Detecting a quantum spin liquid in the cuprate superconductors

Tata Institute of Fundamental Research, Mumbai
January 5, 2026

Subir Sachdev

PHYSICS
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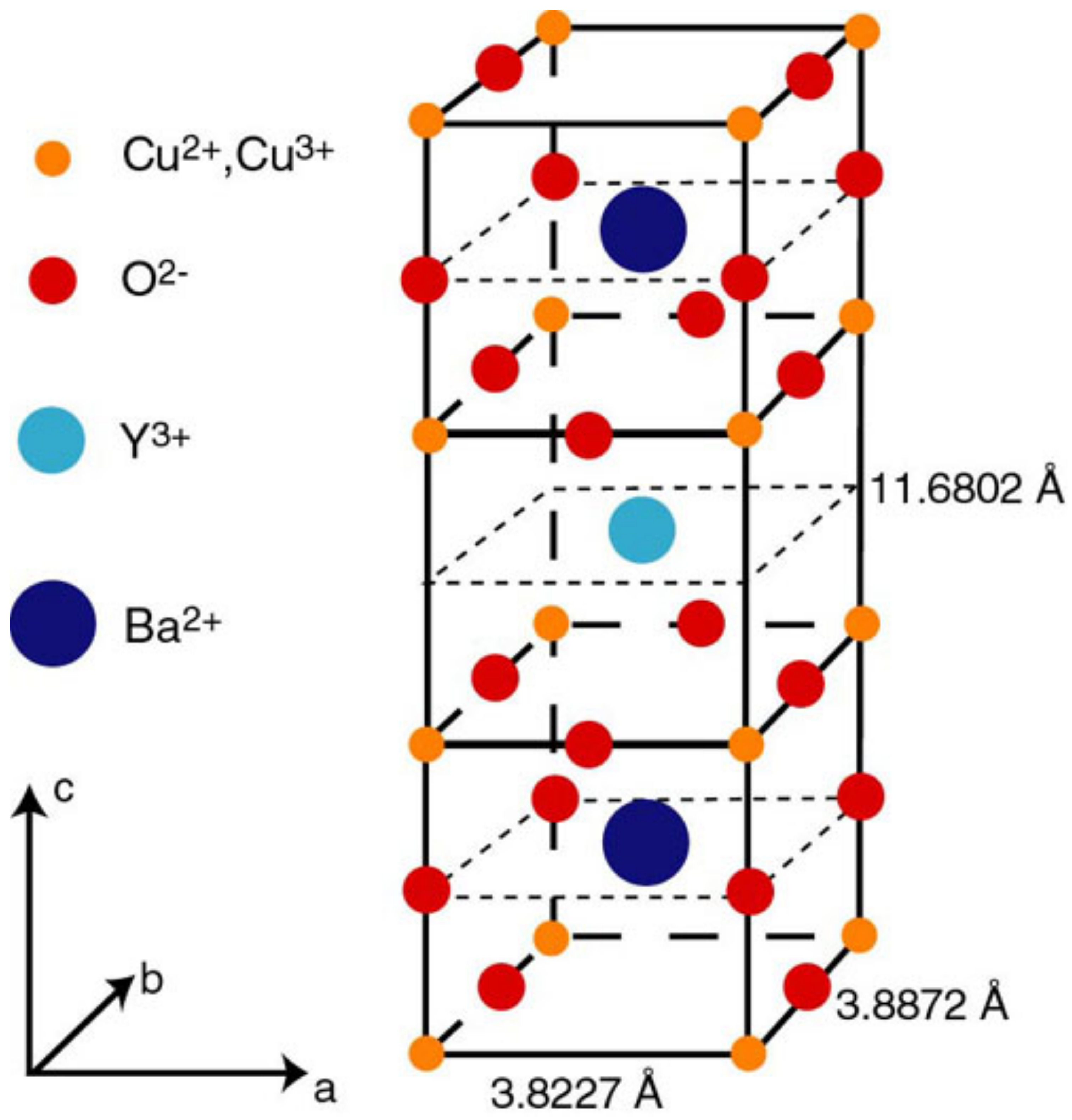

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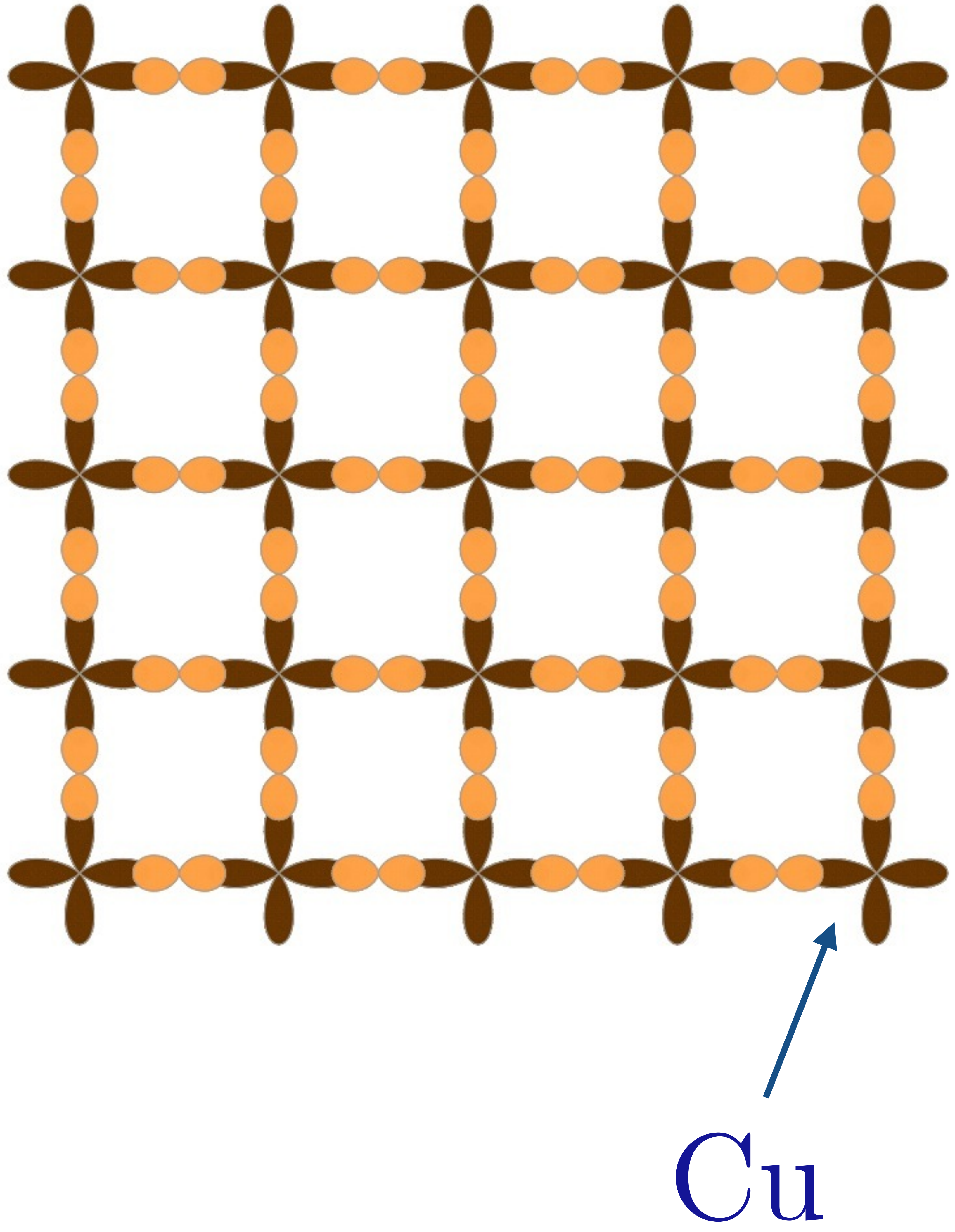

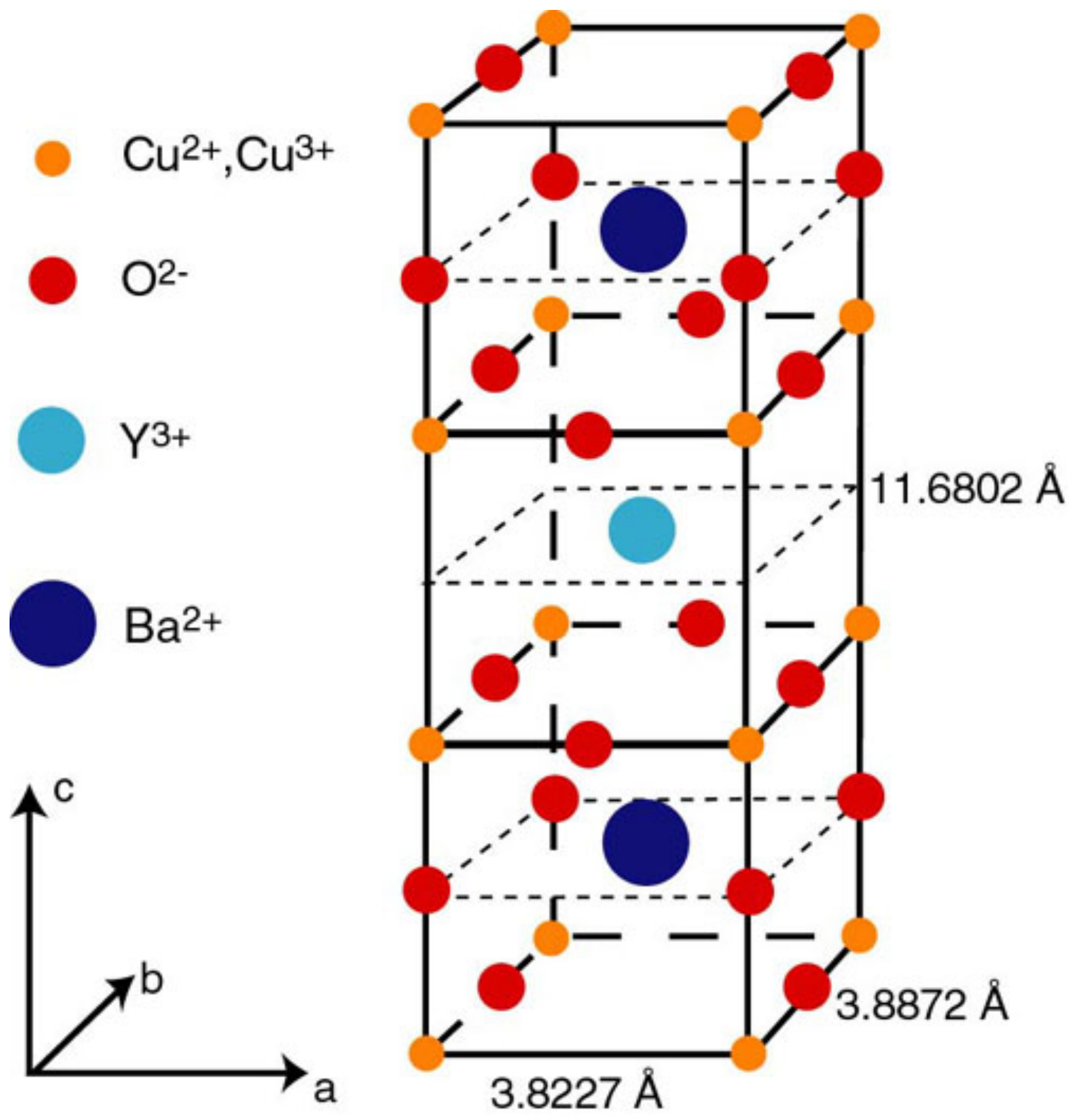

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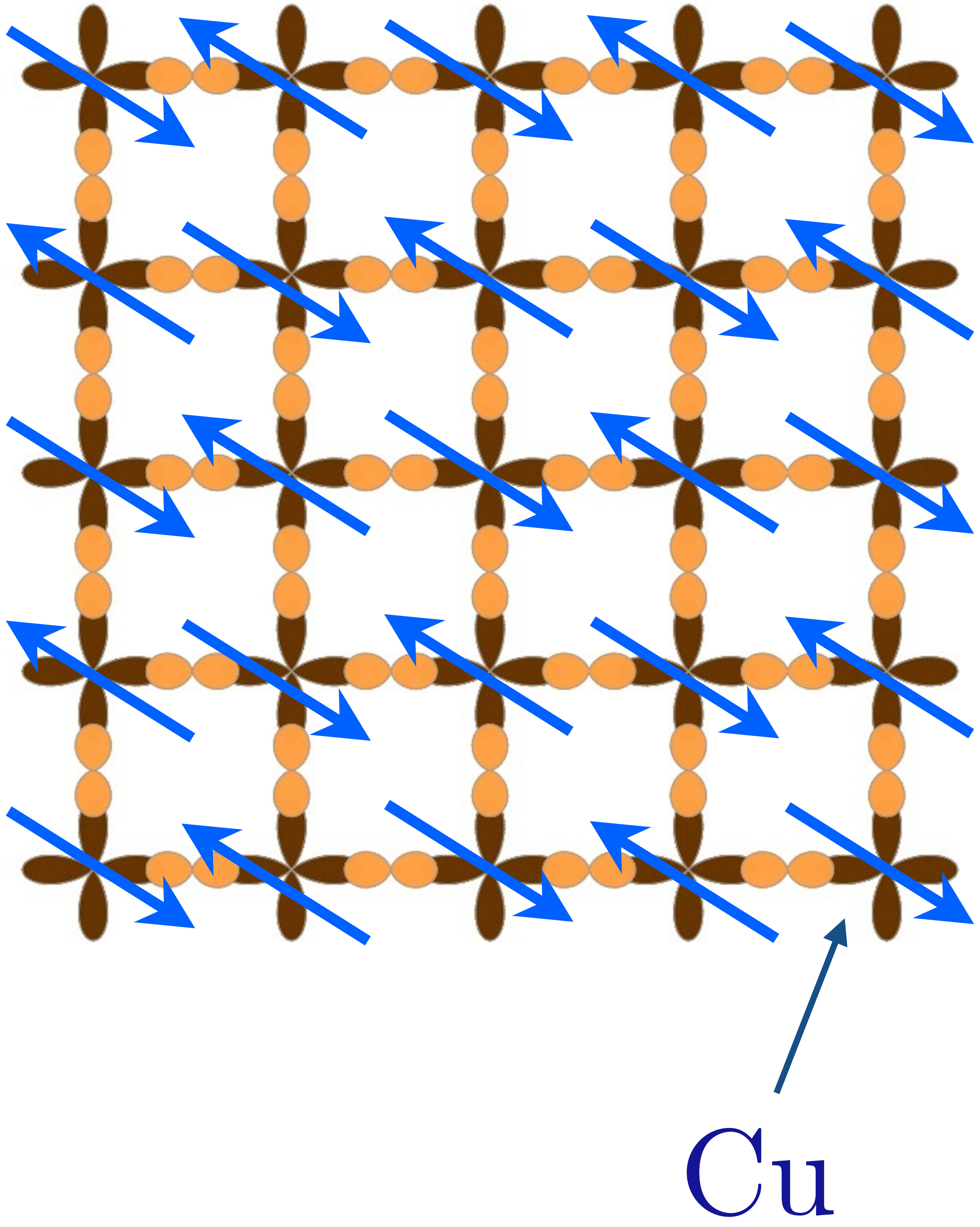
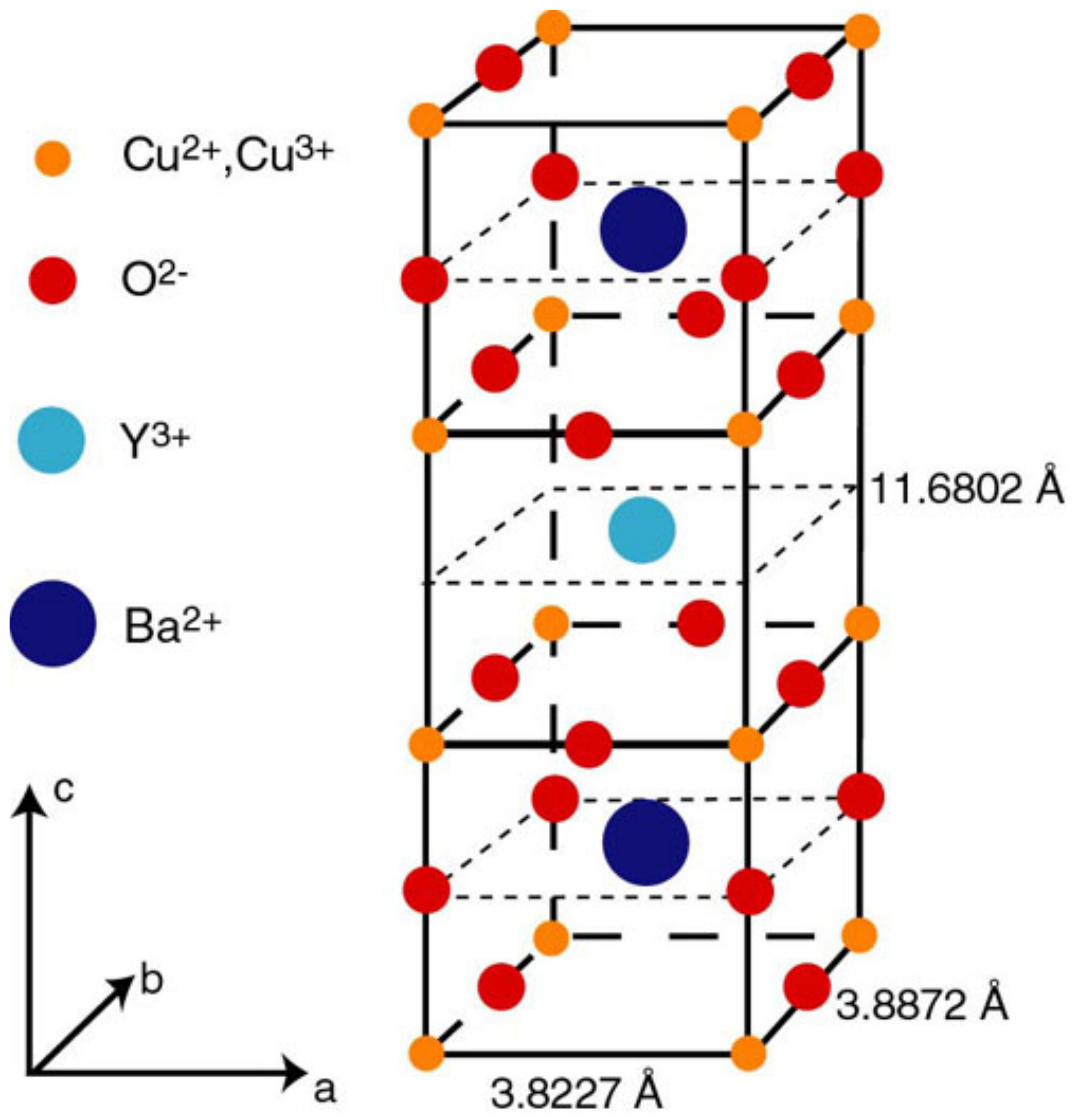


Insulating antiferromagnet with one electron per site



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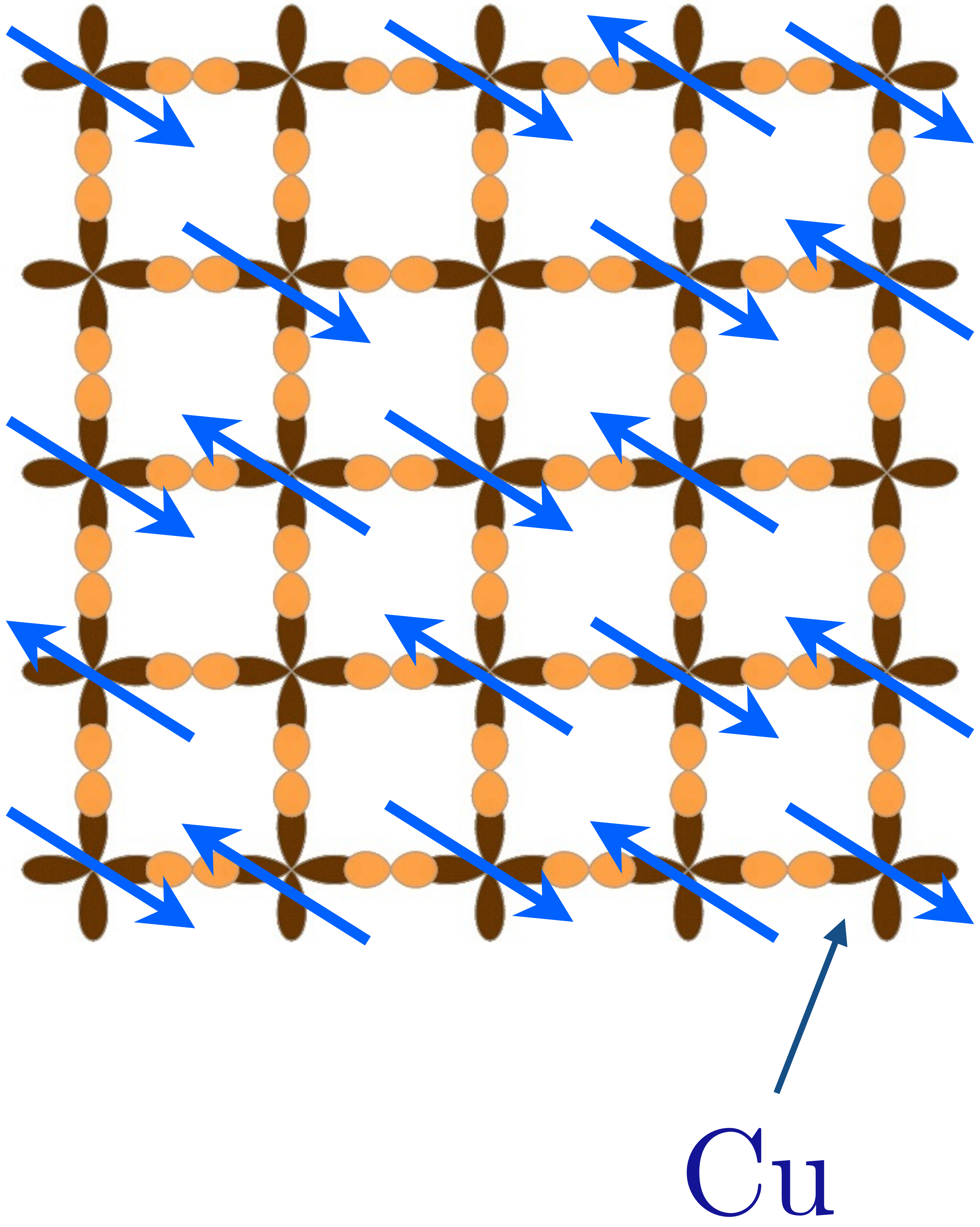
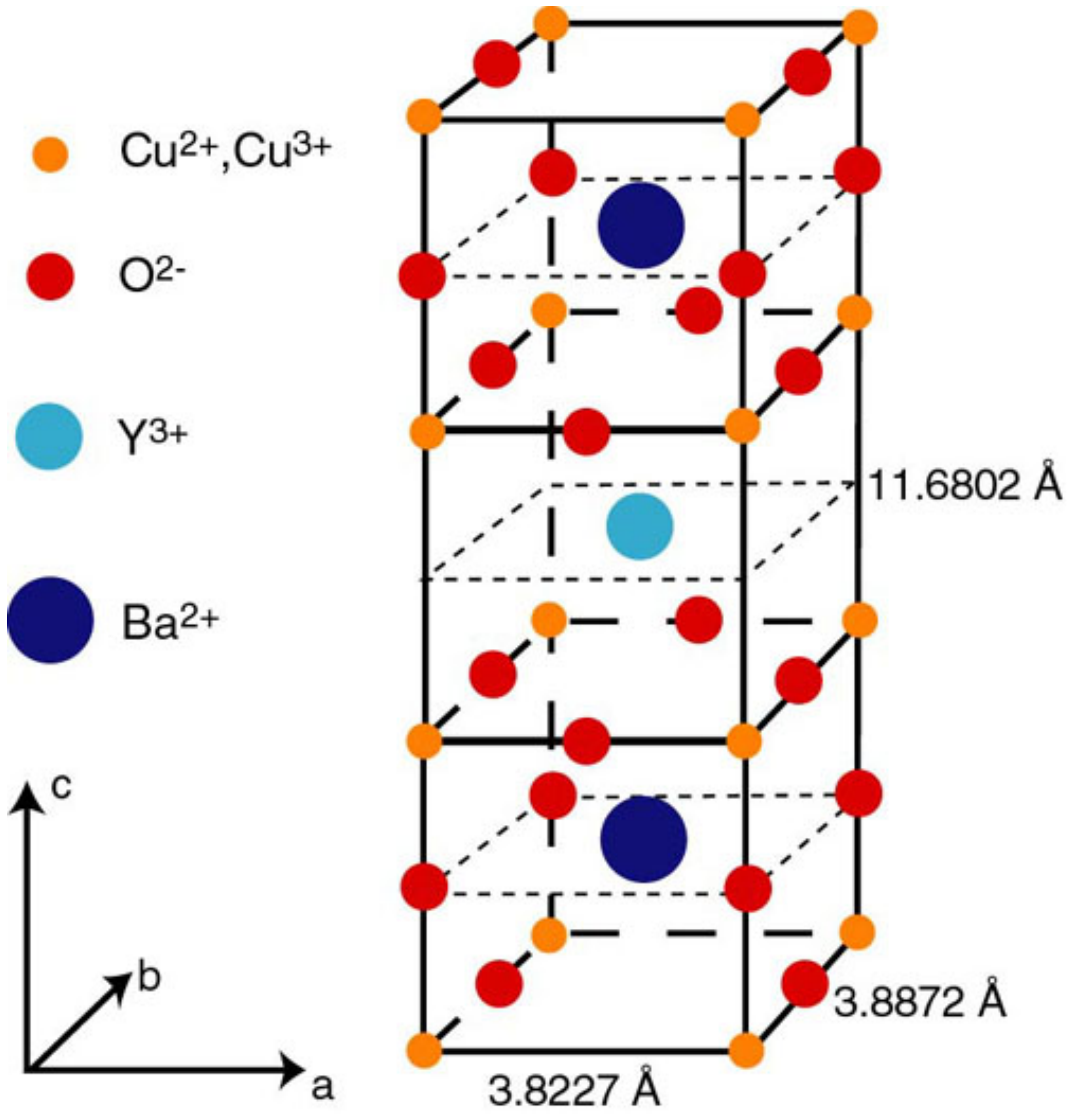


High temperature superconductor obtained upon doping the antiferromagnet with density p holes.



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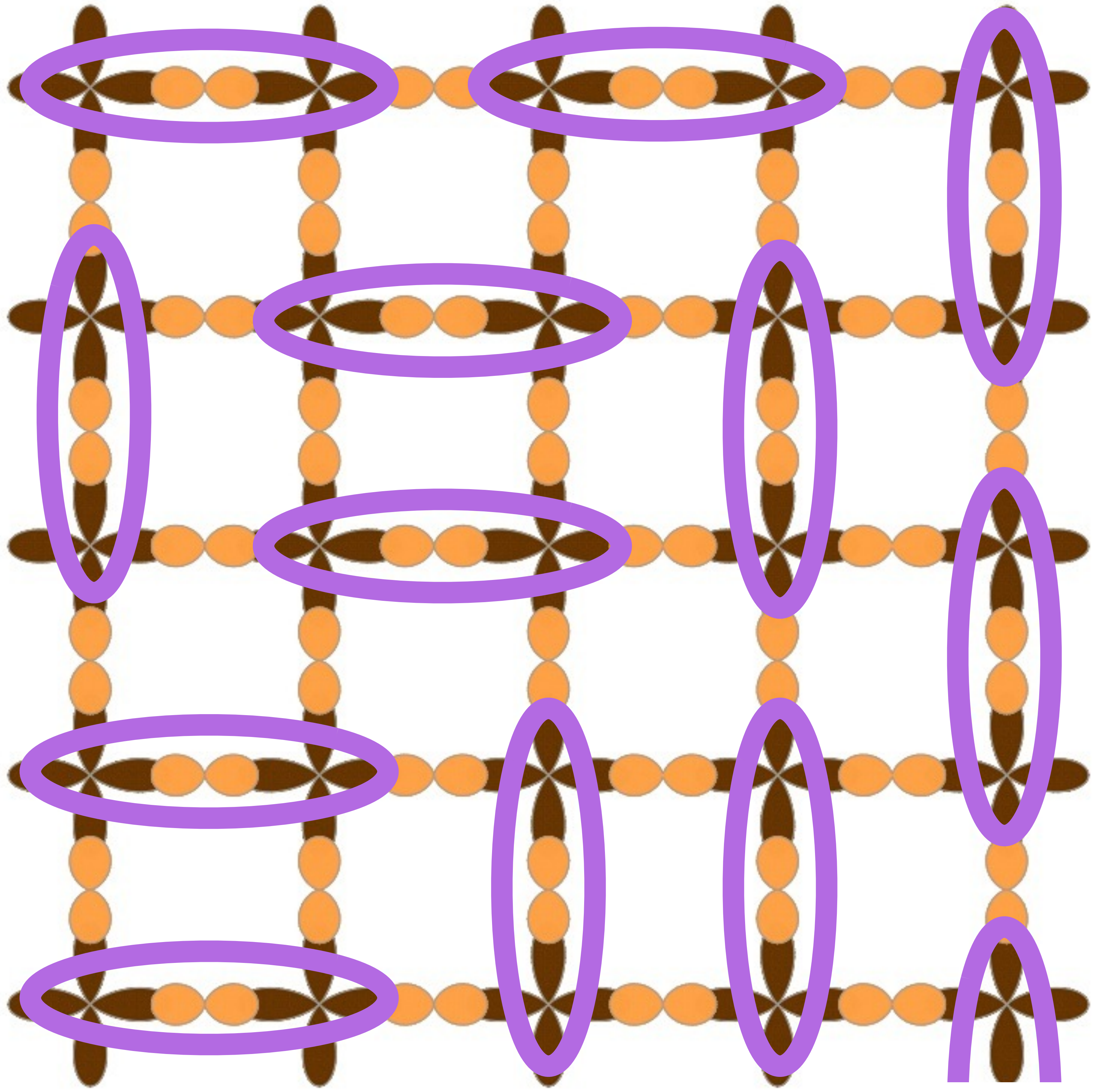
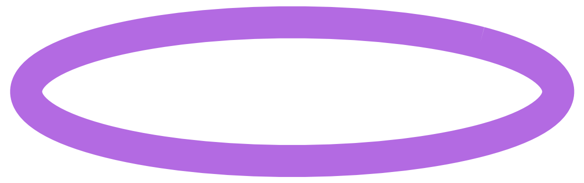


$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

 $= \frac{1}{\sqrt{2}} \left(| \uparrow\downarrow\rangle - | \downarrow\uparrow\rangle \right)$

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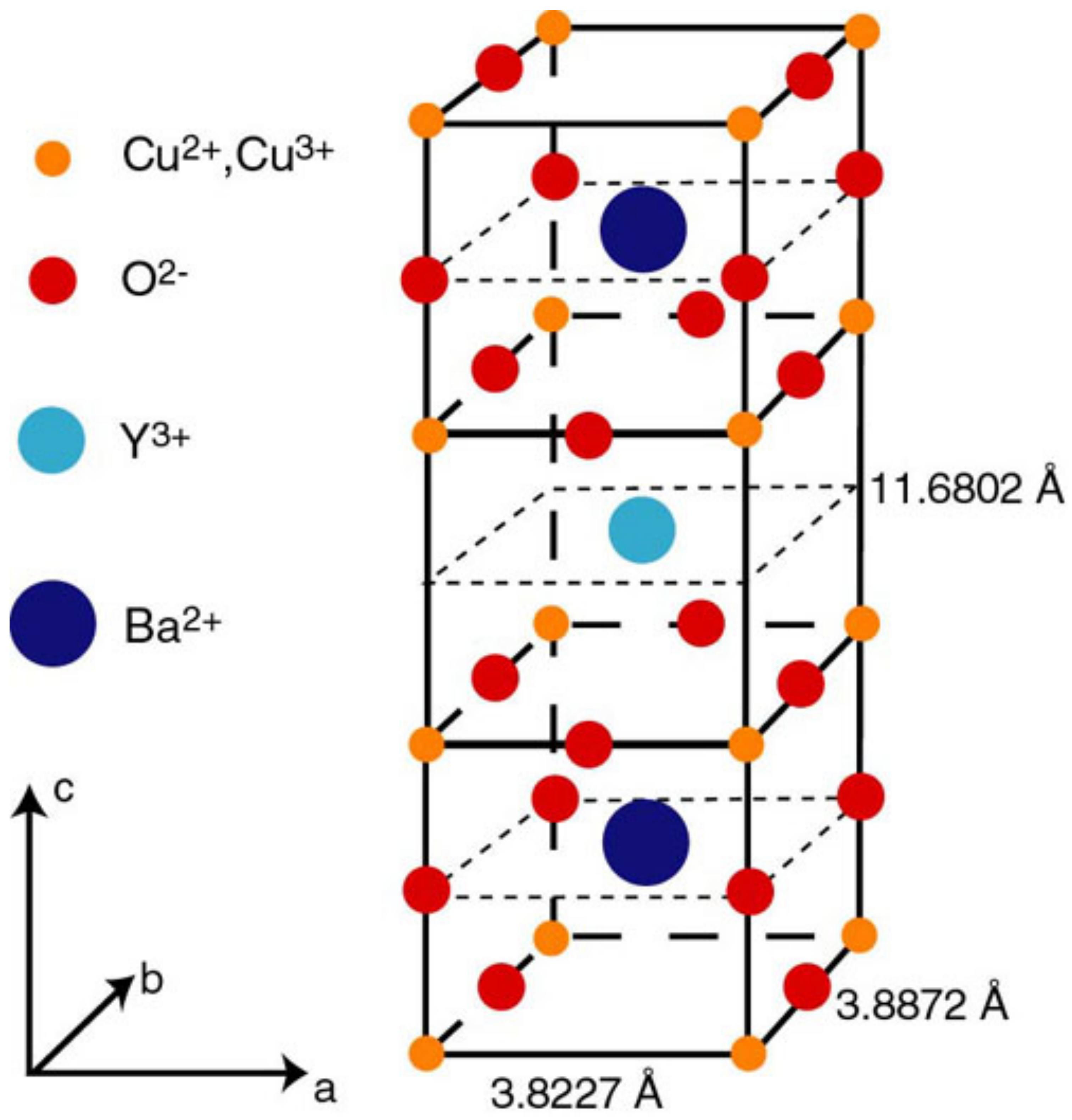

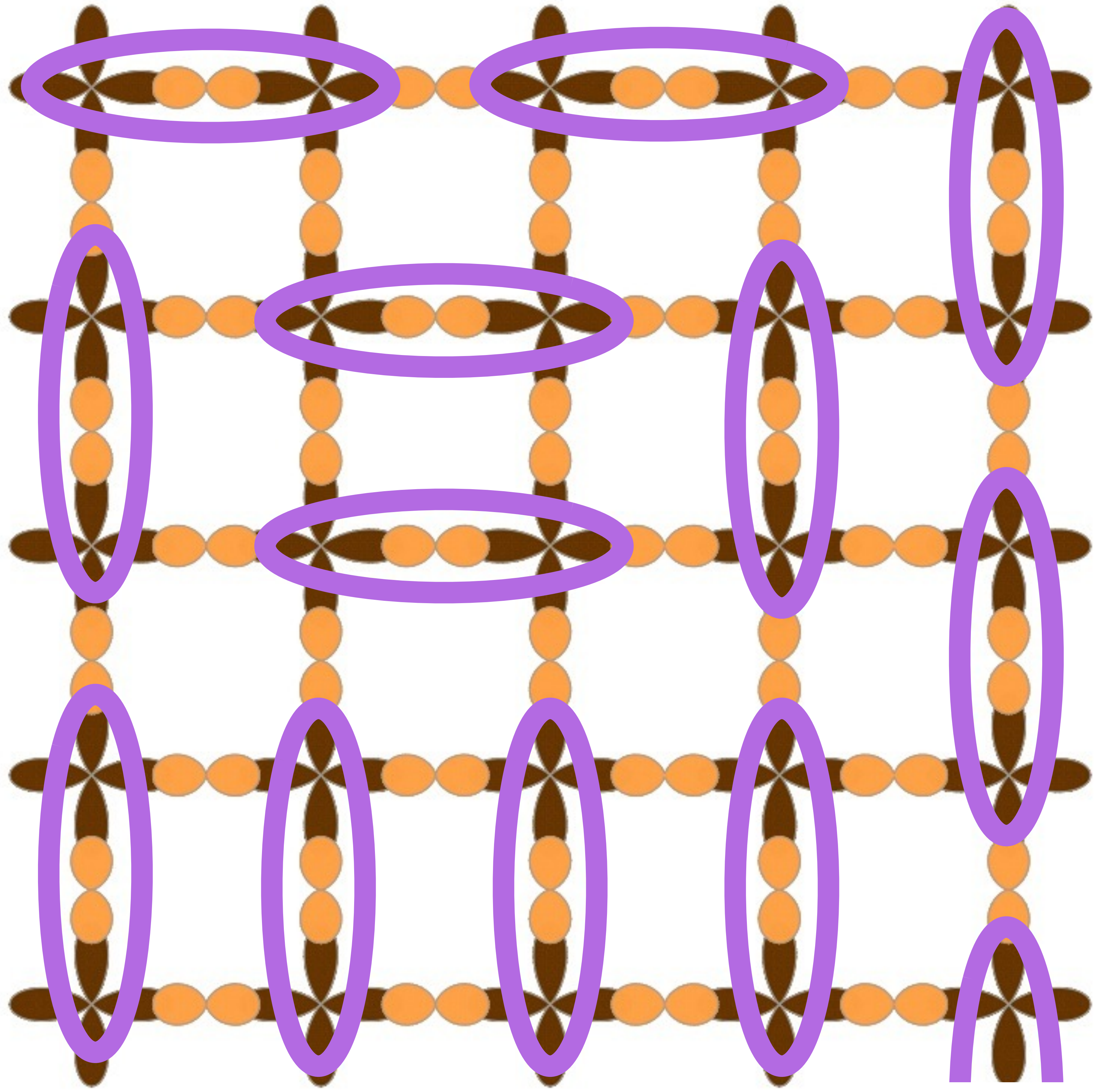

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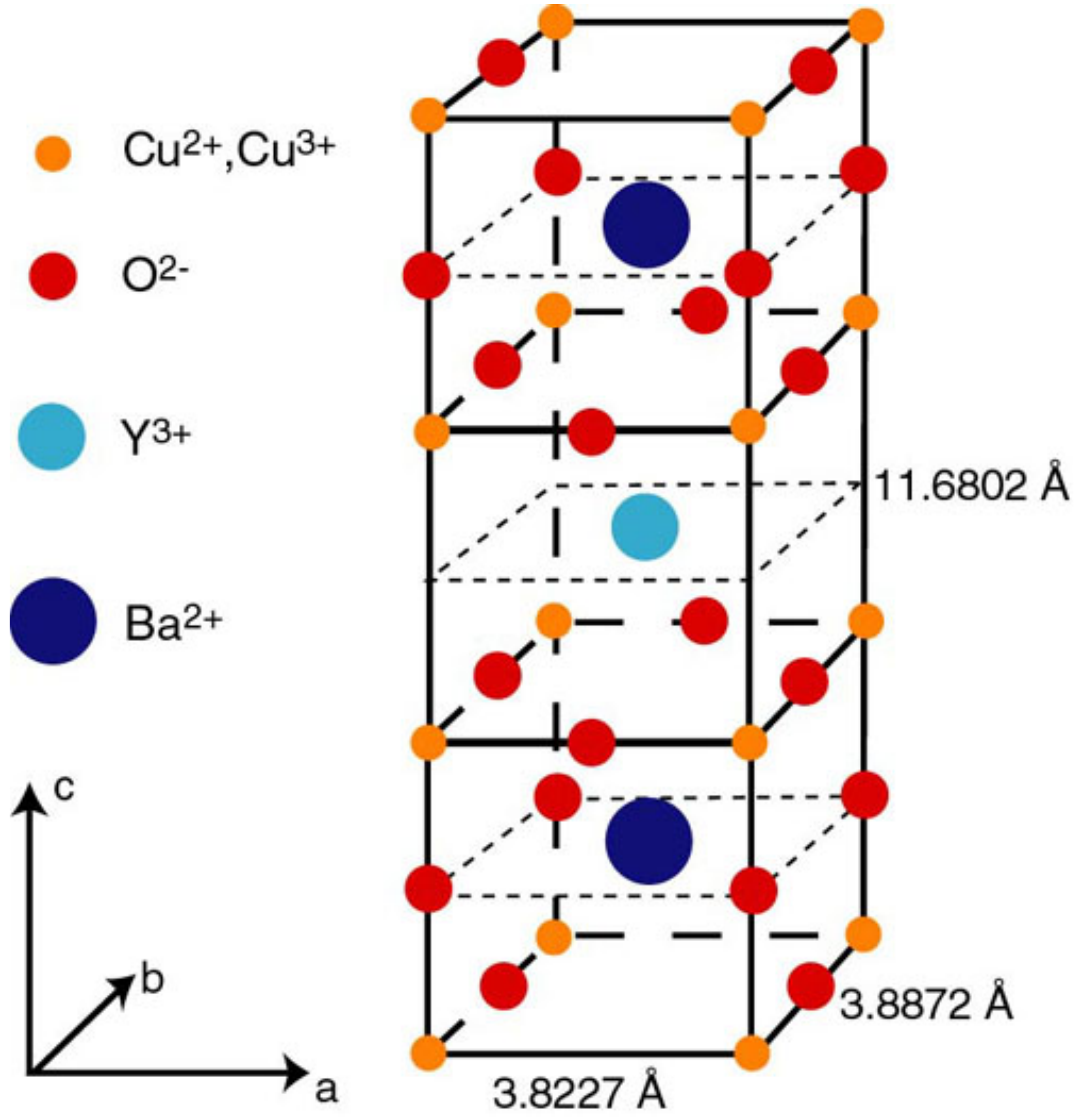

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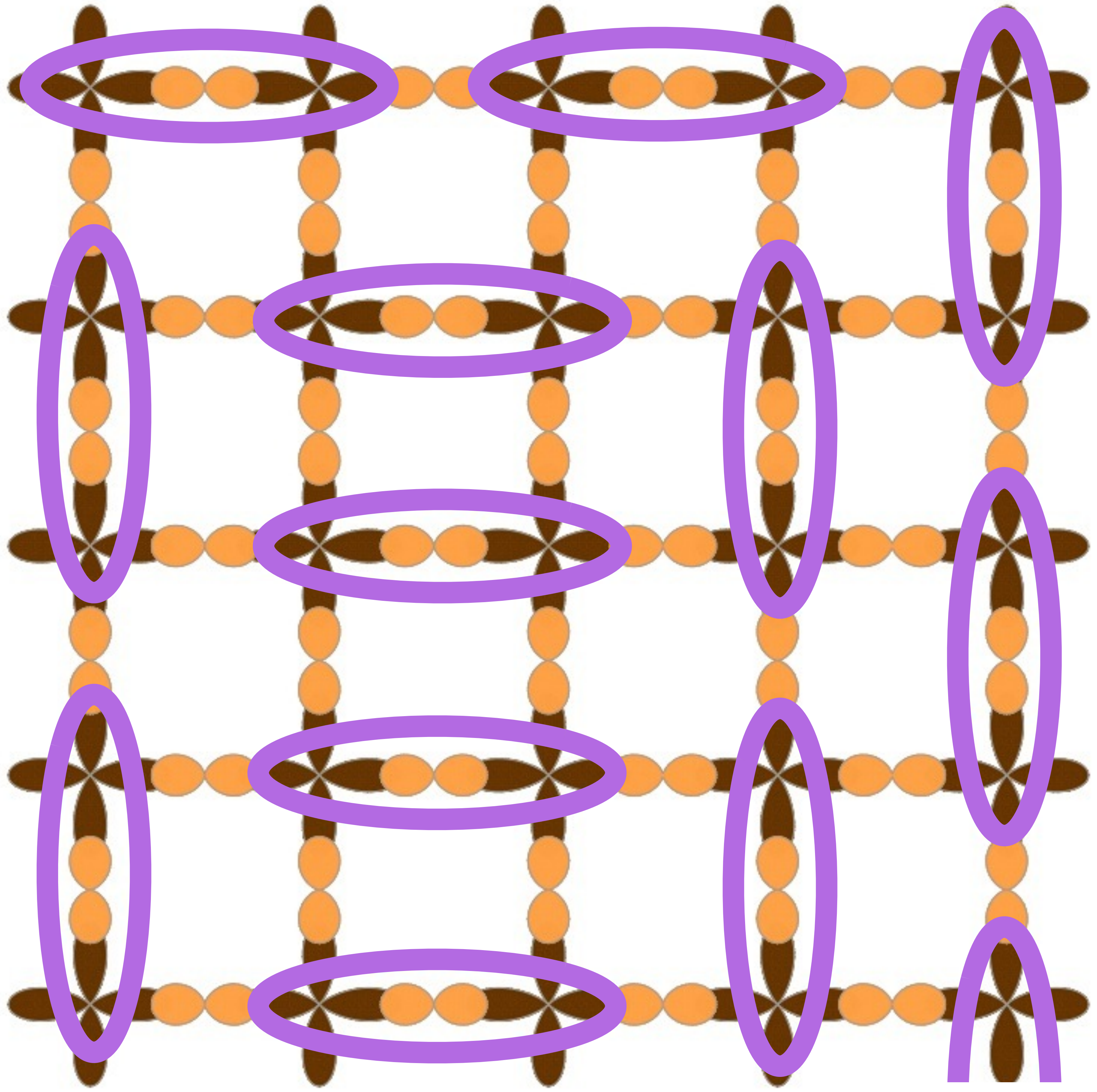

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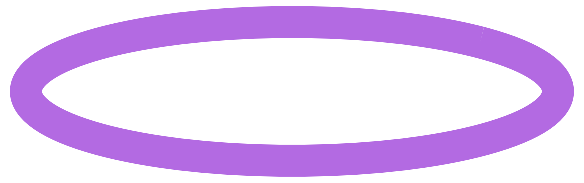 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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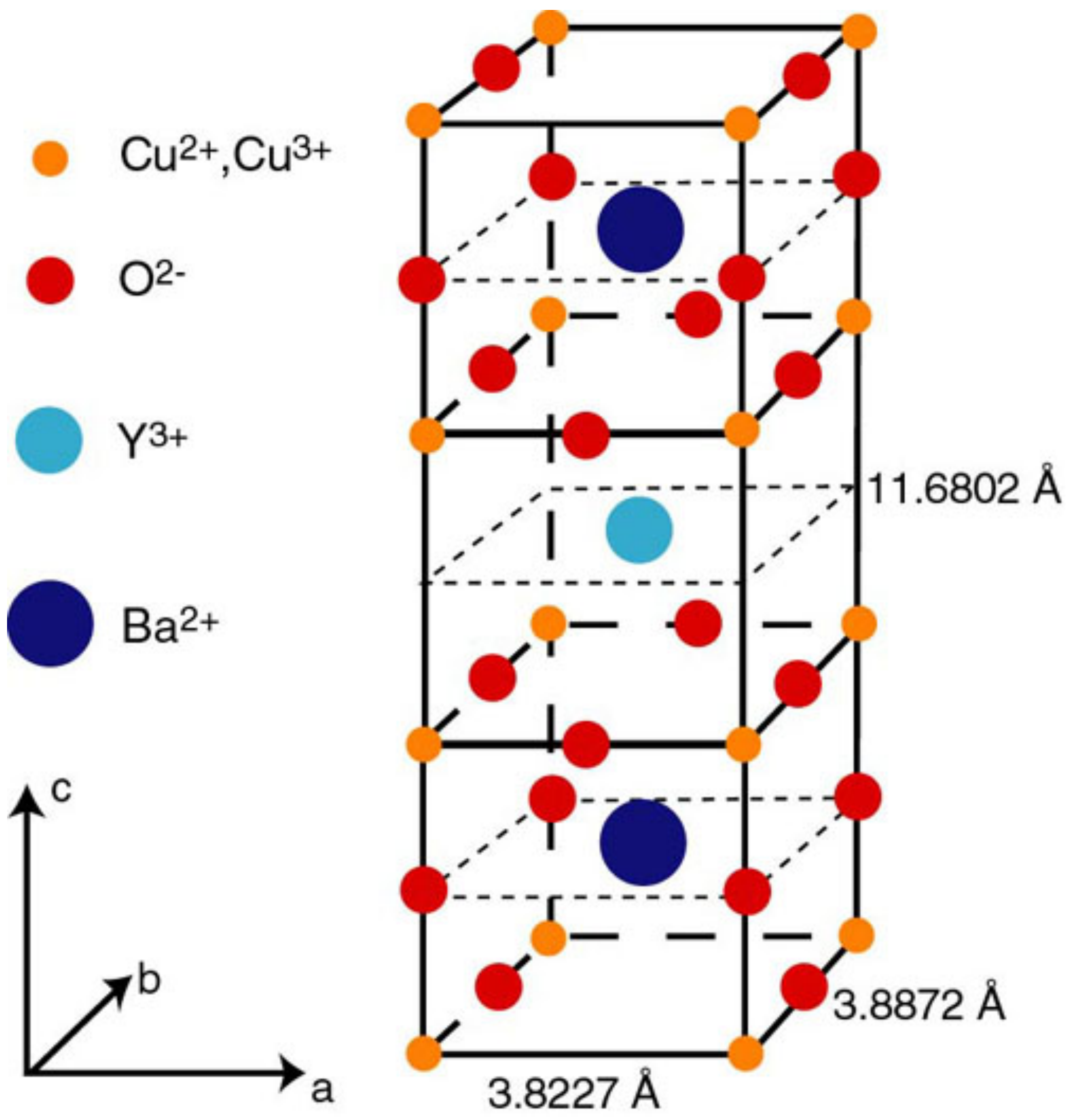

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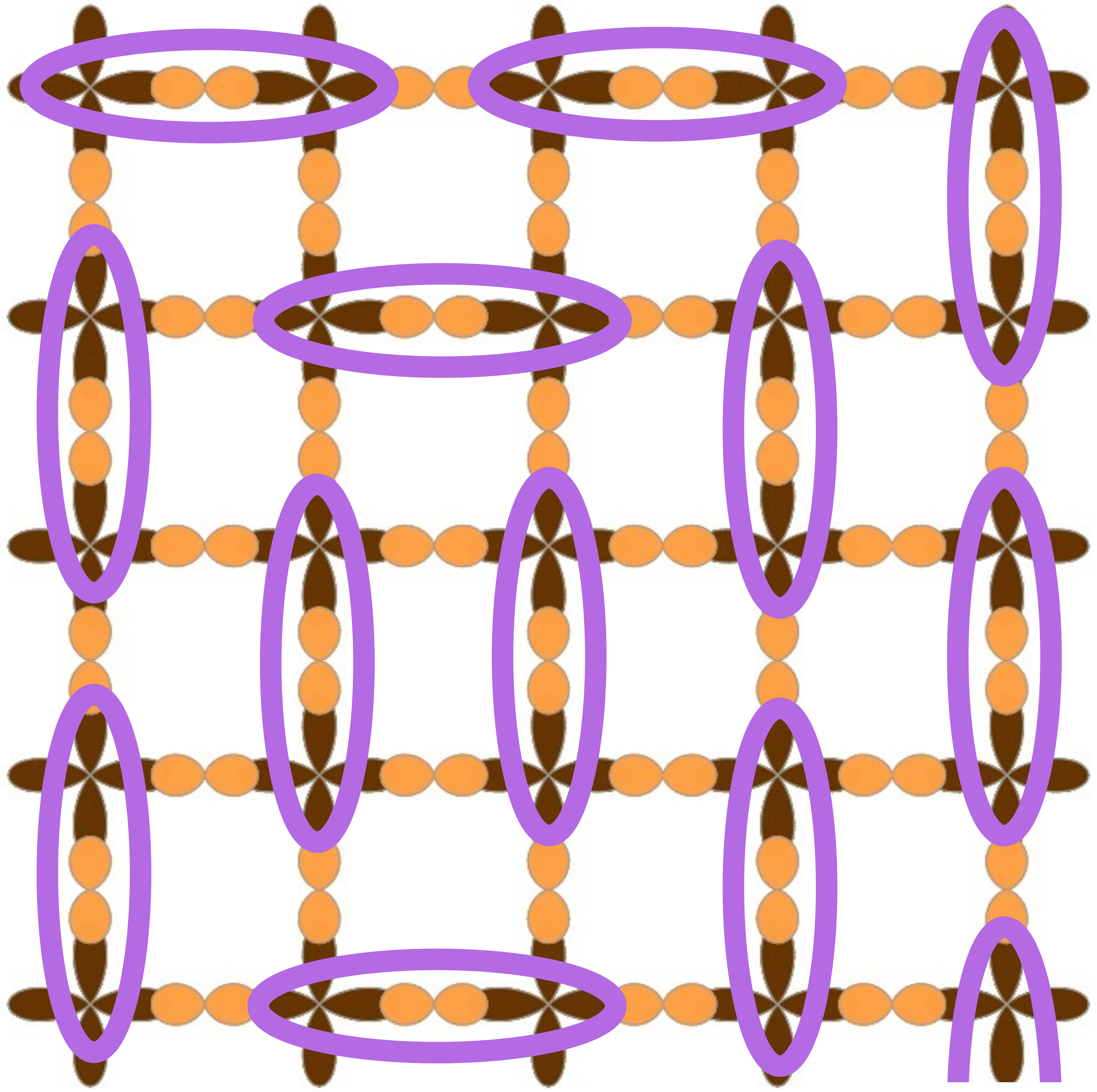

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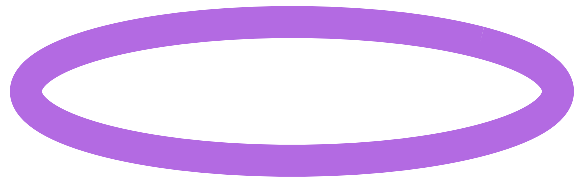 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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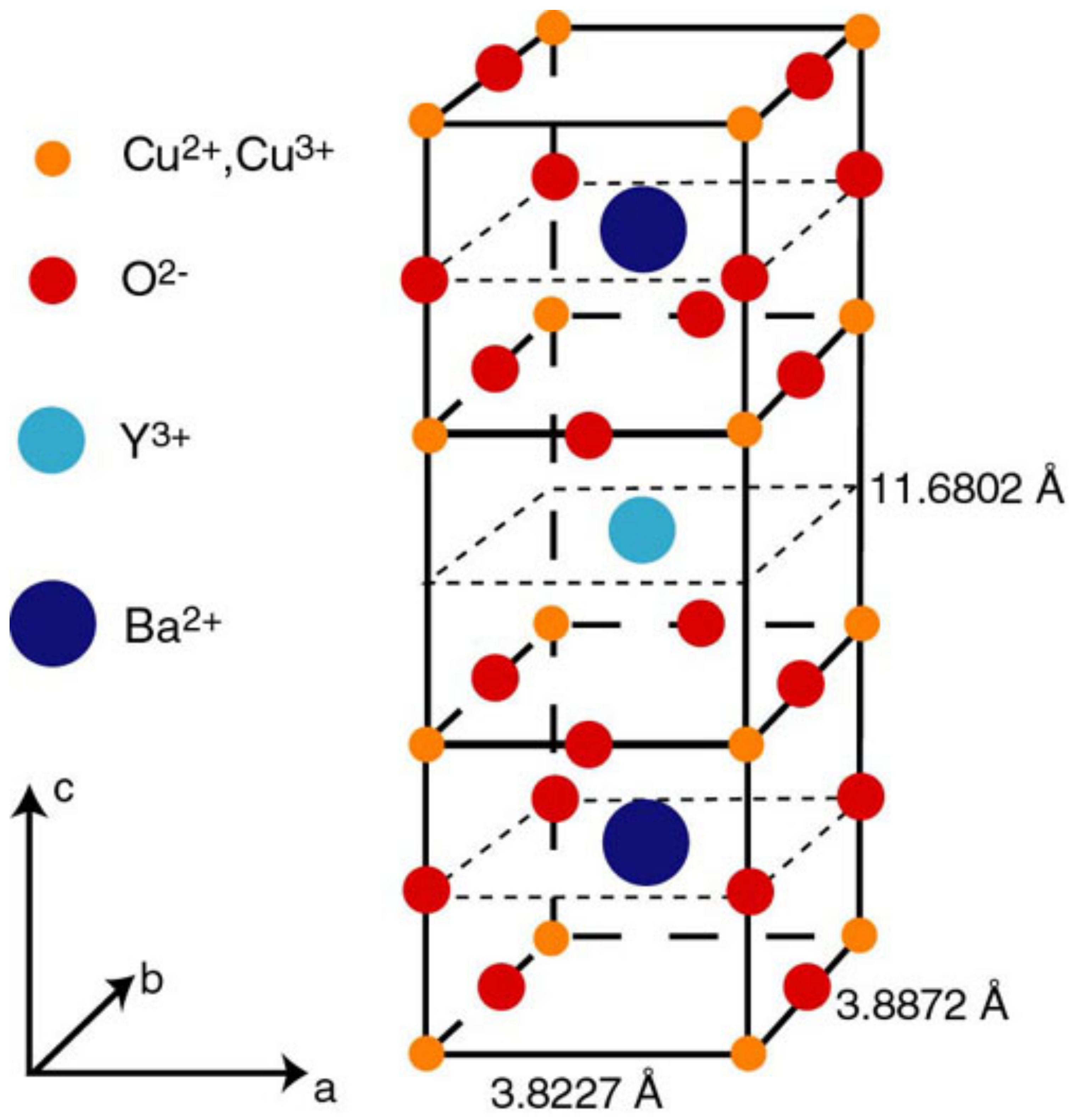

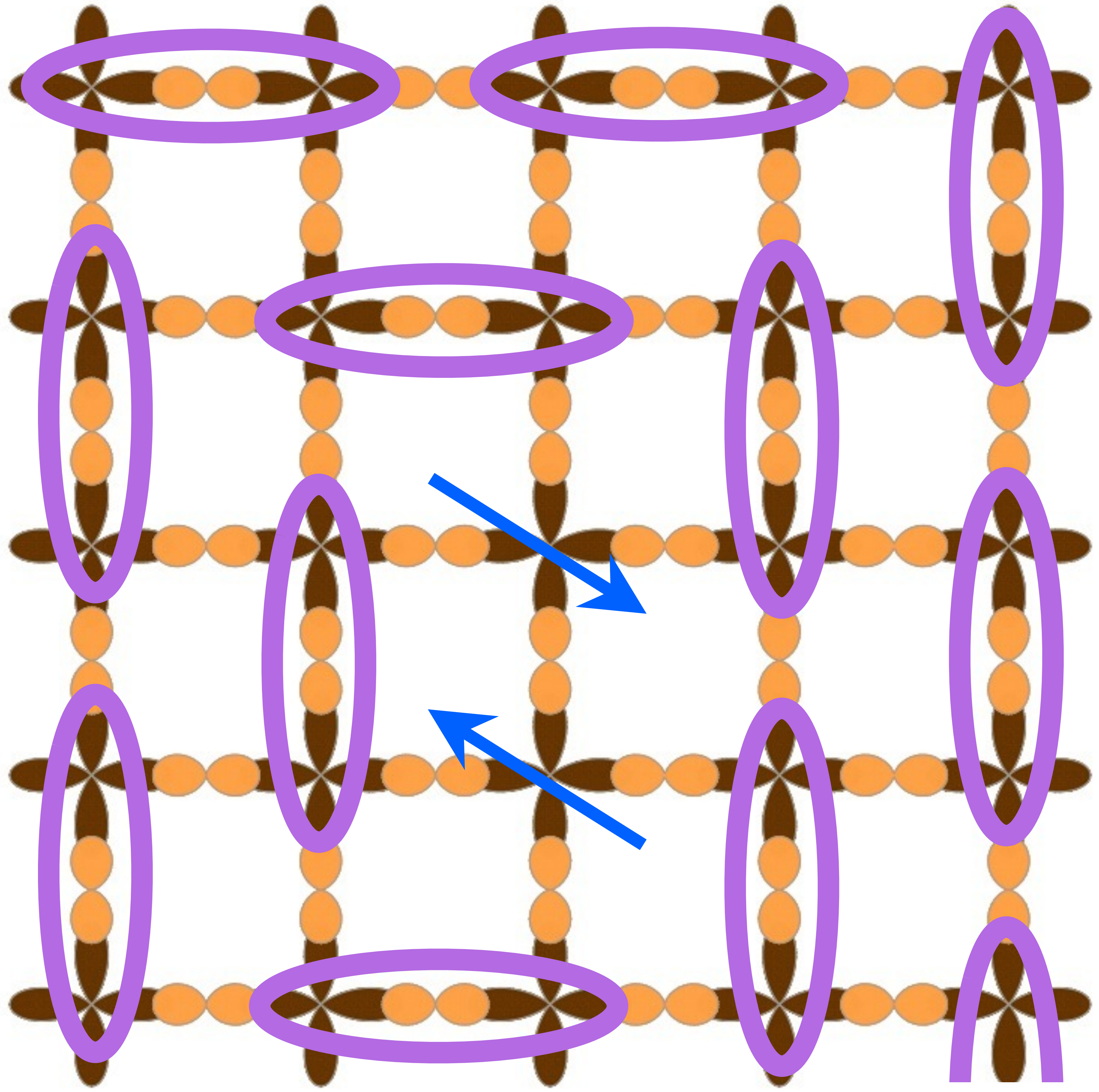

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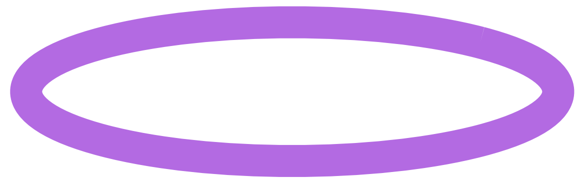
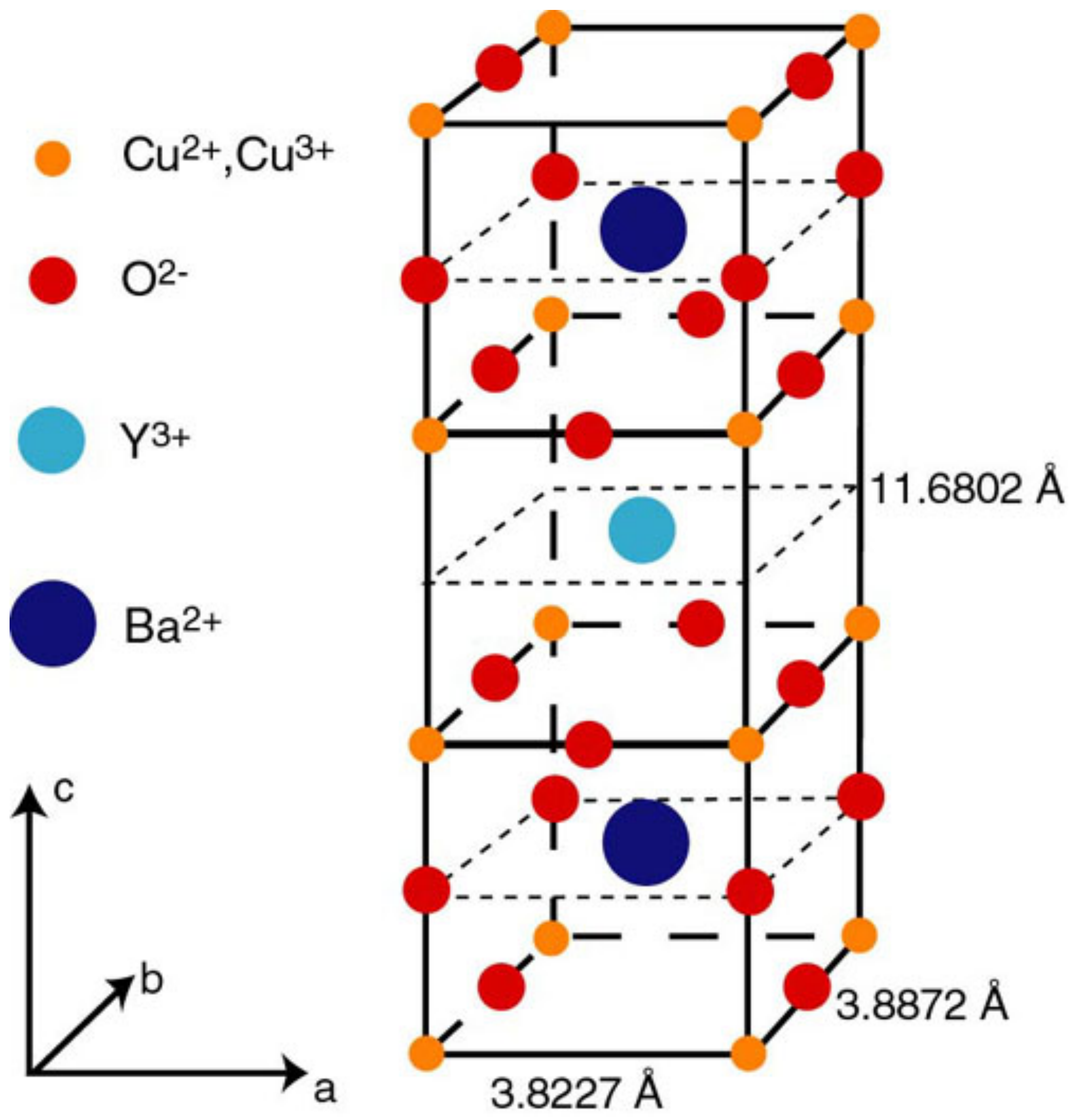
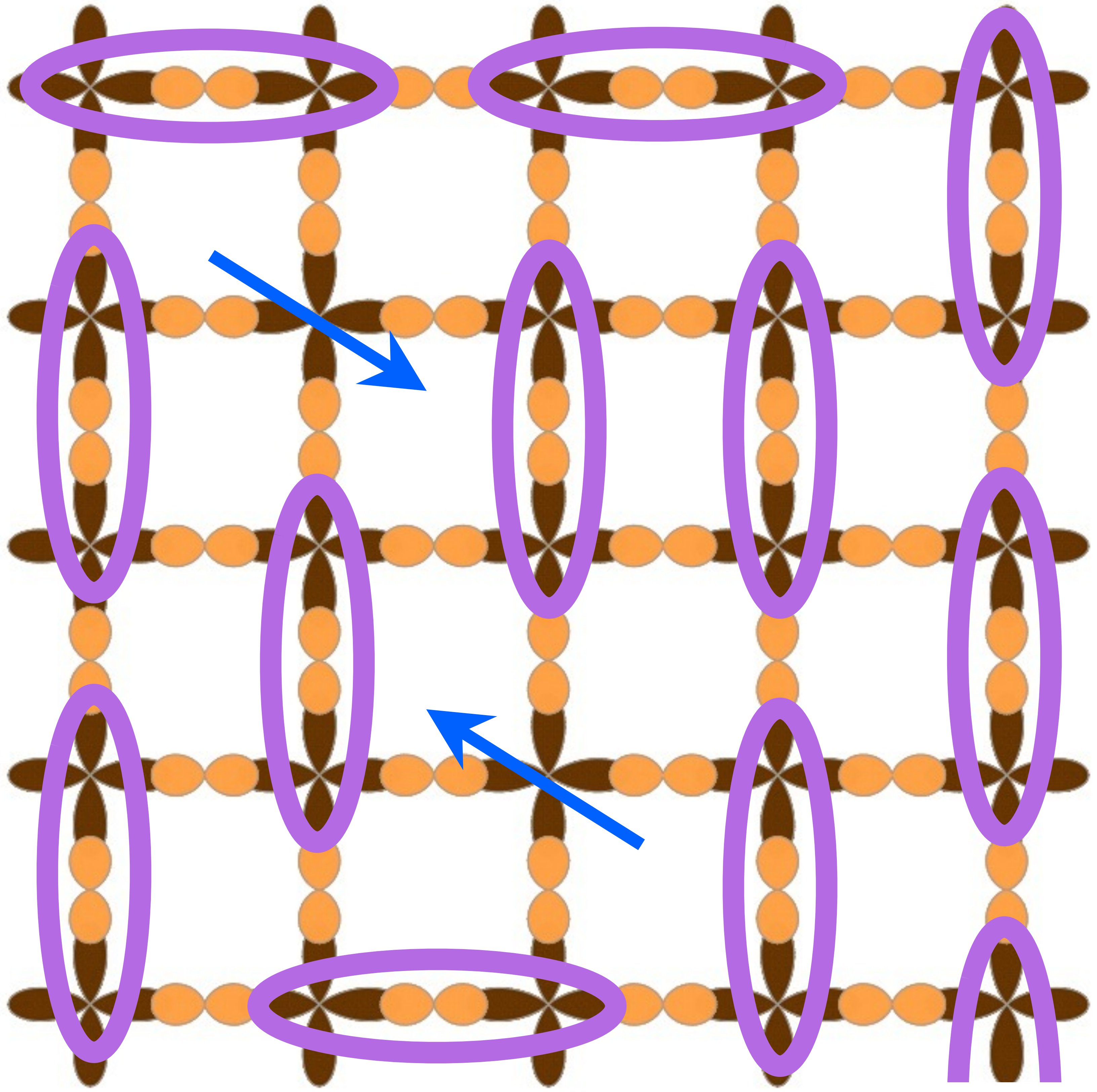
 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

<u>Key feature: fractionalization.</u> Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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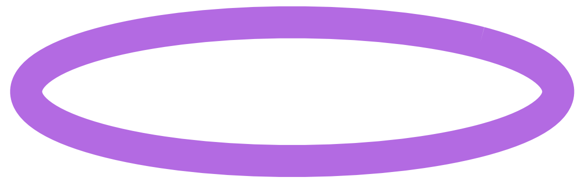 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

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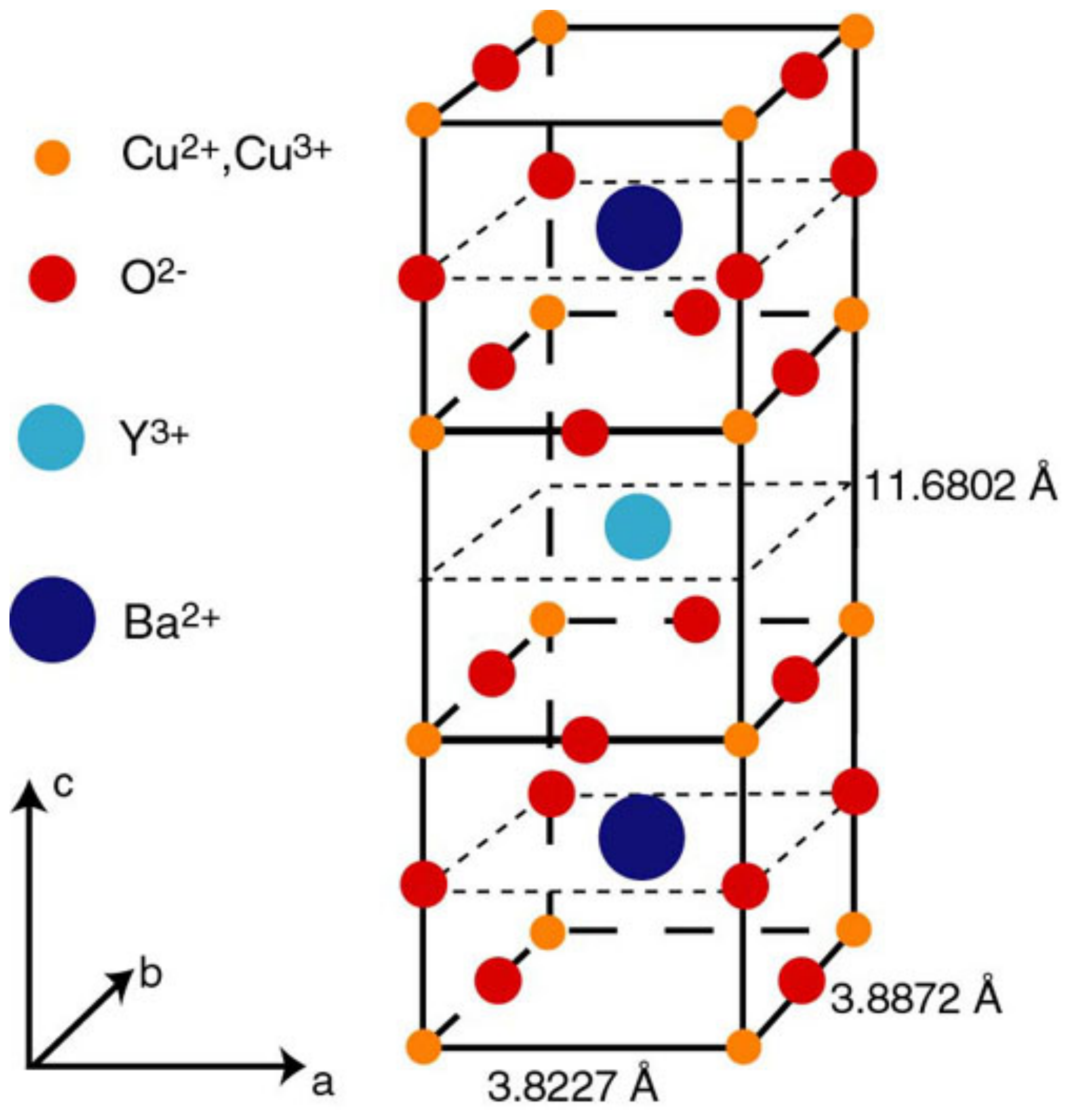

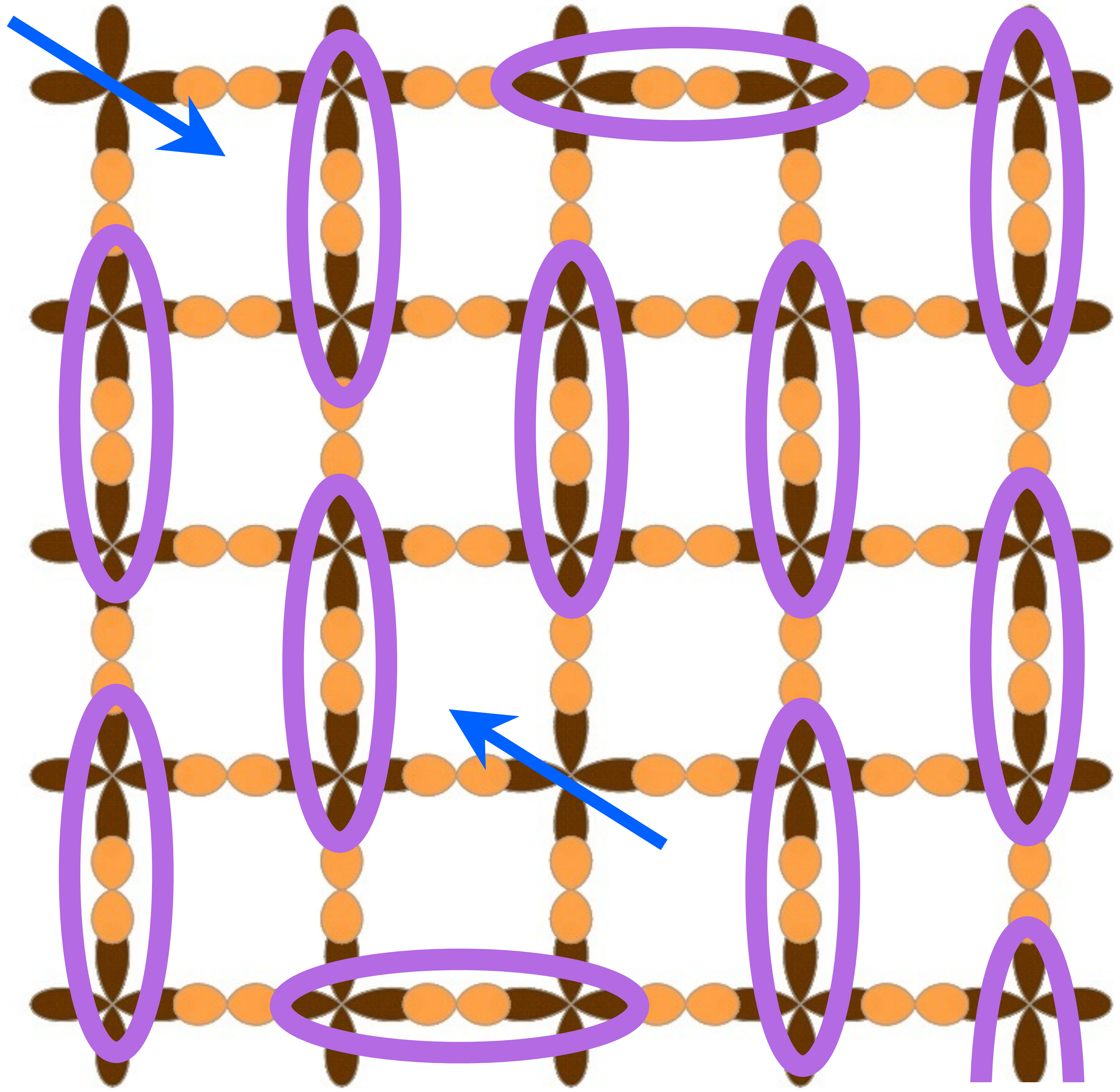

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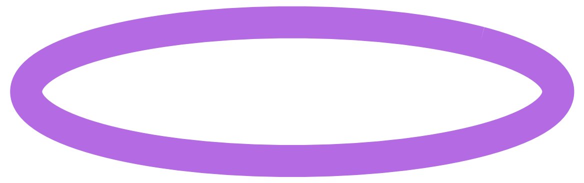

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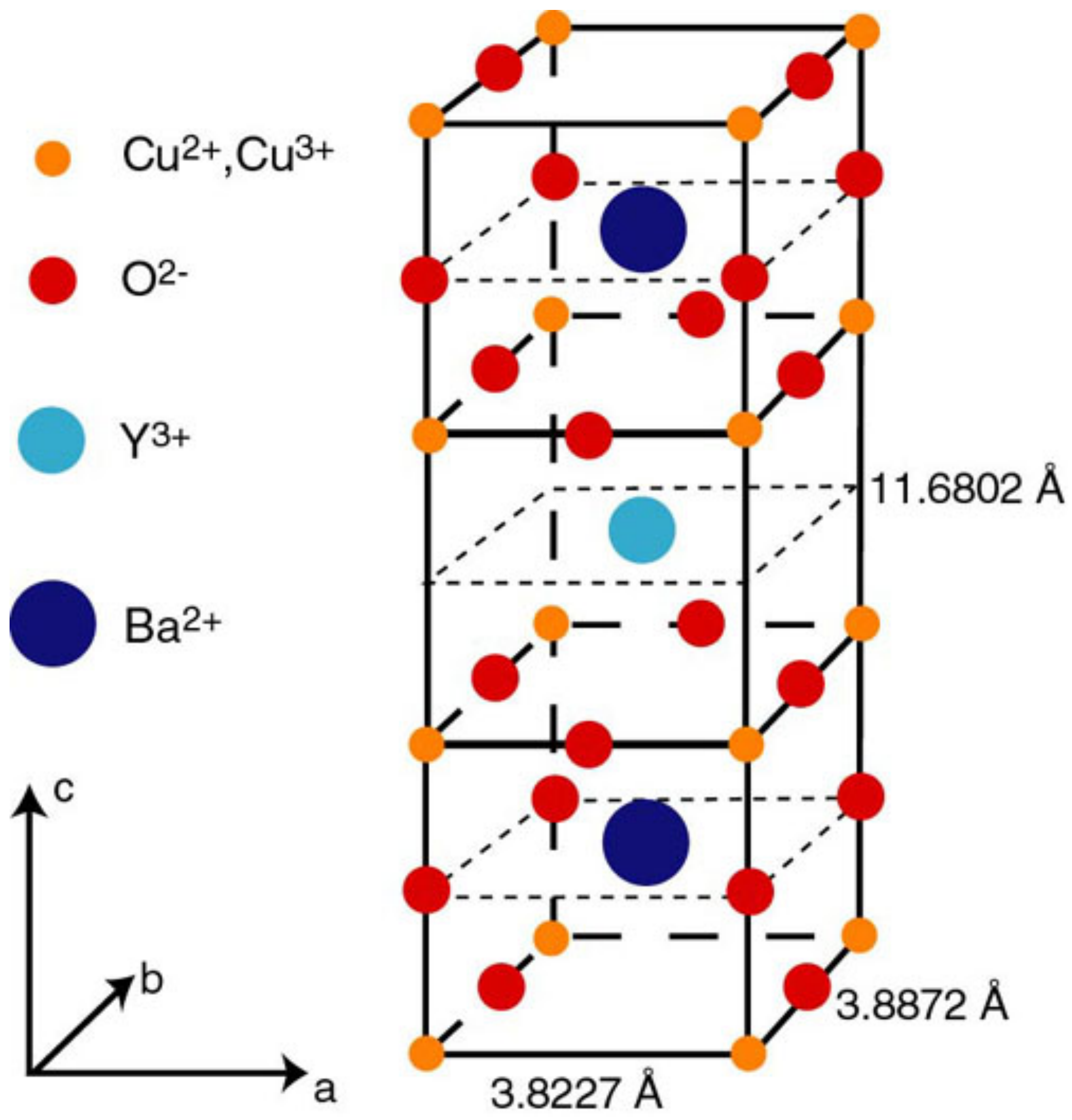

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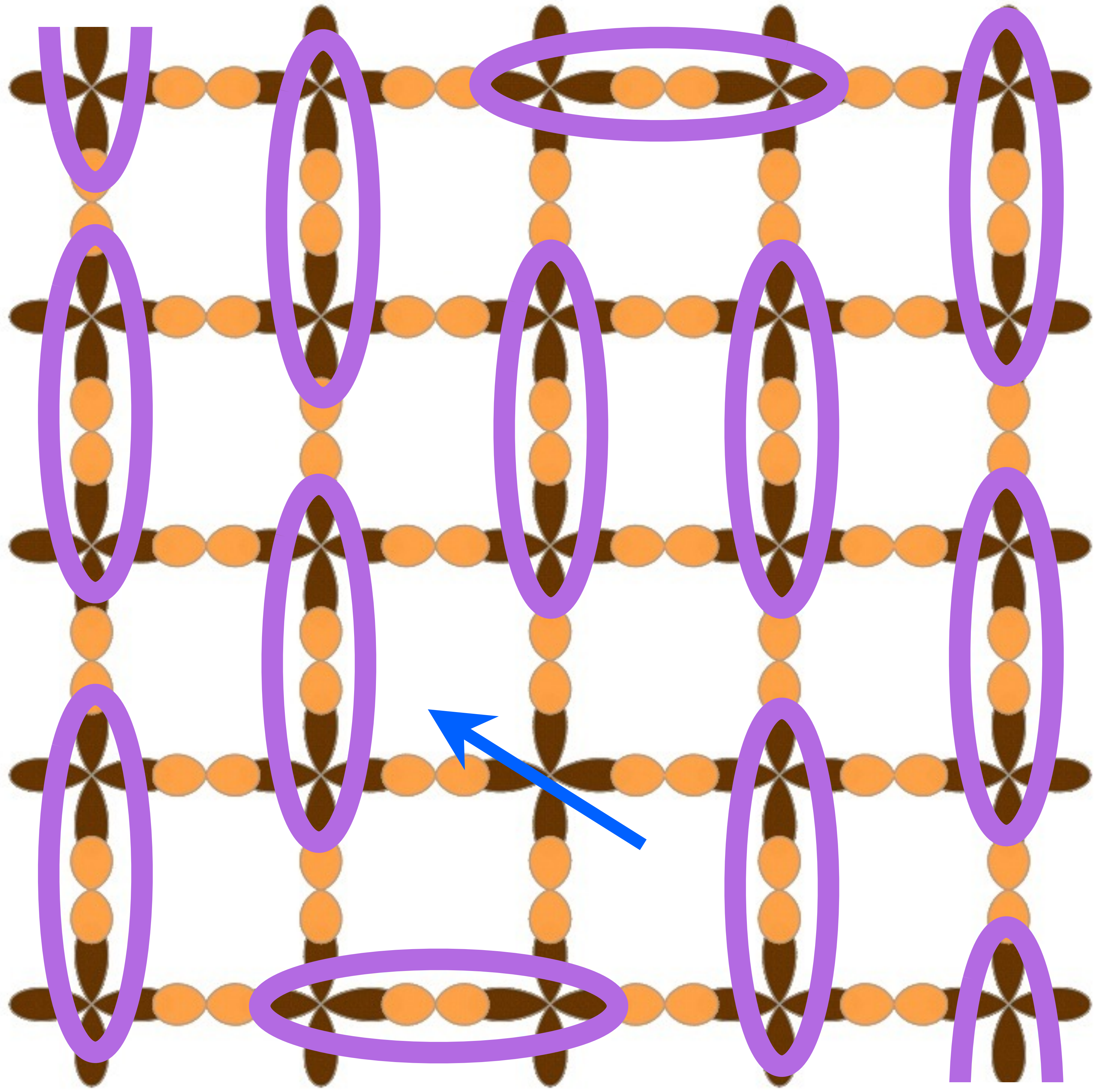

Spin S=1/2, charge neutral spinon

$$\text{(ellipse)} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

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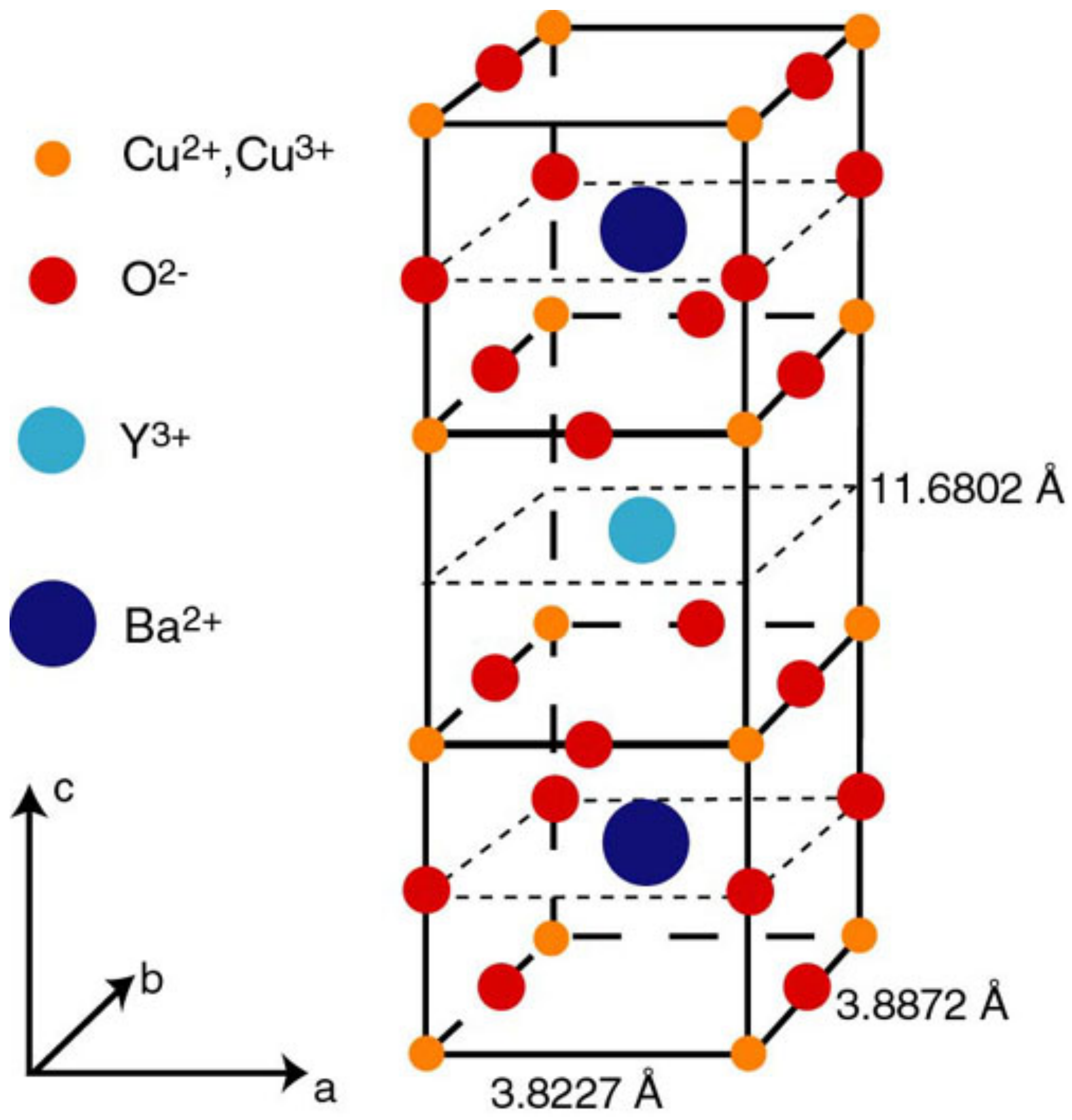

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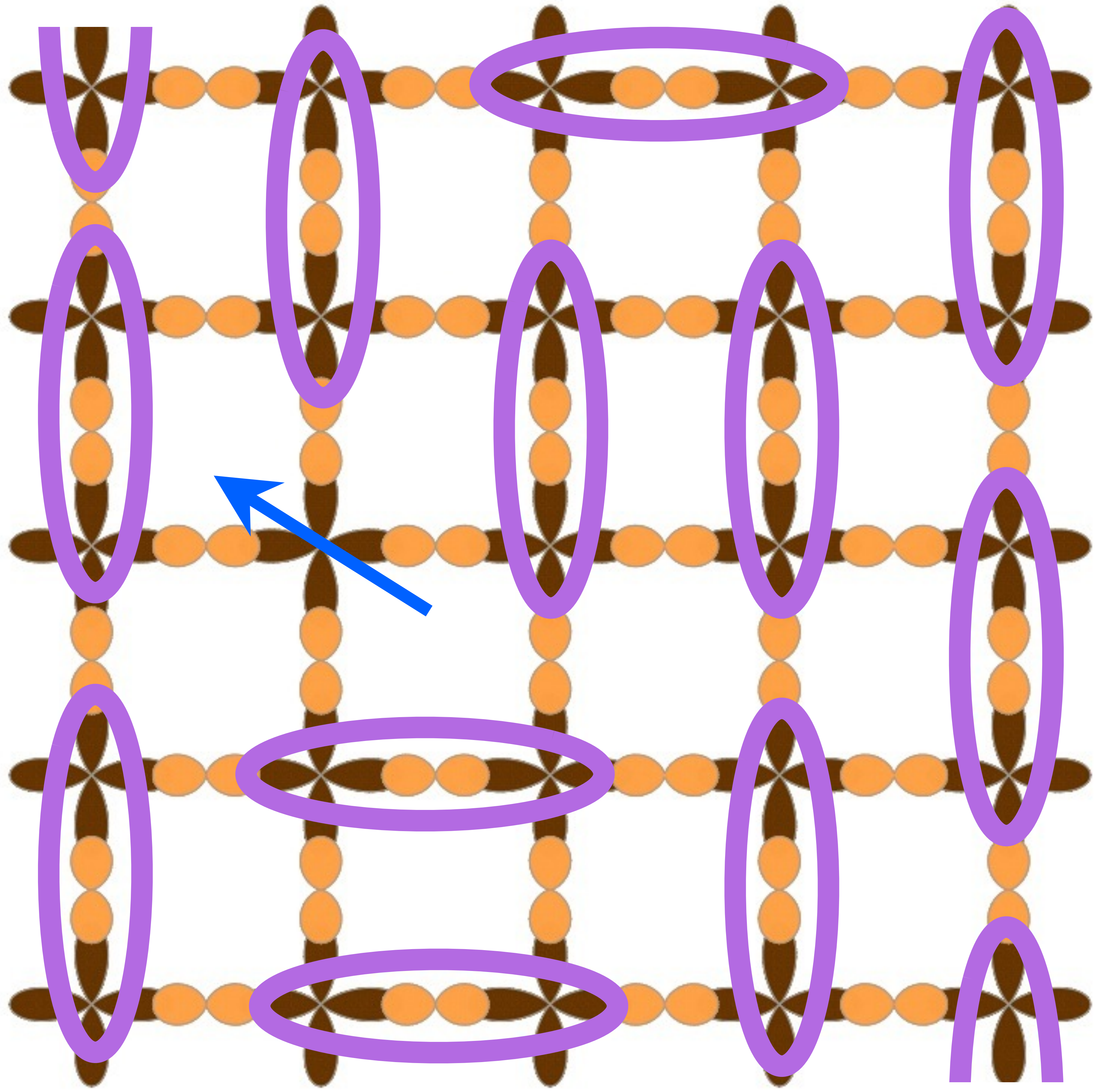

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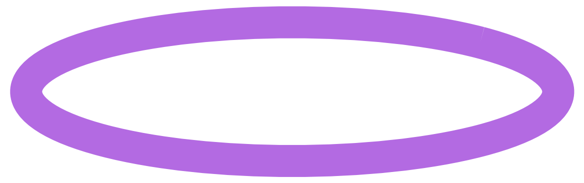

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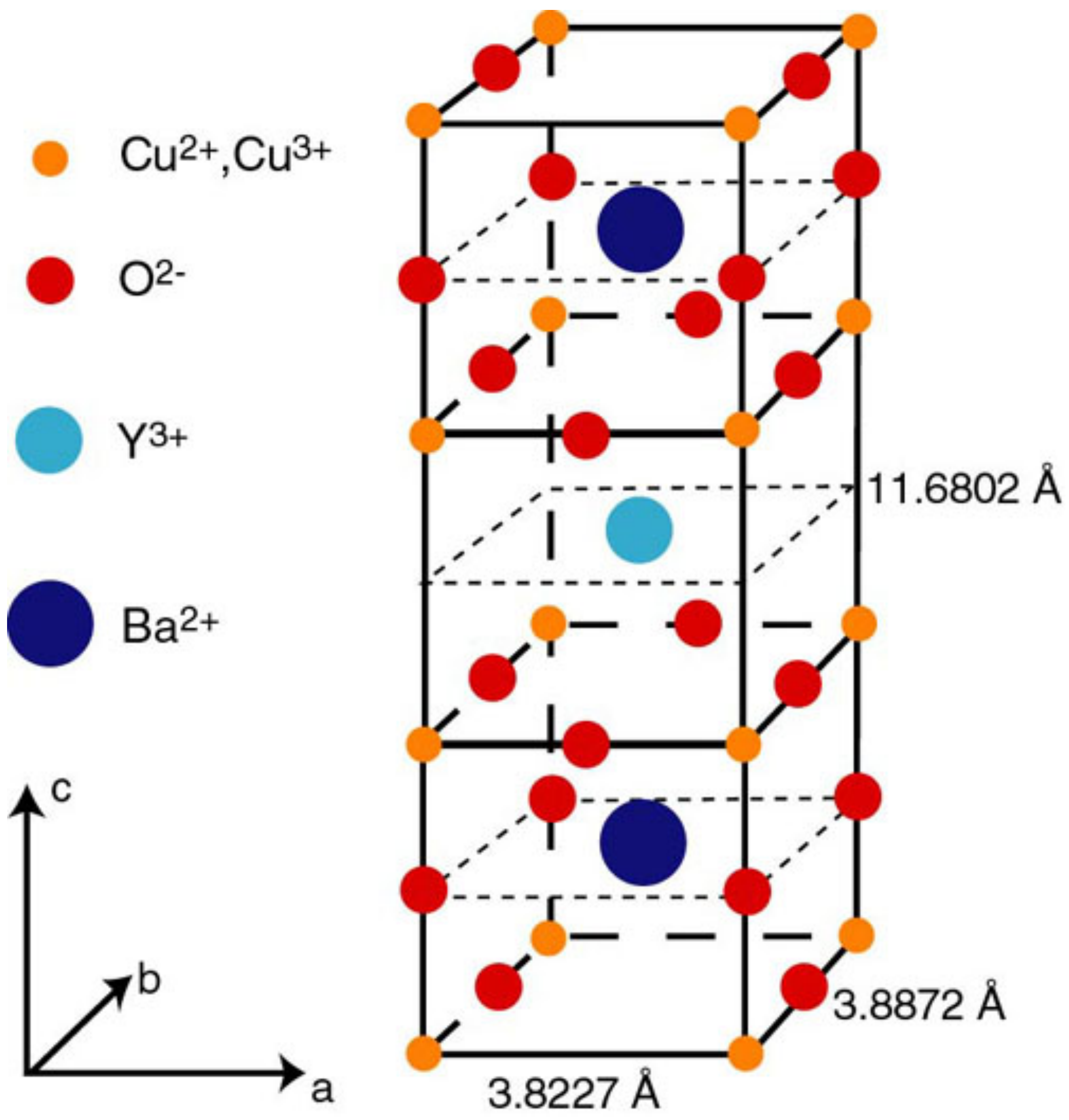

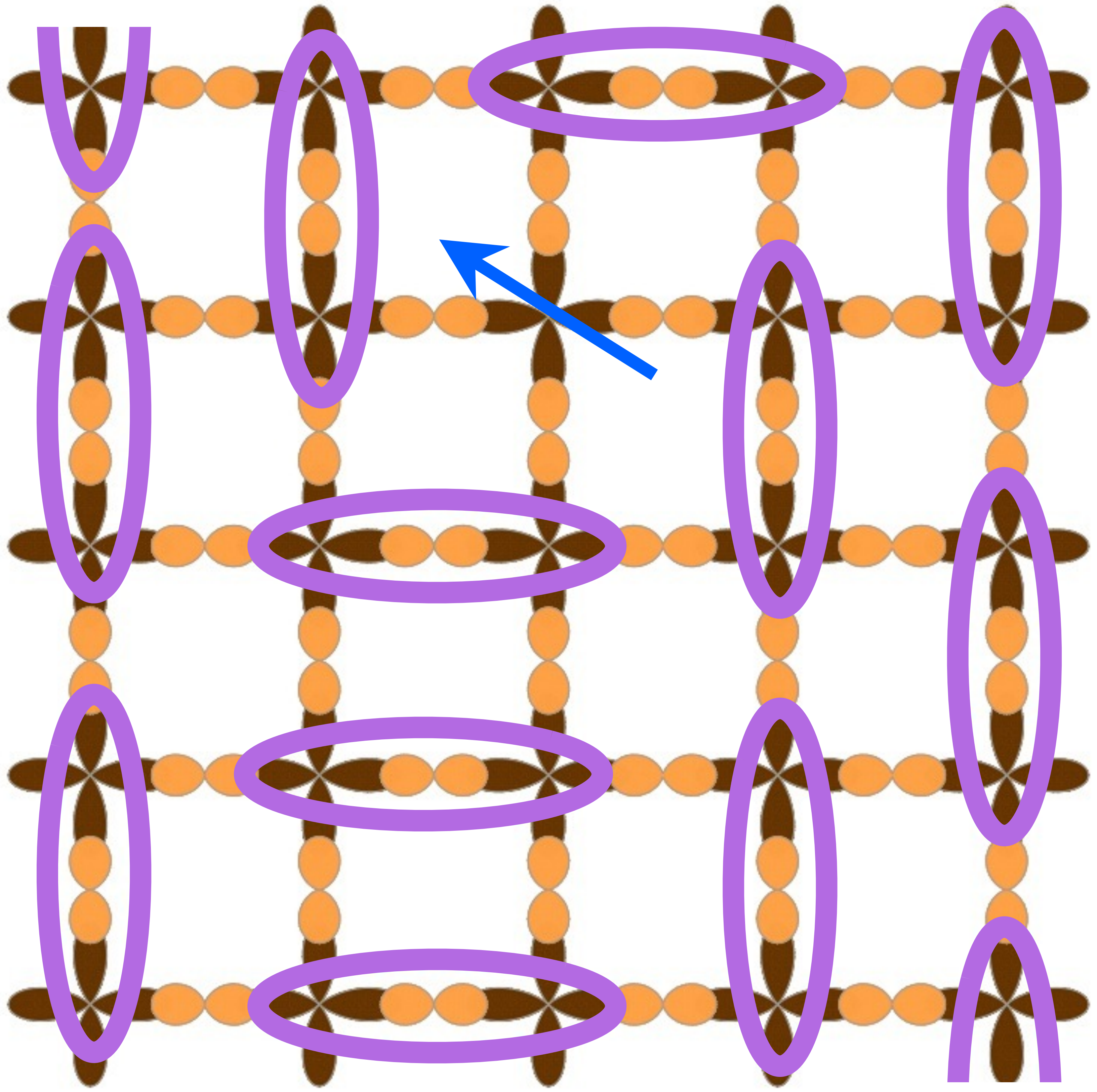

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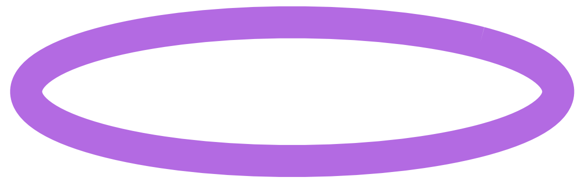

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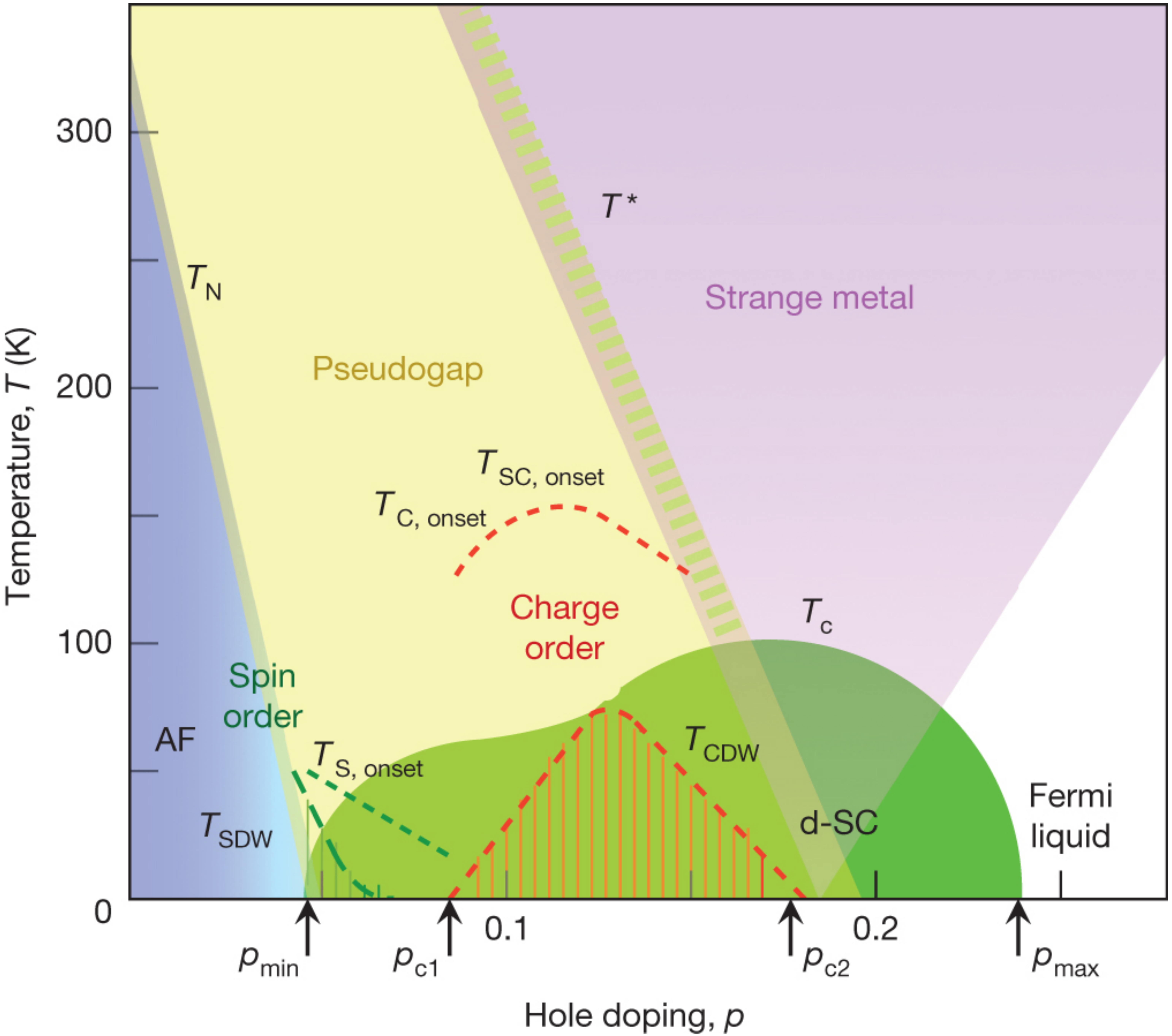
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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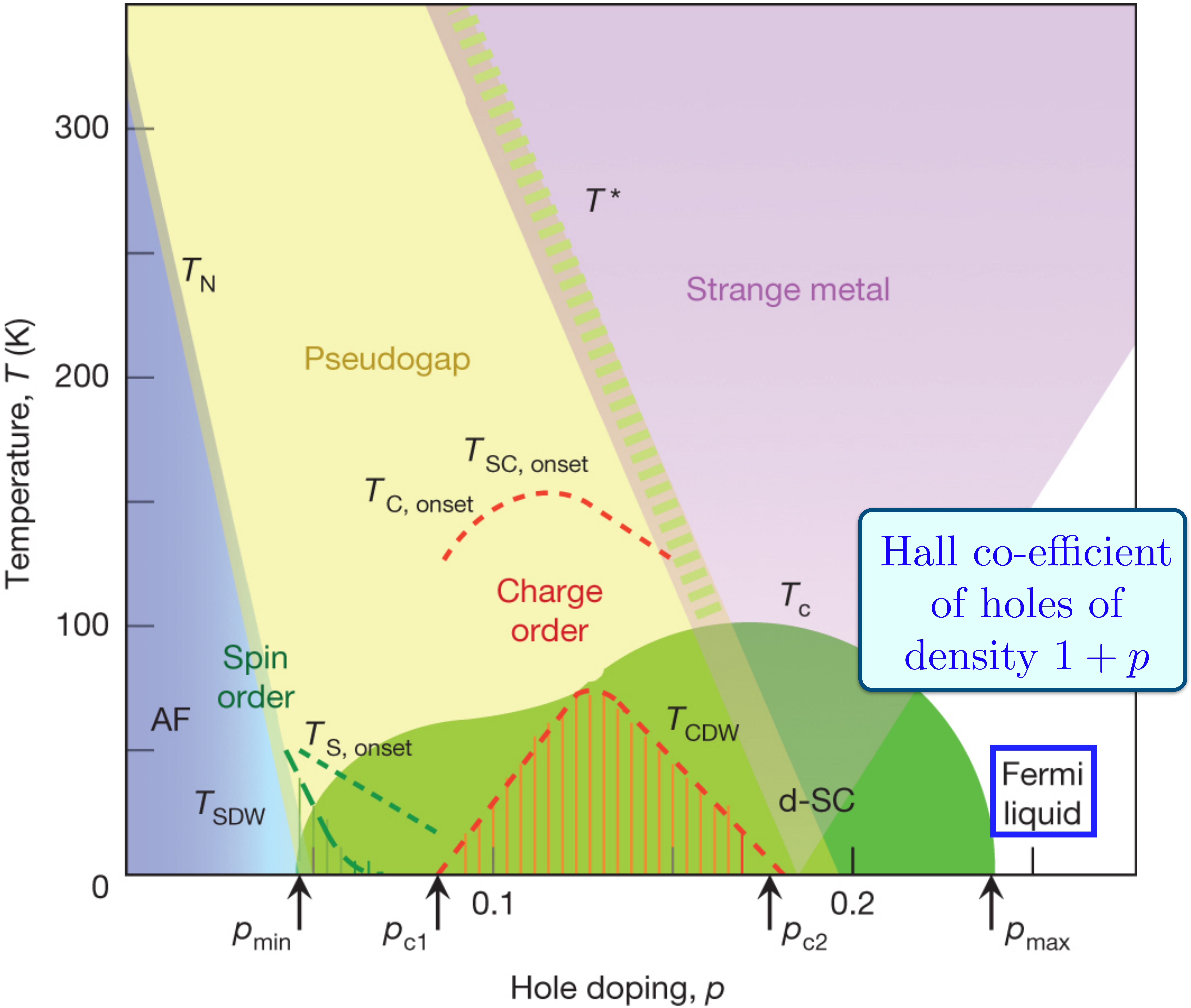

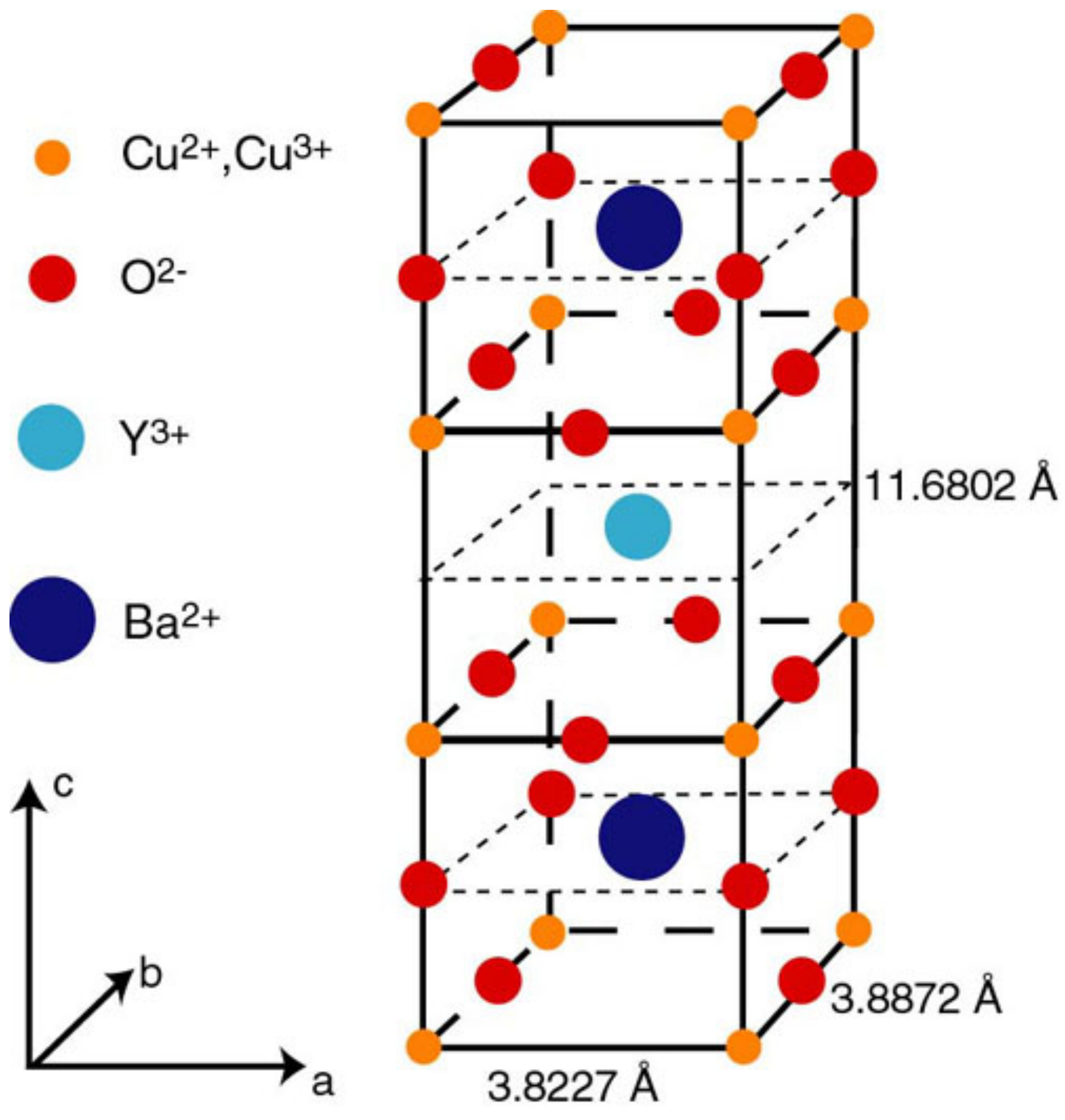

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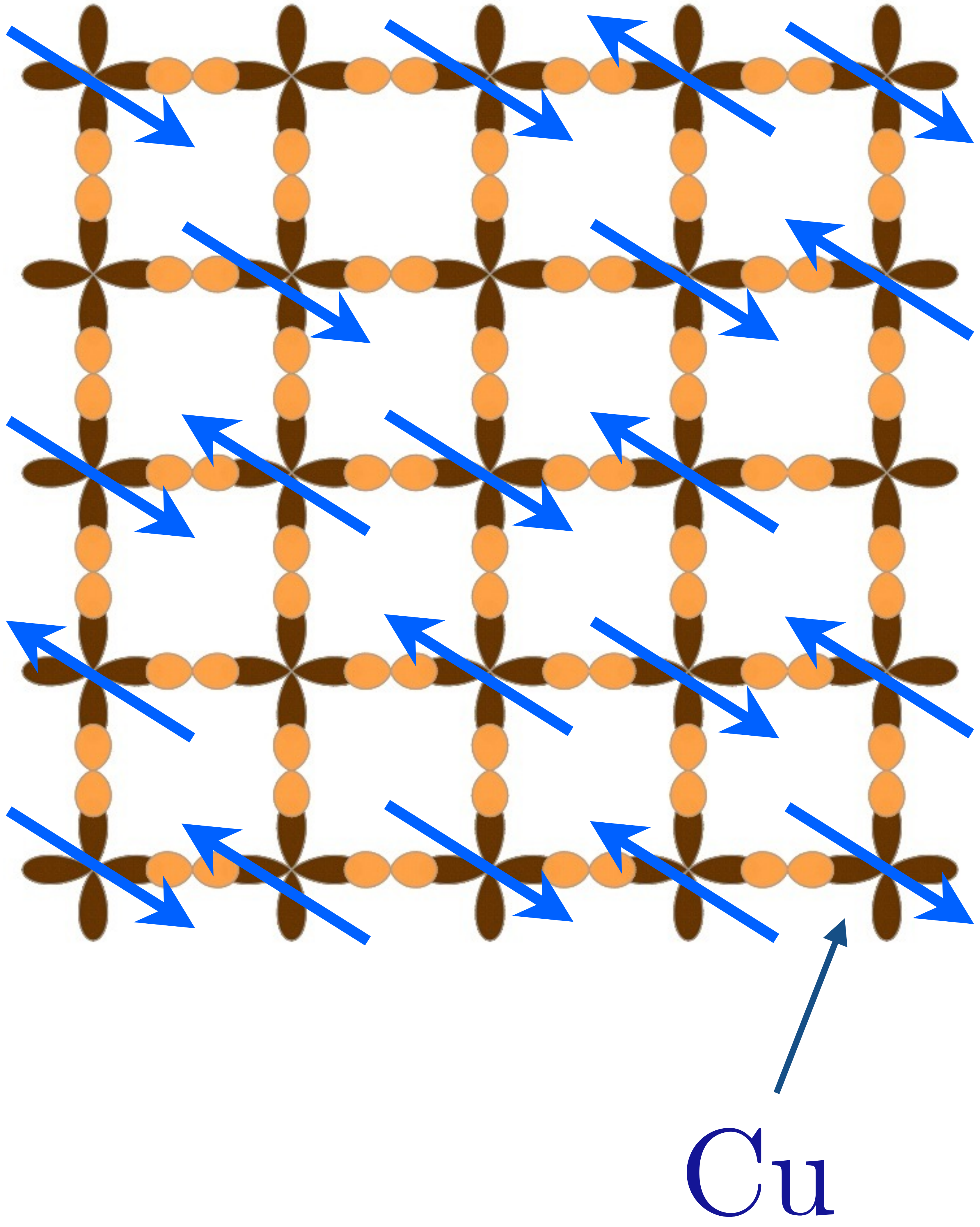
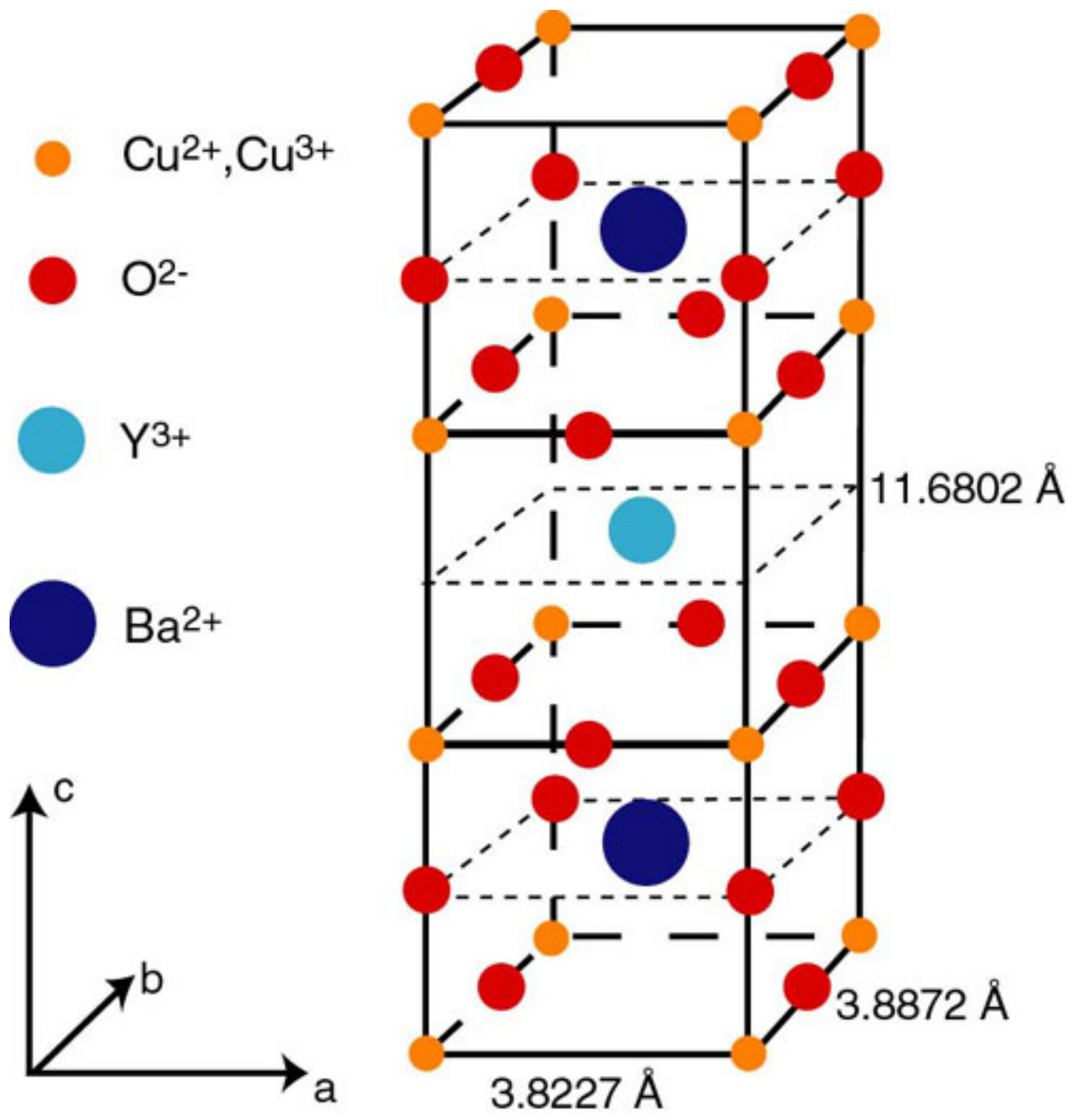


High temperature superconductor obtained upon doping the antiferromagnet with density p holes.



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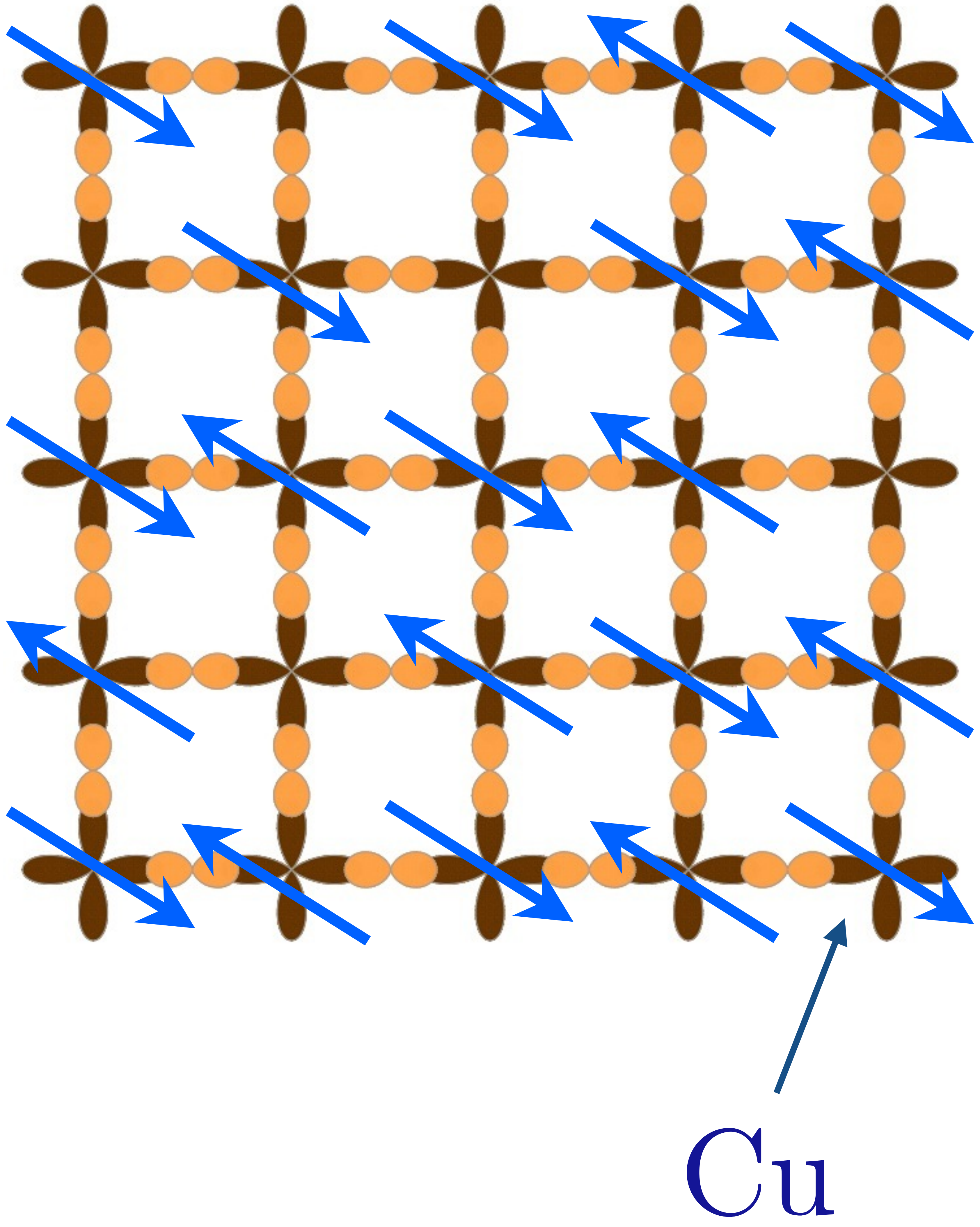

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Hole density relative to the filled band $\rho = 1 + p$.

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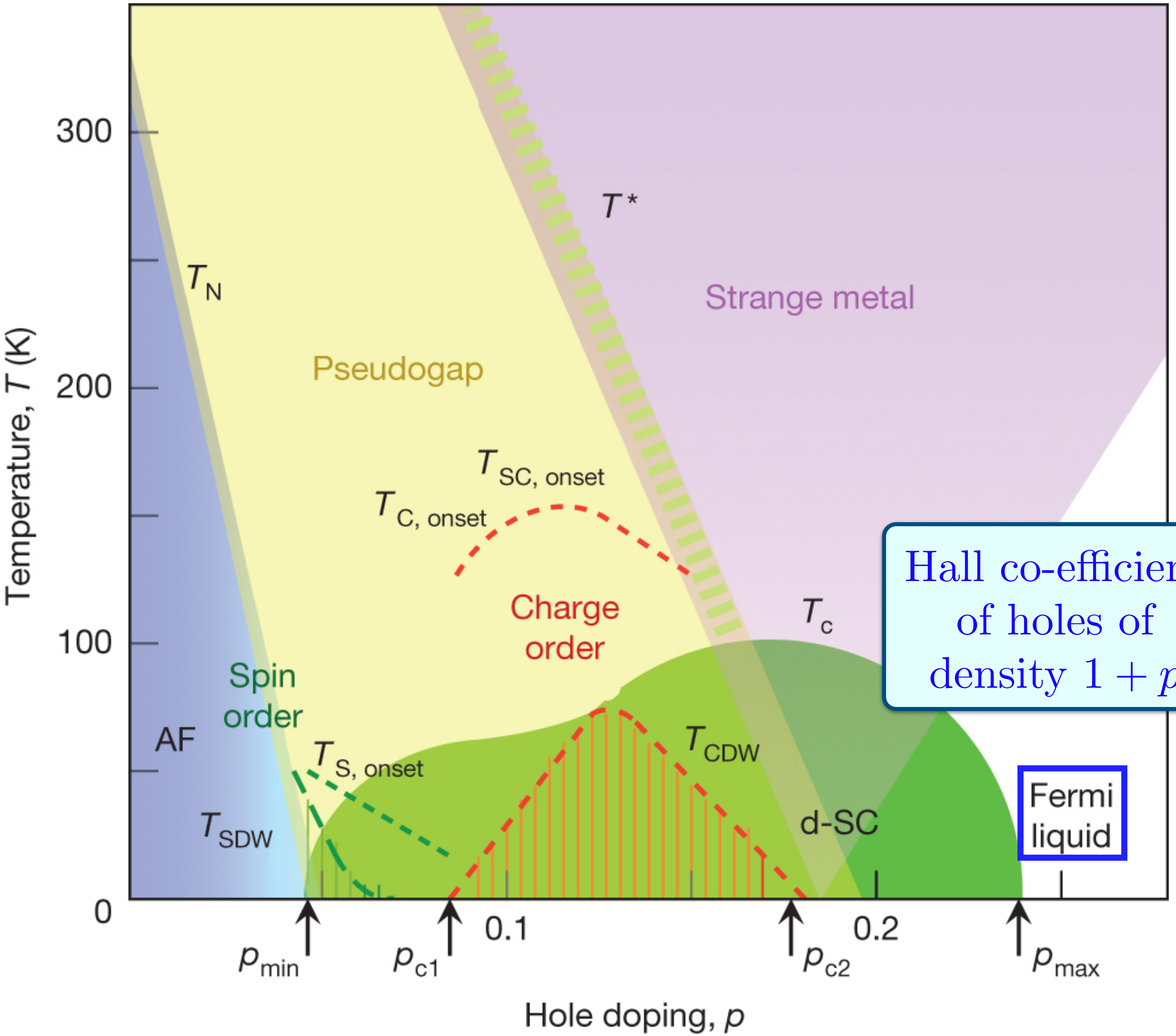
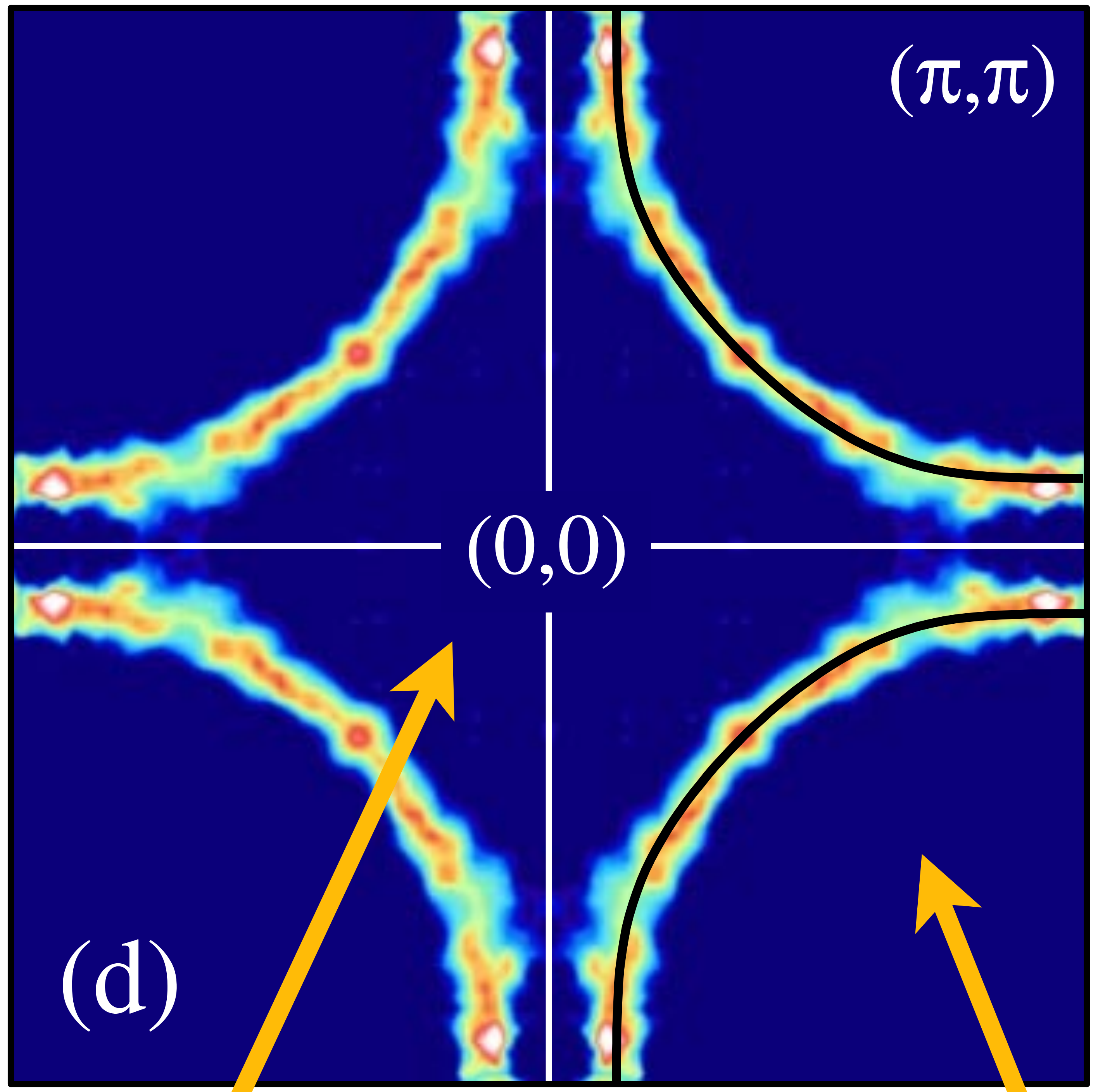


Hall co-efficient of holes of density $1 + p$

M. Platé *et al.*, PRL **95**, 077001 (2005)



1-p electrons

1+p holes

Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

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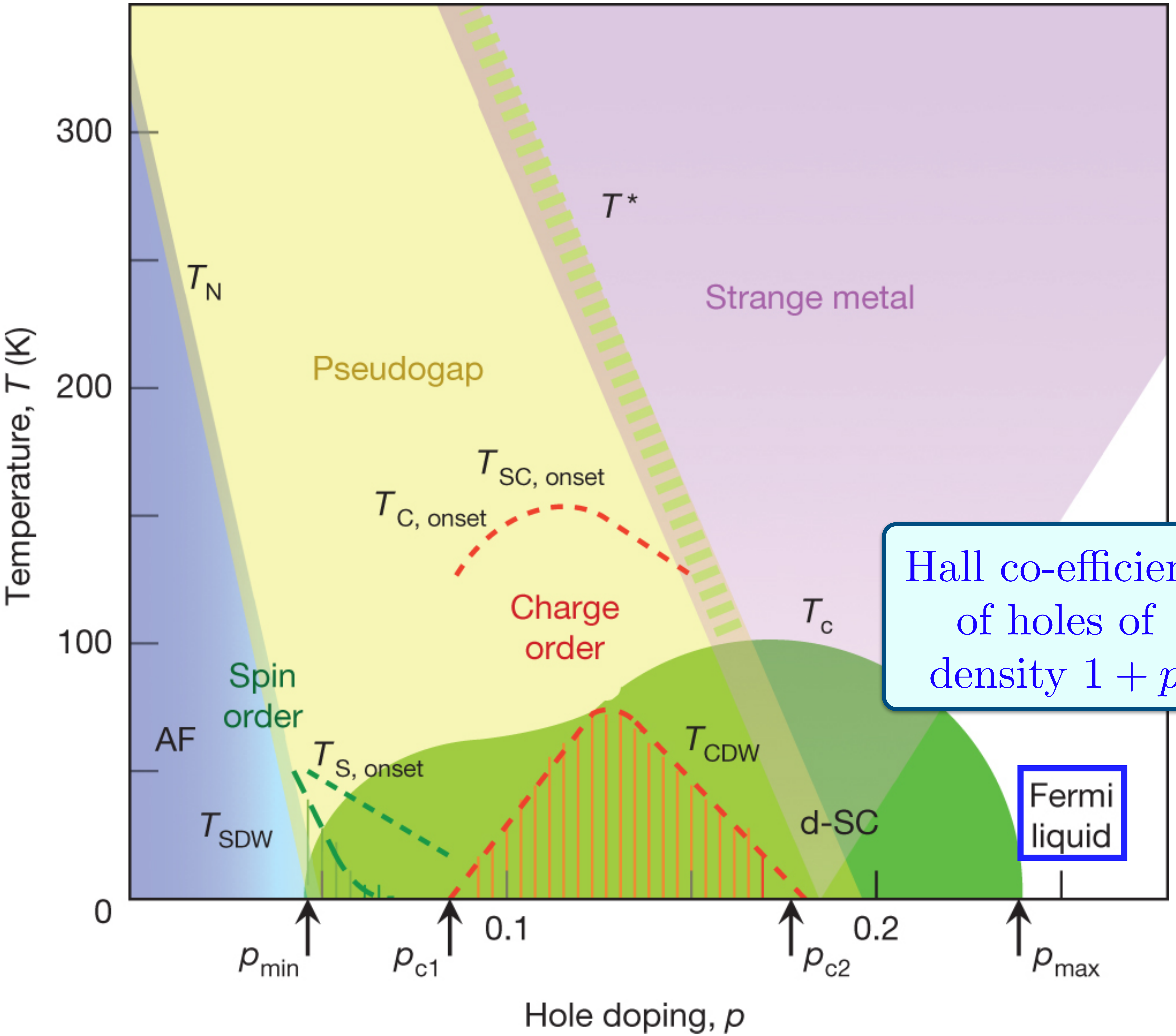

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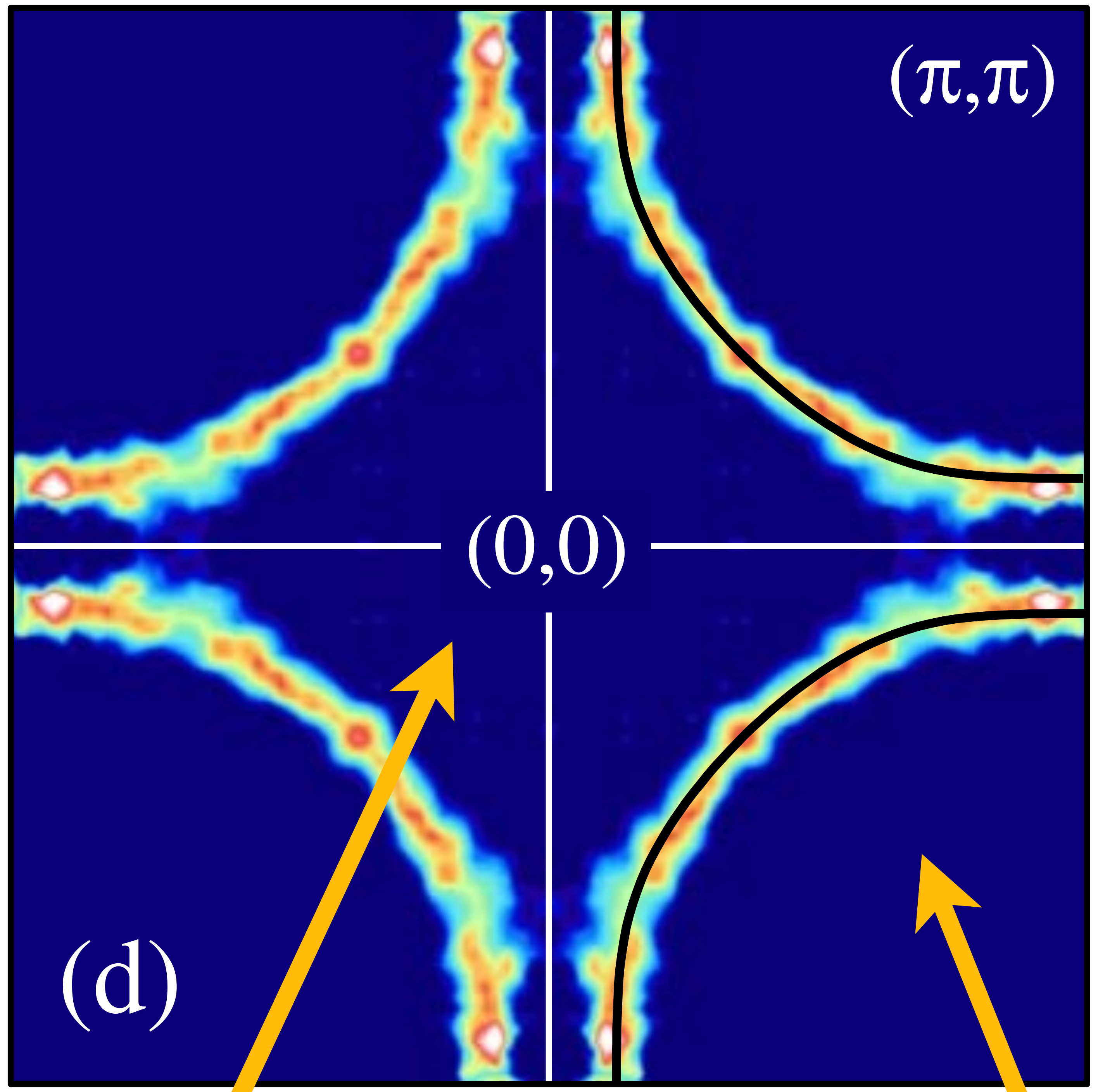

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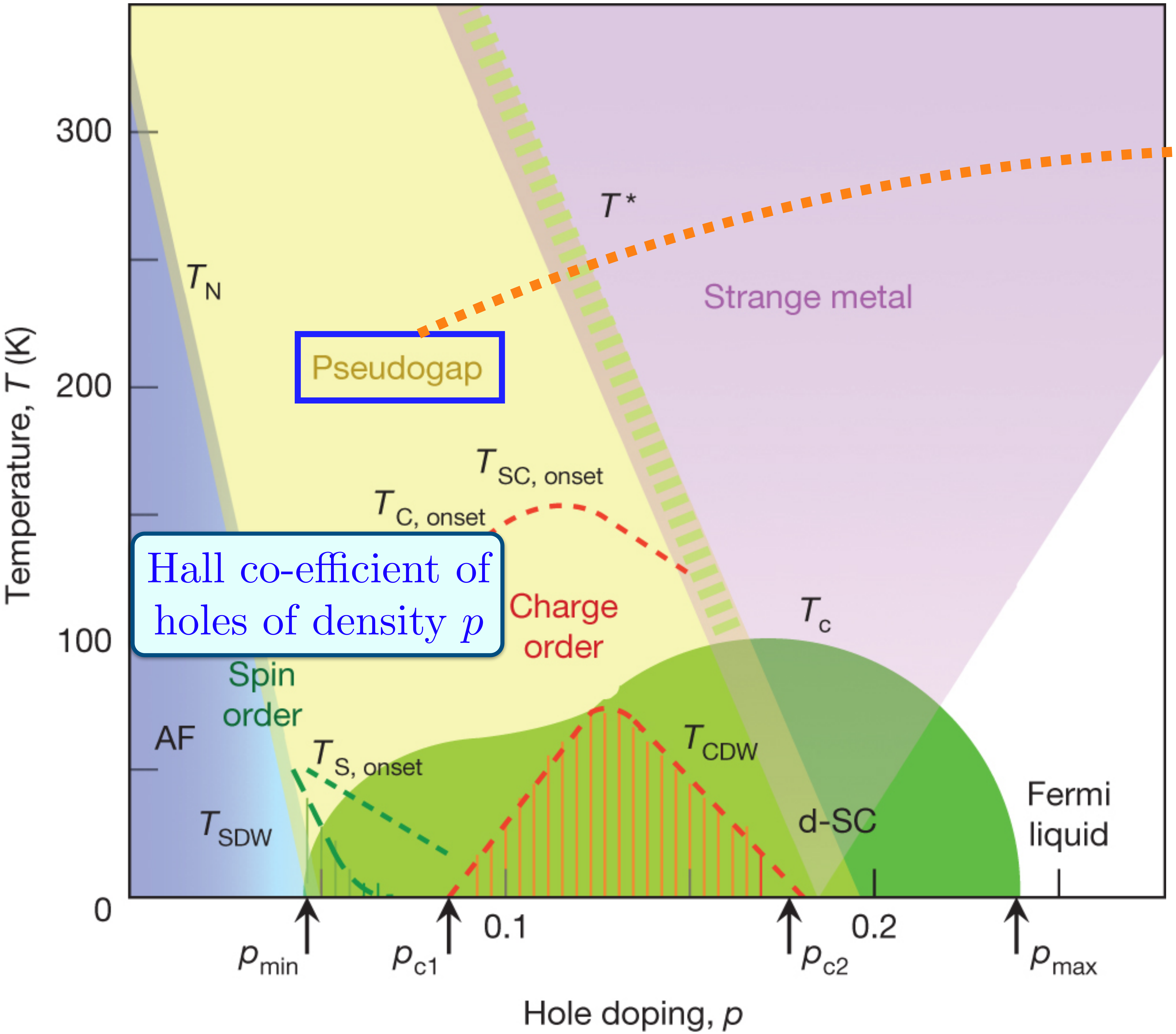

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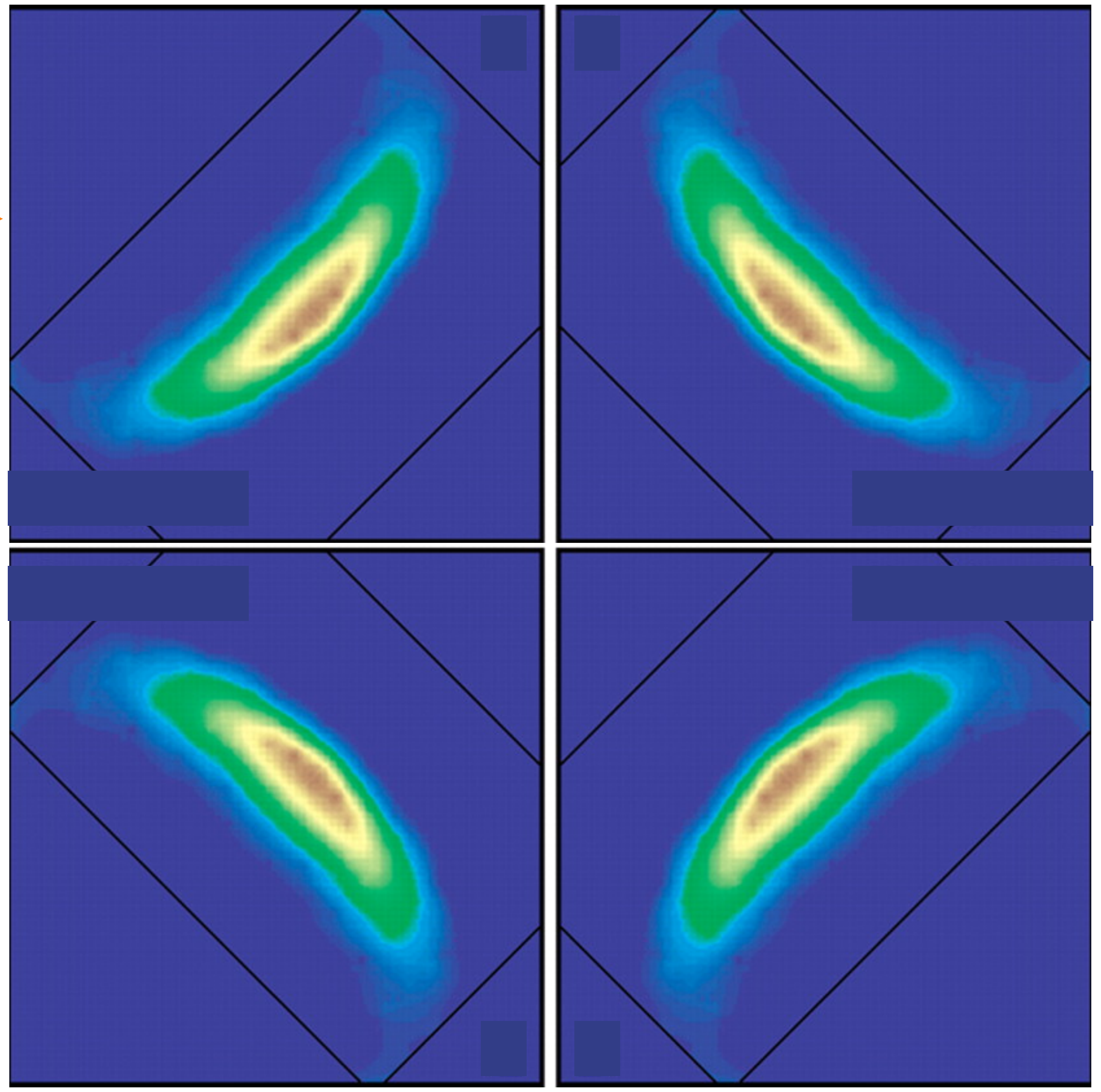

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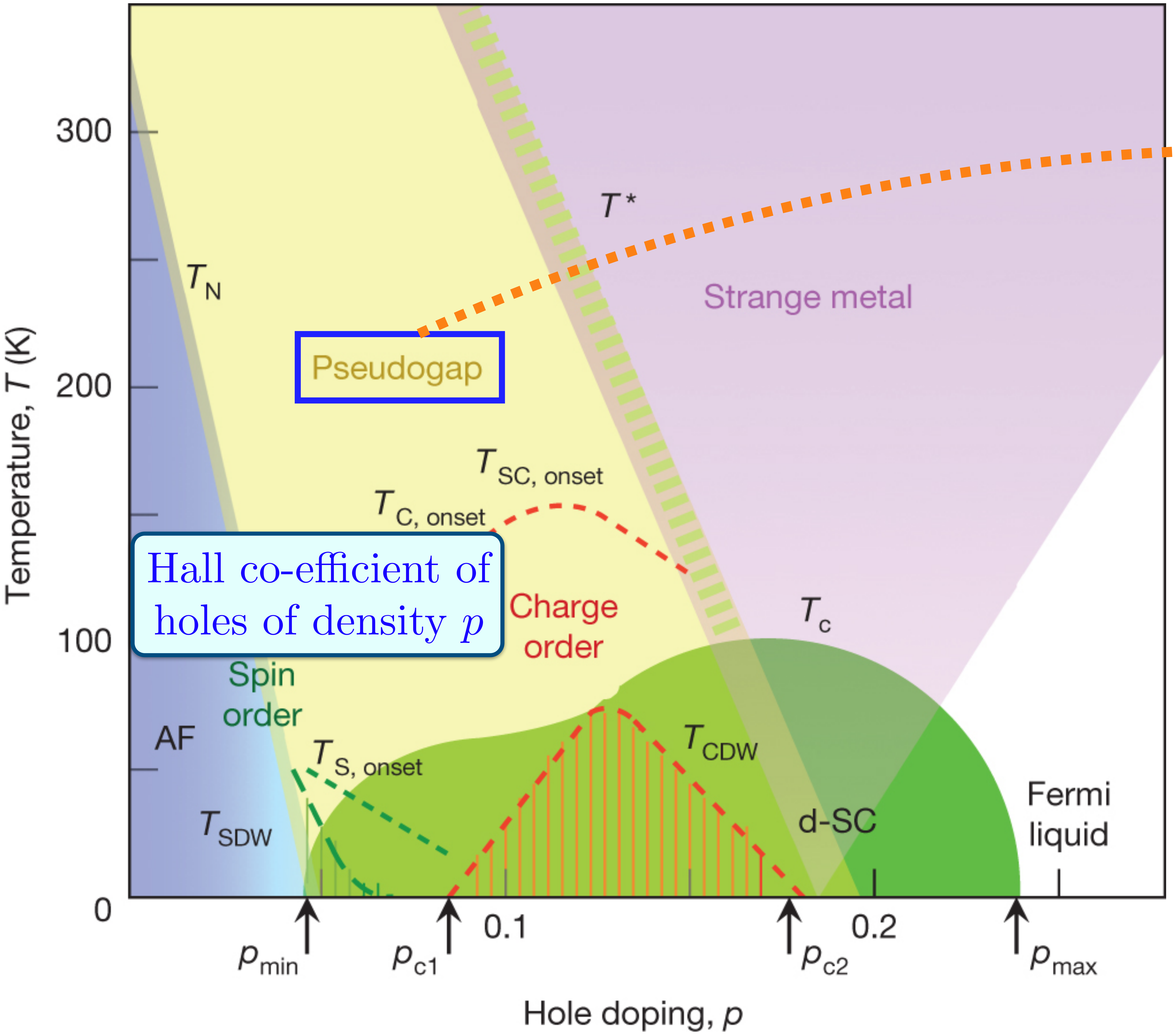

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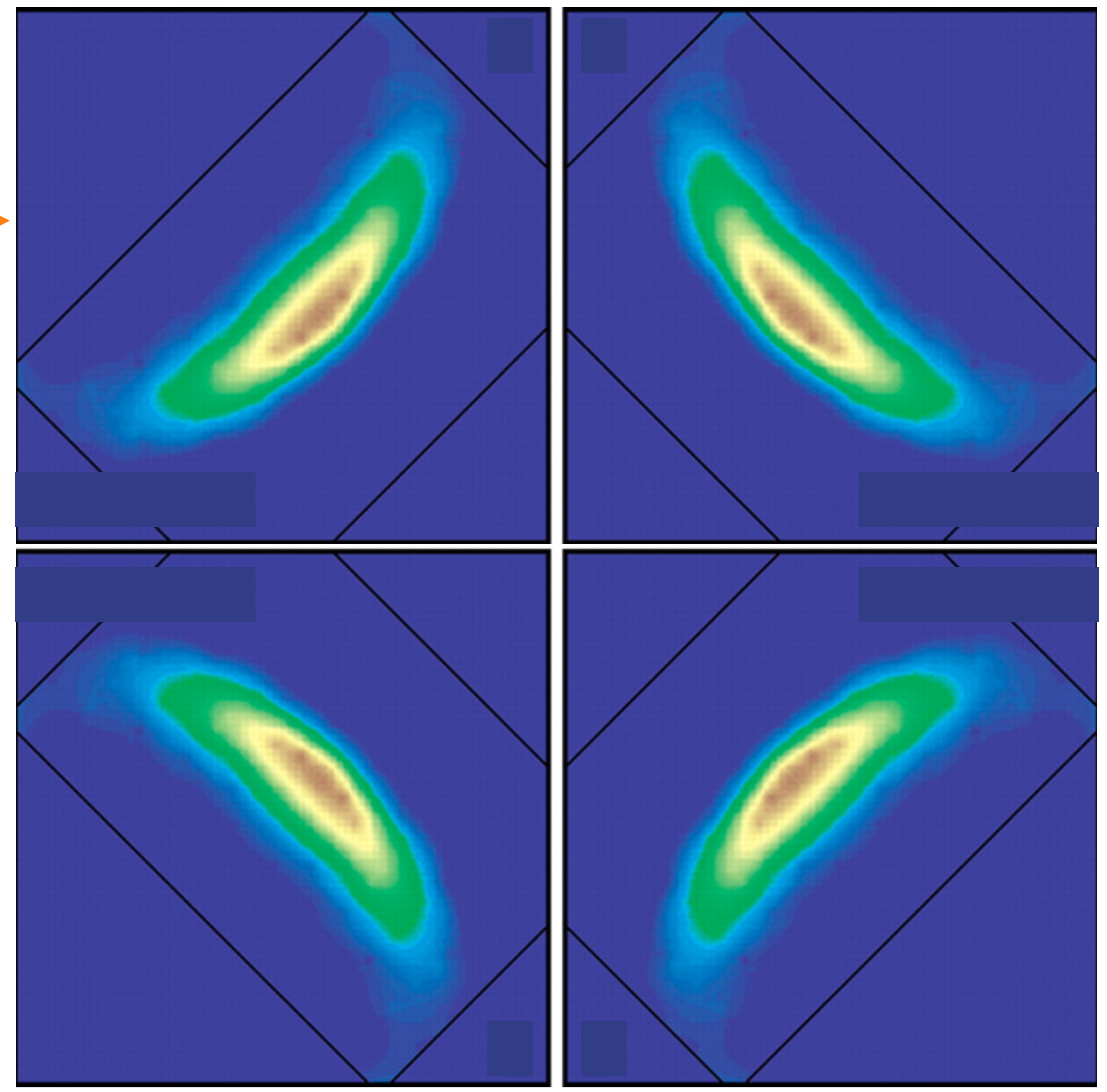
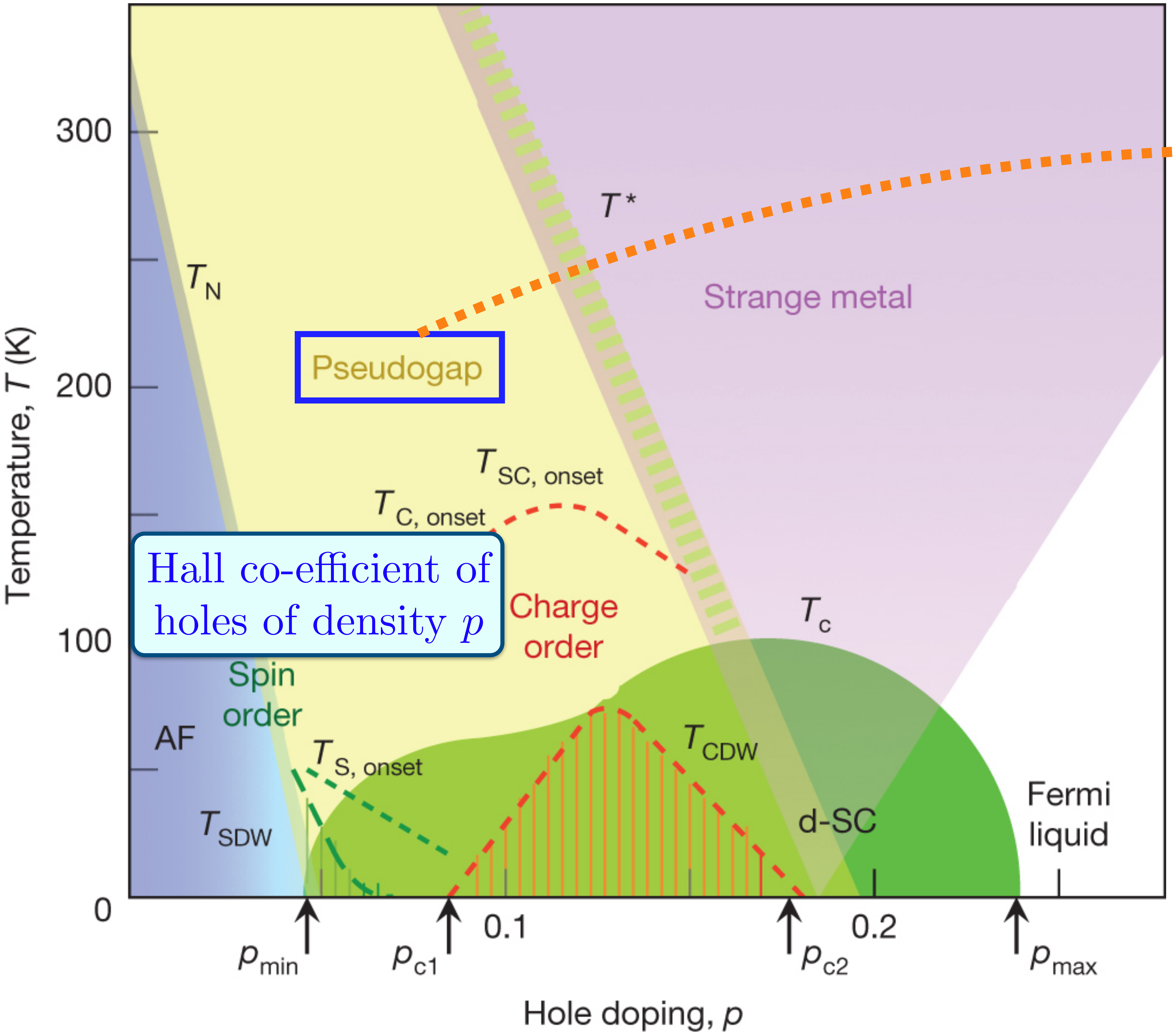


Many theories with fluctuating d-SC and charge orders

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Hall co-efficient of holes of density p

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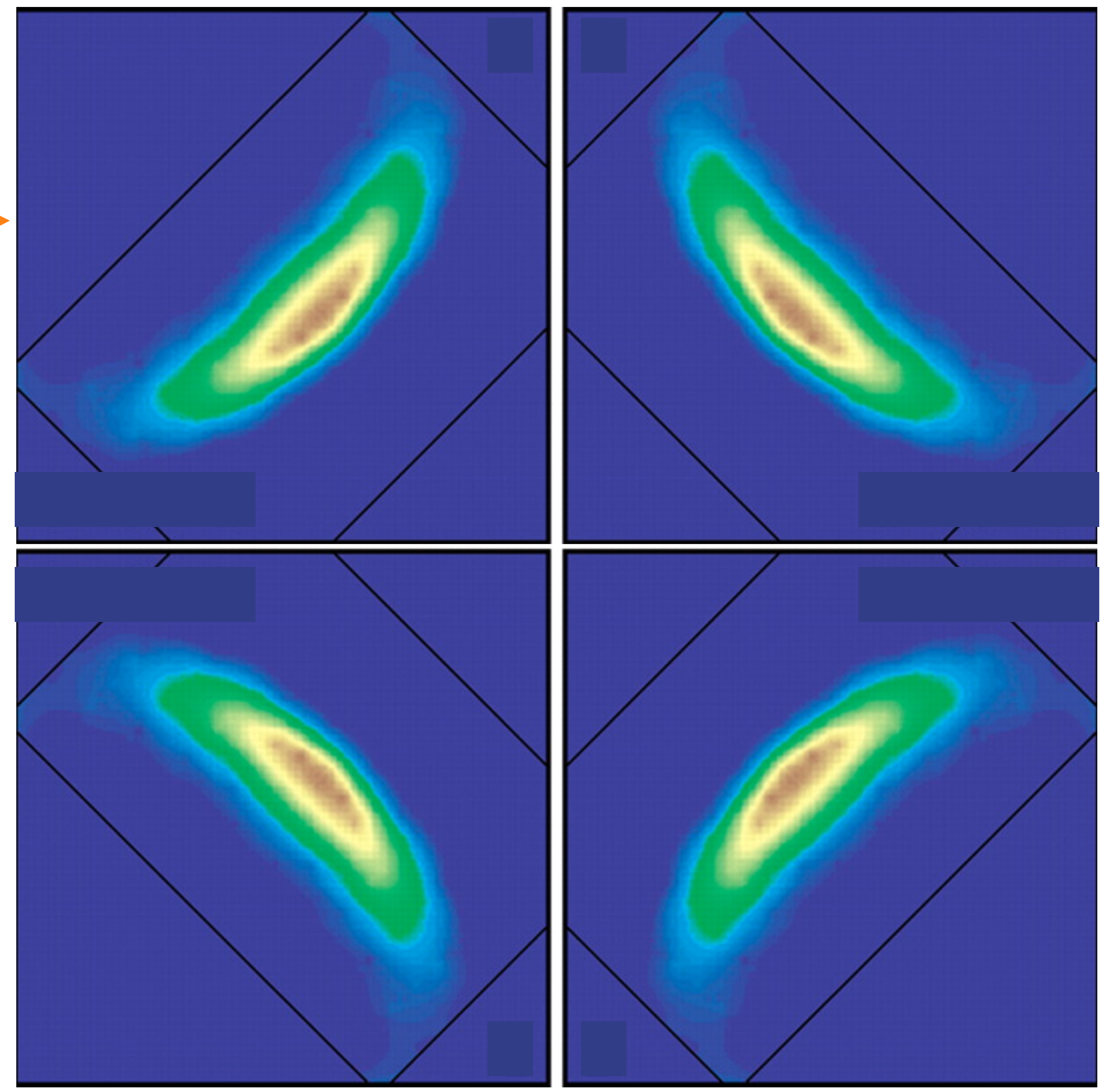
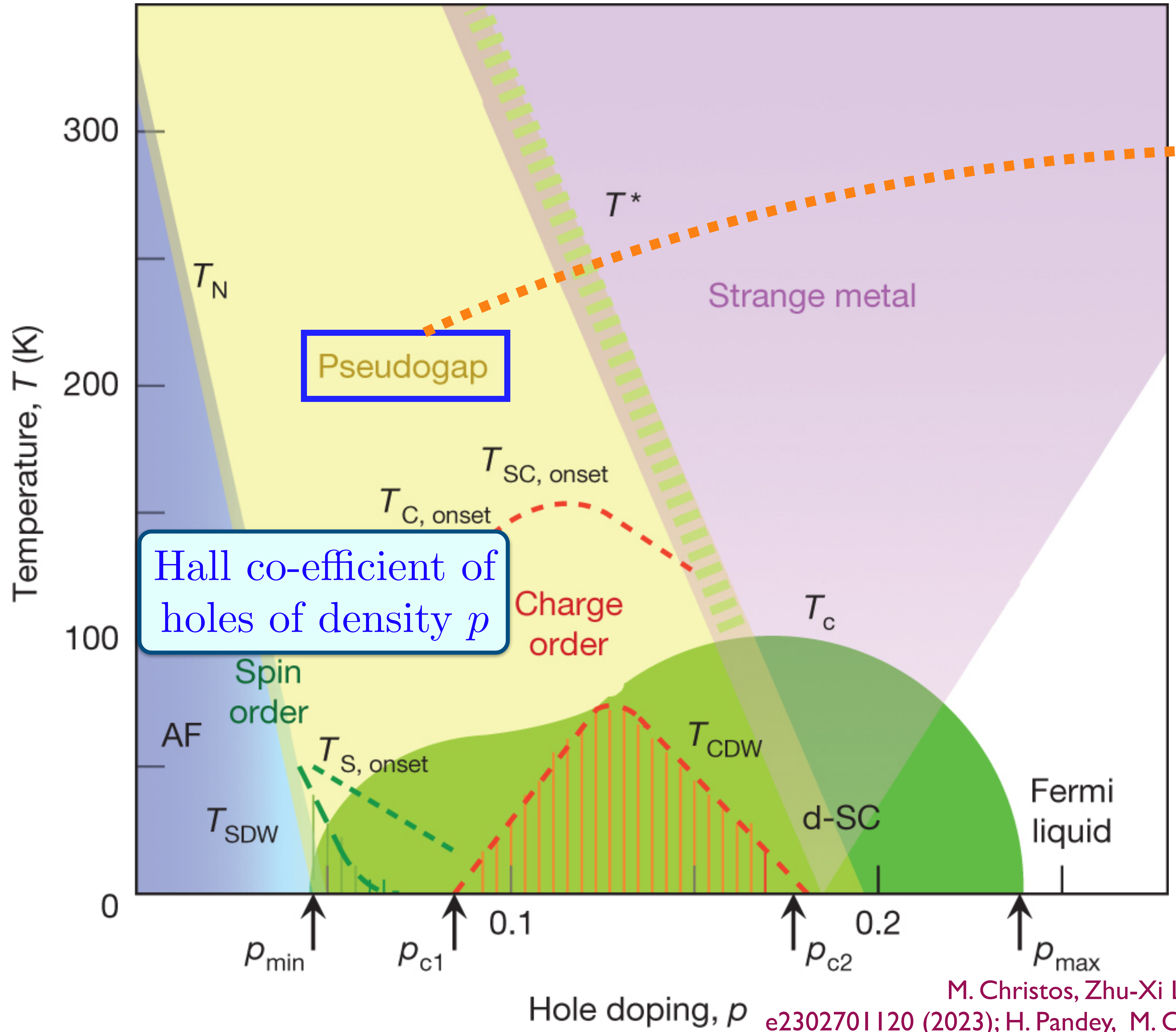


Will explain with a Fractionalized Fermi Liquid (FL*) which evades the Luttinger constraint, but satisfies the Oshikawa anomaly, by a *critical spin liquid*

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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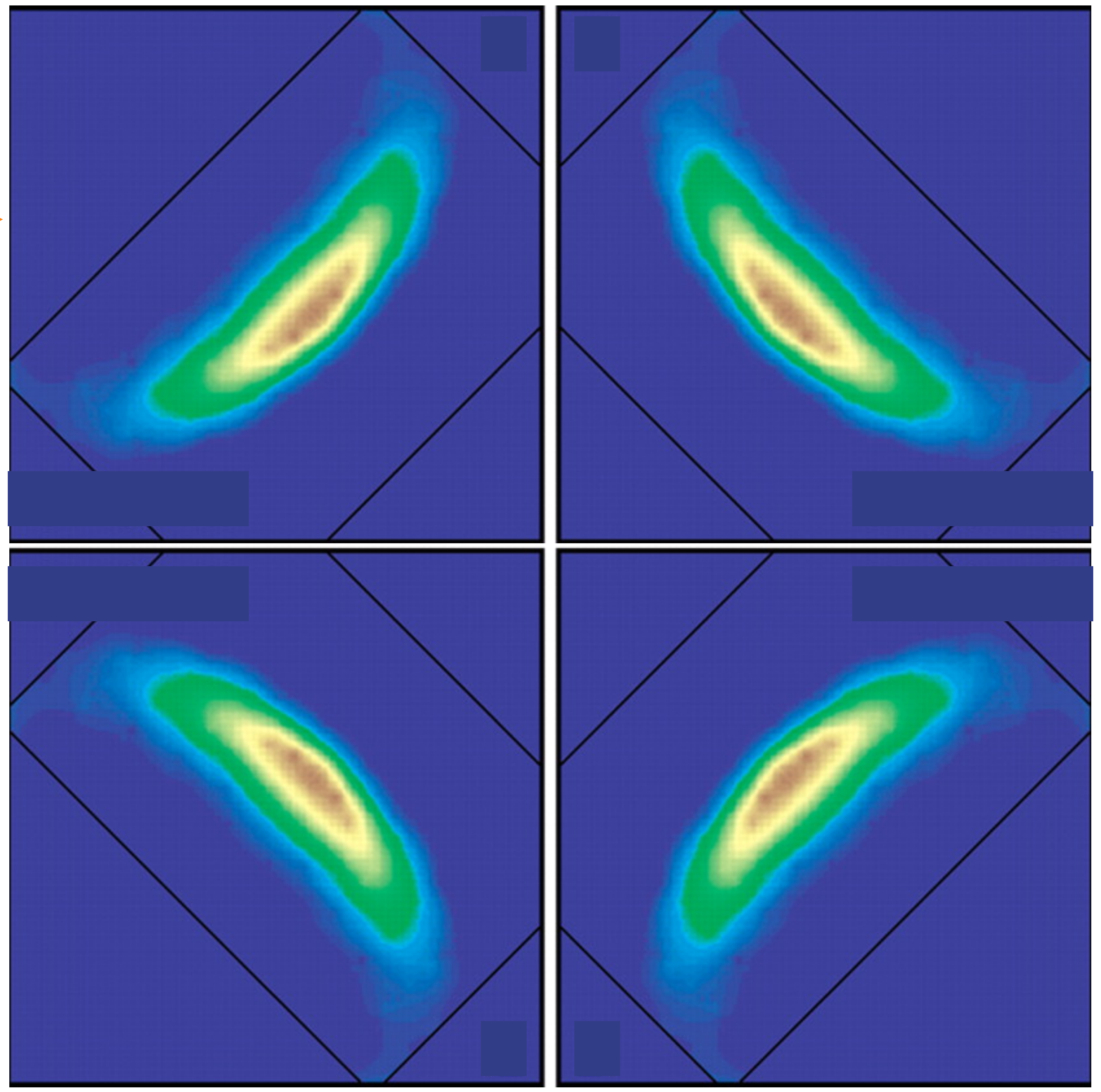


Fluctuating orders appear as composites of a more fundamental fractionalized order parameter, *B*, which carries an emergent SU(2) gauge charge

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

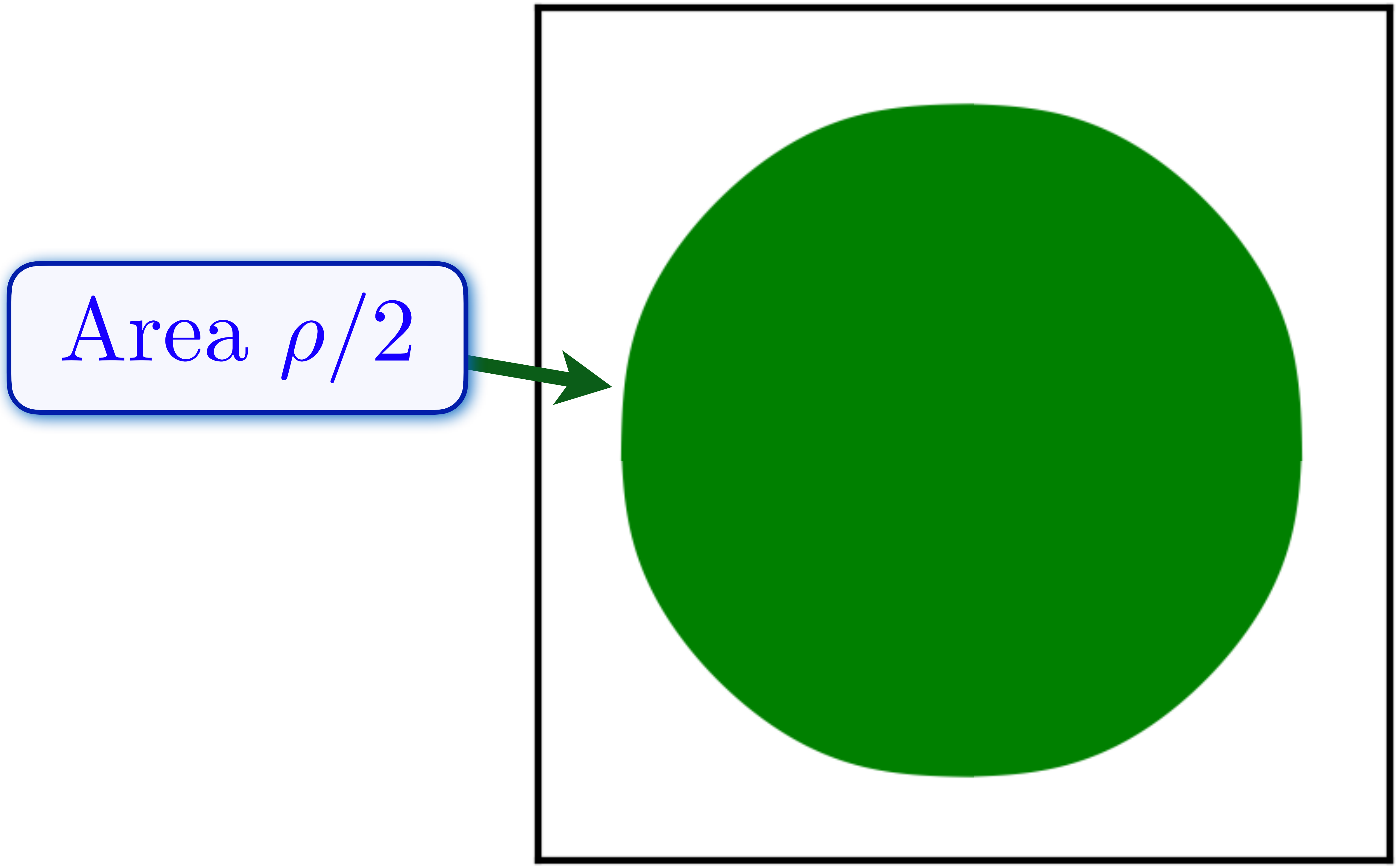
Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density

$$\rho = 1 + p$$

Positive Hall coefficient of carrier density ρ



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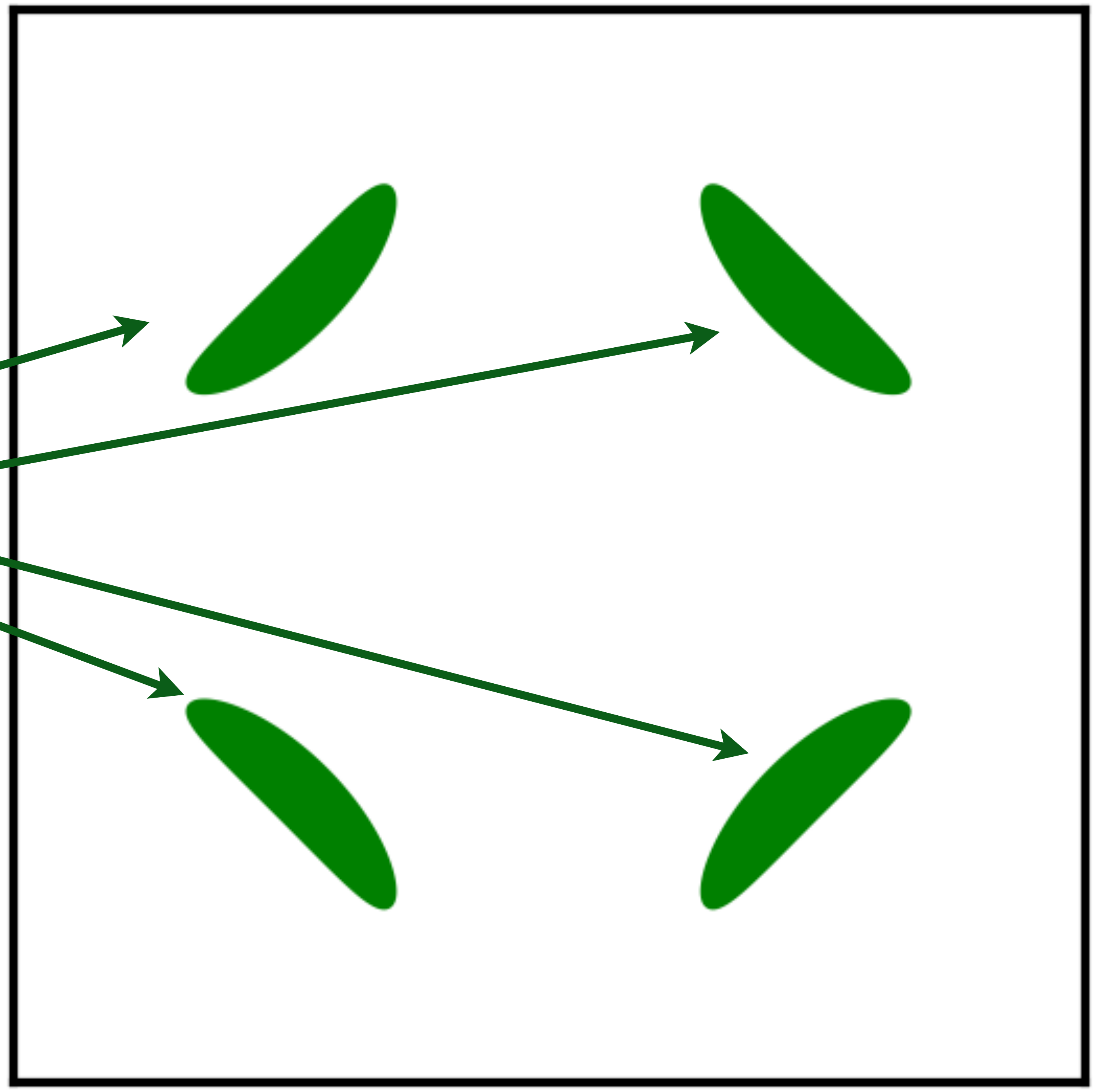
Positive Hall coefficient of carrier density $\rho - 1$

Total area $(\rho - 1)/2$



Oshikawa anomaly is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

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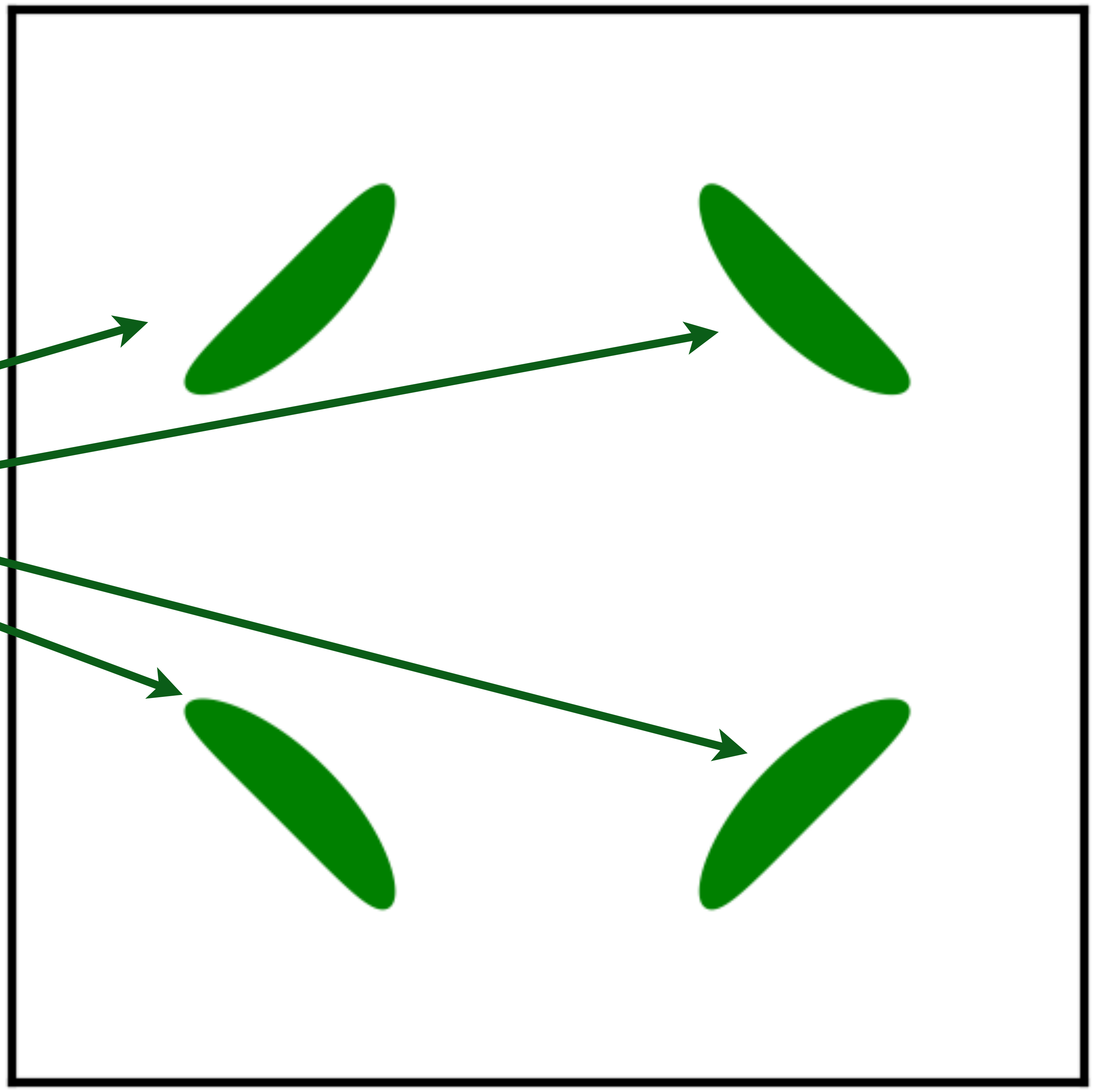

Area of each hole pocket $= p/8$

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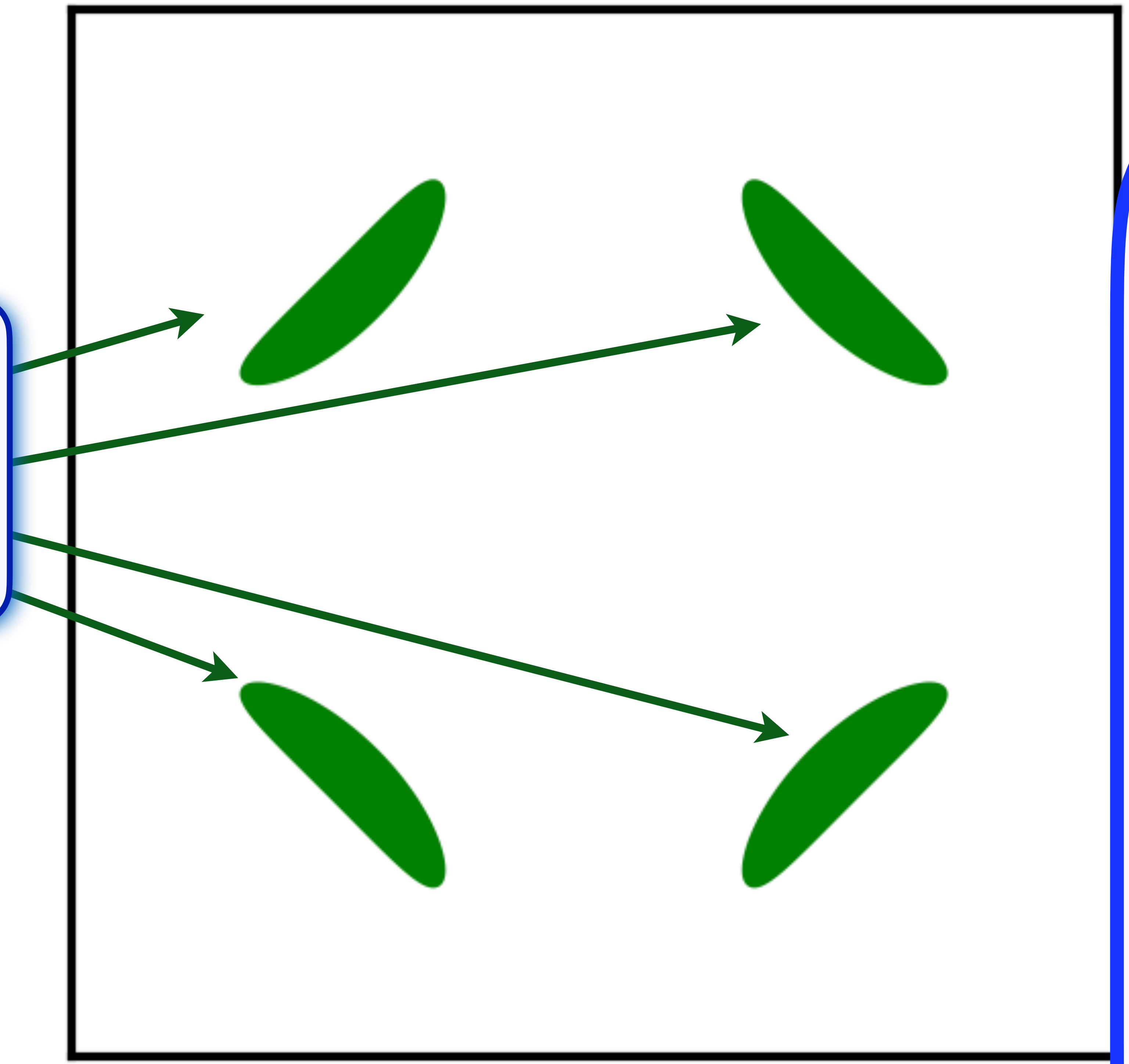
Positive Hall coefficient of carrier density $\rho - 1$

Total area $(\rho - 1)/2$



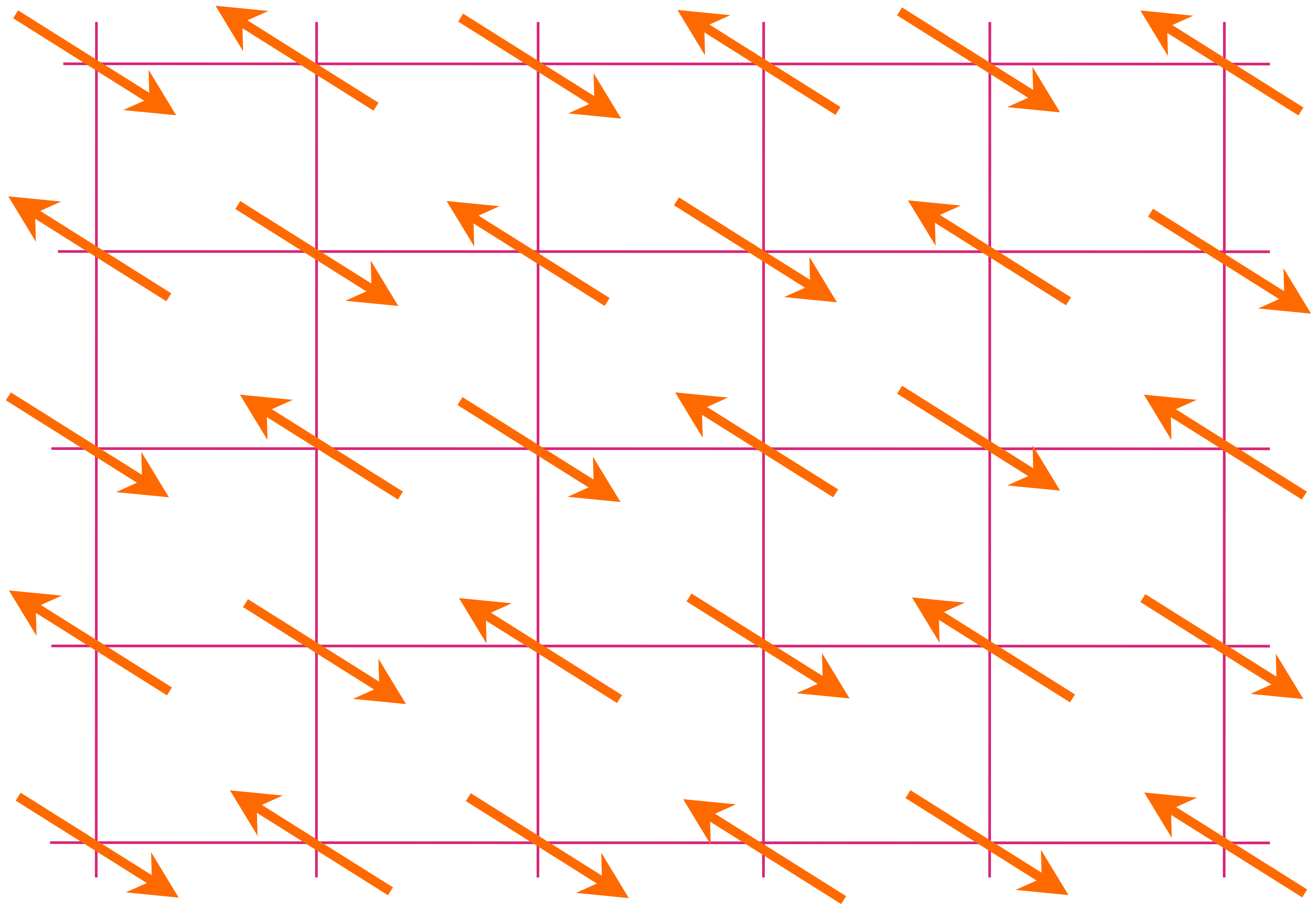
Measuring non-Luttinger Fermi surface area is direct evidence for spin liquid quantum entanglement.

Oshikawa anomaly is satisfied by the sum of
spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

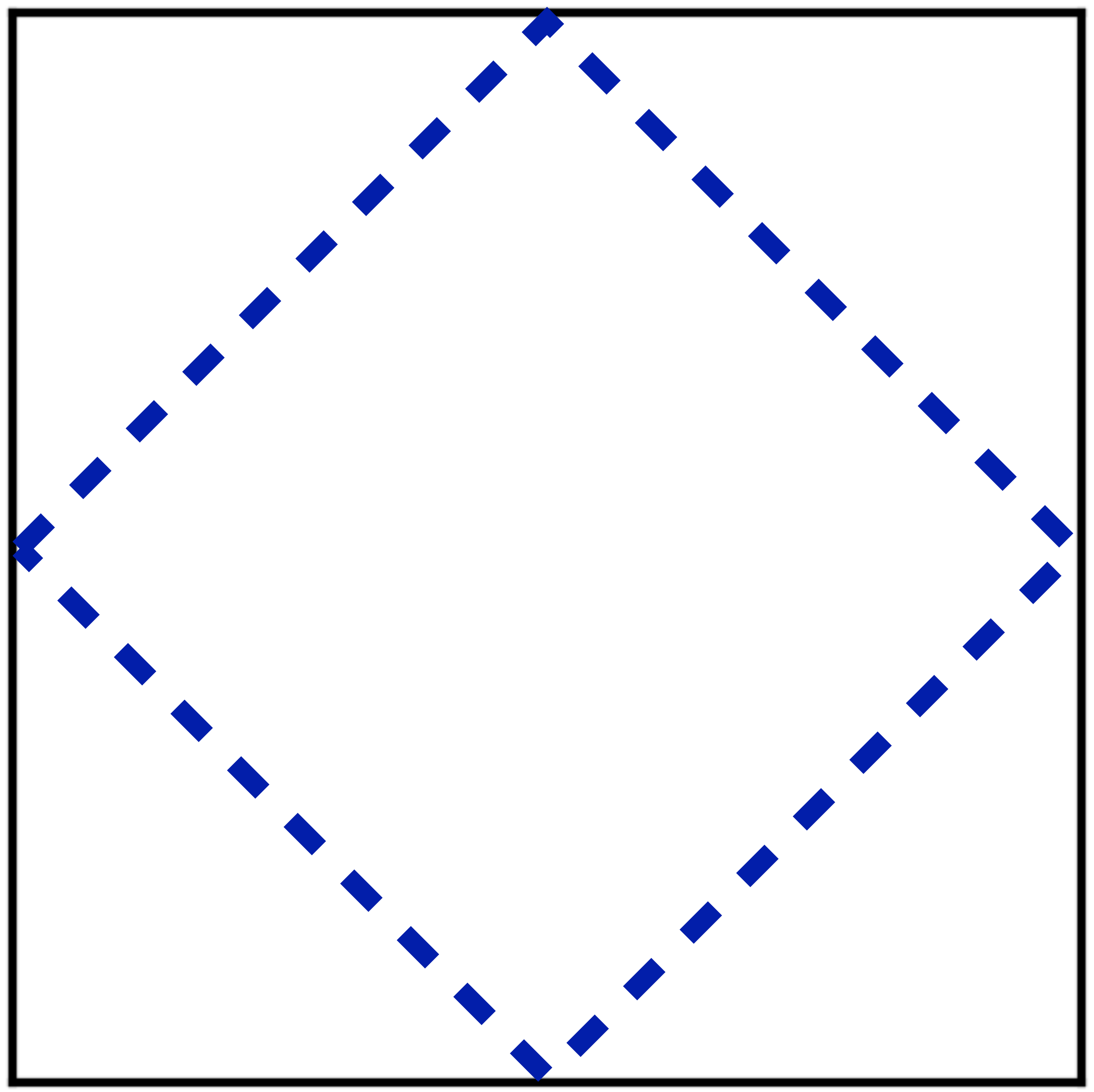
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

FL* in a single-band model on the square lattice

Insulating antiferromagnet



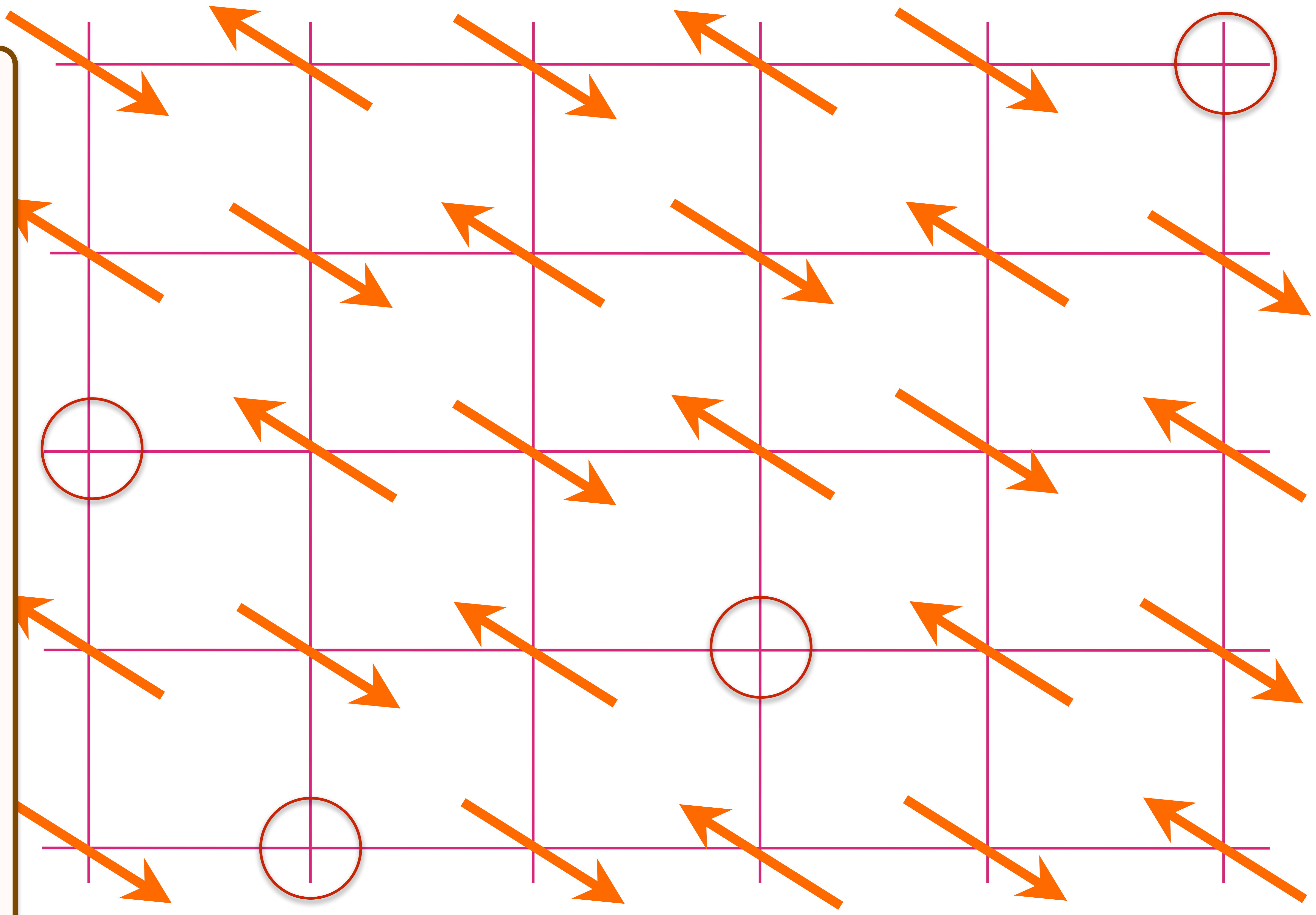
Reduced Brillouin Zone.
Broken symmetry



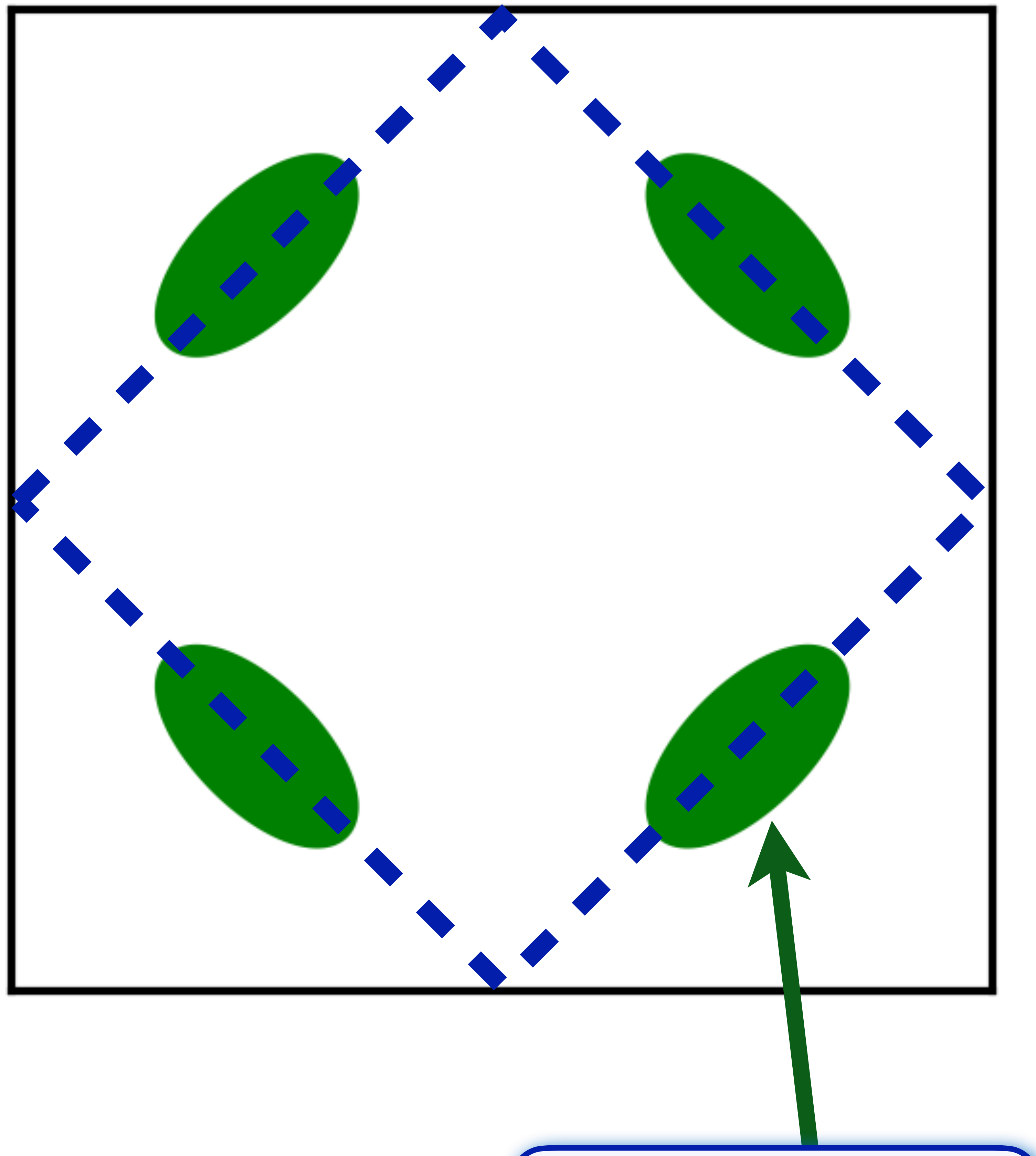
Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Coherent inte-layer transport requires inter-layer spin correlations.



Luttinger area. Broken symmetry



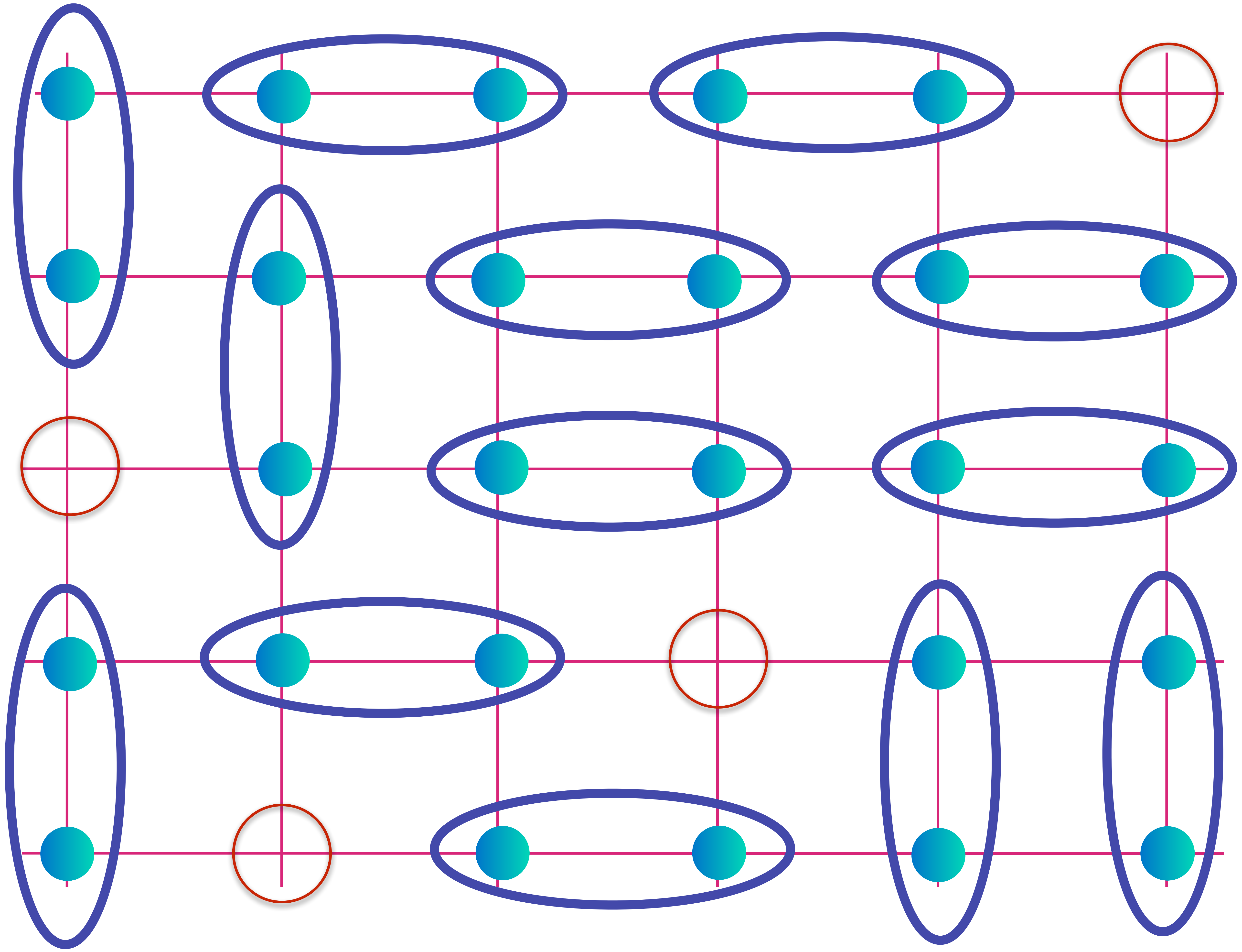
Area $p/4$

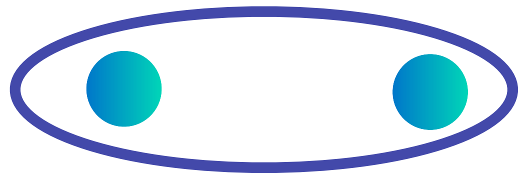
Doping an insulating antiferromagnet with holes of density p

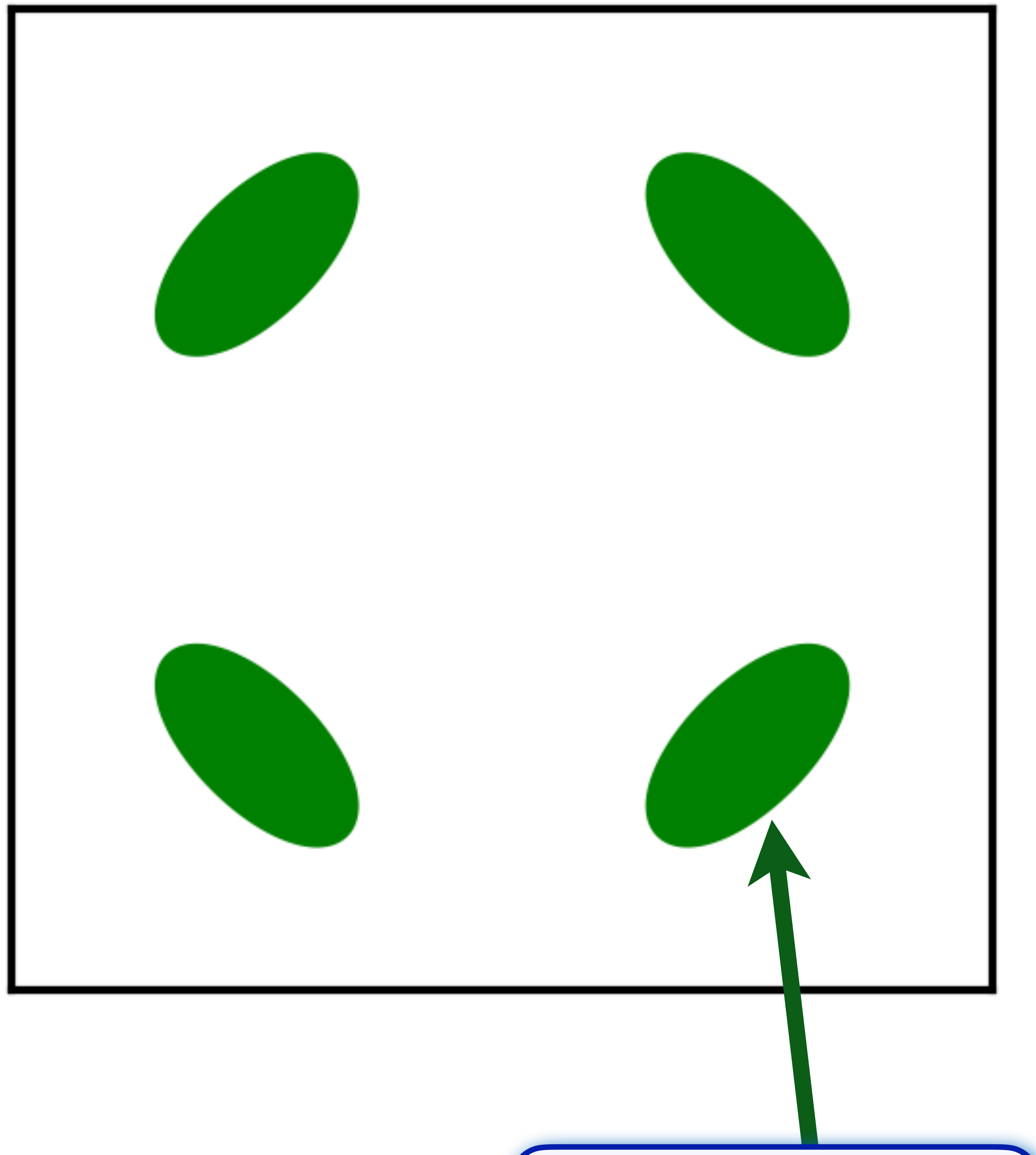
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ holons.
No coherent inter-layer transport.



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

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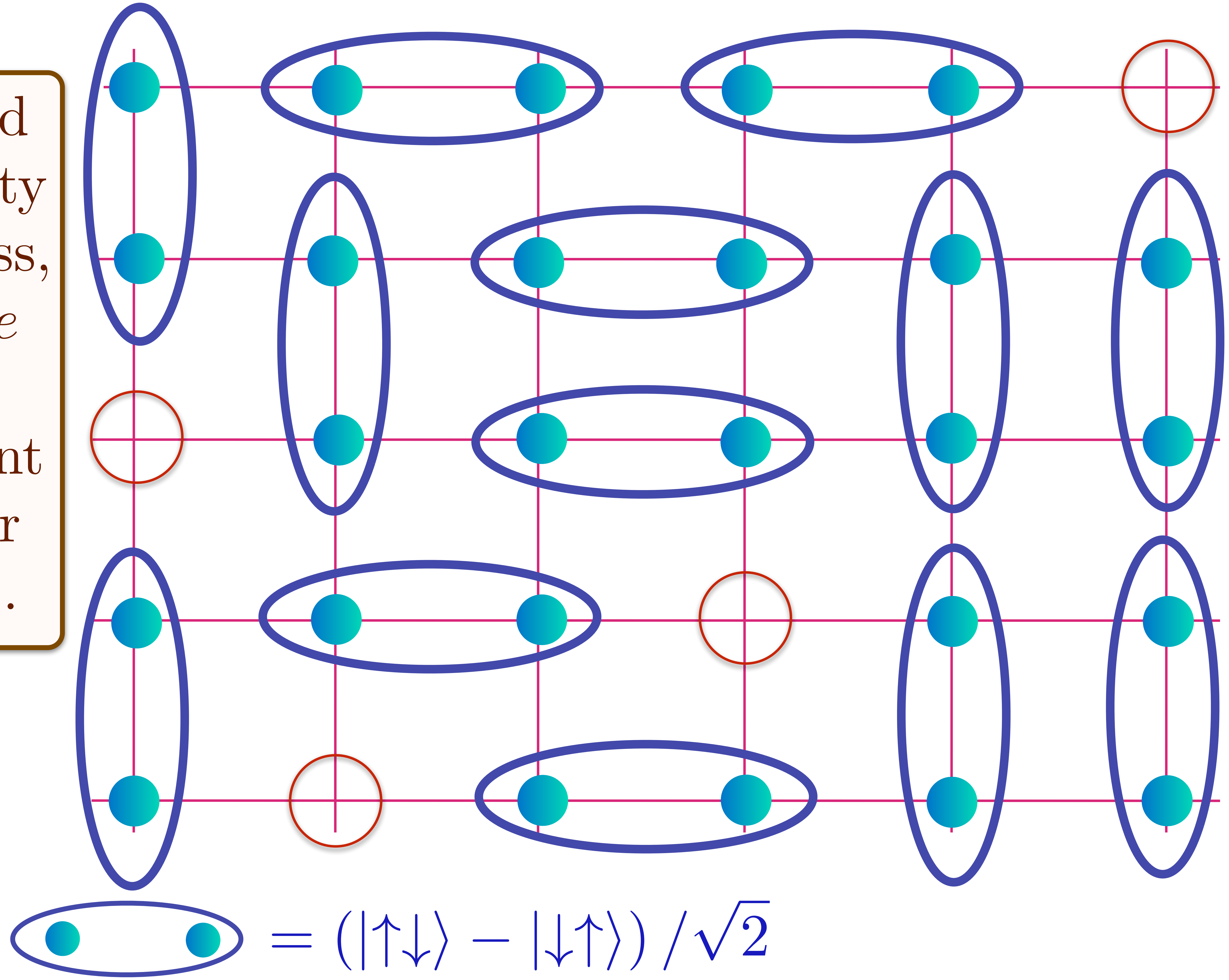
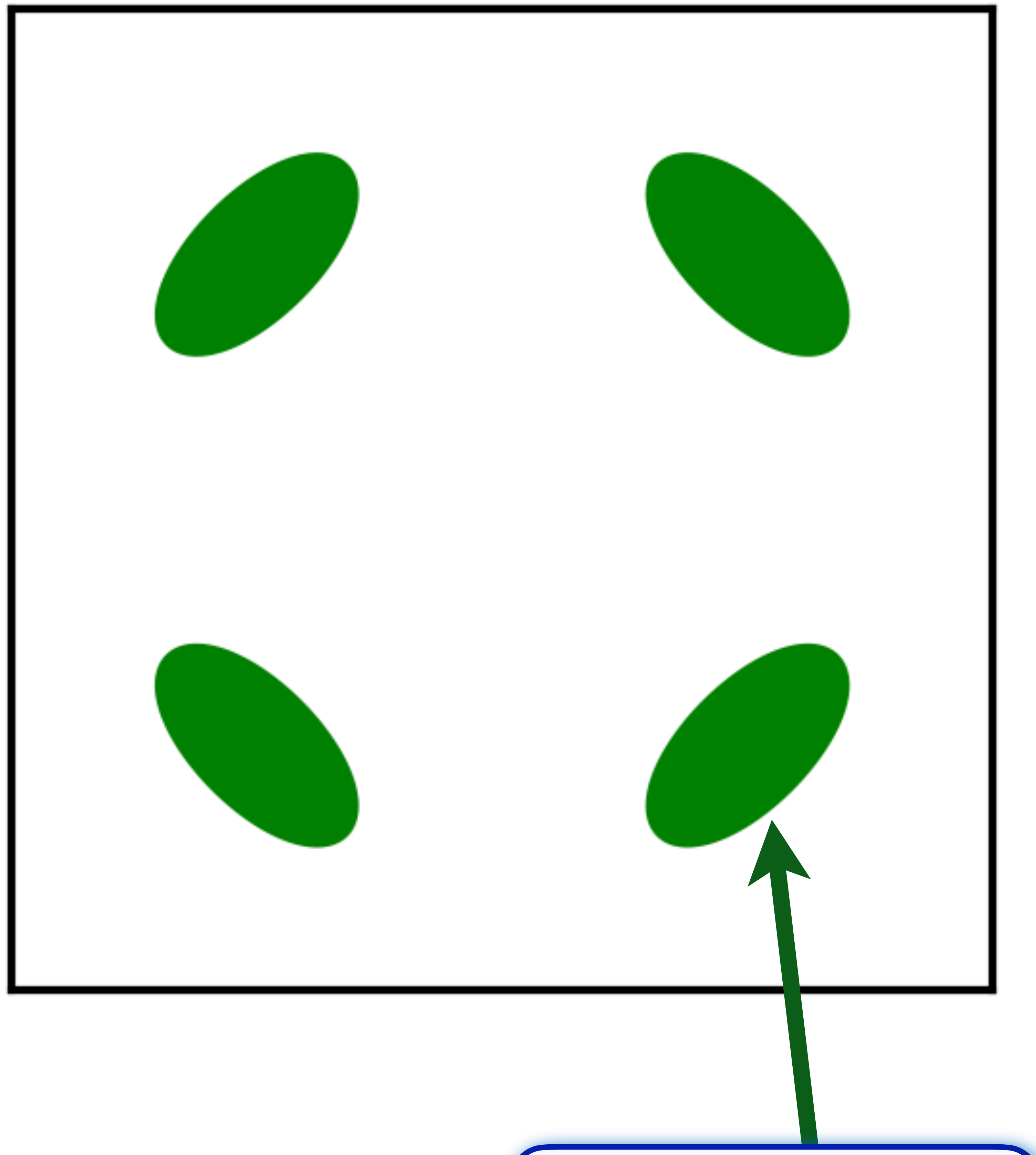


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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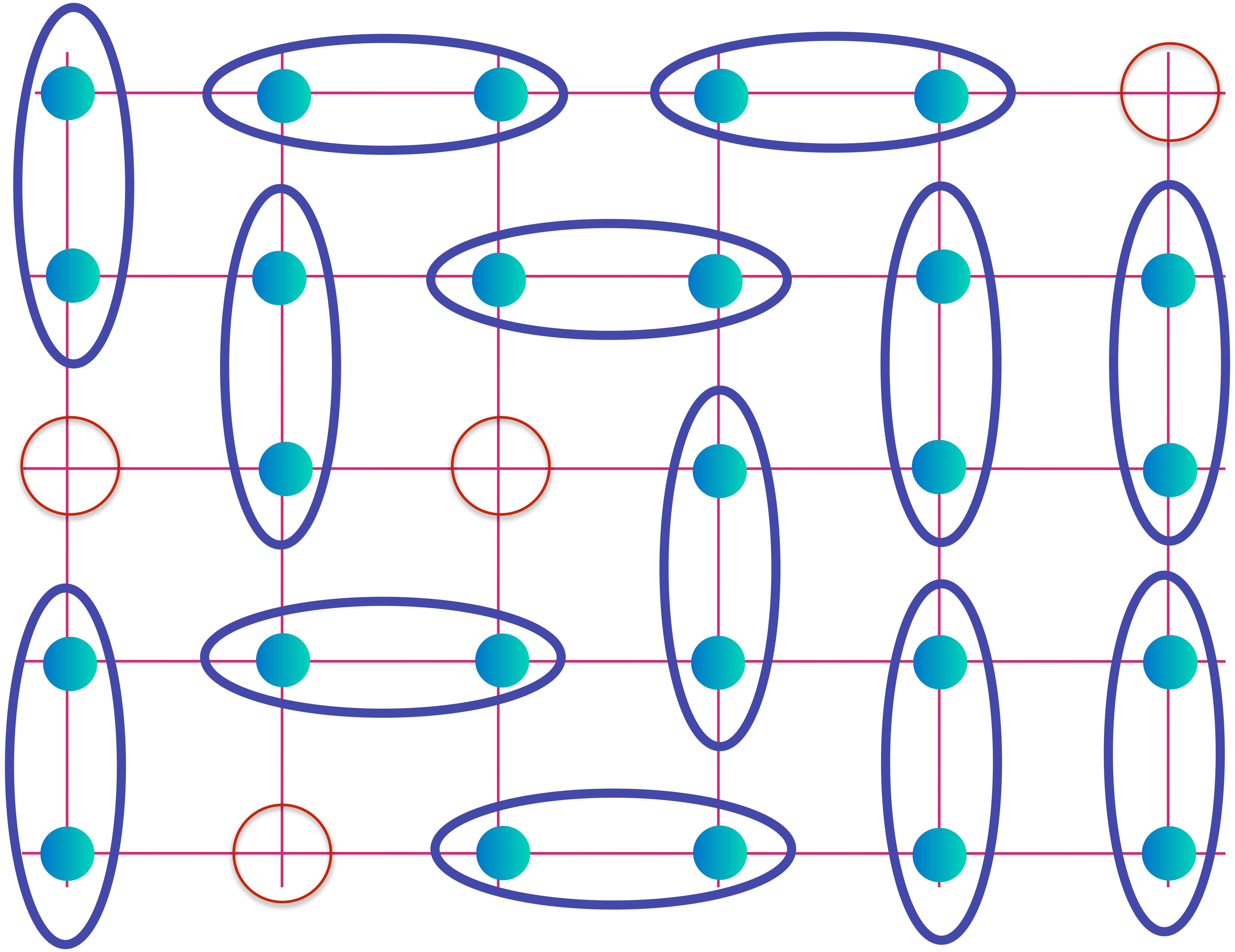

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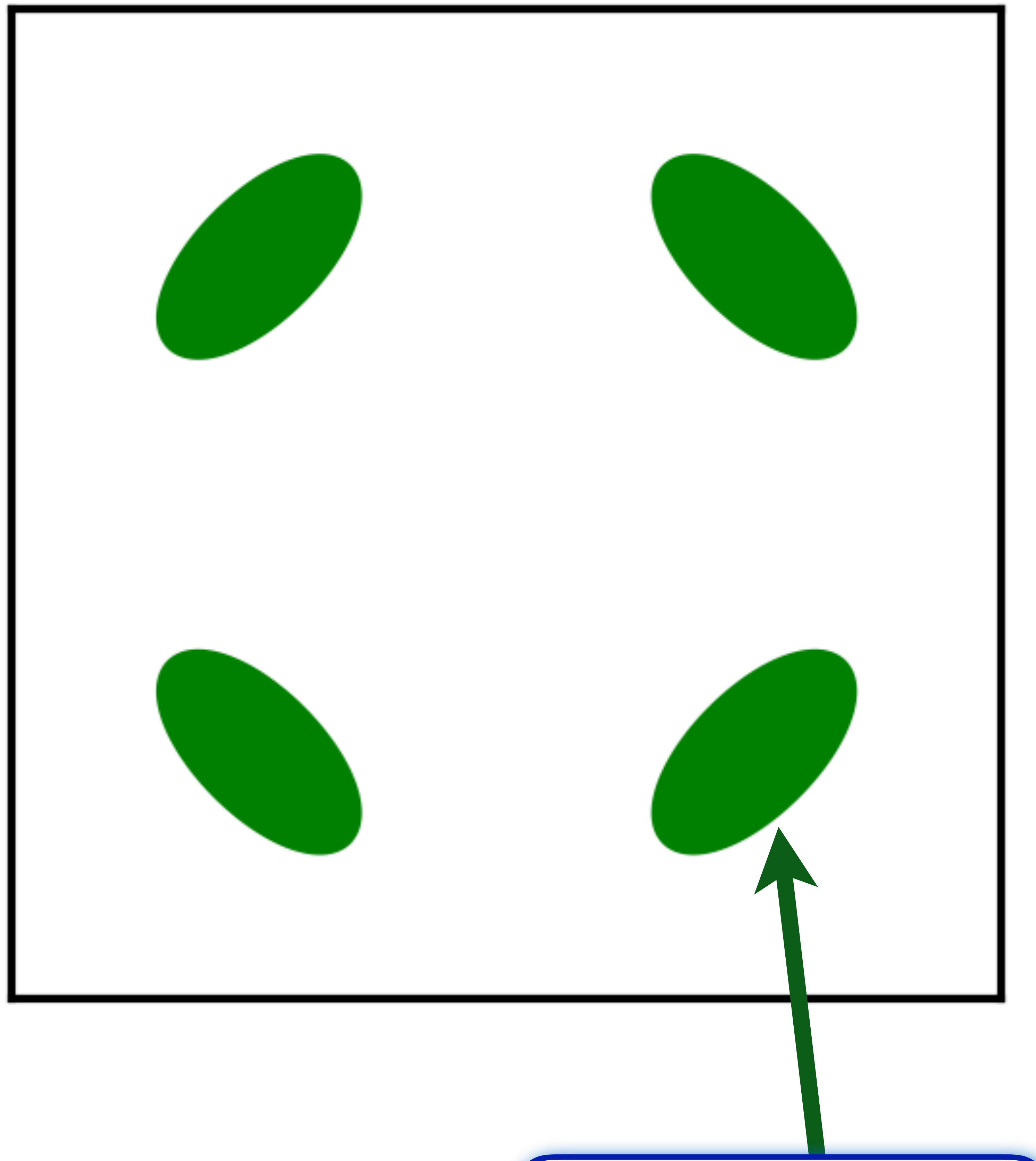

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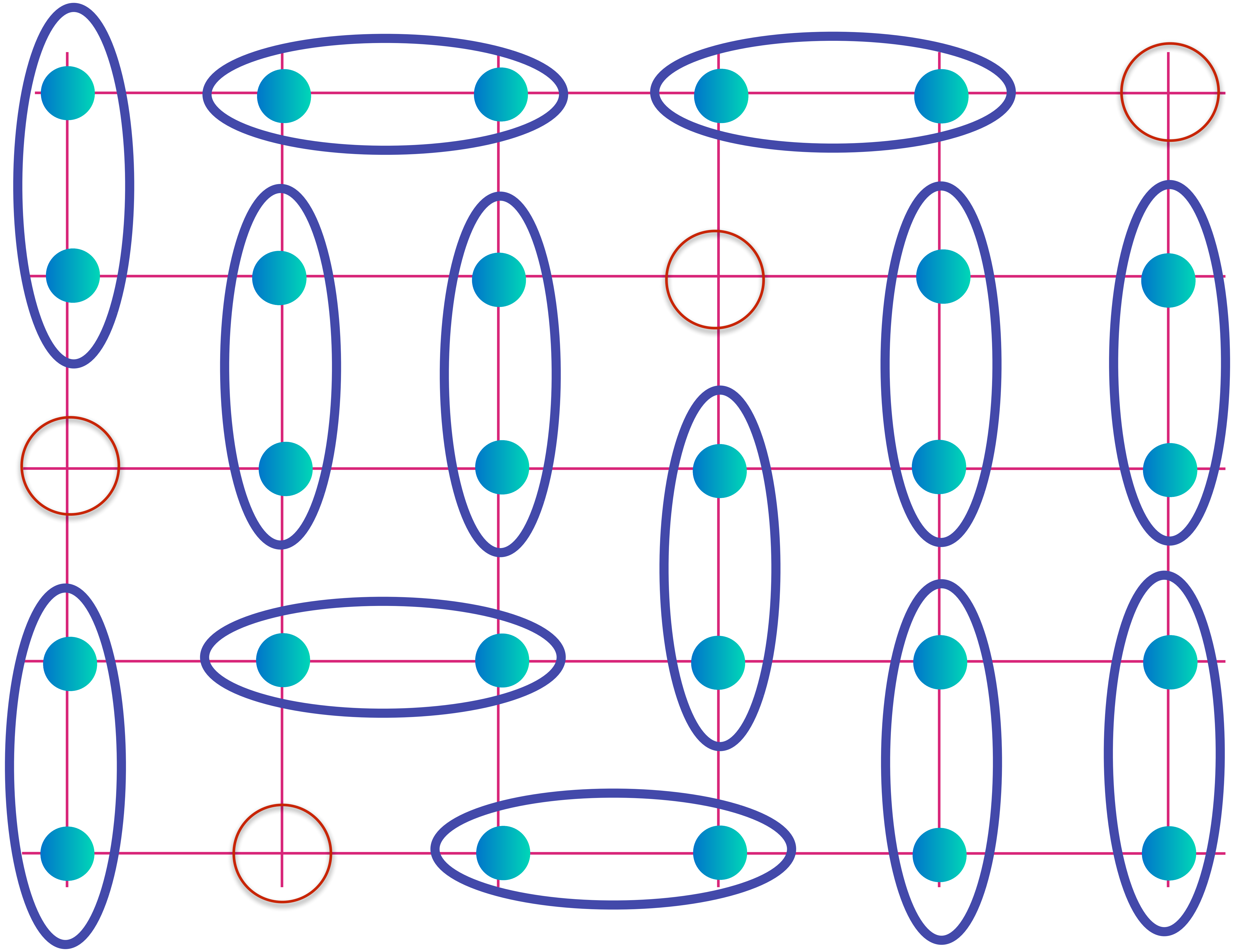

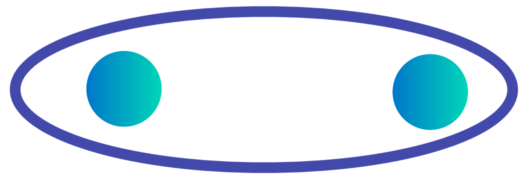
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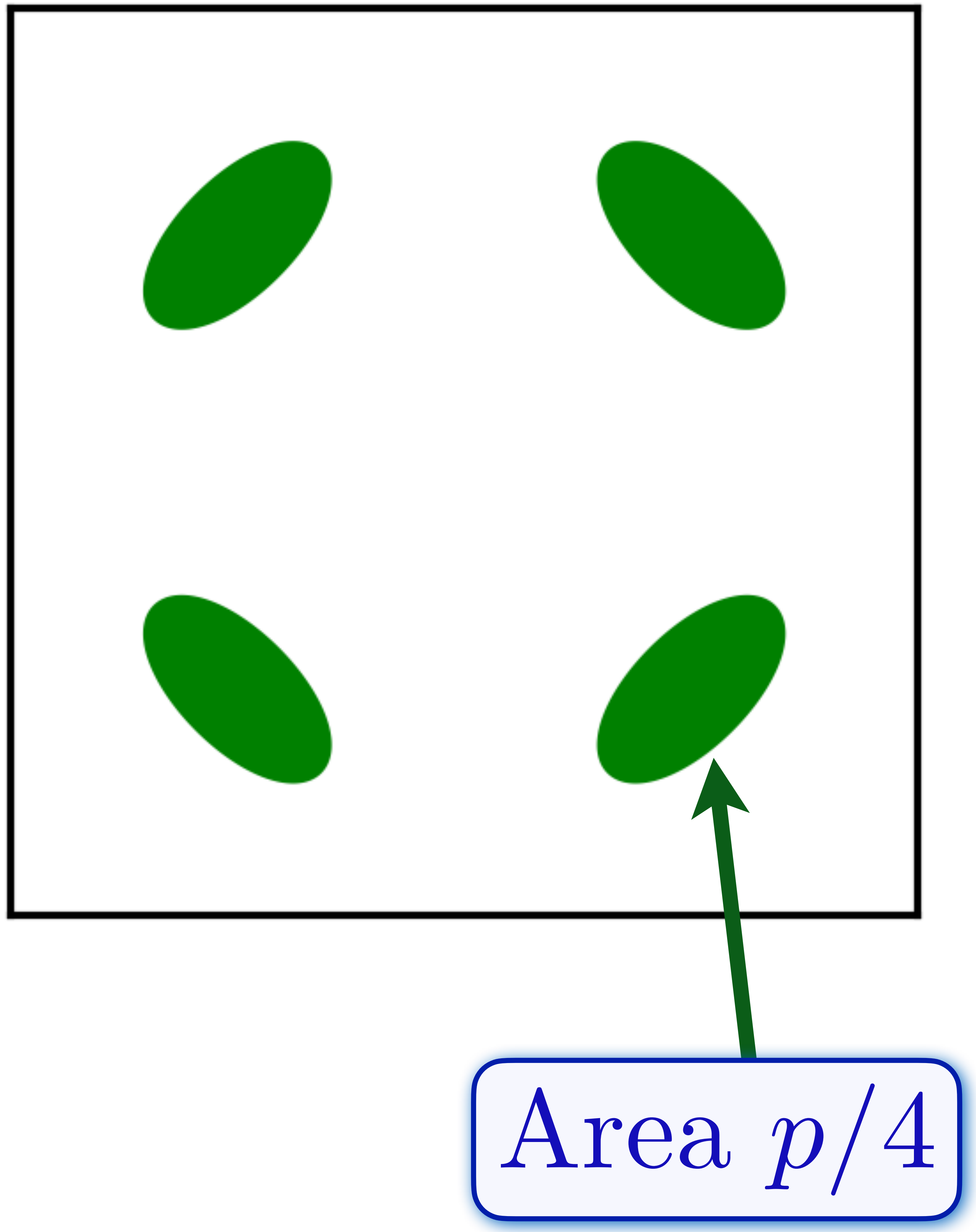

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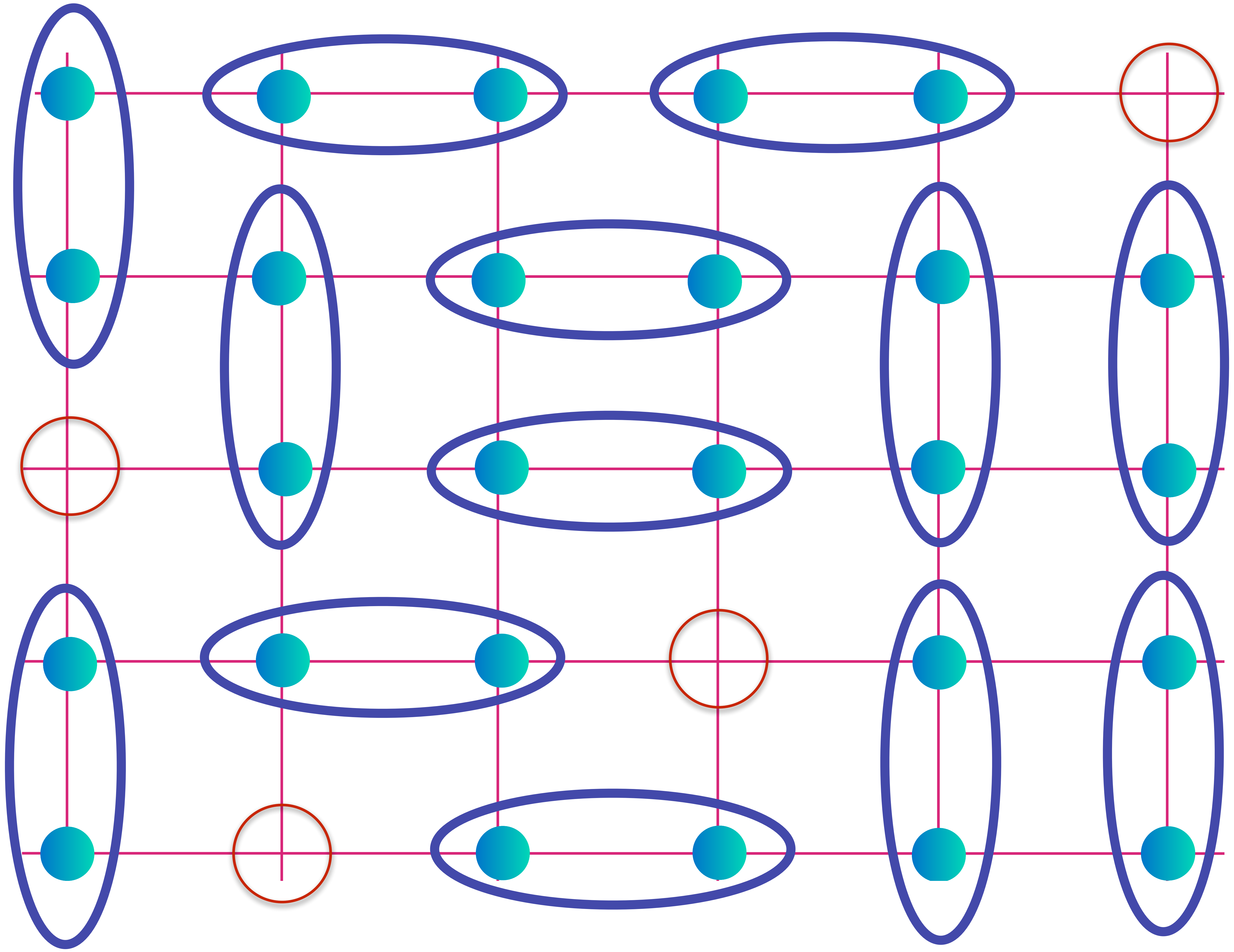
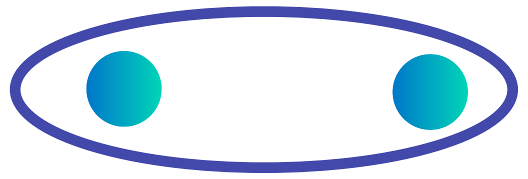
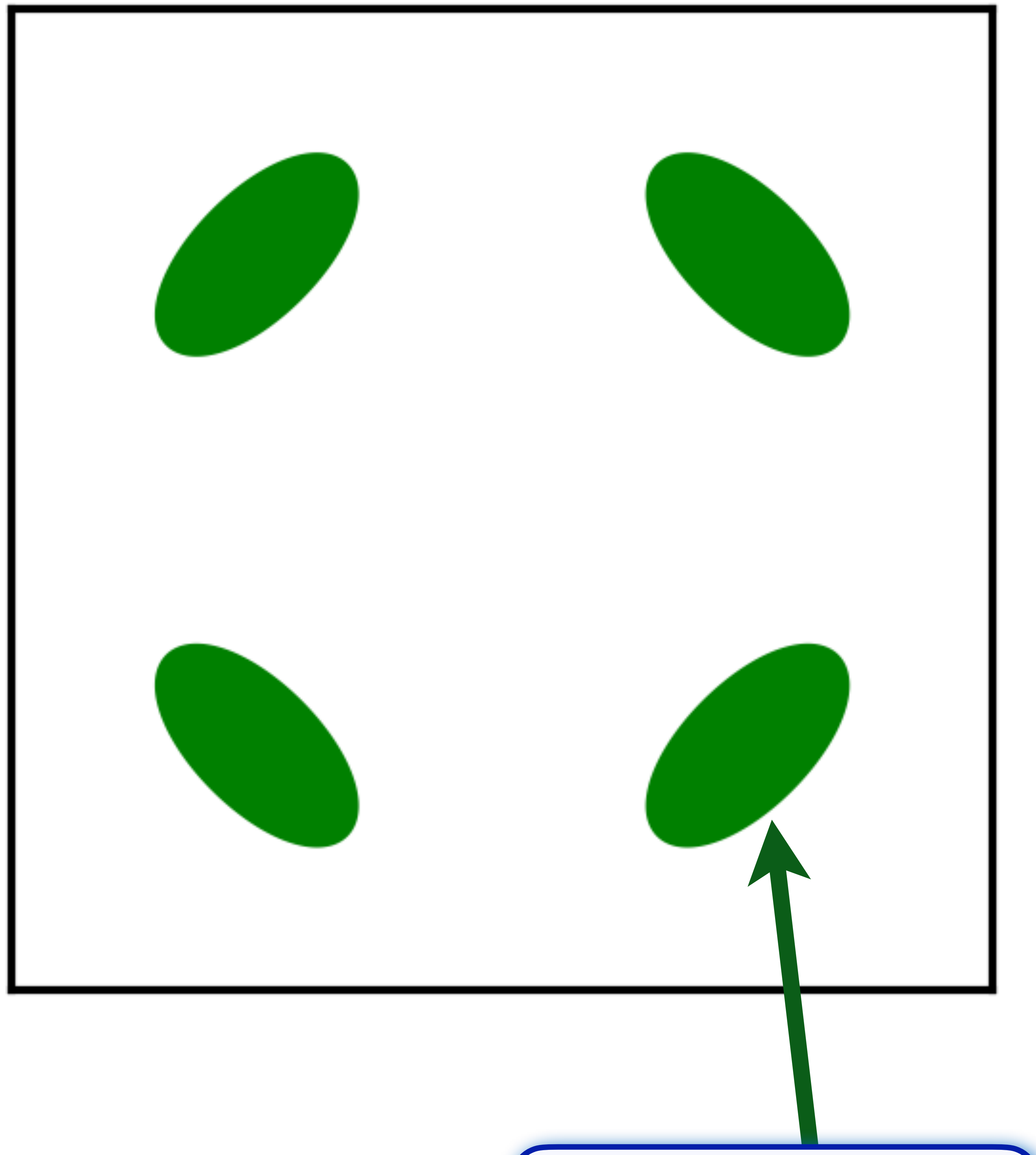


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

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Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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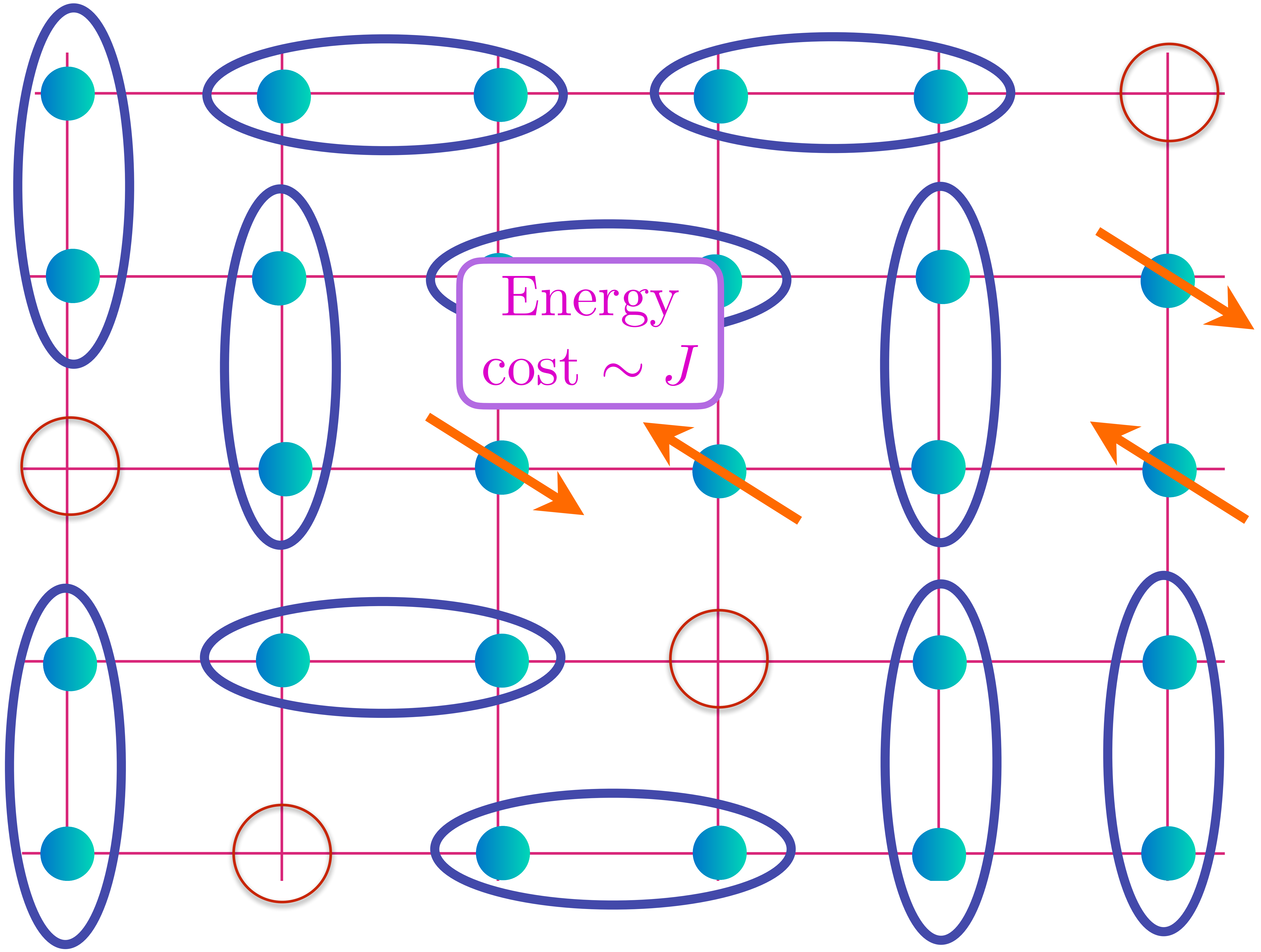
Charge 0, spin-1/2 spinons



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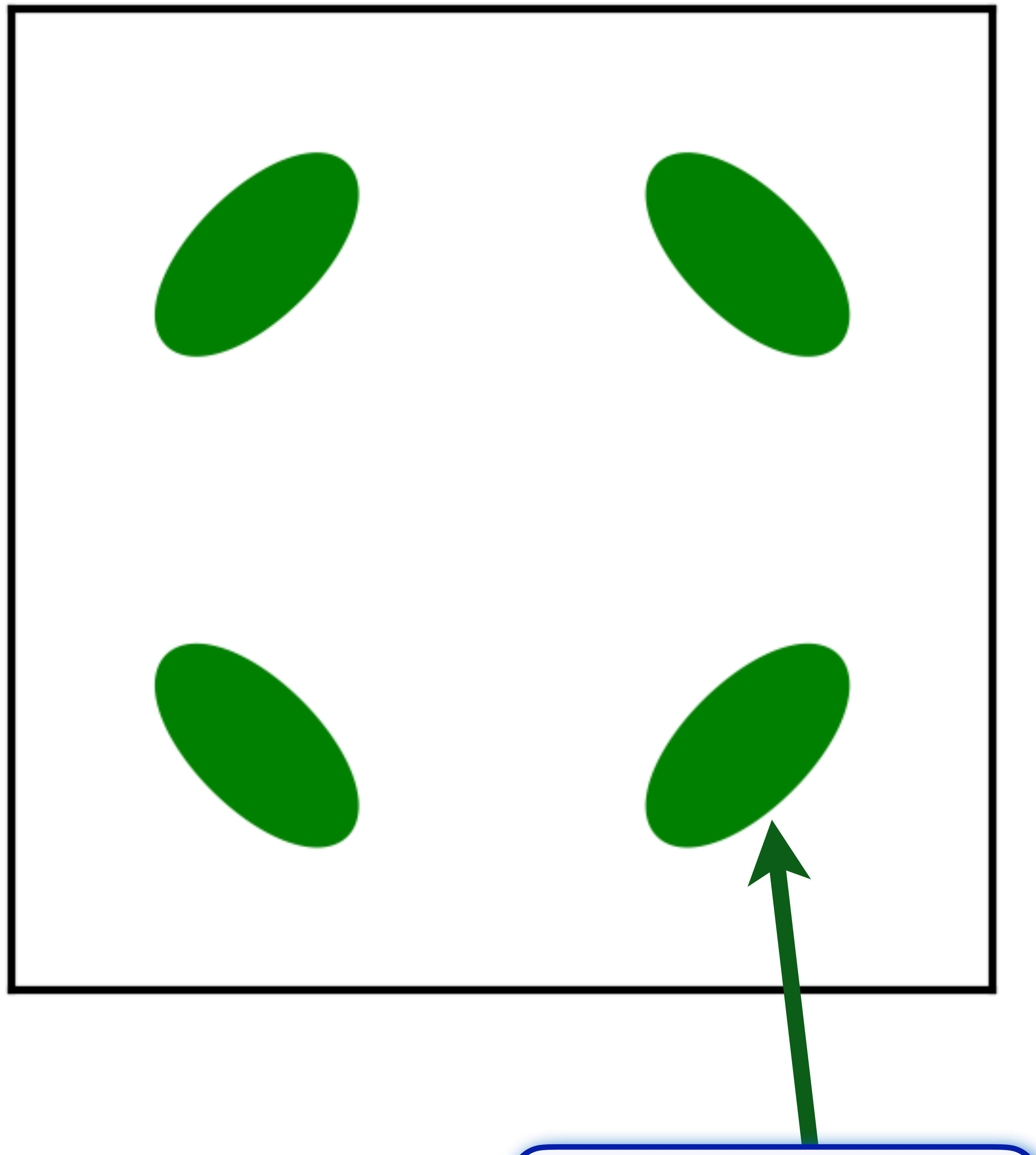

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Doping an insulating antiferromagnet with holes of density p

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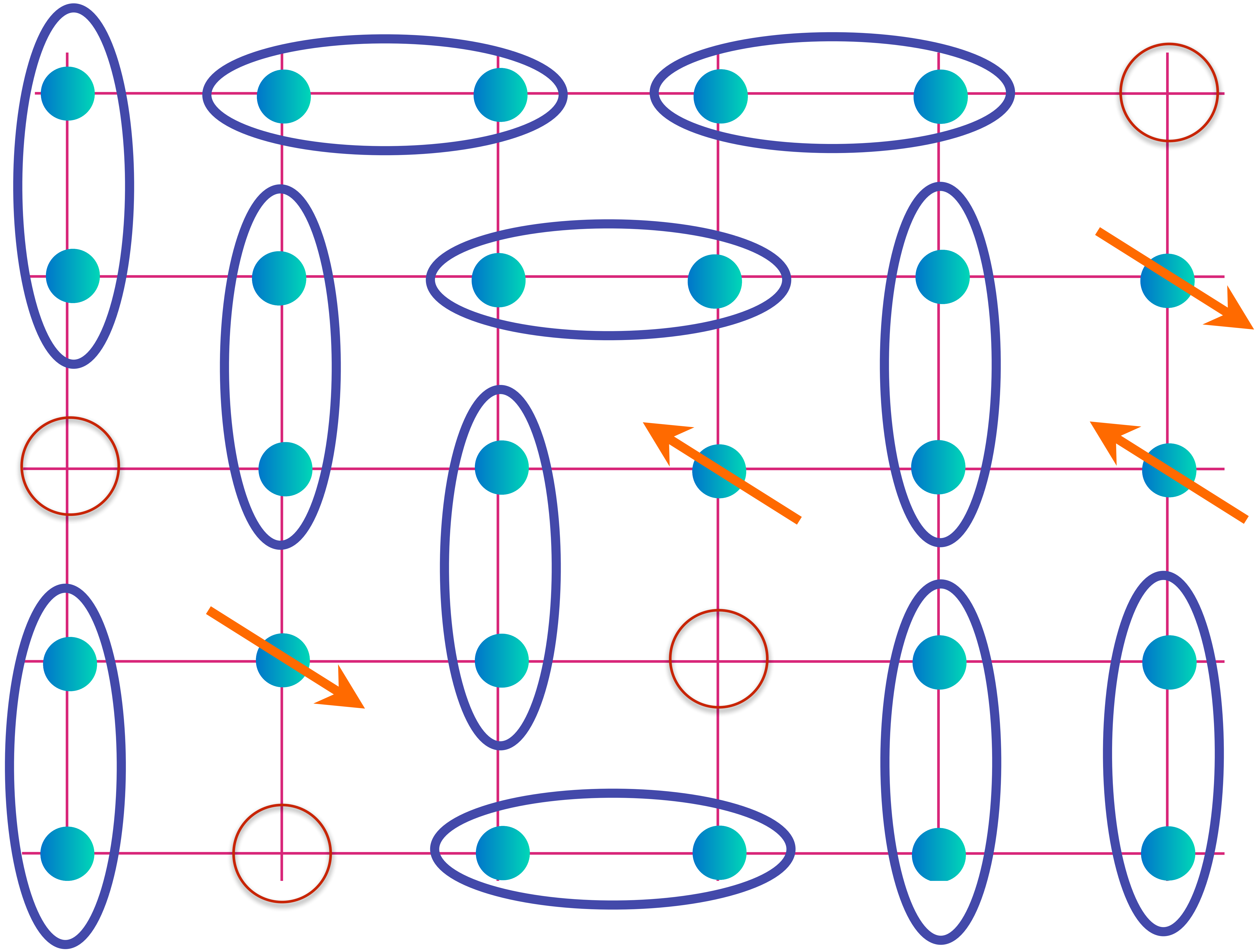

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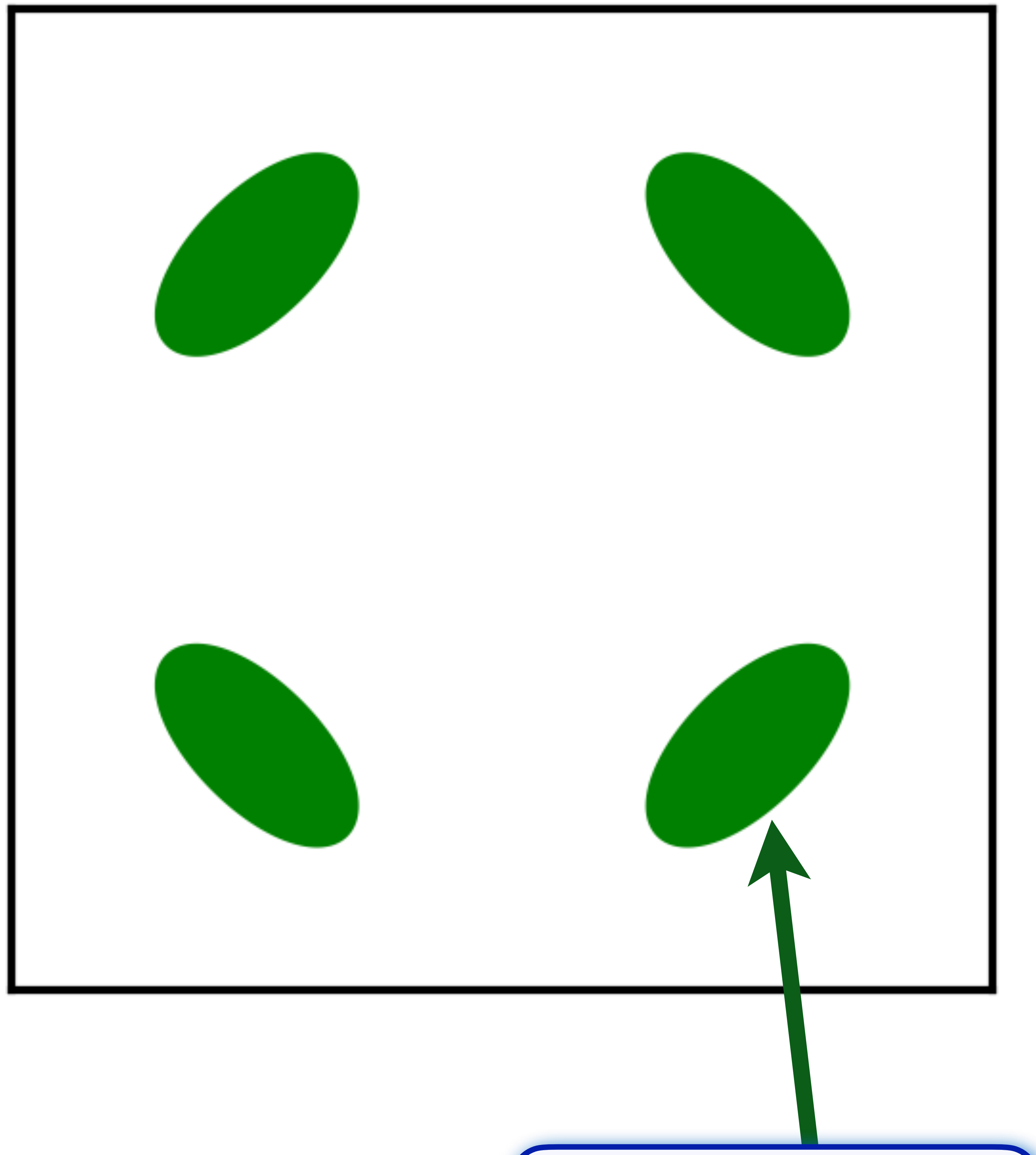

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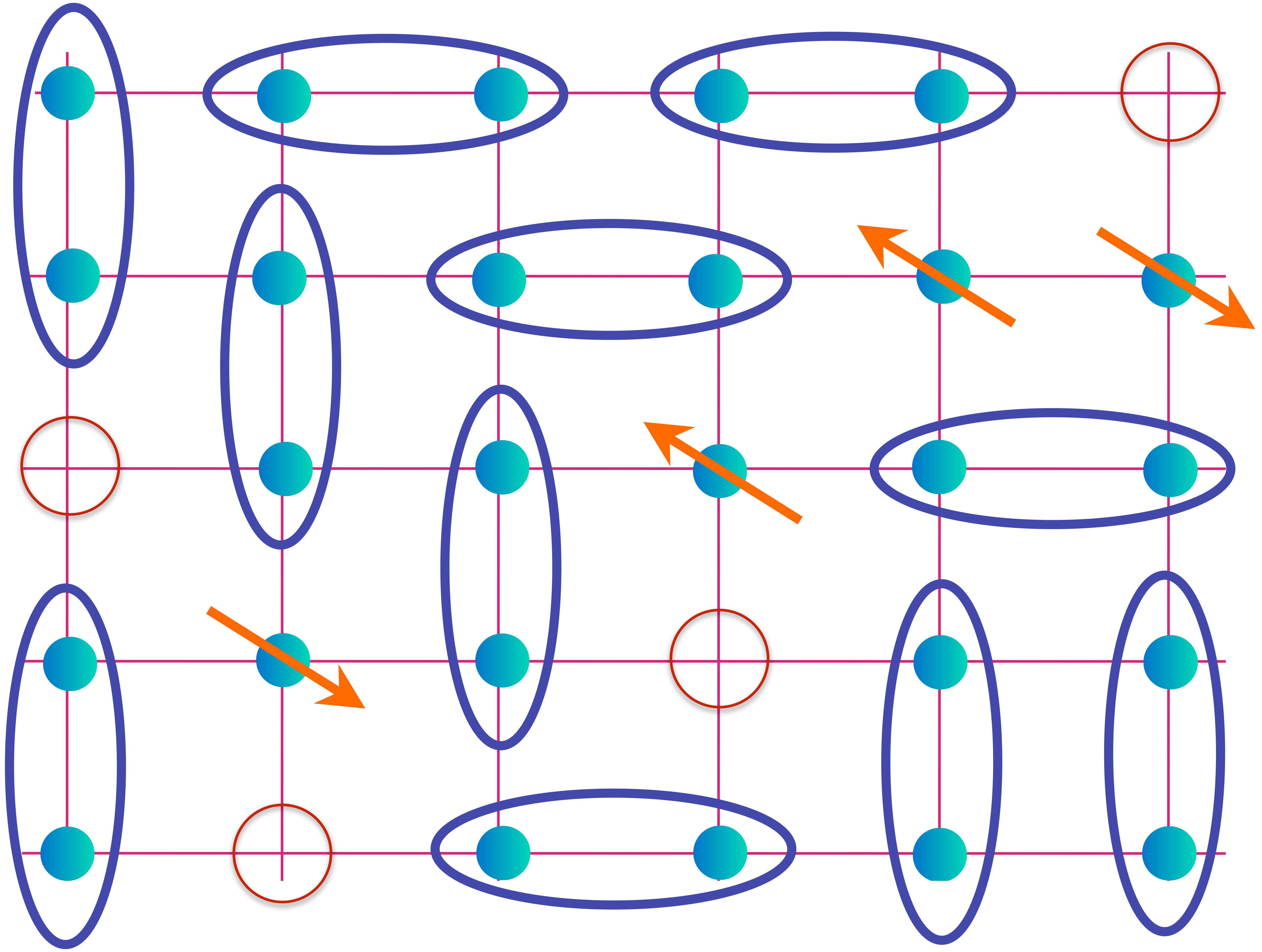
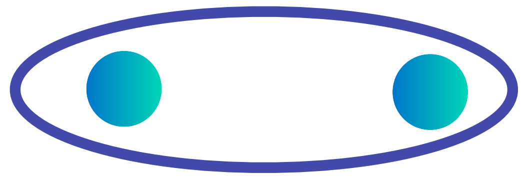
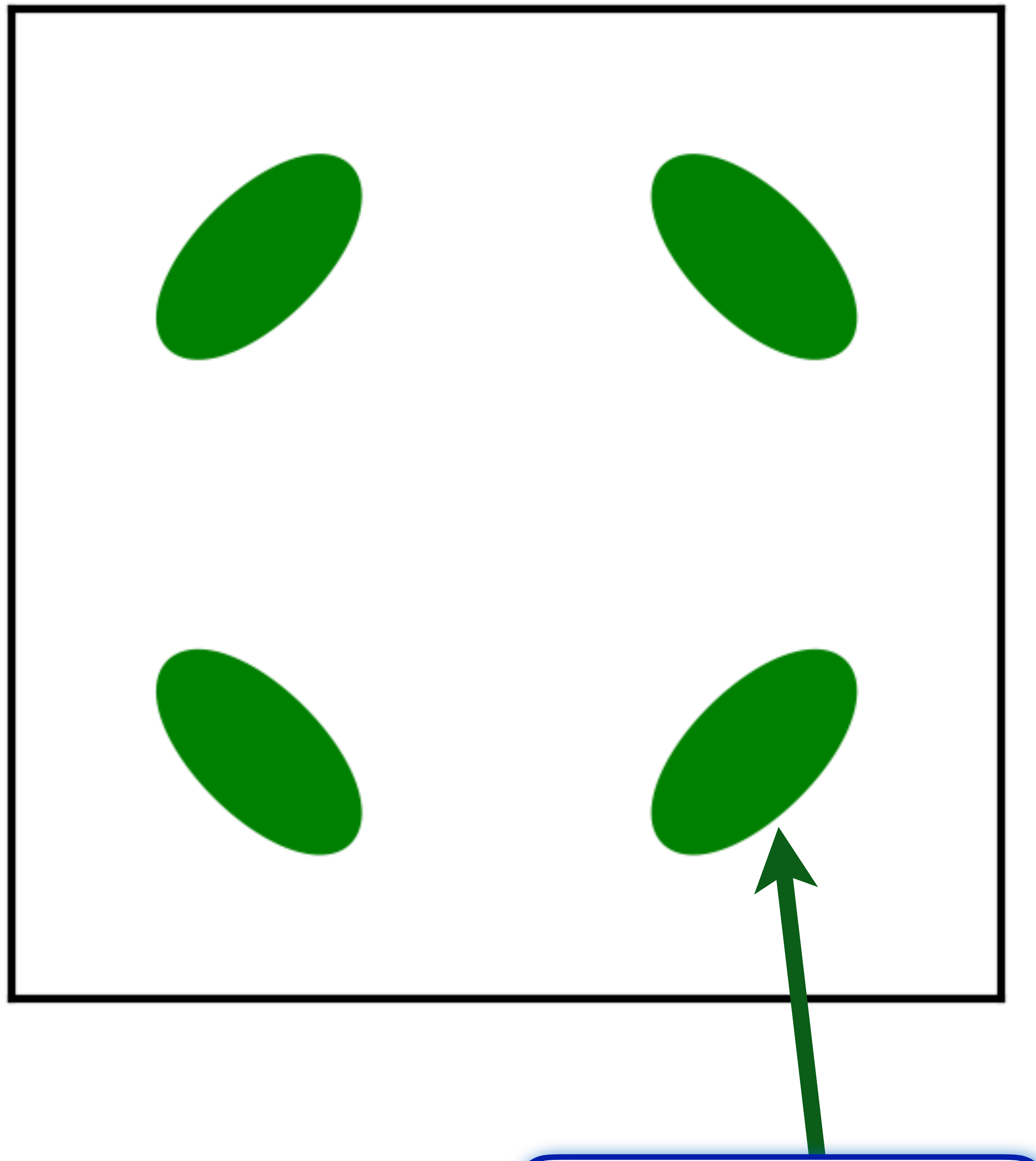


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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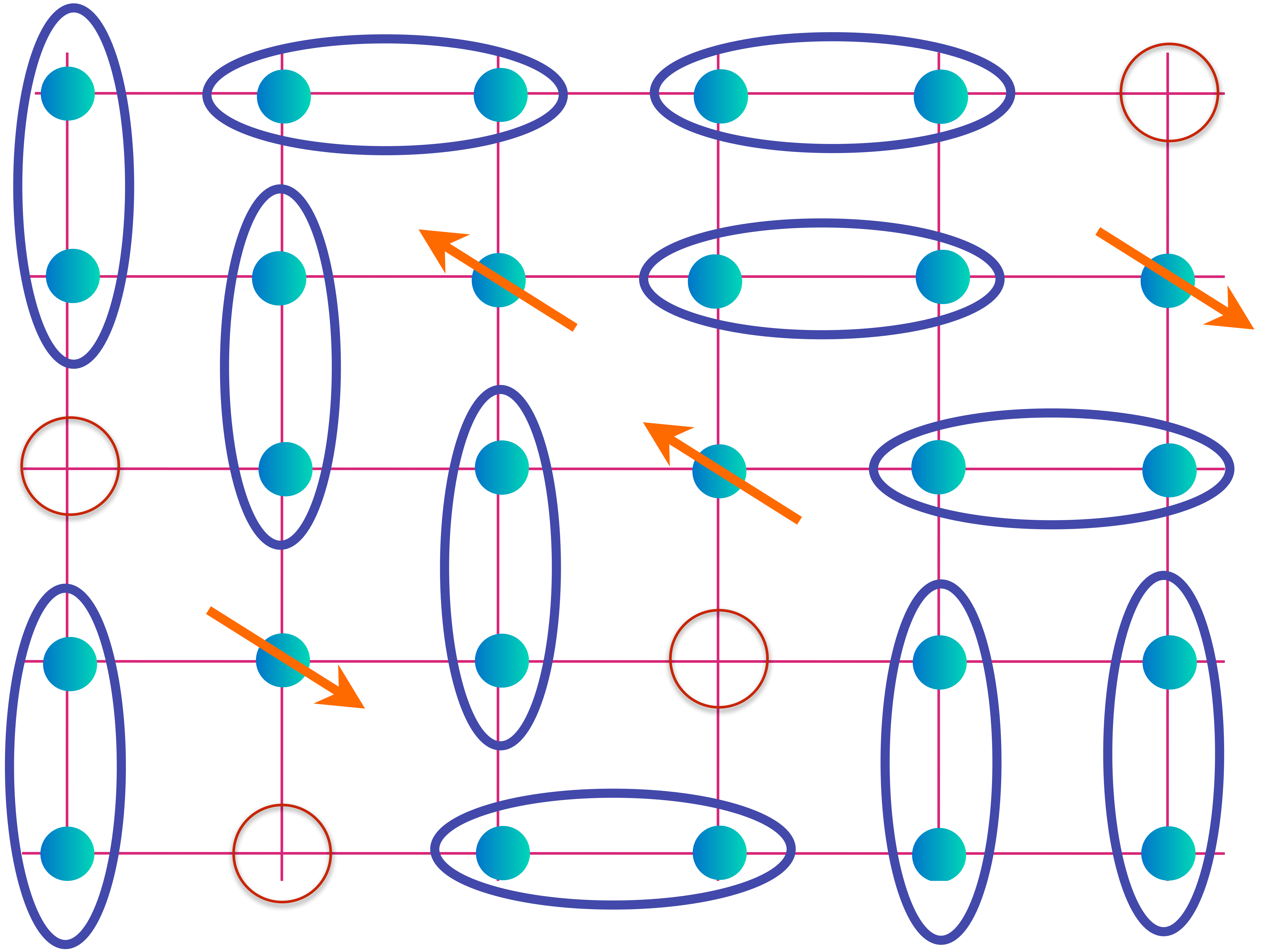
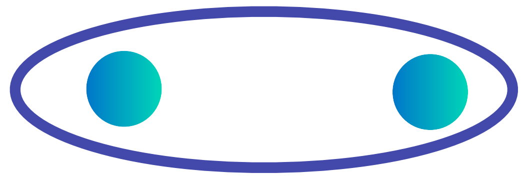
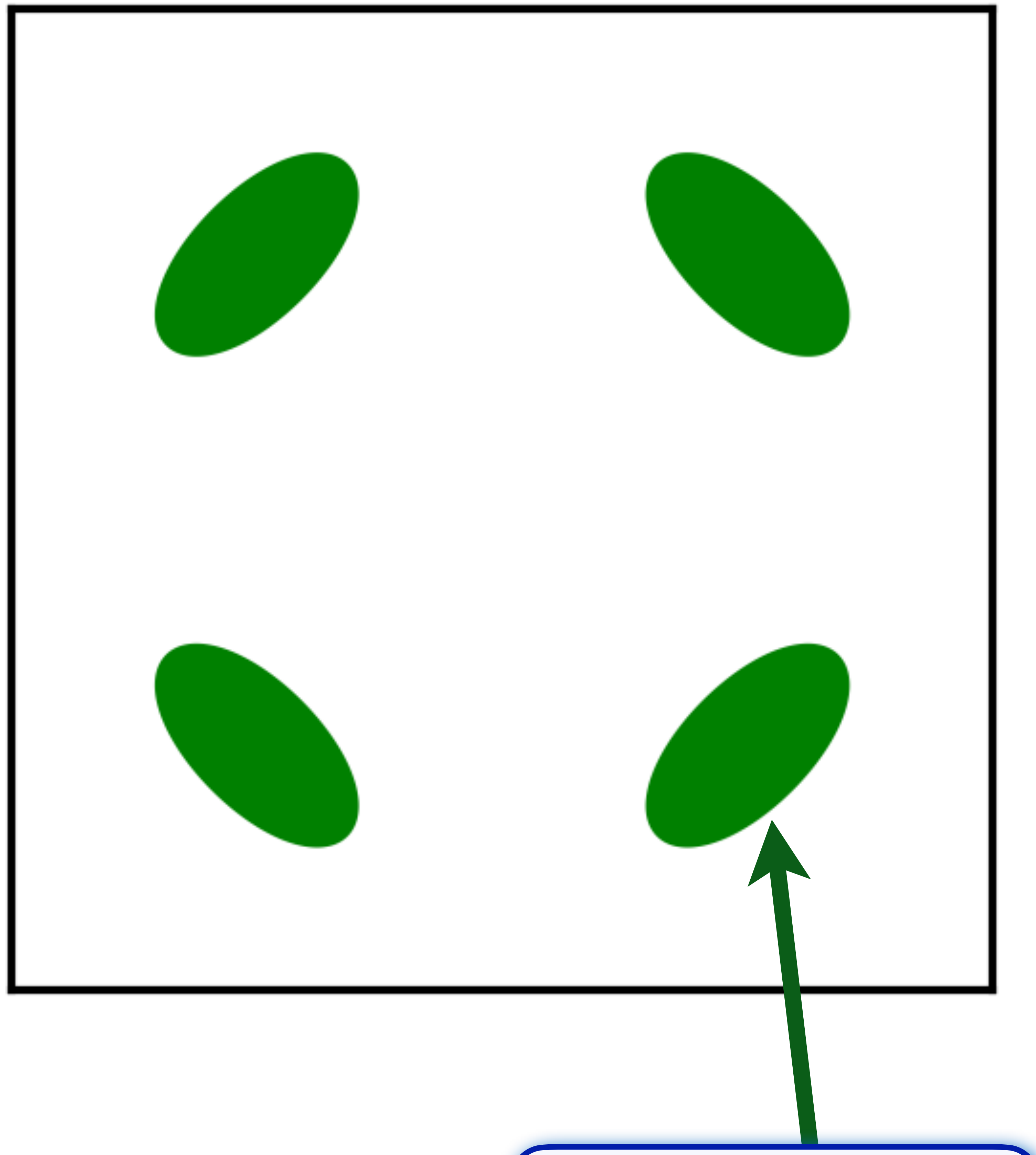


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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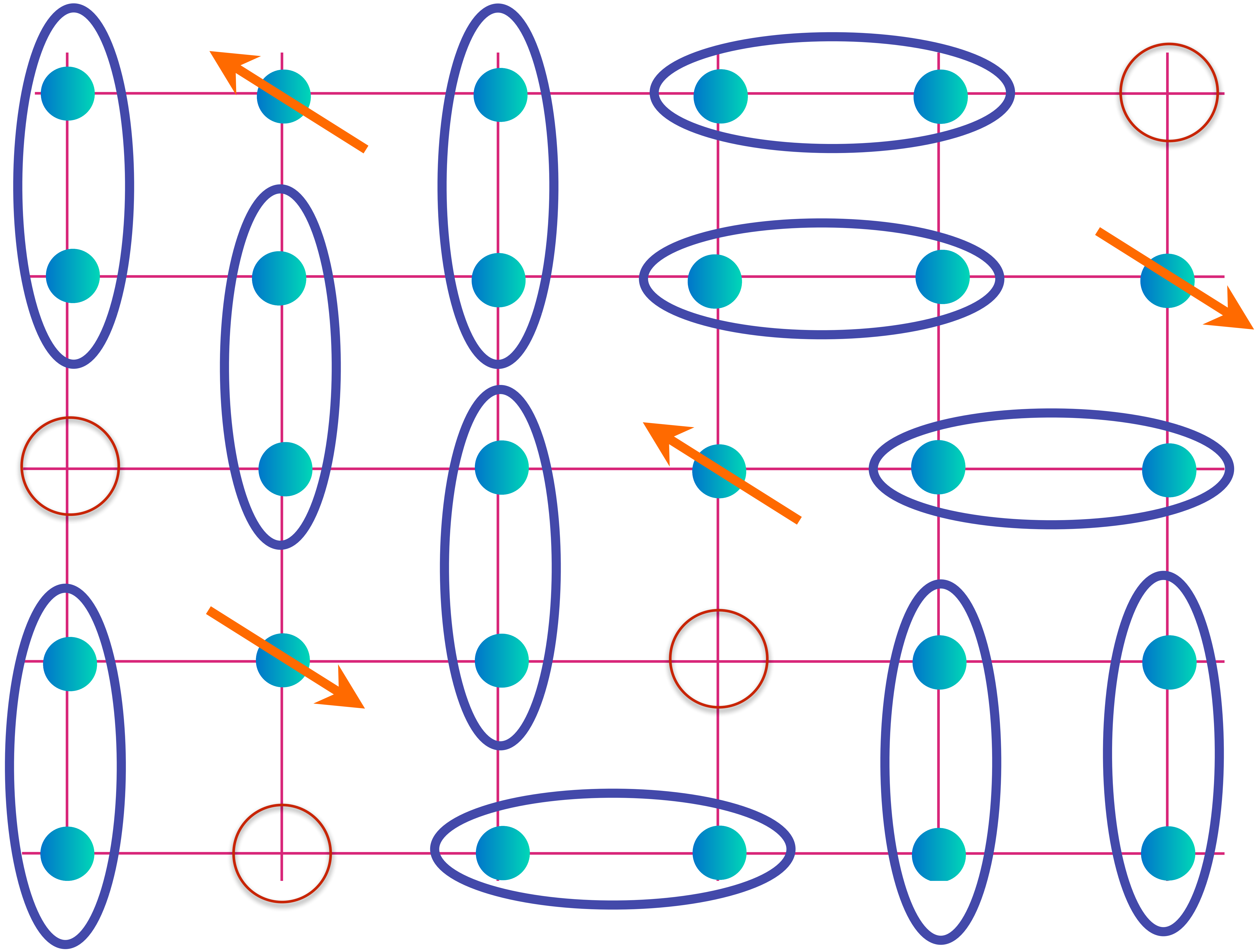
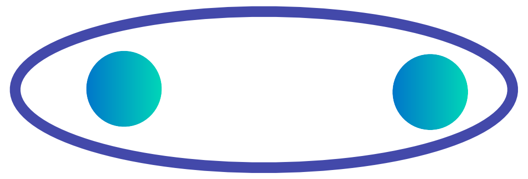
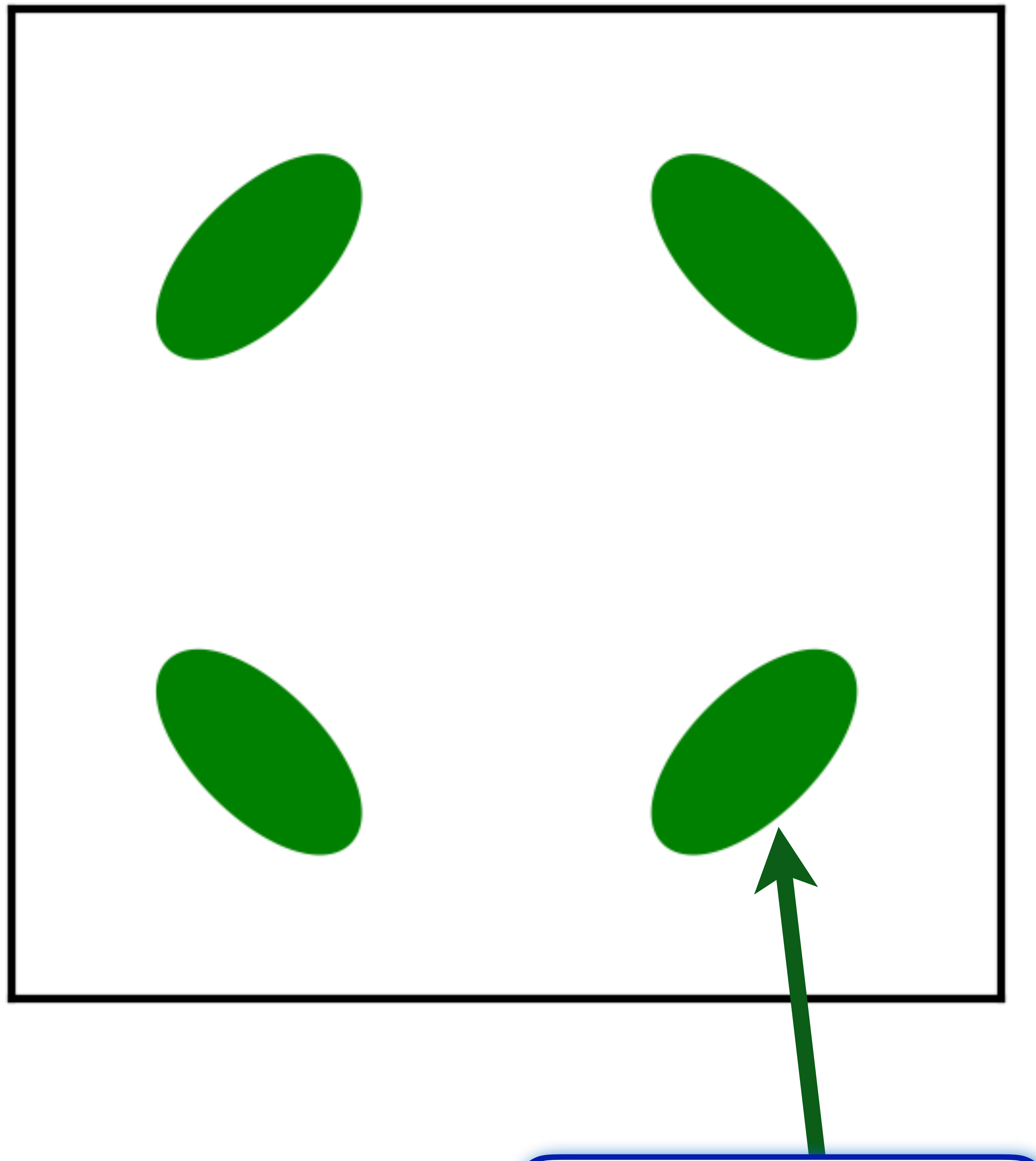


 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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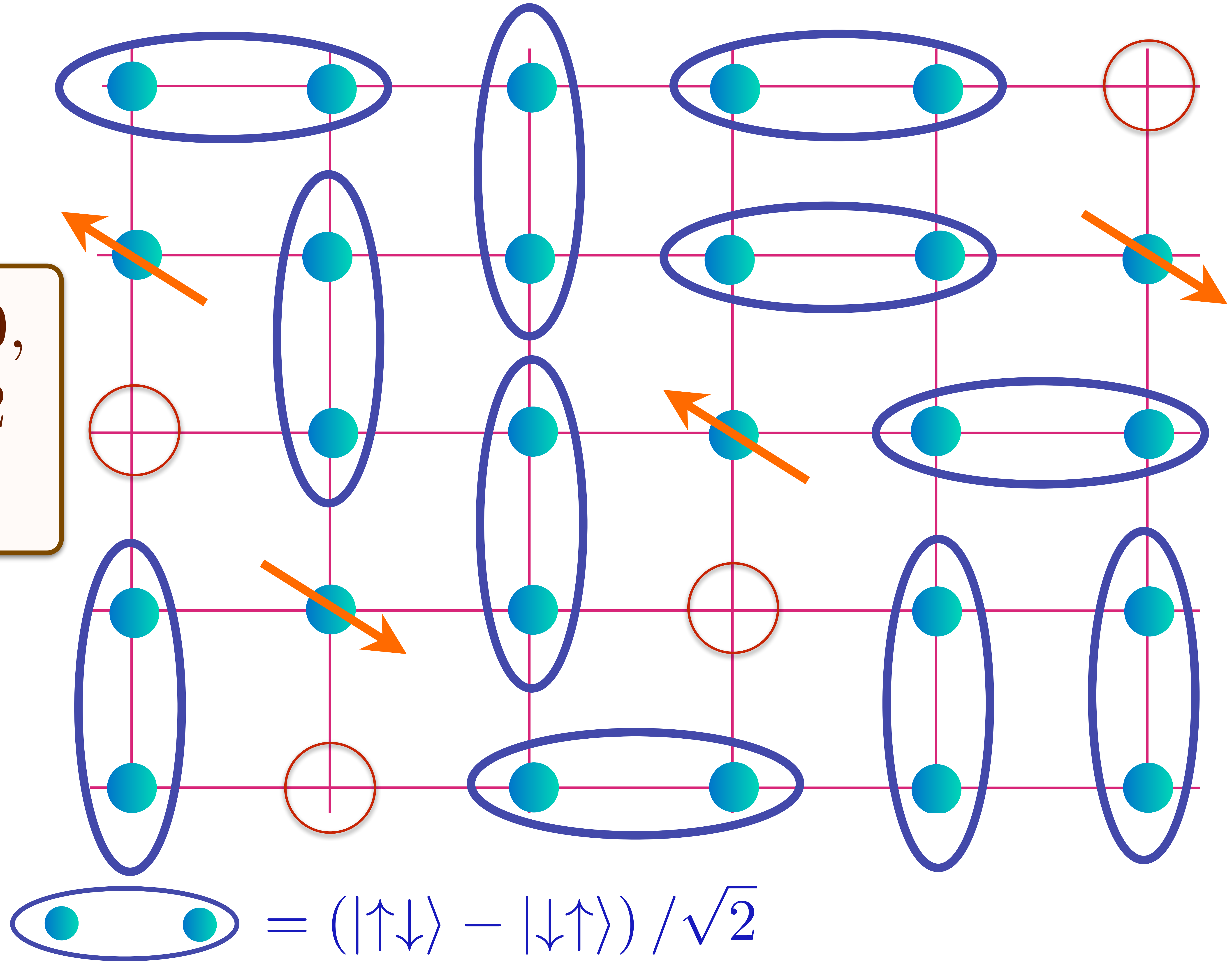

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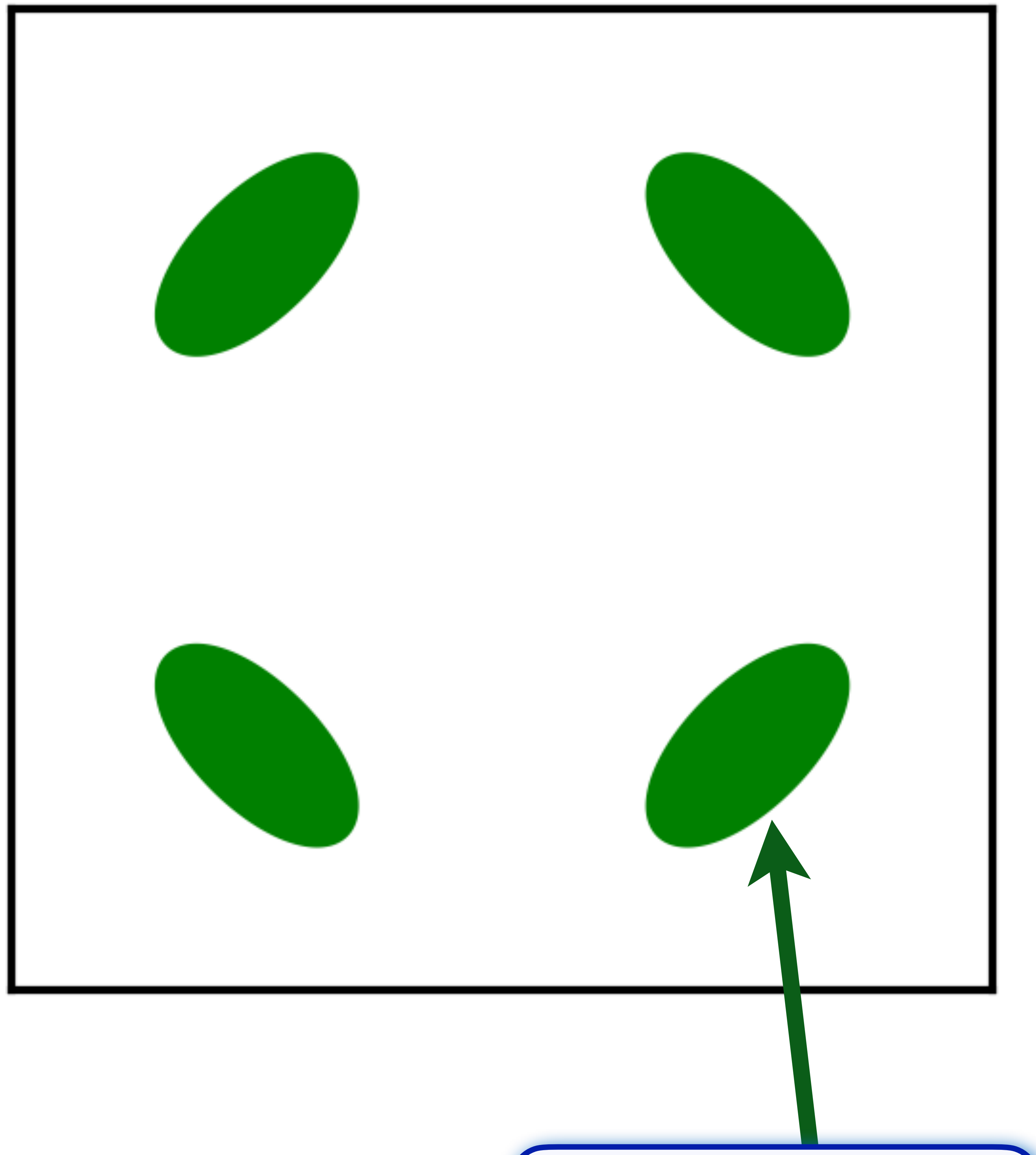

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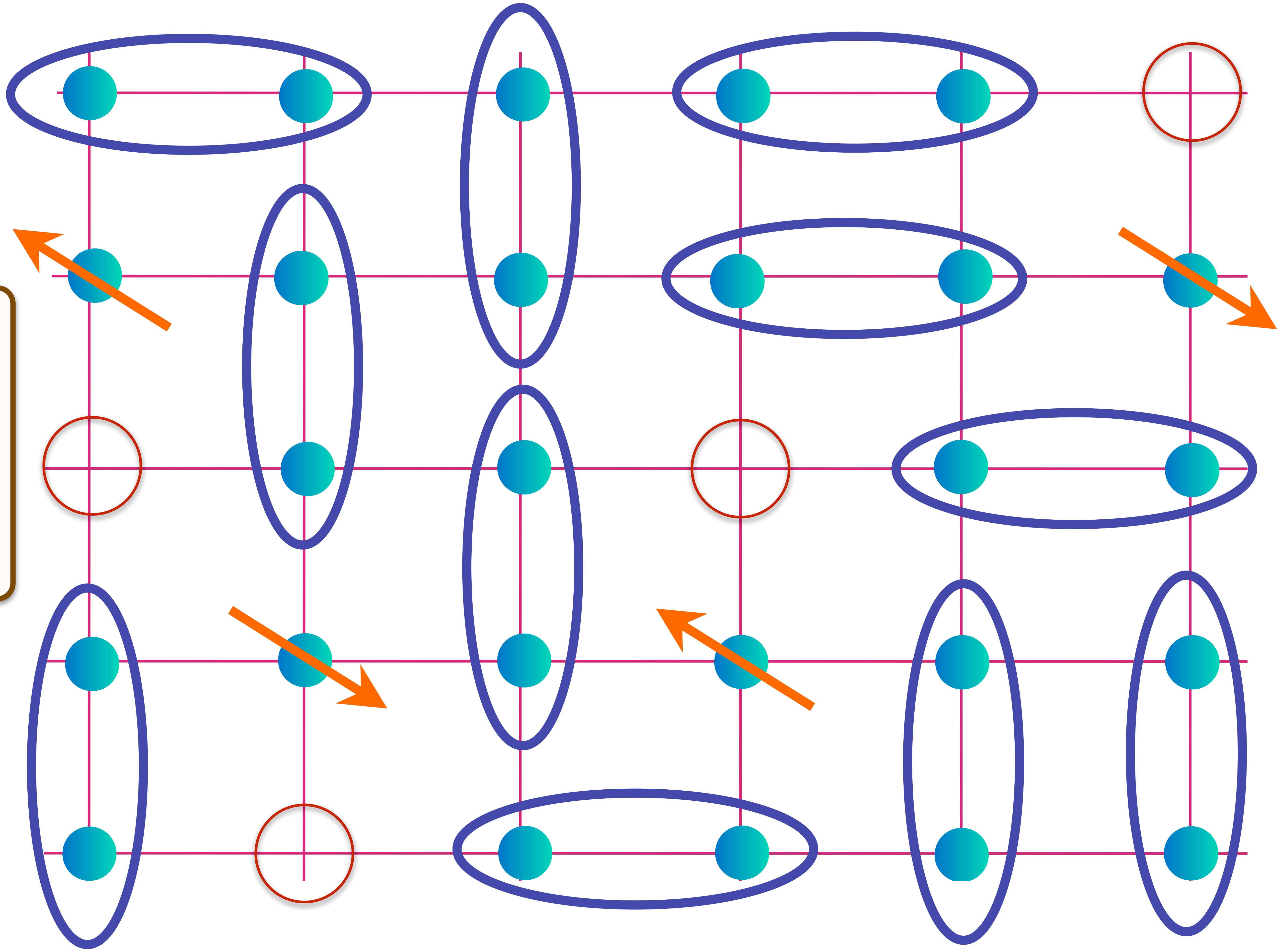

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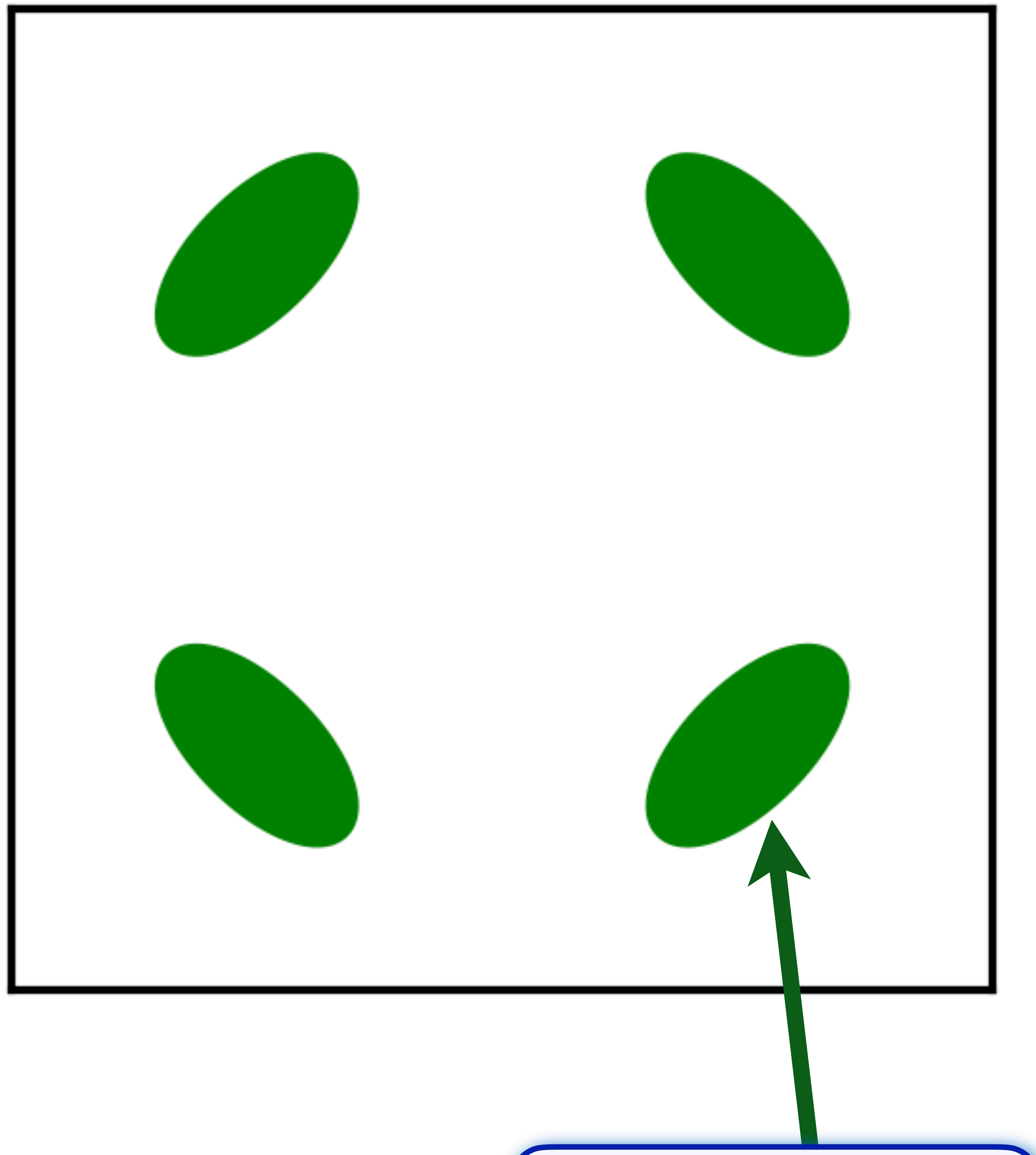

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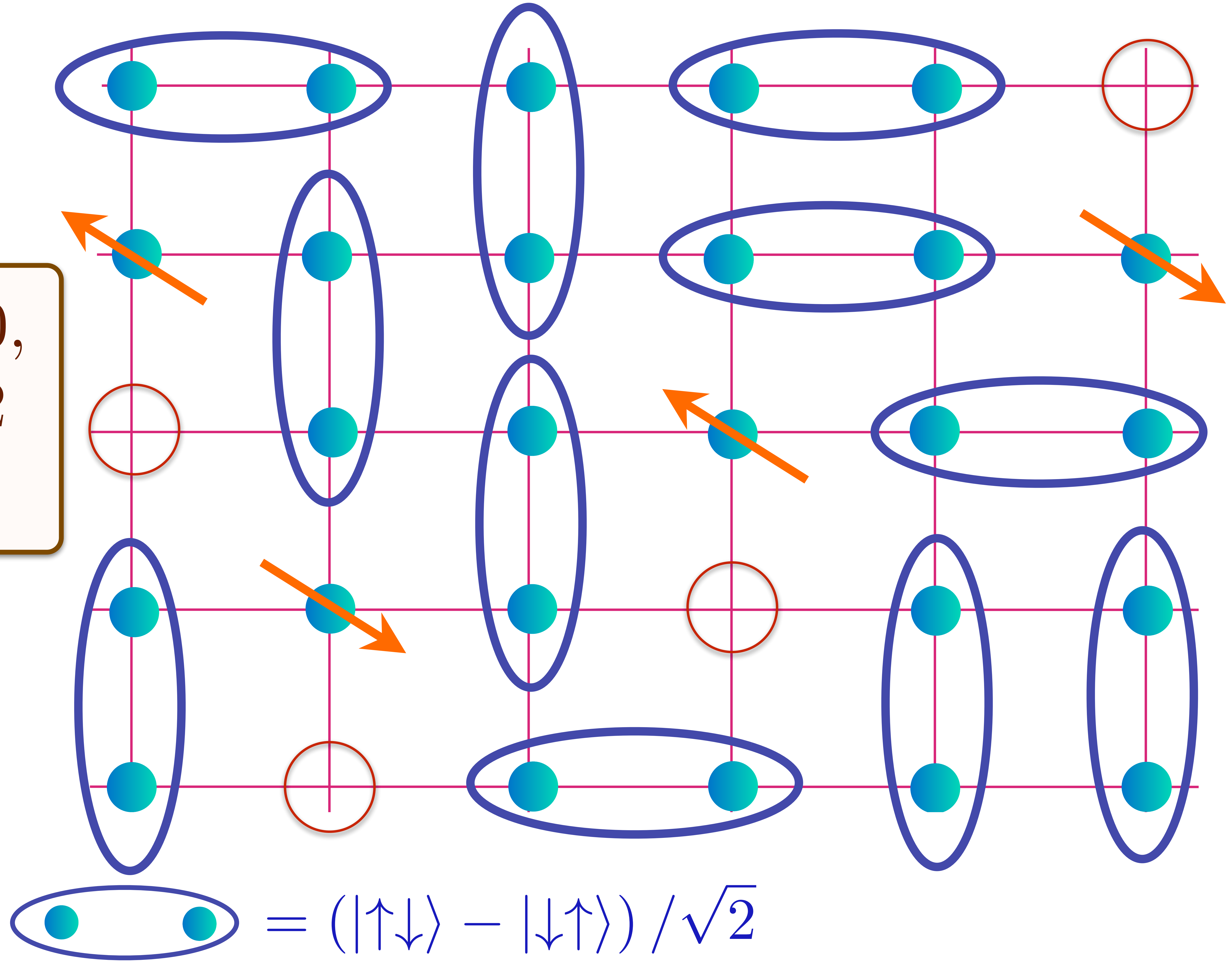

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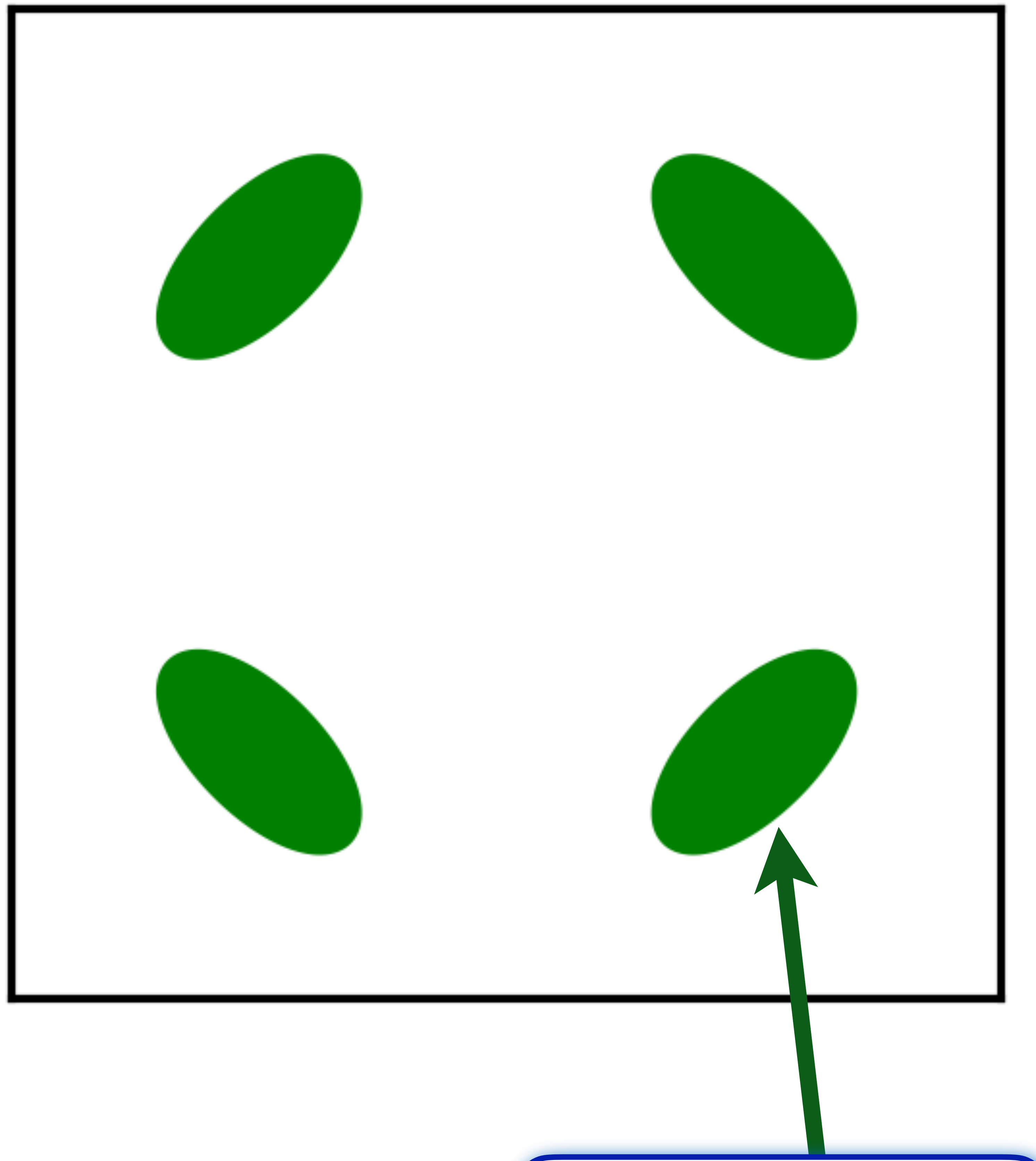

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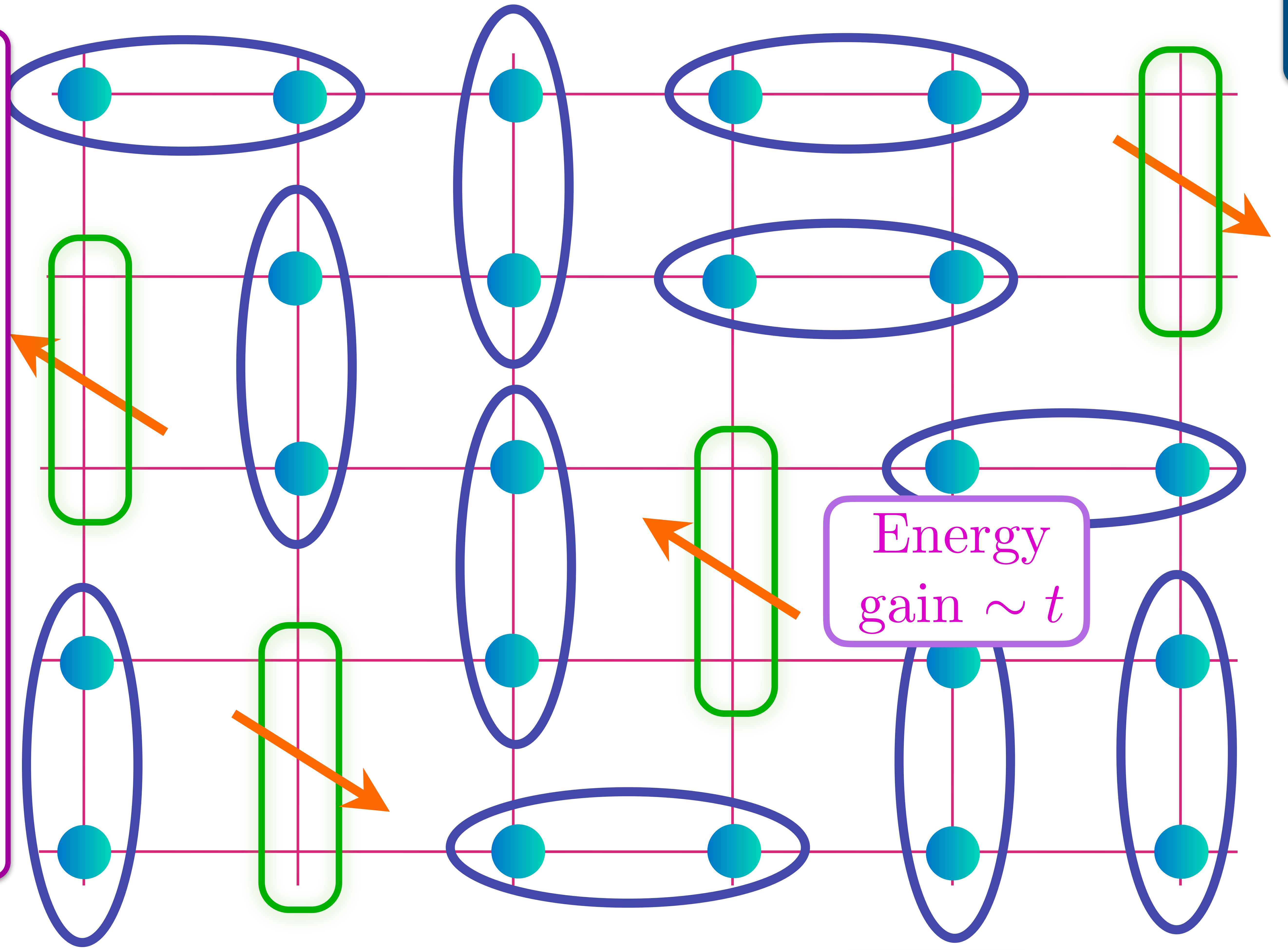
Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.



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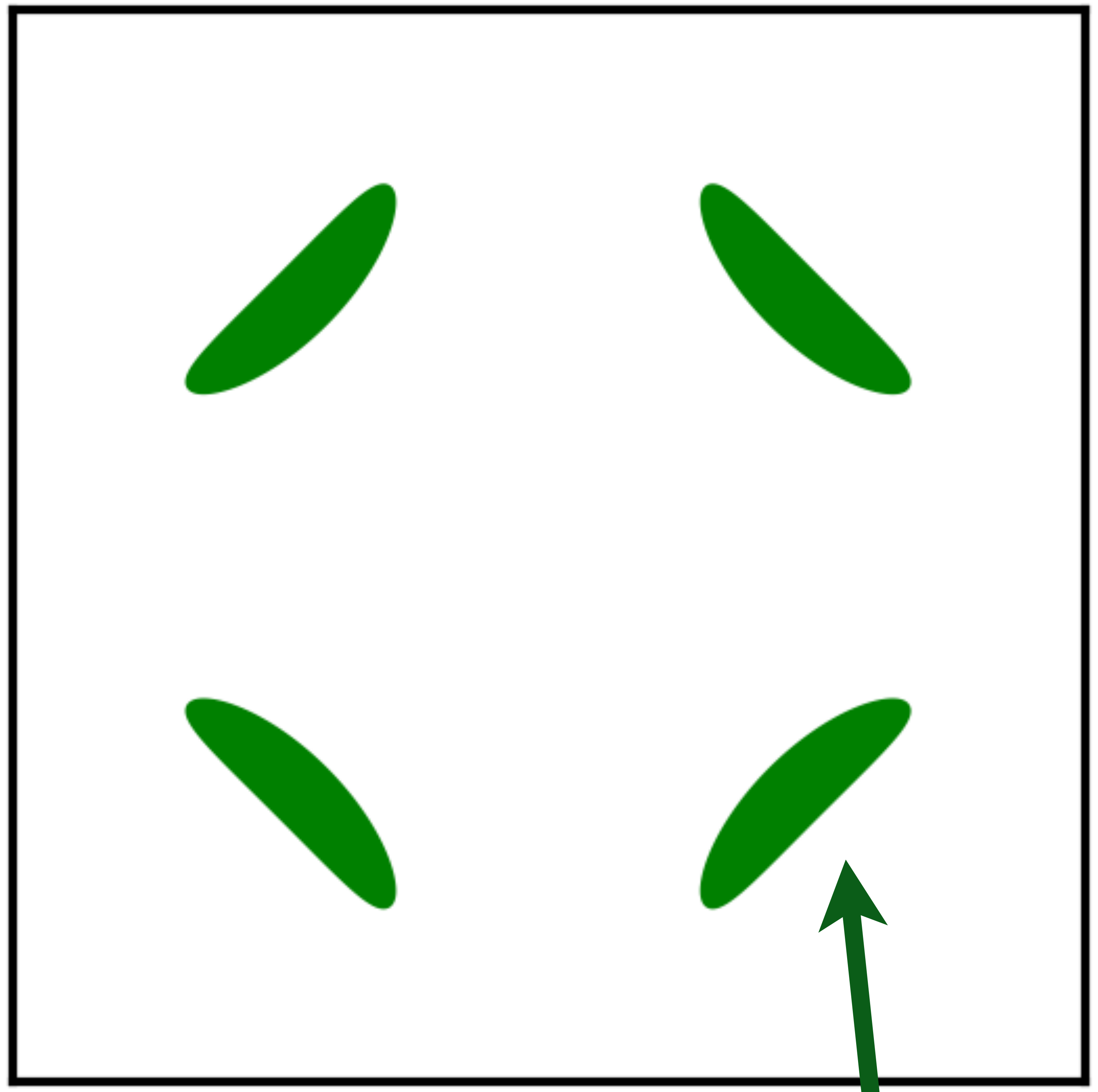
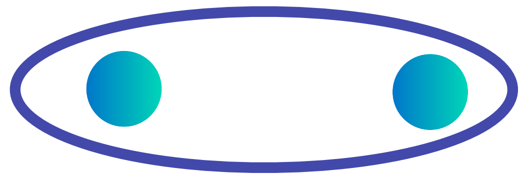
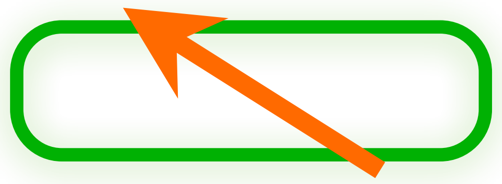


Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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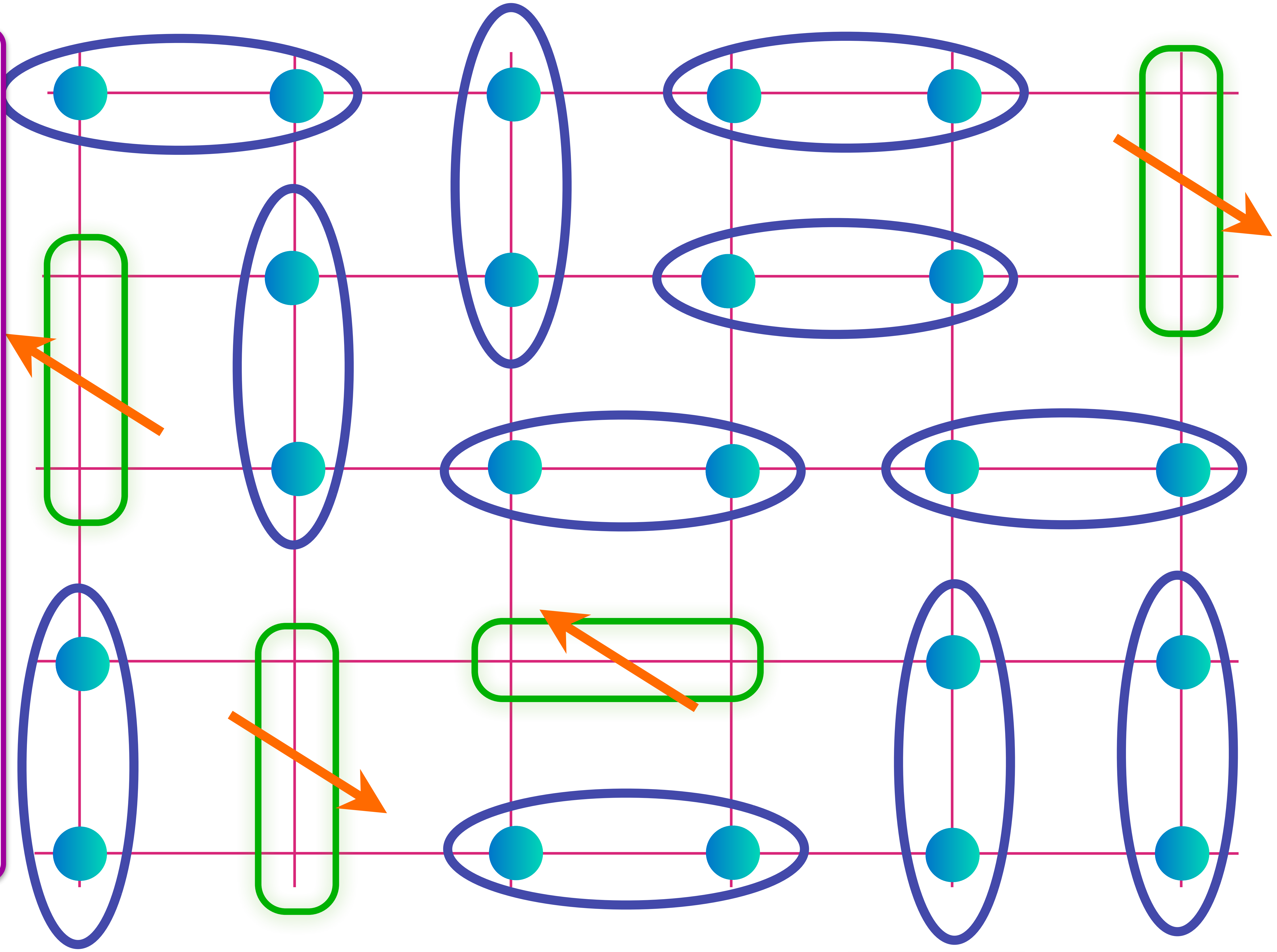

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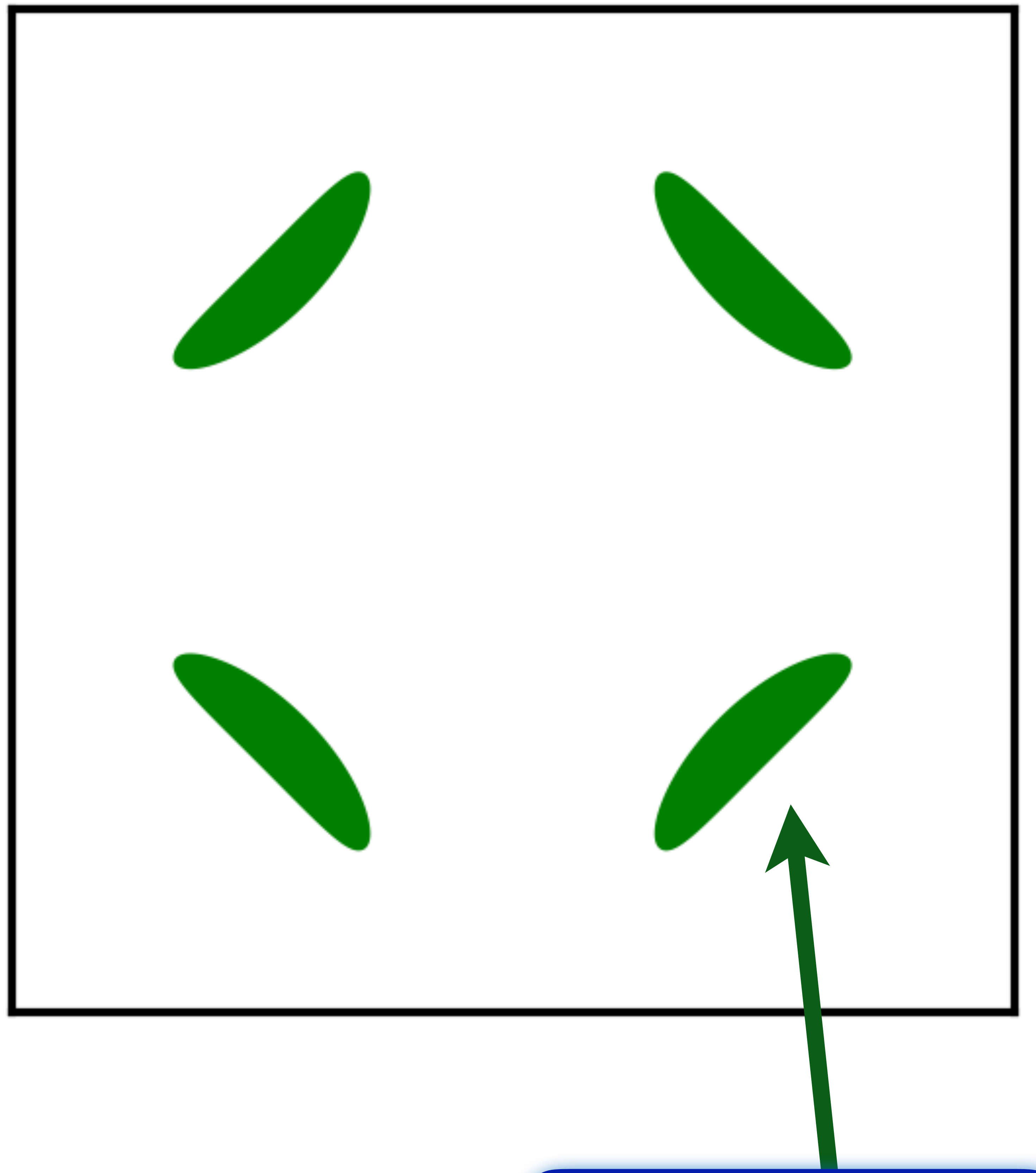

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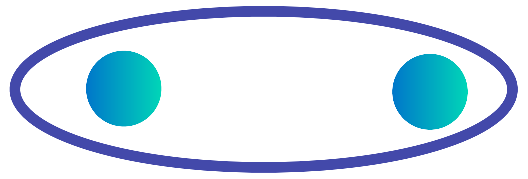
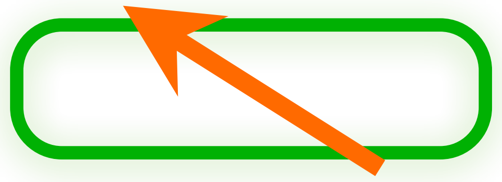
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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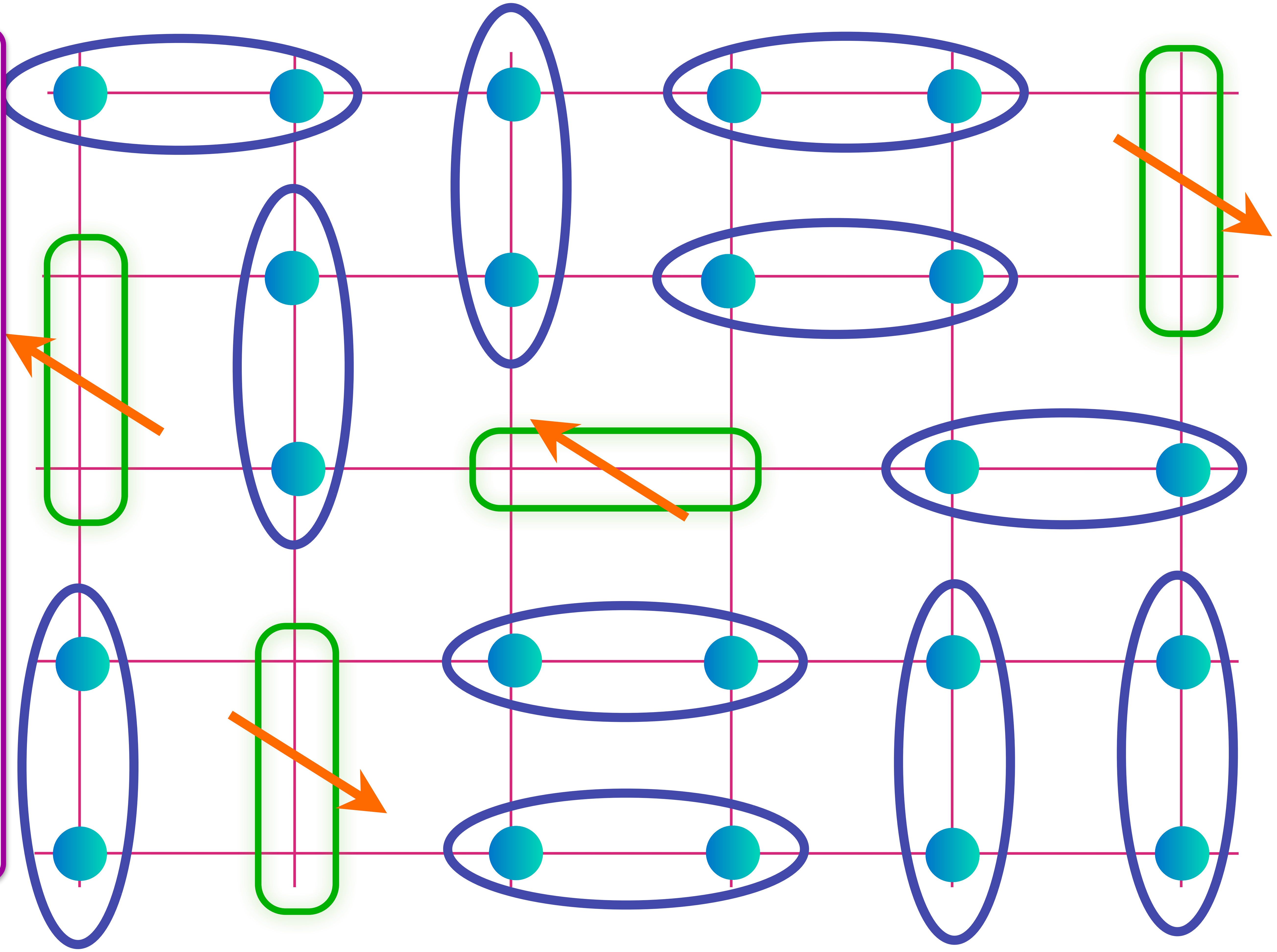

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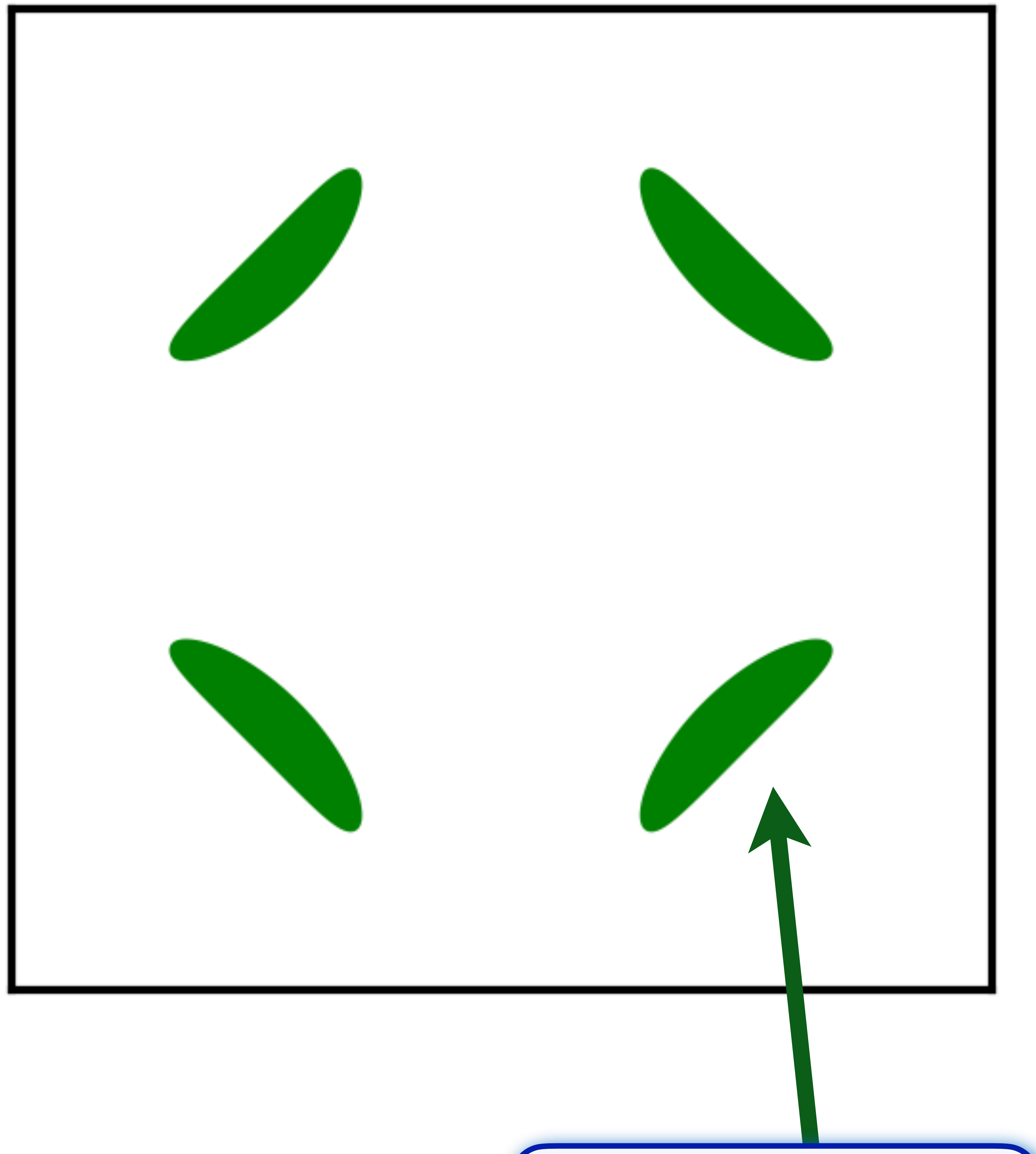
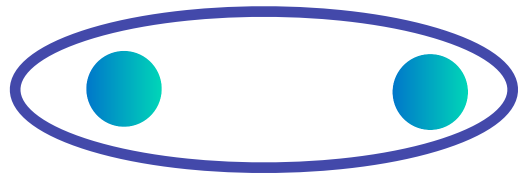
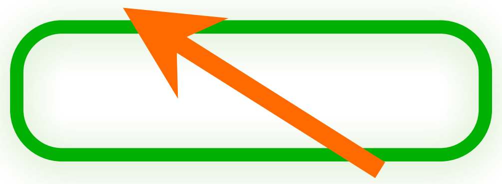


Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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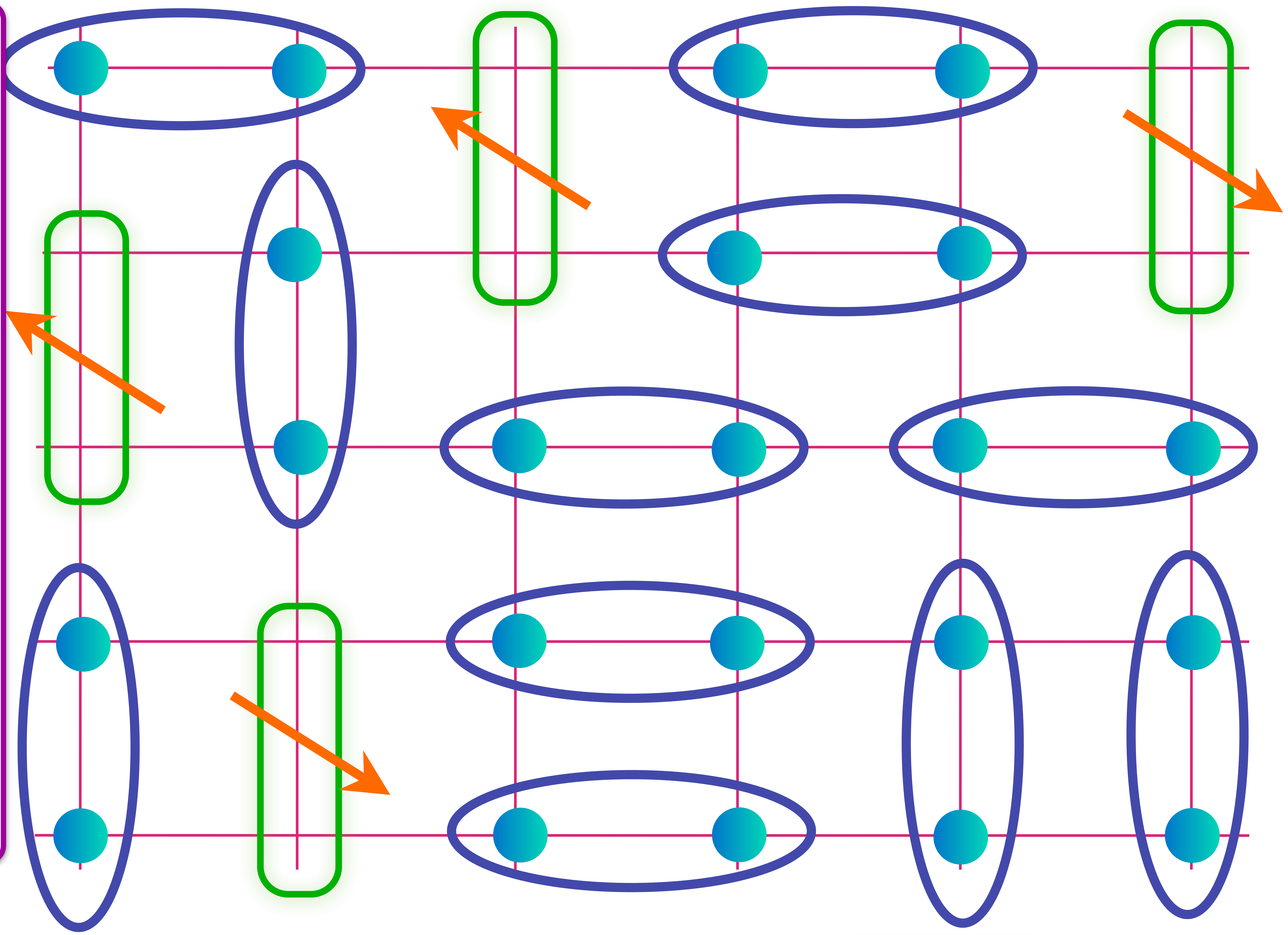

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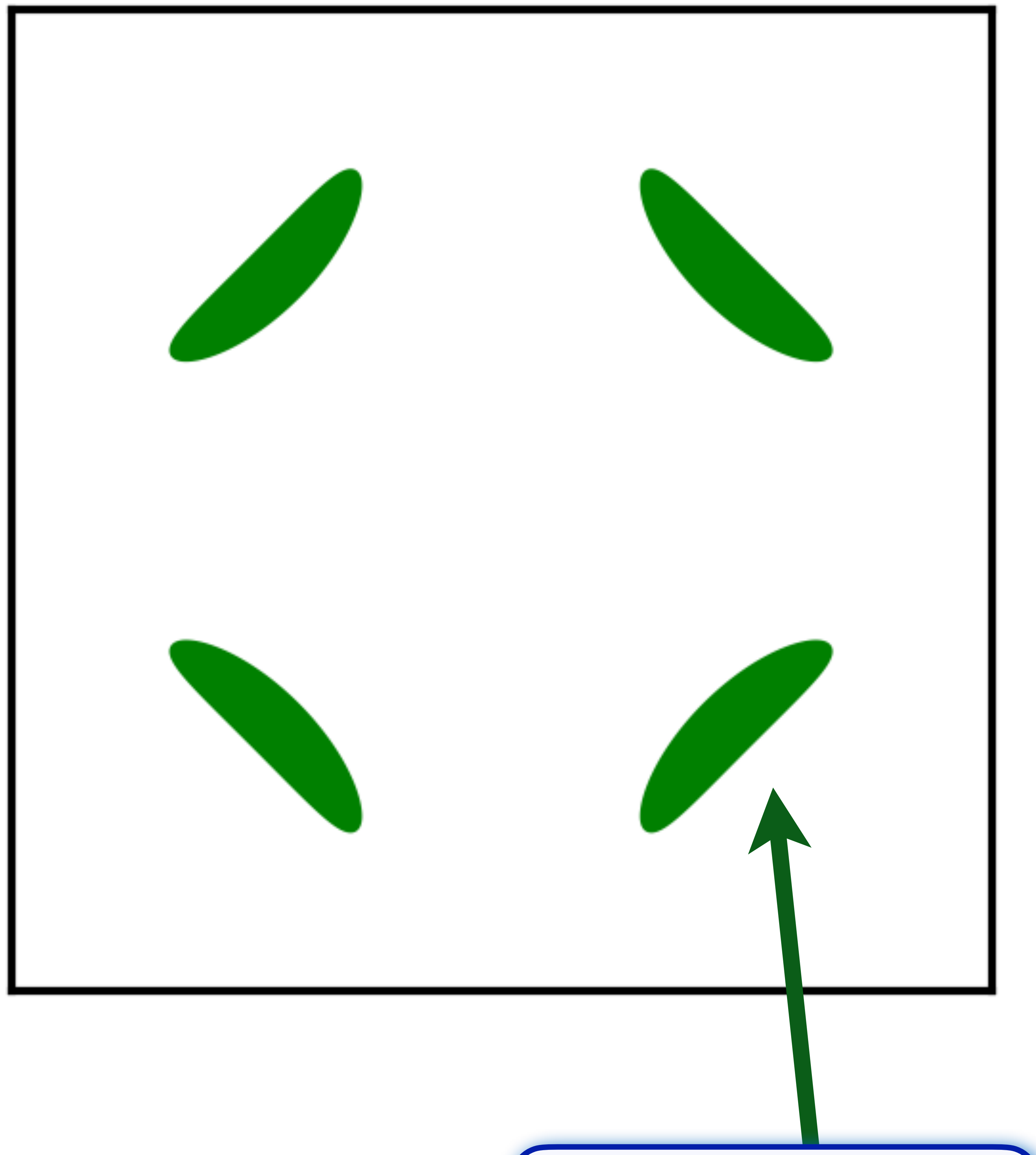

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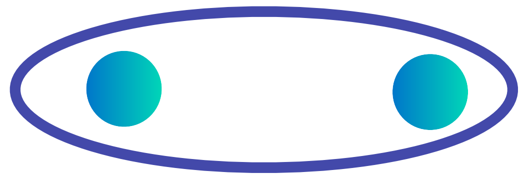
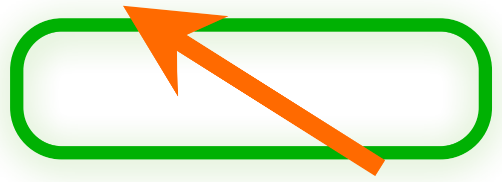
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

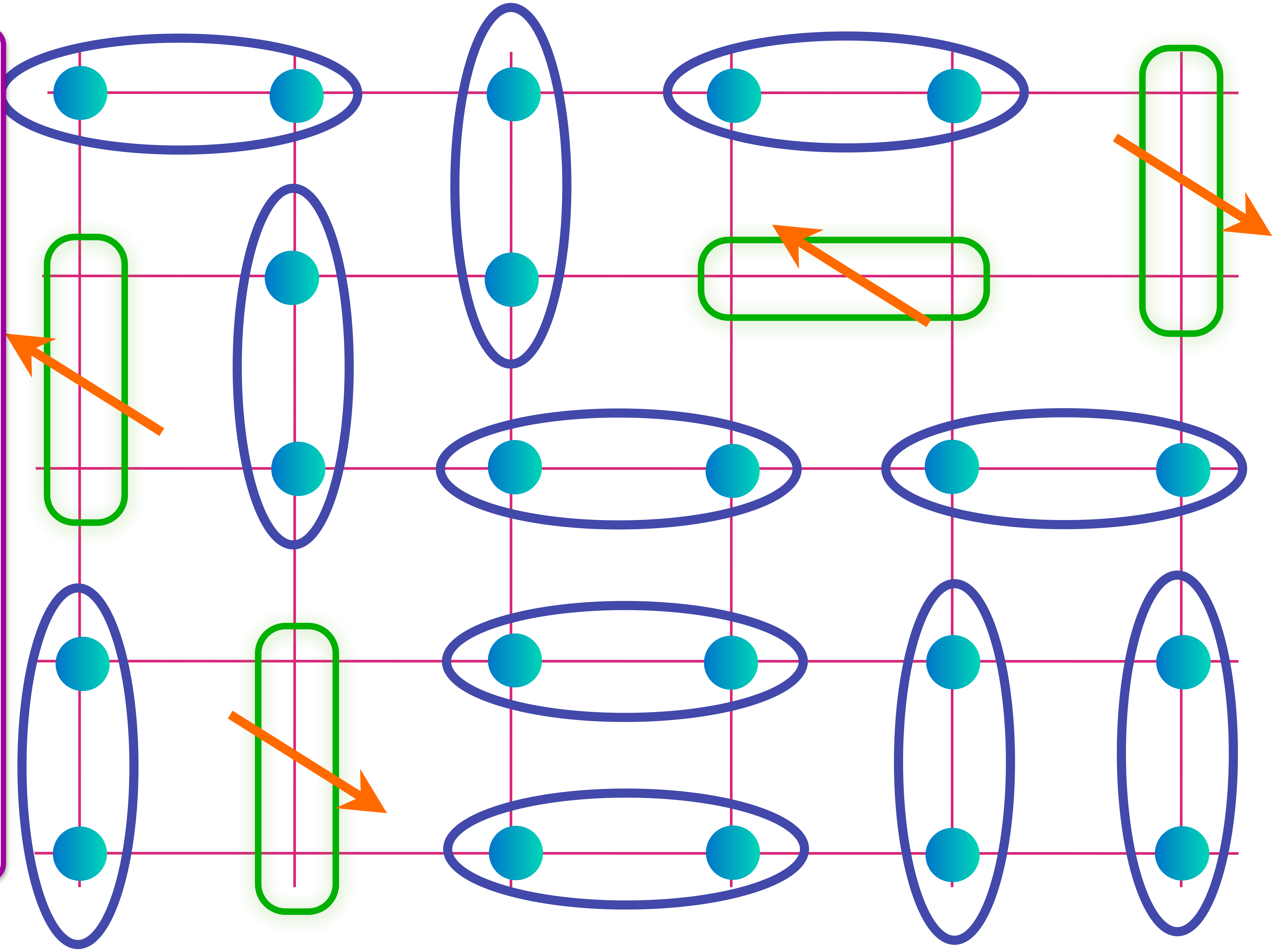
 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

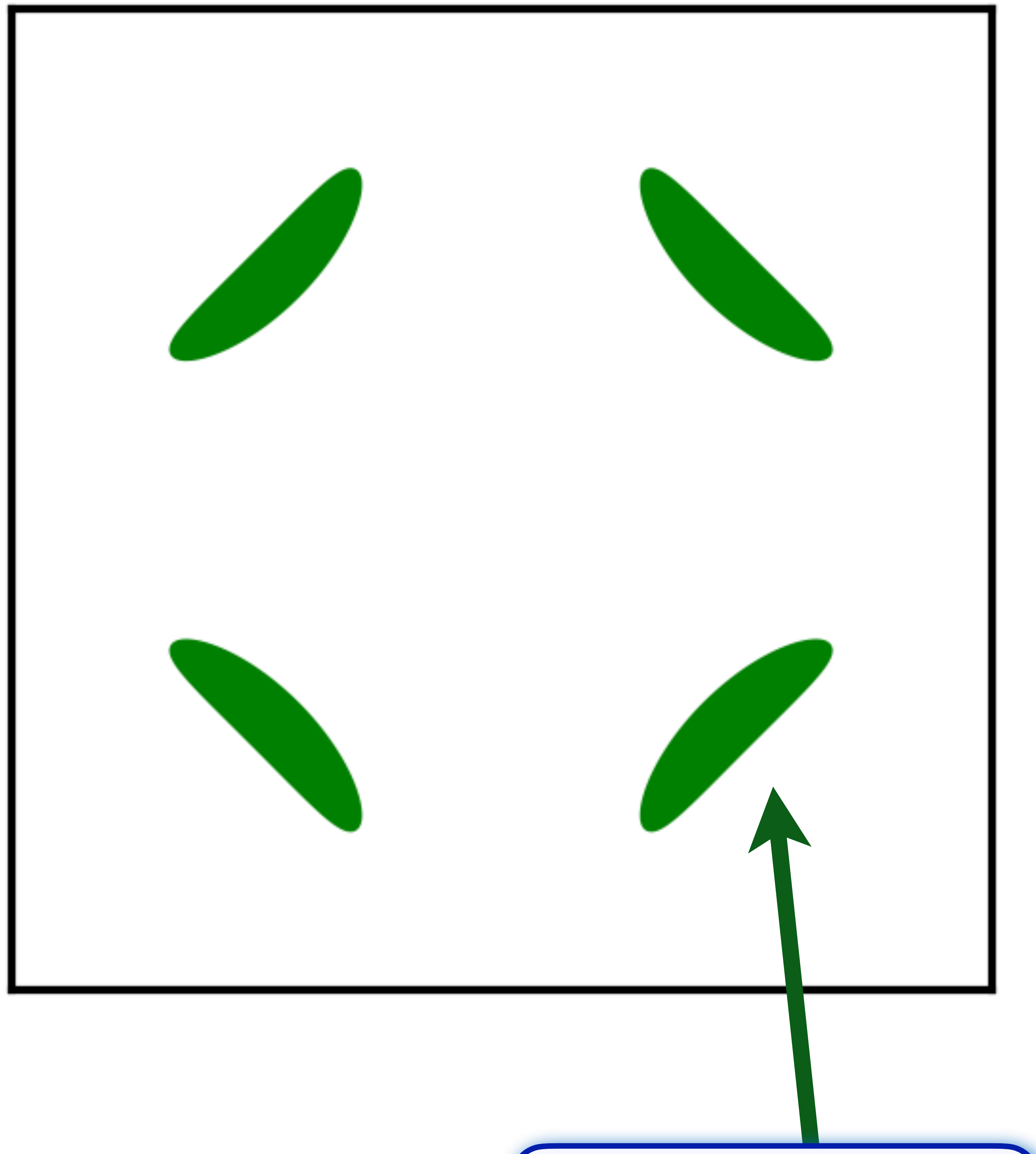
Doping an insulating antiferromagnet with holes of density p

FL*

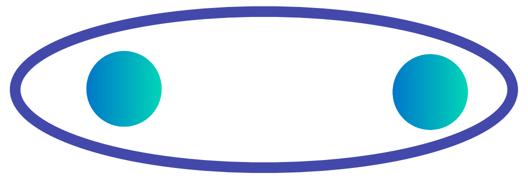
Metal with density p of spin-1/2, charge $+e$ 'holes' (or 'magnetic polarons') with coherent inter-layer transport.

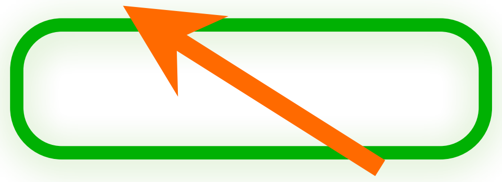


Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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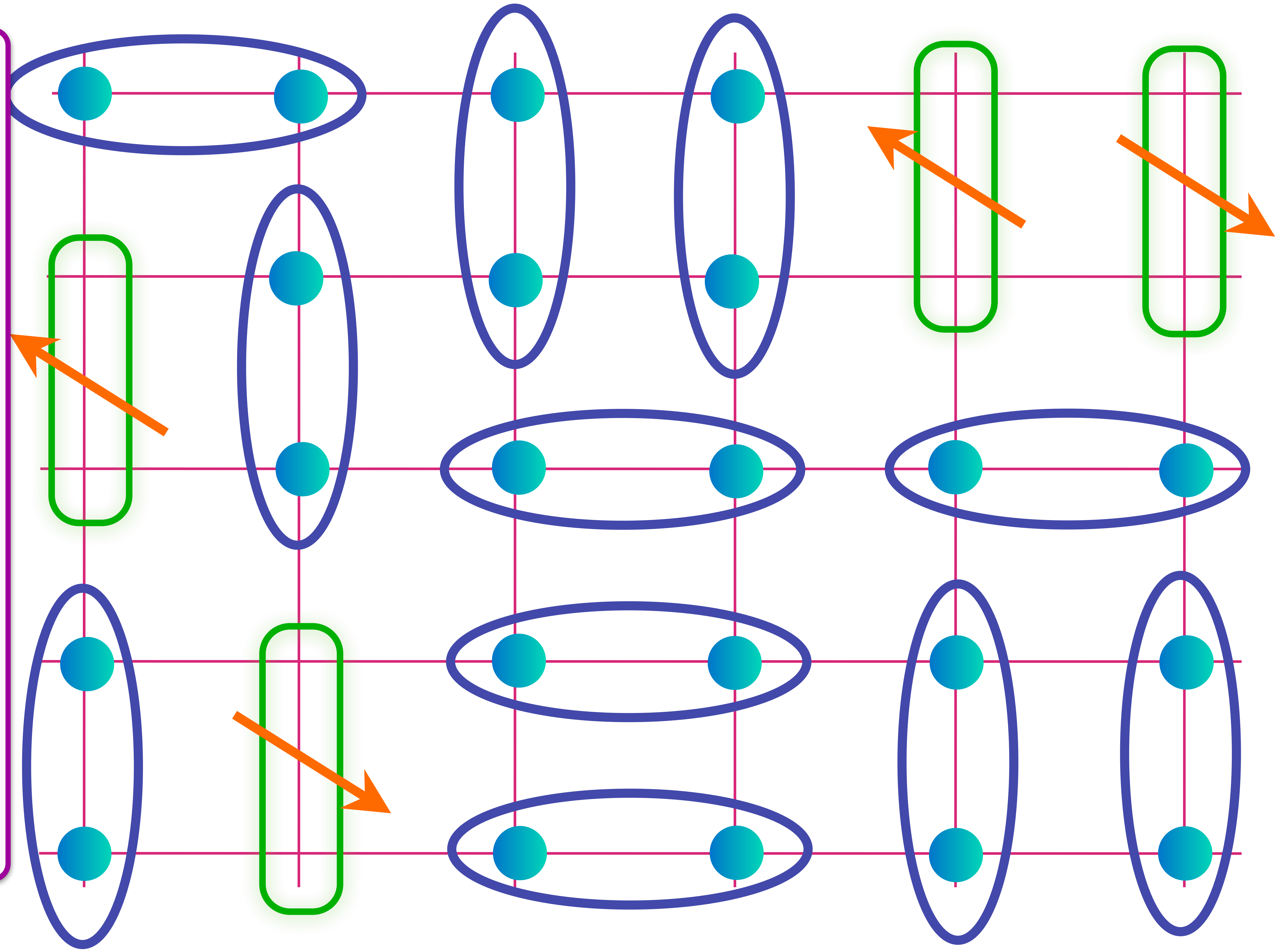

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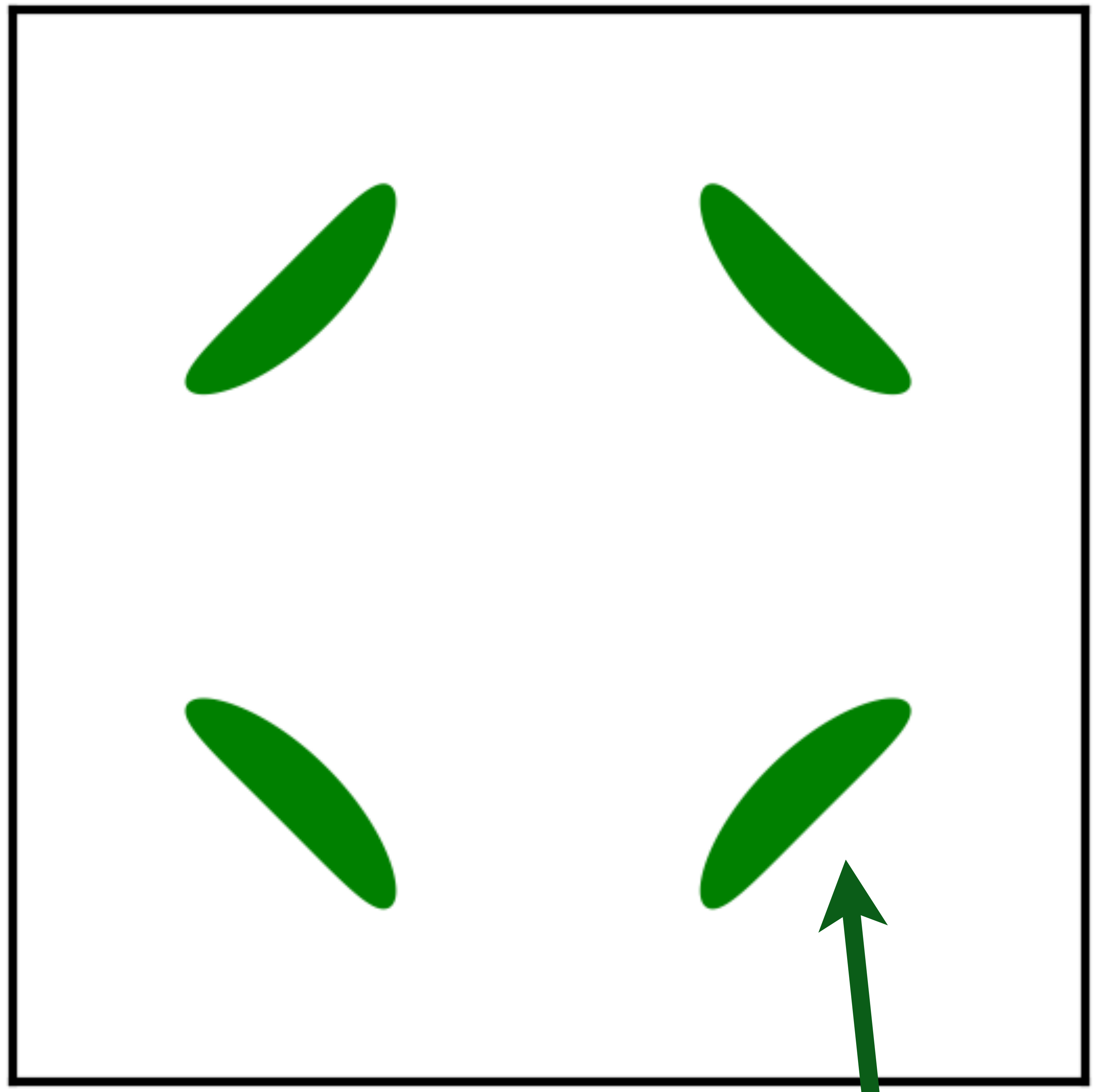

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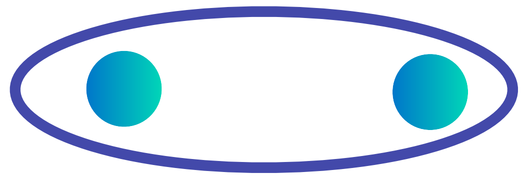
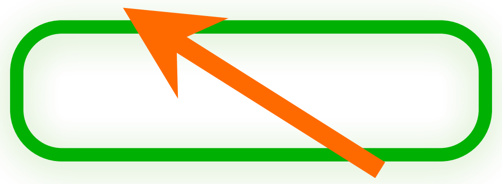
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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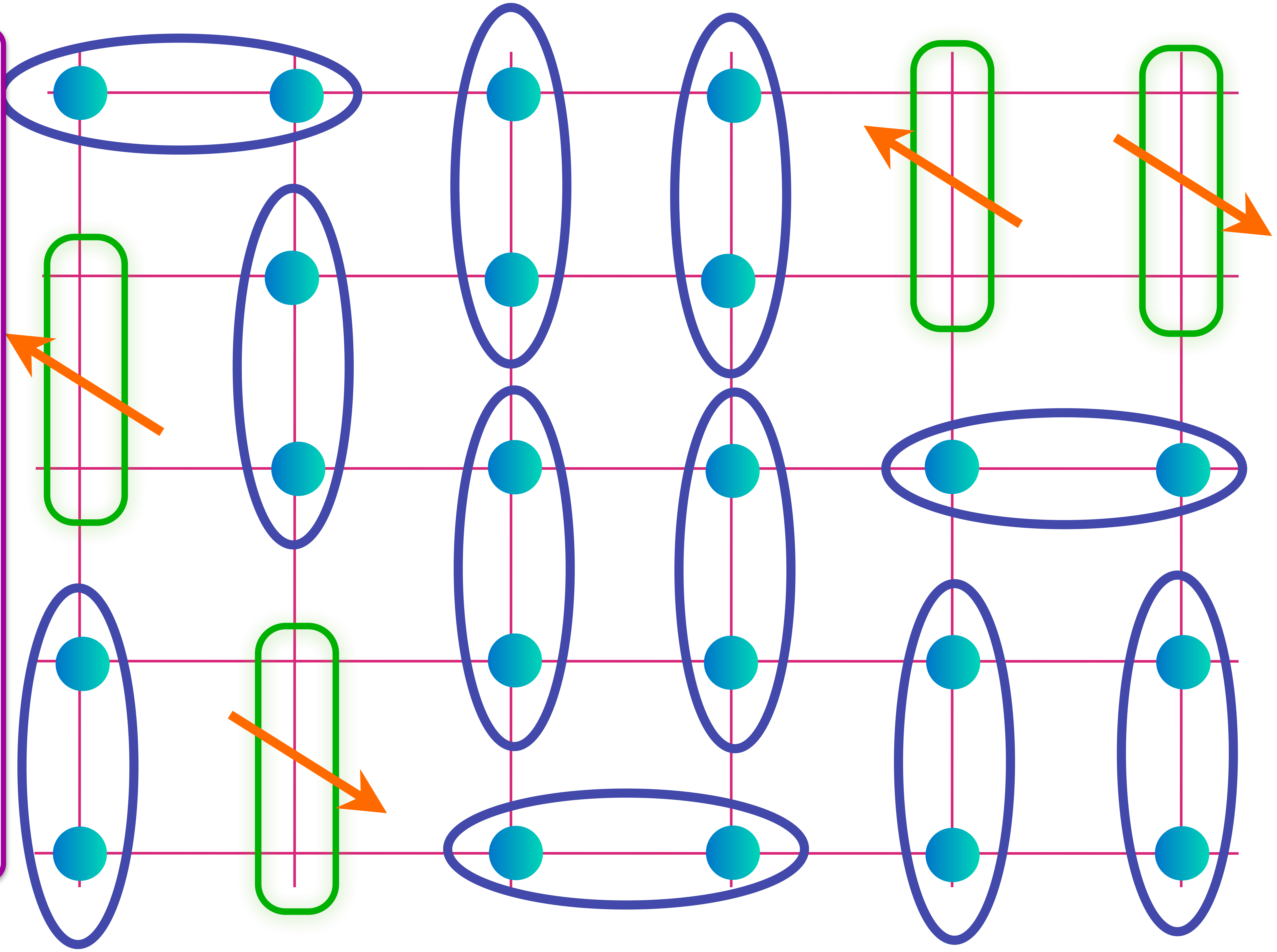

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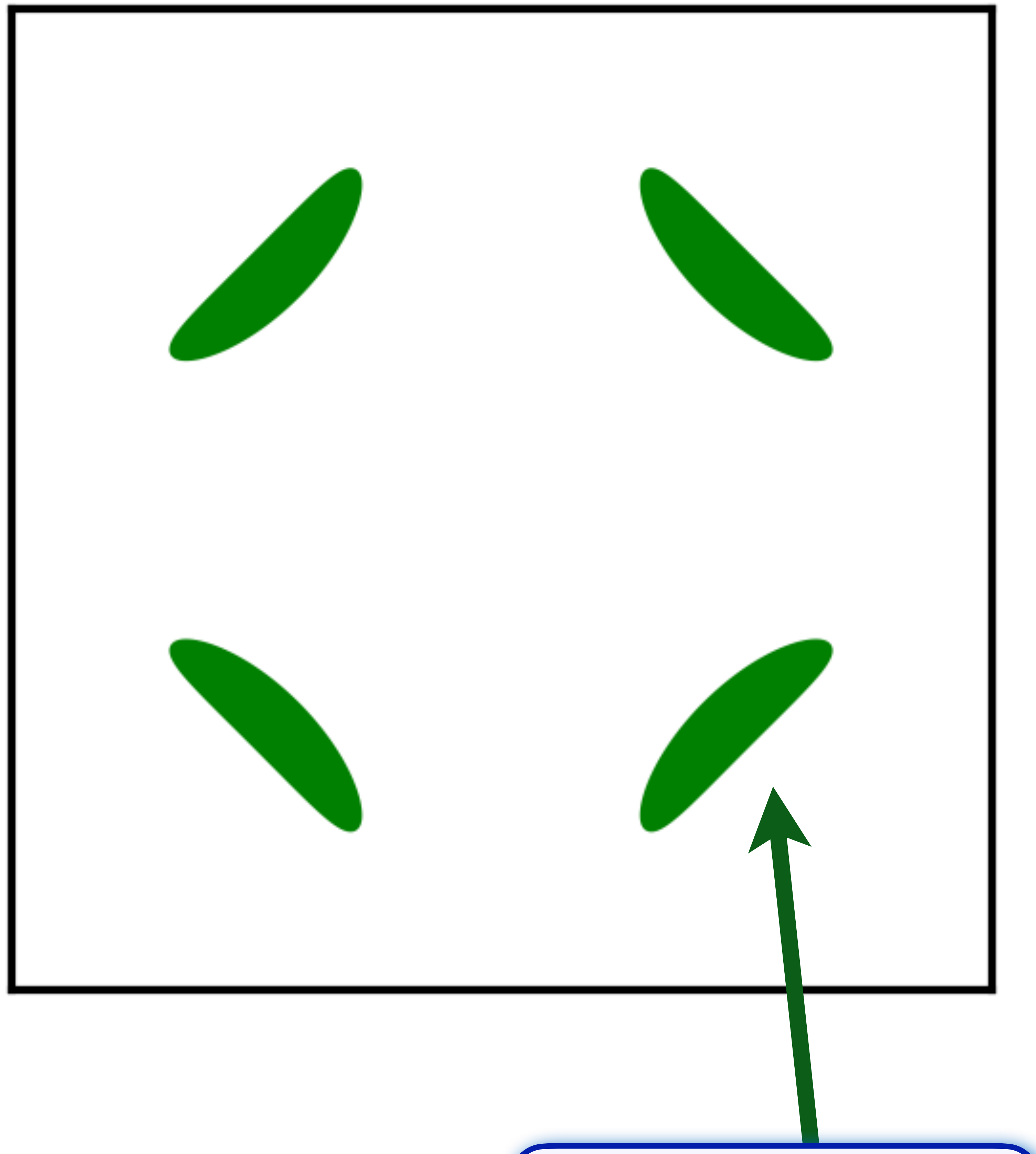

Area $p/8$

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $\quad$ $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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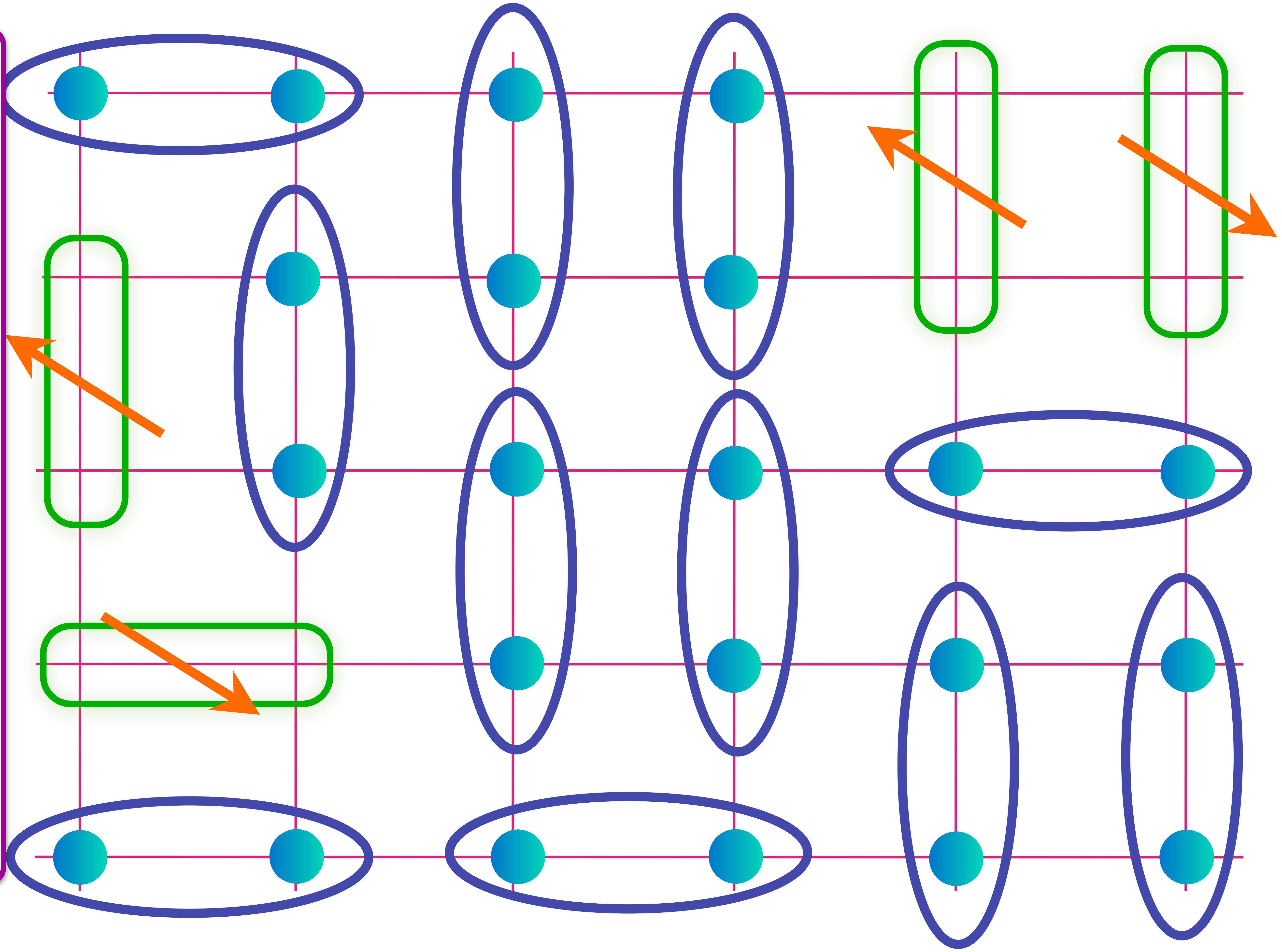

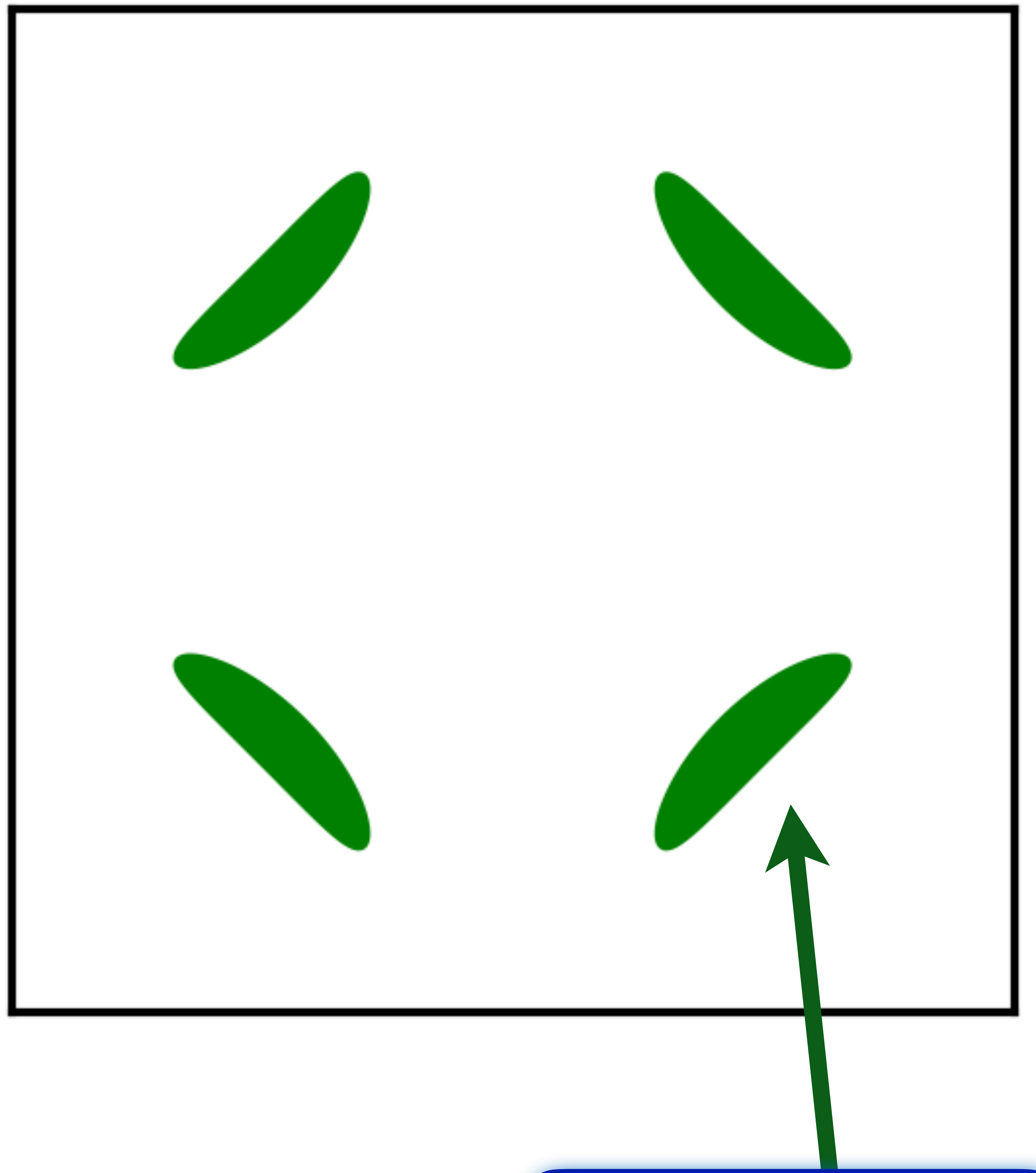

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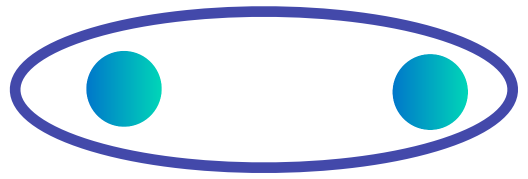
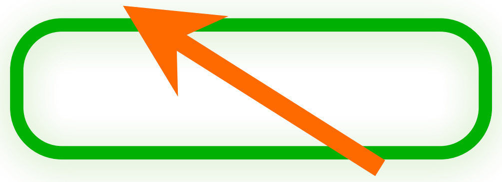
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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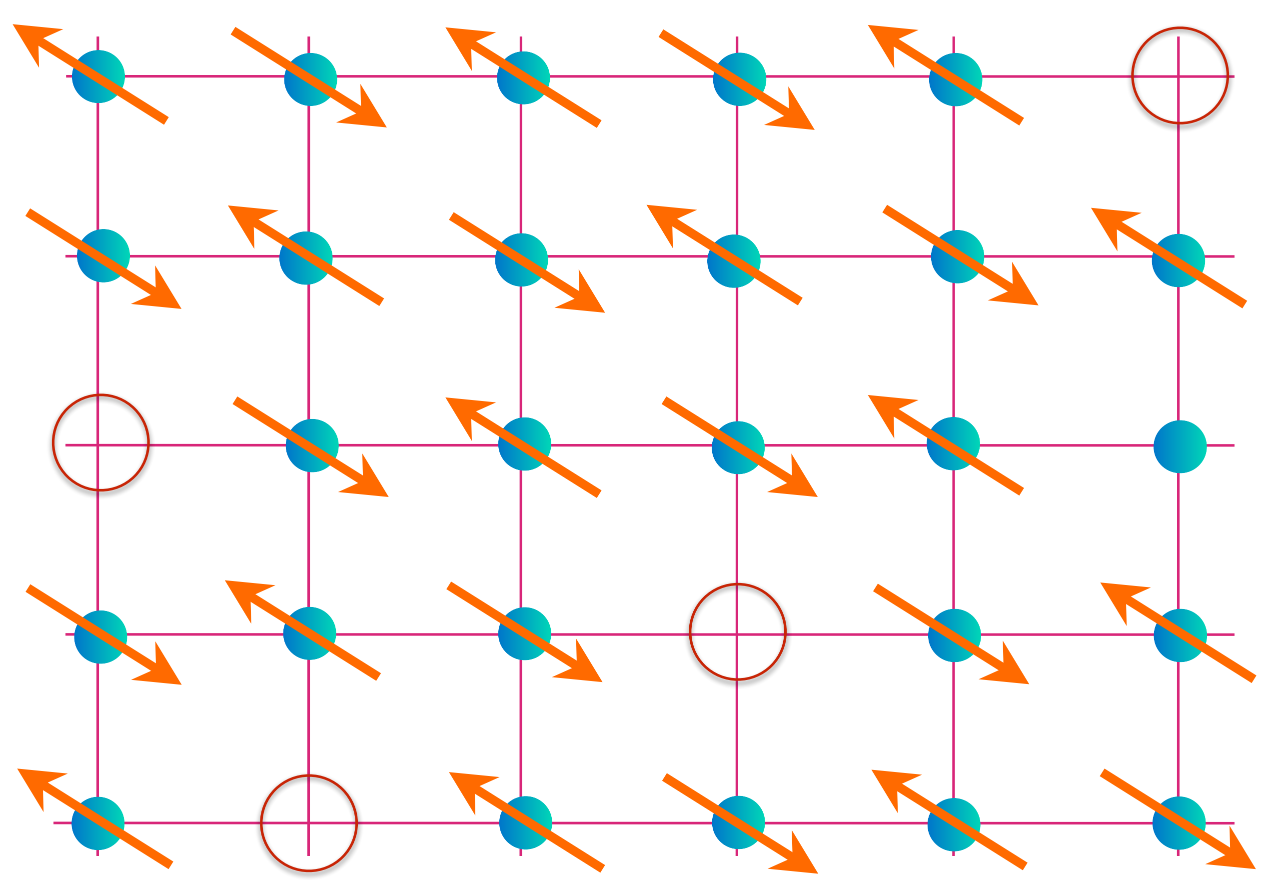
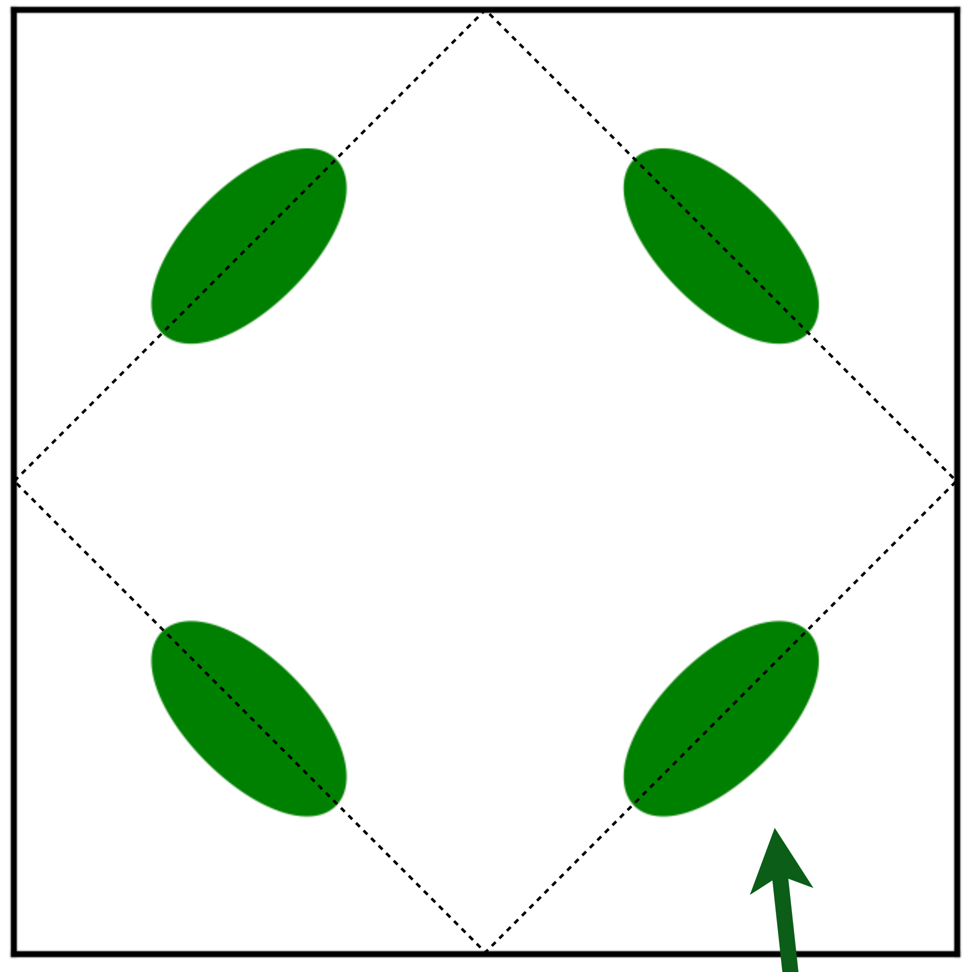
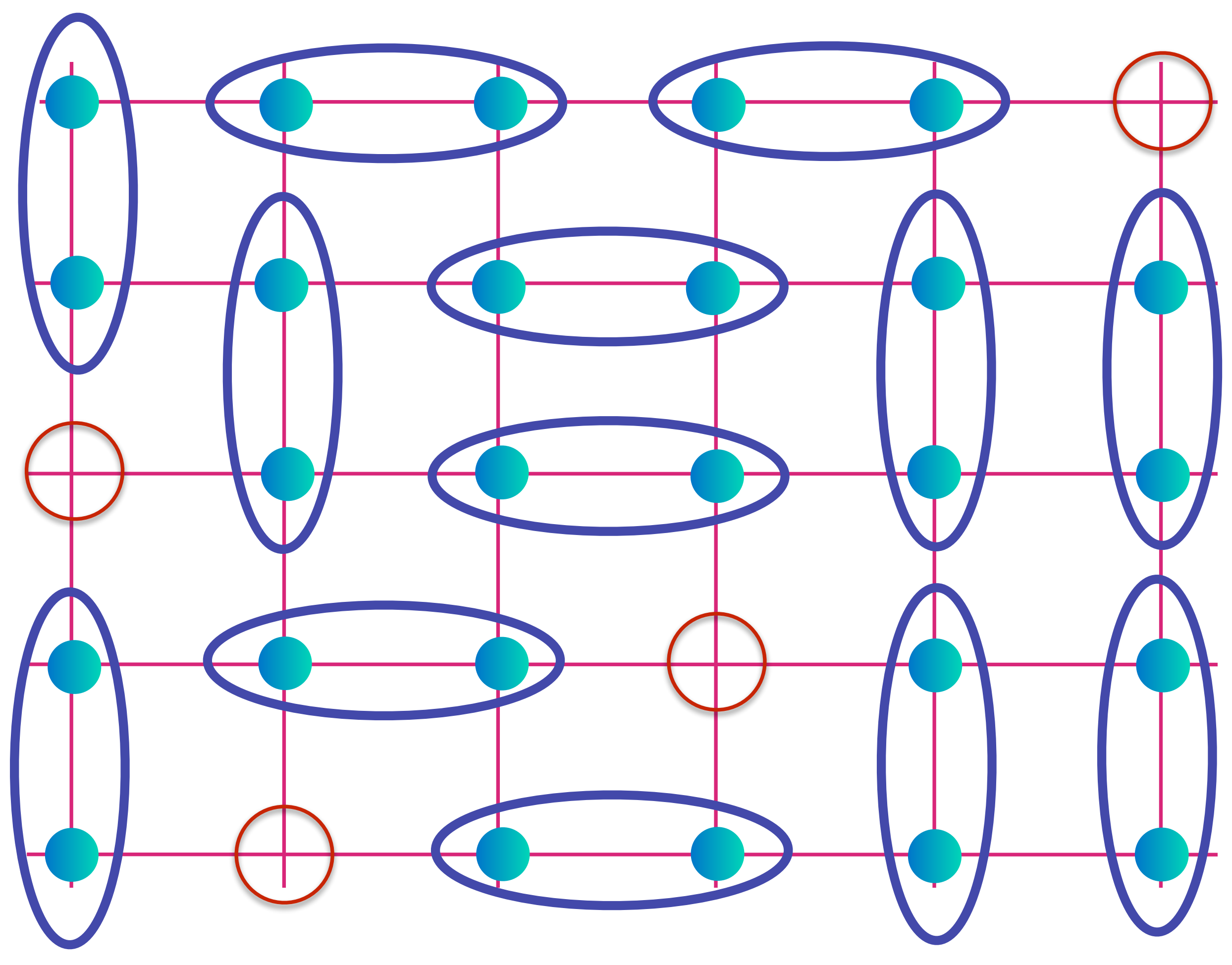


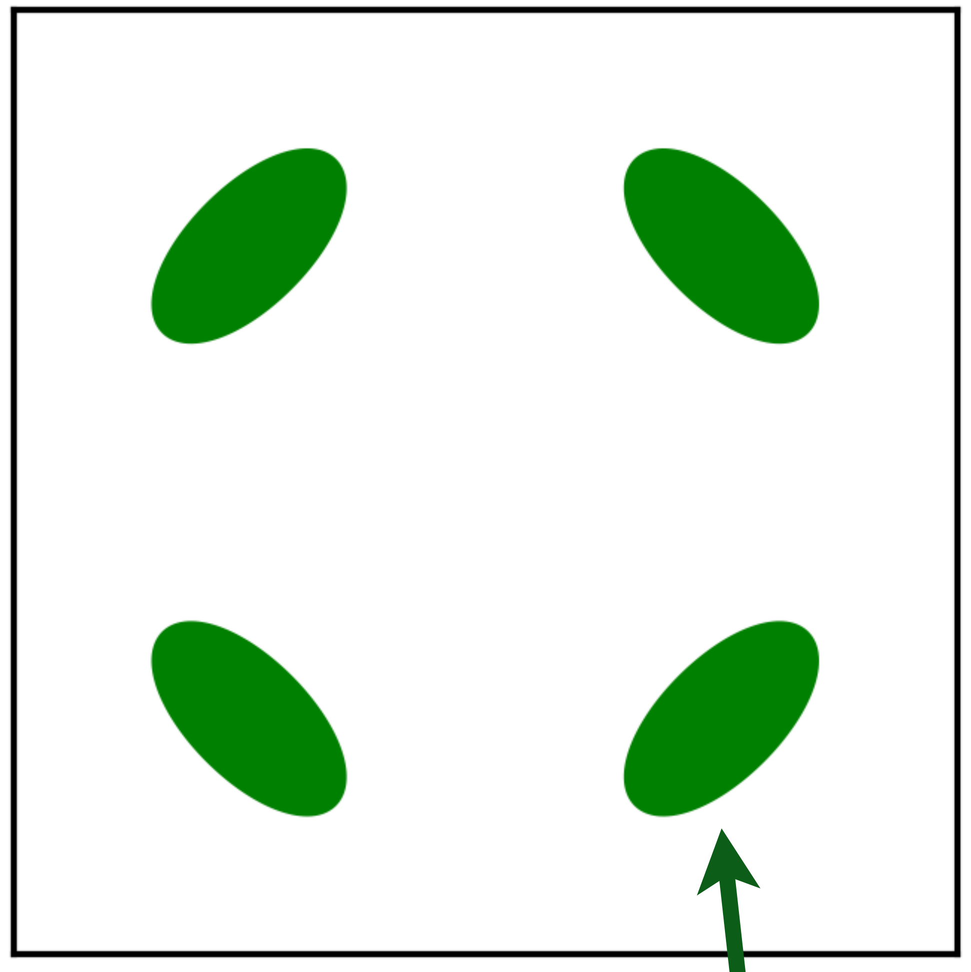


Area $p/4$

AF metal and SDW fluctuation



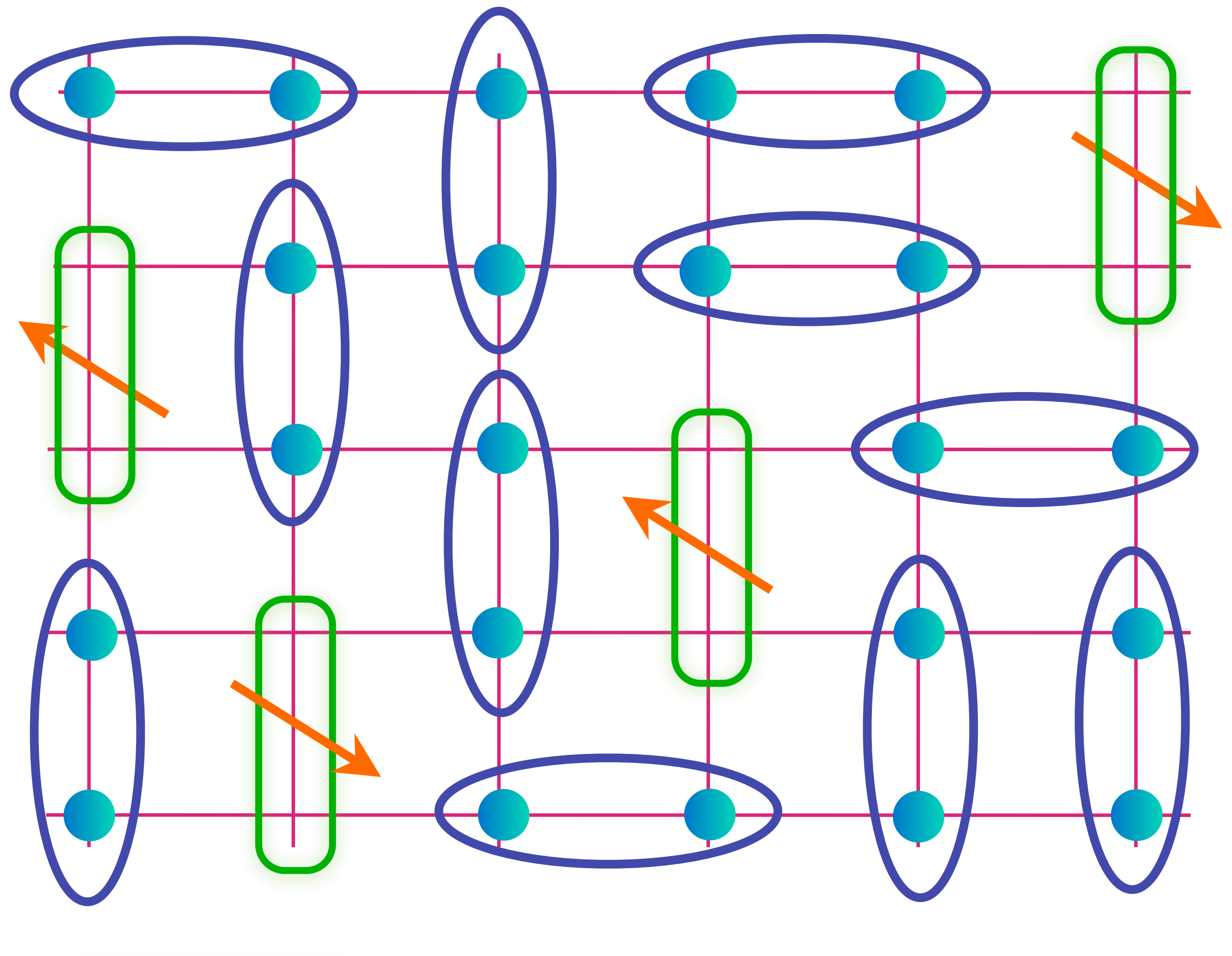
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

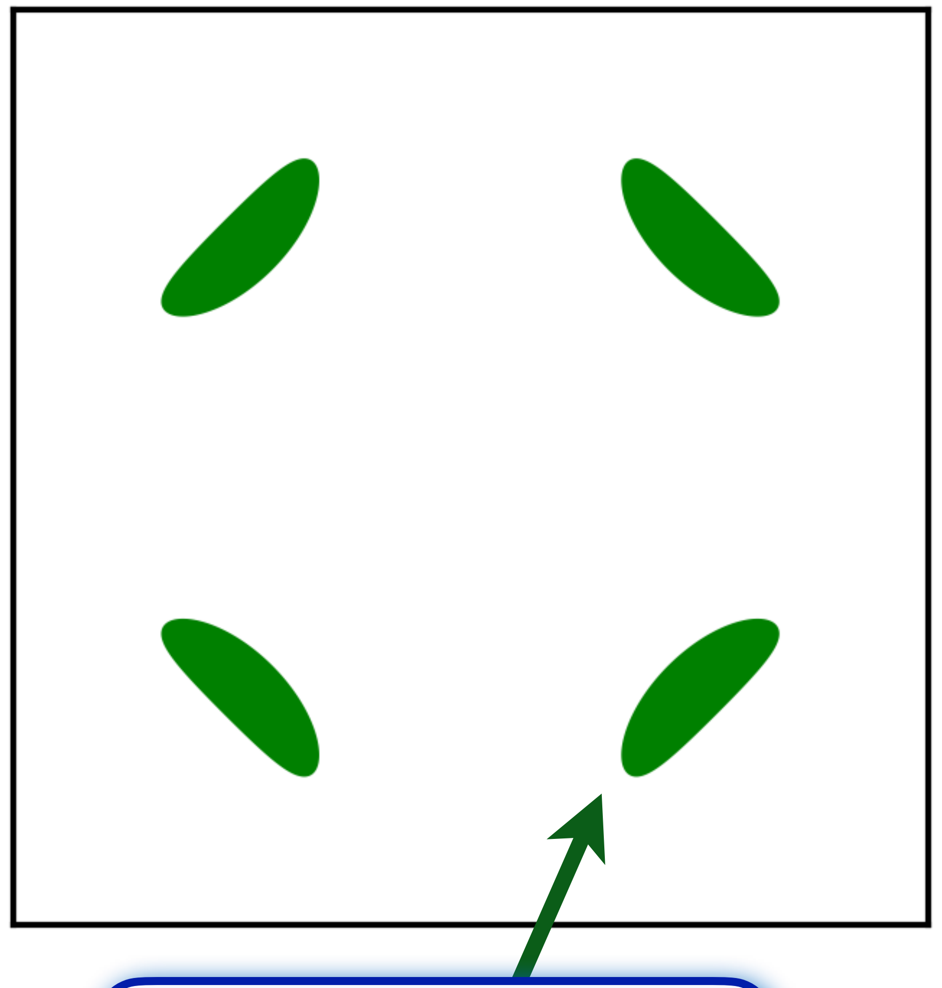
Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

FL*



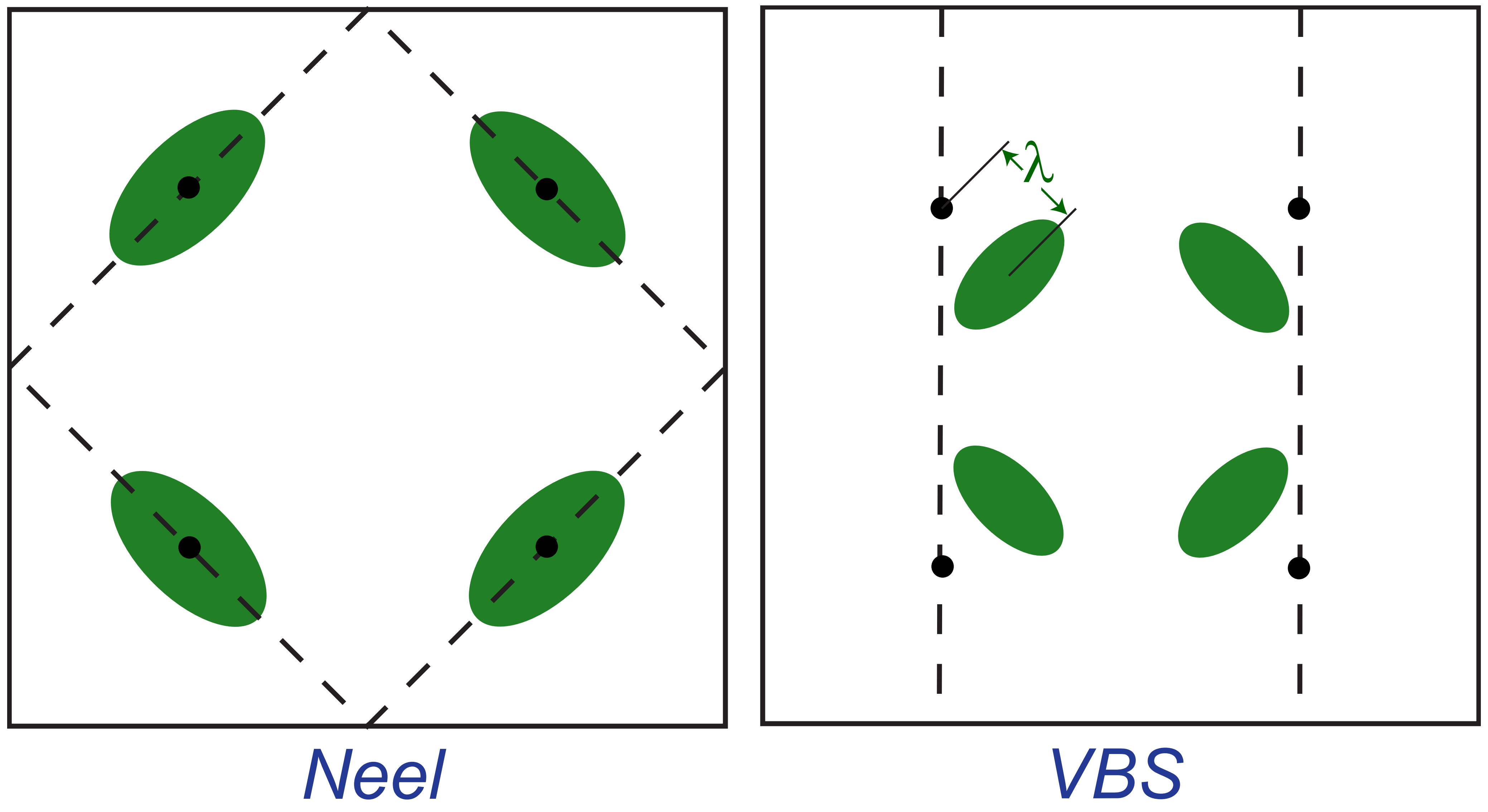
Area $p/8$

Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

Factor of 2 between SDW fluctuation and FL*

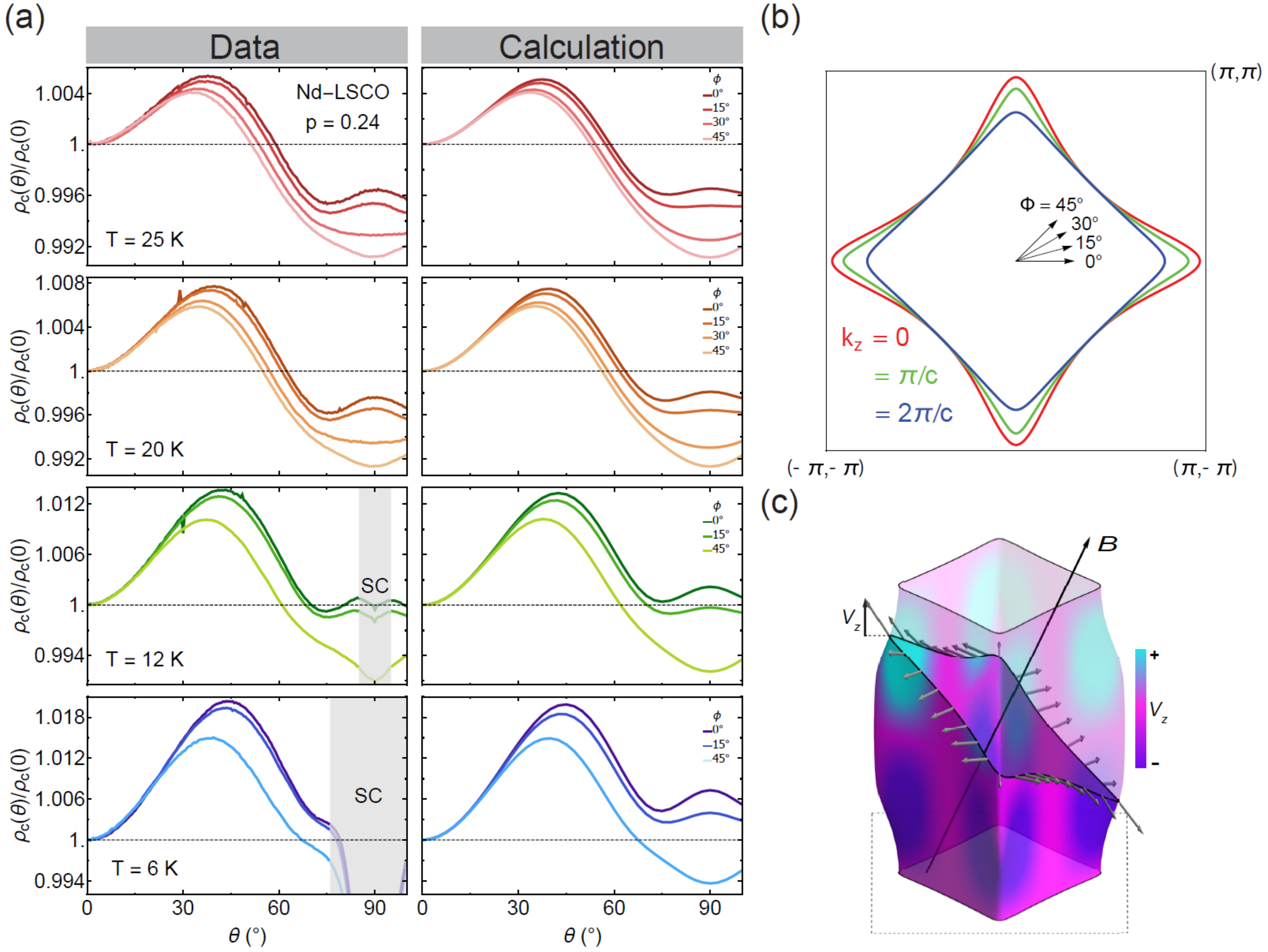
Observation of the Yamaji effect in a cuprate

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

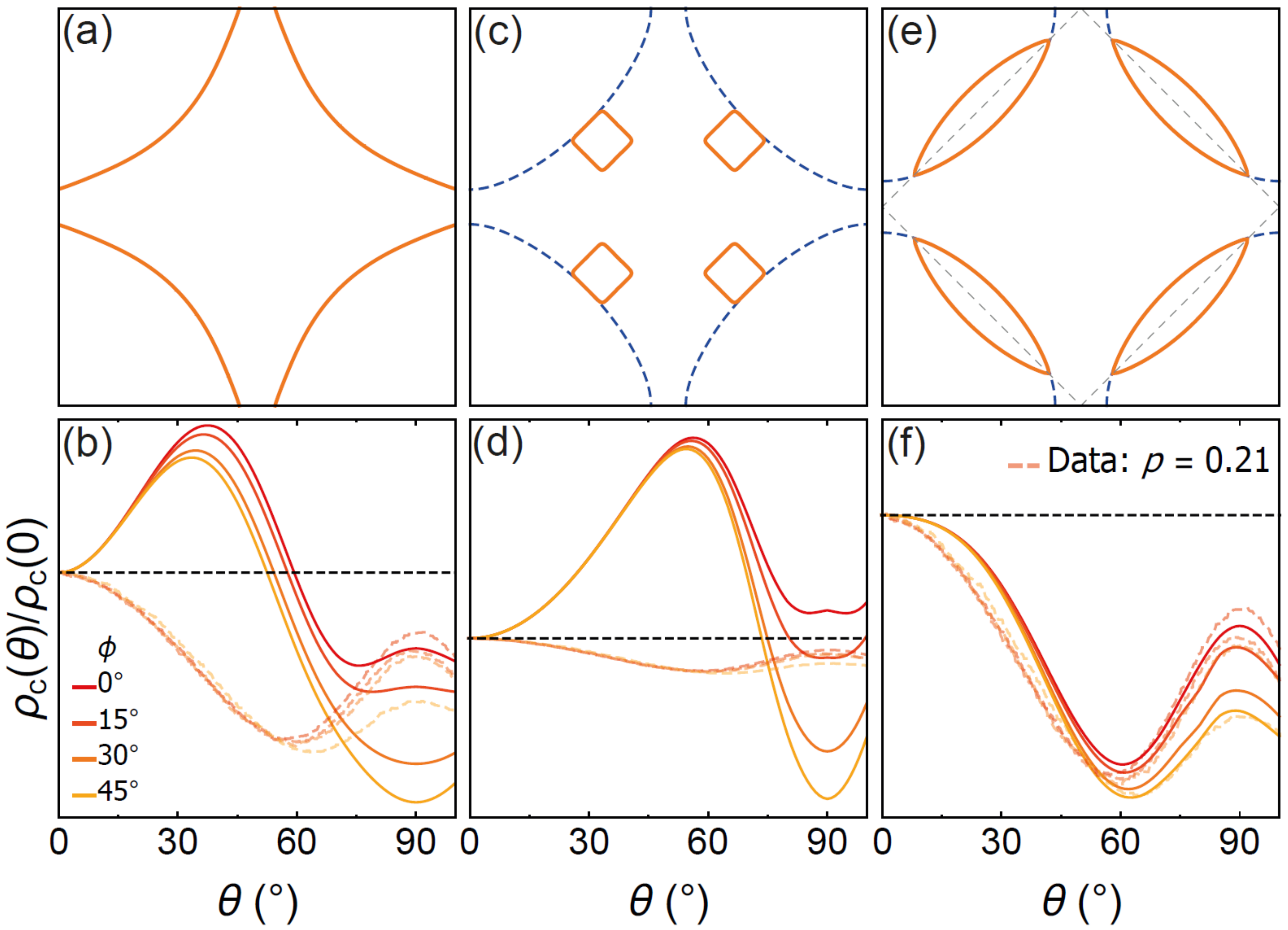
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei,
Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$



$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

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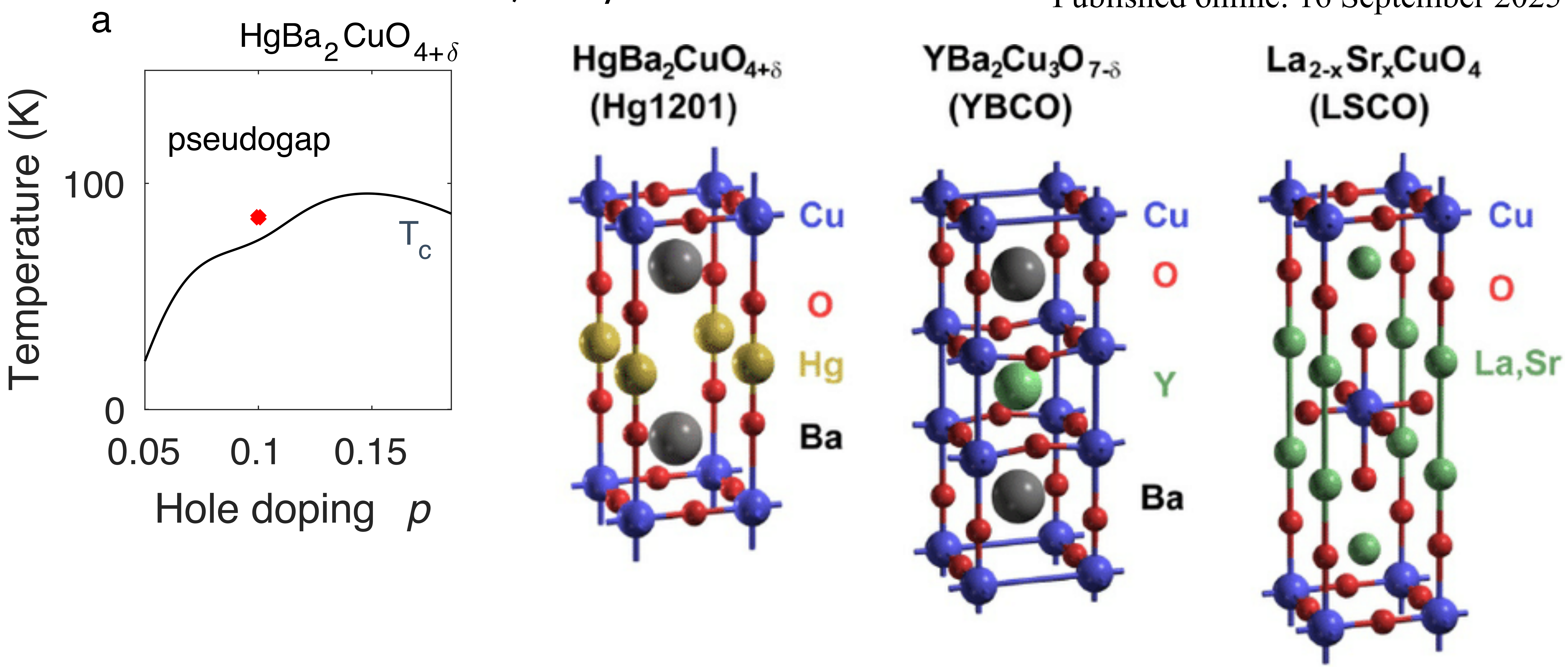
nature physics
21, 1753 (2025)
Published online: 16 September 2025

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]



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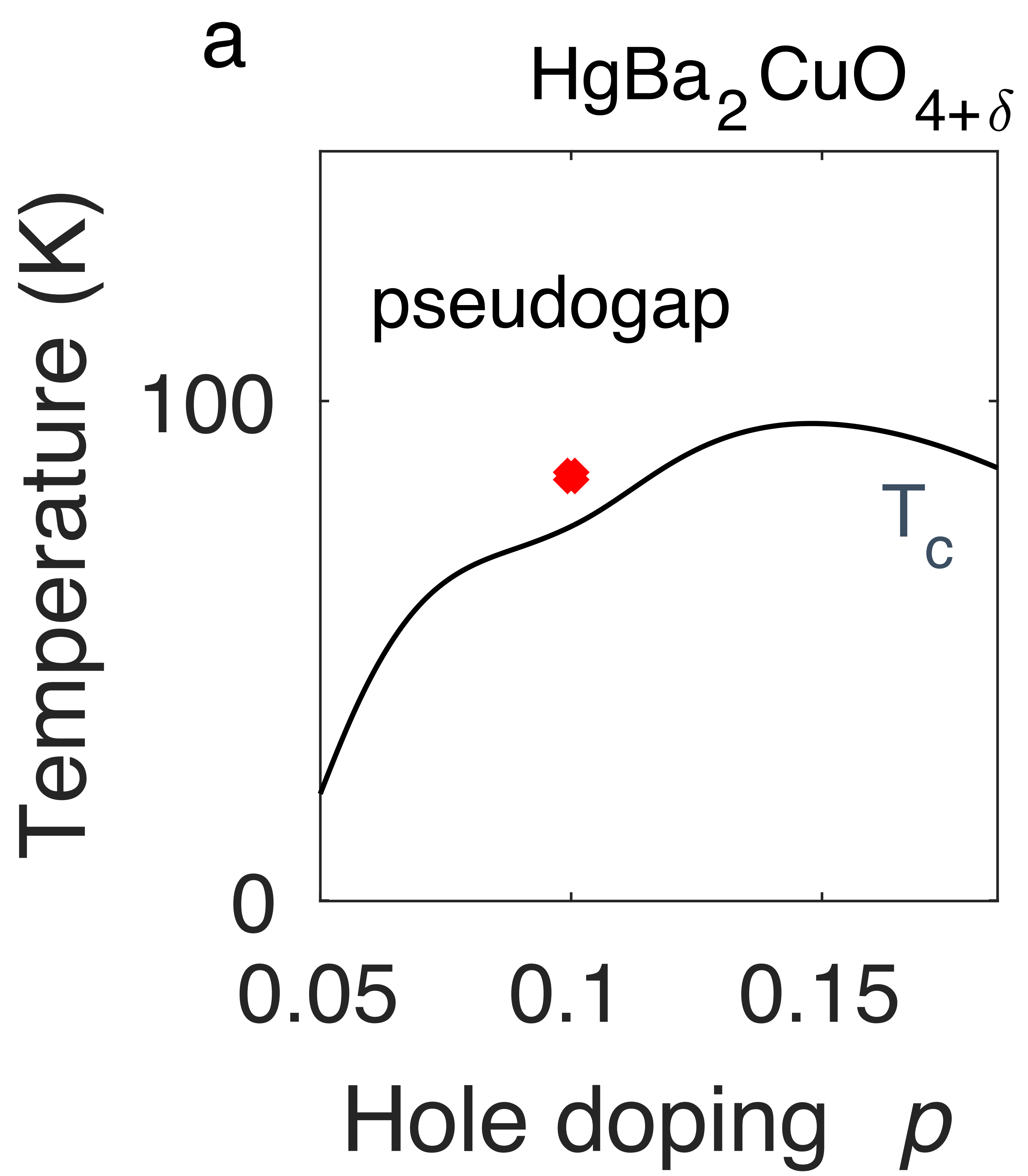

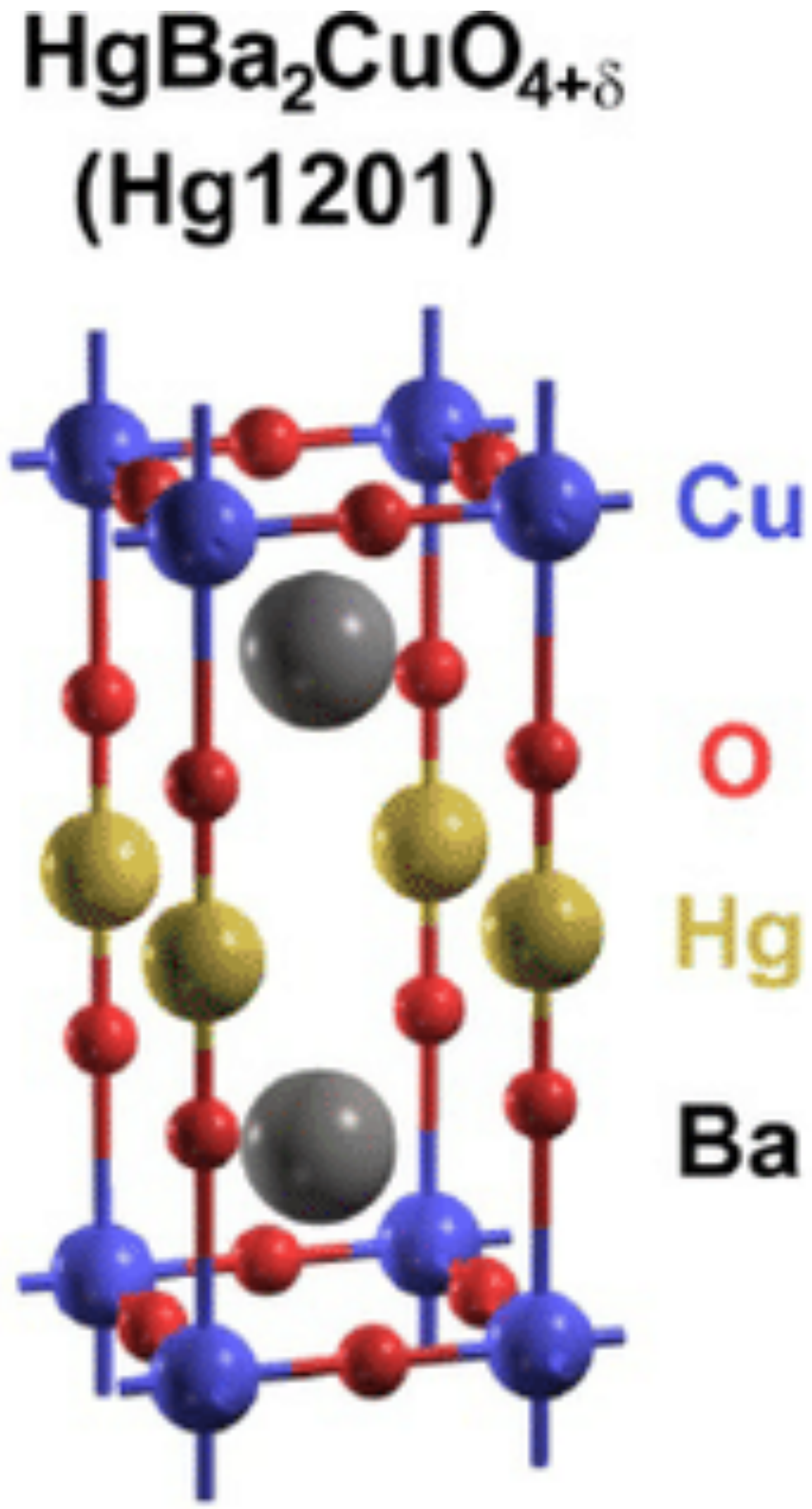

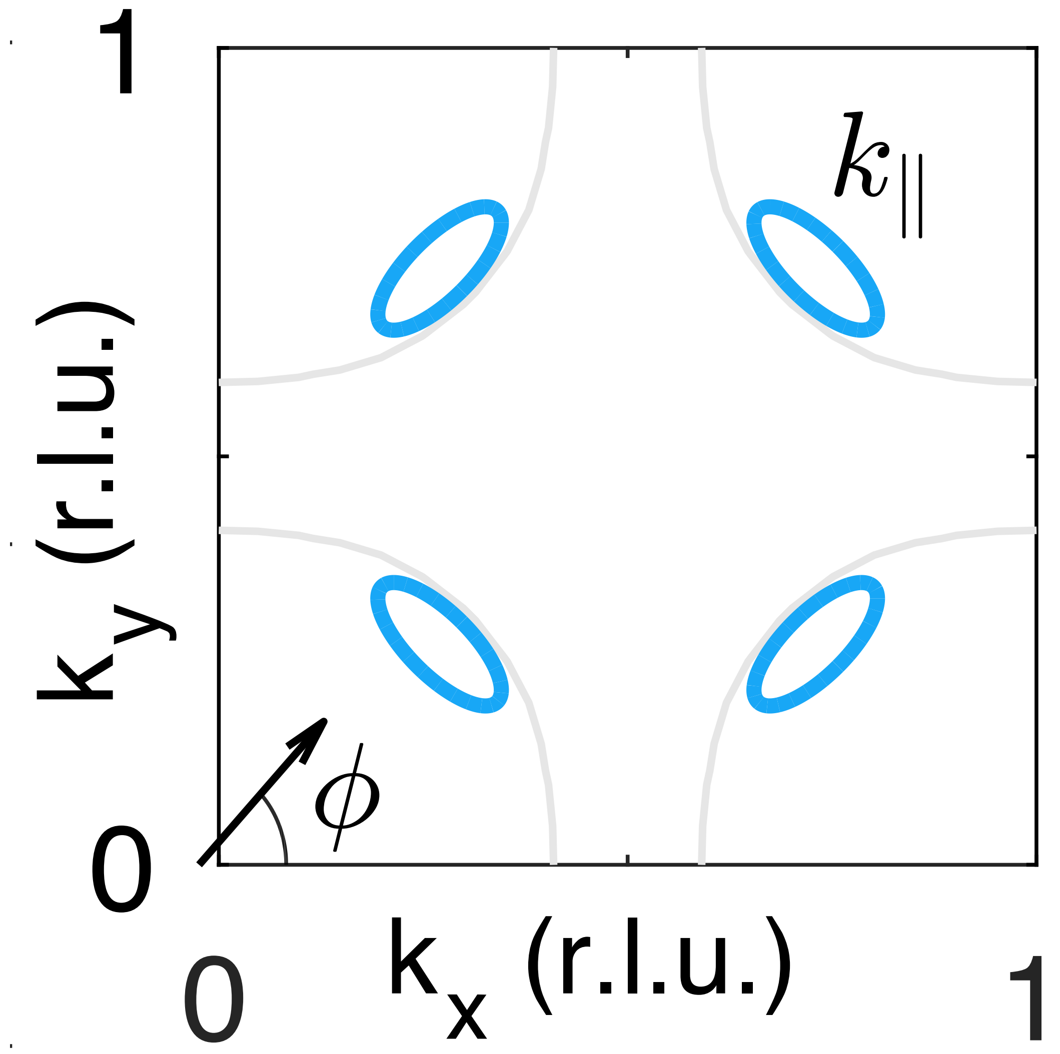

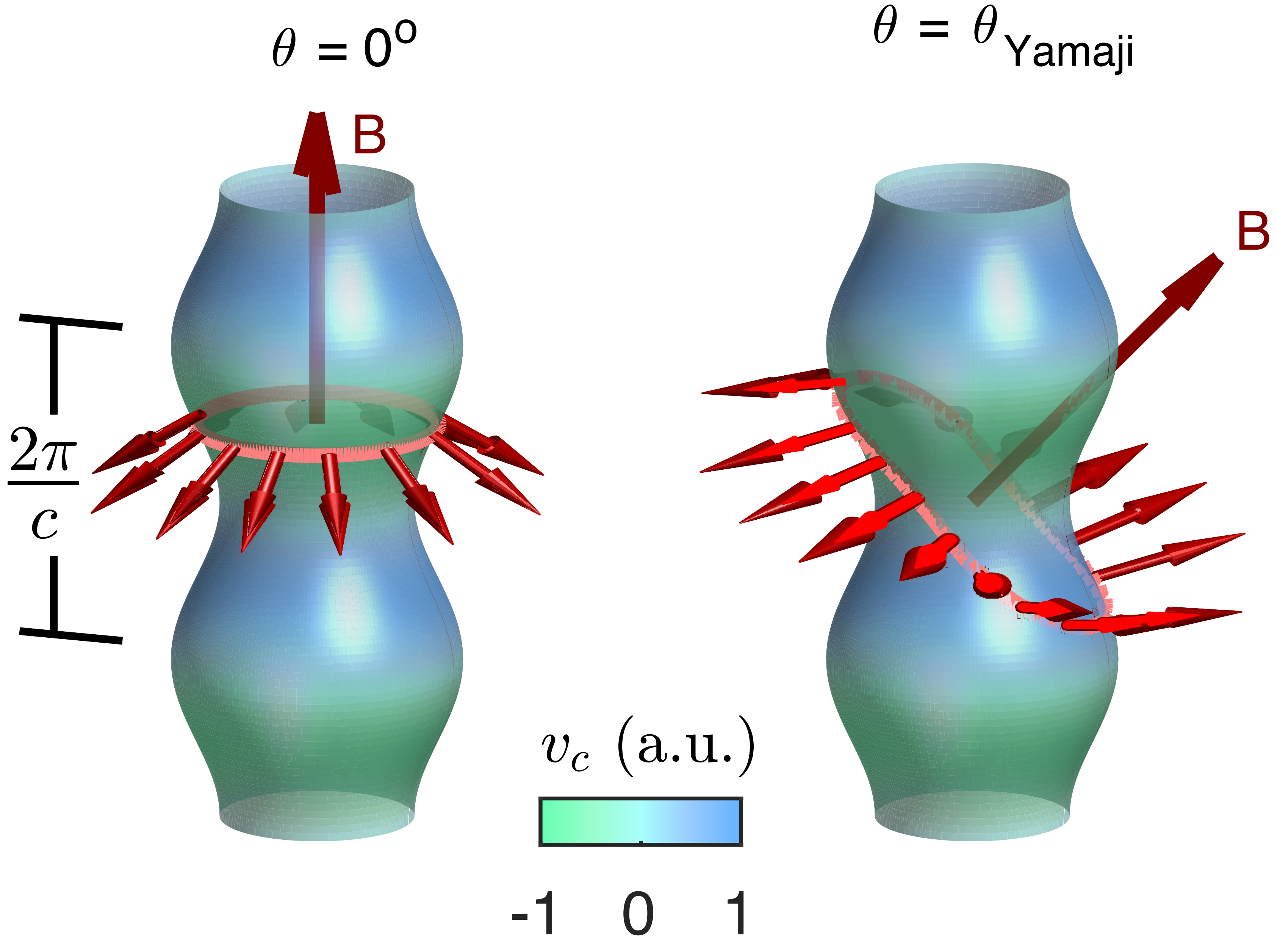

At the Yamaji angle, the orbits in the plane orthogonal to ***B*** have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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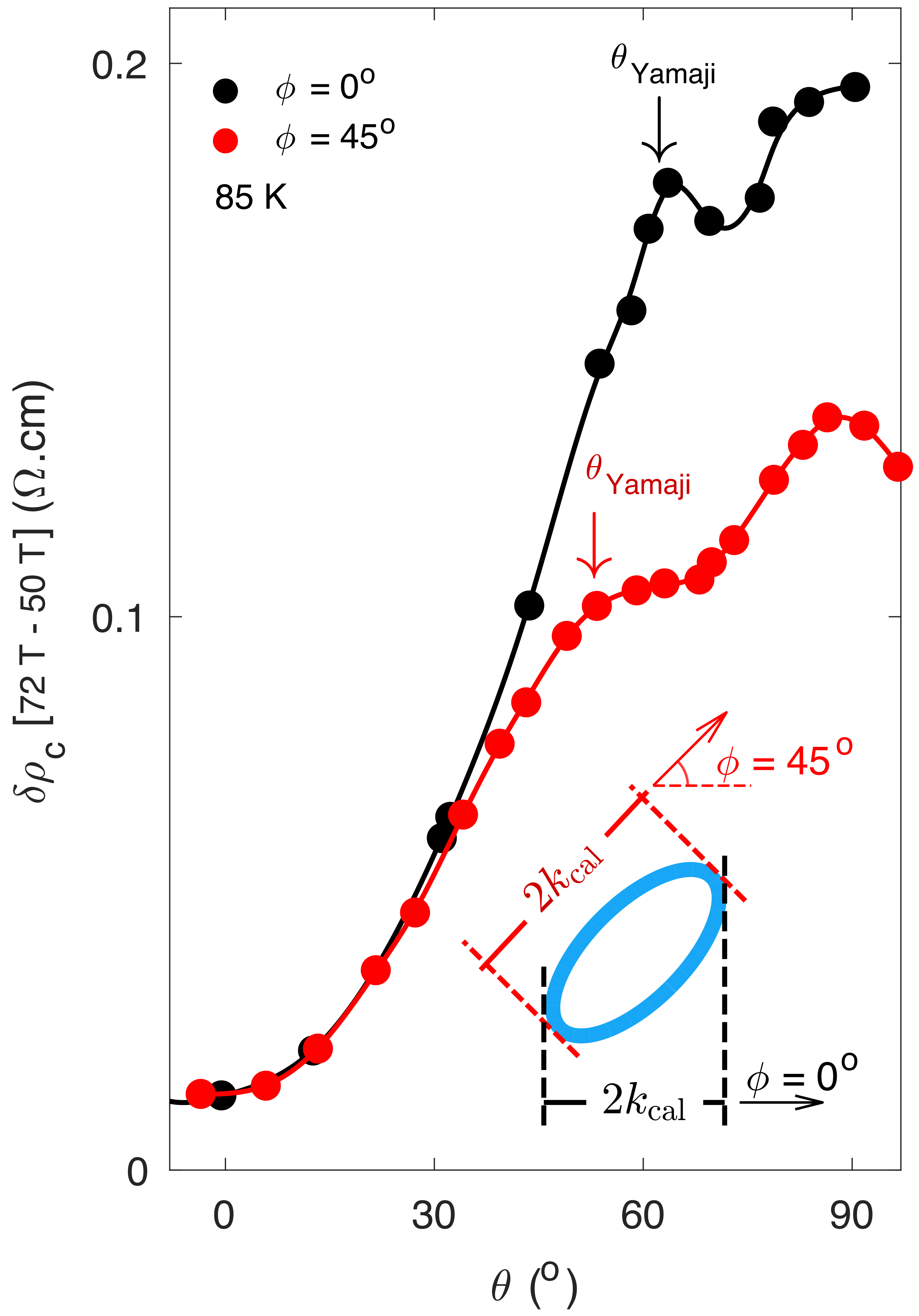

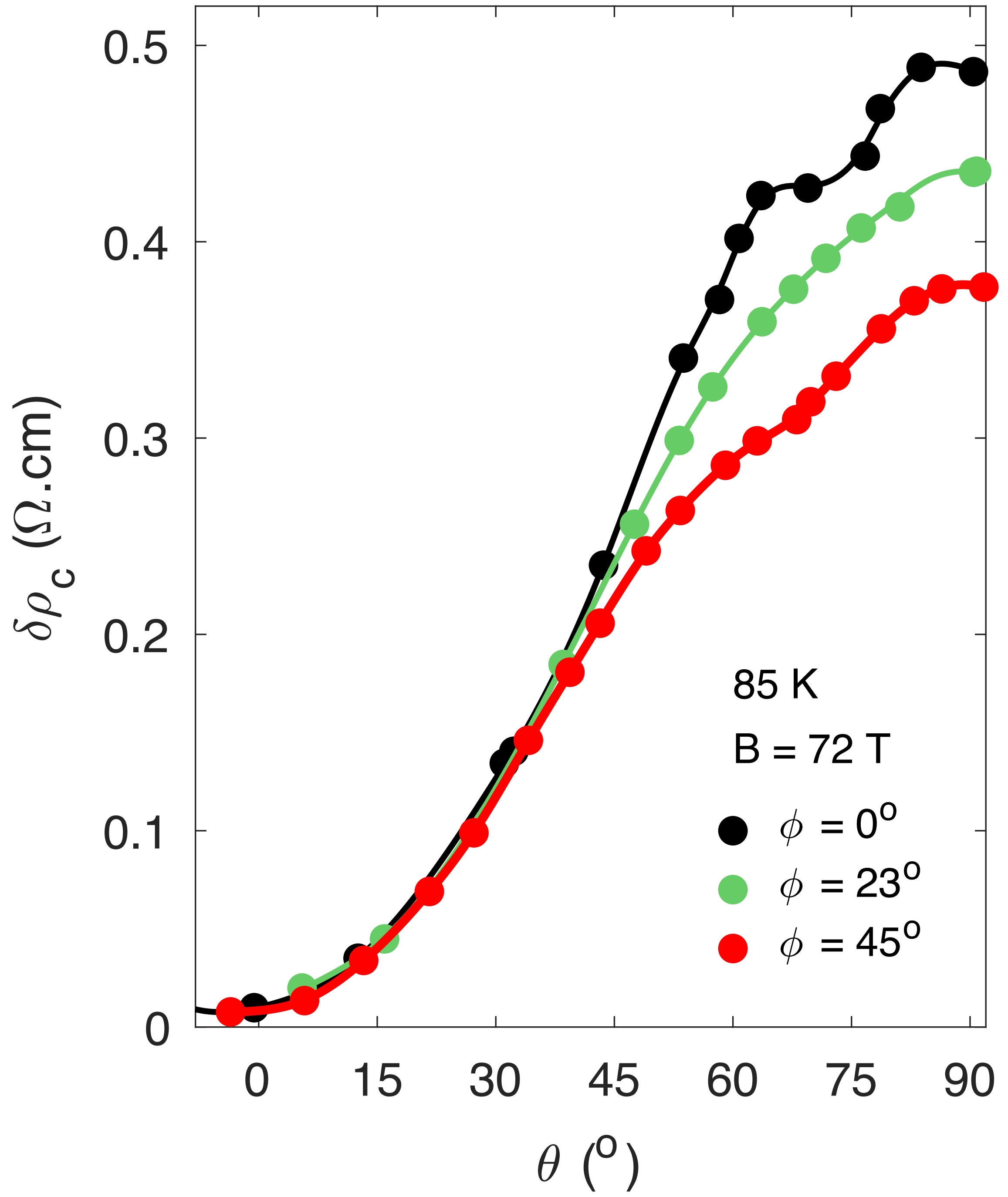

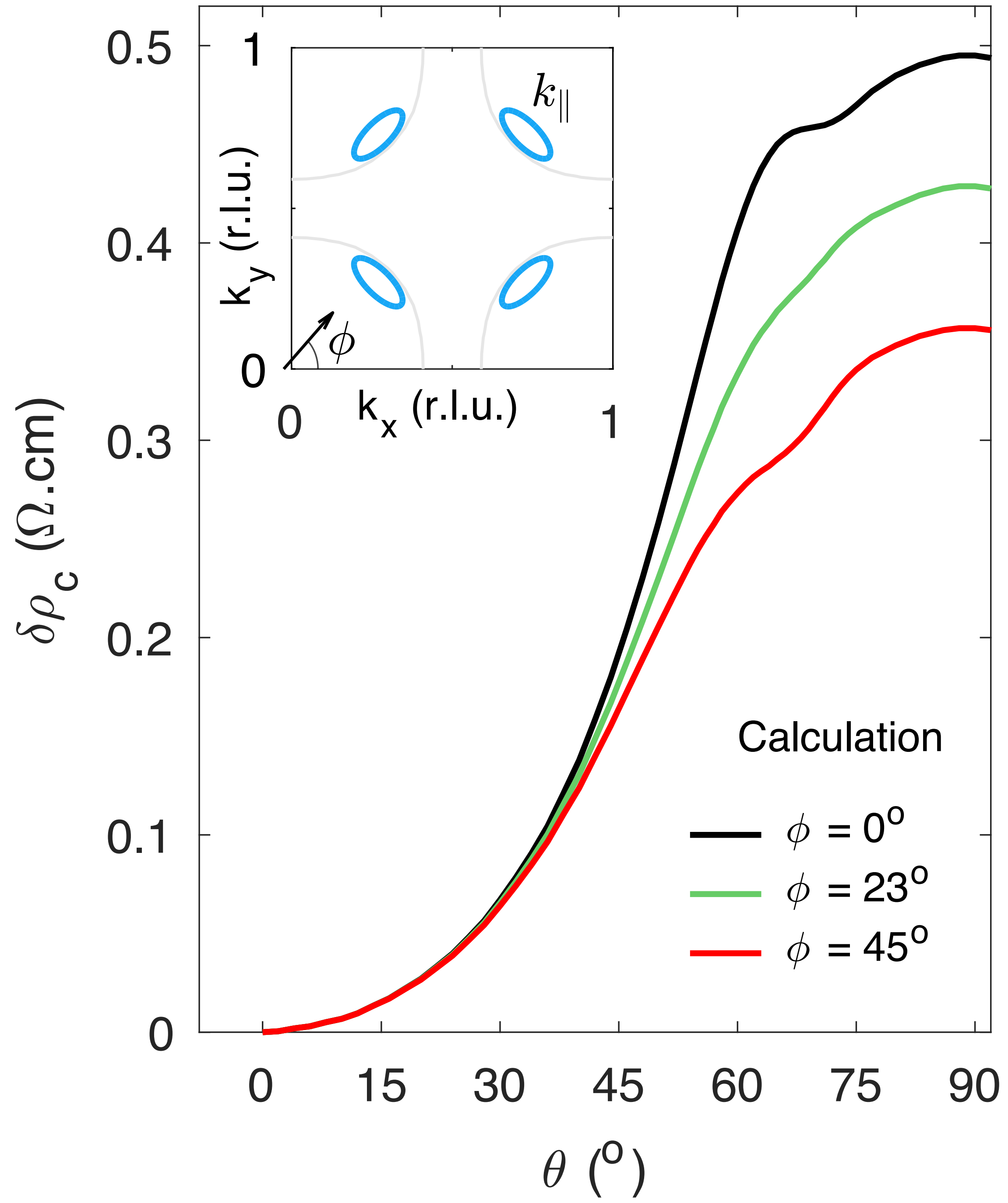

Doping
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The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

Observation of the Yamaji effect in a cuprate superconductor

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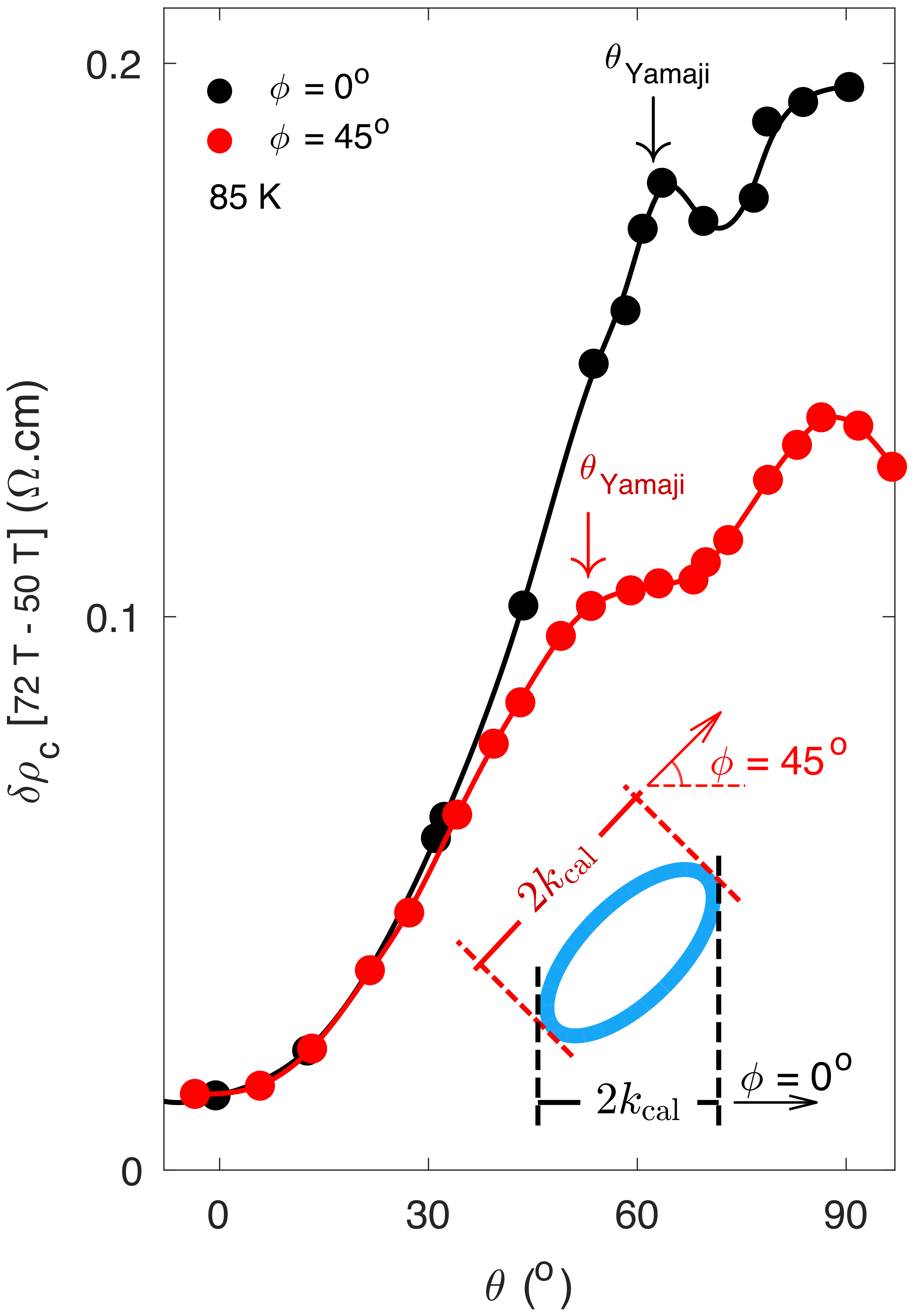

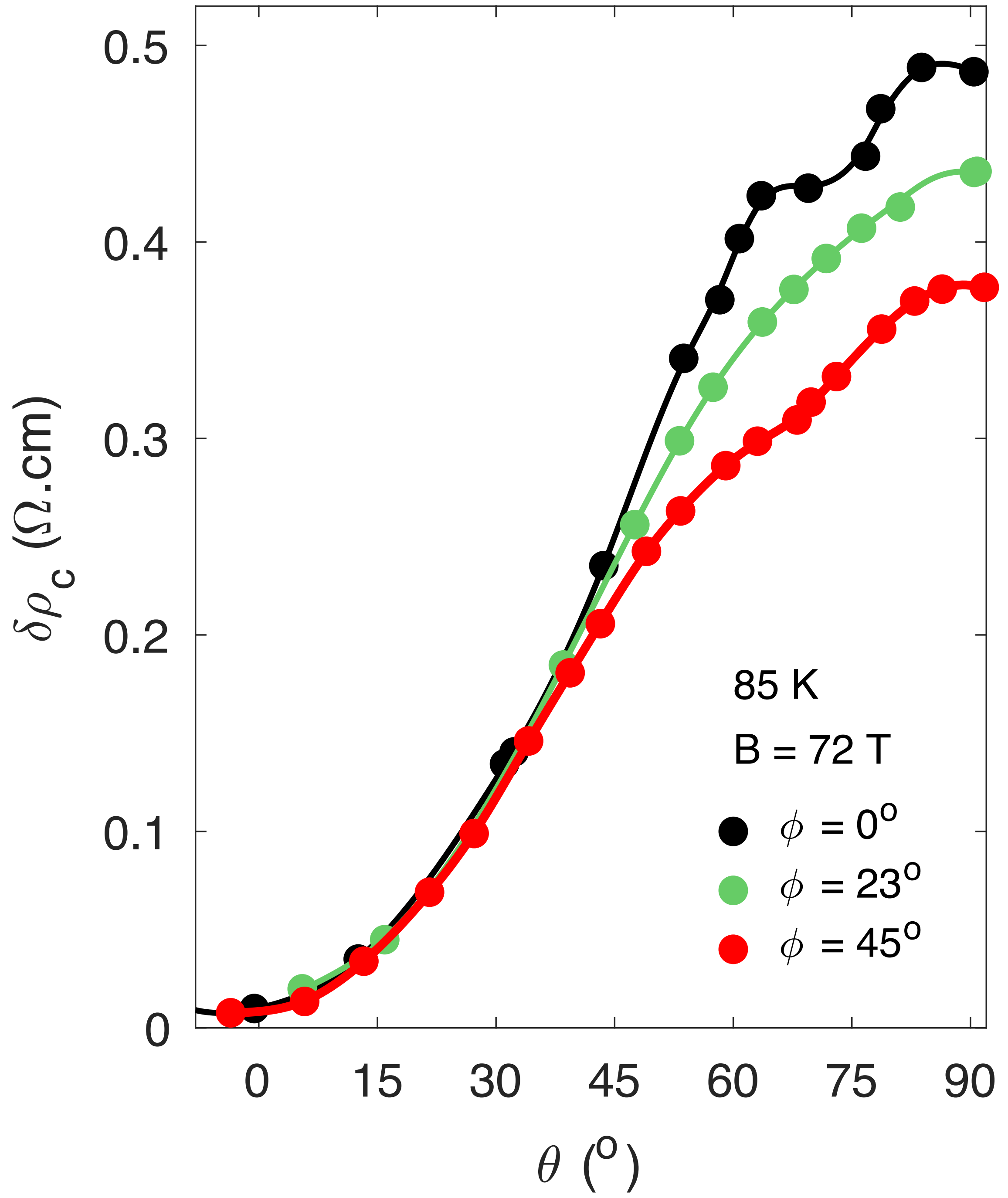

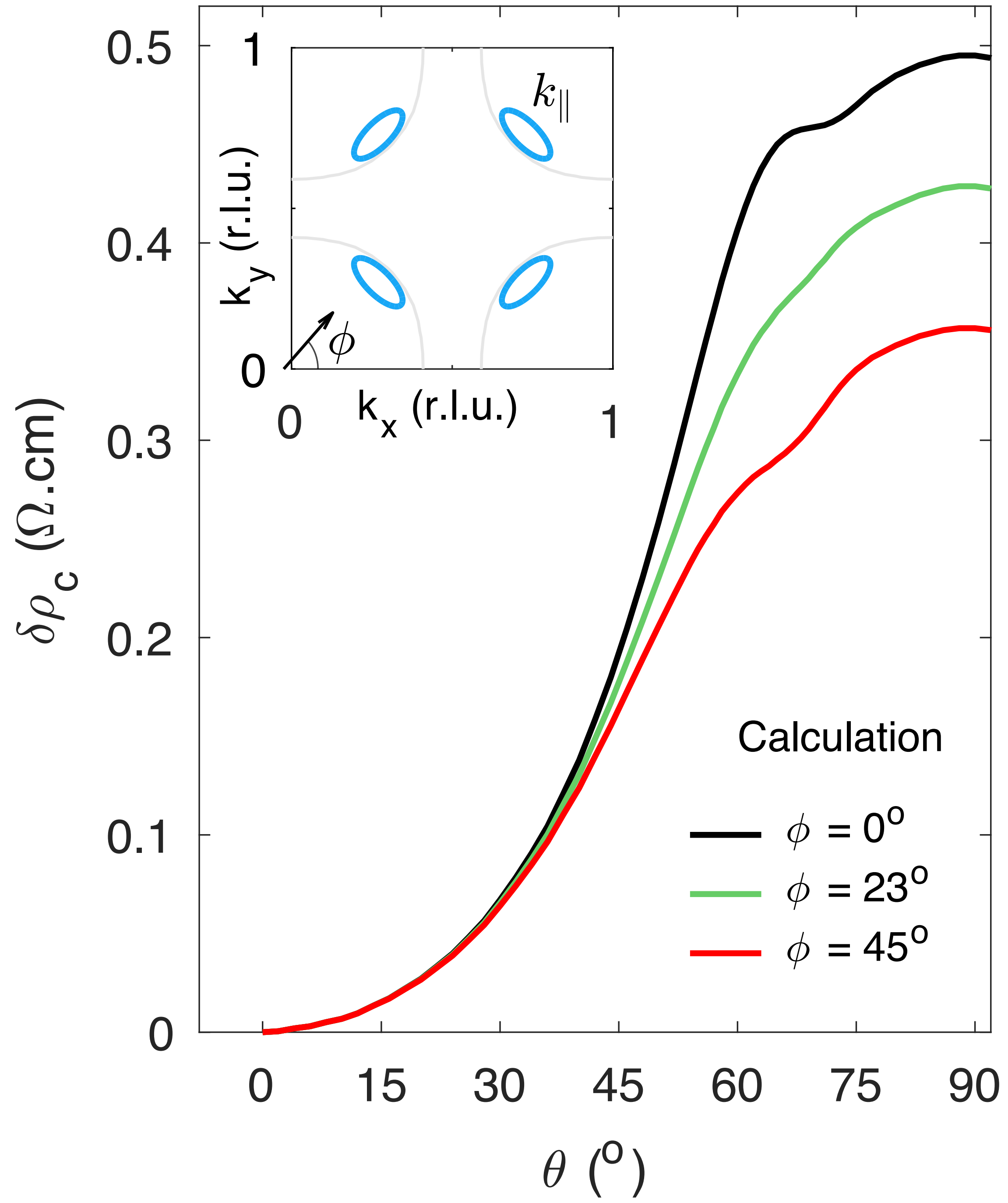

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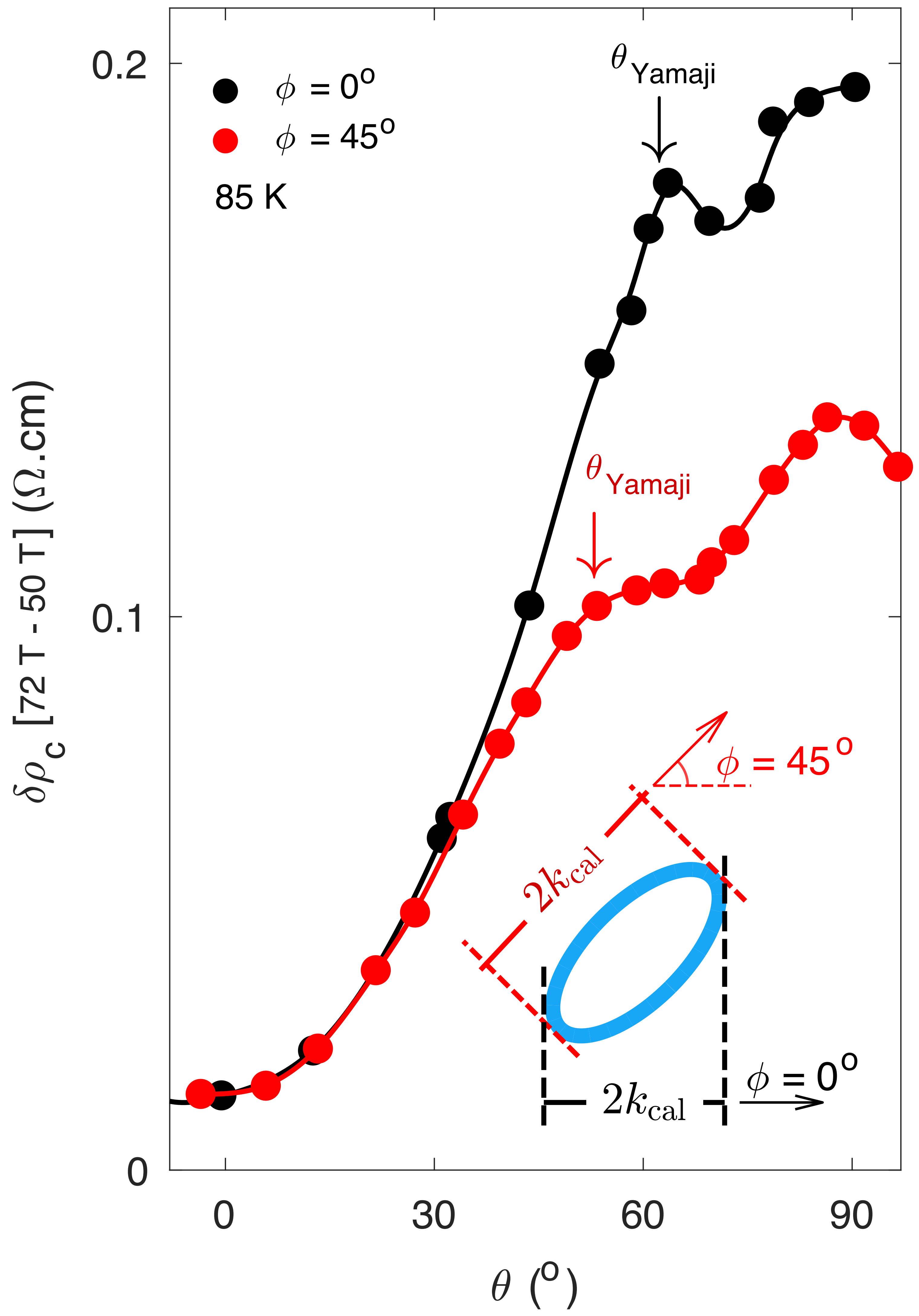

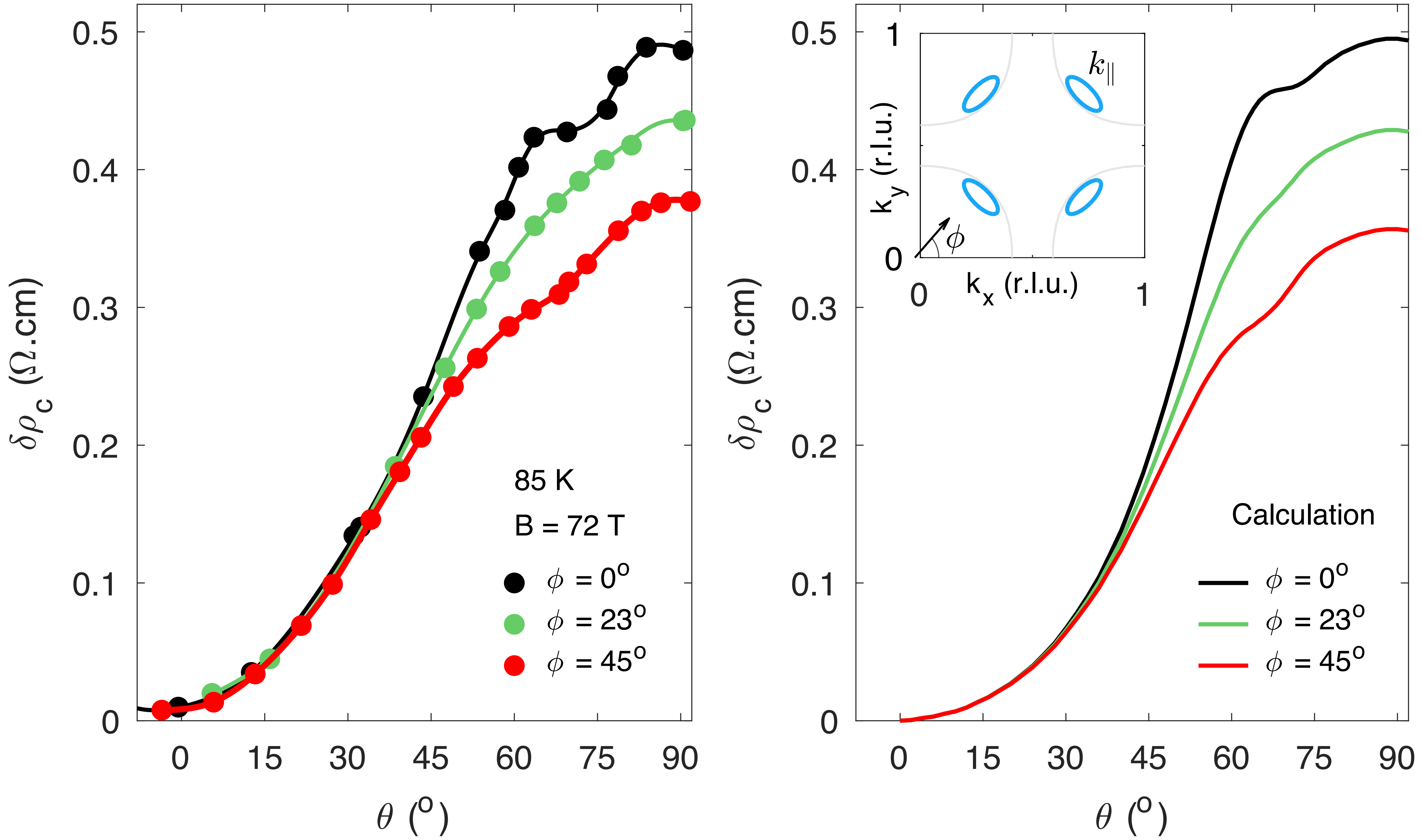

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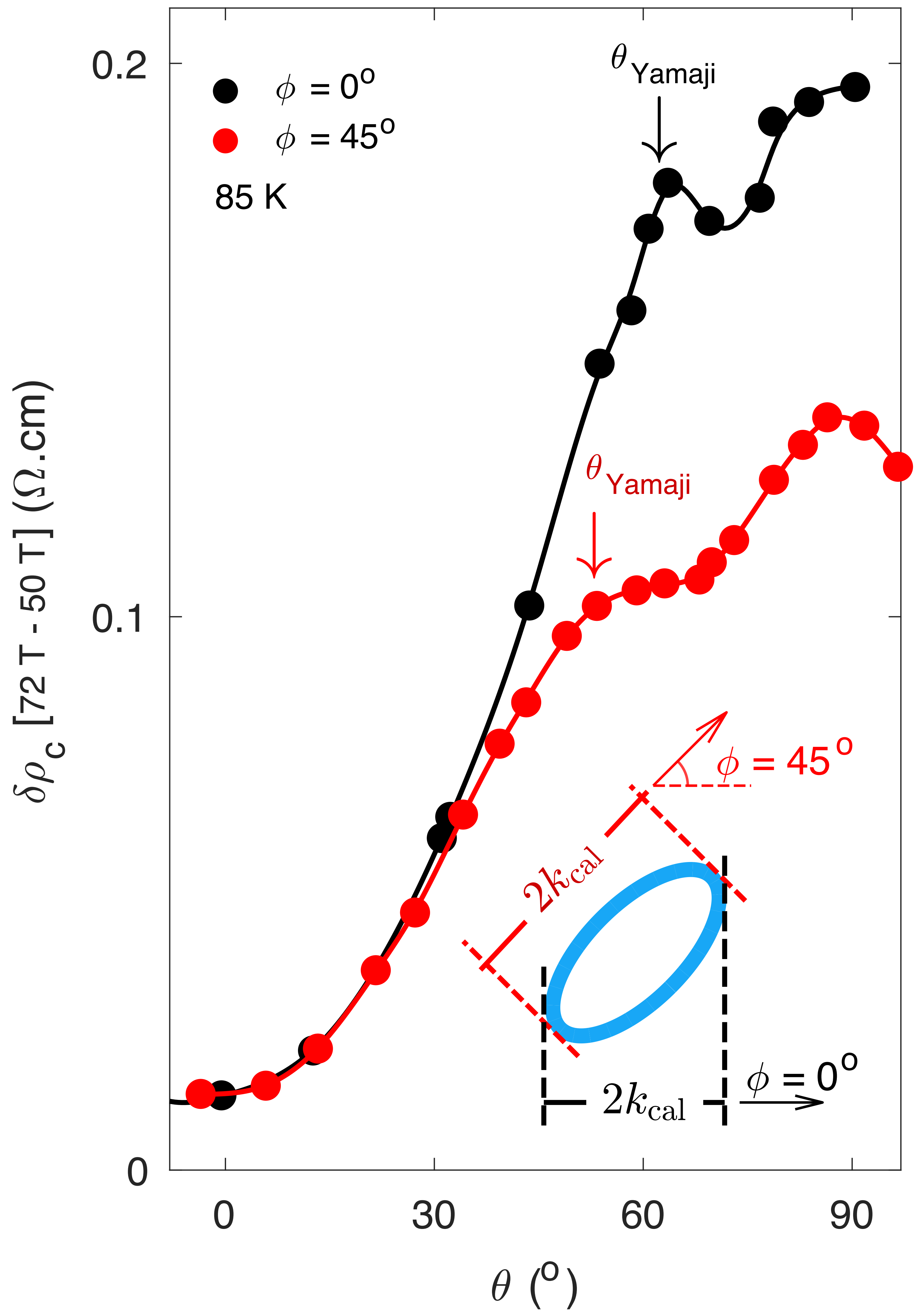

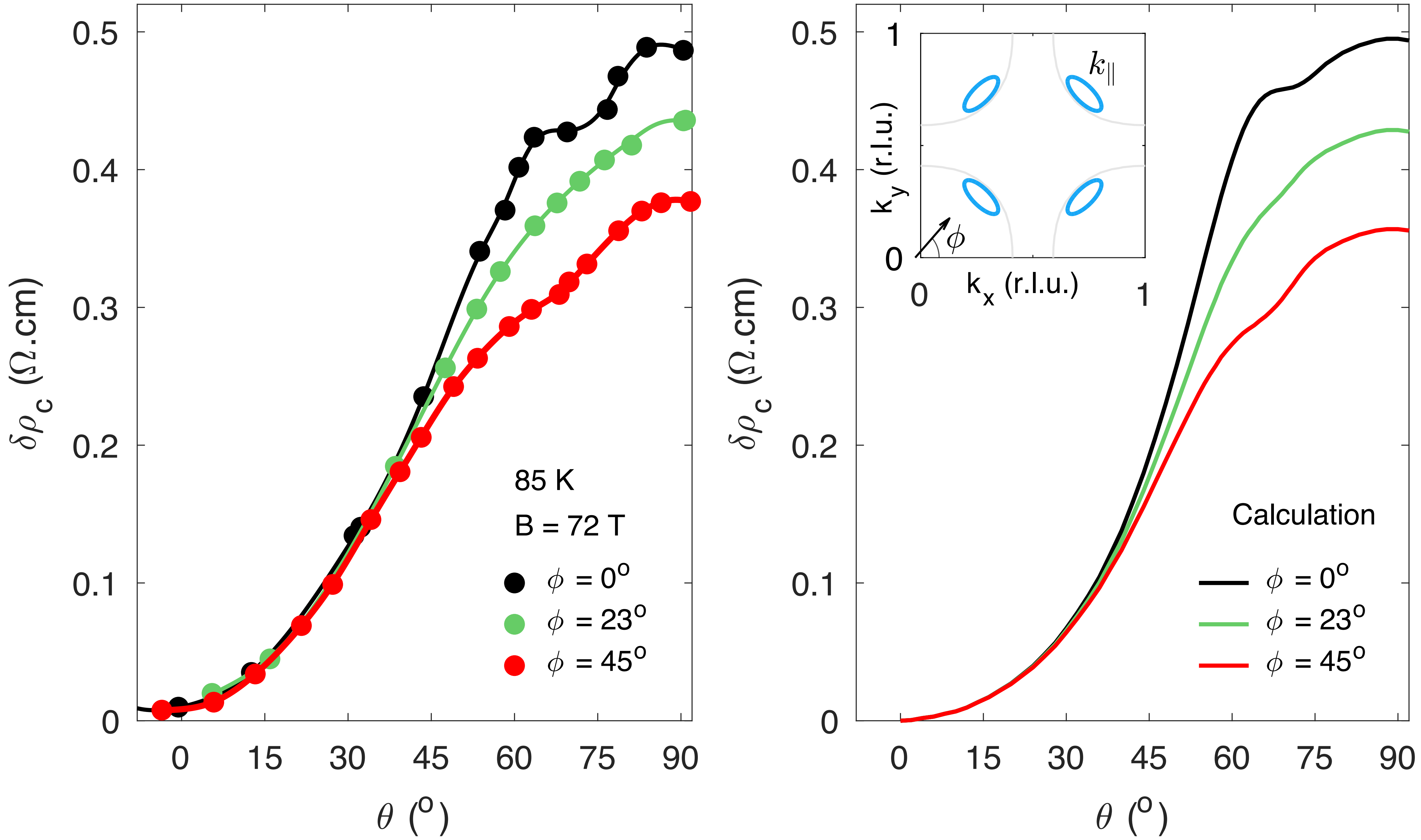

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Rules out holon metal
and possibly SDW metal

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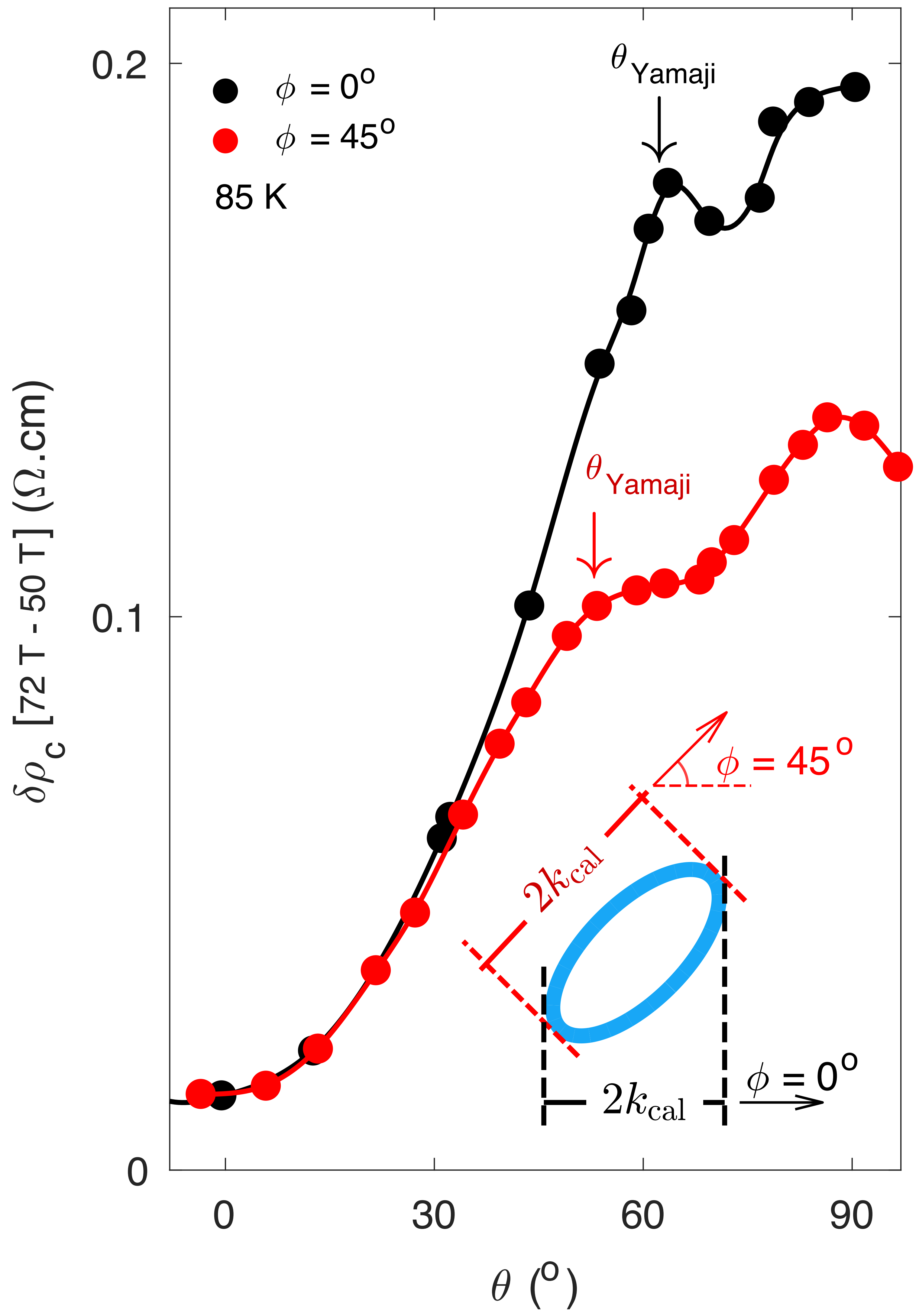

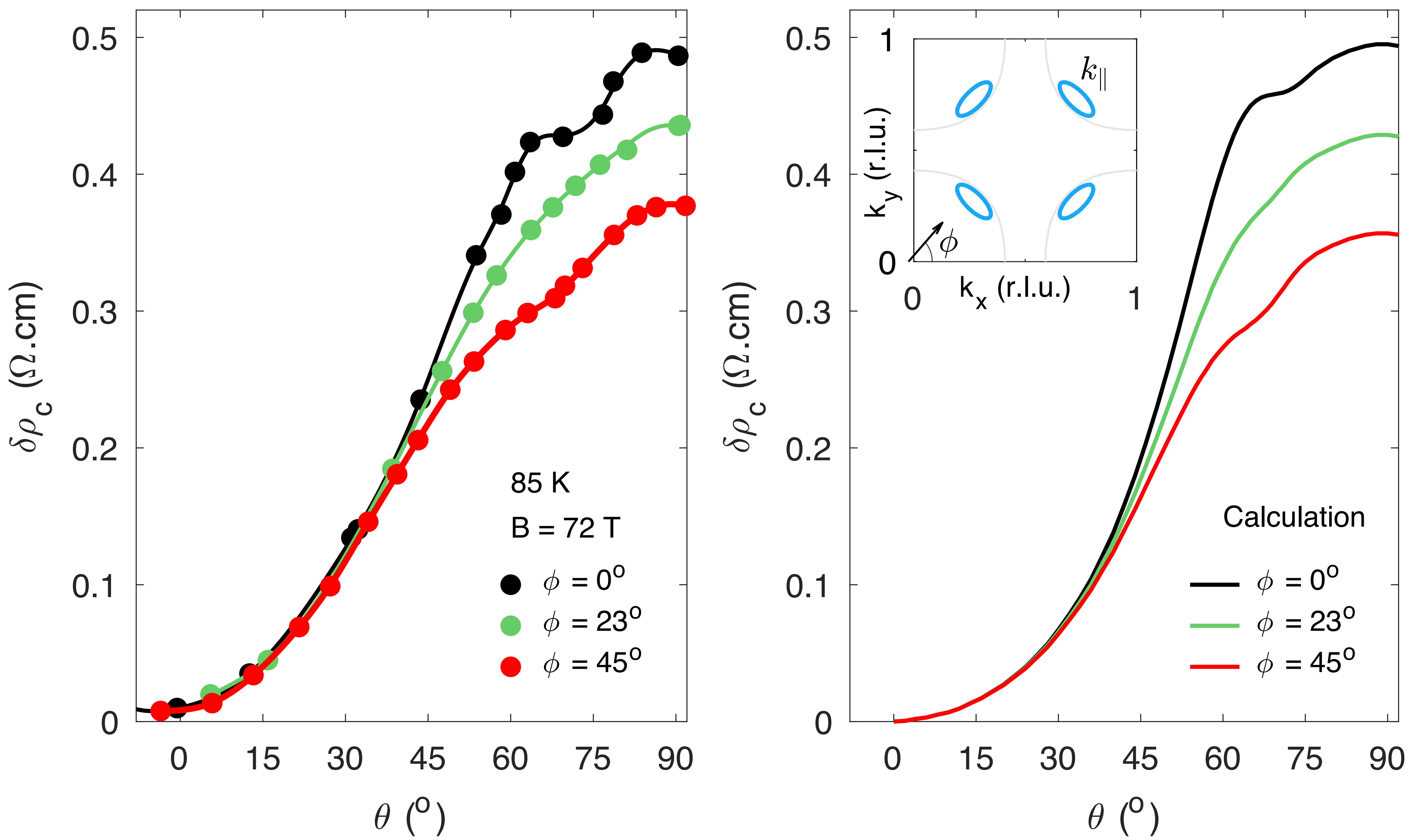

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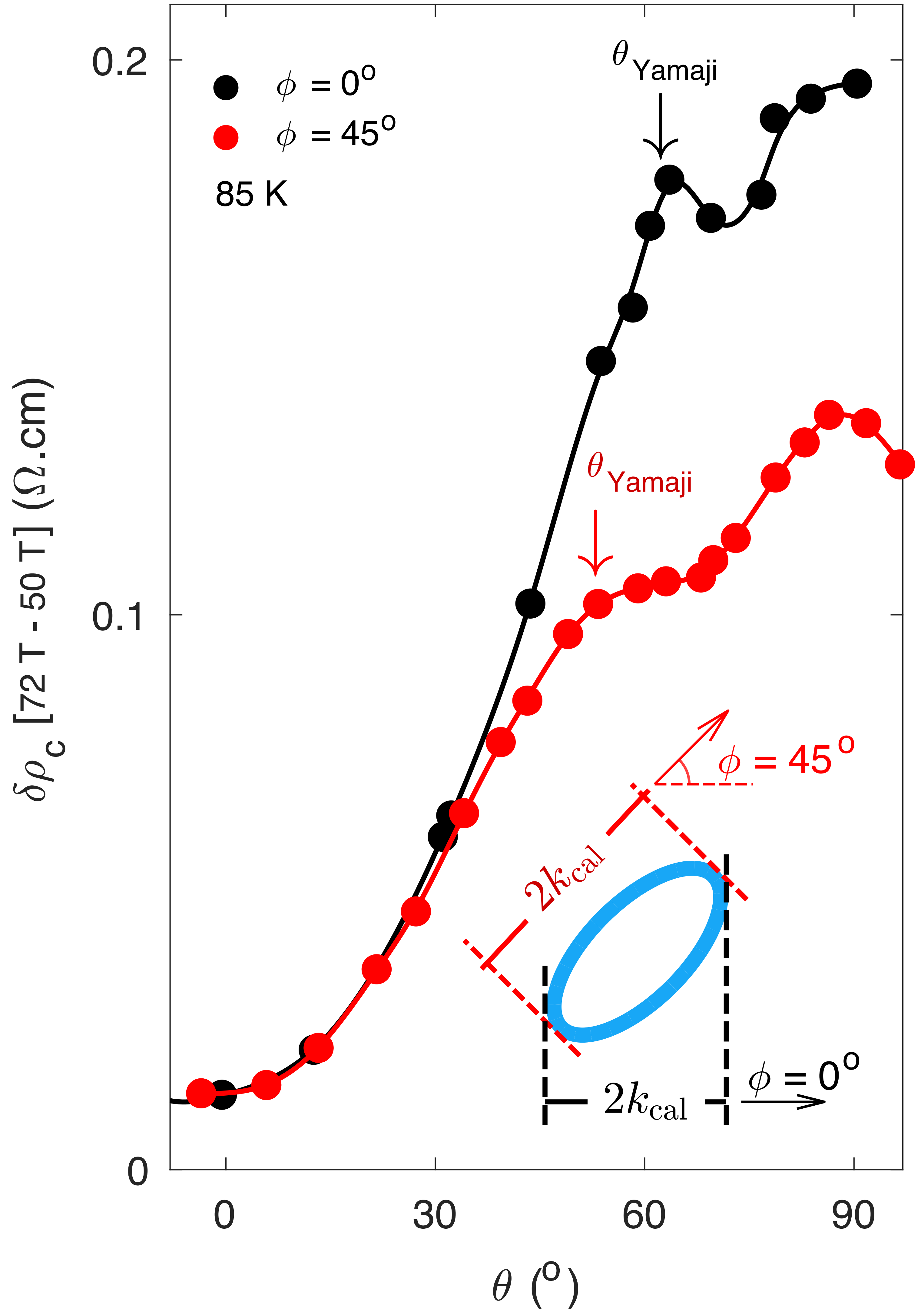

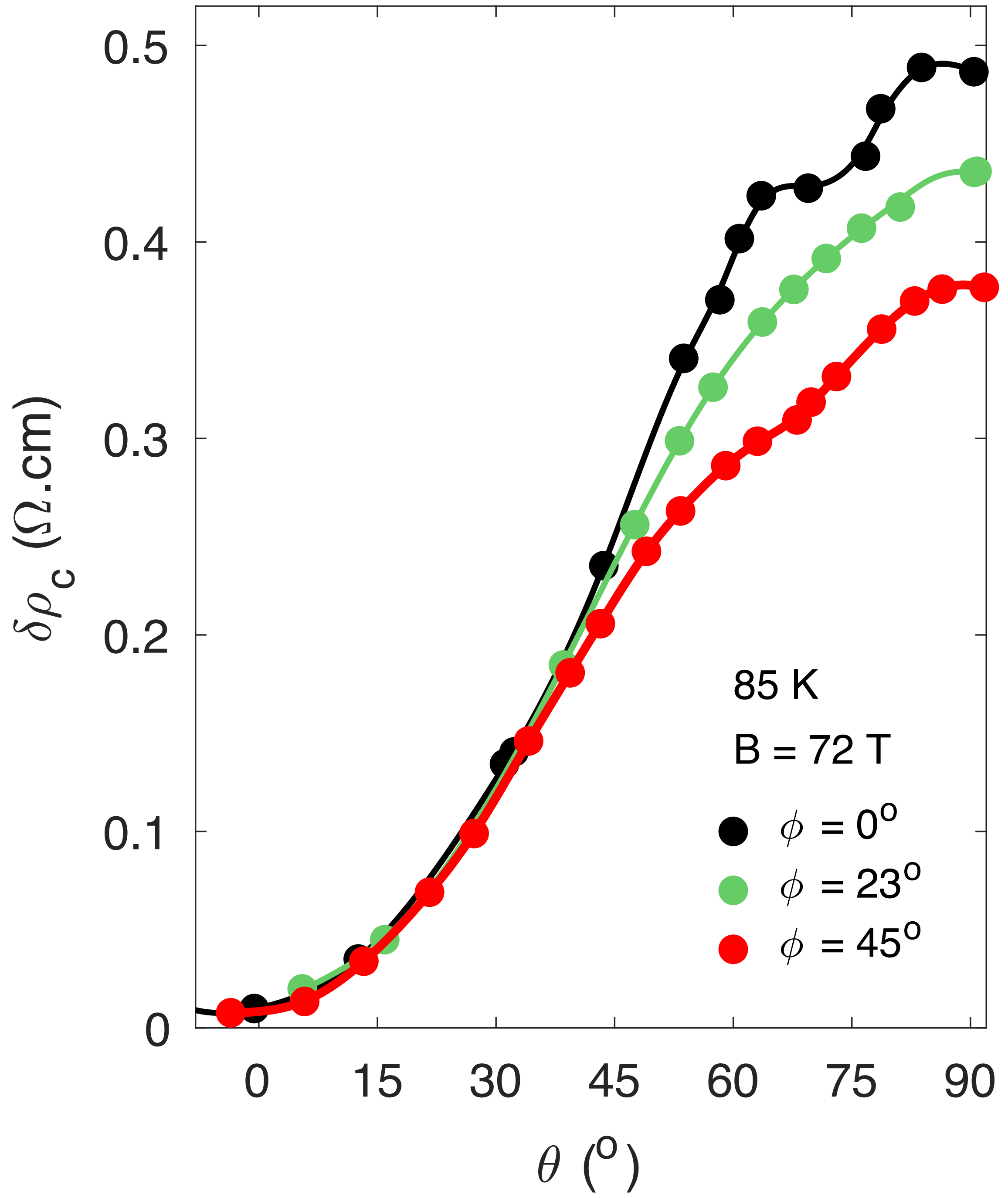

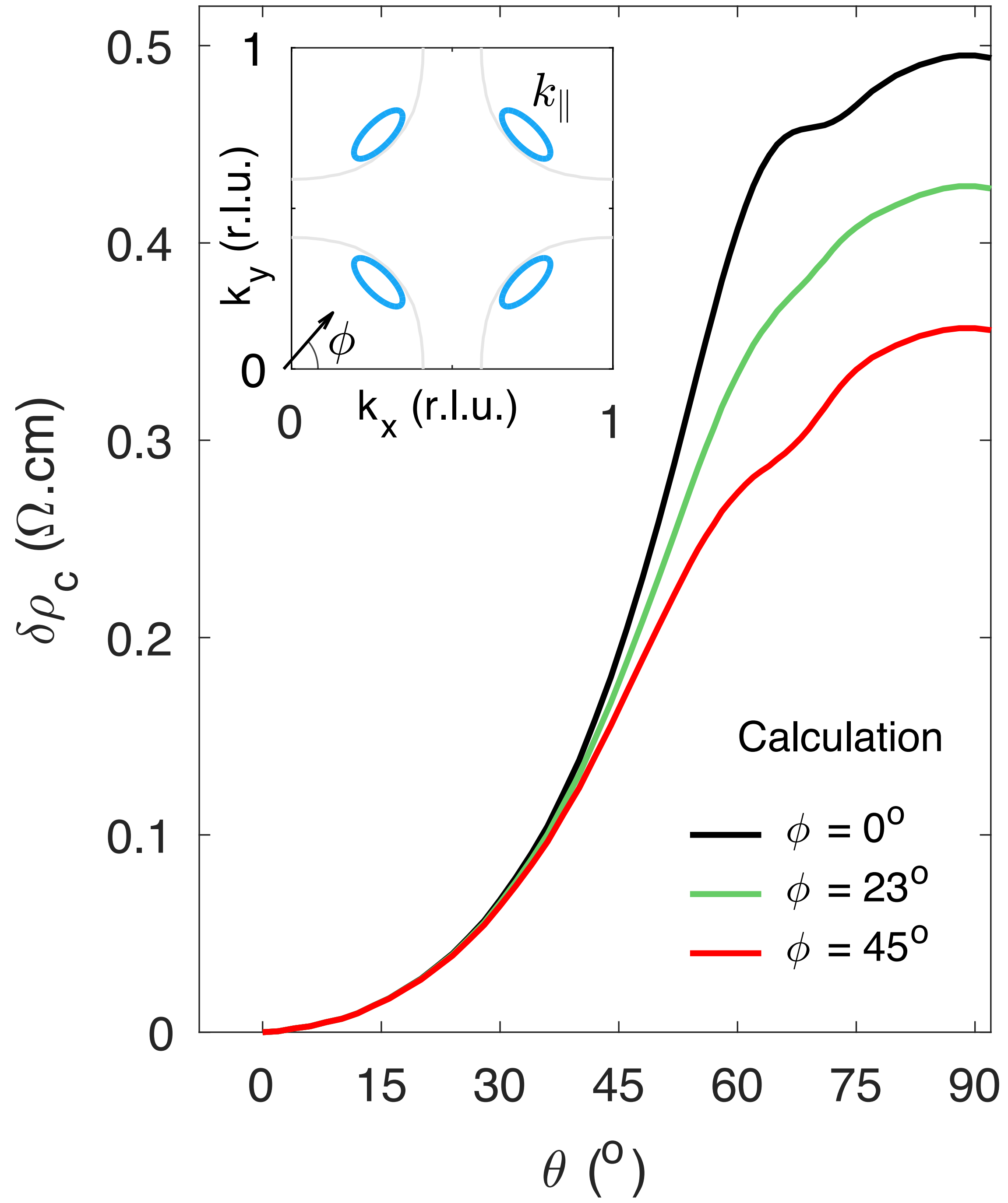

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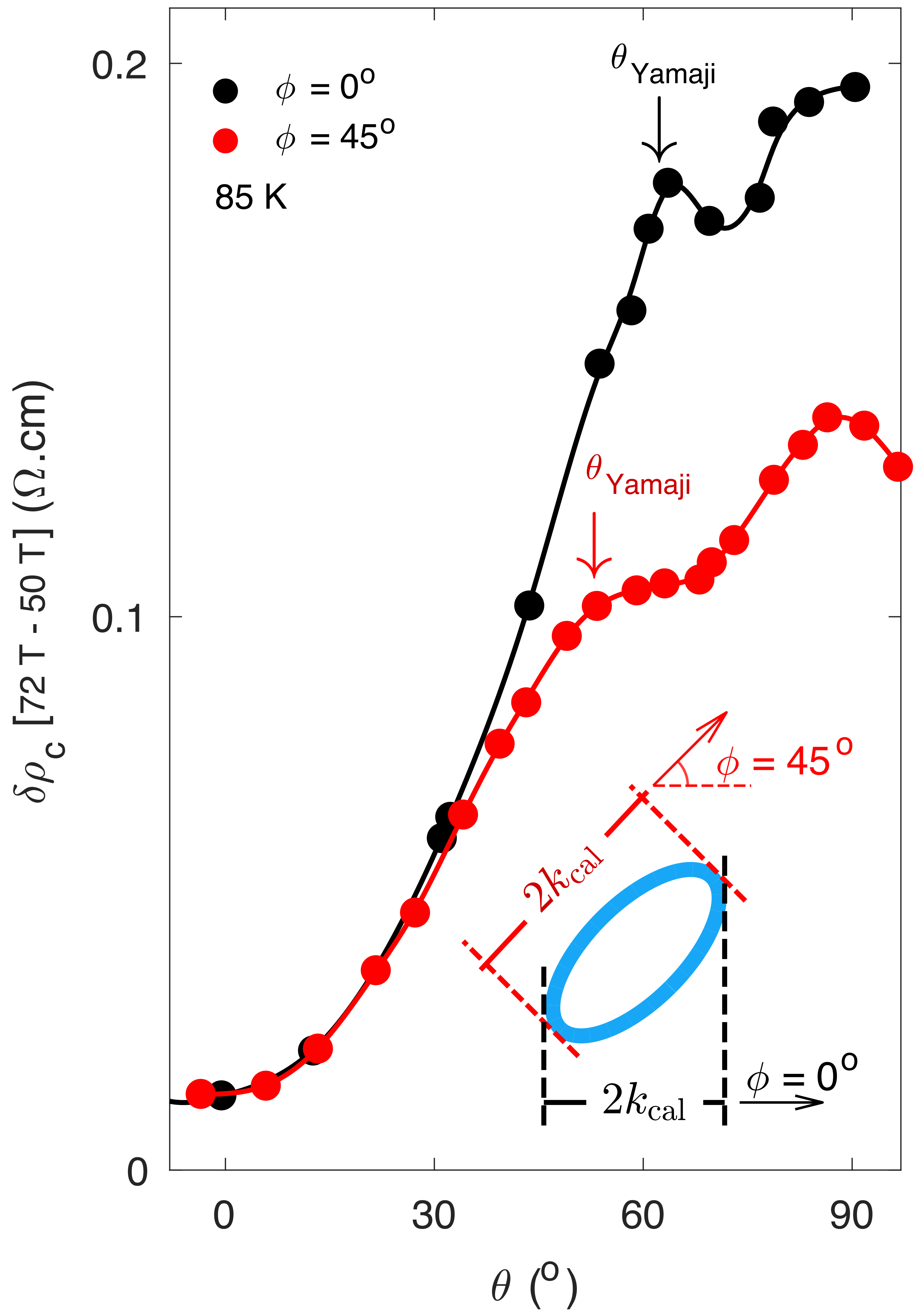

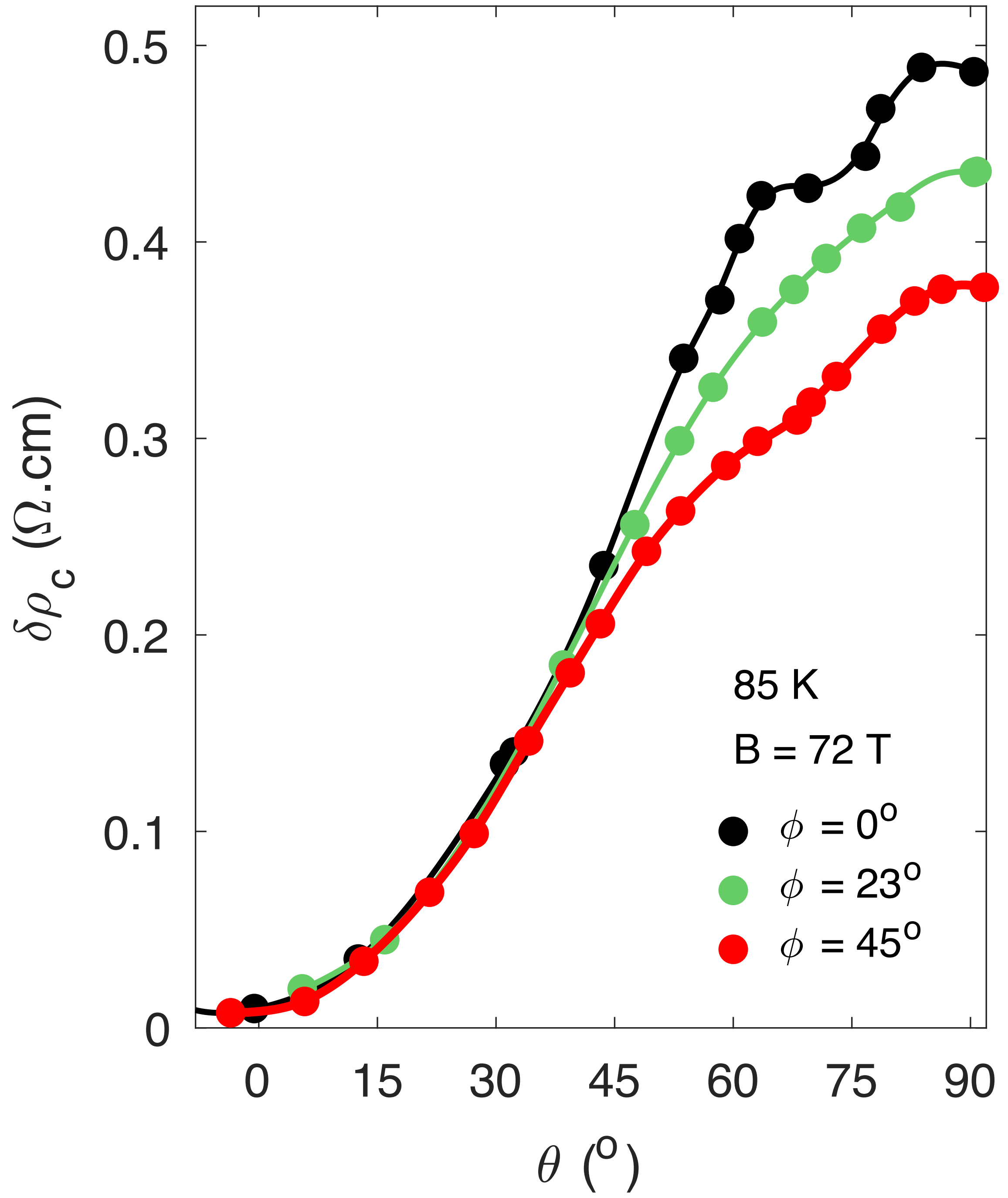

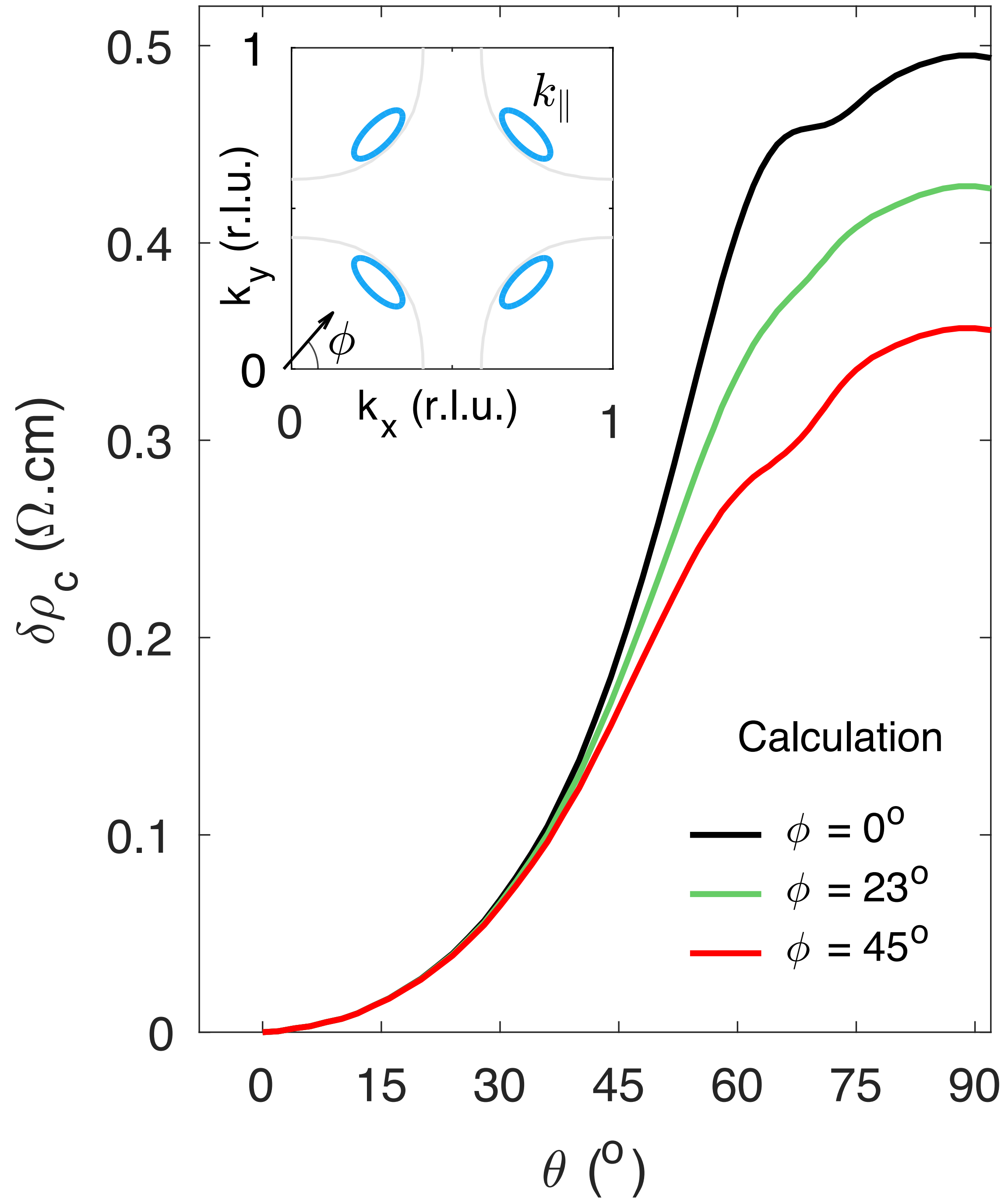

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Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

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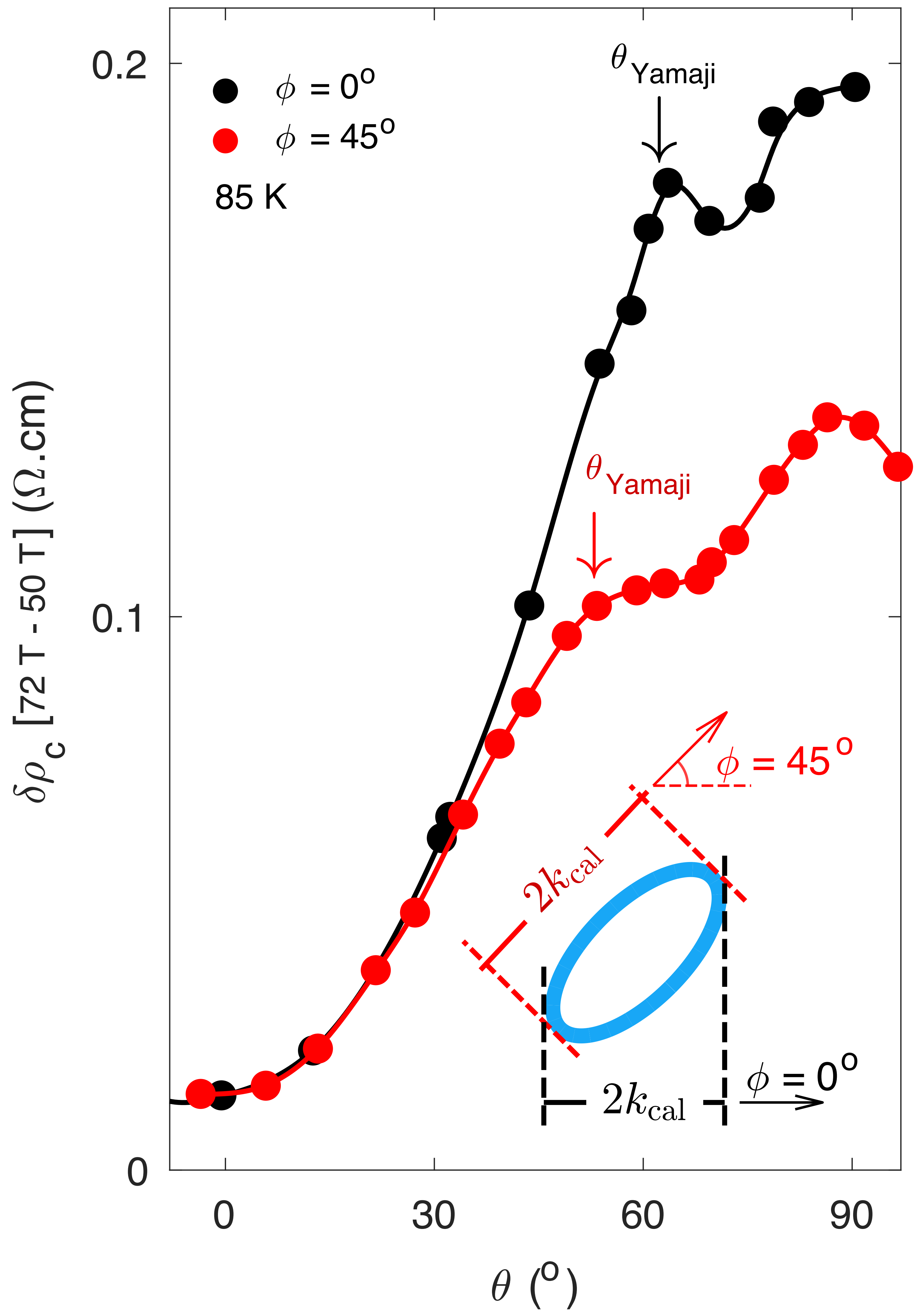
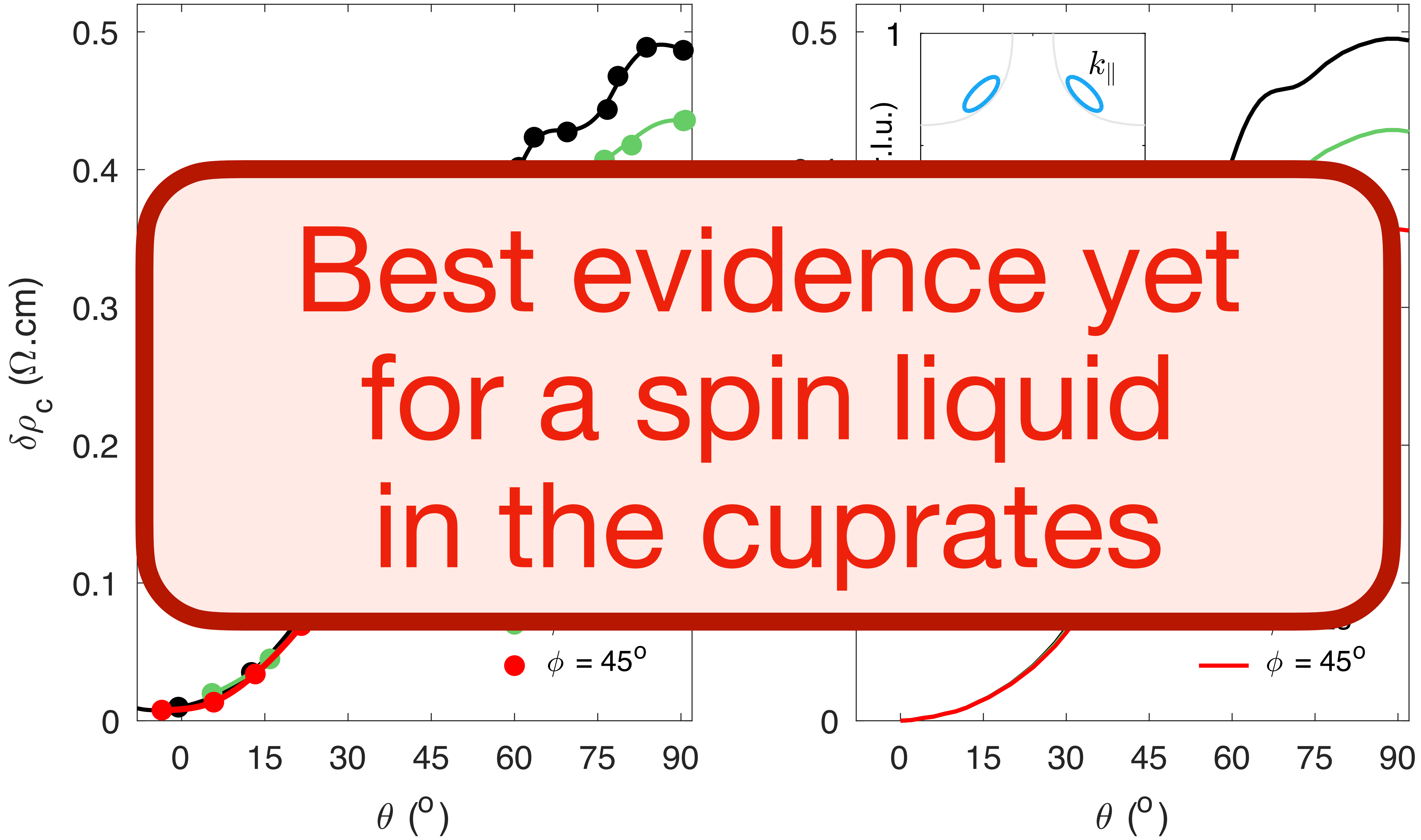




Best evidence yet for a spin liquid in the cuprates

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

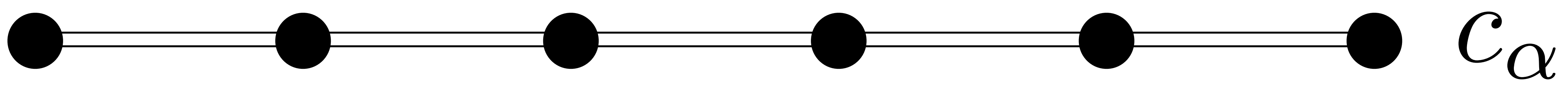
(p/8 also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Ancilla Layer Model (ALM) of single-band FL*

Ancilla Layer Model of the Hubbard model

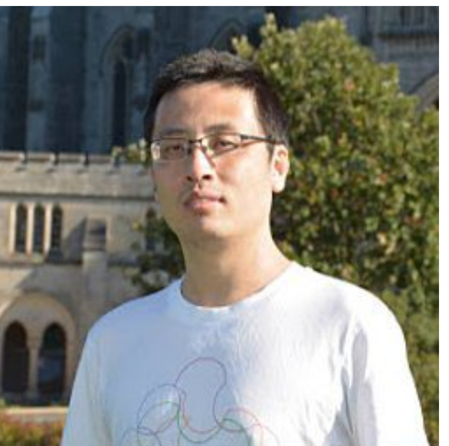
(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + U \sum_{\boldsymbol{i}} (c_{\boldsymbol{i}\uparrow}^\dagger c_{\boldsymbol{i}\uparrow})(c_{\boldsymbol{i}\downarrow}^\dagger c_{\boldsymbol{i}\downarrow})$$

Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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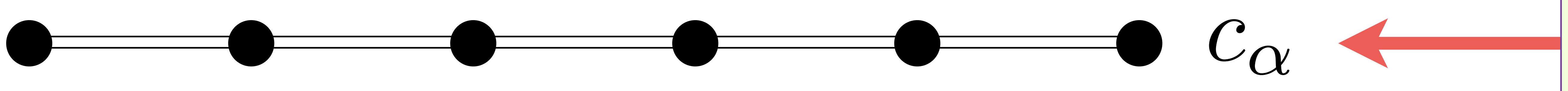

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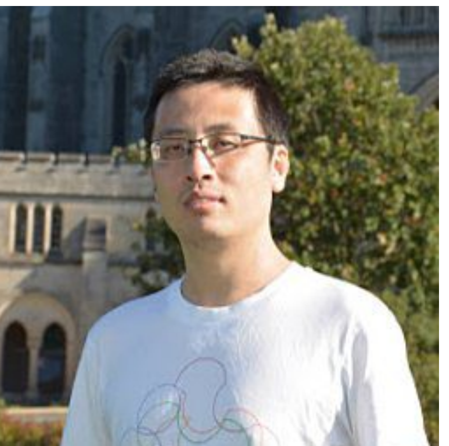
$$\boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \quad \Rightarrow \quad \text{Paramagnon}$$

Ya-Hui Zhang



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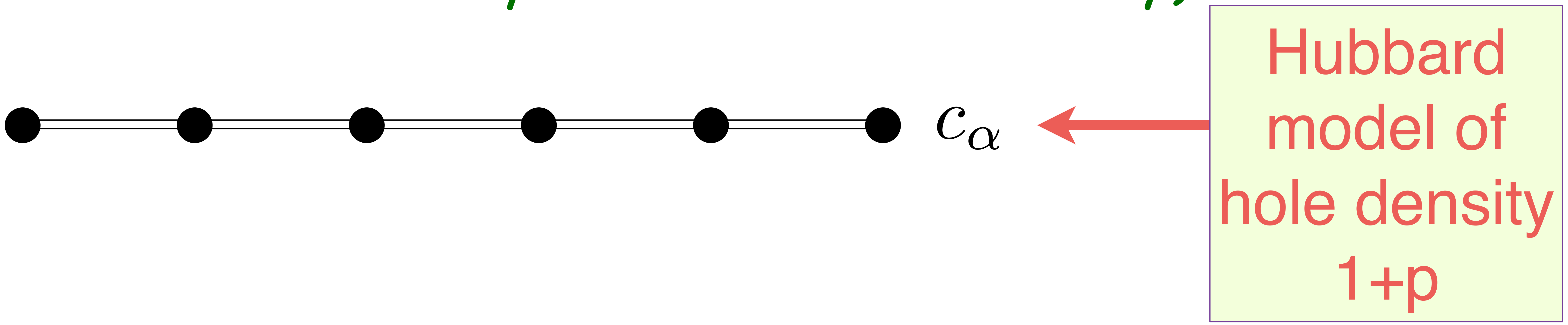

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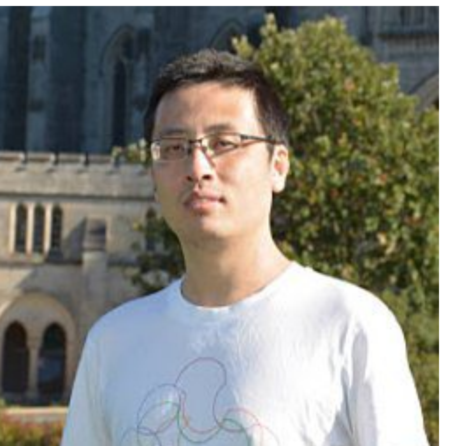
Ya-Hui Zhang



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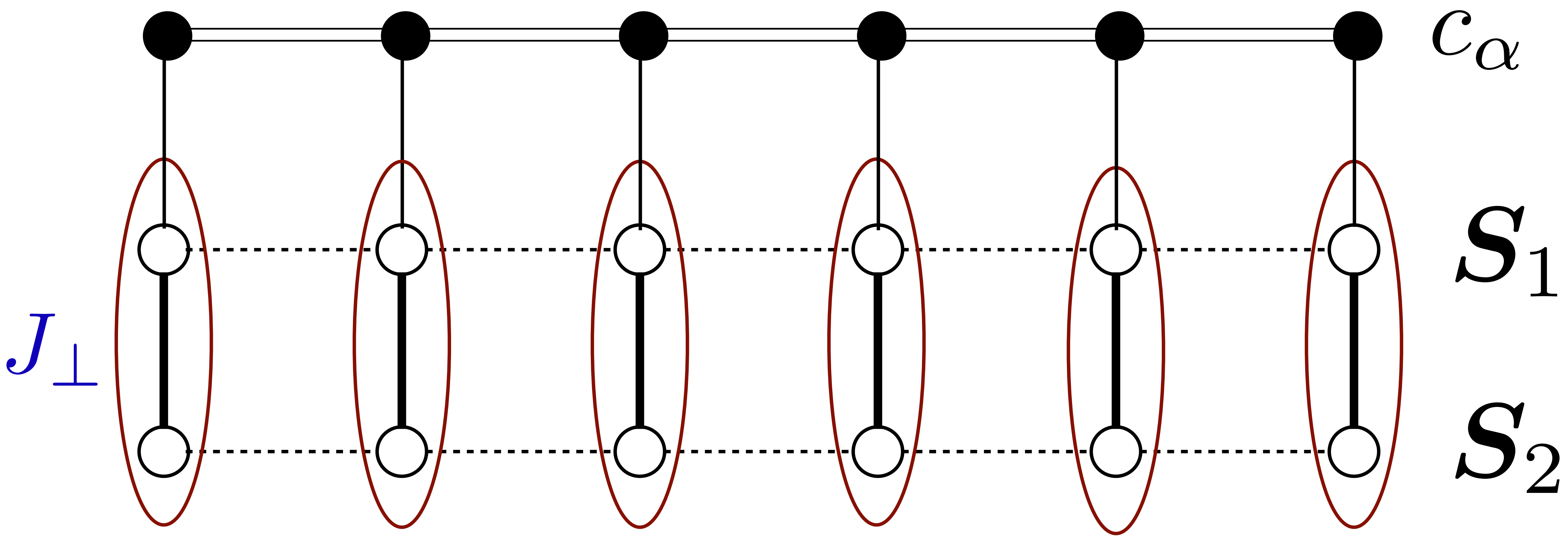
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



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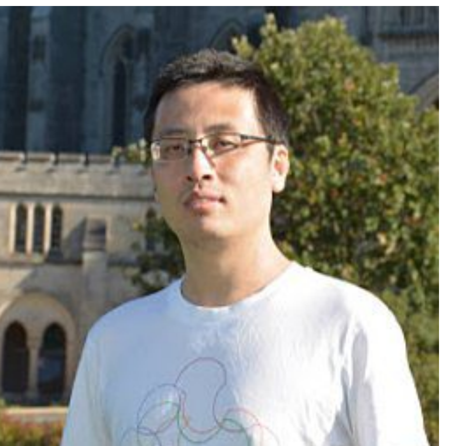
$$3\ \boldsymbol{\mathcal{P}} \text{ oscillators' states: } |0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$$

$$\boldsymbol{S}_{1,2} \text{ ancilla qubits states : } (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang



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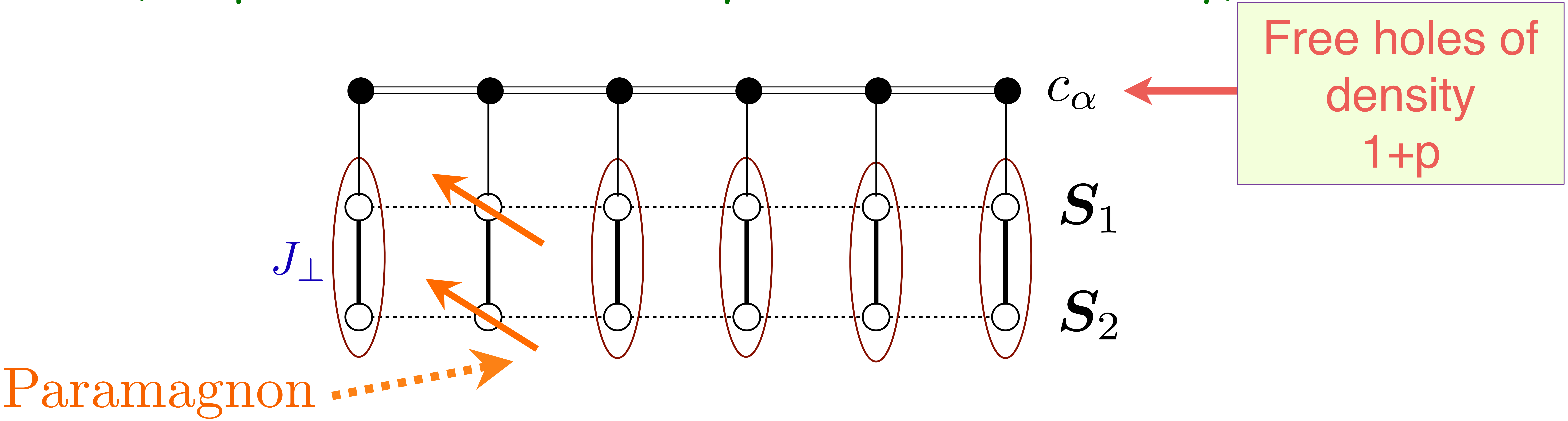

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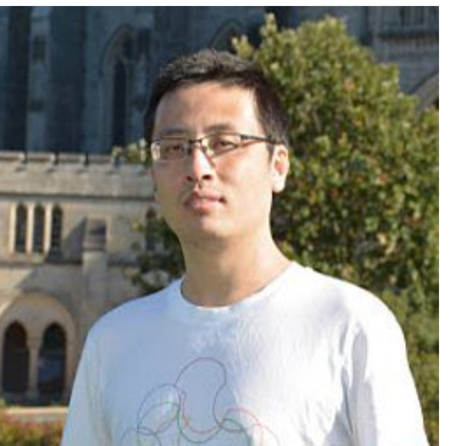

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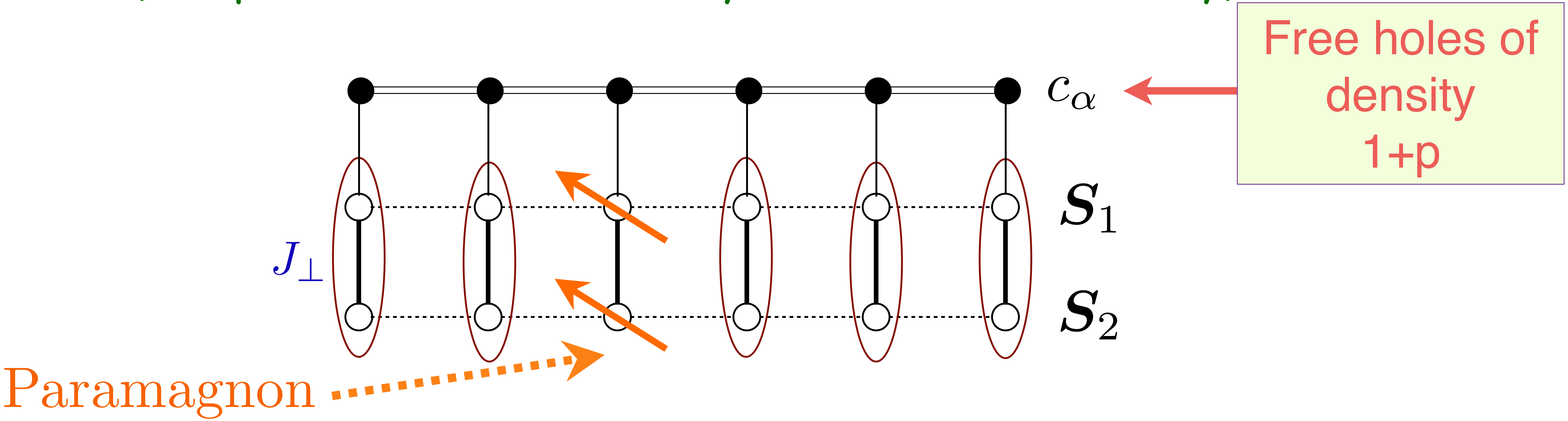

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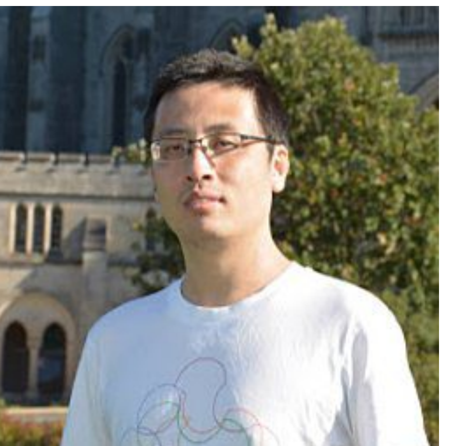
Ya-Hui Zhang



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Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
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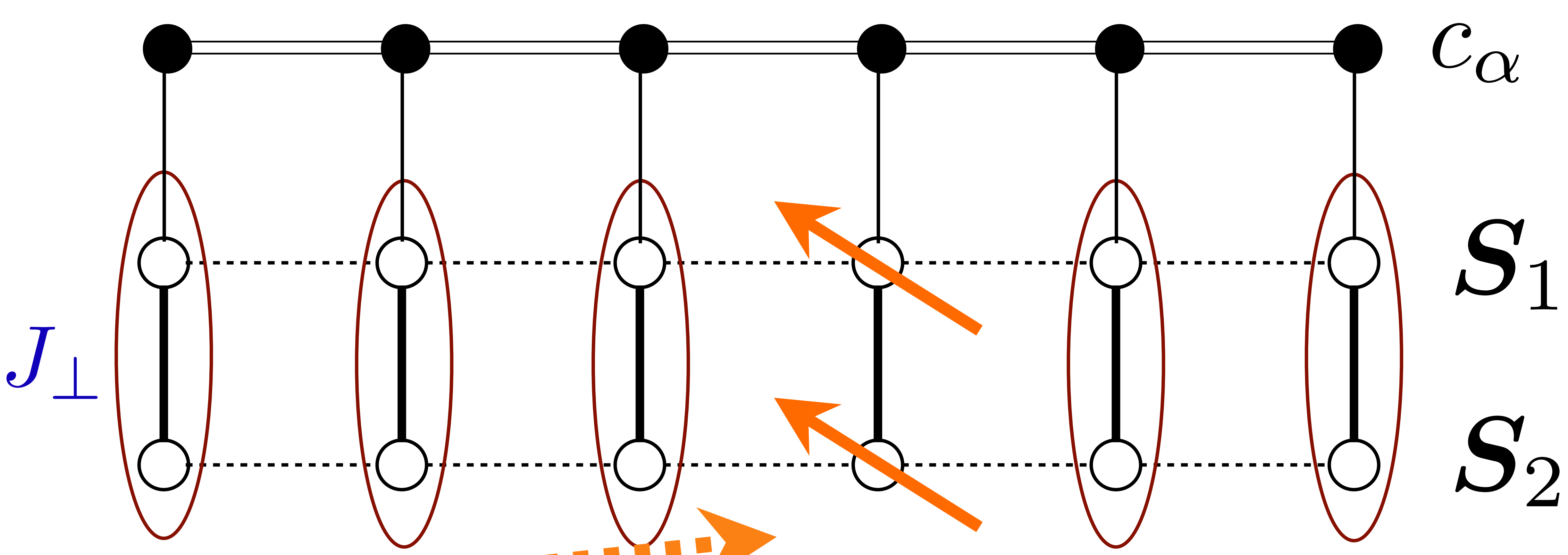
Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



Paramagnon

$$\mathcal{H}_{\text{Hubbard}} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}^2 + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c_{\boldsymbol{i}\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} + \frac{m_{\mathcal{P}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 \right].$$

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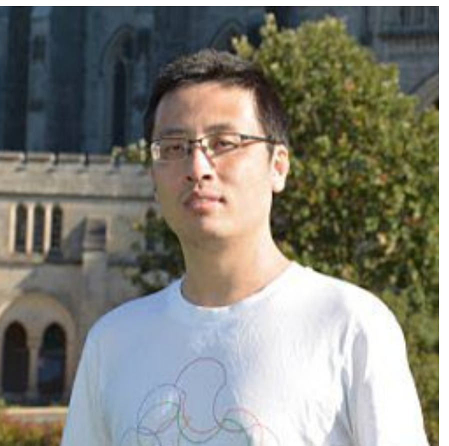

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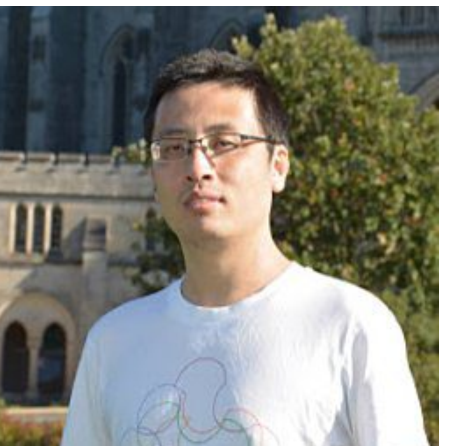
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

$$\mathcal{H}_{\rm ALM} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c^\dagger_{\boldsymbol{i}\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{\boldsymbol{i}\beta} + J_\perp \sum_{\boldsymbol{i}} \boldsymbol{S}_{1\boldsymbol{i}} \cdot \boldsymbol{S}_{2\boldsymbol{i}} \,.$$

Ya-Hui Zhang



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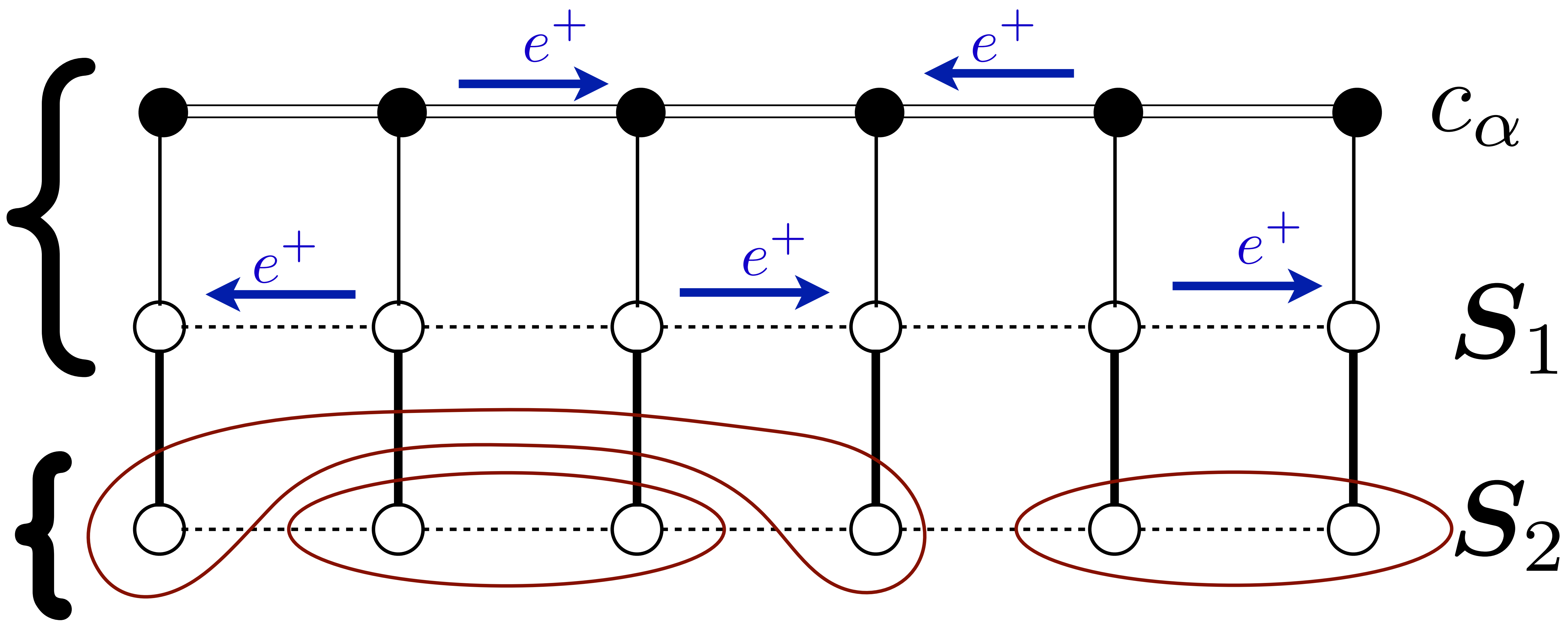
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

ALM of FL* of Hubbard model

Kondo lattice heavy Fermi liquid.
Area $(1 + p + 1)/2$ $= p/2$ (mod 1).
Small Fermi surface!

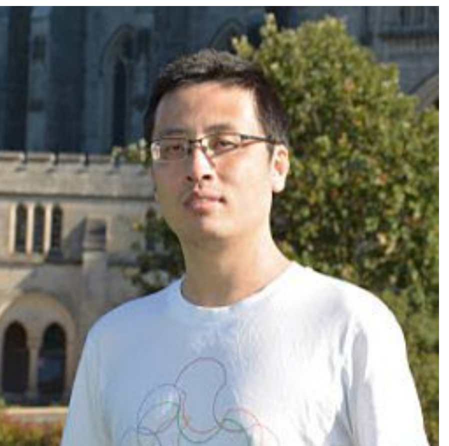


Free holes of density 1+p

Your favorite spin liquid

$$\text{Pseudogap metal} = \begin{array}{c} \text{Kondo Lattice FL of } c_\alpha \text{ and } \boldsymbol{S}_1 \\ \oplus \\ \text{Spin Liquid of } \boldsymbol{S}_2 \end{array}$$

Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
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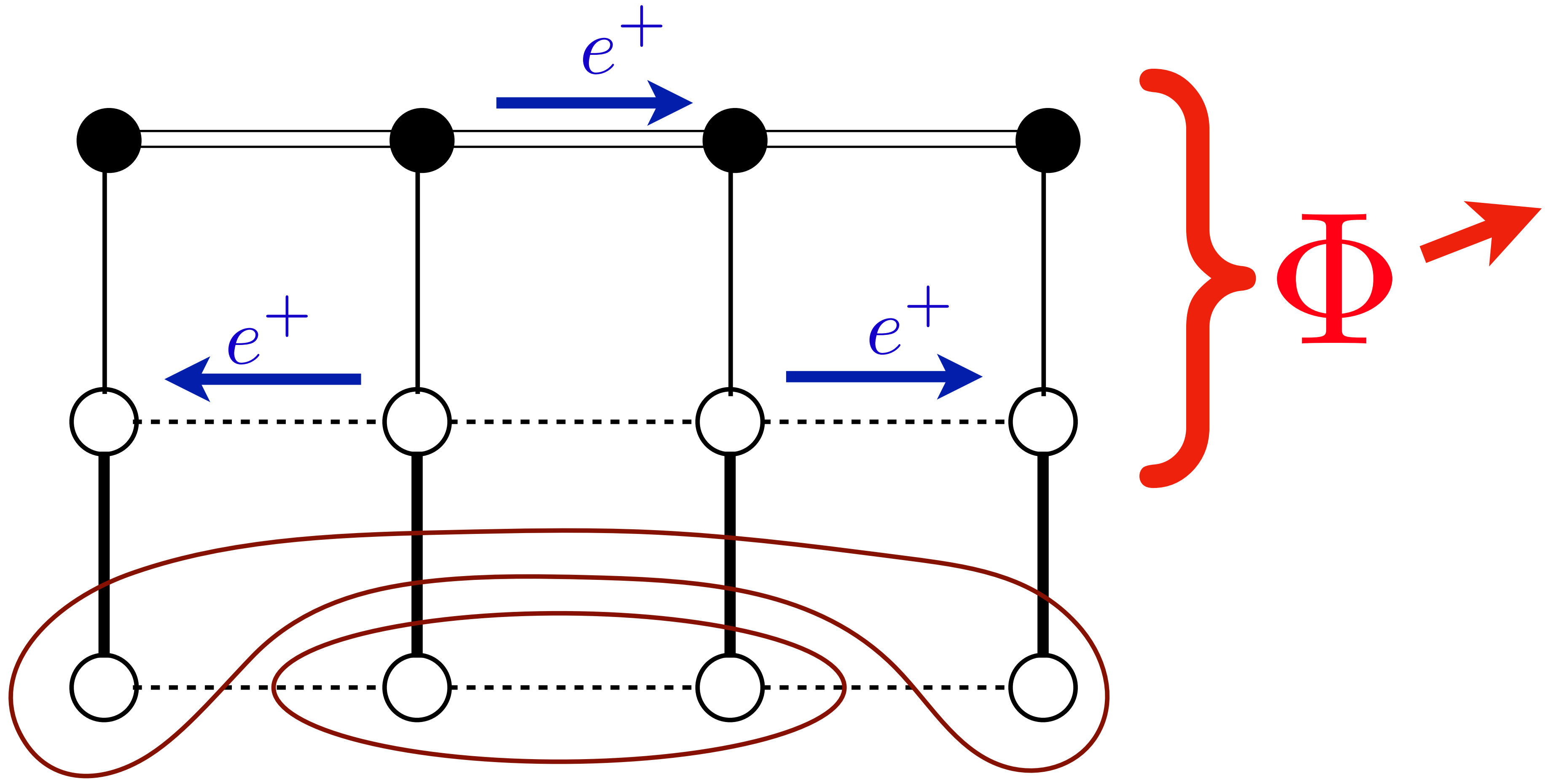
Ancilla Layer Model of the Hubbard model



Higgs field Φ determines the pseudogap.
In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

$$|\text{FL}^*\rangle = |\text{Slater determinant of top two layers with hybridization } \Phi\rangle$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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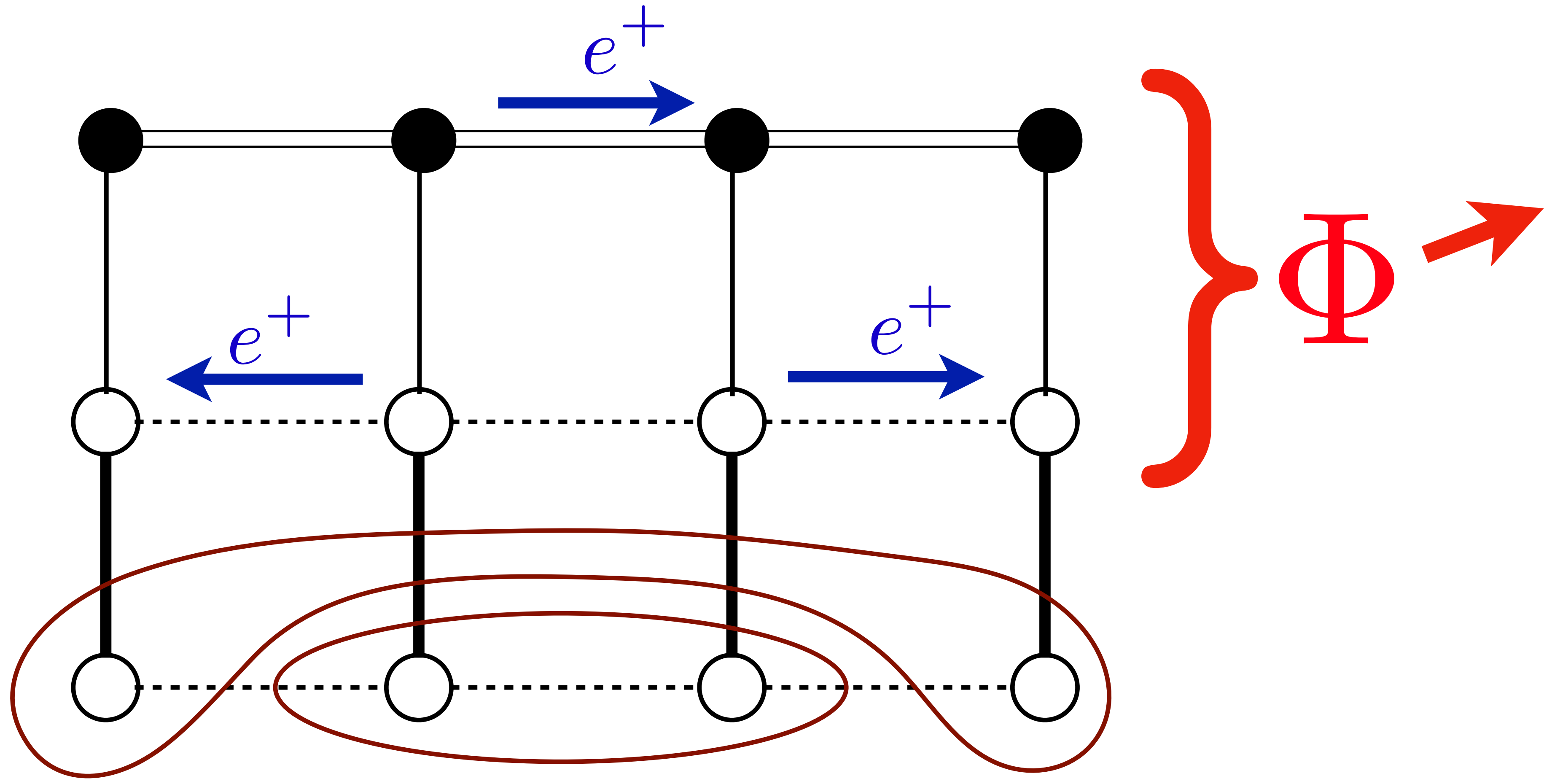

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- Spinons f_α in bottom layer are in a π-flux spin liquid

I. Affleck and J. B. Marston, PRB **37**, 3774 (1988).

$$
|\text{FL}^*\rangle = \bowtie |\text{Slater determinant of top two layers with hybridization } \Phi\rangle \otimes |\text{Slater determinant to spinons } f_\alpha\rangle
$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

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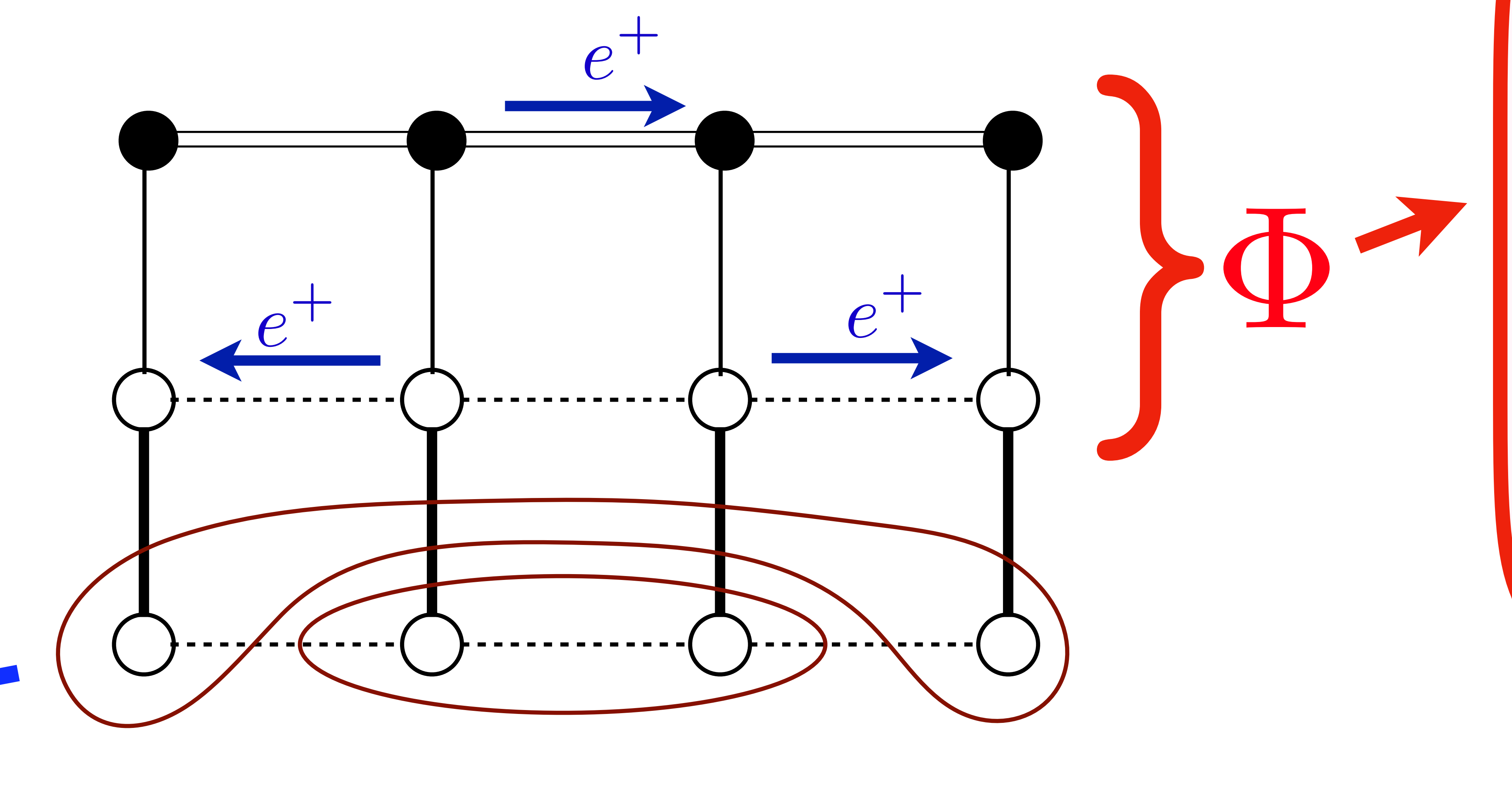

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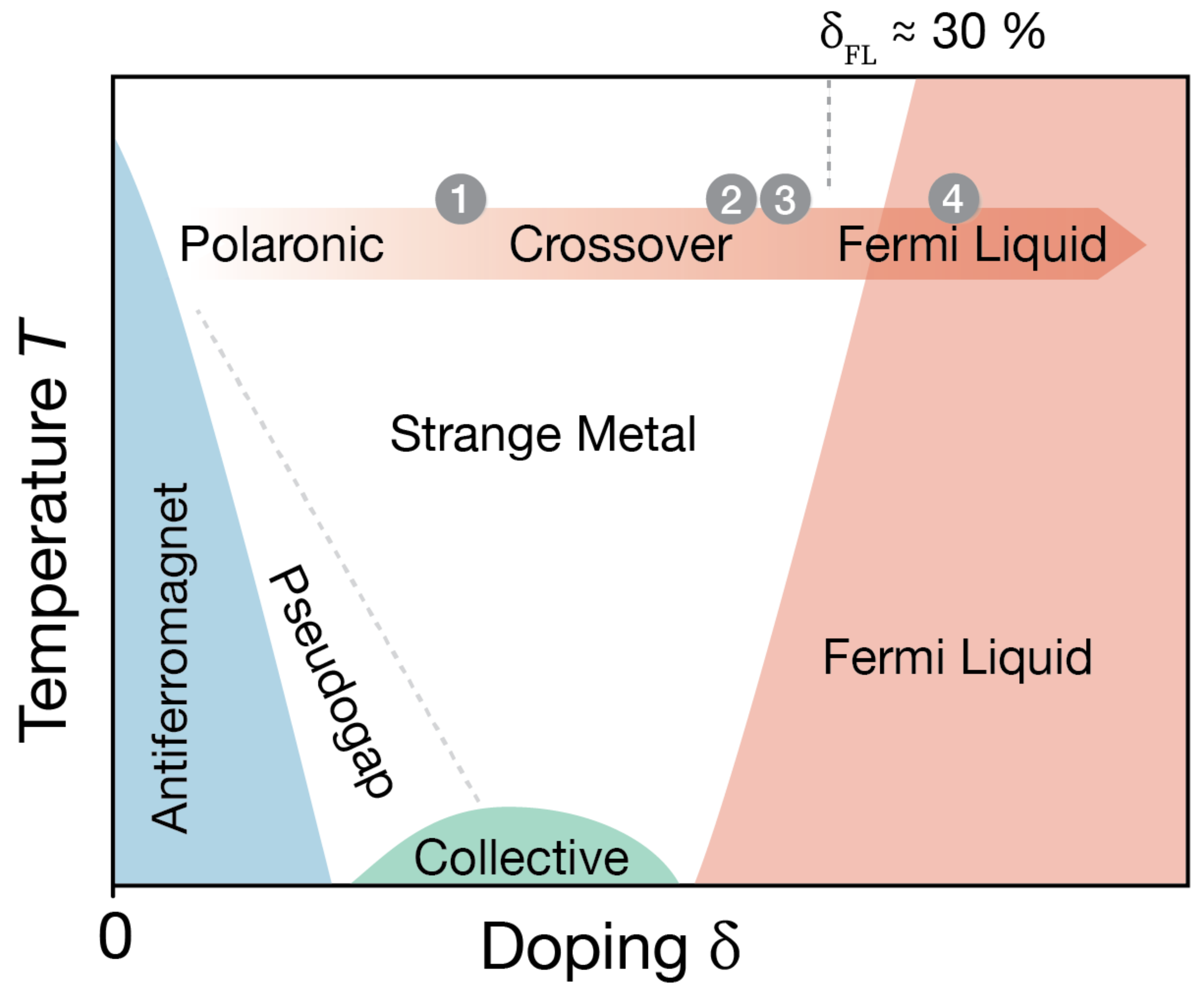
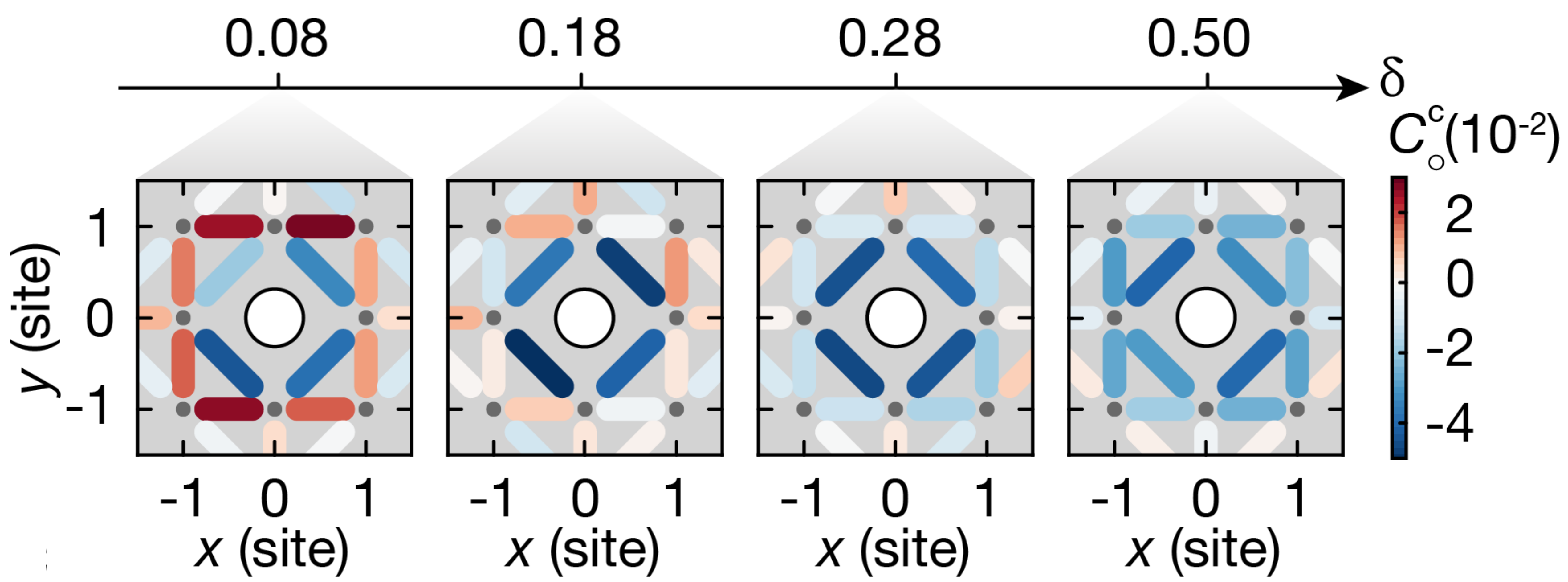


Ultracold fermionic atoms in optical lattices

Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

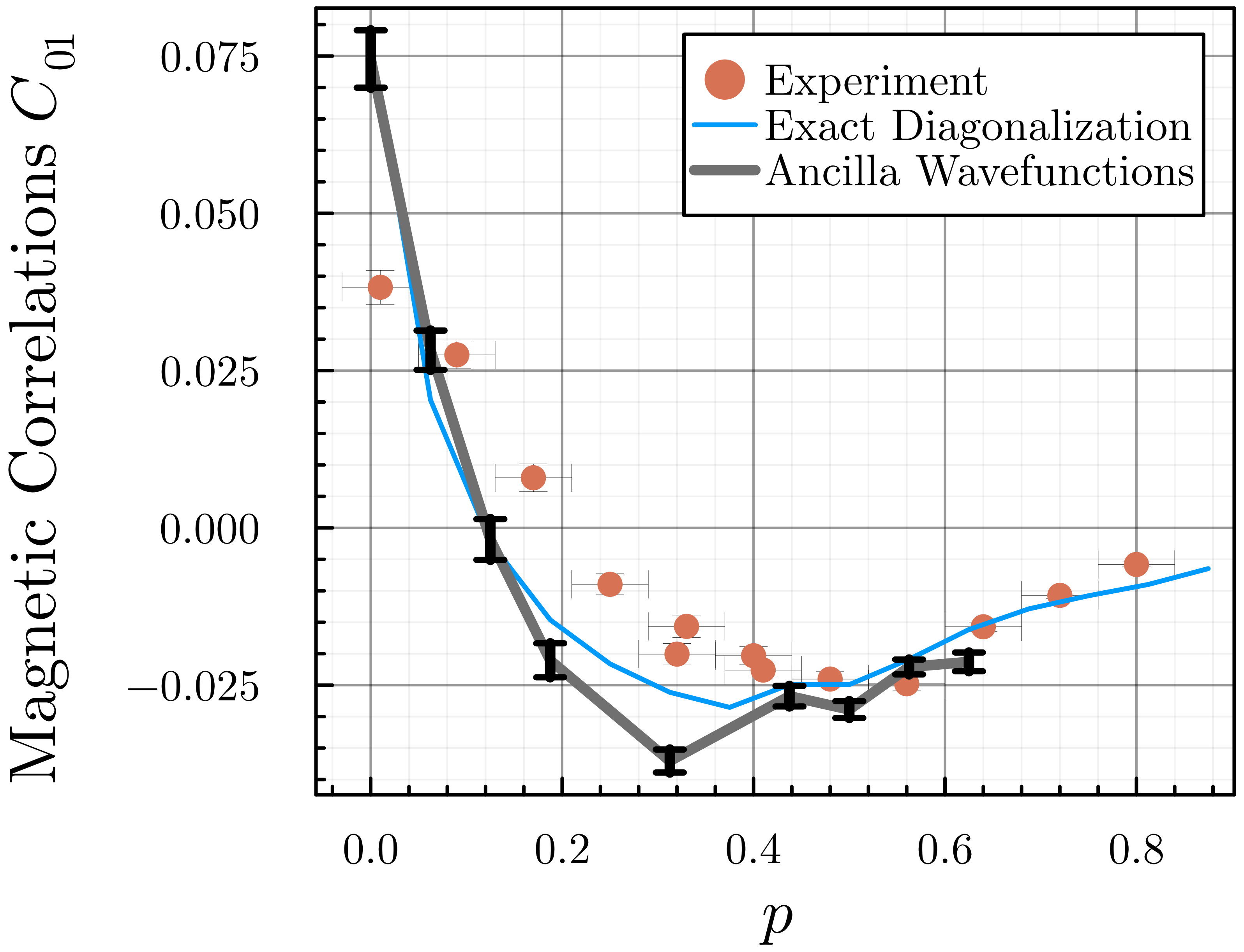
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

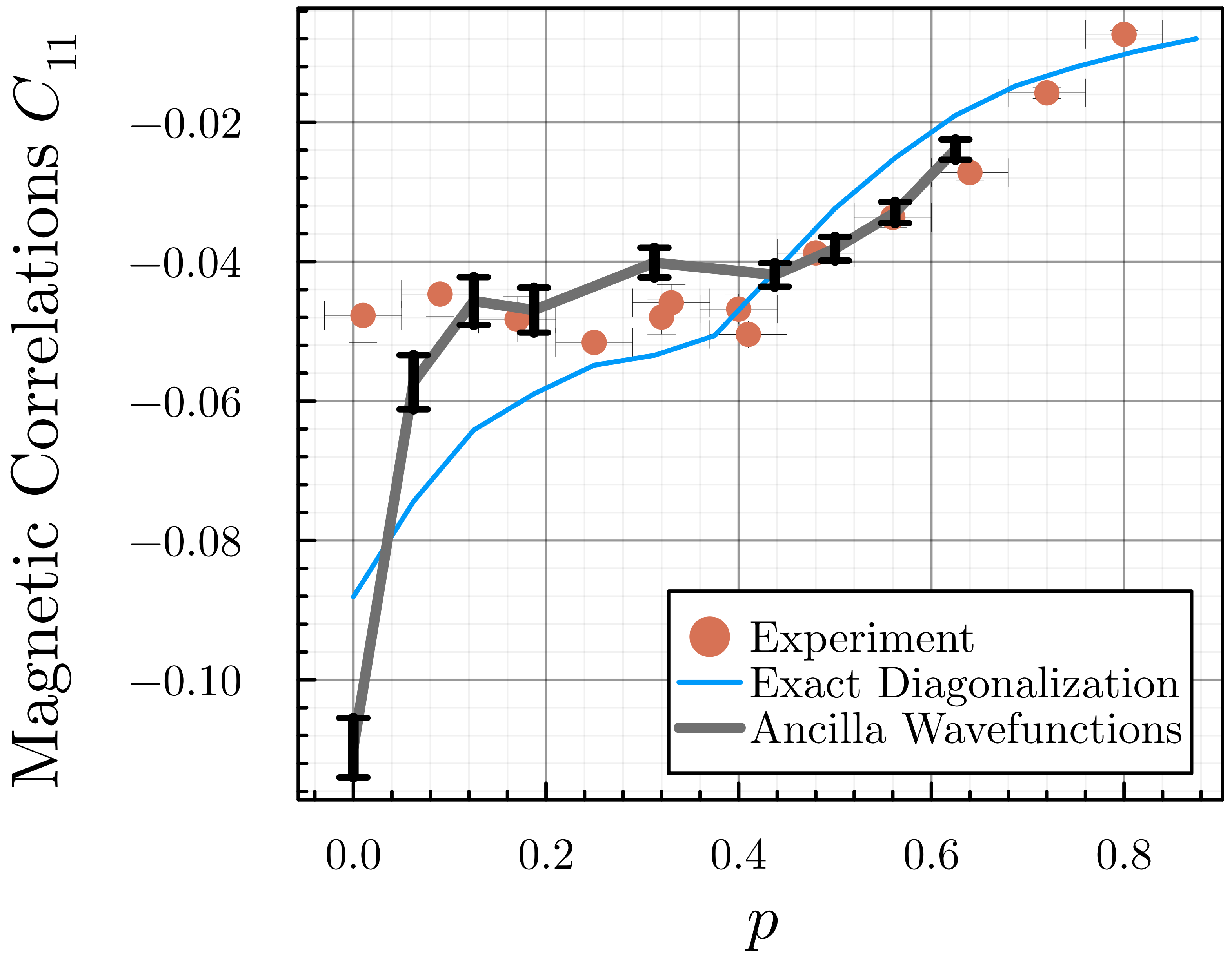
Science **374** (2021) 82

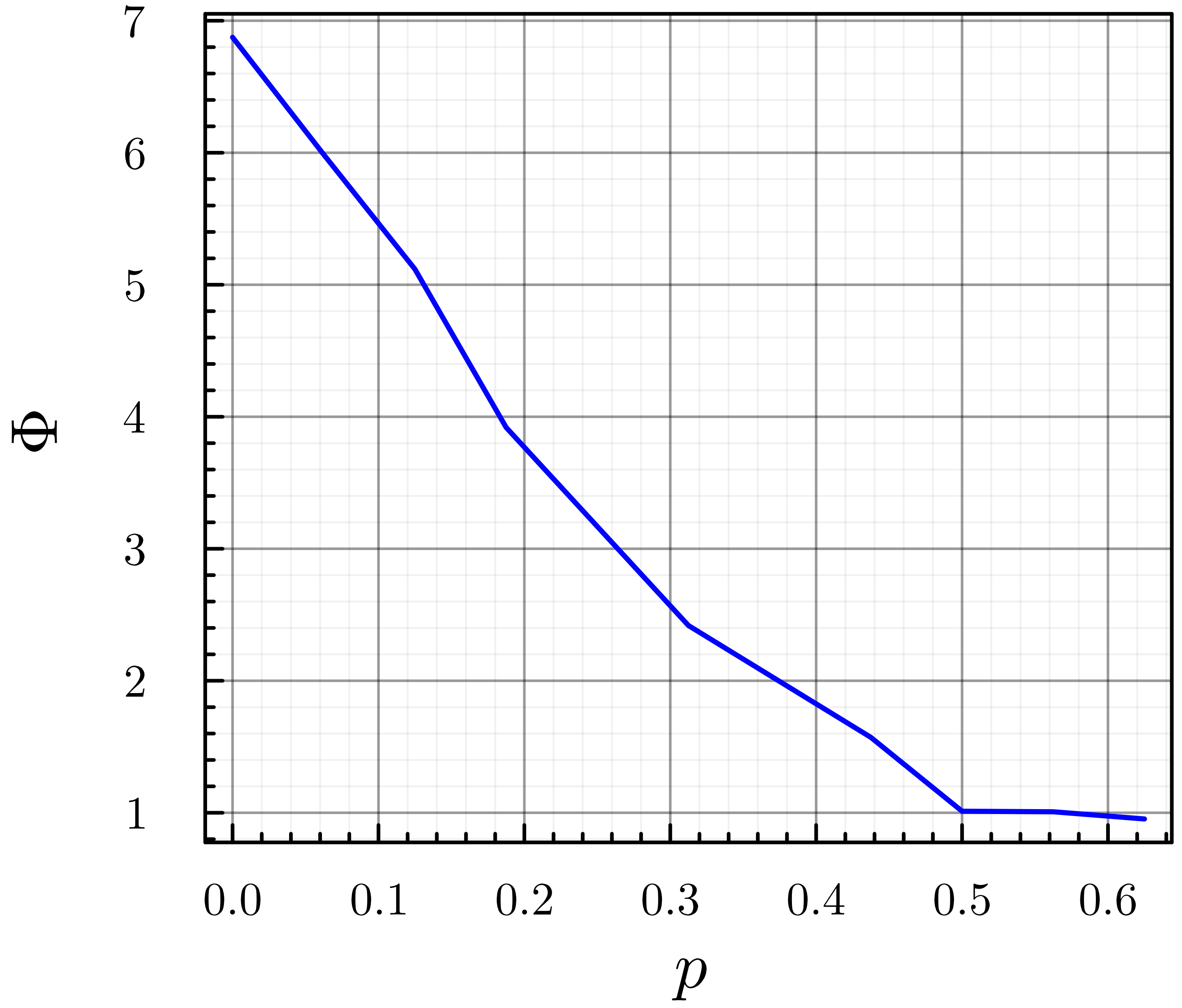


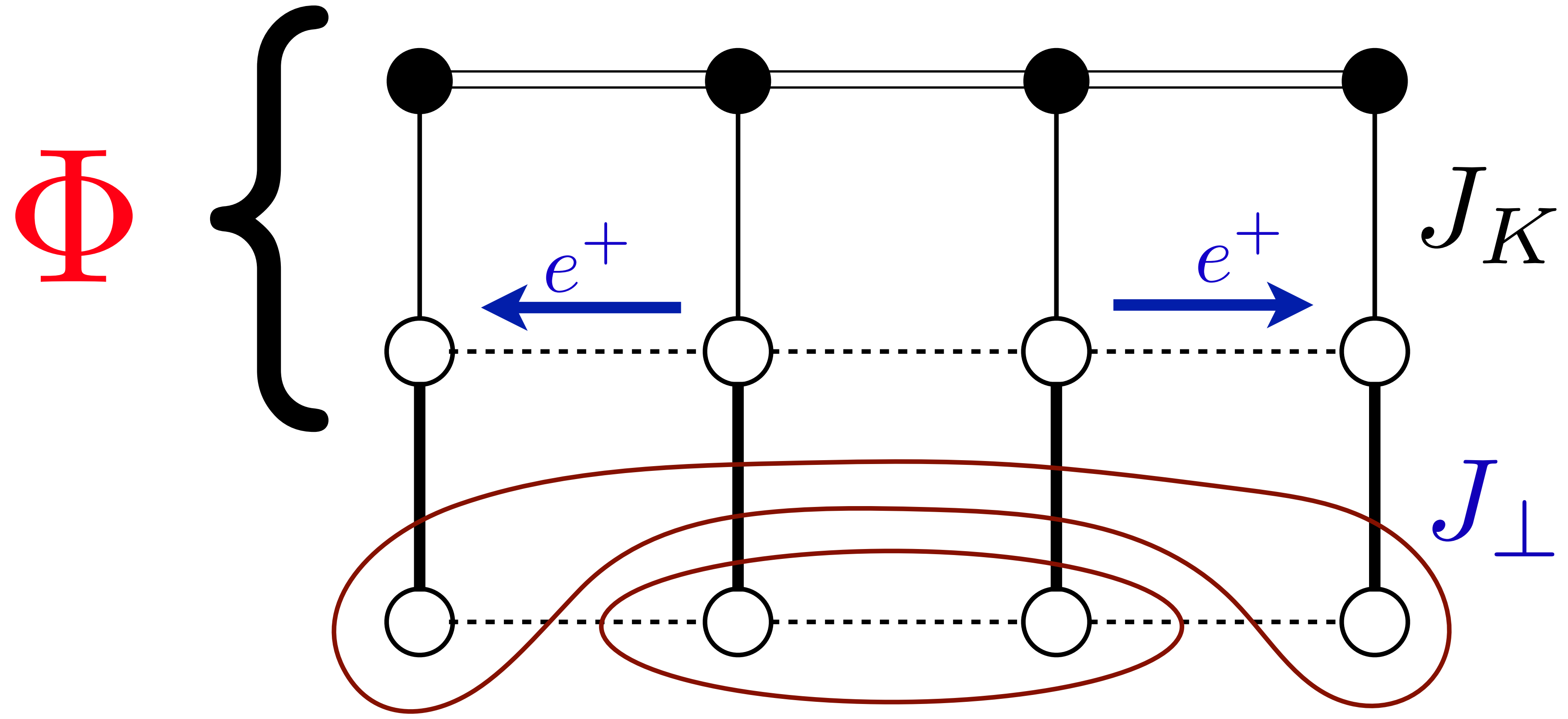
Max Planck Institute of Quantum Optics, Garching

Trial wavefunction for FL* of Hubbard model







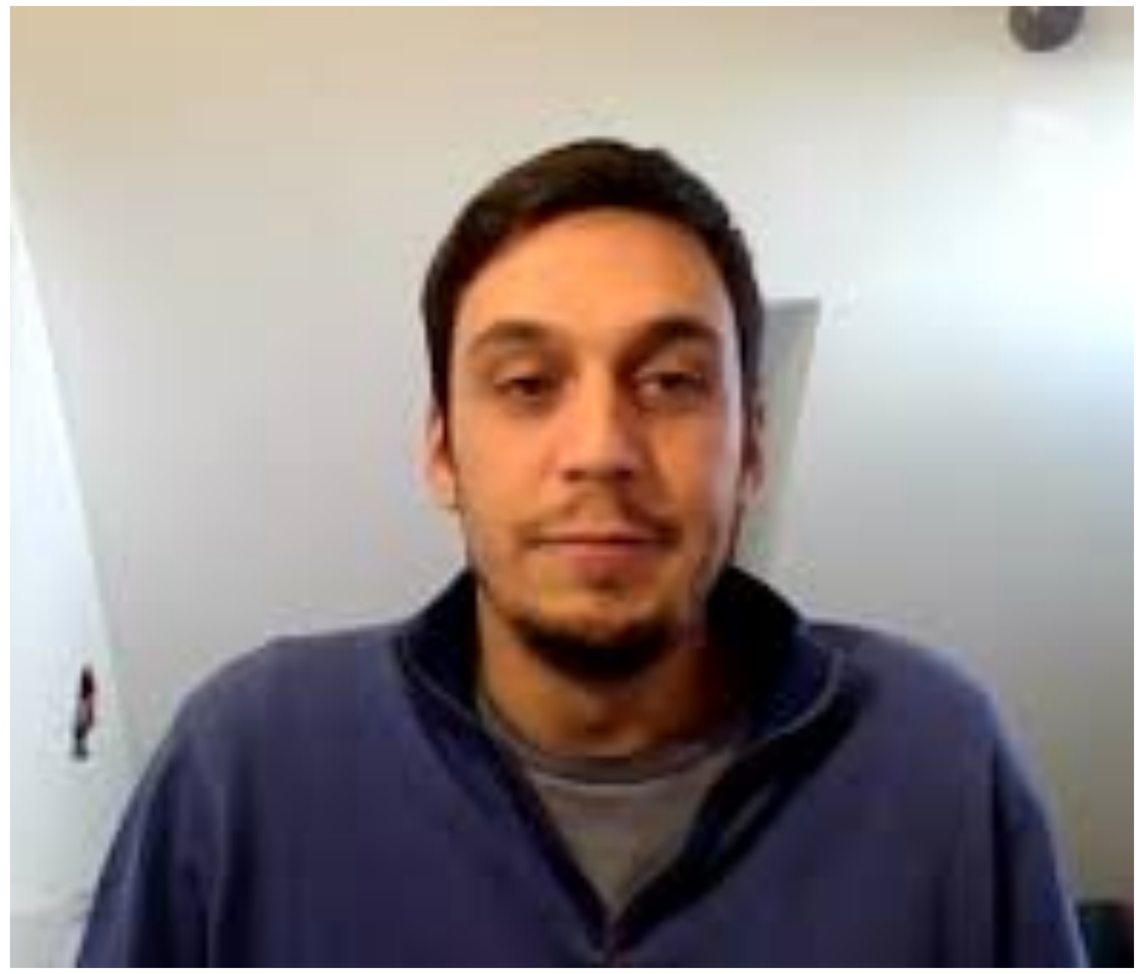


L. Shackleton and Shiwei Zhang, arXiv:2408.02190

Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, PNAS **122**, e2504261122 (2025)

From FL* to d-wave superconductivity and charge order in the ALM

Maine Christos
Caltech



Pietro Bonetti

Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

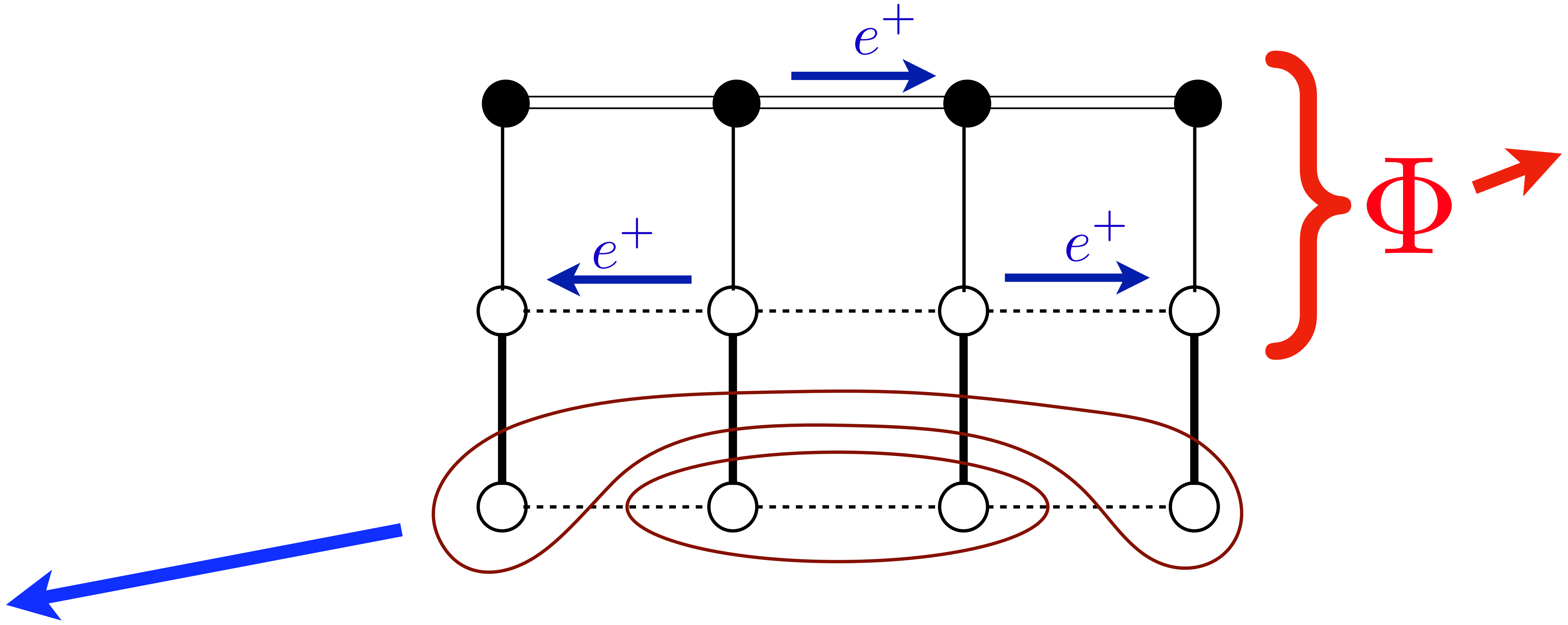
Harshit Pandey

Ravi Shanker

Sayantan Sharma

The Institute of Mathematical Sciences, Chennai

Ancilla Layer Model of the Hubbard model



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M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

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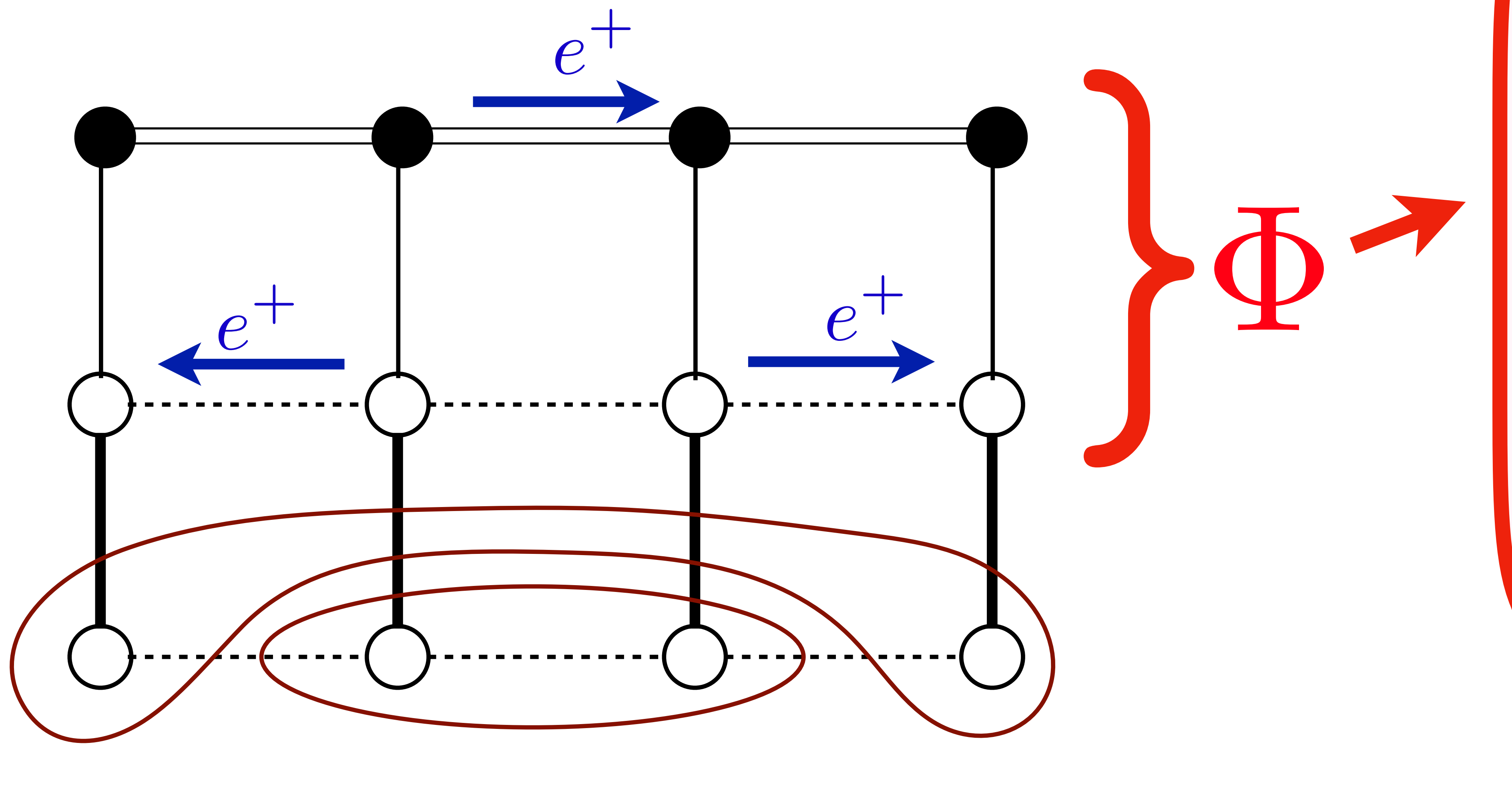

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- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.

I. Affleck and J. B. Marston, PRB **37**, 3774 (1988).
E. Dagotto, E. Fradkin, A. Moreo, PRB **38**, 2926 (1988).

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
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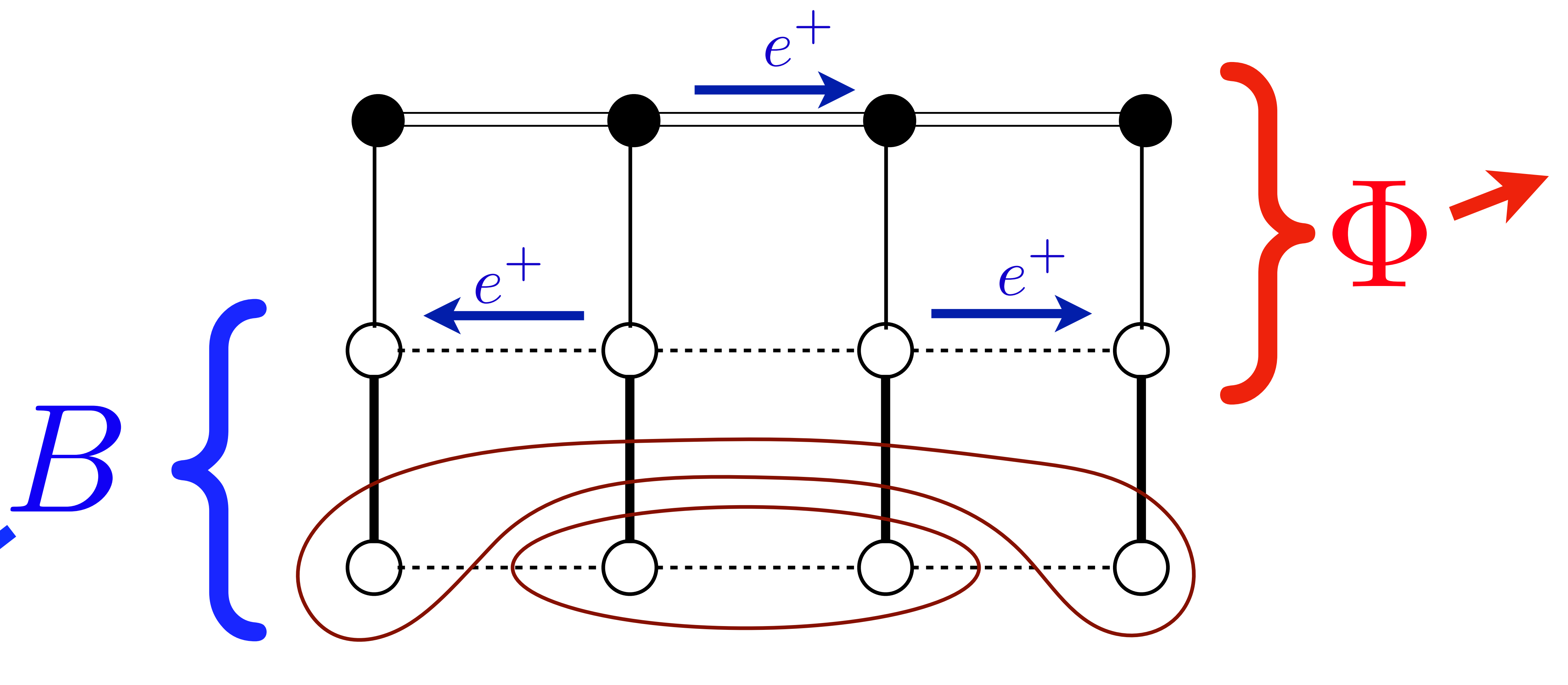

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M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
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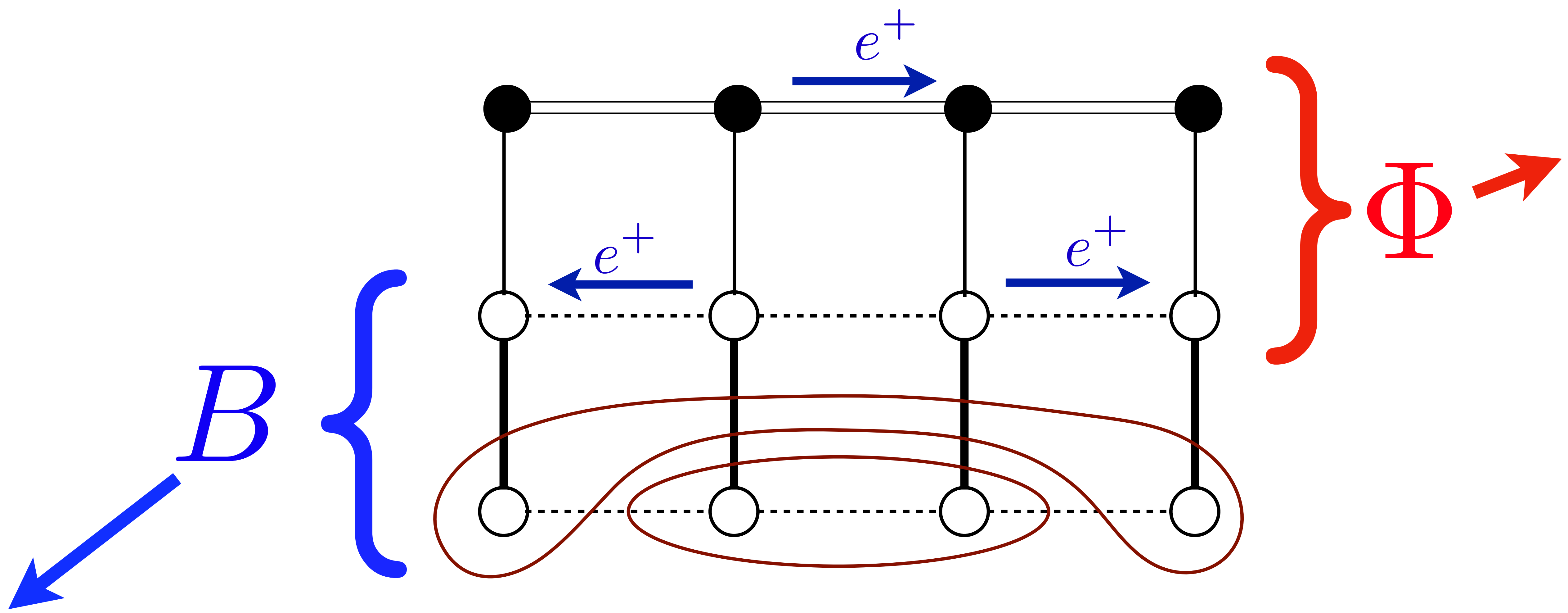

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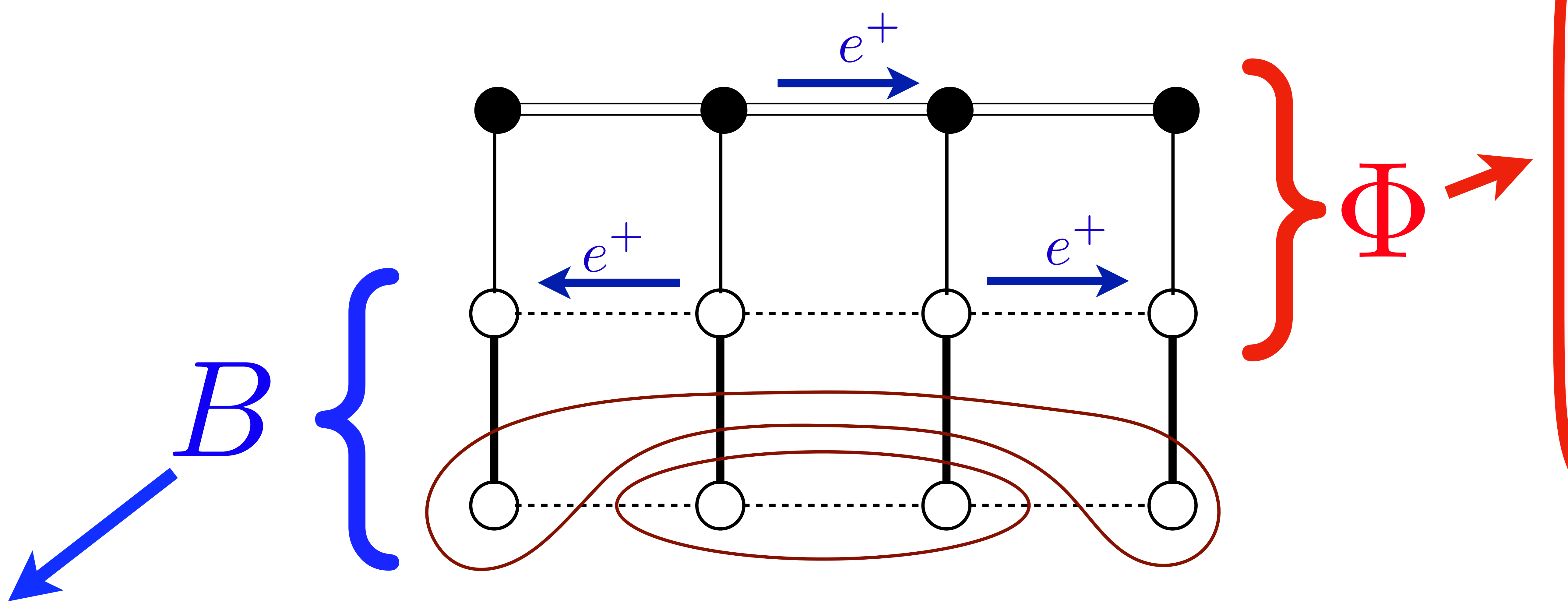
- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Ancilla Layer Model of the Hubbard model



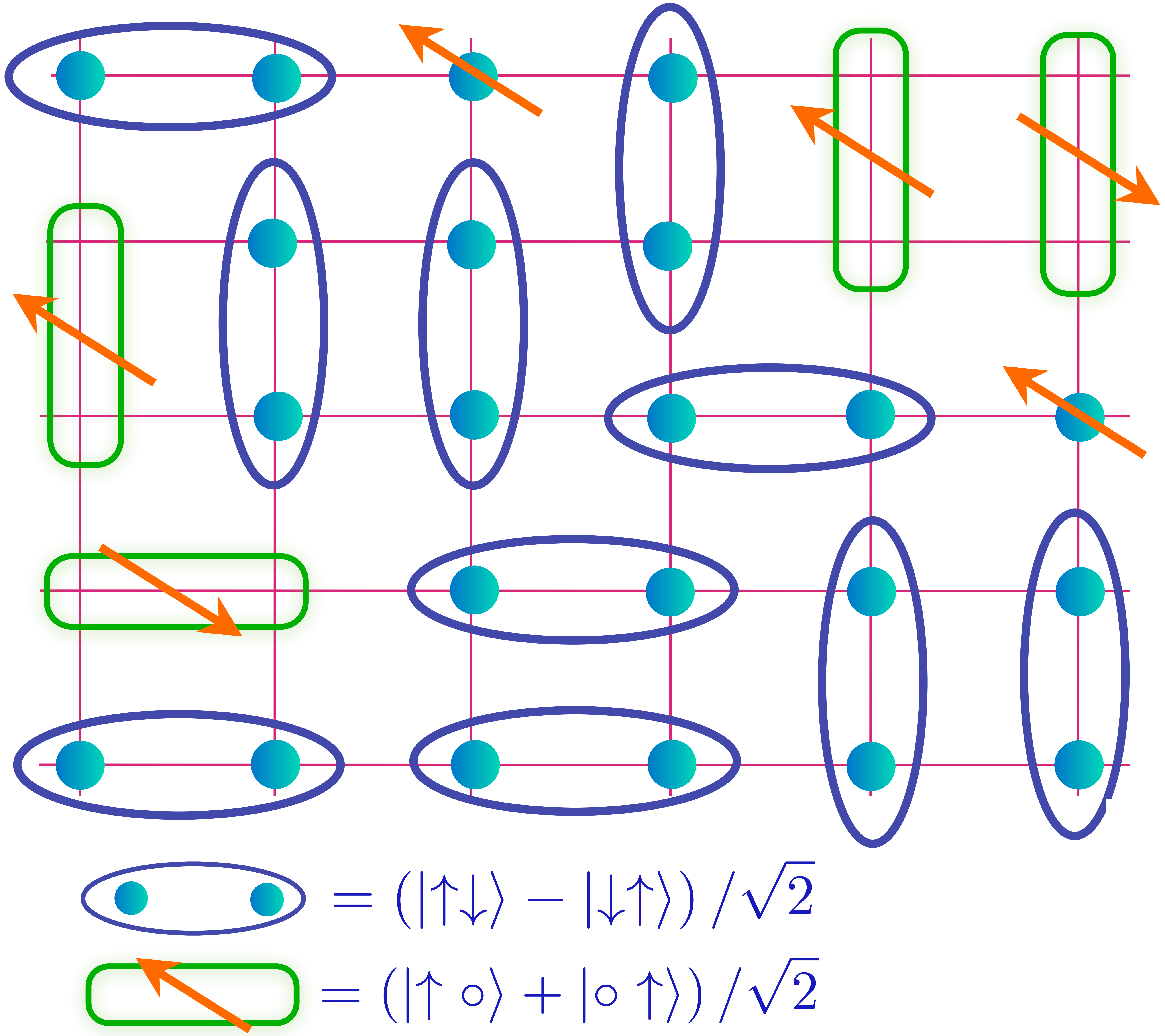
Higgs field Φ determines the pseudogap.
In FL* $\langle \Phi \rangle \neq 0$, antinodal pseudogap is determined by $\langle \Phi \rangle$, and electrons c_α are in 4 area $p/8$ hole pockets.

- Spinons f_α in bottom layer are in a π-flux spin liquid with a SU(2) gauge field U.
- Higgs boson B has charge e, and is a SU(2) fundamental.
- Yukawa coupling between c_α, f_α and B.
- B is a **fractionalized order parameter**, whose composites describe numerous superconducting and charge order parameters!

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

The cuprate phase diagram

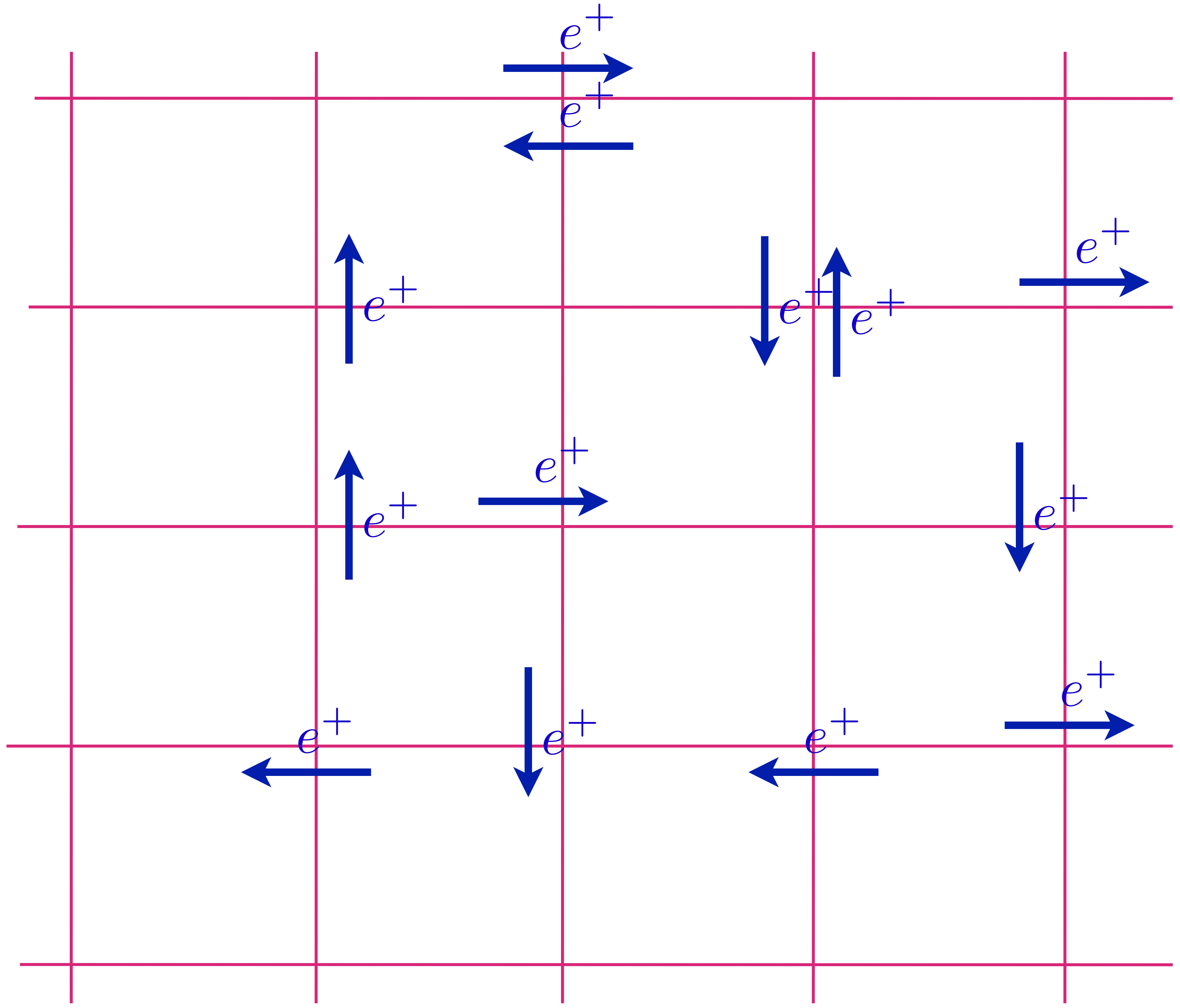
FL*



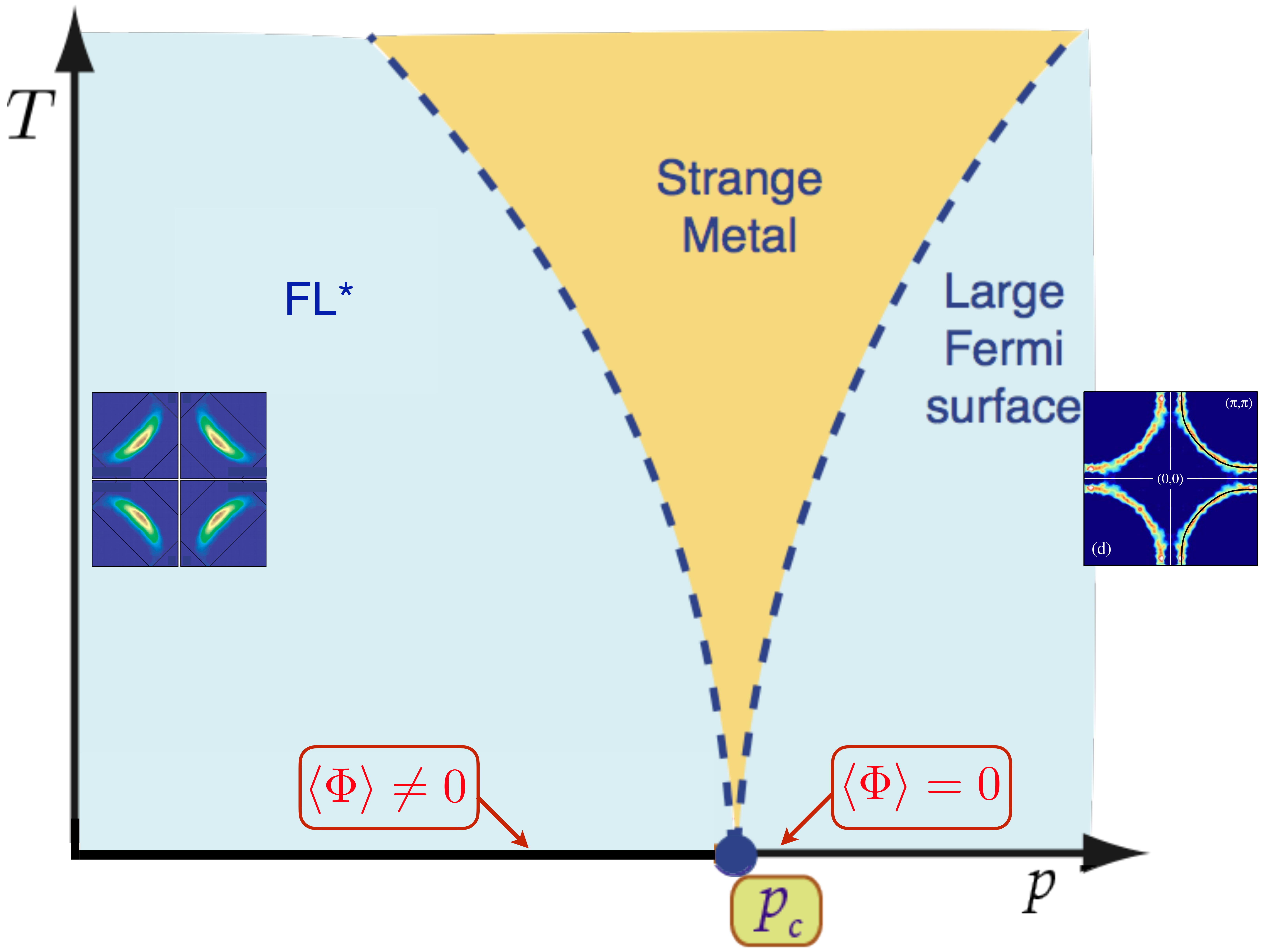
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

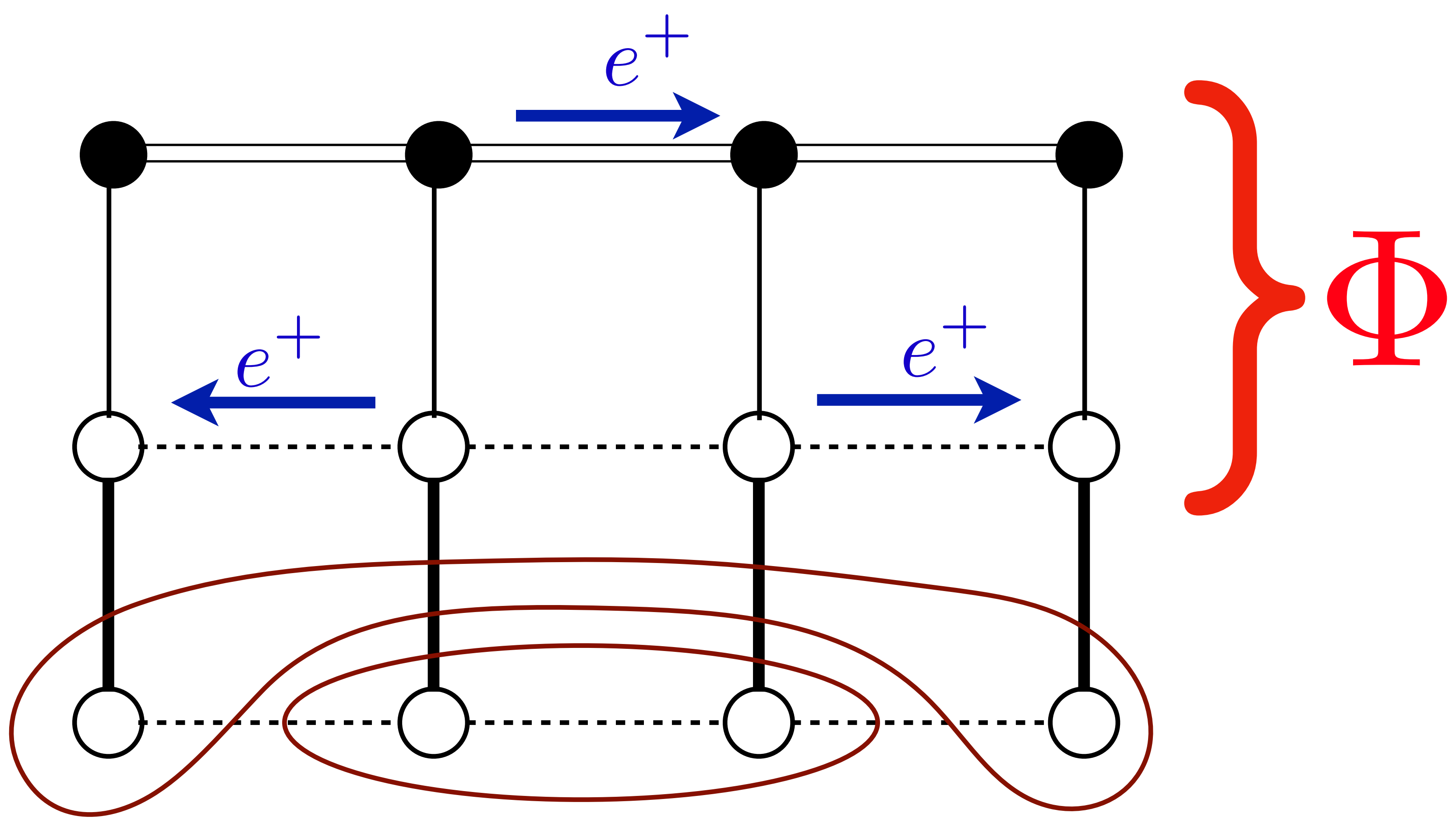
Ordinary metal



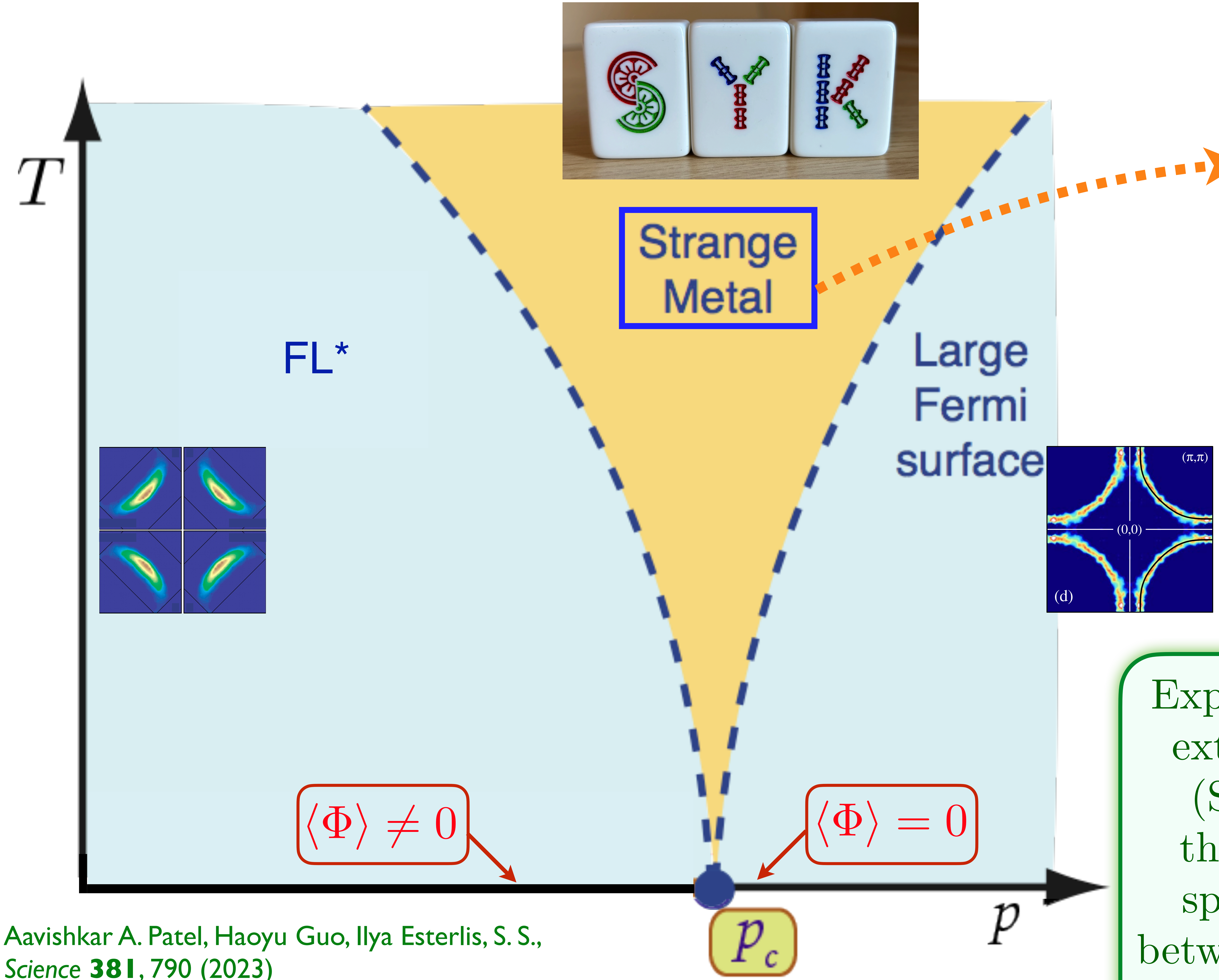
At large p, we obtain a gas of nearly free fermionic holes of density $1+p$ (relative to the filled band with 2 electrons per site)

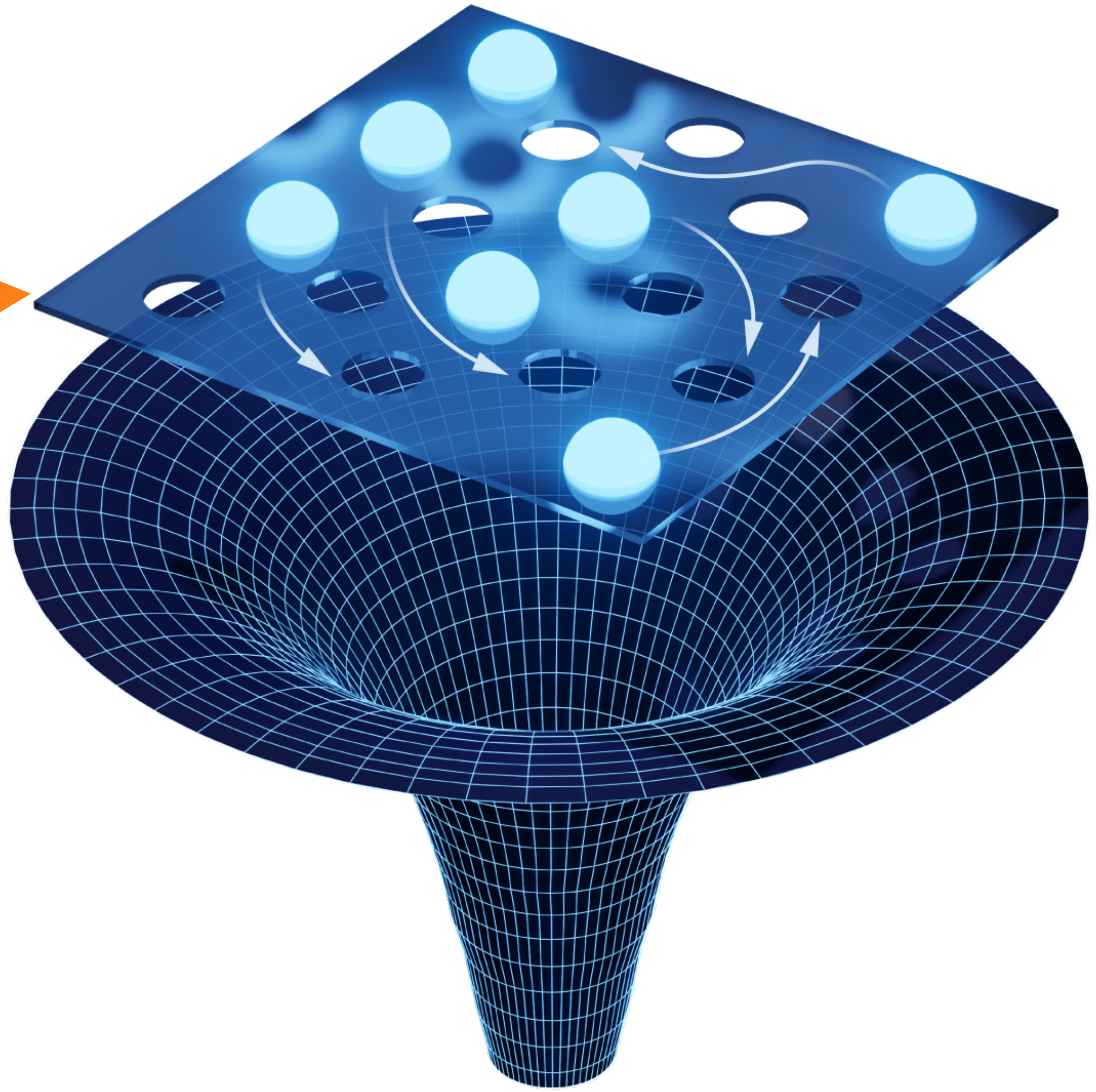


Quantum-criticality
of a quantum phase transition
between two metals
(FL* and FL) at $p = p_c$,
with no symmetry breaking.



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

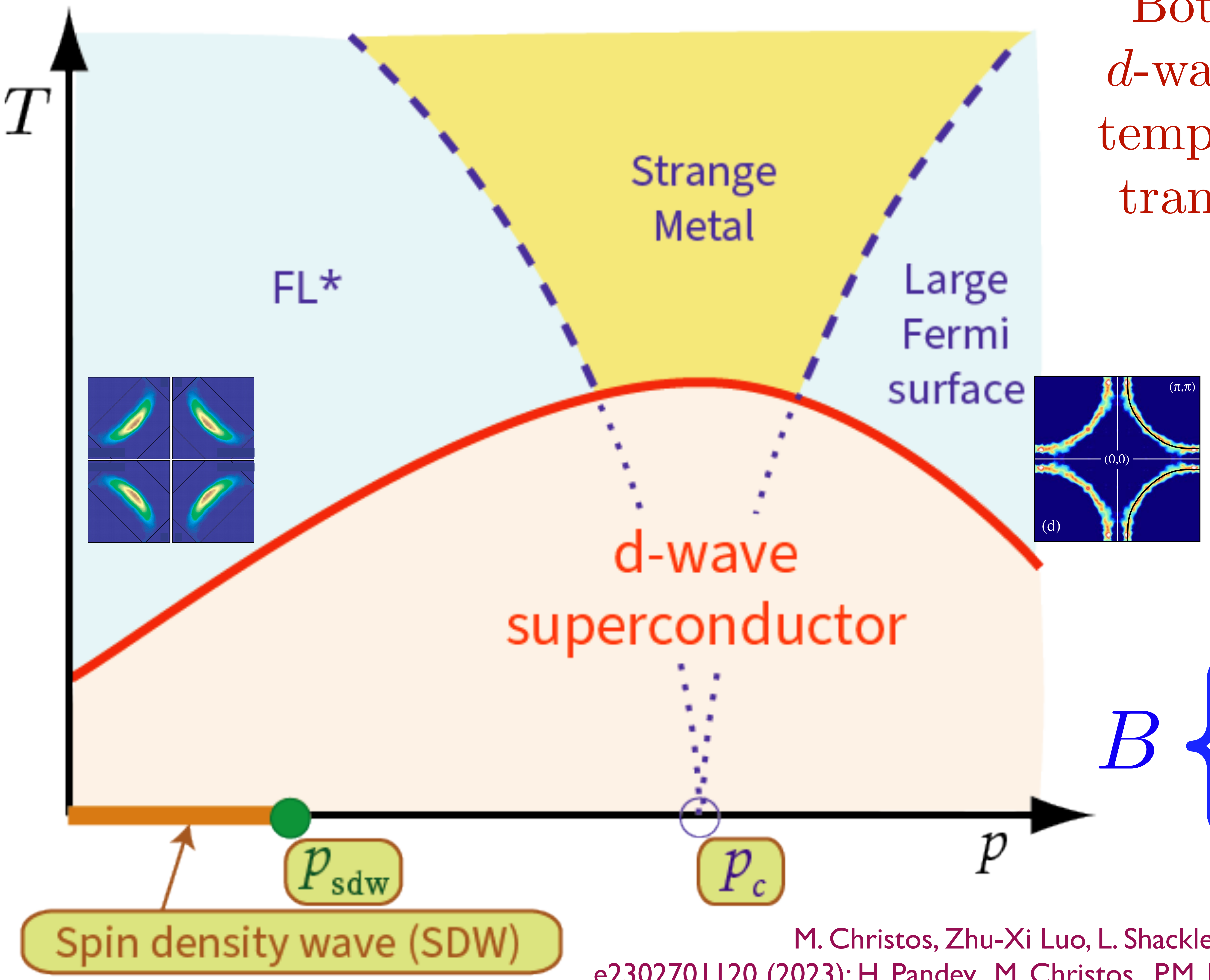




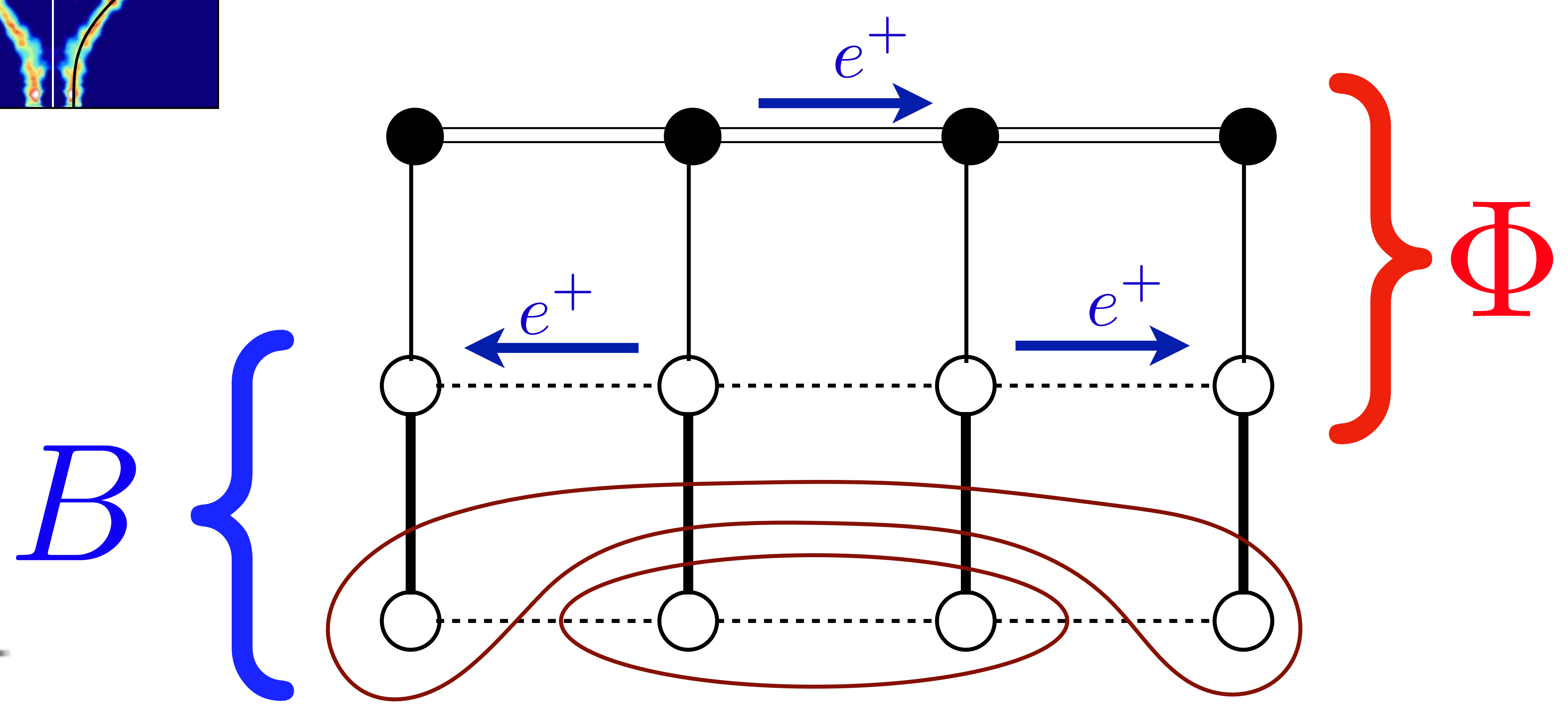
Explain with a local, two-dimensional extension of the Sachdev-Ye-Kitaev (SYK) model of mobile electrons, the 2D-Yukawa-SYK model with a spatially random Yukawa coupling between the large Fermi surface and Φ: a critical charge liquid

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)

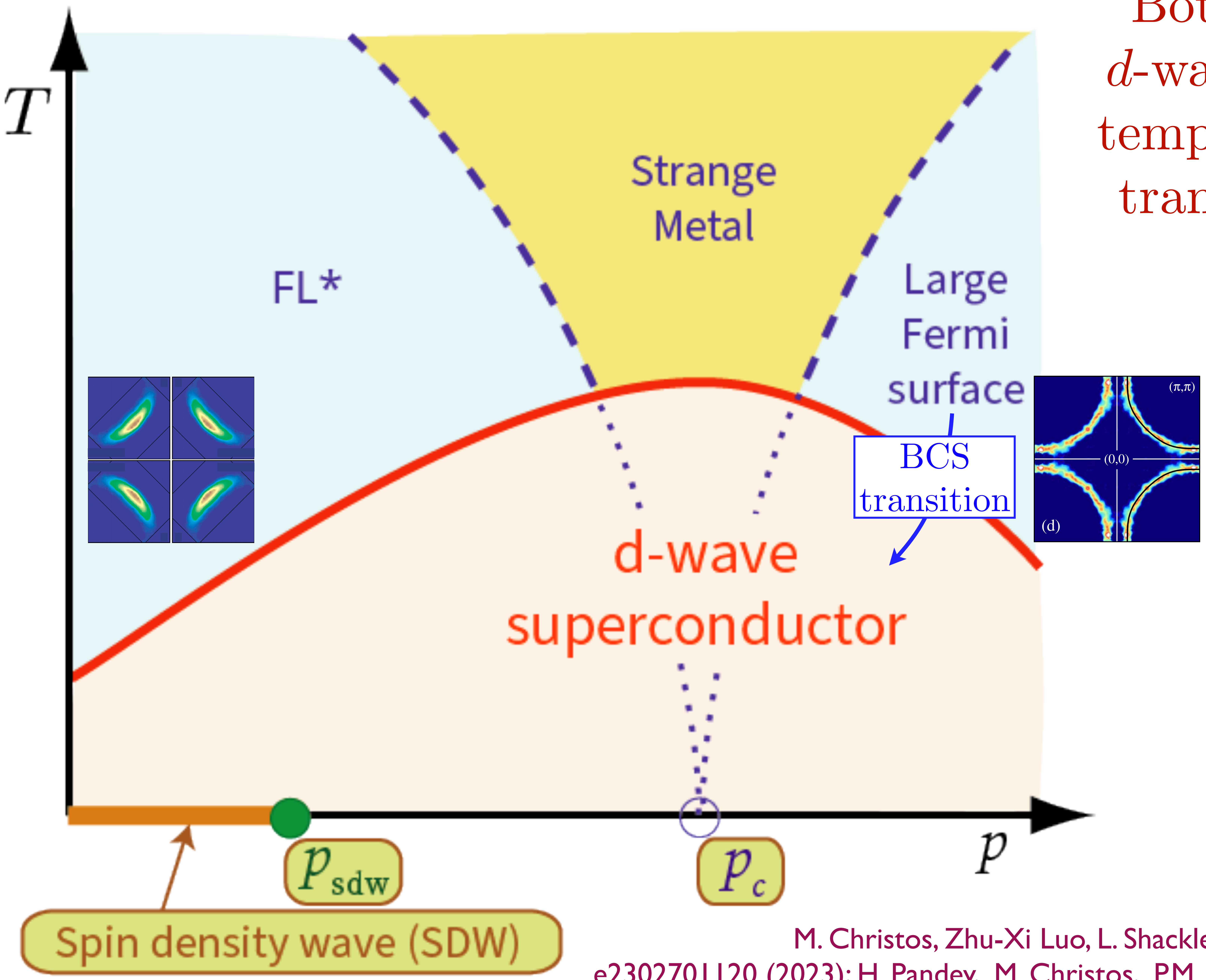


Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.



M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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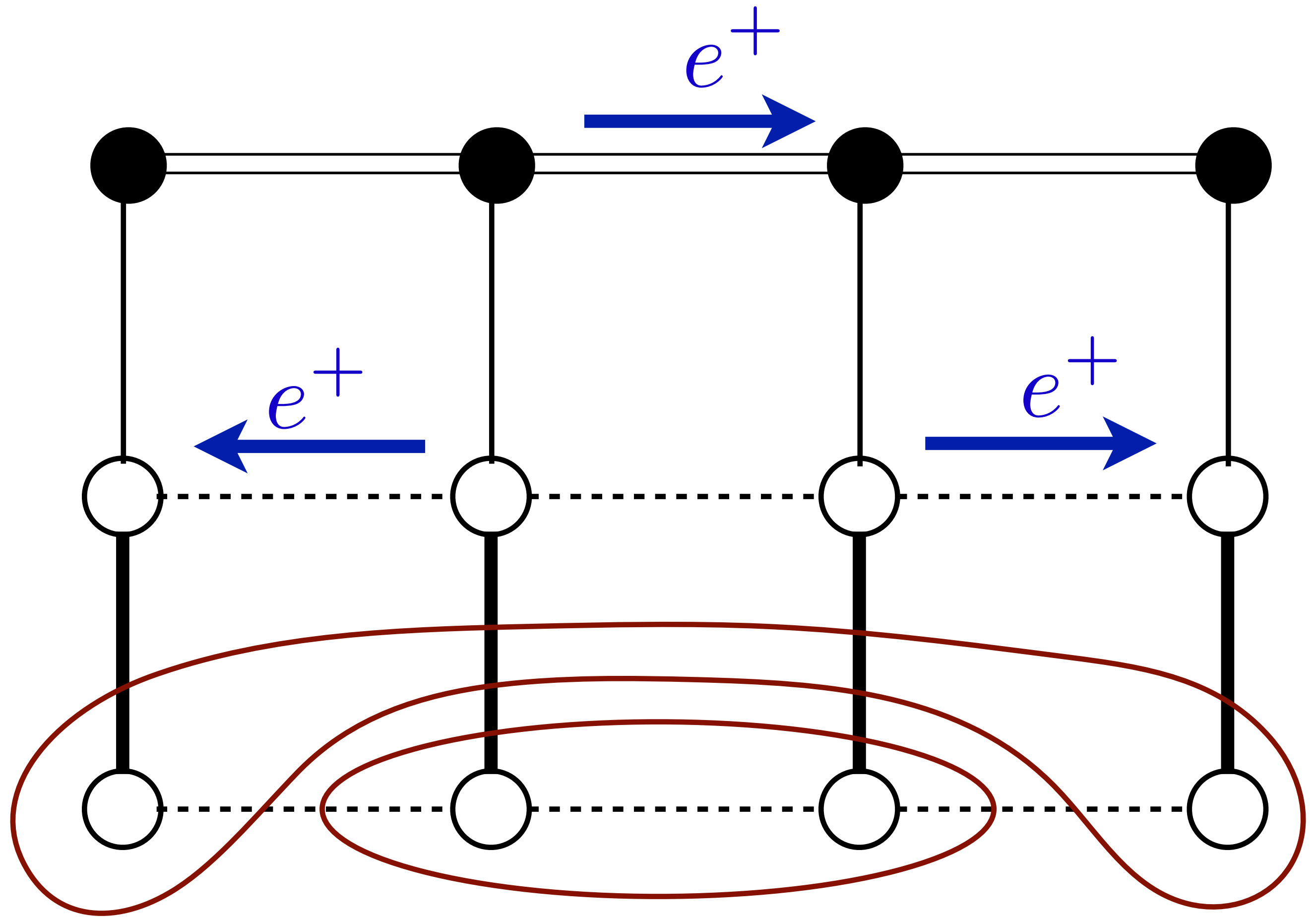
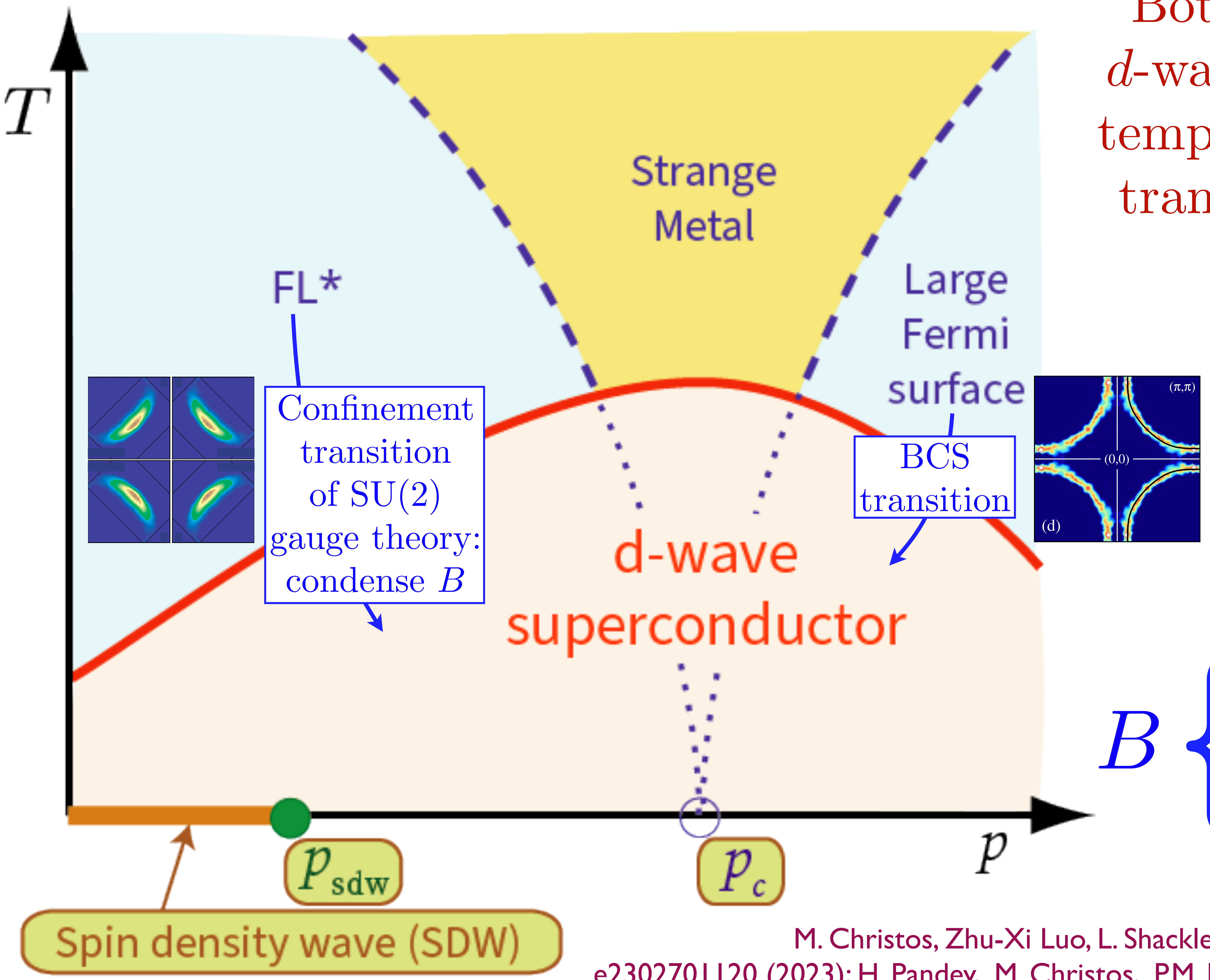


M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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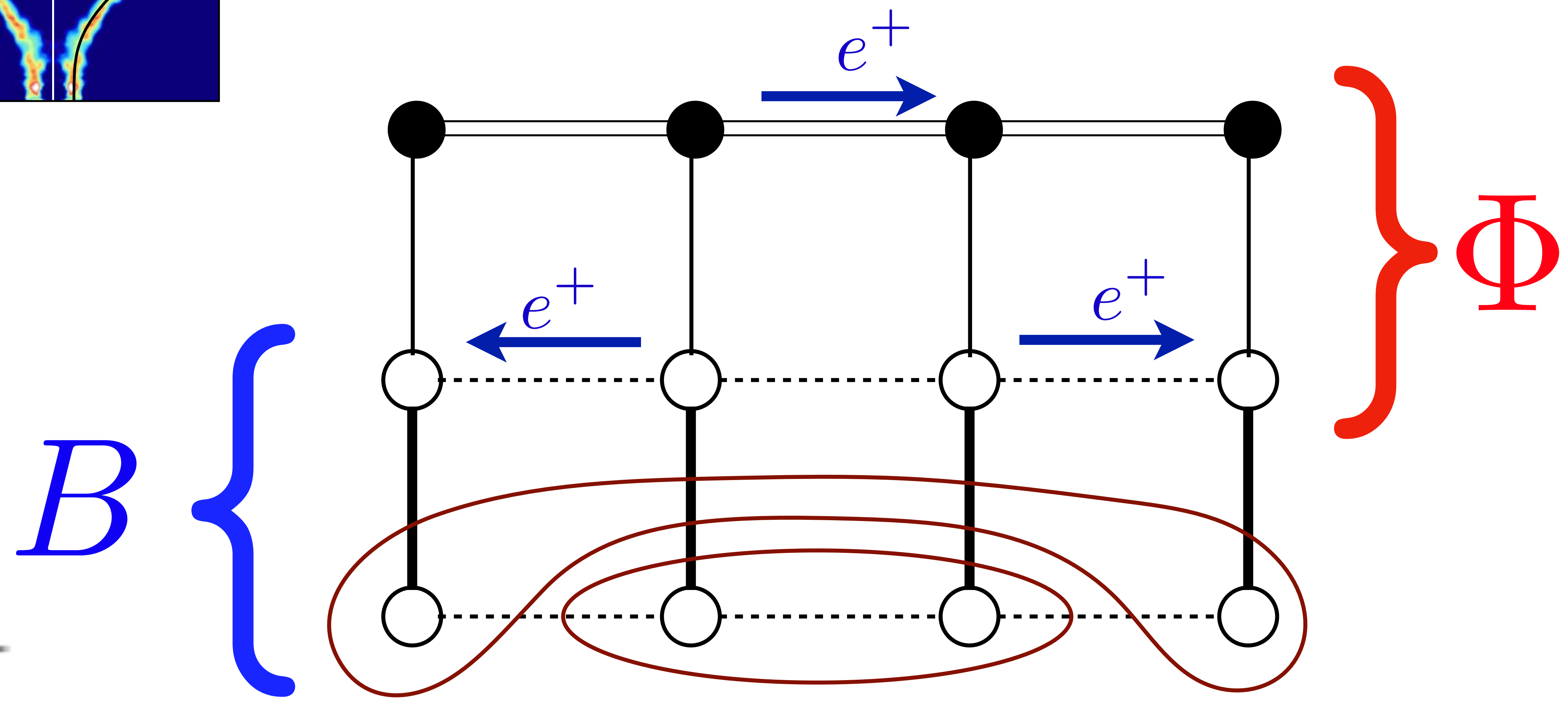
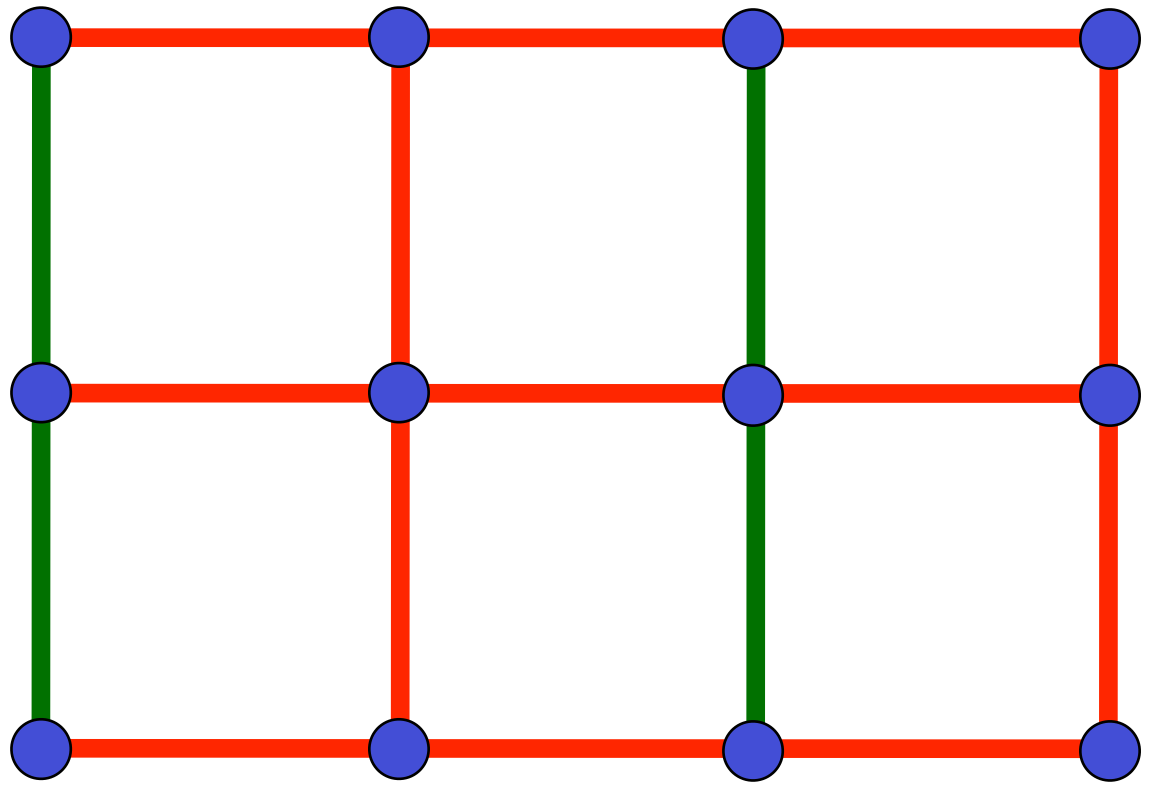


M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Monte-Carlo of Born-Oppenheimer approx.

Classical B, U, quantum c_α, f_α.

f_α and B both move in π-flux



Symmetry	f_α	B_a
T_x	$(-1)^y f_\alpha$	$(-1)^y B_a$
T_y	f_α	B_a
P_x	$(-1)^x f_\alpha$	$(-1)^x B_a$
P_y	$(-1)^y f_\alpha$	$(-1)^y B_a$
P_{xy}	$(-1)^{xy} f_\alpha$	$(-1)^{xy} B_a$
$\mathcal{T}$	$(-1)^{x+y} \varepsilon_{\alpha\beta} f_\beta$	$(-1)^{x+y} B_a$

Projective transformations of the f spinons and B chargons on lattice sites $\boldsymbol{i} = (x, y)$ under the symmetries $T_x : (x, y) \to (x+1, y)$; $T_y : (x, y) \to (x, y+1)$; $P_x : (x, y) \to (-x, y)$; $P_y : (x, y) \to (x, -y)$; $P_{xy} : (x, y) \to (y, x)$; and time-reversal $\mathcal{T}$. The indices α, β refer to global SU(2) spin, while the index $a = 1, 2$ refers to gauge SU(2).

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

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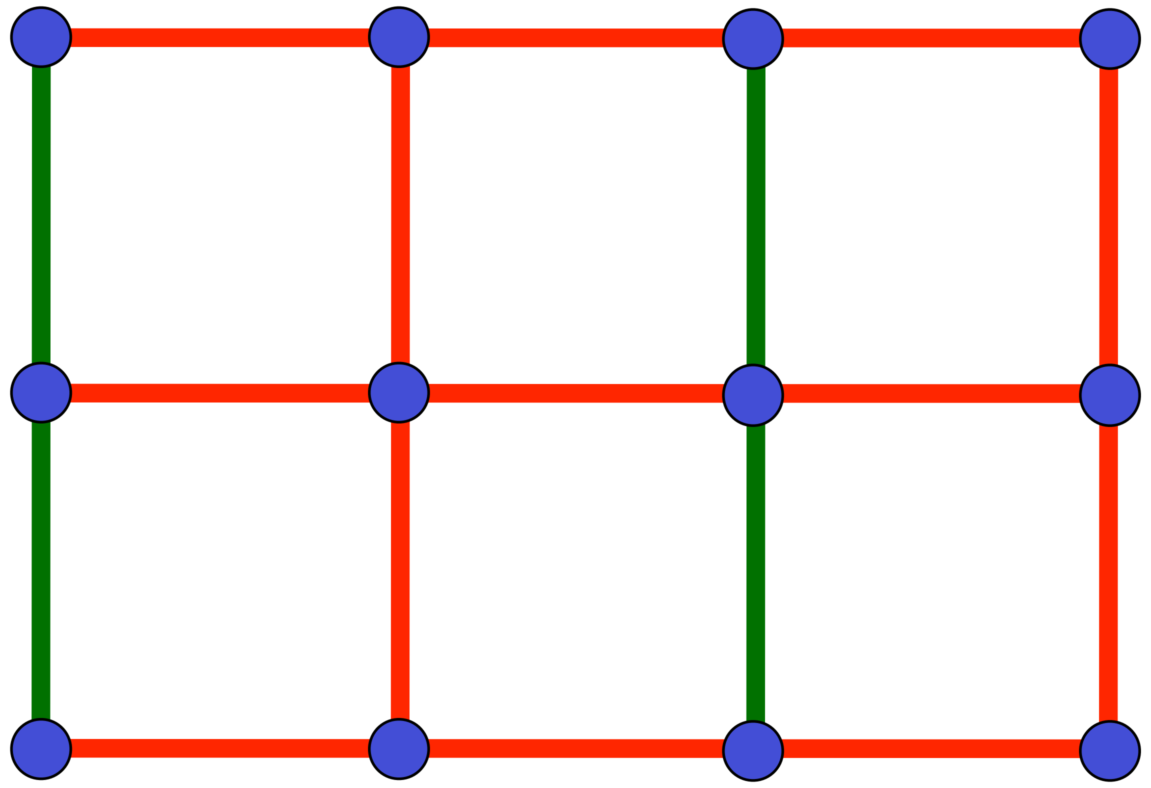


Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim$

$$\Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$$

site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

bond current: $i \left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle$

$$\sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}} \right)$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

Energy functional for B and U: $\mathcal{E}[B,U] = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U] + \mathcal{E}_{YM}[U]$

$$\mathcal{E}_2[B,U] = (r + 2\sqrt{2}w) \sum_{\boldsymbol{i}} B_{\boldsymbol{i}}^{\dagger} B_{\boldsymbol{i}} + iw \sum_{\langle \boldsymbol{i}\boldsymbol{j} \rangle} e_{\boldsymbol{i}\boldsymbol{j}} \left(B_{\boldsymbol{i}}^{\dagger} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}} - B_{\boldsymbol{j}}^{\dagger} U_{\boldsymbol{j}\boldsymbol{i}} B_{\boldsymbol{i}} \right)$$

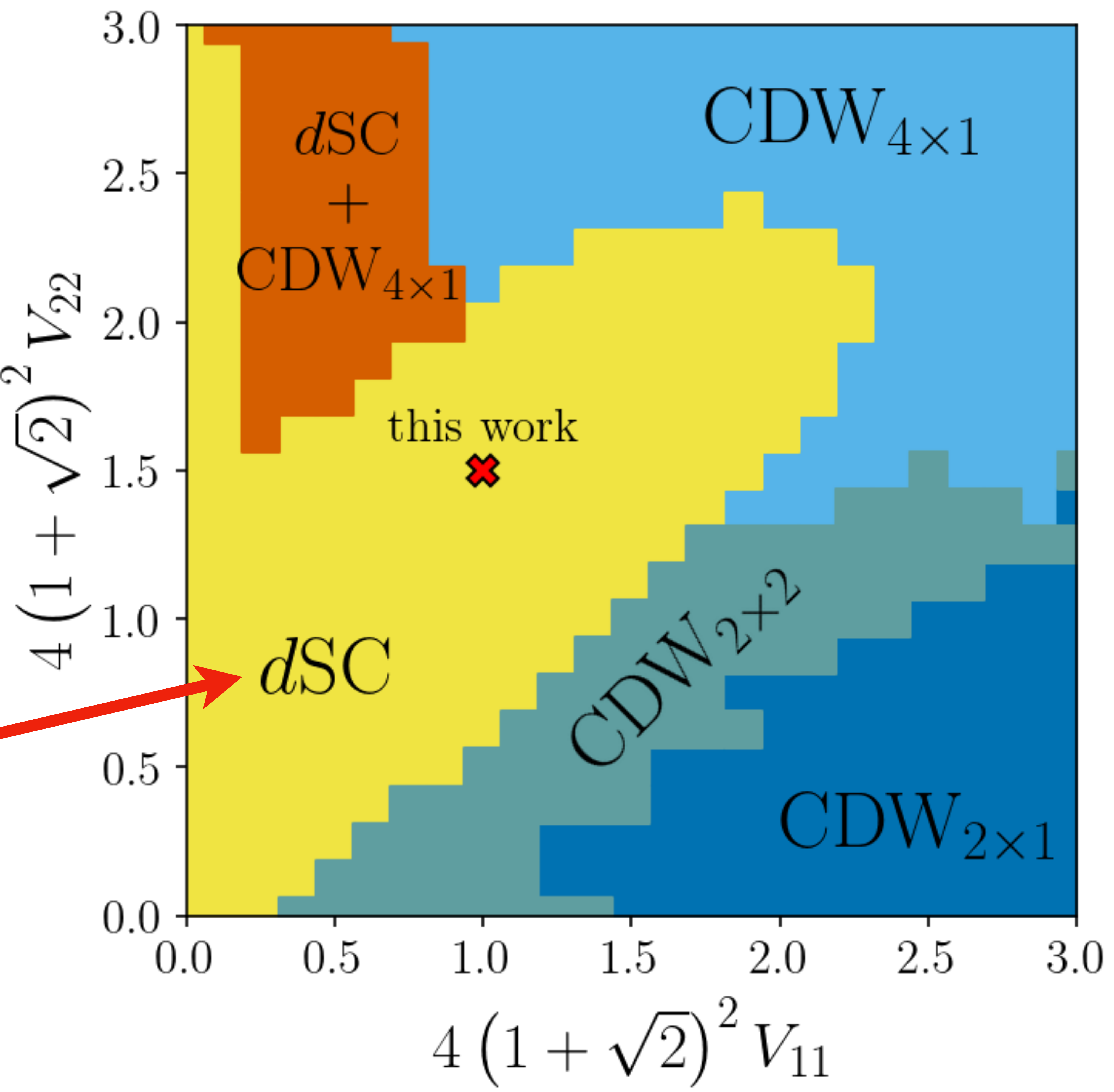
$$\mathcal{E}_4[B,U] = \frac{u}{2} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}} \right) + g \sum_{\langle \boldsymbol{i}\boldsymbol{j} \rangle} |\Delta_{\boldsymbol{i}\boldsymbol{j}}|^2 + J_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j} \rangle} Q_{\boldsymbol{i}\boldsymbol{j}}^2 + K_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j} \rangle} J_{\boldsymbol{i}\boldsymbol{j}}^2$$
$$+ V_{11} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}} \right) + V_{22} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}} \right)$$

$$\mathcal{E}_{YM}[U] = \kappa \sum_{\square} \left[1 - \frac{1}{2} \mathrm{ReTr} \prod_{\boldsymbol{i}\boldsymbol{j} \in \square} U_{\boldsymbol{i}\boldsymbol{j}} \right]$$

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023)

At $T = 0$, minimize $\mathcal{E}[B, U]$.

d-SC with
4 nodal quasiparticles
and $v_F \gg v_\Delta$.



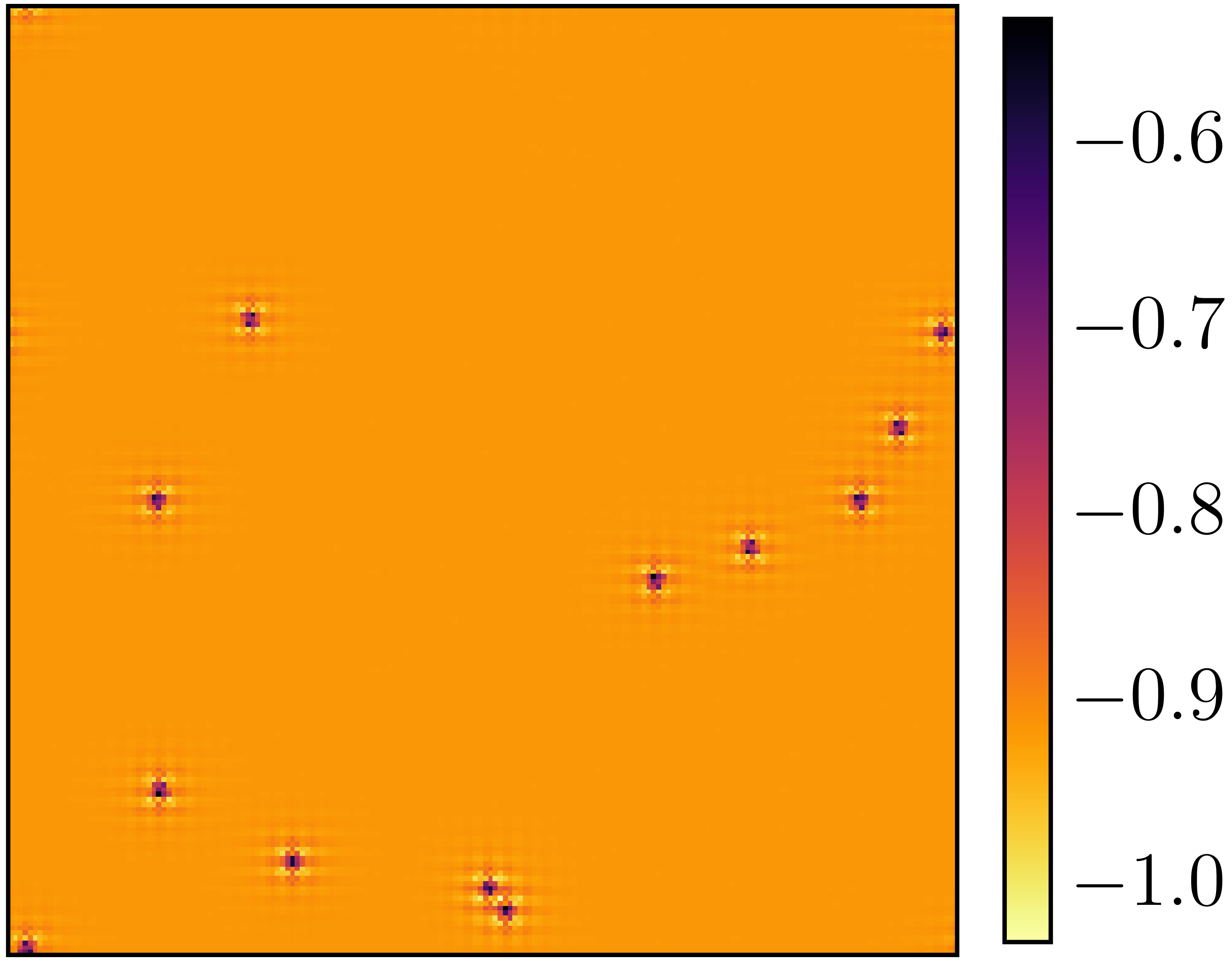
Parameters chosen so that the ground state is a d-wave superconductor,
and second best state is a period-4 stripe.

Monte Carlo at a temperature T

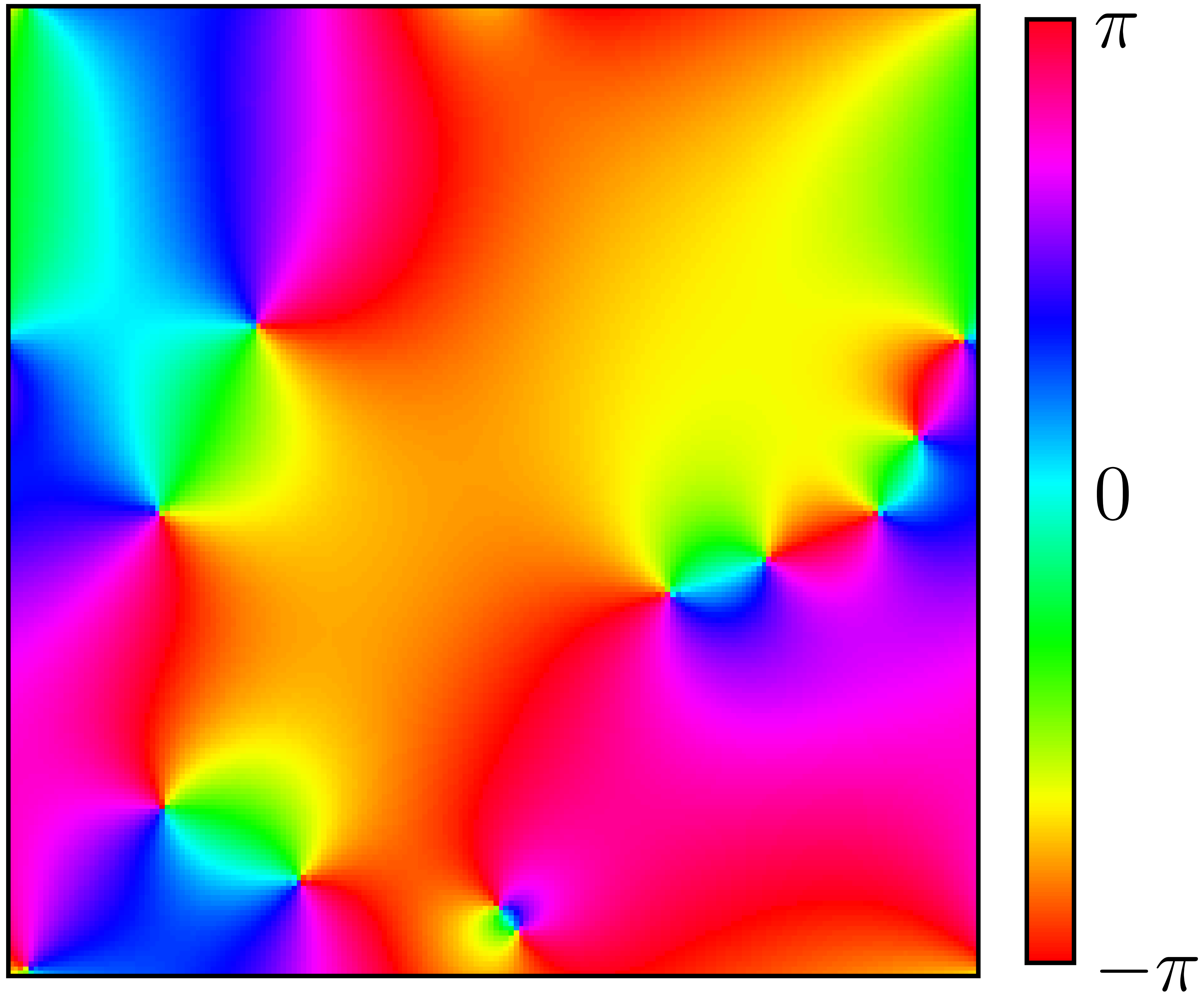
$$\mathcal{Z}_{2+0} = \int \prod_{i} \mathcal{D}B_{\boldsymbol{i}} \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[-\mathcal{E}[B,U]/T\right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.

Monte Carlo at a temperature T

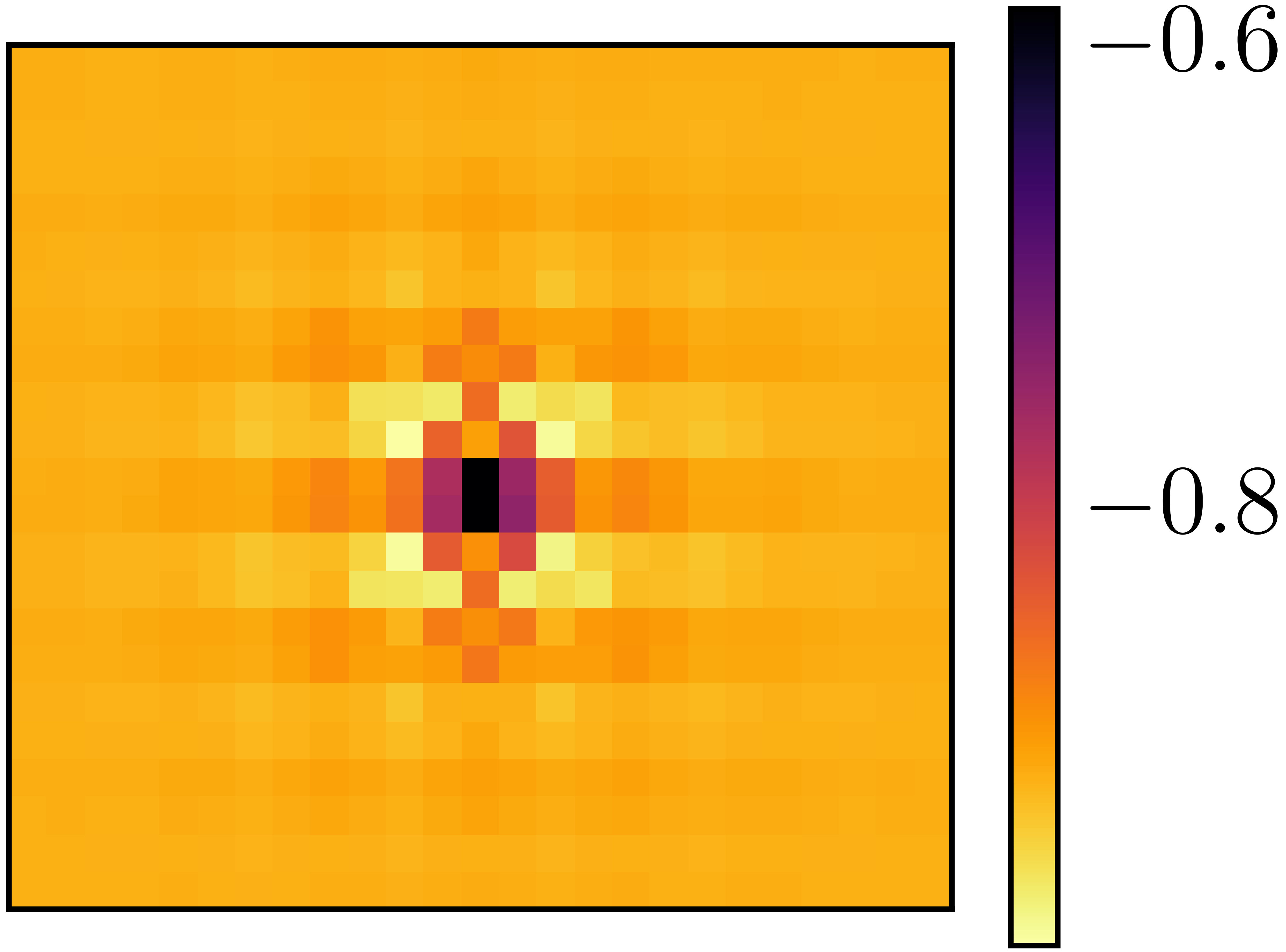


Bond density

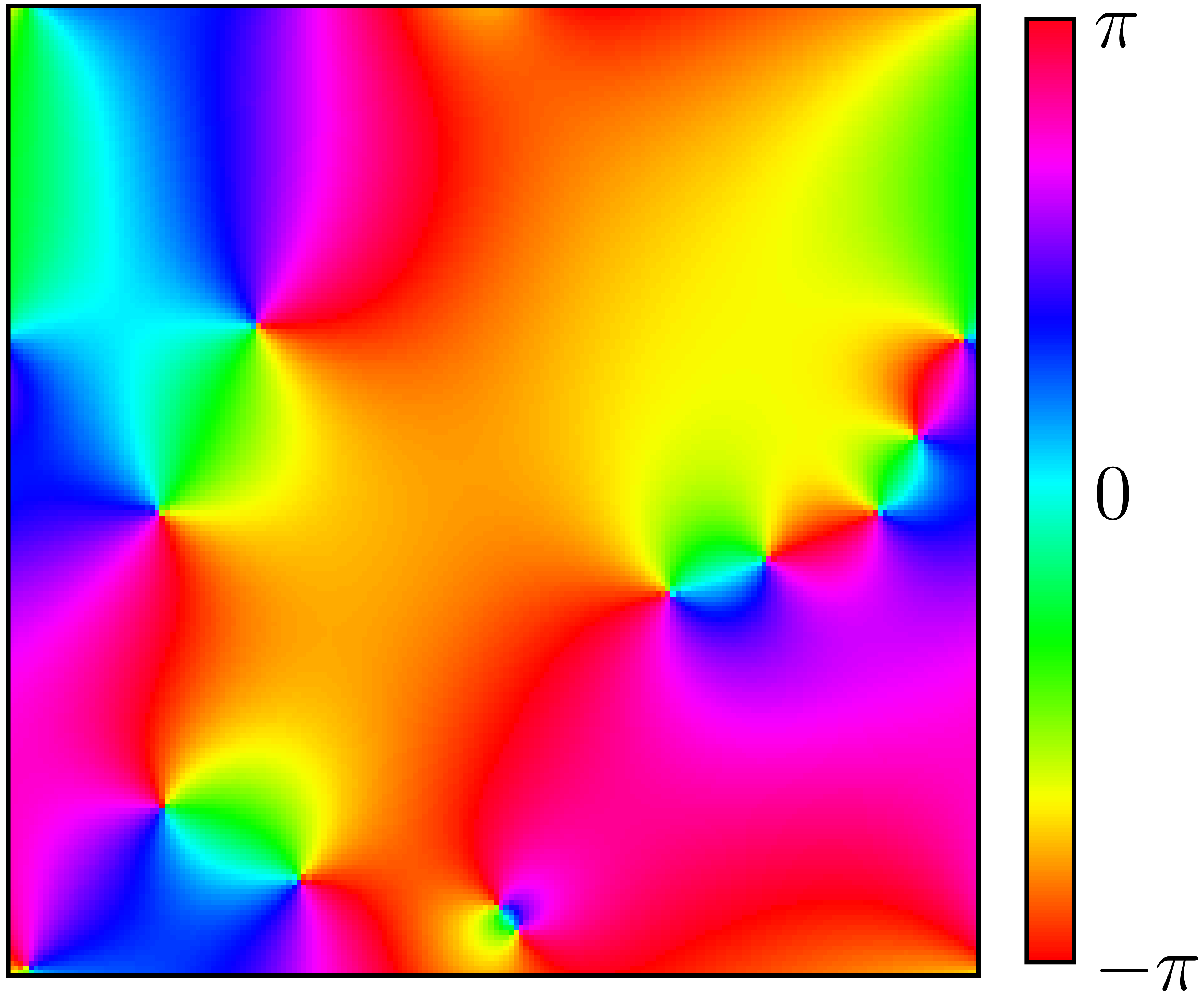


Phase of pairing amplitude

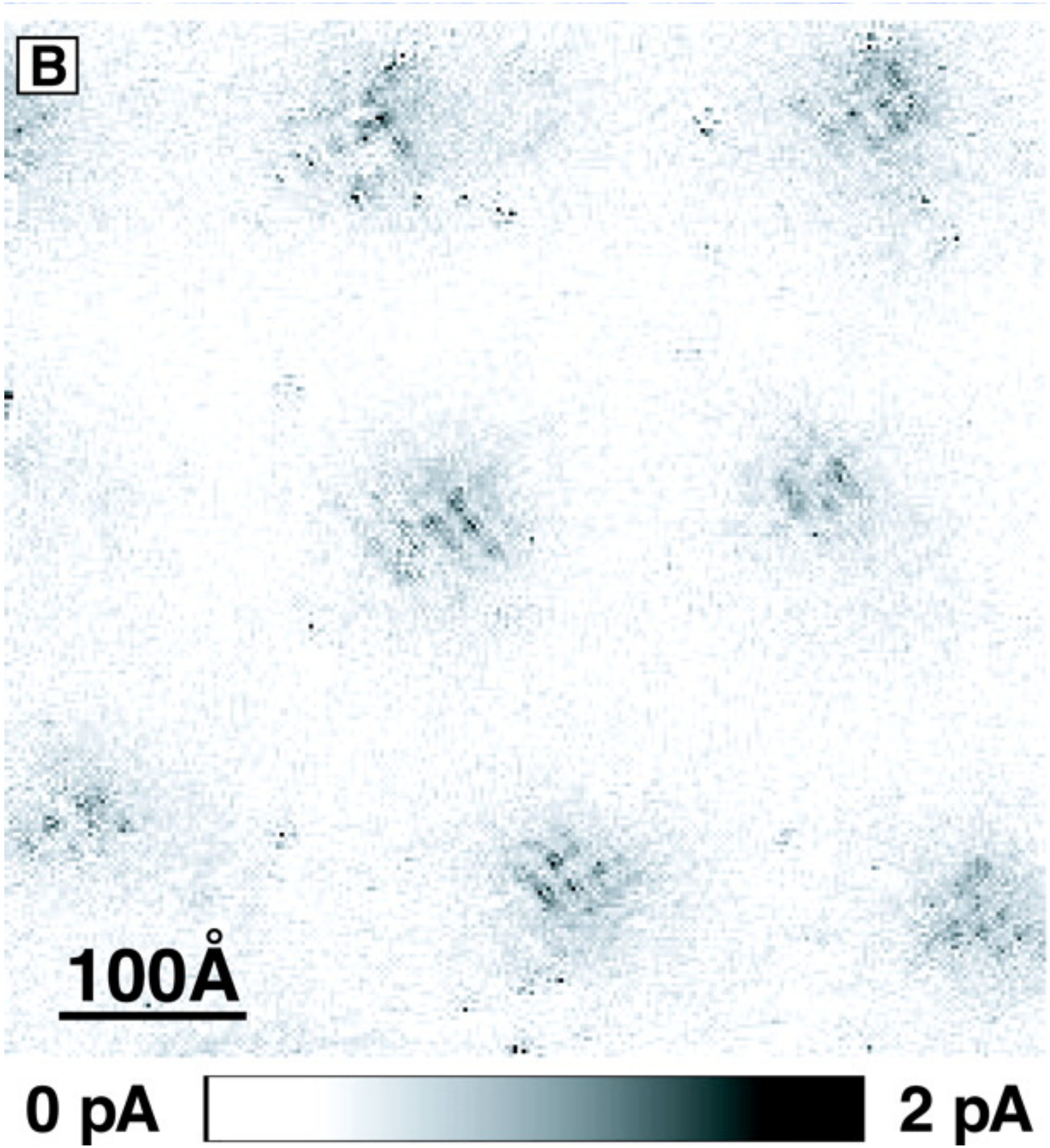
Monte Carlo at a temperature T



Bond density



Phase of pairing amplitude



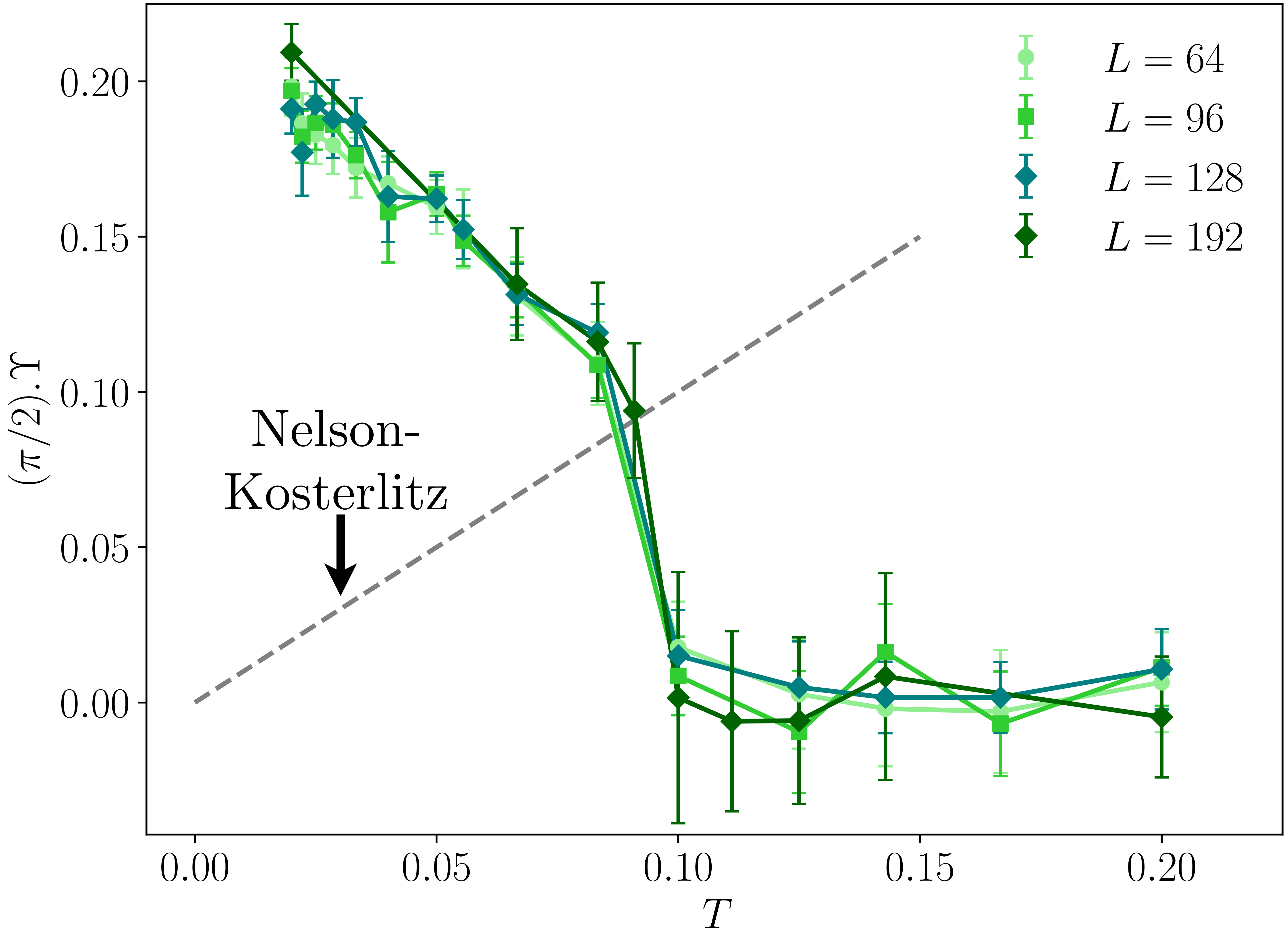
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

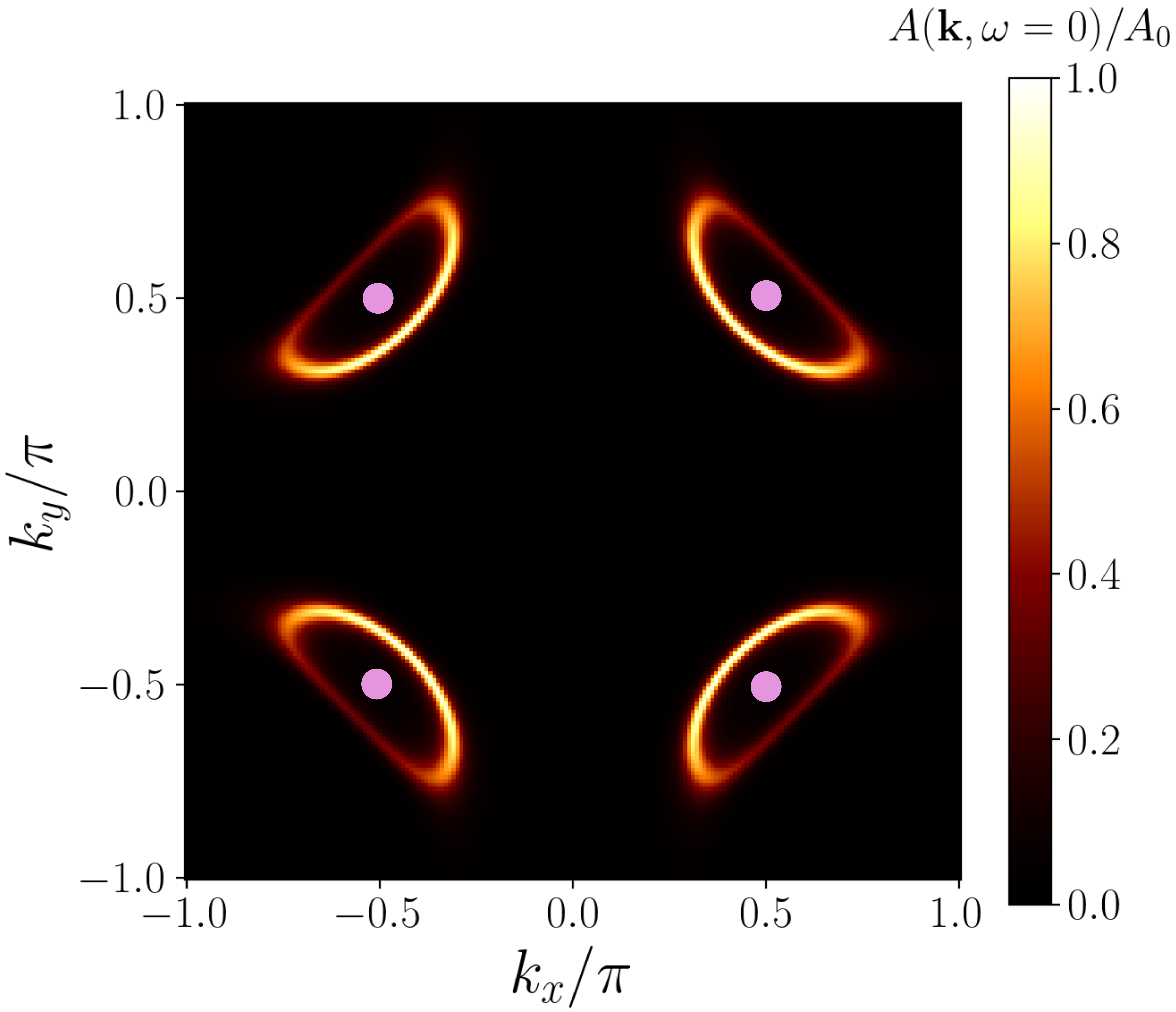
J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, H. Eisaki, S. Uchida, J.C. Davis

Science **295**, 466 (2002)

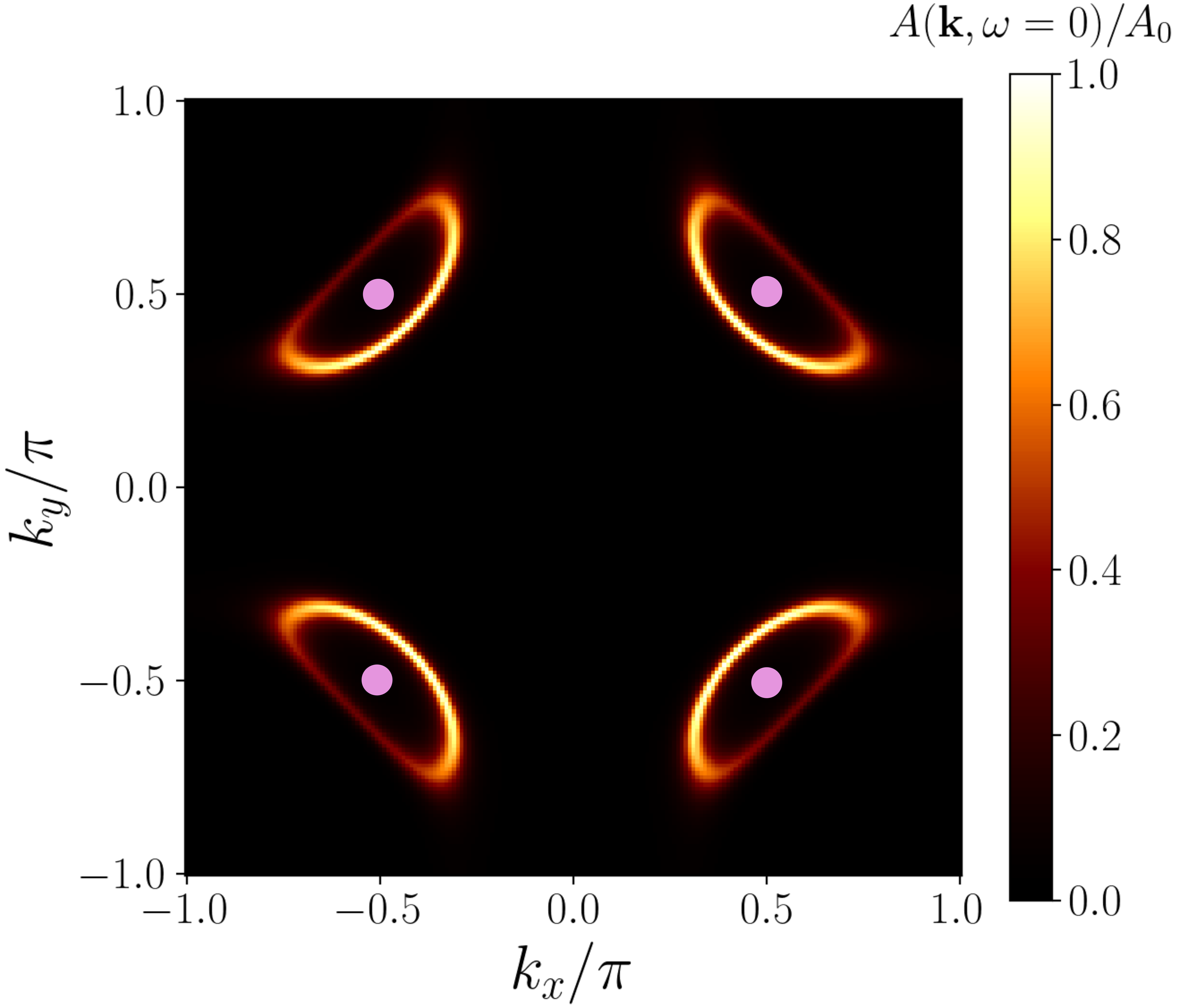
Monte Carlo at a temperature T

$\Upsilon =$
Helicity
Modulus





FL* fermionic spectrum with $B = 0$, $U = 1$
4 holes pockets of size $p/8$;
4 nodal spinons



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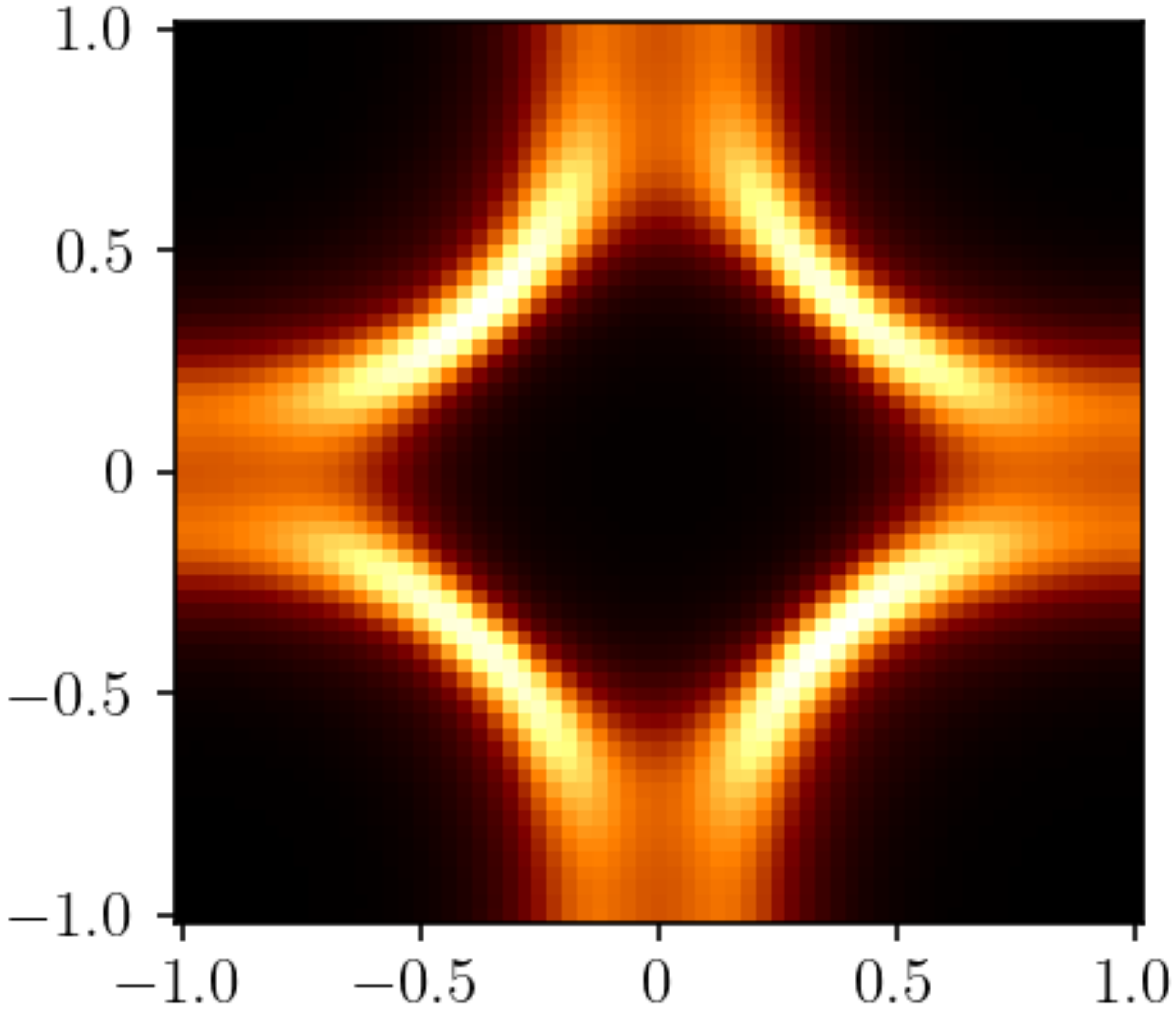
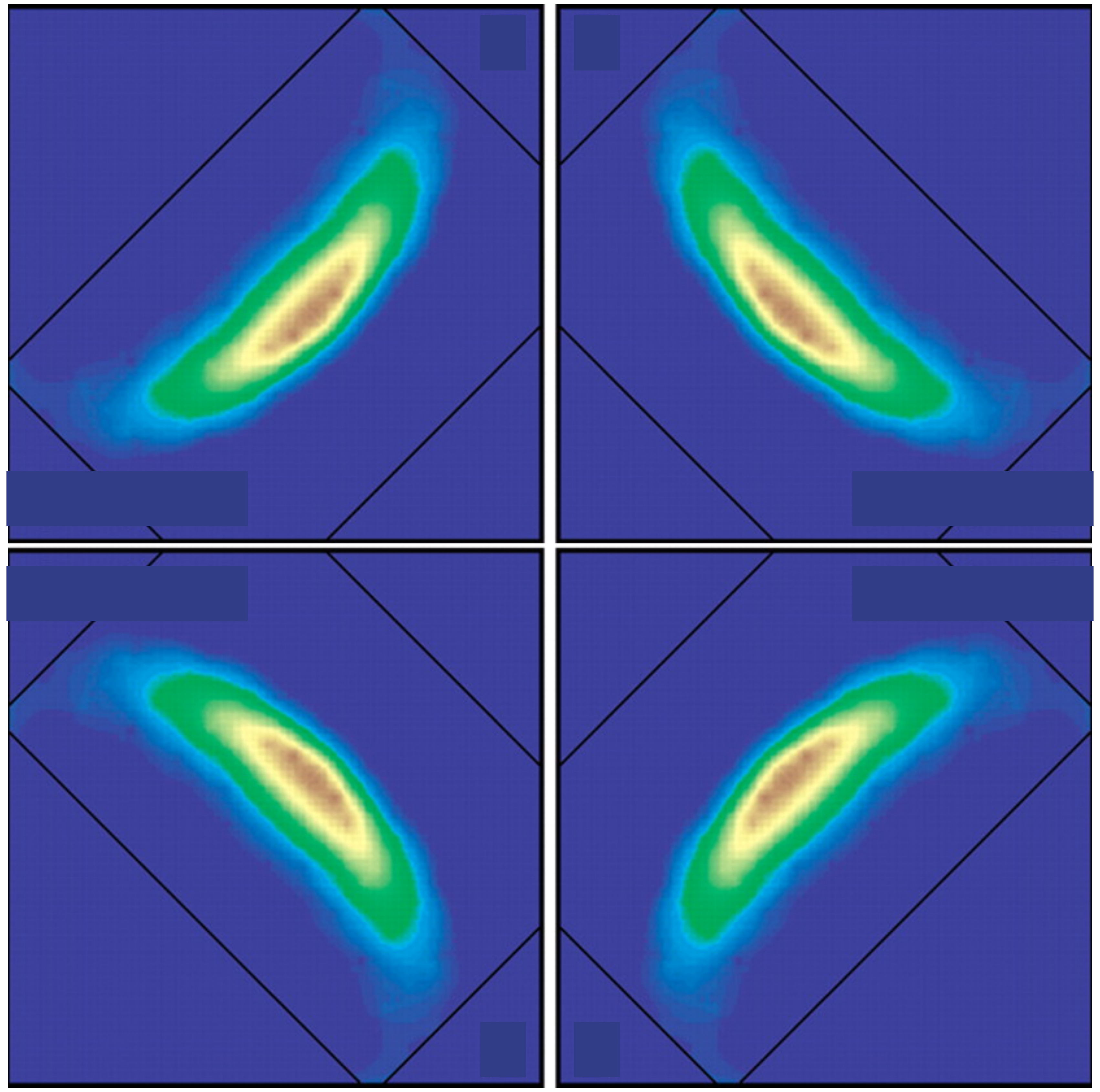
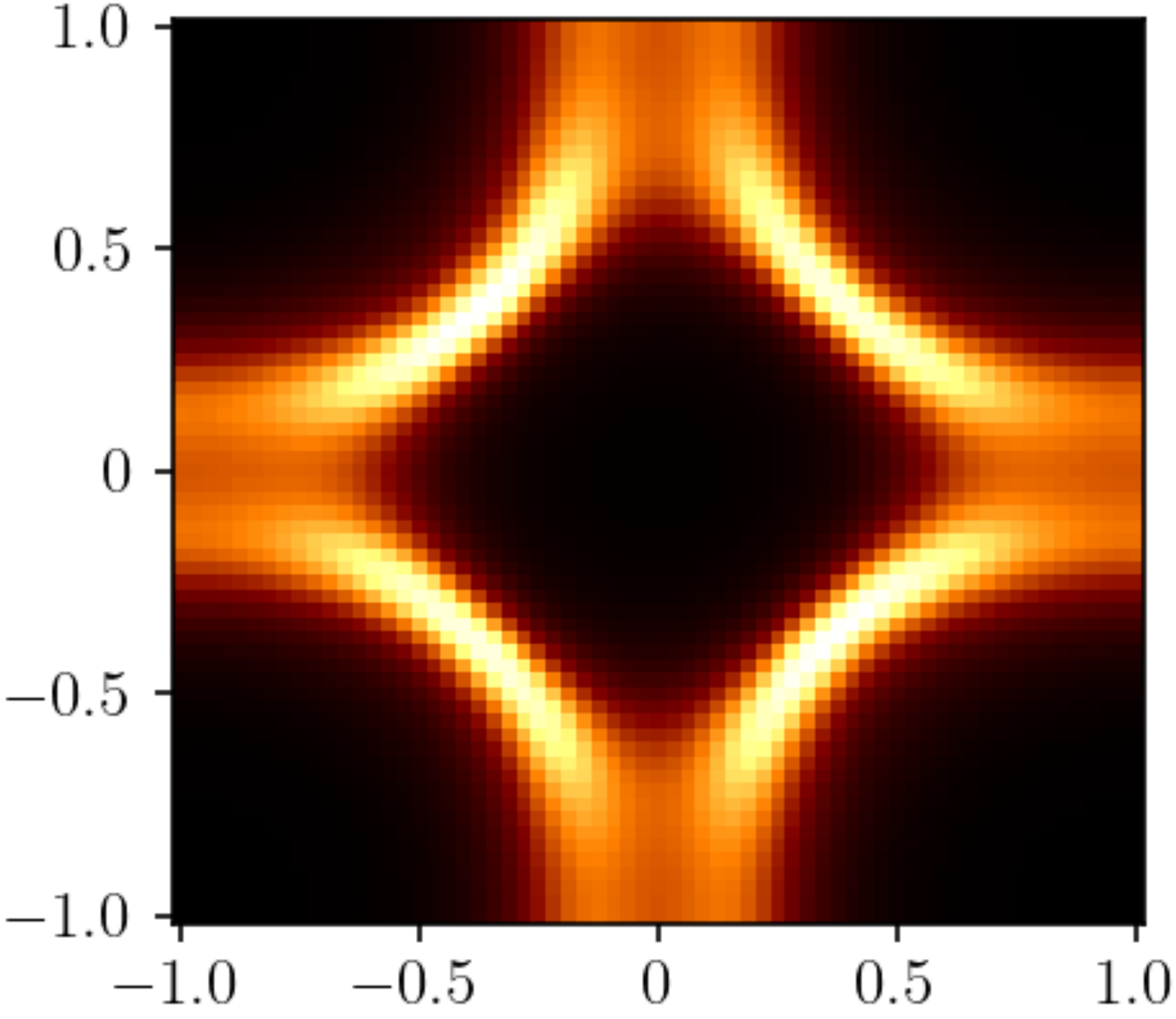


Monte Carlo at a temperature $T > T_{KT}$

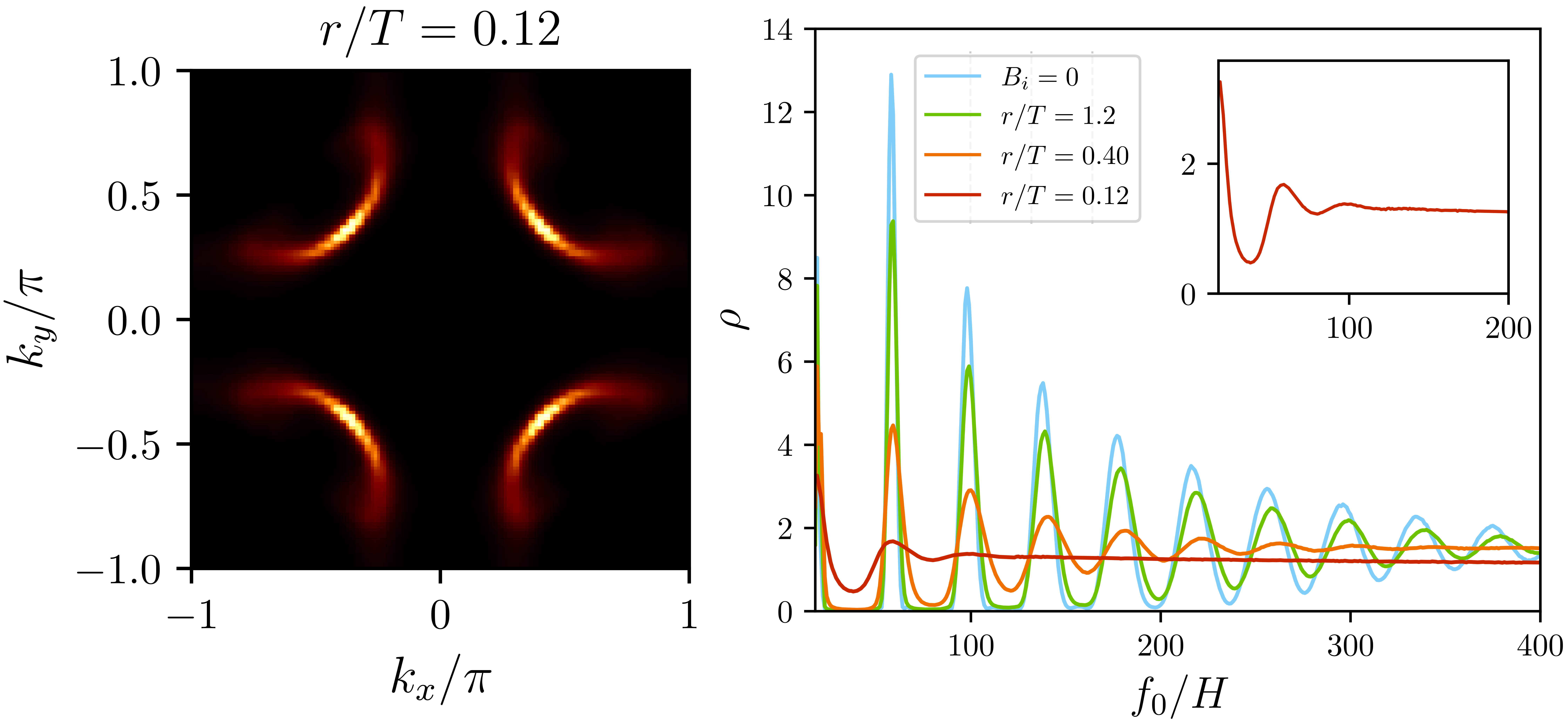


Kyle M. Shen, . . . Z.-X. Shen, Science **307**, 901 (2005)



Photoemission observations

Monte Carlo at a temperature $T > T_{KT}$



Thermal SU(2) gauge theory of mixing between holes and spinons mediated by Yukawa couplings to a SU(2) fundamental, charge $+e$ Higgs boson B.

Quantum oscillations survive even when pockets have turned to arcs in photoemission.