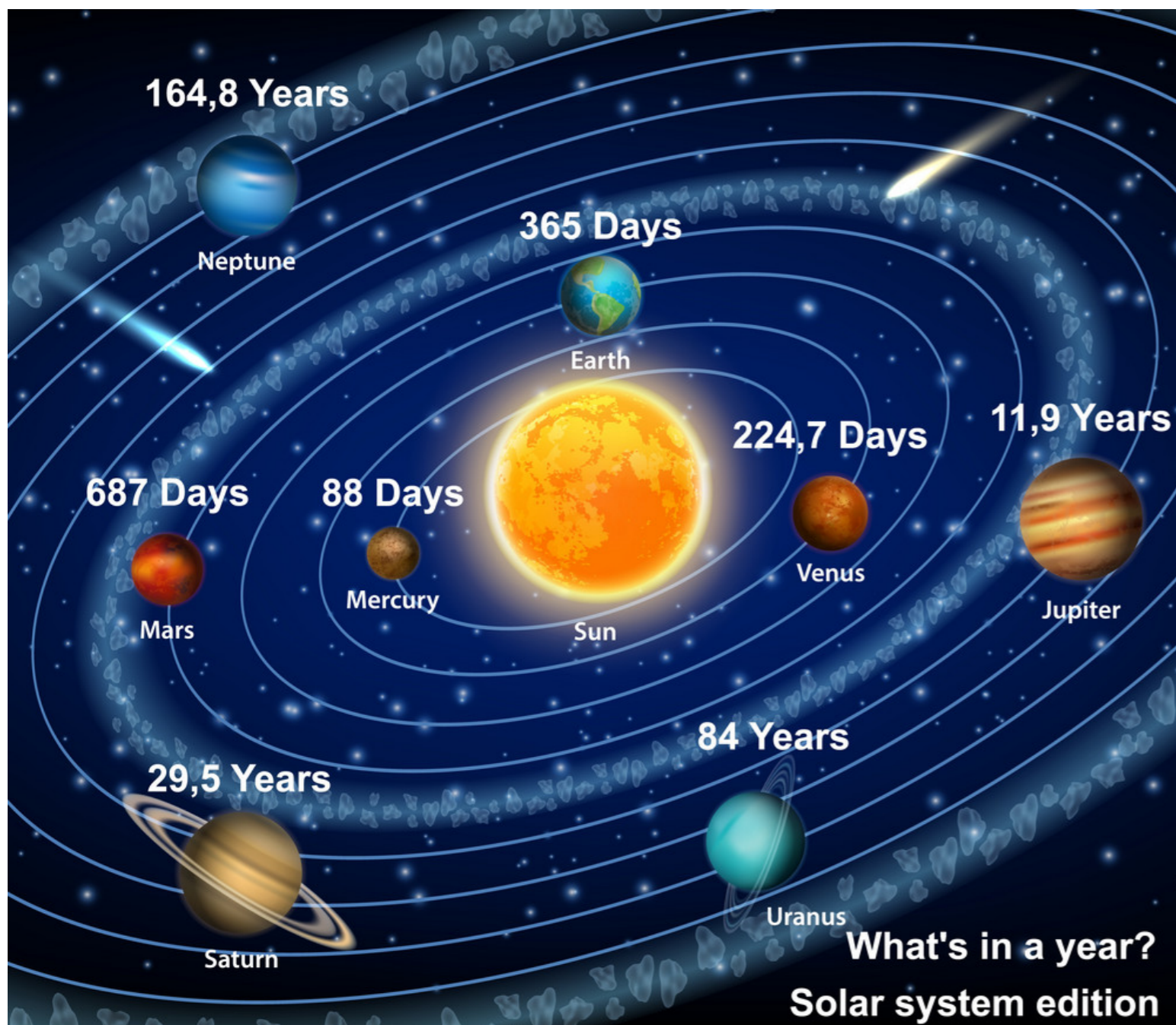


Quantum entanglement at all distances

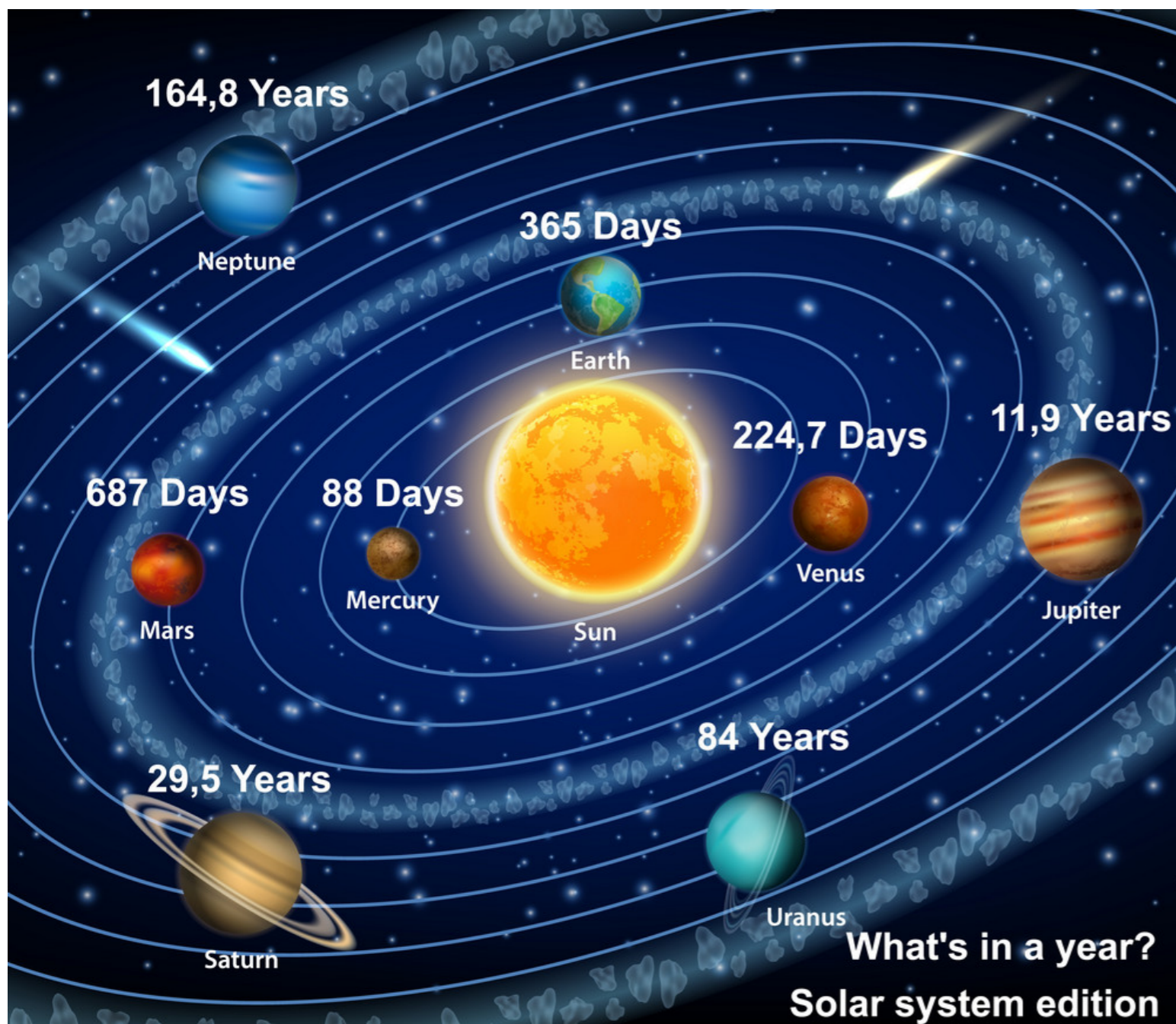
2021 H.L. Welsh Lectures in Physics
University of Toronto
May 6, 2021
Subir Sachdev



Talk online: sachdev.physics.harvard.edu



Newton showed (1687) that the same laws of motion applied on planetary length scales (~ 1 trillion meters) and the length scale of an apple tree (1 meter).

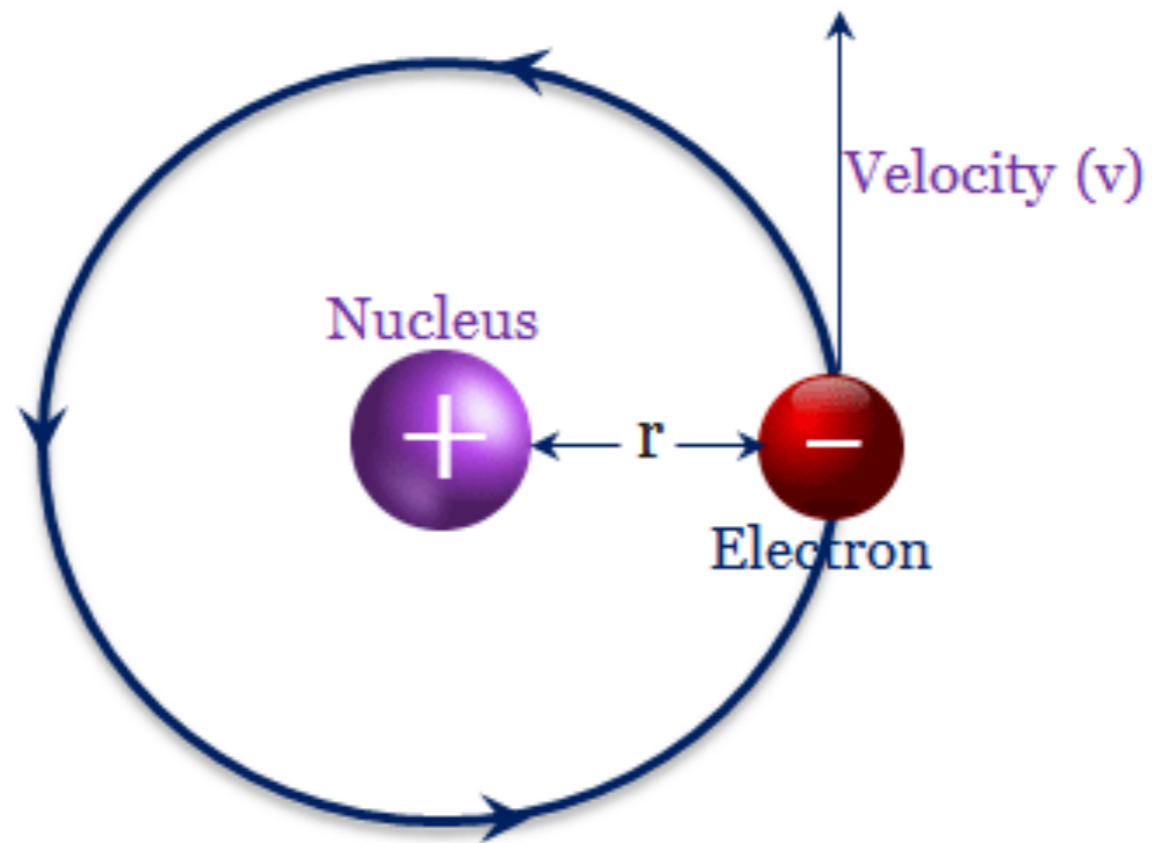


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What happens on smaller and larger distances ?

Going small.....

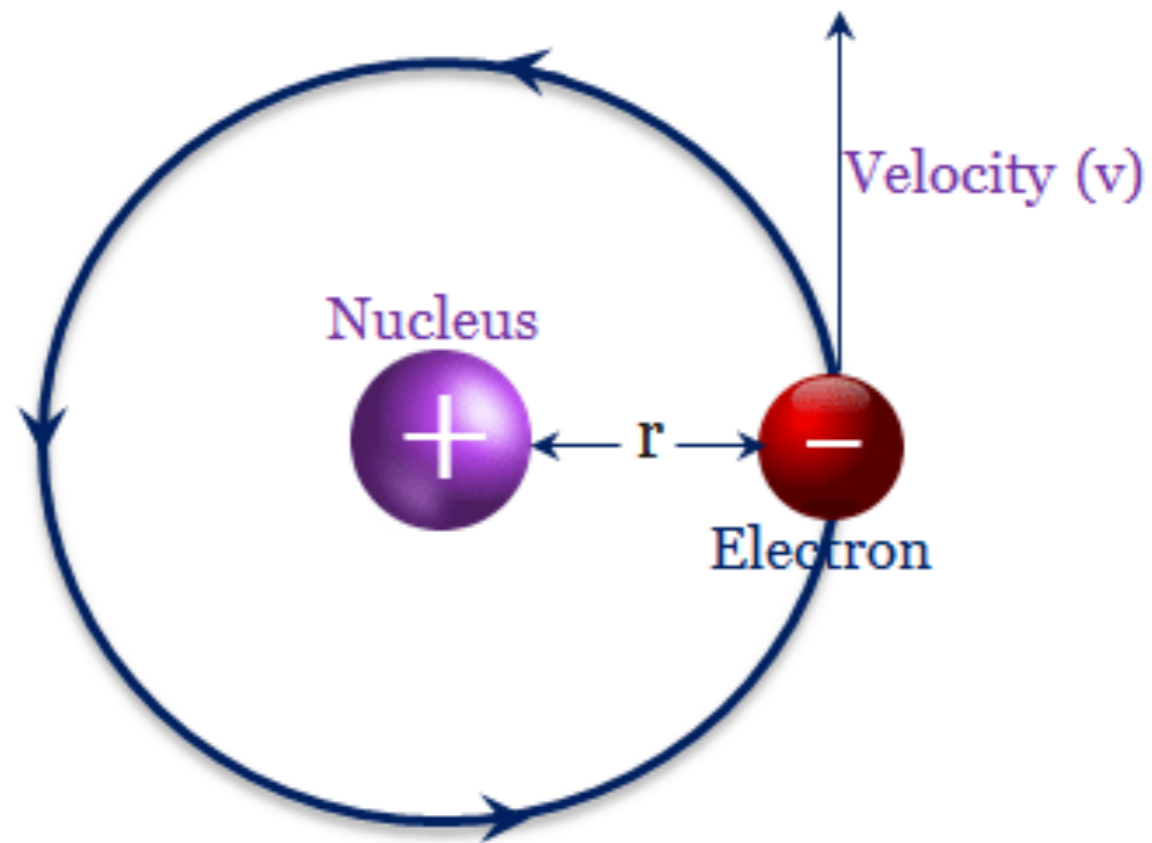
Hydrogen atom



$\Rightarrow 10^{-10}$ meters \Leftarrow

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

Hydrogen atom



$\Rightarrow 10^{-10}$ meters \Leftarrow

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

It is described by the quantum theory of Schrödinger and Heisenberg (1925).

Schrodinger's Cat

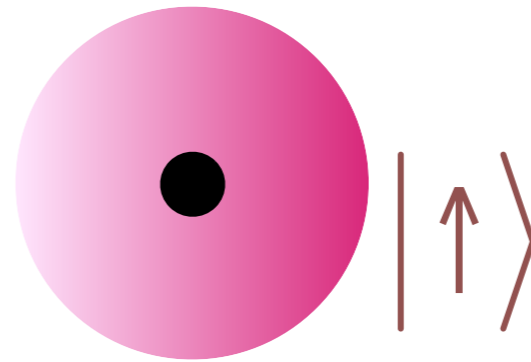


$$\frac{1}{\sqrt{2}} |\text{cat sitting}\rangle + \frac{1}{\sqrt{2}} |\text{cat dead}\rangle$$

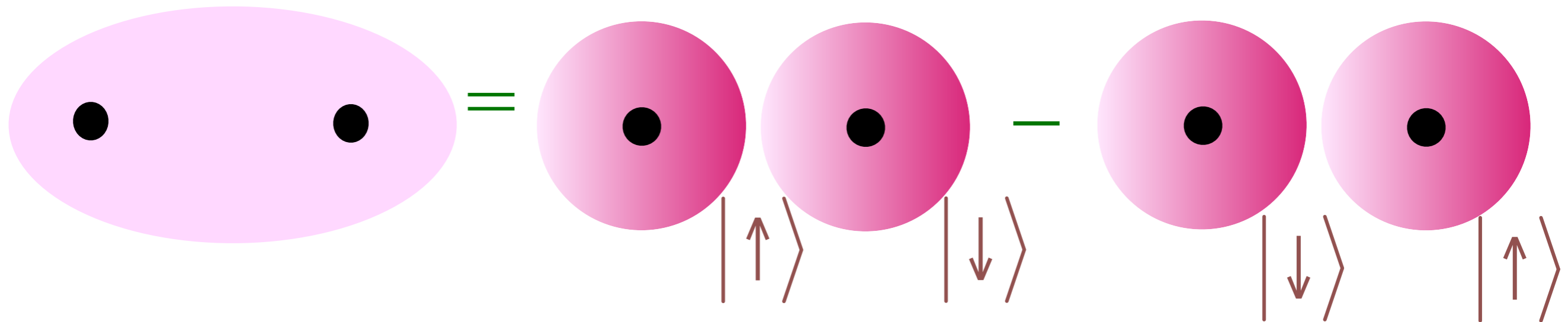
The most remarkable new idea in the quantum theory is the principle of superposition:
a physical system can be in a superposition of two (or more) distinct states.

Quantum superposition in a hydrogen molecule

Hydrogen atom:



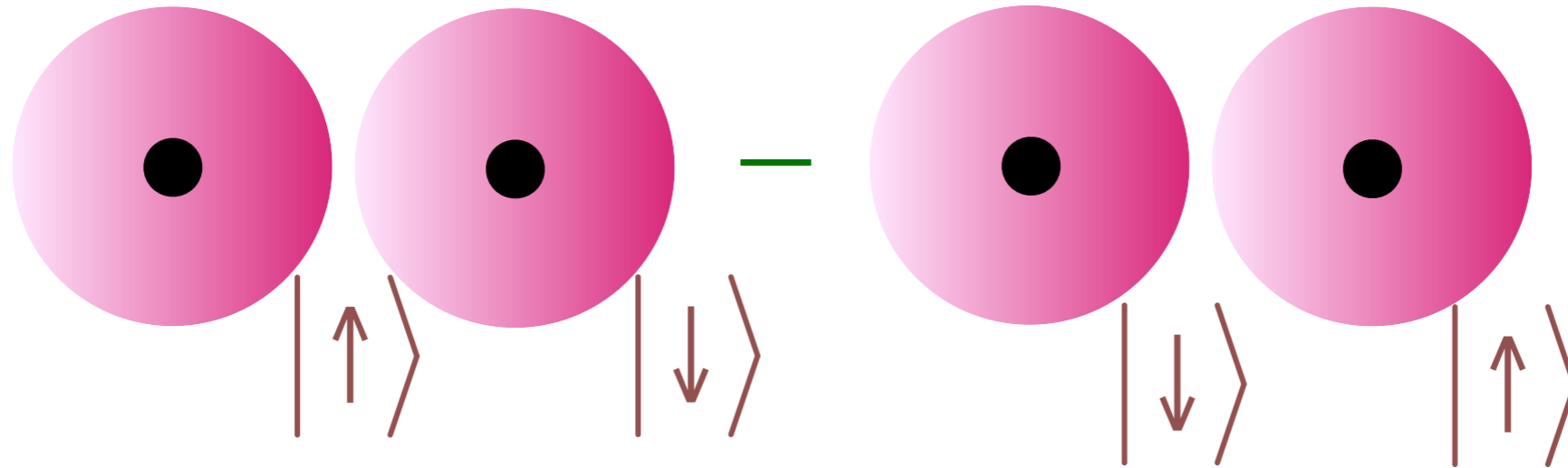
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

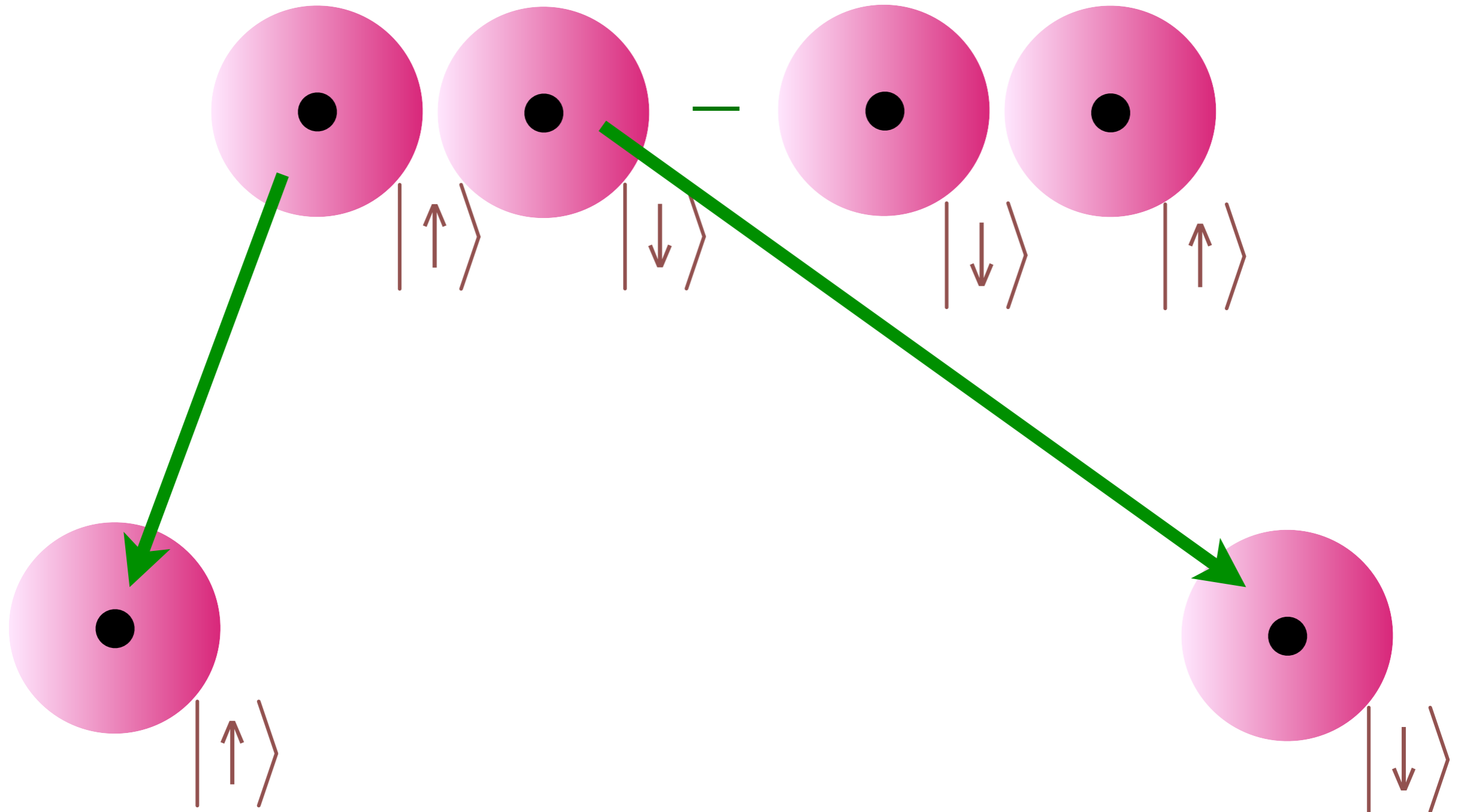
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



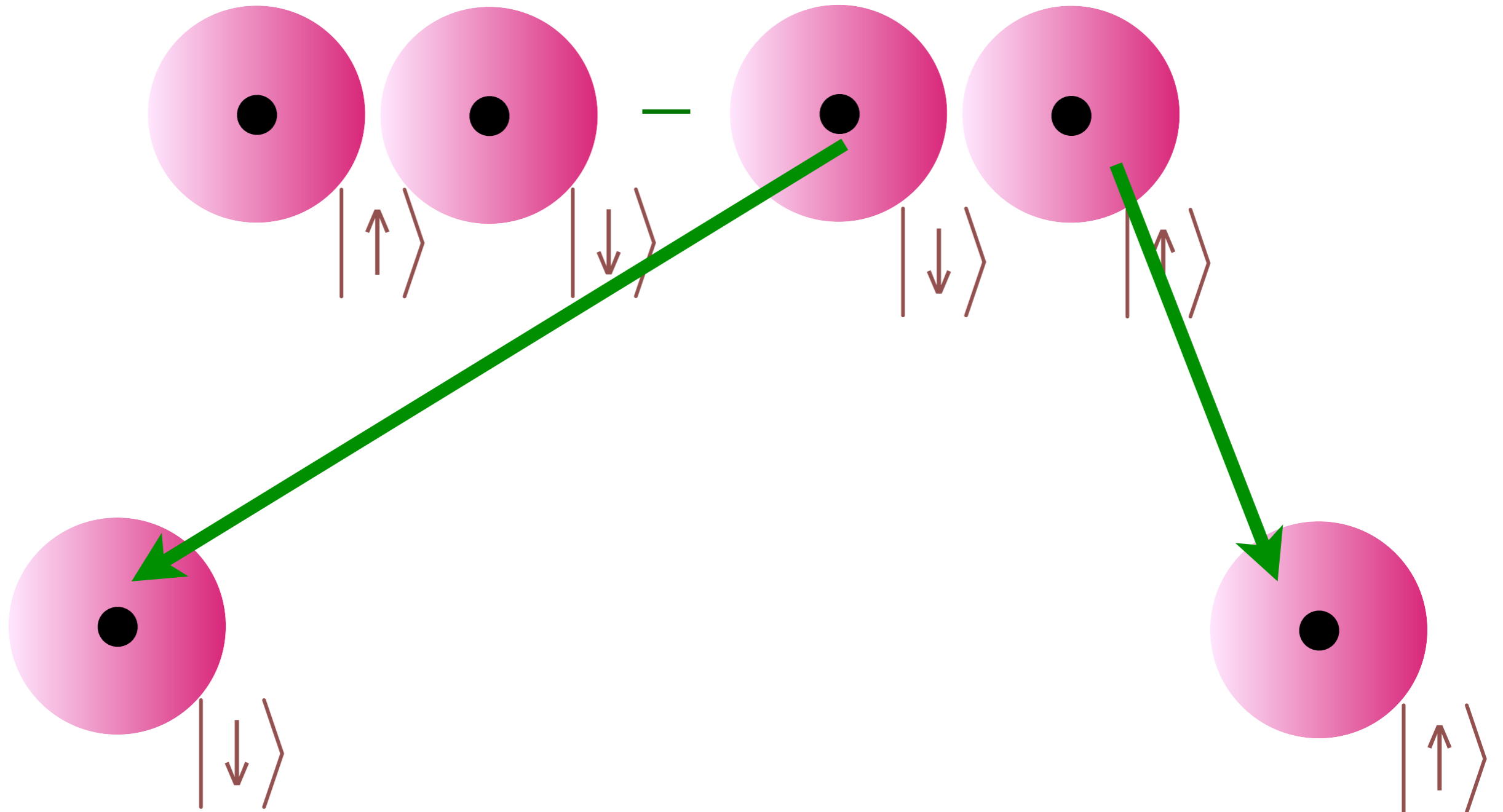
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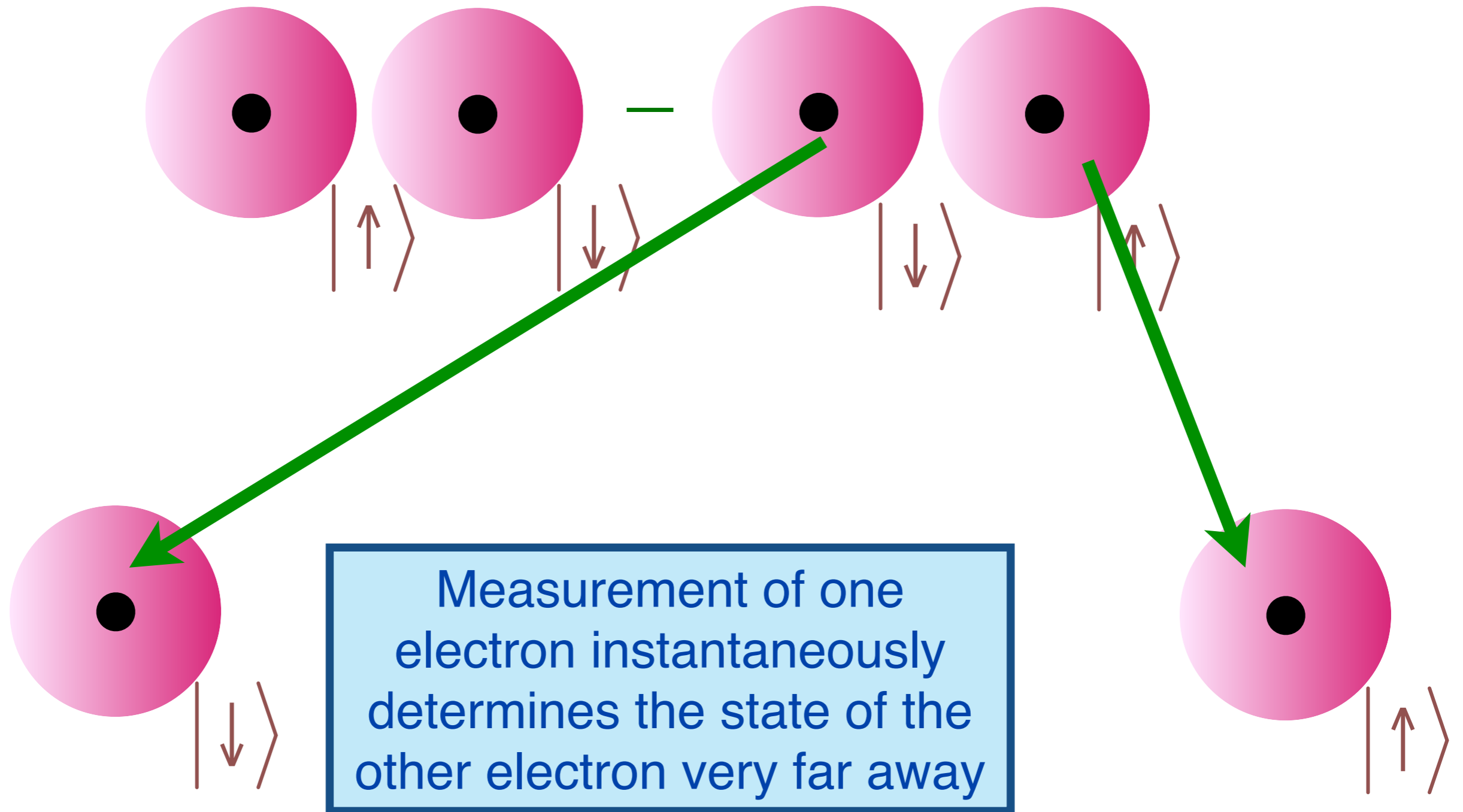
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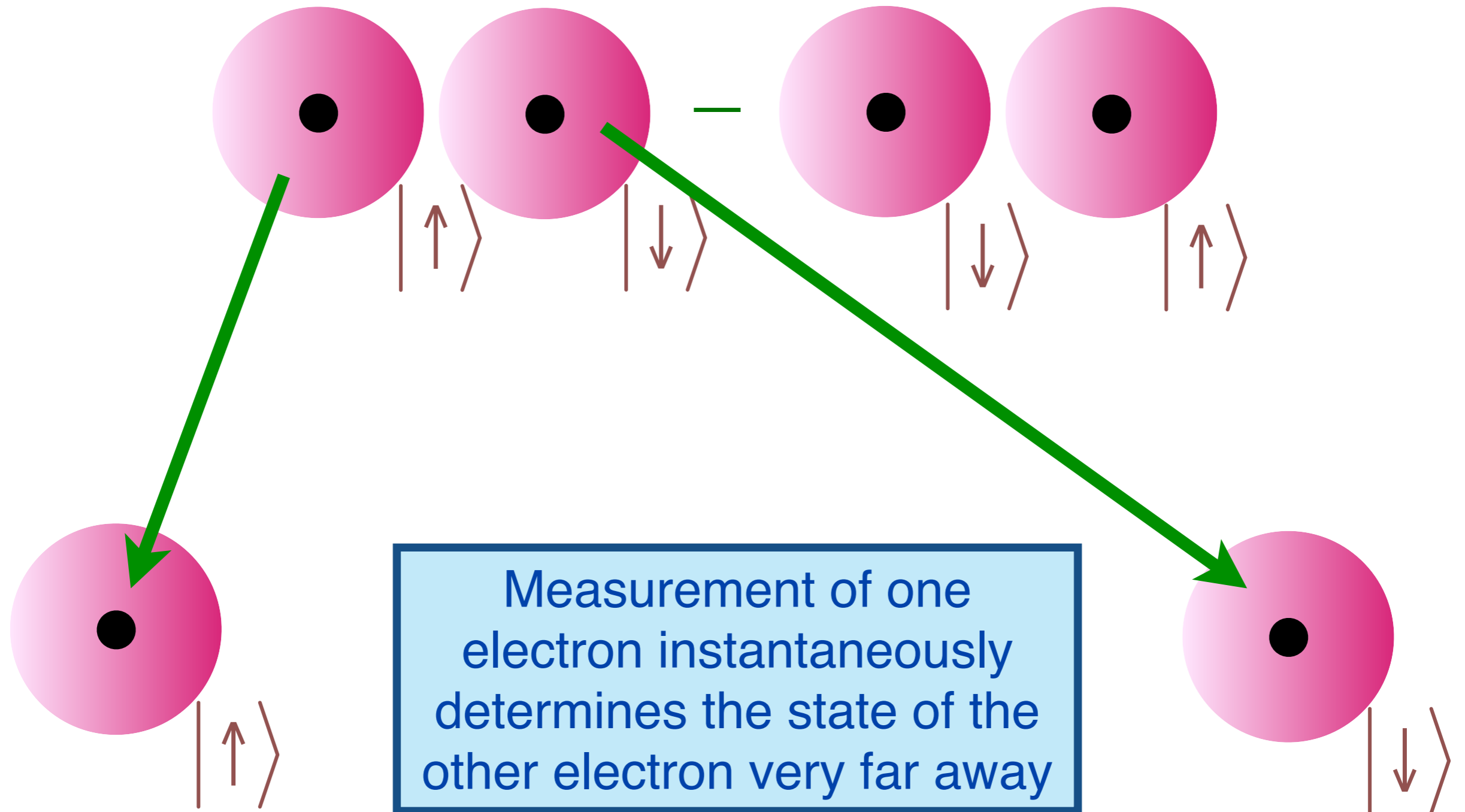
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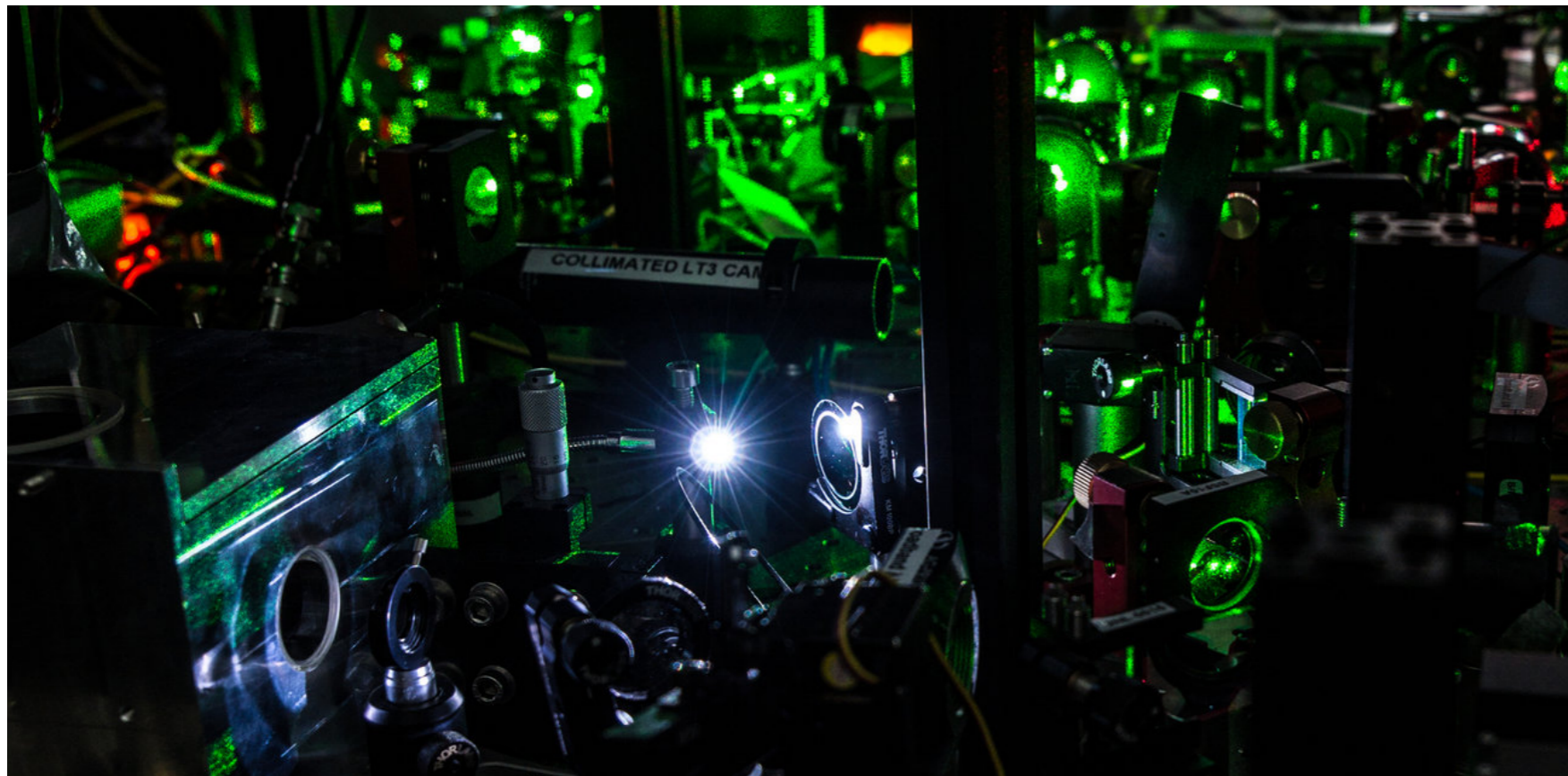
Einstein, Podolsky, Rosen (1935)



Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

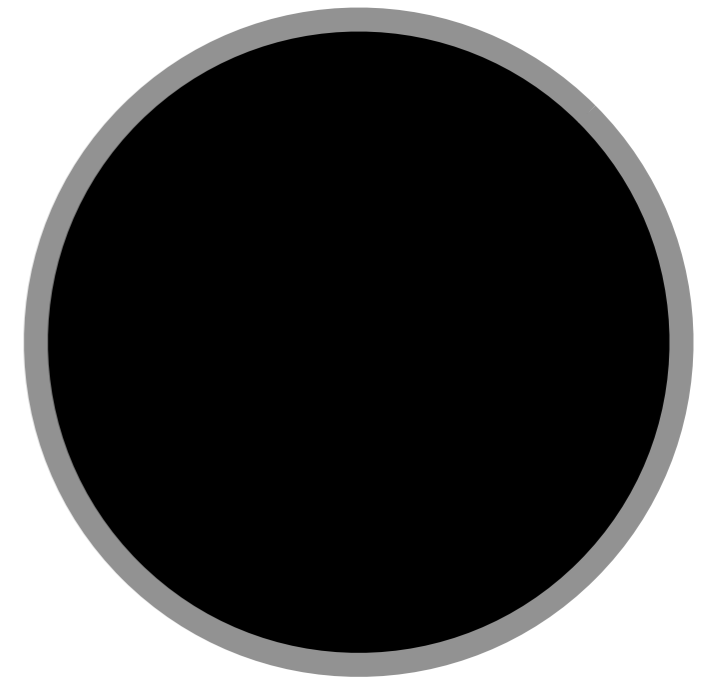


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Going big (and heavy).....

Black Holes

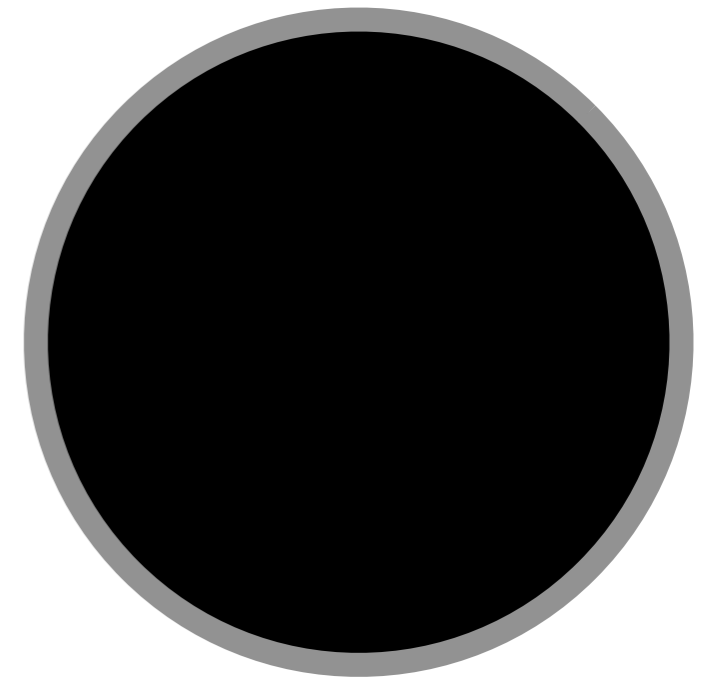
Objects so dense that light is gravitationally bound to them.



Black Holes

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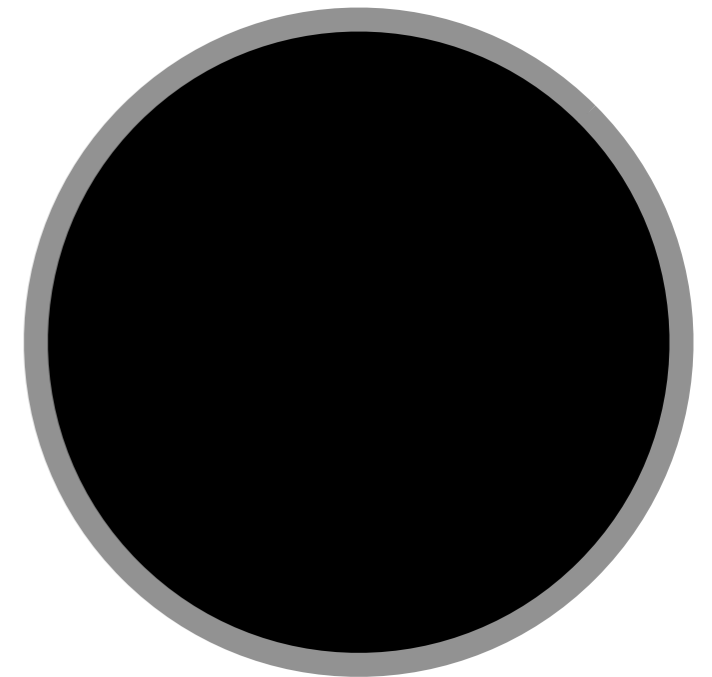
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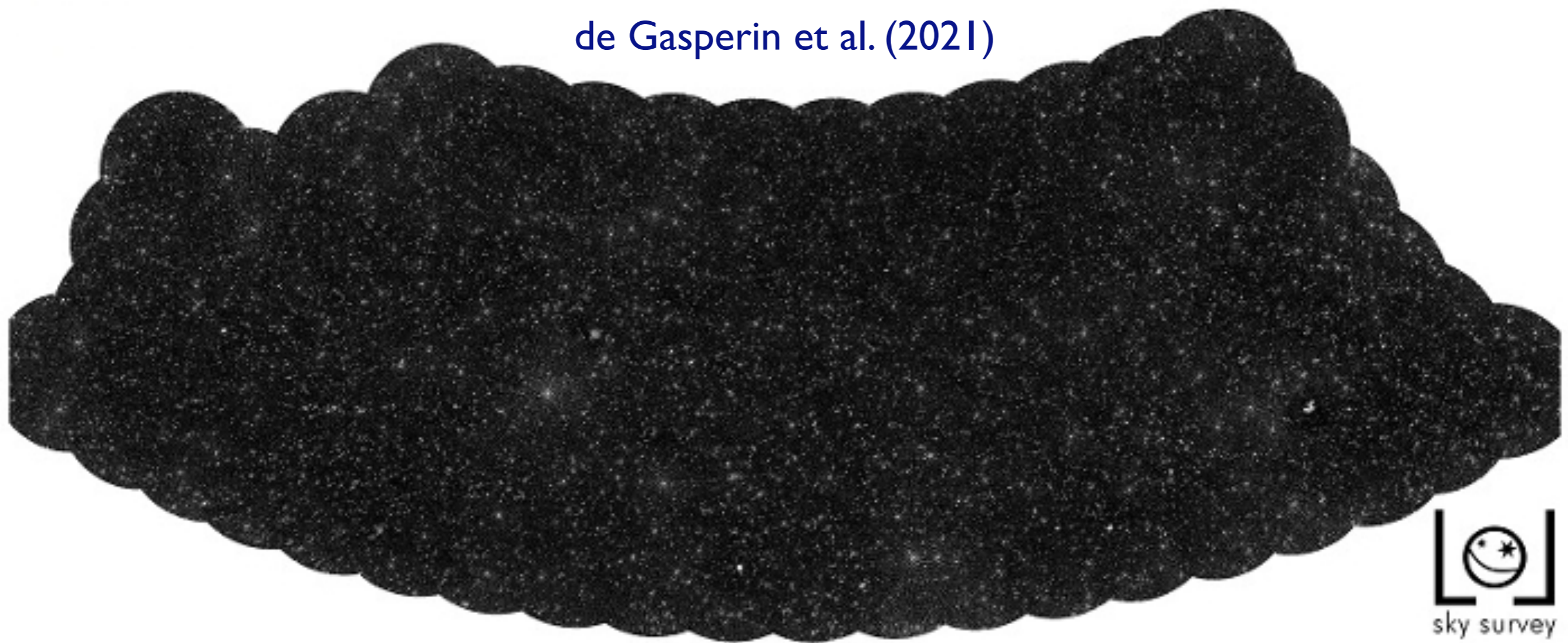
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$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

G Newton's constant, c velocity of light, M mass of black hole

For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

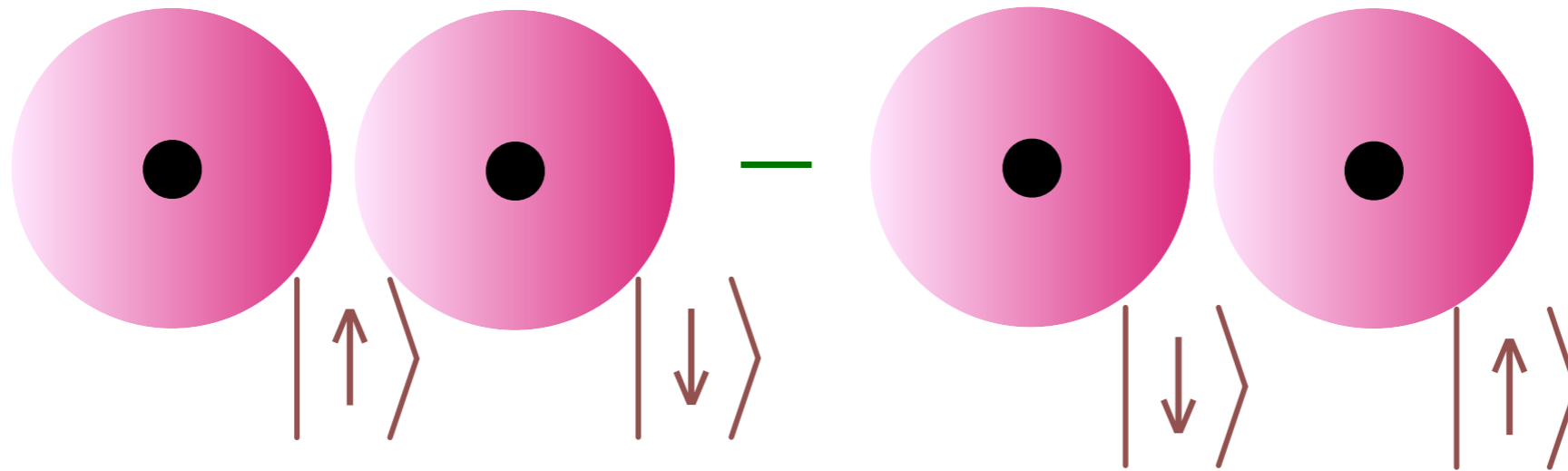


**LOFAR LBA Sky Survey showing 25000 supermassive
black holes on 4% of the northern sky.
Obtained by 52 radio telescopes across Europe**

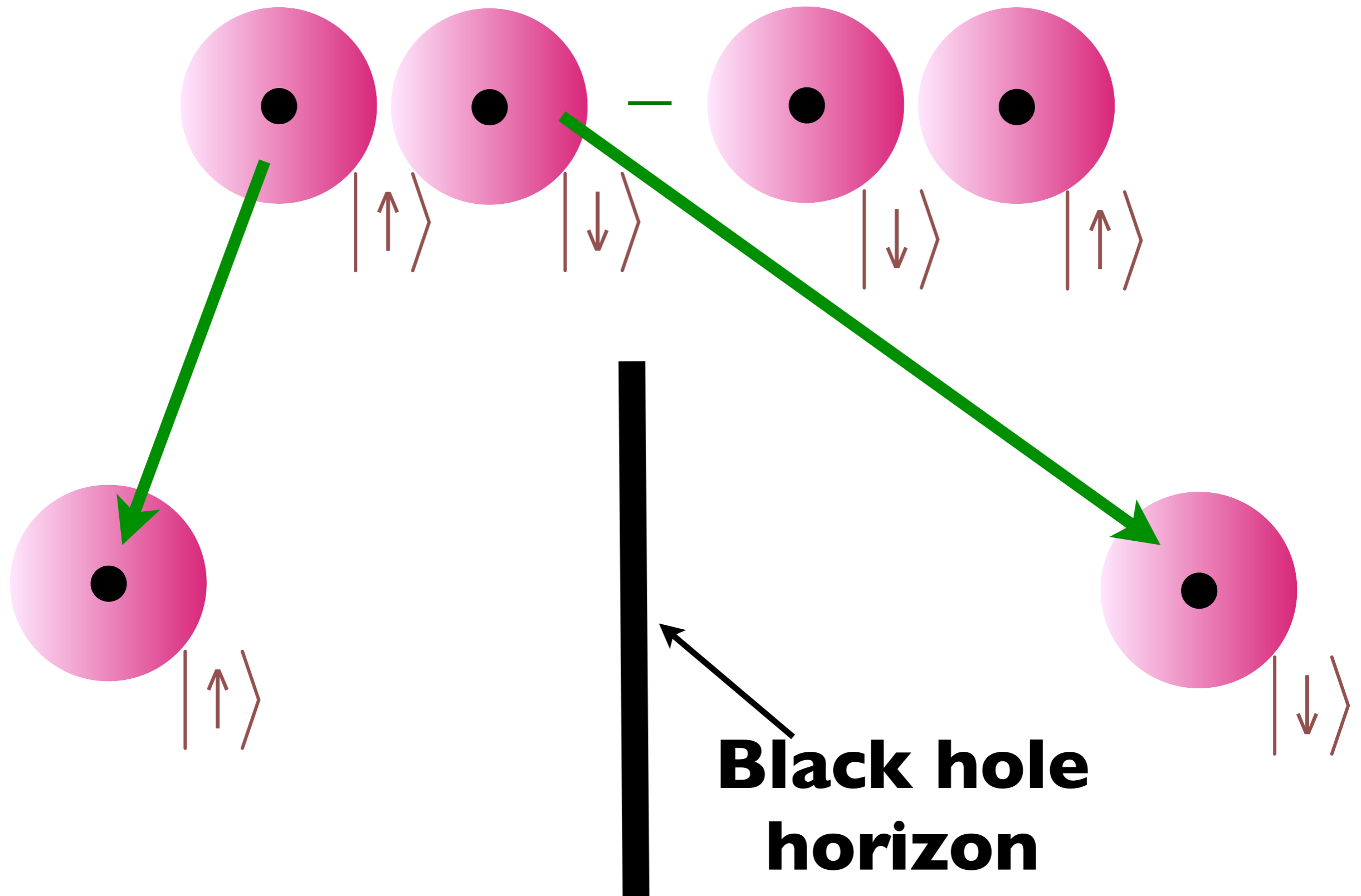
Each black hole has a size $\sim 10^{10}$ meters,
and is at the center of its own galaxy (like the Milky way)

Applying the theory of the
very small
to the
very big (black holes)....

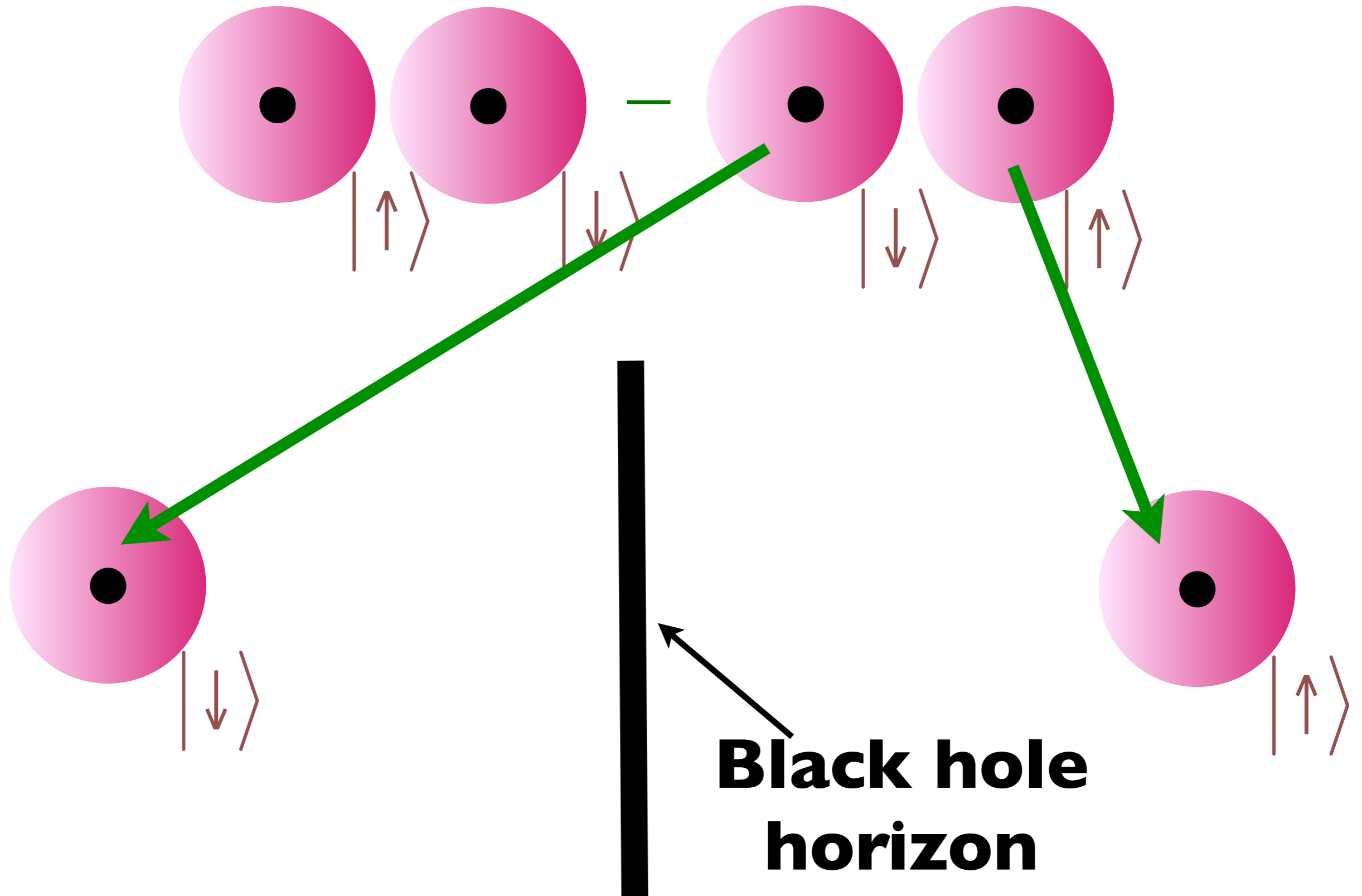
Quantum Entanglement across a black hole horizon



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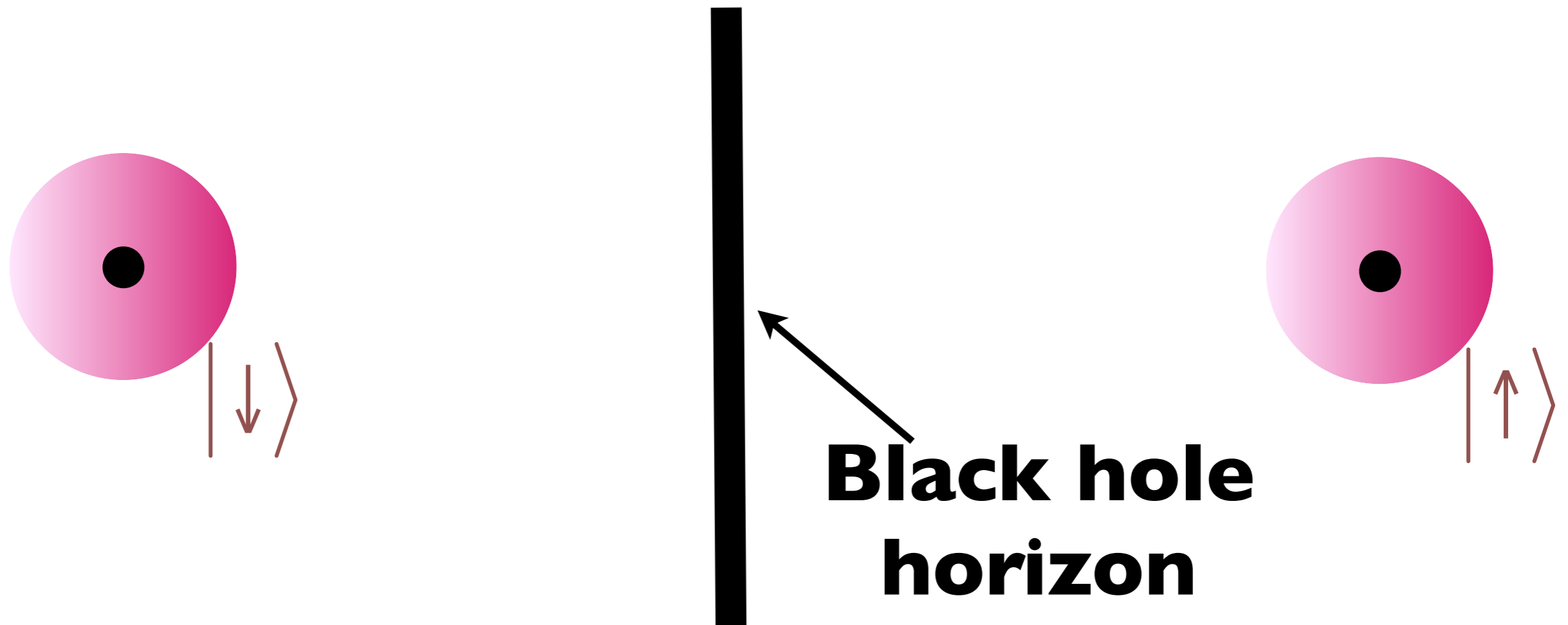


Quantum Entanglement across a black hole horizon



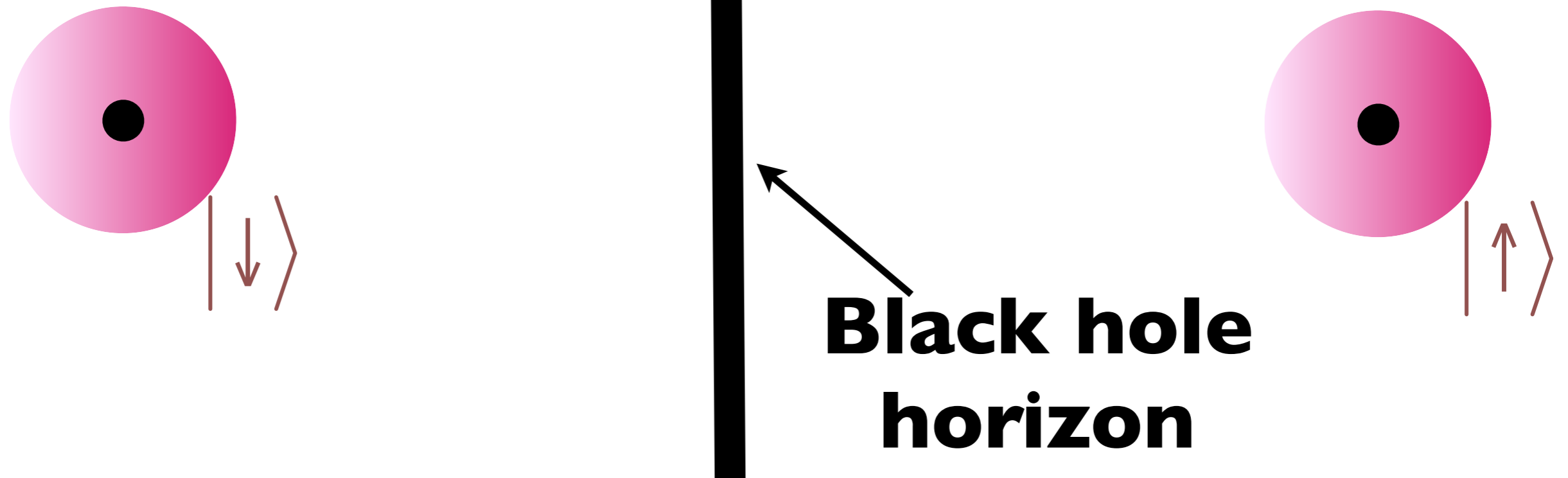
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



Going small and big.....

We don't have a completely general quantum theory of black holes today.

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Such a theory requires that we understand multi-particle entanglement for many-many particles.

The problem of multi-particle entanglement
also appears in other problems
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Quantum computing:

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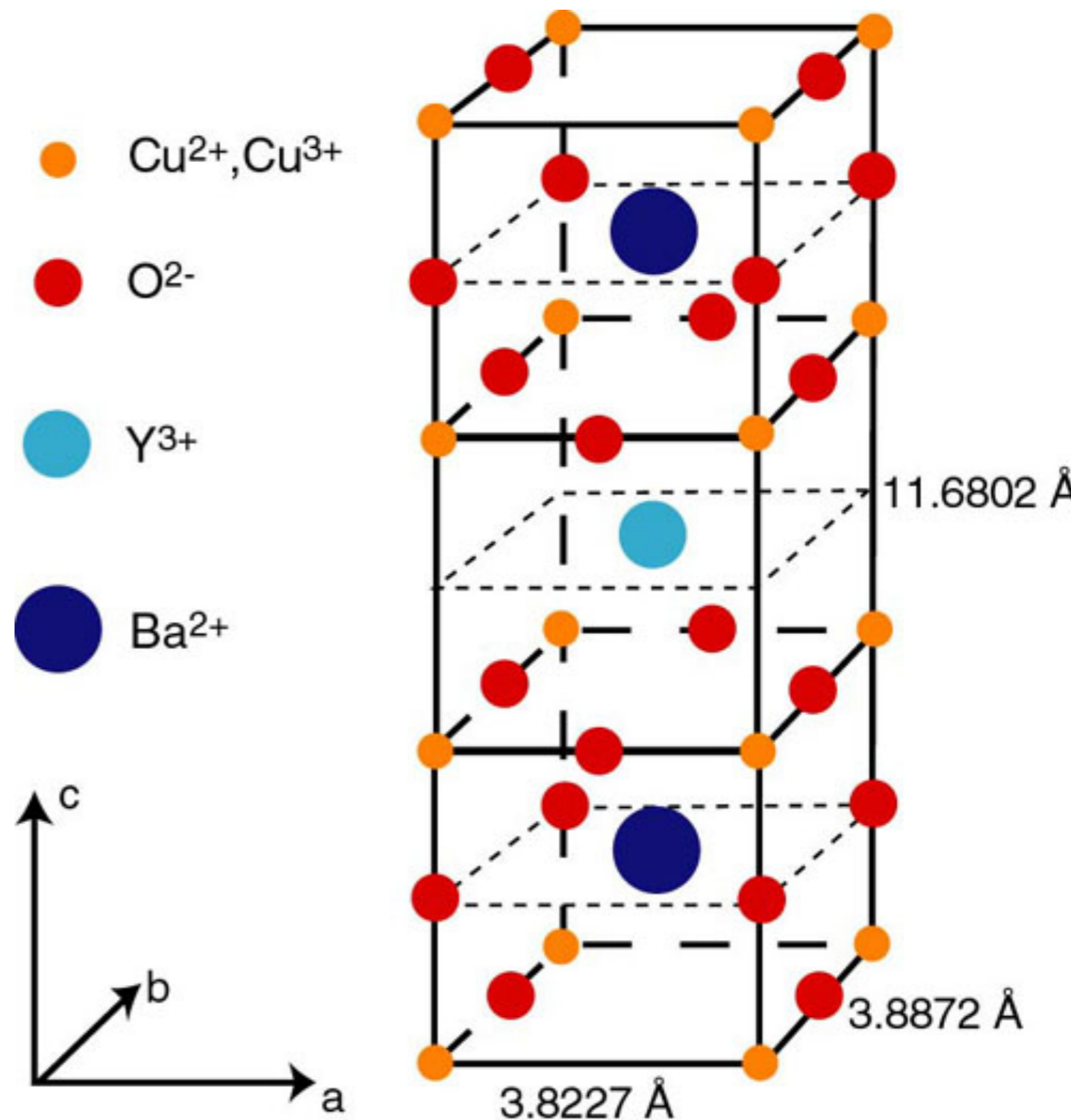
Quantum computing:

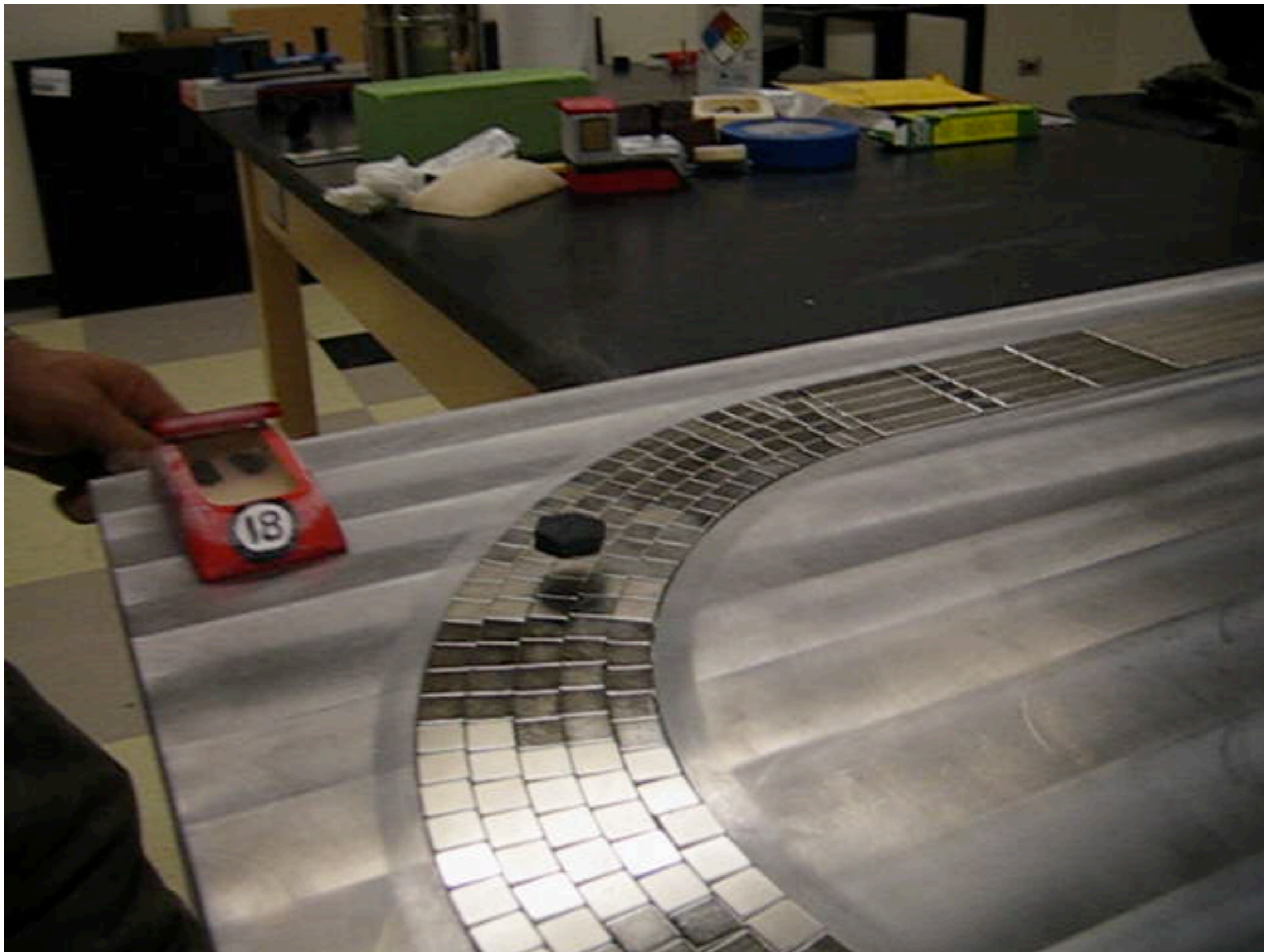
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Quantum materials:

crystals with multiple elements
naturally display phases with
multi-electron entanglement.

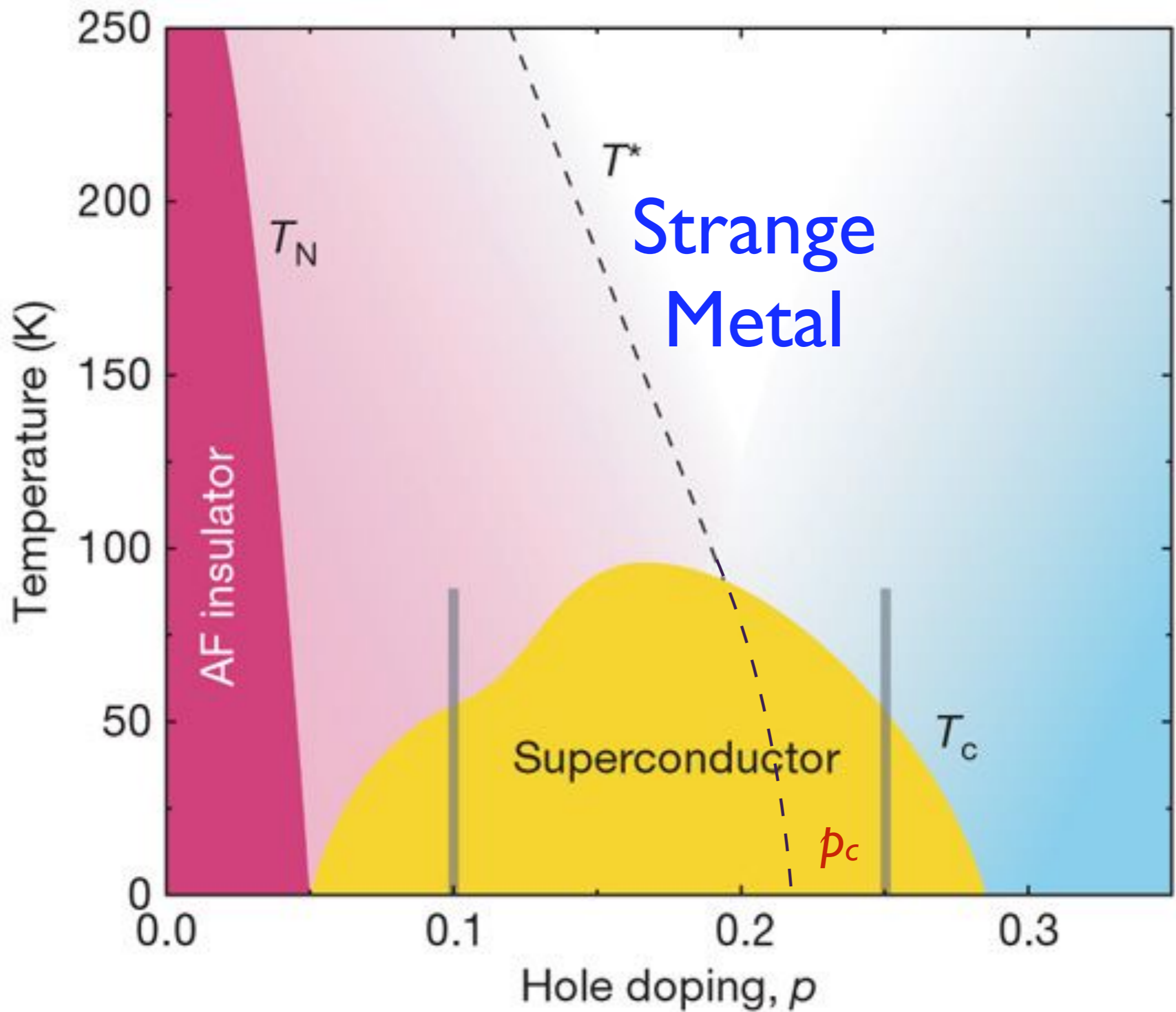
High temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



The SYK model:
a theory of entanglement,
from the very small
to the very big.....

The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

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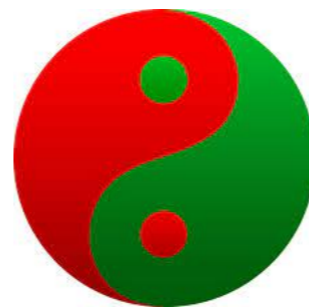
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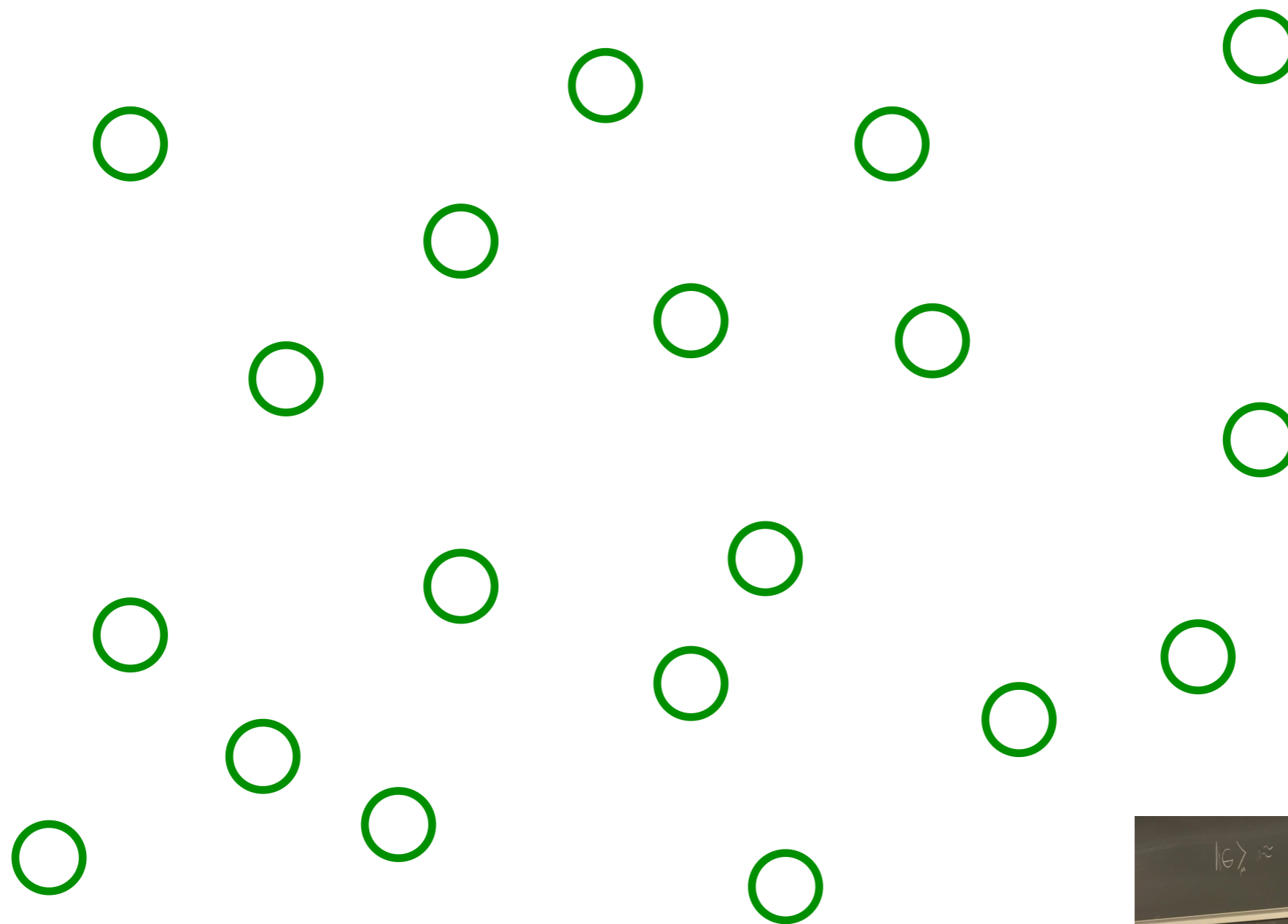


In a ***dual*** set of variables it describes certain ***black holes***

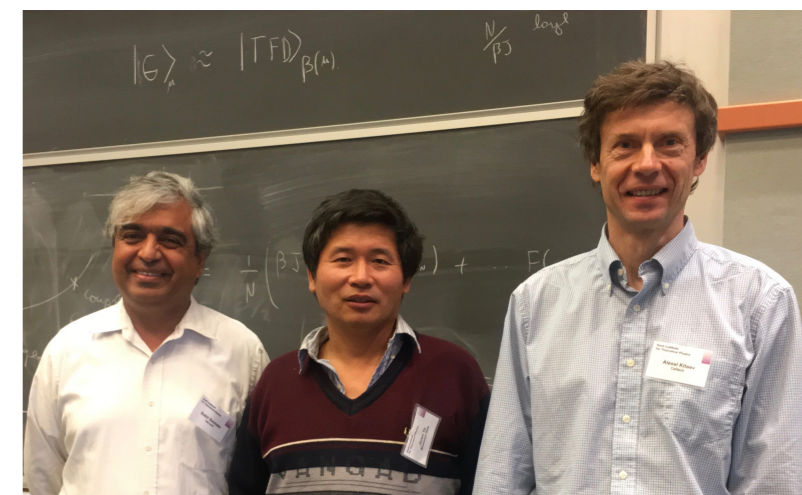
Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

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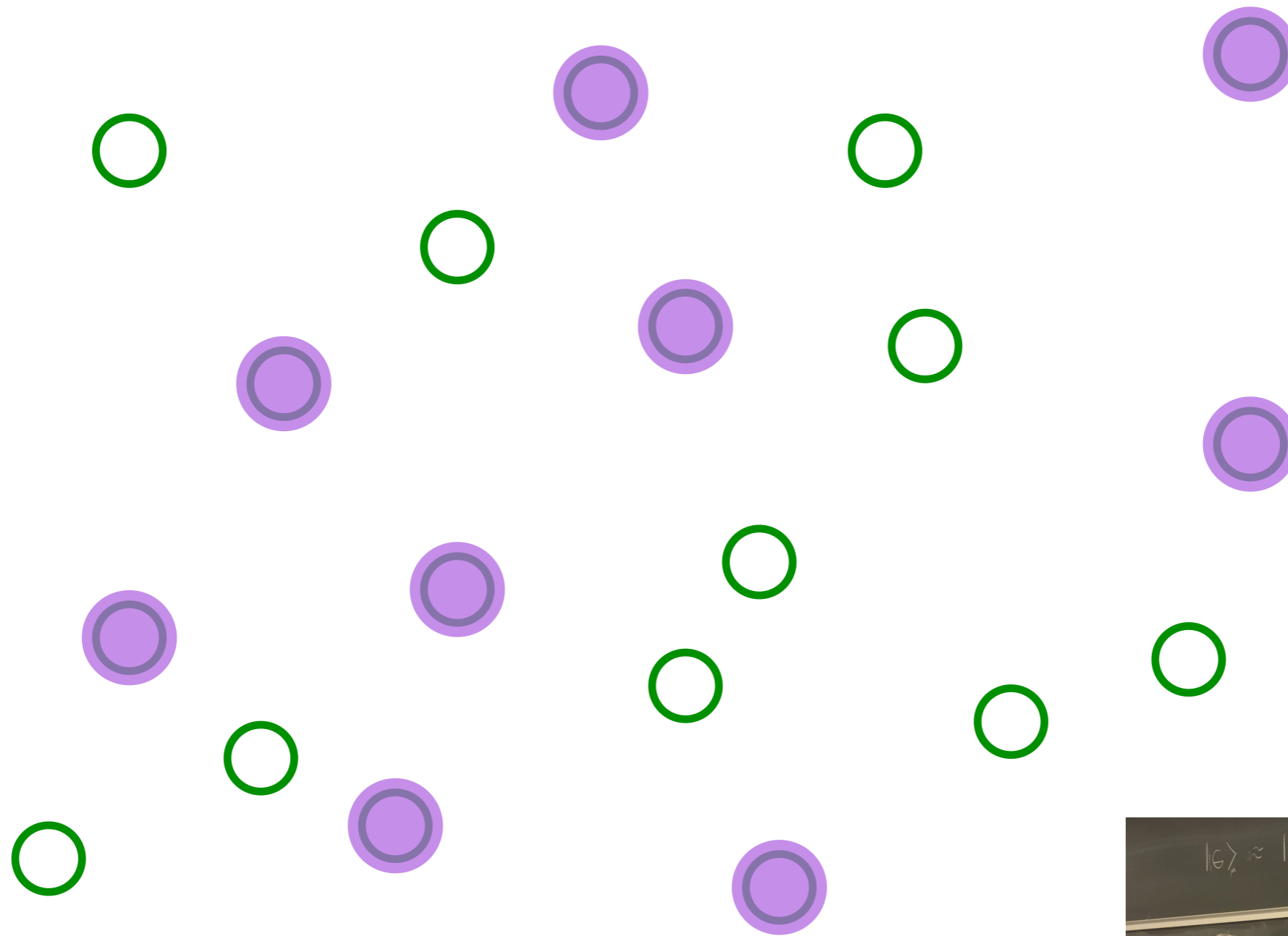


Pick a set of random positions

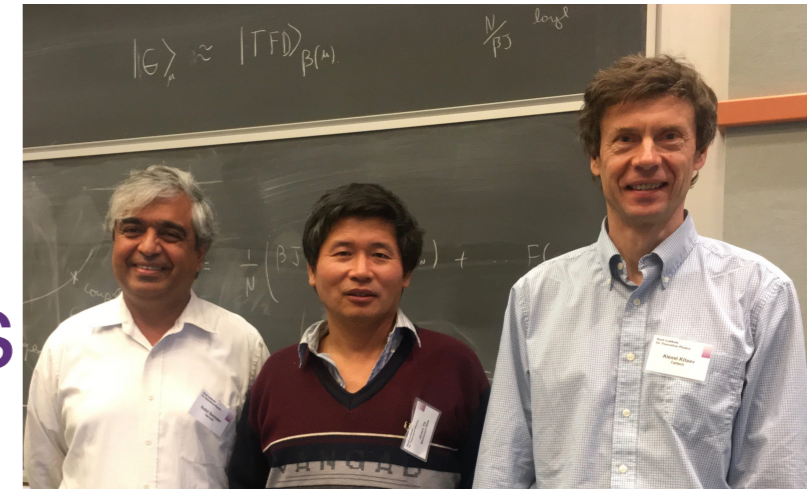


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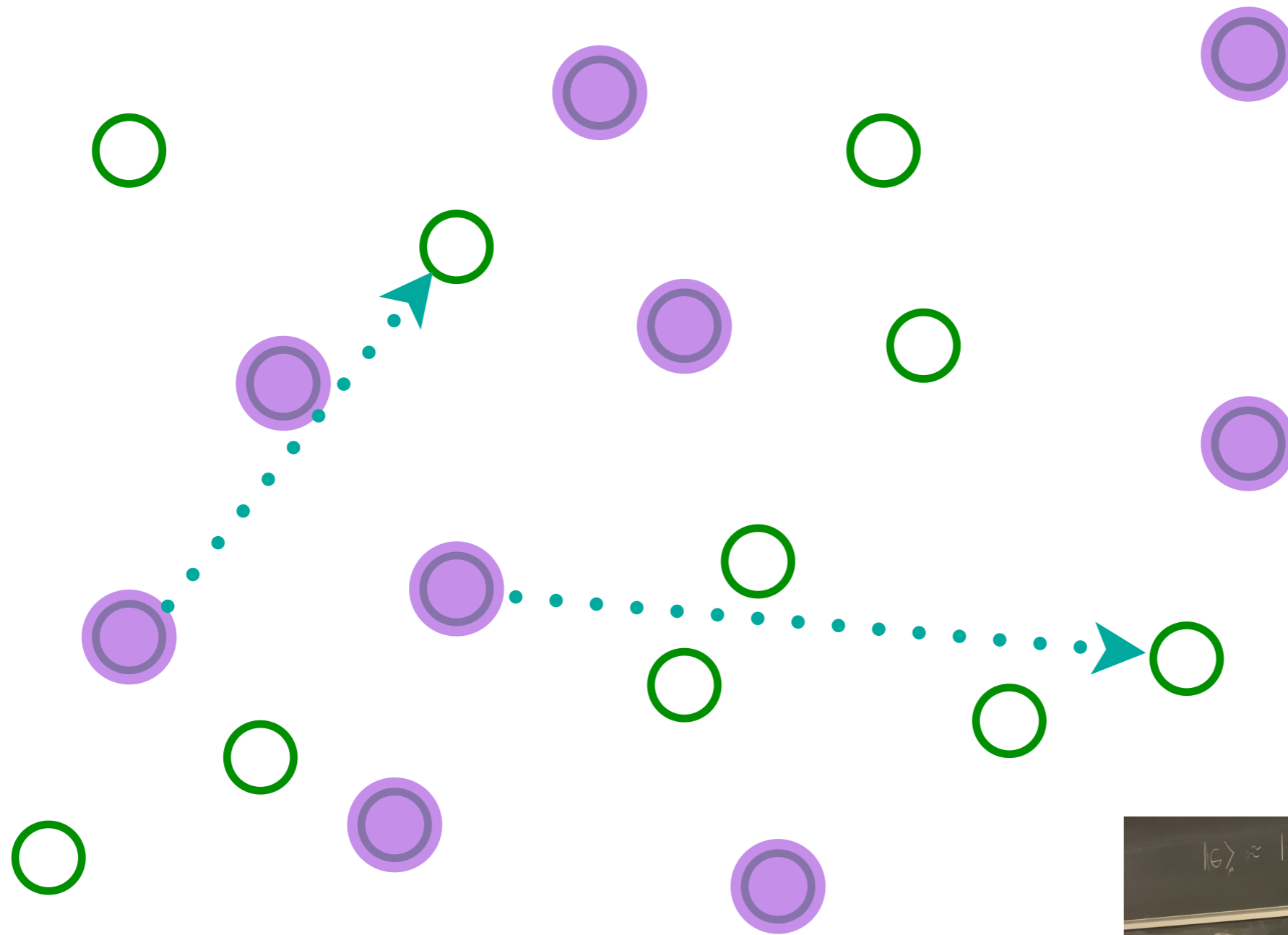


Place electrons randomly on some sites

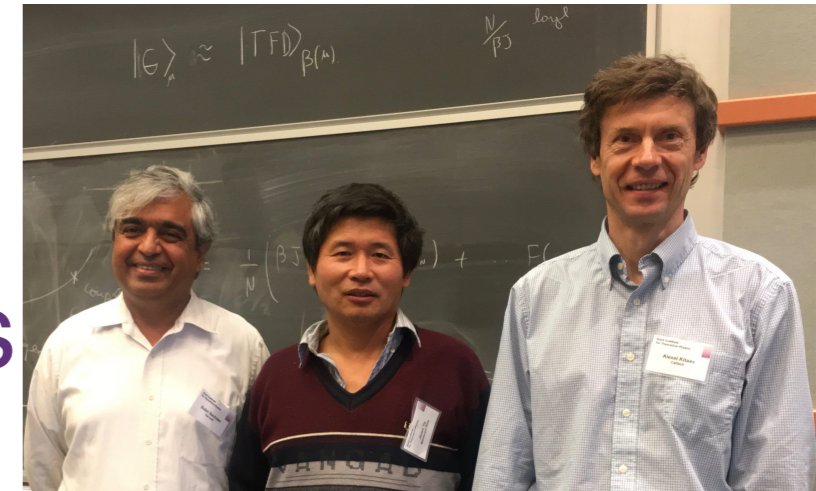


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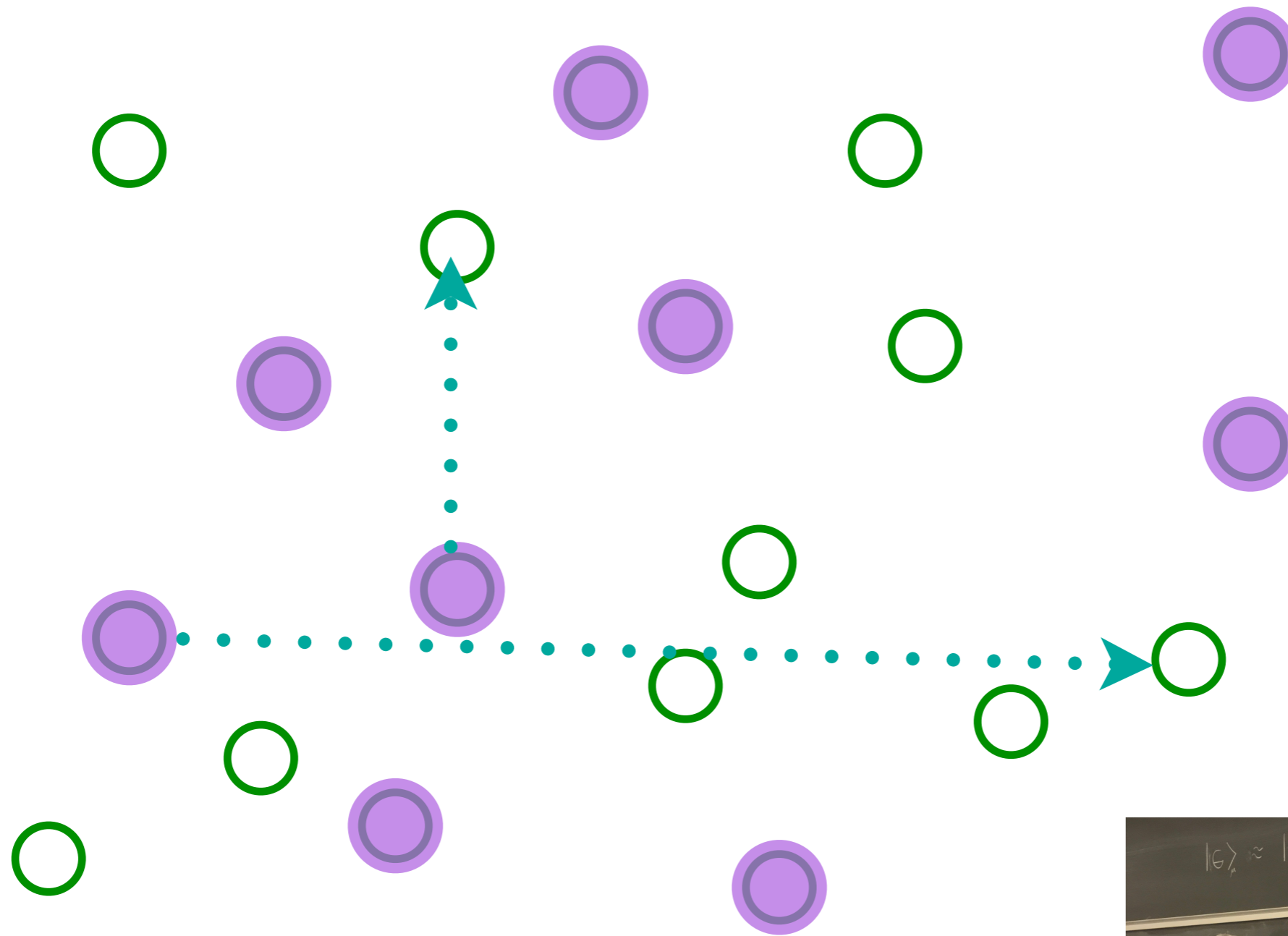


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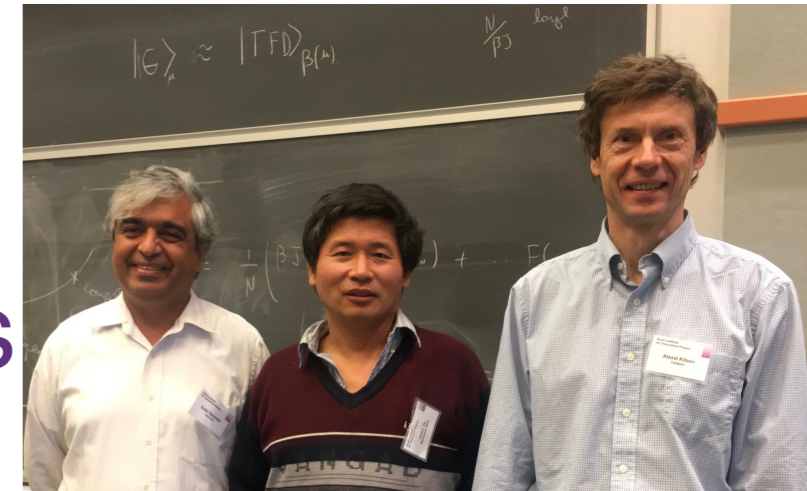


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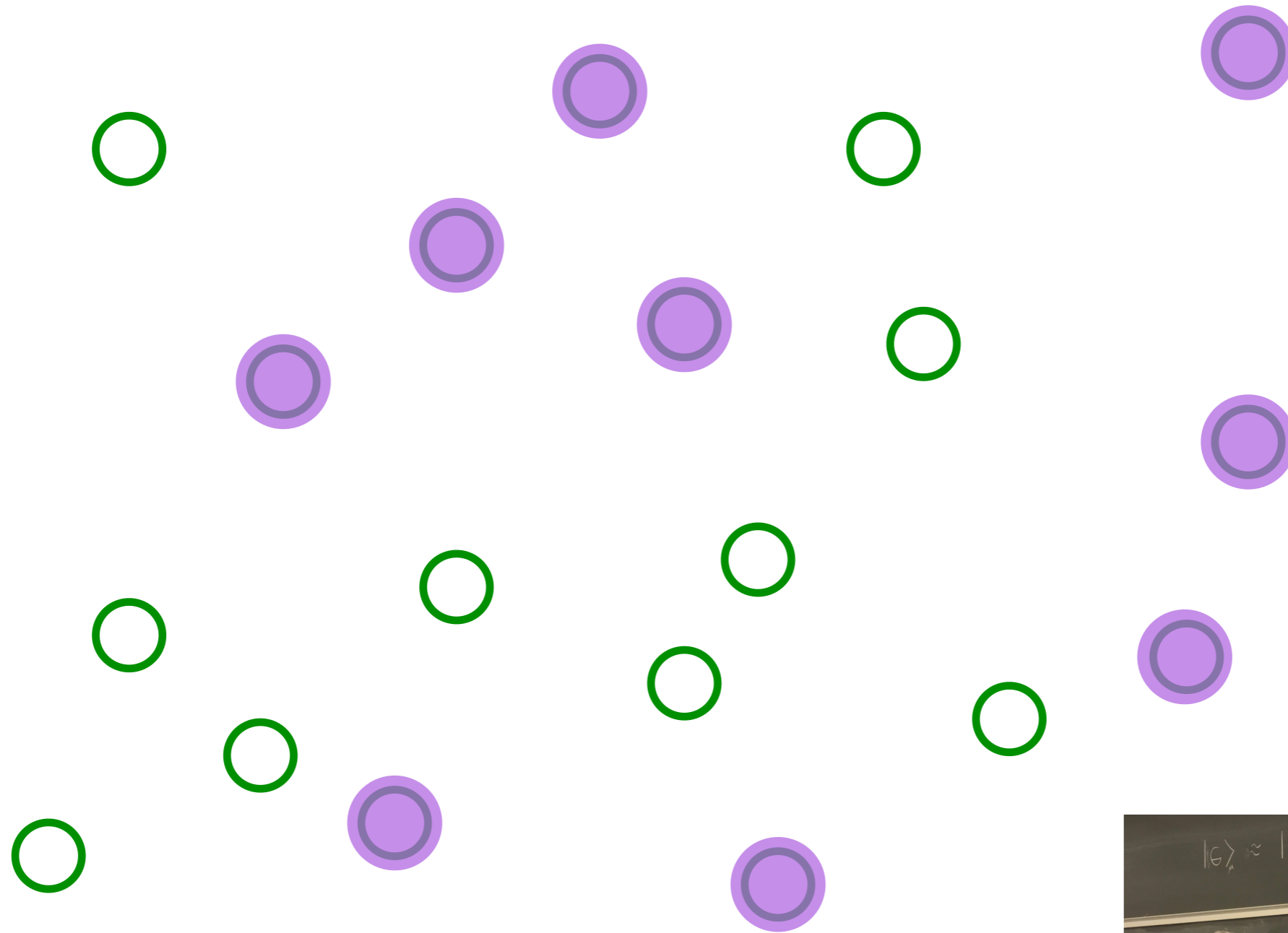


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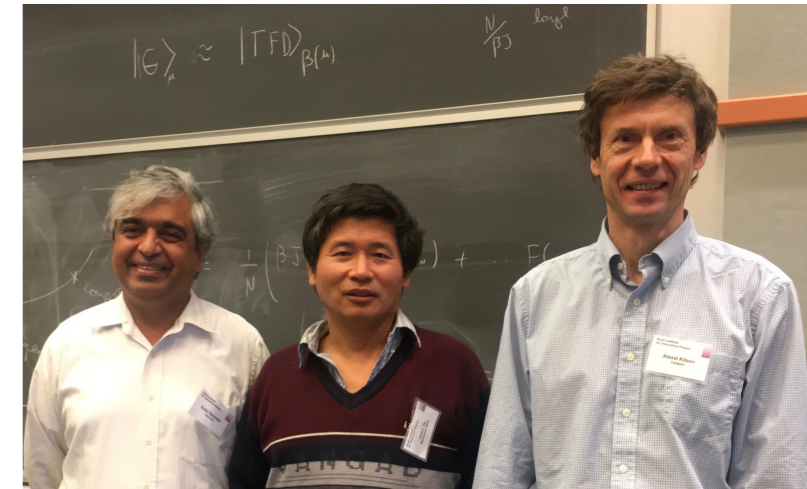


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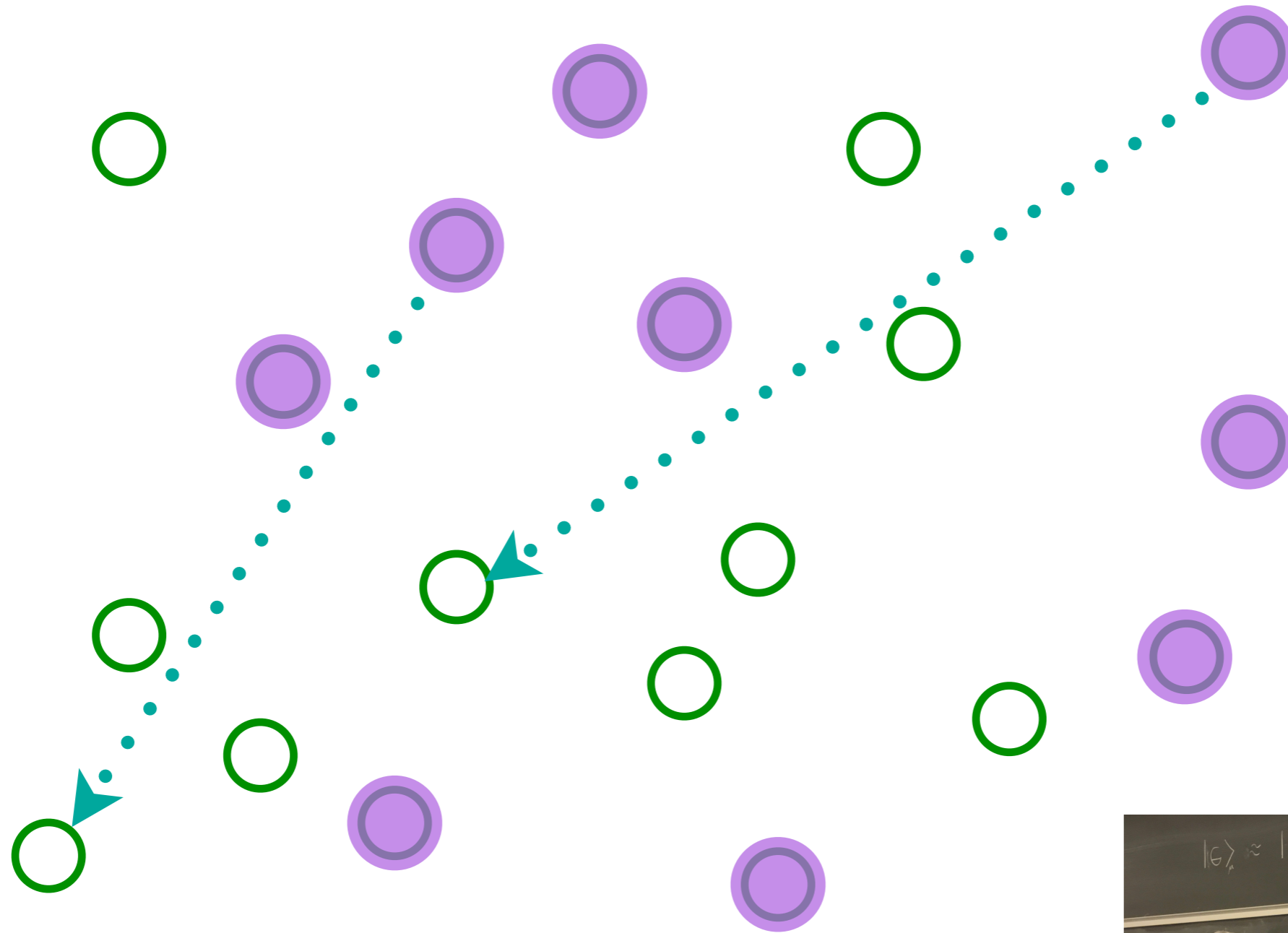


Entangle electrons pairwise randomly

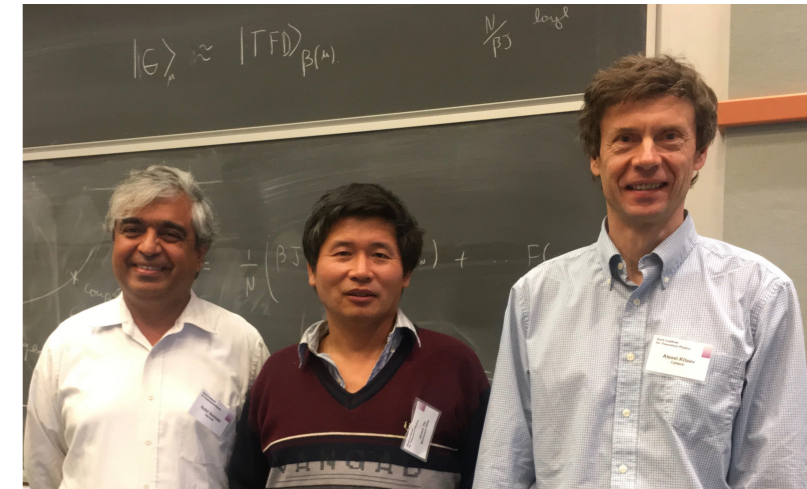


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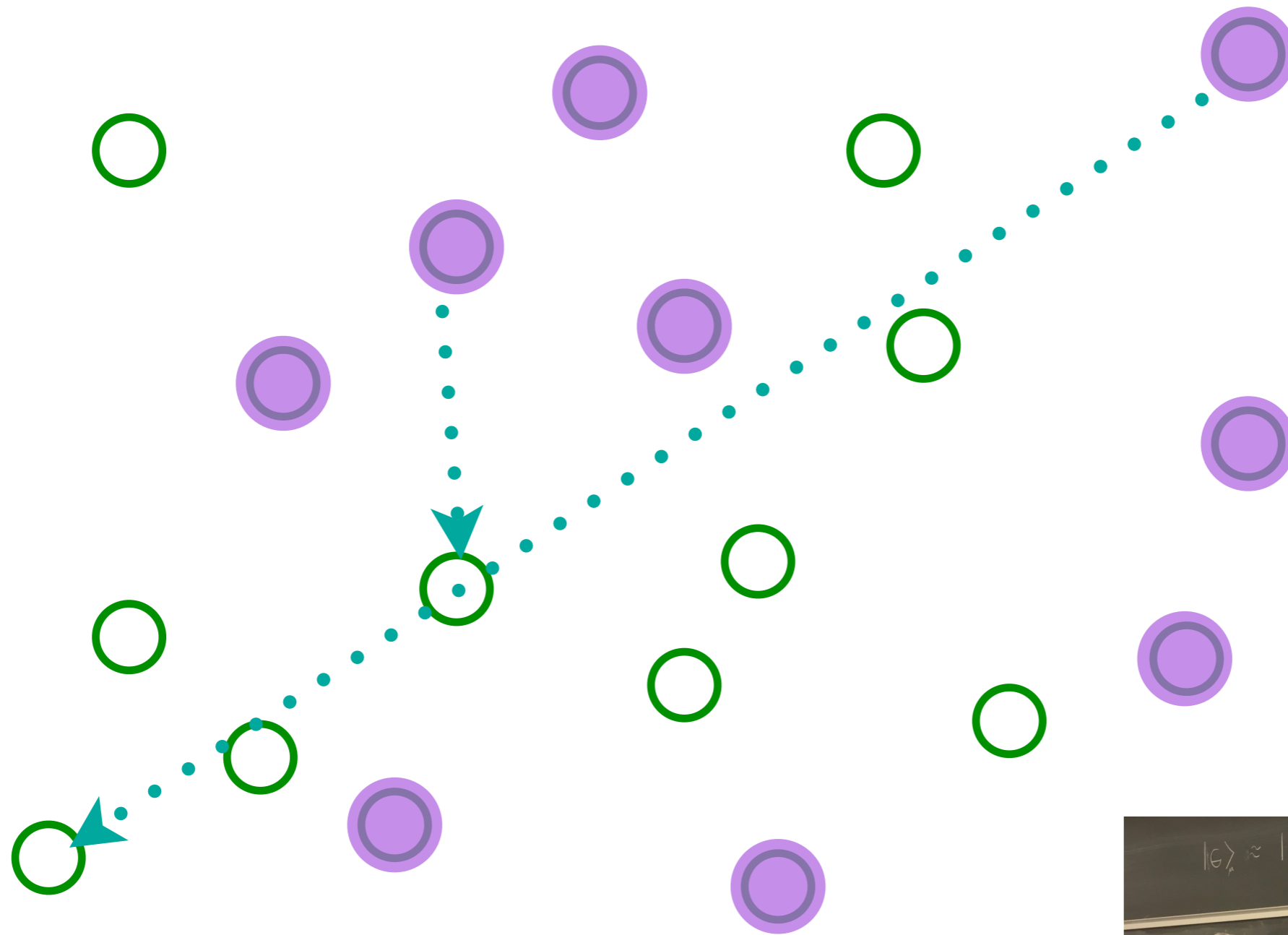


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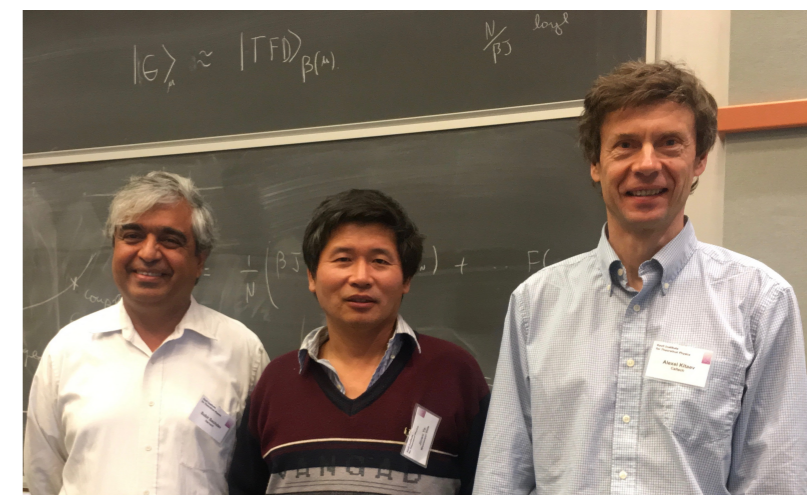


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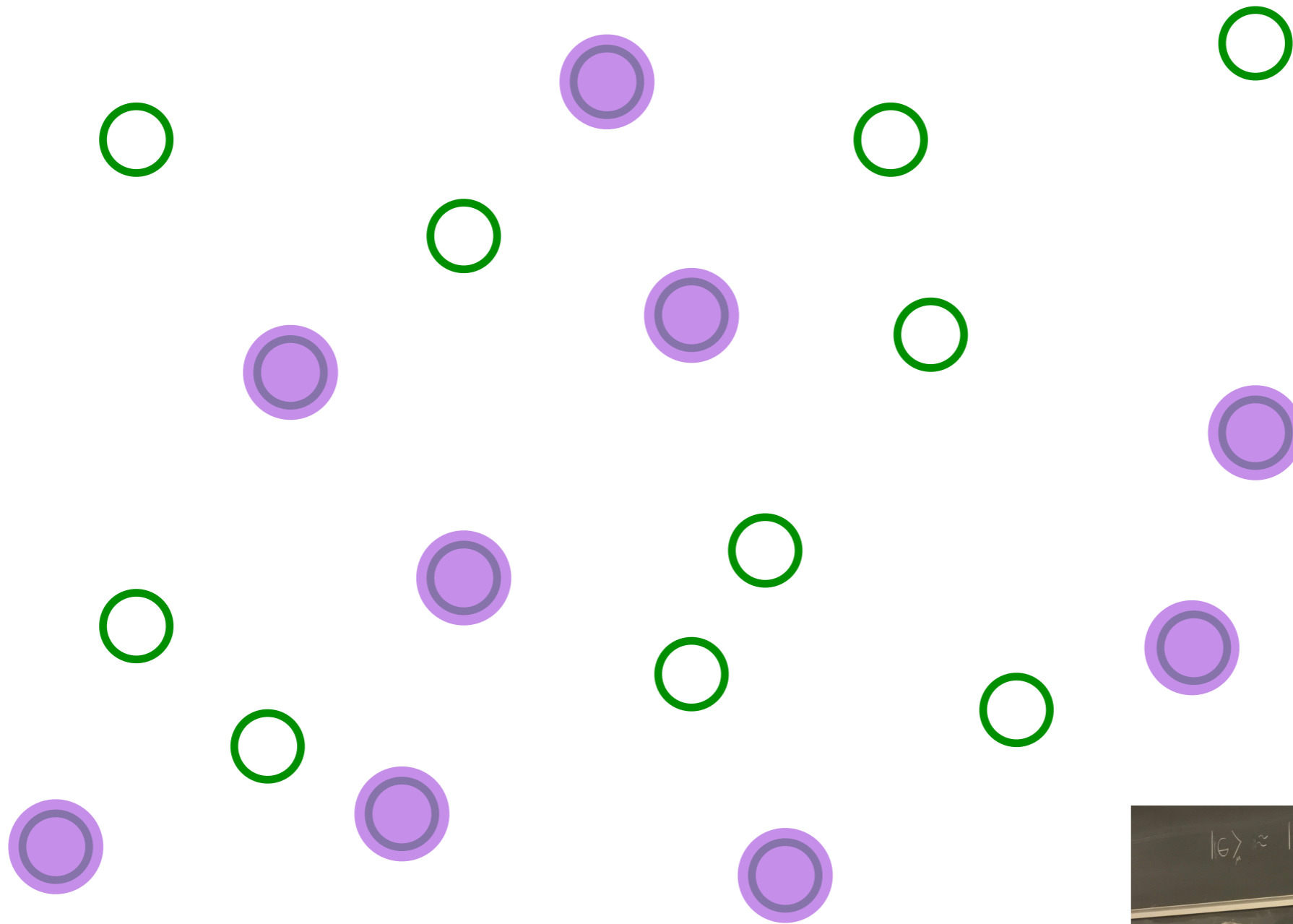


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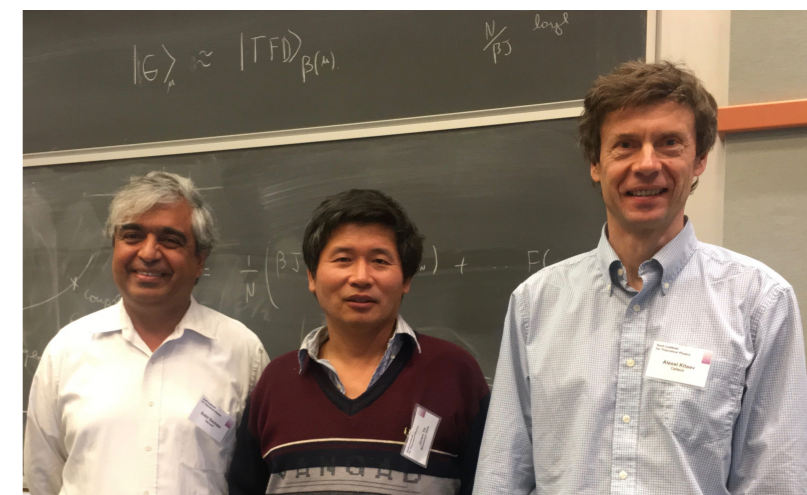


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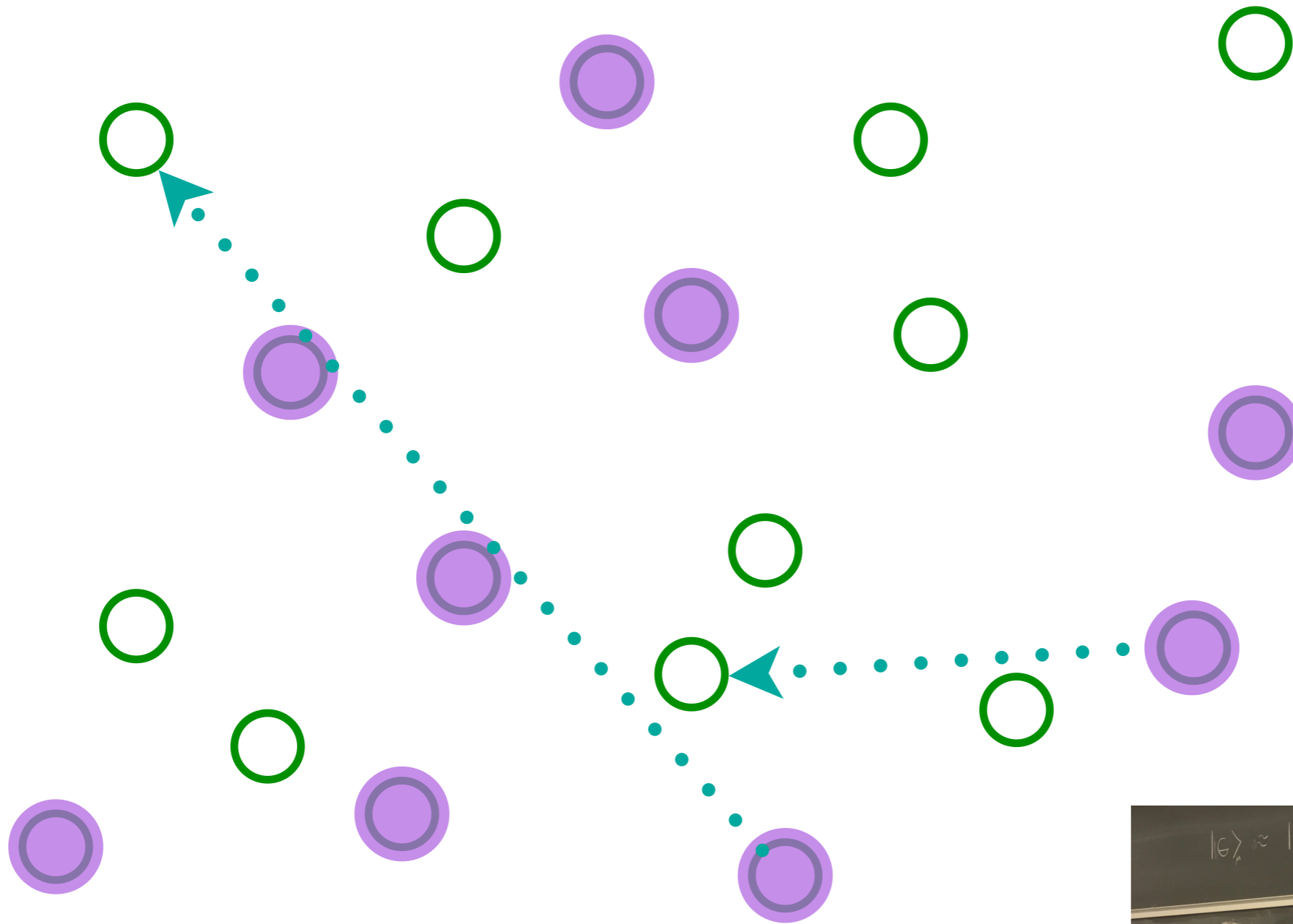


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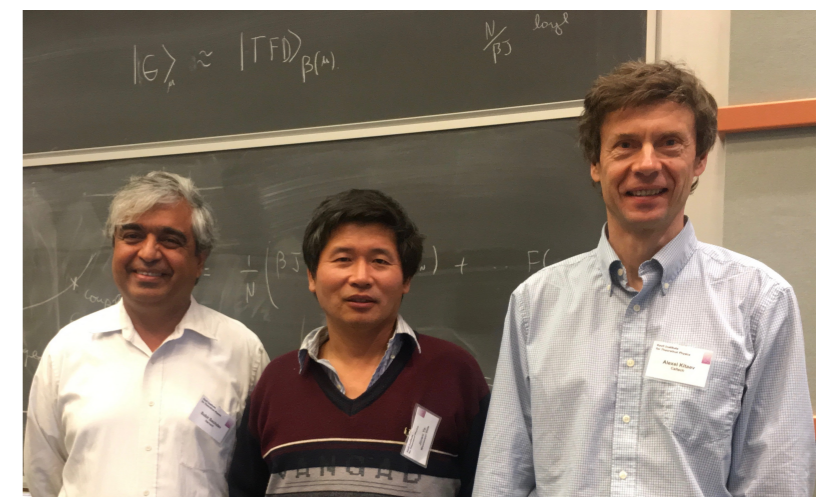


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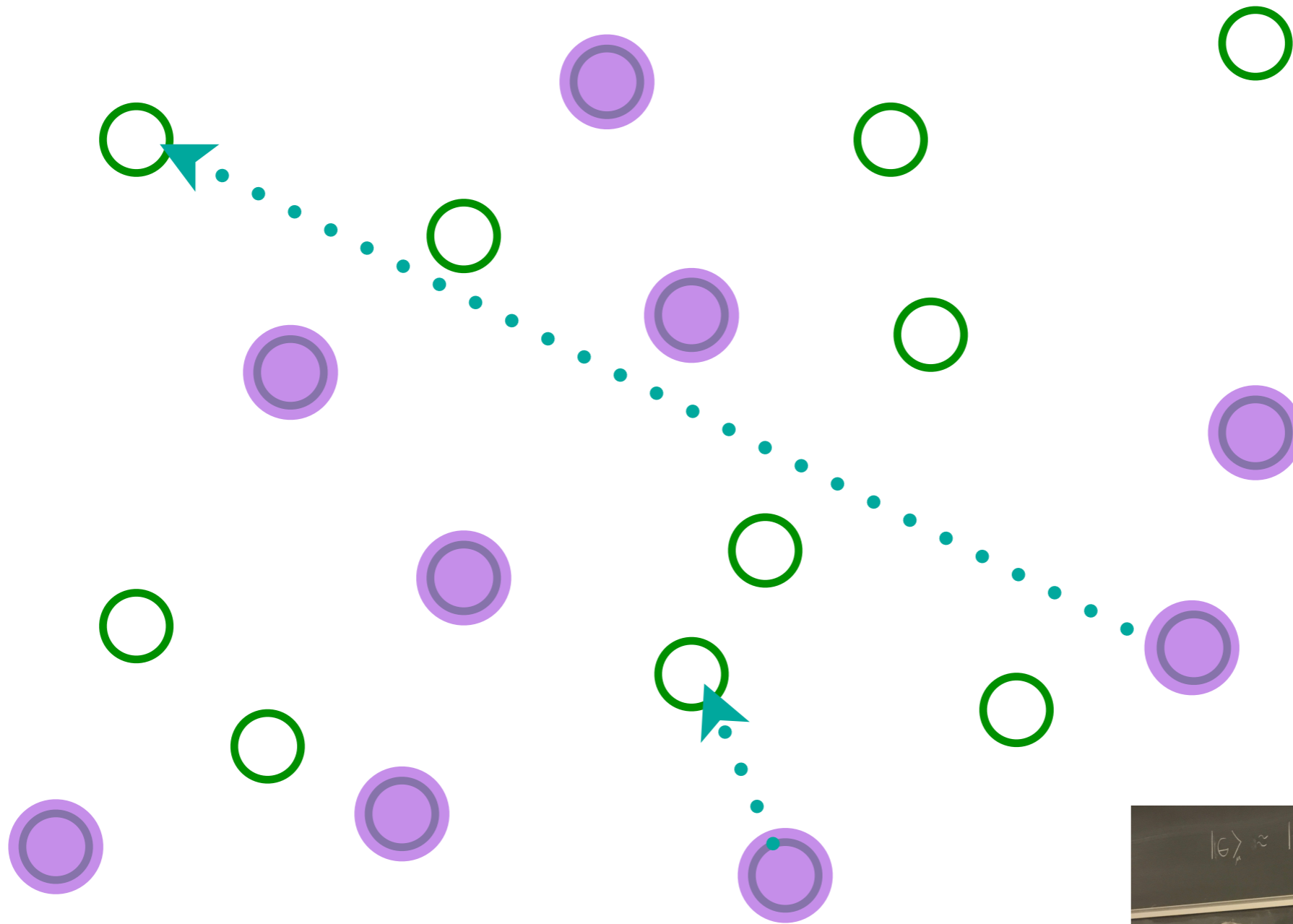


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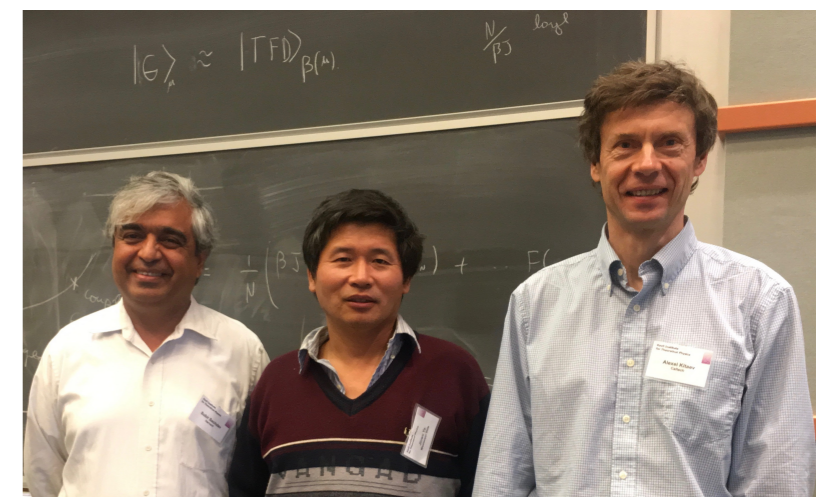


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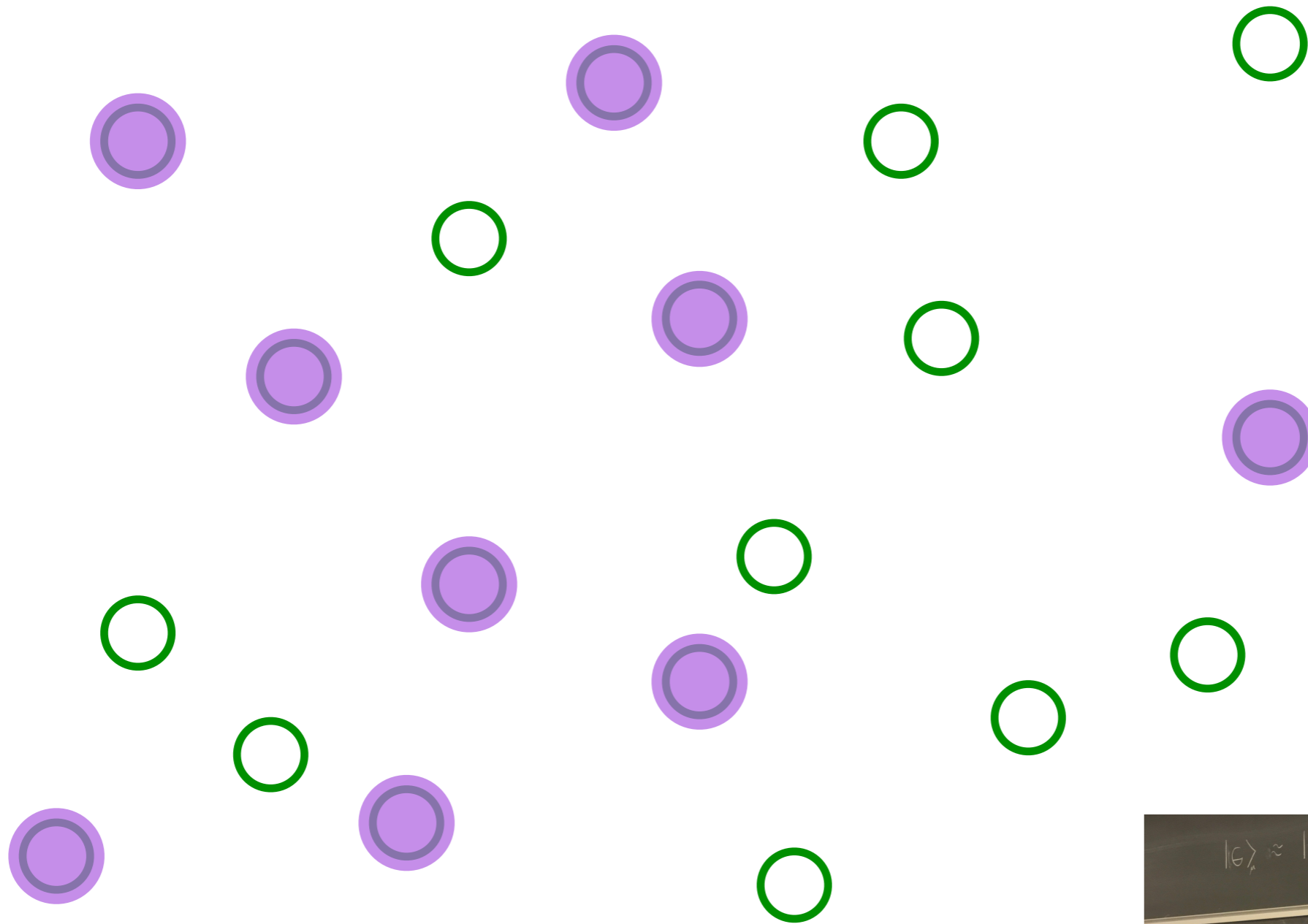


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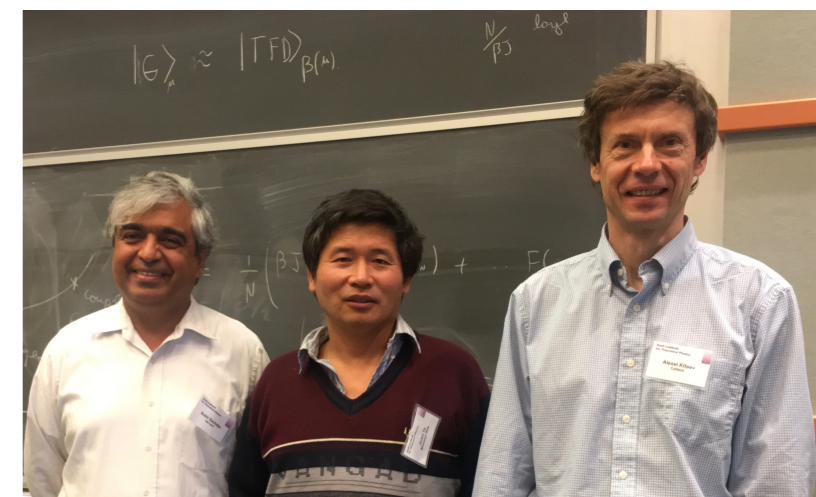


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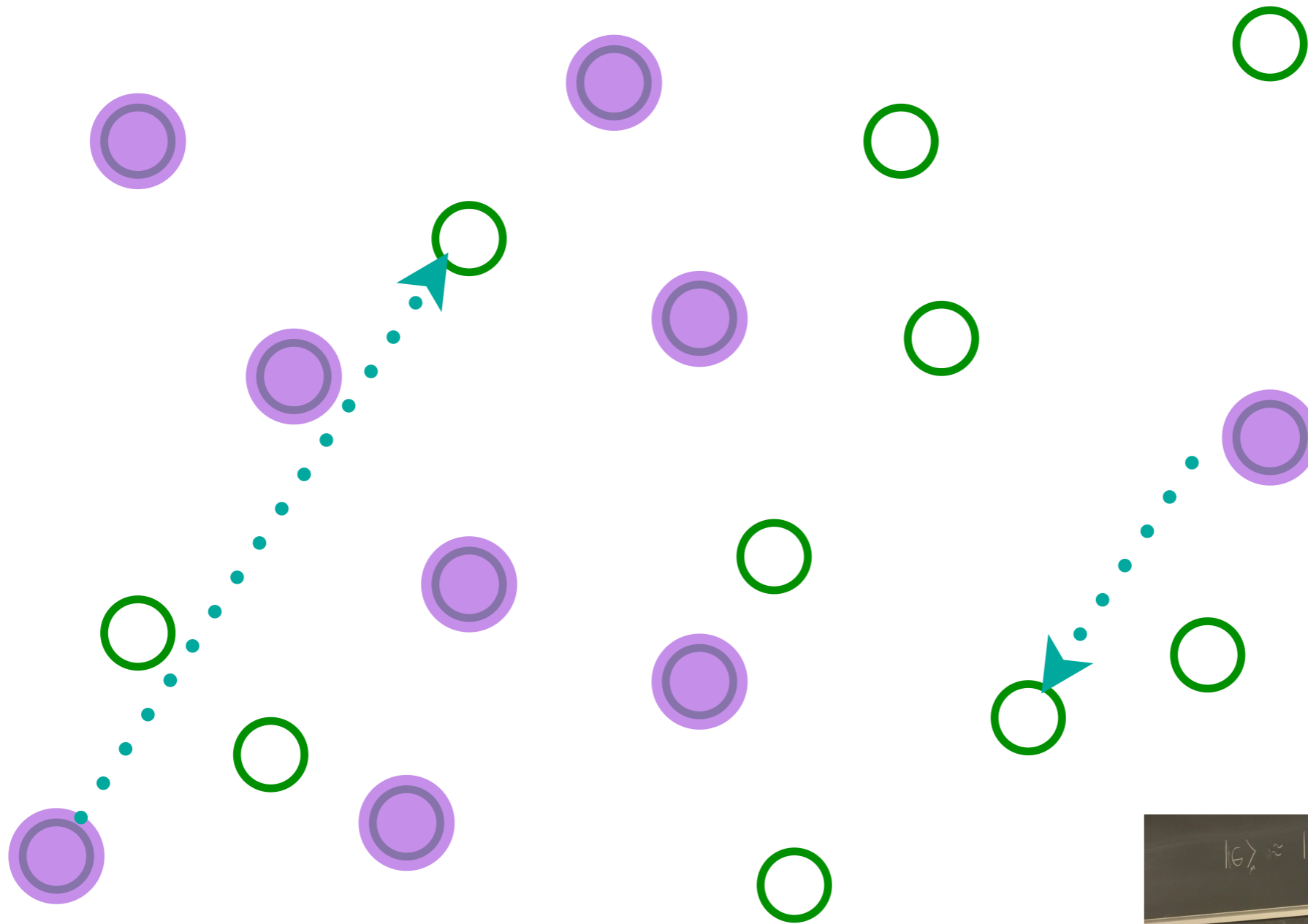


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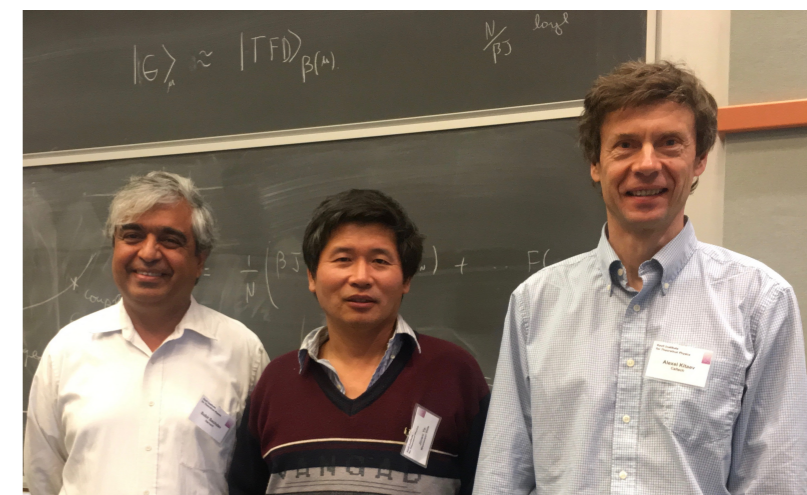


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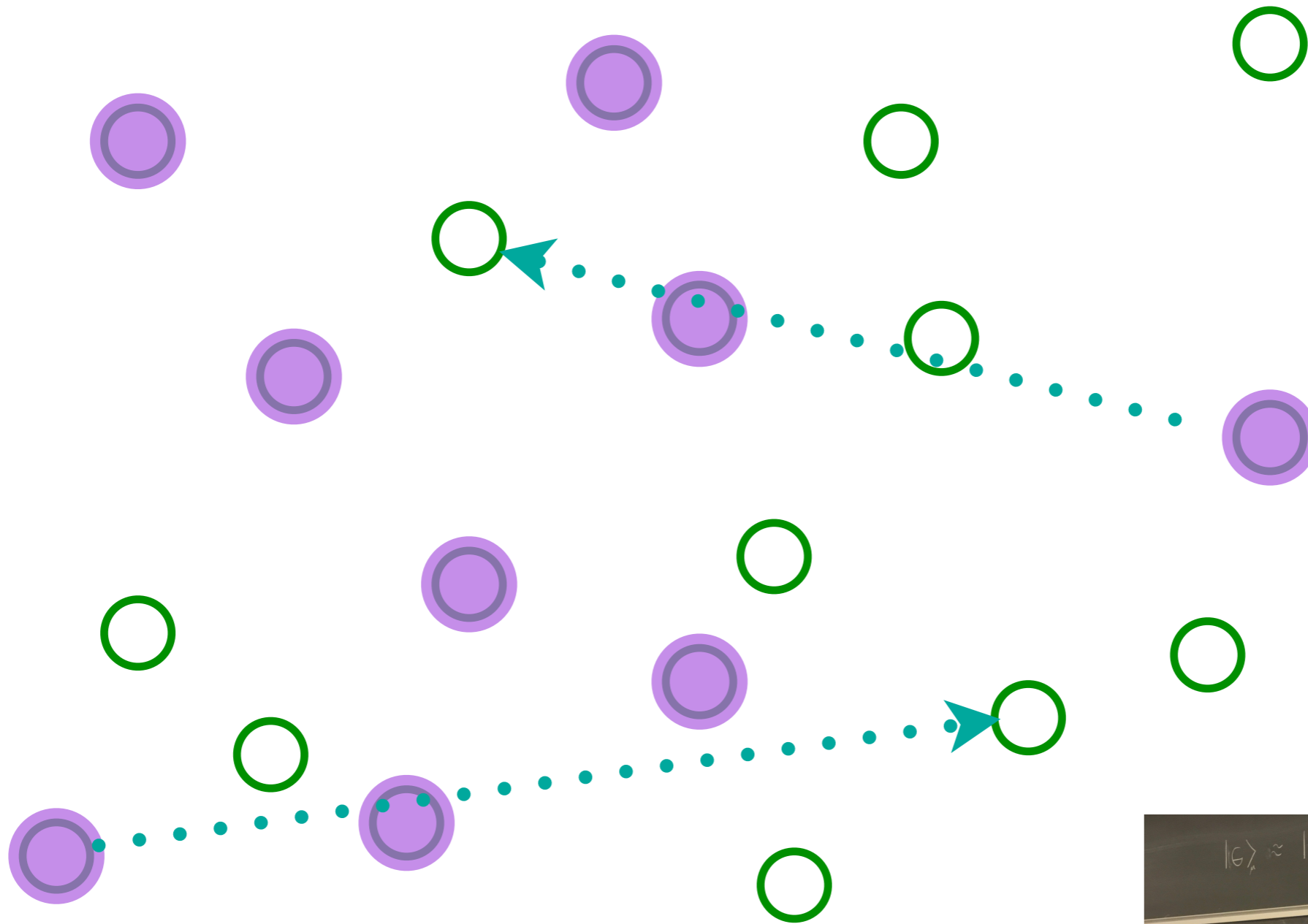


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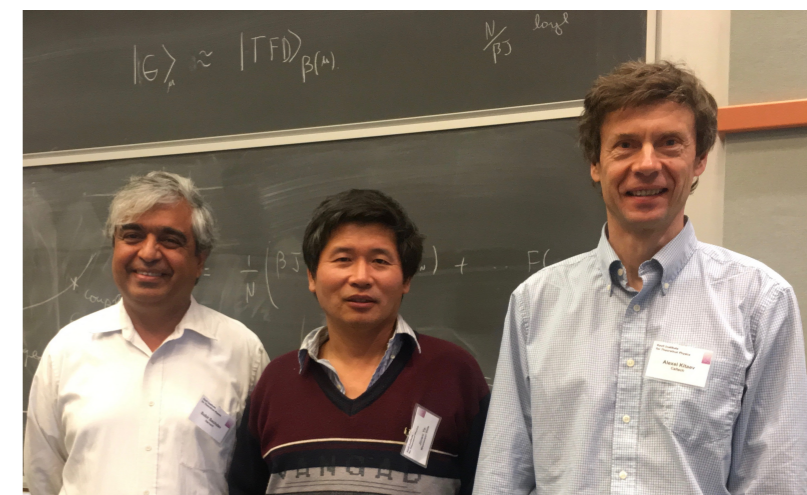


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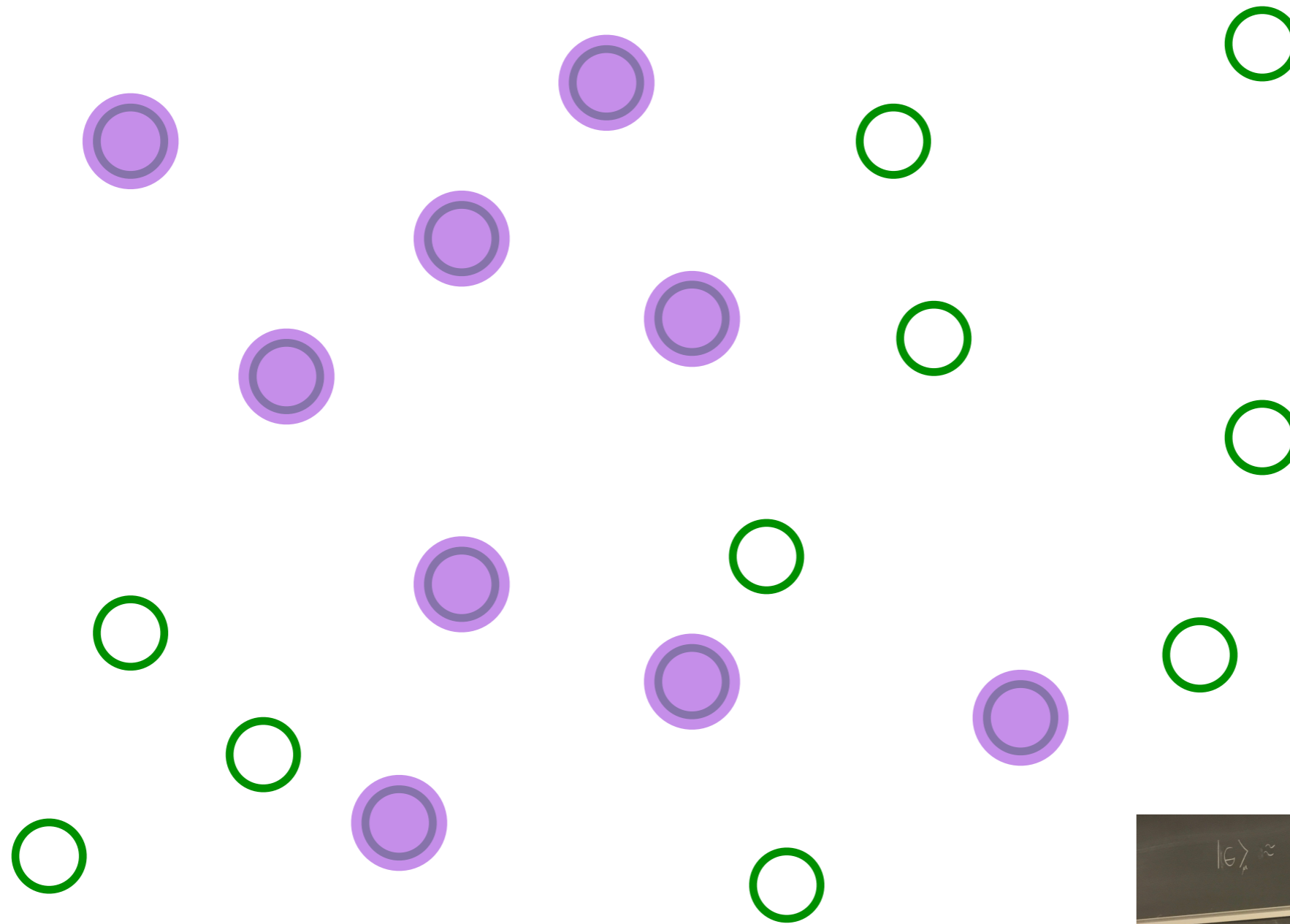


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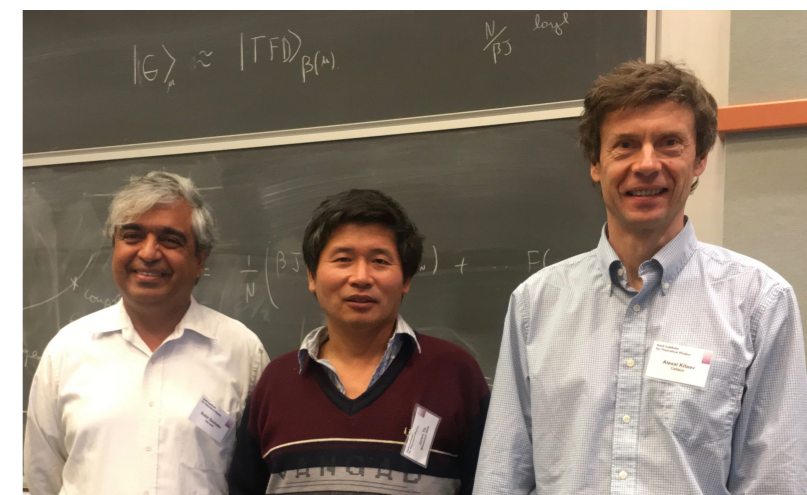


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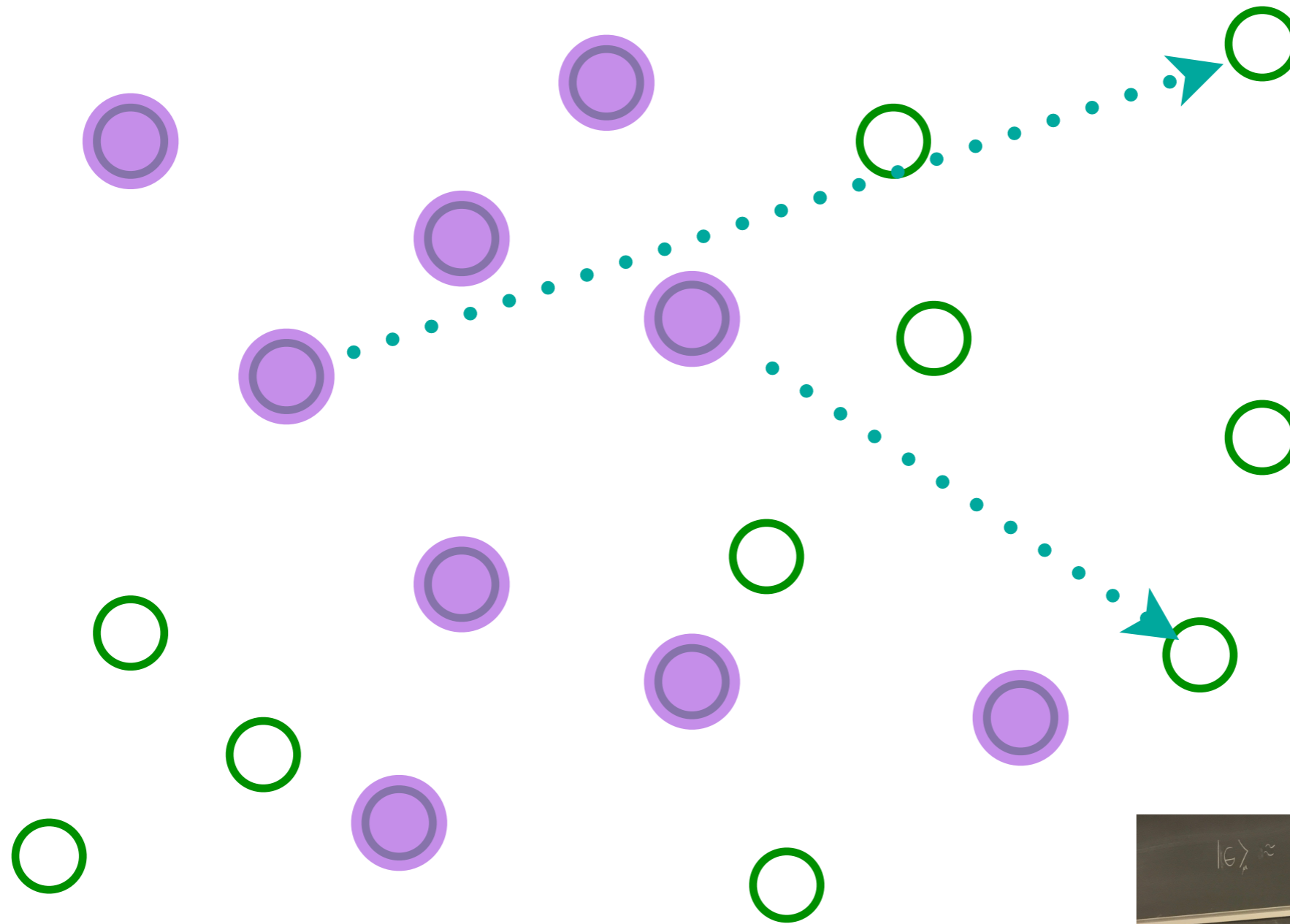


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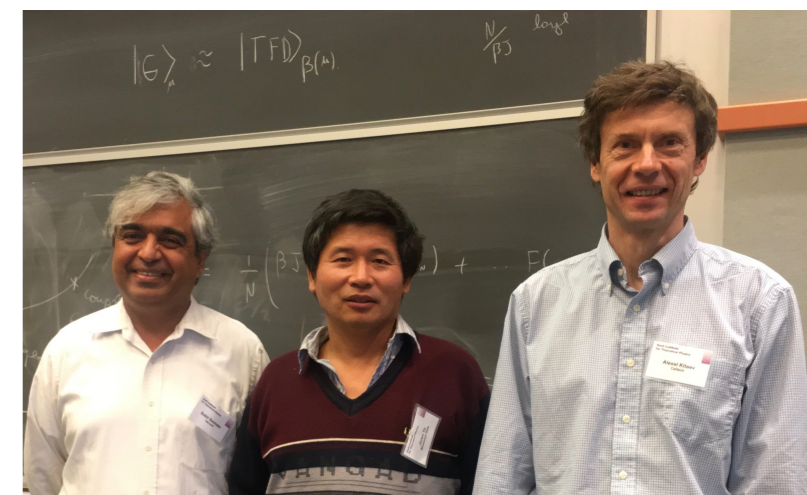


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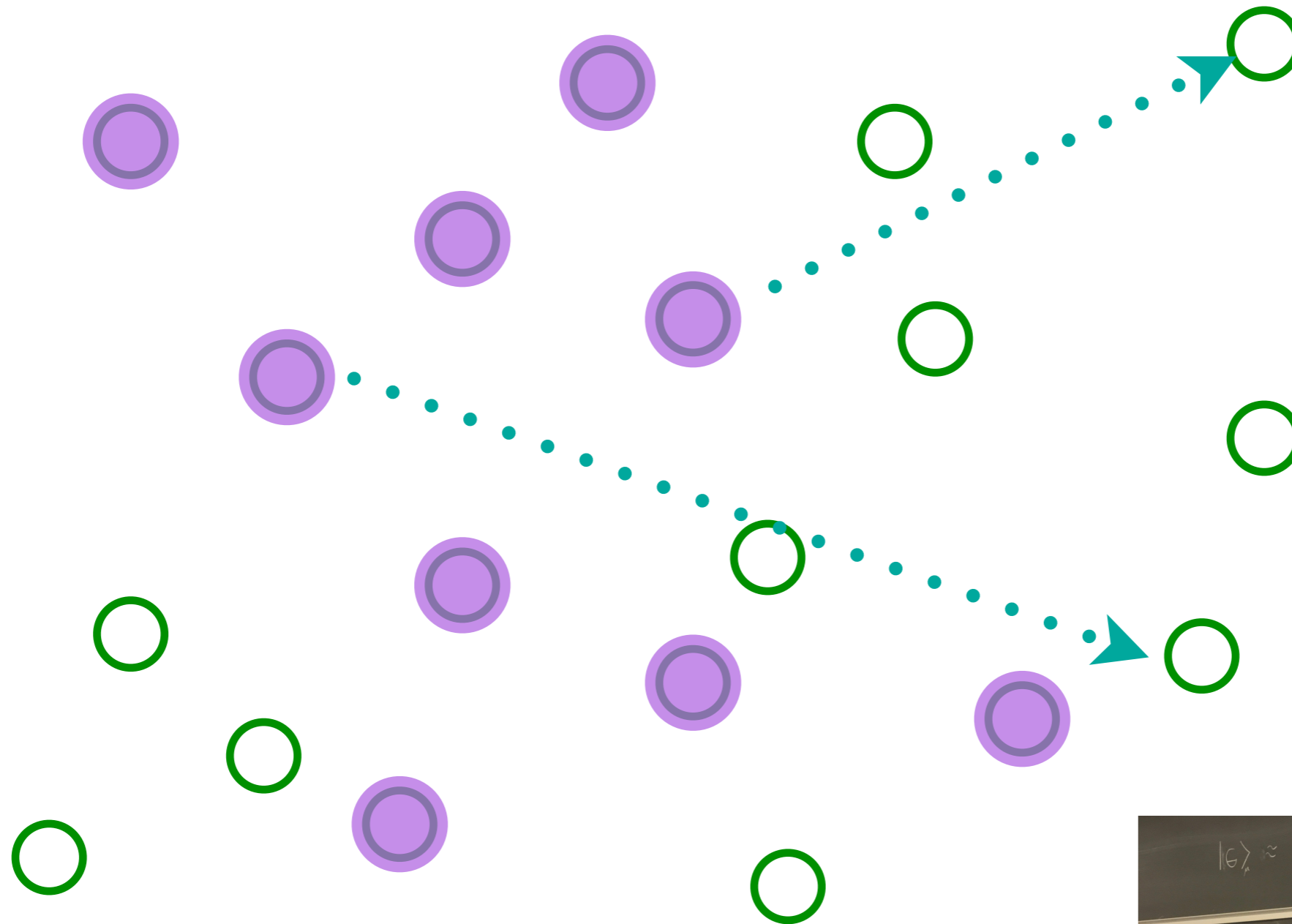


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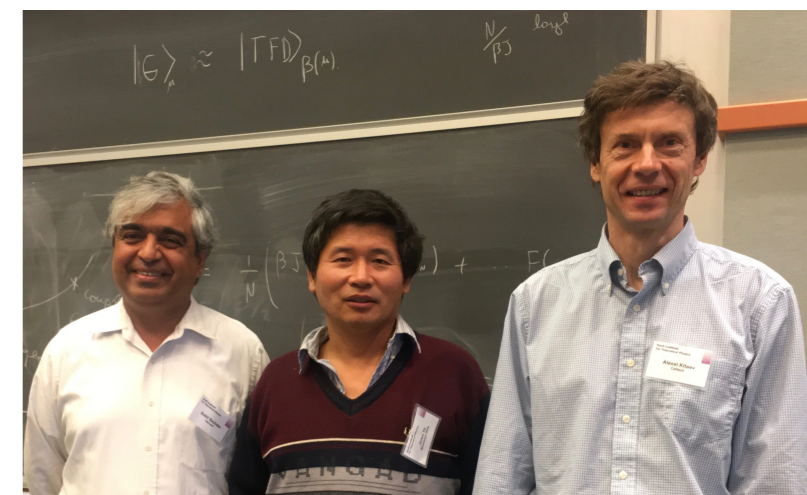


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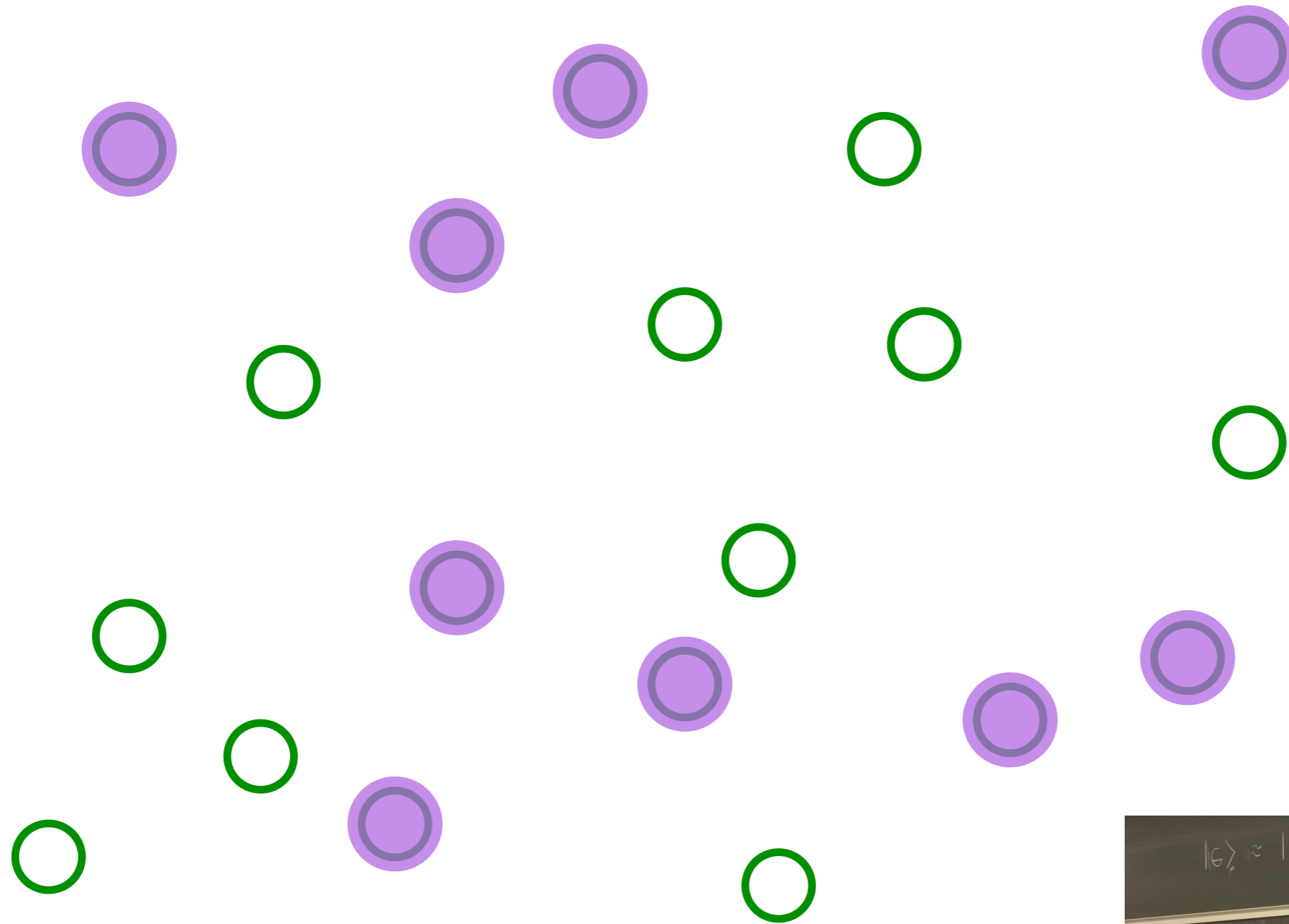


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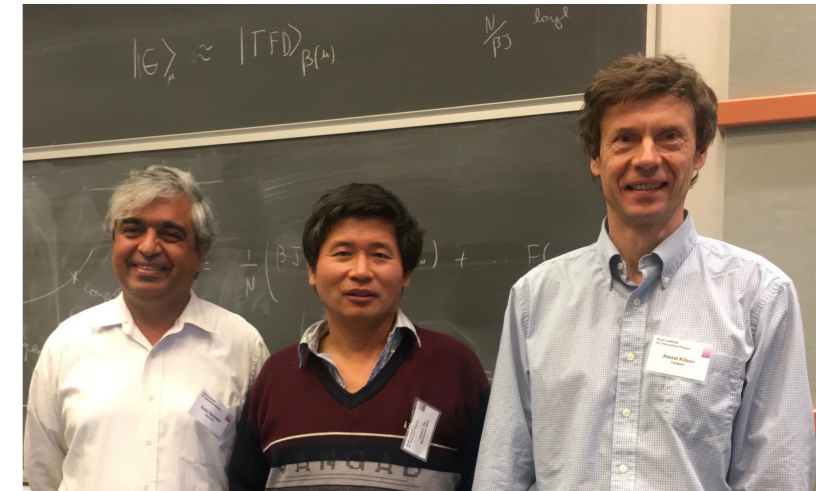


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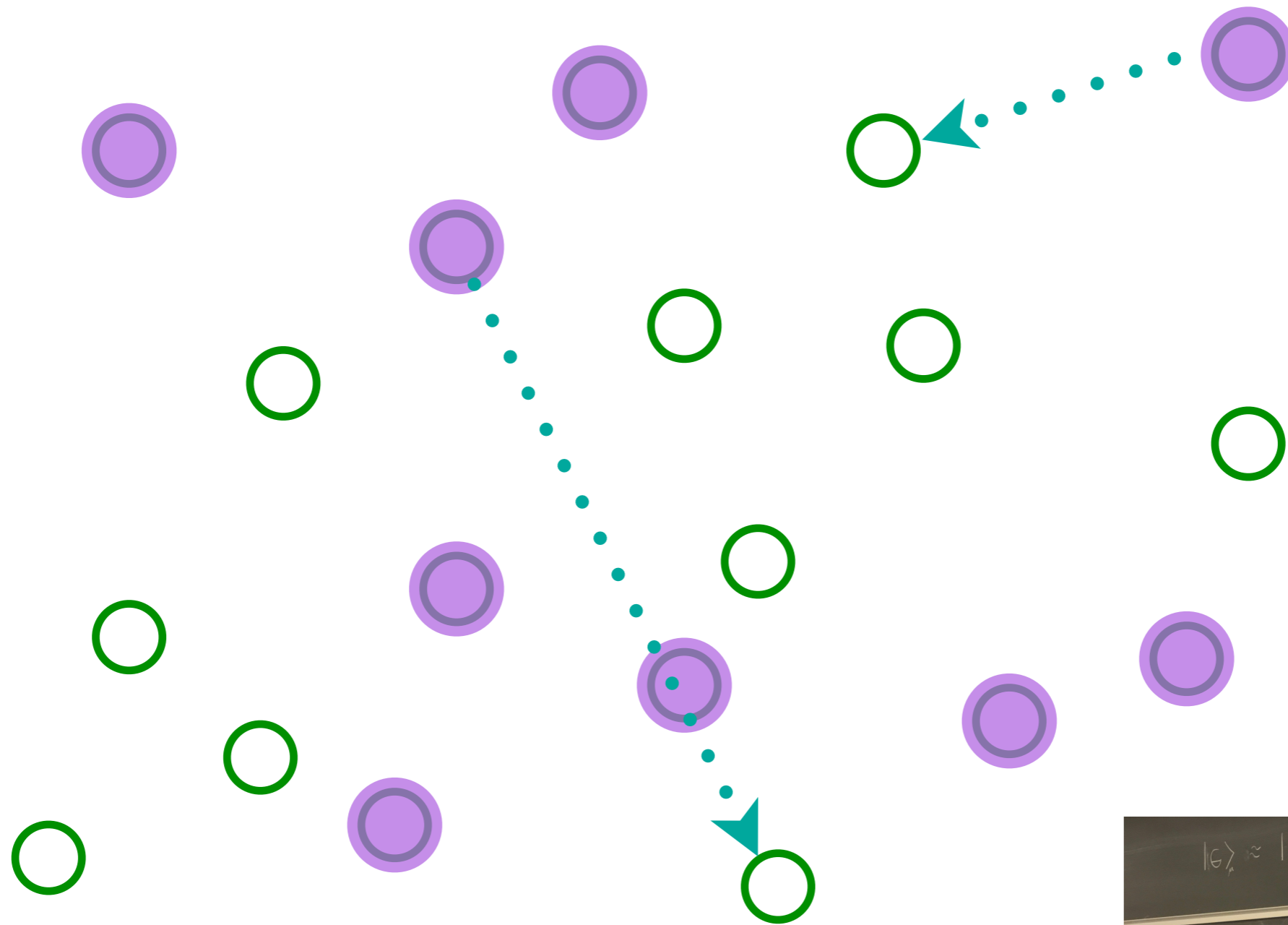


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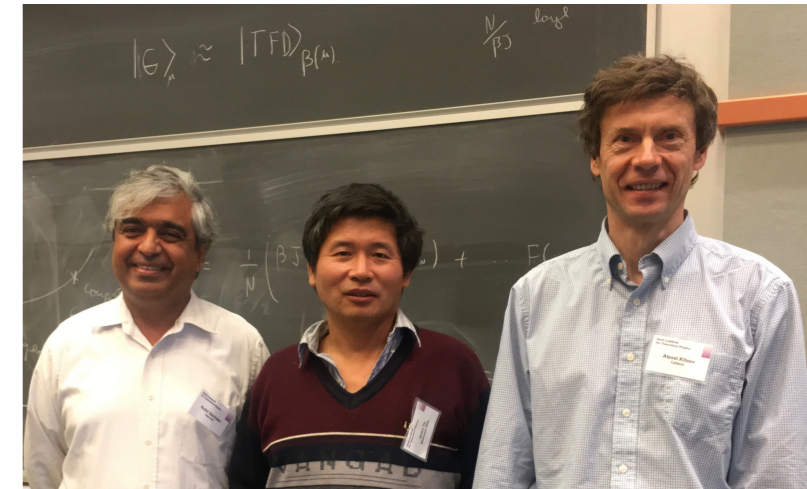


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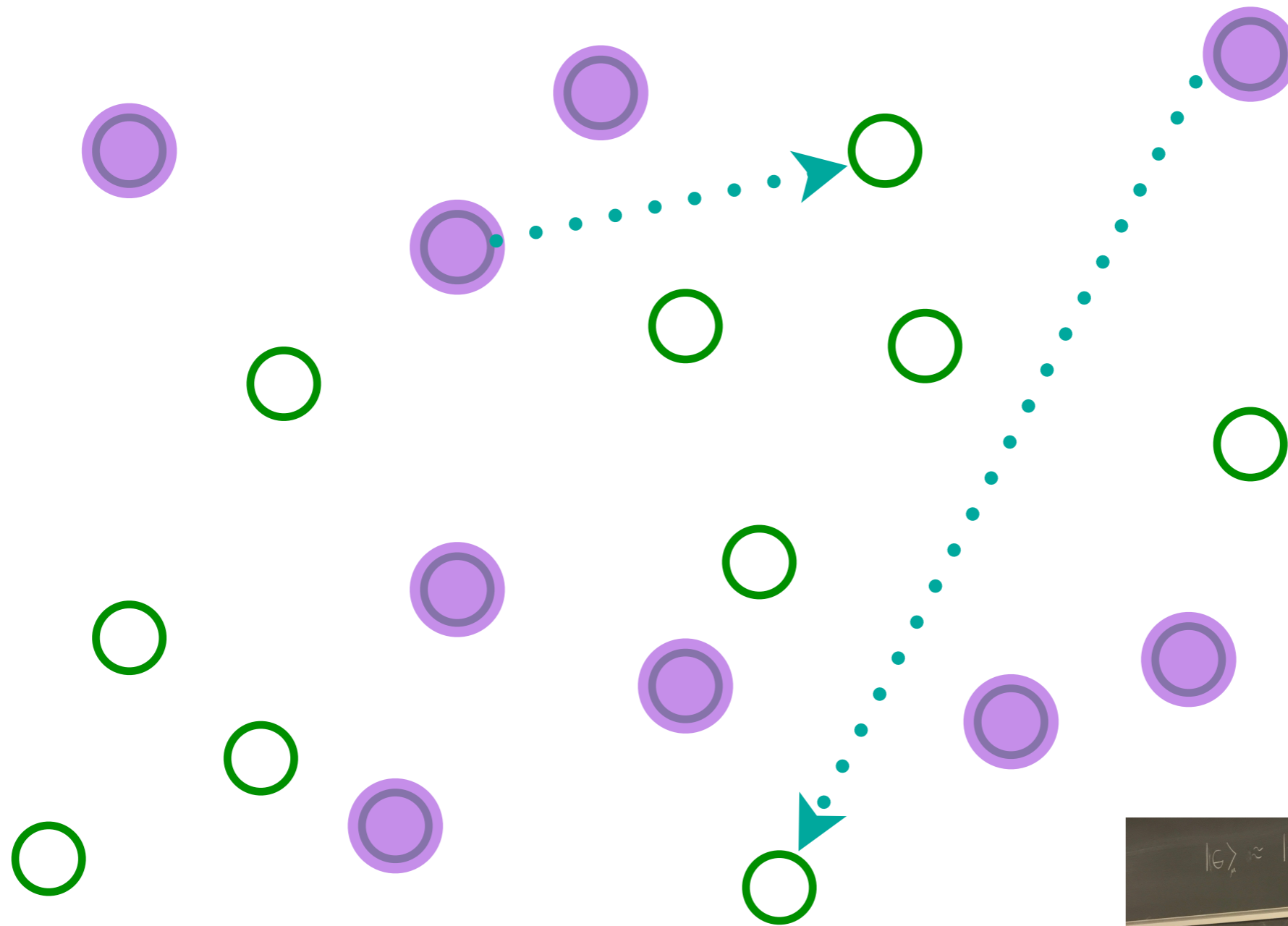


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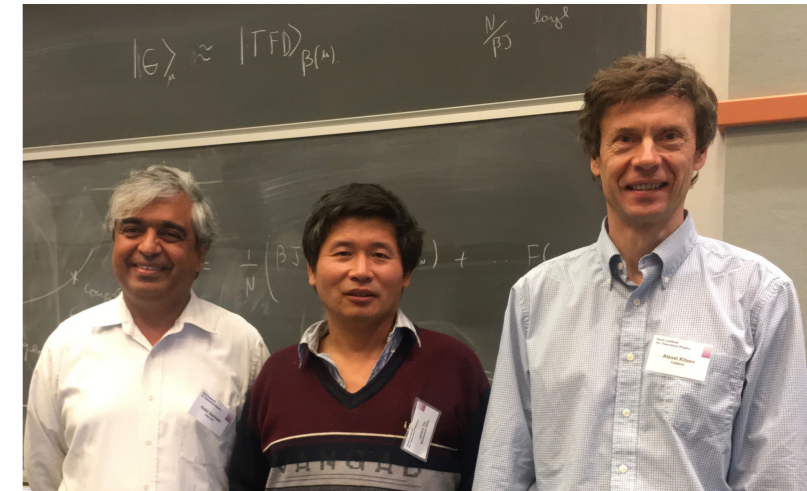


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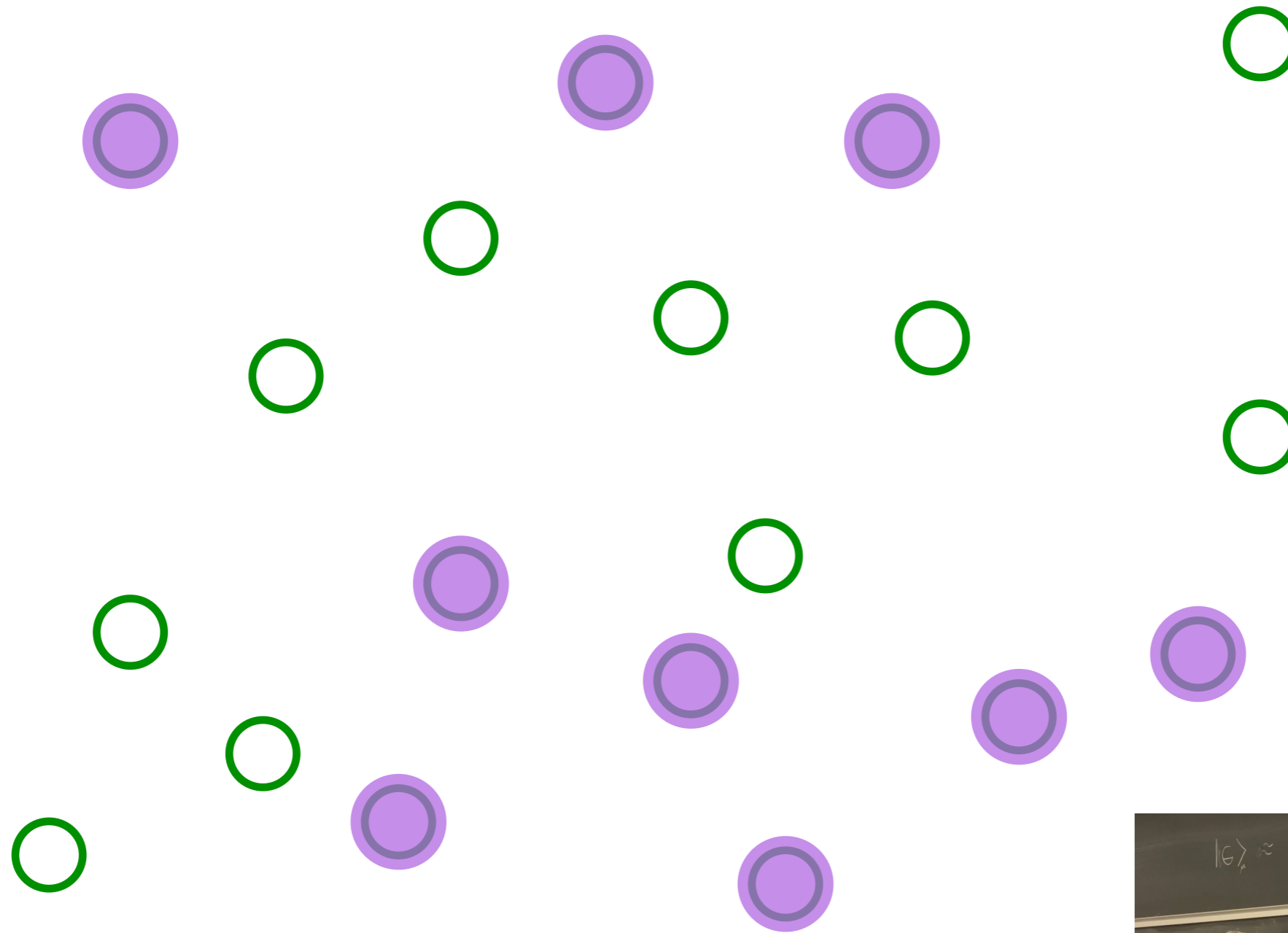


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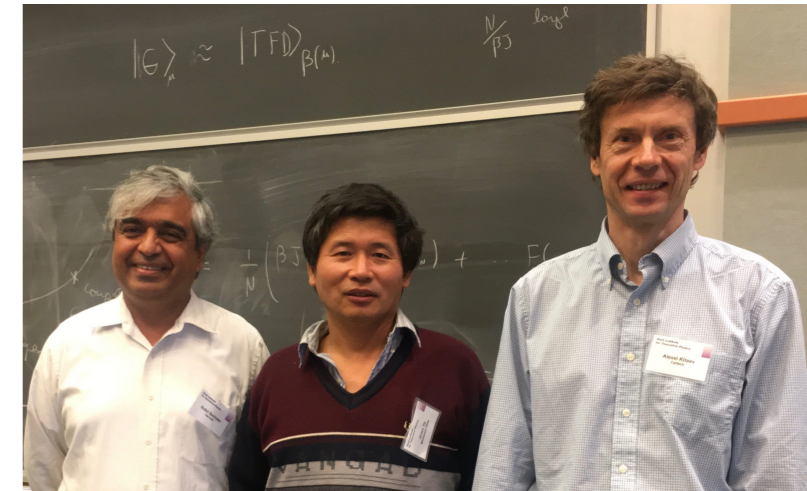


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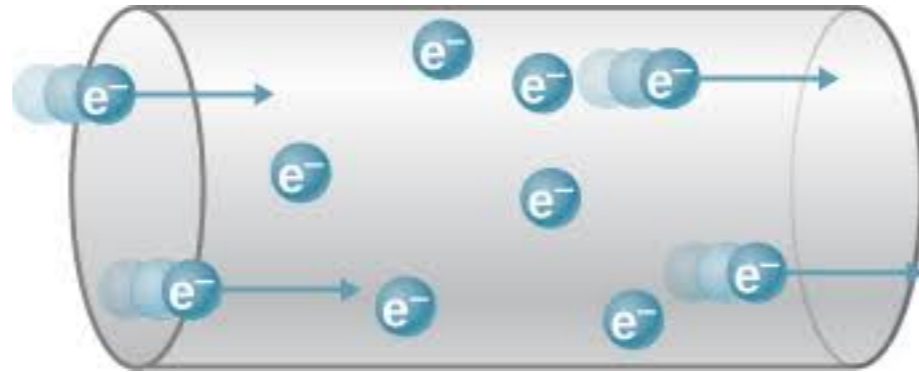
More on
metals,
ordinary and strange)

Ordinary metals



Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Current flow with quasiparticles



Flowing quasiparticles scatter off each other in a typical scattering time τ

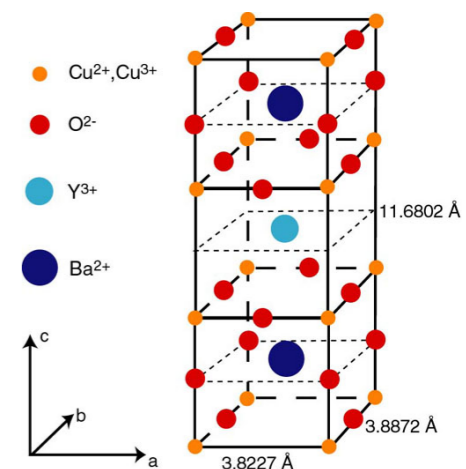
This time is much longer than a limiting ‘Planckian time’ $\frac{\hbar}{k_B T}$.

The long scattering time implies that quasiparticles are well-defined.

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11 \text{ kbar}$	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Electron scattering time τ in 7 different strange metals

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

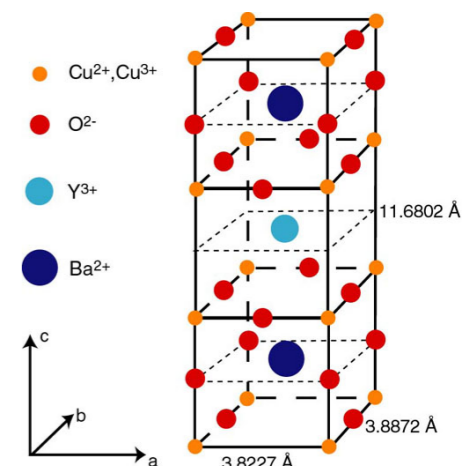


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Current flow without quasiparticles



**More on
quantum
black
holes**

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)

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All many-body quantum systems
(without quantum gravity)
have an entropy
proportional to their volume !?!?

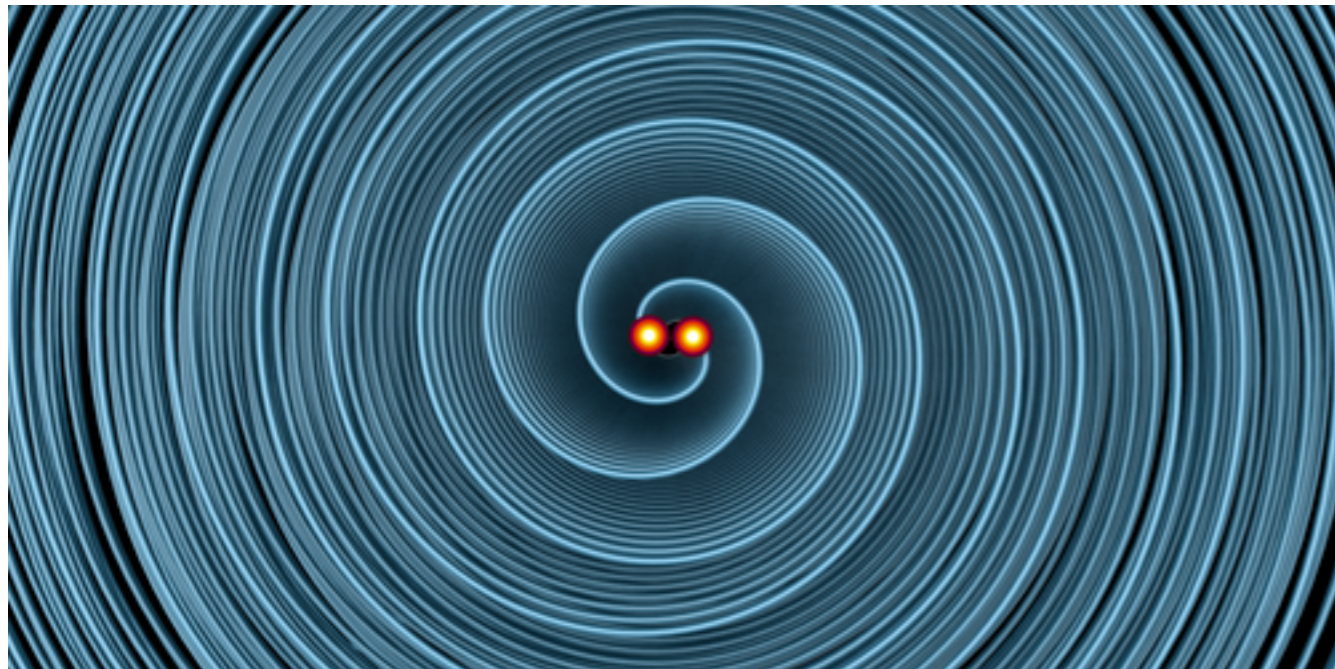
Black Holes Obey Information-Emission Limits

Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

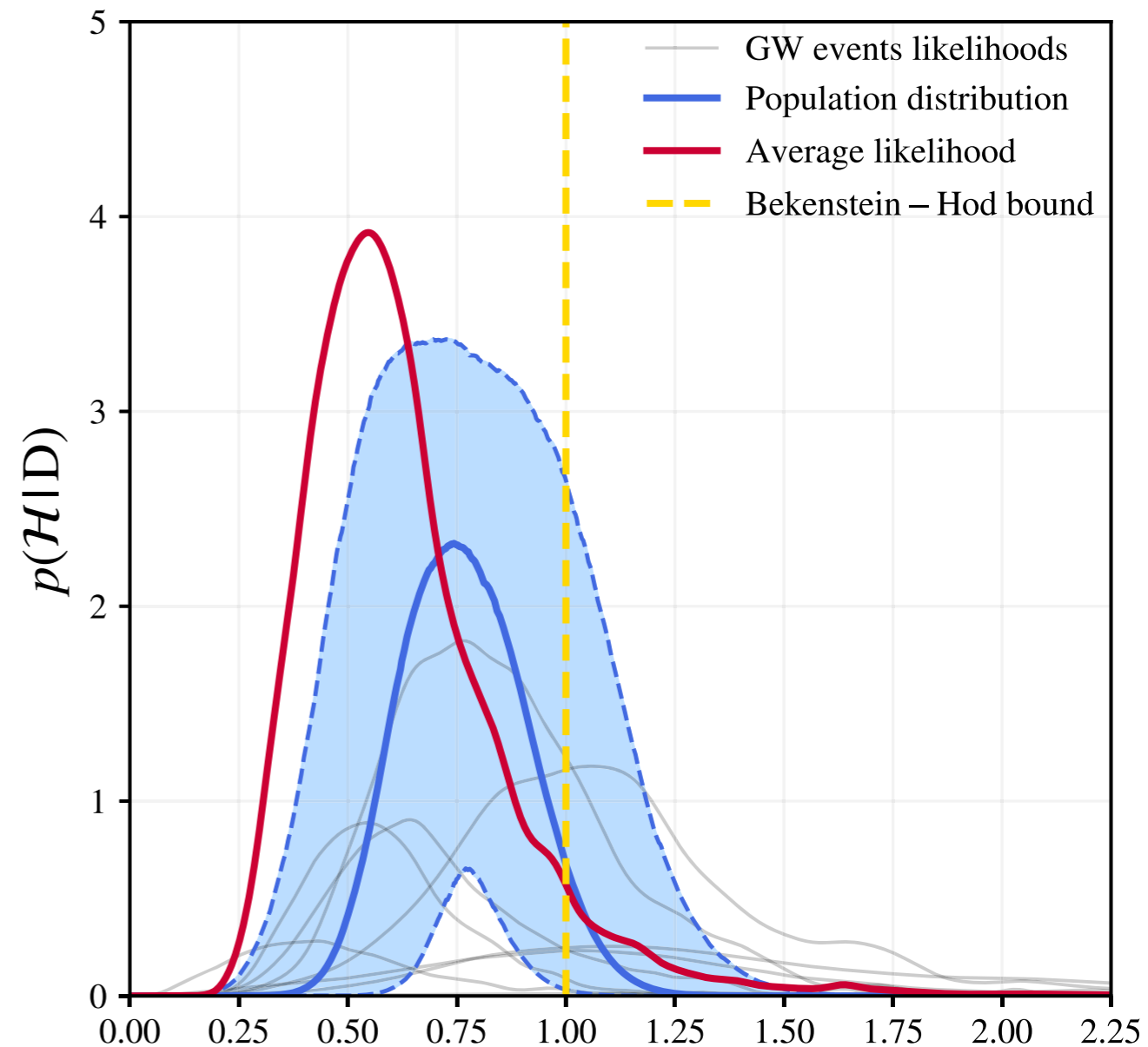
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, *Phys. Rev. Lett.* **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$

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Black holes are represented as a '*hologram*' by a quantum many-body system in one lower dimension.

Duality: a '*change of variables*' between the many-particle configurations and the metric of spacetime

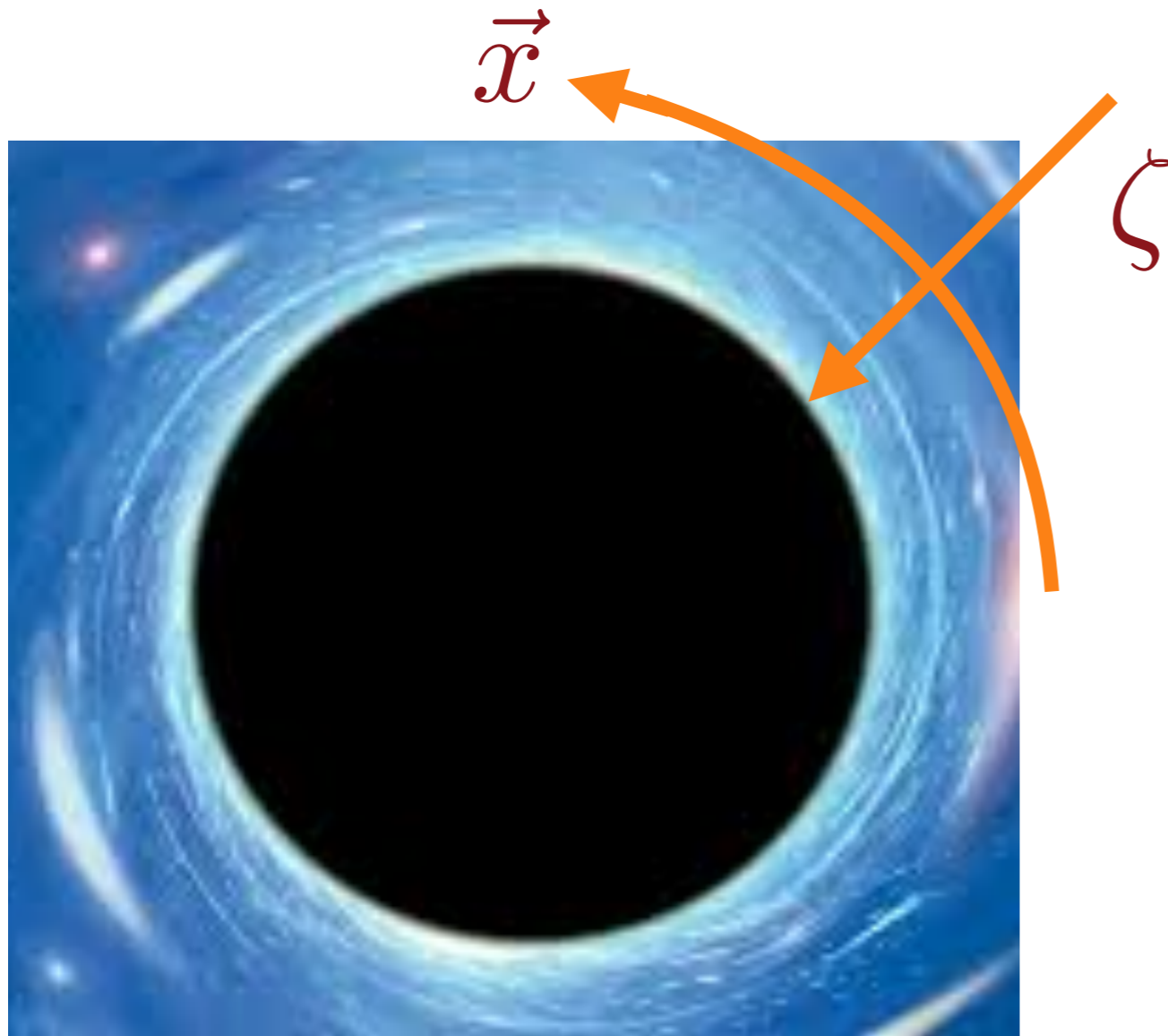
Quantum Black holes

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The hologram of a black hole
in d dimensions
is a quantum many-particle system
in $(d - 1)$ dimensions
which relaxes to thermal equilibrium
in a Planckian time $\sim \hbar/(k_B T)$



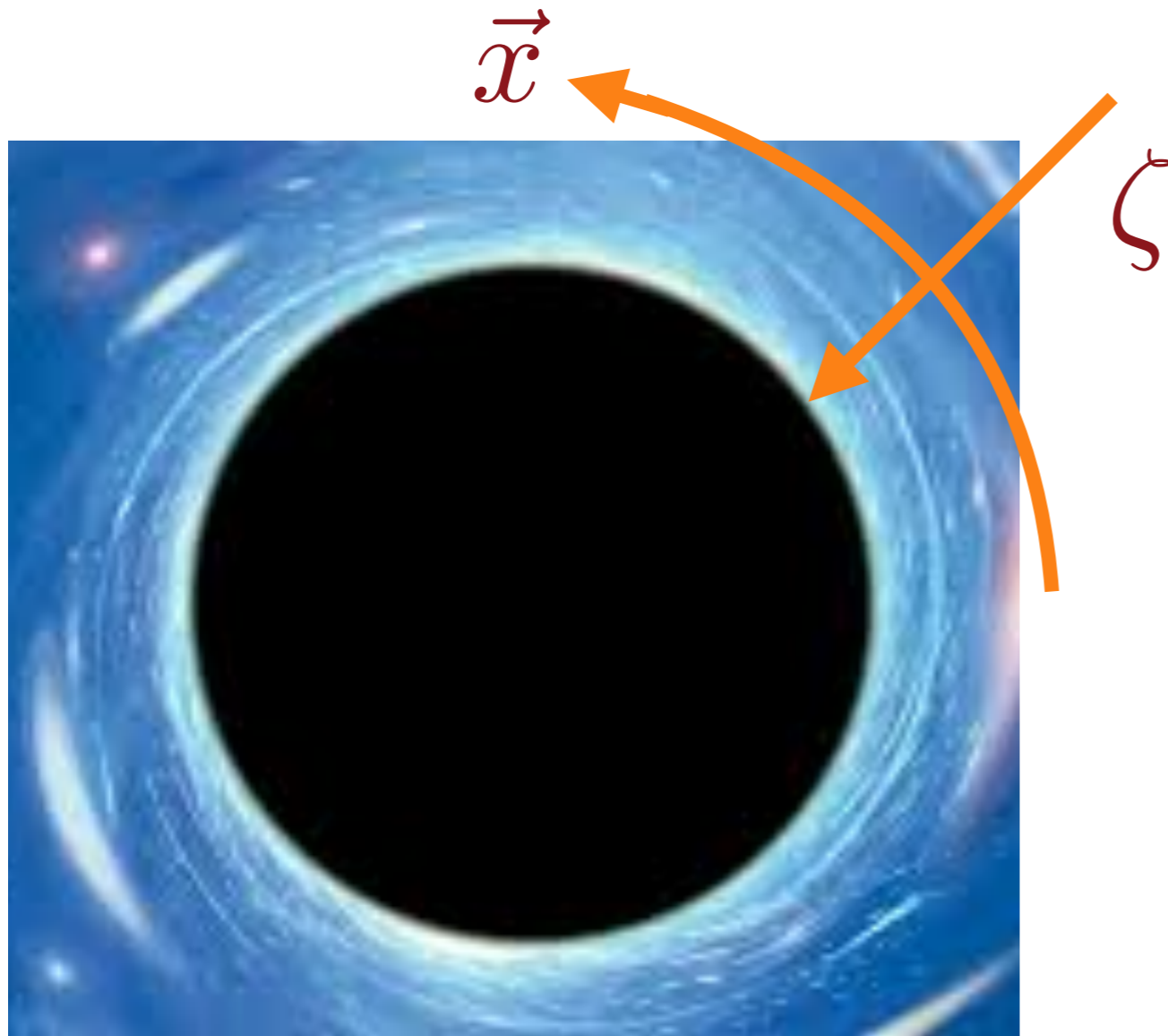
Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The near-horizon
geometry of a
charged black hole is
one-dimensional (ζ)



Maxwell's electromagnetism
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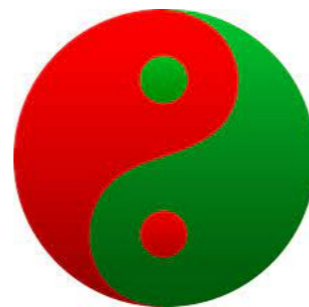
The hologram of the
 $1+1$ dimensional
gravity near the
horizon of a charged
black hole is the
 $0+1$ dimensional
SYK model

The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

In one set of variables, it describes certain ***strange metals***

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes certain ***black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)