

Universal theory of transport in strange metals

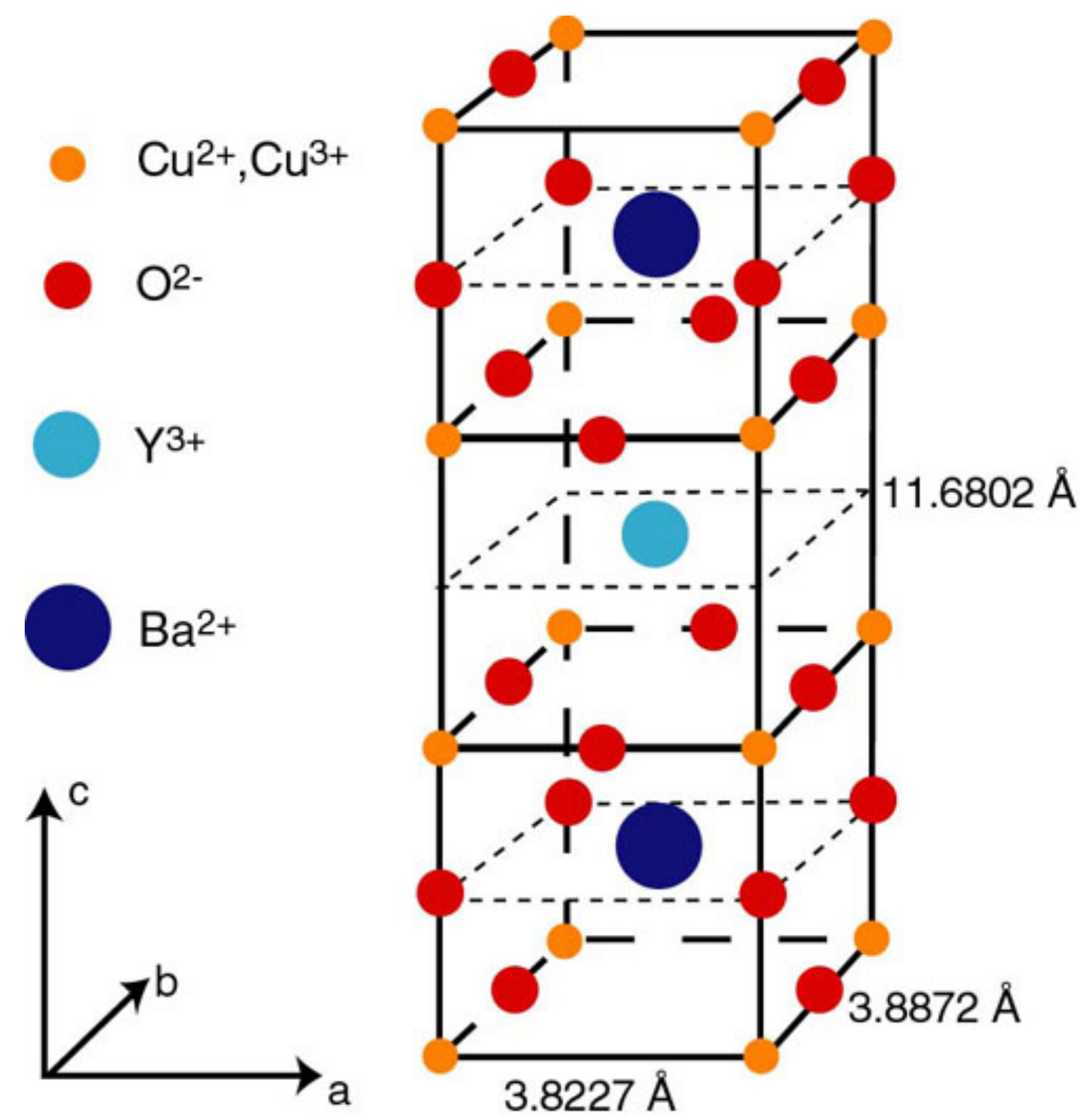
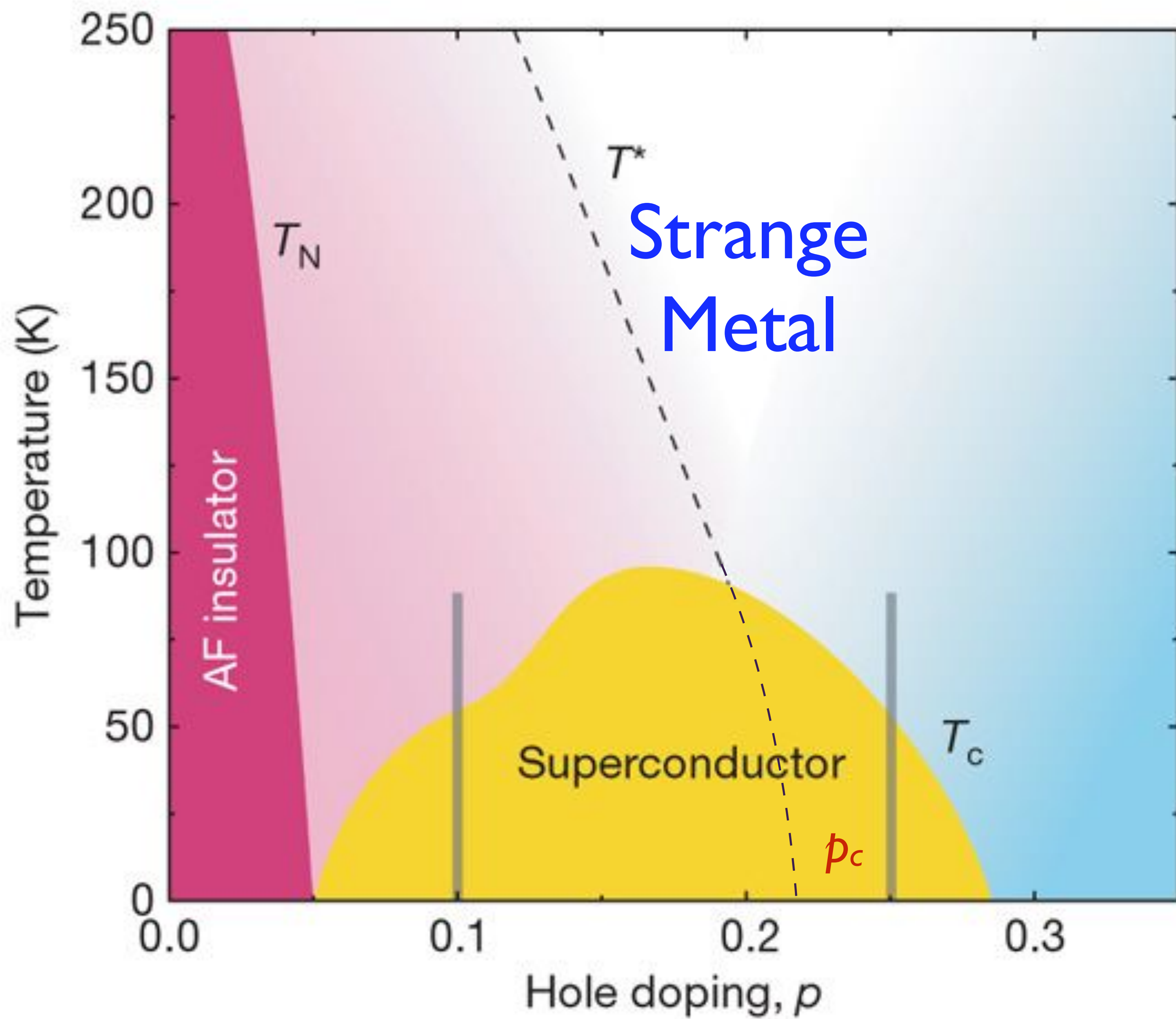
Pan-TIFR condensed matter physics conference
Tata Institute of Fundamental Research, Hyderabad
December 15, 2023

Subir Sachdev



Talk online: sachdev.physics.harvard.edu

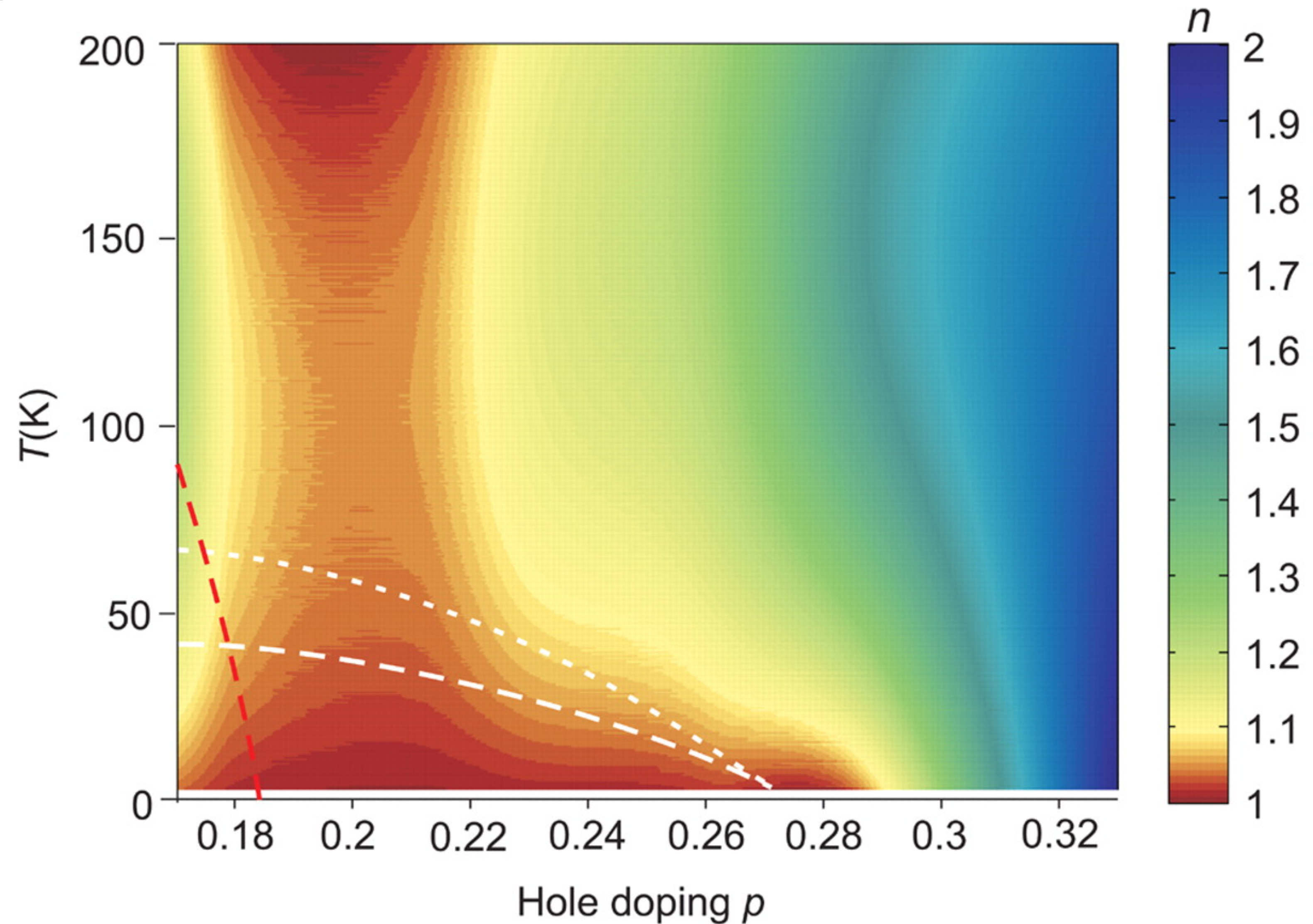
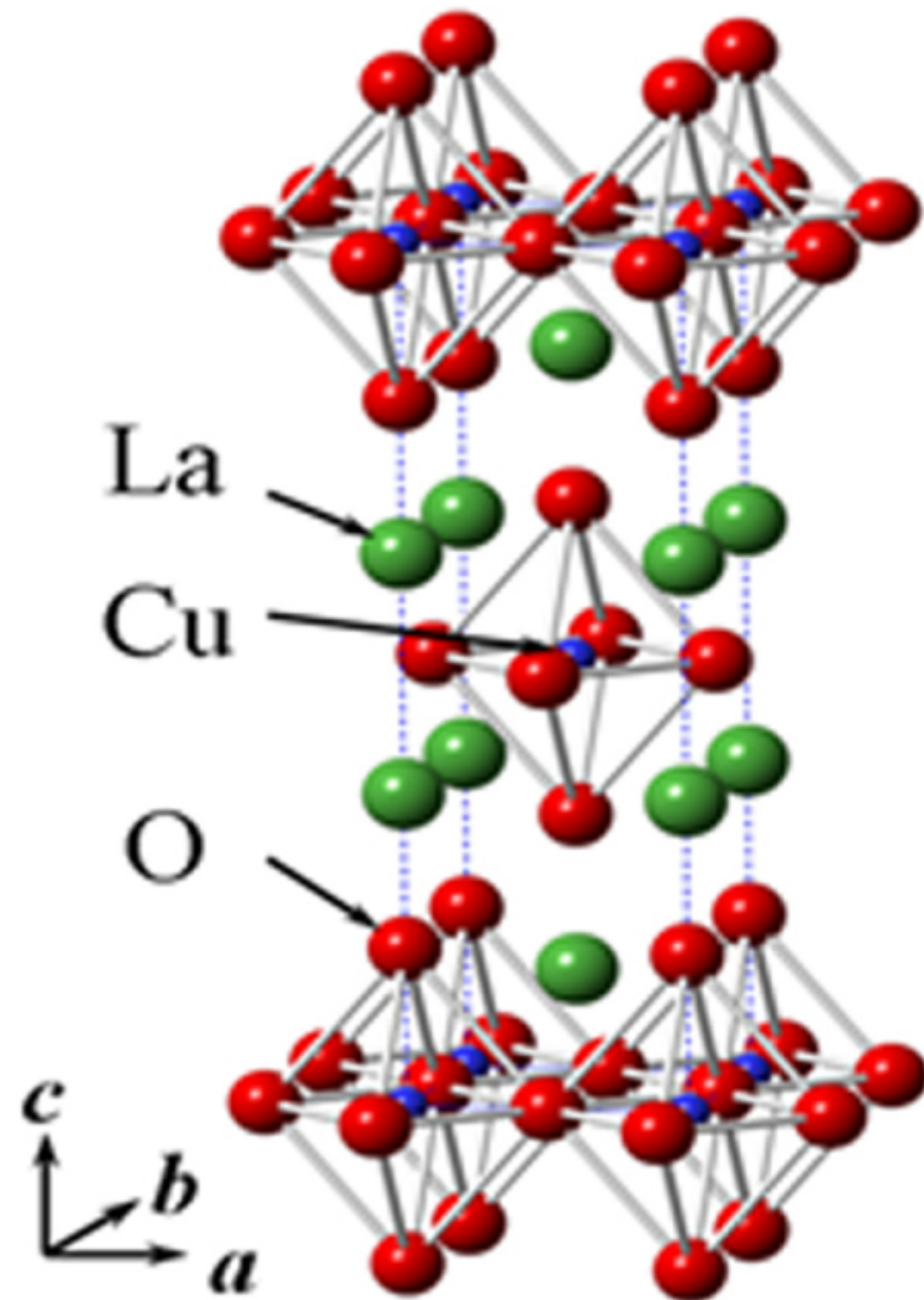




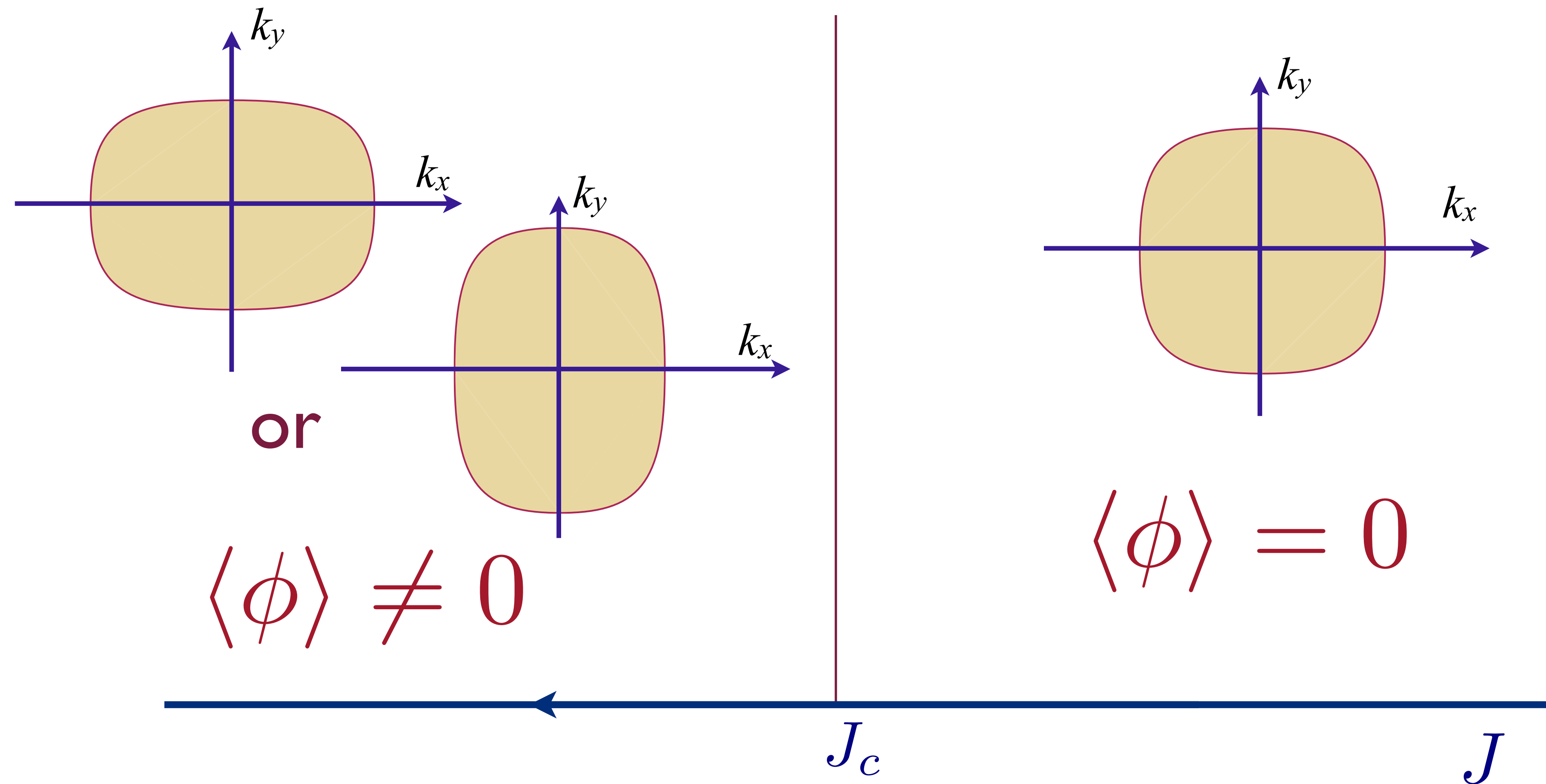
Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

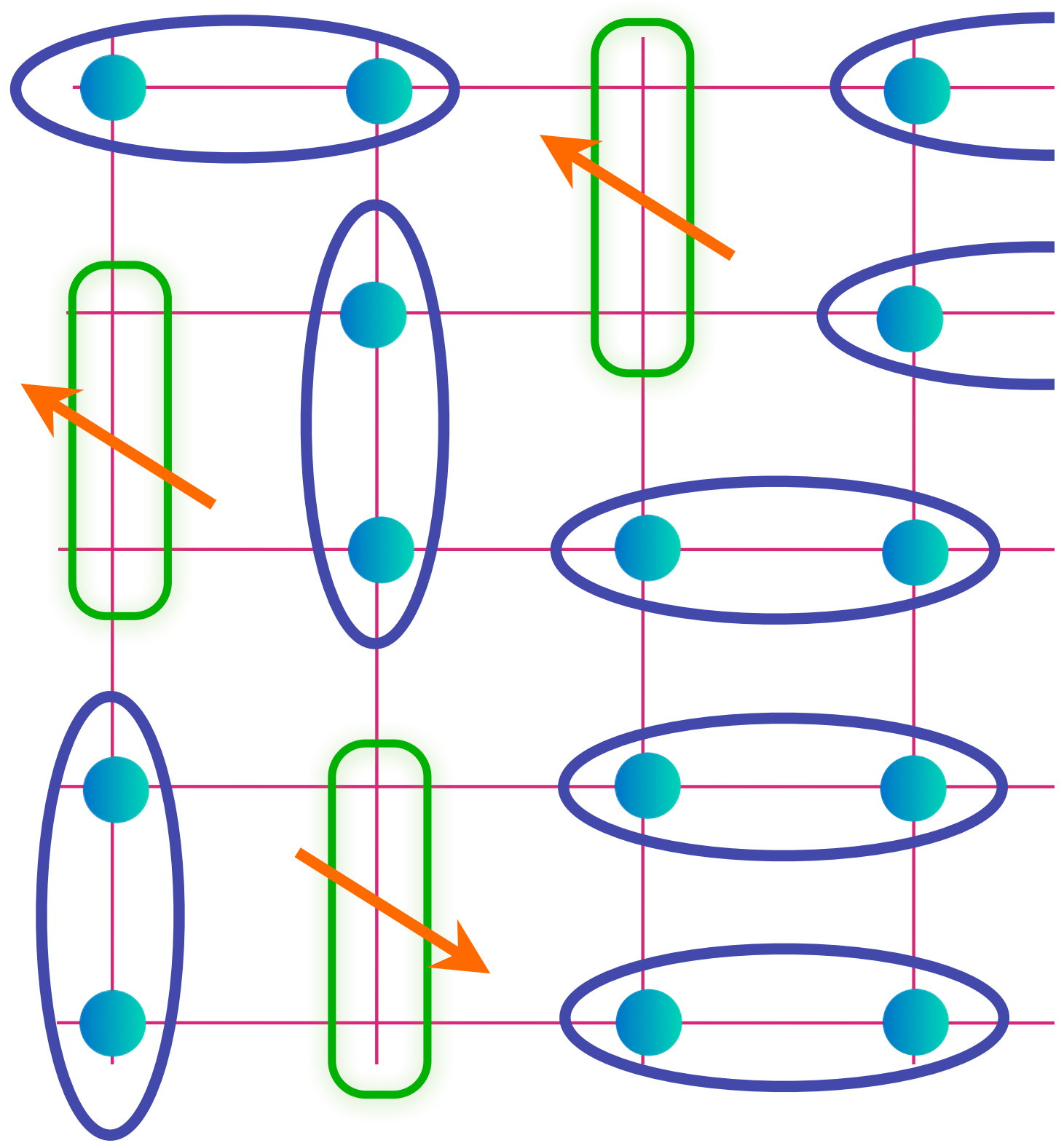


Quantum criticality of Ising-nematic ordering in a metal



Pommeranchuk instability as a function of coupling J

Pseudogap metal to Fermi liquid in single band model



Higgs boson with Φ the fundamental gauge charge of an emergent SU(2) gauge field.

$$\text{Blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{Green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Small Fermi surface of size p

+
spin liquid.

FL*

$$\langle \Phi \rangle \neq 0$$

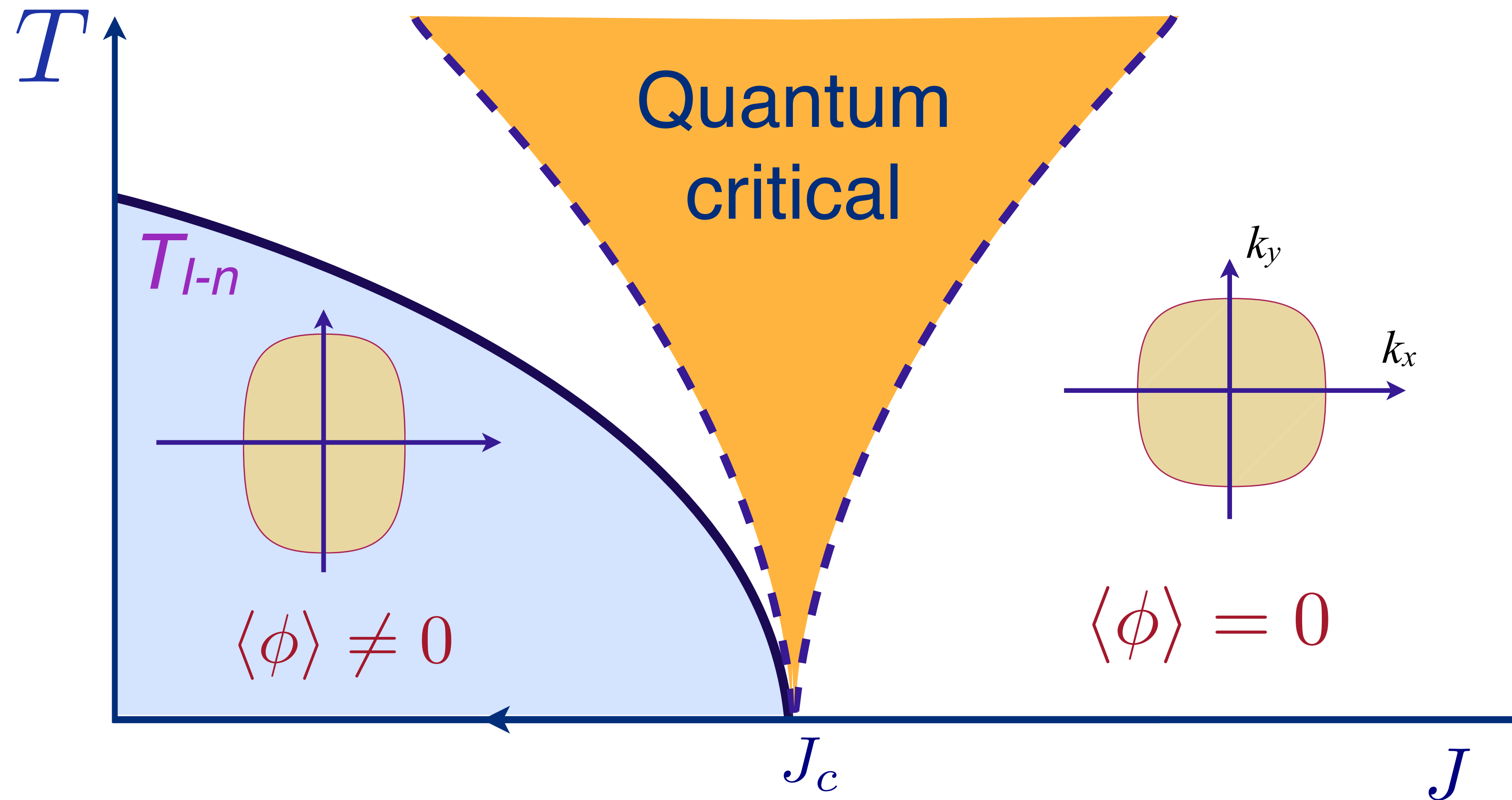
Large Fermi surface of size $1 + p$

FL

$$\langle \Phi \rangle = 0$$

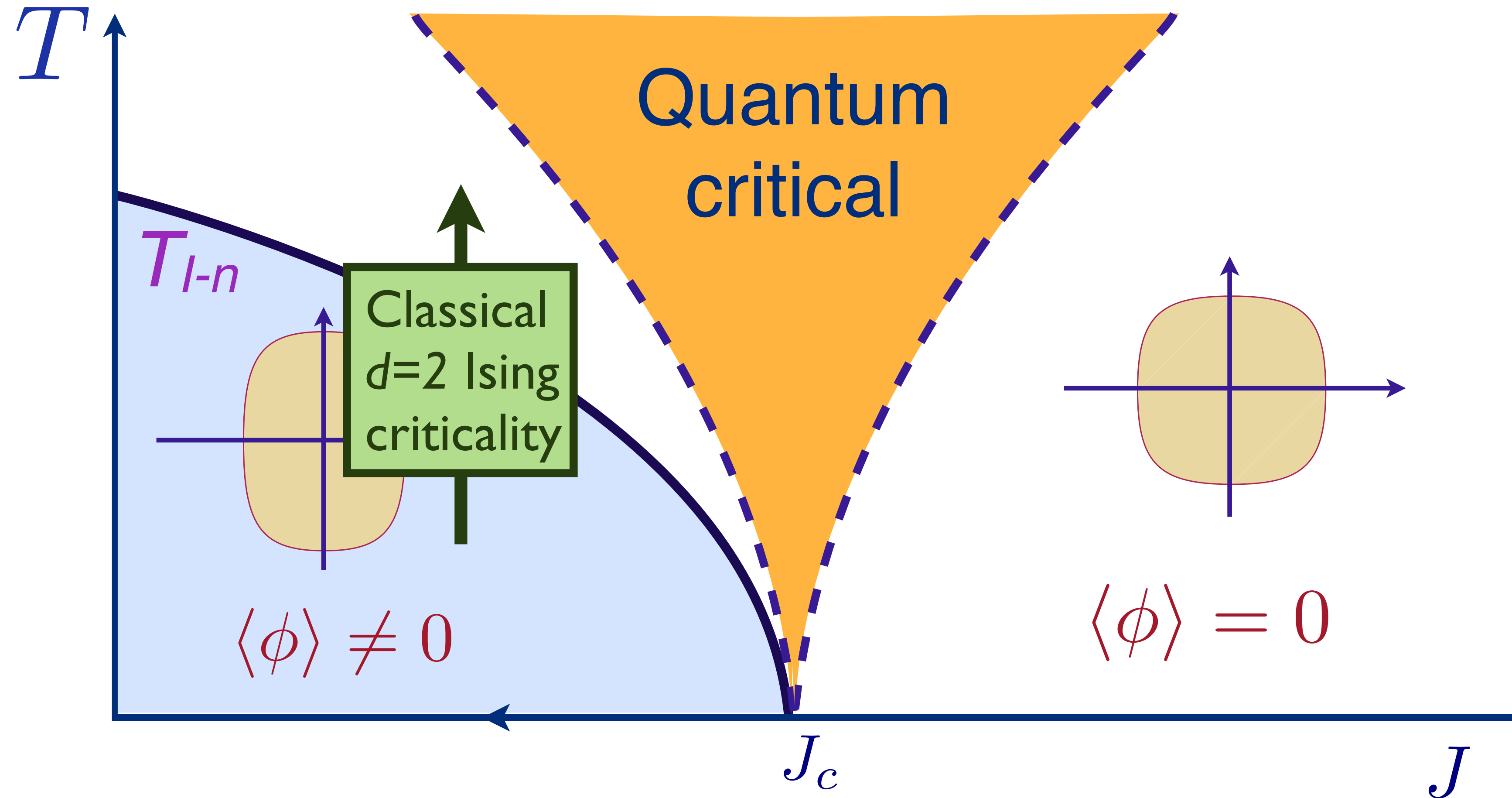
doping p

Quantum criticality of Ising-nematic ordering in a metal



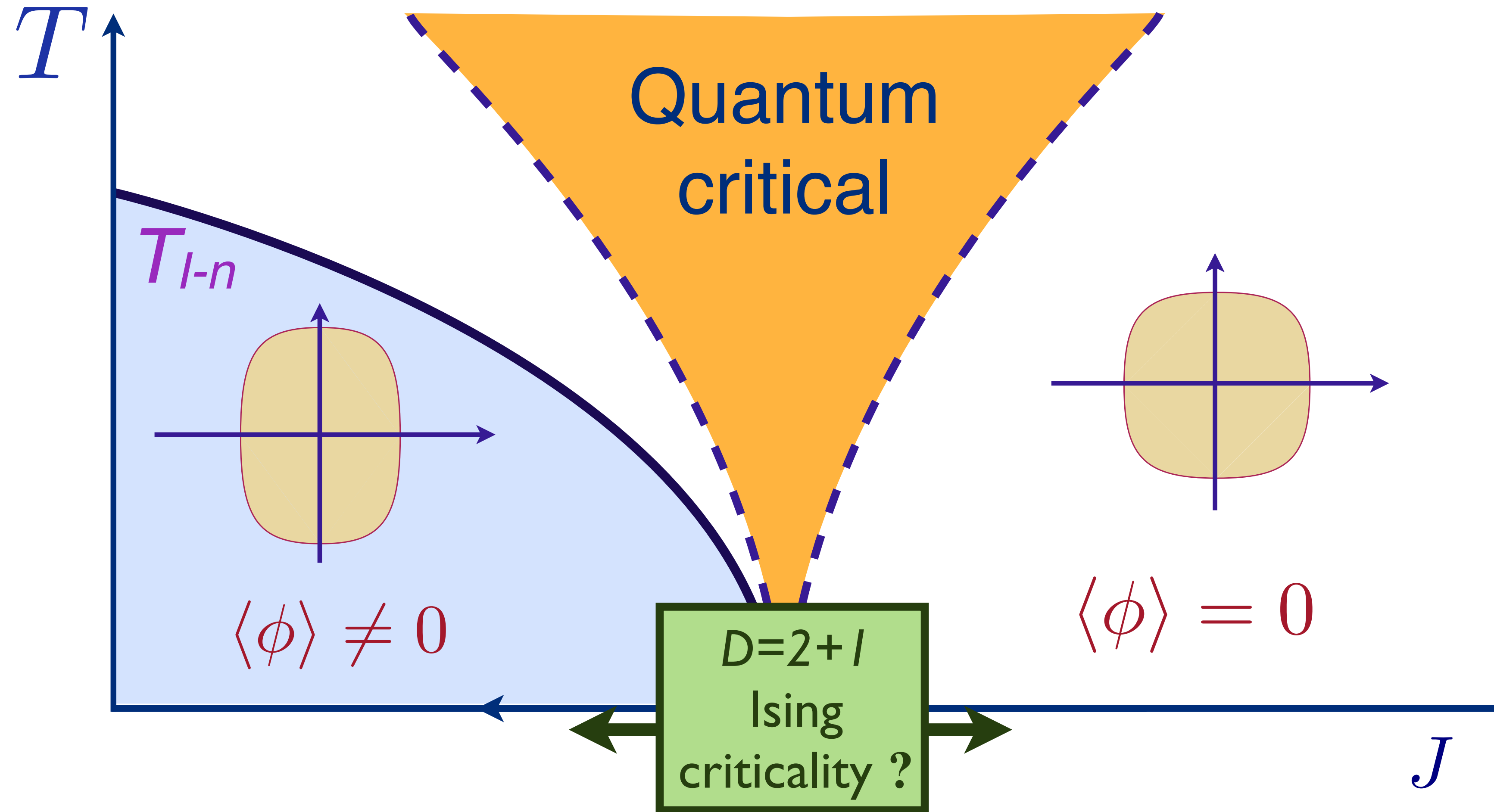
Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



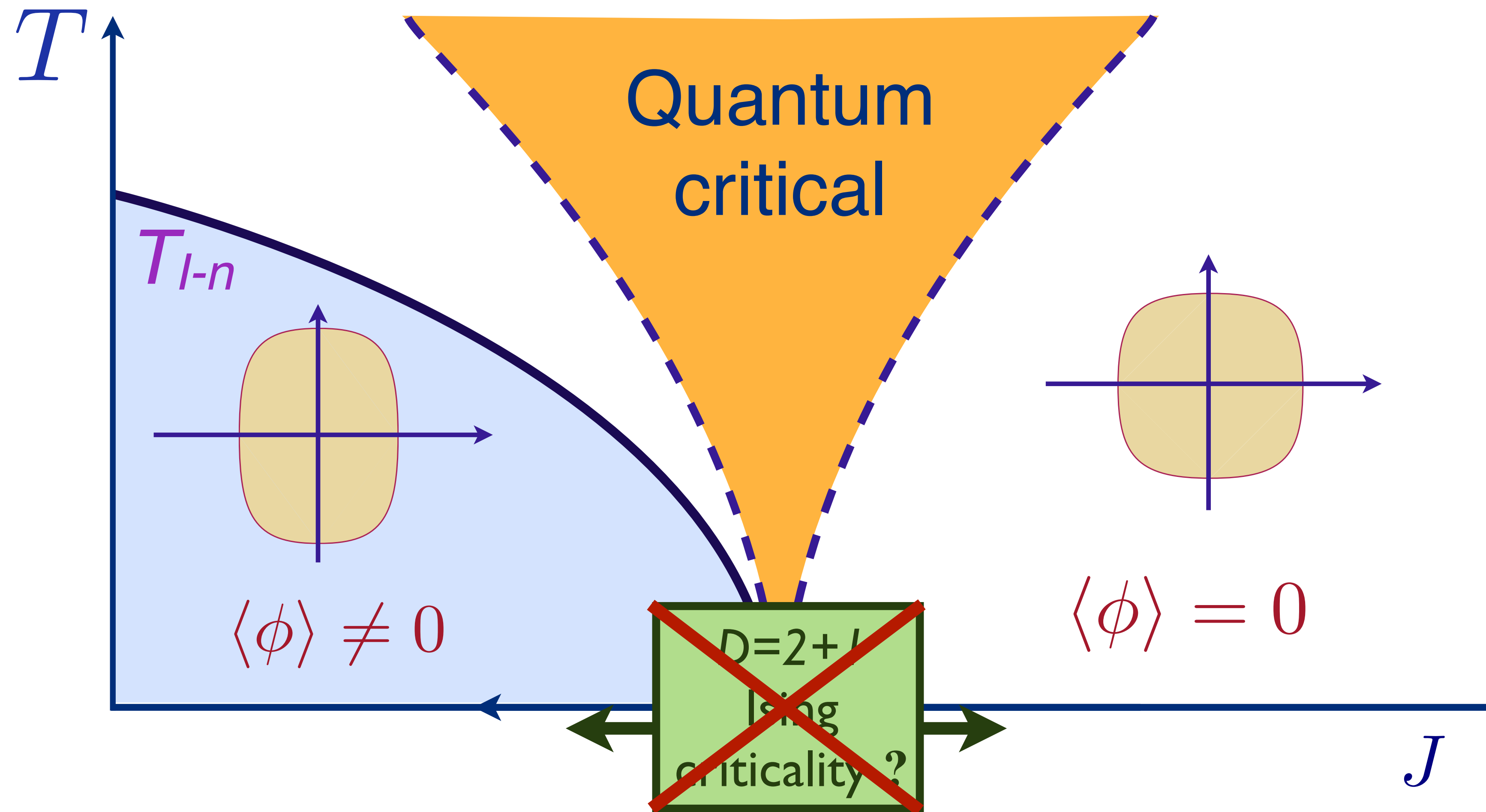
Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



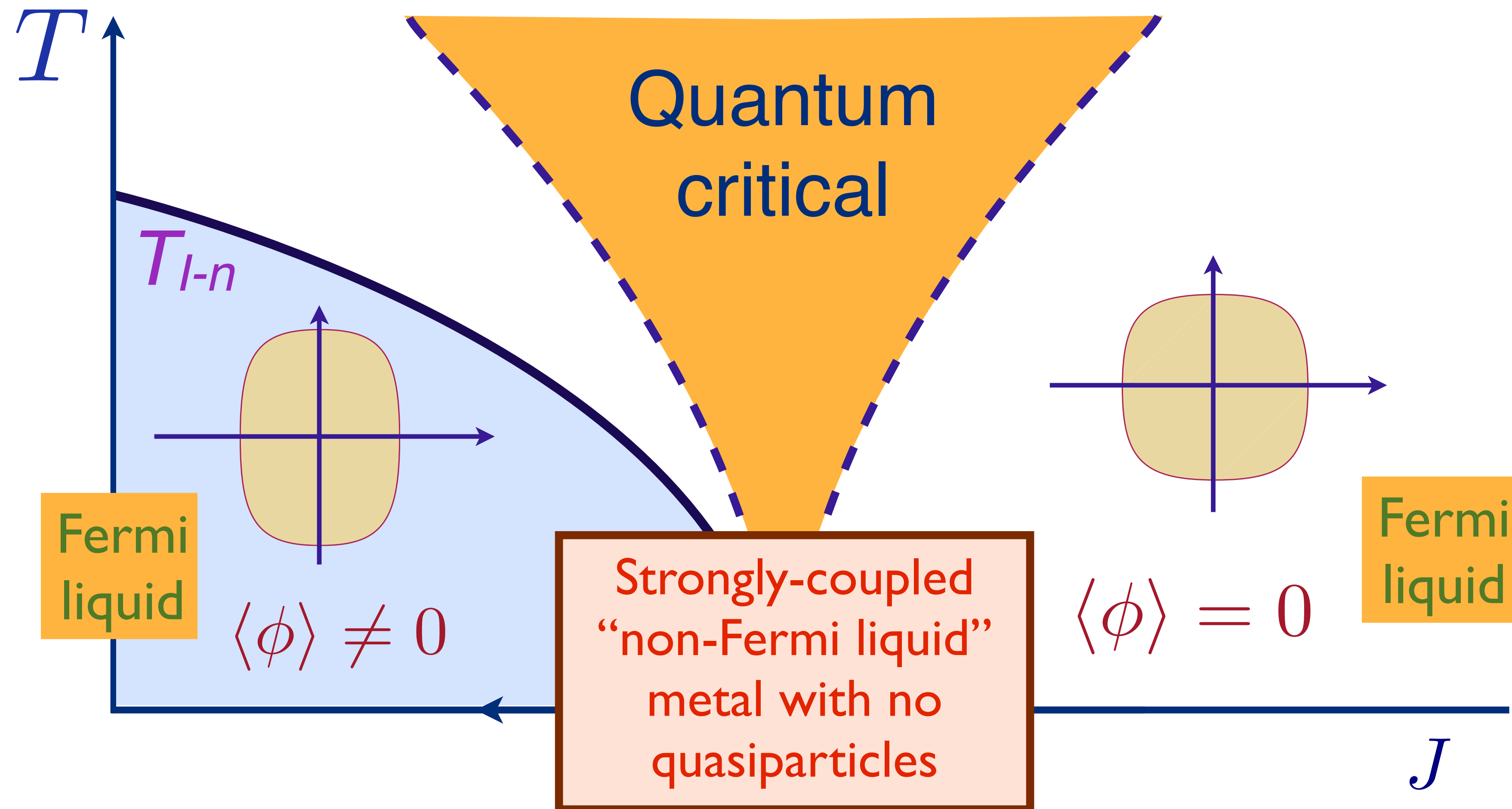
Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



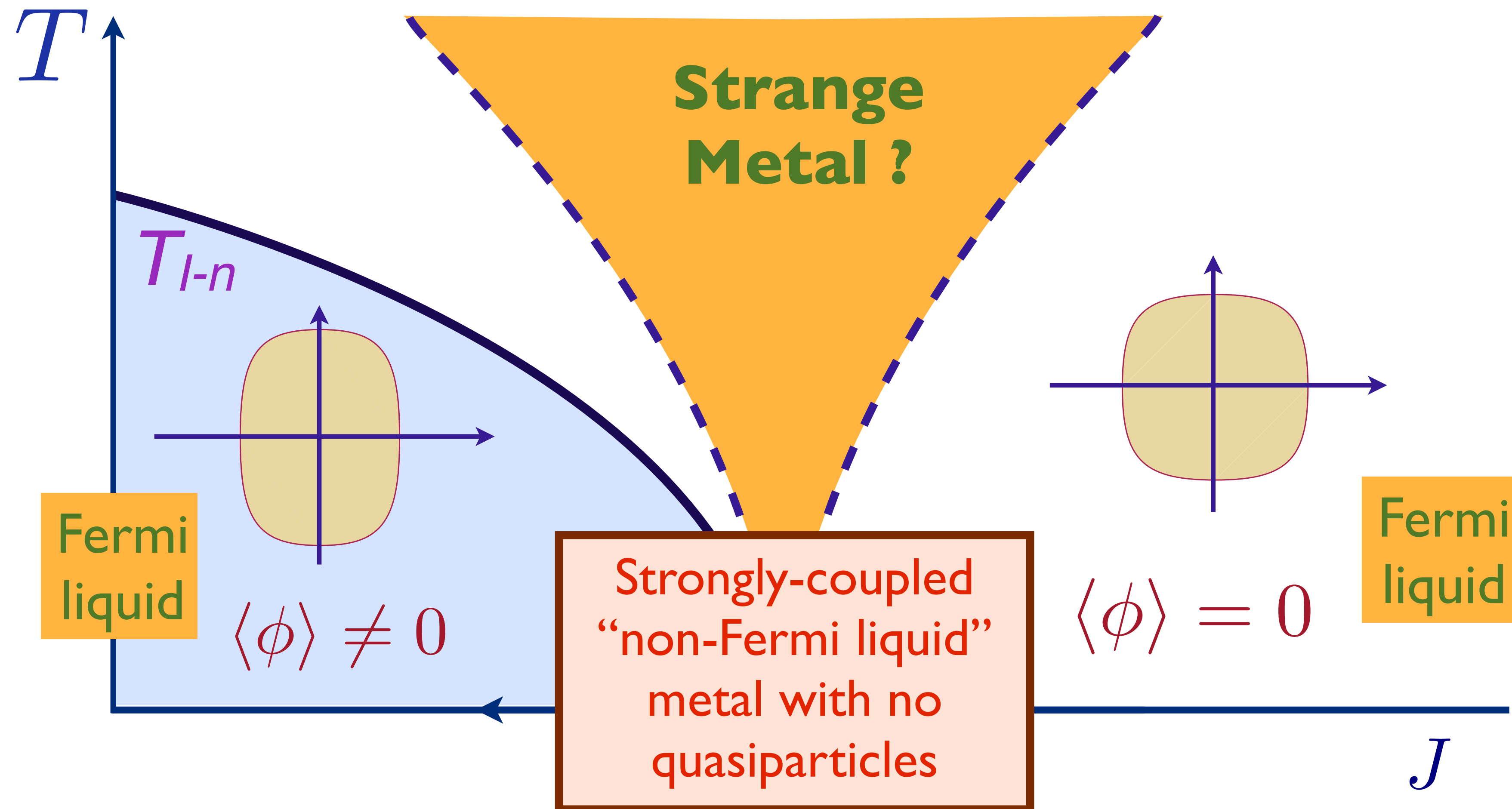
Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



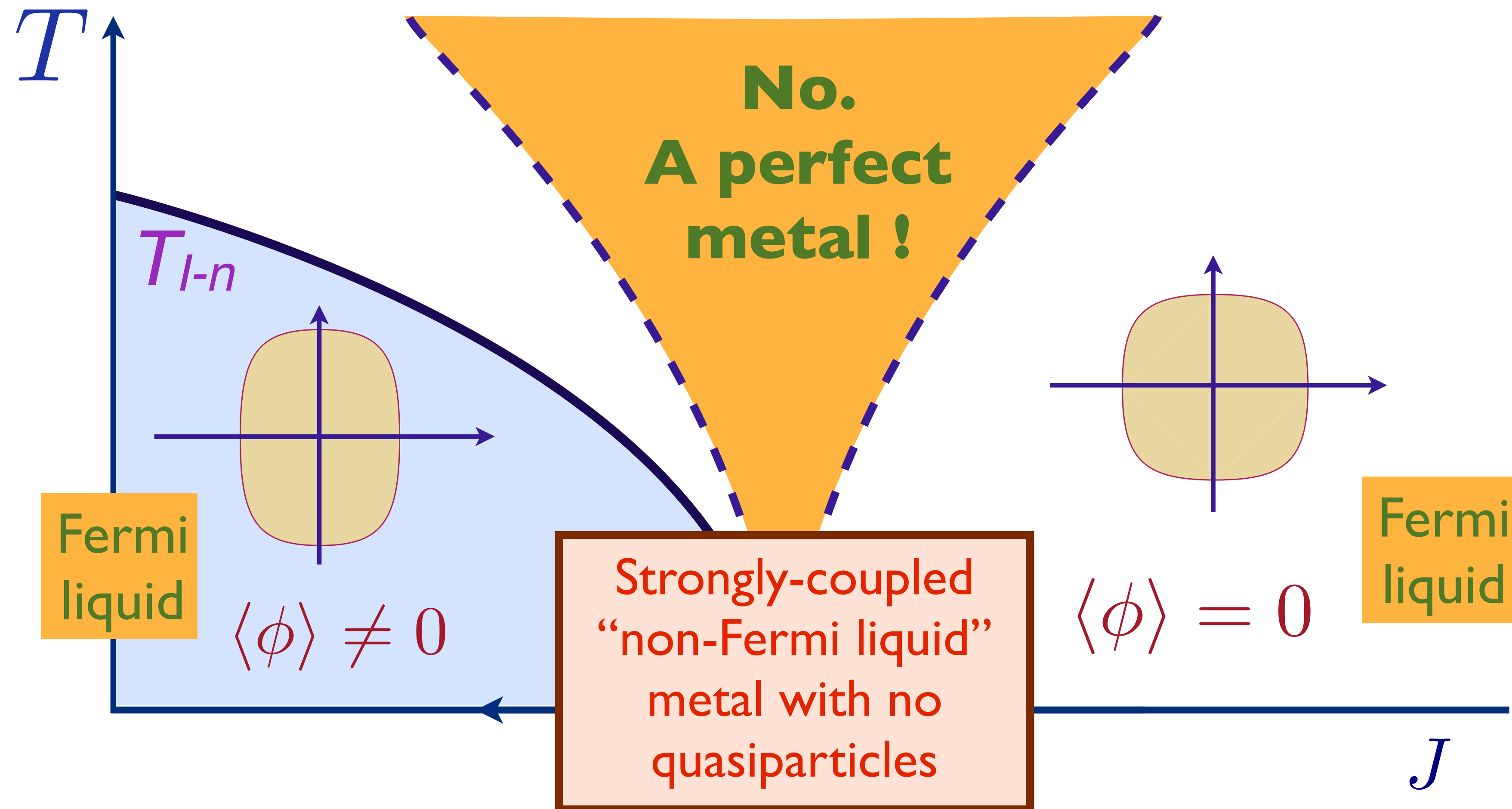
Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Kohn's Theorem

PHYSICAL REVIEW

VOLUME 123, NUMBER 4

AUGUST 15, 1961

Cyclotron Resonance and de Haas-van Alphen Oscillations of an Interacting Electron Gas*

WALTER KOHN

University of California at San Diego, La Jolla, California

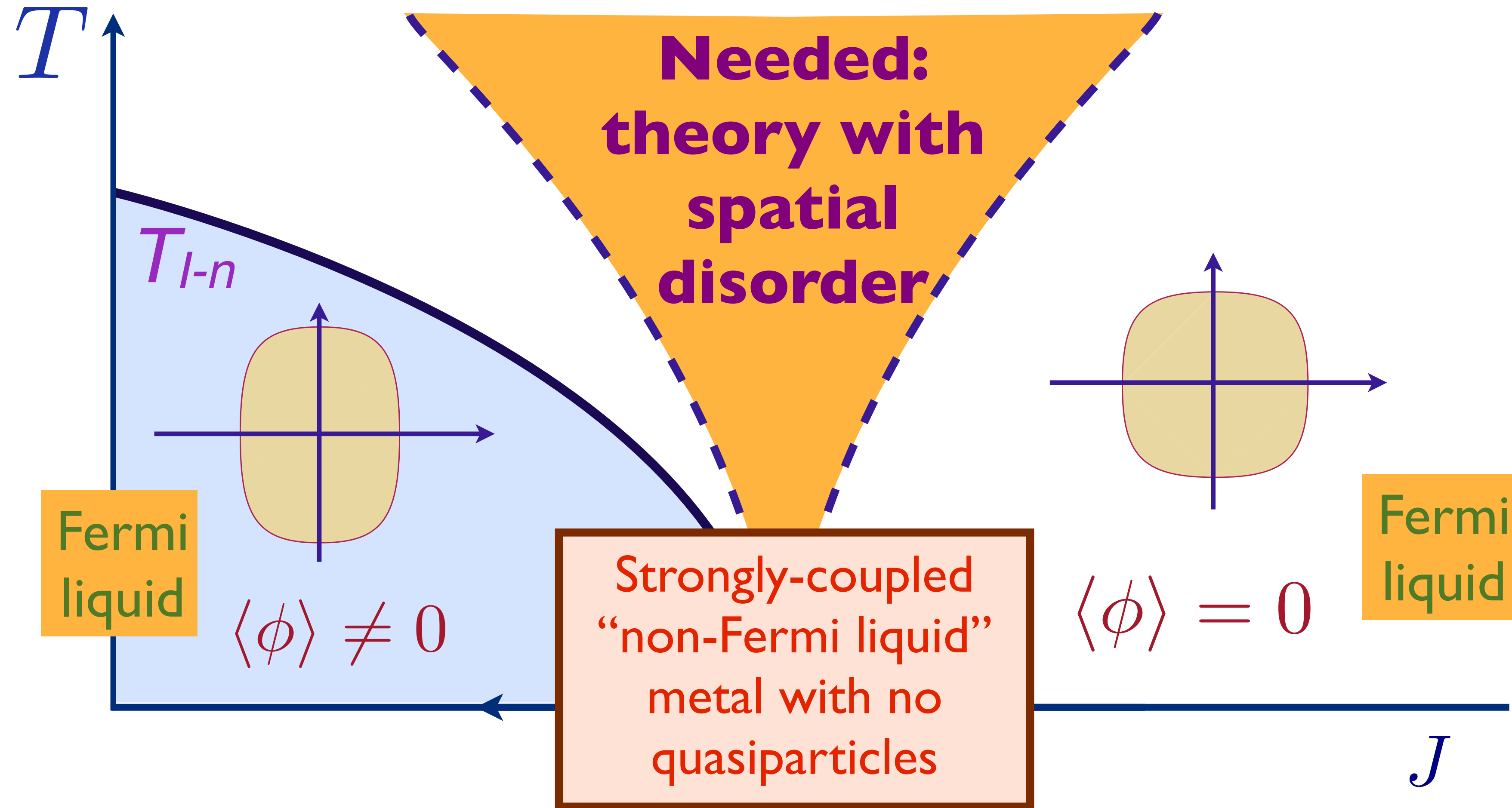
(Received April 5, 1961)

An electron gas with short-range interactions is considered in the presence of a uniform magnetic field. It is shown that (1) the cyclotron resonance frequency is independent of the interaction; (2) for a two-dimensional gas, the de Haas-van Alphen period is independent of the interaction. The low-lying excited states are briefly discussed.

In the absence of umklapp and impurities,
the Fermi liquid is a *perfect metal*.

$$\sigma(\omega) = iD/(\omega - \omega_c)$$

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J



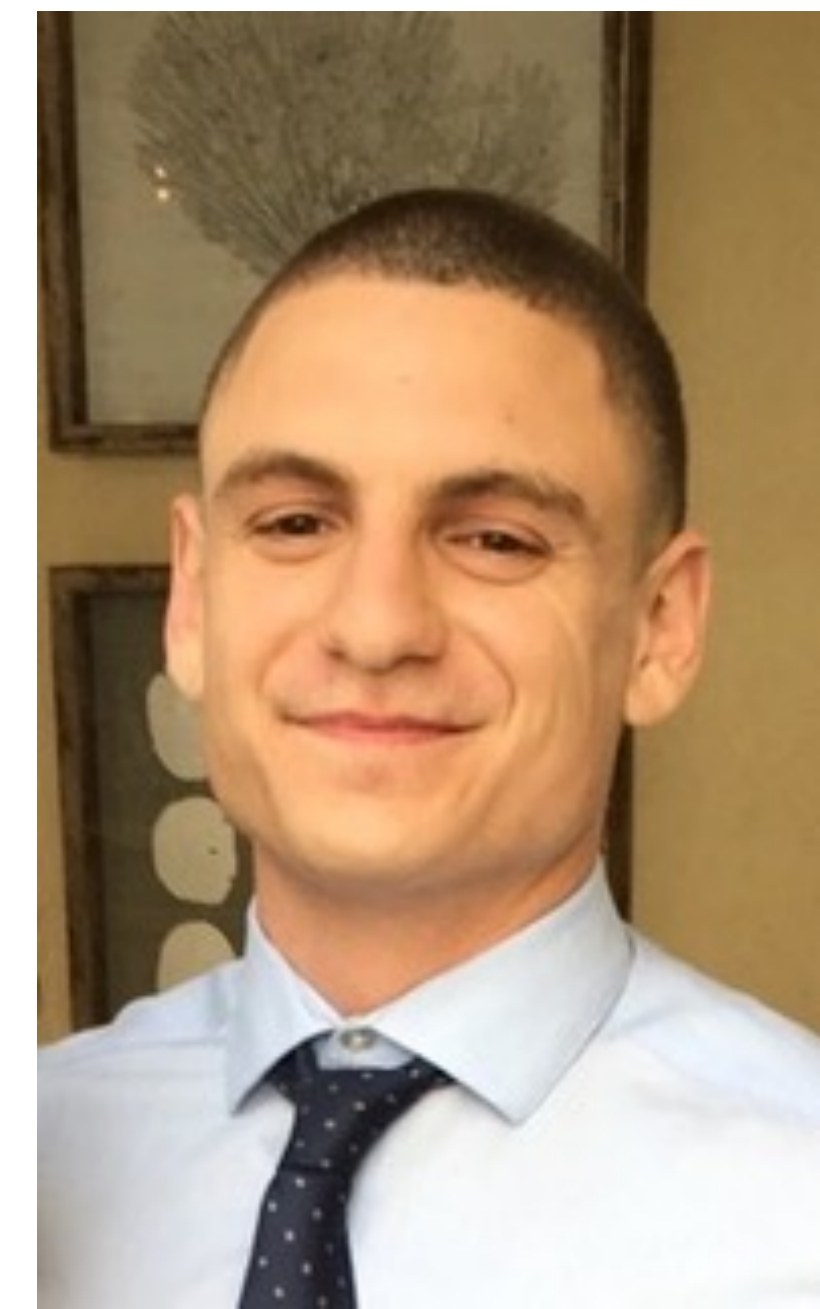
Aavishkar Patel

Flatiron Institute, NYC



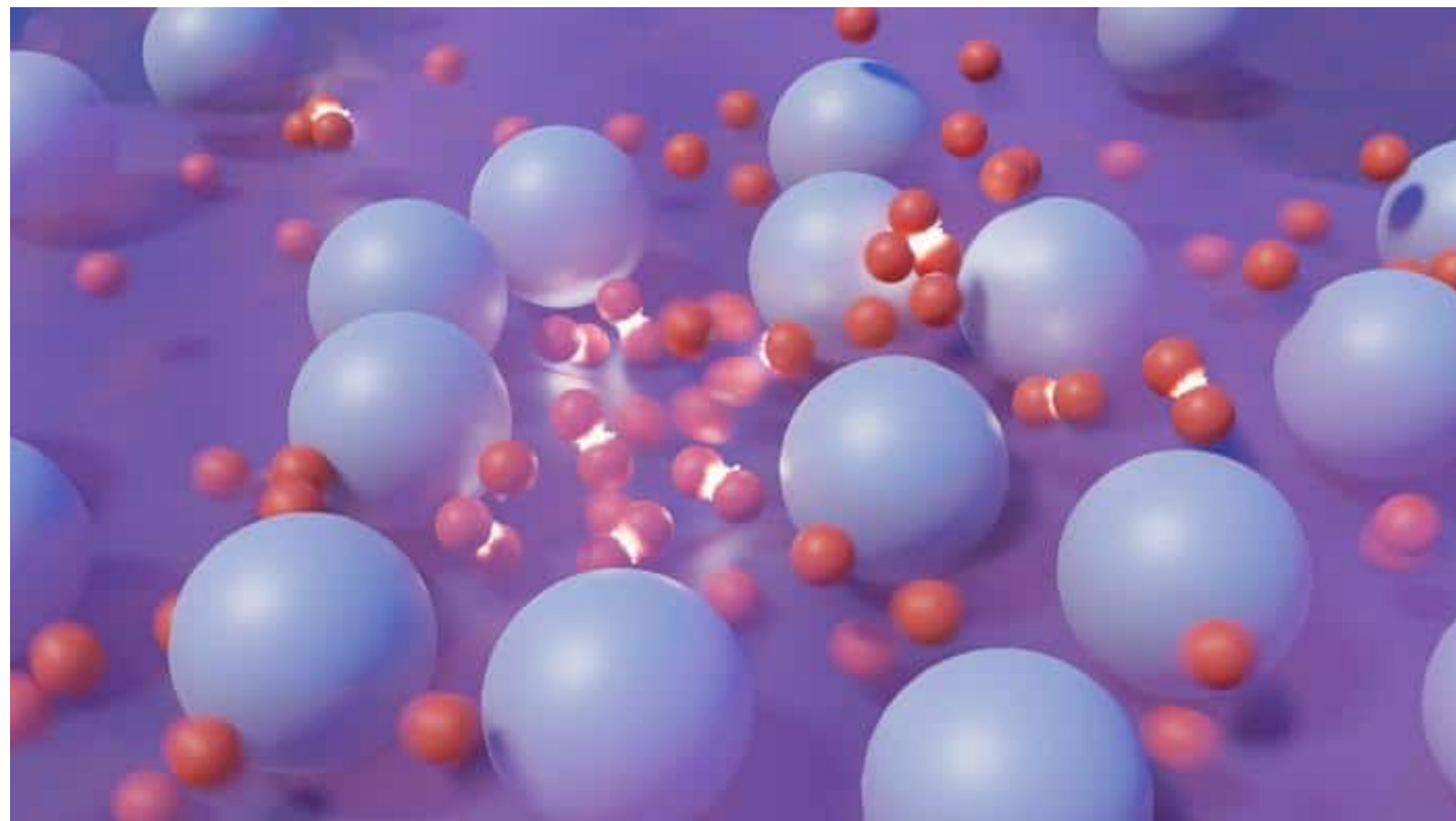
Haoyu Guo

Cornell



Ilya Esterlis

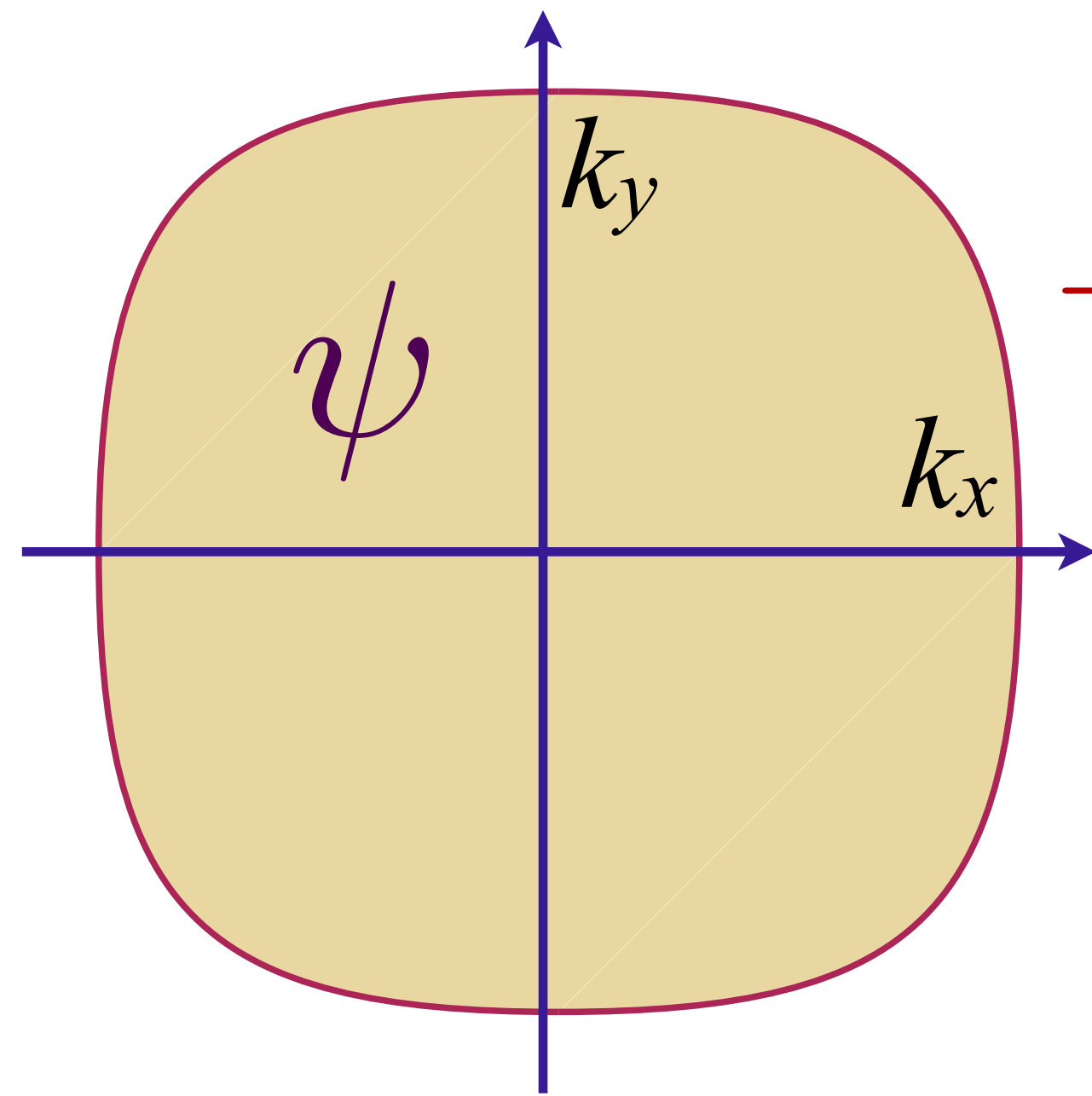
Wisconsin



Universal theory of strange metals from
spatially random interactions,
Aavishkar A. Patel, Haoyu Guo,
Ilya Esterlis, and S. Sachdev,
Science **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

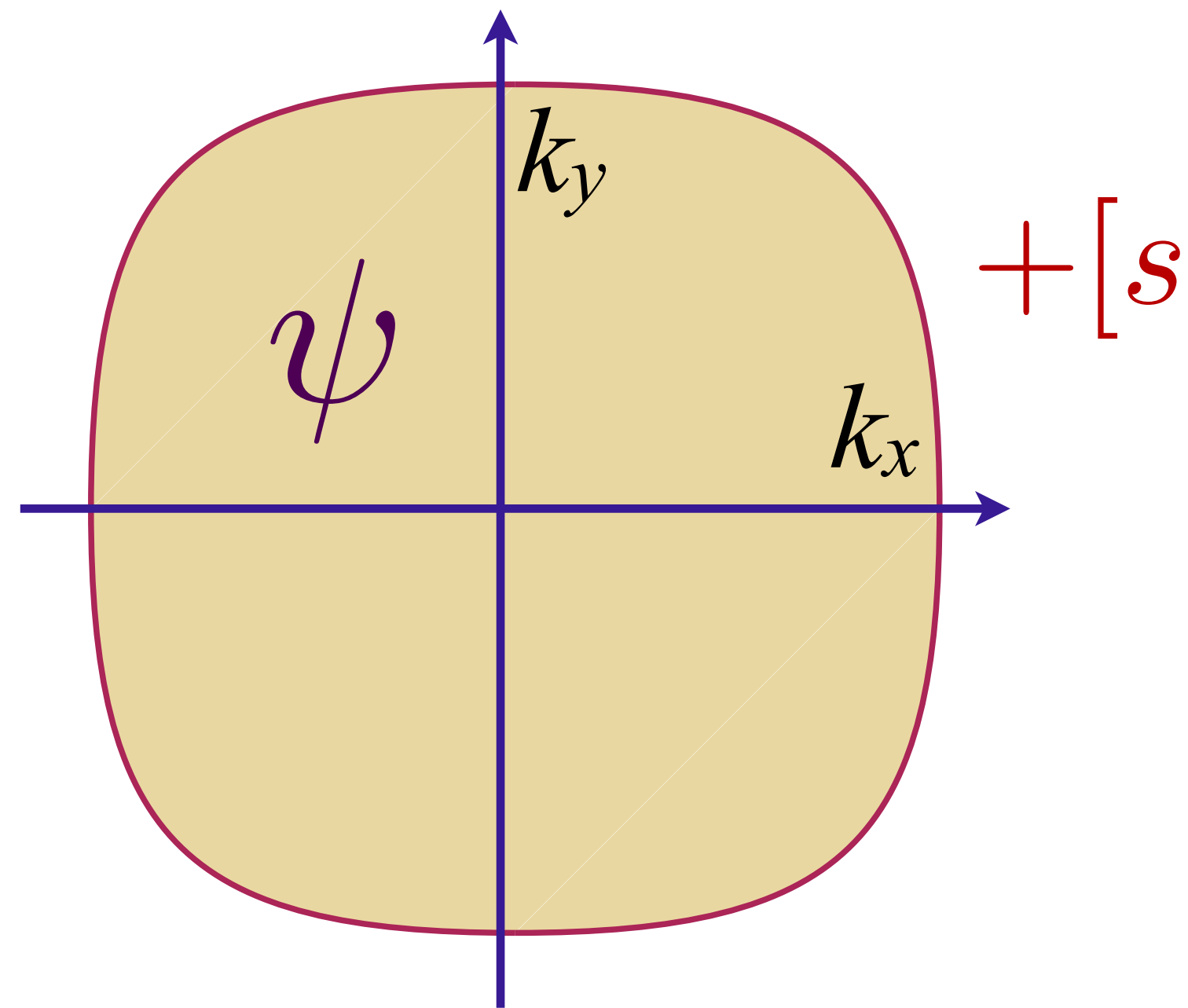
Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

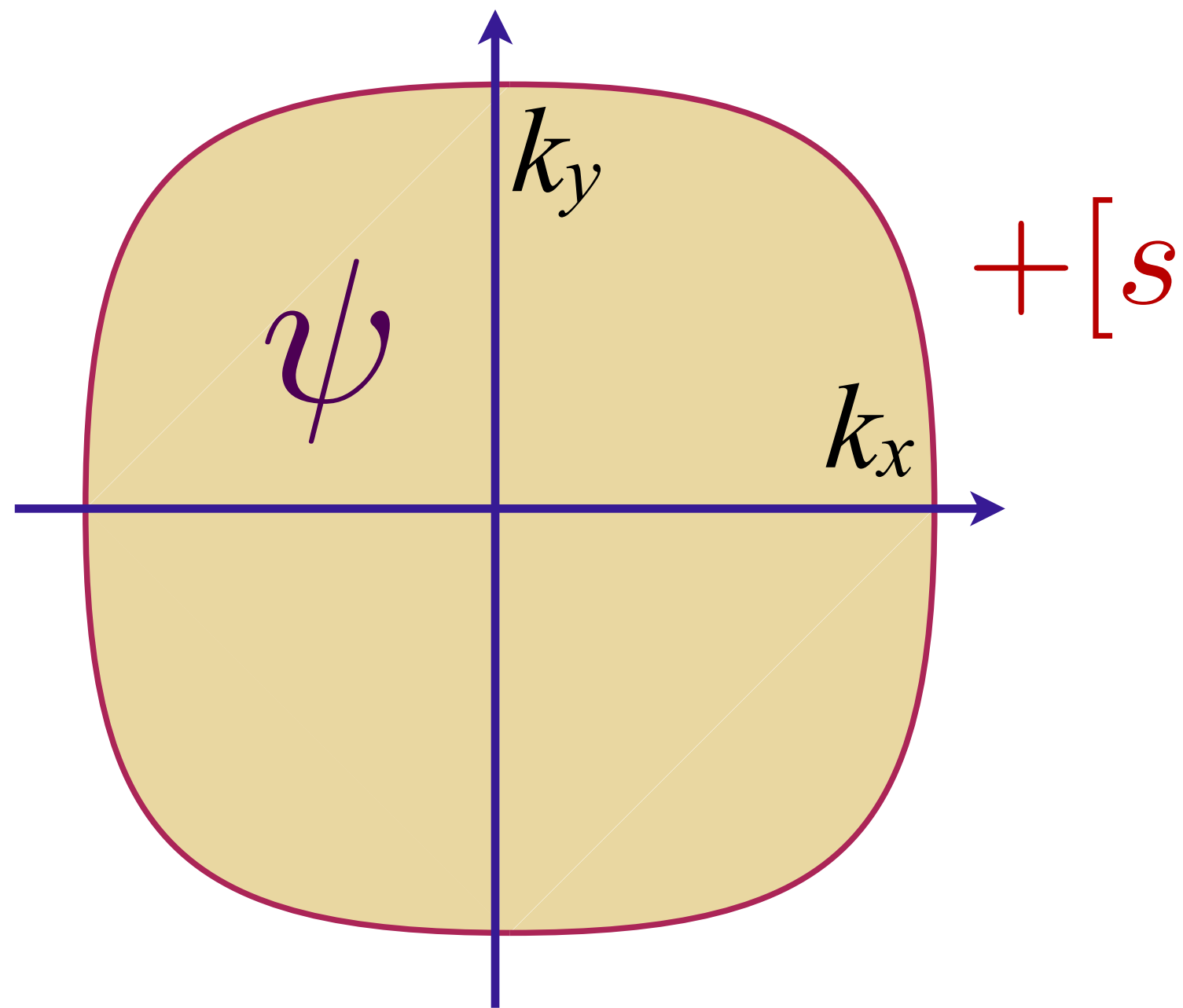
$$+ [s + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Rescale $\phi(\mathbf{r})$ to absorb random mass $\delta s(\mathbf{r})$ into
 the random Yukawa coupling $g'(\mathbf{r})$
 and analyze with self-averaging as in Yukawa-SYK model.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Rescale $\phi(\mathbf{r})$ to absorb random mass $\delta s(\mathbf{r})$ into
 the random Yukawa coupling $g'(\mathbf{r})$

and analyze with self-averaging as in Yukawa-SYK model.

Should be applicable as long as eigenmodes of $\phi(\mathbf{r})$ are extended.

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

Leads to fully self-consistent Migdal-Eliashberg equations
 $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)

A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)

E. Marcus and S. Vandoren, JHEP 01, 166 (2018)

Yuxuan Wang, PRL **124**, 017002 (2020)

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)

W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

See also Yuxuan Wang, PRL **124**, 017002 (2020)

Fermi surface + critical boson with potential and interaction disorder

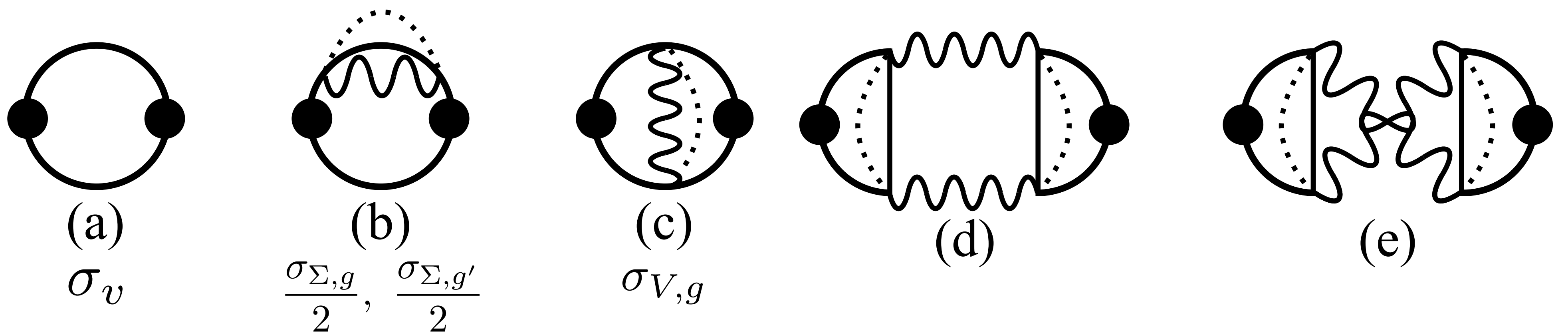
SYK-type self-consistent equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$



+ all ladders and bubbles.....

Conductivity:

Fermi surface + critical boson with potential and interaction disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ; Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Random “mass” disorder at quantum critical points in metals

Aavishkar A. Patel, Peter Lunts, S. Sachdev, *arXiv:2312.06751*



Aavishkar Patel
Flatiron Institute, NYC



Peter Lunts
Harvard

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left(\frac{\lambda + \lambda'_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$

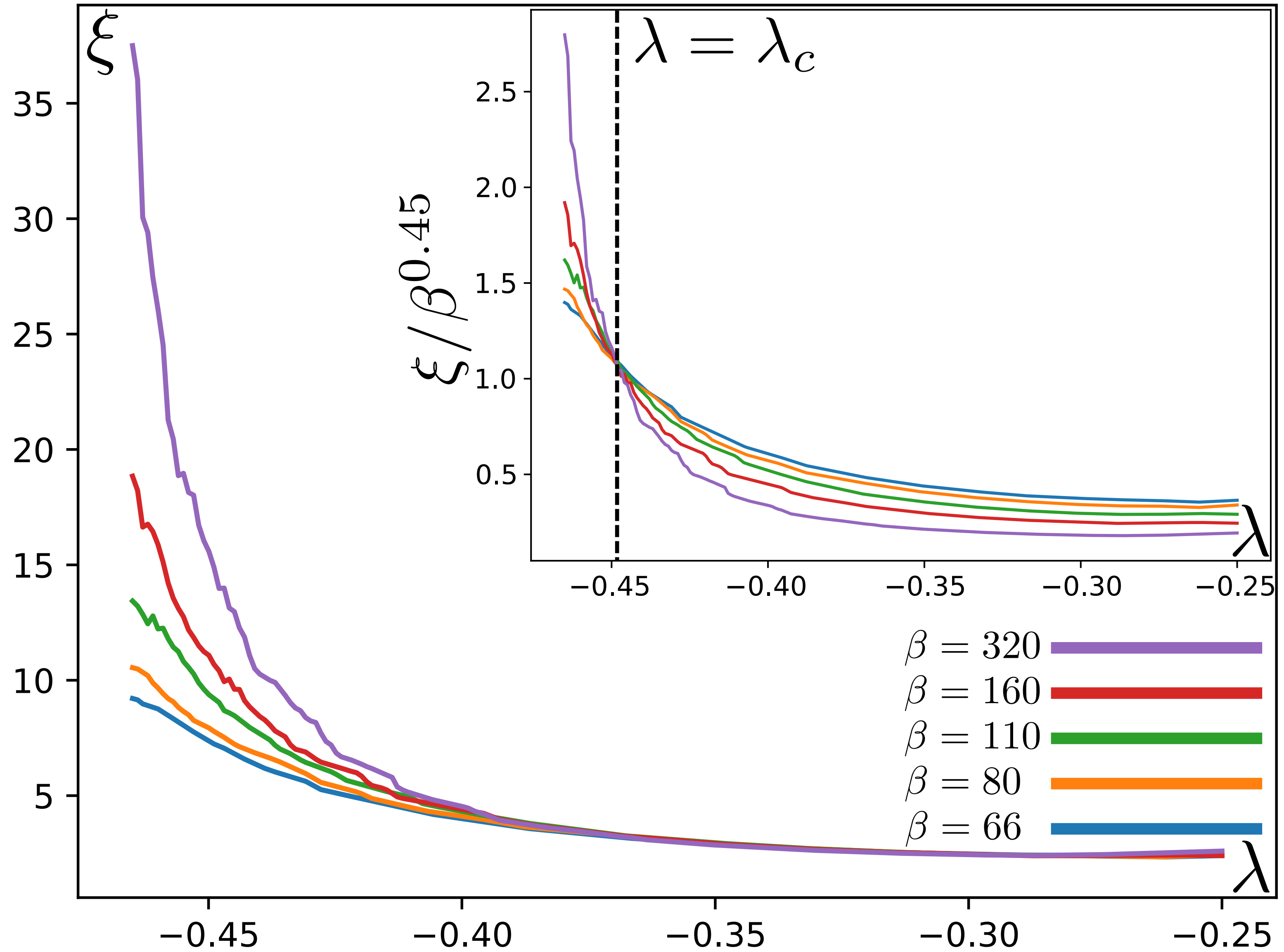
$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma |\Omega| + \Omega^2 / c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots N$ is a flavor index for an order parameter with $O(N)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

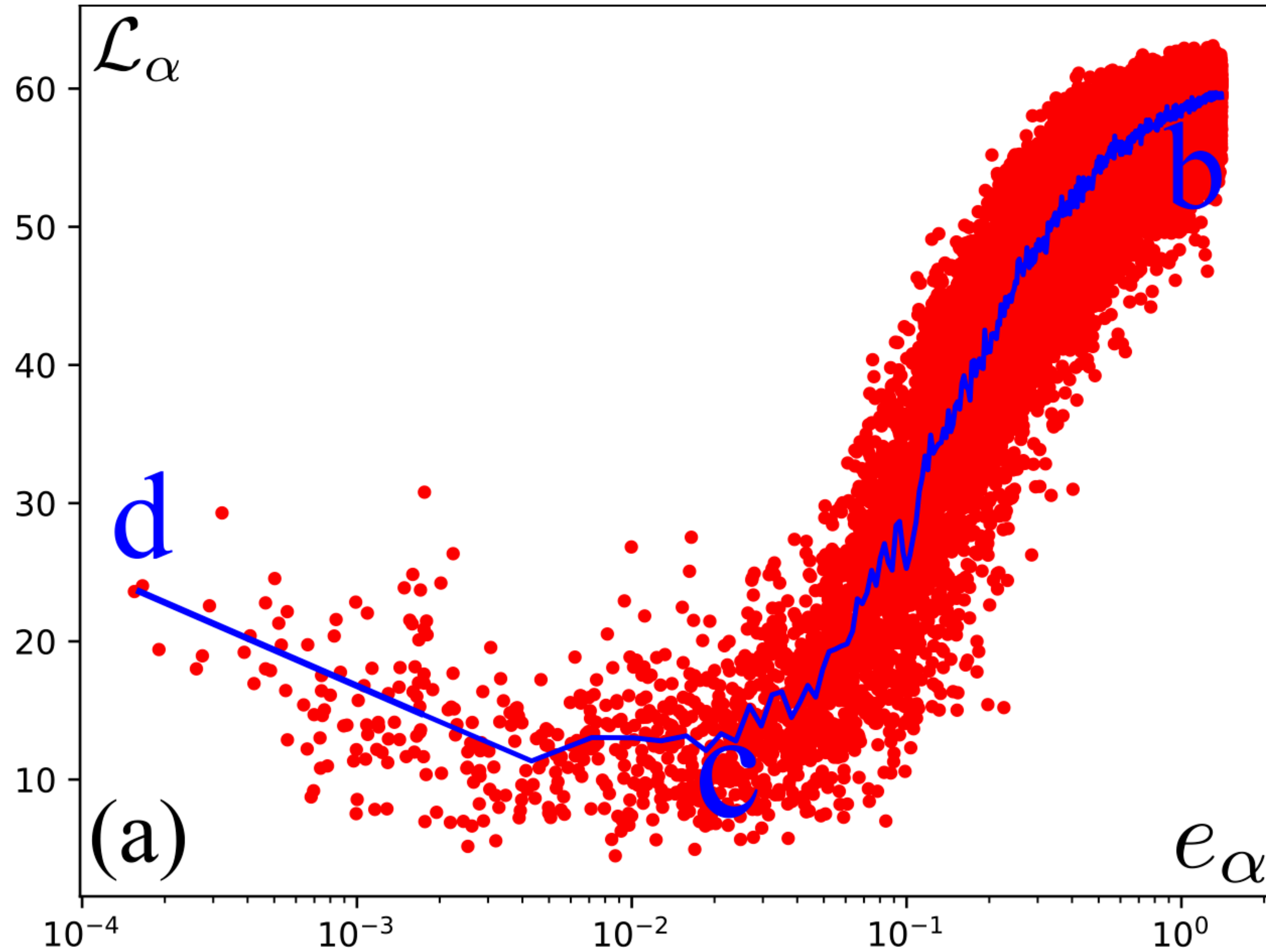
$$\bar{\mathcal{S}}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{\lambda}'_j}{2} \phi_{ja}^2 \right]$$

$$\bar{\lambda}'_j = \lambda + \lambda'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}}$$

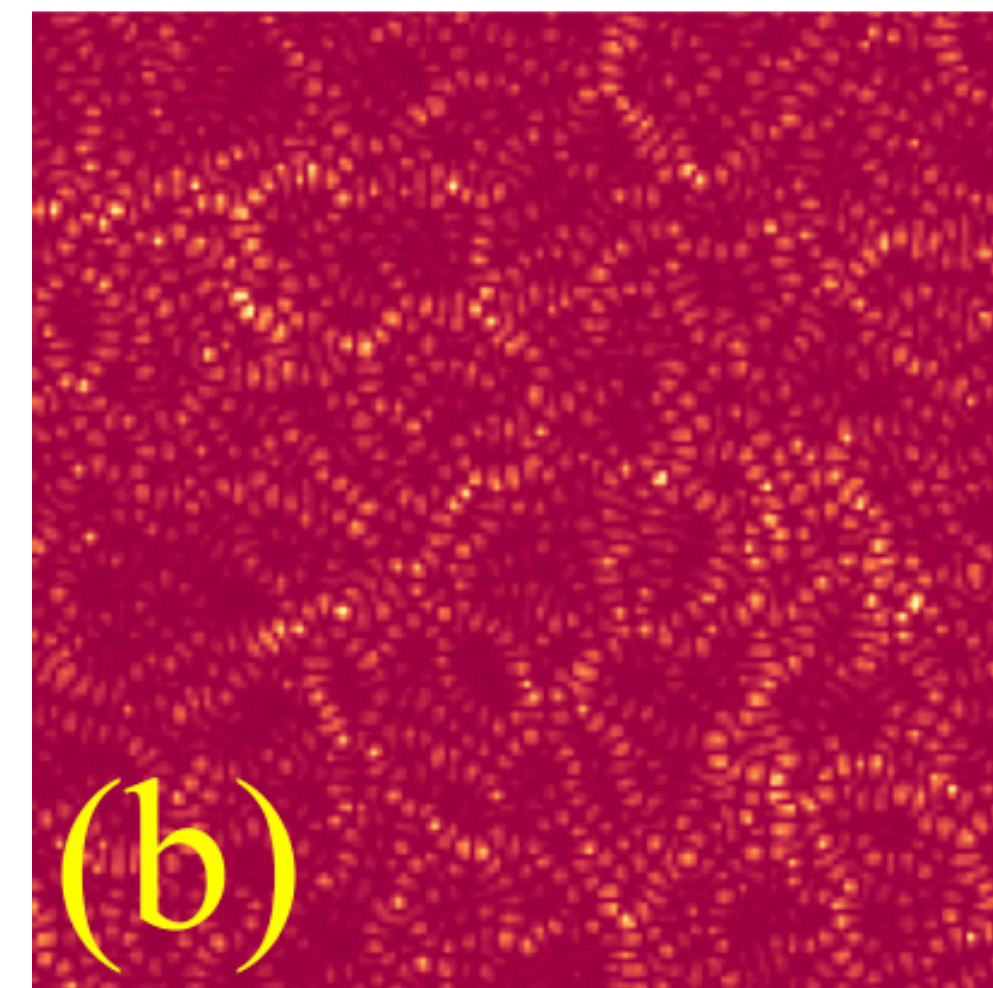
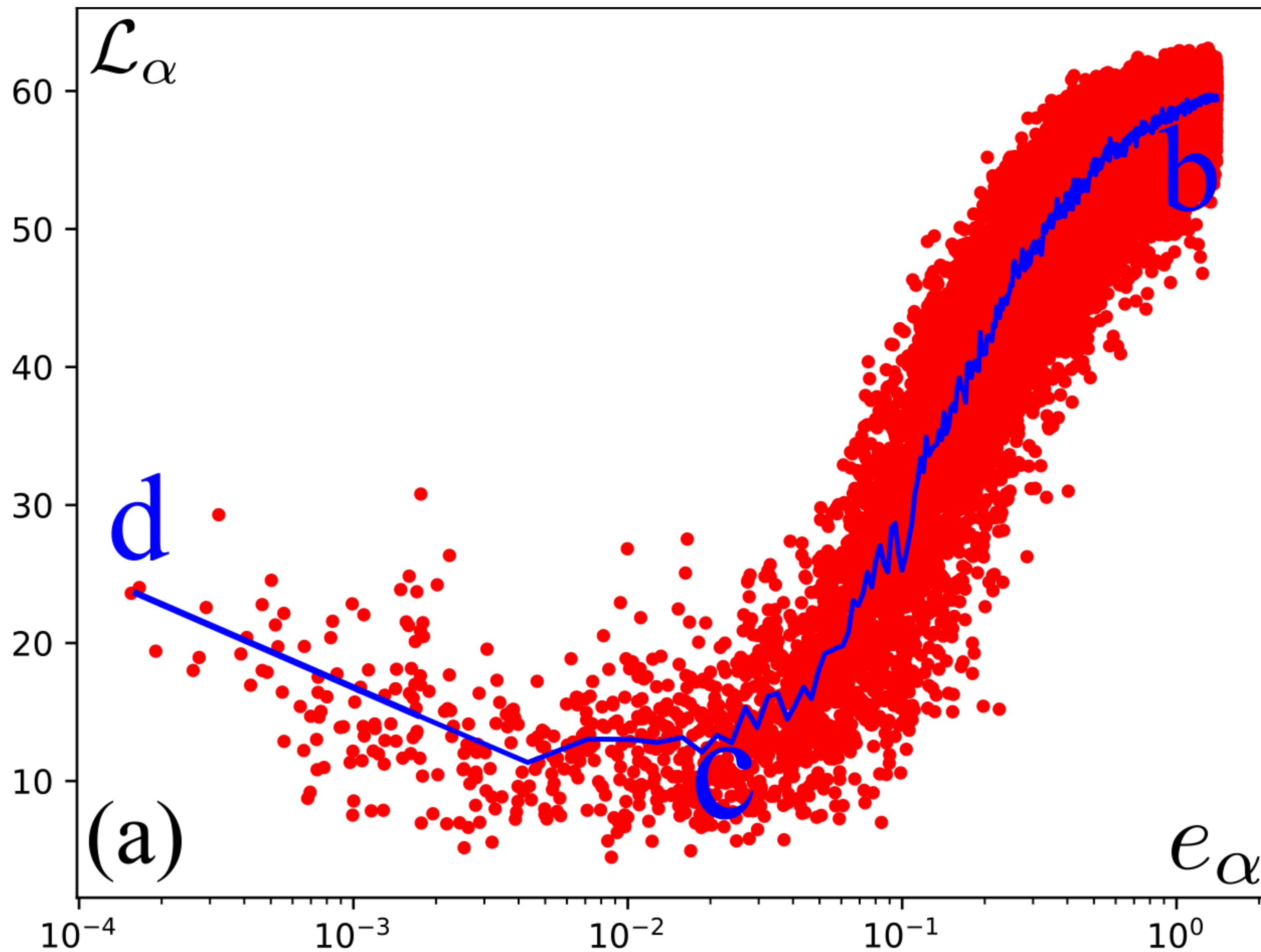
ϕ correlation length ξ



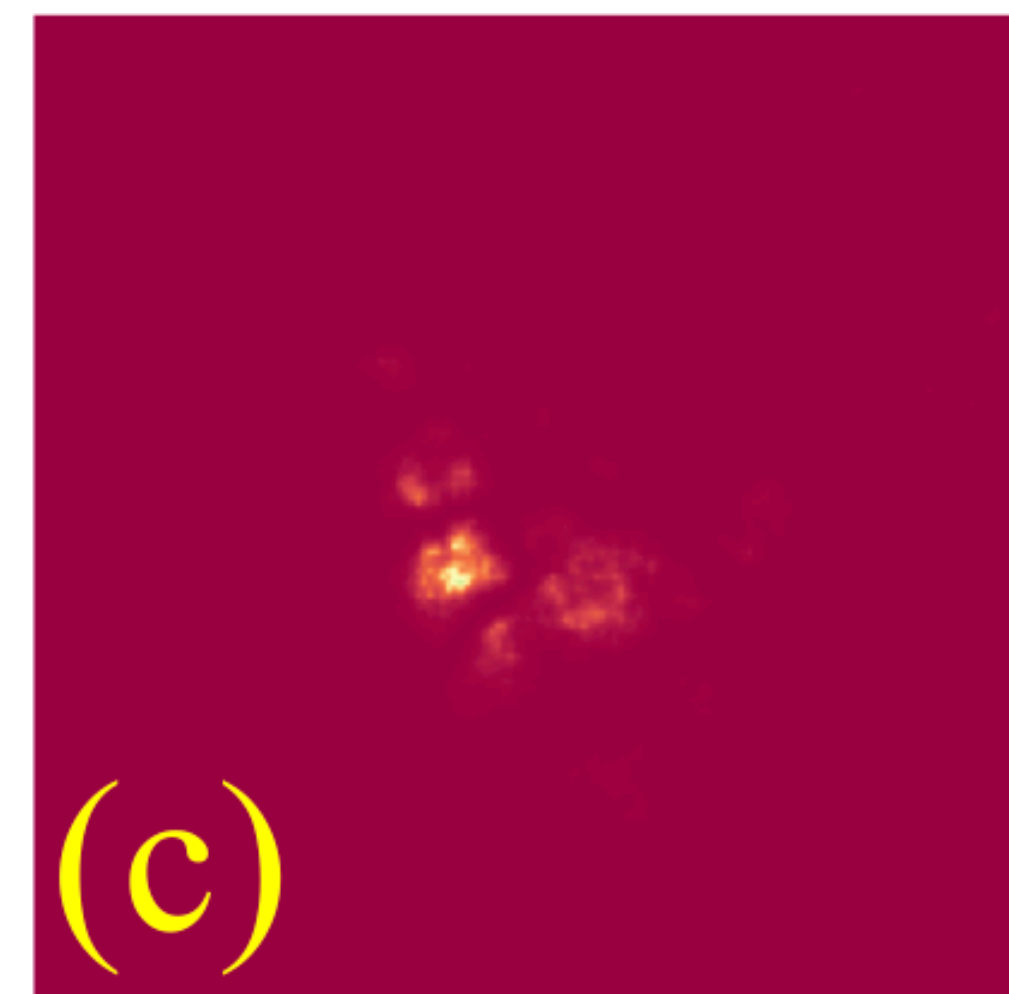
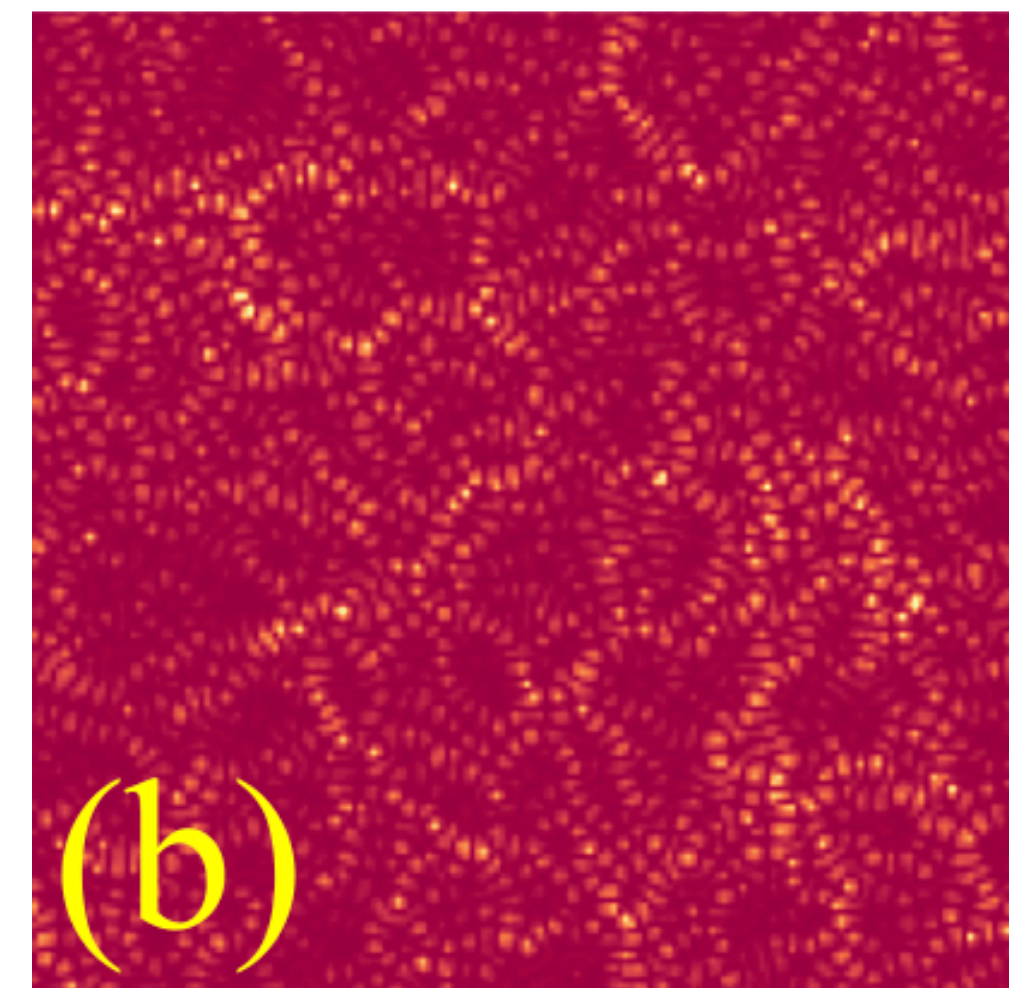
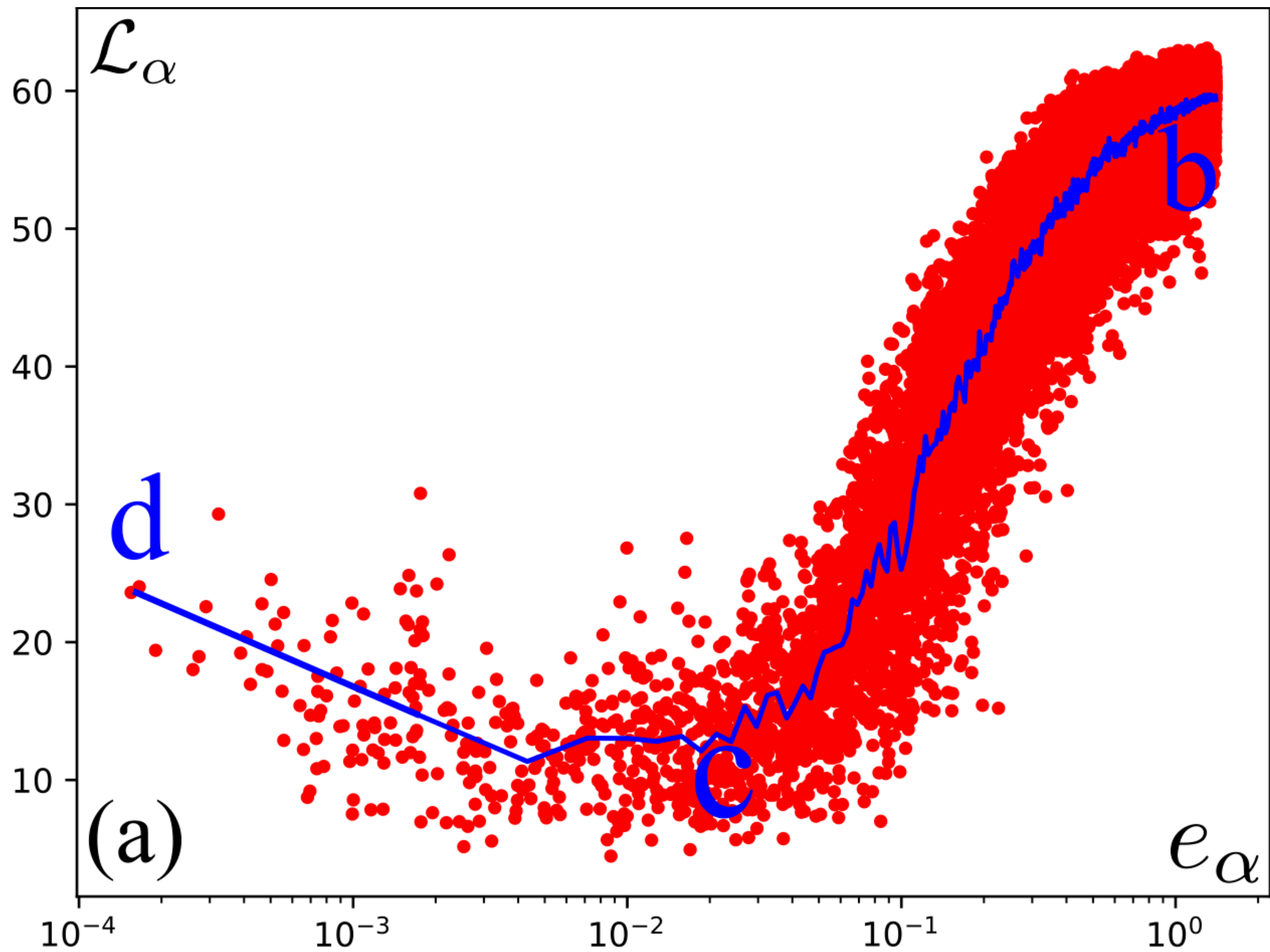
ϕ eigenmodes localization length \mathcal{L}_α



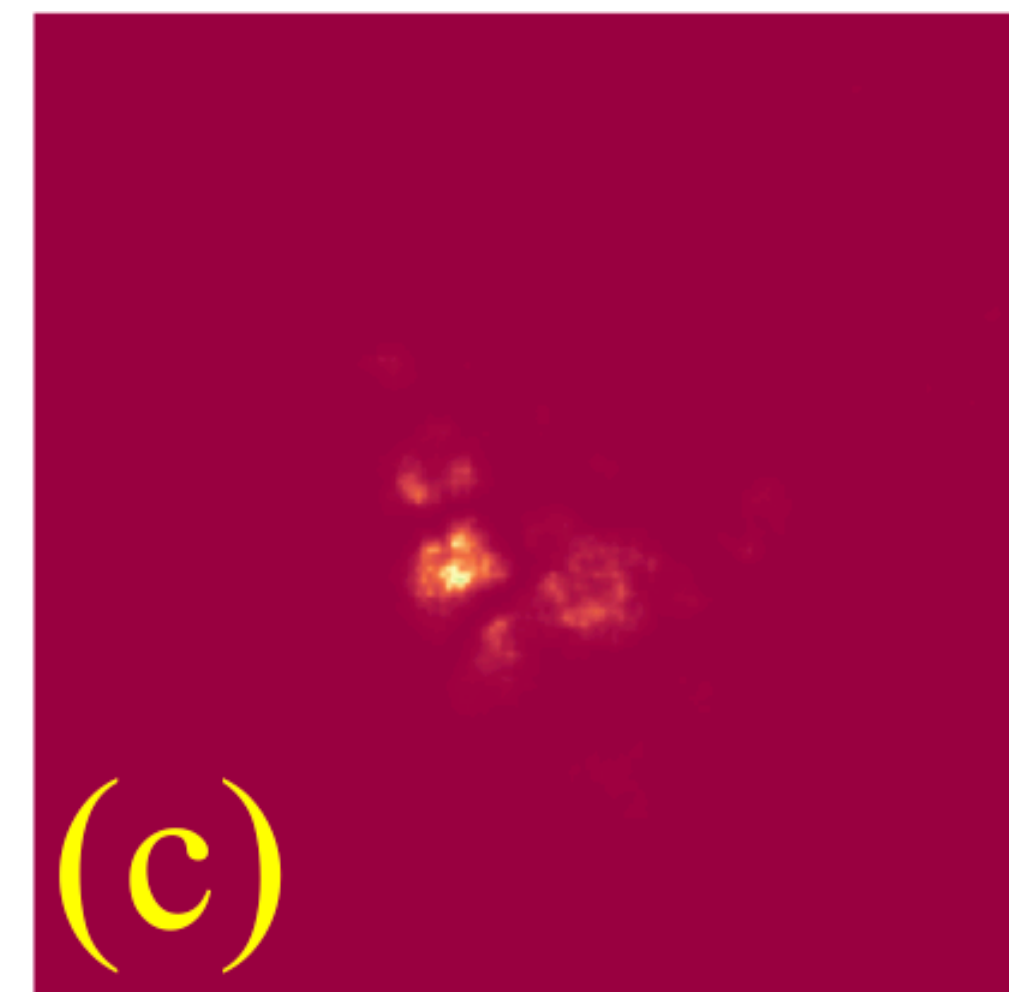
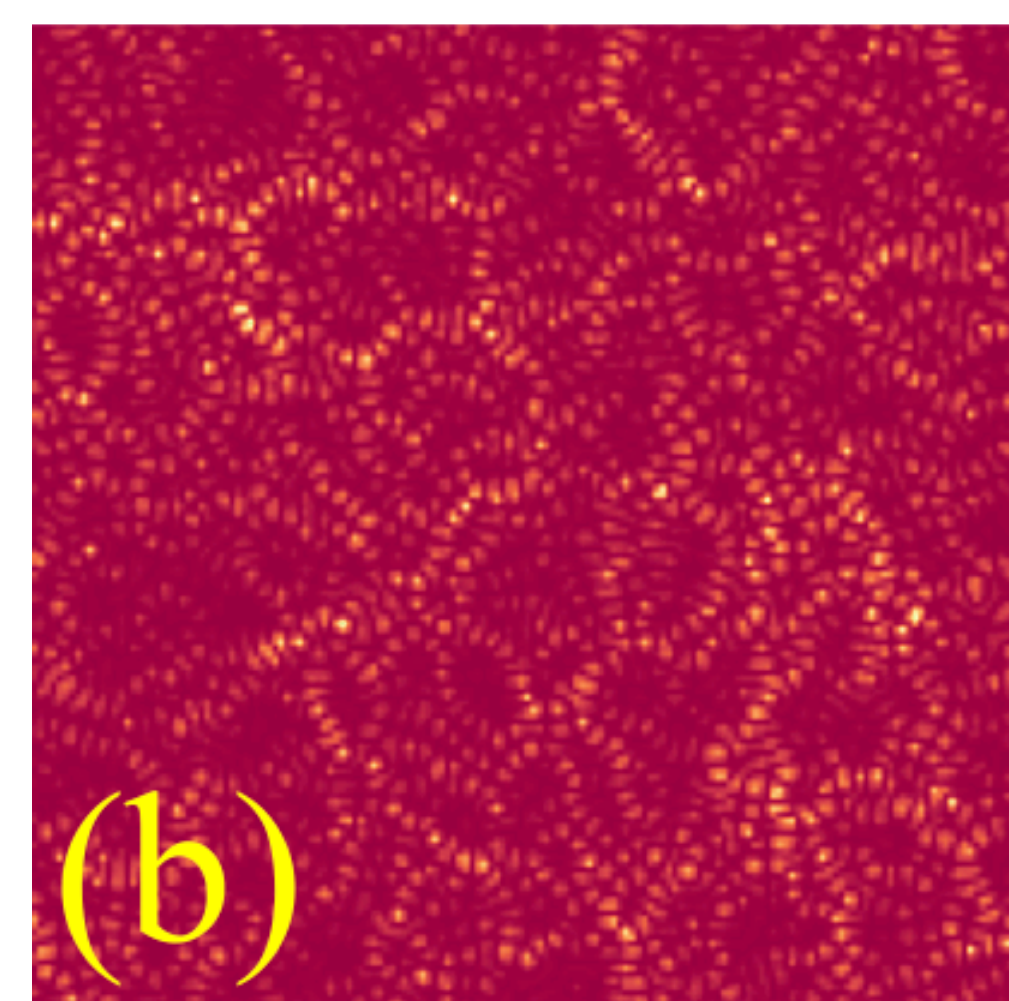
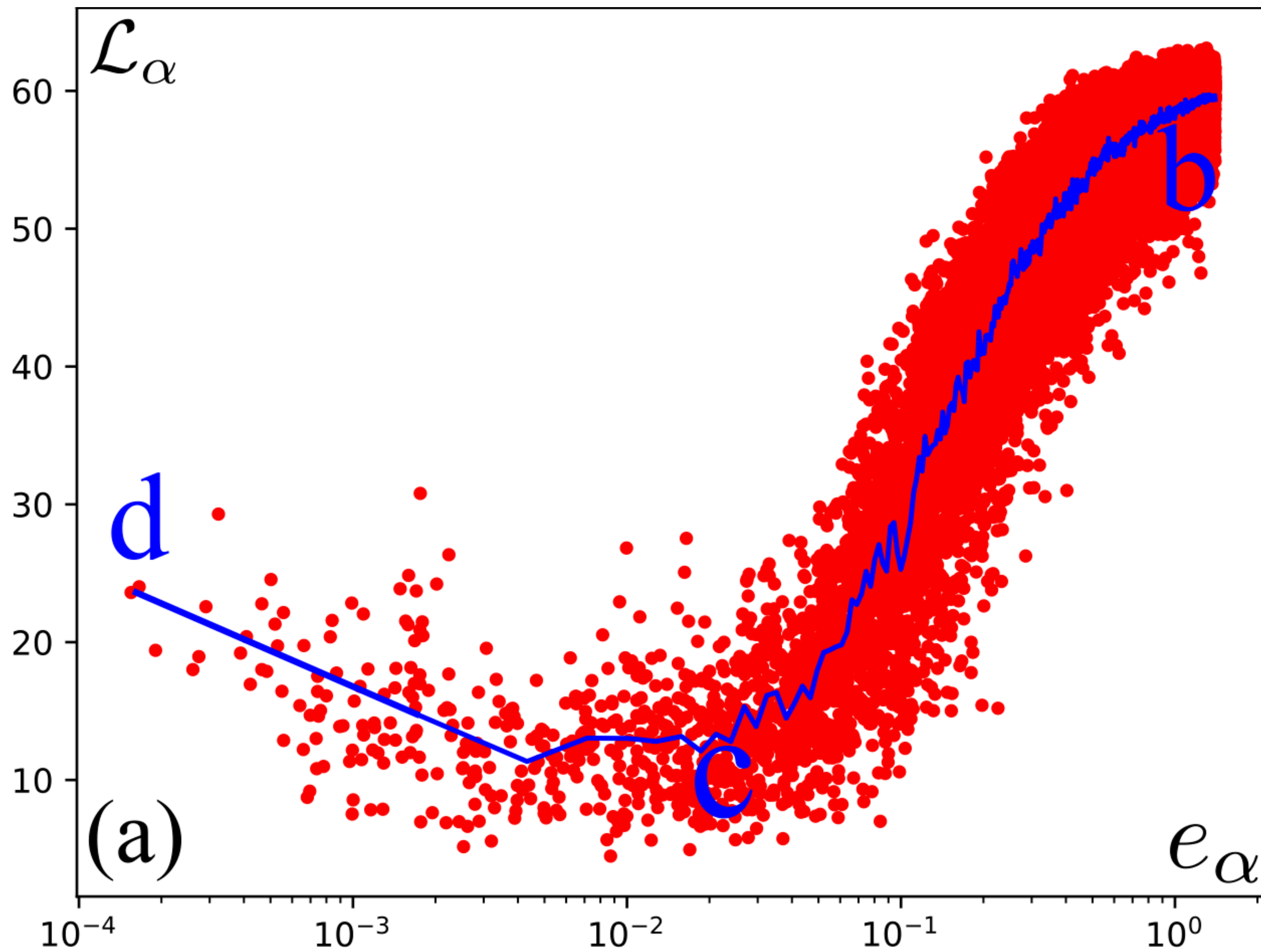
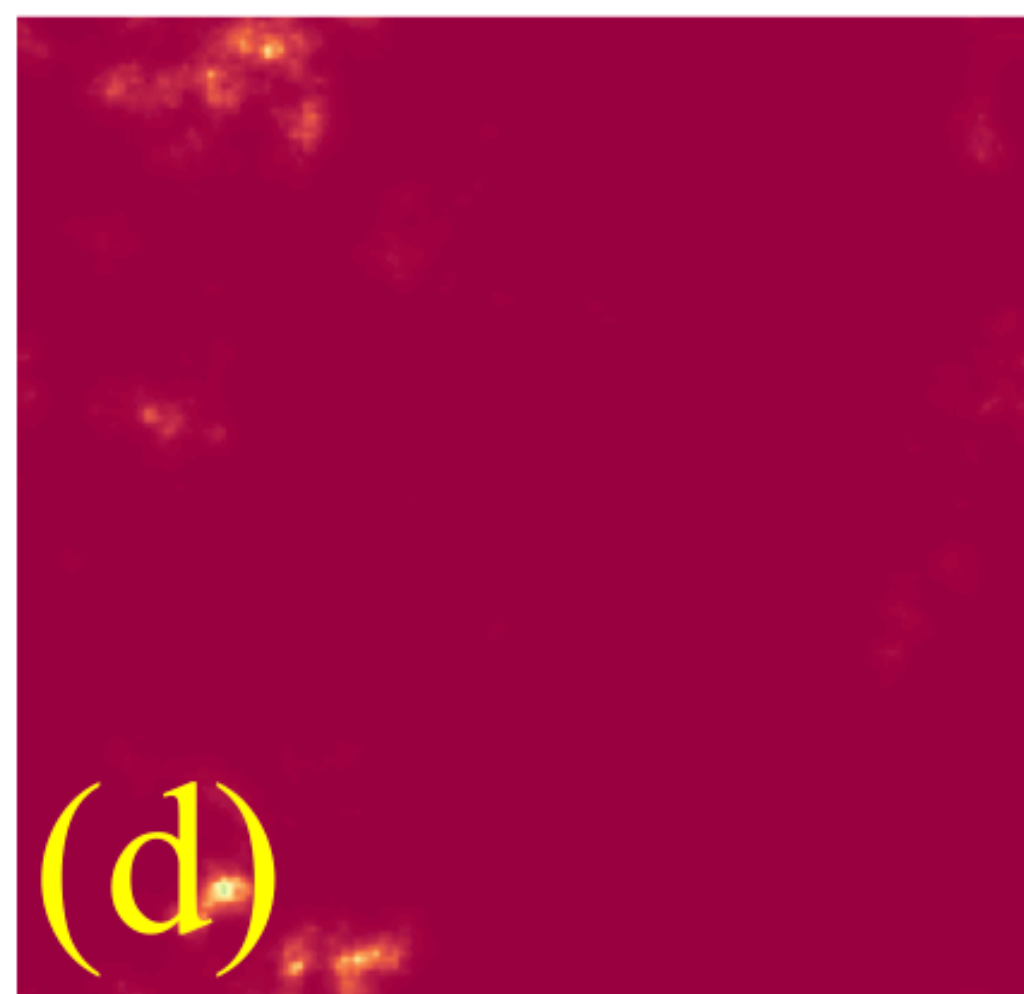
ϕ eigenmodes localization length \mathcal{L}_α



ϕ eigenmodes localization length \mathcal{L}_α



ϕ eigenmodes localization length \mathcal{L}_α



Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

Strong disorder RG identical to that for the RTFIM (D.S. Fisher)

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

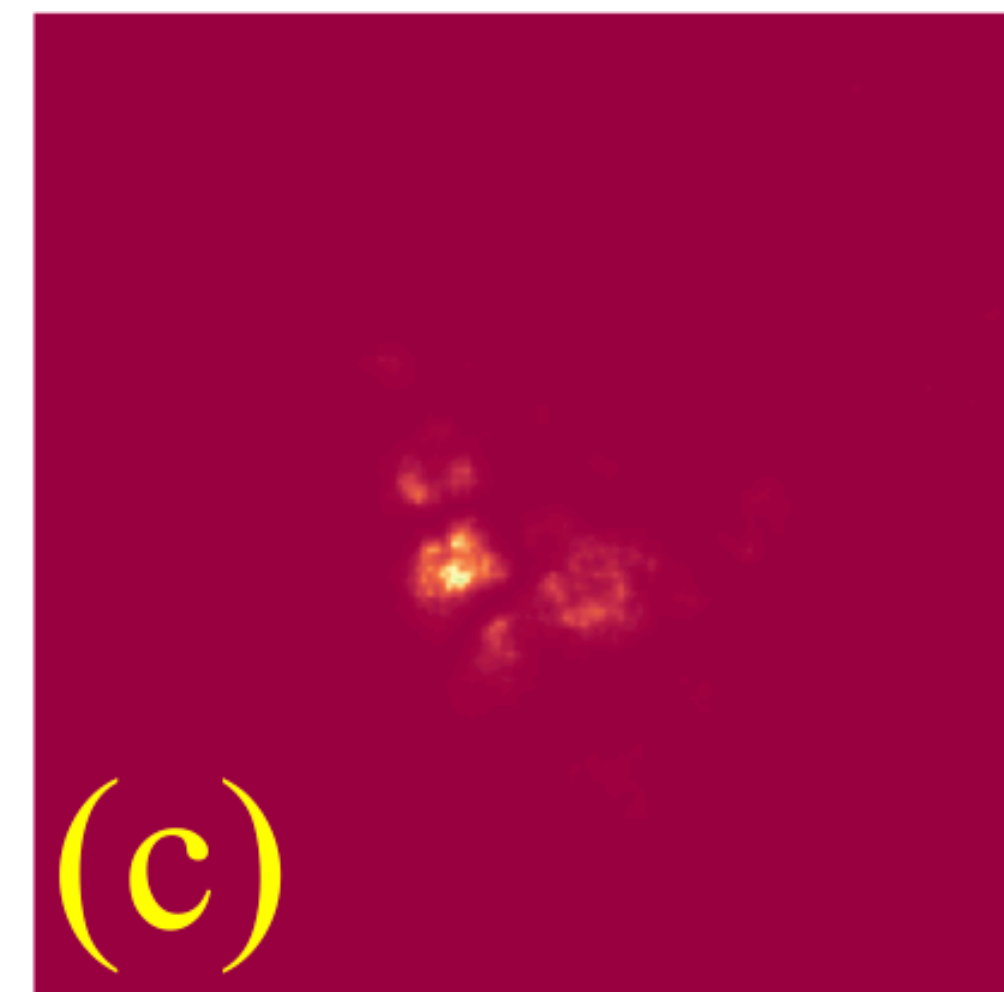
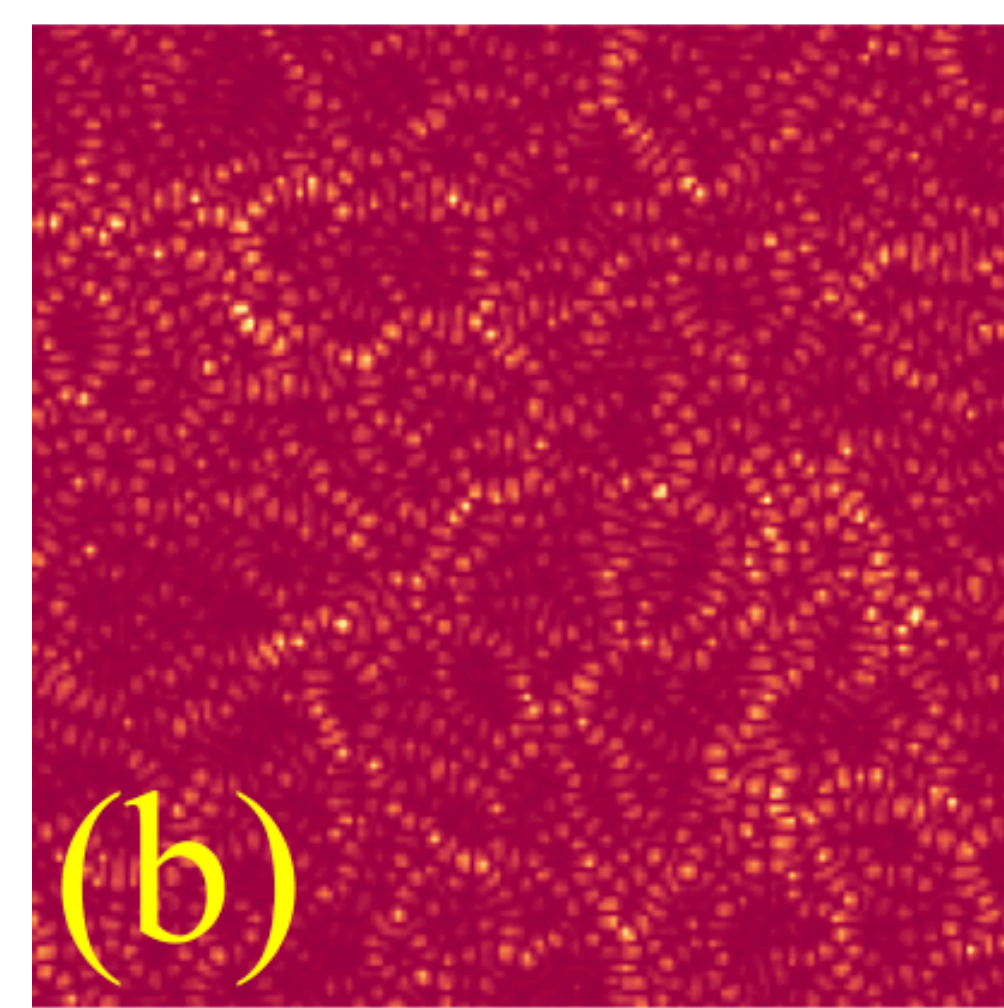
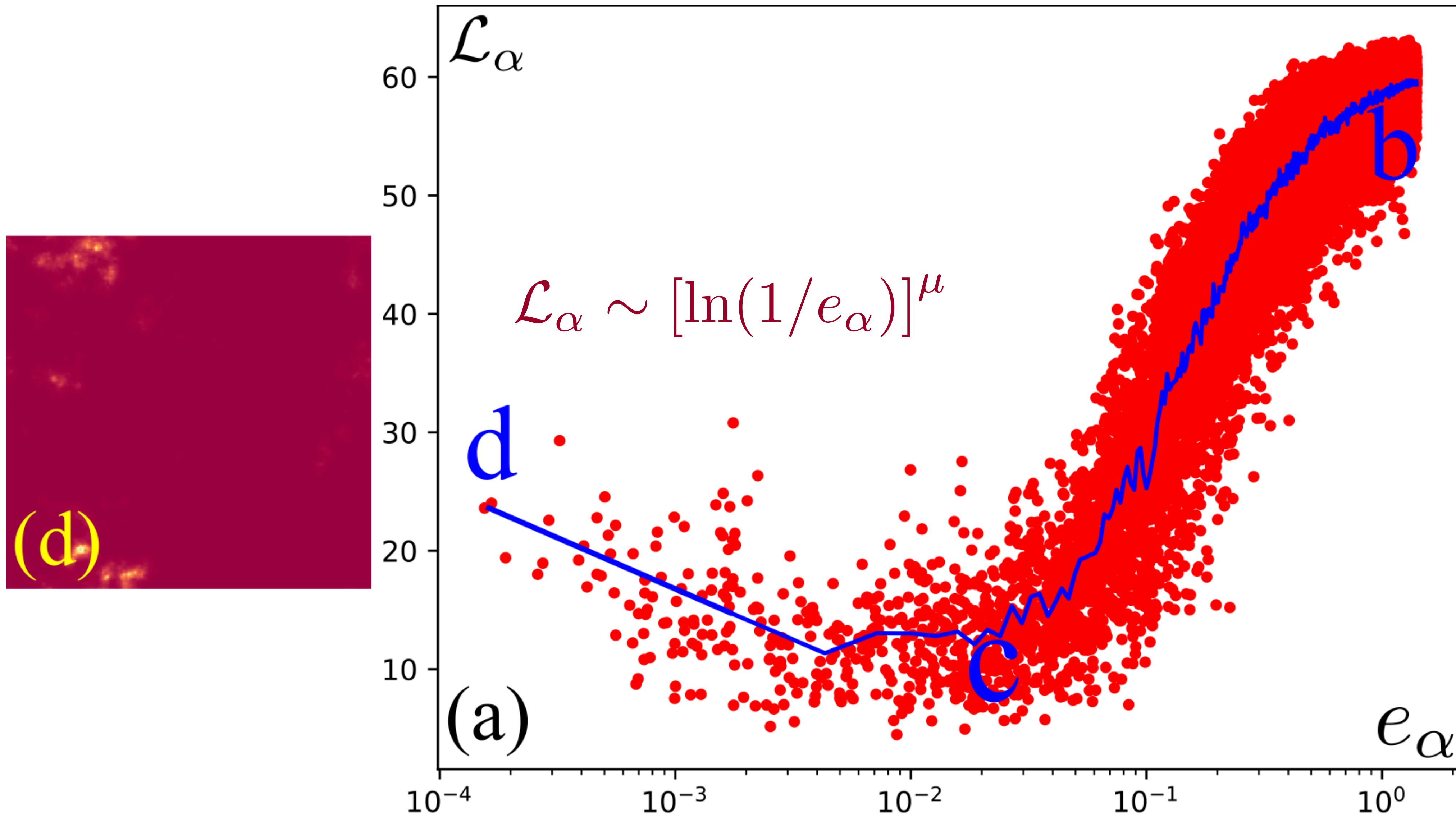
Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

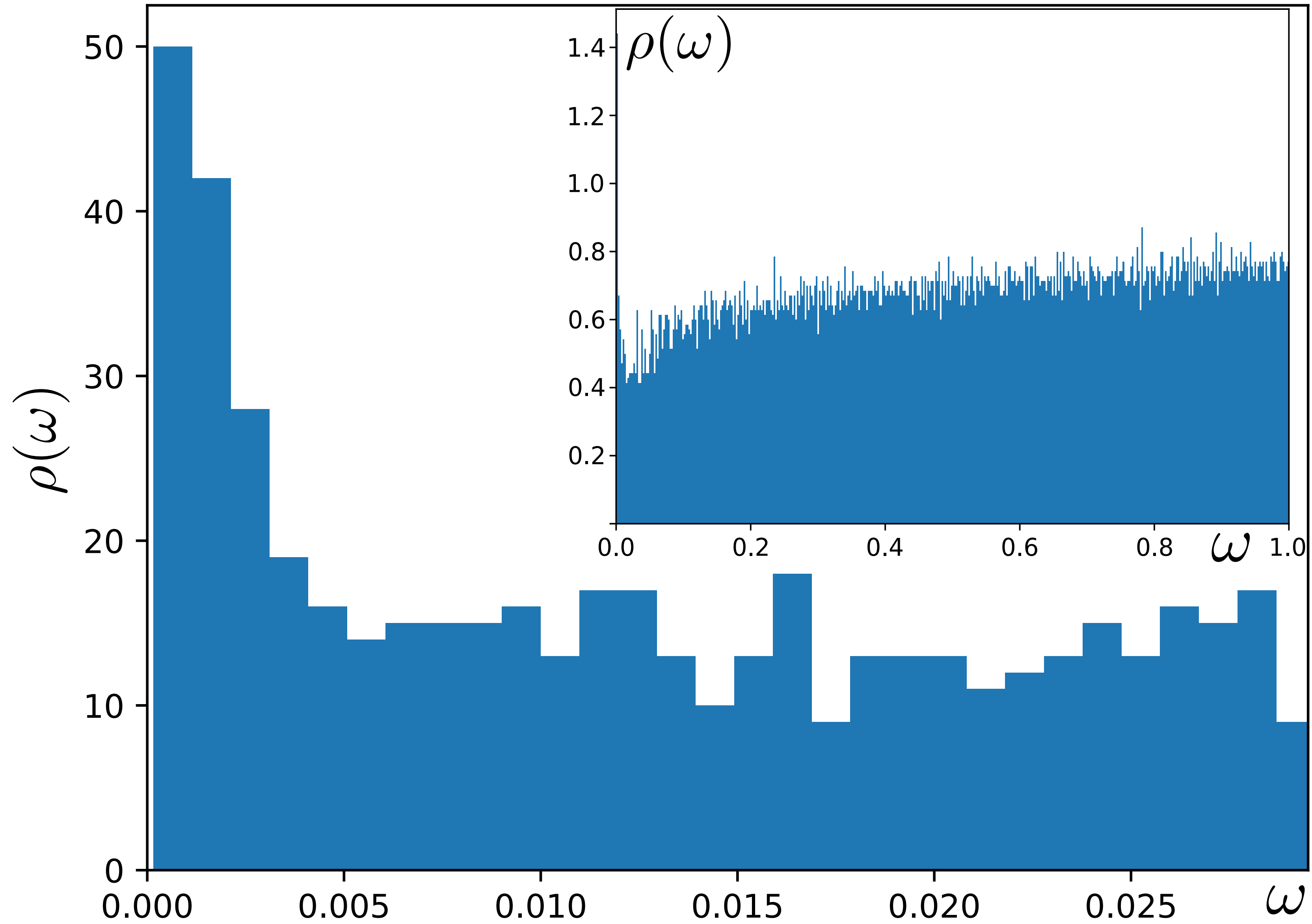
We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

- Each rare region is described by a one-dimensional classical $O(N)$ model with a long-range $1/\tau^2$ interaction.
- For $N \geq 2$, the classical model has an exponentially long correlation time at weak coupling (low ‘temperature’) - Kosterlitz 1976.
- This is similar to the classical Ising chain with short-range interactions.

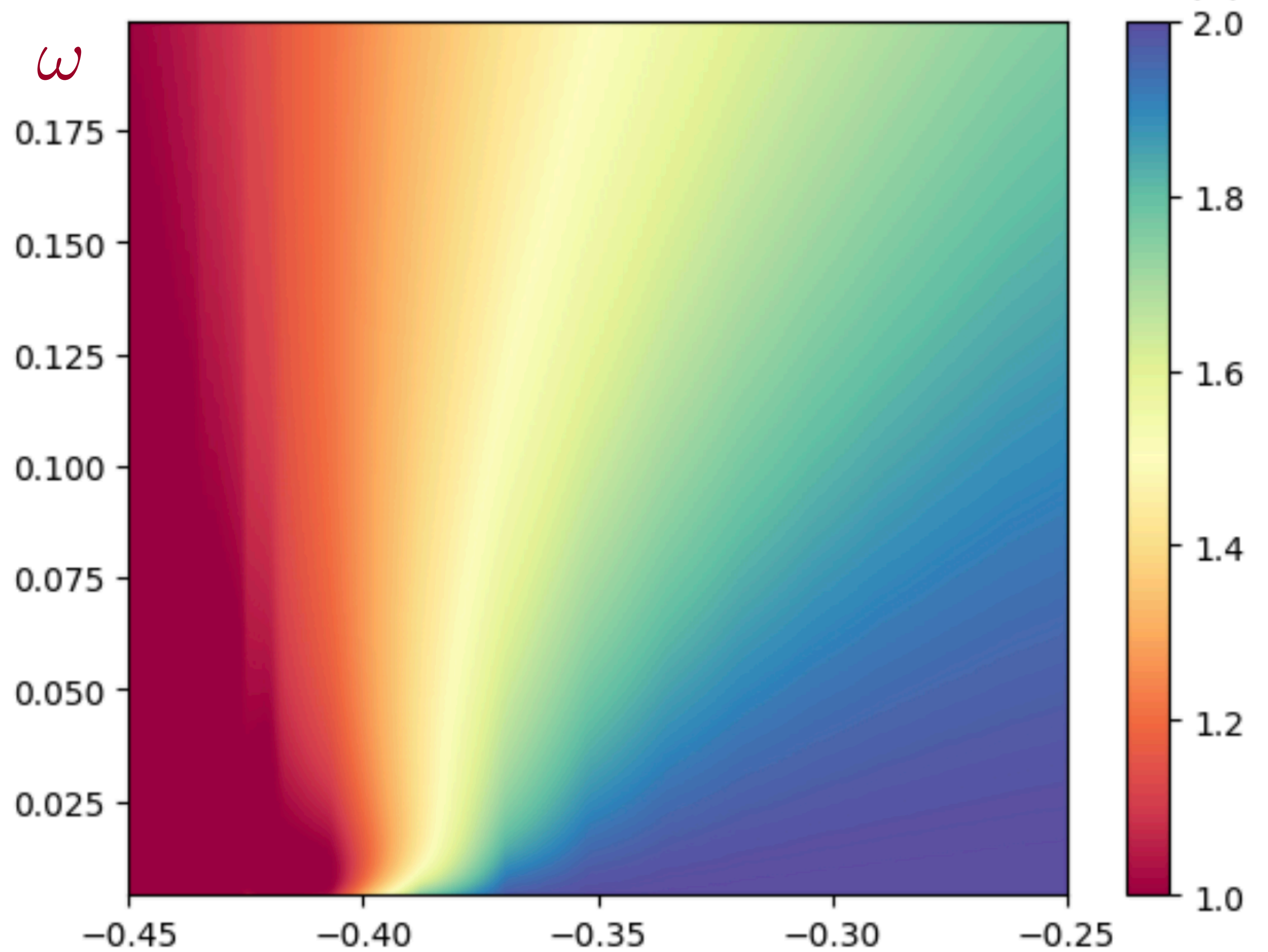
ϕ eigenmodes localization length \mathcal{L}_α



Density of states of ϕ eigenmodes $\rho(\omega)$
Yukawa-SYK theory $\rho(\omega) \sim \text{constant}$.



$$-\text{Im}[\Sigma^R(\omega > 2\pi T)] \sim \text{const.} + |\omega|^n$$

 n 

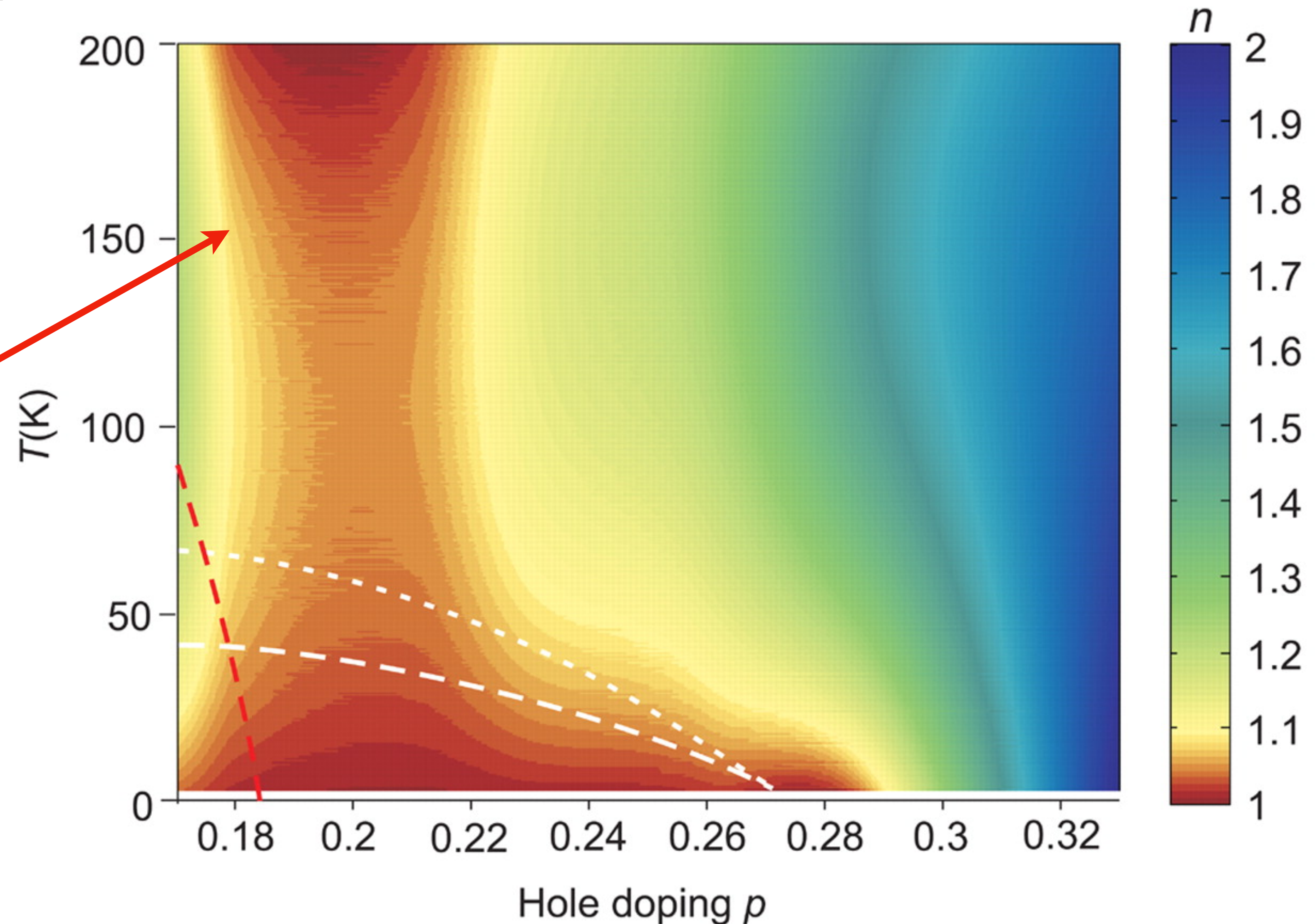
Fermion
self energy

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

Extended bosons:
physics of Yukawa-SYK



Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³
Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

Localized bosons:
physics of RTFIM

