

Graphene flakes, strange metals, and black holes: insights from the SYK model

Tata Institute of Fundamental Research
Mumbai, January 17, 2023

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Talk online: sachdev.physics.harvard.edu



Foundations

by

Boltzmann

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states

$$D(E) = \sum_i \delta(E - E_i) \\ \sim \exp(S(E)/k_B)$$



Ludwig Boltzmann

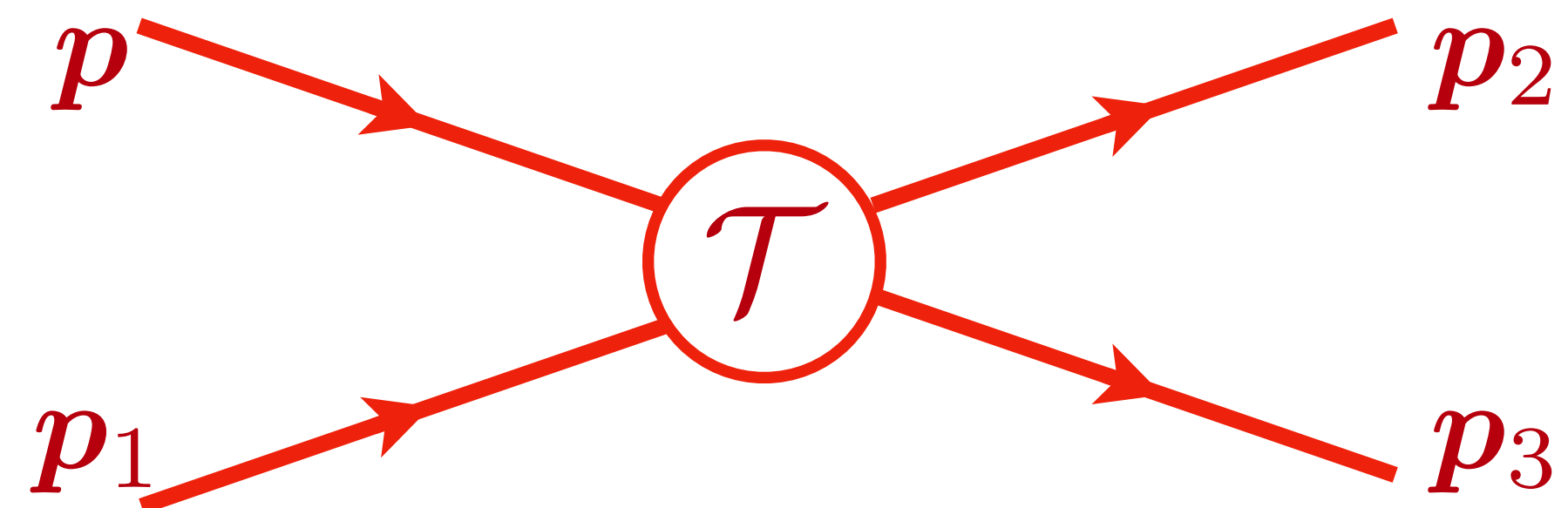
20 February 1844 - September 5, 1906
Vienna, Austria

Boltzmann equation (1872)

Dilute classical gas

Molecular chaos: successive collisions are statistically independent

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} - f_{\mathbf{p}_2} f_{\mathbf{p}_3}]$$



Ludwig Boltzmann

20 February 1844 - September 5, 1906

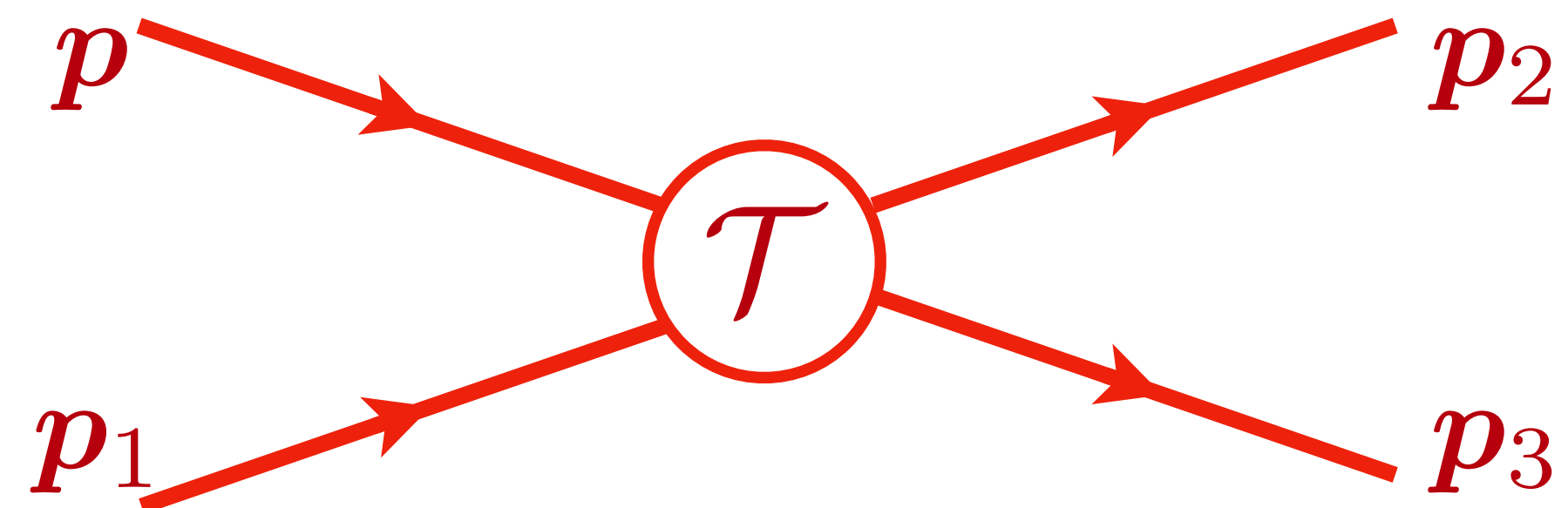
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Quantum Boltzmann equation (Landau)

Dense gas of electrons

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \cdot \nabla_{\mathbf{r}} f_{\mathbf{p}} + \mathbf{F} \cdot \nabla_{\mathbf{p}} f_{\mathbf{p}} =$$
$$- 2\pi \int_{\mathbf{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_2} - \varepsilon_{\mathbf{p}_3}) \delta(\mathbf{p} + \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$
$$\times [f_{\mathbf{p}} f_{\mathbf{p}_1} (1 - f_{\mathbf{p}_2}) (1 - f_{\mathbf{p}_3}) - f_{\mathbf{p}_2} f_{\mathbf{p}_3} (1 - f_{\mathbf{p}}) (1 - f_{\mathbf{p}_1})]$$

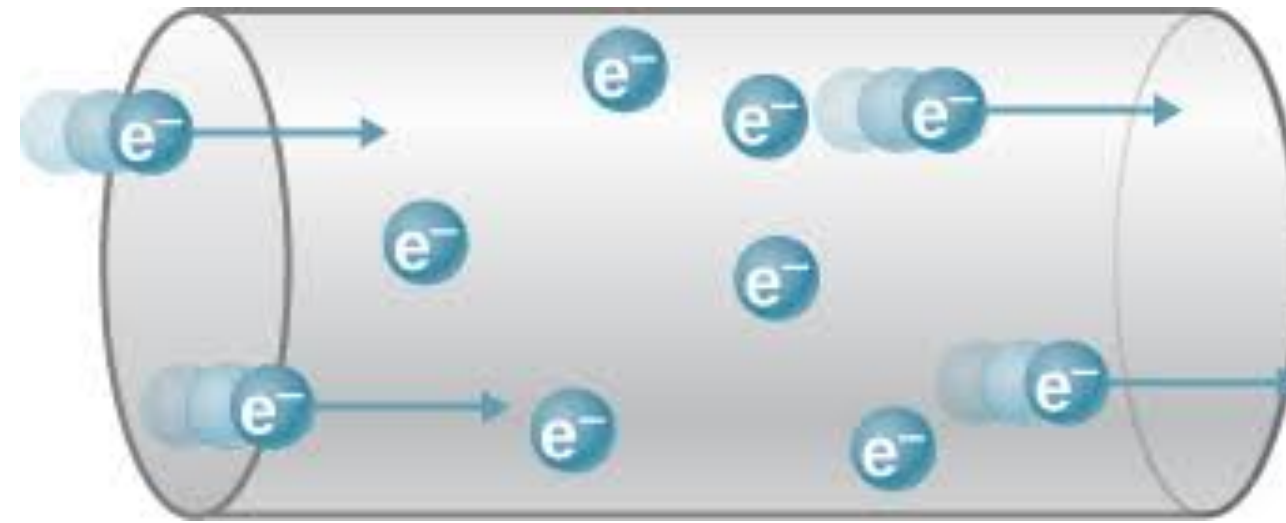


Ludwig Boltzmann

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Current flow with electrons in Copper

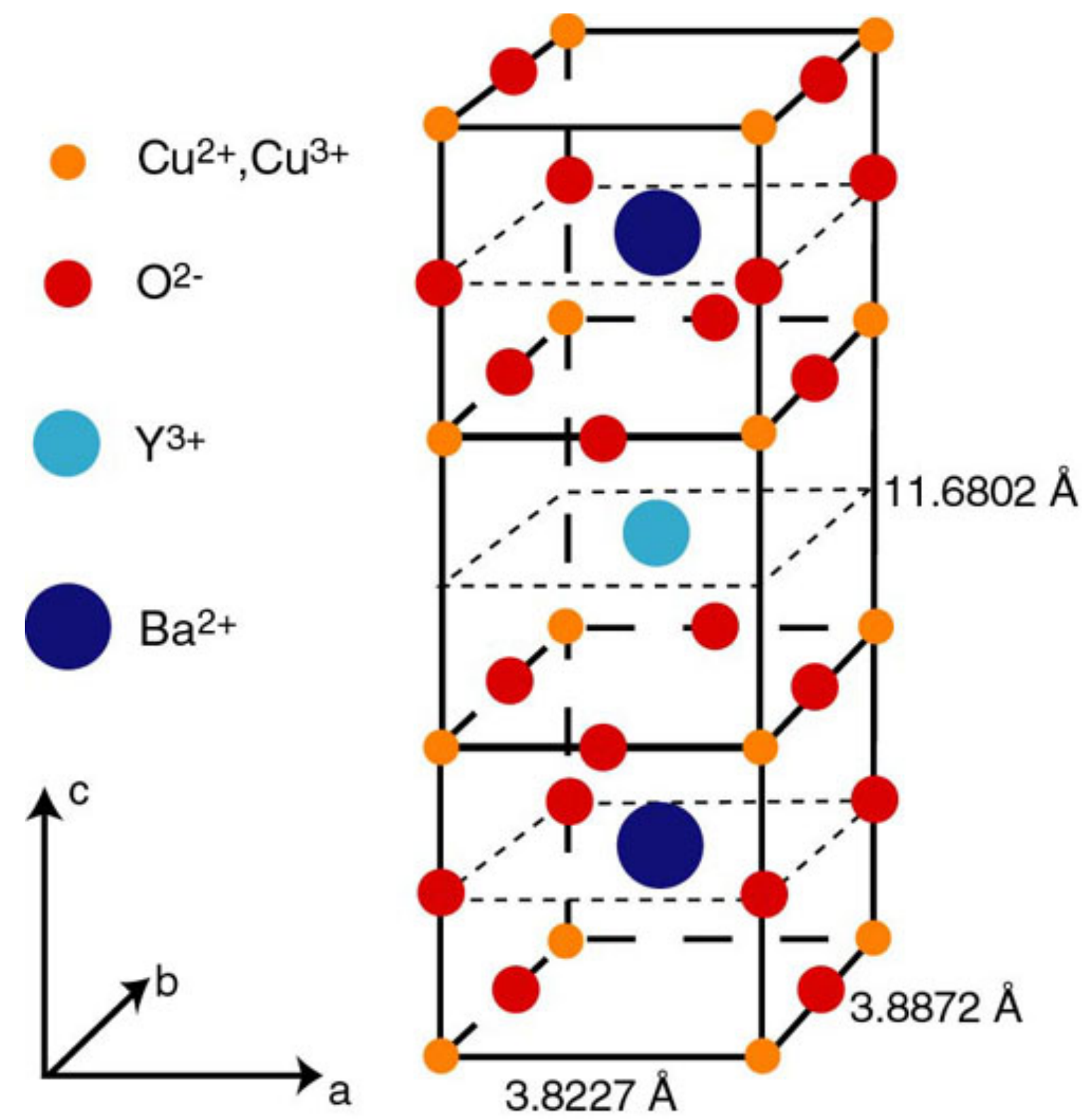
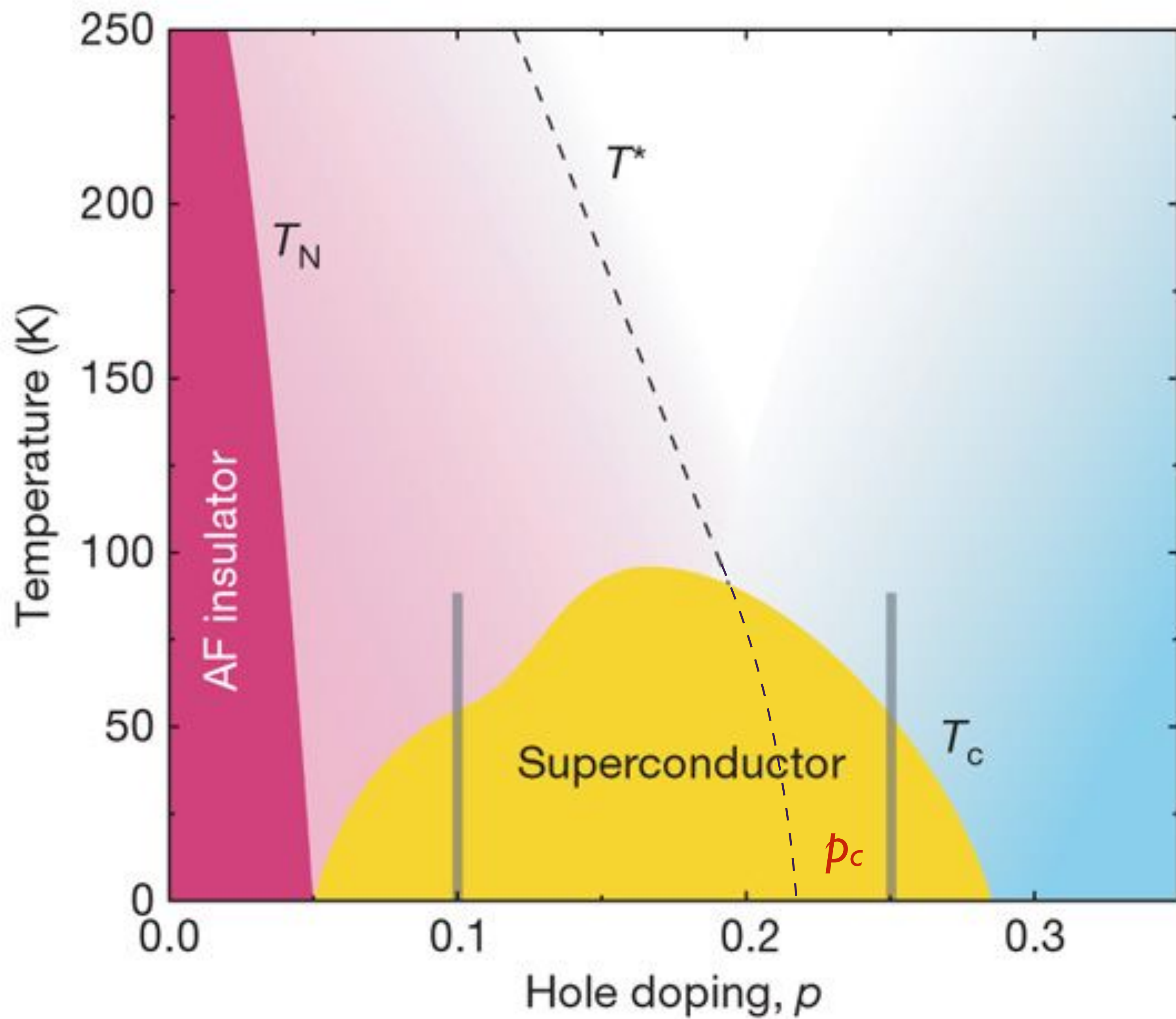


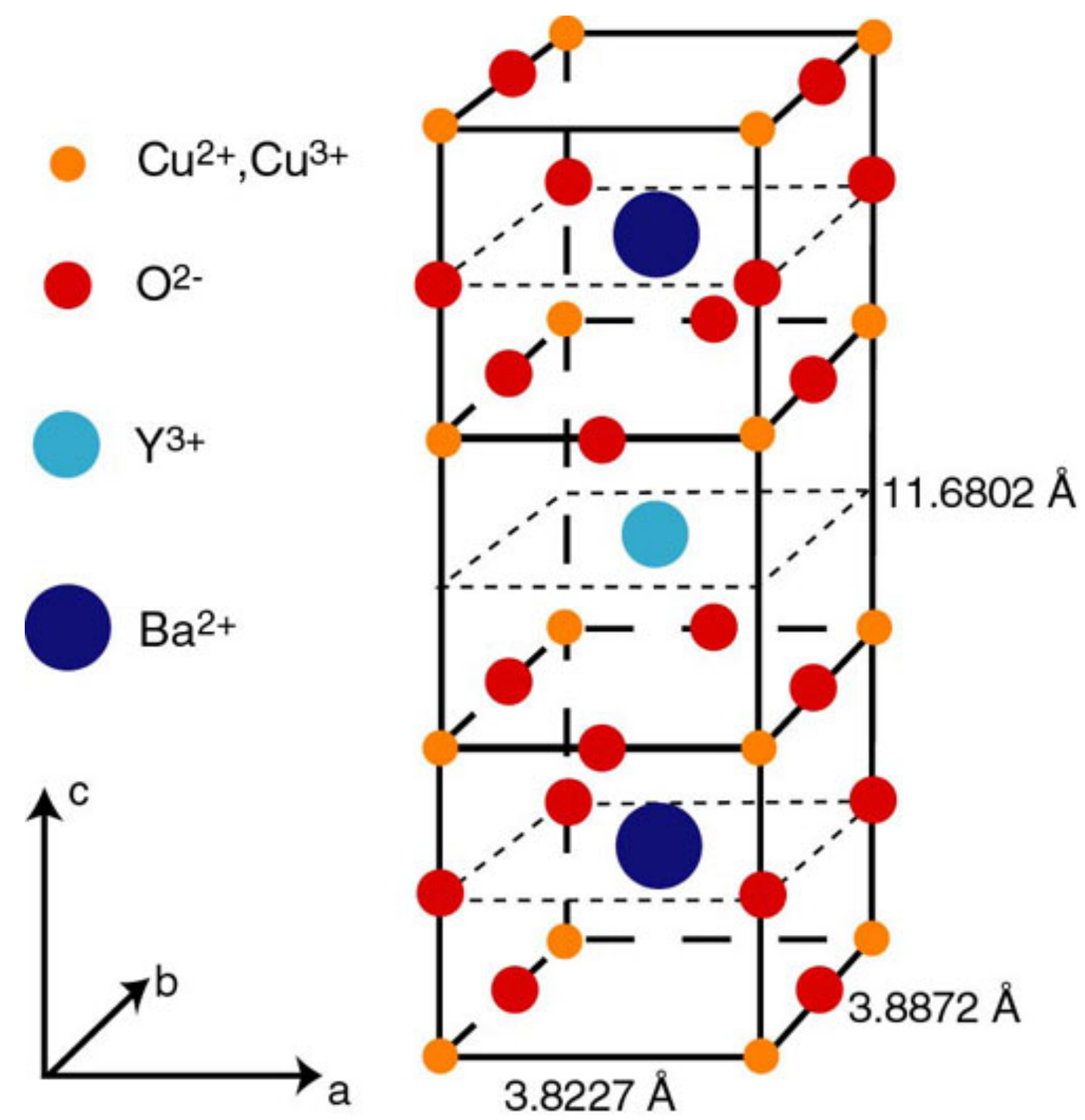
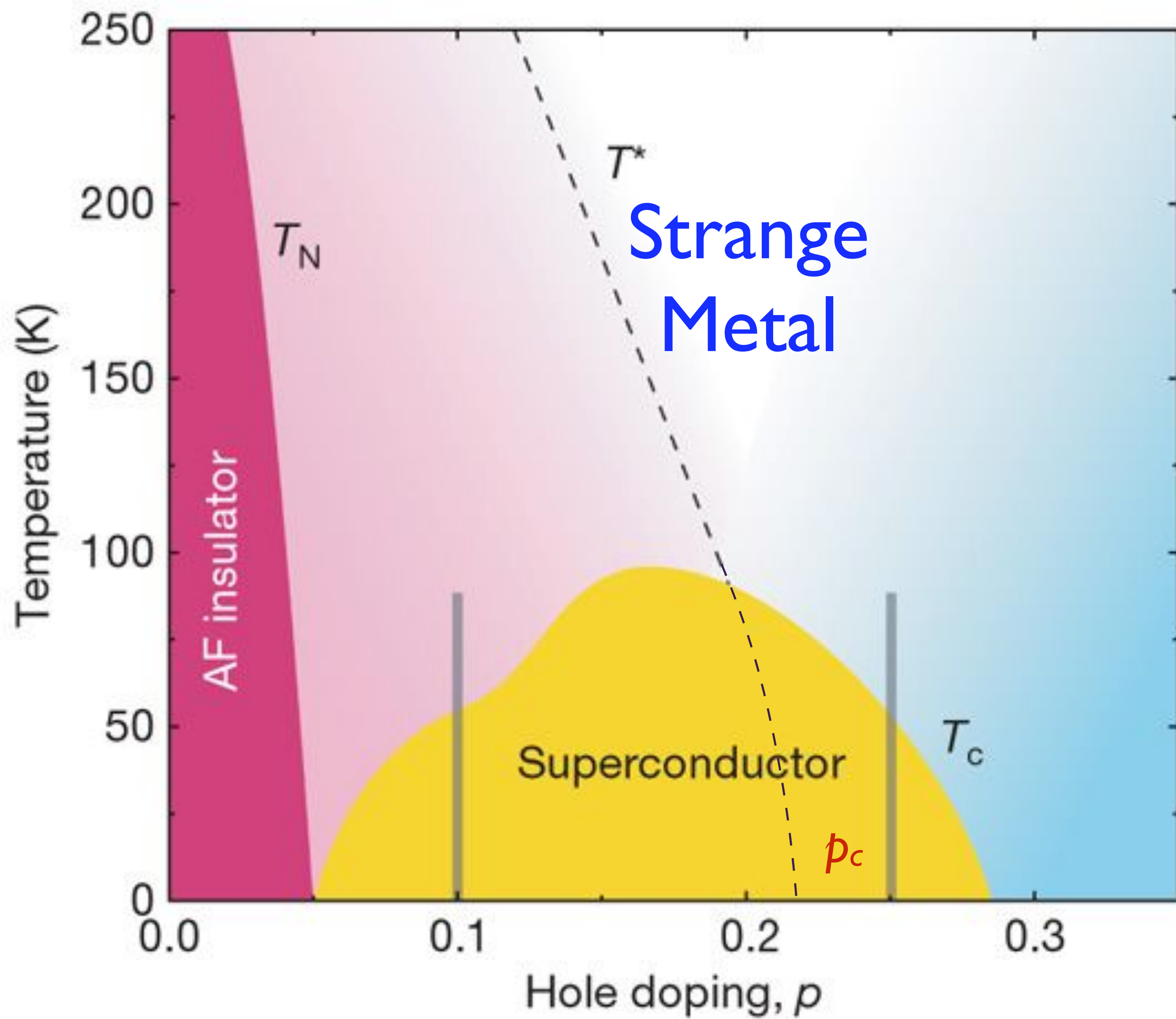
Flow of electrons described by Boltzmann equation \Rightarrow
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

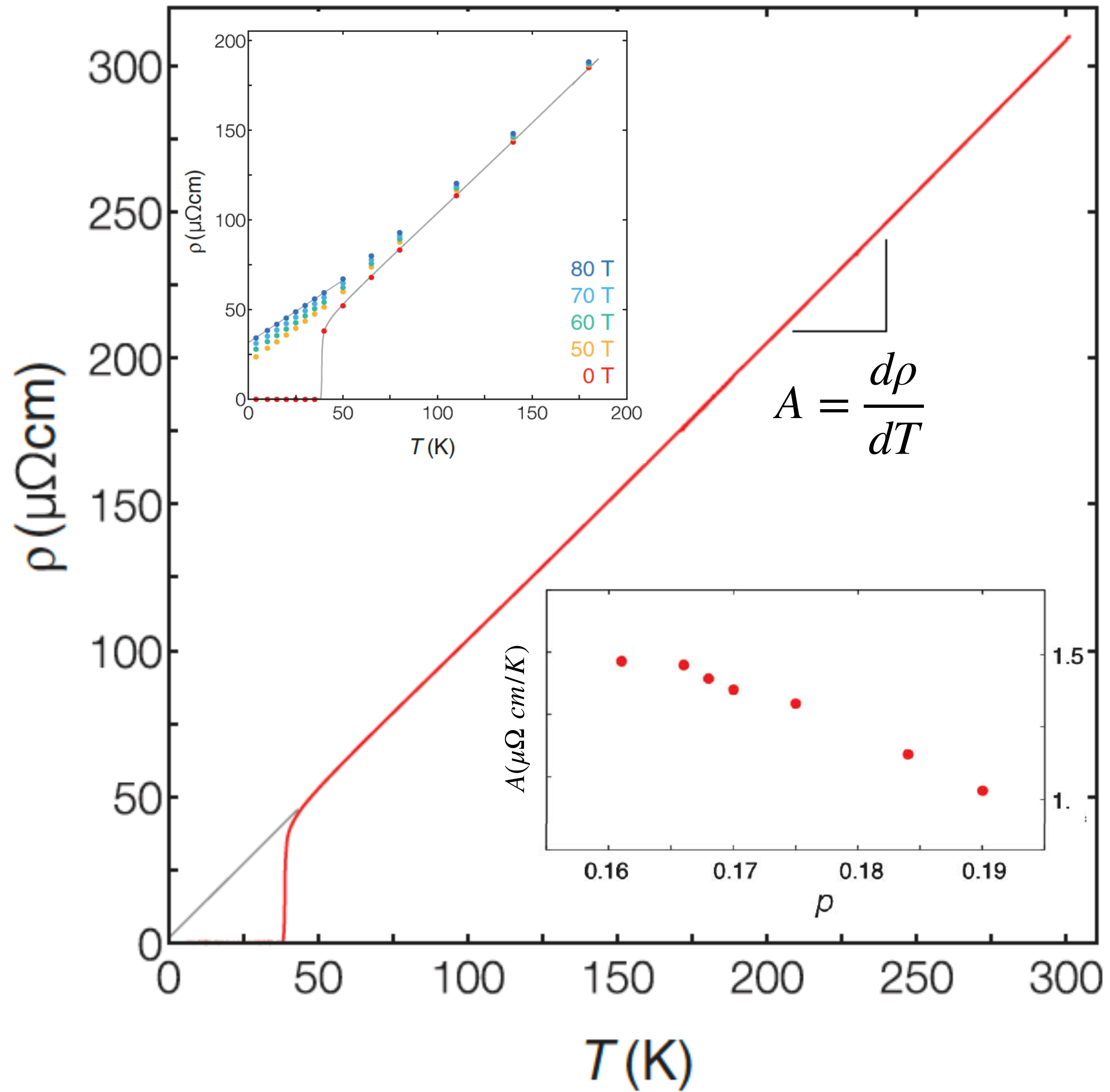
The time τ is much longer than a limiting ‘Planckian time’ $\frac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

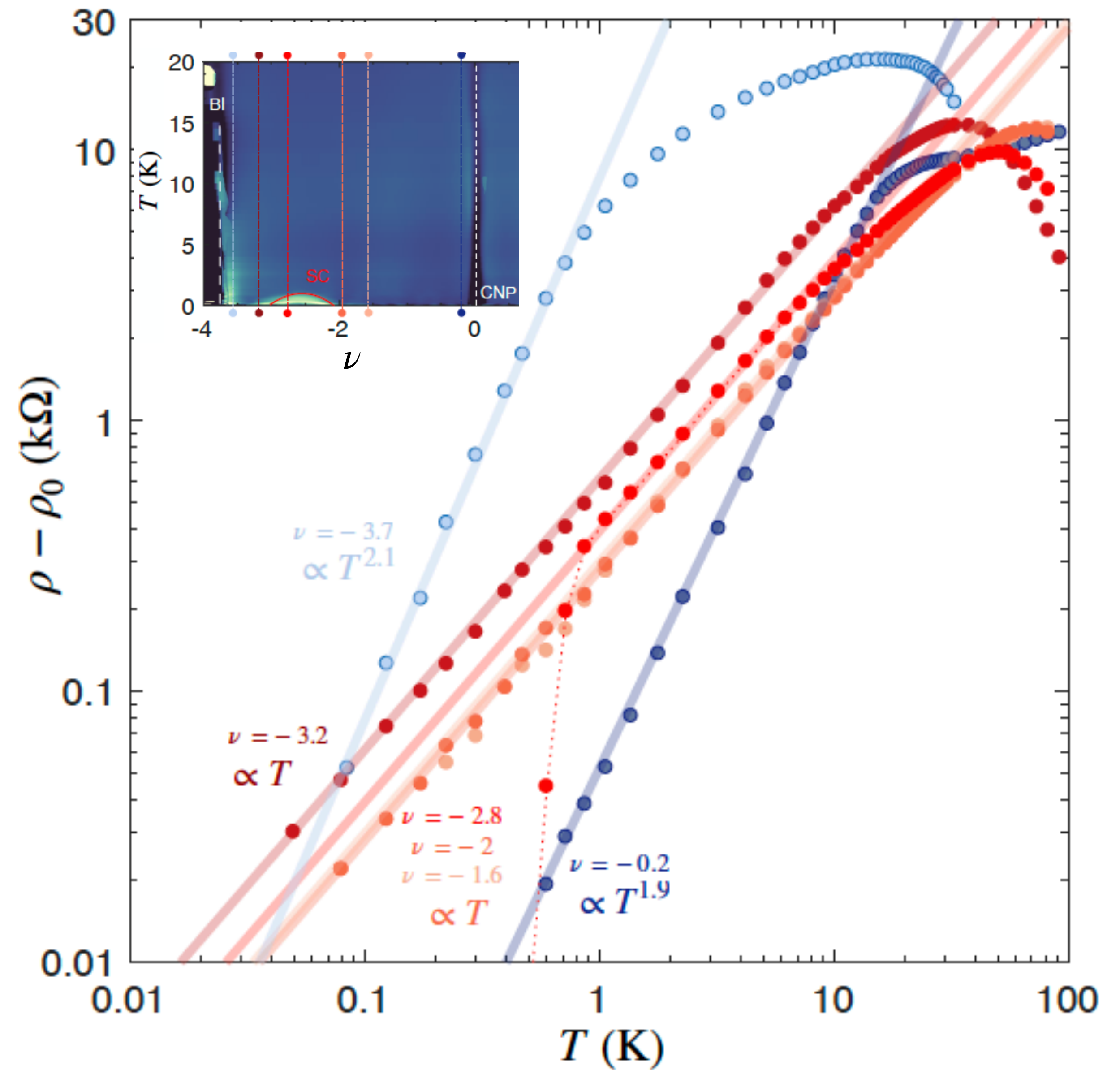
The motion of electrons is ‘ballistic’ or ‘integrable’
up to the long time τ , after which it is chaotic.







LSCO: Giraldo-Gallo et al. 2018

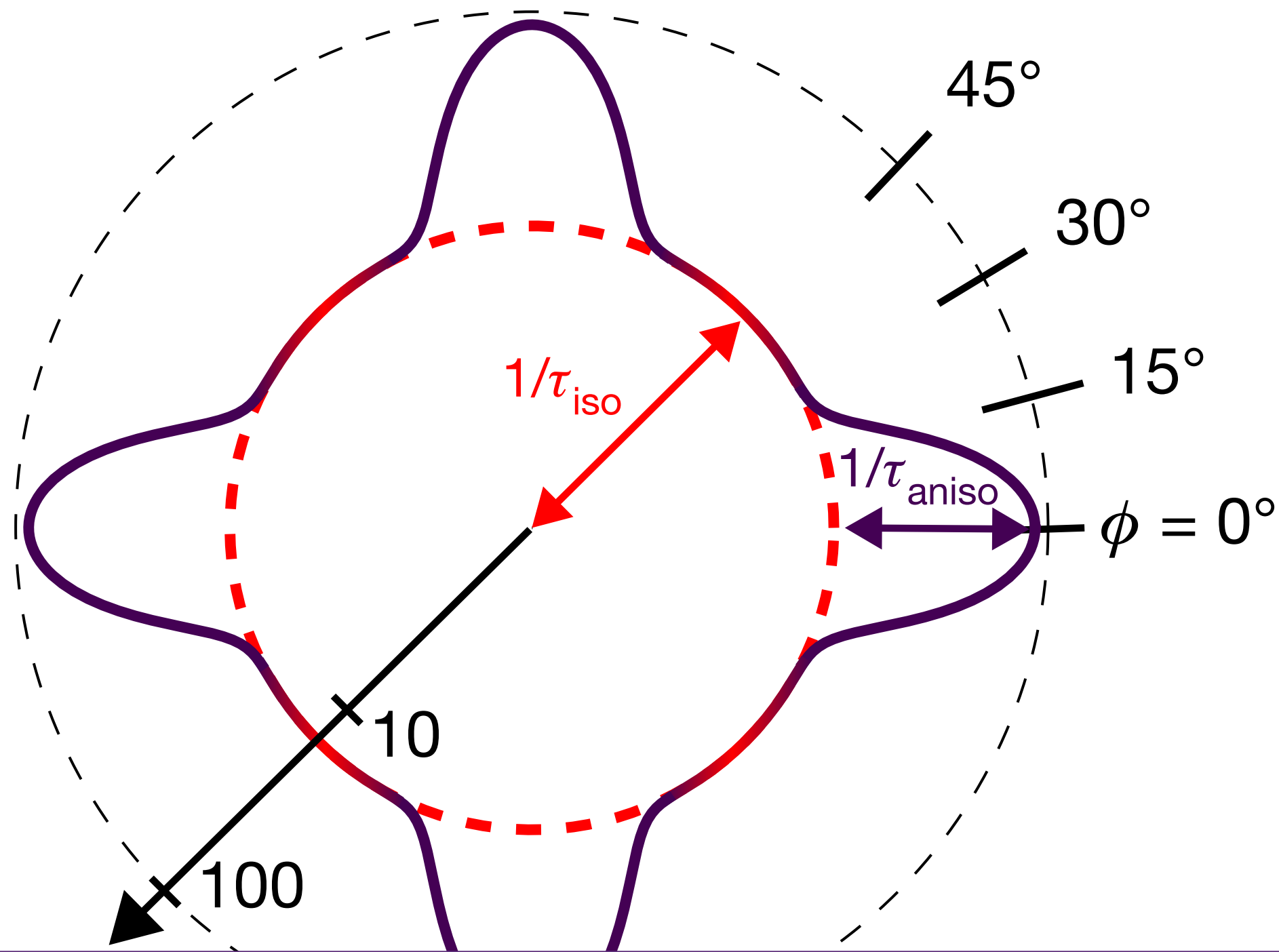


MATBG: Jaoui et al. 2021

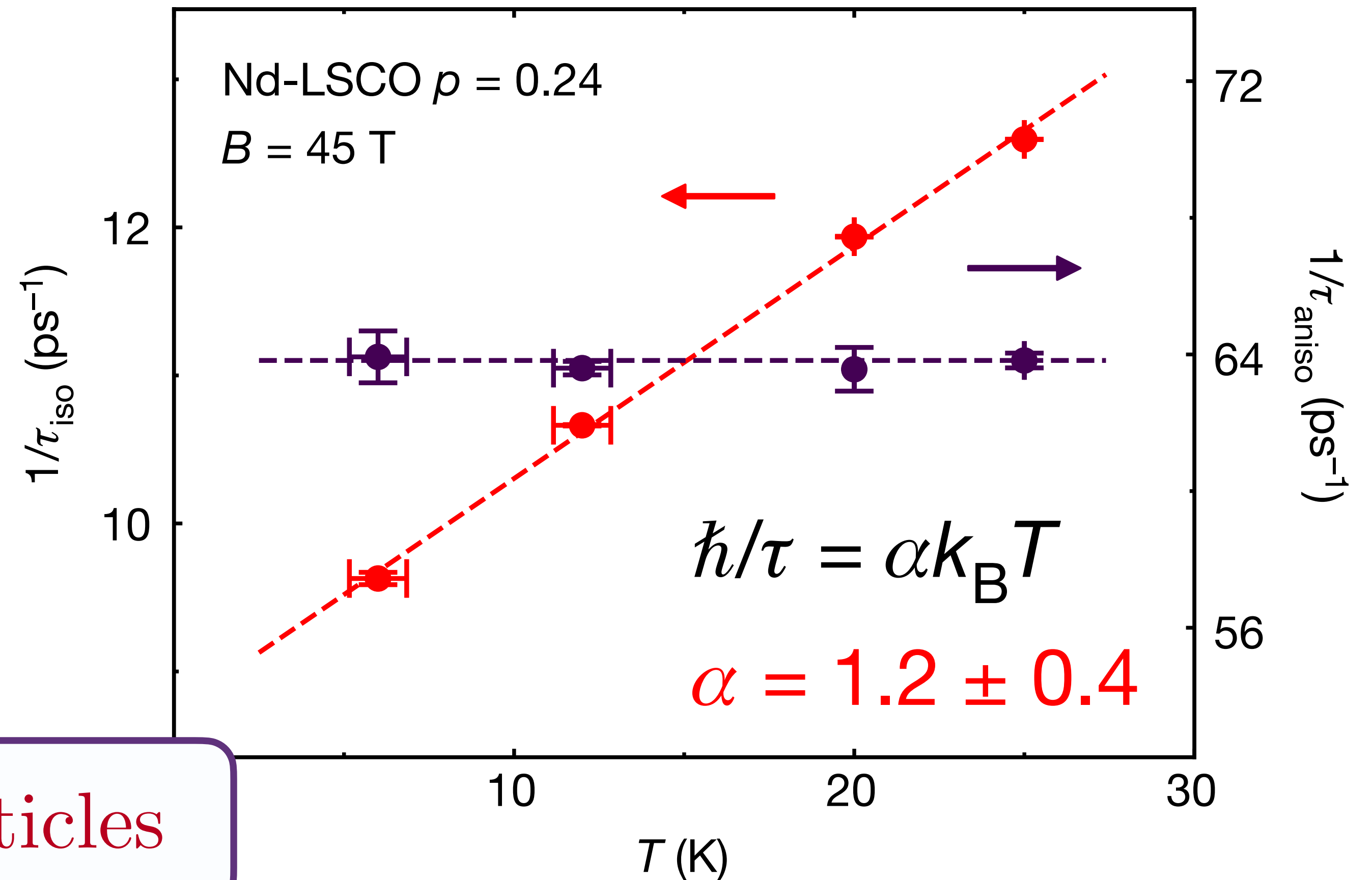
Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw



Current flow without quasiparticles



No Boltzmann-Landau quasiparticle description \Rightarrow
Many particle quantum entanglement
from quantum interference between “collisions”

Sachdev-Ye-Kitaev Model

A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

leading to a metal with no particle-like excitations

The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

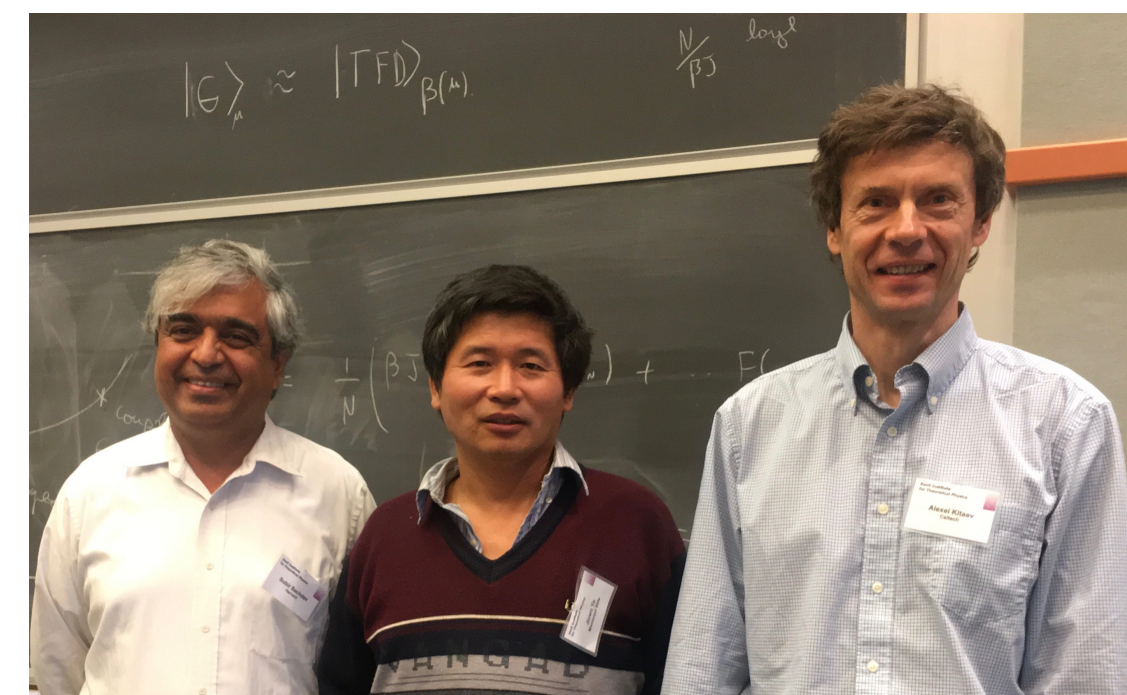
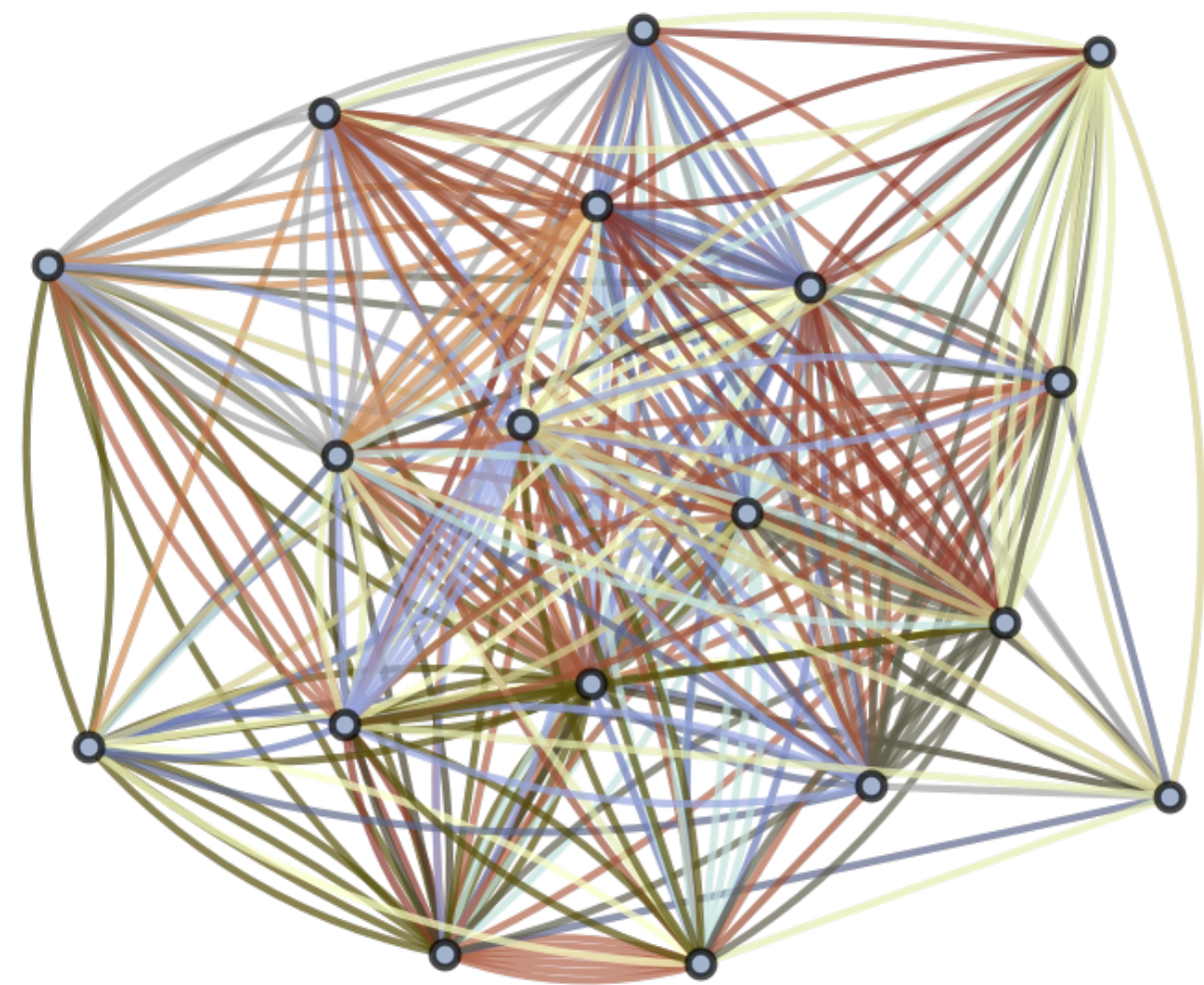
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.
Fermion Green's function $G(\omega) \sim T^{-1/2} F(\hbar\omega/k_B T)$

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- There is an extensive entropy as $T \rightarrow 0$

$$\begin{aligned} s_0/k_B &= \lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} S/(k_B N) \neq 0 \\ &= \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833 \dots \end{aligned}$$

However, the ground state is not extensively degenerate: $D(E) \neq \delta(E) \exp(N s_0/k_B)$.

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- The $D(E) \sim \exp(S/k_B)$ is determined by a time-reparameterization $\tau \rightarrow f(\tau)$ mode (similar to the graviton fluctuations of the spacetime metric), and a phase mode $\phi(\tau)$:

$$\mathcal{Z}_{SYK} = e^{N s_0/k_B} \int \mathcal{D}f \mathcal{D}\phi \exp \left(-\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \mathcal{L}_{SYK}[f, \phi] \right)$$

The SYK model

$$\begin{aligned} \mathcal{Z}(\mathcal{Q}, T) &= \text{Tr}_{\mathcal{Q}} \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T); \text{ Entropy } S = -\frac{\partial F}{\partial T}. \\ &\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}; \quad D(E) = \sum_i \delta(E - E_i); \quad \mathcal{H} |\Psi_i\rangle = E_i |\Psi_i\rangle \end{aligned}$$

Conformal perturbations of saddle-point theory:

$$S = N(s_0 + \gamma T) \quad \Rightarrow \quad D(E) \sim \exp\left(N s_0 + \sqrt{2N\gamma E}\right)$$

$\gamma = \# / U$ is non-universal.

Exact path integral over Schwarzian quantum gravity theory:

$$S = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E})$$

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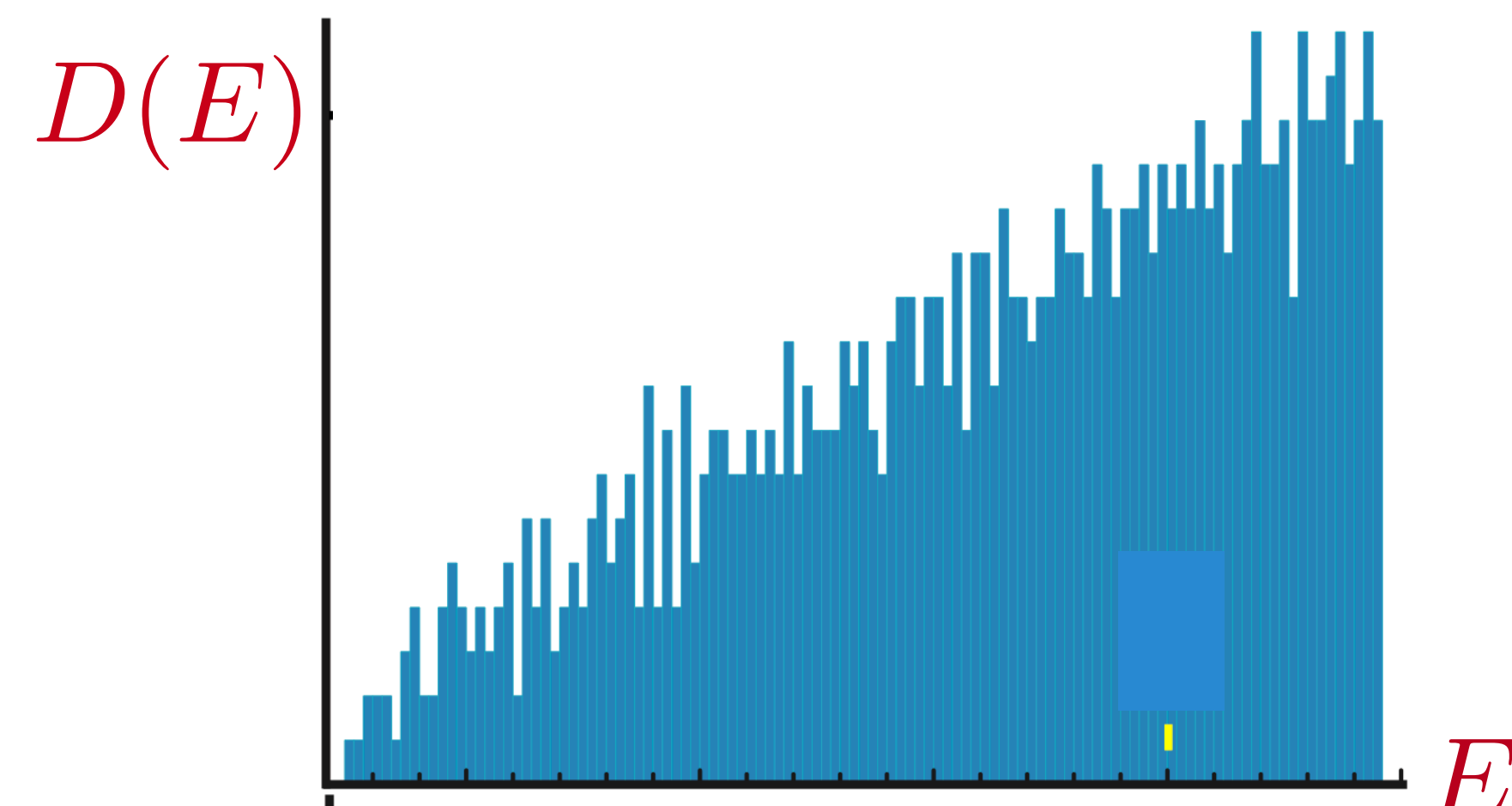
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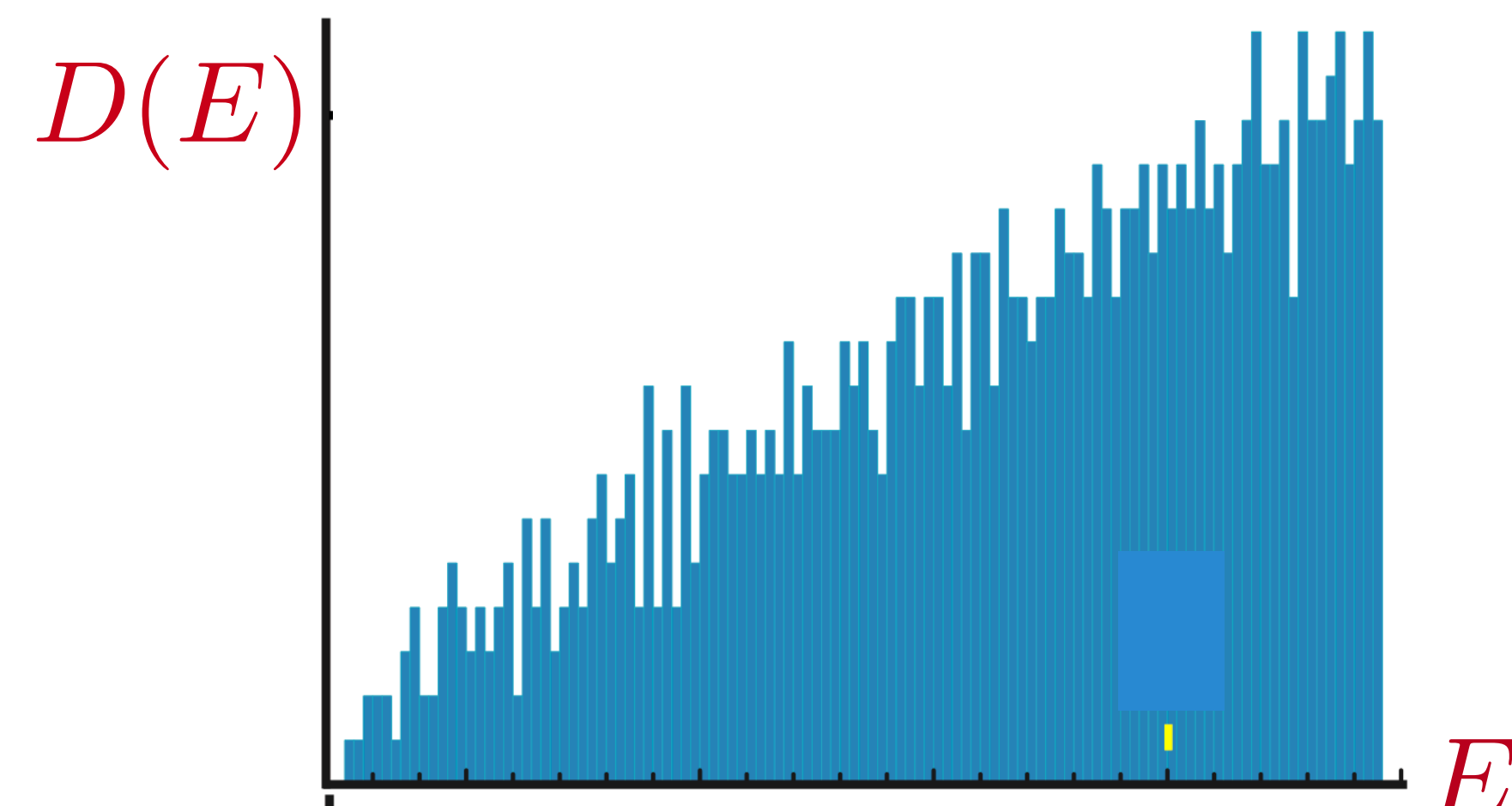
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A. Georges, O. Parcollet, and S. Sachdev,
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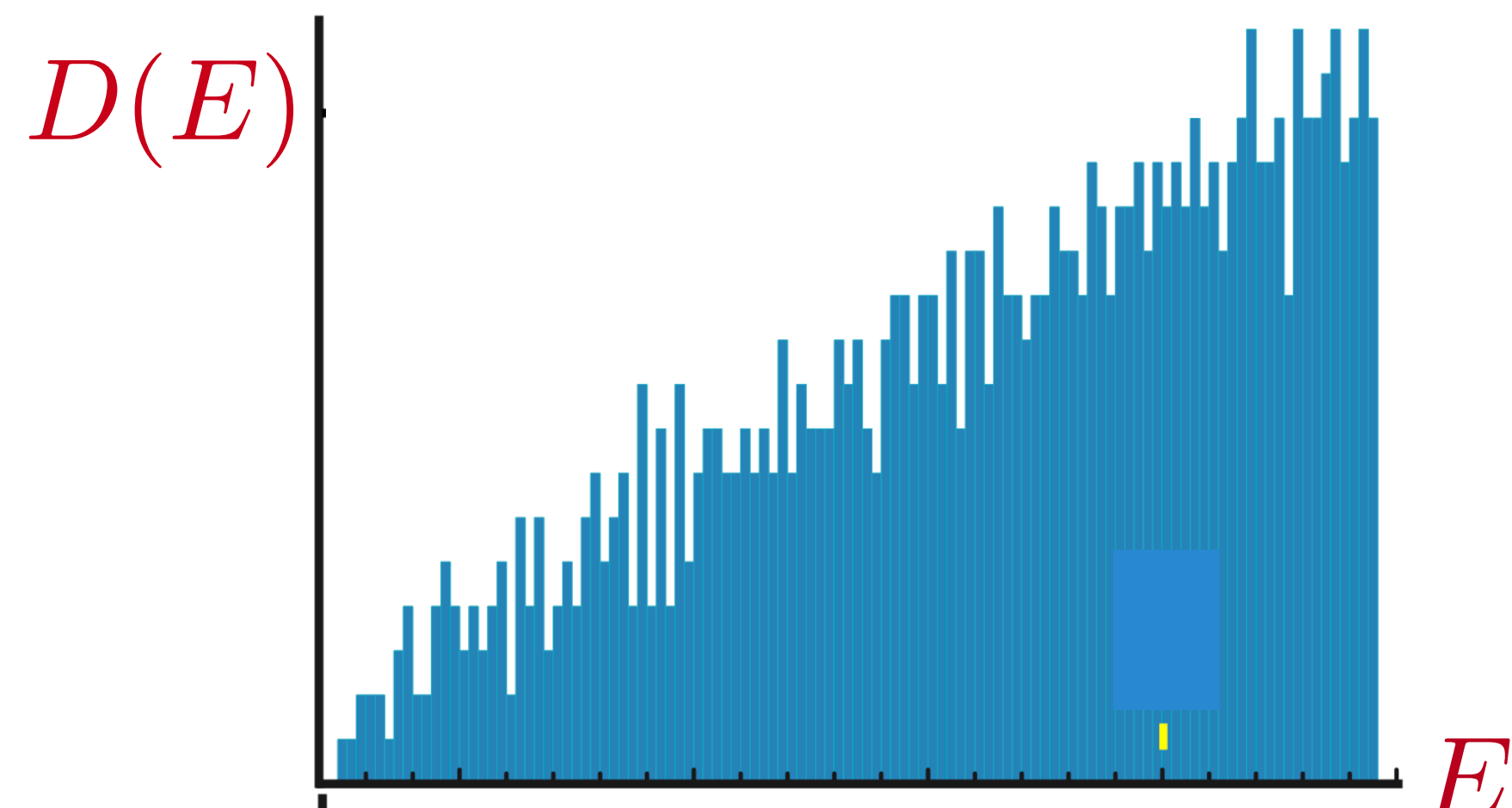
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JHEP 05 (2017) 118

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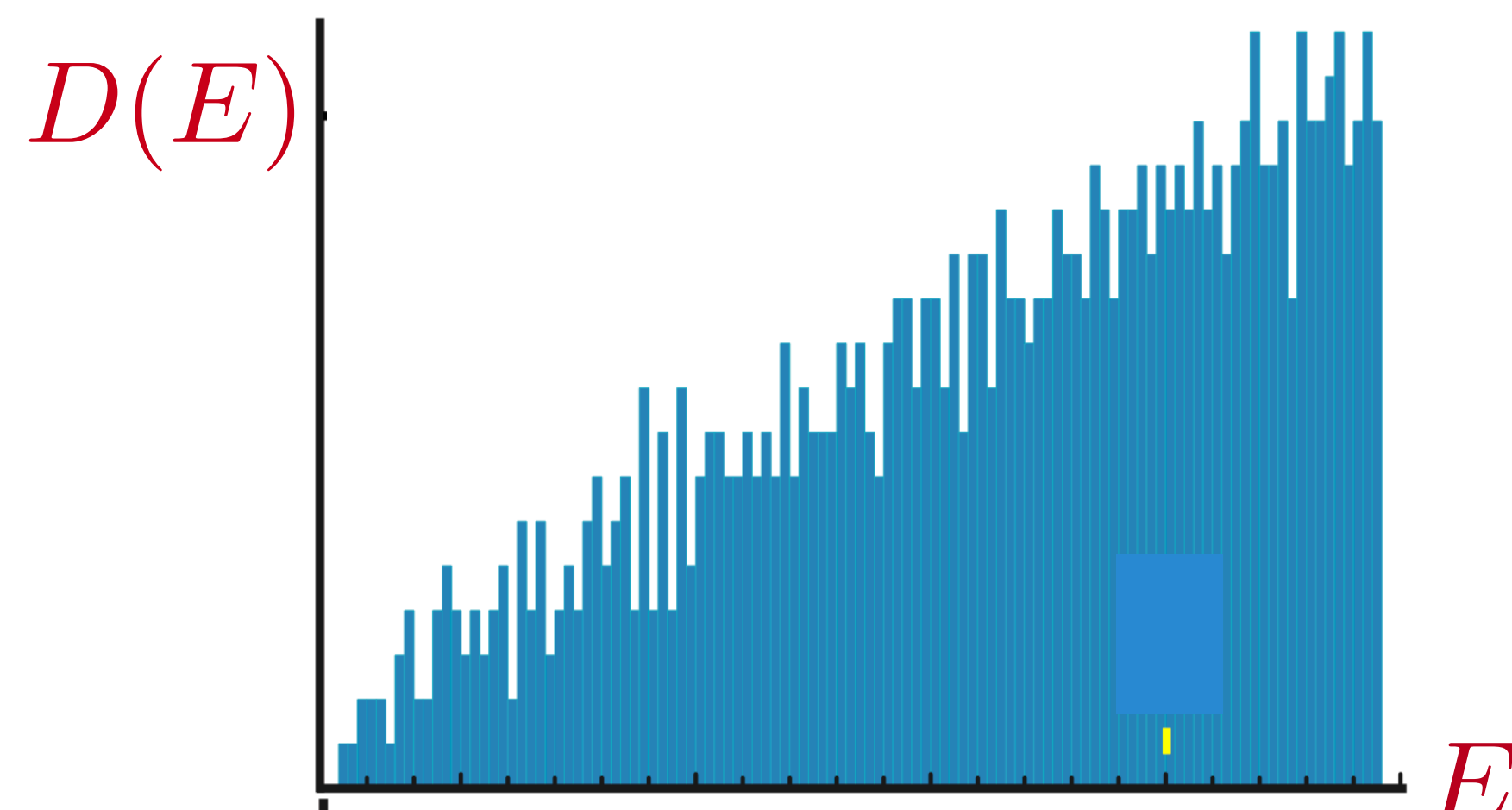
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Yingfei Gu, A. Kitaev, S. Sachdev, and
G. Tarnopolsky, JHEP 02 (2020) 157



Graphene flakes and the SYK model

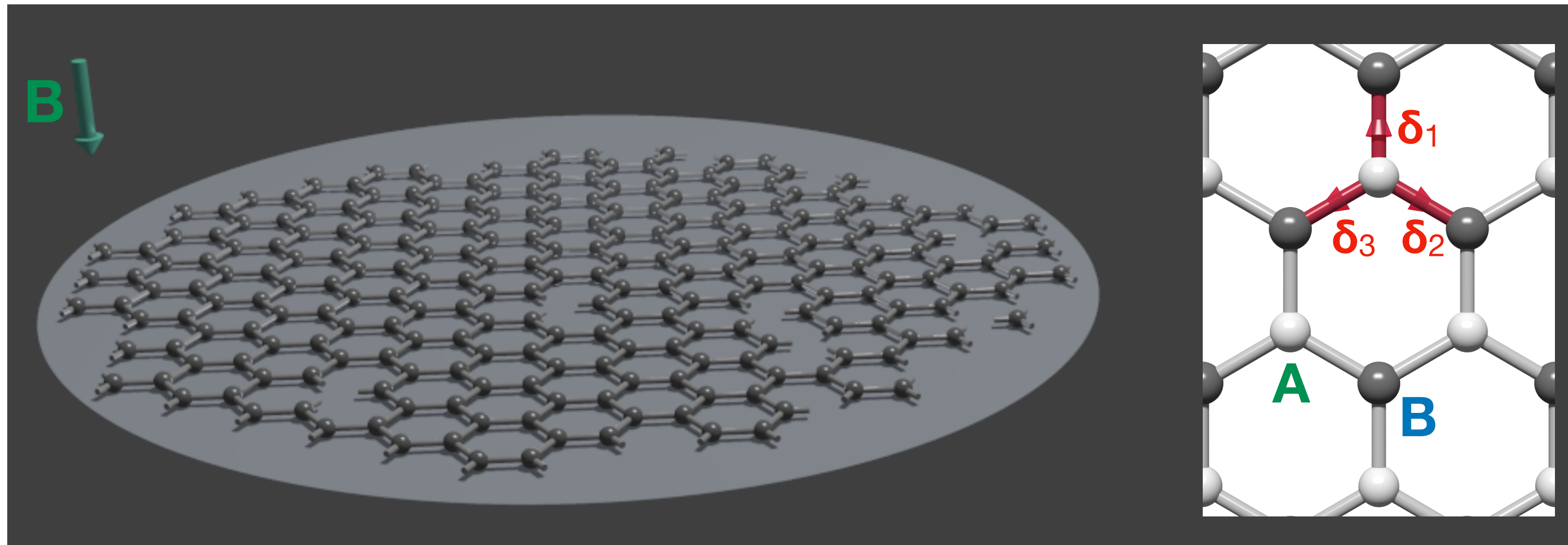
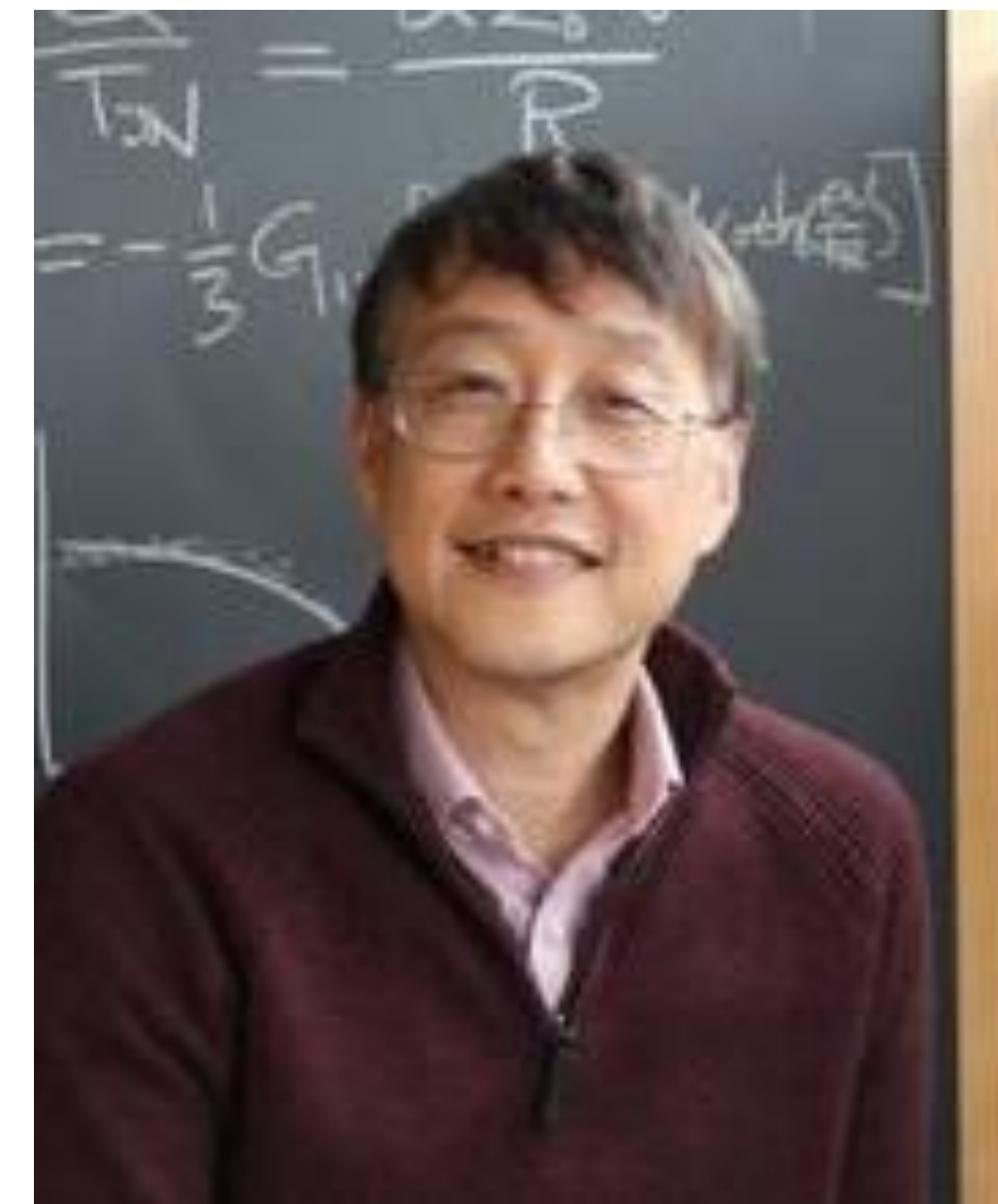
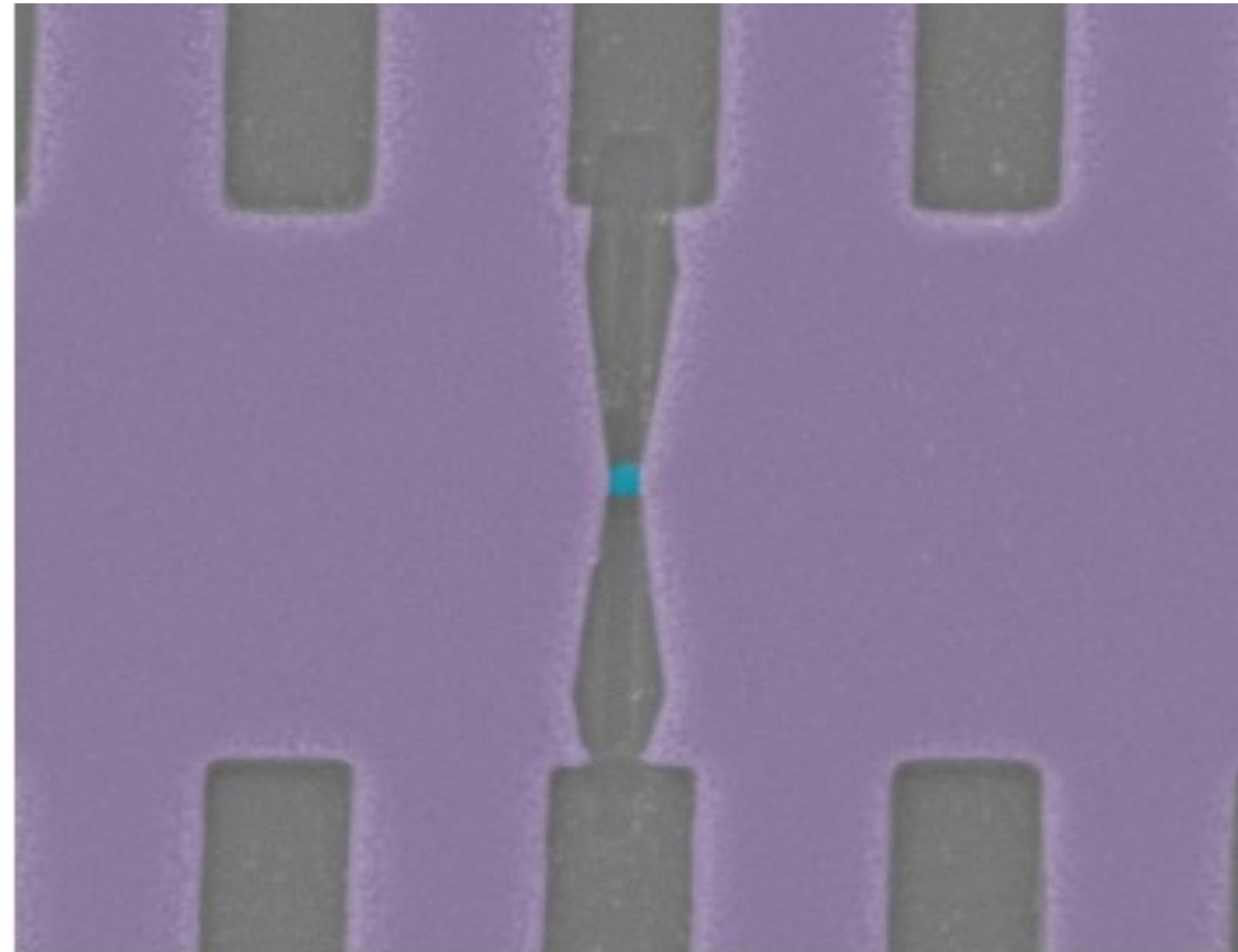
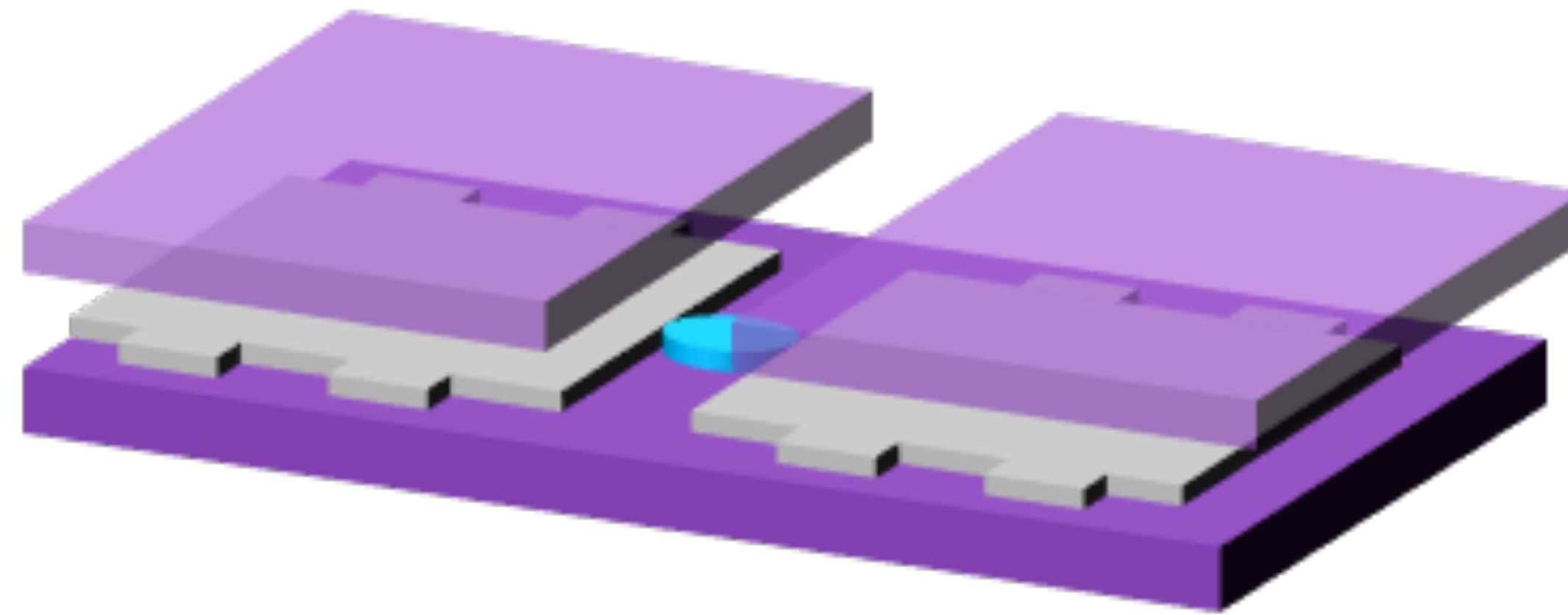


FIG. 1. Schematic depiction of the proposed device. Irregular shaped graphene flake in applied magnetic field B forms the $(0+1)$ dimensional many-body system equivalent to a black hole in $(1+1)$ anti-de Sitter space. Inset: lattice structure of graphene with A and B sublattices marked and nearest neighbor vectors denoted by δ_j .

Laurel Anderson
Philip Kim, to appear.

L. Anderson, Ph.D.
thesis, Harvard, 2022



Thermopower

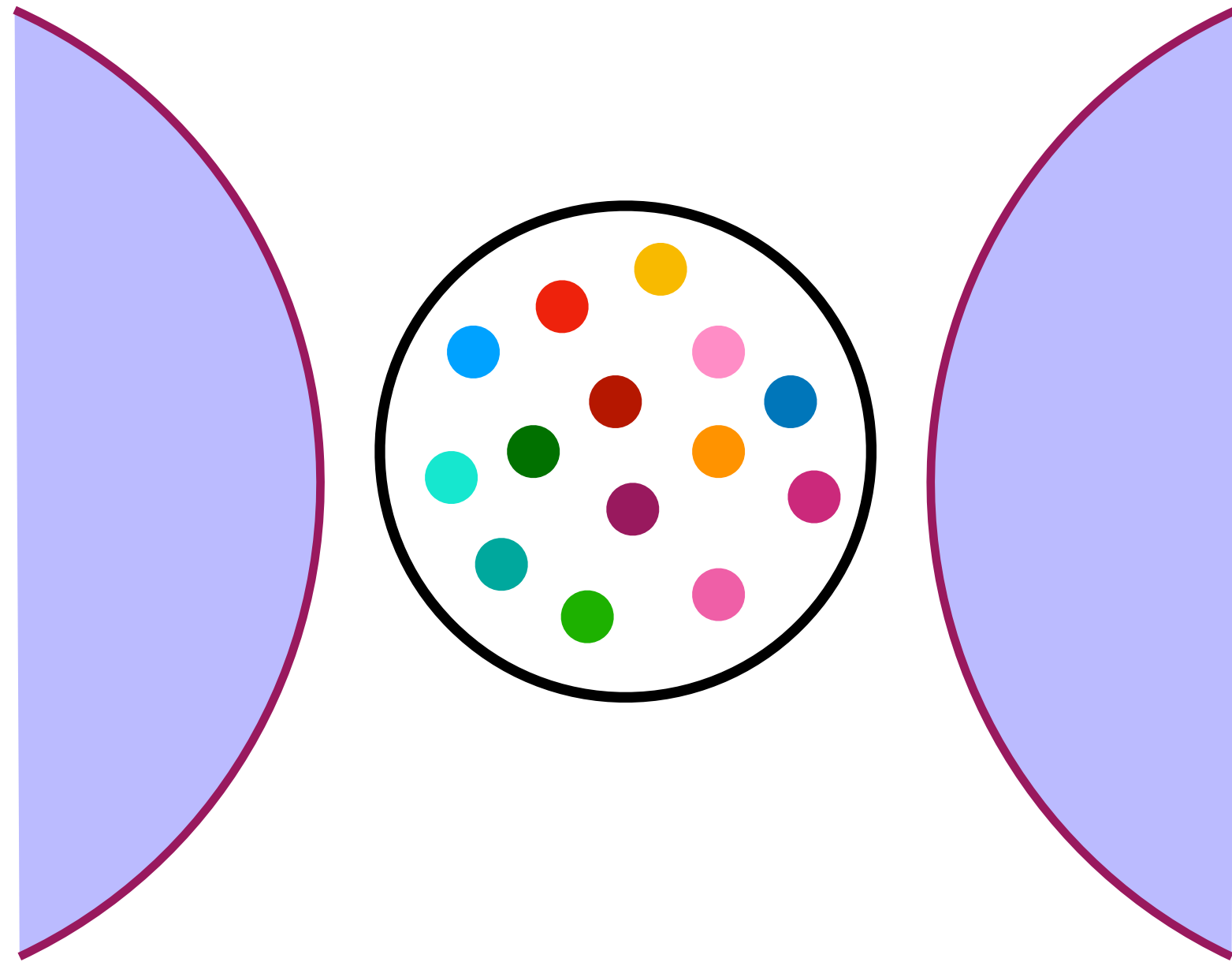
Apply a temperature difference ΔT , and measure the voltage difference ΔV , while no current is flowing.

The thermopower is

$$\Theta = \frac{\Delta V}{\Delta T} = \frac{k_B}{e} \times (\text{a number})$$

Thermopower

A. Kruchkov,
A.A. Patel,
Philip Kim, and
S. Sachdev,
PRB **101**,
205148 (2020)



SYK metal island
weakly coupled to
normal metal leads

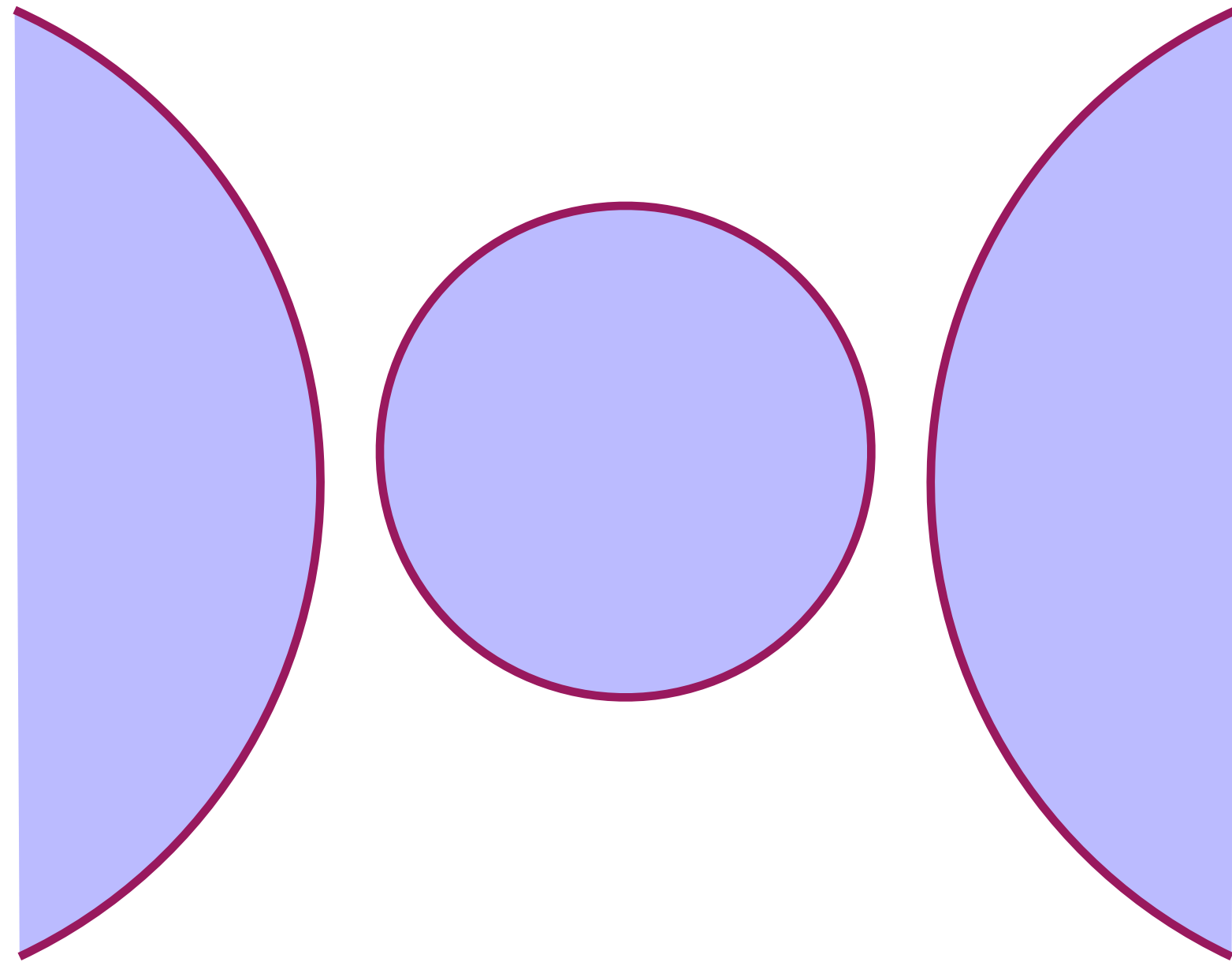
\mathcal{E} is a dimensionless measure of the
particle-hole asymmetry
($\mathcal{E} = 0$ for $Q = 1/2$)

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega \omega (-\partial f / \partial \omega) [\text{Im} \overline{G(\omega)}]}{\int d\omega (-\partial f / \partial \omega) [\text{Im} \overline{G(\omega)}]} = \frac{k_B}{e} \frac{4\pi\mathcal{E}}{3} = \frac{2}{3e} \frac{ds_0}{dQ}$$

Thermopower is non-zero as $T \rightarrow 0$

Thermopower



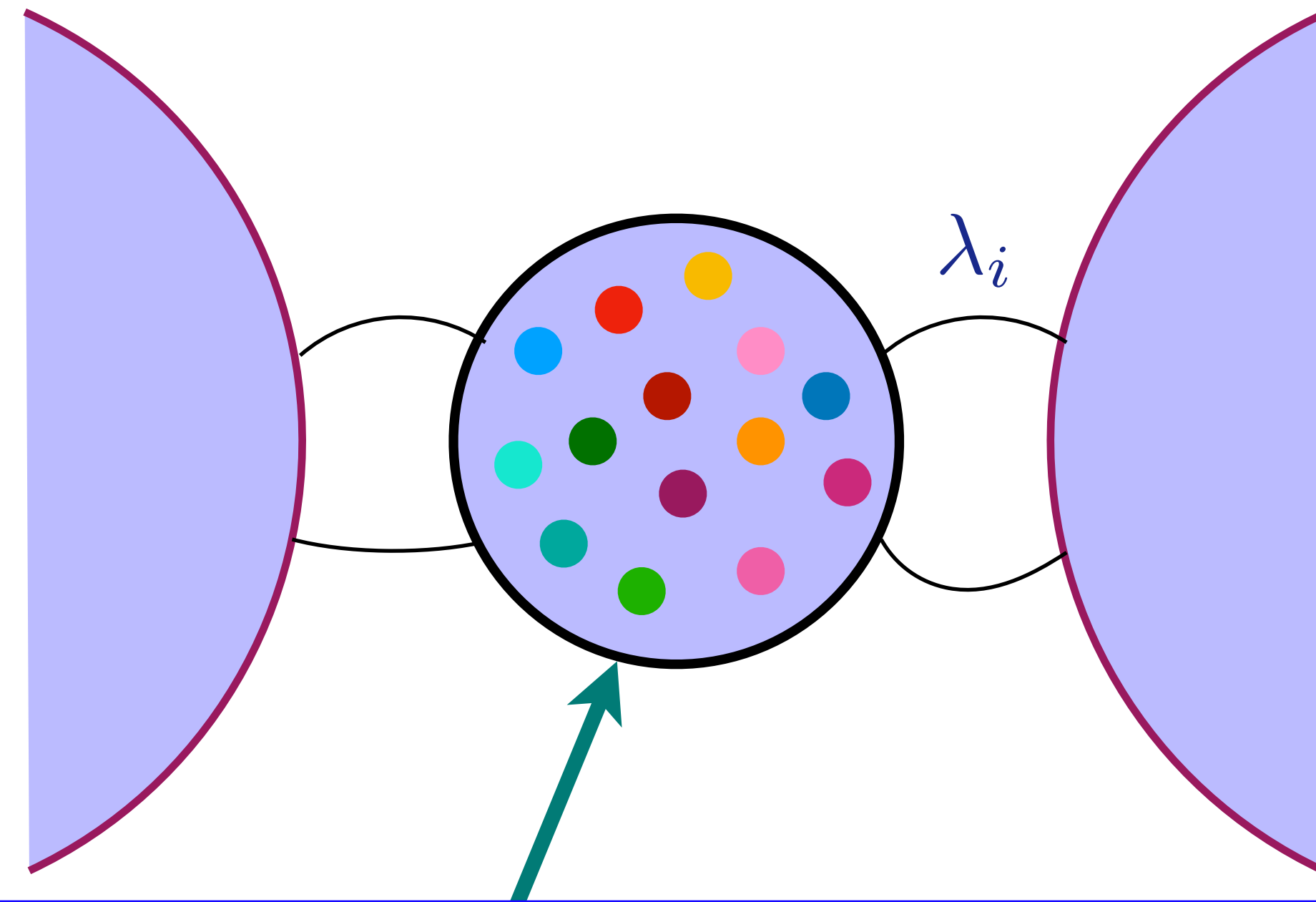
Normal metal island
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$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$

Thermopower vanishes linearly as $T \rightarrow 0$

A more realistic model



$$\Gamma = \pi \rho_{\text{lead}} \sum_i |\lambda_i|^2$$
$$\overline{|h_{\alpha\beta}|^2} = h^2$$

$$H = \frac{1}{\sqrt{N}} \sum_{\alpha\beta=1}^N h_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

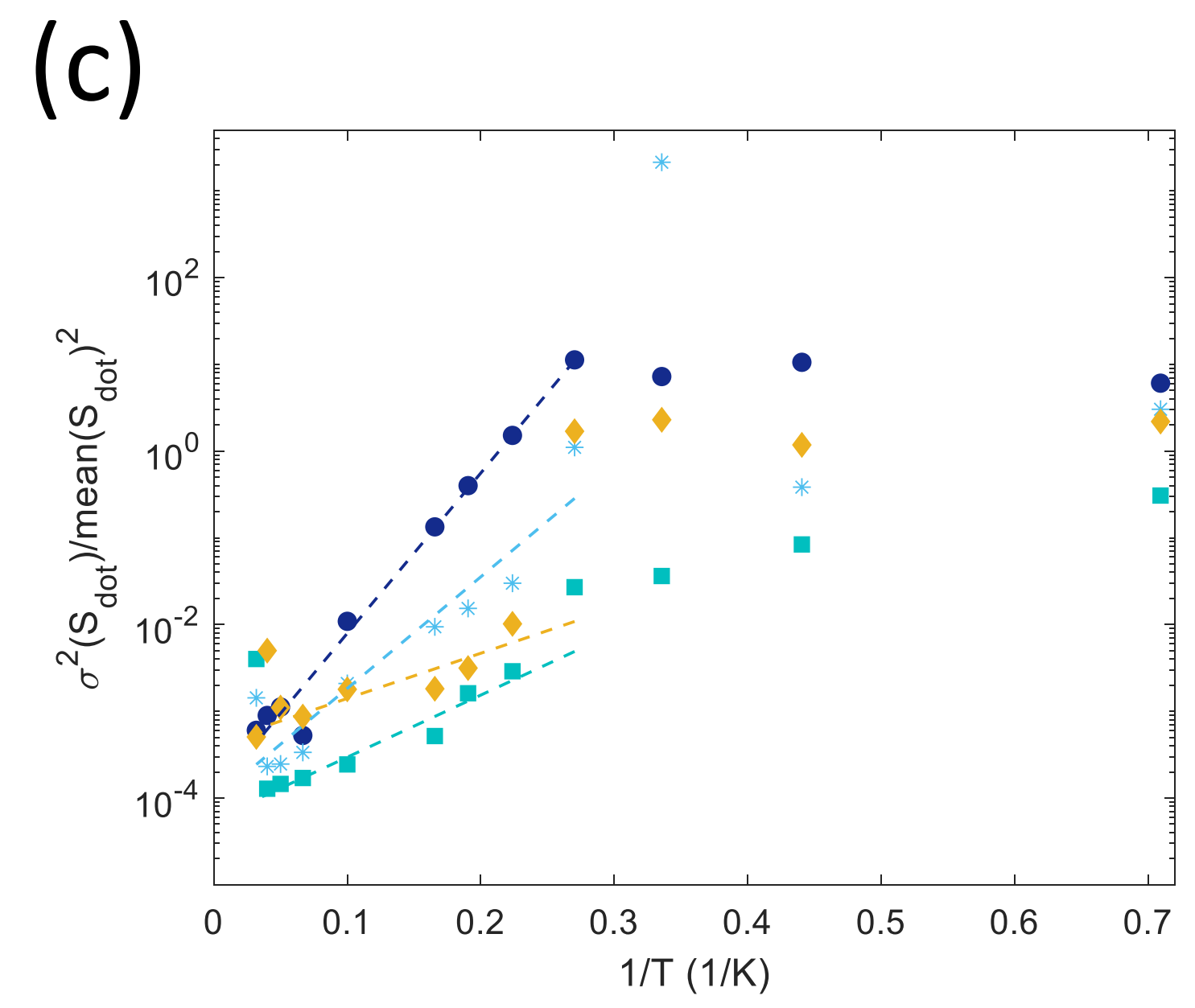
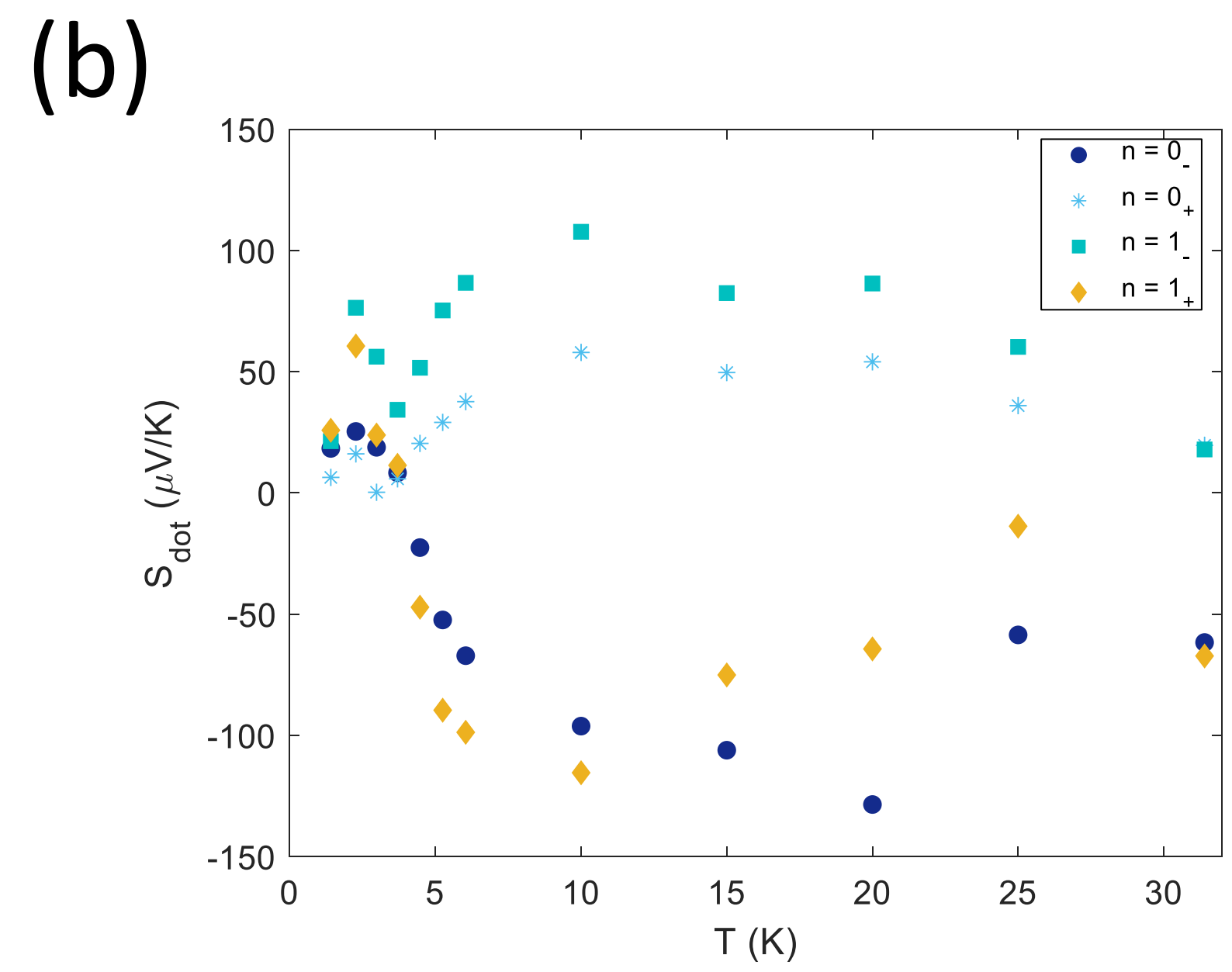
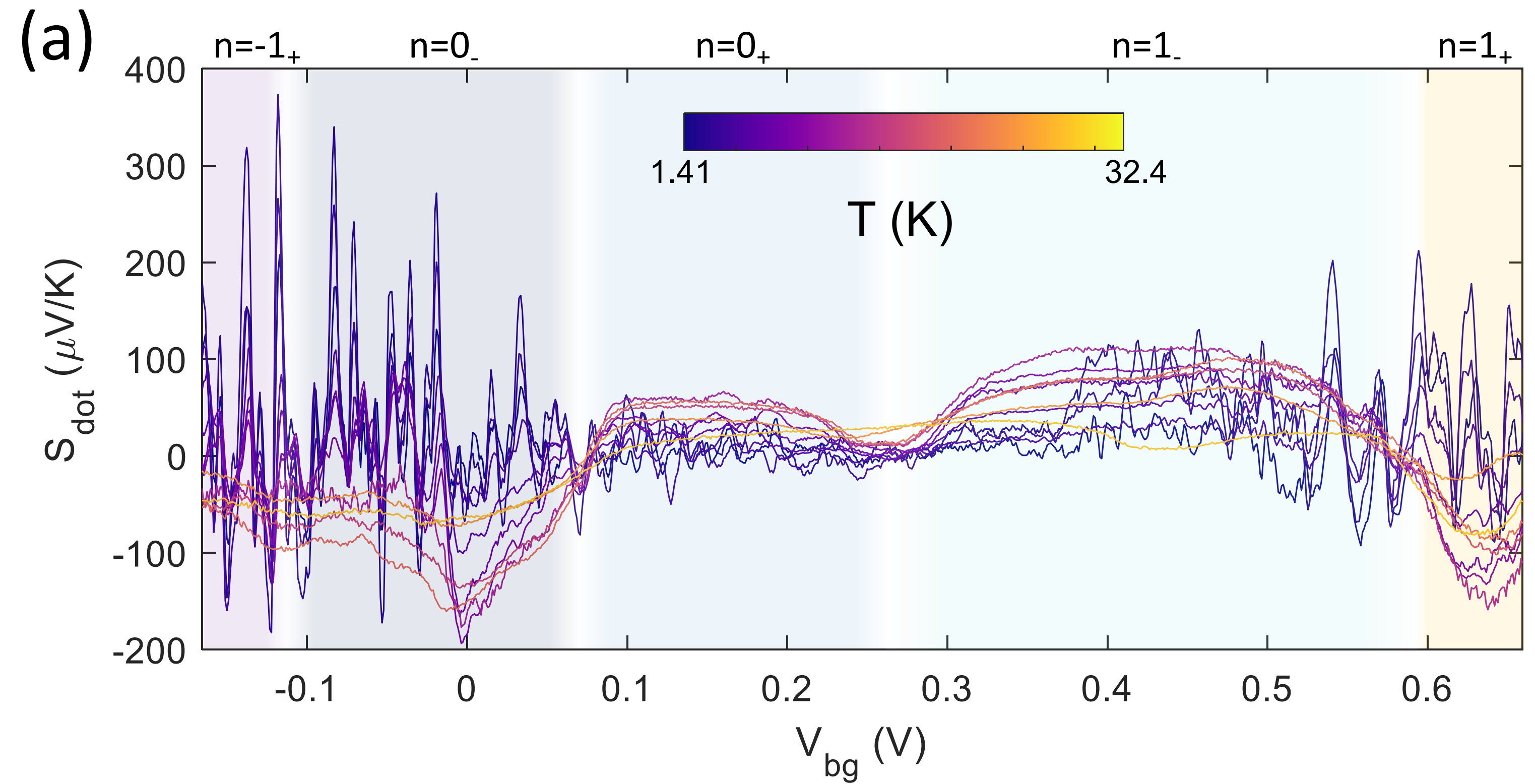
Normal metal behavior for $T < T_{\text{coh}} \sim h^2/U$

SYK behavior for $T > T_{\text{coh}} \sim h^2/U$

Notation: $S \rightarrow \ominus$

Laurel Anderson
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Striking feature of observations at finite N :
fluctuations from random couplings are much larger
for $T < T_{\text{coh}}$ (normal metal regime)
than for $T > T_{\text{coh}}$ (possible SYK regime)

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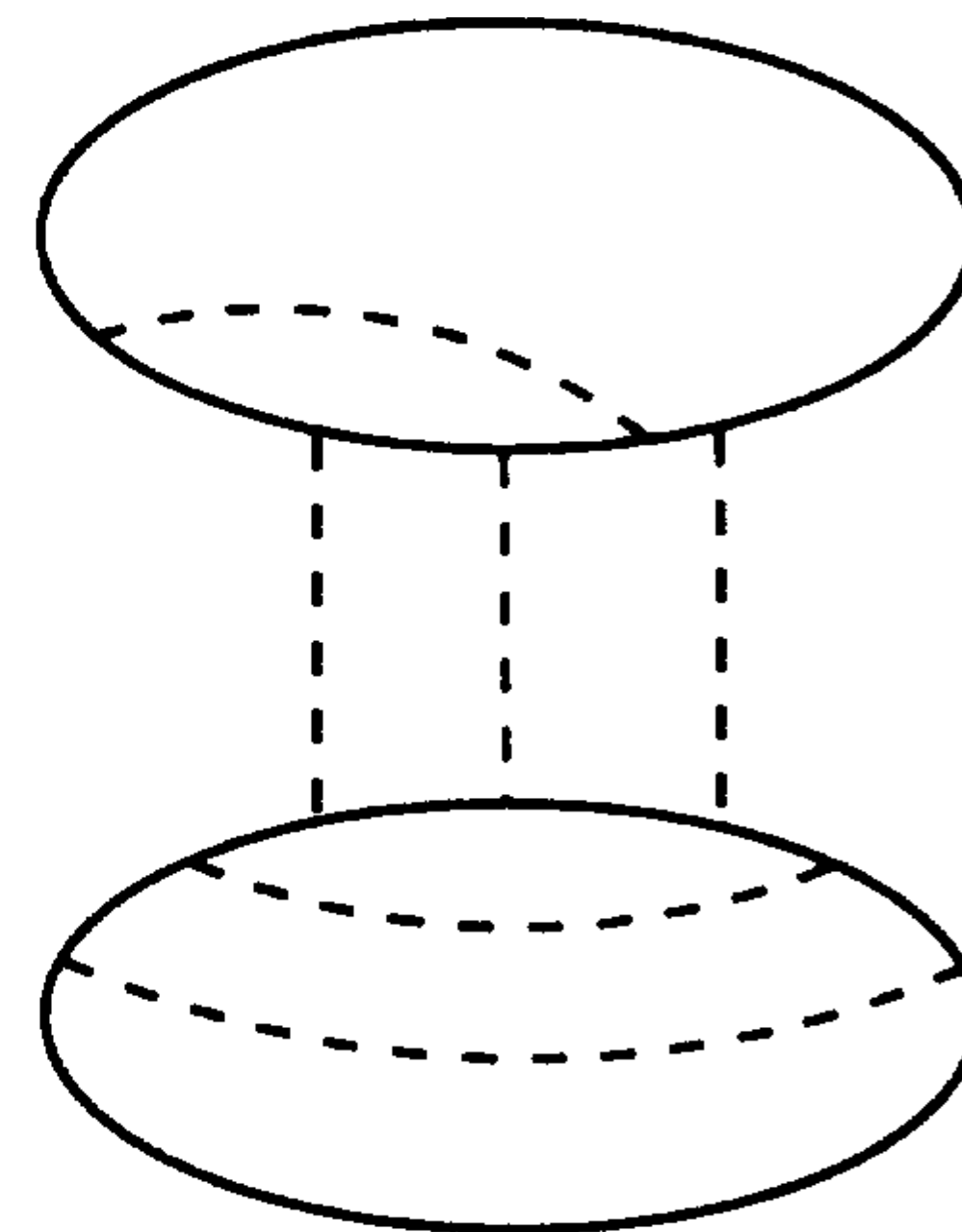
Henry Shackleton

Conductivity σ
 Normal metal regime:

$$\frac{\text{var}(\sigma)}{\bar{\sigma}^2} = \frac{\overline{(\sigma - \bar{\sigma})^2}}{\bar{\sigma}^2} = \frac{3\xi(3)}{16\pi^3} \left(\frac{h}{TN} \right)^2$$

Follows from Dyson-Mehta formula
 for single-particle RMT eigenvalue repulsion

$$\overline{\text{Im}G^r(\omega)\text{Im}G^r(\epsilon)} - \overline{\text{Im}G^r(\omega)}\overline{\text{Im}G^r(\epsilon)} = -\frac{1}{2(\omega - \epsilon)^2}$$



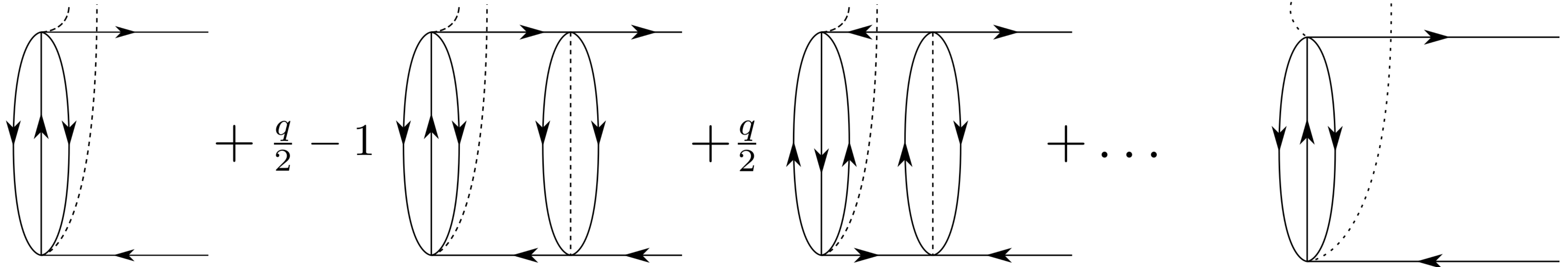
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Henry Shackleton

Conductivity σ
 SYK regime:

$$\frac{\text{var}(\sigma)}{\bar{\sigma}^2} = \frac{(\sigma - \bar{\sigma})^2}{\bar{\sigma}^2} = \frac{1}{8N^4}$$

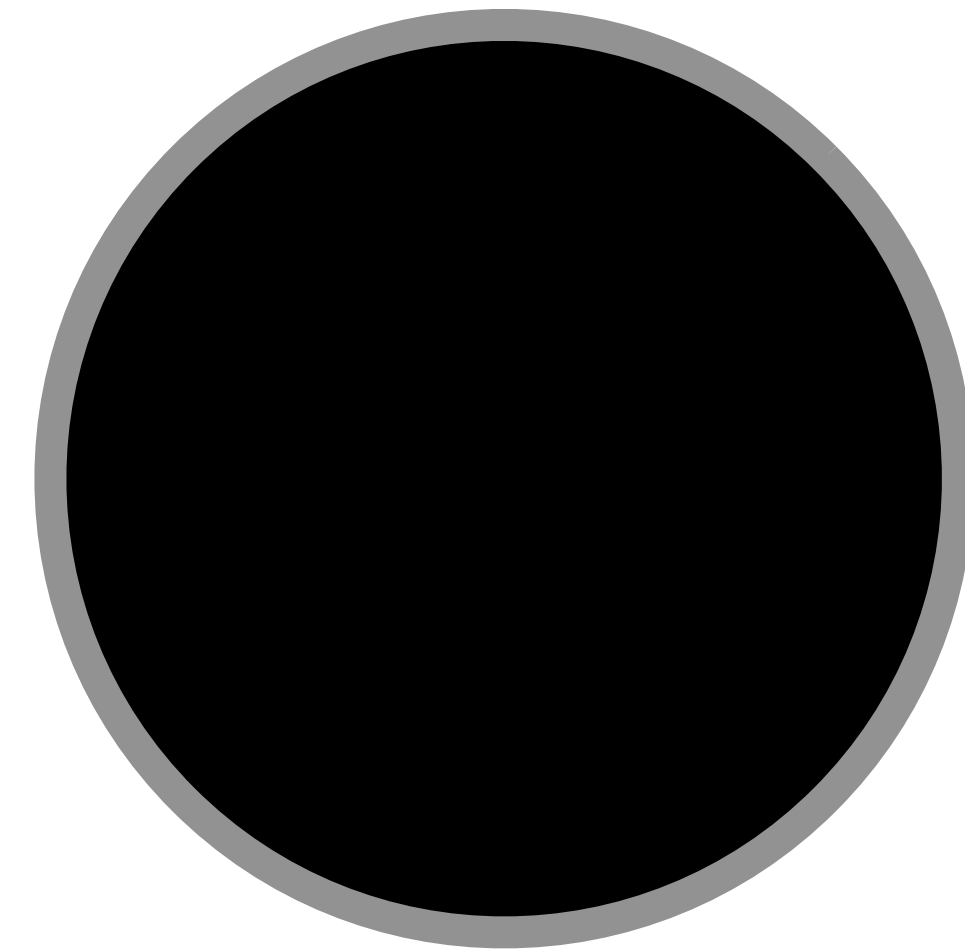


From the SYK model
to a quantum theory of
charged black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

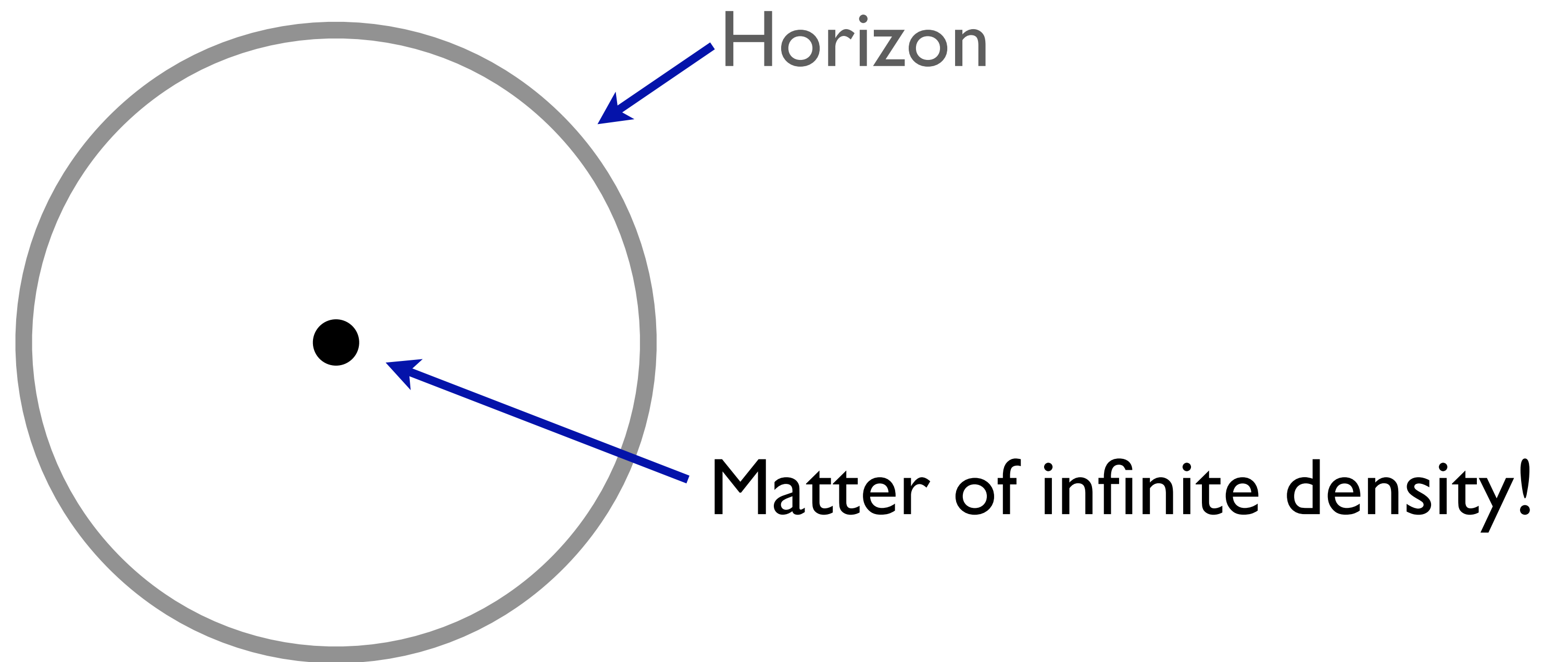
Horizon radius $R = \frac{2GM}{c^2}$



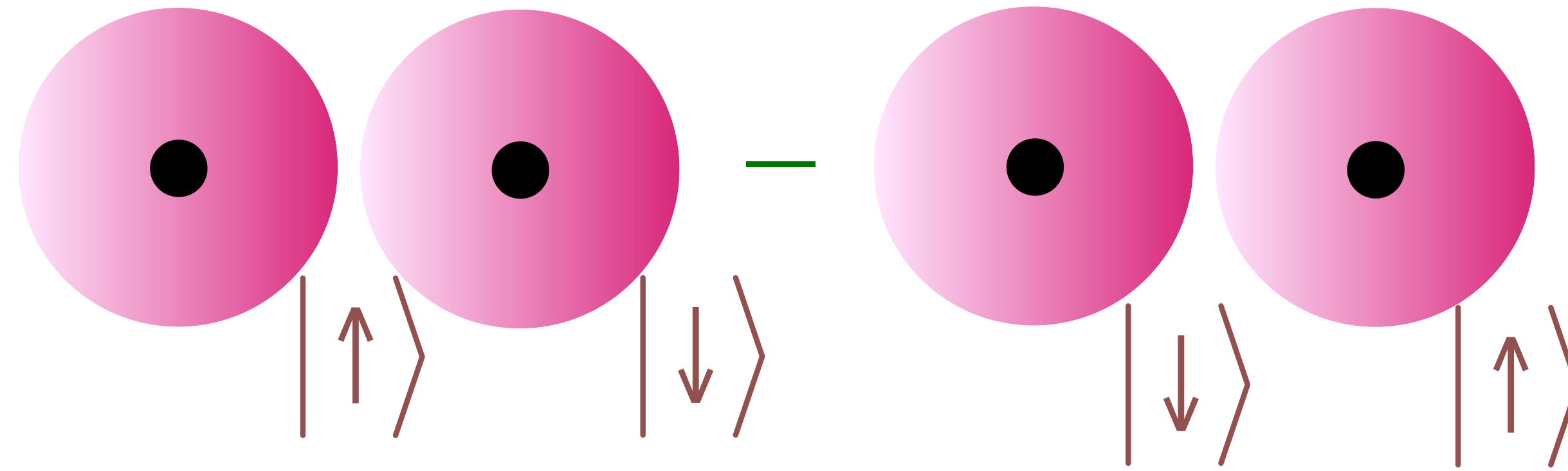
G Newton's constant, c velocity of light, M mass of black hole
For $M = \text{earth's mass}$, $R \approx 9 \text{ mm}$!

What is inside a black hole ???

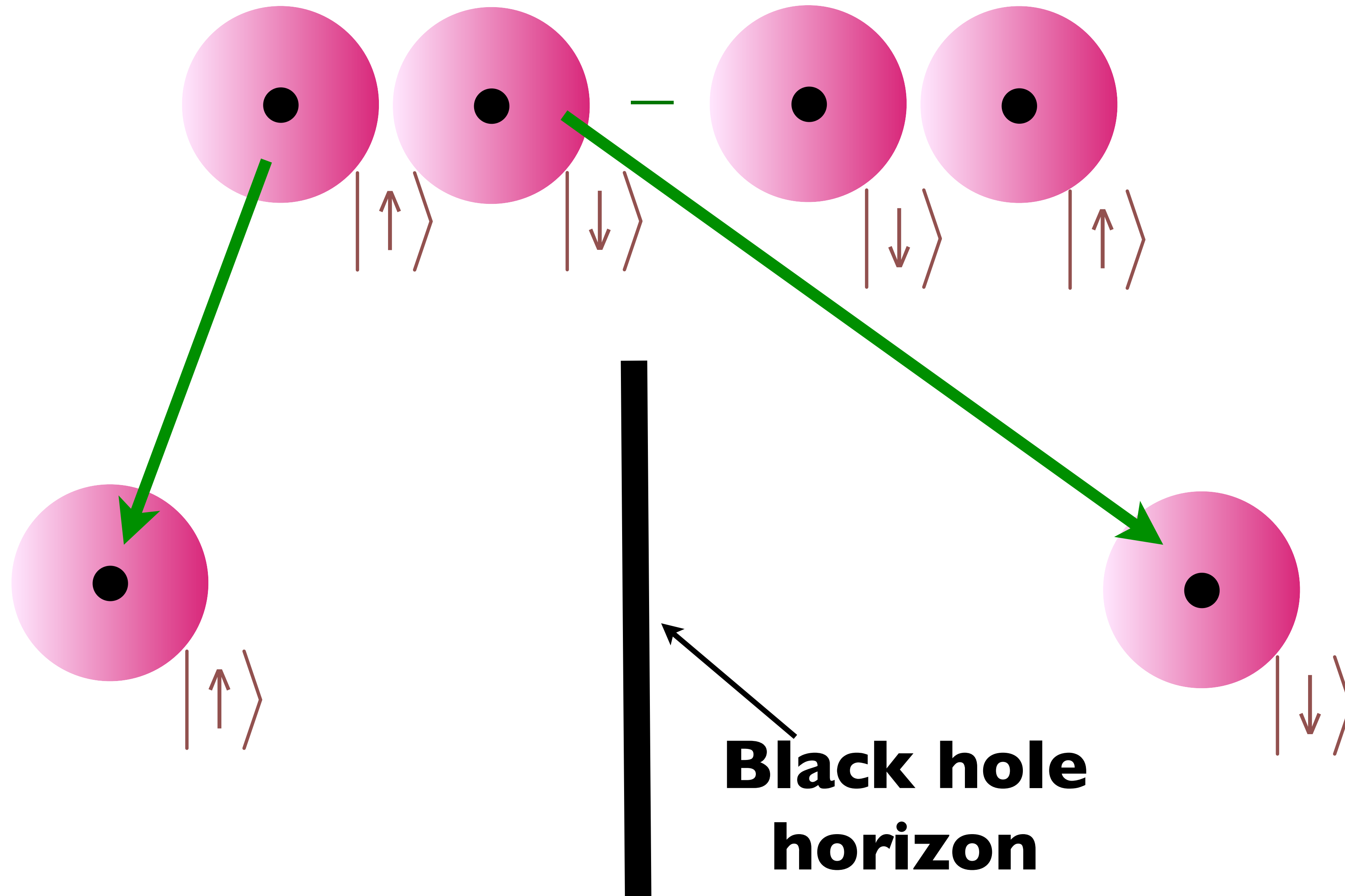
In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.
This singularity must be understood quantum mechanically



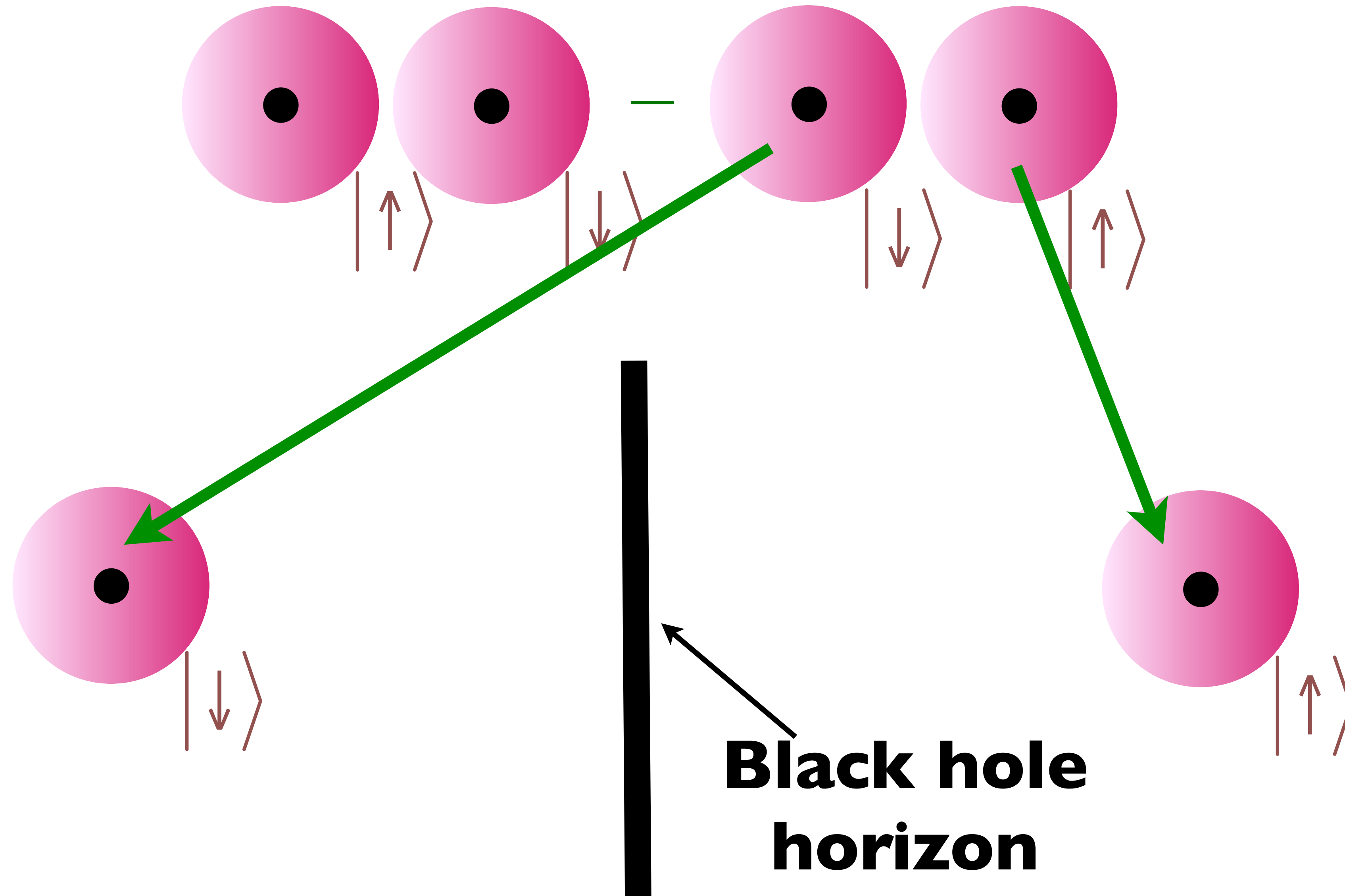
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

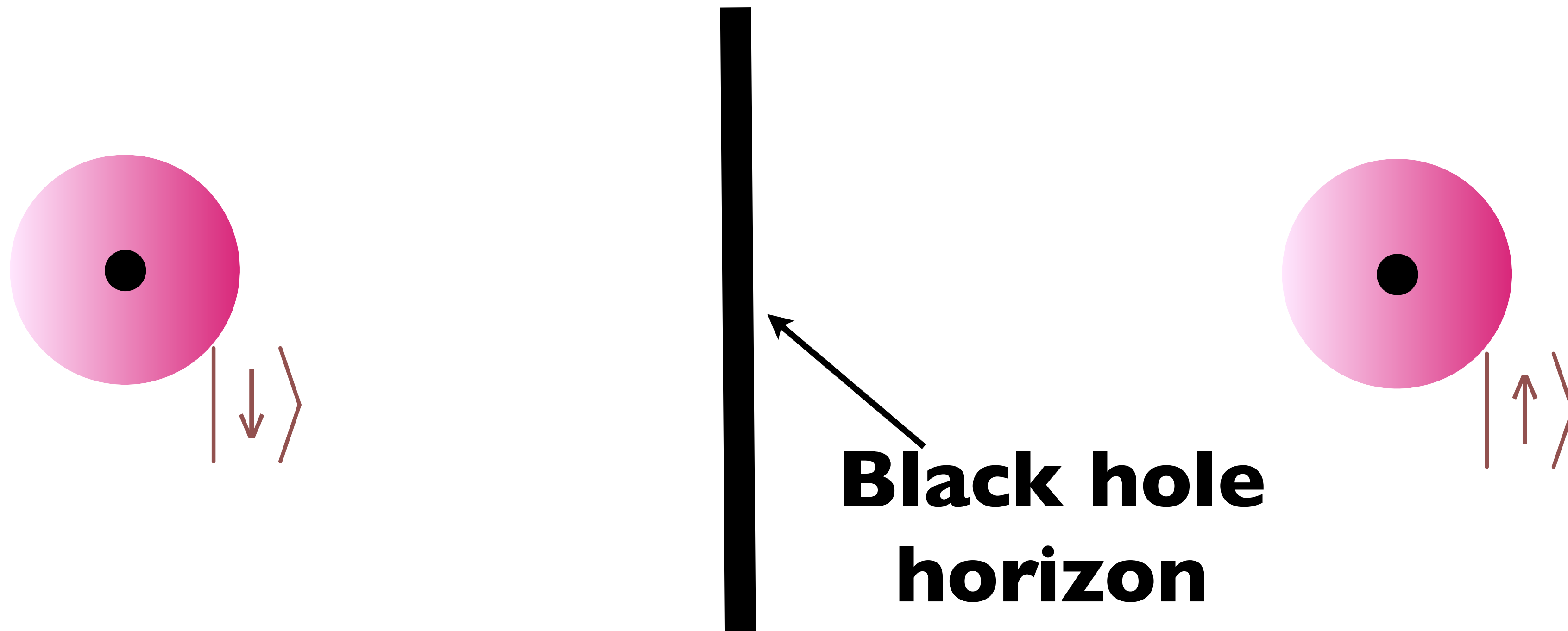


Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Black hole horizons have a temperature and an entropy (Hawking, 1974): to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.

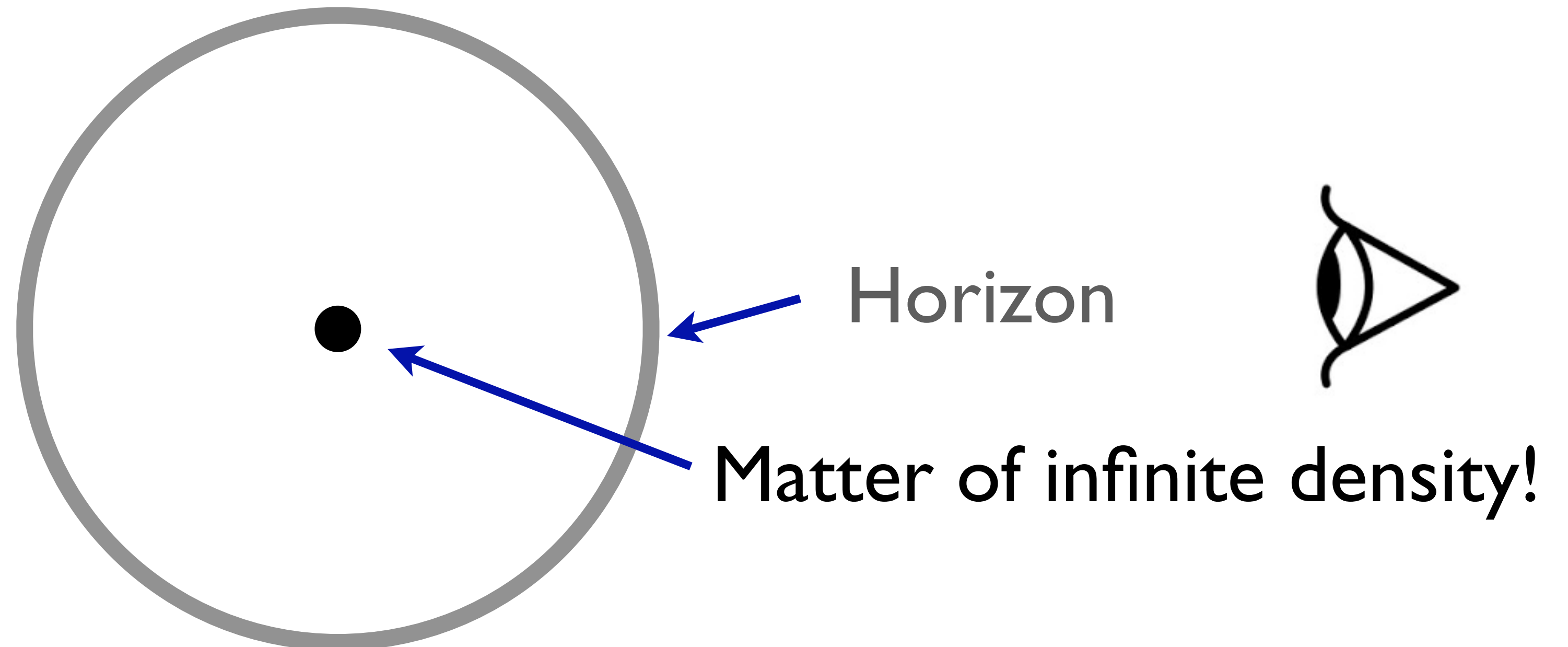


Quantum Black Holes

Hawking obtained the black hole entropy by the saddle-point of the gravitational path integral outside the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.

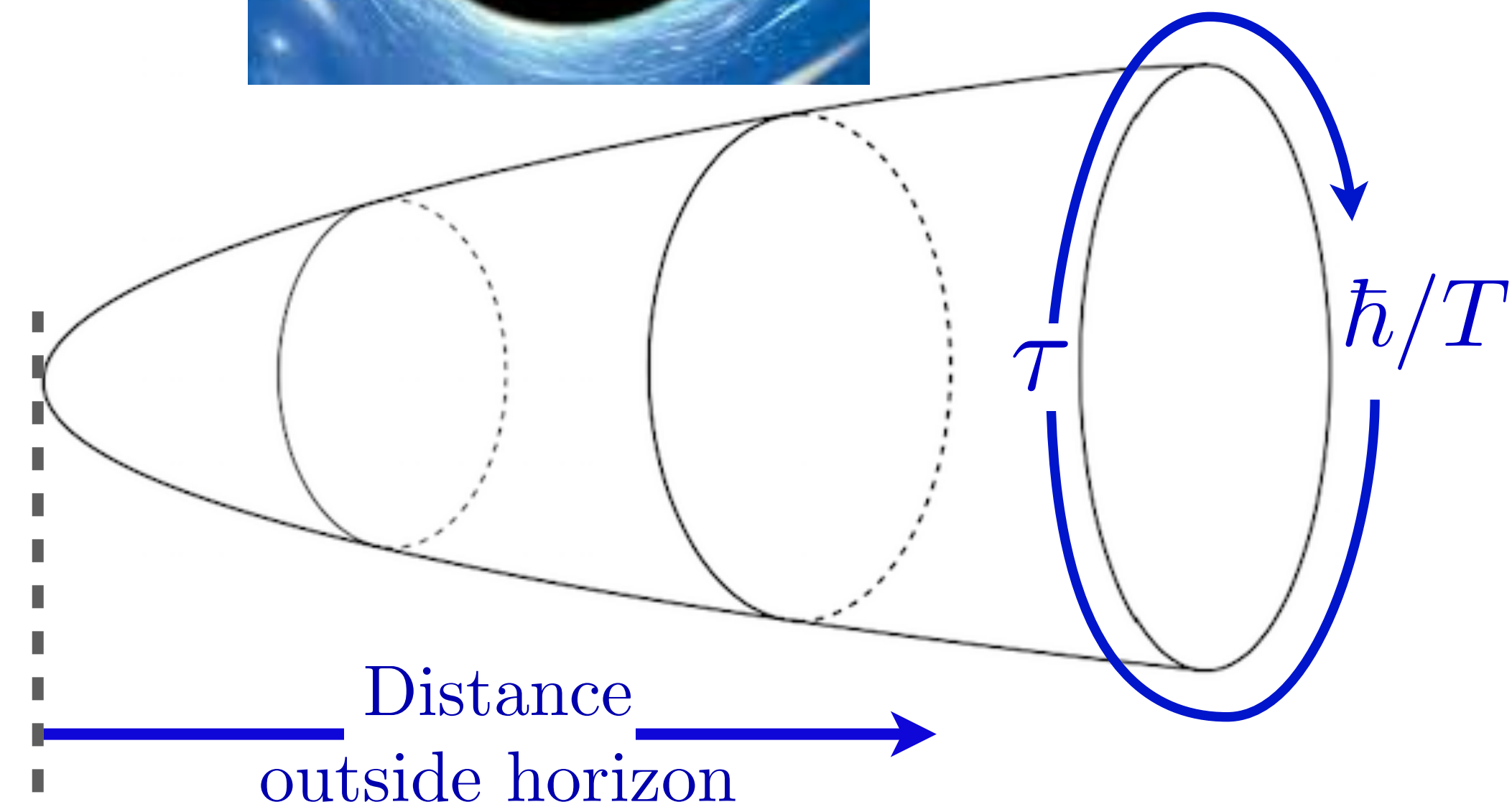
The answer was independent of the nature of the matter inside.



Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



Thermodynamics of quantum black holes with charge Q :



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$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

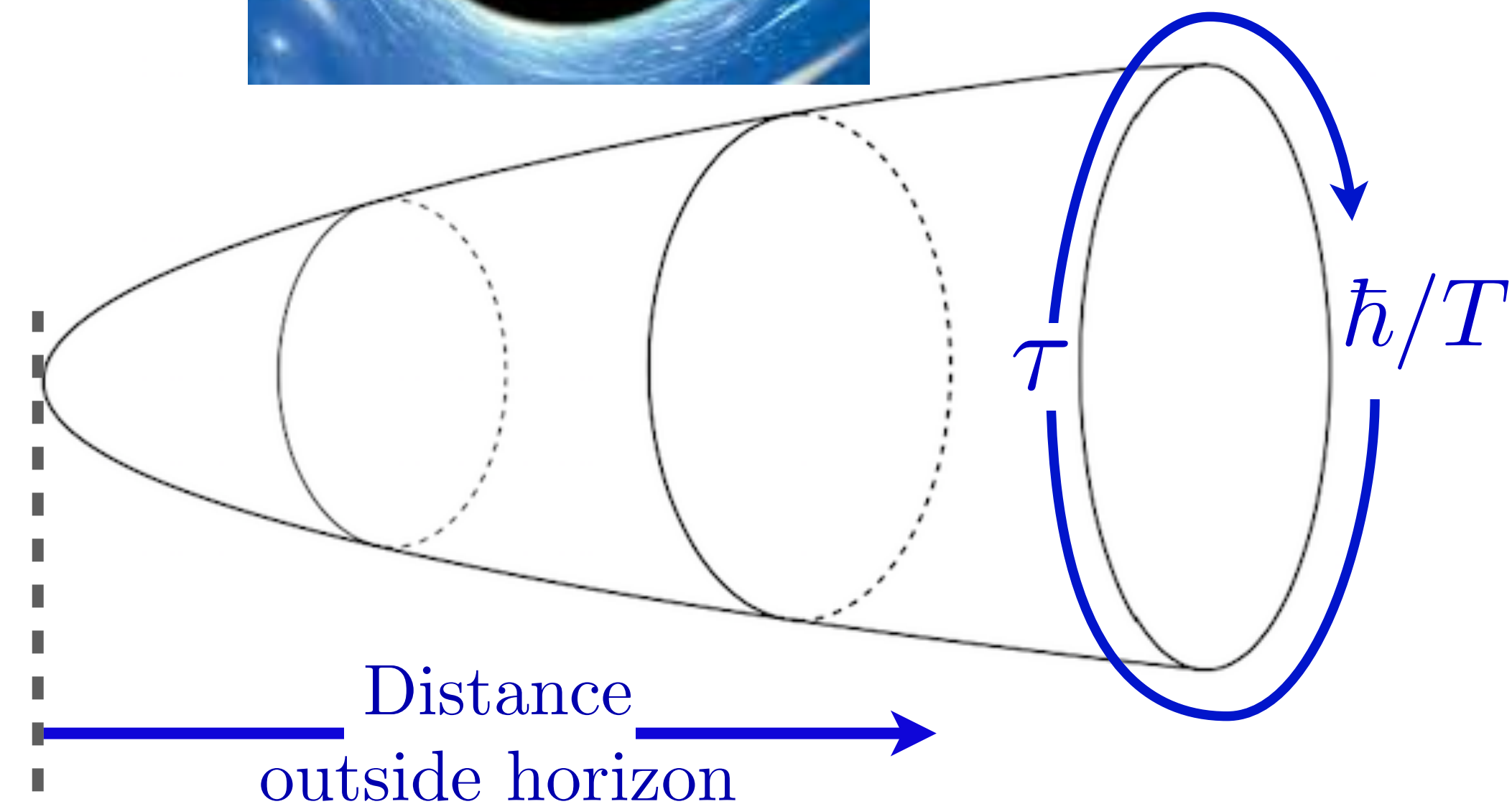
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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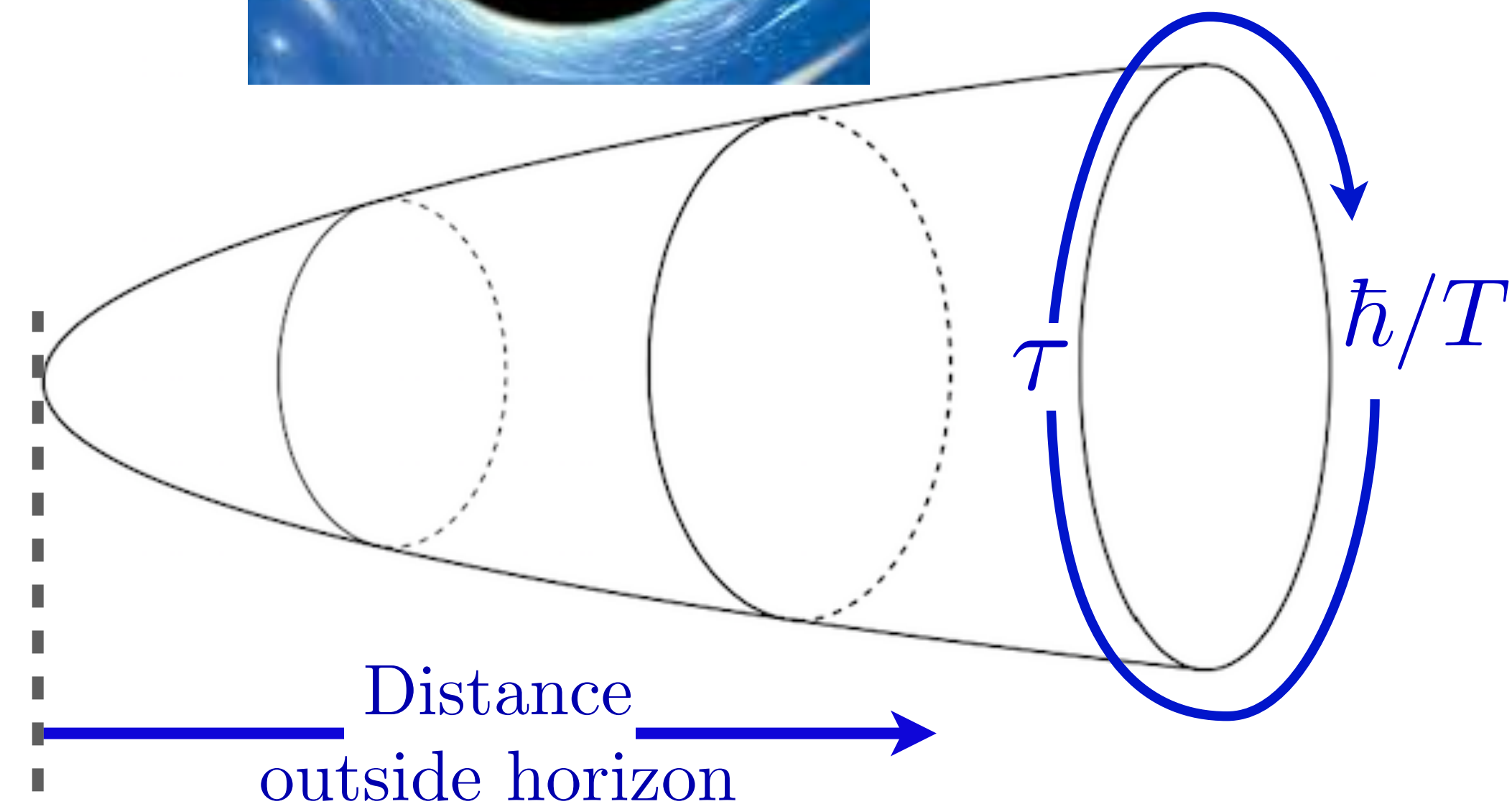


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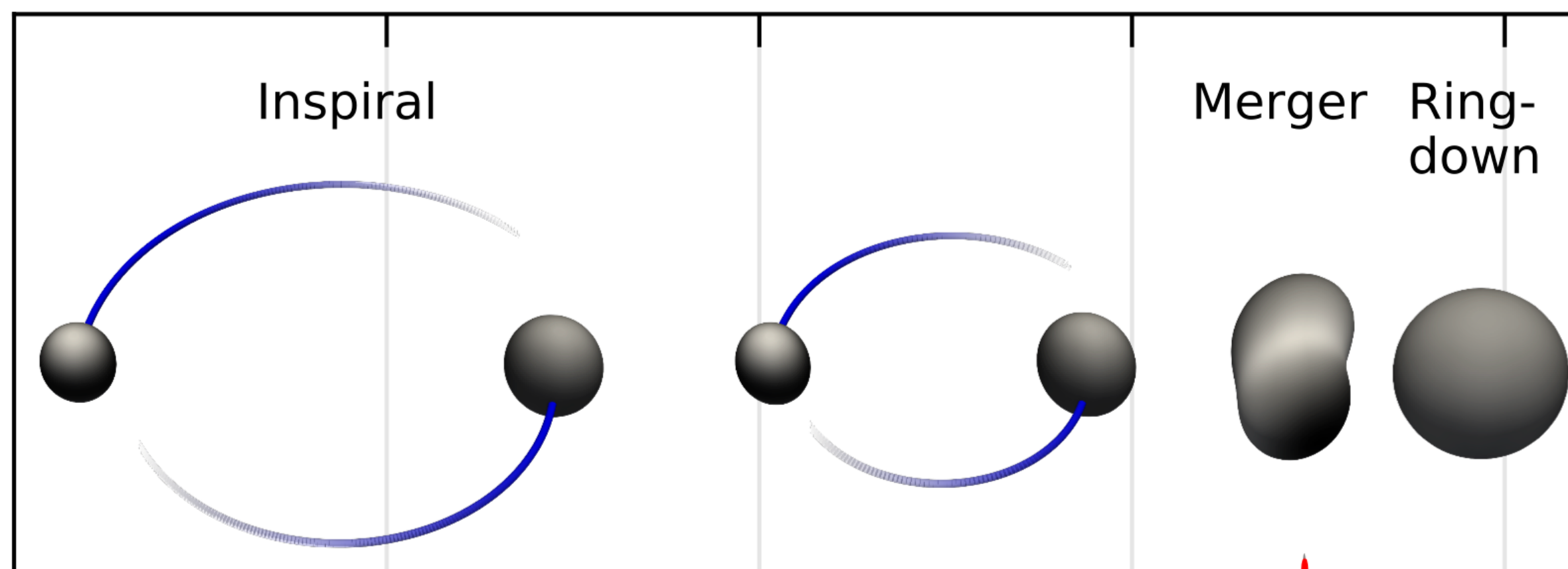
Note the similarity to the large N entropy of the SYK model!
(along with other similarities)

Sachdev PRL 2010



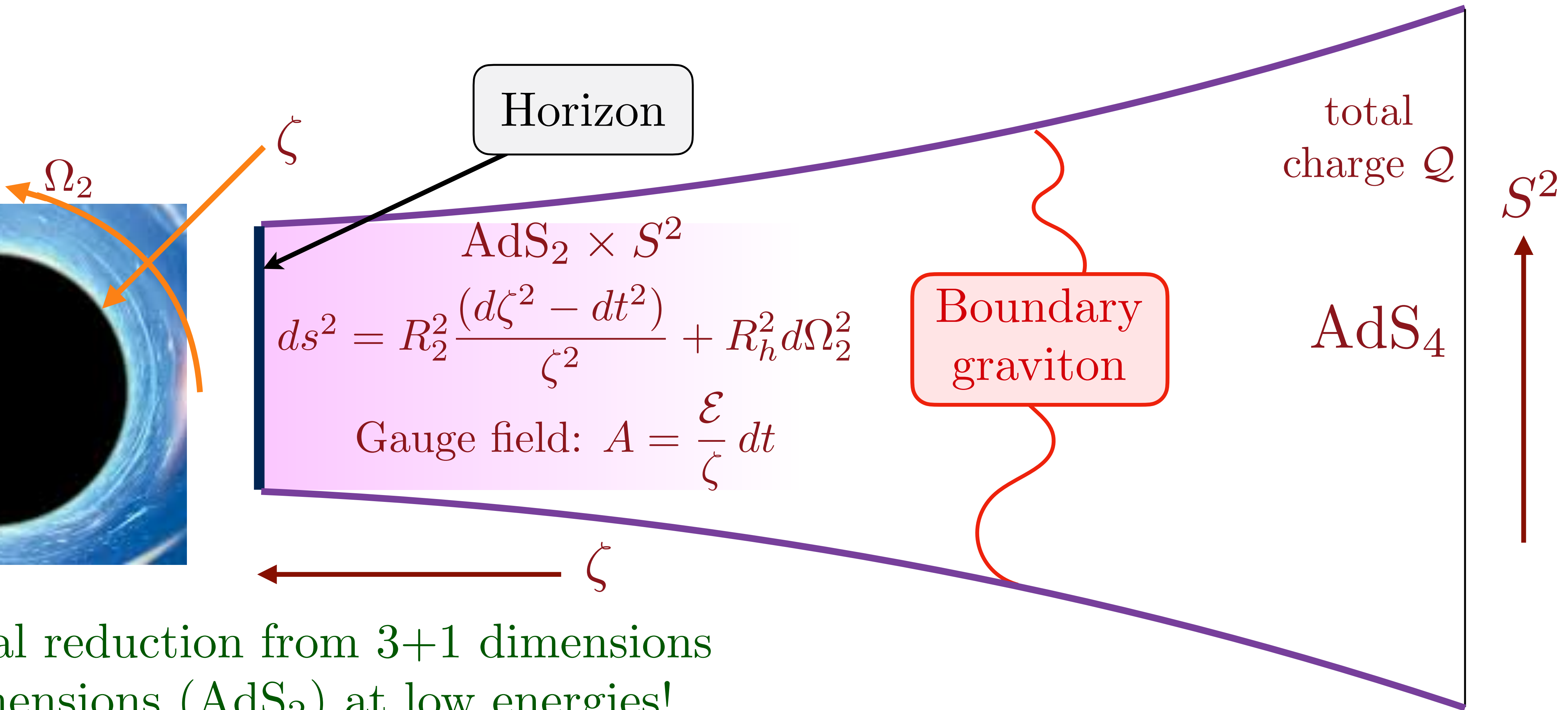
Quantum black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3 / (8\pi GM k_B)$.
- The entropy is proportional to their surface area. $S = Ak_B c^3 / (4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi GM / c^3 = \hbar / (k_B T_H)$ which is Planckian!



J. D. Bekenstein, PRD **7**, 2333 (1973)
S.W. Hawking, Nature **248**, 30 (1974)
C.V. Vishveshwara, Nature **227**, 936 (1970)

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group $SL(2, \mathbb{R})$.

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$

Saddle-point:

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} + \dots \right)$$

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Thermodynamics of quantum black holes with charge Q :

$$\begin{aligned} \mathcal{Z}(Q, T) &= \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_\mu] \right) \\ &\approx \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_\mu \exp \left(-\frac{1}{\hbar} I_{\text{JT gravity of AdS}_2+\text{boundary graviton}}^{(1+1)}[g_{\mu\nu}, A_\mu] \right) \end{aligned}$$

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 &= \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) \exp \left(-\frac{1}{\hbar} I_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)] \right)
 \end{aligned}$$

Saddle-point:

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} + \dots \right)$$

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$$S(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{2} \ln \left(\frac{(\hbar c^5 / G)^{1/2}}{T} \right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a

$$-\frac{559}{180} \ln \left(\frac{A_0 c^3}{\hbar G} \right)$$

term from other massless modes; Sen (2011)
Iliesiu, Murthy, Turiaci (2022)

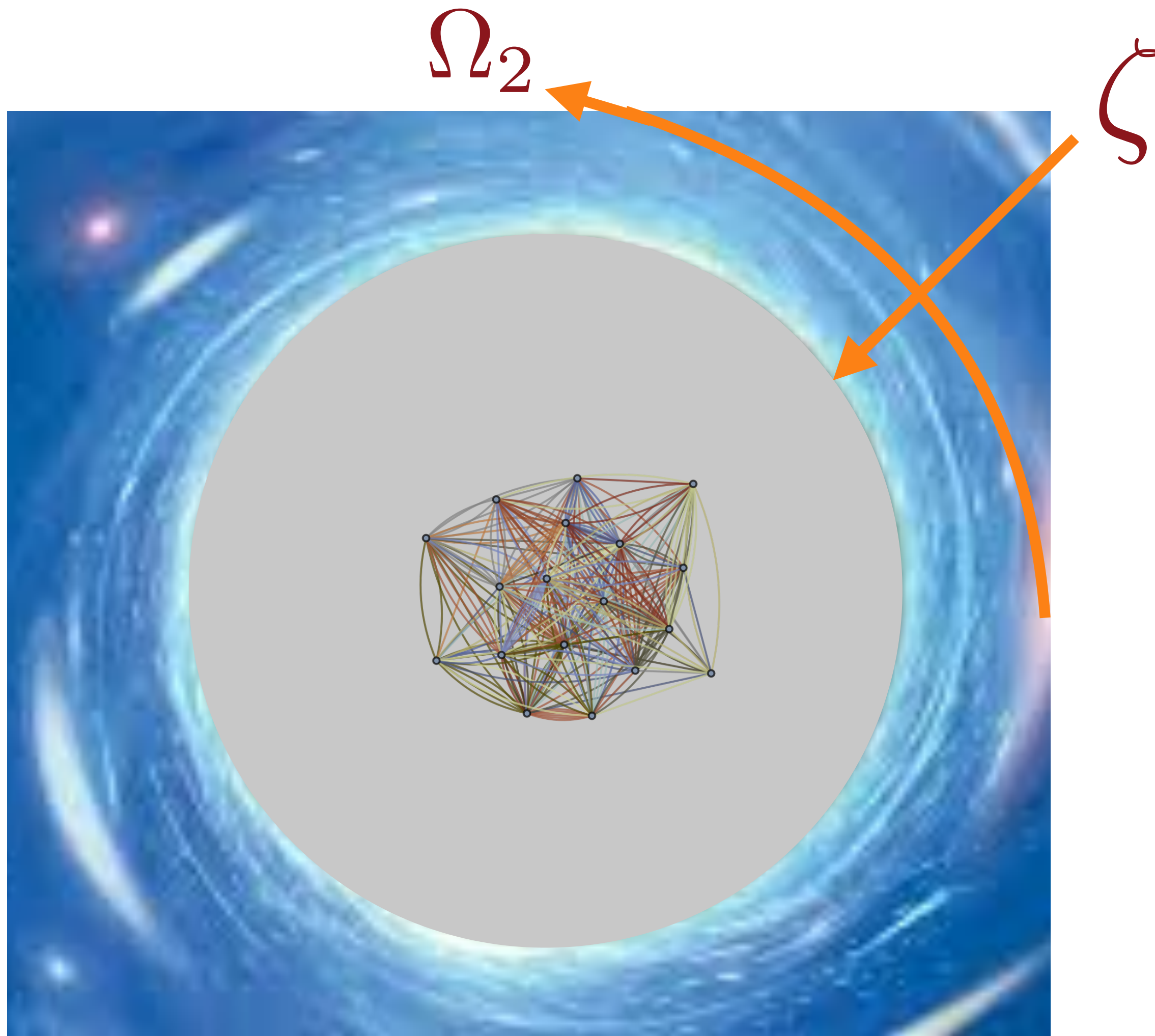
Quantum Black Holes

Hawking obtained the black hole entropy by the saddle-point of the gravitational path integral outside the black hole horizon.

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose density of quantum states matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?



Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The SYK provides a realization of the black hole interior, and its density of quantum states matches gravitational entropy computations for charged black holes !

The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

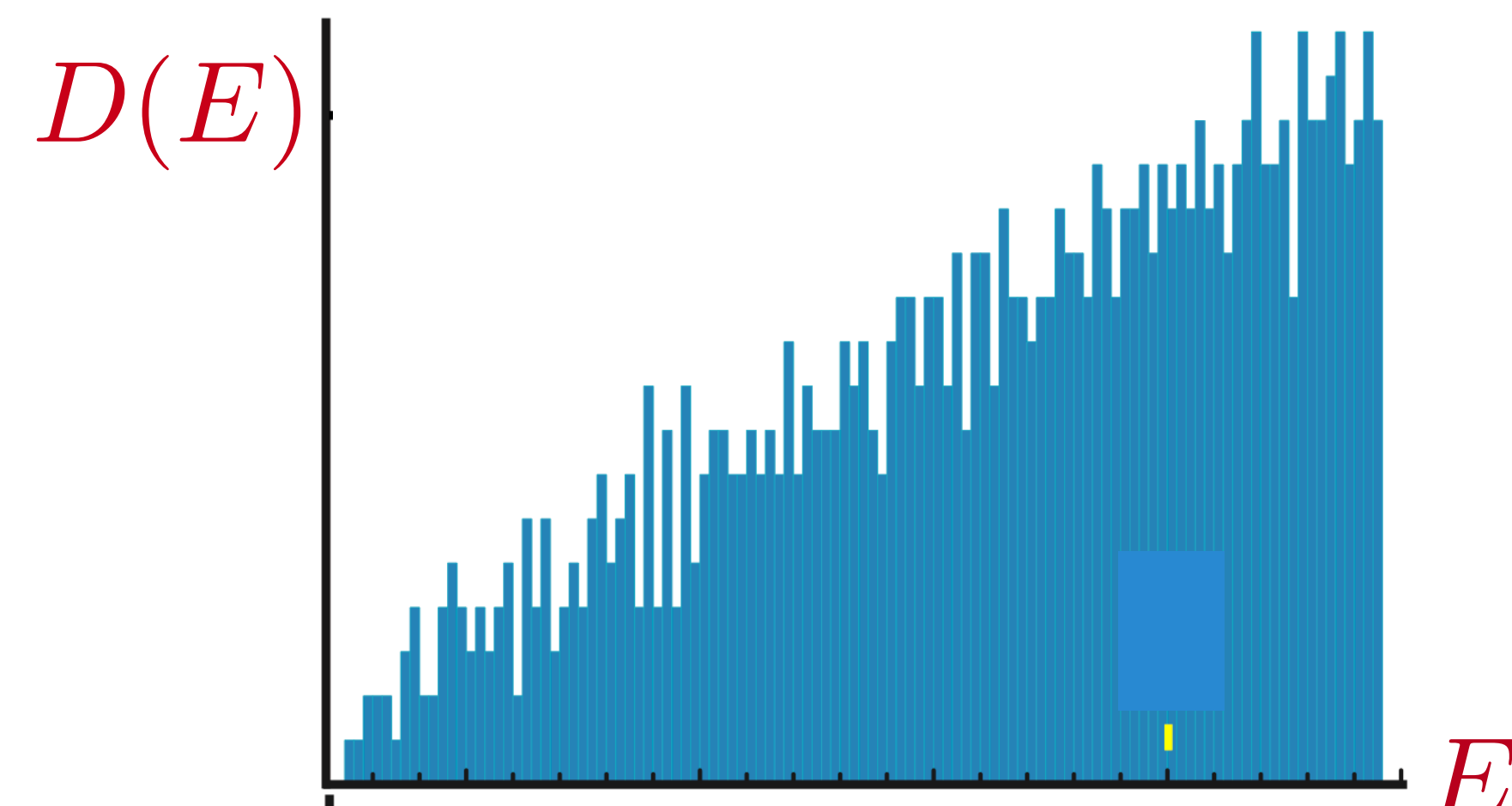
Fermion Green's function $G(\omega) \sim T^{-1/2} F(\hbar\omega/k_B T)$

- There is an extensive entropy as $T \rightarrow 0$

$$\begin{aligned} s_0/k_B &= \lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} S/(k_B N) \neq 0 \\ &= \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833 \dots \end{aligned}$$

However, the ground state is not extensively degenerate: $D(E) \neq \delta(E) \exp(N s_0/k_B)$.

$$D(E) \sim N^{-1} \exp(N s_0/k_B) \sinh(\sqrt{2N\gamma E})$$



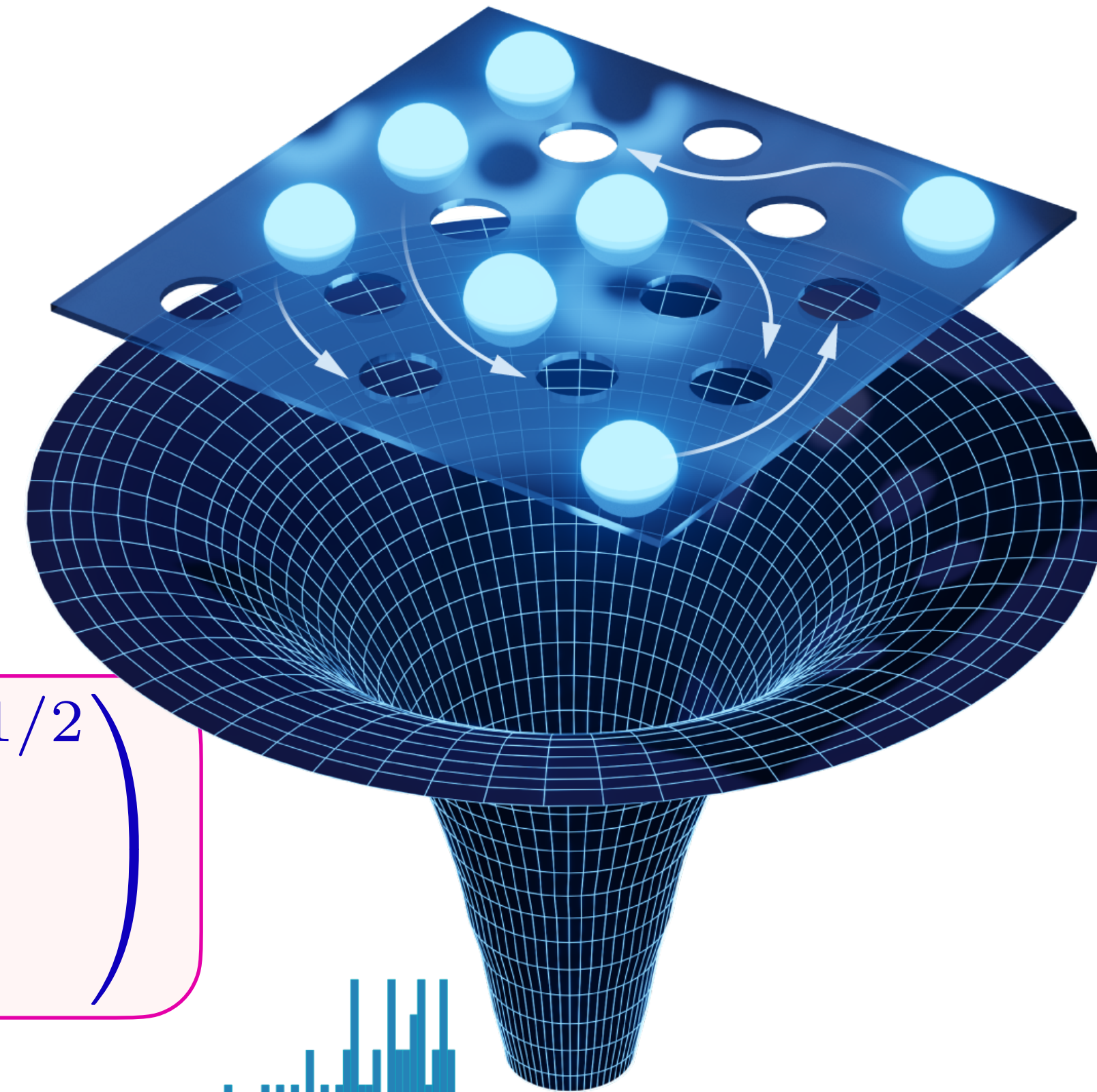
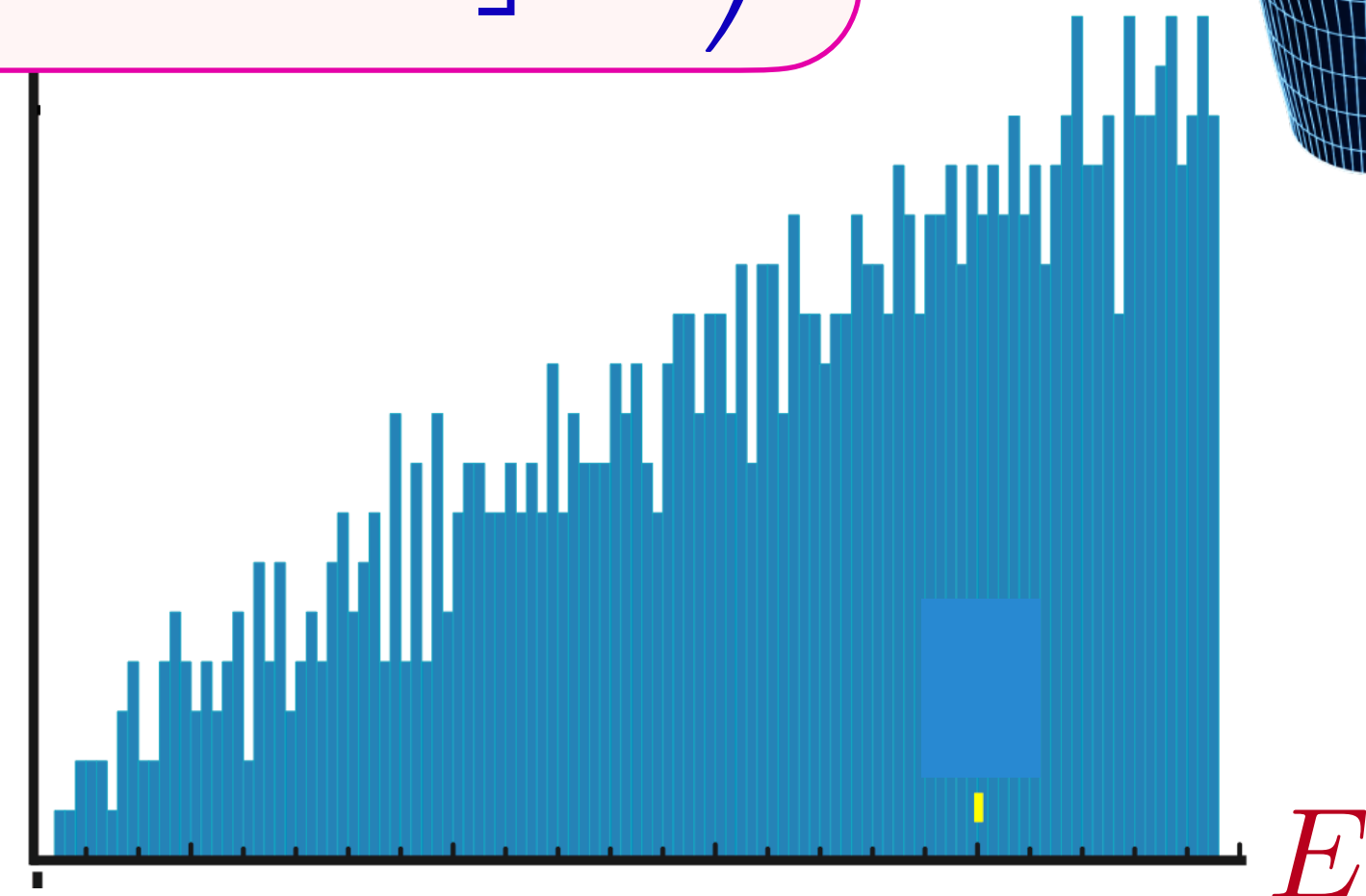
Quantum simulation of charged black holes by the SYK model

- For generic black holes with charge Q in asymptotically Minkowski 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$, the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

$D(E)$



Quantum simulation of charged black holes by the SYK model

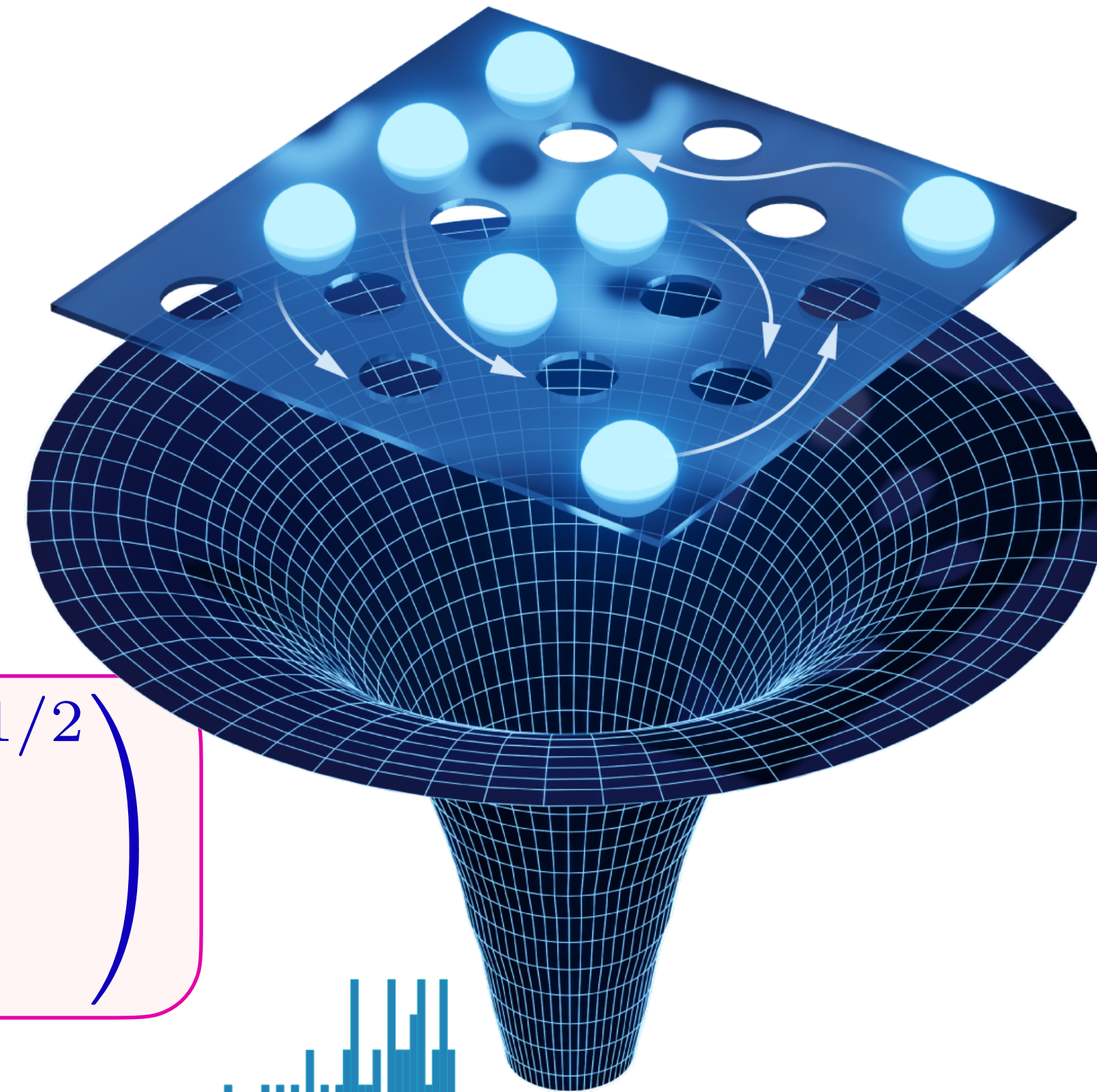
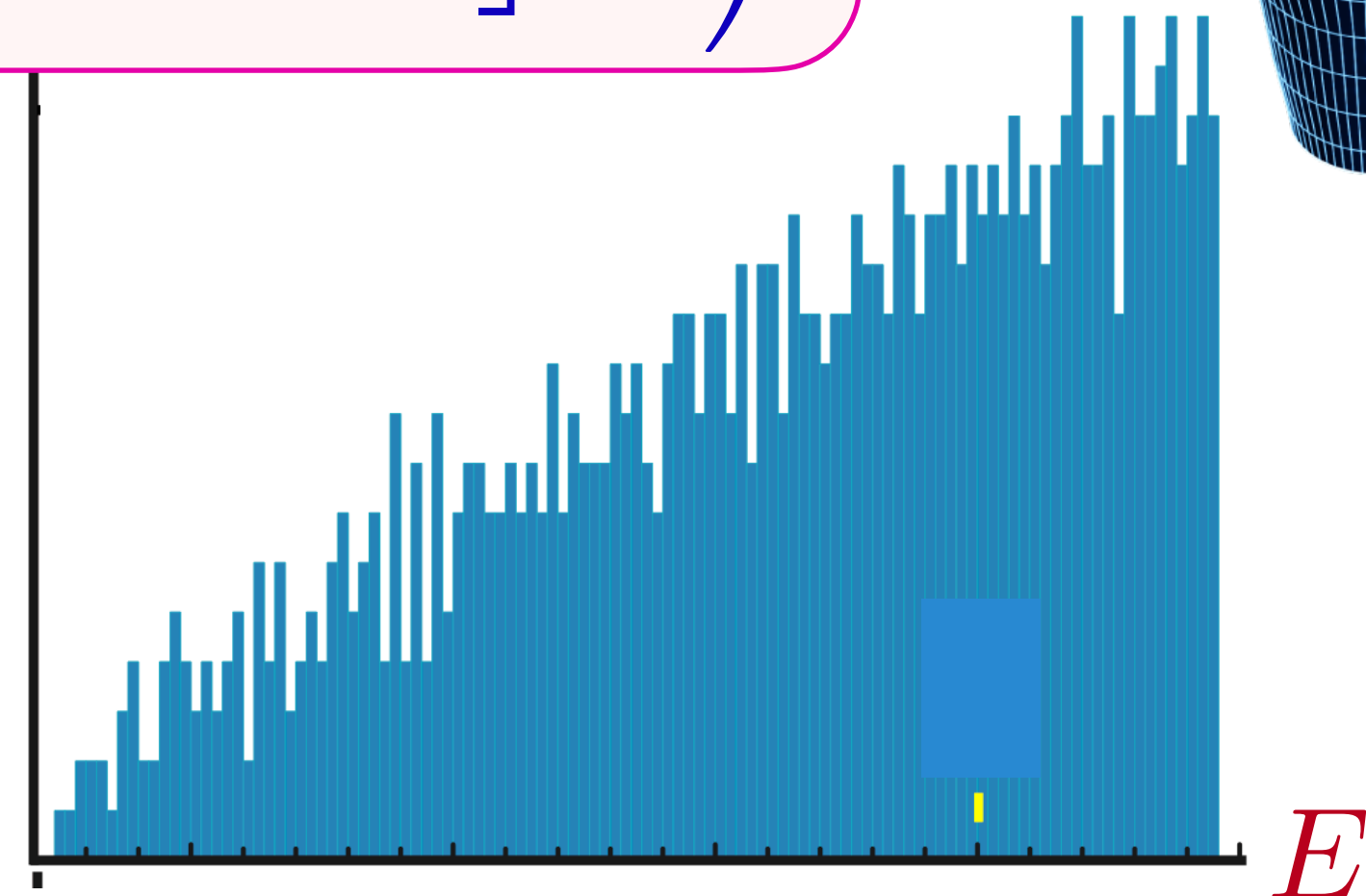
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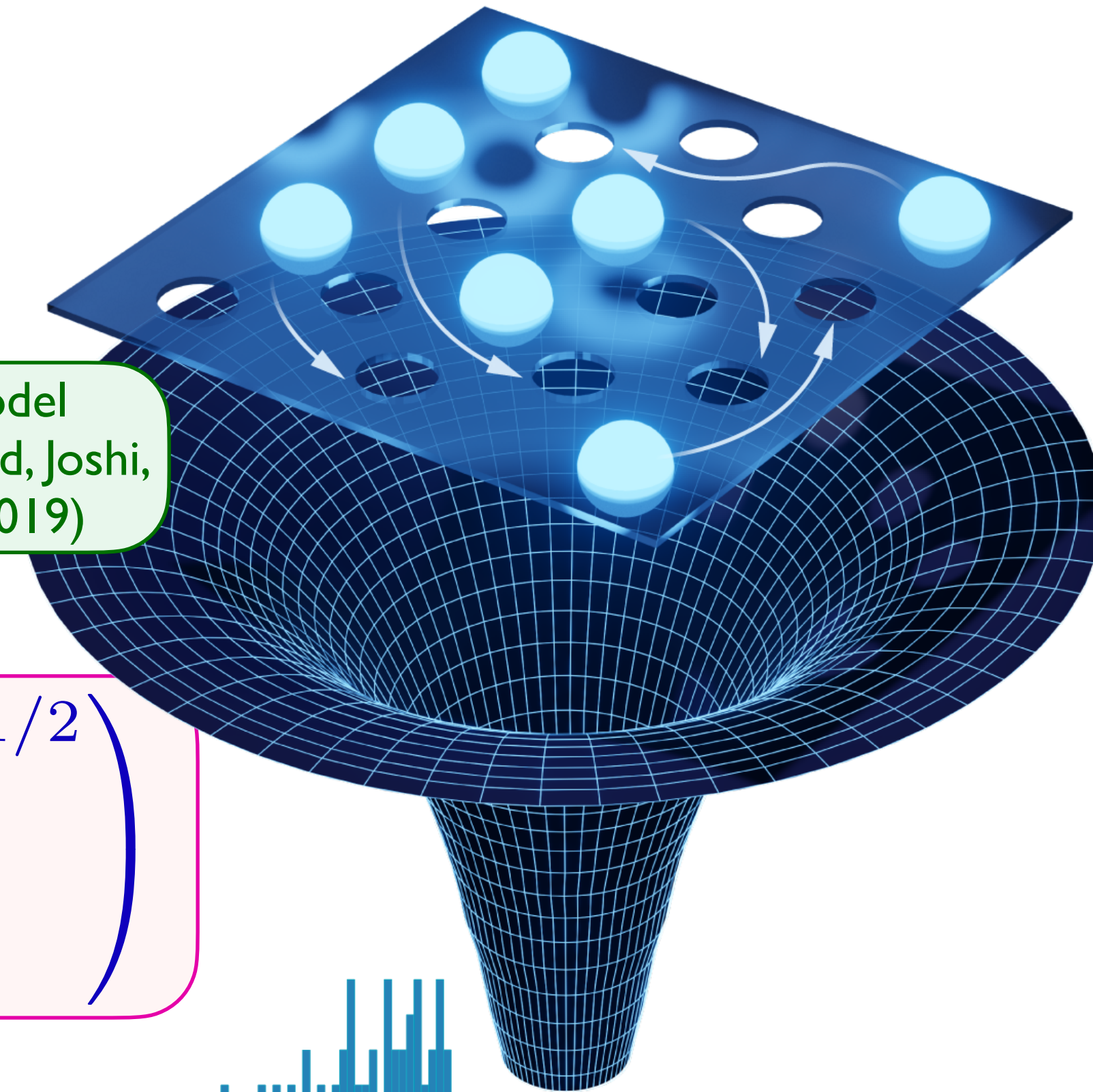
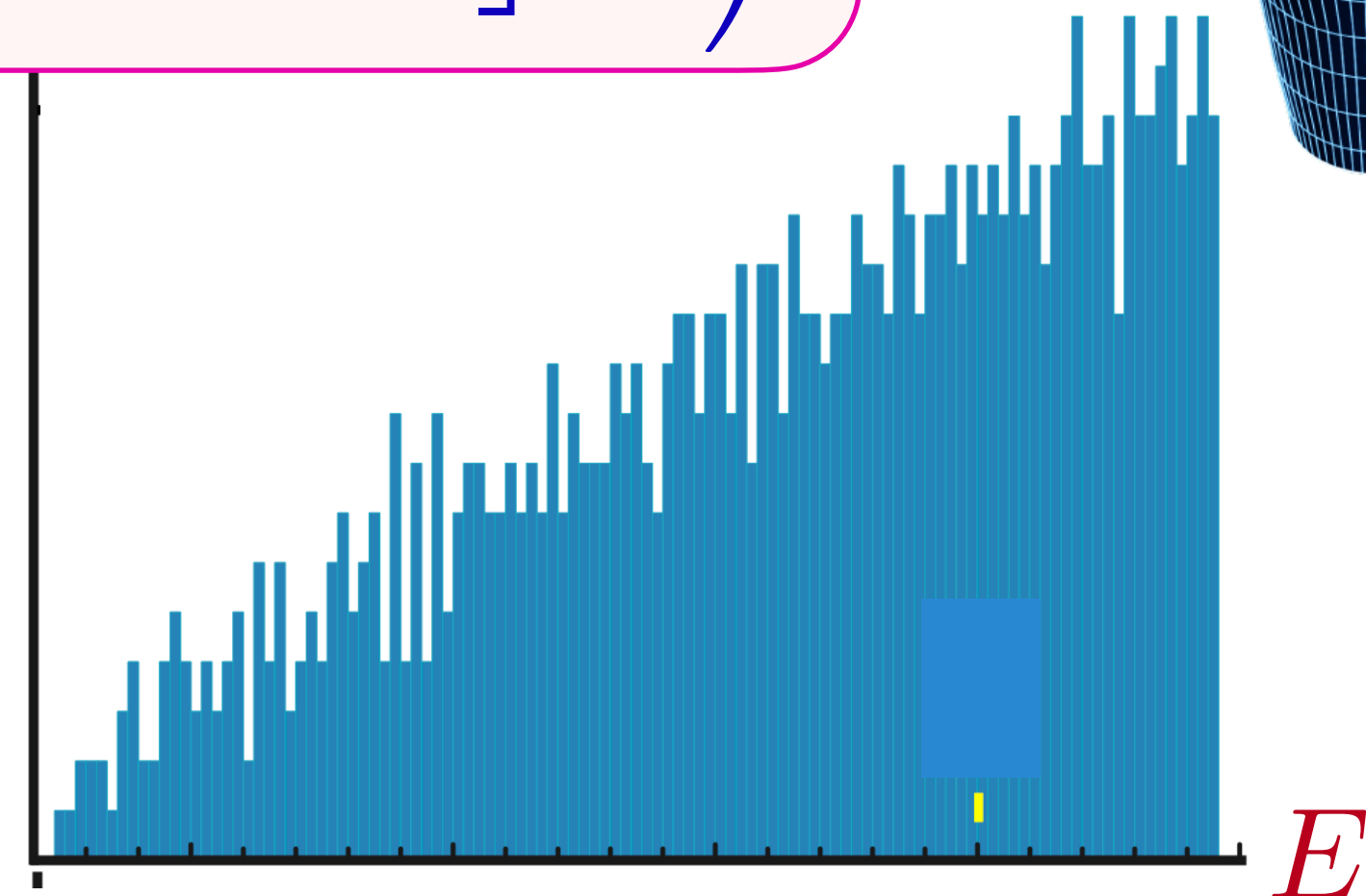
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Bekenstein-Hawking

Developments from the SYK model
Moitra, Trivedi, Vishal (2018); Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019)

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Iliesiu, Murthy, Turiaci (2022)

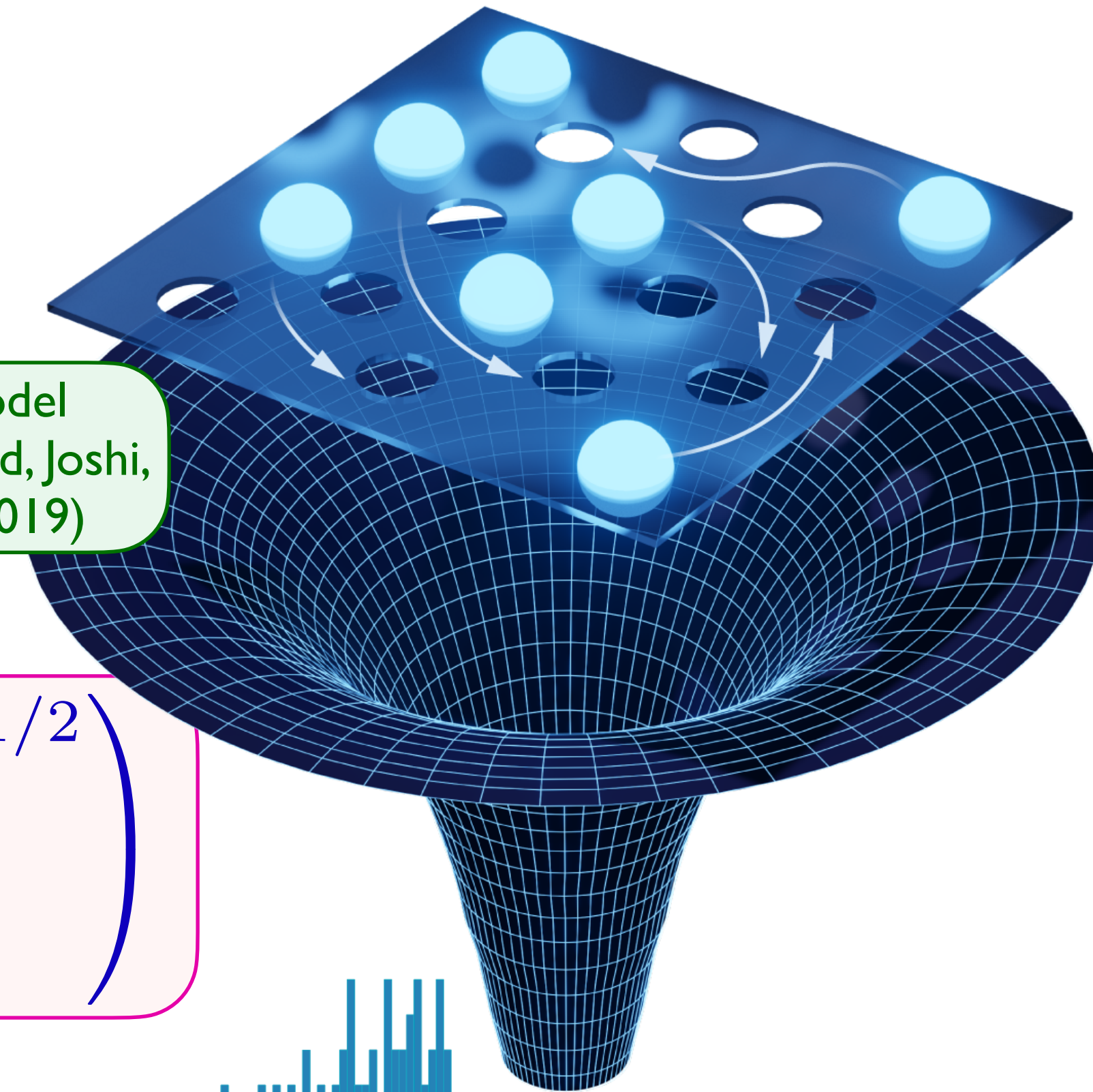
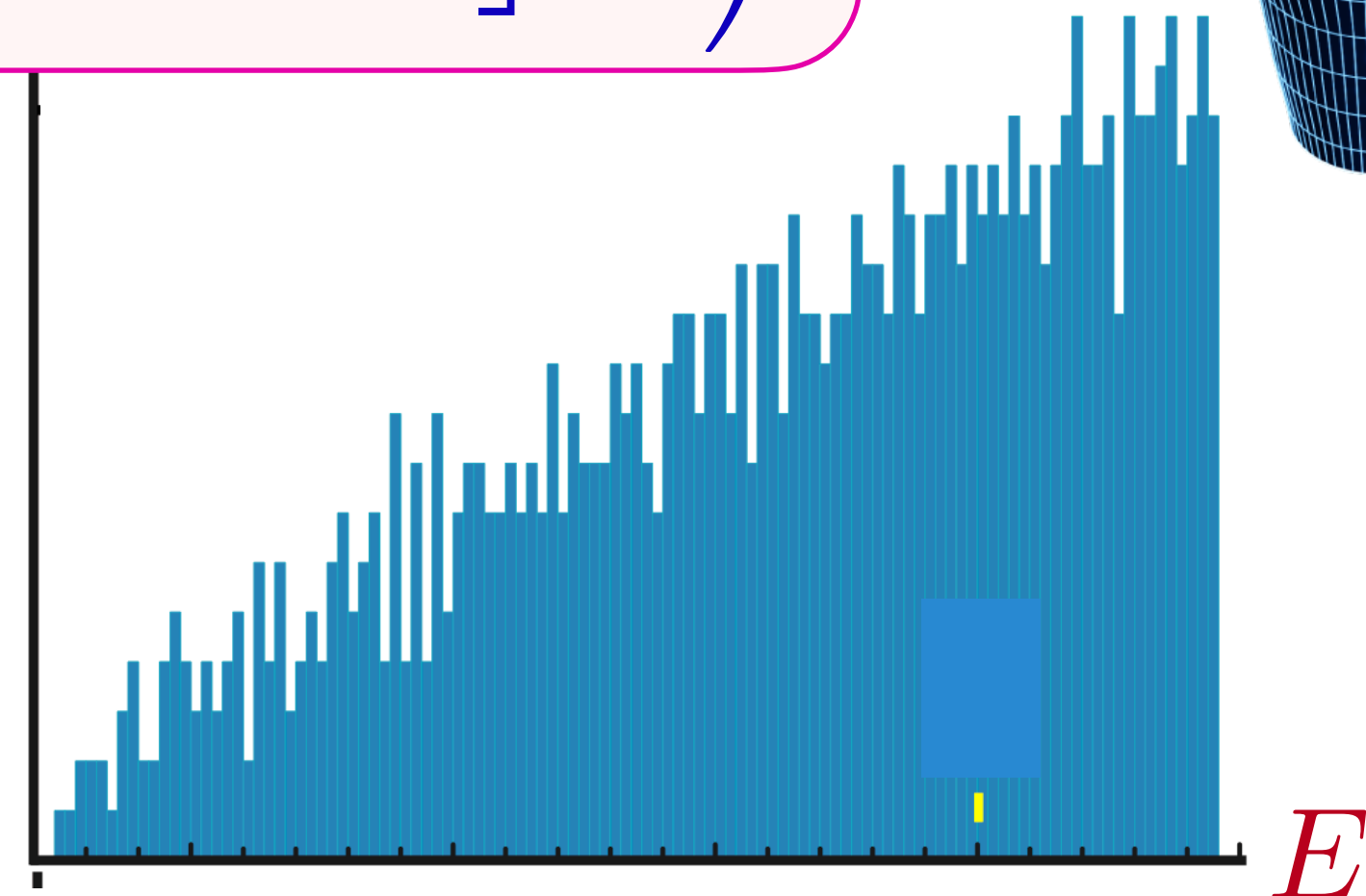
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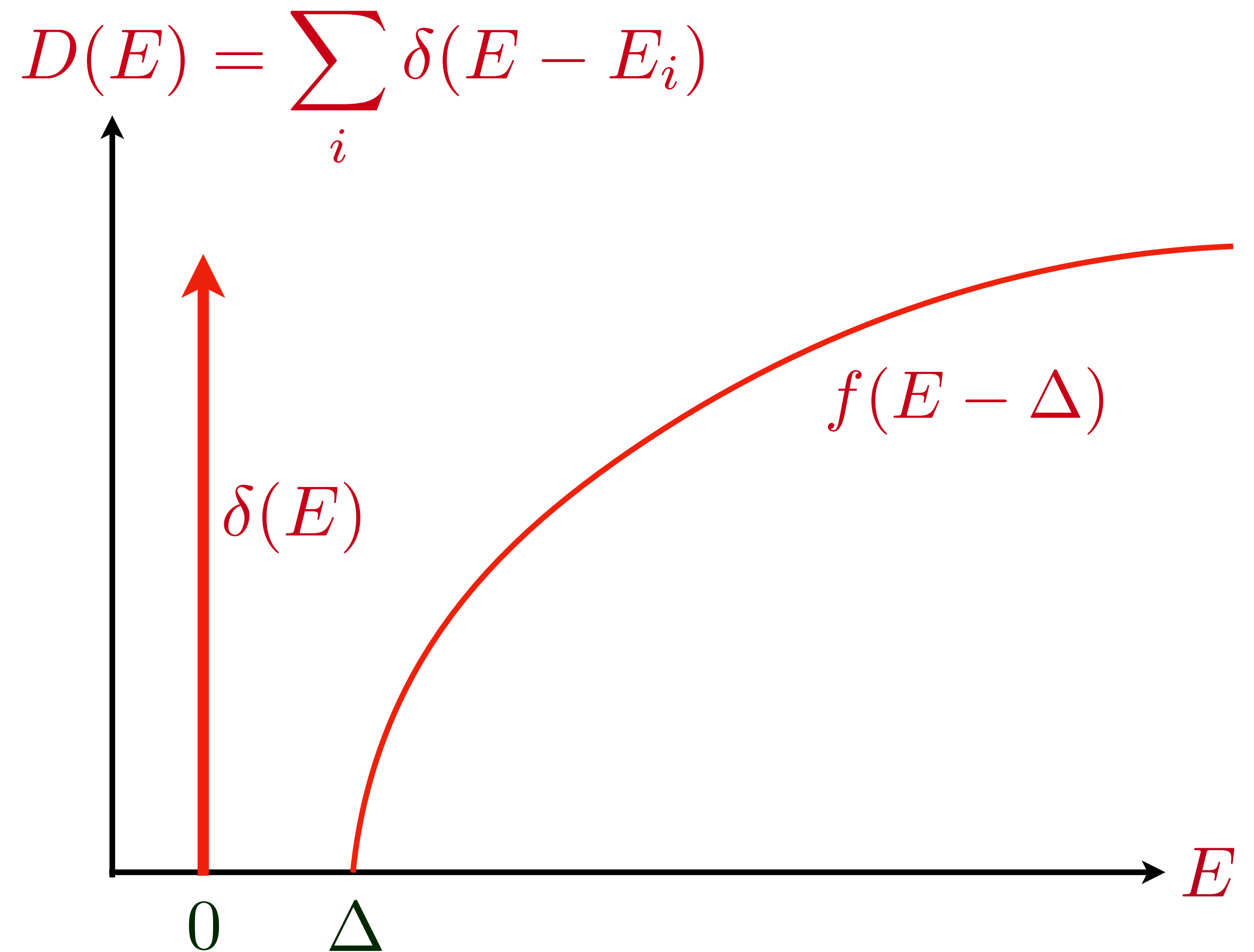


String theory of charged black holes

- With sufficient low energy supersymmetry:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \left[\delta(E) + \theta(E - \Delta) f(E - \Delta) + \dots \right]$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state. Similar results for supersymmetric SYK models



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020
L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

From the SYK model
to a universal theory of
strange metals



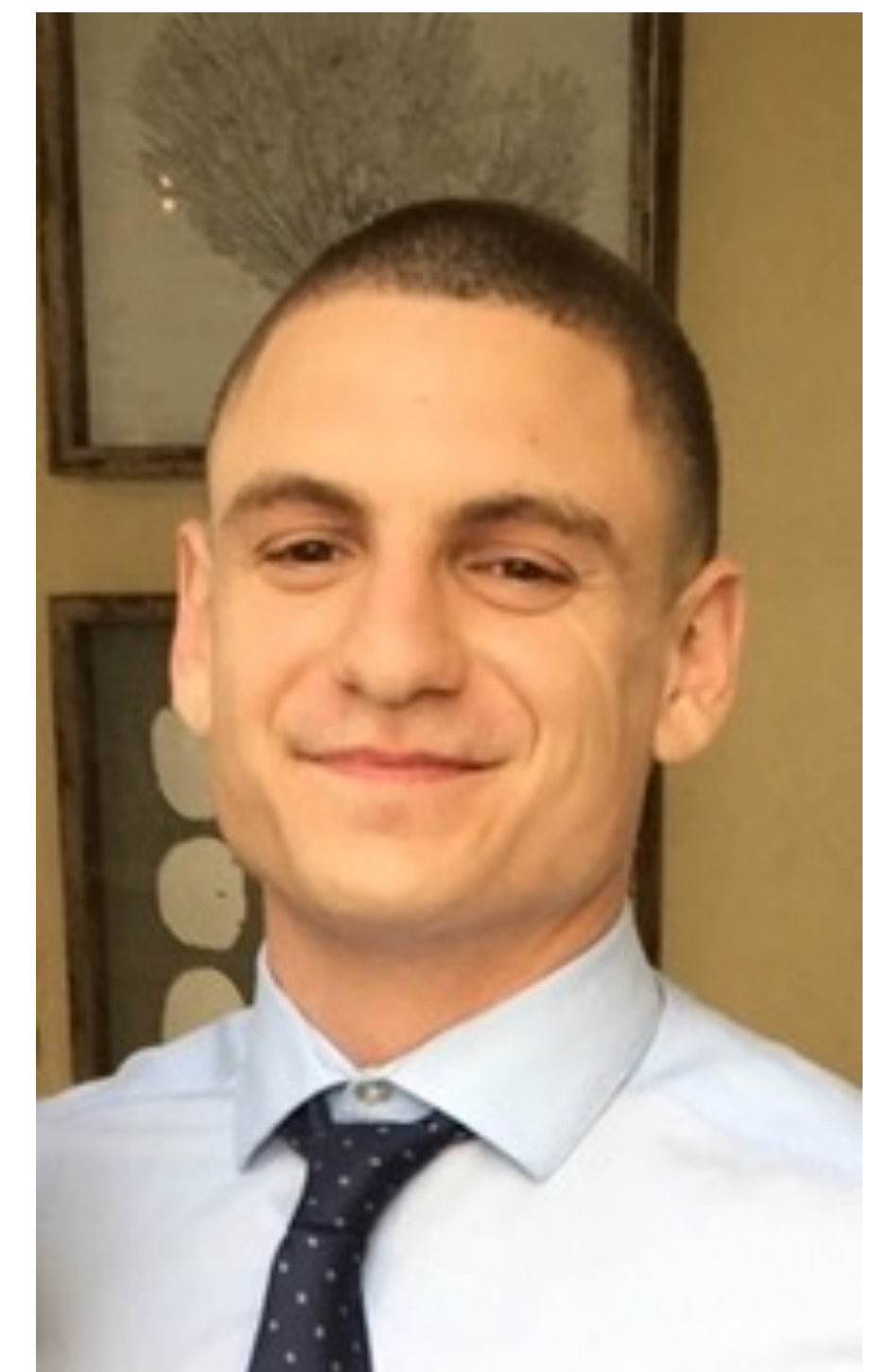
Aavishkar Patel

Flatiron Institute, NYC



Haoyu Guo

Harvard

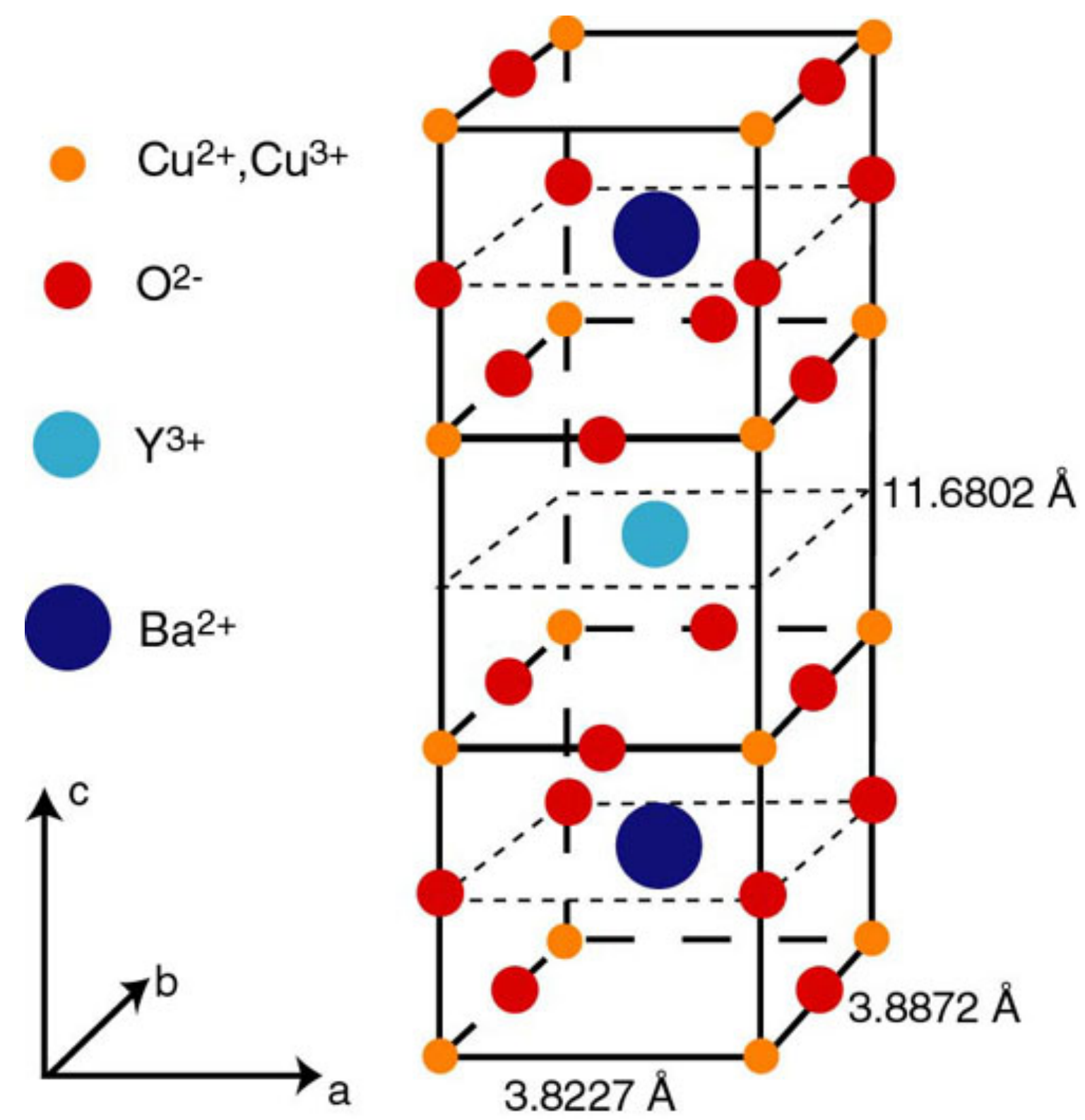
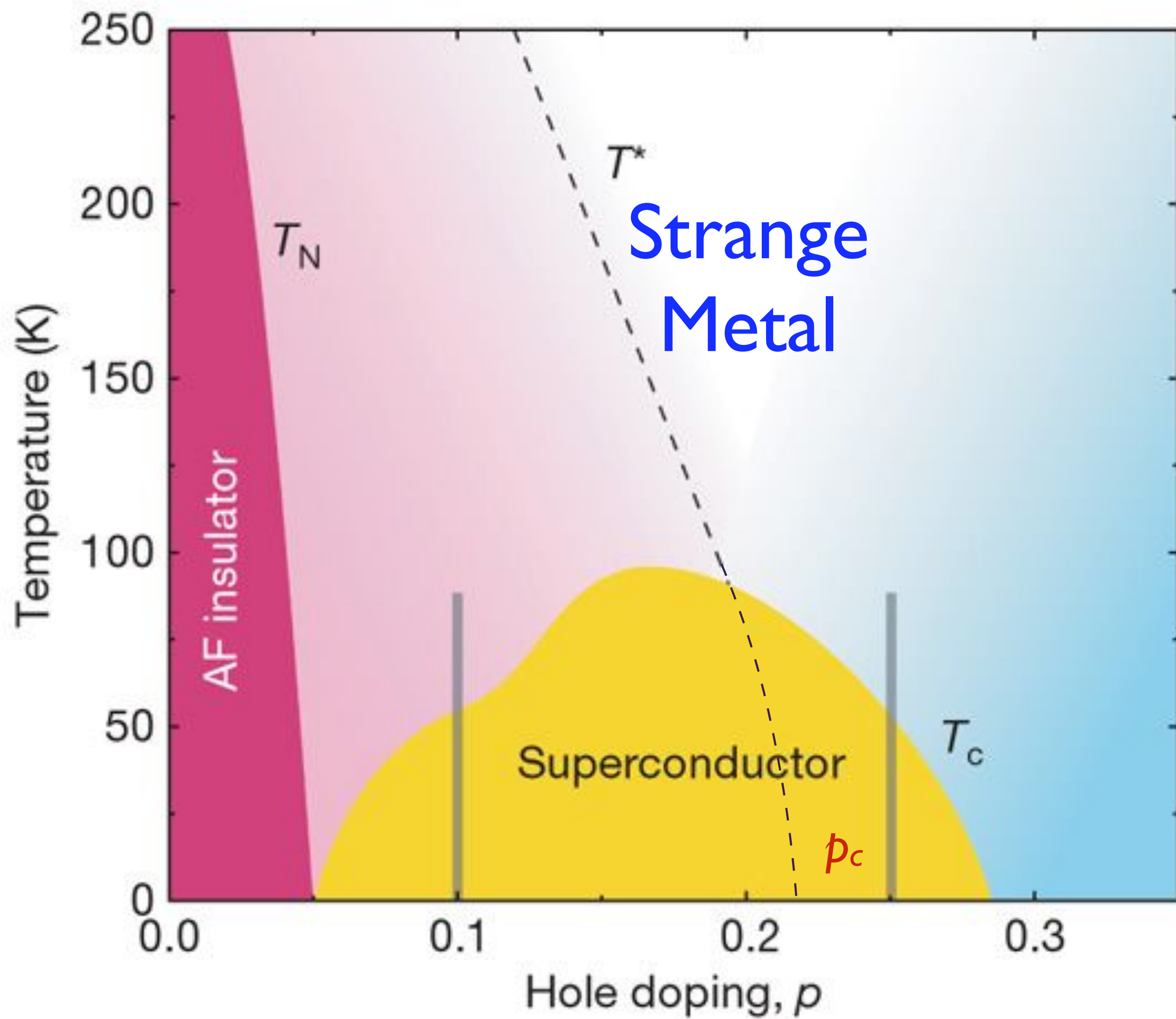


Ilya Esterlis

Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763



Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)

A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)

E. Marcus and S. Vandoren, JHEP 01, 166 (2018)

Yuxuan Wang, PRL **124**, 017002 (2020)

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)

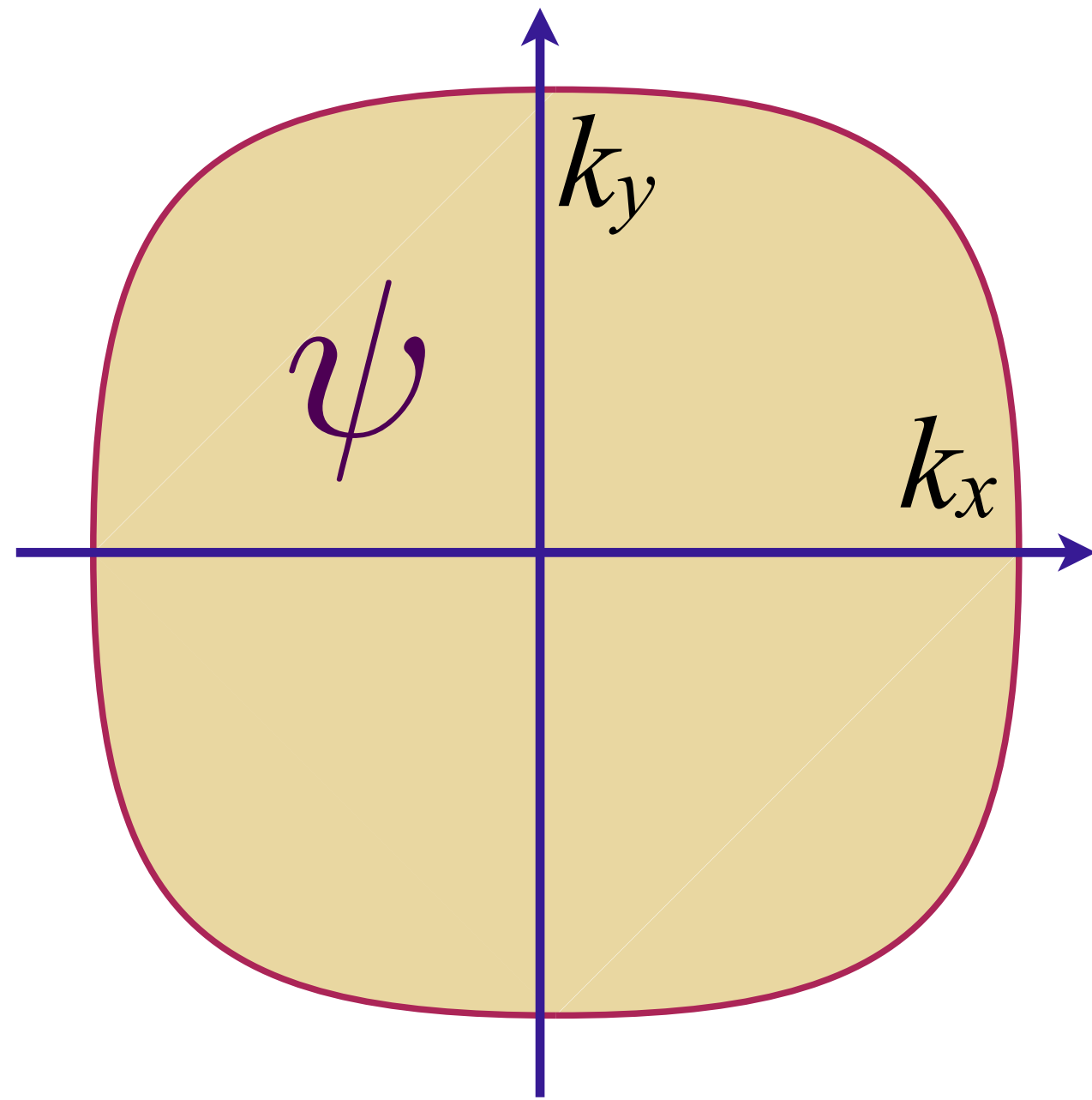
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

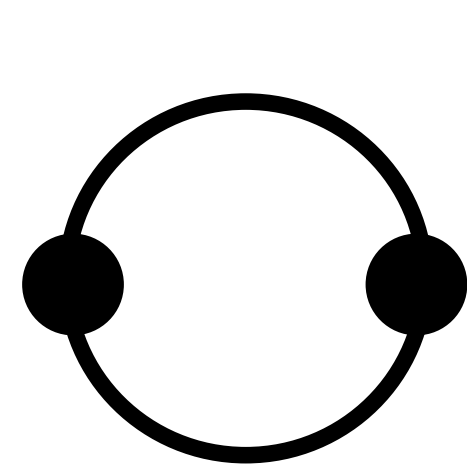
Electron Green's function:
$$G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson with disorder

Conductivity:



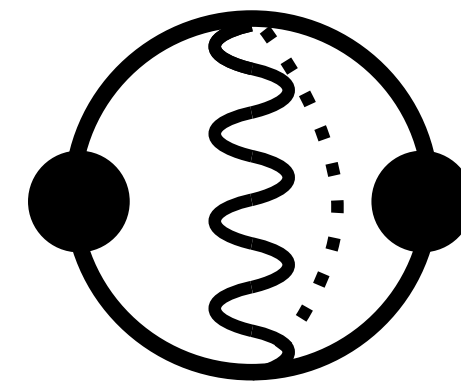
(a)

$$\sigma_v$$



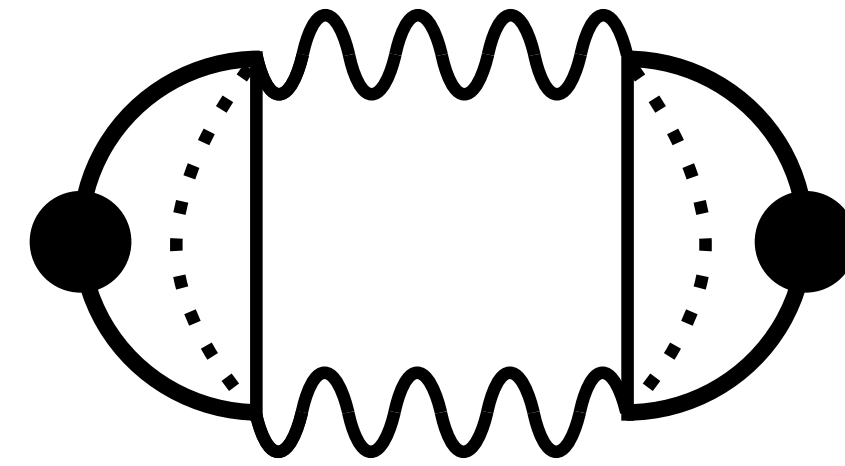
(b)

$$\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$$

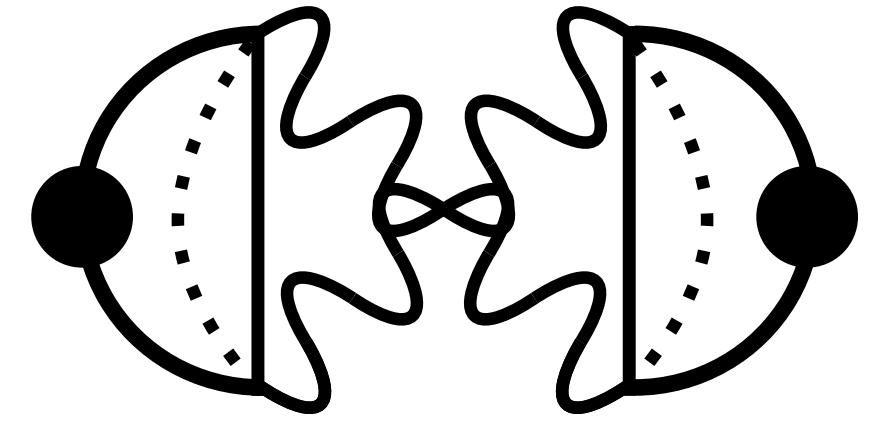


(c)

$$\sigma_{V,g}$$



(d)



(e)

+ all ladders and bubbles.....

Electron Green's function:
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Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson with disorder

$$\text{Conductivity: } \sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

$$\text{Electron Green's function: } G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ; Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Summary

- Graphene flakes display a crossover from quasiparticle behavior with large disorder-induced fluctuations at low T , to a higher T regime with weak fluctuations and a T -independent thermopower. The latter behavior matches that of the SYK model

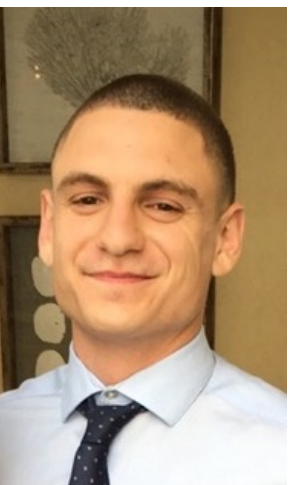
Summary

- Graphene flakes display a crossover from quasiparticle behavior with large disorder-induced fluctuations at low T , to a higher T regime with weak fluctuations and a T -independent thermopower. The latter behavior matches that of the SYK model
- SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a quantum simulation of black hole microstates.

Summary

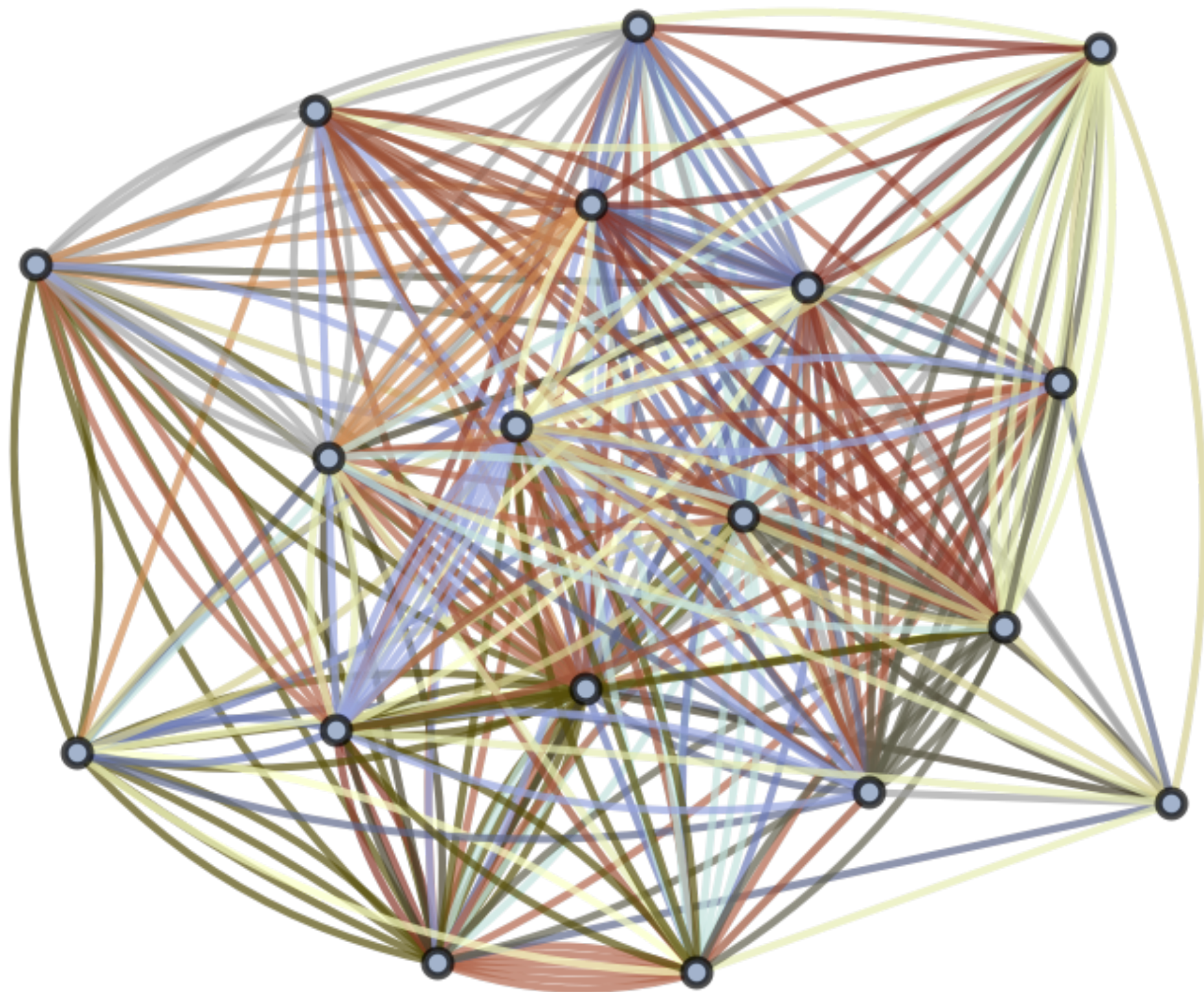
- Graphene flakes display a crossover from quasiparticle behavior with large disorder-induced fluctuations at low T , to a higher T regime with weak fluctuations and a T -independent thermopower. The latter behavior matches that of the SYK model
- SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a quantum simulation of black hole microstates.
- Linear- T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990



The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

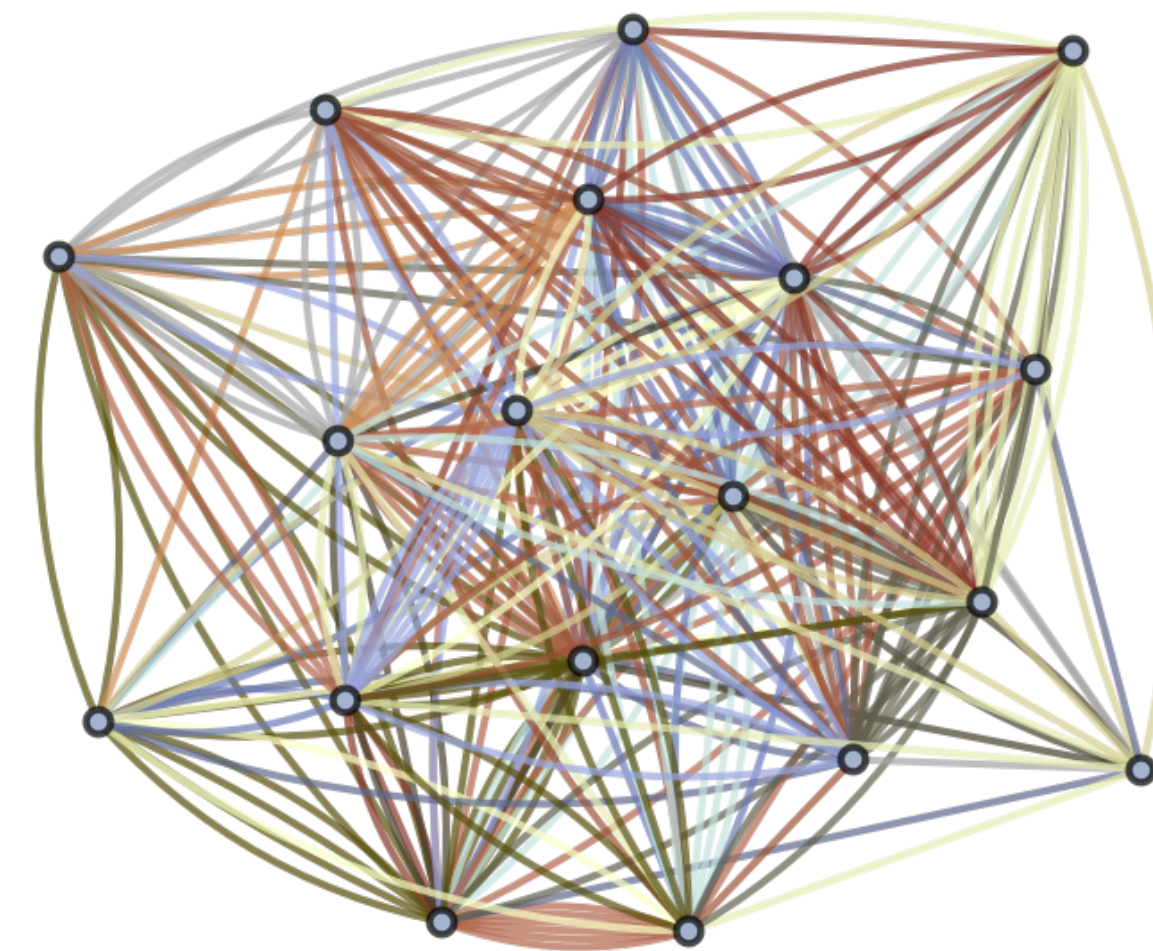
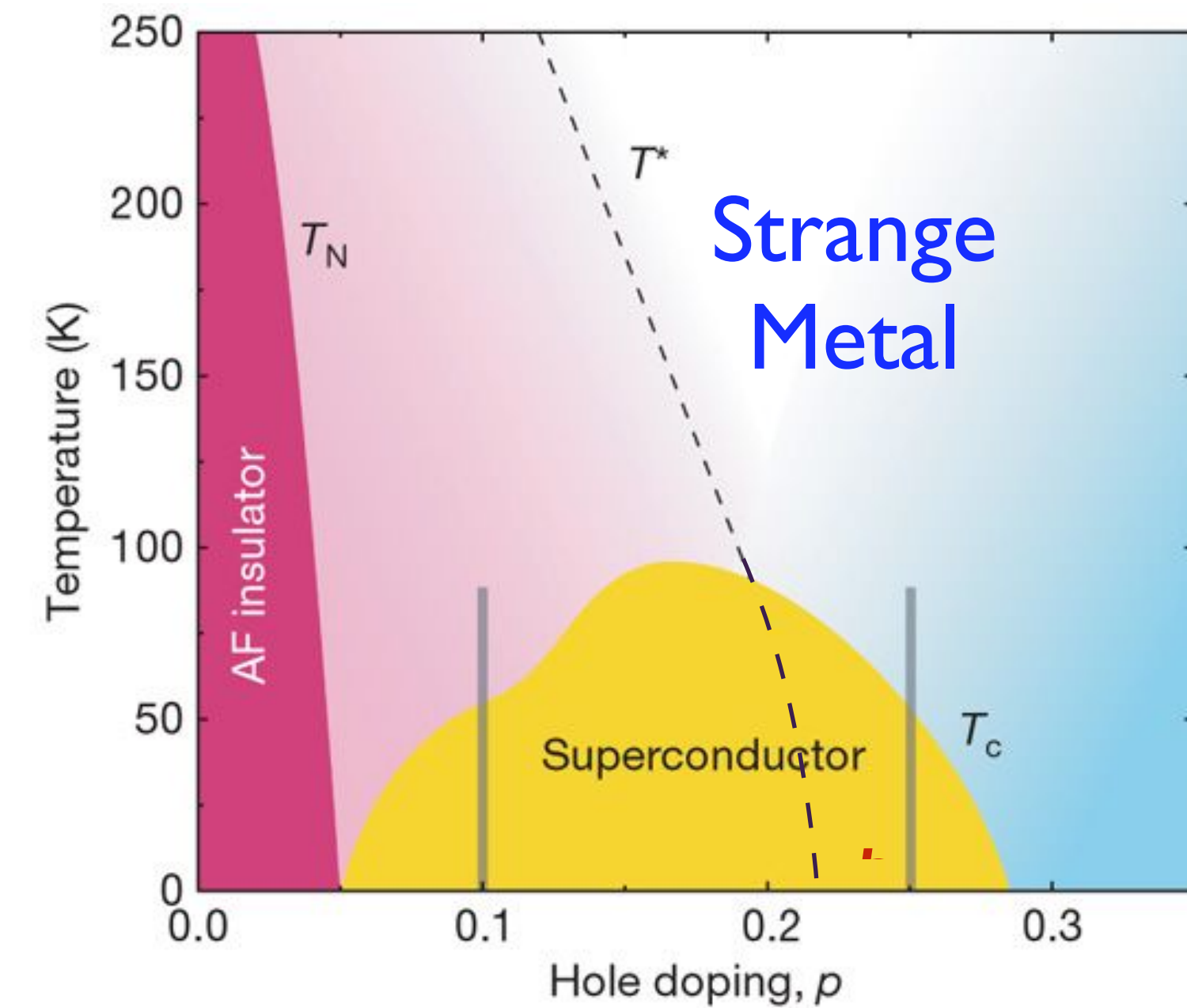


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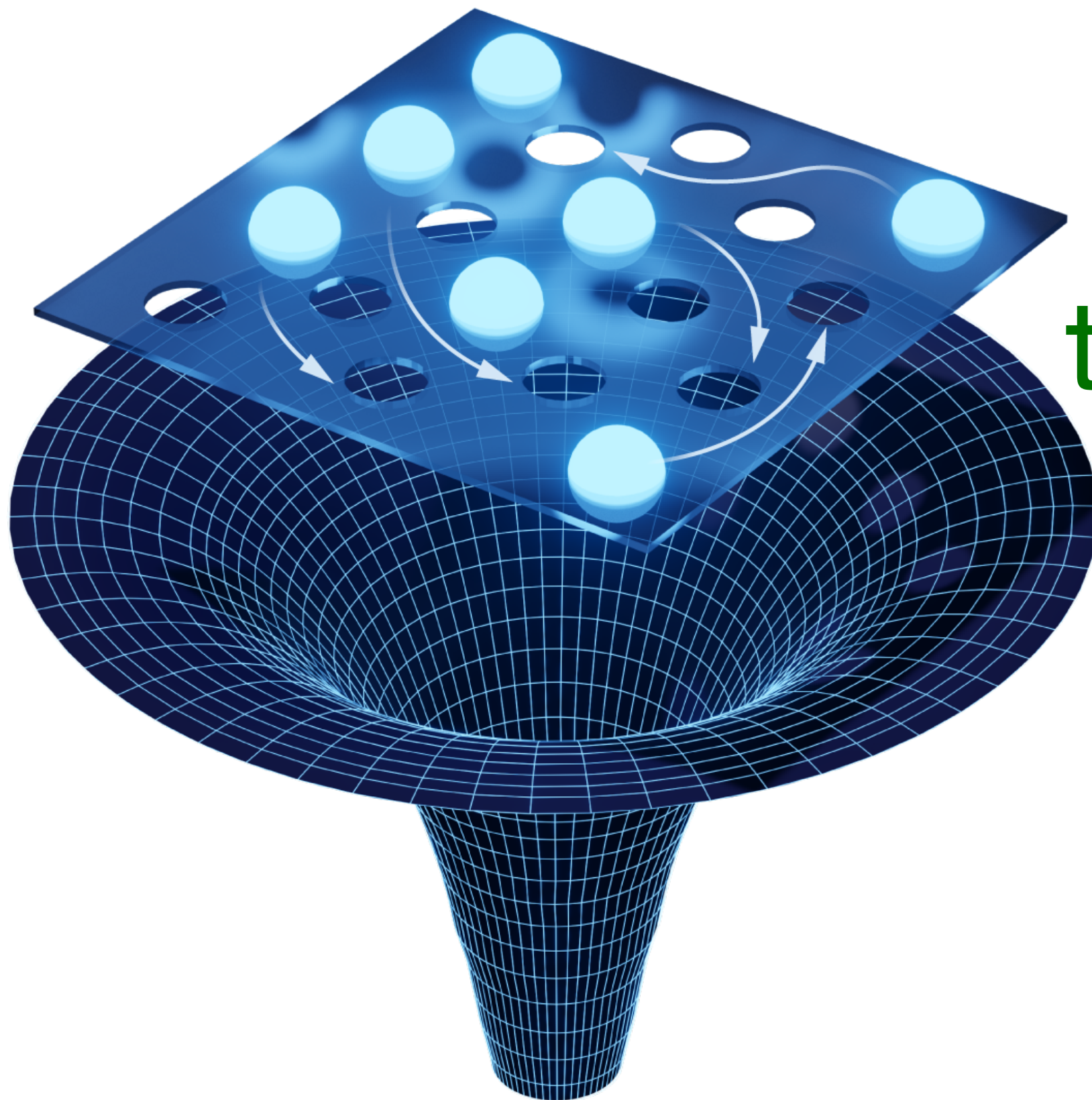
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



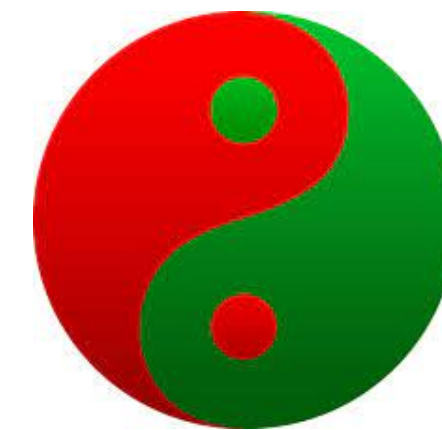
The Sachdev-Ye-Kitaev (SYK) model

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In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)