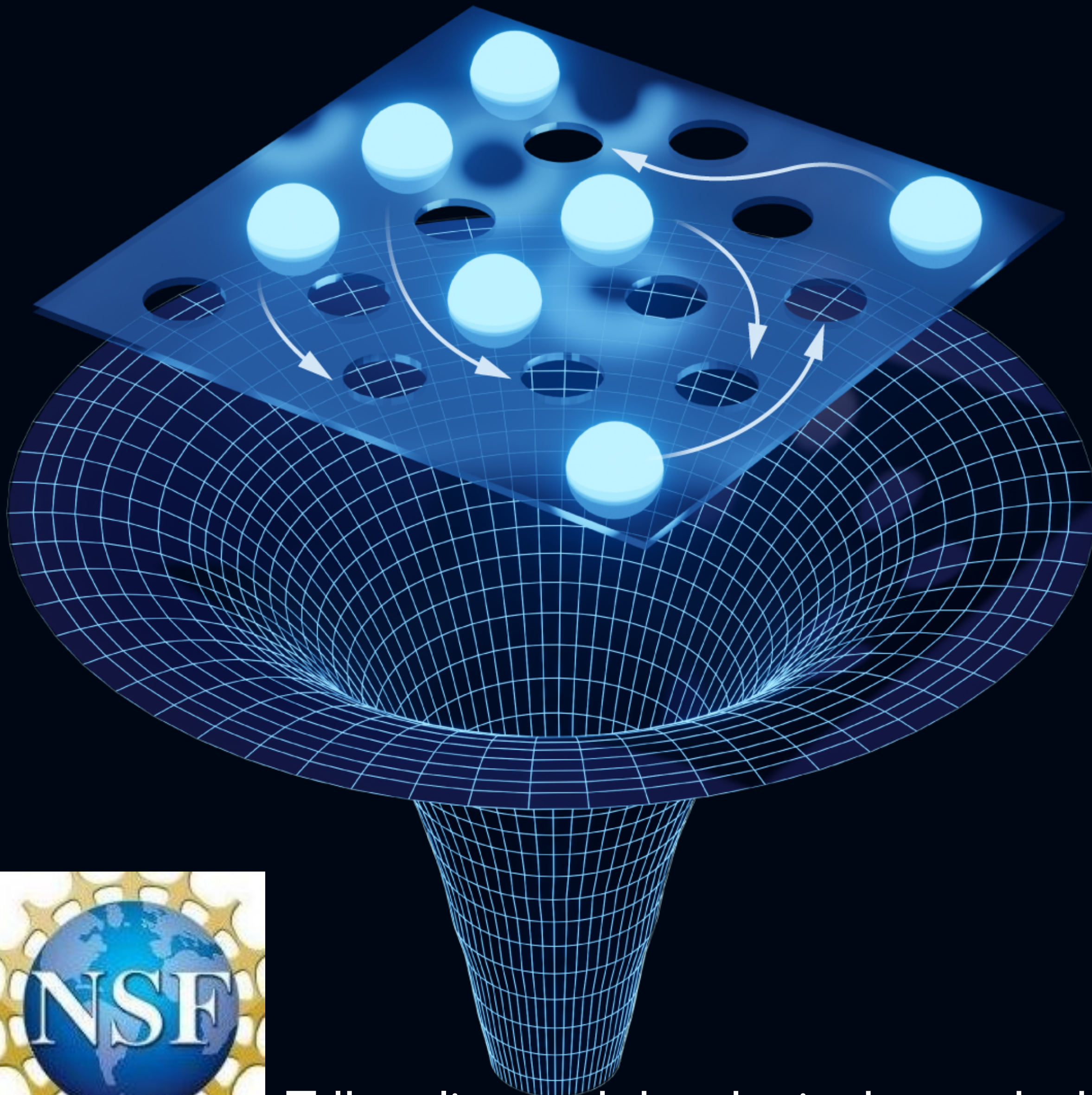


Quantum entanglement in nature: superconductors and black holes



Tata Institute of Fundamental Research

Hyderabad

December 16, 2023

Subir Sachdev



Talk online: sachdev.physics.harvard.edu



Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

What is entropy
and temperature ?

Clausius (1865):

Second Law of Thermodynamics:

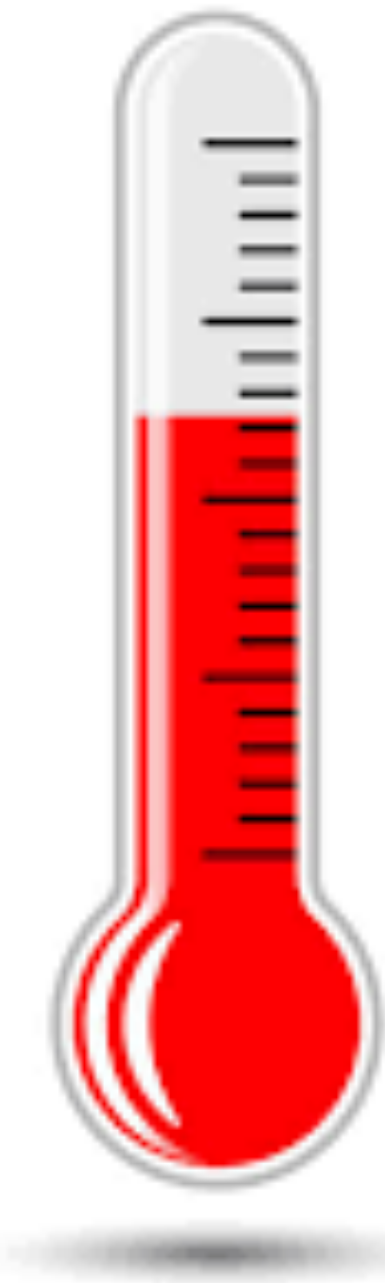
Every macroscopic system has an
“entropy” which cannot decrease.

➔ No perpetual motion machines!



Clausius (1865):
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Temperature

Clausius (1865):

Second Law of Thermodynamics:

Every macroscopic system has an “entropy” which cannot decrease.

➔ No perpetual motion machines!

Heat always flows
from a
hot body
to a
cold body



Statistical interpretation of entropy (1870)

$$S = k_B \log W$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

$$\frac{1}{T} = \frac{dS}{dE}$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

Vienna, Austria

Ordinary conductors
(metals)
and
superconductors



CONDUCTOR

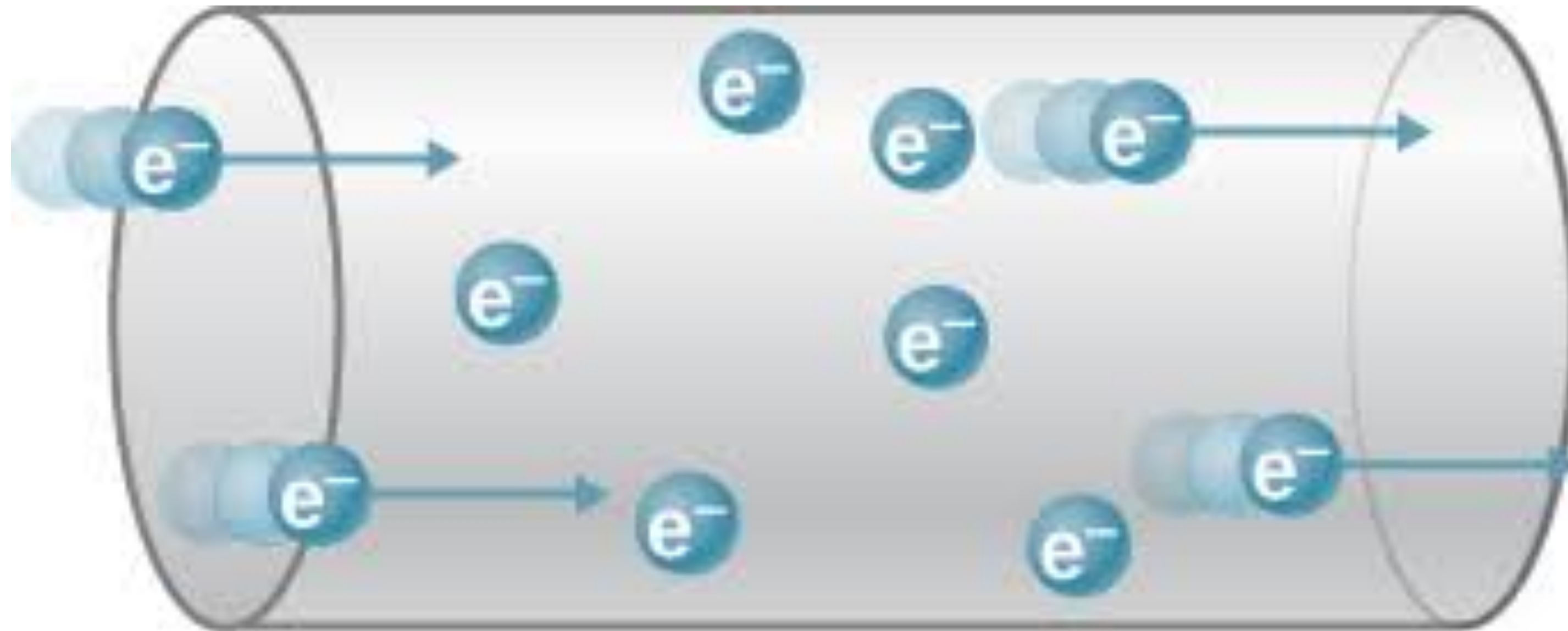


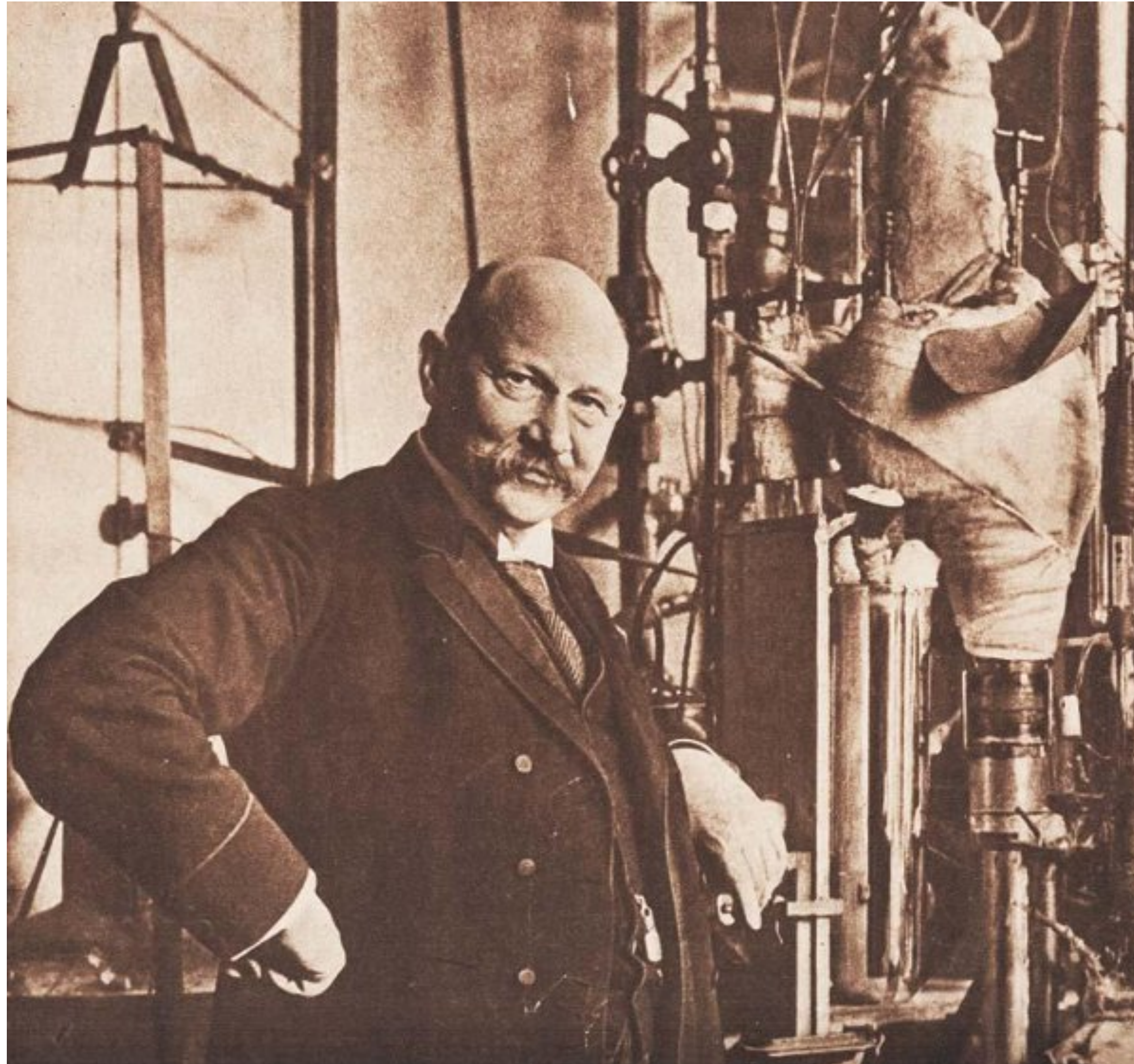
SEMICONDUCTOR



SUPERCONDUCTOR

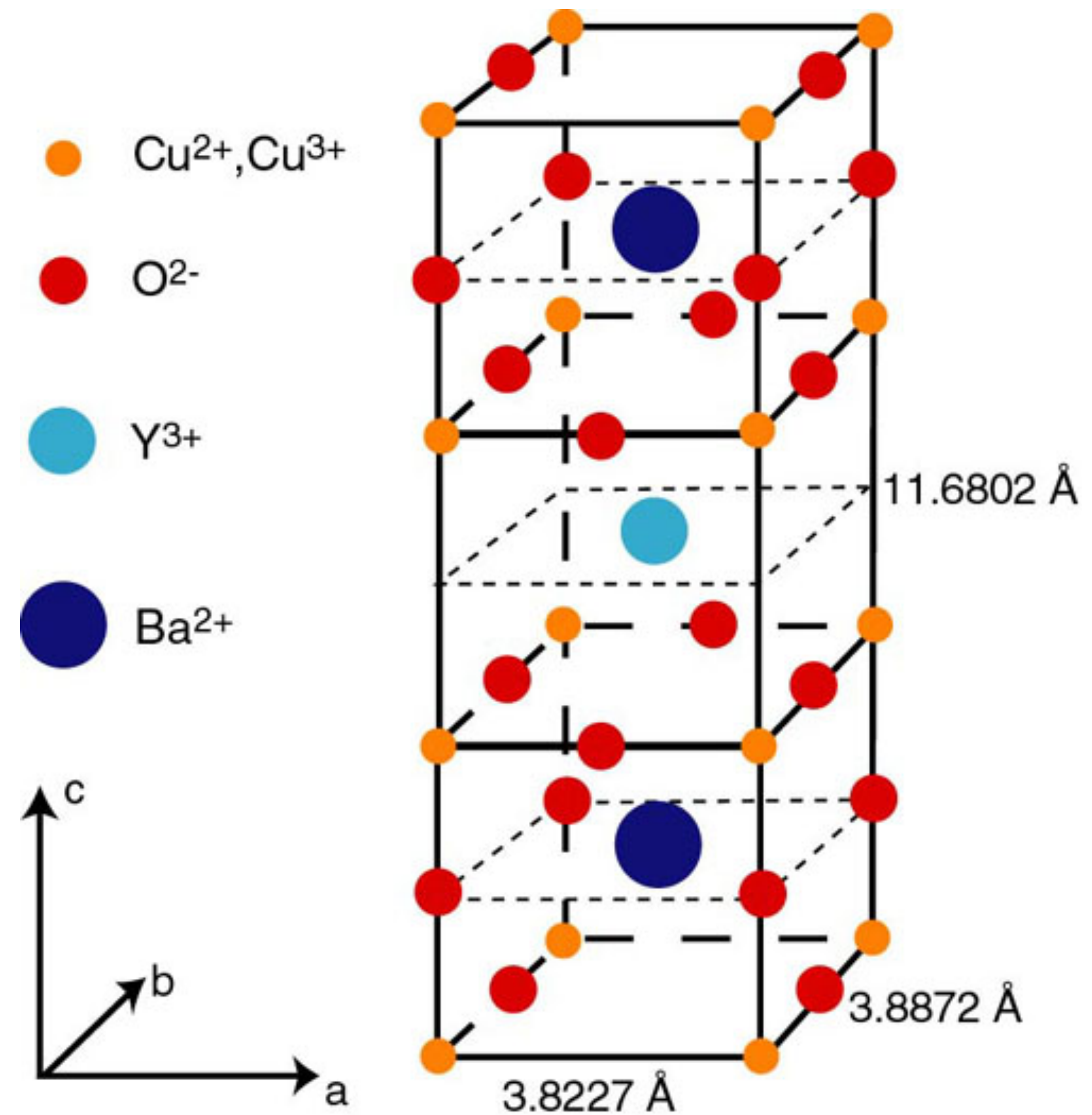
Copper: ordinary metal quantum matter

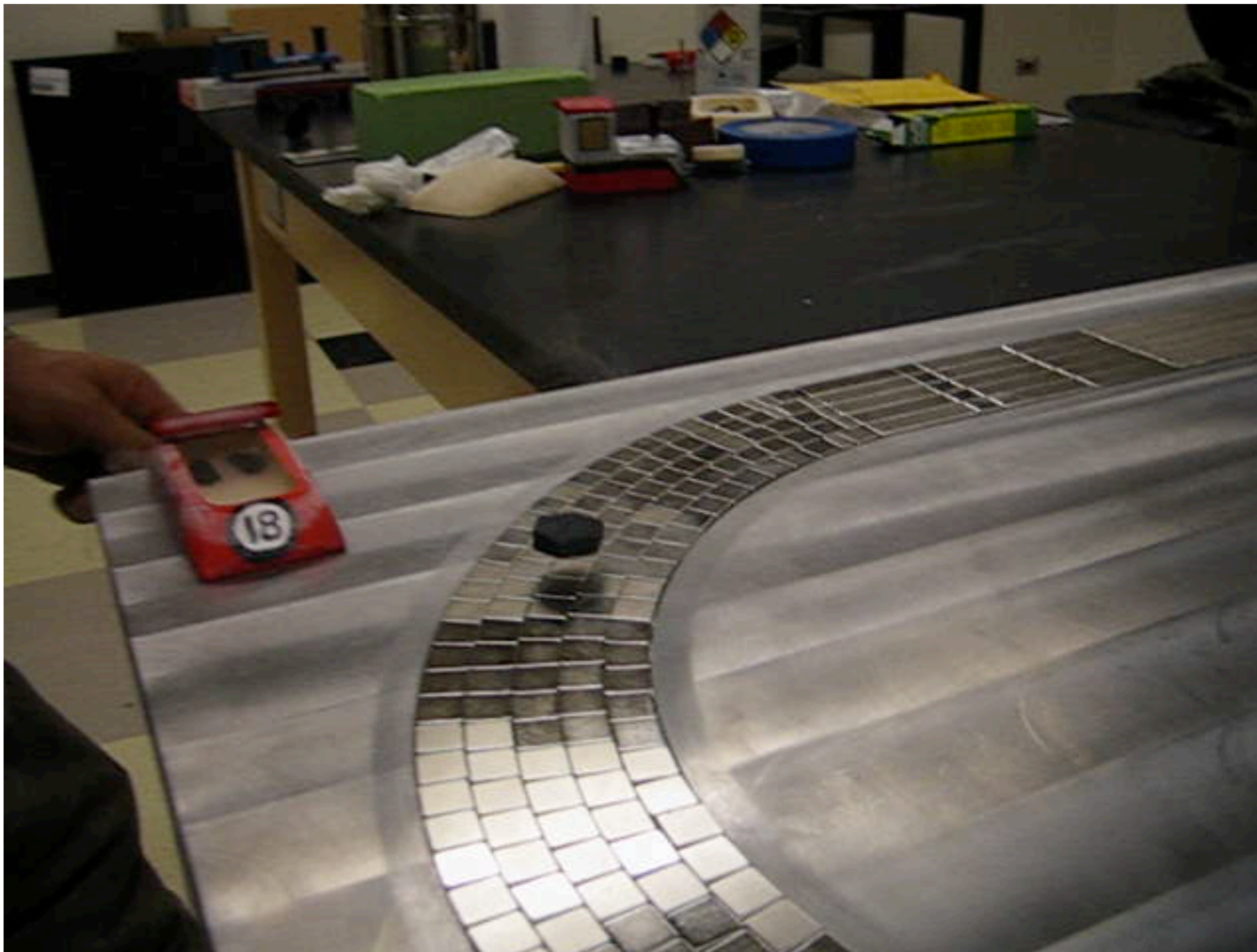




Kamerlingh Onnes 1911:
Mercury is a superconductor below $-269\text{ }^{\circ}\text{C}$

Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

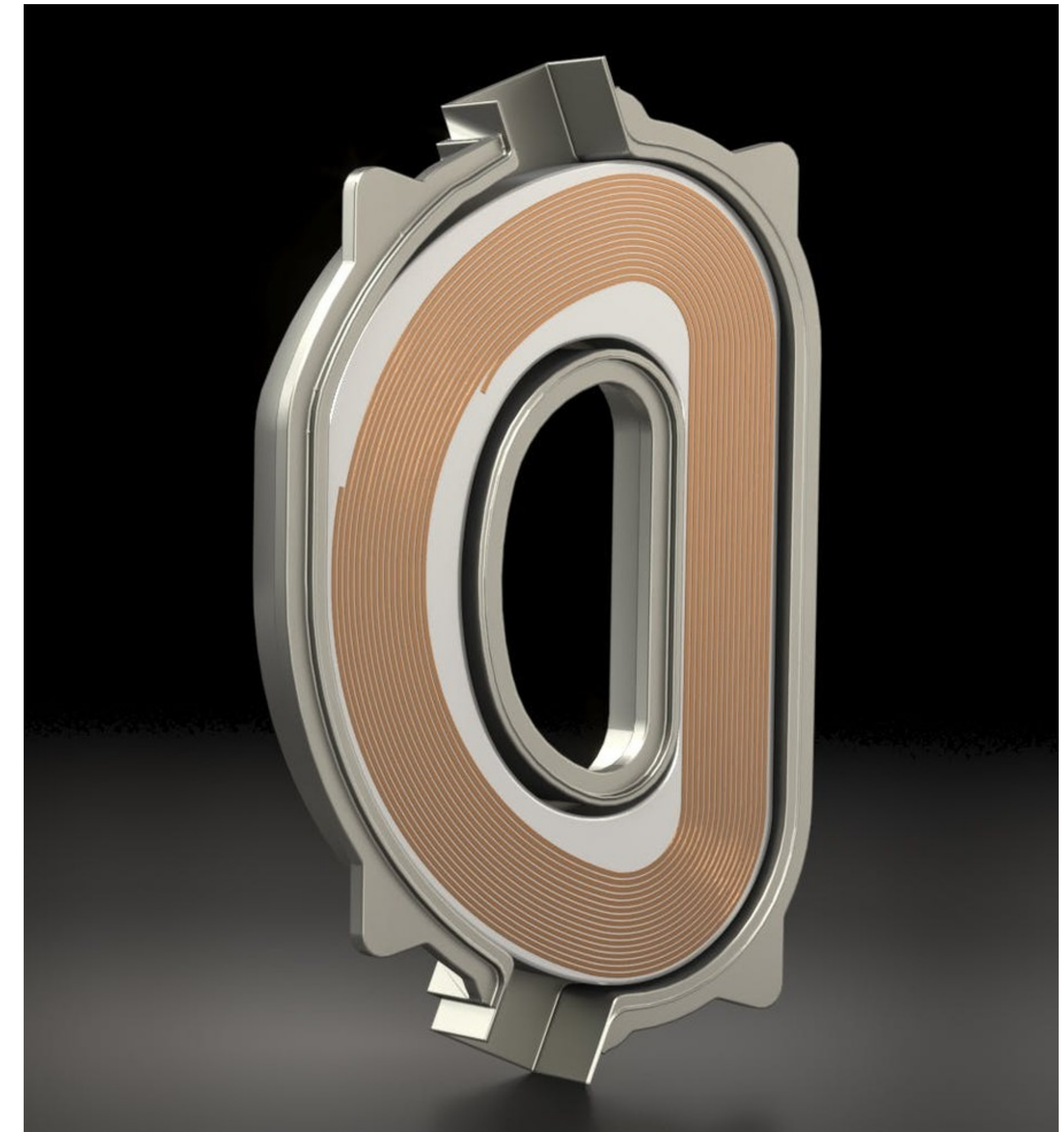
HTS Magnets: Enabling Technology

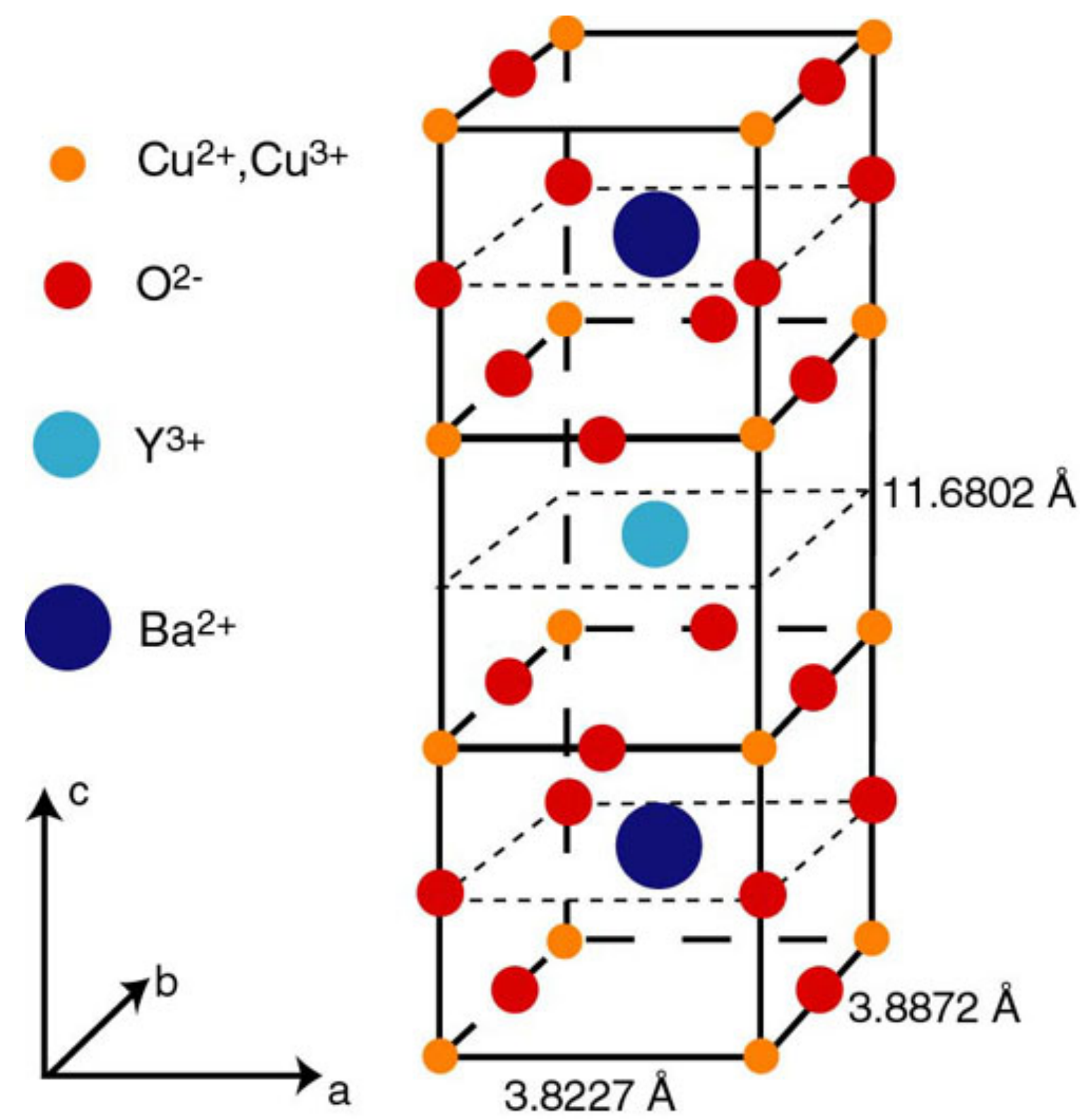
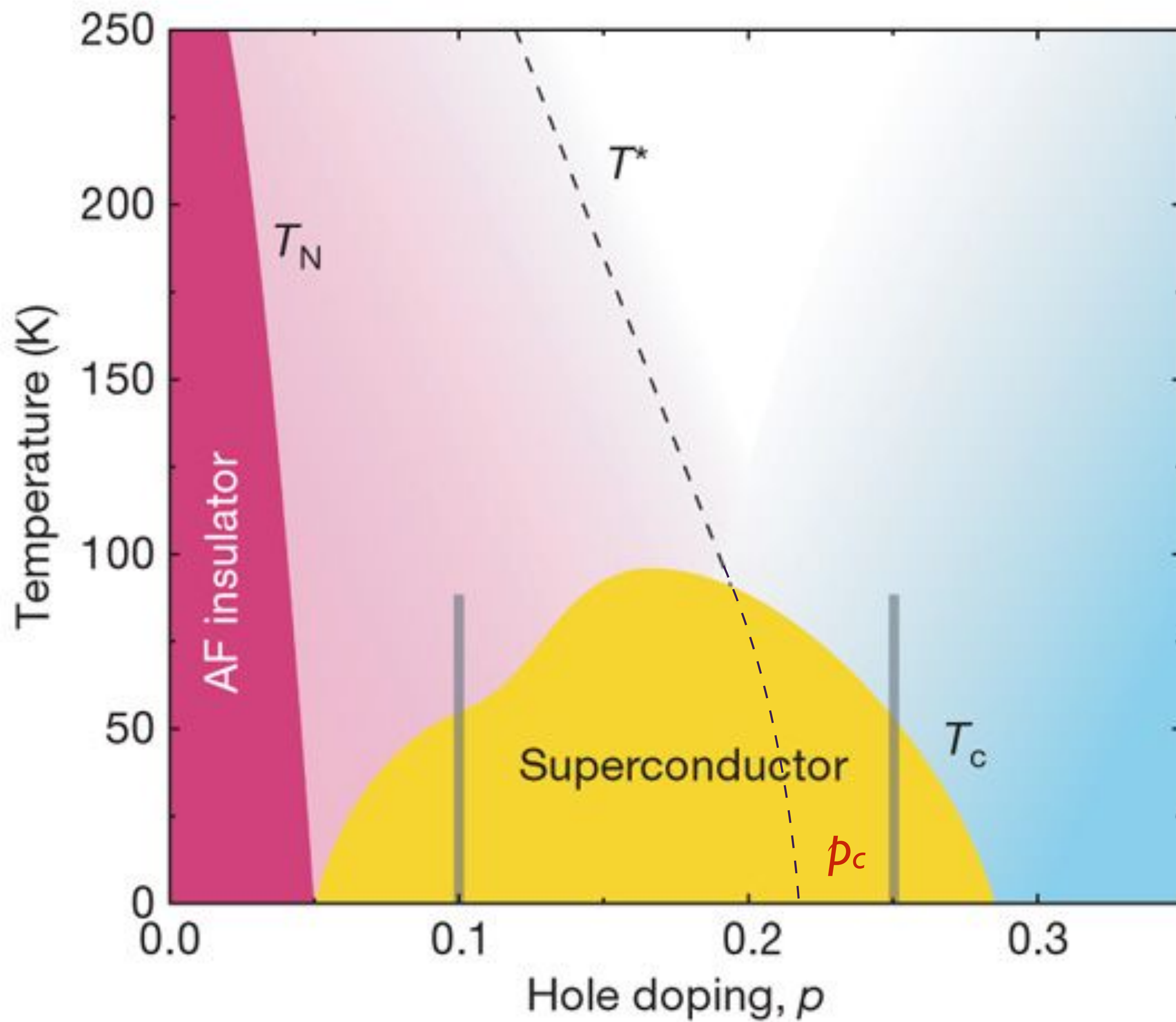
The surest path to limitless,
clean, fusion energy

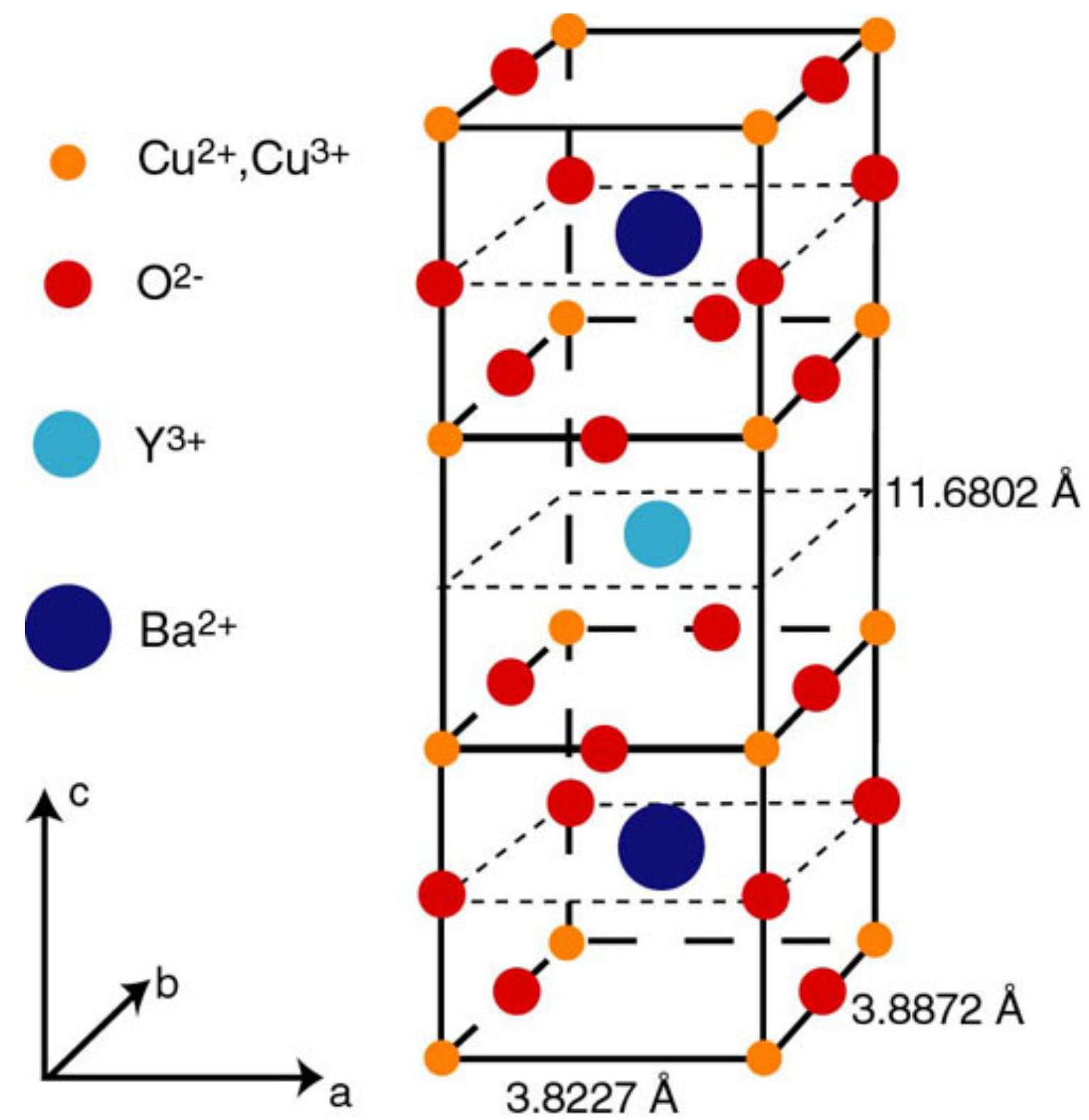
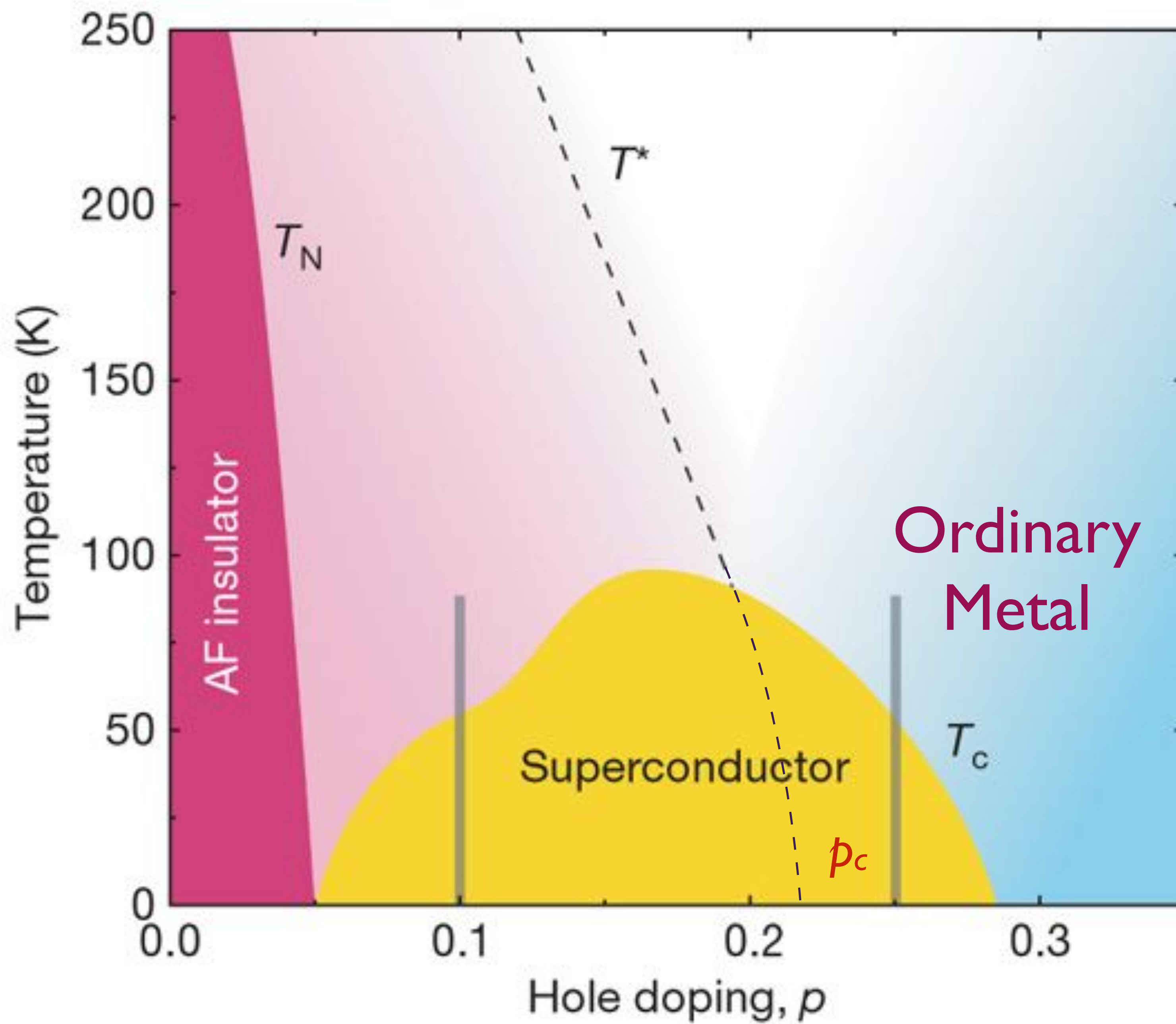
YBCO magnets allow for smaller,
faster, and less expensive
tokamaks for plasma fusion

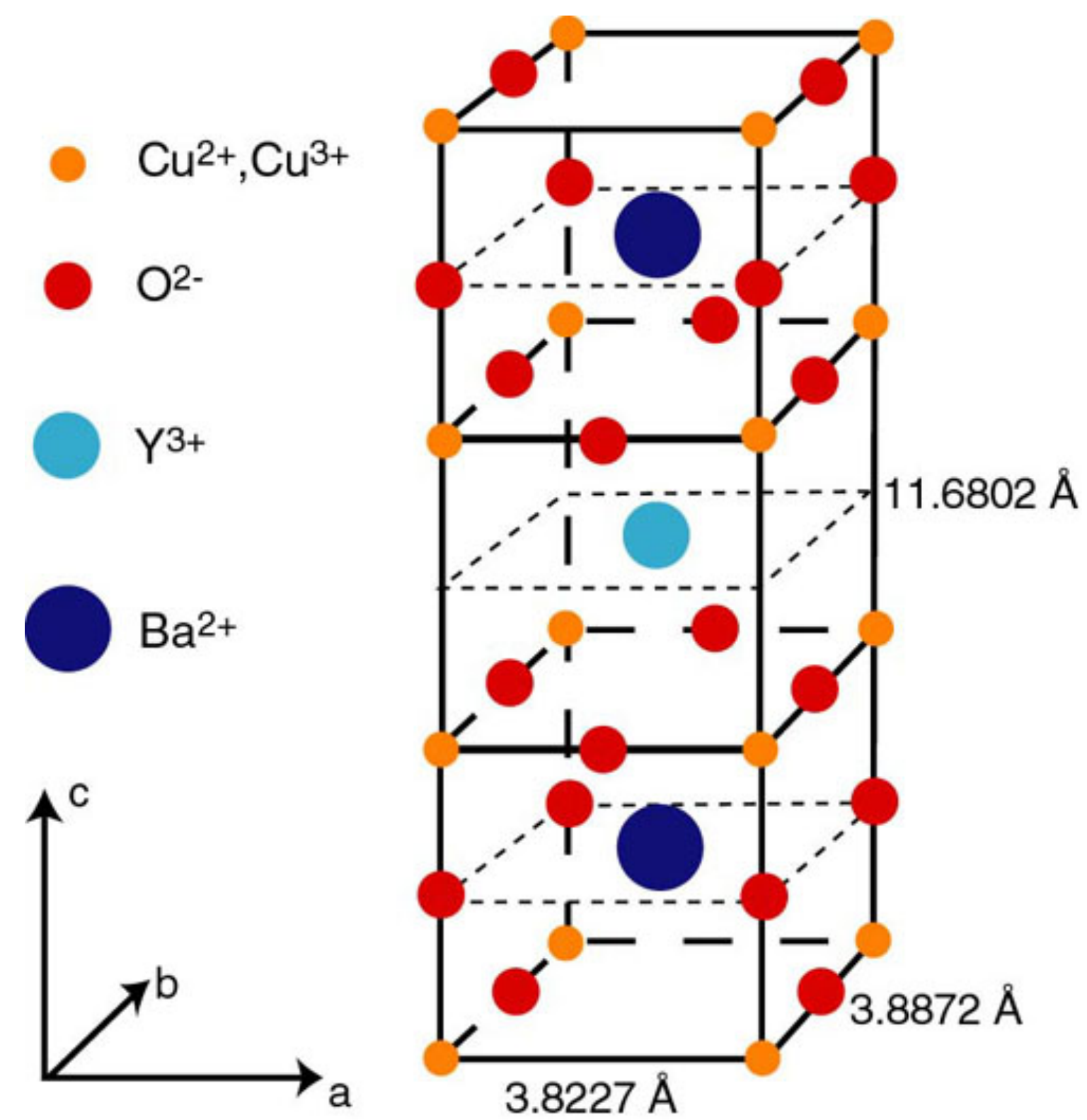
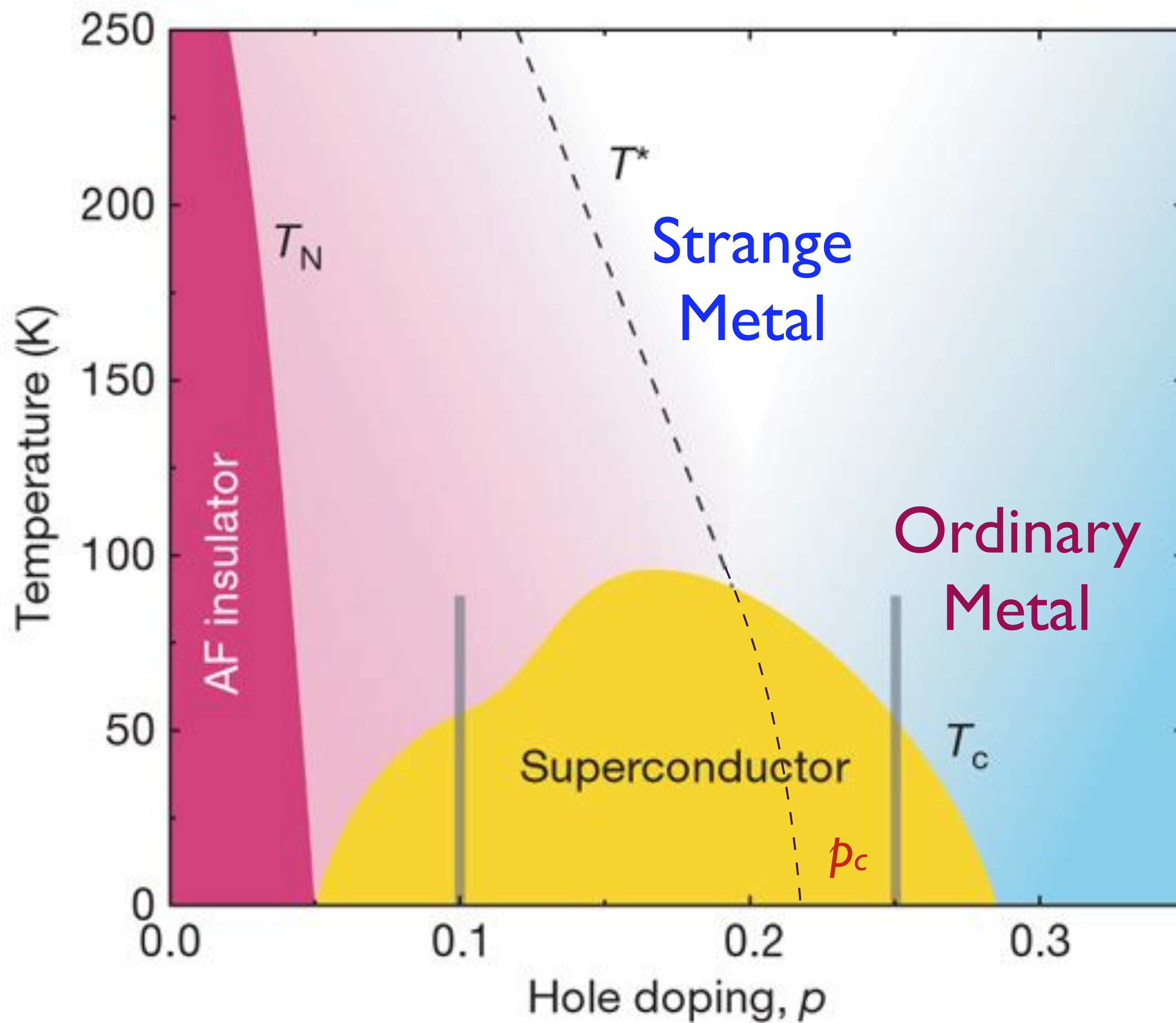


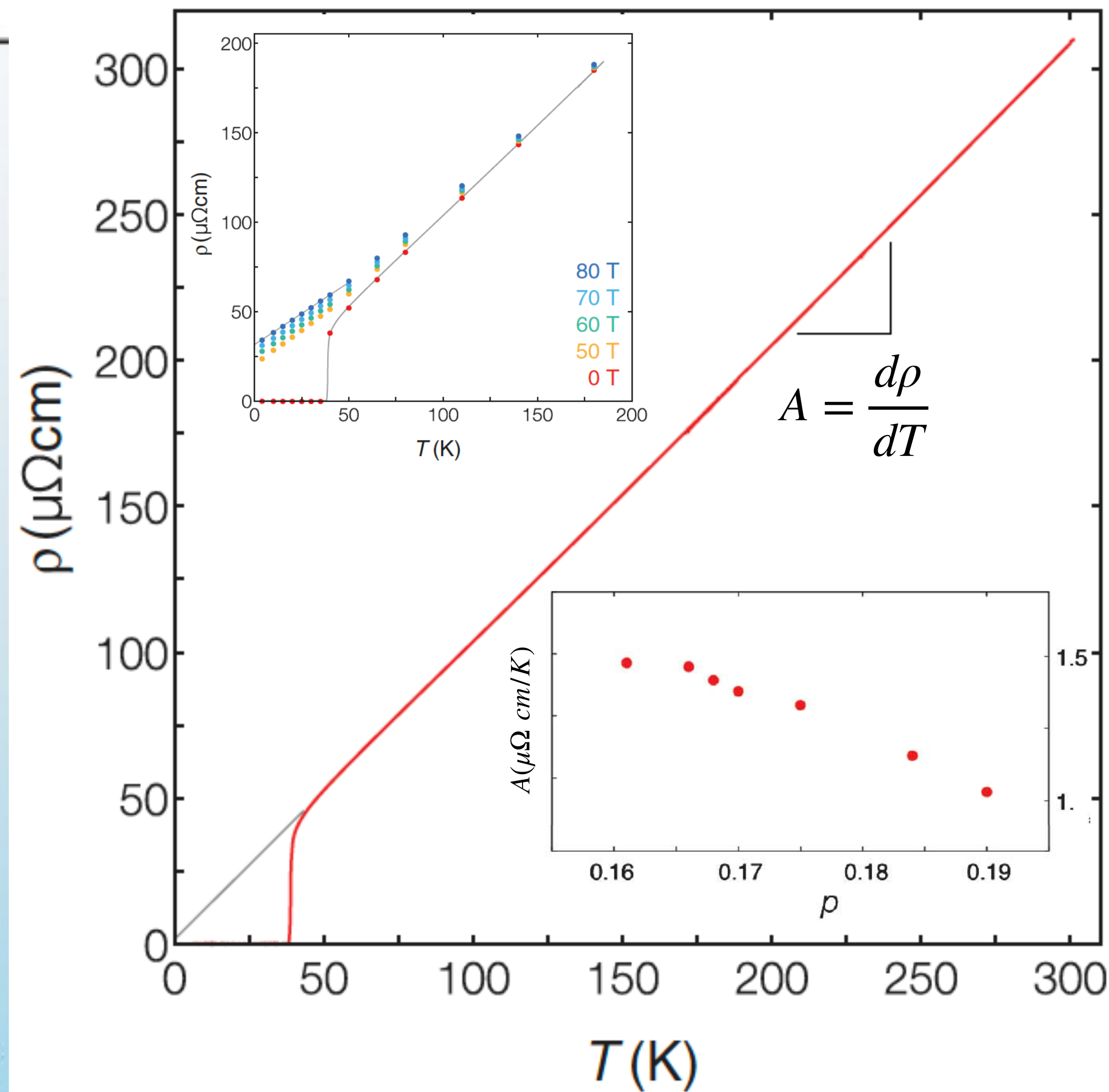
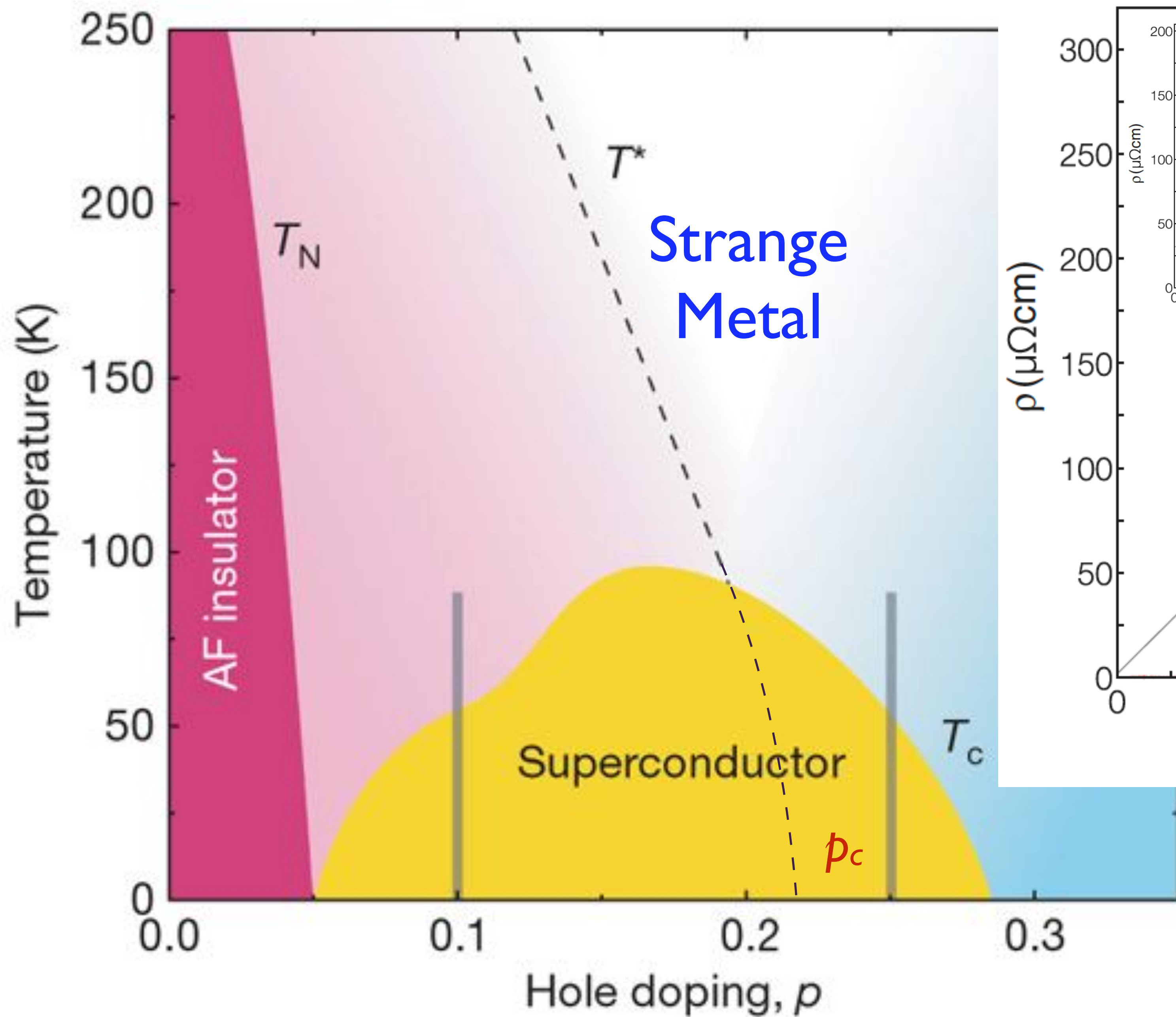
Commonwealth
Fusion Systems











LSCO: Giraldo-Gallo et al. 2018

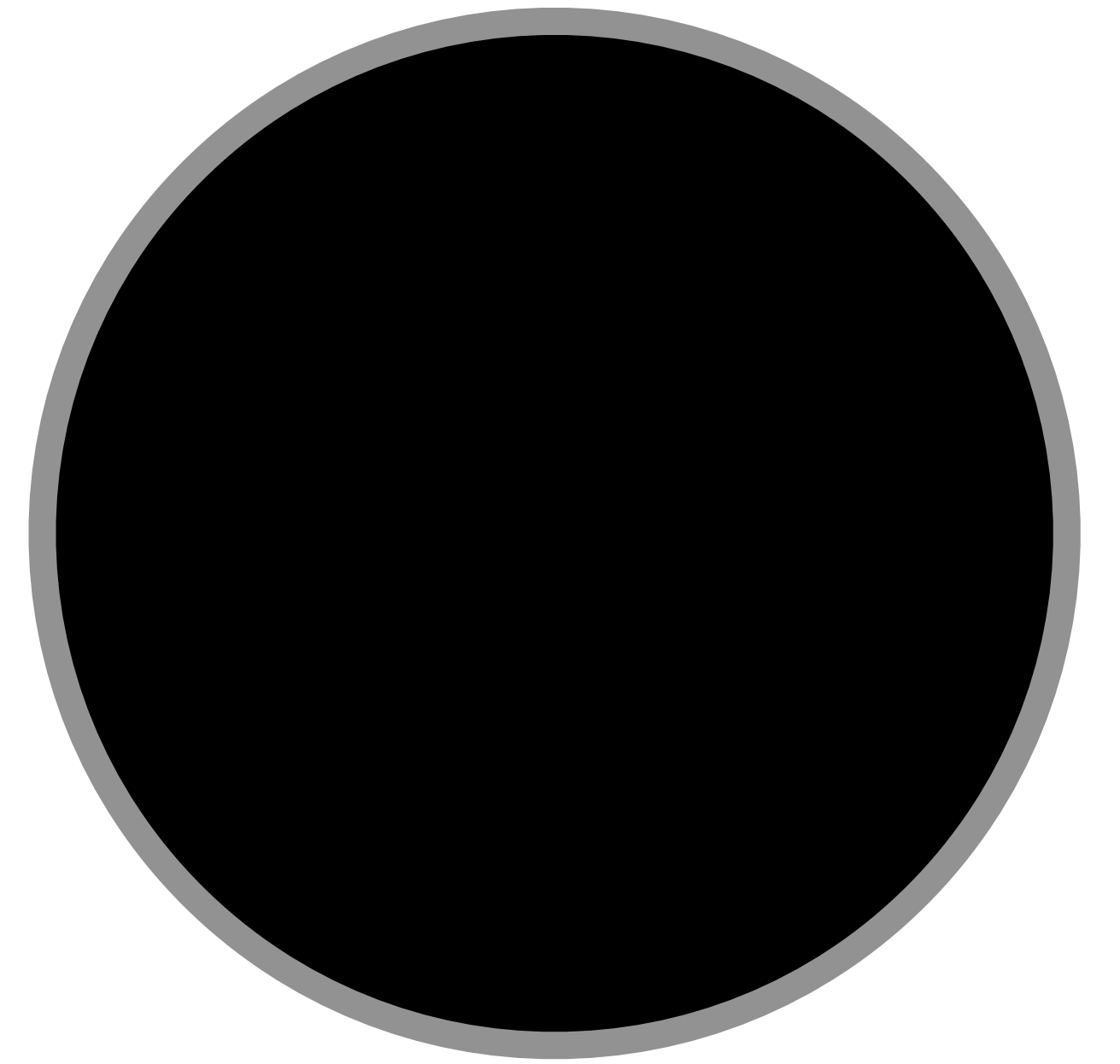
**Black
holes**

Black Holes

Objects so dense that light is gravitationally bound to them.



Horizon radius $R = \frac{2GM}{c^2}$

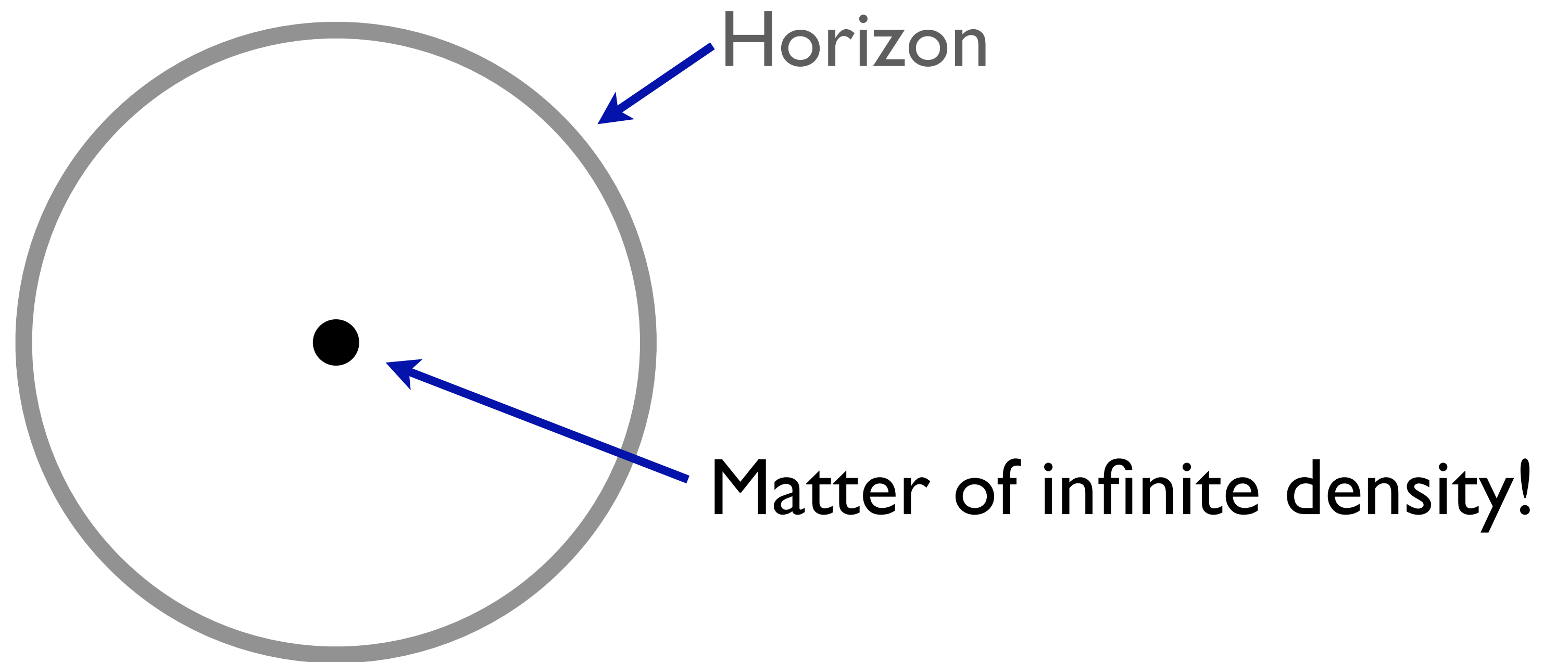


Karl Schwarzschild (1916)

G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm!}$

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



What is inside a black hole ???

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This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

An artist's impression of the supermassive black hole Sagittarius A* at the center of the Milky Way. The image shows a dark, circular event horizon surrounded by a glowing accretion disk. The disk is composed of multiple layers, with the innermost layers being the brightest and most intense, transitioning from white and yellow to orange and red as they move further out. The background is a dark, starry field.

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A* contains about 4.3 million solar masses

$$R = 1.3 \times 10^{11} \text{ m}$$

\approx earth's orbit

An artist's impression of Sagittarius A*, the supermassive black hole at the heart of the Milky Way. Image: International Gemini Observatory/NOIRLab/NSF/AURA/J. da Silva/(Spaceengine); M. Zamani (NSF's NOIRLab)

What is inside a black hole ???

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In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

What is inside a black hole ???

Hawking (1975): when viewed from the outside, black holes have an entropy and a temperature, and slowly evaporate like any thermal object



$$T = \frac{\hbar c^3}{8\pi G M k_B}$$

What is
quantum entanglement?

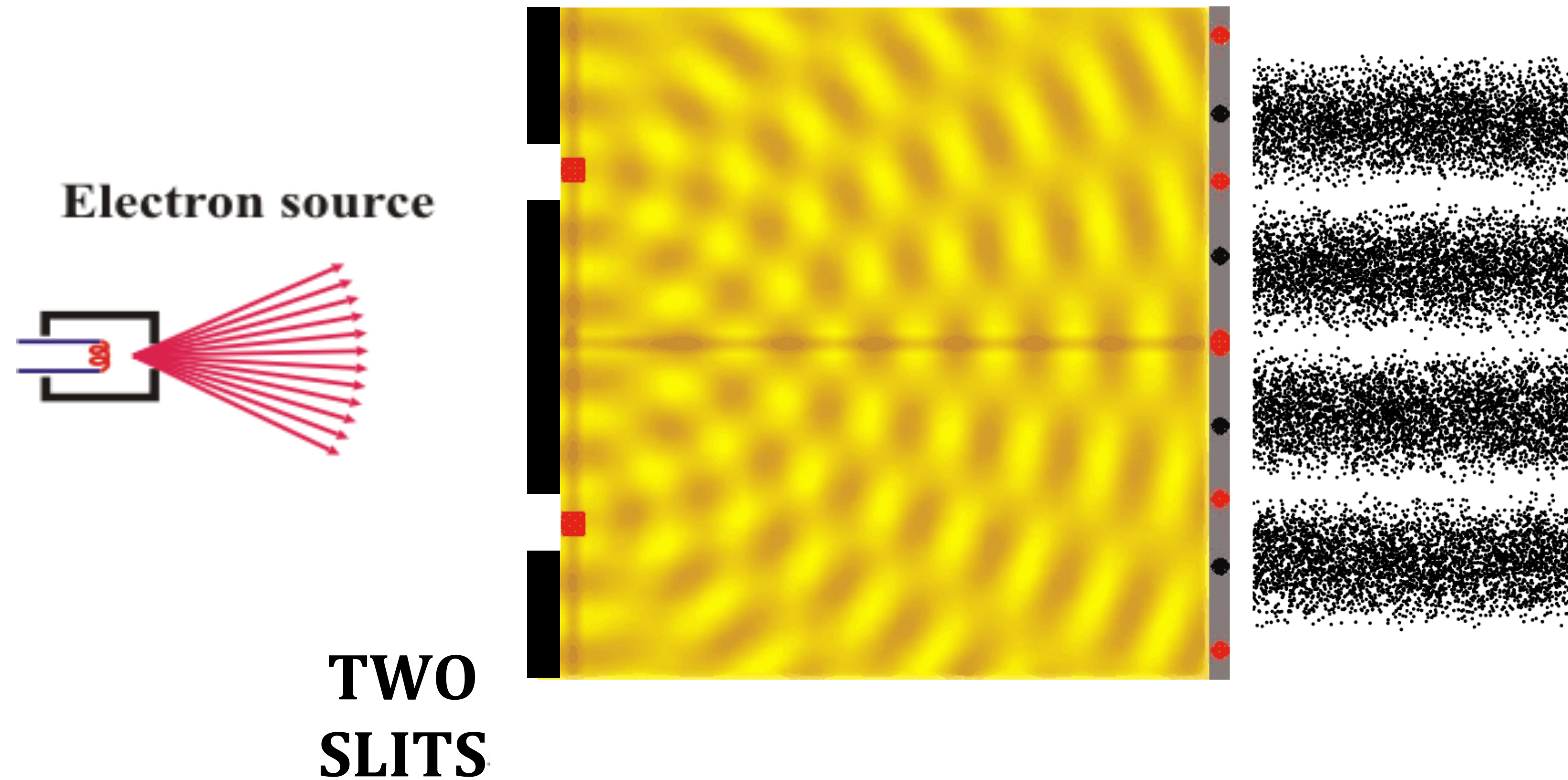


“About your cat, Mr. Schrödinger—I have good news and bad news.”

The most remarkable new idea in the quantum theory is the
principle of superposition:
a physical system can be in a
superposition of two (or more) distinct states.

Principles of Quantum Mechanics: I. Quantum Superposition

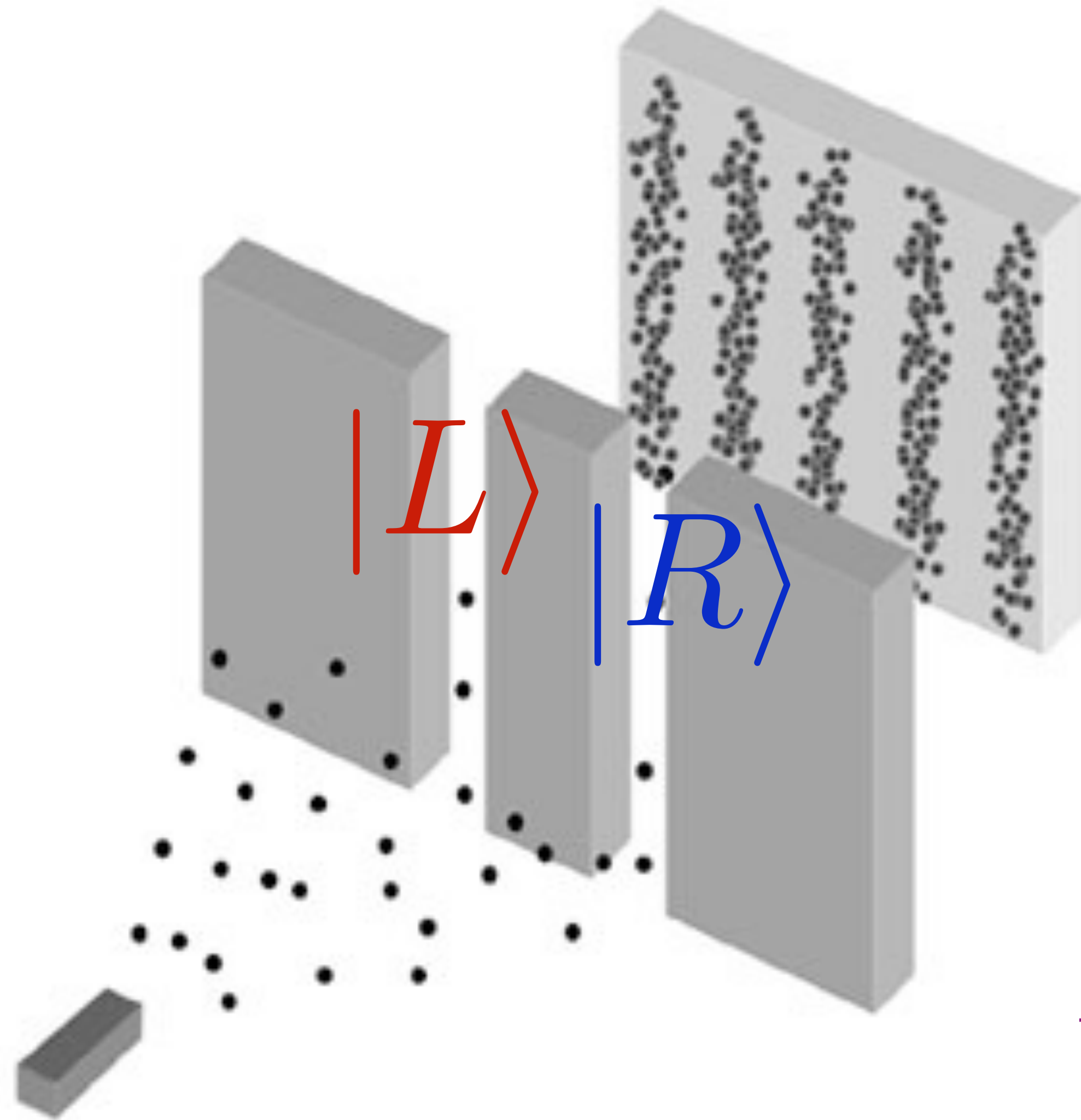
The double slit experiment



Unlike water waves, electrons arrive one-by-one (so is it like a particle ?)

Interference of electrons

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

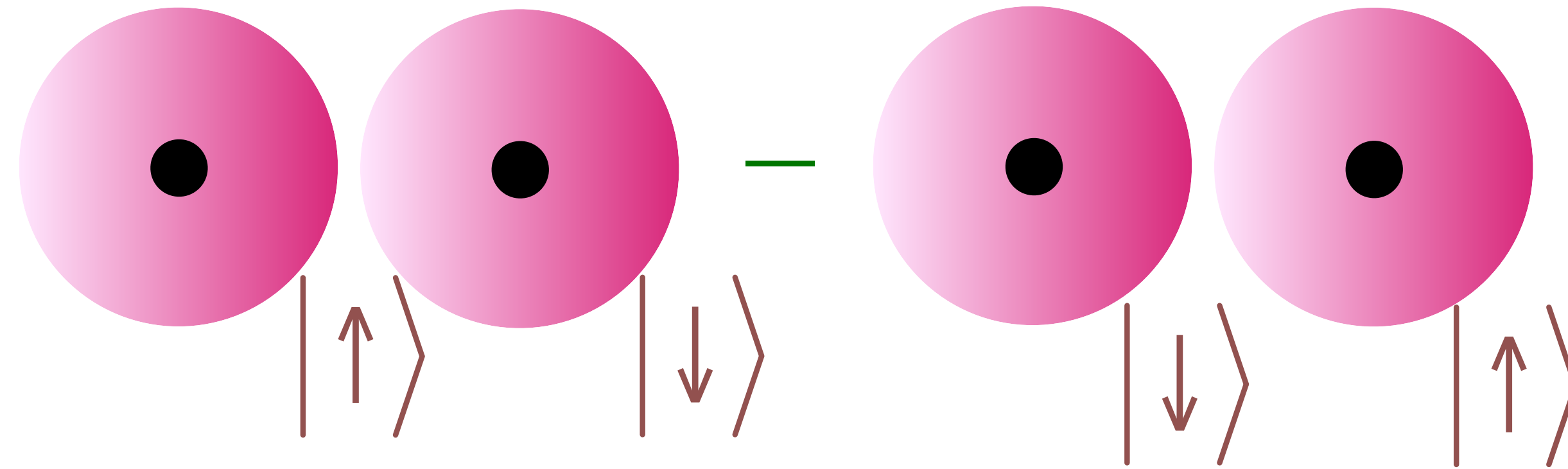
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

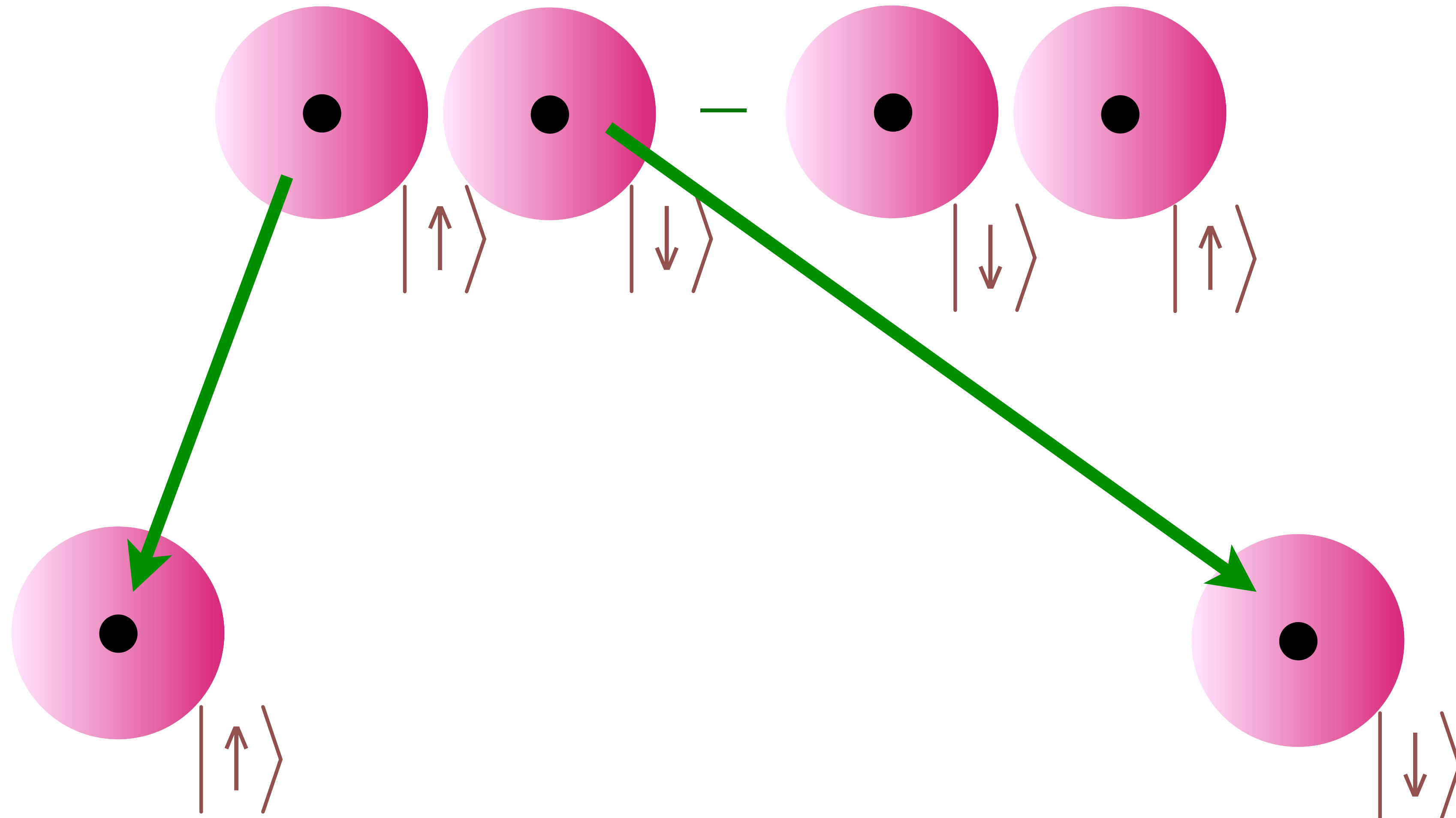
Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



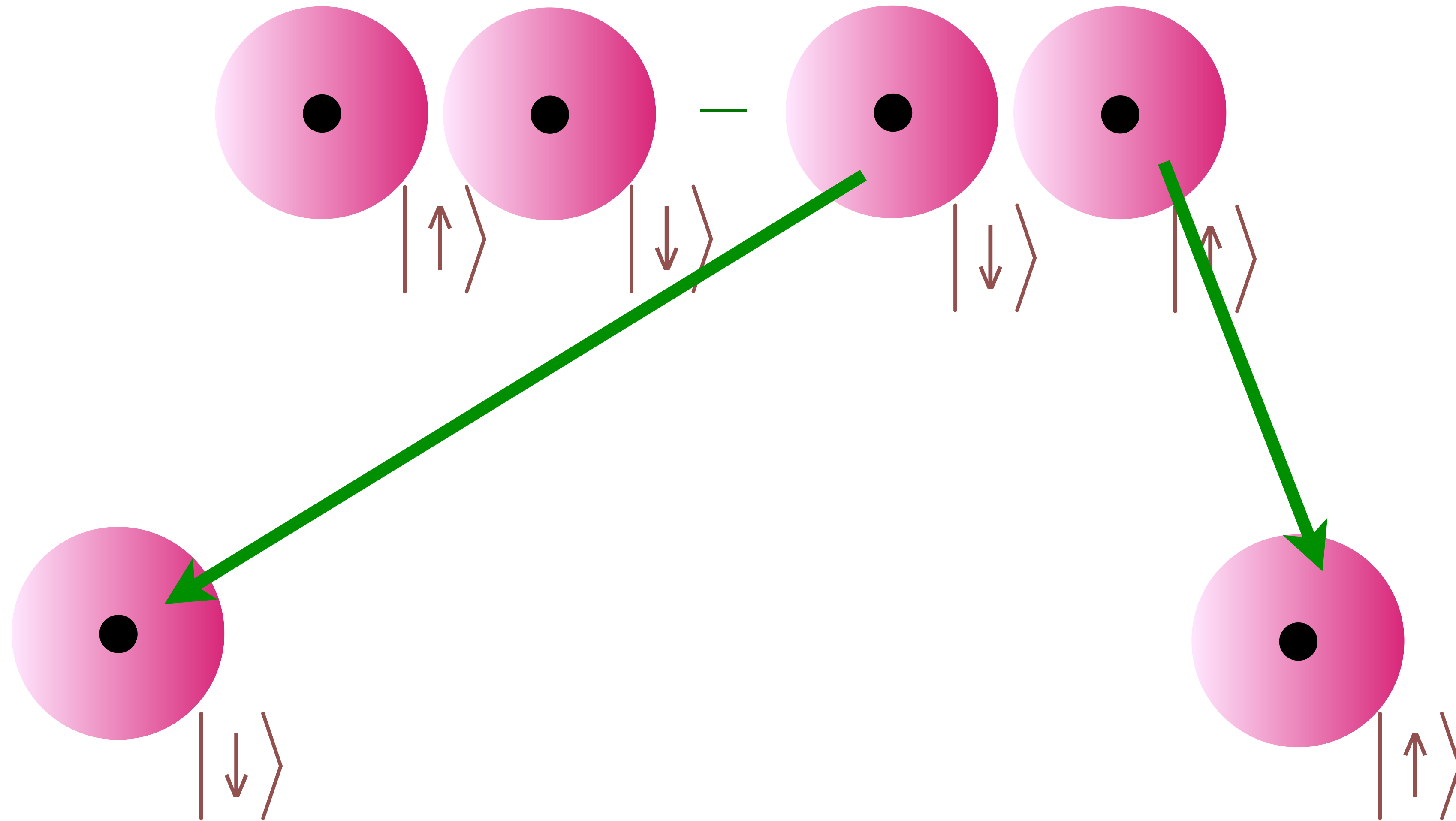
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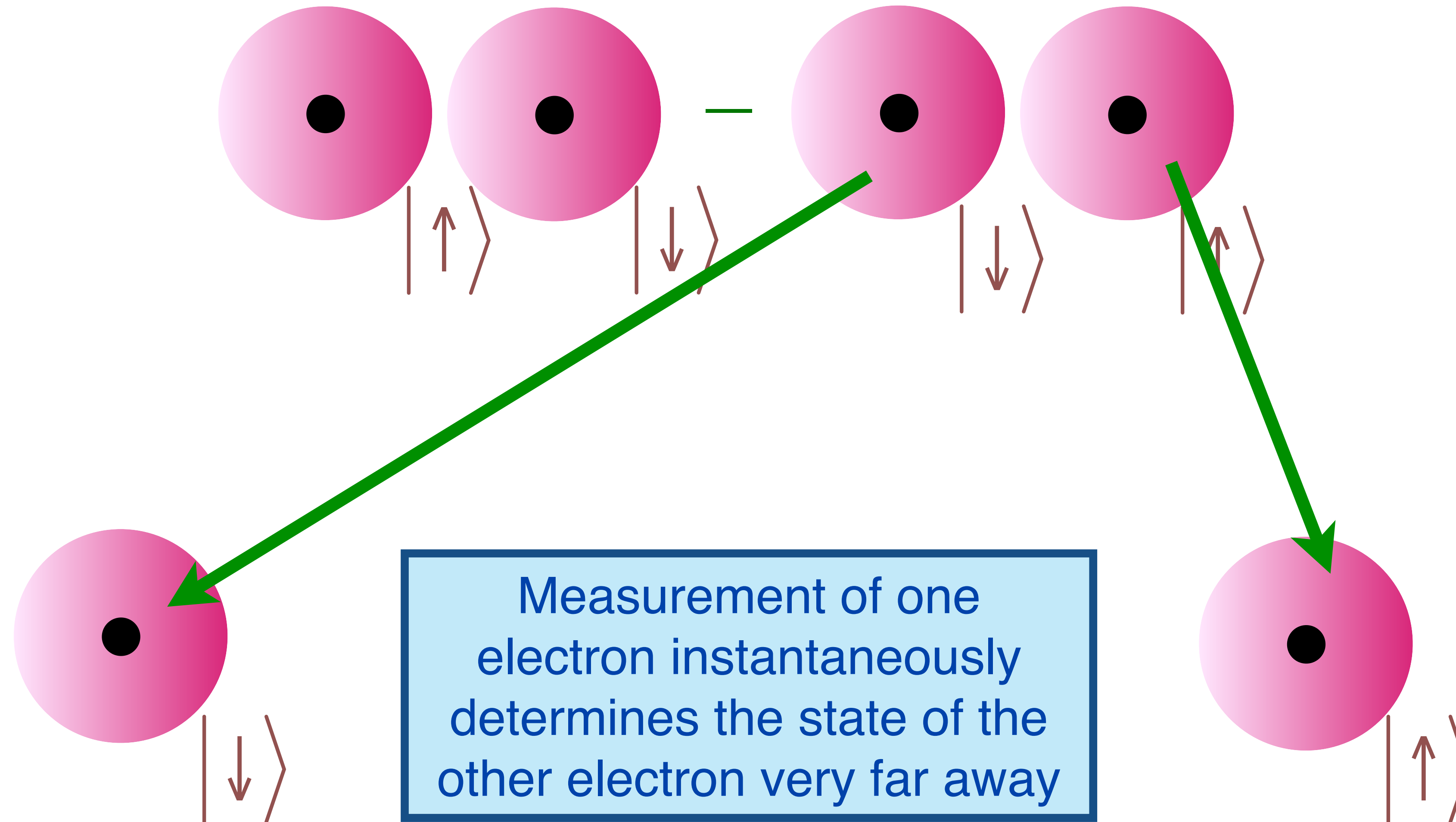
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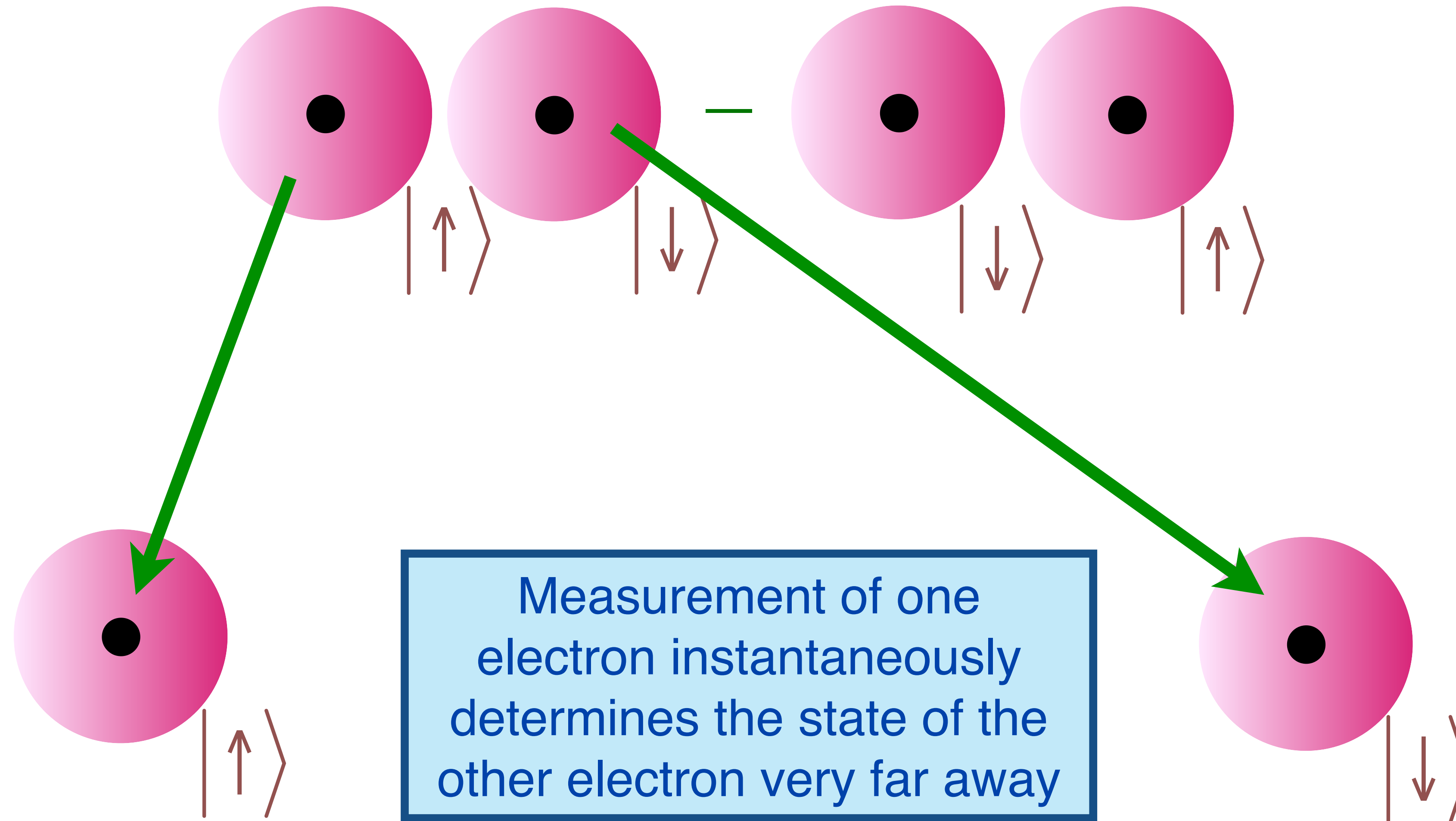
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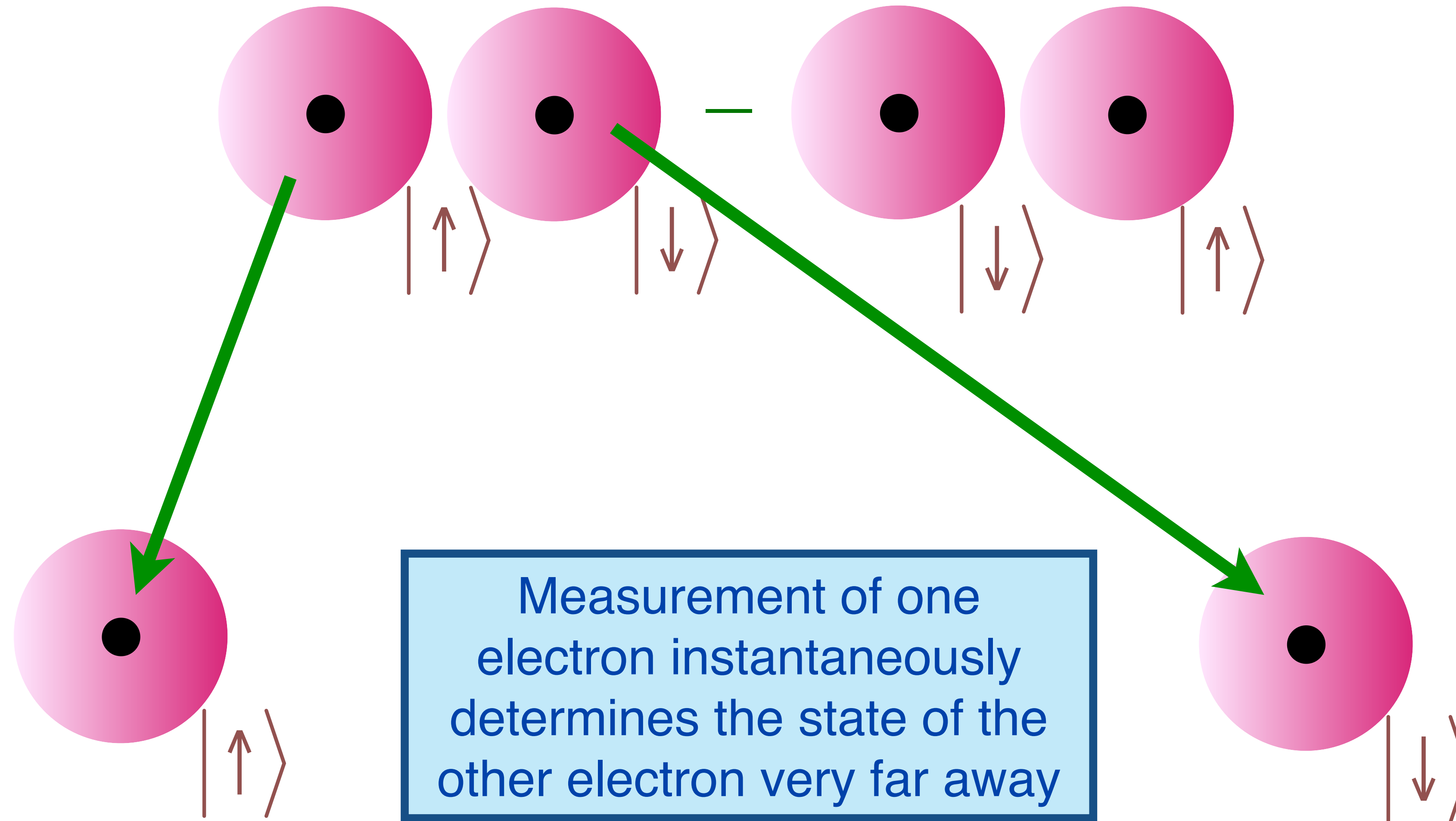
Quantum Entanglement

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Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



Spooky action at a distance !

natürlicher
deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
überzeugt, daß es wirklich mit der Theorie

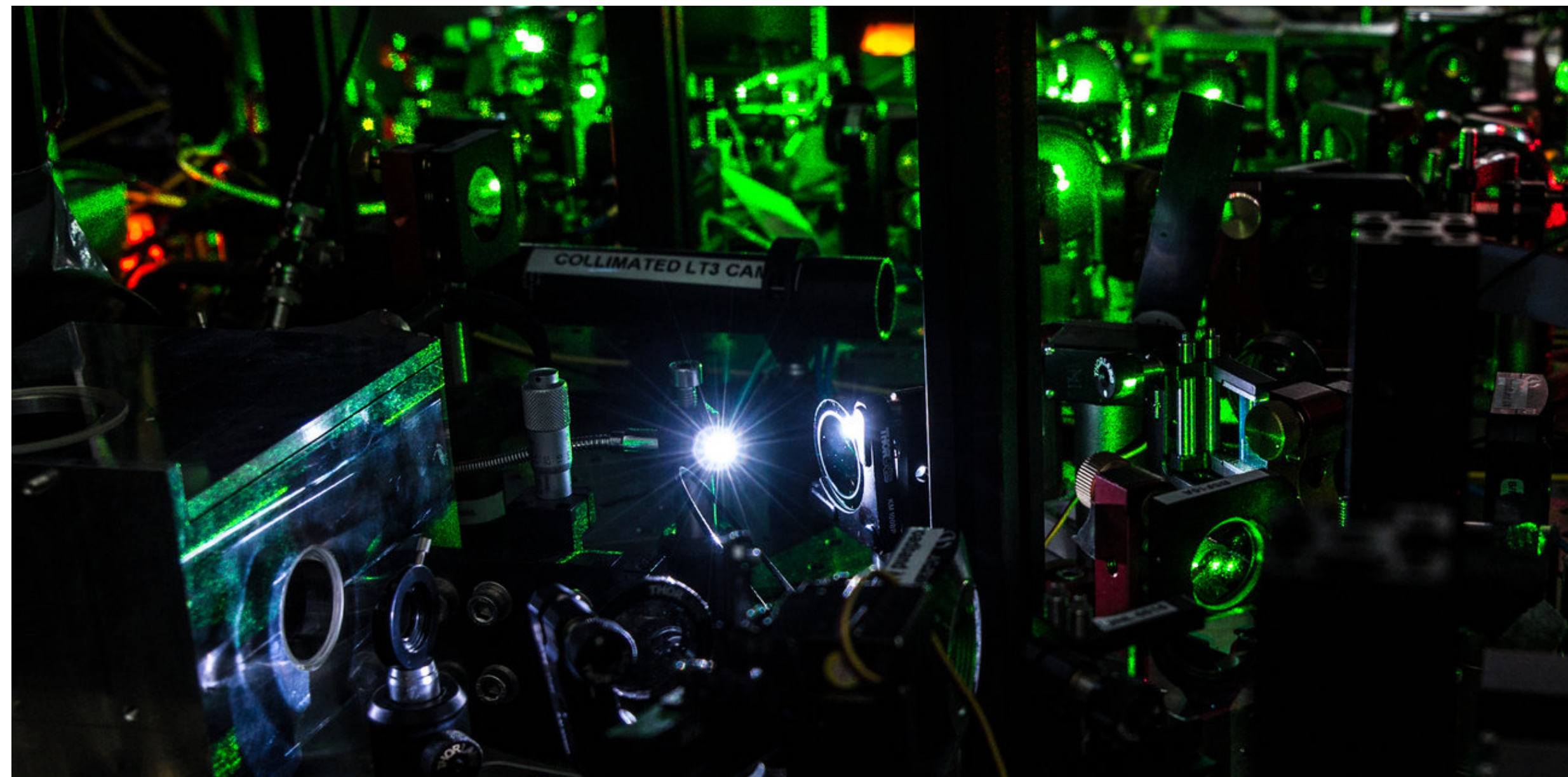
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

The New York Times

Sorry, Einstein. Quantum Study Suggests 'Spooky Action' Is Real.

By **JOHN MARKOFF** OCT. 21, 2015



Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

Great discoveries in physics

Entropy (1870)

Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

Needed,
to solve open problems in the theory of
superconductivity and black holes:

A solvable model of quantum entanglement
of 3, 4, 5, ... ∞ particles

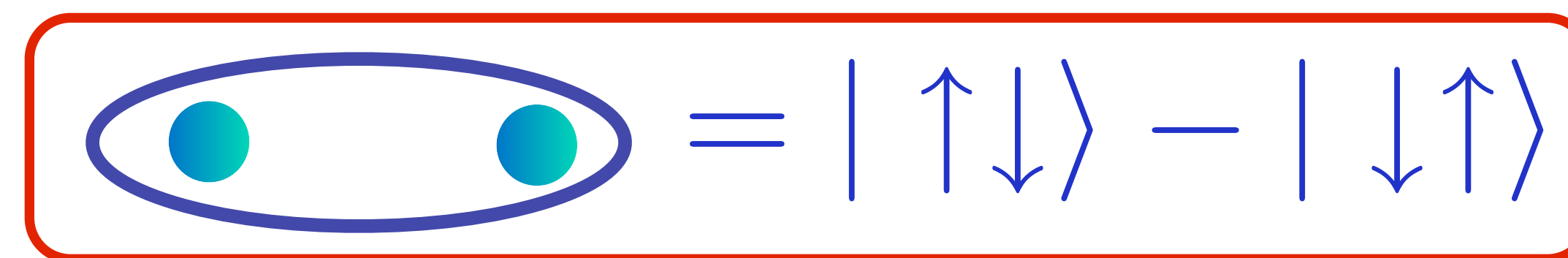
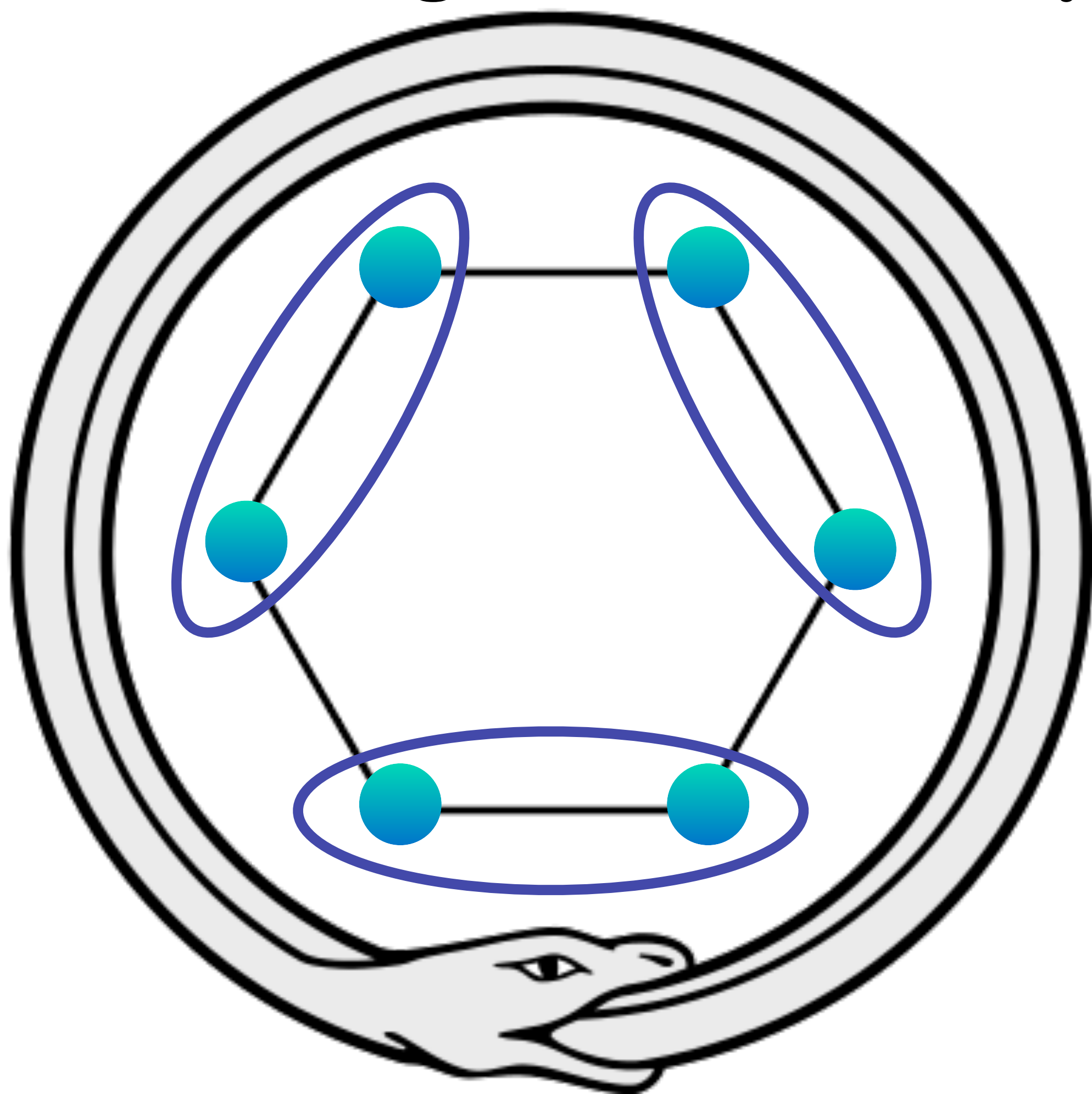
Needed,
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**The Sachdev-Ye-Kitaev model
of many-particle entanglement**

Kekulé's spooky dream (1865)

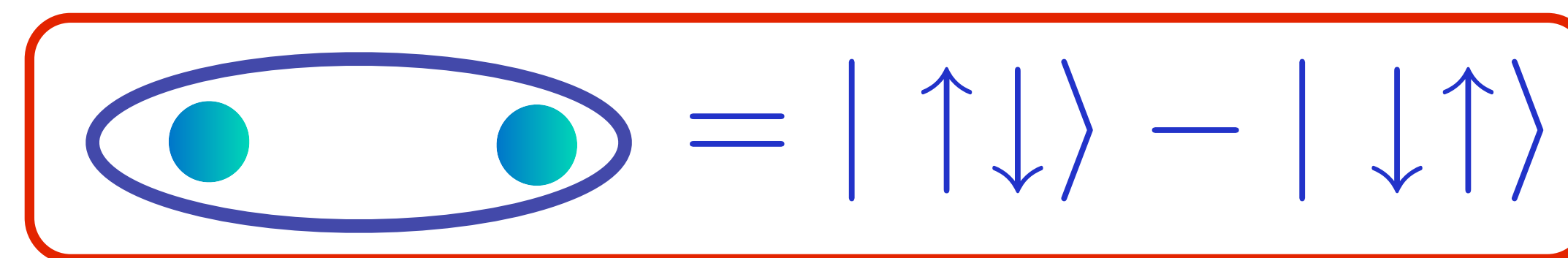
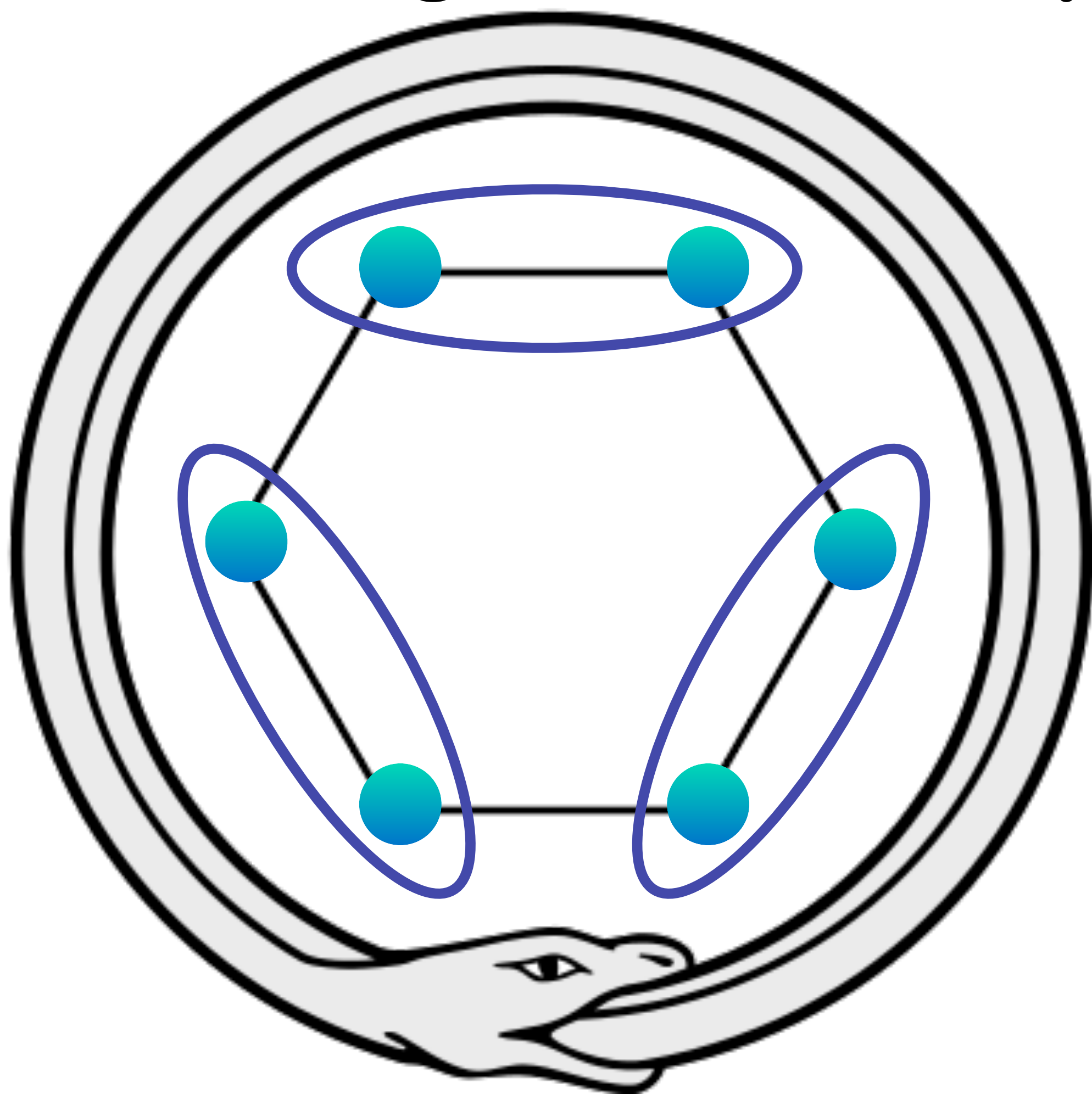
Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



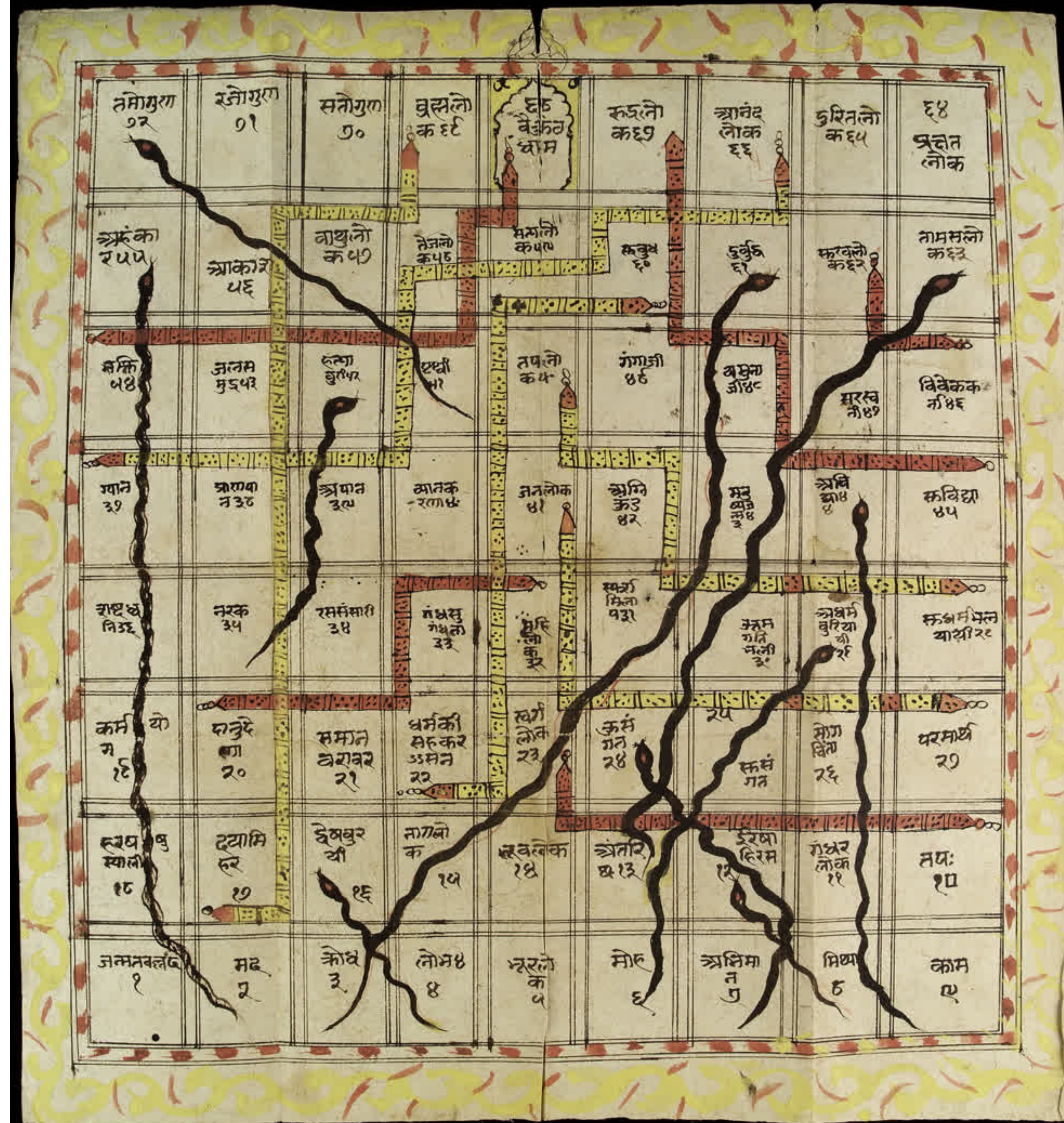
Benzene

Kekulé's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*



Benzene



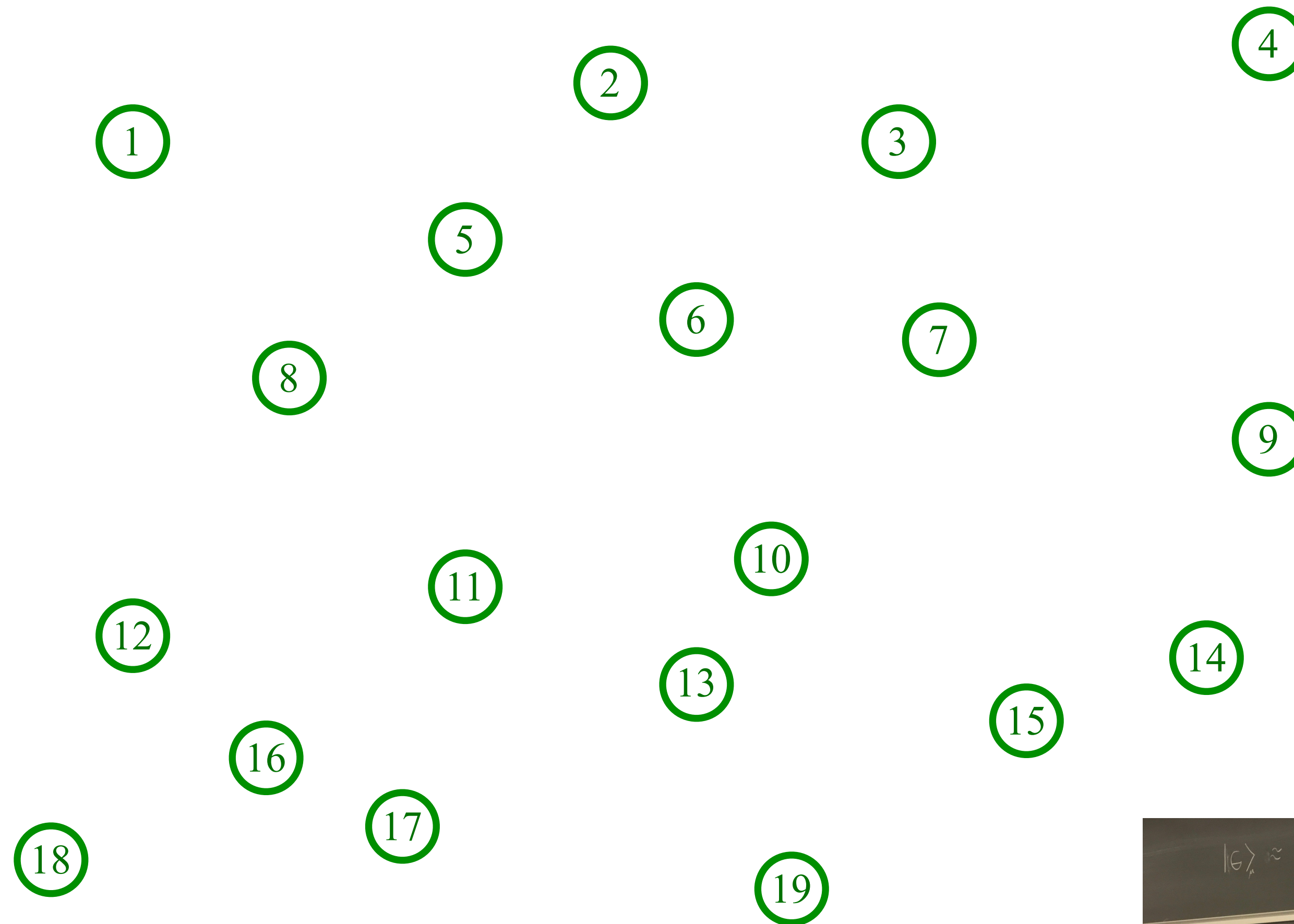
My
spooky
dream*

Ancient
Indian
game of
Snakes
and
Ladders

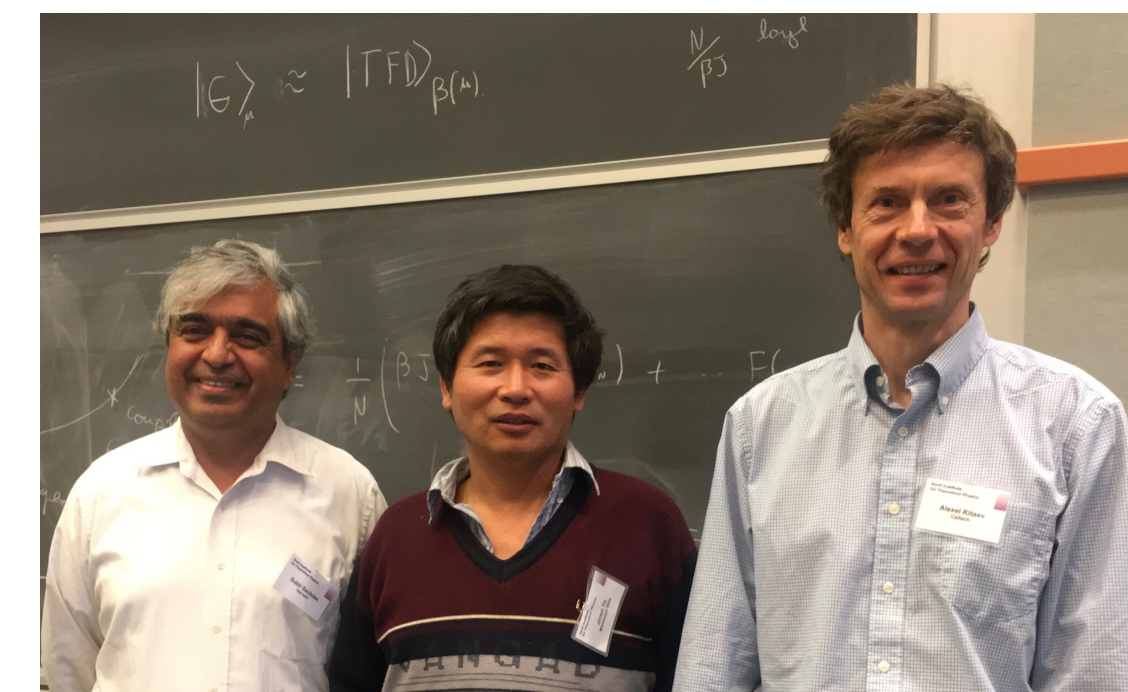
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

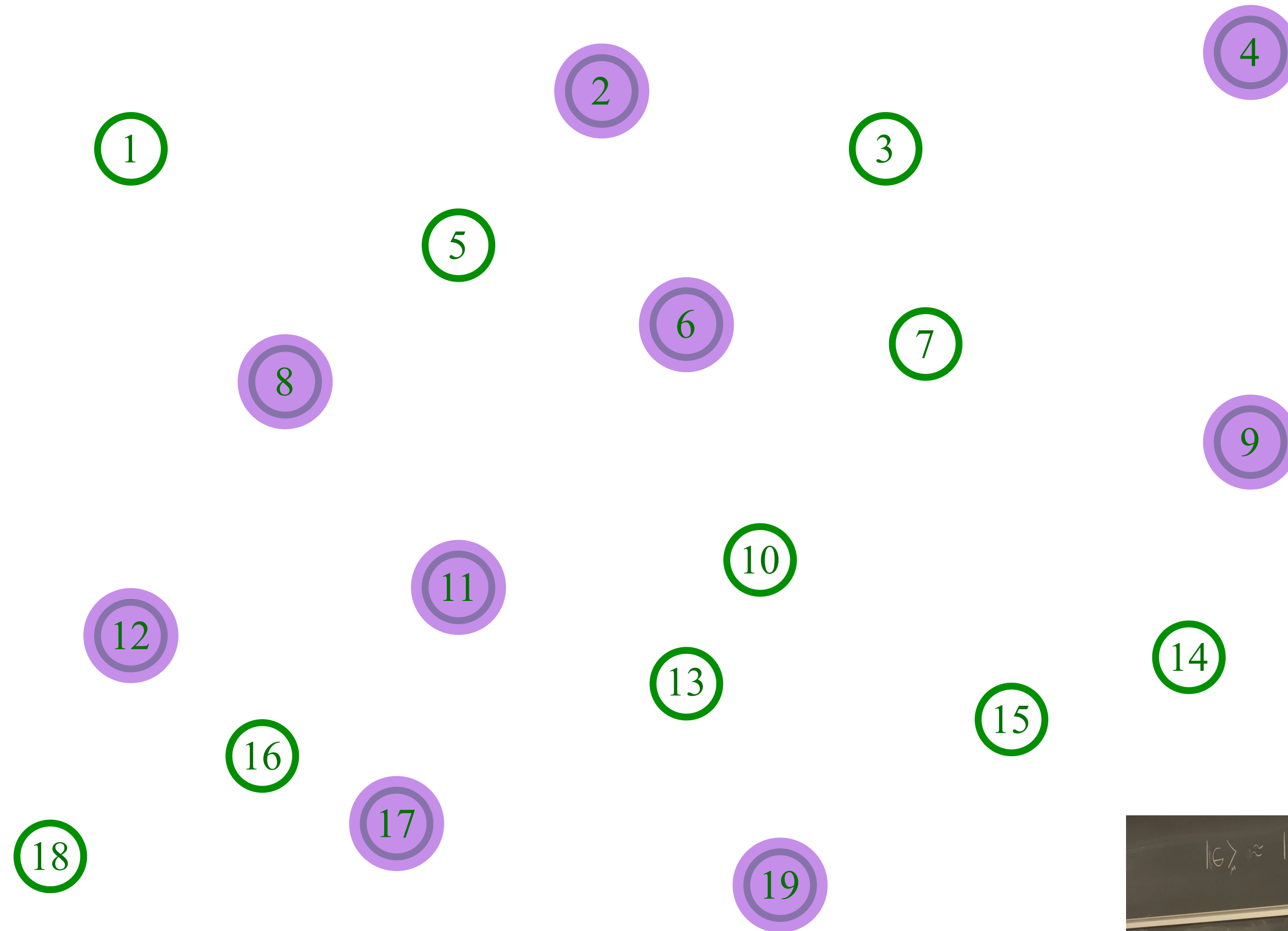


Pick a set of random positions

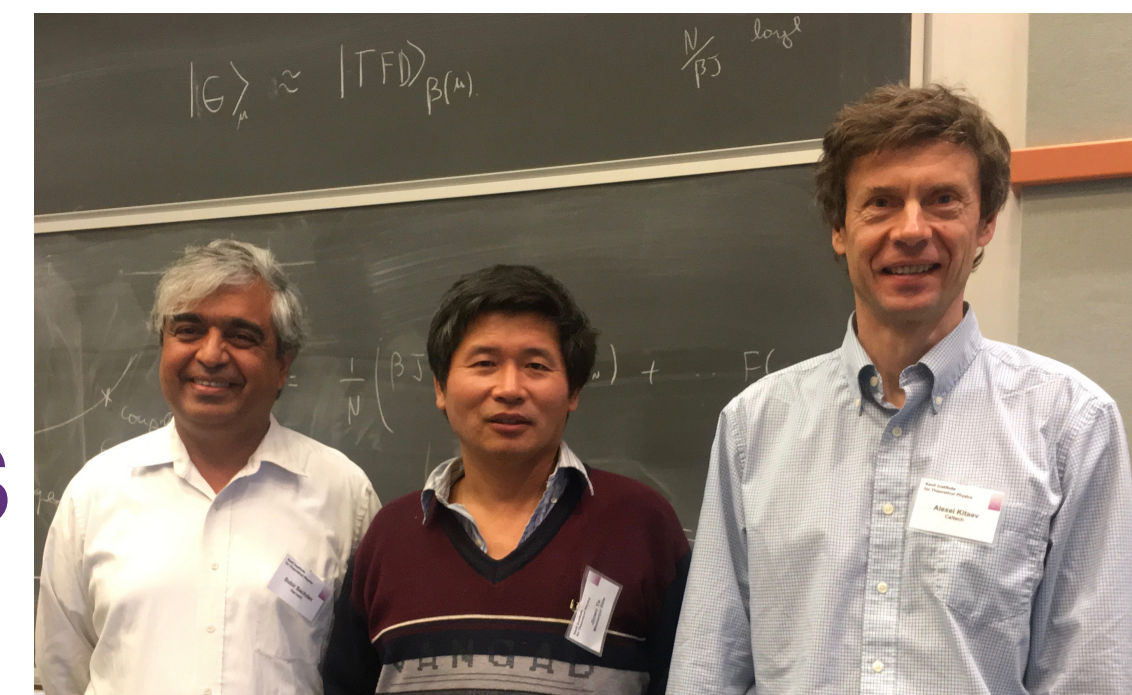


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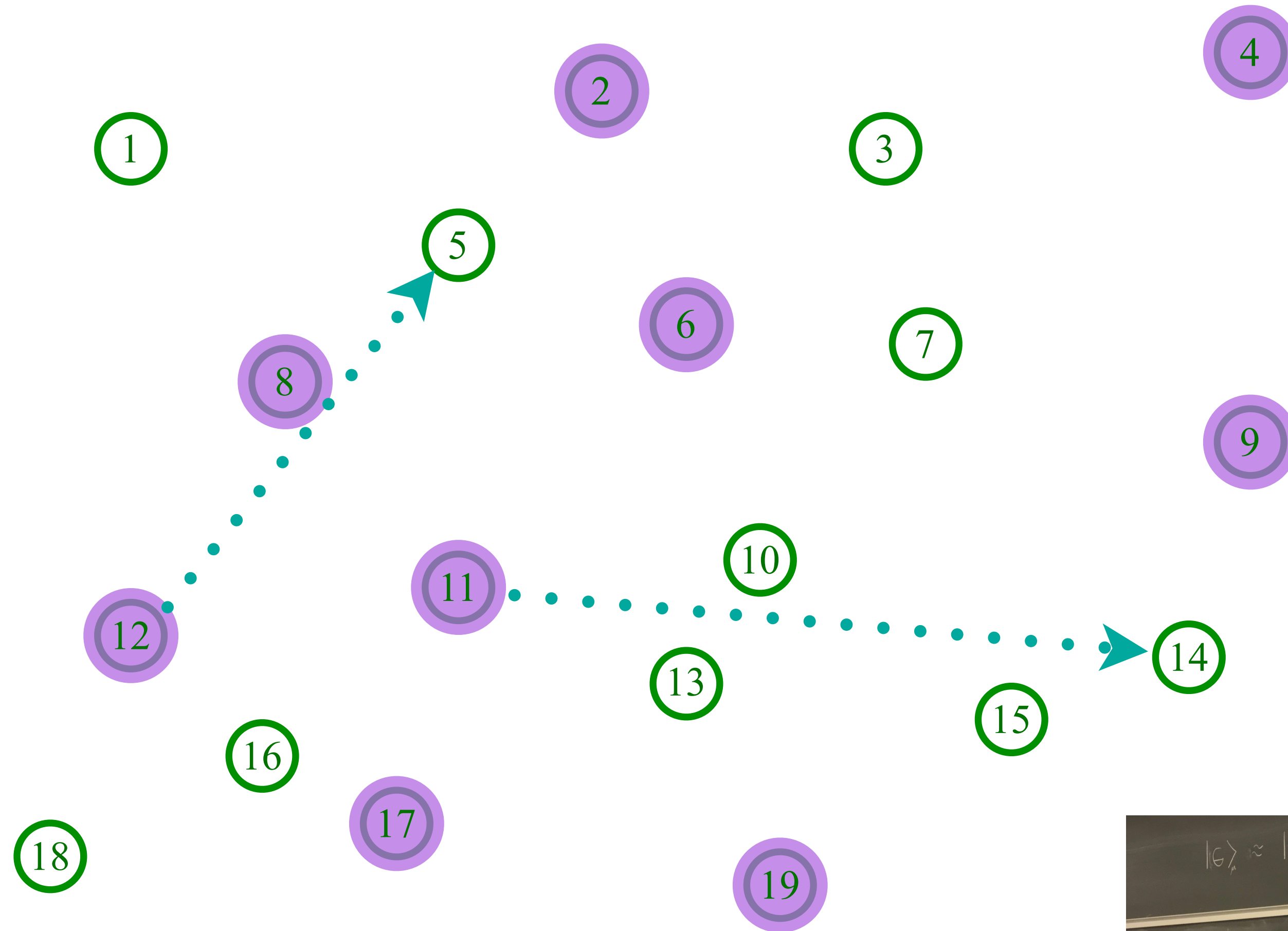
Place electrons randomly on some sites



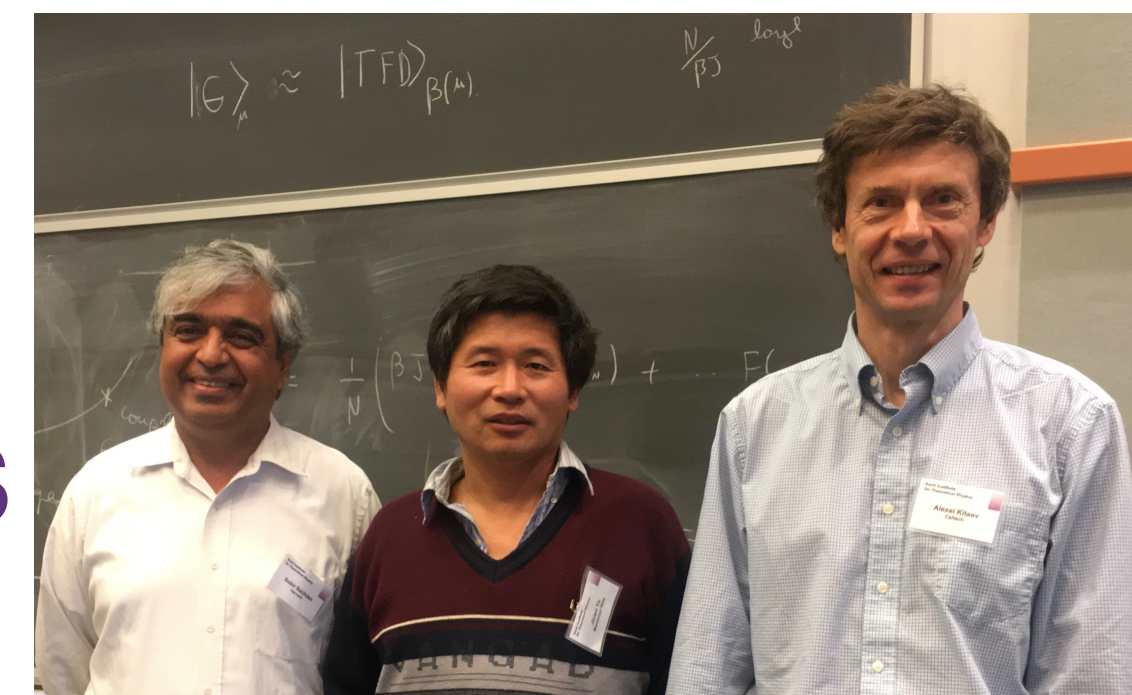
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



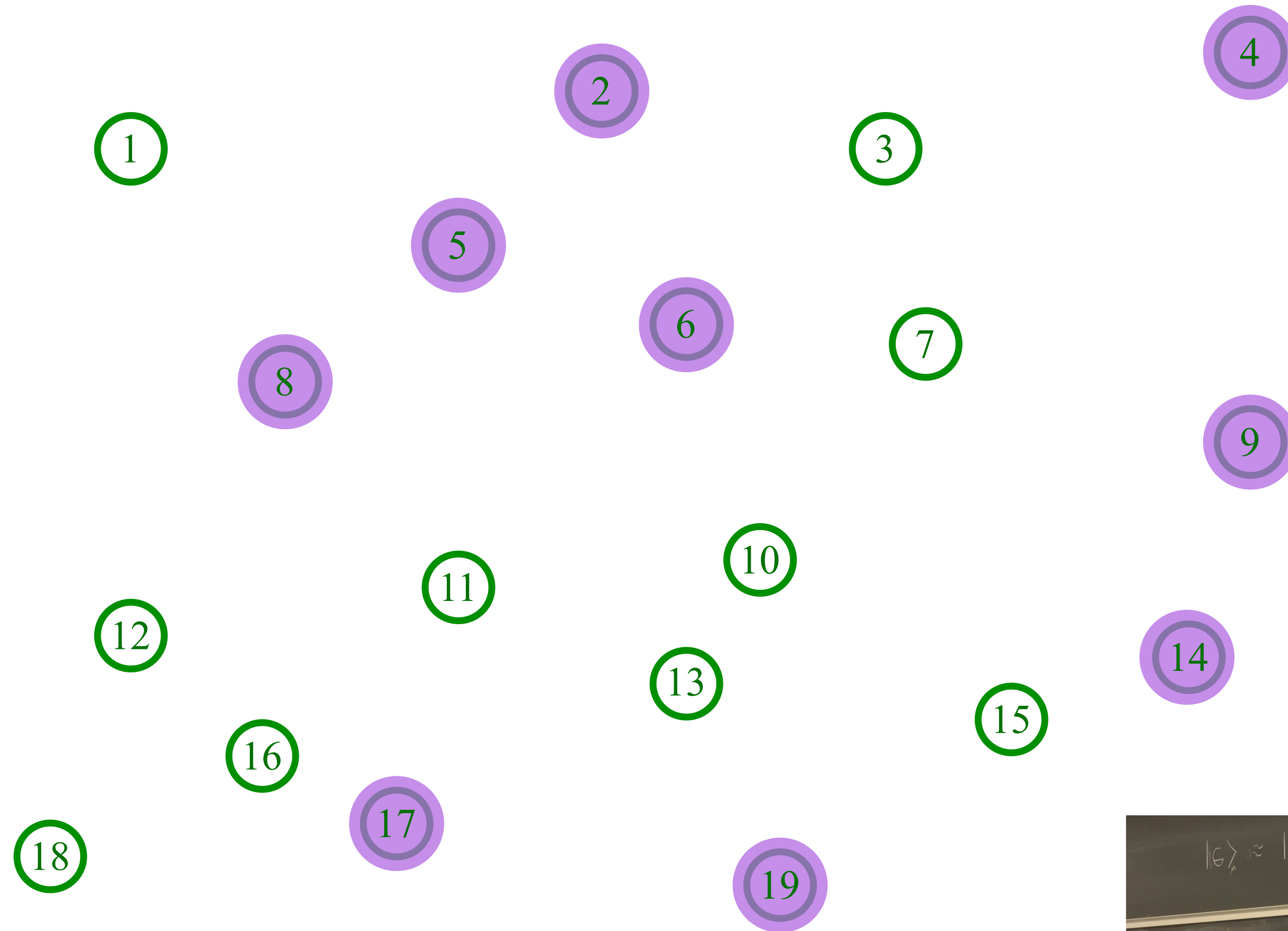
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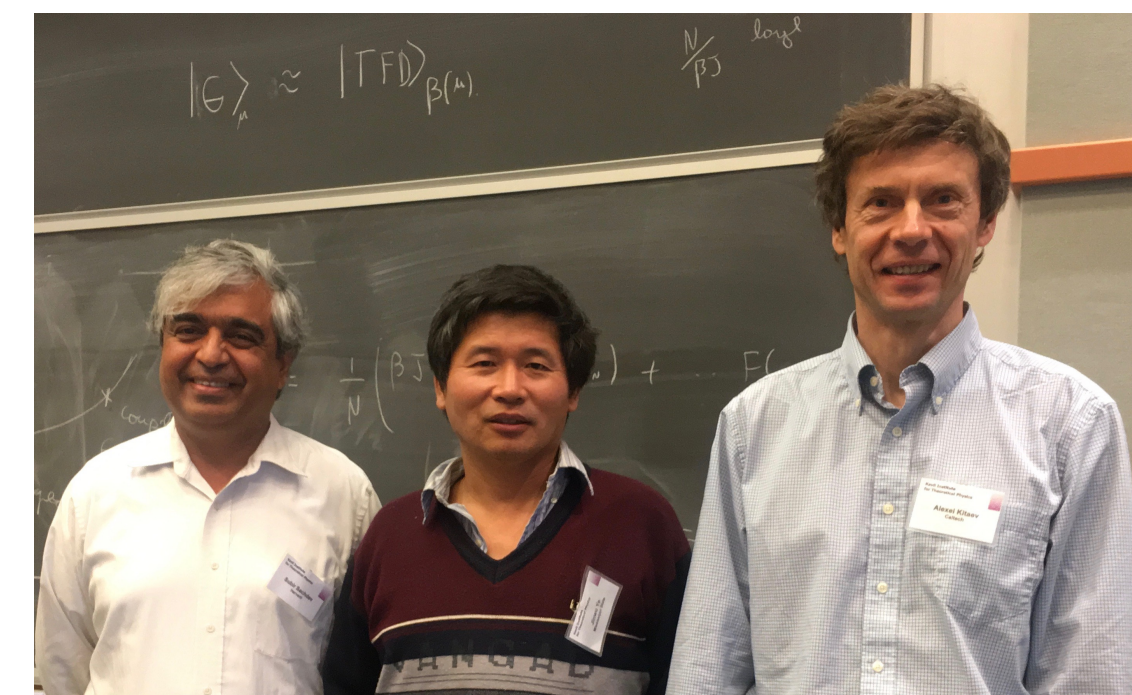
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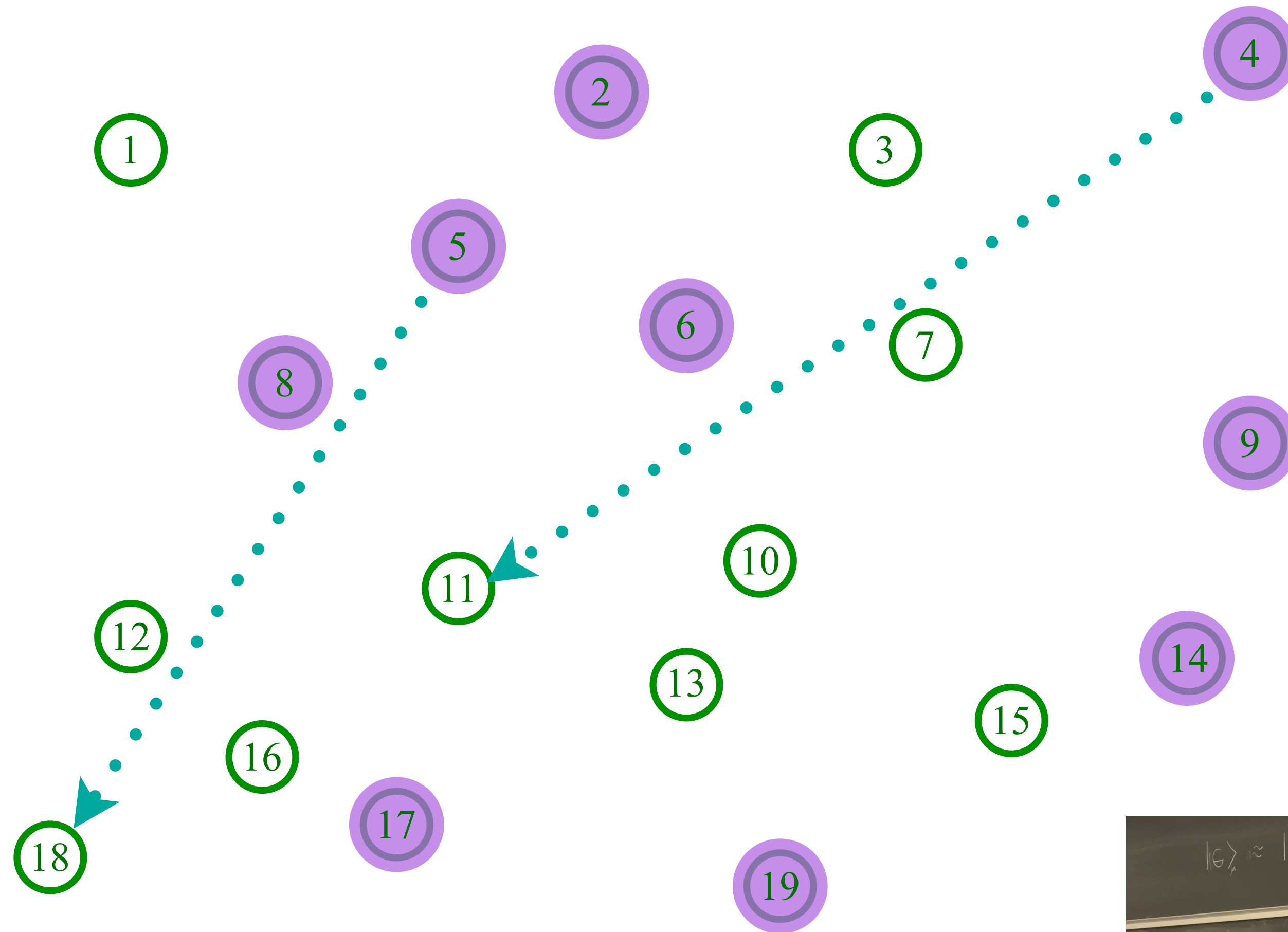
Entangle electrons pairwise randomly



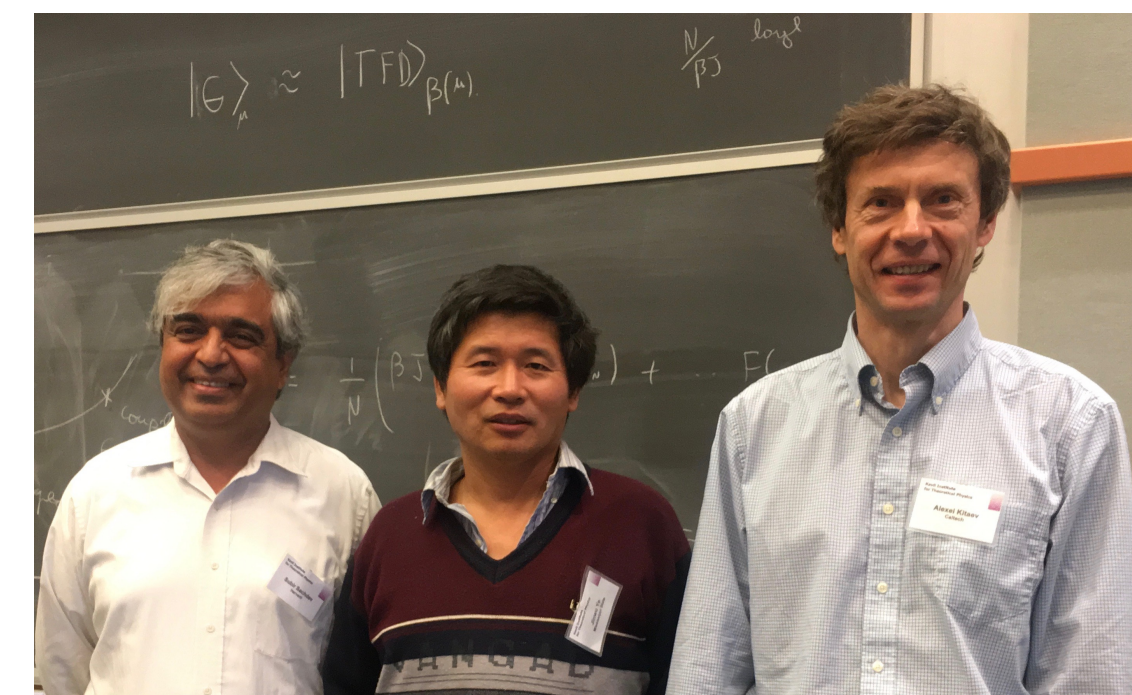
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



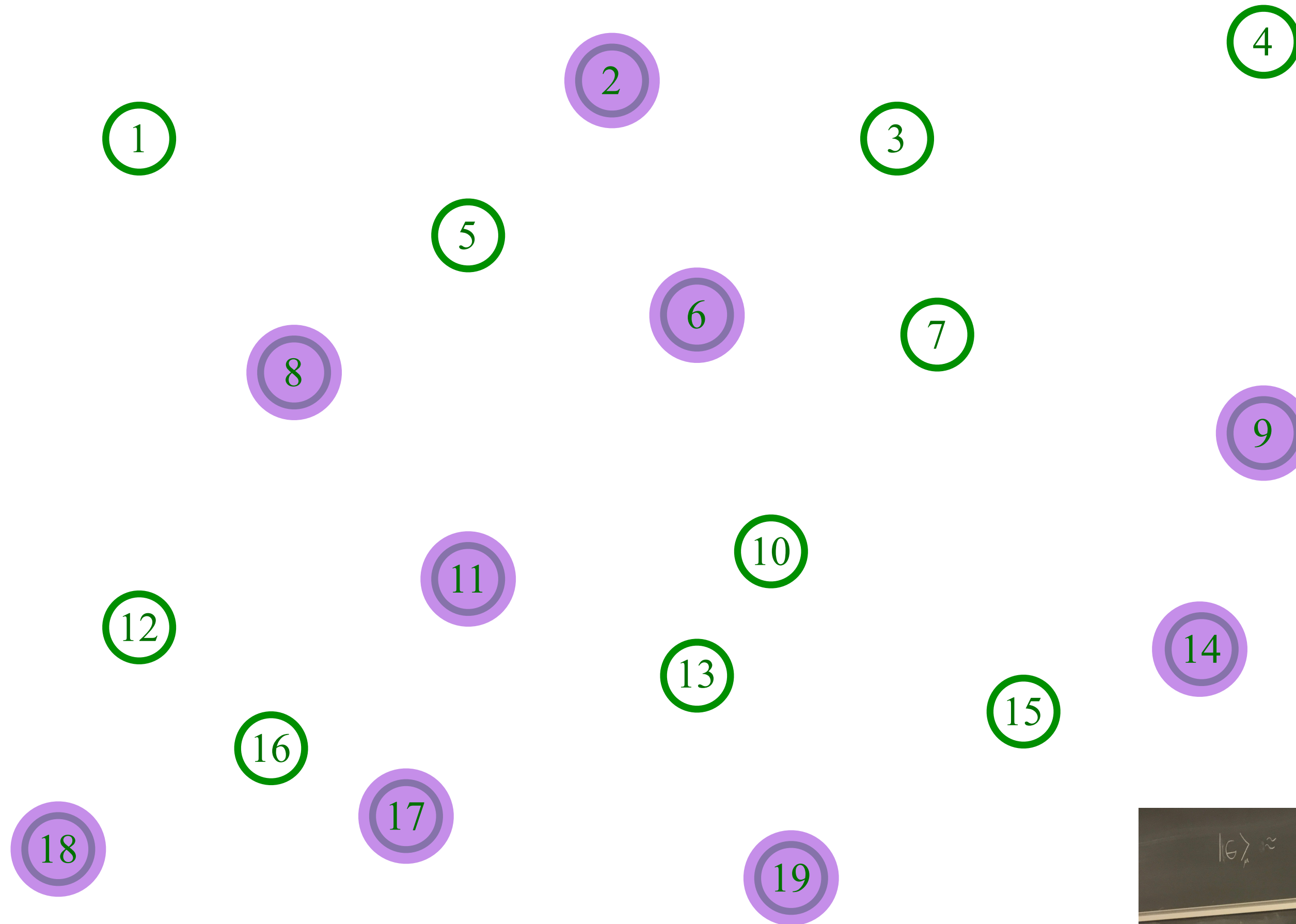
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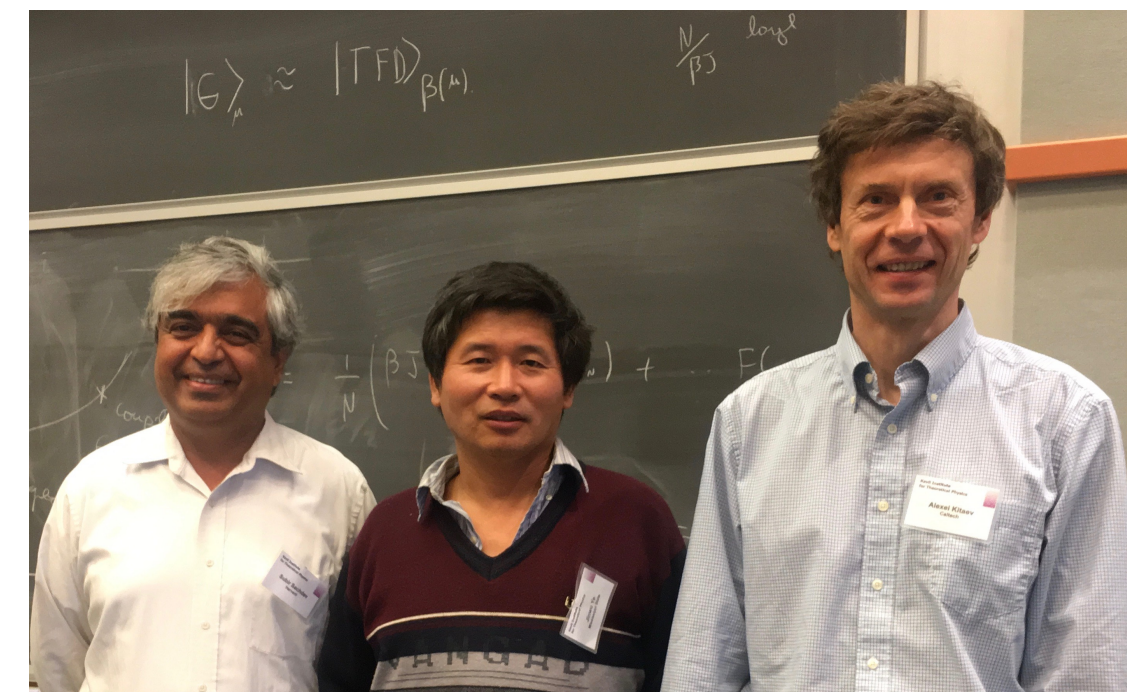
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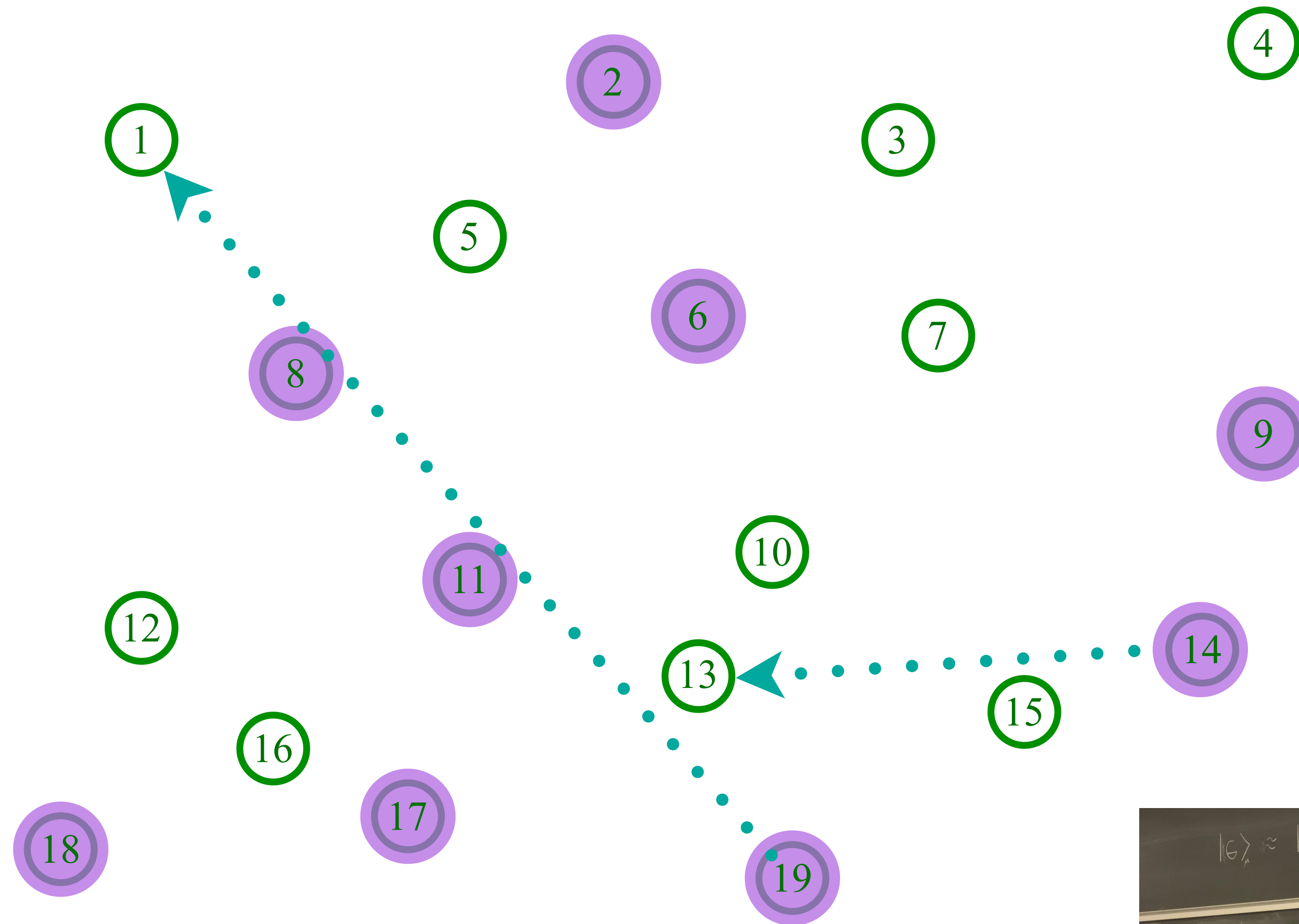
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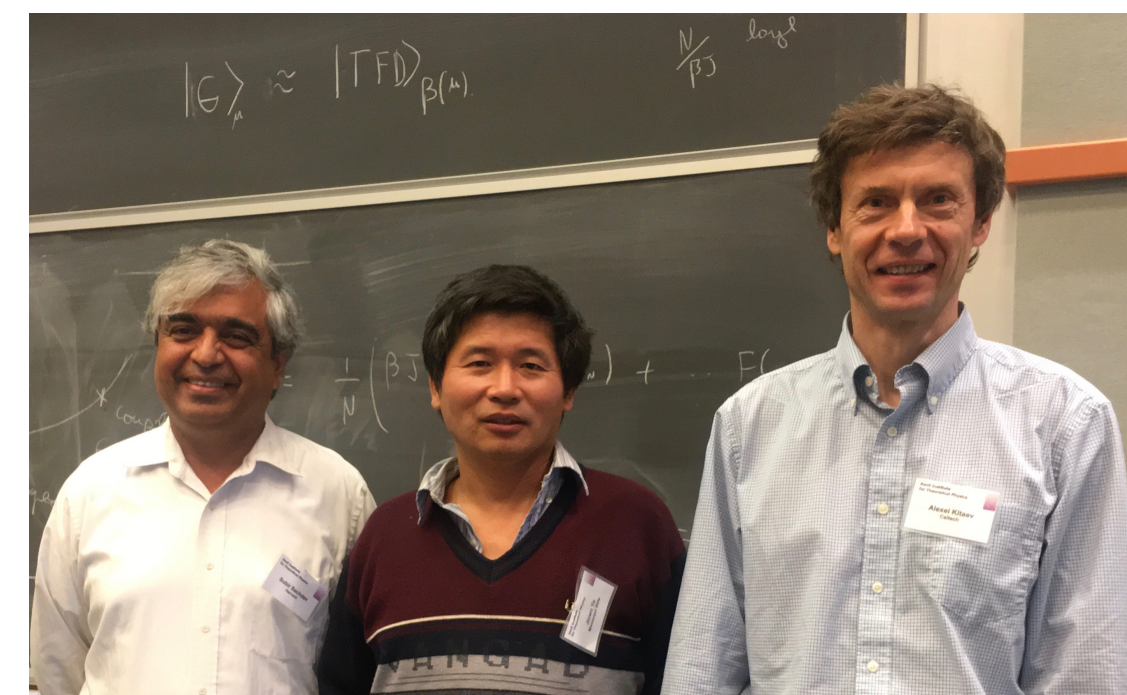
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



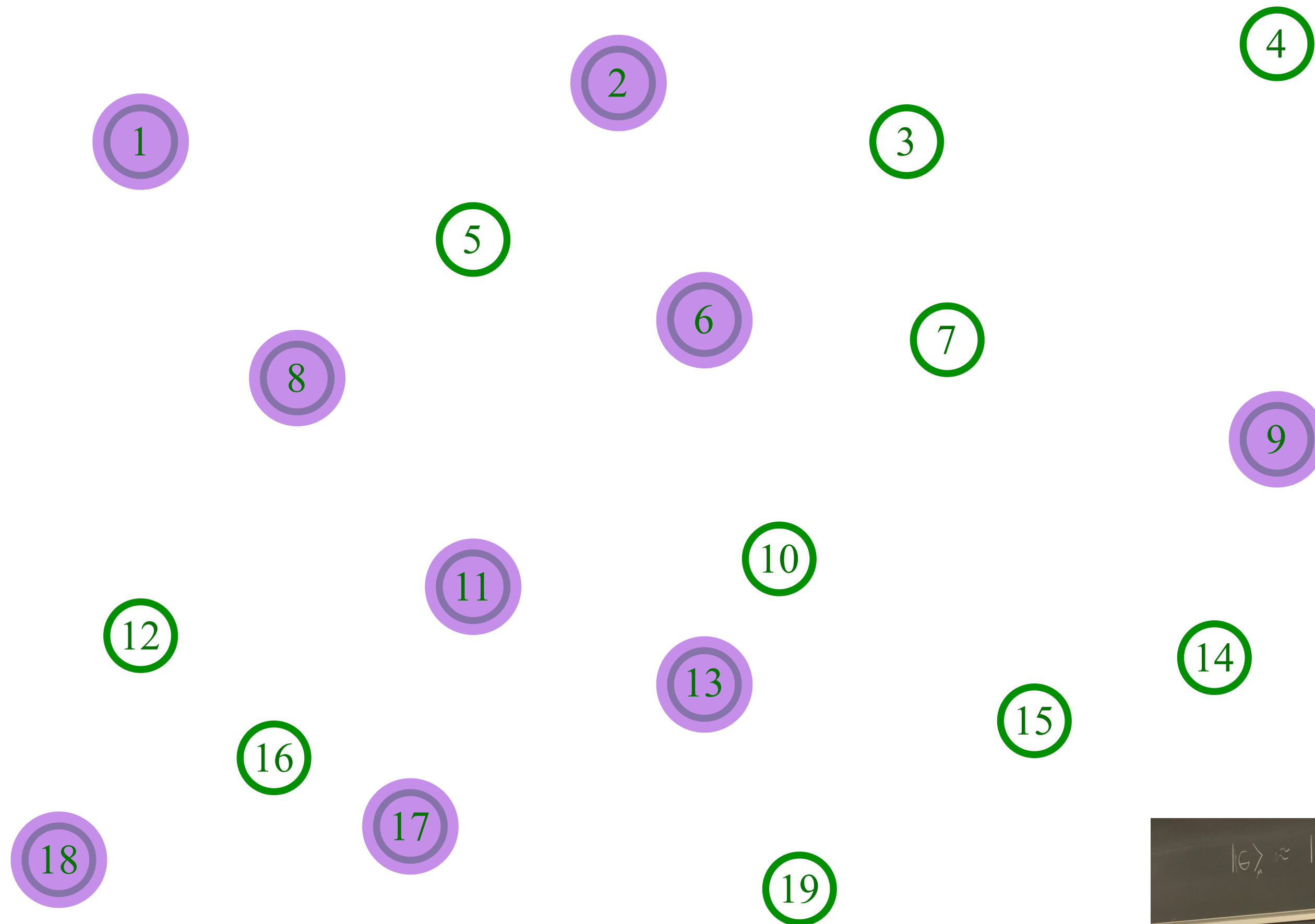
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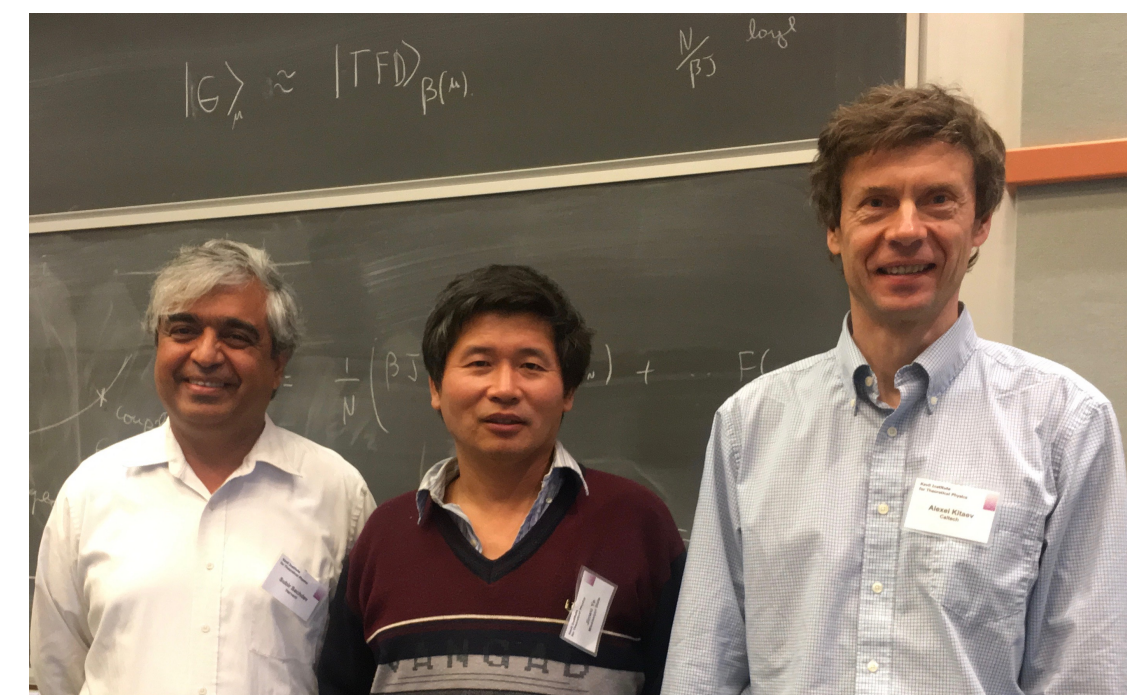
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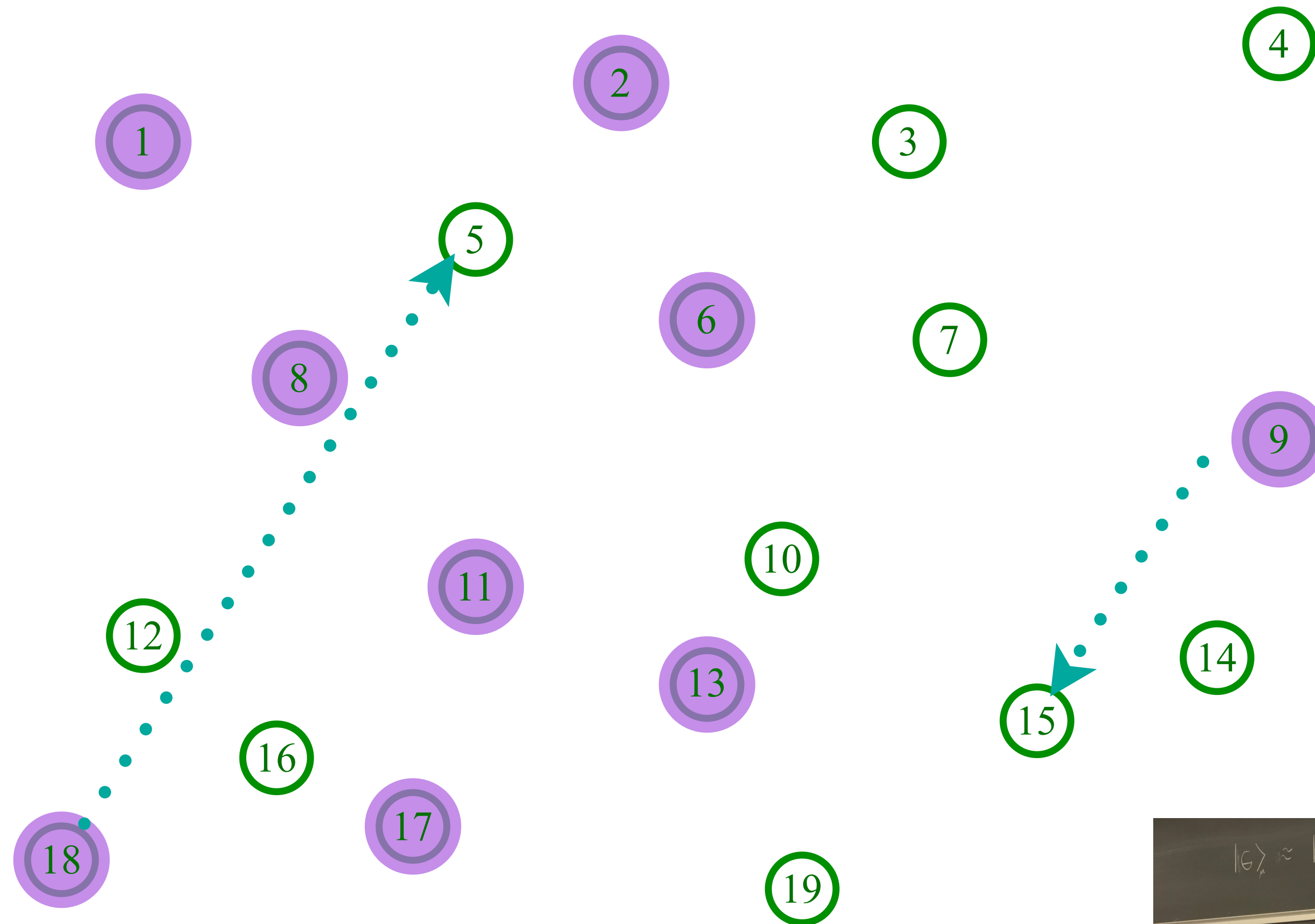
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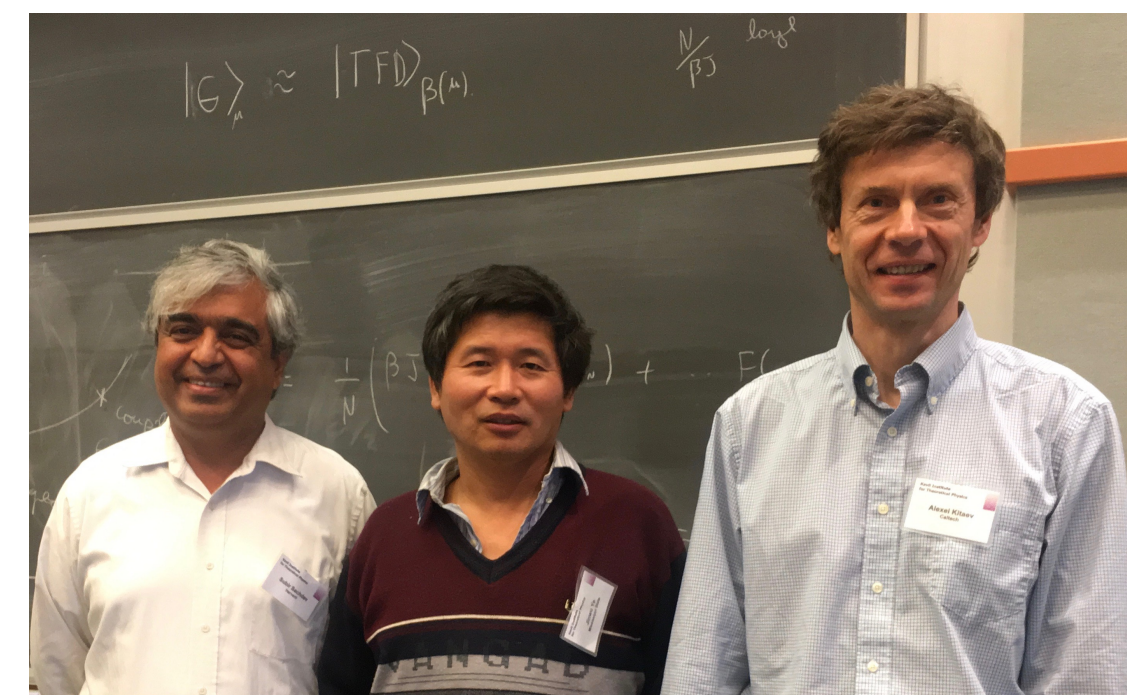
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$$U_{9,18;5,15}$$



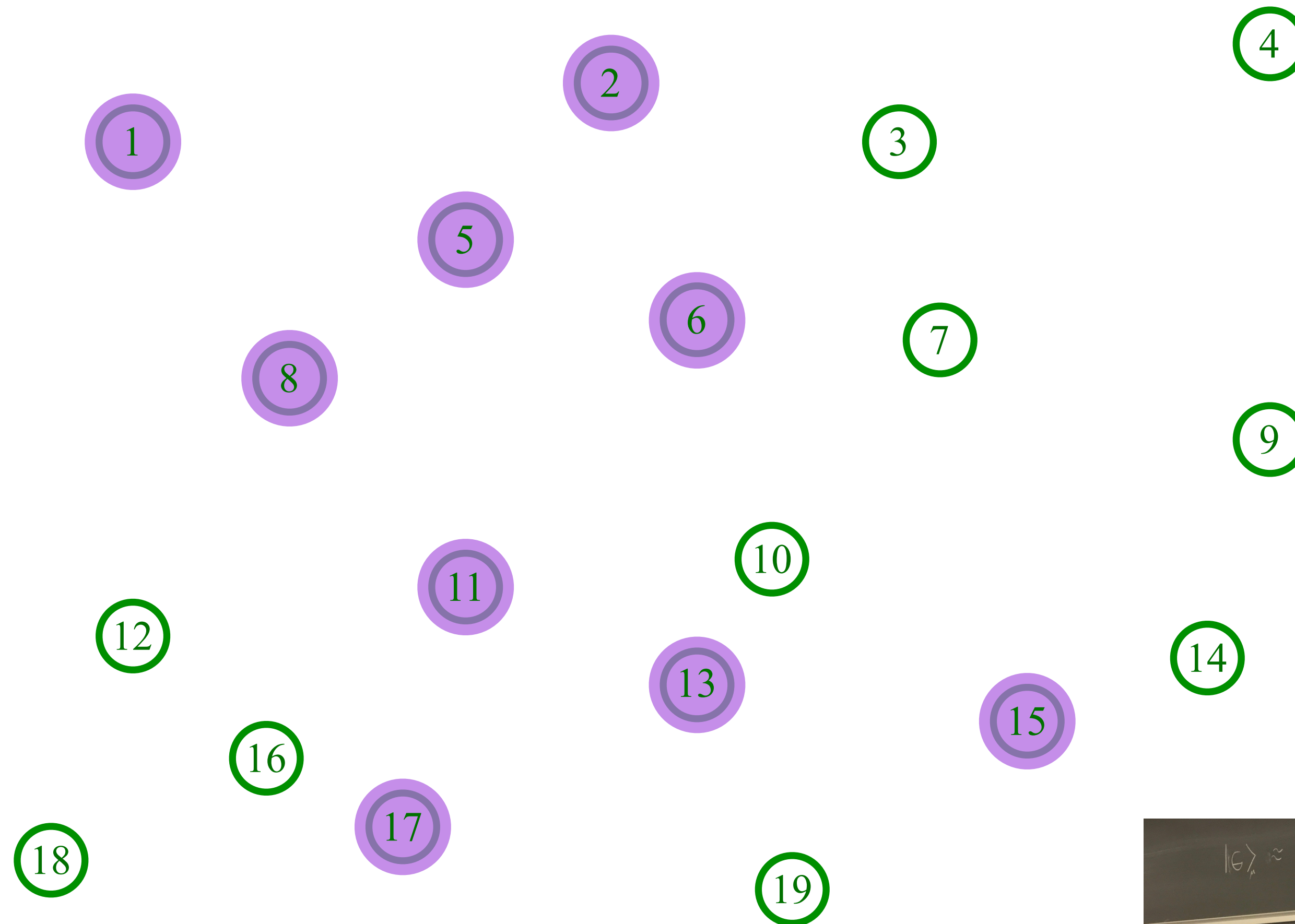
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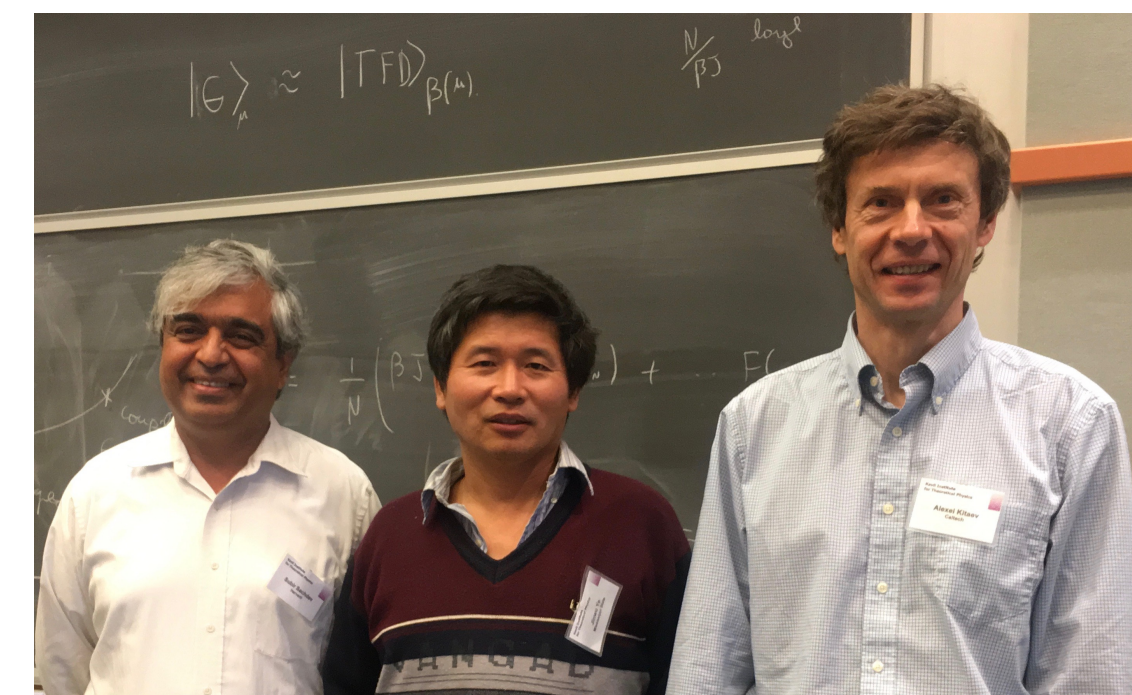
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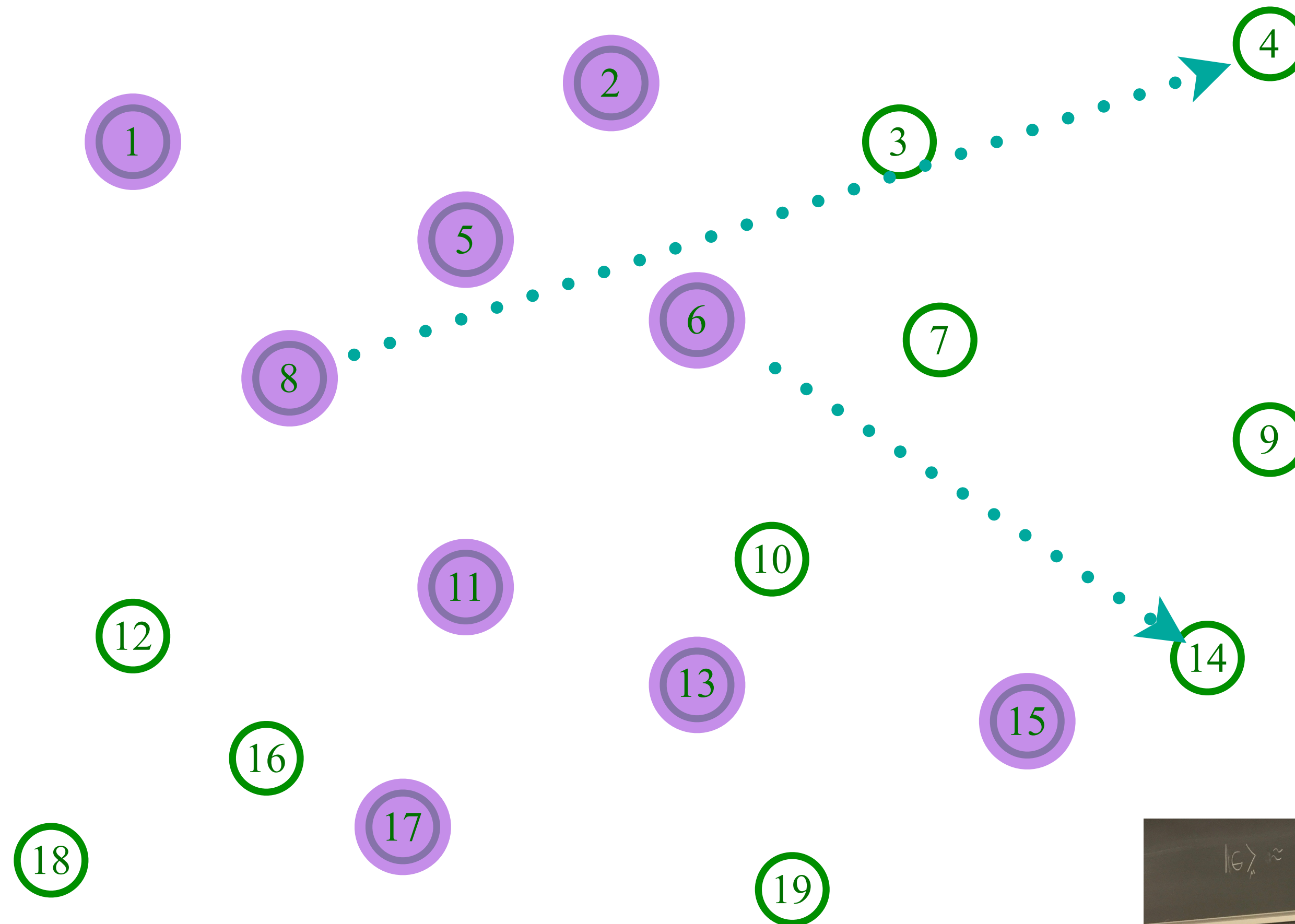
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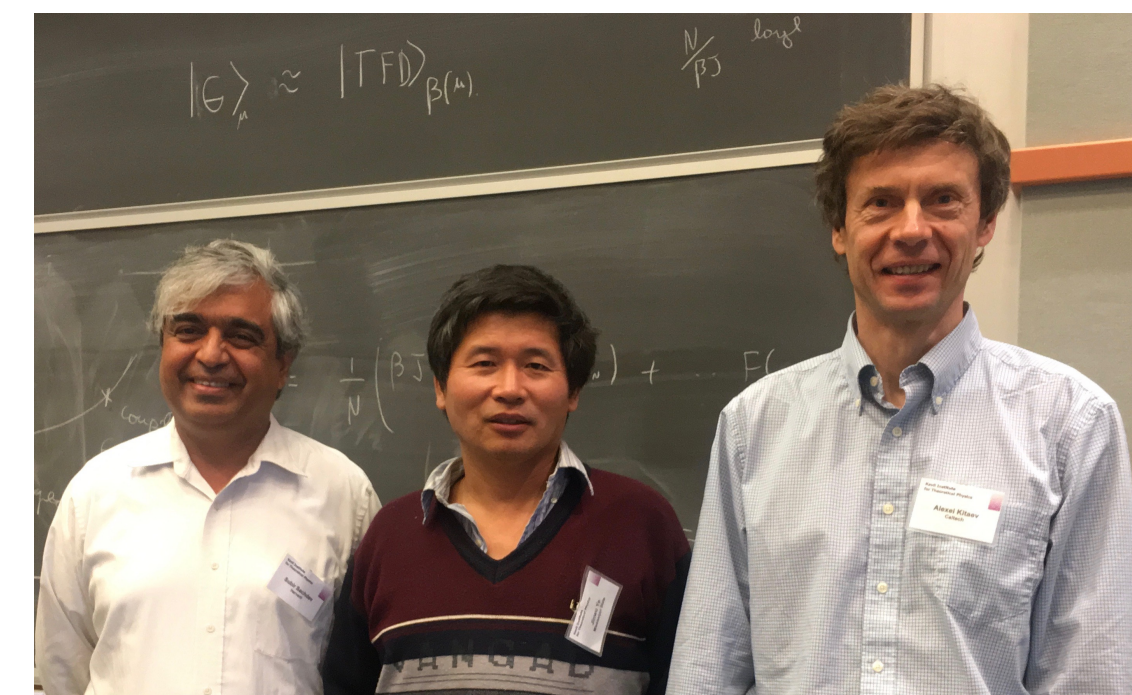
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$$U_{6,8;4,14}$$



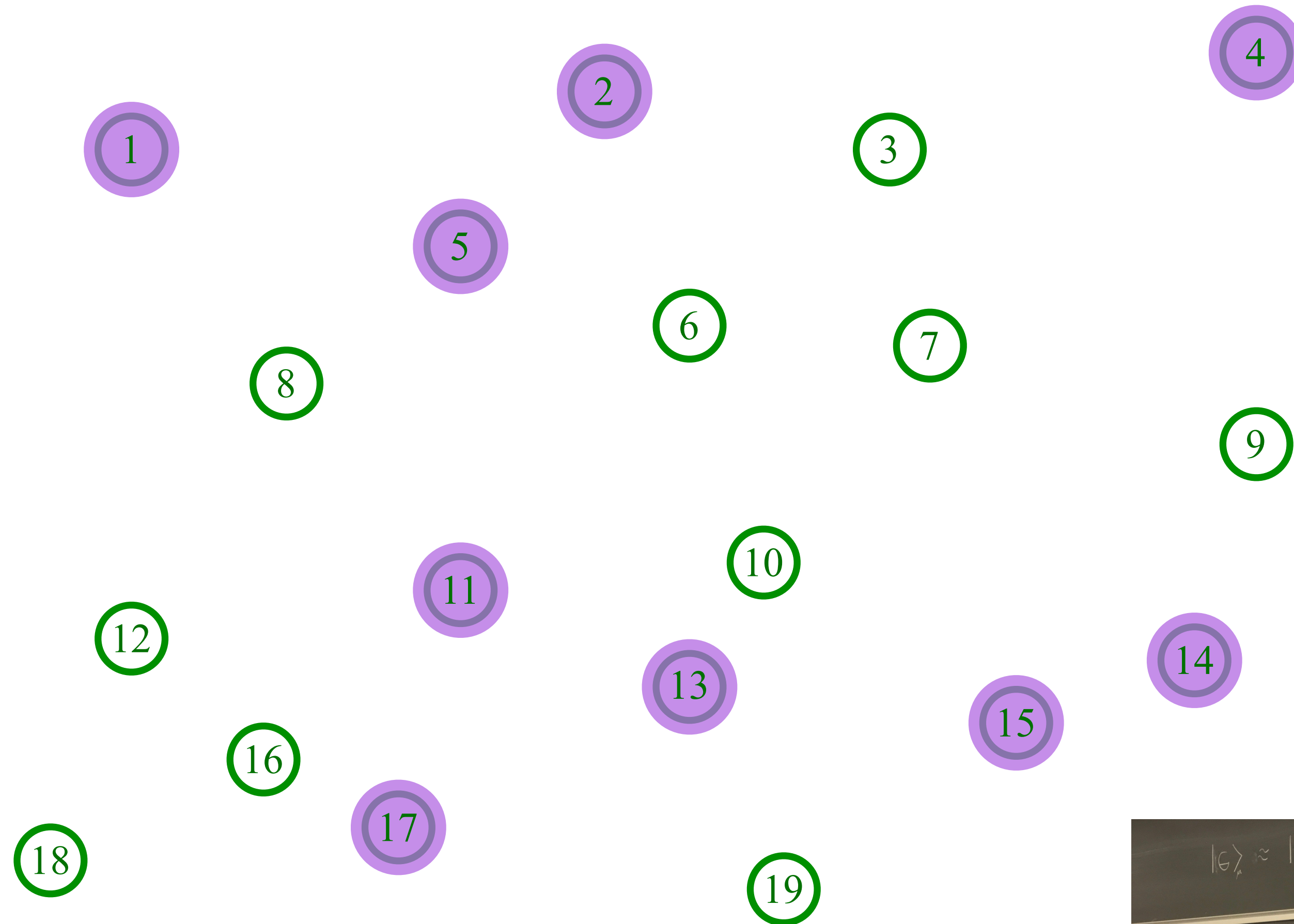
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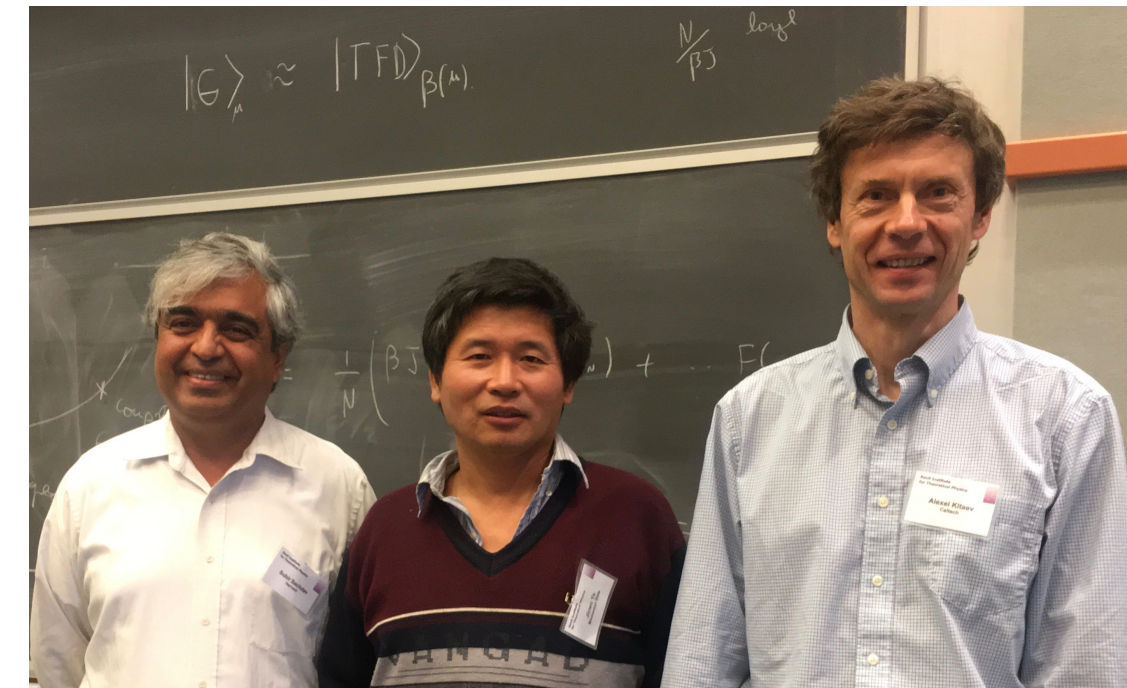
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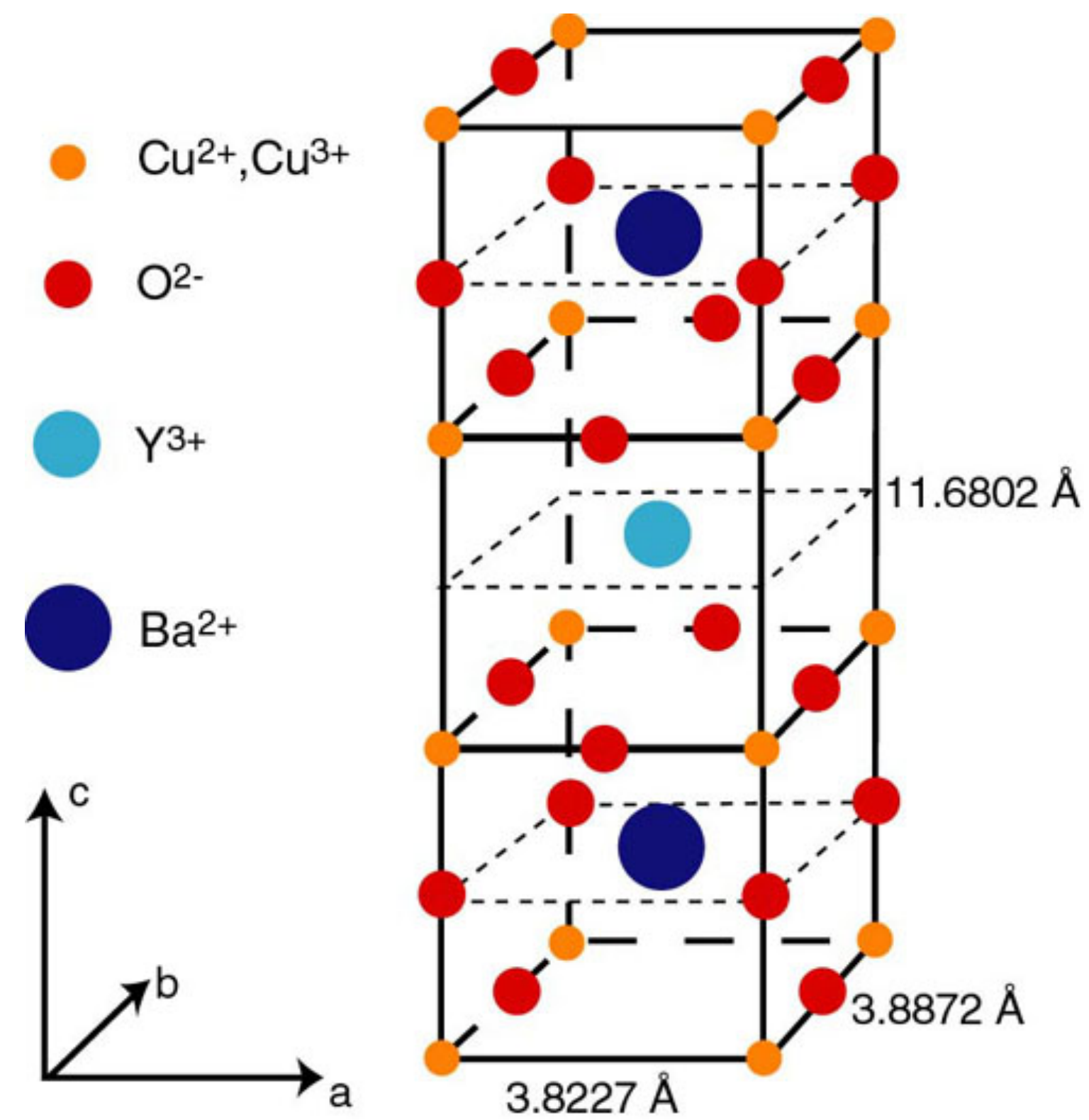
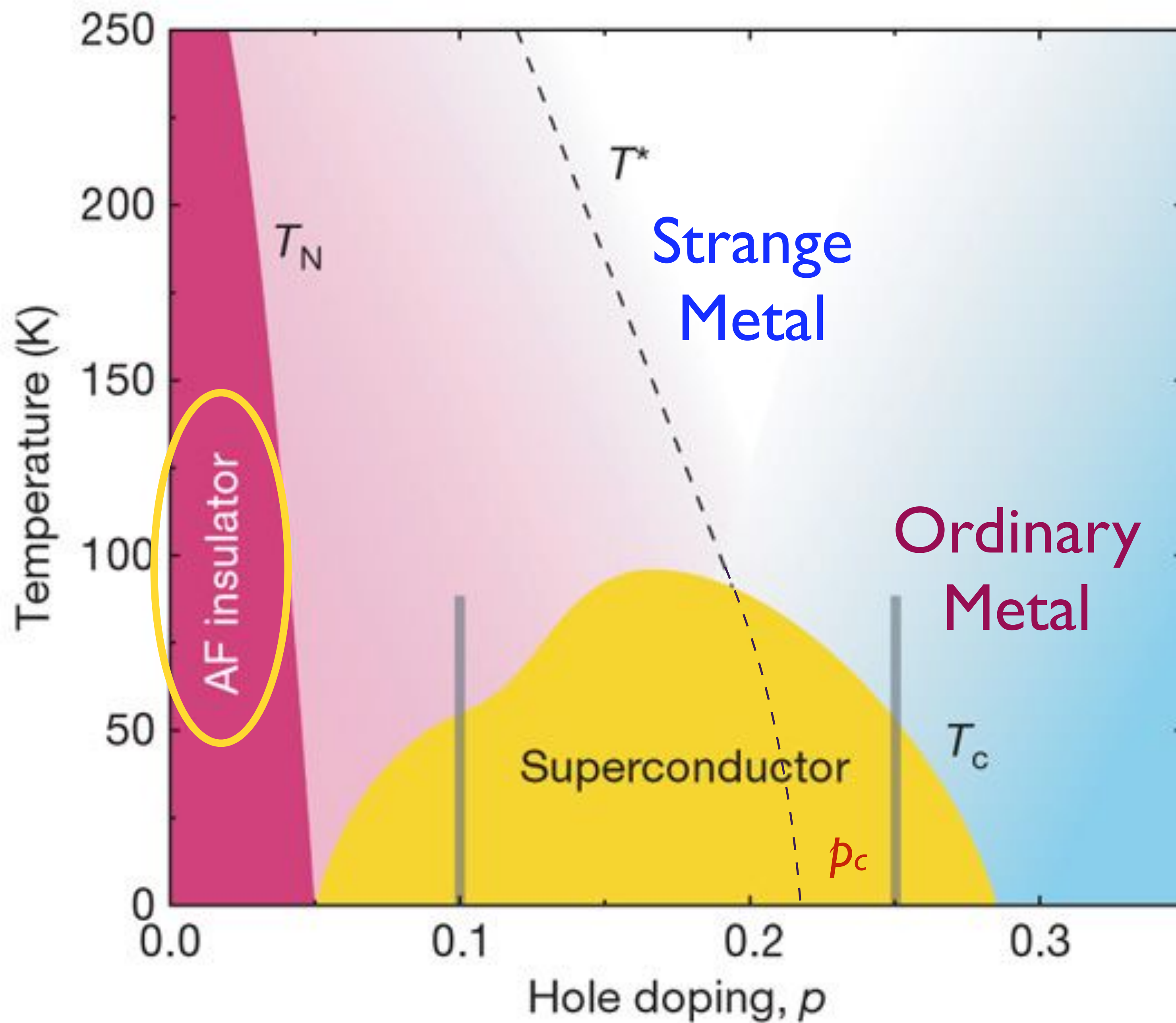
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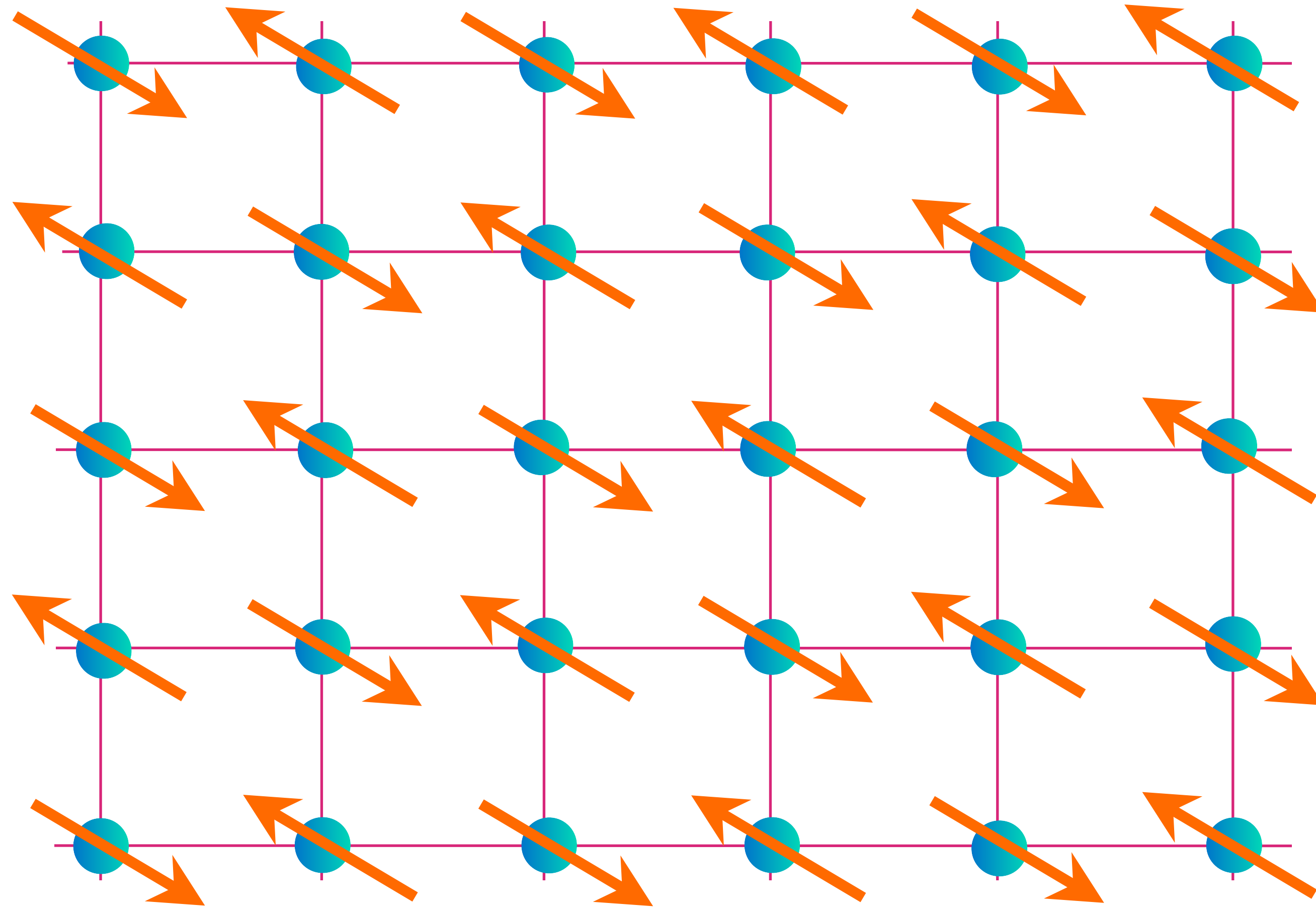
A solvable model of multi-particle
quantum entanglement.

Yields a metal in which current is carried
not by individual electrons,
but by an entangled “quantum soup”

Quantum entanglement and superconductivity

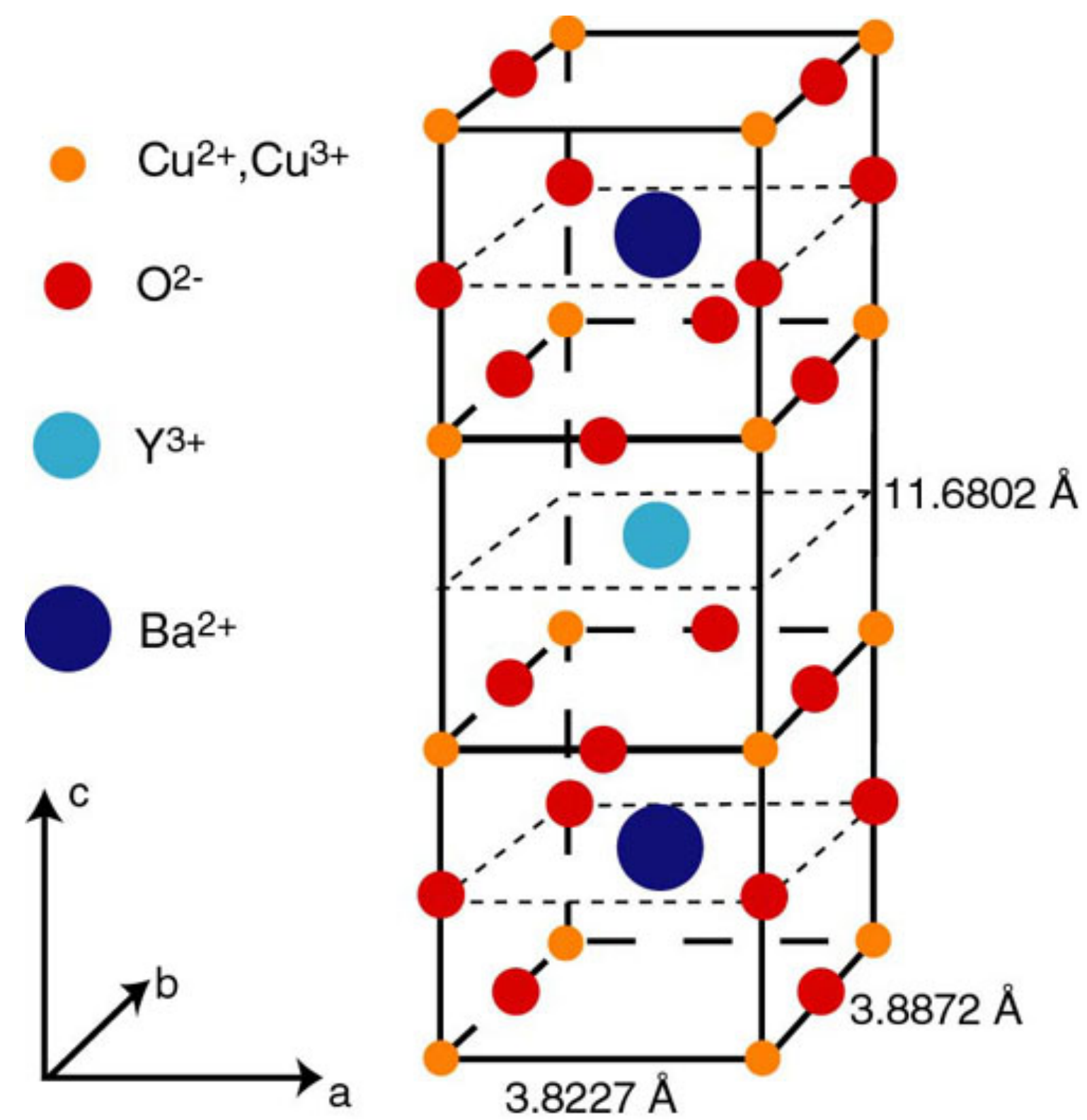
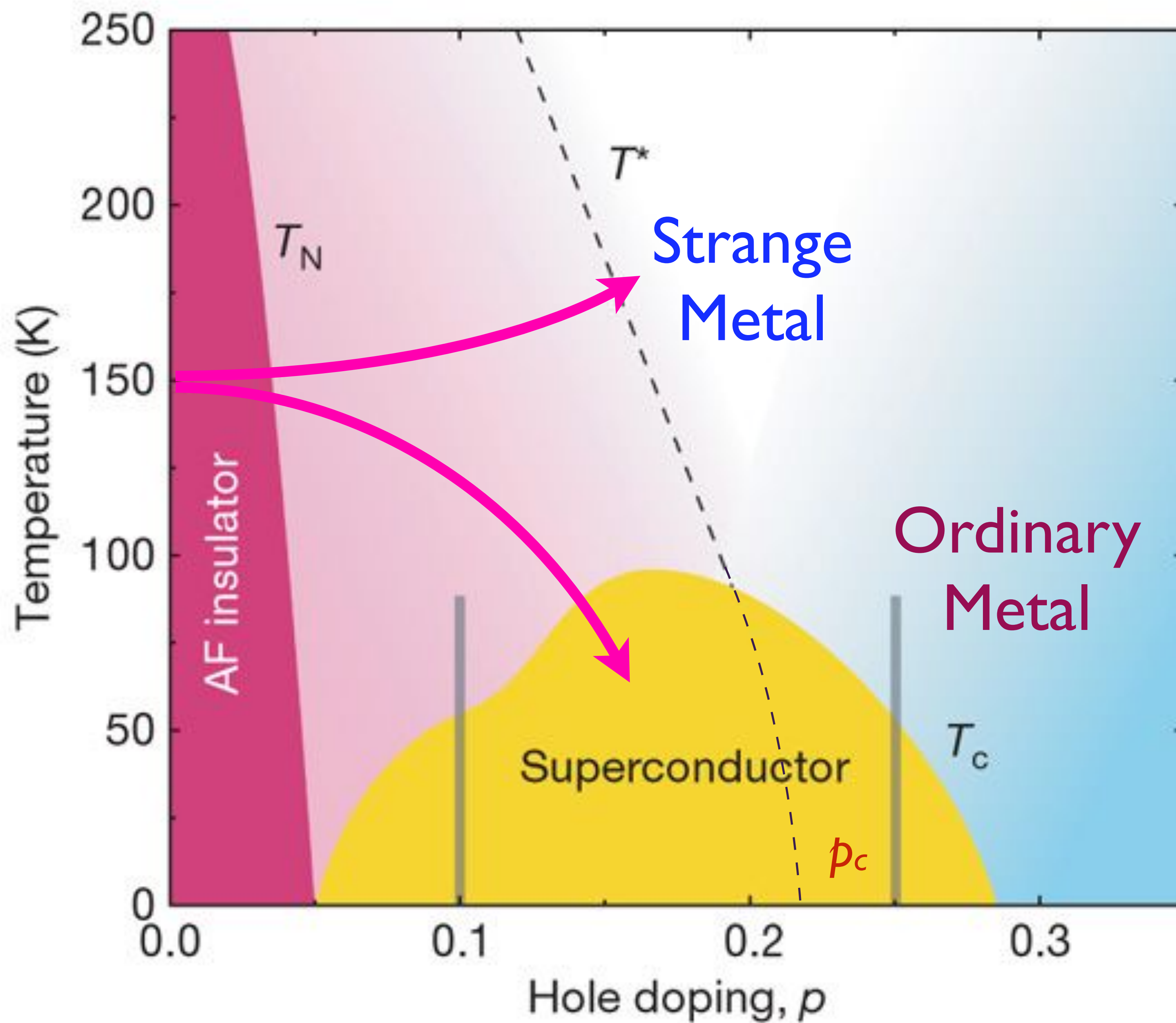


The dance of electrons on Cu atoms in YBCO

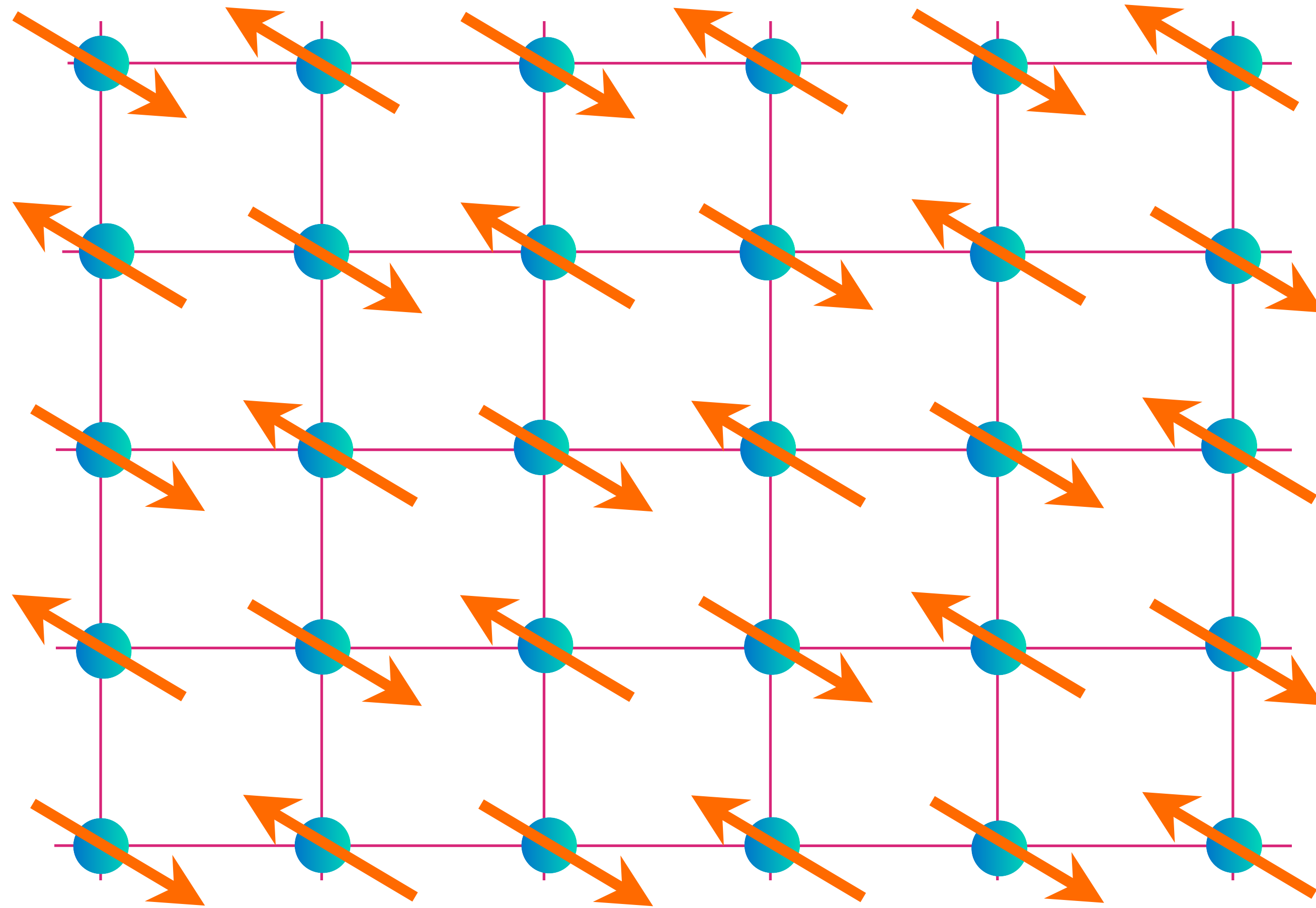


Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins



The dance of electrons on Cu atoms in YBCO



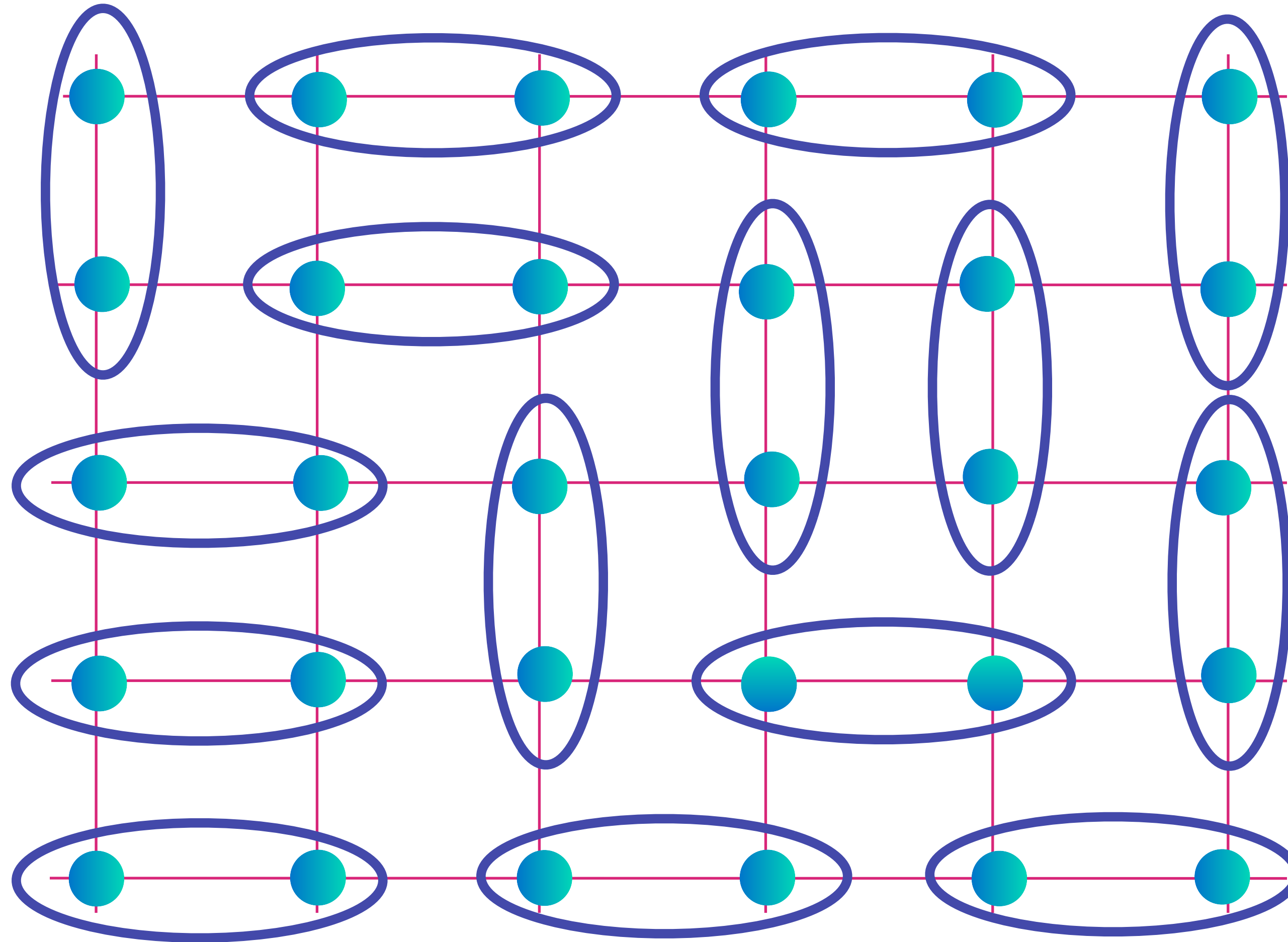
Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



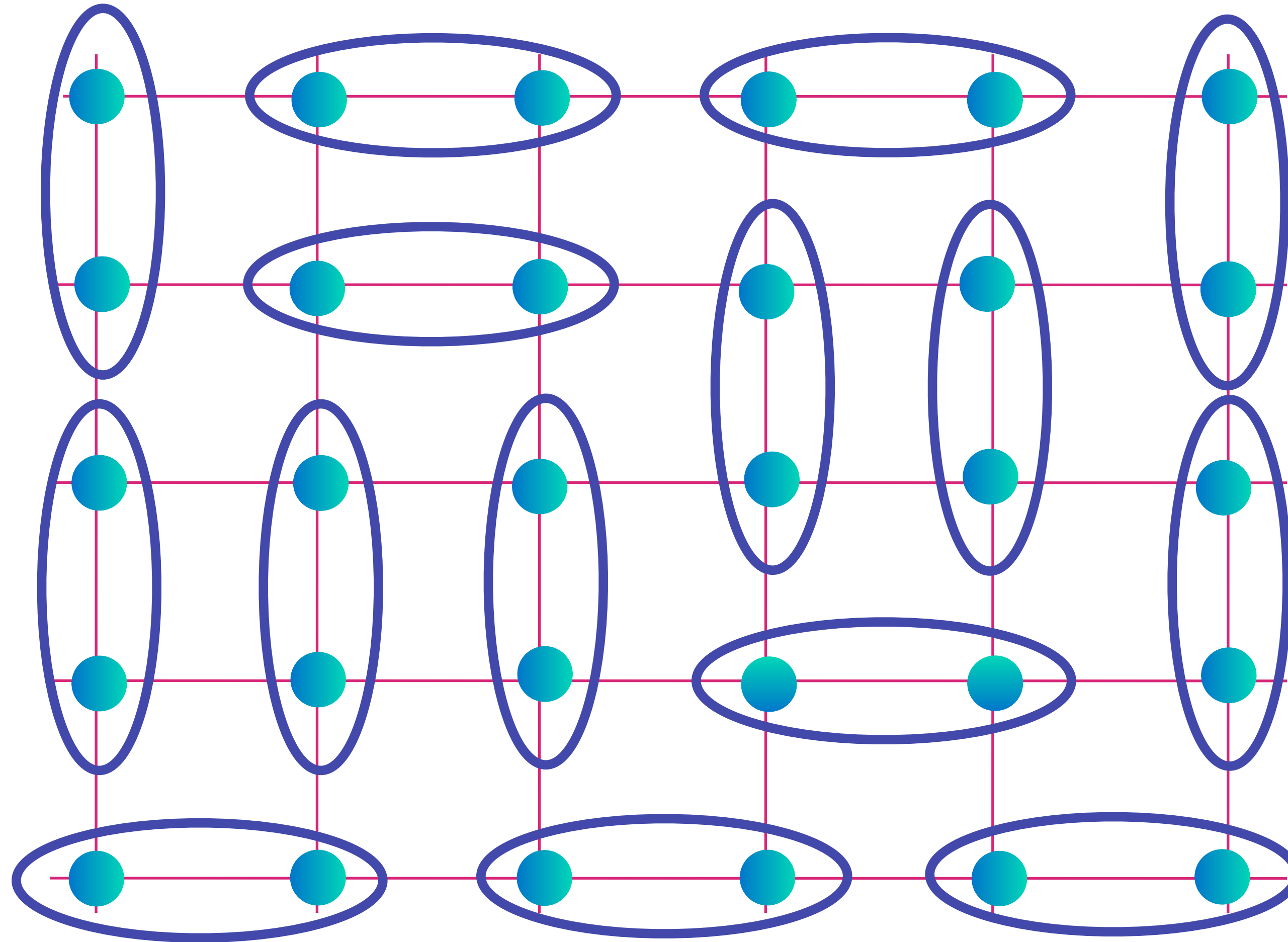
Electrons form entangled pairs, and the pairs entangle across the entire sample

$$\text{[Diagram of two electrons in an oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



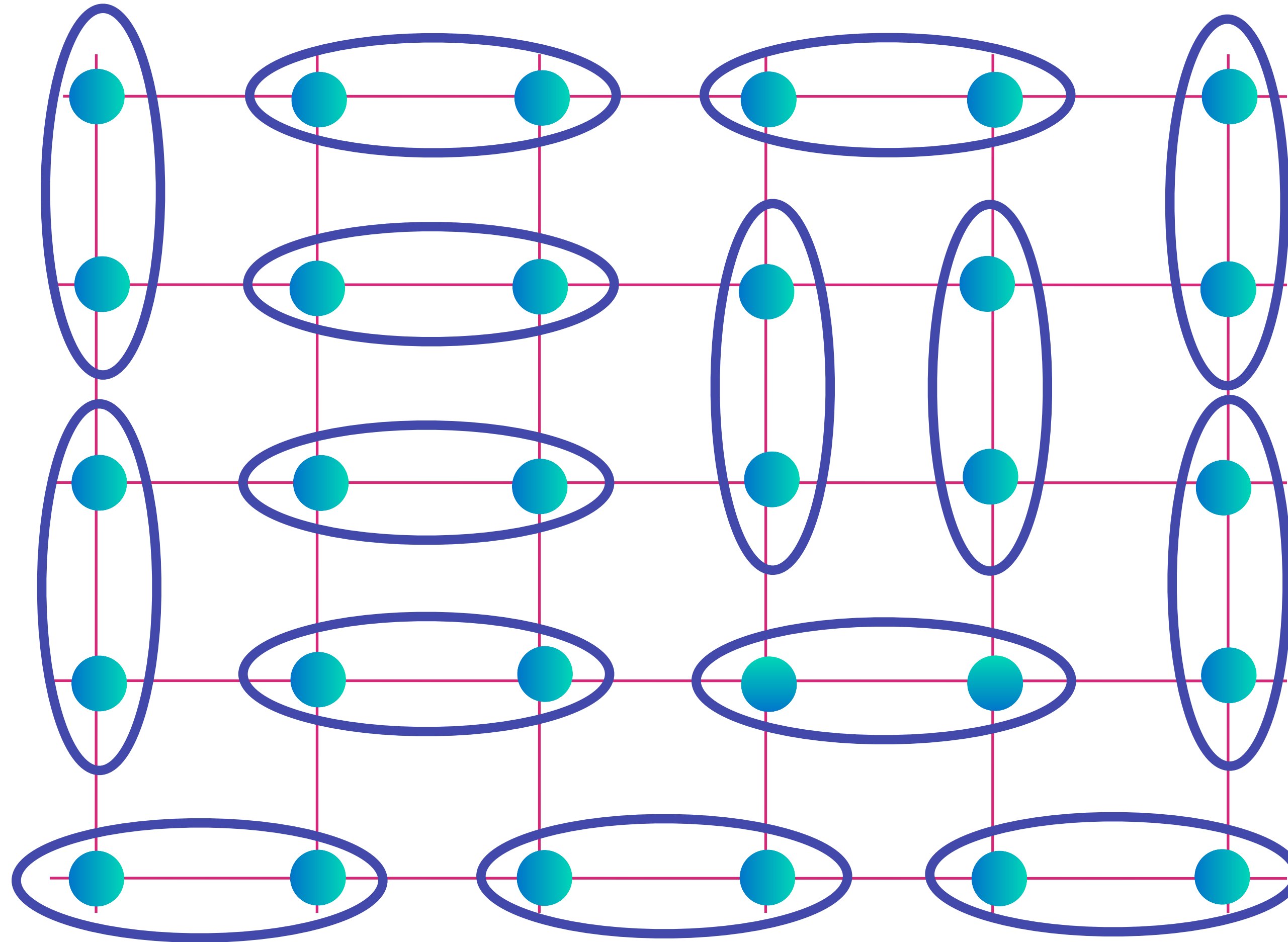
Electrons form entangled pairs, and the pairs entangle across the entire sample

$$\text{[Diagram of two electrons in a pair]} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



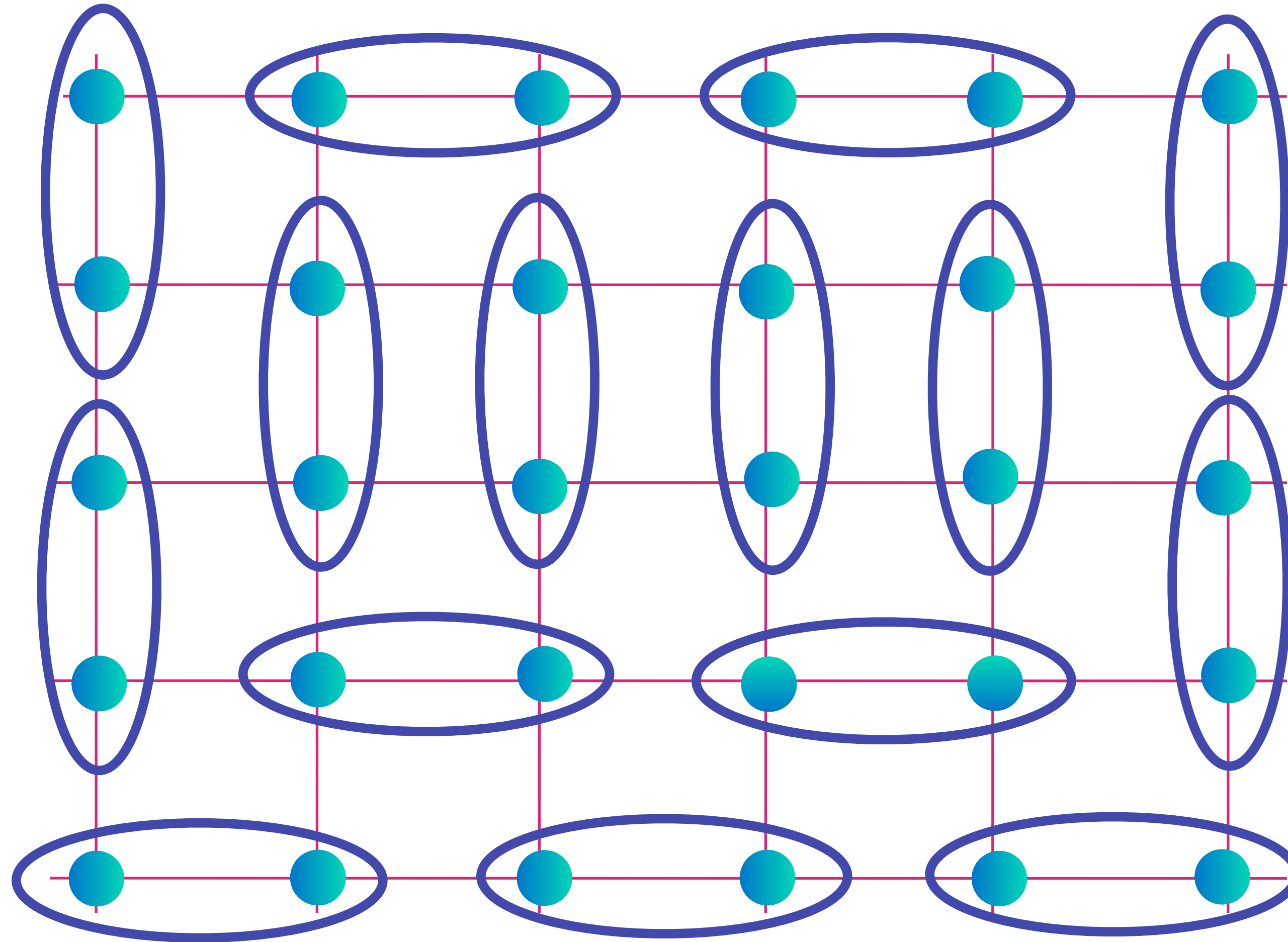
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The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



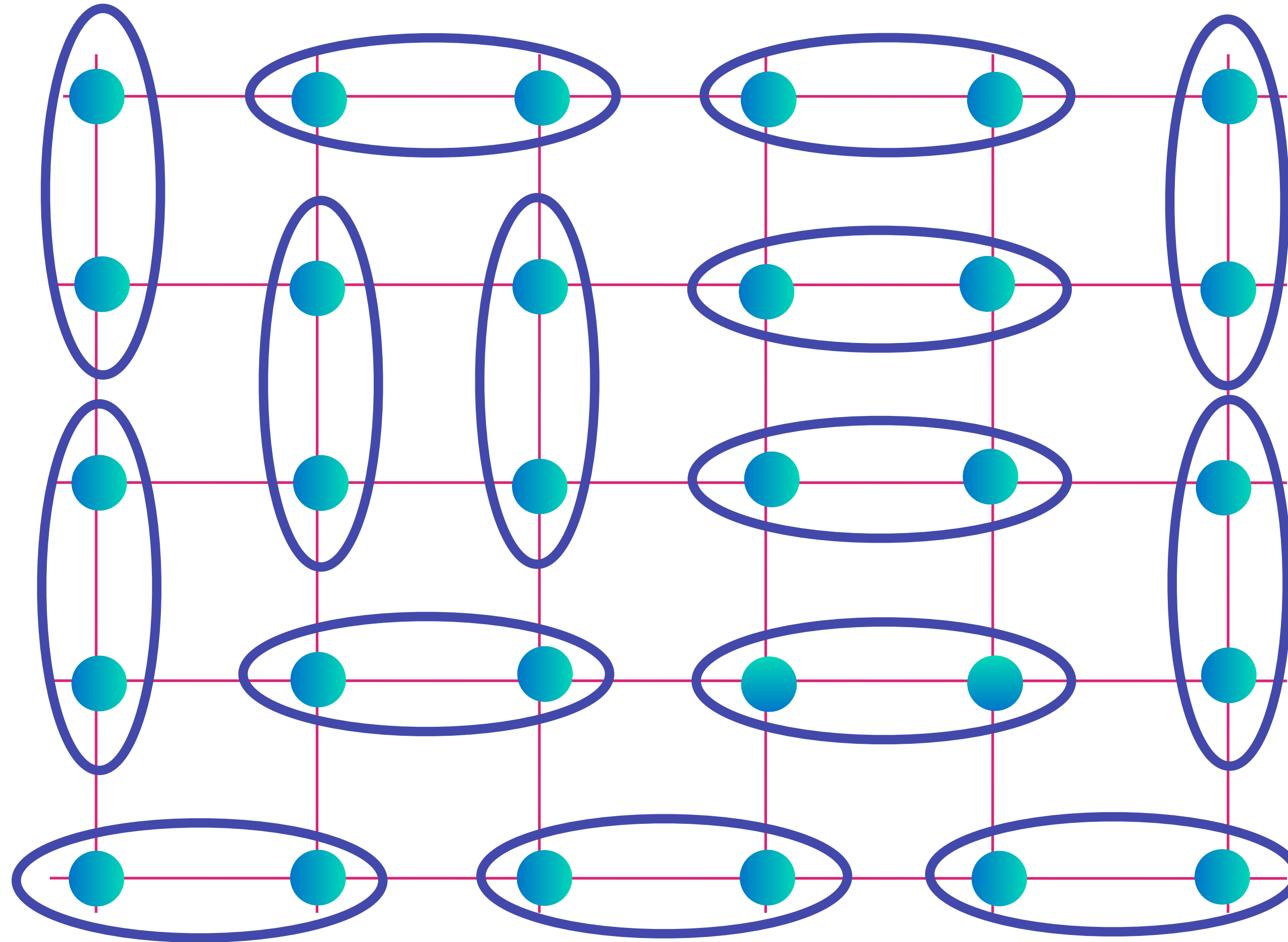
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The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

Spin liquid



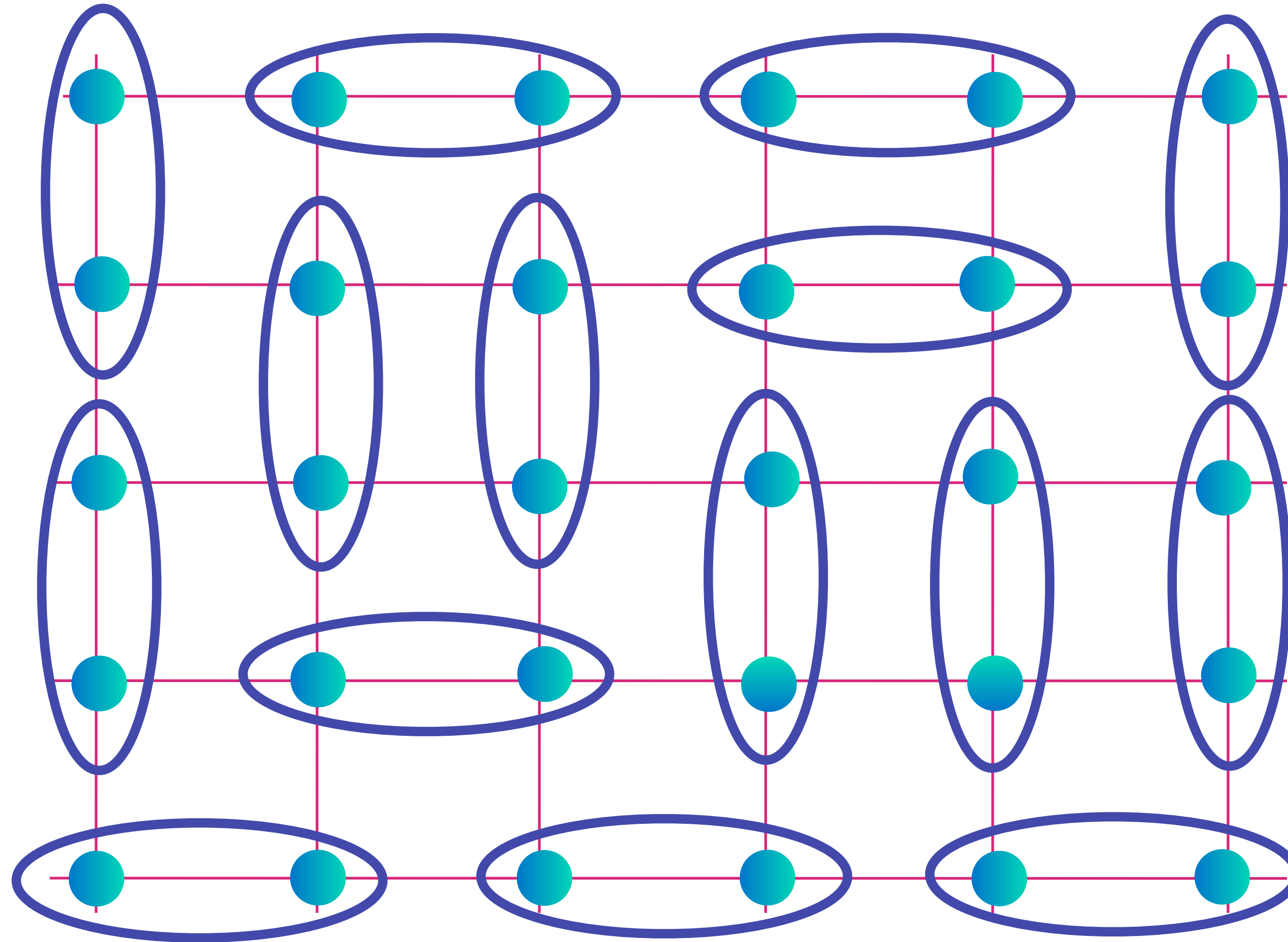
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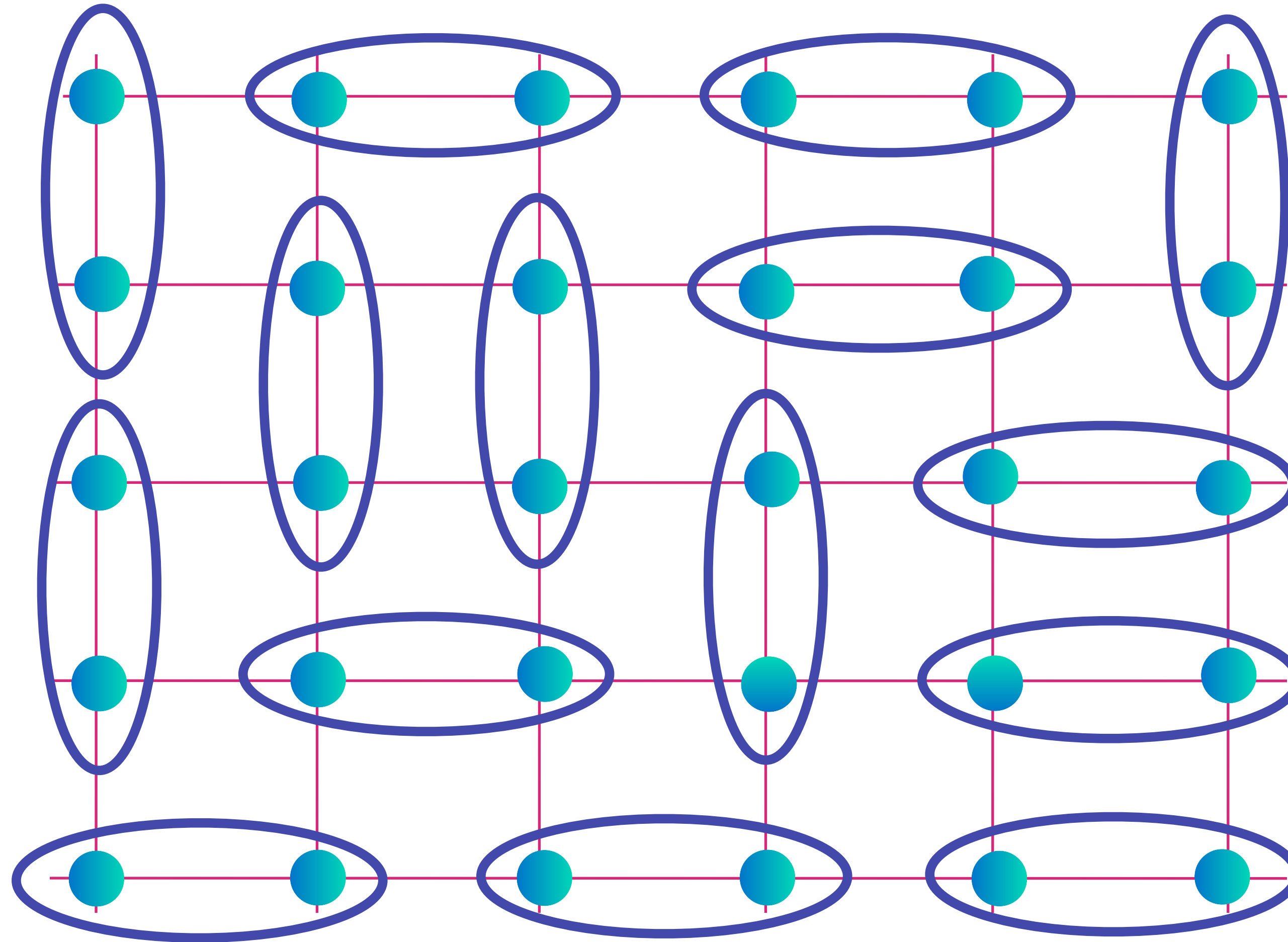
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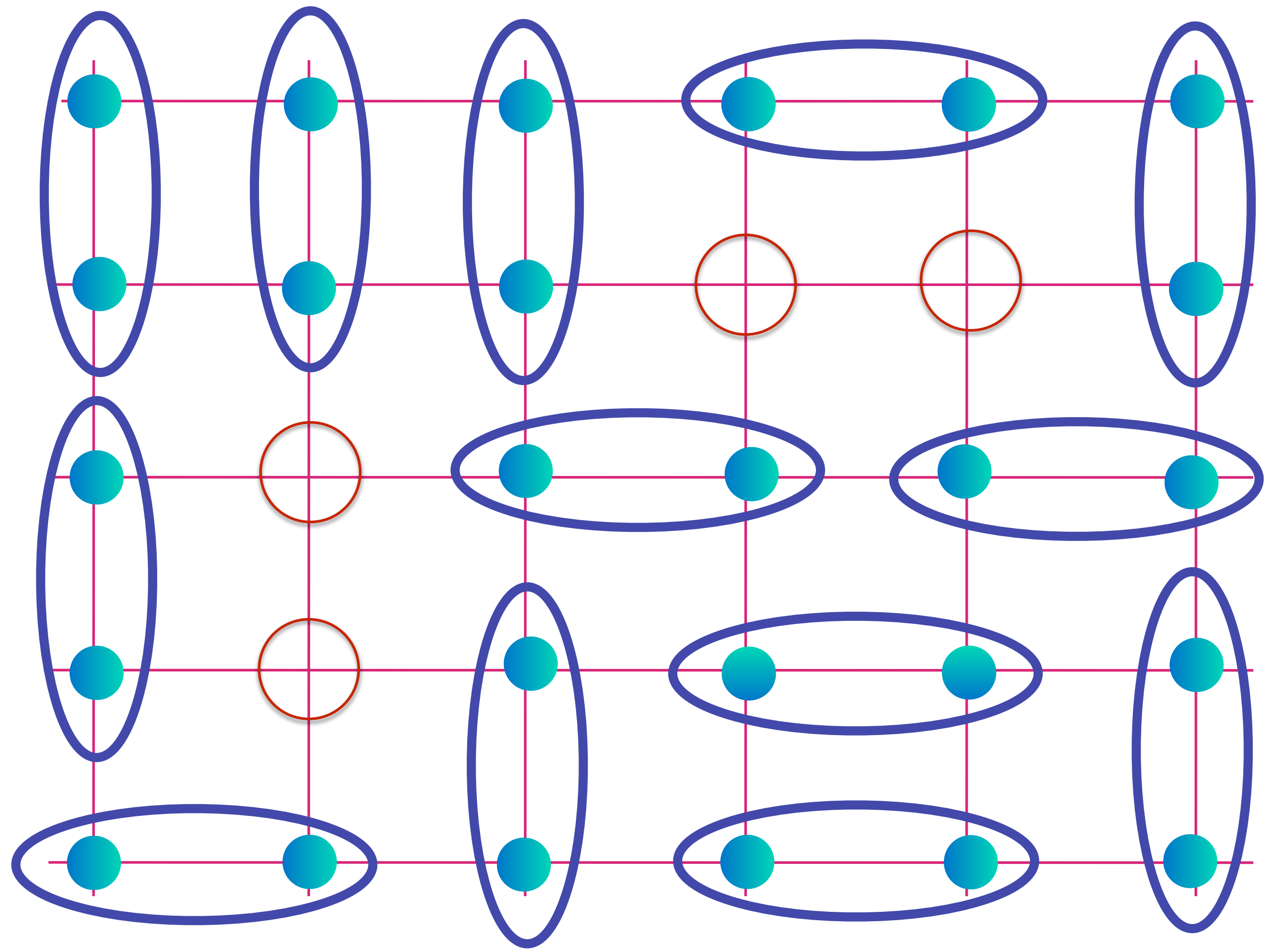
Spin liquid



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The dance of electrons on Cu atoms in YBCO



Superconductivity

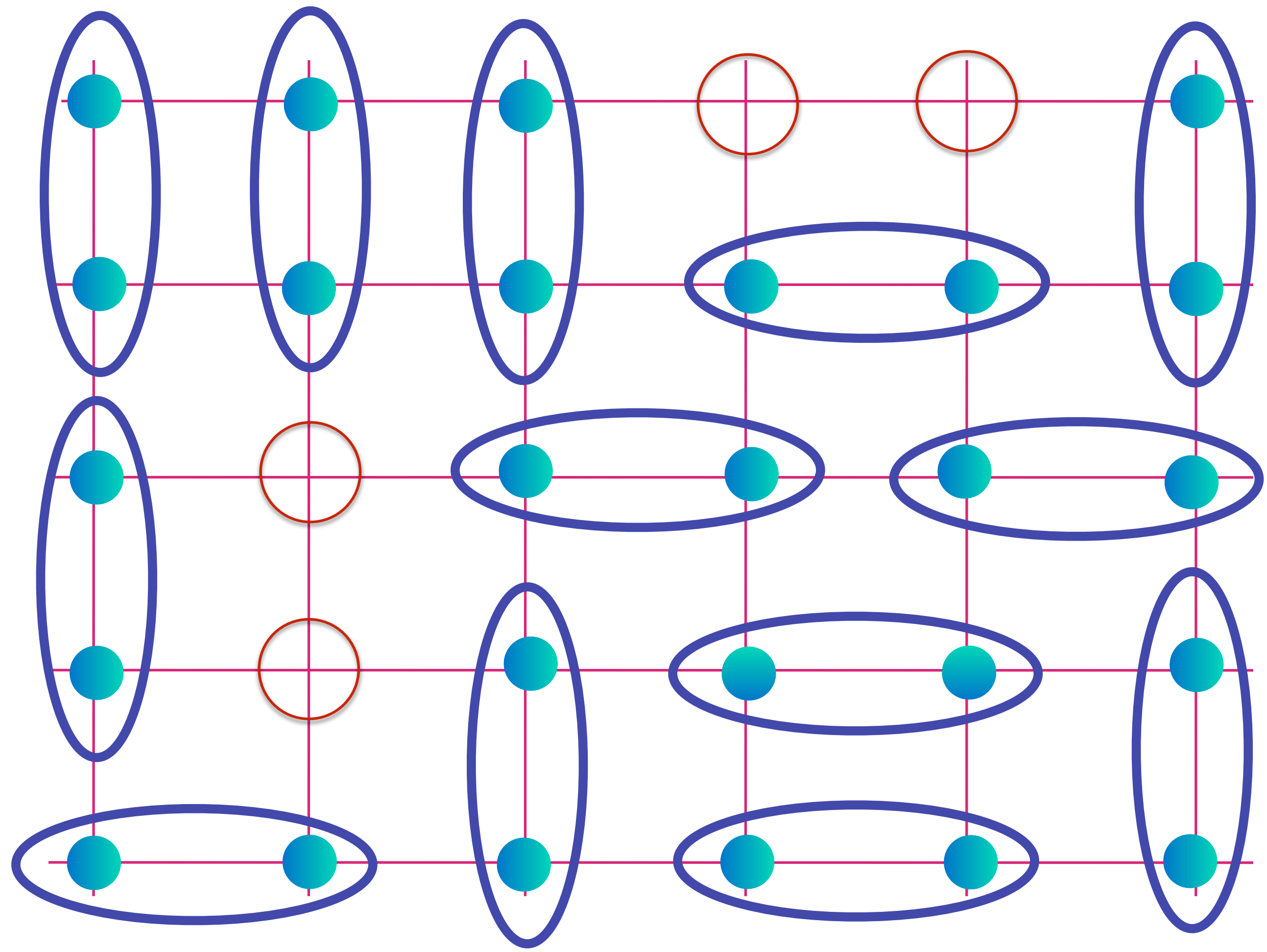
Bose
condensation of
electron pairs

$$\text{[Diagram of a pair of electrons]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



S.N. Bose (1924)

The dance of electrons on Cu atoms in YBCO



Superconductivity

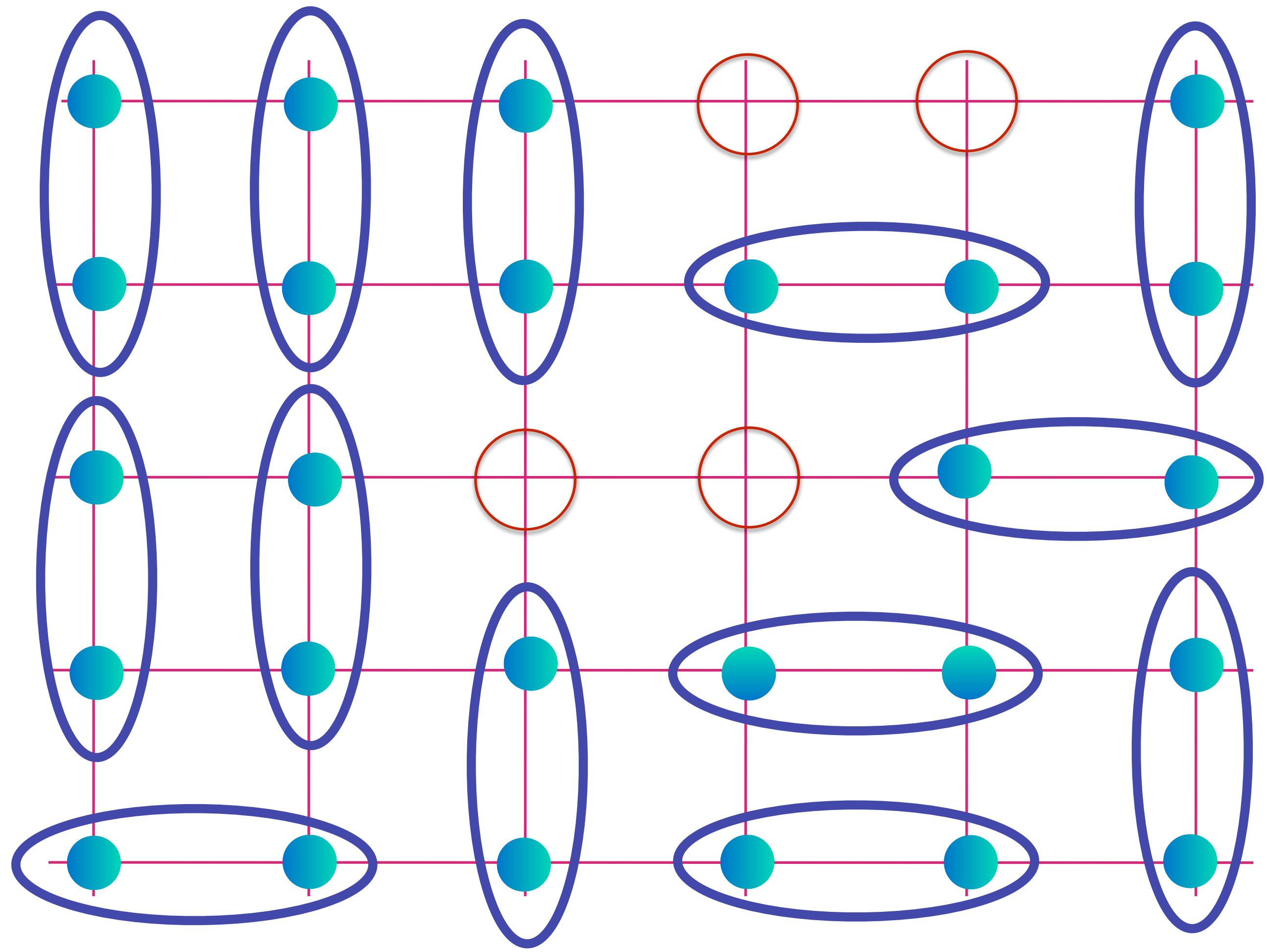
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The dance of electrons on Cu atoms in YBCO



Superconductivity

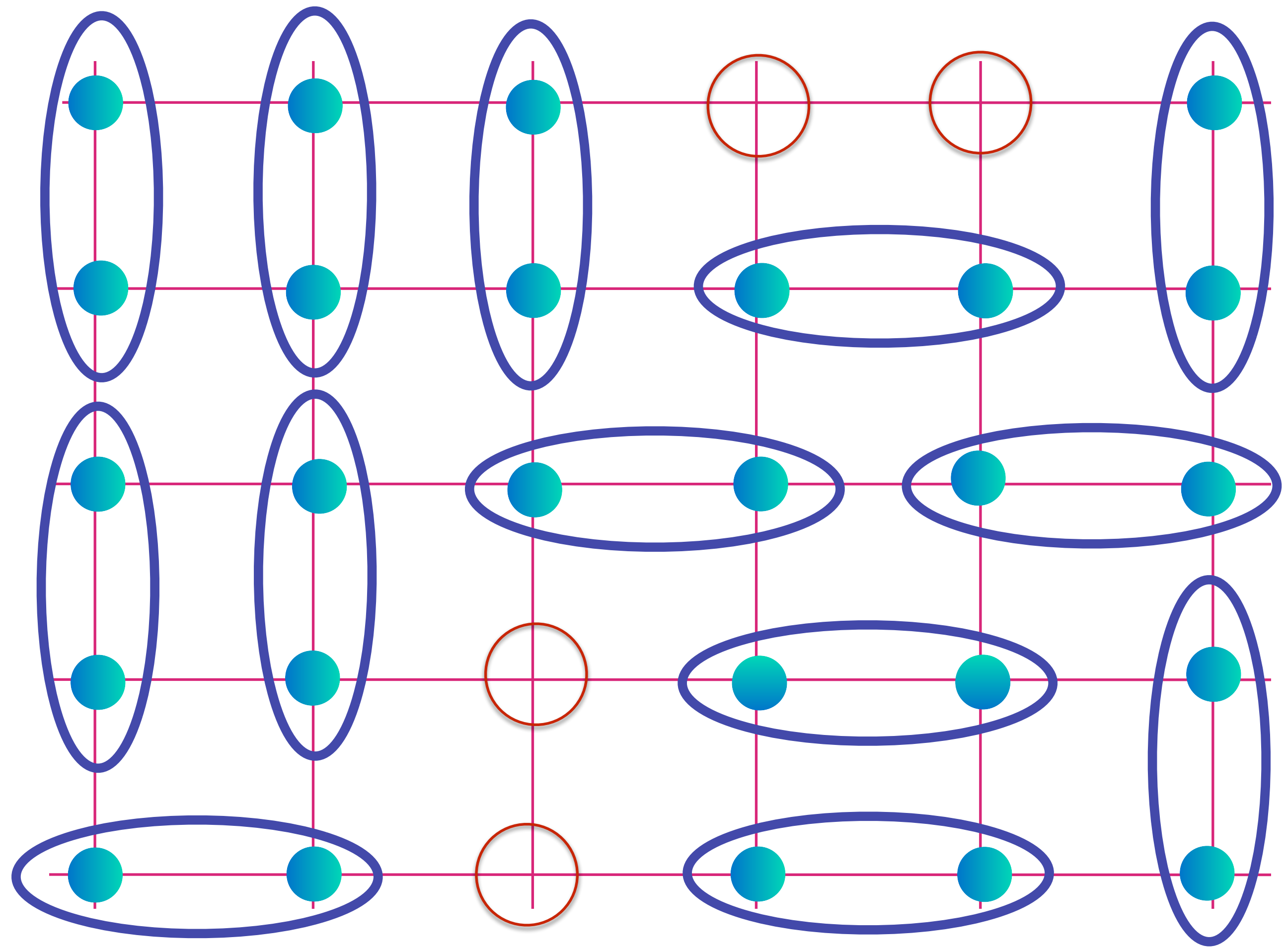
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Superconductivity

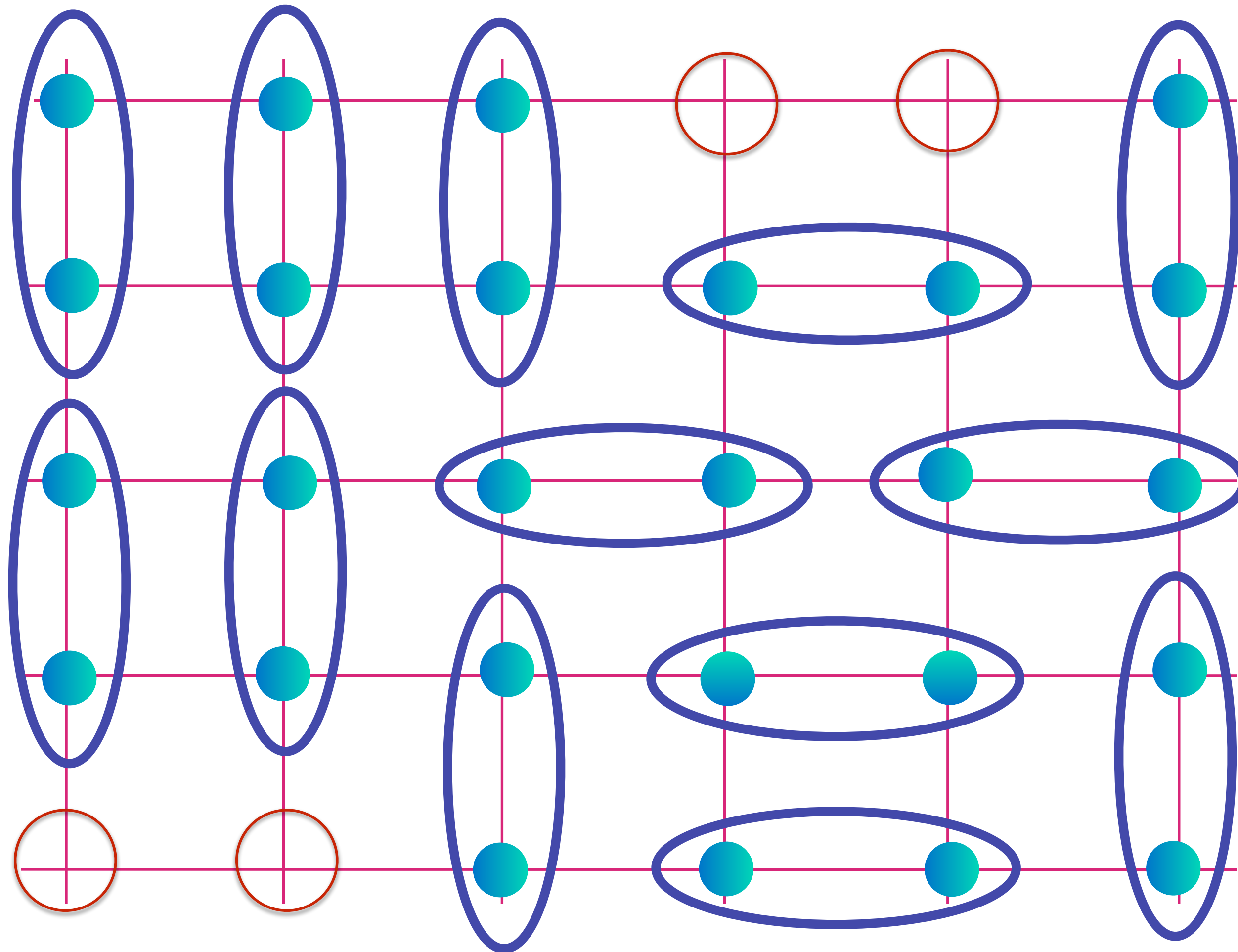
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S.N. Bose (1924)

The dance of electrons on Cu atoms in YBCO



Superconductivity

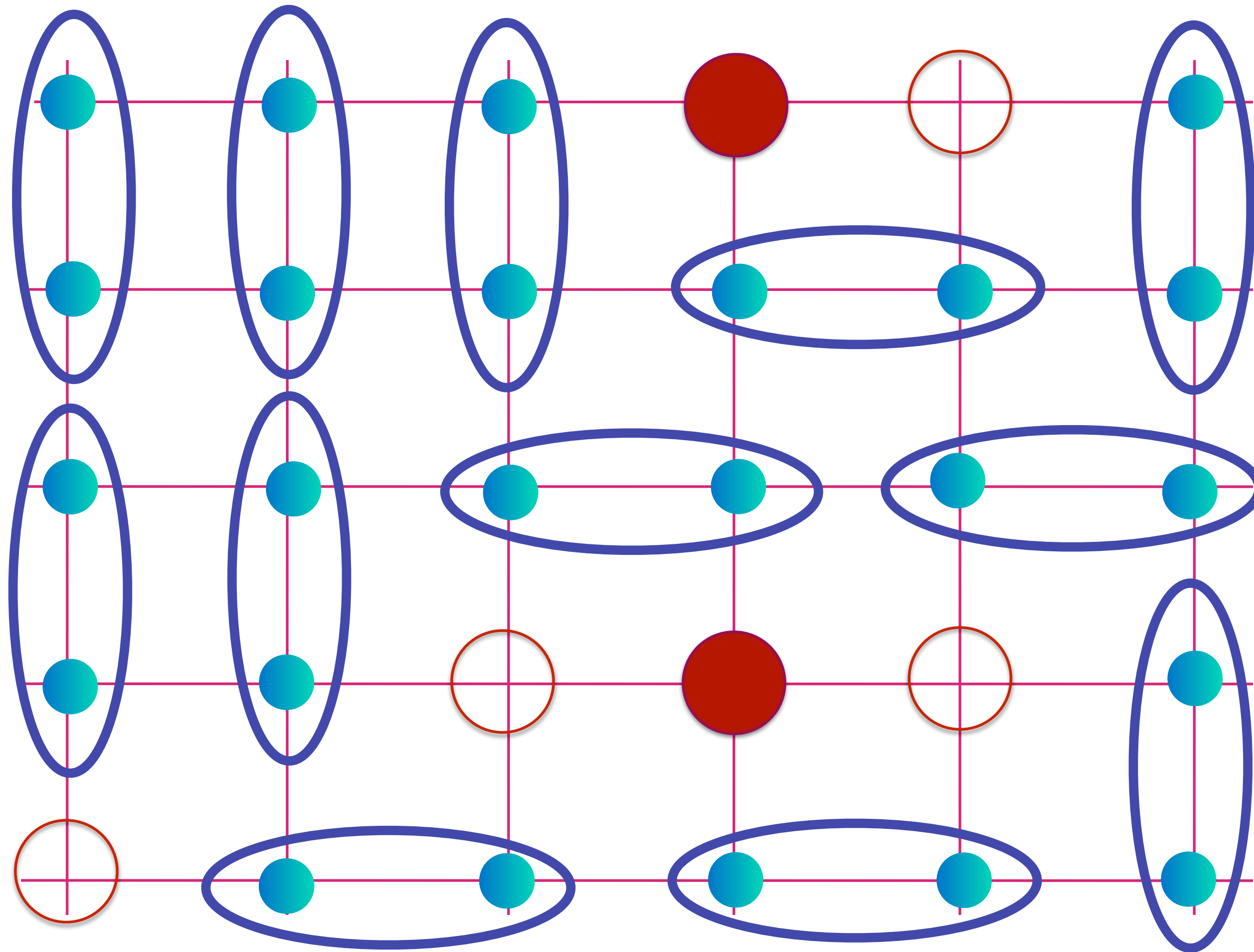
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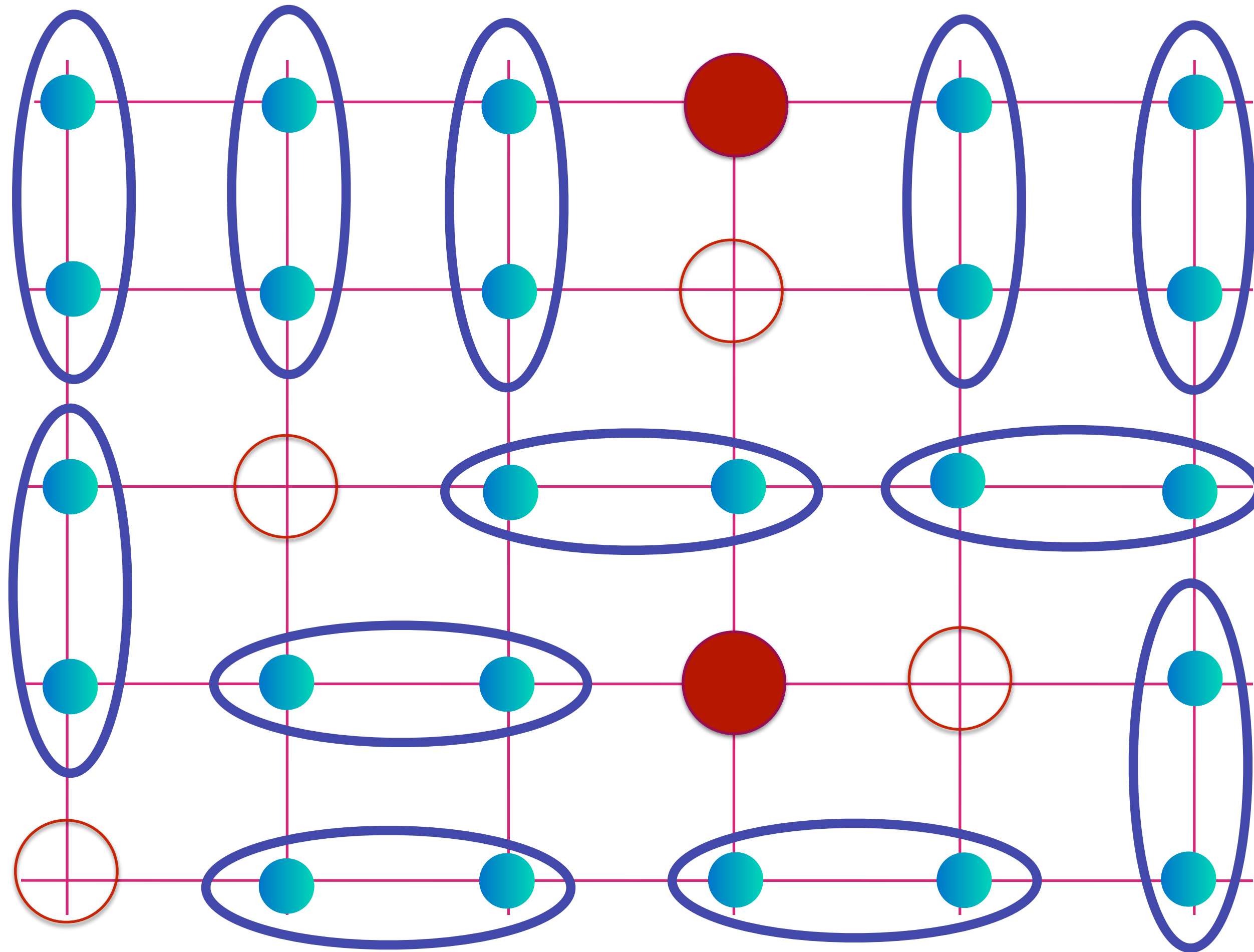


Strange metal

Complex entanglement in the presence of impurities, similar to that in the SYK model

$$\text{[Diagram of two electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO

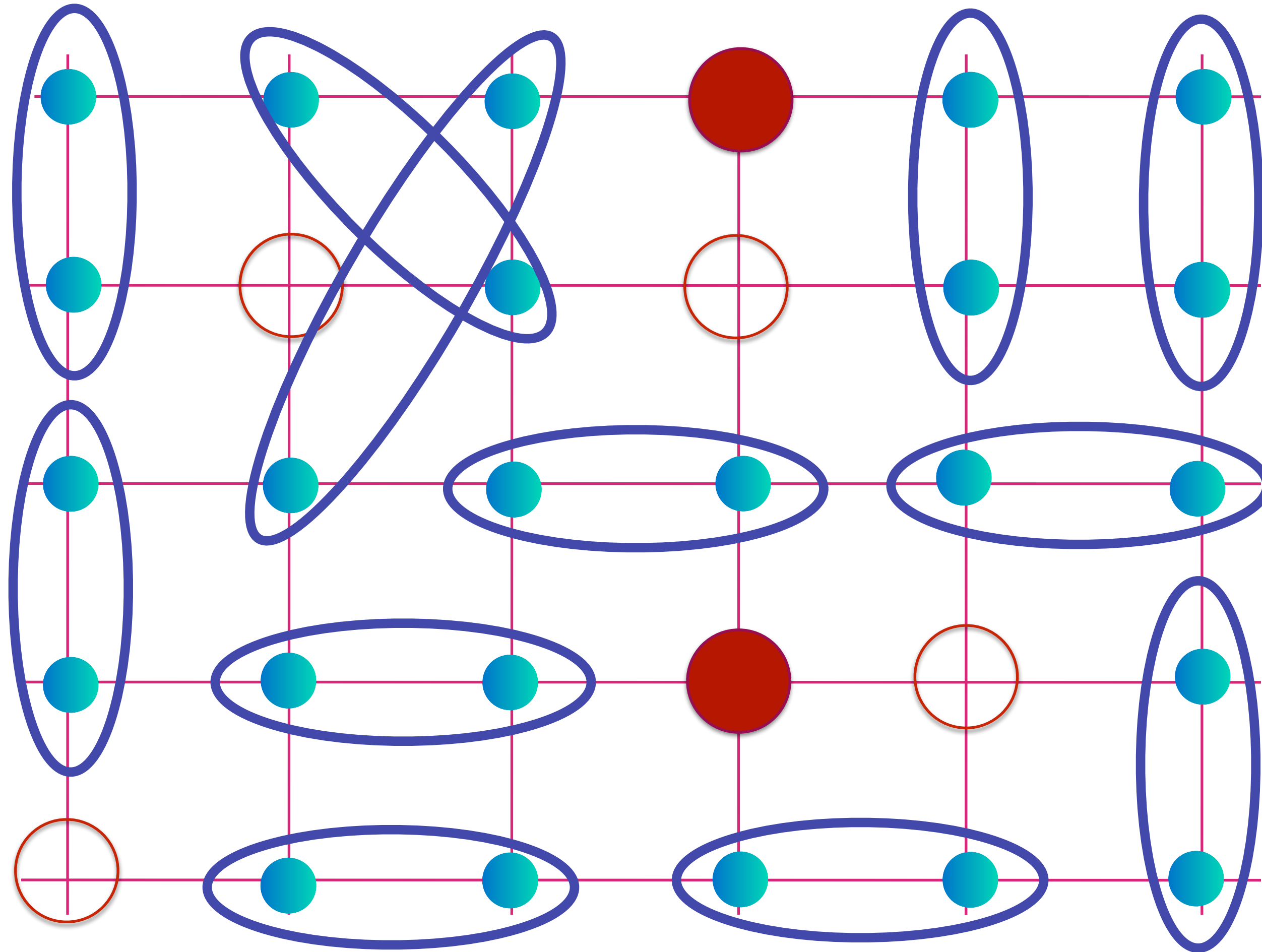


Strange metal

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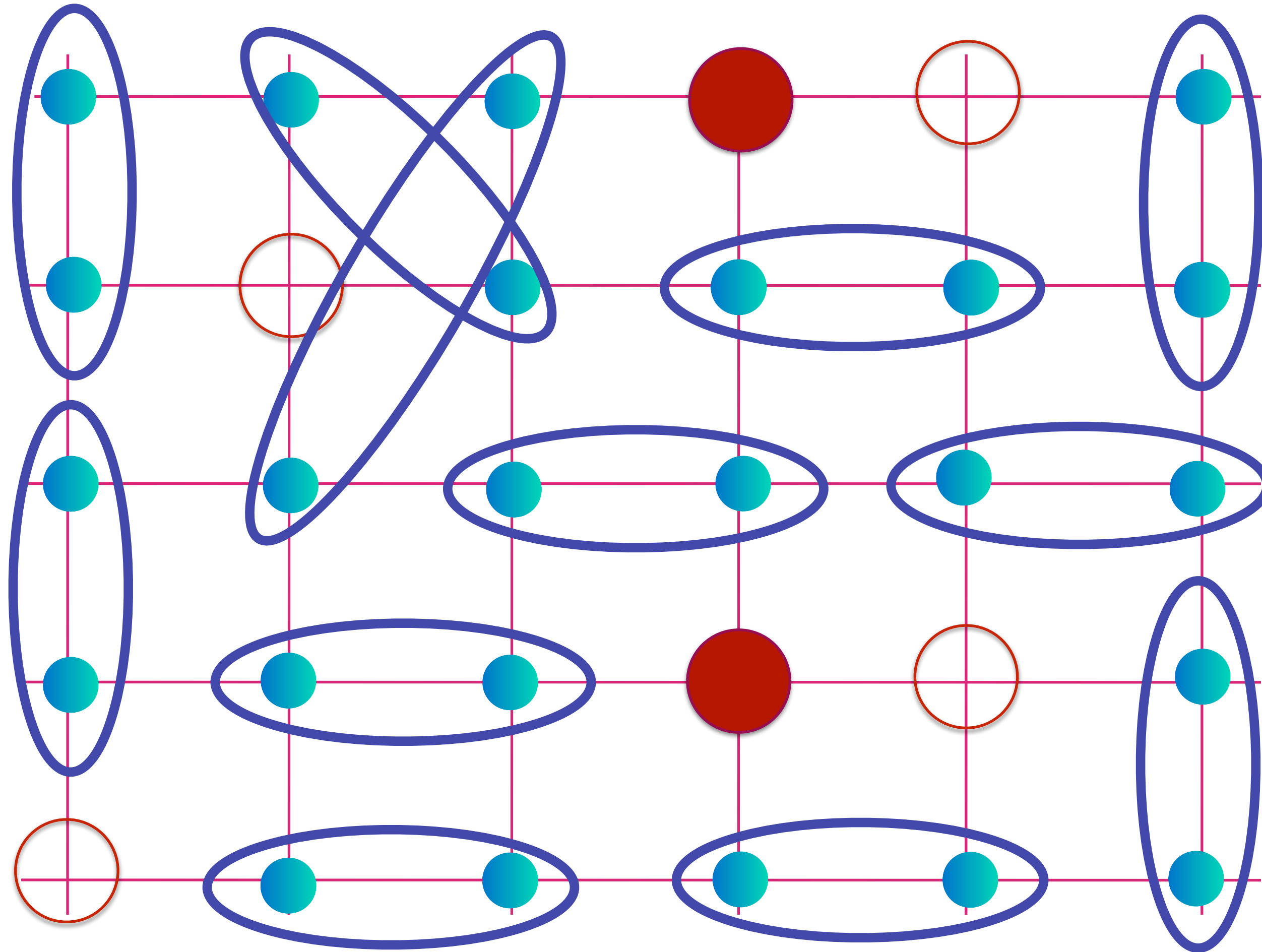


Strange metal

Complex entanglement in the presence of impurities, similar to that in the SYK model

$$\text{Diagram of two cyan dots in a blue oval} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

The dance of electrons on Cu atoms in YBCO



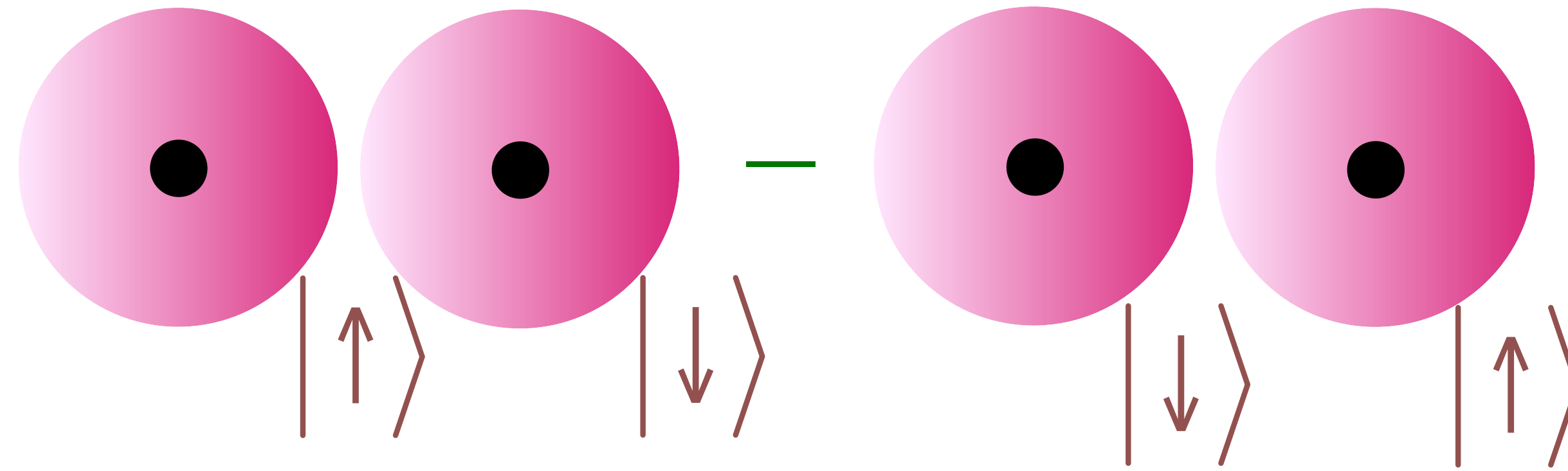
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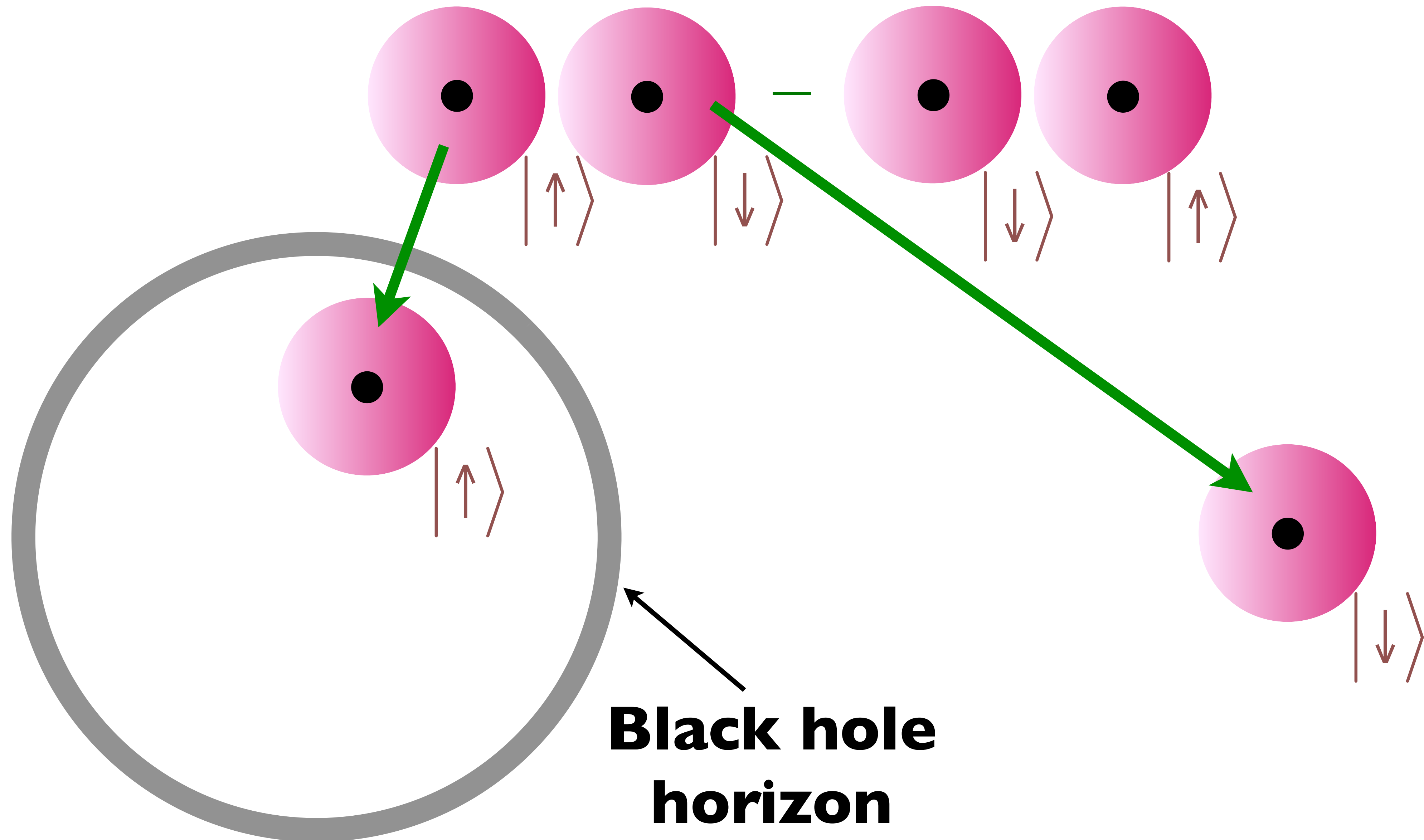
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Quantum entanglement,
the SYK model,
and black holes
holes

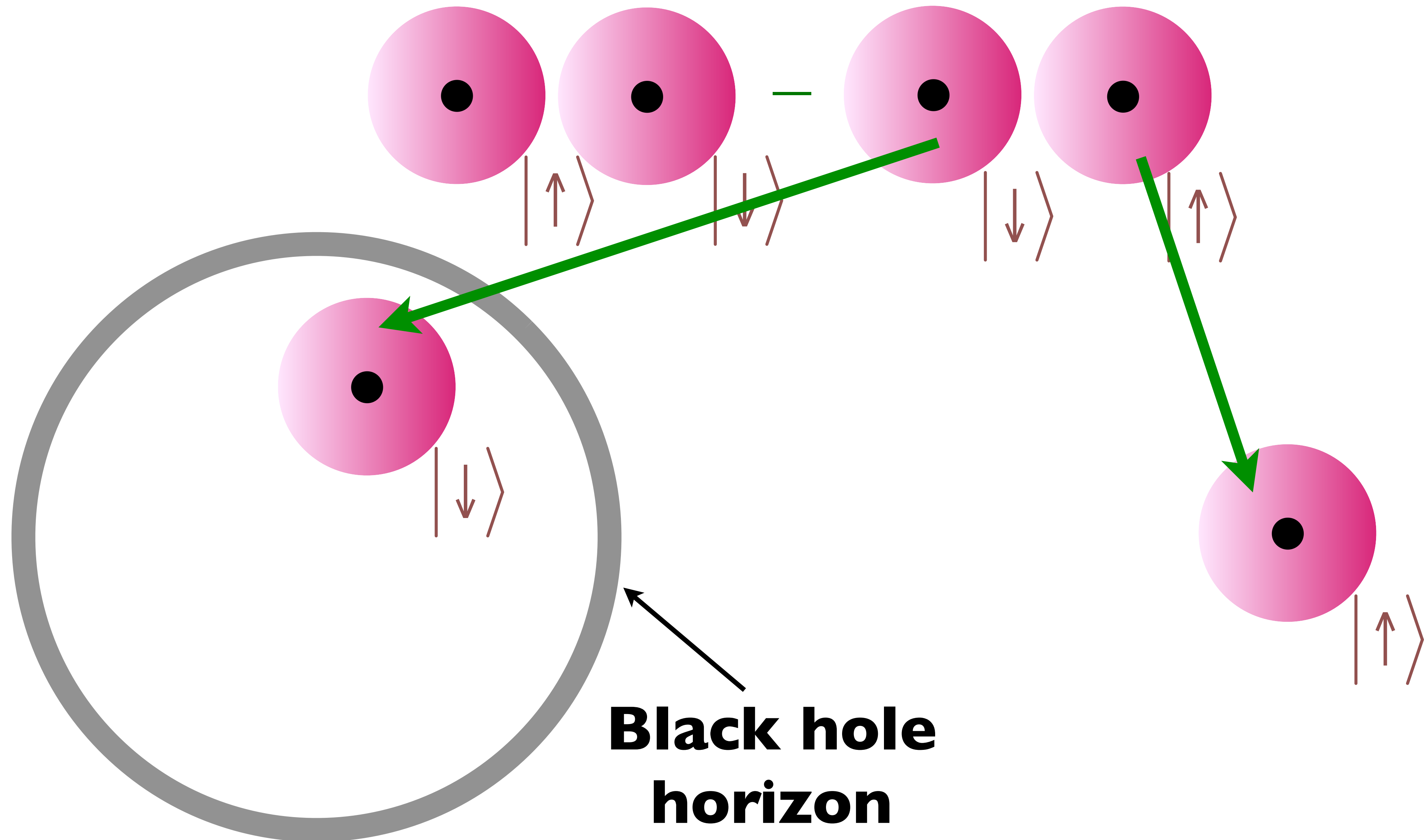
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

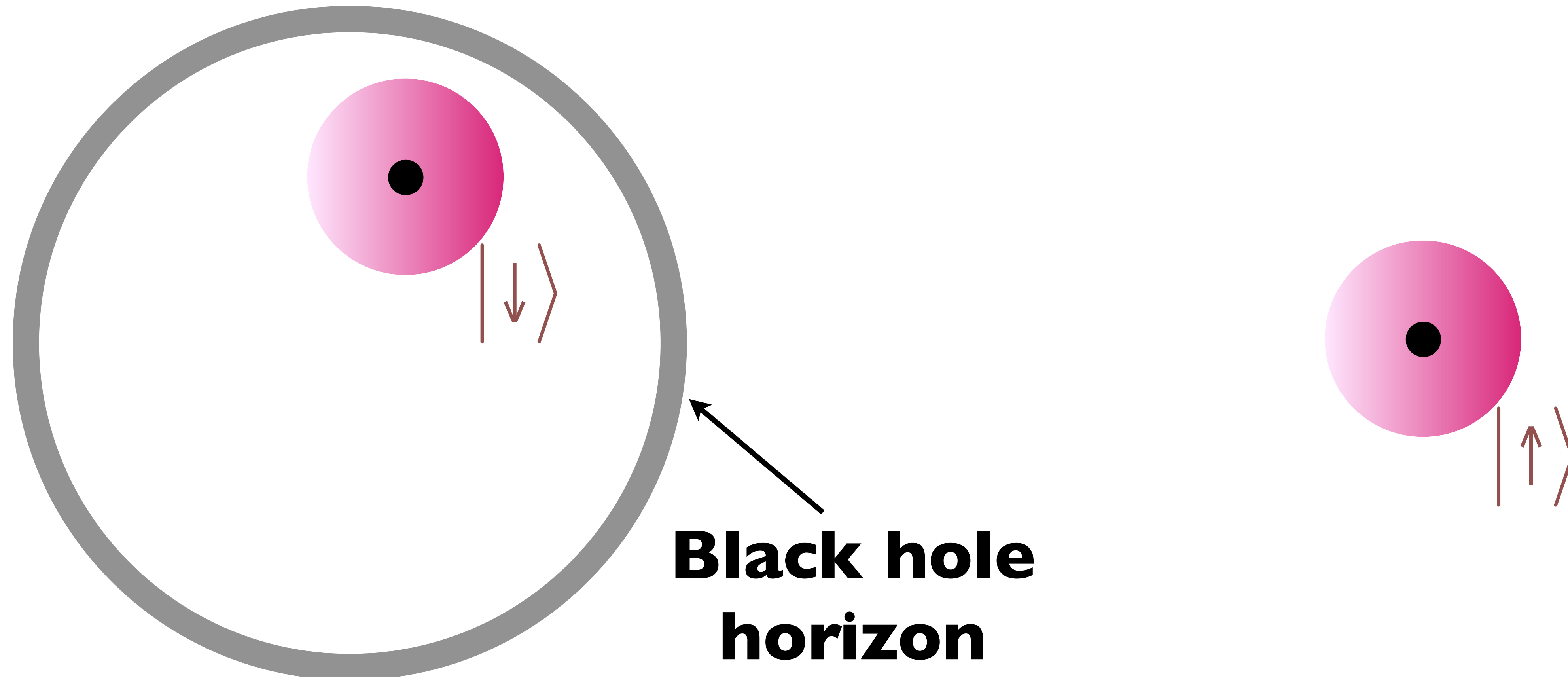


Quantum Entanglement across a black hole horizon



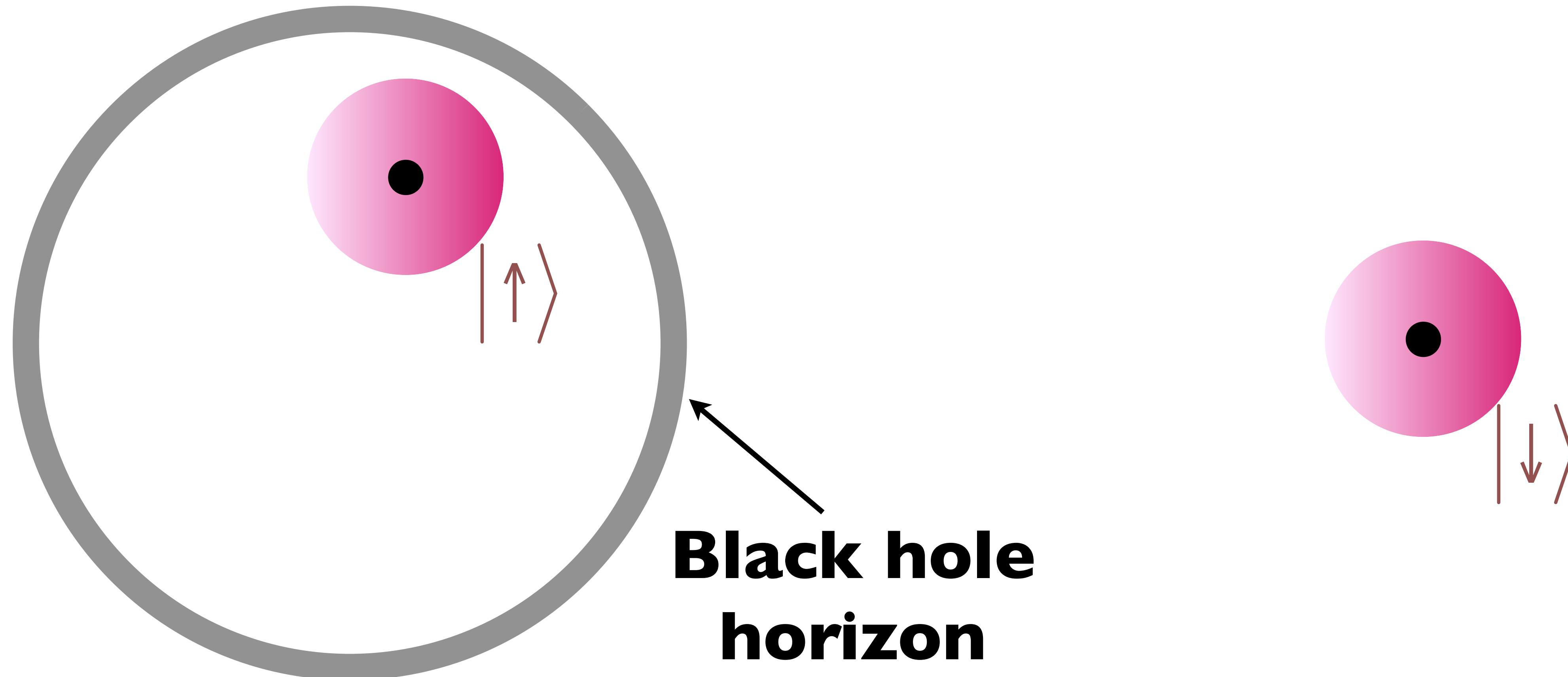
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

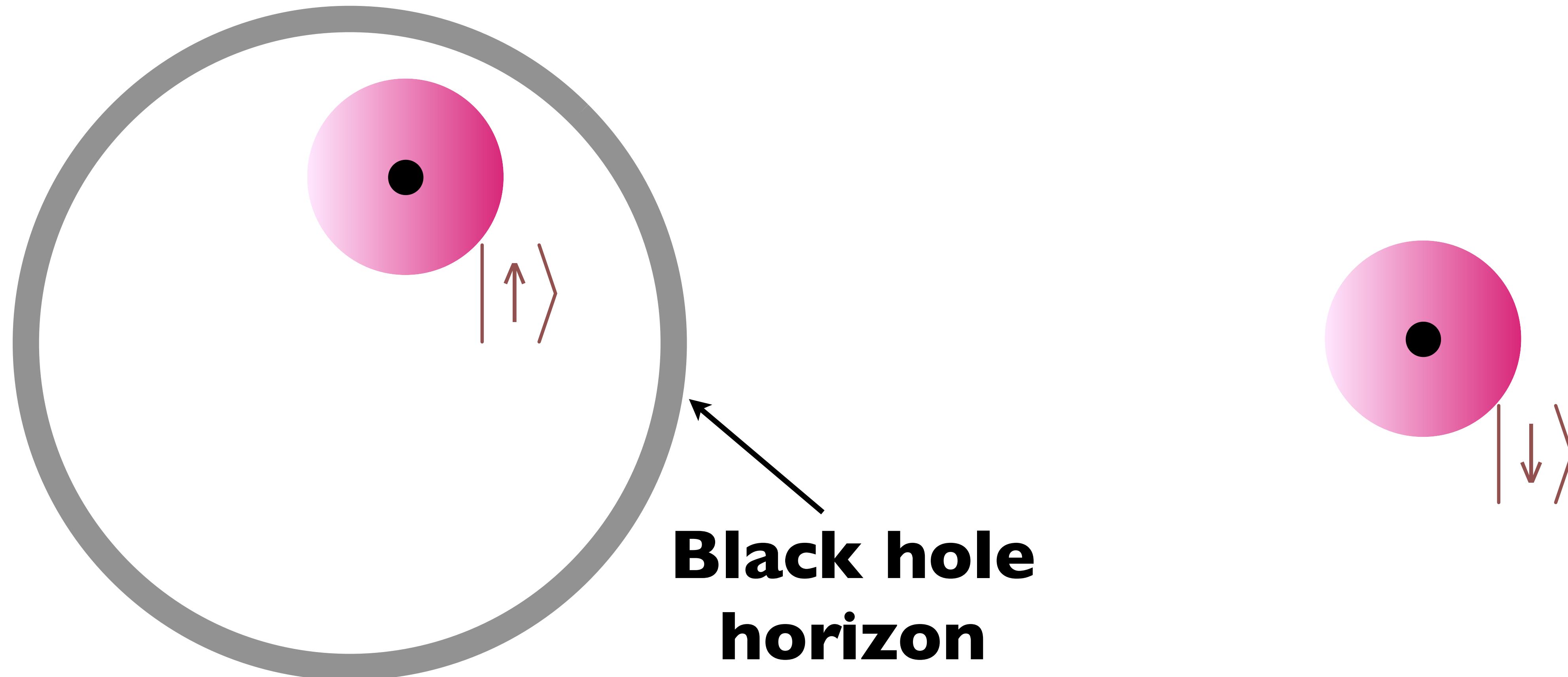
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

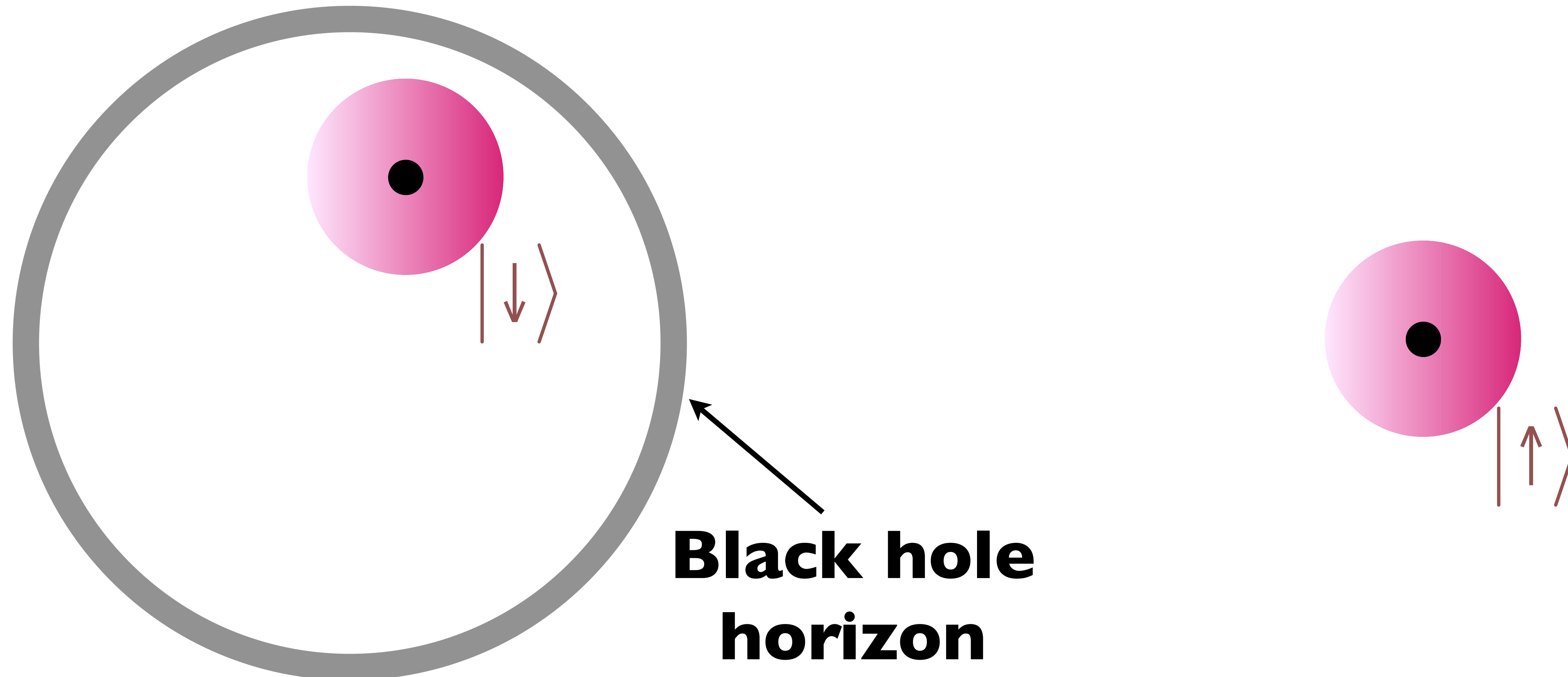
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

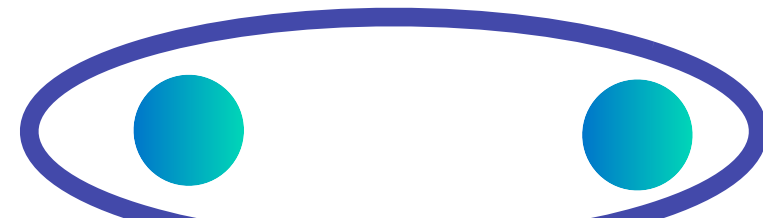
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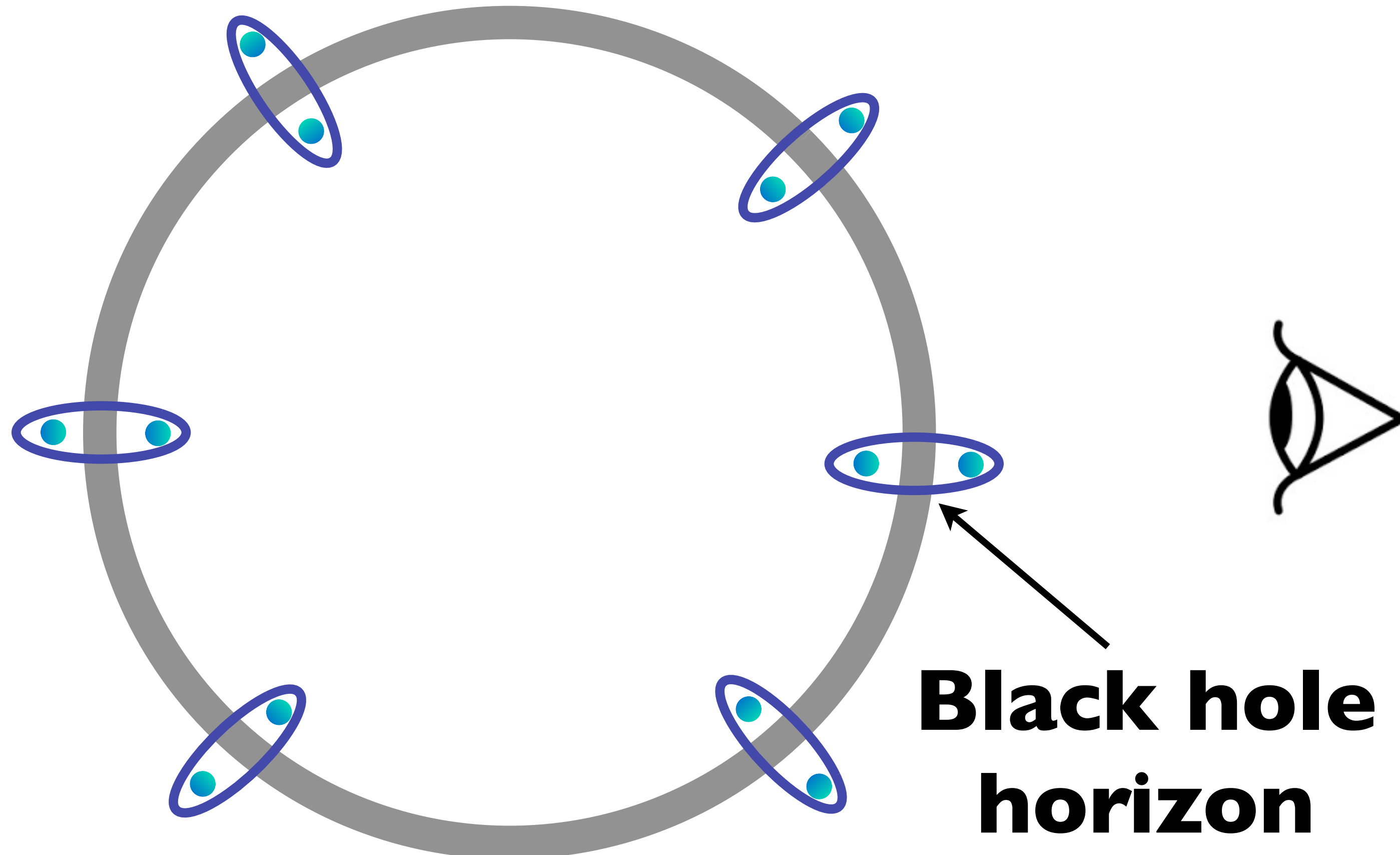
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

Quantum entanglement
on the surface


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside*
the black hole,
Hawking obtained

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the
black hole horizon.

All other systems have
entropy proportional to
their volume.

Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ matches Hawking's entropy, in accordance with Boltzmann's principles of statistical mechanics, $D(E) = e^{S(E)/k_B}$?

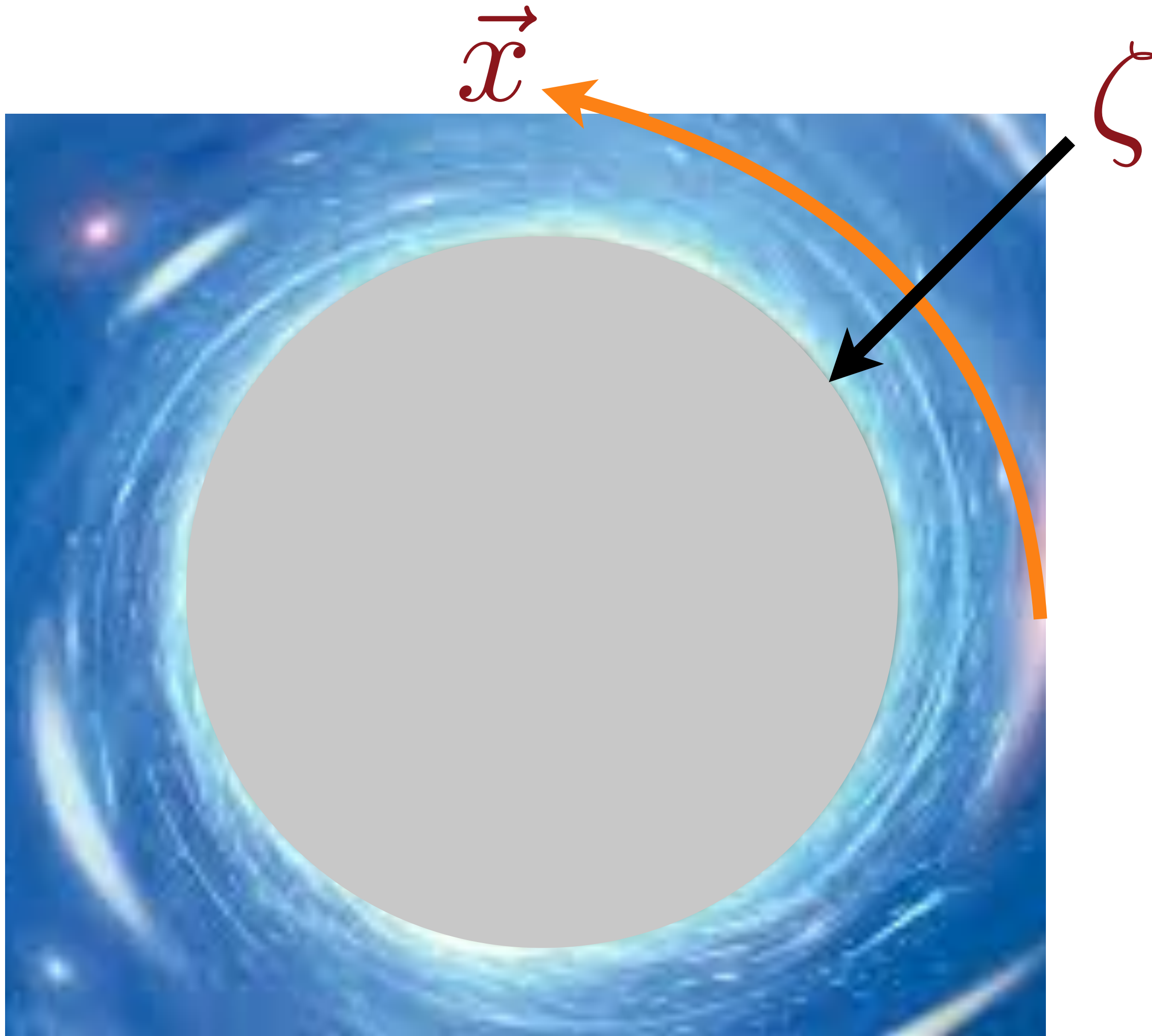
Quantum Black Holes

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- Answer from string theory for 'supersymmetric' charged black holes: $D(E) = e^S \delta(E)$ *i.e.* all the states required by Hawking's entropy have exactly the same energy.

Strominger, Vafa (1996)

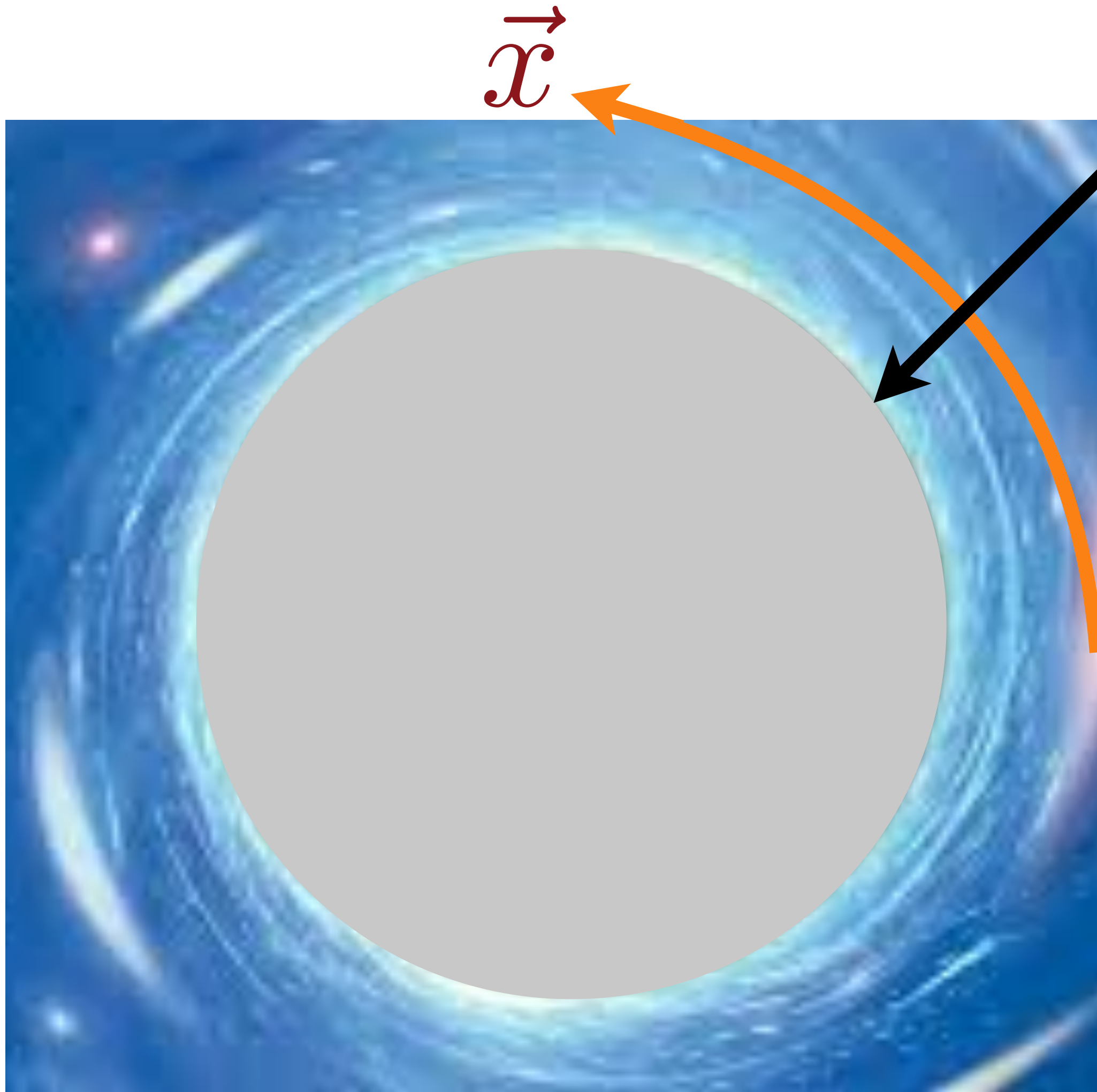


Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge





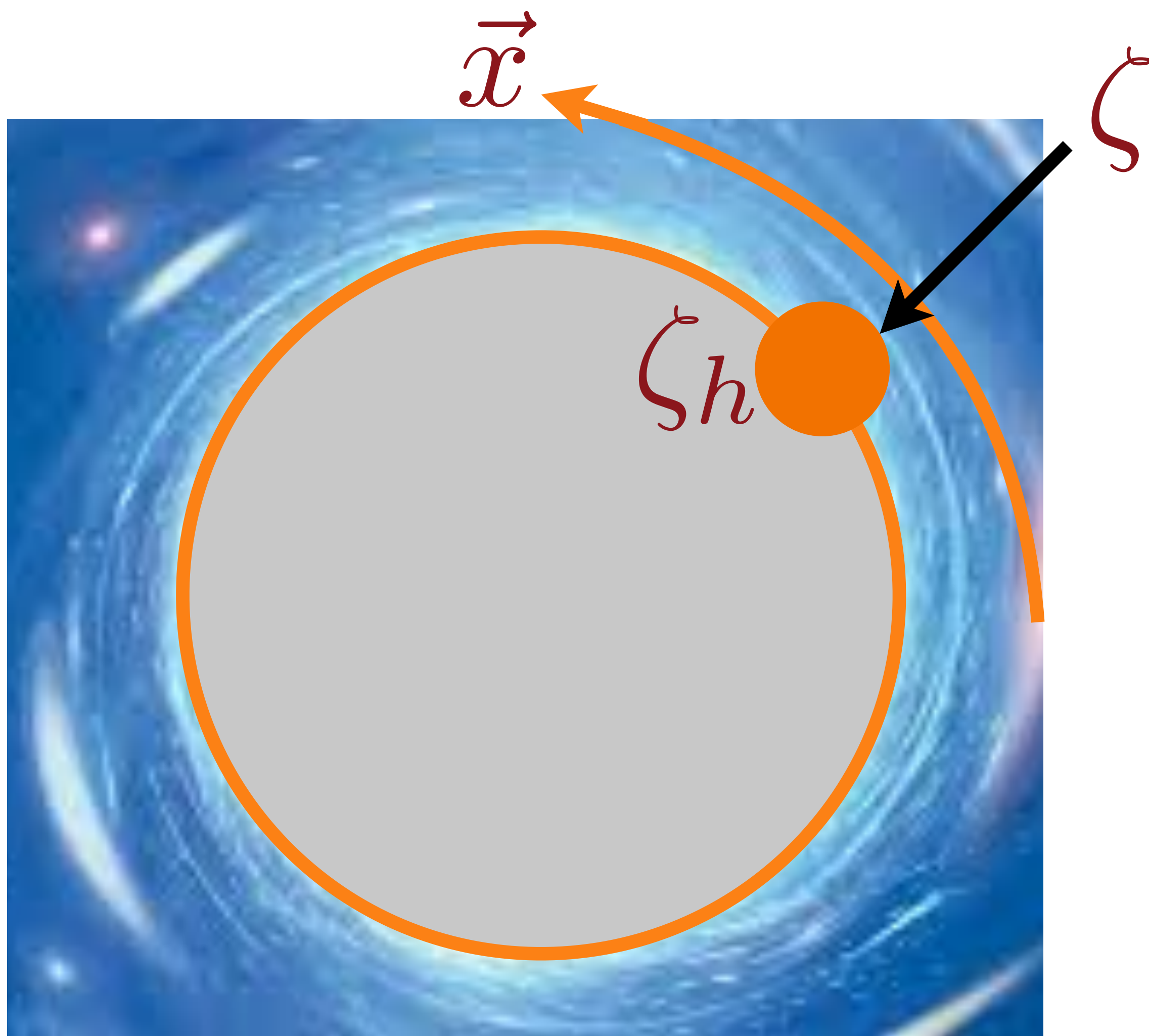
Maxwell's electromagnetism
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Zooming into the
near-horizon region
of a charged black hole
at low temperature,
yields a theory
in one space (ζ) and
one time dimension



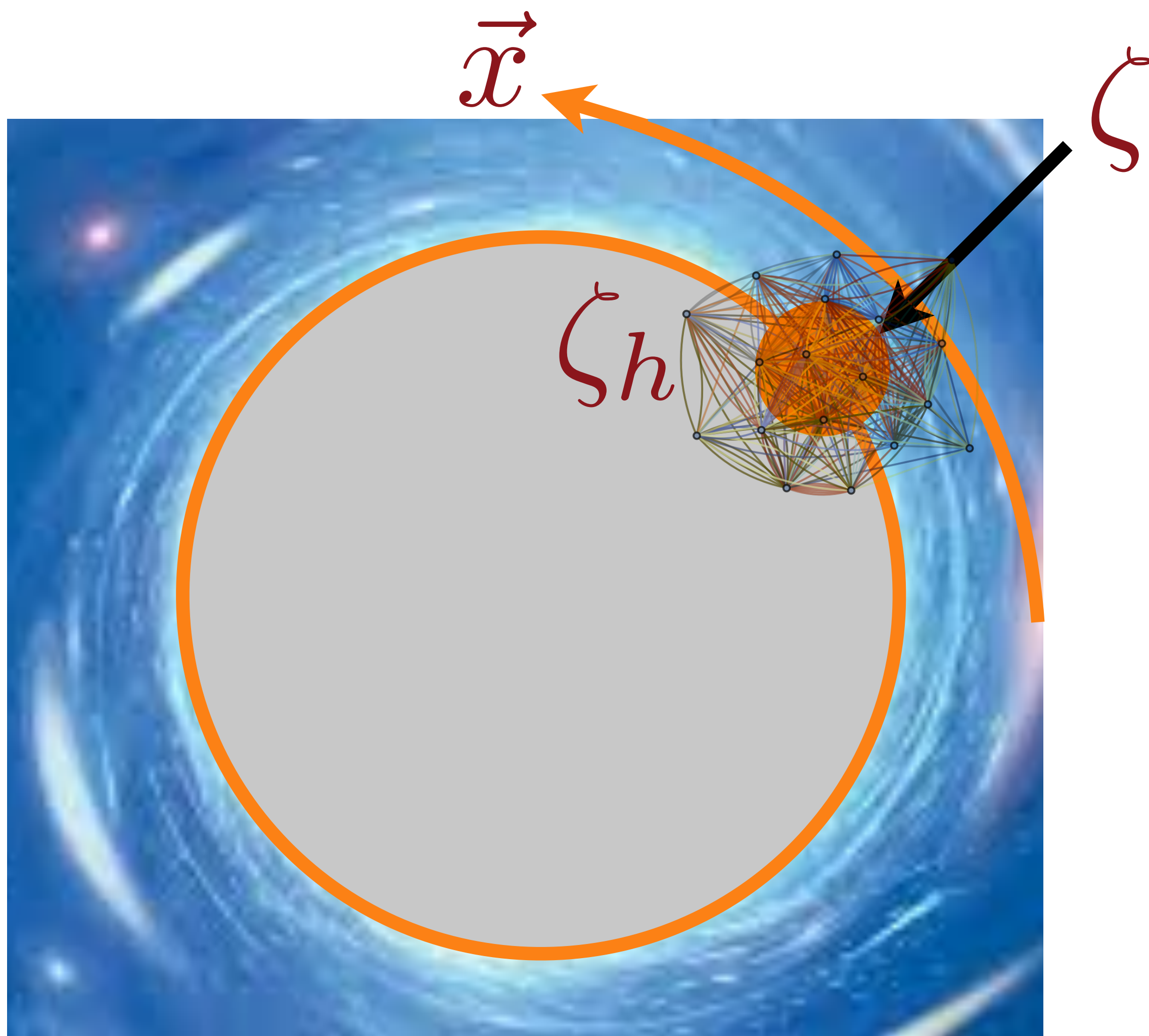
Maxwell's electromagnetism
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So we need only consider
complex entanglement at
one spatial “point”
on the horizon ($\zeta = \zeta_h$),
just as is described
by the SYK model



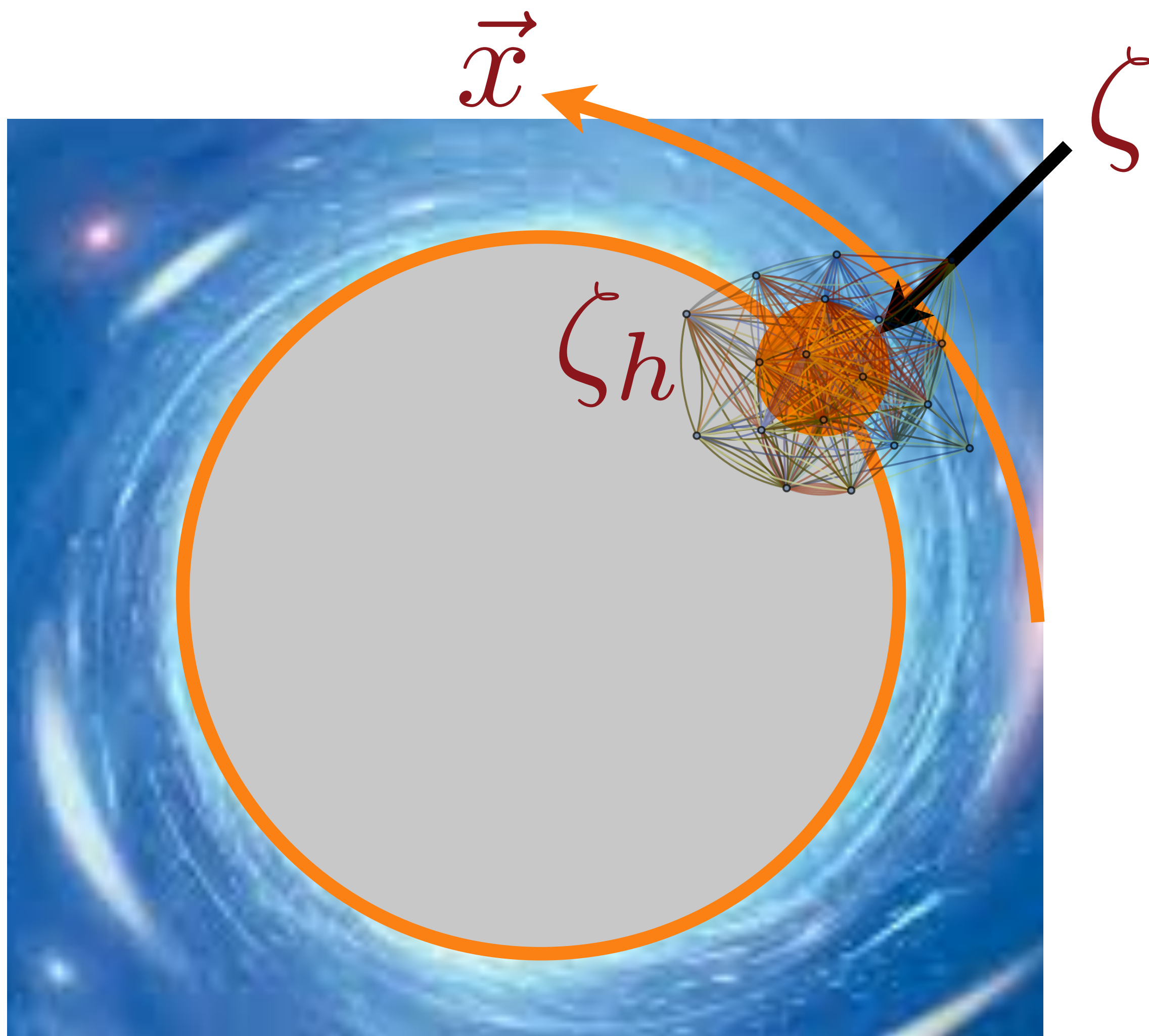
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The quantum versions of
Maxwell's and Einstein's
equations in this
two-dimensional spacetime are
also the equations describing
electron entanglement in the
SYK model!



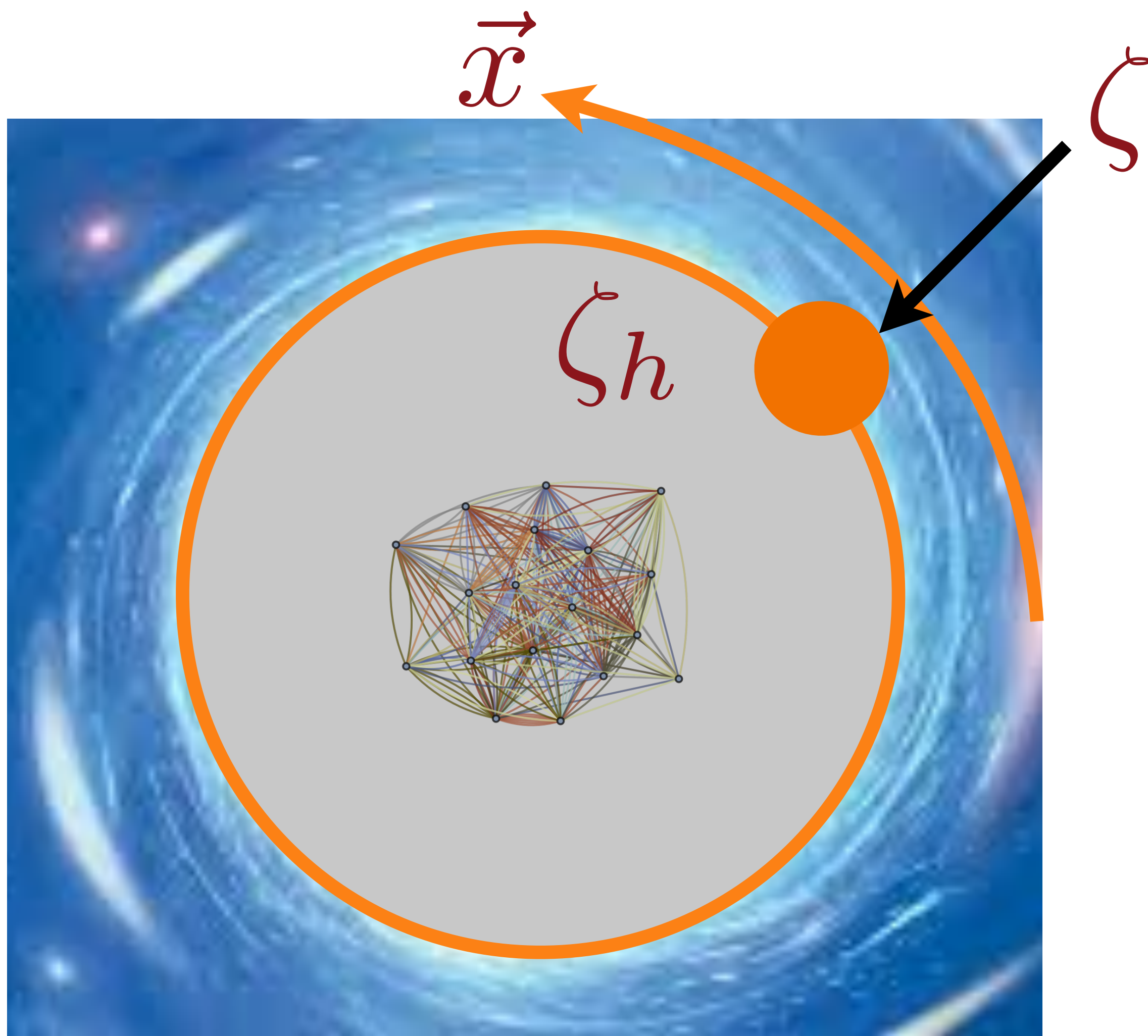
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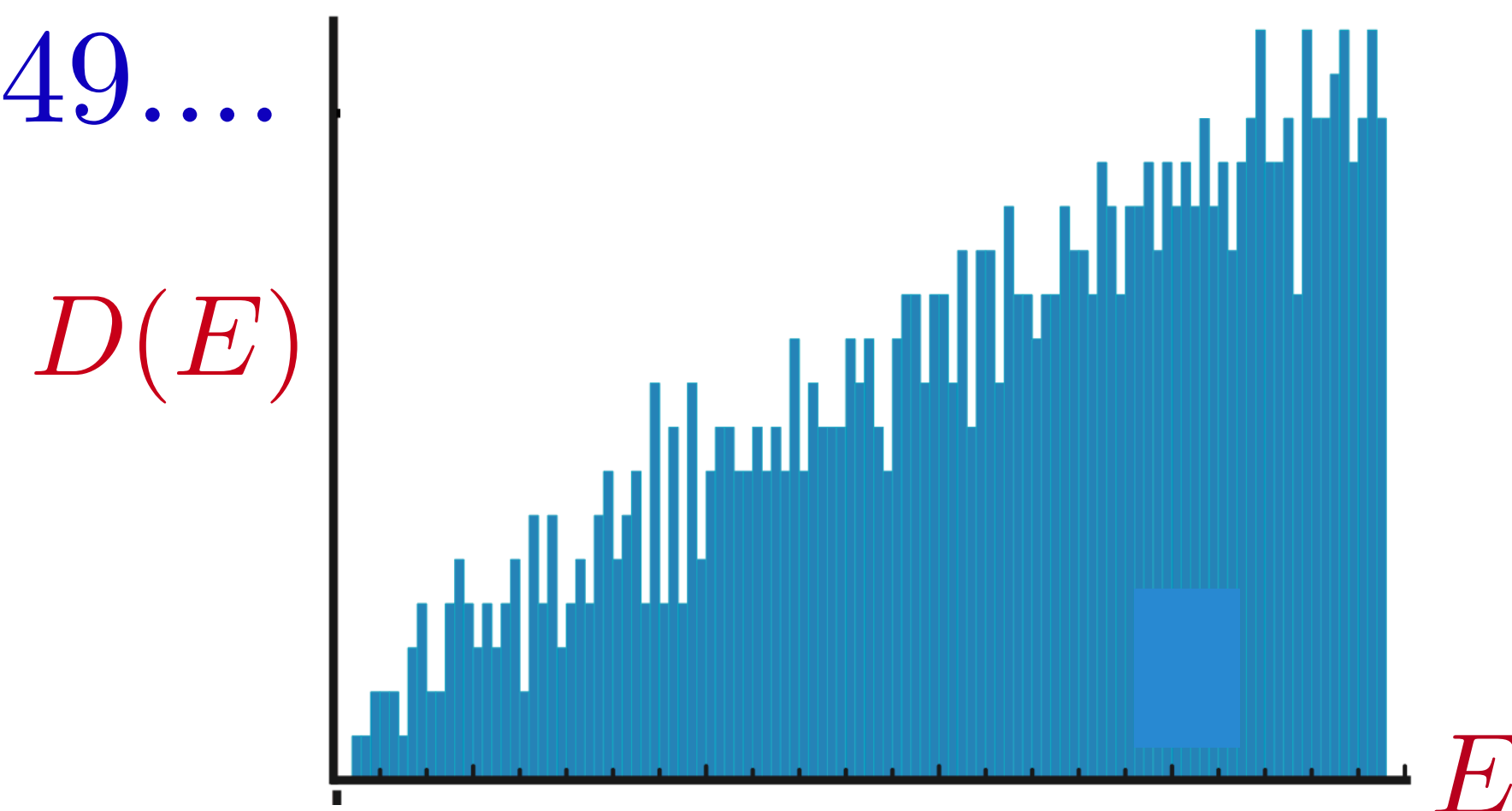
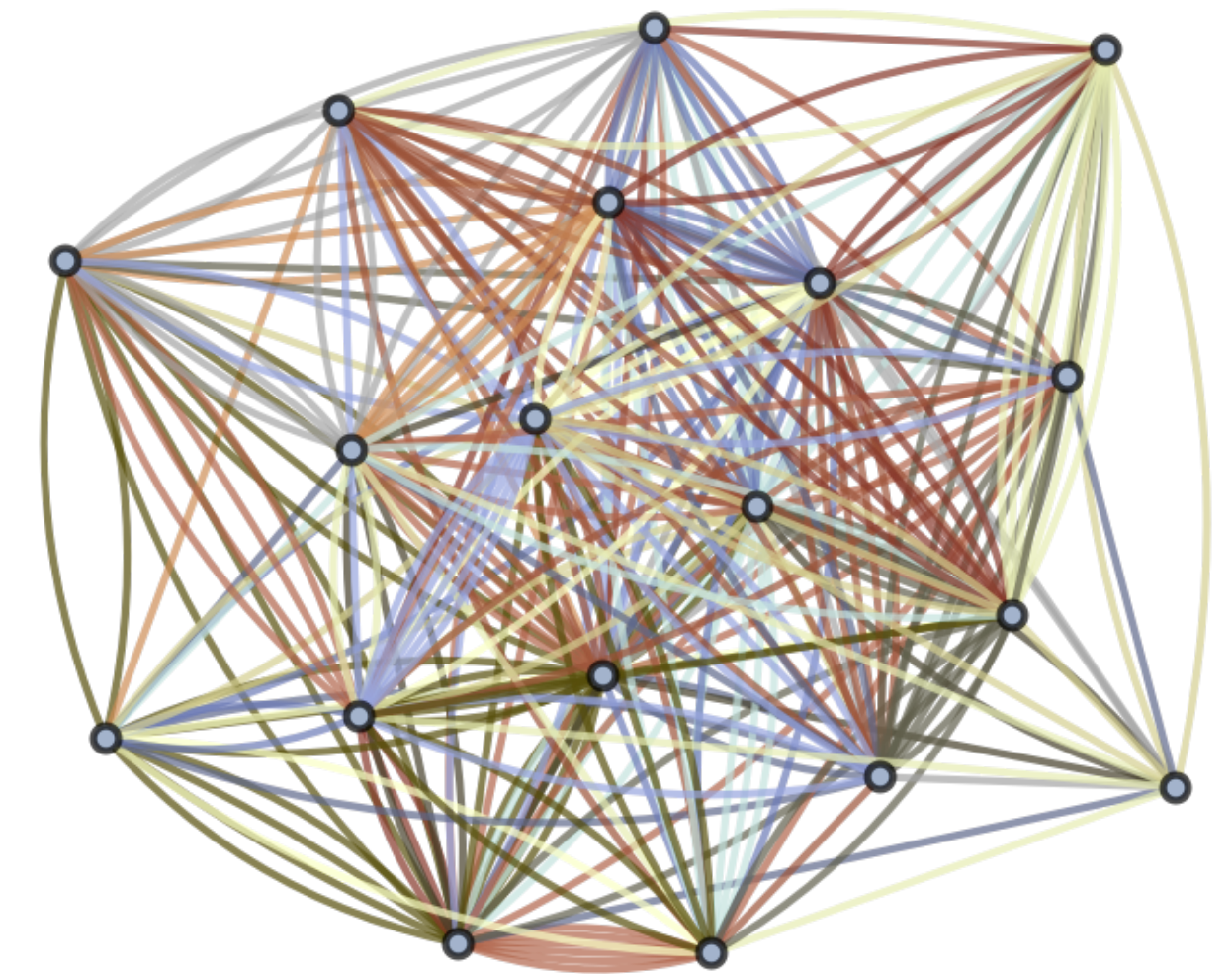
The quantum versions of
Maxwell's and Einstein's
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The Sachdev-Ye-Kitaev (SYK) model

- Density of quantum states of the SYK model with N sites

$$D(E) \sim \frac{1}{N} \exp(N s_0) \sinh\left([2N\gamma E]^{1/2}\right)$$

where $s_0 = 0.46484769917080510749\dots$



D(E) of charged black holes from the SYK model

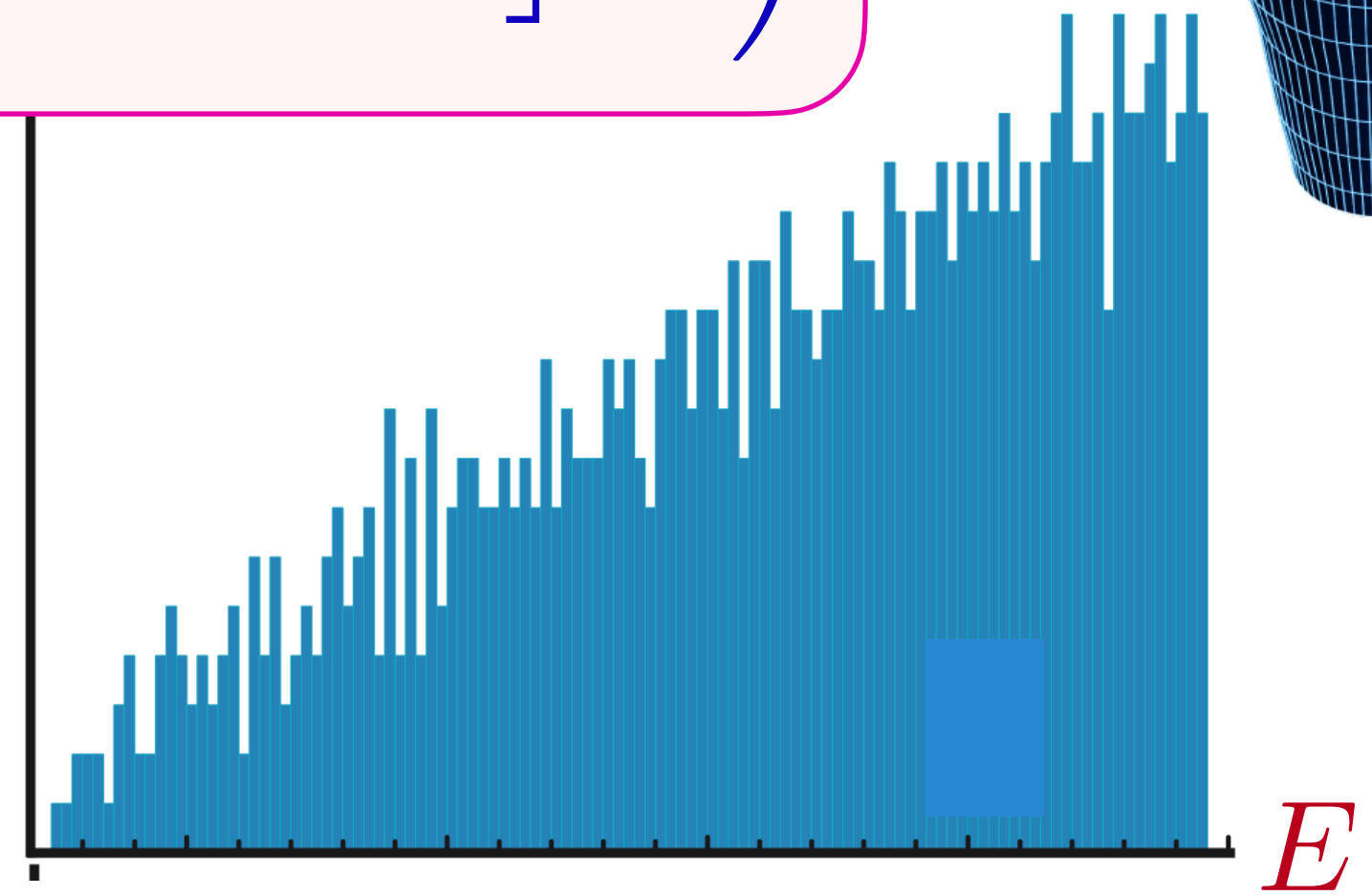
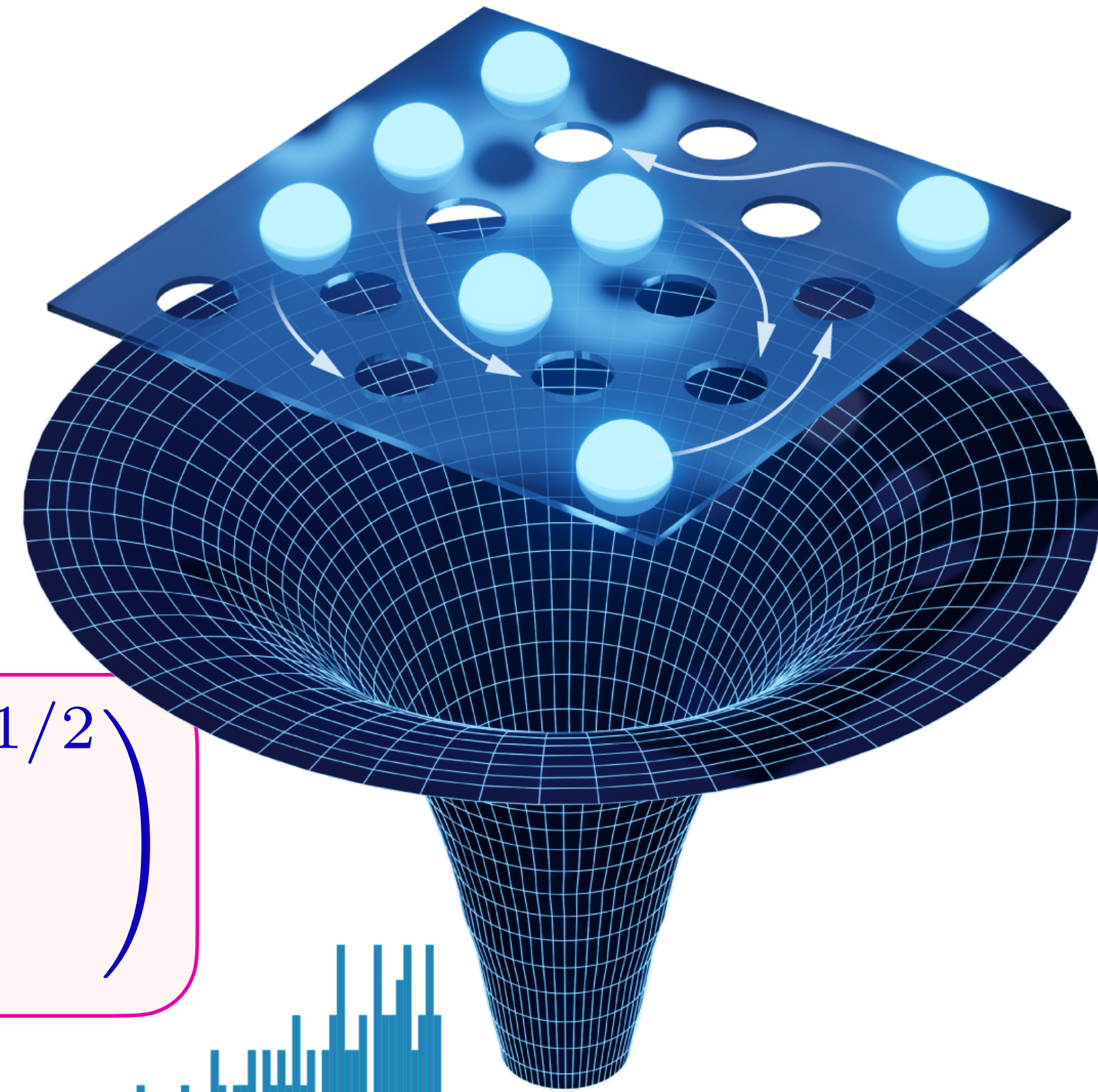
- For generic charged black holes in 3+1 dimensions with horizon area A at $T = 0$ and fixed charge Q ($A = 2GQ^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{Ac^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{Ac^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



Recap

Great discoveries in physics

Entropy (1870)

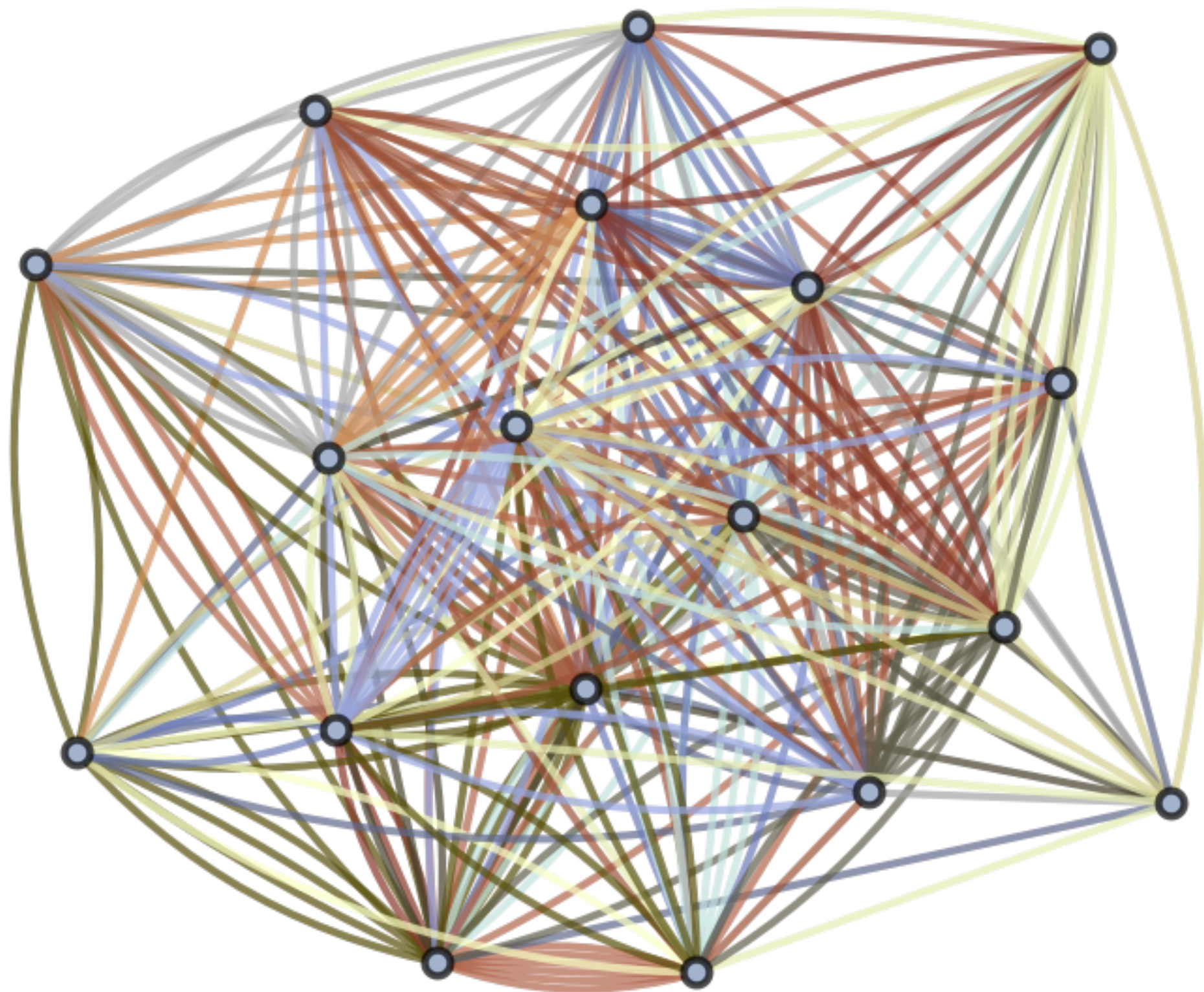
Superconductivity (1911)

Black holes (1916)

Quantum entanglement (1935)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

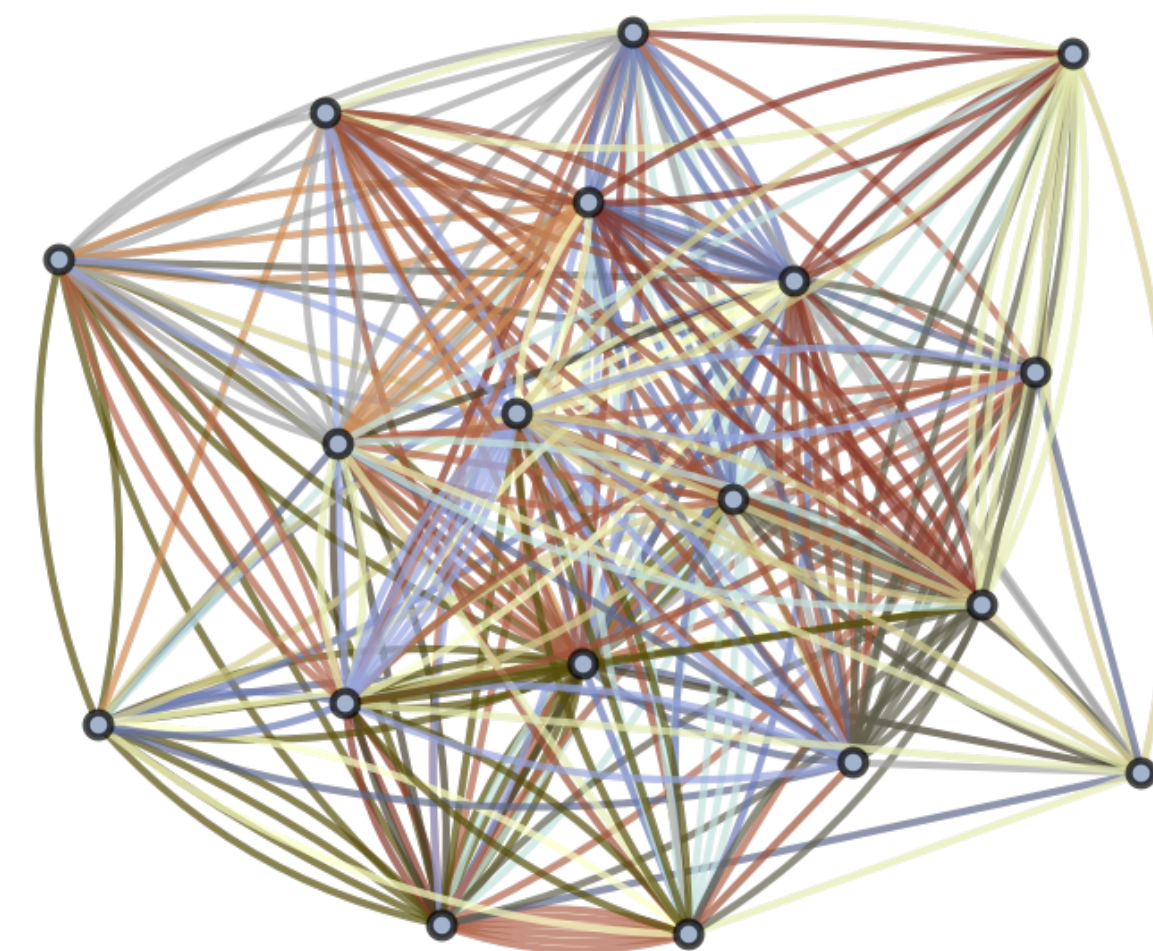
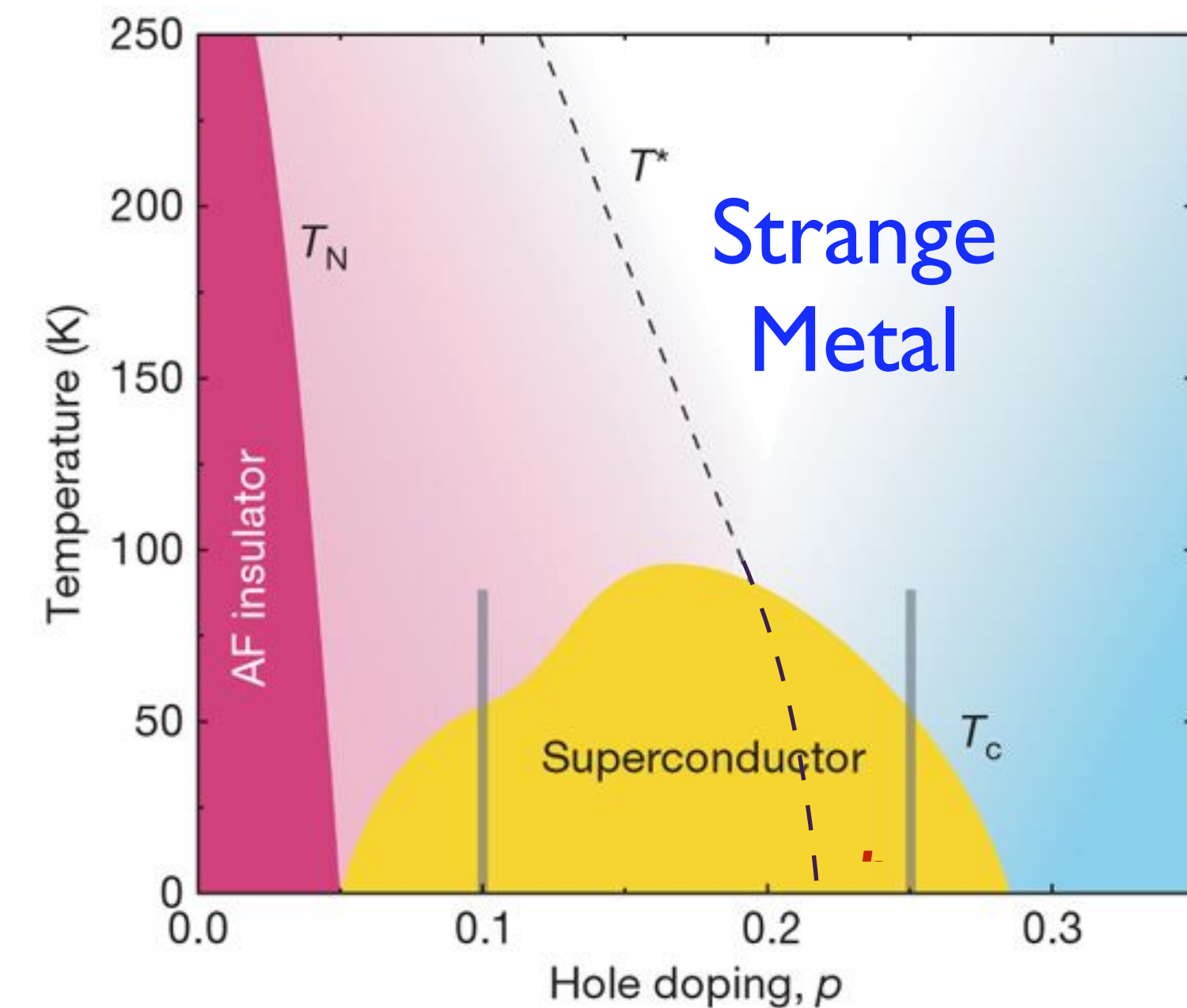


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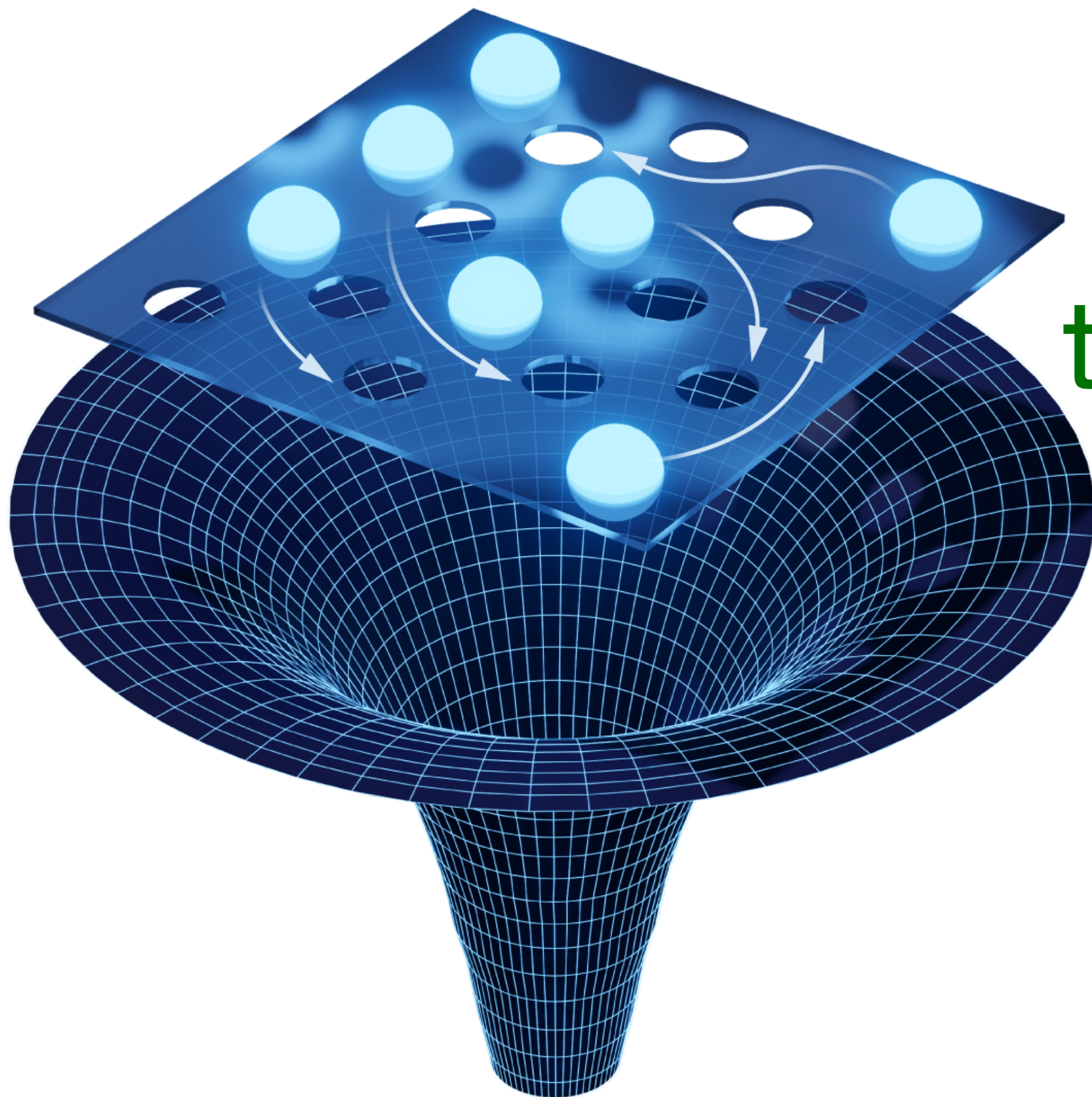
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



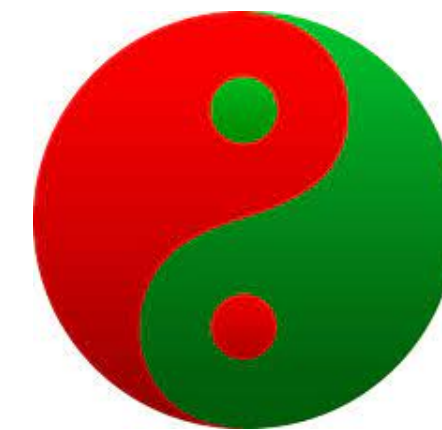
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In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)