

Compressible quantum liquids: field theory vs. holography

TIFR, Mumbai
January 19, 2012

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Compressible quantum matter

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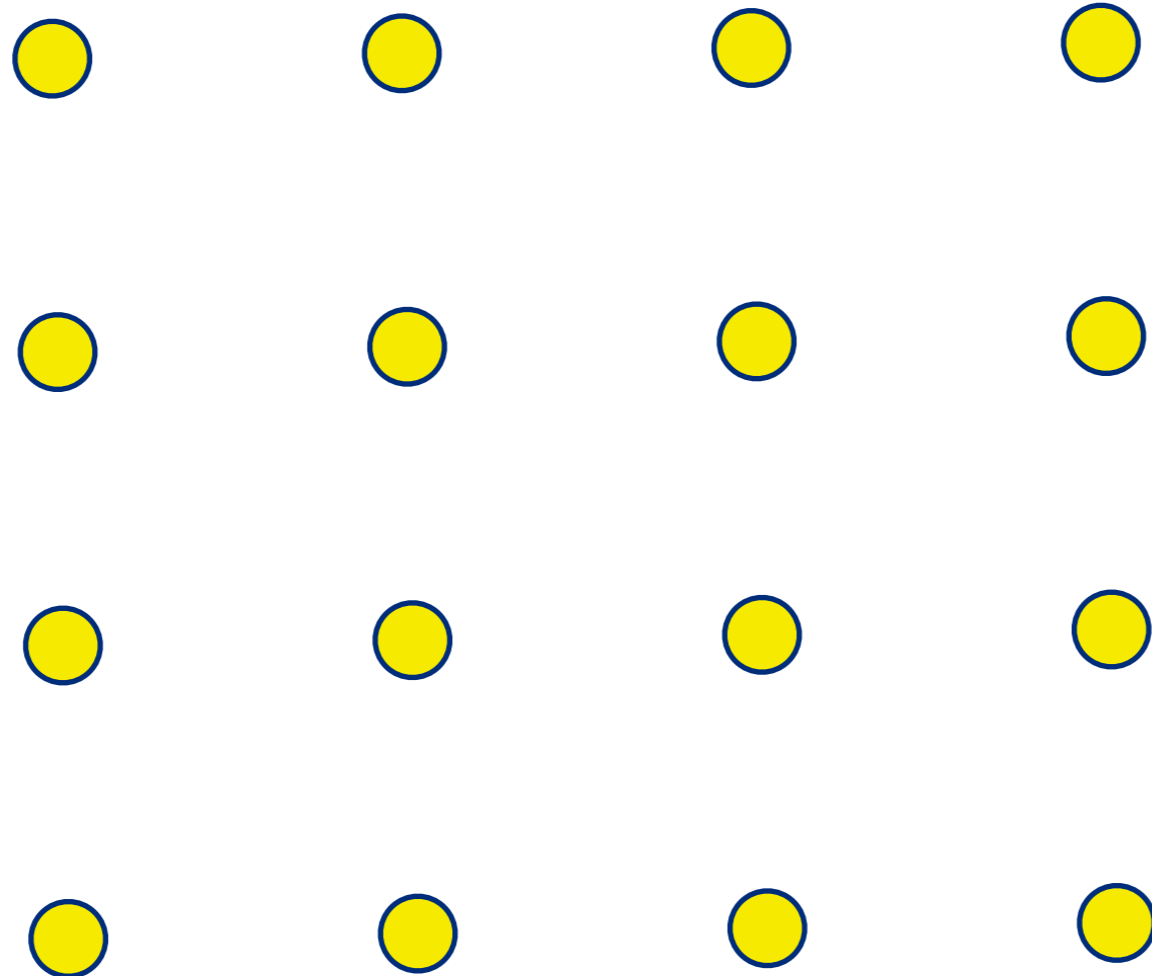
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- Compressible systems must be gapless.
- “Relativistic” quantum critical systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

One compressible state is the **solid** (or “Wigner crystal” or “stripe”).

This state breaks translational symmetry.



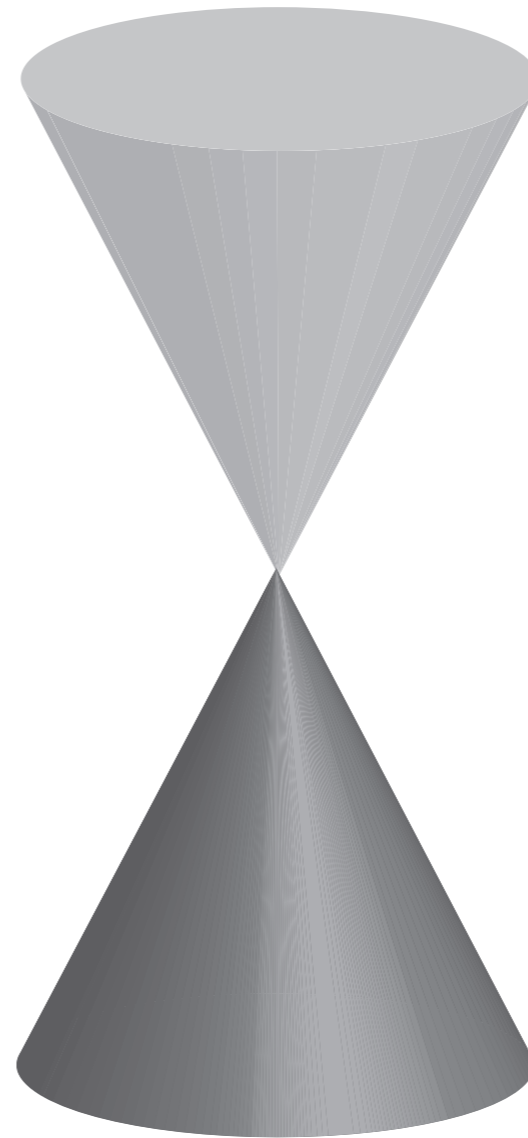
Compressible quantum matter

Another familiar compressible state is
the **superfluid**.

This state breaks the global $U(1)$
symmetry associated with Q

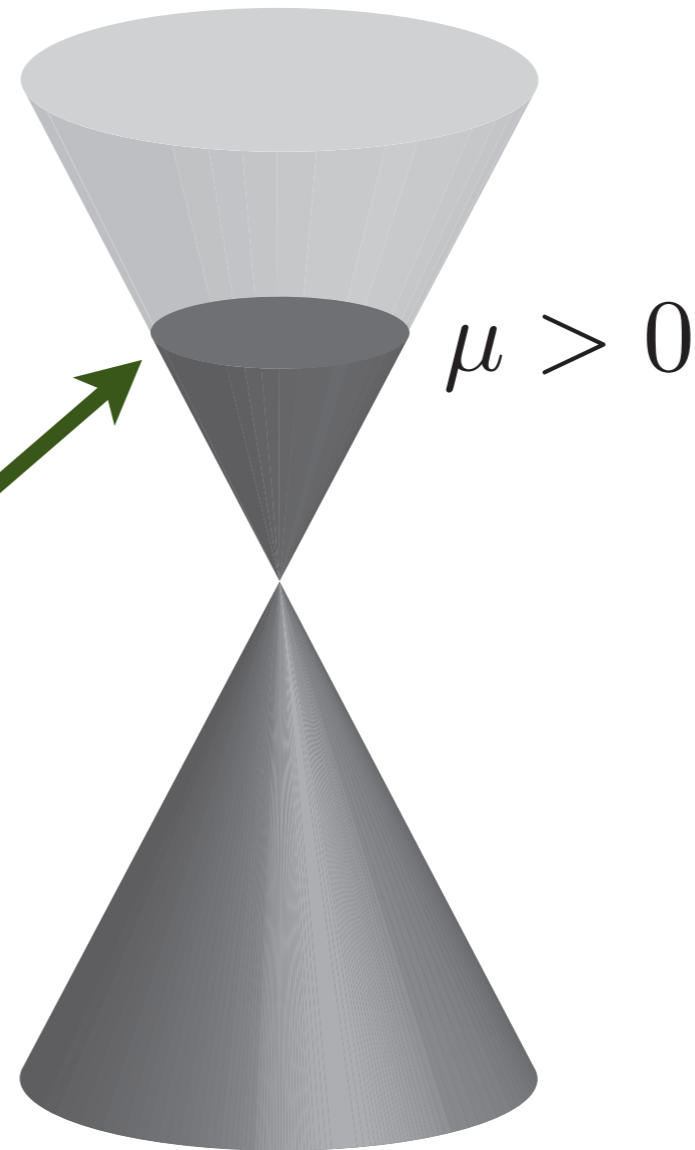


Condensate of
fermion pairs

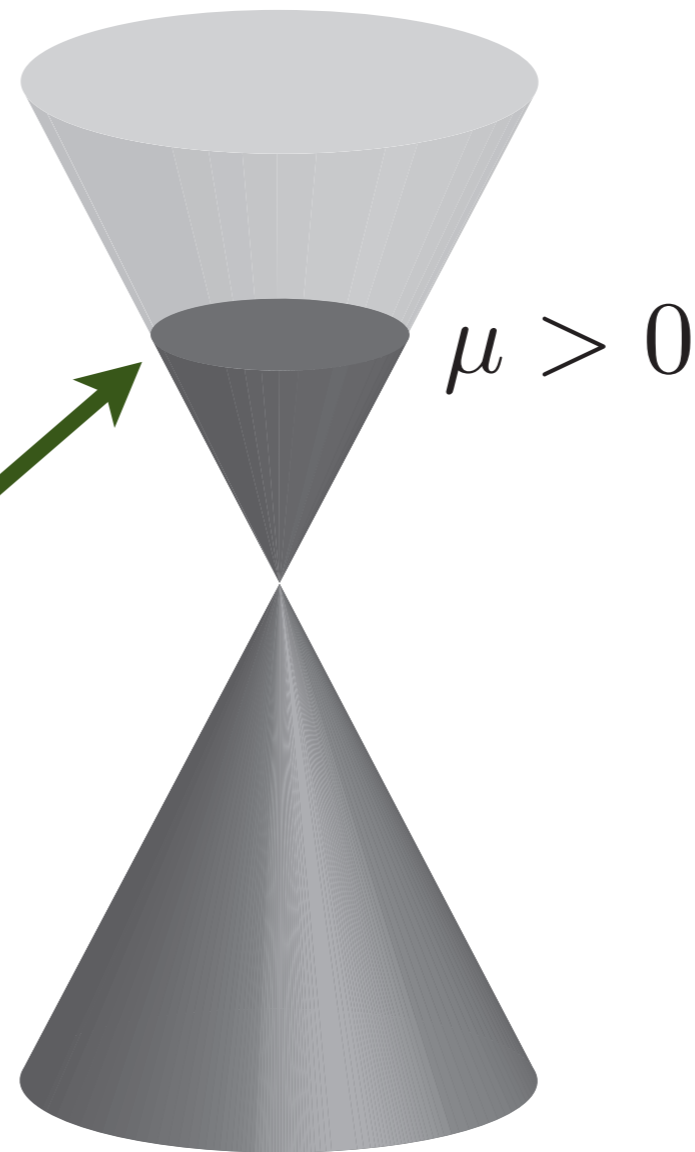


Graphene

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**

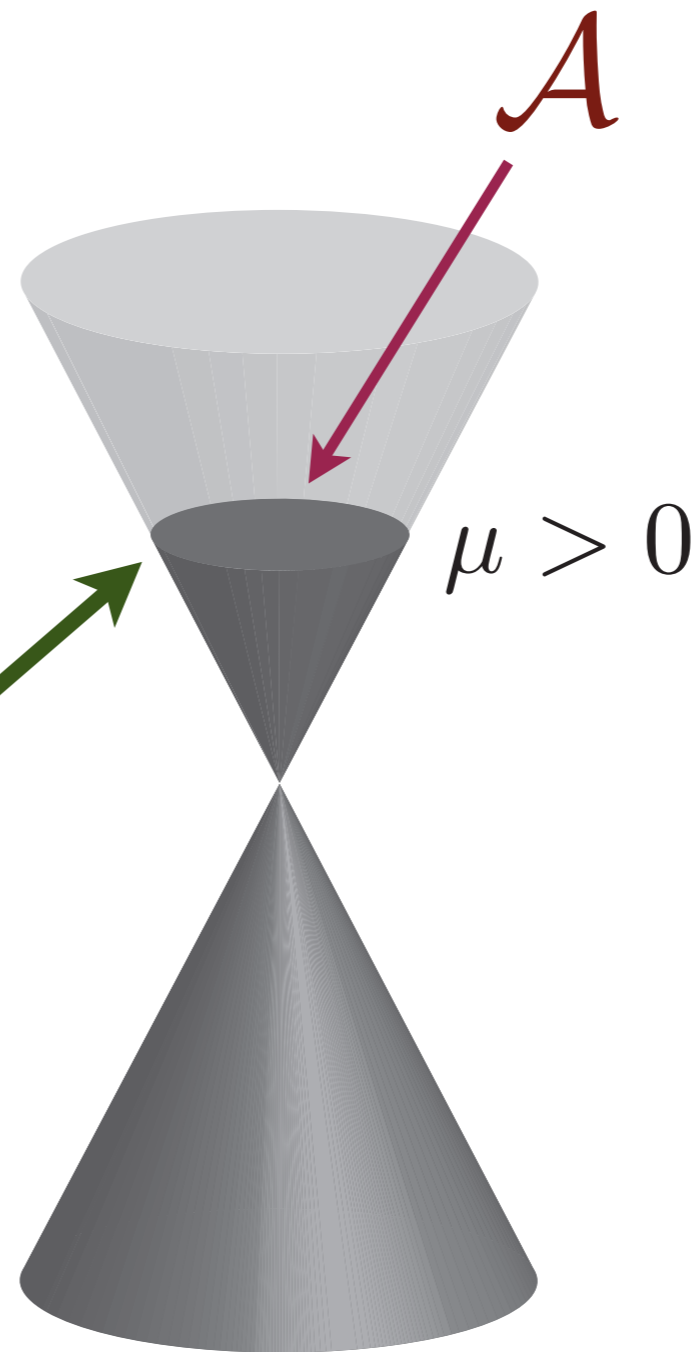


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- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



- **Luttinger relation:** The total “volume (area)” \mathcal{A} enclosed by the Fermi surface is equal to $\langle Q \rangle$.

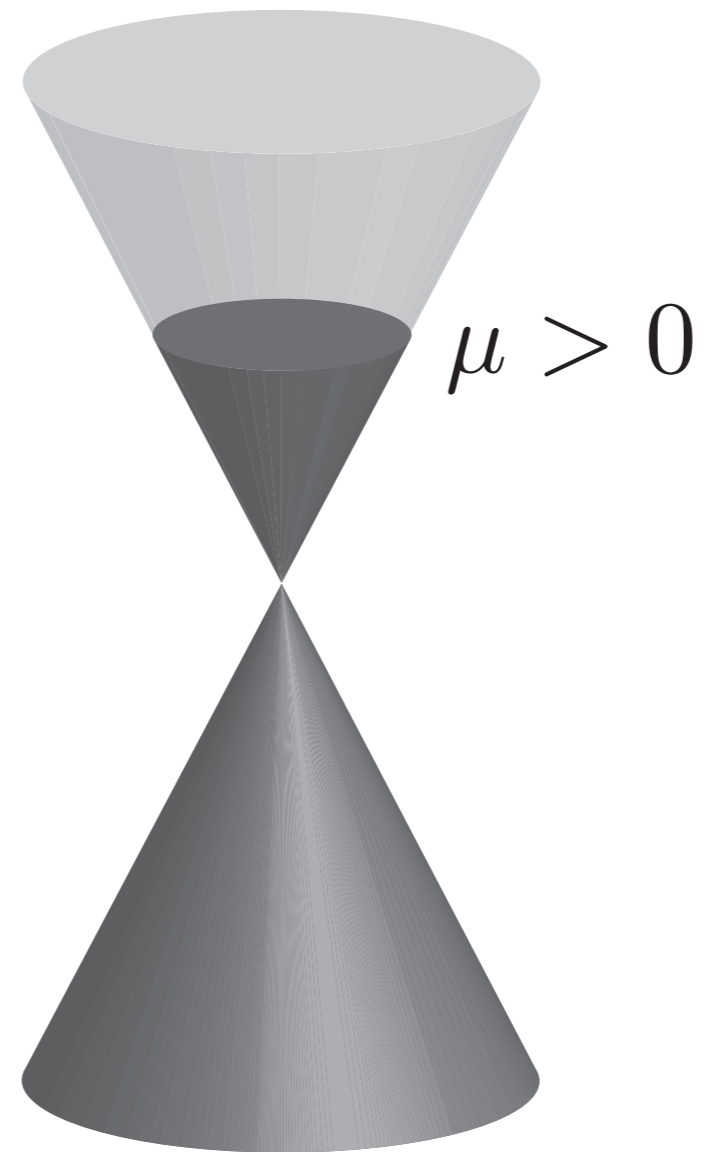
A. Field theory

B. Holography

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The Fermi Liquid (FL)



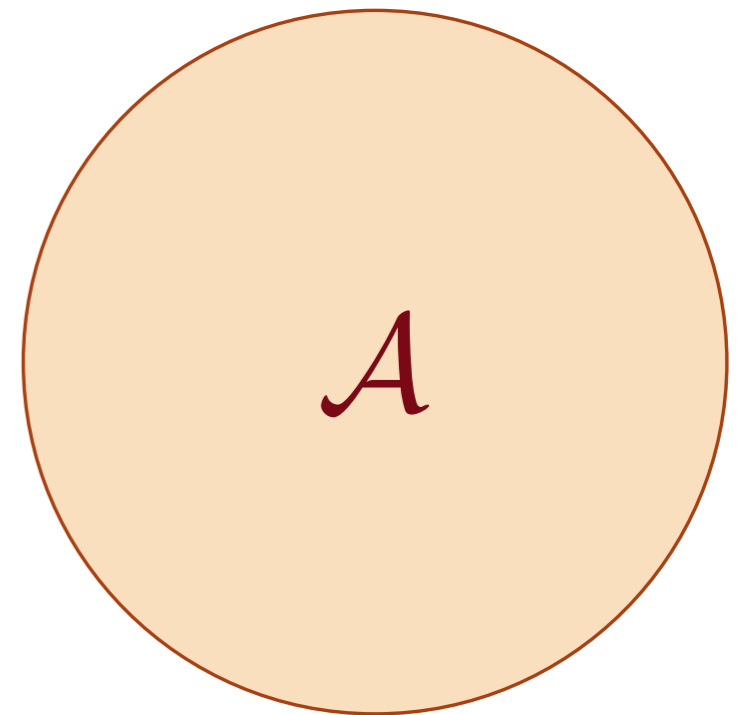
The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function G_f has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms}$$

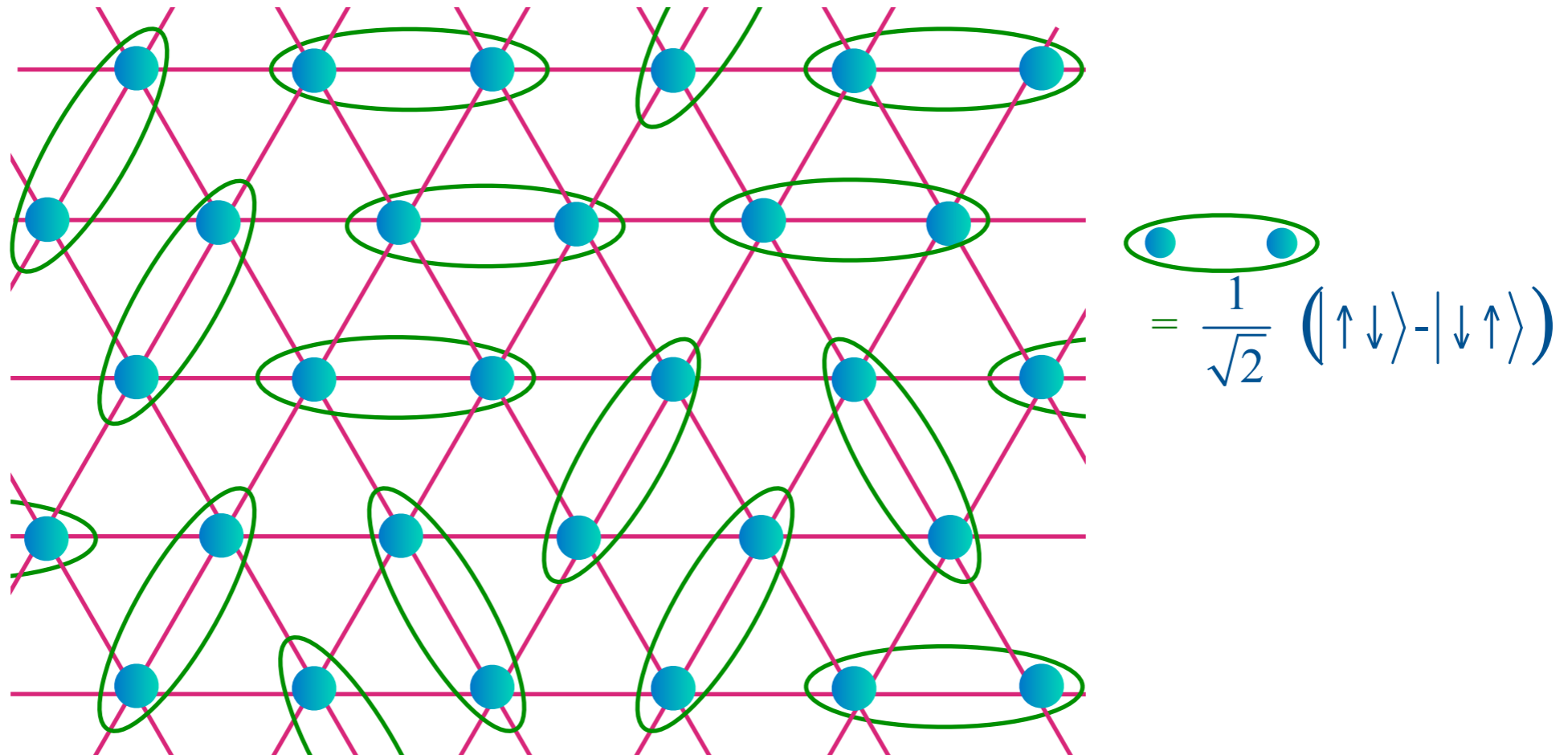
$$A = \langle f_\sigma^\dagger f_\sigma \rangle = \langle Q_\sigma \rangle$$

$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



The Non-Fermi Liquid (NFL)

- Model of a spin liquid (“Bose metal”): couple fermions to a dynamical gauge field A_μ .



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- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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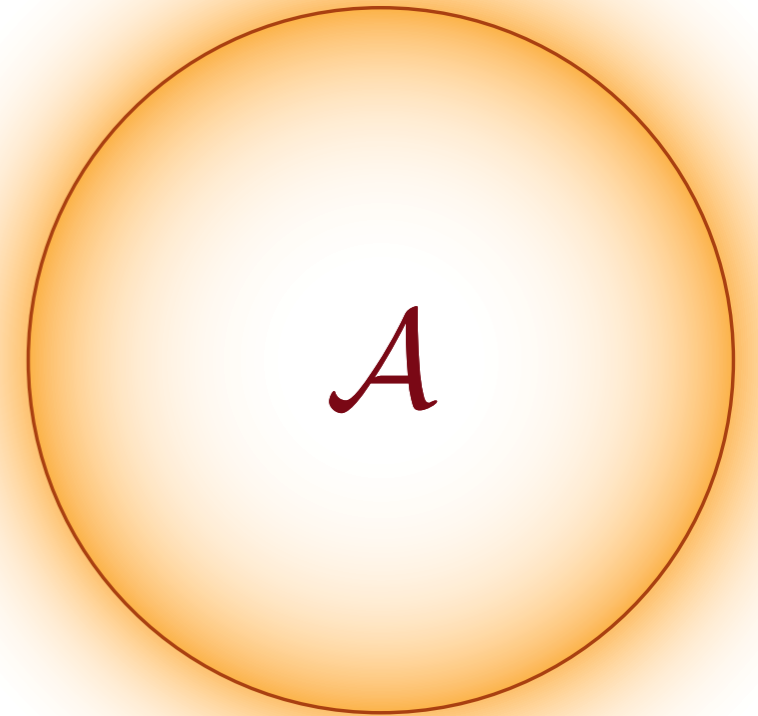
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- The overdamped transverse gauge modes lead to “non-Fermi liquid” broadening of the fermion pole near the Fermi surface.

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The Non-Fermi Liquid (NFL)

- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*

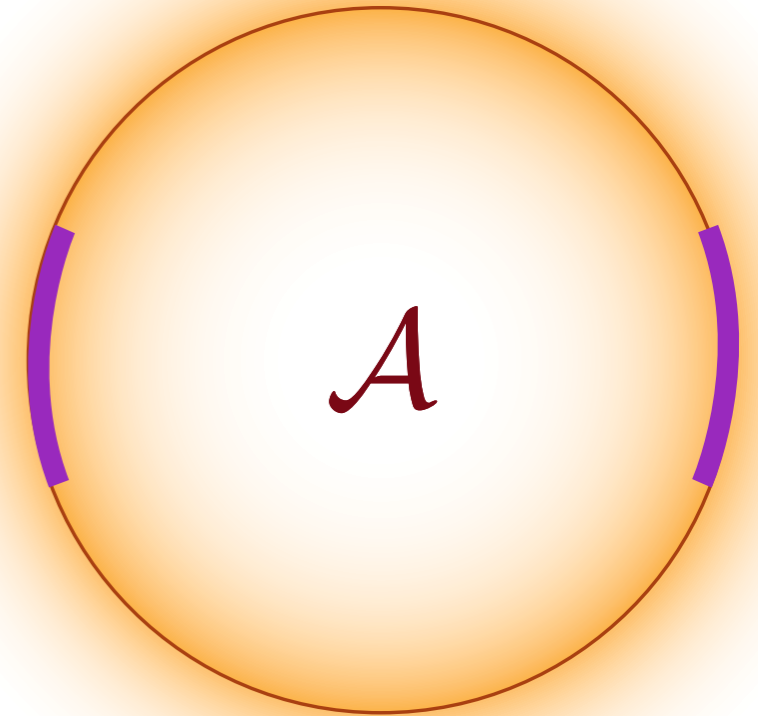


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- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*
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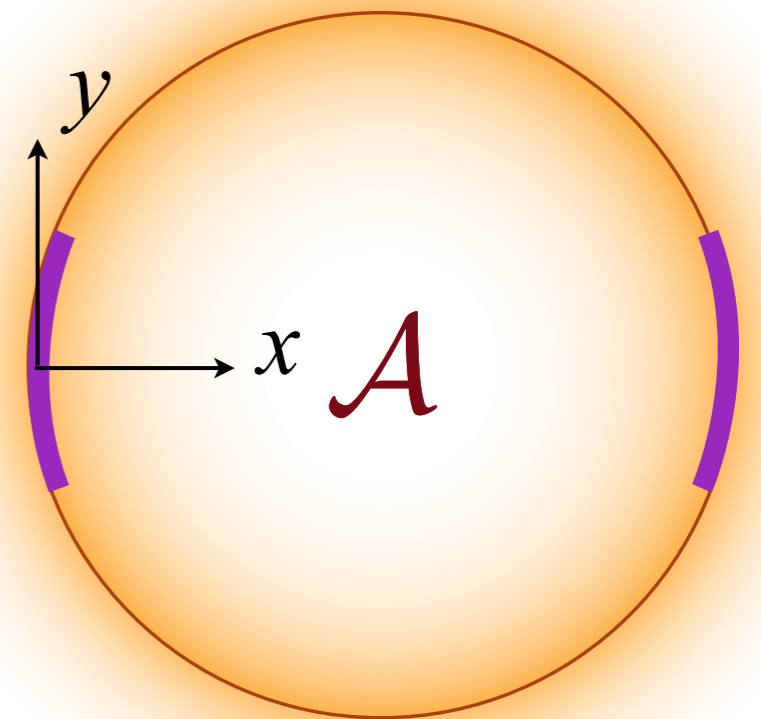
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- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega/q^z)$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2/(2k_F)$, and η and z are anomalous exponents. To three-loop order, we find $\eta \neq 0$ and $z = 3/2$.

$$\text{One-loop order: } G_f^{-1} \sim v_F q + i\omega^{2/3}$$



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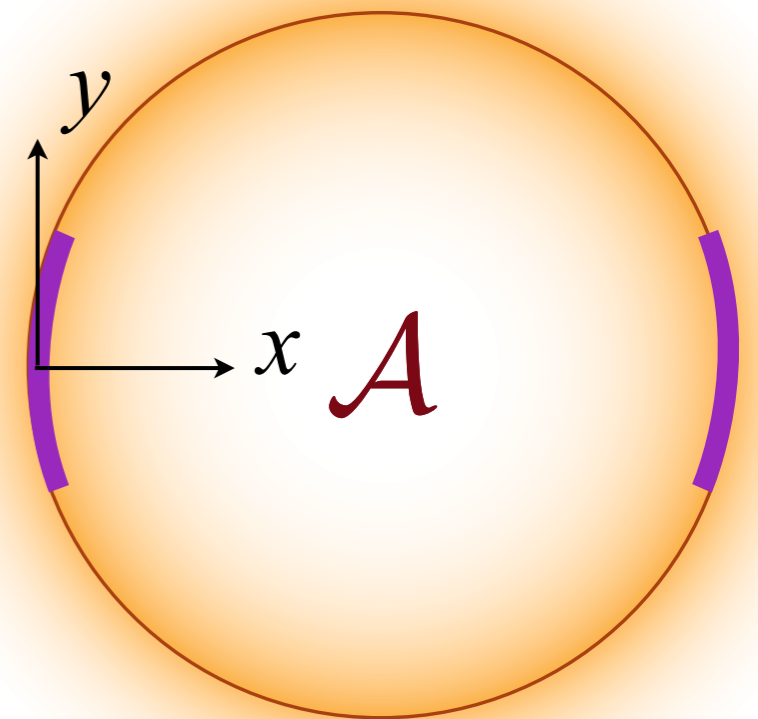
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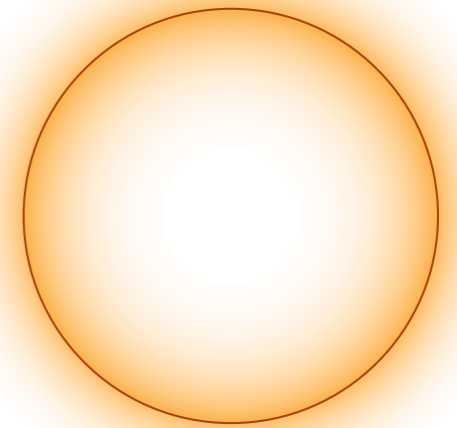


$$A = \langle f_\sigma^\dagger f_\sigma \rangle = \langle Q_\sigma \rangle$$

Key question:

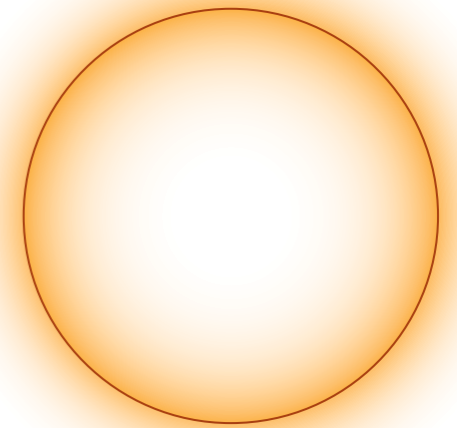
How do we detect the
“hidden Fermi surfaces”
of fermions with gauge charges
in the non-Fermi liquid phases ?

These are not directly visible in the
gauge-invariant fermion correlations
computable via holography



One promising answer:

How do we detect the
“hidden Fermi surfaces”
of fermions with gauge charges
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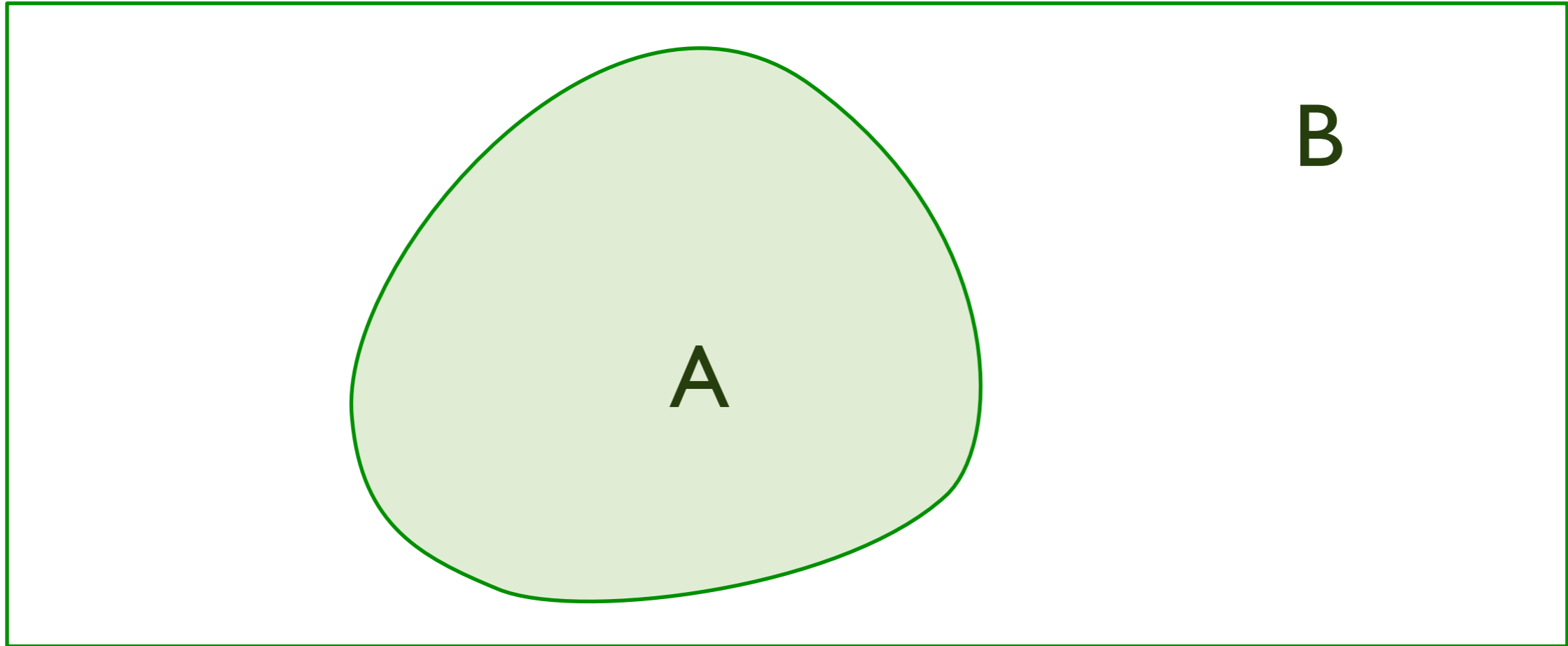


Compute
entanglement entropy

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023

L. Huijse, B. Swingle, and S. Sachdev arXiv:1112.0573

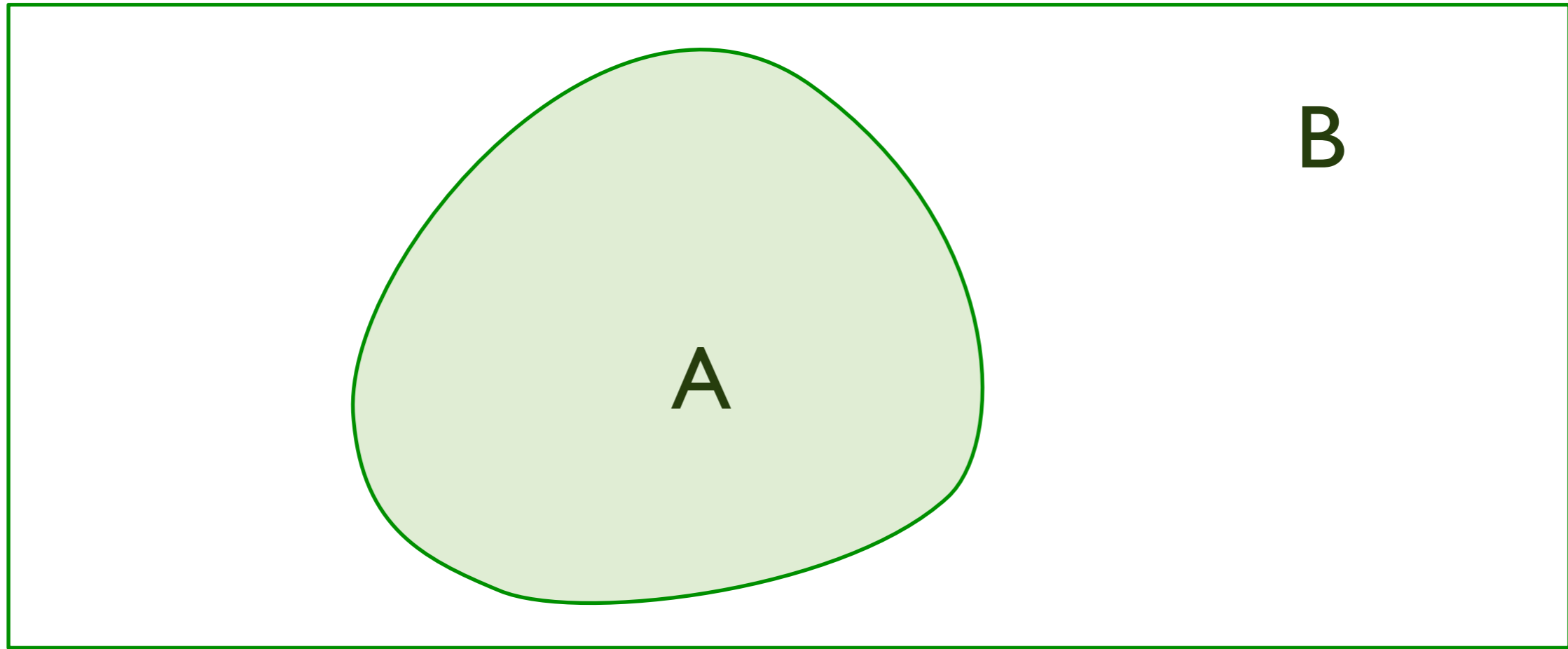
Entanglement entropy of Fermi surfaces



$\rho_A = \text{Tr}_B \rho =$ density matrix of region A

Entanglement entropy $S_{EE} = -\text{Tr} (\rho_A \ln \rho_A)$

Entanglement entropy of Fermi surfaces



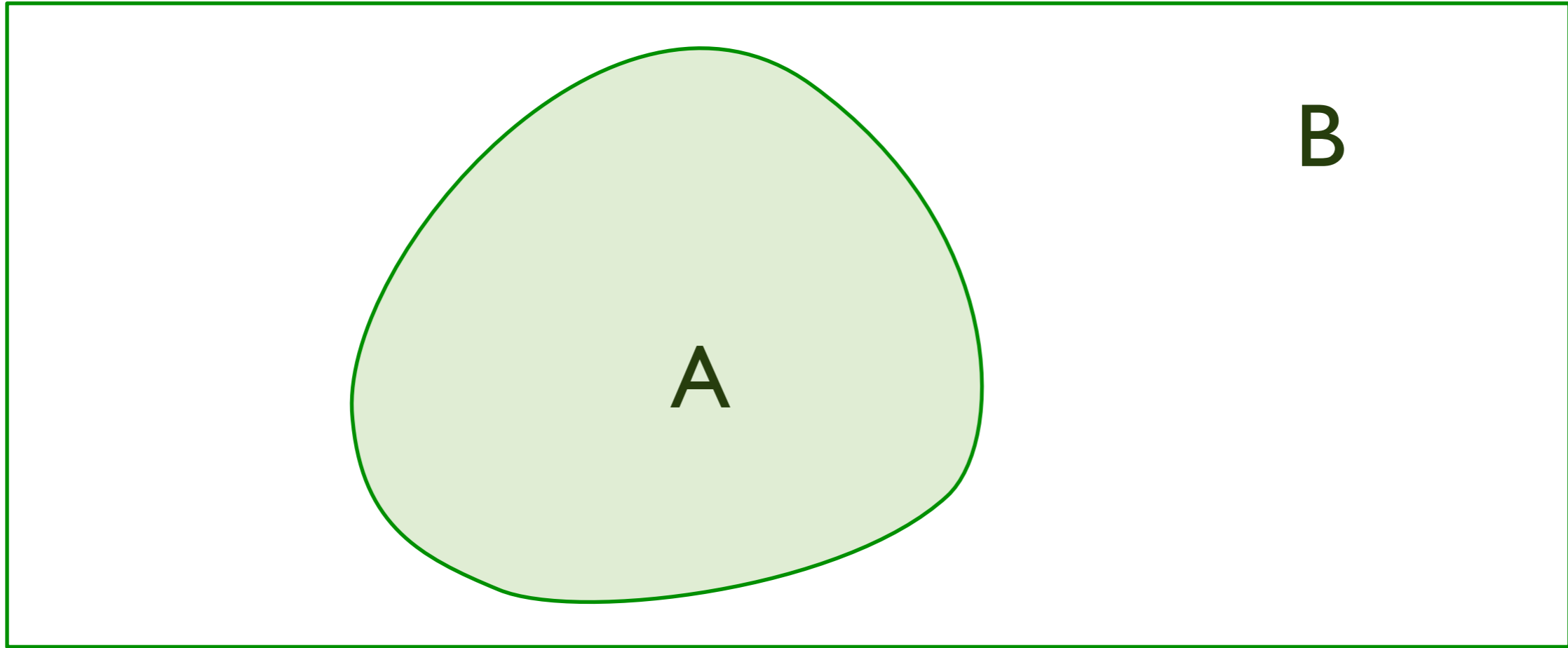
Logarithmic violation of “area law”: $S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F ,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)

B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Entanglement entropy of Fermi surfaces



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Non-Fermi liquids have, at most, the “1/12” prefactor modified.

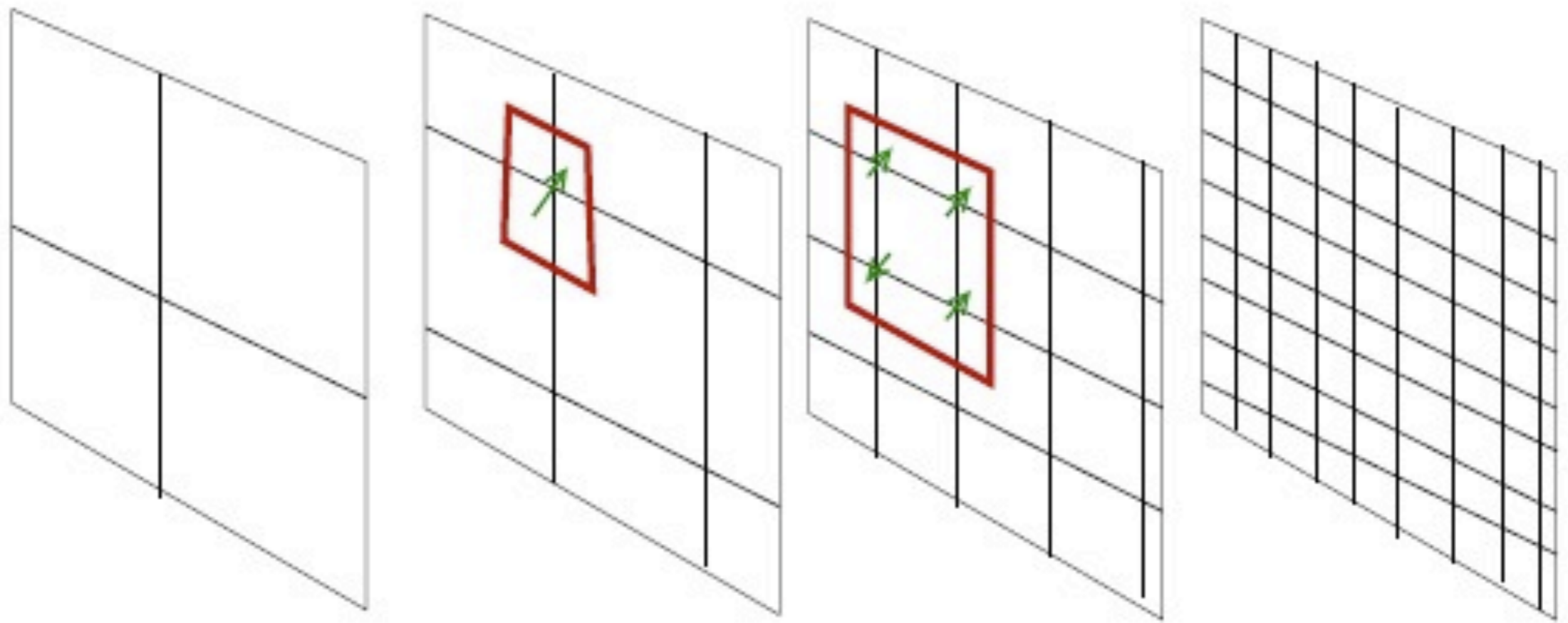
Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

A. Field theory

B. Holography

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r ←

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

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This gives the unique metric

$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \rightarrow \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2} .

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

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This metric transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta x_i \\ t &\rightarrow \zeta^z t \\ ds &\rightarrow \zeta^{\theta/d} ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for “relativistic” quantum critical points).

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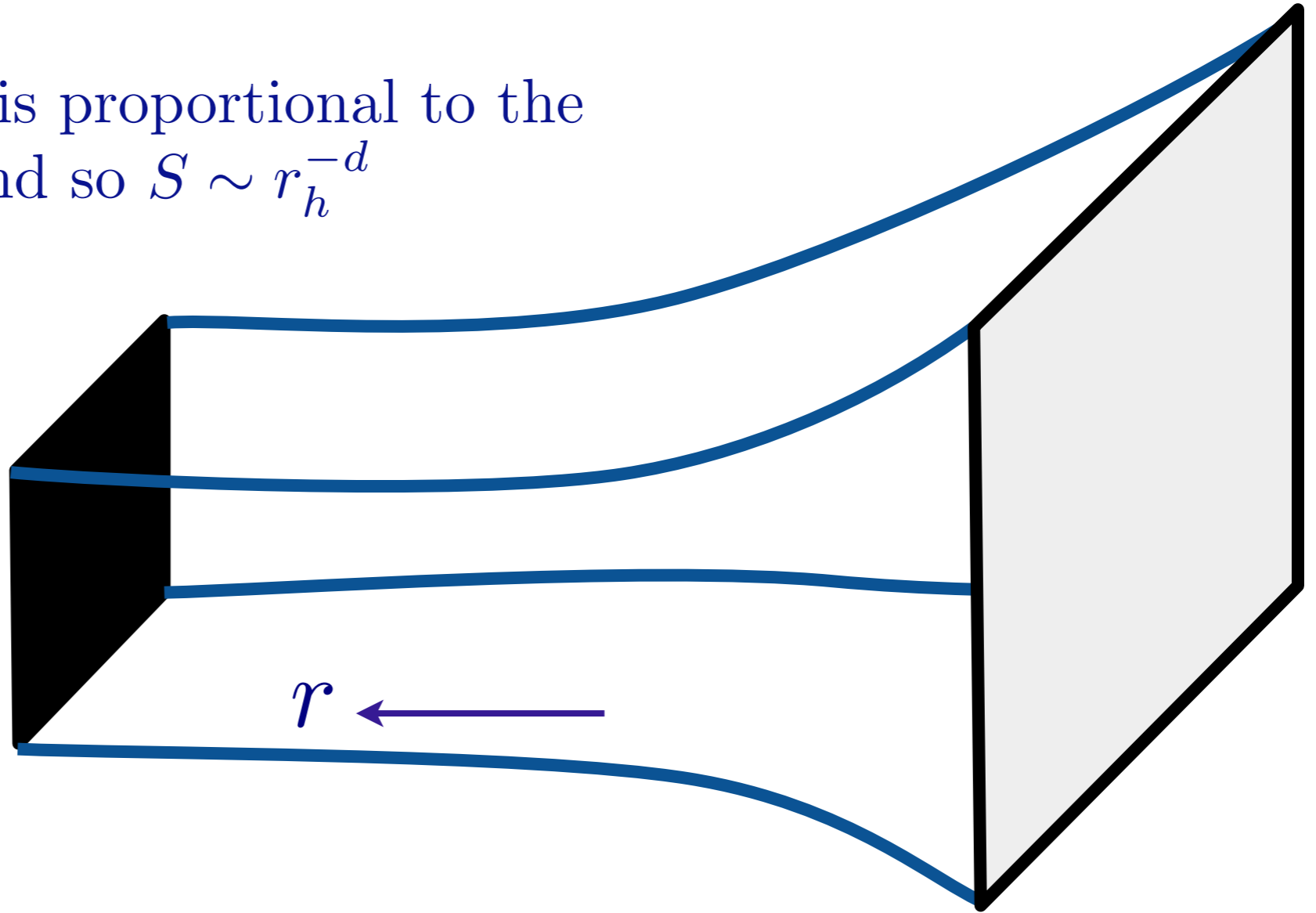
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What is θ ? ($\theta = 0$ for “relativistic” quantum critical points).

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Bekenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

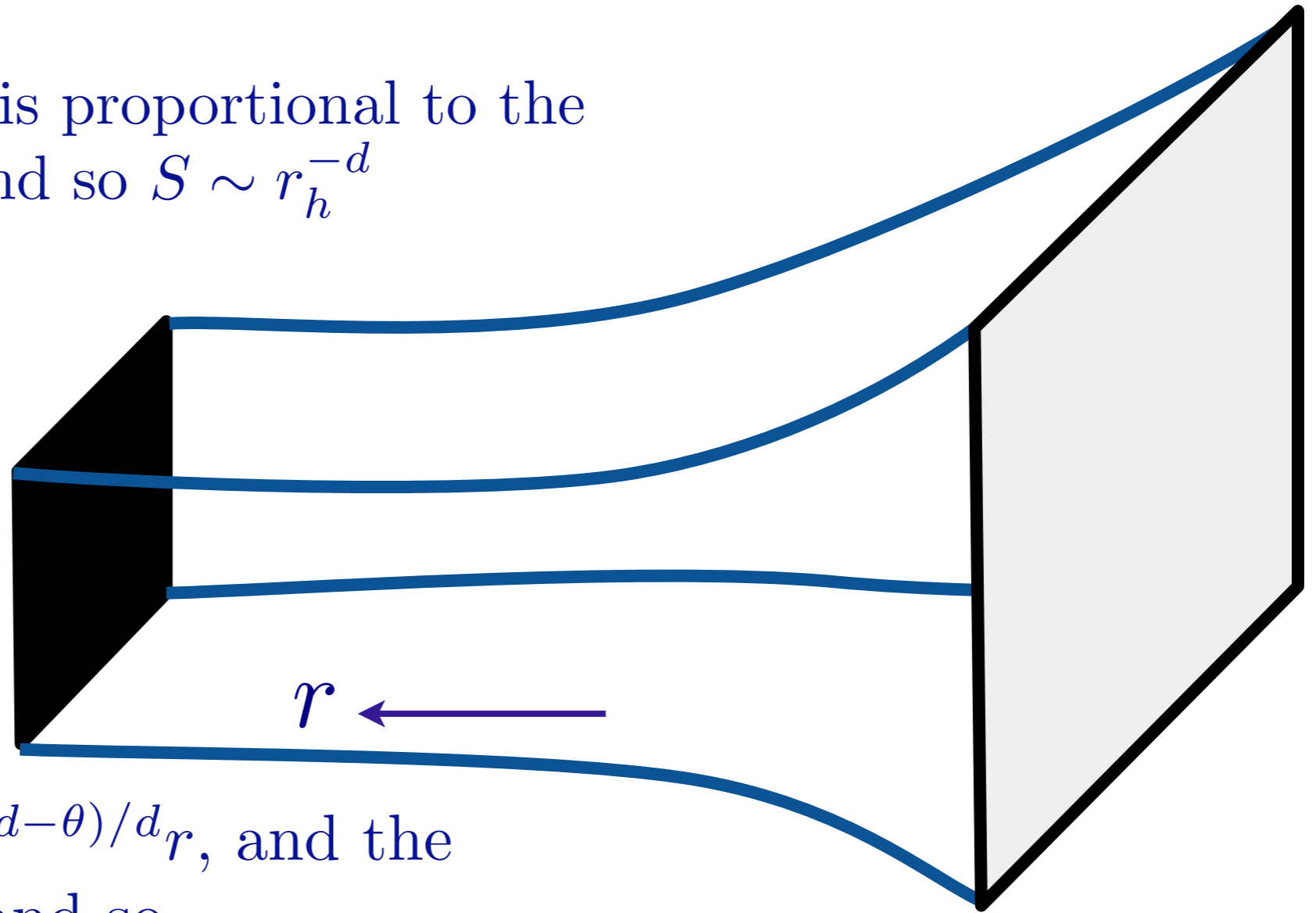
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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z}$$

So θ is the “violation of hyperscaling” exponent.

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z . So we expect compressible quantum states to have an effective dimension $d - \theta$ with

$$\theta = d - 1$$

Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A .

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Entanglement entropy of the metric

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The area law is obeyed for

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For $\theta = d - 1$, the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

$$S_E = \Xi \mathcal{Q}^{(d-1)/d} \Sigma \ln \left(\mathcal{Q}^{(d-1)/d} \Sigma \right).$$

- Σ is the $(d - 1)$ -dimensional surface area of entangling region (in $d = 2$, $\Sigma = P$ is the perimeter). Note S_E is otherwise independent of the shape of the entangling region, unlike other gapless systems. This is a characteristic property of a Fermi surface

Entanglement entropy of the metric

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$$S_E = \Xi Q^{(d-1)/d} \Sigma \ln \left(Q^{(d-1)/d} \Sigma \right).$$

- The dependence of the entanglement entropy on the boundary charge density, Q , is computed by realizing the metric in an Einstein-Maxwell-dilaton theory.

K. Goldstein, S. Kachru, S. Prakash, and S. P. Trivedi JHEP **1008**, 078 (2010)

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- Ξ is a dimensionless constant which is *independent* of Q and of any property of the entangling region.

Entanglement entropy of the metric

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$$S_E = \Xi Q^{(d-1)/d} \Sigma \ln \left(Q^{(d-1)/d} \Sigma \right).$$

- The metric has a complicated dependence on Q , but S_E is just proportional to $Q^{(d-1)/d}$. Many UV details are irrelevant, and S_E flows to the universal Q dependence in the IR. By Luttinger's relation $Q \sim k_F^d$, and so the prefactor is the area of the Fermi surface, as expected from field theory.

Inequalities

$$ds^2 = \frac{1}{r^2} \left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

The “null energy condition” of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

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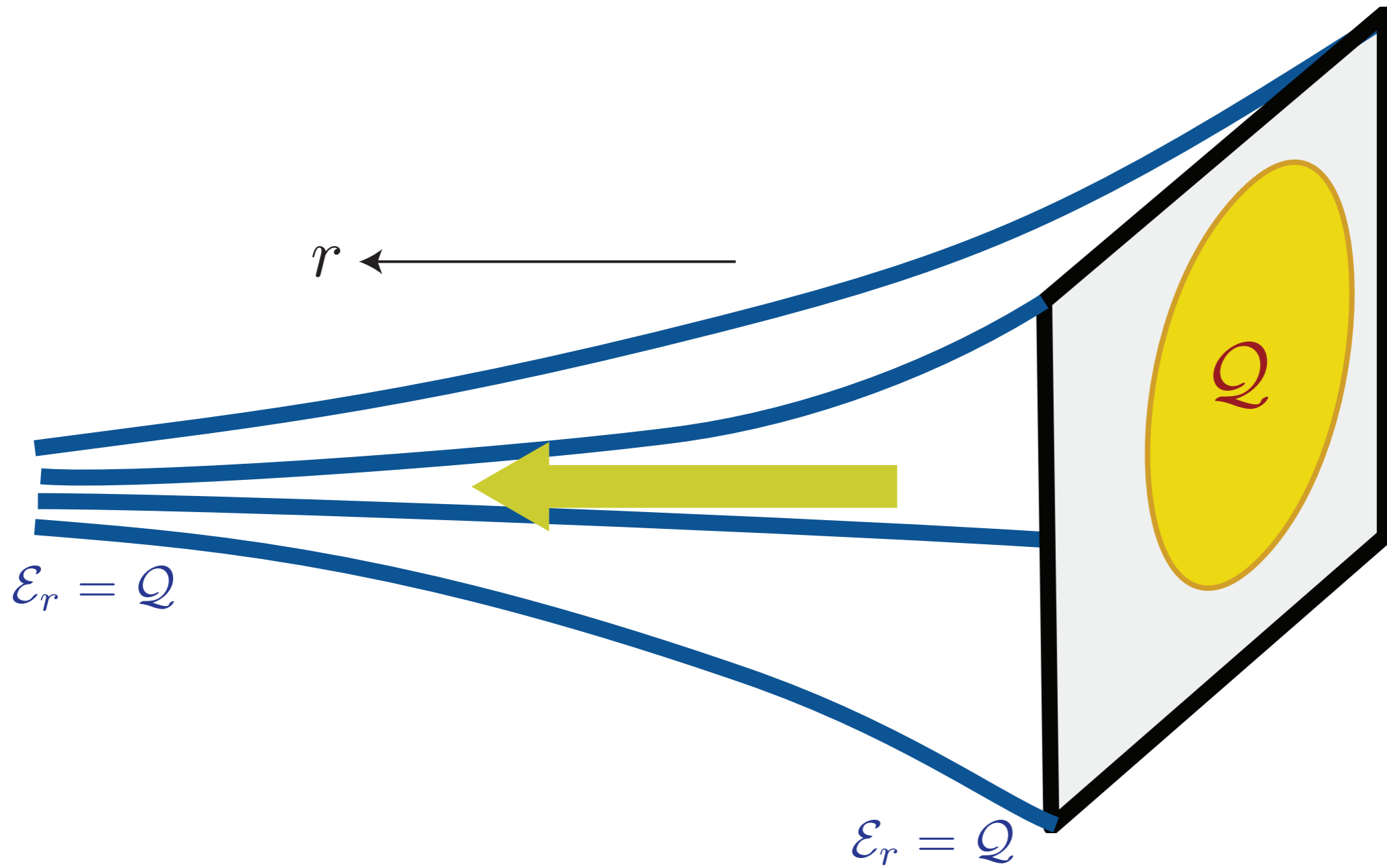
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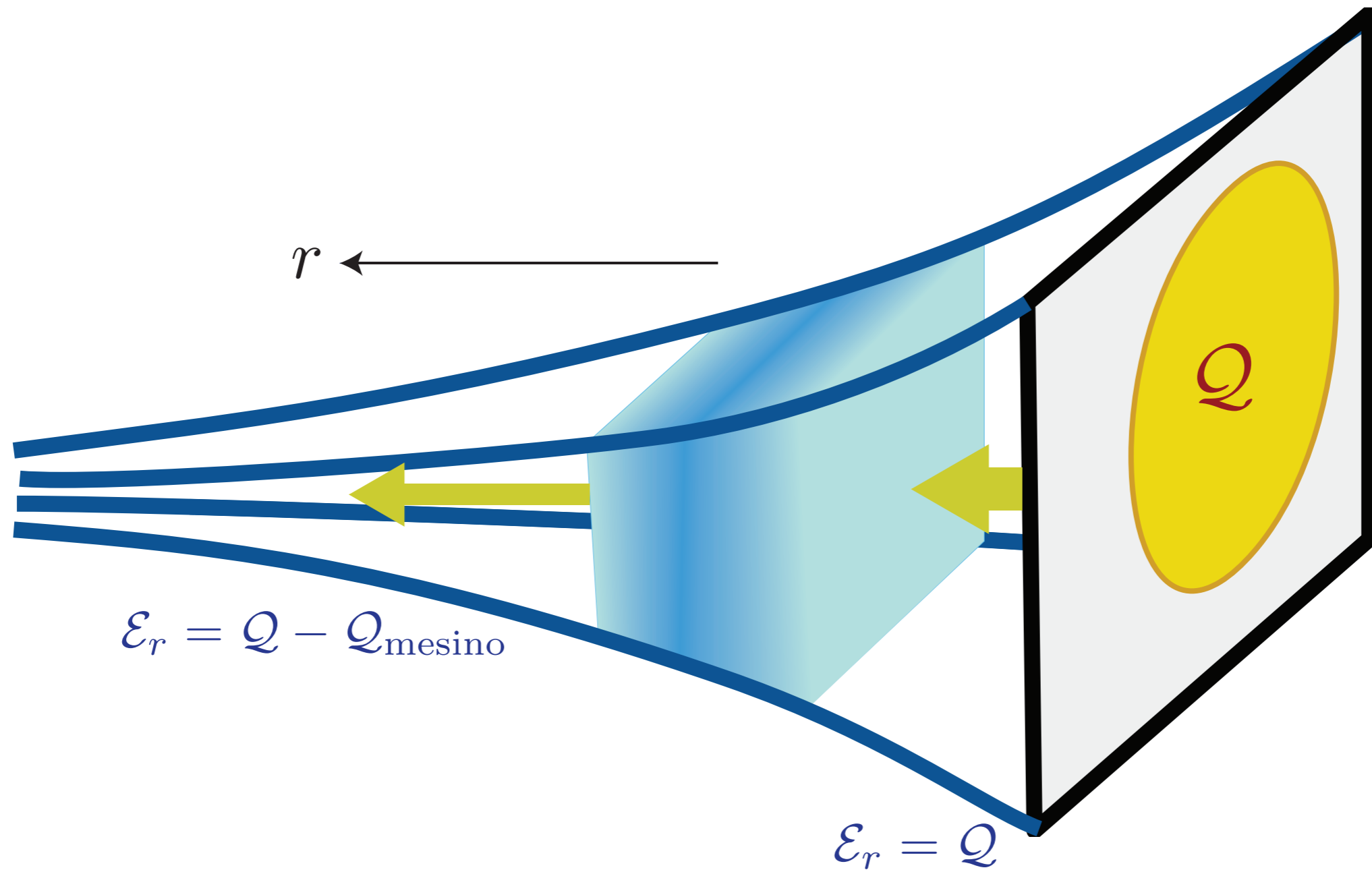
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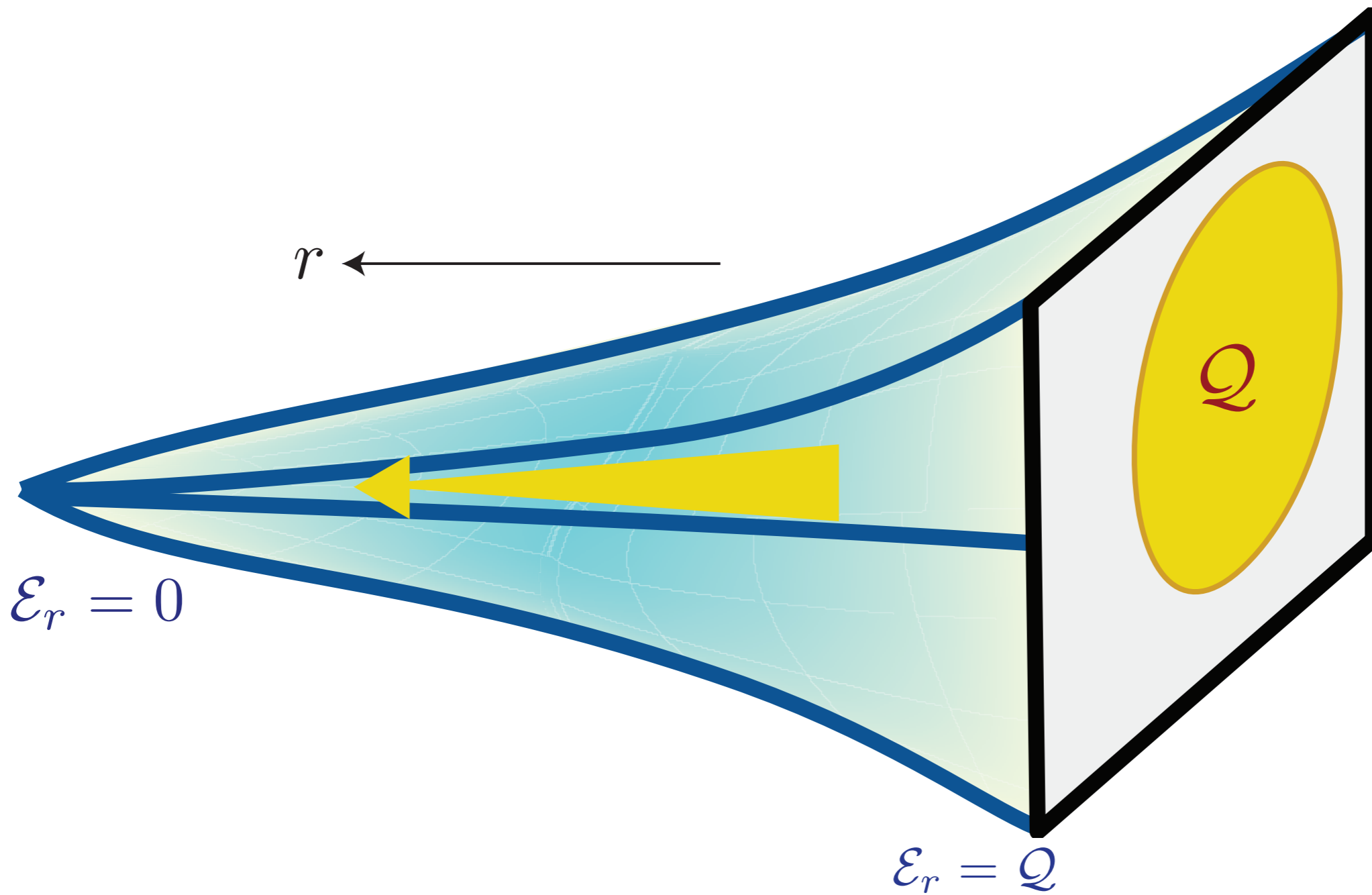
Holographic theory of a non-Fermi liquid (NFL)



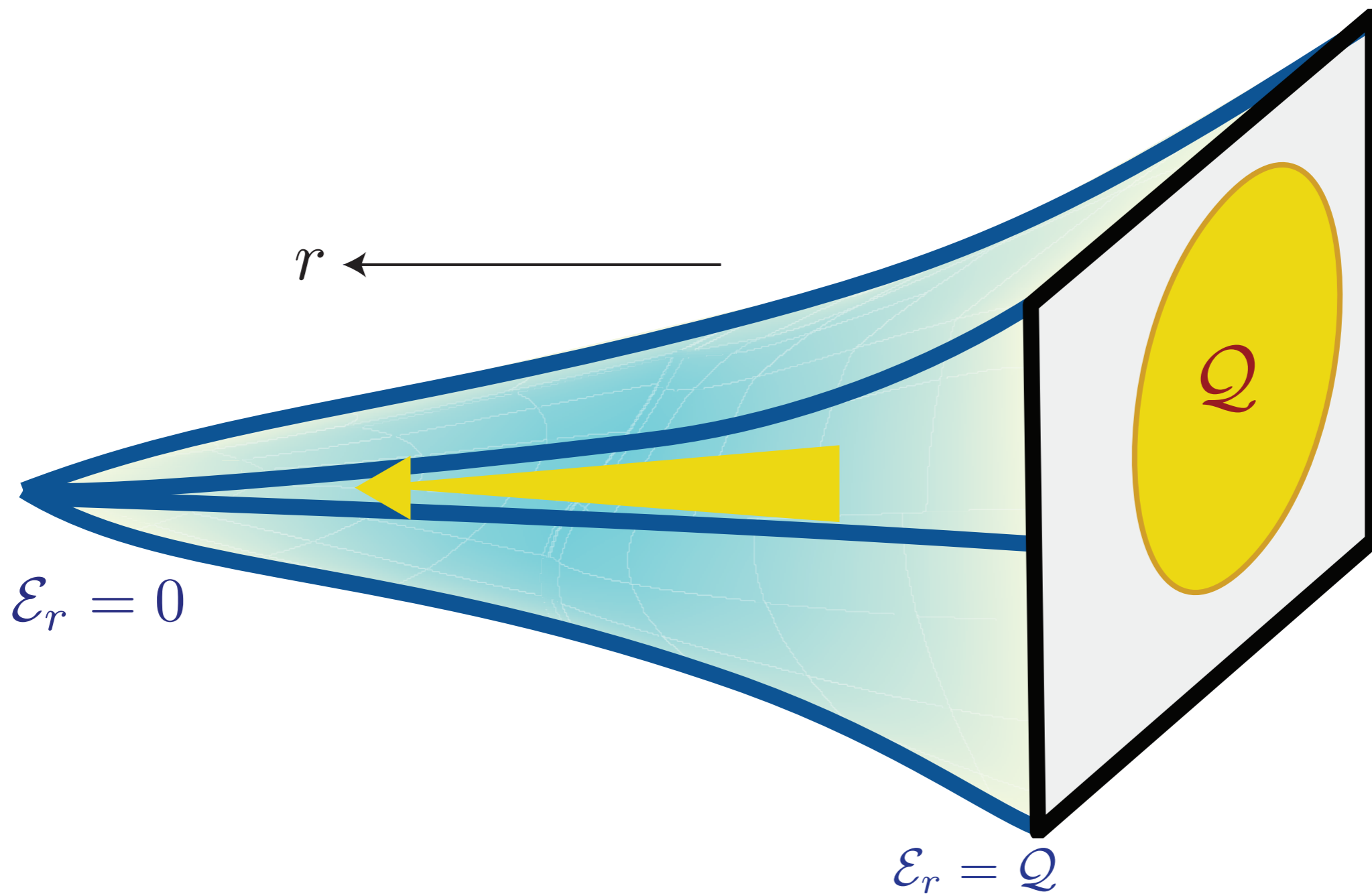
Holographic theory of a fractionalized-Fermi liquid (FL*)



Holographic theory of a Fermi liquid (FL)



Holographic theory of a Fermi liquid (FL)



Gauss Law in the bulk

\Leftrightarrow Luttinger theorem on the boundary

Theory of a non-Fermi liquid (NFL)

Field theory

Holography

A gauge-dependent Fermi surface of overdamped gapless fermions.

Fermi surface is hidden.

Theory of a non-Fermi liquid (NFL)

Field theory

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Thermal entropy density $S \sim T^{1/z}$ in $d = 2$, where z is the dynamic critical exponent.

Holography

Fermi surface is hidden.

Thermal entropy density $S \sim T^{1/z}$ in all d for hyperscaling violation exponent $\theta = d - 1$, and z the dynamic critical exponent.

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Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $Q^{(d-1)/d}$ and the boundary area of the entangling region.

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Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $Q^{(d-1)/d}$ and the boundary area of the entangling region.

Theory of a non-Fermi liquid (NFL)

Field theory

Three-loop analysis shows
 $z = 3/2$ in $d = 2$.

Holography

Existence of gravity dual implies $z \geq 1 + \theta/d$; leads to $z \geq 3/2$ for $\theta = d-1$ in $d = 2$.

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Fermi surface encloses a volume proportional to \mathcal{Q} , as demanded by the Luttinger relation.

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Fermi surface encloses a volume proportional to Q , as demanded by the Luttinger relation.

Gauge neutral ‘mesinos’ reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $Q - Q_{\text{mesino}}$.

Holography

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The value of k_F obtained from the entanglement entropy implies the Fermi surface encloses a volume proportional to Q , as demanded by the Luttinger relation.

Gauge neutral ‘mesinos’ reduce the volume enclosed by hidden Fermi surfaces to $Q - Q_{\text{mesino}}$.

Compressible quantum matter

- Strongly interacting non-Fermi liquid states (with “hidden” Fermi surfaces of gauge-dependent particles) are realized by holographic theories with dynamic critical exponent z , and violation of hyperscaling exponent $\theta = d-1$

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- Strongly interacting non-Fermi liquid states (with “hidden” Fermi surfaces of gauge-dependent particles) are realized by holographic theories with dynamic critical exponent z , and violation of hyperscaling exponent $\theta = d-1$
- The entanglement entropy of this theory has all the properties expected for a Fermi surface enclosing the volume expected by the Luttinger relation.
- The thermal entropy also has the properties expected by the presence such a Fermi surface.

Compressible quantum matter

- Strongly interacting non-Fermi liquid states (with “hidden” Fermi surfaces of gauge-dependent particles) are realized by holographic theories with dynamic critical exponent z , and violation of hyperscaling exponent $\theta = d-1$
- The entanglement entropy of this theory has all the properties expected for a Fermi surface enclosing the volume expected by the Luttinger relation.
- The thermal entropy also has the properties expected by the presence such a Fermi surface.
- “Visible” Fermi surfaces of gauge-neutral particles are realized in a confining metric.