosonic spinons.

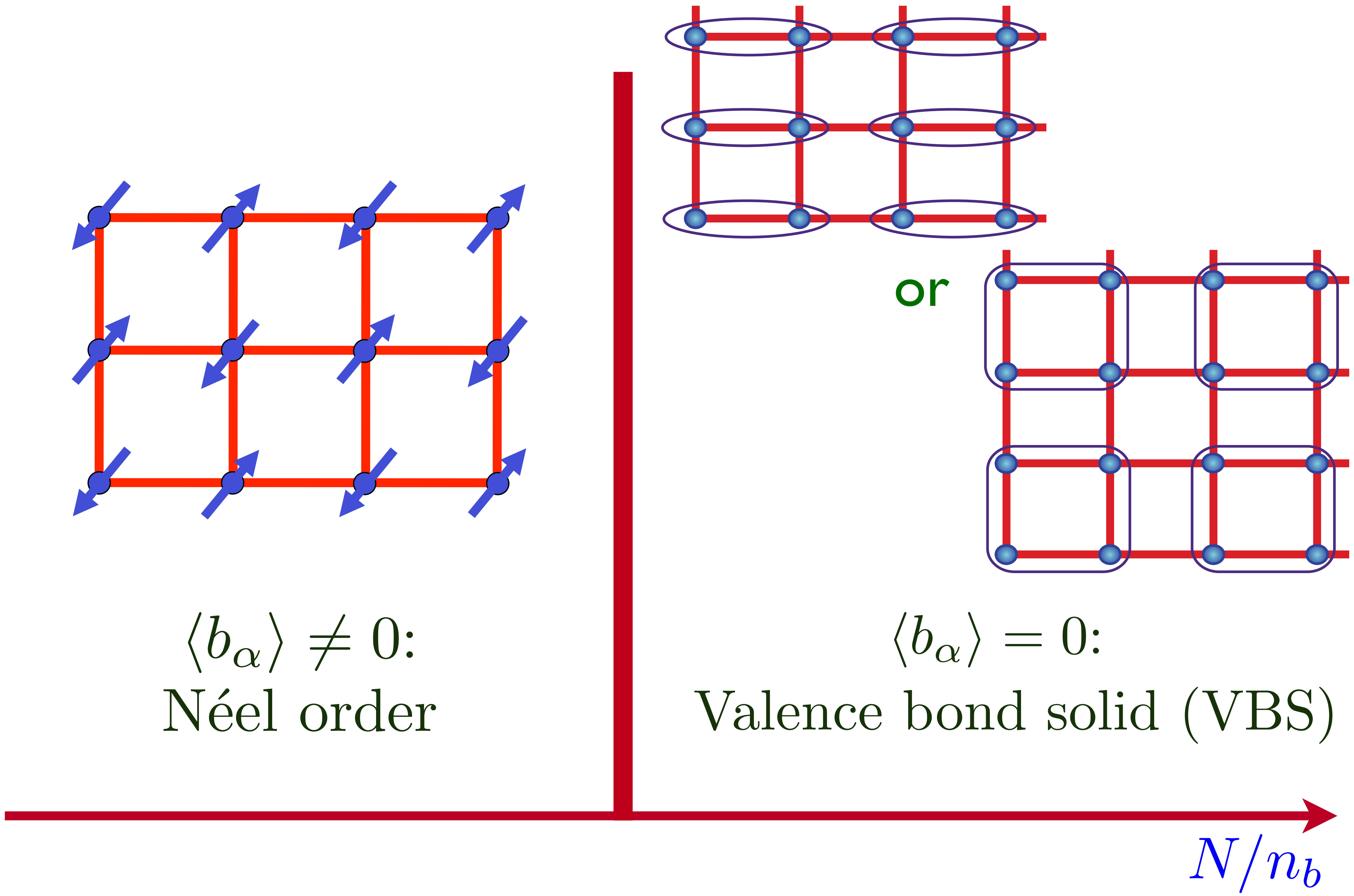


$\langle b_\alpha \rangle \neq 0$:
Néel order

$\langle b_\alpha \rangle = 0$:
Spin liquid

D.P. Arovas and A. Auerbach, *Phys. Rev. B* **38**, 316 (1988)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \, , \quad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$

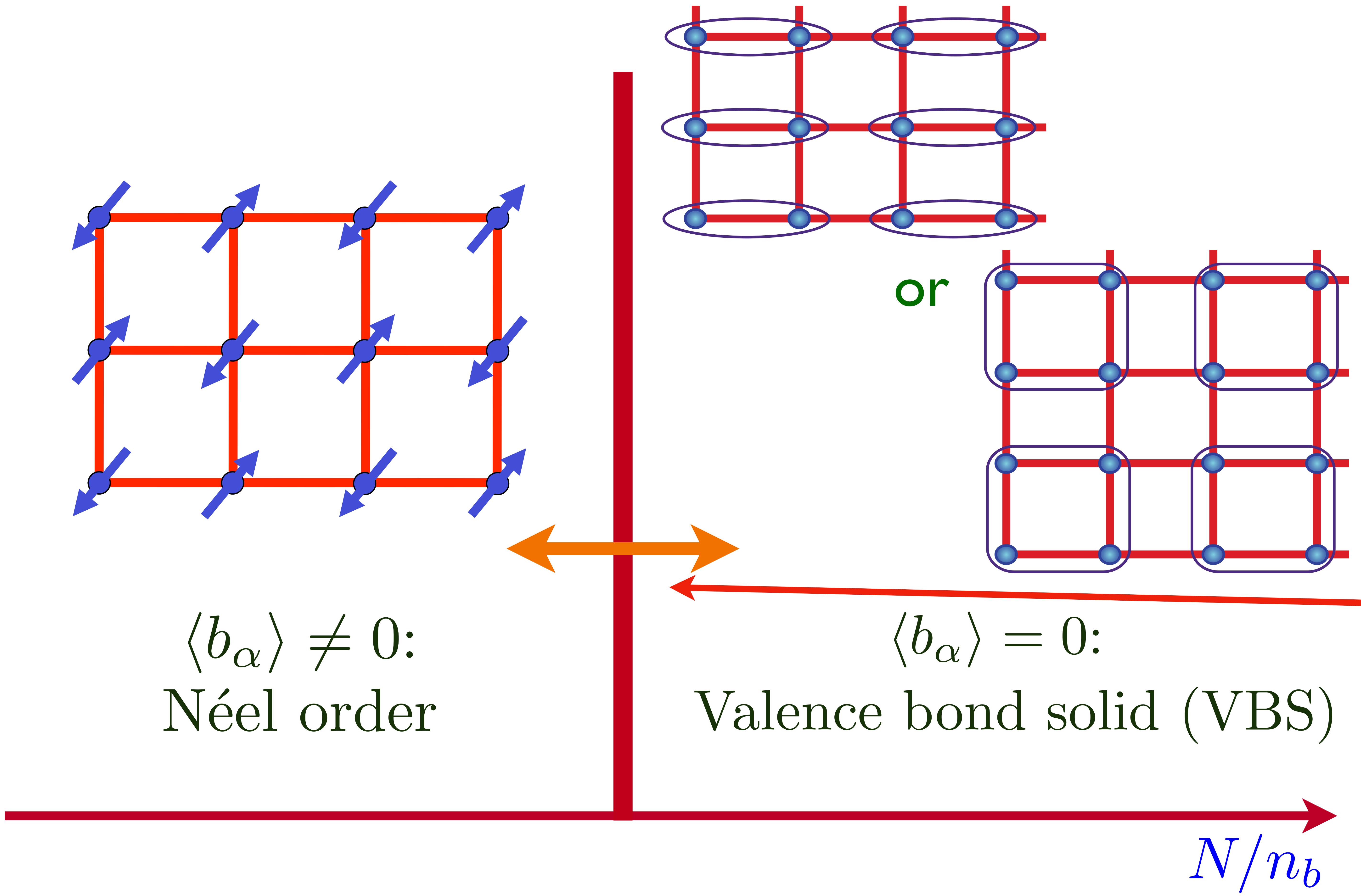
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

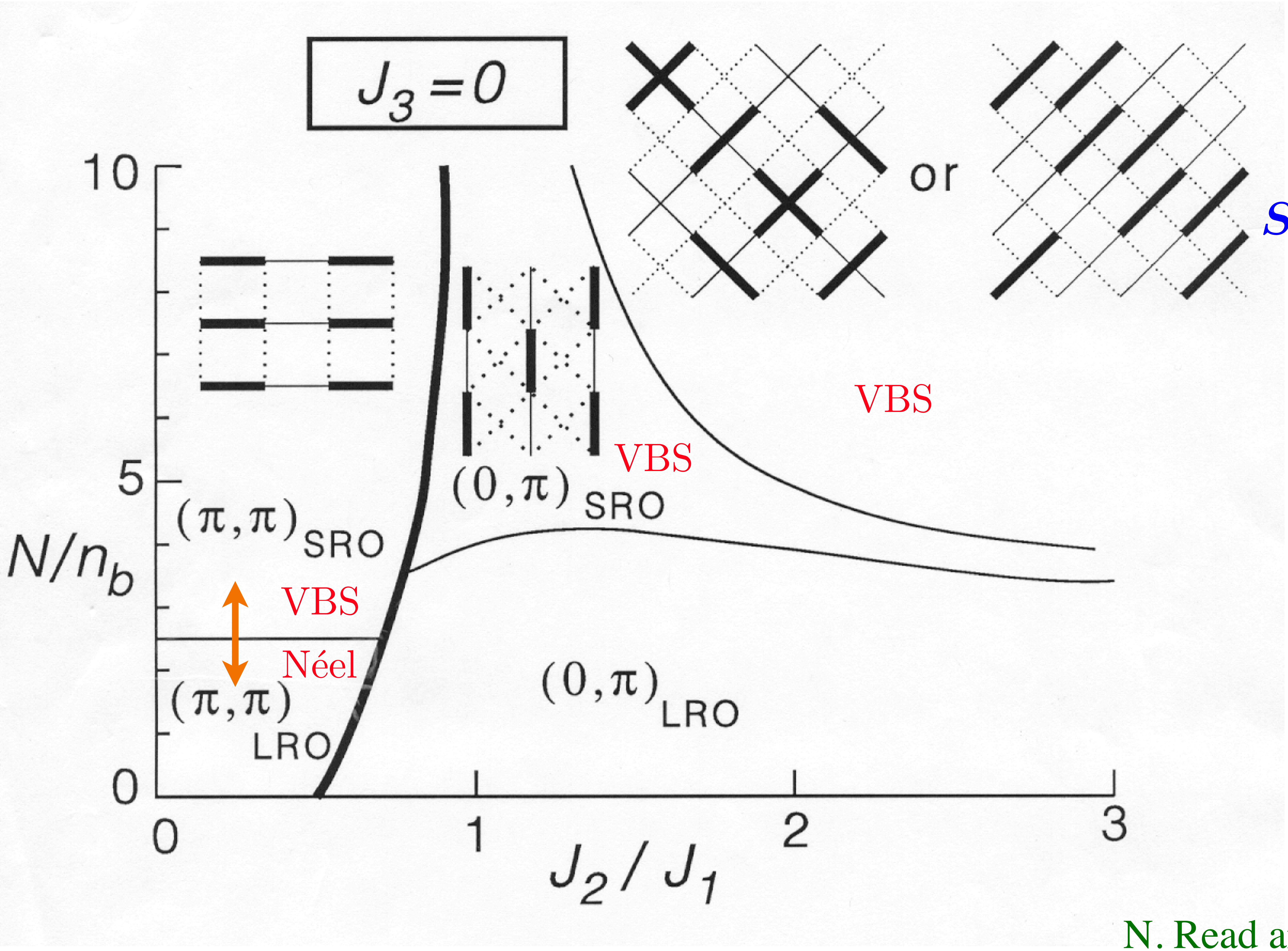
$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \quad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$



DQCP
(first sighting)

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

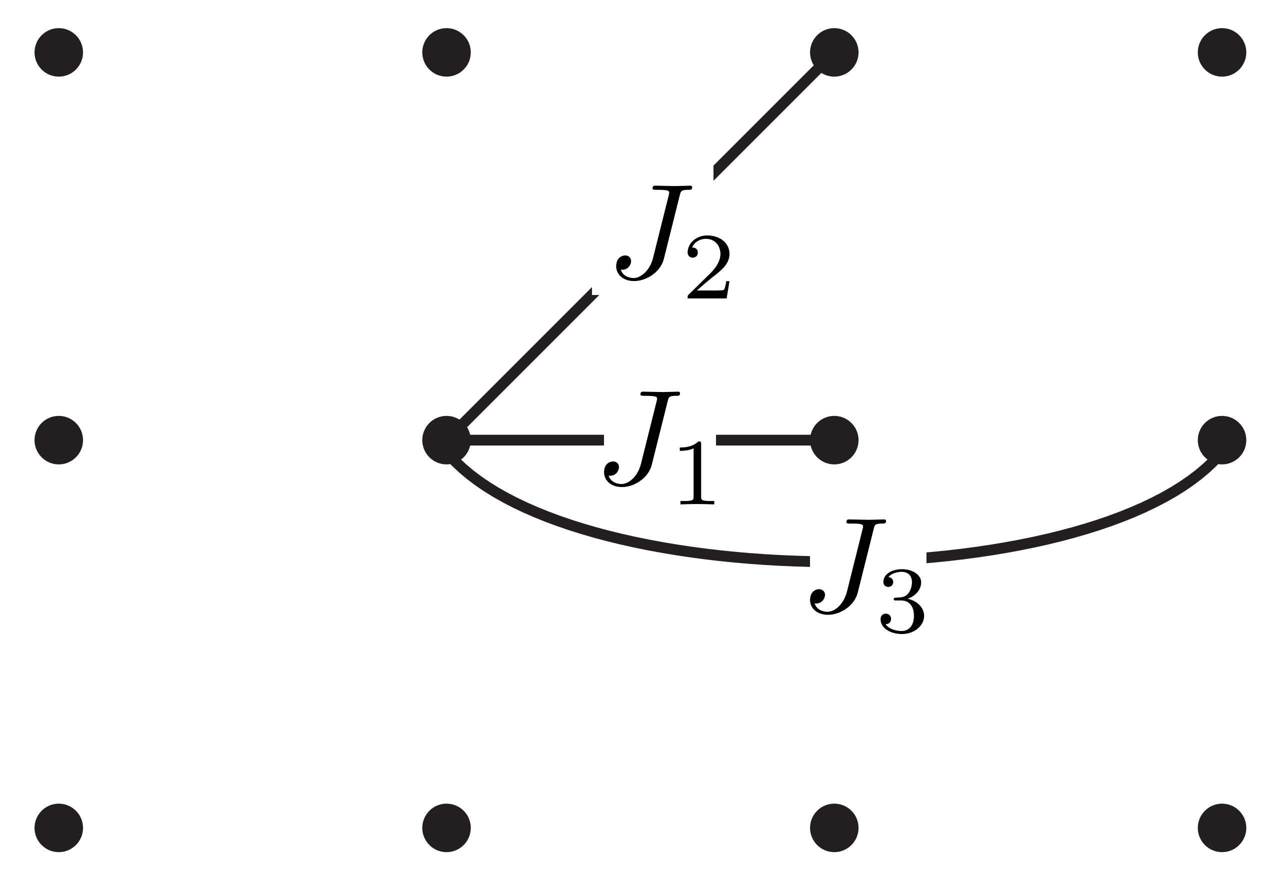
Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

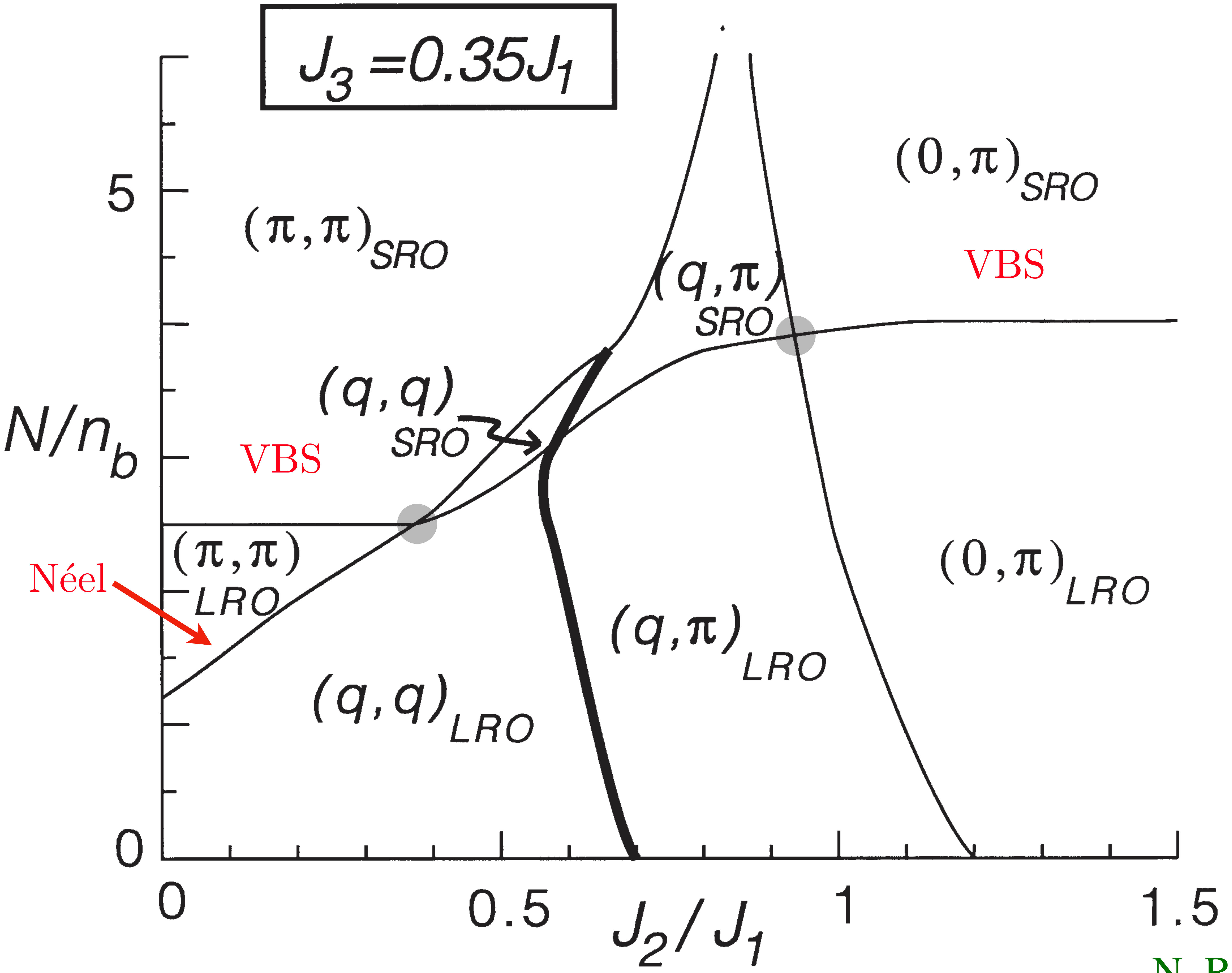
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$



N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

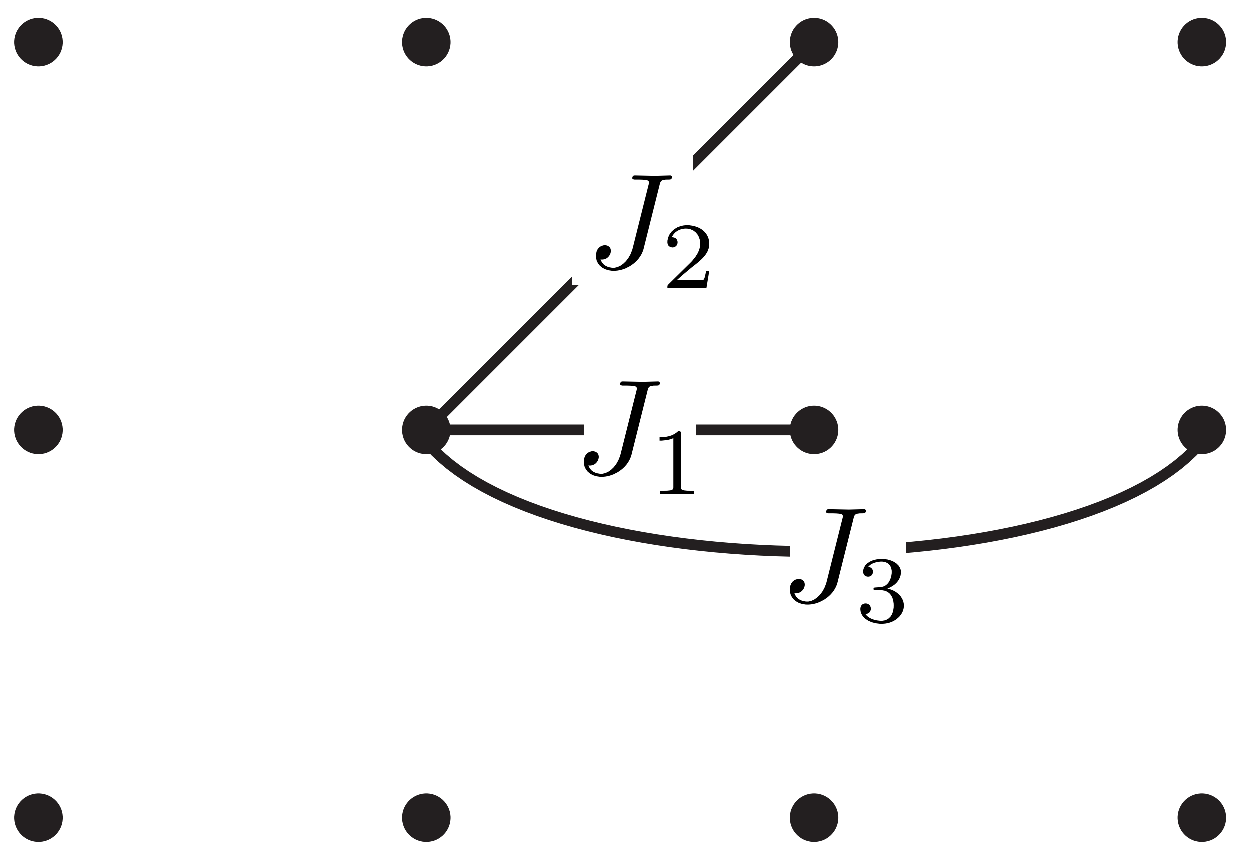
Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

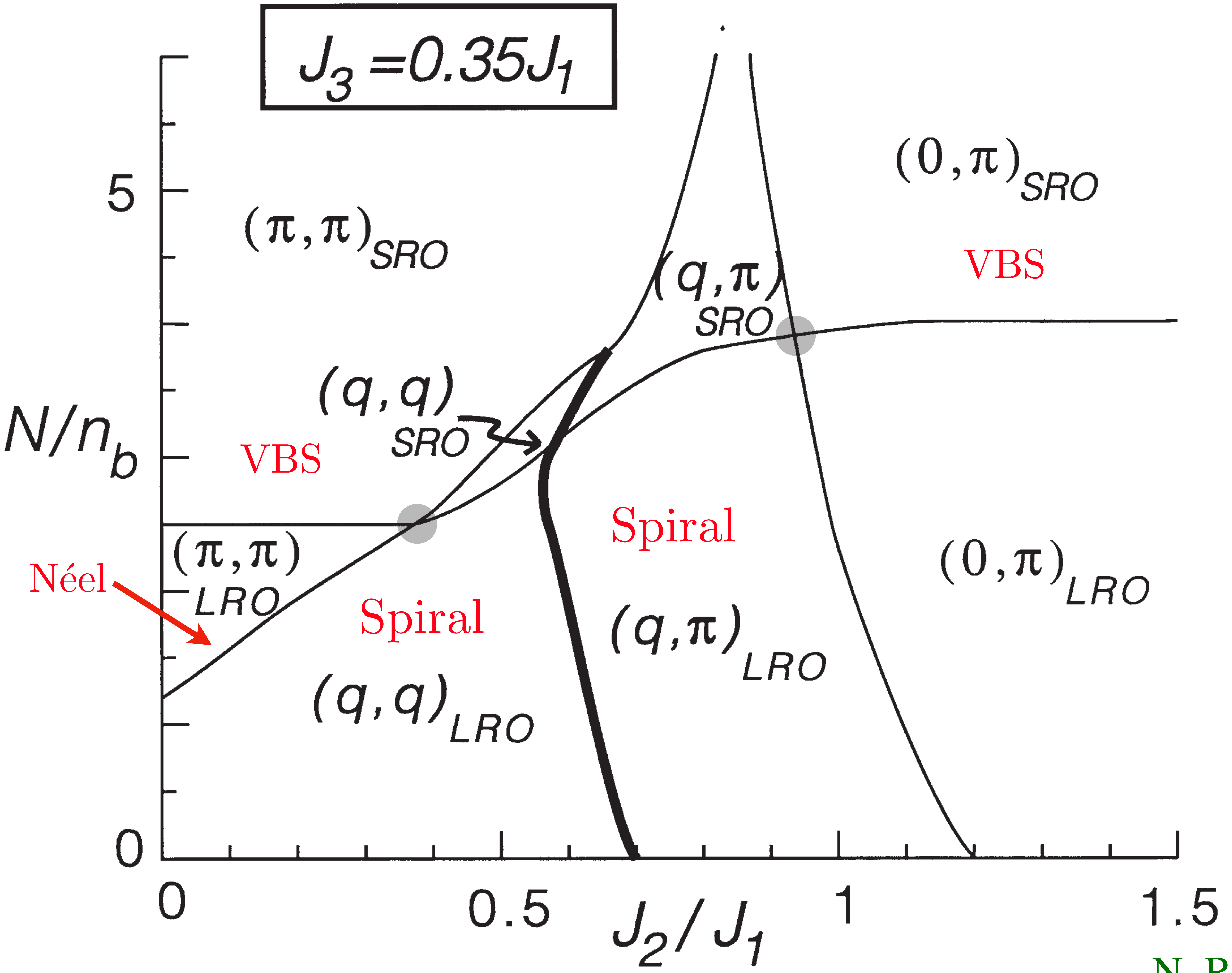
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$



N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

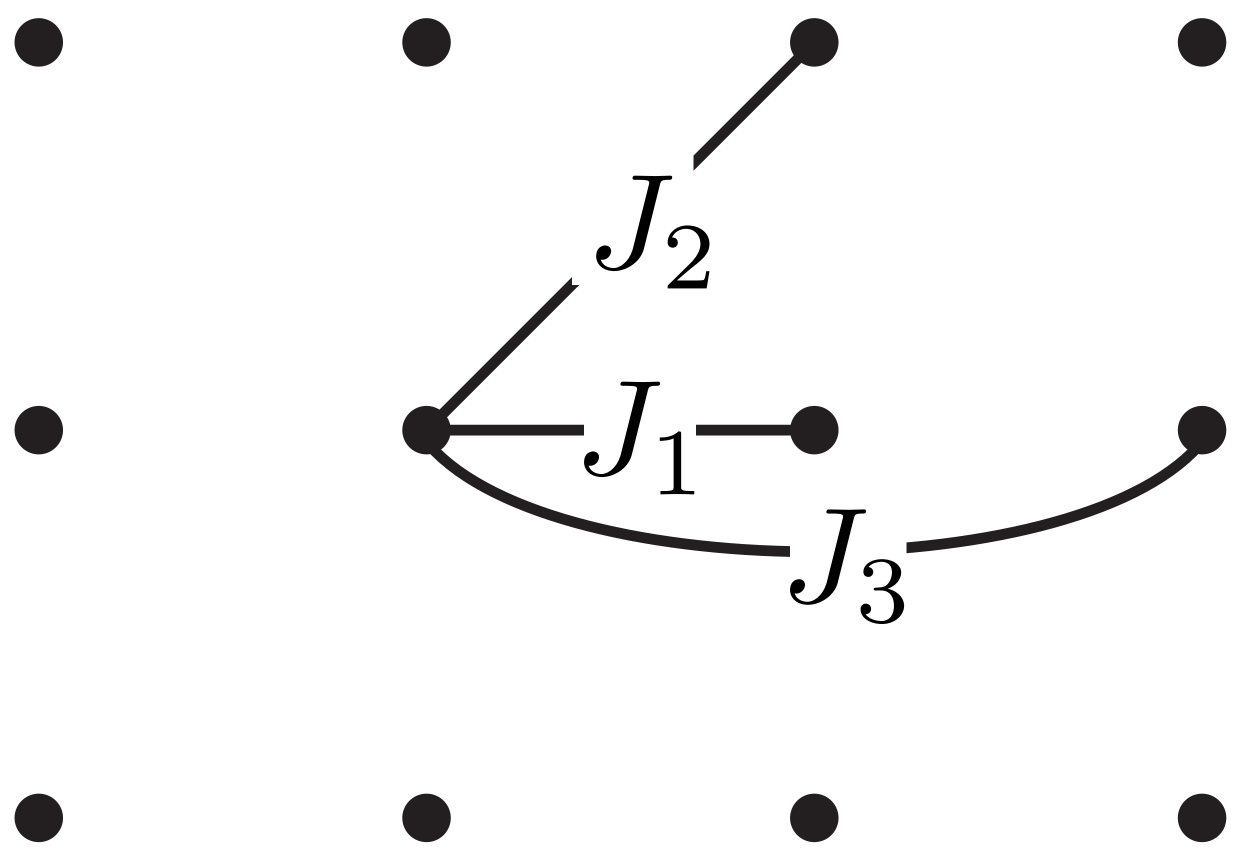
Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

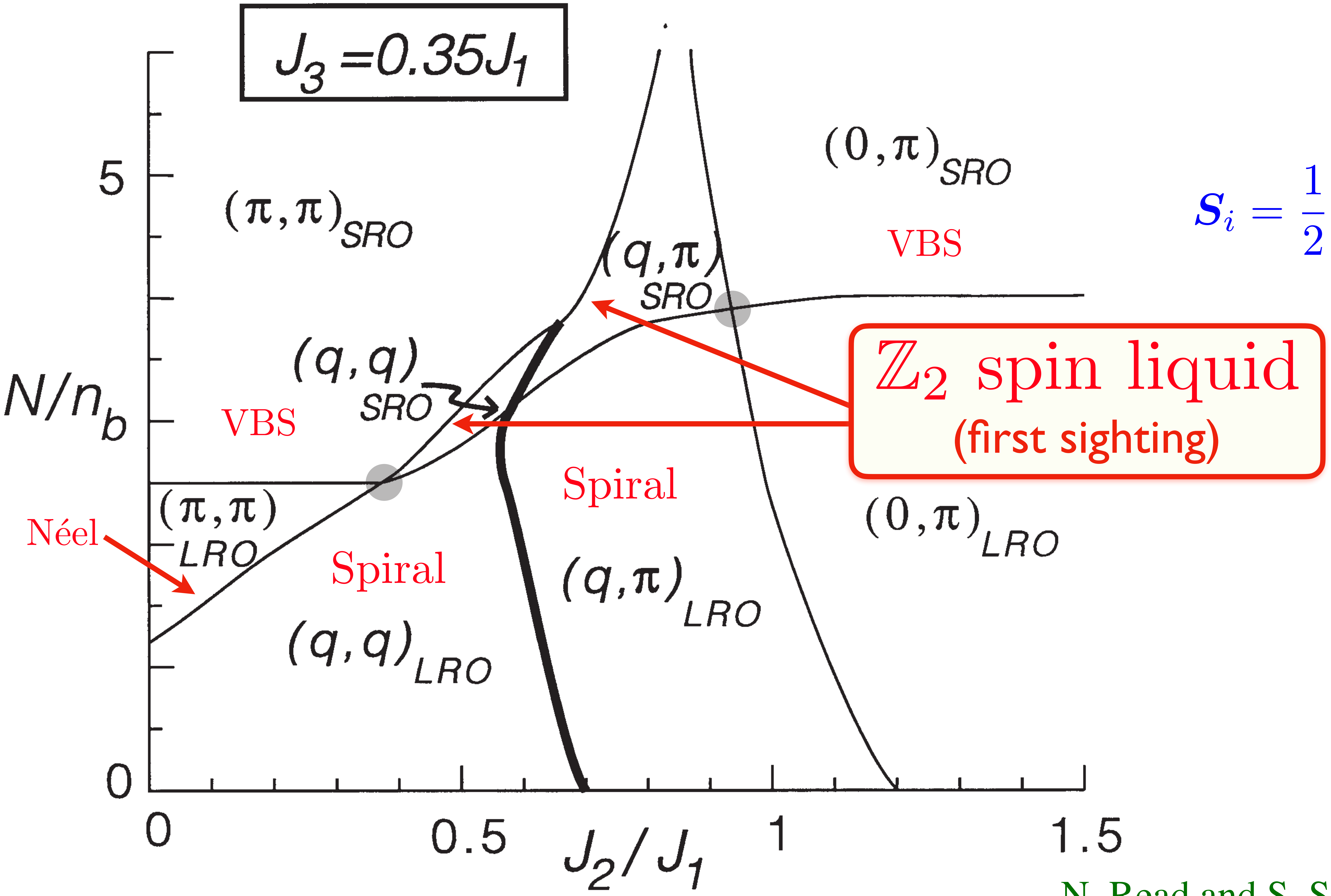
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \quad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$



N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

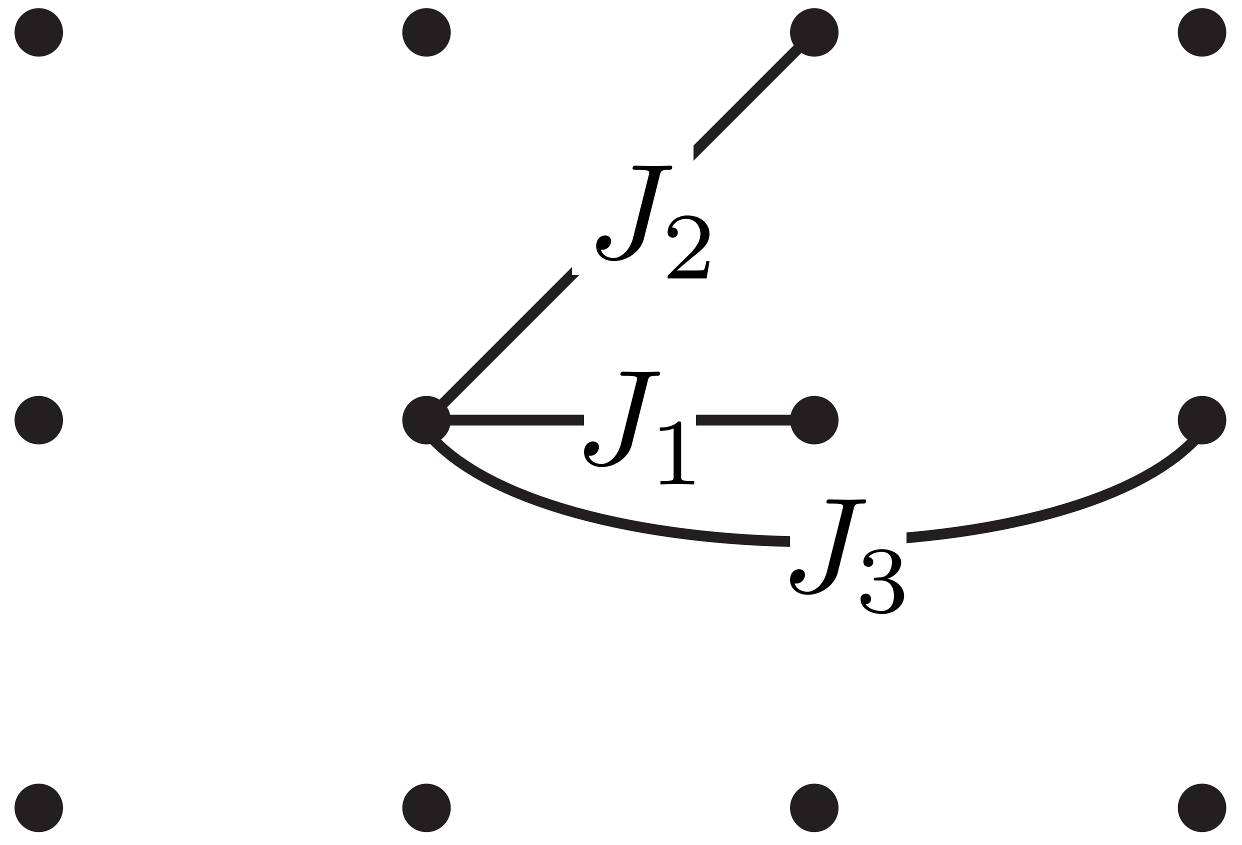
Insulating $S=1/2$ antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

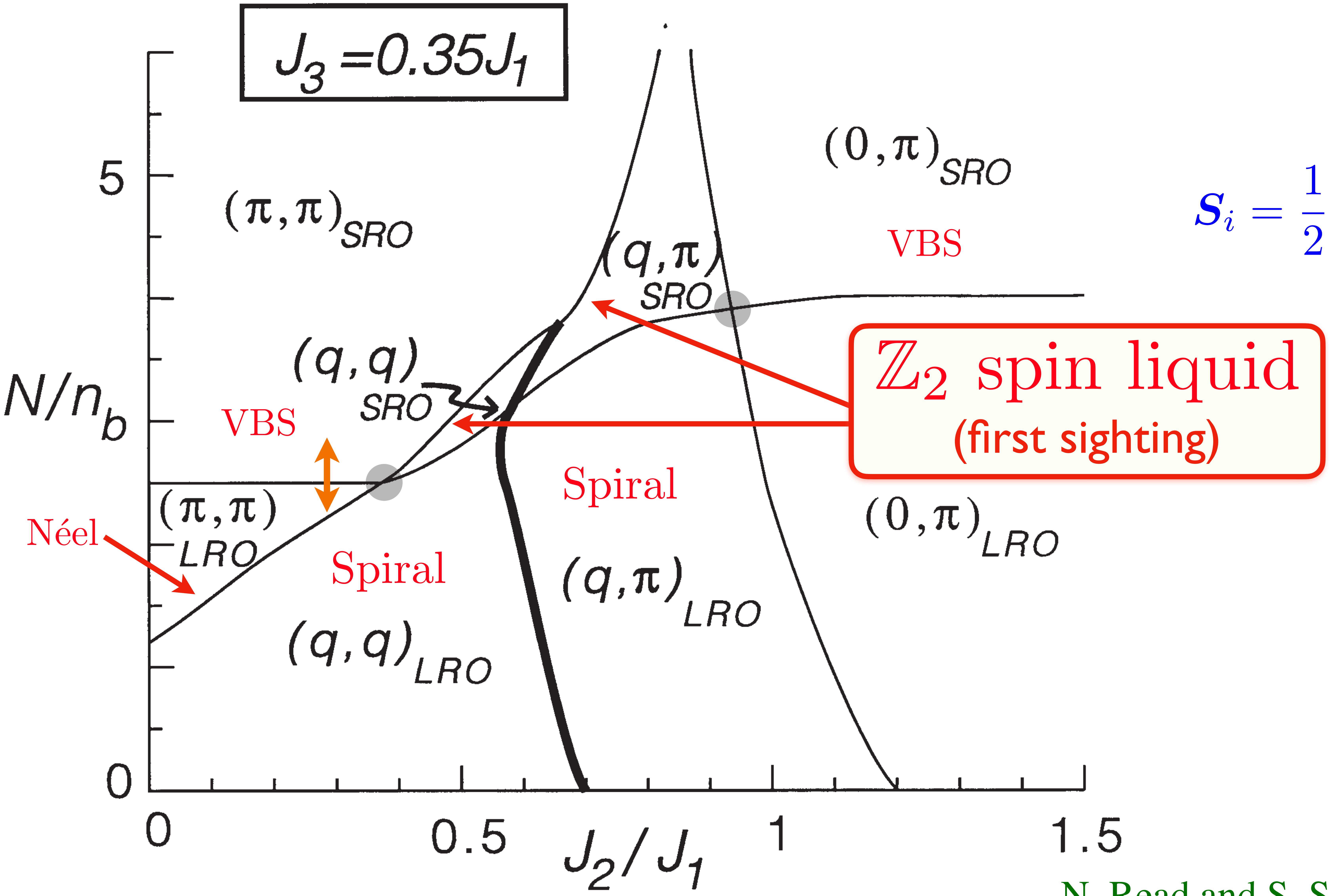
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \qquad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$



N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

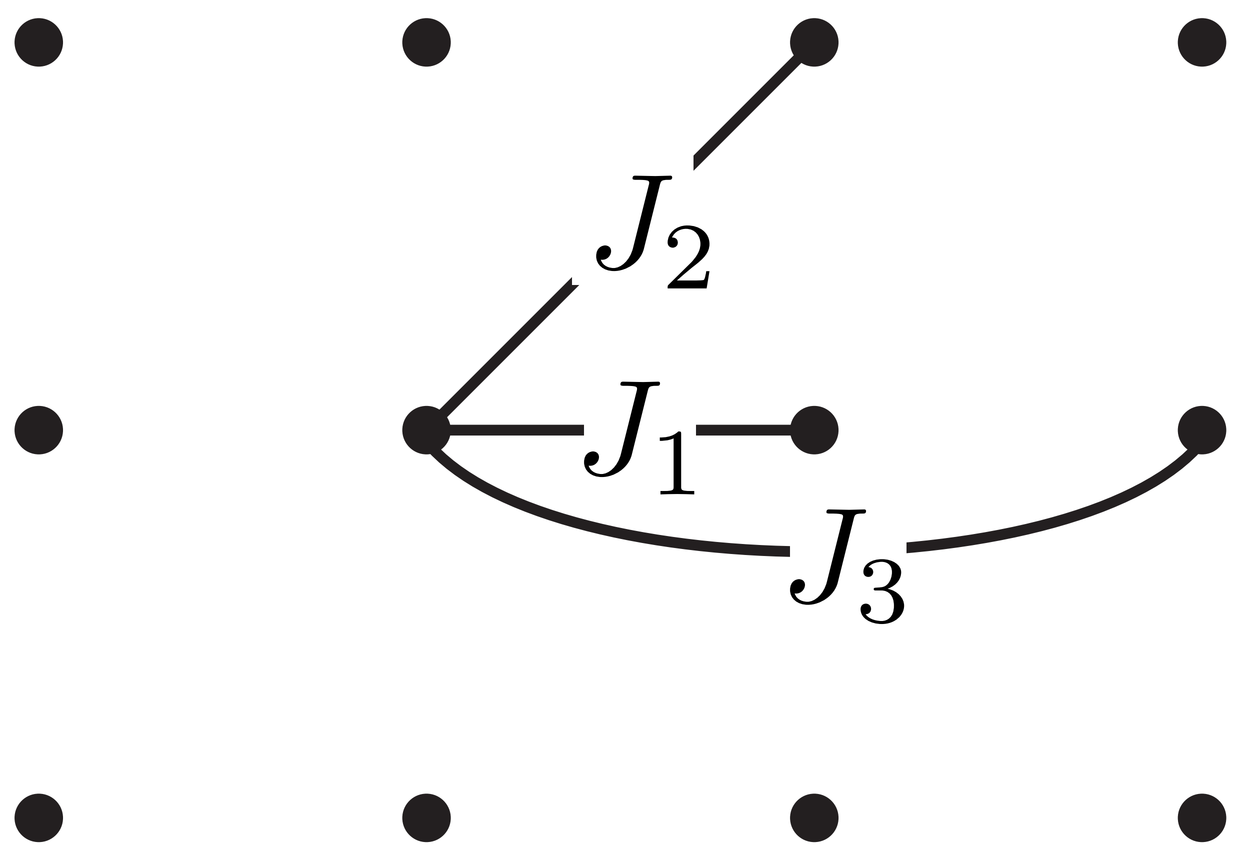
Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

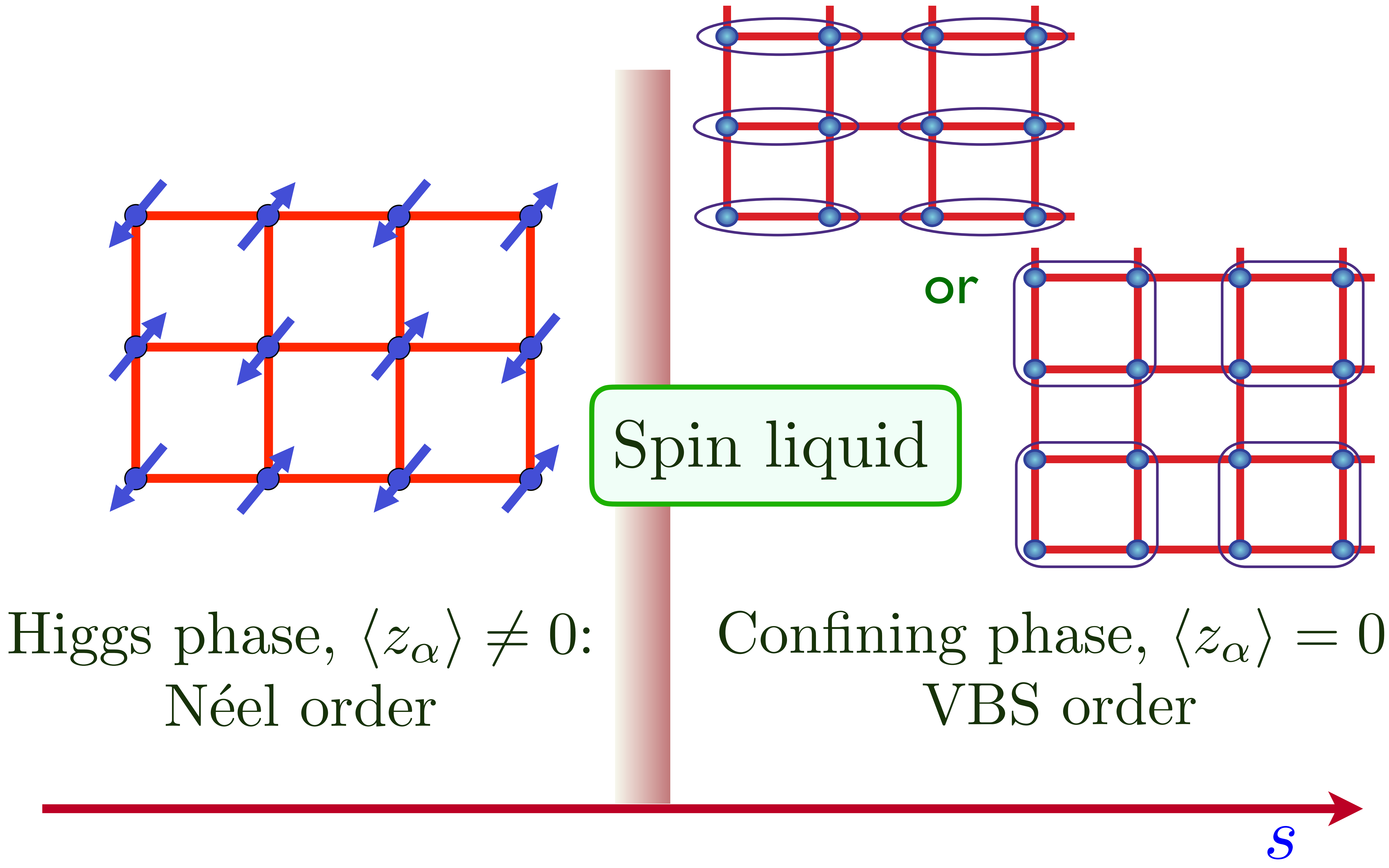
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \qquad \sum_{\alpha=1}^{N=2} b_{i\alpha}^\dagger b_{i\alpha} = n_b = 2S$$



N. Read and S. Sachdev, Phys. Rev. Lett. **66,** 1773 (1991)
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b^\dagger_{i\alpha} b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b^\dagger_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989)
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)

Insulating S=1/2 antiferromagnet

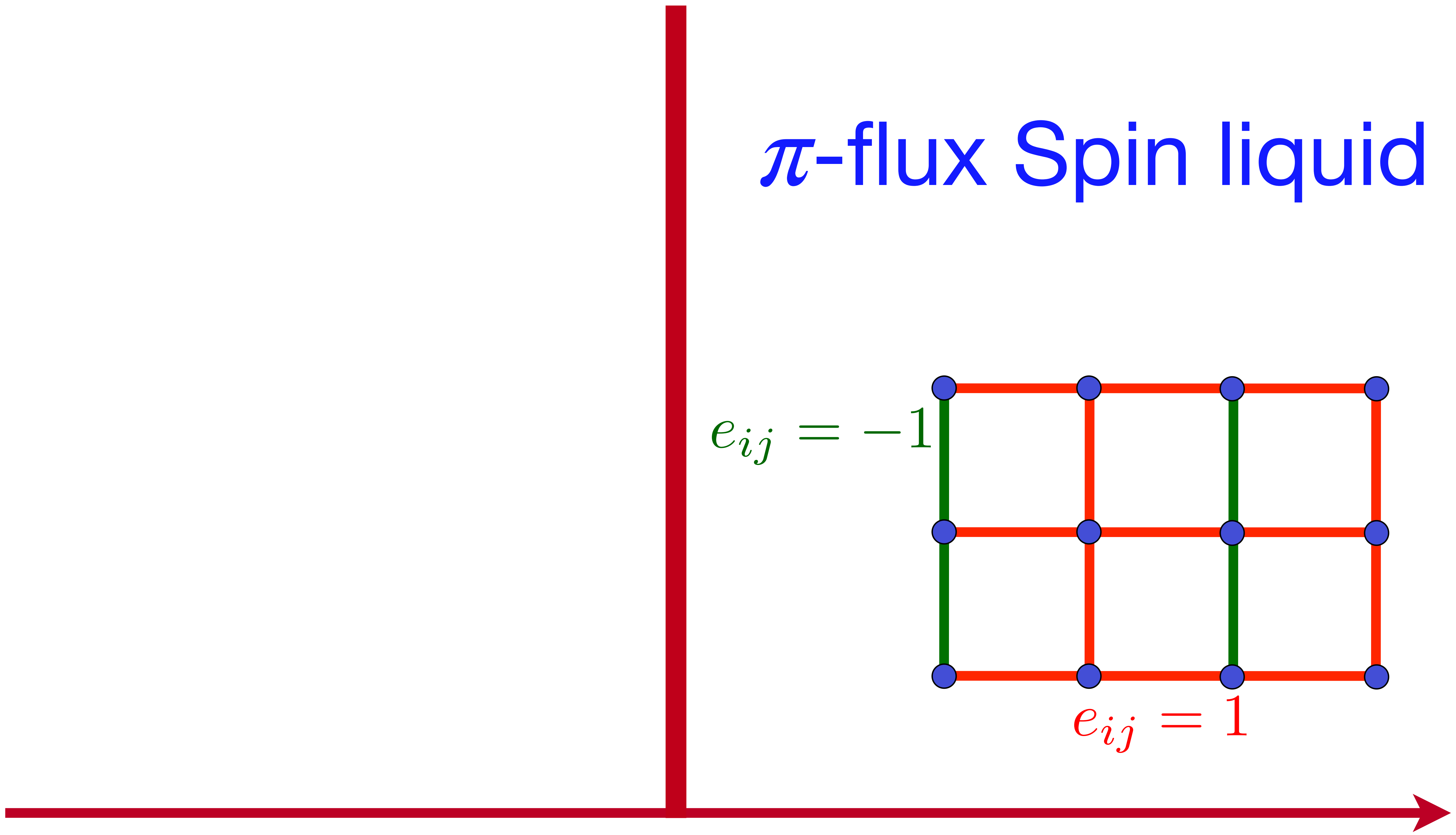
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)



$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

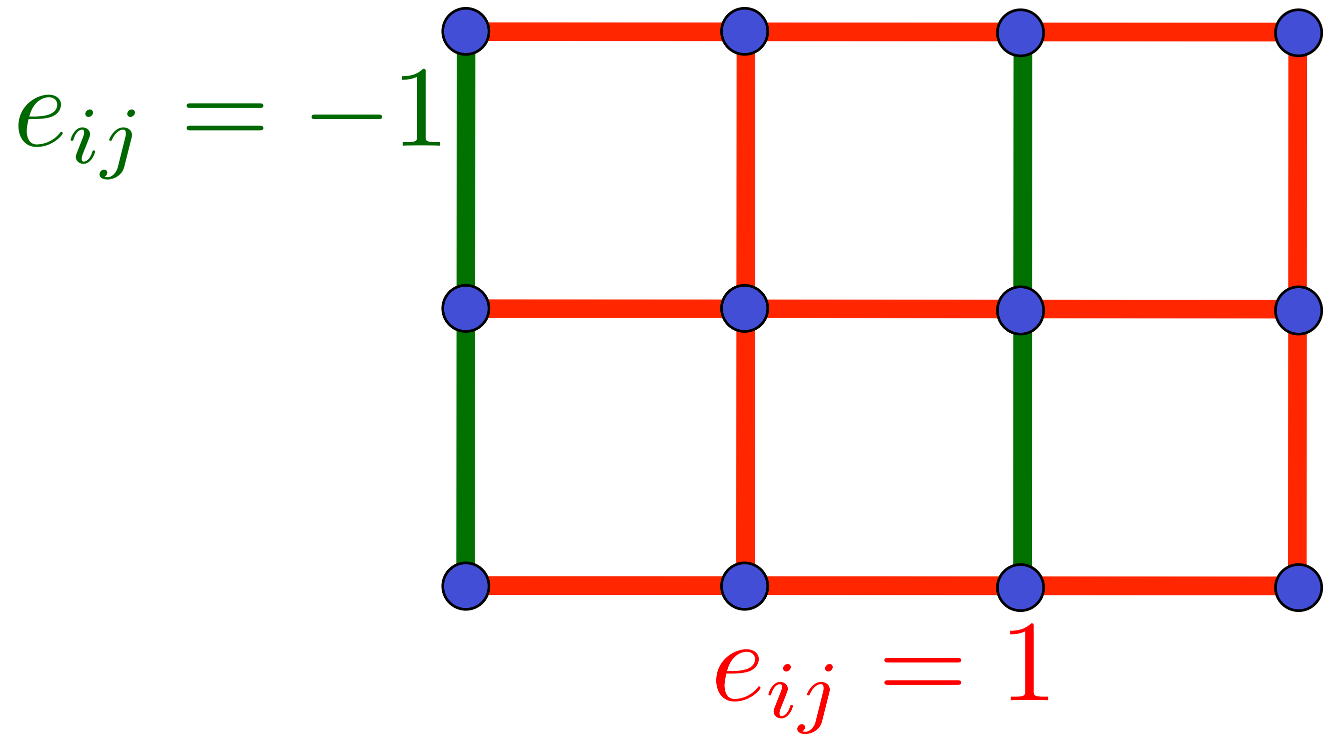
Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

π-flux Spin liquid



$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) \quad , \quad \varepsilon_{\mathbf{k}} = 2J\sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

SU(2)$_N$ QCD with $N_f = 2$ massless fermions; $\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2 square lattice anti-ferromagnet

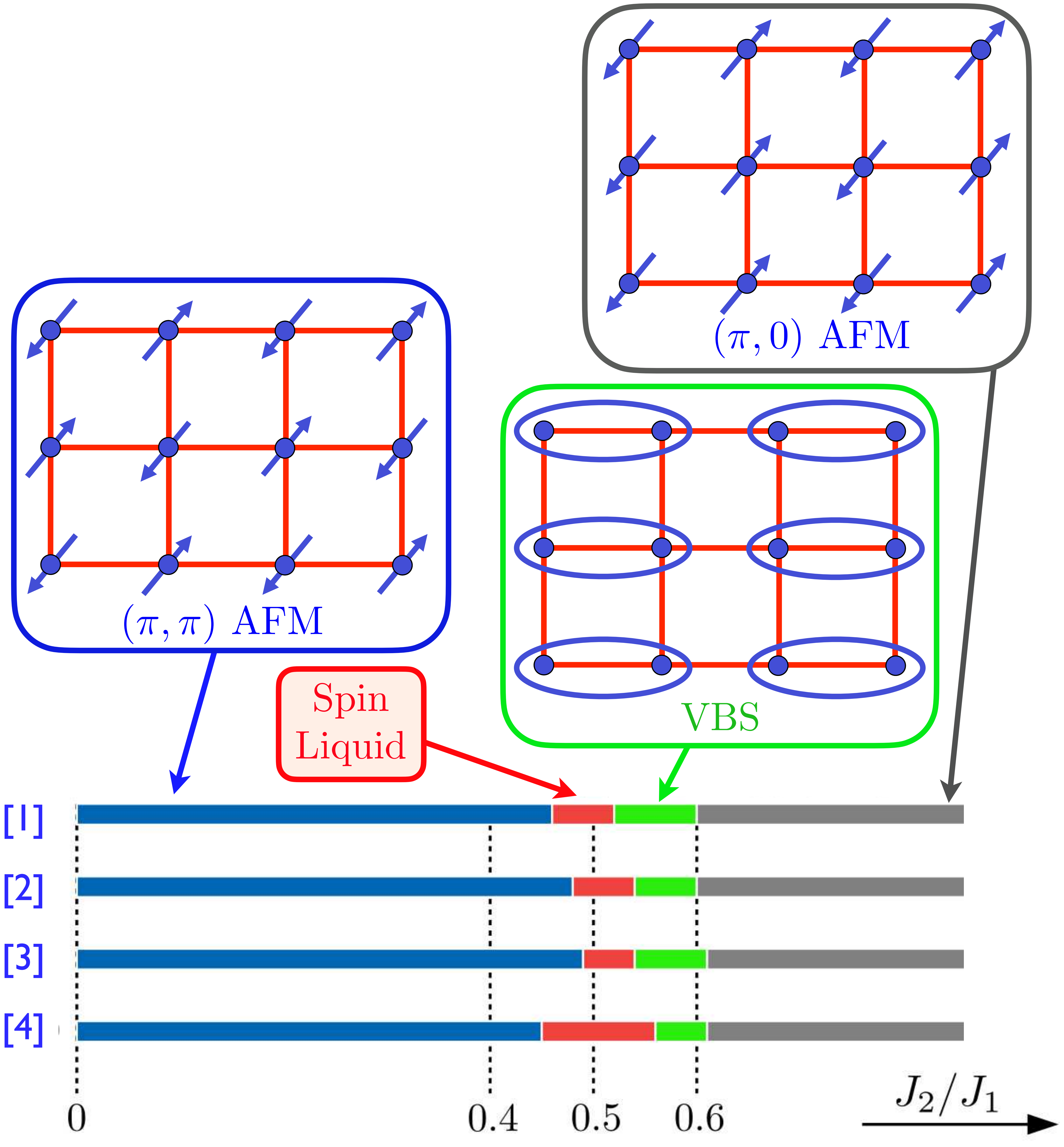
SU(2)$_N$ gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

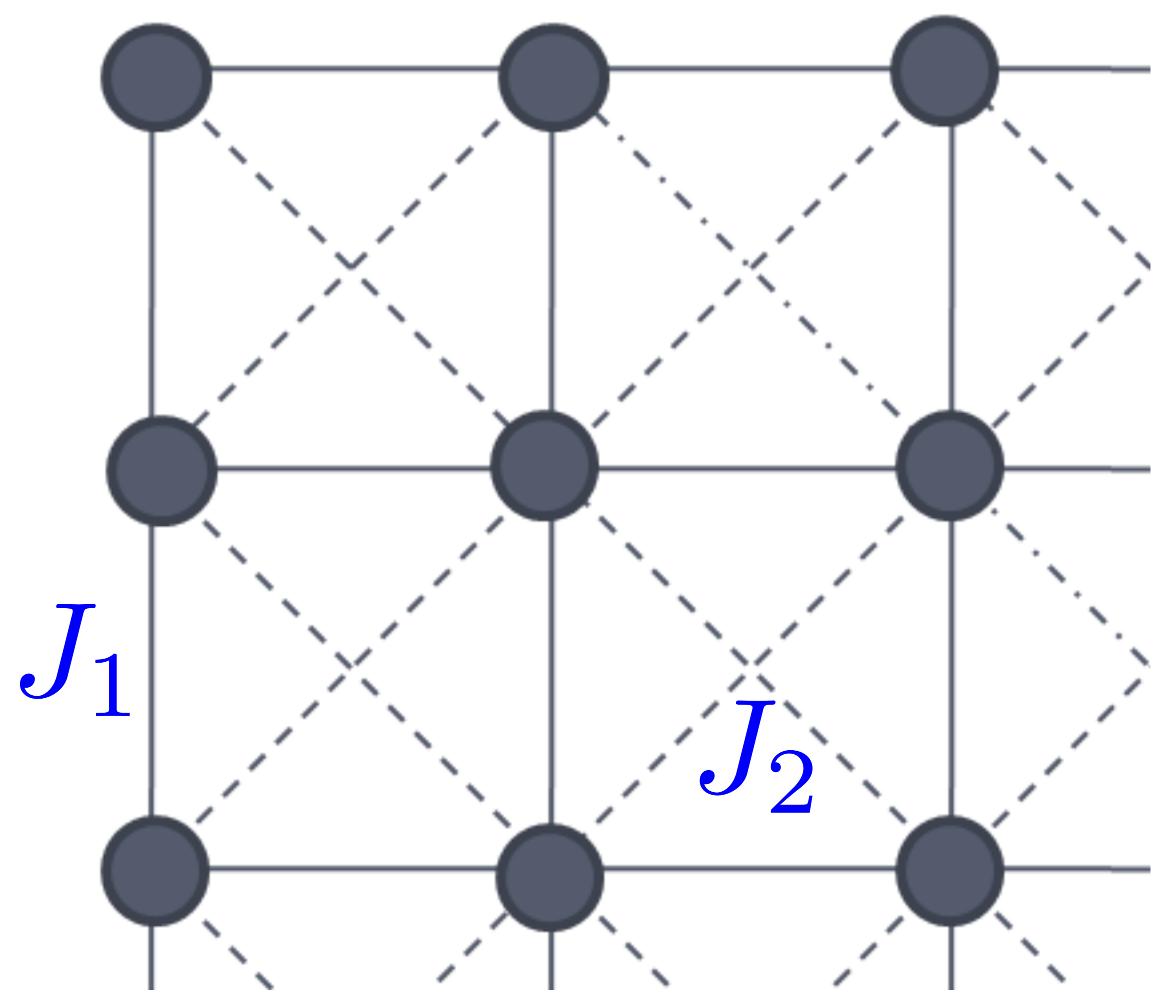
Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620; C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67,** 1034 (2022)

$\mathbb{CP}^1$ U(1) gauge theory

S=1/2 square lattice anti-ferromagnet

SU(2)$_N$ gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

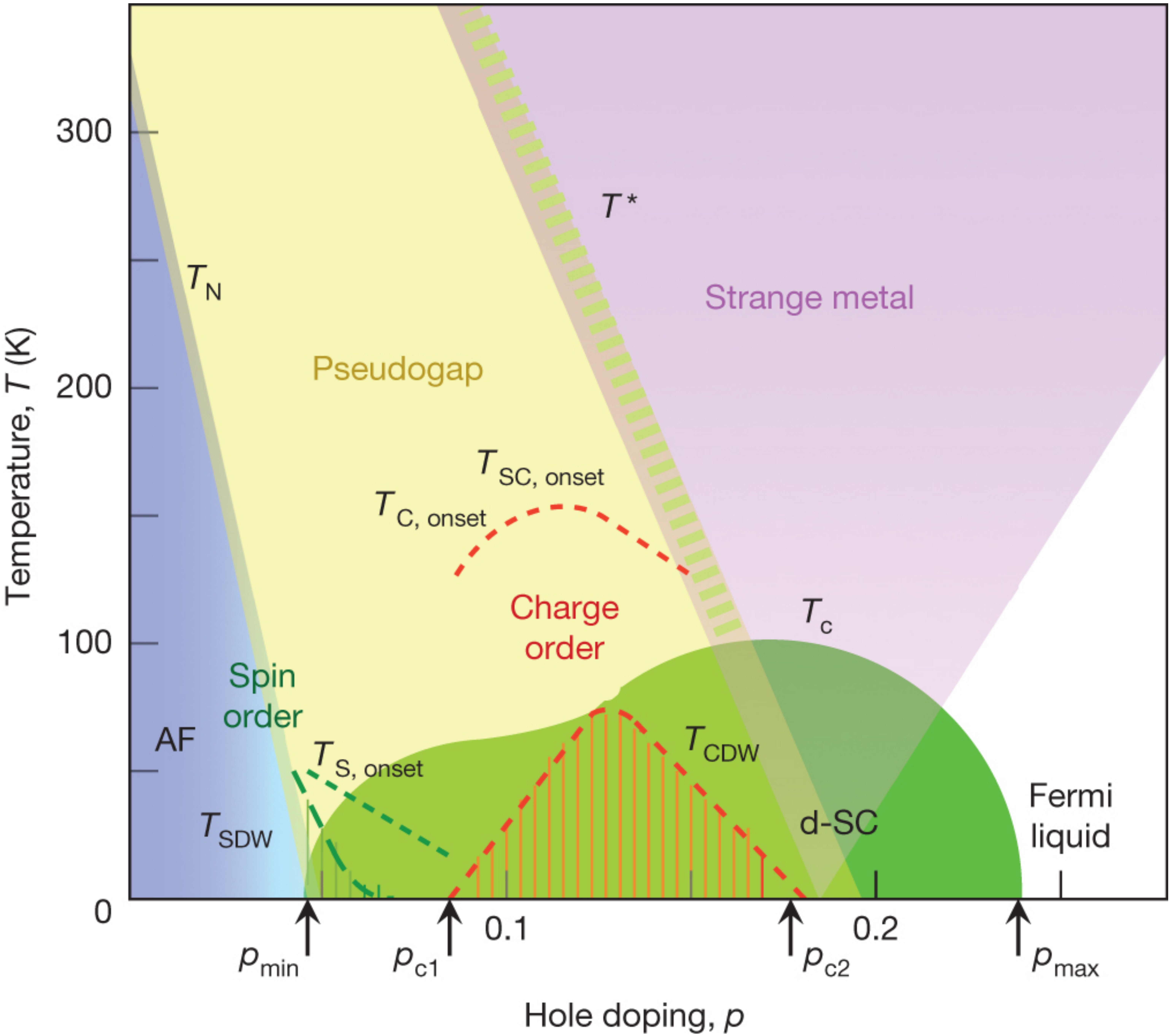
Anders W. Sandvik *Phys. Rev. Lett.* **98**, 227202 (2007)
Adam Nahum, P. Serna, J. T. Chalker, M. Ortuño, and A. M. Somoza, *Phys. Rev. Lett.* **115**, 267203 (2015)
Z. Zhou, L. Hu, W. Zhu, and Yin-Chen He, arXiv:2306.16435

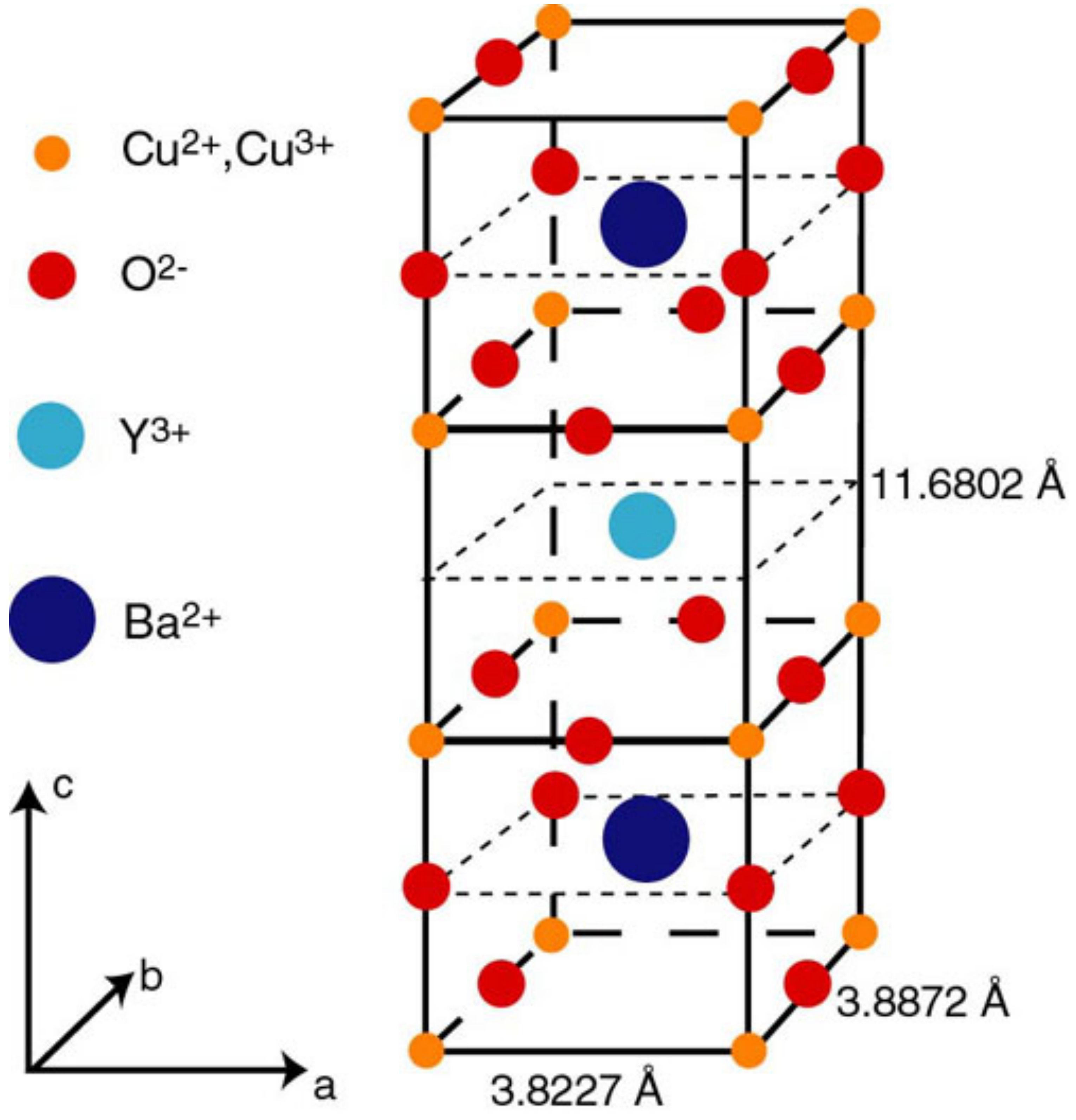
S=1/2 square lattice antiferromagnet

From the pseudogap metal to the d-wave superconductor

Confinement of the π-flux/CP^1 spin liquid, d-wave superconductivity, and charge order

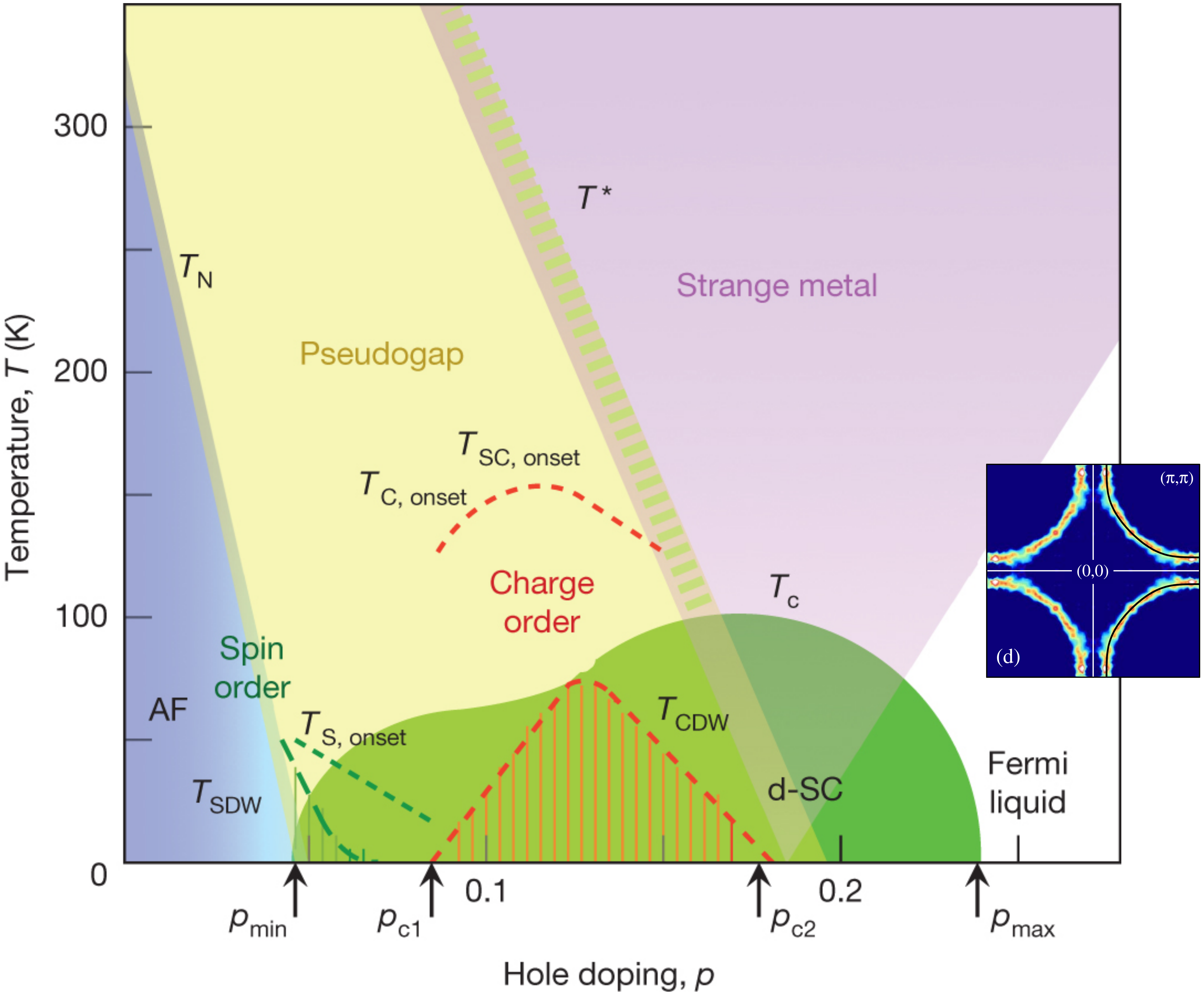
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





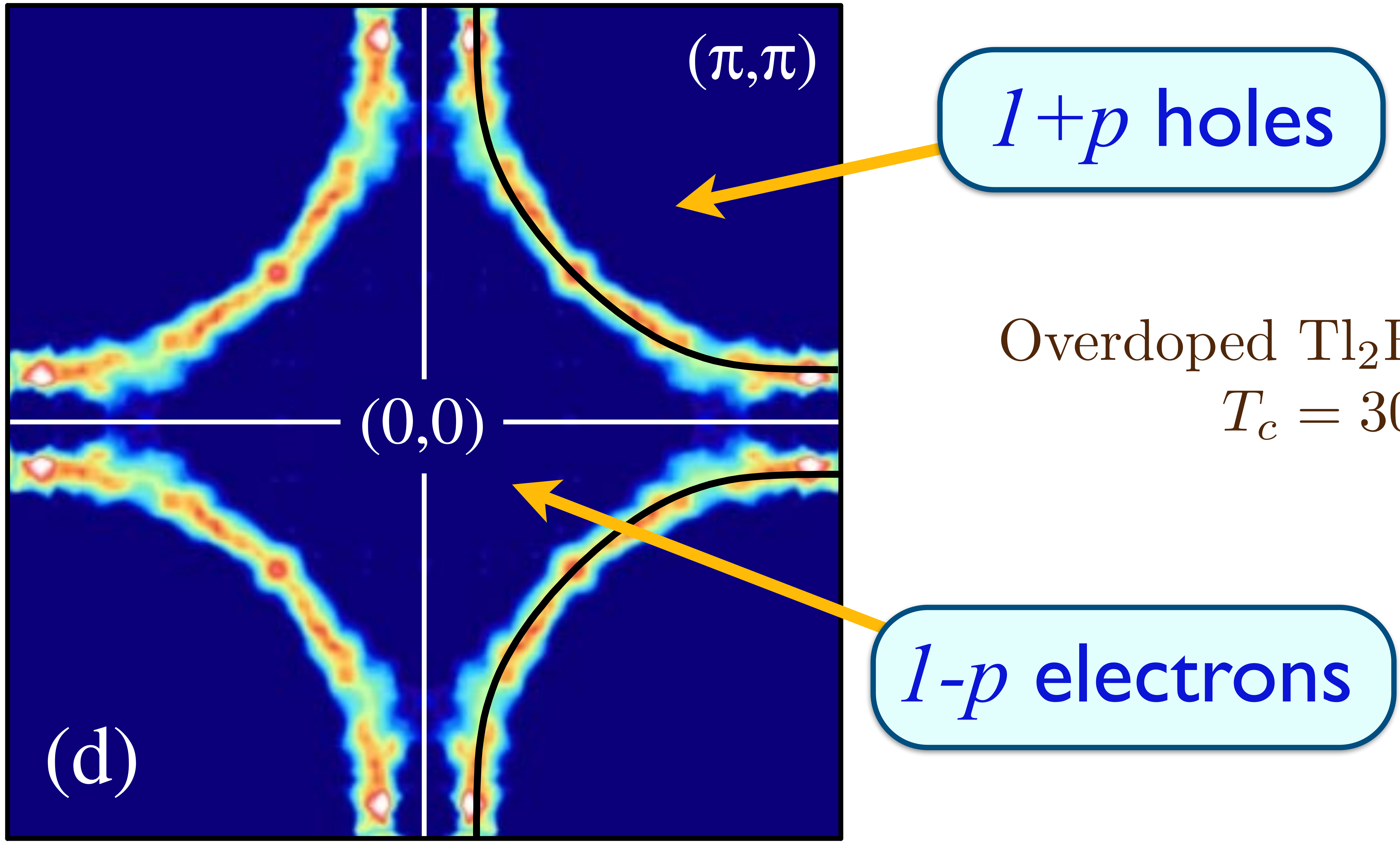
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi liquid in the overdoped metal

Photoemission at large *p*

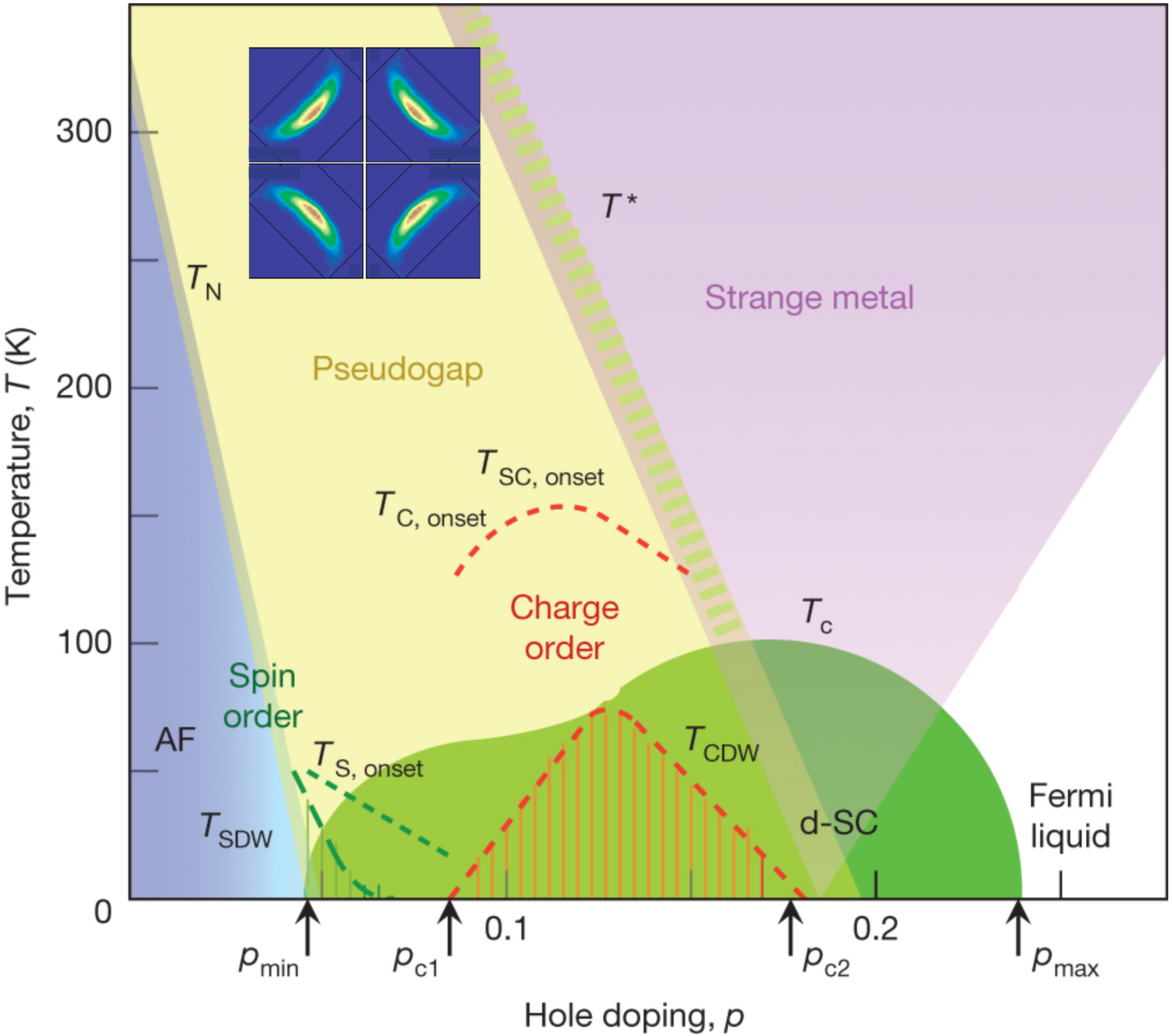


Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

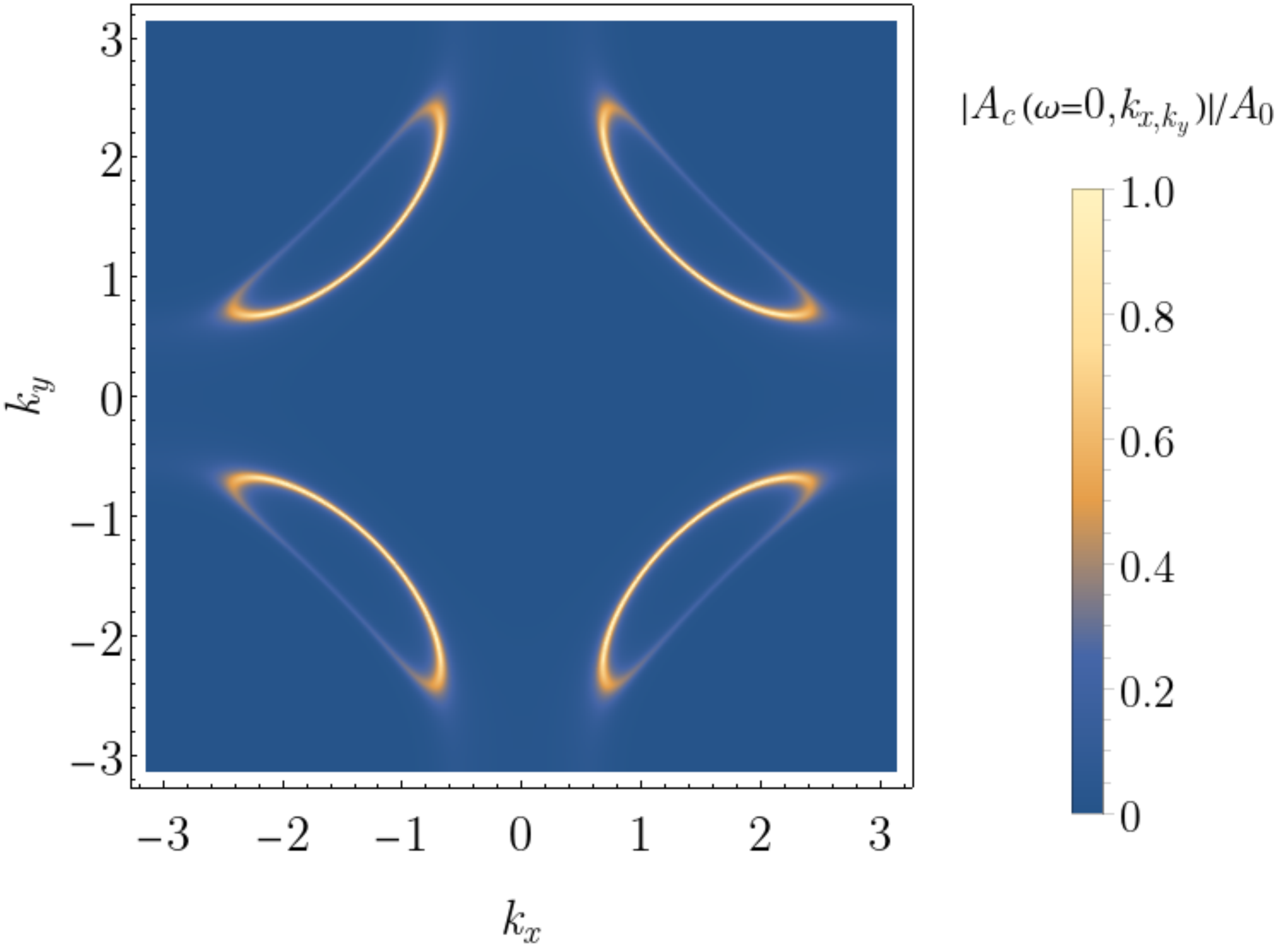
$1+p$ mobile holes in a filled band of 2 electrons per site

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Pseudogap metal with "Fermi arcs"



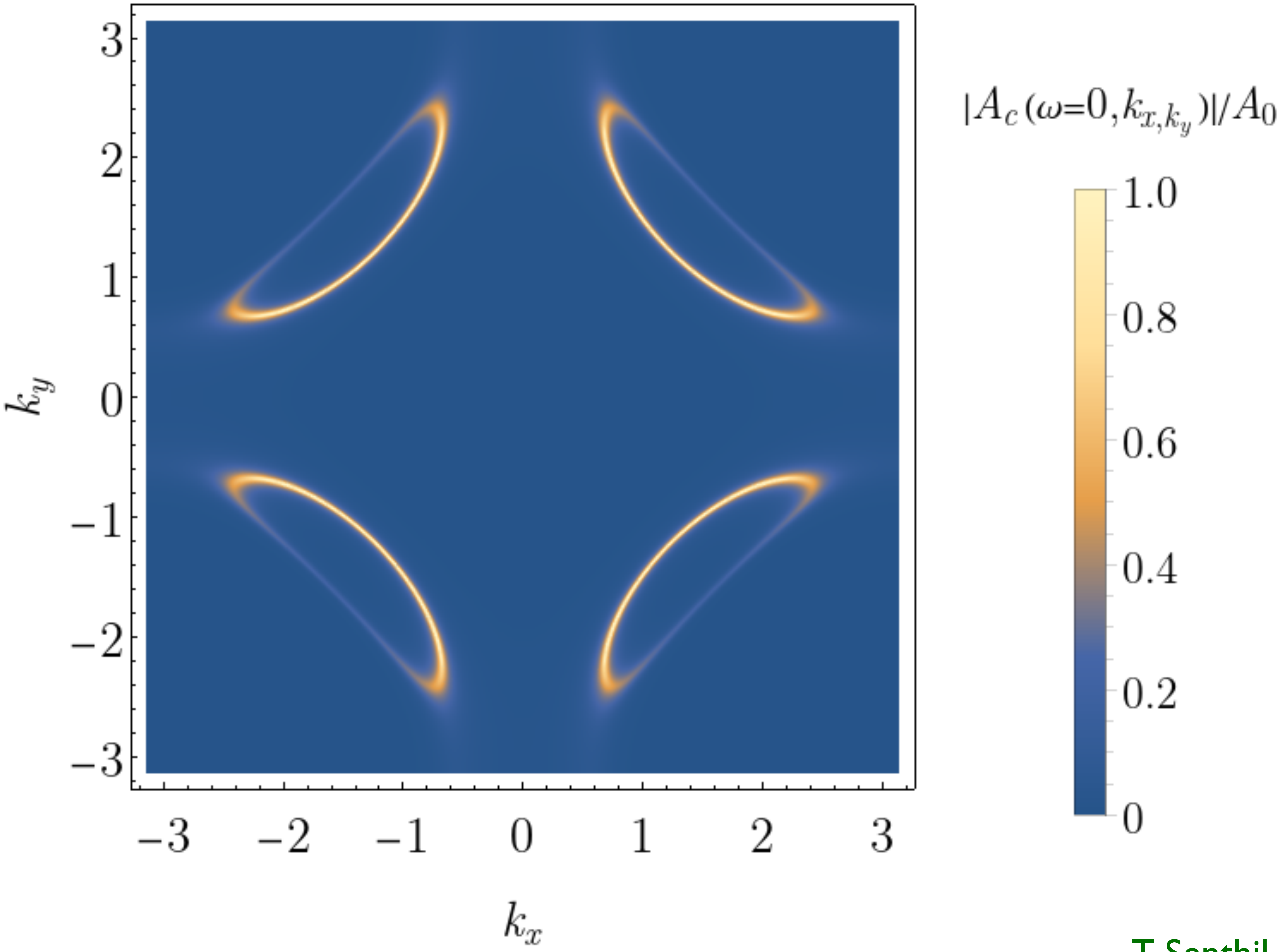
Y. Qi and S. Sachdev *Phys. Rev. B* **81**, 115129 (2010)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *Phys. Rev. B* **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, *Phys. Rev. B* **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *Phys. Rev. Lett.* **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, *Phys. Rev. Lett.* **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022).

The Fermi surface does not enclose the Luttinger volume $(1 + p)$.



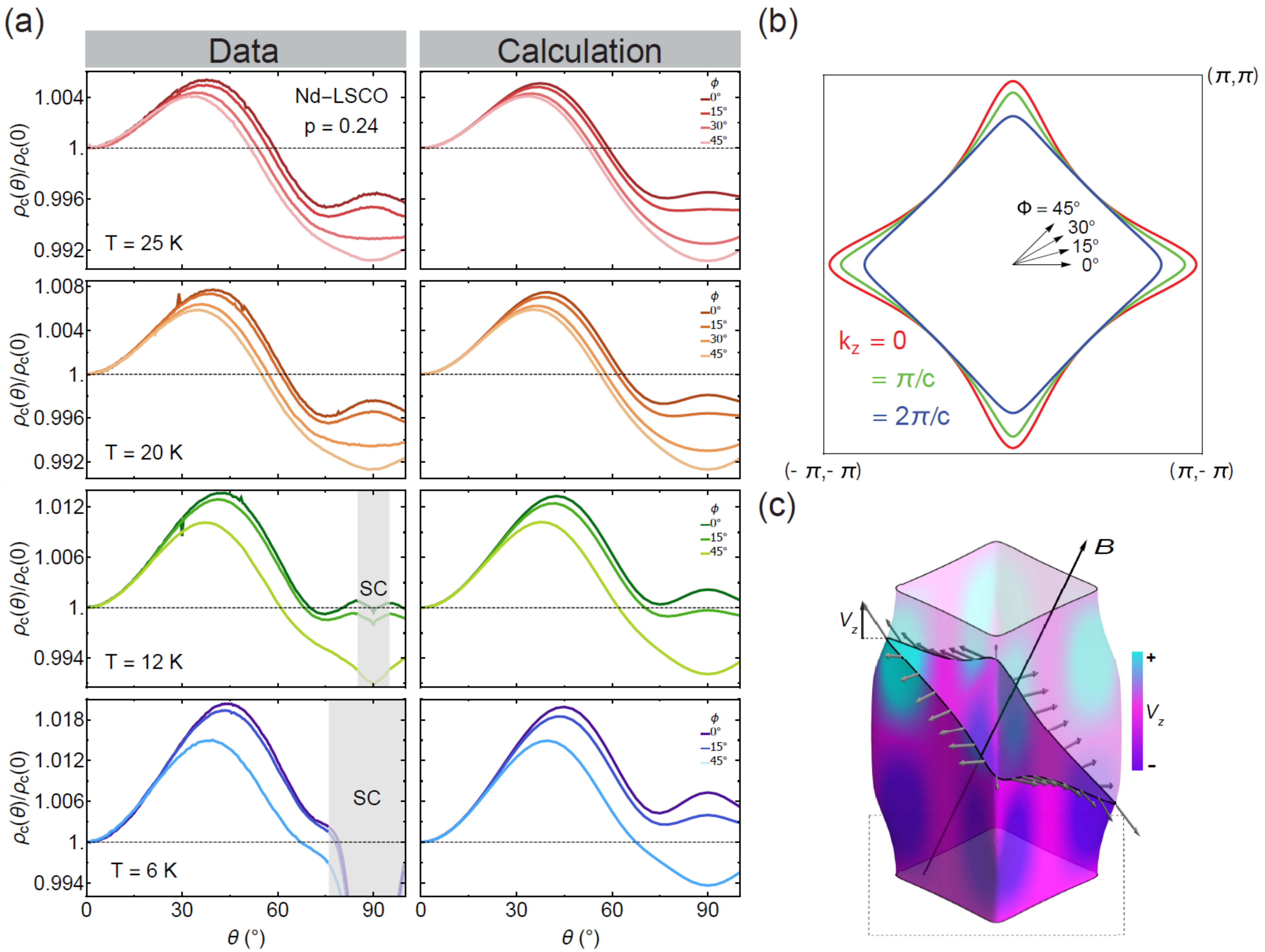
Hole pocket Fermi surfaces
of size p with
charge e, spin-1/2 quasiparticles
$+$
'spectator'
square lattice spin liquid
at half-filling.

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1+p)$.

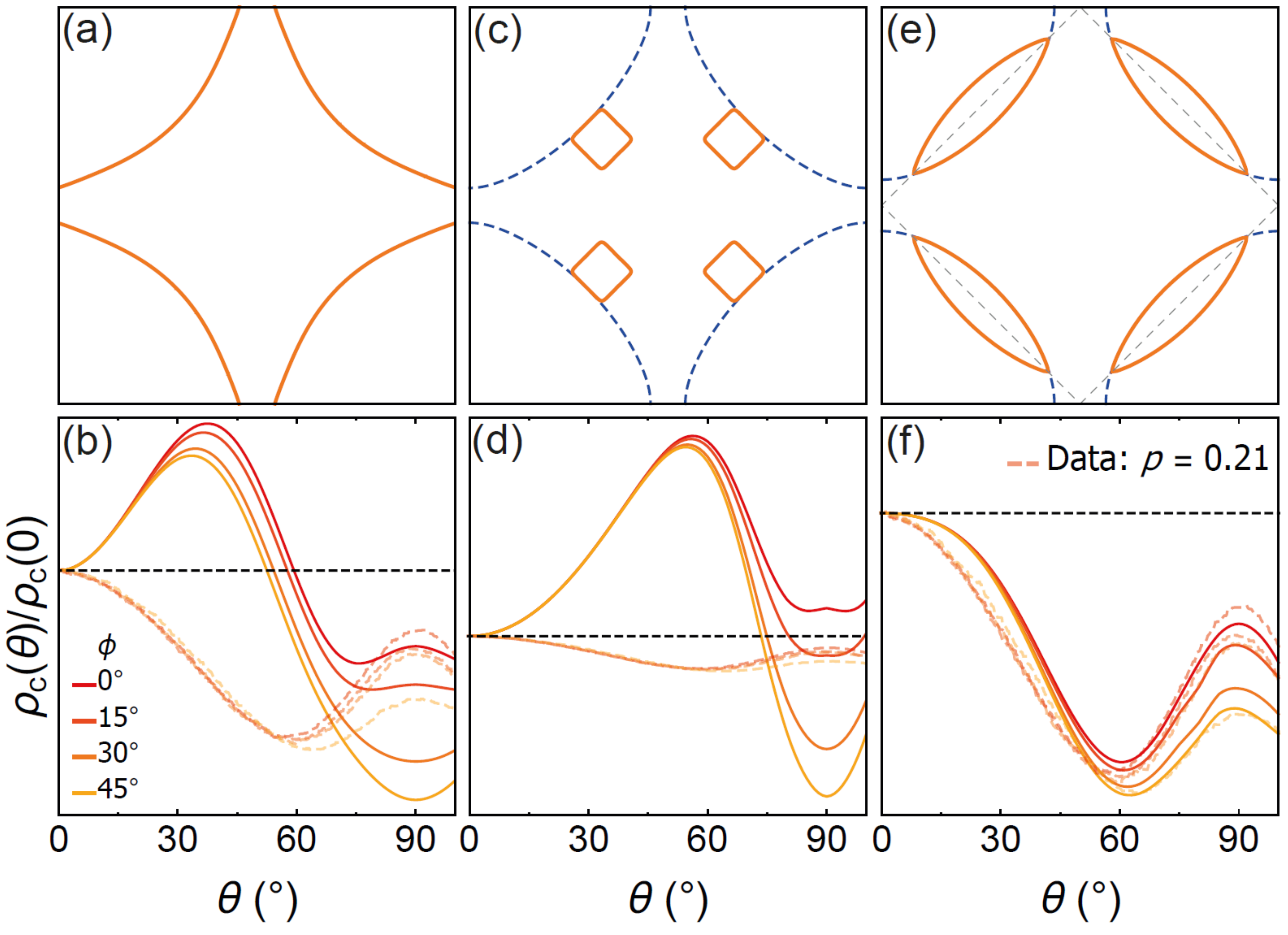
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)
A. Paramekanti and A. Vishwanath, *Phys. Rev. B* **70**, 245118 (2004)

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

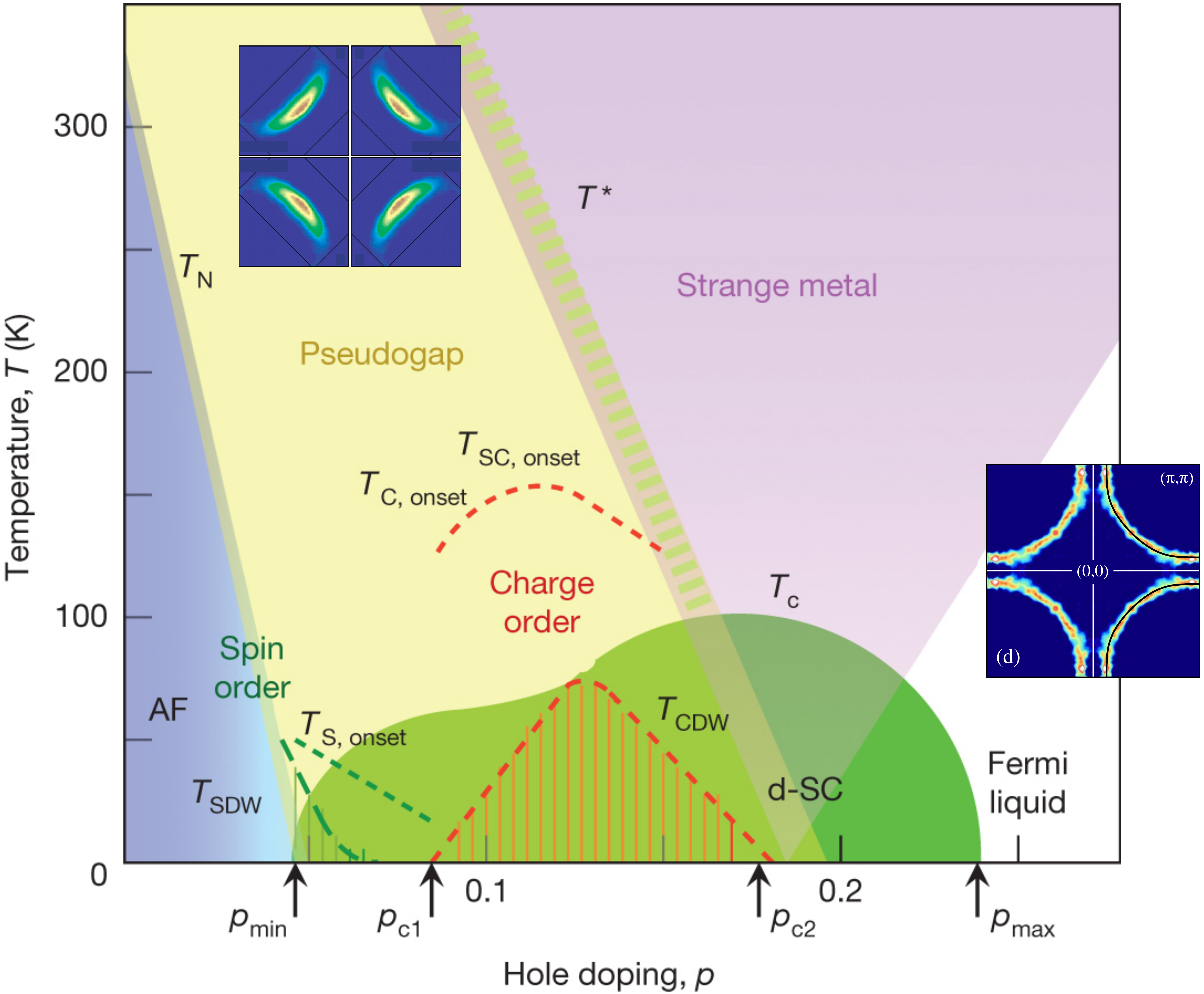


$p > p_c$ Large Fermi surface



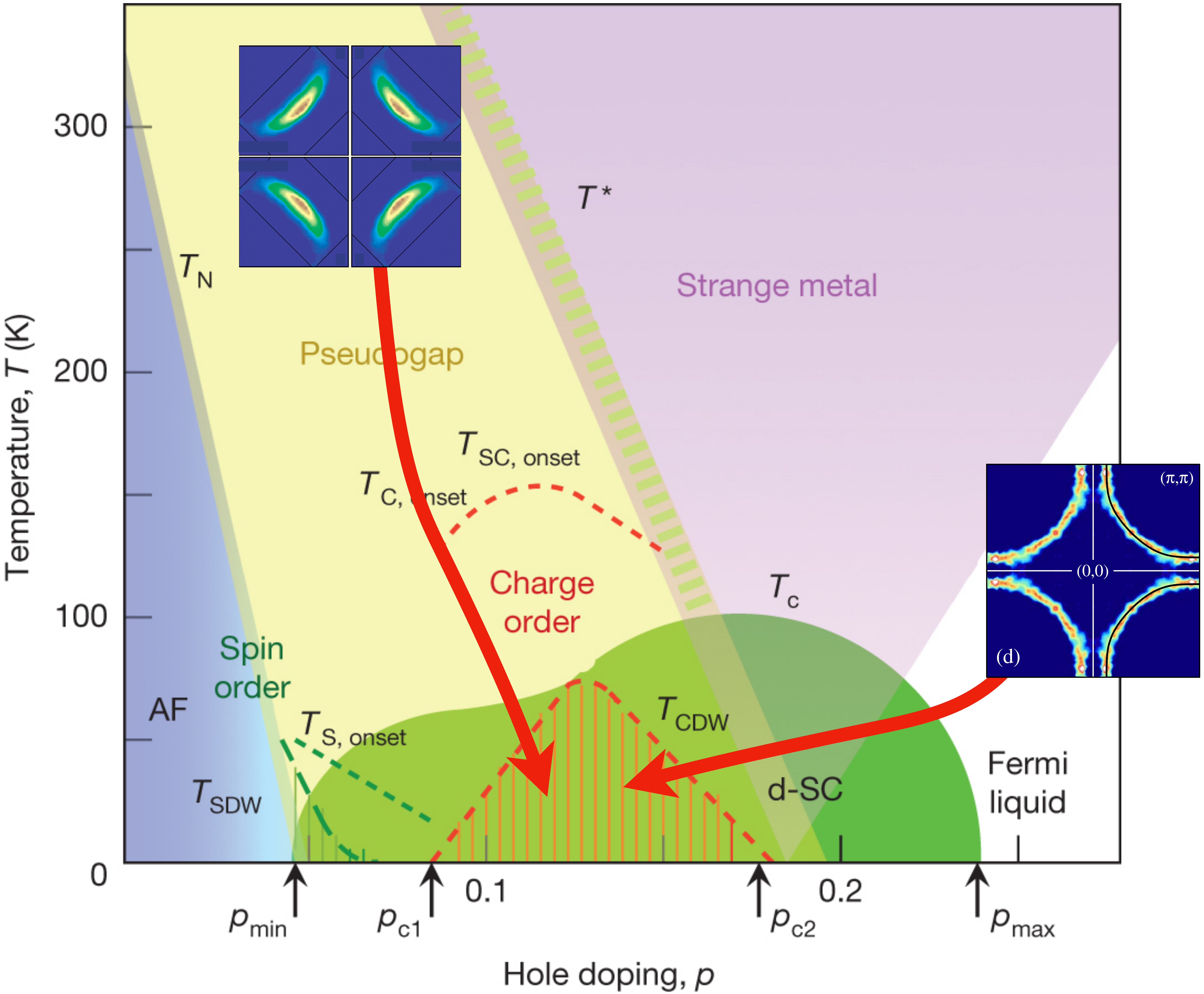
$p < p_c$ Reconstructed Fermi surface

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

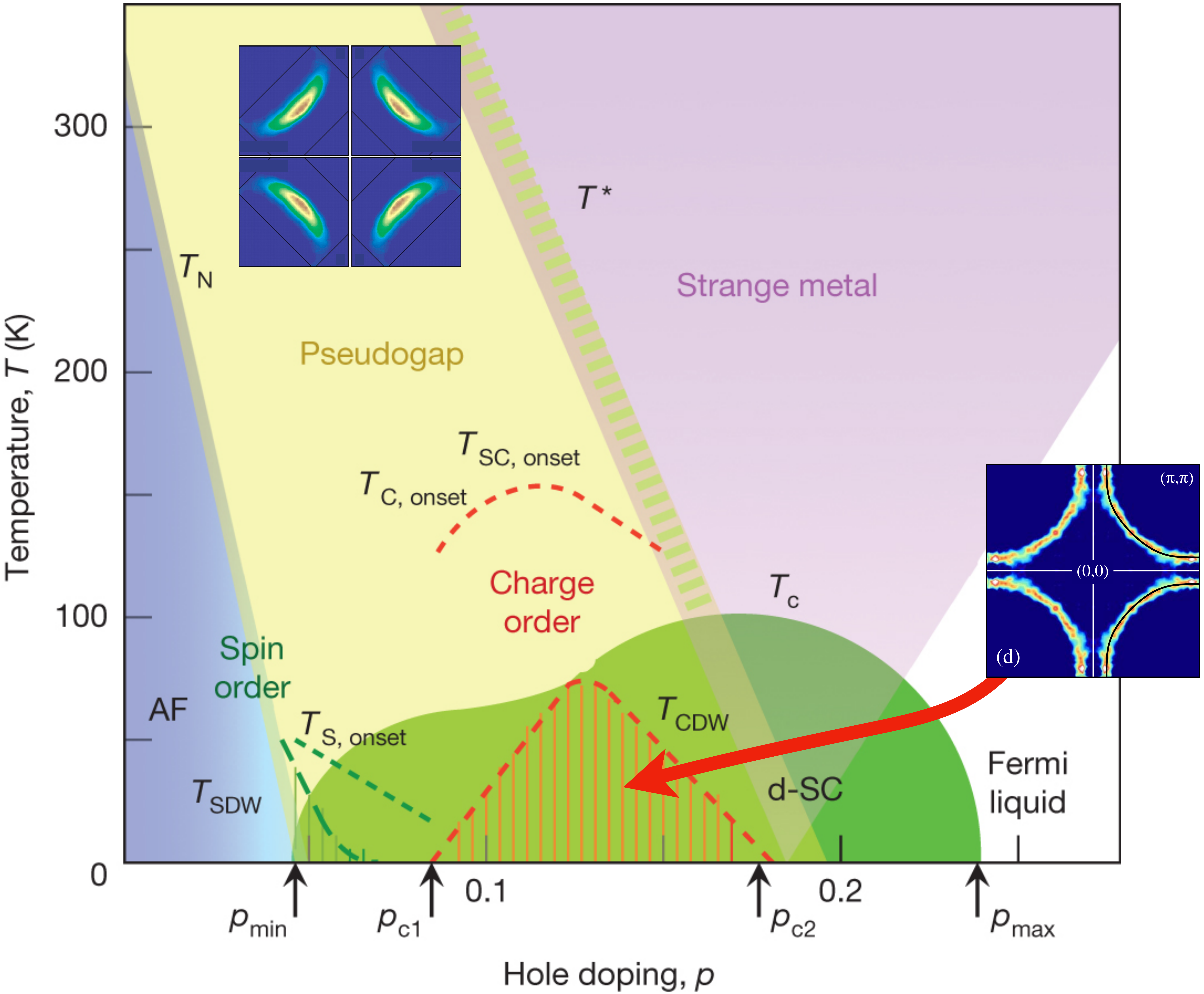
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.

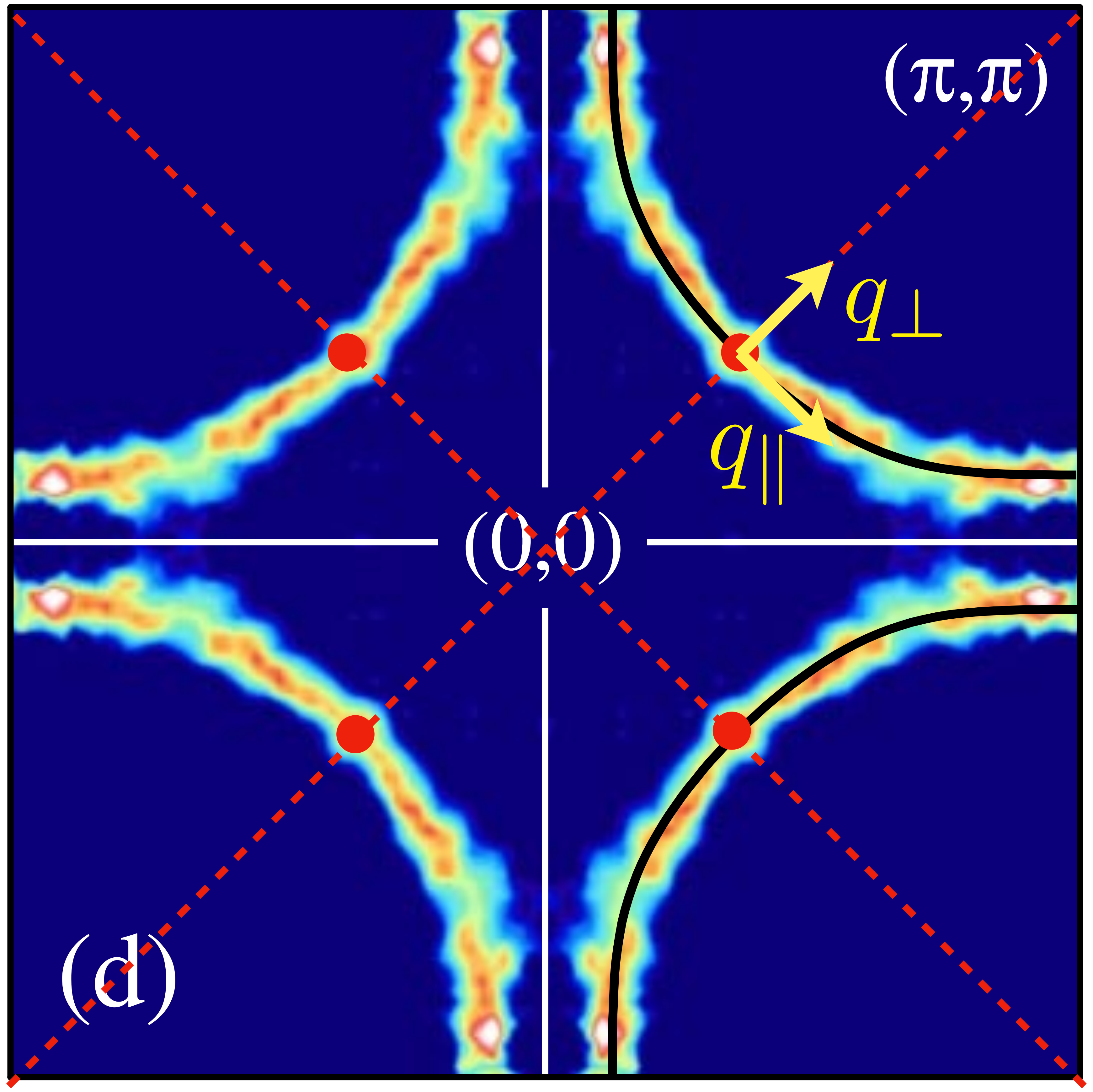
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.

BCS/Bogoliubov quasiparticles in a d-wave superconductor

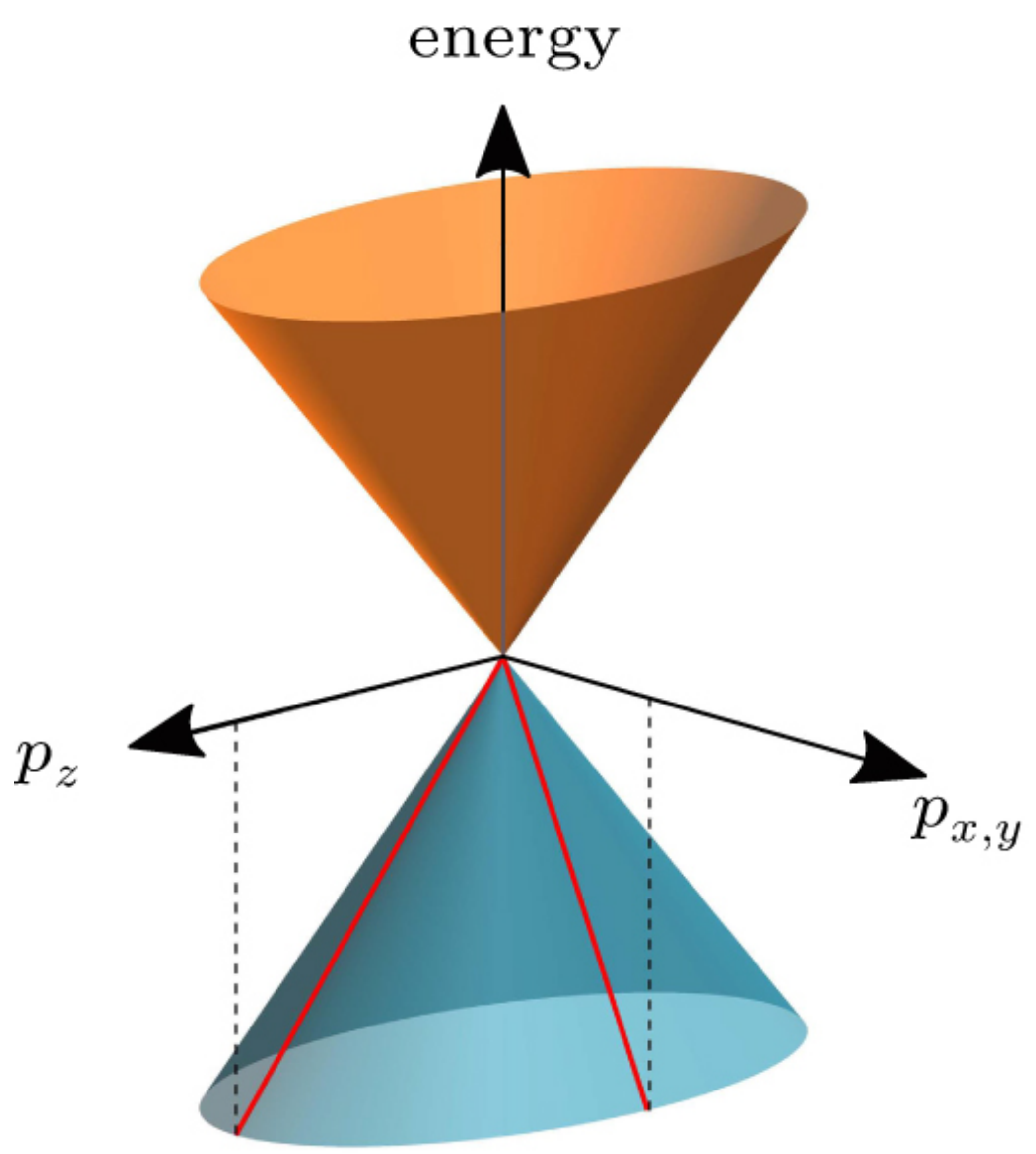


$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

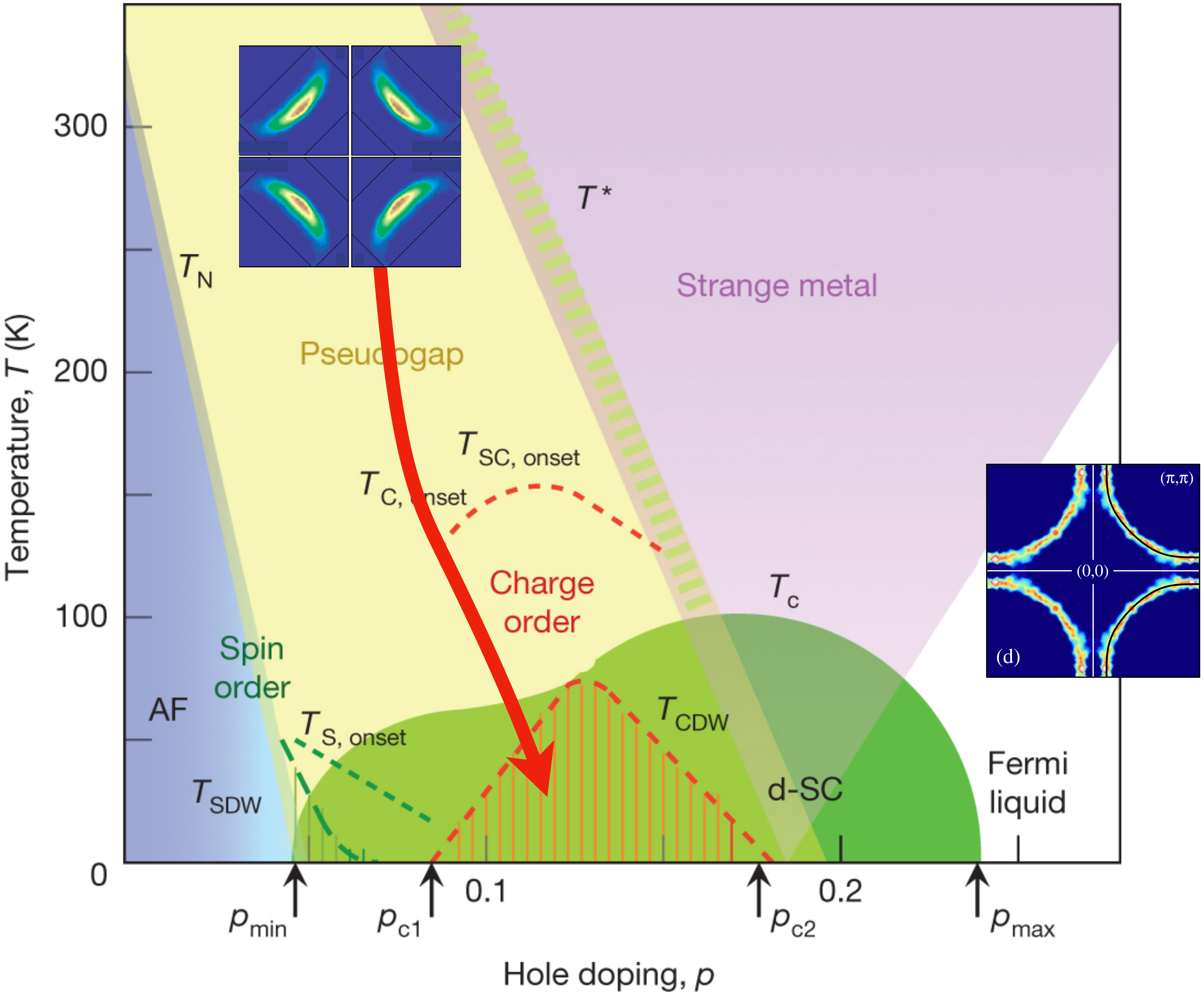
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_\|^2\right)^{1/2}$$

with $v_F \gg v_\Delta$.

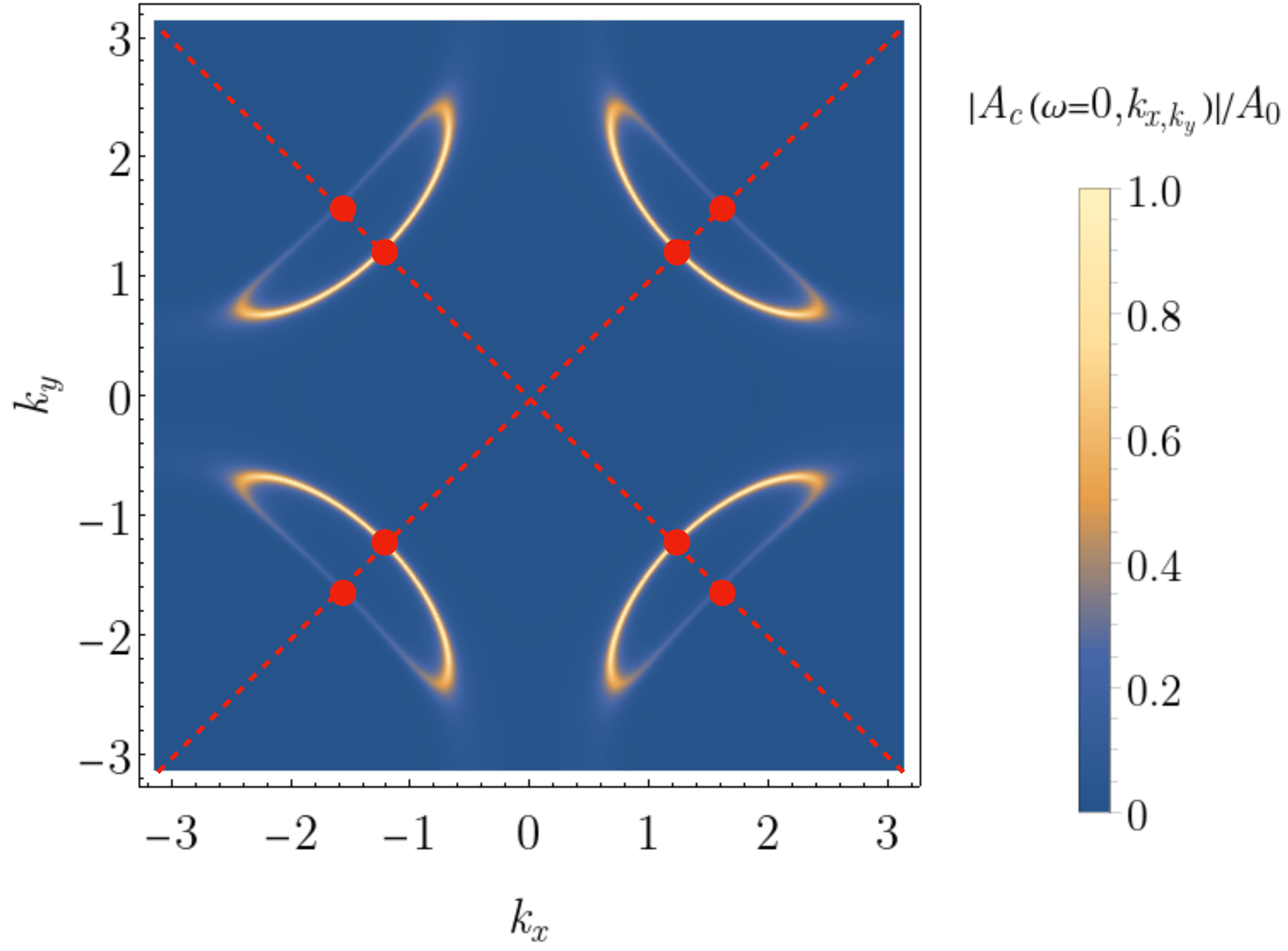


Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



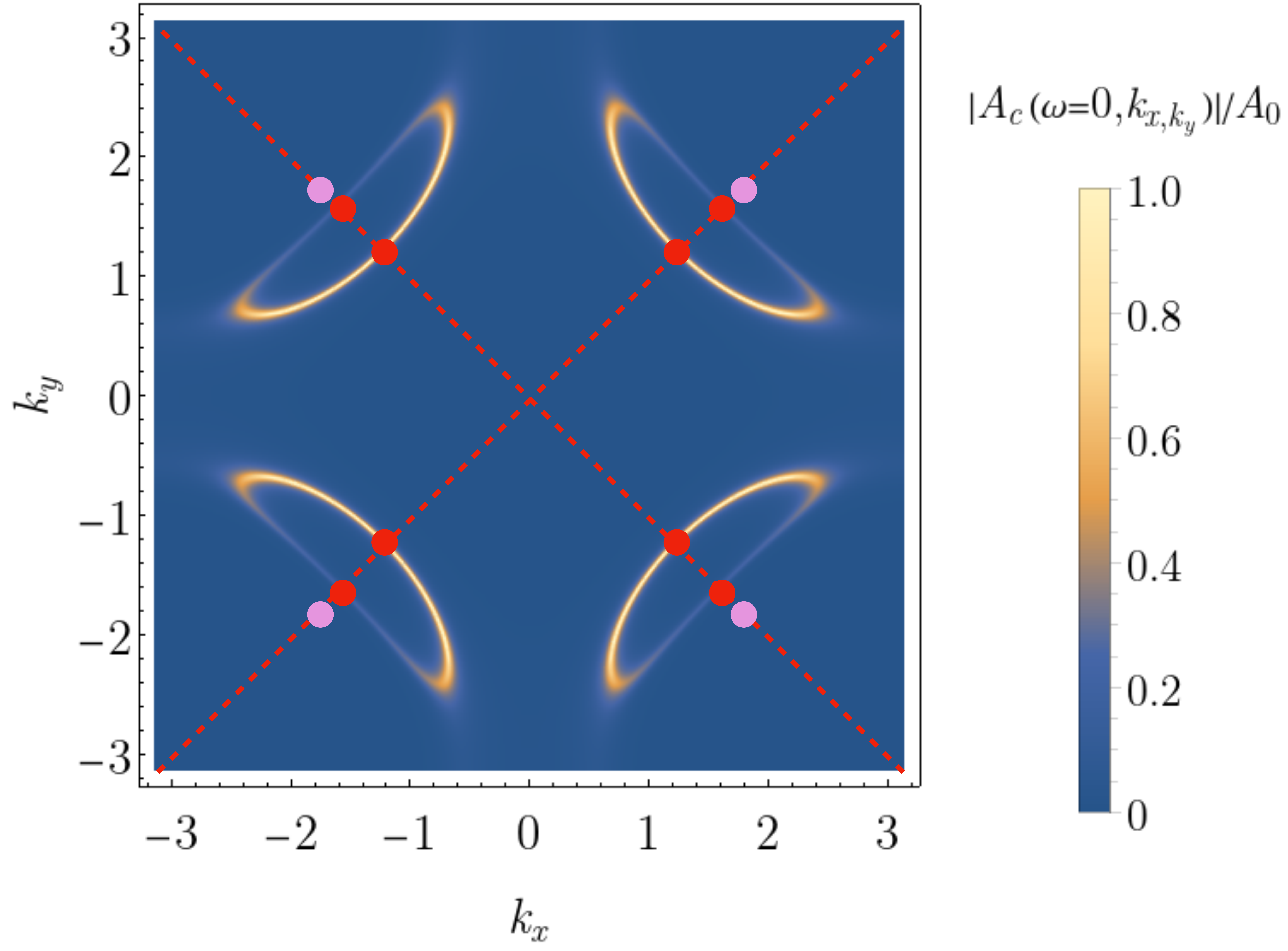
Build a theory for the phase diagram from a theory of the pseudogap metal as a 'metastable' $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.



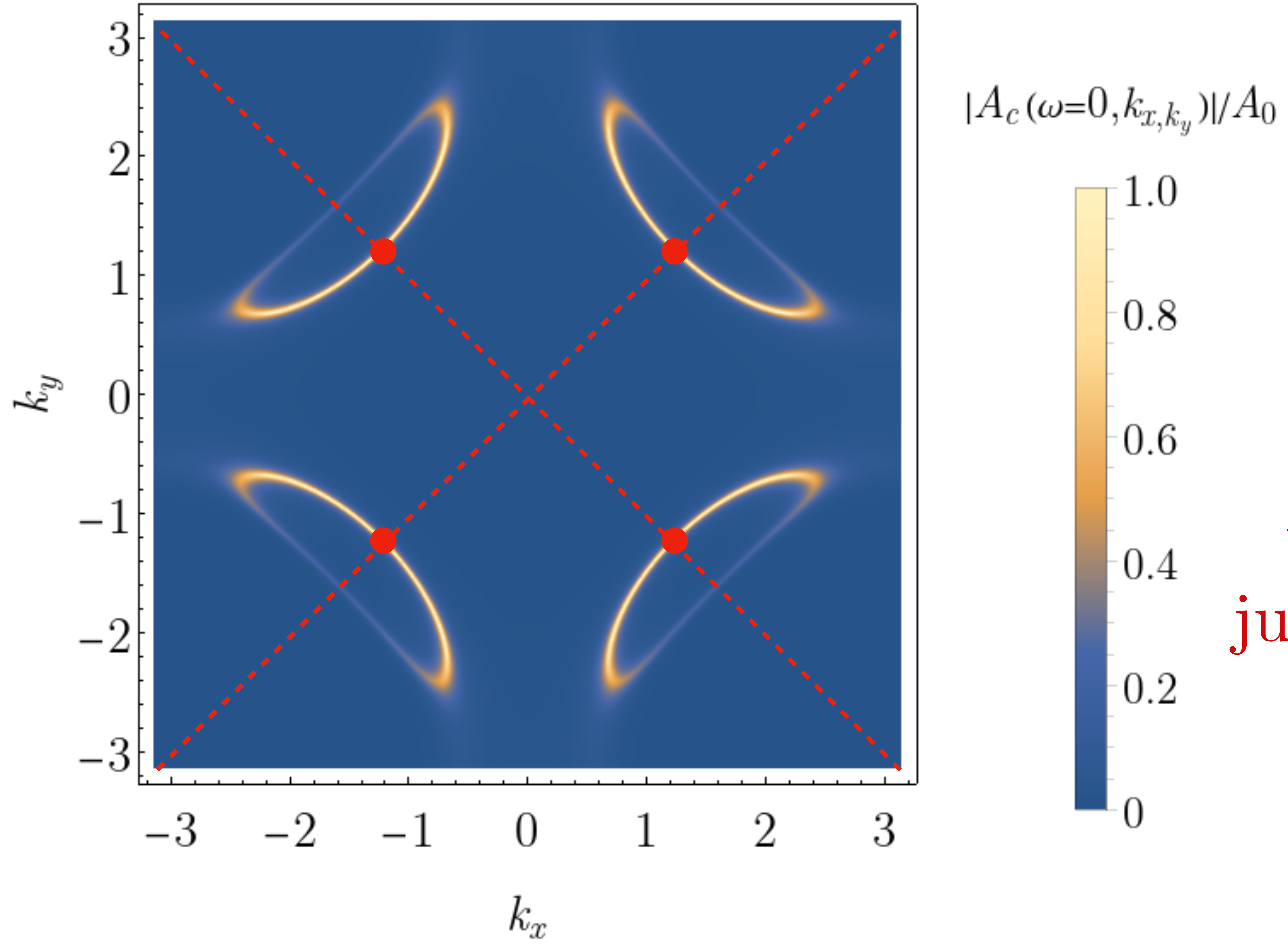
$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

Adding d-wave pairing
to the hole pockets
leads to 8 nodal points???



8 nodal points from the Fermi pockets
and
4 nodal points from the π-flux spin liquid

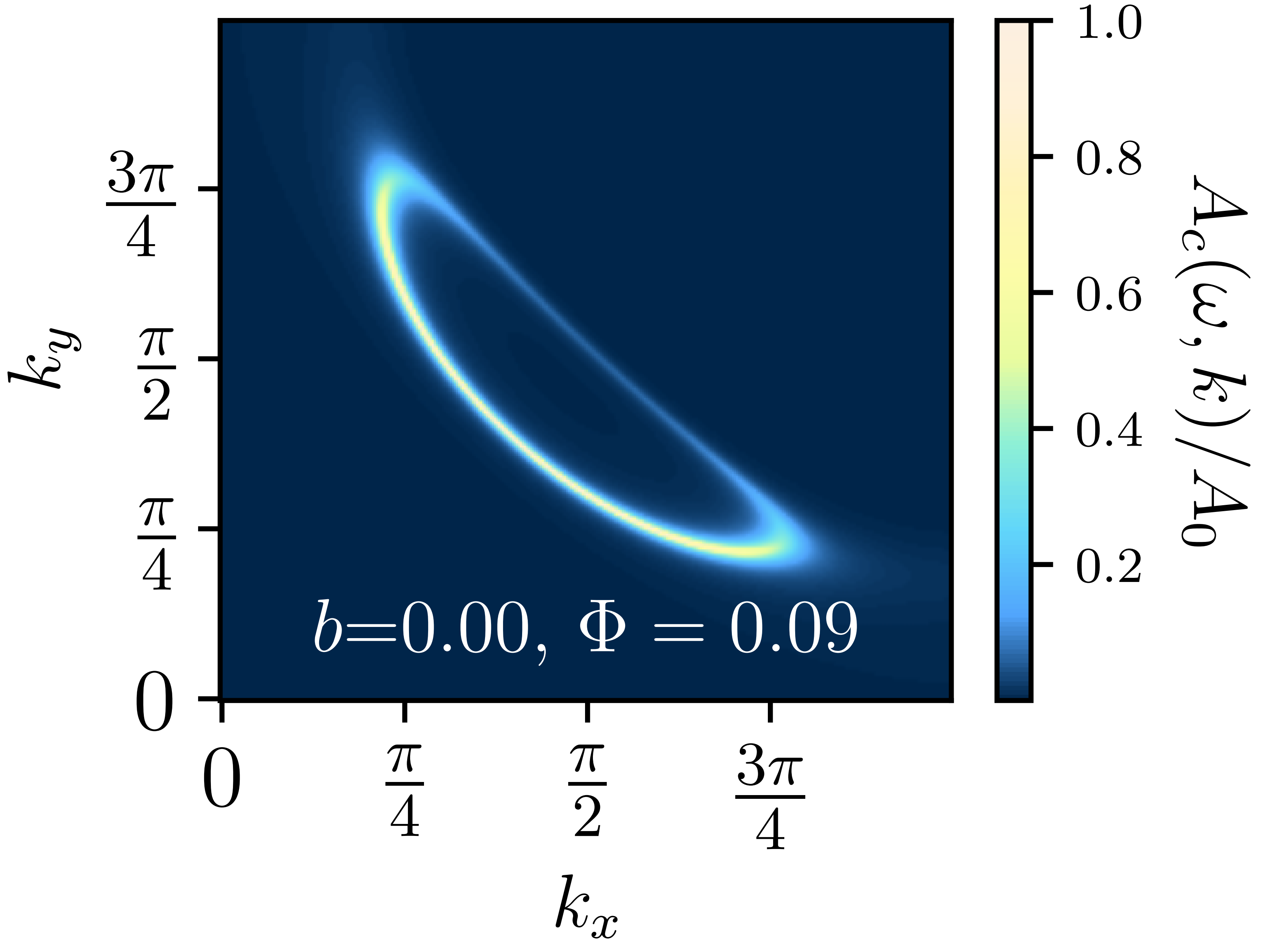
Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)



8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS *d*-wave state.

Shubhayu Chatterjee and S. Sachdev, PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

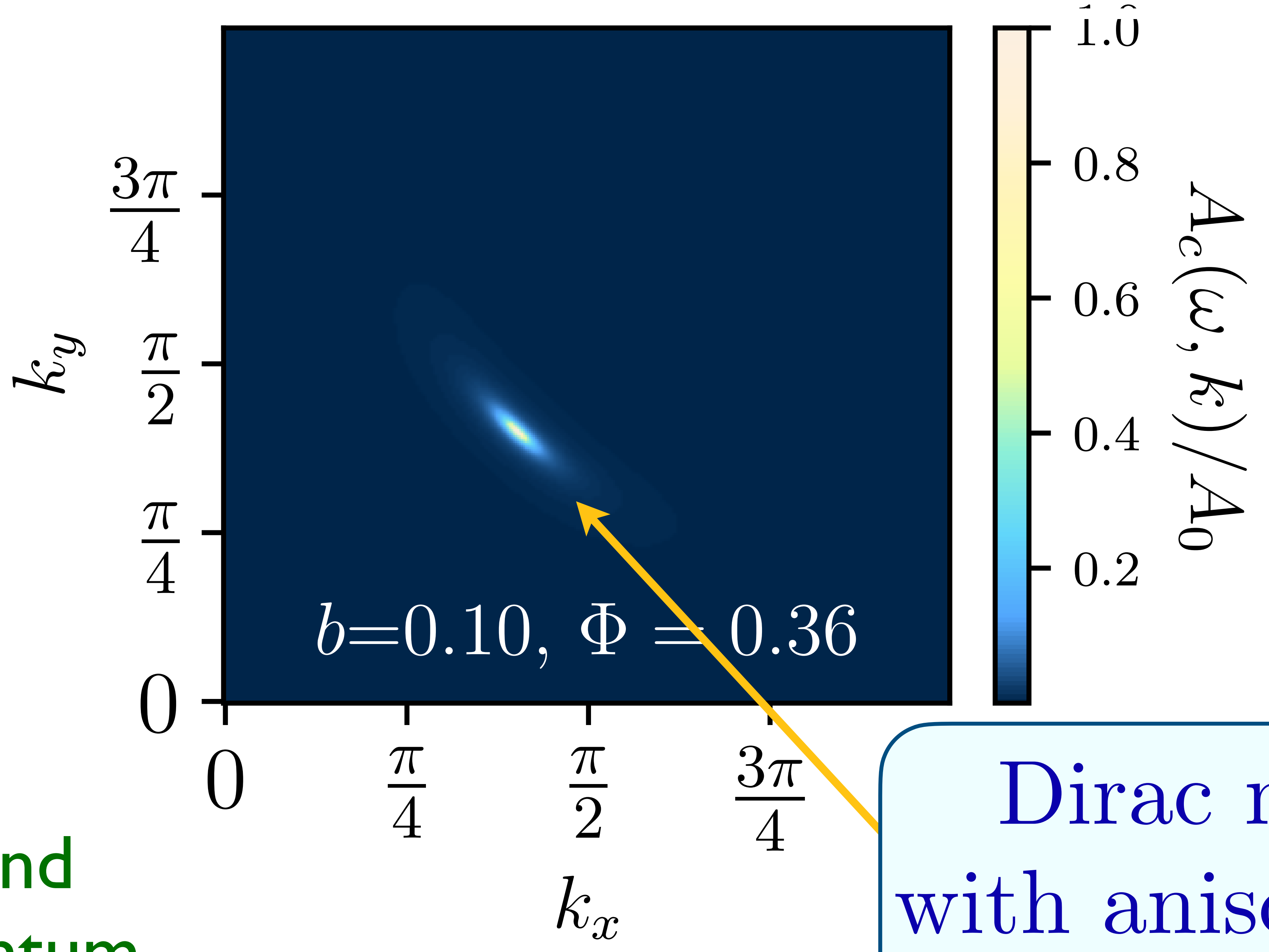
Electron spectral density in hole-doped cuprates



FL*

d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson.

Spinons of the π-flux state annihilate the extra nodes in the d-wave superconductor.

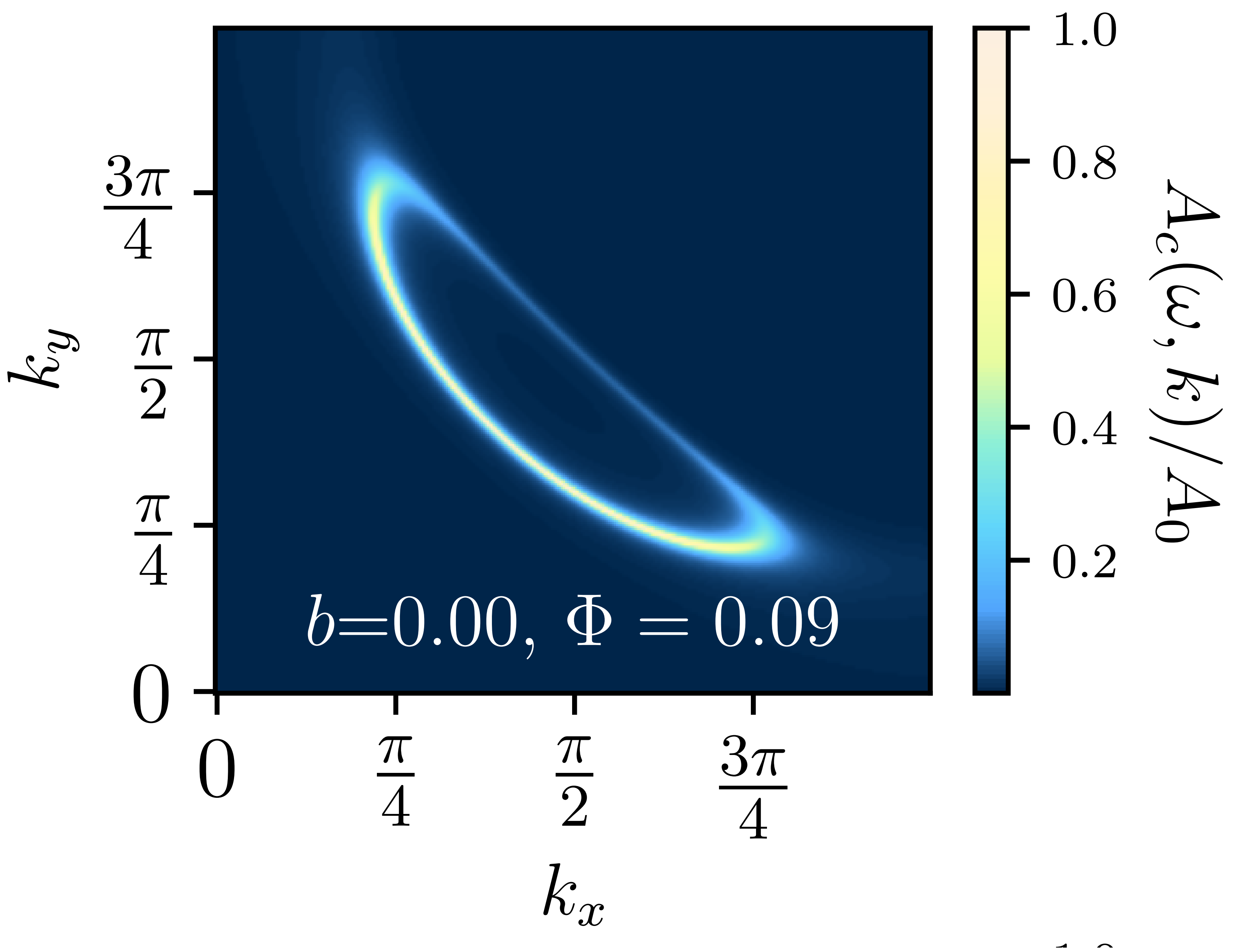


dSC

Dirac node with anisotropic velocities

Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

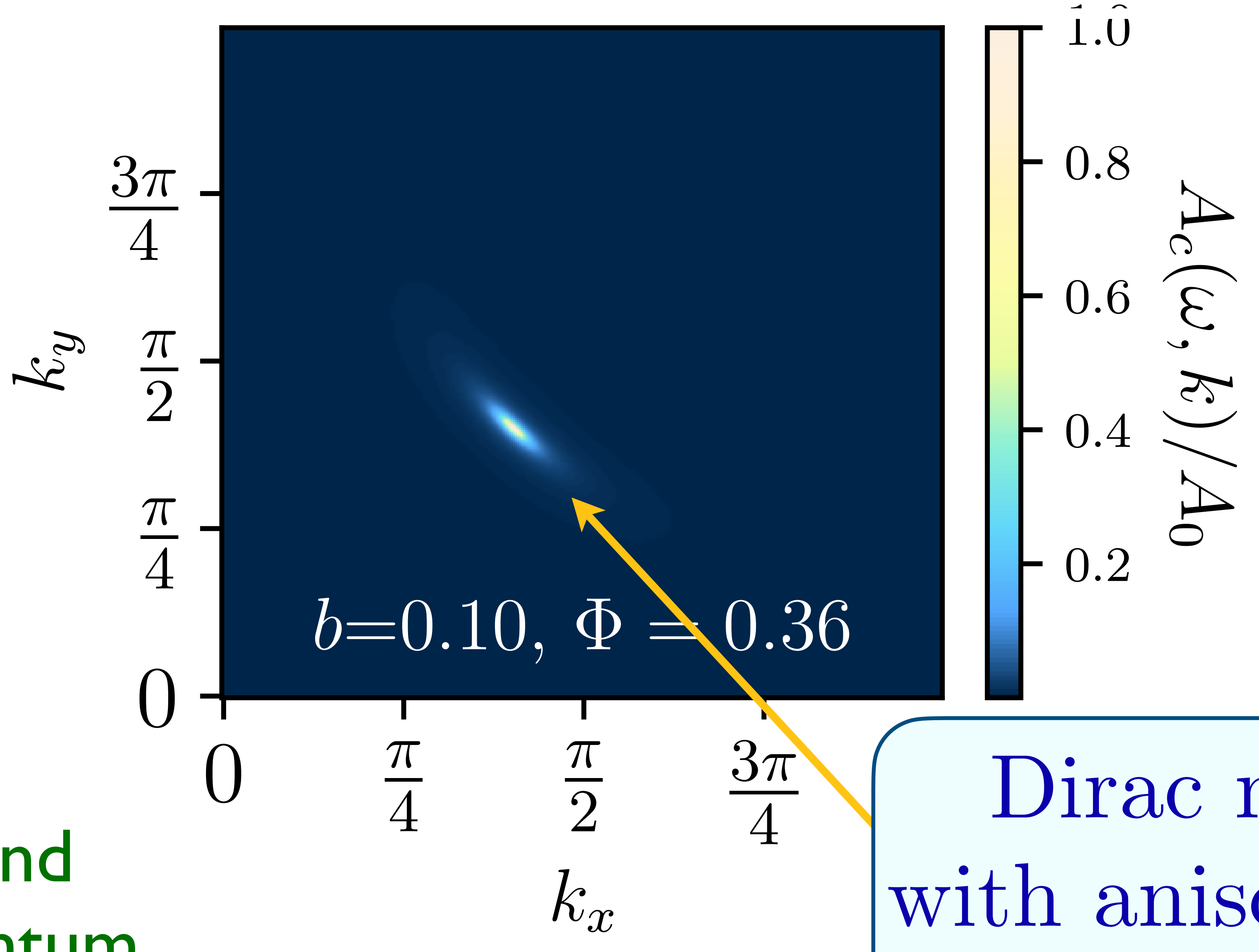
Electron spectral density in hole-doped cuprates



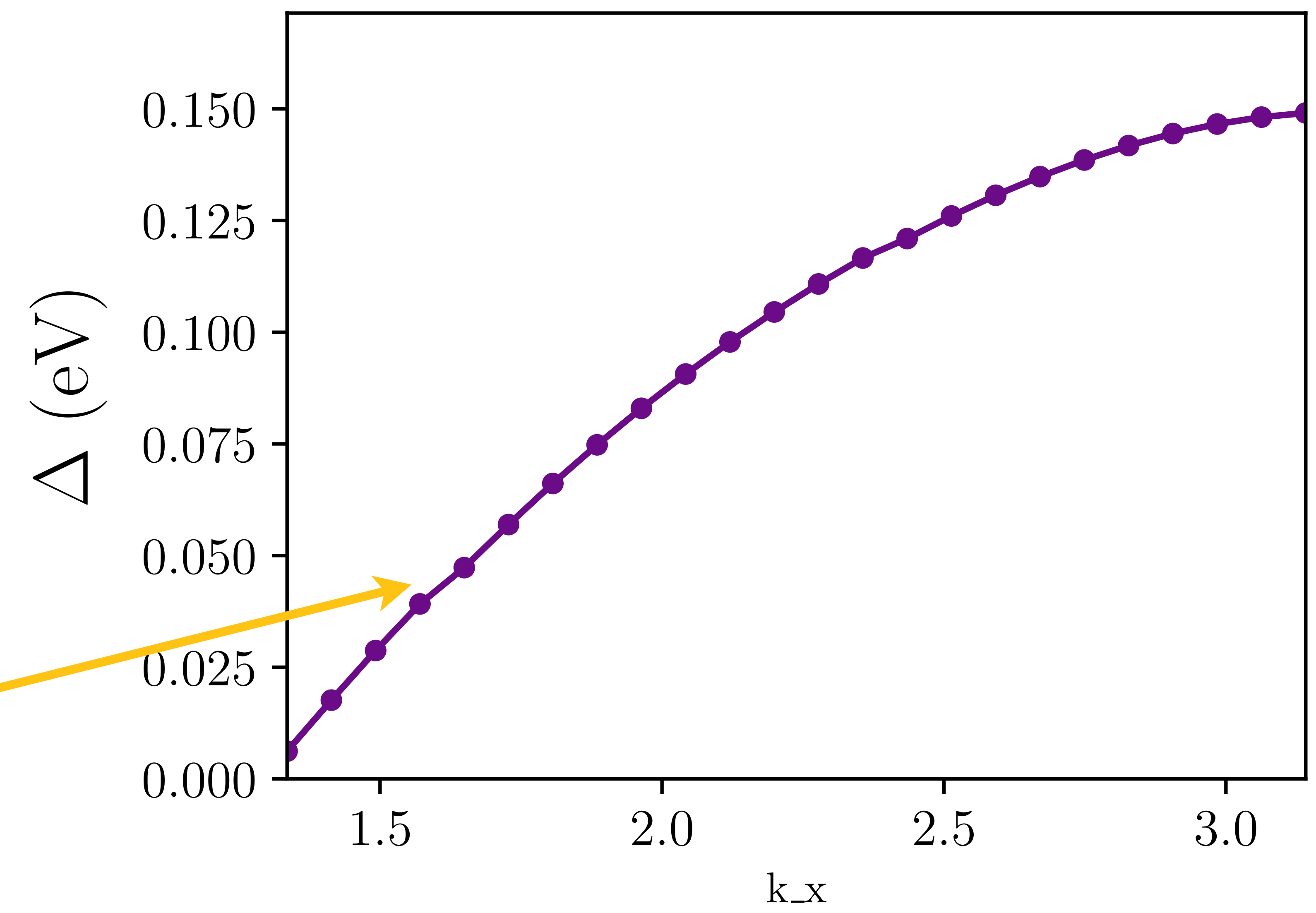
FL*

d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson.

Spinons of the π-flux state annihilate the extra nodes in the d-wave superconductor.



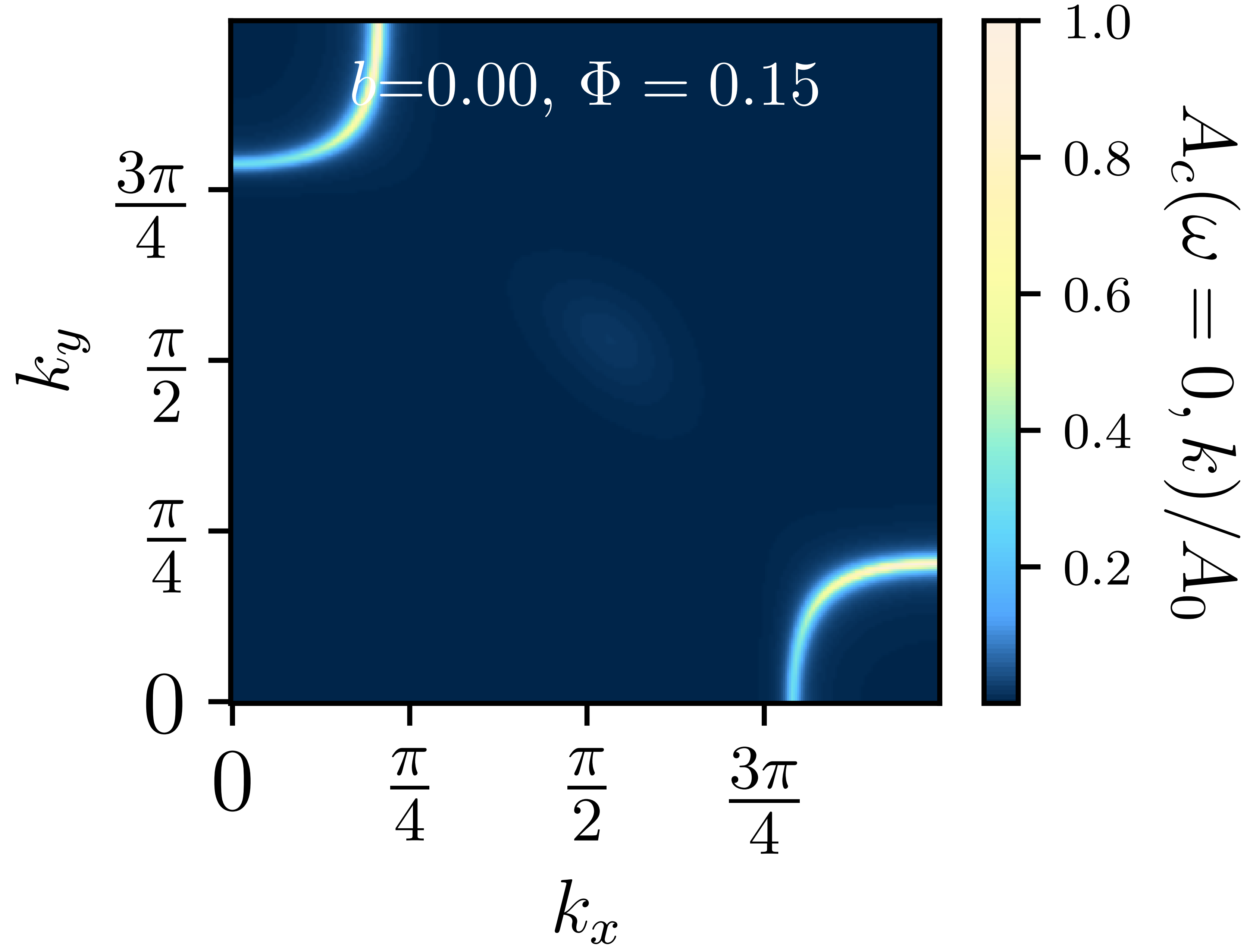
dSC



Dirac node with anisotropic velocities

Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

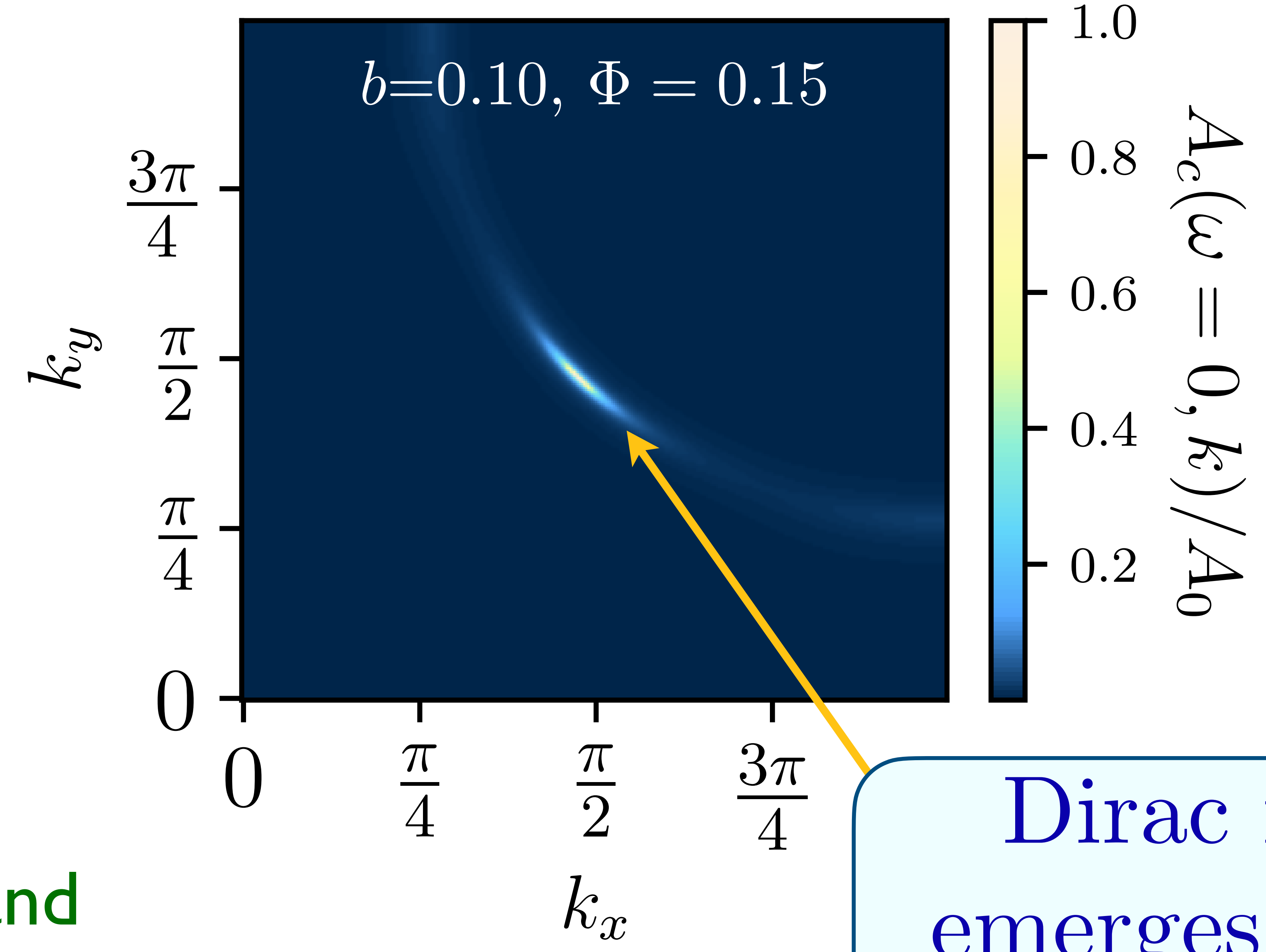
Electron spectral density in electron-doped cuprates



FL*

d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson.

Nodes of the d-wave superconductor are remnants of the spinons of the π-flux state.

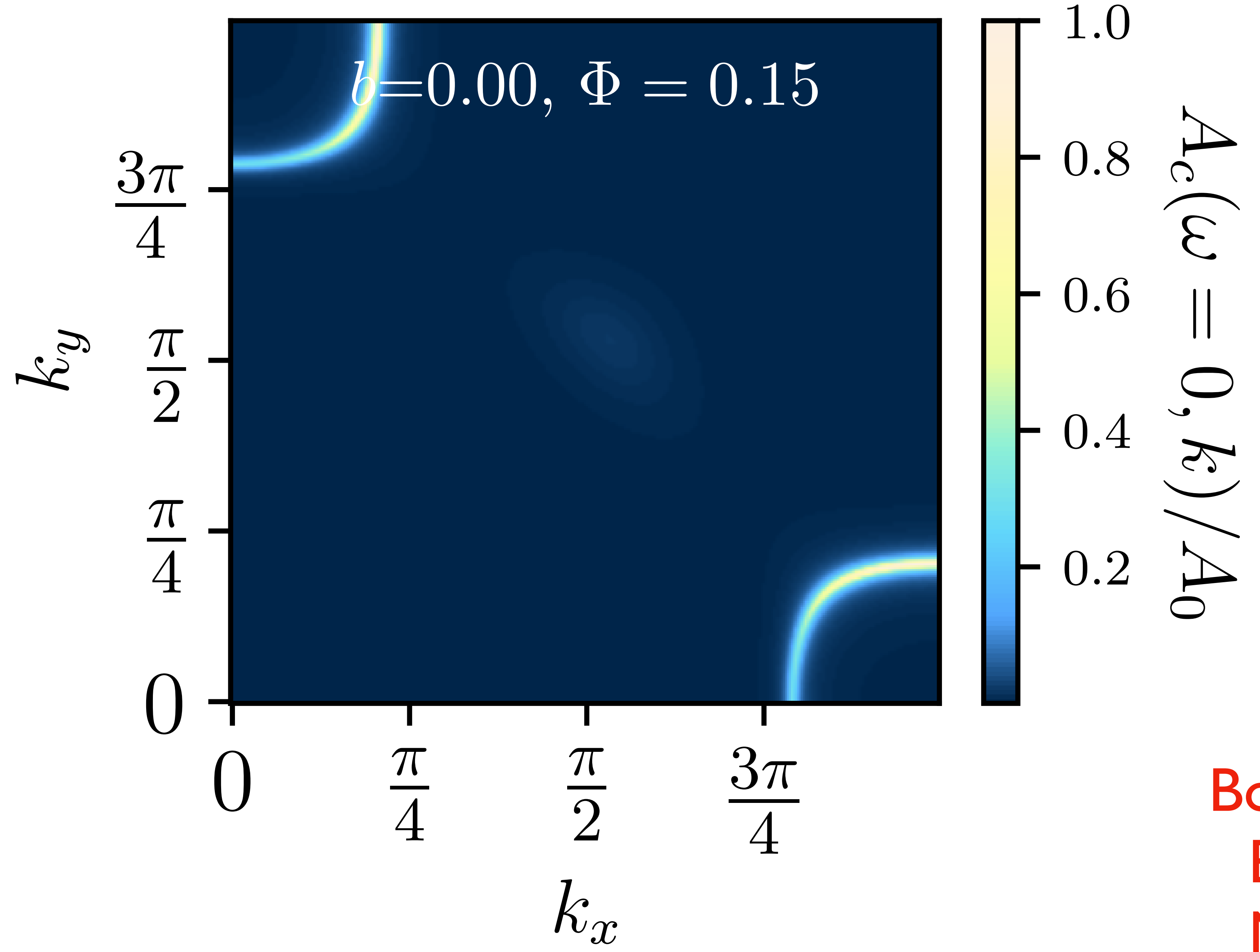


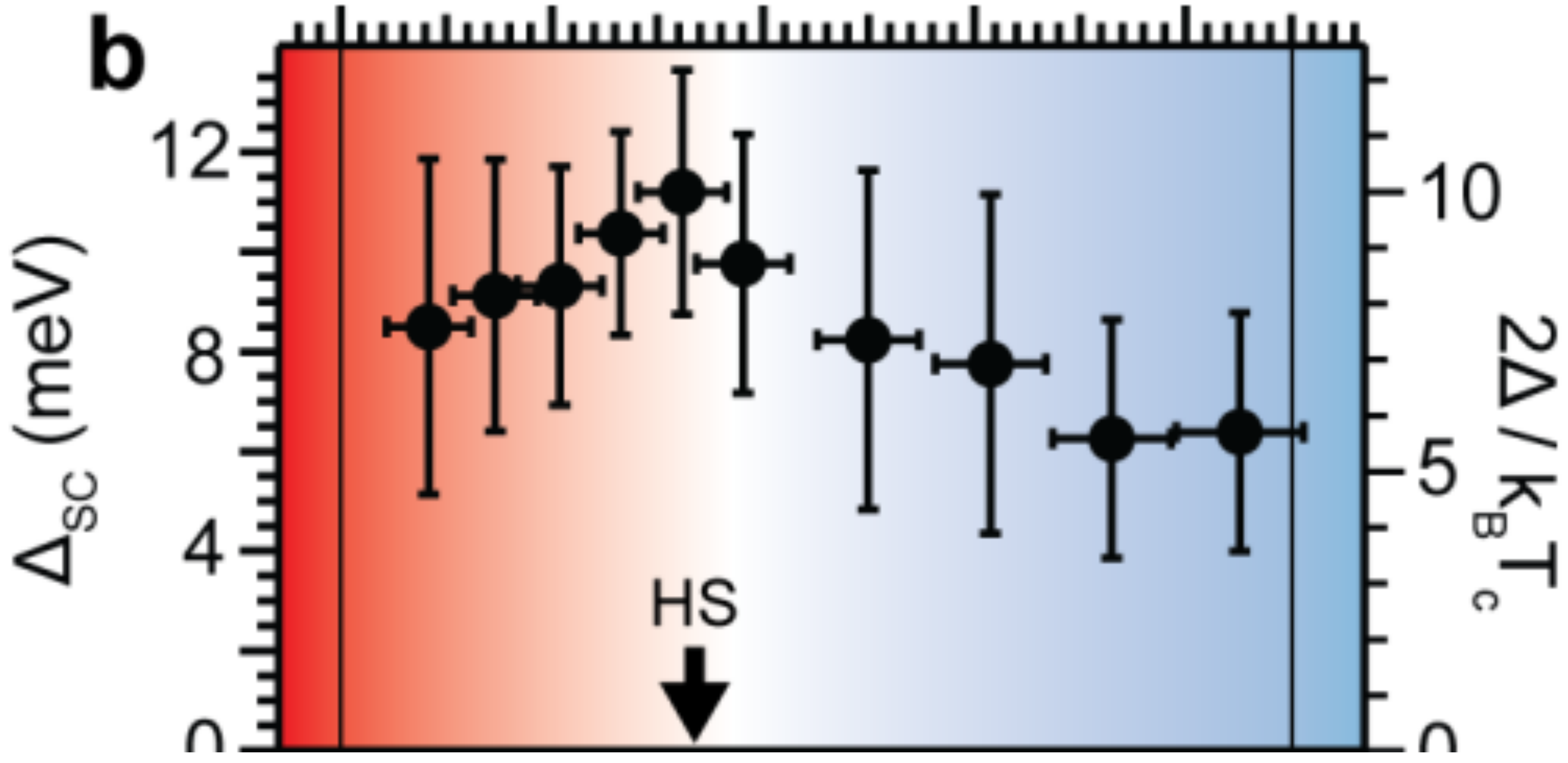
dSC

Dirac node emerges inside normal state gap

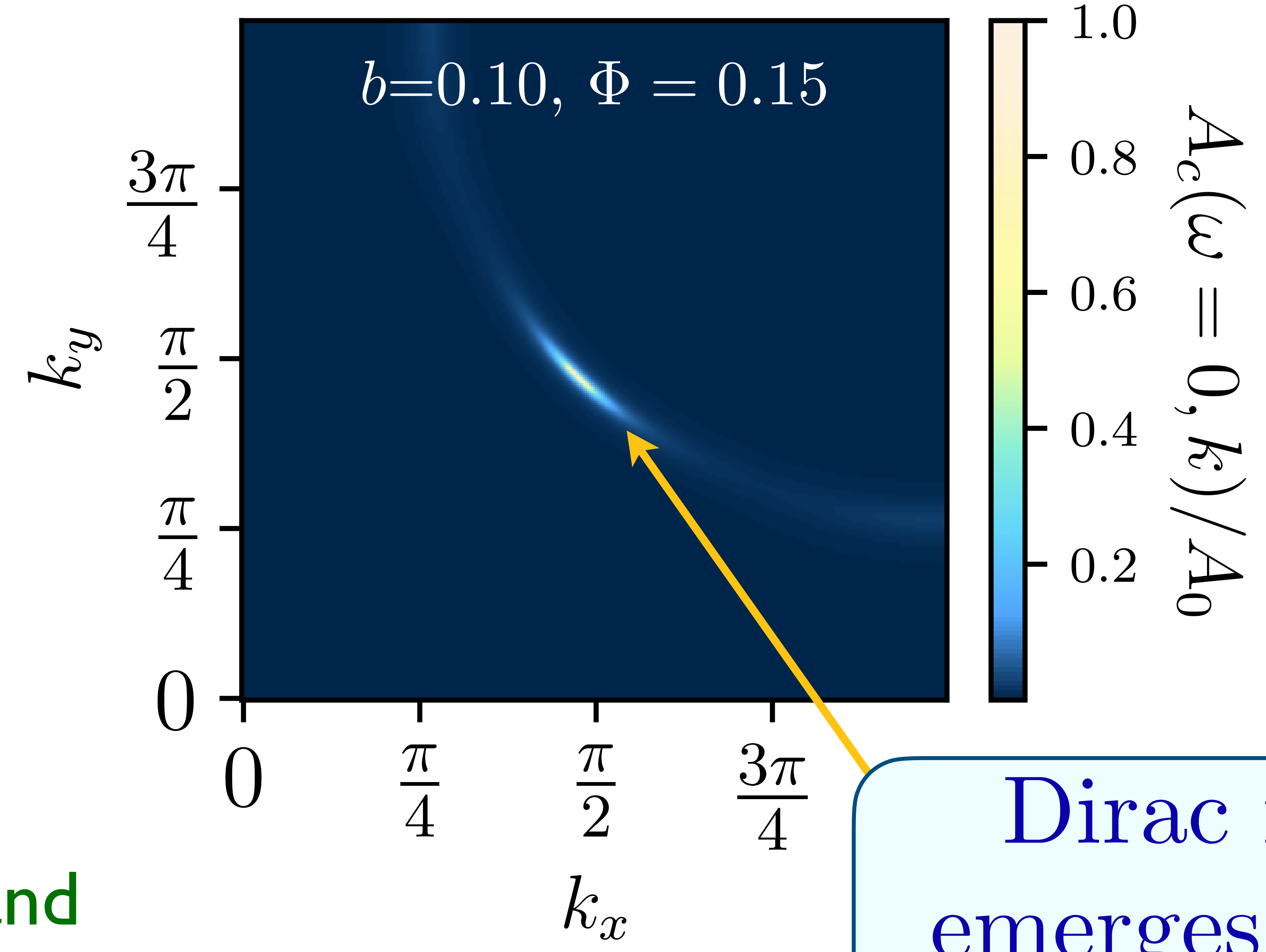
Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

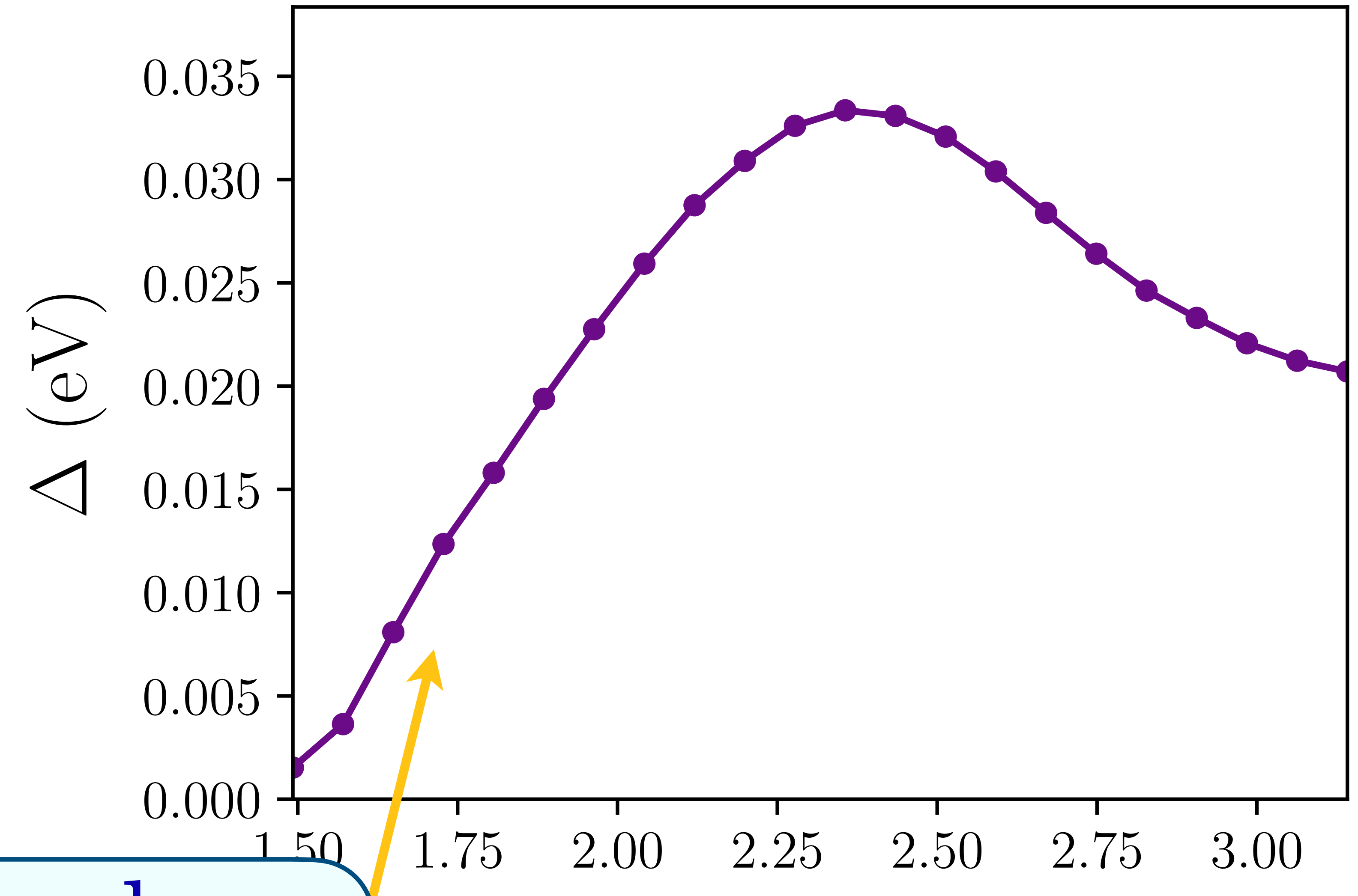
Electron spectral density in electron-doped cuprates





Bogoliubov Quasiparticle on the Gossamer Fermi Surface in Electron-Doped Cuprates, Ke-Jun Xu……Z.-X. Shen, arXiv:2308.05313; Nature Physics





Dirac node emerges inside normal state gap

Maine Christos and S.Sachdev, npj Quantum Materials **9**, 4 (2024)

S=1/2 square lattice antiferromagnet

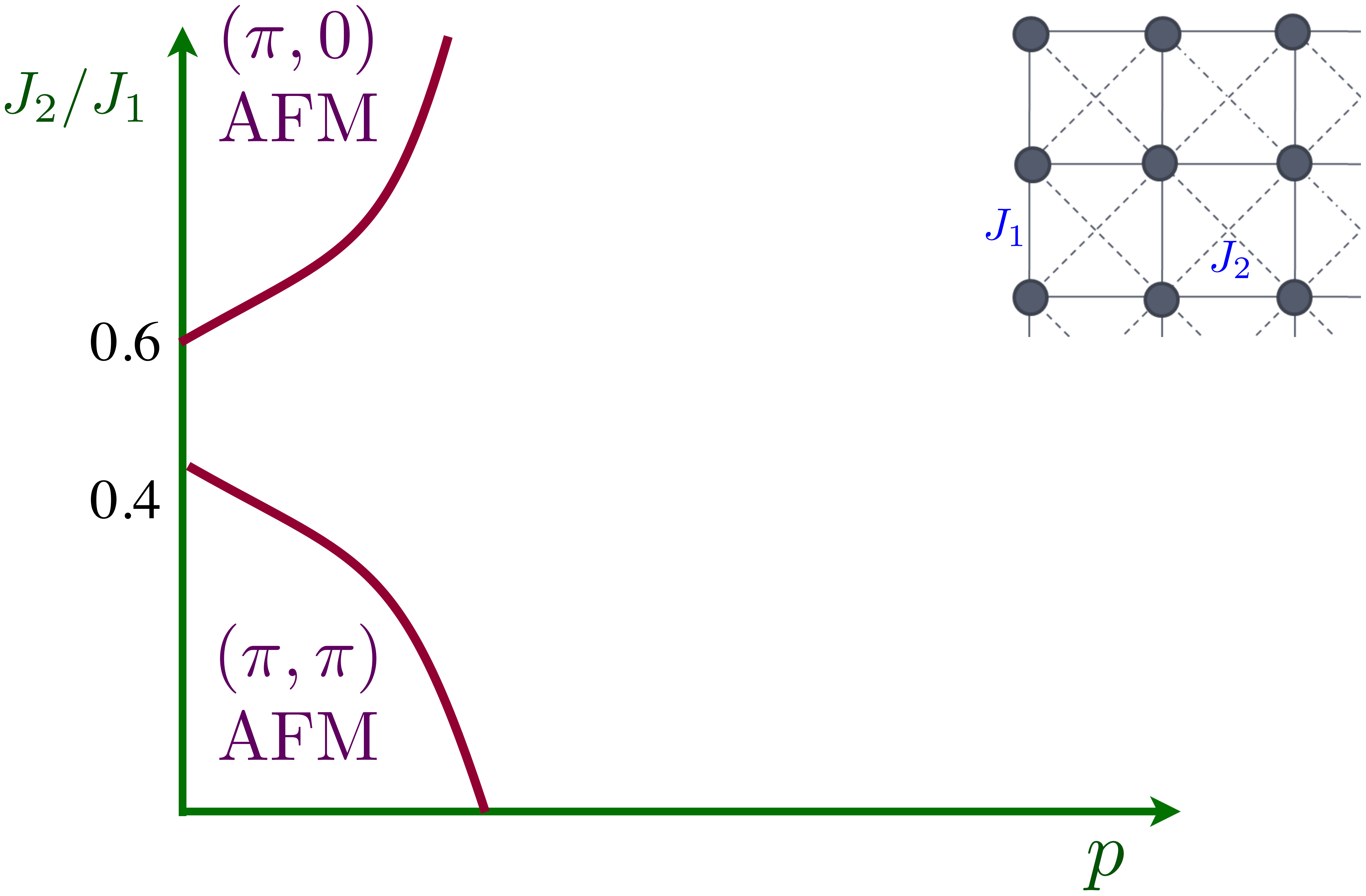
From the pseudogap metal to the d-wave superconductor

Confinement of the π-flux/CP^1 spin liquid, d-wave superconductivity, and charge order

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

Phys. Rev. Lett. **127**, 097002 (2021)



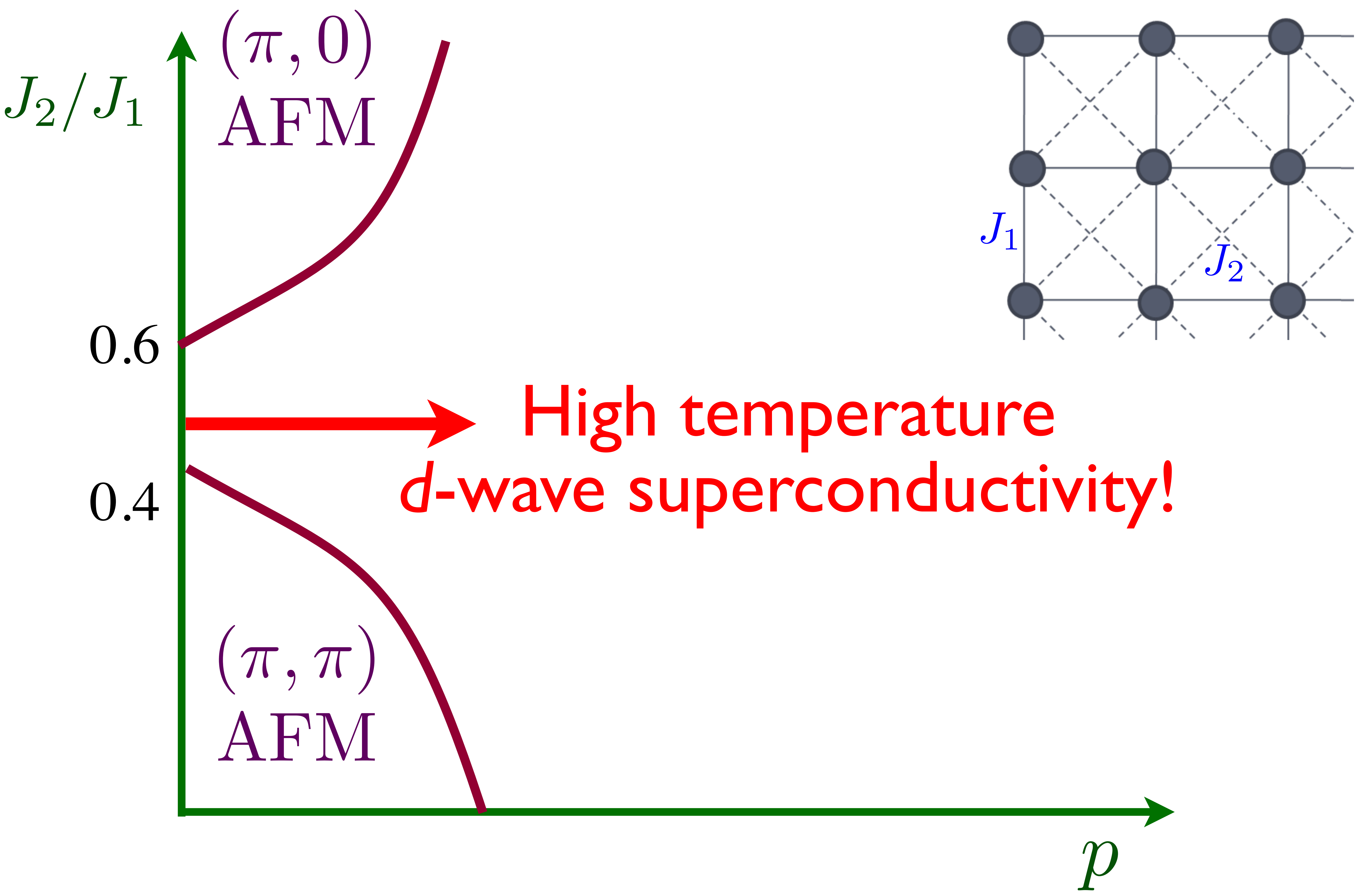
Superconducting valence bond fluid in lightly doped 8-leg *t*-*J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, *Phys. Rev. B* **108**, 054505 (2023)

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

Phys. Rev. Lett. **127**, 097002 (2021)



Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders

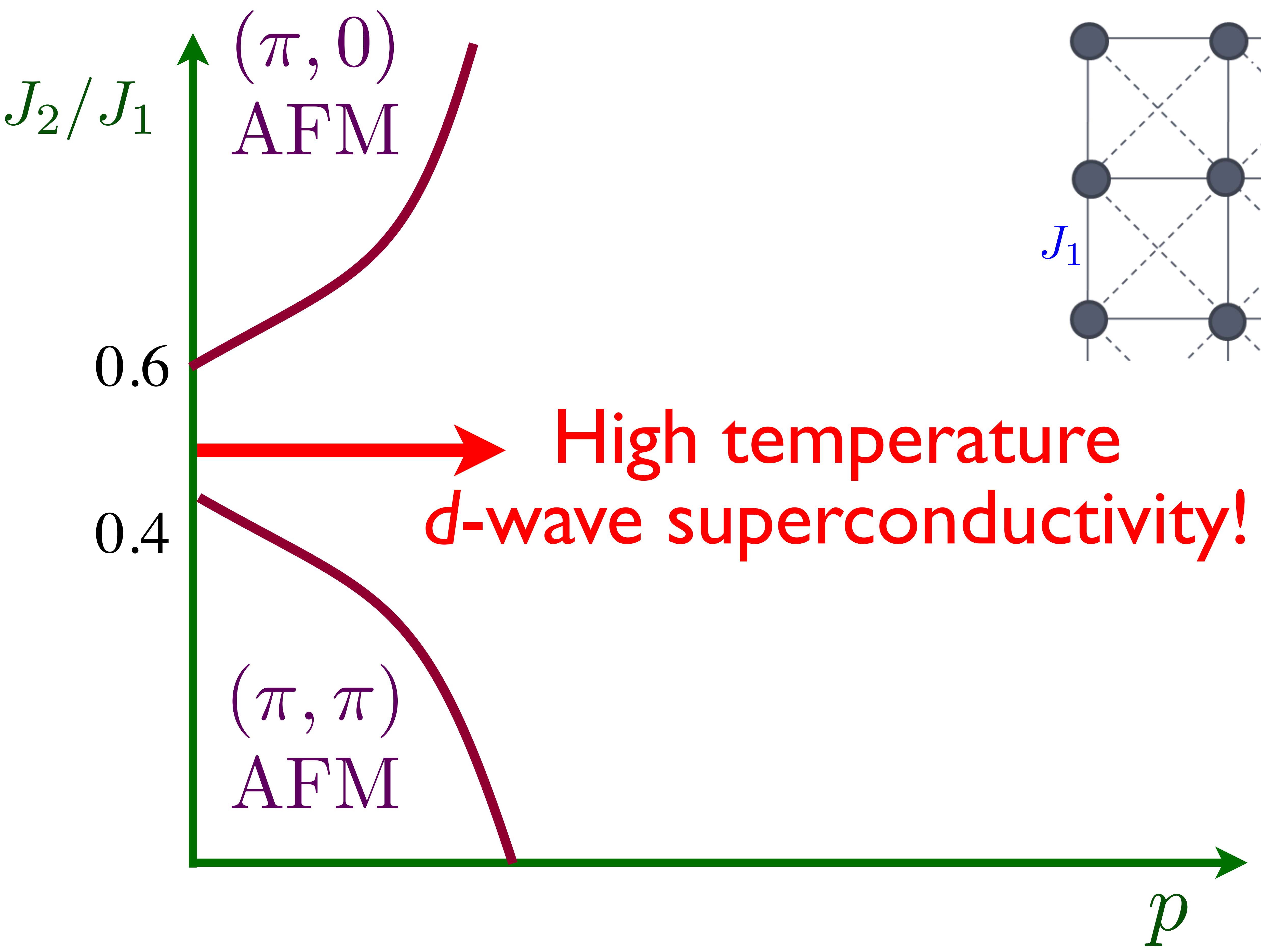
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, *Phys. Rev. B* **108**, 054505 (2023)

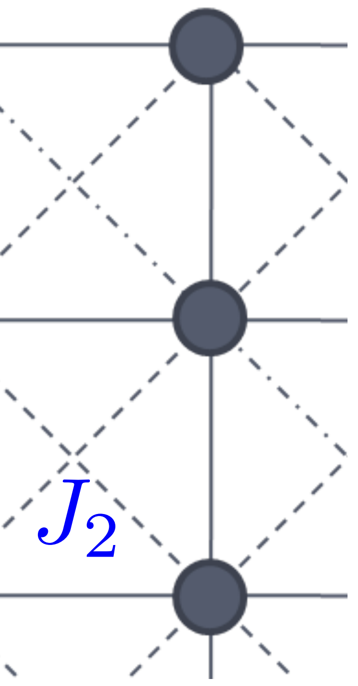
Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

Phys. Rev. Lett. **127**, 097002 (2021)





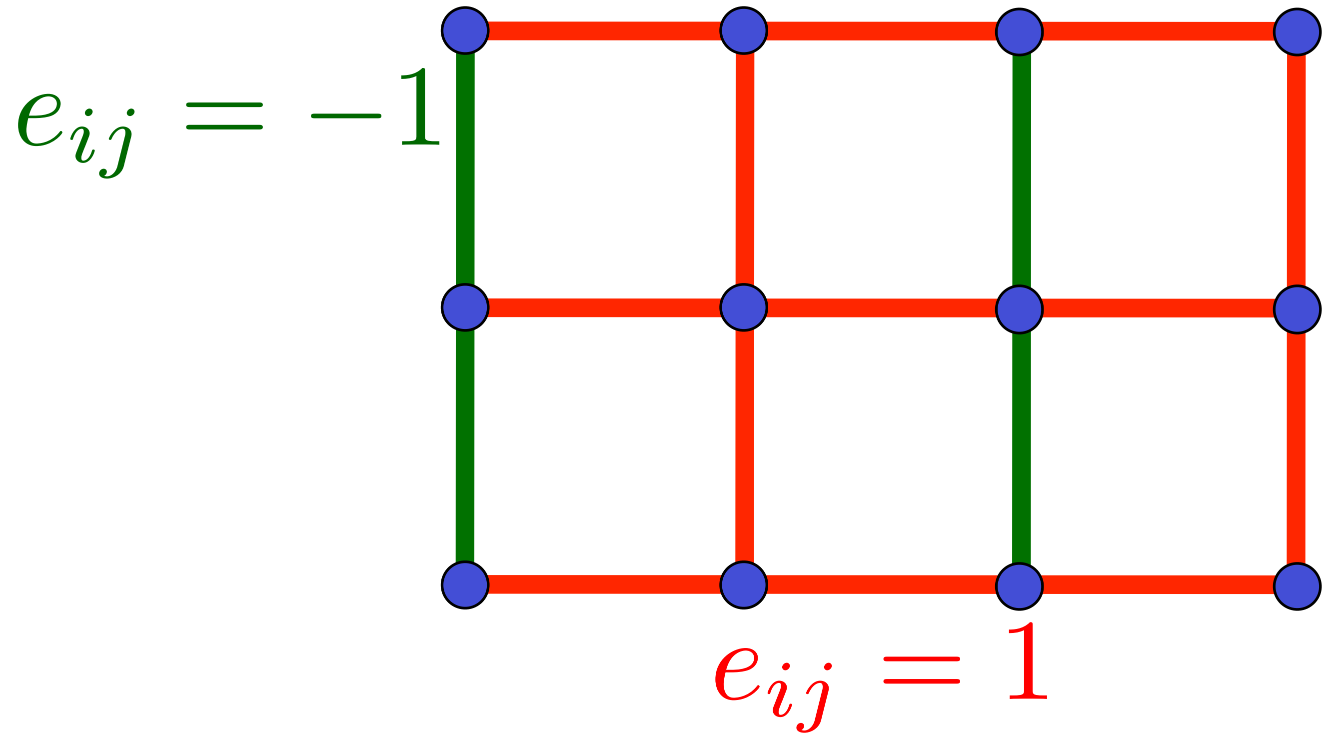
Superconducting valence bond fluid in lightly doped 8-leg *t*-*J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, *Phys. Rev. B* **108**, 054505 (2023)

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. ~~Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.~~

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$

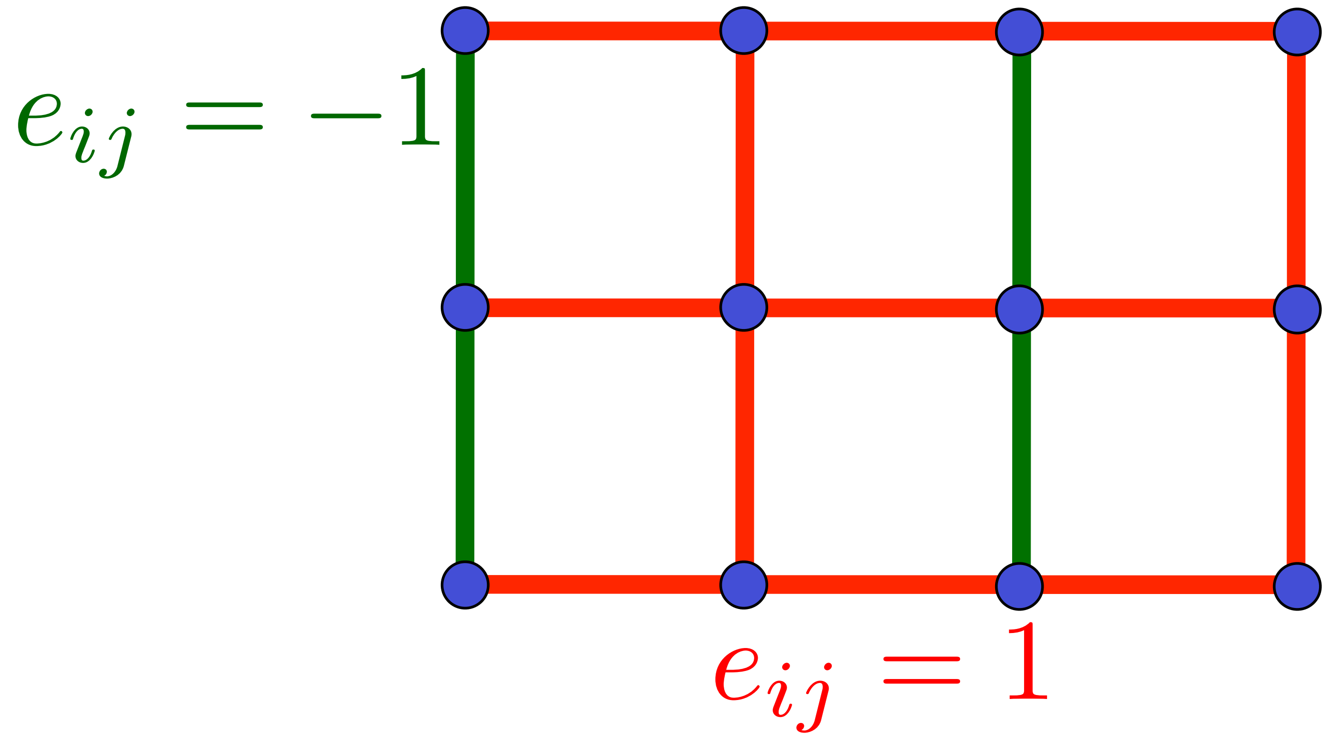


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = \boxed{iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right)} \; ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.

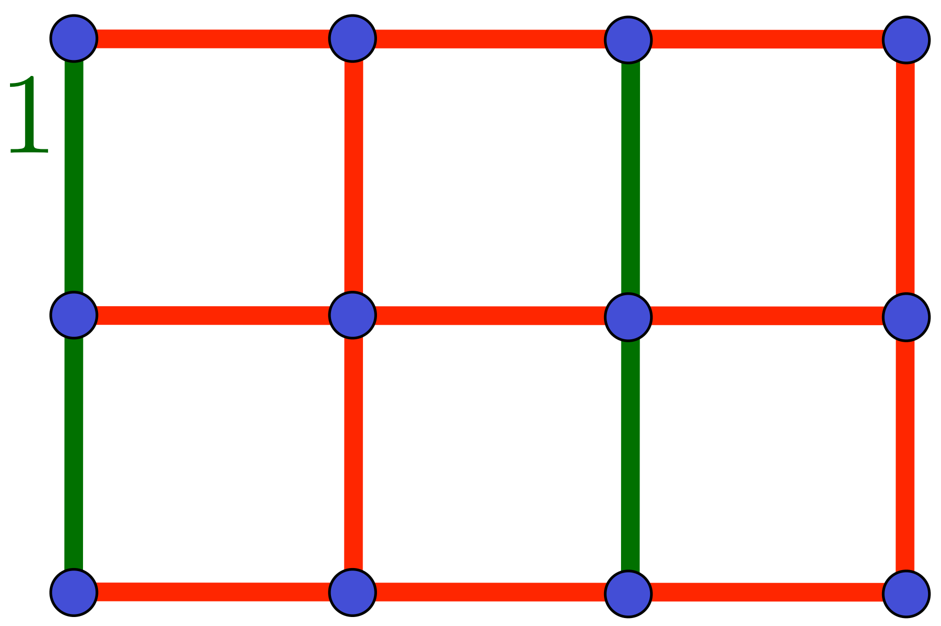


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = \boxed{iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right)} \; ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



- Introduce a charge e, SU(2)$_N$ fundamental boson B_i such that the composite of B_i and Ψ_i is an electron.

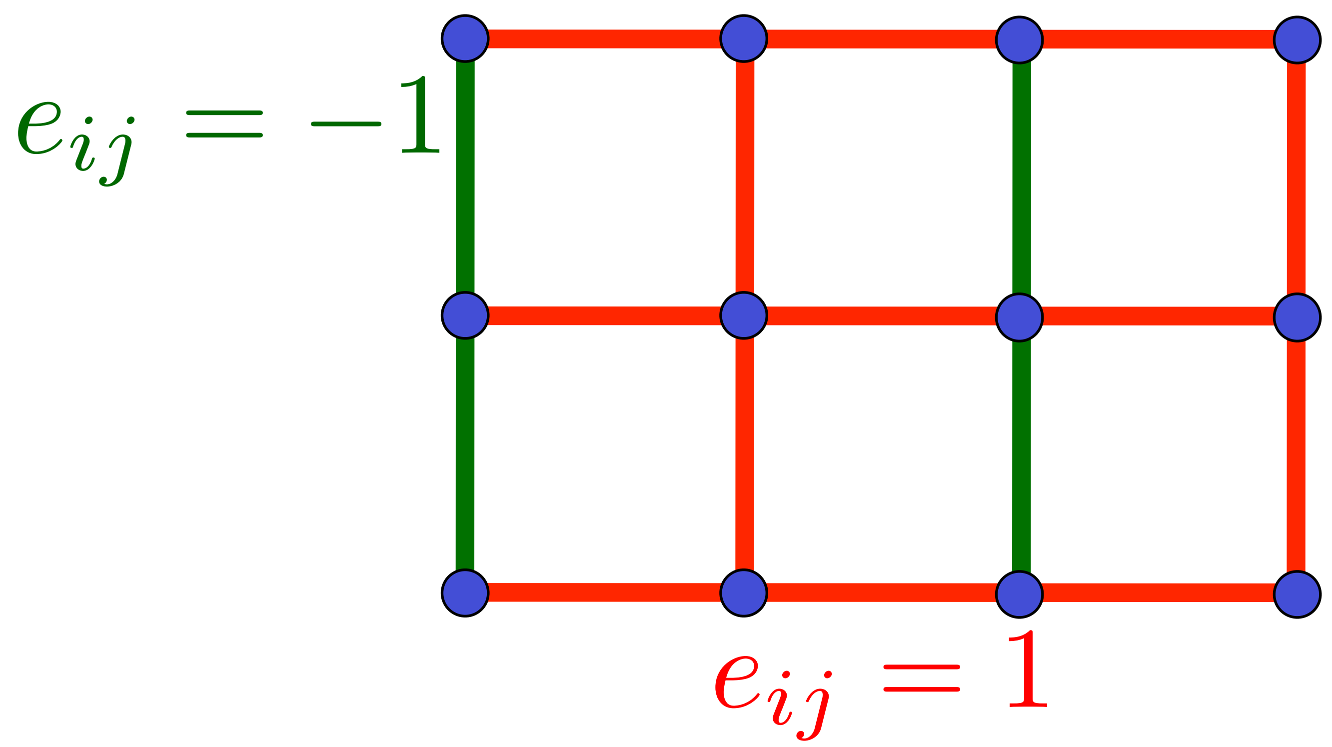
X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right) = \boxed{iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi^\dagger_i U_{ij} \Psi_j - \Psi^\dagger_j U_{ji} \Psi_i \right)} \; ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



- Introduce a charge e, SU(2)$_N$ fundamental boson B_i such that the composite of B_i and Ψ_i is an electron. The projective symmetries require

$$\boxed{H_B = r \sum_i B^\dagger_i B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B^\dagger_i U_{ij} B_j - B^\dagger_j U_{ji} B_i \right) + \ldots}$$

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij} \rangle} |\Delta_{\boldsymbol{ij}}|^2$$
$$+ J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2 .$$

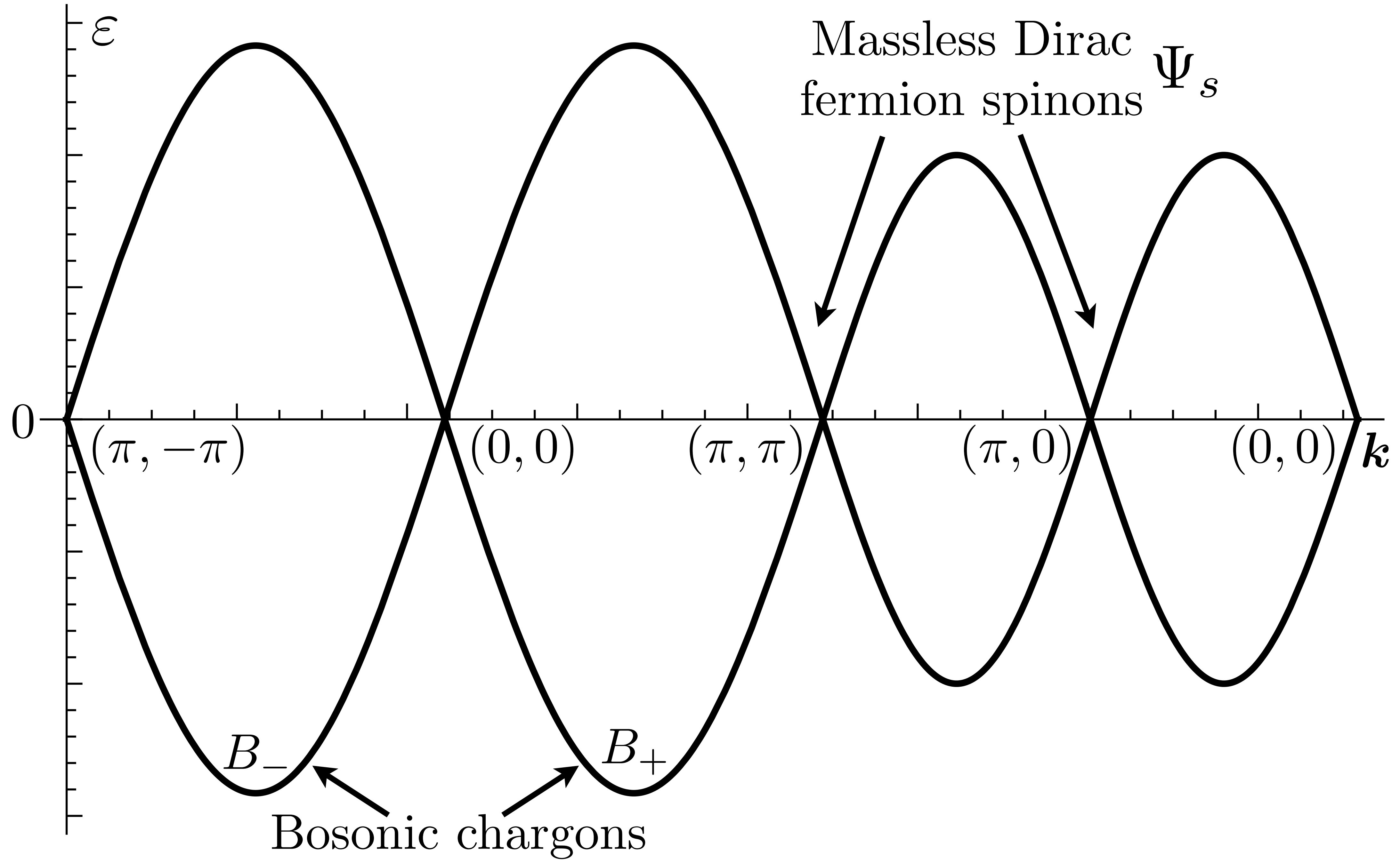
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

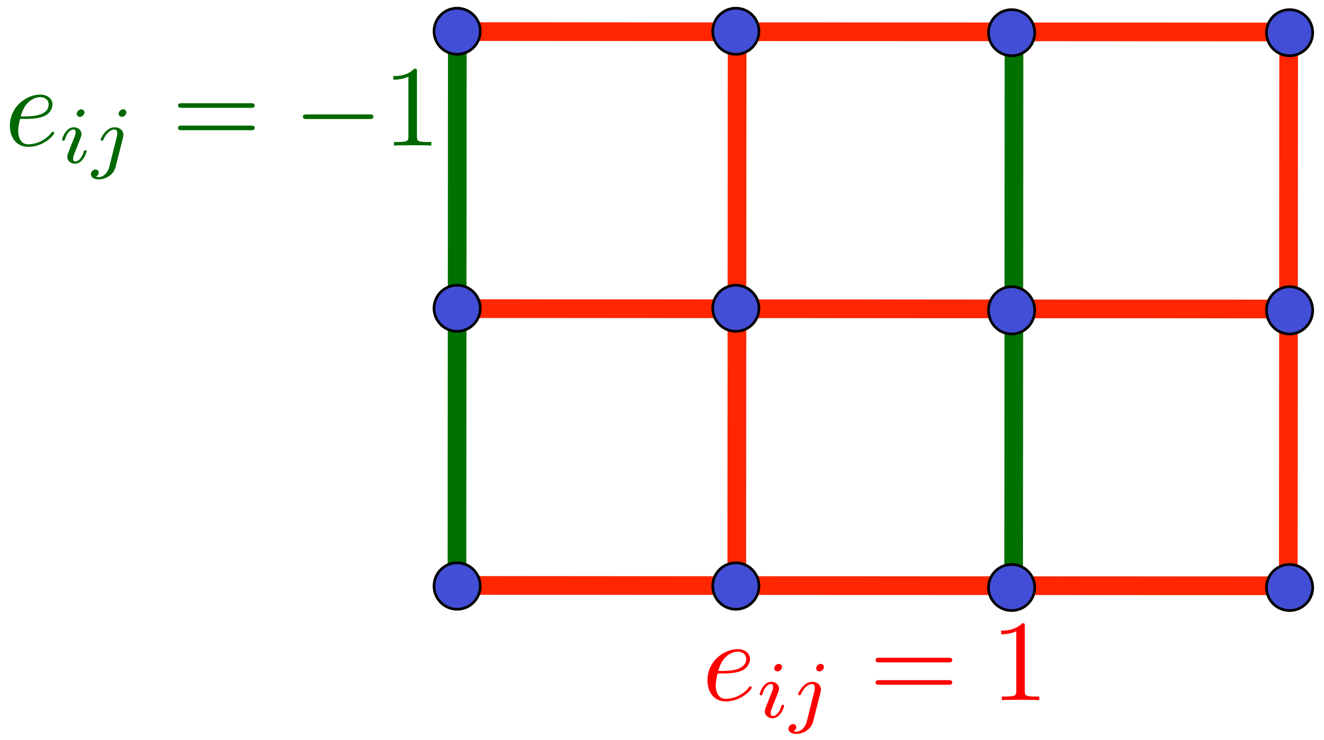
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i \left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

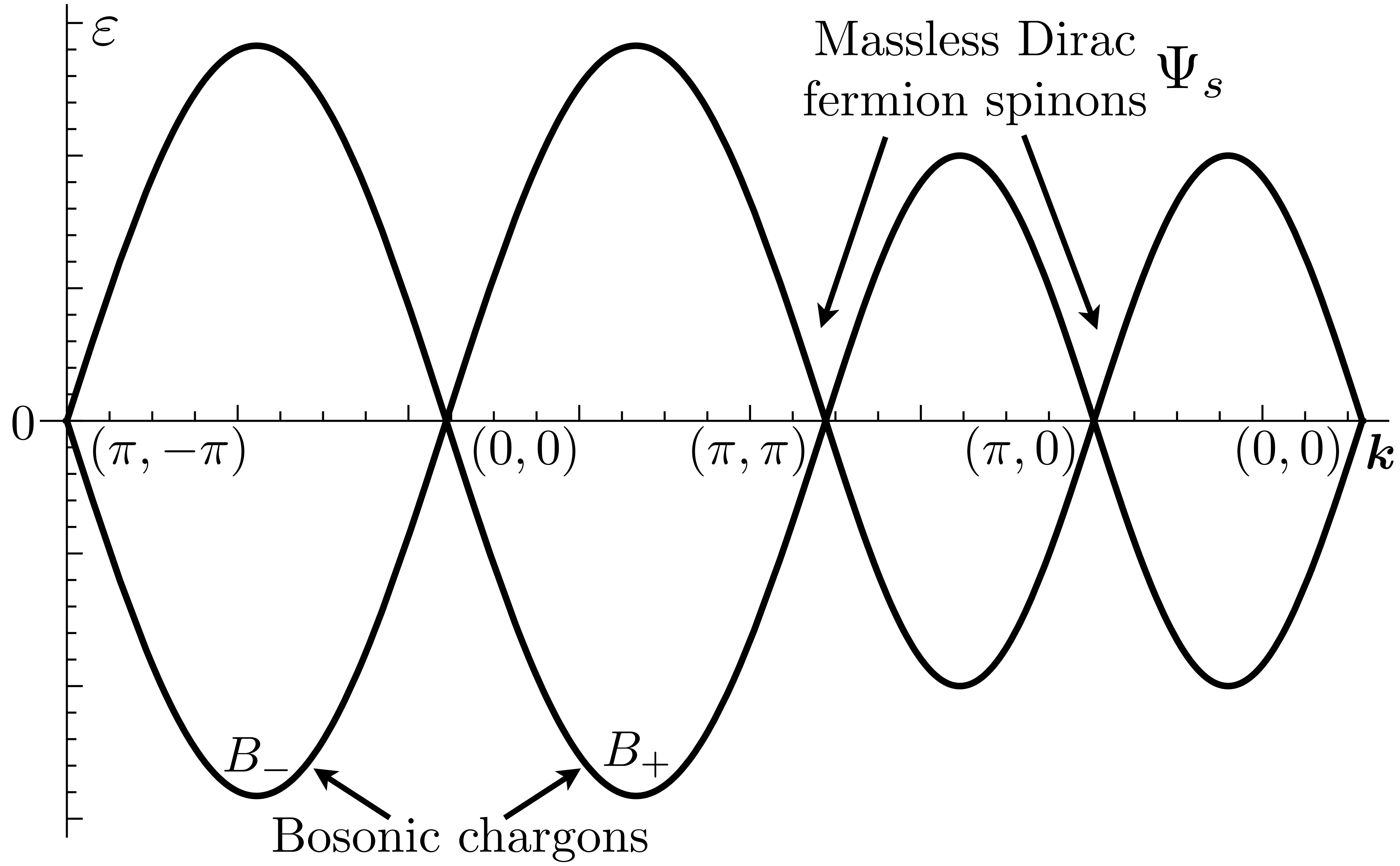
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

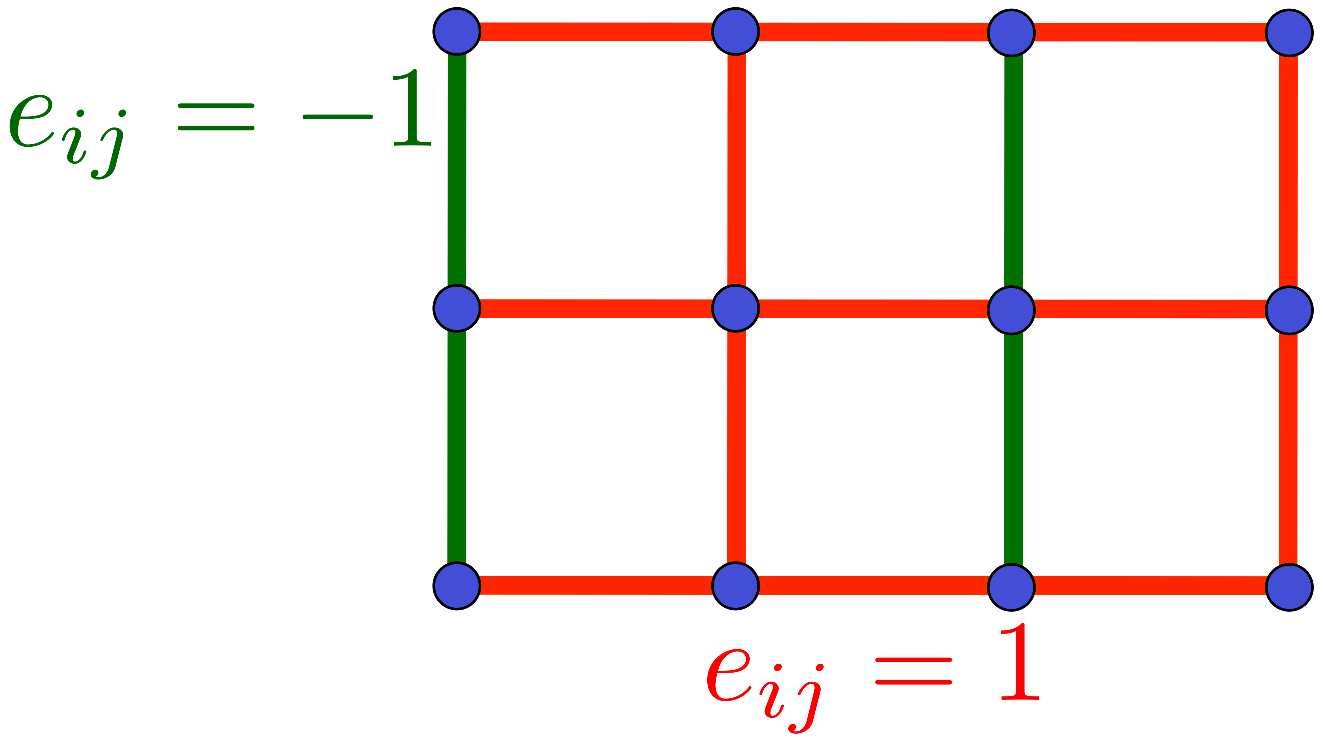
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





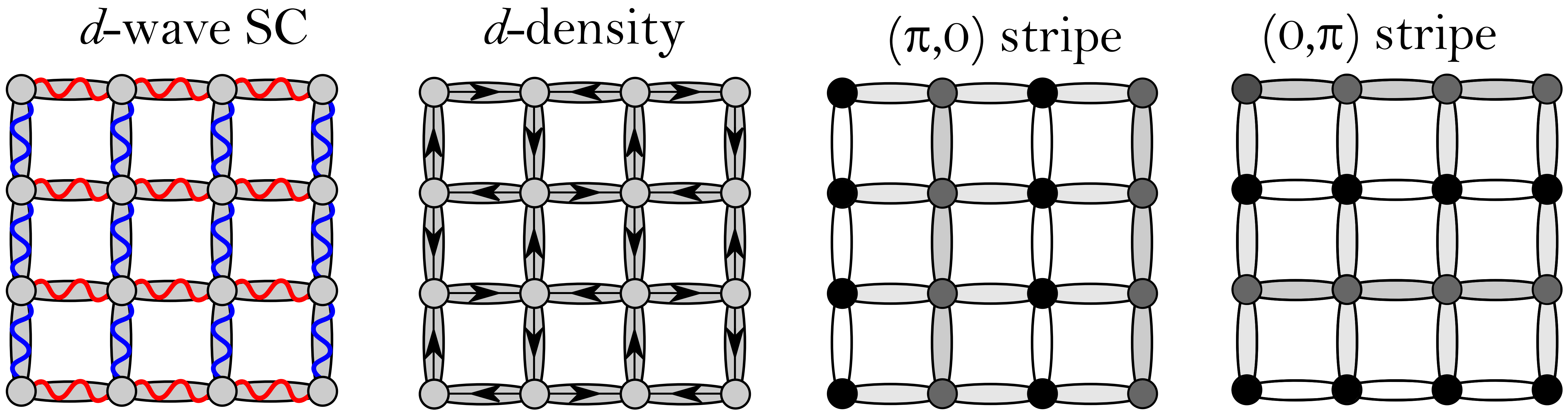
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





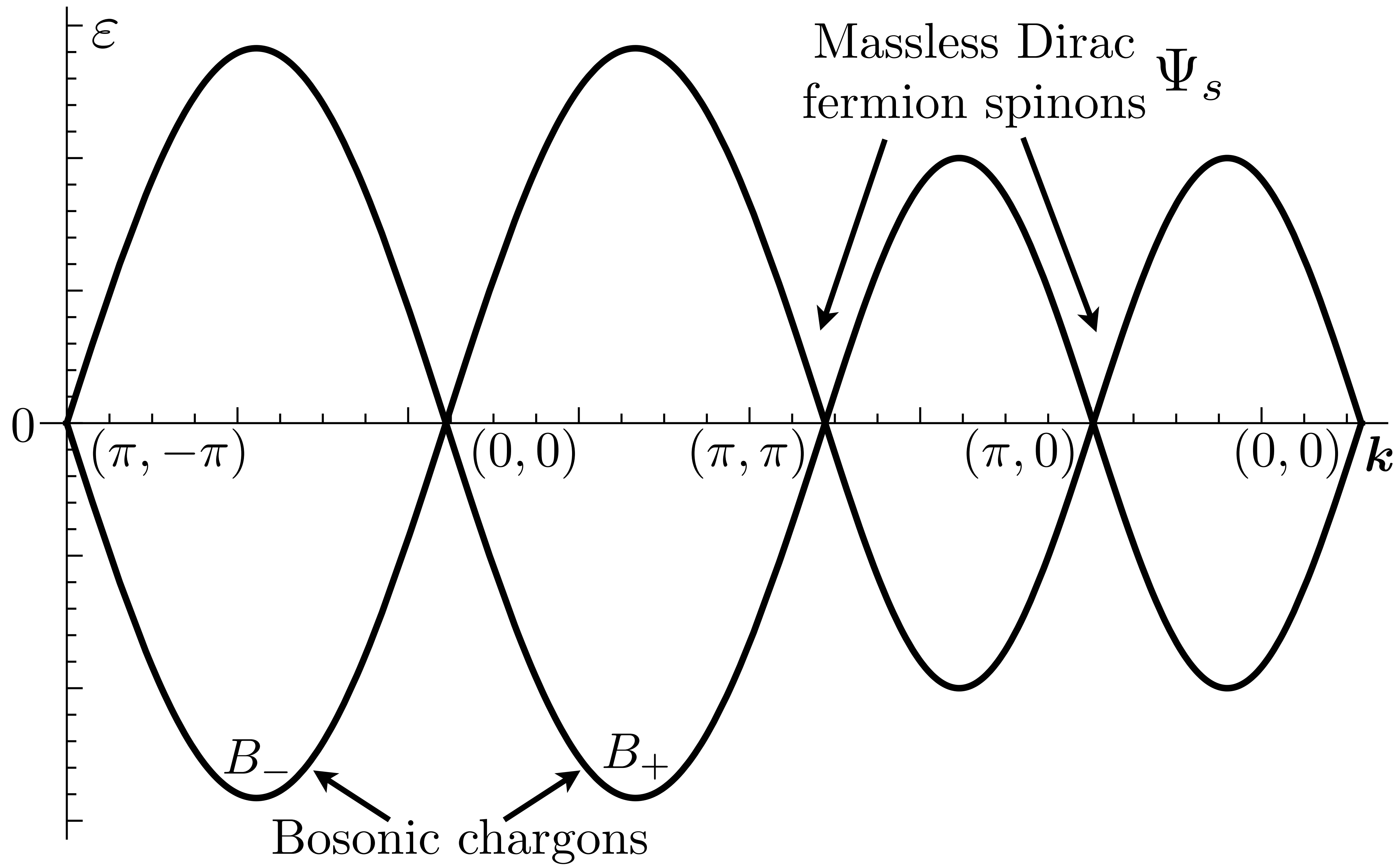
SU(2)$_N$ gauge-invariant and SU(2) spin invariant order parameters of Higgs phases:

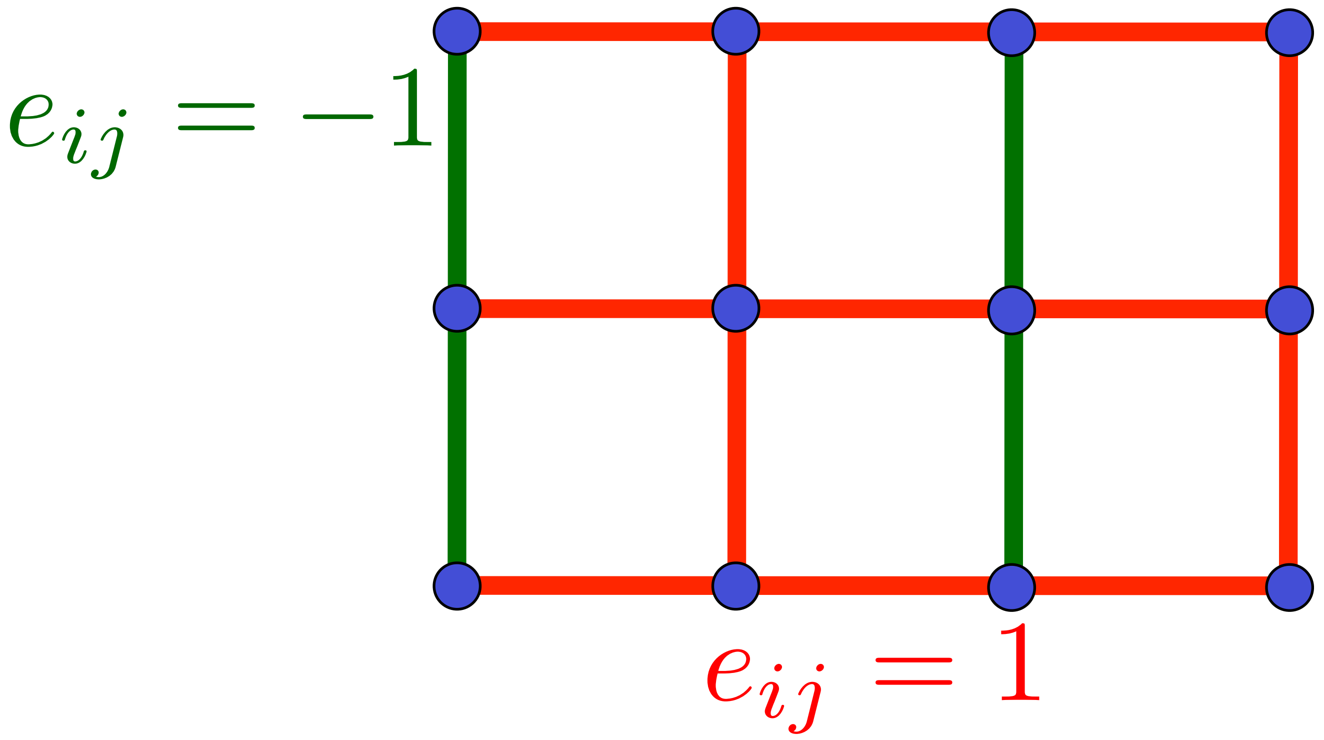
$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$



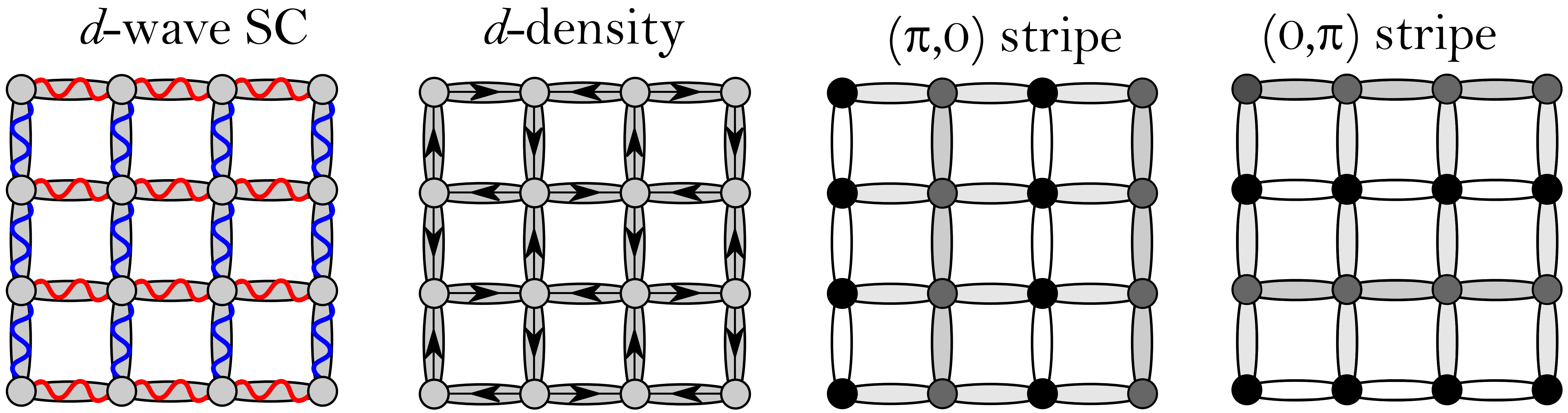
The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a SO(5)$_b$ rotation symmetry between these orders.

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

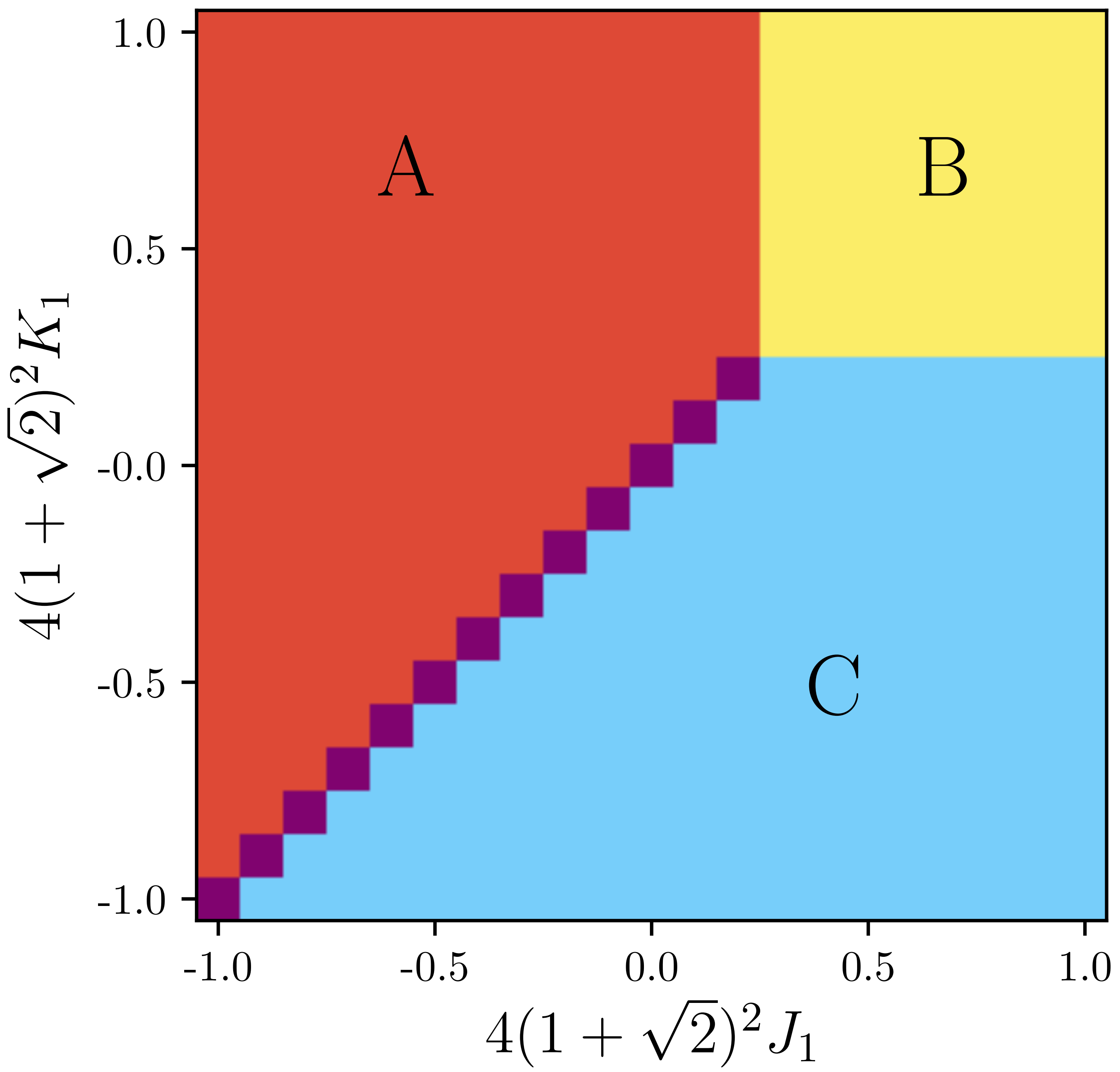




The B_{av}
$(a \to \mathrm{SU}(2)_N$ gauge, $v \to$ valley$)$
are the "square roots" of
conventional
d-wave superconductor,
charge density wave,
pair density wave
$\ldots$

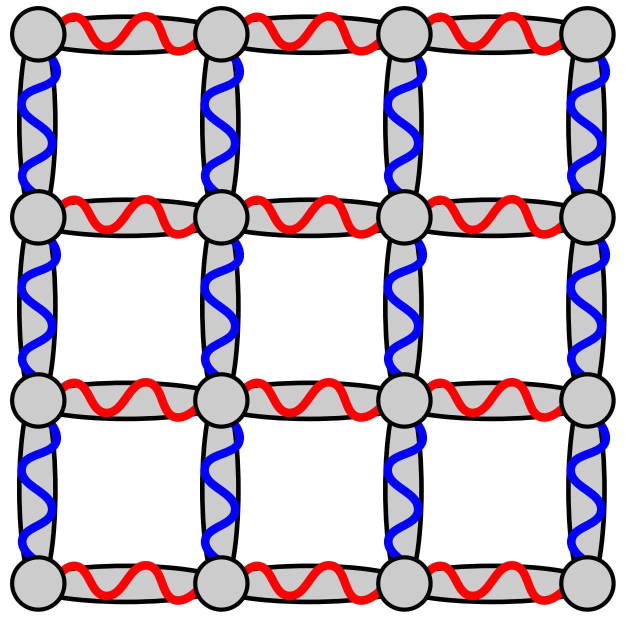


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

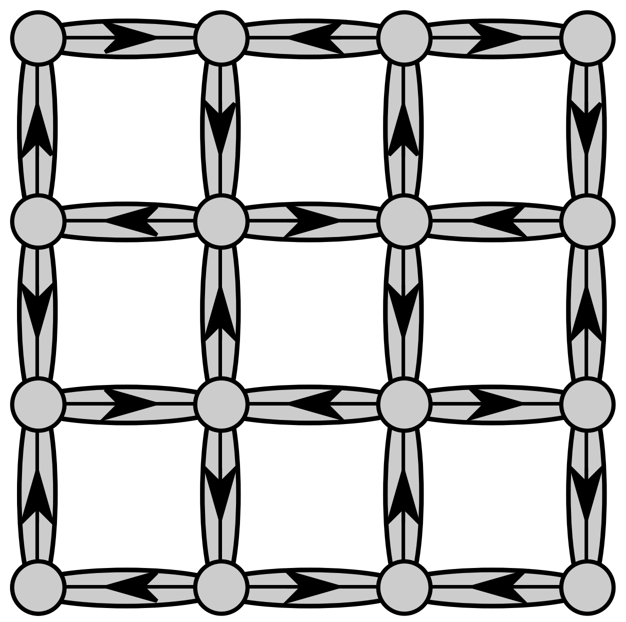


$$\langle B \rangle \neq 0$$

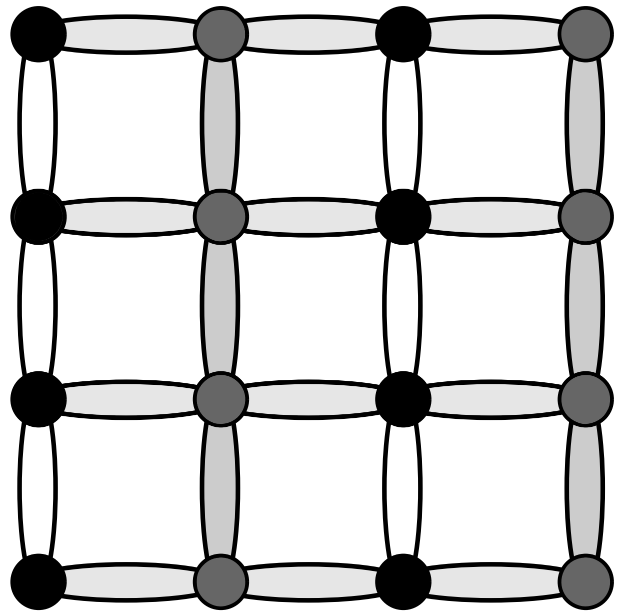
Phase B
d-wave SC



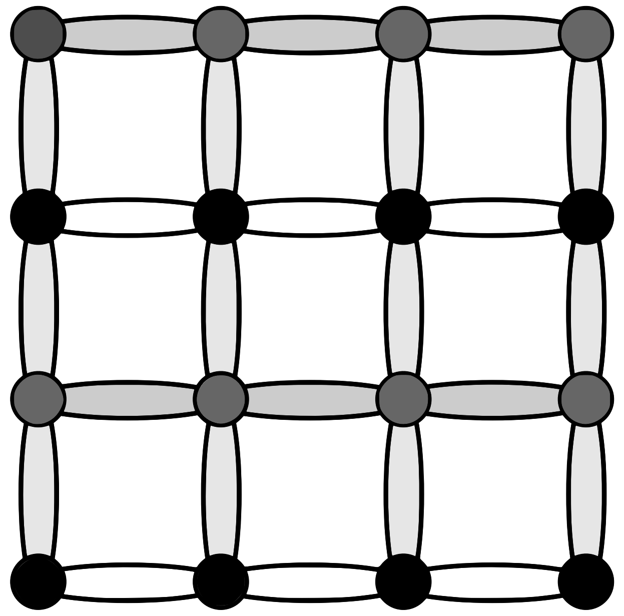
Phase C
d-density



Phase A
(π,0) stripe



Phase A
(0,π) stripe



Global phase diagram of SU(2)$_N$ gauge theory

Global phase diagram of SU(2)$_N$ gauge theory

Summary

Ancilla theory of pseudogap metal with hole pockets and underlying π-flux spin liquid yields:

- Theory for Fermi arcs in hole-doped pseudogap metal.
- ADMR in pseudogap.
- Anti-nodal and nodal electronic dispersion.
- d-wave superconductor with 4 nodal points in both electron- and hole-doped cuprates.
- Near-equality of dSC and charge order onset temperatures
- Multipoint correlators measured by cold atom experiments
- Theory for strange metal in the crossover from FL* to FL