Gauge theory for the cuprates across optimal doping

Topological Aspects of Quantum Matter
Tata Institute of Fundamental Research, Mumbai
December 20, 2018

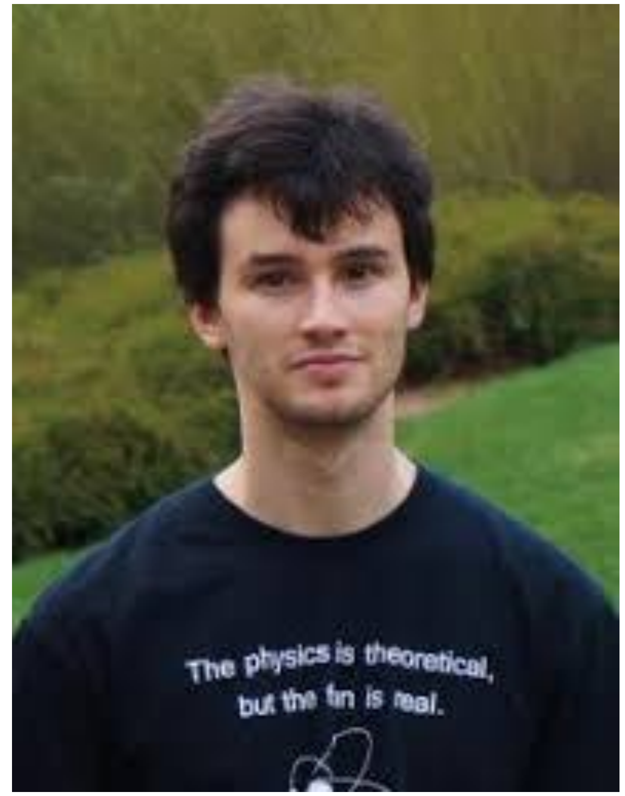
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS

HARVARD

Mathias Scheurer

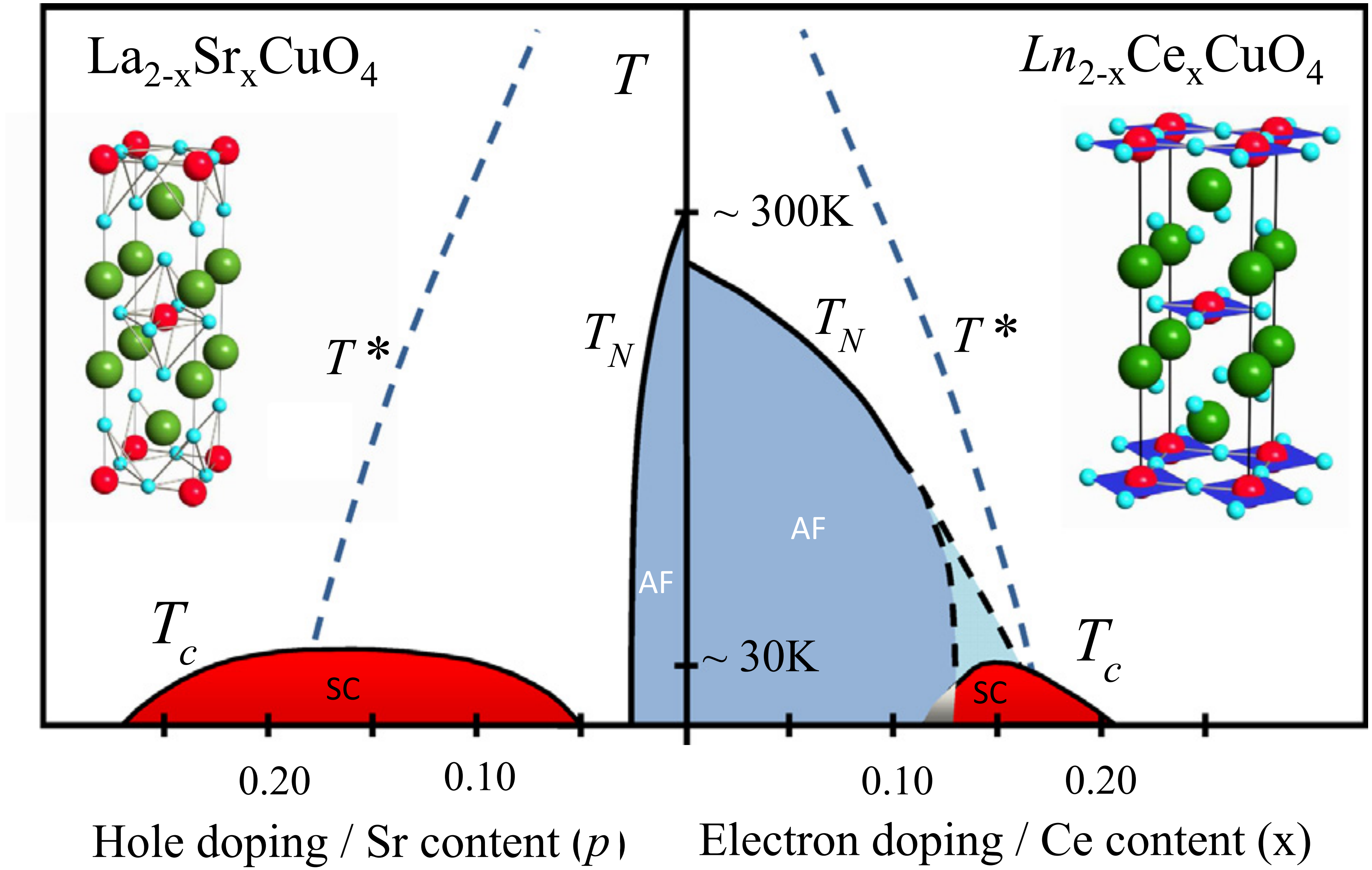


Grigory Tarnopolsky

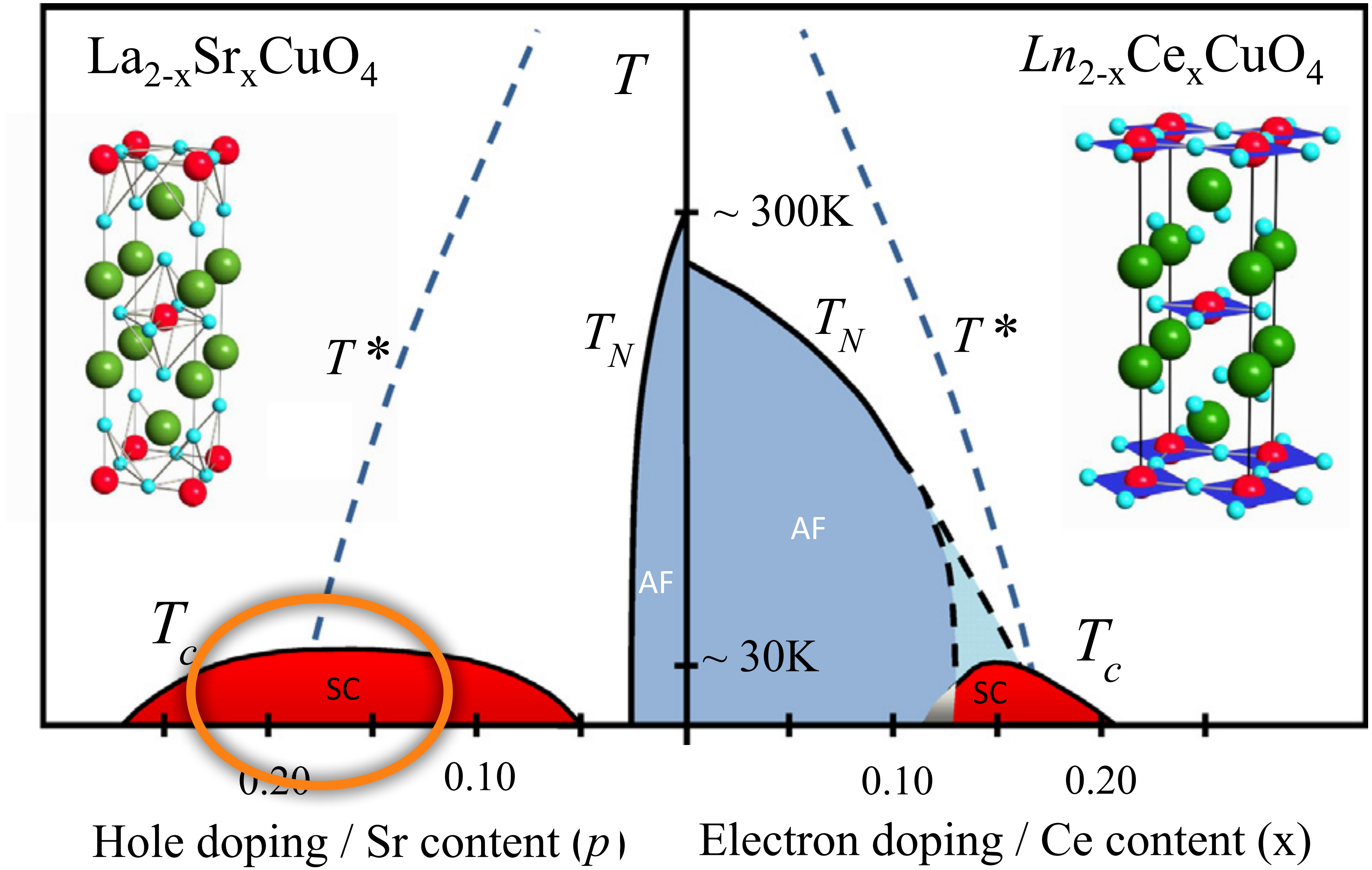
Harley Scammell

arXiv:1811.04930

PHYSICS
HARVARD



La2-xSrxCuO4
Ln2-xCexCuO4
T
~ 300K
~ 30K
T*
T*
TN
TN
AF
AF
Tc
Tc
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

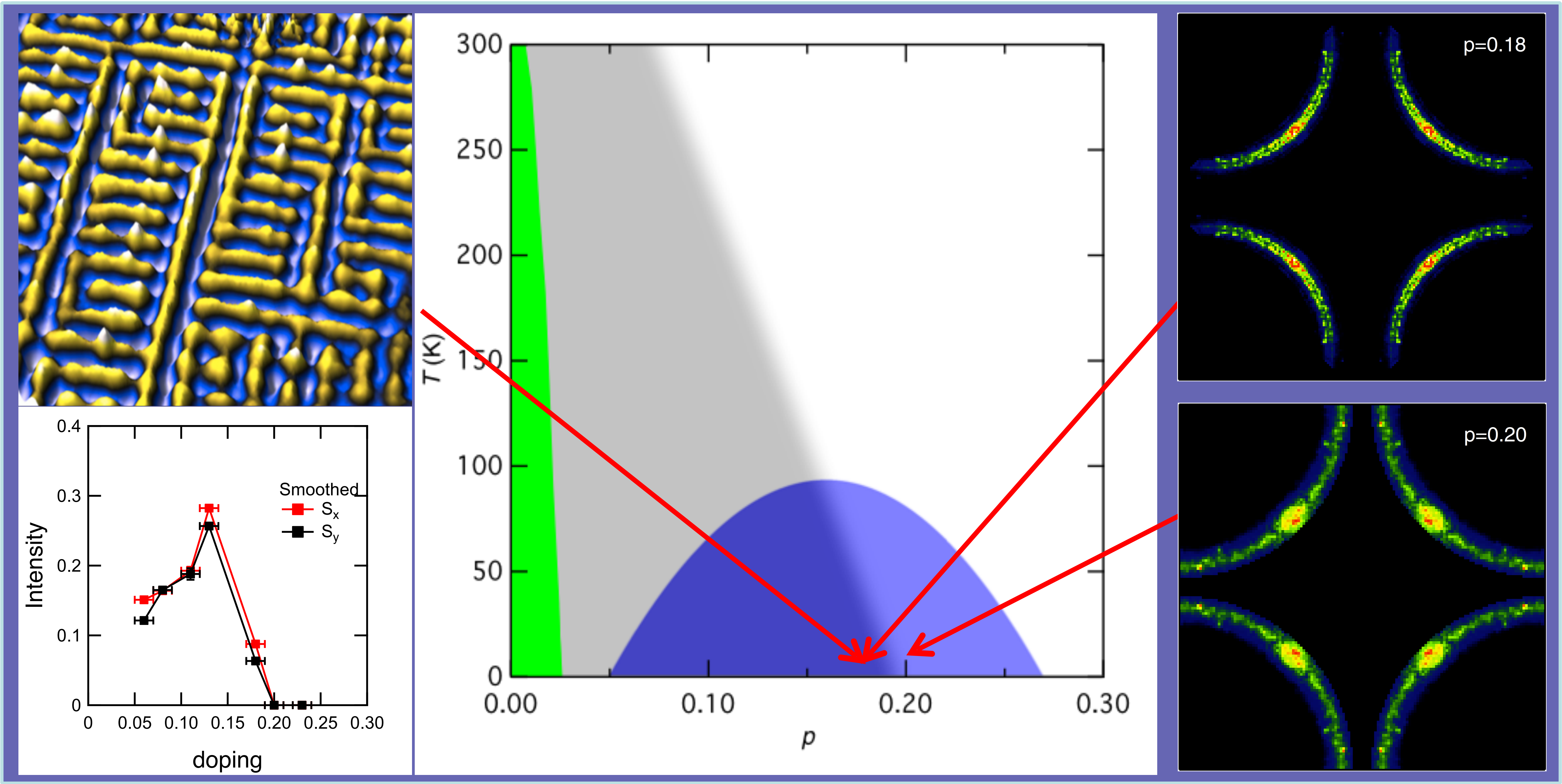


$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
T^*
T^*
T_N
T_N
T_c
T_c
~ 300K
~ 30K
AF
AF
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

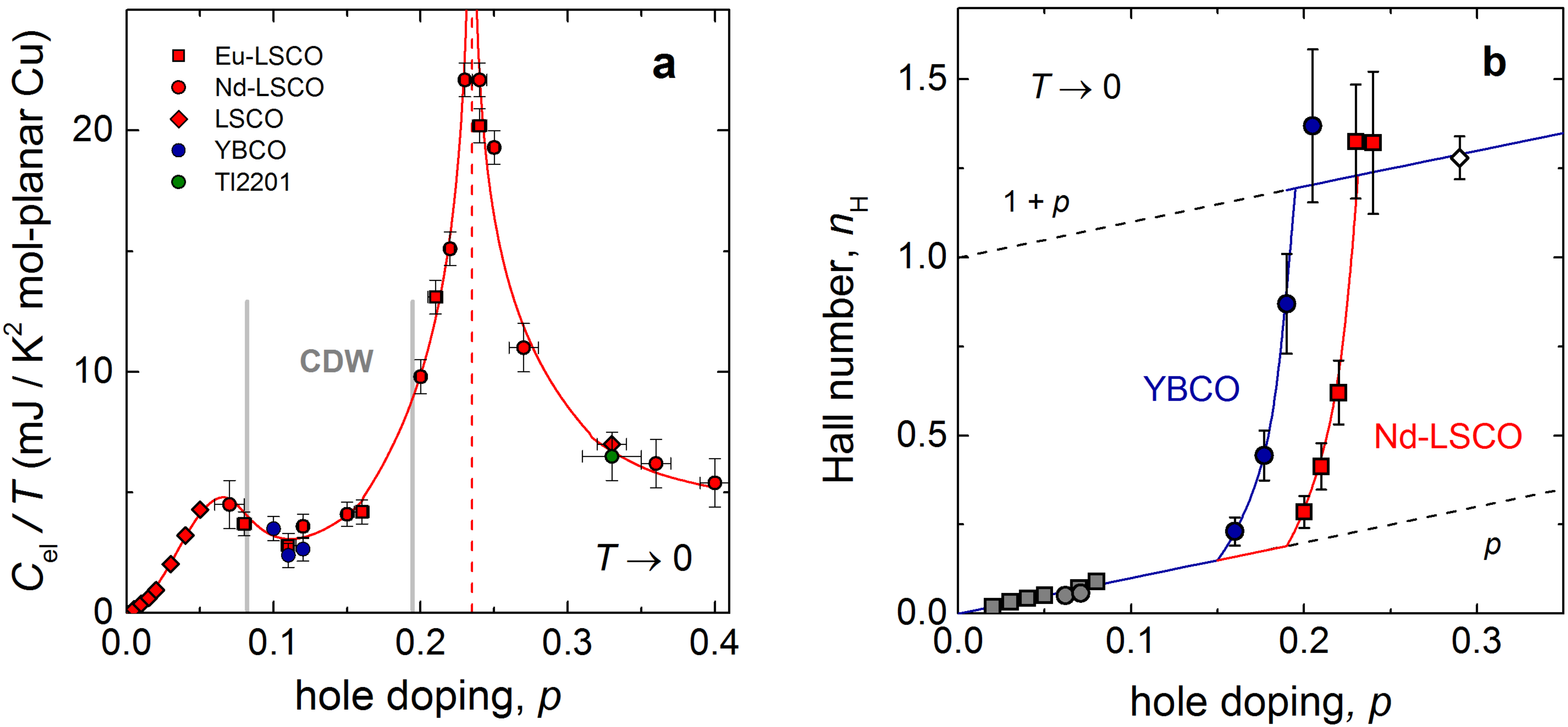
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

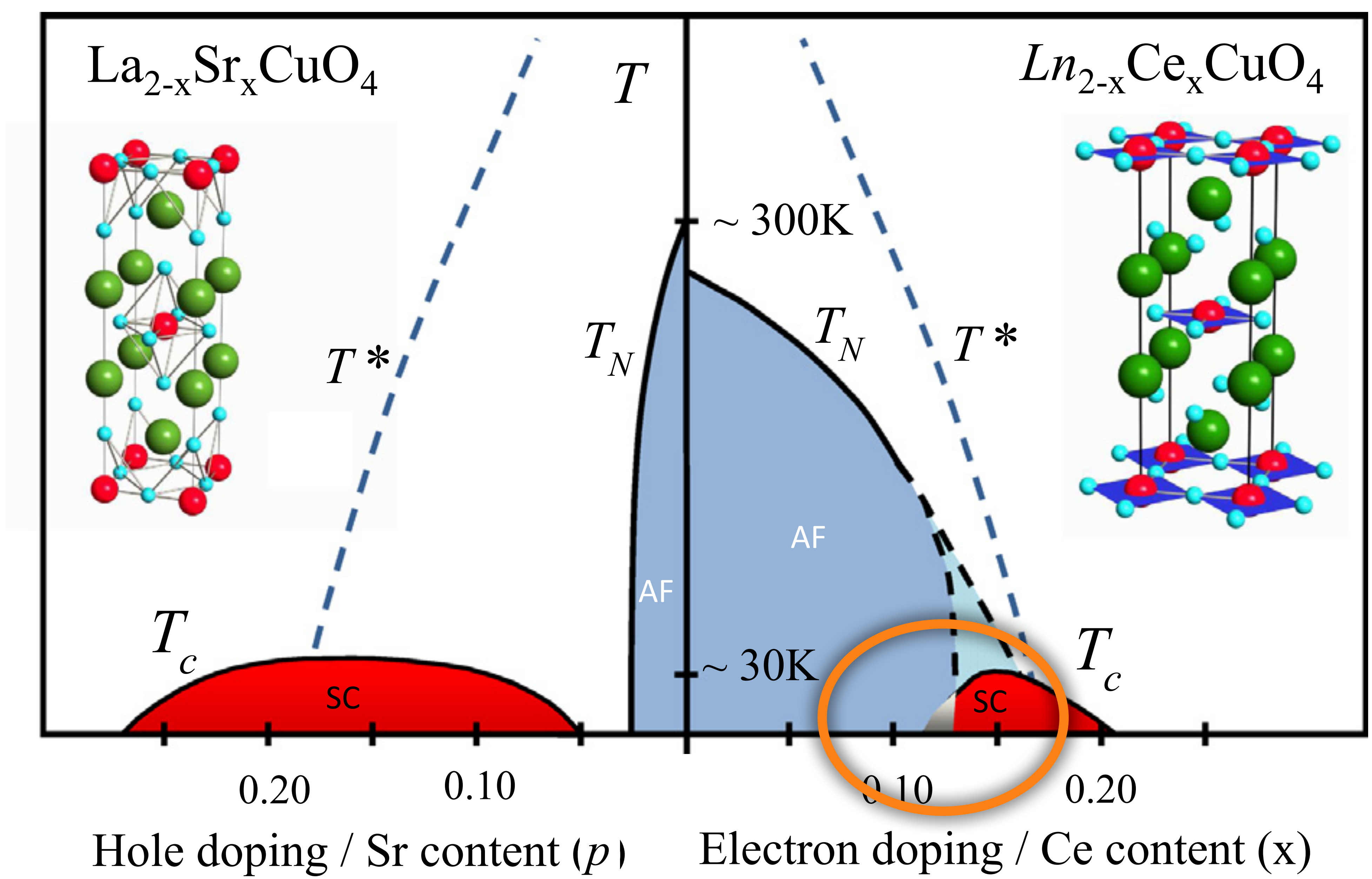


Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507





$La_{2-x}Sr_xCuO_4$
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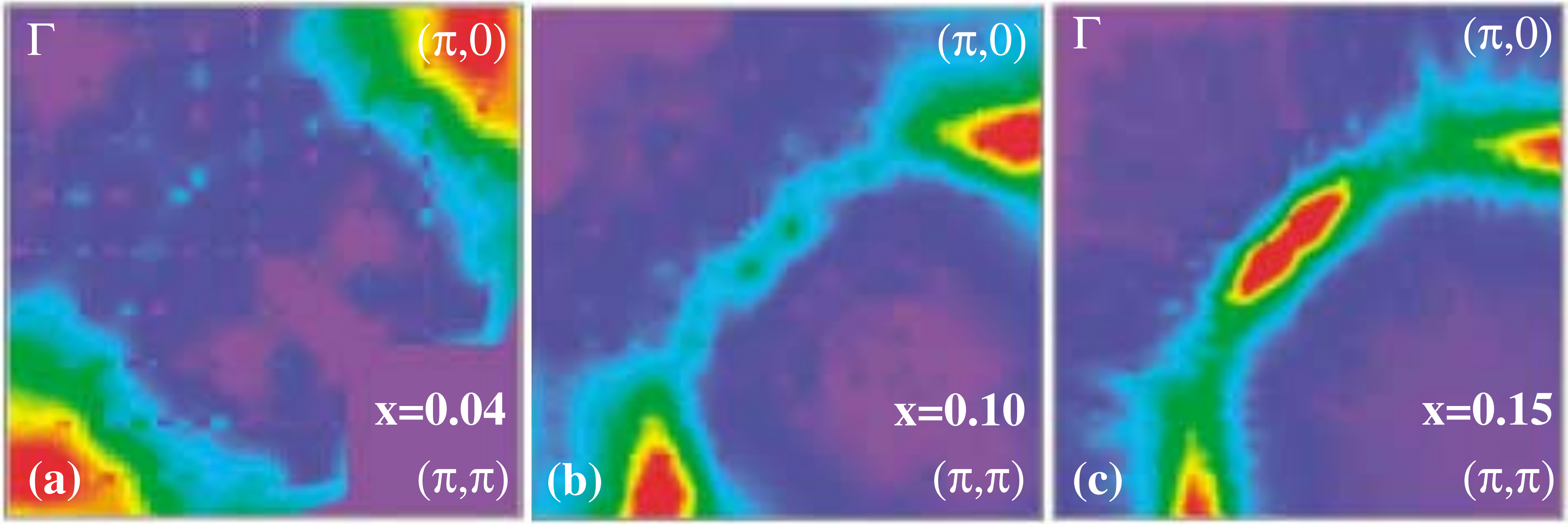
~ 30K
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Electron doped cuprates

$Nd_{2-x}Ce_xCuO_{4\pm\delta}$

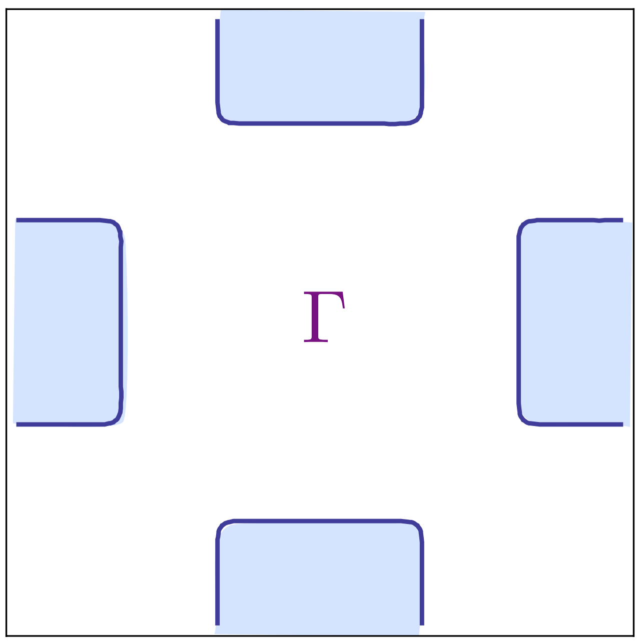
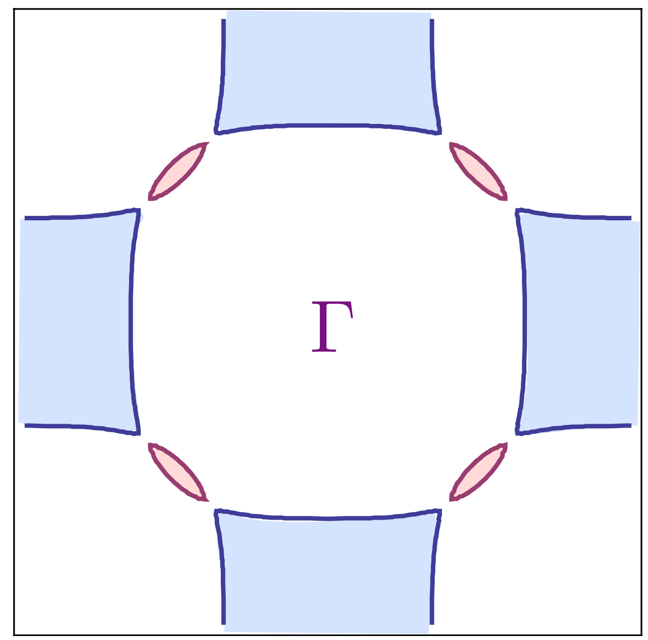
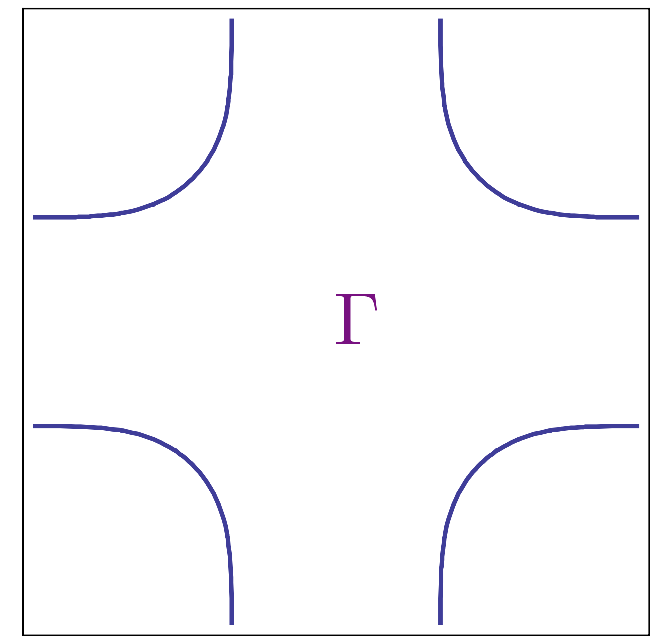


Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

Phys. Rev. Lett. **88**, 257001 (2002)

Square lattice Hubbard model with electron doping

$\langle \Phi^a \rangle \neq 0$ and large	$\langle \Phi^a \rangle \neq 0$ and small	$\langle \Phi^a \rangle = 0$
		
Metal with electron pockets	Metal with electron and hole pockets	Metal with "large" Fermi surface

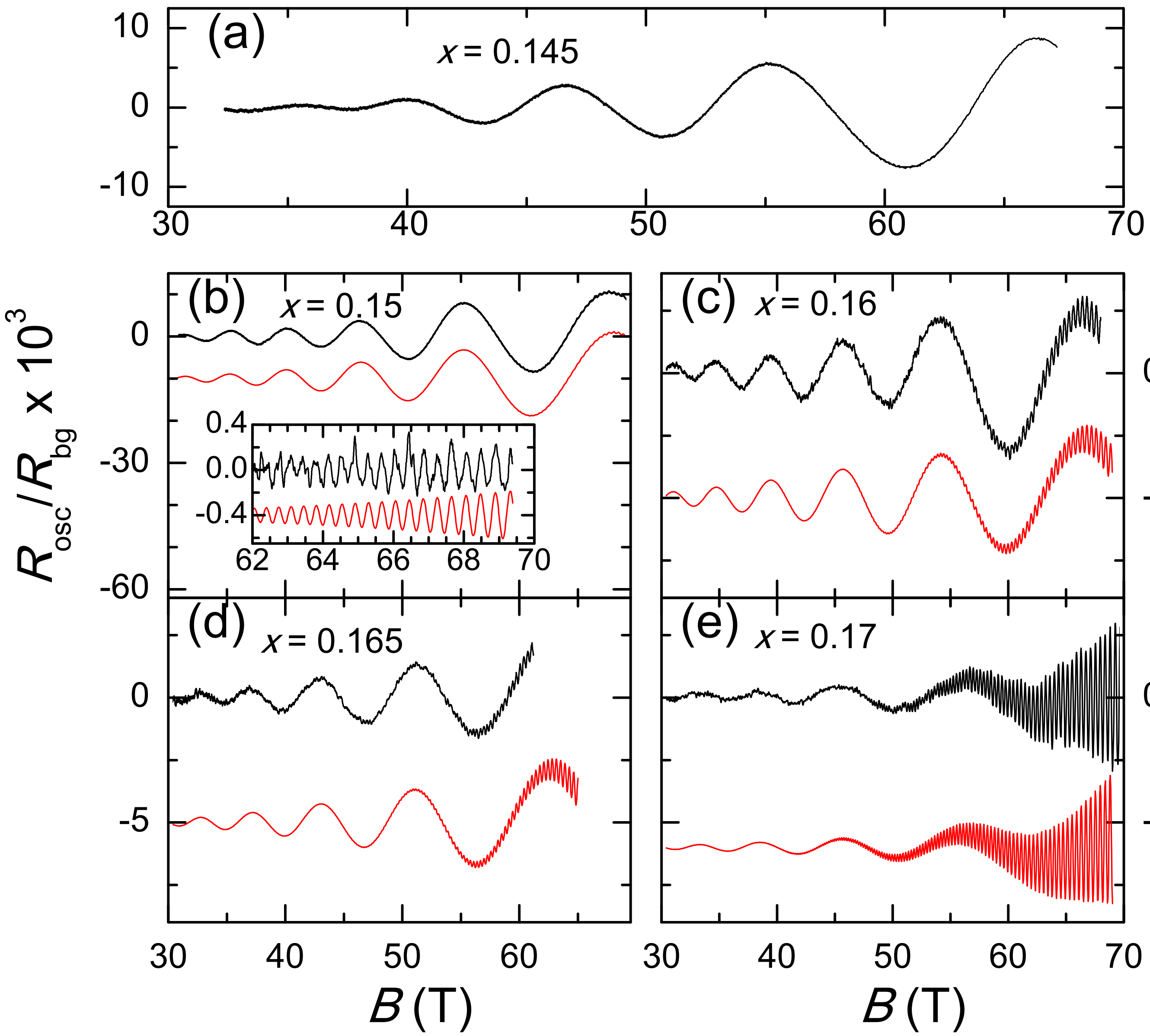
s

$\Phi^a \Rightarrow$ Antiferromagnetism at (π, π)

PHYSICAL REVIEW B **92**, 094501 (2015)

Correlation between Fermi surface transformations and superconductivity in the electron-doped high-T_c superconductor $Nd_{2-x}Ce_xCuO_4$

T. Helm,[1,*] M. V. Kartsovnik,[1,†] C. Proust,[2] B. Vignolle,[2] C. Putzke,[3,‡] E. Kampert,[3] I. Sheikin,[4] E.-S. Choi,[5] J. S. Brooks,[5] N. Bittner,[1,§] W. Biberacher,[1] A. Erb,[1,6] J. Wosnitza,[3] and R. Gross[1,6,‖]



- Quantum oscillations show the presence of small hole pockets up to a doping $x = 0.175$ although anti-ferromagnetism disappears near $x = 0.14$

arXiv:1811.04992

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

- New photoemission measurements at zero magnetic field show Fermi surfaces in quantitative agreement with quantum oscillation measurements.

- The energy gap between the electron and hole pockets collapses near $x = 0.17$ like an order parameter.

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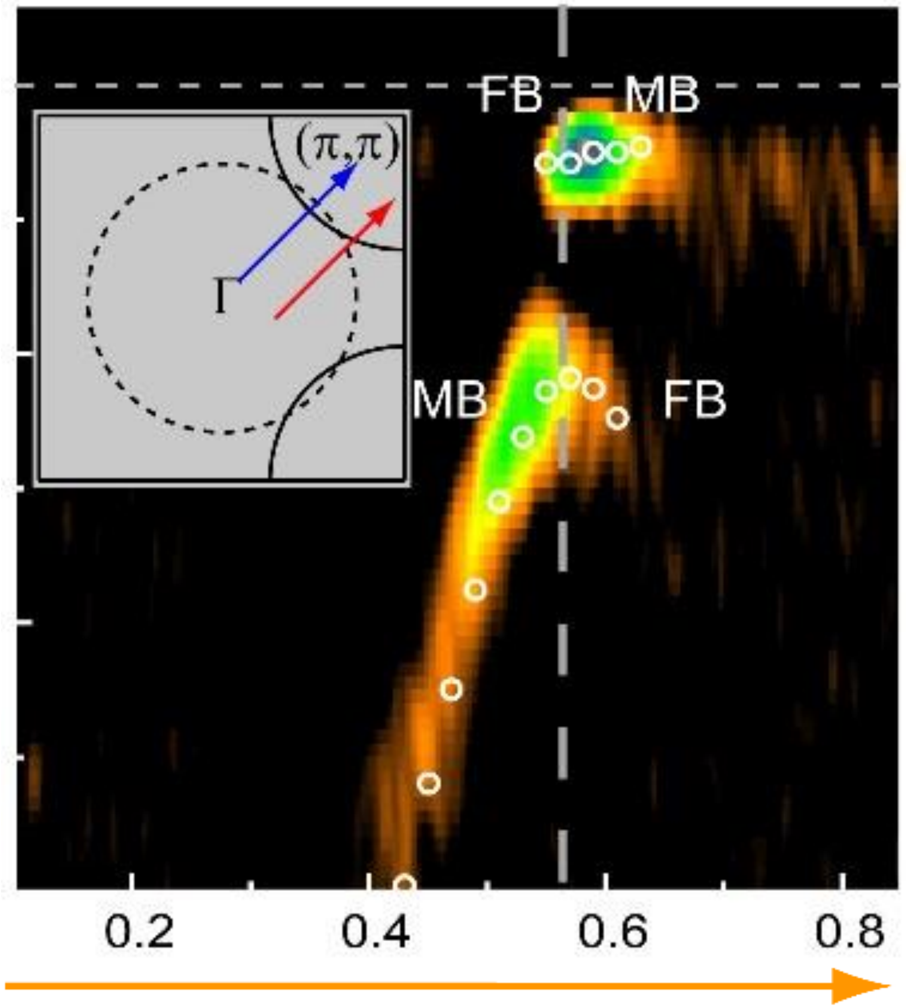
- "The totality of the data points to a mysterious order between $x = 0.14$ and $x = 0.17$, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin."

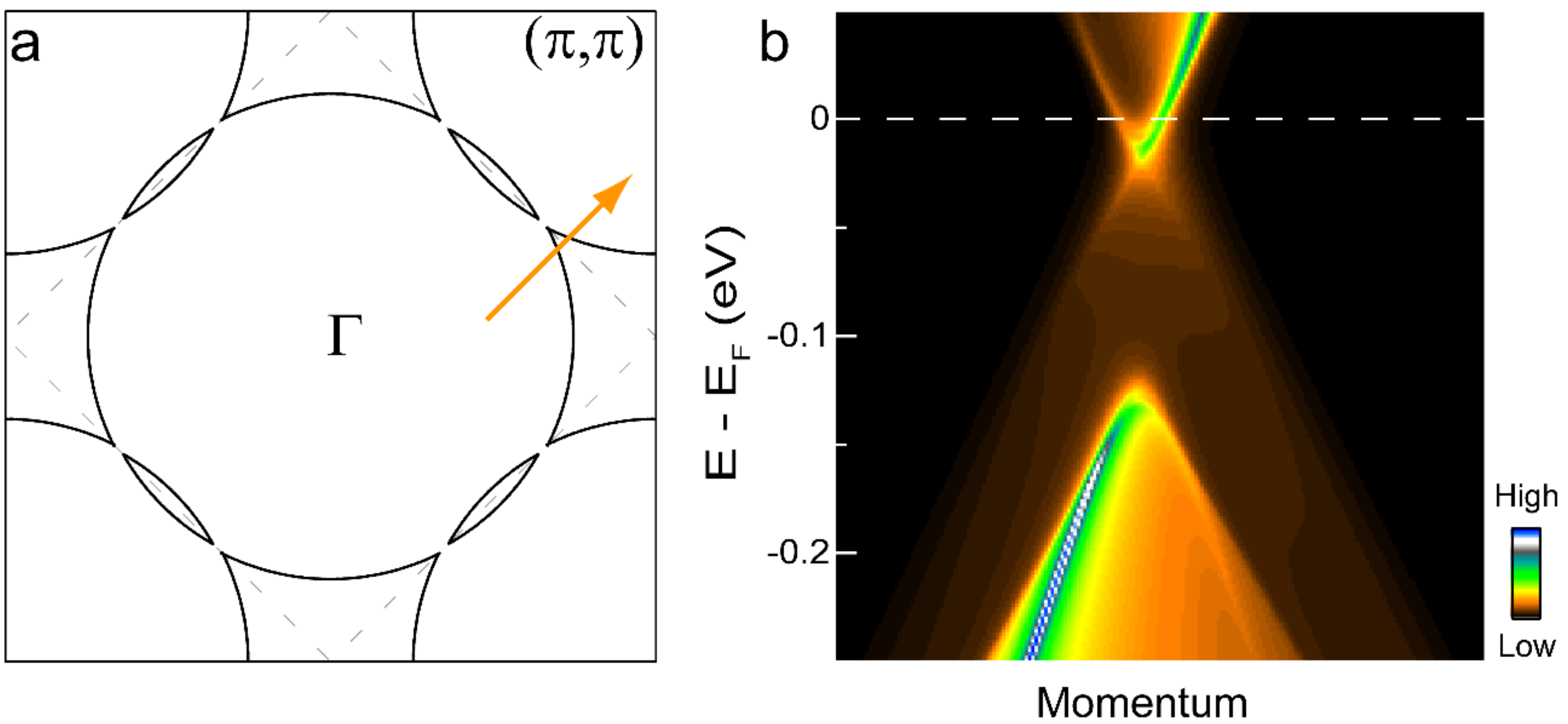
Mathias Scheurer

S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)



Junfeng He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen, arXiv:1811.04992



Square lattice Hubbard model with electron doping

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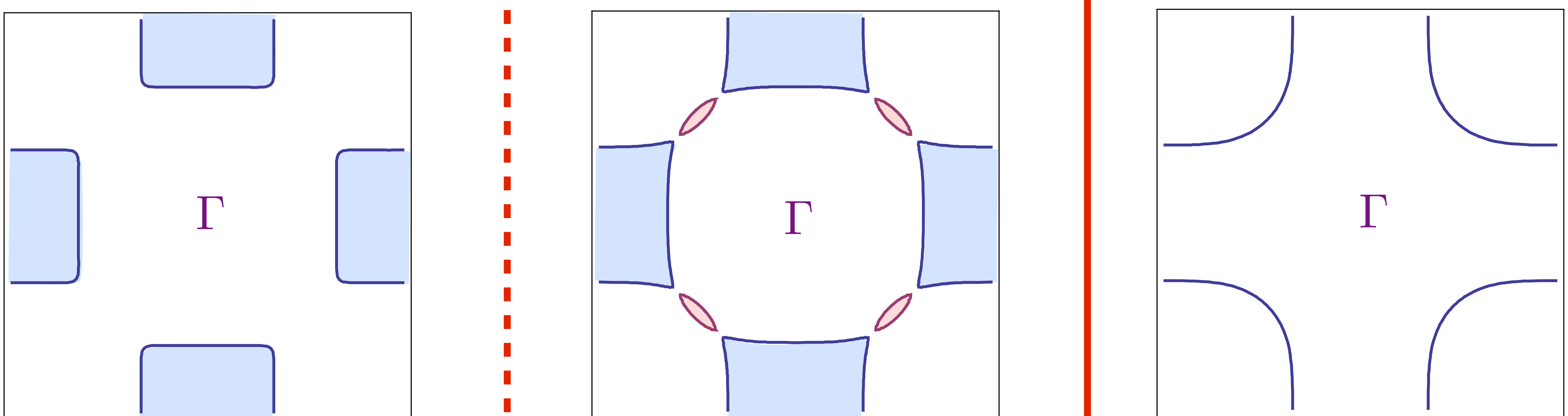

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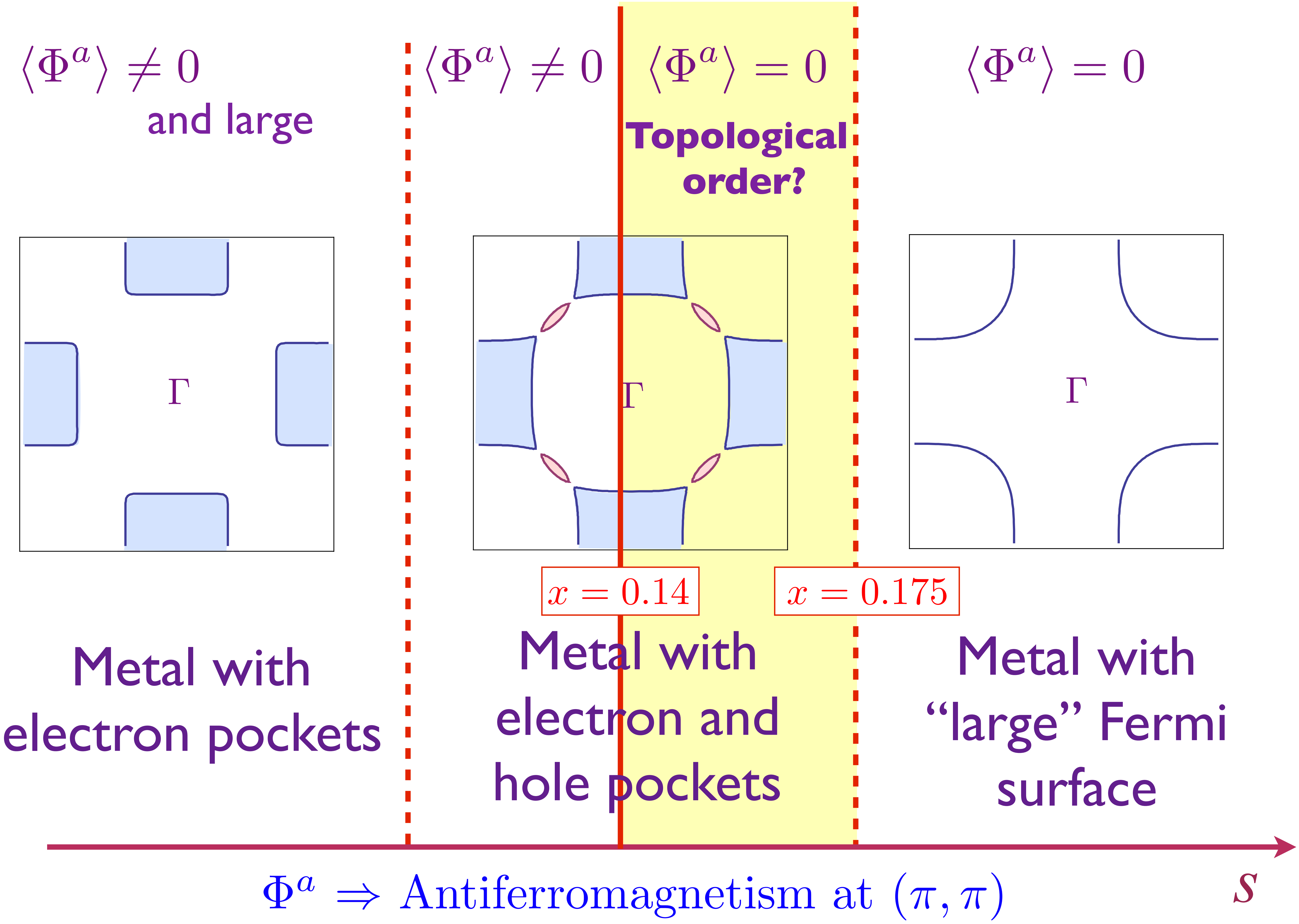

1. Simple models of metals with intrinsic topological order

*ACL and FL**

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Higgs-confinement transition to a Fermi liquid

SU(2) gauge theory with N_h adjoint Higgs fields

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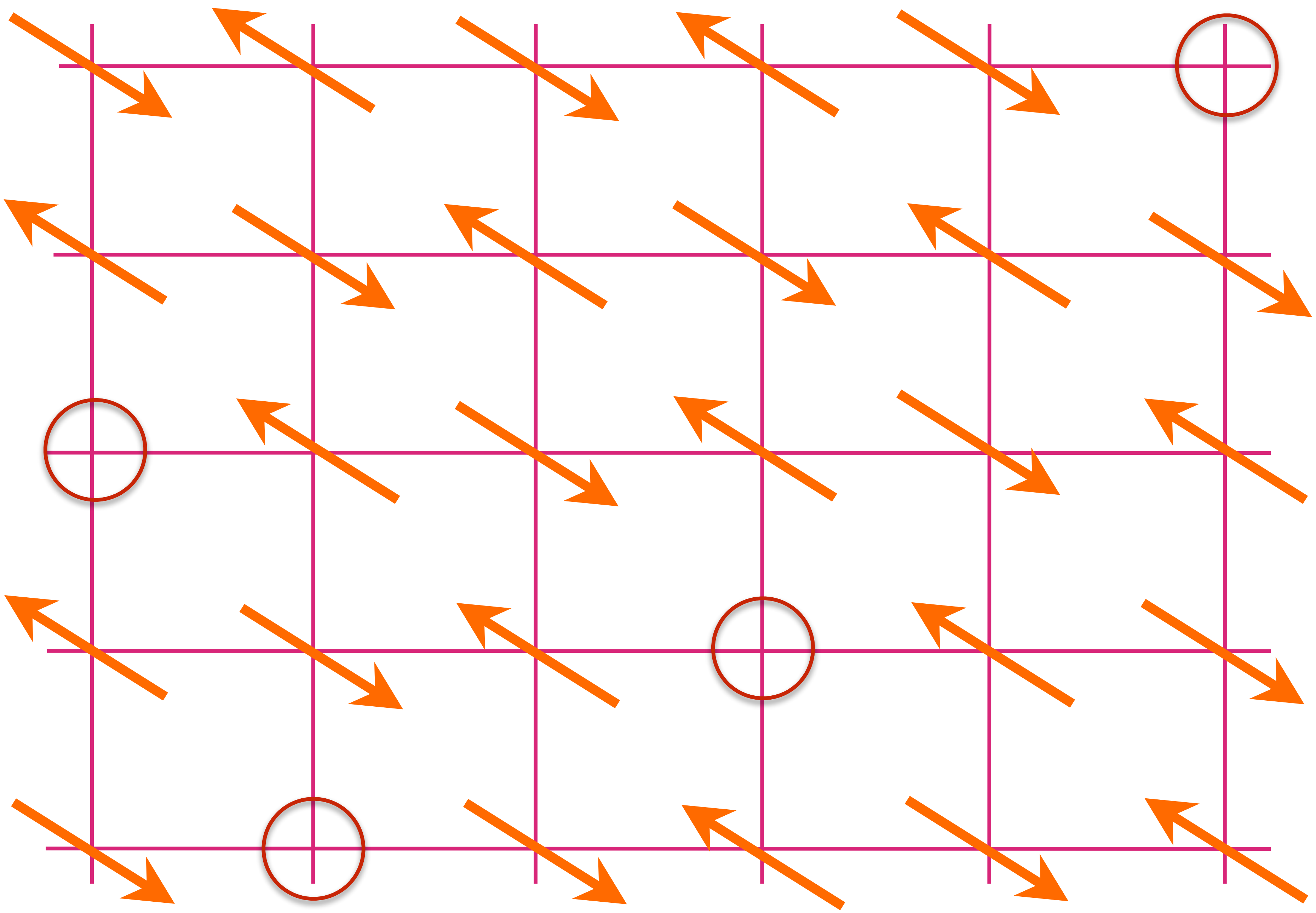
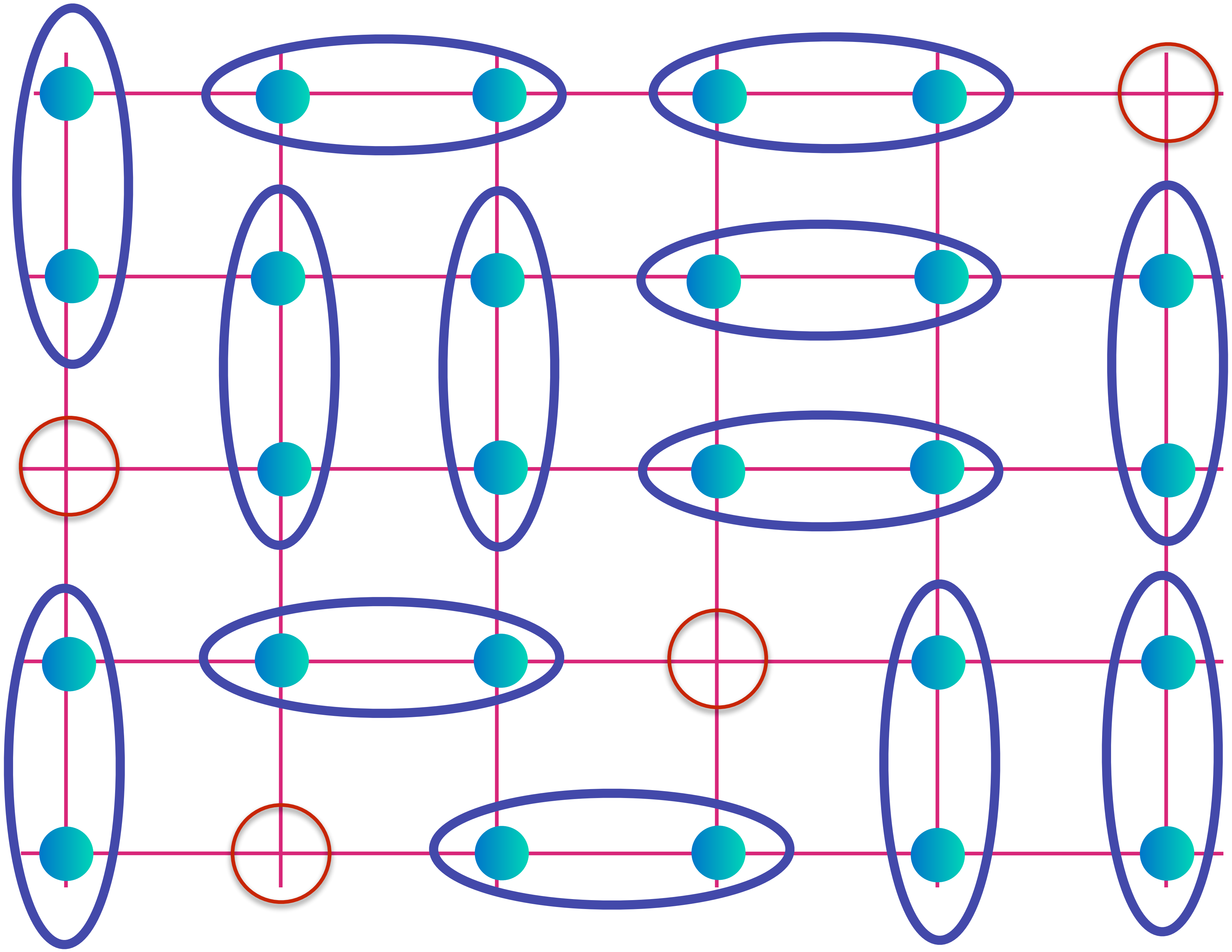
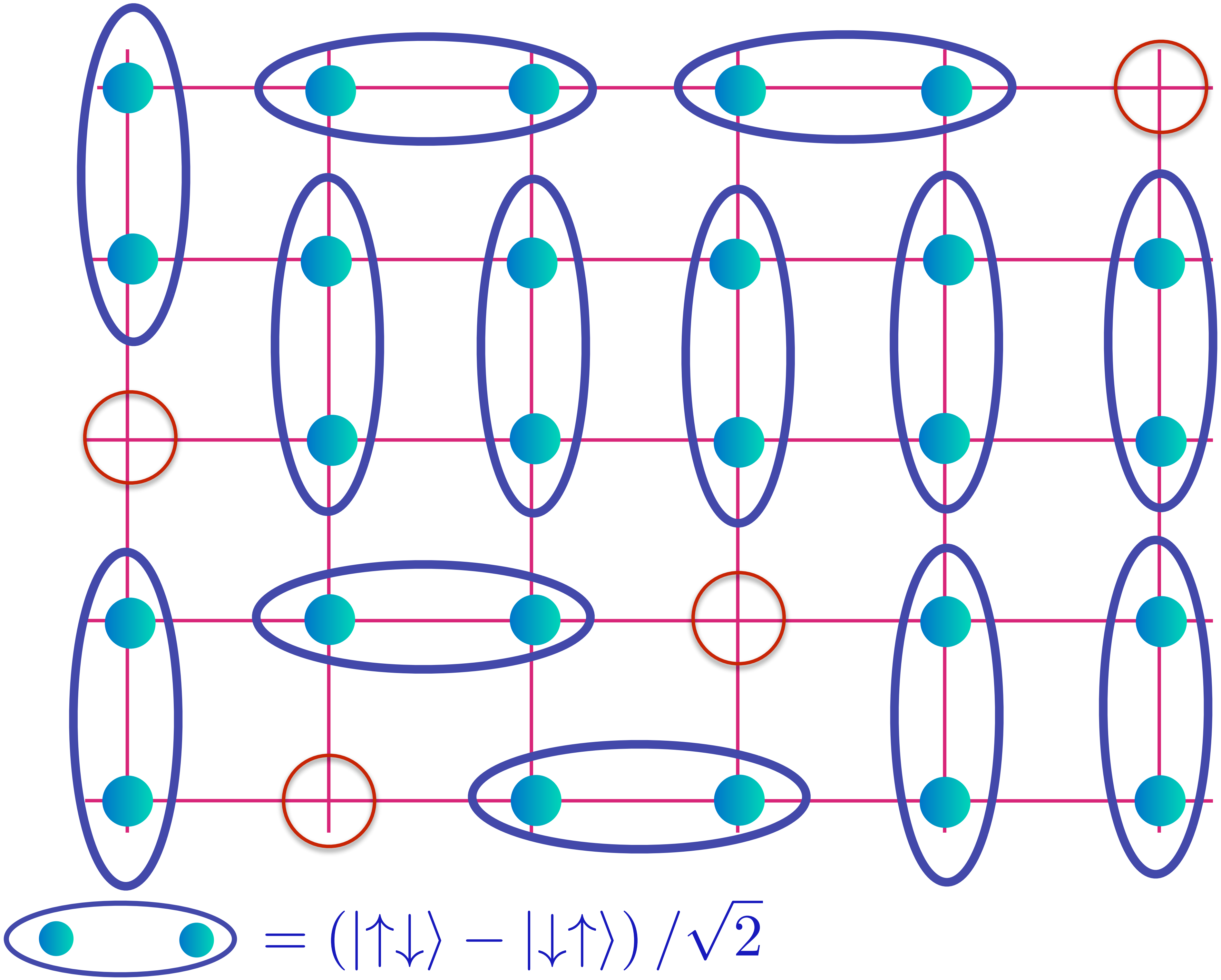


Anti-ferromagnet with p holes per square



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),



ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

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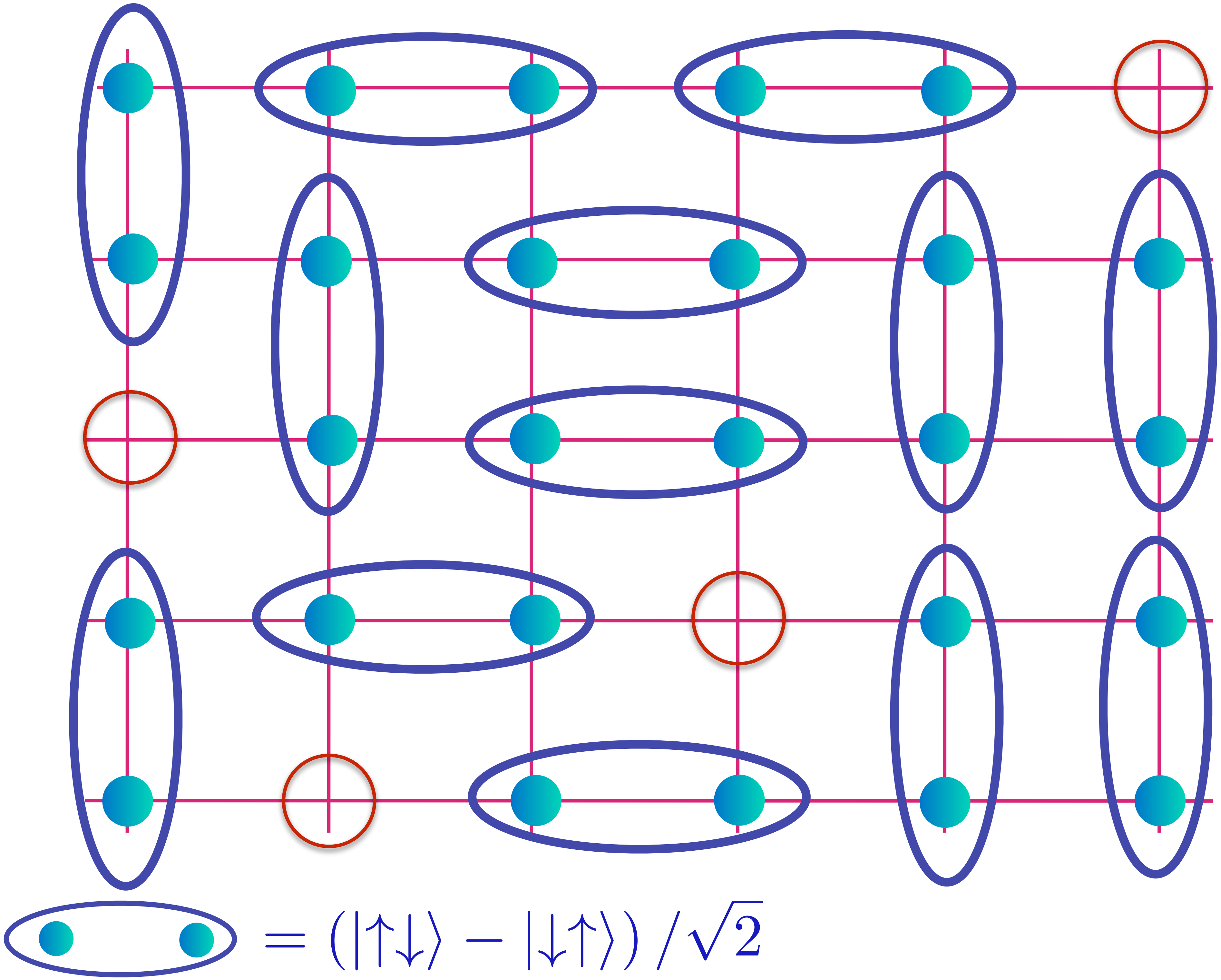

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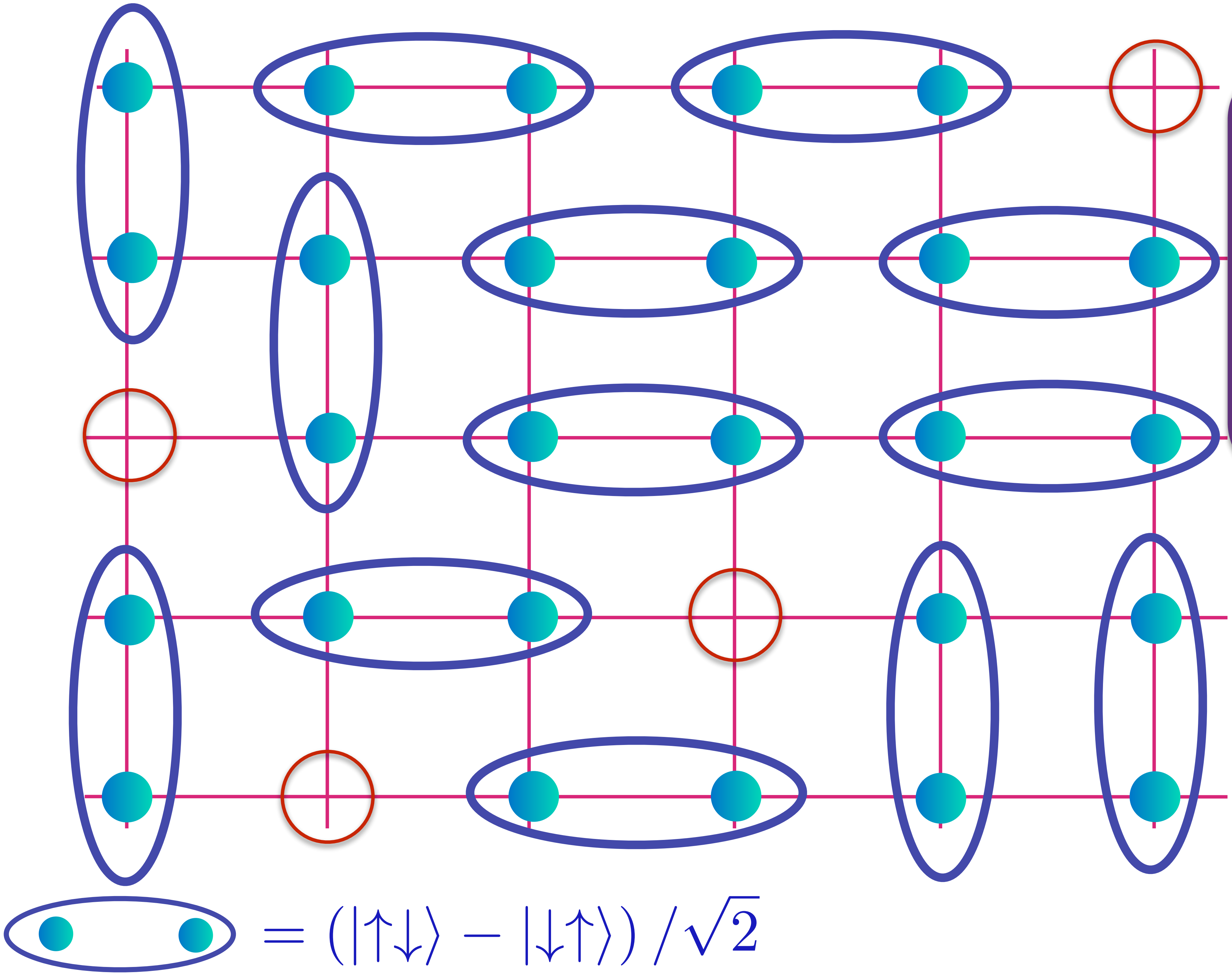

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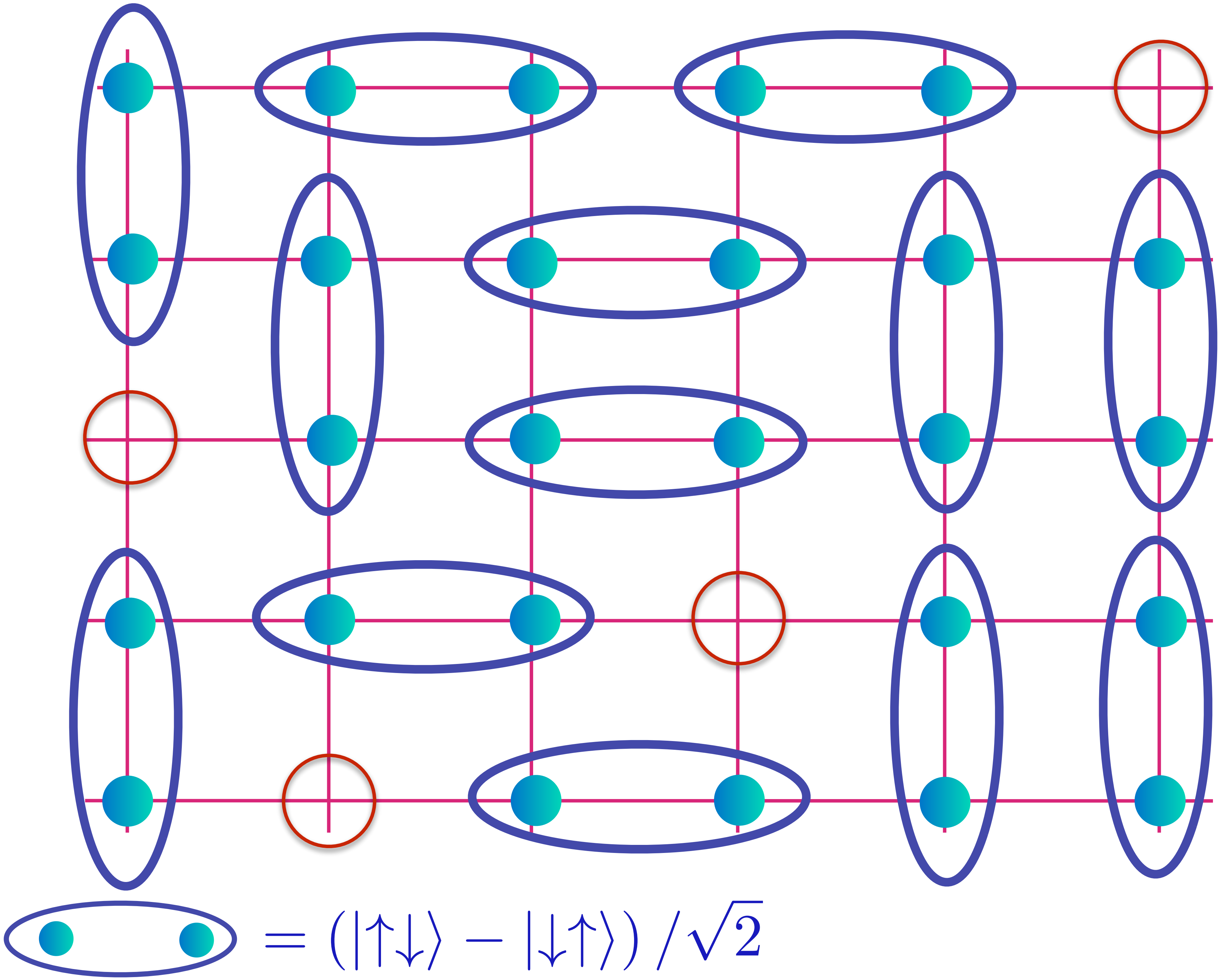

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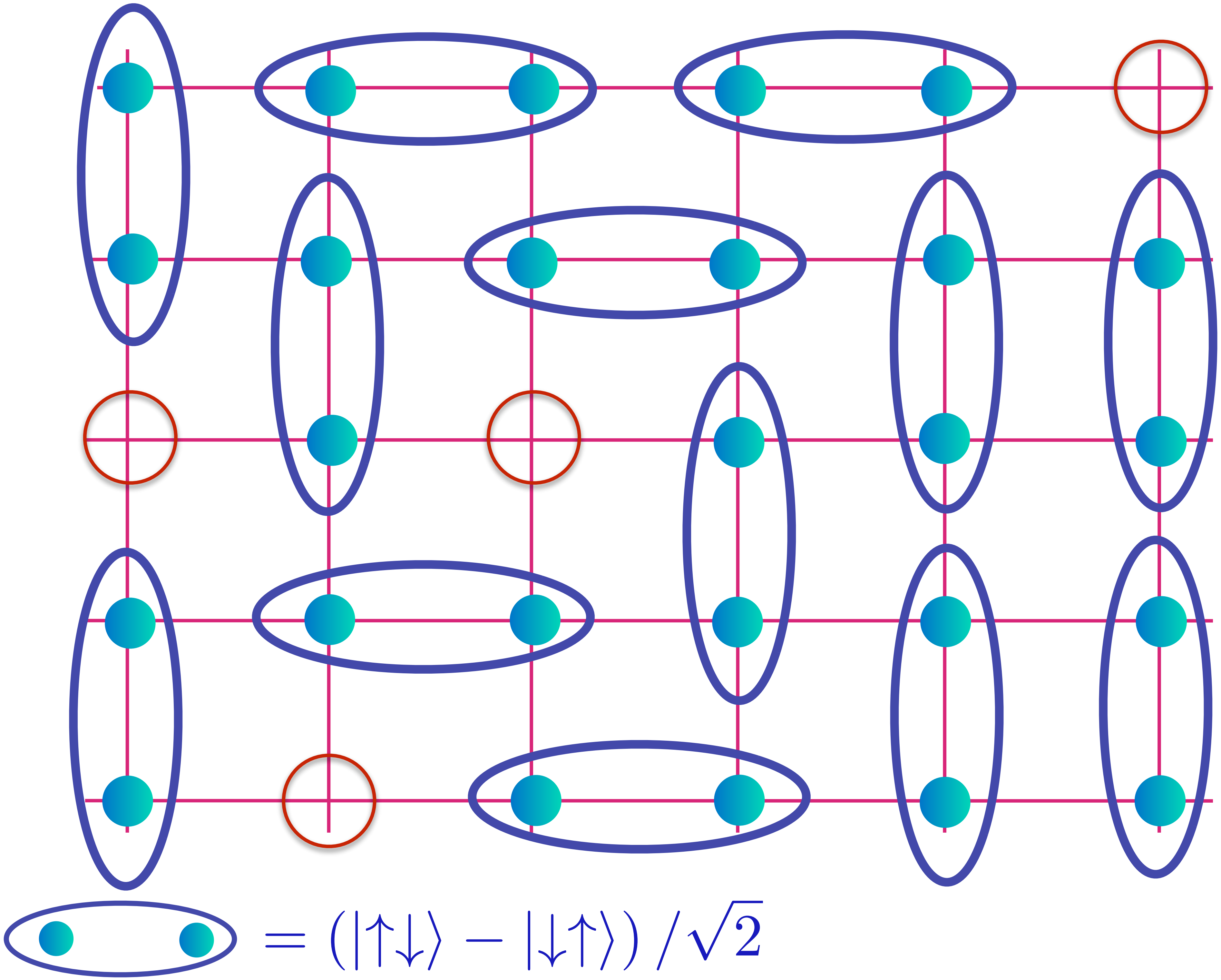

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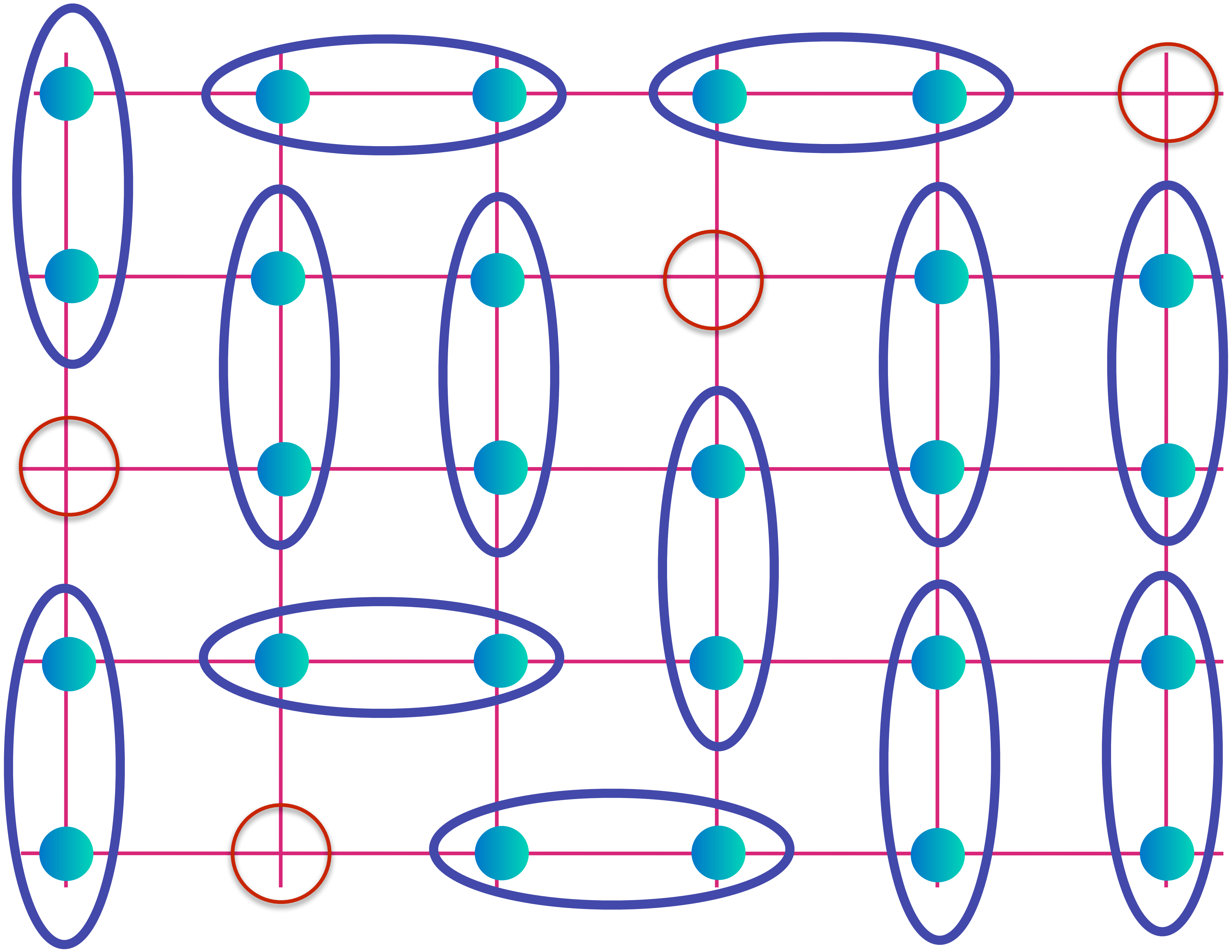
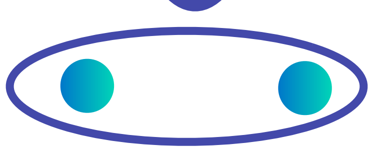
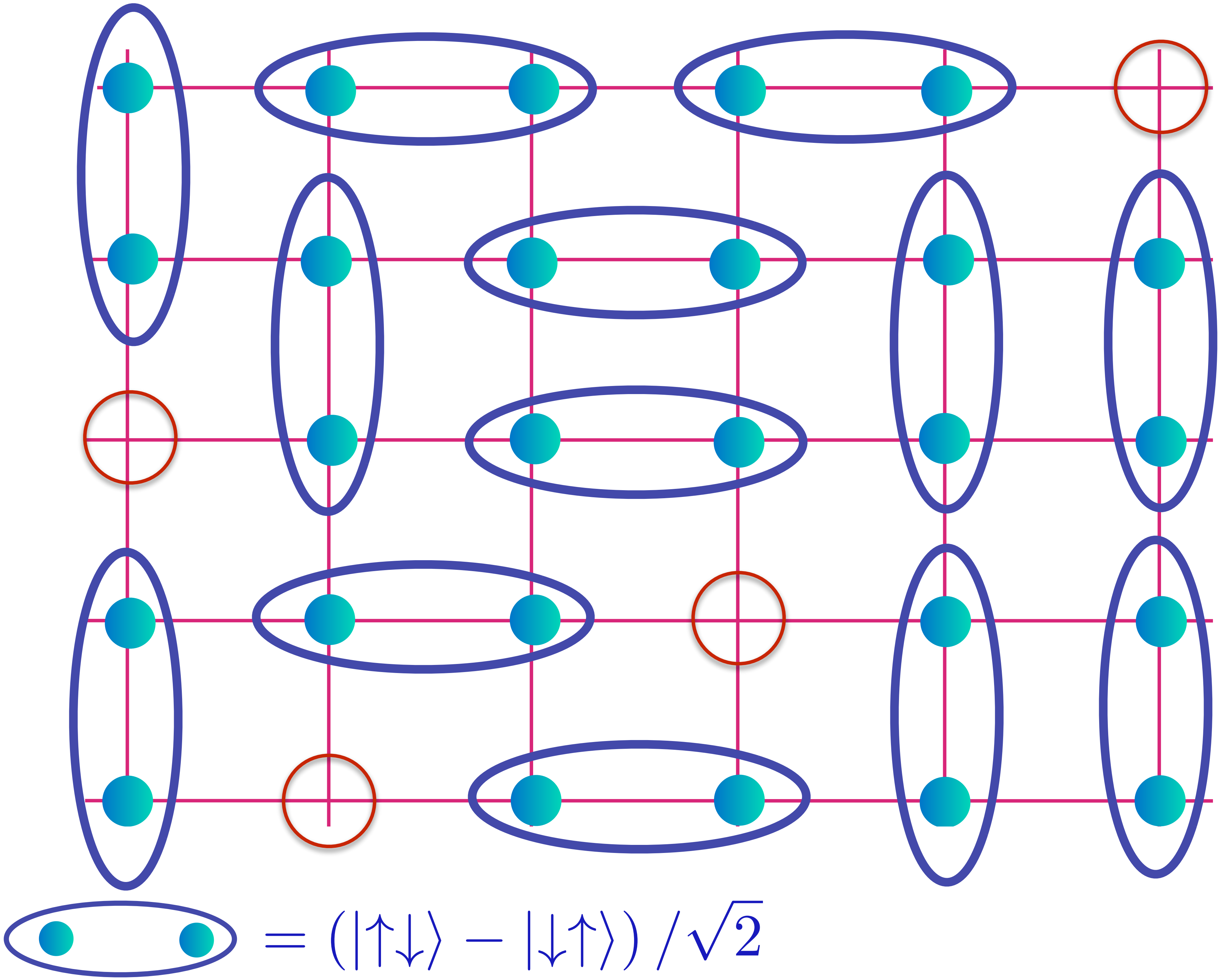


ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers),

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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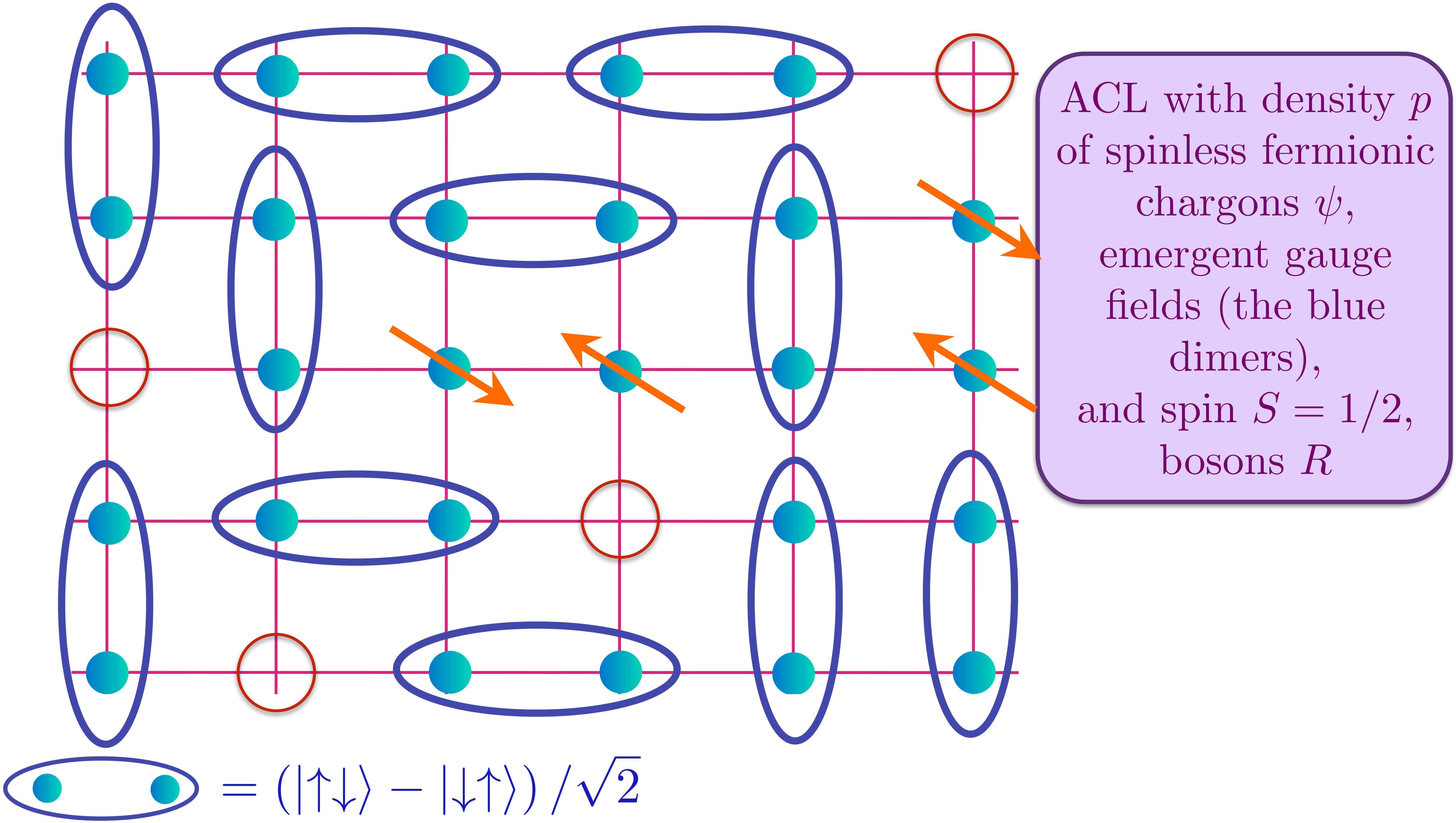
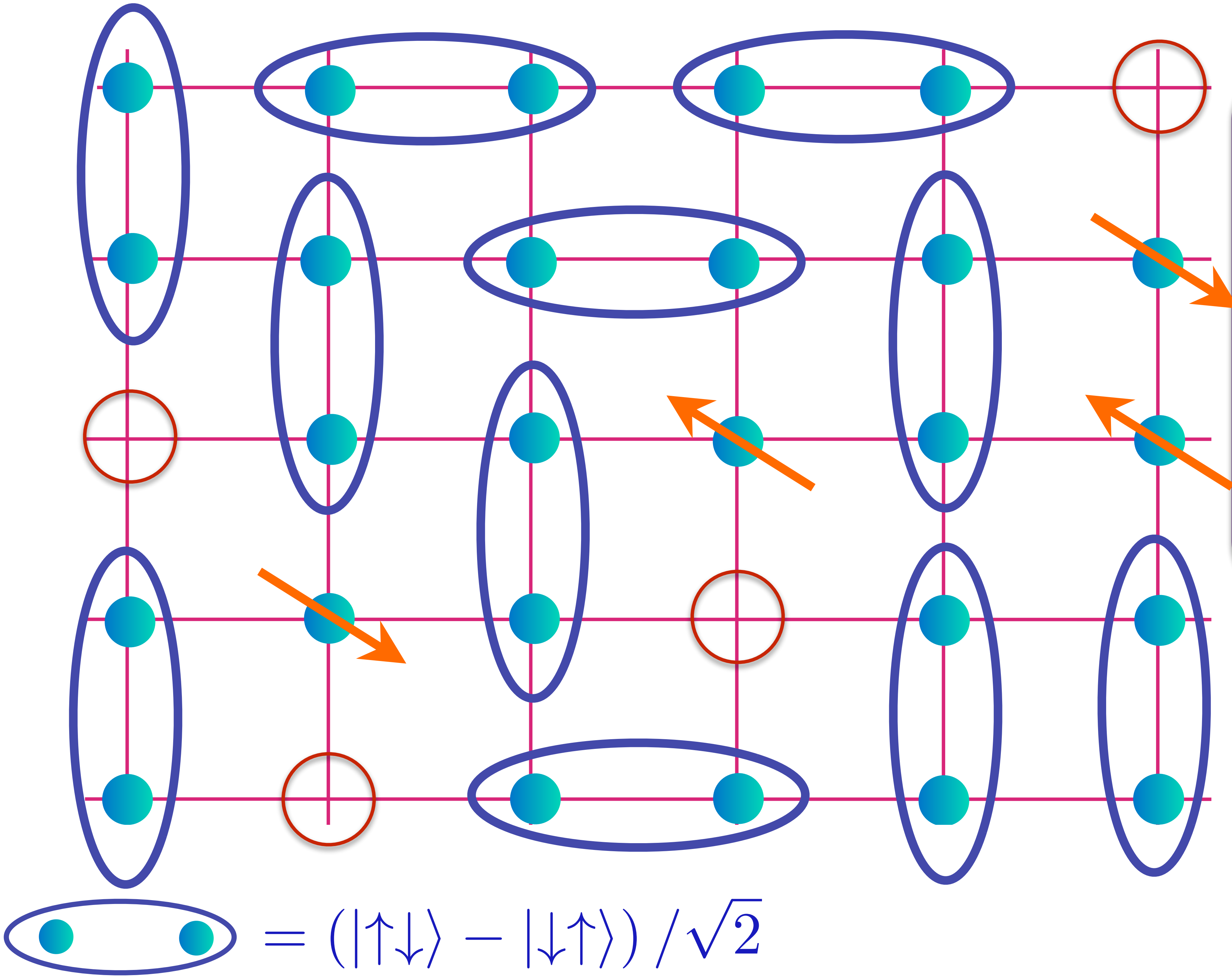

ACL with density p of spinless fermionic chargons ψ, emergent gauge fields (the blue dimers), and spin $S = 1/2$, bosons R
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



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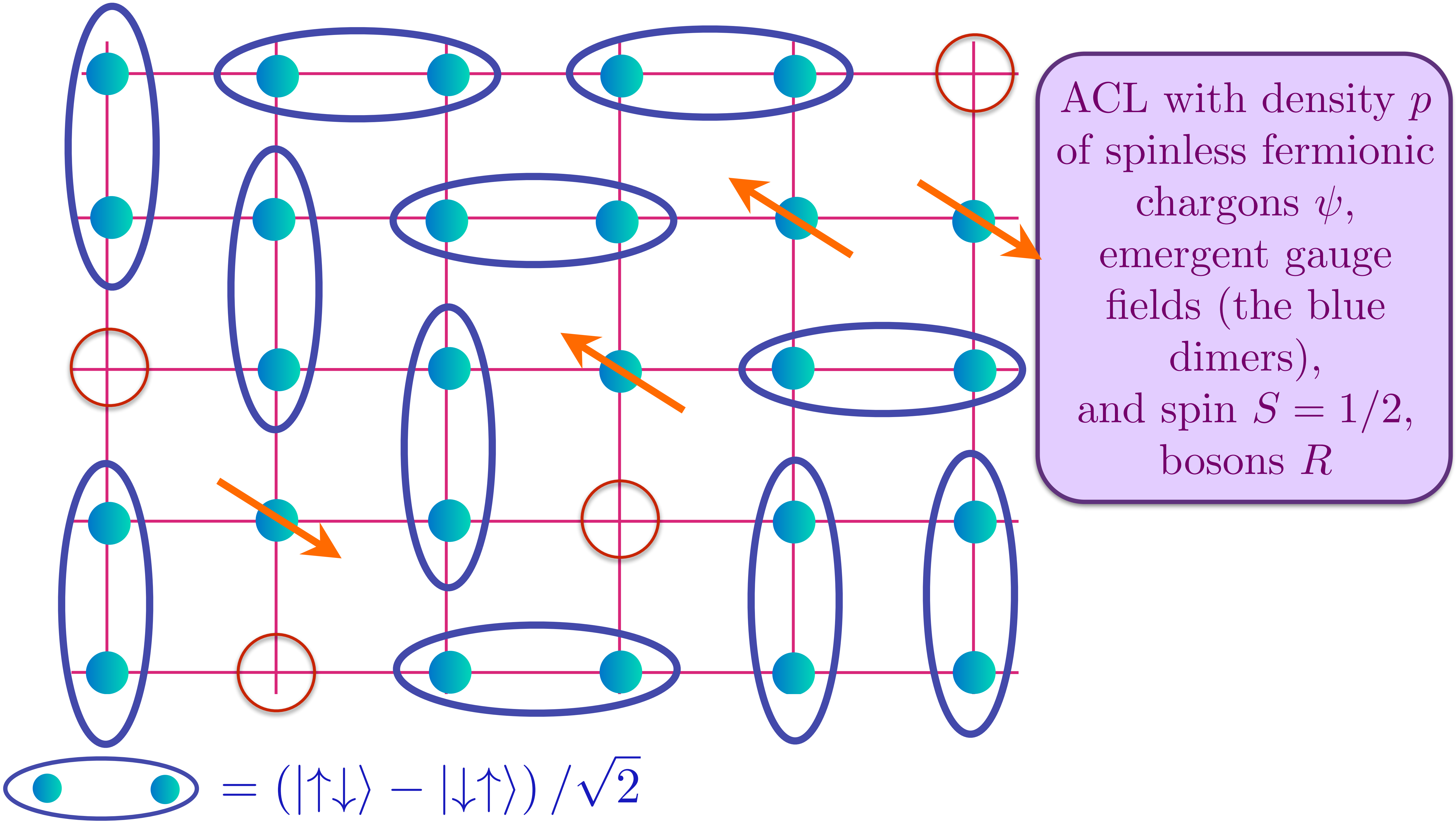
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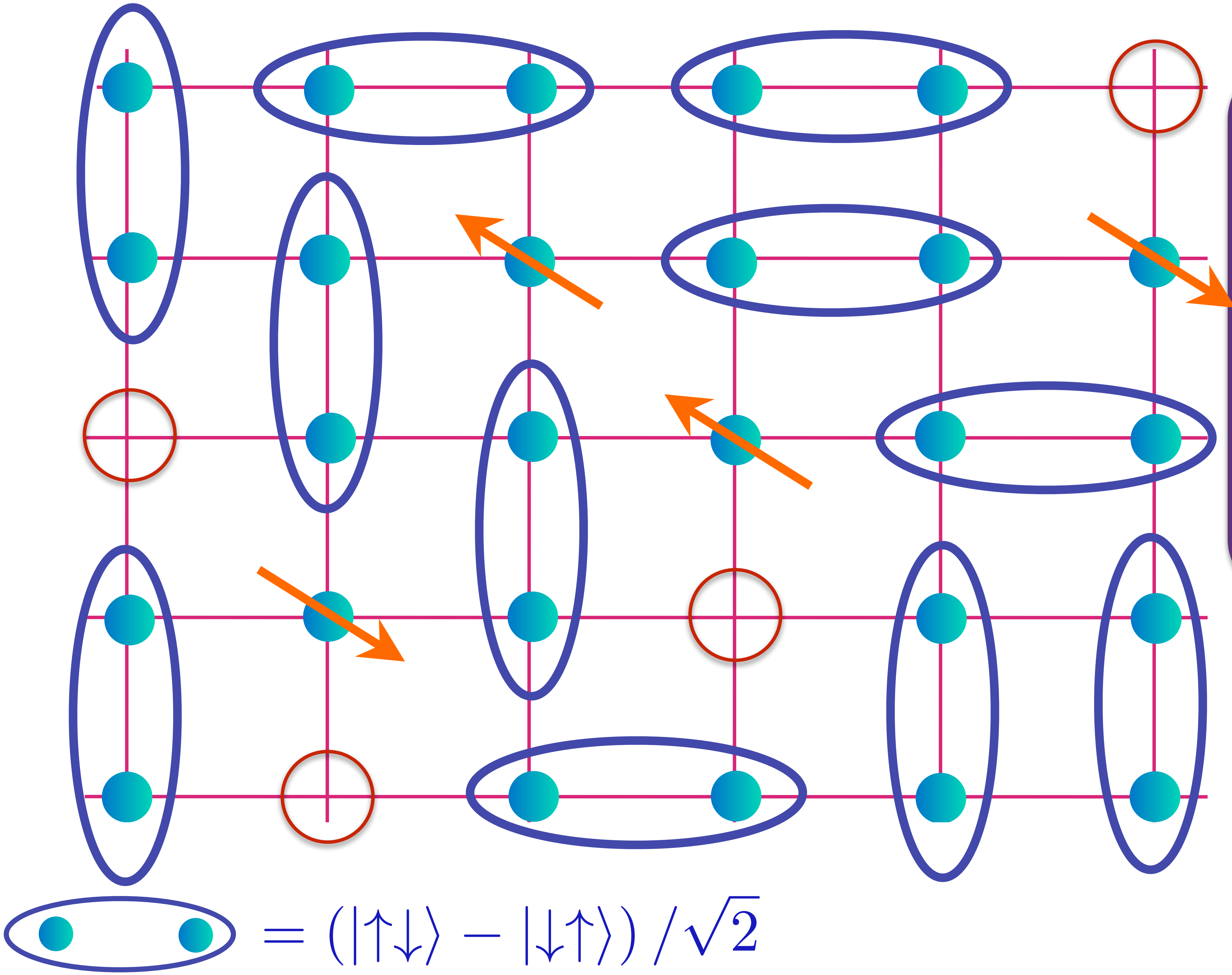

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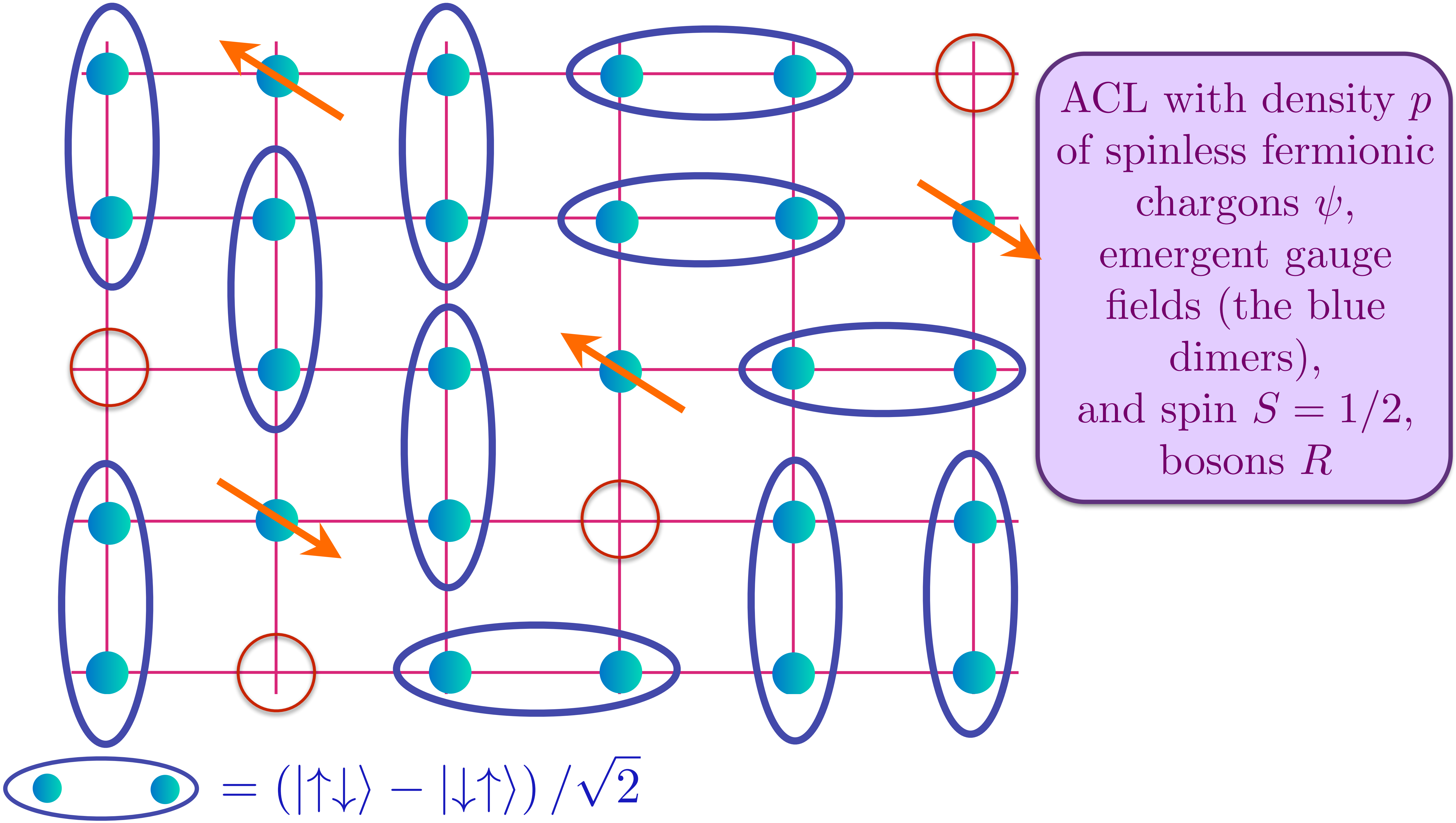

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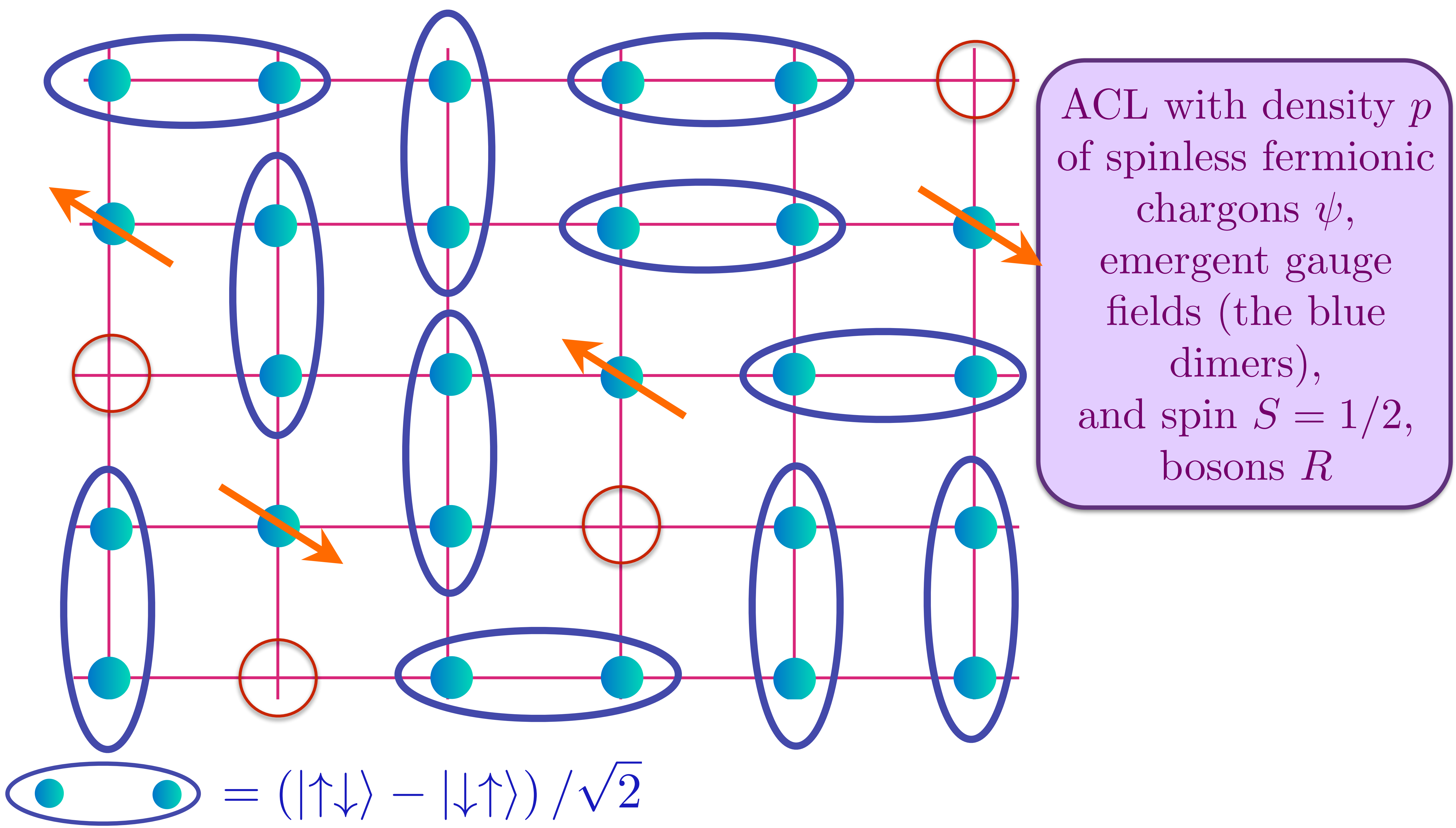

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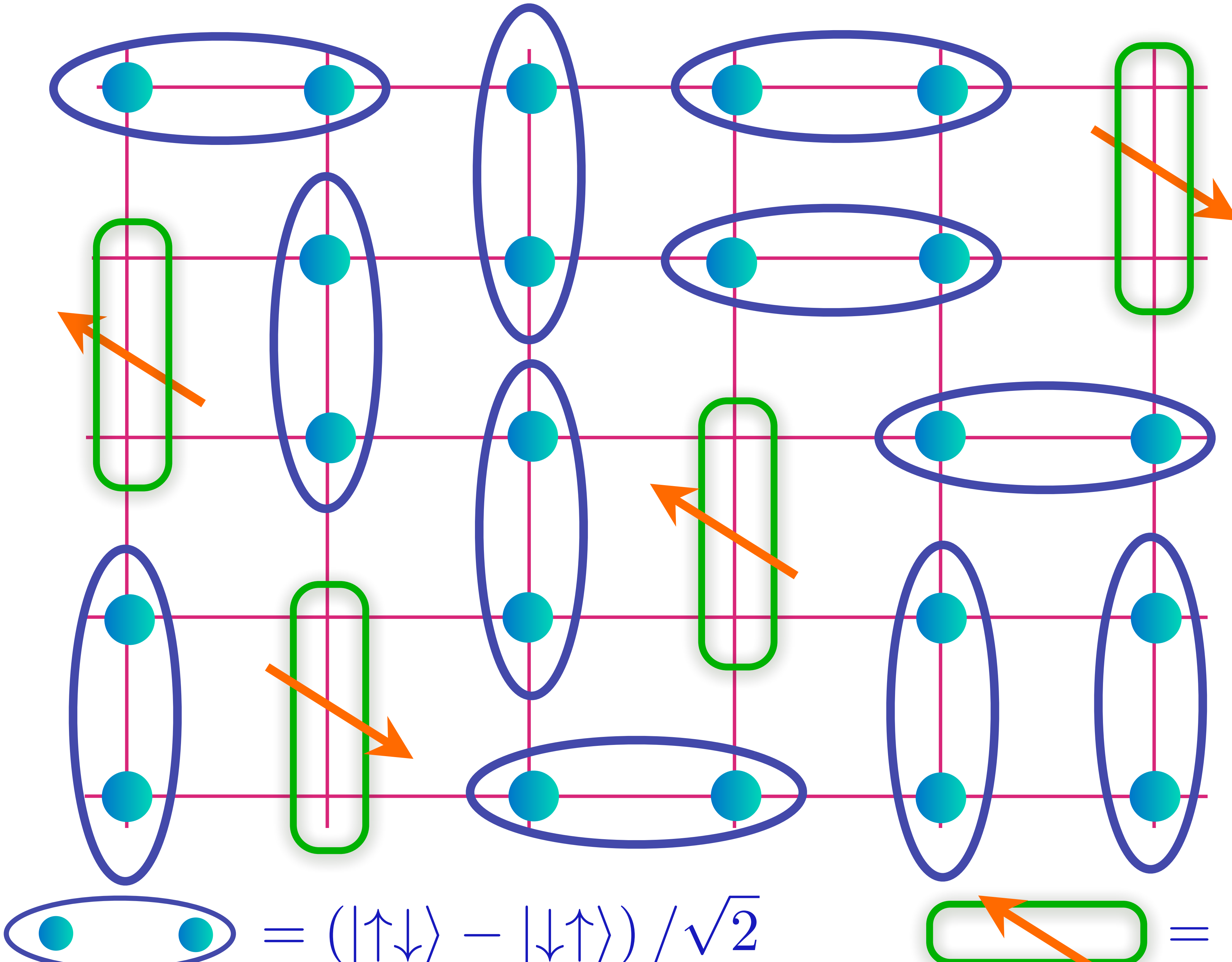
FL*



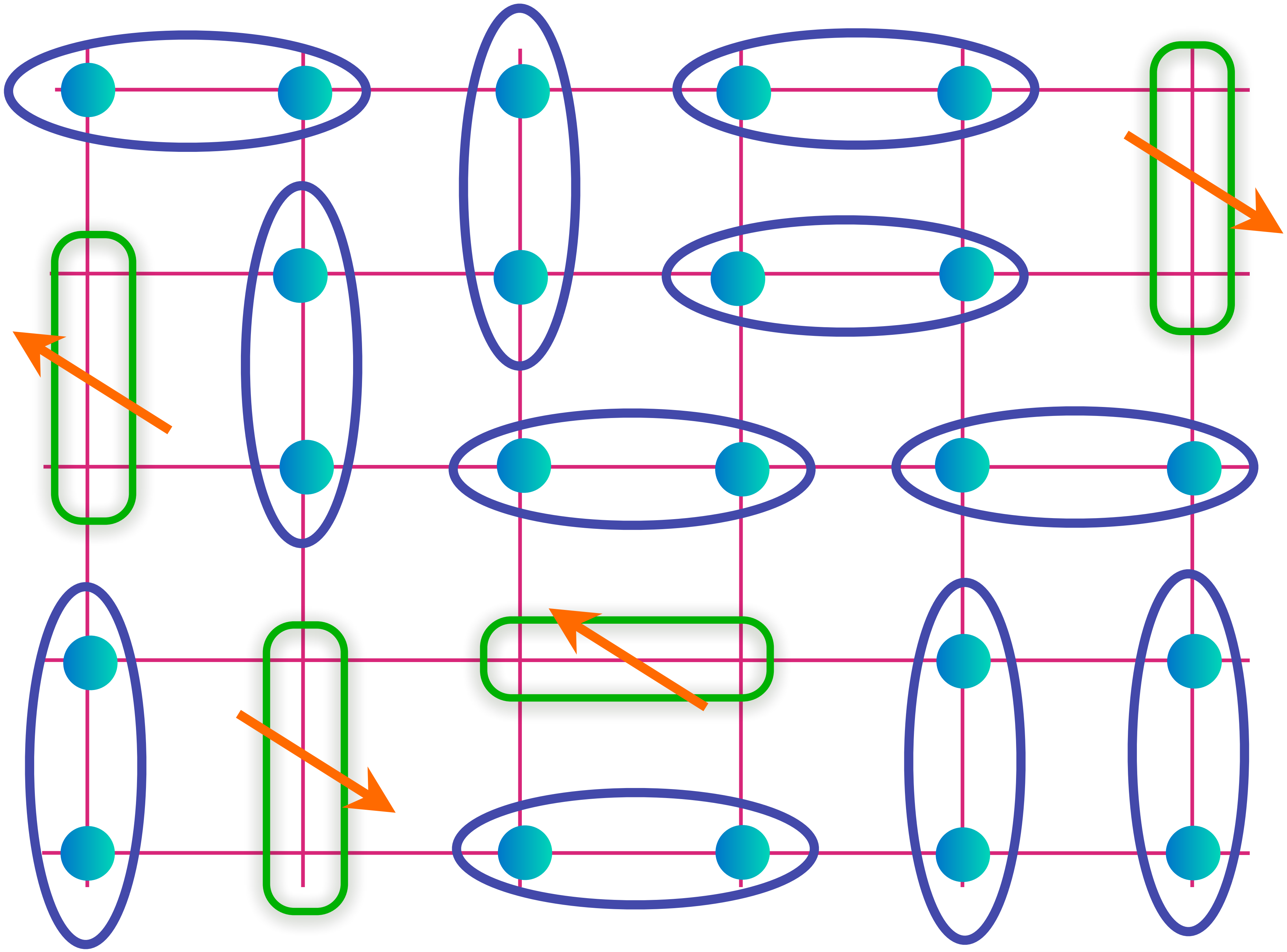
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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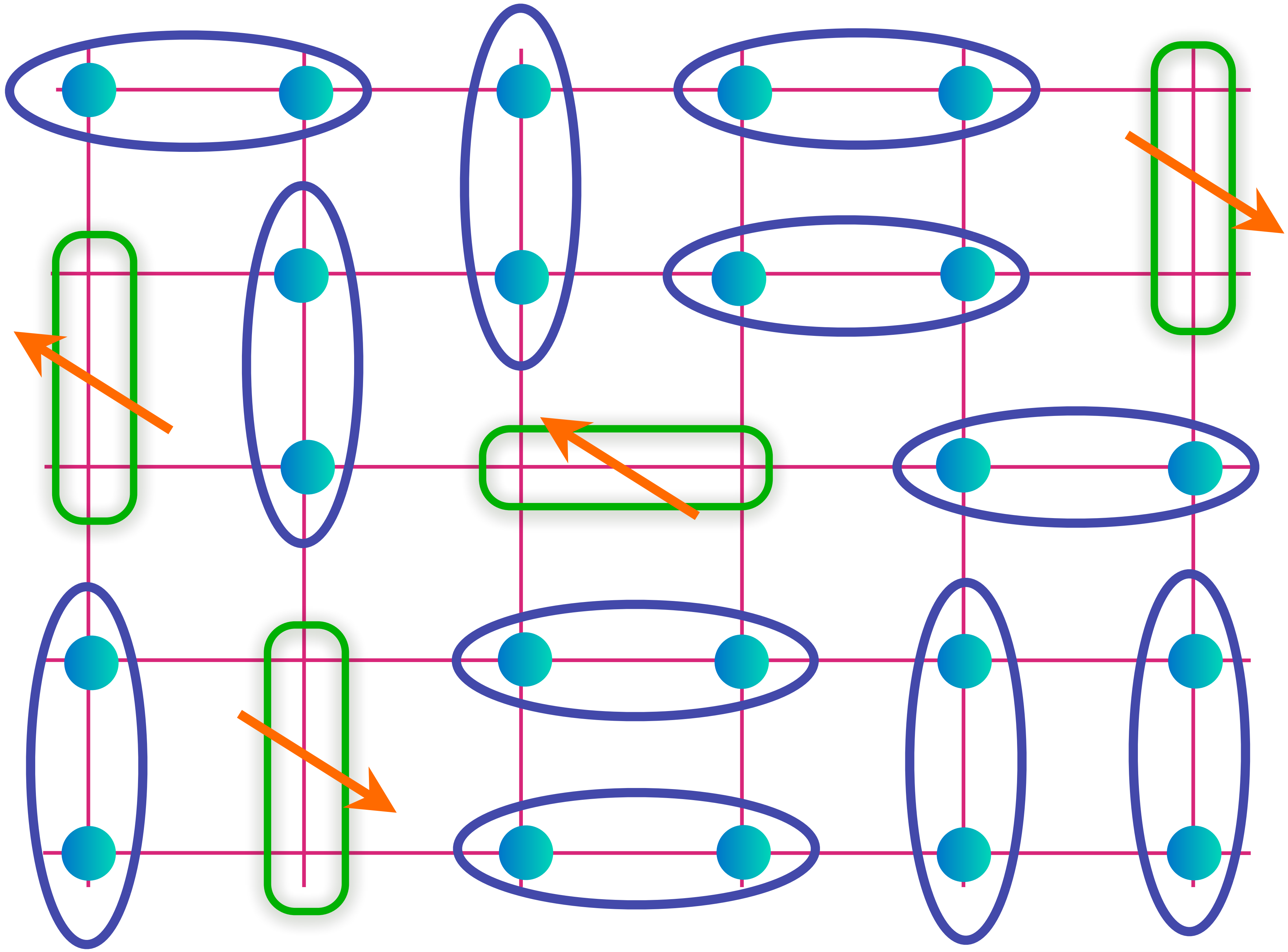

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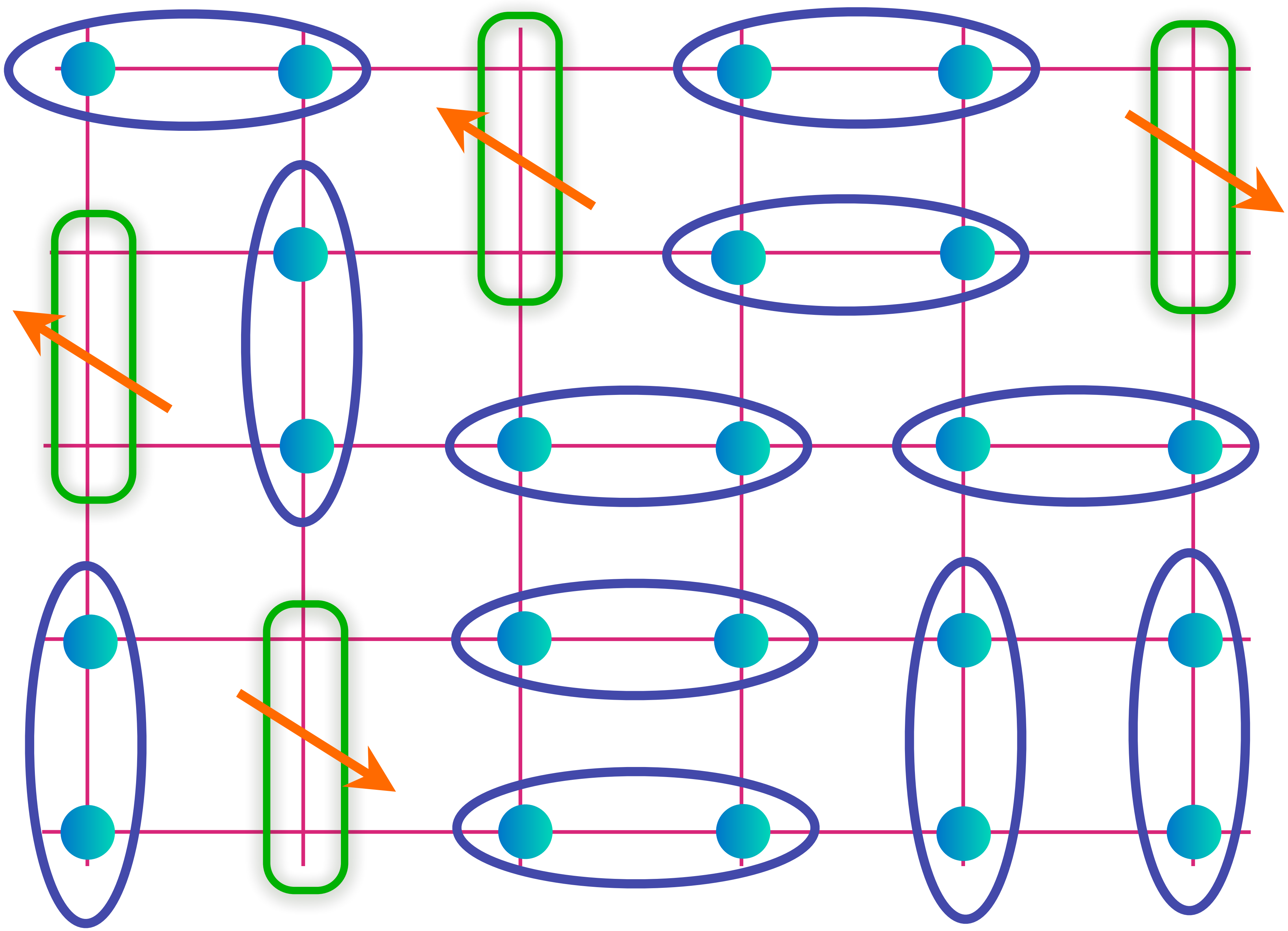

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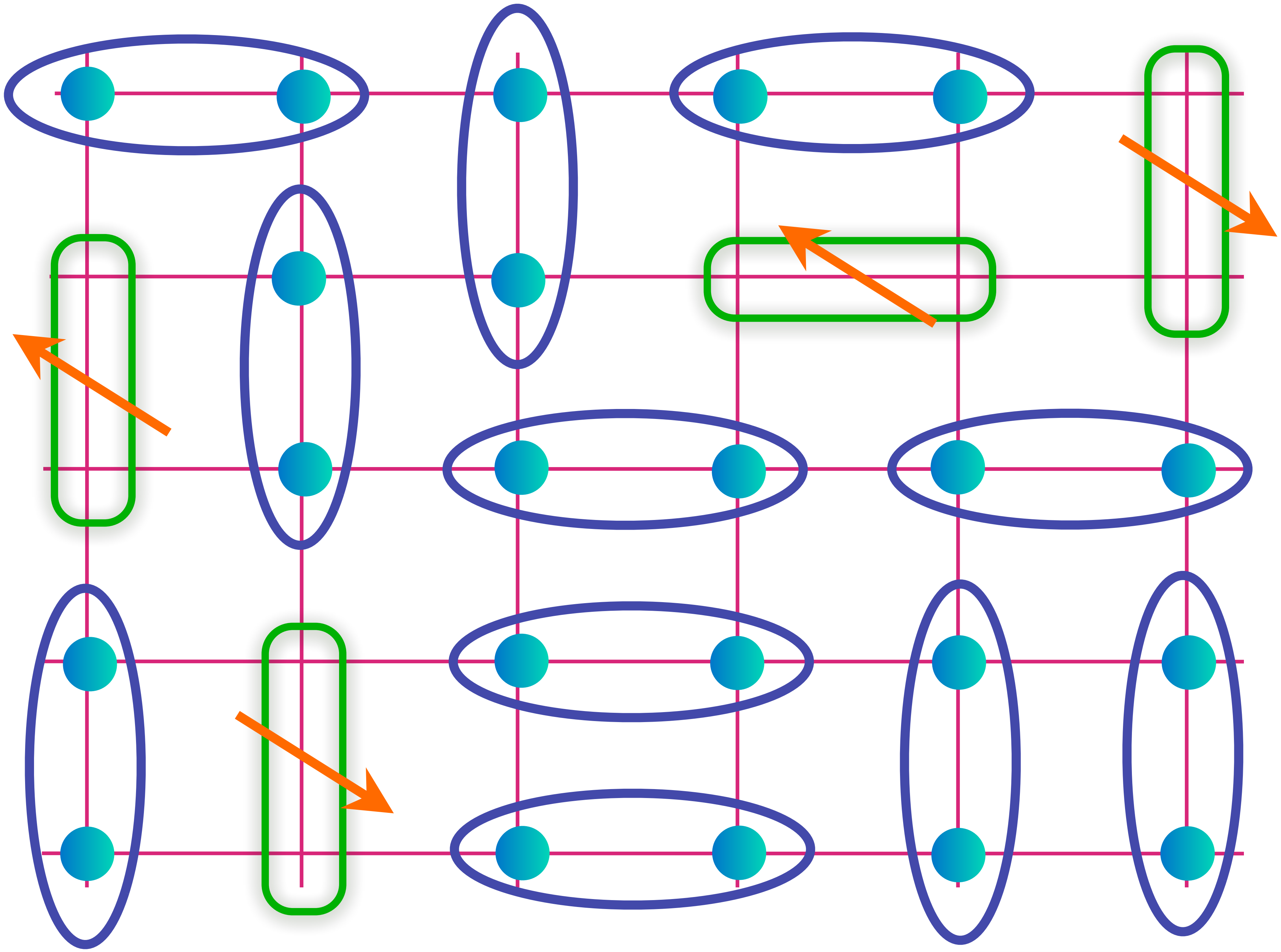

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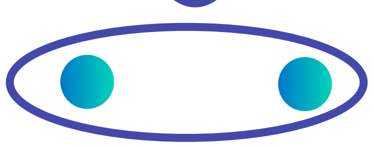
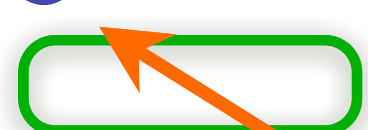
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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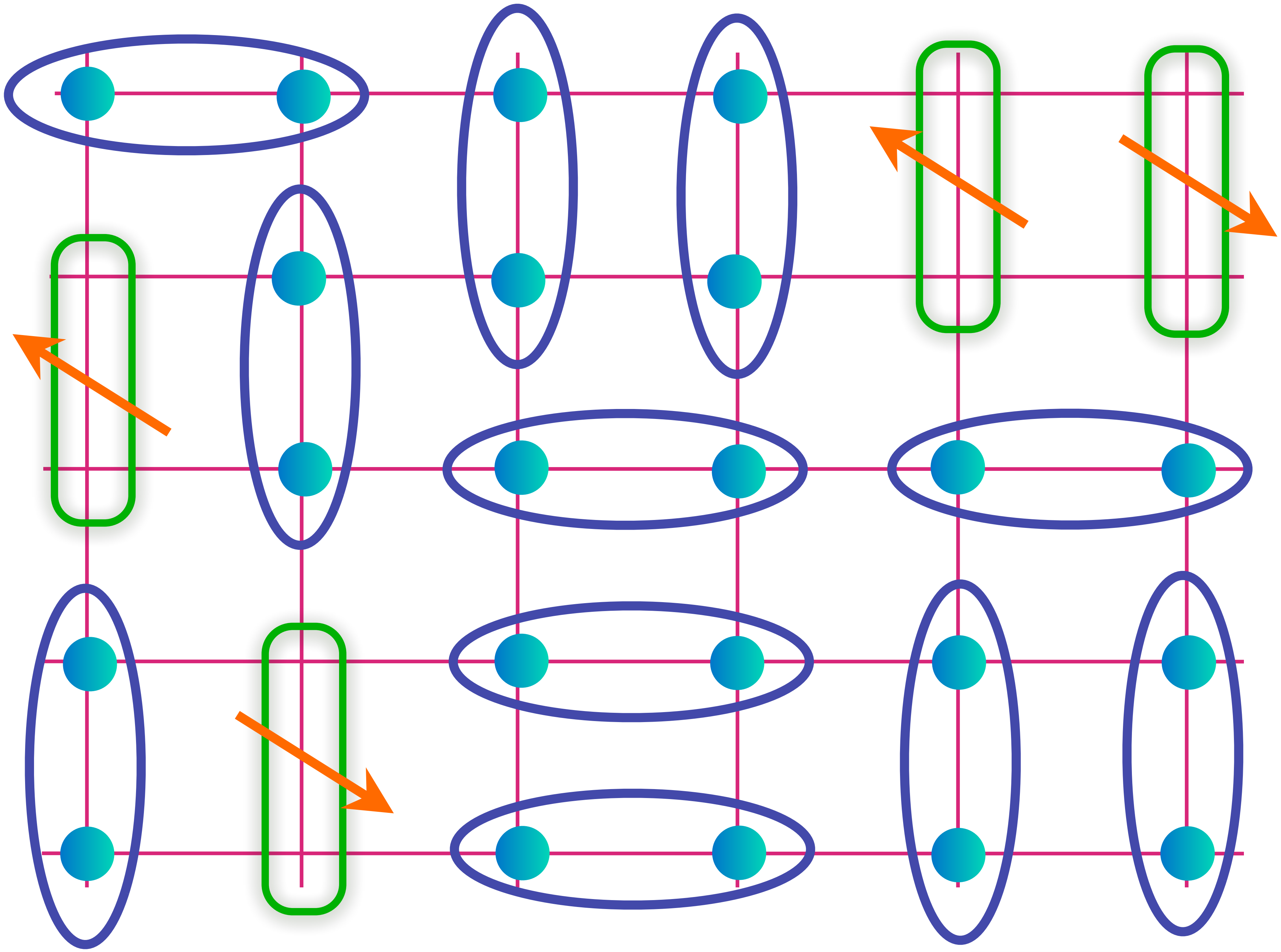

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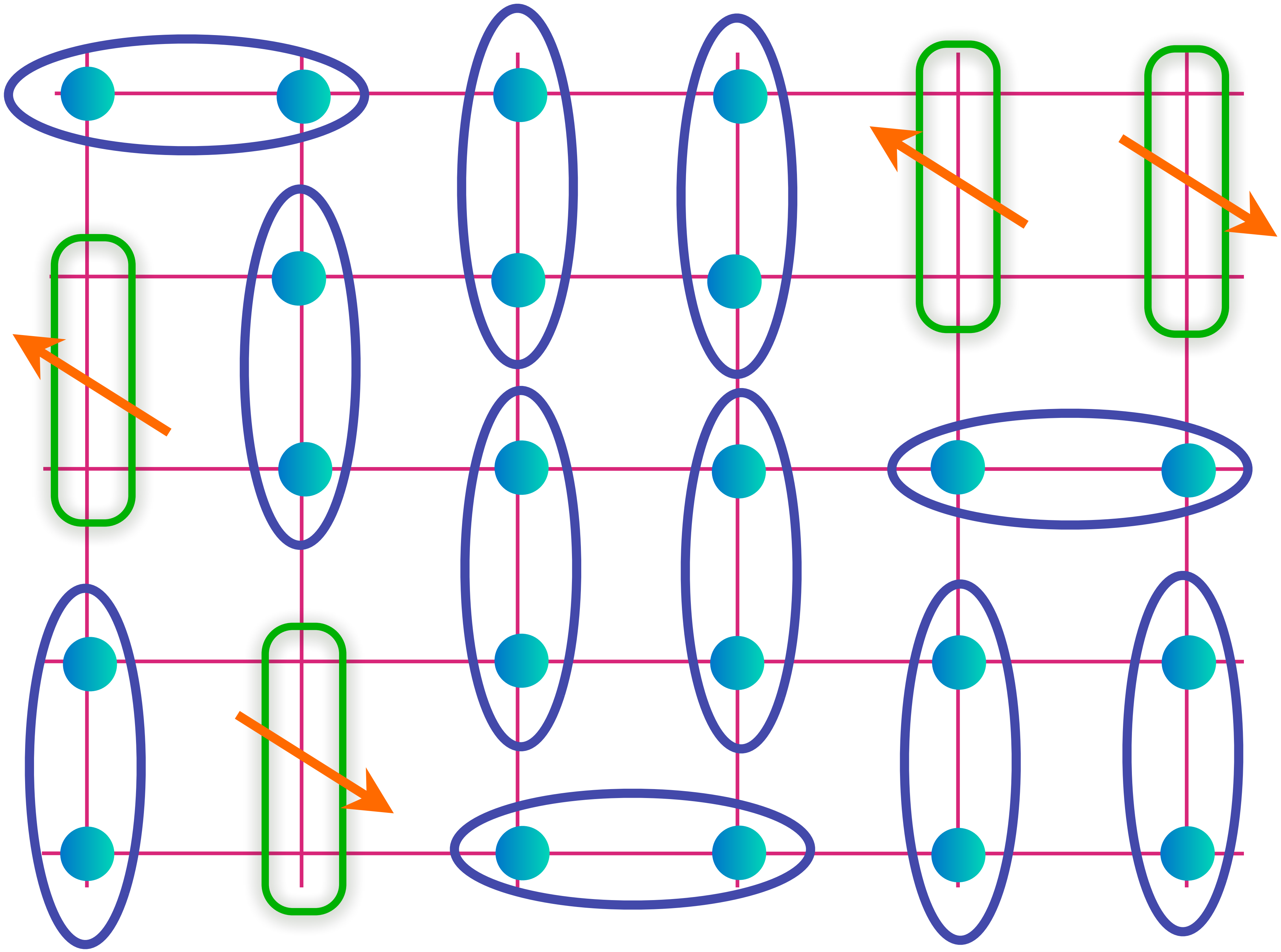

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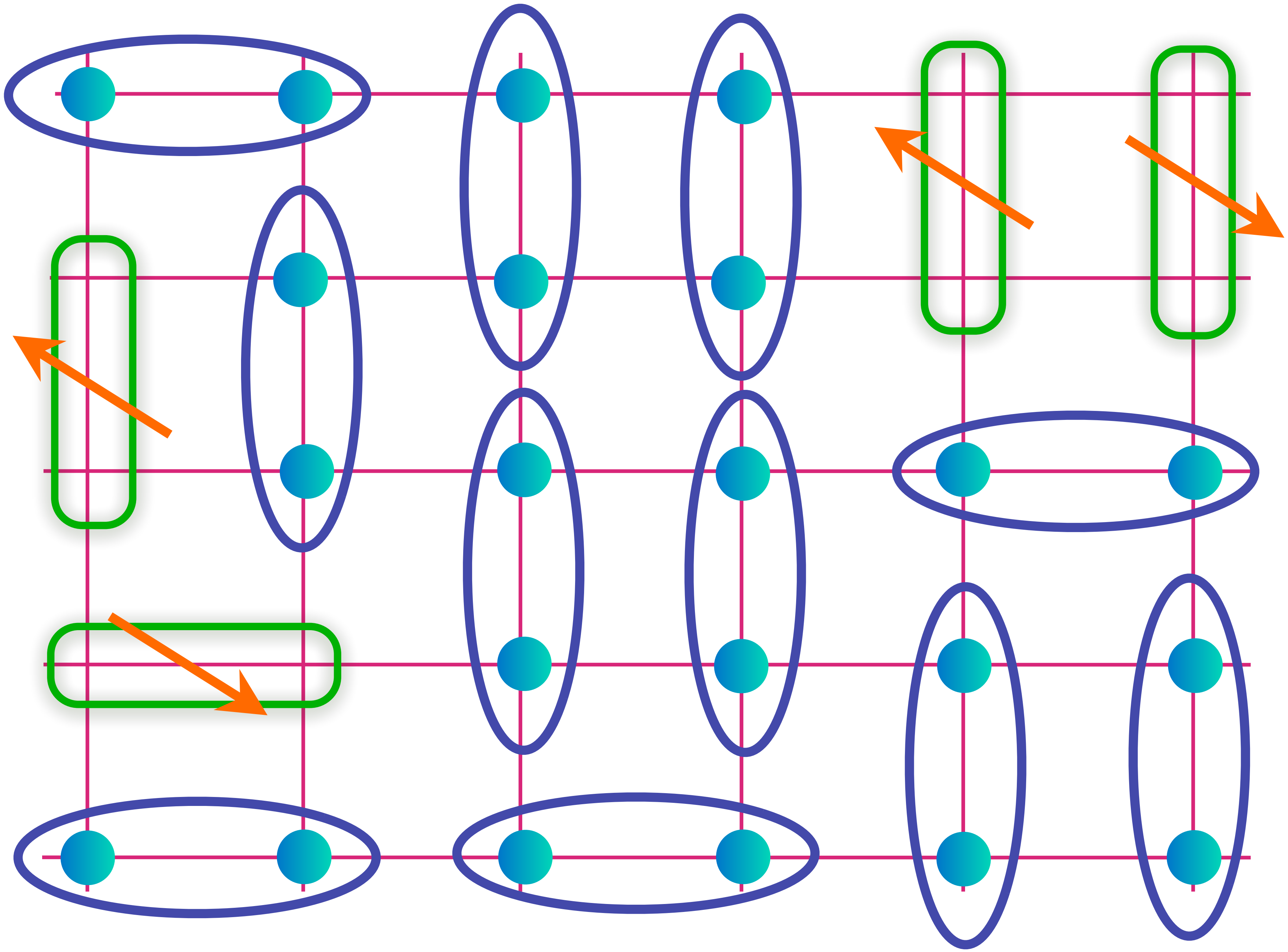
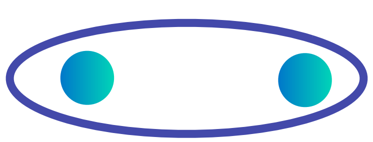
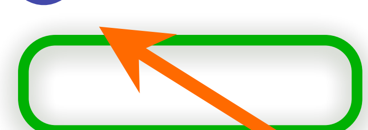


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Constraints on volume enclosed by the Fermi surface

- In a conventional Fermi liquid state, Fermi volume must equal *(1-p) (mod 2)*.
- When the unit cell is doubled by SDW order, total Fermi volume must equal *(1-p) (mod 1)*.
- A state with Fermi volume *(-p) (mod 2)*, but no translational symmetry breaking, must have non-quasiparticle excitations with vanishing energy on a torus *i.e.* emergent gauge fields (bulk topological order)

M. Oshikawa, *Phys. Rev. Lett.* **84**, 3370 (2000)
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

1. Simple models of metals with intrinsic topological order

   *ACL and FL**

2. SU(2) gauge theory of fluctuating antiferromagnetism

3. Higgs-confinement transition to a Fermi liquid

   SU(2) gauge theory with N_h adjoint Higgs fields

Transforming to a rotating reference frame

We can (exactly) transform the Hubbard model to the "spin-fermion" model:
electrons $c_{i\alpha}$ on the square lattice with dispersion

$$\begin{aligned}\mathcal{H}_c &= -\sum_{i,\rho} t_\rho \left(c^\dagger_{i,\alpha} c_{i+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{i+\boldsymbol{v}_\rho,\alpha} c_{i,\alpha} \right) \\ &\quad -\mu \sum_i c^\dagger_{i,\alpha} c_{i,\alpha} + \mathcal{H}_{\rm int}\end{aligned}$$

are coupled to a magnetic moment order parameter $\Phi^a(i)$, $a = x, y, z$

$$\begin{aligned}\mathcal{H}_{\rm int} &= -\lambda \sum_i \Phi^a(i) c^\dagger_{i,\alpha} \sigma^a_{\alpha\beta} c_{i,\beta} + V(\Phi^a) \\ V(\Phi^a) &= s \Phi^a \Phi^a + u \Phi^a \Phi^a \Phi^b \Phi^b\end{aligned}$$

Transforming to a rotating reference frame

For fluctuating antiferromagnetism (spin density waves (SDW)), we transform to a rotating reference frame using the SU(2) rotation R_i

$$\left(\begin{array}{c} c_{i\uparrow} \\ c_{i\downarrow} \end{array} \right) = R_i \left(\begin{array}{c} \psi_{i,+} \\ \psi_{i,-} \end{array} \right),$$

in terms of fermionic "chargons" ψ_s and a **Higgs field** $H^a(i)$

$$\sigma^a \Phi^a(i) = R_i \, \sigma^a H^a(i) \, R_i^\dagger$$

The Higgs field is the SDW order in the rotating reference frame.

Transforming to a rotating reference frame

The simplest effective Hamiltonian for the fermionic chargons is the same as that for the electrons, with the SDW order replaced by the Higgs field.

$$\mathcal{H}_\psi = -\sum_{i,\rho} t_\rho \left(\psi_{i,s}^\dagger \psi_{i+\boldsymbol{v}_\rho,s} + \psi_{i+\boldsymbol{v}_\rho,s}^\dagger \psi_{i,s} \right) - \mu \sum_i \psi_{i,s}^\dagger \psi_{i,s} + \mathcal{H}_{\rm int}$$

$$\mathcal{H}_{\rm int} = -\lambda \sum_i \eta_i H^a(i) \psi_{i,s}^\dagger \sigma_{ss'}^a \psi_{i,s'} + V(H^a)$$

IF we can transform to a rotating reference frame in which $H^a(i) =$ a constant independent of i and time, **THEN** the ψ fermions in the presence of (fluctuating) SDW SRO will inherit the small Fermi surfaces of the electrons in the presence of SDW LRO.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Gauge theory of fluctuating antiferromagnetism

Field	Symbol	Statistics	$SU(2)_{gauge}$	$SU(2)_{spin}$	$U(1)_{e.m.charge}$
Electron	c	fermion	$\mathbf{1}$	$\mathbf{2}$	-1
AF order	Φ	boson	$\mathbf{1}$	$\mathbf{3}$	0
Chargon	ψ	fermion	$\mathbf{2}$	$\mathbf{1}$	-1
Spinon	R or z	boson	$\bar{\mathbf{2}}$	$\mathbf{2}$	0
Higgs	H	boson	$\mathbf{3}$	$\mathbf{1}$	0

Note that the transformation to a rotating reference frame is ambiguous up to a SU(2) gauge transformation, V_i

$$\begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix} \to V_i \begin{pmatrix} \psi_{i,+} \\ \psi_{i,-} \end{pmatrix}$$

$$R_i \to R_i V_i^\dagger$$

$$\sigma^a H^a(i) \to V_i \, \sigma^b H^b(i) \, V_i^\dagger .$$

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SU(2) gauge theory: fractionalize the SDW order parameter into the Higgs field (H) and the spinons (R); fractionalize the electron (c) into chargons (ψ) and spinons (R). When the Higgs field is condensed, the ψ fermions and R bosons are deconfined particles in an algebraic charge liquid (ACL) state, but with a strong residual attractive interaction from the t_{ij}.

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SU(2) gauge theory: The ACL was used to model the photoemission and the cluster DMFT results in the intermediate phase.

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)

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Effective theory for the Higgs field and the electrons

Gauge theory of fluctuating antiferromagnetism

We obtain different numbers of adjoint Higgs scalars, N_h, depending upon the spatial dependence of the local spin correlations:

Neel correlations (electron doped cuprates):
$N_h = 1$,
$\boldsymbol{K} = (\pi, \pi)$,
$H^a(i) = H^a_1(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}$

Bidirectional incommensurate correlations (hole doped cuprates):
$N_h = 4$,
$\boldsymbol{K}_y = (\pi, \pi - \delta)$, $\boldsymbol{K}_x = (\pi - \delta, \pi)$,
$H^a(i) = \mathrm{Re}\left\{ \left[H^a_1(\boldsymbol{r}) + i H^a_2(\boldsymbol{r}) \right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H^a_3(\boldsymbol{r}) + i H^a_4(\boldsymbol{r}) \right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i} \right\}$

Optimal doping for electron-doped cuprates

SU(2) gauge theory

SU(2) gauge theory with $N_h = 1$ adjoint real Higgs fields $\mathcal{H}^a$ $(a = 1, 2, 3)$, and gauge-invariant, electron-like fermions c_α with a large Fermi surface.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \mathcal{H}^a - \epsilon_{abc} A^b_\mu \mathcal{H}^c\right)^2 + \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + V(\mathcal{H}^a)$$

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\ V(H^a) &= s\, H^a H^a + u_0\, H^a H^a H^b H^b \end{aligned}$$

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$$
\begin{aligned}
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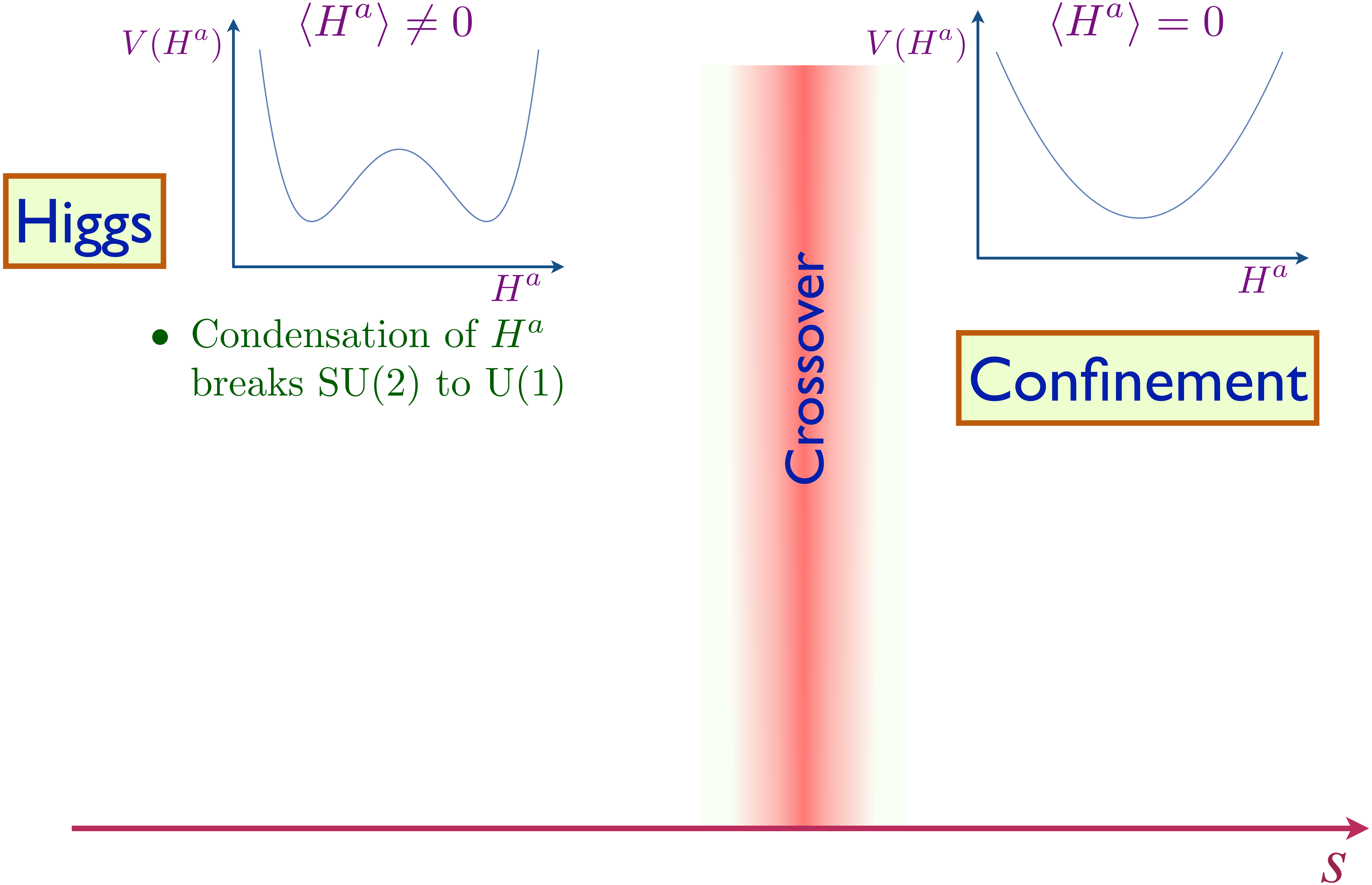
&\quad - \sum_{j,\rho} t_\rho \left(c^\dagger_{j,\alpha} c_{j+\boldsymbol{v}_\rho,\alpha} + c^\dagger_{j+\boldsymbol{v}_\rho,\alpha} c_{j,\alpha} \right) - \mu \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} \\
&\quad + \lambda \sum_j c^\dagger_{j,\alpha} c_{j,\alpha} H^a(j) H^a(j) \\
F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon_{abc} A^b_\mu A^c_\nu \\
V(H^a) &= s\, H^a H^a + u_0\, H^a H^a H^b H^b
\end{aligned}
$$

The fermions do not have Yukawa coupling to the Higgs fields, or a minimal coupling to the gauge fields: both are prohibited by gauge invariance. We treat the quartic coupling, λ perturbatively.

$N_h = 1$

Phase diagrams of SU(2) gauge theory



A. C. Davis, A. Hart, T.W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

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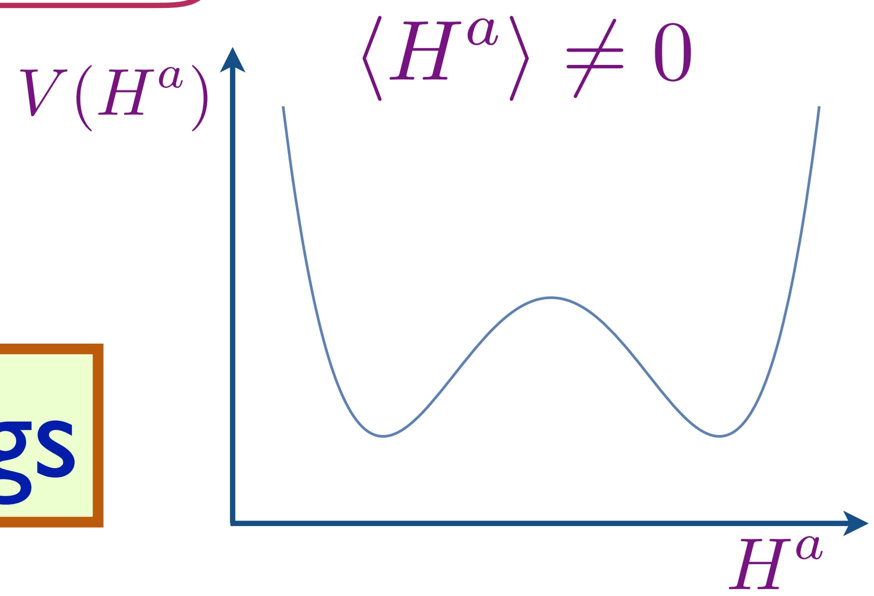
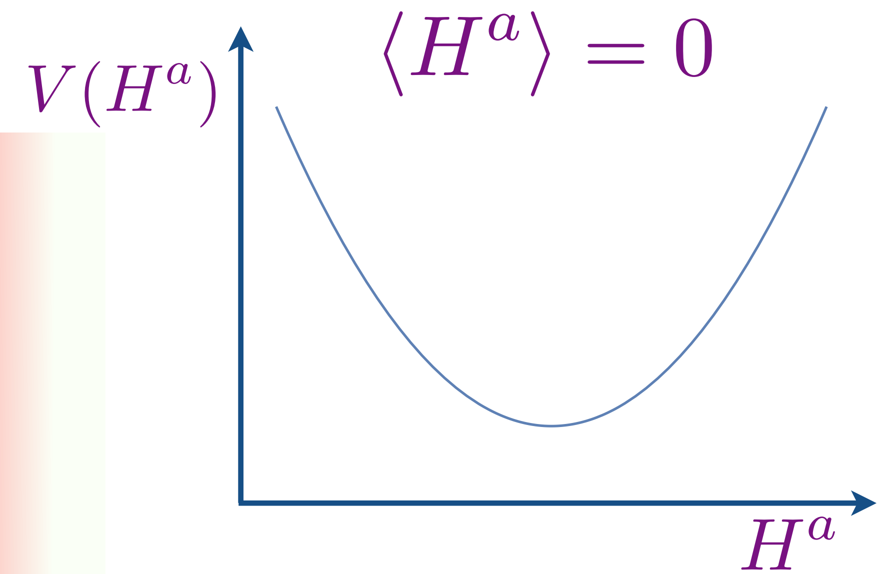
Phase diagrams of SU(2) gauge theory



Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles

Crossover



Confinement

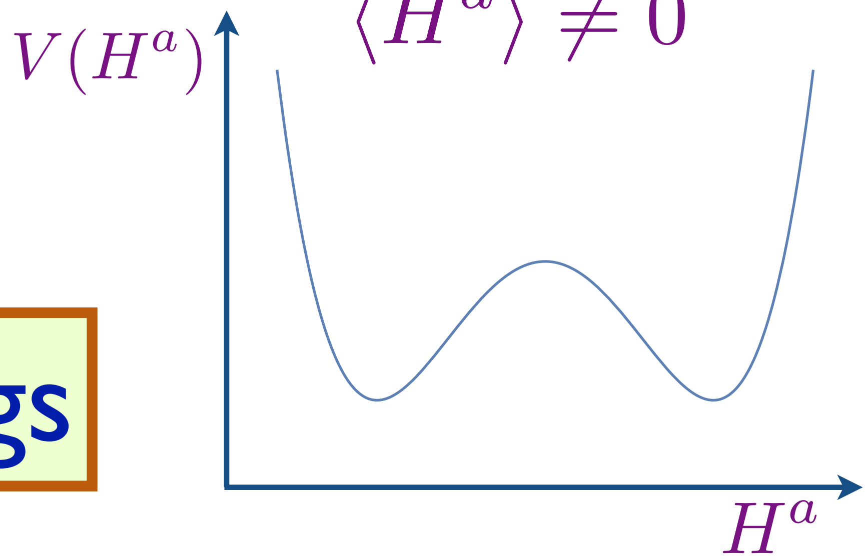
s

A. C. Davis, A. Hart, T. W. B. Kibble, and A. Rajantie, Phys. Rev. D 65, 125008 (2002)

$N_h = 1$

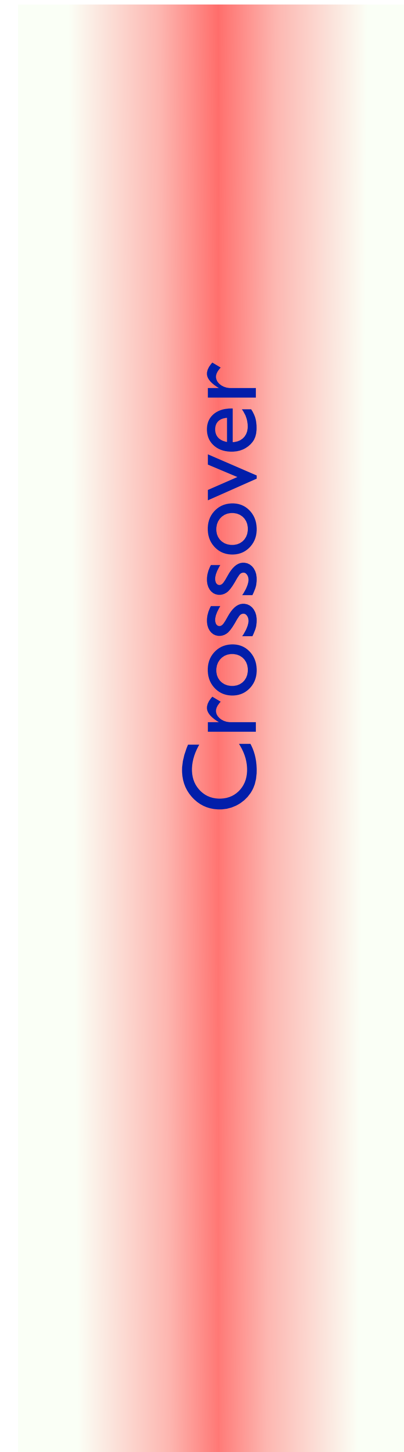
Phase diagrams of SU(2) gauge theory

$\langle H^a \rangle \neq 0$

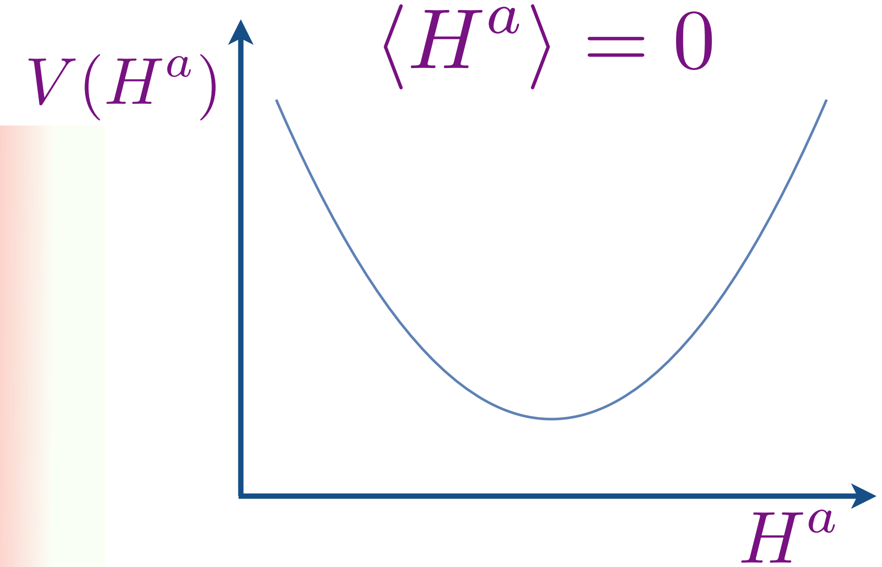


Higgs

- Condensation of H^a breaks SU(2) to U(1)
- U(1) confines because of proliferation of 'tHooft-Polyakov monopoles
- Monopole action $\sim \sqrt{-s}$, leading to an exponentially large confinement scale



$\langle H^a \rangle = 0$



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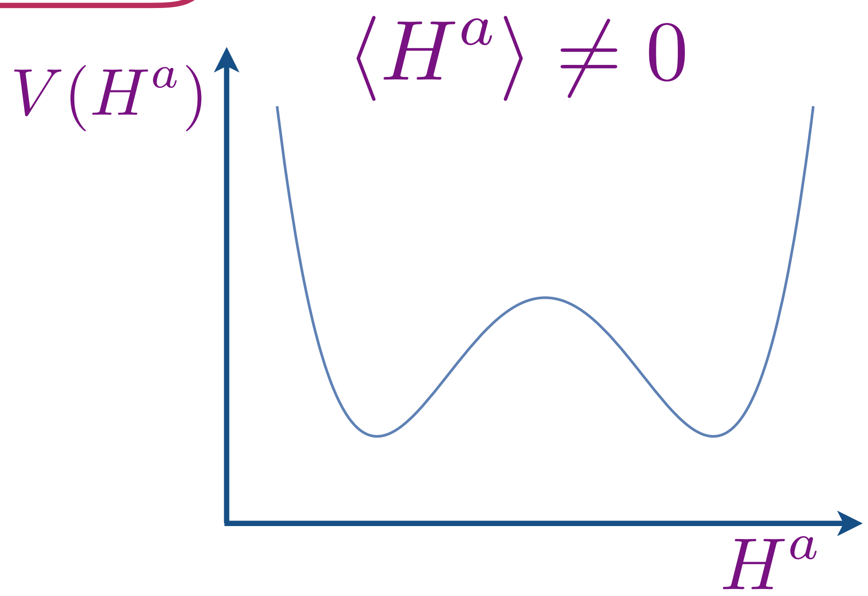

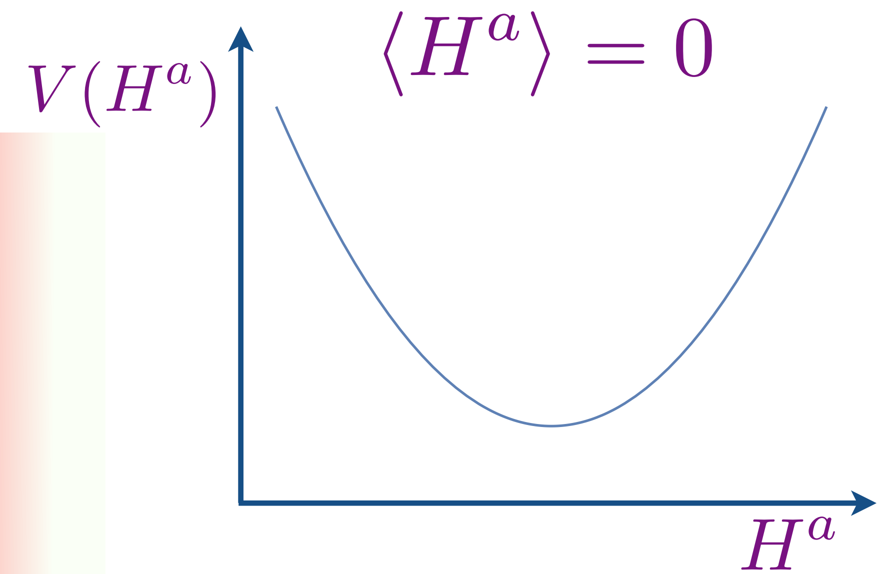

Higgs/U(1) confinement

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in a U(1) gauge theory, leading to re-emergence of large Fermi surface

Crossover

Confinement

Fermi liquid with large Fermi surface

S

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$H^a(i) = \mathrm{Re}\left\{\left[H_1^a(\boldsymbol{r}) + iH_2^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_x\cdot\boldsymbol{r}_i} + \left[H_3^a(\boldsymbol{r}) + iH_4^a(\boldsymbol{r})\right] e^{i\boldsymbol{K}_y\cdot\boldsymbol{r}_i}\right\}$

Optimal doping for hole-doped cuprates

SU(2) gauge theory

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + i H_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + i H_4^a \, .$$

The SU(2) gauge theory is

$$\mathcal{L} = \frac{1}{2} \left| \partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c \right|^2 + \frac{1}{2} \left| \partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c \right|^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a$$

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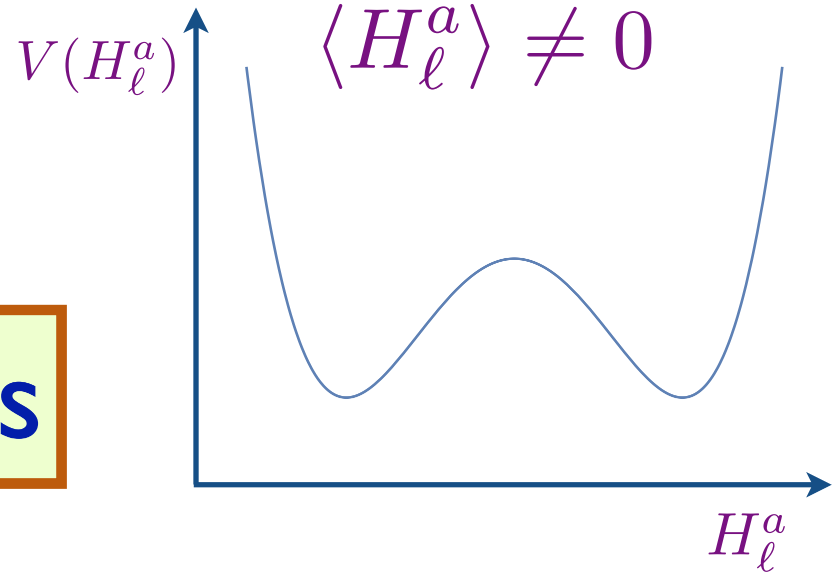
$$+ V(\mathcal{H}_{x,y}^a) + \text{coupling to electrons with large Fermi surface}$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\ V(\mathcal{H}_{x,y}^a) &= s \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a + \mathcal{H}_y^{a*} \mathcal{H}_y^a \right) + u_0 \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a + \mathcal{H}_y^{a*} \mathcal{H}_y^a \right)^2 \\ & \quad \frac{u_1}{4} \left(\mathcal{H}_x^{a*} \mathcal{H}_x^a - \mathcal{H}_y^{a*} \mathcal{H}_y^a \right)^2 + \frac{u_2}{2} \left[\left| \mathcal{H}_x^a \mathcal{H}_x^a \right|^2 + \left| \mathcal{H}_y^a \mathcal{H}_y^a \right|^2 \right] \\ & \quad + u_3 \left(\left| \mathcal{H}_x^a \mathcal{H}_y^a \right|^2 + \left| \mathcal{H}_x^a \mathcal{H}_y^{a*} \right|^2 \right) . \end{aligned}$$

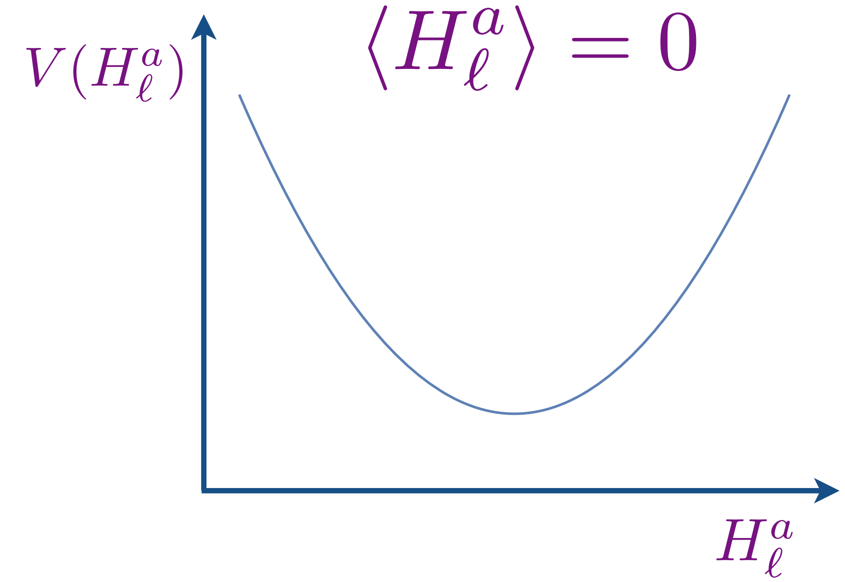
$N_h = 4$

Phase diagrams of SU(2) gauge theory

Higgs



- Condensation of H^a breaks $\mathrm{SU}(2)$ to $\mathrm{U}(1)$ or $\mathbb{Z}_2$.



Confinement

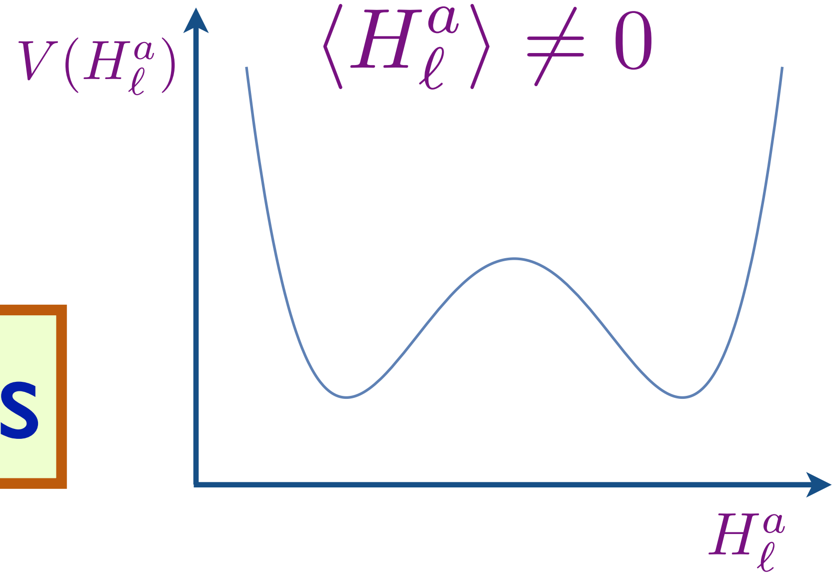
Possible Deconfined critical SU(2) gauge theory

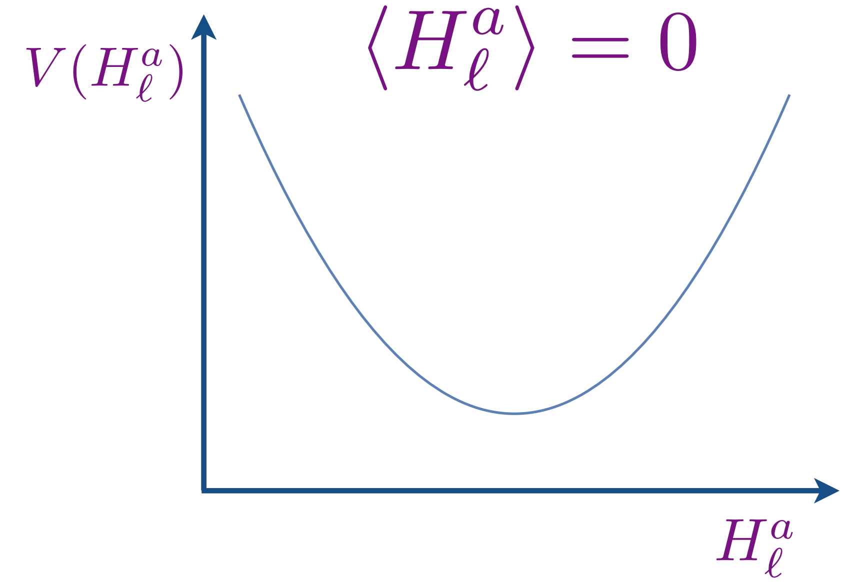
s

$N_h = 4$

Phase diagrams of SU(2) gauge theory

Higgs





Confinement

- Condensation of H^a breaks SU(2) to U(1) or $\mathbb{Z}_2$.
- The U(1) cases confine, while the $\mathbb{Z}_2$ cases have $\mathbb{Z}_2$ (toric code) topological order.

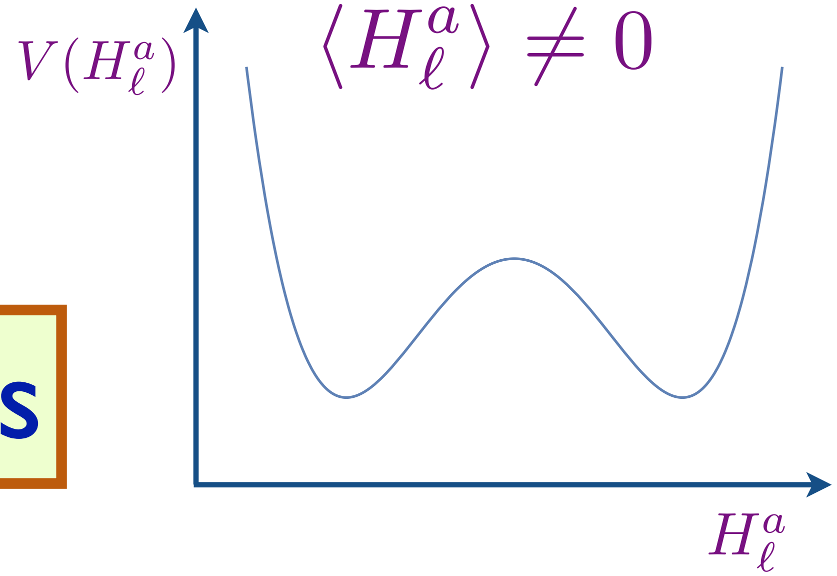
Possible Deconfined critical SU(2) gauge theory

s

$N_h = 4$

Phase diagrams of SU(2) gauge theory

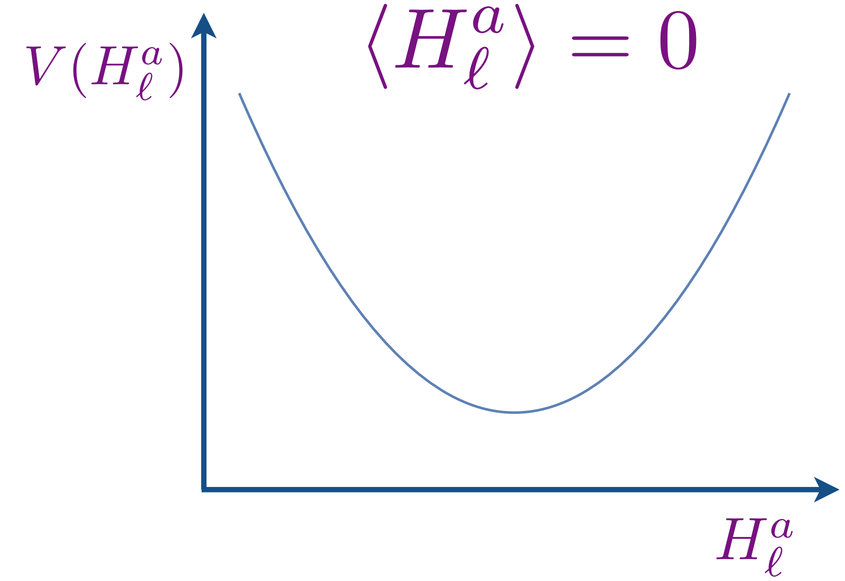
Higgs



- Condensation of H^a breaks SU(2) to U(1) or $\mathbb{Z}_2$.
- The U(1) cases confine, while the $\mathbb{Z}_2$ cases have $\mathbb{Z}_2$ (toric code) topological order.
- One or more global symmetries are broken in all cases.

Possible Deconfined critical SU(2) gauge theory

Confinement

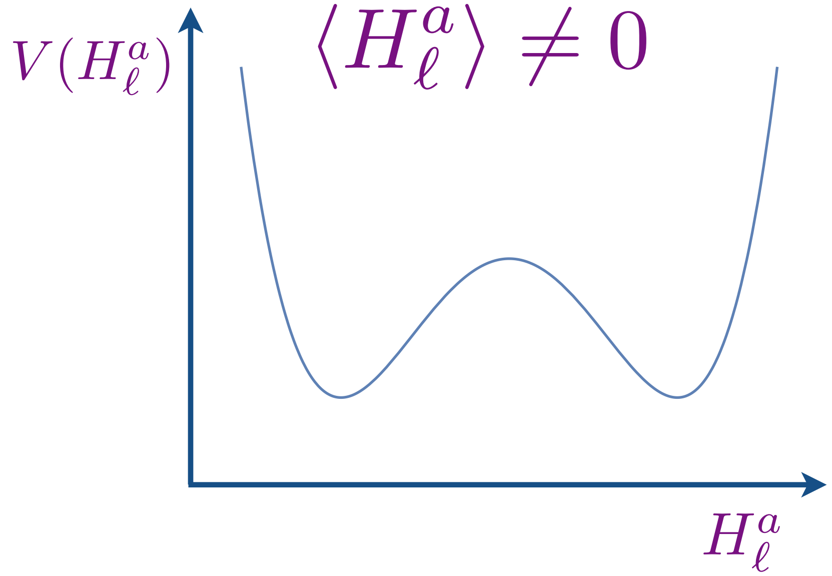


s

$N_h = 4$

Phase diagrams of SU(2) gauge theory

$\langle H_\ell^a \rangle \neq 0$



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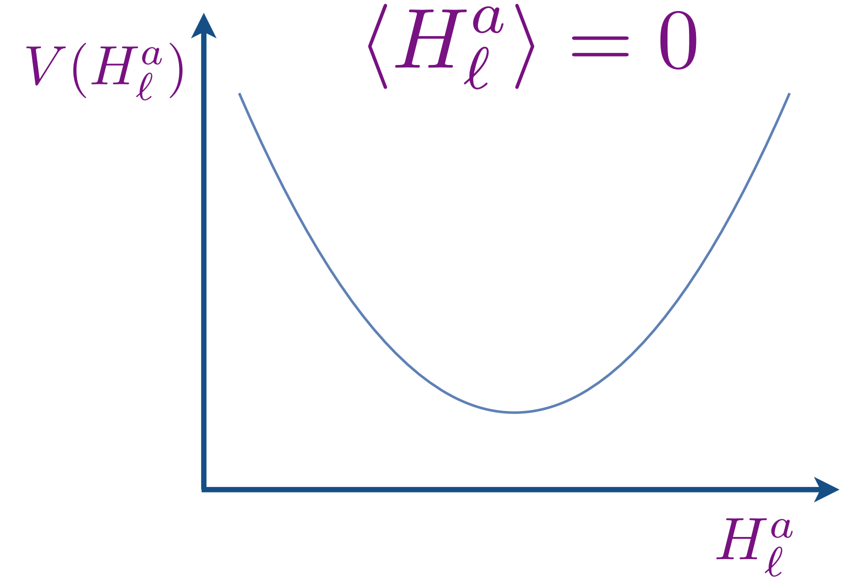


Higgs/U(1) confinement /Z_2 deconfined

One or more of Ising-nematic, CDW, scalar spin chirality, and $\mathbb{Z}_2$ topological orders

Reconstructed (FL*) Fermi surfaces, with large length scale confinement in the U(1) cases

Possible Deconfined critical SU(2) gauge theory

Confinement

Fermi liquid with large Fermi surface

s

Optimal doping for hole-doped cuprates

There are multiple order parameters for different broken symmetries (Note: spin rotations are preserved and there is no SDW order)

- Ising nematic order

$$\phi = \mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $2\boldsymbol{K}_{x,y}$

$$\Phi_x = \mathcal{H}_x^a\mathcal{H}_x^a \quad , \quad \Phi_y = \mathcal{H}_y^a\mathcal{H}_y^a$$

- Charge density wave (CDW) order at wavevectors $\boldsymbol{K}_x \pm \boldsymbol{K}_y$

$$\Phi_+ = \mathcal{H}_x^a\mathcal{H}_y^a \quad , \quad \Phi_- = \mathcal{H}_x^a\mathcal{H}_y^{a*}$$

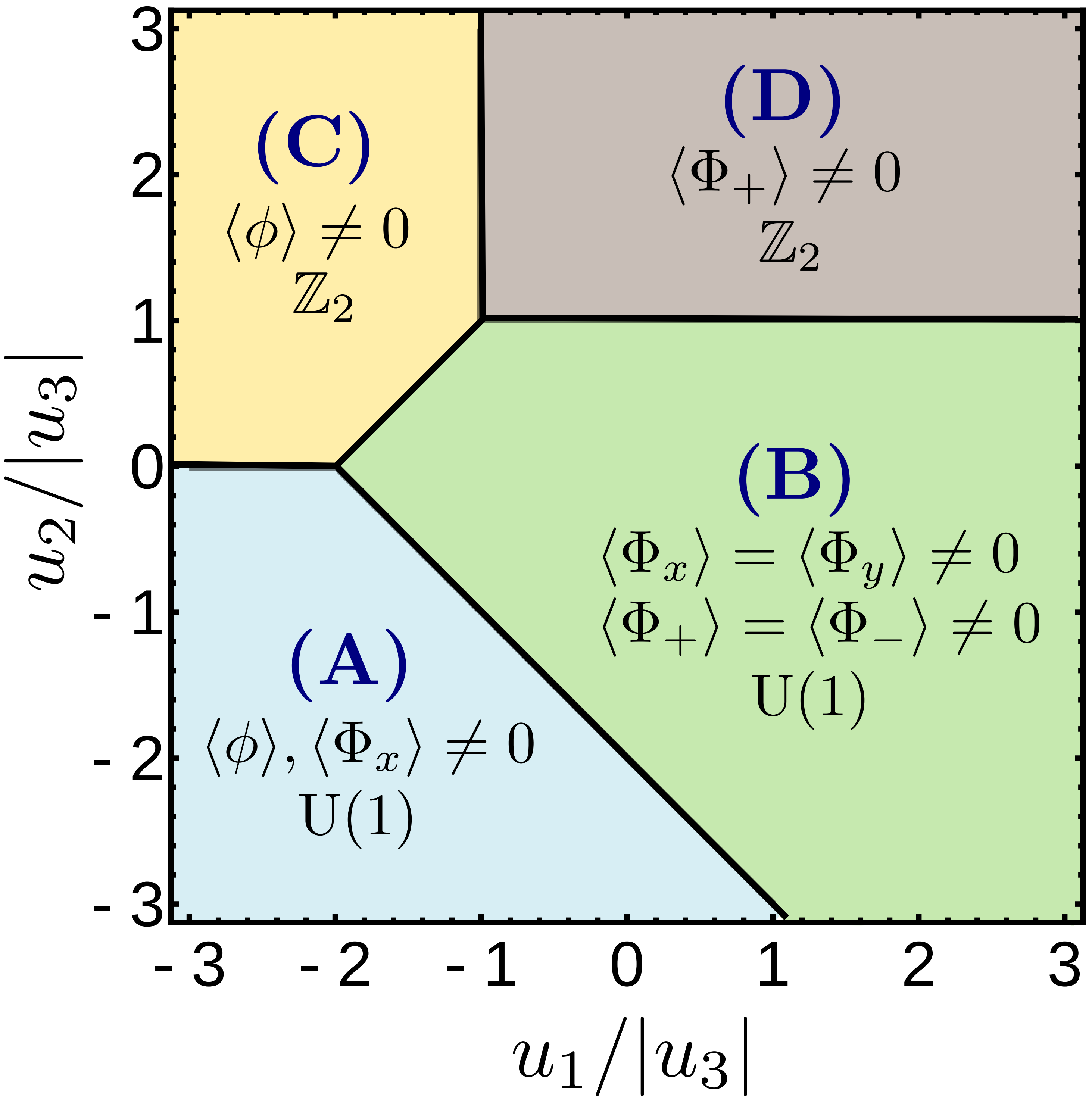
- (Modulated) scalar spin chirality

$$\chi_{ijk} = \epsilon_{abc} H^a(\boldsymbol{r}_i) H^b(\boldsymbol{r}_j) H^c(\boldsymbol{r}_k)$$

Optimal doping for hole-doped cuprates

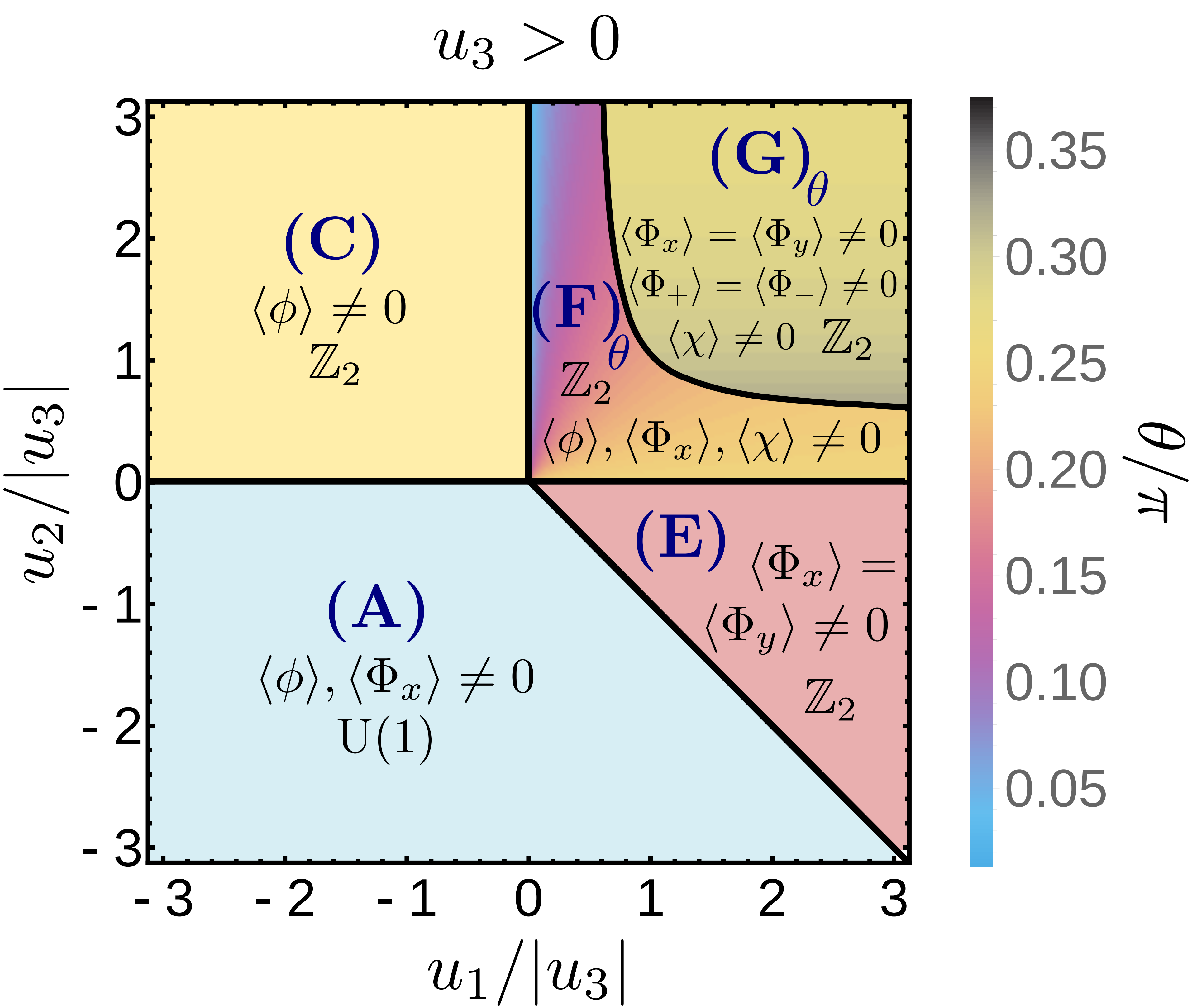
Broken symmetries and topological order in the Higgs phase

$u_3 < 0$



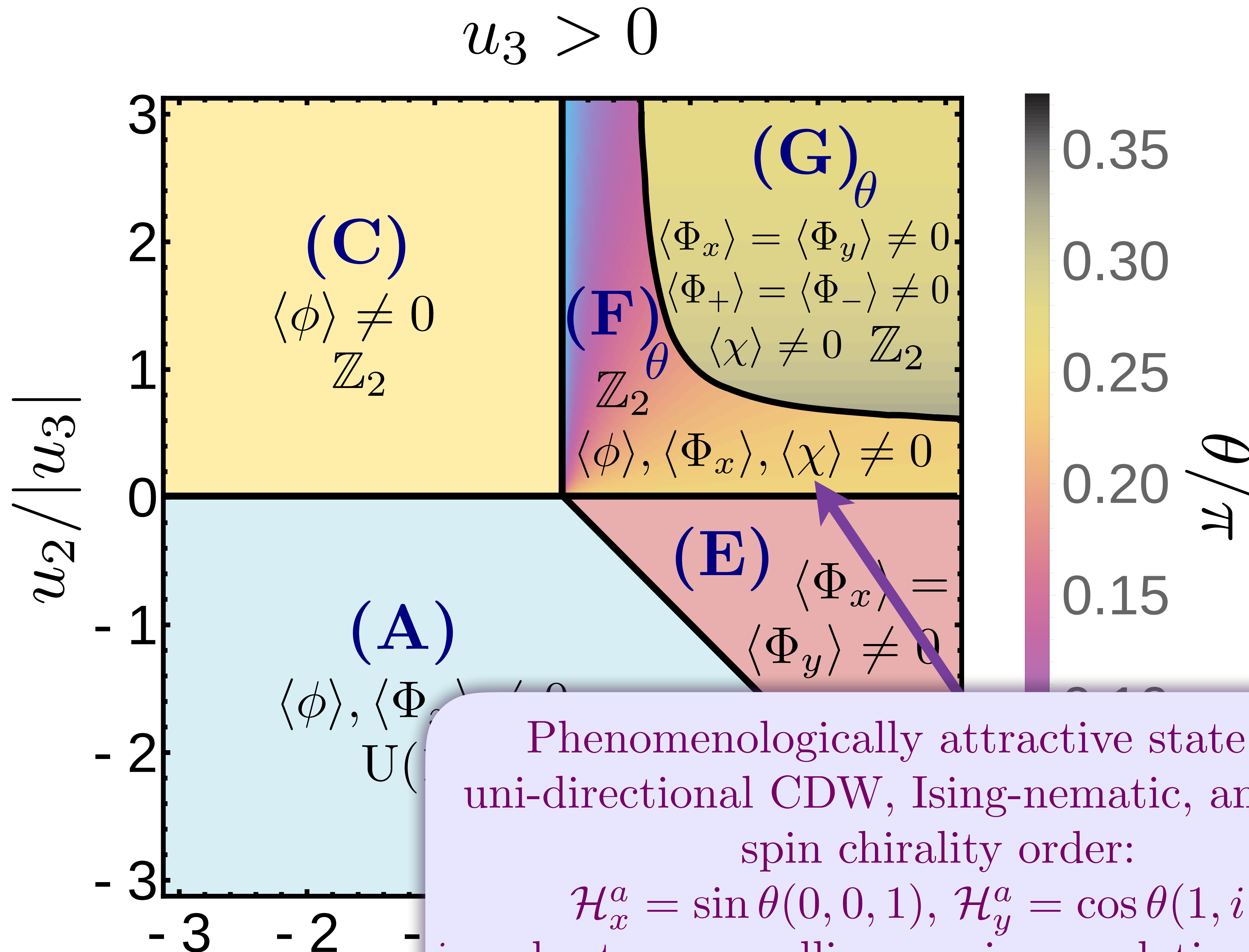
Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Optimal doping for hole-doped cuprates

Broken symmetries and topological order in the Higgs phase



Phenomenologically attractive state with uni-directional CDW, Ising-nematic, and scalar spin chirality order:
$\mathcal{H}^a_x = \sin\theta(0,0,1)$, $\mathcal{H}^a_y = \cos\theta(1,i,0)$
i.e. short-range collinear spin correlations along x, and short-range spiral spin correlations along y.

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- Hole doped cuprates are described by $N_h = 4$. In this case, a phase transition must occur, at least in the absence of disorder. The Higgs phase is characterized by:
  - Stable $\mathbb{Z}_2$ topological order, or U(1) topological up to an exponentially large length scale
  - One or more broken symmetries involving Ising-nematic, CDW, and scalar spin chirality.

Optimal doping for hole-doped cuprates

SU(2) gauge theory

For the hole-doped cuprates, $N_h = 4$, we define complex Higgs fields

$$\mathcal{H}_x^a = H_1^a + iH_2^a \quad , \quad \mathcal{H}_y^a = H_3^a + iH_4^a \,.$$

The SU(2) gauge theory is

$$\mathcal{L} = \frac{1}{2}\left|\partial_\mu \mathcal{H}_x^a - \epsilon_{abc} A_\mu^b \mathcal{H}_x^c\right|^2 + \frac{1}{2}\left|\partial_\mu \mathcal{H}_y^a - \epsilon_{abc} A_\mu^b \mathcal{H}_y^c\right|^2 + \frac{1}{4g^2} F_{\mu\nu}^a F_{\mu\nu}^a$$

$$+V(\mathcal{H}_{x,y}^a) + \text{coupling to electrons with large Fermi surface}$$

$$\begin{aligned}
F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - \epsilon_{abc} A_\mu^b A_\nu^c \\
V(\mathcal{H}_{x,y}^a) &= s\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right) + u_0\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a + \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 \\
&\quad \frac{u_1}{4}\left(\mathcal{H}_x^{a*}\mathcal{H}_x^a - \mathcal{H}_y^{a*}\mathcal{H}_y^a\right)^2 + \frac{u_2}{2}\left[\left|\mathcal{H}_x^a\mathcal{H}_x^a\right|^2 + \left|\mathcal{H}_y^a\mathcal{H}_y^a\right|^2\right] \\
&\quad +u_3\left(\left|\mathcal{H}_x^a\mathcal{H}_y^a\right|^2 + \left|\mathcal{H}_x^a\mathcal{H}_y^{a*}\right|^2\right) \,.
\end{aligned}$$