# Quantum phases of graphene

Emergent Phenomena in Quantum Hall Systems,
Tata Institute for Fundamental Research, Mumbai
January 9, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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PHYSICS
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1. Competing quantum orders in the zeroth Landau level of graphene

2. Dirac liquid in graphene: quantum matter without quasiparticles

1. Competing quantum orders in the zeroth Landau level of graphene

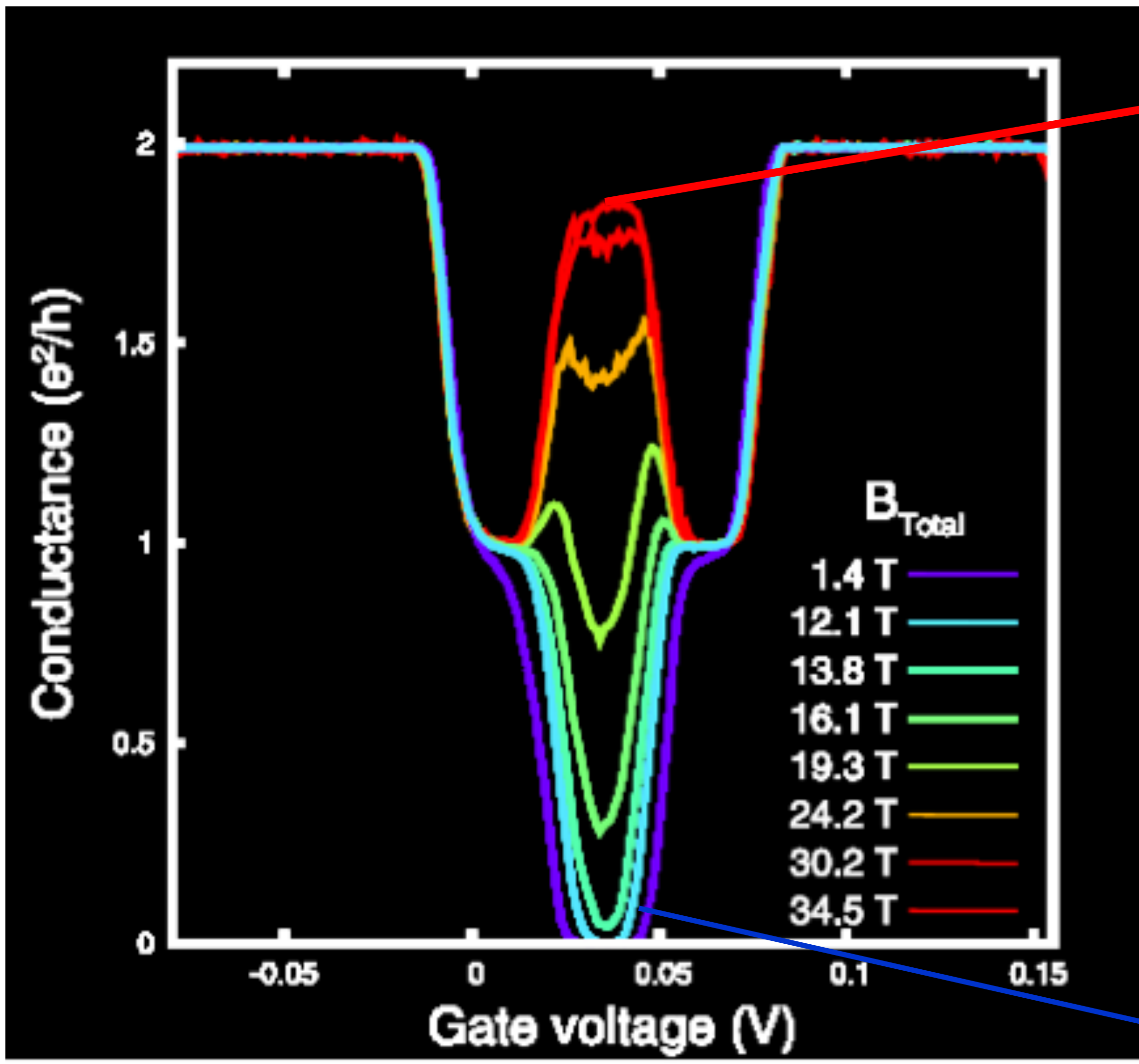
2. Dirac liquid in graphene: quantum matter without quasiparticles

**Junhyun Lee** and Subir Sachdev,
"Wess-Zumino-Witten Terms in Graphene Landau Levels,"
*Physical Review Letters* **114**, 226801 (2015)

# Experimentally: single layer graphene

# at $\nu = 0$ is an INSULATOR

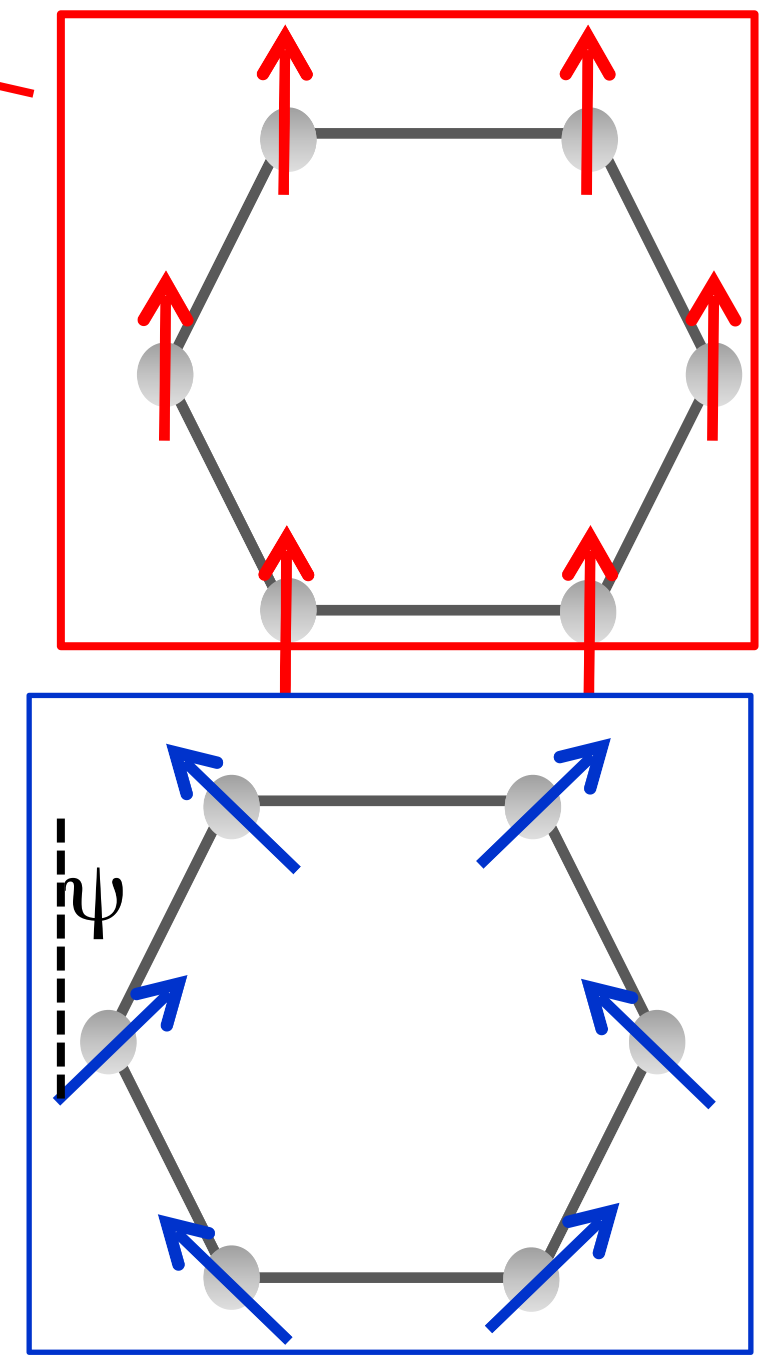
Kharitonov (2012): Canted Anti-Ferromagnet → Ferromagnet





$$E_z^c = 2g_{xy}$$

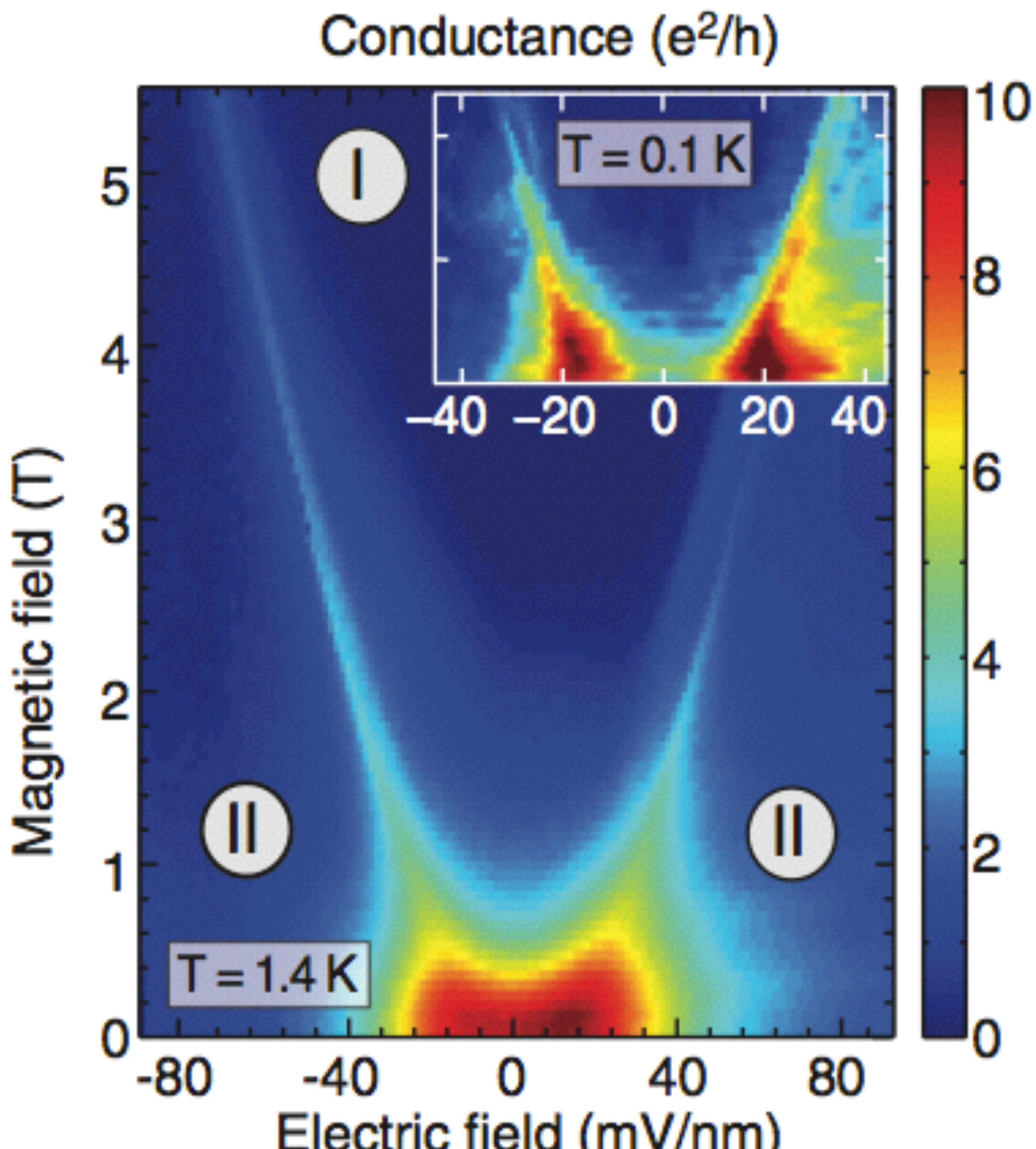
$$\cos\psi = \frac{E_z}{2g_{xy}}$$




A. F. Young *et al., Nature 505* (2014):
tilted magnetic field

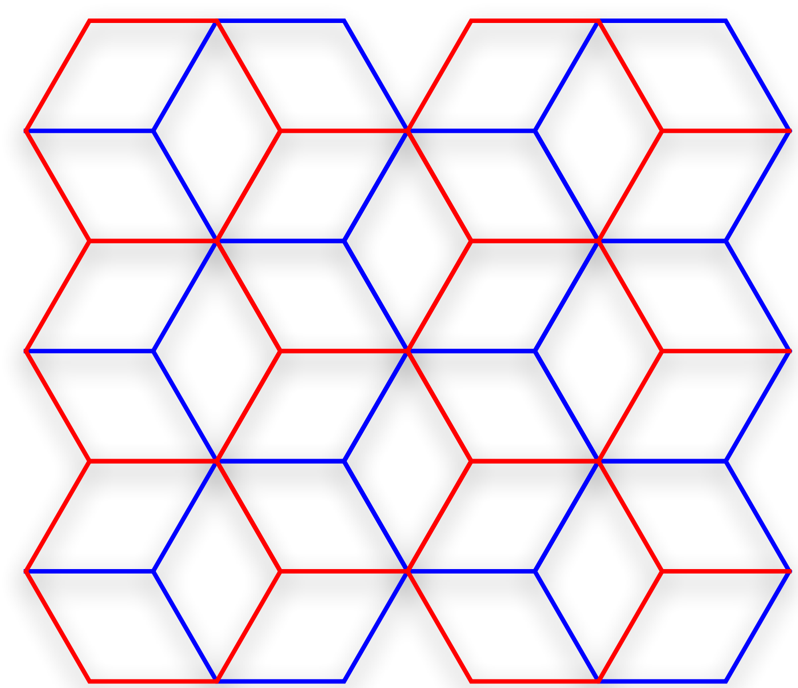
Figure: Efrat Shimshoni

# Conductance in bilayer graphene

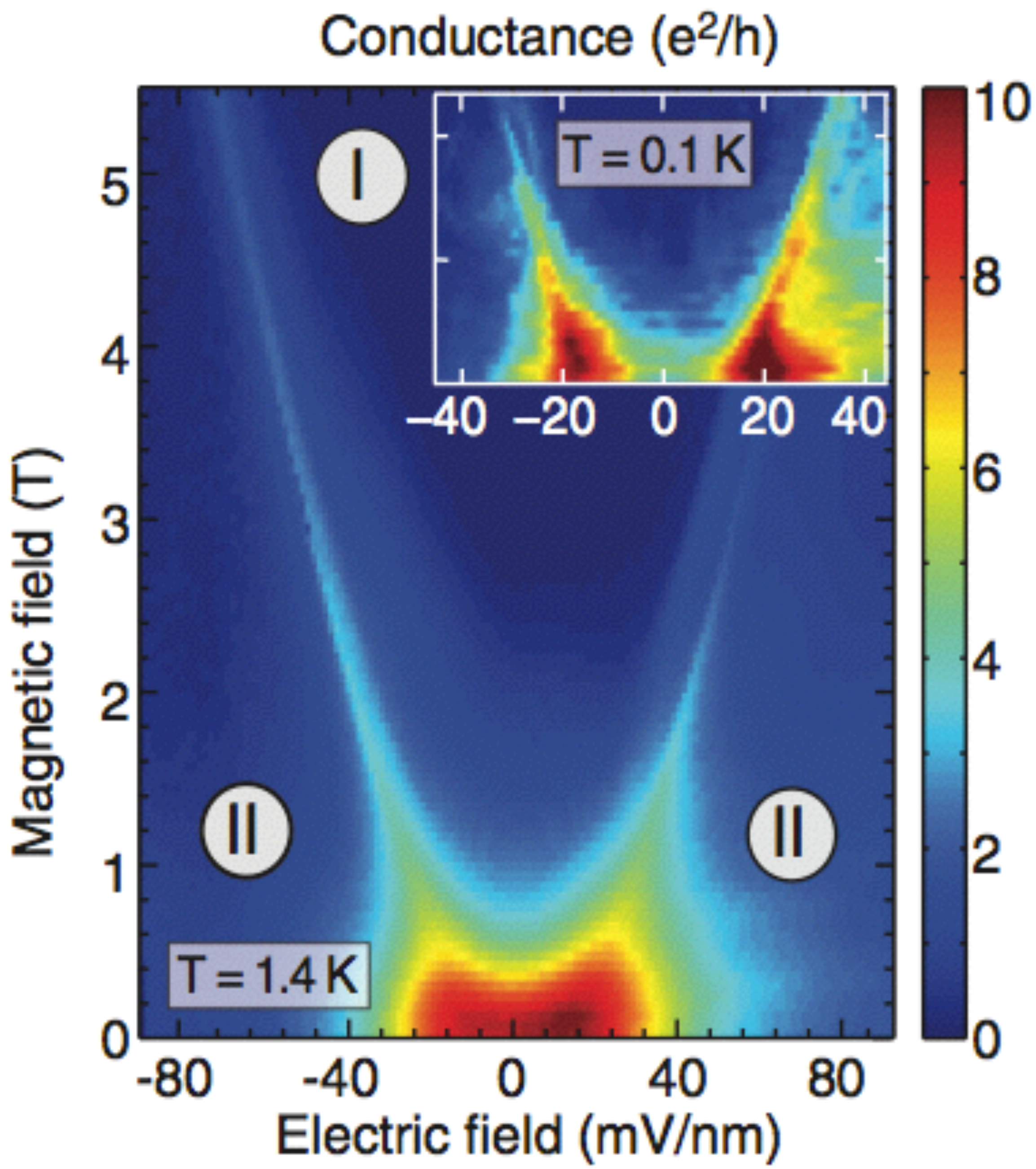



R. T. Weitz *et al.*, *Science*, **330**, 812 (2010)
P. Maher *et al.*, Nature Physics **9**, 154 (2013)

- AB stacked bilayer graphene suspended in setup
- Electric and magnetic field perpendicular to the plane
- $\nu = 0$ filling



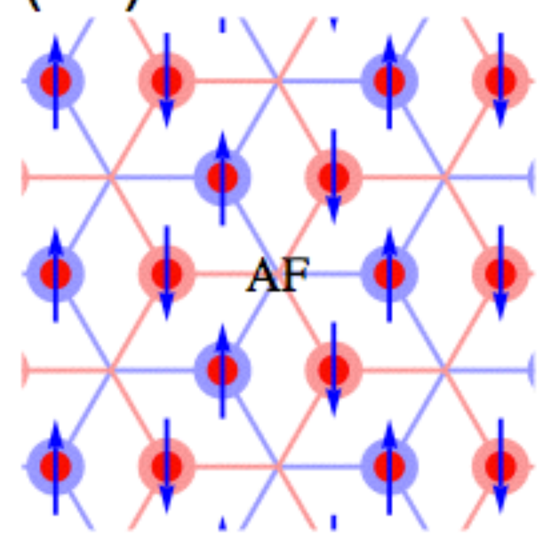
# Conductance in bilayer graphene




R. T. Weitz *et al.*, *Science*, **330**, 812 (2010)
P. Maher *et al.*, Nature Physics **9**, 154 (2013)

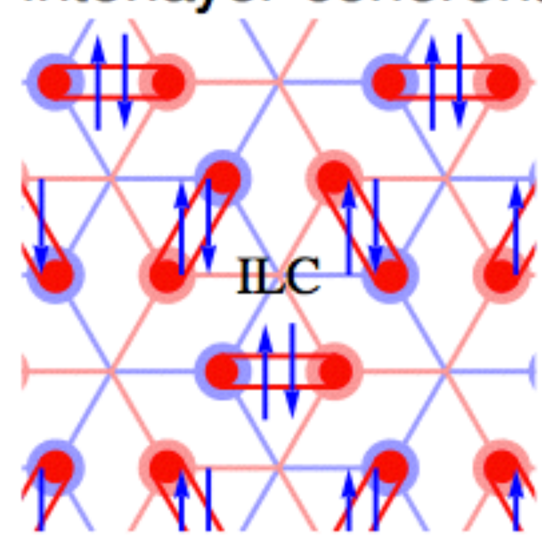
Phase I :

spin antiferromagnetic (AF)



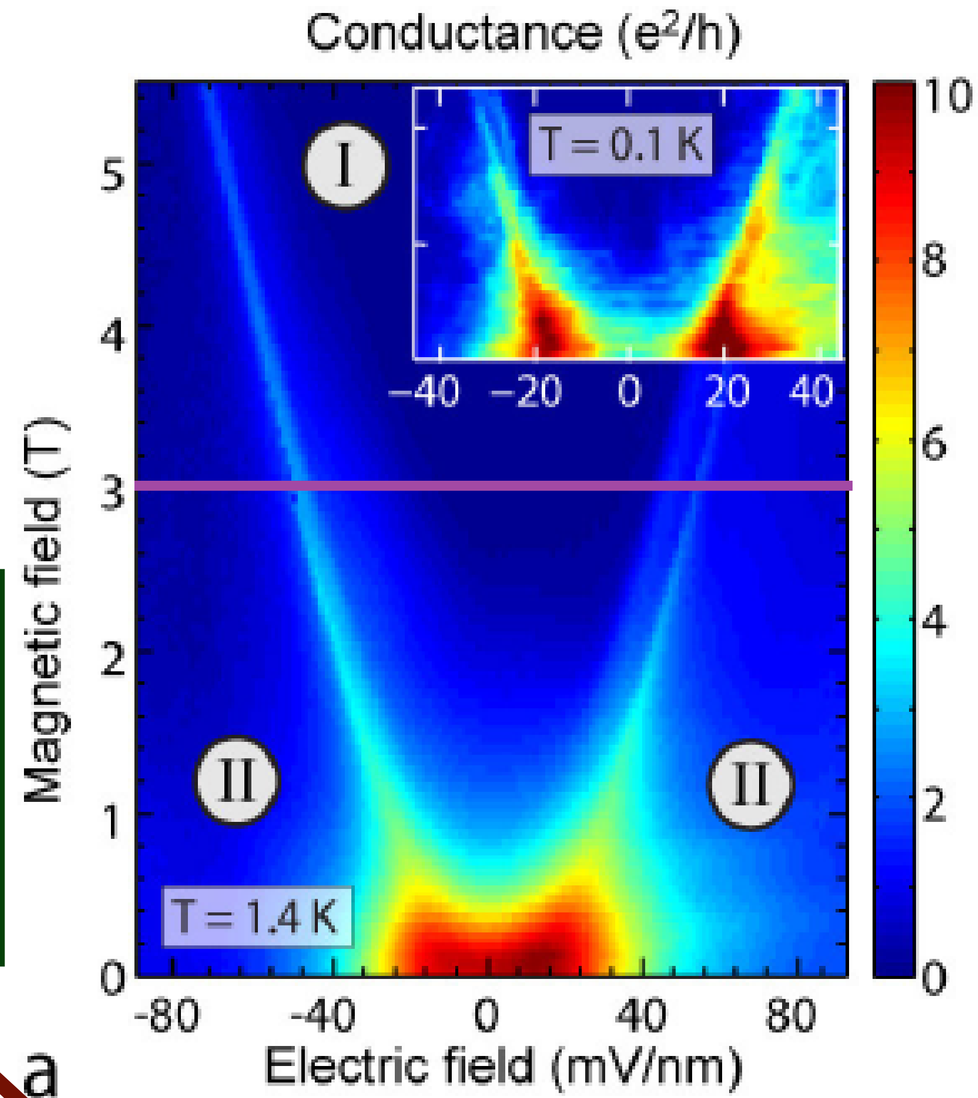

Phase II :

valence bond solid (VBS) or interlayer-coherent (ILC)



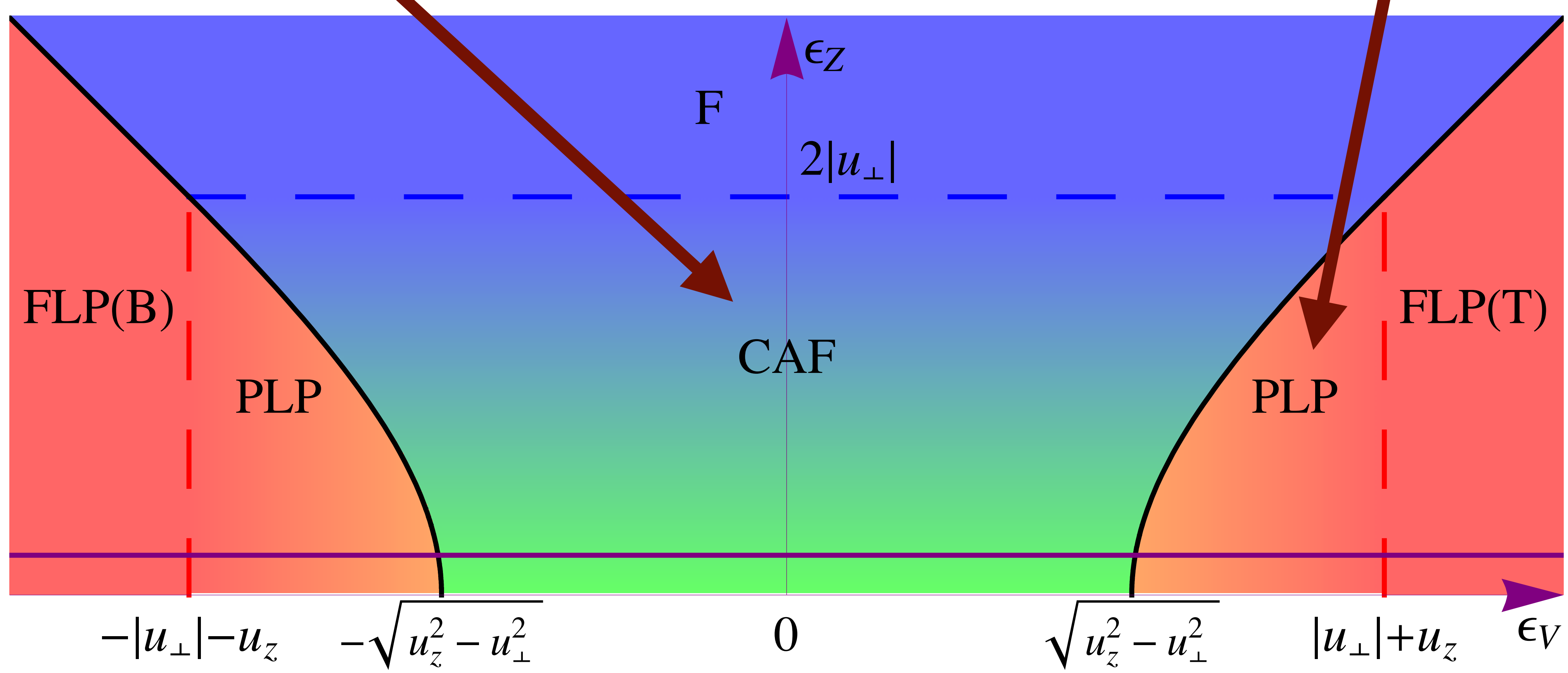

Maxim Kharitonov, *PRL*, **109**, 046803 (2012)

I.
Canted Antiferromagnet (CAF)




II.
Partial layer polarized (PLP) = Interlayer-coherent (ILC) = valence bond solid (VBS)

Maxim Kharitonov,
*PRL*, **109**, 046803 (2012)



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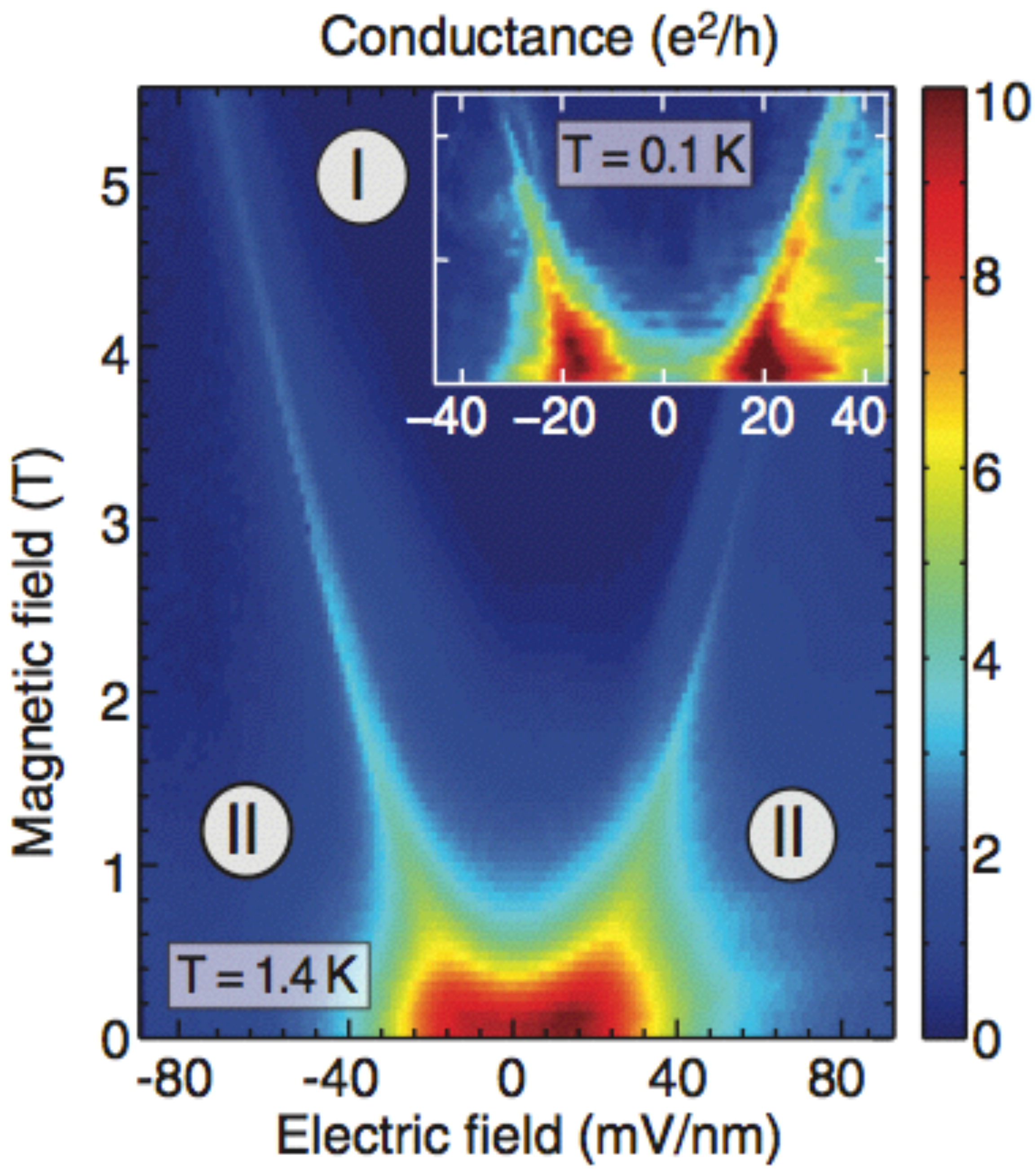


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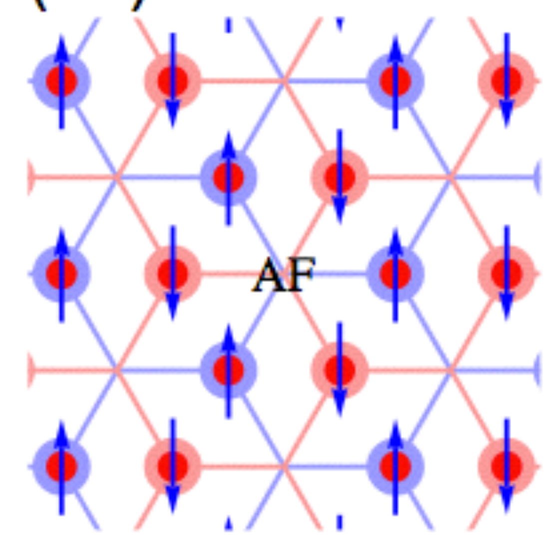
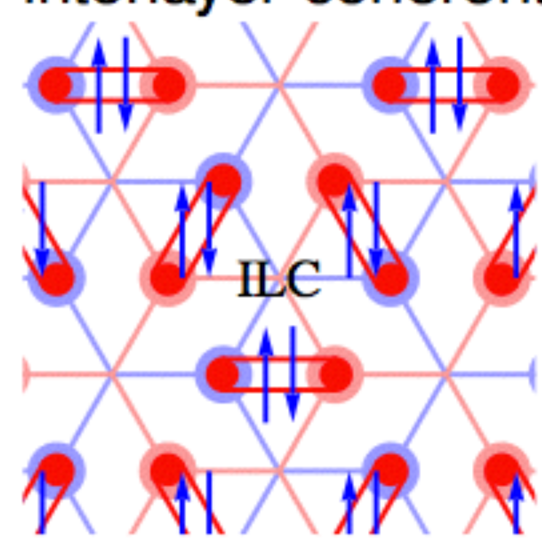



Phase II :

interlayer-coherent (ILC)

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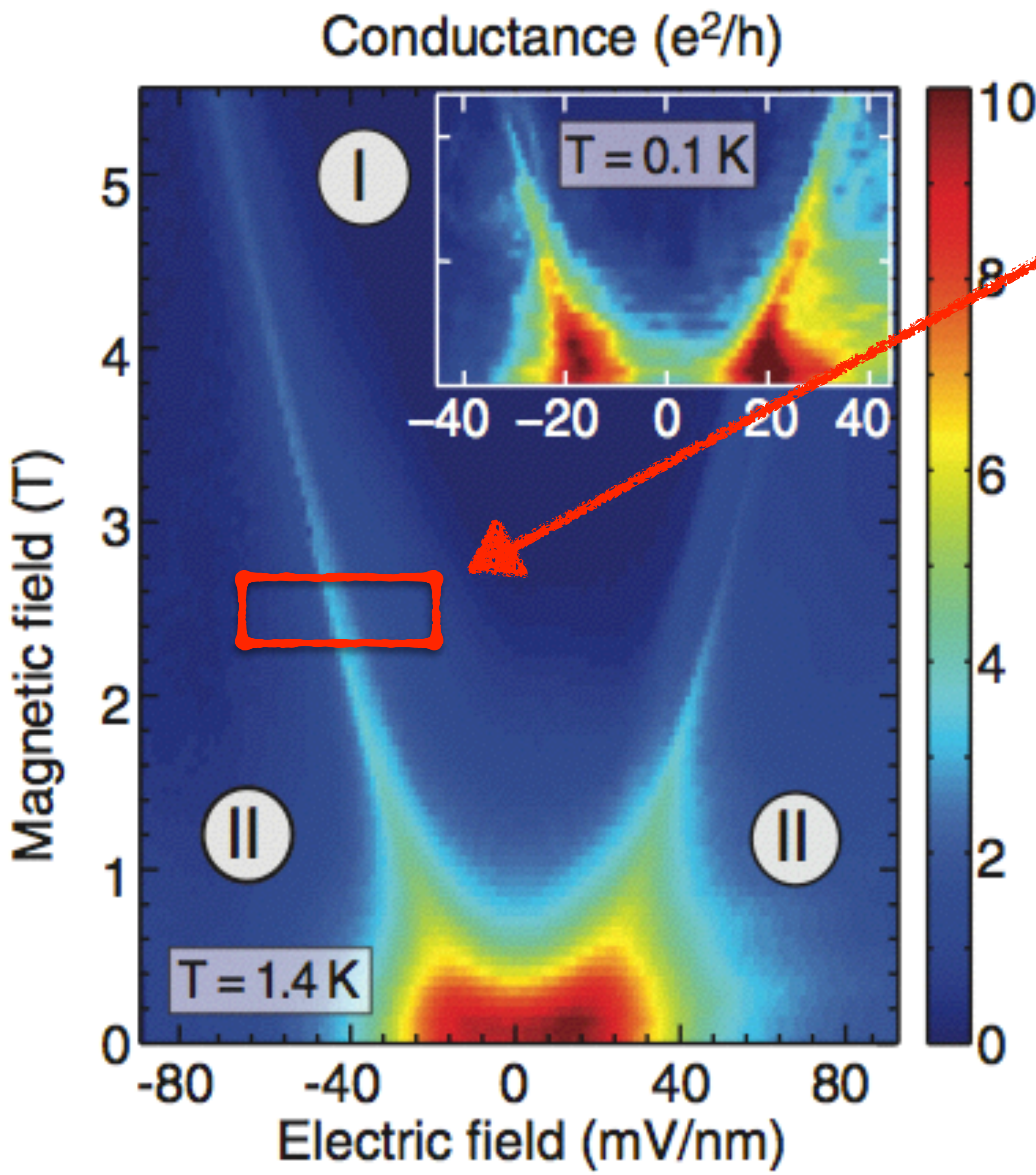
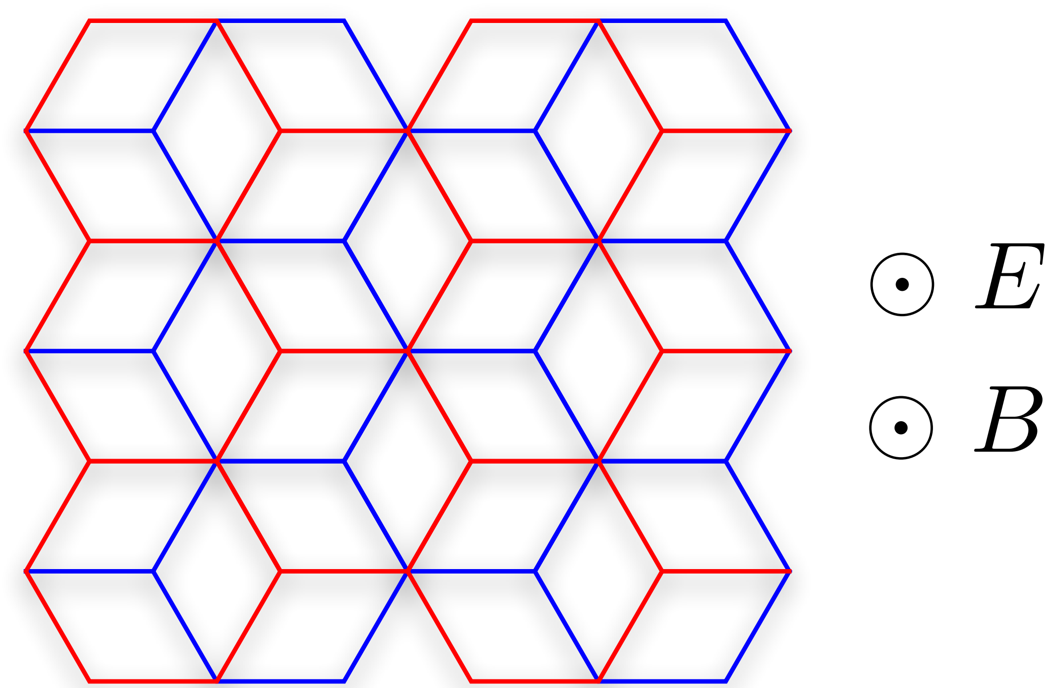
# Conductance in bilayer graphene

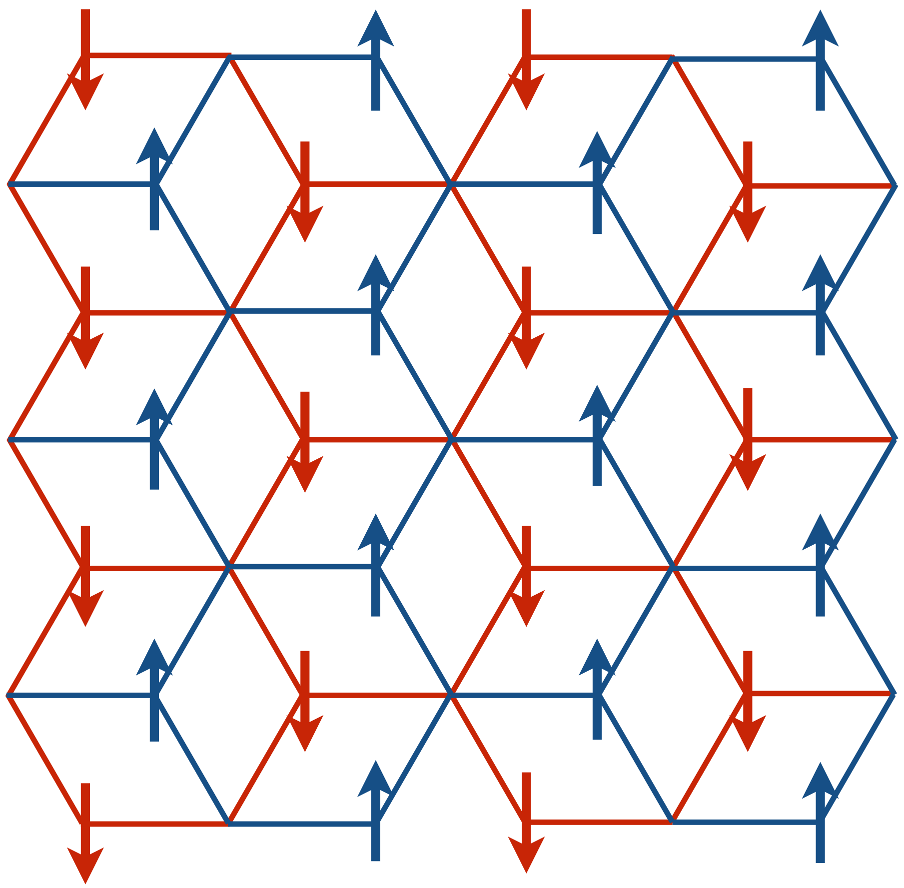


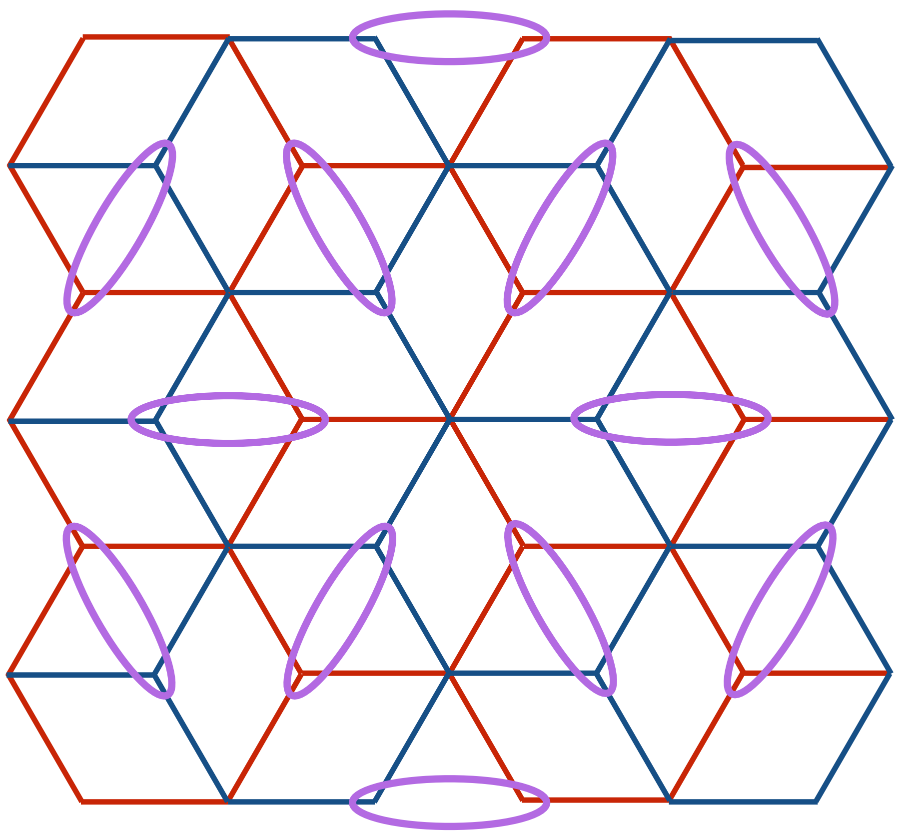

- What is the nature of this transition?
- What is the origin of the conducting transition point?




R. T. Weitz *et al*, *Science*, **330**, 812 (2010)

# Main result





Neel

$$\Gamma_1 = \rho^z \sigma^x$$
$$\Gamma_2 = \rho^z \sigma^y$$
$$\Gamma_3 = \rho^z \sigma^z$$

Valence bond solid

$$\Gamma_4 = \rho^x$$
$$\Gamma_5 = \rho^y$$

# Main result

$$\mathcal{L} = \sum_{\ell=1}^{qN_\Phi} c_\ell^\dagger \frac{\partial c_\ell}{\partial \tau} - \lambda \int d^2\boldsymbol{r}\, n_a(\boldsymbol{r},\tau)\, \Psi^\dagger(\boldsymbol{r},\tau)\Gamma_a \Psi(\boldsymbol{r},\tau)$$

5 component unit vector $n_a(\boldsymbol{r},\tau)$ couples as $n_a\Gamma_a$

<u>Neel</u>

$$\Gamma_1 = \rho^z\sigma^x$$
$$\Gamma_2 = \rho^z\sigma^y$$
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<u>Valence bond solid</u>

$$\Gamma_4 = \rho^x$$
$$\Gamma_5 = \rho^y$$

# Main result

$$\mathcal{S}_{\rm WZW} = 2\pi i q\, W[n_a]$$

$$W[n_a] = \frac{3}{8\pi^2}\int_0^1 du \int d^2\boldsymbol{r}\, d\tau\, \epsilon_{abcde} n_a \partial_x n_b \partial_y n_c \partial_\tau n_d \partial_u n_e$$

5 component unit vector $n_a(\boldsymbol{r},\tau)$ couples as $n_a \Gamma_a$

**Neel**

$$\Gamma_1 = \rho^z \sigma^x$$
$$\Gamma_2 = \rho^z \sigma^y$$
$$\Gamma_3 = \rho^z \sigma^z$$

**Valence bond solid**

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$q = 1, 2$ for mono-,bi-layer graphene

# Main result

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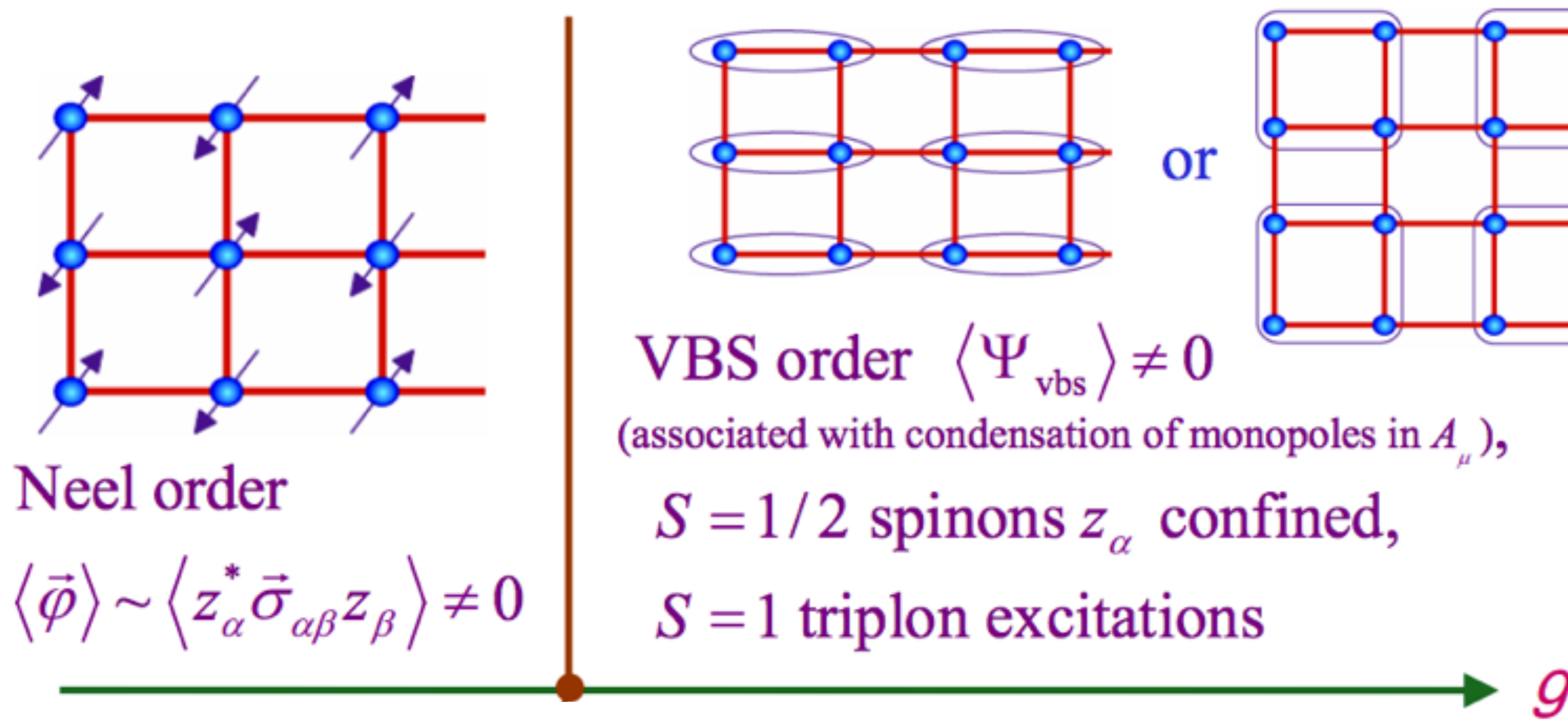
$$W[n_a] = \frac{3}{8\pi^2}\int_0^1 du \int d^2\boldsymbol{r}\, d\tau\, \epsilon_{abcde} n_a \partial_x n_b \partial_y n_c \partial_\tau n_d \partial_u n_e$$

$W[n_a]$ is the Wess-Zumino-Witten term with a quantized coefficient: it computes a Berry phase linking together spatial and temporal textures in the AF and VBS orders. It is a higher dimensional generalization of the Berry phase of a single spin $S$, which is equal to $S$ times the area enclosed by the spin world-line on the unit sphere. Similarly, the WZW term measures the area on the surface of the sphere in the five-dimensional AF and VBS order parameter space.

$q = 1, 2$ for mono-,bi-layer graphene

# Deconfined criticality

- Second order transition between two ordered phases, different from Landau-Ginzburg-Wilson picture
- Gapless 'photon' excitation of a emergent U(1) at critical point
- Typical example is the Neel to VBS transition in square lattice




T. Senthil *et al*, *Science*, **303**, 1490 (2004)

# Connection to deconfined criticality

$O(5)$ non-linear sigma model + level 1 WZW term

↓

$CP^1$ model

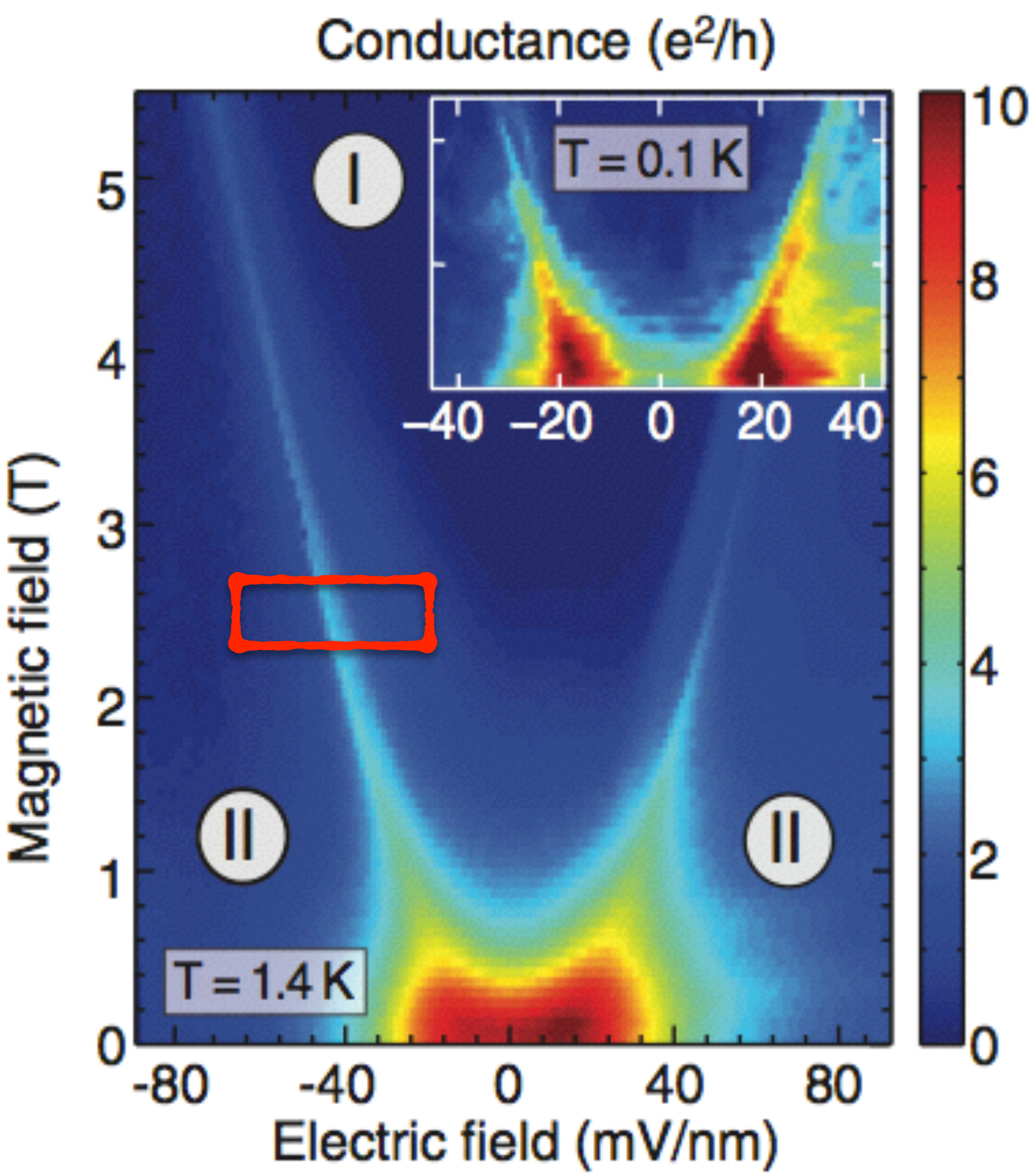
$O(5)$ non-linear sigma model + level 2 WZW term

↓

$CP^2$ model


T. Senthil and M. P.A. Fisher, *Phys. Rev. B*, **74**, 064405 (2006)
T. Grover and T. Senthil, *Phys. Rev. Lett.*, **100**, 156804 (2008)
M. Levin and T. Senthil, *Phys. Rev. B*, **70**, 220403(R) (2004)
T. Grover and T. Senthil, *Phys. Rev. Lett.*, **98**, 247202 (2007)

# Experimental consequencces




- The AF Skyrmion $\vec{N} = (n_1, n_2, n_3)$ in bilayer graphene carries interlayer charge, and so there is a counterflow "critical superfluid".
- With a finite electric field, the interlayer symmetry is broken, and so there is enhanced conductivity, a <u>vestige of the counterflow superfluidity.</u>

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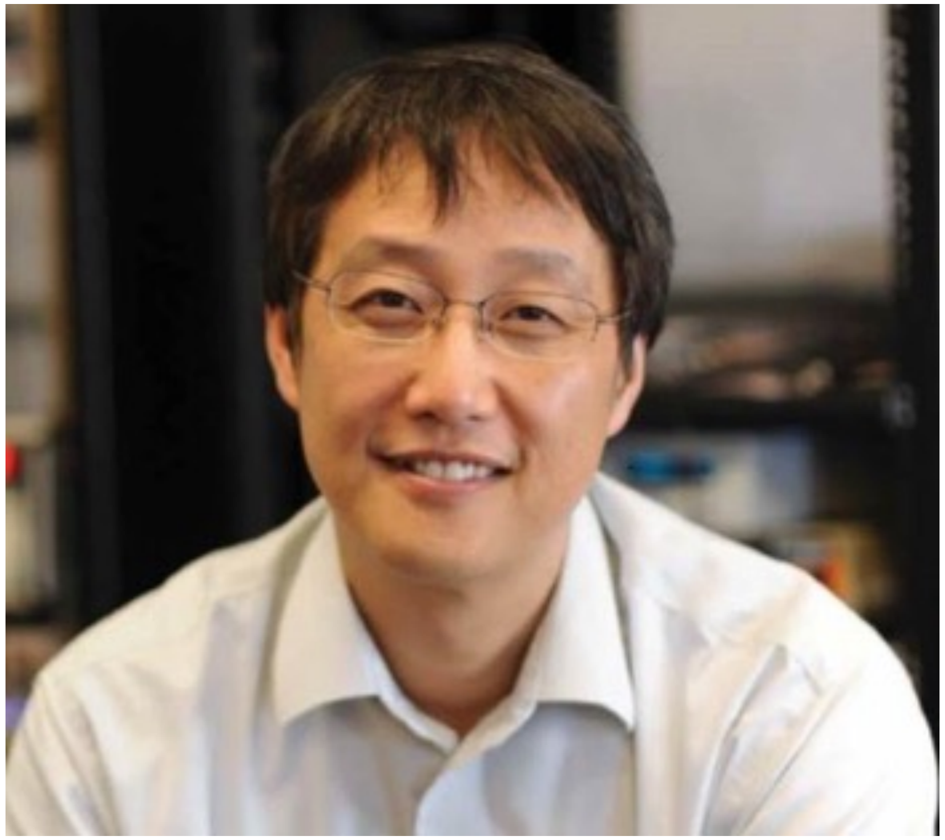
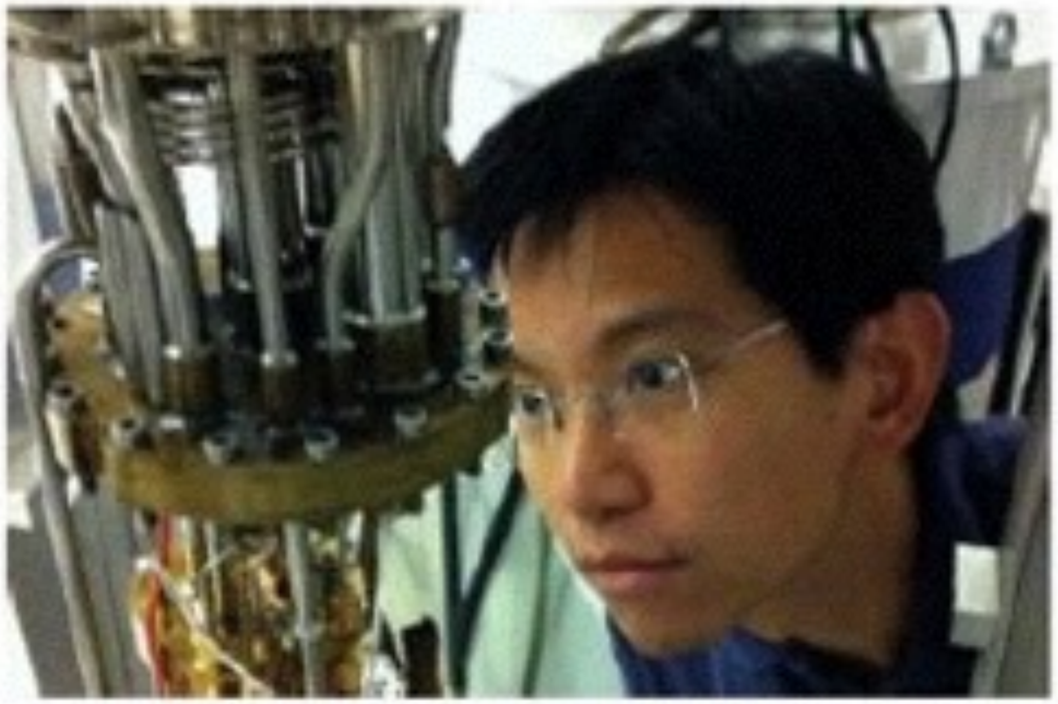
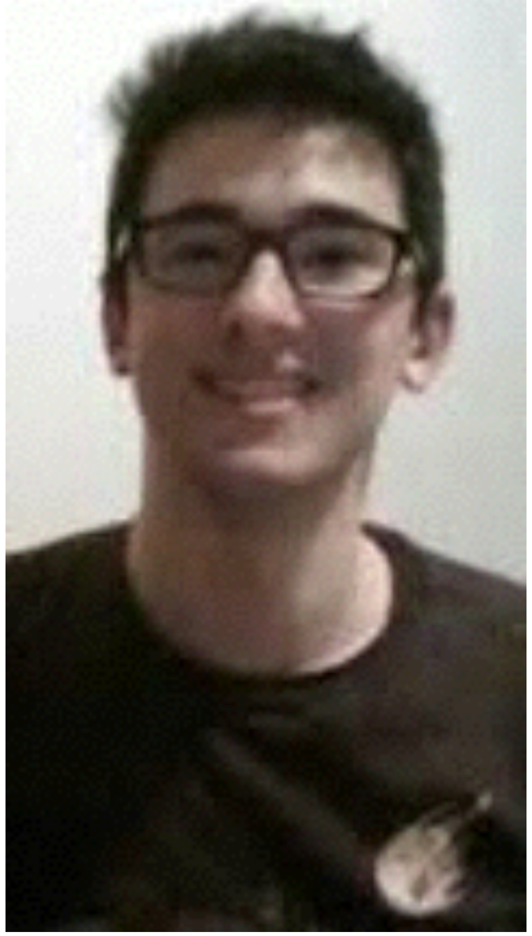
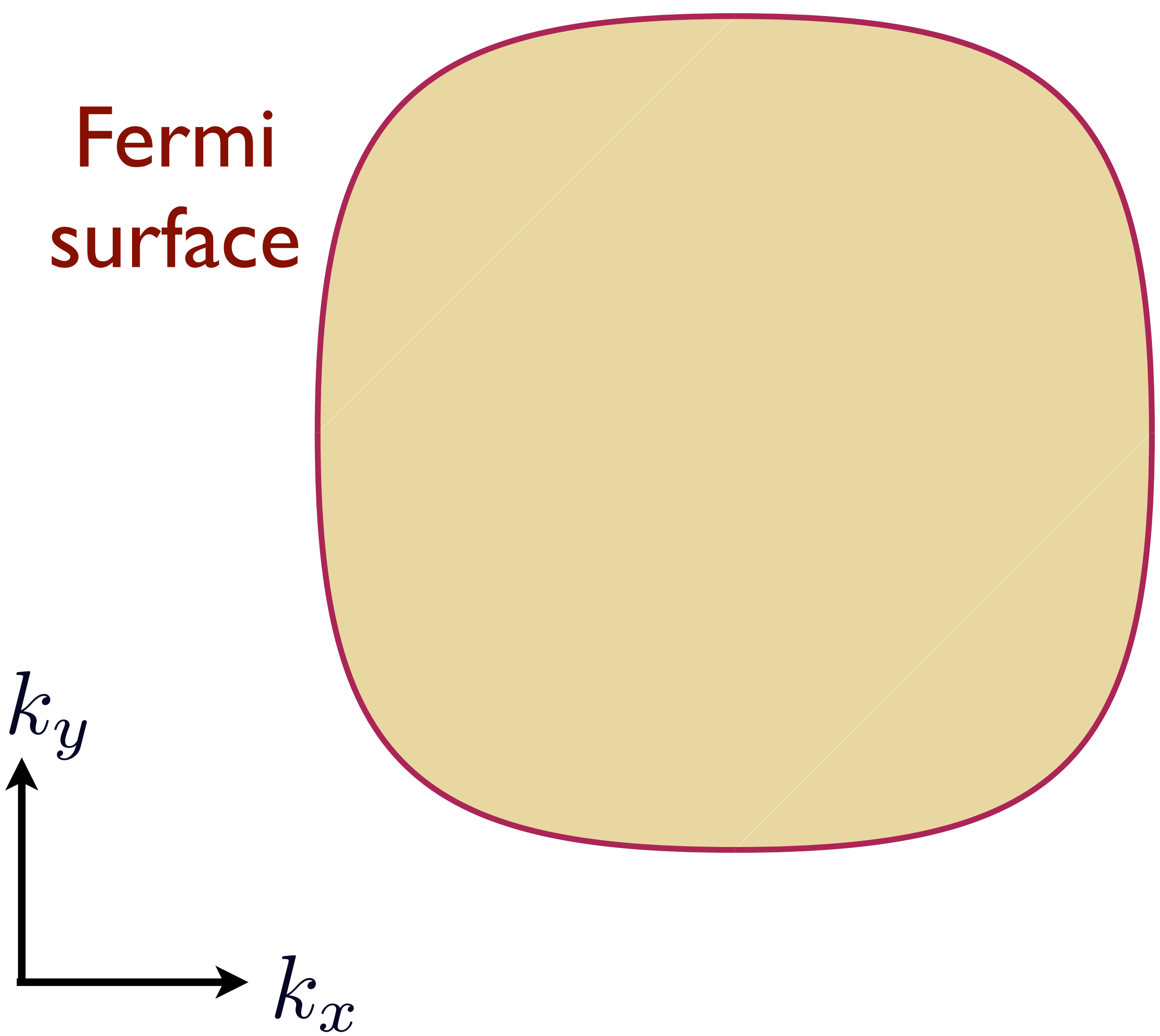

Philip Kim


Jesse Crossno


Kin Chung Fong


Andrew Lucas

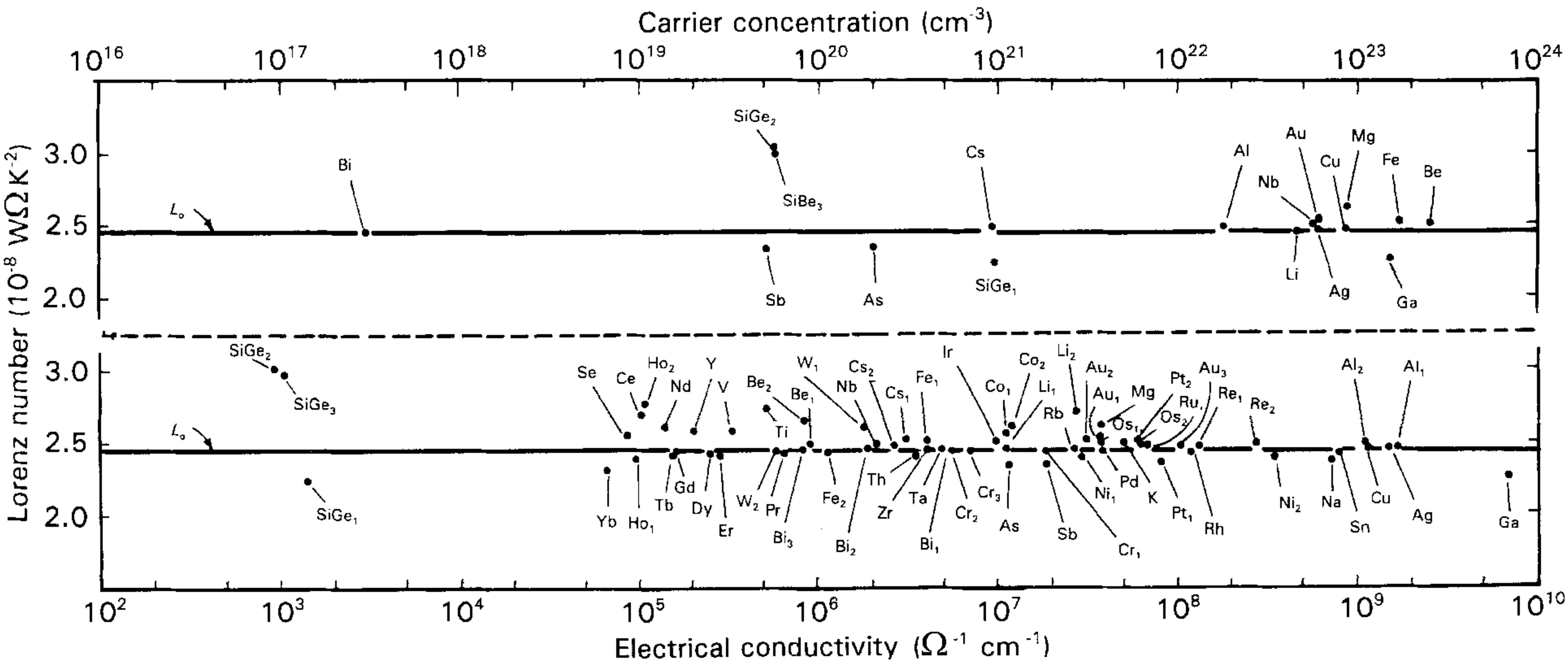
# Ordinary metals: the Fermi liquid




- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface $= \mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.
- $$\frac{(\text{Thermal conductivity})}{T\,(\text{Electrical conductivity})} = \frac{\pi^2 k_B^2}{3e^2} \equiv L_0$$

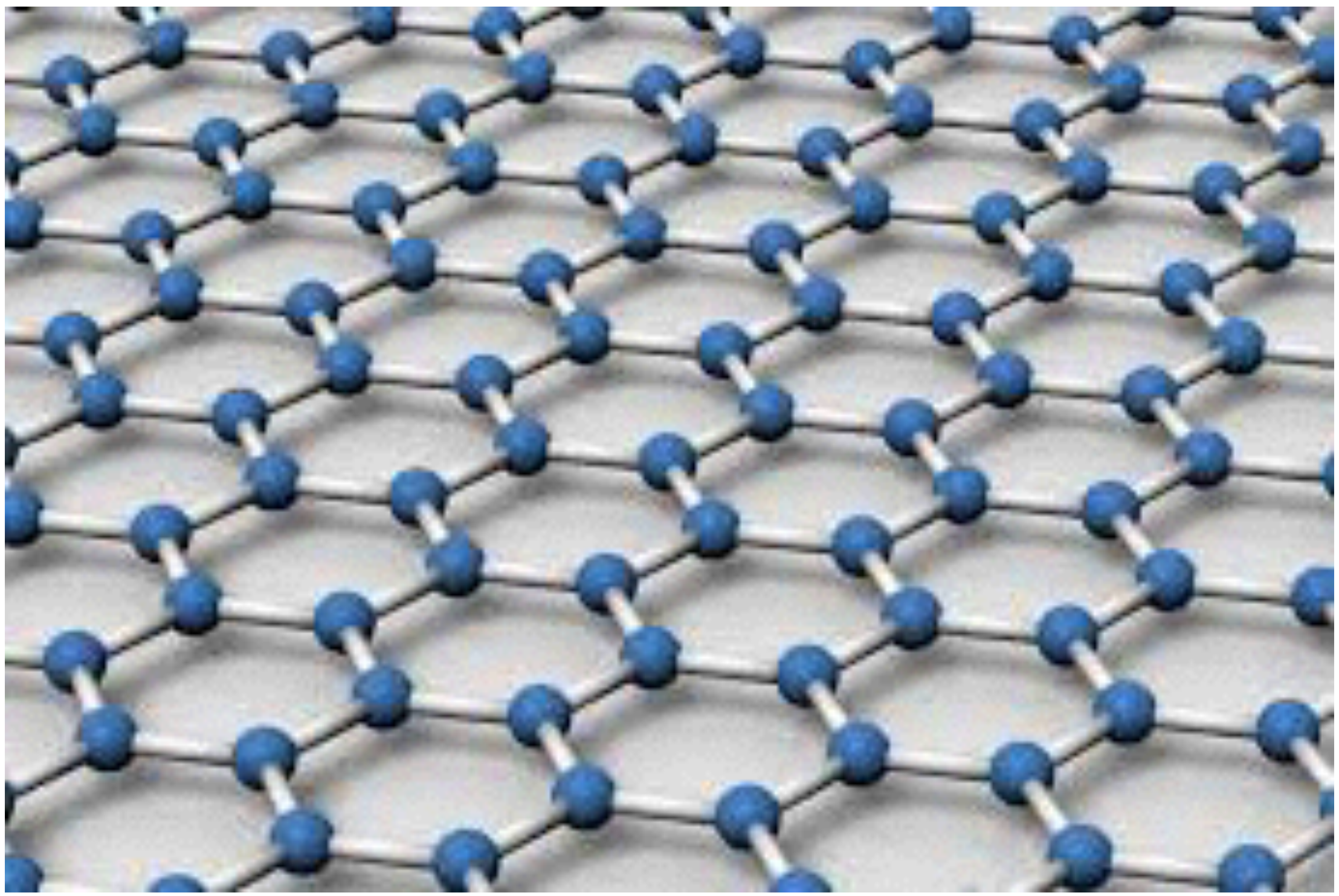
- Wiedemann-Franz law in a Fermi liquid:

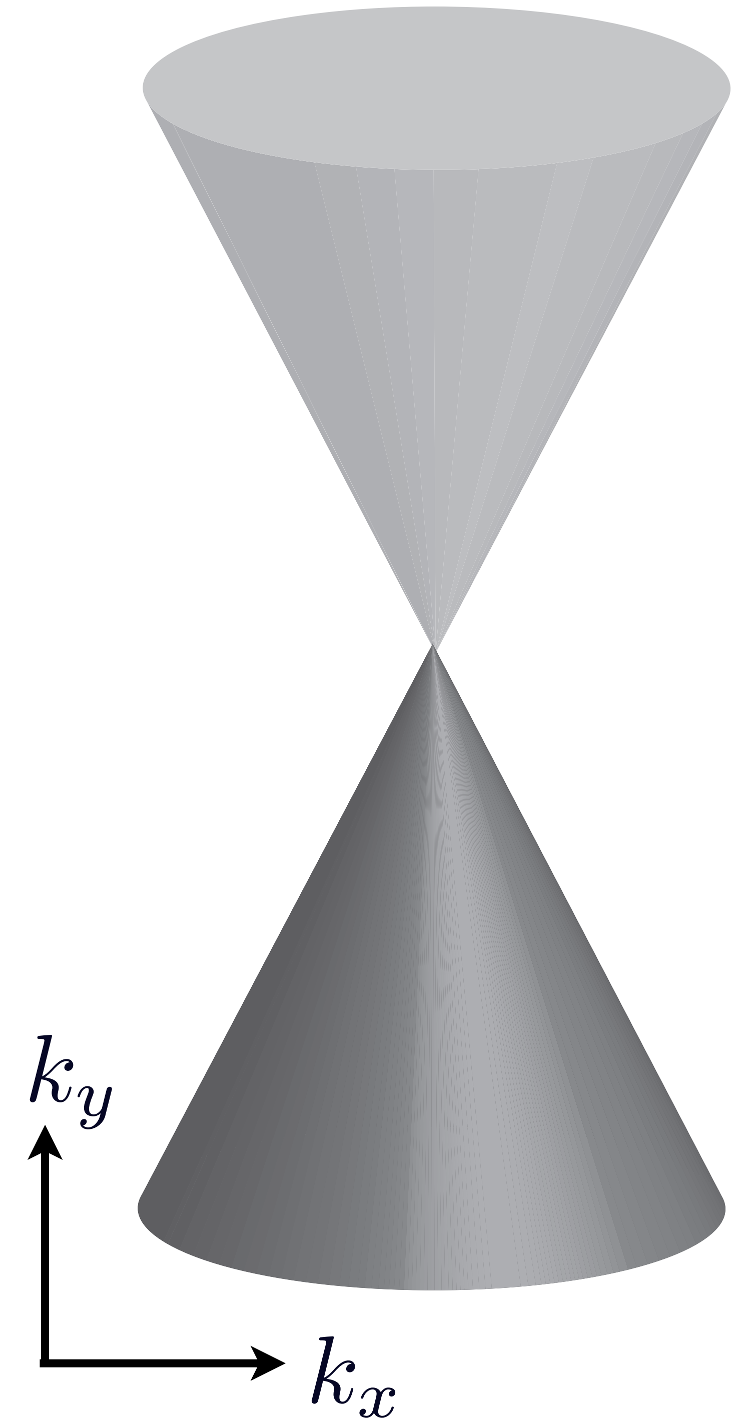
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



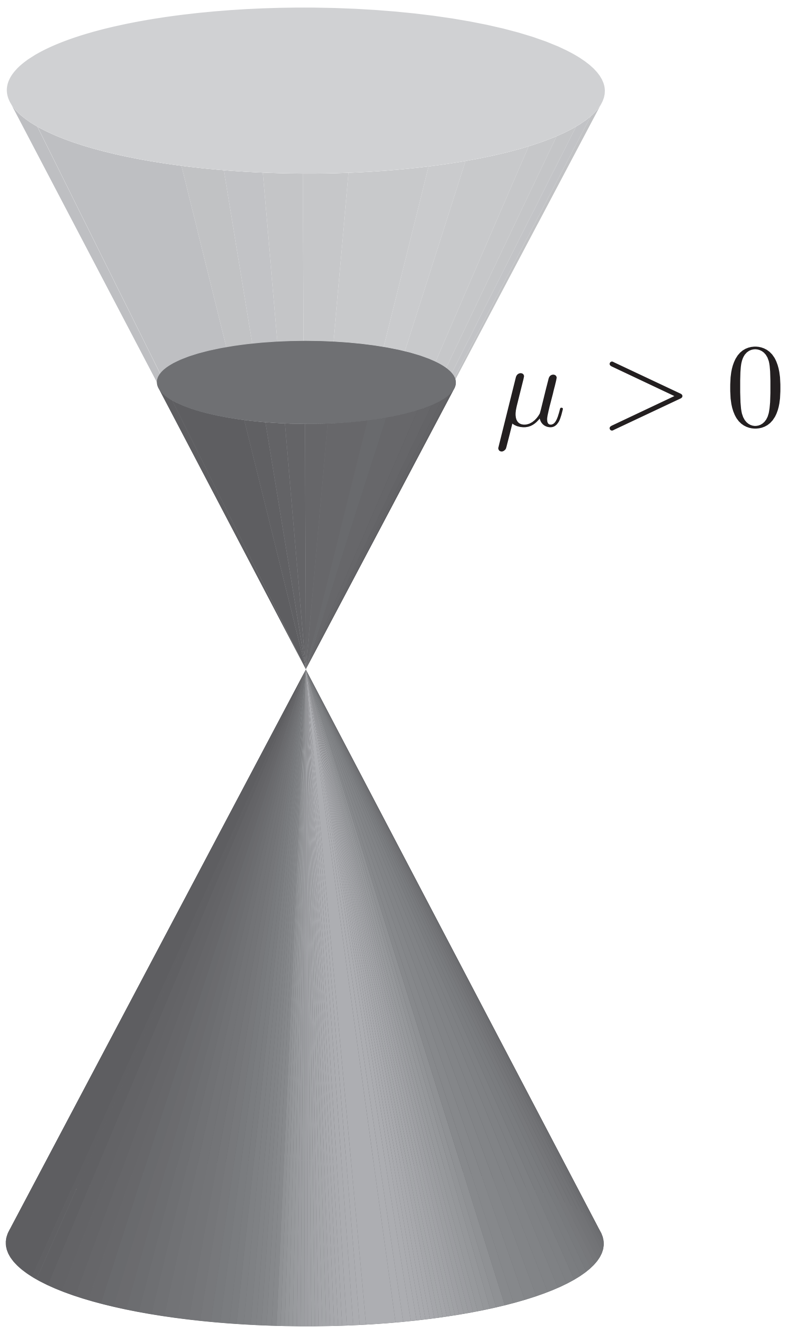

G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

# Graphene





# Graphene



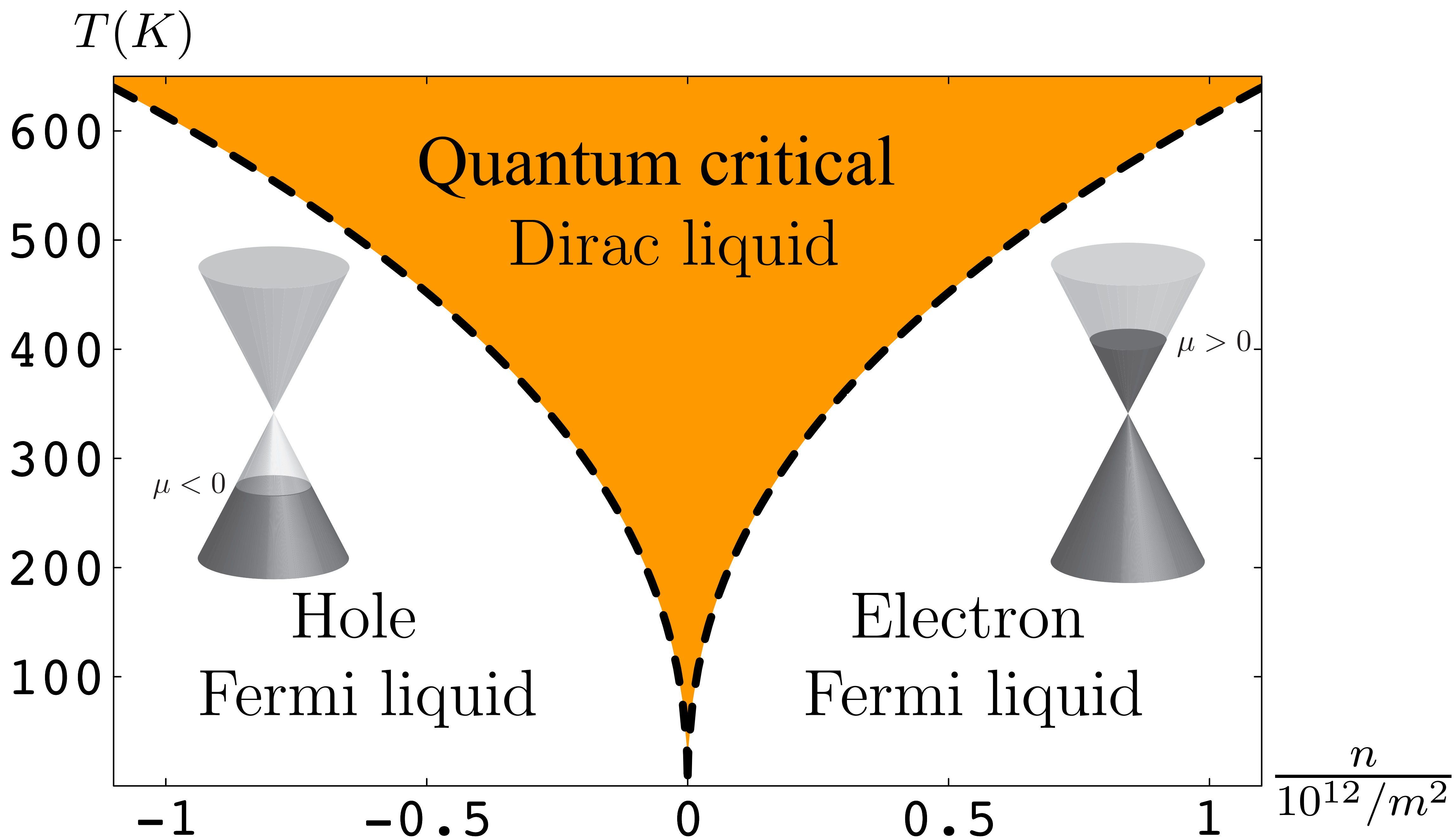

Electron
Fermi surface

# Graphene


Hole
Fermi surface

Electron
Fermi surface

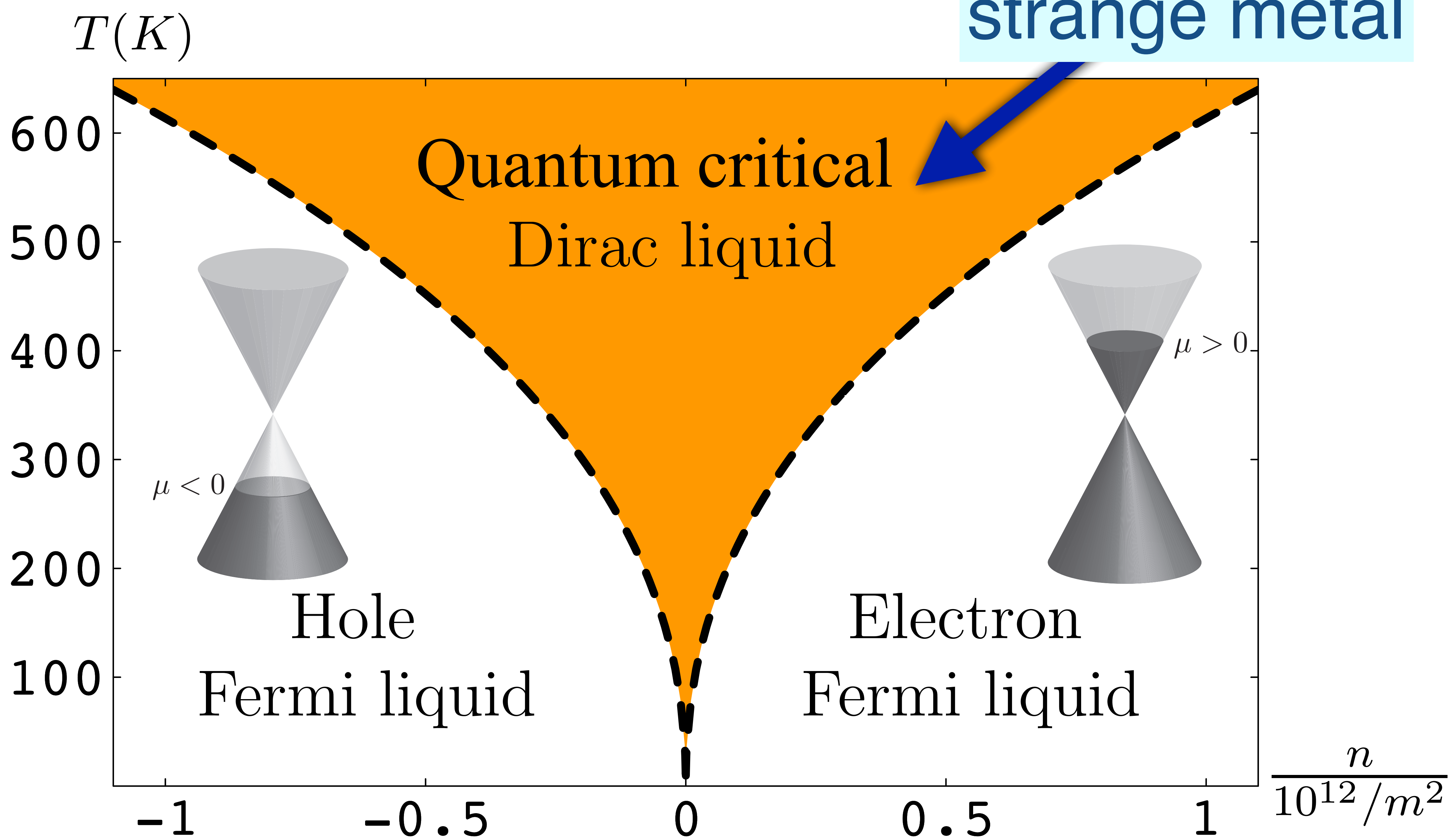
# Graphene




D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

# Graphene

Predicted strange metal

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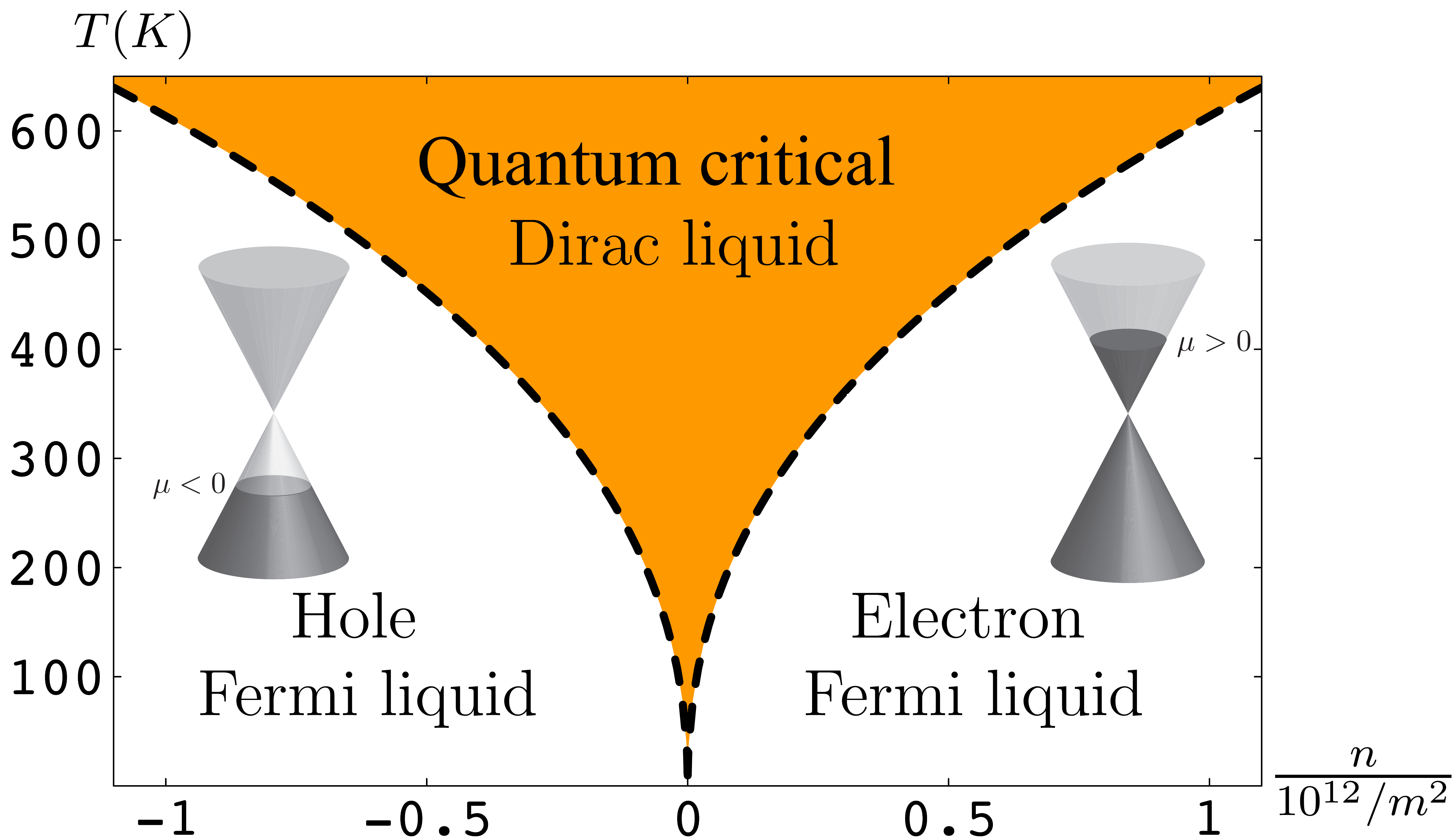
# Key properties of a strange metal

- No quasiparticle excitations

- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

# Graphene

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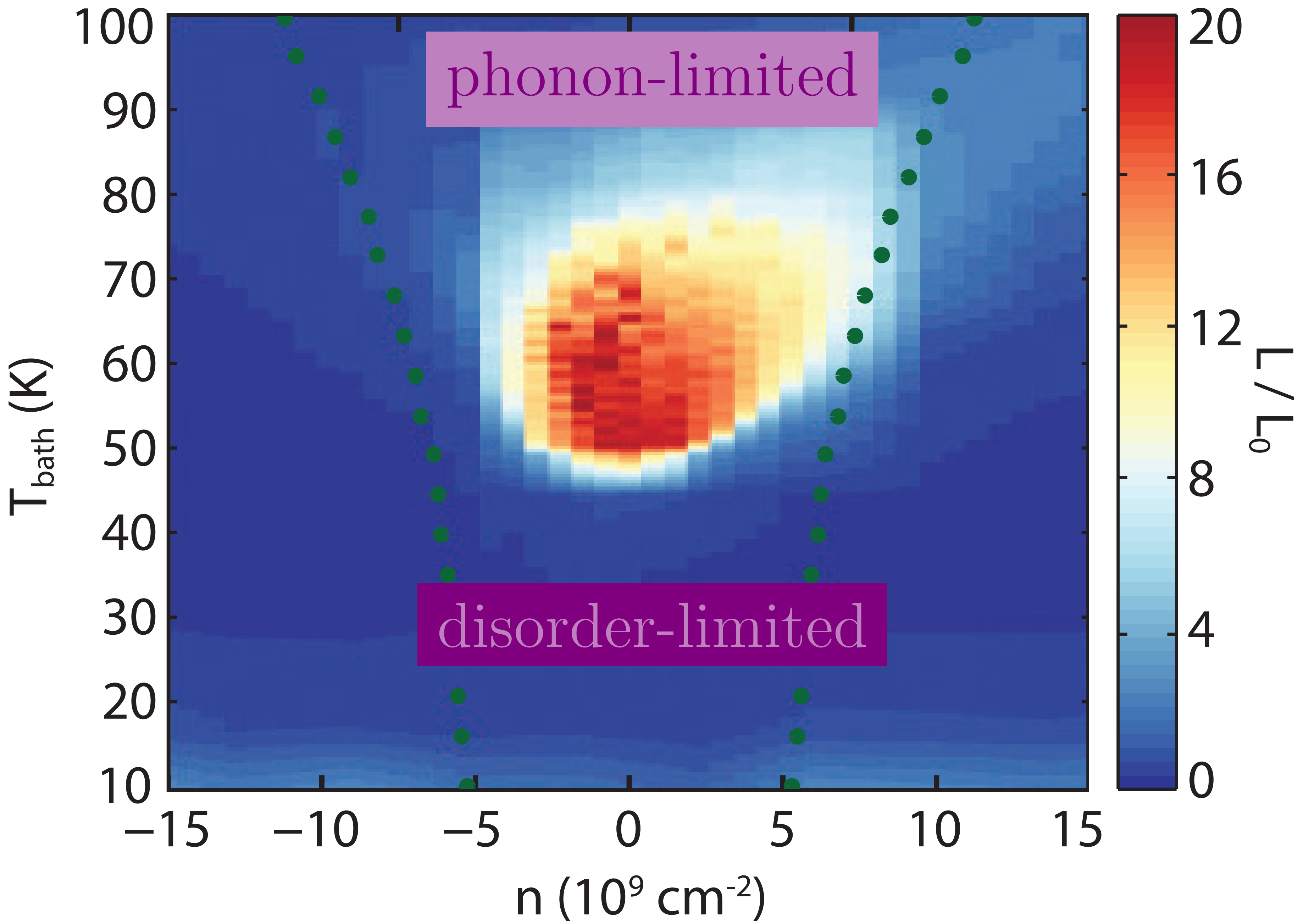
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

# Strange metal in graphene




$$L = \frac{(\text{Thermal conductivity})}{T\,(\text{Electrical conductivity})}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

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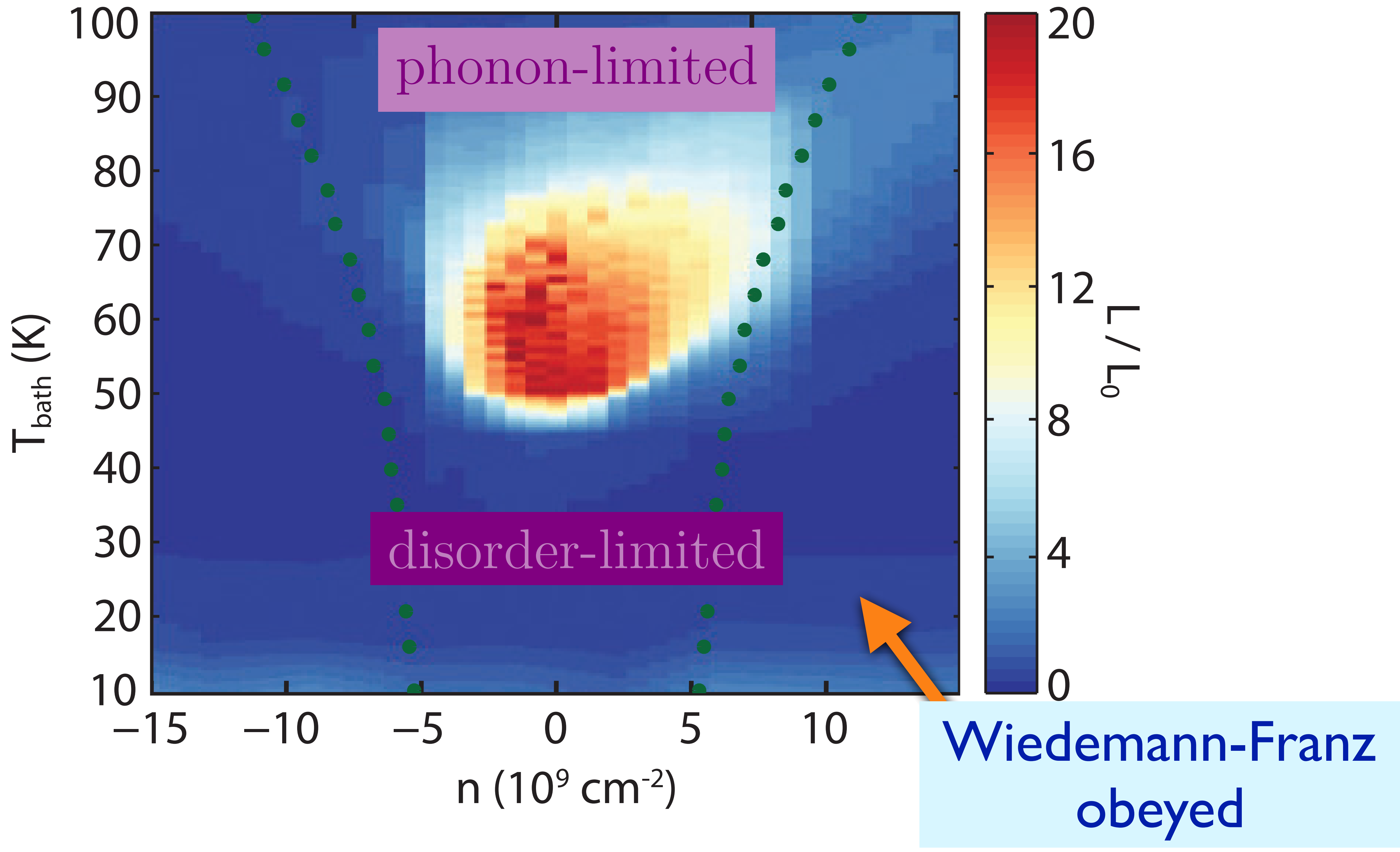


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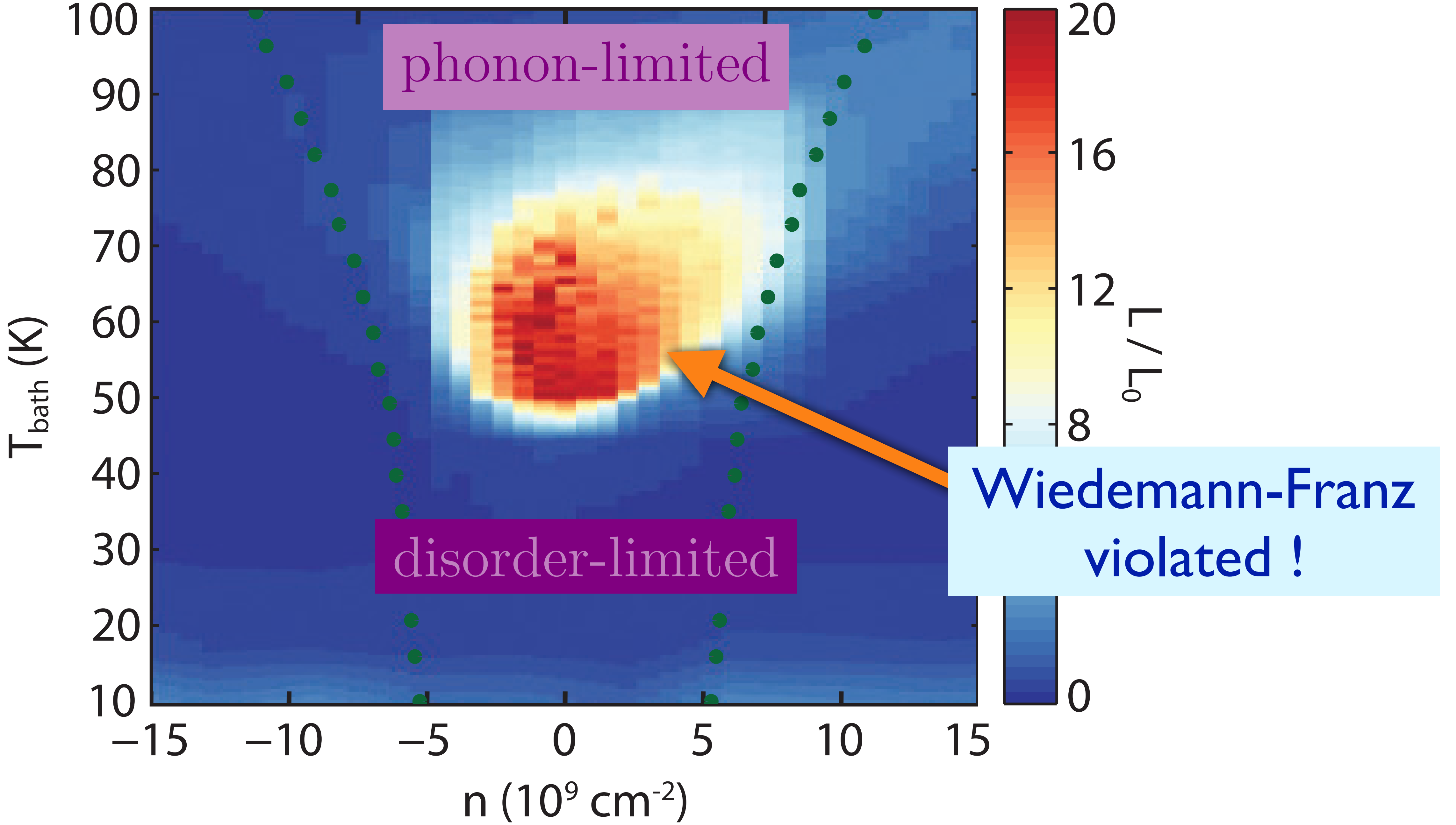


$$L = \frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

# Quasiparticle transport in metals:

- Focus on infinite number of (near) conservation laws (momenta of quasiparticles on the Fermi surface) and compute how they are slowly violated by the lattice or impurities

# Transport in strange metals

- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

# Transport in strange metals

- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

- Focus on relaxation of *total* momentum (including contributions of the Fermi surface (if present) and all critical bosons) by the lattice or impurities

# Transport in Strange Metals

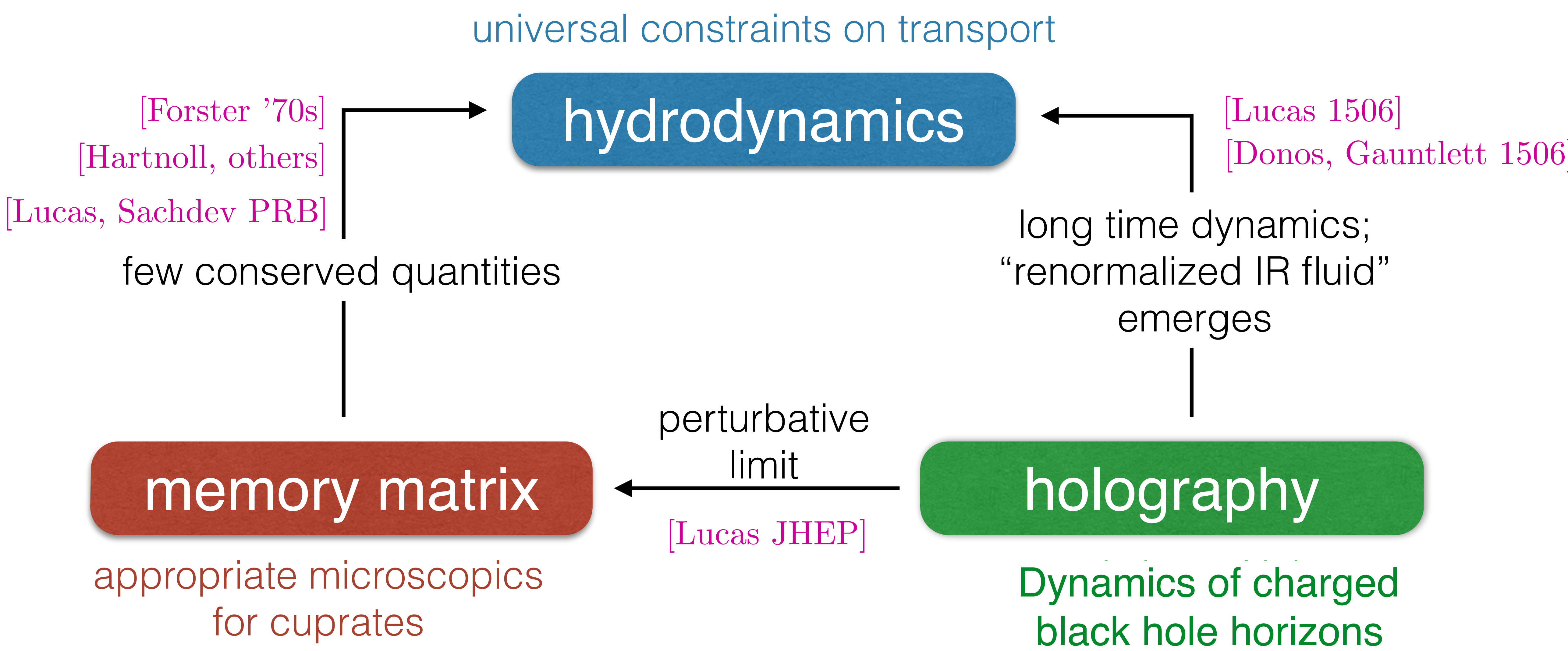


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

# Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute.

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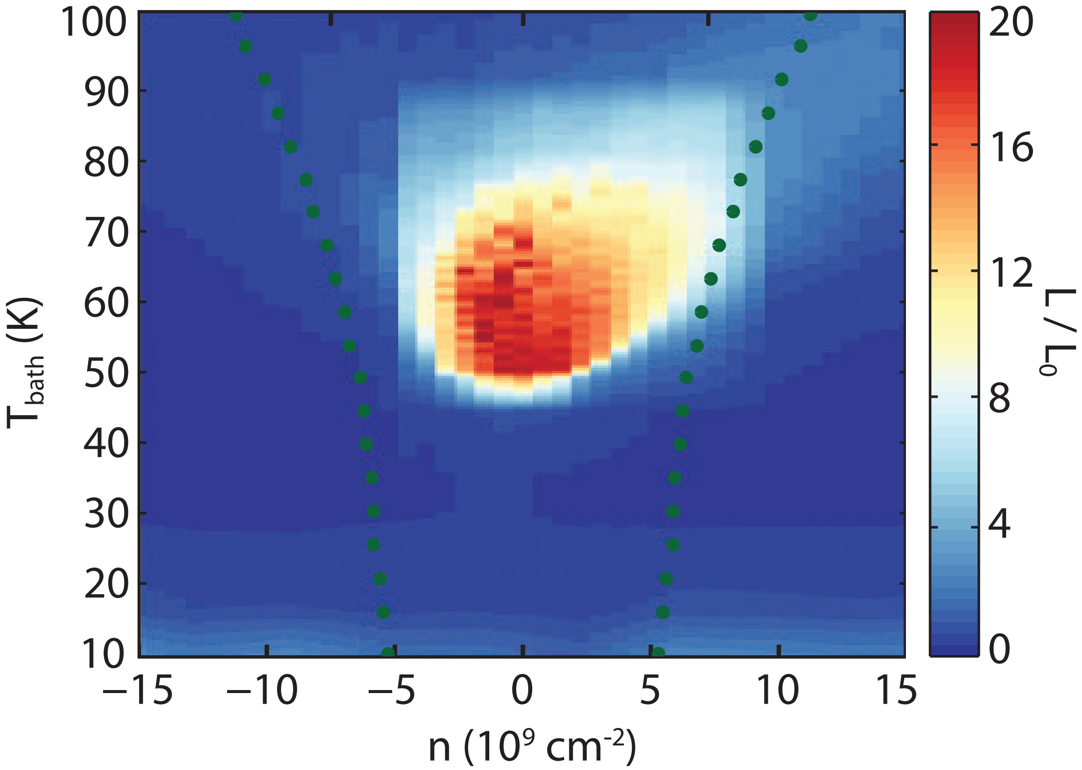


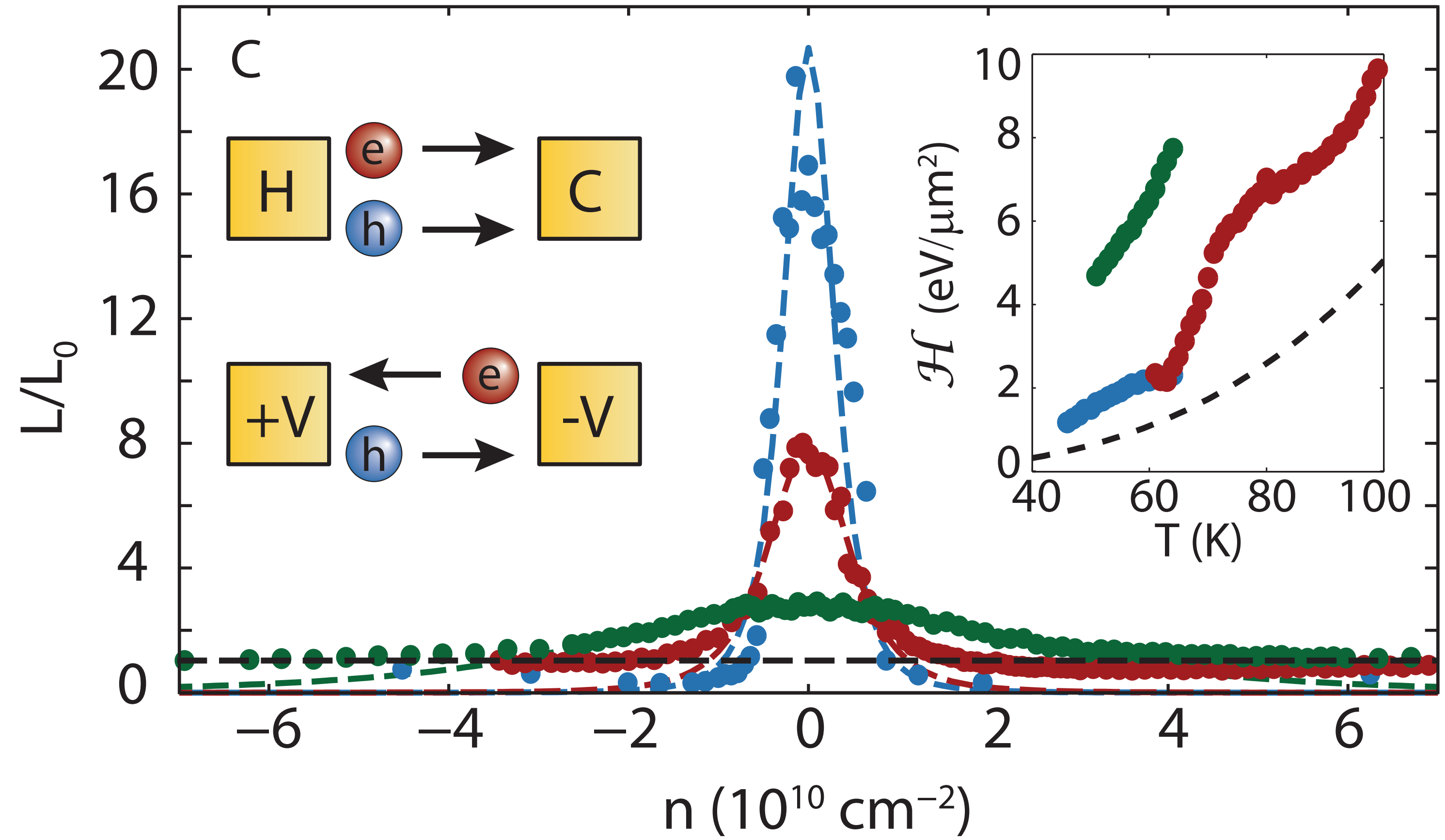


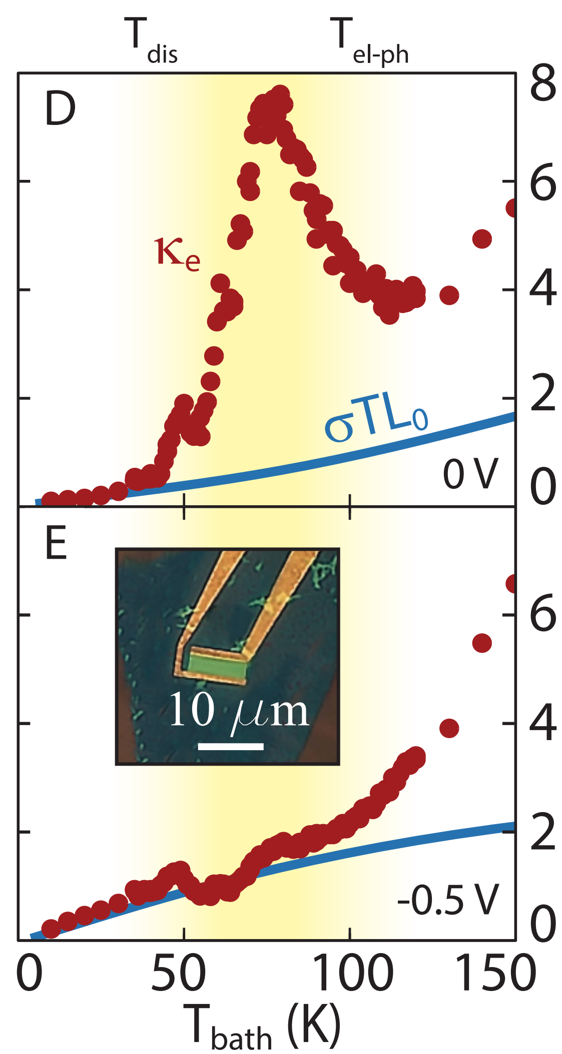


$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$
$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

J. Crossno *et al.* arXiv:1509.04713; Science, to appear

# Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{\rm ee}$, $t \gg t_{\rm ee}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\rm ext}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, $J^\mu$ in perturbative parameter $l_{\rm ee}\partial_\mu$ :

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\rm Q}\mathcal{P}^{\mu\rho}\left(\partial_\rho \mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\rm ext}_{\rho\nu}\right) + \cdots ,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

- New (and only) transport co-efficient, $\sigma_Q$: "quantum critical" conductivity at $\mathcal{Q} = 0$.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

# Translational symmetry breaking

Momentum relaxation by an external source $h$ coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\text{imp}}} + \ldots$$

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A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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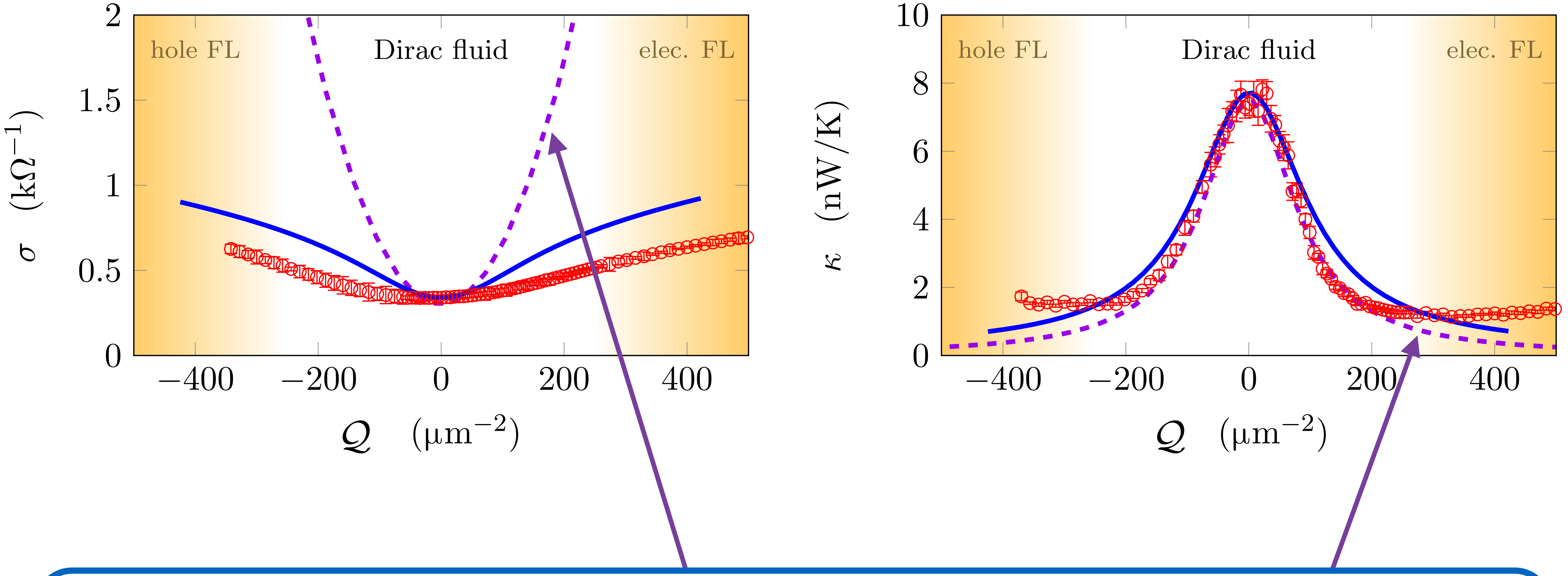
Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\text{imp}}} + \ldots$$

"Memory function" methods yield an explicit expression for $\tau_{\text{imp}}$:

$$\frac{\mathcal{M}}{\tau_{\text{imp}}} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\text{Im}\left( G^{\text{R}}_{\mathcal{O}\mathcal{O}}(q,\omega) \right)_{H_0}}{\omega} + \ldots$$

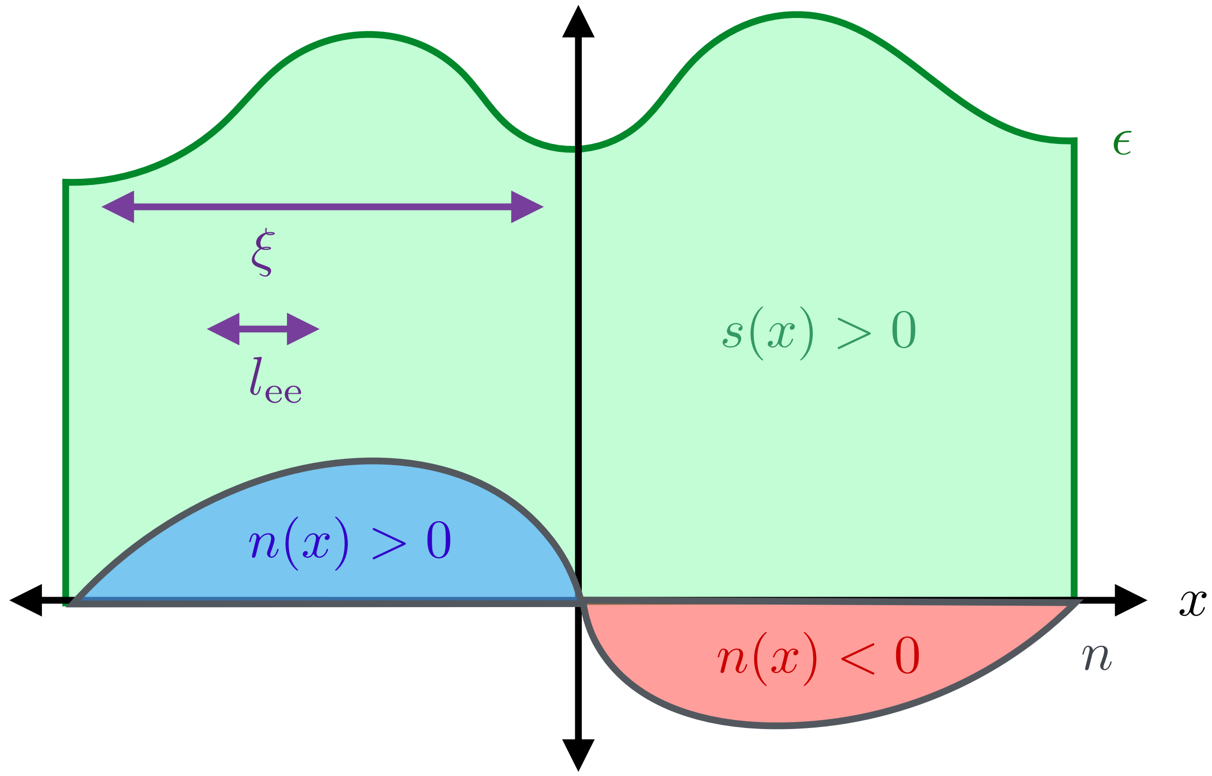
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)




Comparison to theory with a single momentum relaxation time $\tau_{imp}$. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

# Non-perturbative treatment of disorder

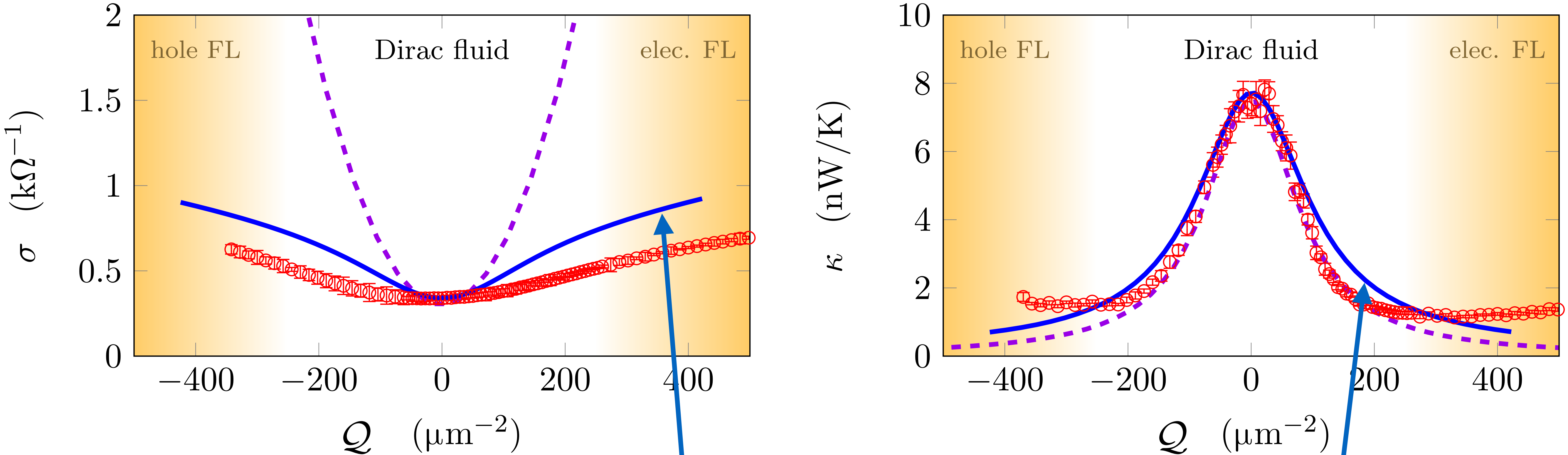



Note
$n \equiv \mathcal{Q}$

**Figure 3:** A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\text{B}}T$, and so the entropy density $s/k_{\text{B}}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density $\epsilon$ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder $\xi$ is much larger than $l_{\text{ee}}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a $x$-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, $\eta$.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear




Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

# Quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
  (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
  /memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.

1. Competing quantum orders in the zeroth Landau level of graphene

2. Dirac liquid in graphene: quantum matter without quasiparticles