SYK criticality at metal-metal transitions in the Hubbard and Kondo lattice models

Theoretical & Experimental Magnetism Meeting (TEMM 2021)
Cosener's House in Abingdon, Oxfordshire
July 13, 2021
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

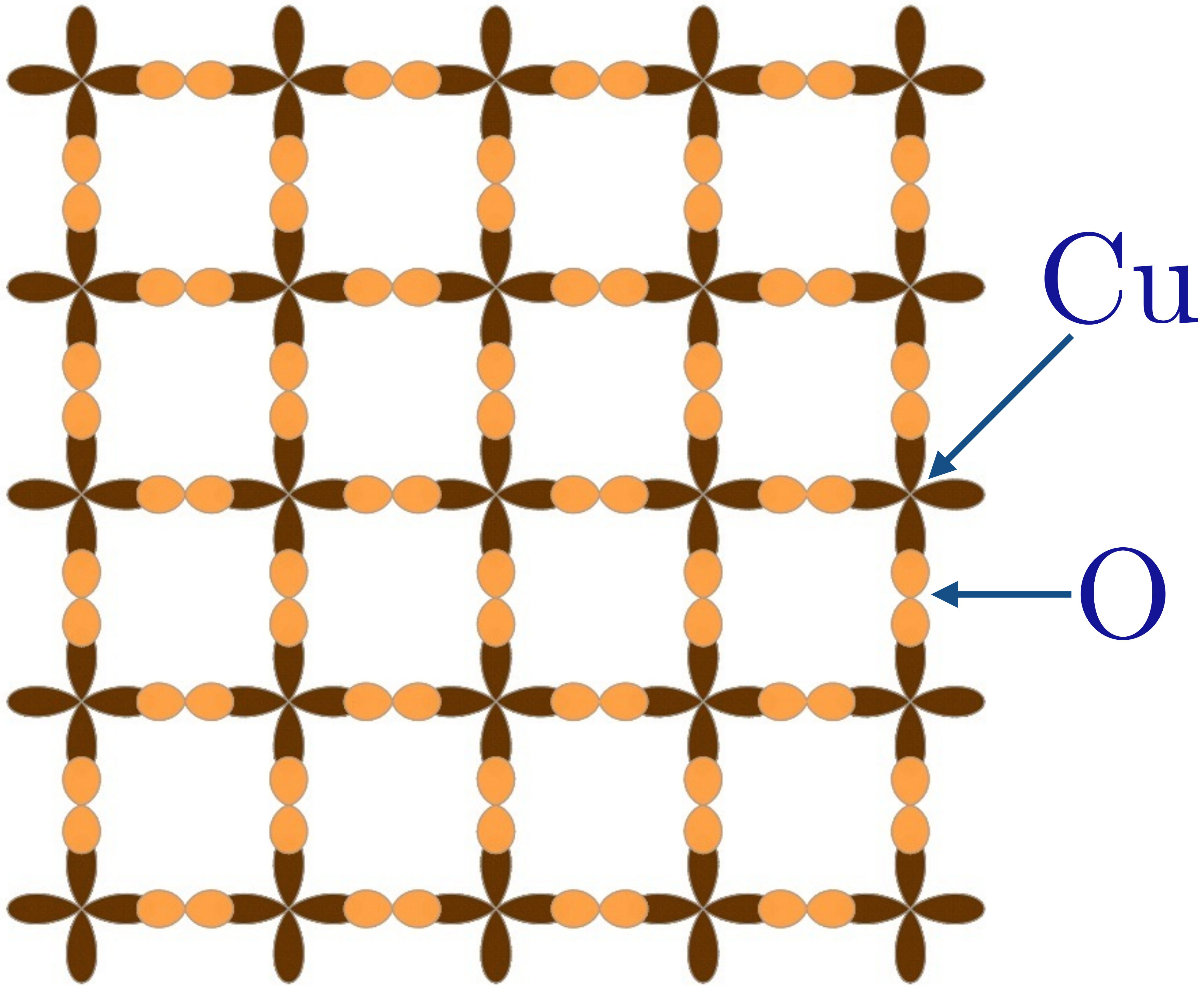
PHYSICS
HARVARD

1. Experiments in cuprates and CeCoIn_5
2. Deconfined criticality at metal-metal transitions
3. Dynamic mean field theory:
   self-consistent single site theory
4. Numerical results on the Hubbard Model
5. RG results on the SY(K) random magnet
6. RG results on the Kondo model
7. RG results on the *t*-*J* Model

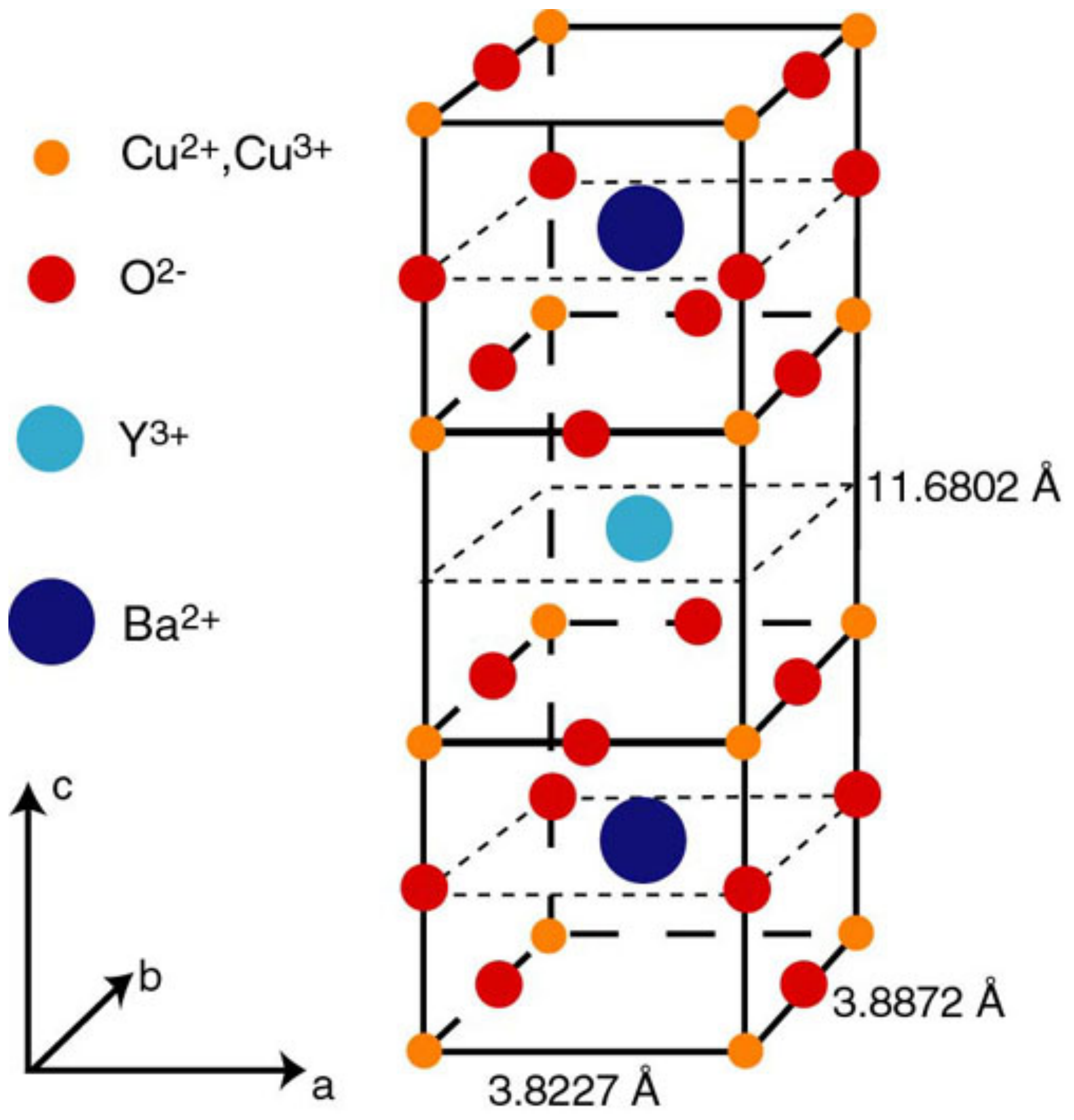
$$\mathcal{H}_H = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

Hubbard model

High temperature superconductors



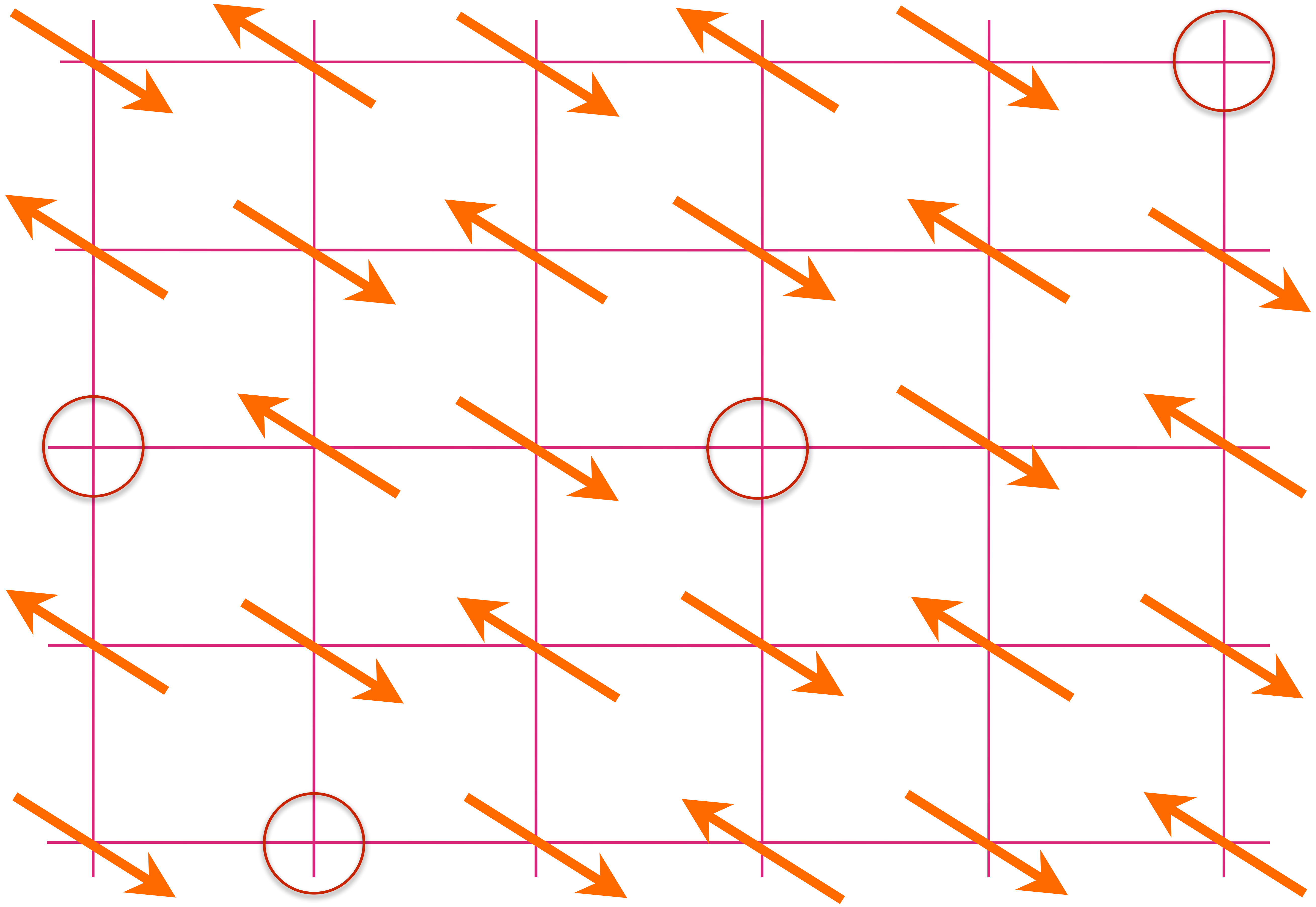
CuO_2 plane



$YBa_2Cu_3O_{6+x}$

$$\mathcal{H}_H = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Hubbard model



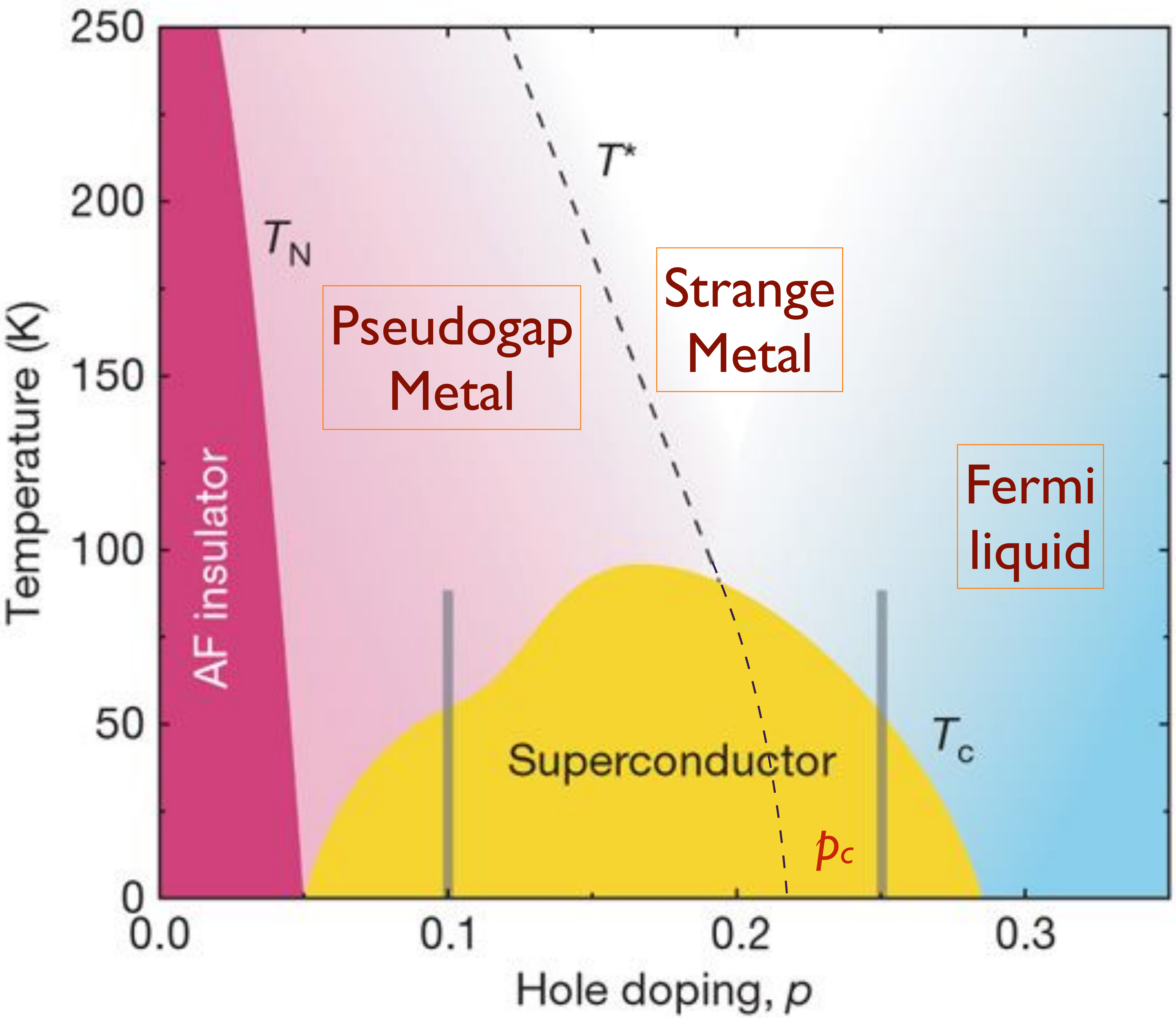
Antiferromagnet doped with hole density *p*

or

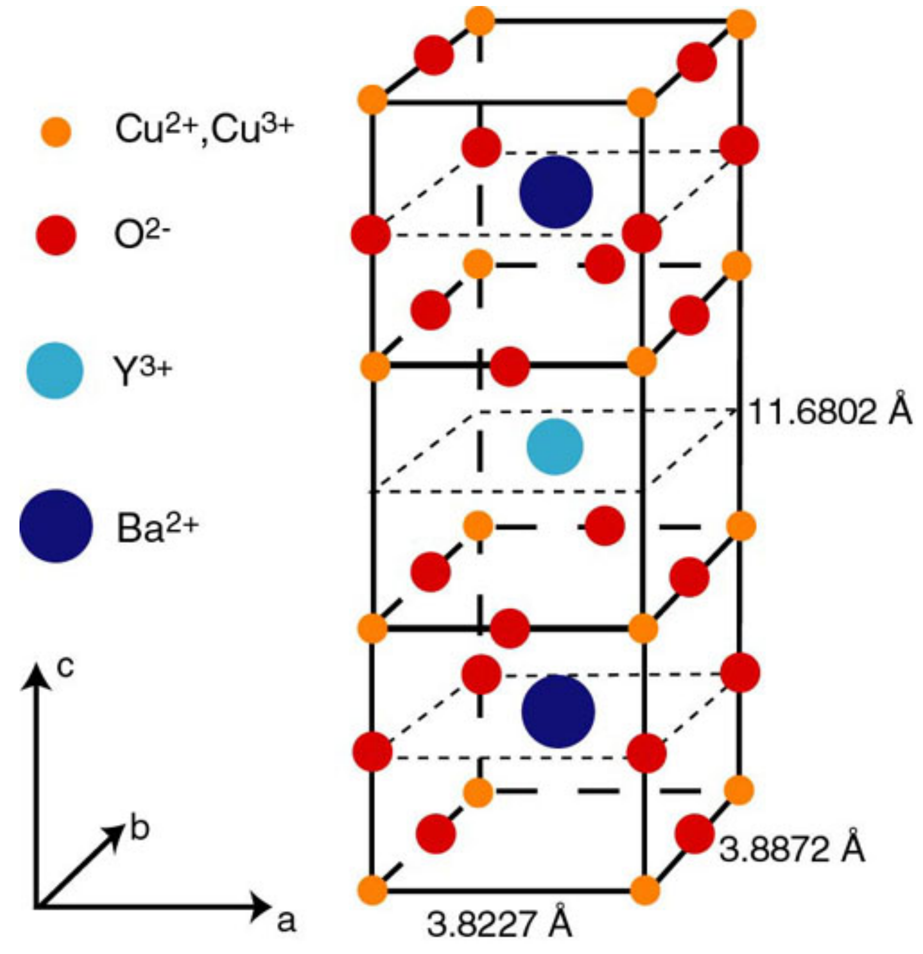
Filled band with hole density *1+p*

=

Empty band with electron density *1-p*

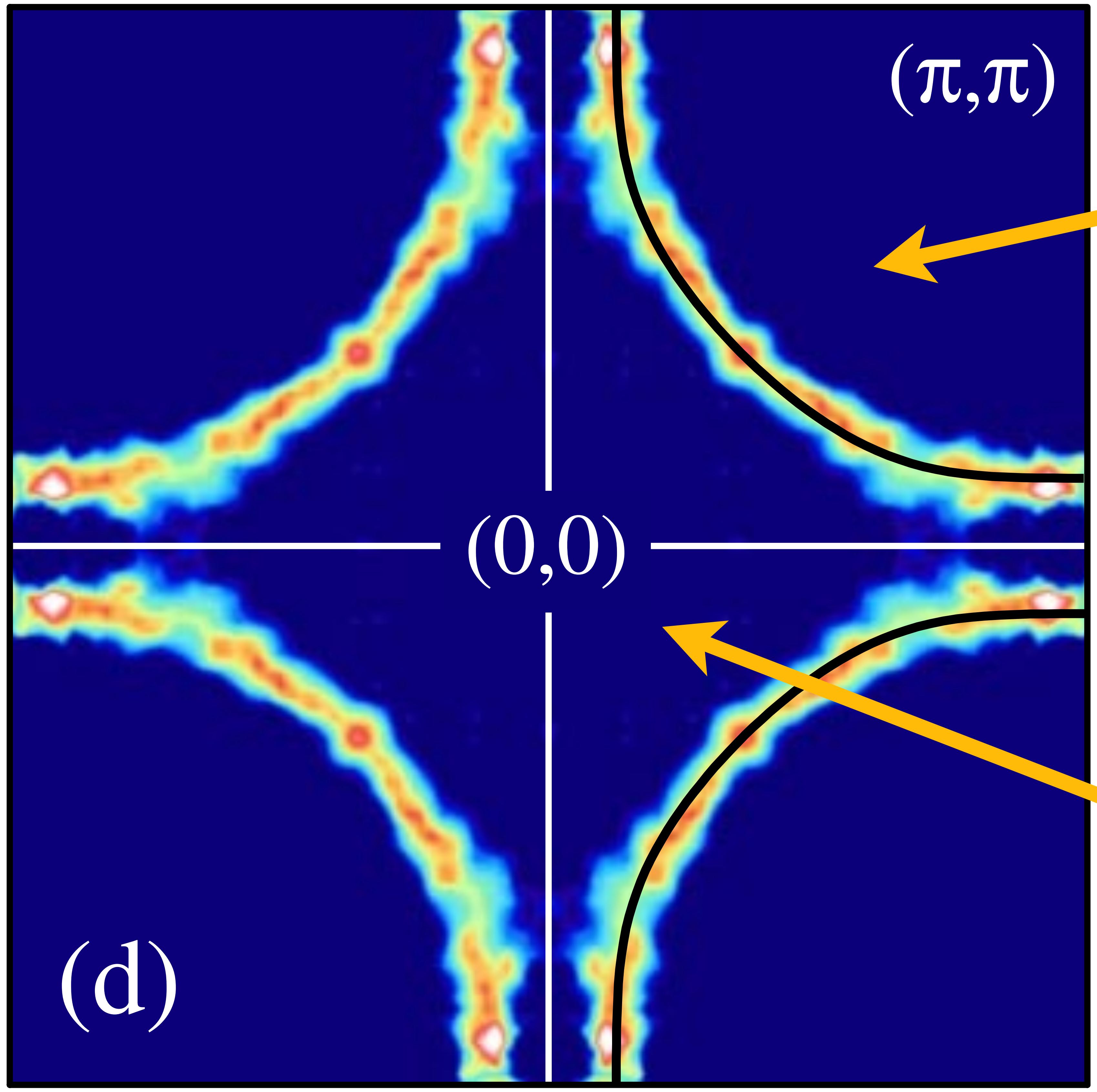


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Pseudogap Metal
Strange Metal
Fermi liquid
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at large p



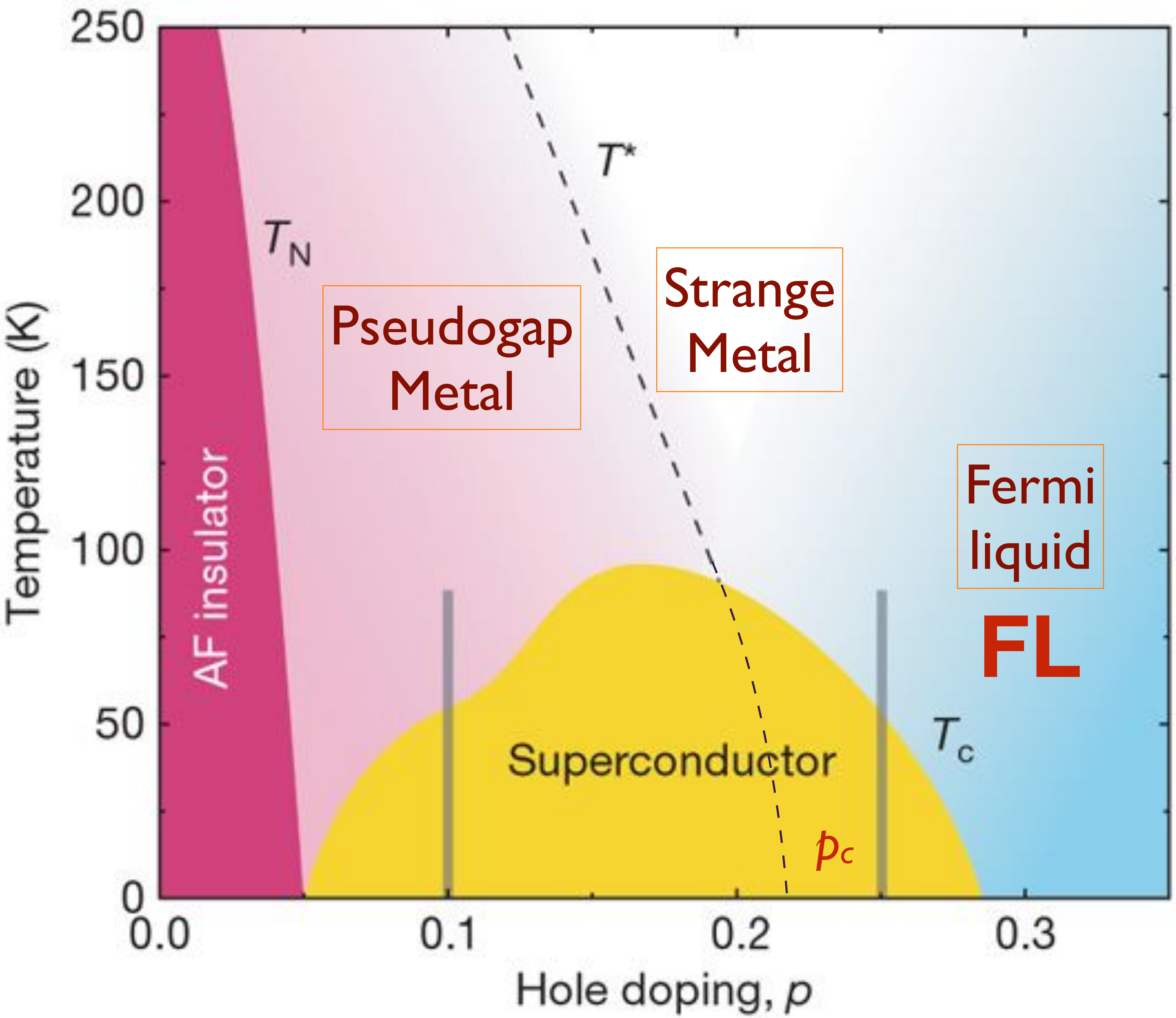
$1+p$ holes

1-p electrons

Overdoped $\mathrm{Tl_2Ba_2CuO_{6+\delta}}$
$T_c = 30\mathrm{K}$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)


250
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Hole doping, p
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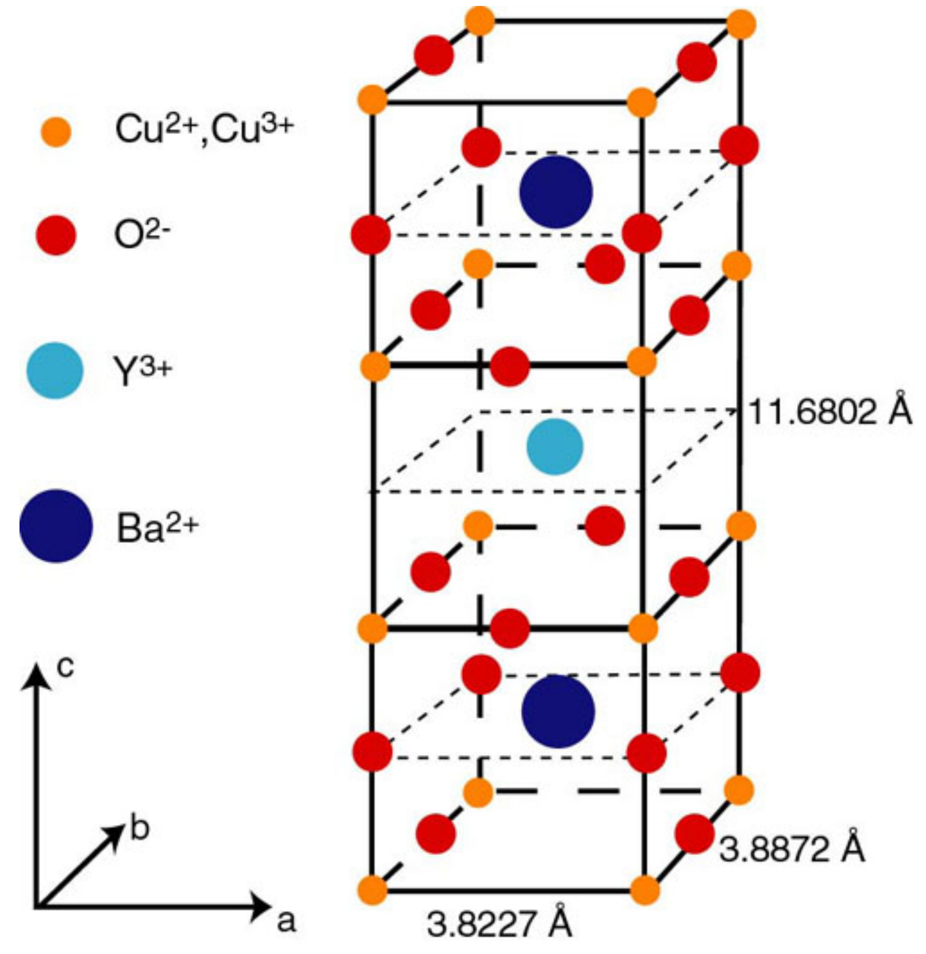
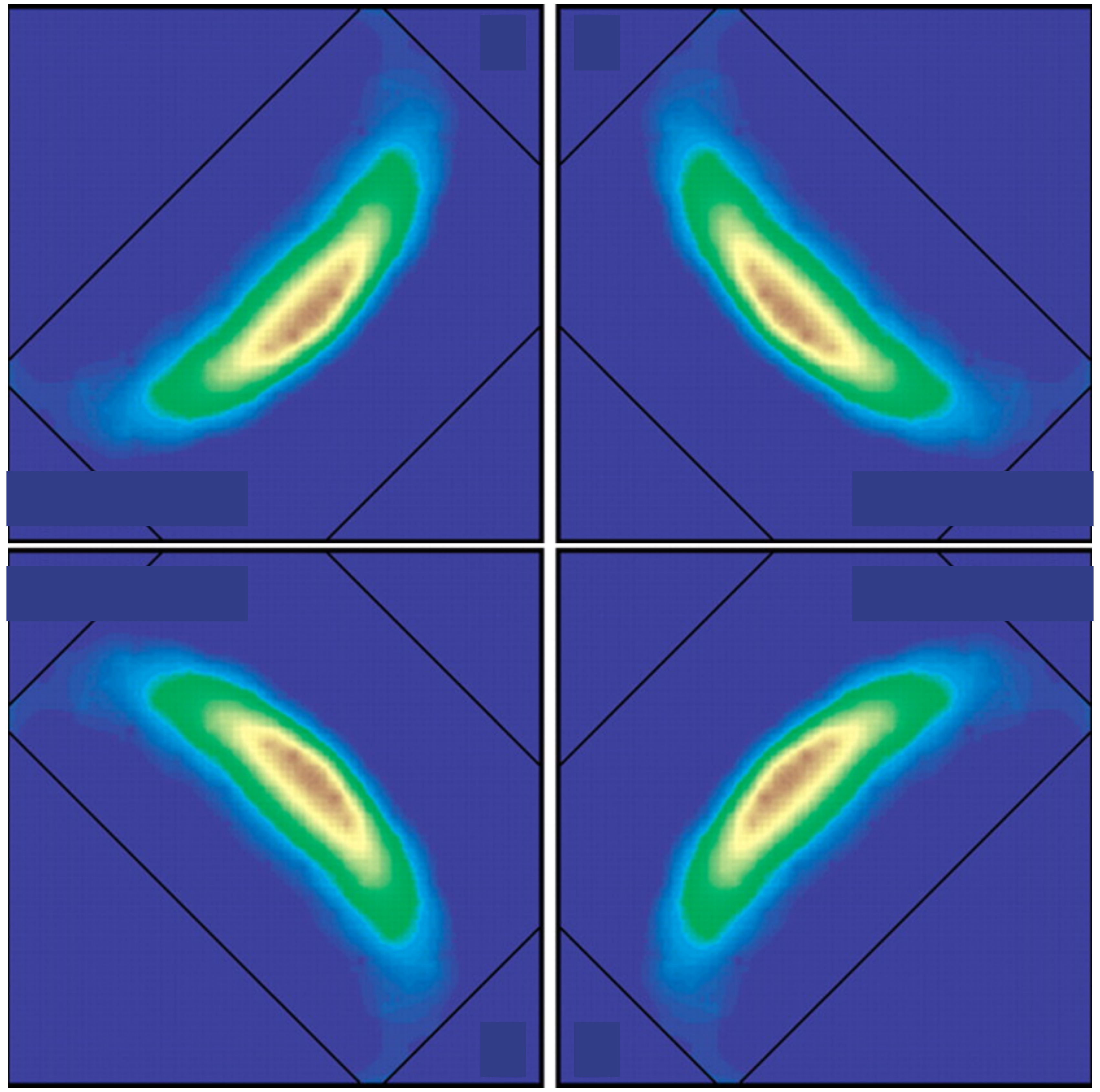

Cu^{2+}, Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

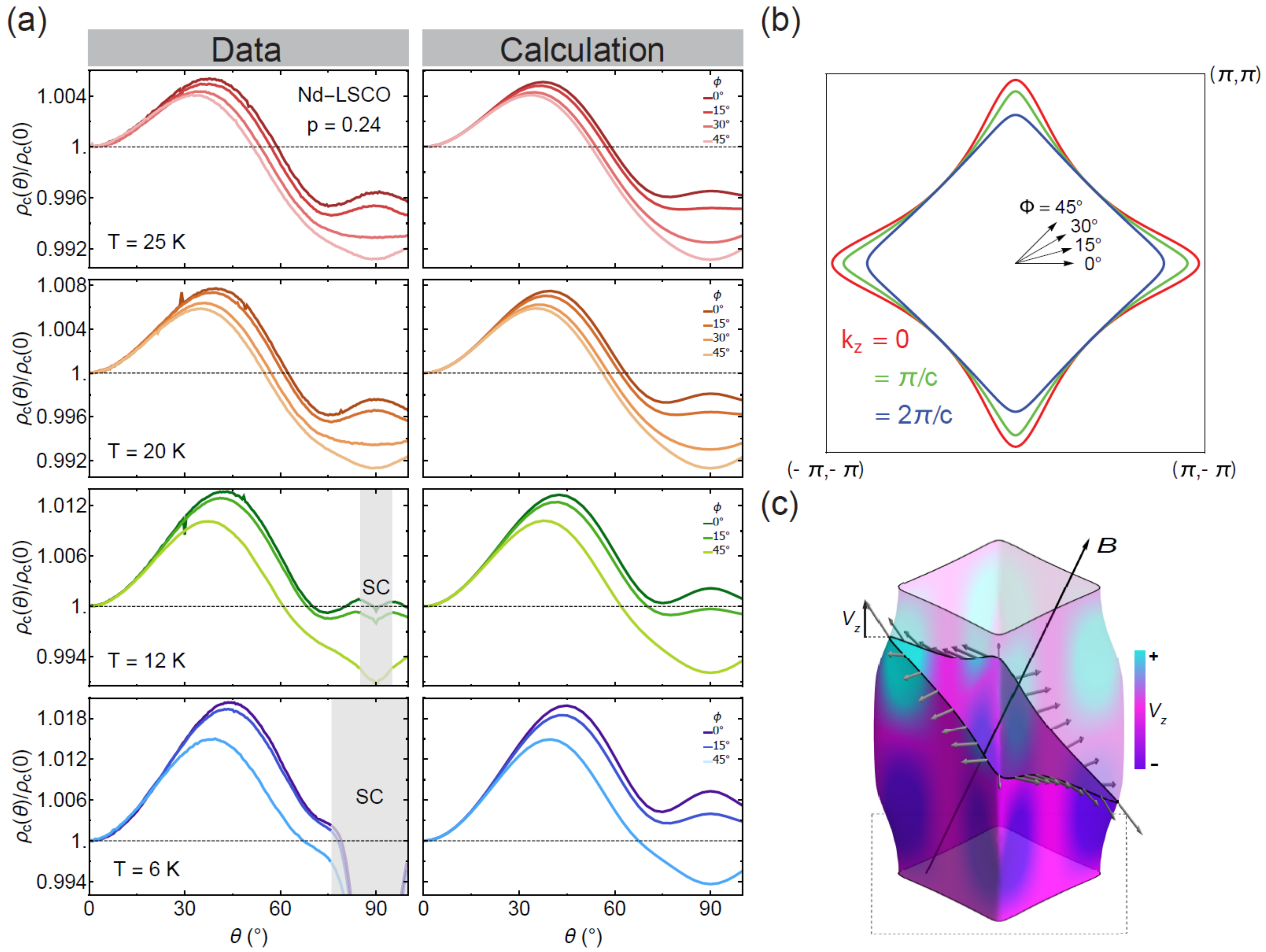
“Fermi arcs”

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

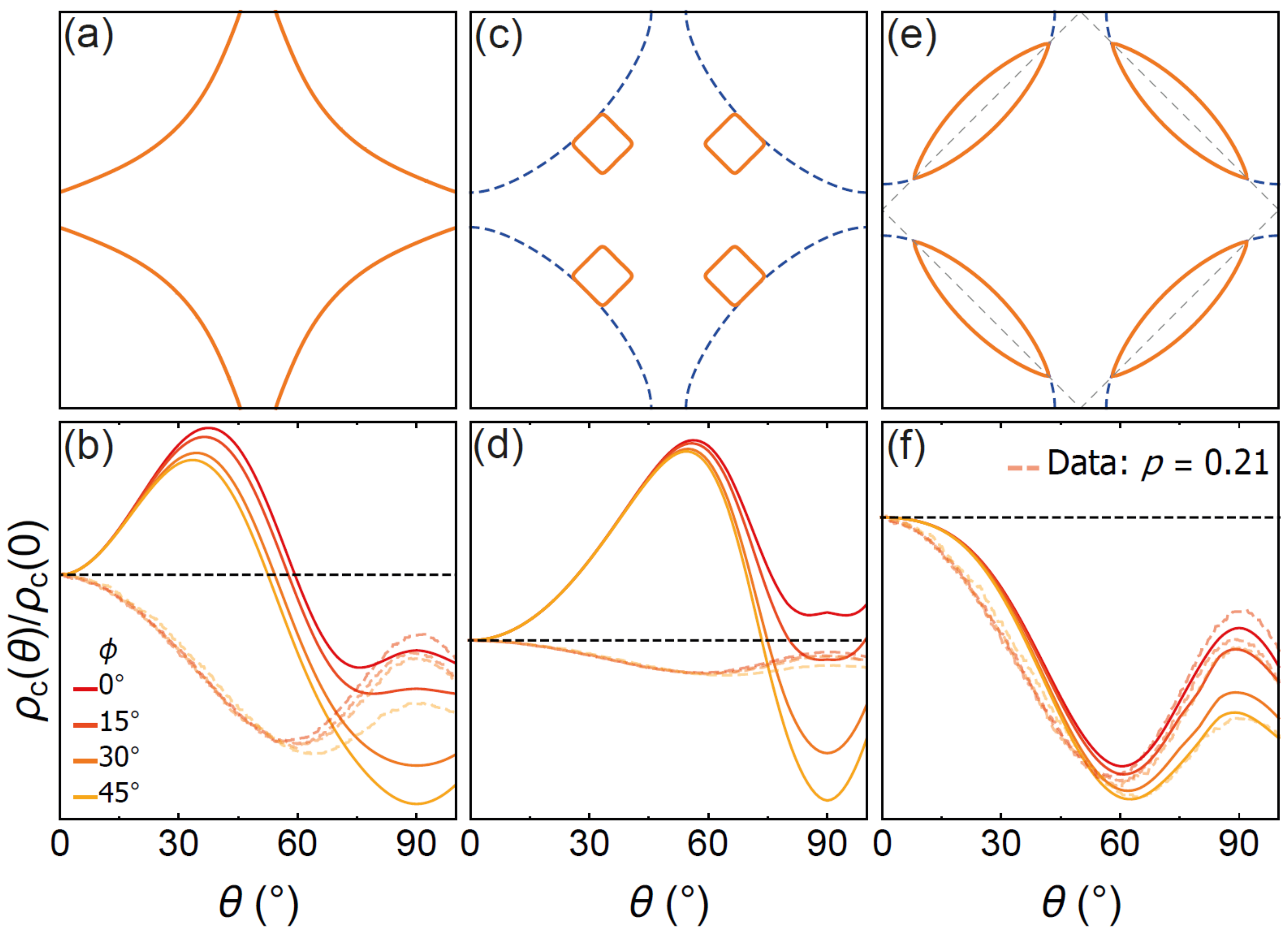
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$. Above the critical doping p*—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p*, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a Q = (π,π) wavevector. While static Q = (π,π) antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

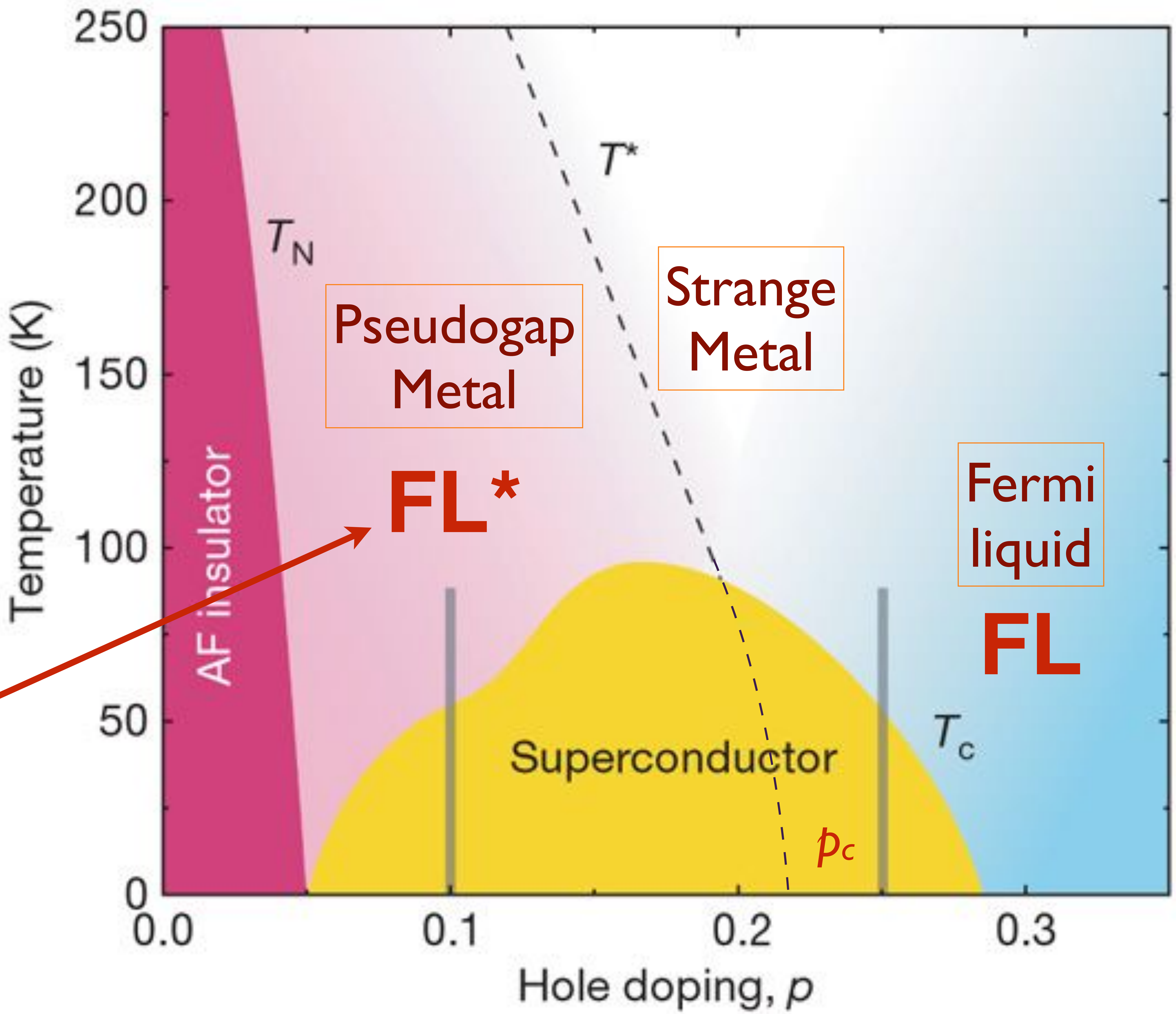


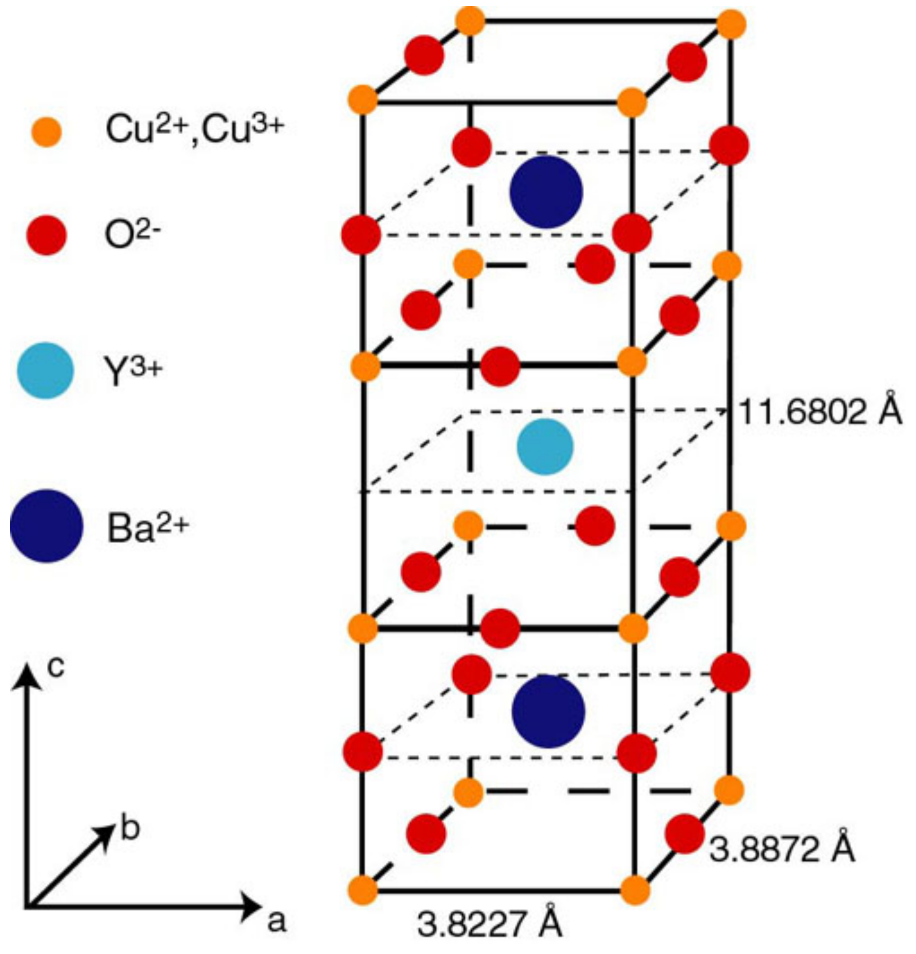
$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

Fermi surface of size p without any order which breaks translational symmetry (violates Luttinger theorem)

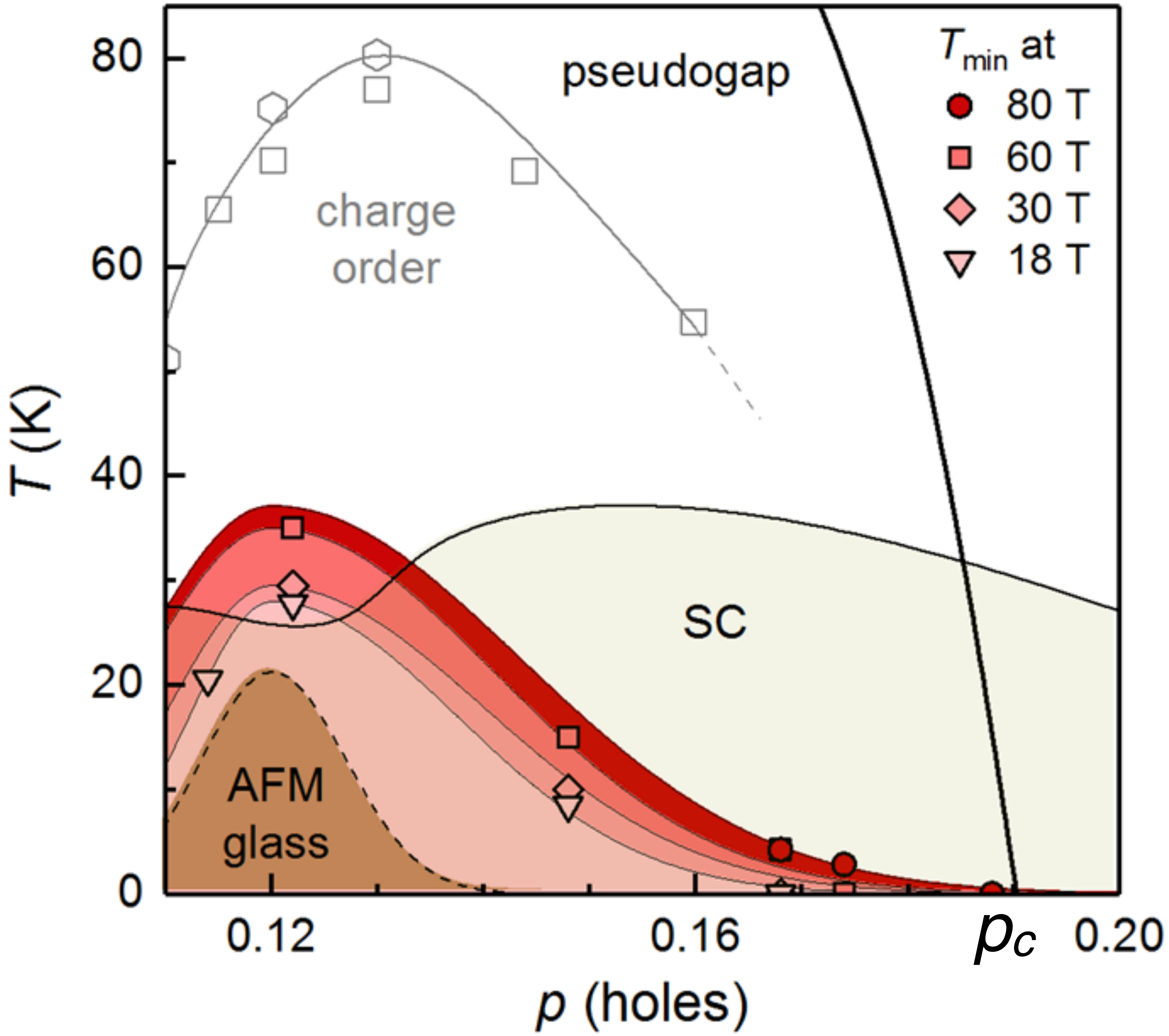




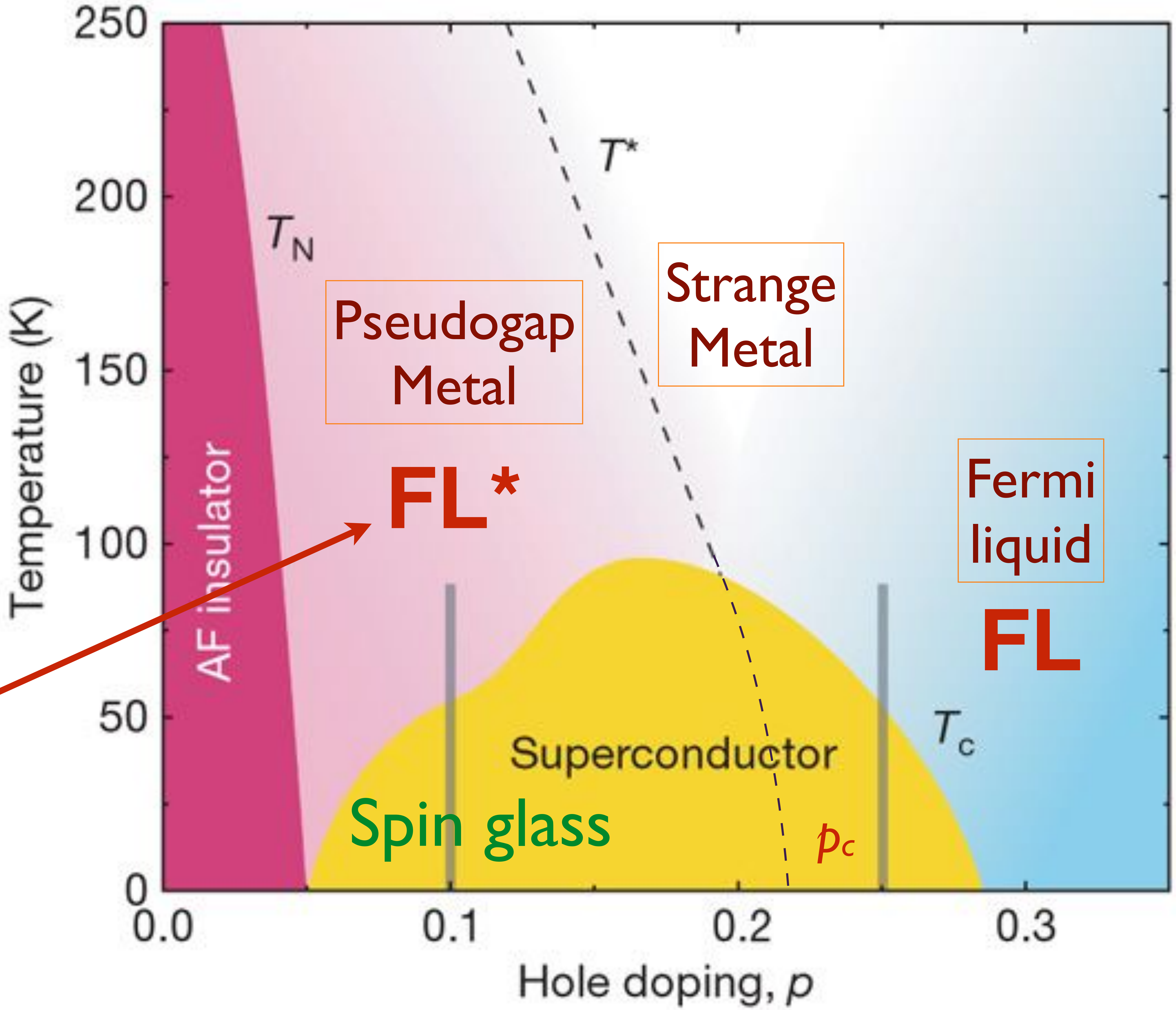
Hidden magnetism at the pseudogap critical point of a high temperature superconductor

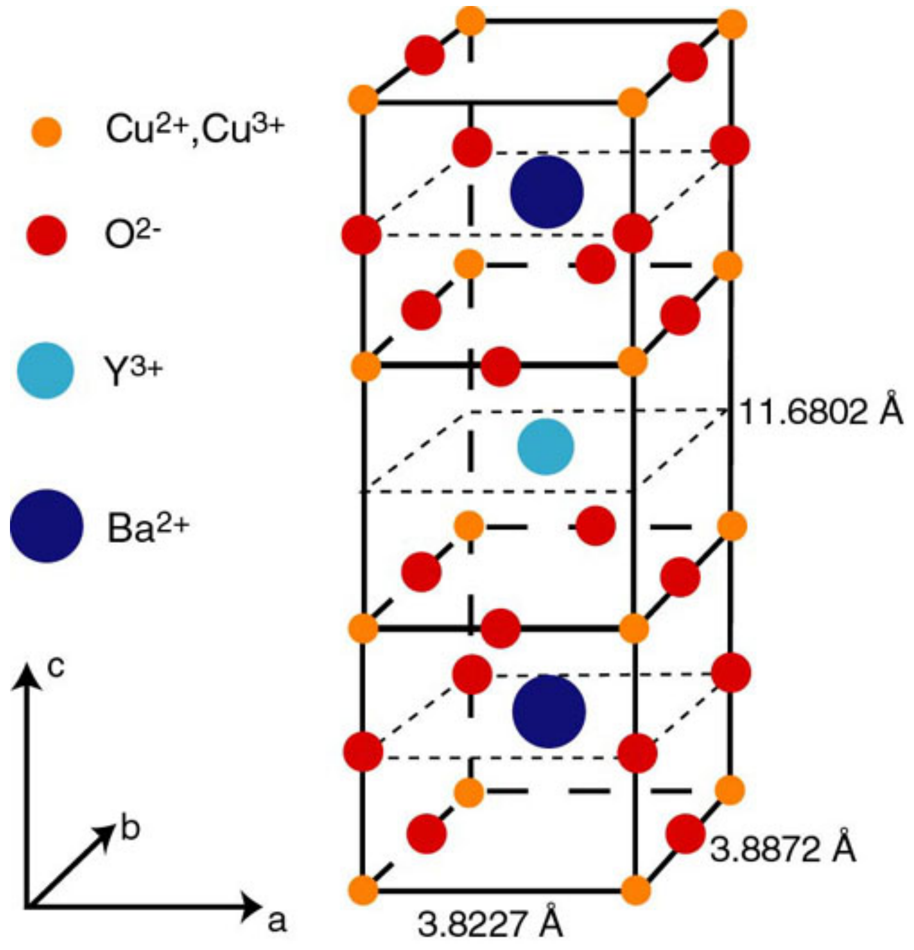
Nature Physics **16**, 1064 (2020)

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1]*, Marc-Henri Julien[1]*

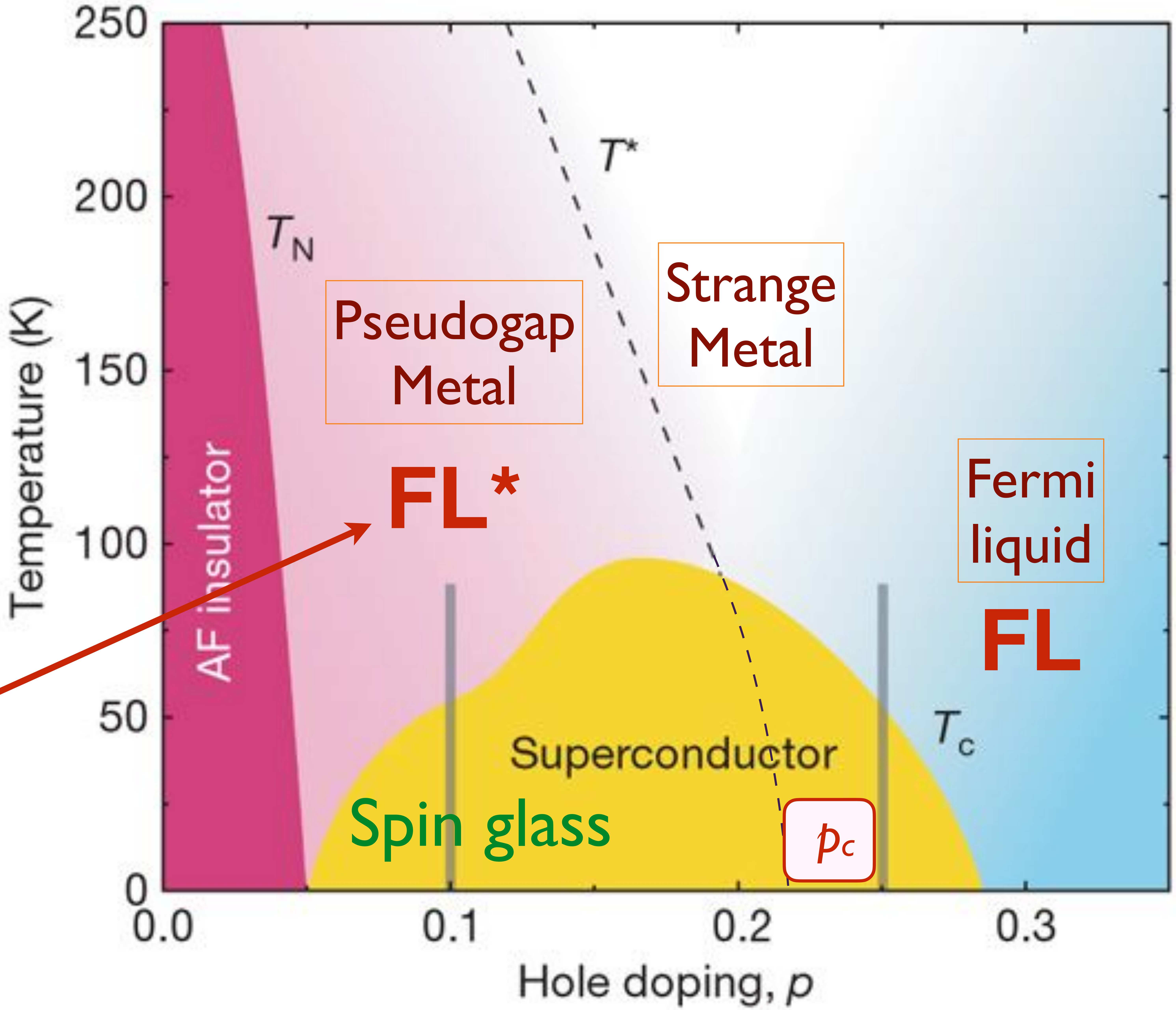


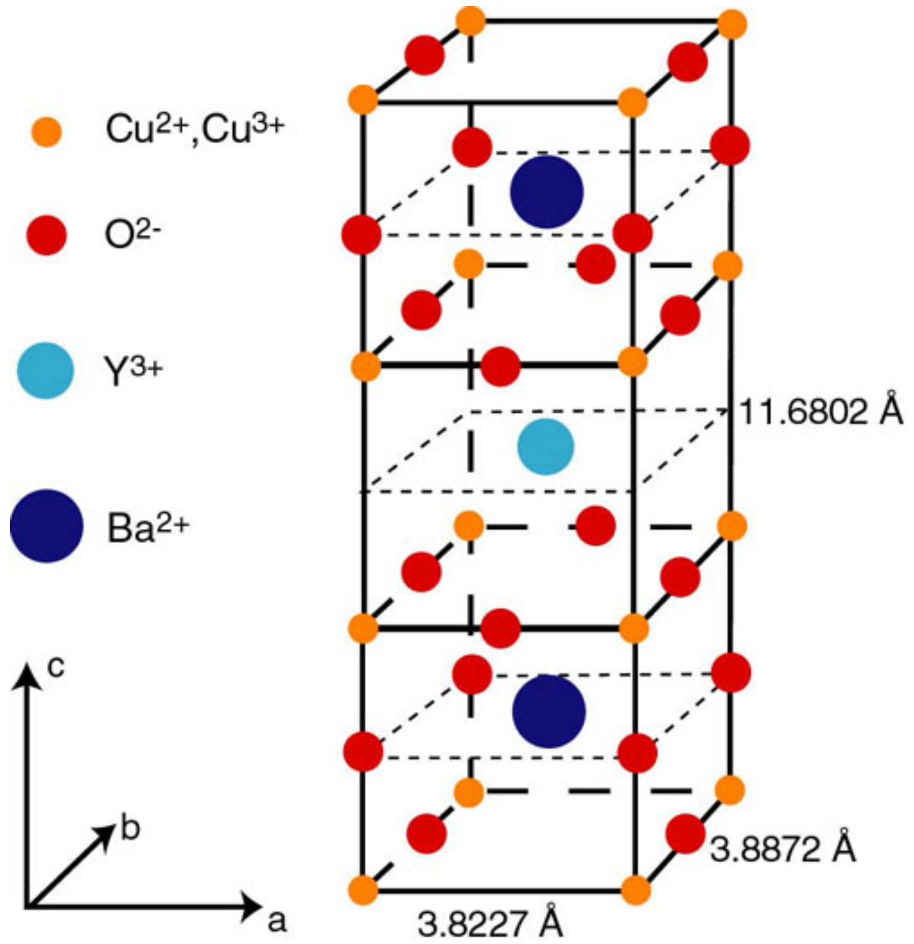
Fermi surface of size p without any order which breaks translational symmetry (violates Luttinger theorem)





Fermi surface of size p without any order which breaks translational symmetry (violates Luttinger theorem)



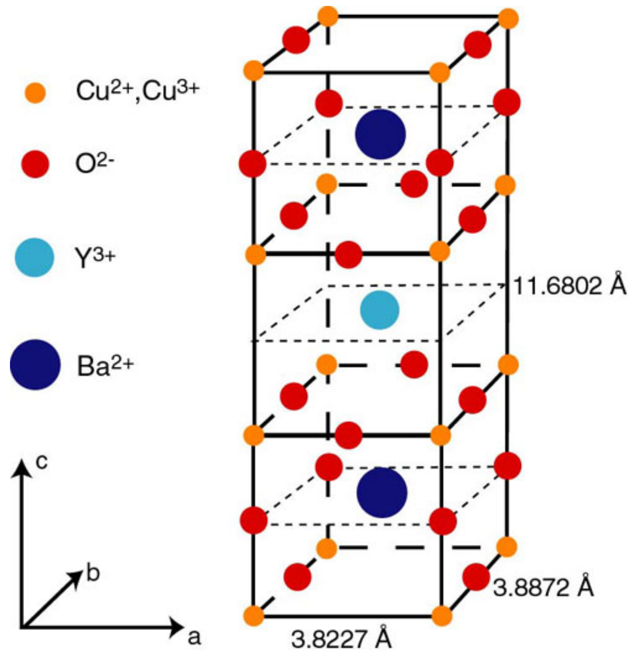


A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Electron scattering time τ in 6 cuprates near p_c

$$\frac{1}{\tau} = \alpha \, \frac{k_B T}{\hbar}$$

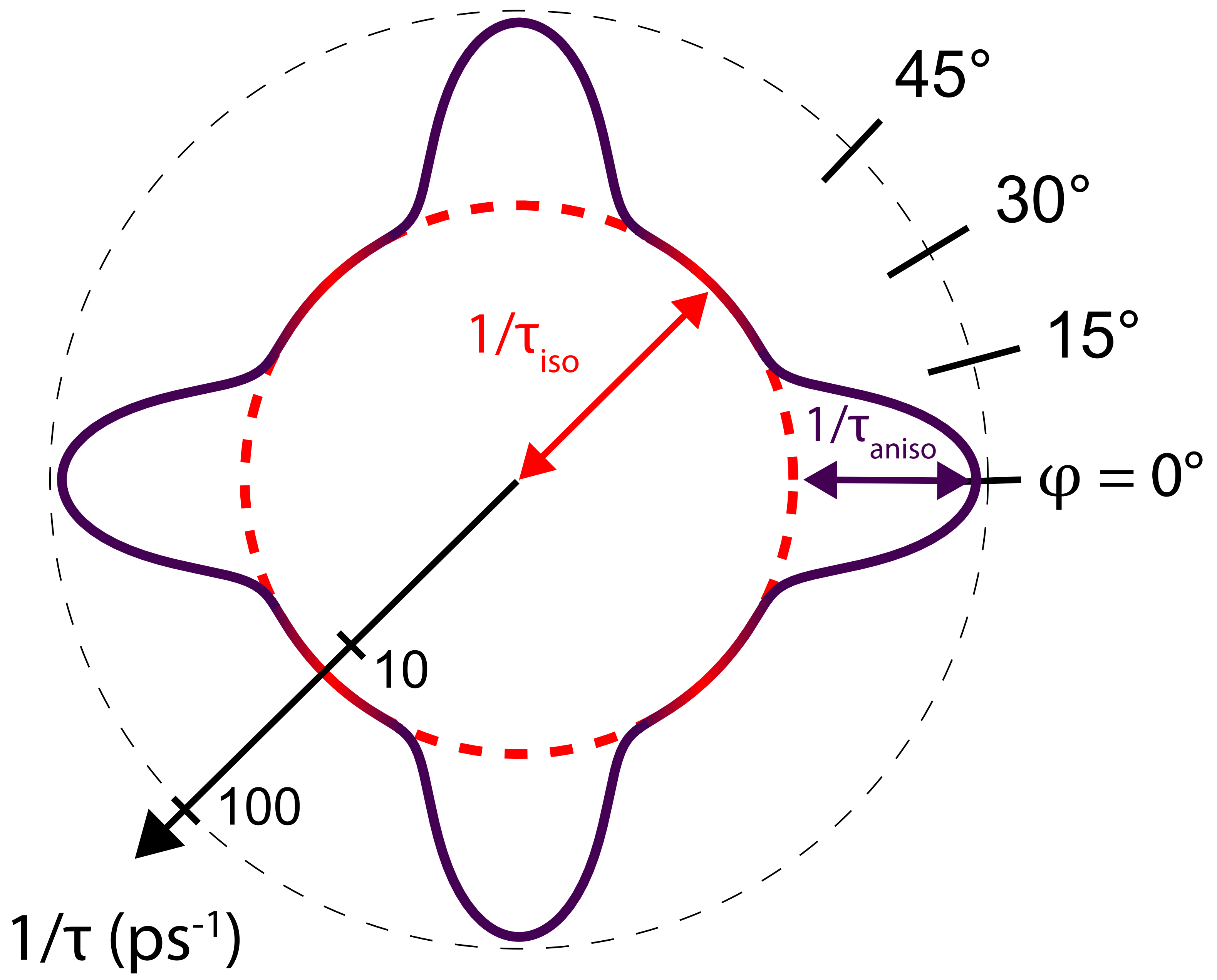


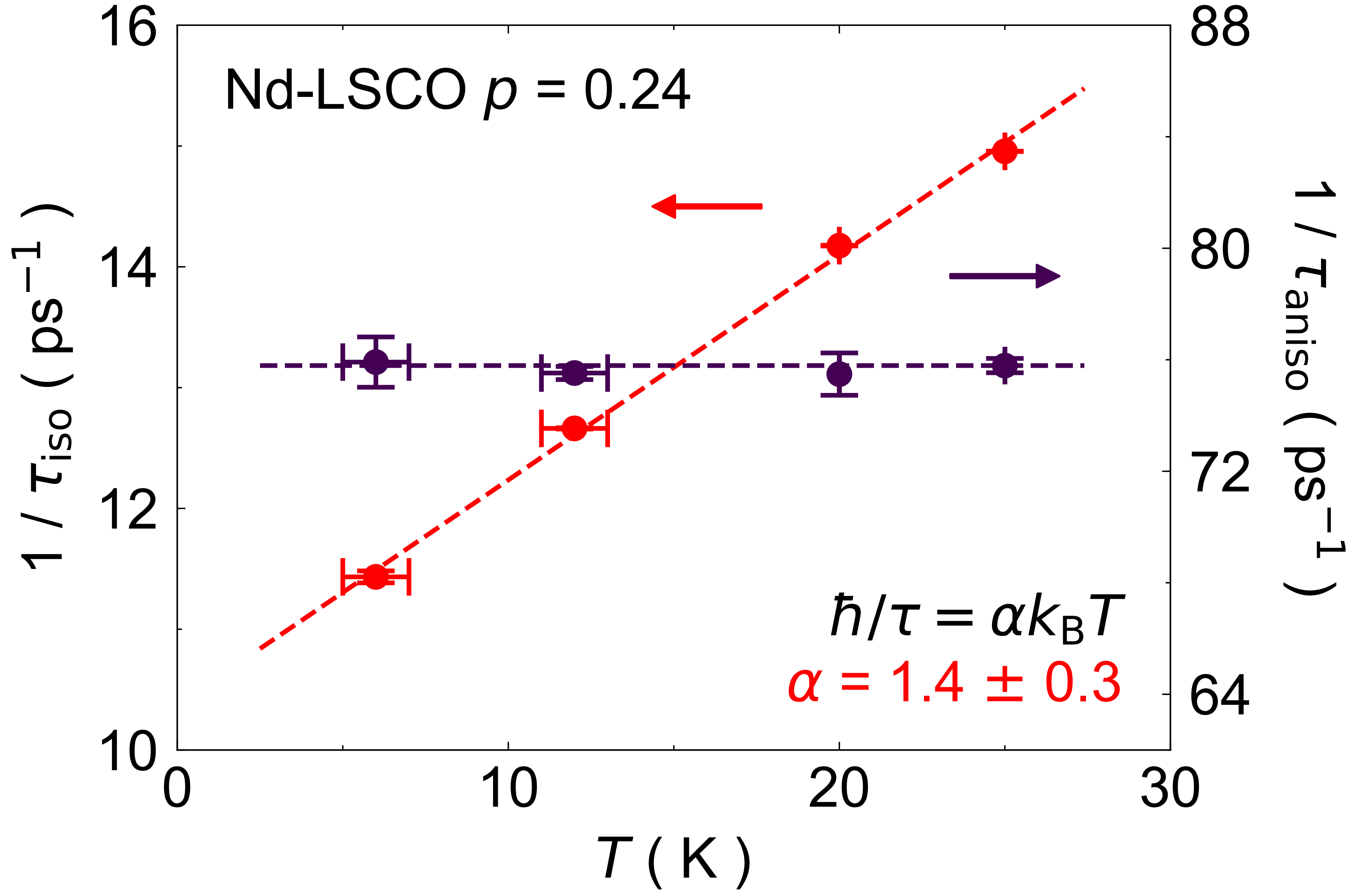
Current flow without quasiparticles

Measurement of the Planckian Scattering Rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw, arXiv:2011.13054

Angle-dependent magnetoresistance in Nd-LSCO near $p = p_c \approx 0.23$.

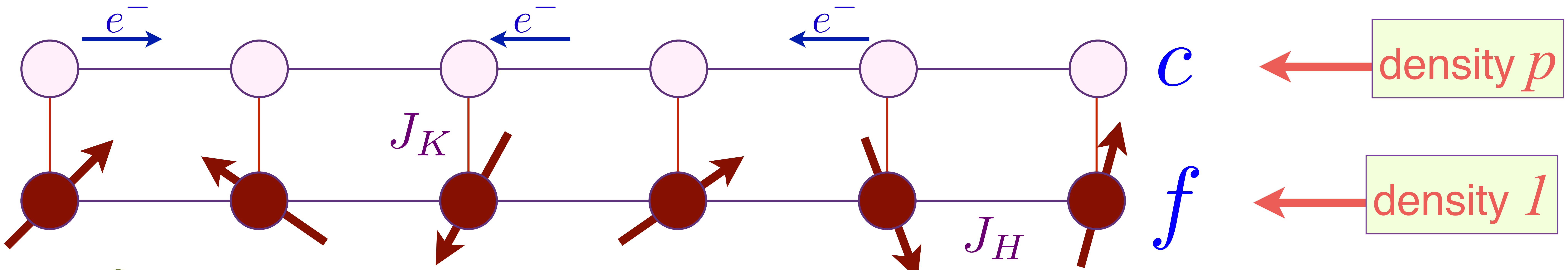


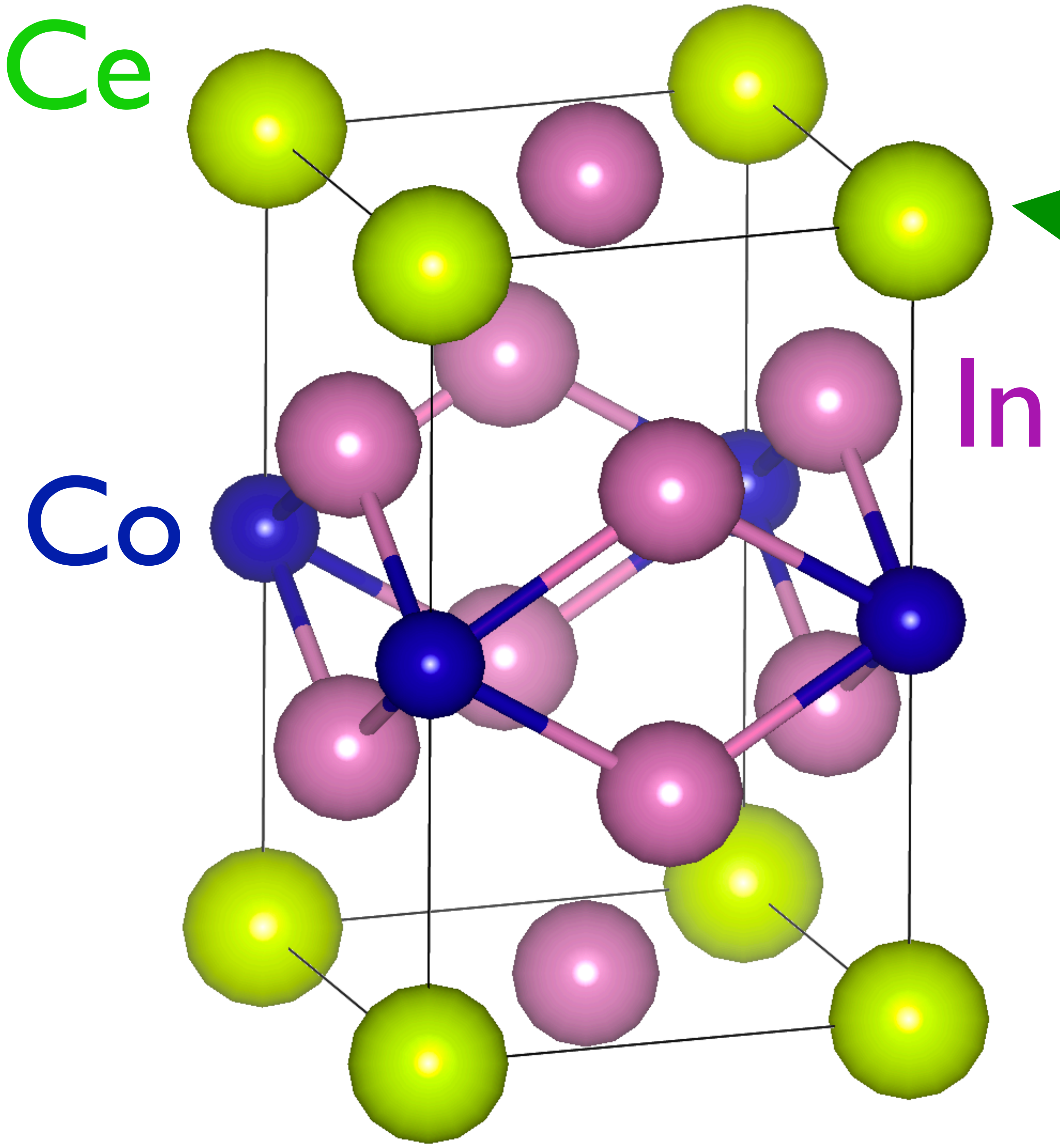


$$\frac{1}{\tau} = \frac{1}{\tau_{\mathrm{aniso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T$$

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_{i} J_K \, c^{\dagger}_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H \, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Kondo lattice



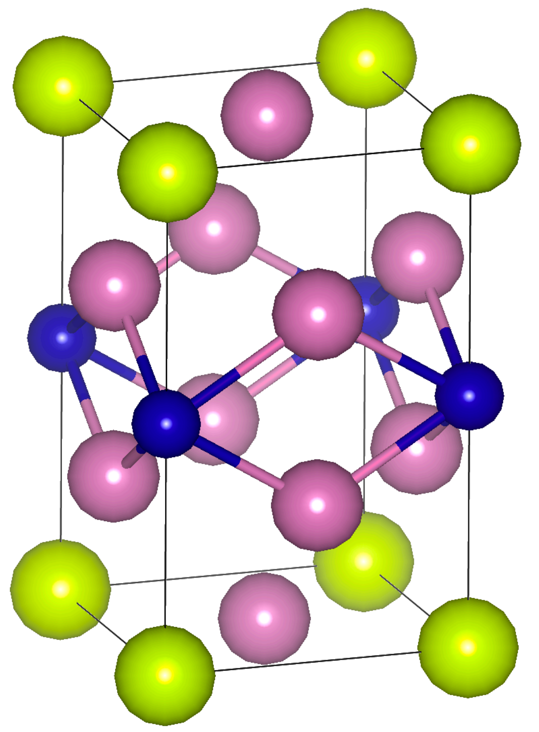


Strong on-site repulsion U only on Ce sites

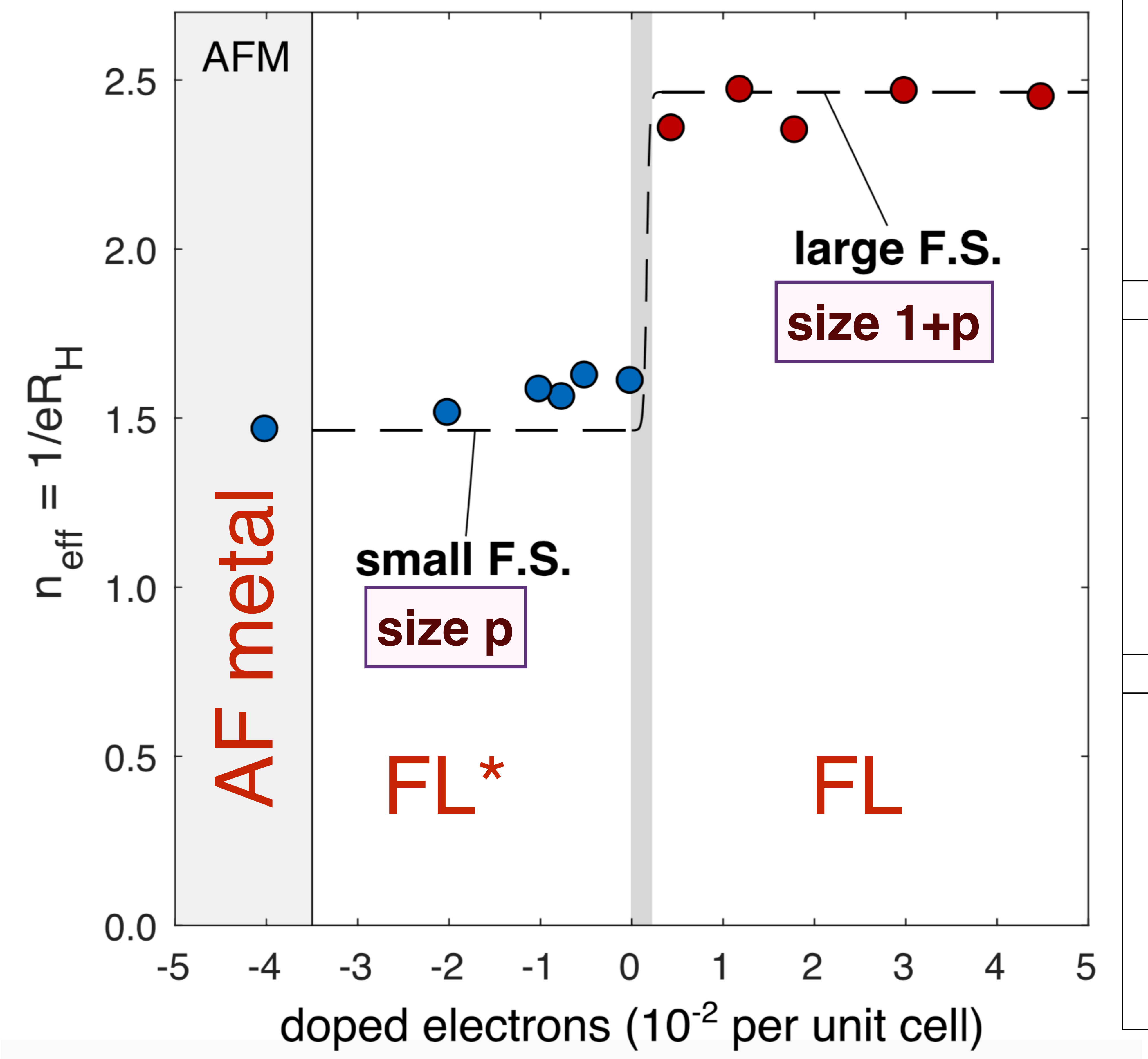
CeCoIn$_5$

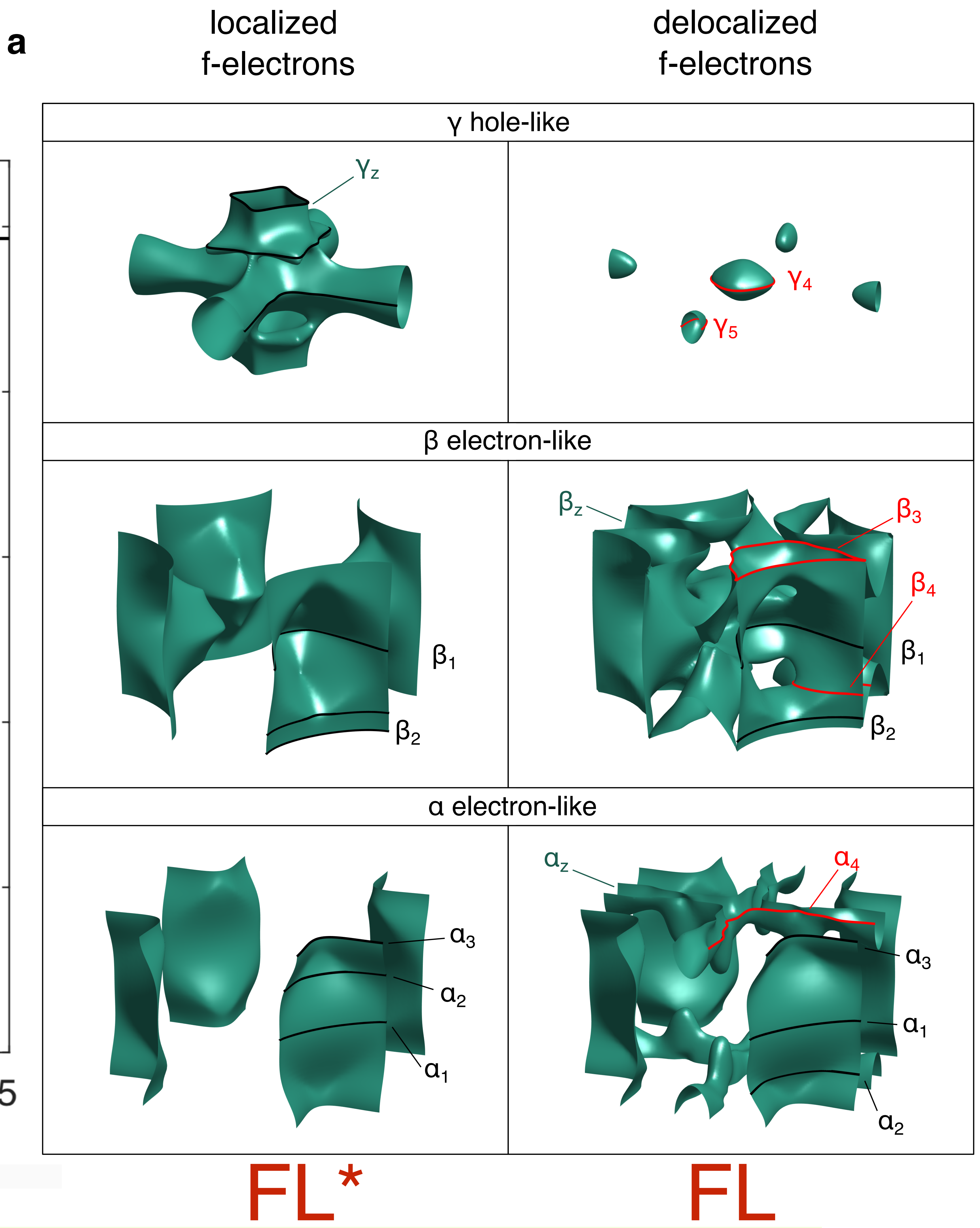
Evidence for freezing of charge degrees of freedom across a critical point in $CeCoIn_5$

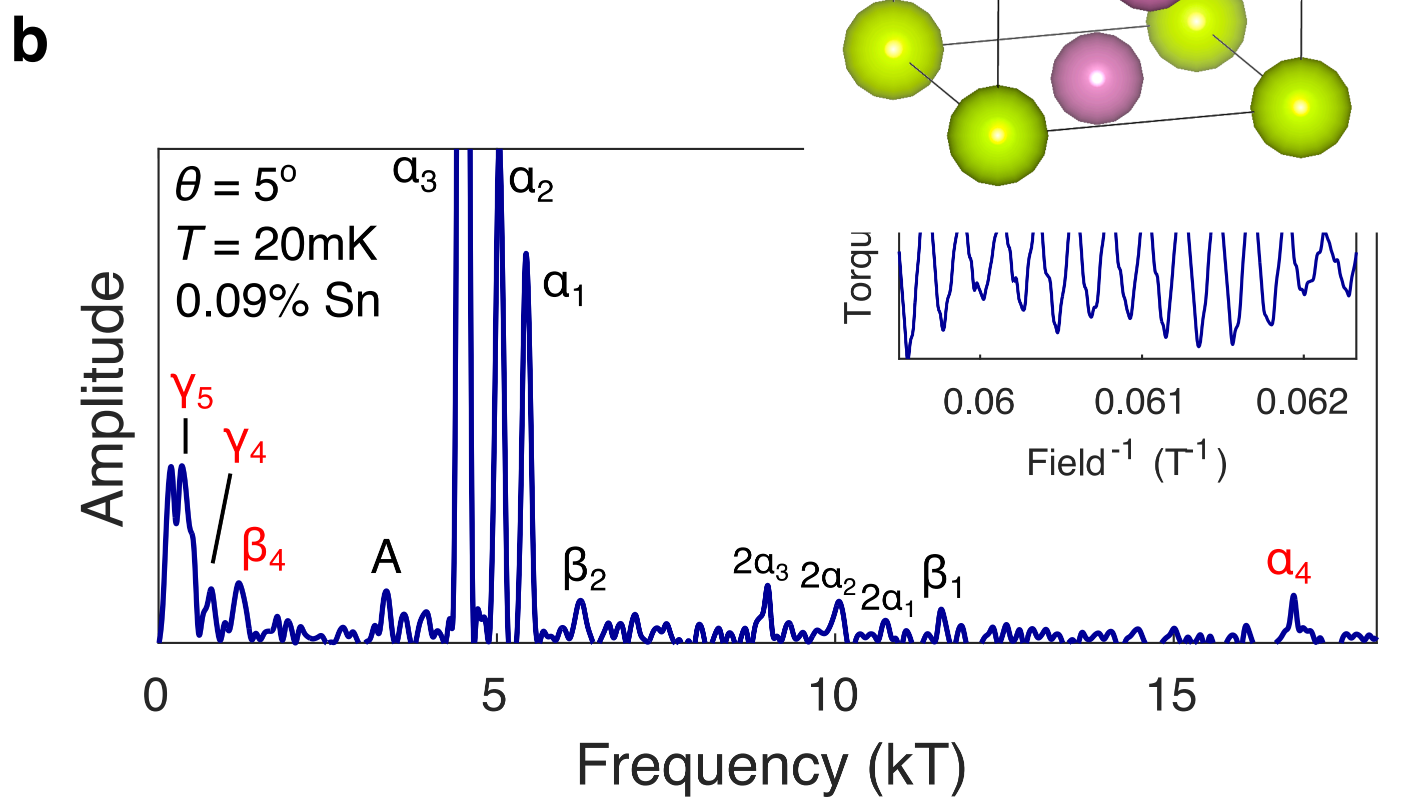
Nikola Maksimovic, Taylor Cookmeyer, Jan Rusz, Vikram Nagarajan, Amanda Gong, Fanghui Wan,
Stefano Faubel, Ian M. Hayes, Sooyoung Jang, Yochai Werman, Peter M. Oppeneer, Ehud Altman, James G. Analytis

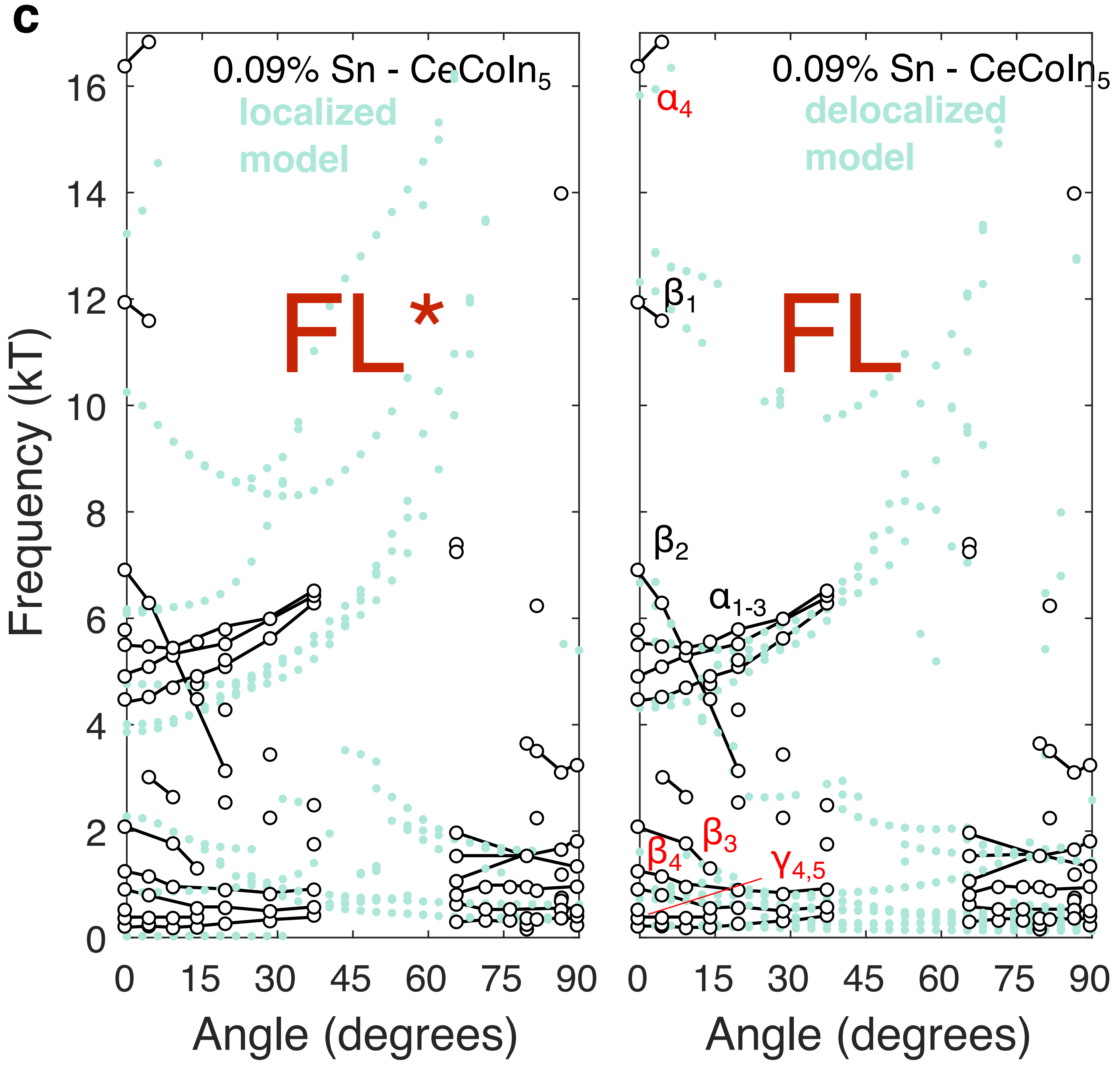


arXiv:2011.12951









See also H. Zhao, J. Zhang, M. Lyu, S. Bachus, Y. Tokiwa, P. Gegenwart, S. Zhang, J. Cheng, Y.-f. Yang, G. Chen, Y. Isikawa, Q. Si, F. Steglich, and P. Sun, Nature Physics 15, 1261 (2019) for CePdAl

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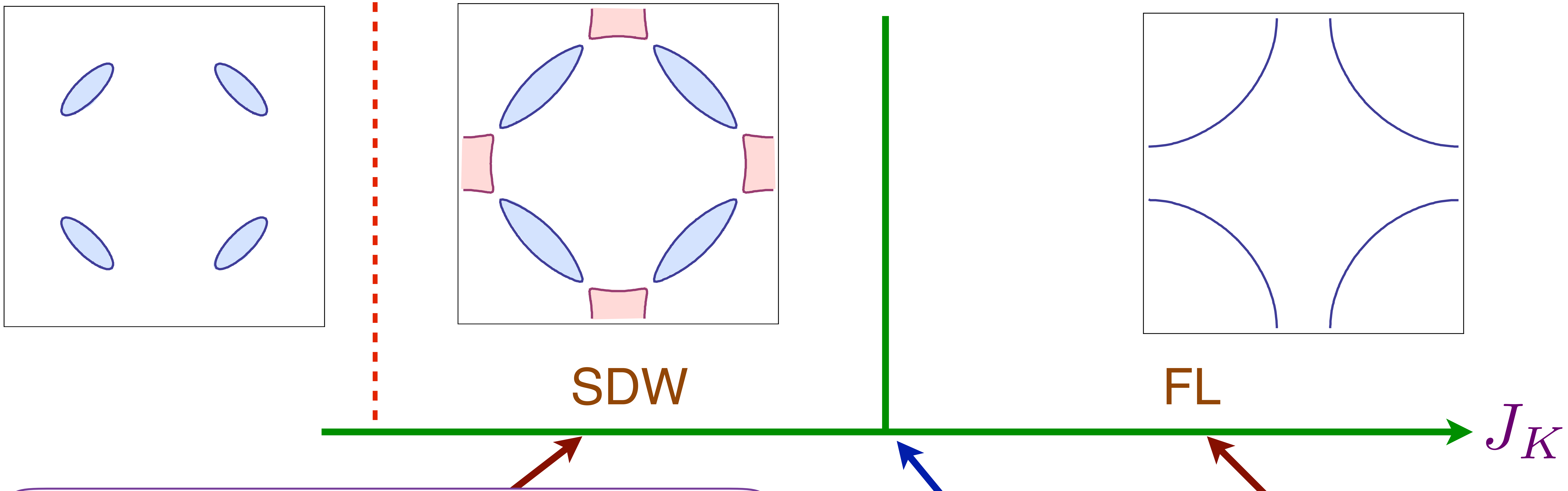
Coventional LGW theory of metal-metal transitions

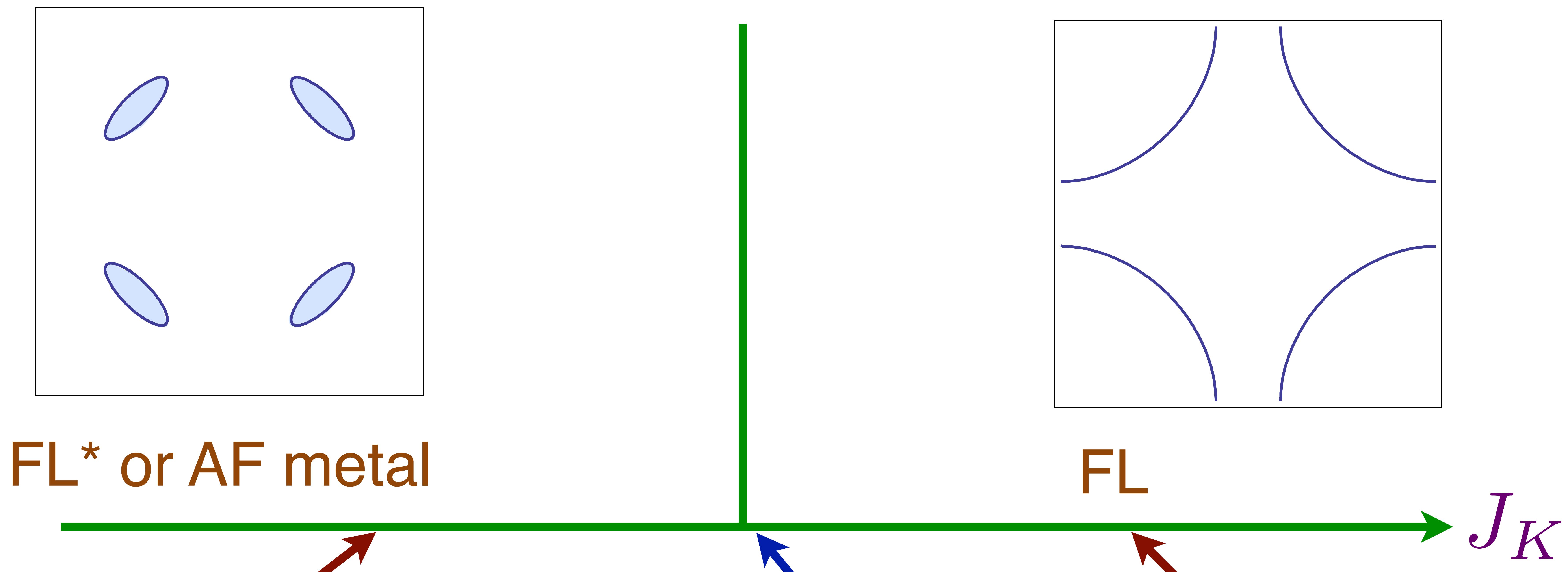


Fermi surface of size p obtained by reconstruction of large Fermi surface by SDW order. Conventional Luttinger theorem obeyed.

Large Fermi surface of size $1 + p$

Hertz-Millis theory of order parameter, with Landau damping from excitations of the large Fermi surface.

Deconfined critical theory of metal-metal transitions



Fermi surface of size p with shape unrelated to magnetic order.
In the AF metal, the conventional Luttinger theorem is obeyed.
In FL*, the conventional Luttinger theorem is evaded by fractionalization and emergent gauge fields

Deconfined criticality with fractionalization and emergent gauge fields

Large Fermi surface of size $1 + p$

T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)
Ya-Hui Zhang, S. Sachdev, PRR **2**, 023172; PRB **102**, 155124 (2020)

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Sachdev-Ye-Kitaev Models and Beyond: A Window into Non-Fermi Liquids

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USA

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USA

Review article, to appear soon…….

Dynamic mean-field theory of metal-metal transition in Hubbard model

$$H_{tUJ} = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} \sum_{\alpha=1}^{2} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} - \mu \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} + \frac{U}{2} \sum_{i} \left(\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} - 1 \right)^{2} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \,.$$

Dynamic mean-field theory of metal-metal transition in Hubbard model

$$H_{tUJ} = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} \sum_{\alpha=1}^{2} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} - \mu \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} + \frac{U}{2} \sum_{i} \left(\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} - 1 \right)^{2} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \,.$$

Dynamic mean field theory reduces everything to a self-consistent single-site problem.
This reduction is exact for the fully-connected random model with t_{ij} and J_{ij} independent random numbers with rms variance t and J.

Dynamic mean-field theory of metal-metal transition in Hubbard model

$$H_{tUJ} = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} \sum_{\alpha=1}^{2} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} - \mu \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} + \frac{U}{2} \sum_{i} \left(\sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} - 1 \right)^{2} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j \,.$$

Dynamic mean field theory reduces everything to a self-consistent single-site problem.
This reduction is exact for the fully-connected random model with t_{ij} and J_{ij} independent random numbers with rms variance t and J.

$$\begin{aligned}\mathcal{S}_{\rm tUJ} &= \int d\tau \sum_{\alpha} c_{\alpha}^{\dagger}(\tau) \left(\frac{\partial}{\partial \tau} - \mu \right) c_{\alpha}(\tau) + \frac{U}{2} \int d\tau \left(\sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha} - 1 \right)^{2} \\ &+ \int d\tau d\tau' \Delta(\tau - \tau') \sum_{\alpha} c_{\alpha}^{\dagger}(\tau) c_{\alpha}(\tau') - \frac{1}{2} \int d\tau d\tau' \mathcal{J}(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau')\end{aligned}$$

From this action we have to determine the Green's function and spin correlator:

$$G(\tau - \tau') = -\frac{1}{2} \sum_{\alpha} \left\langle c_{\alpha}(\tau) c_{\alpha}^{\dagger}(\tau') \right\rangle_{\mathcal{S}_{tUJ}} , \quad \chi(\tau - \tau') \equiv \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{S}_{tUJ}} ,$$

and impose the self-consistency condition that results from the cavity construction:

$$\Delta(\tau) = t^2\, G(\tau) \ , \ \ \mathcal{J}(\tau) = J^2\, \chi(\tau).$$

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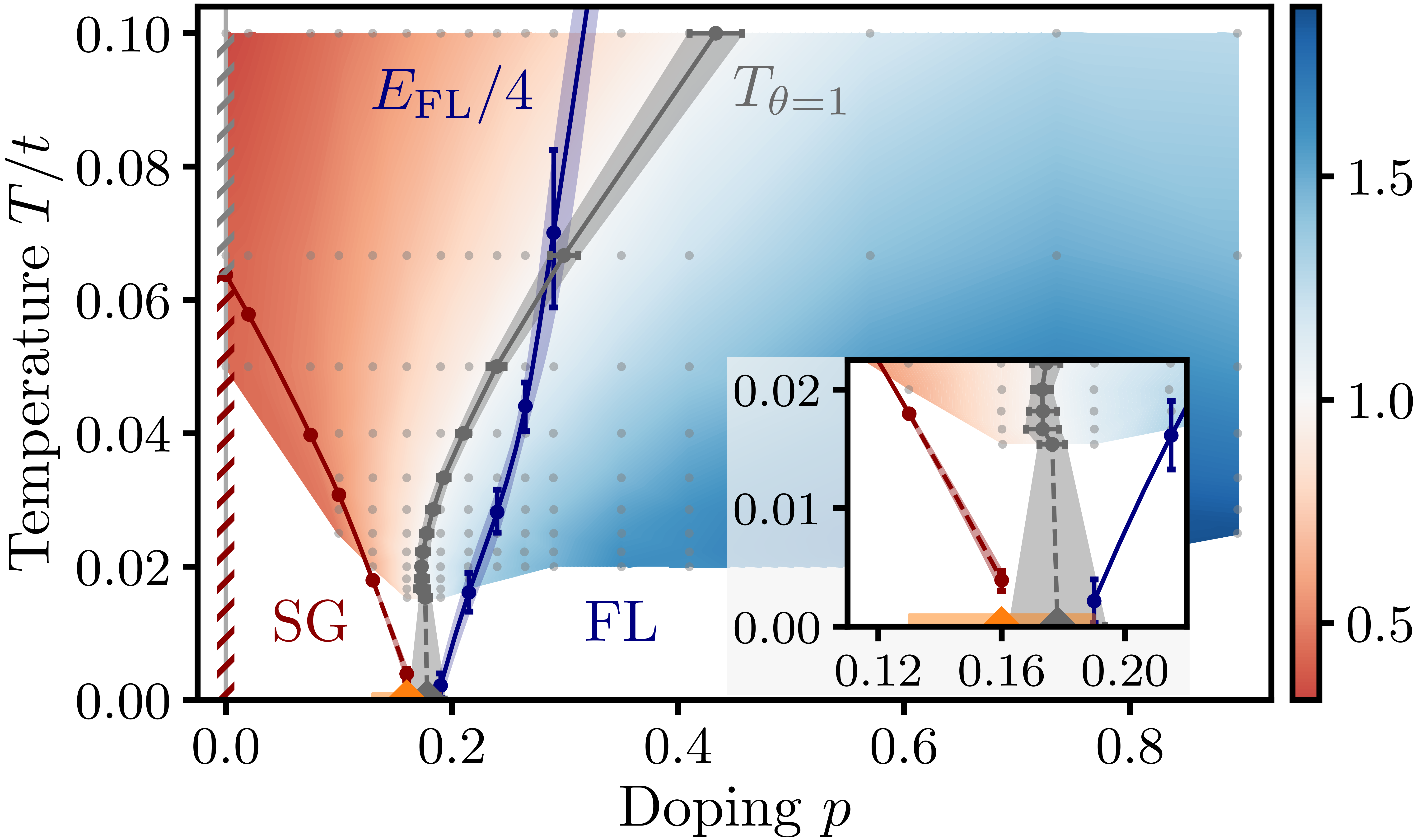
Phase diagram (doping driven QCP)

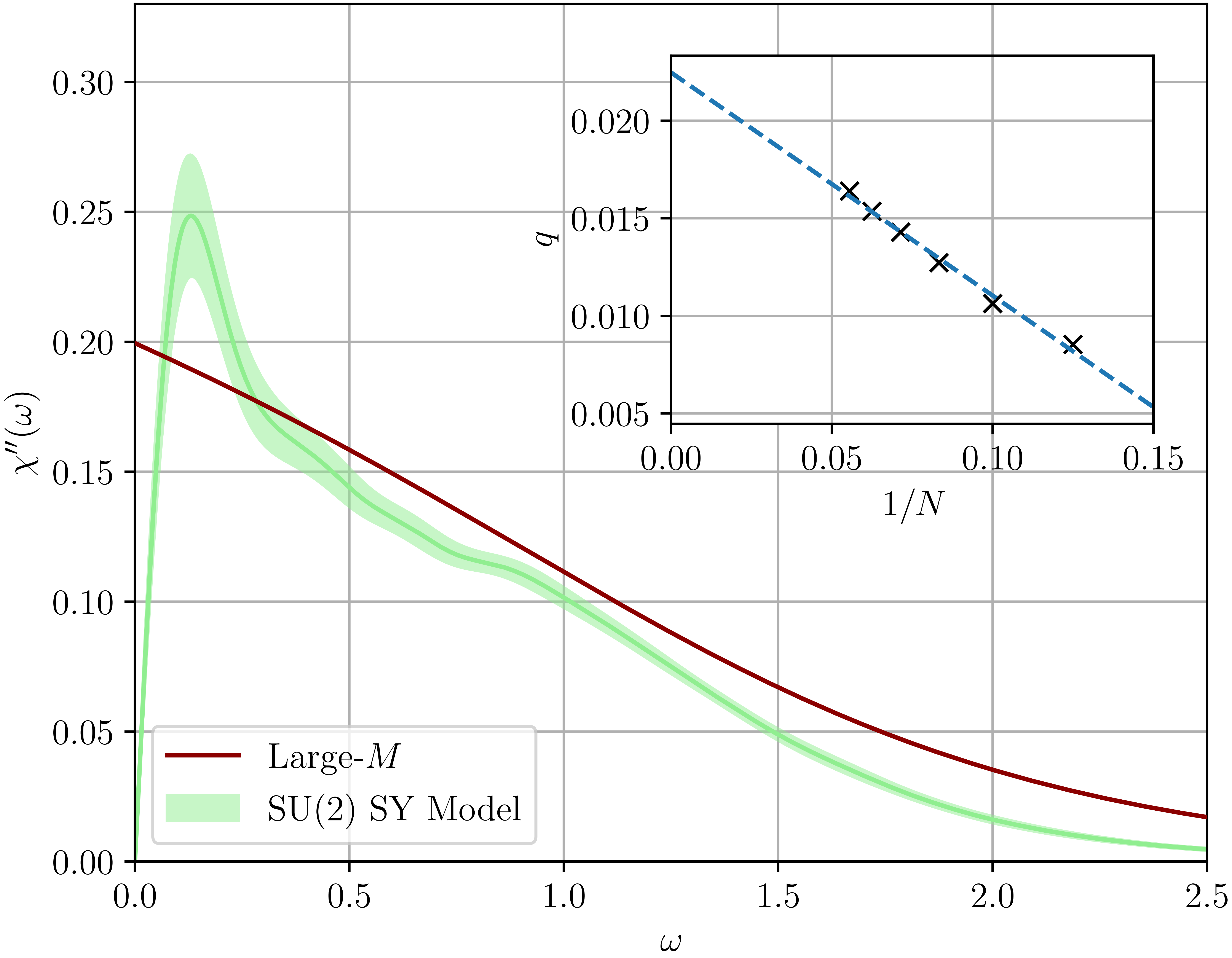
- J = 0.5t, U = 4t



Numerical solutions of DMFT equations

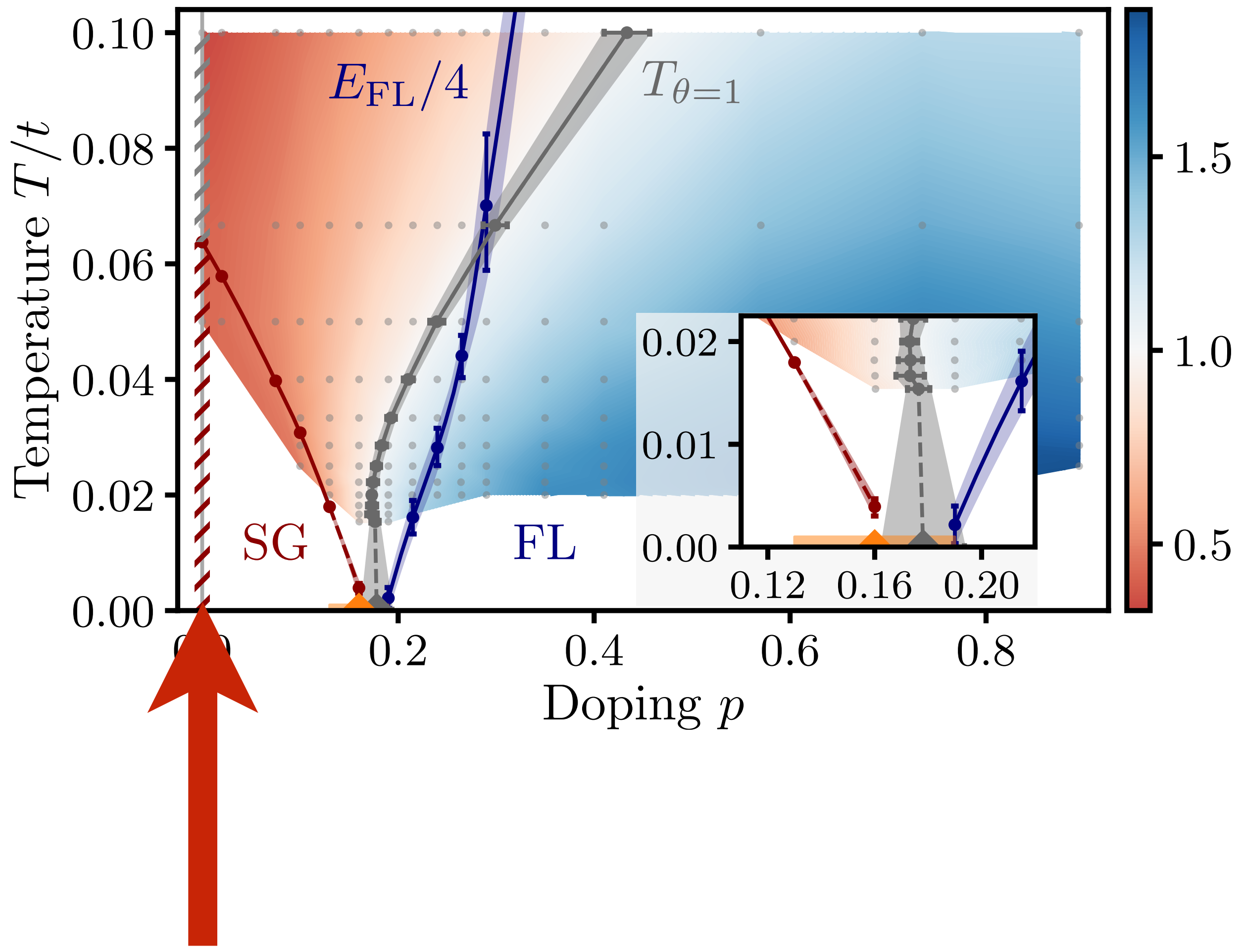
P. T. Dumitrescu,
N. Wentzell,
A. Georges,
O. Parcollet
arXiv:2103.08607

Numerical solution for Hubbard model



Exact diagonalization of random t-J model.

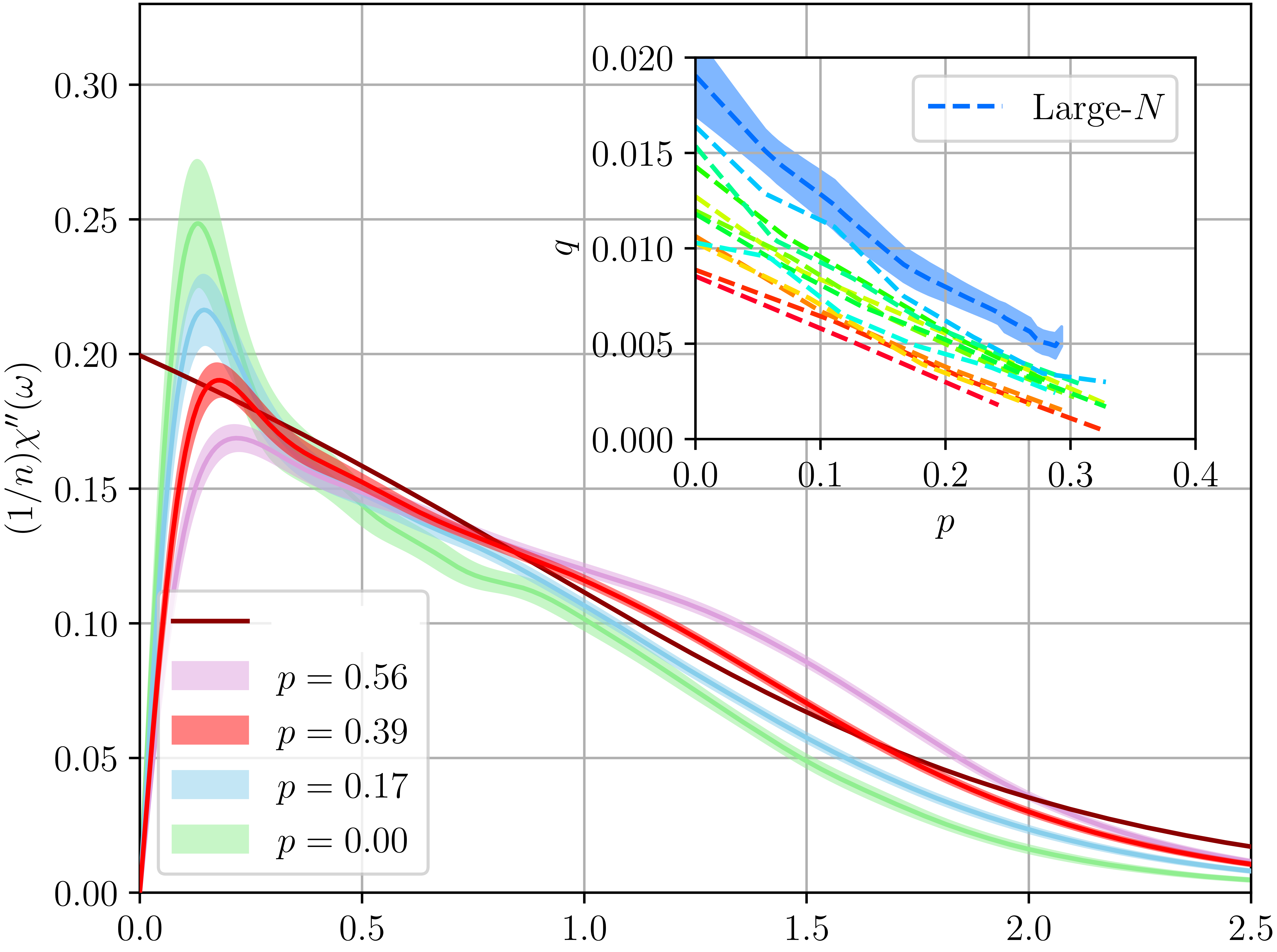
$$p = 0$$



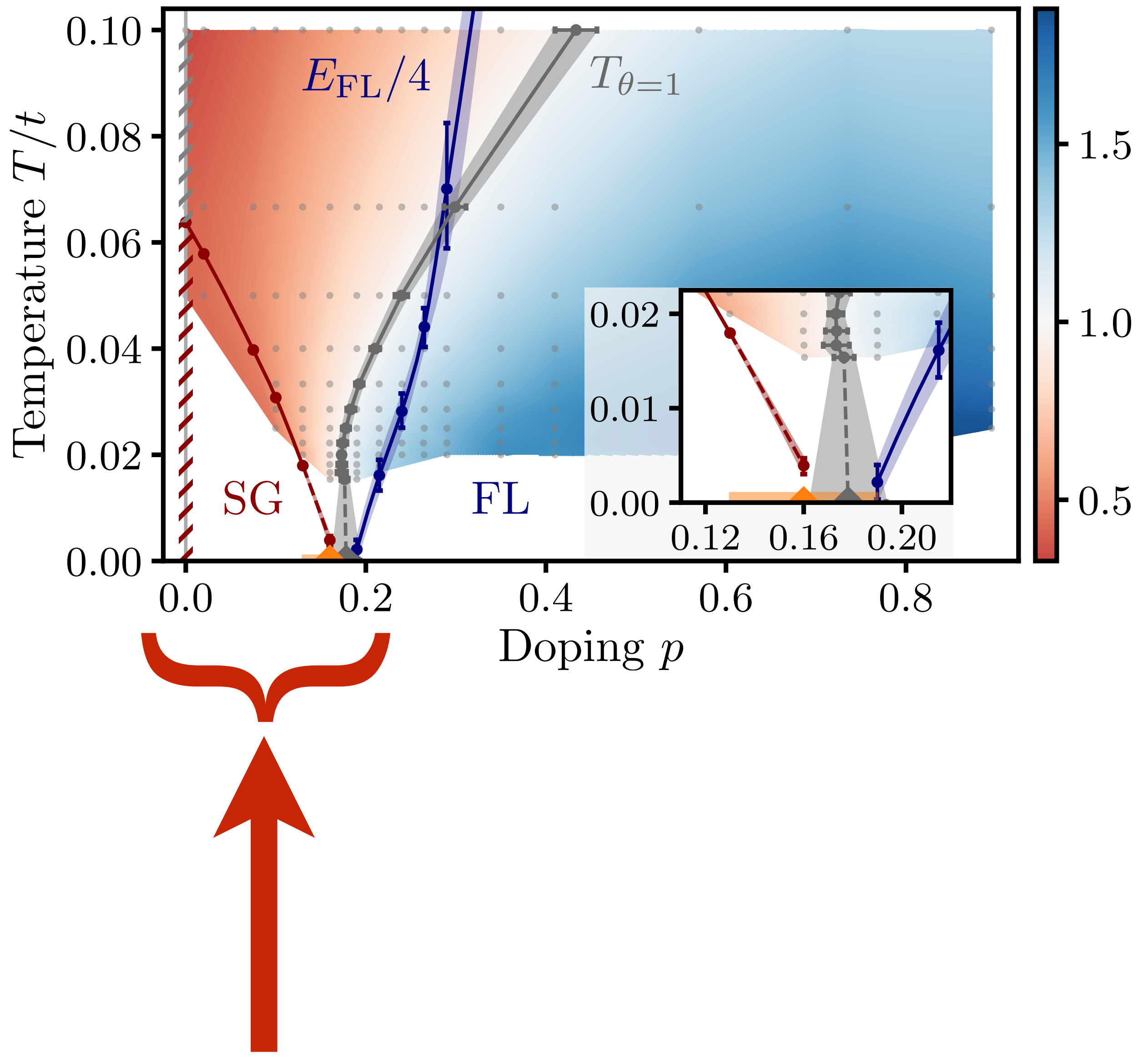
The peak at small ω indicates the presence of spin glass order.
The large M SYK theory predicts $\chi''(\omega) \sim \mathrm{sgn}(\omega)\,[1 - c|\omega| + \ldots]$

H. Shackleton,
A. Wietek,
A. Georges, and
S. Sachdev,
PRL **126**,
136602 (2021)

Numerical solution for Hubbard model



Exact diagonalization of random t-J model.



H. Shackleton,
A. Wietek,
A. Georges, and
S. Sachdev,
PRL **126**,
136602 (2021)

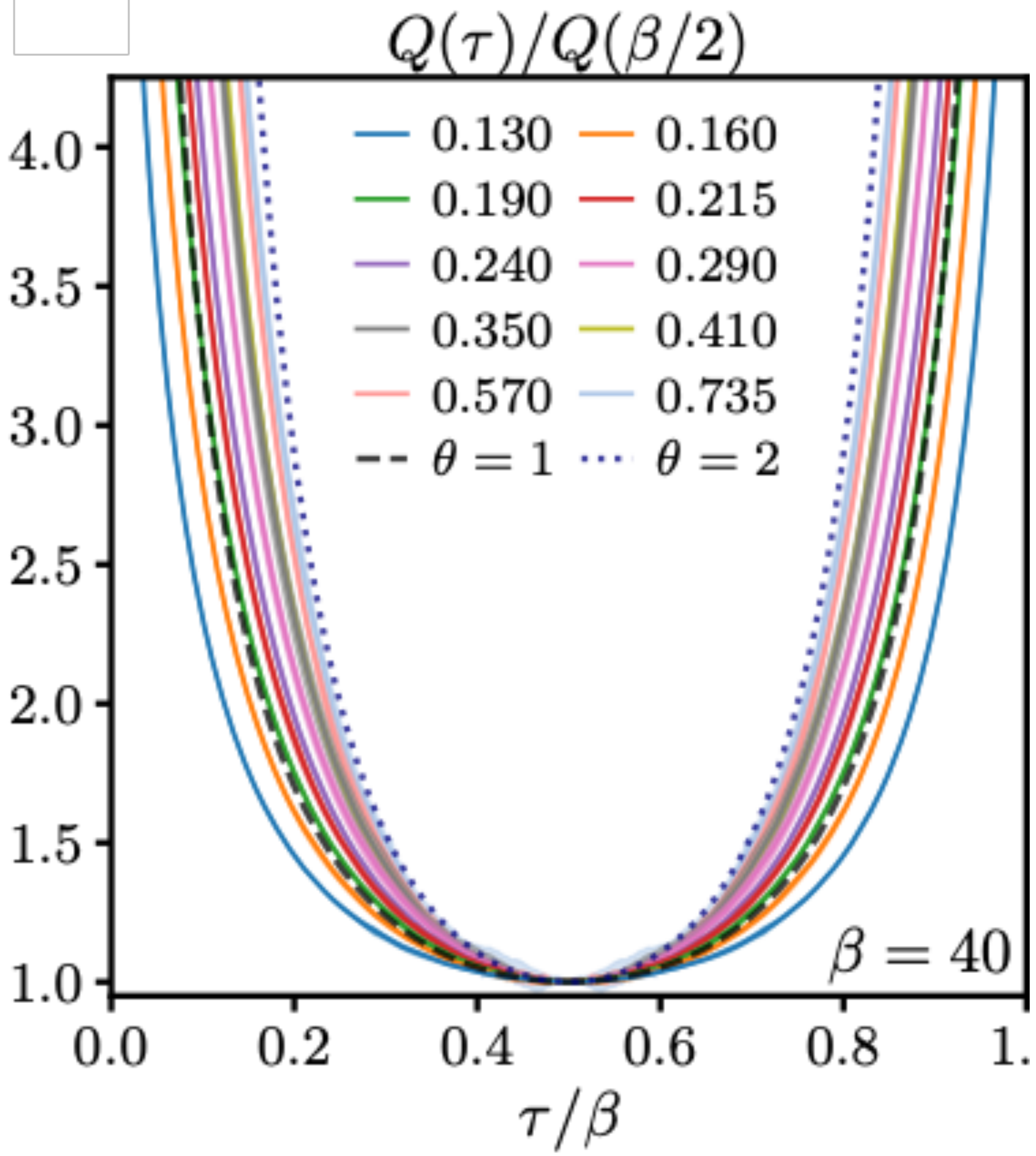
Spin glass order disappears above $p \approx 1/3$.
The large M SYK theory predicts $\chi''(\omega) \sim \mathrm{sgn}(\omega)\,[1 - c|\omega| + \ldots]$
and this fits well near criticality

Critical scaling : spin dynamics

- Match conformal invariant form

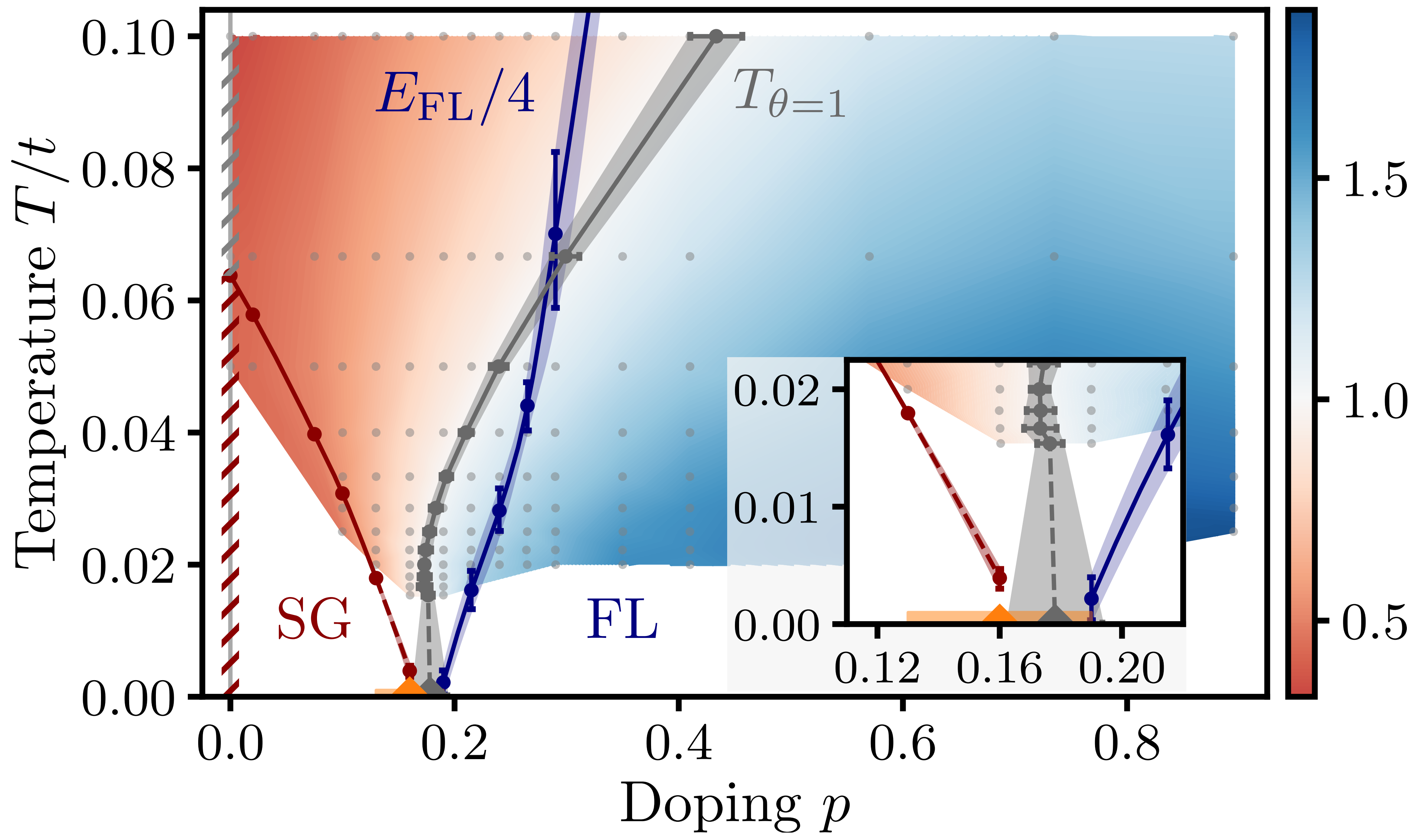
$$Q(\tau - \tau') = \frac{1}{3}\langle \boldsymbol{S}(\tau) \cdot \boldsymbol{S}(\tau') \rangle$$

$$Q(\tau) \sim \frac{1}{[\sin(\pi\tau/\beta)]^{\theta}}$$



- θ=2 (Fermi liquid), θ=1(QCP)
- Phase diagram color map : θ

$\theta = 1$ is SYK value.



Numerical solutions of DMFT equations

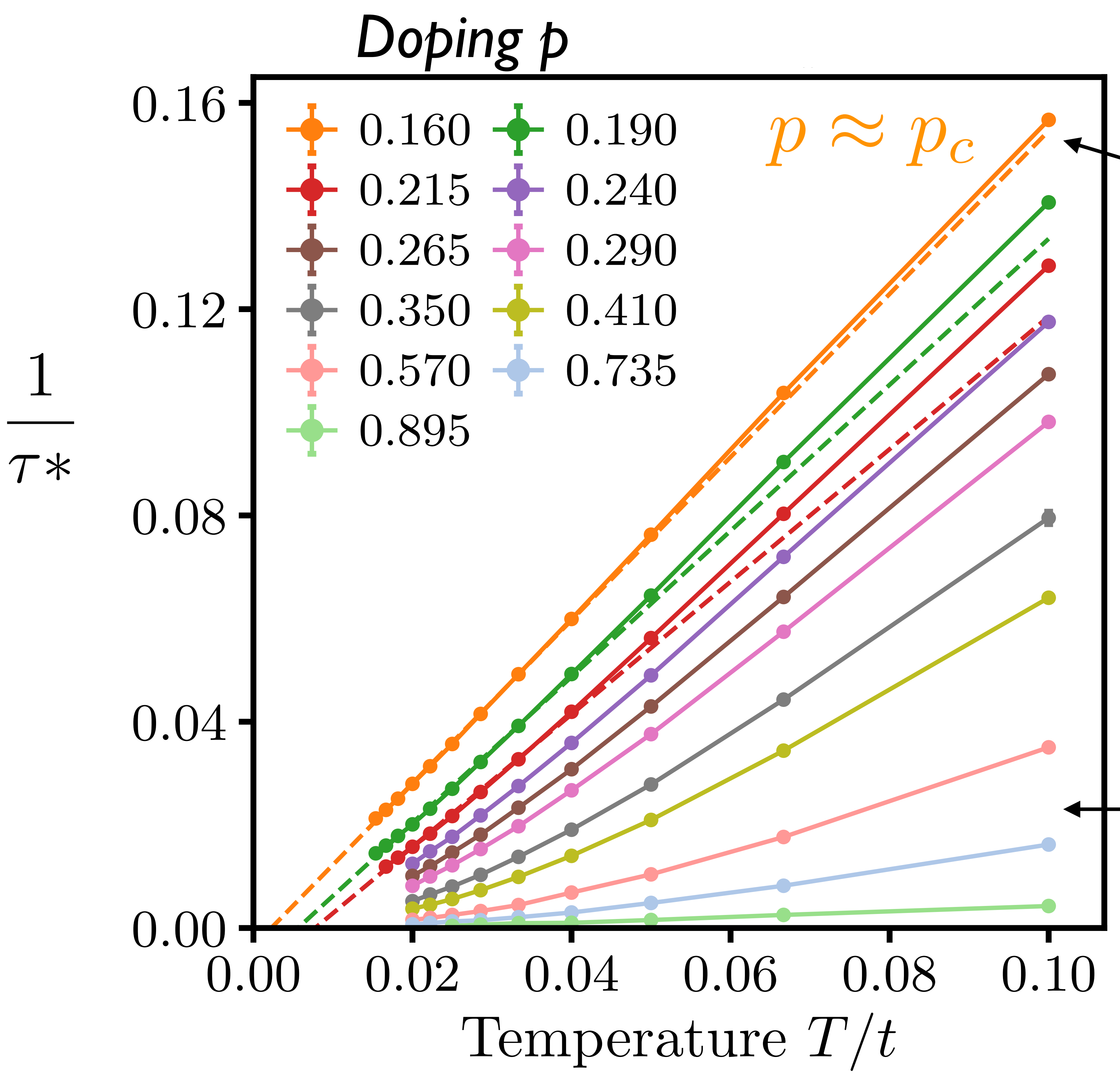
P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet arXiv:2103.08607

Single particle lifetime

- Quasiparticle lifetime in the Fermi liquid

$$\frac{1}{\tau*} = -Z Im\Sigma(\omega = 0)$$

Extrapolated to ω=0

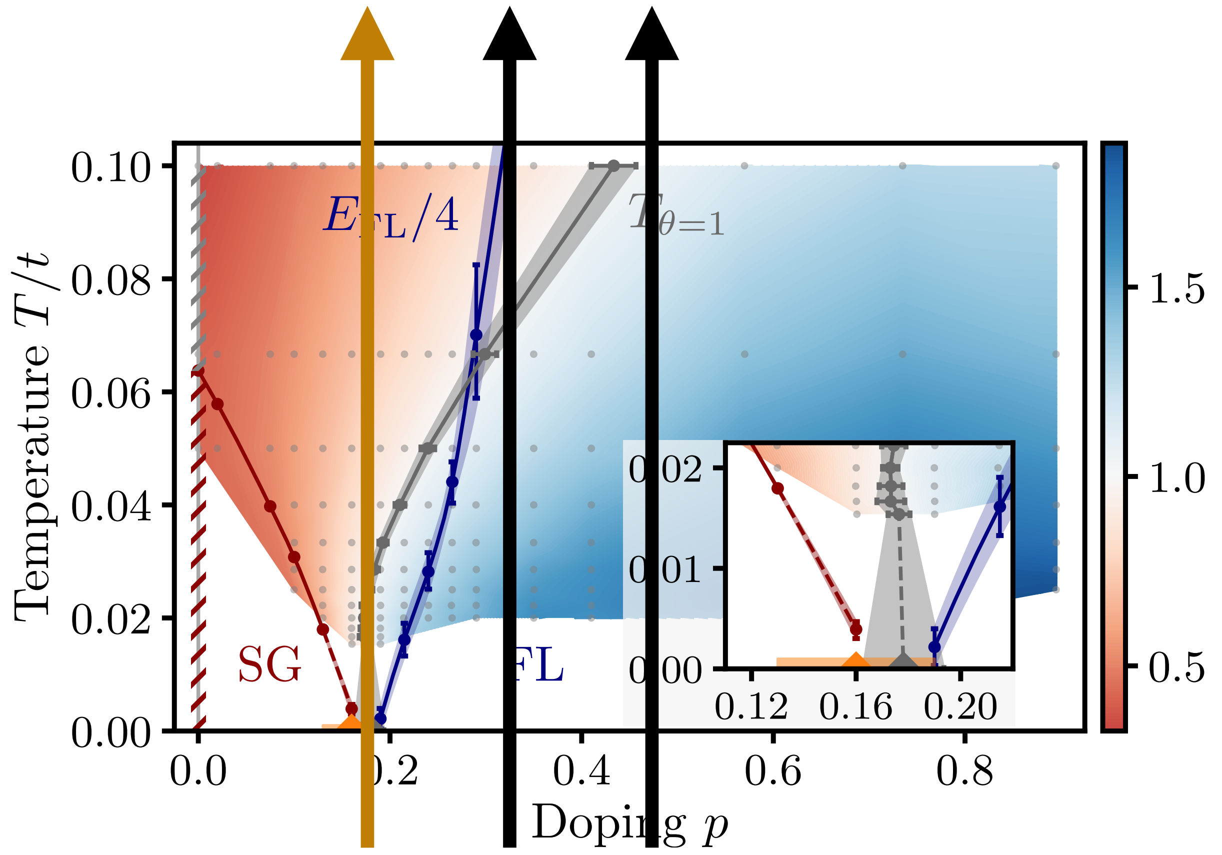


$$\frac{1}{\tau*} \simeq c\,\frac{k_B T}{\hbar}$$

p = p_c
Planckian

$$\frac{1}{\tau*} \propto T^2$$

p>>p_c
Fermi Liquid



- NB : Z factor is important to get the constant *c* of order 1

Numerical solutions of DMFT equations
P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet arXiv:2103.08607

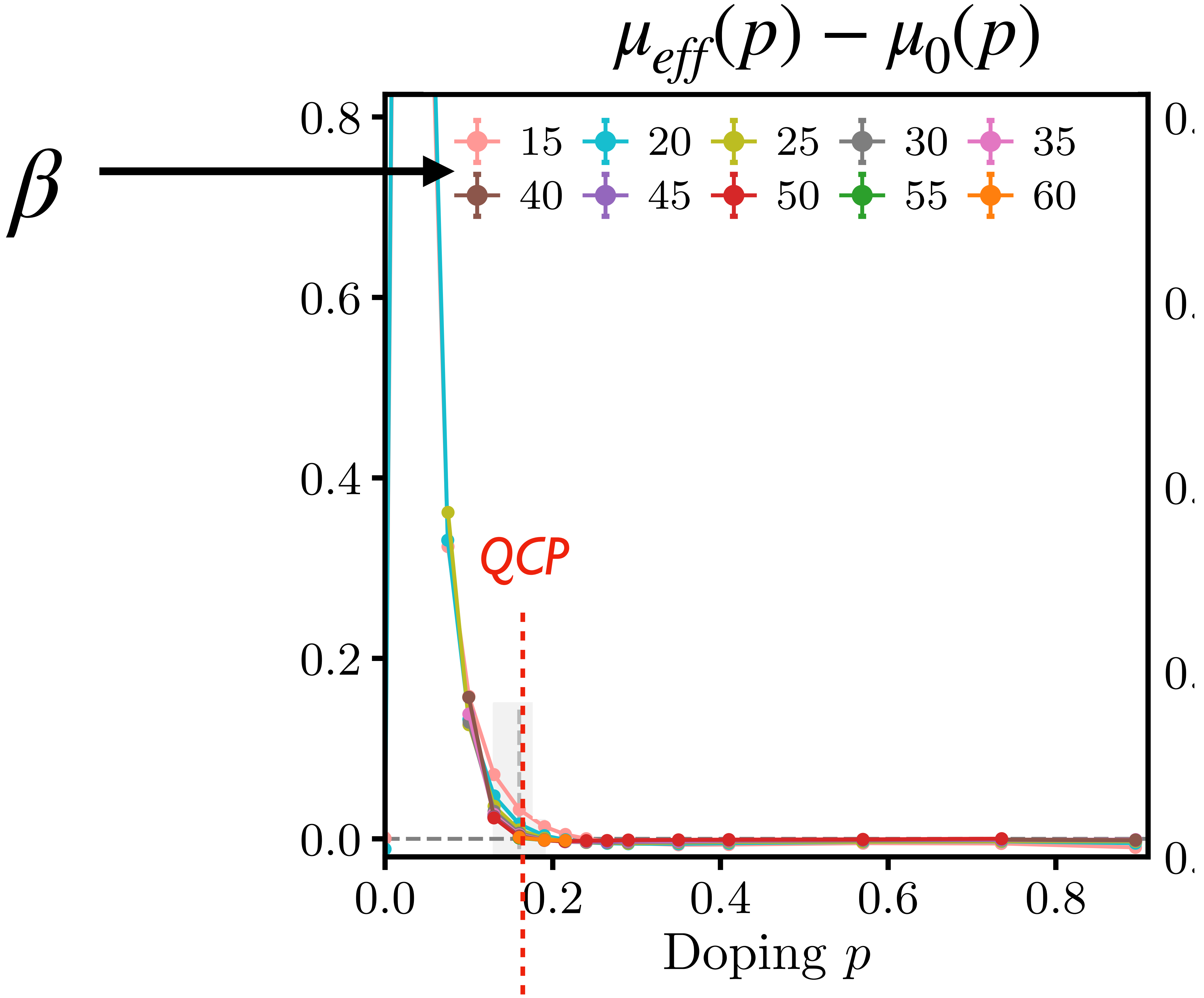
Fermi surface reconstruction at the QCP

See also Otzuki,Vollhardt

- Luttinger theorem : volume of Fermi surface independent of interaction
- Takes a simple form here, as Σ is local

$$\begin{aligned}\mu_{\rm eff}(p) &\equiv \mu - Re\Sigma(\omega=0, T=0)\\ &= \mu_0(p)\end{aligned}$$

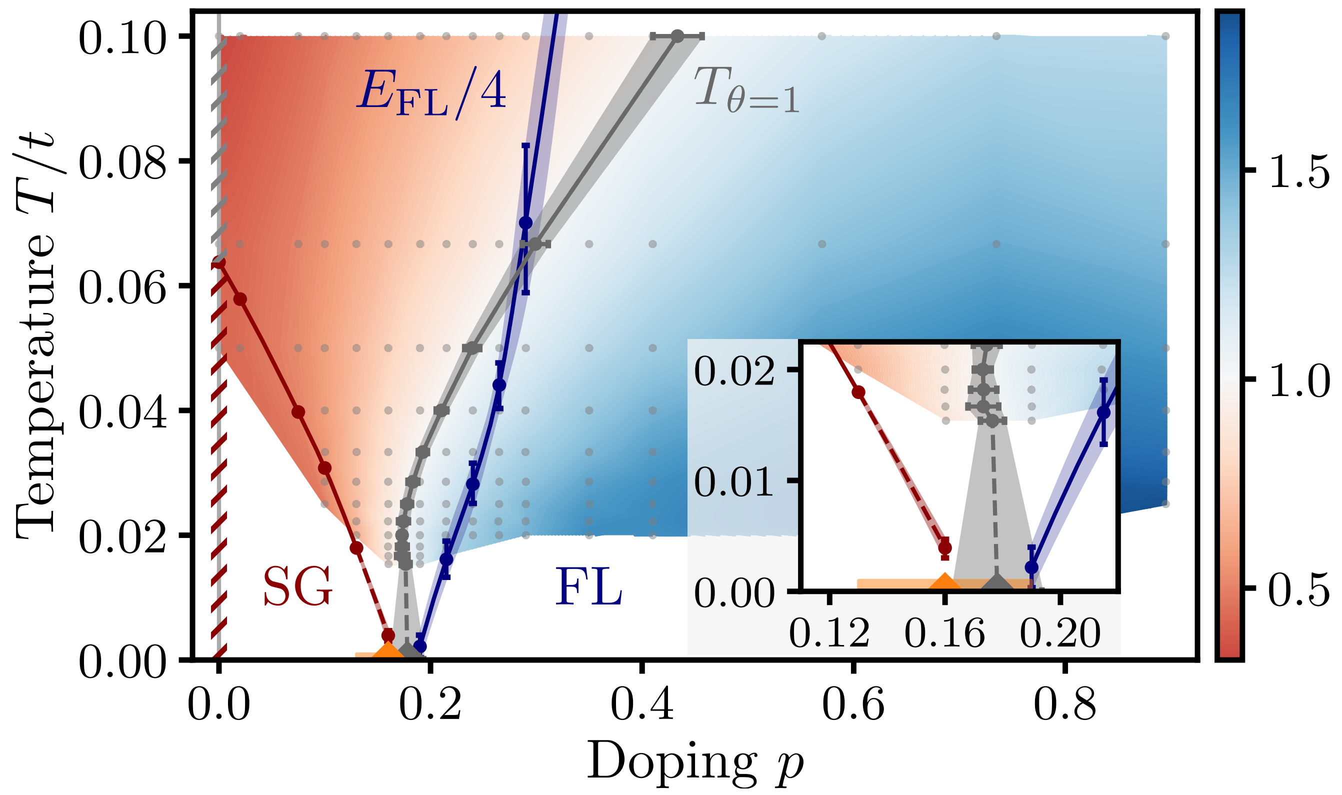
Chemical potential of non interacting model



Luttinger theorem violated

Reconstruction
of the Fermi surface

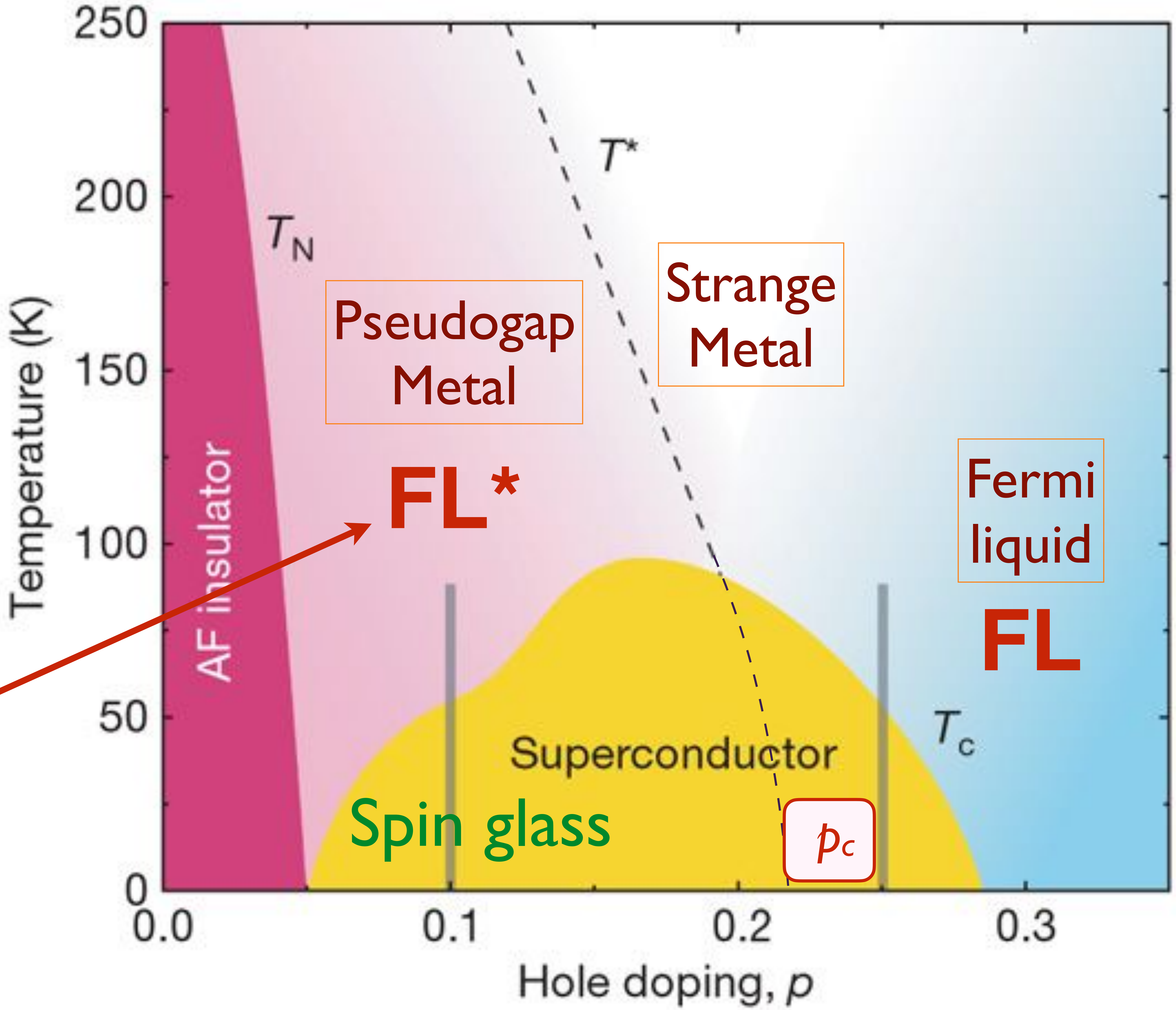
Luttinger
theorem ok



Numerical solutions of DMFT equations

P. T. Dumitrescu, N. Wentzell, A. Georges, O. Parcollet arXiv:2103.08607

Fermi surface of size p without any order which breaks translational symmetry (violates Luttinger theorem)



Pseudogap Metal

FL*

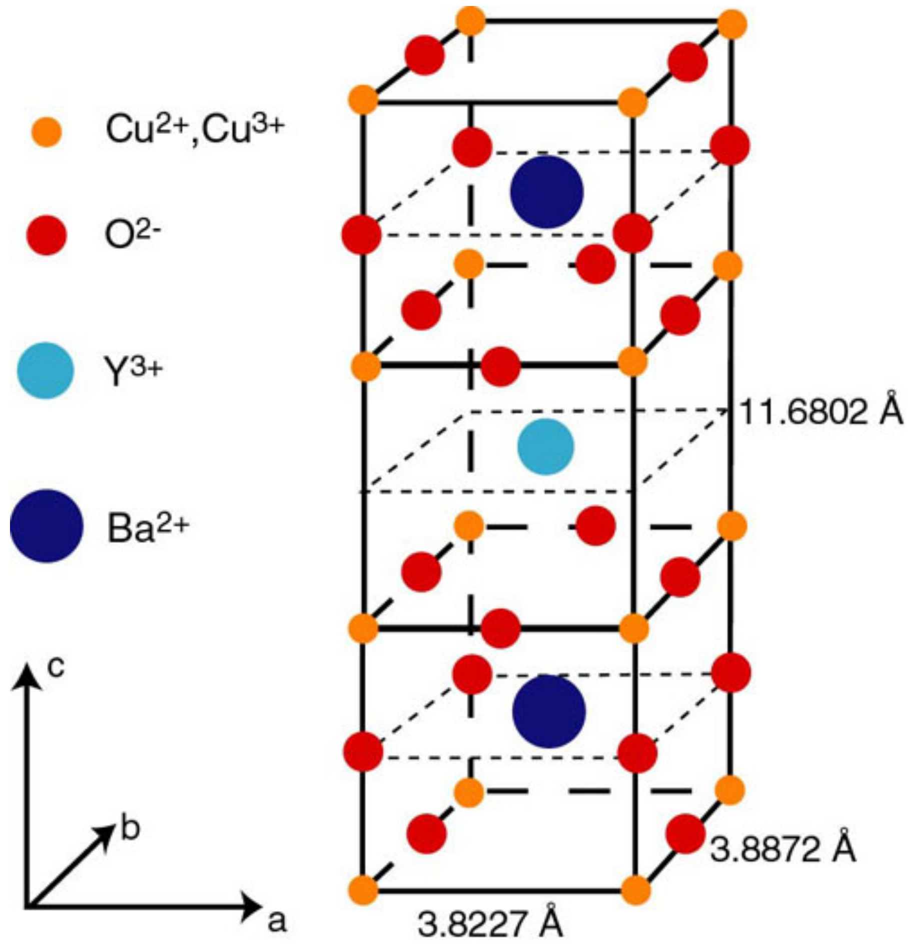
Strange Metal

Fermi liquid

FL

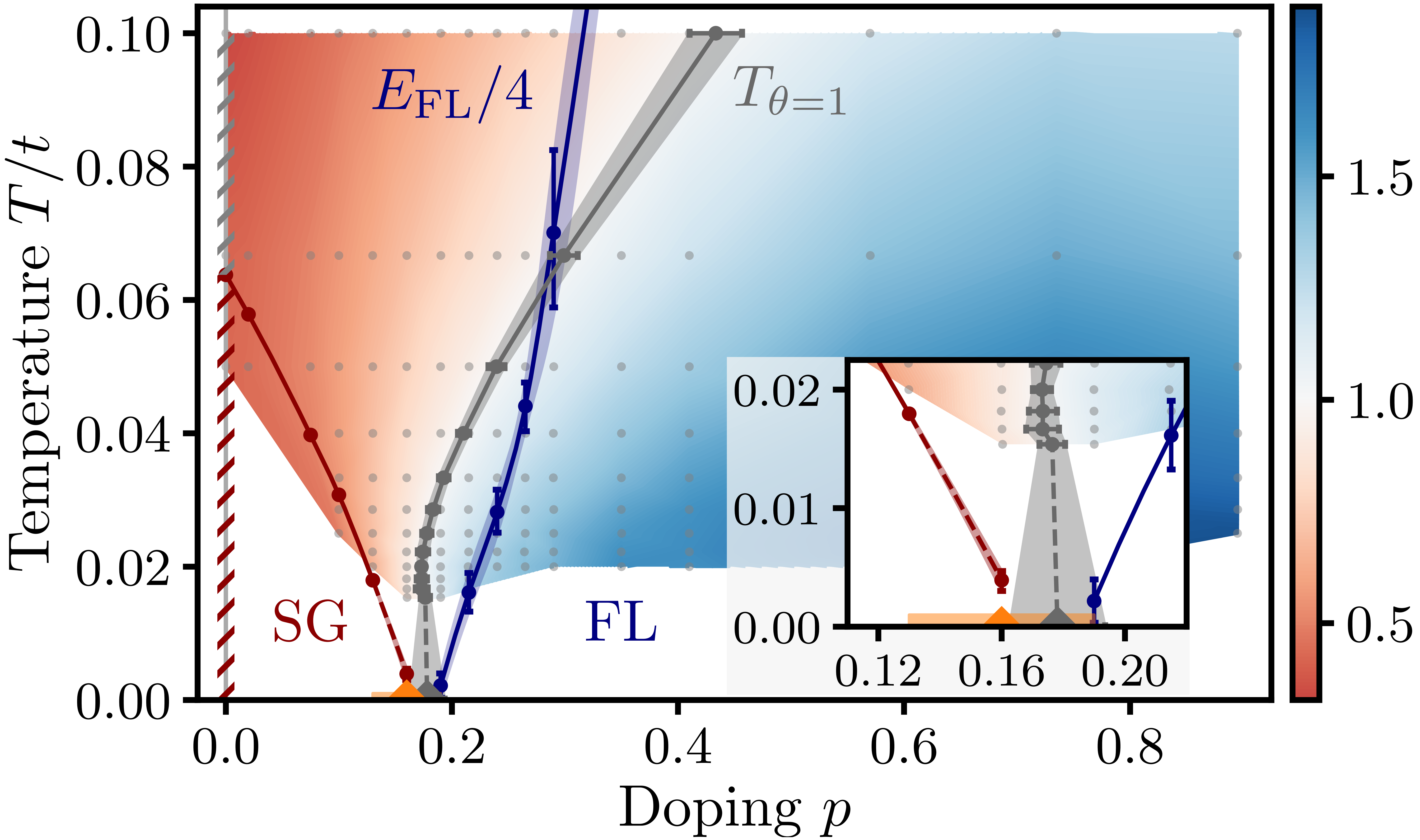
Spin glass

p_c



Phase diagram (doping driven QCP)

- J = 0.5t, U = 4t



Numerical solutions of DMFT equations

P. T. Dumitrescu,
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7. RG results on the *t-J* Model

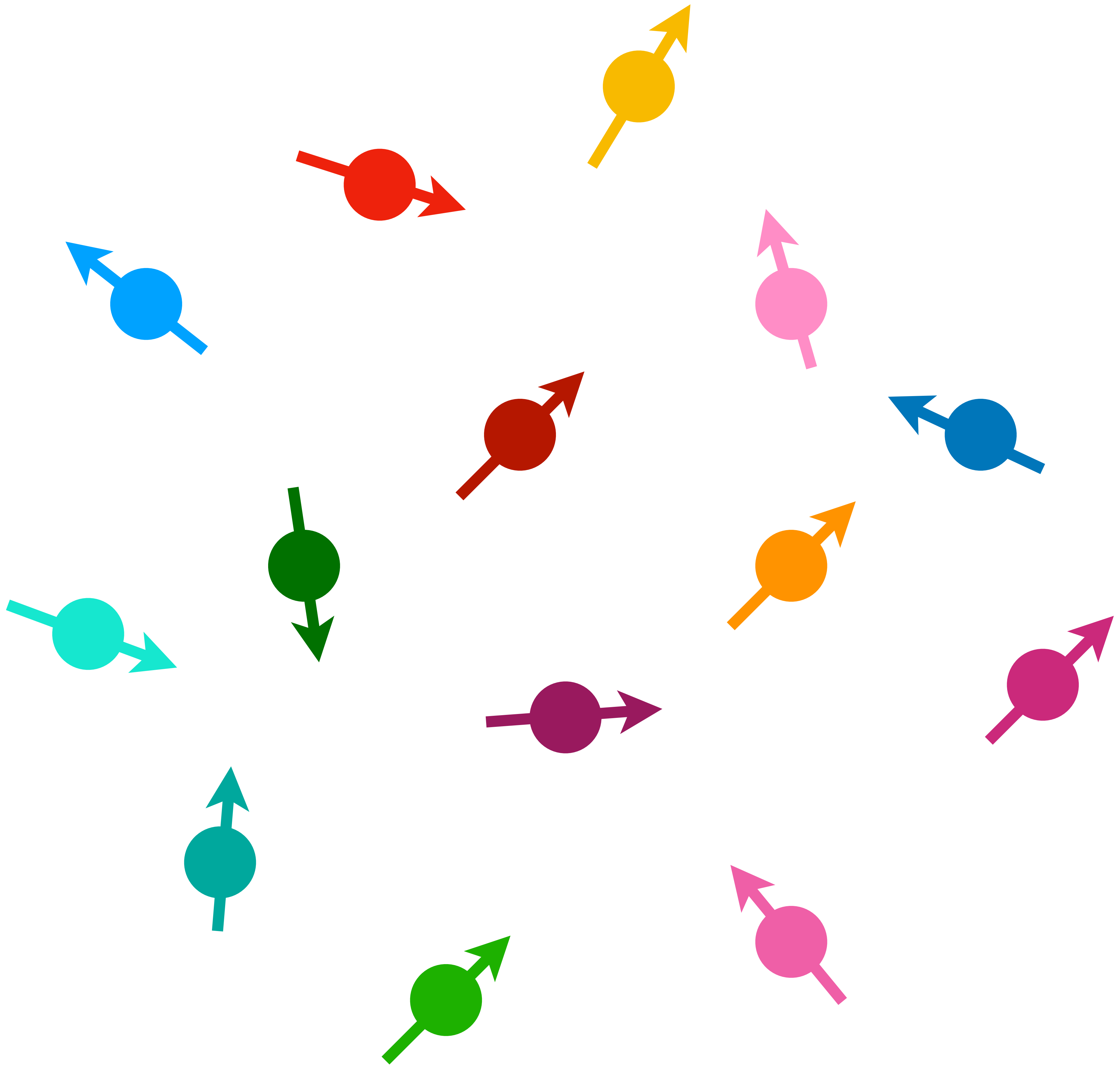
Dynamic mean-field theory of metal-metal transition in Hubbard model

$$H_{tUJ} = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} \sum_{\alpha=1}^{2} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} - \mu \sum_{i\alpha} c^\dagger_{i\alpha} c_{i\alpha} + \frac{U}{2} \sum_i \left(\sum_\alpha c^\dagger_{i\alpha} c_{i\alpha} - 1 \right)^2 + \boxed{\frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j\,.}$$

First examine $p=0$

Random *J* model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random *J* model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha} f_{i\alpha}^{\dagger} f_{i\alpha} = 1$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

Fermionic spinons

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random *J* model (insulator)

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} \mathfrak{b}_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \mathfrak{b}_{i\beta}, \quad \sum_{\alpha} \mathfrak{b}_{i\alpha}^\dagger \mathfrak{b}_{i\alpha} = 1$$

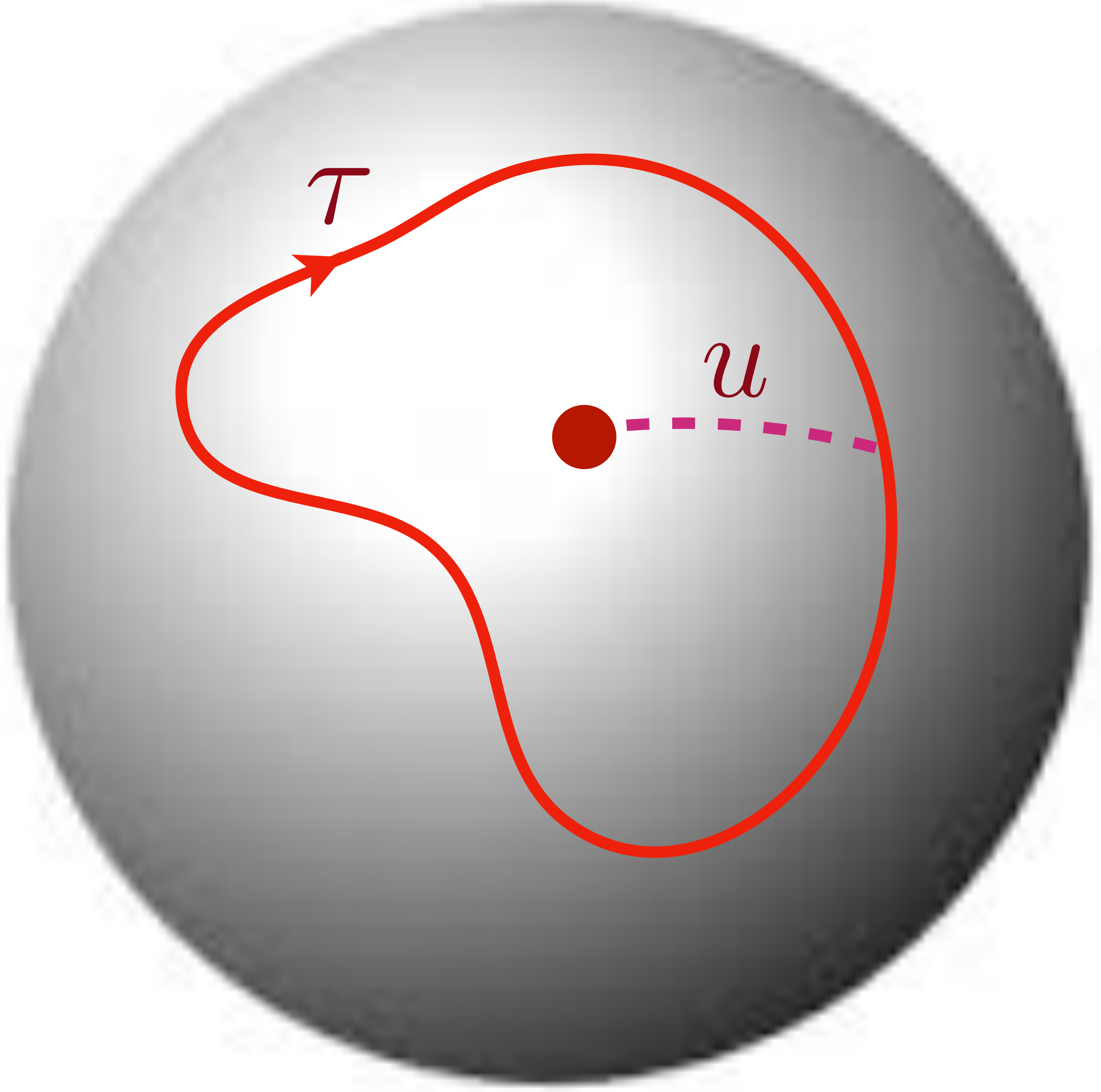
$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

Bosonic spinons

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random *J* model (insulator)

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J} \\
\mathcal{S}_B &= \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right) \\
\mathcal{S}_J &= -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.
\end{aligned}
$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random J model (insulator)

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J} \\ \mathcal{S}_B &= \frac{i}{2}\int_0^1 du \int d\tau \vec{S}\cdot\left(\frac{\partial\vec{S}}{\partial\tau}\times\frac{\partial\vec{S}}{\partial u}\right) \\ \mathcal{S}_J &= -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,. \end{aligned}$$

From this action we compute

$$\overline{Q}(\tau-\tau') = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$\boxed{Q(\tau) = \overline{Q}(\tau).}$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random J model:RG

We assume a power-law decay

$$Q(\tau) \sim \frac{\gamma^2}{|\tau|^\alpha}\,.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau)\cdot\vec{S}(0)$ interaction by introducing a bosonic (ϕ_a, $a = 1\ldots3$) bath. Then the problem reduces to the 'Bose-Kondo' Hamiltonian

$$H_{\rm imp} \quad = \quad \gamma S_a\,\phi_a(0) + \frac{1}{2}\int d^dx\,\left[\pi_a^2 + (\partial_x\phi_a)^2\right]$$

where π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x=0)$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and then we need $\alpha = d - 1$.

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
S. Sachdev, Physica C **357**, 78 (2001)

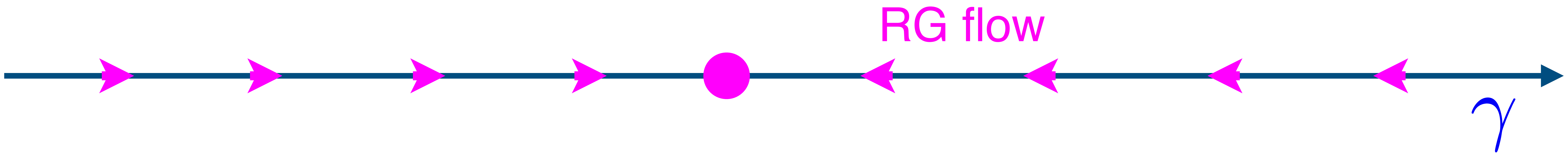
Random J model:RG

- The β-function of γ can by computed order-by-order in $\epsilon = 2 - \alpha$

$$\frac{d\gamma}{d\ell} = \epsilon \frac{\gamma}{2} - \gamma^3 \, .$$

- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.

- Because of the quantized Berry phase (Wess-Zumino-Witten) term, the renormalization of the coupling γ is given only by the wavefunction renormalization. We can then prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ.



M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
S. Sachdev, Physica C **357**, 78 (2001)

Random *J* model:RG

- The β-function of γ can by computed order-by-order in $\epsilon = 2 - \alpha$

$$\frac{d\gamma}{d\ell} = \epsilon\frac{\gamma}{2} - \gamma^3\,.$$

- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.
- Because of the quantized Berry phase (Wess-Zumino-Witten) term, the renormalization of the coupling γ is given only by the wavefunction renormalization. We can then prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ.
- The self-consistency condition therefore yields

$$\boxed{\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}\,.}$$

to all orders in ϵ. The same exponent is obtained in a SU(M) theory at large M with fractionalization.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
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Dynamic mean-field theory of metal-metal transition in Kondo lattice model

$$H_{KH} = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} - \mu \sum_{i} c_{i\alpha}^{\dagger} c_{i\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + \frac{J_K}{2} \sum_{i} \vec{S}_i \cdot \left(c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta} \right)$$

Single site action for single spin *and* a single electron

$$\mathcal{S}_{KH} = \frac{i}{2} \int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right) + \int d\tau \left[c_{\alpha}^{\dagger} \frac{\partial c_{\alpha}}{\partial \tau} - \mu \, c_{\alpha}^{\dagger} c_{\alpha} + \frac{J_K}{2} \vec{S} \cdot (c_{\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{\beta}) \right]$$
$$- \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') - t^2 \int d\tau d\tau' R(\tau - \tau') c_{\alpha}^{\dagger}(\tau) c_{\alpha}(\tau') + \text{H.c.}$$

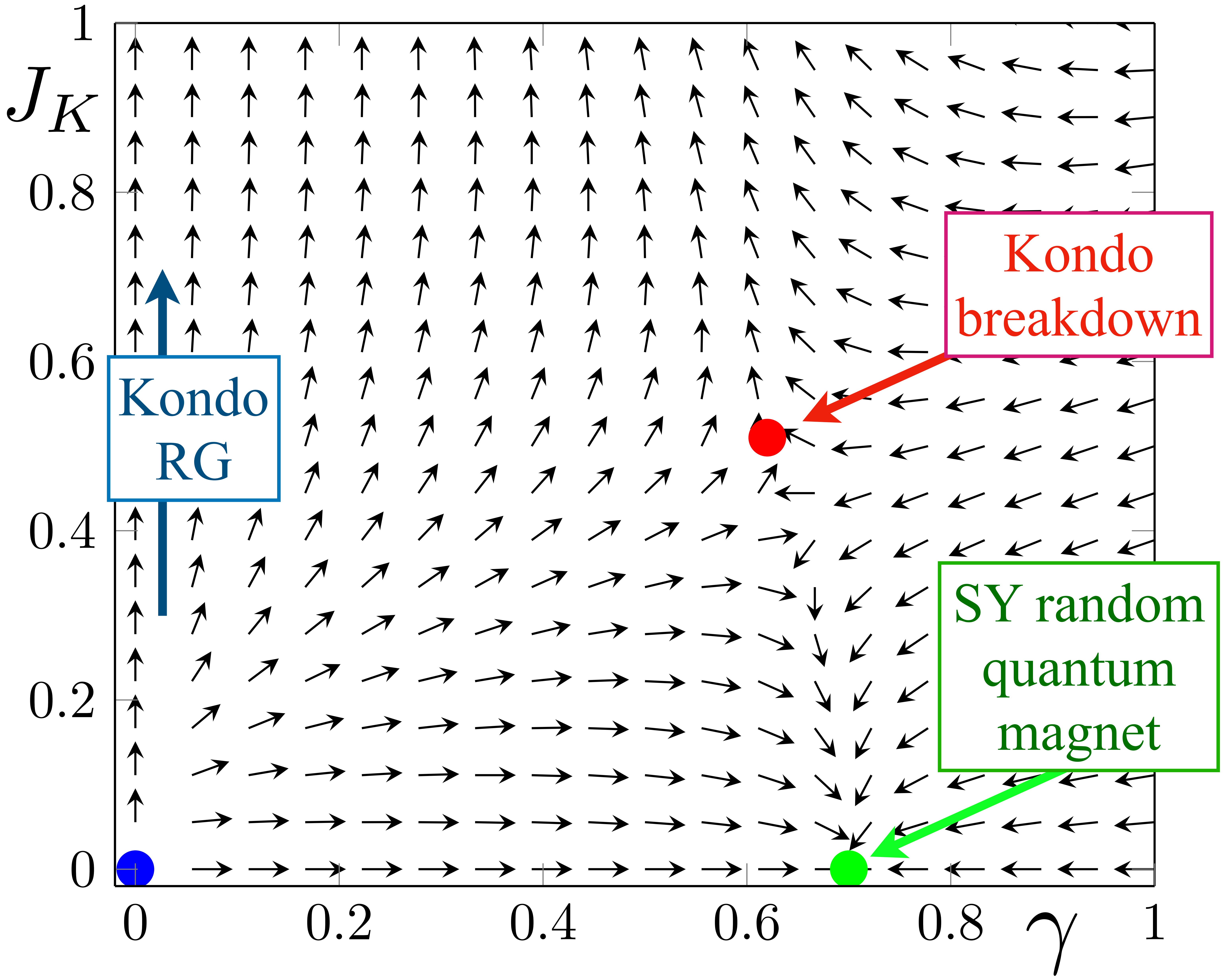
From this action we determined the correlators

$$\overline{R}(\tau - \tau') = -\frac{1}{2} \left\langle c_{\alpha}(\tau) c_{\alpha}^{\dagger}(\tau') \right\rangle_{\mathcal{Z}_{KH}} , \quad \overline{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}_{KH}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).$$

Kondo model:RG



The electron forms a Fermi liquid with small density p, and we need only consider a single spin interacting with fermionic and bosonic baths (the Bose-Fermi Kondo model). As in the random J model, the self-consistency condition leads to exponents that can be determined to all orders

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

The spin exponent can be understood from a large M SU(M) SY(K) theory with fractionalization.

A.M. Sengupta, PRB **61**, 4041 (2000)
S. Burdin, D. R. Grempel, and A. Georges, PRB **66**, 045111 (2002)
Lijun Zhu and Qimiao Si, PRB **66**, 024426 (2002)

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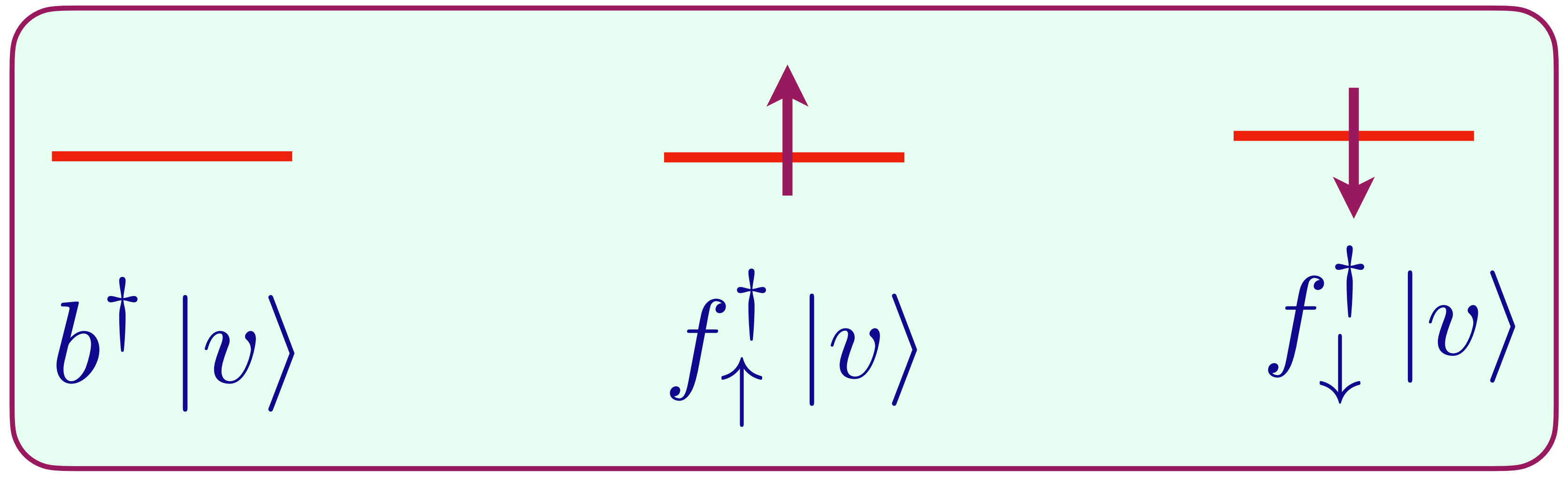
Parton theory I

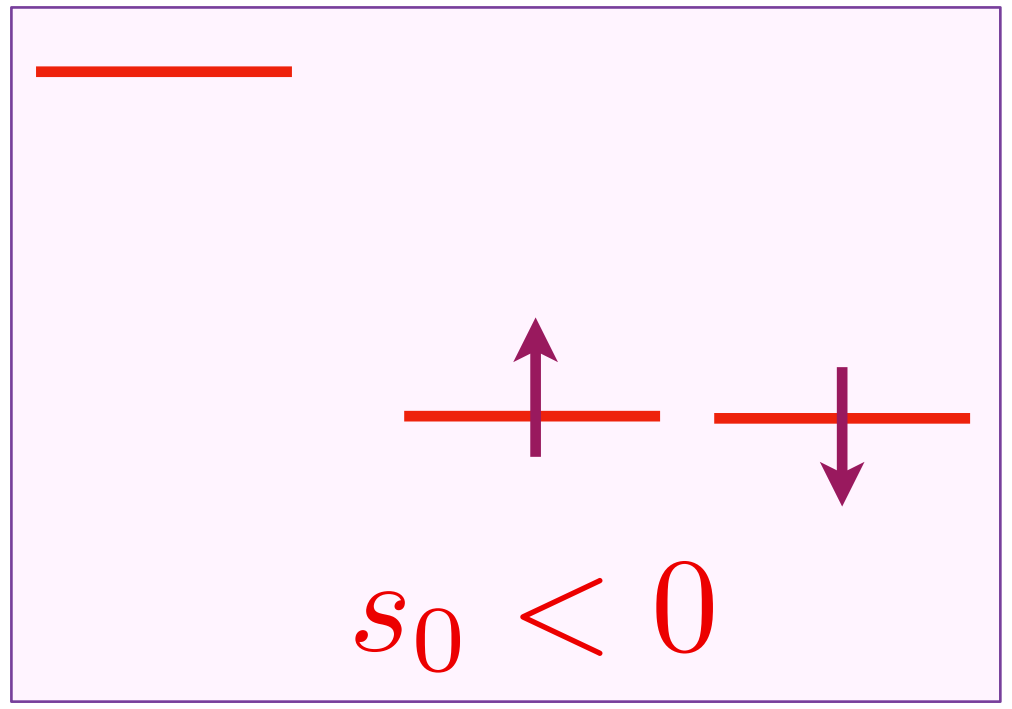
$$H = -\frac{1}{\sqrt{N}}\sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}}\sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

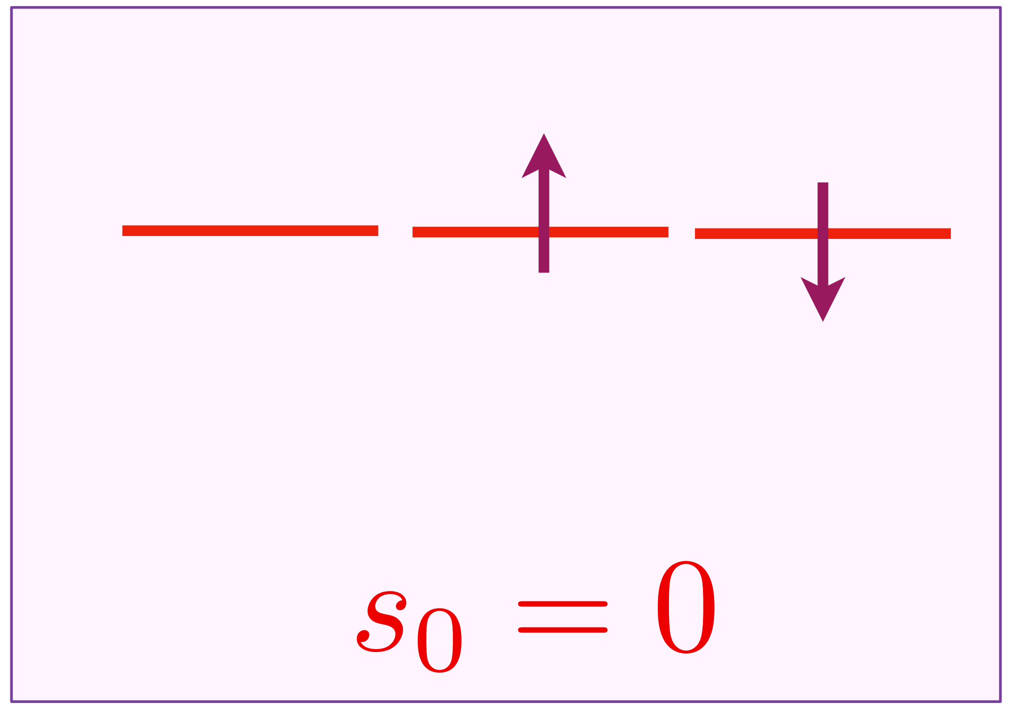
Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

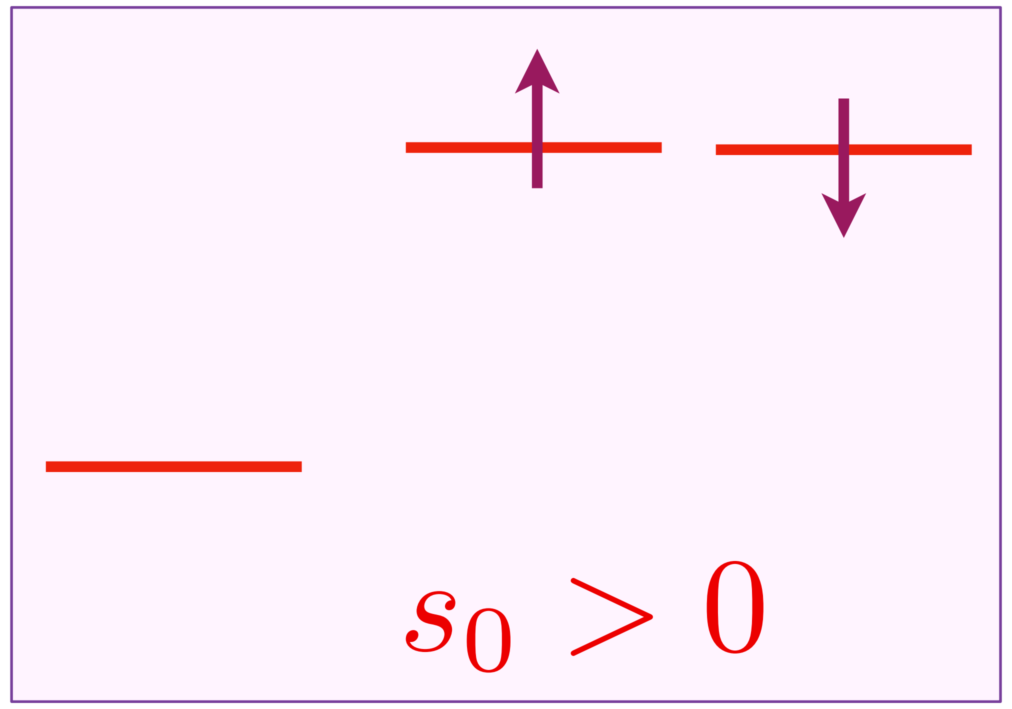


"Bose-Fermi Superspin Kondo!"

Then we can map the problem to that of a 'superspin' interaction with a bosonic bath (with coupling γ), and a fermionic bath (with coupling g). There is also a 'Zeeman' field $s_0(f_\alpha^\dagger f_\alpha - b^\dagger b)$ which splits the degeneracy between the holon and spinon states. Note $s_0 = 0$ for $p = 1/3$.



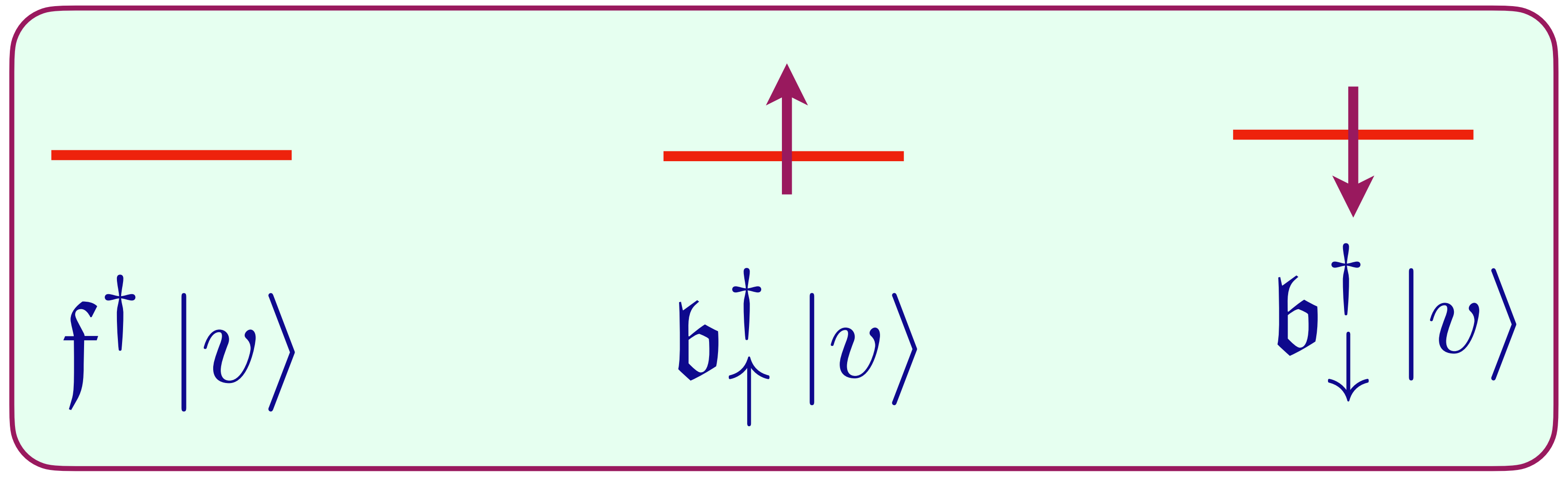




Parton theory II

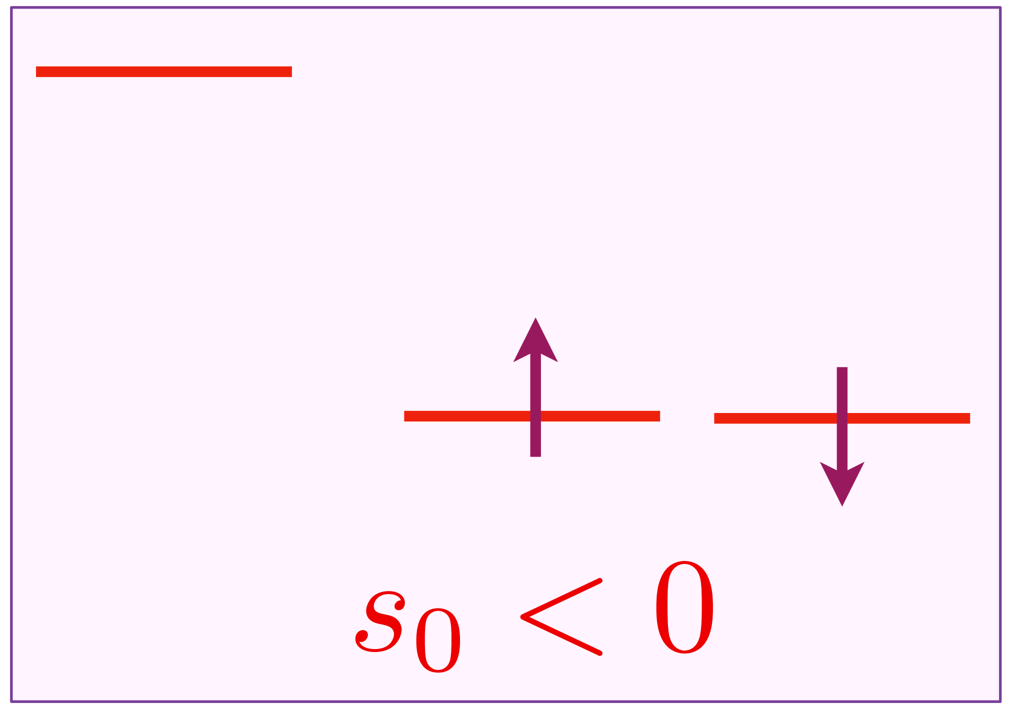
$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

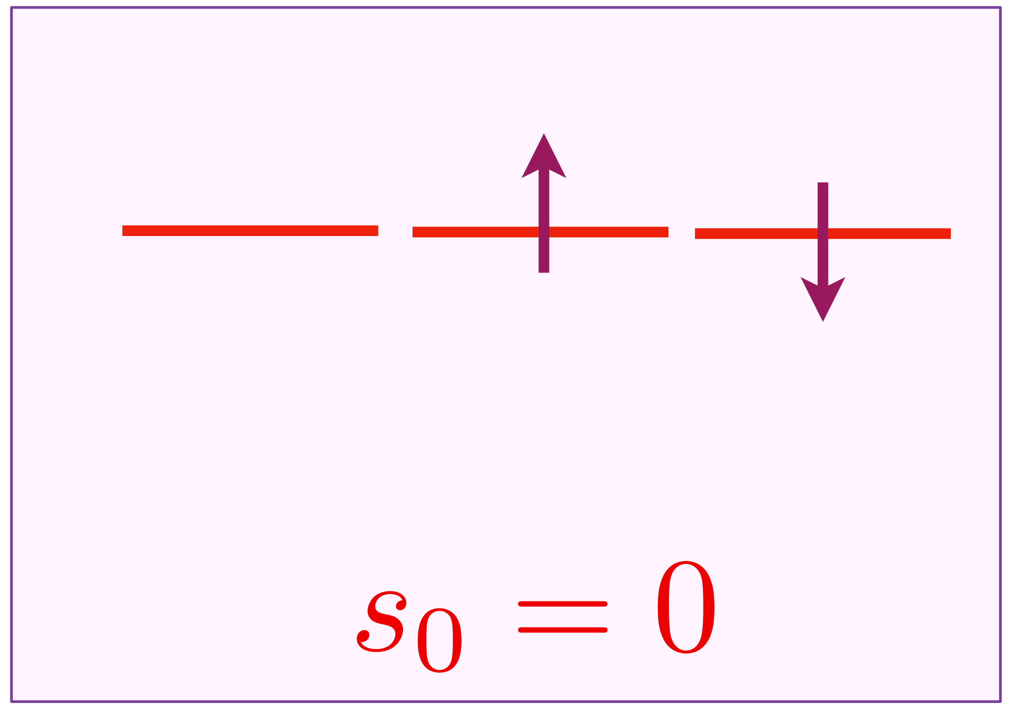
Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{f}_\alpha$ (the spinon):

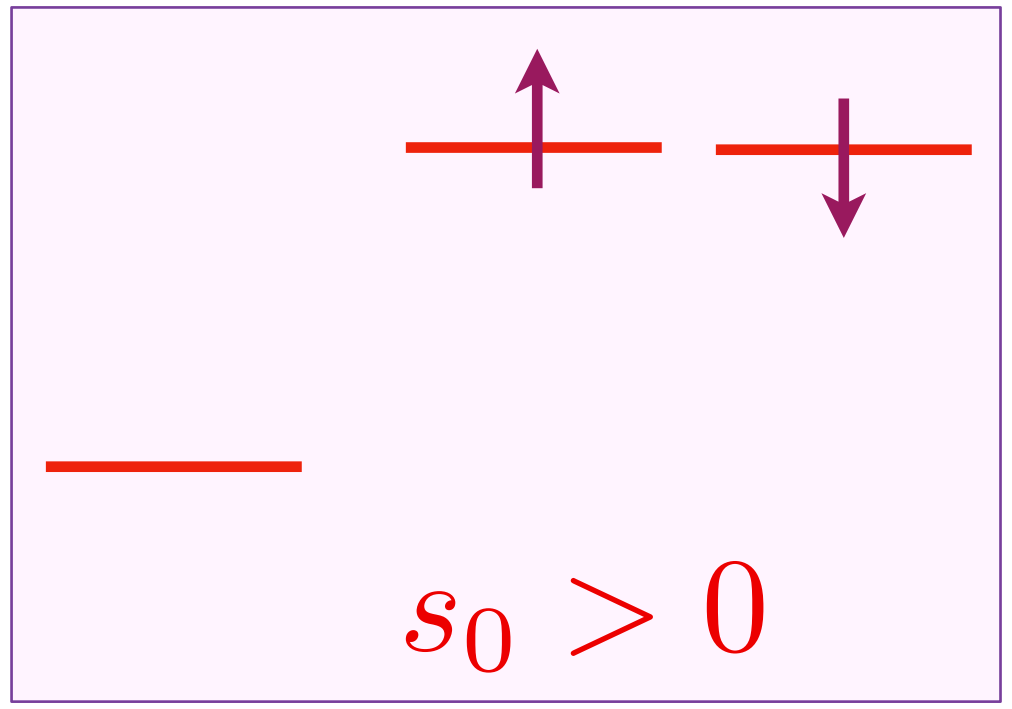


"Bose-Fermi Superspin Kondo!"

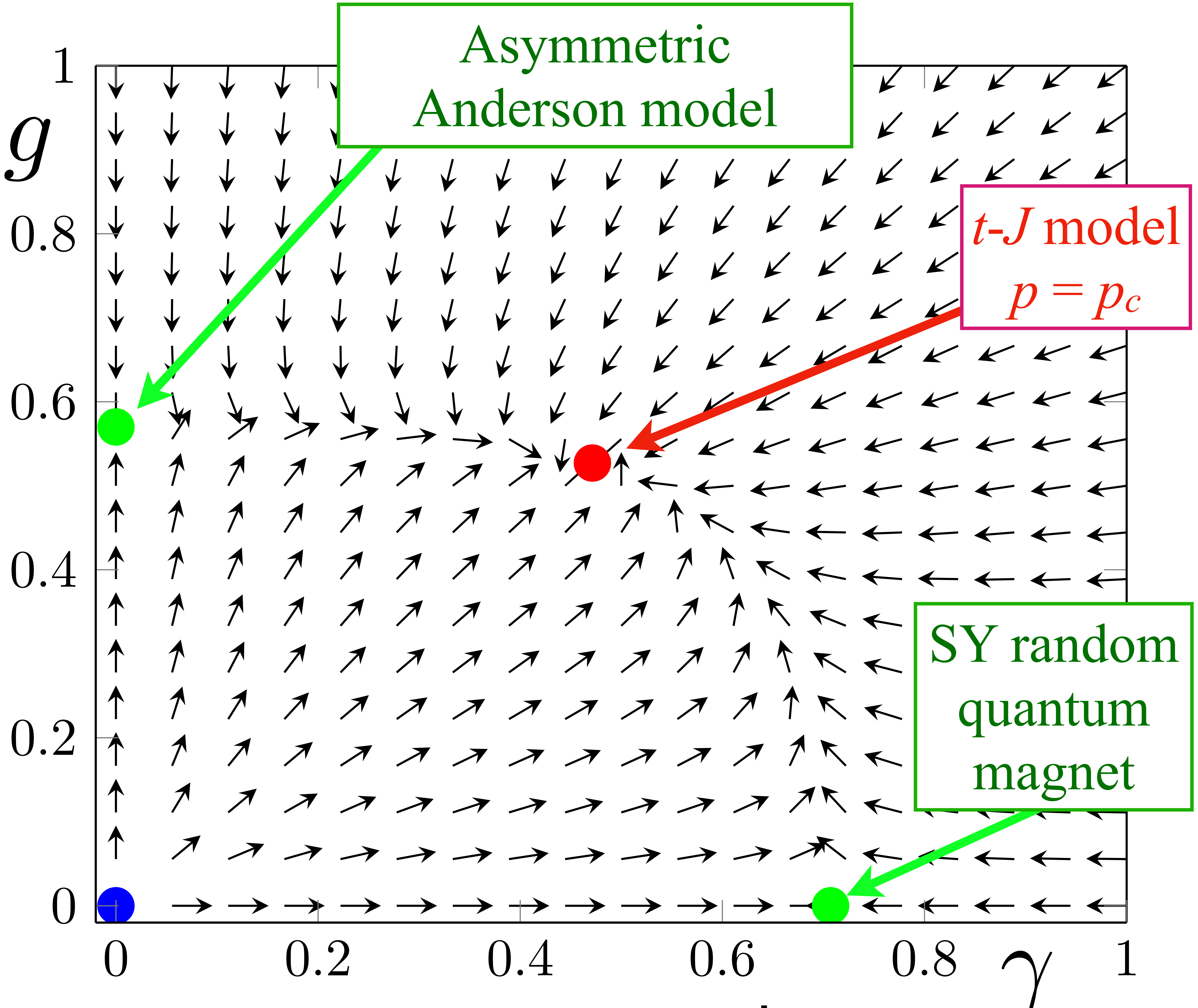
Then we can map the problem to that of a 'superspin' interaction with a bosonic bath (with coupling γ), and a fermionic bath (with coupling g). There is also a 'Zeeman' field $s_0(\mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha - \mathfrak{f}^\dagger \mathfrak{f})$ which splits the degeneracy between the holon and spinon states. Note $s_0 = 0$ for $p = 1/3$.

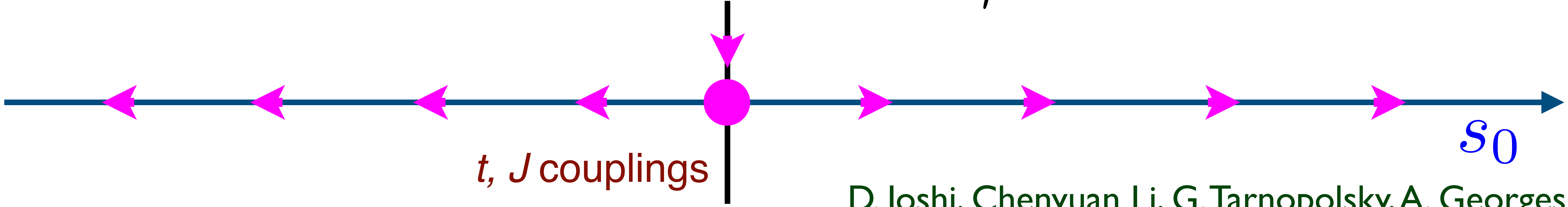






Random t-*J* model:RG





As in the random J model, the self-consistency condition leads to exponents that can be determined to all orders

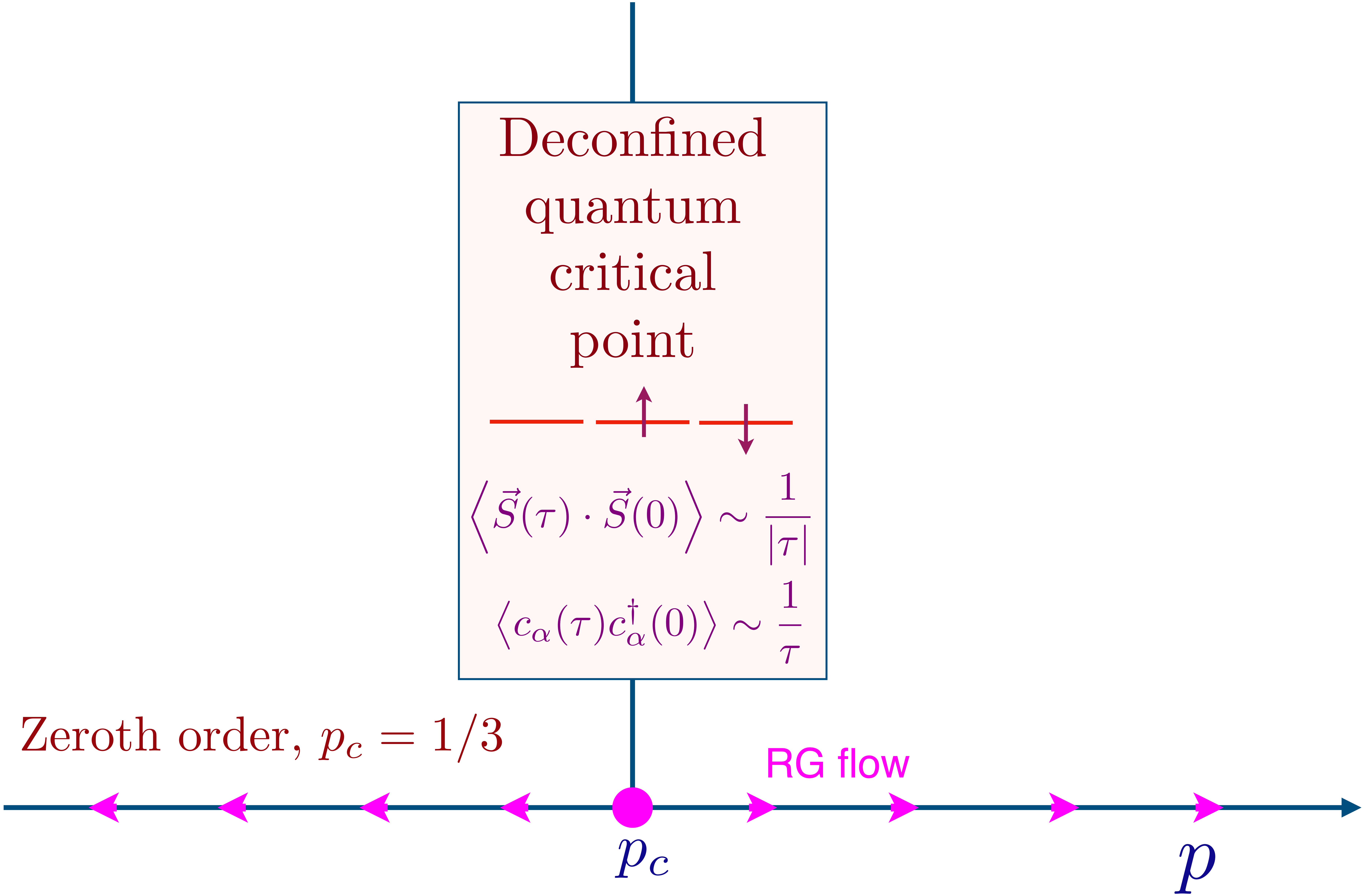
$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

These exponents can be understood from a large M SU(M) SYK theory with fractionalization: the partons have correlators $\sim 1/\sqrt{\tau}$, the spin and electron Green's functions are products of 2 parton correlators.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model: phase diagram



Deconfined quantum critical point

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

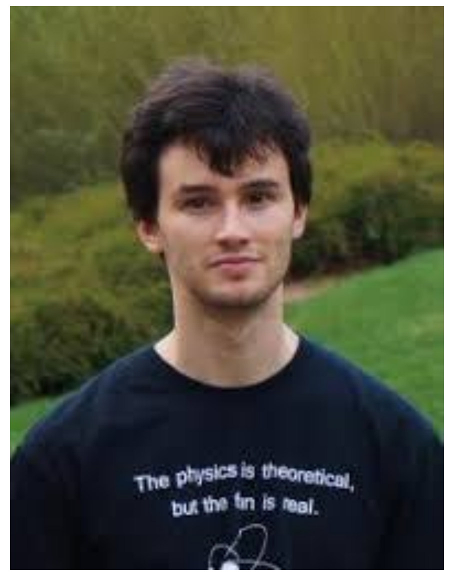
$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$

RG flow

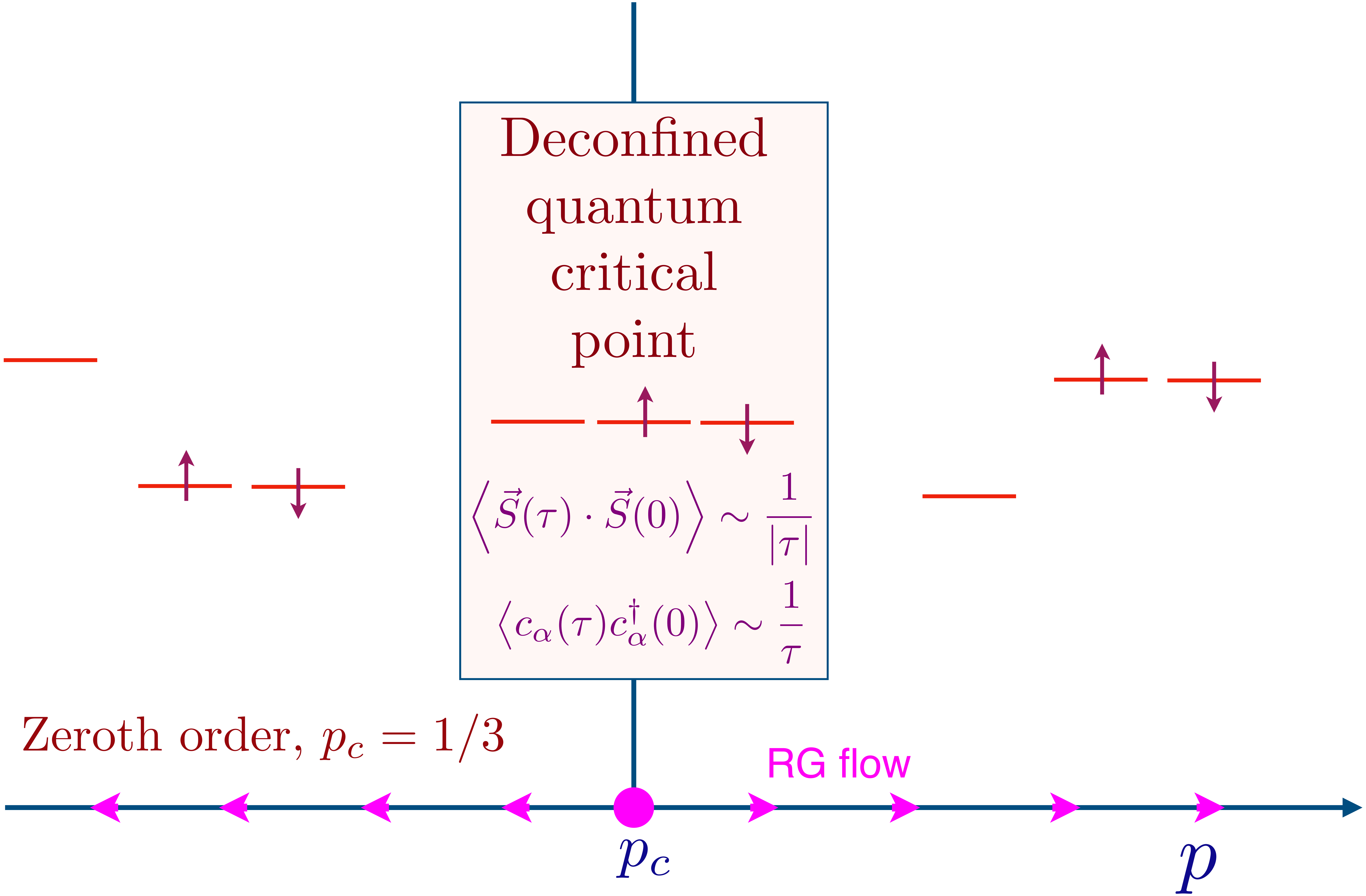
p_c

p



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model: phase diagram

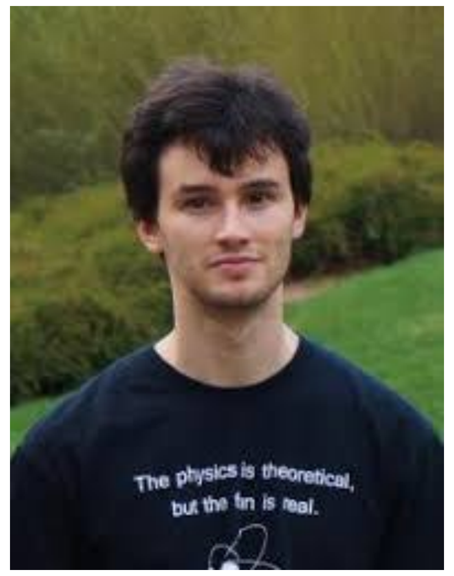


Deconfined quantum critical point

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$



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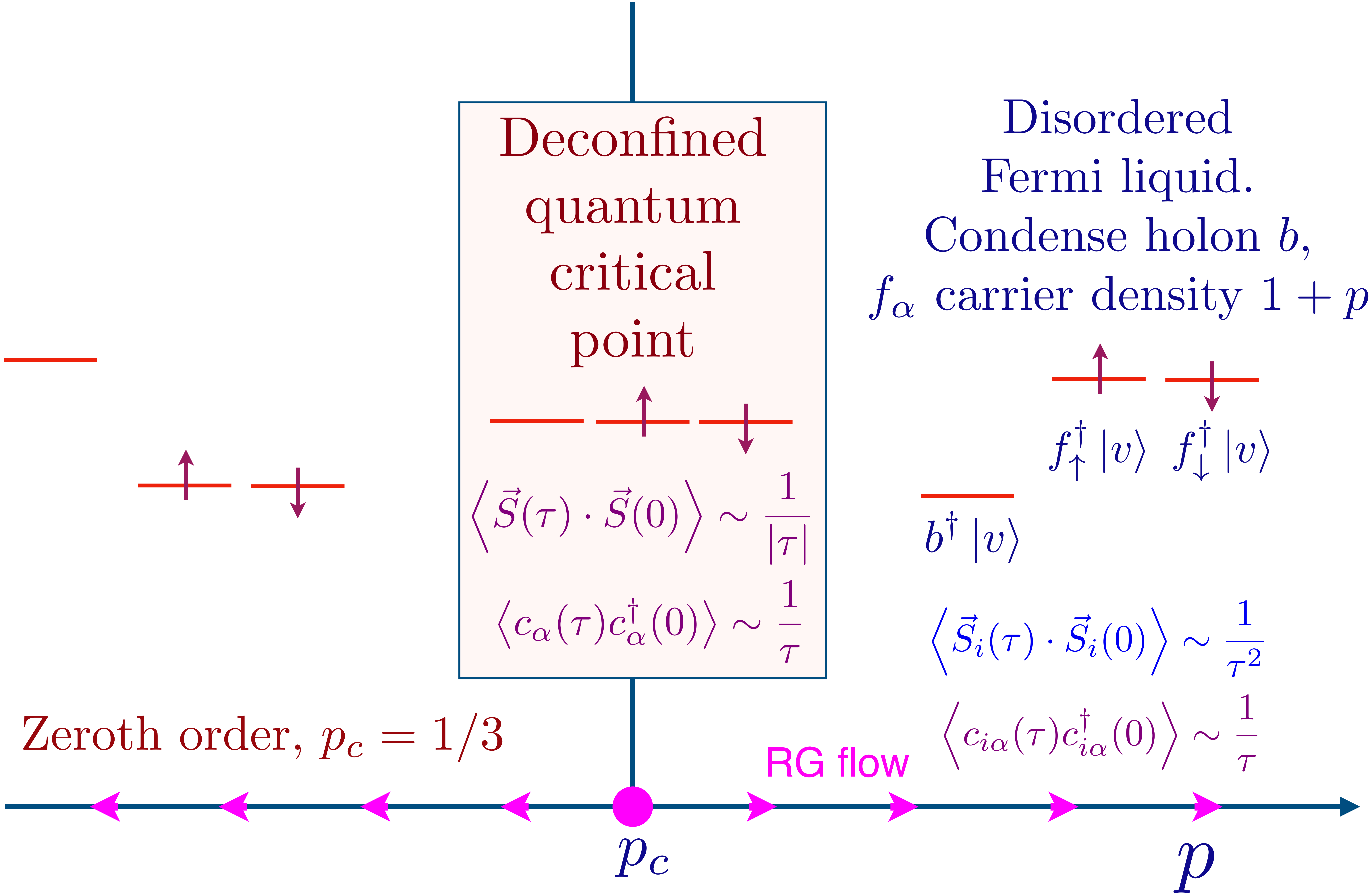
Random t-J model: phase diagram



Deconfined quantum critical point

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Disordered Fermi liquid. Condense holon b, f_α carrier density $1 + p$

$f_\uparrow^\dagger \left|v\right\rangle$ $f_\downarrow^\dagger \left|v\right\rangle$

$b^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

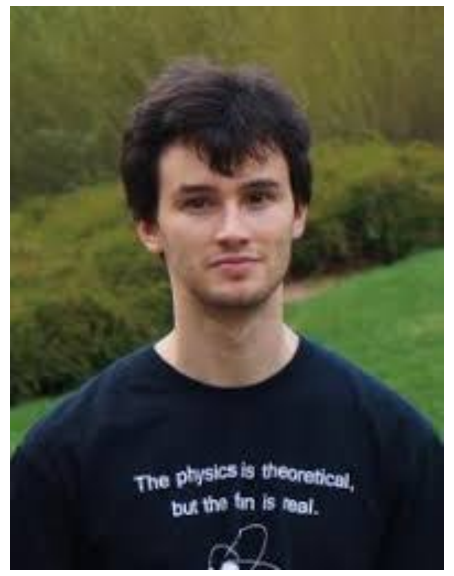
$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Zeroth order, $p_c = 1/3$

RG flow

p_c

p



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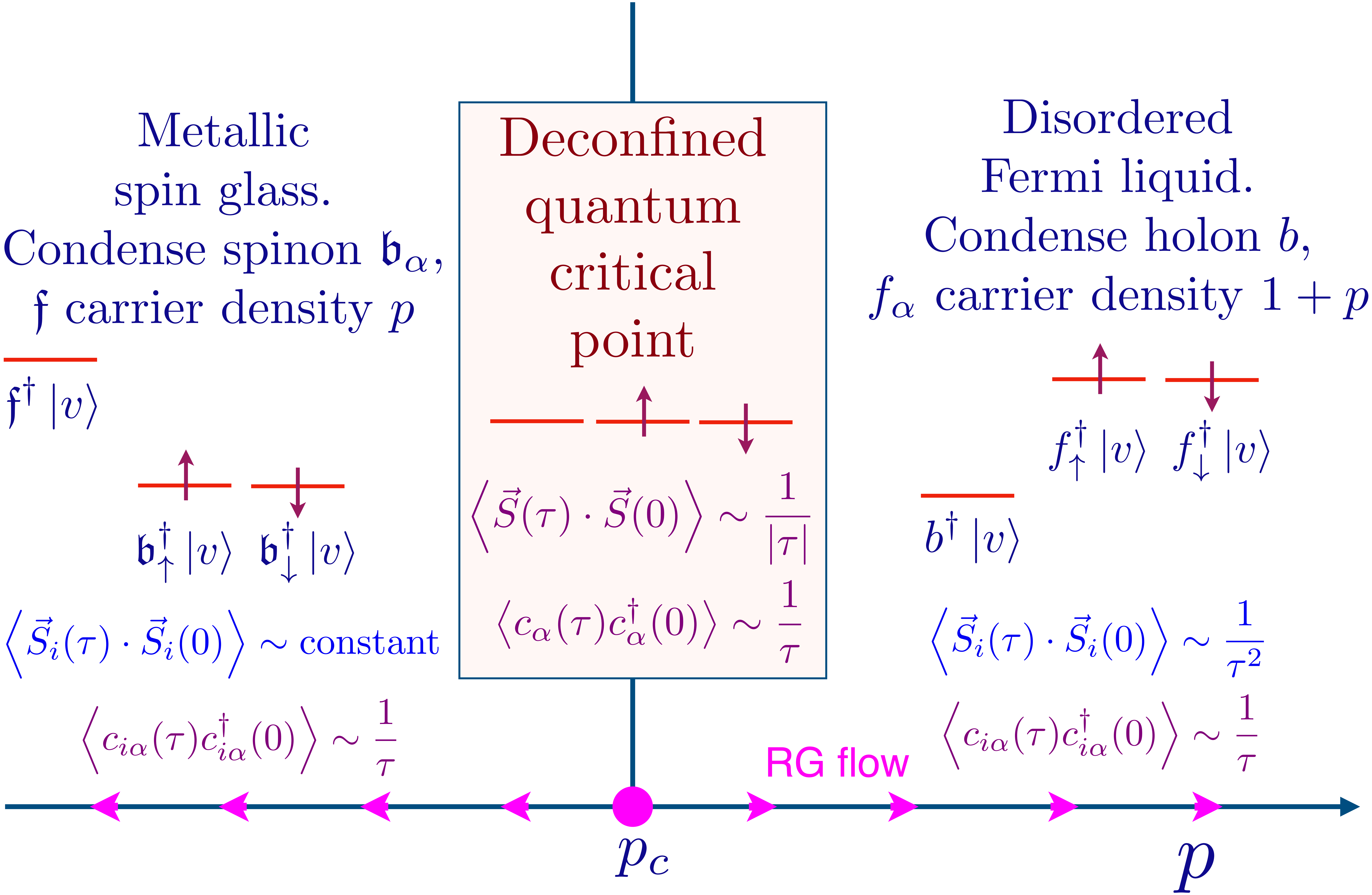
Random t-J model: phase diagram



Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger \left|v\right\rangle$

$\mathfrak{b}_\uparrow^\dagger \left|v\right\rangle$ $\mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Deconfined quantum critical point

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f_\uparrow^\dagger \left|v\right\rangle$ $f_\downarrow^\dagger \left|v\right\rangle$

$b^\dagger \left|v\right\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

RG flow

p_c

p

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Summary

Dynamic mean field theory of metal-metal transitions

Numerical solutions of Hubbard model capture key aspects of the cuprate phase diagram:
the FL* pseudogap metal,
spin glass order at low T for small p,
and Planckian metal near p_c

RG analysis yields connections to SYK criticality of fractionalized partons