

# Fermi surfaces, large and small

2021 H.L. Welsh Lectures in Physics  
University of Toronto  
May 7, 2021  
Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)





**Maria Tikhanovskaya**



**Yahui Zhang**



**Alexander Nikolaenko**

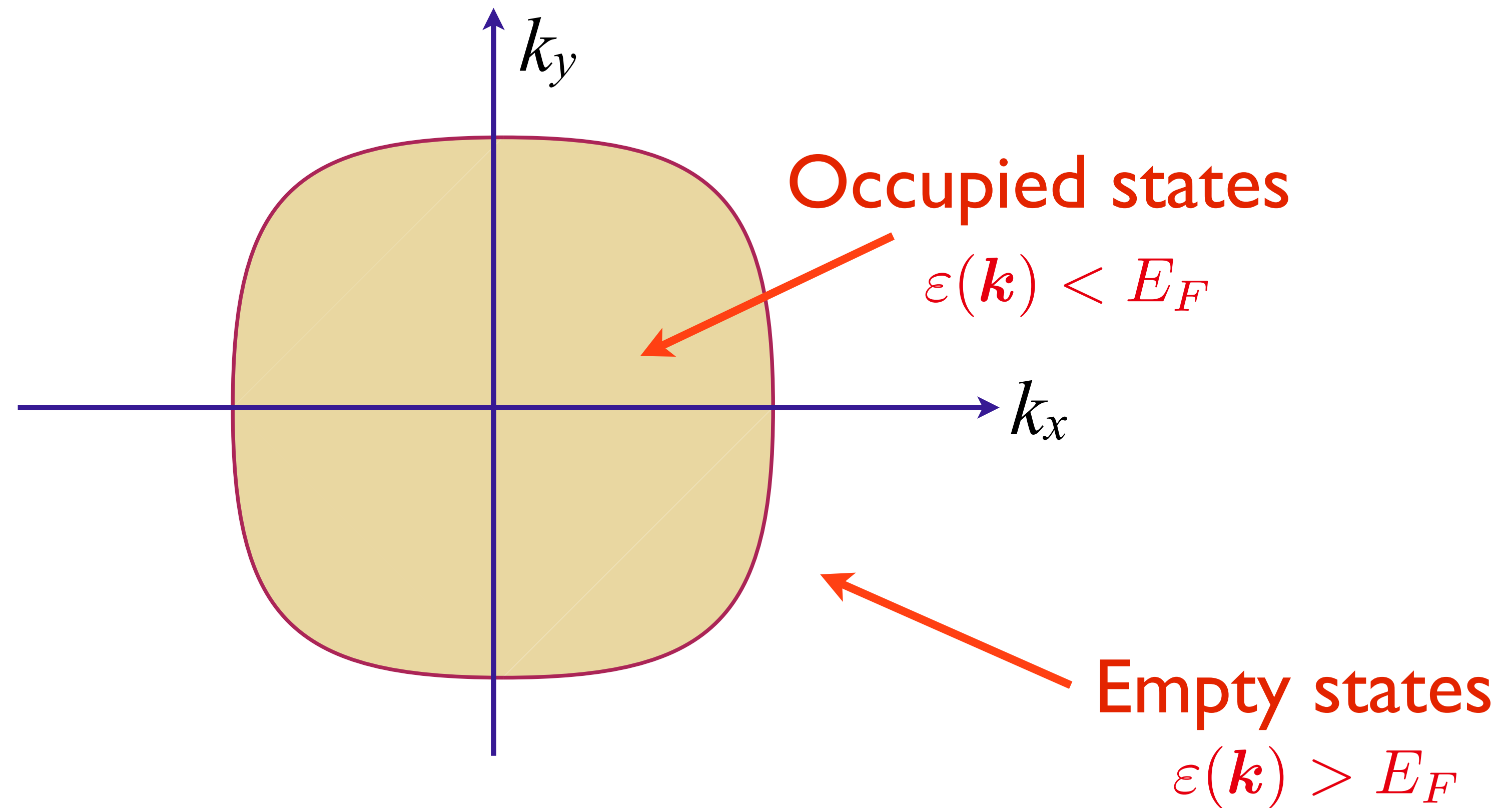
## Ordinary metals



Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

# Theory of metals

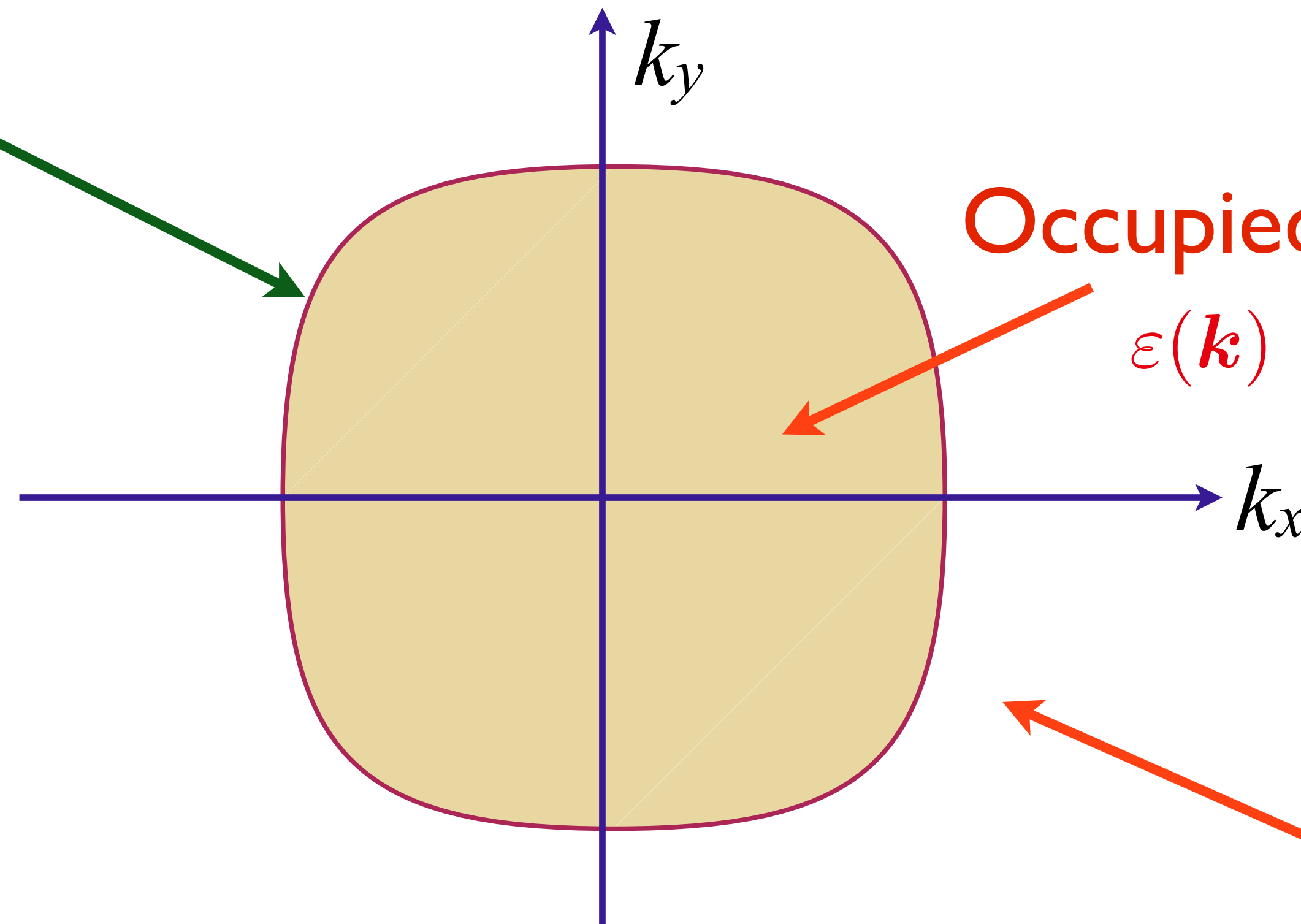
Electrons move with momentum  $\mathbf{k}$  through the lattice with dispersion  $\varepsilon(\mathbf{k})$



# Theory of metals

Electrons move with momentum  $\mathbf{k}$  through the lattice with dispersion  $\varepsilon(\mathbf{k})$

**Fermi surface**



**Occupied states**

$$\varepsilon(\mathbf{k}) < E_F$$

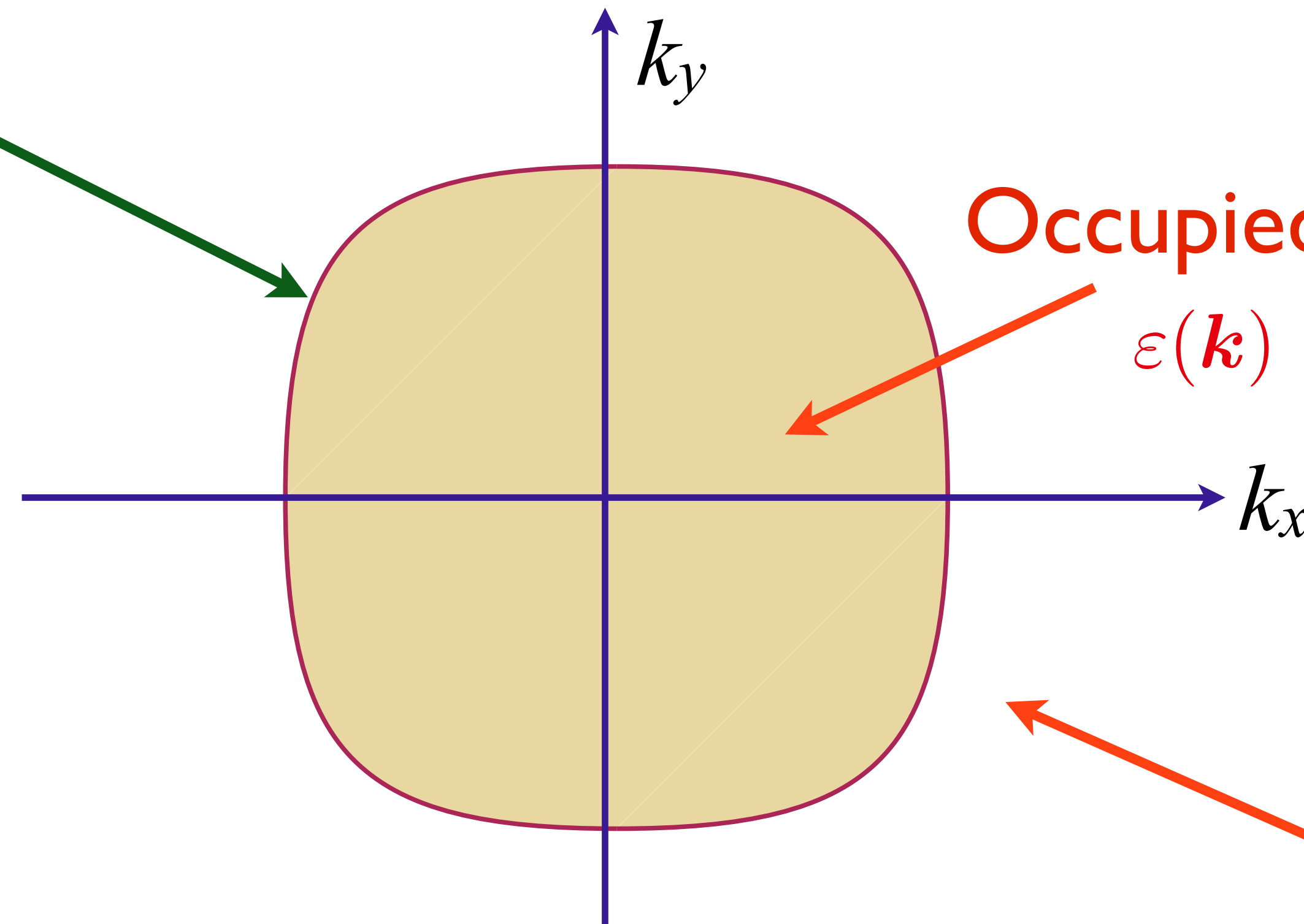
**Empty states**

$$\varepsilon(\mathbf{k}) > E_F$$

# Theory of metals

Electrons move with momentum  $\mathbf{k}$  through the lattice with dispersion  $\varepsilon(\mathbf{k})$

**Fermi surface**



$$\frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons per spin (mod 1)}$$

# Luttinger theorem

$$\frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons per spin (mod 1)}$$

- Continues to hold in the presence of electron-electron interactions, to all orders in perturbation theory.

# Luttinger theorem

$$\frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons per spin (mod 1)}$$

- Continues to hold in the presence of electron-electron interactions, to all orders in perturbation theory.
- Thoroughly confirmed in experiments.

# Luttinger theorem

$$\frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons per spin (mod 1)}$$

- Continues to hold in the presence of electron-electron interactions, to all orders in perturbation theory.
- Thoroughly confirmed in experiments.
- Consequence of a global U(1) symmetry,  $c_\alpha \rightarrow c_\alpha e^{i\lambda}$ , associated the electron number conservation.

# Luttinger theorem

$\frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons per spin (mod 1)}$

- Continues to hold in the presence of electron-electron interactions, to all orders in perturbation theory.
- Thoroughly confirmed in experiments.
- Consequence of a global U(1) symmetry,  $c_\alpha \rightarrow c_\alpha e^{i\lambda}$ , associated the electron number conservation.
- (Can be understood by an ‘anomaly’ argument, under which we gauge the U(1) symmetry to  $c_\alpha \rightarrow c_\alpha e^{i\Omega t}$ , so that the electron Green’s function  $G(\mathbf{k}, \omega) \rightarrow G(\mathbf{k}, \omega + \Omega)$ . The effect of interactions are accounted for by the Luttinger-Ward functional, which remains invariant  $\Phi_{LW}[G(\mathbf{k}, \omega)] = \Phi_{LW}[G(\mathbf{k}, \omega + \Omega)]$ .)

What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ?

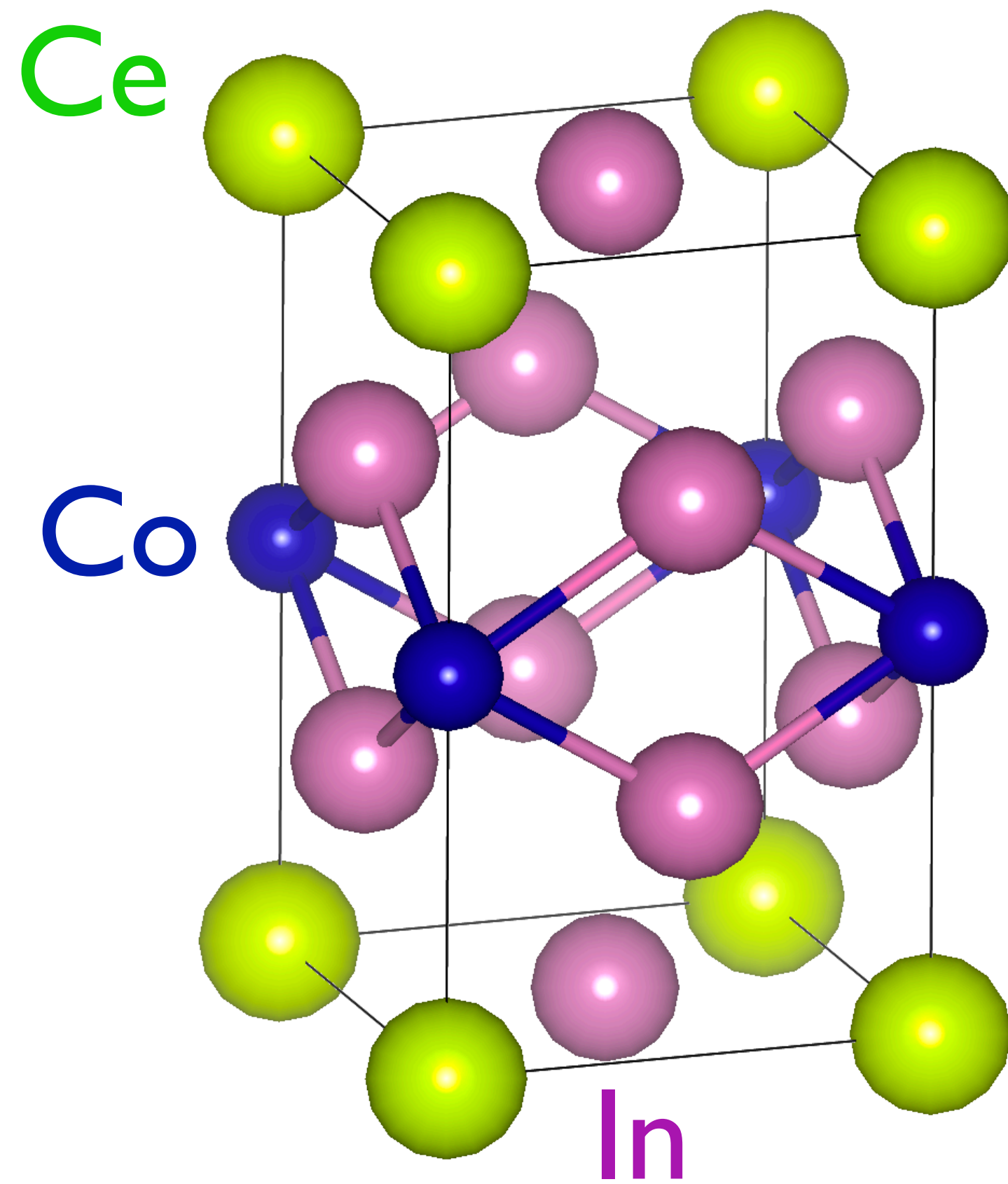
(without translational symmetry breaking)



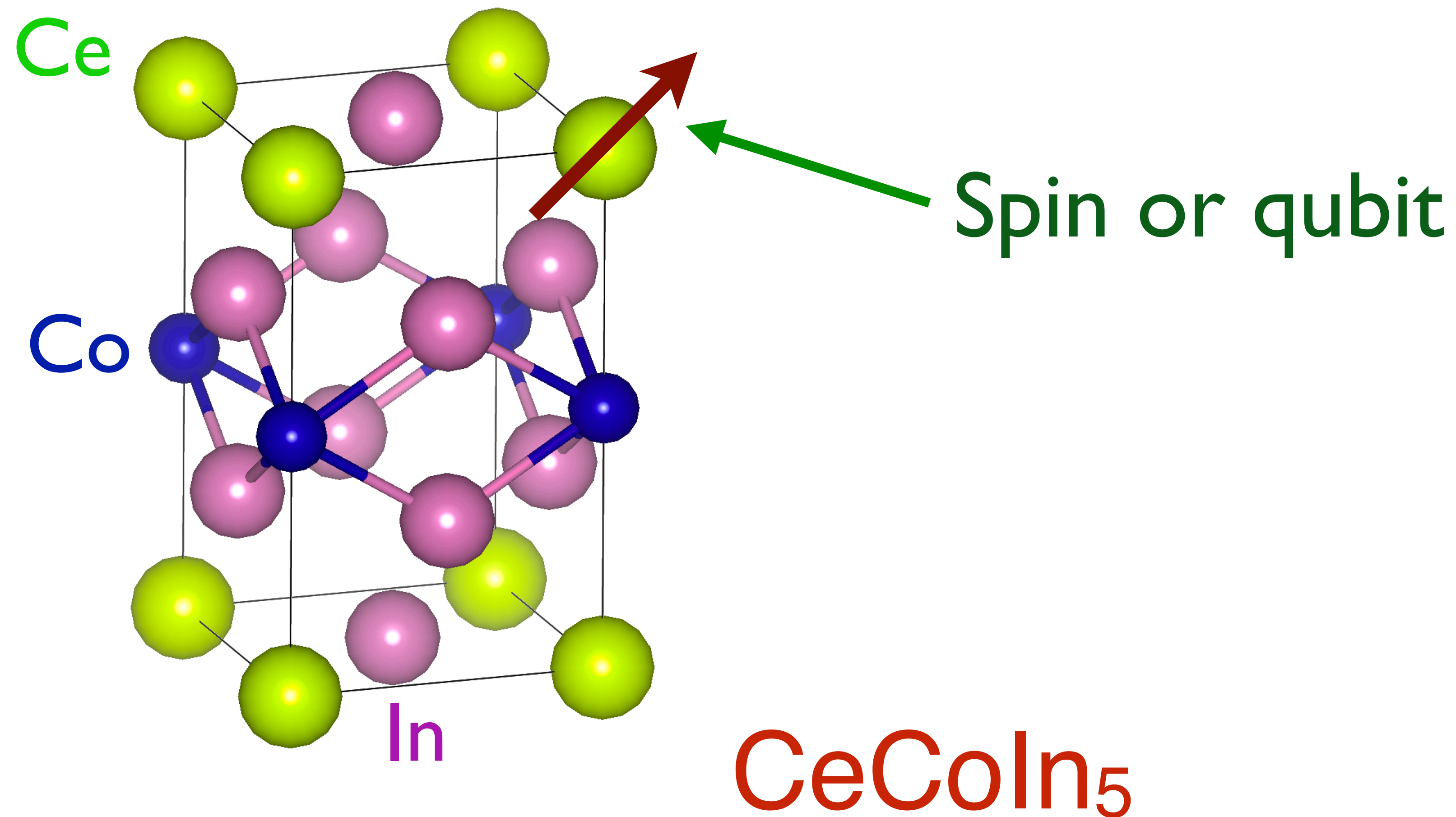
What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)

---

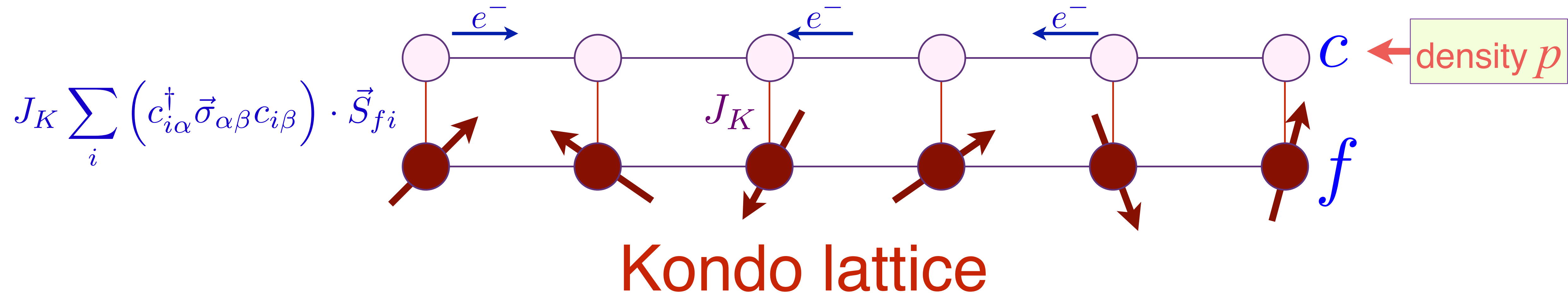
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In “**Kondo lattice**” systems, the electron charge is localized on some sites: these sites are fully described by a spin degree of freedom *i.e.* a “qubit”.



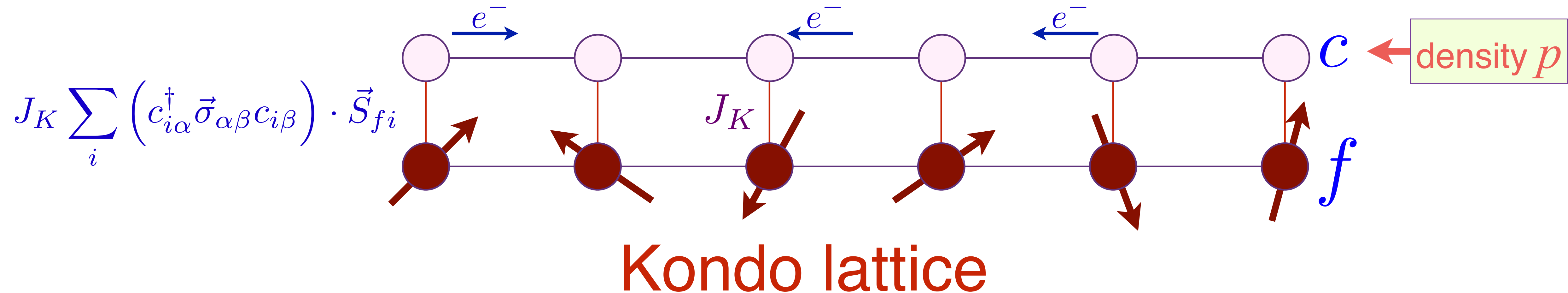
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In “**Kondo lattice**” systems, the electron charge is localized on some sites: these sites are fully described by a spin degree of freedom *i.e.* a “qubit”.



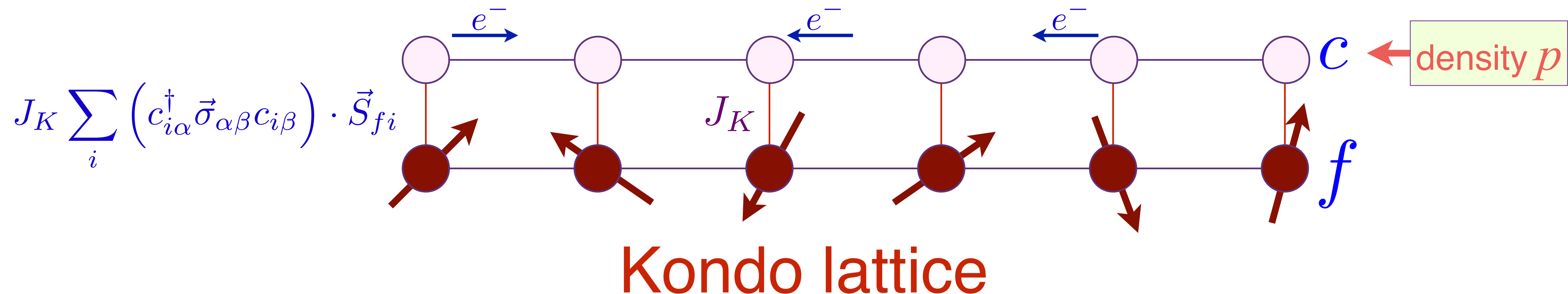
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In “**Kondo lattice**” systems, the electron charge is localized on some sites: these sites are fully described by a spin degree of freedom *i.e.* a “qubit”.



- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- Does the Fermi surface volume count the electron qubits (this is the large Fermi surface, of size  $l+p$ ) ?

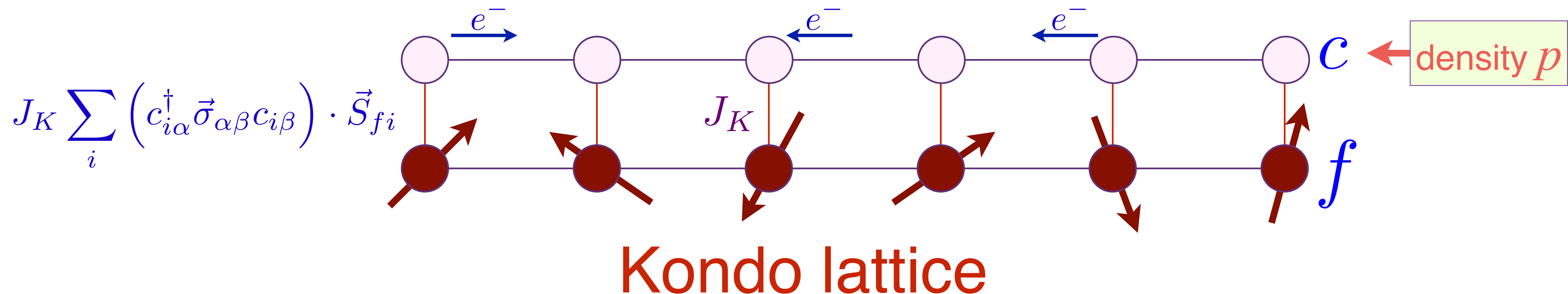


- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- Does the Fermi surface volume count the electron qubits (this is the large Fermi surface, of **size  $l+p$** ) ?
- Or does it not, so that the small Fermi surface (of **size  $p$** ) only includes the mobile electrons ?



- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- Does the Fermi surface volume count the electron qubits (this is the large Fermi surface, of size  $l+p$ ) ?
- Or does it not, so that the small Fermi surface (of size  $p$ ) only includes the mobile electrons ?

For non-infinite interactions, Luttinger's theorem states the Fermi surface should be large.



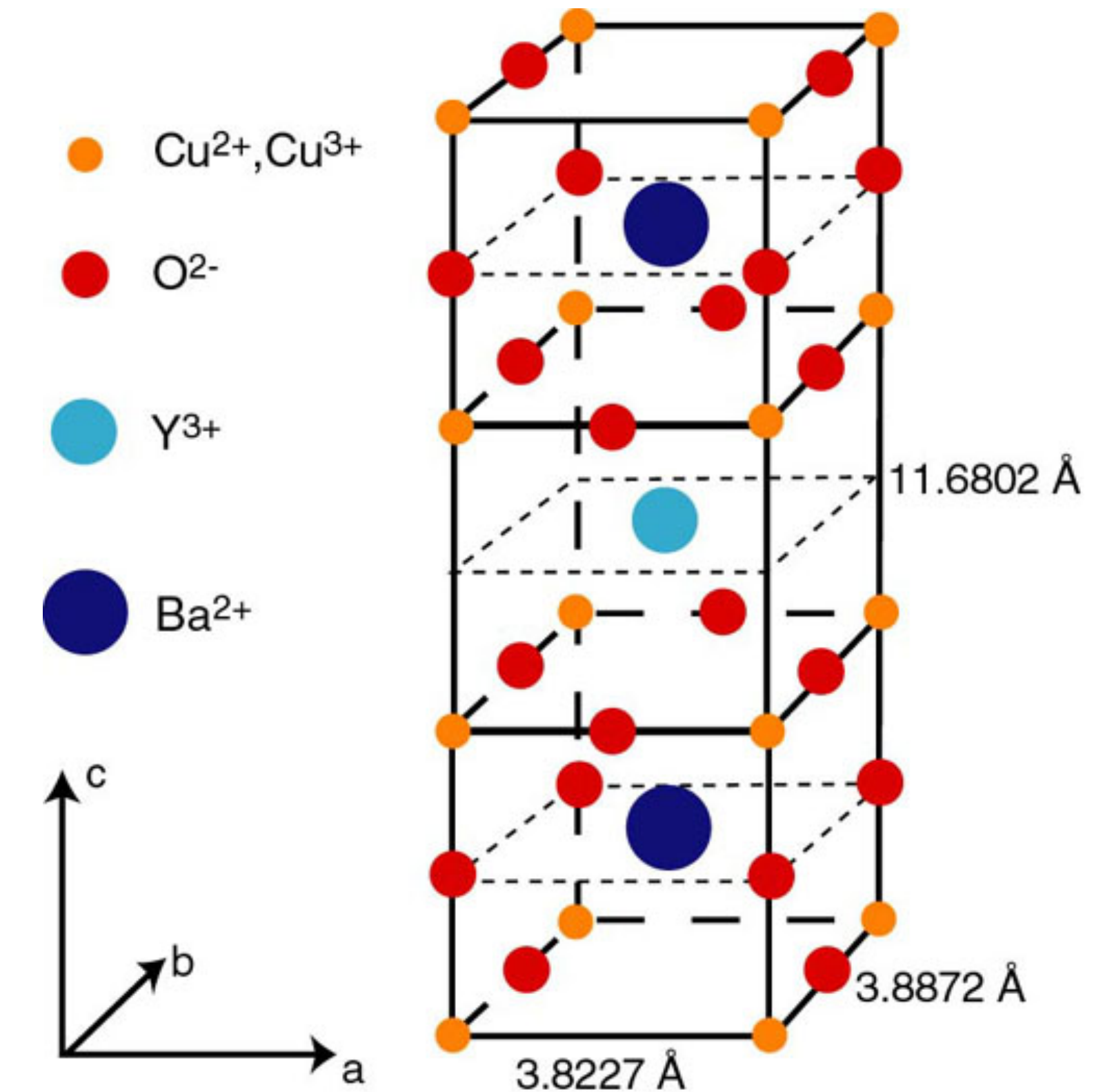
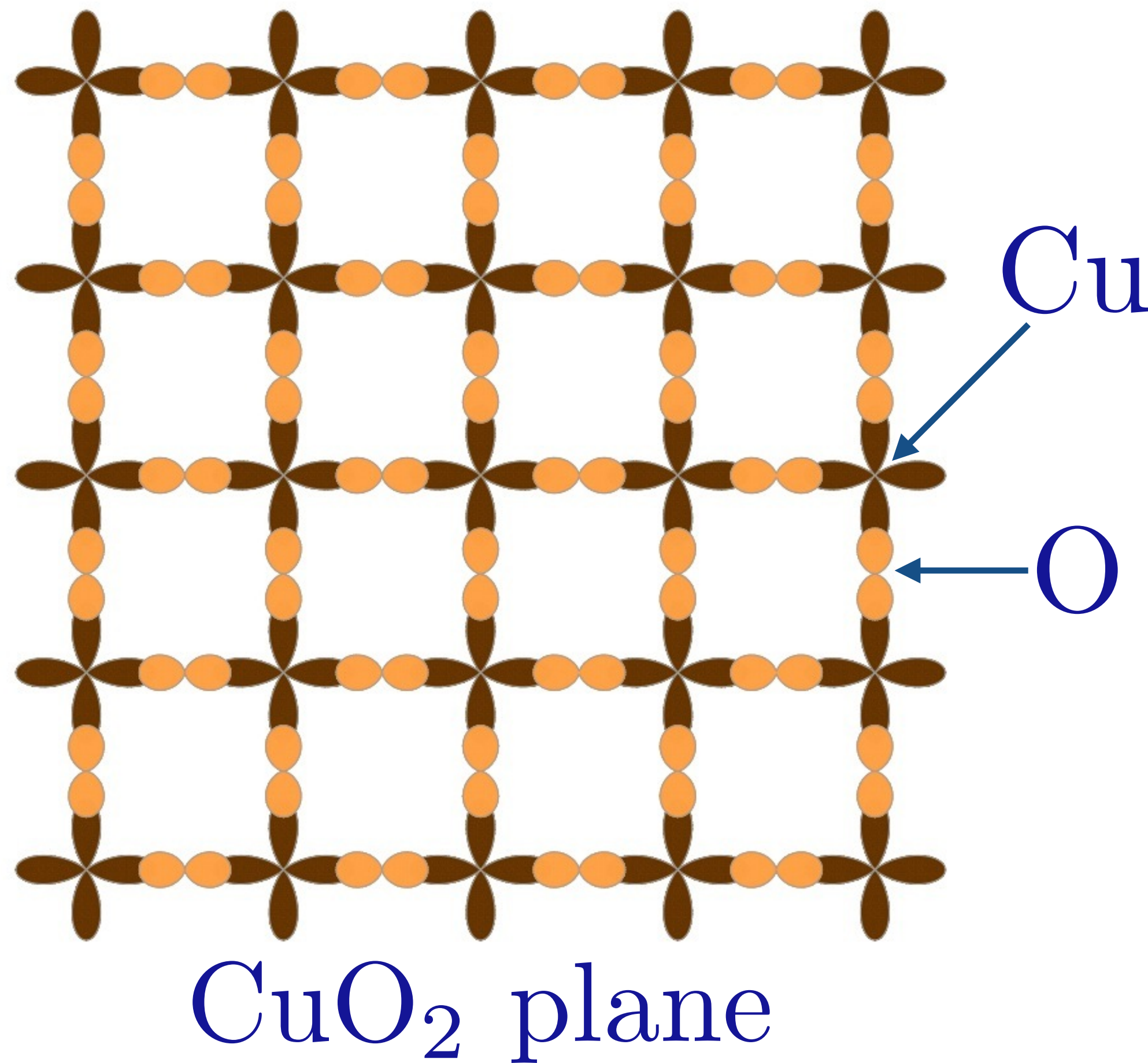


What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)

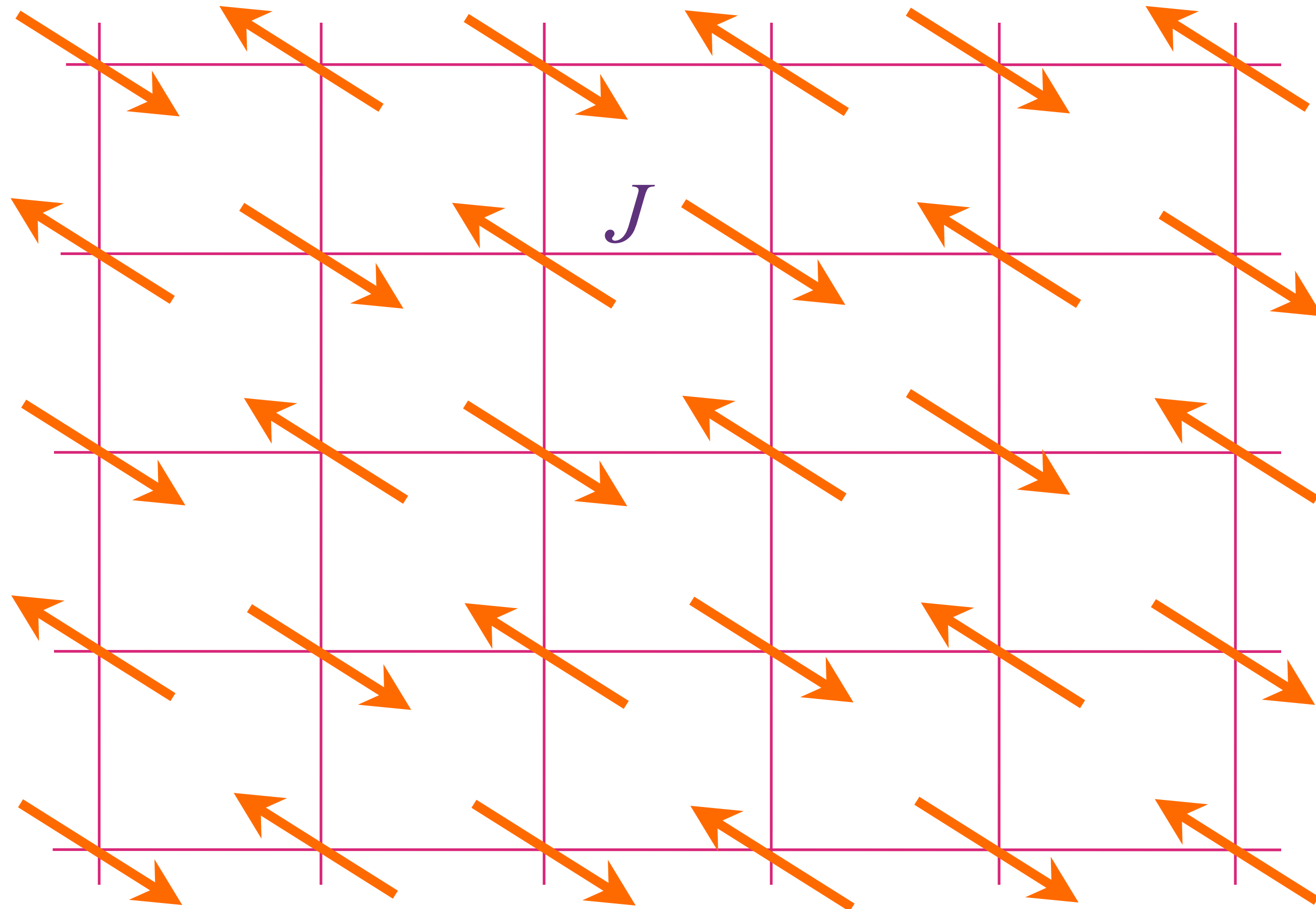
---

- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.

High temperature superconductors

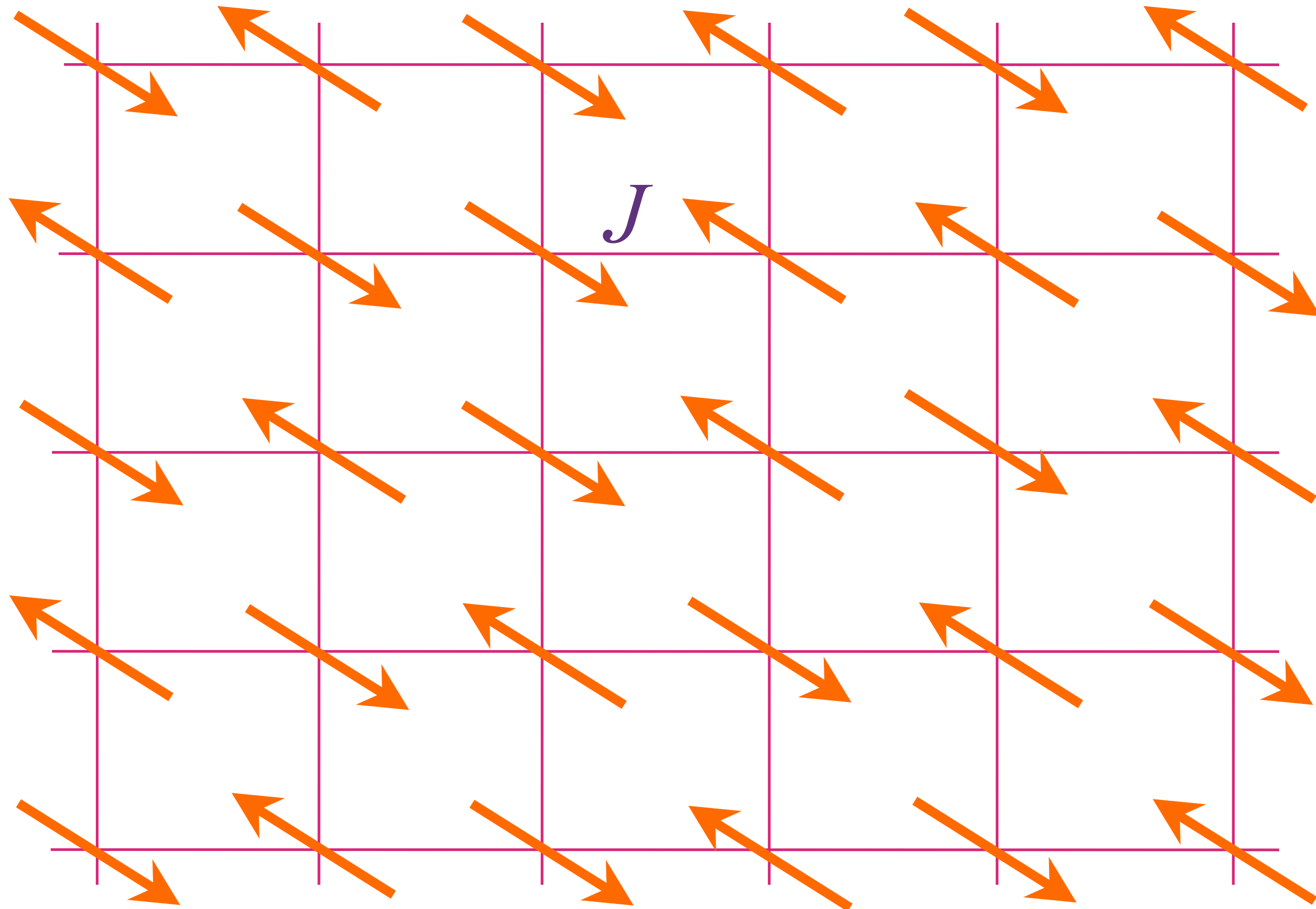


- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



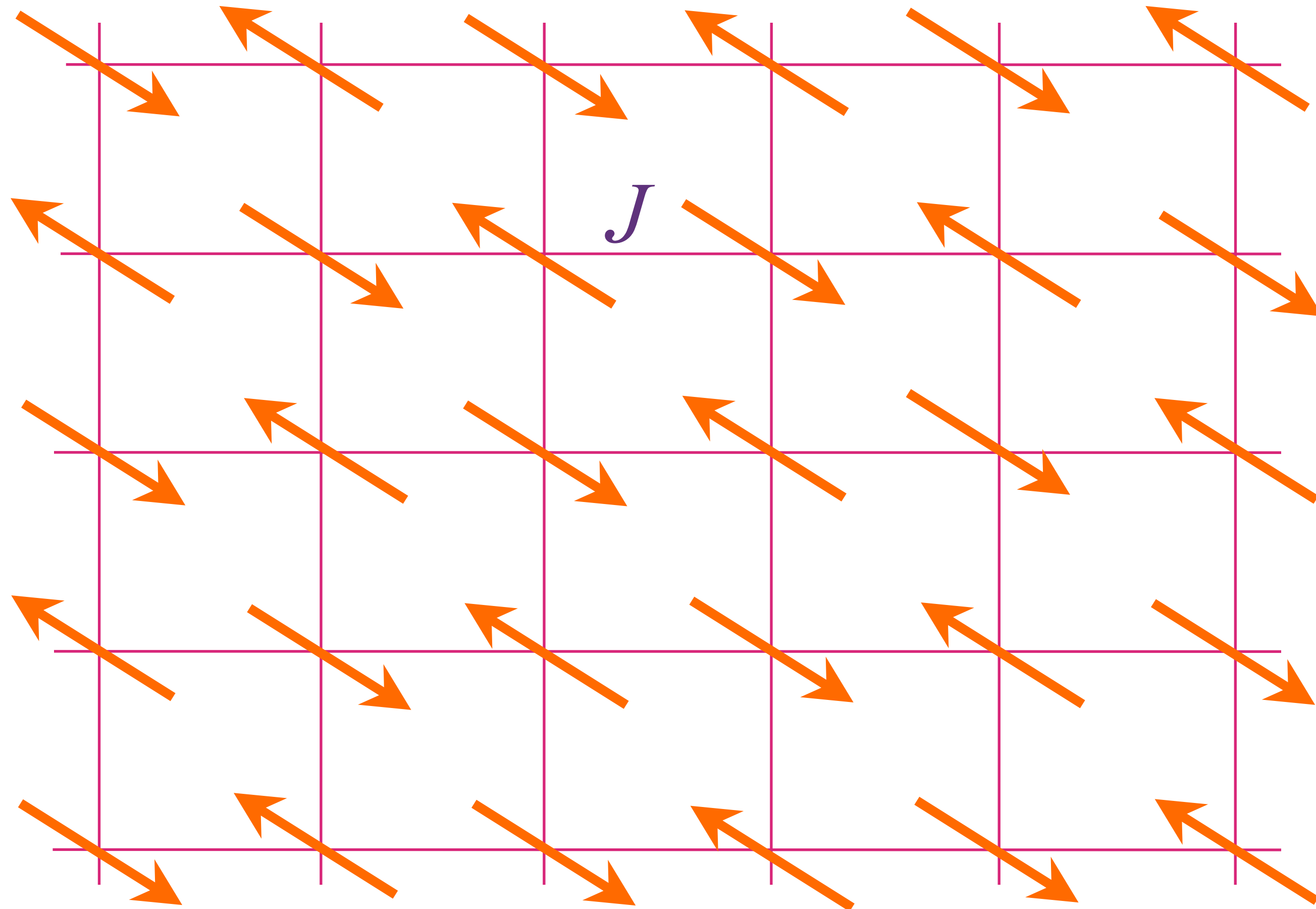
Insulating  
antiferromagnet

- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



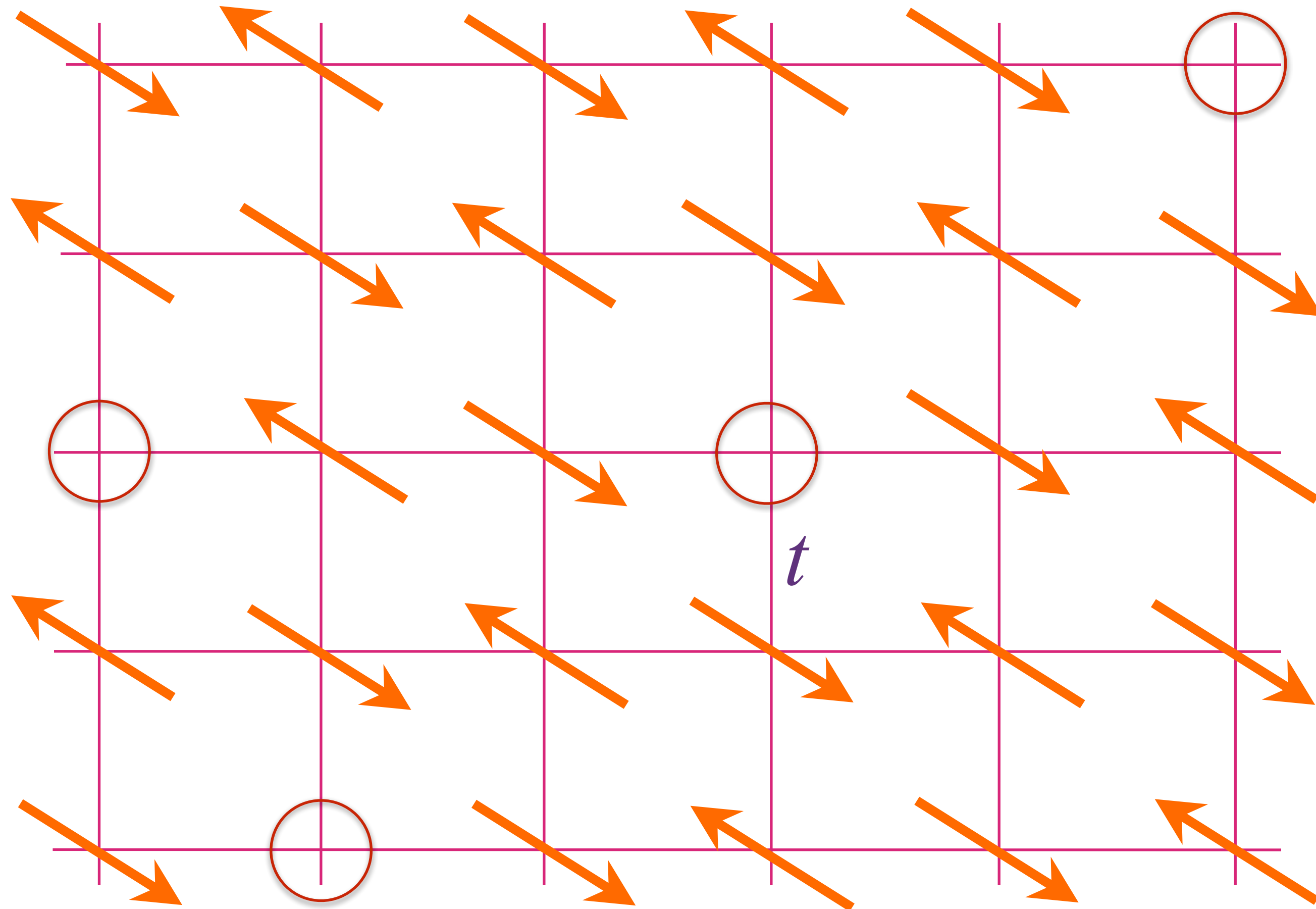
Insulating  
antiferromagnet

- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



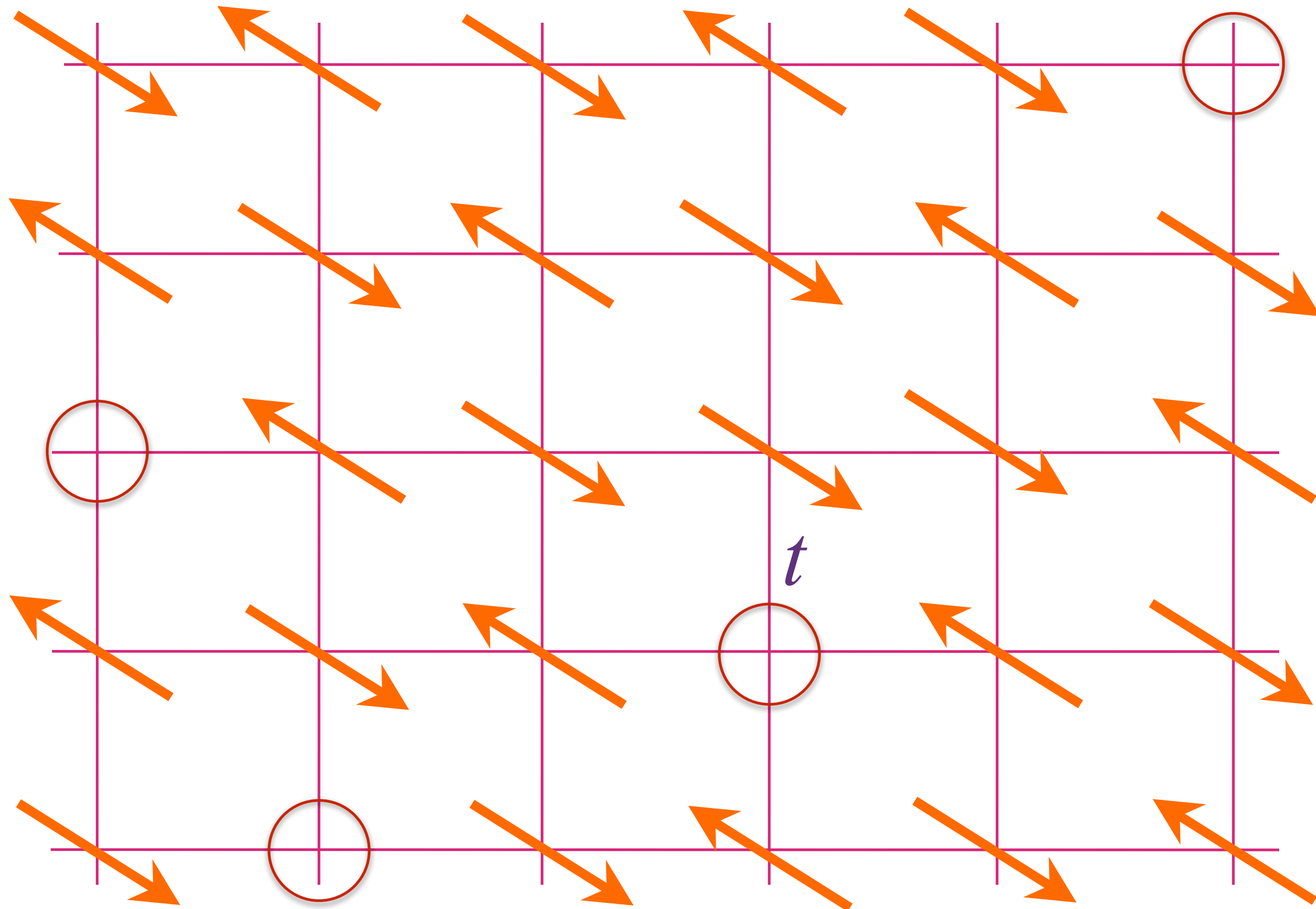
Insulating  
antiferromagnet

- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

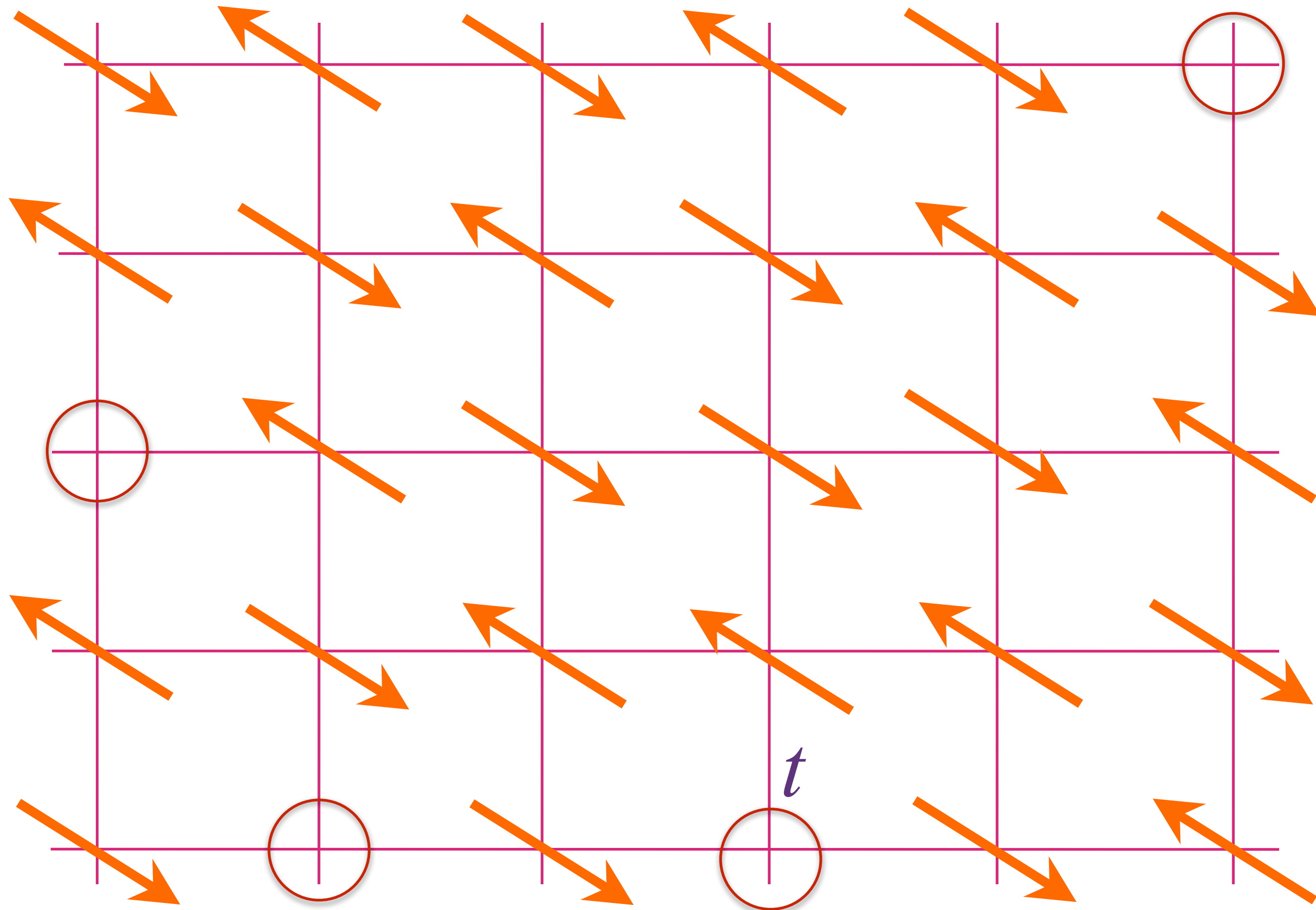
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

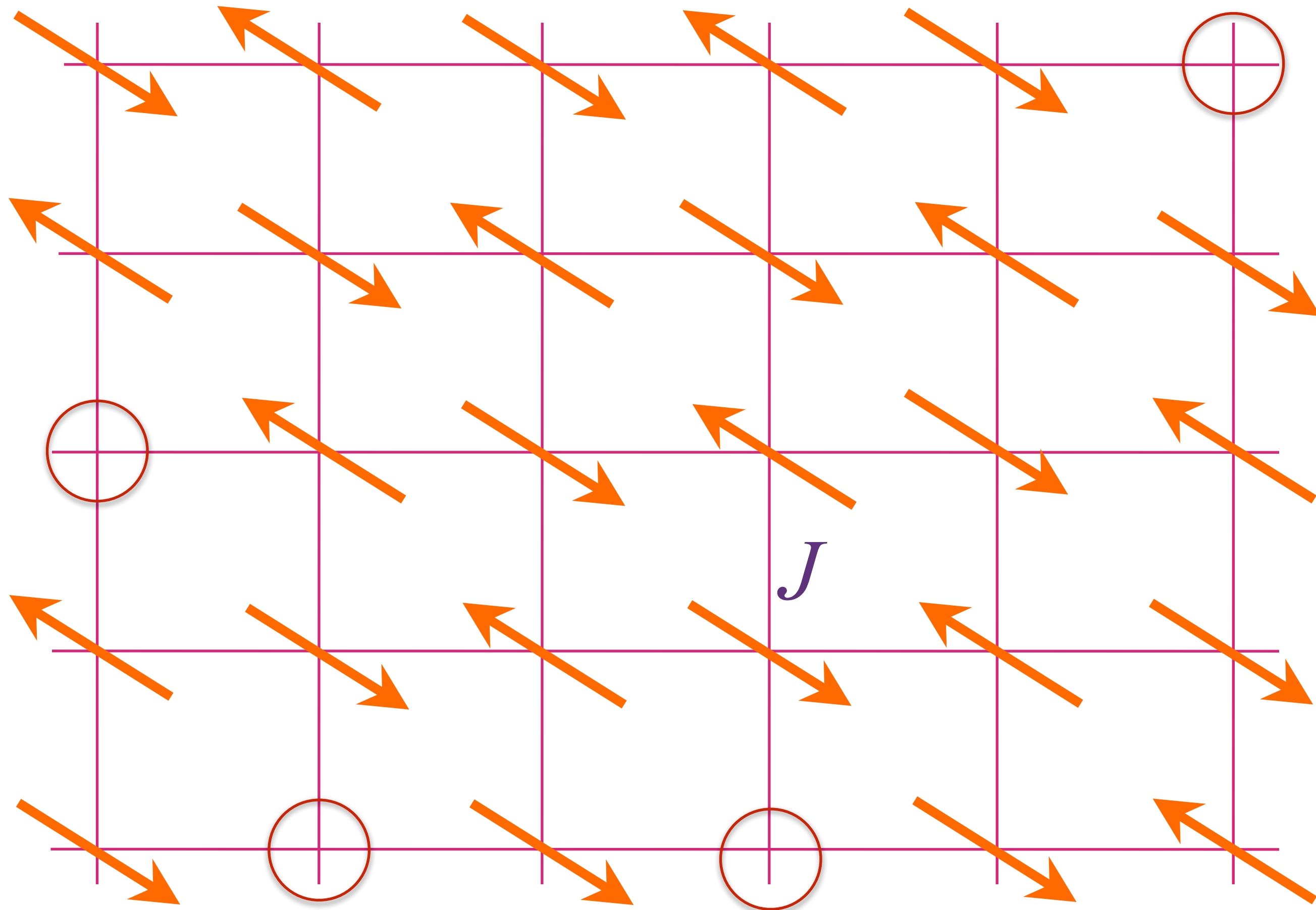
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

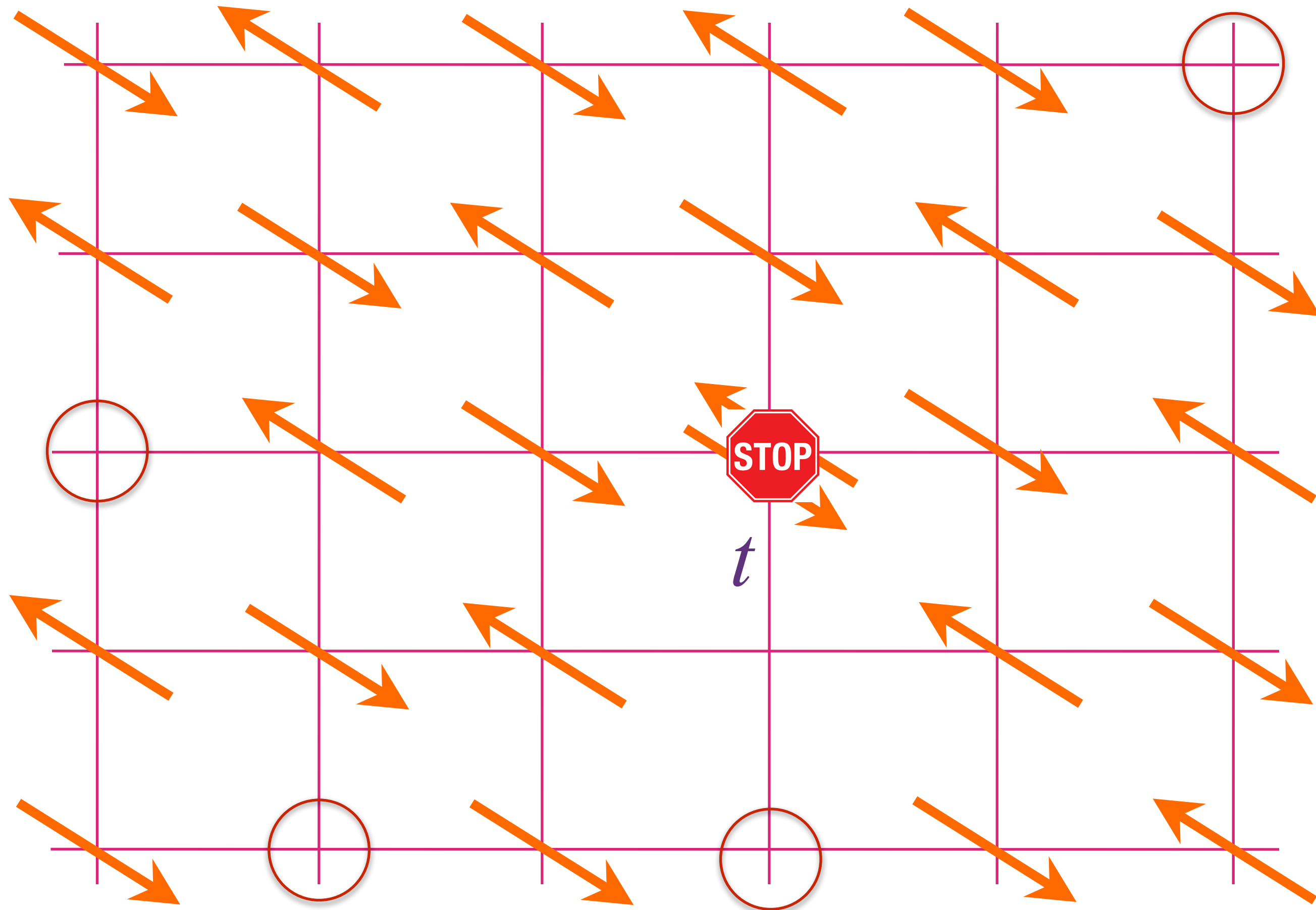
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

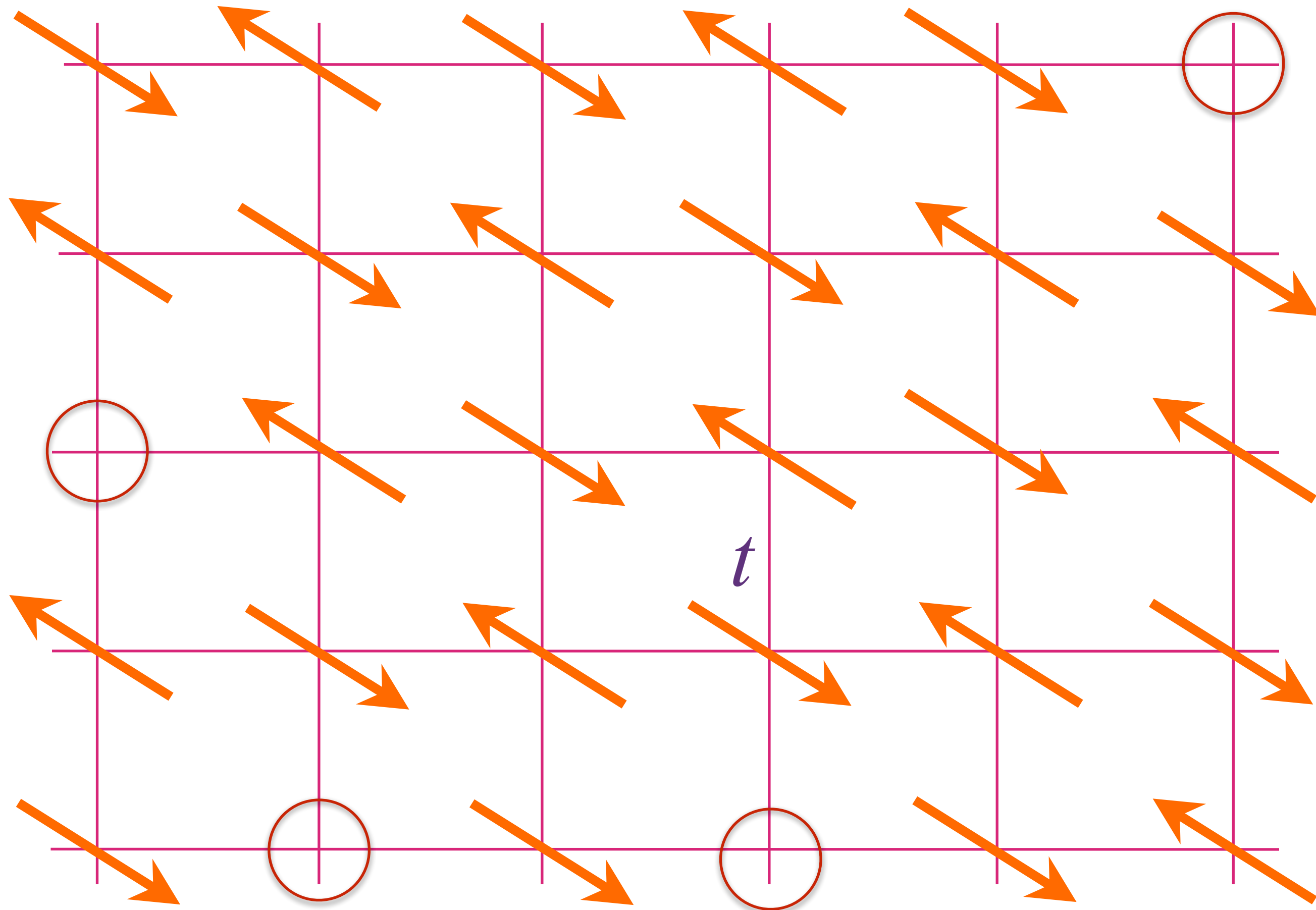
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

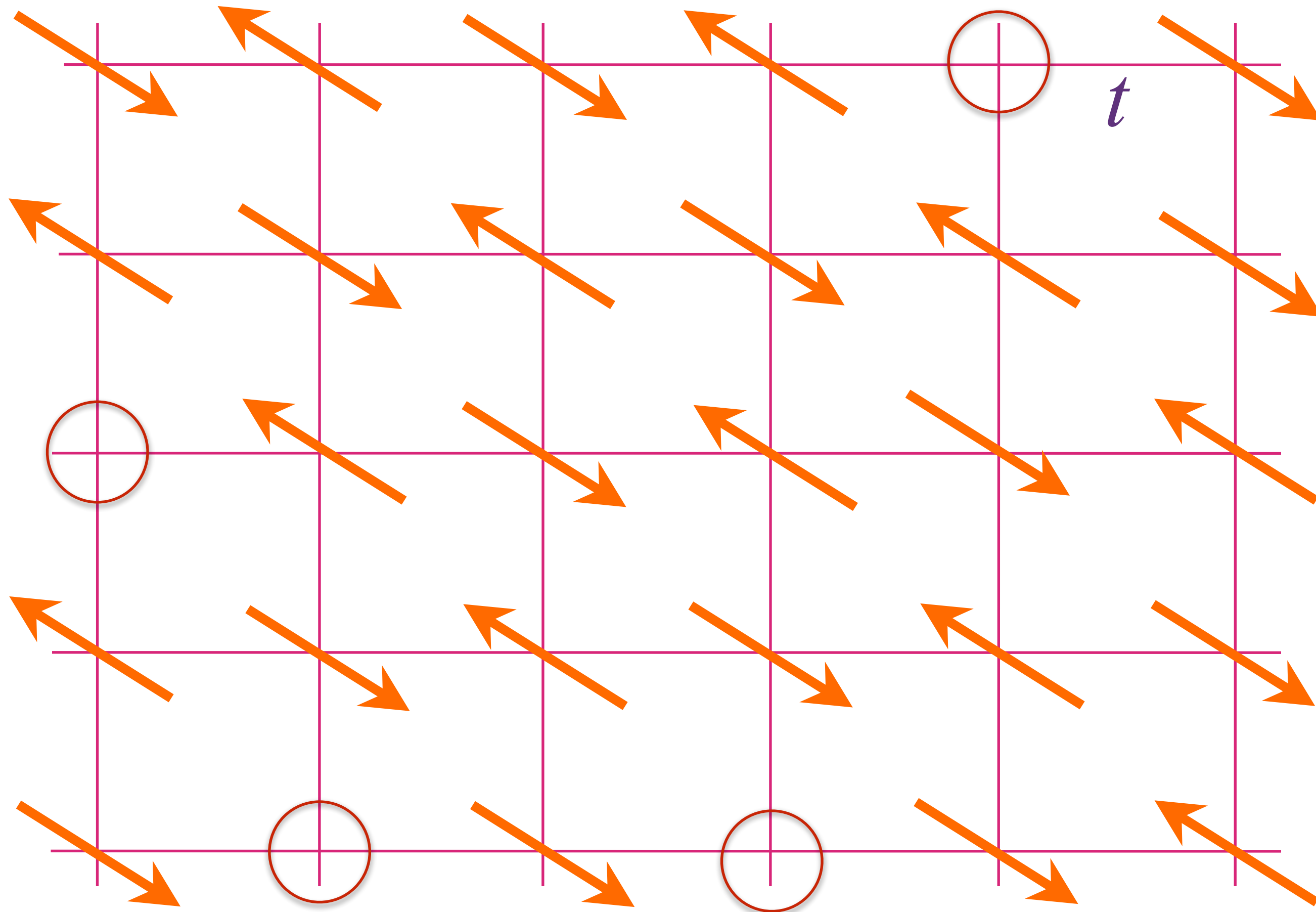
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

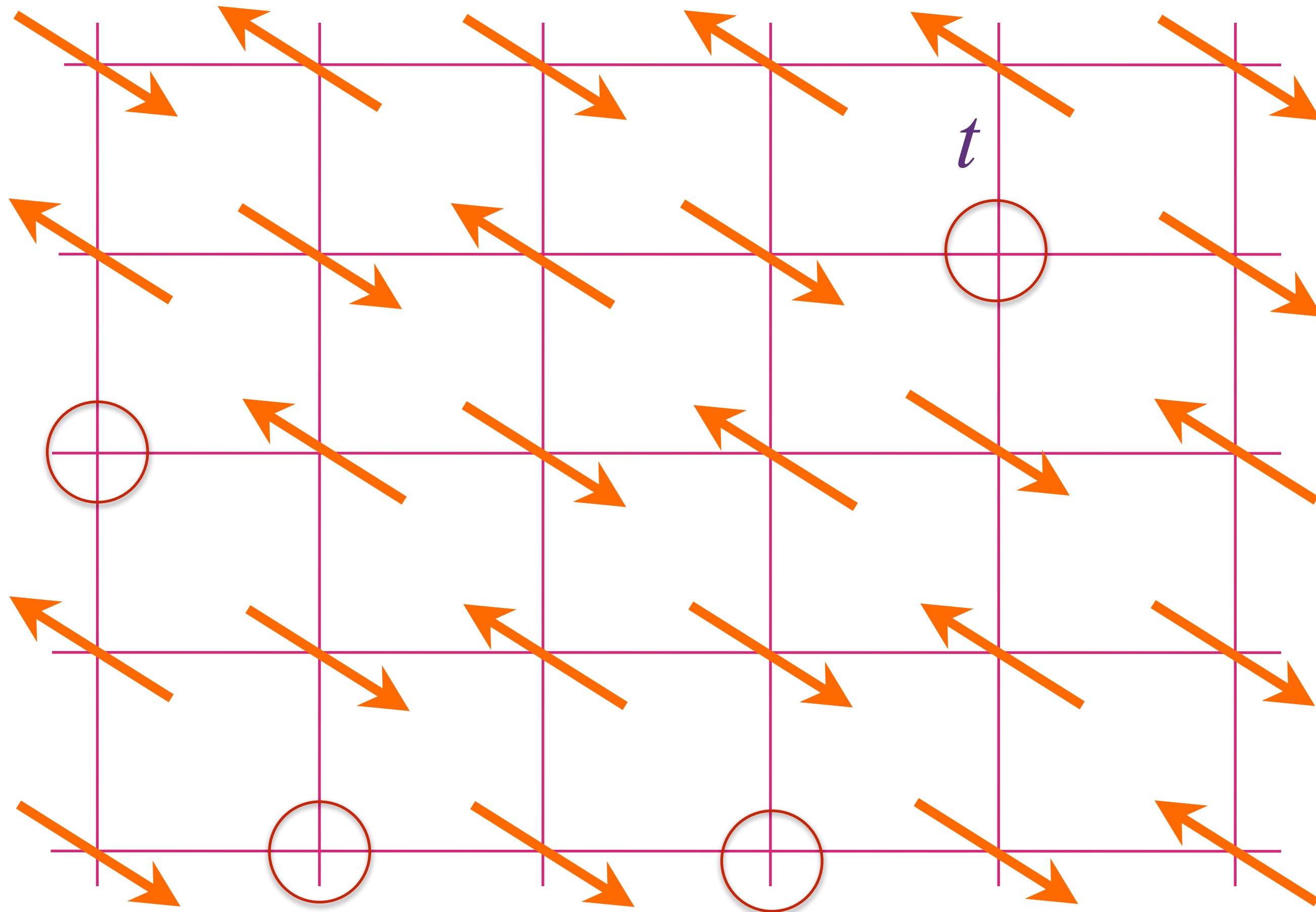
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

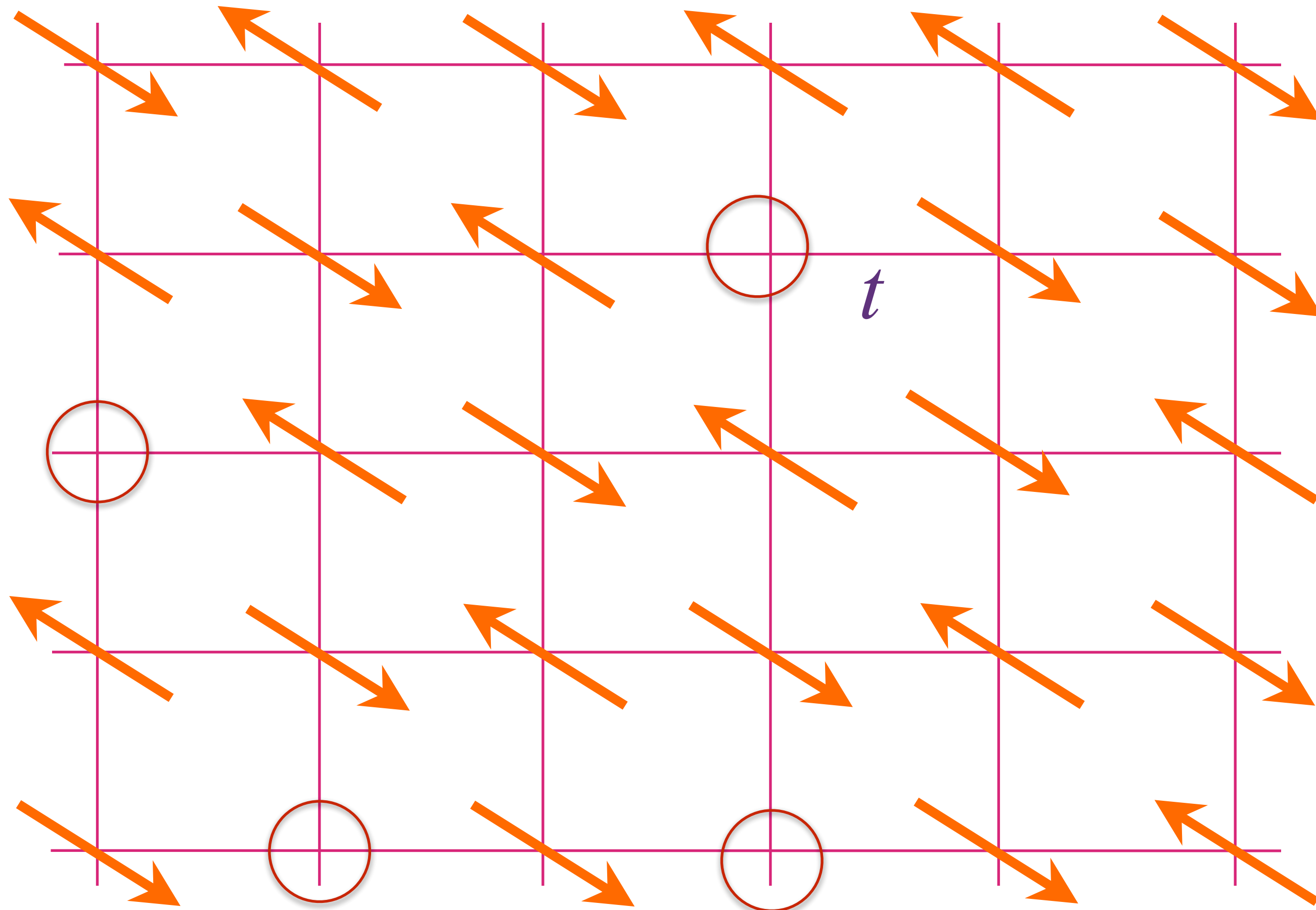
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

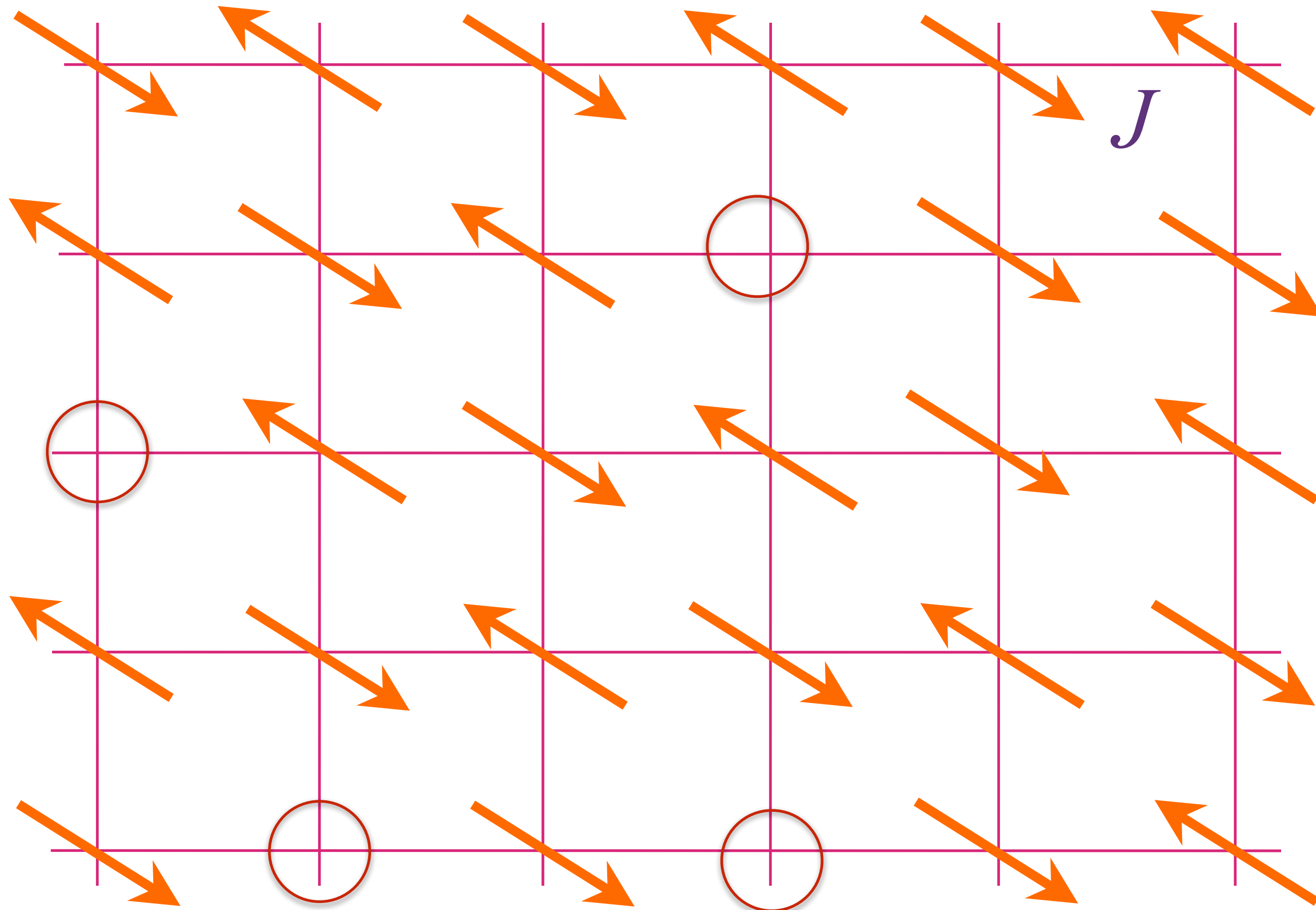
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

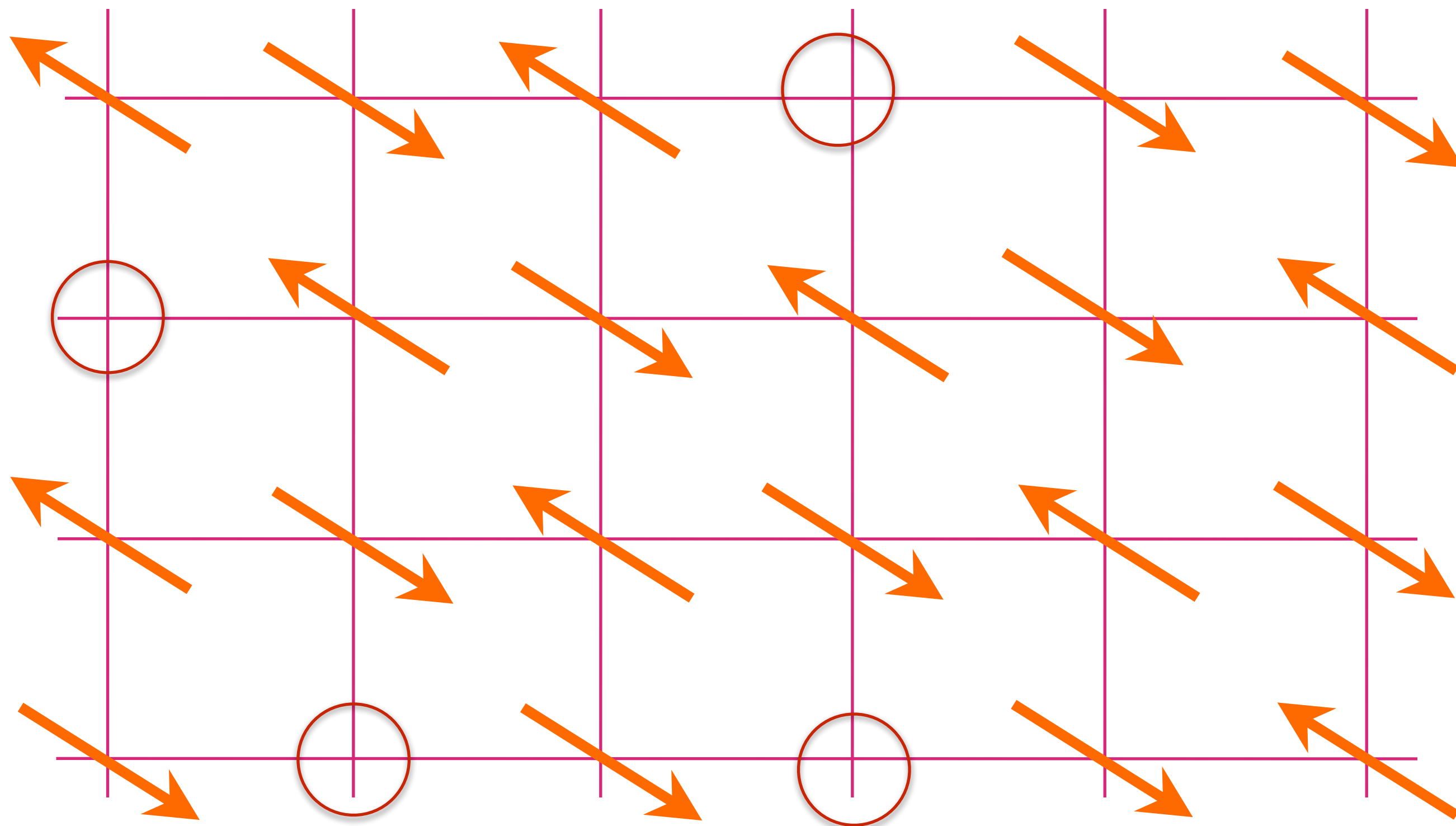
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

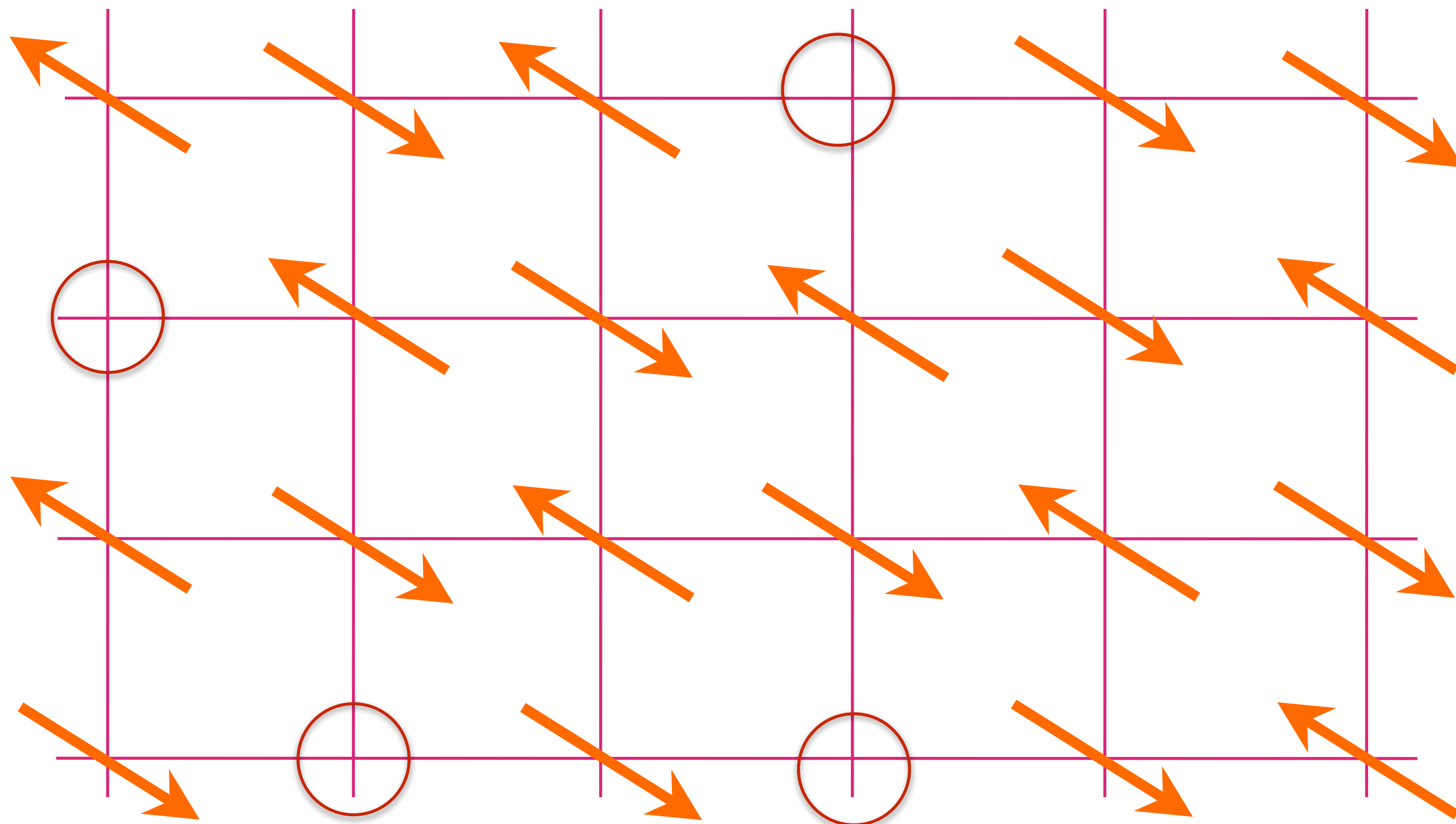
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.
- Luttinger's theorem states the Fermi surface should have a large size  $1-p$  electrons (equivalent to  $1+p$  holes).



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ? (without translational symmetry breaking)
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.
- Luttinger's theorem states the Fermi surface should have a large size  $1-p$  electrons (equivalent to  $1+p$  holes).
- Or should there be a small Fermi surface of size  $p$  holes ?



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

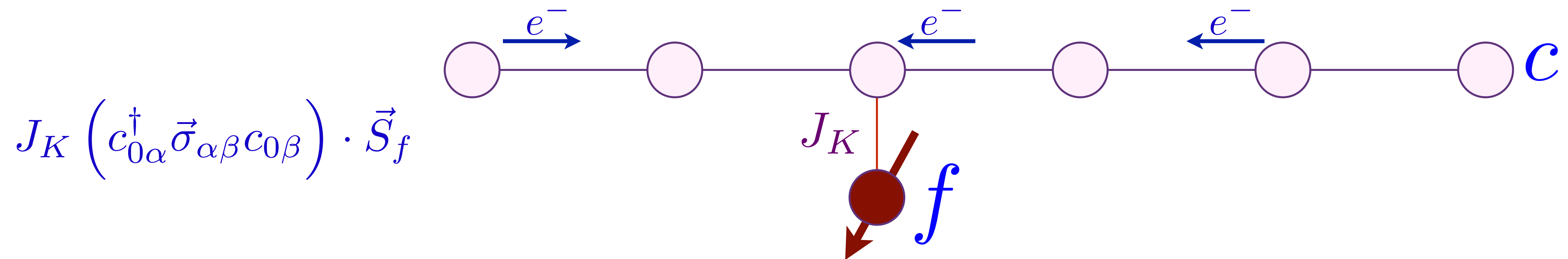
1. Luttinger volume violation in  
Kondo lattice models  
*The FL\* phase and CeCoIn<sub>5</sub>*

2. Luttinger volume violation a  $t$ - $J$  model  
*Ancilla qubits and a small to  
large Fermi surface transition*

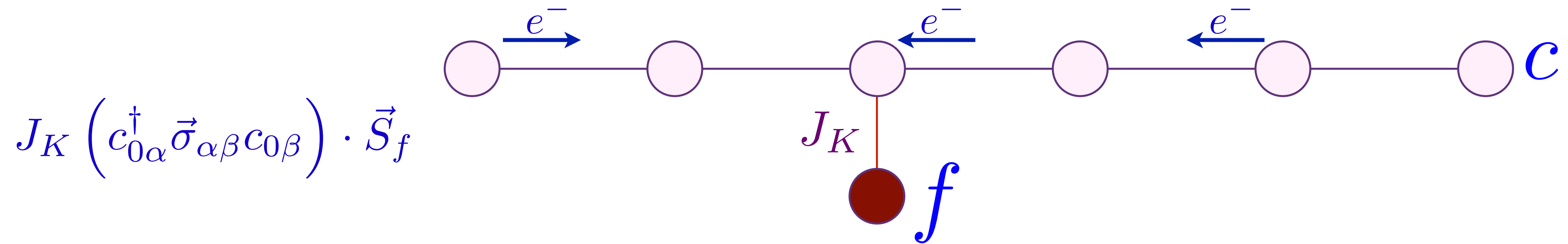
1. Luttinger volume violation in  
Kondo lattice models  
*The FL\* phase and CeCoIn<sub>5</sub>*

2. Luttinger volume violation a  $t$ - $J$  model  
*Ancilla qubits and a small to  
large Fermi surface transition*

# Kondo model



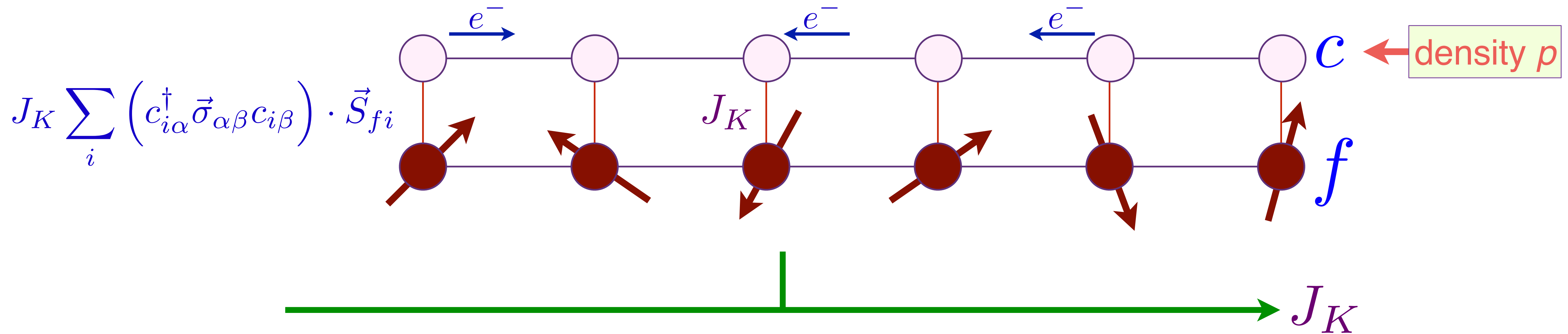
# Kondo model



The  $c$  electrons ‘Kondo screen’ the  $f$  spin at low energies:  
The  $f$  electron ‘dissolves’ into the Fermi sea.

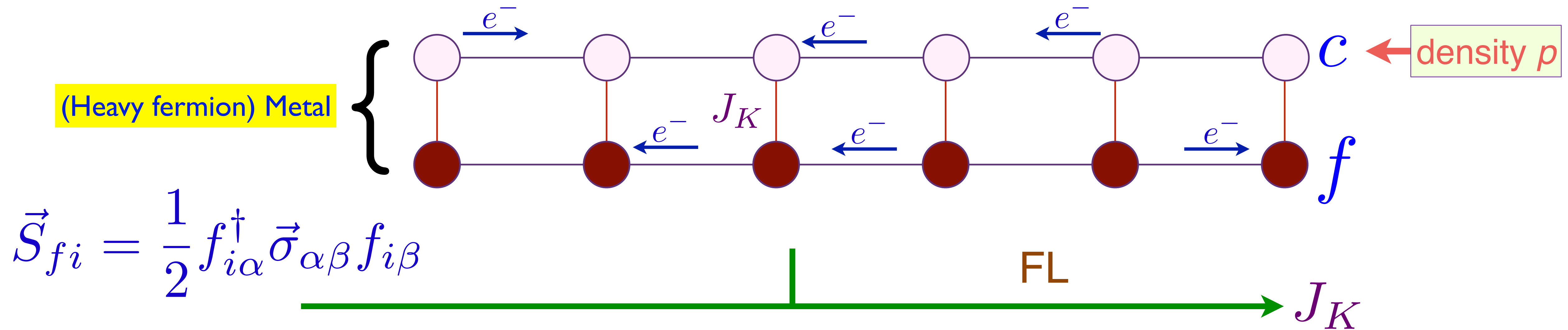
# Luttinger volume in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



# Luttinger volume in **Kondo lattice** models

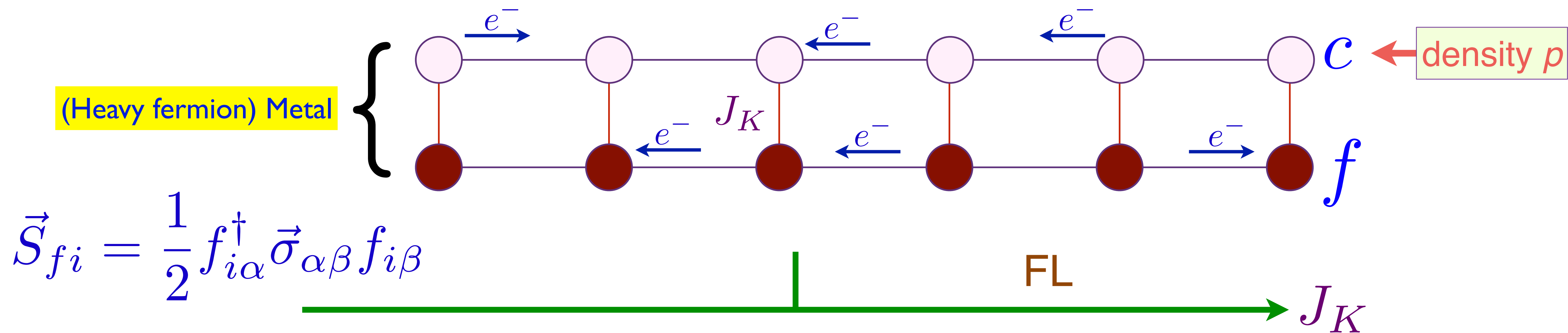
Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



The  $c$  electrons ‘Kondo screen’ the  $f$  spins in the FL phase:  
The  $f$  electrons ‘dissolve’ into the Fermi sea.

# Luttinger volume in **Kondo lattice** models

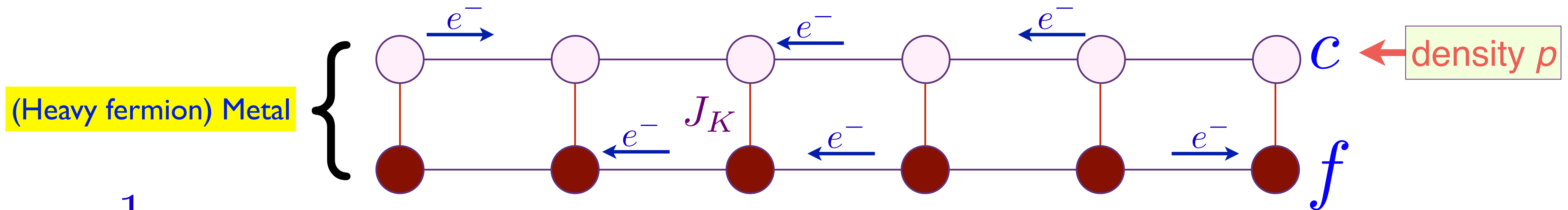
Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



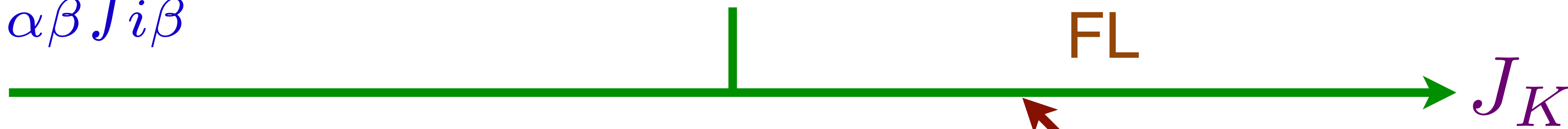
The  $c$  electrons ‘Kondo screen’ the  $f$  spins in the FL phase:  
 The  $f$  electrons ‘dissolve’ into the Fermi sea.  
 The Fermi surface is large: encloses volume of  $1 + p$  electrons.

# Luttinger volume in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

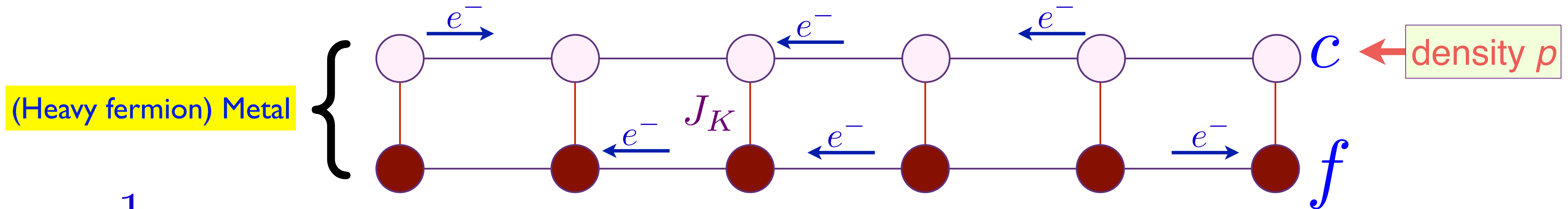


Large Fermi surface of size  $1 + p$

$|\Phi\rangle = [\text{Projection onto one } f \text{ per site}]$   
 $\otimes |\text{Slater determinant of } (c, f)\rangle$

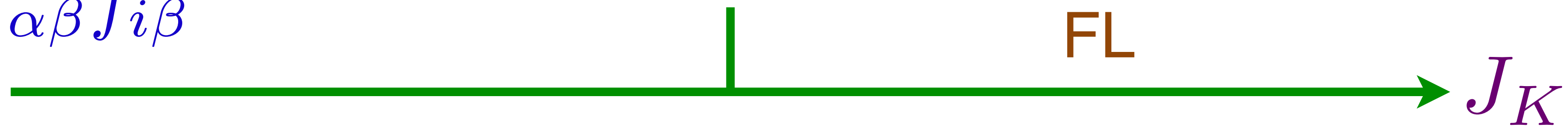
# Luttinger volume in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

$$\langle c_\alpha^\dagger f_\alpha \rangle \neq 0$$

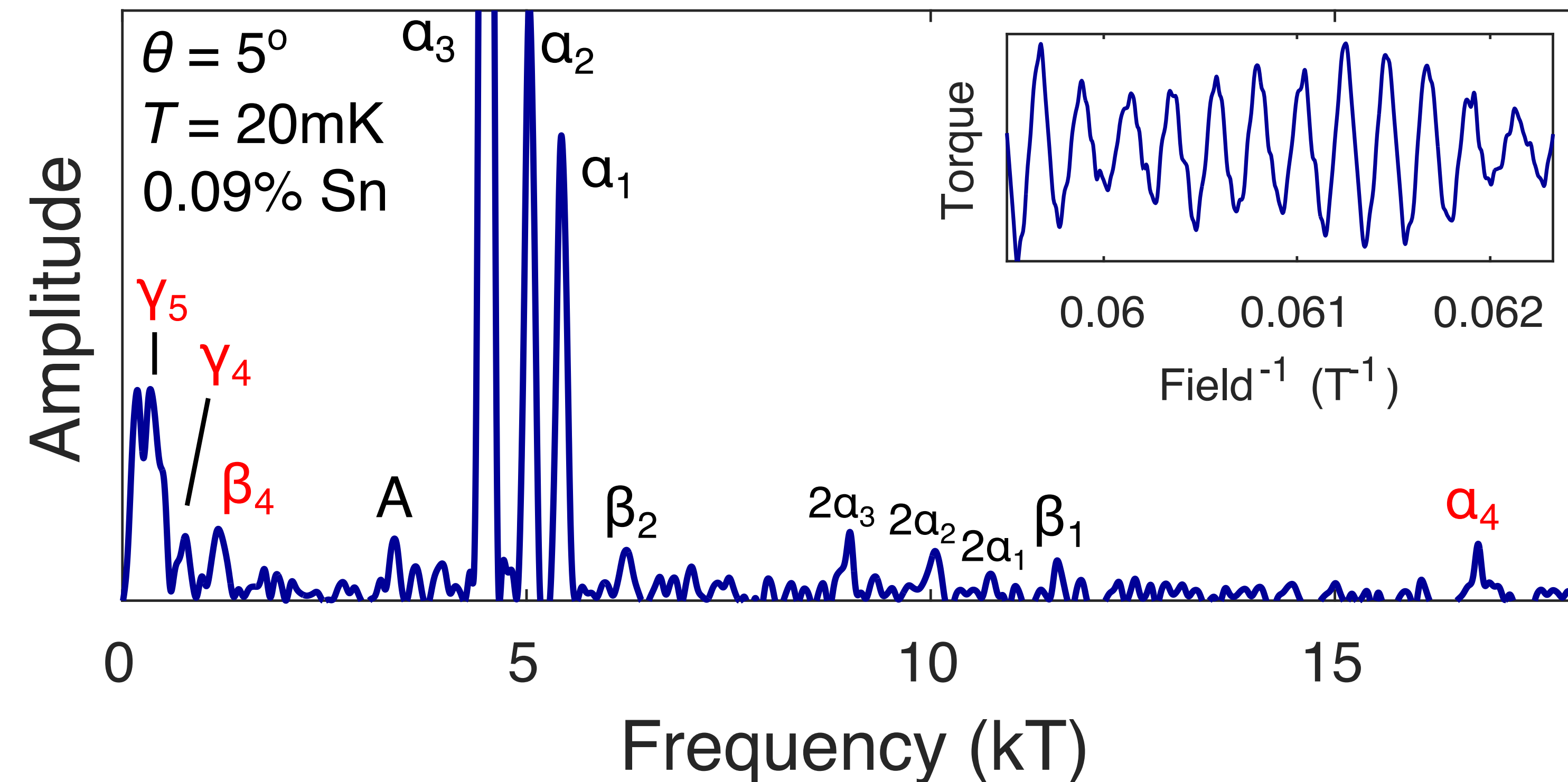
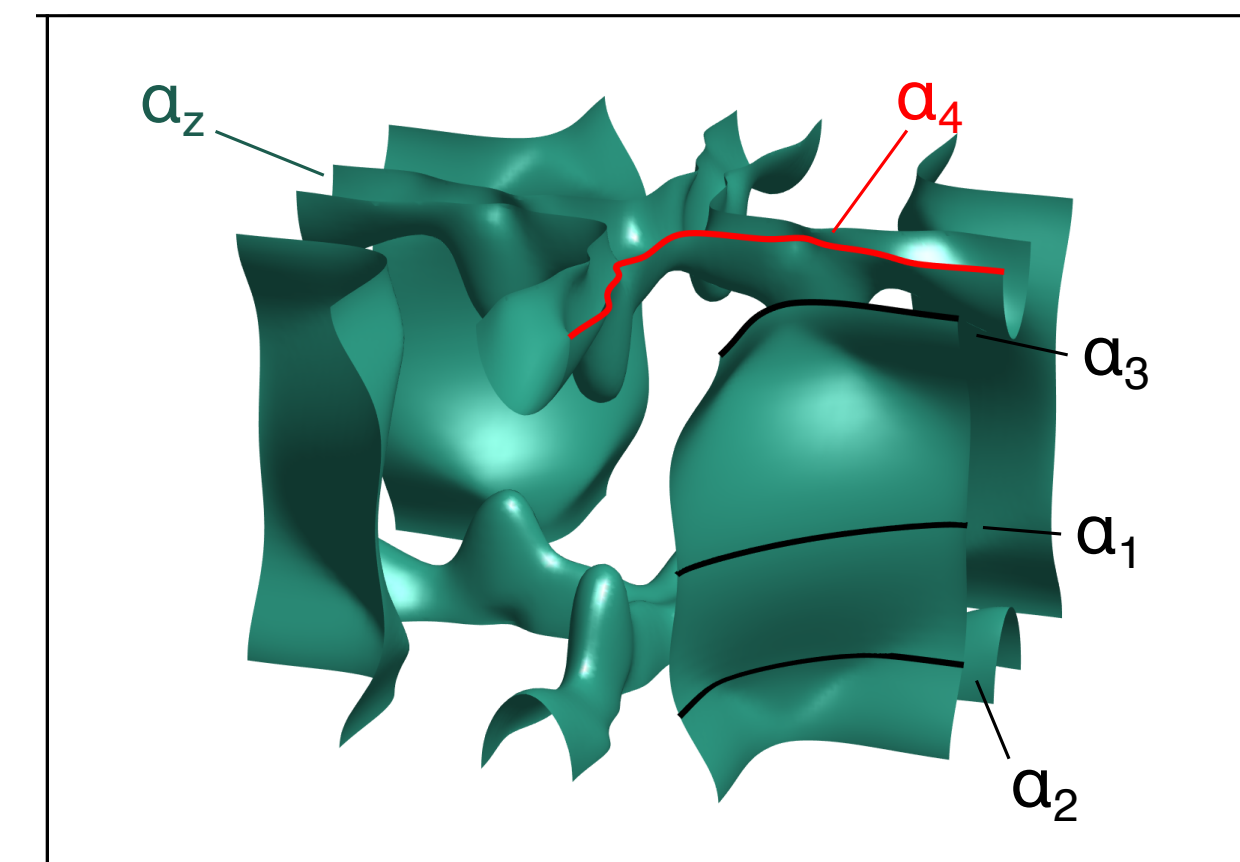
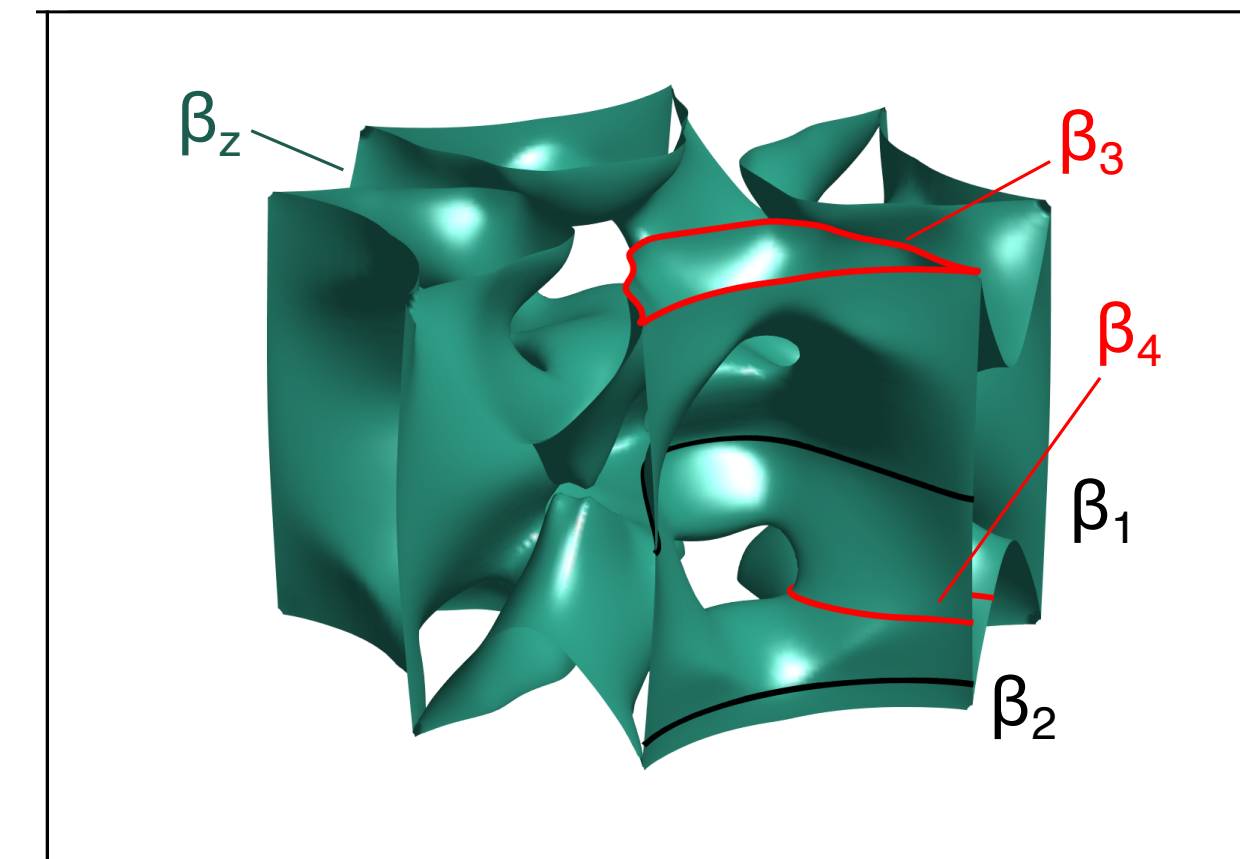
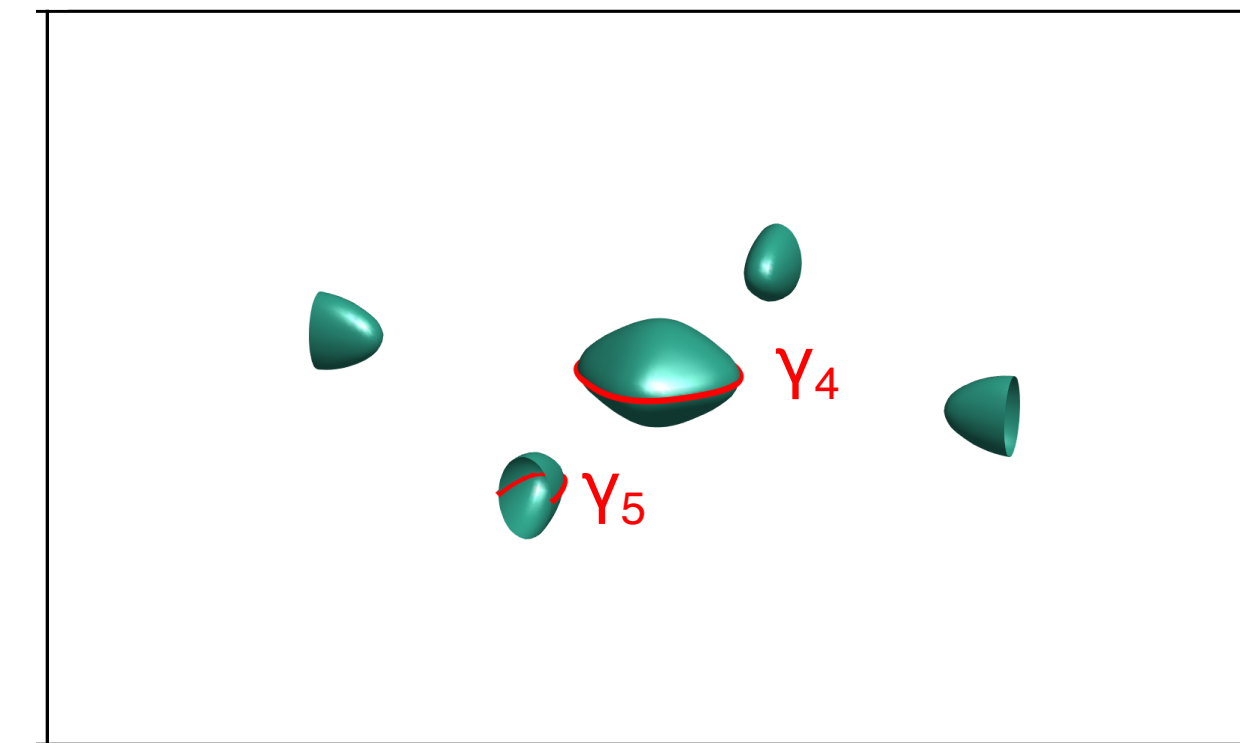
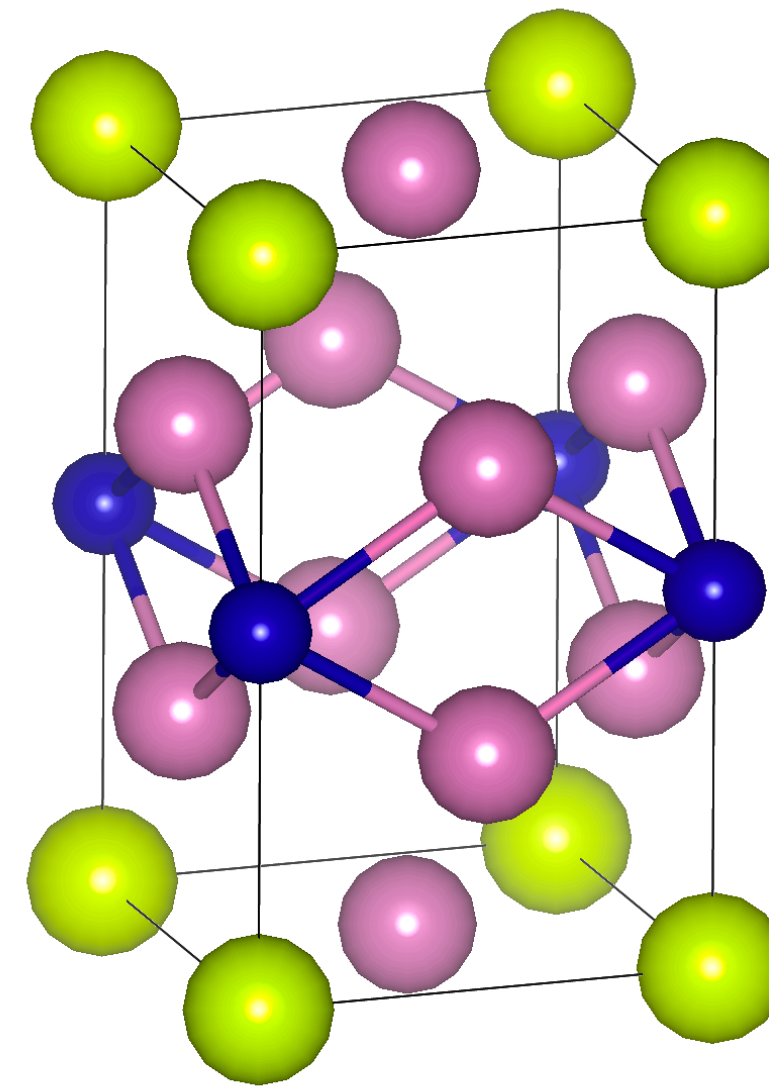


The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$ , because  $\sum_i f_{i\alpha}^\dagger f_{i\alpha} = 1$  for all  $i$ .

This gauge symmetry is fully broken by a Higgs condensate  $\langle c_\alpha^\dagger f_\alpha \rangle$  in the FL phase.

# Large Fermi surface in CeCoIn<sub>5</sub>

Nikola Maksimovic, Taylor Cookmeyer, Jan Rusz, Vikram Nagarajan, Amanda Gong, Fanghui Wan, Stefano Faubel, Ian M. Hayes, Sooyoung Jang, Yochai Werman, Peter M. Oppeneer, Ehud Altman, James G. Analytis [arXiv:2011.12951](https://arxiv.org/abs/2011.12951)



# Luttinger volume violation

- The Luttinger volume can be violated in a metal when there is a deconfined emergent gauge field and fractionalized excitations *i.e.* ‘bulk topological order’.

T. Senthil, M.Vojta, and S. Sachdev, PRB **69**, 035111 (2004)

A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

# Luttinger volume violation

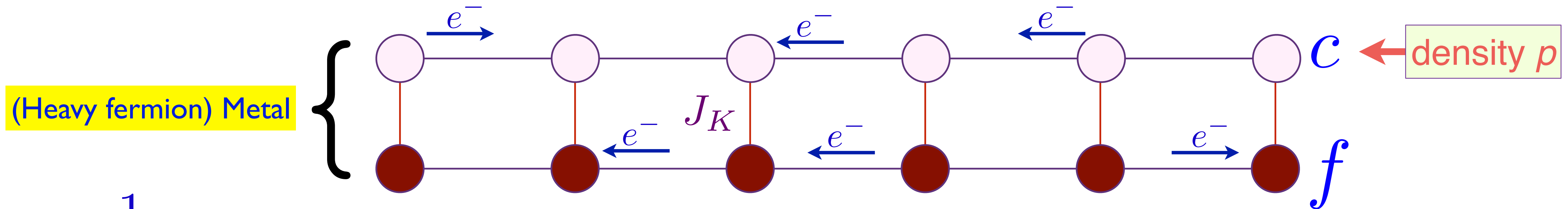
- The Luttinger volume can be violated in a metal when there is a deconfined emergent gauge field and fractionalized excitations *i.e.* ‘bulk topological order’.
- When the gauge field is fully Higgsed, the gauge charge effectively becomes identical to the electron number, and there is only a single Luttinger theorem for *all* electrons: *i.e.* a large Fermi surface.

# Luttinger volume violation

- The Luttinger volume can be violated in a metal when there is a deconfined emergent gauge field and fractionalized excitations *i.e.* ‘bulk topological order’.
- When the gauge field is fully Higgsed, the gauge charge effectively becomes identical to the electron number, and there is only a single Luttinger theorem for *all* electrons: *i.e.* a large Fermi surface.
- When the gauge field is deconfined, there are separate Luttinger anomalies for the gauge and global charges. In this manner we obtain the **FL\* phase**: a metallic phase with a Fermi surface of Fermi-liquid-like electronic quasiparticles, enclosing a non-Luttinger small volume.

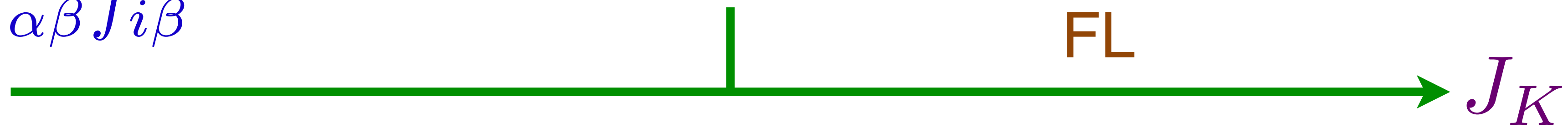
# Luttinger volume in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

$$\langle c_\alpha^\dagger f_\alpha \rangle \neq 0$$

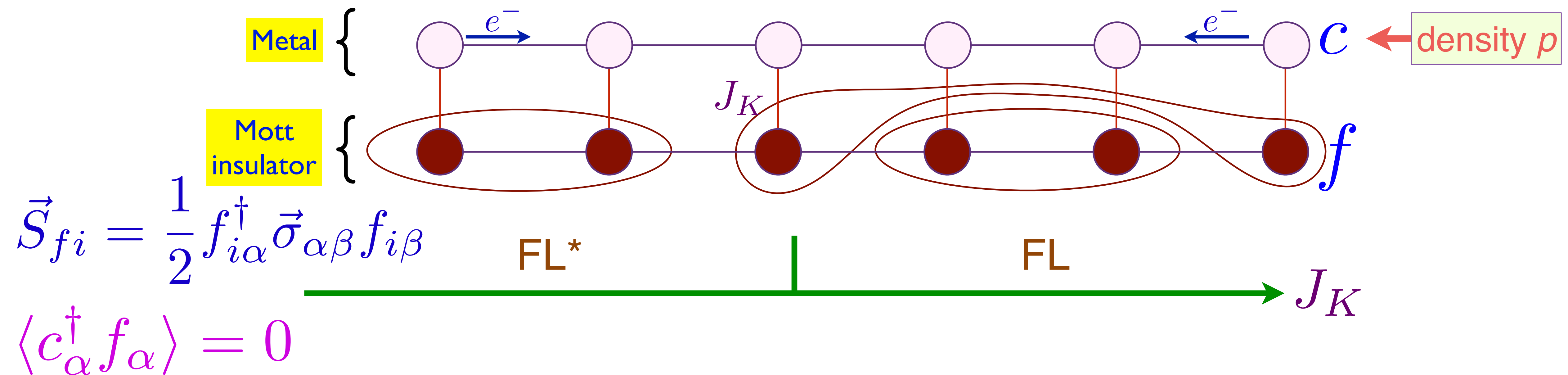


The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$ ,  
because  $\sum_i f_{i\alpha}^\dagger f_{i\alpha} = 1$  for all  $i$ .

This gauge symmetry is fully broken by a Higgs condensate  $\langle c_\alpha^\dagger f_\alpha \rangle$  in the FL phase.

# FL\* phase in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .

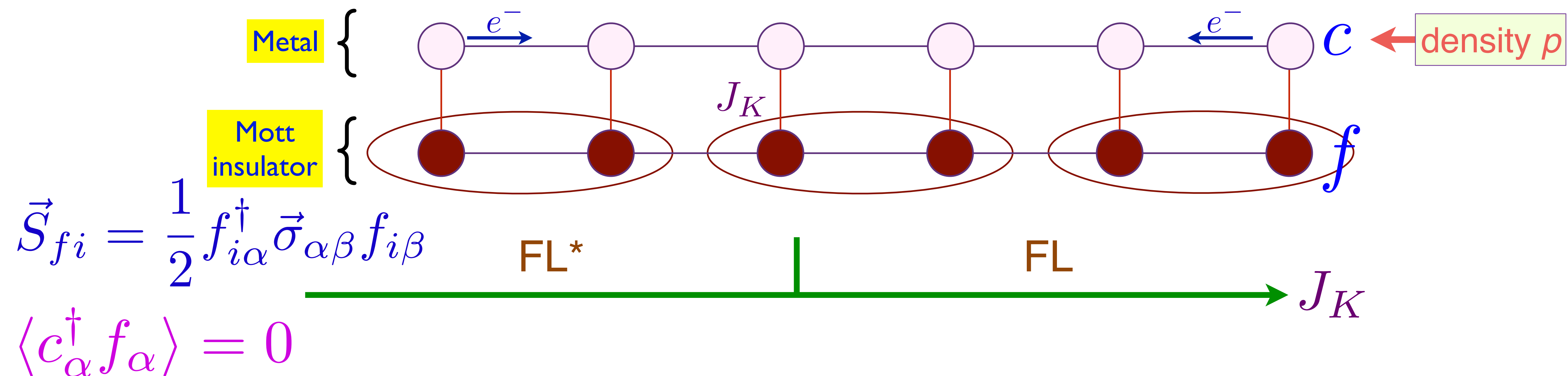


The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$ , because  $\sum_i f_{i\alpha}^\dagger f_{i\alpha} = 1$  for all  $i$ .

This gauge symmetry is unbroken in the FL\* phase, and there are gauge-charged spin excitations, and an emergent gauge field.

# FL\* phase in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .

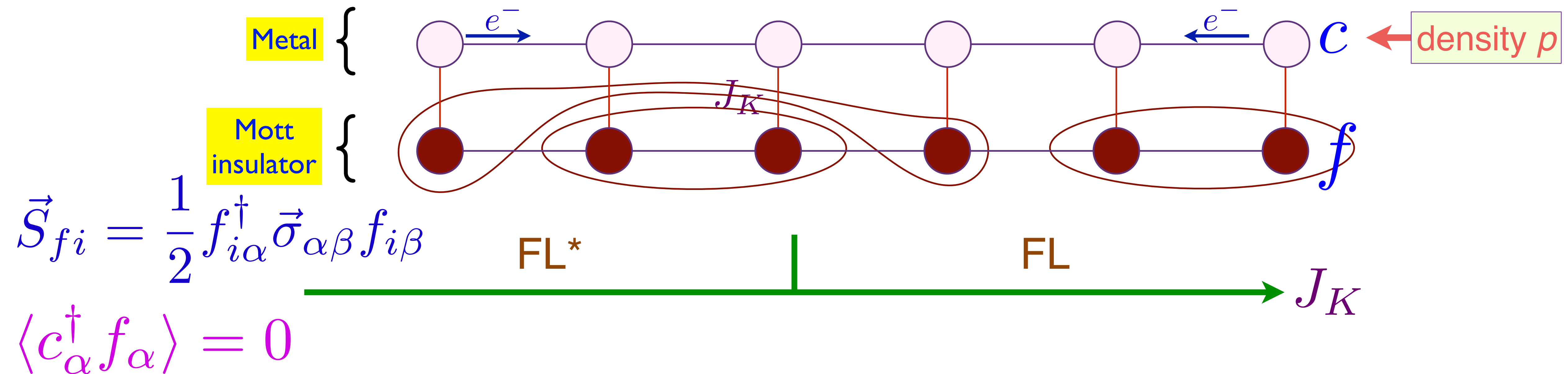


The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$ , because  $\sum_i f_{i\alpha}^\dagger f_{i\alpha} = 1$  for all  $i$ .

This gauge symmetry is unbroken in the FL\* phase, and there are gauge-charged spin excitations, and an emergent gauge field.

# FL\* phase in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



The Kondo lattice model has a gauge symmetry:  $f_{i\alpha} \rightarrow e^{i\theta_i} f_{i\alpha}$ , because  $\sum_i f_{i\alpha}^\dagger f_{i\alpha} = 1$  for all  $i$ .

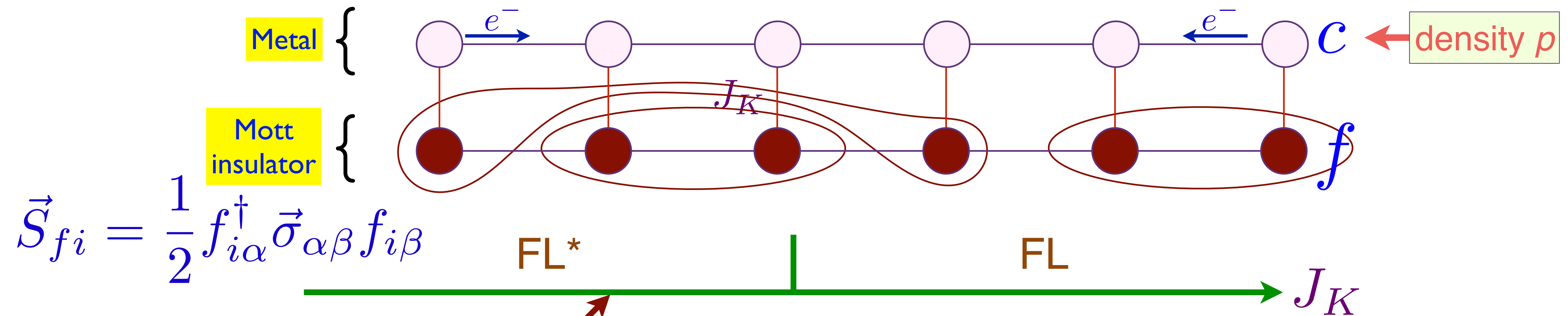
This gauge symmetry is unbroken in the  $FL^*$  phase, and there are gauge-charged spin excitations, and an emergent gauge field.

S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004); A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

# FL\* phase in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .



$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

Small Fermi surface of size  $p$

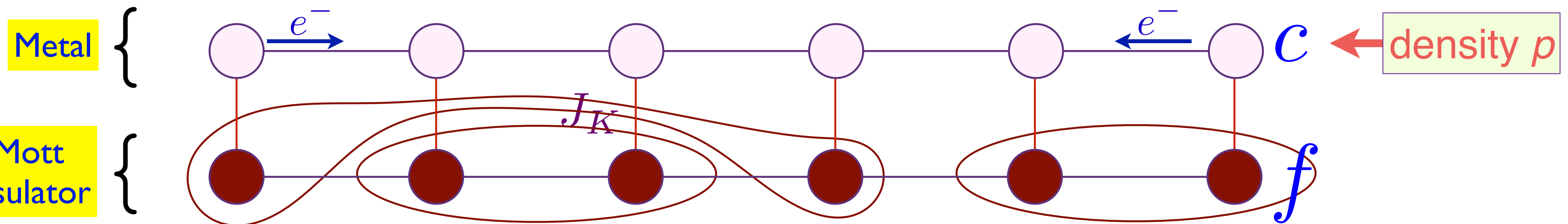
$|\Phi\rangle = [\text{Projection onto one } f \text{ per site}]$   
 $\otimes |\text{Slater determinant of } f\rangle$   
 $\otimes |\text{Slater determinant of } c\rangle$

# FL\* phase in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .

V.I. Anisimov, I.A. Nekrasov,  
D.E. Kondakov, T.M. Rice & M. Sigrist,  
EPJB **25**, 191 (2002)  
L. de' Medici, A. Georges, S. Biermann,  
PRB **72**, 205124 (2005)

Kondo-breakdown or 'selective Mott' transition



$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

FL\*

FL

$J_K$

Small Fermi surface of size  $p$

$|\Phi\rangle = [\text{Projection onto one } f \text{ per site}]$   
 $\boxtimes |\text{Slater determinant of } f\rangle$   
 $\otimes |\text{Slater determinant of } c\rangle$

S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)  
 T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)  
 A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

# Metal-metal transitions in **Kondo lattice** models

Kondo lattice of  $f$  electron spins coupled to a conduction band of  $c$  electrons of density  $p$ .

Kondo-breakdown or ‘selective Mott’ transition

U(1) gauge theory of a ‘hybridization-Higgs’ boson  $b \sim f_{\alpha}^{\dagger} c_{\alpha}$  which condenses on the ‘Large Fermi surface’ side.

$$\vec{S}_{fi} = \frac{1}{2} f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i\beta}$$

FL\*

FL

$J_K$

Small Fermi surface of size  $p$

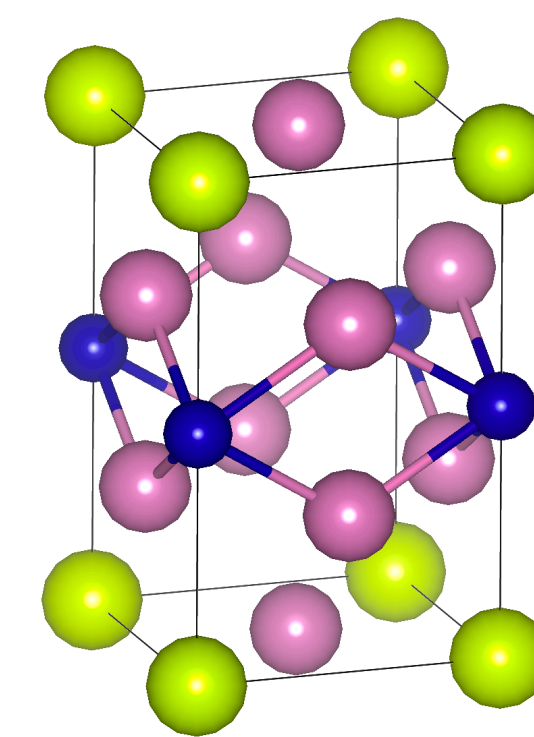
Deconfined gauge fields and fractionalized spin excitations

Large Fermi surface of size  $1 + p$

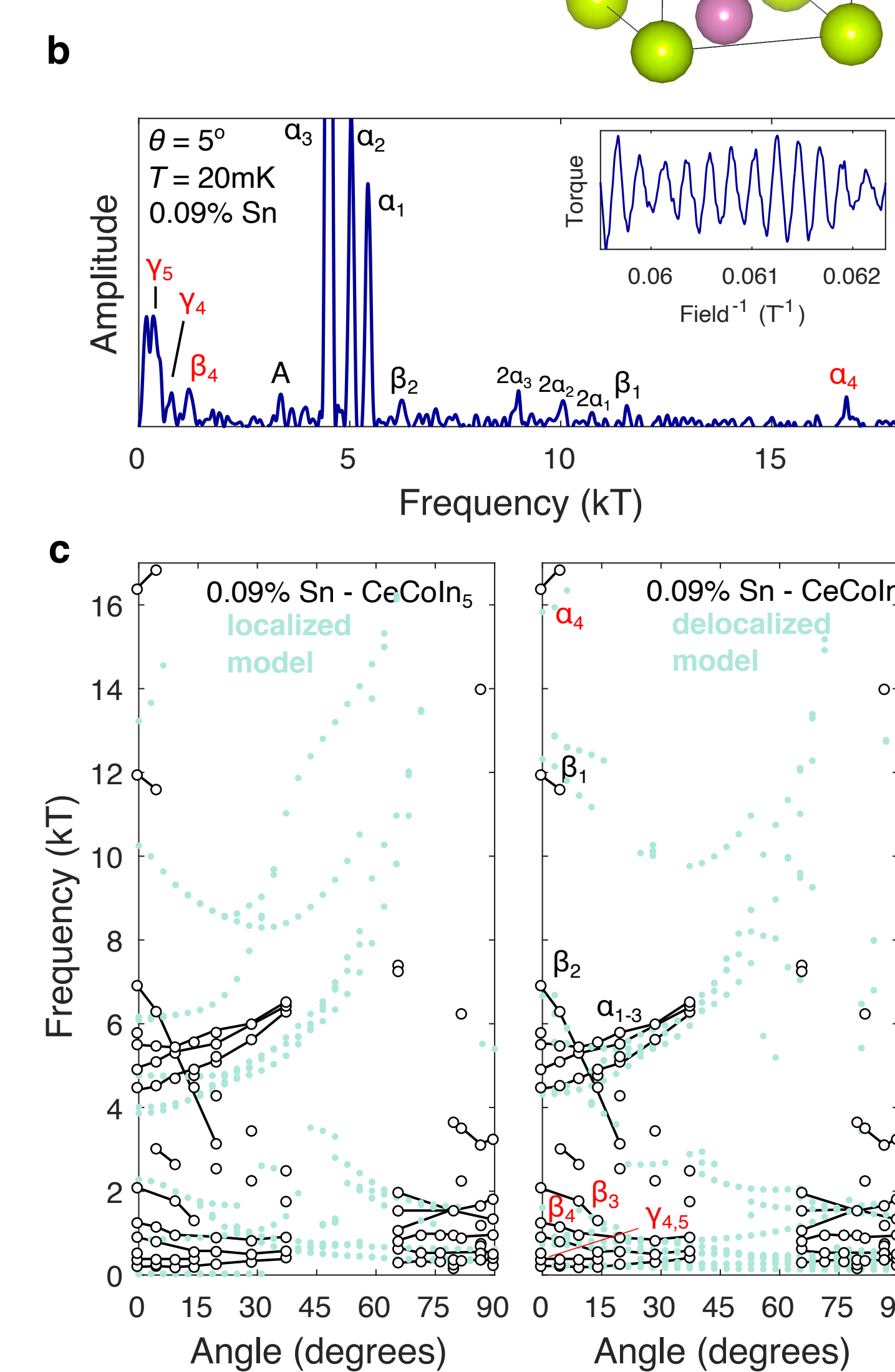
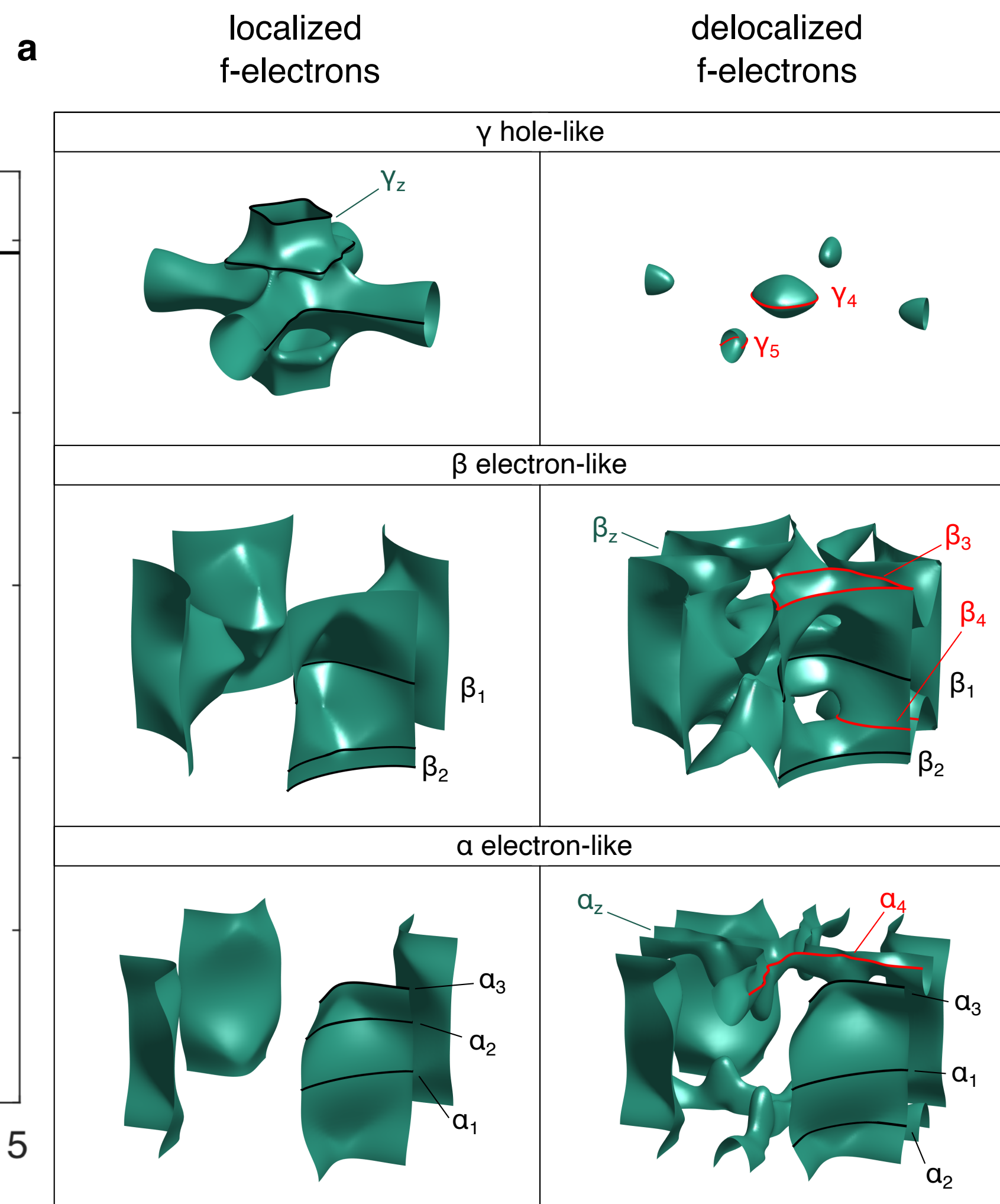
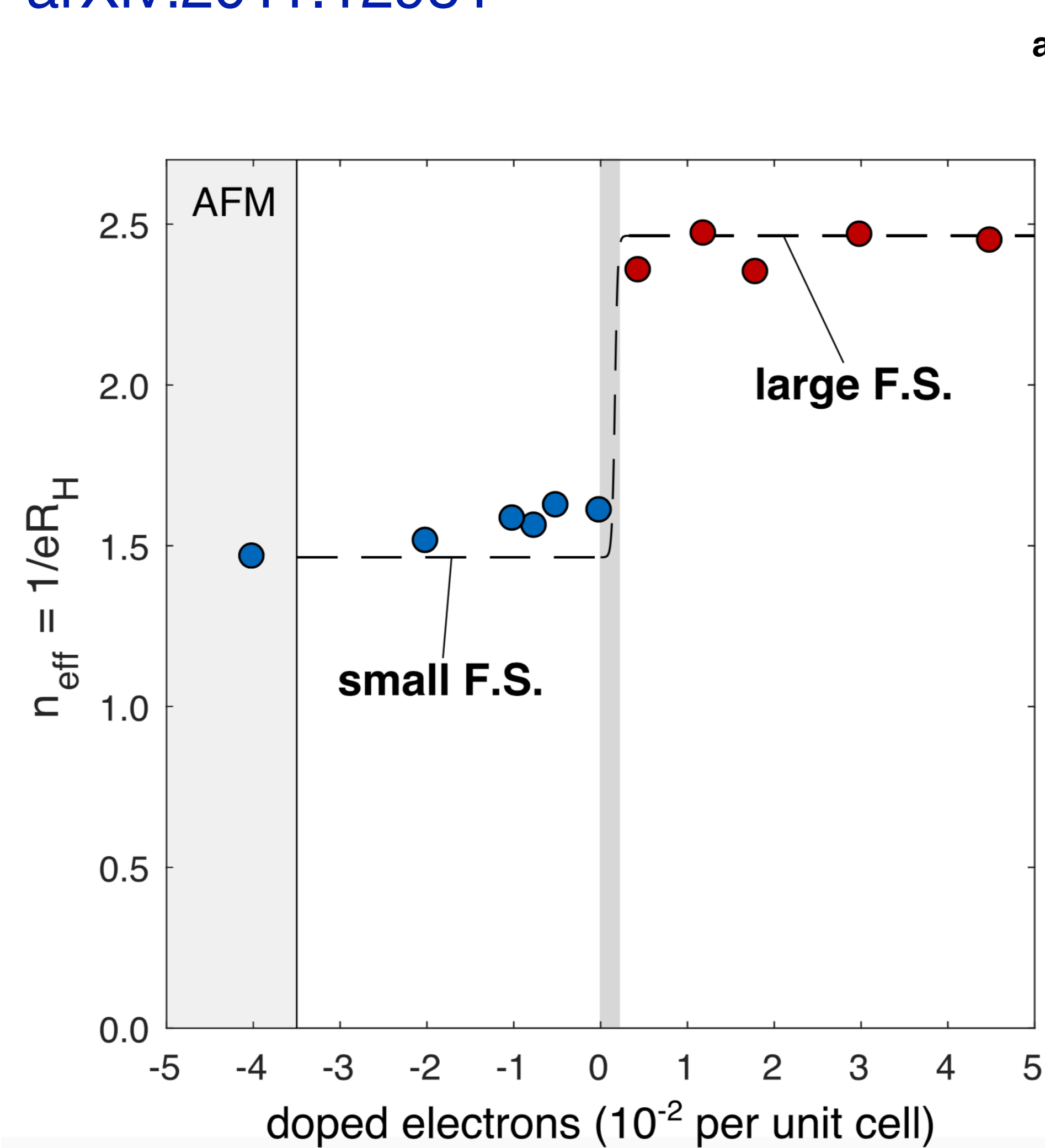
All gauge symmetry is Higgsed.  
No fractionalization: excitations are electron-like or composites.

# Evidence for freezing of charge degrees of freedom across a critical point in $\text{CeCoIn}_5$

Nikola Maksimovic, Taylor Cookmeyer, Jan Ruzs, Vikram Nagarajan, Amanda Gong, Fanghui Wan, Stefano Faubel, Ian M. Hayes, Sooyoung Jang, Yochai Werman, Peter M. Oppeneer, Ehud Altman, James G. Analytis

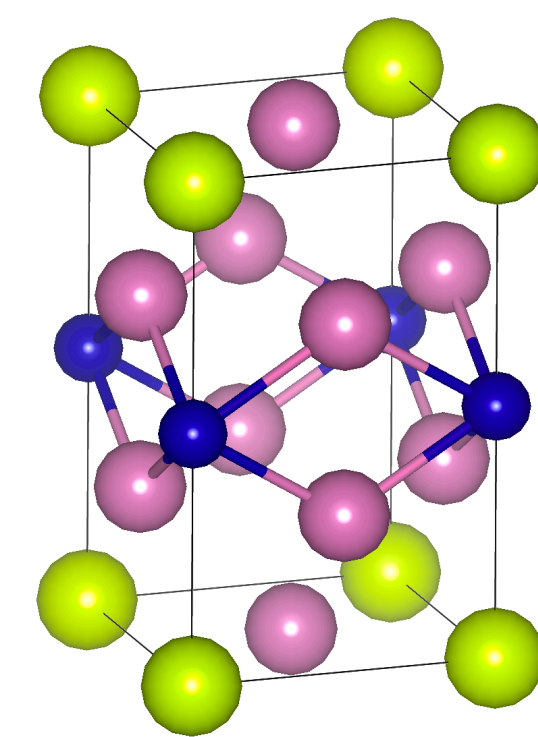


arXiv:2011.12951

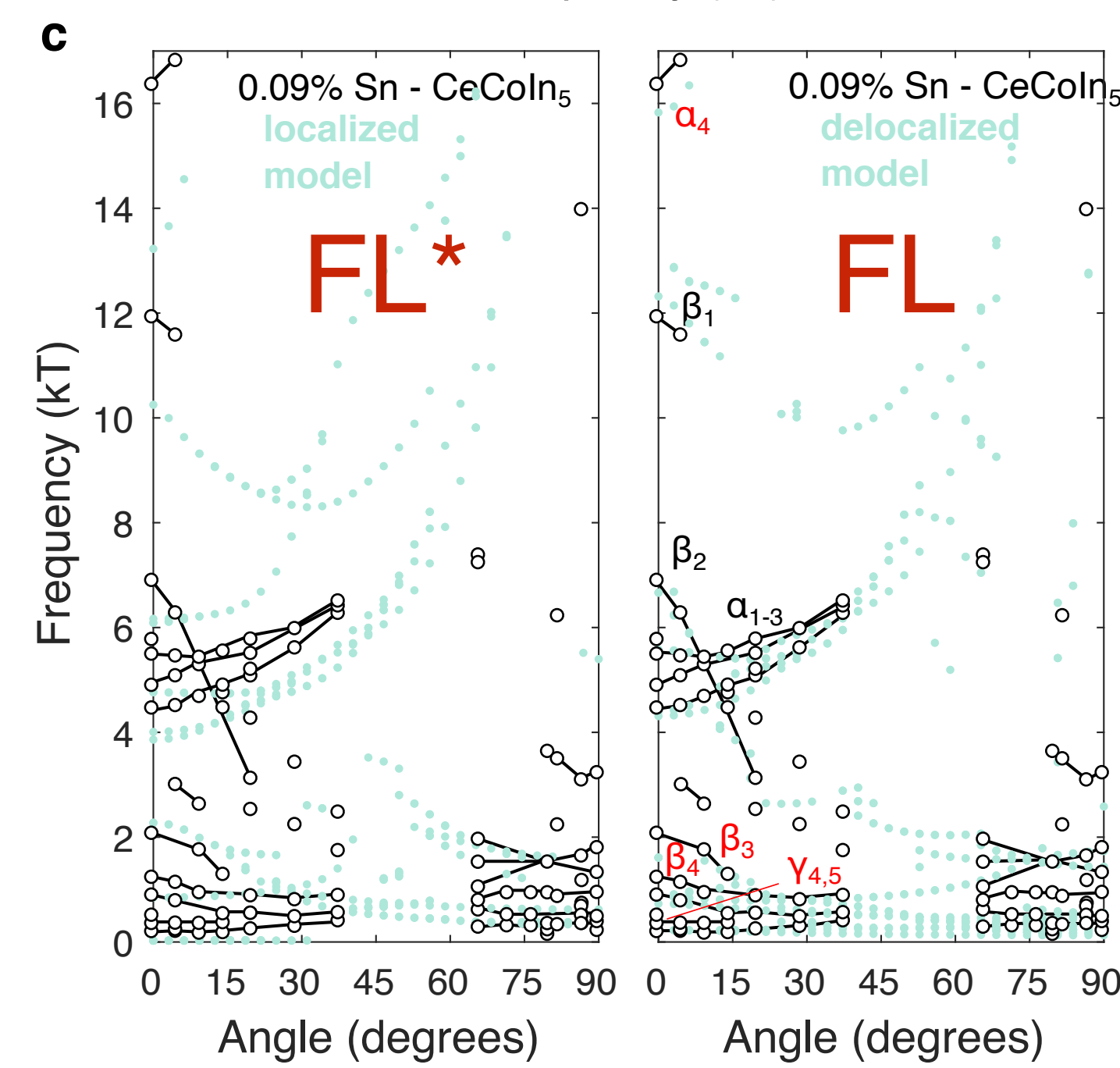
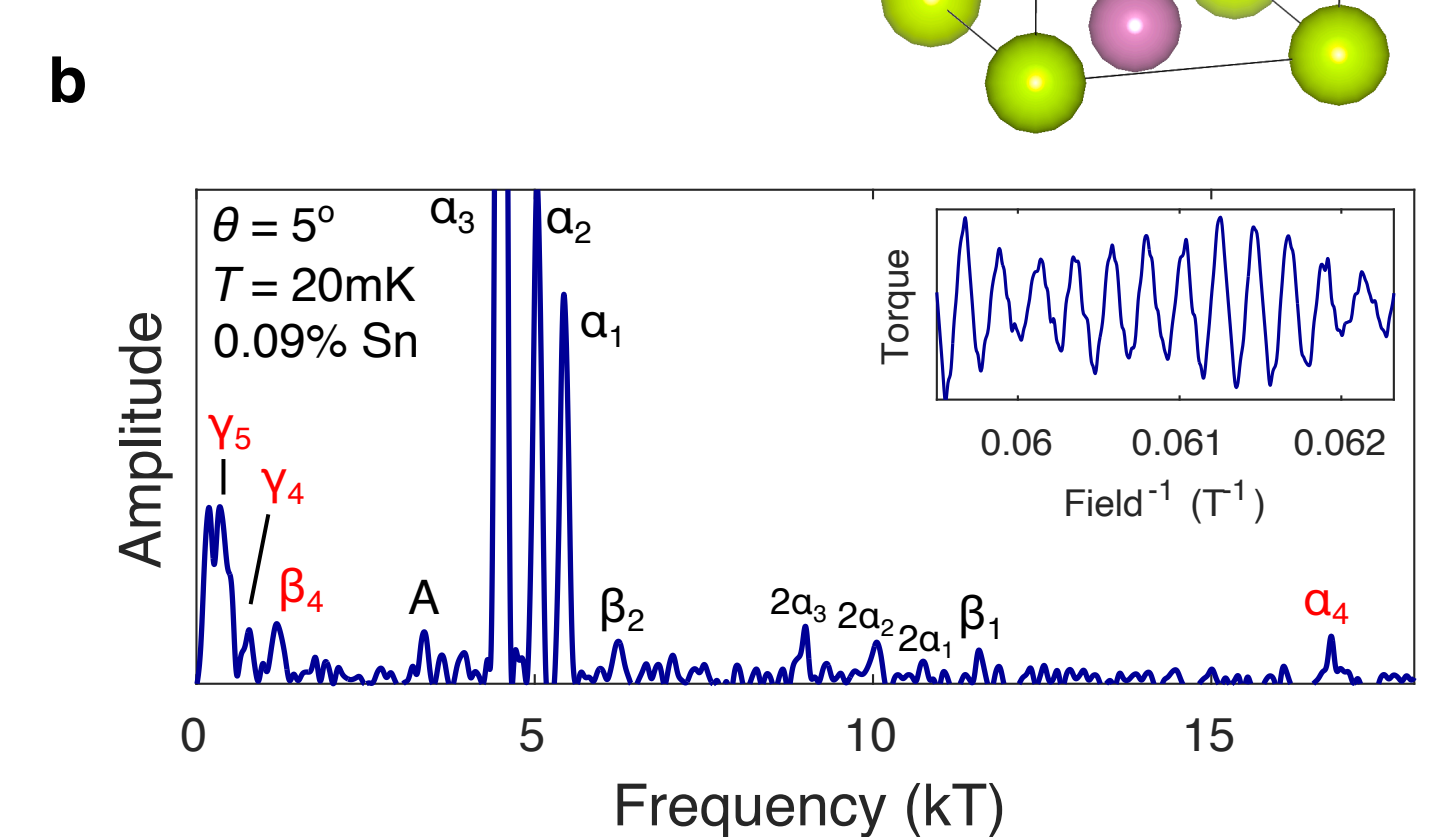
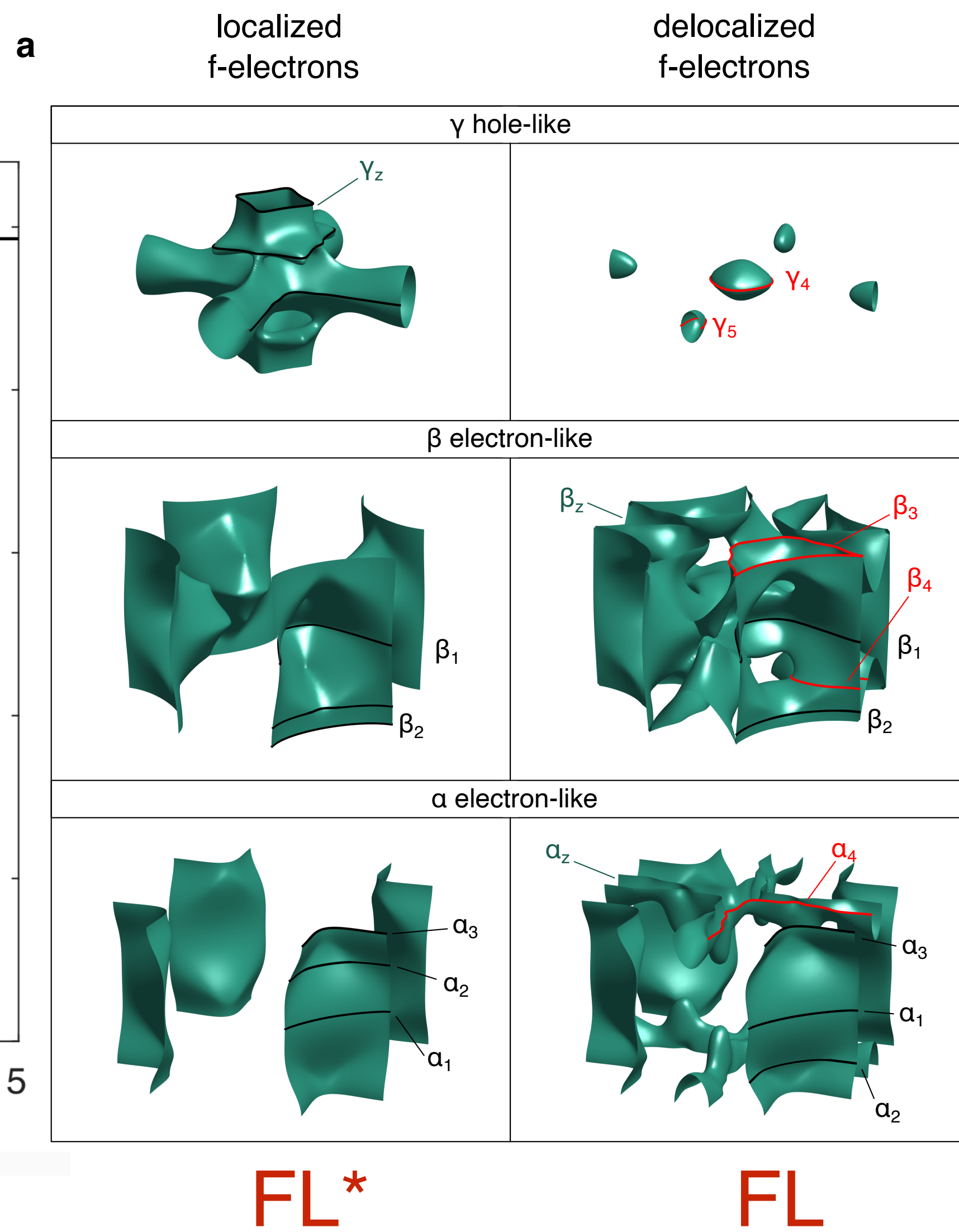
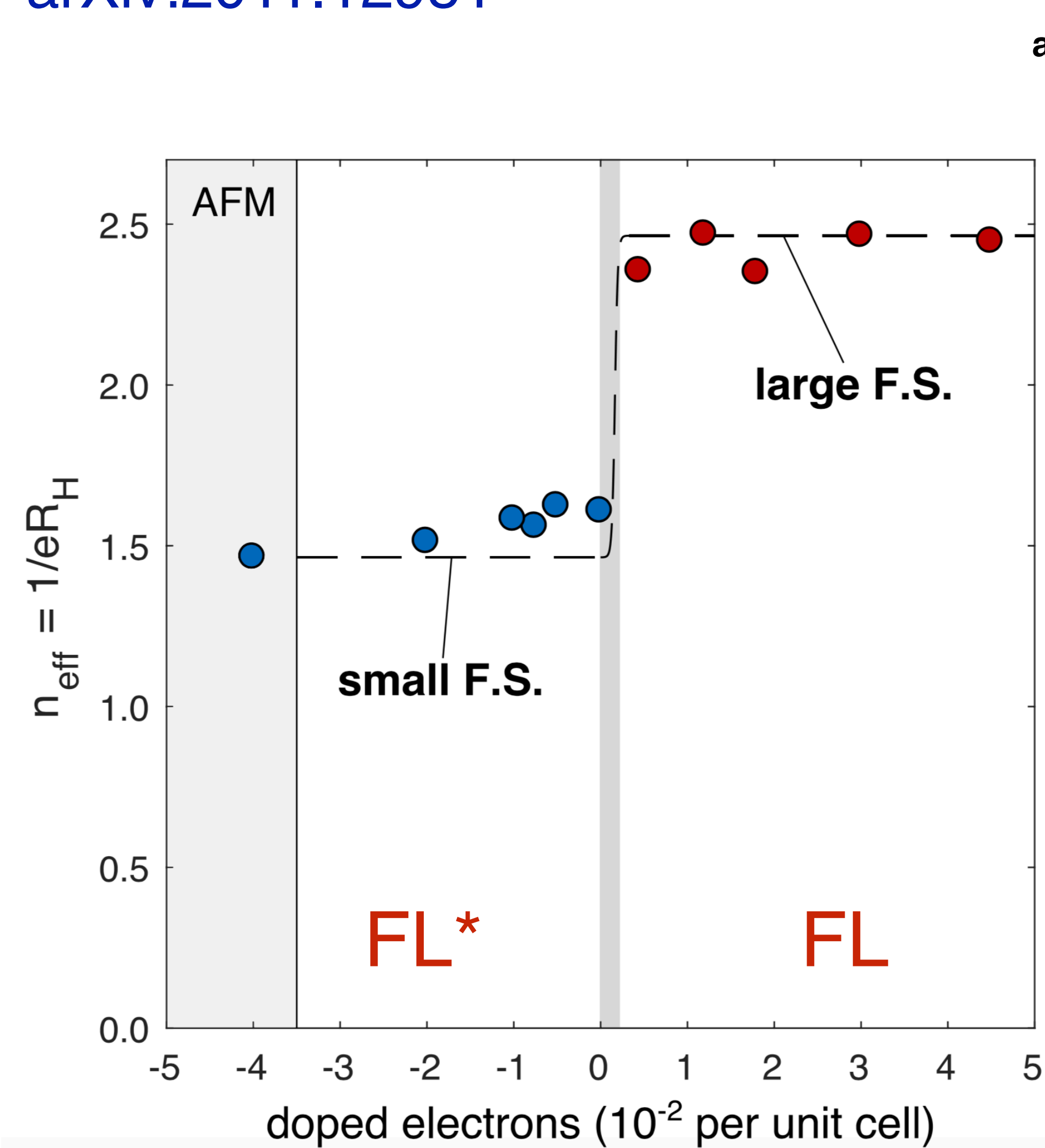


# Evidence for freezing of charge degrees of freedom across a critical point in $\text{CeCoIn}_5$

Nikola Maksimovic, Taylor Cookmeyer, Jan Ruzs, Vikram Nagarajan, Amanda Gong, Fanghui Wan, Stefano Faubel, Ian M. Hayes, Sooyoung Jang, Yochai Werman, Peter M. Oppeneer, Ehud Altman, James G. Analytis



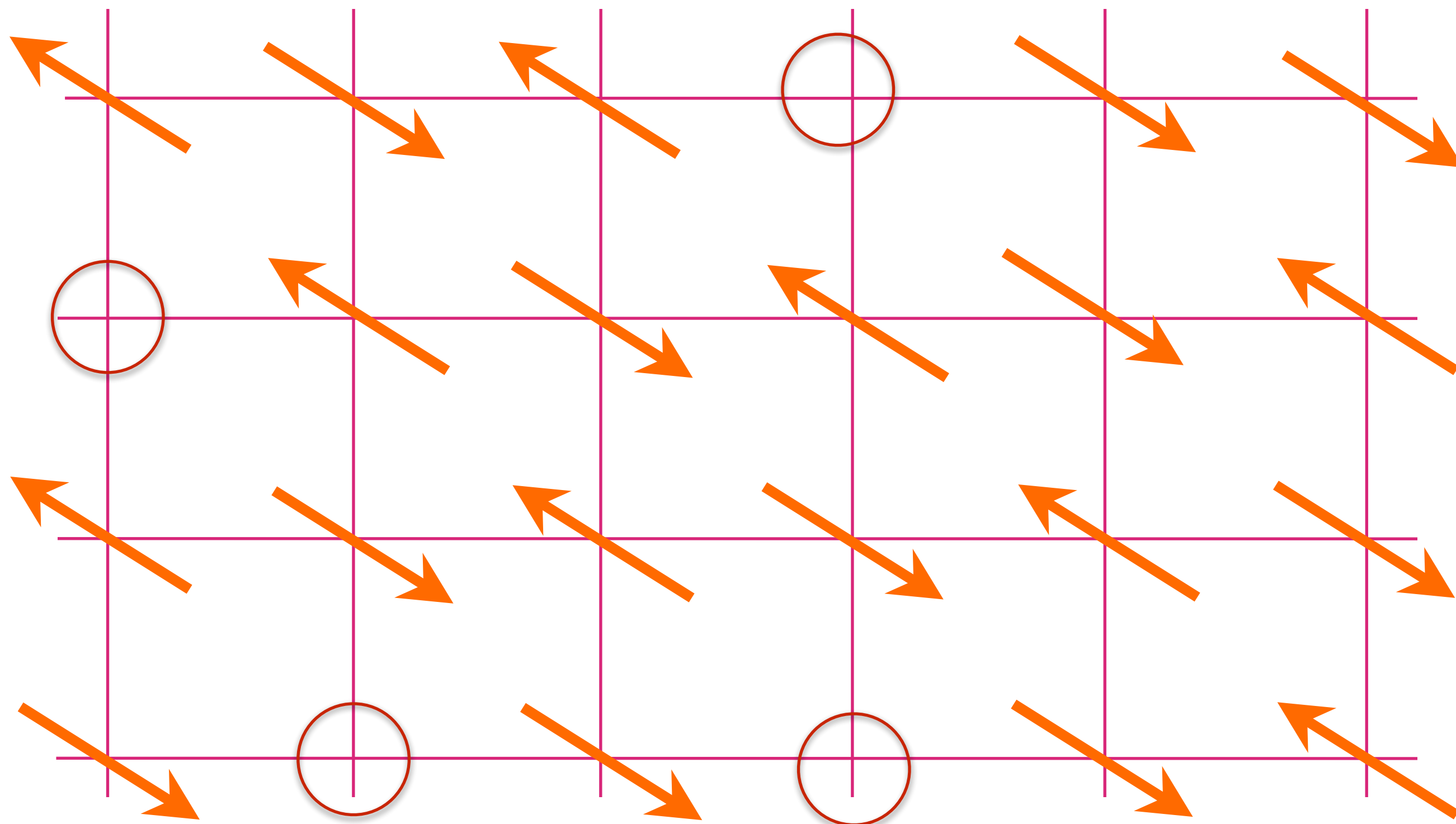
arXiv:2011.12951



1. Luttinger volume violation in  
Kondo lattice models  
*The FL\* phase and CeCoIn<sub>5</sub>*

2. Luttinger volume violation a  $t$ - $J$  model  
*Ancilla qubits and a small to  
large Fermi surface transition*

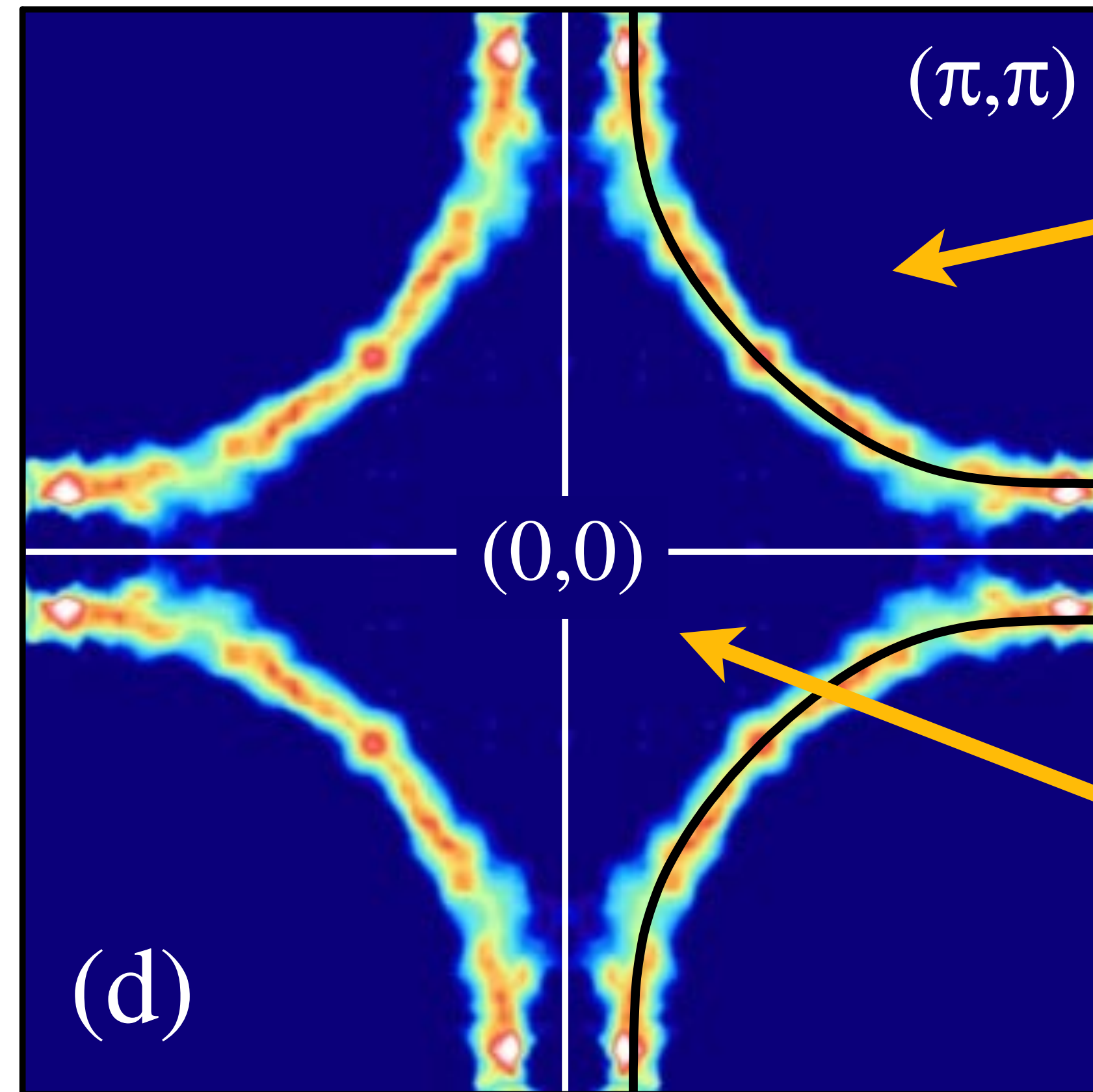
- What happens if the repulsive interaction between electrons becomes essentially infinite on all or some sites ?
- In the cuprates, all Cu sites prohibit more than one electron: the  $t$ - $J$  model.
- Luttinger's theorem states the Fermi surface should have a large size  $1-p$  electrons (equivalent to  $1+p$  holes).
- Or should there be a small Fermi surface of size  $p$  holes ?



Antiferromagnet  
doped with hole  
density  $p$

$p$  mobile holes in a  
background of  
fluctuating spins

# Momentum-space view at large $p$



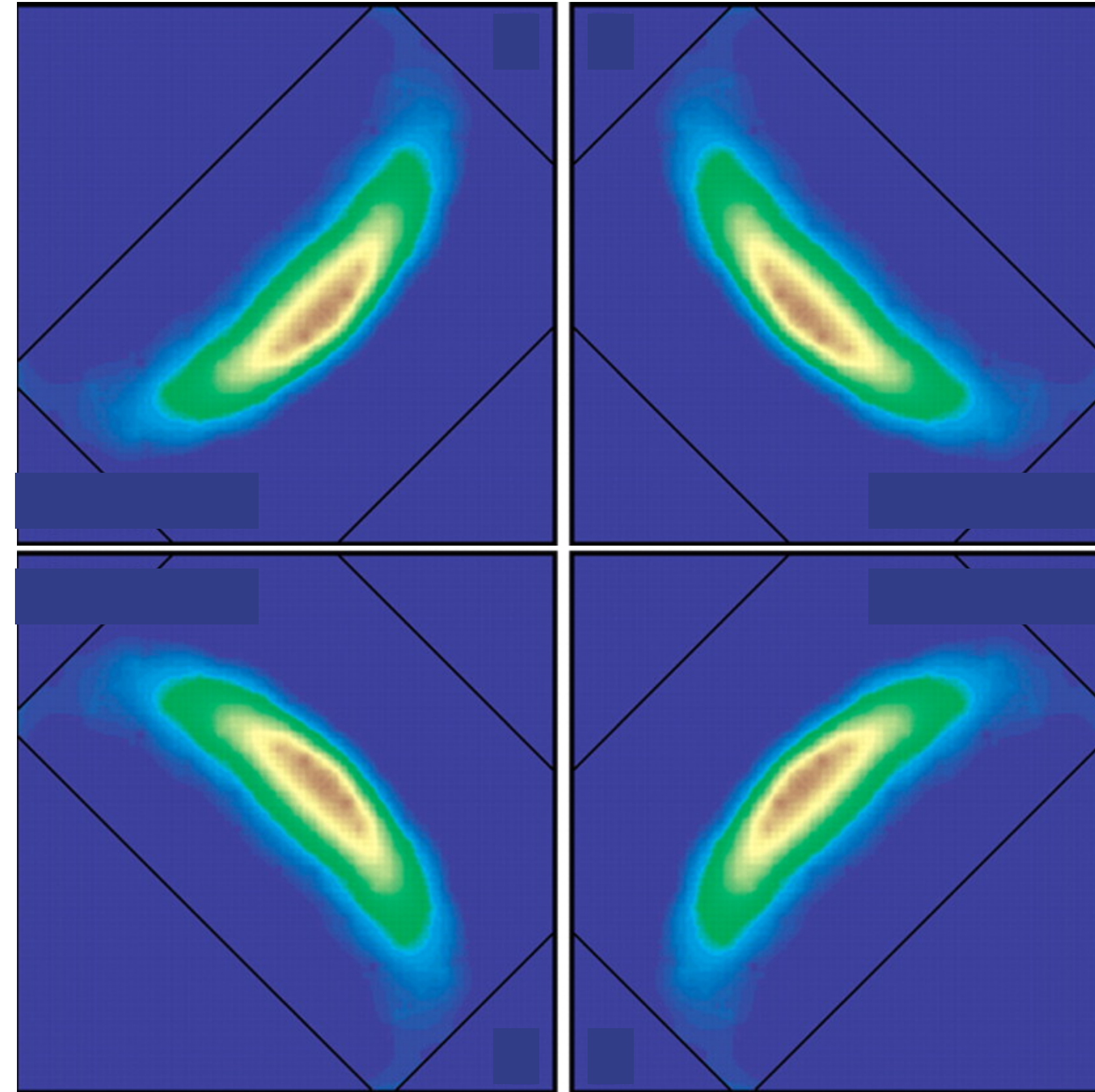
$l+p$  holes

Overdoped  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$   
 $T_c = 30\text{K}$

$l-p$  electrons

$l+p$  mobile holes in a filled band

# Momentum-space view at small $p$



$\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$   
at  $x = 0.10$

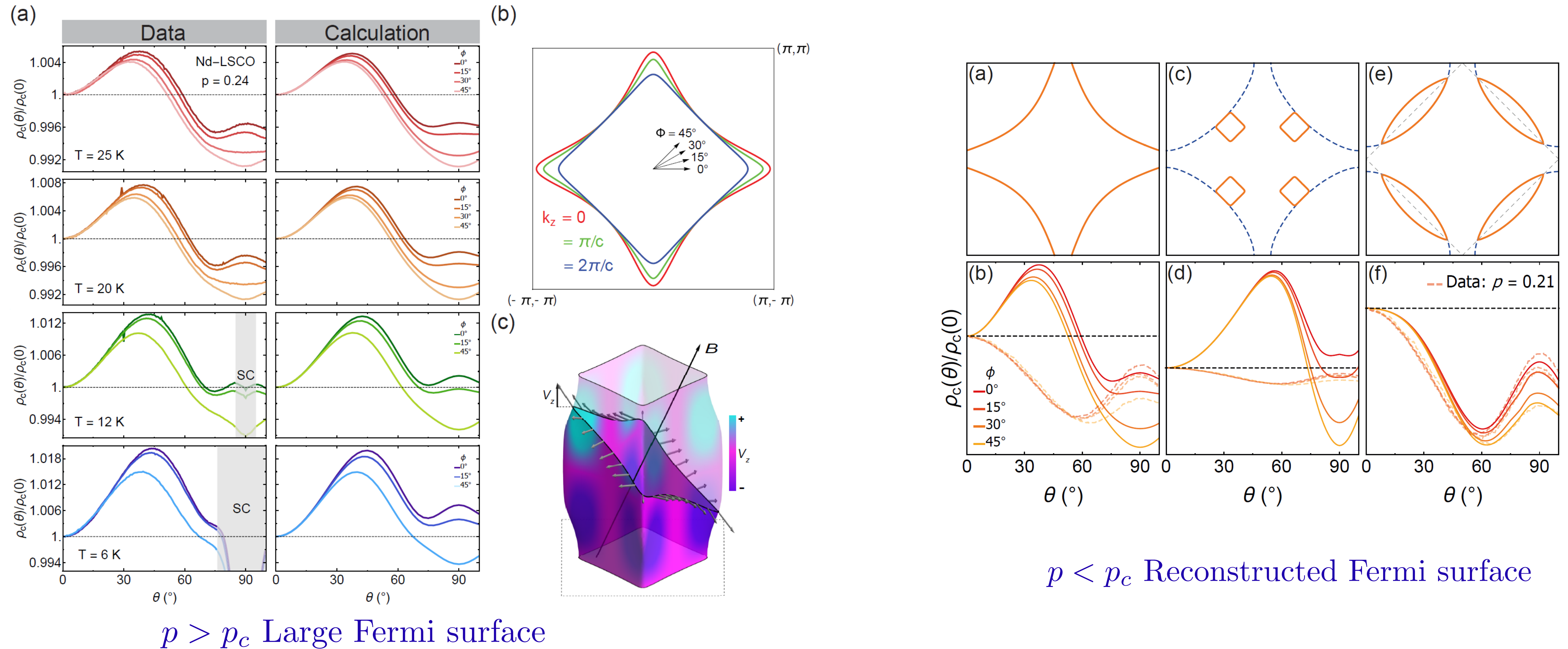
*“Fermi arcs”*

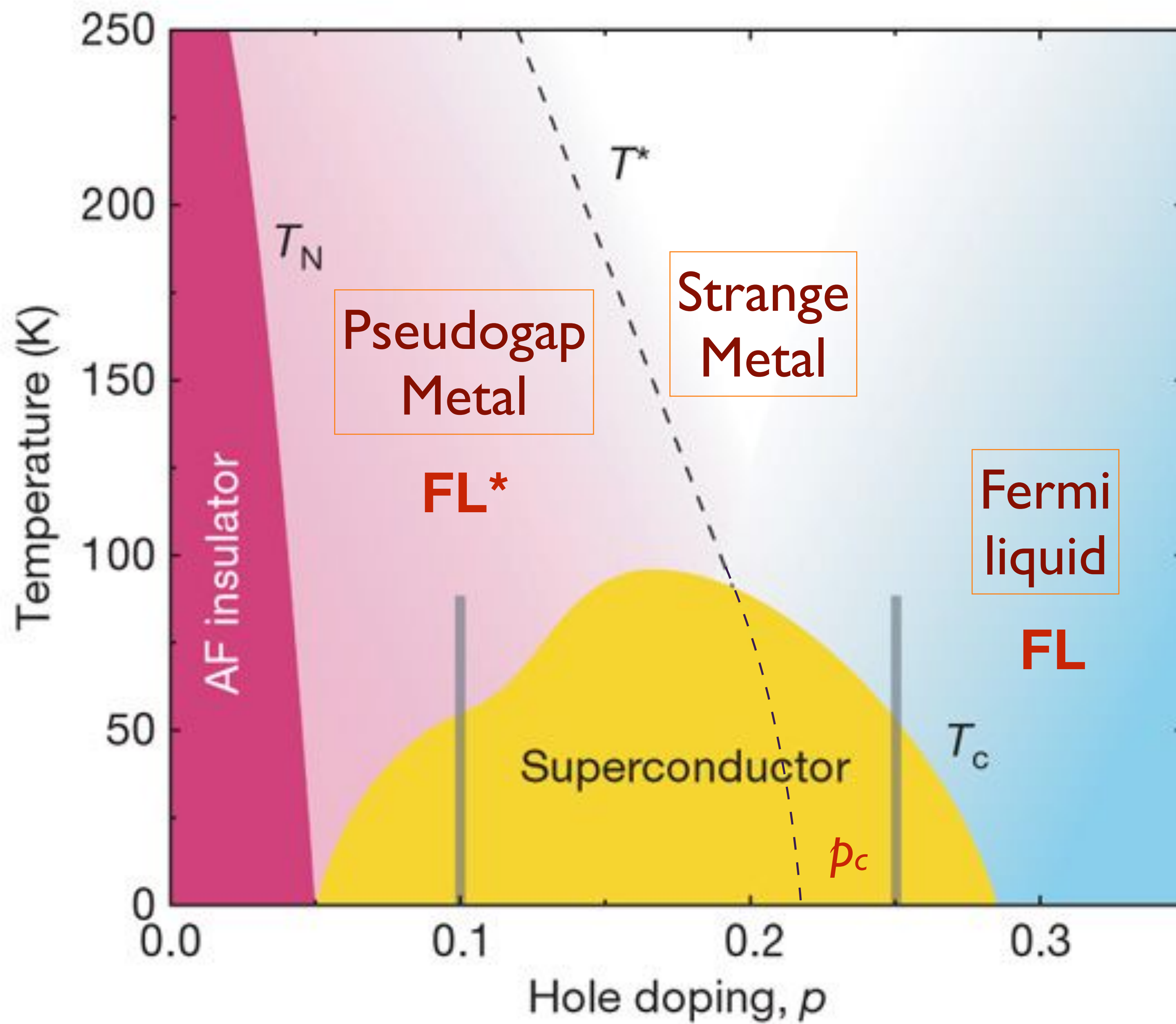
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, *Science* **307**, 901 (2005)

# Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

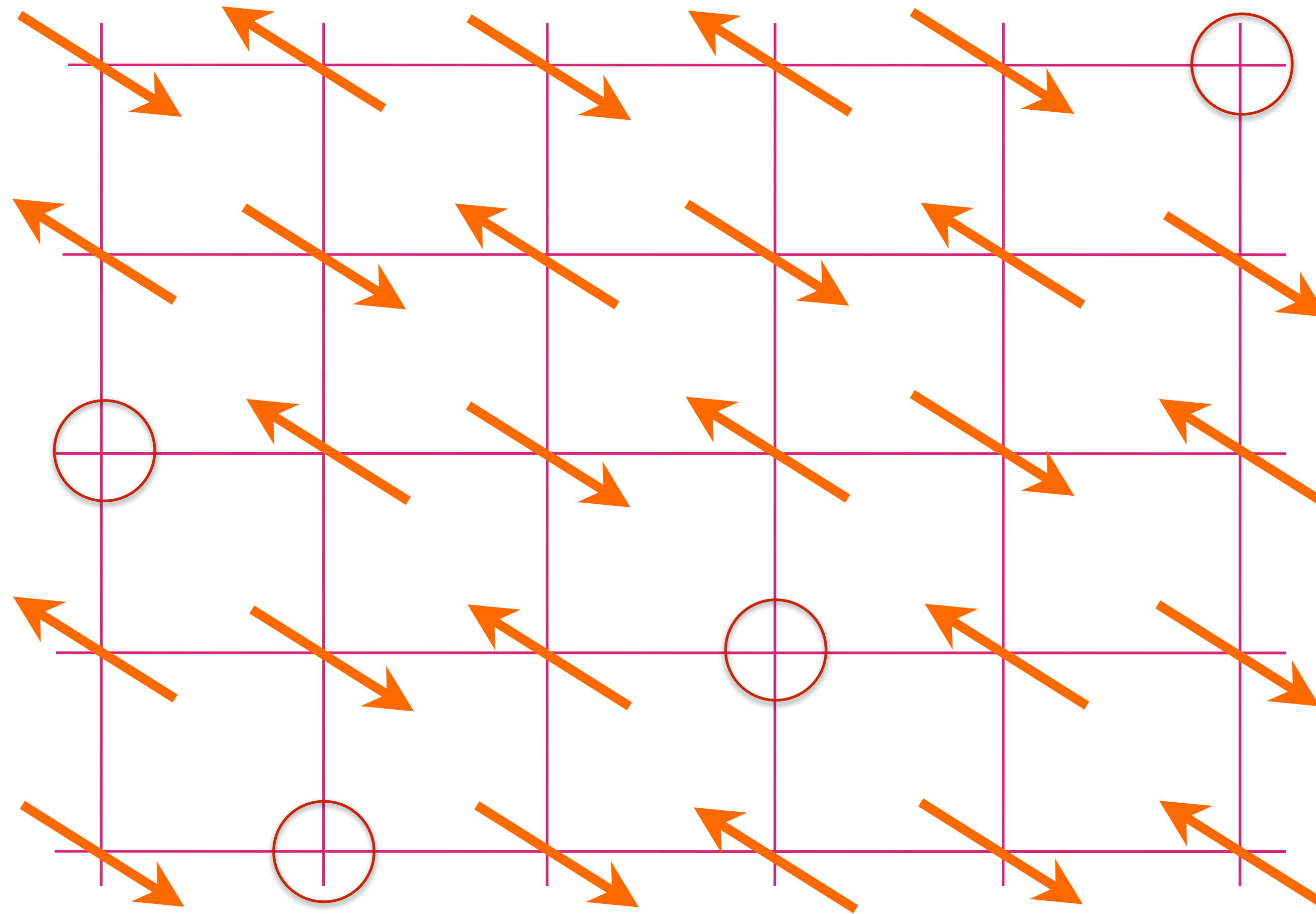
Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate  $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$ . Above the critical doping  $p^*$  — outside of the pseudogap phase — we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below  $p^*$ , however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a  $Q = (\pi, \pi)$  wavevector. While static  $Q = (\pi, \pi)$  antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.





# Earlier approach to FL\* in a **one-band** model

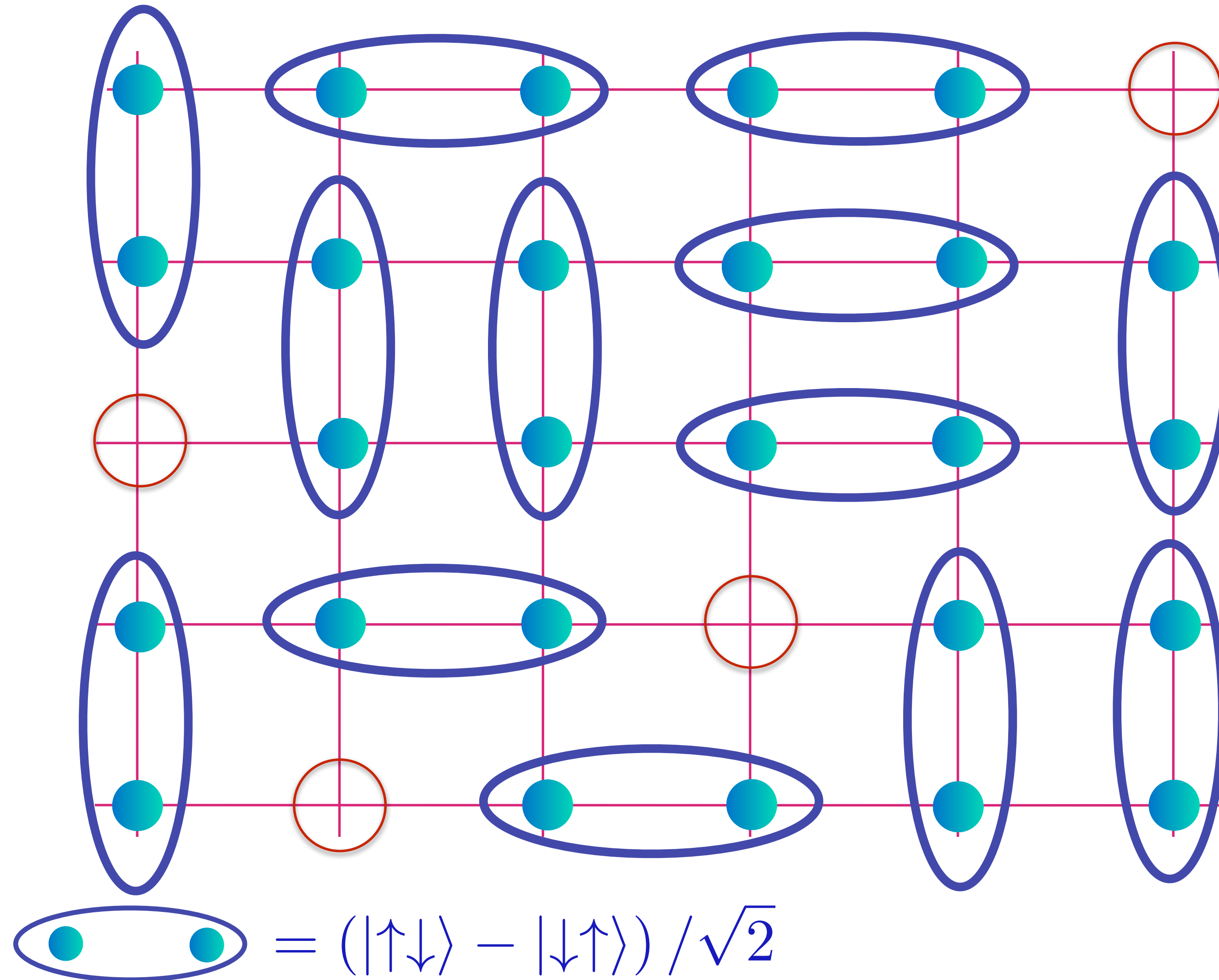


Anti-ferromagnet with  $p$  holes per square

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

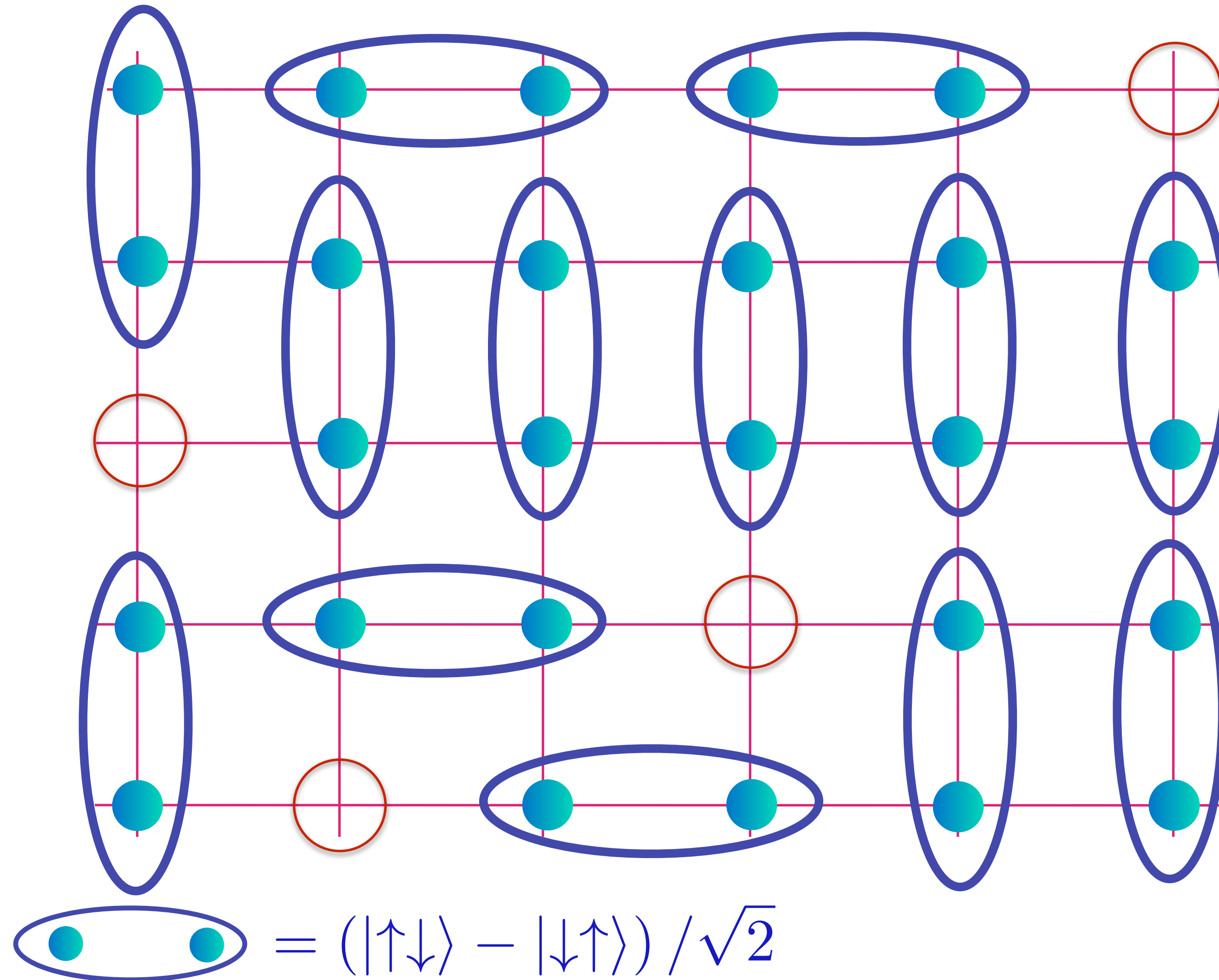


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

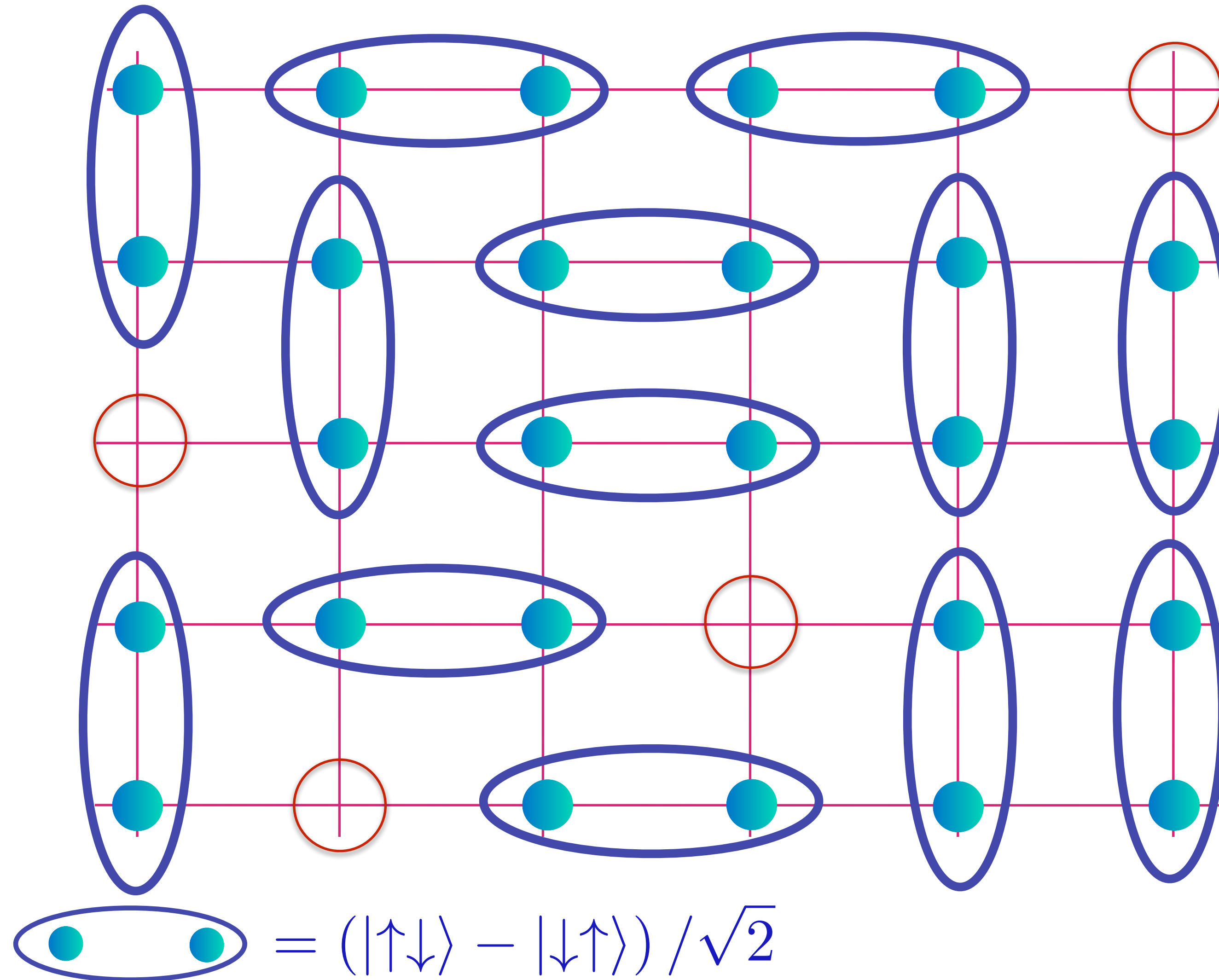


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

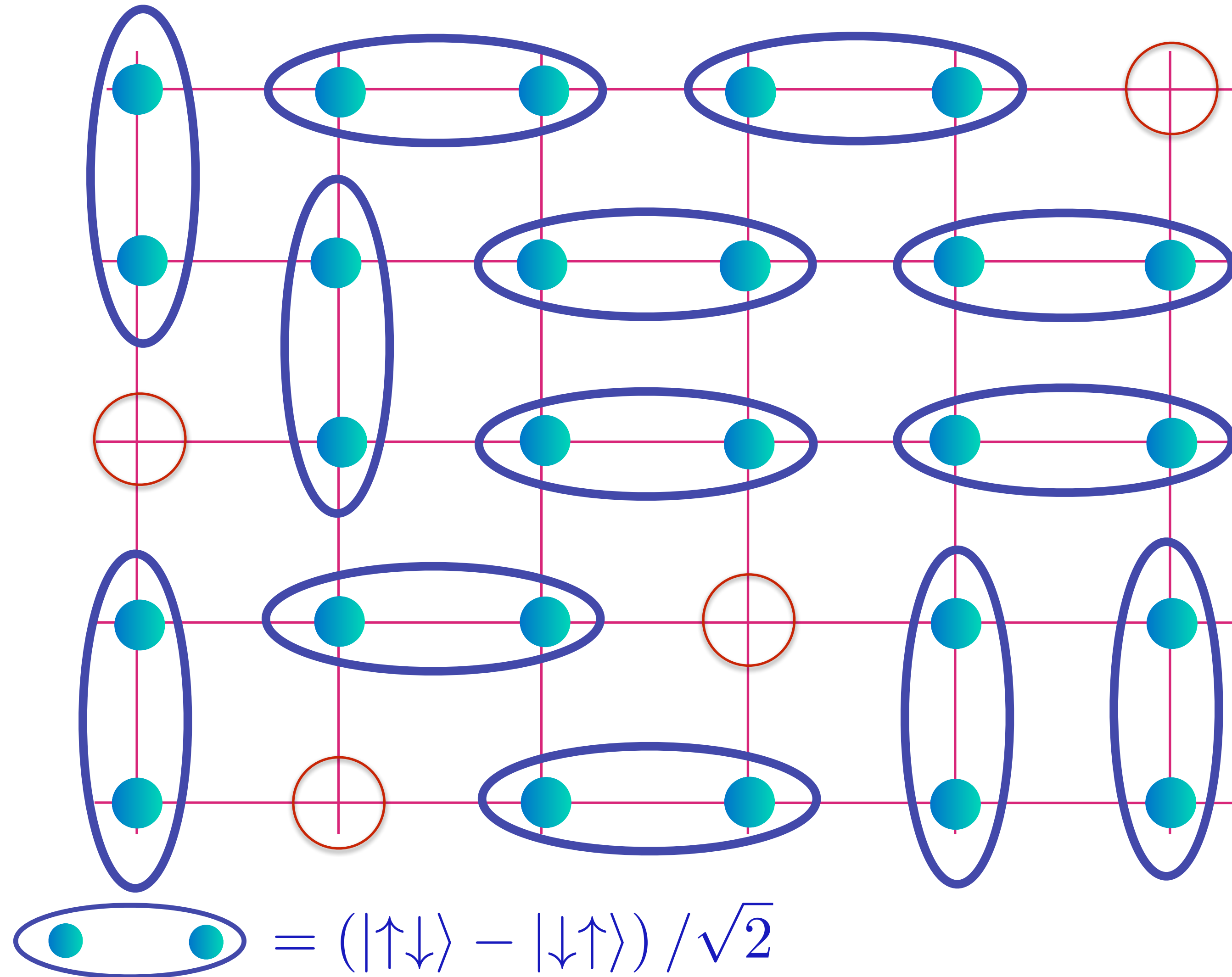


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

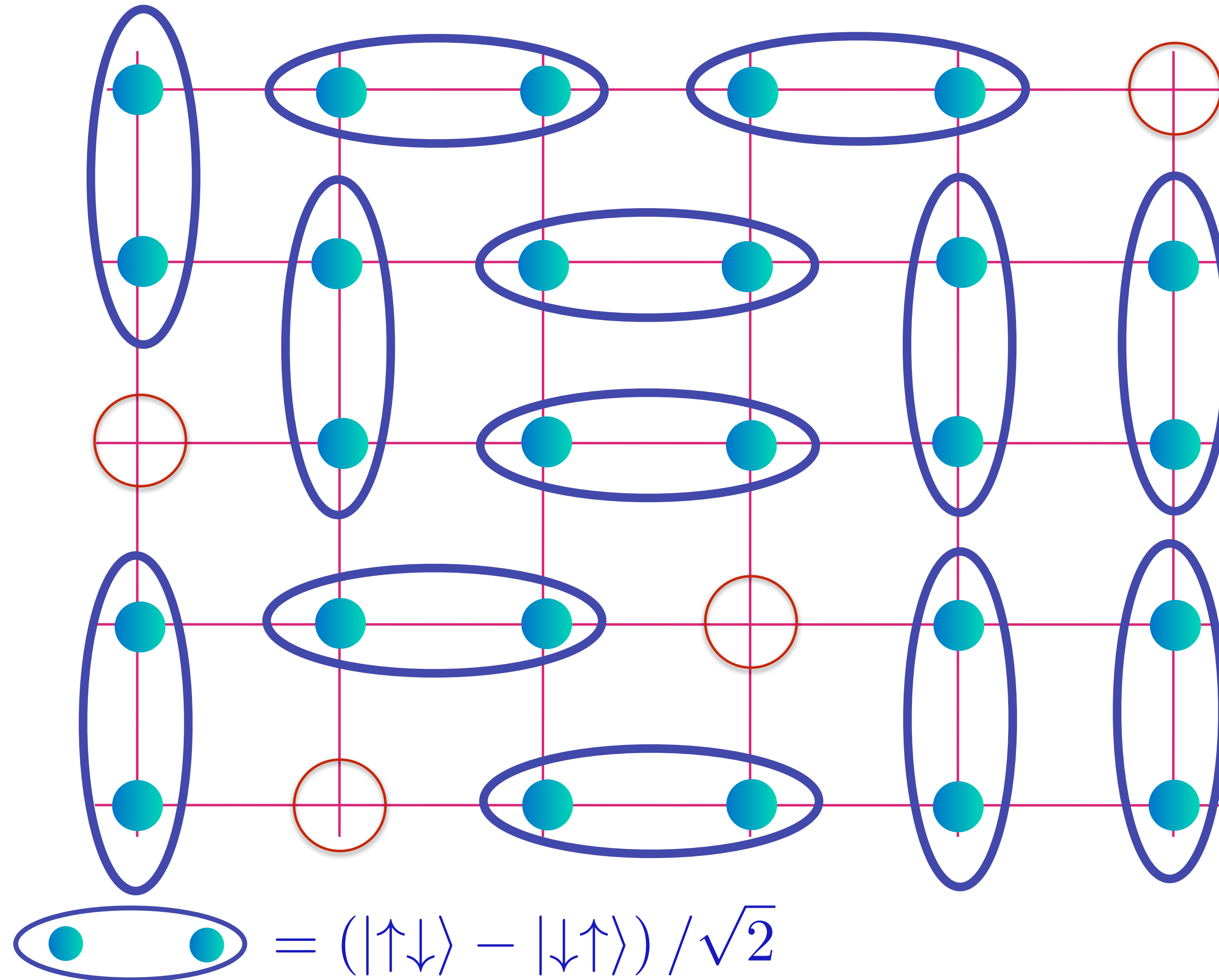


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

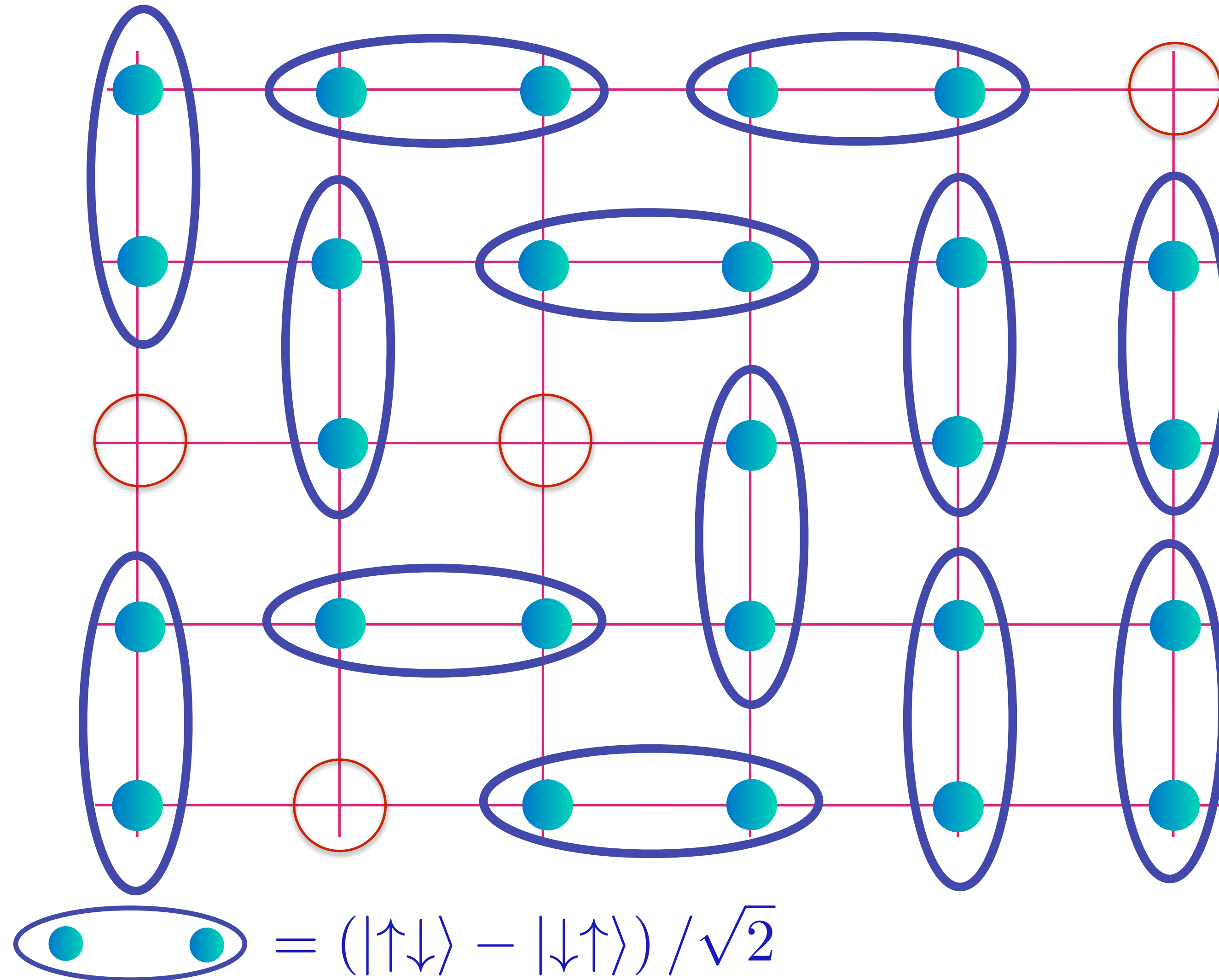


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

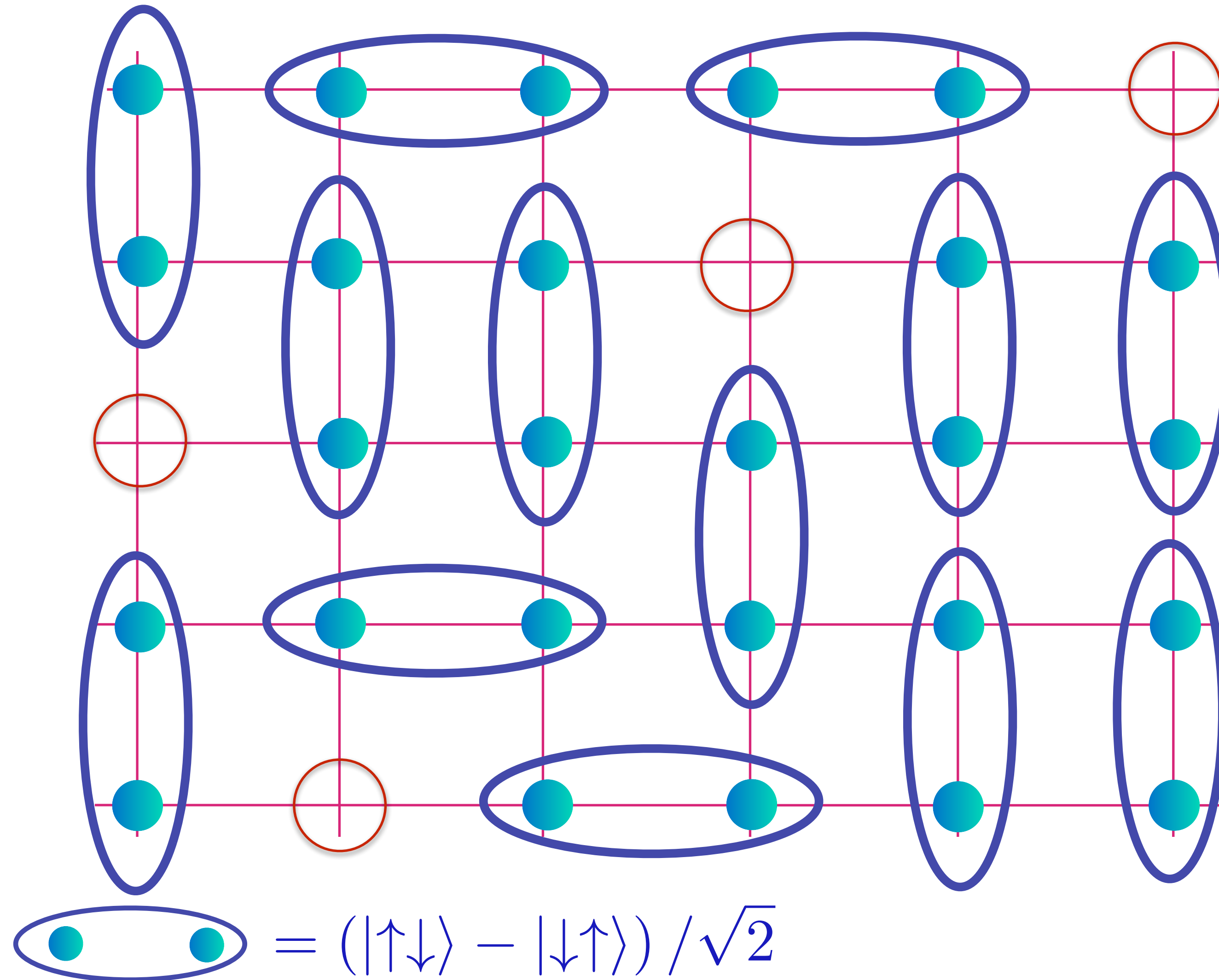


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

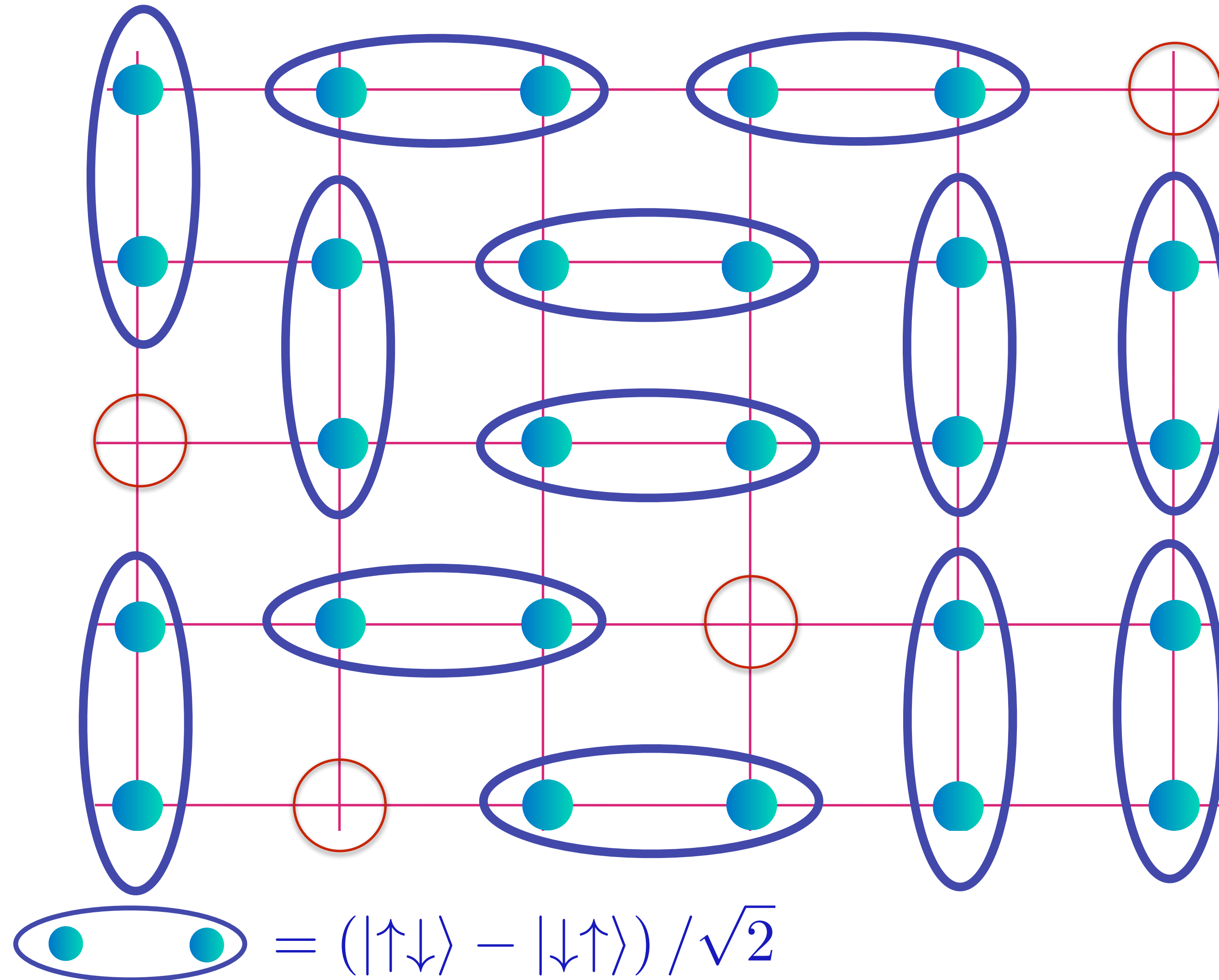


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

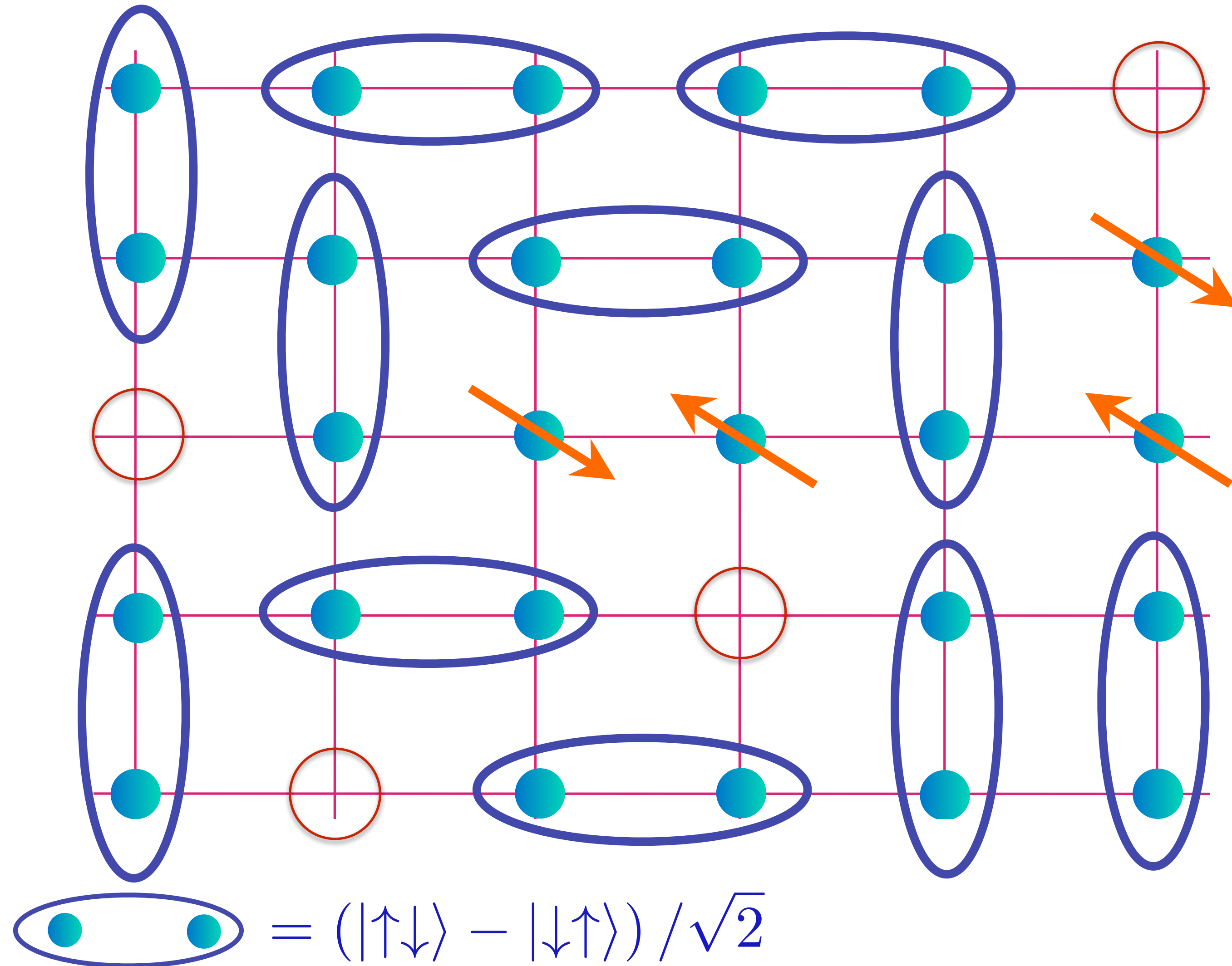


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

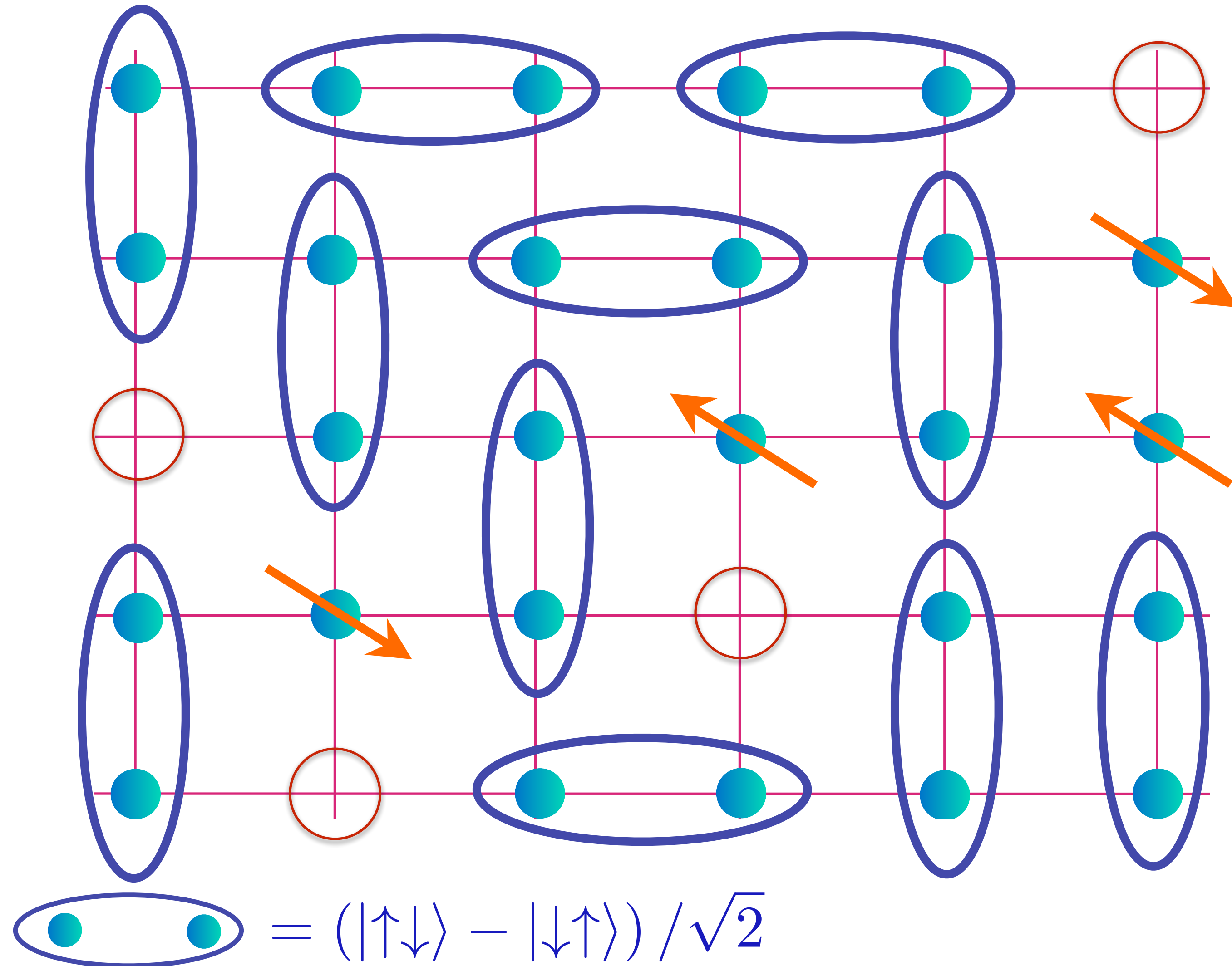


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

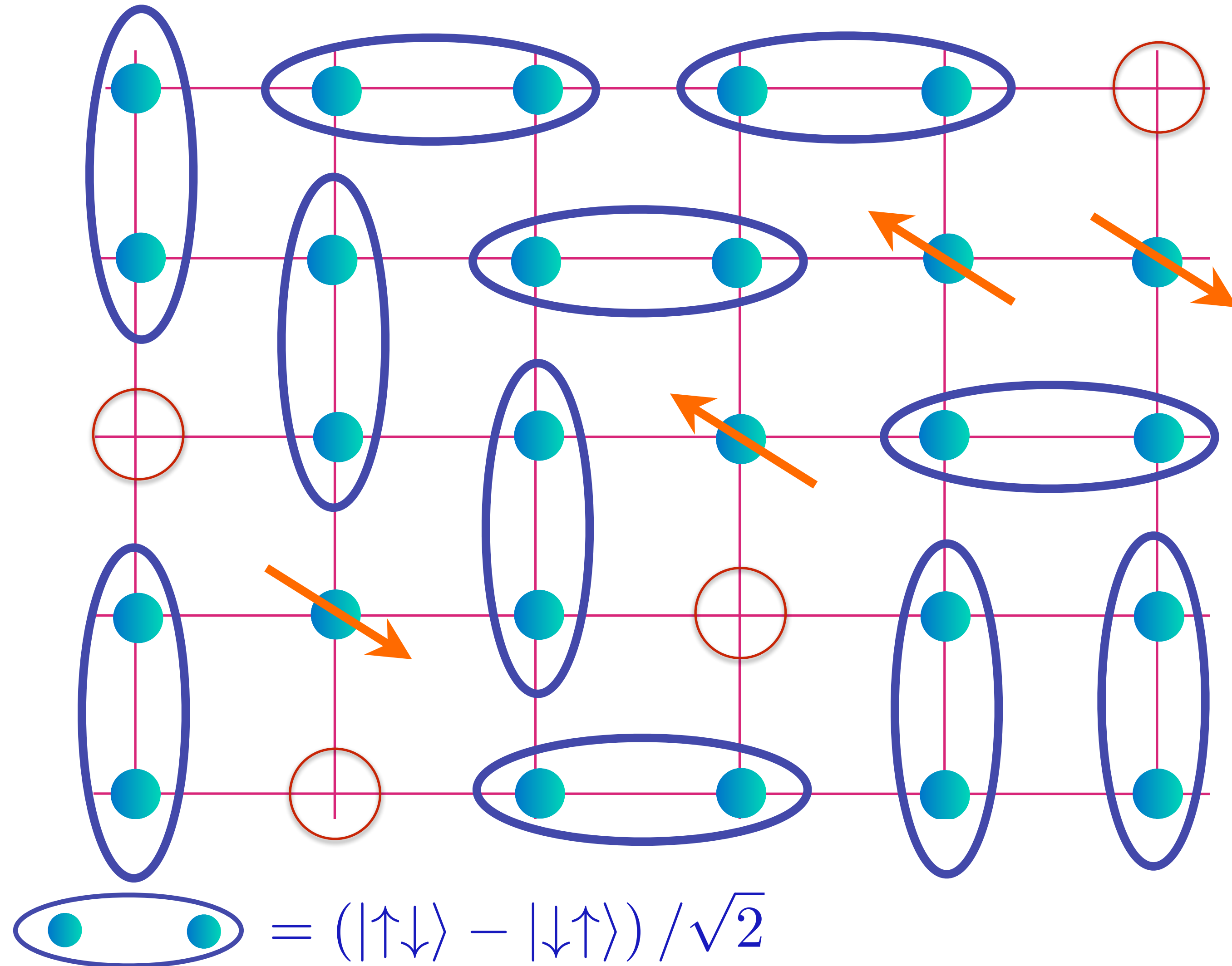


Spin liquid  
with density  
 $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

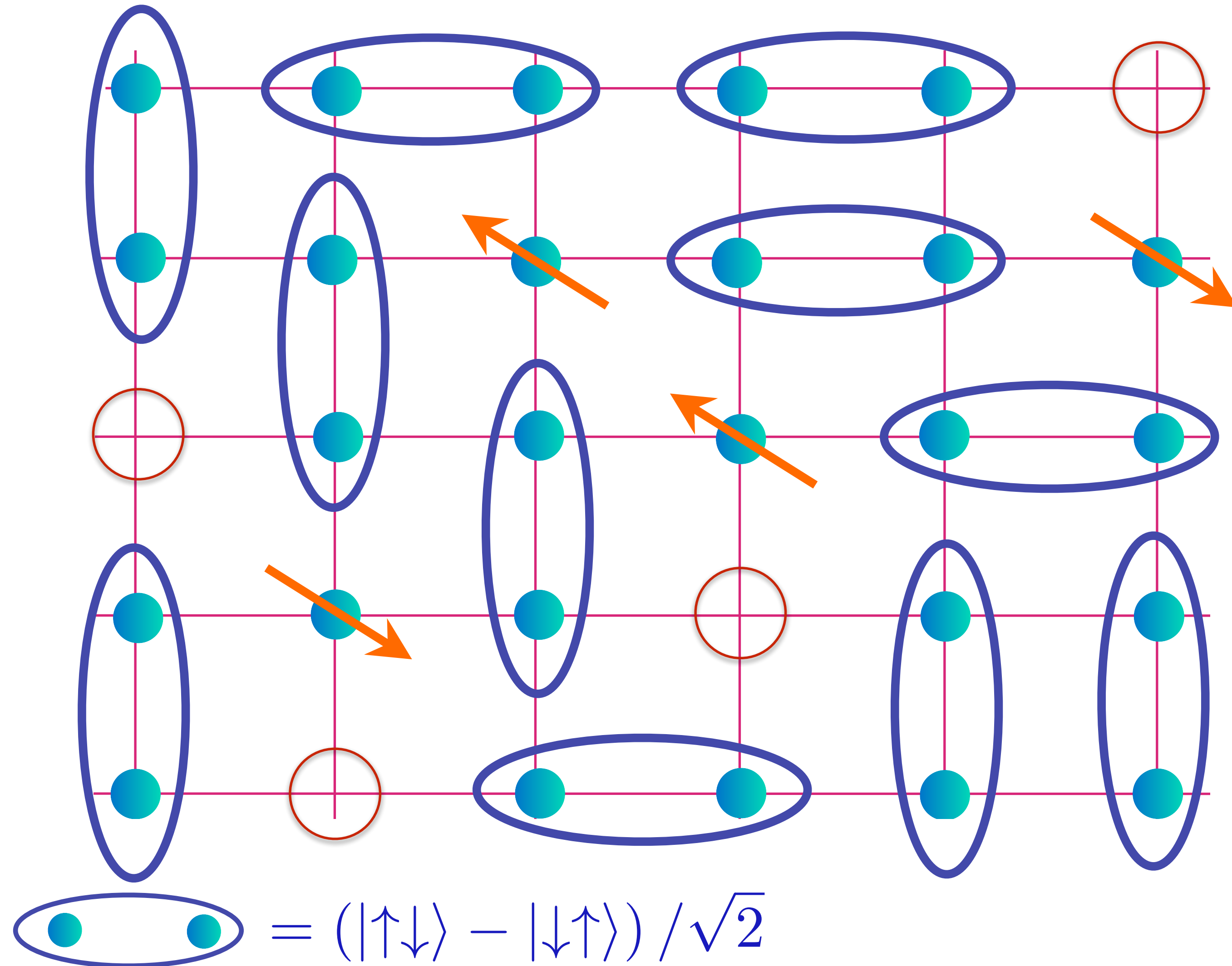


Spin liquid  
with density  
 $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

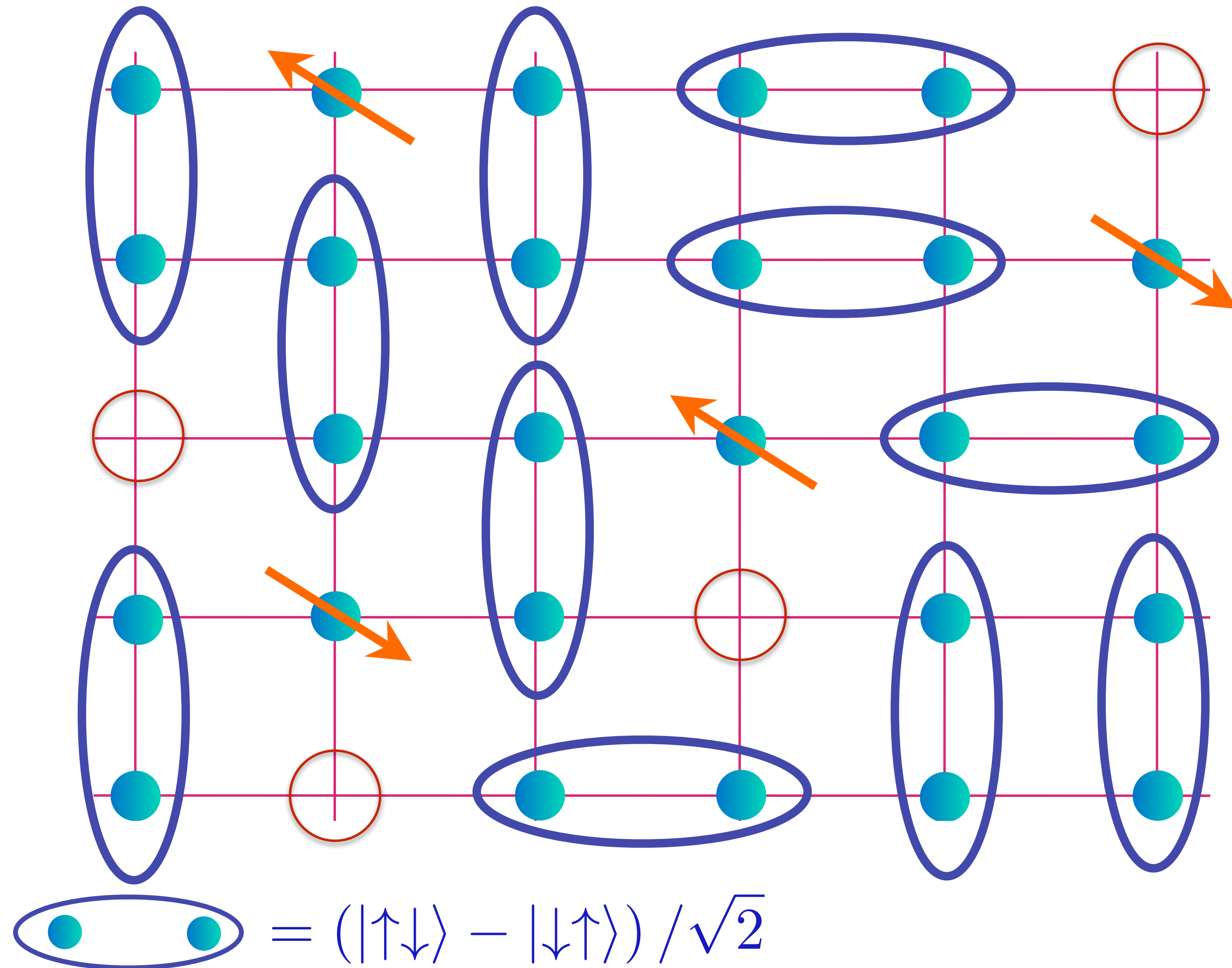


Spin liquid  
with density  
 $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

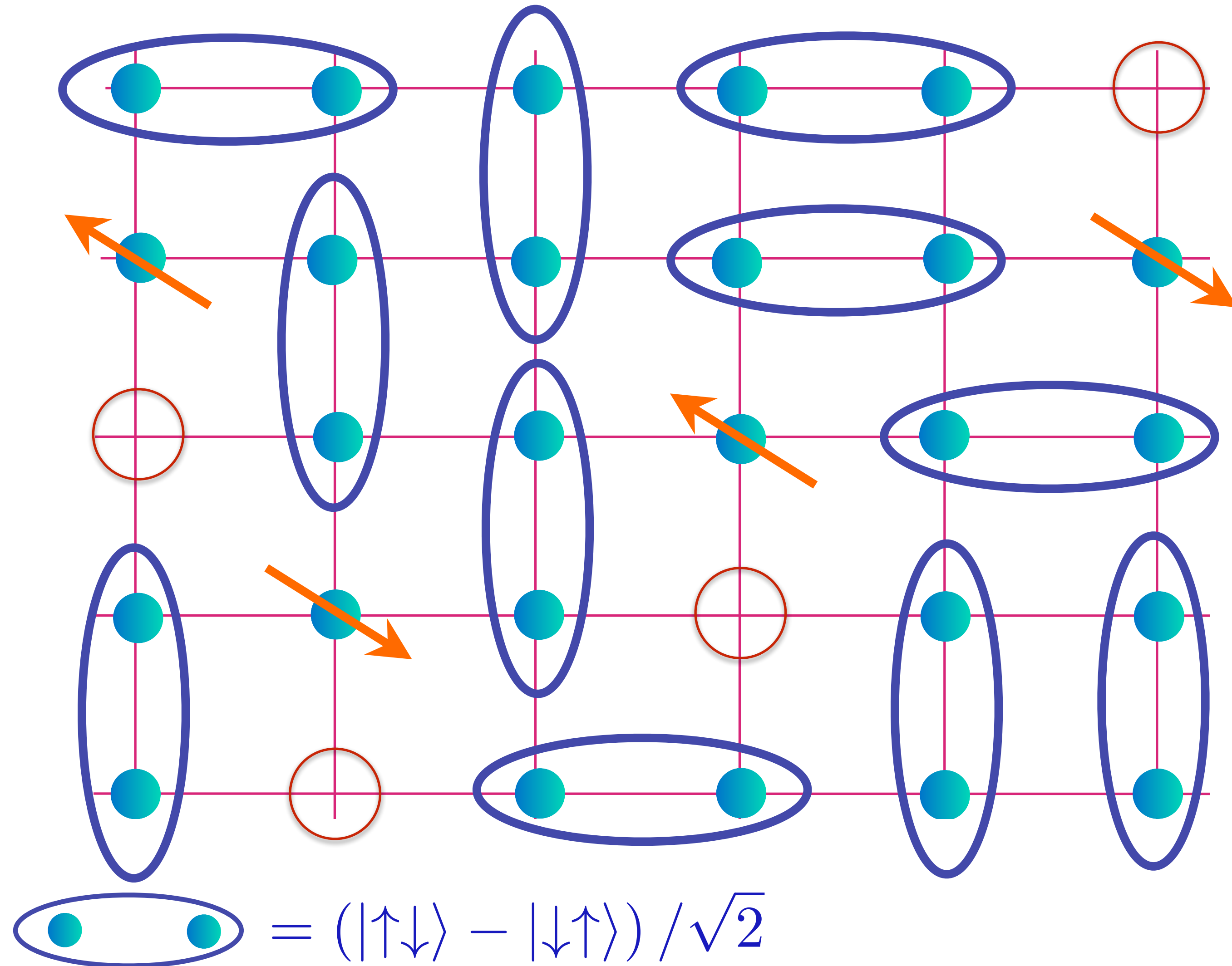


Spin liquid  
with density  
 $\rho$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

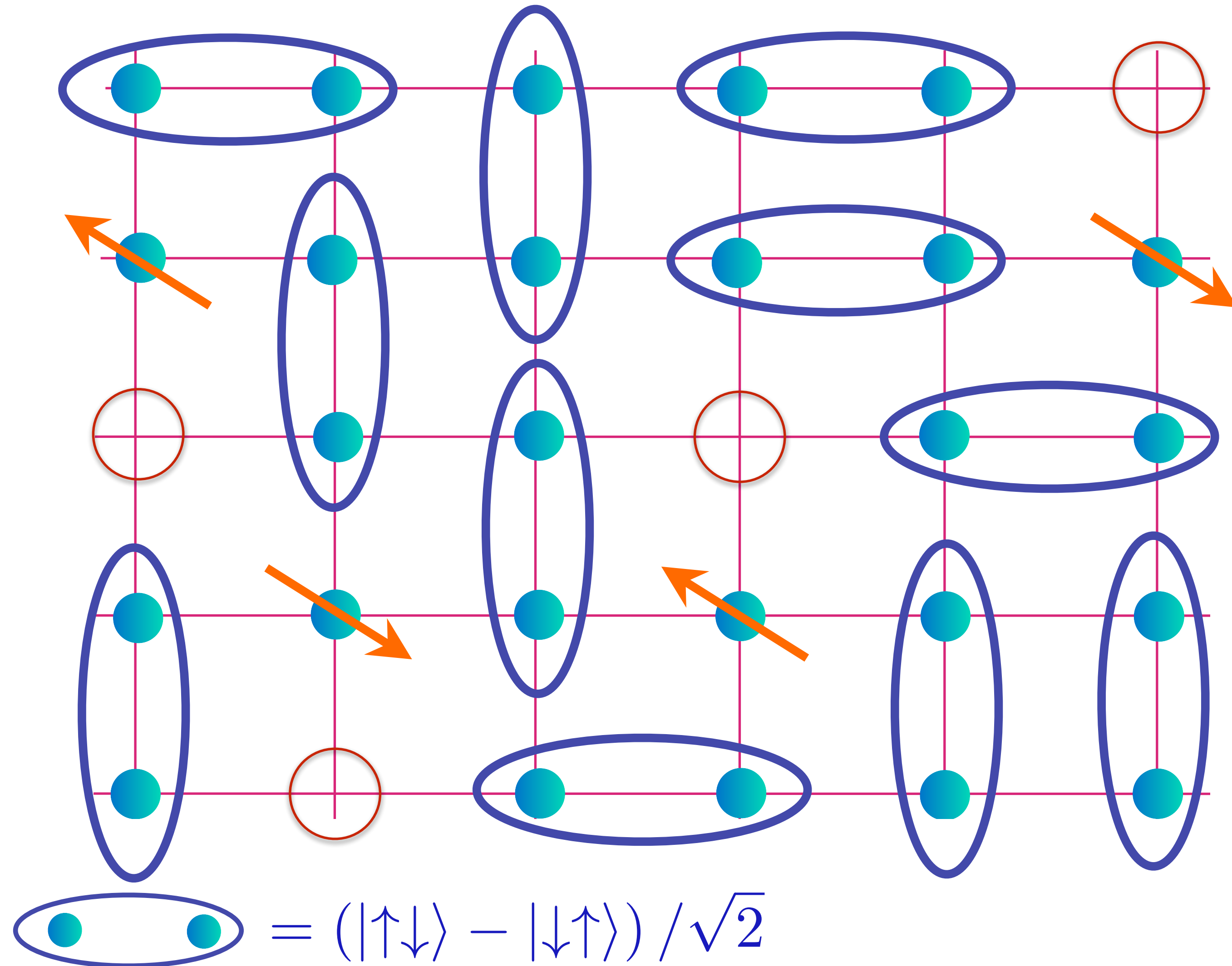


Spin liquid  
with density  
 $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

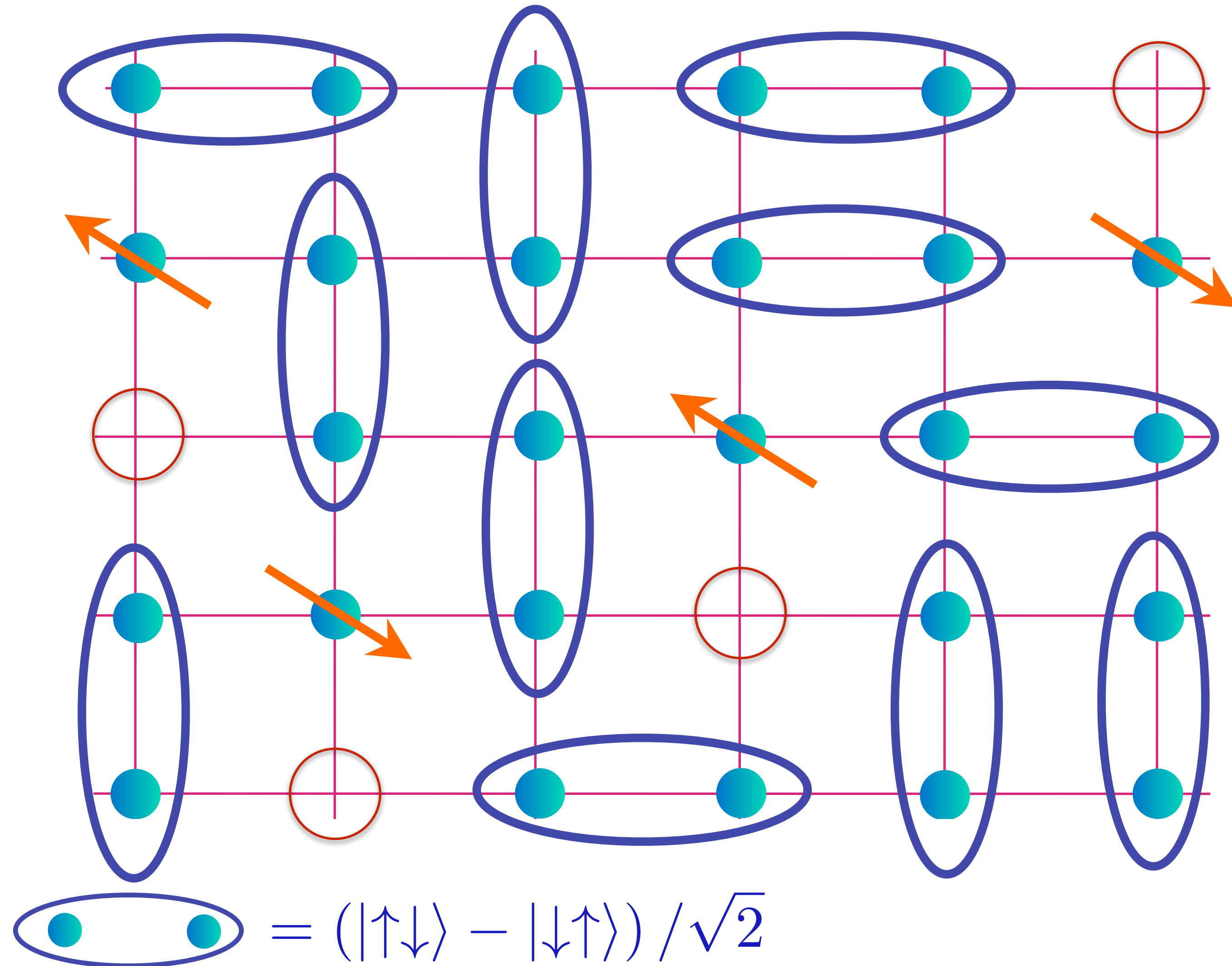


Spin liquid with density  $p$  of spinless, charge  $+e$  "holons" and charge 0, spin-1/2 "spinons".

# Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

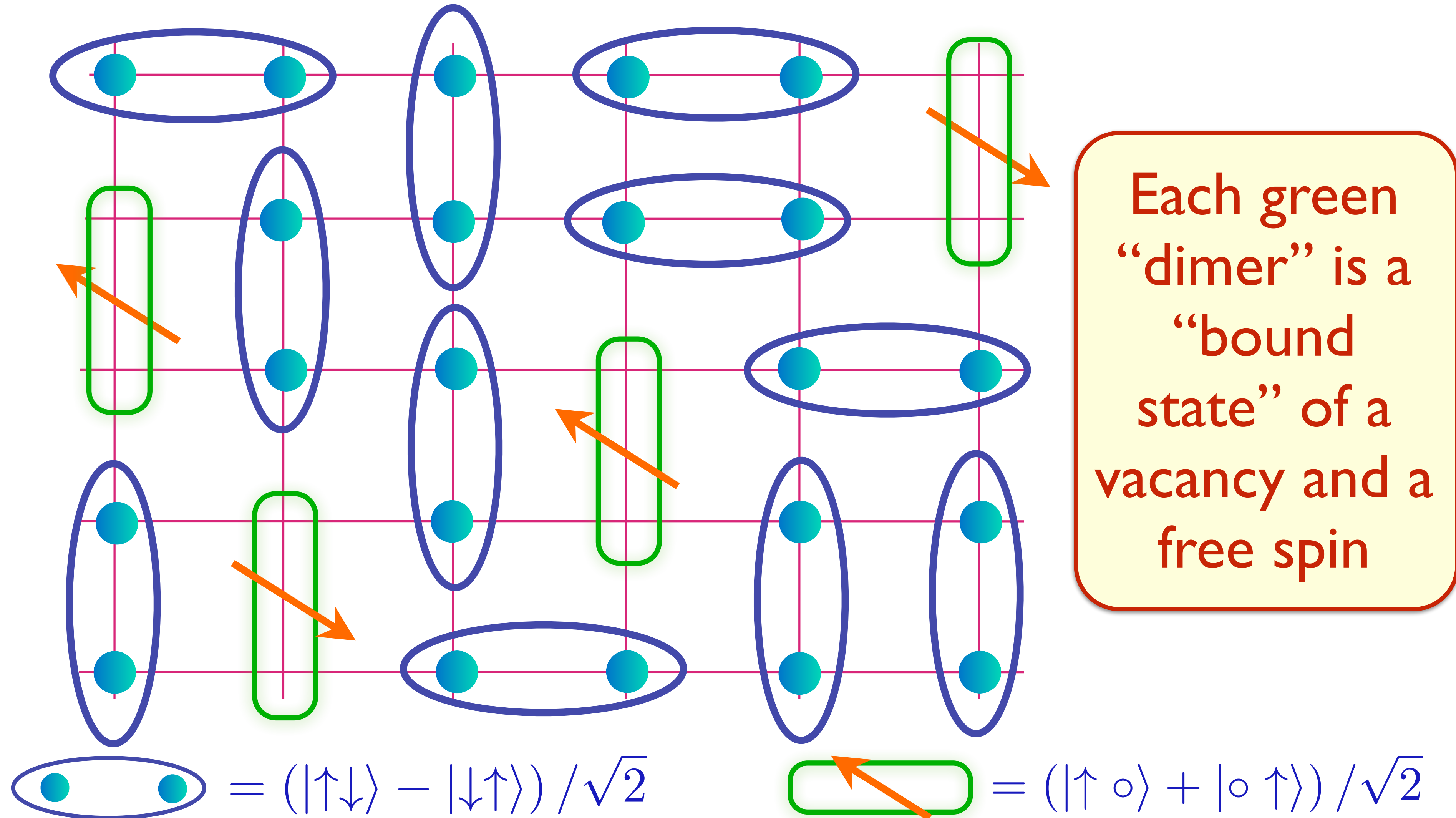


Spin liquid  
with density  
 $p$  of spinless,  
charge  $+e$   
“holons” and  
charge 0, spin-1/2  
“spinons”.

# Earlier approach to FL\* in a *one-band* model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

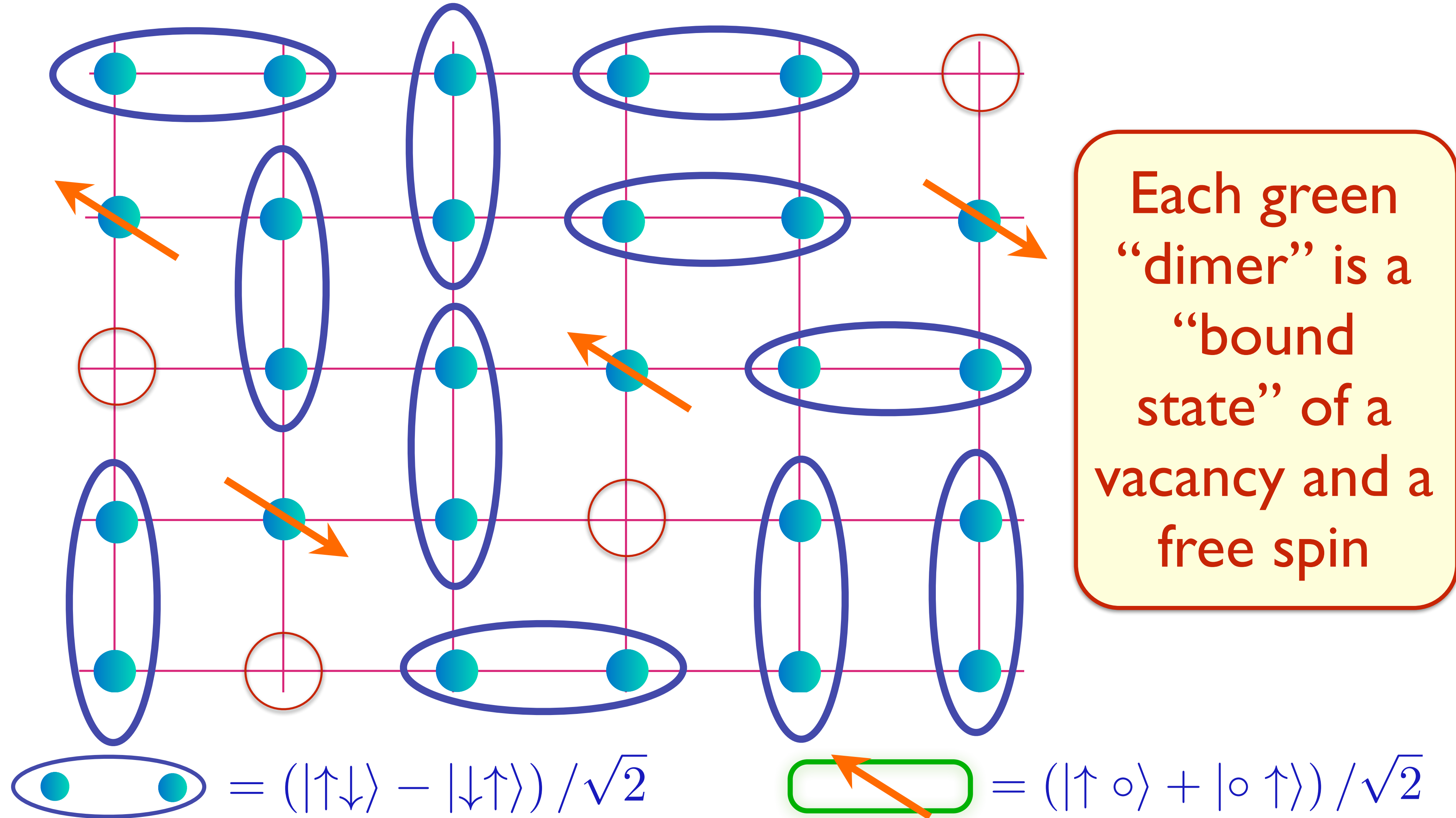


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

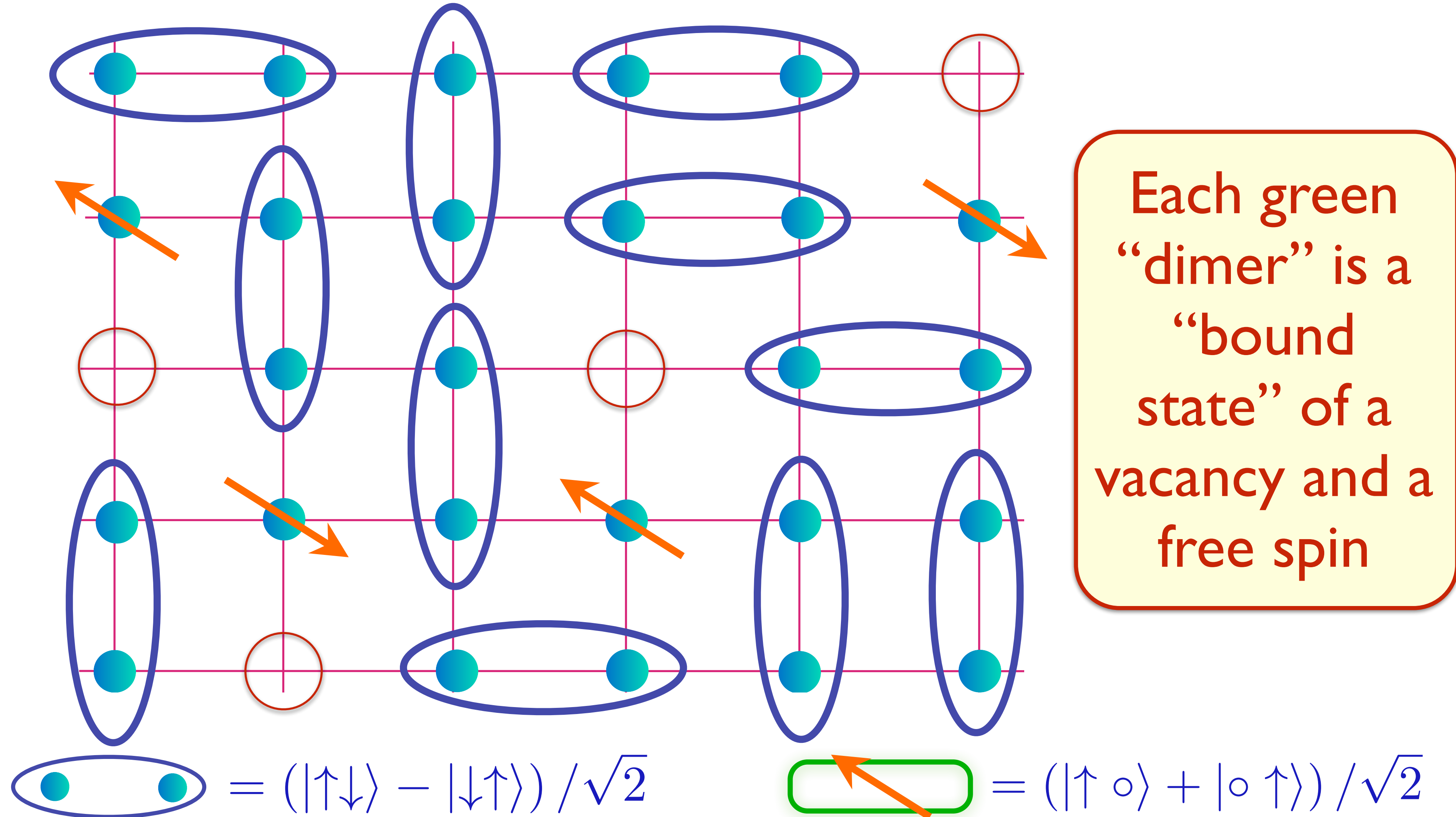


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

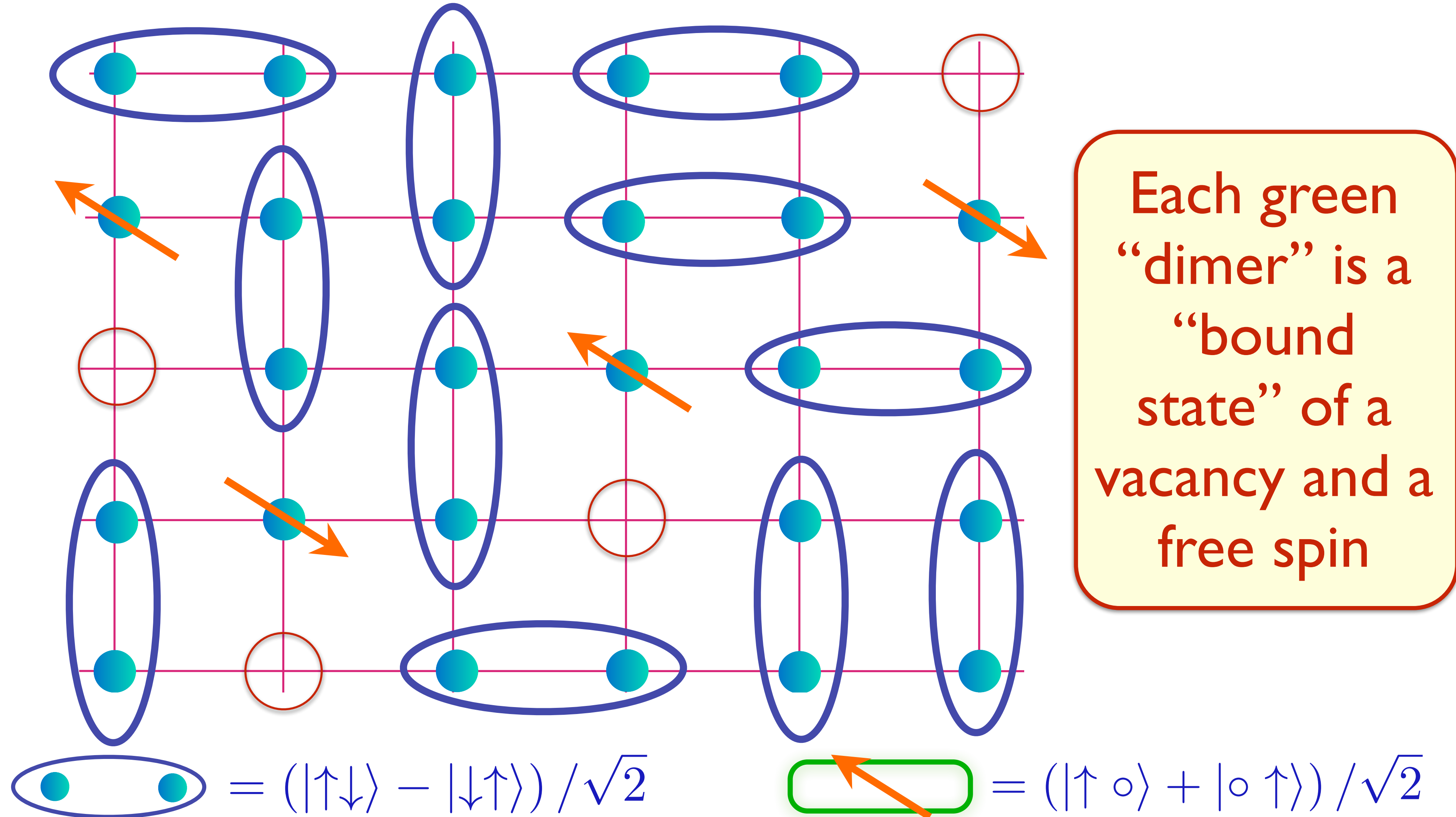
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)



# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

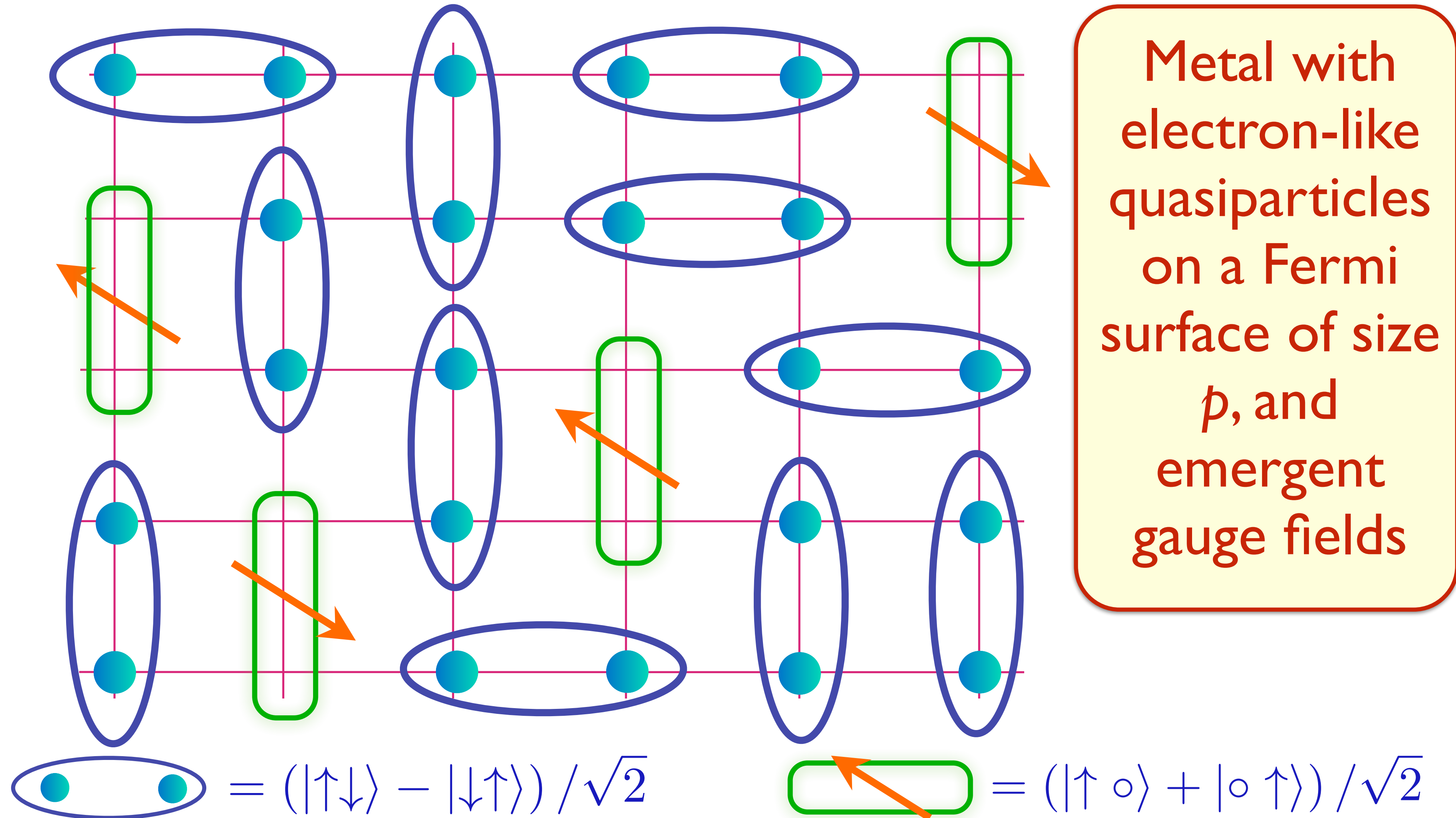
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)



# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

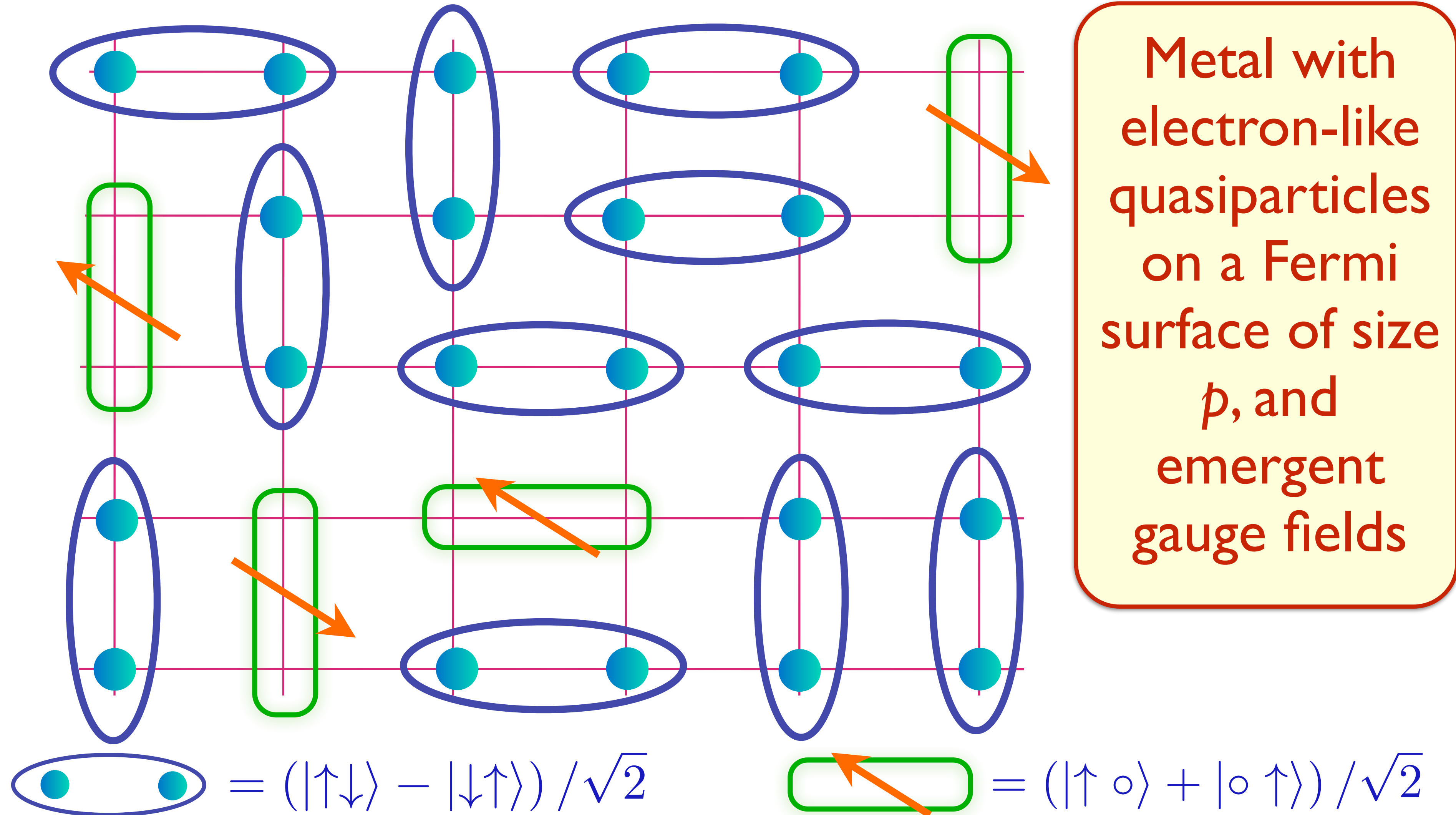


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

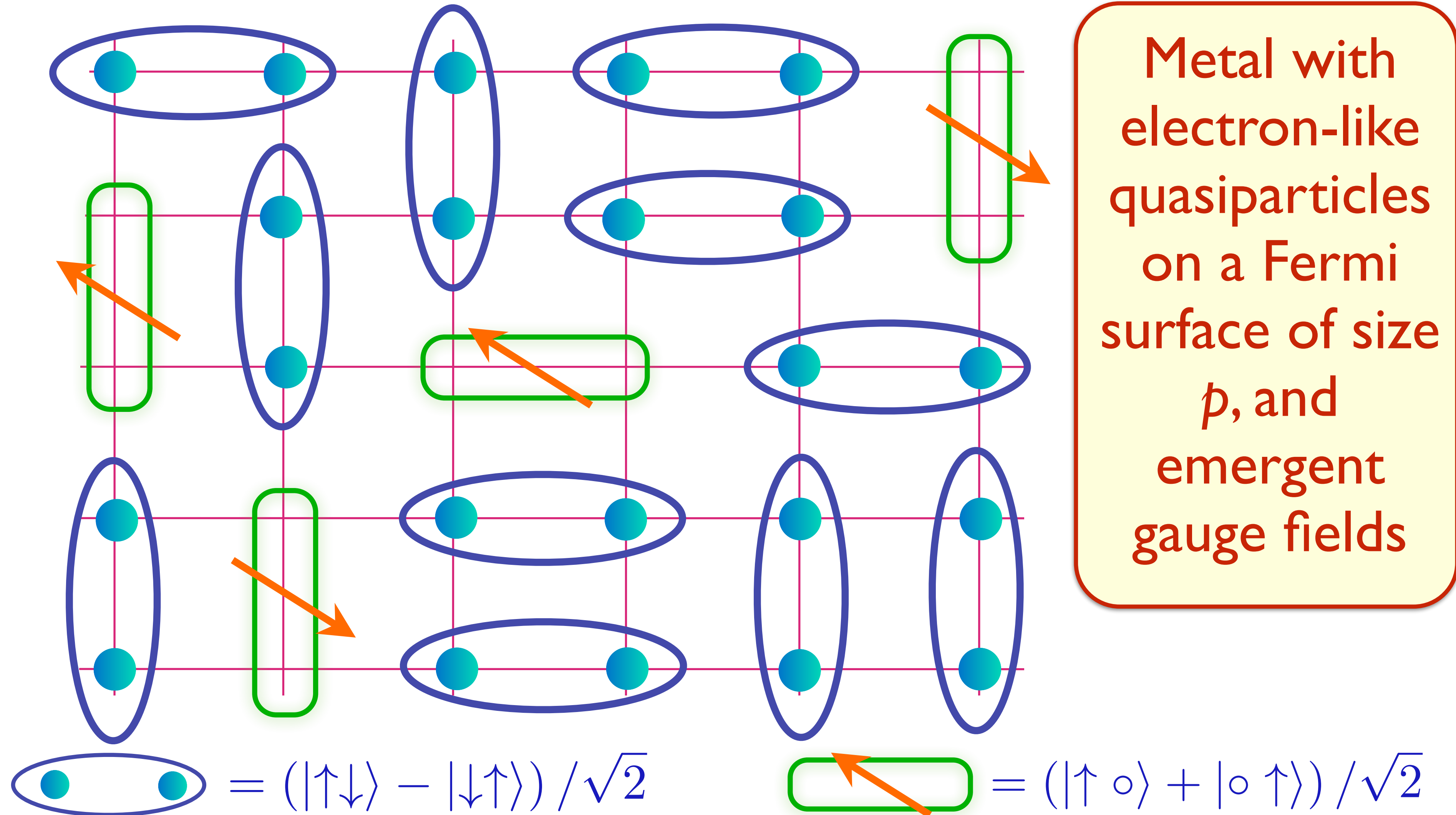


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

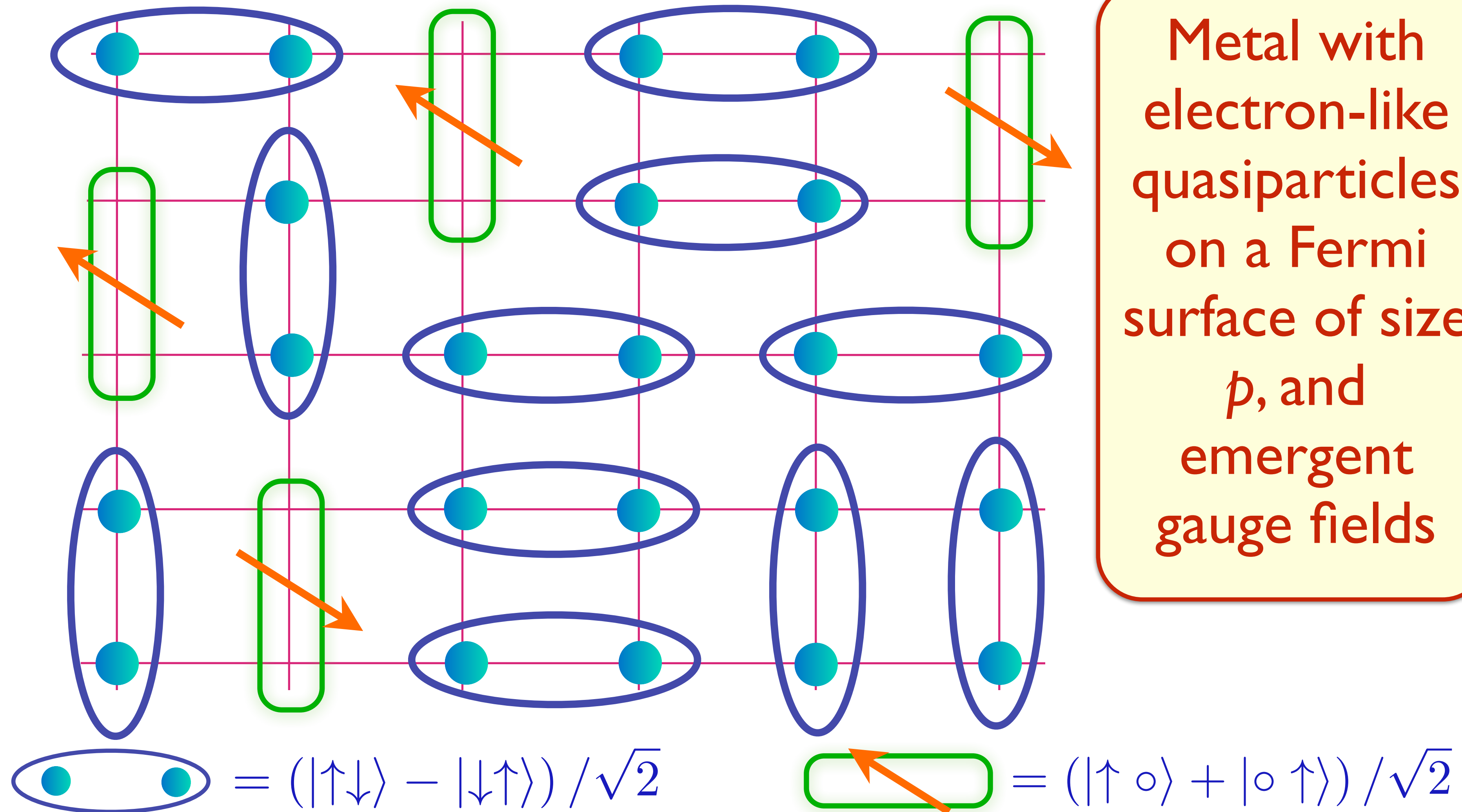


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

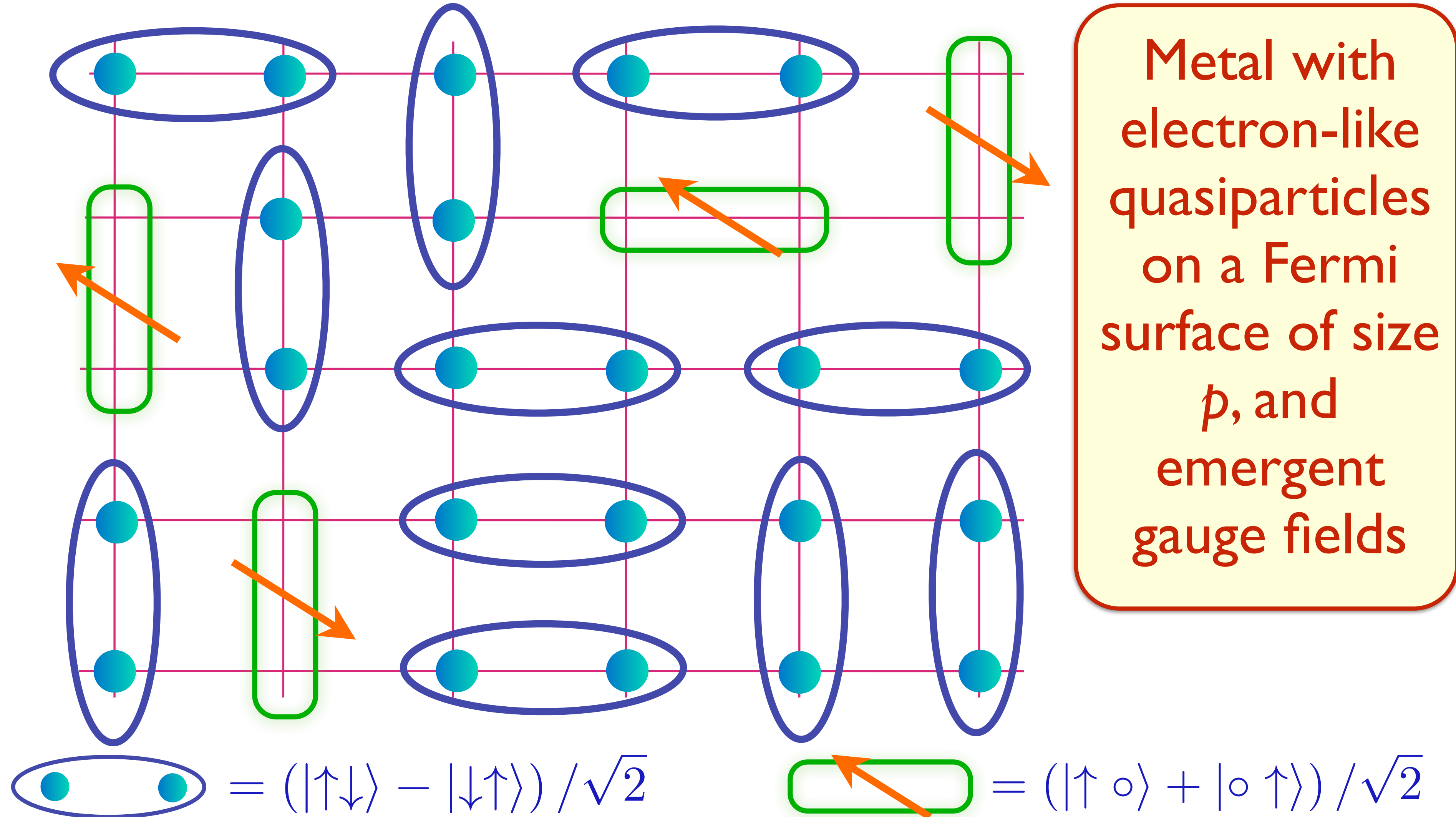


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

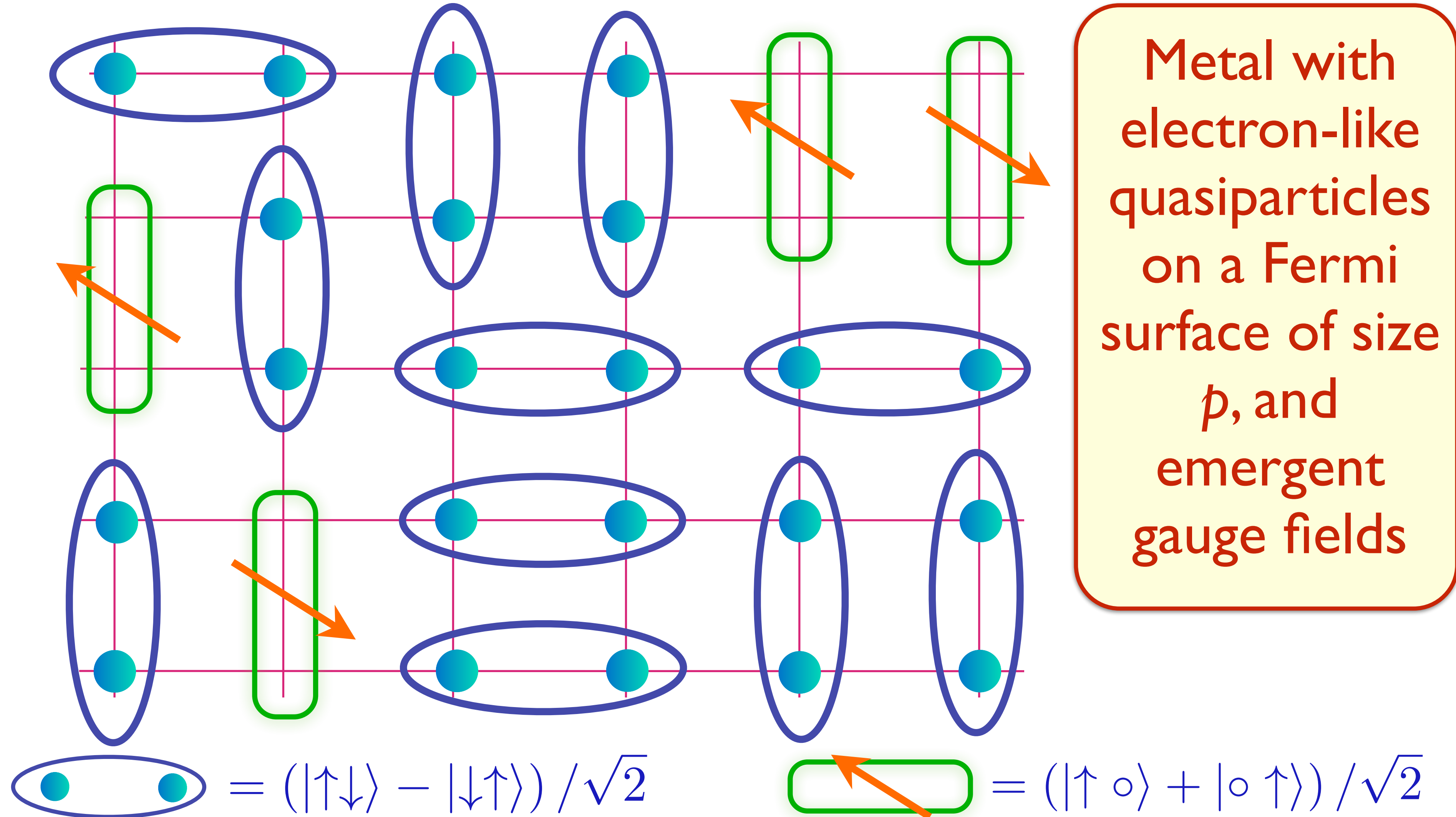


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

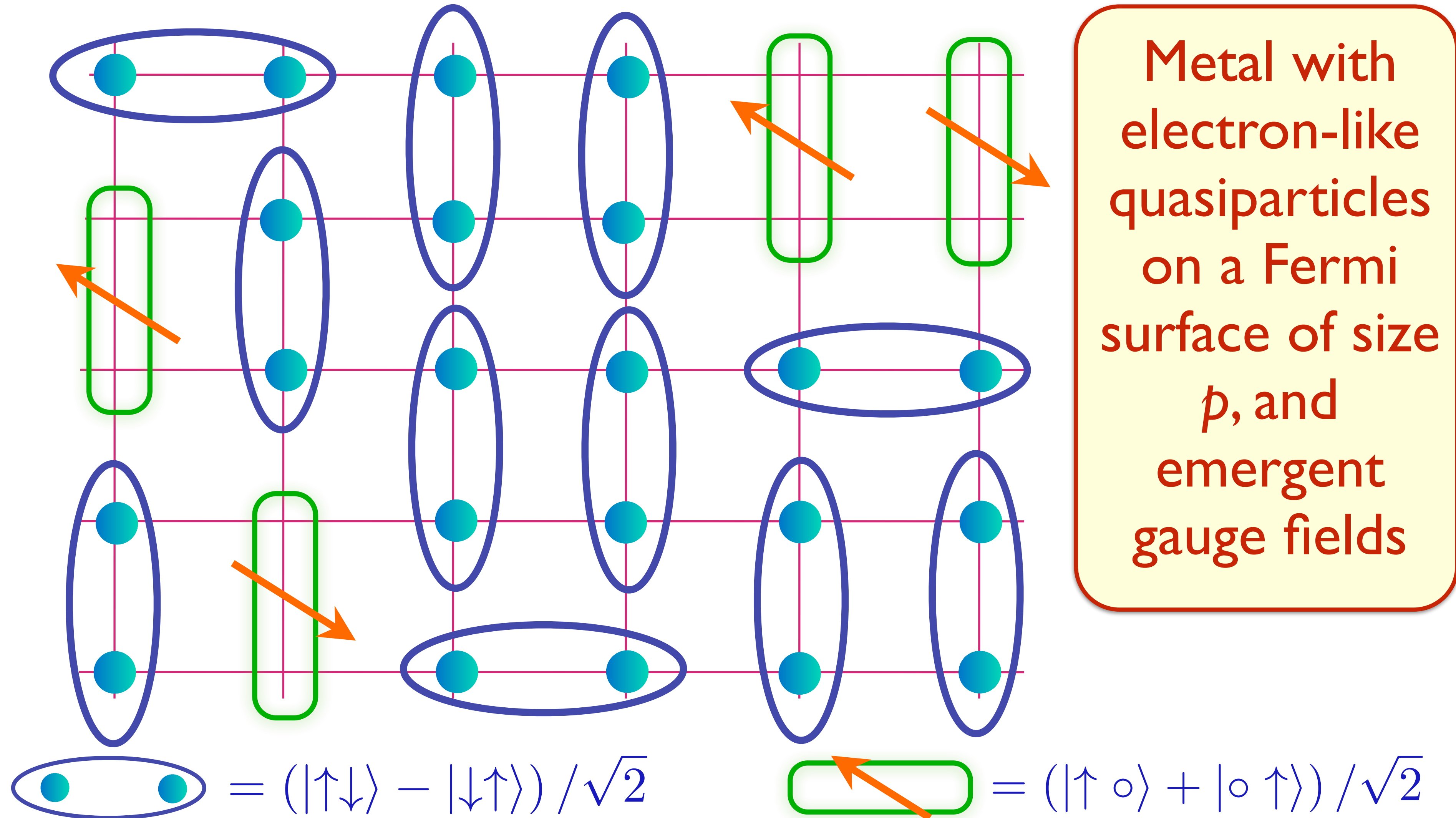


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

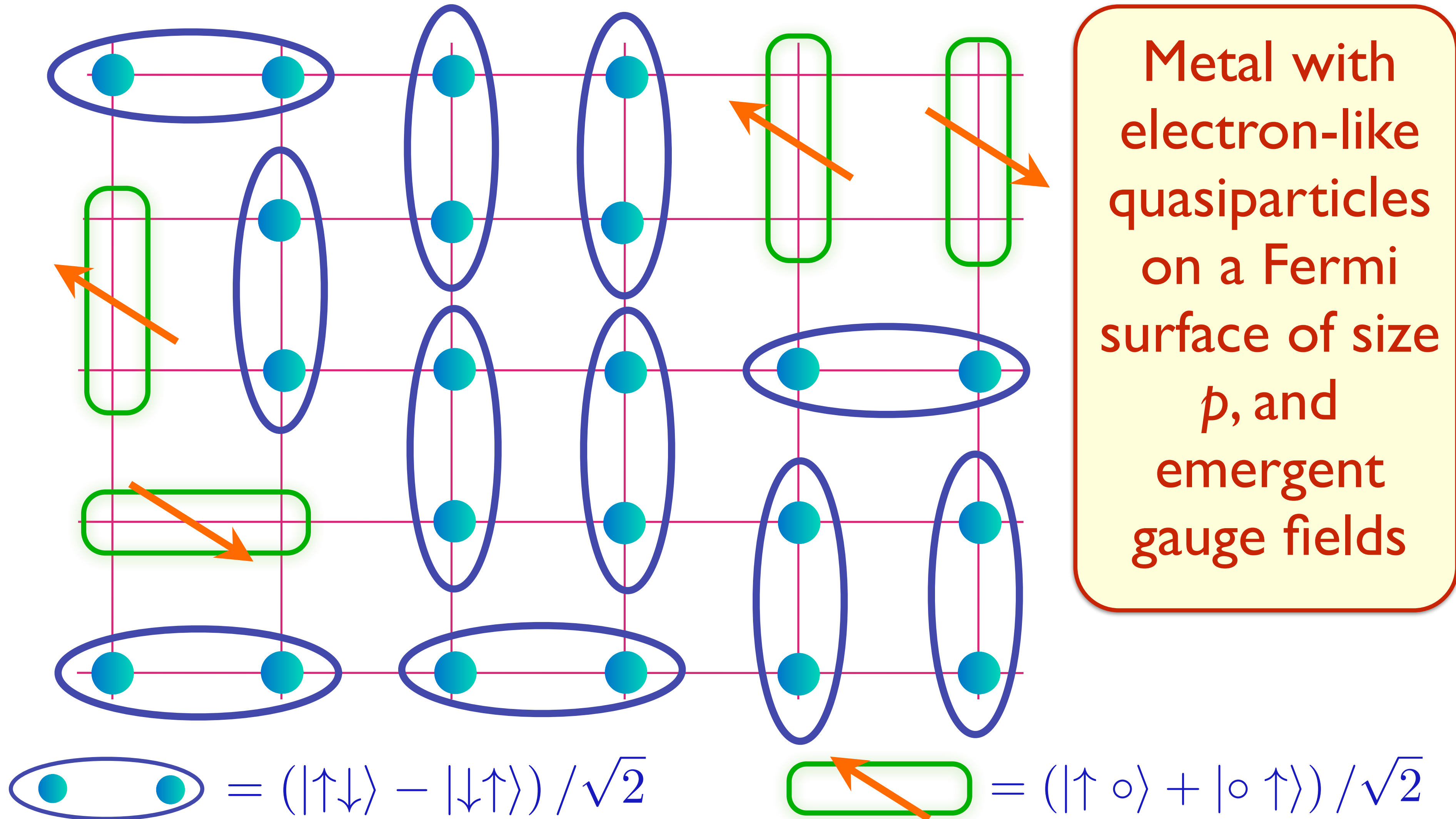


E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

# Earlier approach to FL\* in a **one-band** model

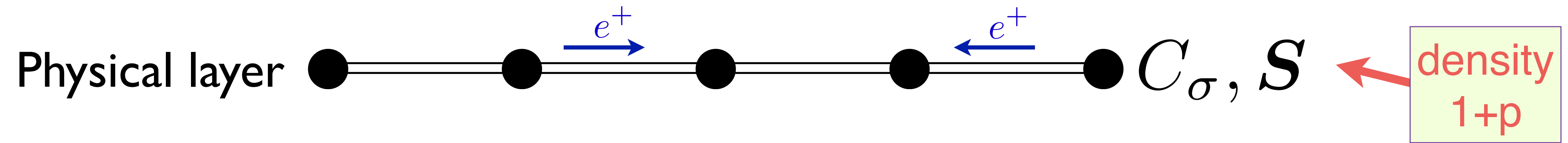
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, Nature Physics **4**, 28 (2008)

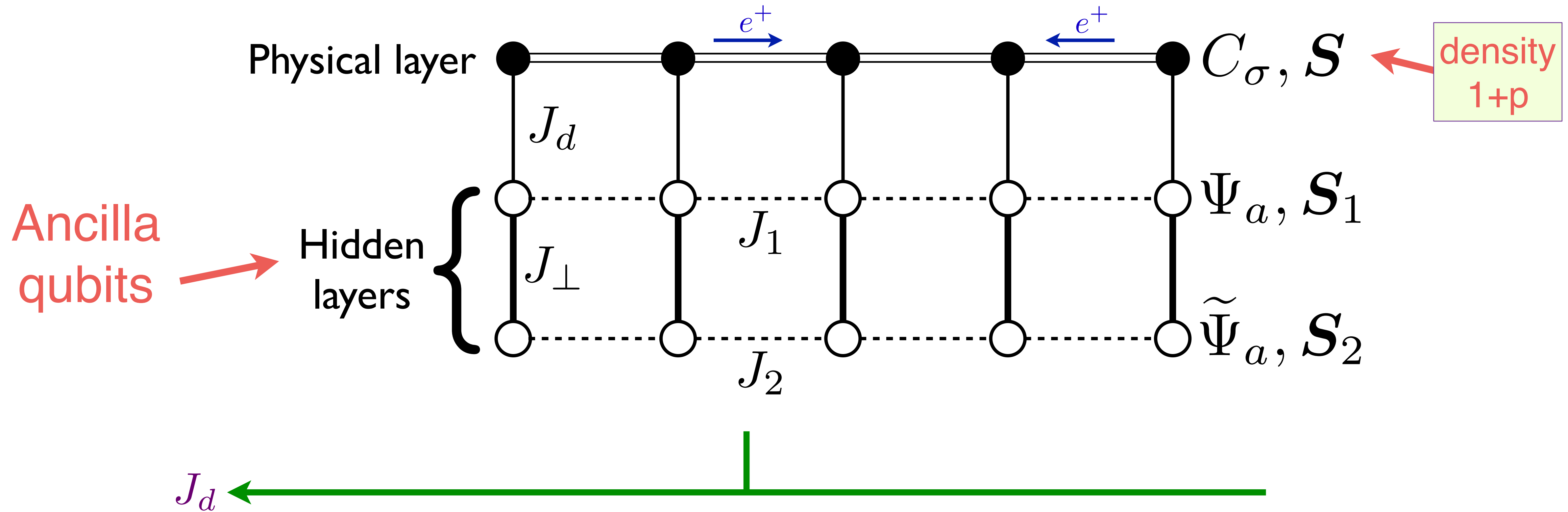


Metal with  
 electron-like  
 quasiparticles  
 on a Fermi  
 surface of size  
 $p$ , and  
 emergent  
 gauge fields

# FL\* and FL in a **one-band** model



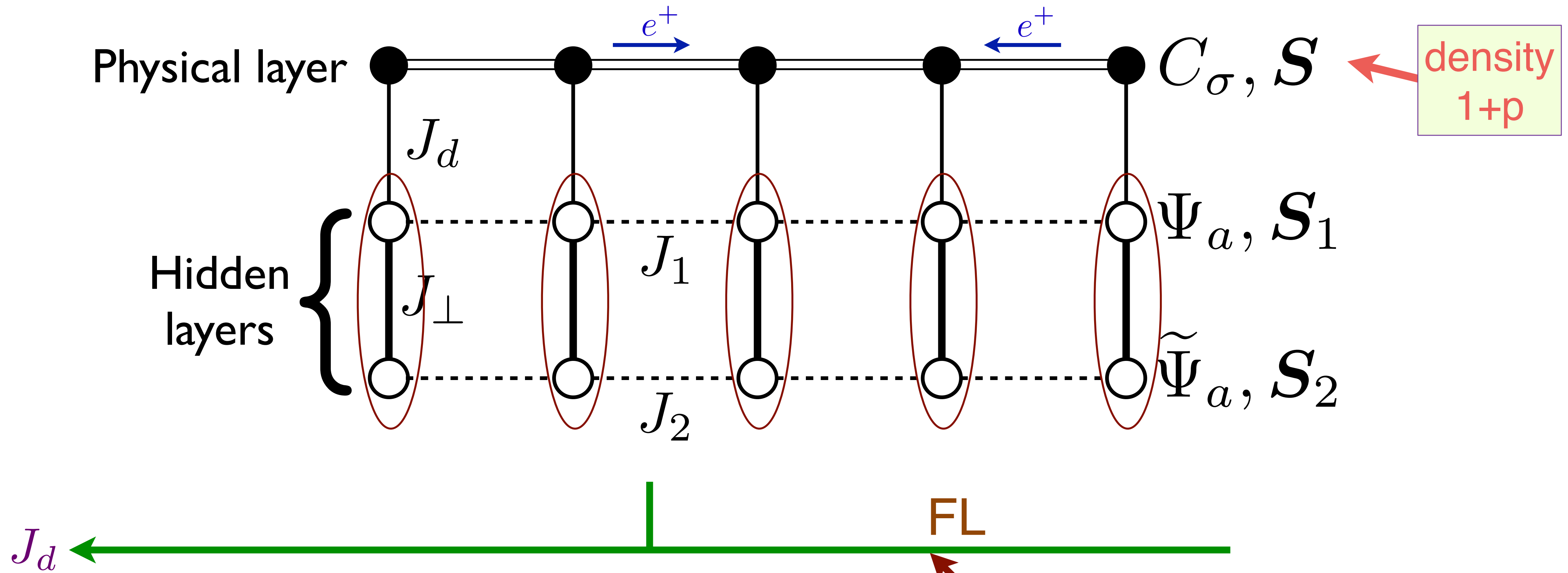
# FL\* and FL in a **one-band** model



Ya-Hui Zhang

Precursor with one extra band of fermions in FL\* phase:  
 Y. Qi and S. Sachdev, PRB **81**, 115129 (2010)  
 E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011)

# FL\* and FL in a **one-band** model

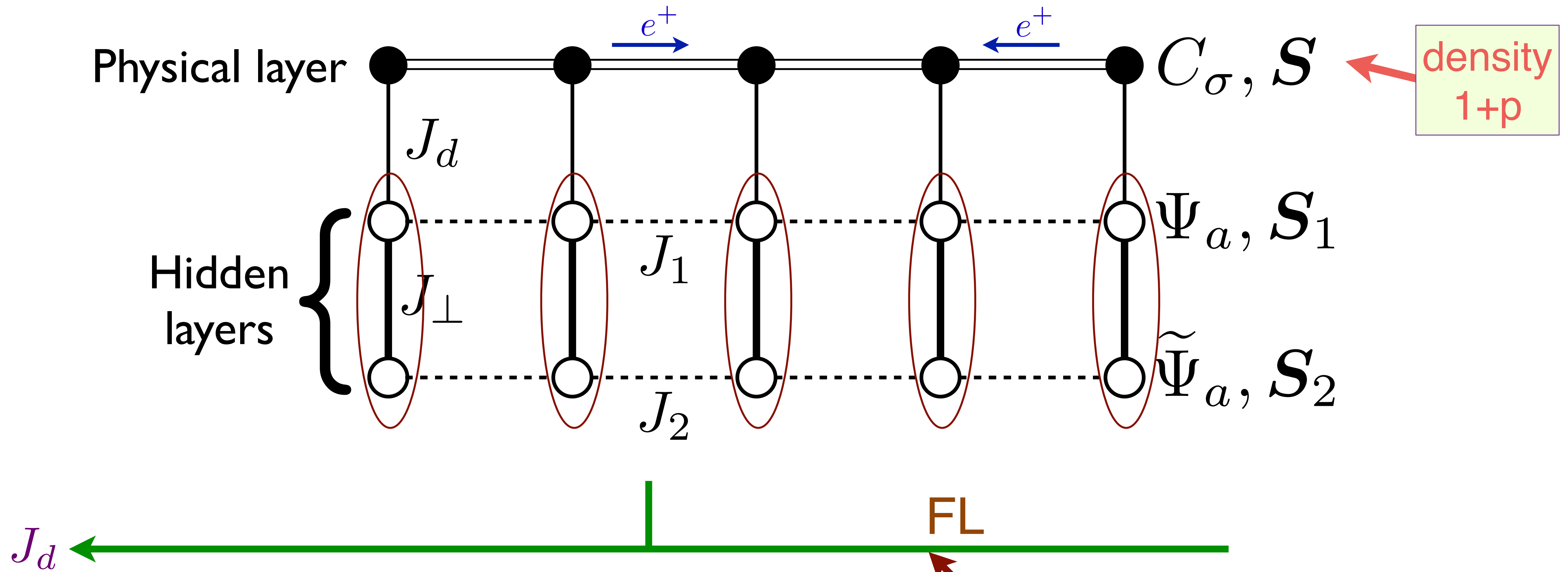


Ya-Hui Zhang

Large Fermi surface of size  $1 + p$

$$|\Phi\rangle = \left| \text{Rung singlets of } \Psi, \tilde{\Psi} \right\rangle \otimes \left| \text{Slater determinant of } C \right\rangle$$

# FL\* and FL in a **one-band** model



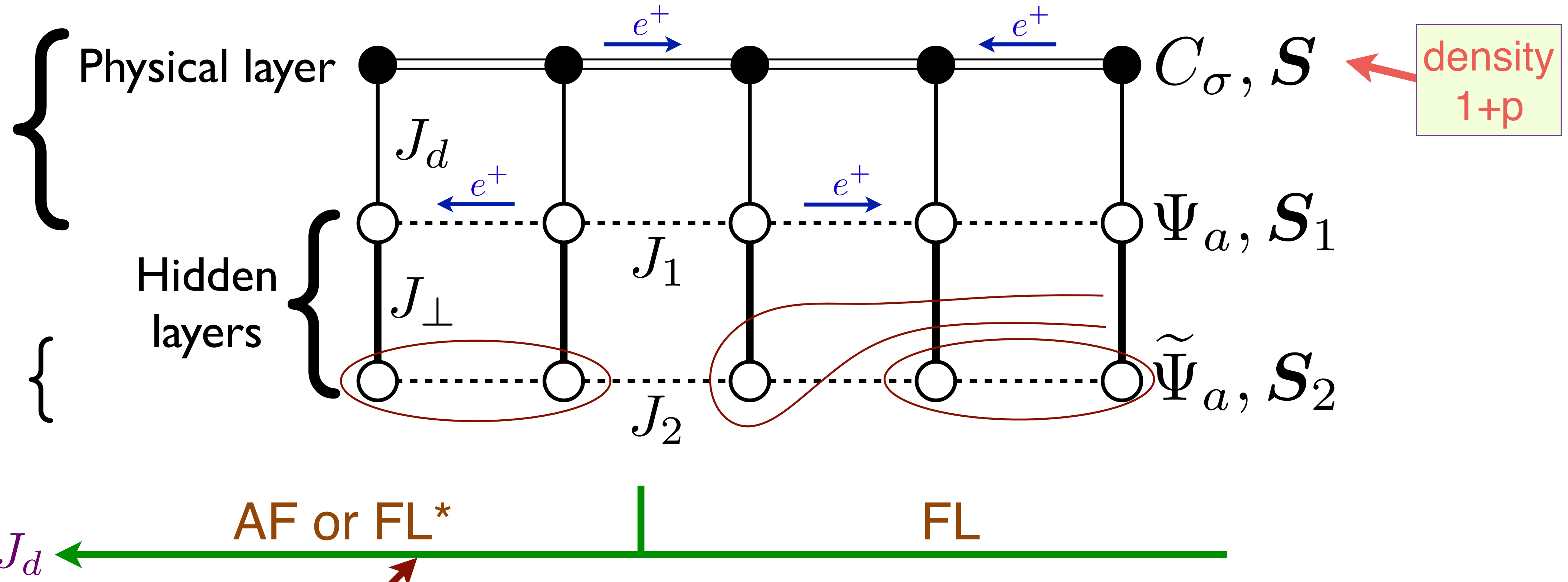
Ya-Hui Zhang

Large Fermi surface of size  $1 + p$

All gauge symmetries are confined.  
No fractionalization.

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$



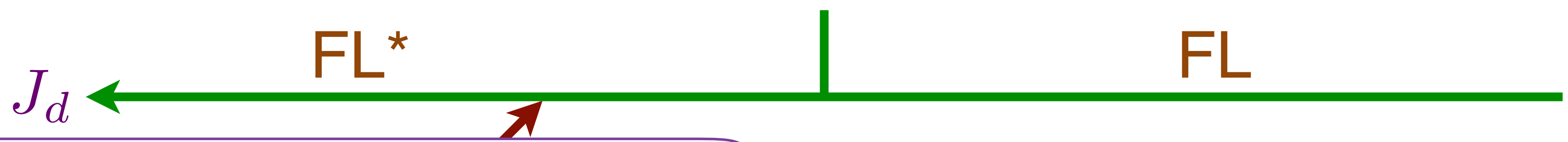
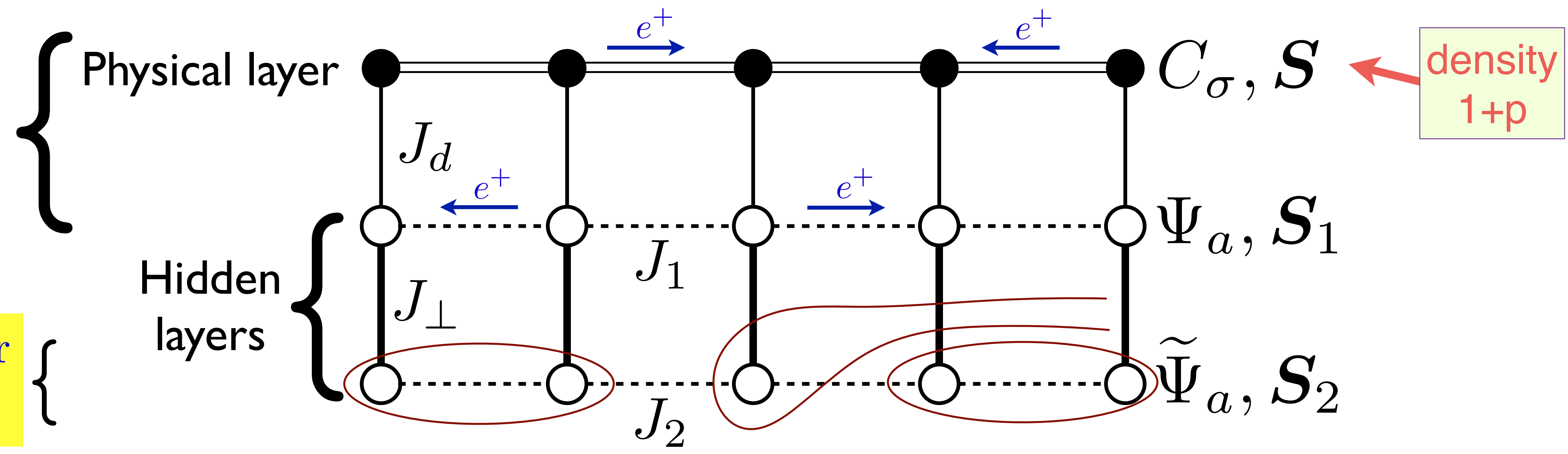
Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$

Mott insulator  
Spin liquid



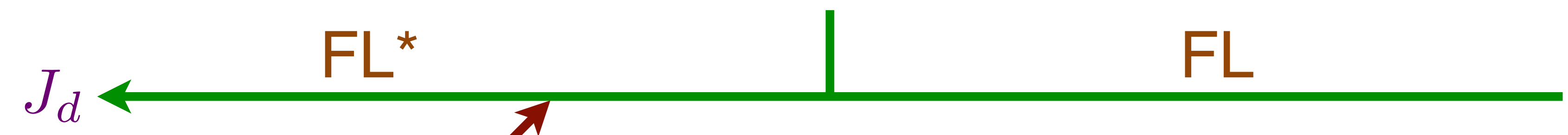
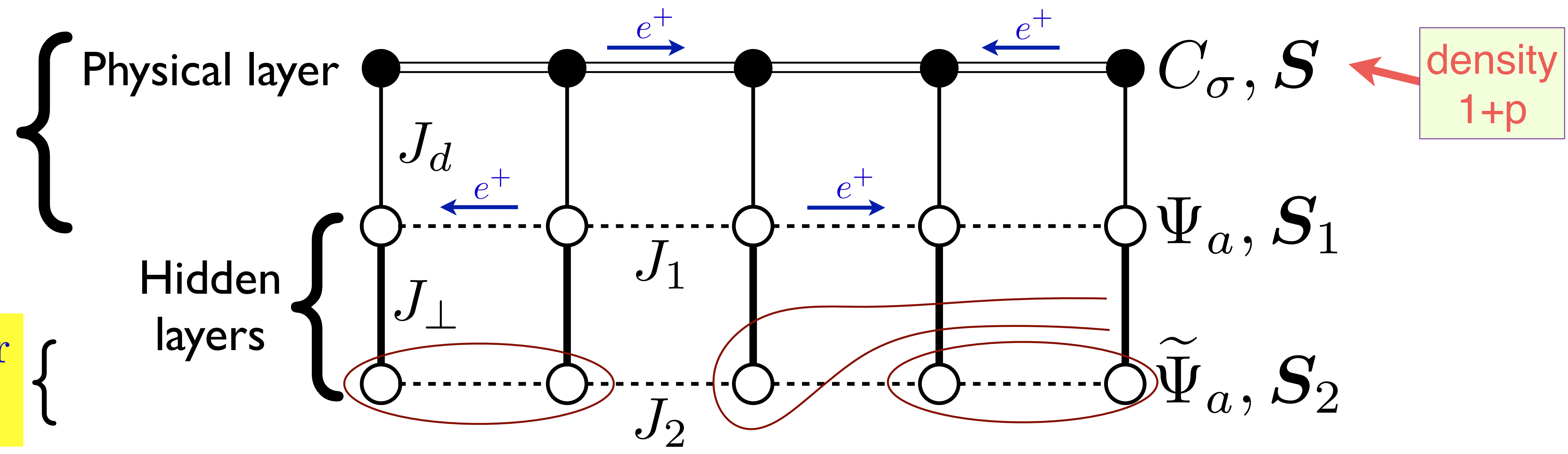
Small Fermi surface of size  $p$

$$|\Phi\rangle = \left[ \text{Projection onto rung singlets of } \Psi, \tilde{\Psi} \right] \otimes |\text{Slater determinant of } (C, \Psi)\rangle \otimes |\text{Slater determinant of } \tilde{\Psi}\rangle$$

# FL\* and FL in a **one-band** model

Metal.  
Density  
 $2 + p \cong p$

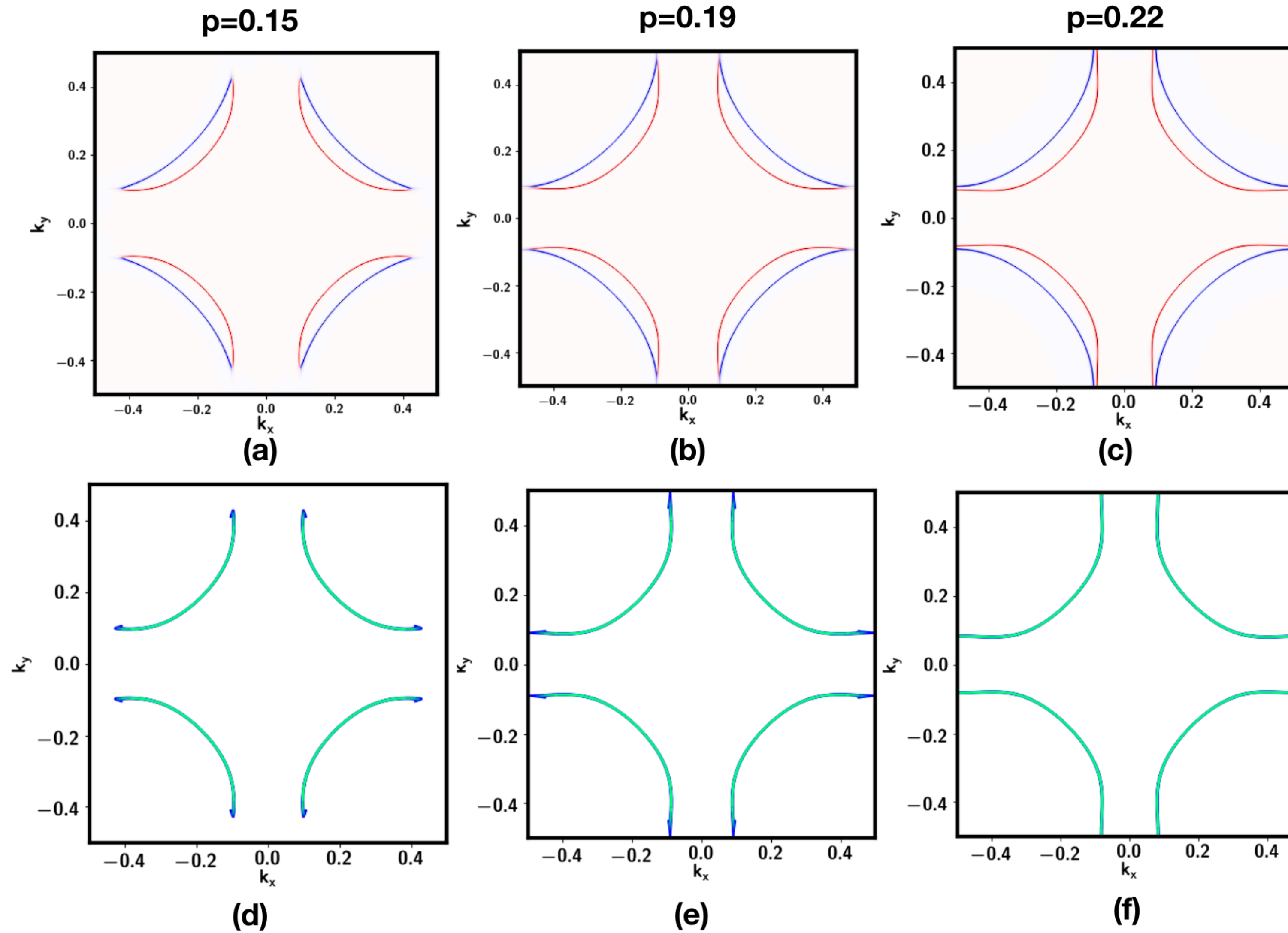
Mott insulator  
Spin liquid



Small Fermi surface of size  $p$

Gauge symmetry on  $\Psi$  layer is Higgsed.  
Gauge symmetry on  $\tilde{\Psi}$  layer is deconfined.  
Fractionalization is present.

# FL\* in a **one-band** model



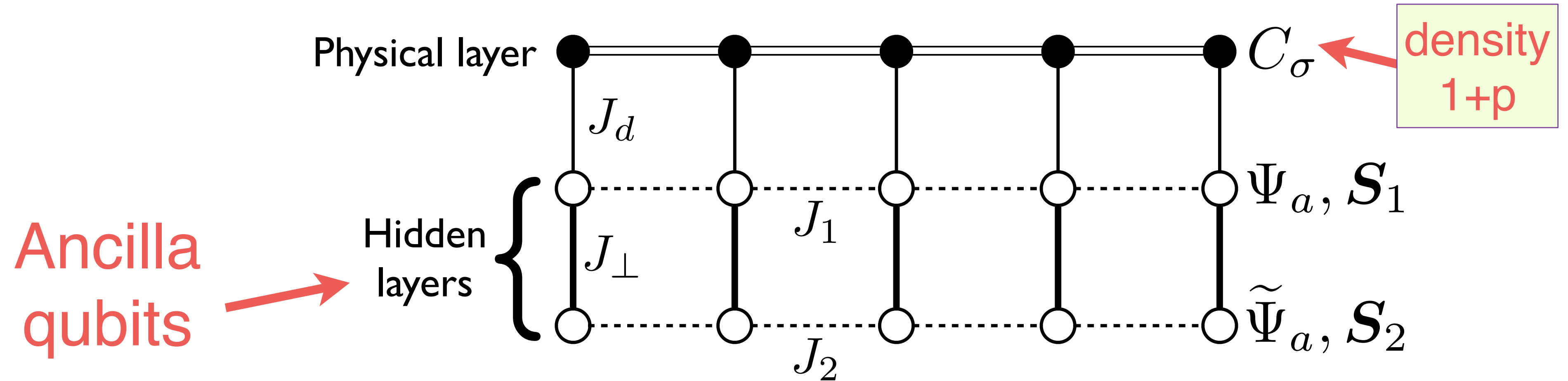
“Fermi arc”  
spectral functions  
in the FL\* phase



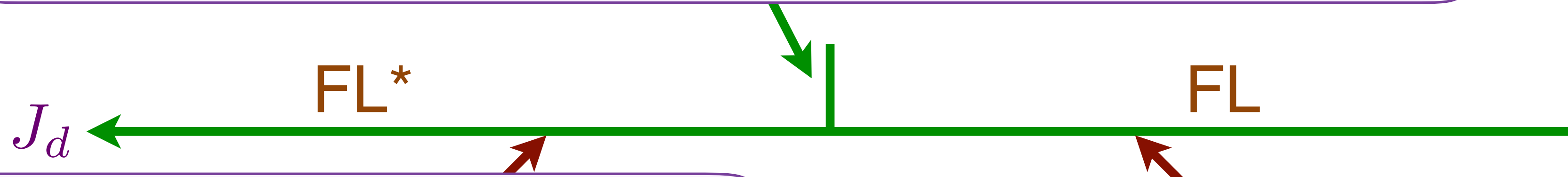
Ya-Hui Zhang

Zero frequency spectral density of electrons (red) and ghosts (blue)

# Metal-metal transitions in a **one-band** model



$(SU(2)_S \times U(1)_1)/Z_2$  gauge theory of a  $\Psi$  ghost Fermi surface and a ‘hybridization-Higgs’ boson  $\sim C_\sigma^\dagger \Psi_a$  which condenses on the ‘Small Fermi surface’ side.



Small Fermi surface of size  $p$

Gauge symmetry on  $\Psi$  layer is Higgsed.  
 Gauge symmetry on  $\tilde{\Psi}$  layer is deconfined.  
 Fractionalization is present.

Large Fermi surface of size  $1 + p$

All gauge symmetries are confined.  
 No fractionalization.

## FL\*

- Evidence for a FL\* phase in a Kondo lattice: CeCoIn<sub>5</sub>. (Also in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019))

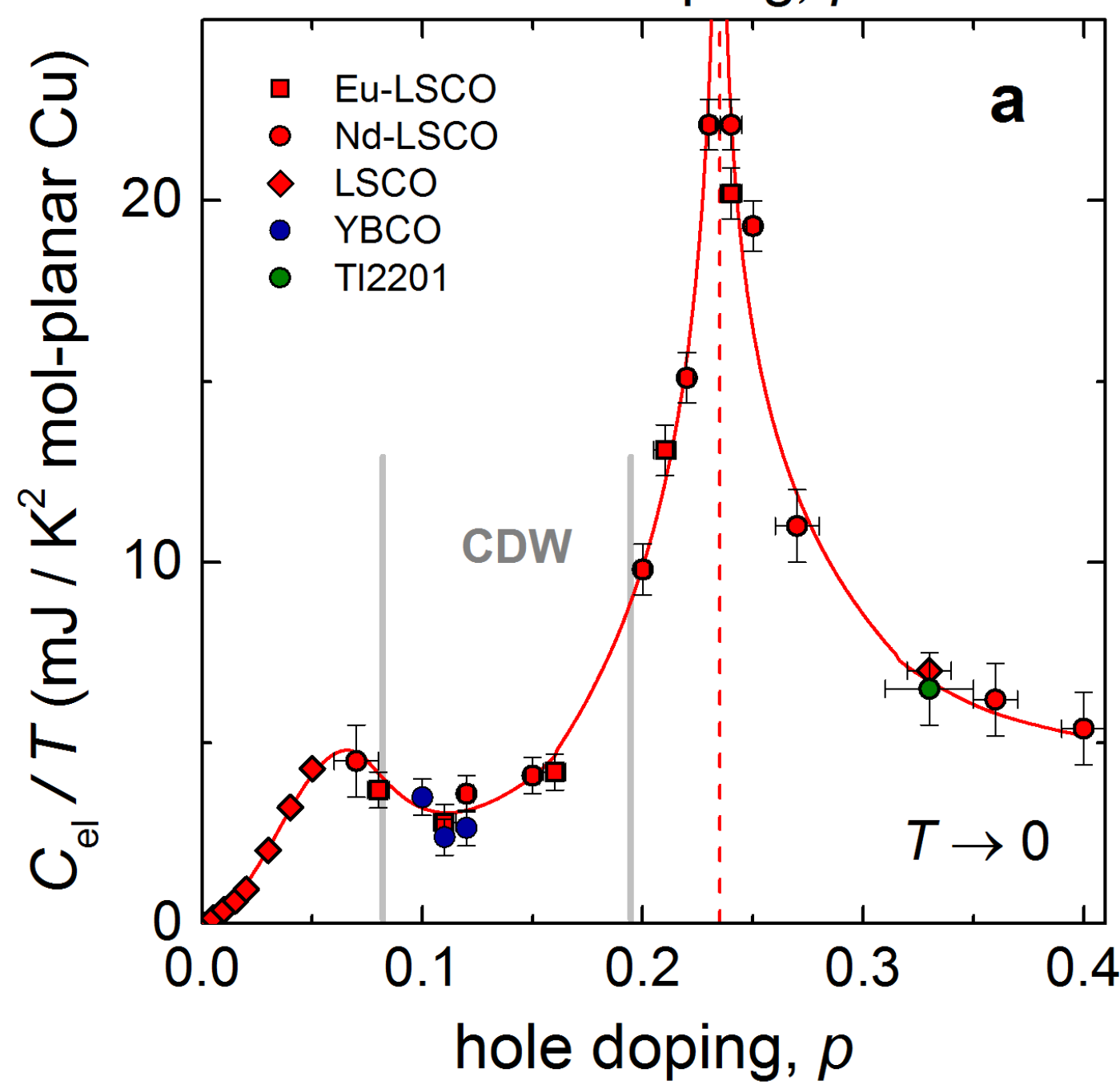
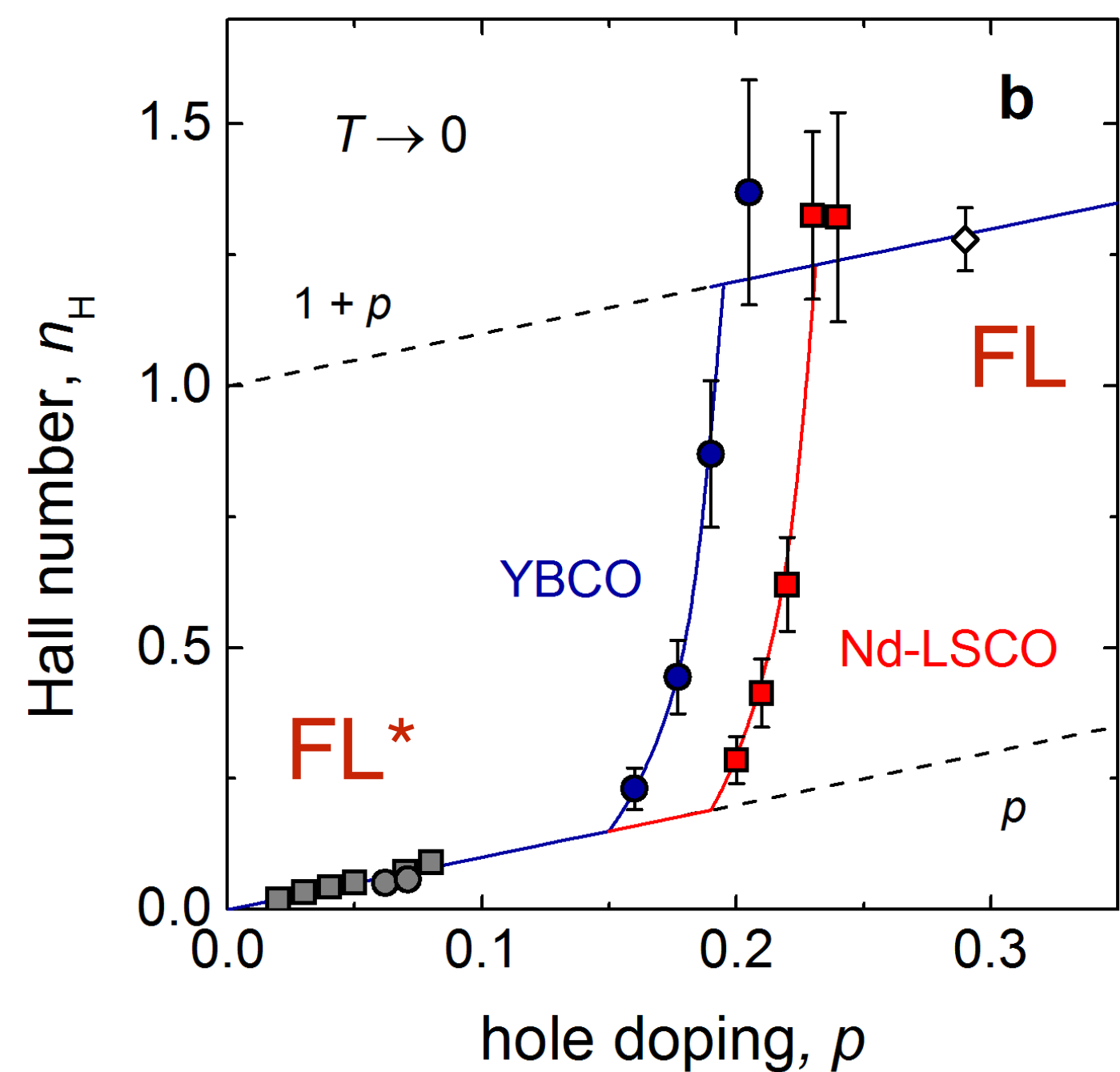
## FL\*

- Evidence for a FL\* phase in a Kondo lattice: CeCoIn<sub>5</sub>. (Also in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019))
- Theory for a FL\* phase in a  $t$ - $J$  model for cuprates: ancilla qubits. This achieves a ‘democratic’ localization of some of the electrons into an insulator of spins, while the others form a Luttinger-volume-violating Fermi surface.

## FL\*

- Evidence for a FL\* phase in a Kondo lattice: CeCoIn<sub>5</sub>. (Also in CePdAl, Zhao *et al.*, Nature Physics **15**, 1261 (2019))
- Theory for a FL\* phase in a  $t$ - $J$  model for cuprates: ancilla qubits. This achieves a ‘democratic’ localization of some of the electrons into an insulator of spins, while the others form a Luttinger-volume-violating Fermi surface.
- Ancilla method yields a  $SU(2) \times U(1)$  gauge theory for a metal-metal quantum phase transition with a change in carrier density from  $p$  in a FL\* phase, to  $1 + p$  in a FL phase. Prediction: critical ‘ghost’ Fermi surfaces near the transition.

# Cuprates



Evidence for ghost Fermi surfaces in the  $FL^*$ - $FL$  transition in a single-band model ?

# CeCoIn<sub>5</sub>

