Strange Metals and Black Holes

Quantum Spacetime Seminar
Tata Institute for Fundamental Research, Mumbai
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Talk online: sachdev.physics.harvard.edu

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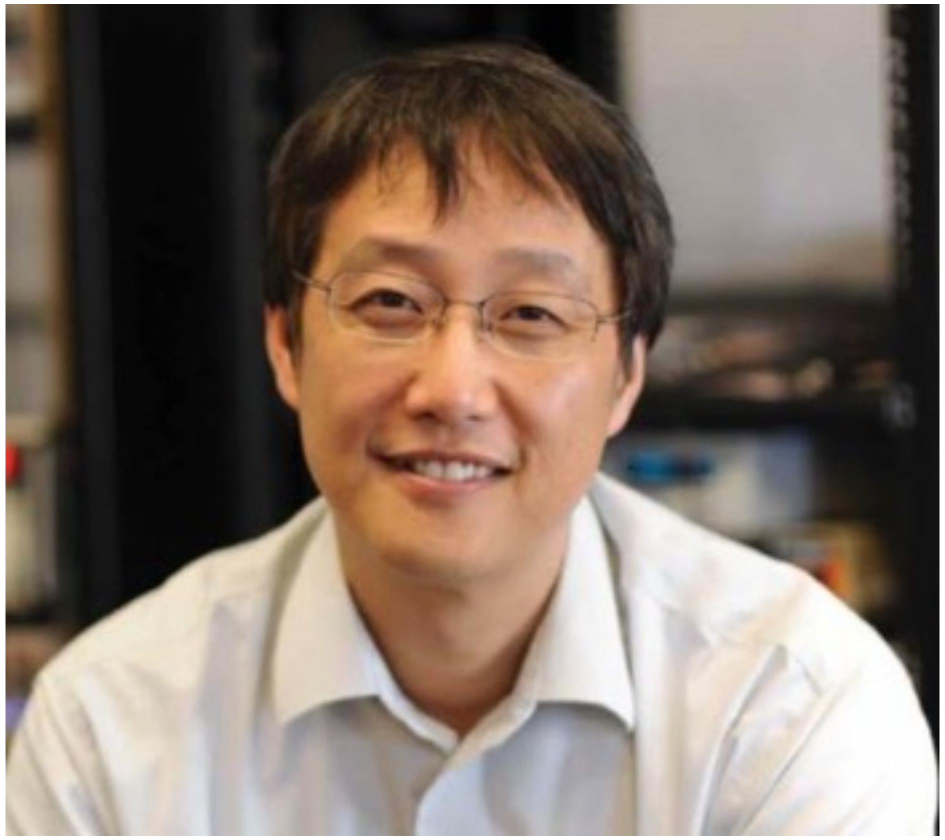

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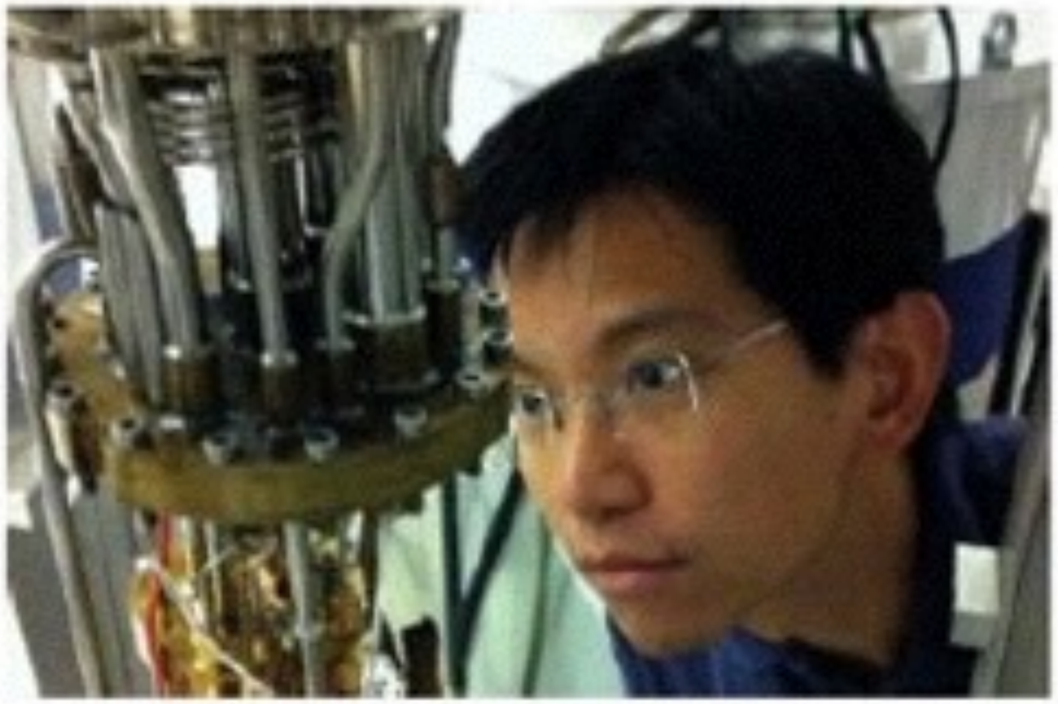
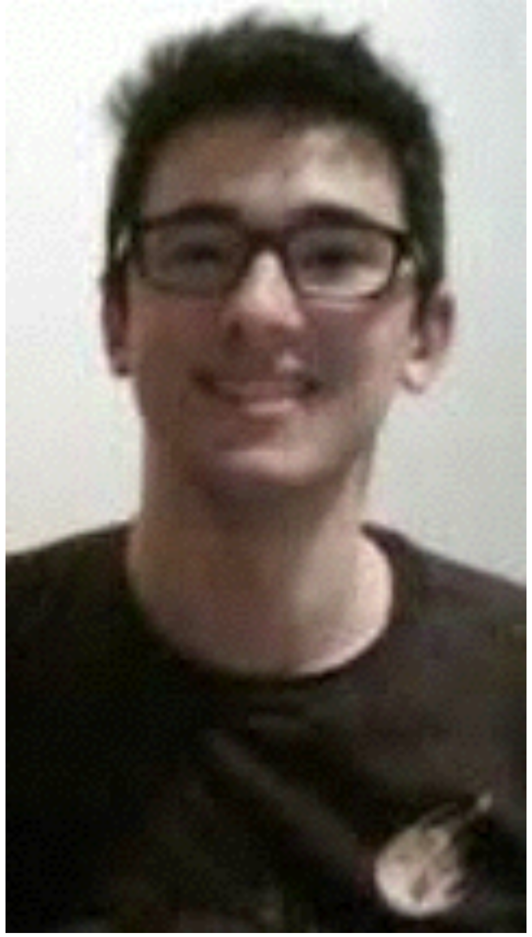
Jesse Crossno

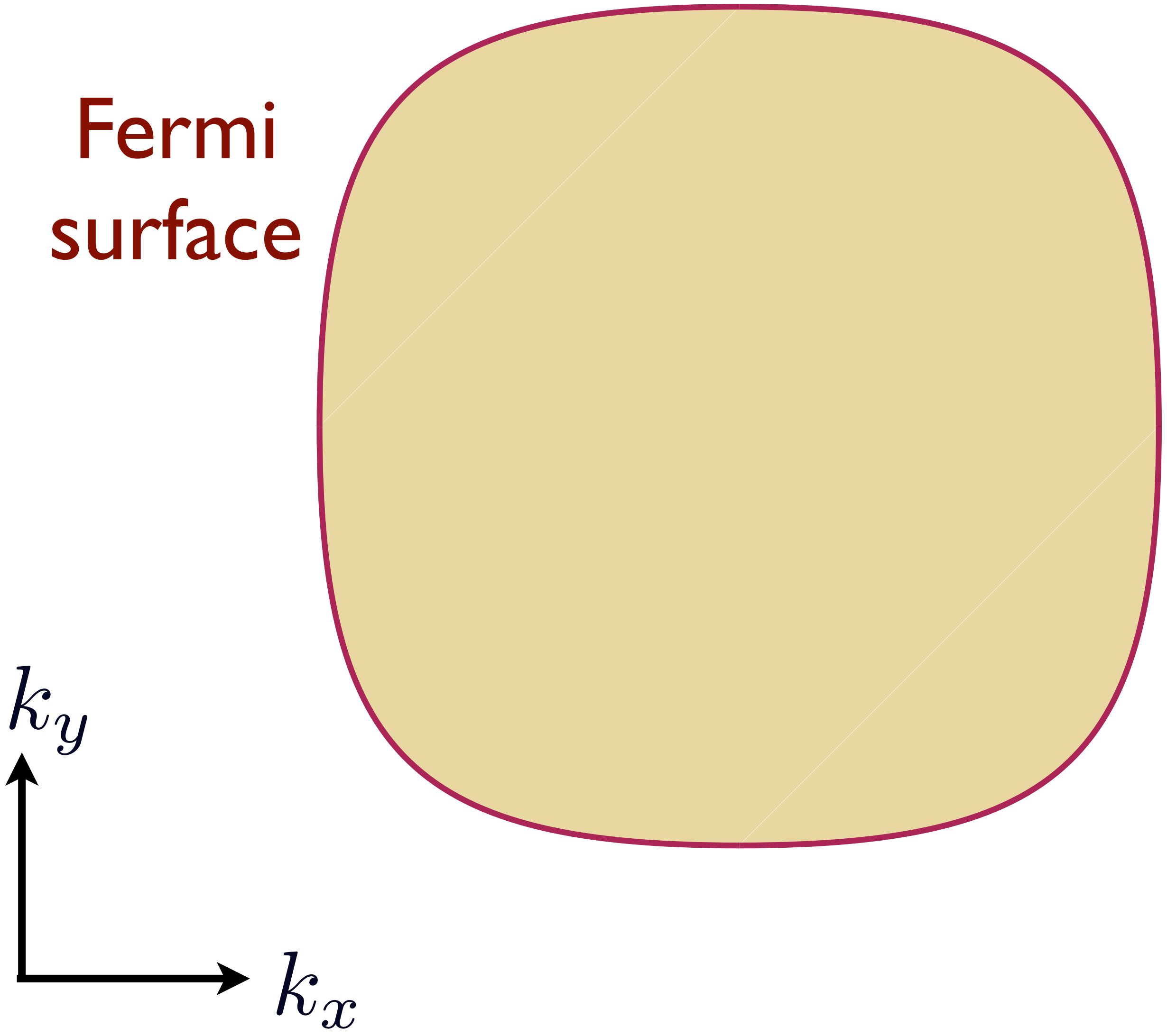


Kin Chung Fong



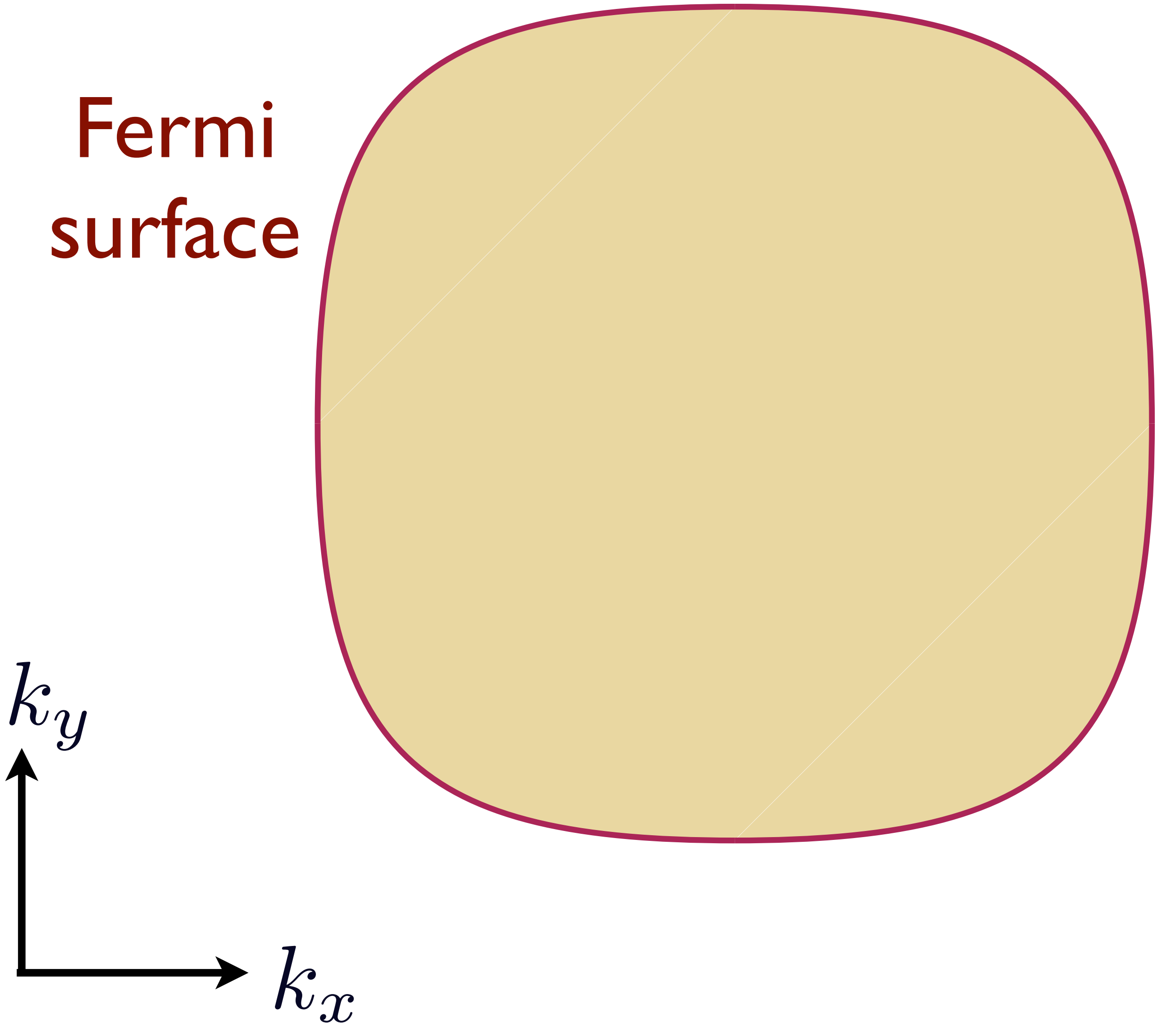
Andrew Lucas

Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = $\mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

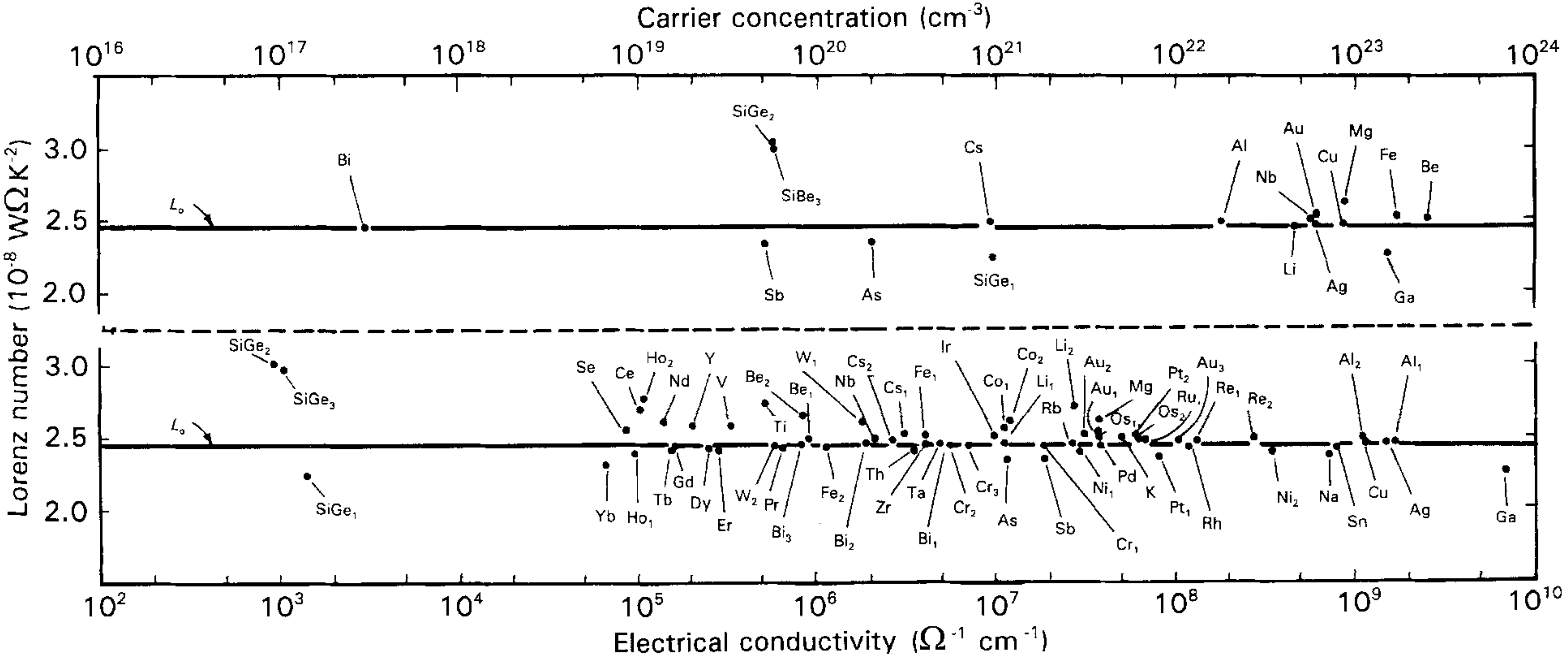
Ordinary metals: the Fermi liquid



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface $= \mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.
- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2} \equiv L_0$$

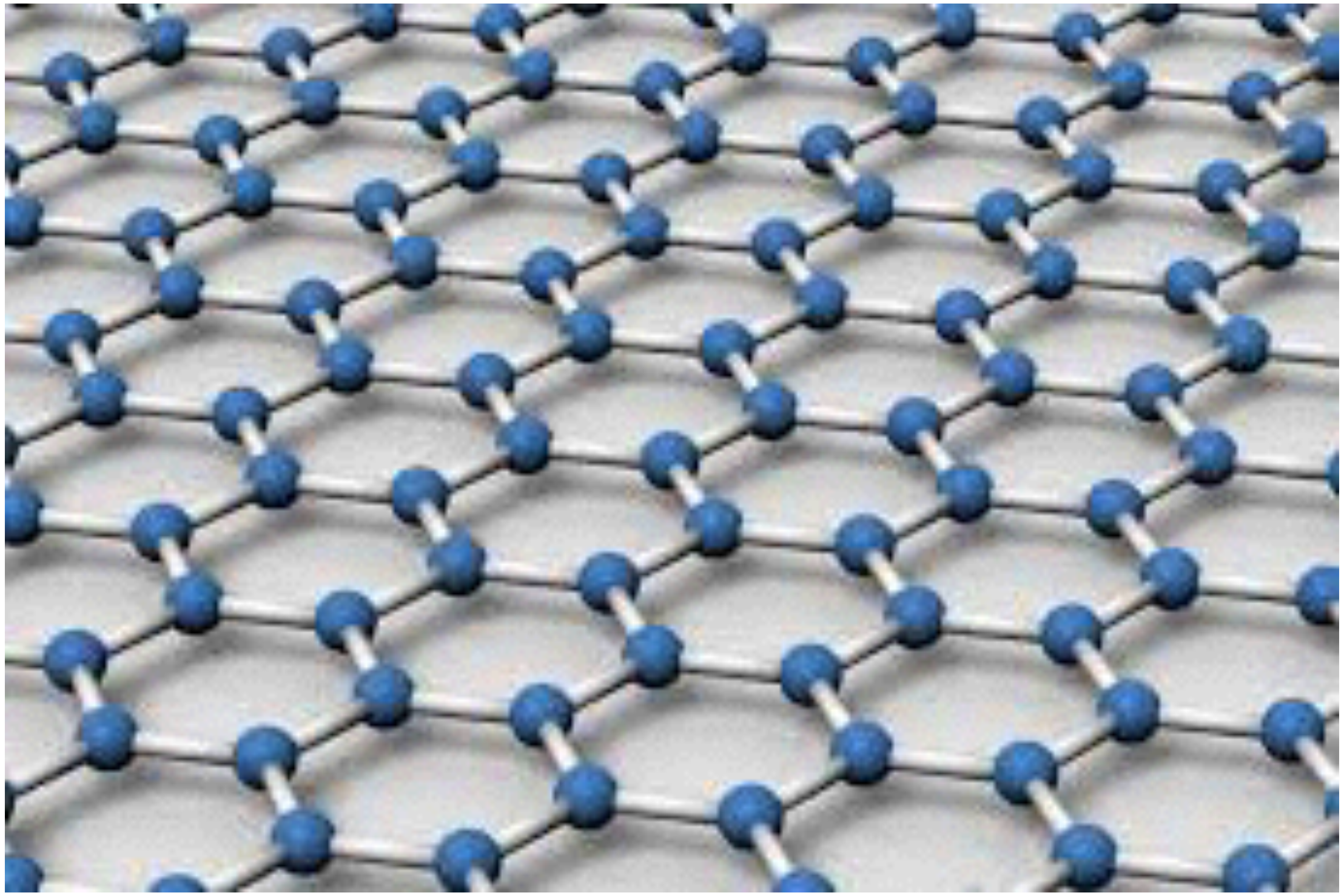
- Wiedemann-Franz law in a Fermi liquid:

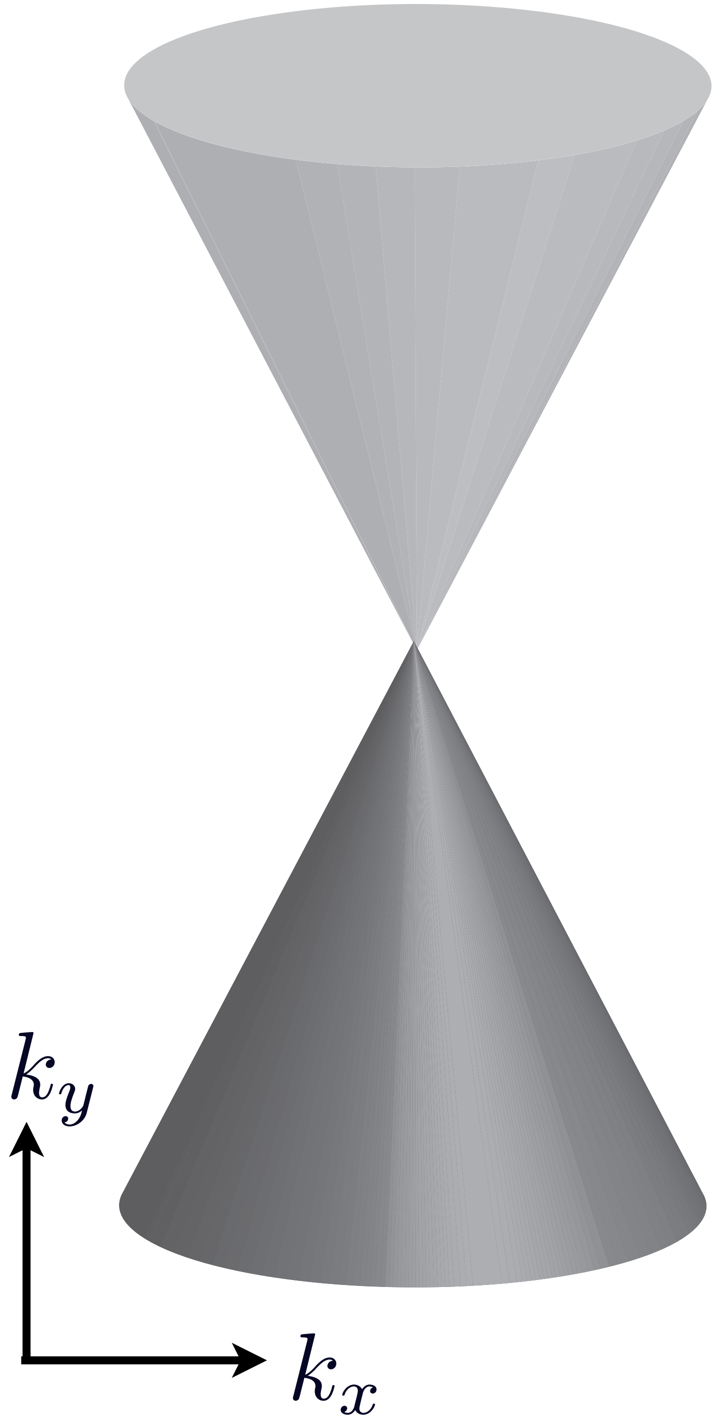
$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_\mathrm{B}^2}{3e^2} \approx 2.45 \times 10^{-8}\,\frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



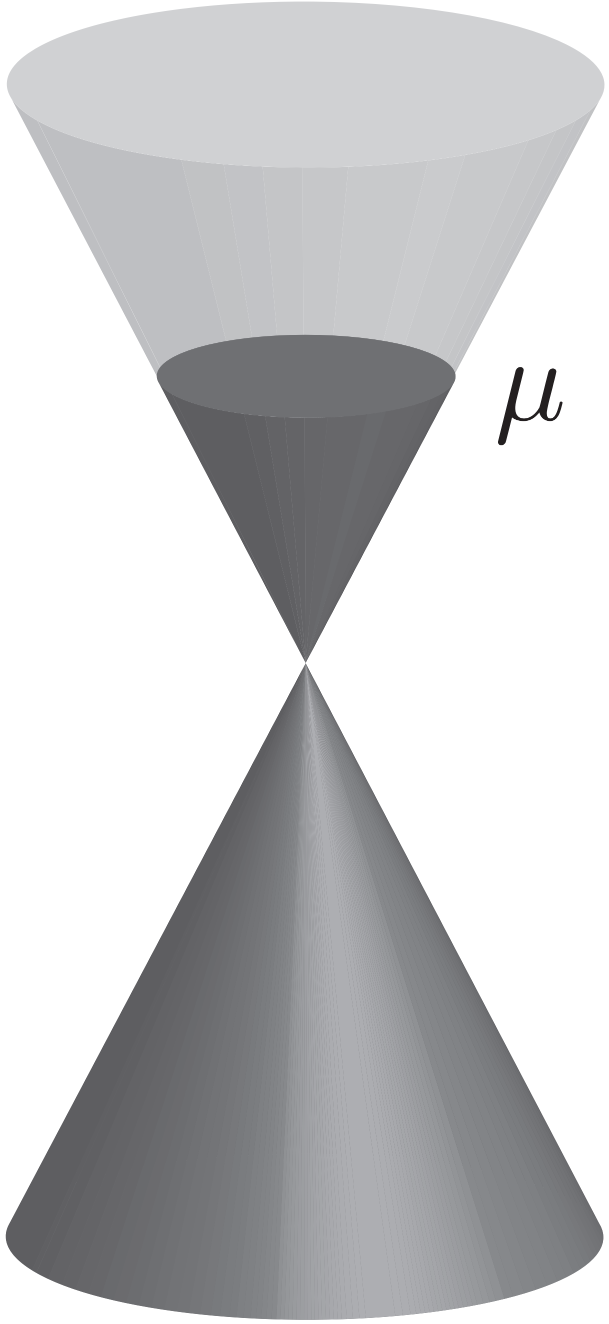
G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Graphene





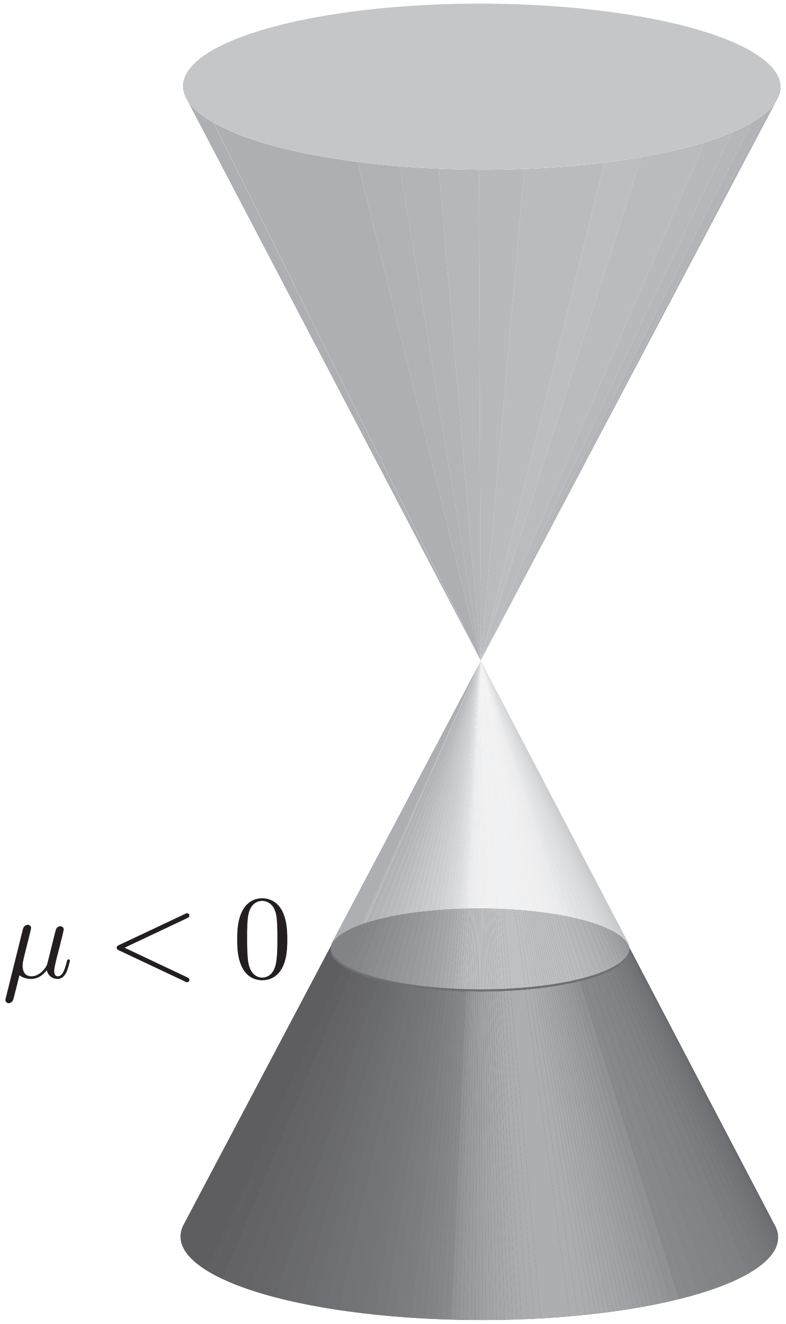
Graphene



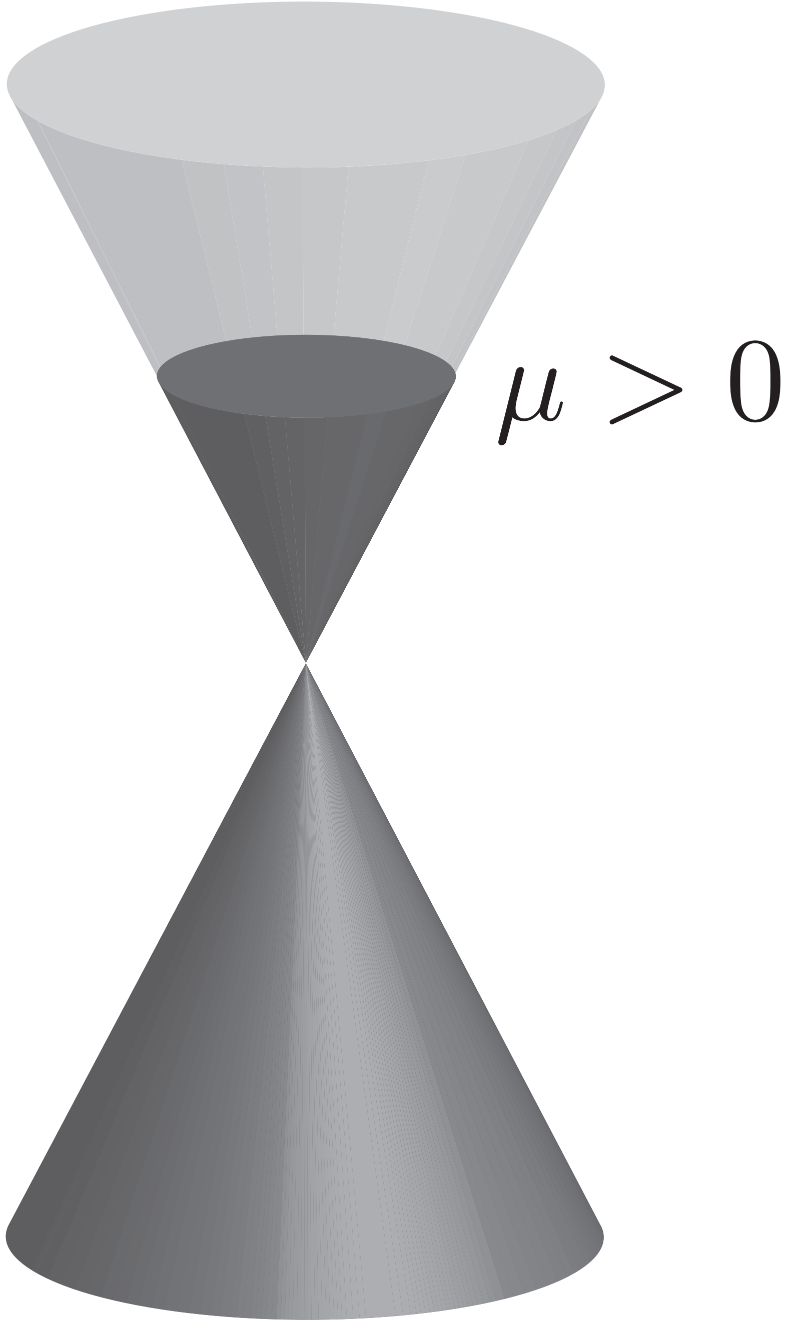
$\mu > 0$

Electron
Fermi surface

Graphene

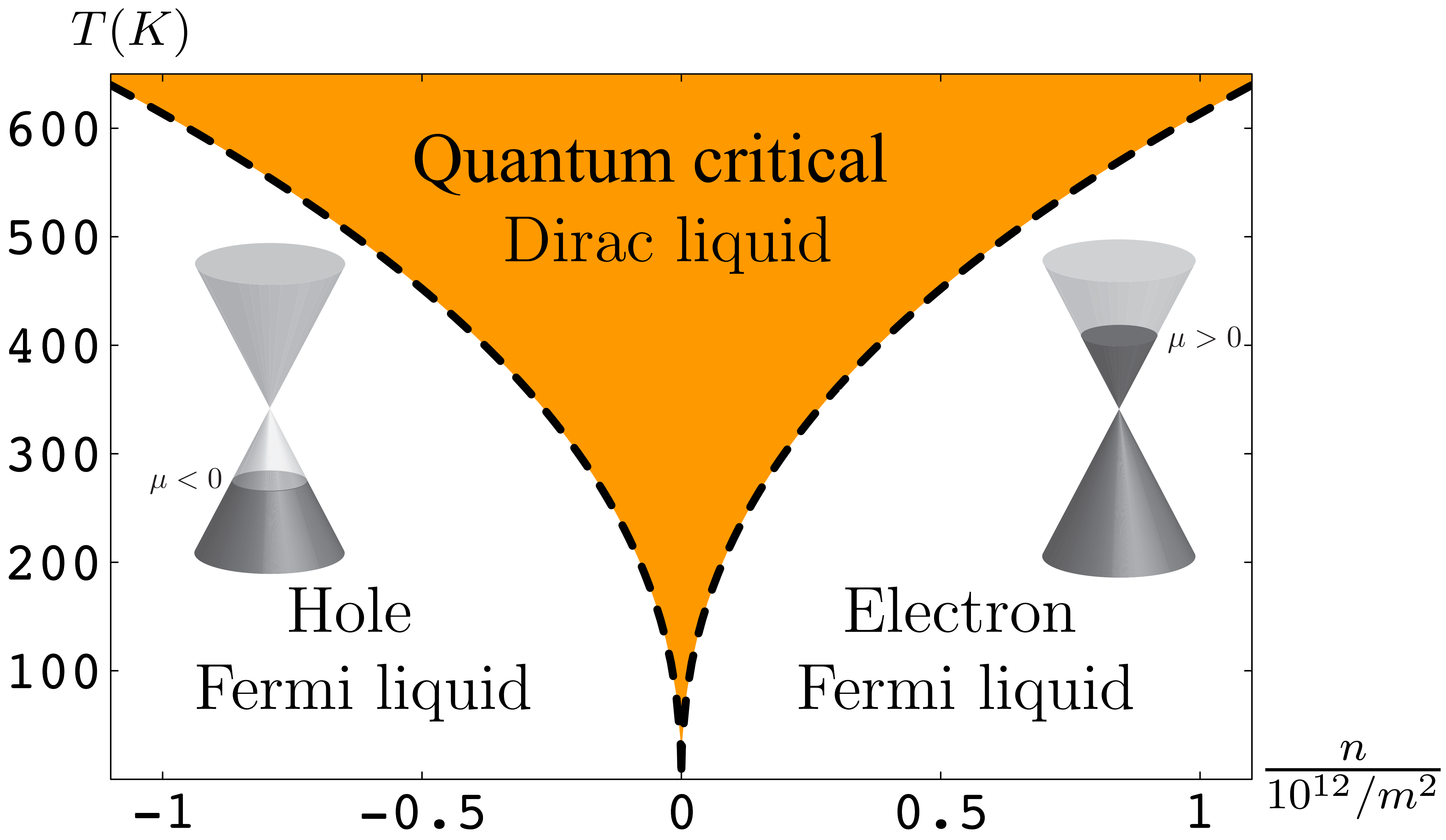


Hole
Fermi surface



Electron
Fermi surface

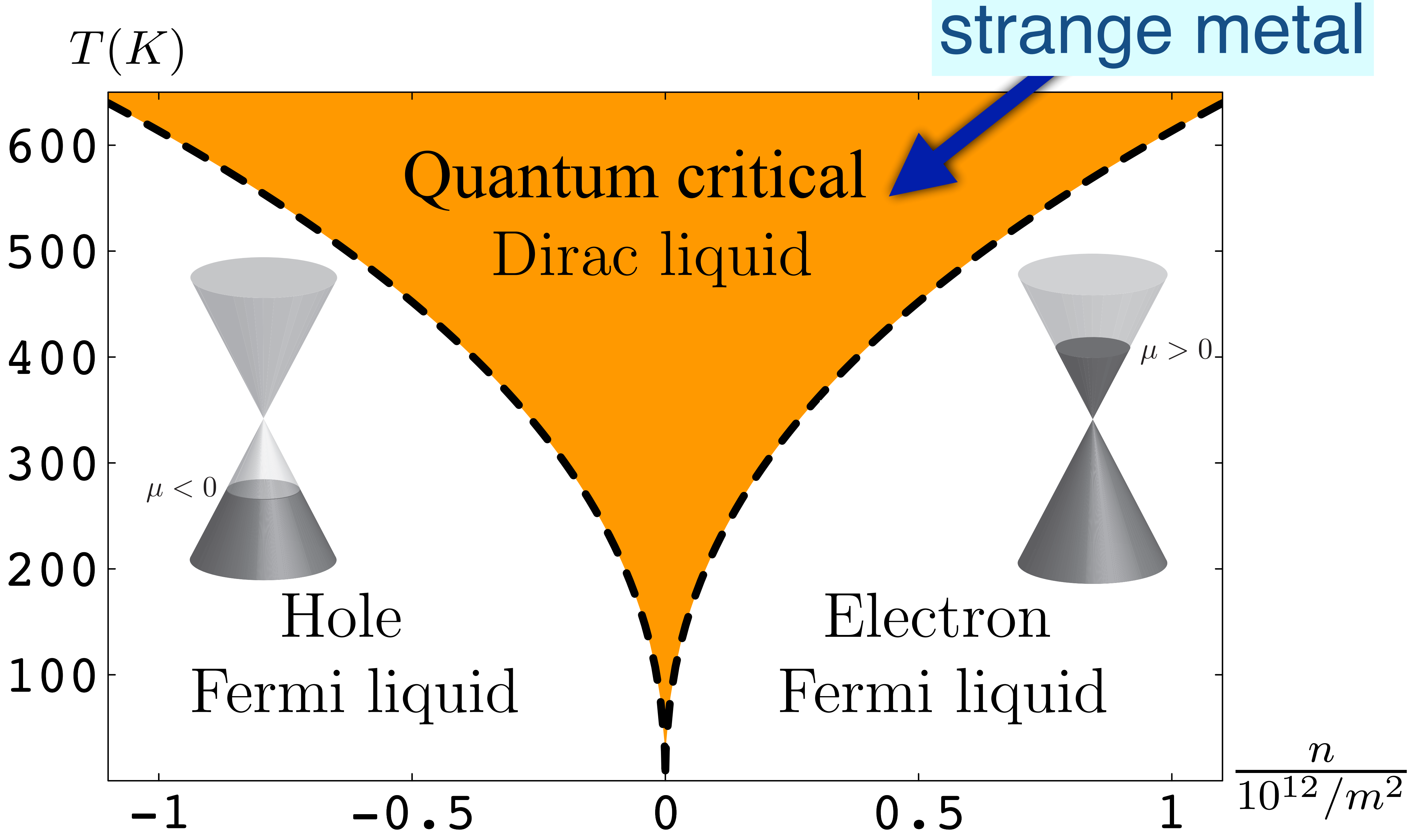
Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted strange metal



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Key properties of a strange metal

- No quasiparticle excitations

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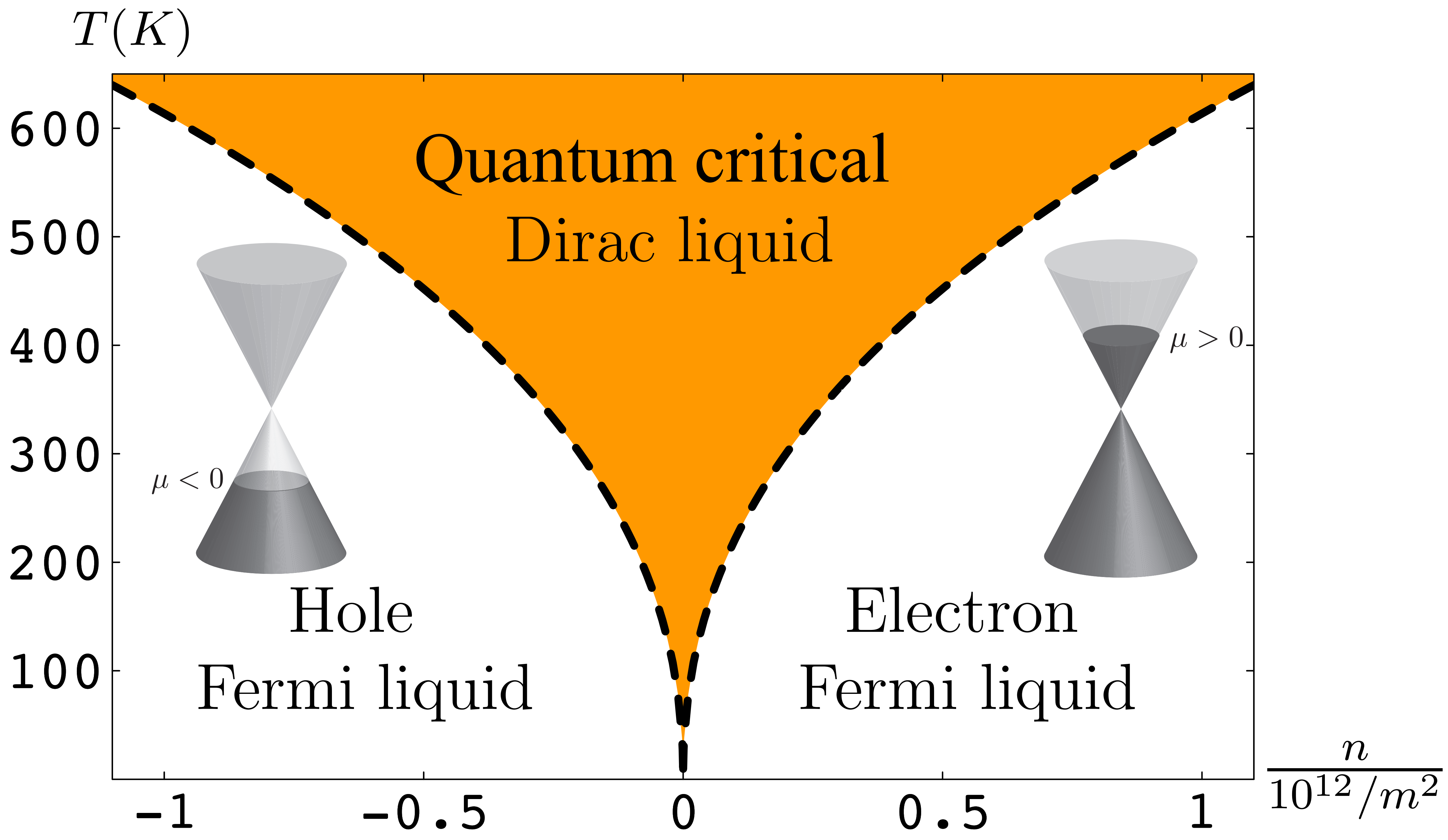
- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$

S. Sachdev, Quantum Phase Transitions (1999)

Key properties of a strange metal

- No quasiparticle excitations
- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

Graphene



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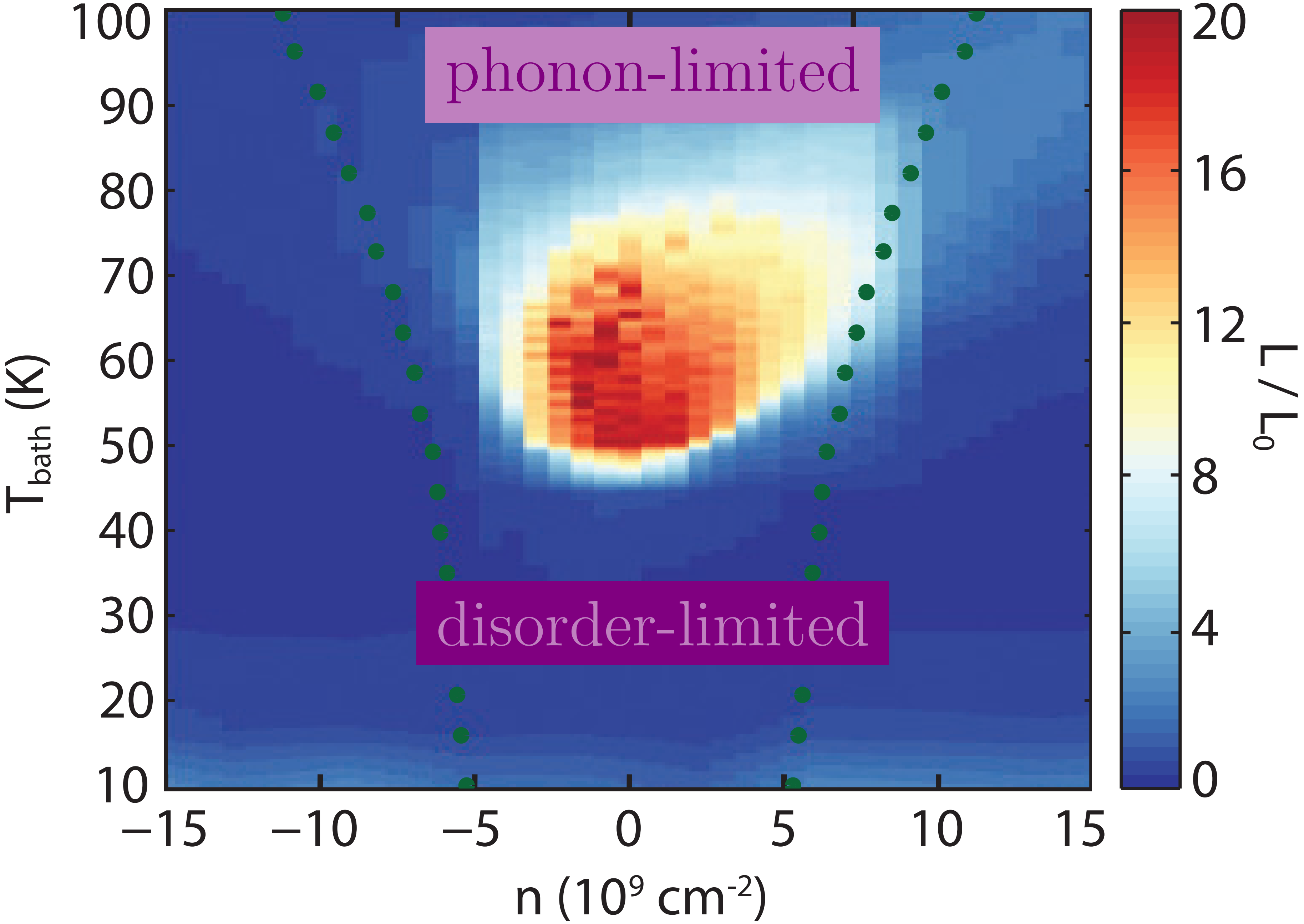
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

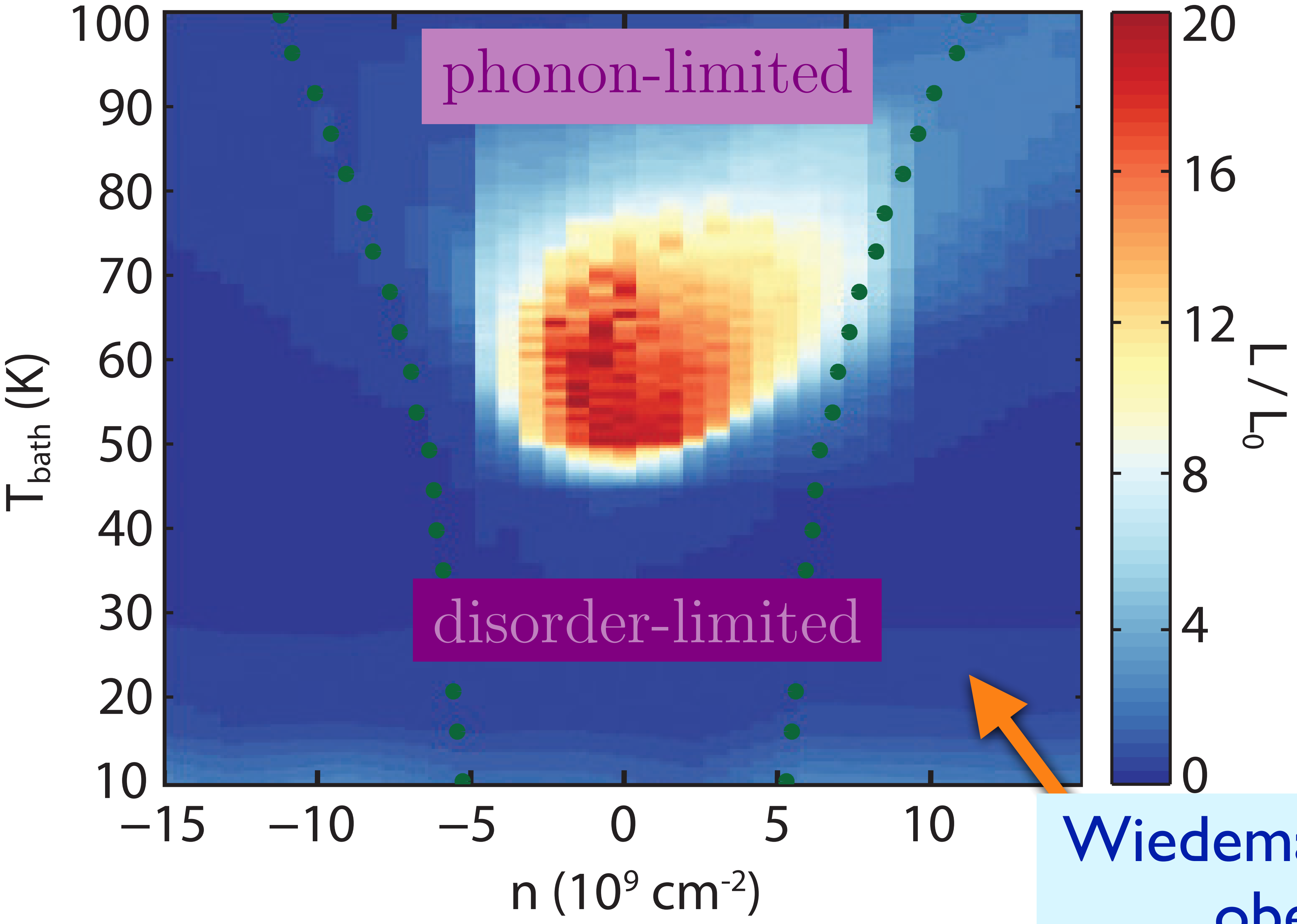
Strange metal in graphene



$$L = \frac{(\text{Thermal conductivity})}{T\,(\text{Electrical conductivity})}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

J. Crossno *et al.* arXiv:1509.04713; Science, to appear

Strange metal in graphene

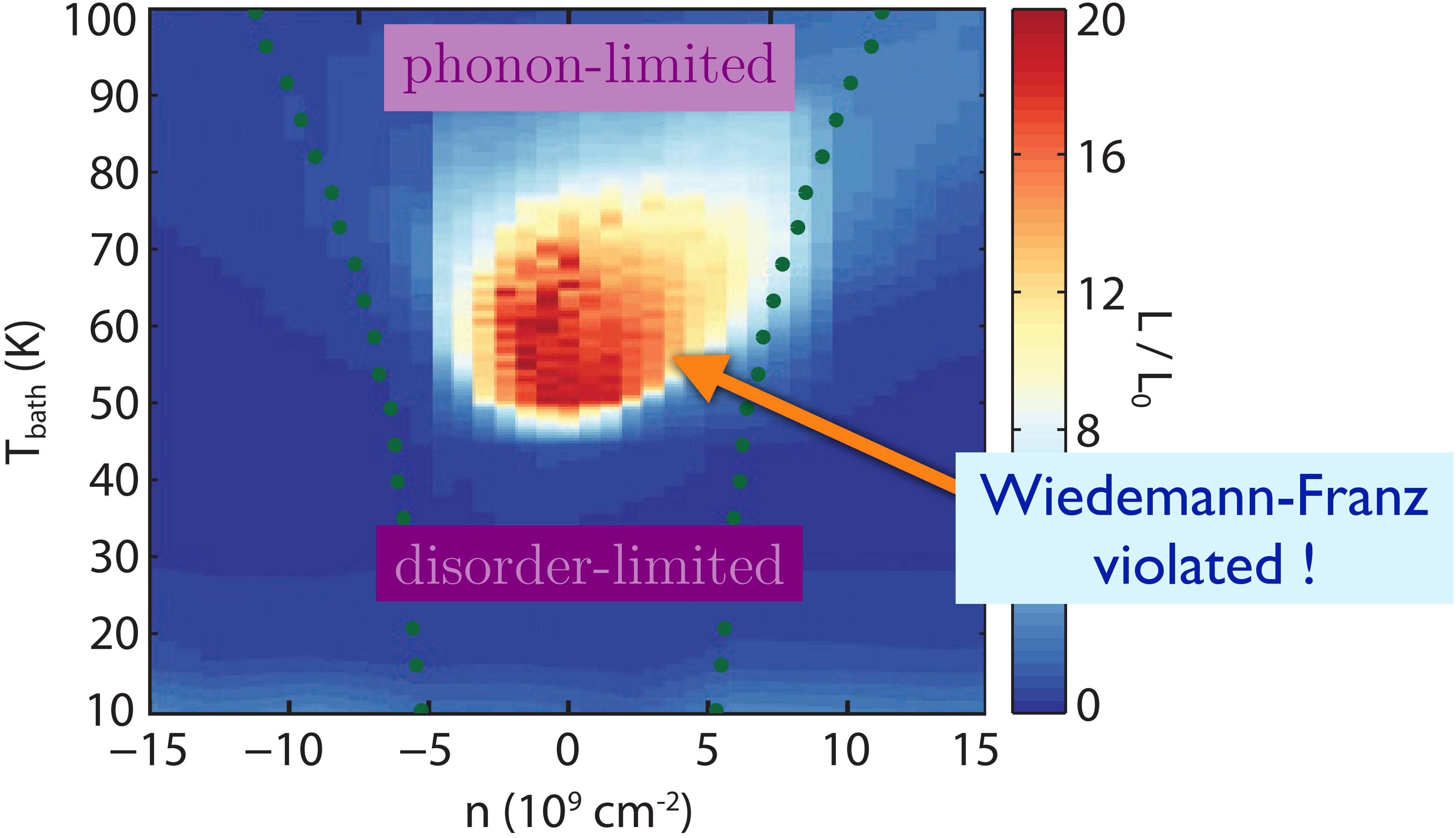


Wiedemann-Franz obeyed

$$L = \frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} ; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

J. Crossno *et al.* arXiv:1509.04713; Science, to appear

Strange metal in graphene



Wiedemann-Franz violated !

$$L = \frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

Quantum matter without quasiparticles

1. A solvable model of a strange metal
2. Holography and charged black holes
3. Theory of transport in strange metals
4. The (slightly less) strange metal in graphene

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Infinite-range model of a Fermi liquid

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

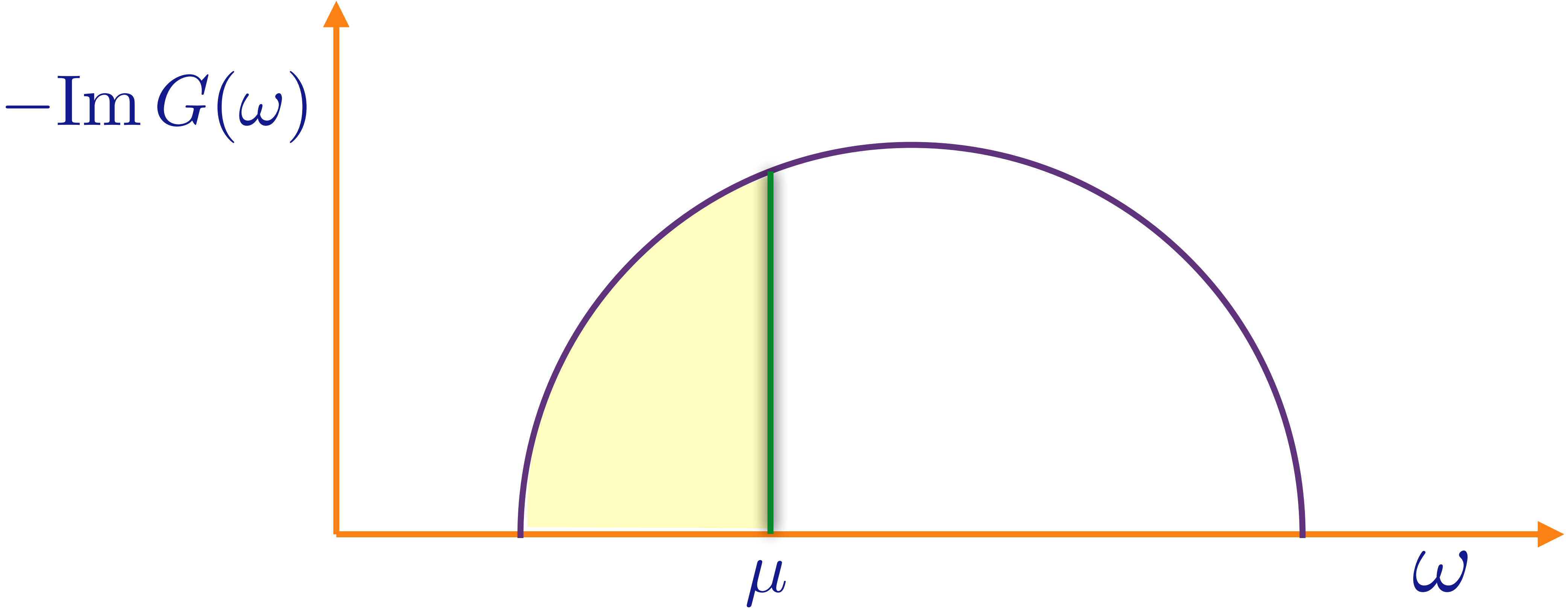
Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



Fermions occupying eigenstates with a “semi-circular” density of states

Infinite-range model of a strange metal

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^\dagger_{j\beta} + c^\dagger_{j\beta} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^\dagger_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

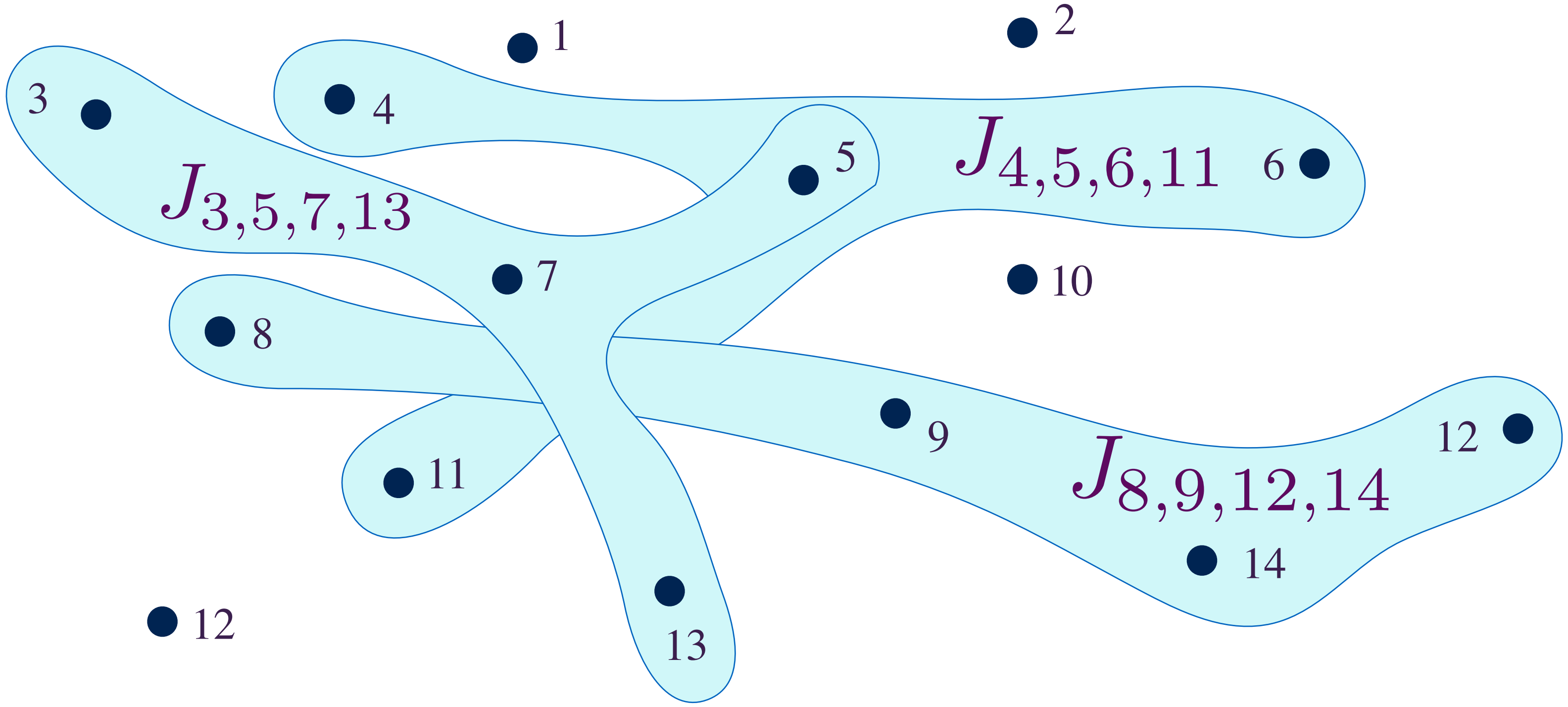
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields <u>same</u> critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

At frequencies $\ll J$, the equations for G and Σ can be written as

$$\int d\tau_2 G(\tau_1,\tau_2)\widetilde{\Sigma}(\tau_2,\tau_3) = -\delta(\tau_1-\tau_3)$$

$$\widetilde{\Sigma}(\tau_1,\tau_2) = -J^2\left[G(\tau_1,\tau_2)\right]^2 G(\tau_2,\tau_1)$$

These equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1,\tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4}\frac{g(\sigma_1)}{g(\sigma_2)}\,G(\sigma_1,\sigma_2)$$

$$\widetilde{\Sigma}(\tau_1,\tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4}\frac{g(\sigma_1)}{g(\sigma_2)}\,\widetilde{\Sigma}(\sigma_1,\sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

These equations and invariances have similarities to those of the large N limit of quantum spins at the spatial boundary of a CFT2 (multi-channel Kondo problems)

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
PRB **58**, 3794 (1998)

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\mathrm{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2} & , \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

Analog of the relationship between $\mathcal{Q}$ and k_F in a Fermi liquid.

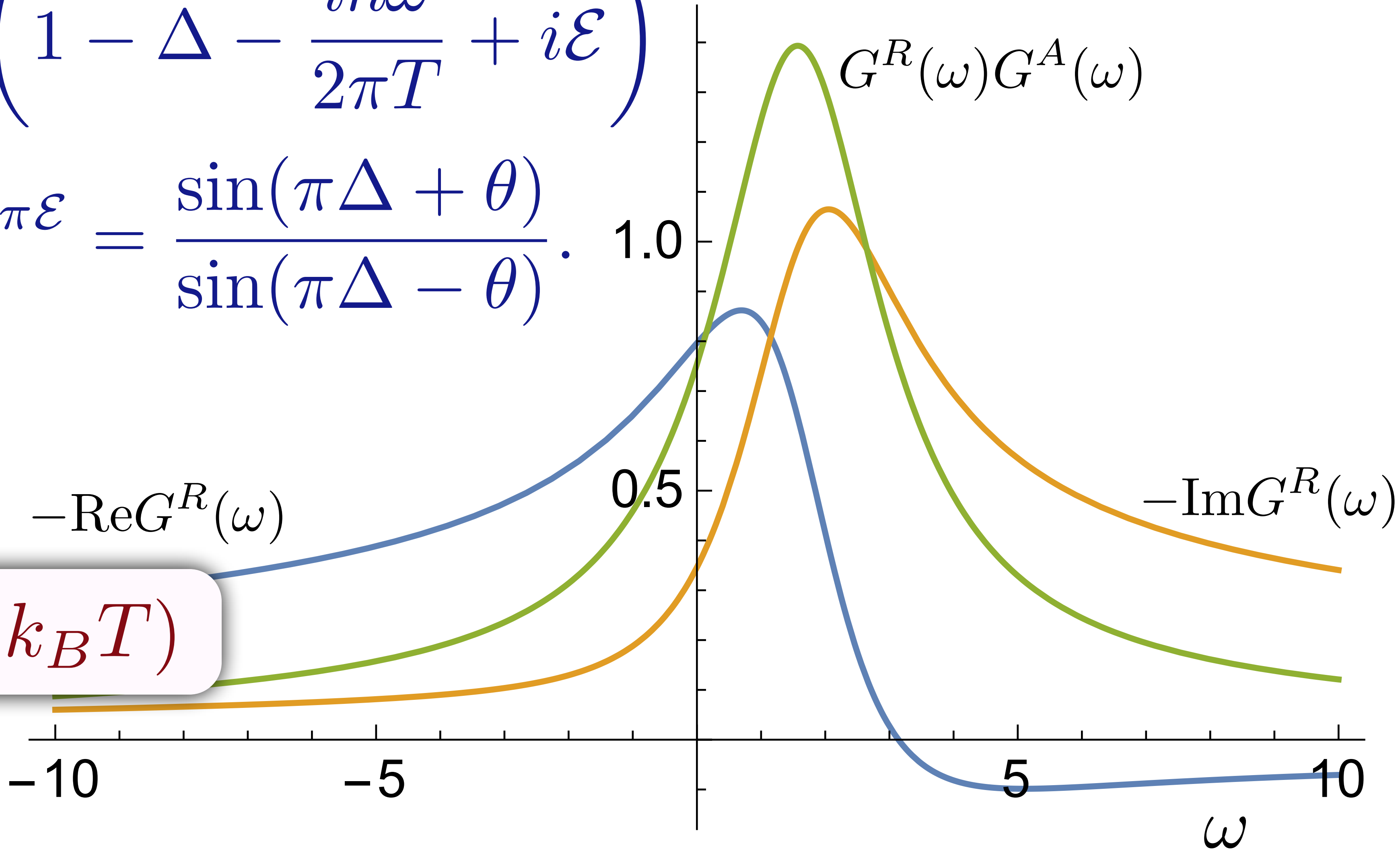
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}.$

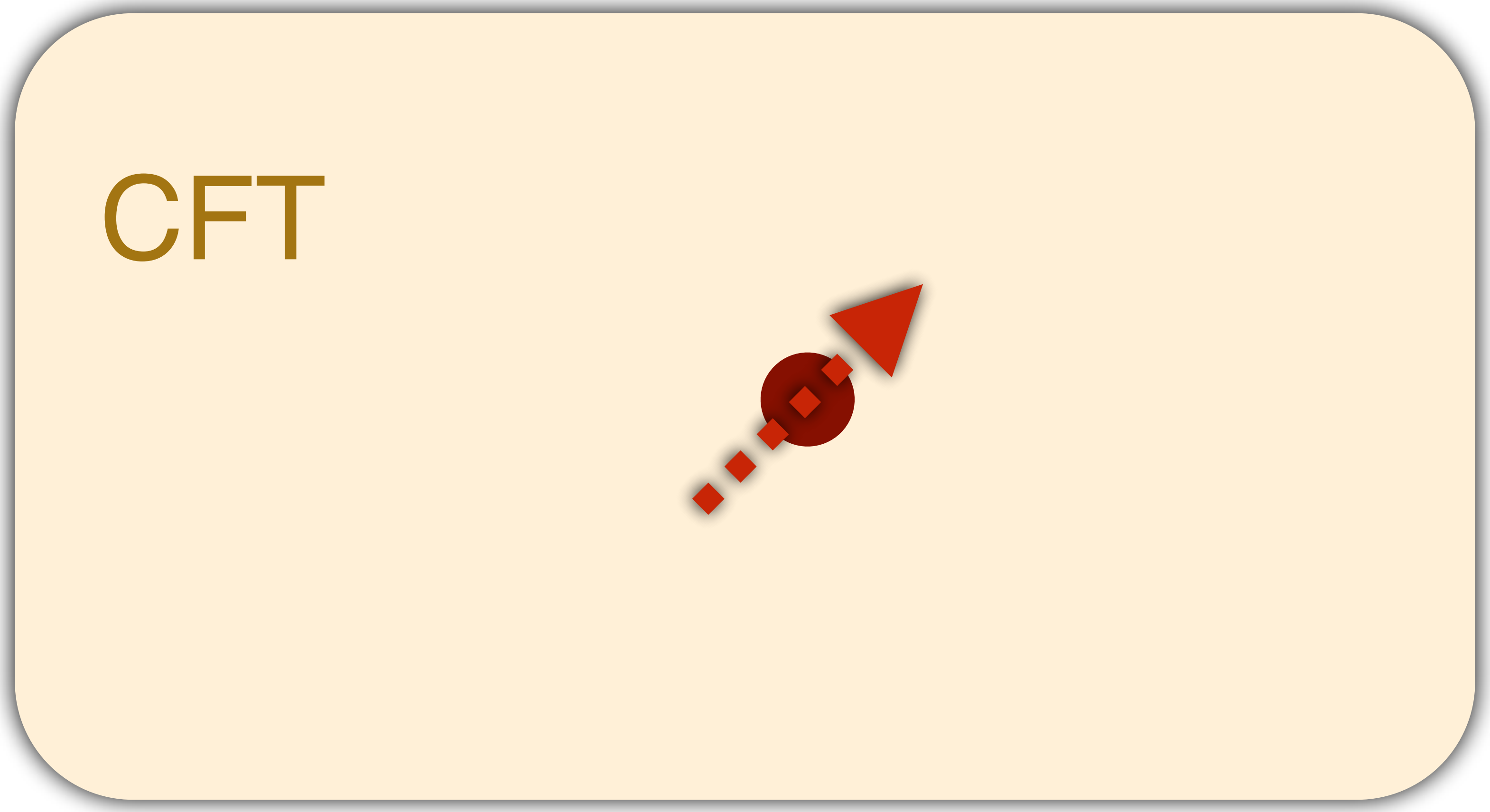


Note $G(\omega) \equiv f(\hbar\omega/k_BT)$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

“Critically-screened” spin has “irrational” entropy



Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

This entropy obeys

$$\boxed{\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

Note that $\mathcal{S}$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

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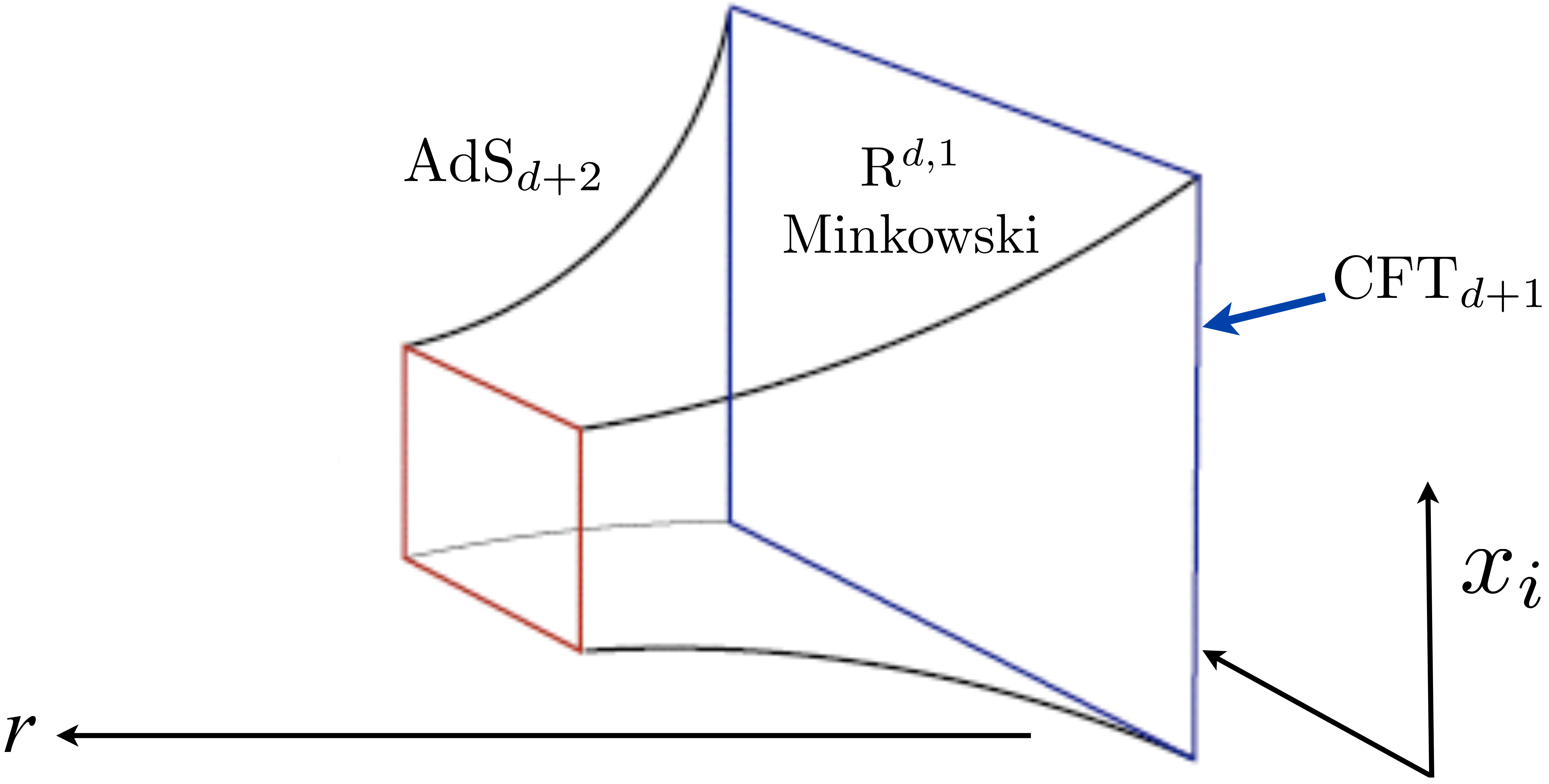
AdS/CFT correspondence at zero temperature

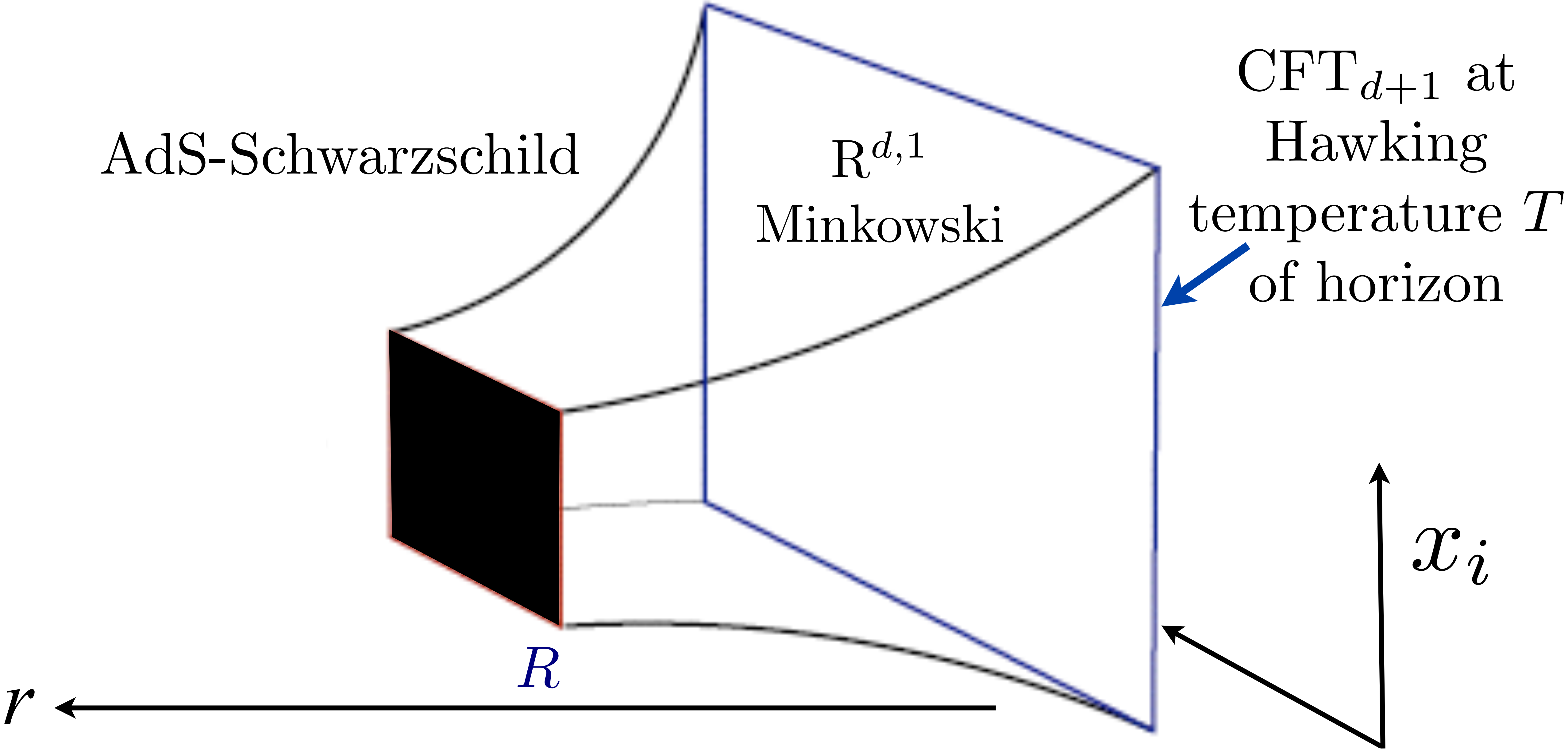
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}+\frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2\left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$

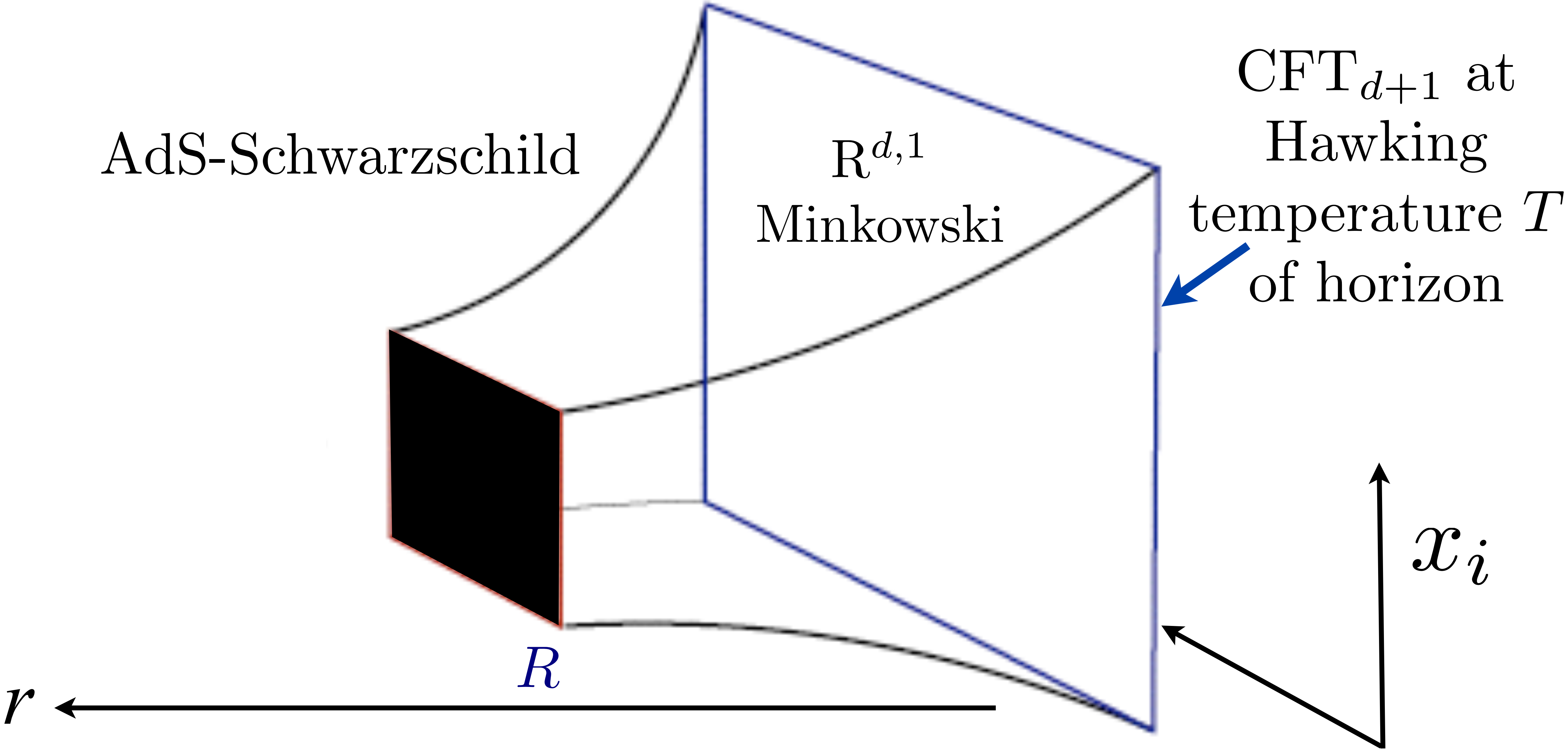


Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



AdS-Schwarzschild

$\mathrm{R}^{d,1}$

Minkowski

CFT_{d+1} at Hawking temperature T of horizon

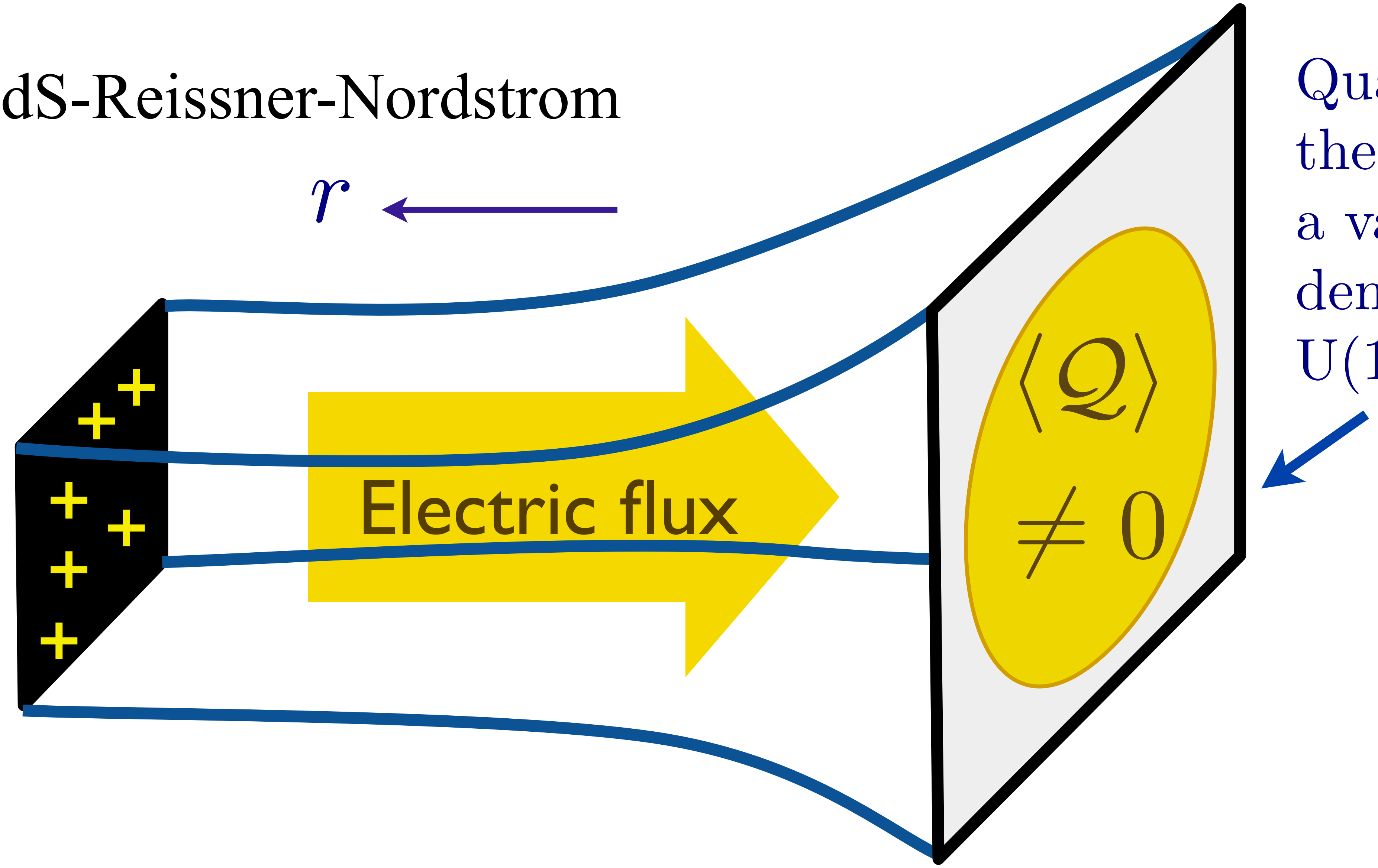
For SU(N) SYM in $d = 3$, $\mathcal{S}_{\mathrm{BH}} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

S. S. Gubser, I. R. Klebanov, A. W. Peet PRD **54**, 3915 (1996)

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

AdS-Reissner-Nordstrom

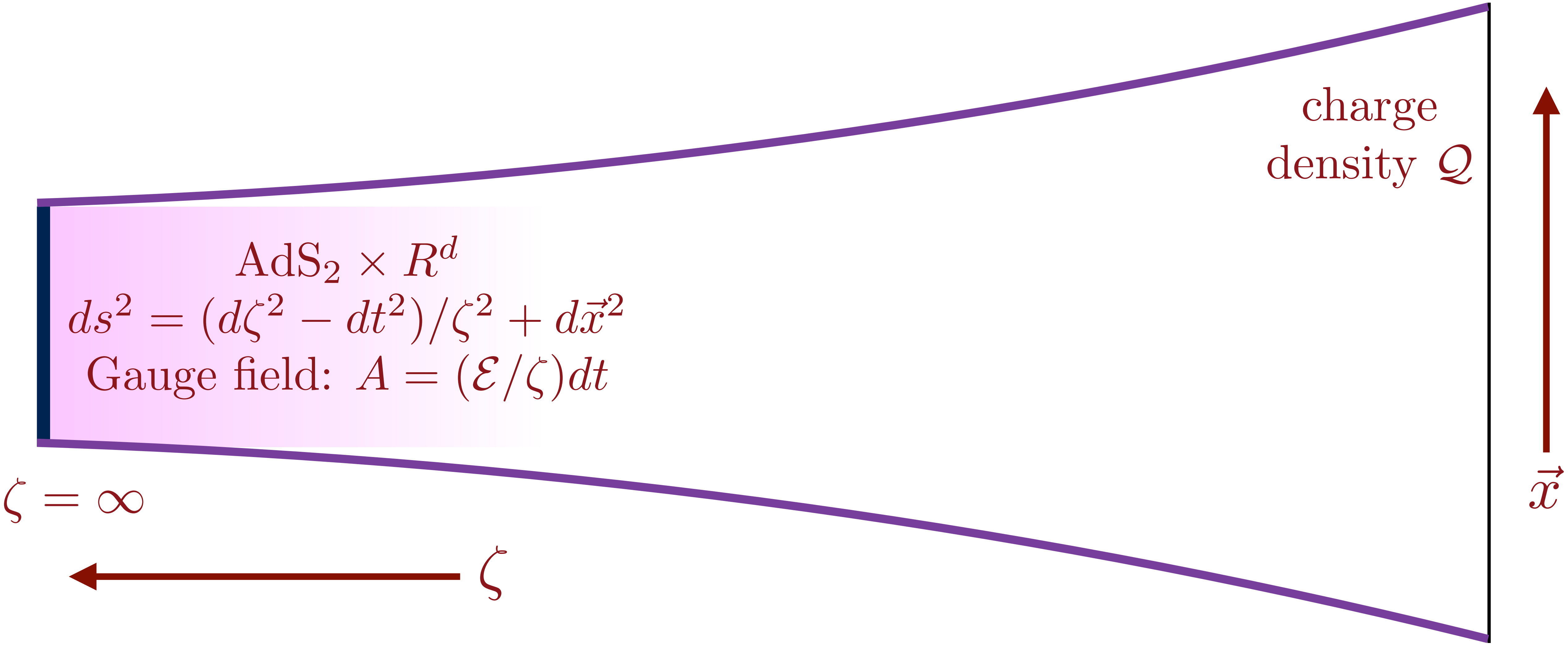


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, PRD **60**, 064018 (1999)

General Relativity of charged black branes



$\mathrm{AdS}_2 \times R^d$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

charge
density $\mathcal{Q}$

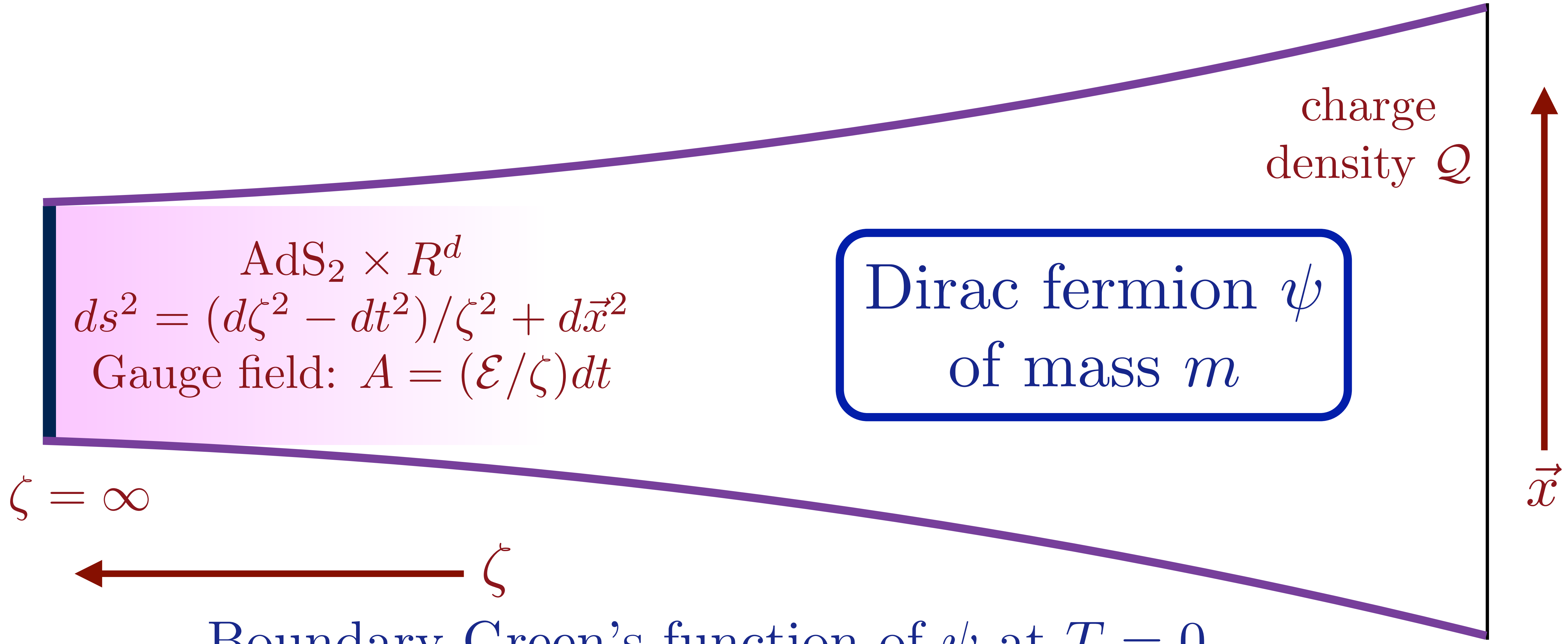
$\vec{x}$

$\zeta = \infty$

ζ

- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.

Quantum fields on charged black branes



$\mathrm{AdS}_2 \times R^d$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

charge density $\mathcal{Q}$

Dirac fermion ψ of mass m

$\zeta = \infty$

ζ

$\vec{x}$

Boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \ \omega < 0. \end{cases}$$

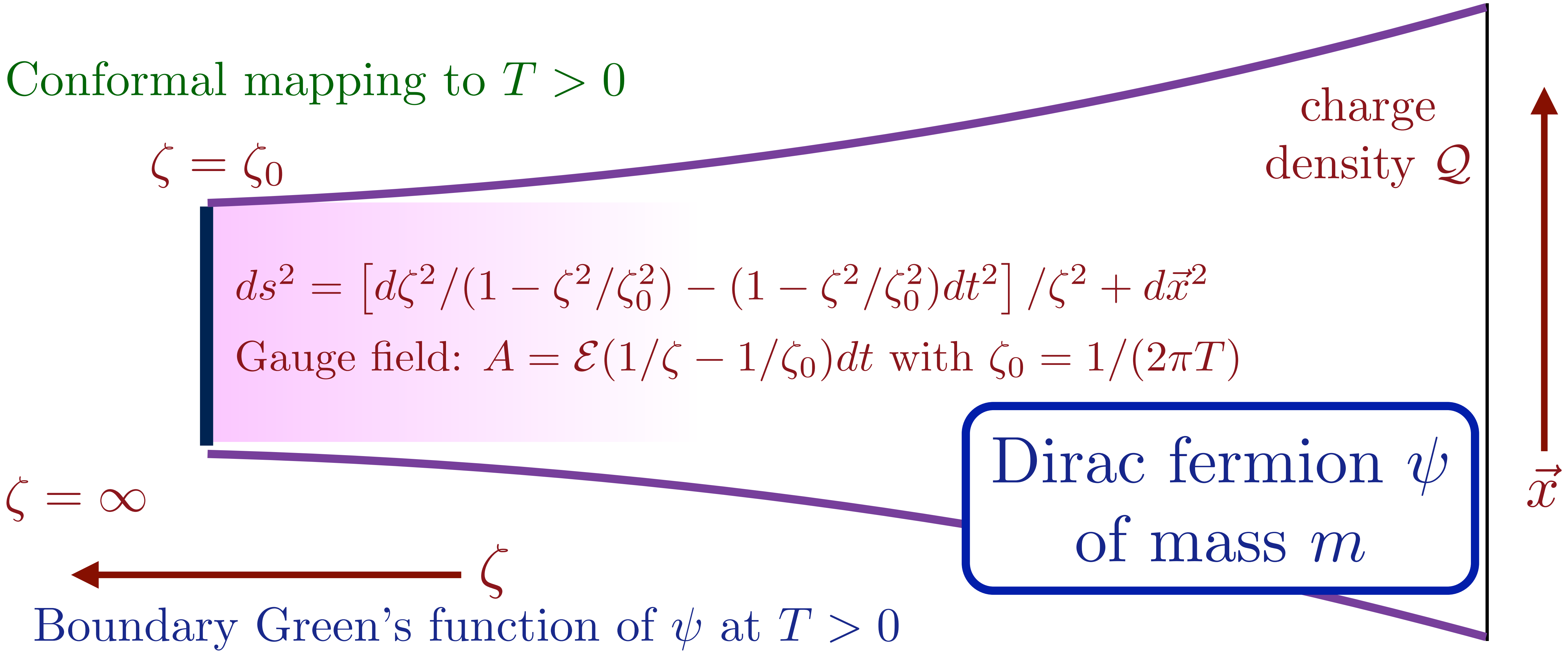
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

Quantum fields on charged black branes

Conformal mapping to $T > 0$



$\zeta = \zeta_0$

charge density $\mathcal{Q}$

$$ds^2 = \left[d\zeta^2/(1-\zeta^2/\zeta_0^2) - (1-\zeta^2/\zeta_0^2)dt^2 \right]/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)dt$ with $\zeta_0 = 1/(2\pi T)$

Dirac fermion ψ of mass m

$\zeta = \infty$

ζ

$\vec{x}$

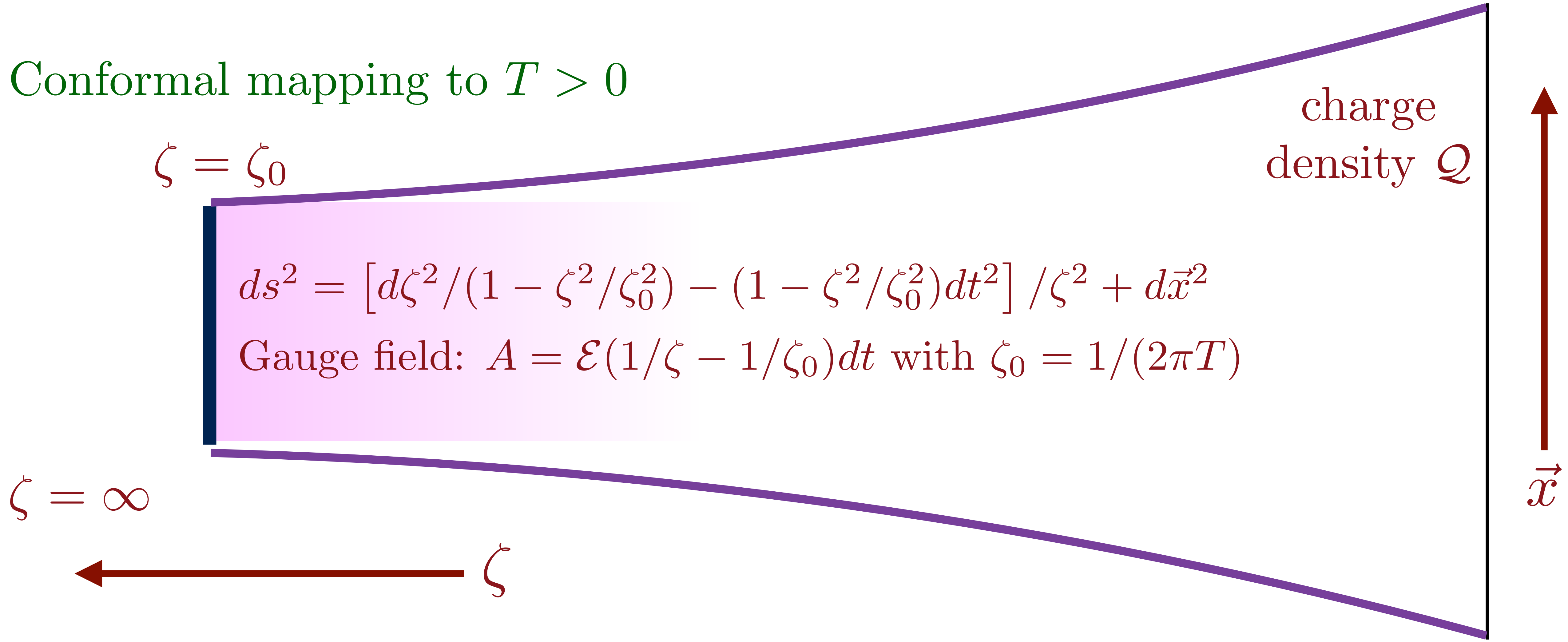
Boundary Green's function of ψ at $T > 0$ is fully determined by $\mathcal{E}$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\hbar\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

General Relativity of charged black branes



Conformal mapping to $T > 0$

$\zeta = \zeta_0$

charge density $\mathcal{Q}$

$$ds^2 = \left[d\zeta^2/(1-\zeta^2/\zeta_0^2) - (1-\zeta^2/\zeta_0^2)dt^2\right]/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)dt$ with $\zeta_0 = 1/(2\pi T)$

$\zeta = \infty$

ζ

$\vec{x}$

- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
PRX 5, 041025 (2015)

$$\boxed{\left(\frac{\partial\mathcal{S}_{\rm BH}}{\partial\mathcal{Q}}\right)_T = -\left(\frac{\partial\mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

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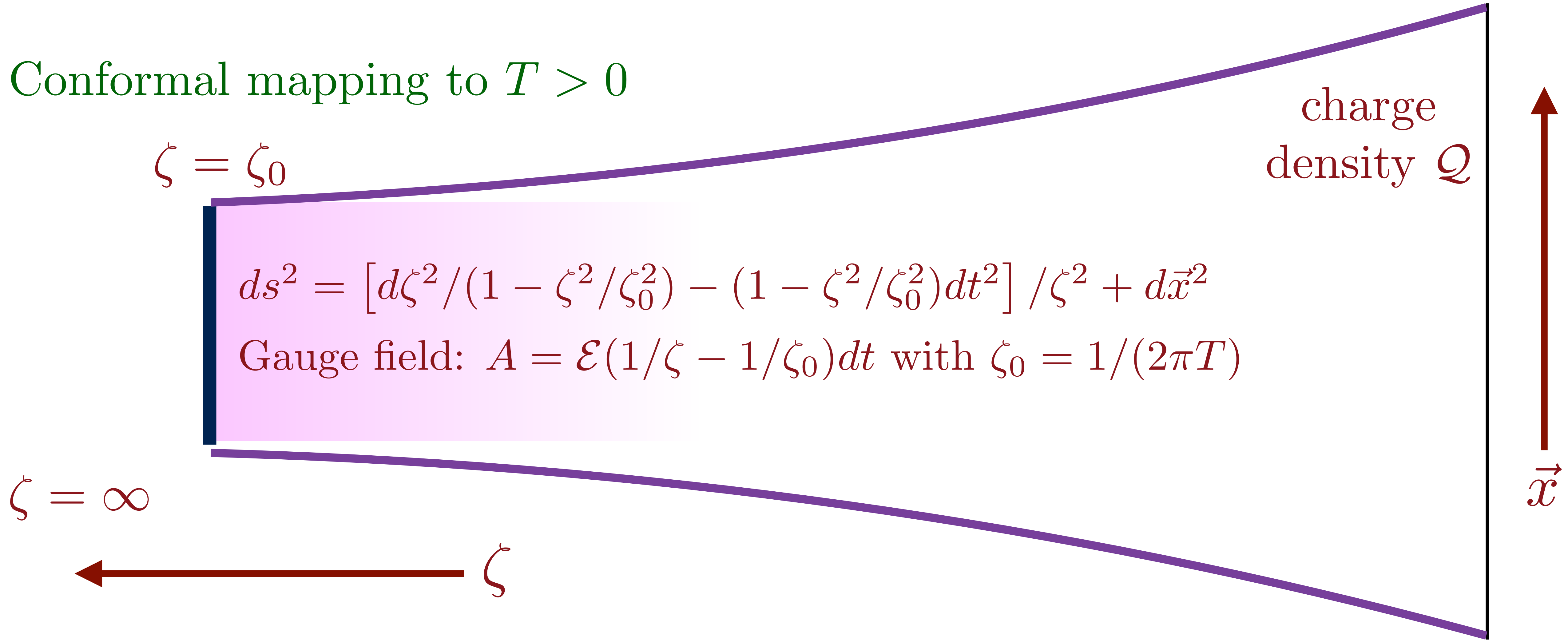

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A. Sen
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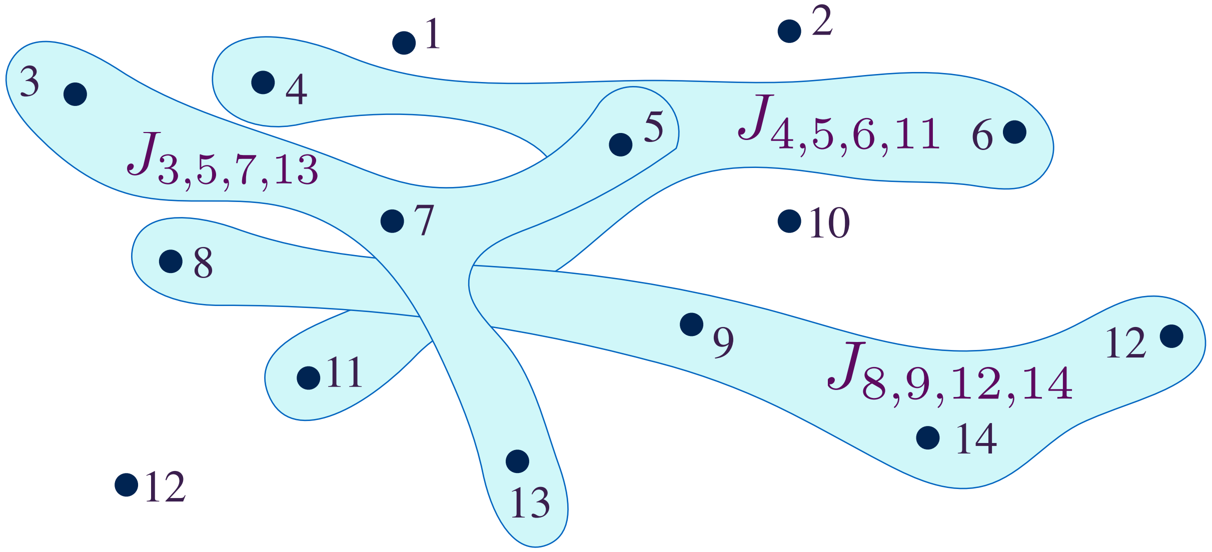
PRX **5**, 041025 (2015)

$$\boxed{\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

Also obeyed by Wald entropy in higher-derivative gravity.

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state'
determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature
entropy density, $\mathcal{S}$, obeys

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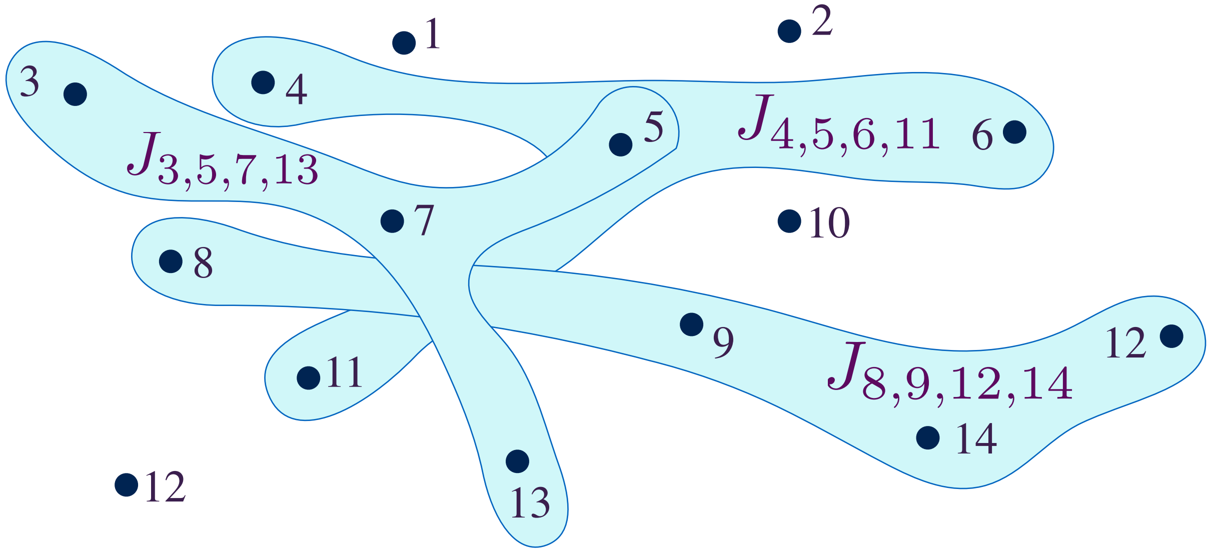

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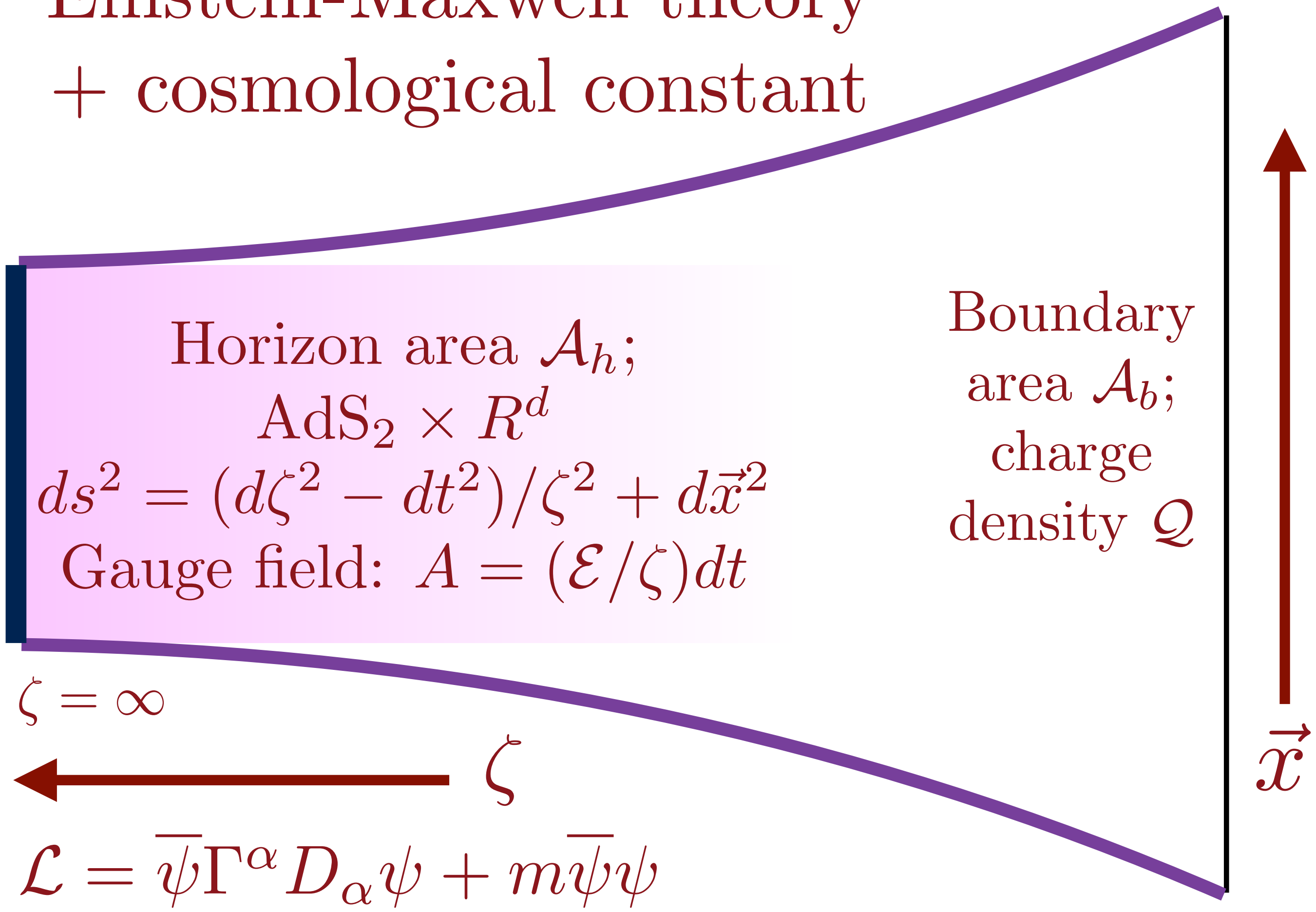
Einstein-Maxwell theory
+ cosmological constant



Horizon area $\mathcal{A}_h$;
AdS$_2 \times R^d$
$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
Gauge field: $A = (\mathcal{E}/\zeta) dt$

Boundary area $\mathcal{A}_b$;
charge density $\mathcal{Q}$

$\zeta = \infty$

ζ

$\vec{x}$

$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

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'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS$_2$

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen hep-th/0506177; S. Sachdev PRX **5**, 041025 (2015)

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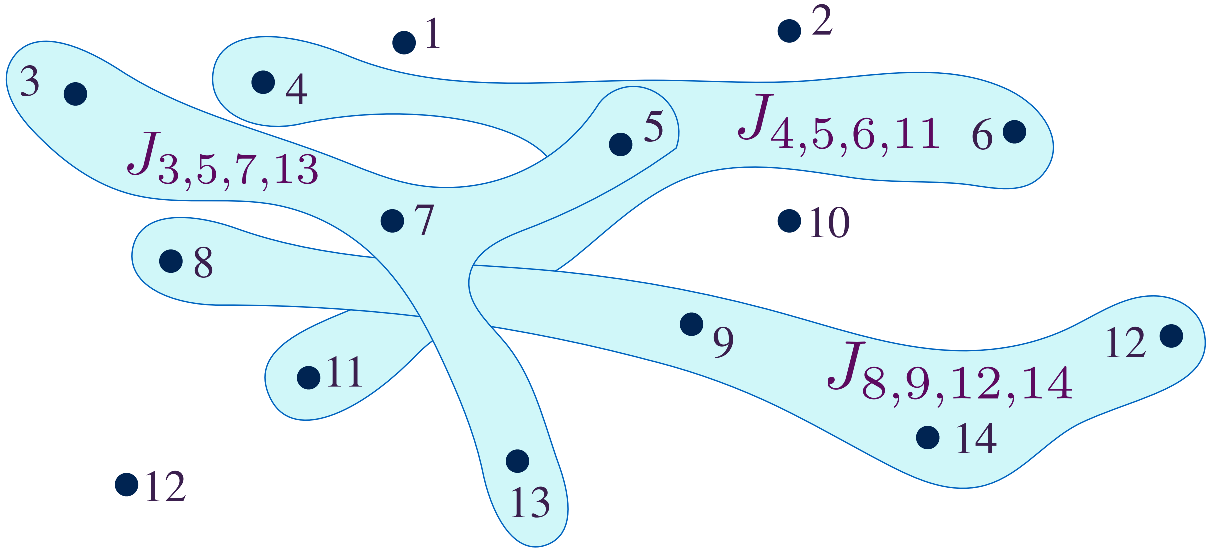

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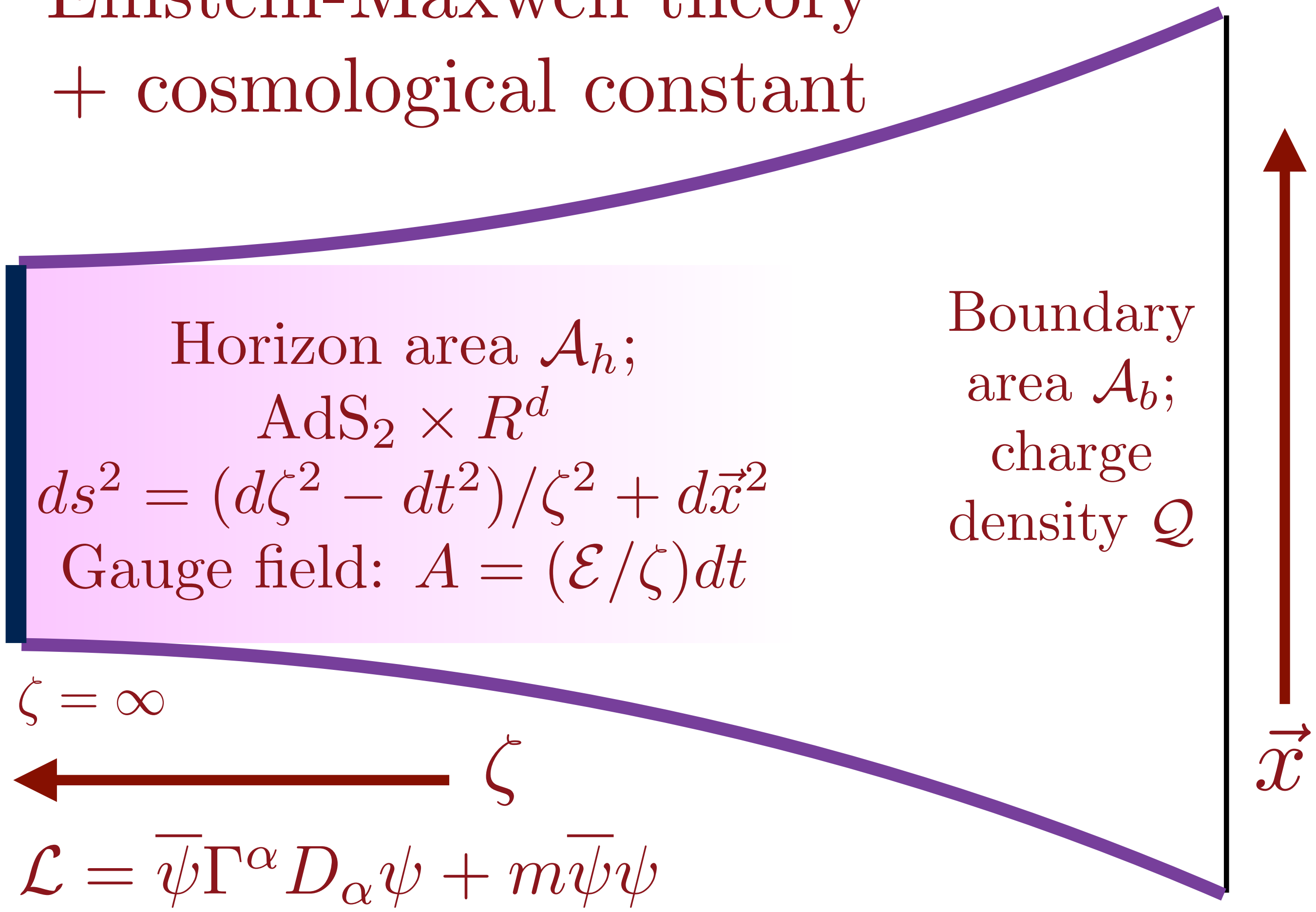

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Evidence for
AdS_2 gravity
dual of H

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S. Sachdev, PRX 5, 041025 (2015)

Quantum matter without quasiparticles

1. A solvable model of a strange metal
2. Holography and charged black holes
3. Theory of transport in strange metals
4. The (slightly less) strange metal in graphene

Quantum matter without quasiparticles

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Quasiparticle transport in metals:

- Focus on infinite number of (near) conservation laws (momenta of quasiparticles on the Fermi surface) and compute how they are slowly violated by the lattice or impurities

Transport in strange metals

- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

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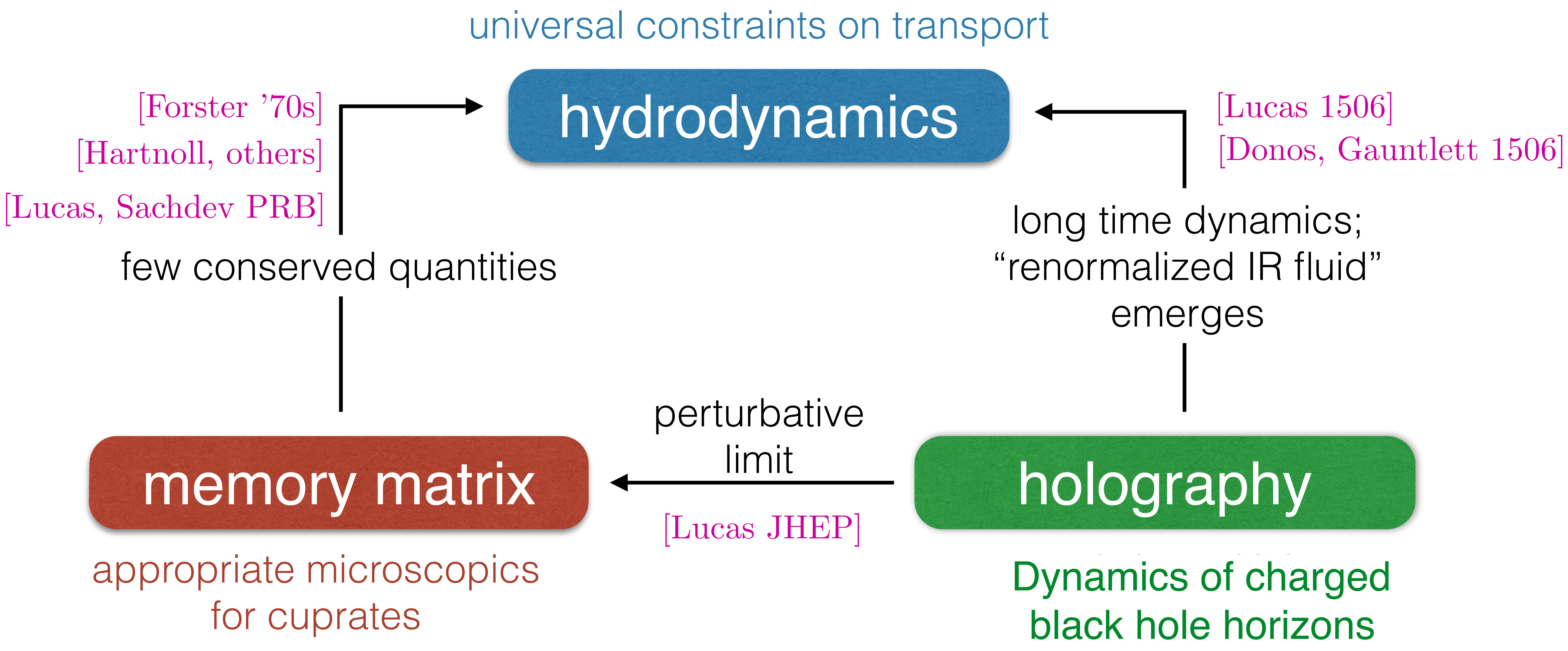
- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

- Focus on relaxation of *total* momentum (including contributions of the Fermi surface (if present) and all critical bosons) by the lattice or impurities

Transport in Strange Metals



universal constraints on transport

hydrodynamics

[Forster ’70s]
[Hartnoll, others]
[Lucas, Sachdev PRB]

few conserved quantities

[Lucas 1506]
[Donos, Gauntlett 1506]

long time dynamics;
“renormalized IR fluid”
emerges

memory matrix

appropriate microscopics
for cuprates

perturbative
limit

[Lucas JHEP]

holography

Dynamics of charged
black hole horizons

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{\mathrm{ee}}$, $t \gg t_{\mathrm{ee}}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\mathrm{ext}}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\mathrm{ee}}\partial_\mu$:

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\mathrm{Q}}\mathcal{P}^{\mu\rho}\left(\partial_\rho \mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\mathrm{ext}}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

- New (and only) transport co-efficient, σ_Q:
  "quantum critical" conductivity at $\mathcal{Q} = 0$.

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Translational symmetry breaking

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\rm imp}} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
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"Memory function" methods yield an explicit expression for τ_{imp}:

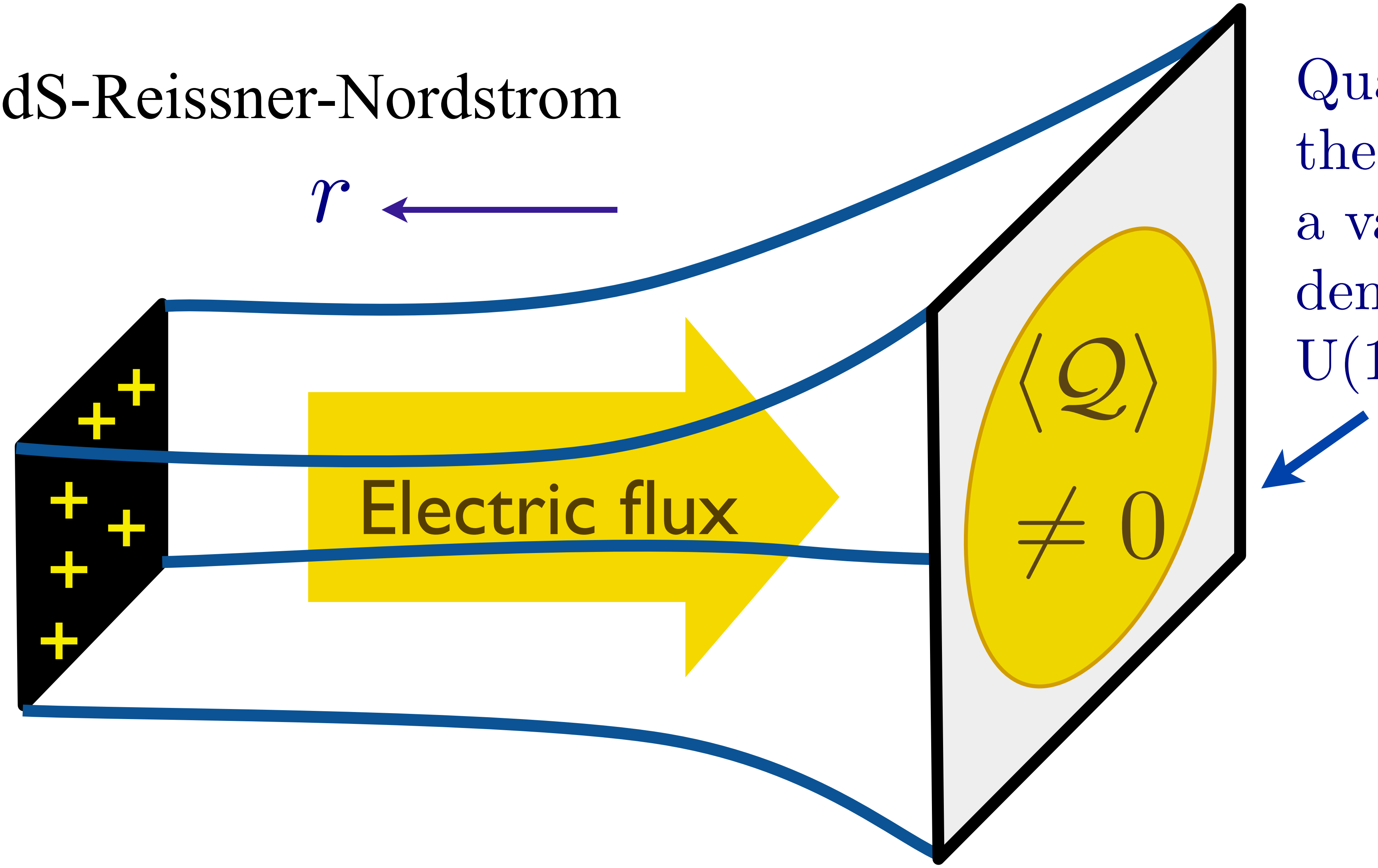
$$\boxed{\frac{\mathcal{M}}{\tau_{\text{imp}}} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\text{Im}\left(G^{\text{R}}_{\mathcal{O}\mathcal{O}}(q,\omega)\right)_{H_0}}{\omega} + \ldots}$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

Charged black branes

Einstein-Maxwell theory
$$\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$$

AdS-Reissner-Nordstrom

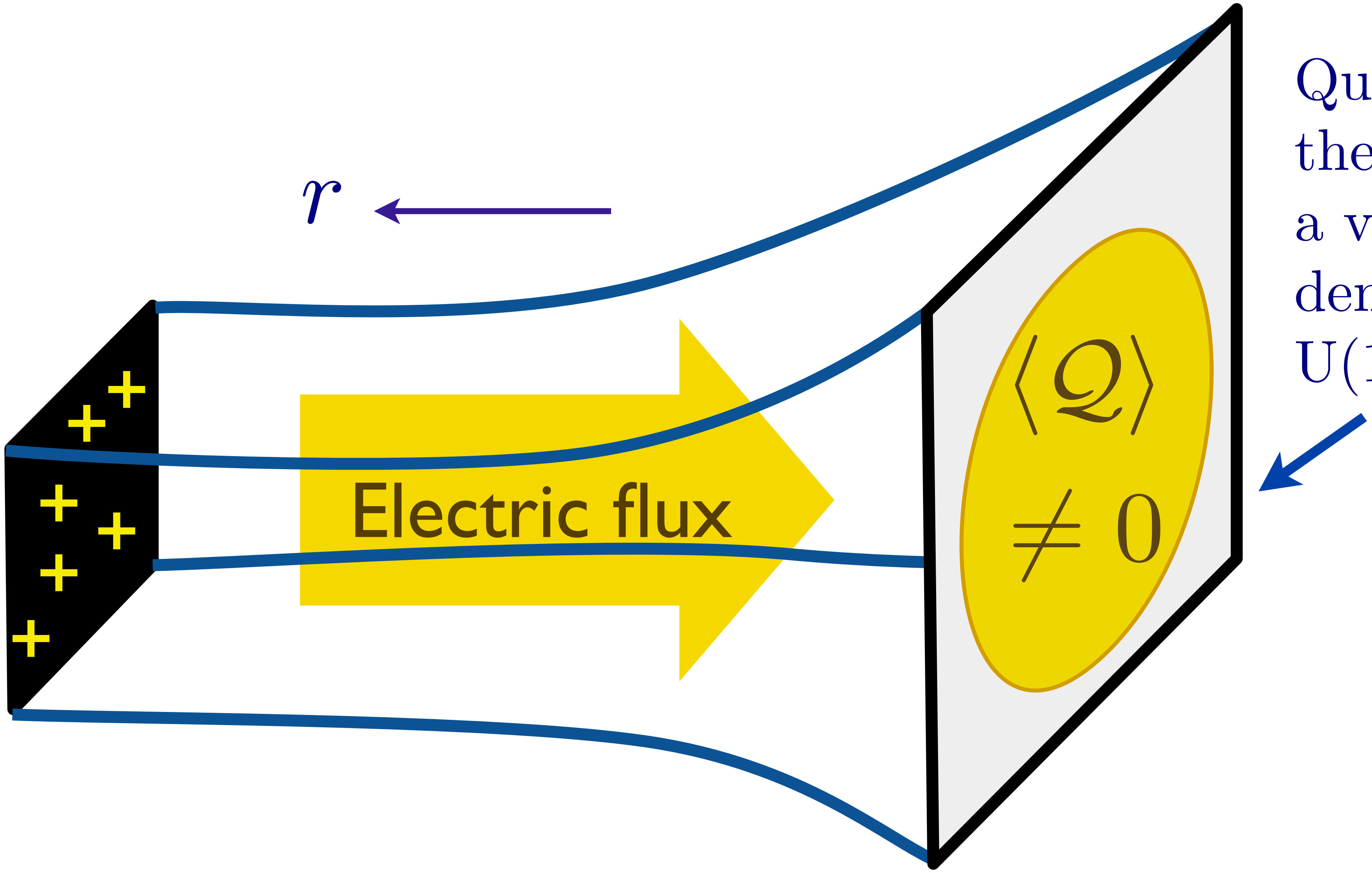


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, PRD **60**, 064018 (1999)

Charged black branes



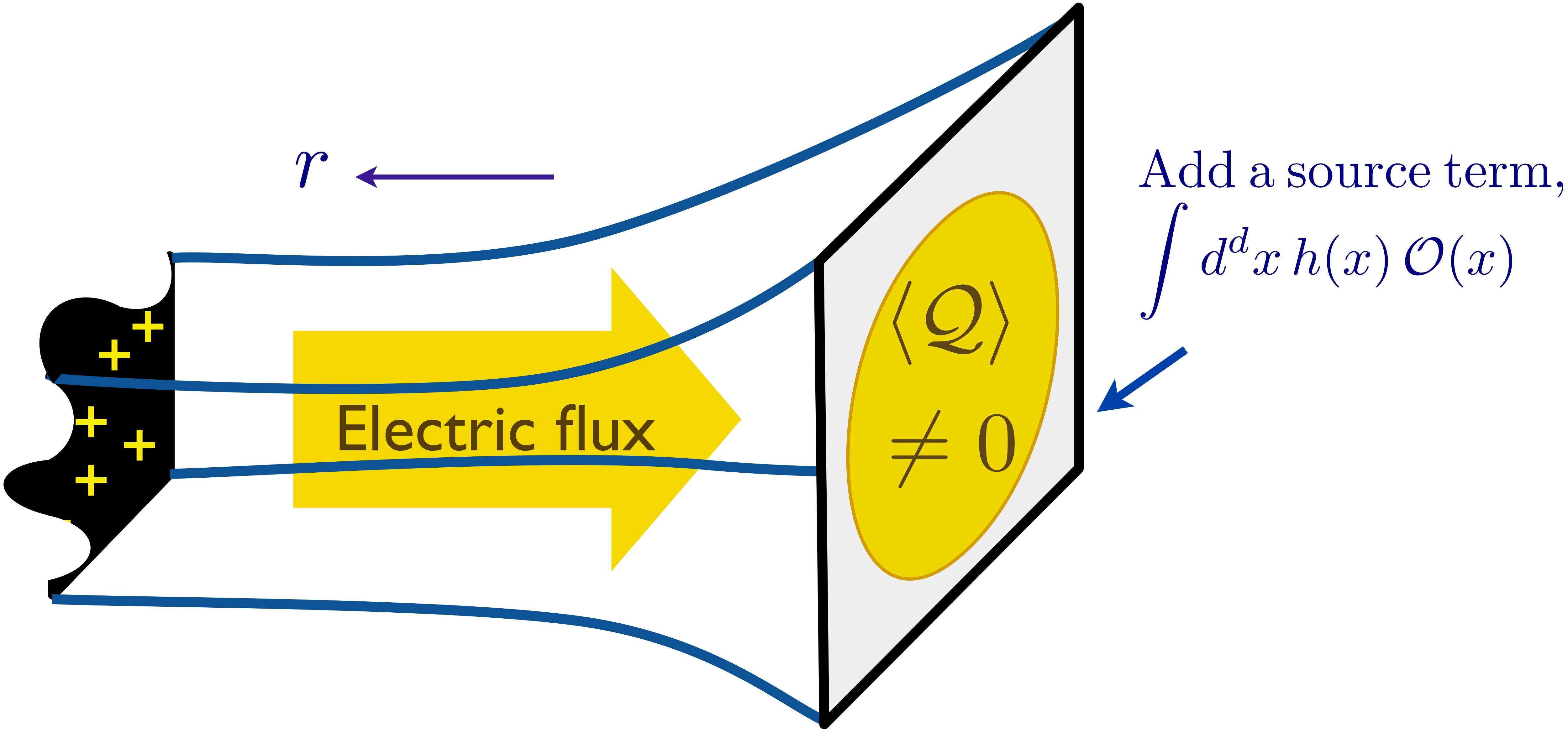
Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

More general theories have "hyperscaling violating metric"

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \qquad \text{at } T{=}0$$

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 94 (2012).
L. Huijse, S. Sachdev, B. Swingle, Phys. Rev. B **85**, 035121 (2012)

Inhomogeneous charged black branes



Add a source term,
$$\int d^d x \, h(x) \, \mathcal{O}(x)$$

Weakly disordered charged black branes yield results identical to those obtained from memory functions and hydrodynamics

- G.T. Horowitz, J.E. Santos, and D. Tong, JHEP **1207**, 168 (2012), JHEP **1211**, 102 (2012).
- D. Vegh, arXiv:1301.0537.
- M. Blake, D. Tong, and D. Vegh, PRL **112**, 071602 (2013).
- M. Blake and D. Tong, PRD **88**, 106004 (2013).
- A. Lucas, S. Sachdev, and K. Schalm, PRD 89, 066018 (2014).
- A. Lucas, JHEP **1503**, 071 (2015).
- R. A. Davison and B. Goutéraux, arXiv:1505.05092; arXiv:1507.07137.
- M. Blake, arXiv:1505.06992; arXiv:1507.04870.

Quantum matter without quasiparticles

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Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right)^{-1}$$

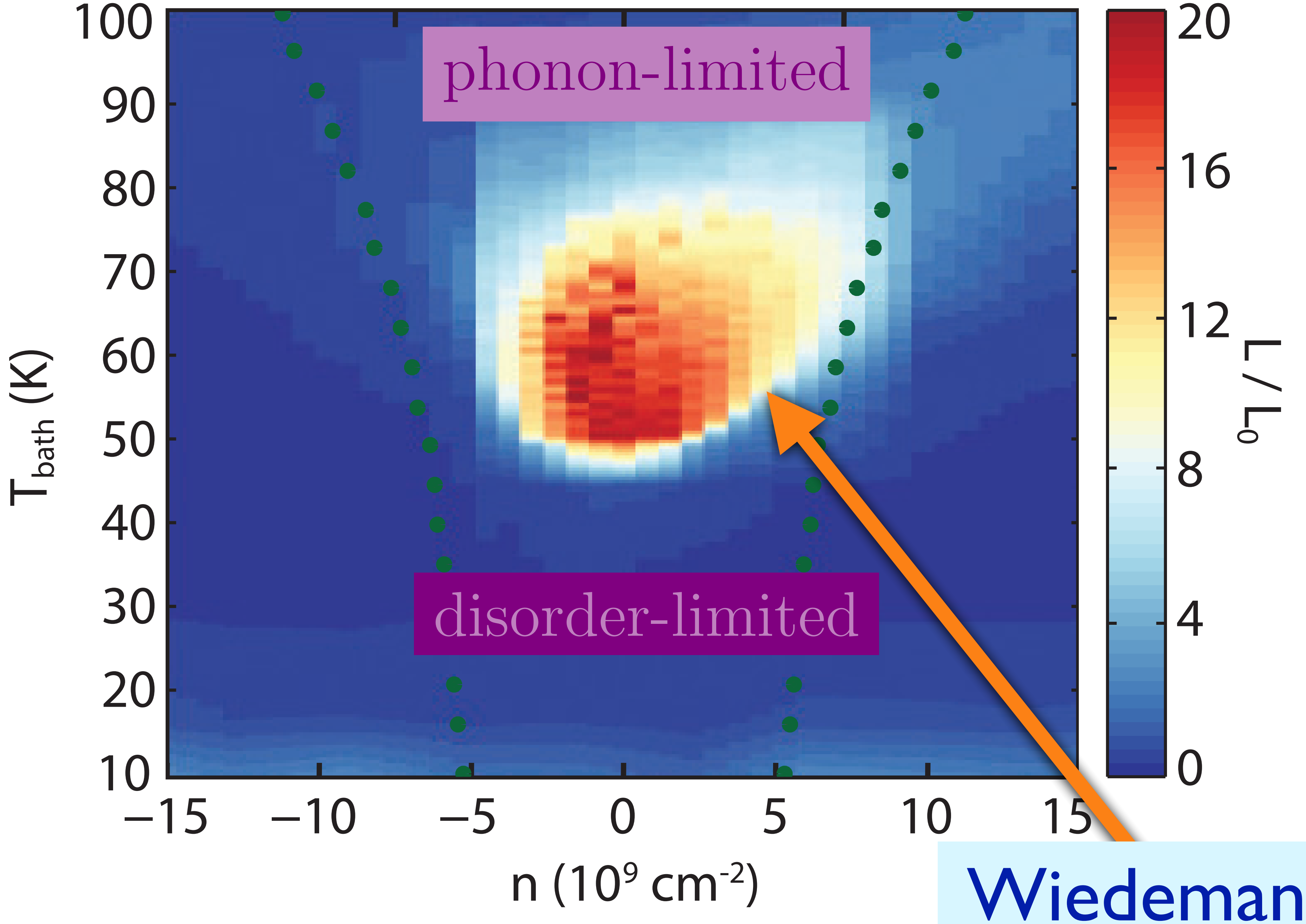
$$L = \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H} \sigma_Q} \right)^{-2} ,$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \rightarrow 0$ and $\tau_{\rm imp} \rightarrow \infty$ do not commute.

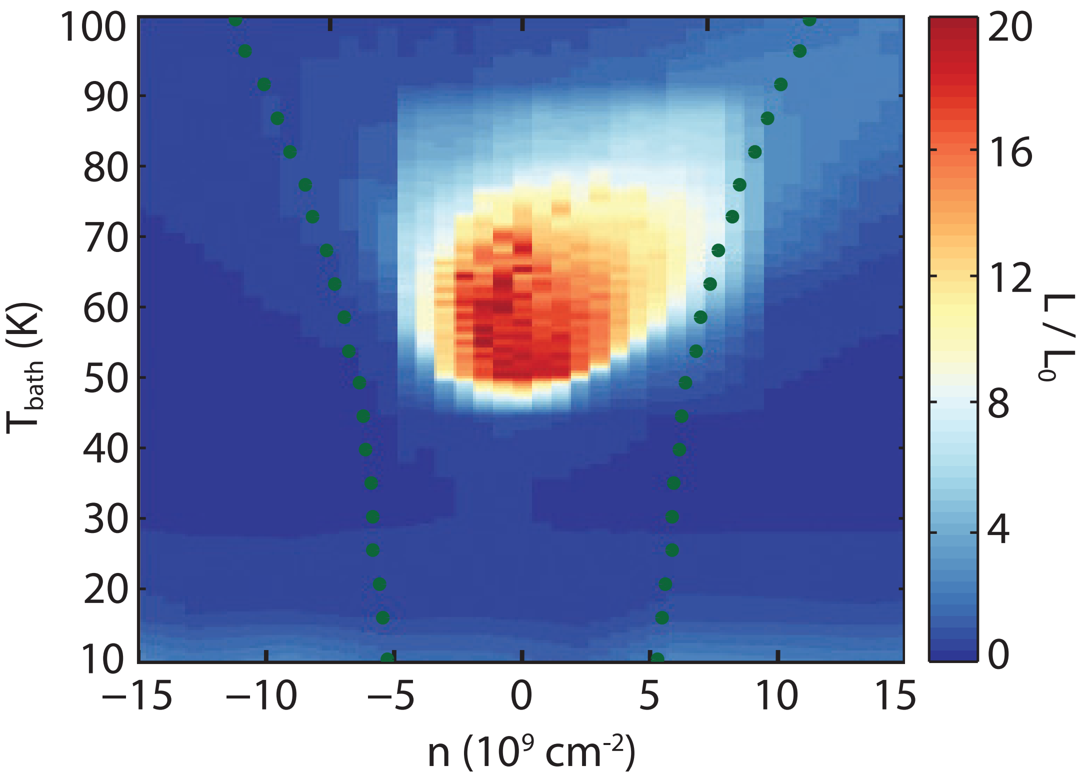
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

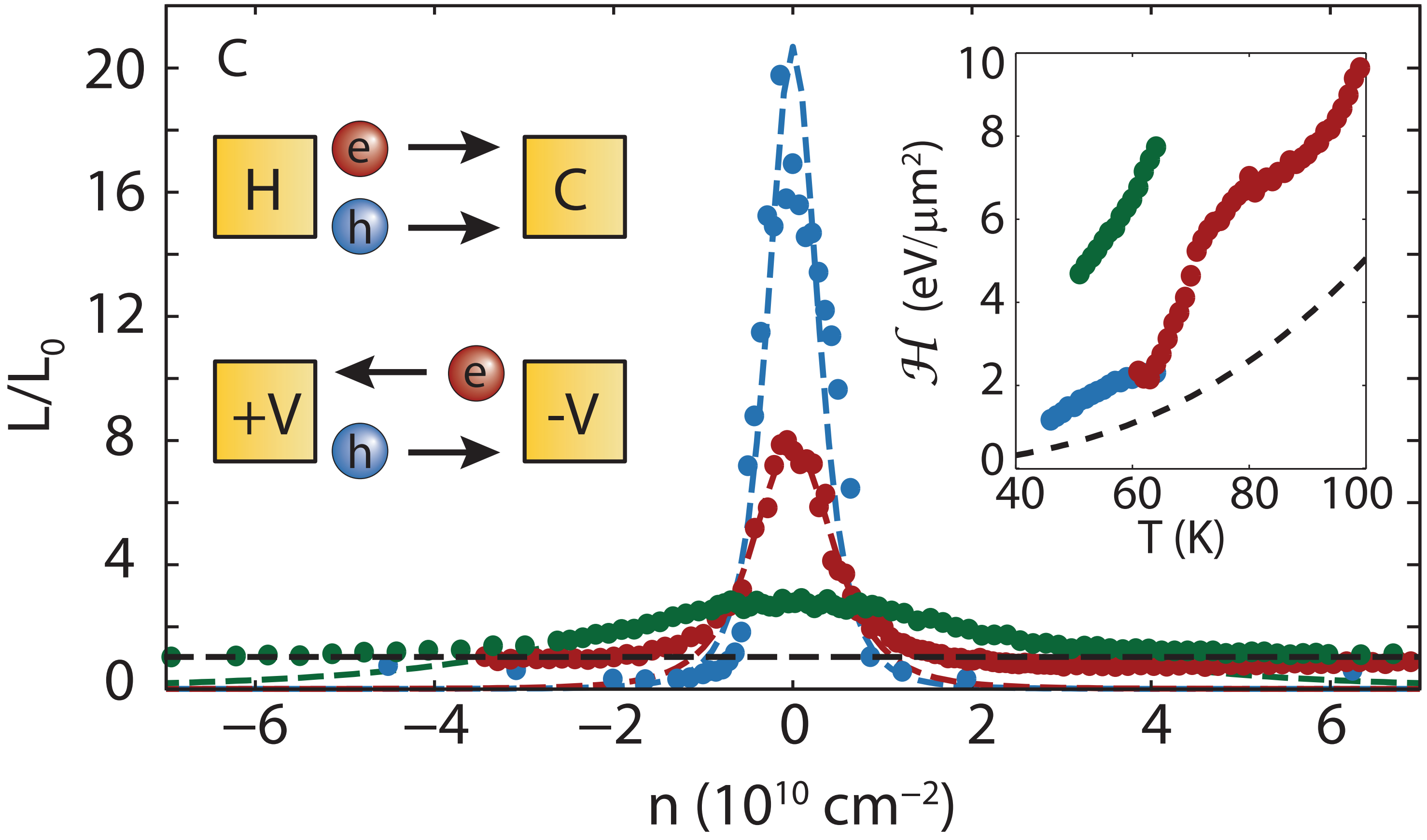
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

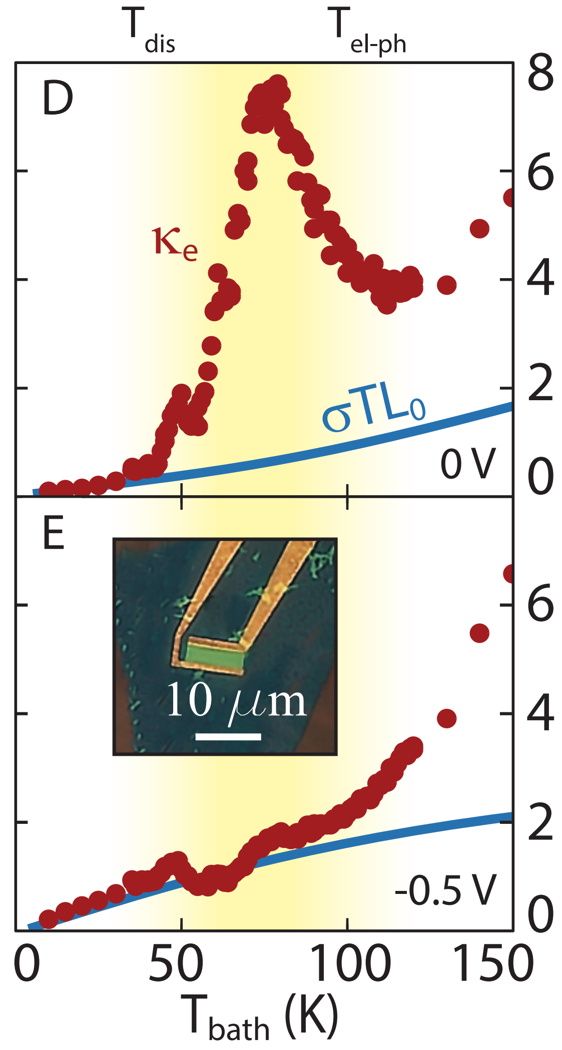
Strange metal in graphene



Wiedemann-Franz violated !

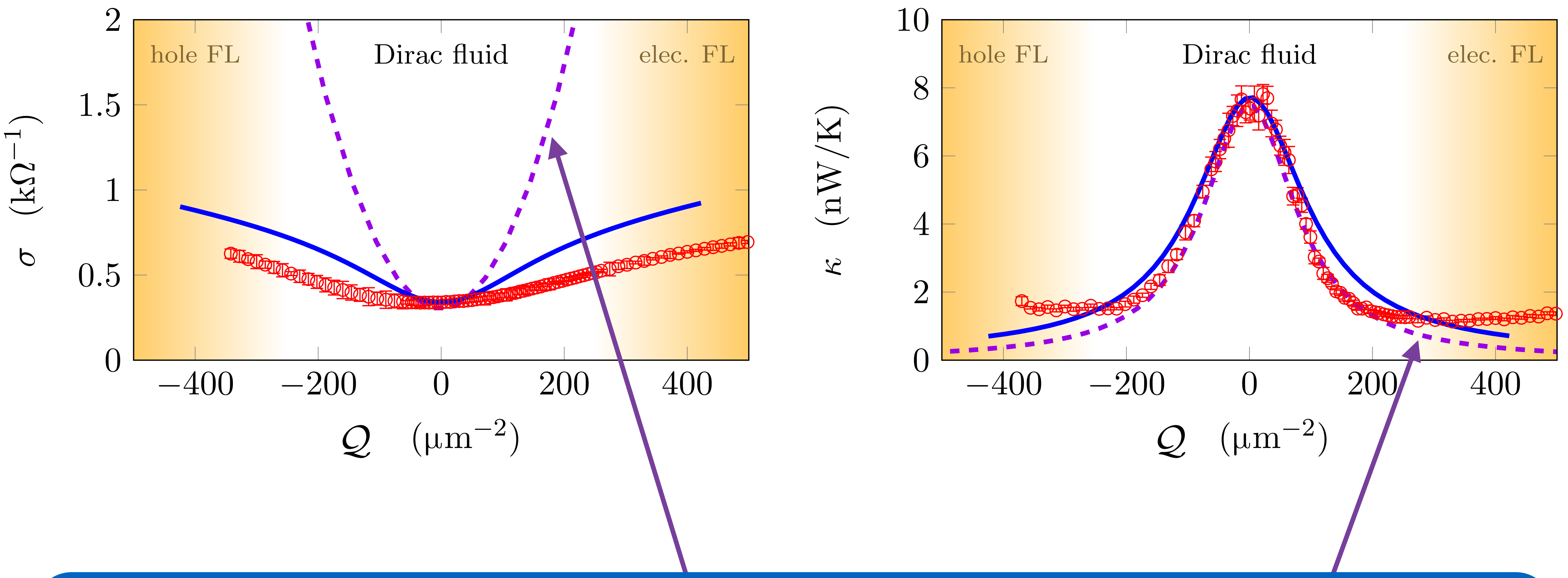






$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$
$$= \frac{v_F^2 \mathcal{H}\tau_{\rm imp}}{T^2\sigma_Q}\,\frac{1}{\left(1+e^2v_F^2\mathcal{Q}^2\tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

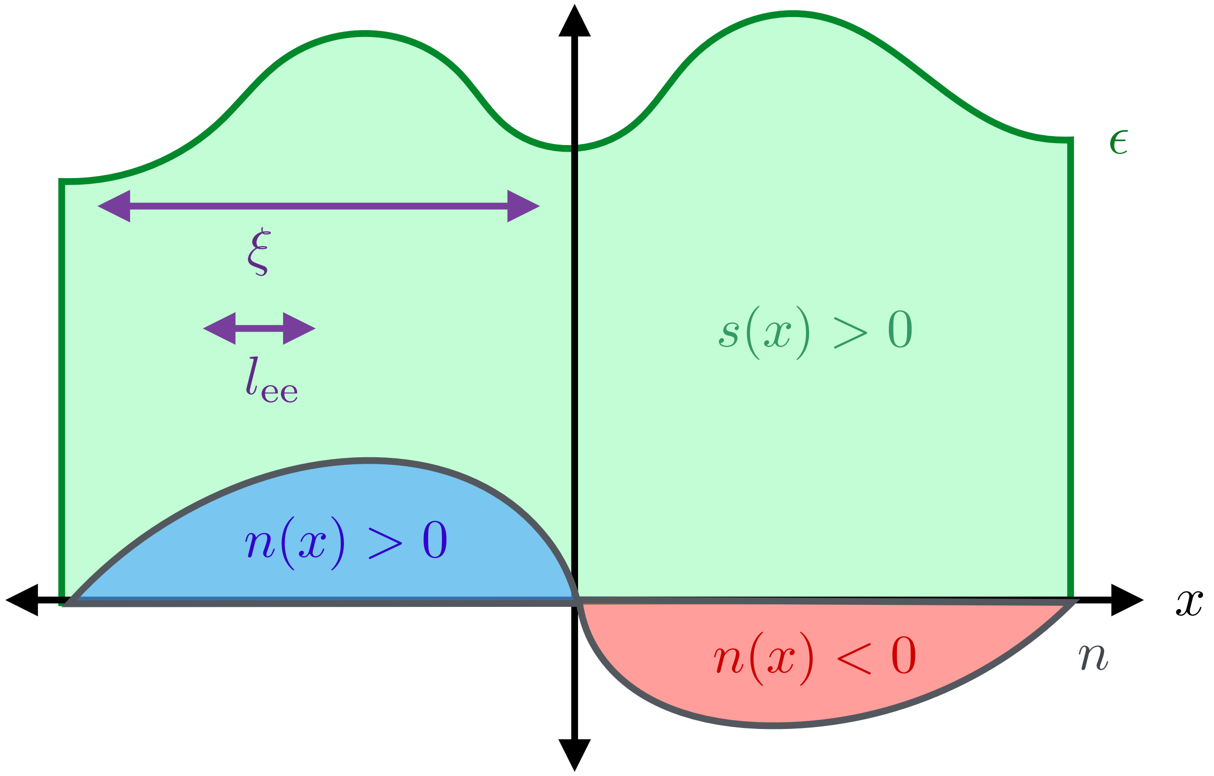
J. Crossno *et al.* arXiv:1509.04713; Science, to appear



Comparison to theory with a single momentum relaxation time τ_{imp}.
Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

Non-perturbative treatment of disorder

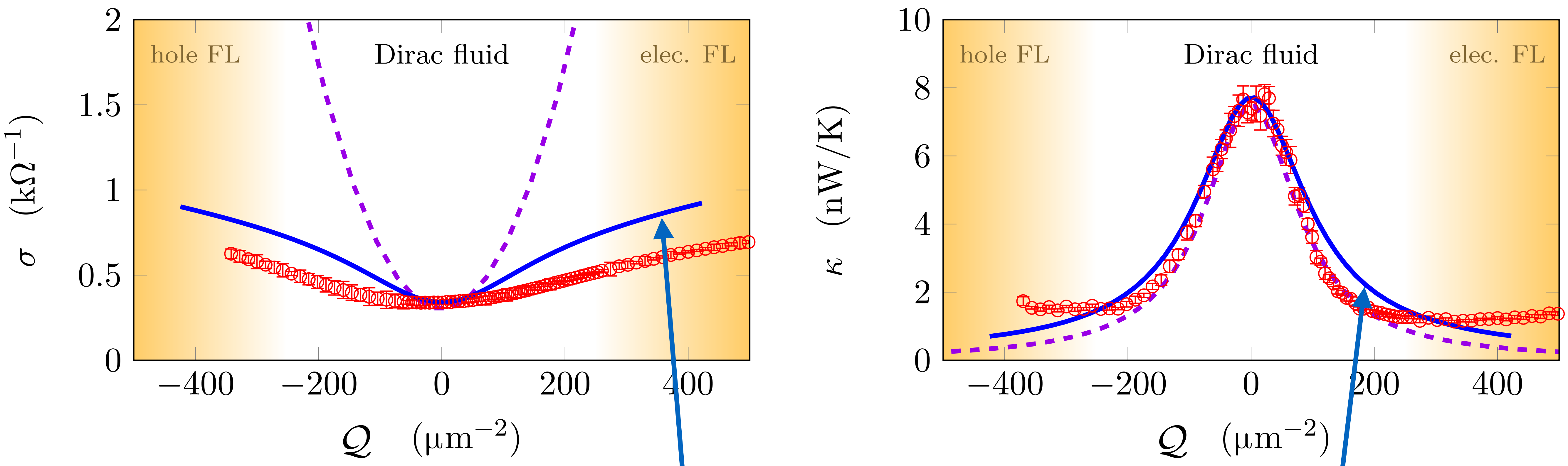


Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\text{B}}T$, and so the entropy density s/k_{B} is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than l_{ee}, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear



Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential. Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

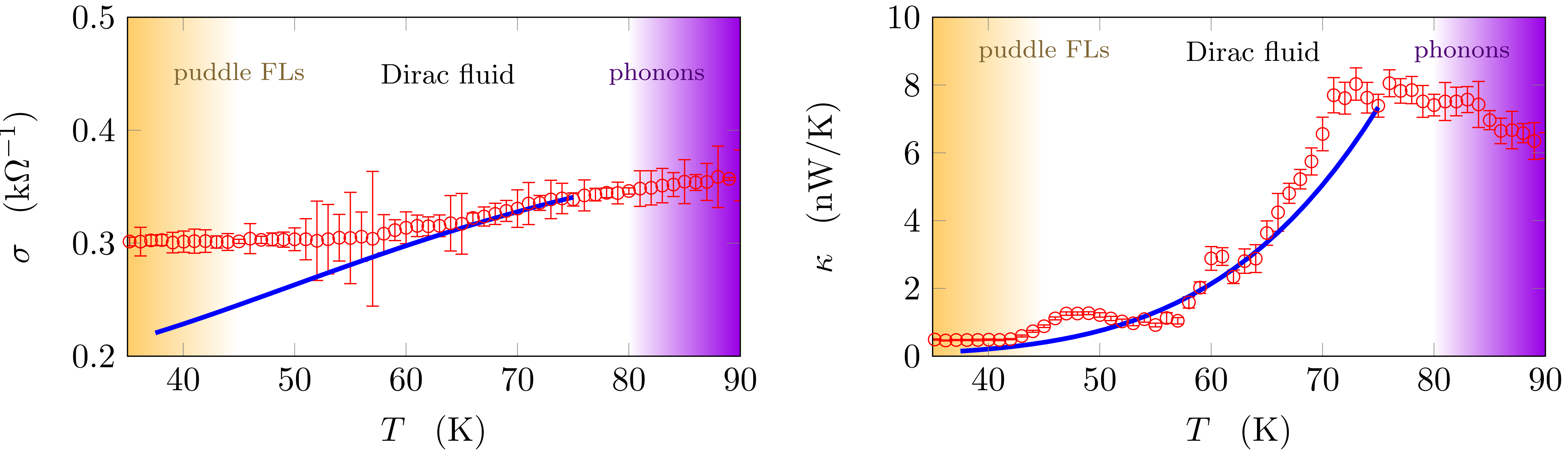


Figure 2: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at the charge neutrality point ($n = 0$). We use no new fit parameters co $\mathcal{Q}$ red to Figure 1. The yellow shaded region denotes $\mathcal{Q}$ re Fermi liquid behavior is observed; the purple shaded region denotes the likely onset of electron-phonon coupling.

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A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

Quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
  (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
  /memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.