Paramagnon fractionalization theory of the cuprate pseudogap and phase diagram

The 13th TOYOTA RIKEN International Workshop
Integrated Spectroscopy for Strong Electron Correlation
-Theory, Computation and Experiment
University of Tokyo,
December 7, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

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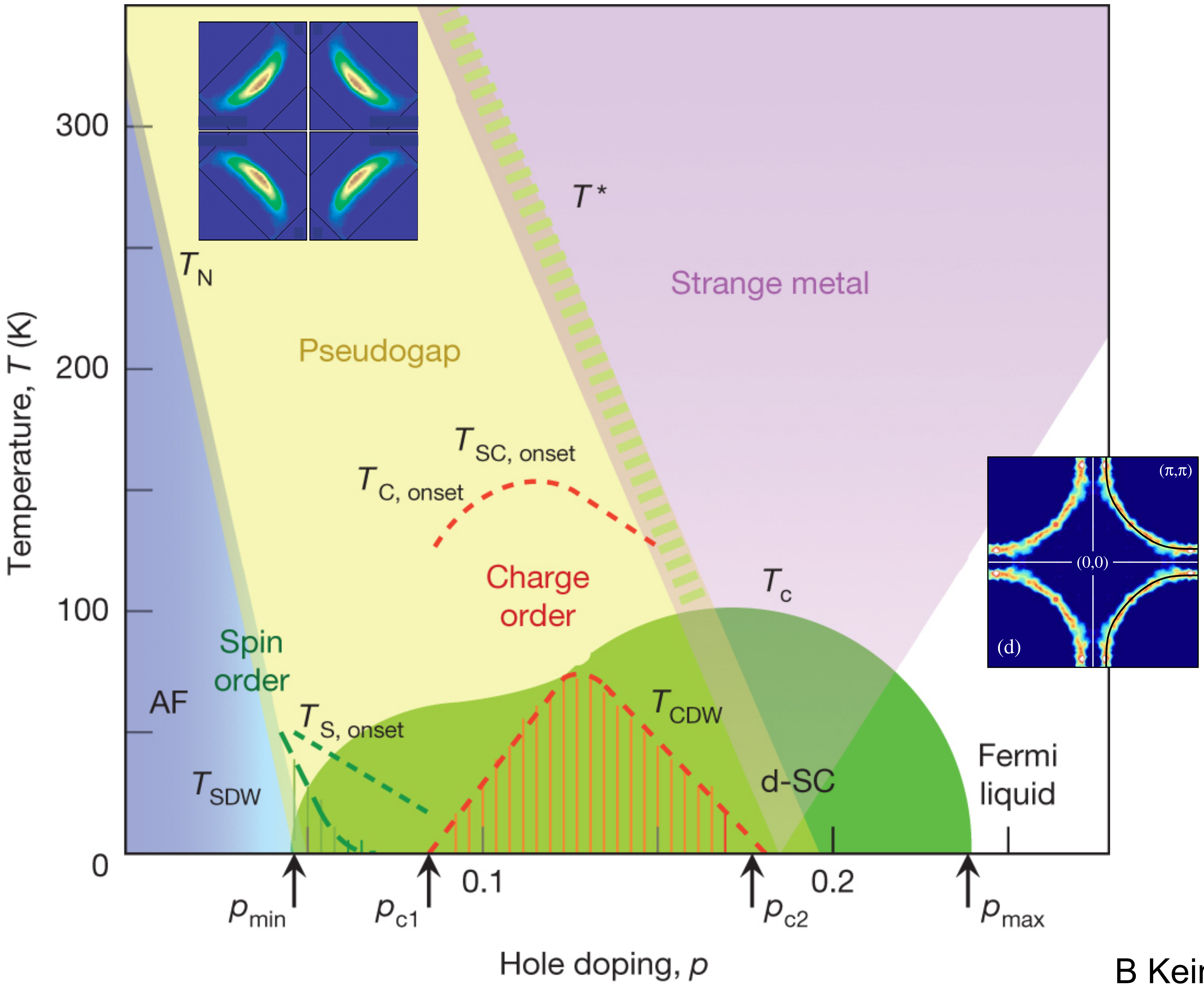
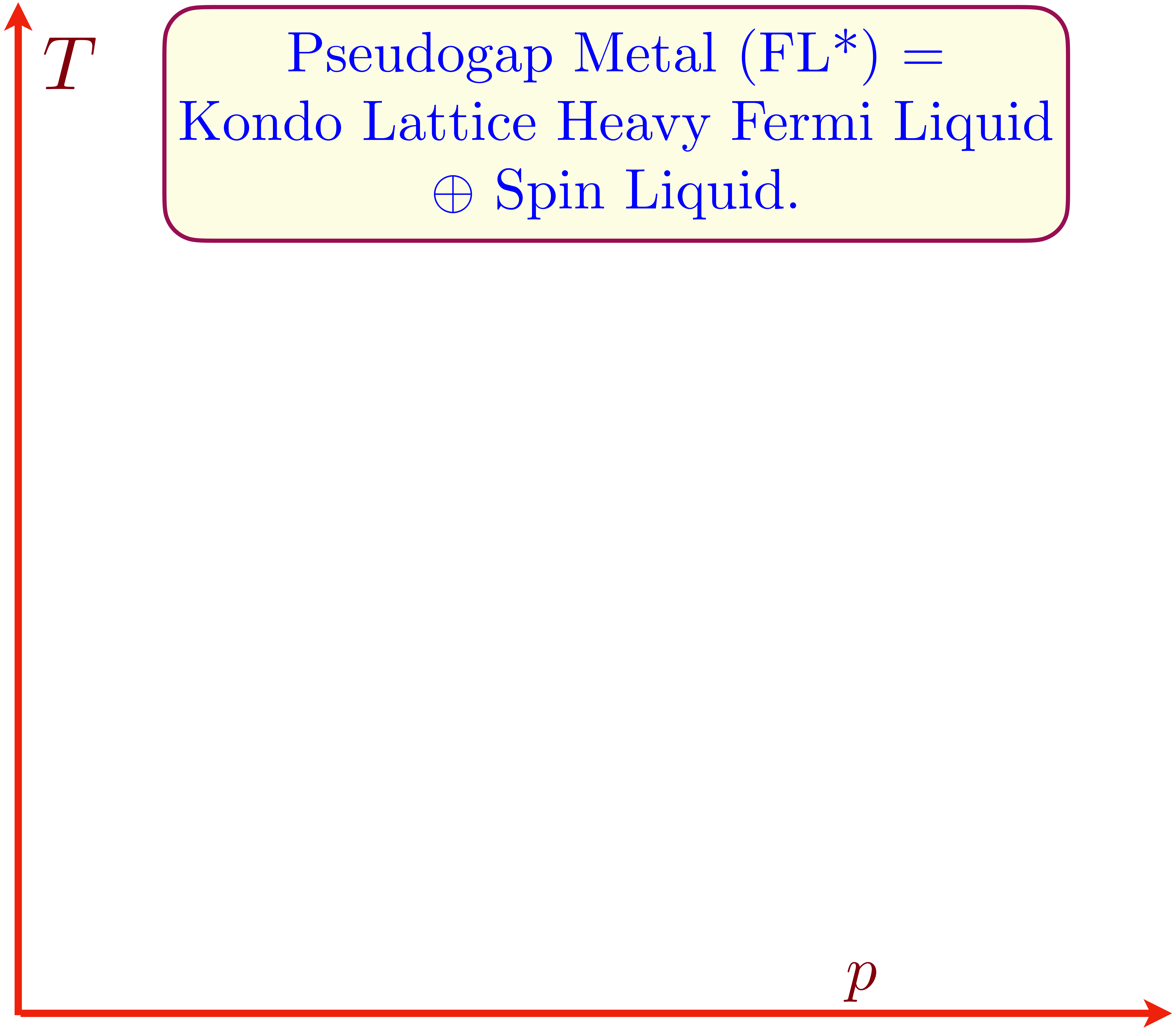
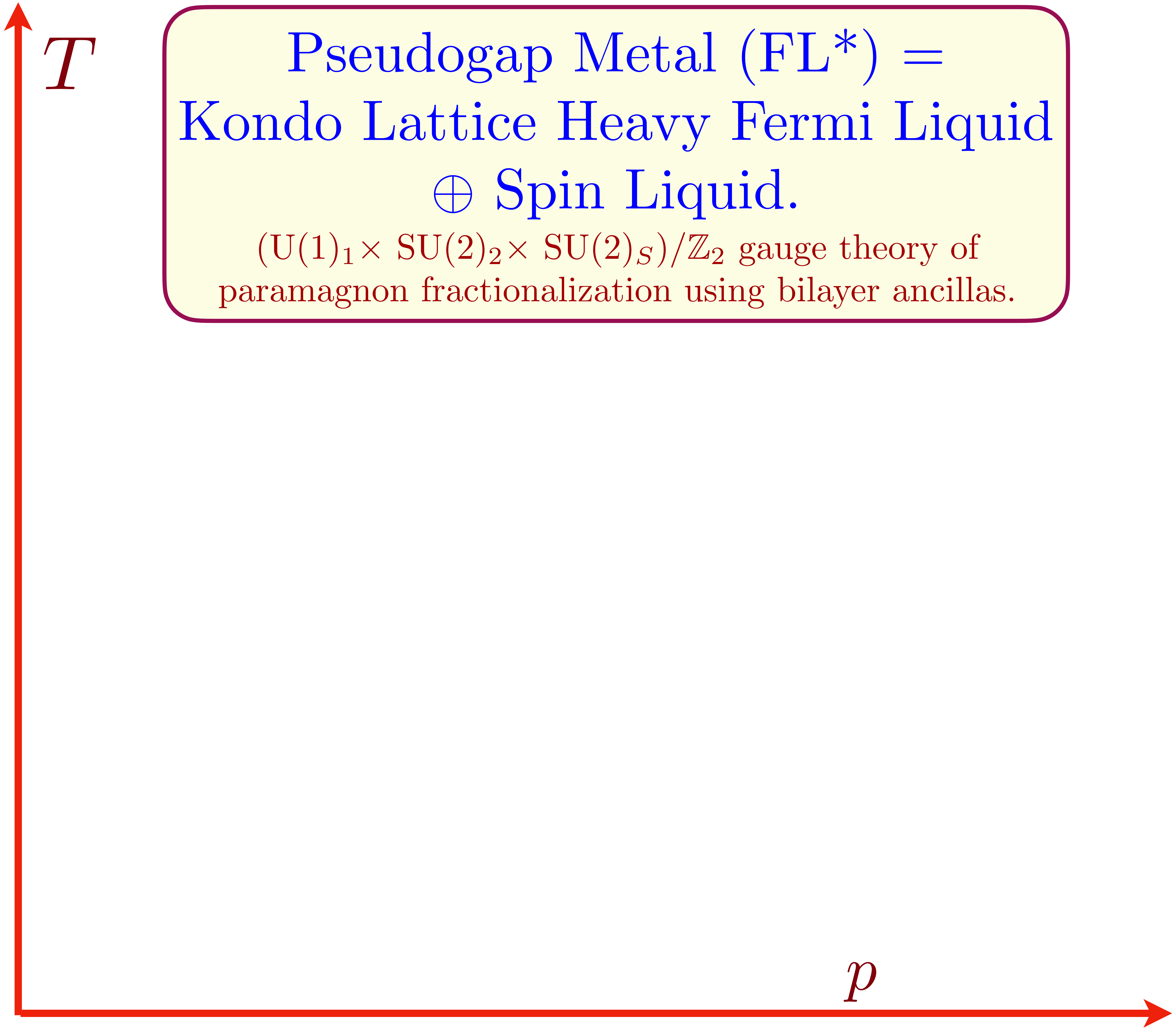


B Keimer *et al.* *Nature* **518**, 179-186 (2015)


Pseudogap Metal (FL*) =
Kondo Lattice Heavy Fermi Liquid
⊕ Spin Liquid.
T
p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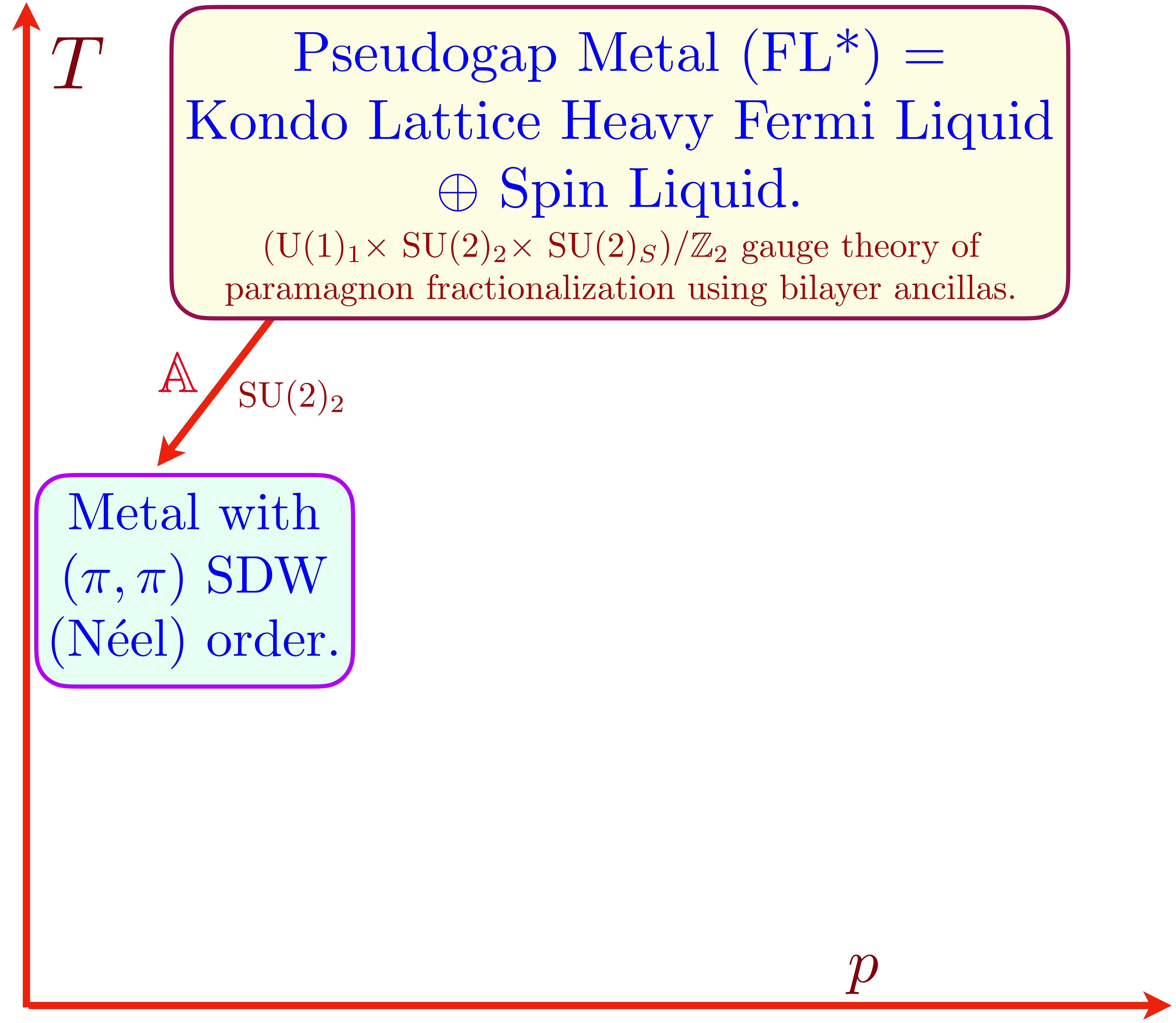
$(\mathrm{U}(1)_1\times\ \mathrm{SU}(2)_2\times\ \mathrm{SU}(2)_S)/\mathbb{Z}_2$ gauge theory of
paramagnon fractionalization using bilayer ancillas.
p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$\mathbb{A}$
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Metal with
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(Néel) order.
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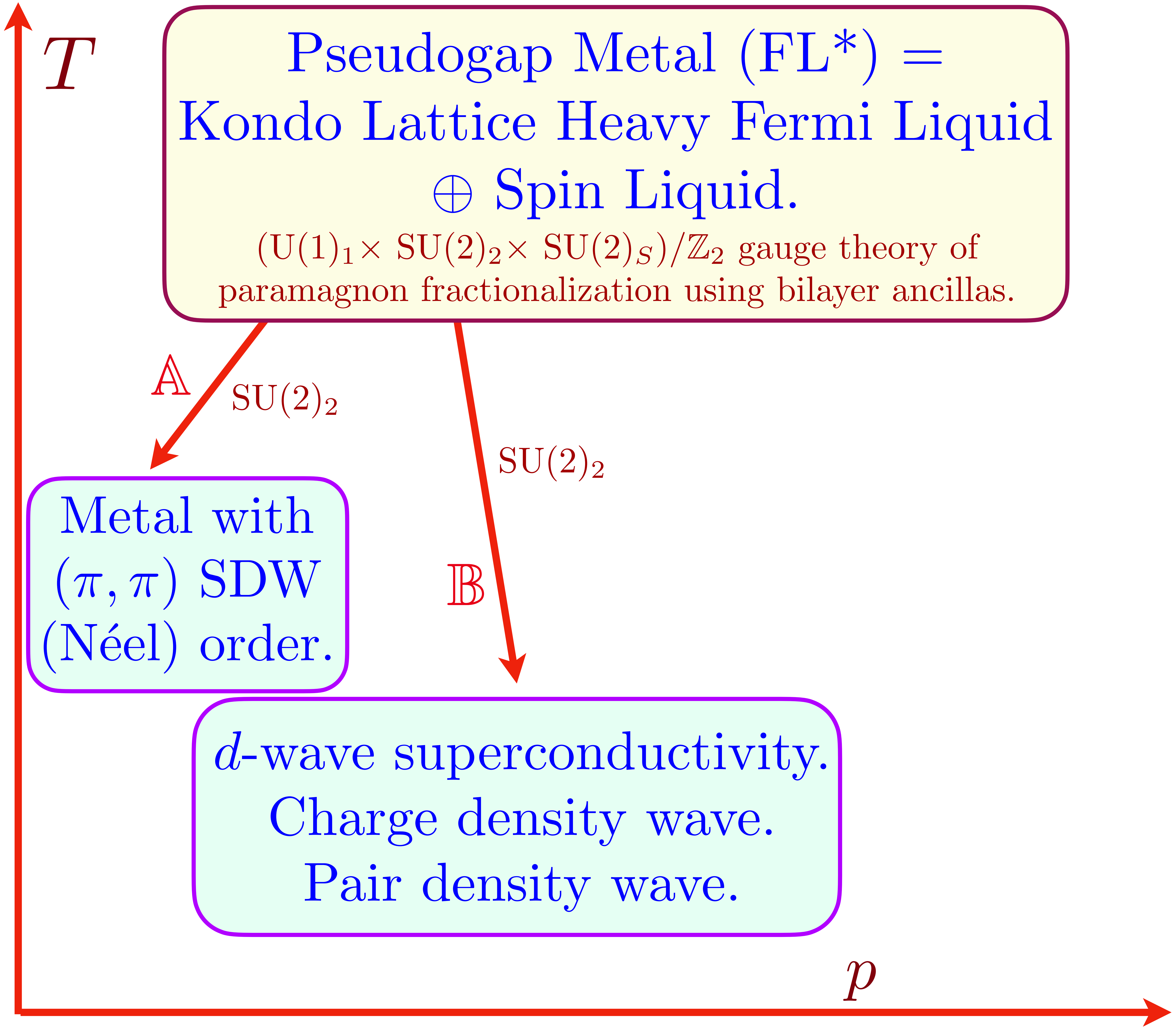

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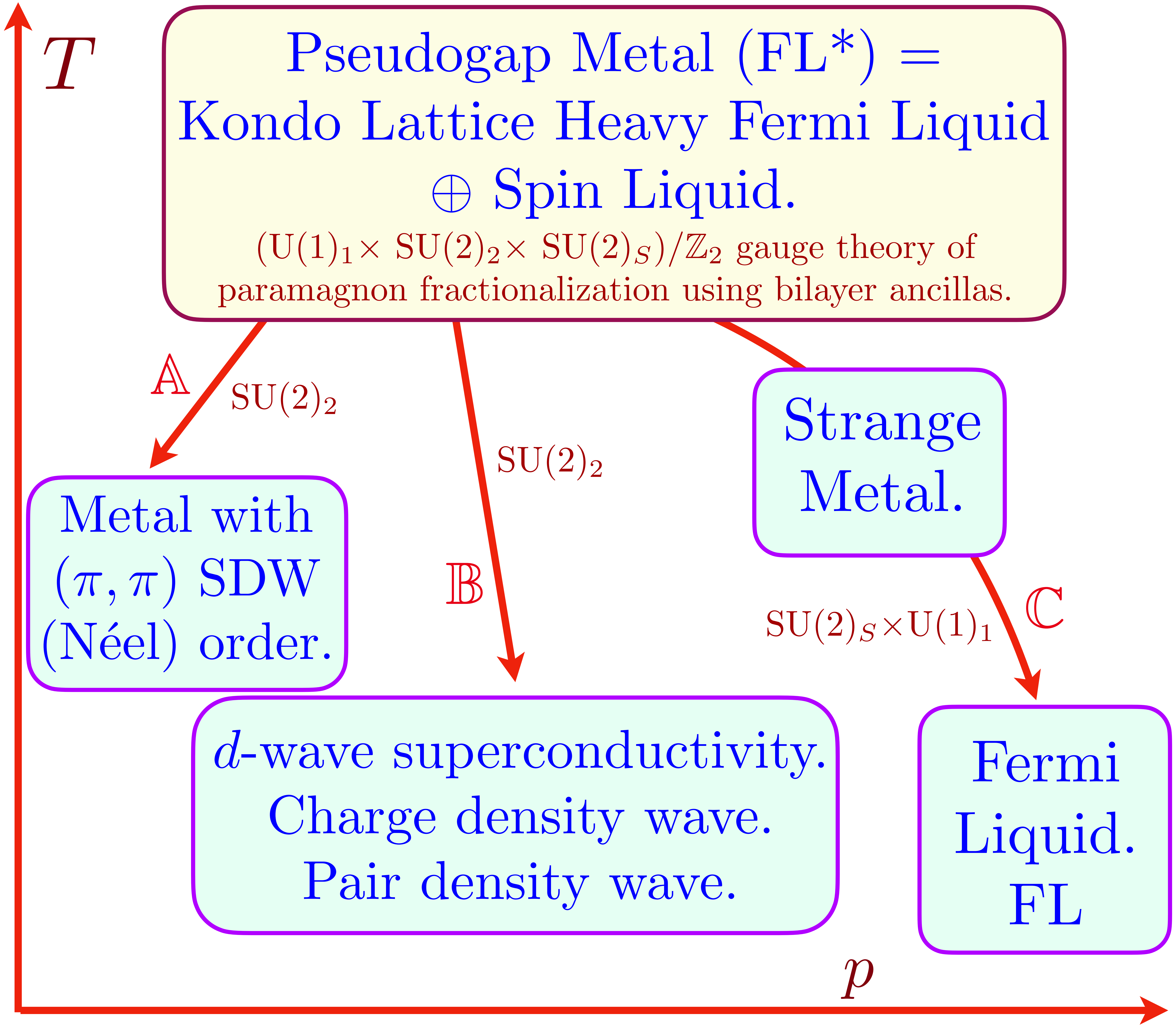
d-wave superconductivity.
Charge density wave.
Pair density wave.
p



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Strange Metal.
Metal with (π,π) SDW (Néel) order.
$\mathrm{SU}(2)_S\times\mathrm{U}(1)_1$
$\mathbb{C}$
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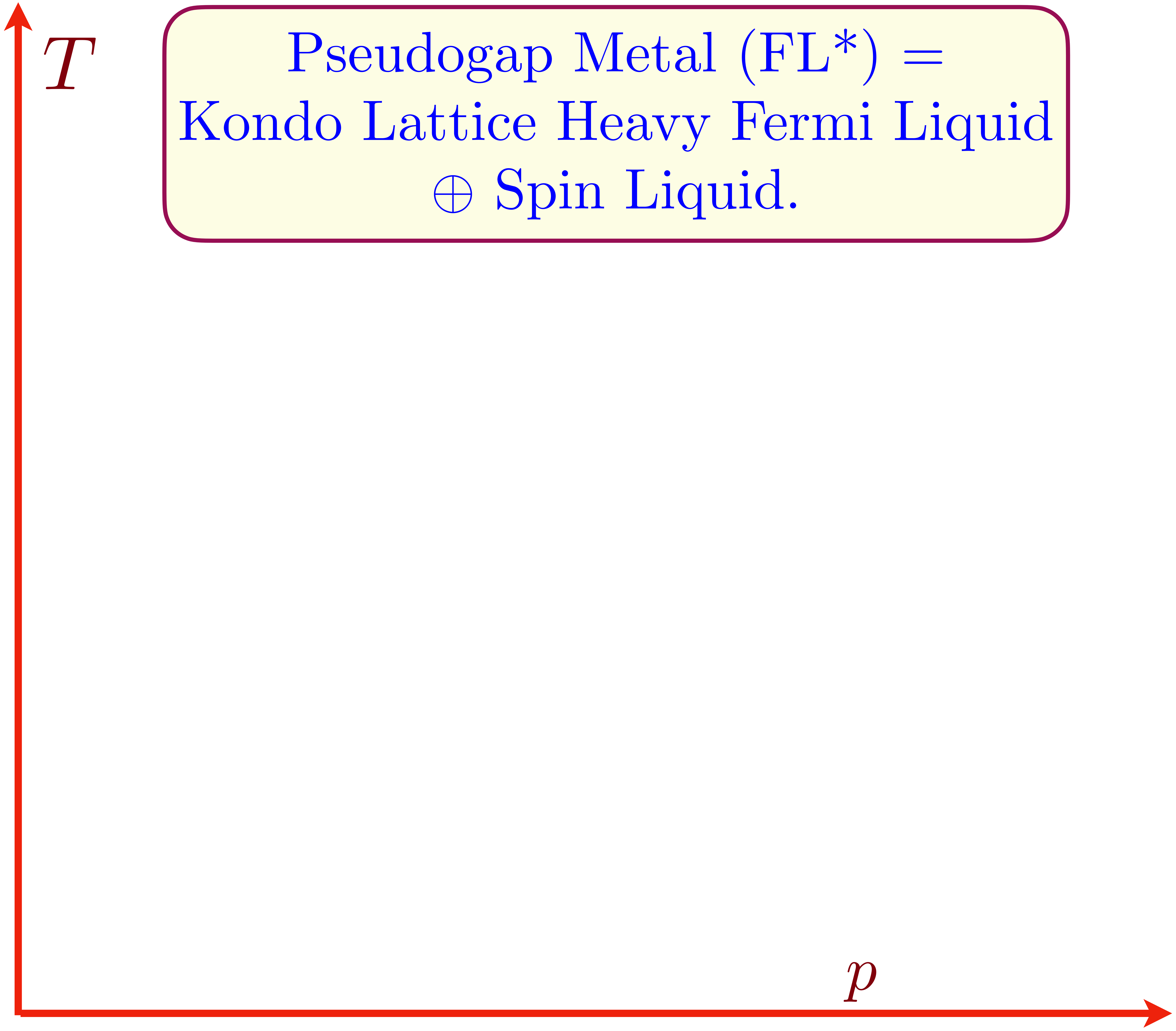
Charge density wave.
Pair density wave.
Fermi Liquid. FL
p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T
p

Yahui Zhang

Alexander Nikolaenko

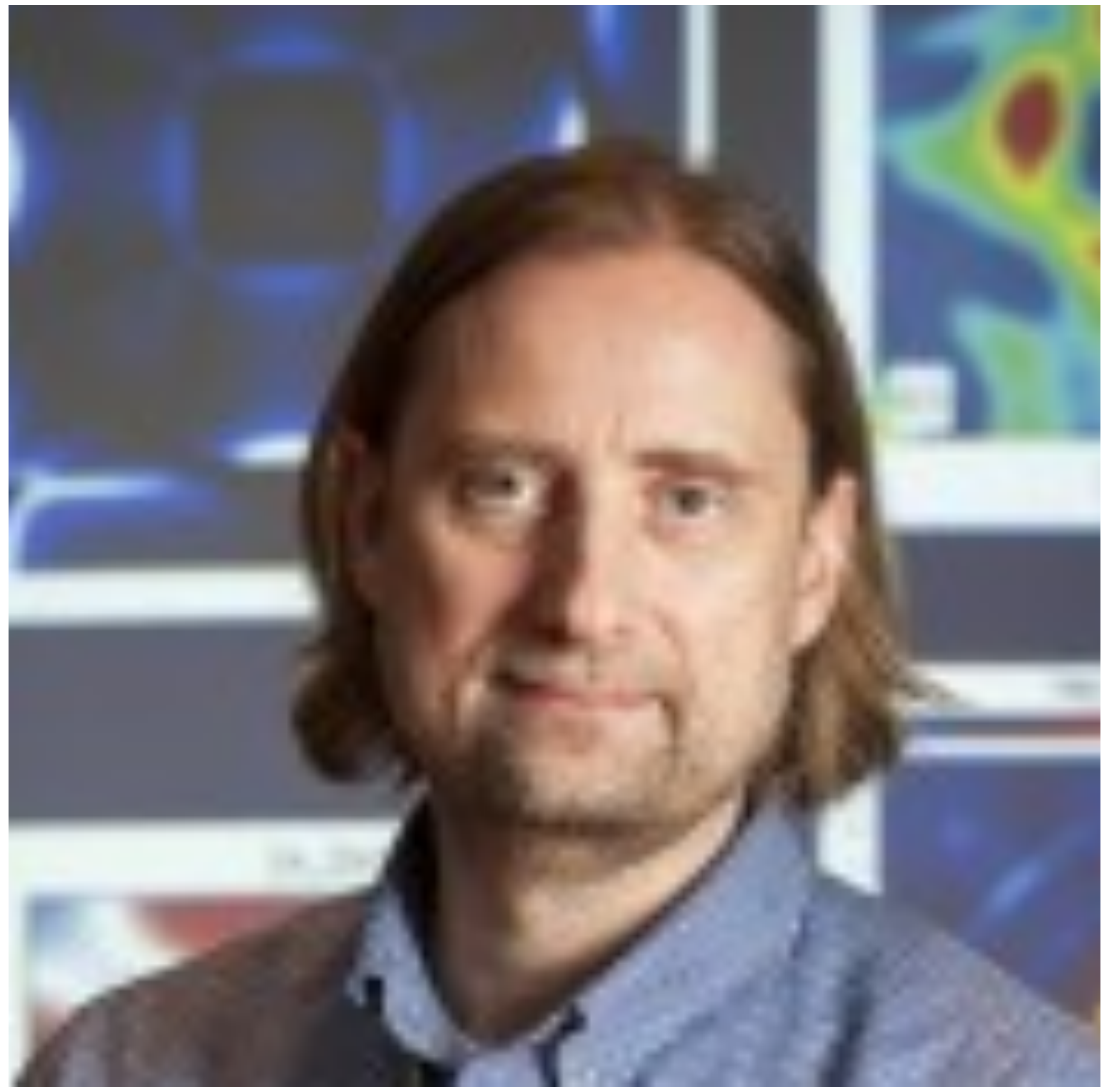
Maria Tikhanovskaya

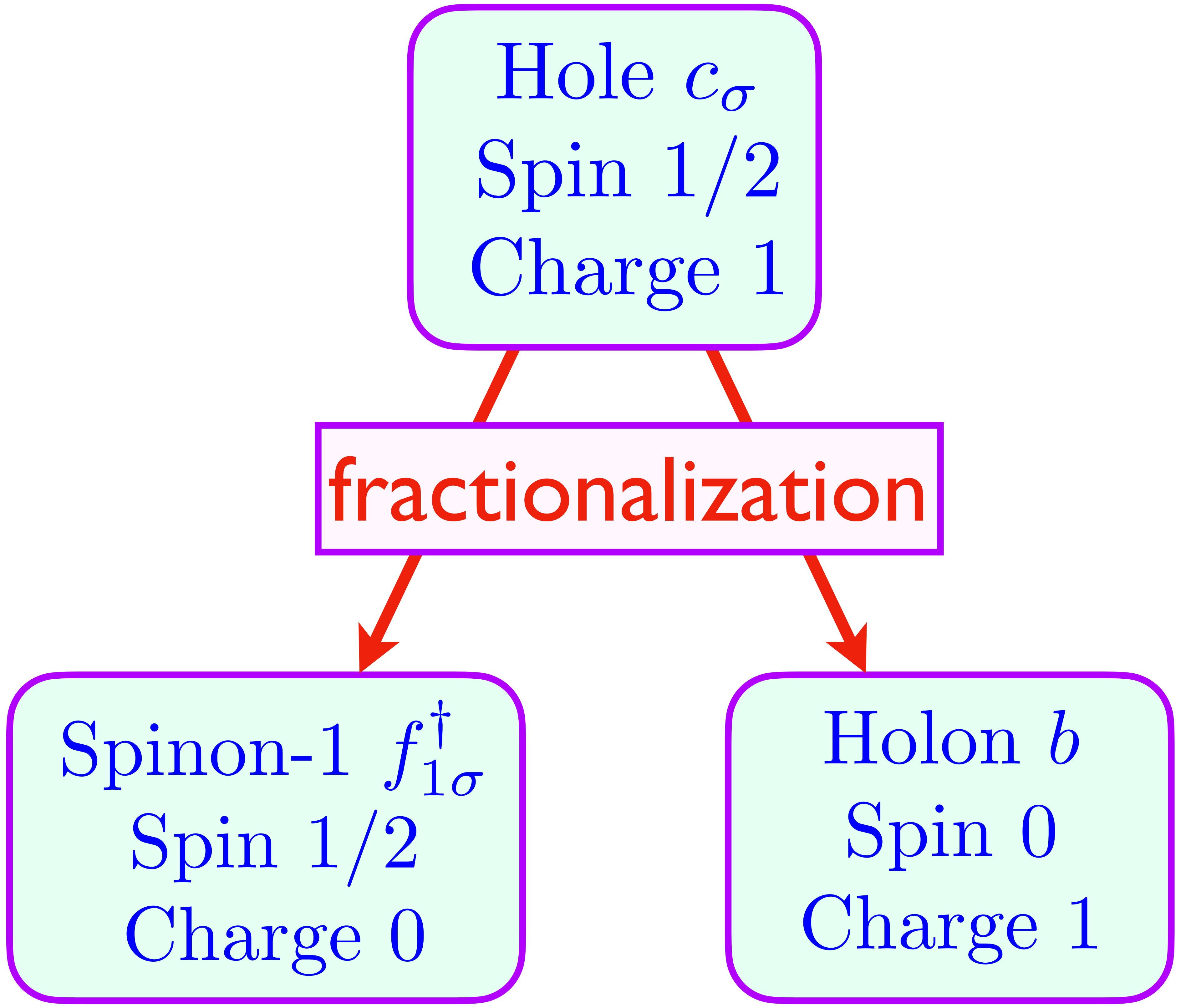
Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

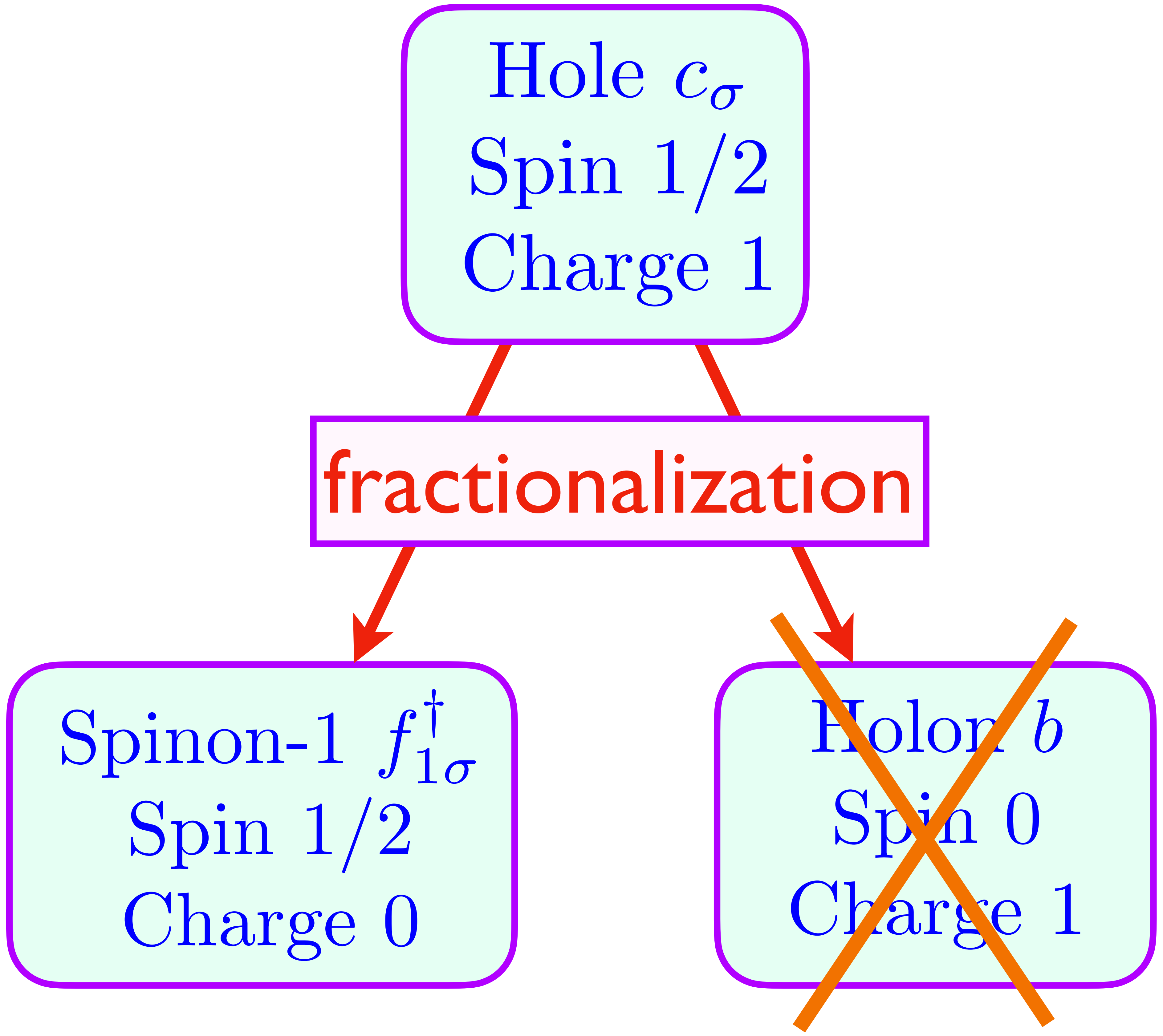
arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr



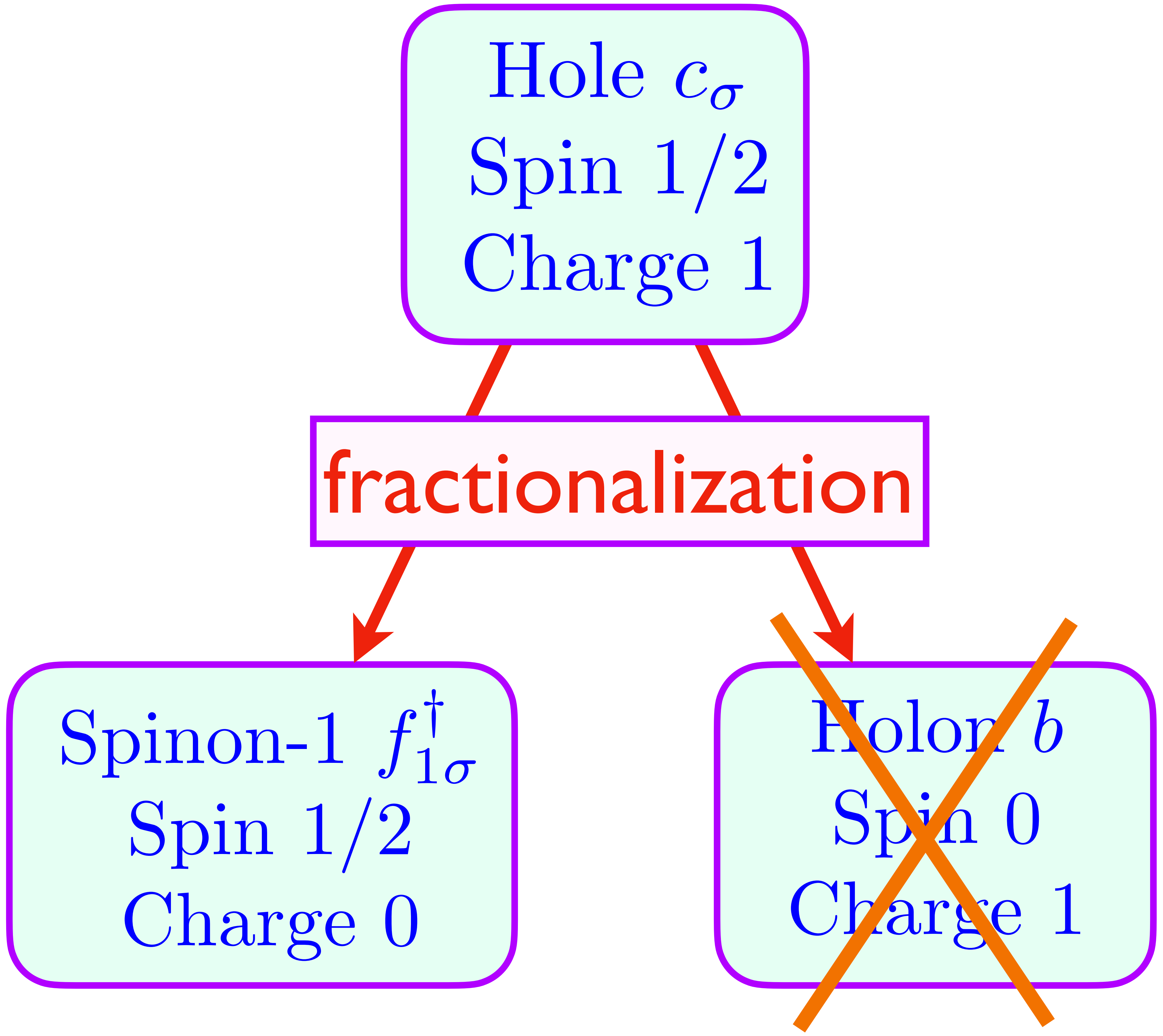


Electron fractionalization



Electron fractionalization

$$\text{Fractionalization} \sim J$$
$$\text{Holon-spin attraction} \sim t$$
$$\text{Holon does not exist for } t \gg J.$$



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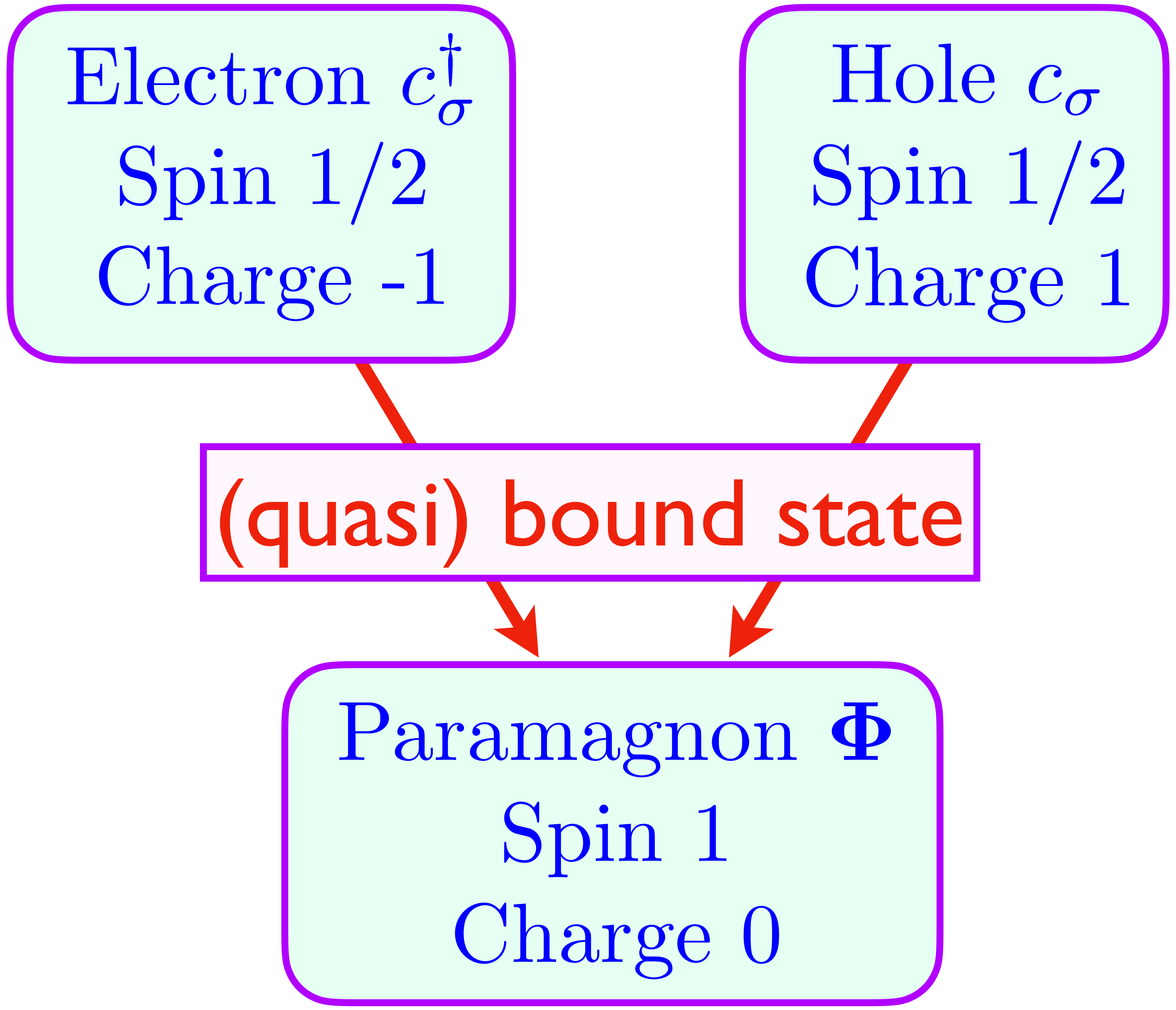
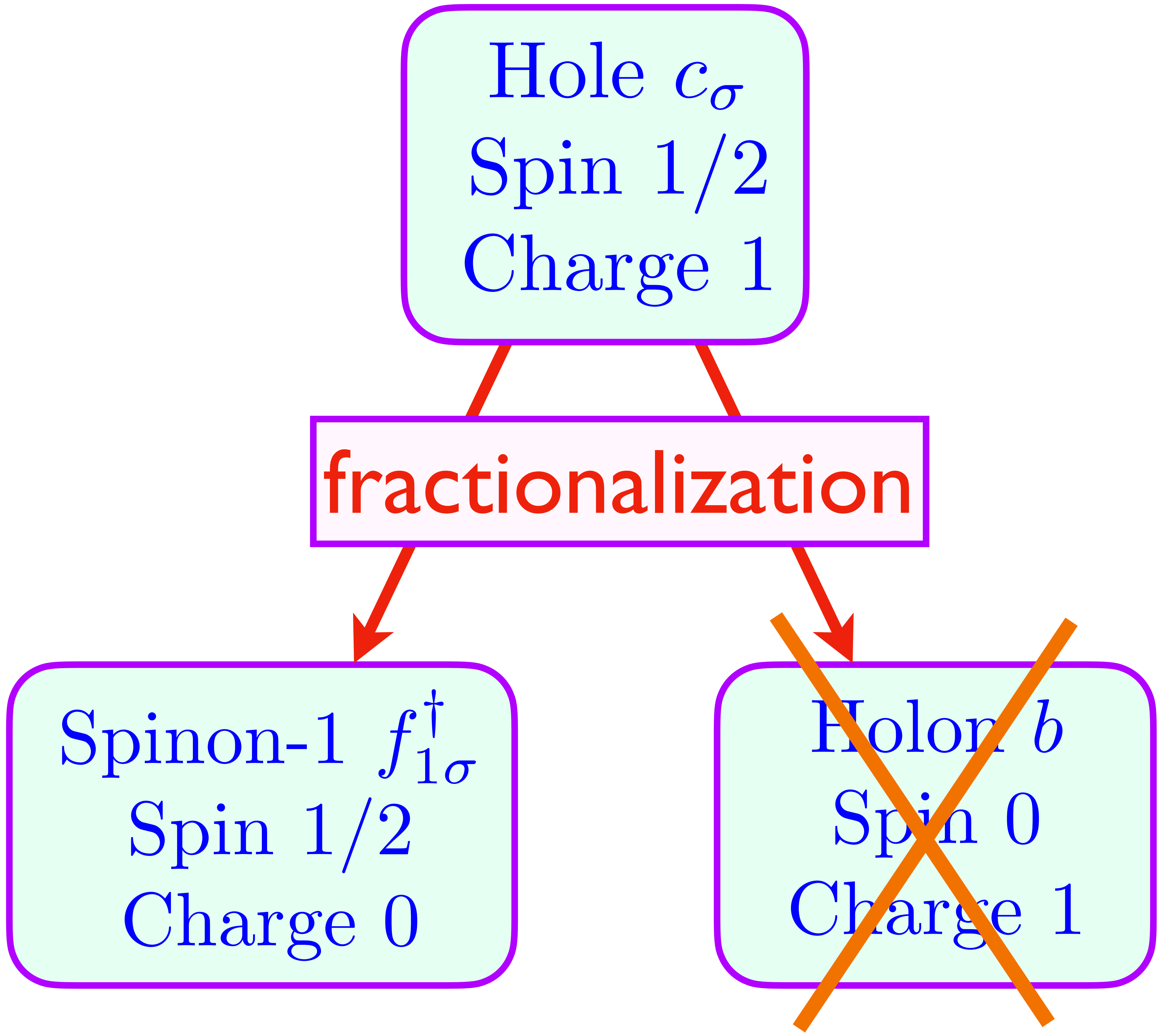
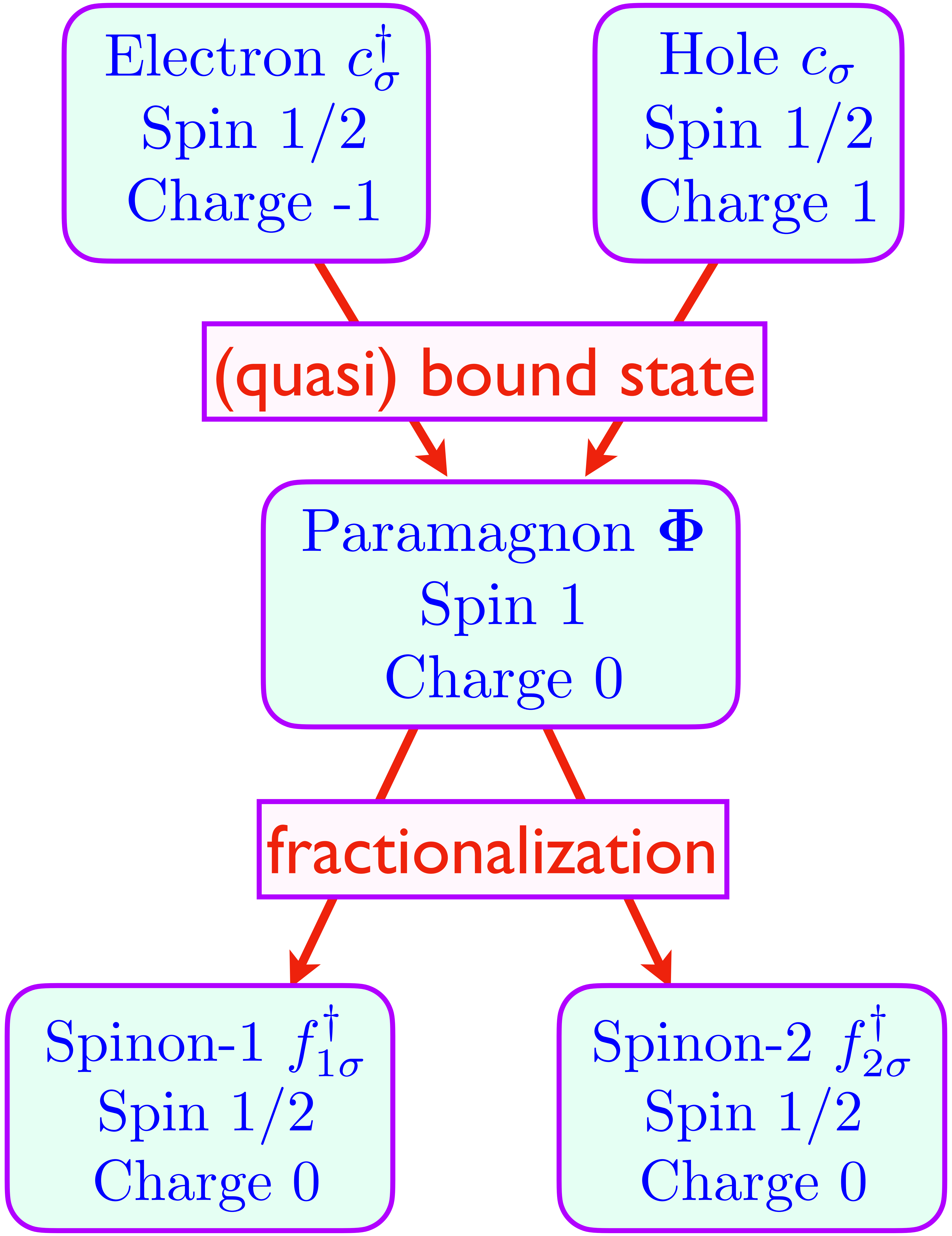


Don’t fractionalize the electron;
fractionalize the paramagnon!



Electron fractionalization



Paramagnon fractionalization

Don’t fractionalize the electron; fractionalize the paramagnon!

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

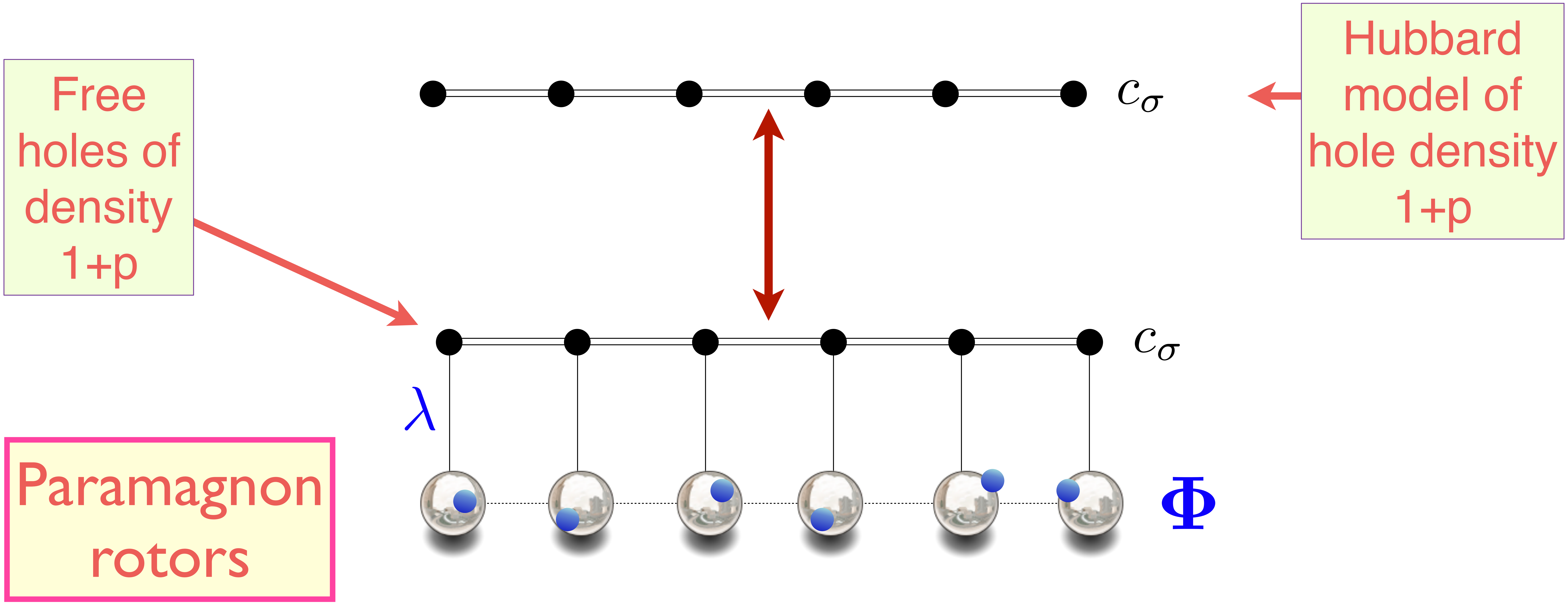
$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

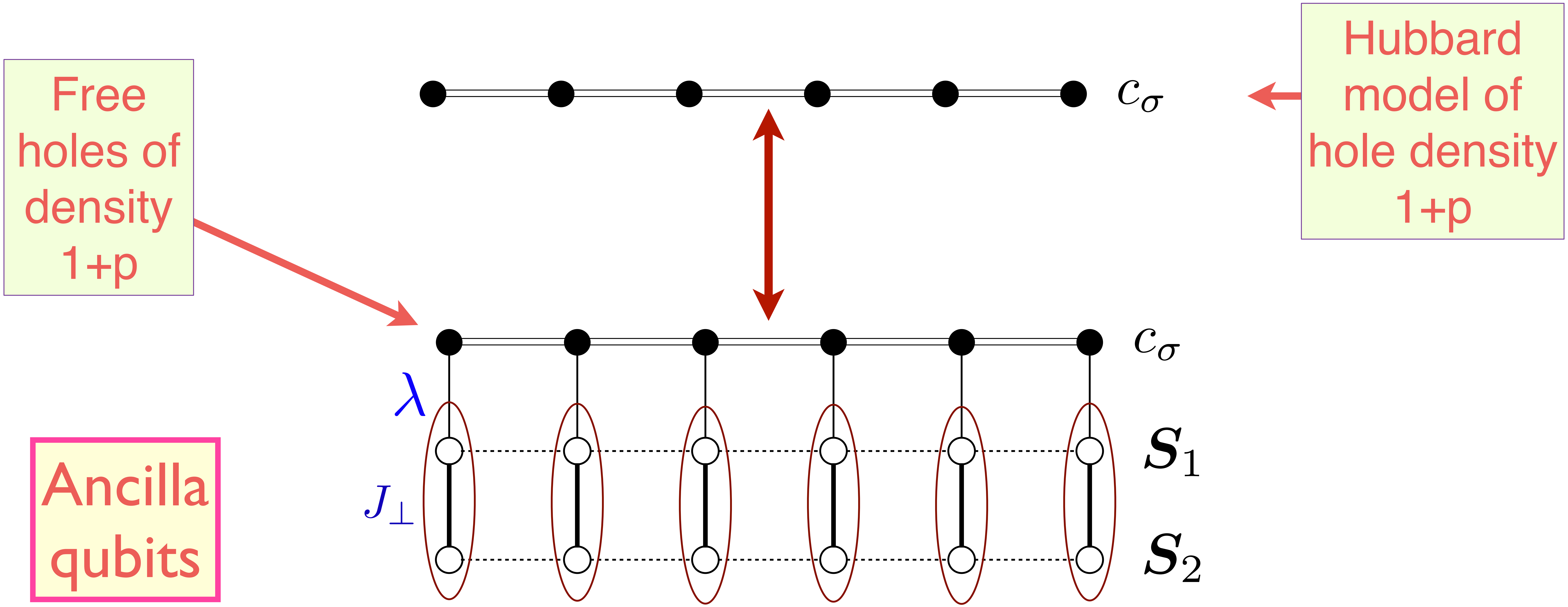
This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model



$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^{\dagger} c_{\mathbf{p}\sigma} - \lambda \sum_{i} c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_{i} + \frac{J_{\perp}}{2} \sum_{i} \boldsymbol{P}_{\boldsymbol{\Phi}i}^{2} + \sum_{i} V(\boldsymbol{\Phi}_{i}) + \ldots$$

Paramagnon theory of the Hubbard model



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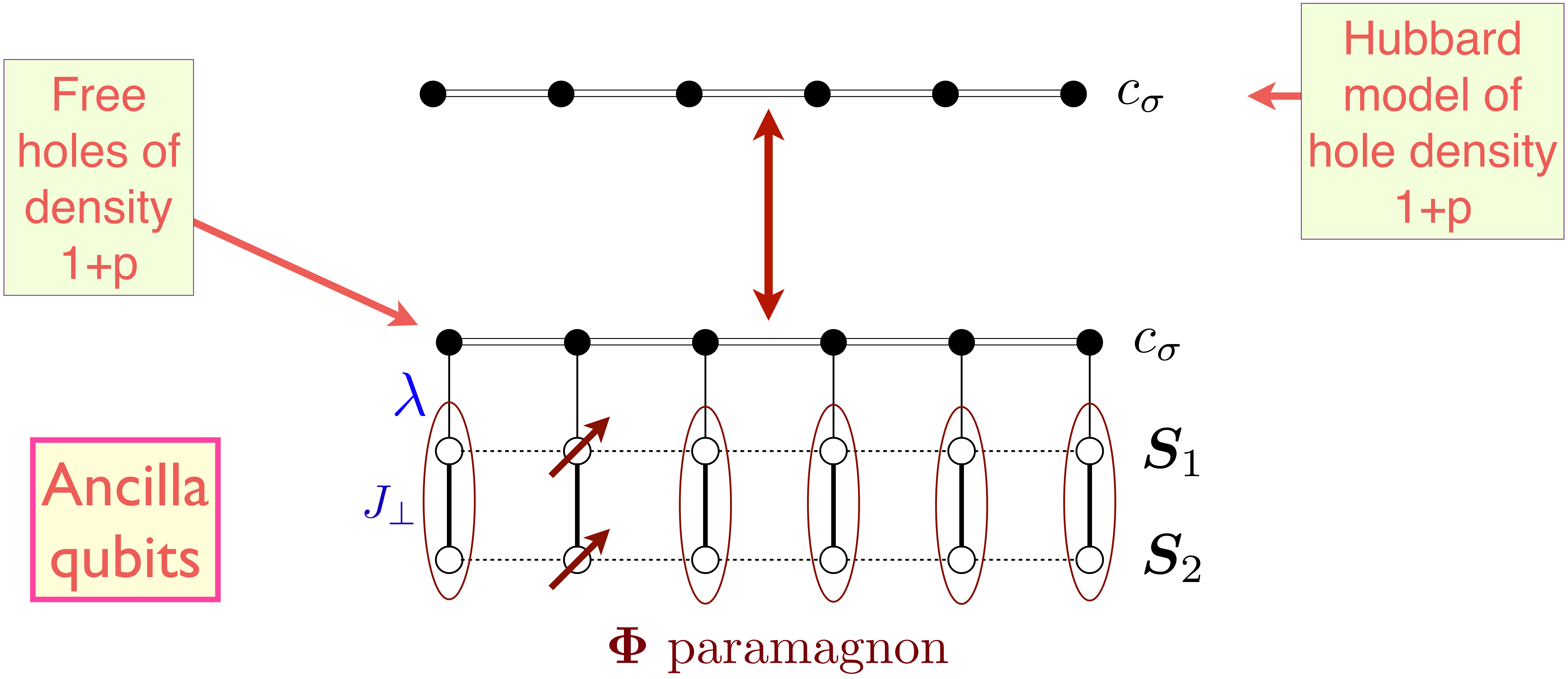

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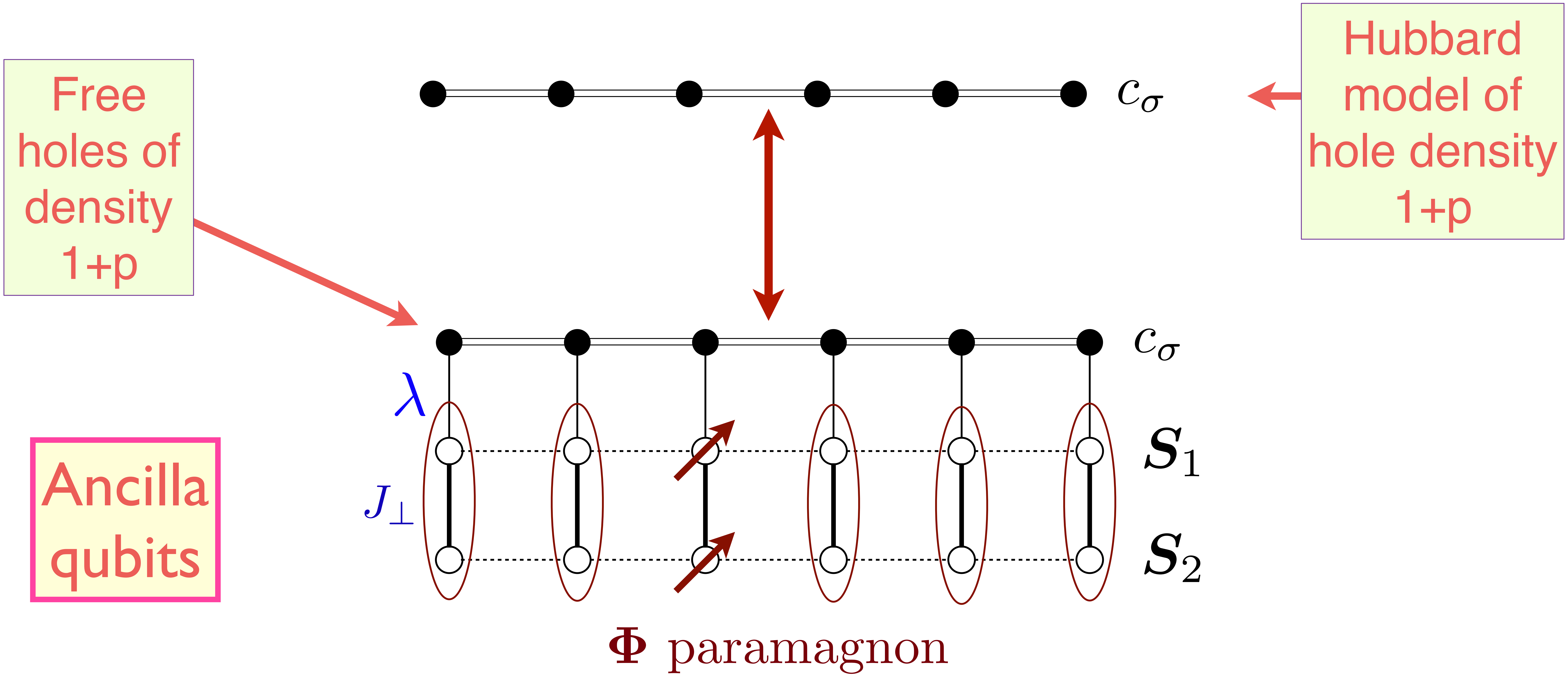

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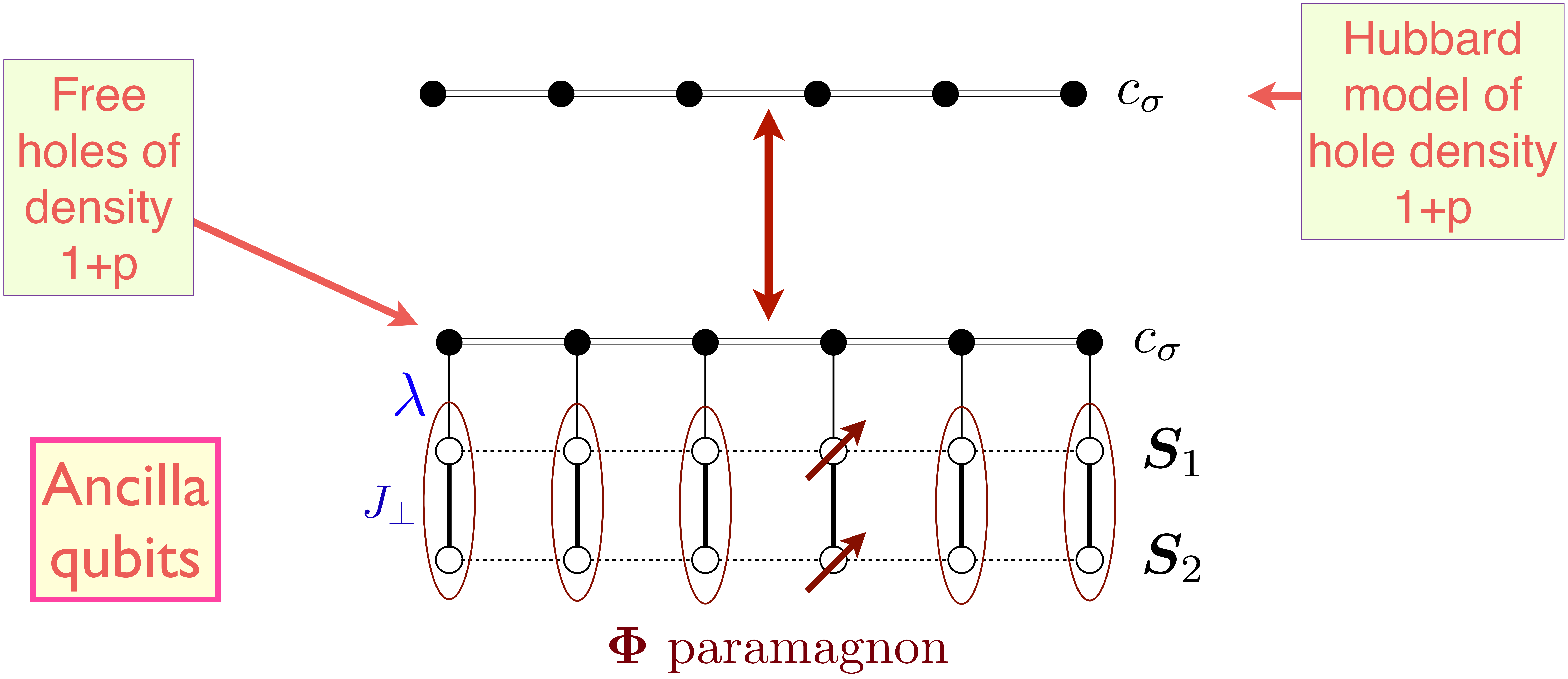

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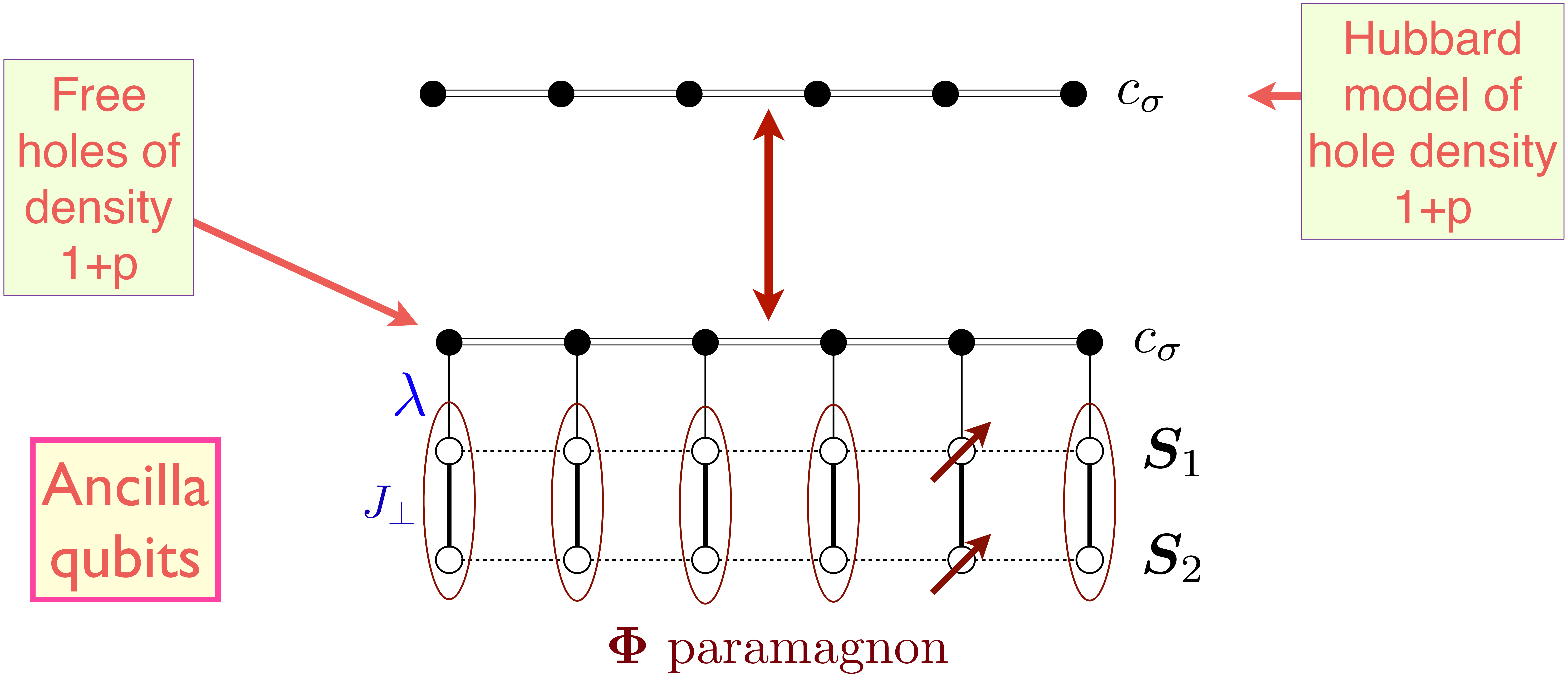

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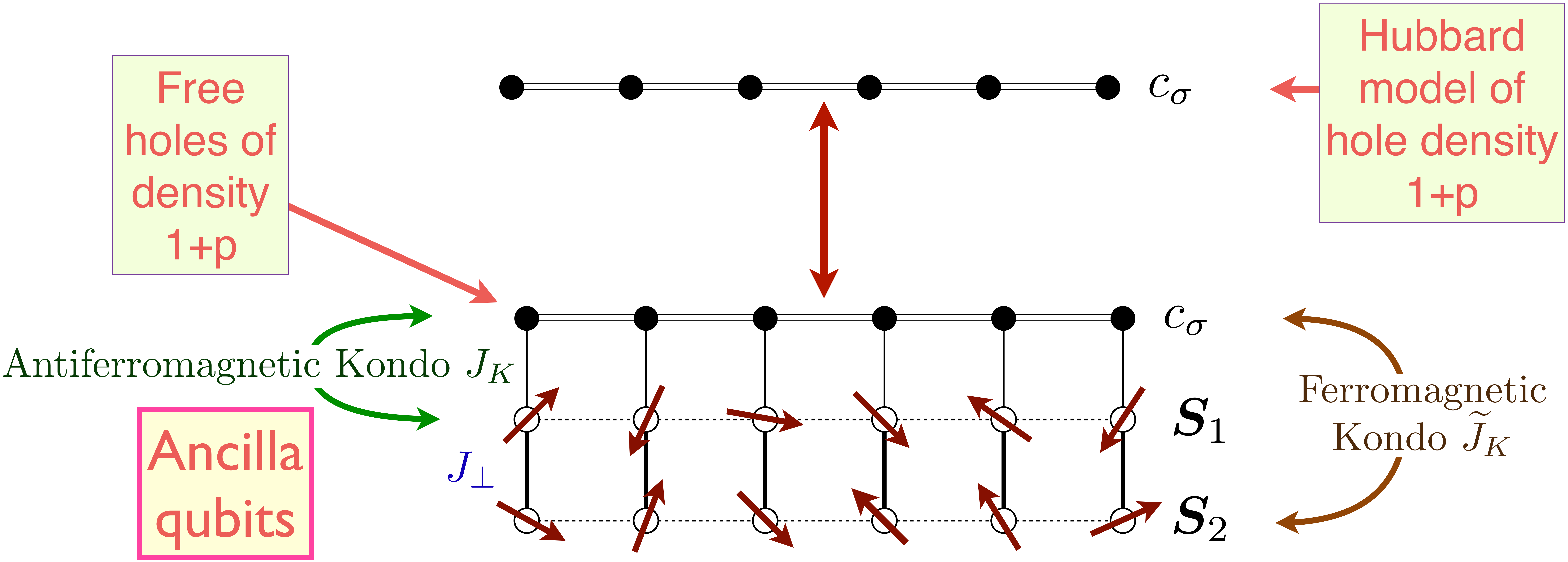
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

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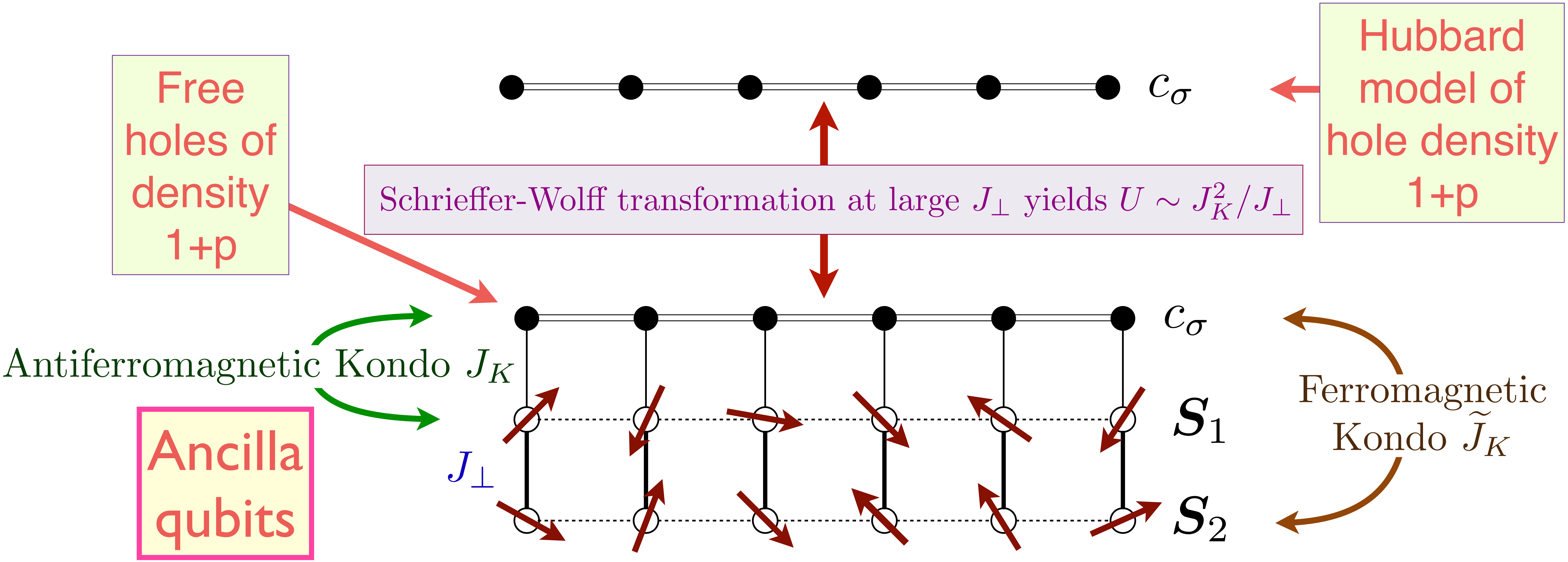

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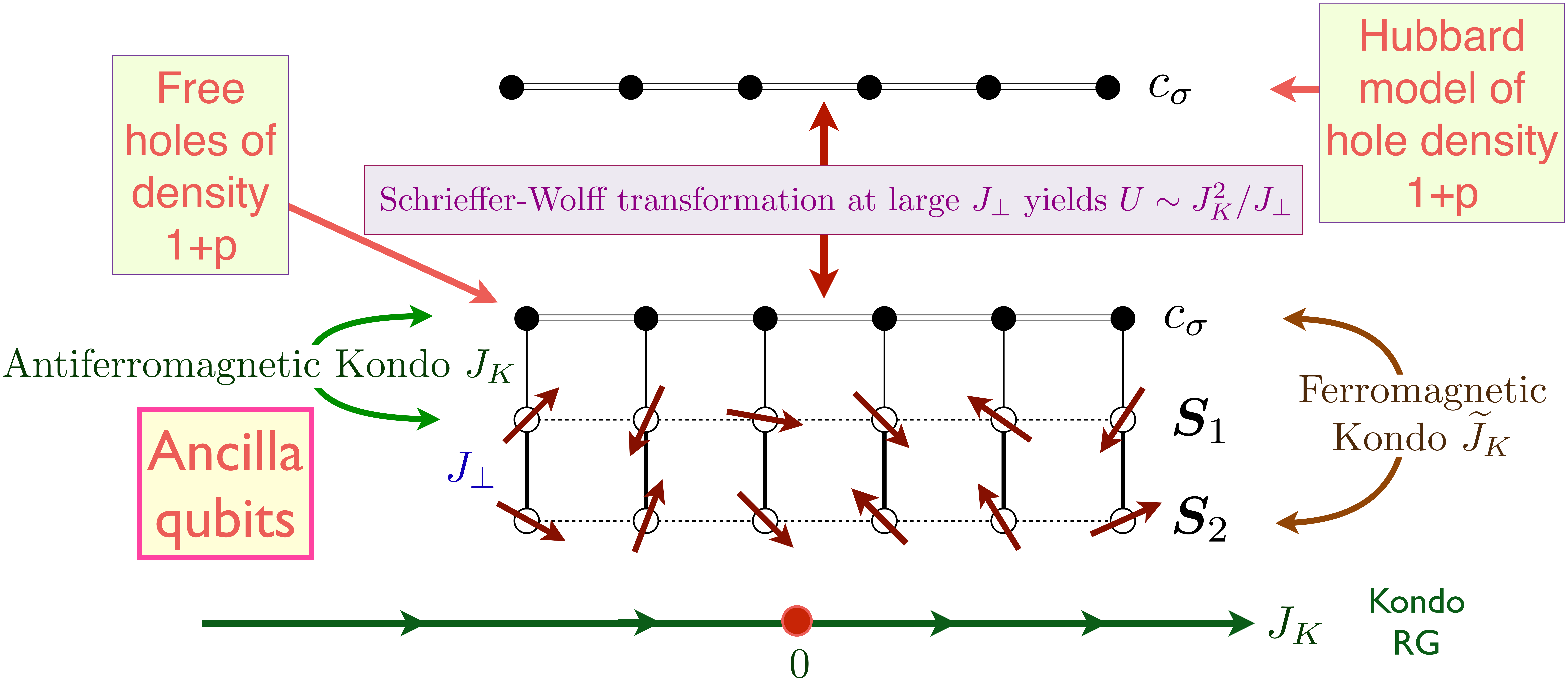

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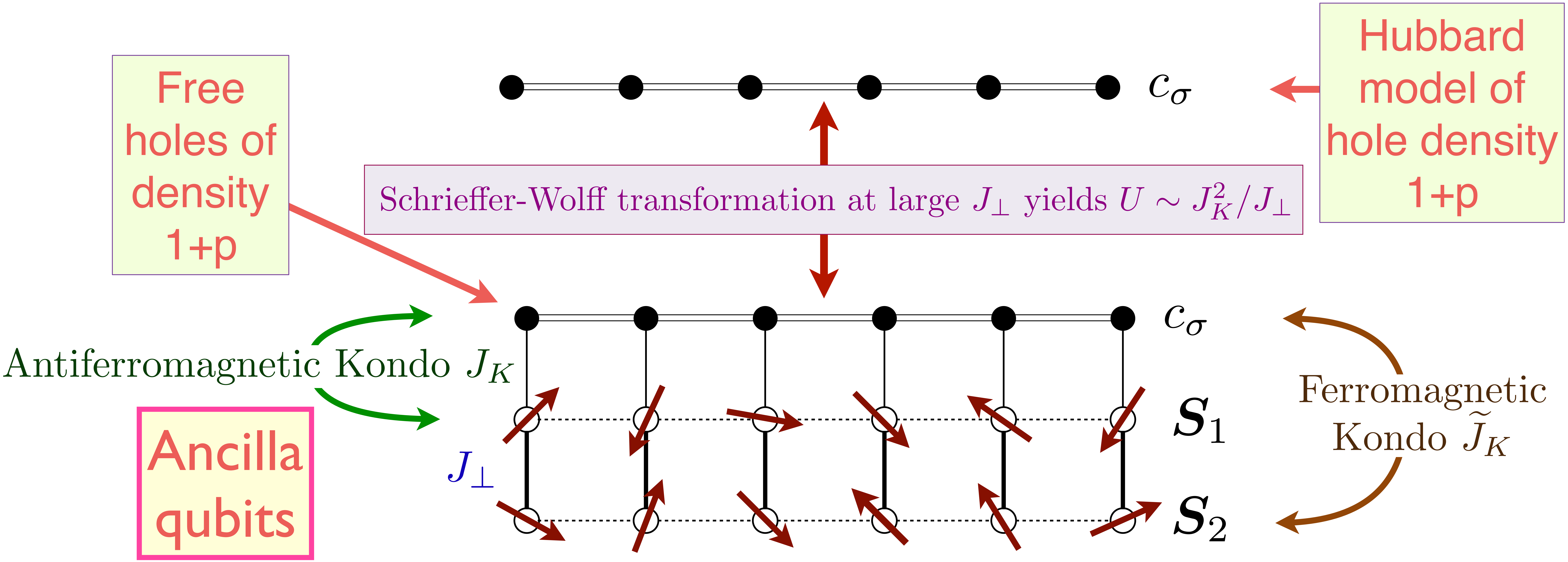
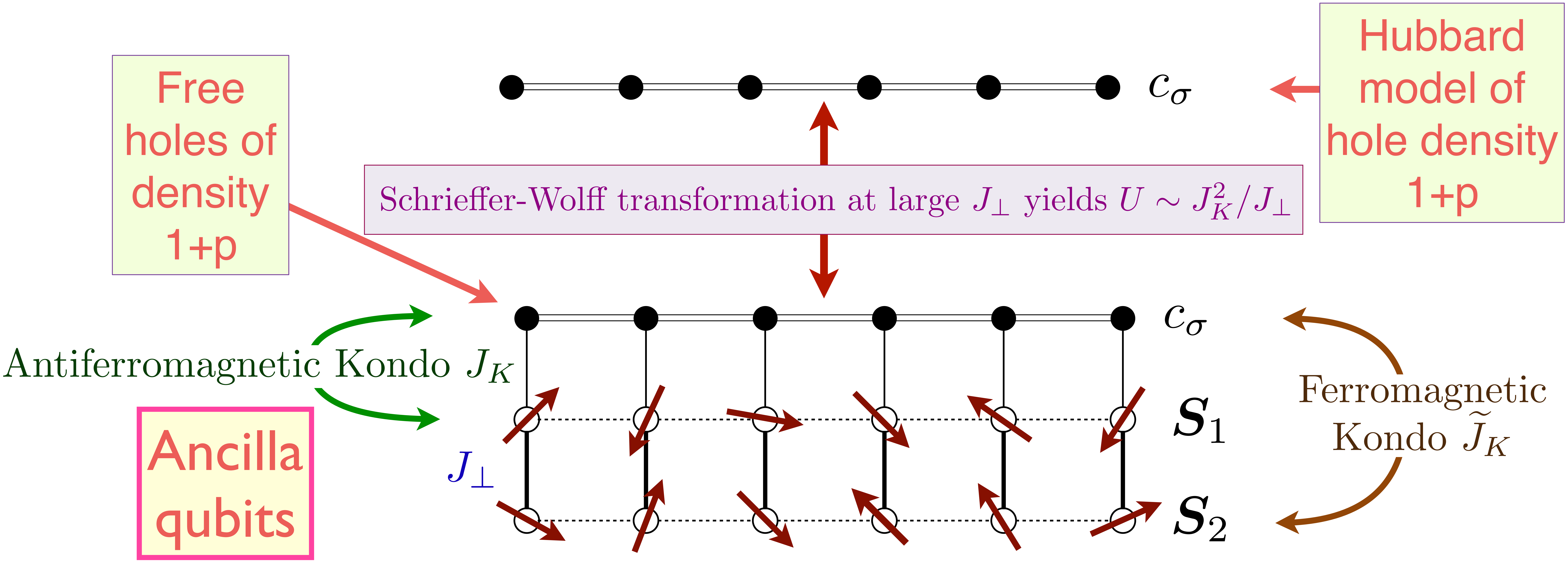


A FL* state is realized when the antiferromagnetic Kondo coupling dominates over $J_\perp$, and the c_σ and $\boldsymbol{S}_1$ form a heavy Fermi liquid state (as found in the heavy fermion compounds) of hole density $(1+p)+1 = 2+p = p$ mod 2!

Paramagnon fractionalization theory of the Hubbard model



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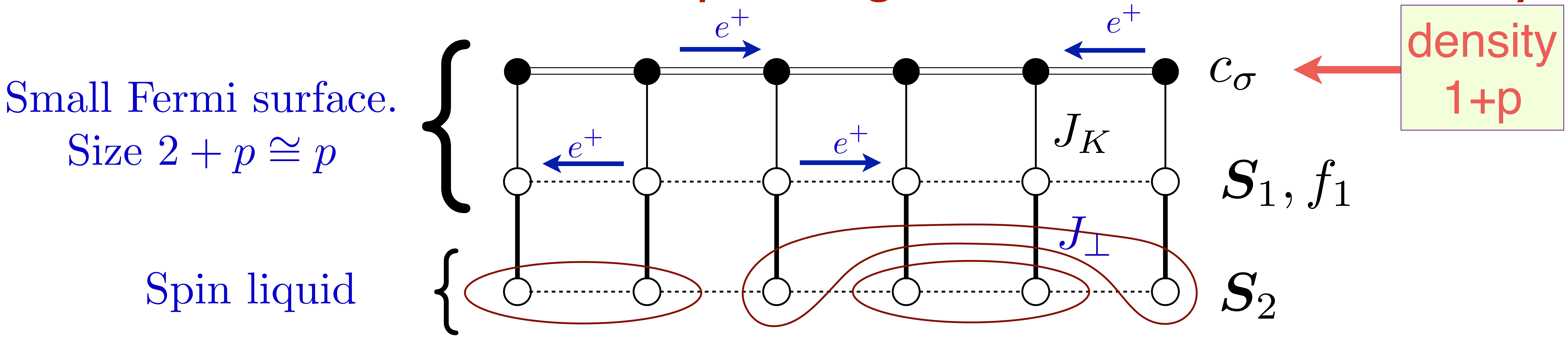
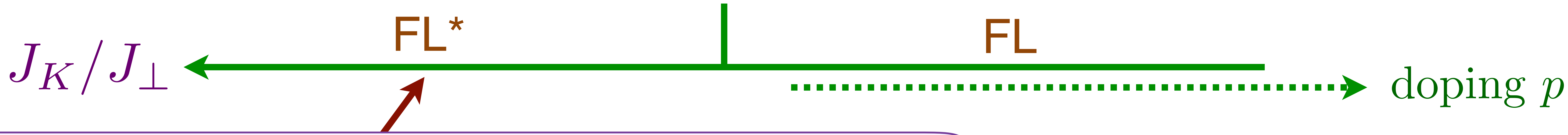
The $\boldsymbol{S}_2$ must form an 'odd' spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

Trial wavefunctions in the paramagnon fractionalization theory

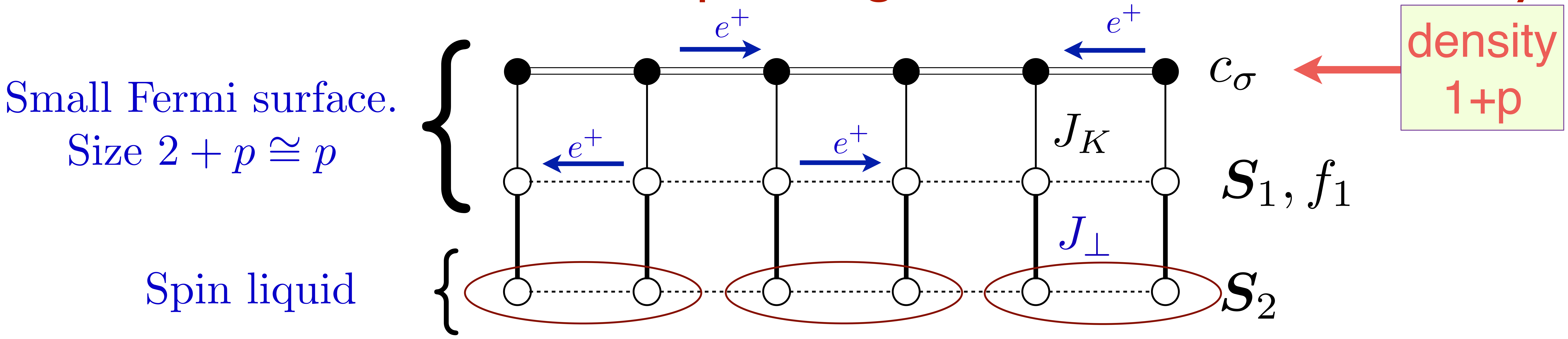


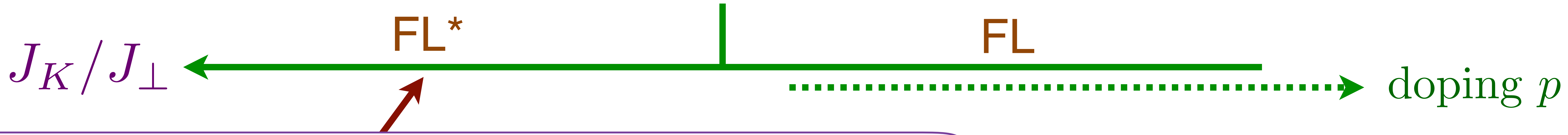


Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$$
$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
$$\otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory





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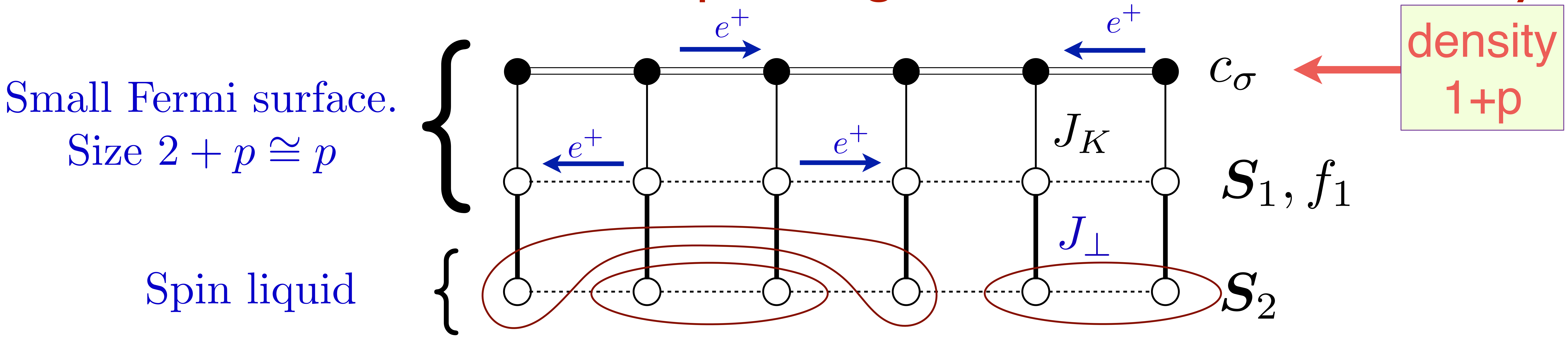

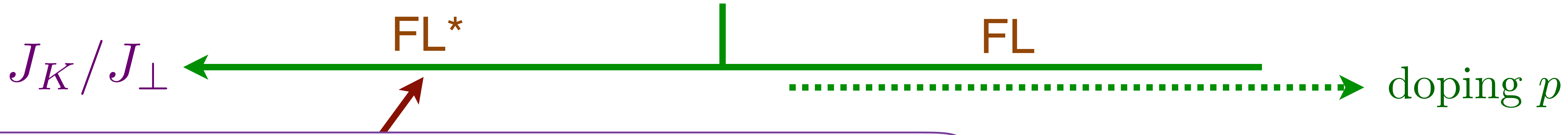

Small Fermi surface of size p

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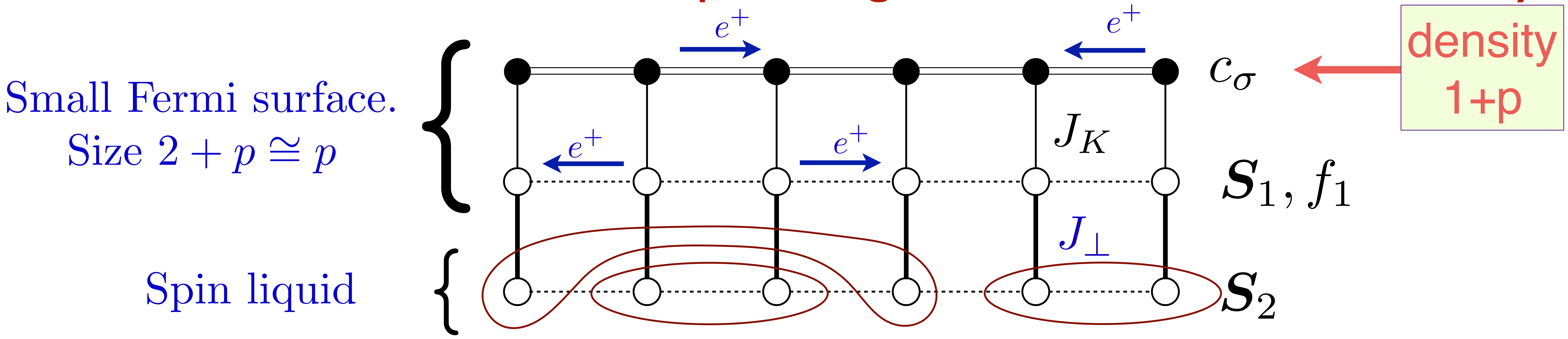
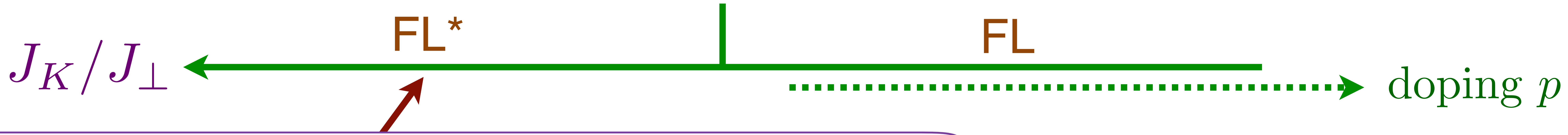
Trial wavefunctions in the paramagnon fractionalization theory





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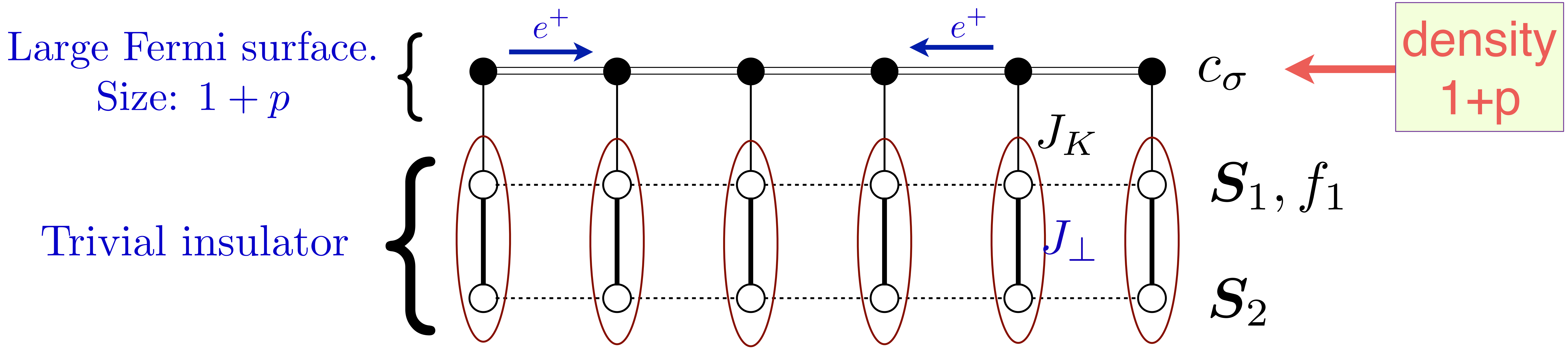
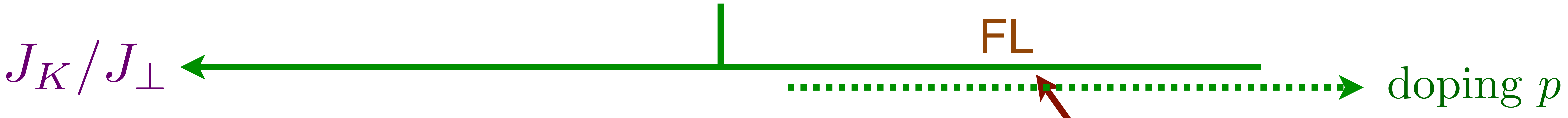


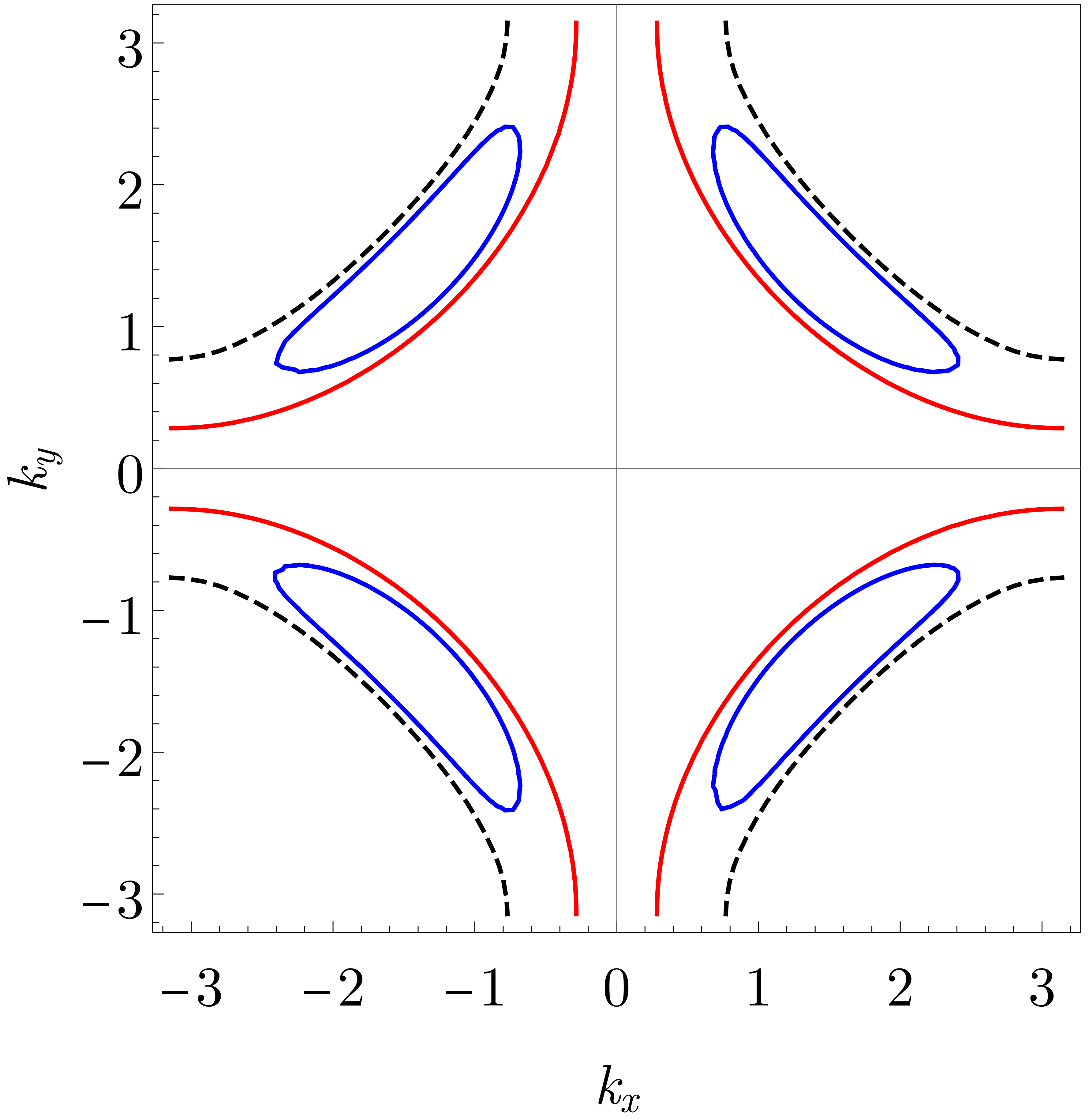


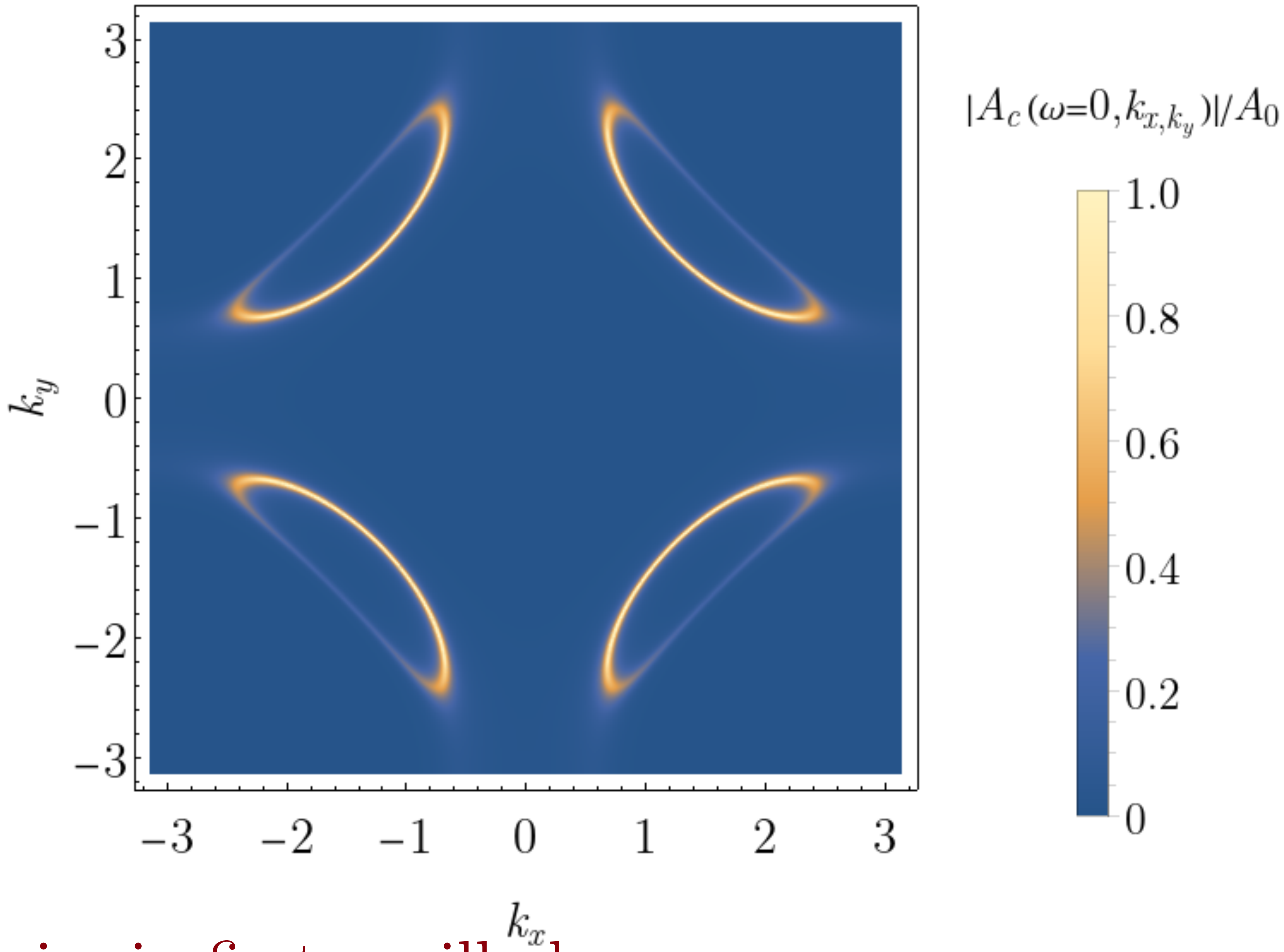
Large Fermi surface of size $1 + p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

FL* in a ***one-band*** model

"Fermi arc" spectral functions

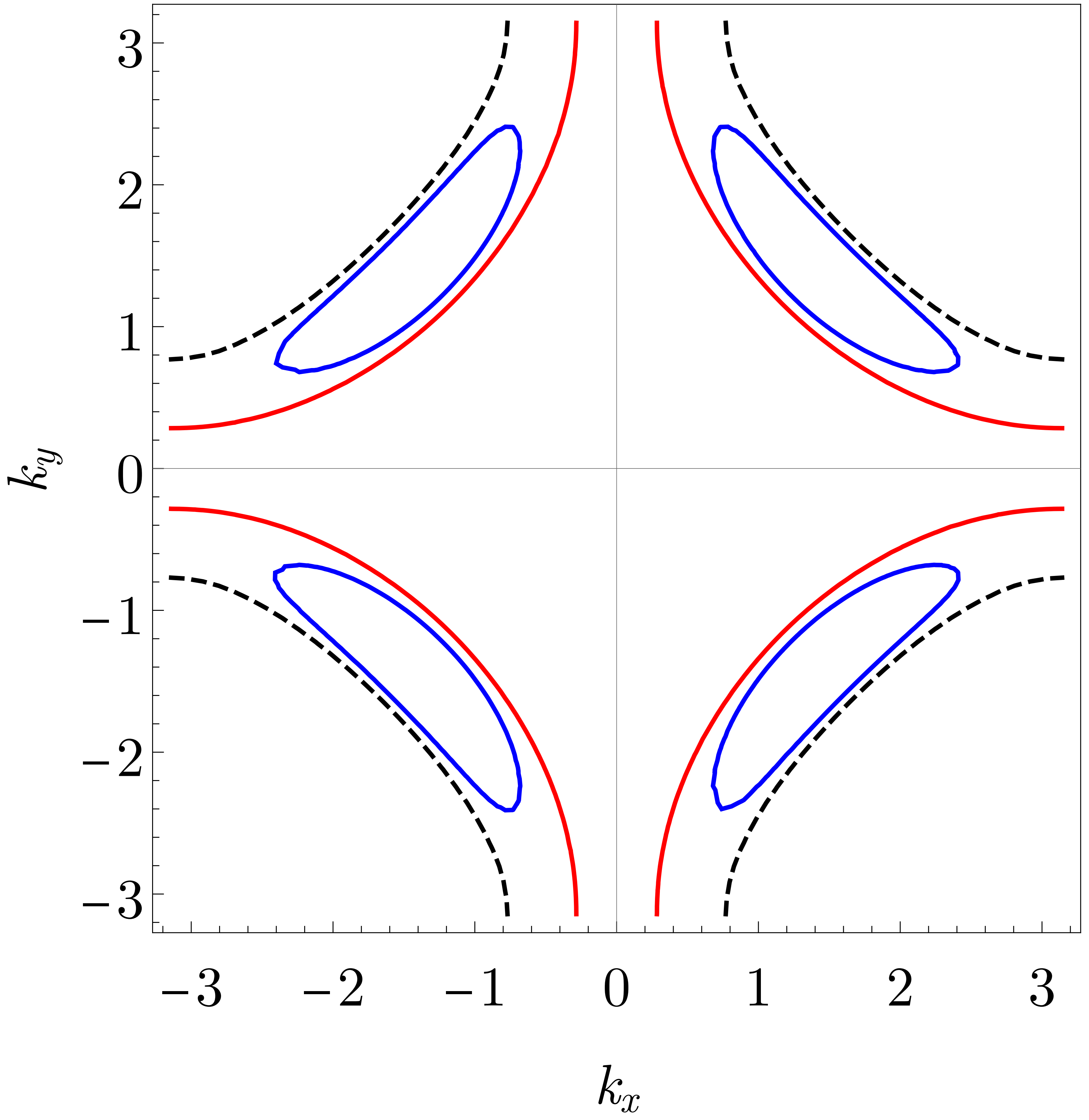




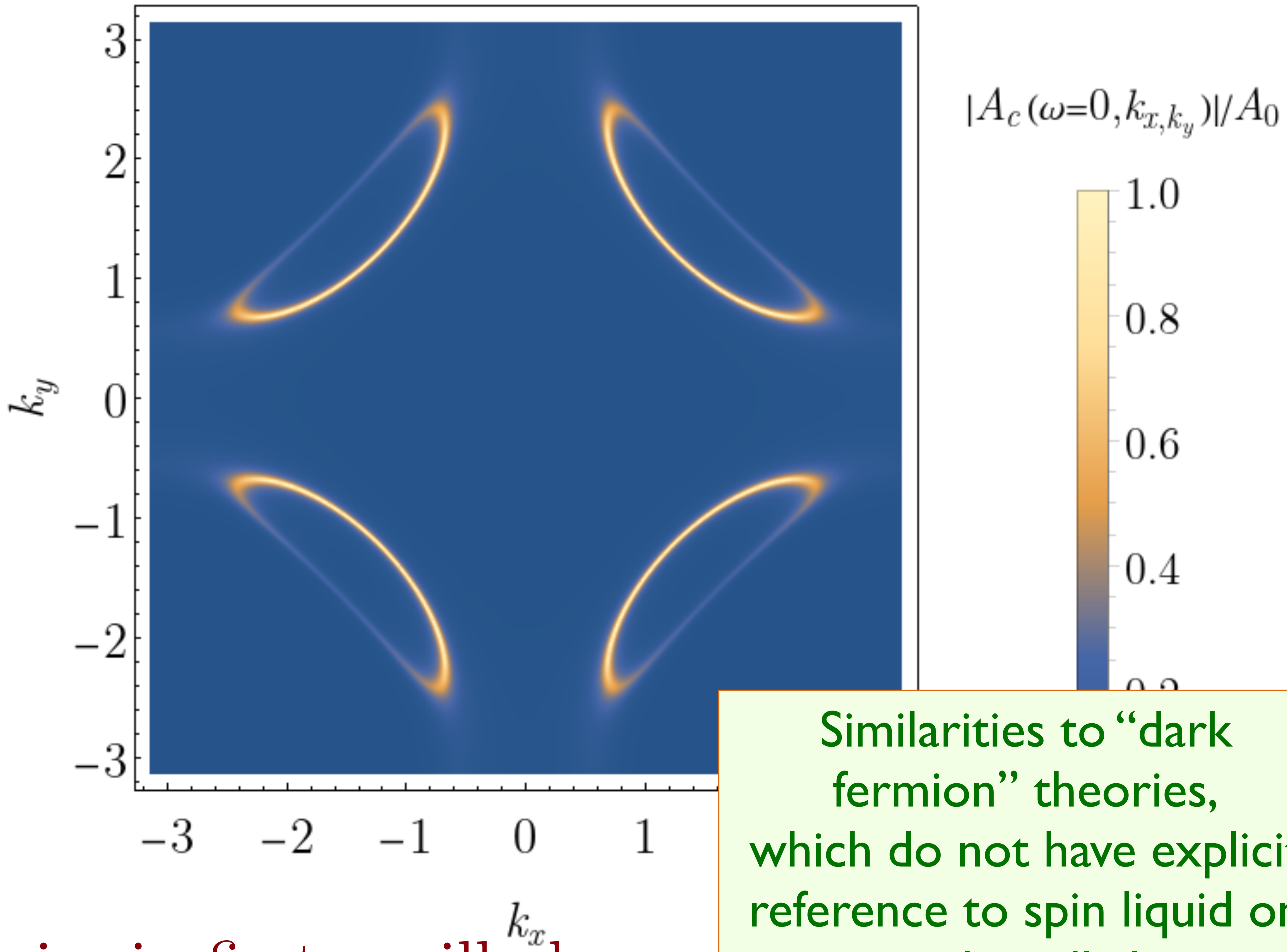
FL*: Condensate B breaks gauge symmetries in first ancilla layer.

$$H = -\sum_{i,j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{1i\sigma}^{\dagger} f_{1j\sigma} + \sum_{i} B \left(c_{i\sigma}^{\dagger} f_{1i\sigma} + f_{1i\sigma}^{\dagger} c_{\sigma} \right)$$

FL* in a ***one-band*** model



“Fermi arc” spectral functions



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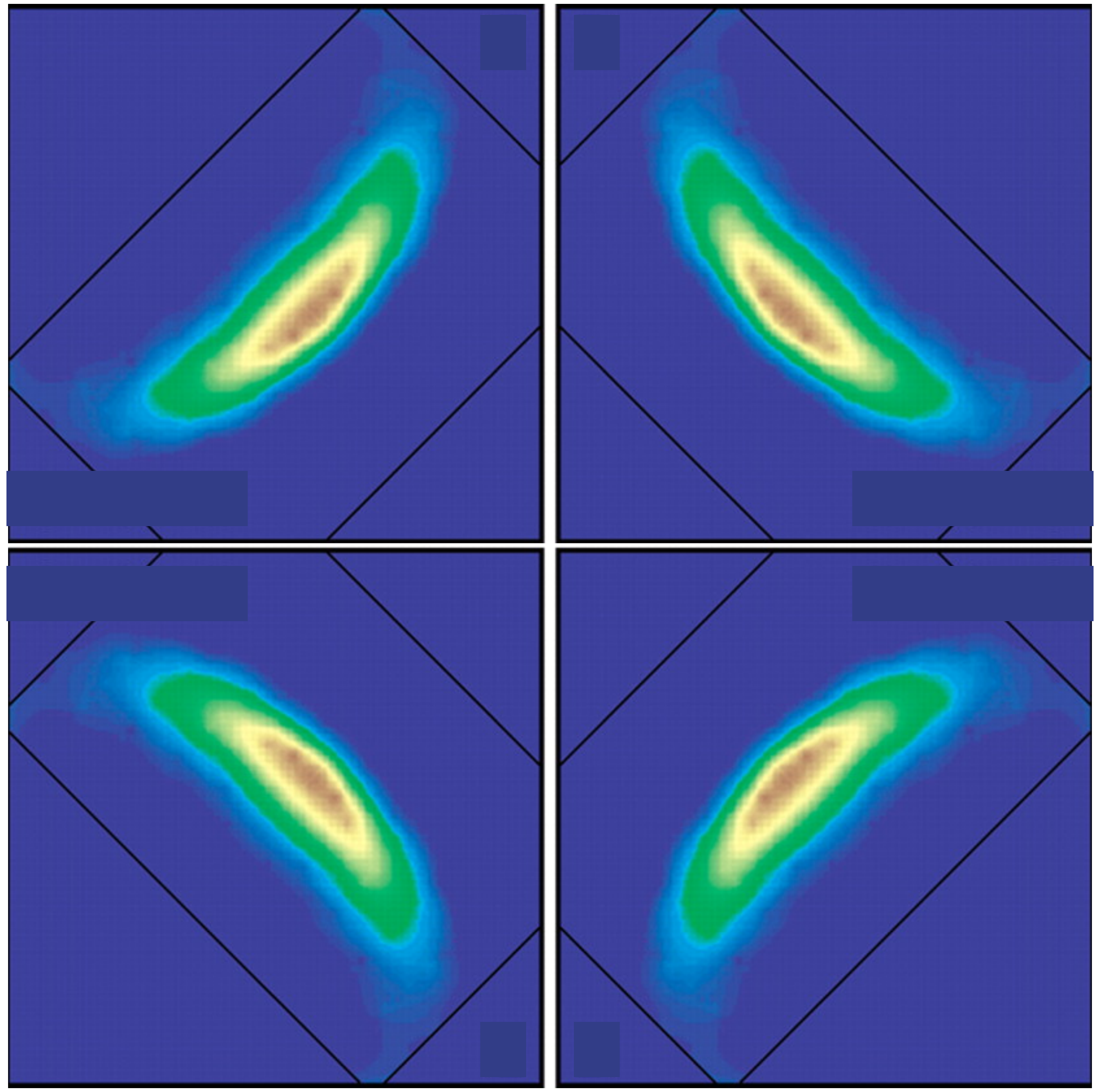
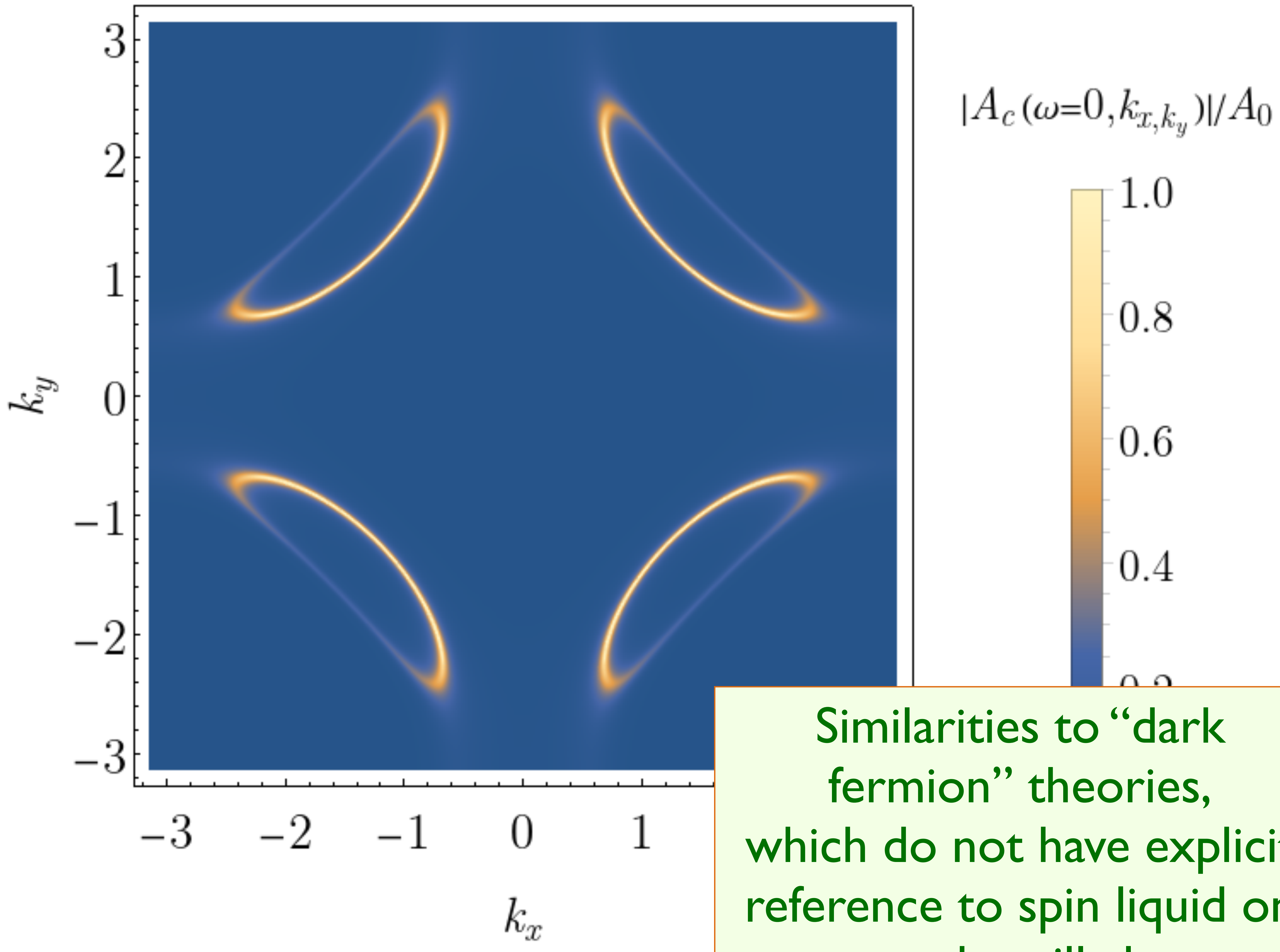
Similarities to “dark fermion” theories, which do not have explicit reference to spin liquid on second ancilla layer

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$



"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

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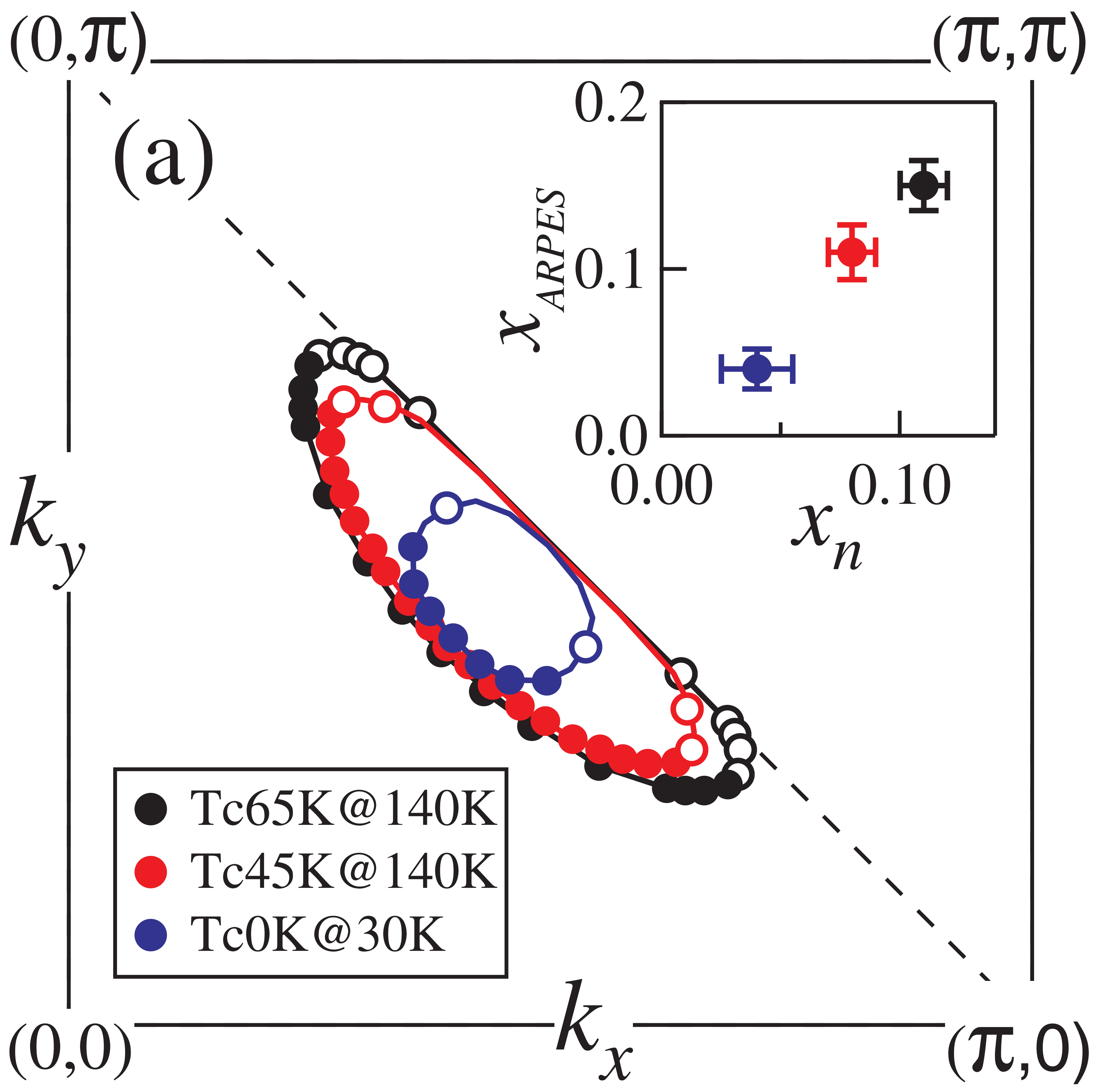
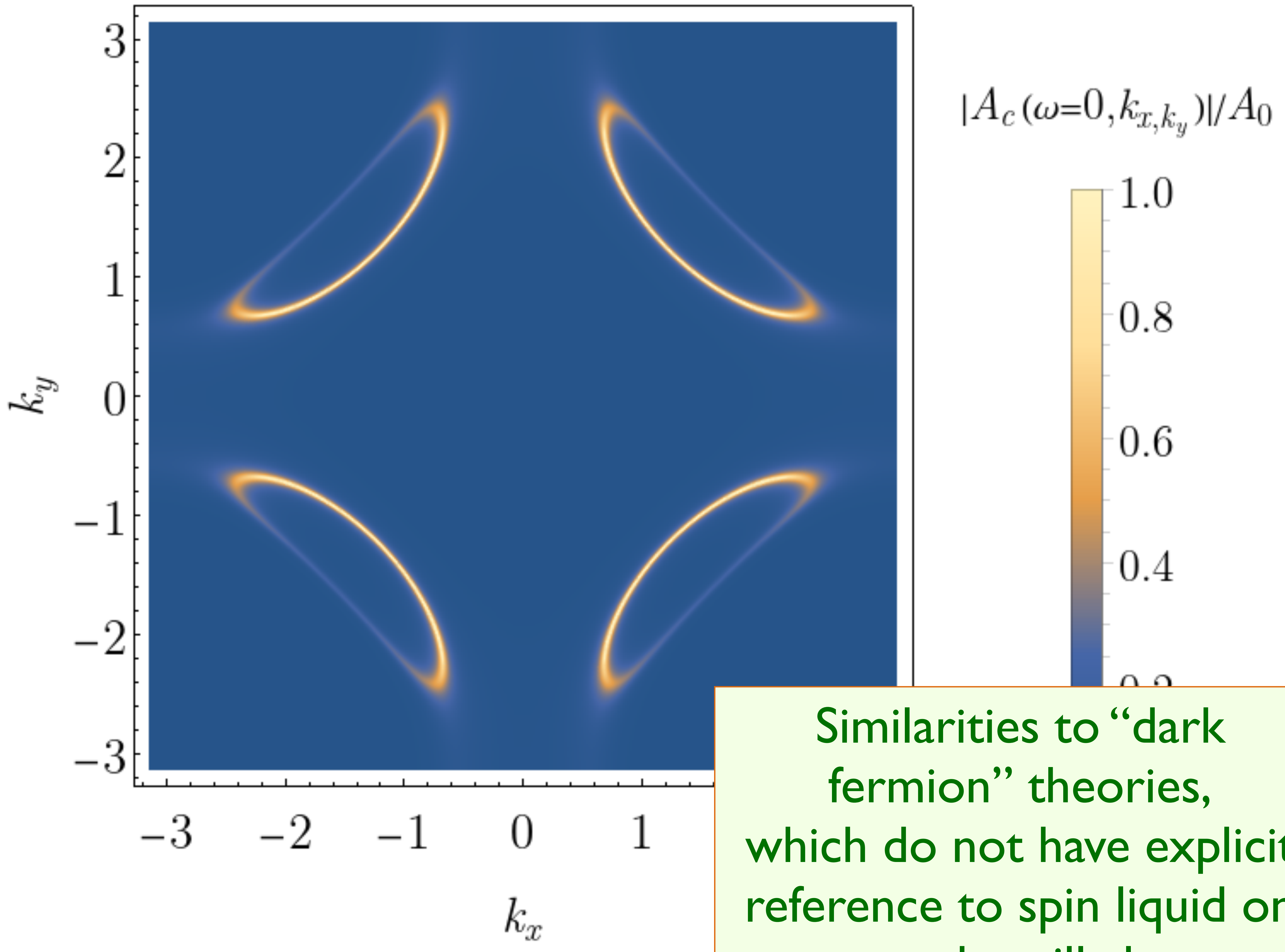
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)

Photoemission at small p



“Fermi pockets”



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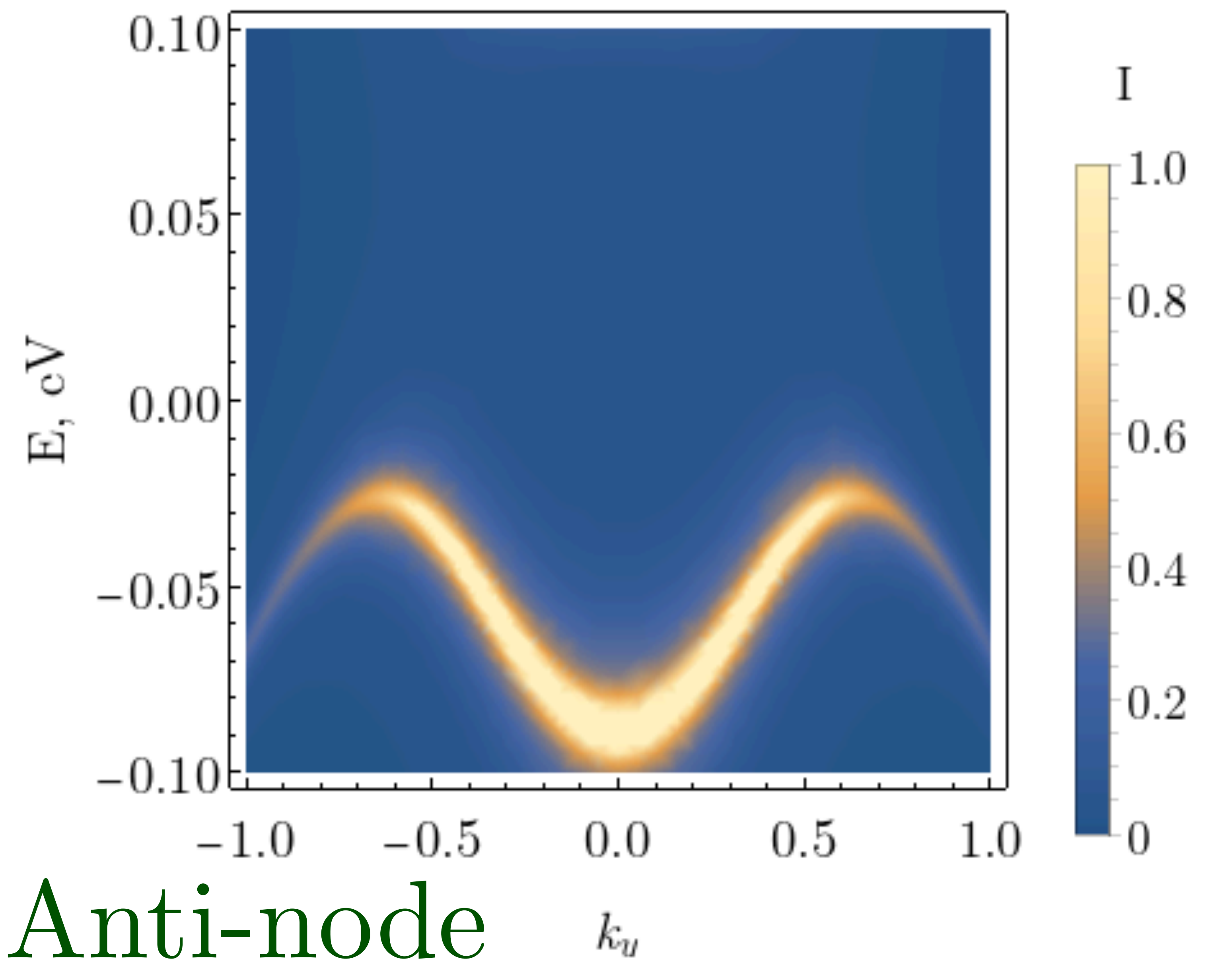
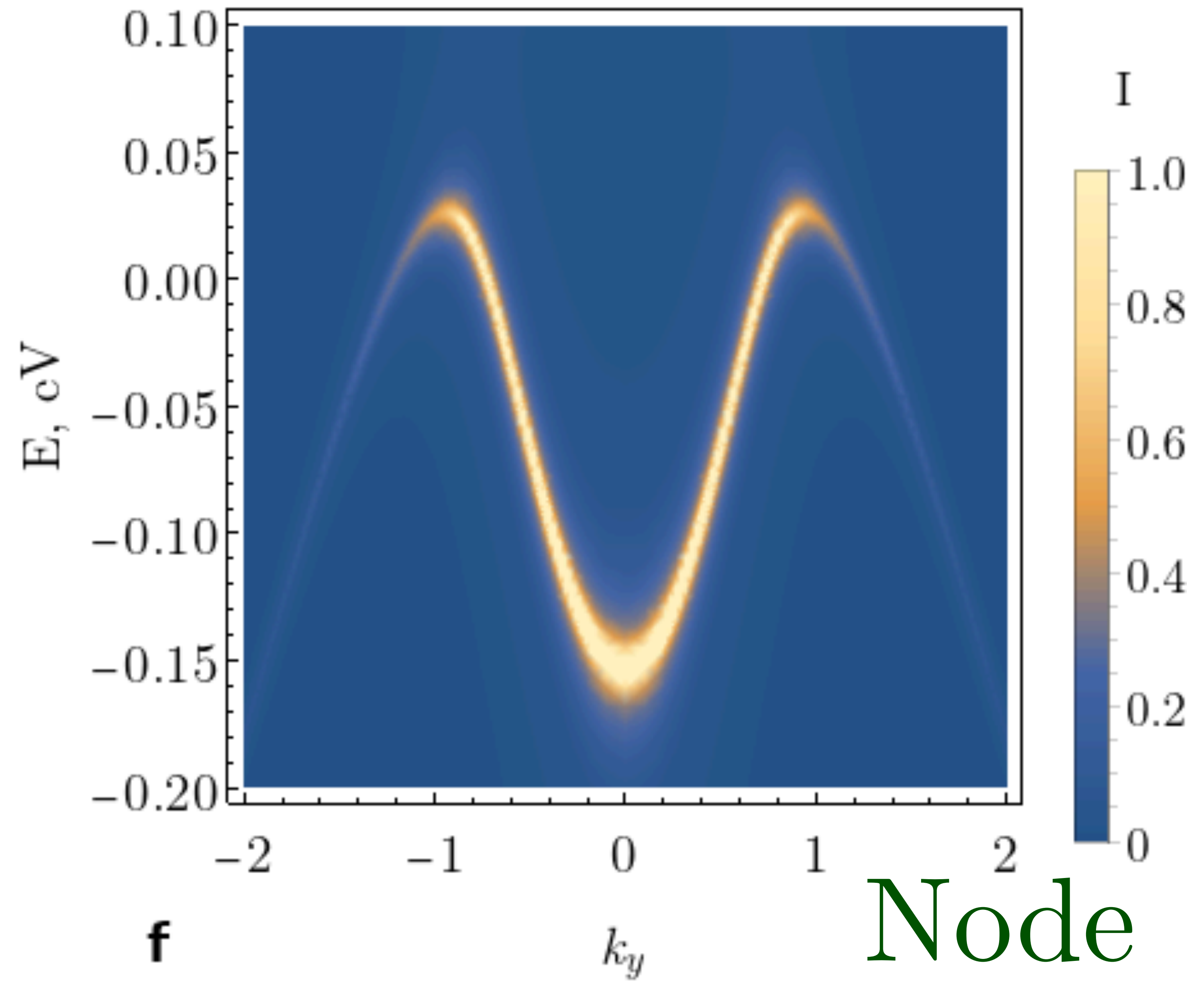
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

FL* in a *one-band* model

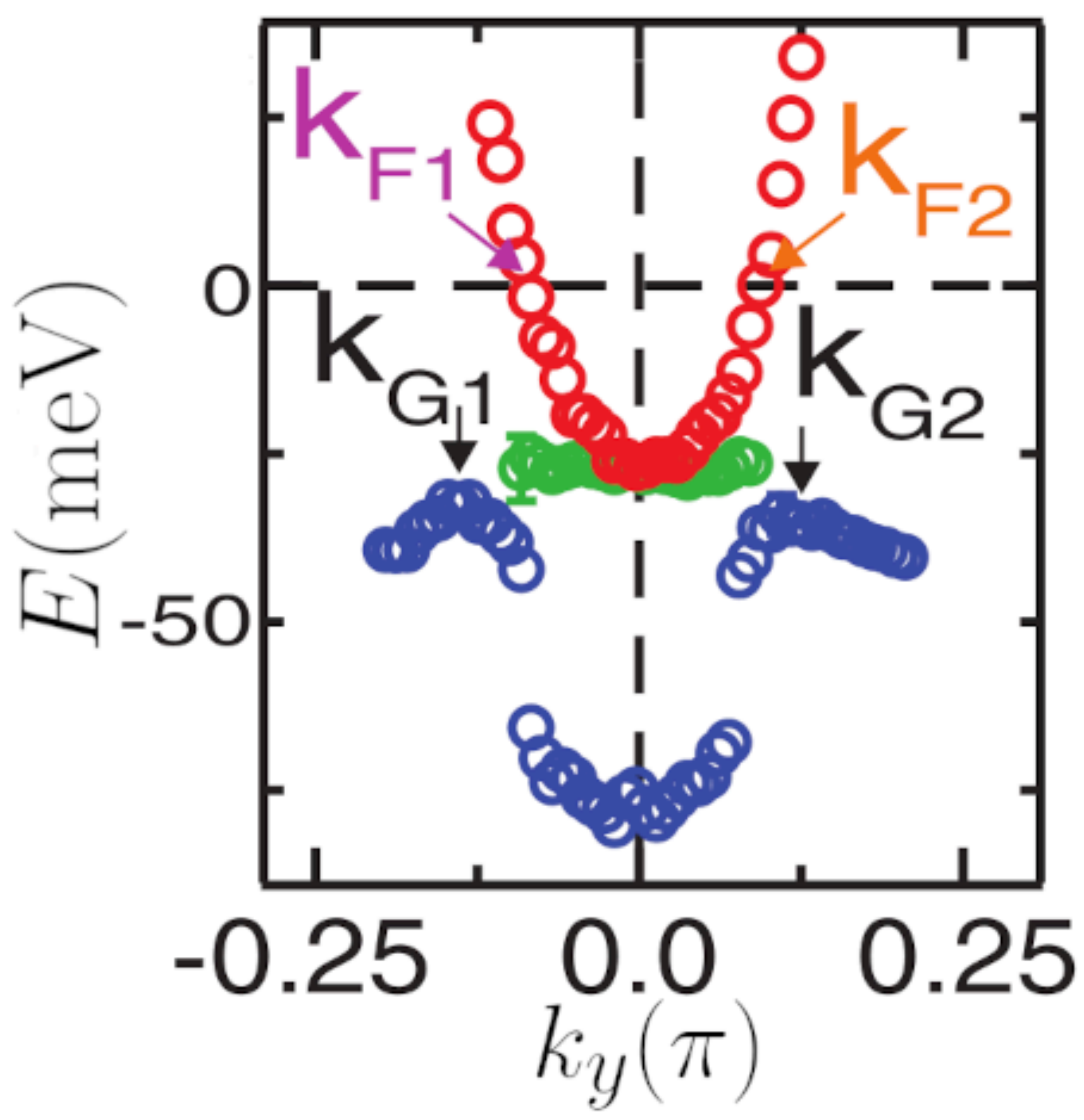
Second ancilla layer is needed to describe MDC and EDC

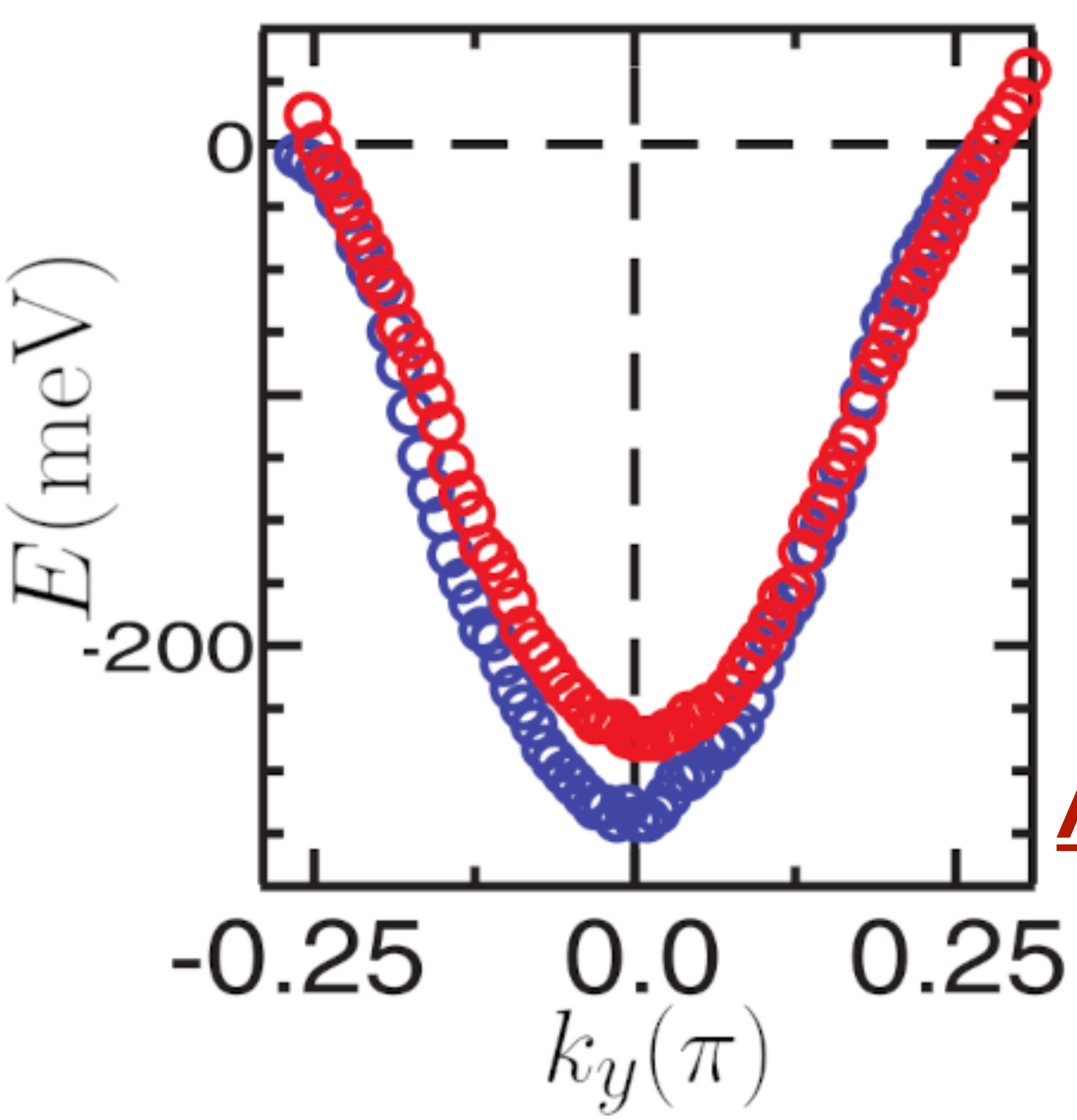


Anti-node



Node





R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

ARPES on Bi2201

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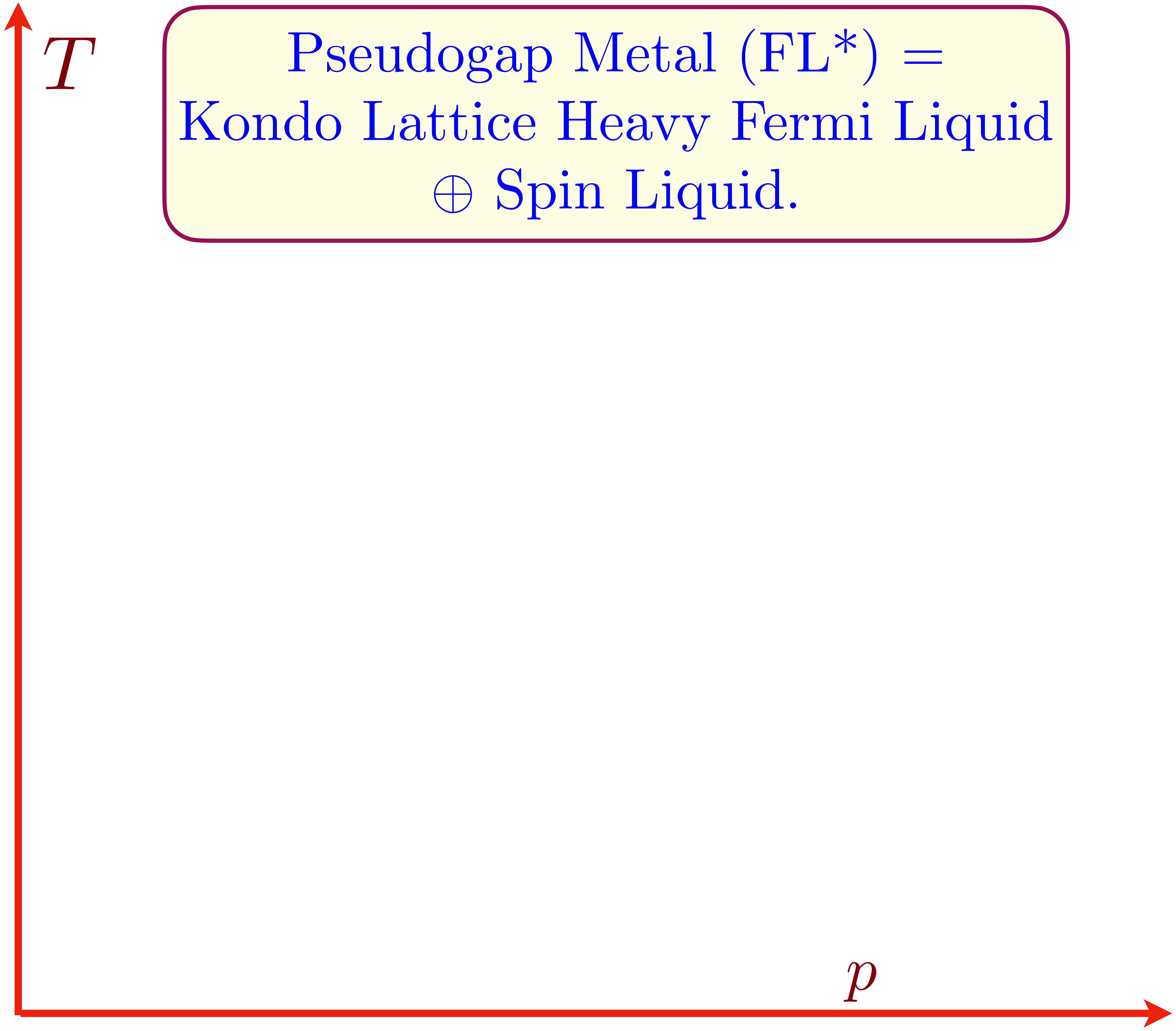
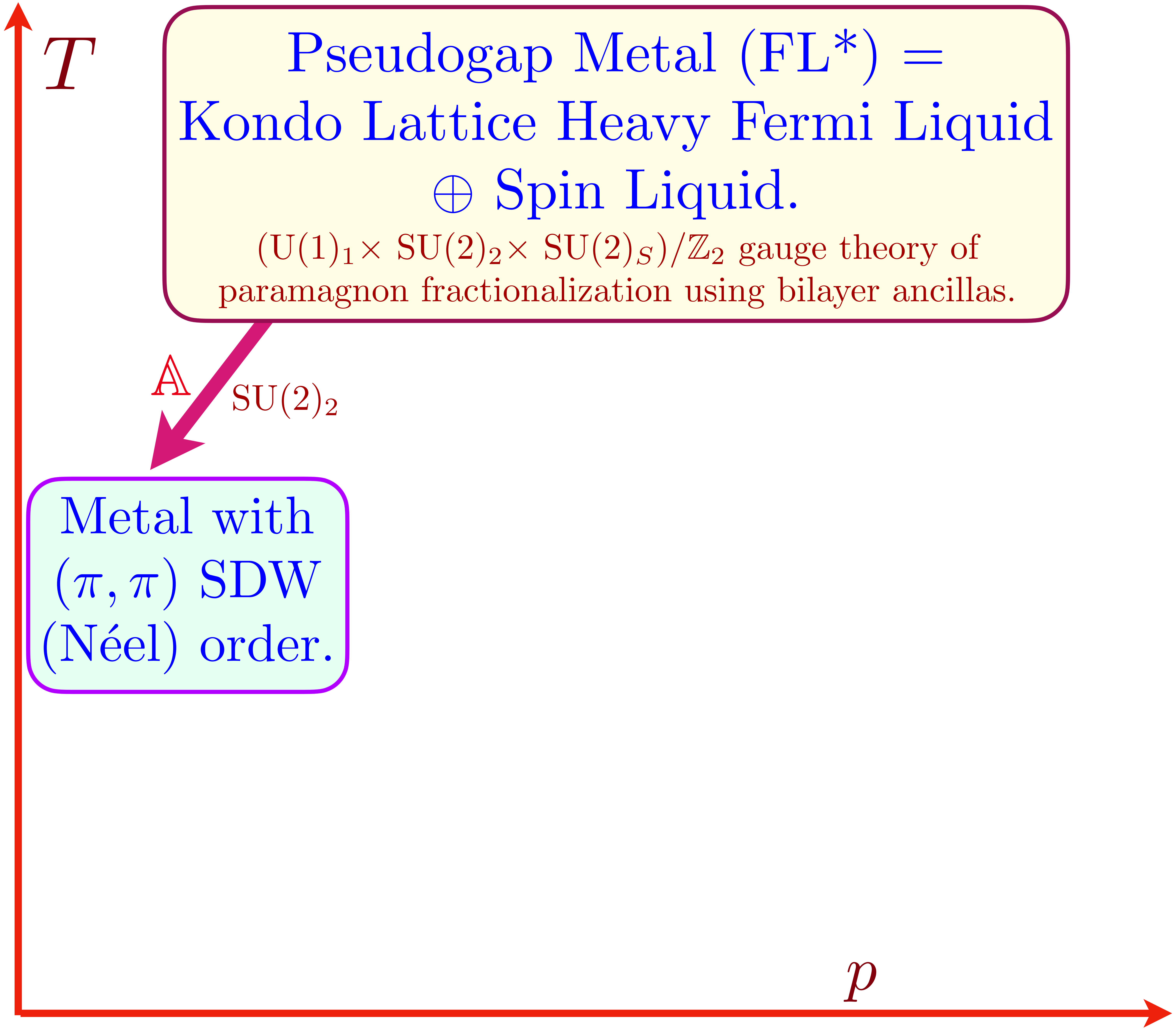

Pseudogap Metal (FL*) =
Kondo Lattice Heavy Fermi Liquid
⊕ Spin Liquid.
T
p

Alexander Nikolaenko

Jonas von Milczewski

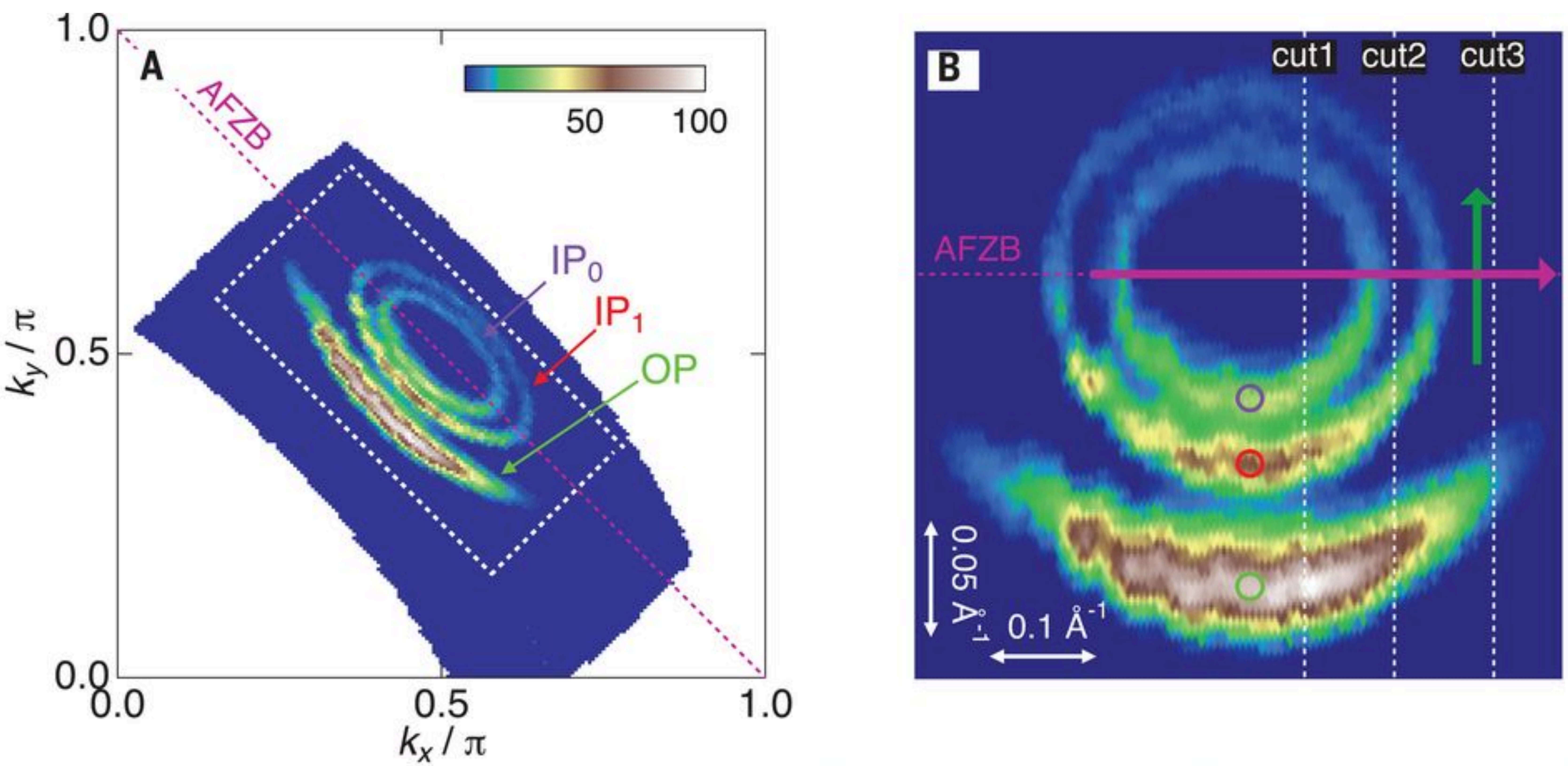
Darshan Joshi

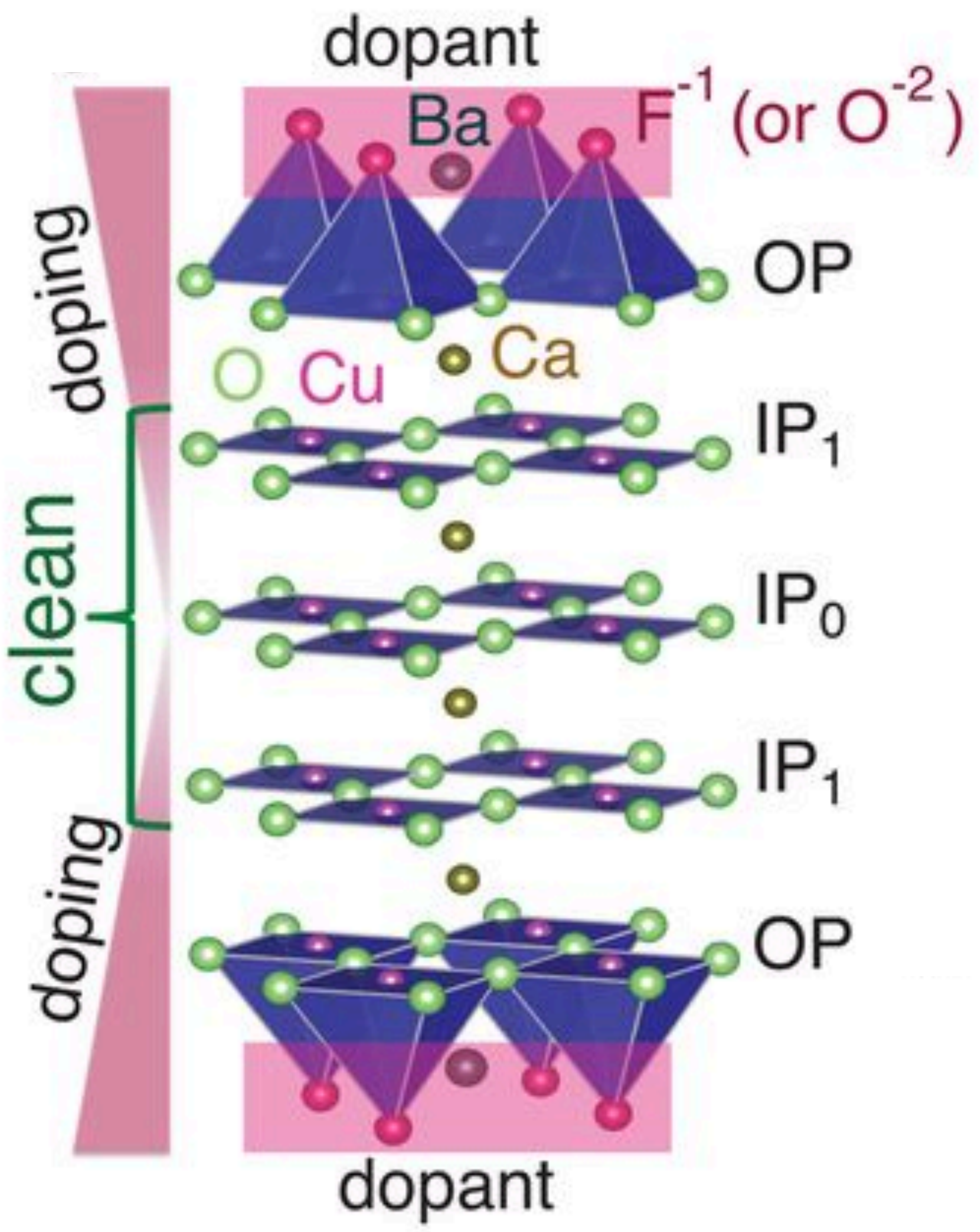


A. Nikolaenko, J. v. Milczewski, D. G. Joshi, S.S., arXiv:2211.10452

Observation of small Fermi pockets protected by clean CuO_2 sheets of a high-T_c superconductor

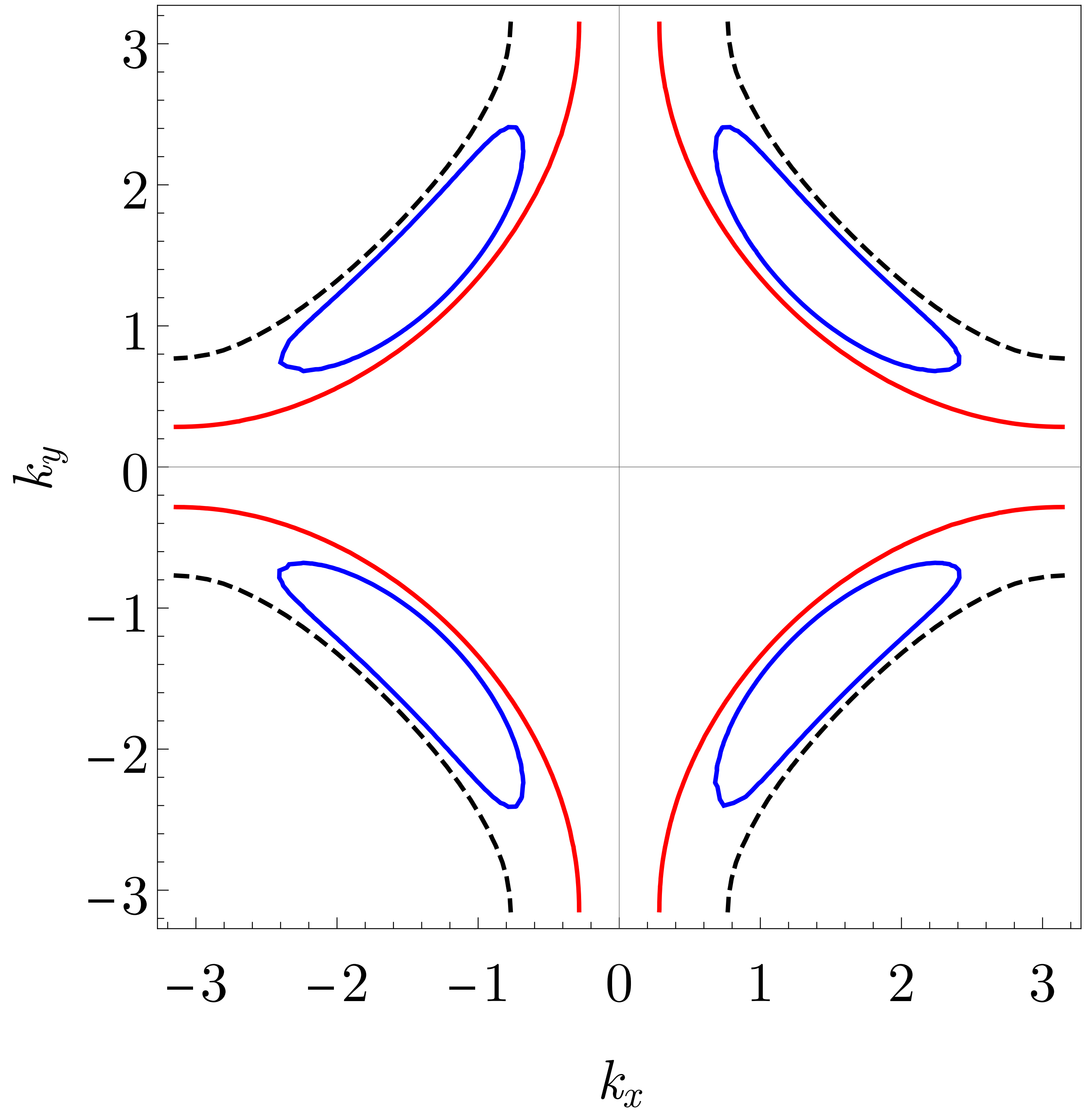
So Kunisada[1], Shunsuke Isono[2], Yoshimitsu Kohama[1,3], Shiro Sakai[4], Cédric Bareille[1], Shunsuke Sakuragi[1], Ryo Noguchi[1], Kifu Kurokawa[1], Kenta Kuroda[1], Yukiaki Ishida[1], Shintaro Adachi[5], Ryotaro Sekine[2], Timur K. Kim[6], Cephise Cacho[6], Shik Shin[1,7], Takami Tohyama[8], Kazuyasu Tokiwa[2]*, Takeshi Kondo[1,3]*





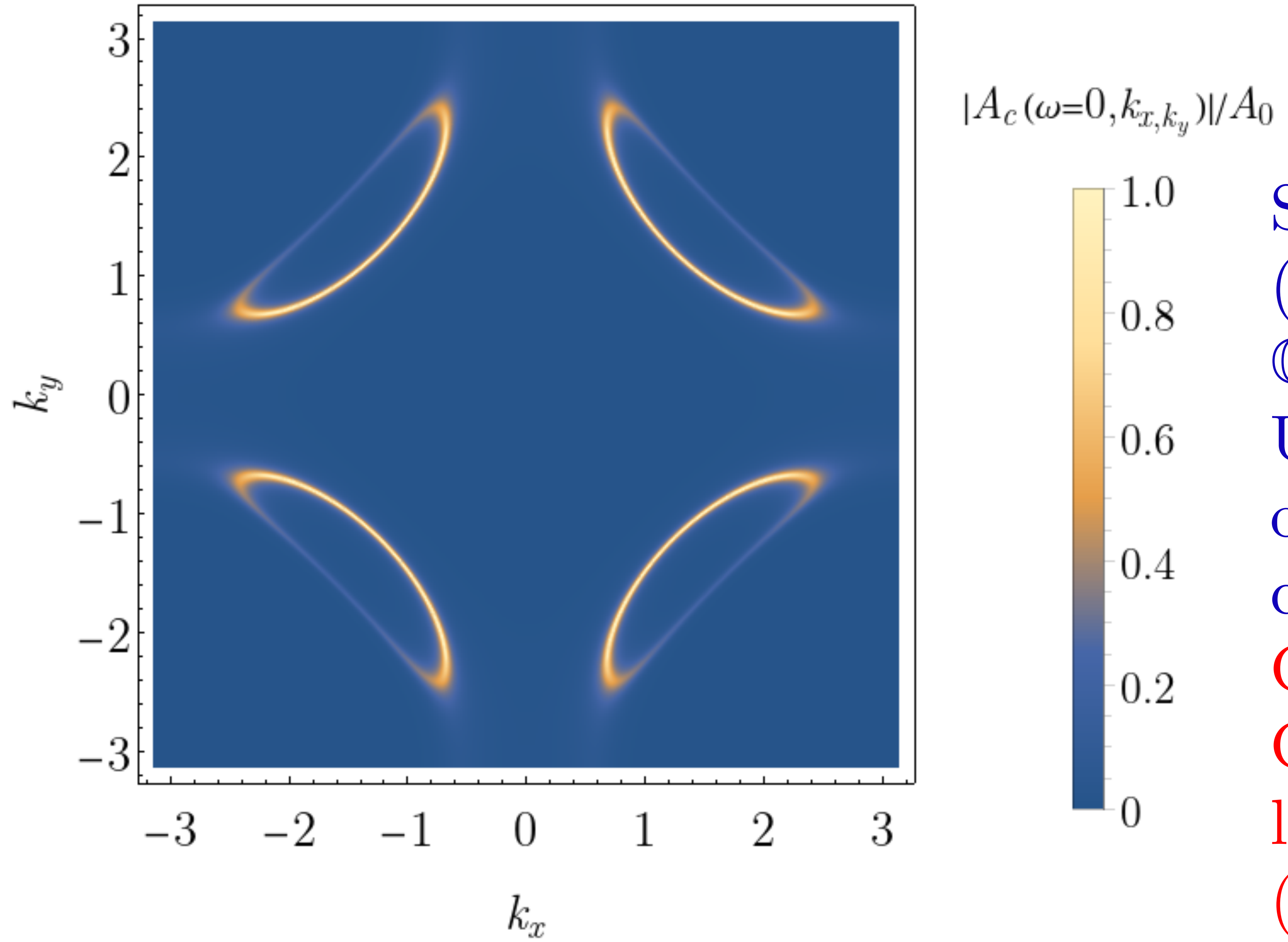
Hole pockets
in a metallic SDW state
with Néel order at (π, π).

Science **369**, 833 (2020).

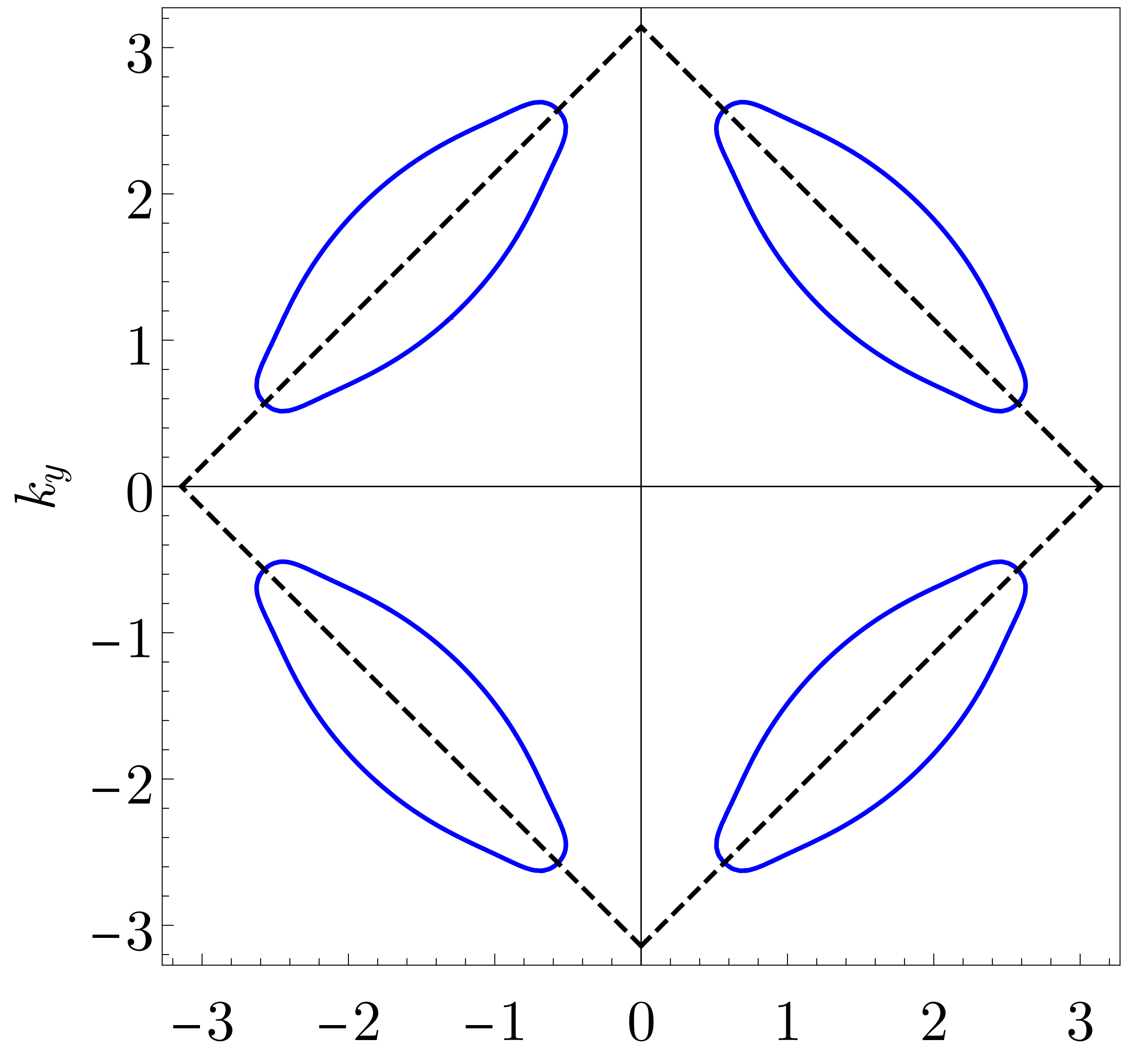


FL*

$\langle Z \rangle = 0$



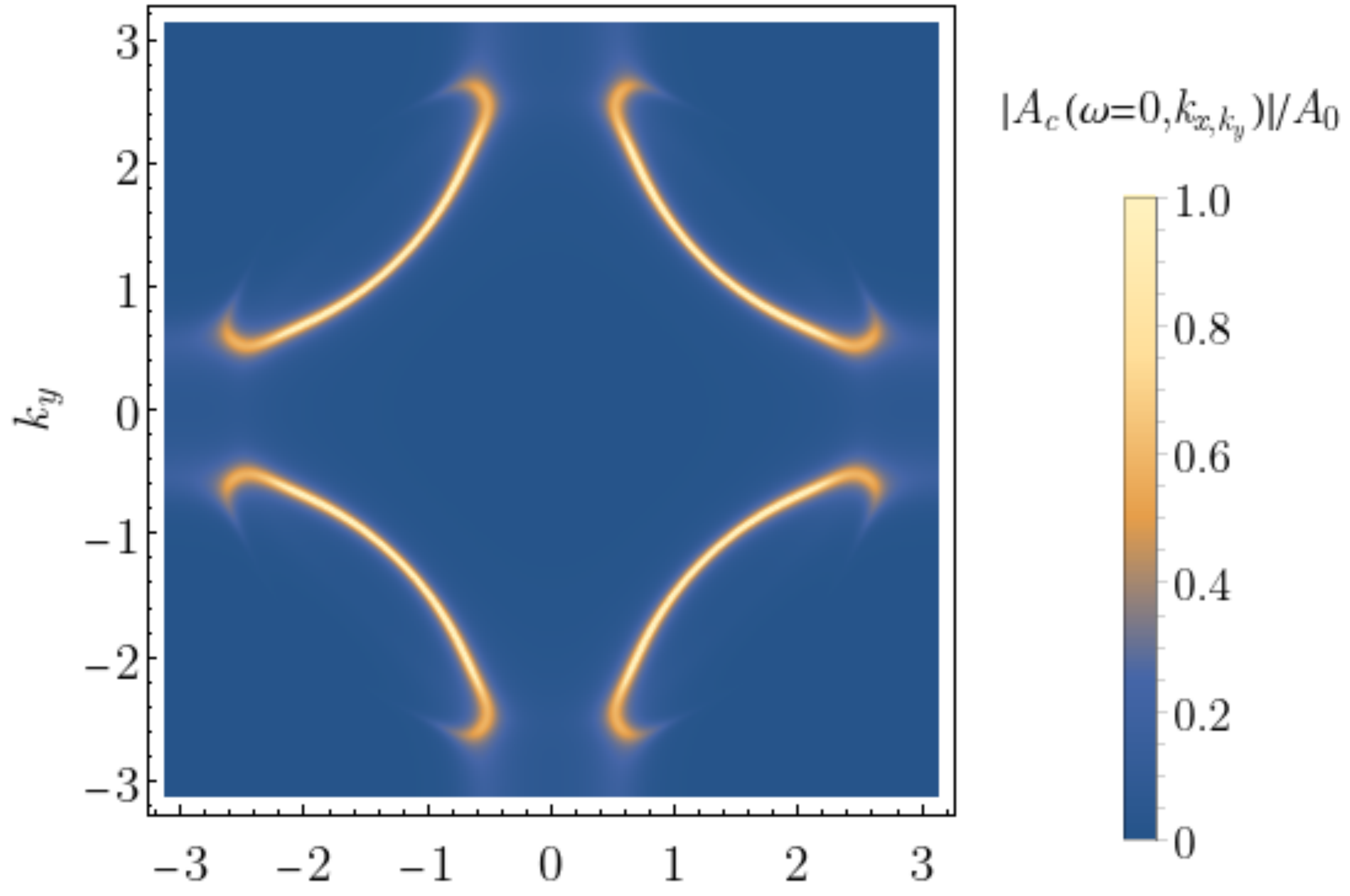
Schwinger bosons ($\boldsymbol{S}_{2i} = b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}$).
$\mathbb{CP}^1$ theory:
U$(1)_2$ gauge theory of 2 relativistic complex scalars, Z.
Confinement:
Condensation of Z leads to
(π, π) Néel order



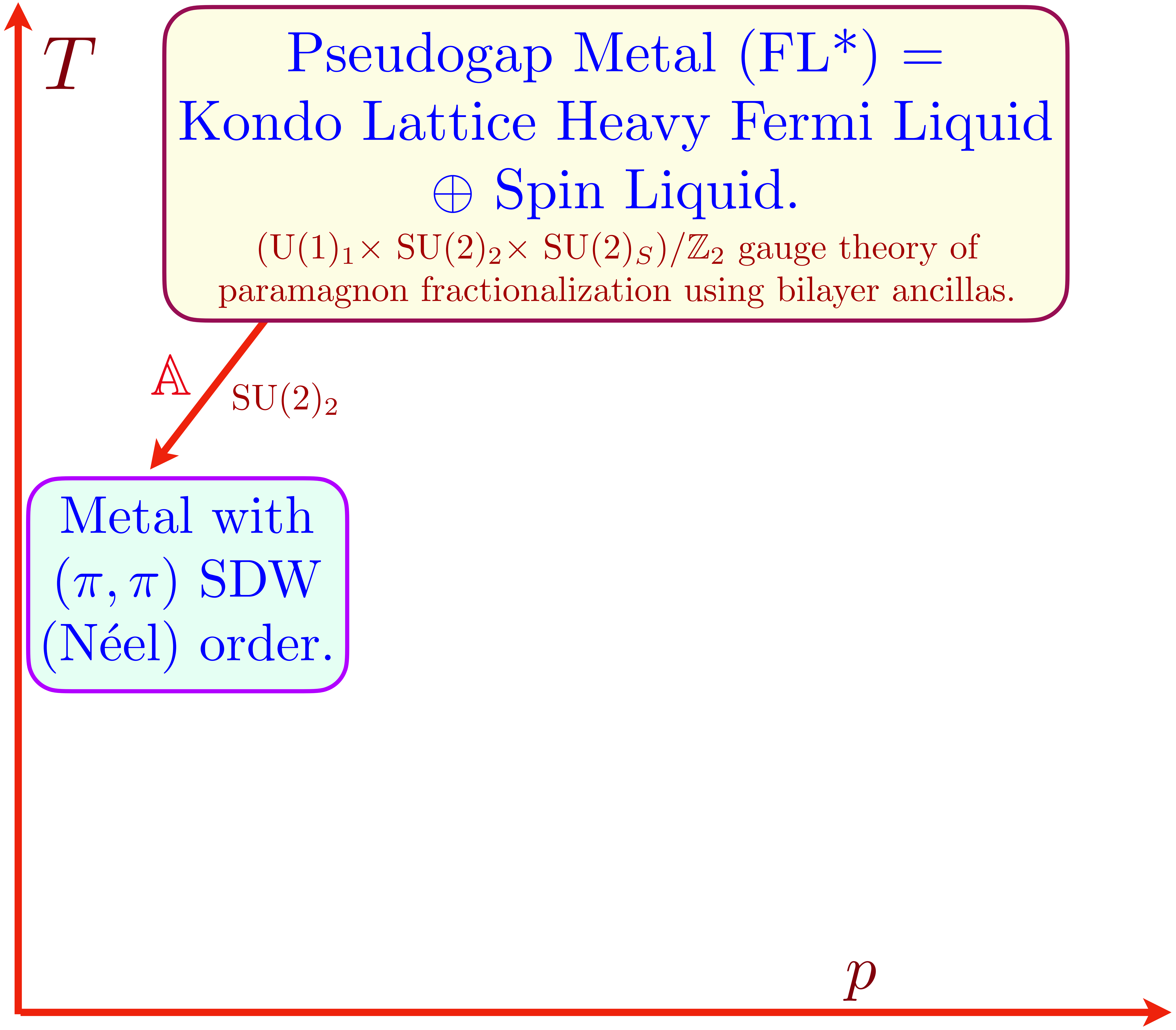
Néel

(π, π) SDW

$\langle Z \rangle \neq 0$

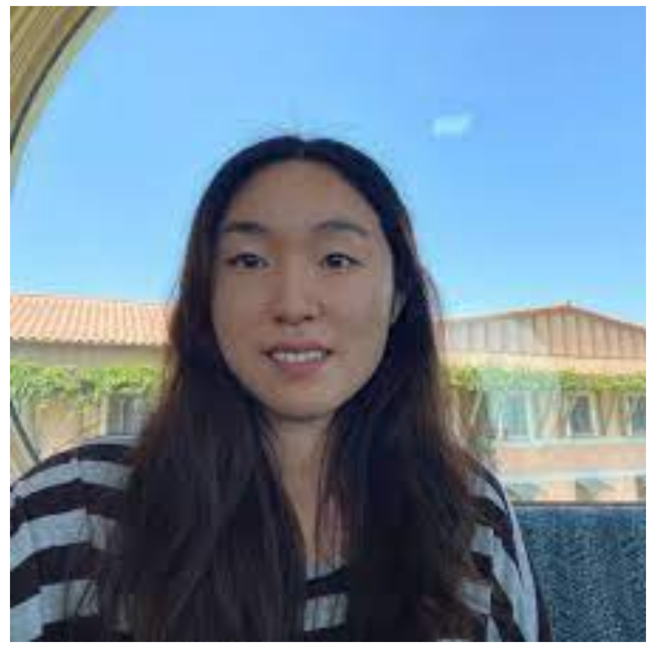


A. Nikolaenko,
J. v. Milczewski,
D. G. Joshi,
S.S.,
arXiv:2211.10452



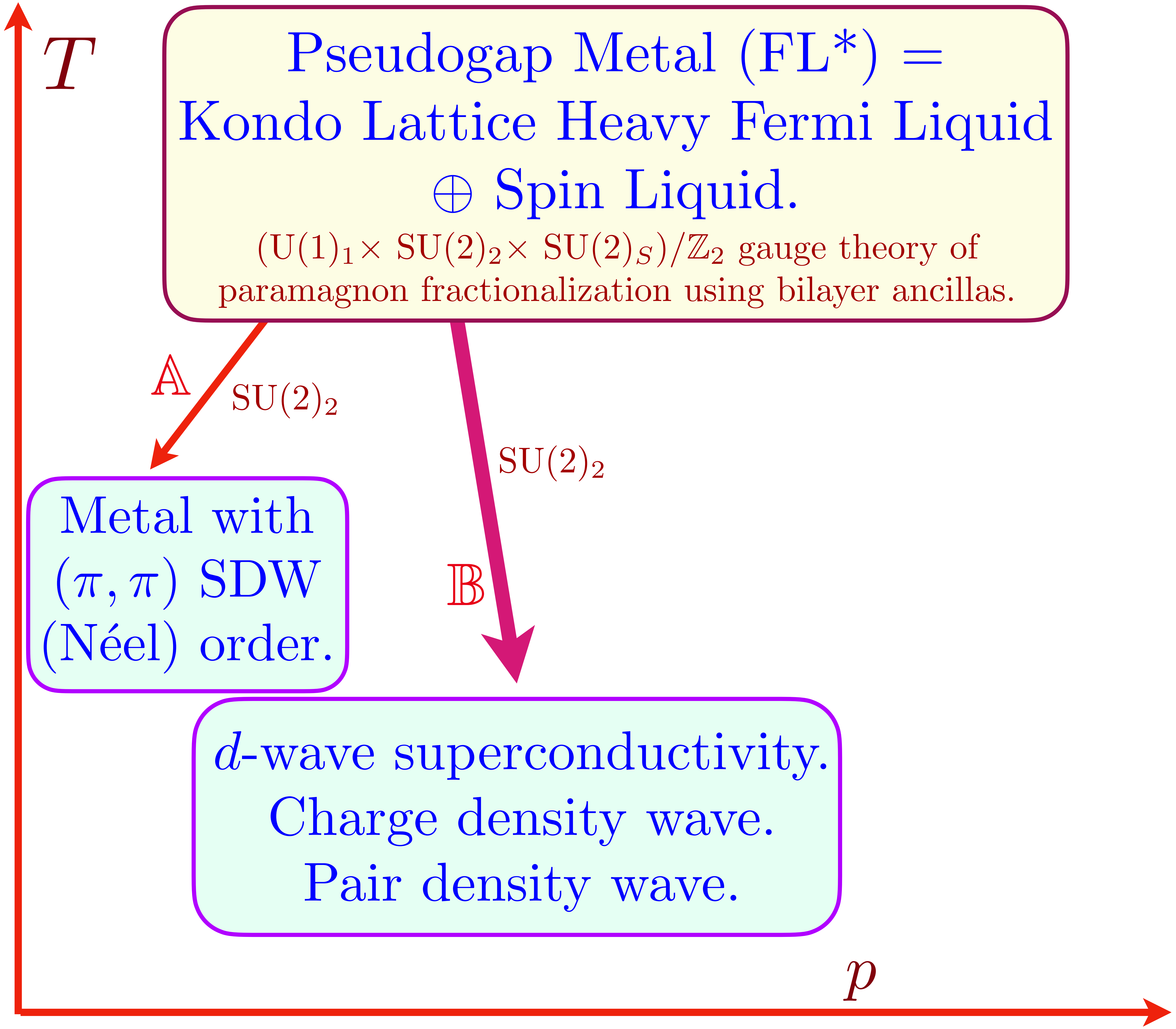
T
Pseudogap Metal (FL*) =
Kondo Lattice Heavy Fermi Liquid
$\oplus$ Spin Liquid.
$(\mathrm{U}(1)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ gauge theory of
paramagnon fractionalization using bilayer ancillas.
$\mathbb{A}$
$\mathrm{SU}(2)_2$
Metal with
(π, π) SDW
(Néel) order.
p

Maine Christos



Zhu-Xi Luo

Henry Shackleton



Schwinger fermion representation ($\boldsymbol{S}_{2\boldsymbol{i}} = f^\dagger_{2\boldsymbol{i}\alpha}\boldsymbol{\sigma}_{\alpha\beta} f_{2\boldsymbol{i}\beta}$) and π-flux mean field theory leads to spin liquid described by a $SU(2)_2$ gauge theory with $N_f = 2$ massless Dirac fermions. This is dual to the CP^1 theory of Schwinger bosons

Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

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A. Nahum,
M.A. Metlitski,
Cenke Xu, and
T. Senthil, PRX **7**,
031051 (2017)

$\mathrm{SU}(2)_2$ Confinement:

Condensation of a charge e boson $B_{\boldsymbol{i}} = \begin{pmatrix} B_{1\boldsymbol{i}} \\ B_{2\boldsymbol{i}} \end{pmatrix} \sim \begin{pmatrix} f^{\dagger}_{1\boldsymbol{i}\alpha} f_{2\boldsymbol{i}\alpha} \\ \varepsilon^{\alpha\beta} f^{\dagger}_{1\boldsymbol{i}\alpha} f^{\dagger}_{2\boldsymbol{i}\beta} \end{pmatrix}$ which is a $\mathrm{SU}(2)_2$ fundamental.

Landau energy for $B_{\boldsymbol{i}}$ in π-flux ($e_{\boldsymbol{ij}} = \pm 1$ with π-flux).

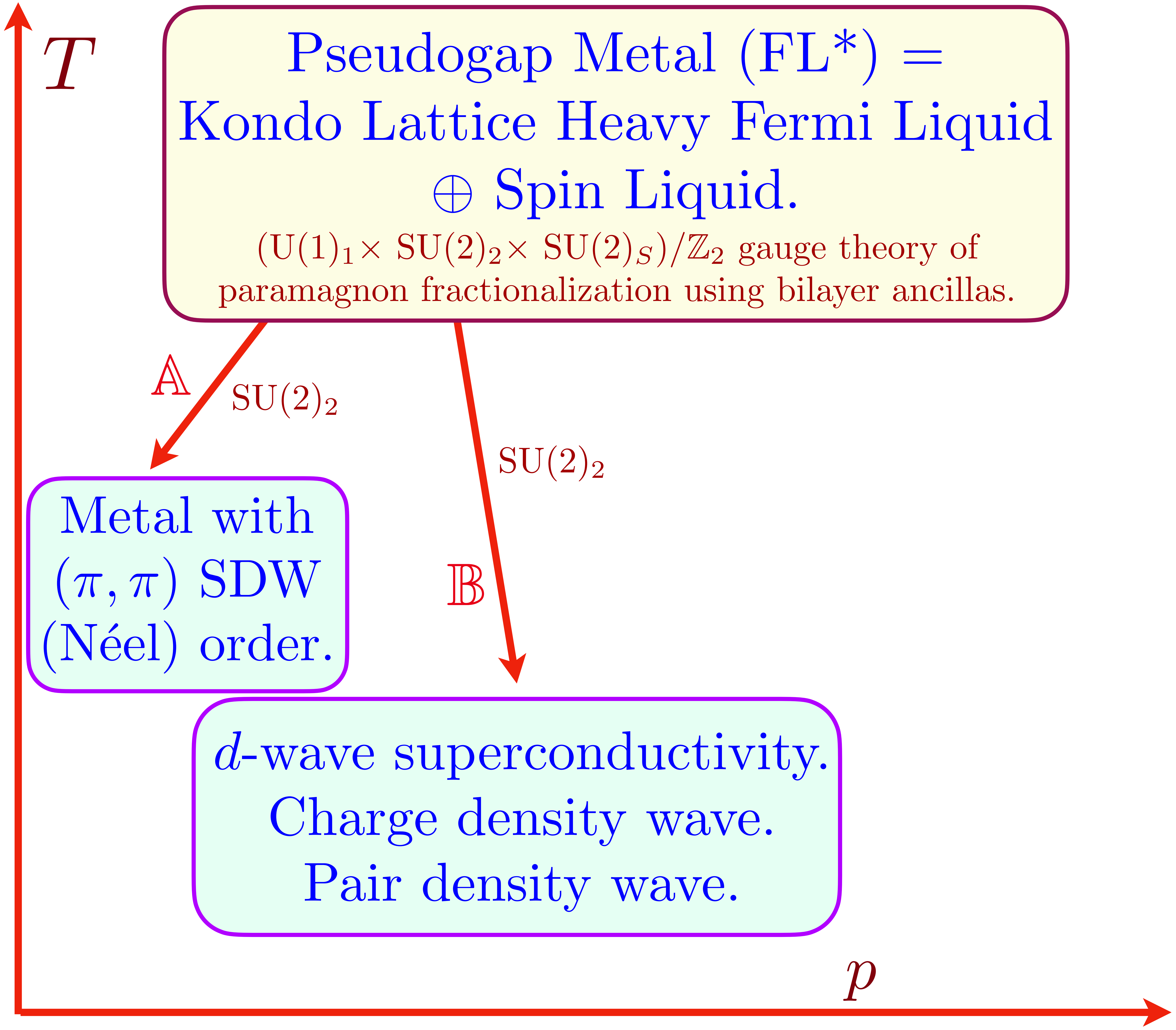
$$F(B) = r \sum_{\boldsymbol{i}} B^{\dagger}_{\boldsymbol{i}} B_{\boldsymbol{i}} - iw \sum_{\boldsymbol{ij}} B^{\dagger}_{\boldsymbol{i}} e_{\boldsymbol{ij}} B_{\boldsymbol{j}} + \frac{u}{2} \sum_{\boldsymbol{i}} \left(B^{\dagger}_{\boldsymbol{i}} B_{\boldsymbol{i}} \right)^2 + \frac{v}{2} \sum_{\langle \boldsymbol{ij} \rangle} \left| \varepsilon_{ab} B_{a\boldsymbol{i}} B_{b\boldsymbol{j}} \right|^2 + \ldots ,$$

CDW: $\rho_i = B^{\dagger}_{\boldsymbol{i}} B_{\boldsymbol{i}}$

VBS: $Q_{\boldsymbol{ij}} = Q_{ji} = \mathrm{Im} \left(B^{\dagger}_{\boldsymbol{i}} e_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$

Current: $J_{\boldsymbol{ij}} = -J_{ji} = \mathrm{Re} \left(B^{\dagger}_{\boldsymbol{i}} e_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.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Metal with
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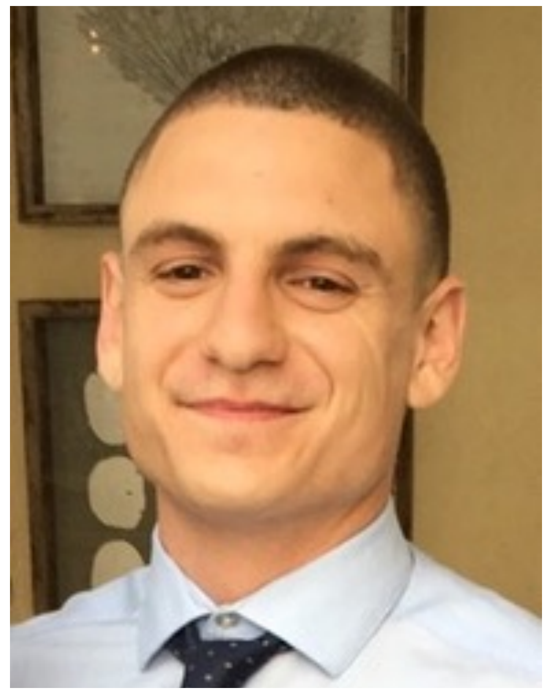
(Néel) order.
d-wave superconductivity.
Charge density wave.
Pair density wave.
p

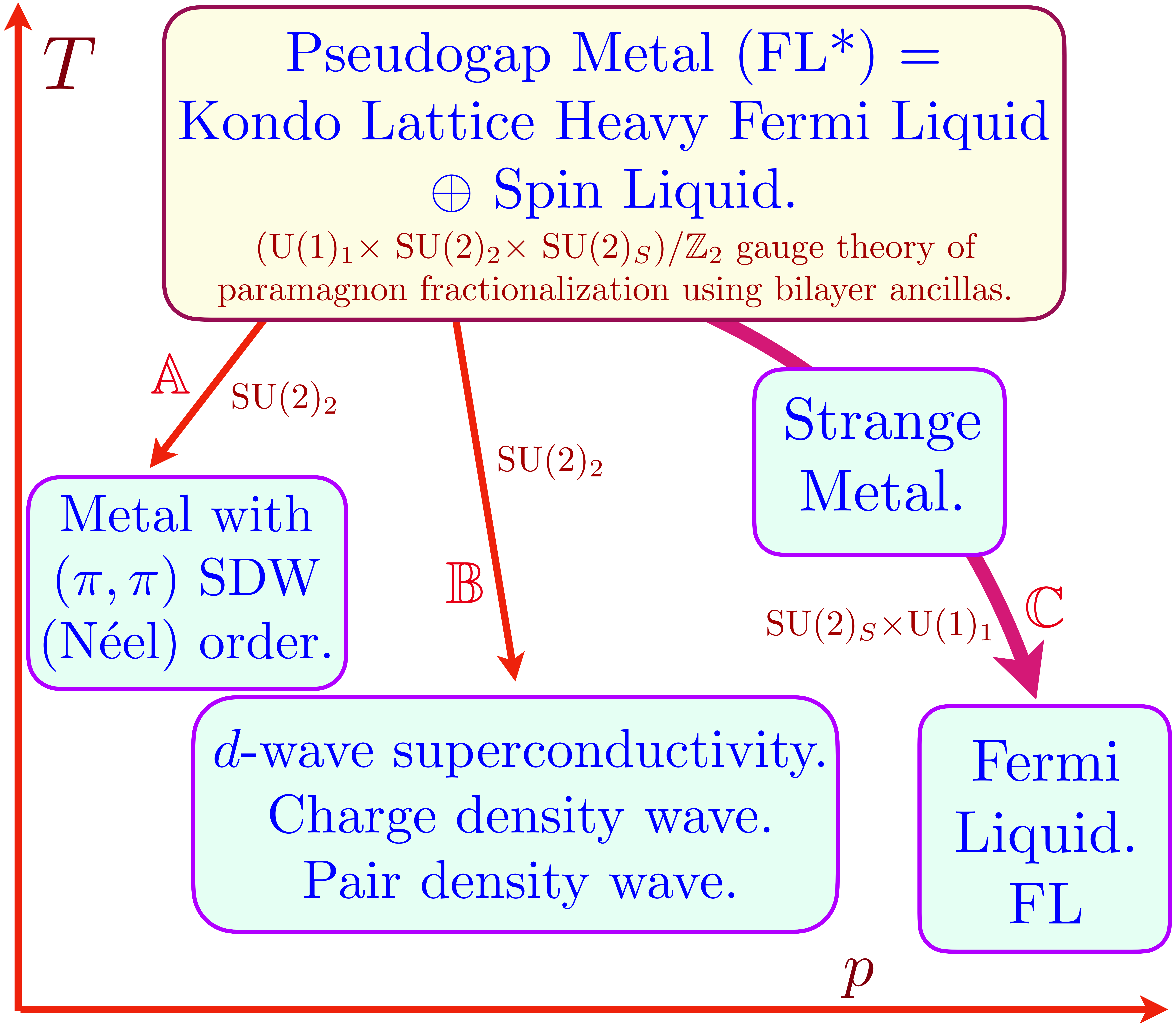
Yahui Zhang

Aavishkar Patel

Haoyu Guo



Ilya Esterlis



$\mathrm{SU}(2)_S\times\mathrm{U}(1)_1$ gauge theory with 'ghost' Fermi surfaces and Higgs fields.

Ya-Hui Zhang and S. S., PRR **2**, 023172
PRB **102** , 155124 (2020)

Yields strange metal with spatially random Yukawa couplings.

A. A. Patel, Haoyu Guo, I. Esterlis, and S. S., arXiv:2203.04990

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

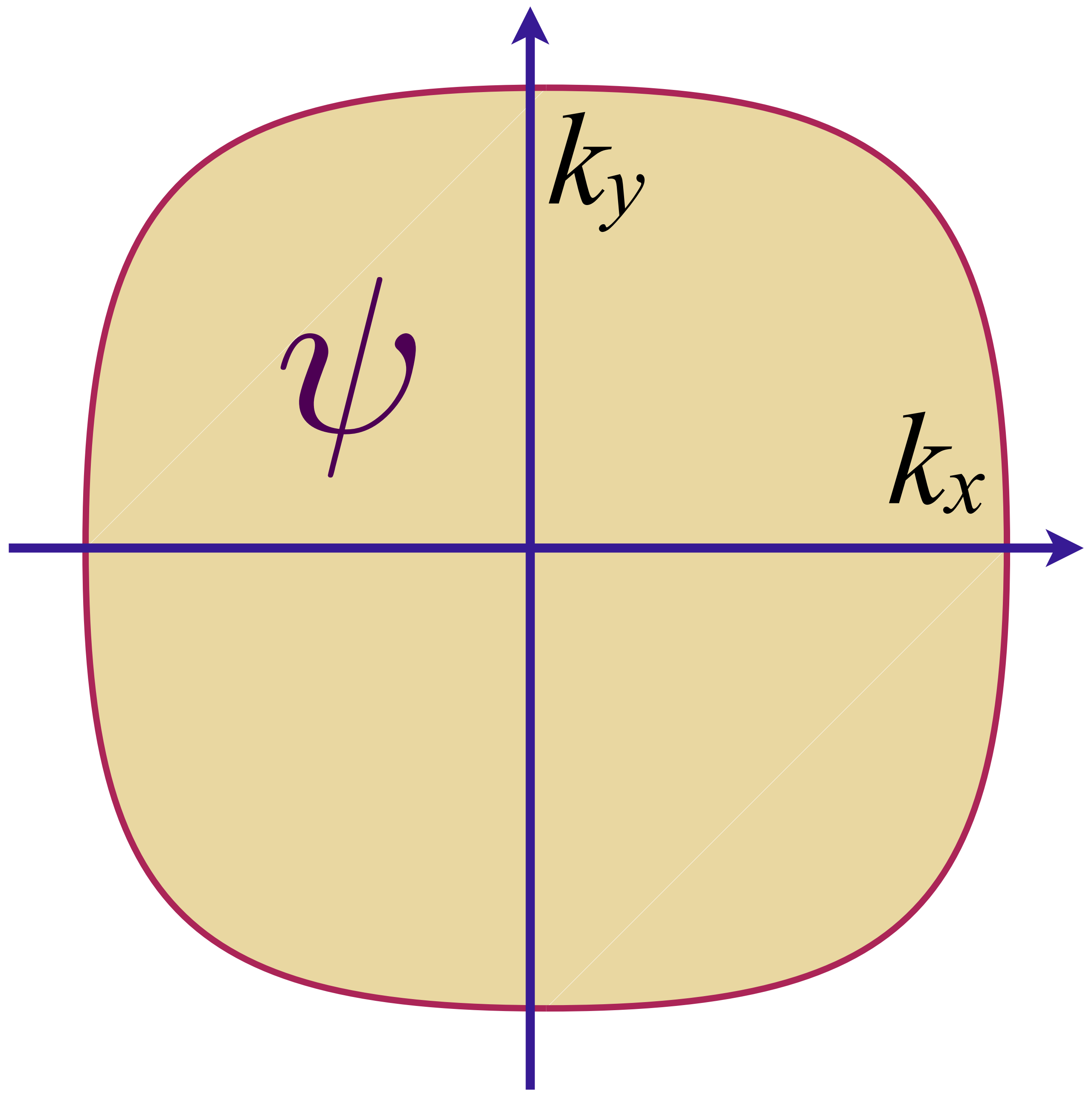
4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber….D. Dessau, Nature Communications **10**, 5737 (2019)

Fermi surface

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

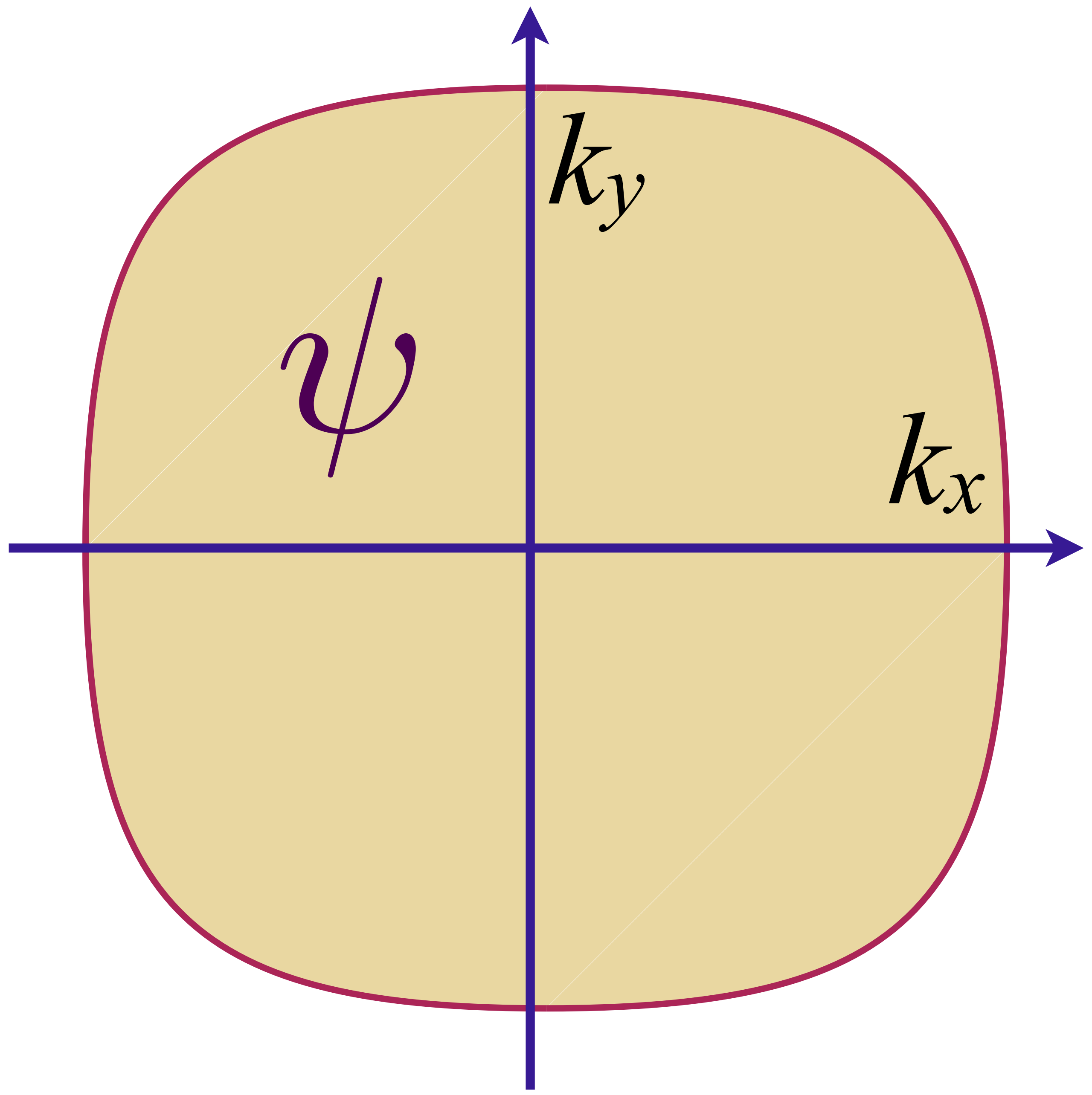


$$- J \, \psi^\dagger(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r}) \psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

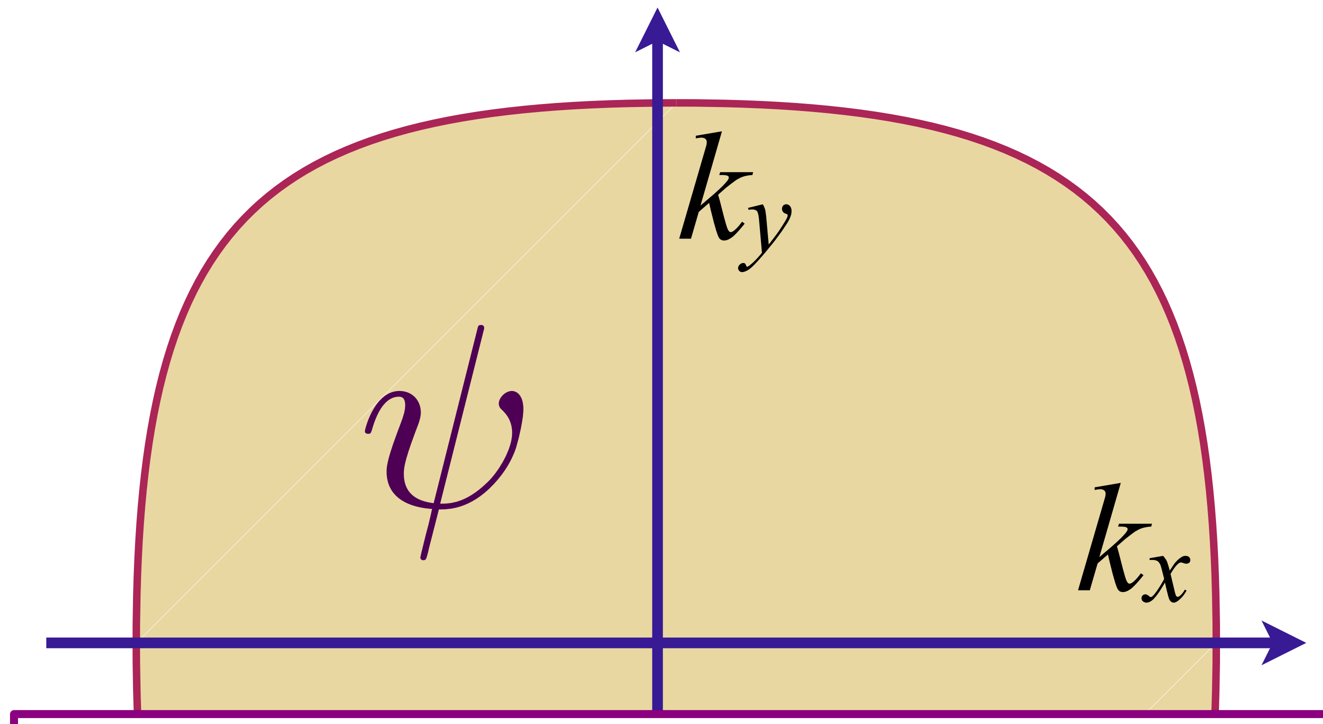
a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...



$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order, Higgs field ...

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Solve in a large N limit with Yukawa coupling

$$\frac{g_{ij\ell}}{N} \int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

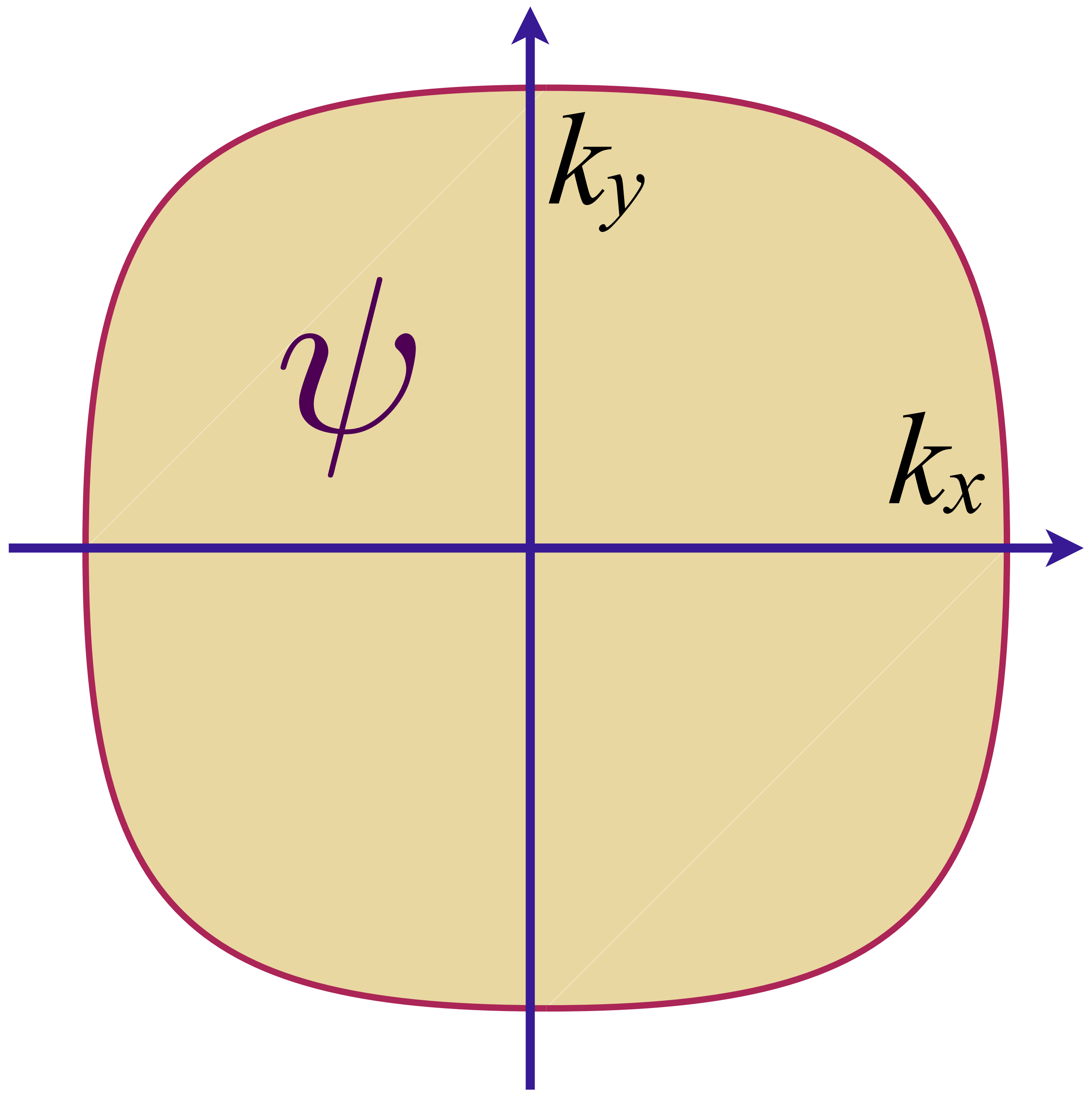
to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{2/3}, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

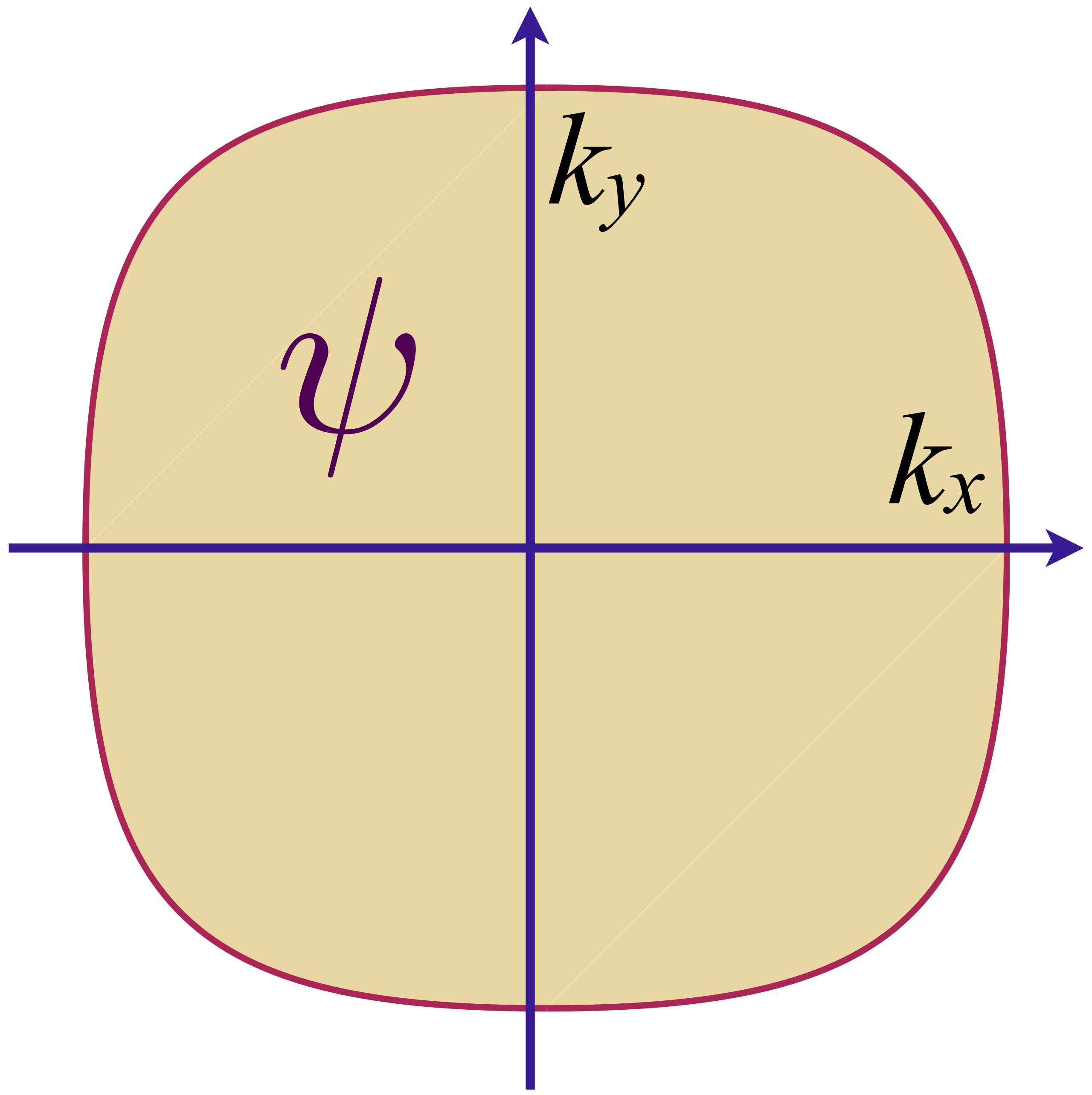
S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

<u>Transport—a perfect metal!</u>
Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\left[\sigma(\omega)\right] = D\delta(\omega) + \ldots$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
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$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Transport—a perfect metal!

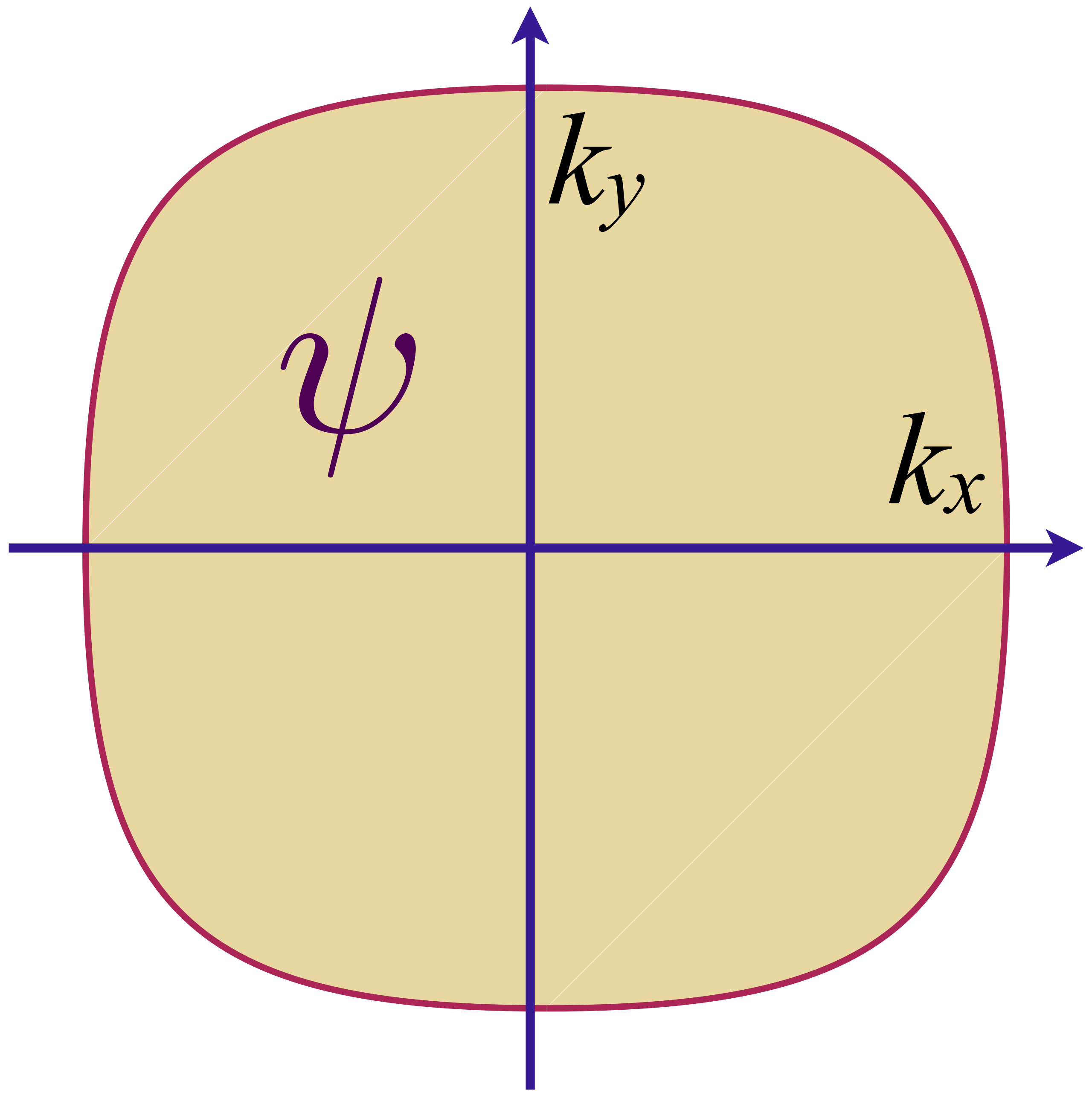
Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \cdots \quad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field ...

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

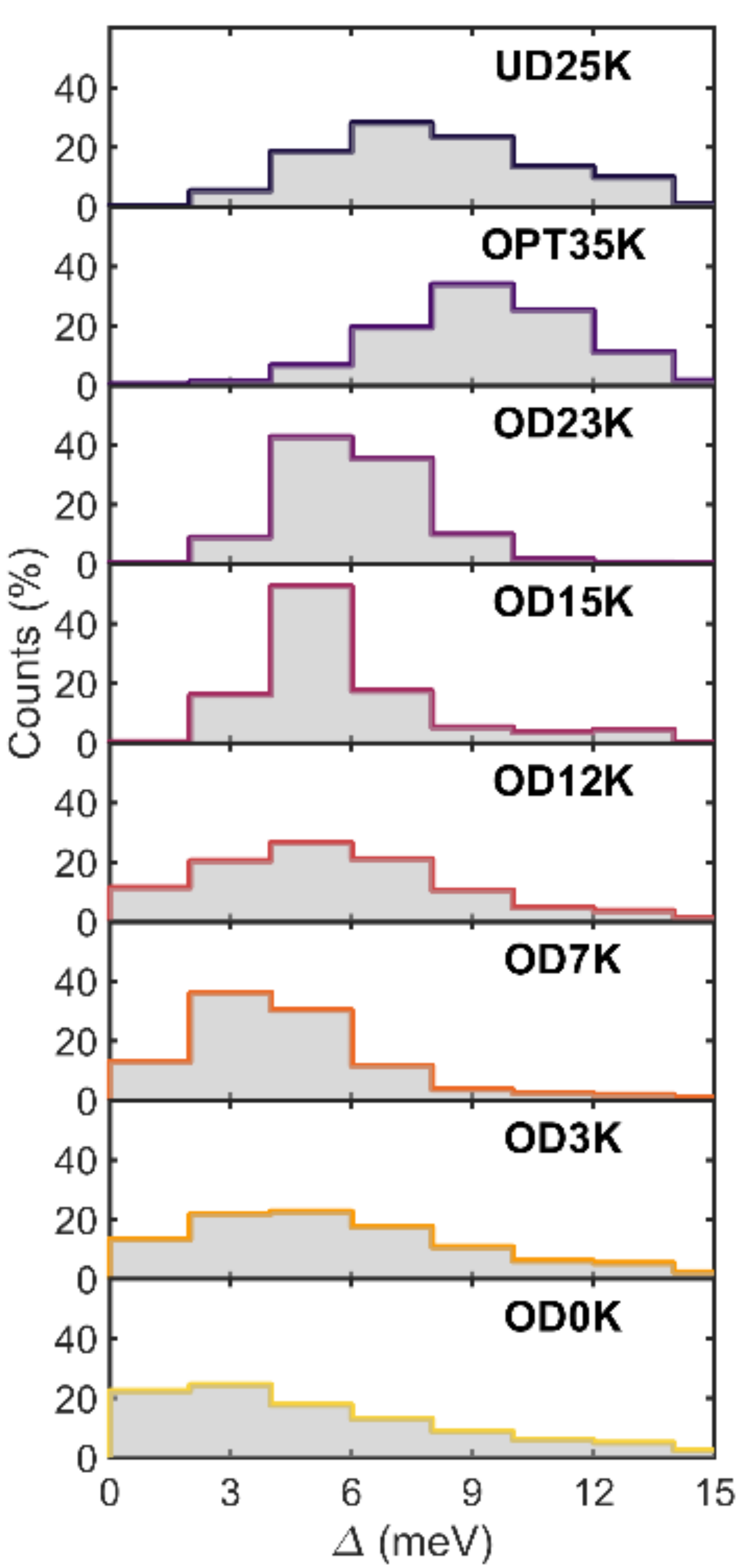
Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

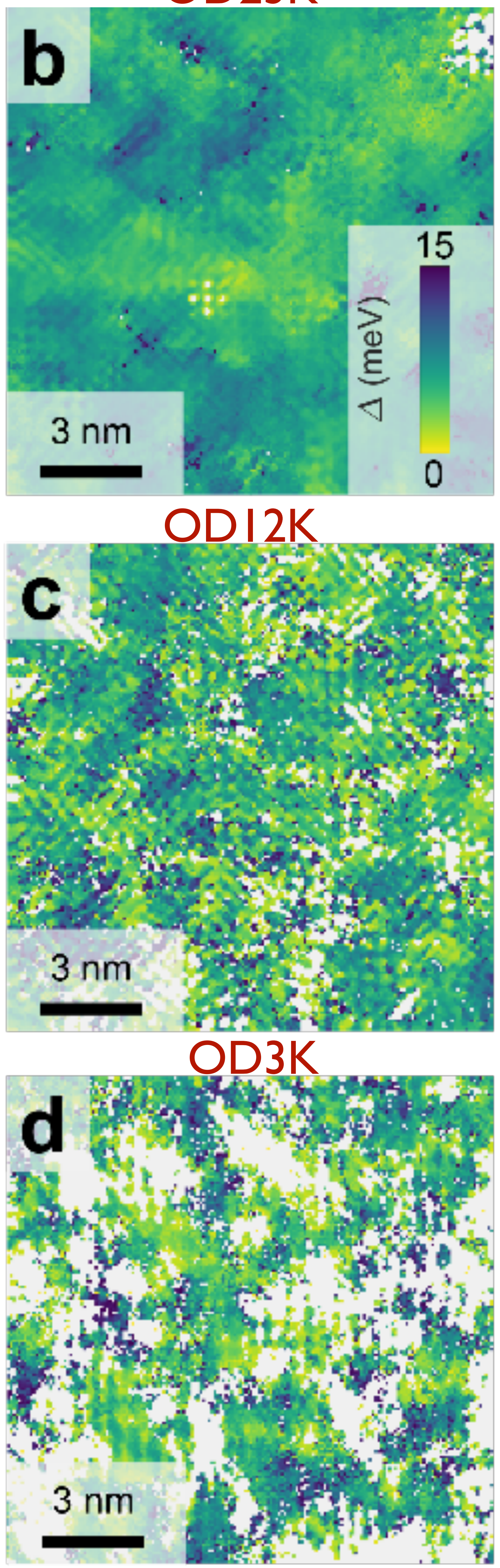
arXiv:2205.09740



OD23K

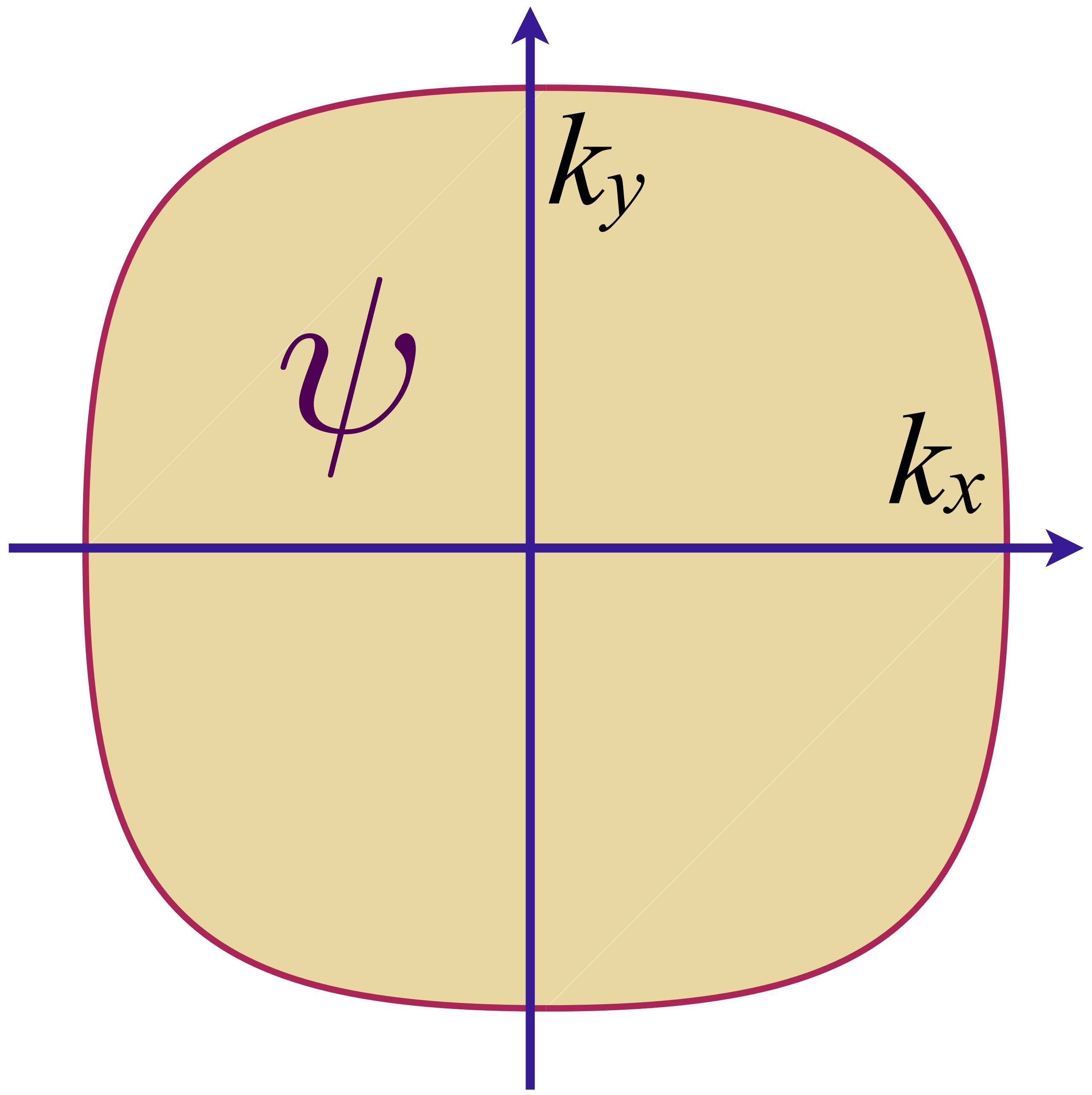
OD12K

OD3K



Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order,
Higgs field . . .

$$\frac{[\phi(\boldsymbol{r})]^2}{J + J'(\boldsymbol{r})} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$$+v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order,
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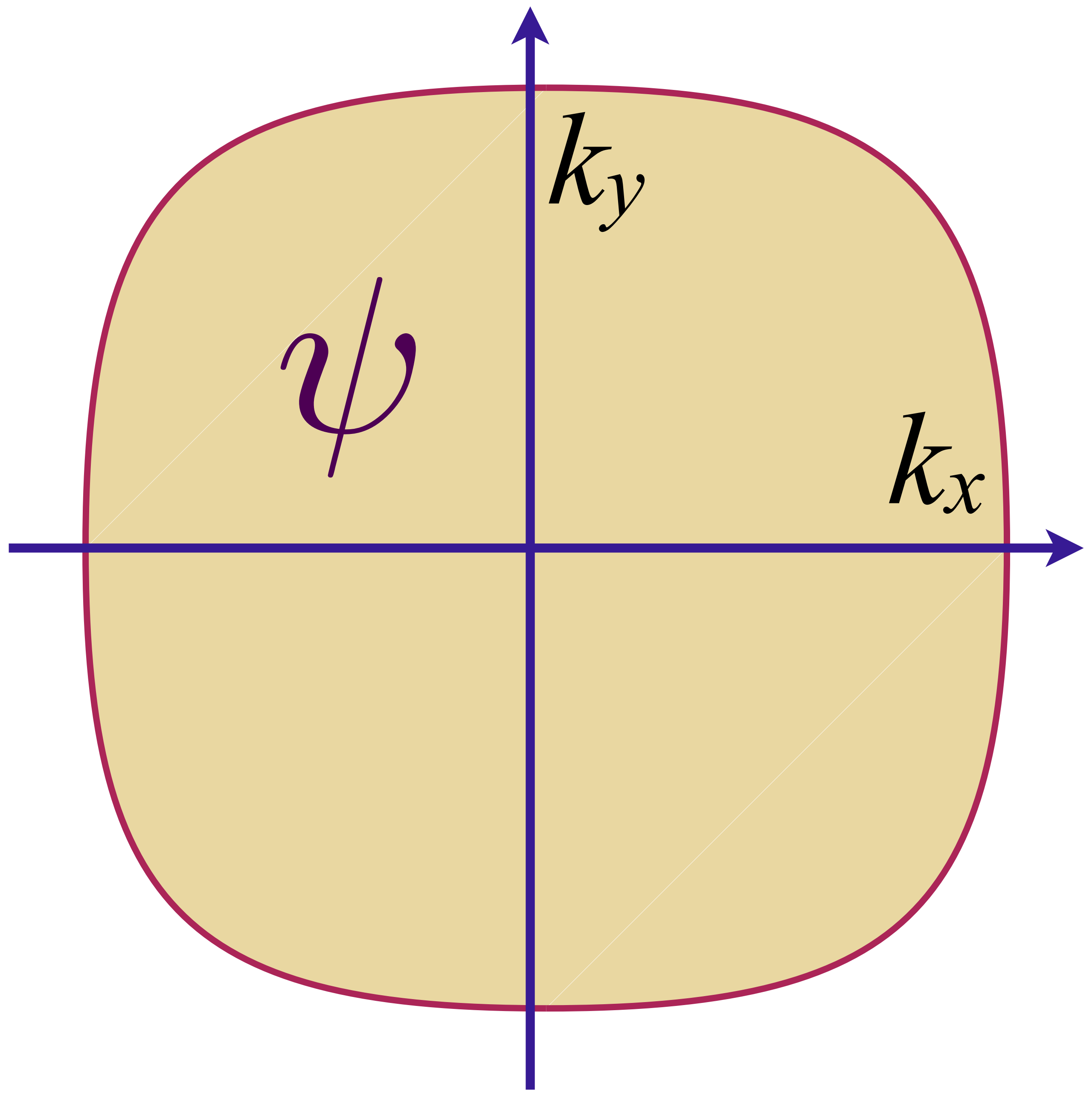

$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Fermi surface coupled to a critical boson with disorder

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a critical boson ϕ
e.g. Ising-nematic order,
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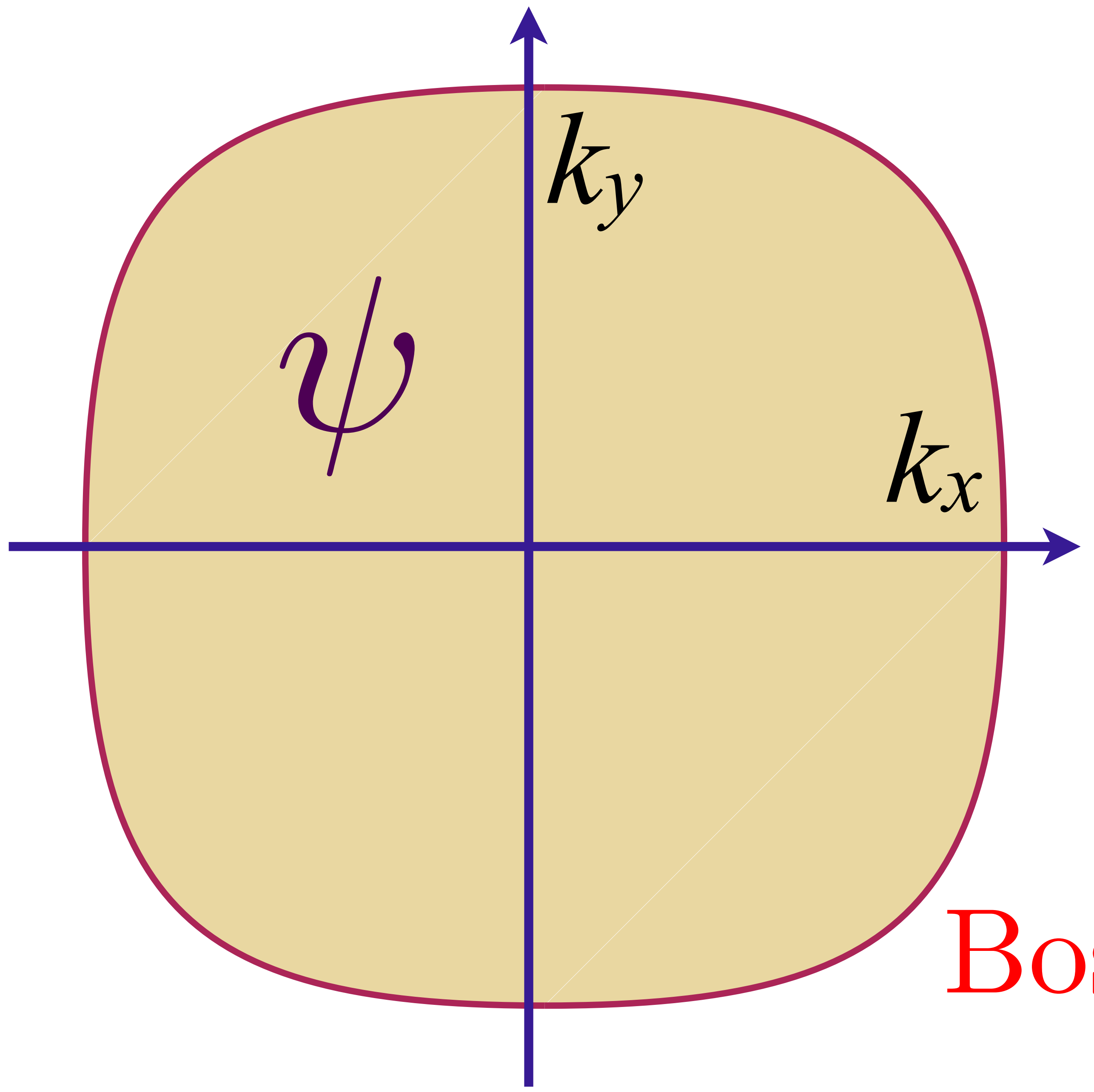


$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

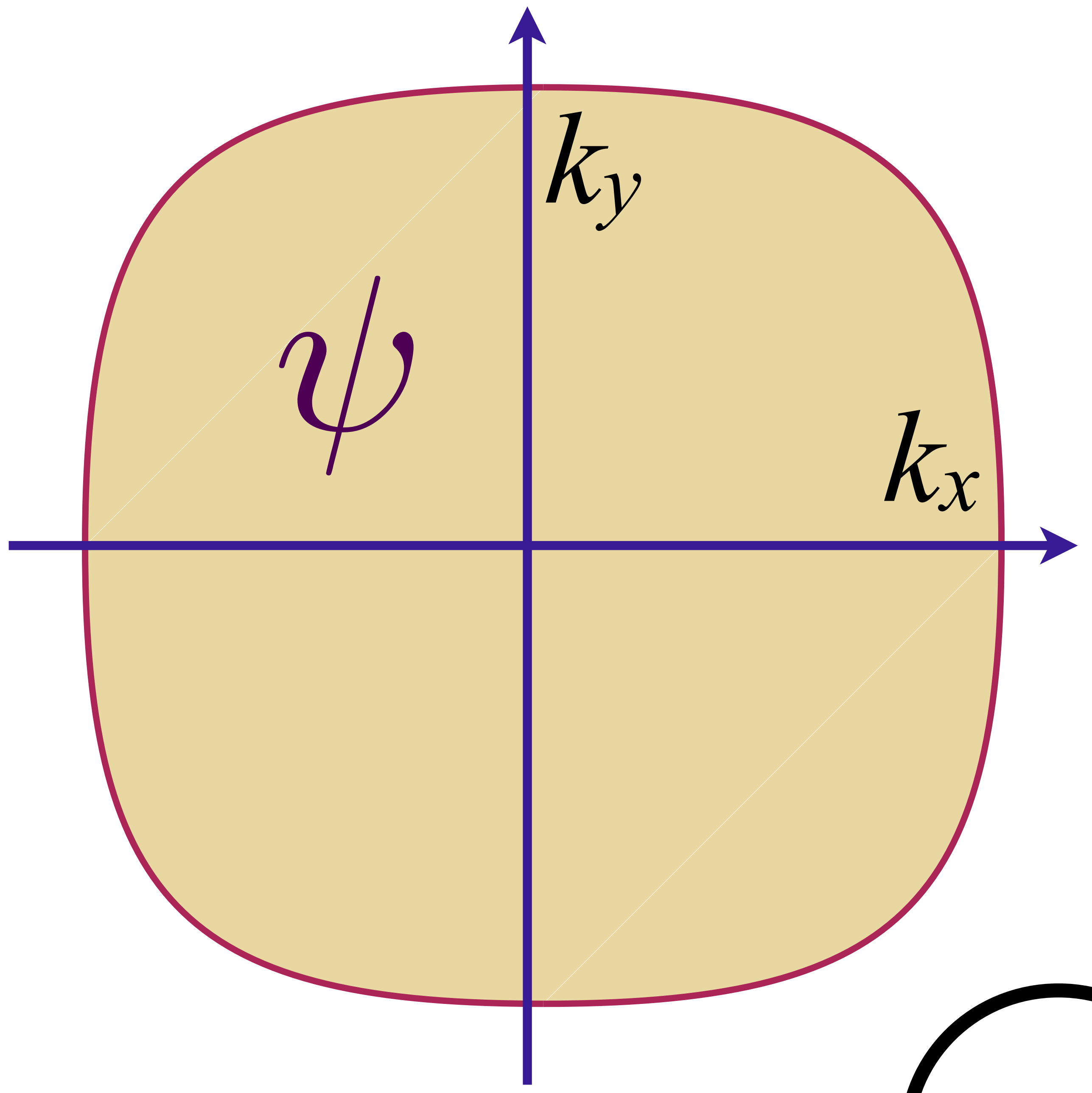
$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

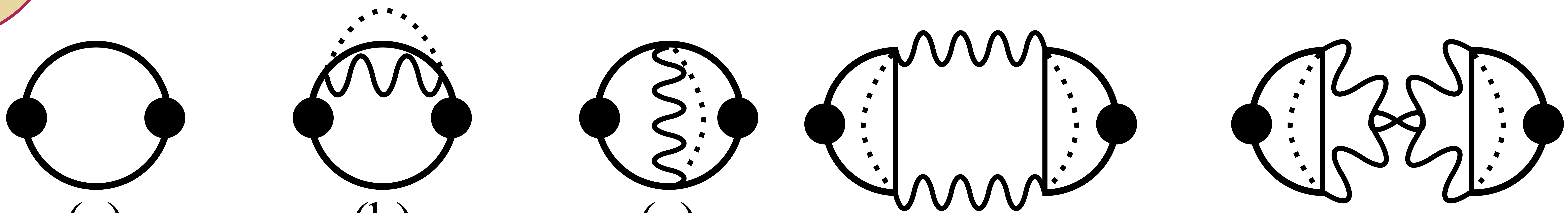
a critical boson ϕ
e.g. Ising-nematic order,
Higgs field …



$$[\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Conductivity:



(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}$, $\frac{\sigma_{\Sigma,g'}}{2}$

(c) $\sigma_{V,g}$

(d)

(e)

+ all ladders and bubbles…..

Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega\,\dfrac{m^*_{\text{trans}}(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g

Fermi surface coupled to a critical boson with disorder

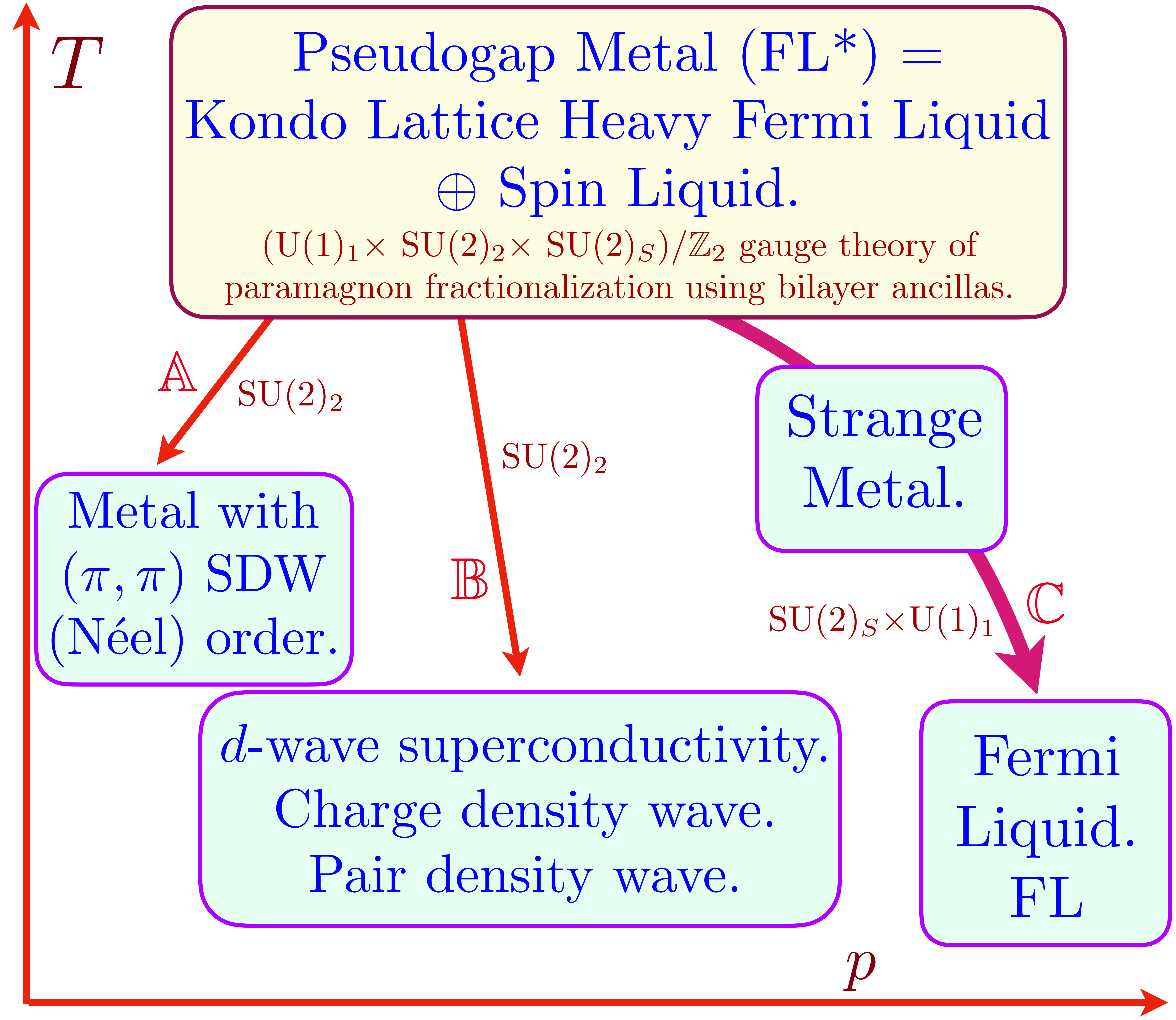
Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \, \dfrac{m^*_{\rm trans}(\omega)}{m}}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \, \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i \left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)} \right) {\rm sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2;
Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.



$\mathrm{SU}(2)_S \times \mathrm{U}(1)_1$ gauge theory with 'ghost' Fermi surfaces and Higgs fields.

Ya-Hui Zhang and S. S., PRR **2**, 023172 PRB **102** , 155124 (2020)

Yields strange metal with spatially random Yukawa couplings.

A. A. Patel, Haoyu Guo, I. Esterlis, and S. S., arXiv:2203.04990