Non-abelian dualities and square lattice antiferromagnets

Tata Institute for Fundamental Research
Mumbai, January 8, 2020

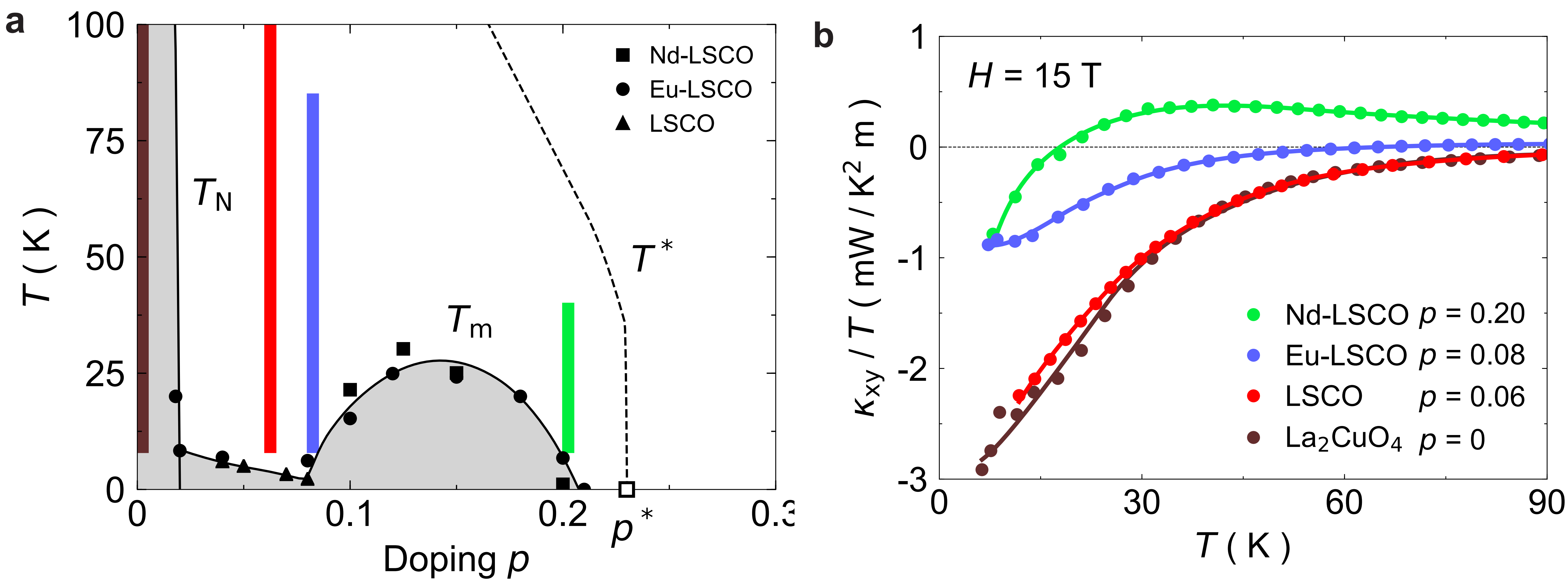
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

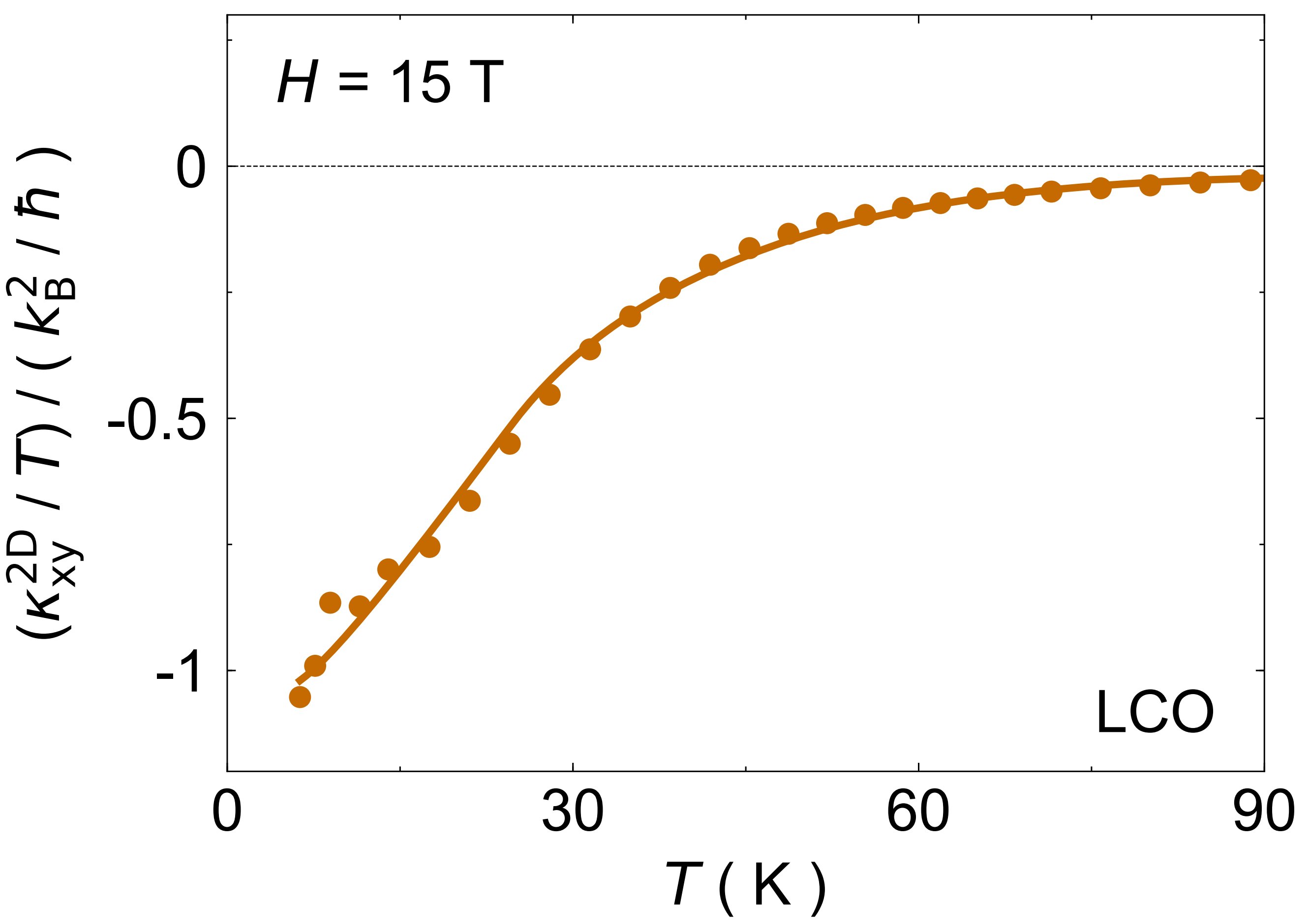
Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

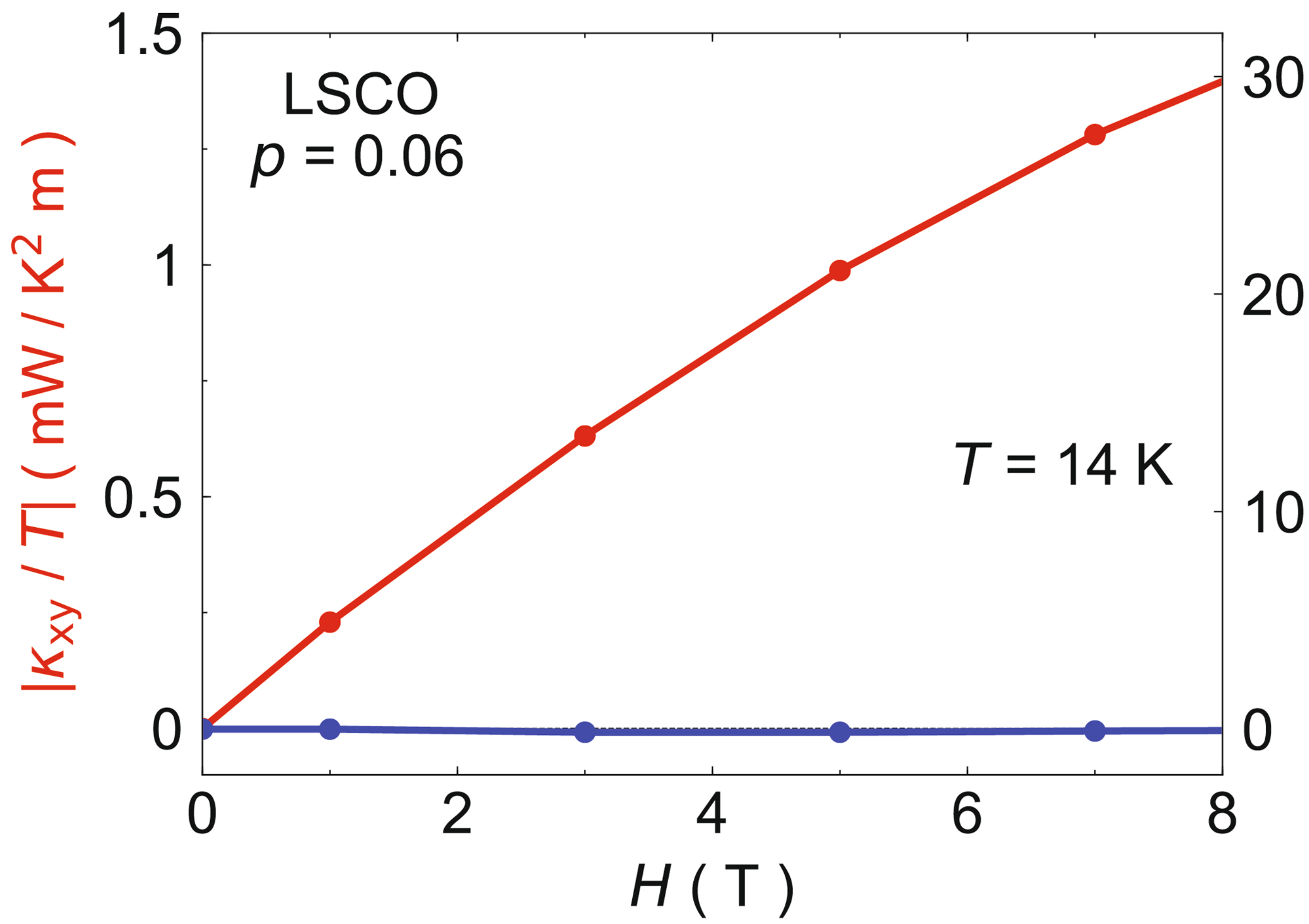
G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer,
Nature **571**, 376 (2019)



Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer,
Nature **571**, 376 (2019)

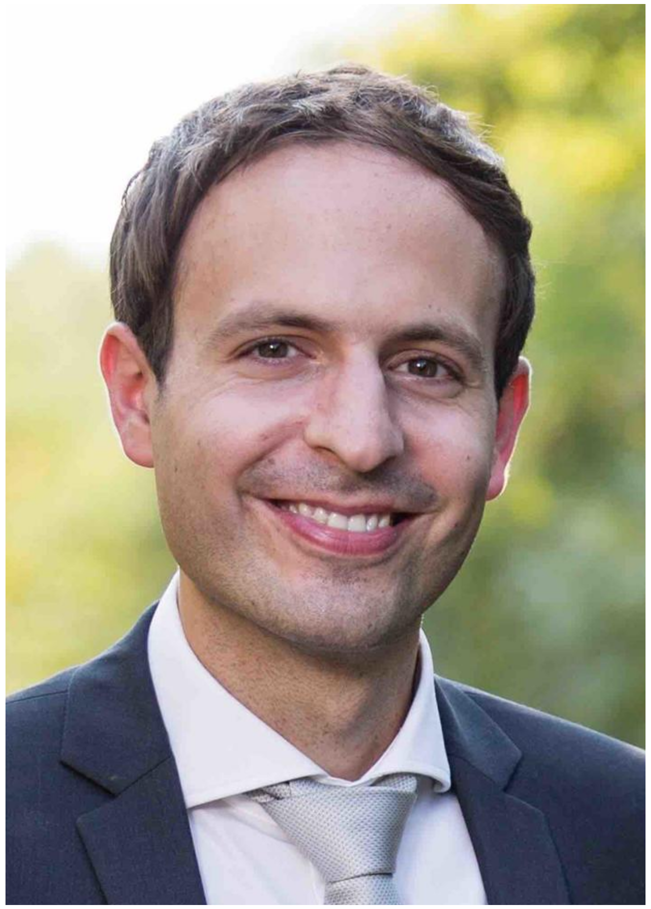




Enhanced thermal Hall effect in the square-lattice Néel state

Nature Physics **15**, 1290 (2019)

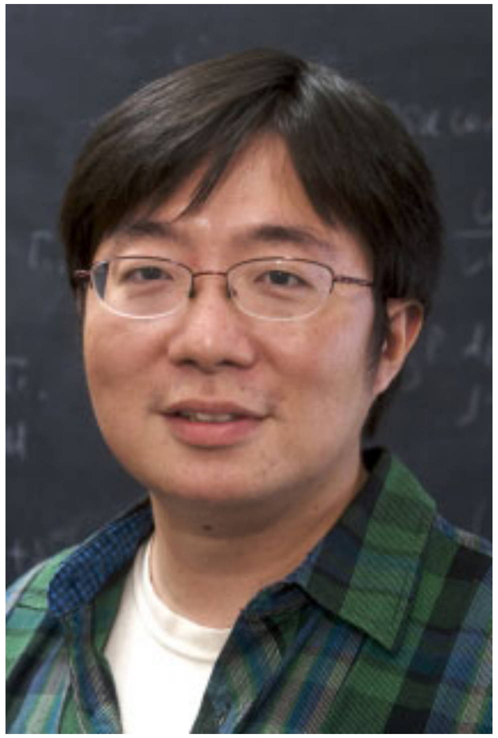
Rhine Samajdar



Mathias Scheurer

Shubhayu Chatterjee

Haoyu Guo



Cenke Xu

Enhanced thermal Hall effect in the square-lattice Néel state

Nature Physics **15**, 1290 (2019)

- The ground state of the square lattice antiferromagnet is a conventional Neel state.
- In a sufficiently large orbital magnetic field, there is a quantum transition to a "chiral spin liquid" (CSL) co-existing with conventional Neel order.
- Proximity to this quantum transition can enhance the thermal Hall effect at non-zero temperatures, even though the ground state is conventional.

Enhanced thermal Hall effect in the square-lattice Néel state

Nature Physics **15**, 1290 (2019)

- The quantum transition between Neel and Neel+CSL involves no symmetry breaking and is purely topological. It is described by a strongly-coupled CFT3 with a global SO(3) symmetry which realizes a *quadrilatity* between 4 different abelian and non-abelian gauge theories

1. Insulator

A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

1. Insulator

A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

The Néel state of square lattice antiferromagnets described by

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

has zero transverse thermal conductivity $\kappa_{xy}/T = 0$.

The Néel state of square lattice antiferromagnets described by

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

has zero transverse thermal conductivity $\kappa_{xy}/T = 0$.

In the presence of a magnetic field

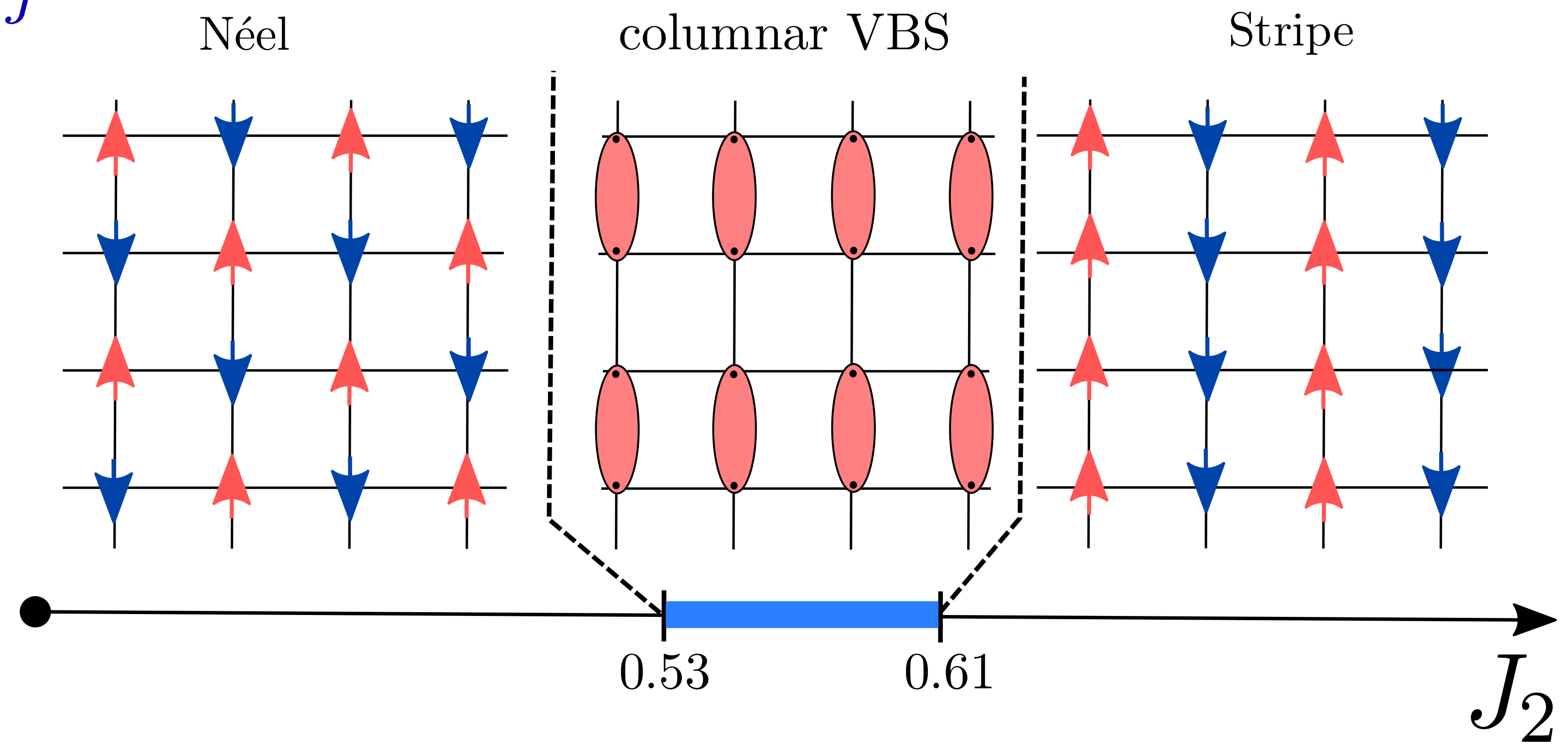
$$H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

The *orbital coupling* $J_\chi \propto \boldsymbol{B}_\perp$ induces a non-zero Berry curvature in the spin-wave dispersion, which induces a non-zero κ_{xy}/T.

However, this Berry curvature is small at long wavelengths, and consequently the thermal Hall conductivity is very small $|\kappa_{xy}/T| \ll k_B^2/\hbar$, and vanishes rapidly as $T \to 0$.

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



$U(1)$-symmetric infinite projected entangled-pair states study of the spin-1/2 square J_1-J_2 Heisenberg model

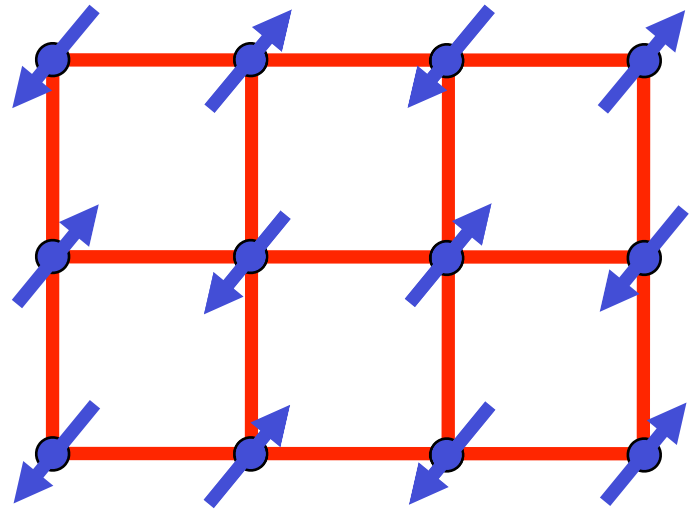
PHYSICAL REVIEW B **97**, 174408 (2018)

R. Haghshenas and D. N. Sheng

By studying the finite-D scaling of the magnetically order parameter, we find a Néel phase for $J_2/J_1 < 0.53$. For $0.53 < J_2/J_1 < 0.61$, a nonmagnetic columnar valence bond solid (VBS) state is established as observed by the pattern of local bond energy. The divergent behavior of correlation length $\xi \sim D^{1.2}$ and vanishing order parameters are consistent with a deconfined Néel-to-VBS transition at $J_2^{c1}/J_1 = 0.530(5)$, where estimated critical anomalous exponents are $\eta_s \sim 0.6$ and $\eta_d \sim 1.9$ for spin and dimer correlations, respectively.

Quantum criticality in a frustrated square lattice antiferromagnet

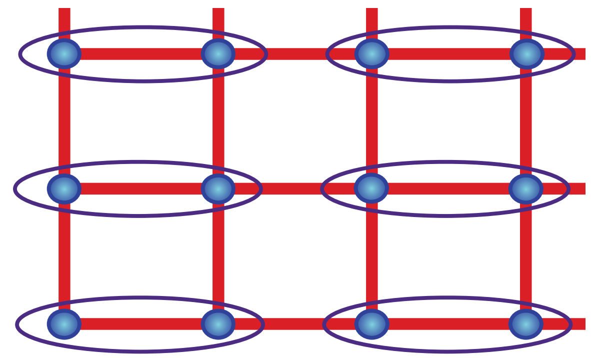
N. Read and S. Sachdev, PRL **62**, 1694 (1989)



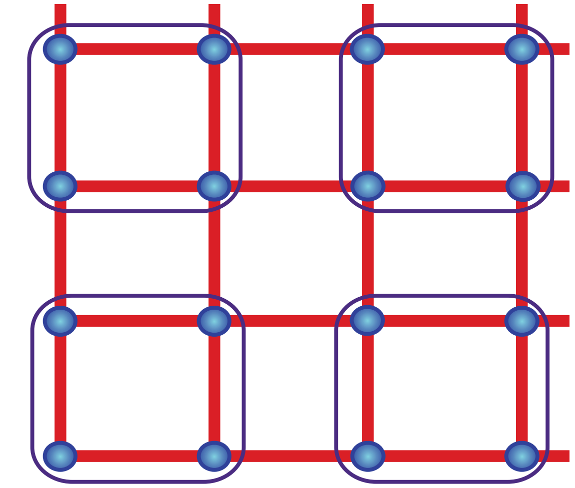
$\langle z_\alpha \rangle \neq 0$

Néel state

$\vec{N} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$



or



$\langle z_\alpha \rangle = 0$

Valence bond solid (VBS) state, V_x, V_y
with a nearly gapless, emergent "photon"

s_c $\qquad$ s

Critical $\mathbb{CP}^1$ theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si}\Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

We Higgs SU(2) to U(1) by condensing $(-1)^i \Psi_i^\dagger \sigma^z \Psi_i$, and the chargons are then gapped, fully filling the lower band. Then we obtain a U(1) gauge theory for the bosonic spinons z_α.

Neel-VBS deconfined criticality

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):
Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = F_i R_{ci}$$

$$F_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow}^\dagger \\ f_{i\downarrow} & f_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad R_{ci} = \begin{pmatrix} b_{i1} & b_{i2} \\ -b_{i2}^* & b_{i1}^* \end{pmatrix}$$

F are fermionic spinons, R_c is a SU(2) rotation. Pseudospin rotations are *right* multiplication of R_c, while *left* multiplication is an emergent SU(2) gauge symmetry:

$$F \to FU, \quad R_c \to U^\dagger R_c \,.$$

The F spinons move in a π-flux background, while the $b_{1,2}$ are gapped chargons. The low energy theory is a SU(2) gauge theory of $N_f = 2$ Dirac fermions, f

M. Scheurer and S. Sachdev, PRB **98**, 235126 (2018)

Neel-VBS deconfined criticality

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

Chong Wang, A. Nahum, M. A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

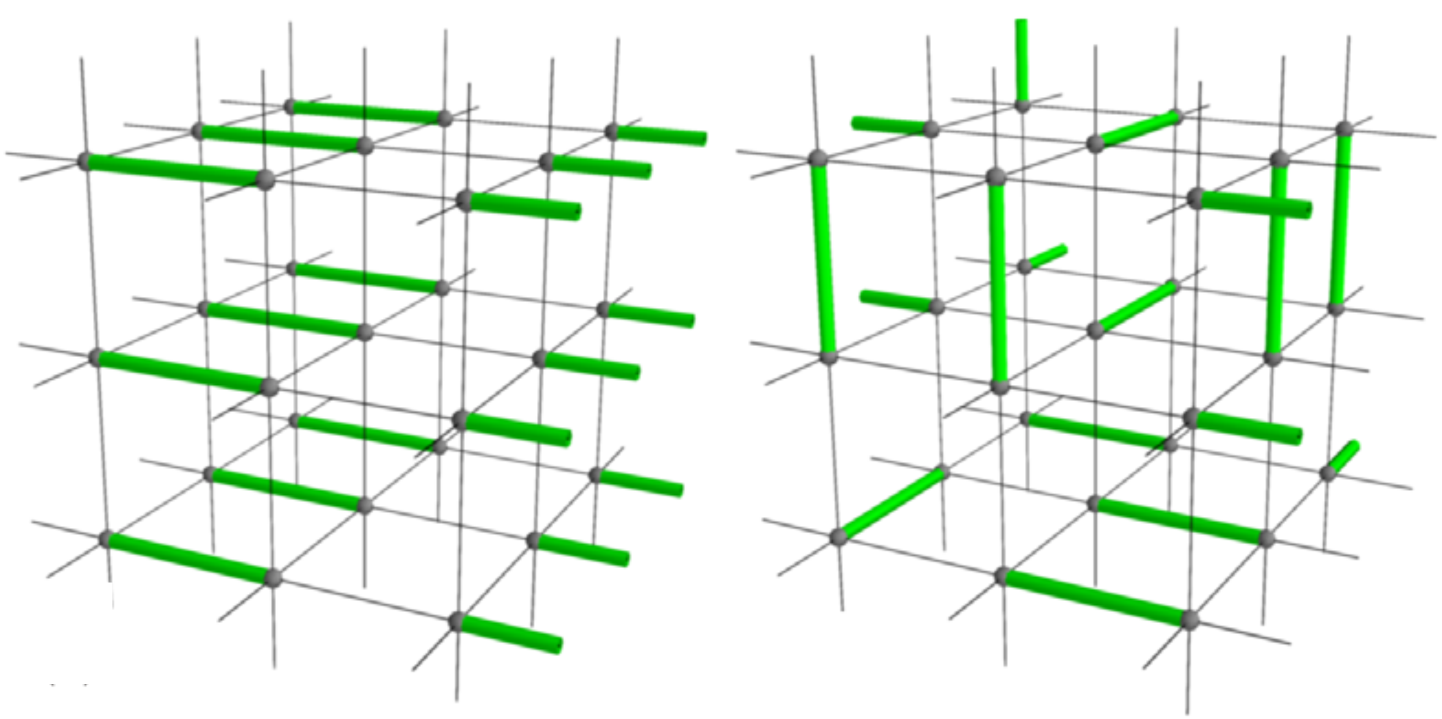
The fermion theory has a SO(5) global flavor symmetry, and the gauge-invariant fermion bilinears form a SO(5) vector which transforms as the Néel and VBS order parameters!

$$(N_x, N_y, N_z, V_x, V_y)$$

Akihiro Tanaka and Xiao Hu, PRL. **95**, 036402 (2005).
T. Senthil and M.P.A. Fisher, PRB **74**, 064405 (2006)
Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

Emergent SO(5) Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

"Studying linear system sizes up to L=96, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that SO(5) emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the SO(5) symmetry that has been proposed for the noncompact CP^1 field theory. We describe an interpretation for these results in terms of a consistent hypothesis for the renormalization-group flow structure, allowing for the possibility that SO(5) may ultimately be a near-symmetry rather than exact."



G.J. Sreejith, Stephen Powell, and Adam Nahum
PRL **122**, 080601 (2019)

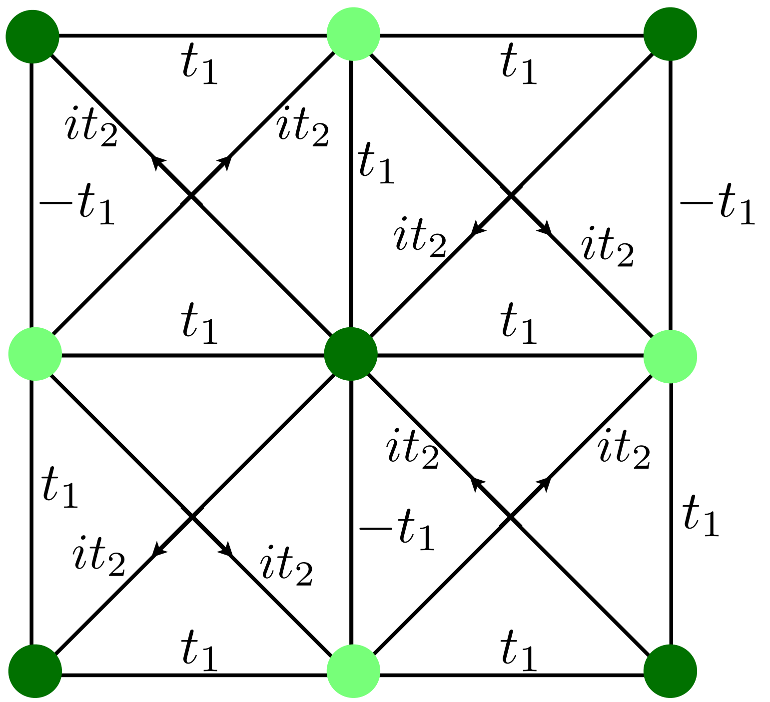
Stephen Powell and John T. Chalker,
PRB **80**, 134413 (2009)

Quantum criticality in a frustrated square lattice antiferromagnet

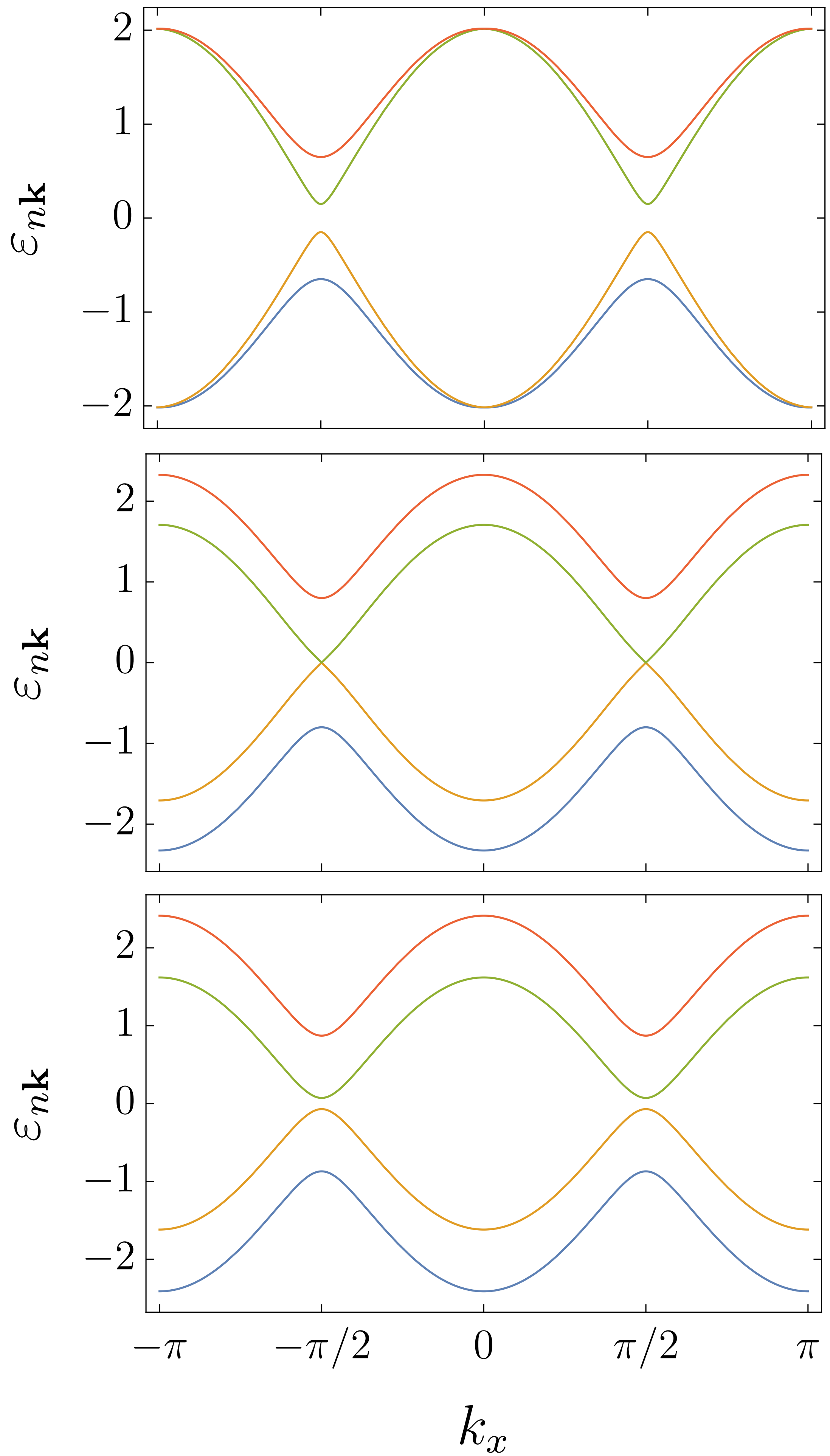
SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):

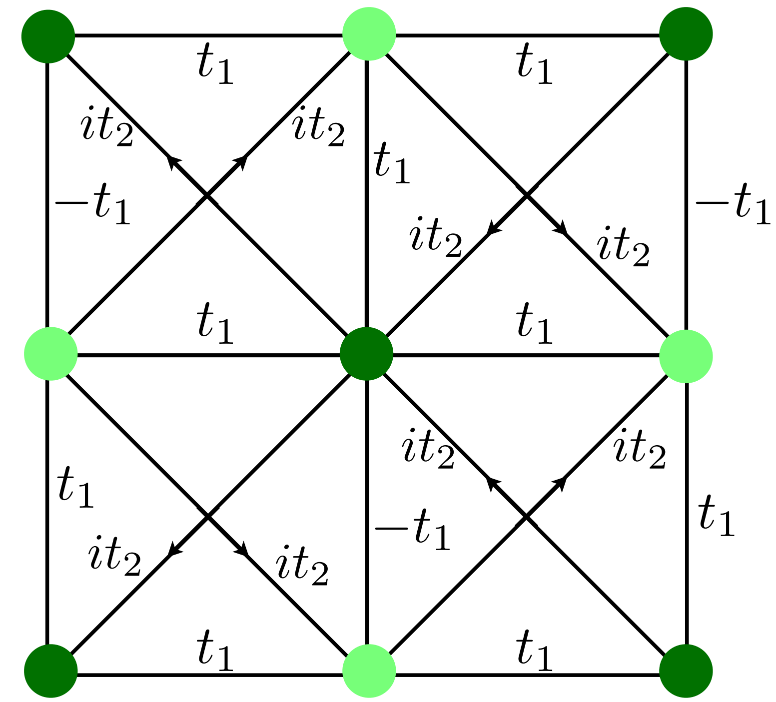
The lattice Hamiltonian of the Schwinger fermions $f_{i\alpha}$ is of the form

$$H_f = - \sum_{i<j} \left(t_{ij} f_{i\alpha}^\dagger f_{j\alpha} + t_{ij}^* f_{j\alpha}^\dagger f_{i\alpha} \right) - \frac{1}{2} \sum_i \left(\boldsymbol{B}_Z + \eta_i \, \boldsymbol{N} \right) \cdot f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \, .$$



t_2 is induced by the spin chirality J_χ, $\boldsymbol{B}_Z$ is the Zeeman coupling to the applied field, and $\boldsymbol{N}$ is proportional to the Néel order. With $t_2 = 0$ and $\boldsymbol{N} = 0$ we obtain 4 massless Dirac fermions from 2 "valleys" in the Brillouin zone. Non-zero t_2 and $\boldsymbol{N}$ induce *different* Dirac mass terms m_χ and m_N.

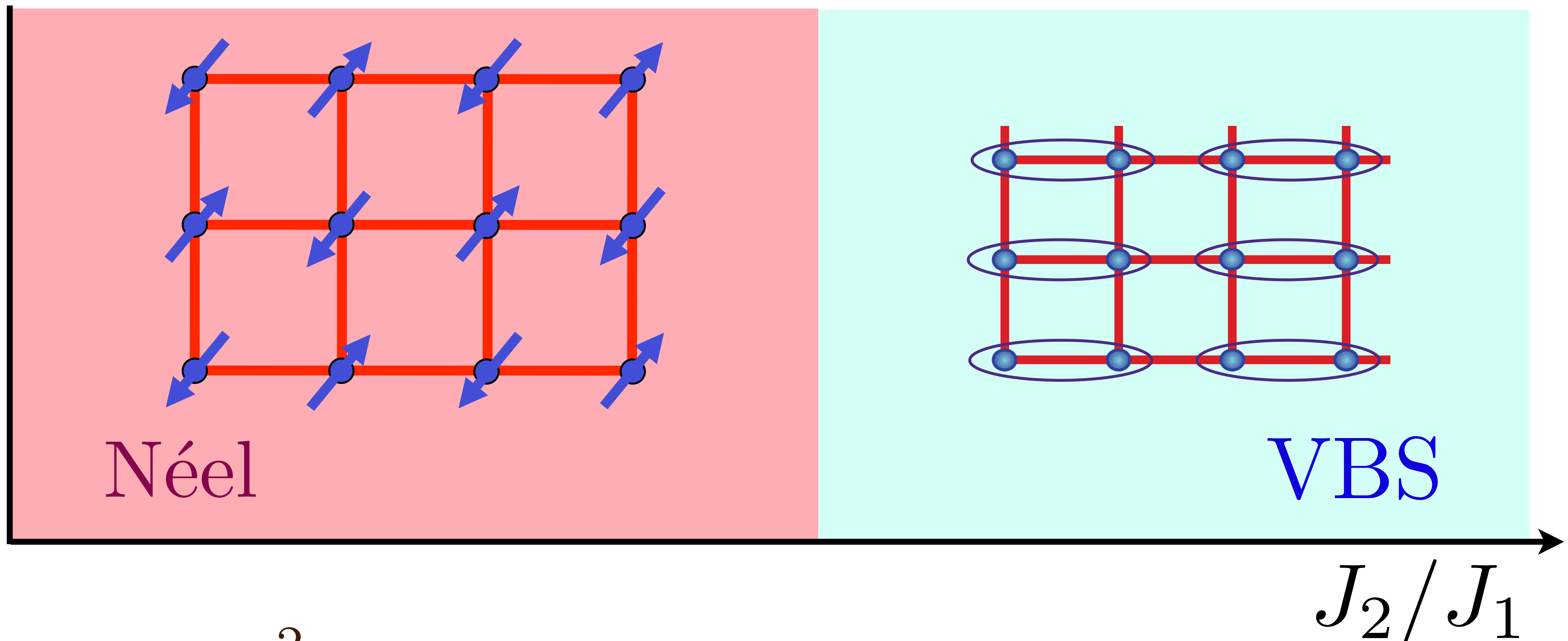




We focus on two possible mass terms for the Dirac fermions: m_χ, from the spin chirality term, and m_N induced by Néel order. There is a line in the m_χ, m_N plane where the occupied bands switch from Chern numbers $\{1, -1\}$ (the Néel state) to $\{1, 1\}$ (Néel order co-existing with semion topological order).

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

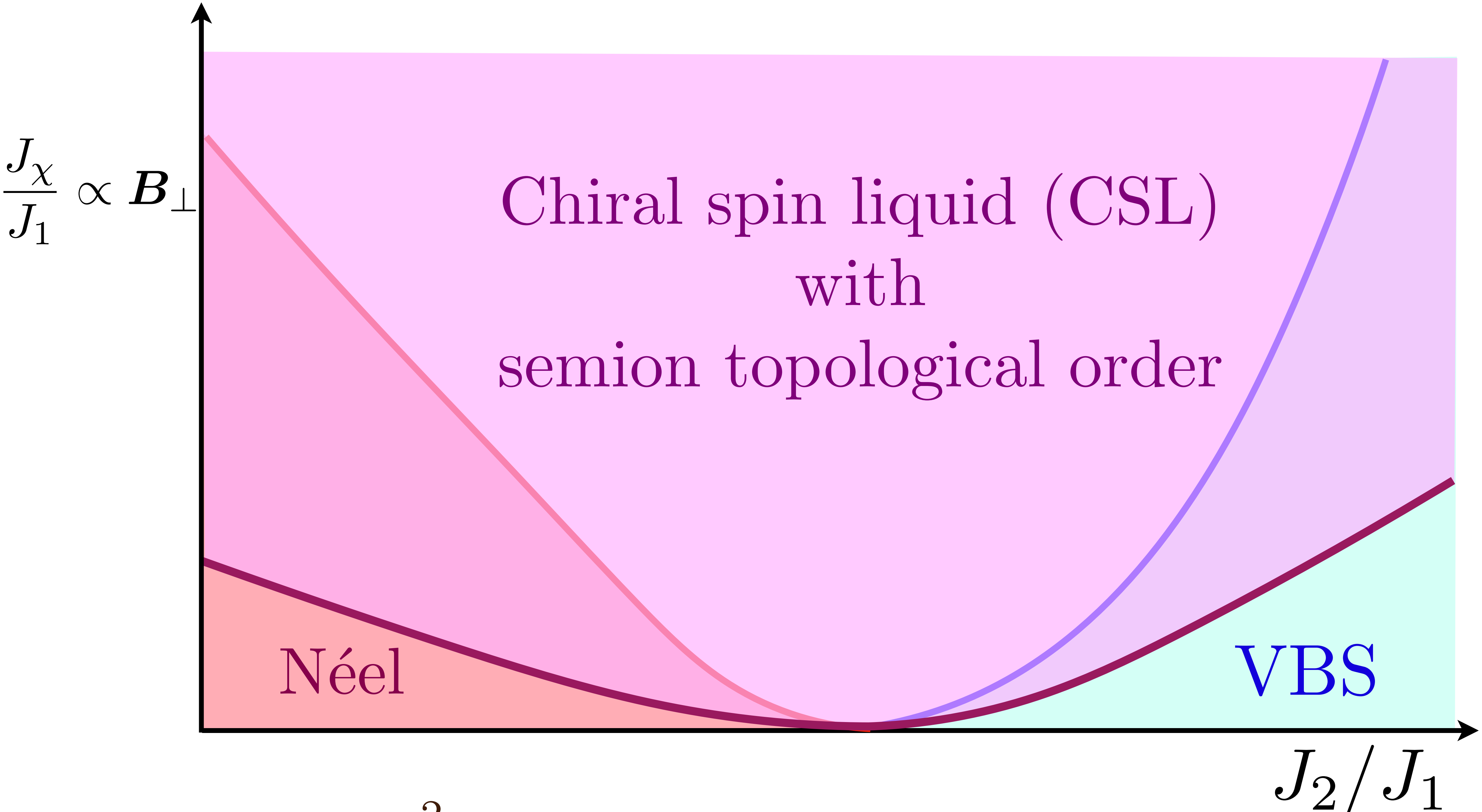
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha \right]$$

Quantum critical theory

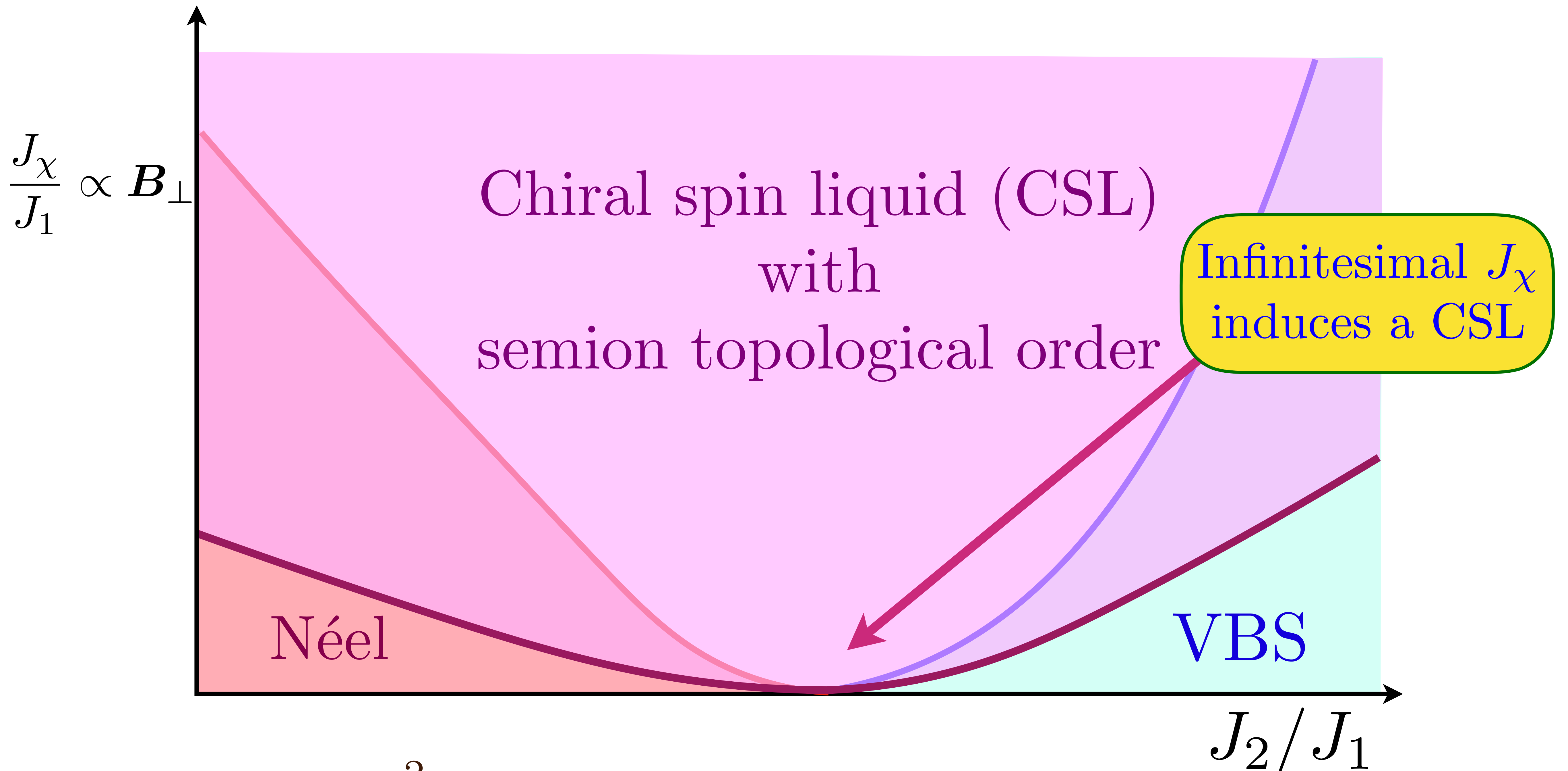
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha \right]$$

Quantum critical theory + J_χ

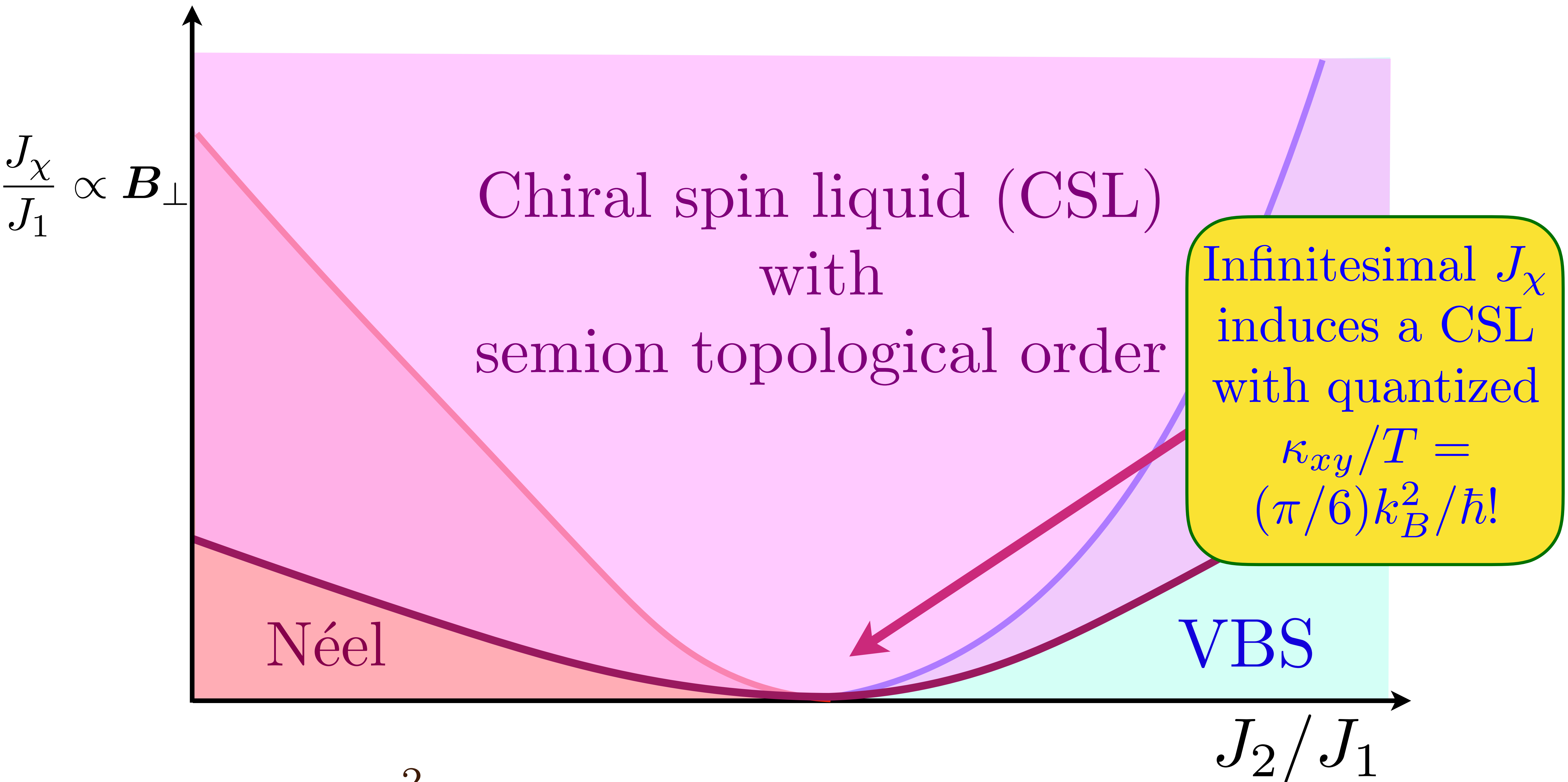
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha \right]$$

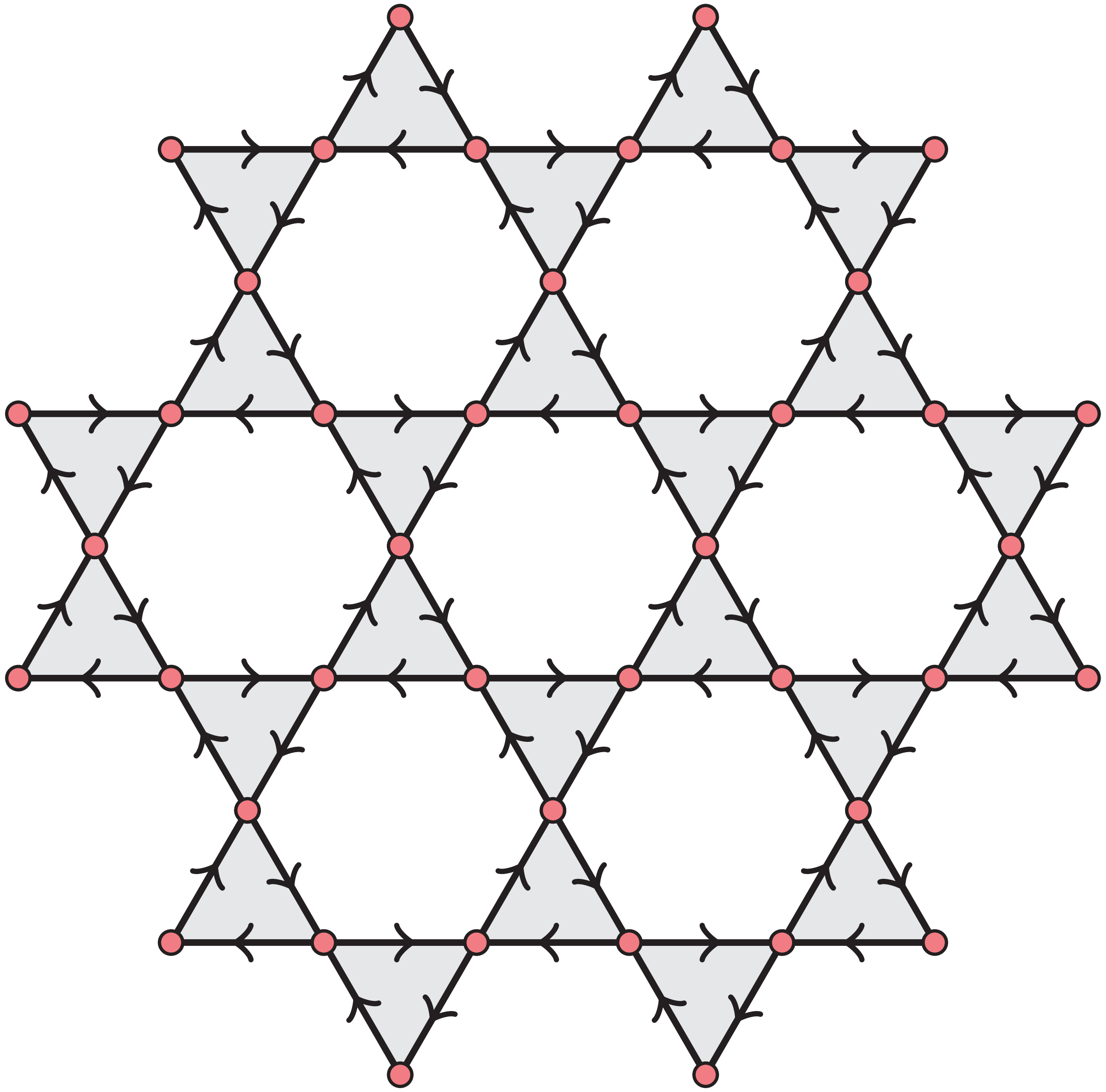
Quantum critical theory + J_χ

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha \right]$$

Quantum critical theory + J_χ



$$H = H_1 + H_\chi$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

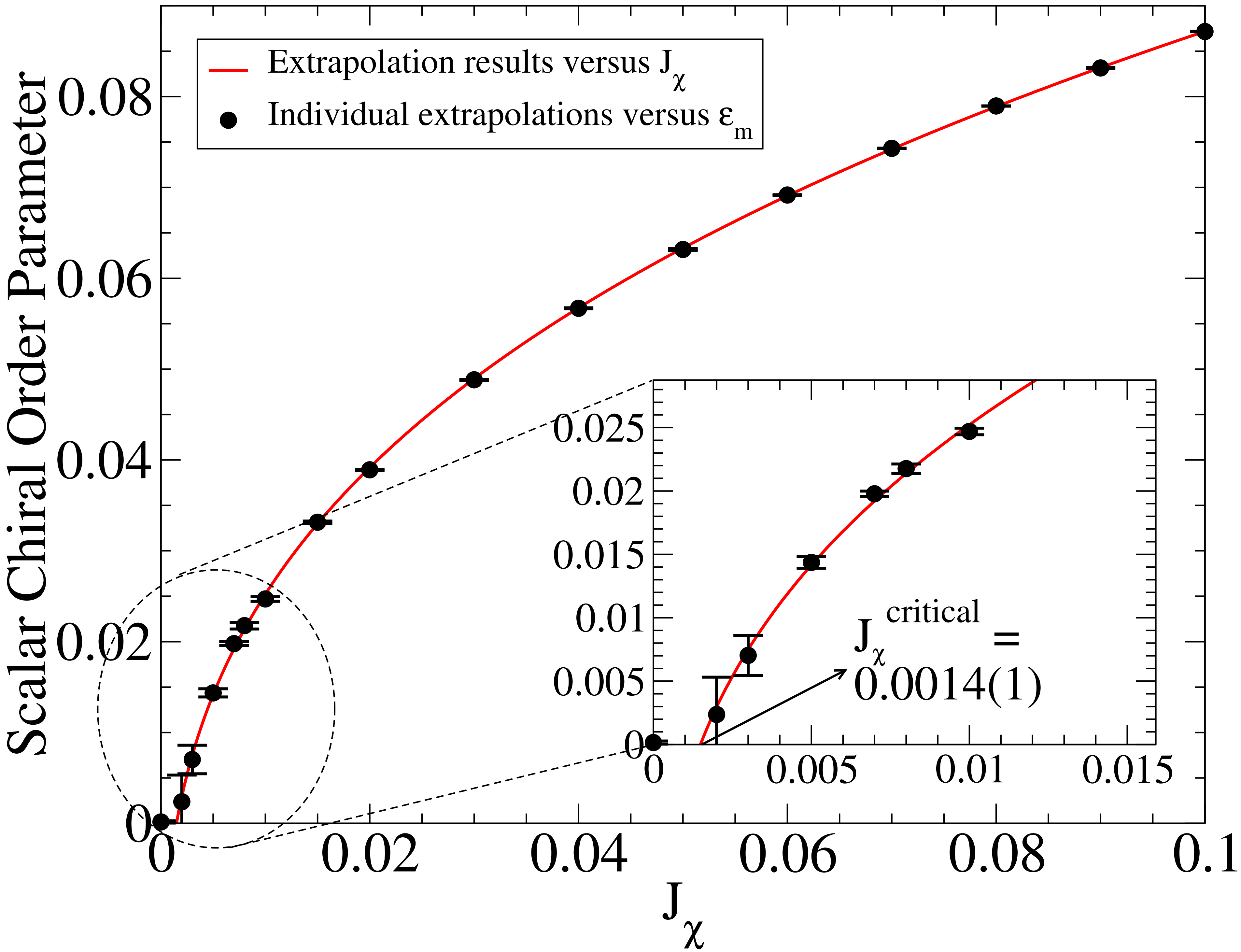
$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

B. Bauer, L. Cincio, B.P. Keller, M. Dolfi, G. Vidal, S. Trebst and A.W.W. Ludwig, Nature Communications **5**, 5137 (2014)

Semion topological order,
i.e. the Kalmeyer-Laughlin chiral spin liquid,
appears for $J_\chi / J > 0.01$.

R. Haghshenas, Shou-Shu Gong, and D.N. Sheng, arXiv:1812.11436

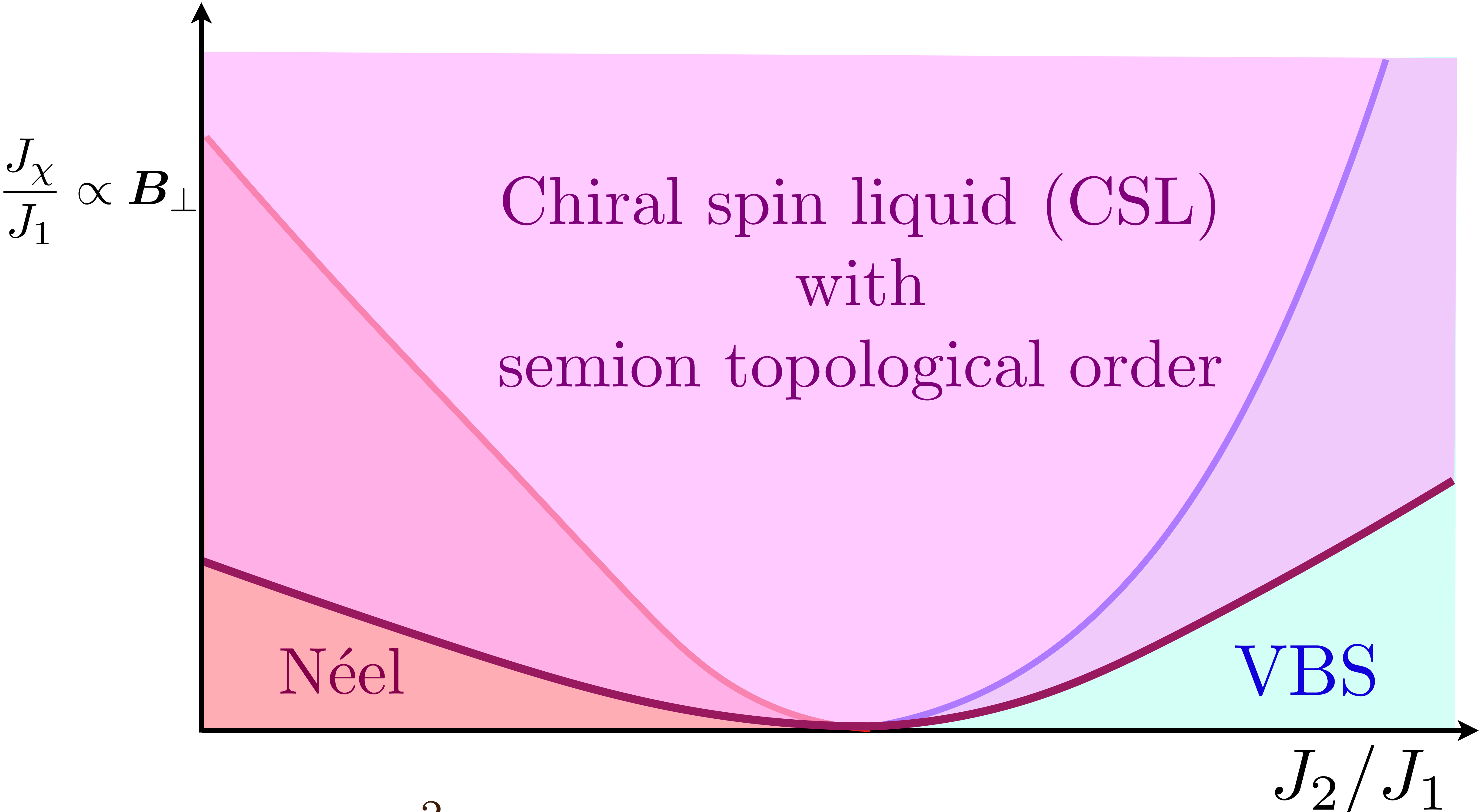
Triangular lattice antiferromagnet



$J_2/J_1 = 1/8$; critical $J_\chi = 0.0014$

S. N. Saadatmand and I. P. McCulloch, PRB **96**, 075117 (2017)

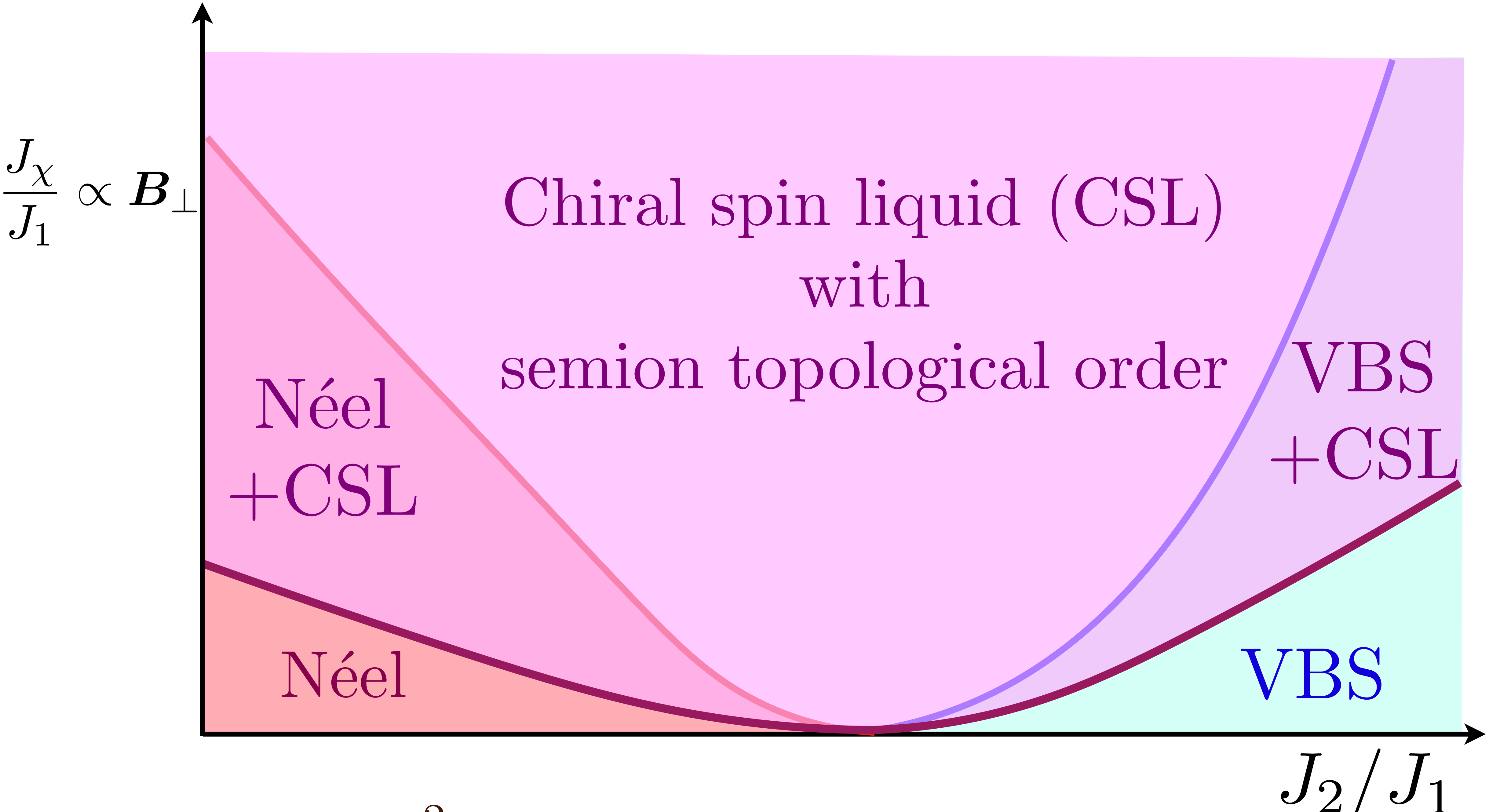
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha \right]$$

Quantum critical theory + J_χ

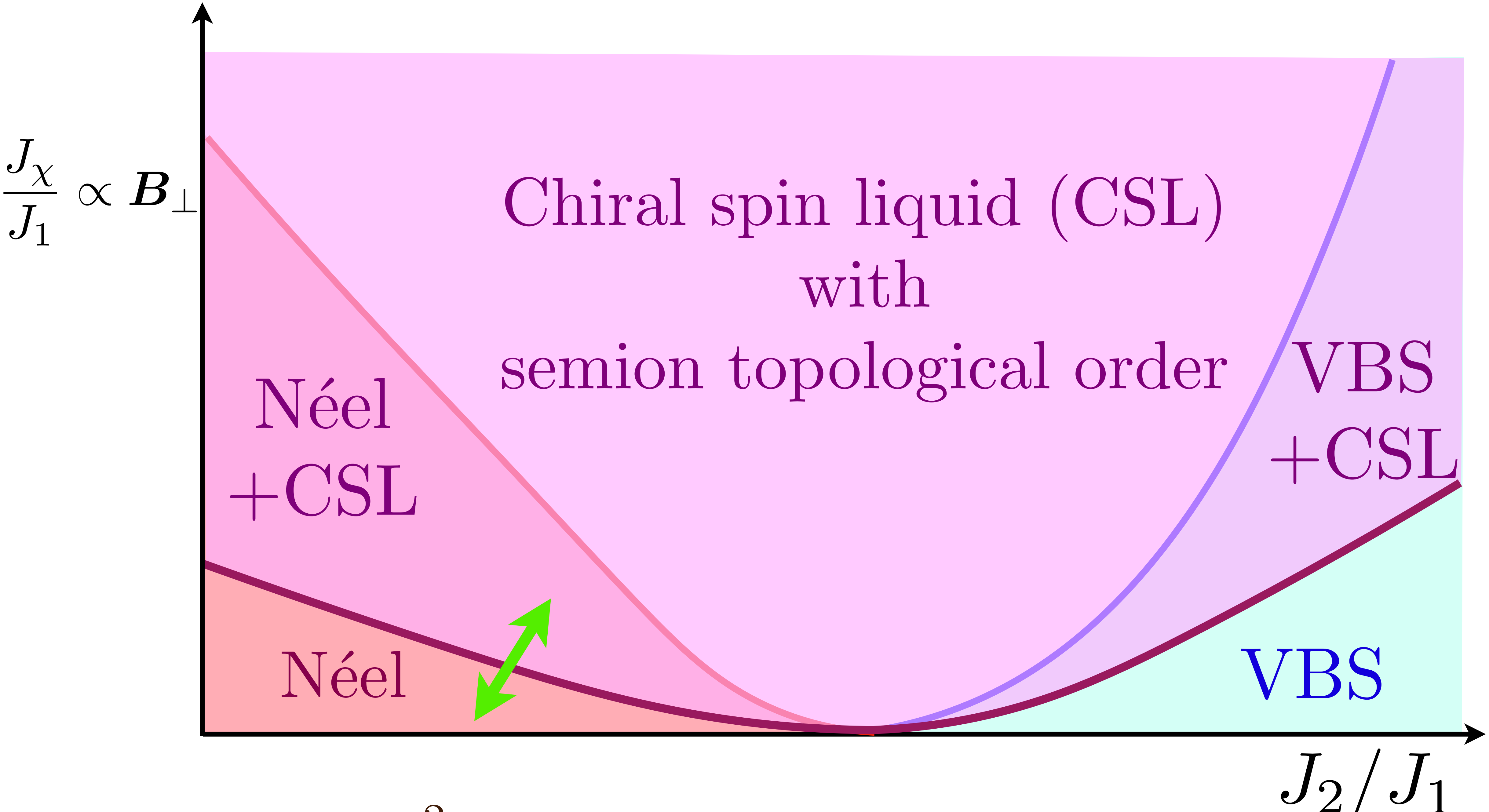
$$H=\sum_{\text{n.n.}} J_1 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+J_\chi\sum_{\triangle}\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)$$



$$\mathcal{S}_f=\int d^2r d\tau \sum_{\alpha=1}^{2}\left[\overline{f}_\alpha\gamma^\mu(\partial_\mu-iA_\mu)f_\alpha+m_\chi\overline{f}_\alpha f_\alpha+m_N\overline{f}_\alpha\Gamma f_\alpha\right]$$

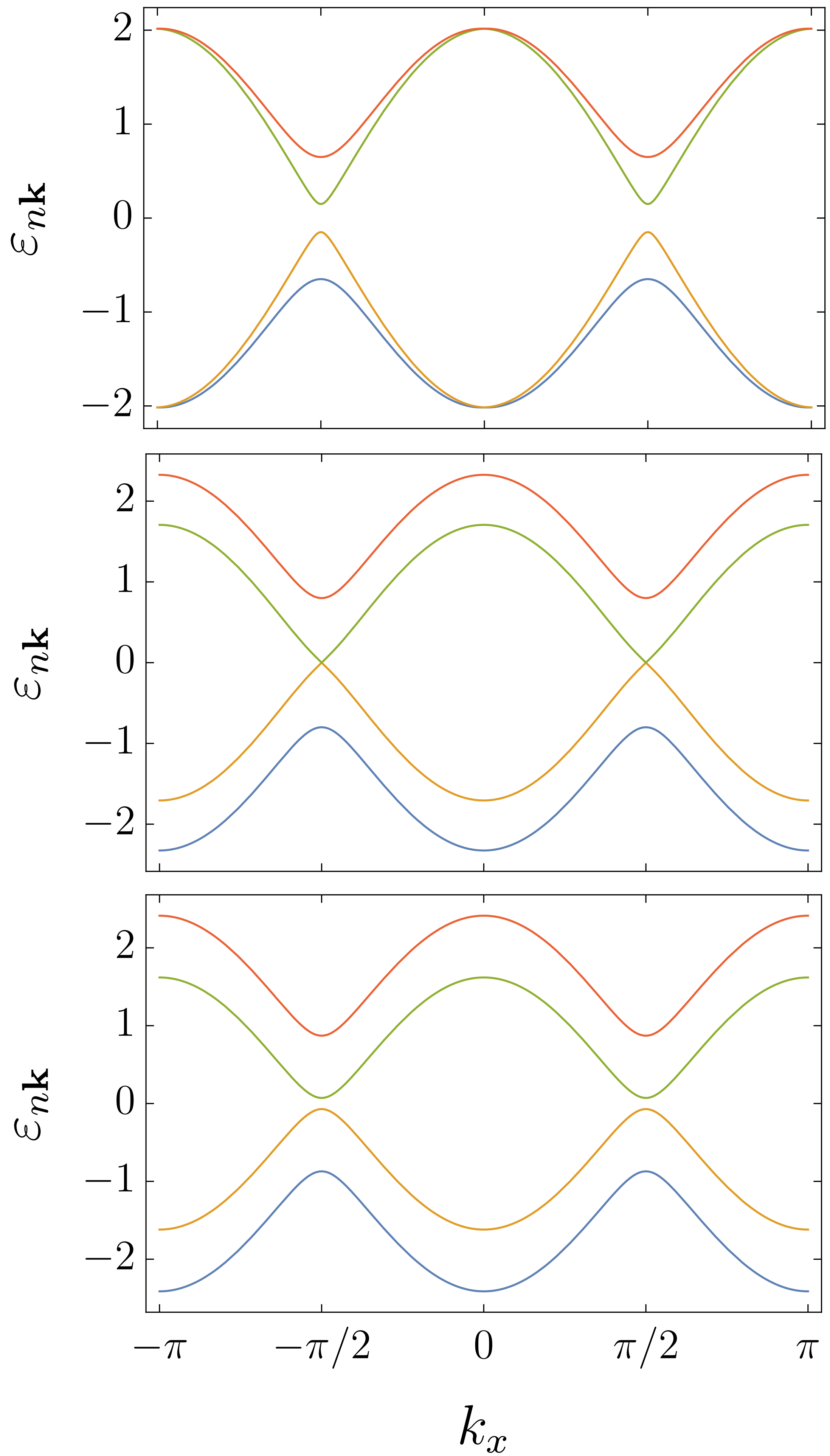
Quantum critical theory + J_χ + Néel order

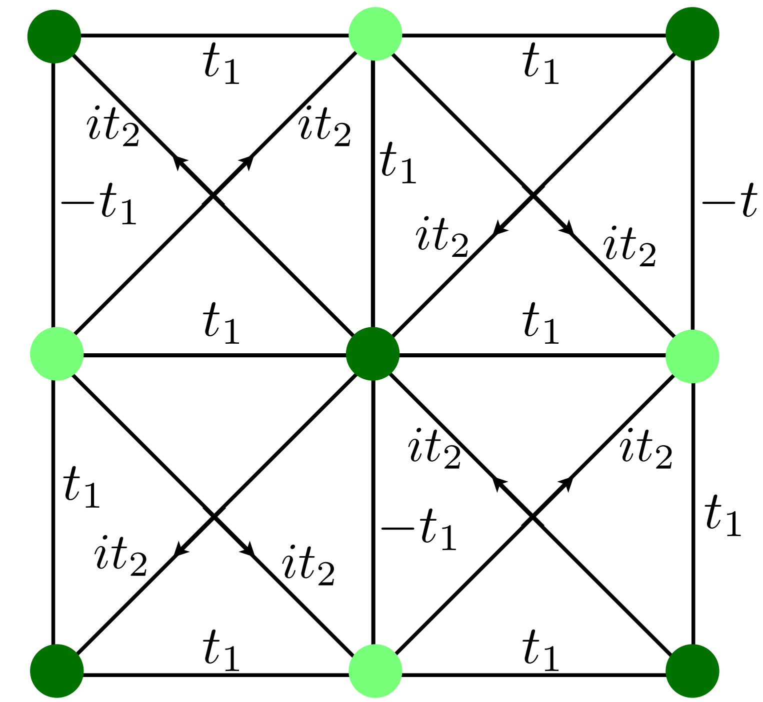
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha + m_N \overline{f}_\alpha \Gamma f_\alpha \right]$$

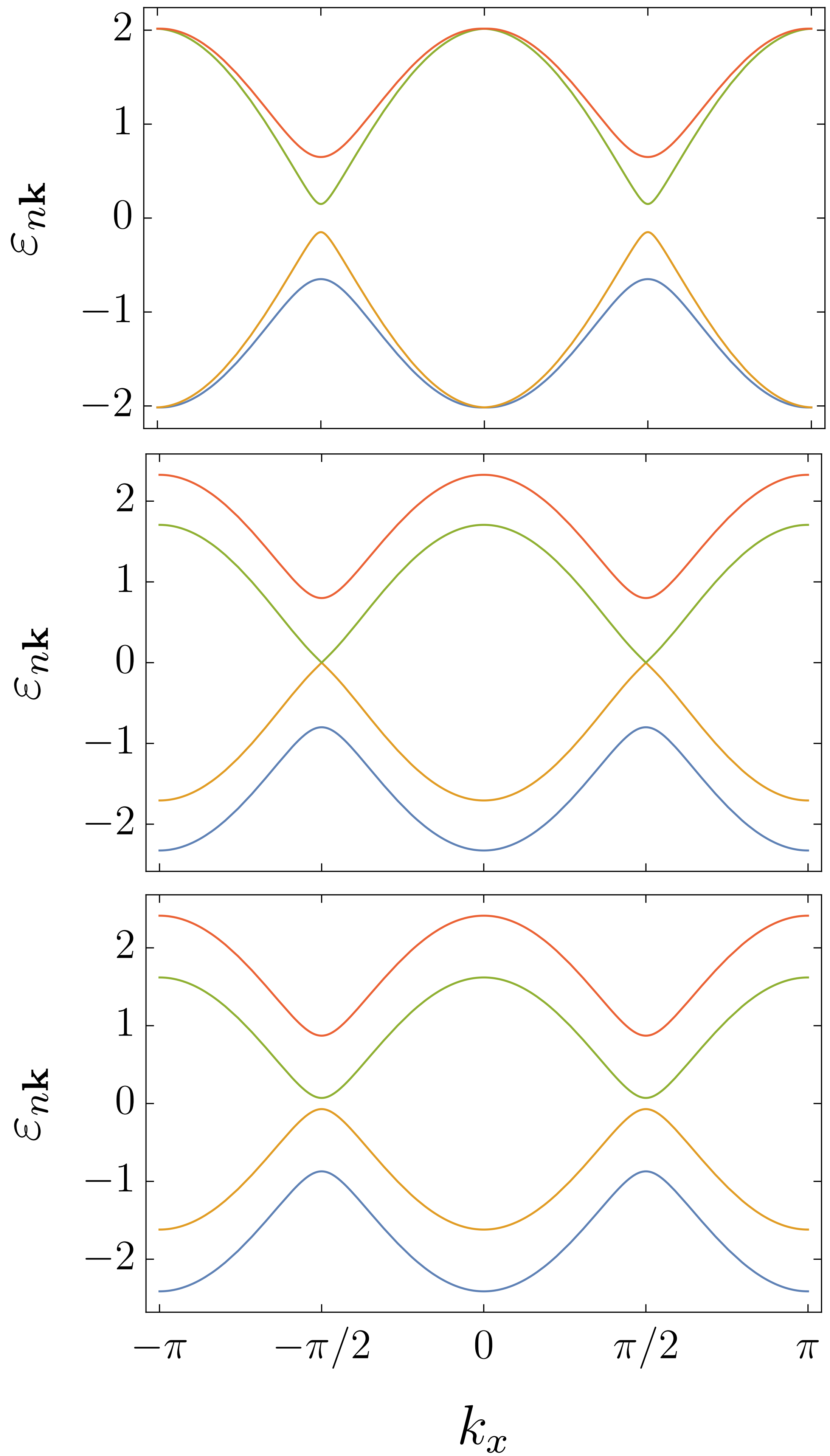
Quantum critical theory + J_χ + Néel order

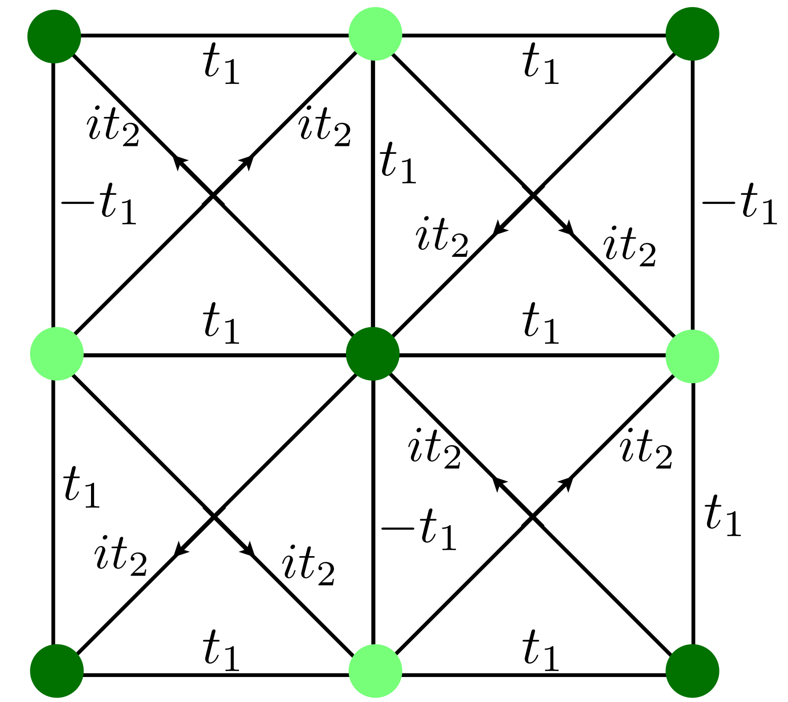




We focus on two possible mass terms for the Dirac fermions: m_χ, from the spin chirality term, and m_N induced by Néel order. There is a line in the m_χ, m_N plane where the occupied bands switch from Chern numbers $\{1, -1\}$ (the Néel state) to $\{1, 1\}$ (Néel order co-existing with semion topological order).

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



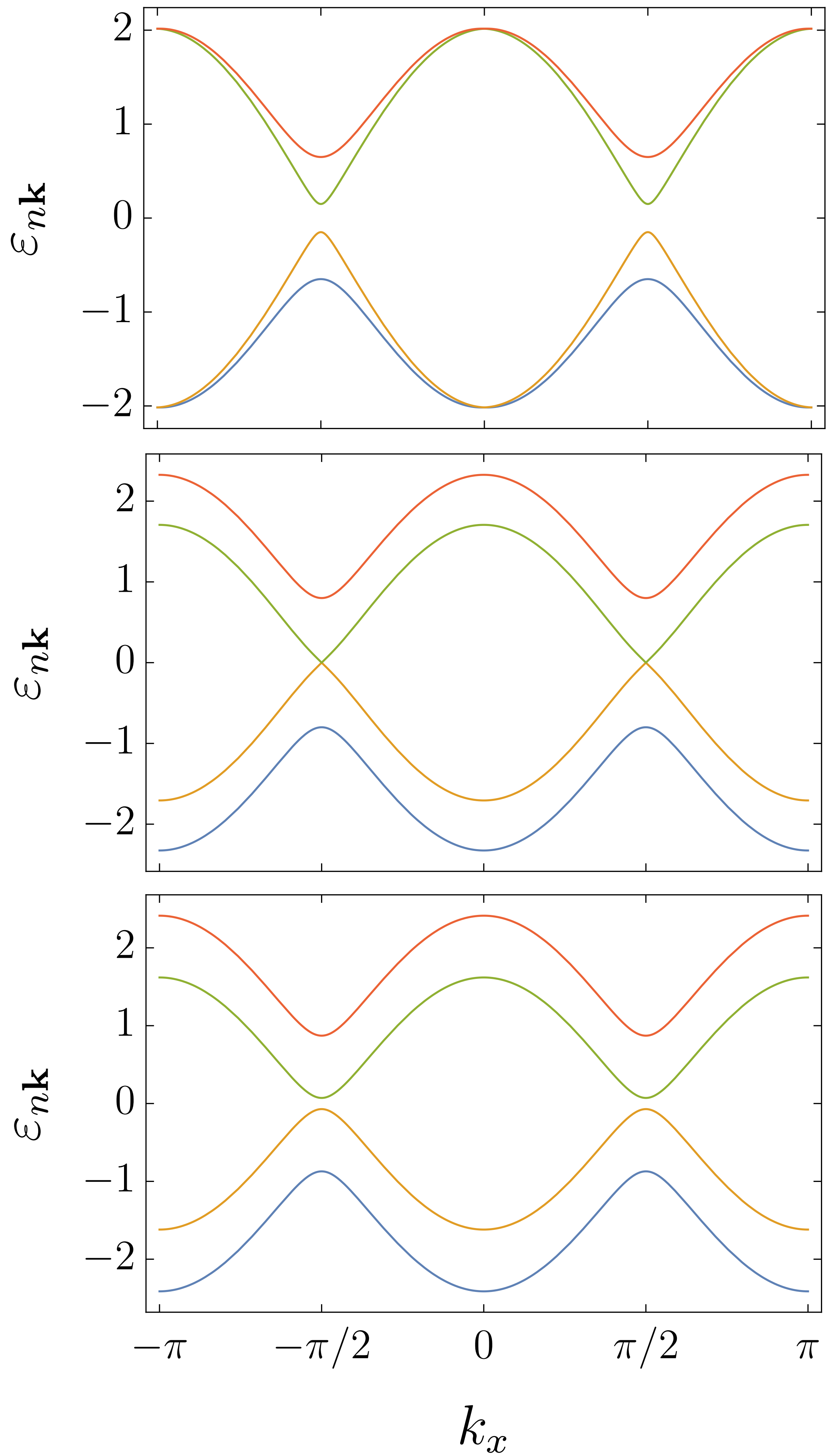


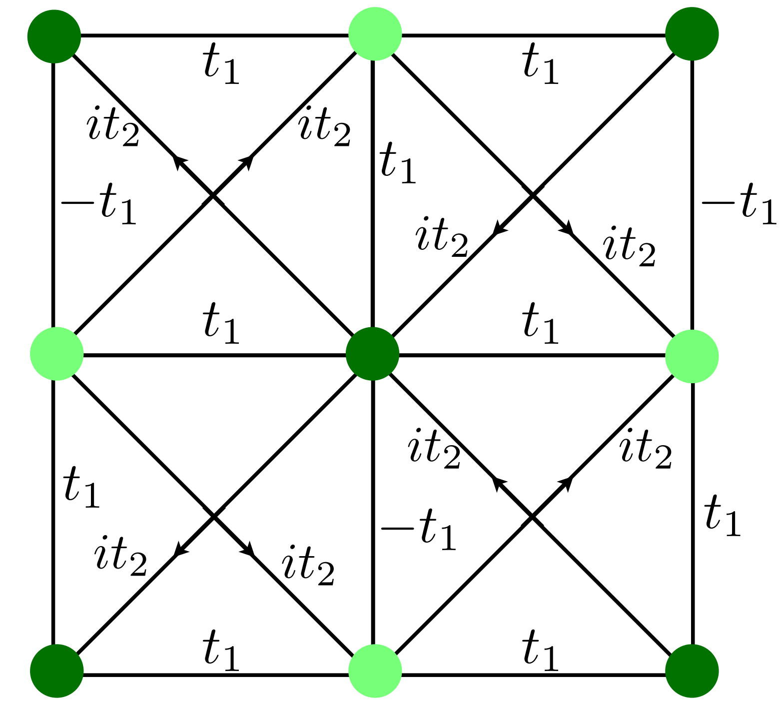
The vicinity of the critical point is described by $N_f = 1$ Dirac fermion coupled to a SU(2) gauge field A_μ at level $-1/2$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

The transition is tuned by the change in sign of m.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

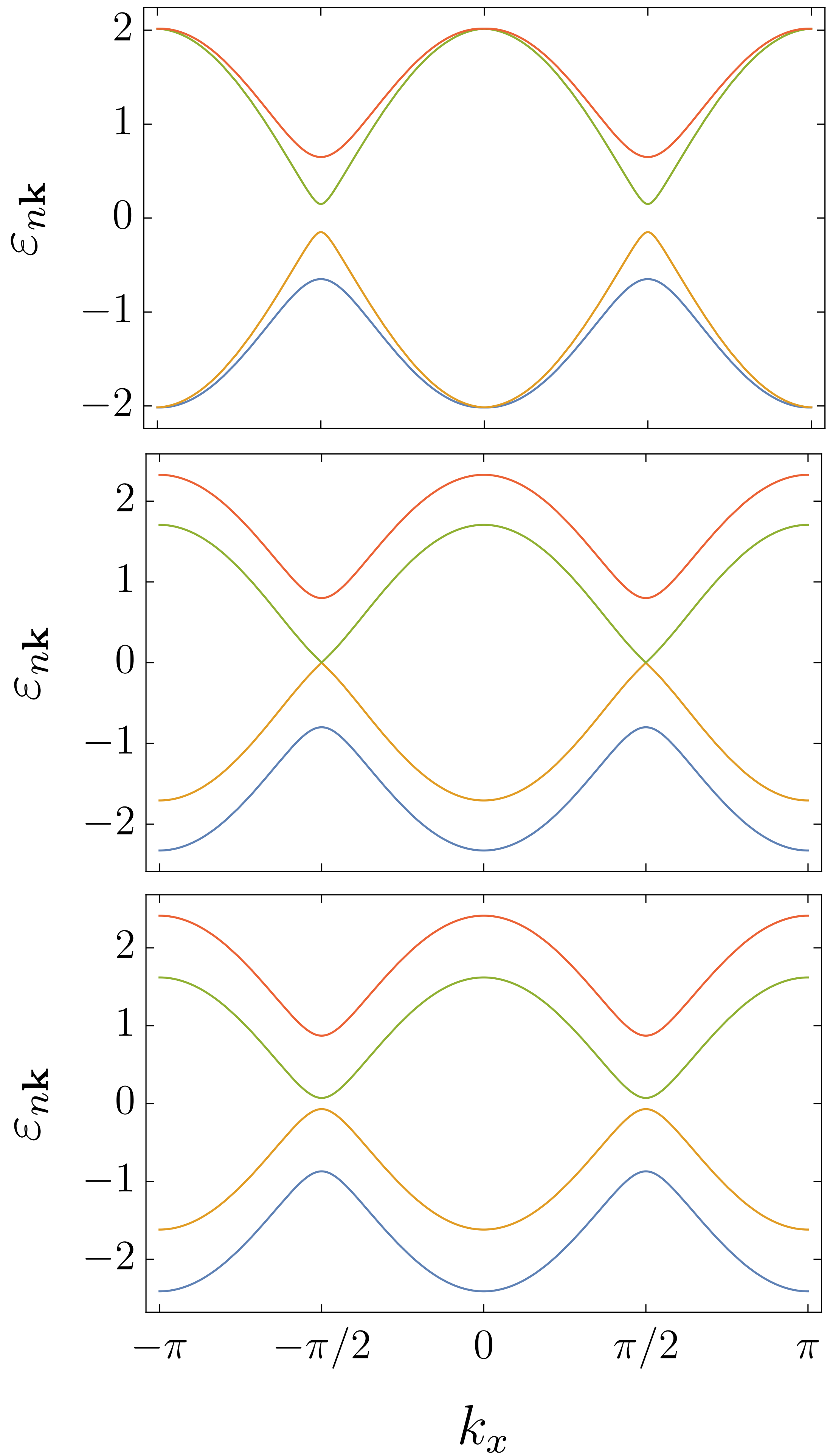


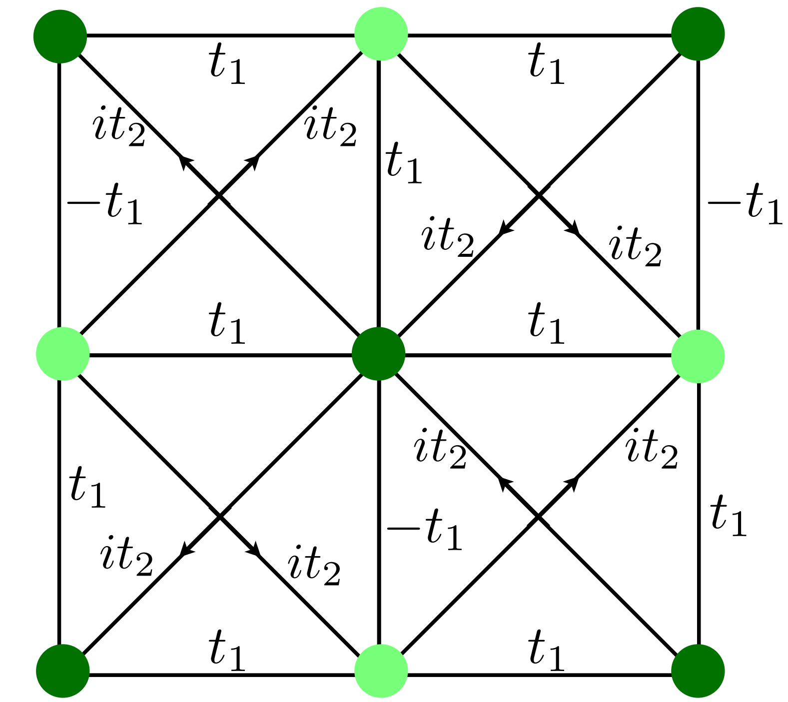


$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

When $m > 0$, we can integrate out f and there is no net CS term. The SU(2) gauge theory confines, and we obtain Néel order.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992



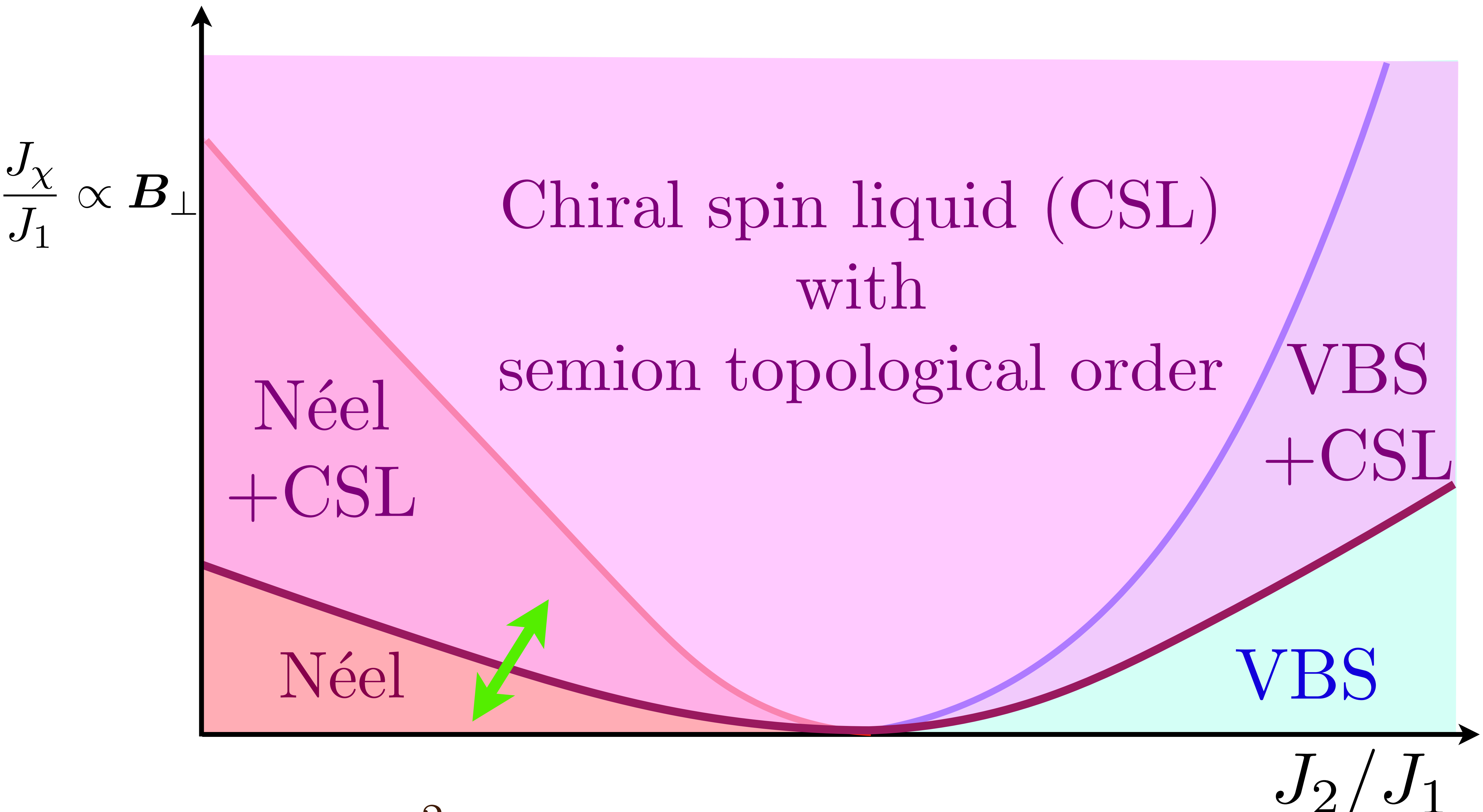


$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\text{CS}[A_\mu]$$

When $m < 0$, we can integrate out f and we obtain a net CS term at level -1. The SU(2) gauge theory at level -1 describes the semion topological phase.

S. Chatterjee, Haoyu Guo, S. Sachdev, R. Samajdar, M. S. Scheurer, N. Seiberg, and Cenke Xu, arXiv:1903.01992

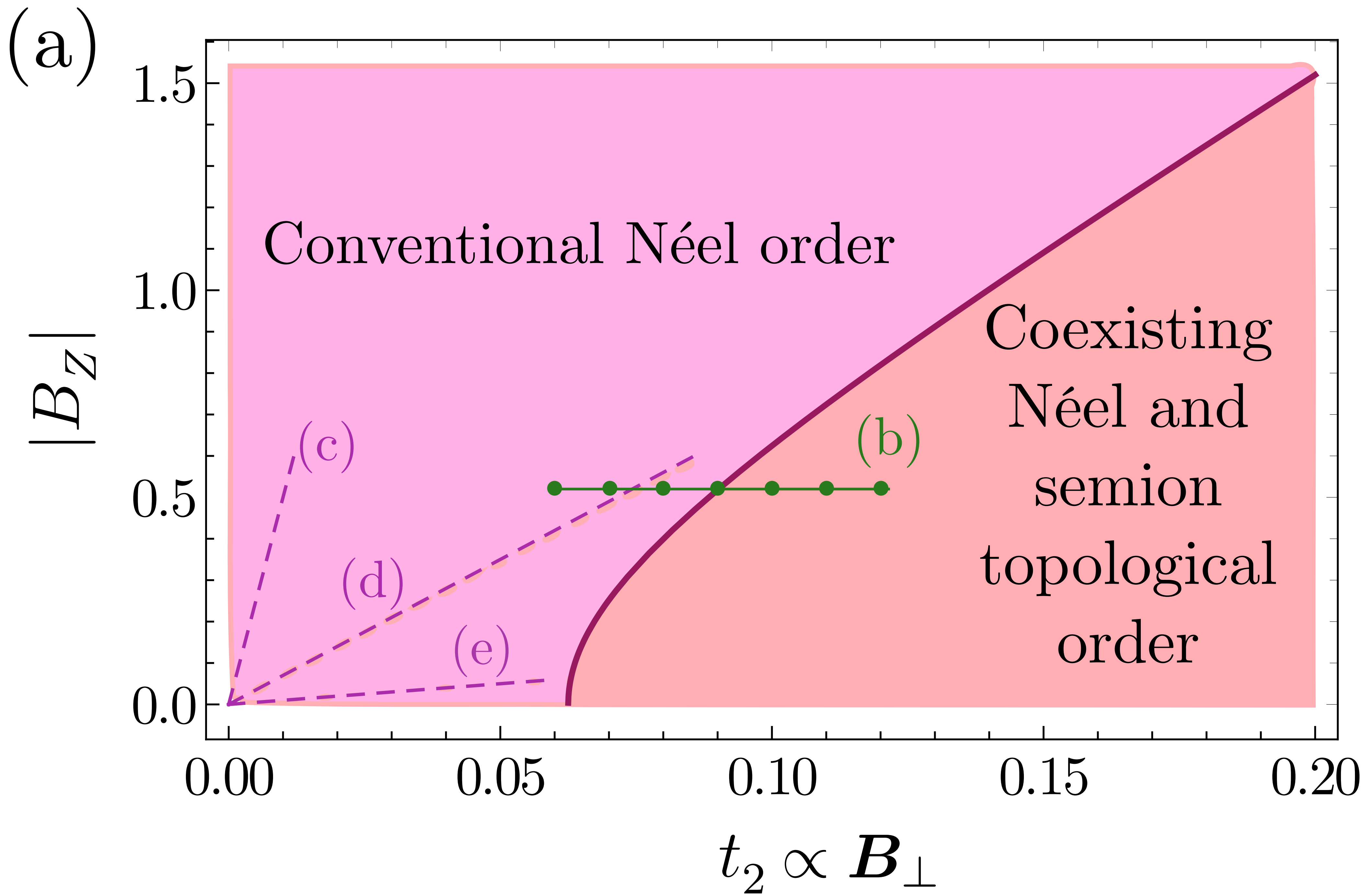
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



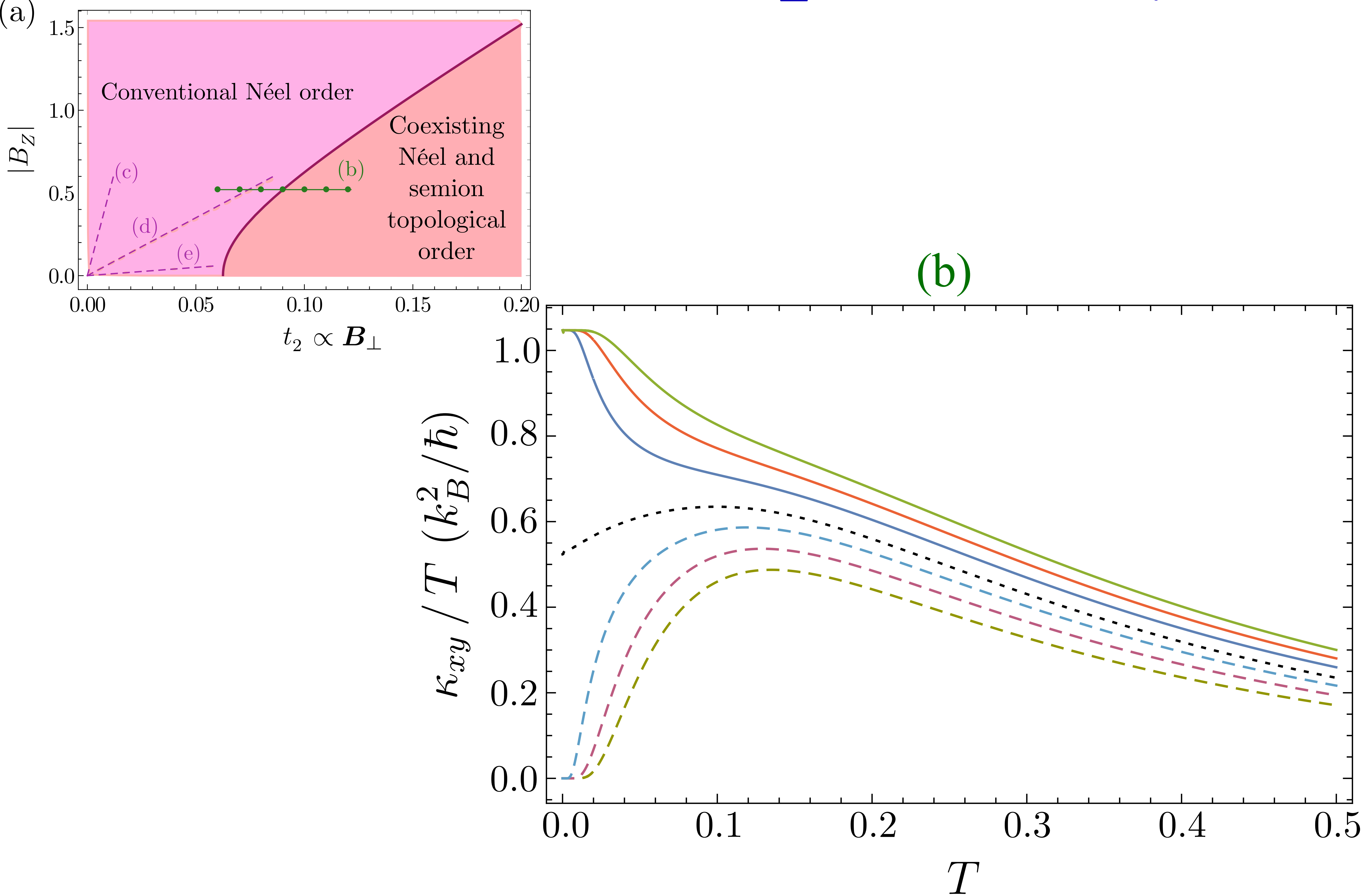
$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha + m_N \overline{f}_\alpha \Gamma f_\alpha \right]$$

Quantum critical theory + J_χ + Néel order

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \, .$$

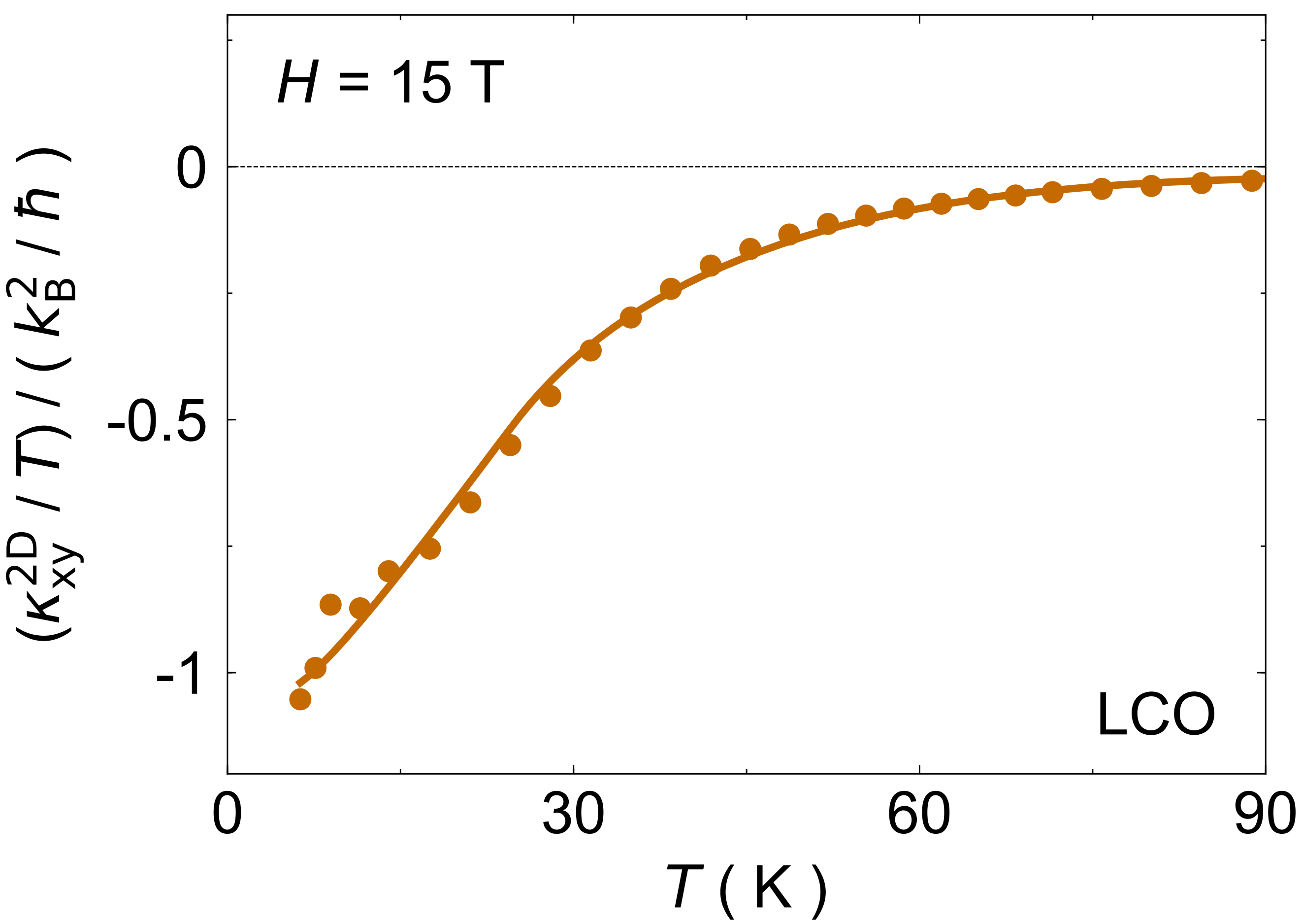


$$H=\sum_{\mathrm{n.n.}} J_1 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\sum_{\mathrm{n.n.n.}} J_2 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+J_\chi\sum_{\triangle}\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)-\sum_i \boldsymbol{B}_Z\cdot\boldsymbol{S}_i\,.$$

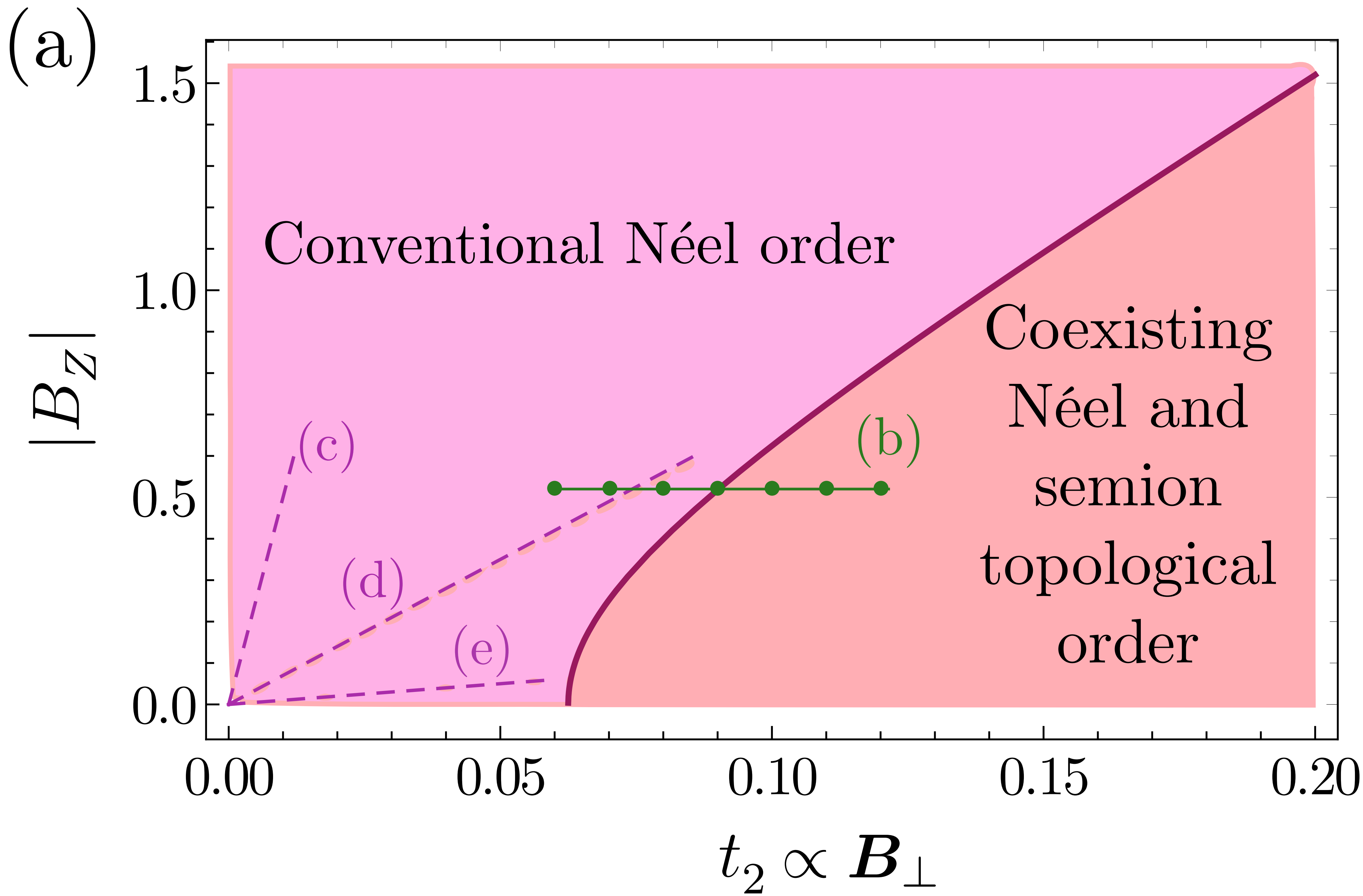


Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

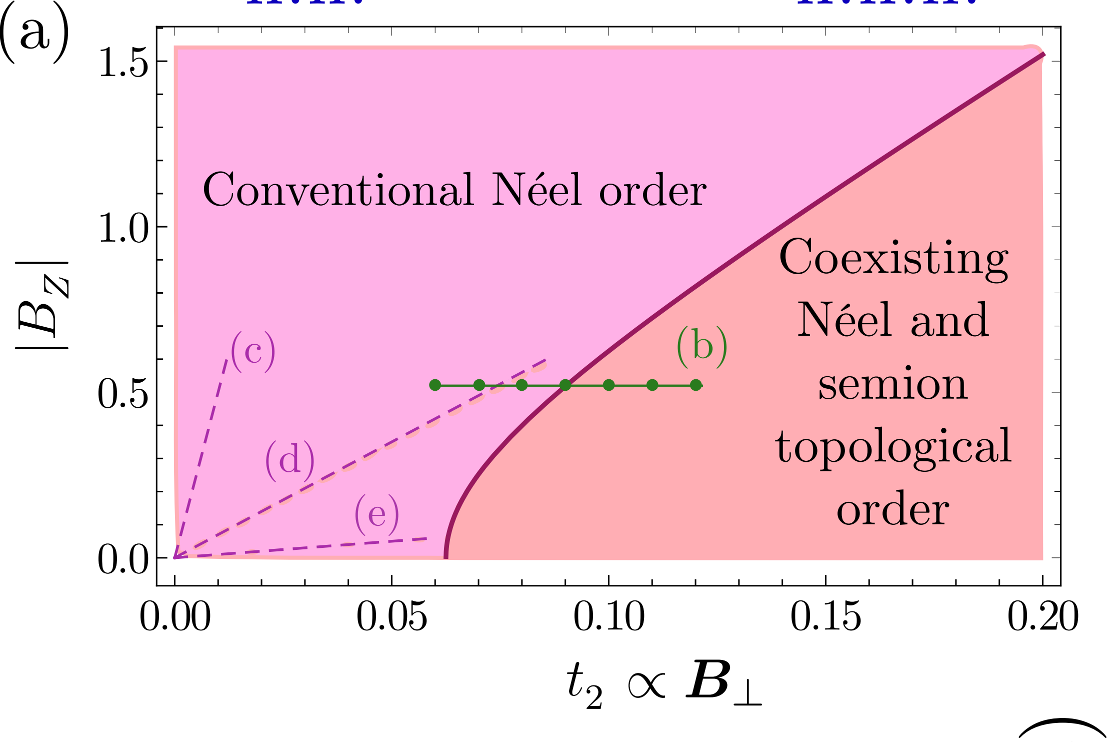
G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko, M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer, Nature **571**, 376 (2019)

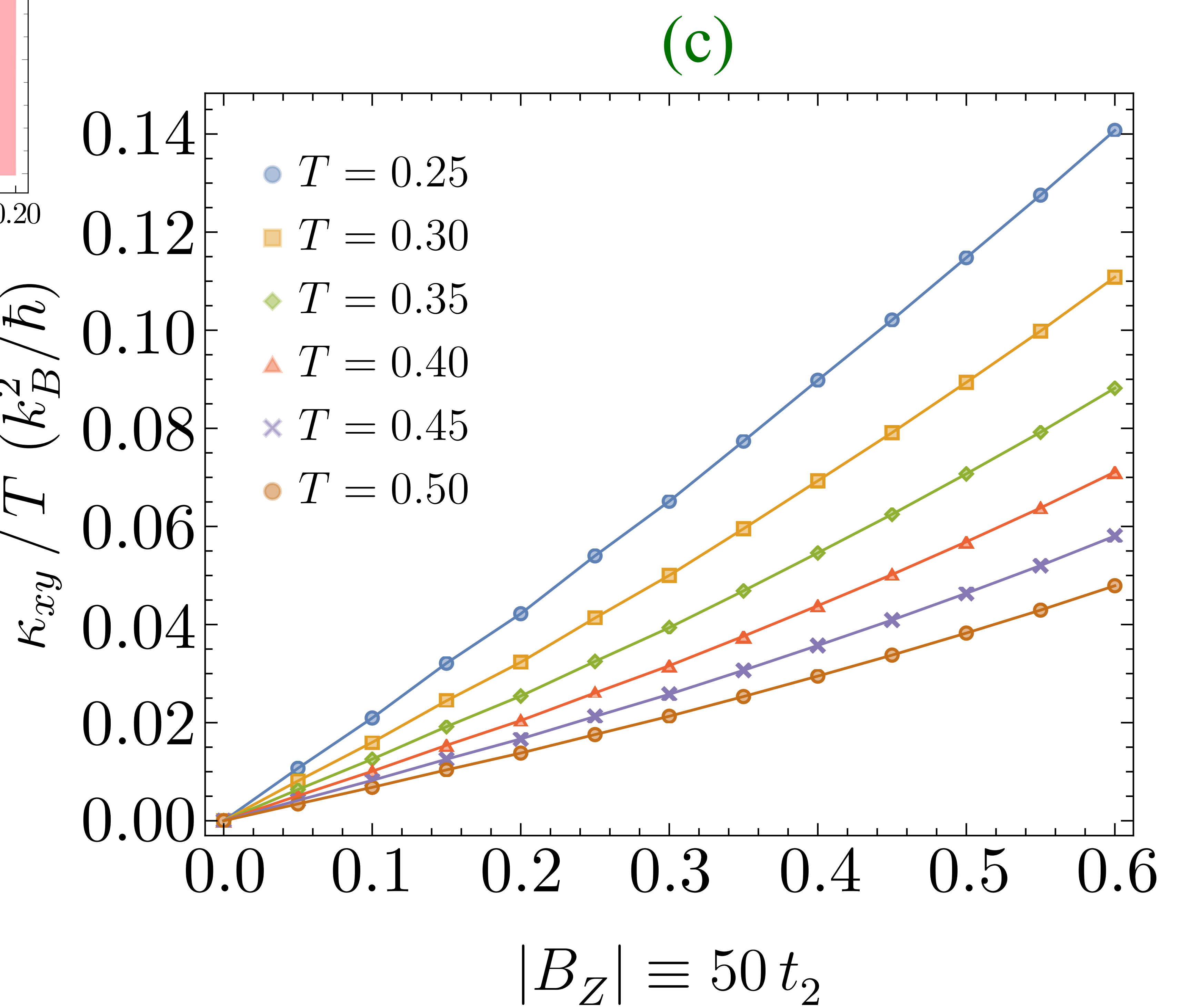


$$H=\sum_{\text{n.n.}} J_1 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+J_\chi\sum_{\triangle}\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)-\sum_i \boldsymbol{B}_Z\cdot\boldsymbol{S}_i\,.$$

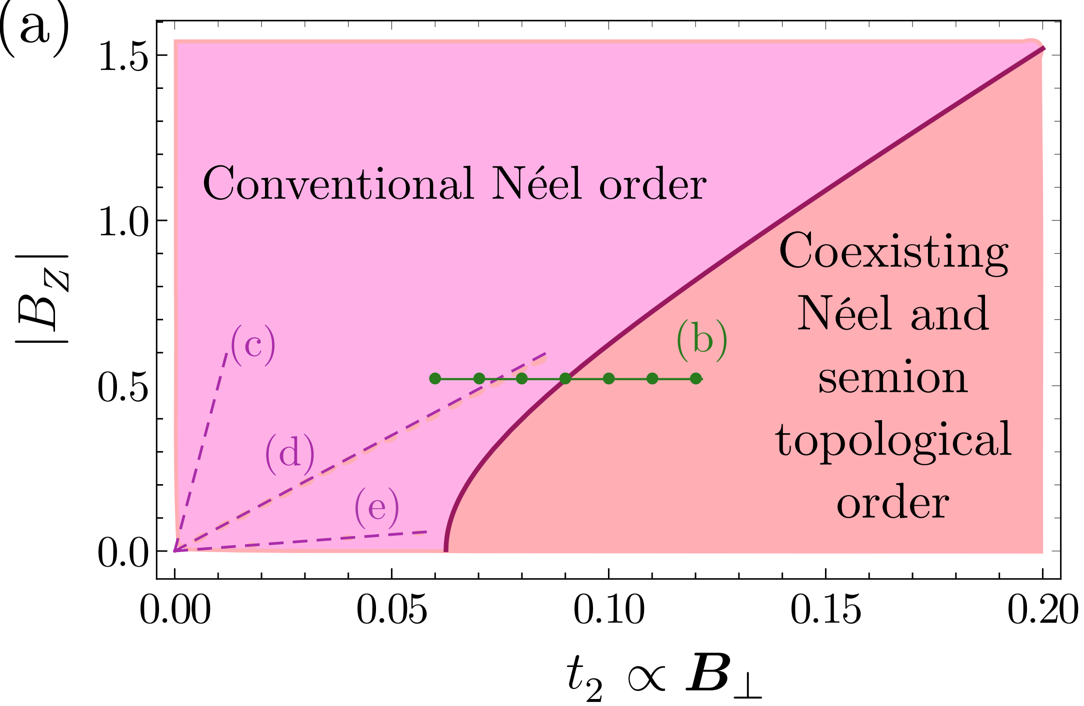


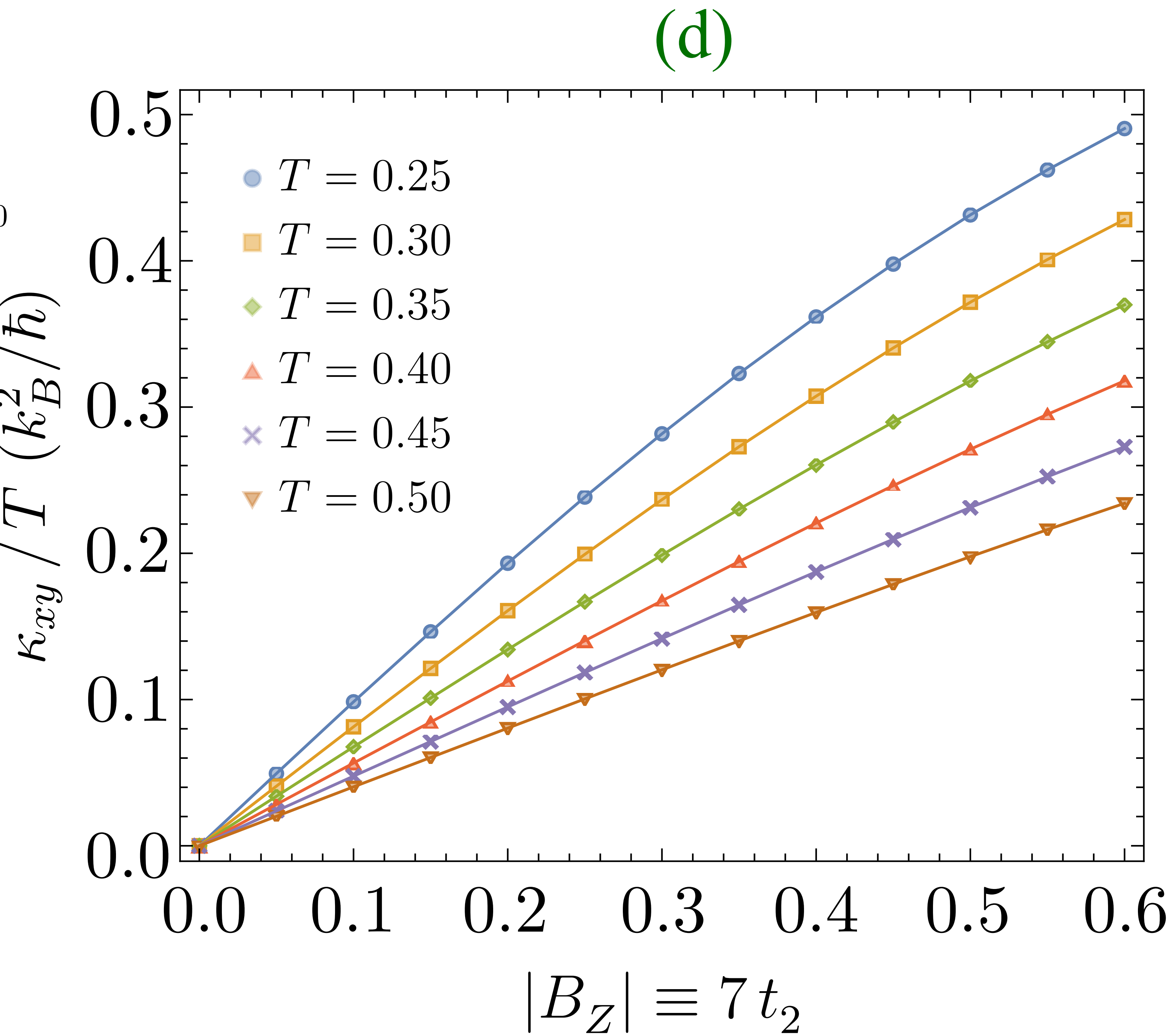
$$H=\sum_{\text{n.n.}} J_1 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+\sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i\cdot\boldsymbol{S}_j+J_\chi\sum_{\triangle}\boldsymbol{S}_i\cdot(\boldsymbol{S}_j\times\boldsymbol{S}_k)-\sum_i \boldsymbol{B}_Z\cdot\boldsymbol{S}_i\,.$$



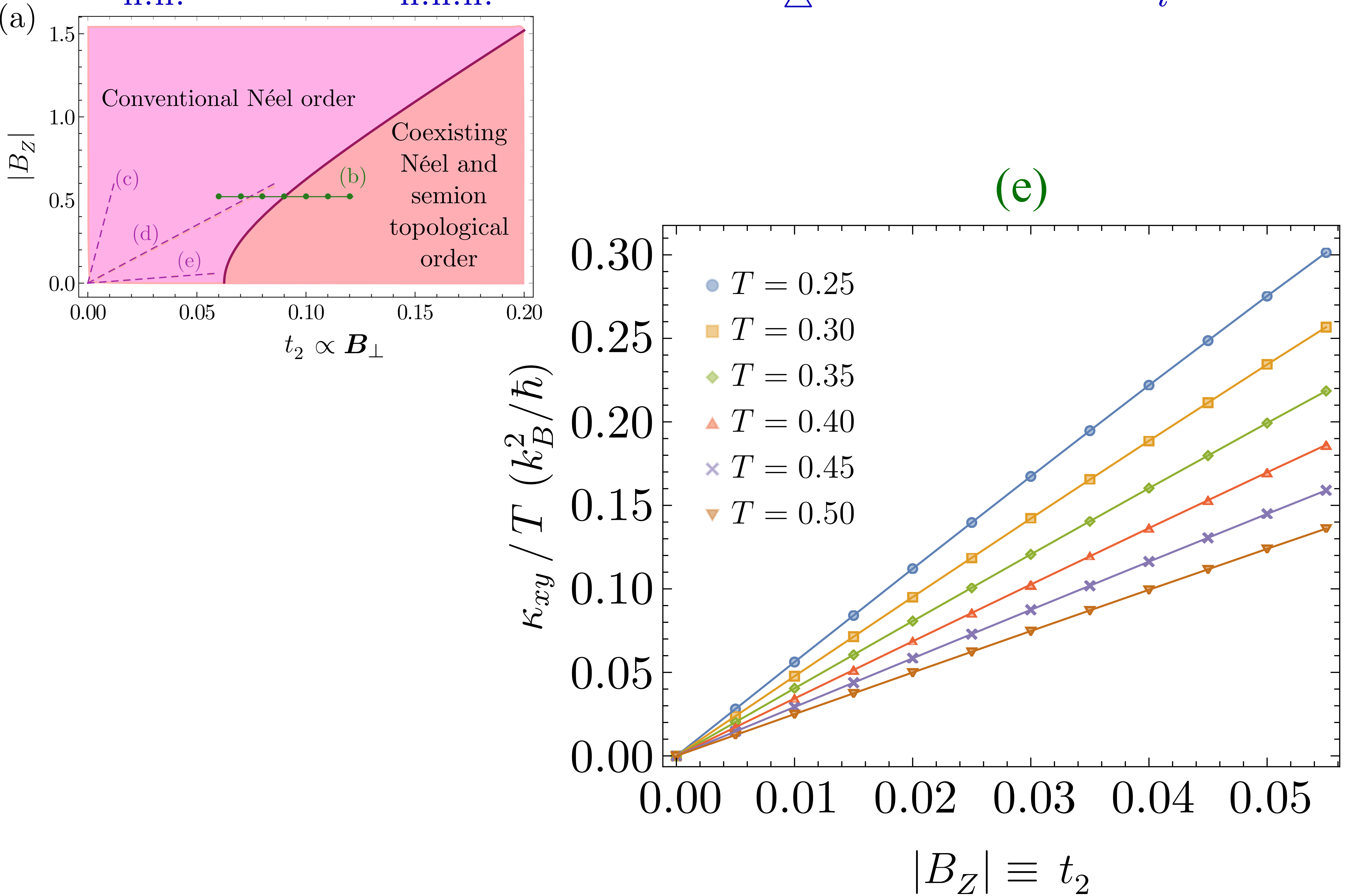


$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \, .$$



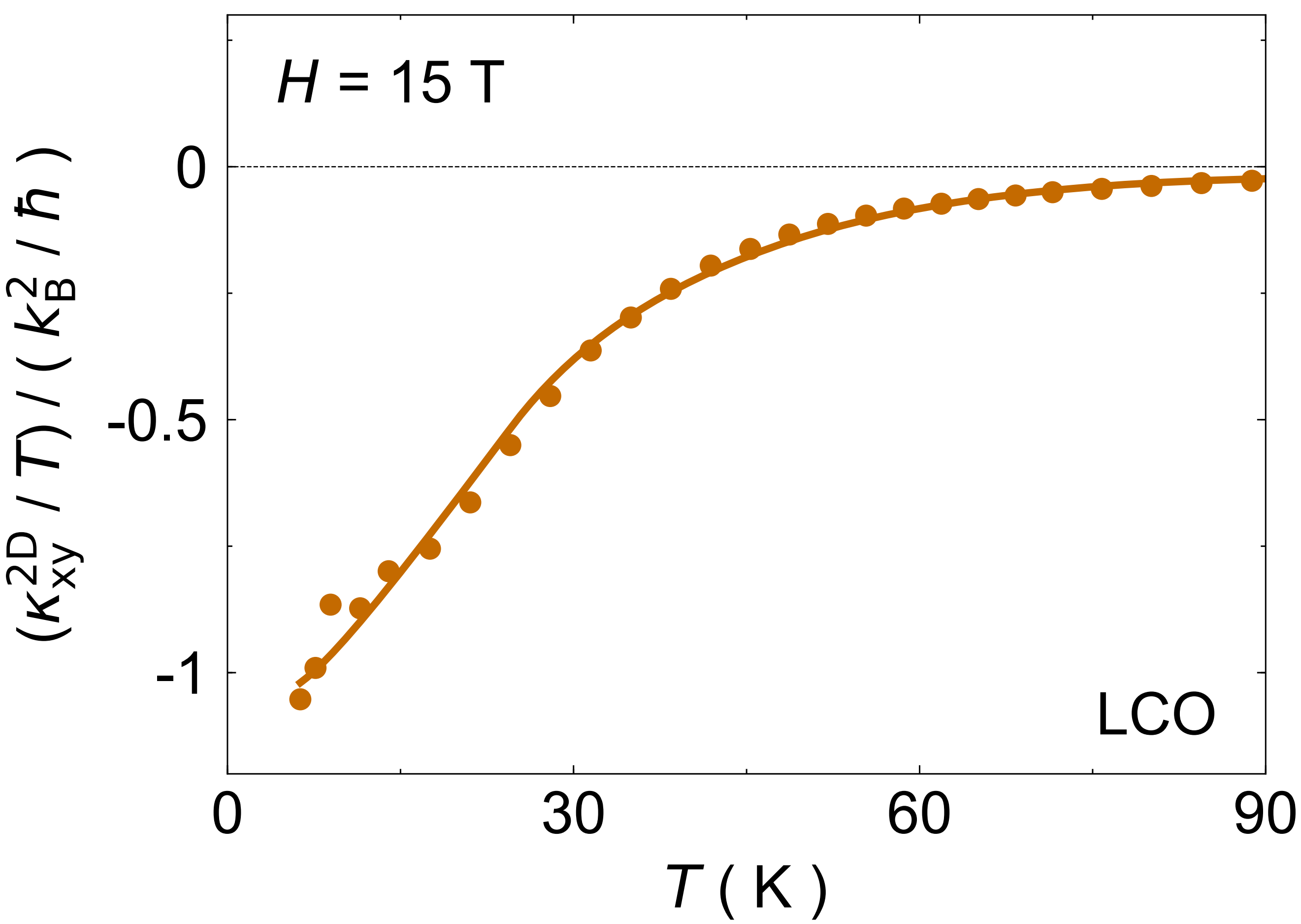


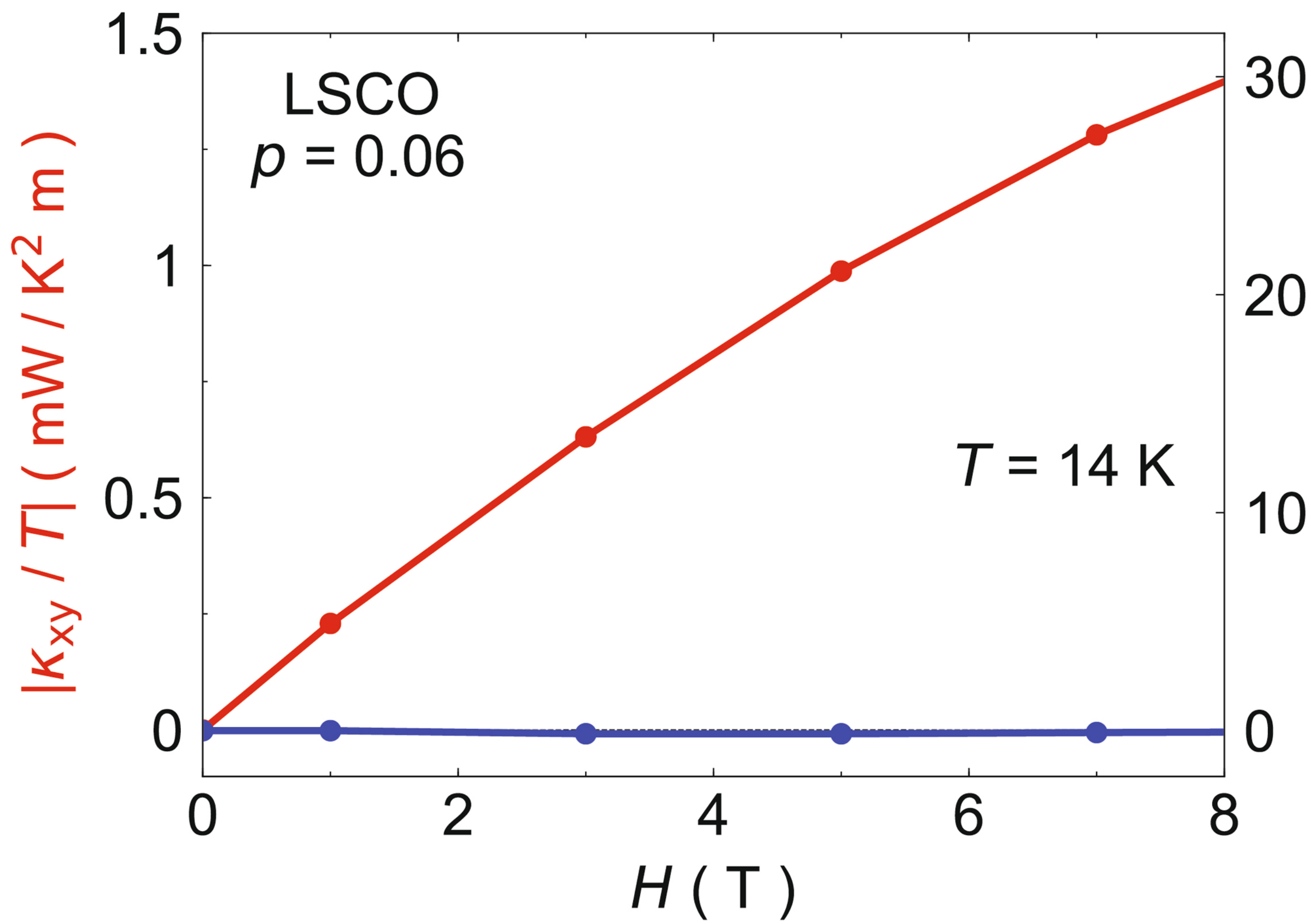
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$



Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko, M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer,
Nature **571**, 376 (2019)





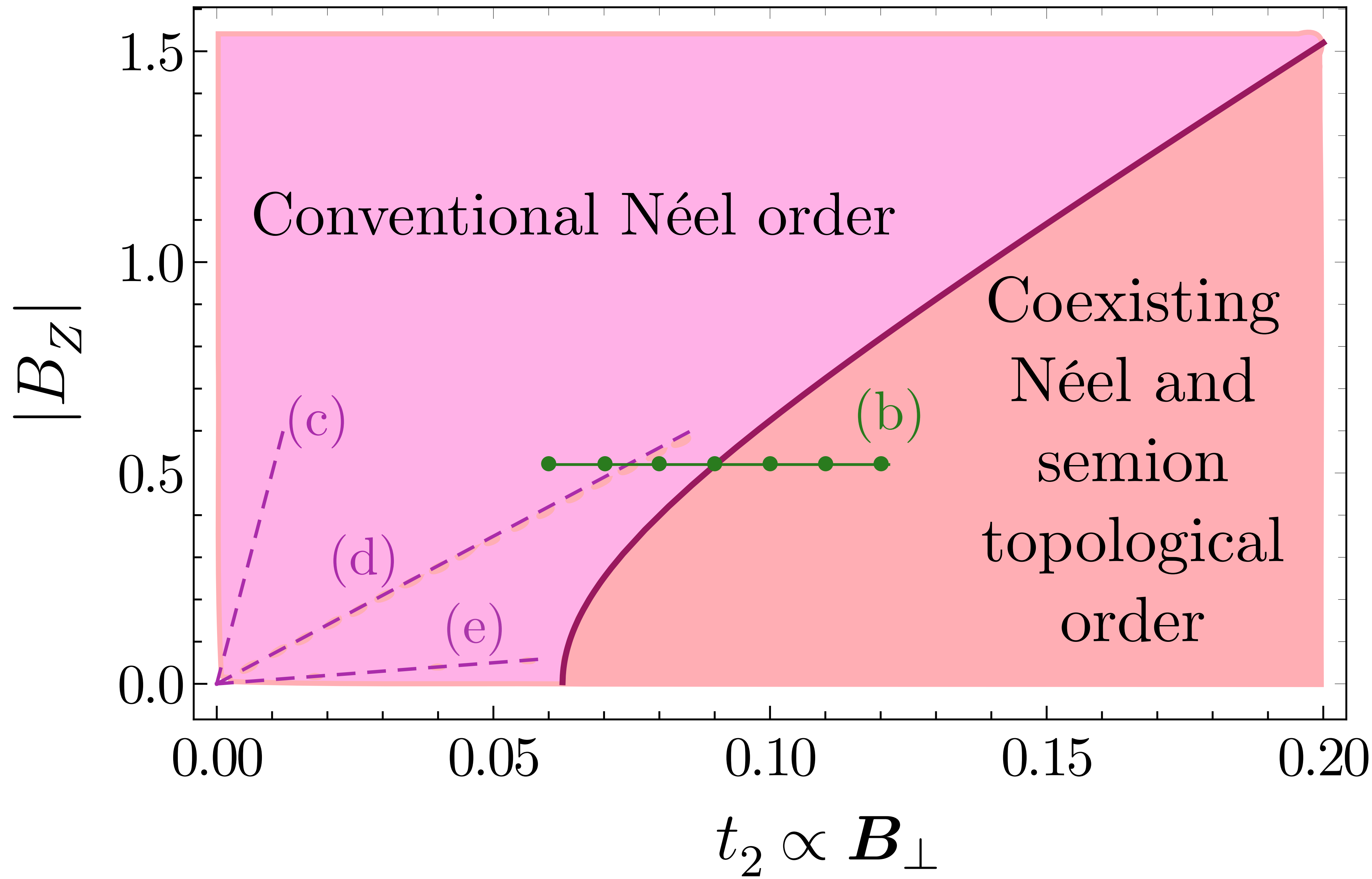
1. Insulator

A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

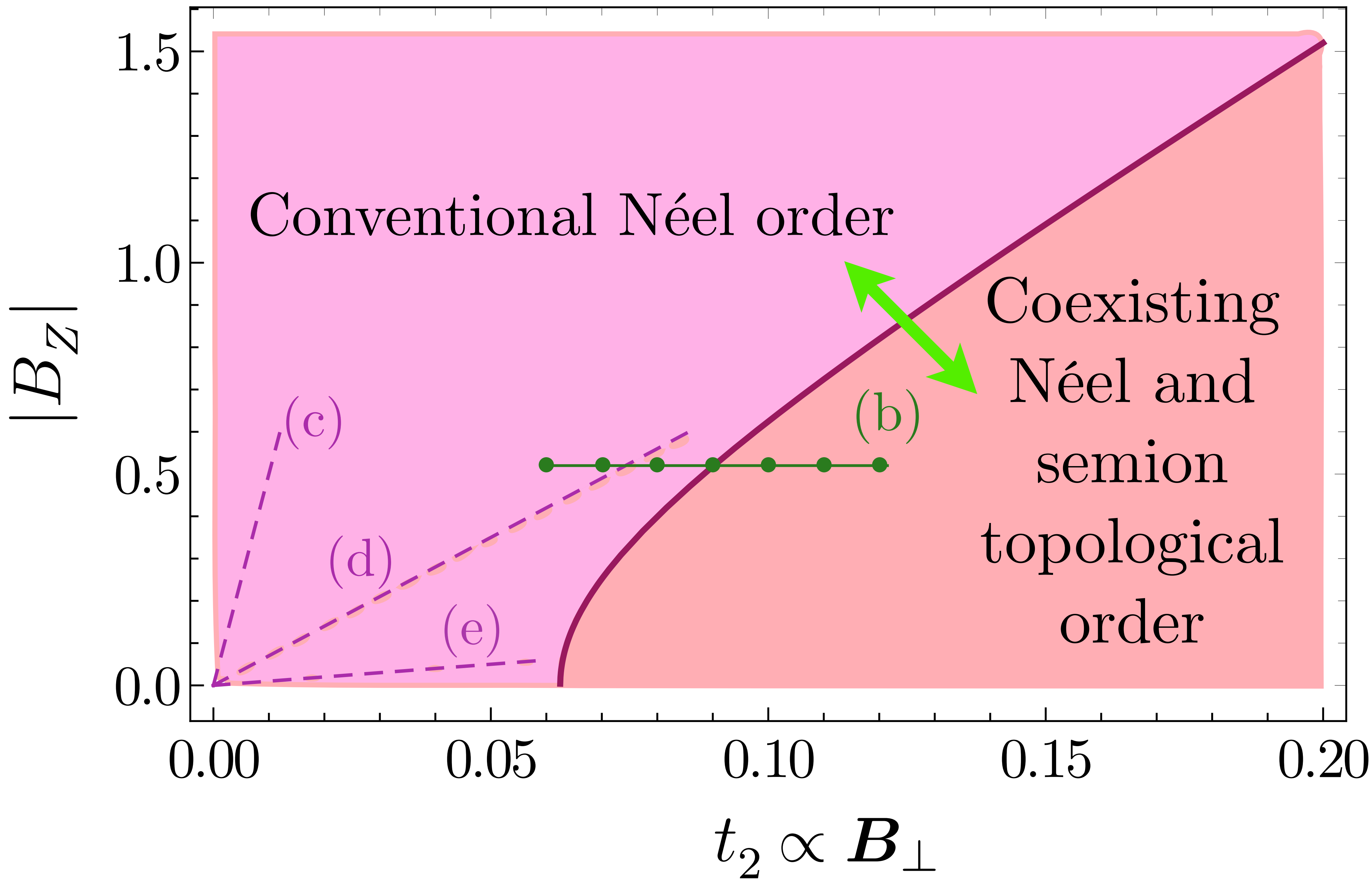
B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

$$H=\sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

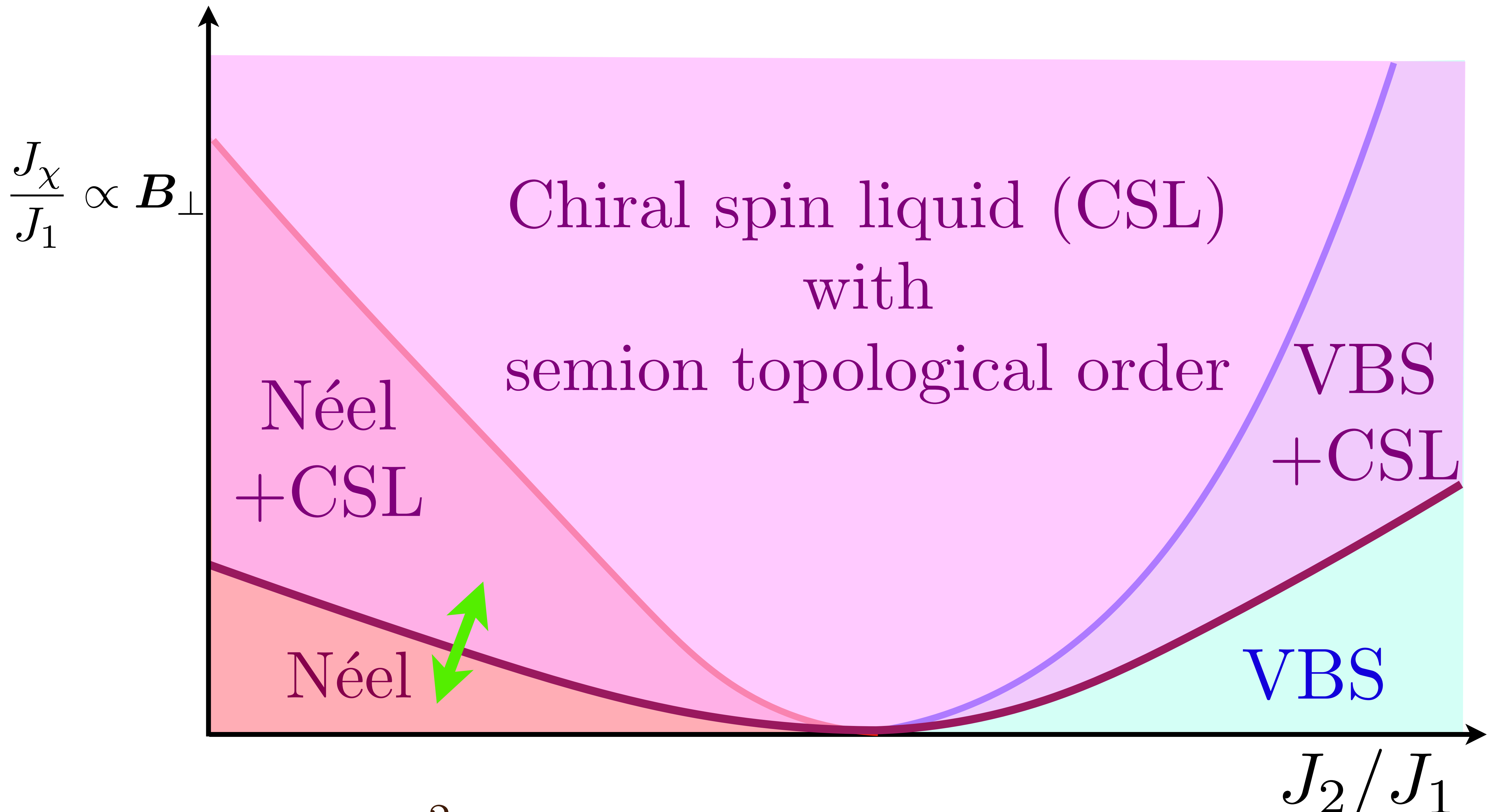


$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$



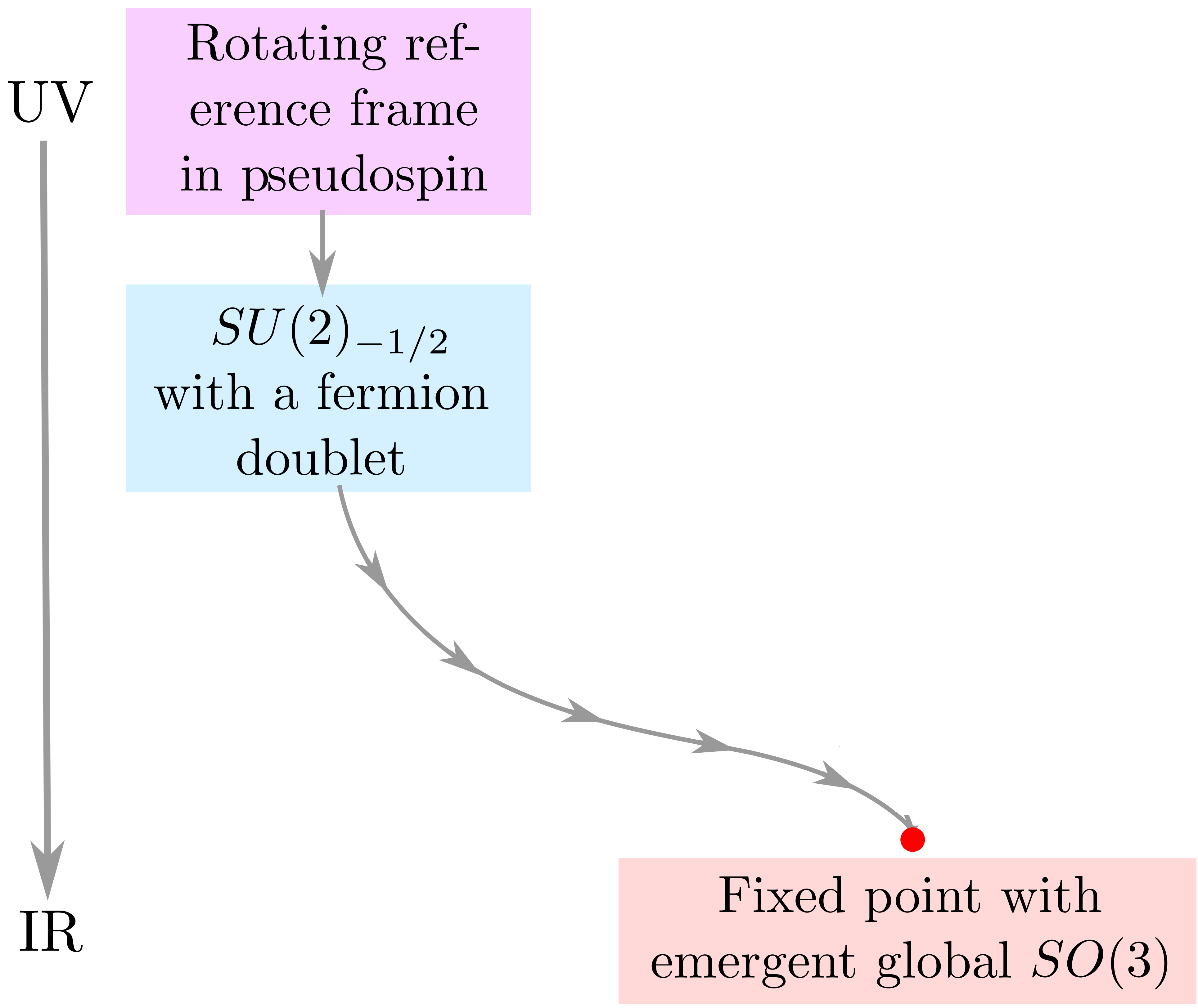
$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\text{CS}[A_\mu]$$

$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$



$$\mathcal{S}_f = \int d^2 r d\tau \sum_{\alpha=1}^{2} \left[\overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m_\chi \overline{f}_\alpha f_\alpha + m_N \overline{f}_\alpha \Gamma f_\alpha \right]$$

Quantum critical theory + J_χ + Néel order


UV
IR
Rotating reference frame in pseudospin
$SU(2)_{-1/2}$ with a fermion doublet
Fixed point with emergent global $SO(3)$

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in pseudospin space (similar to Schwinger fermions):
Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = F_i R_{ci}$$

$$F_i = \begin{pmatrix} f_{i\uparrow} & -f_{i\downarrow}^\dagger \\ f_{i\downarrow} & f_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad R_{ci} = \begin{pmatrix} b_{i1} & b_{i2} \\ -b_{i2}^* & b_{i1}^* \end{pmatrix}$$

F are fermionic spinons, R_c is a SU(2) rotation. Pseudospin rotations are *right* multiplication of R_c, while *left* multiplication is an emergent SU(2) gauge symmetry:

$$F \to FU, \quad R_c \to U^\dagger R_c \,.$$

Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):

Write the lattice electron operator $c_{i\alpha}$ as

$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si}\Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

Ψ are fermionic 'chargons', R_s is a SU(2) rotation. Spin rotations are *left* multiplication of R_s, while *right* multiplication is an emergent SU(2) gauge symmetry:

$$\Psi \to U\Psi, \quad R_s \to R_s U^\dagger .$$

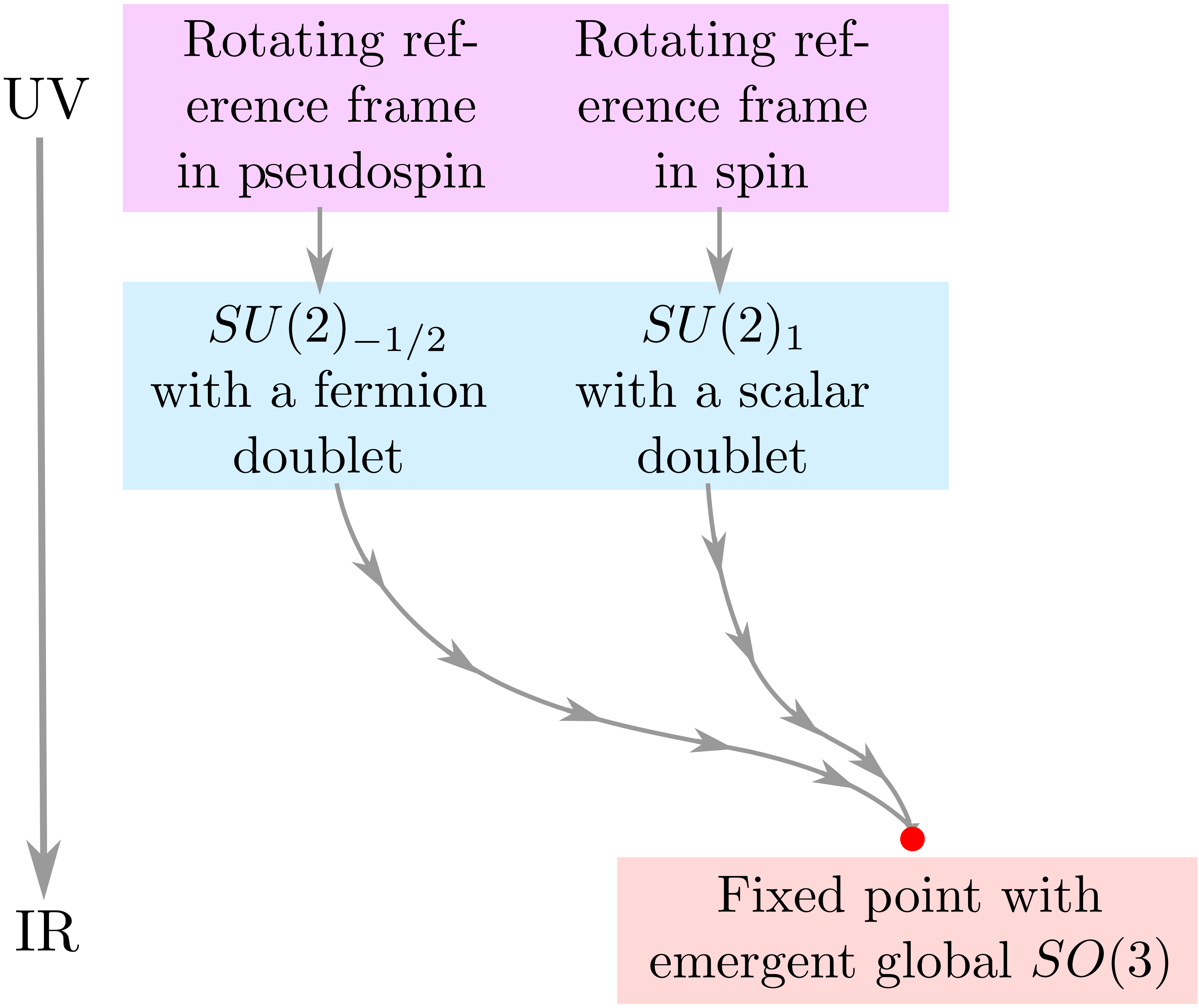
Quantum criticality in a frustrated square lattice antiferromagnet

SU(2) gauge theory of rotating reference frame in spin space (similar to Schwinger bosons):
Write the lattice electron operator $c_{i\alpha}$ as

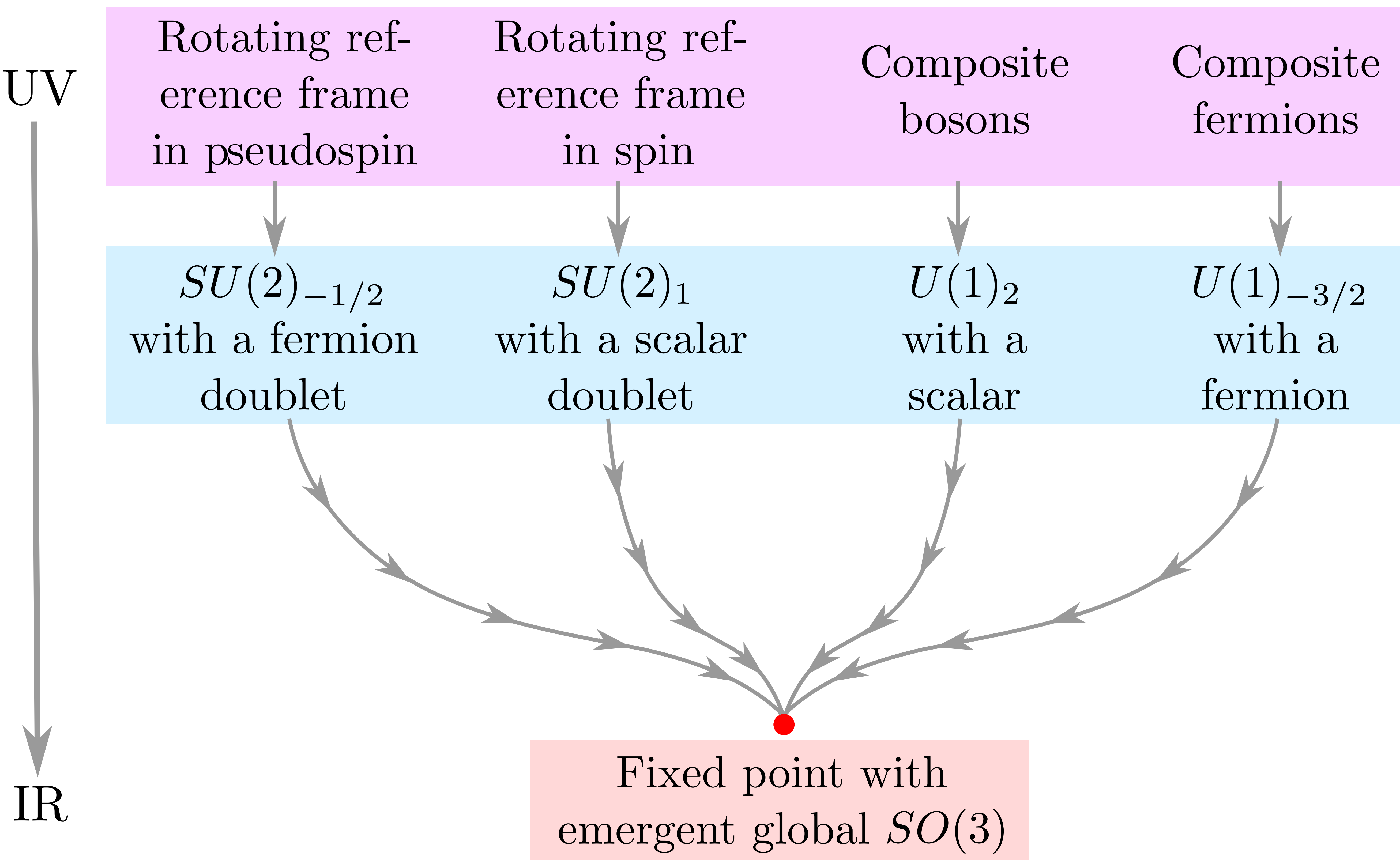
$$C_i = \begin{pmatrix} c_{i\uparrow} & -c_{i\downarrow}^\dagger \\ c_{i\downarrow} & c_{i\uparrow}^\dagger \end{pmatrix} \quad , \quad C_i = R_{si}\Psi_i$$

$$\Psi_i = \begin{pmatrix} \psi_{i+} & -\psi_{i-}^\dagger \\ \psi_{i-} & \psi_{i+}^\dagger \end{pmatrix} \quad , \quad R_{si} = \begin{pmatrix} z_{i\uparrow} & -z_{i\downarrow}^* \\ z_{i\downarrow} & z_{i\uparrow}^* \end{pmatrix}$$

We assume that the band structure of the ψ_β fermions is such that both species are in a filled band with unit Chern number; then, integrating out these gapped fermions yields the Chern-Simons terms for the $SU(2)_1$ gauge field. Now, the needed transition is obtained by the Higgs transition of the scalar: the topological phase has R gapped, while the trivial phase has R condensed.



Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035



Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

A quadrilarity

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

$$\mathcal{L}_\phi = |(\partial_\mu - ia_\mu)\phi|^2 + s|\phi|^2 + u(|\phi|^2)^2 + 2\mathrm{CS}[a_\mu]$$

$$\mathcal{L}_g = \overline{g}\gamma^\mu(\partial_\mu - ia_\mu)g + m\overline{g}g - \frac{3}{2}\mathrm{CS}[a_\mu]$$

Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \dots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k \, \mathrm{CS}[A_\mu]$$

The thermal Hall conductivity is a universal function of m/T, which can be computed in an expansion in $1/N_f$ with $k \propto N_f$.

$$\kappa_{xy} = \frac{k_B^2 T}{\hbar} \mathcal{F}(m/T)$$

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \ldots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k\, \mathrm{CS}[A_\mu]$$

The thermal Hall conductivity is a universal function of m/T, which can be computed in an expansion in $1/N_f$ with $k \propto N_f$.

$$\kappa_{xy} = \frac{k_B^2 T}{\hbar} \mathcal{F}(m/T)$$

- In the limit $T \ll |m|$, we obtain two gapped topological phases in which $\mathcal{F}$ is a rational number. These are described by SU(2) gauge theory at integer level $k - (N_f/2)\mathrm{sgn}(m)$.

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \ldots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k \, \mathrm{CS}[A_\mu]$$

$$\kappa_{xy} = -\frac{\pi k_B^2 T}{6\hbar} \mathrm{sgn}(\hat{k}) \left[2|\hat{k}| - \frac{3|\hat{k}|}{|\hat{k}| + 2} \right] \quad , \quad \frac{|m|}{T} \to \infty$$

where the integer $\hat{k}$ is defined by

$$\hat{k} = k - \frac{N_f}{2} \mathrm{sgn}(m)$$

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \ldots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k\,\mathrm{CS}[A_\mu]$$

$$\kappa_{xy} = -\frac{\pi k_B^2 T}{6\hbar} \mathrm{sgn}(\hat{k}) \left[2|\hat{k}| - \frac{3|\hat{k}|}{|\hat{k}|+2} \right] \quad , \quad \frac{|m|}{T} \to \infty$$

where the integer $\hat{k}$ is defined by

$\mathcal{O}(N_f)$ fermion contribution

$$\hat{k} = k - \frac{N_f}{2} \mathrm{sgn}(m)$$

In the large N_f limit, the first term is the fermion contribution, while the gauge field contributes the second term with the opposite sign.

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \dots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k\, \mathrm{CS}[A_\mu]$$

$$\kappa_{xy} = -\frac{\pi k_B^2 T}{6\hbar} \mathrm{sgn}(\hat{k}) \left[2|\hat{k}| - \frac{3|\hat{k}|}{|\hat{k}| + 2} \right] \quad , \quad \frac{|m|}{T} \to \infty$$

where the integer $\hat{k}$ is defined by

$\mathcal{O}(N_f)$ fermion contribution

$$\hat{k} = k - \frac{N_f}{2} \mathrm{sgn}(m)$$

$\mathcal{O}(N_f^0)$ gauge field contribution

In the large N_f limit, the first term is the fermion contribution, while the gauge field contributes the second term with the opposite sign.

Thermal Hall conductivity of N_f fermions f_α, $\alpha = 1 \dots N_f$ coupled to SU(2) gauge field A_μ at Chern-Simons level k

$$\mathcal{L}_f = \overline{f}_\alpha \gamma^\mu (\partial_\mu - i A_\mu) f_\alpha + m \overline{f}_\alpha f_\alpha + k \, \mathrm{CS}[A_\mu]$$

The thermal Hall conductivity is a universal function of m/T, which can be computed in an expansion in $1/N_f$ with $k \propto N_f$.

$$\kappa_{xy} = \frac{k_B^2 T}{\hbar} \mathcal{F}(m/T)$$

- In the limit $T \ll |m|$, we obtain two gapped topological phases in which $\mathcal{F}$ is a rational number. These are described by SU(2) gauge theory at integer level $k - (N_f/2)\mathrm{sgn}(m)$.

- For $T \gg |m|$, we have the quantum critical region, where $\mathcal{F}$ is not rational, and κ_{xy} requires a bulk 2+1 dimensional description.

1. Insulator

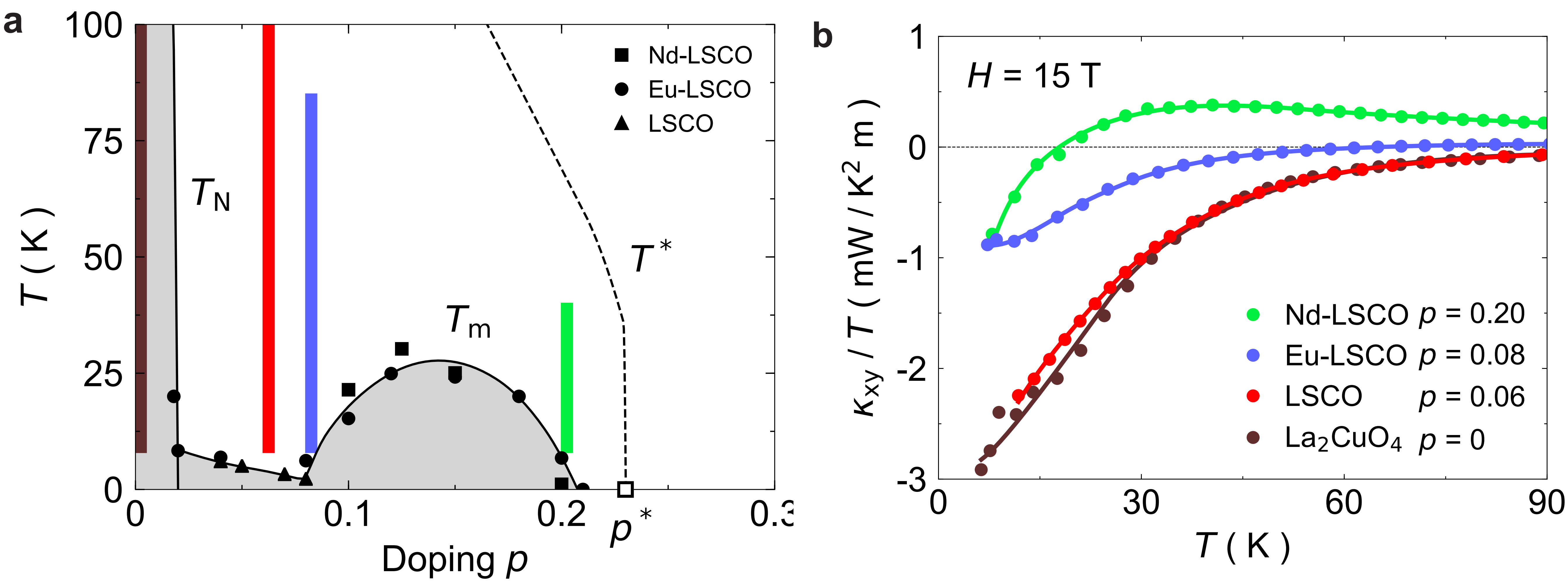
A. Thermal Hall conductivity across the Neel/Neel+CSL quantum transition

B. Quantum criticality and non-Abelian dualities

2. Pseudogap at non-zero doping

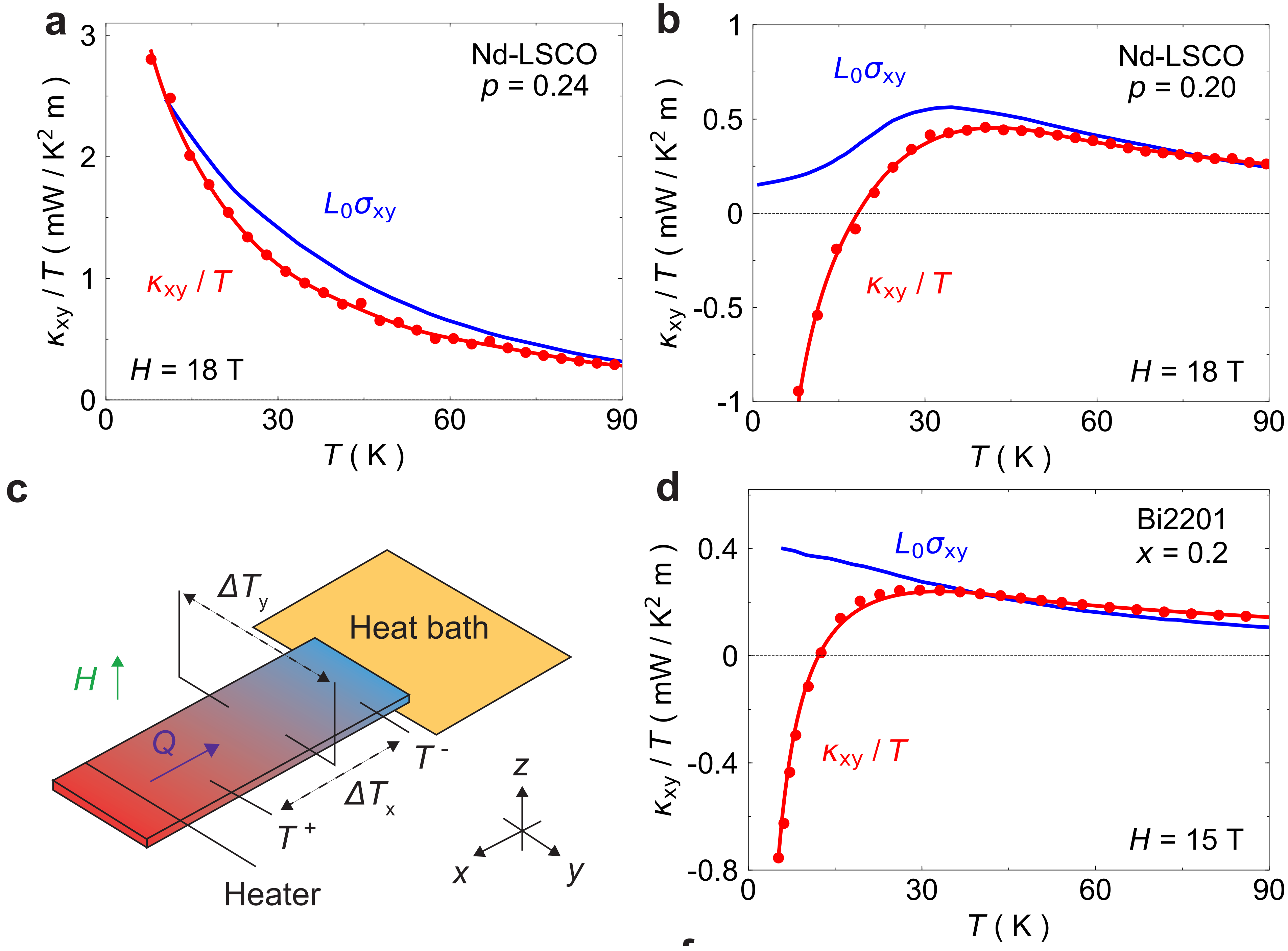
Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko, M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer,
Nature **571**, 376 (2019)



Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

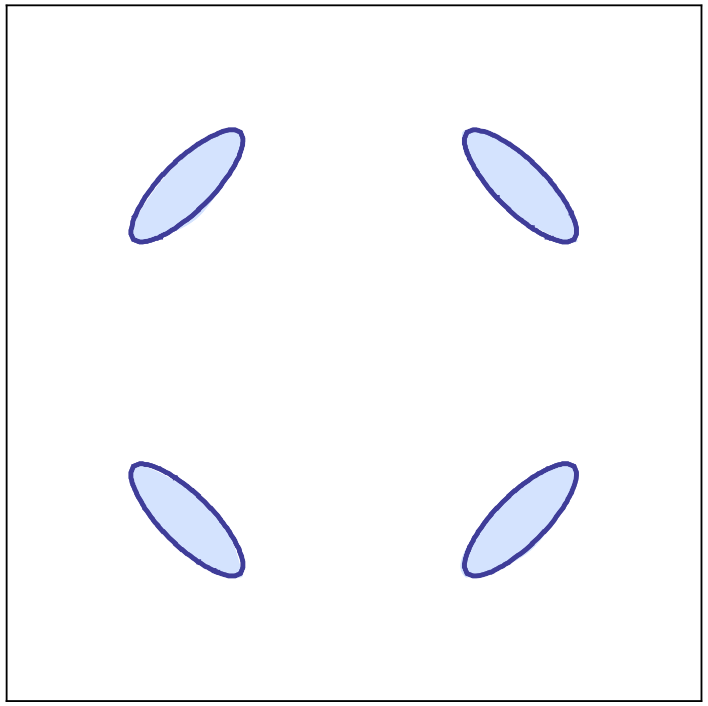
G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko, M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer, Nature **571**, 376 (2019)



Model for the pseudogap

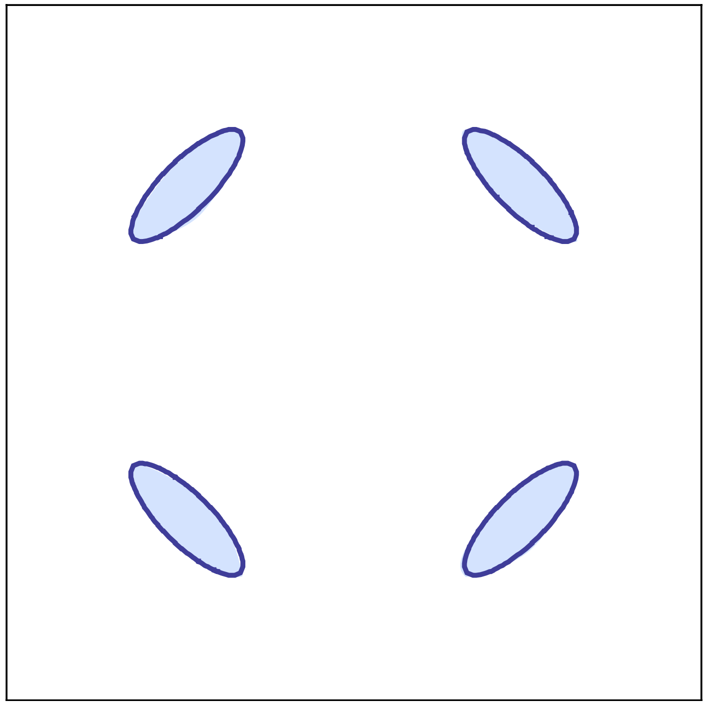
Fermionic 'chargons' of density δ in hole pockets.
The fermions carry electromagnetic gauge charge $+e$,
and charges $p = \pm 1$ under an emergent U(1) gauge field.
v is a valley index, $v_{\rm dis}$ is an impurity potential.

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - i p a_\tau - \frac{(\boldsymbol{\nabla} - i p \boldsymbol{a} - i e \boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv}$$
$$+ v_{\rm dis}(\boldsymbol{r}) f_{pv}^\dagger f_{pv}$$



Thermal Hall conductivity

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f_{pv}^\dagger \left(\frac{\partial}{\partial \tau} - \mu - i p a_\tau - \frac{(\boldsymbol{\nabla} - i p \boldsymbol{a} - i e \boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv} + v_{\rm dis}(\boldsymbol{r}) f_{pv}^\dagger f_{pv}$$



Leading order fermionic contribution is that implied by the Wiedemann-Franz law.

$$\sigma_{xy} = \left(\frac{\delta e^2 \tau}{m^*} \right) \omega_c \tau$$

$$\kappa_{xy}^0 = \frac{\pi^2 T}{3} \left(\frac{k_B}{e} \right)^2 \sigma_{xy}$$

Thermal Hall conductivity

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f^\dagger_{pv} \left(\frac{\partial}{\partial \tau} - \mu - ipa_\tau - \frac{(\boldsymbol{\nabla} - ip\boldsymbol{a} - ie\boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv} + v_{\rm dis}(\boldsymbol{r}) f^\dagger_{pv} f_{pv}$$

Integrating out the fermions leads to an effective action for the emergent U(1) gauge field

$$\mathcal{S}_a = \int d^2x d\tau \left[\frac{K_1(\boldsymbol{x})}{2} (\boldsymbol{\nabla} \times \boldsymbol{a})^2 + \frac{K_2(\boldsymbol{x})}{2} (\boldsymbol{\nabla} a_\tau - \partial_\tau \boldsymbol{a})^2 - \frac{i\sigma_{xy}(\boldsymbol{x})}{2e^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right] + \int \frac{d^2k d\omega}{8\pi^3} \gamma_k |\omega| \left[\boldsymbol{a}^T(k,\omega)\right]^2$$

Thermal Hall conductivity

$$\mathcal{L}_f = \sum_{v=1,2} \sum_{p=\pm 1} f^\dagger_{pv} \left(\frac{\partial}{\partial \tau} - \mu - i p a_\tau - \frac{(\boldsymbol{\nabla} - i p \boldsymbol{a} - i e \boldsymbol{A}_{\rm em})^2}{2m^*} \right) f_{pv}$$
$$+ v_{\rm dis}(\boldsymbol{r}) f^\dagger_{pv} f_{pv}$$

Integrating out the fermions leads to an effective action for the emergent U(1) gauge field

$$\mathcal{S}_a = \int d^2x d\tau \left[\frac{K_1(\boldsymbol{x})}{2} (\boldsymbol{\nabla} \times \boldsymbol{a})^2 + \frac{K_2(\boldsymbol{x})}{2} (\boldsymbol{\nabla} a_\tau - \partial_\tau \boldsymbol{a})^2 - \frac{i \sigma_{xy}(\boldsymbol{x})}{2e^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right] + \int \frac{d^2k d\omega}{8\pi^3} \gamma_k |\omega| \left[\boldsymbol{a}^T(k,\omega) \right]^2$$

The gauge field contributes a thermal Hall conductivity, κ^1_{xy}, which has the *opposite sign* from the Wiedemann-Franz term determined from σ_{xy}.

Summary: Insulator

- The ground state of the square lattice antiferromagnet is a conventional Neel state.

- In a sufficiently large orbital magnetic field, there is a quantum transition to a "chiral spin liquid" co-existing with conventional Neel order.

- Proximity to this quantum transition can enhance the thermal Hall effect at non-zero temperatures, even though the ground state is conventional.

- Can identify two contributions to the thermal Hall effect: from the fermionic matter (spinons) , and from the emergent gauge field with the opposite sign.

Summary: Pseudogap at non-zero doping

- Model the pseudogap by pockets of fermionic "chargons" carrying gauge charges of an emergent U(1) gauge field

- One contribution to the thermal Hall effect is the Wiedemann-Franz law of a disordered Fermi liquid

- There is an additional emergent gauge field contribution which has the opposite sign.