A photograph of a large, multi-story building with a red roof and white walls, illuminated at night. The building is situated on a hillside overlooking a body of water. The lights from the building and surrounding area are reflected in the water. In the background, a white tower with a green dome is visible against the dark sky.

Stringing together the quantum phases of matter

Talk online: sachdev.physics.harvard.edu



The phases of matter:

The phases of matter:

ice



water



steam

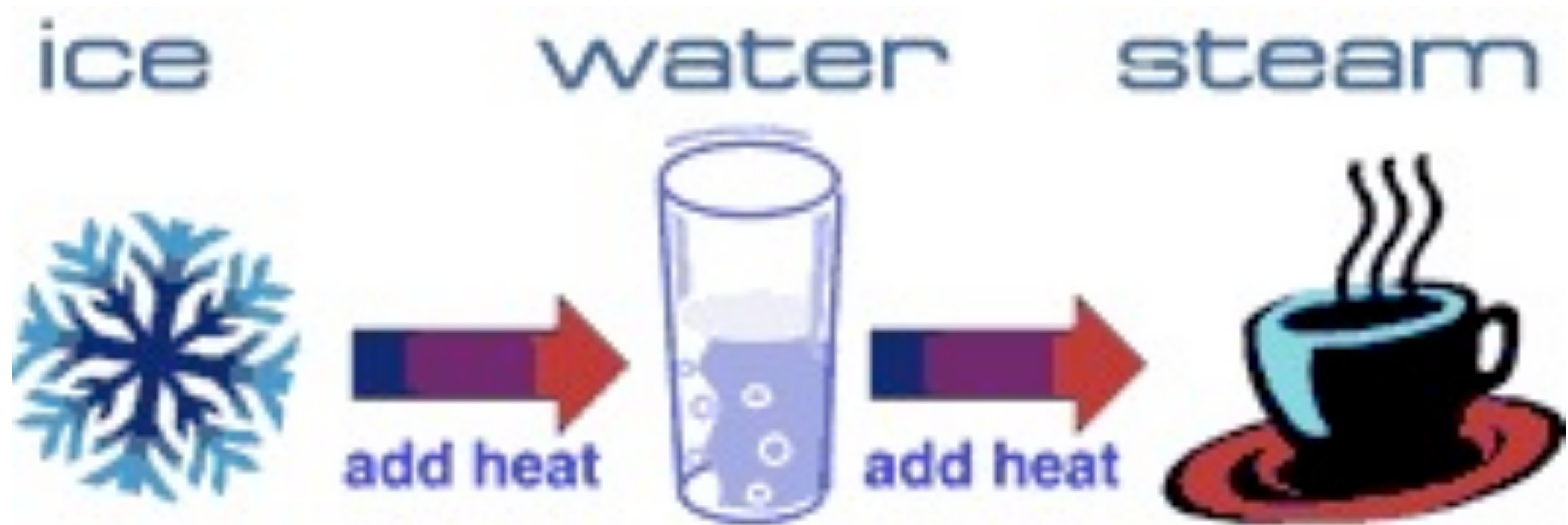


Solids

Liquids

Gases

The phases of matter:



Solids

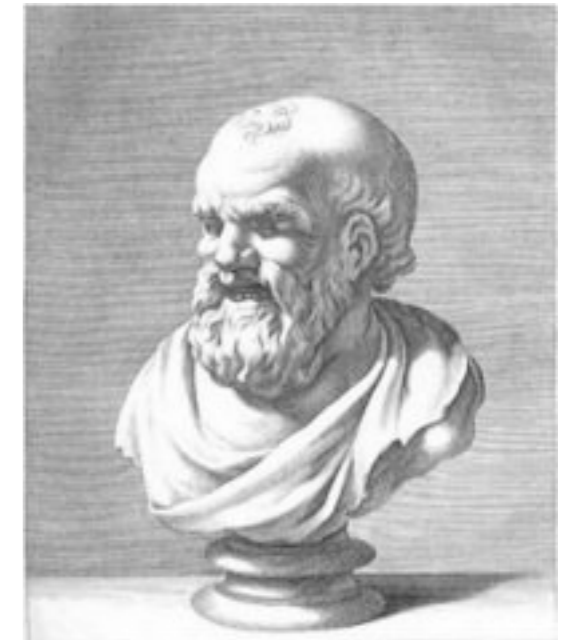
Liquids

Gases

Theory of the phases of matter:

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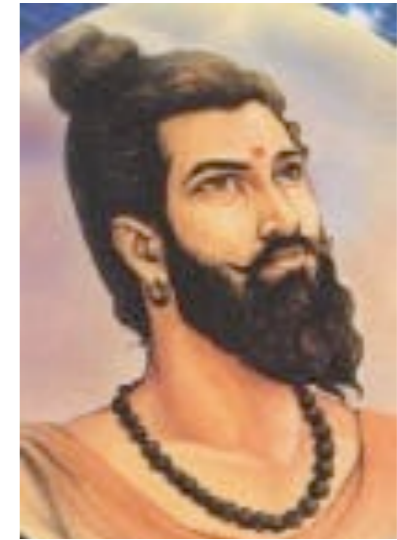
1. Matter is made of atoms



Democritus (4th century B.C.)

Theory of the phases of matter:

1. Matter is made of atoms

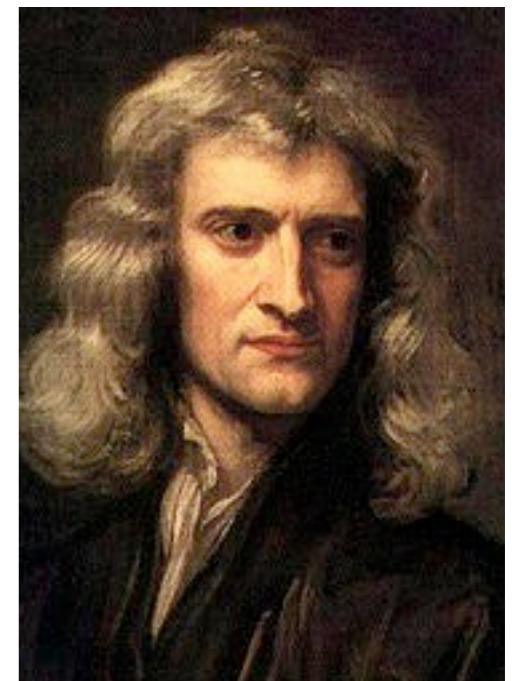


Acharya Kanad (6th century B.C.)

Theory of the phases of matter:

1. Matter is made of atoms

2. The atoms move because of forces acting between them, just like the moon or an apple



Newton (1687)

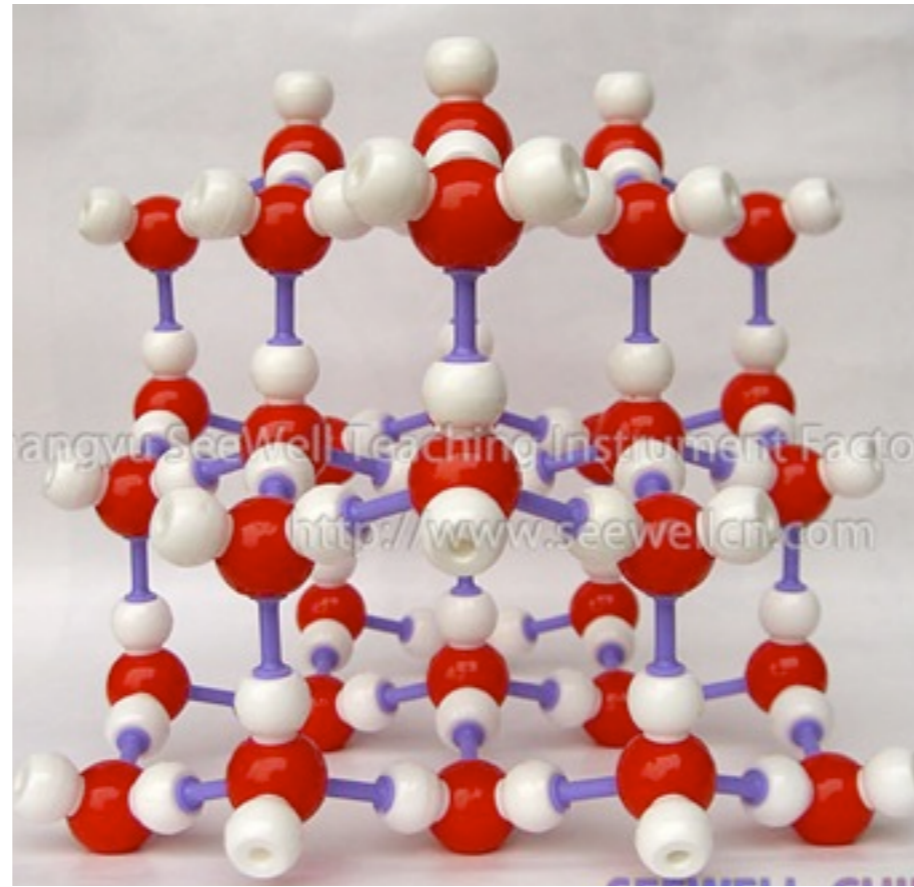
Theory of the phases of matter:

1. Matter is made of atoms
2. The atoms move because of forces acting between them, just like the moon or an apple
3. The phases of matter are determined by the spatial arrangements of atoms



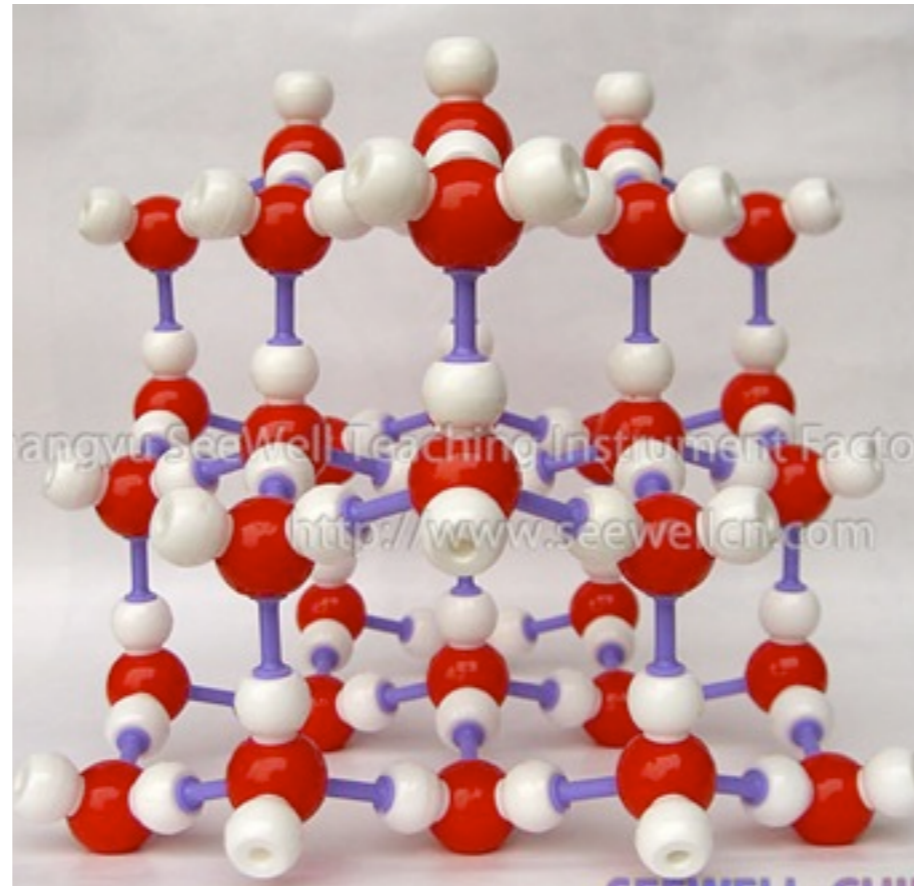
Boltzmann (1877)

Solids



Ice

Solids



Ice

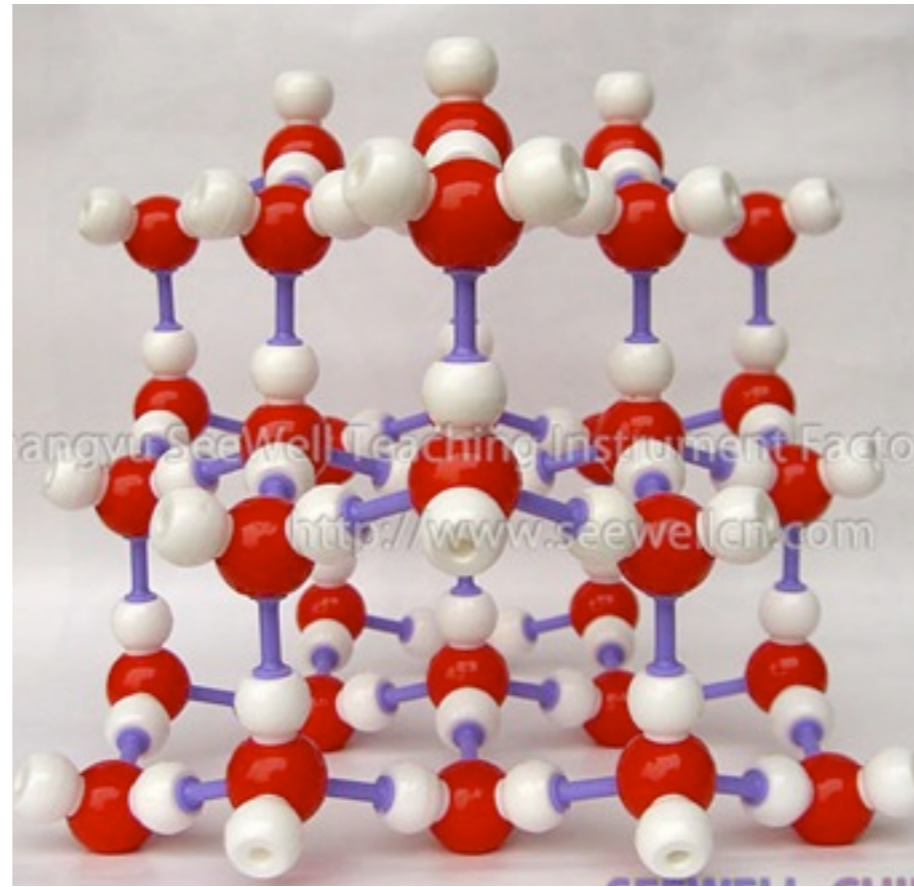


Salt



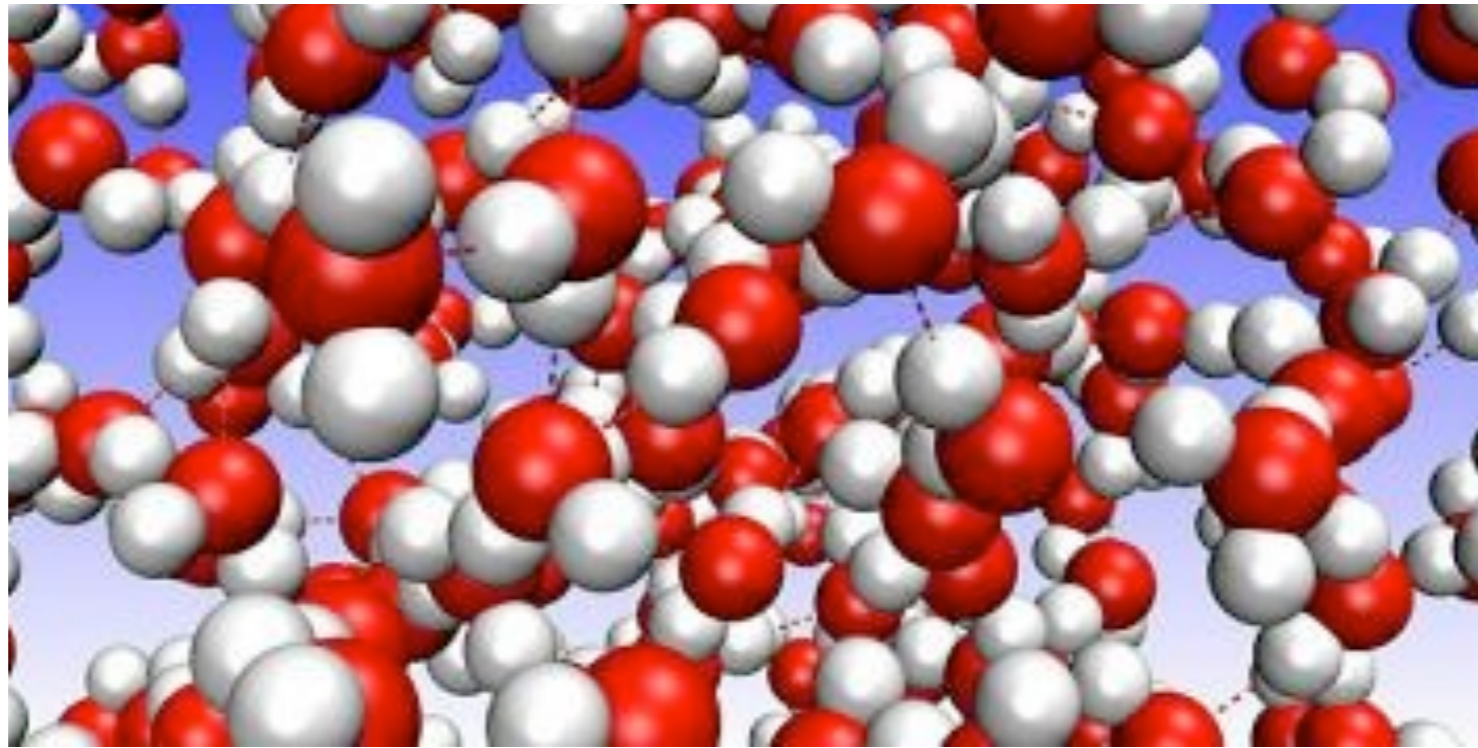
Silicon

Solids



Ice

Liquids



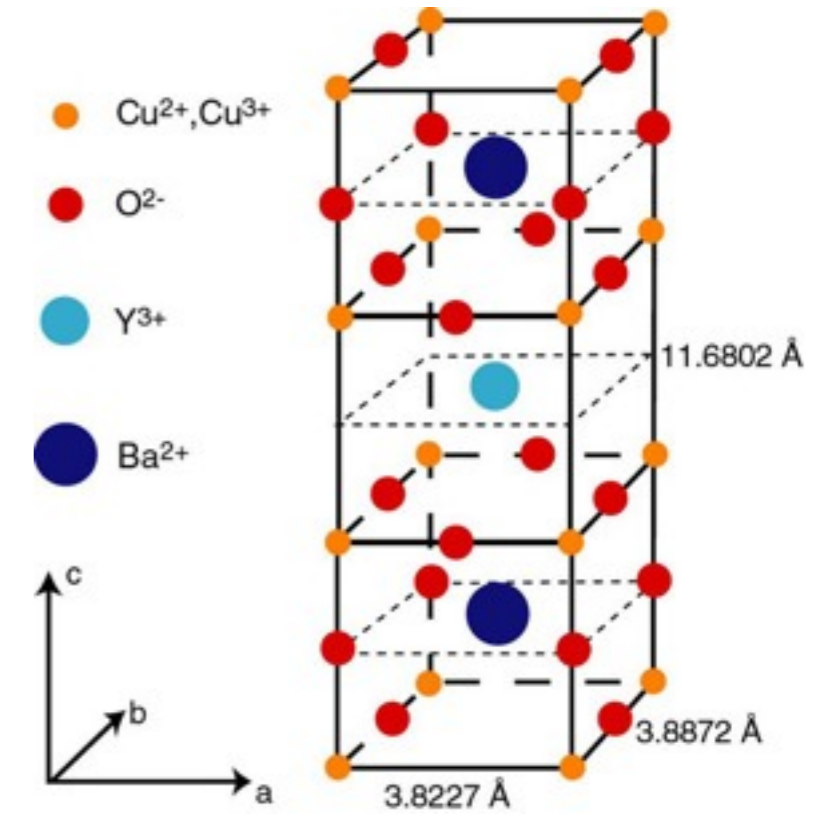
Water

Gases



Steam

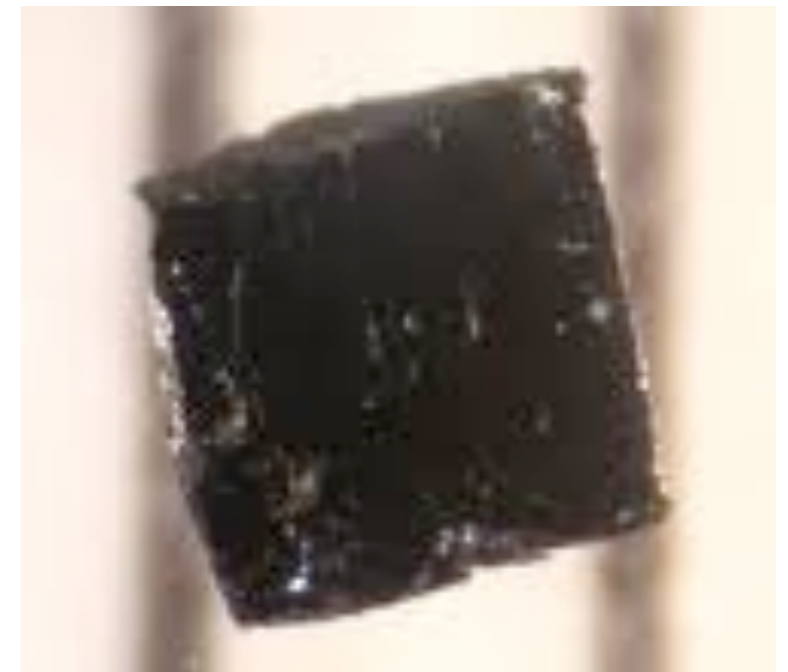
Solids



Copper

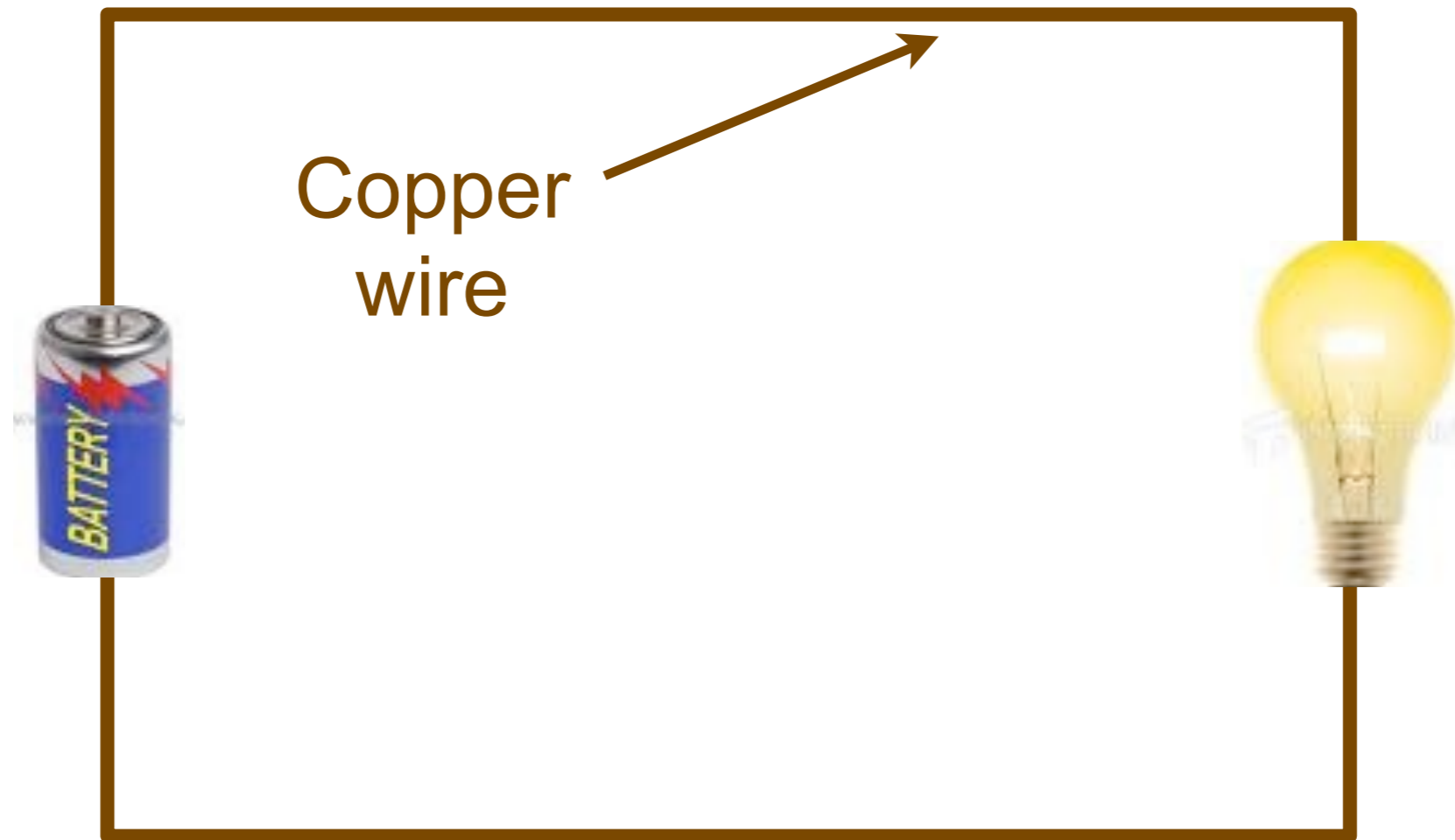


Silicon



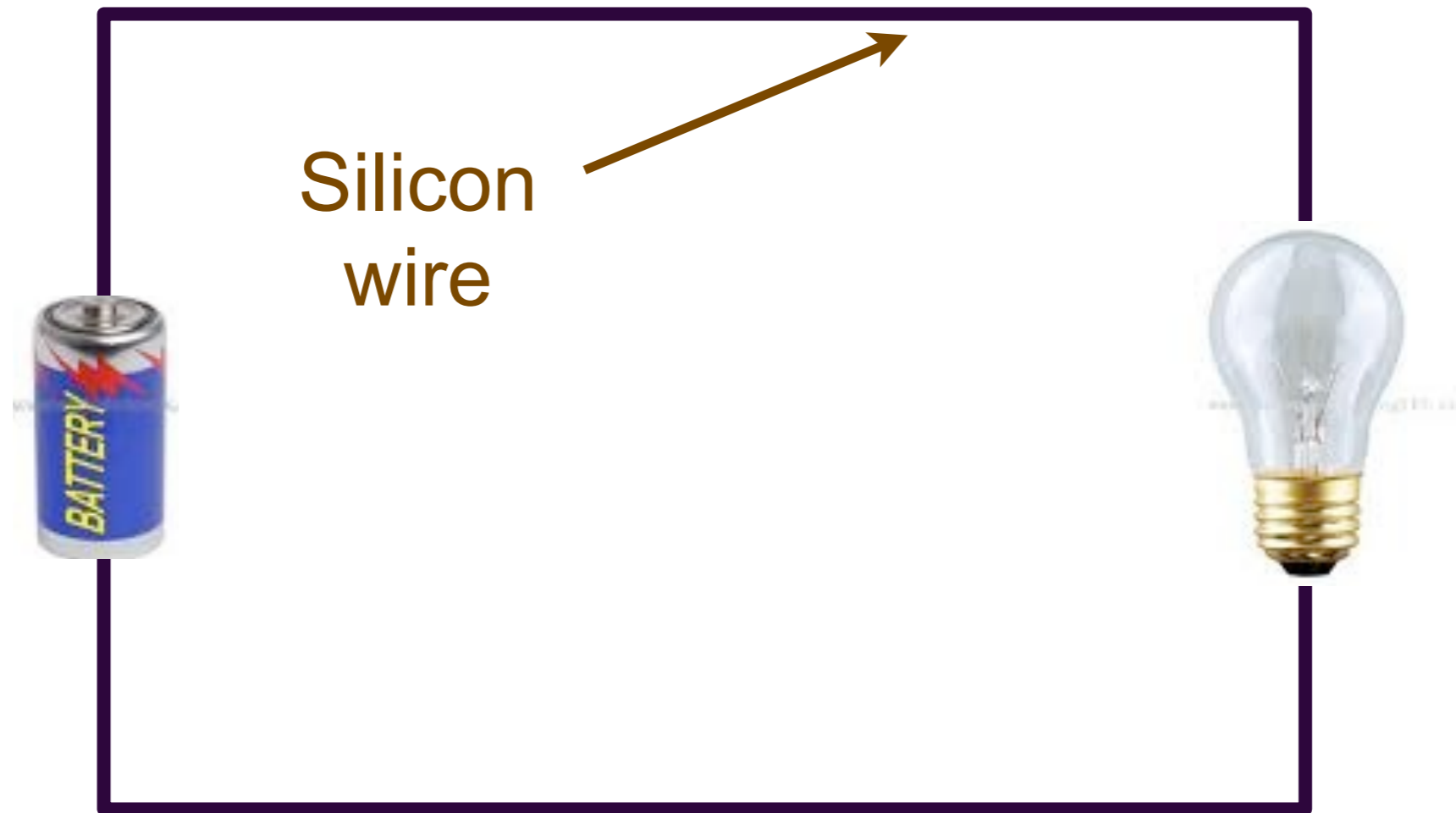
YBCO

These solids have very different electrical and magnetic properties



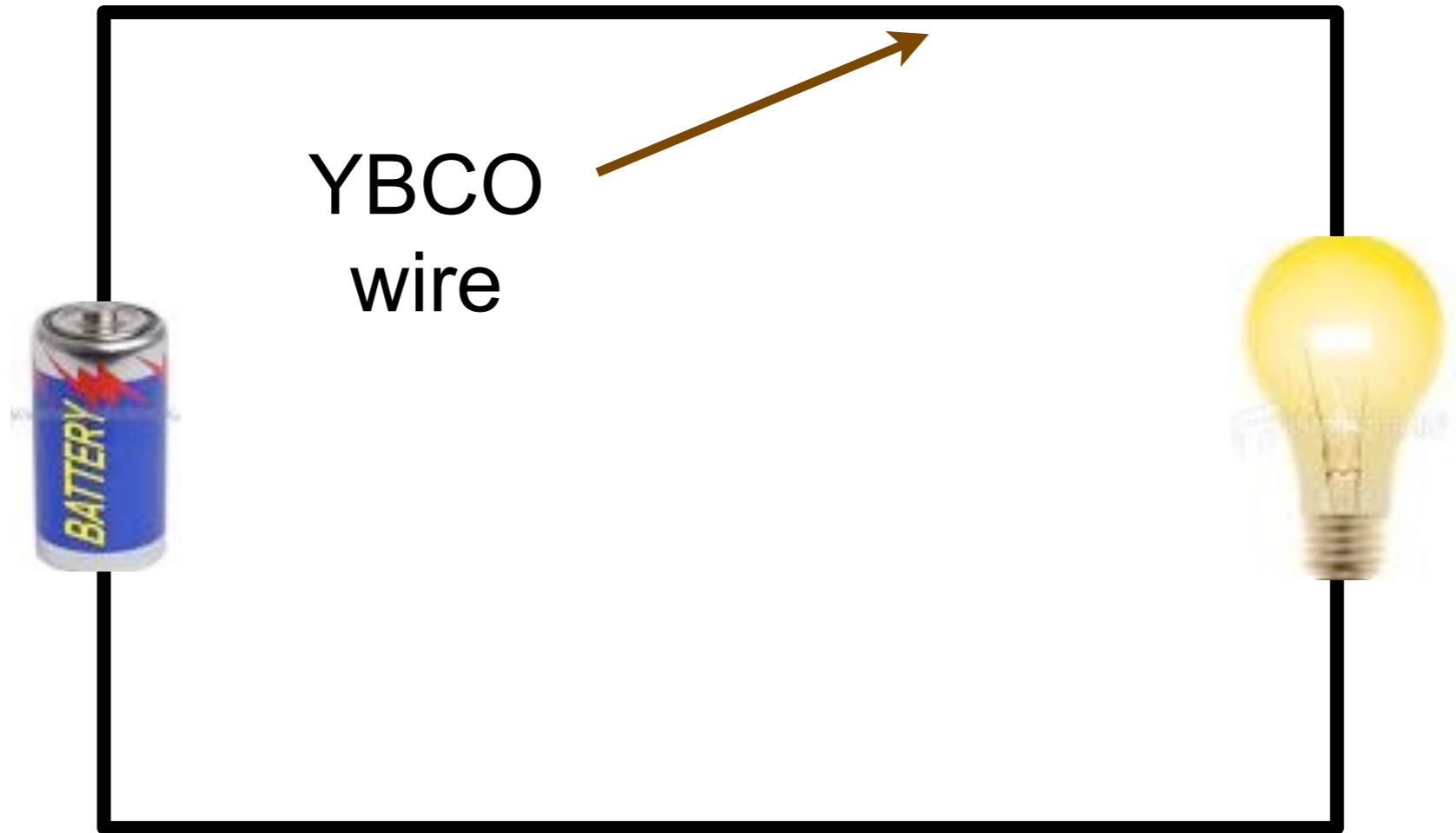
Copper is a conductor of electricity

These solids have very different electrical and magnetic properties



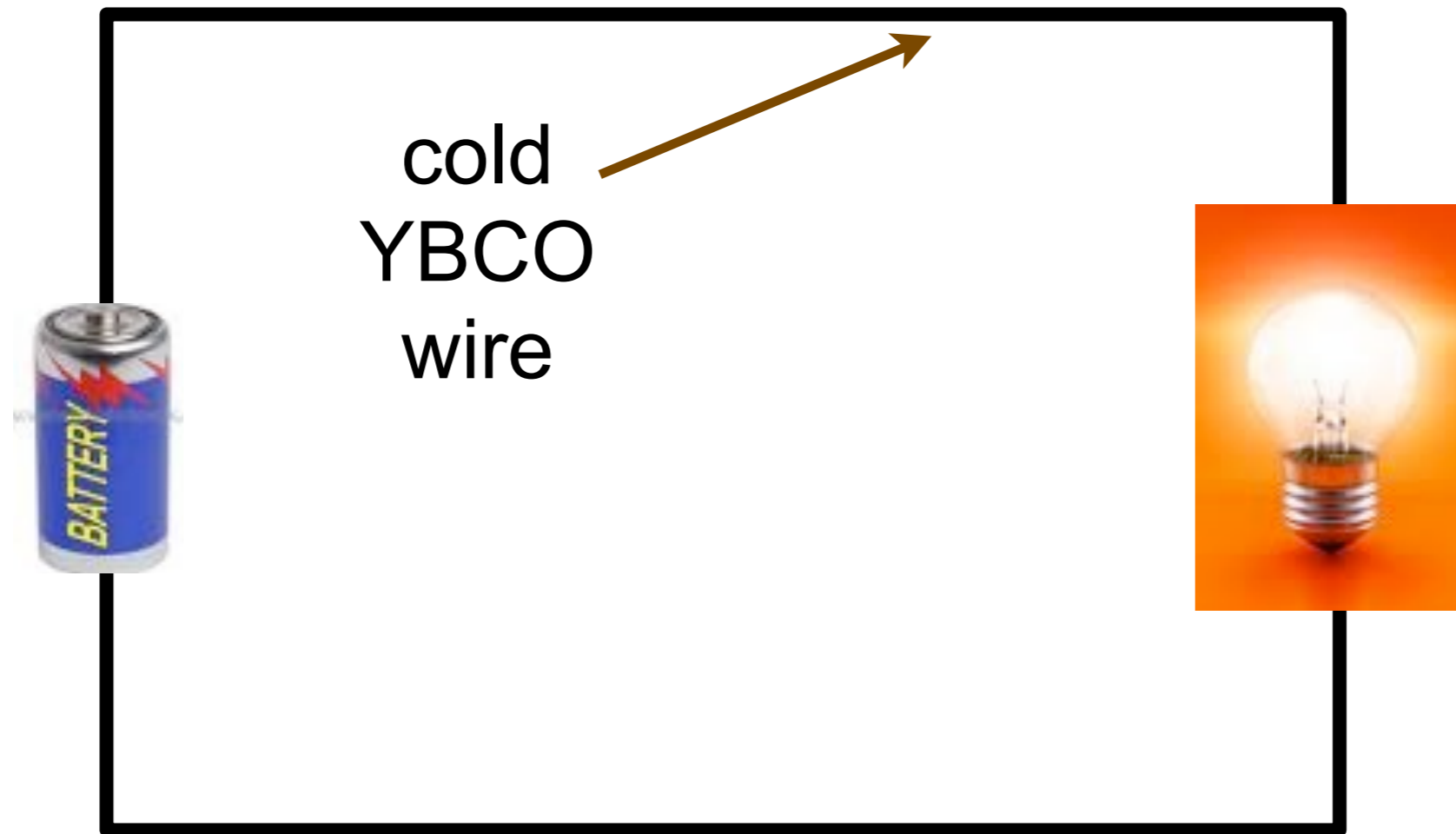
Silicon is an insulator

These solids have very different electrical and magnetic properties



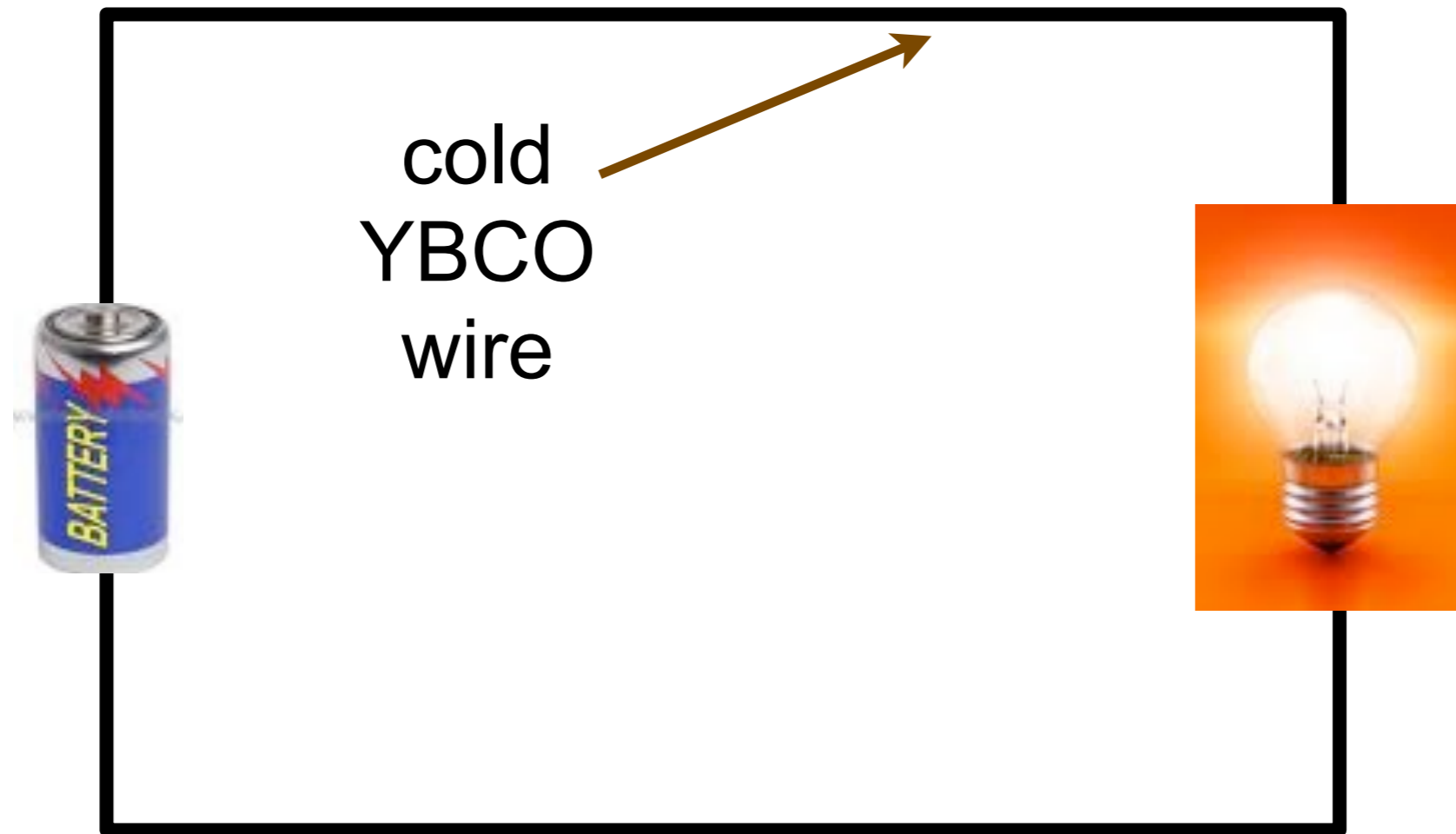
At room temperature, YBCO conducts electricity (but not very well)

These solids have very different electrical and magnetic properties



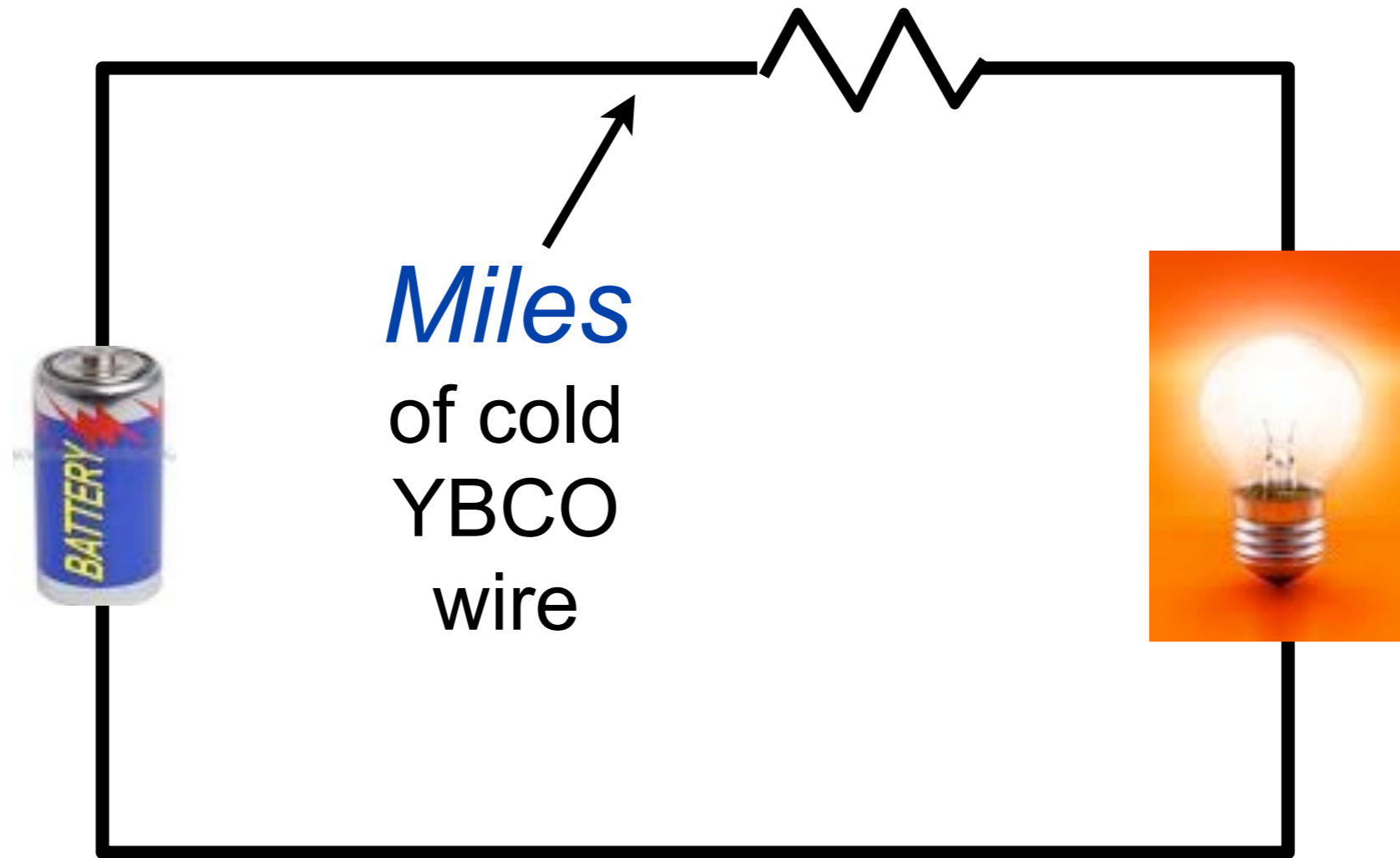
When cooled by liquid nitrogen, YBCO conducts electricity without resistance

These solids have very different electrical and magnetic properties



When cooled by liquid nitrogen,
YBCO is a **SUPERCONDUCTOR** !

These solids have very different electrical and magnetic properties

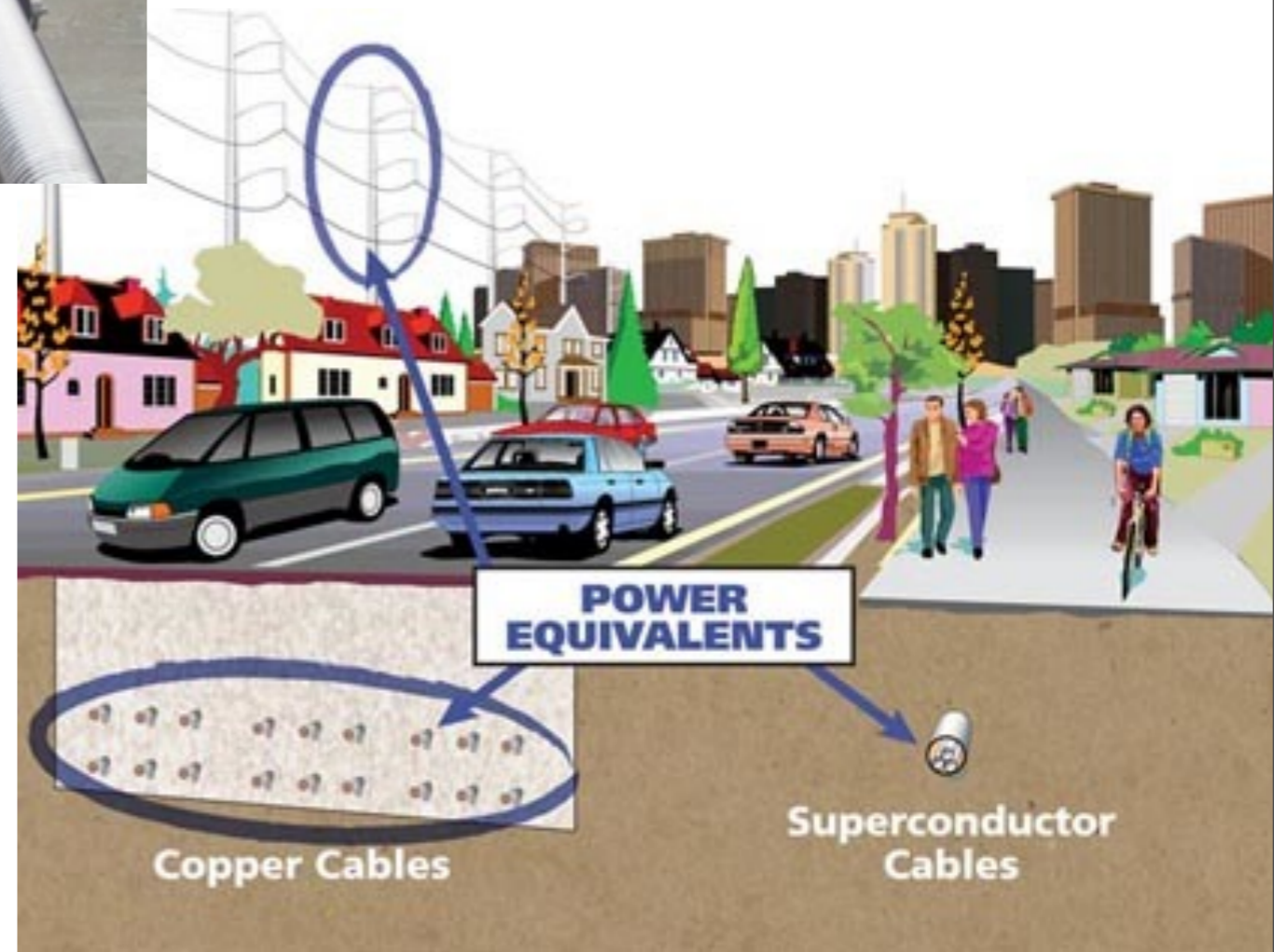


When cooled by liquid nitrogen,
YBCO is a **SUPERCONDUCTOR** !



YBCO cables

**American
Superconductor
Corporation**

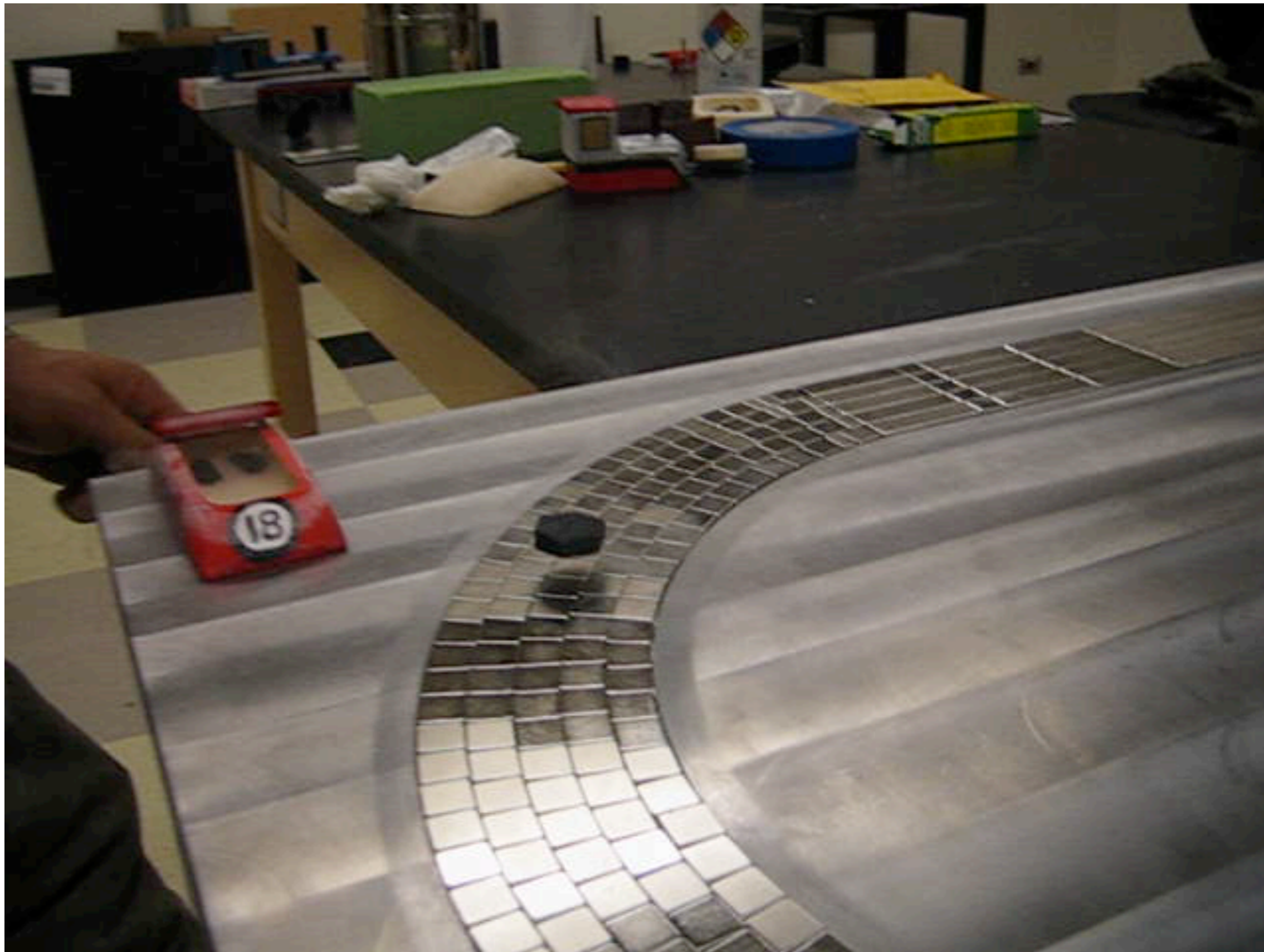


YBCO cables

American Superconductor Corporation

Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

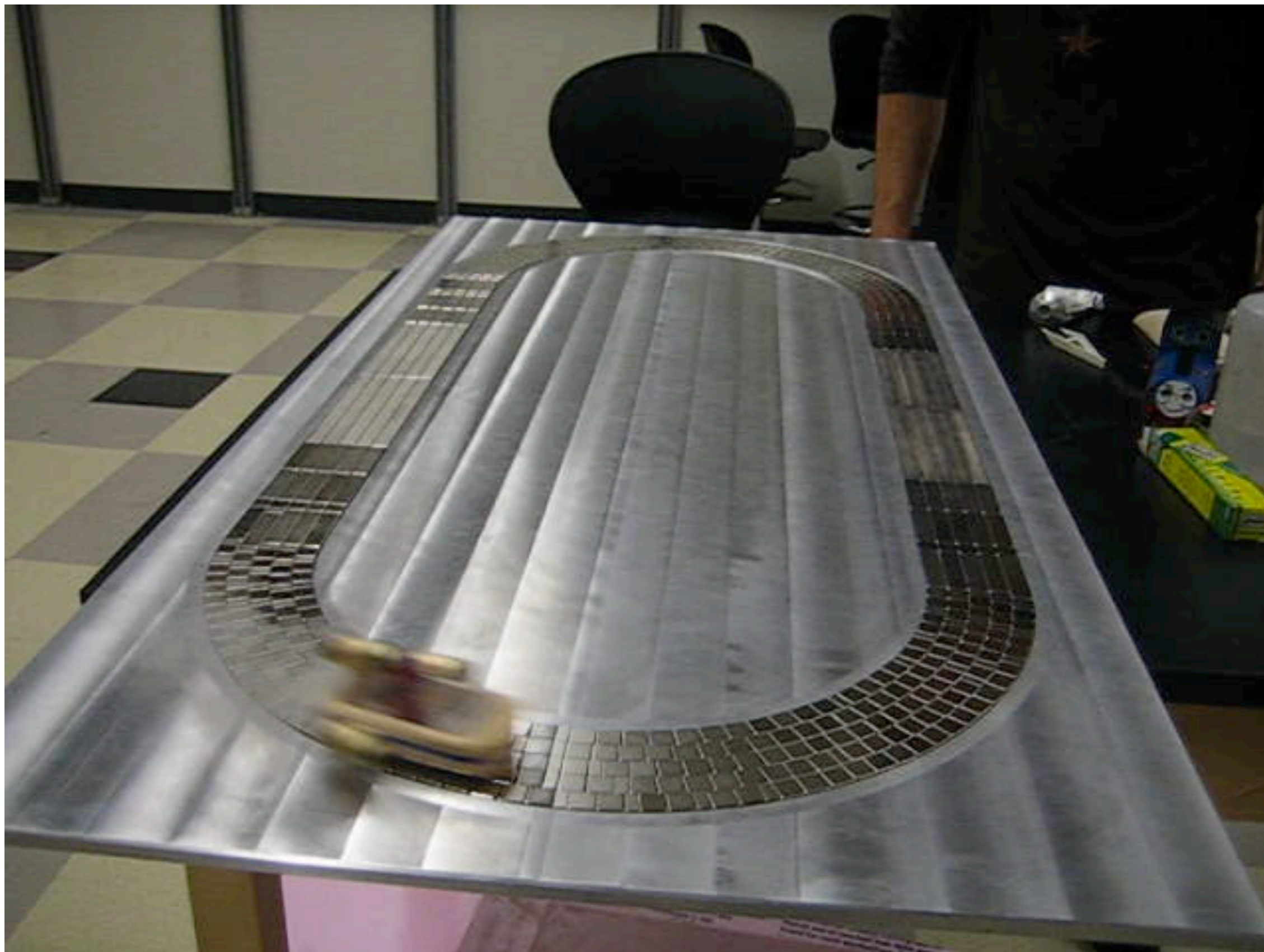


Nd-Fe-B magnets, YBaCuO superconductor

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Nd-Fe-B magnets, YBaCuO superconductor

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Nd-Fe-B magnets, YBaCuO superconductor

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Theory of the electrical phases of matter:

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Theory of the electrical phases of matter:

1. In solids, electrons separate

Needed:

A theory for the
quantum phases of
matter

2.

3. and Schroedinger determines the electrical properties of solids at macroscopic scales

**Quantum
superposition and
entanglement**

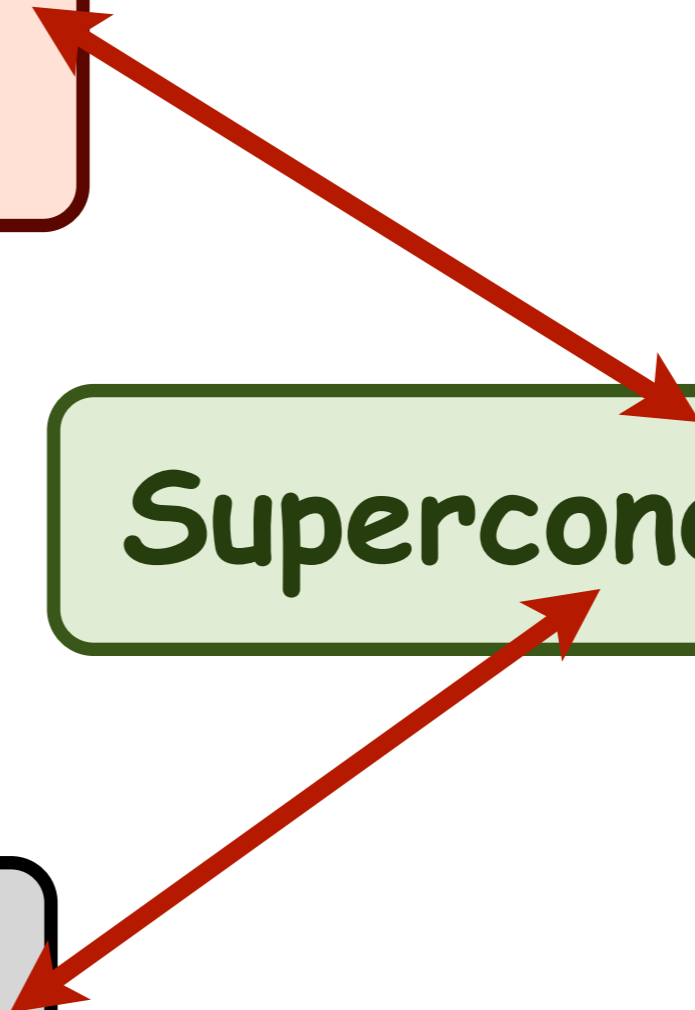
Superconductivity

**Black Holes and
String Theory**

**Quantum
superposition and
entanglement**

Superconductivity

**Black Holes and
String Theory**



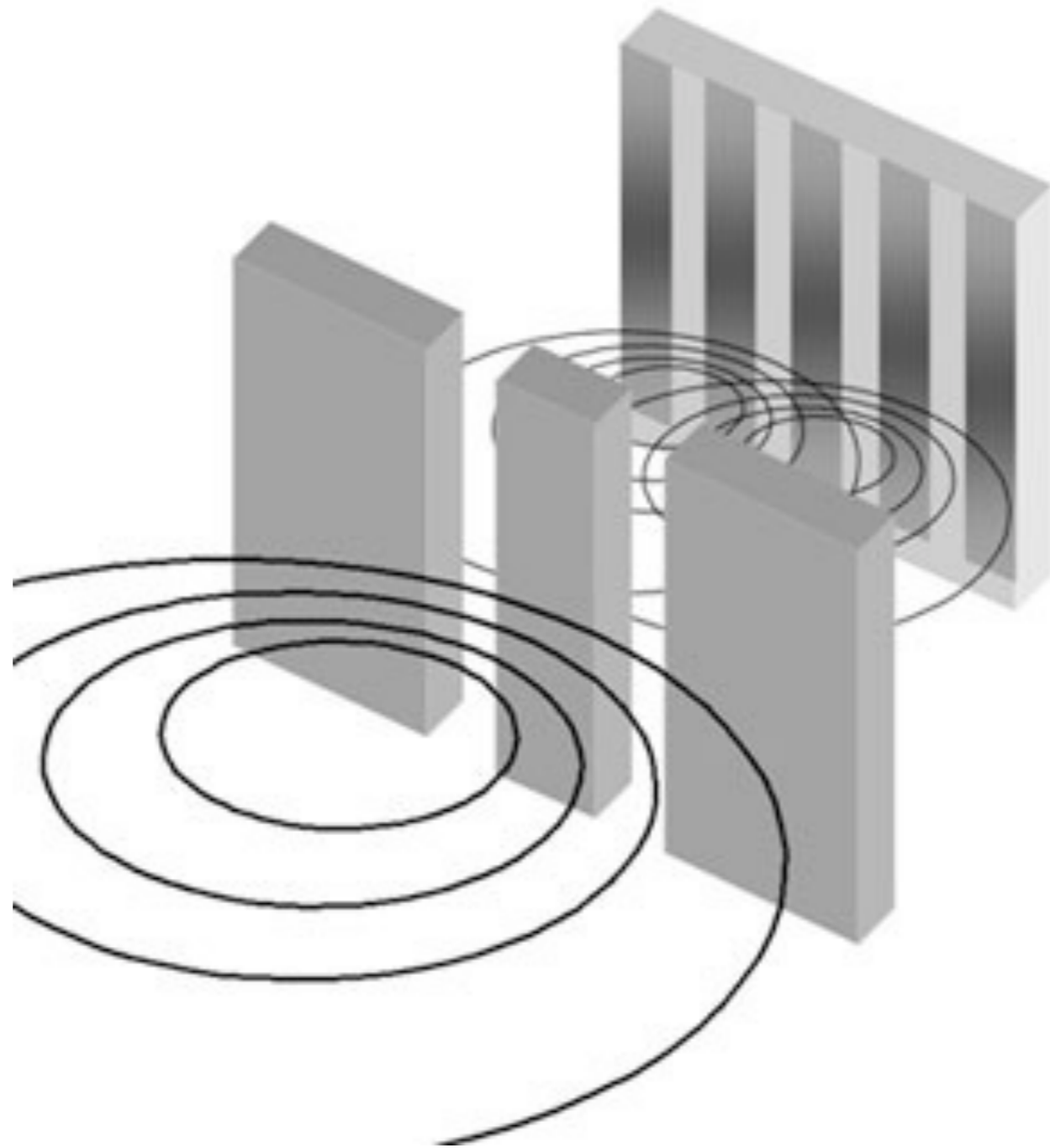
**Quantum
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Quantum Superposition

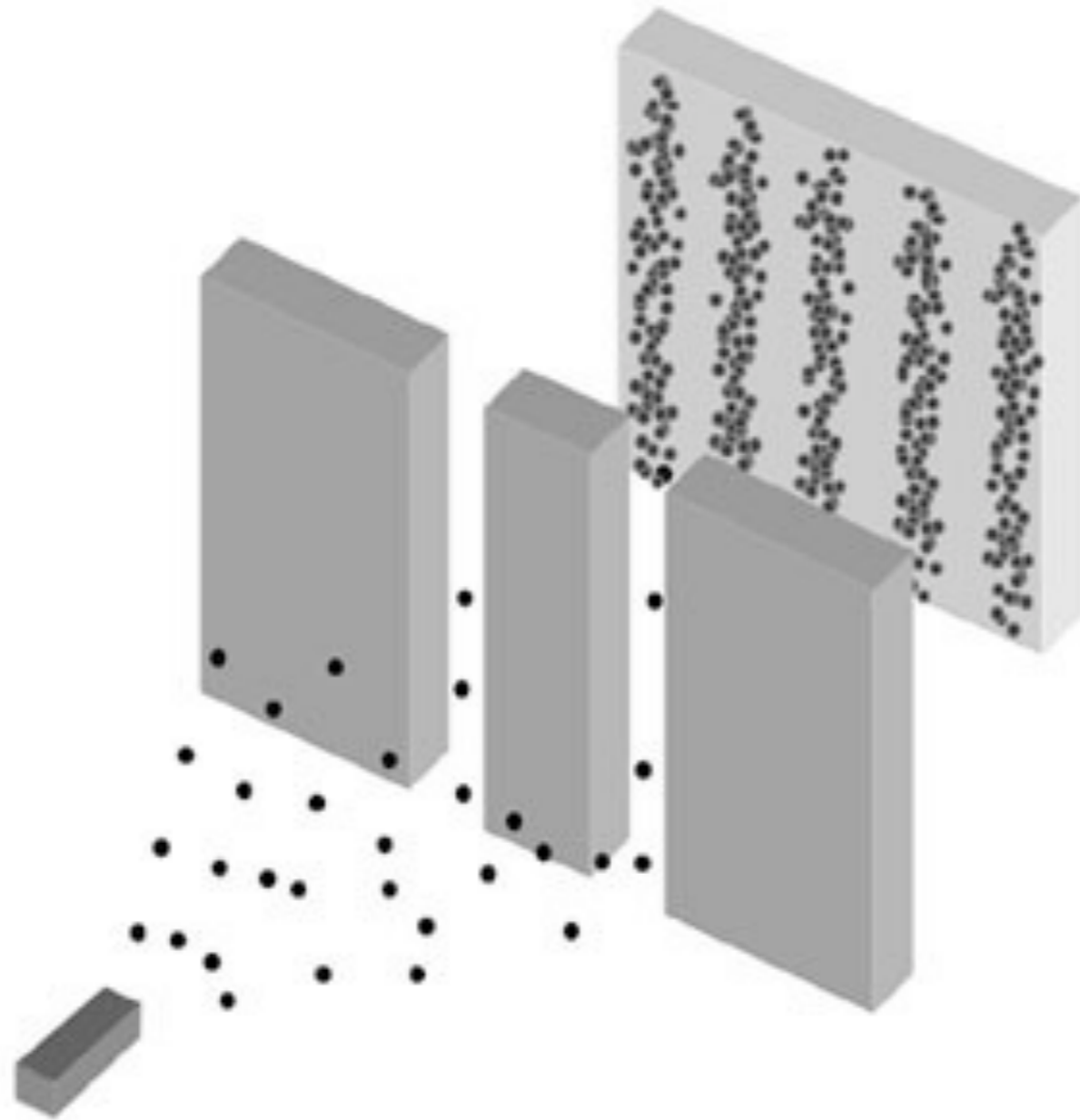
The double slit experiment



Interference of water waves

Quantum Superposition

The double slit experiment

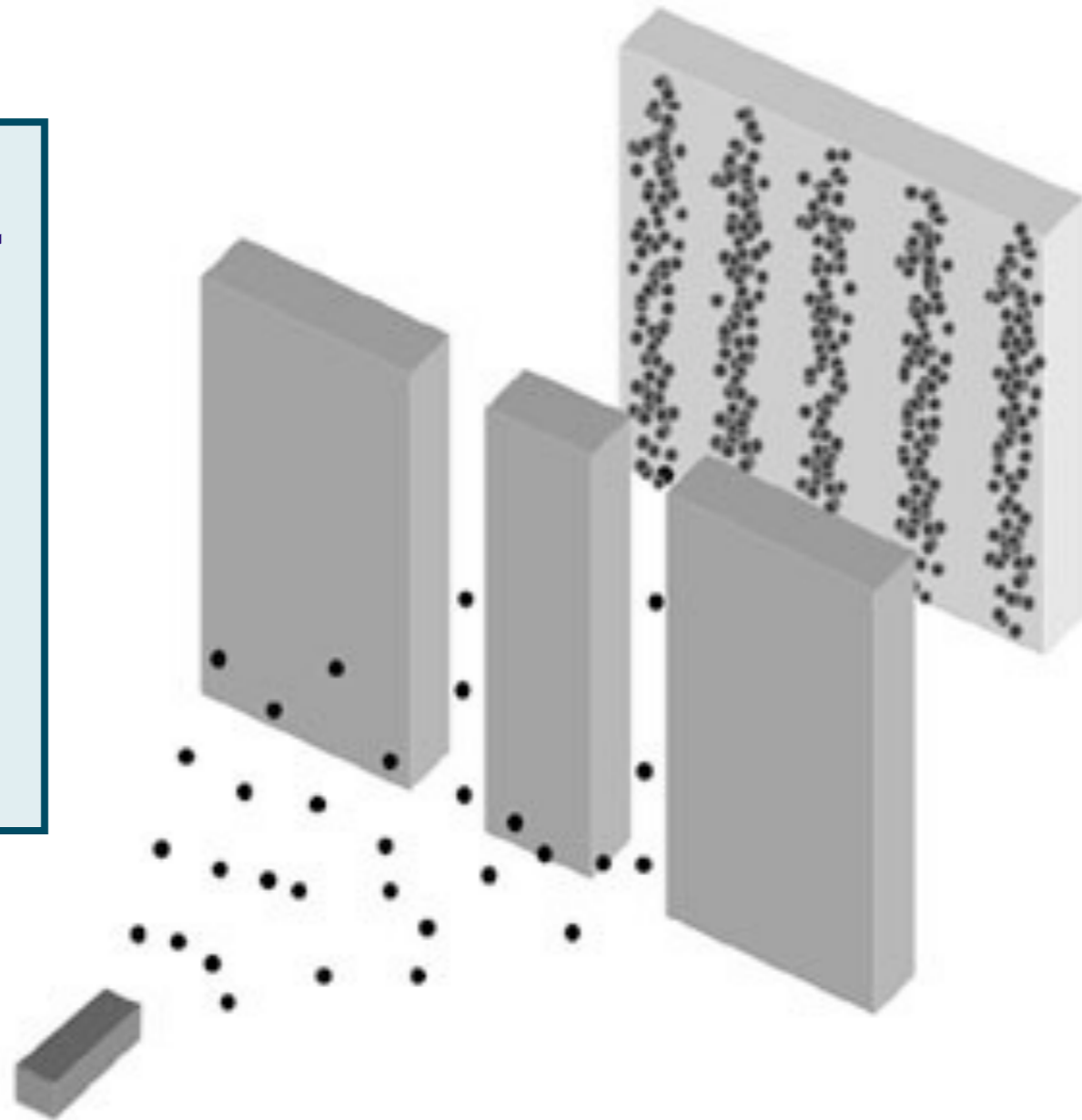


Interference of electrons

Quantum Superposition

The double slit experiment

Which slit
does an
electron
pass
through ?

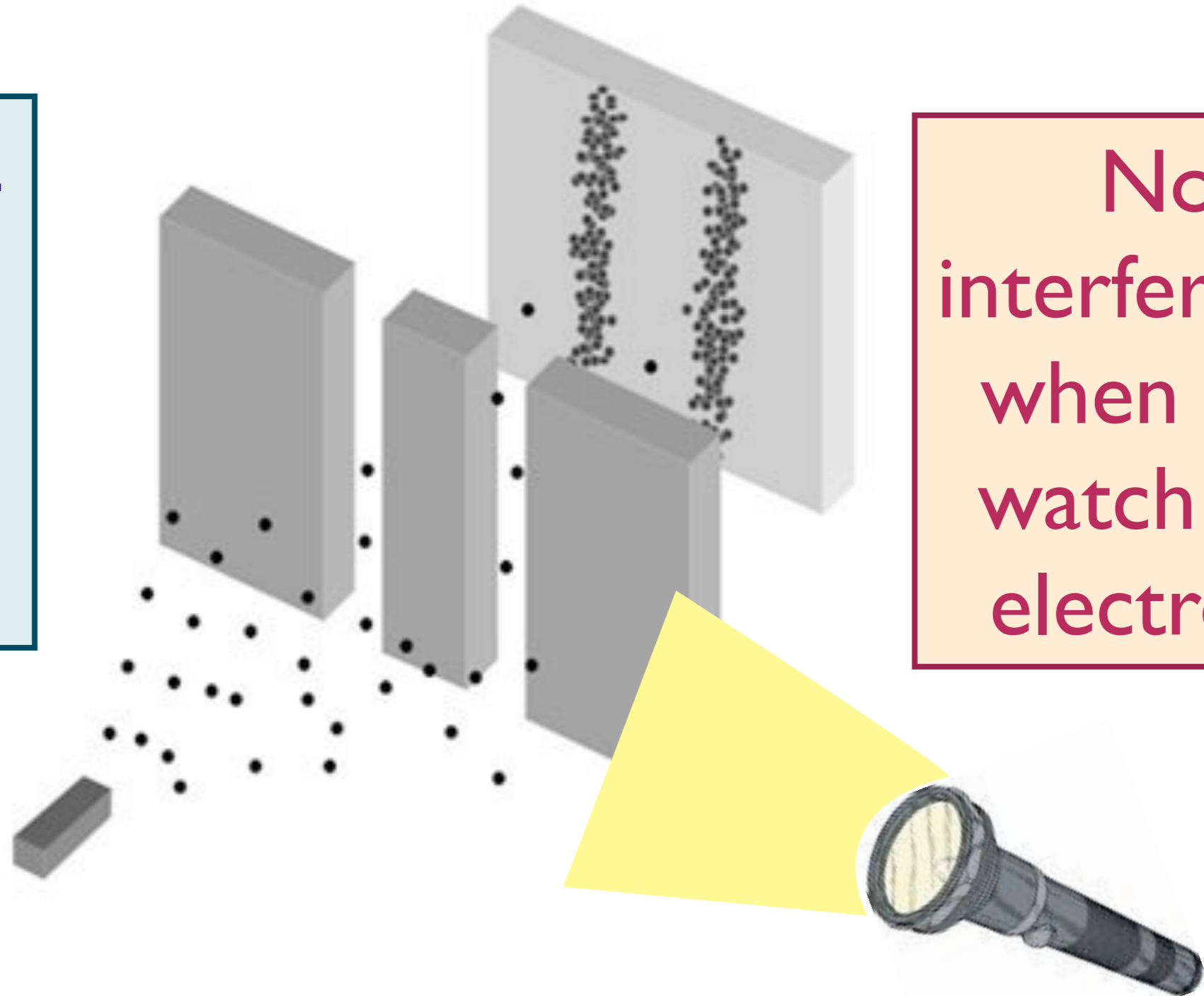


Interference of electrons

Quantum Superposition

The double slit experiment

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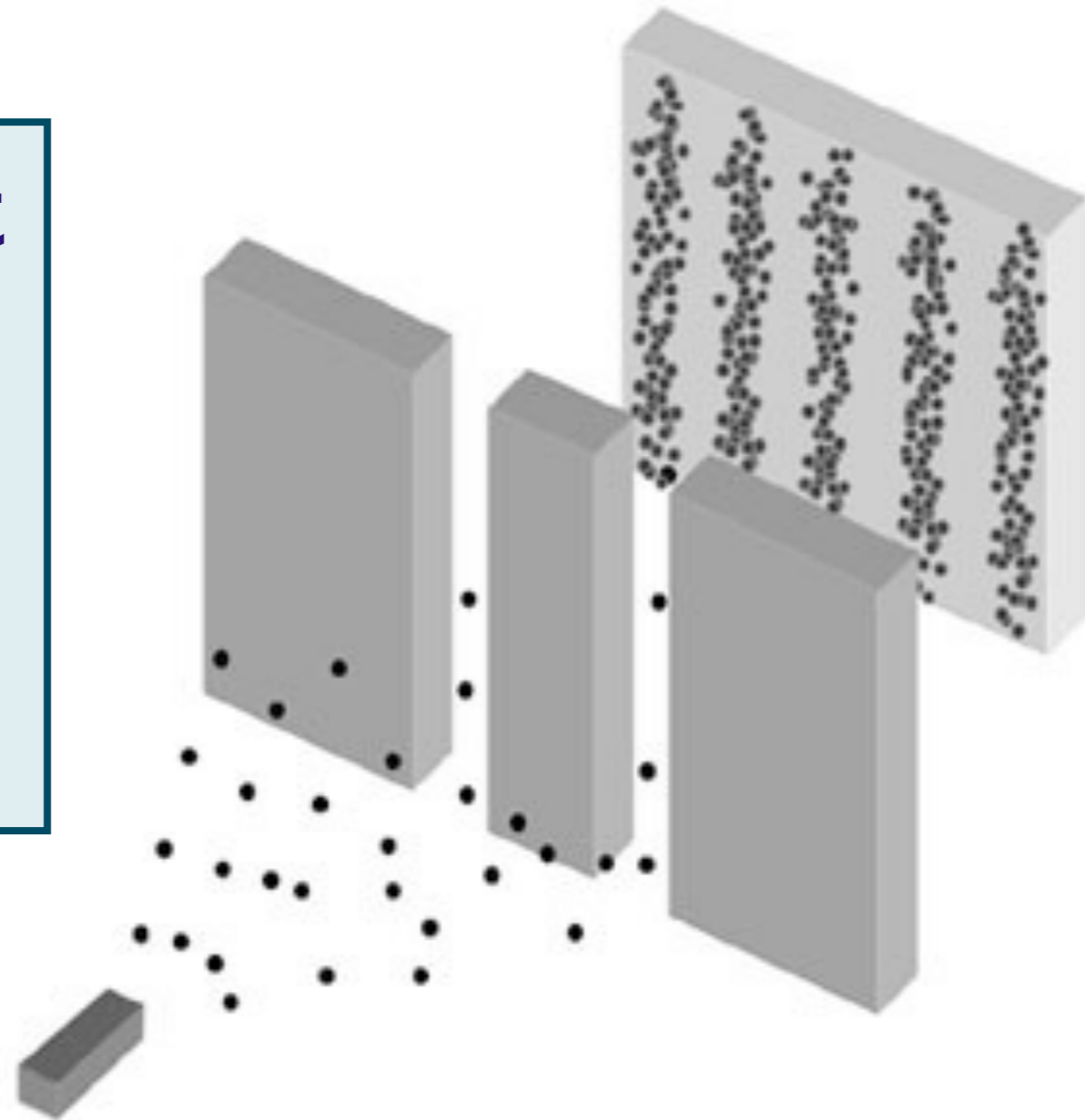
No
interference
when you
watch the
electrons

Interference of electrons

Quantum Superposition

The double slit experiment

Which slit
does an
electron
pass
through ?

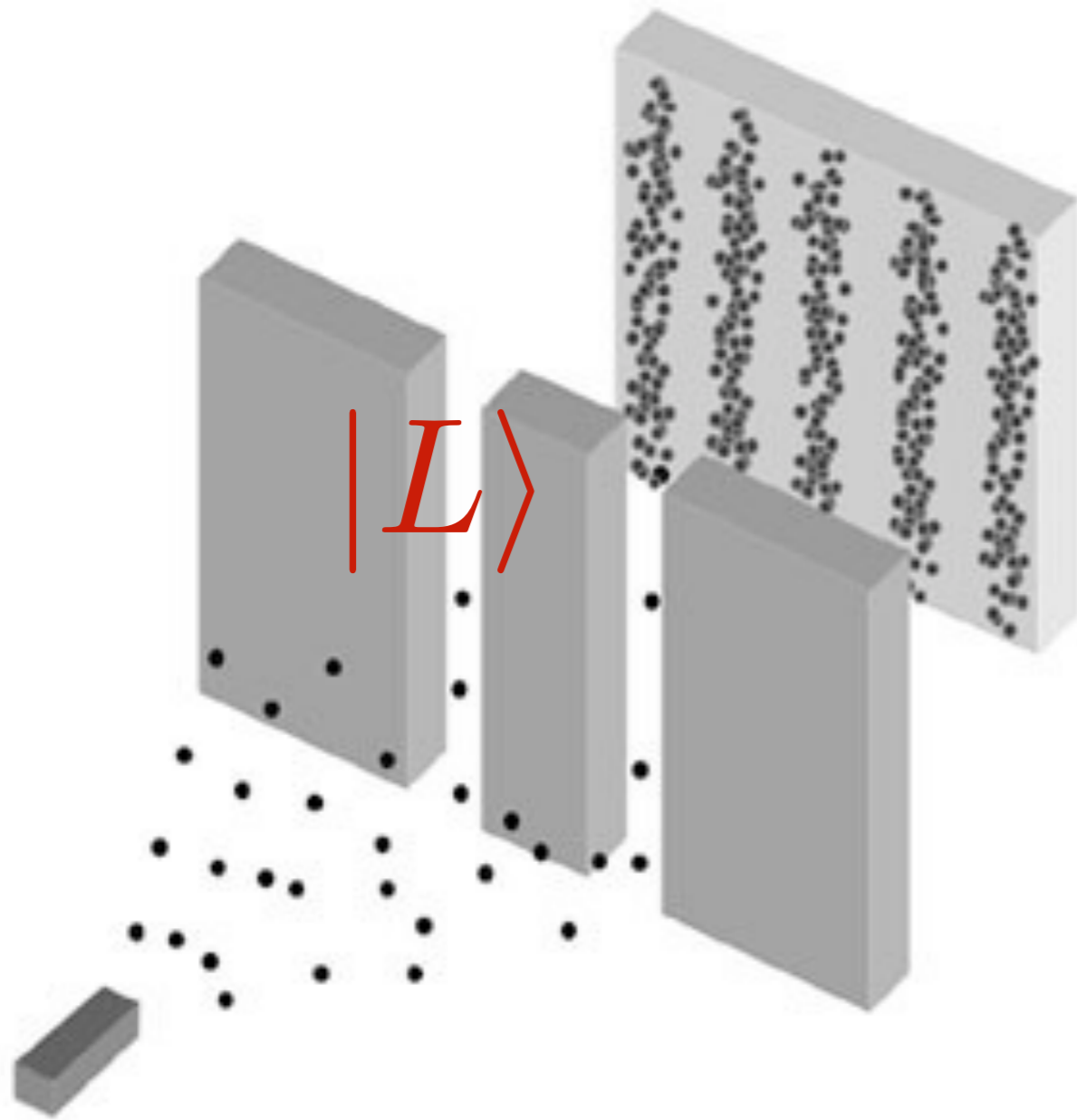


Each
electron
passes
through
both slits !

Interference of electrons

Quantum Superposition

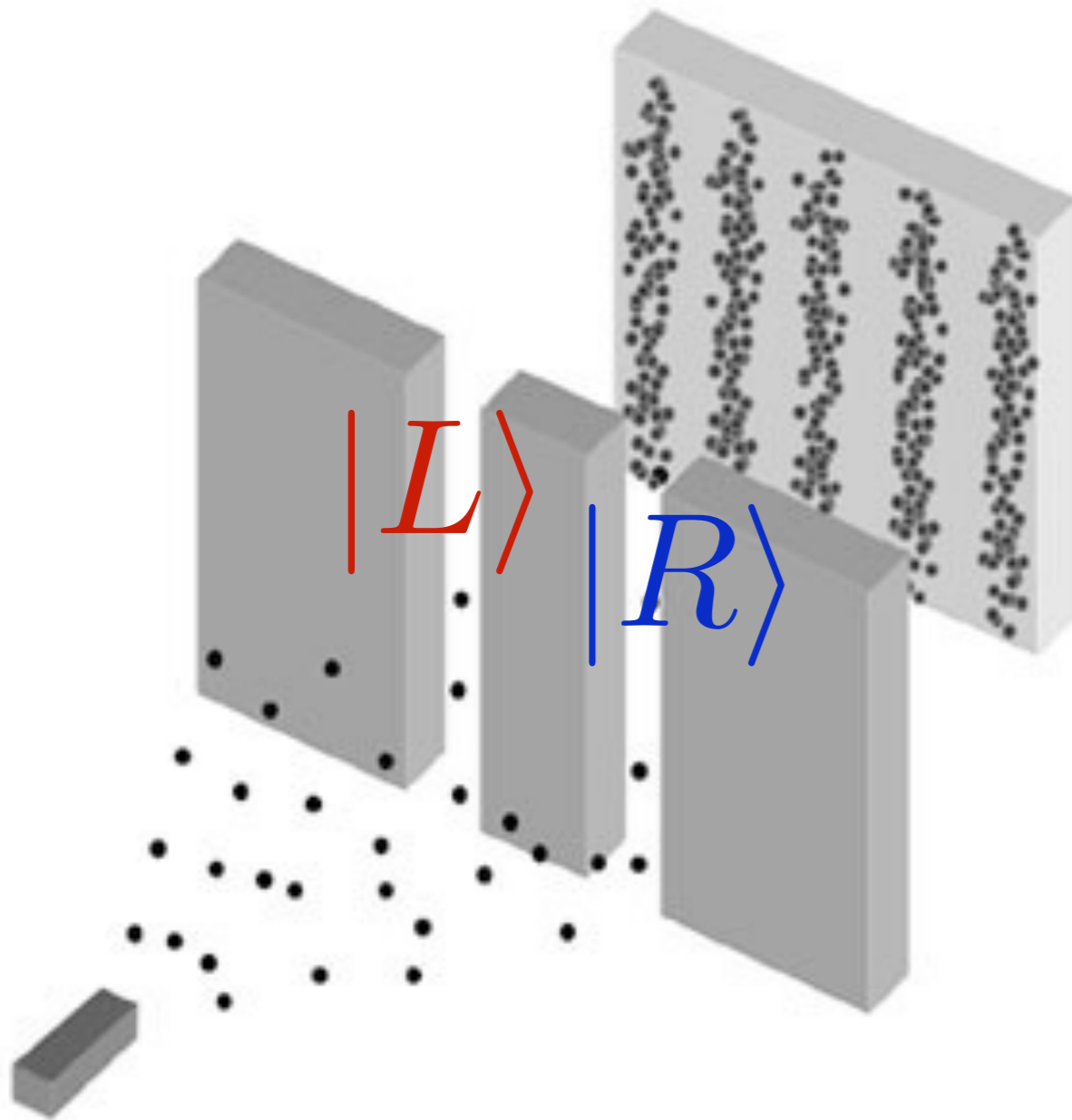
The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

Quantum Superposition

The double slit experiment

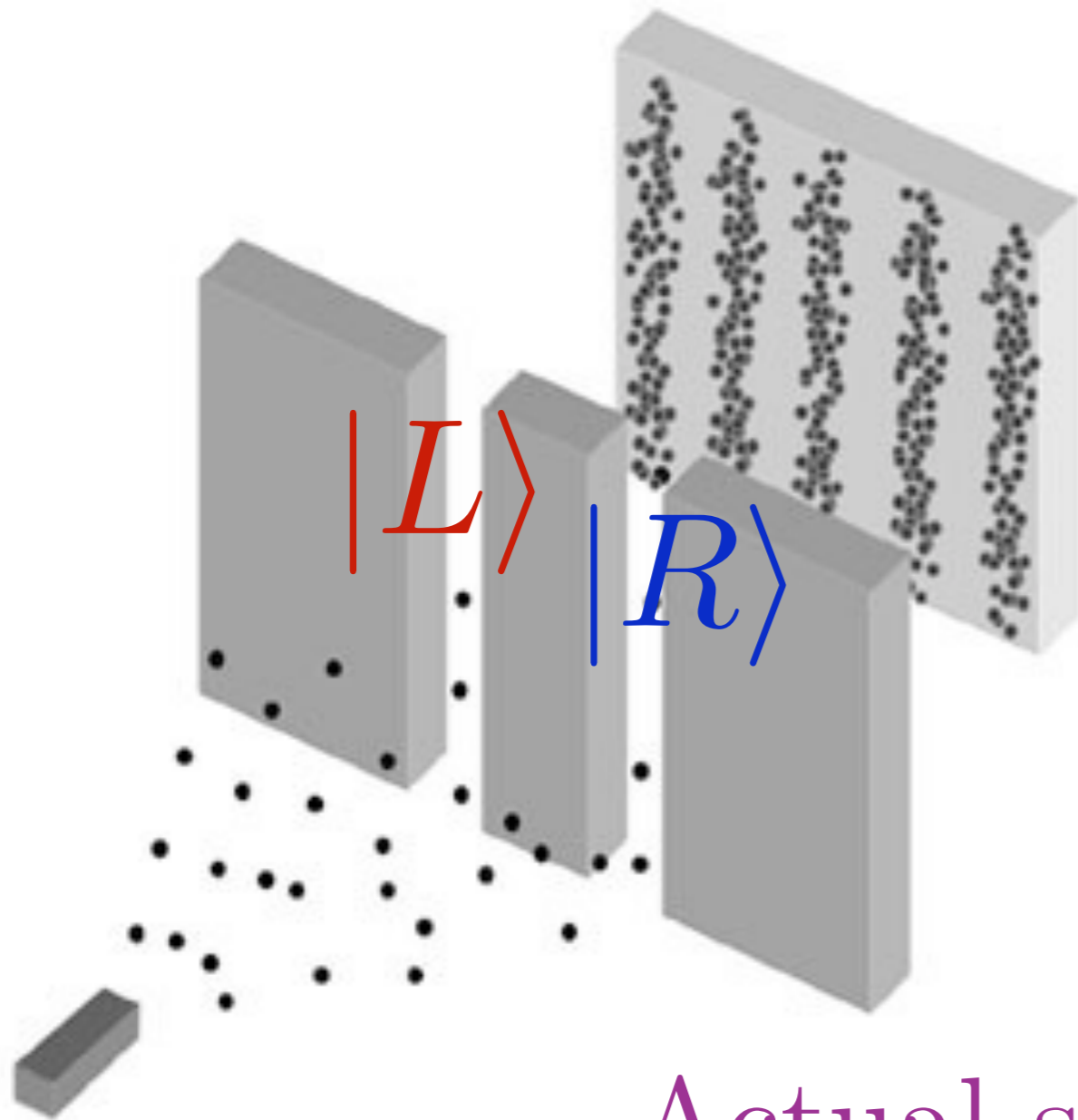


Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Quantum Superposition

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of the electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

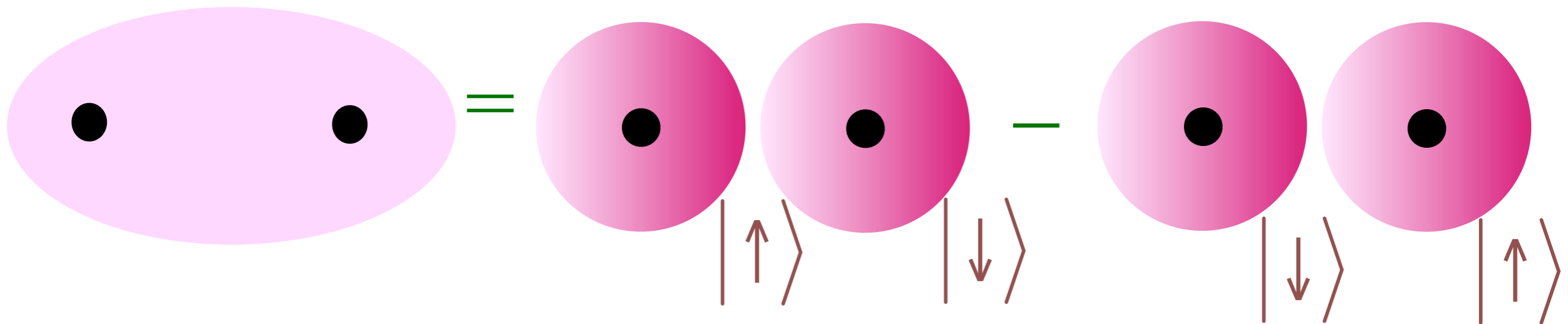


Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



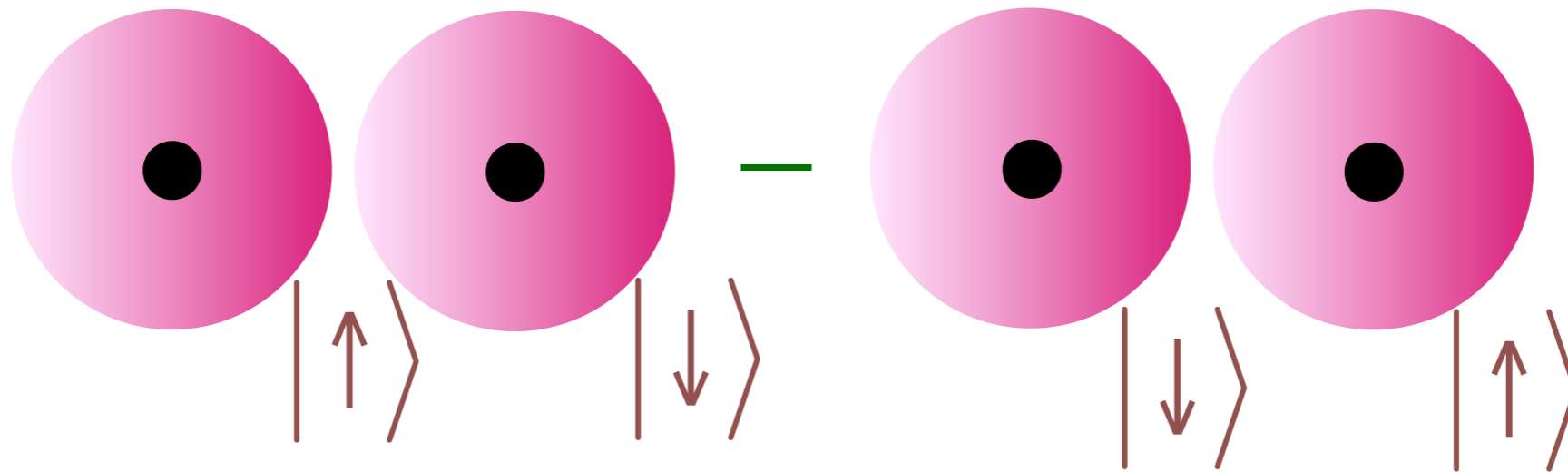
Hydrogen molecule:



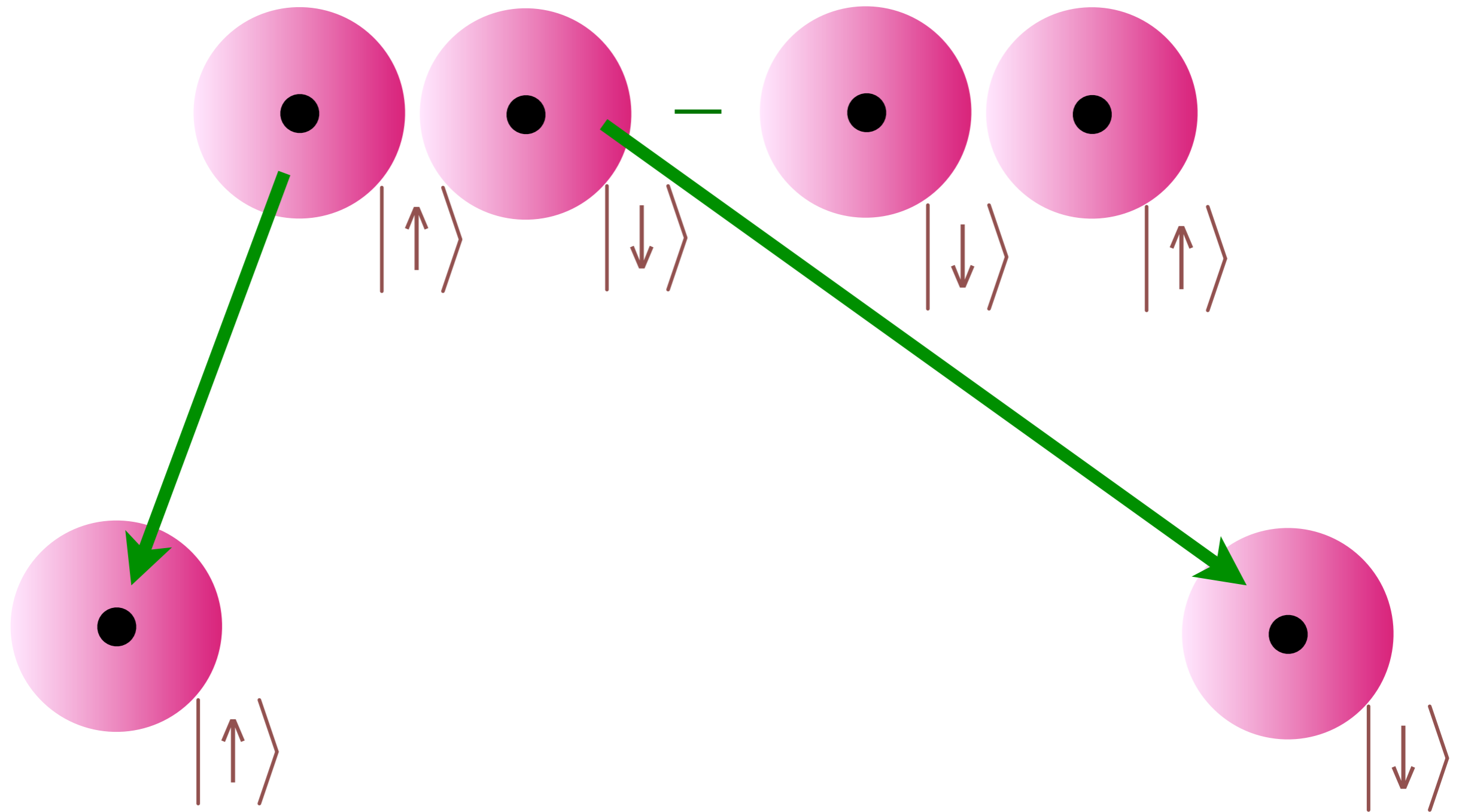
$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Superposition of two electron states leads to non-local
correlations between spins

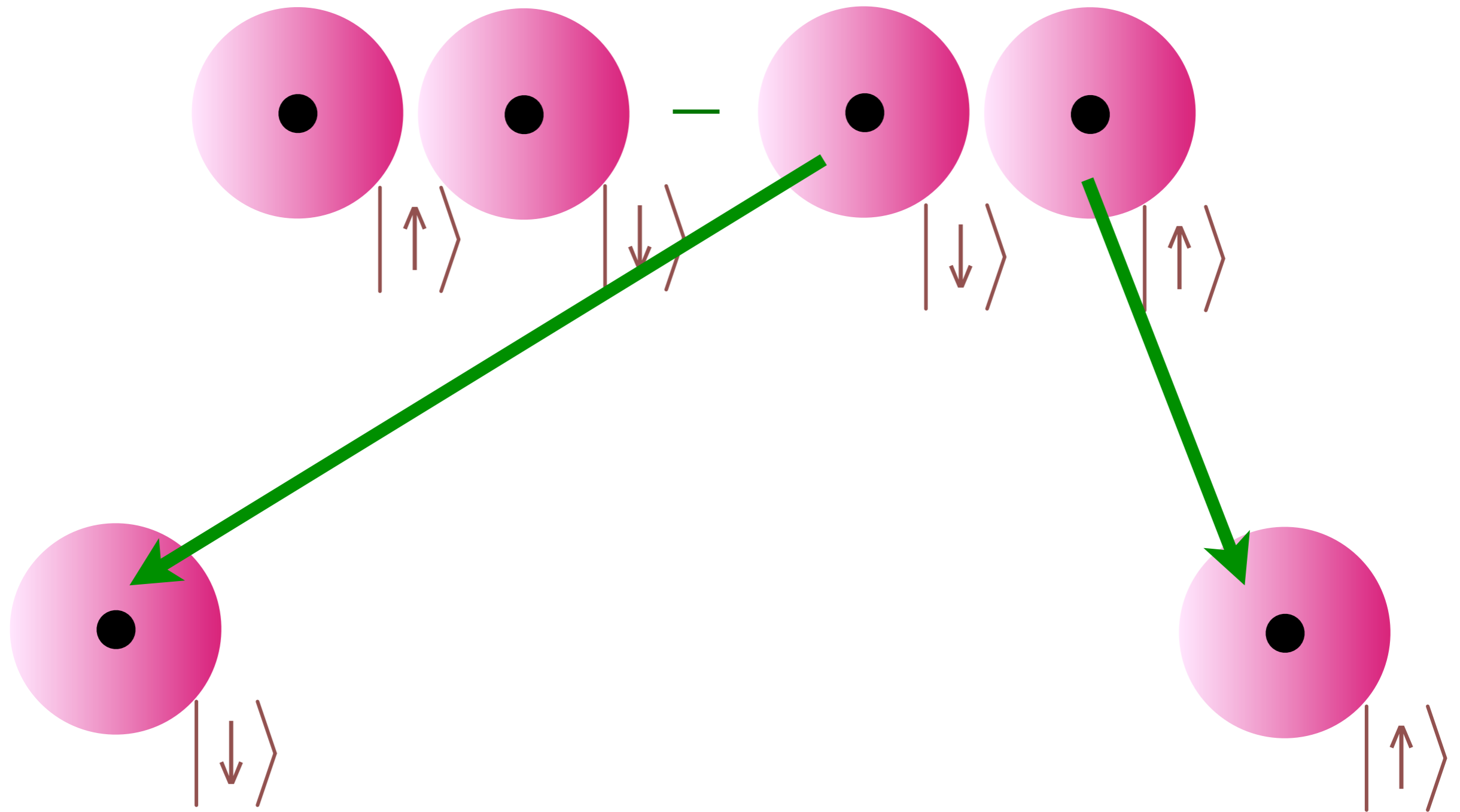
Quantum Entanglement: quantum superposition with more than one particle



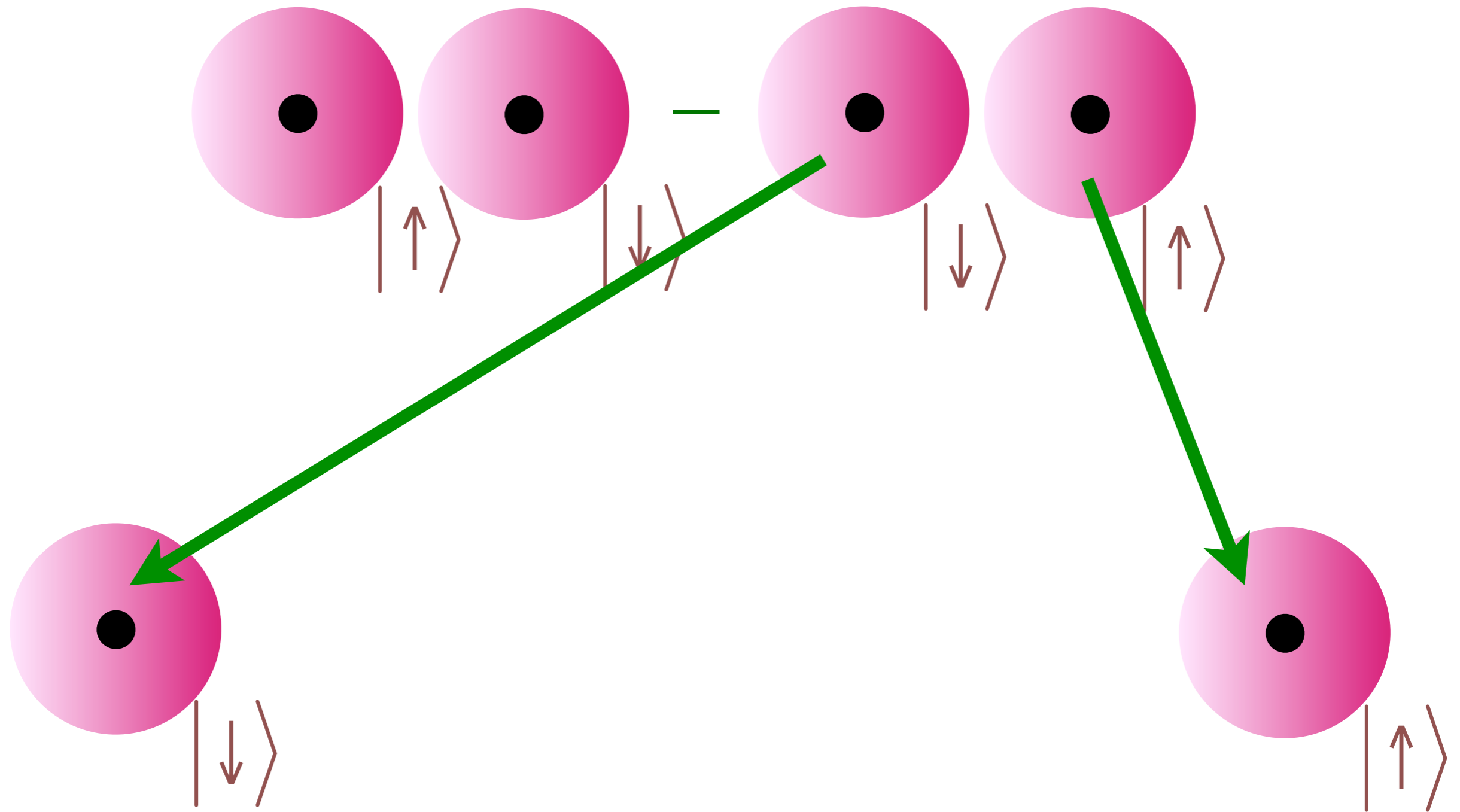
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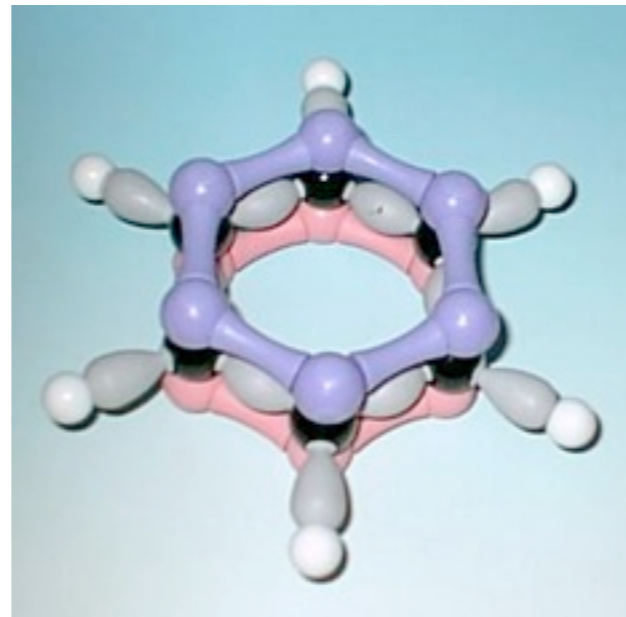
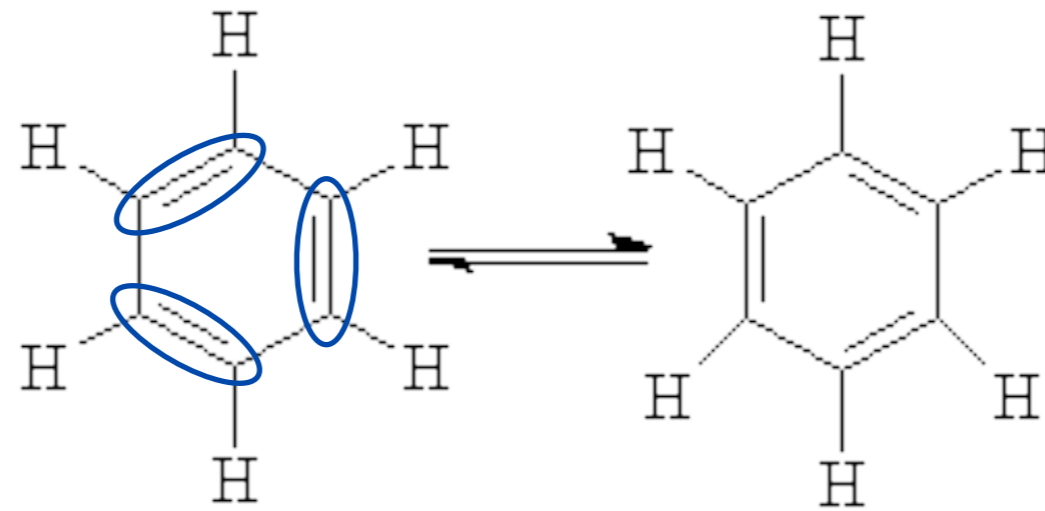


Quantum Entanglement: quantum superposition with more than one particle



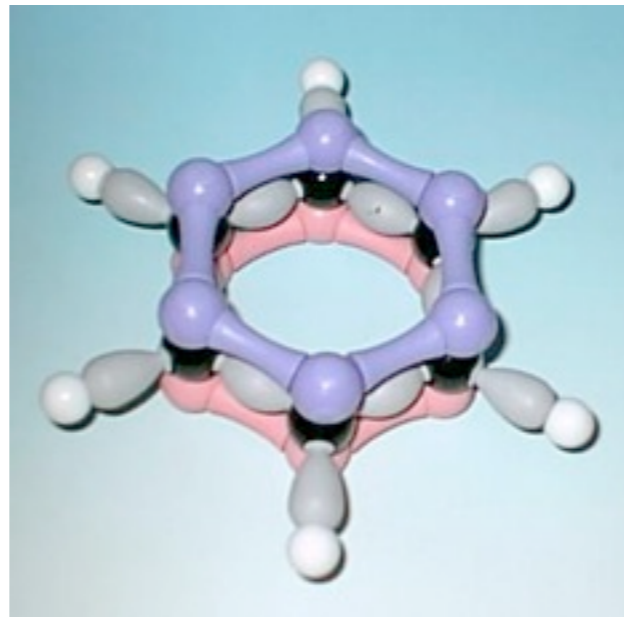
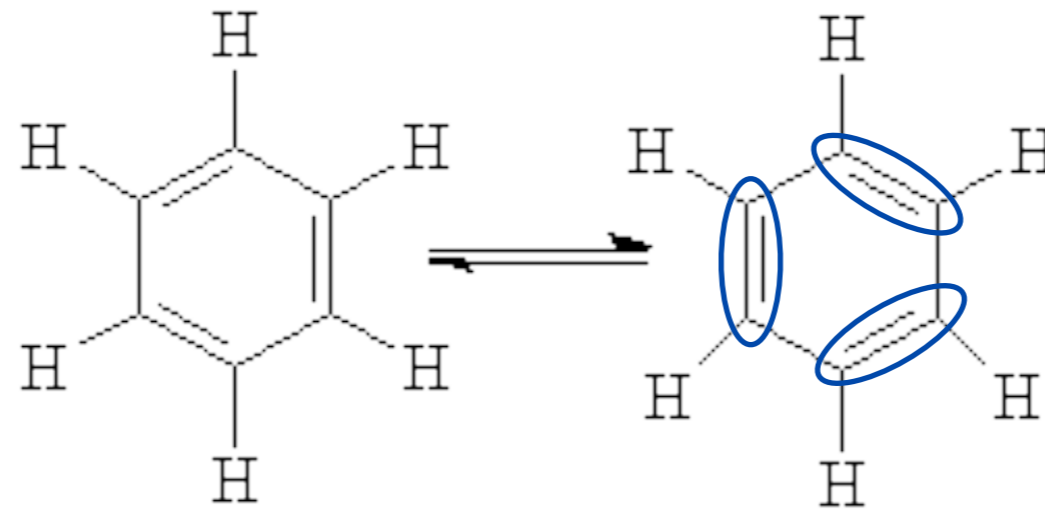
Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart

Entanglement of chemical bonds



Resonance in benzene leads to a symmetric configuration of chemical bonds
(*F. Kekulé, L. Pauling*)

Entanglement of chemical bonds



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**Quantum
superposition and
entanglement**

Superconductivity

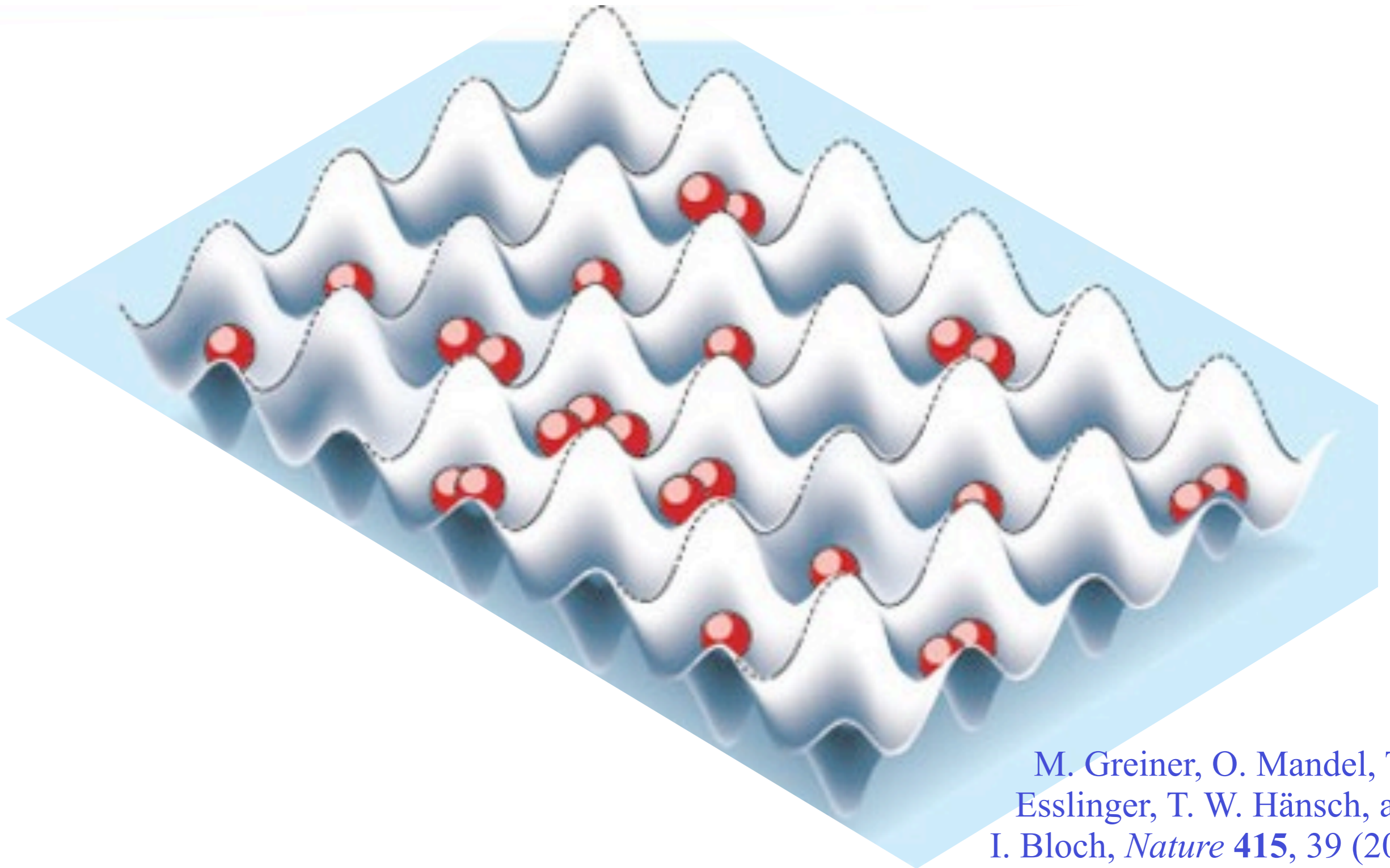
**Black Holes and
String Theory**

**Quantum
superposition and
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Superconductivity

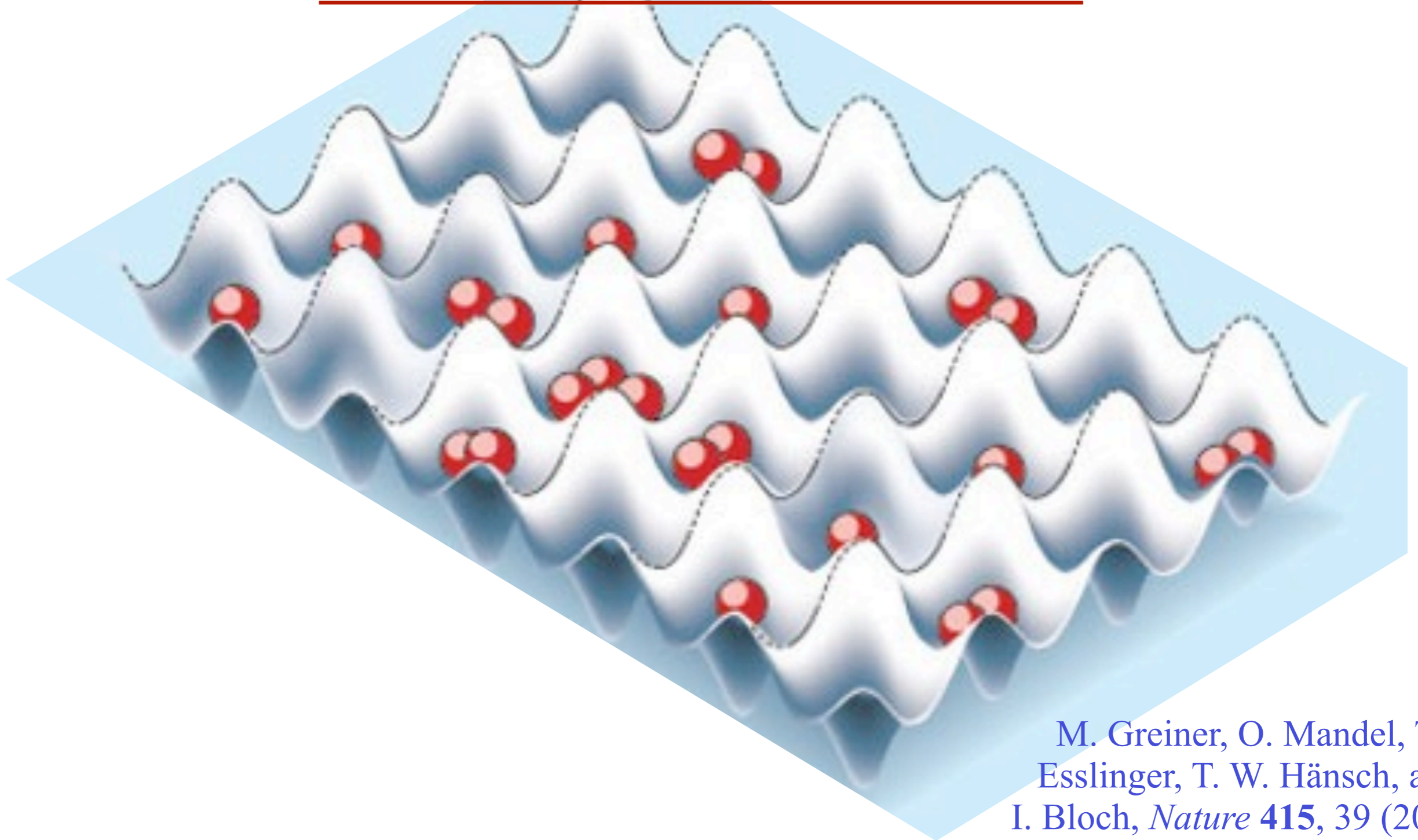
**Black Holes and
String Theory**

Rubidium atoms in a magnetic trap and standing waves of laser light



M. Greiner, O. Mandel, T.
Esslinger, T. W. Hänsch, and
I. Bloch, *Nature* **415**, 39 (2002).

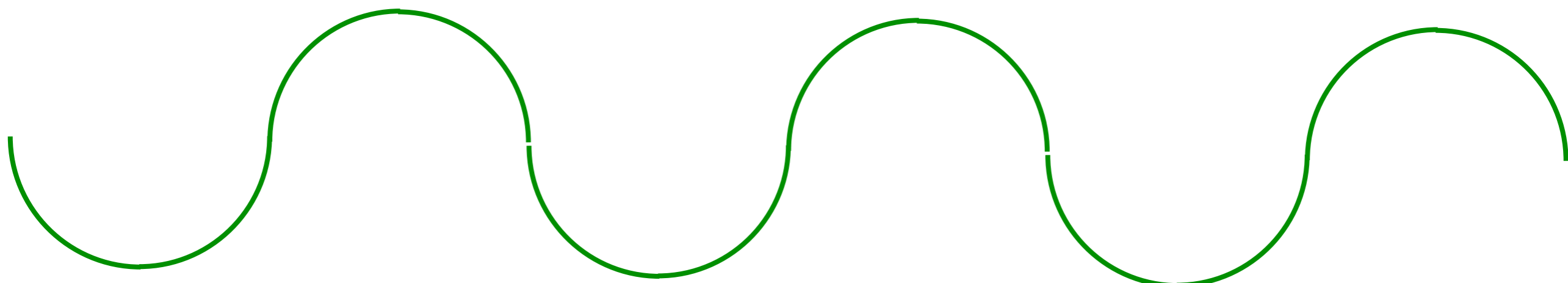
At very low temperatures and for a weak laser light, the Rubidium atoms obey quantum mechanics and form a Bose-Einstein condensate

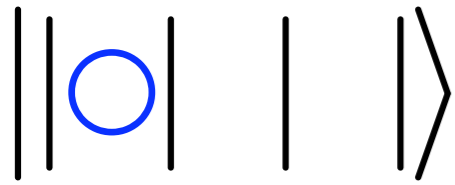
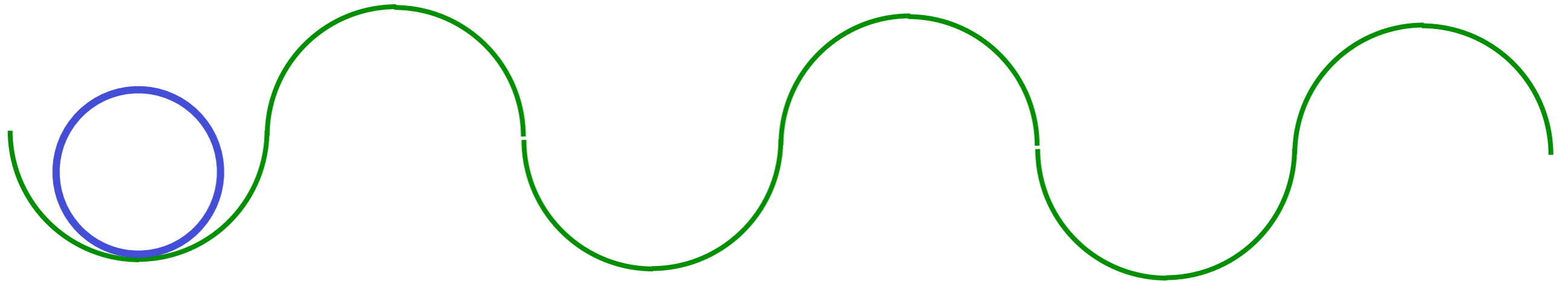


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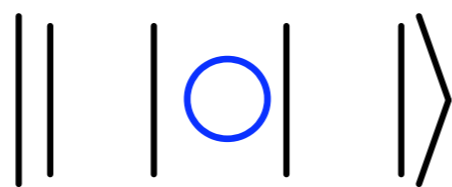
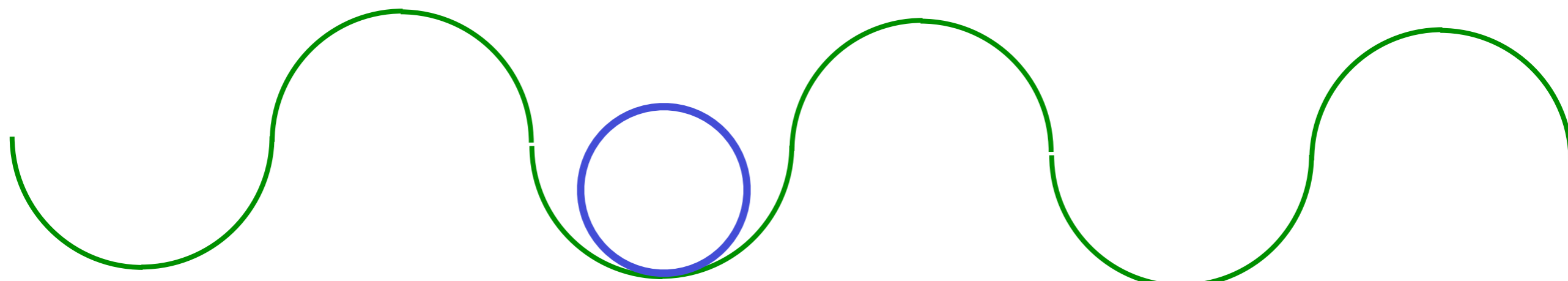
A Bose-Einstein condensate:
An quantum superposition of all
the atoms in all positions

A liquid which flows without
resistance (a superfluid)

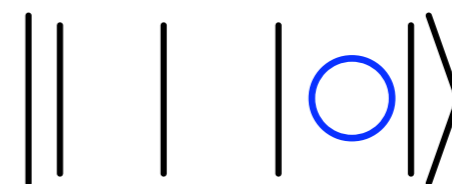
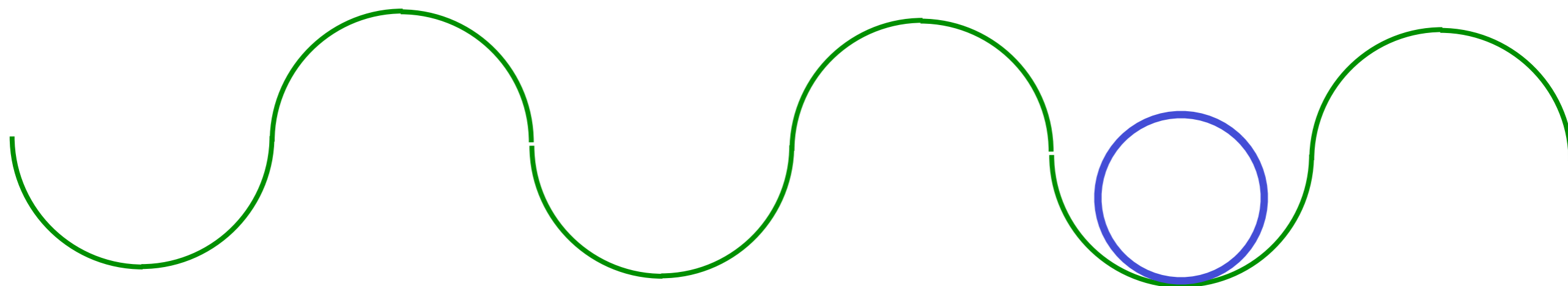




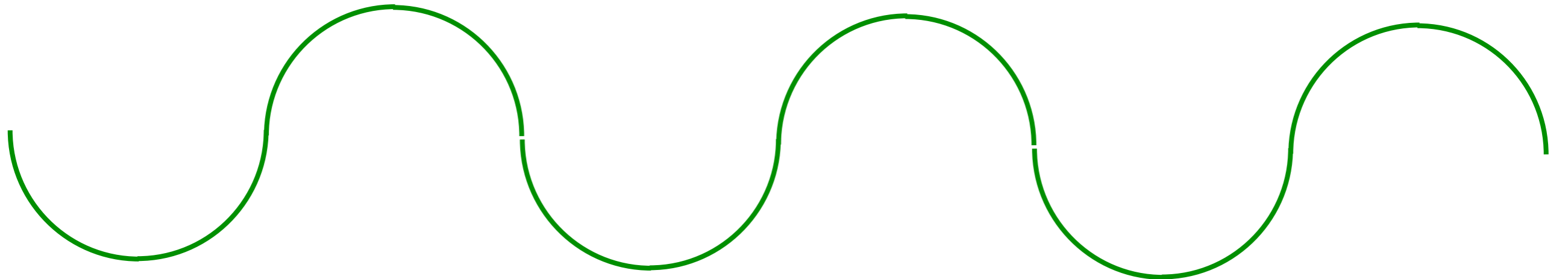
A single atom is superposed
between all positions



A single atom is superposed
between all positions

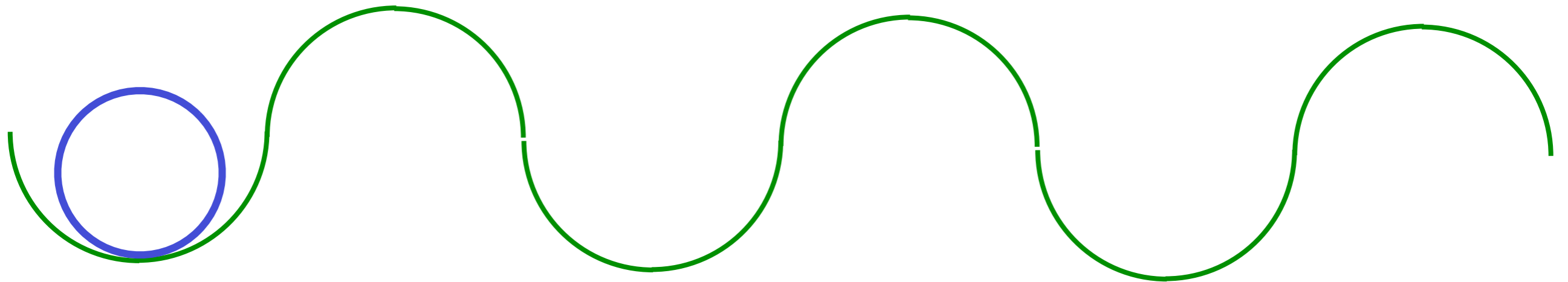


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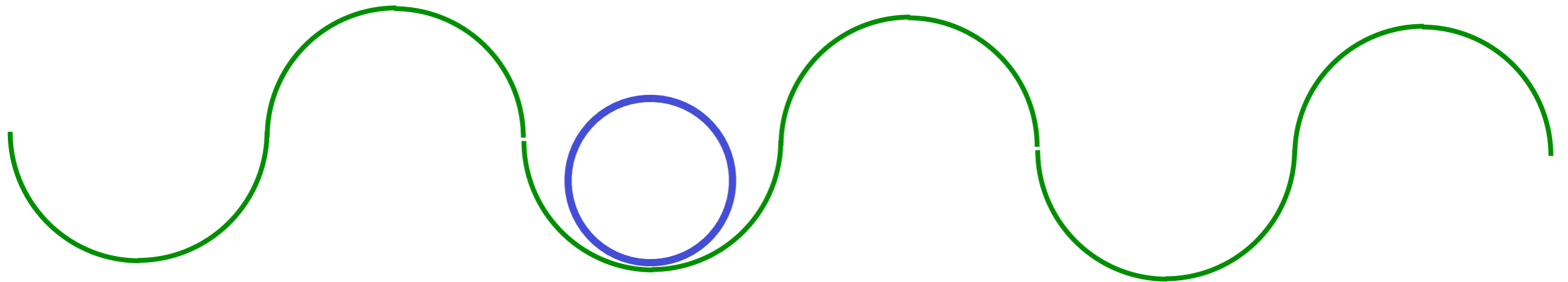
$$|G\rangle = \left(| \circ | | \rangle + | | \circ | | \rangle + | | | \circ | \rangle \right)$$

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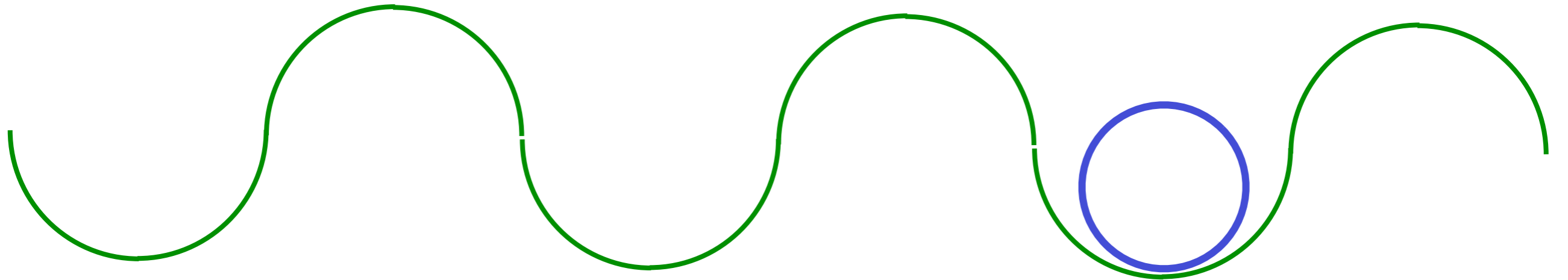
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Bose-Einstein condensate: superposition between all atoms

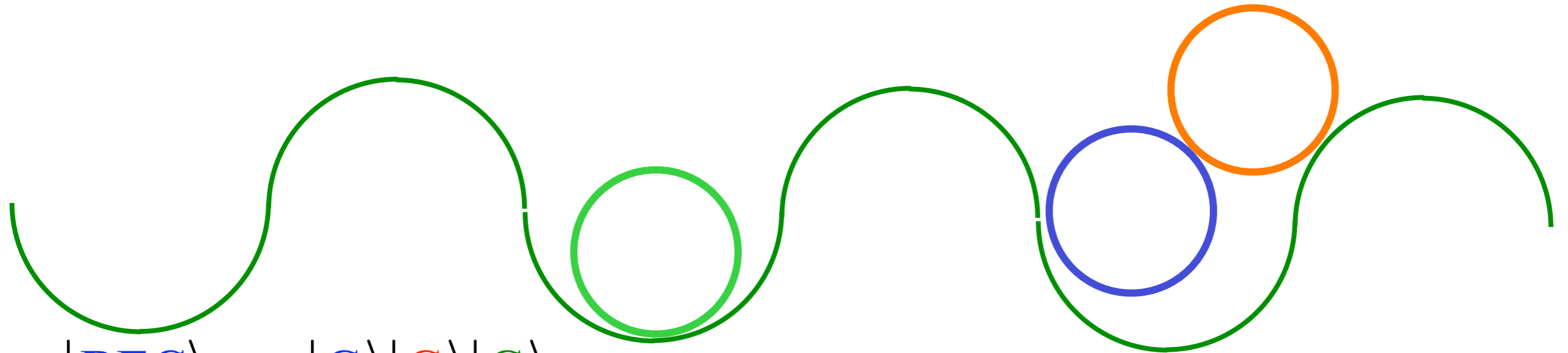
$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

Bose-Einstein condensate: superposition between all atoms

$$\begin{aligned}
 |\text{BEC}\rangle &= |G\rangle|G\rangle|G\rangle \\
 &= \left(\begin{aligned}
 &||\text{blue}\rangle|\text{red}\rangle|\text{green}\rangle + ||\text{red}\rangle|\text{blue}\rangle|\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{blue} \end{array}|\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{green} \end{array}|\text{blue}\rangle \\
 &+ \begin{array}{|c|} \hline \text{red} \\ \hline \text{green} \end{array}|\text{blue}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{blue} \\ \hline \text{green} \end{array} + \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \\ \hline \text{green} \end{array} + \dots 27 \text{ terms}
 \end{aligned} \right)
 \end{aligned}$$

Large fluctuations in number of atoms in each site –
superfluidity (atoms can “flow” without dissipation)

Bose-Einstein condensate: superposition between all atoms

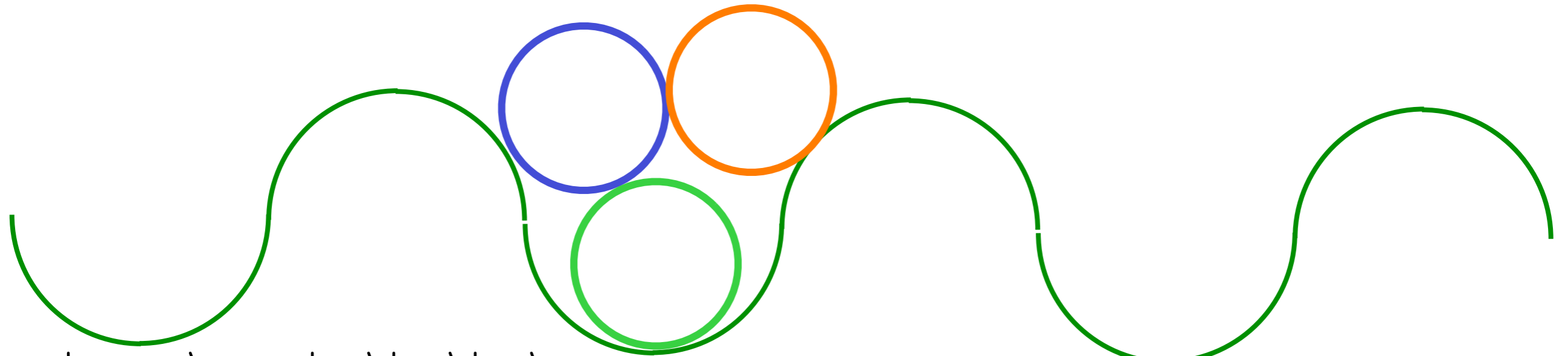


$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

$$= \left(\begin{aligned} &||\text{blue}\rangle|\text{red}\rangle|\text{green}\rangle + ||\text{red}\rangle|\text{blue}\rangle|\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{blue} \\ \hline \end{array} |\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{green} \\ \hline \end{array} |\text{blue}\rangle \\ &+ \begin{array}{|c|} \hline \text{red} \\ \hline \text{green} \\ \hline \end{array} |\text{blue}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \text{blue} \\ \hline \text{green} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{blue} \\ \hline \text{red} \\ \hline \text{green} \\ \hline \end{array} + \dots 27 \text{ terms} \end{aligned} \right)$$

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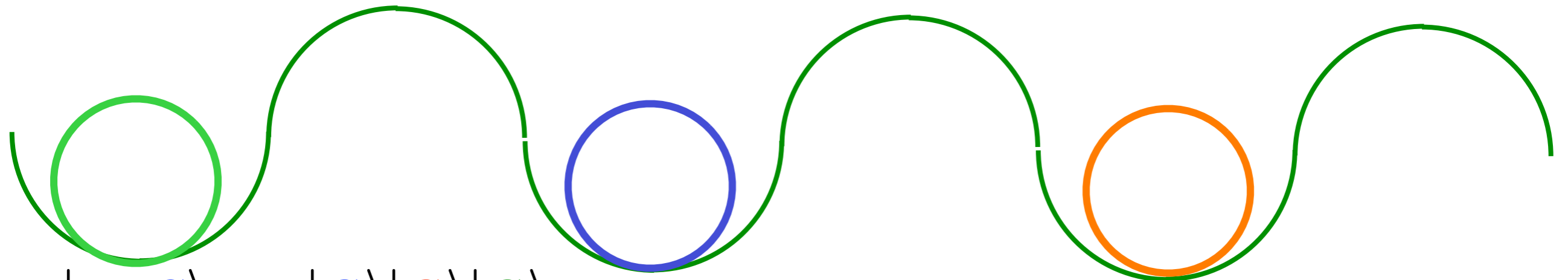


$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

$$= \left(\begin{aligned} &||\text{blue}| \text{orange}| \text{green}\rangle + ||\text{orange}| \text{blue}| \text{green}\rangle + \begin{array}{|c|} \hline \text{orange} \\ \hline \text{blue} \\ \hline \end{array} | \text{green}\rangle + \begin{array}{|c|} \hline \text{orange} \\ \hline \end{array} | \begin{array}{|c|} \hline \text{blue} \\ \hline \text{green} \\ \hline \end{array} \rangle \\ &+ \begin{array}{|c|} \hline \text{orange} \\ \hline \text{green} \\ \hline \end{array} | \text{blue}\rangle + \begin{array}{|c|} \hline \text{orange} \\ \hline \text{blue} \\ \hline \text{green} \\ \hline \end{array} \rangle + \begin{array}{|c|} \hline \text{blue} \\ \hline \text{orange} \\ \hline \text{green} \\ \hline \end{array} | \rangle + \dots 27 \text{ terms} \end{aligned} \right)$$

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Bose-Einstein condensate: superposition between all atoms

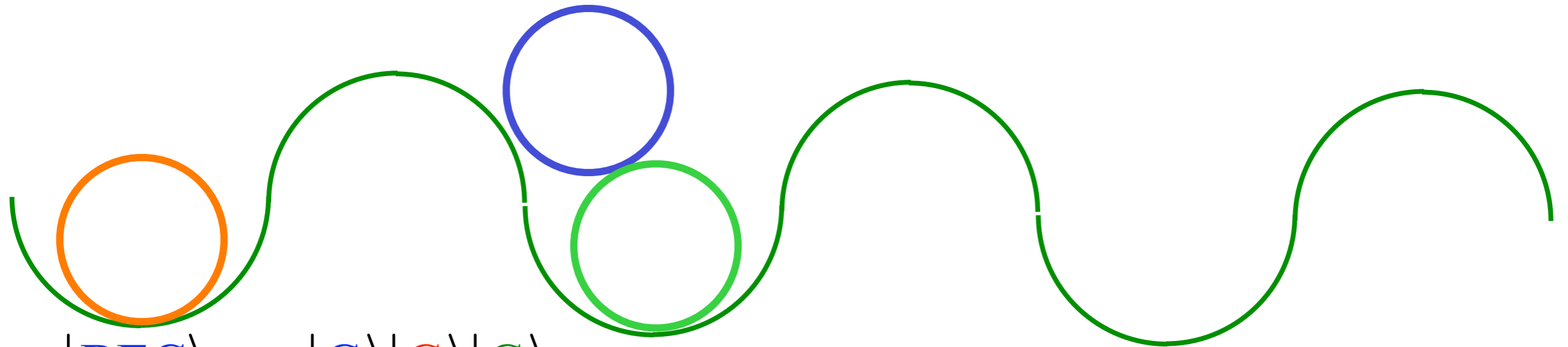


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$$= \left(\begin{aligned} &||\text{blue}\rangle|\text{red}\rangle|\text{green}\rangle + ||\text{red}\rangle|\text{blue}\rangle|\text{green}\rangle + \left| \begin{array}{l} \text{red} \\ \text{blue} \end{array} \right\rangle|\text{green}\rangle + \left| \begin{array}{l} \text{red} \\ \text{green} \end{array} \right\rangle|\text{blue}\rangle \\ &+ \left| \begin{array}{l} \text{red} \\ \text{green} \end{array} \right\rangle|\text{blue}\rangle + \left| \begin{array}{l} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \begin{array}{l} \text{blue} \\ \text{red} \\ \text{green} \end{array} \right\rangle + \dots 27 \text{ terms} \end{aligned} \right)$$

Large fluctuations in number of atoms in each site –
superfluidity (atoms can “flow” without dissipation)

Bose-Einstein condensate: superposition between all atoms

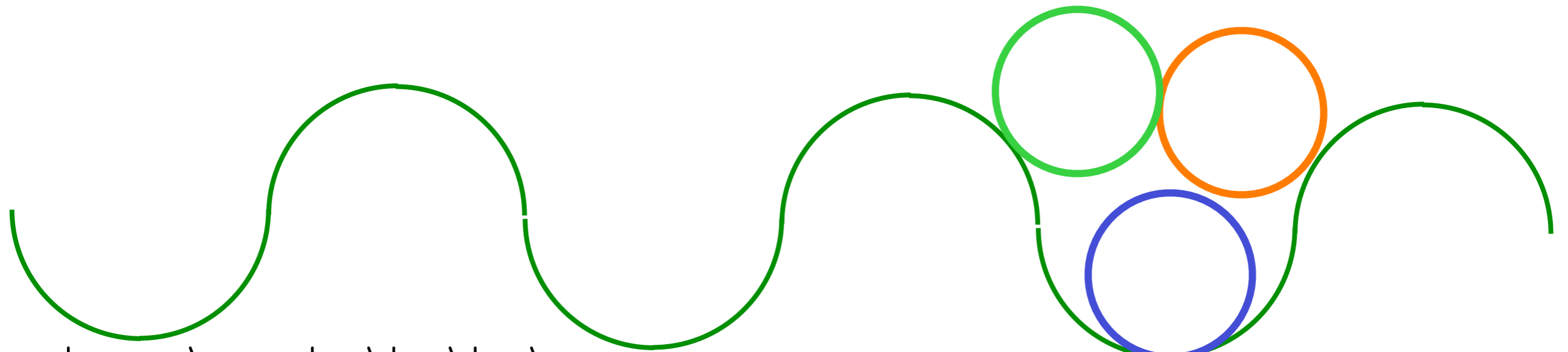


$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

$$= \left(\begin{aligned} & \left| \left| \begin{array}{c} \text{blue} \\ \text{red} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle \right. \\ & \left. + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{red} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \dots 27 \text{ terms} \right. \end{aligned} \right)$$

Large fluctuations in number of atoms in each site –
superfluidity (atoms can “flow” without dissipation)

Bose-Einstein condensate: superposition between all atoms

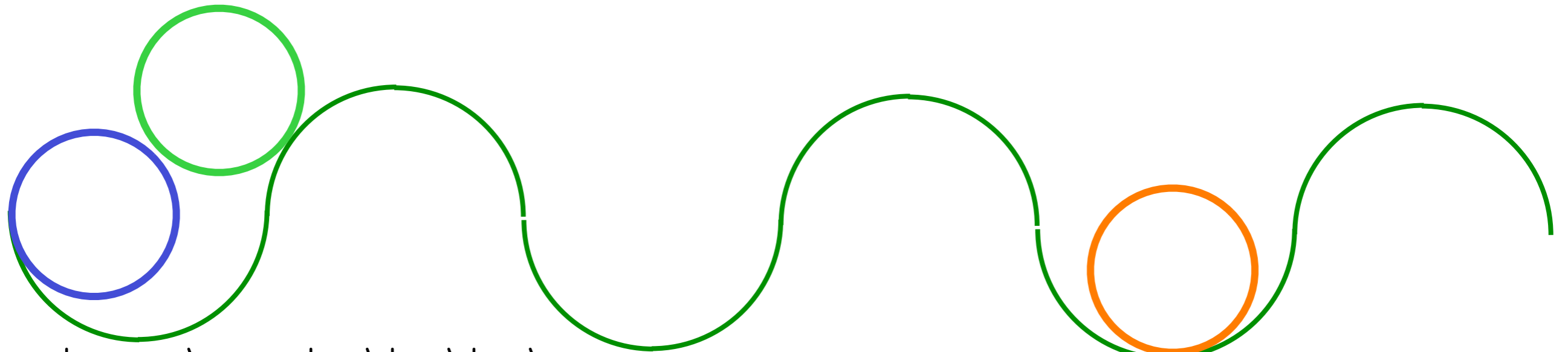


$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

$$= \left(\begin{aligned} & \left| \left| \begin{array}{c} \text{blue} \\ \text{orange} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{orange} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{orange} \\ \text{green} \\ \text{blue} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{orange} \\ \text{blue} \\ \text{green} \end{array} \right\rangle \right. \\ & \left. + \left| \left| \begin{array}{c} \text{orange} \\ \text{green} \\ \text{blue} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{orange} \\ \text{blue} \\ \text{green} \end{array} \right\rangle + \left| \left| \begin{array}{c} \text{blue} \\ \text{orange} \\ \text{green} \end{array} \right\rangle + \dots 27 \text{ terms} \right. \end{aligned} \right)$$

Large fluctuations in number of atoms in each site –
superfluidity (atoms can “flow” without dissipation)

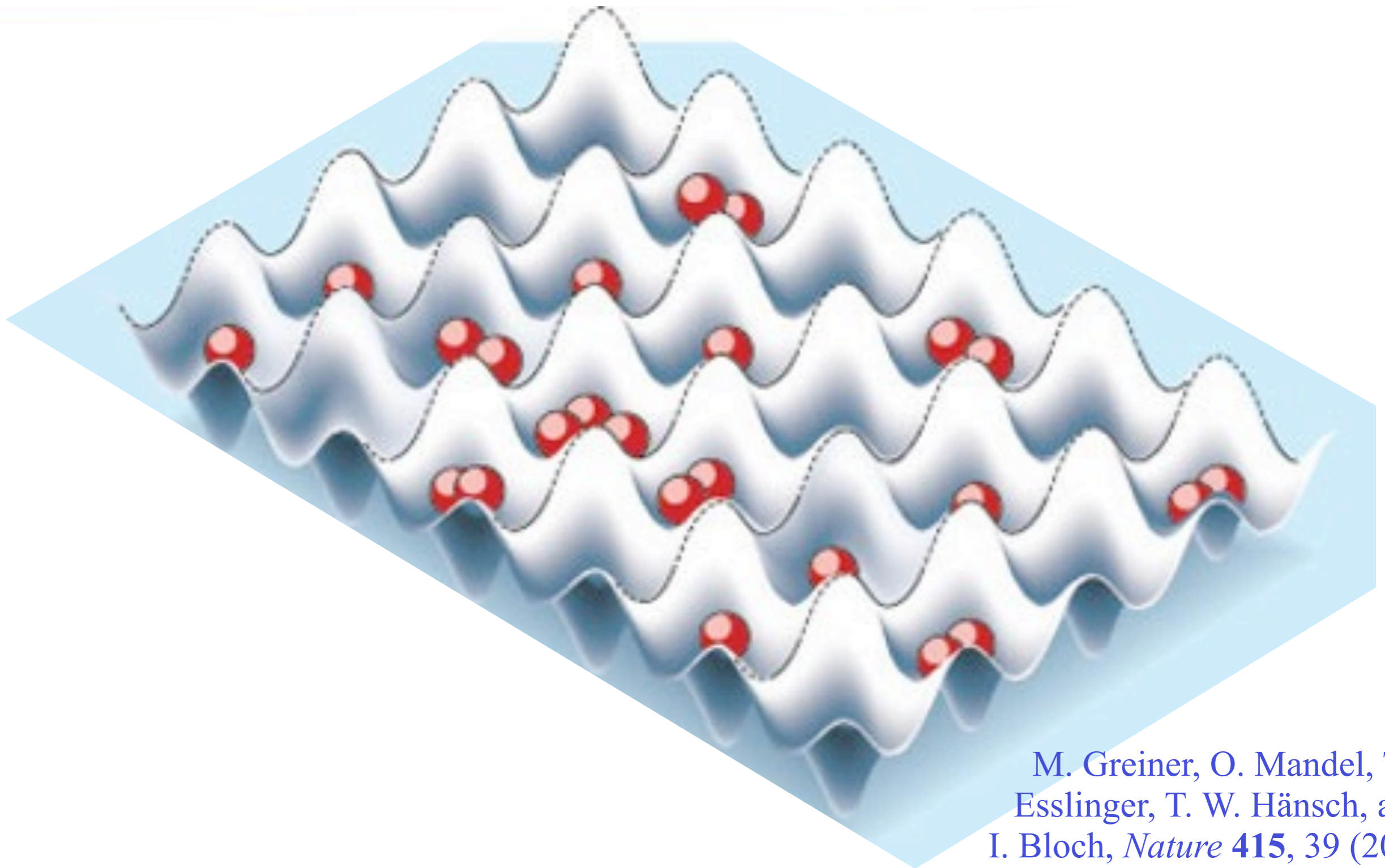
Bose-Einstein condensate: superposition between all atoms



$$\begin{aligned}
 |\text{BEC}\rangle &= |G\rangle|G\rangle|G\rangle \\
 &= \left(\begin{aligned}
 &||\text{blue}|\text{red}|\text{green}\rangle + ||\text{red}|\text{blue}|\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \end{array}|\text{green}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \end{array}|\text{blue}|\text{green}\rangle \\
 &+ \begin{array}{|c|} \hline \text{red} \\ \hline \end{array}|\text{blue}\rangle + \begin{array}{|c|} \hline \text{red} \\ \hline \end{array}|\text{green}\rangle + \begin{array}{|c|} \hline \text{blue} \\ \hline \end{array}|\text{red}\rangle + \begin{array}{|c|} \hline \text{blue} \\ \hline \end{array}|\text{green}\rangle + \dots 27 \text{ terms}
 \end{aligned} \right)
 \end{aligned}$$

Large fluctuations in number of atoms in each site –
superfluidity (atoms can “flow” without dissipation)

At very low temperatures and for a weak laser light, the Rubidium atoms form a Bose-Einstein condensate



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bose-Einstein condensate: superposition between all atoms

$$|\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

(Strictly speaking: this is not entanglement between the atoms because the BEC is a product of simple “wave” states of the atoms)

A superconductor: a Bose condensate of pairs of electrons in a “chemical bond” in a metal

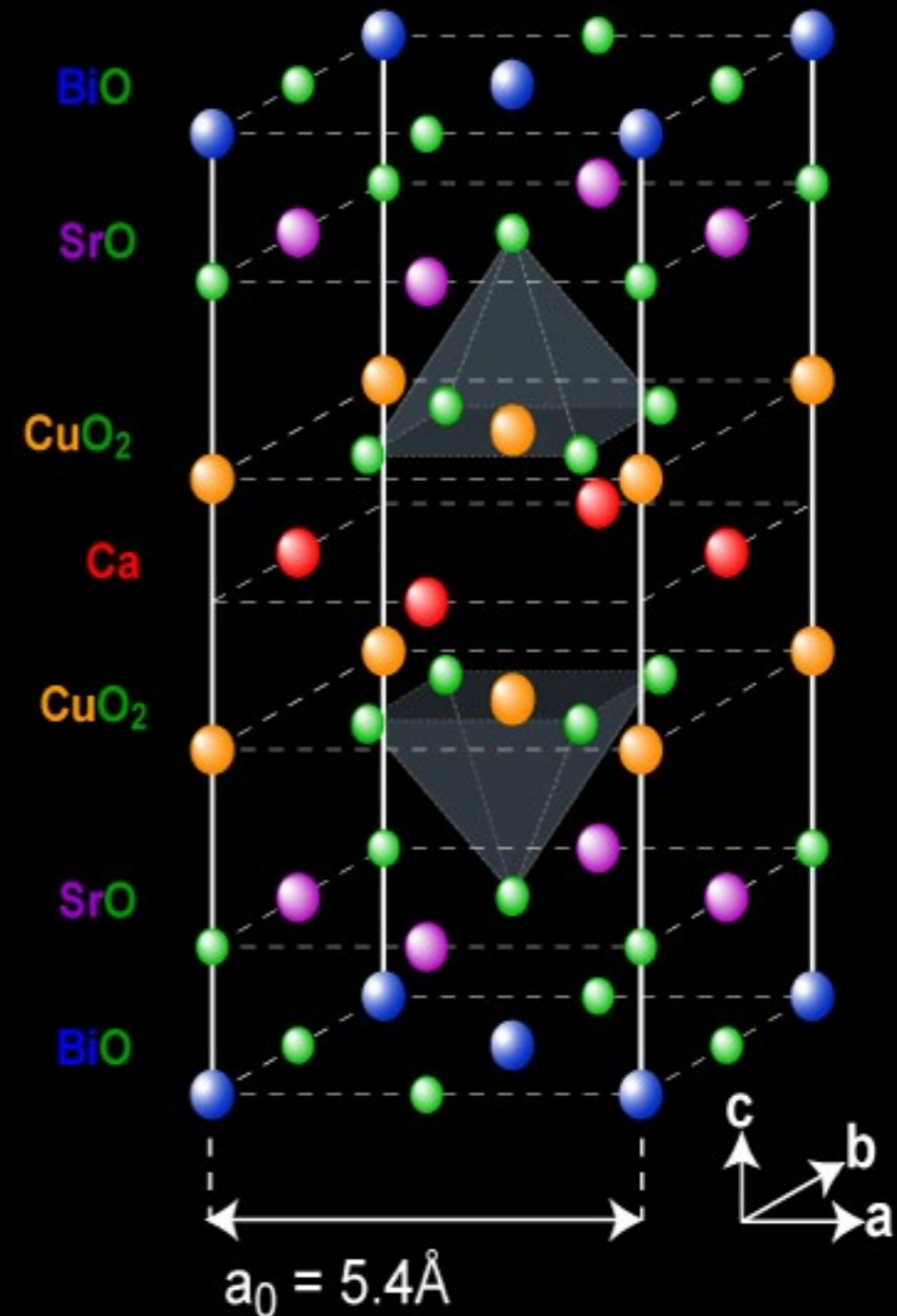
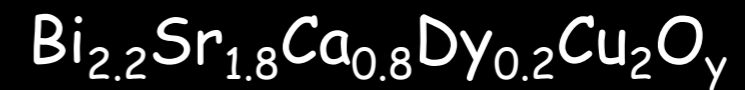
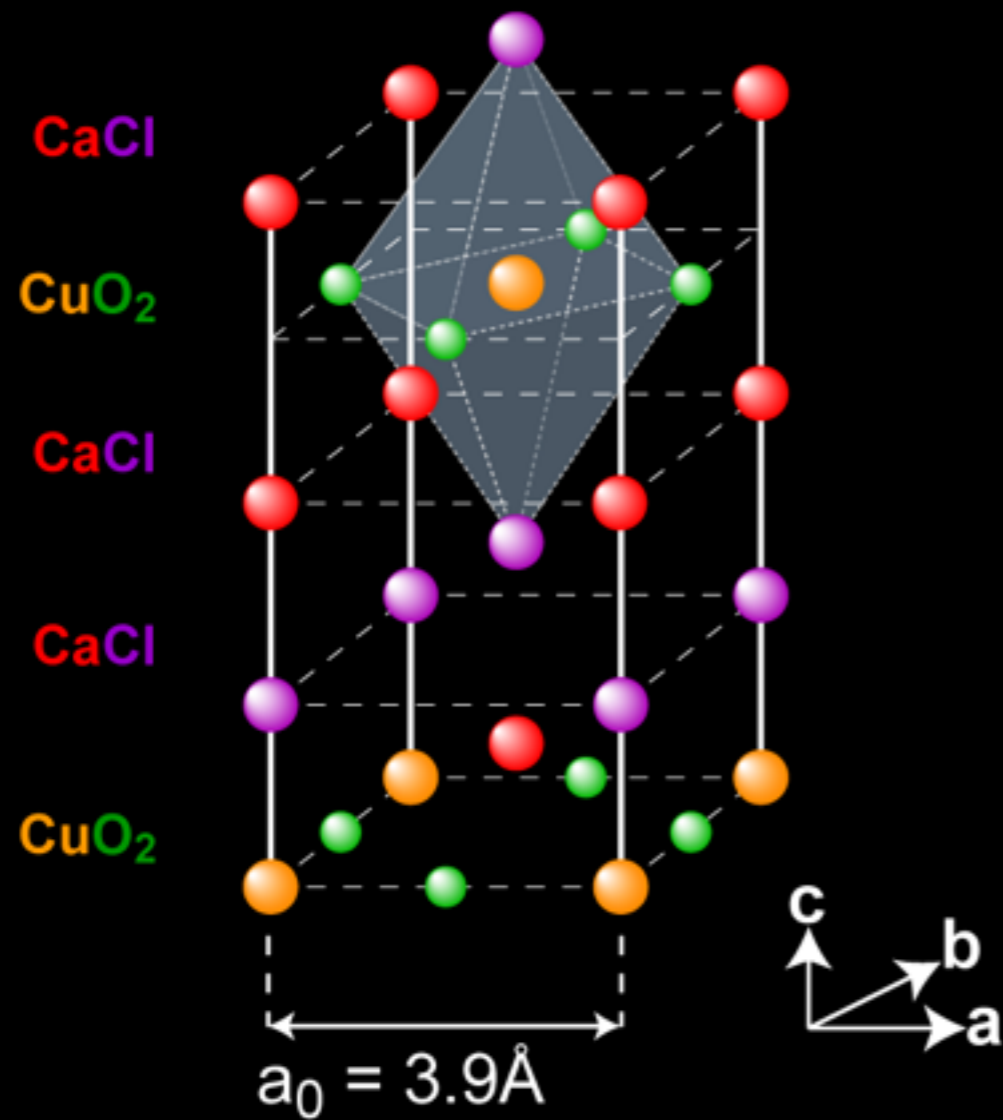
$$|\text{BEC}\rangle = |G\rangle |G\rangle |G\rangle$$

$$|G\rangle \equiv |\uparrow\downarrow - \downarrow\uparrow\rangle$$

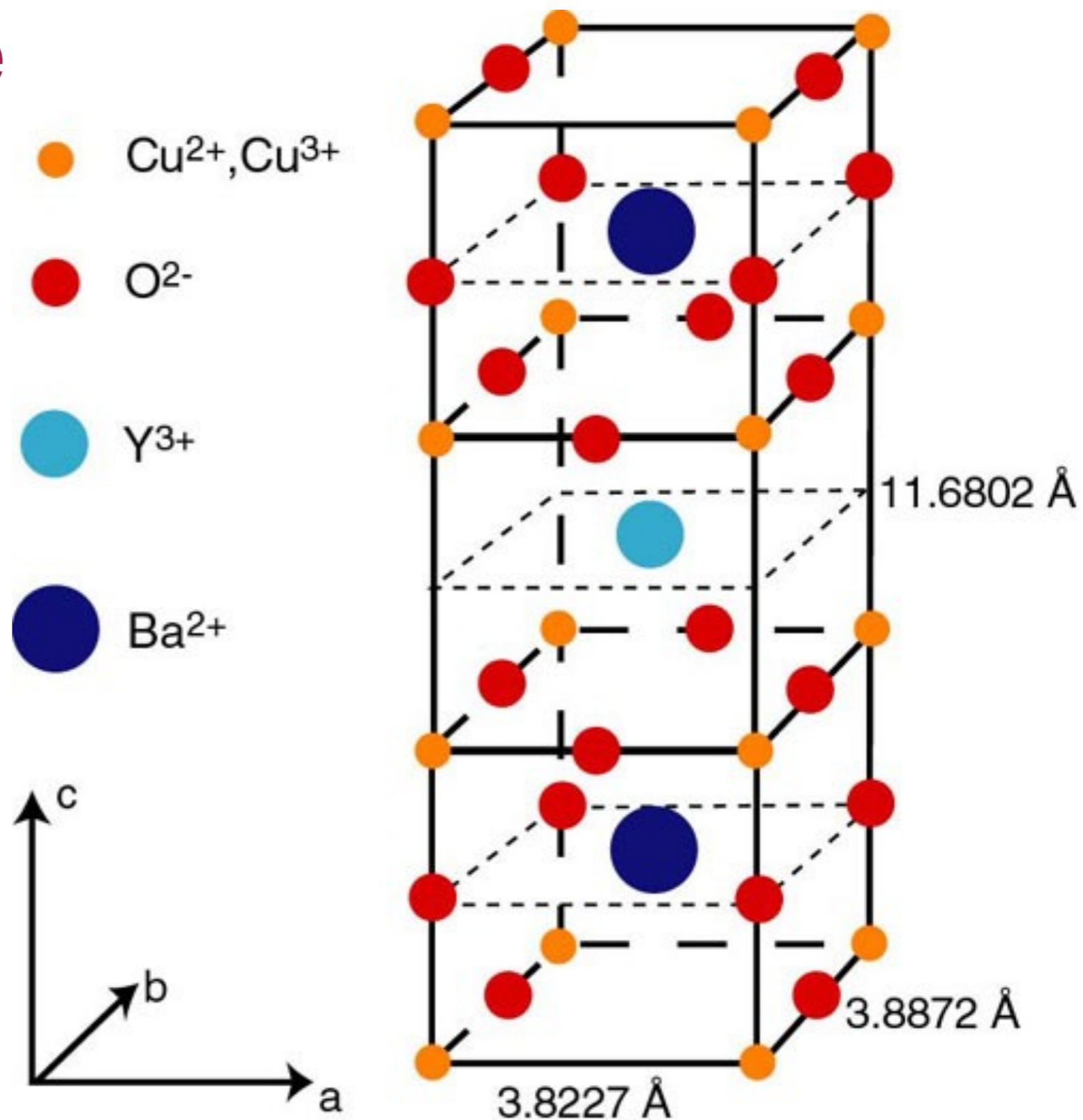
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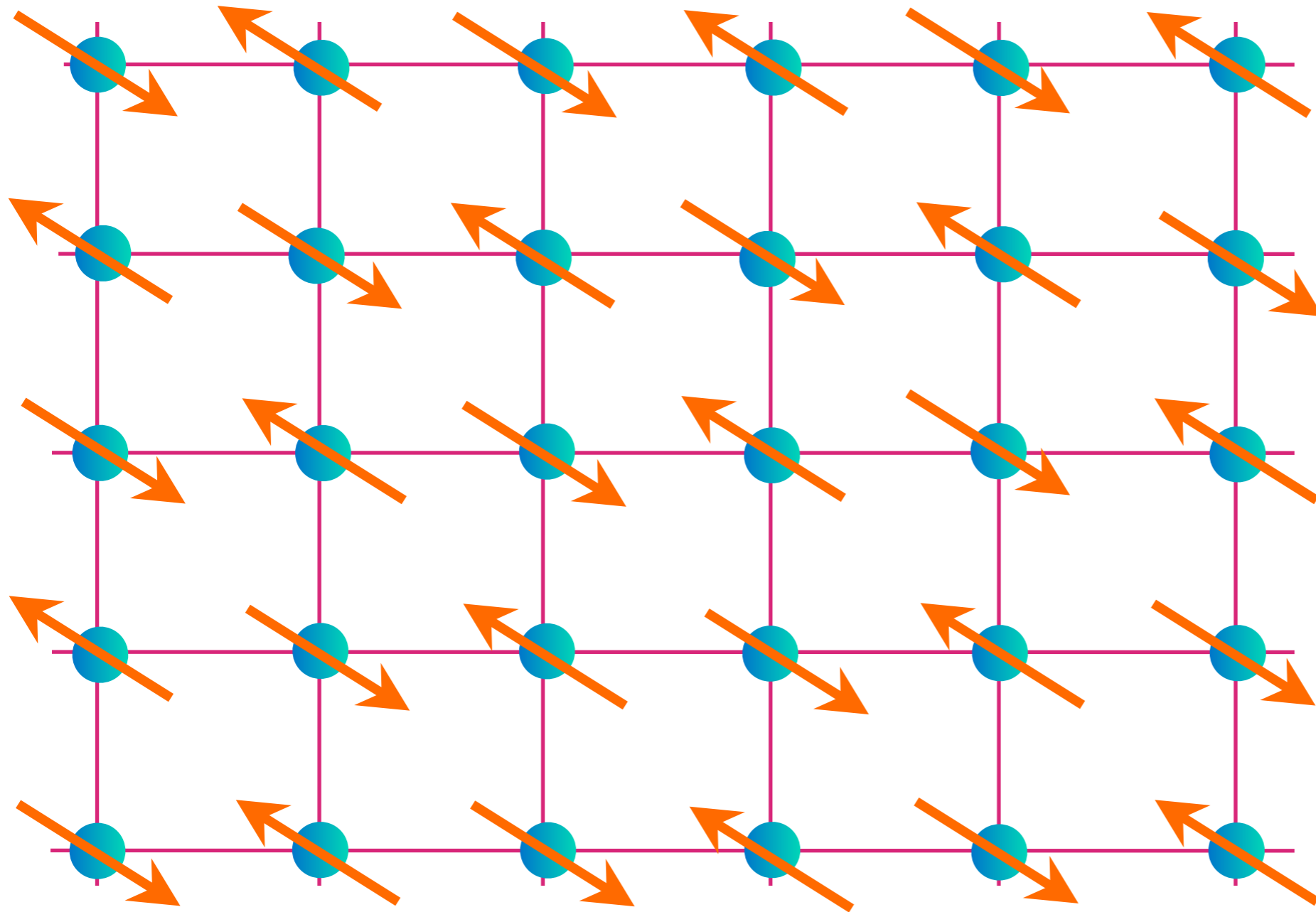
High temperature superconductors



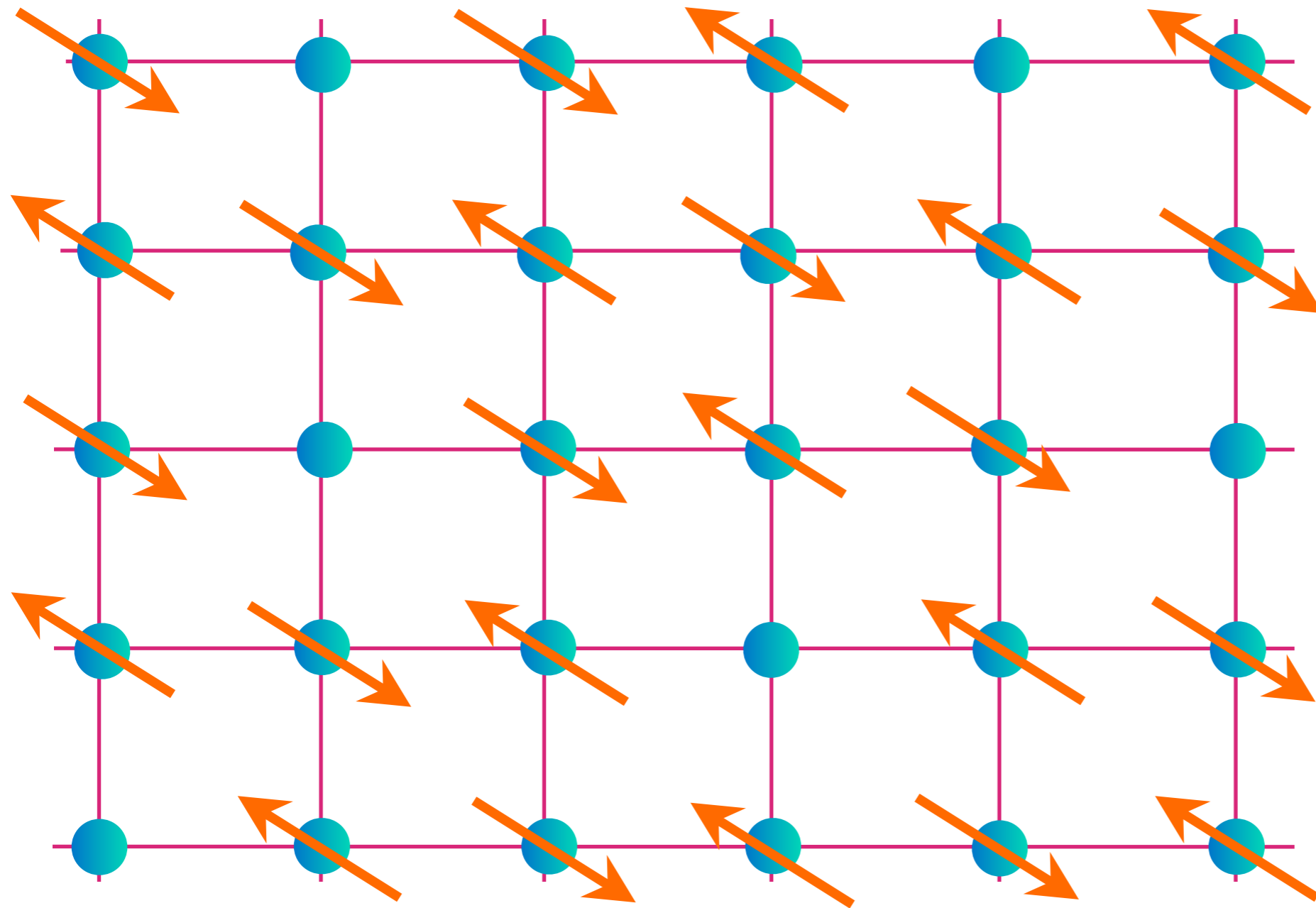
High temperature superconductors



Square lattice of Cu sites

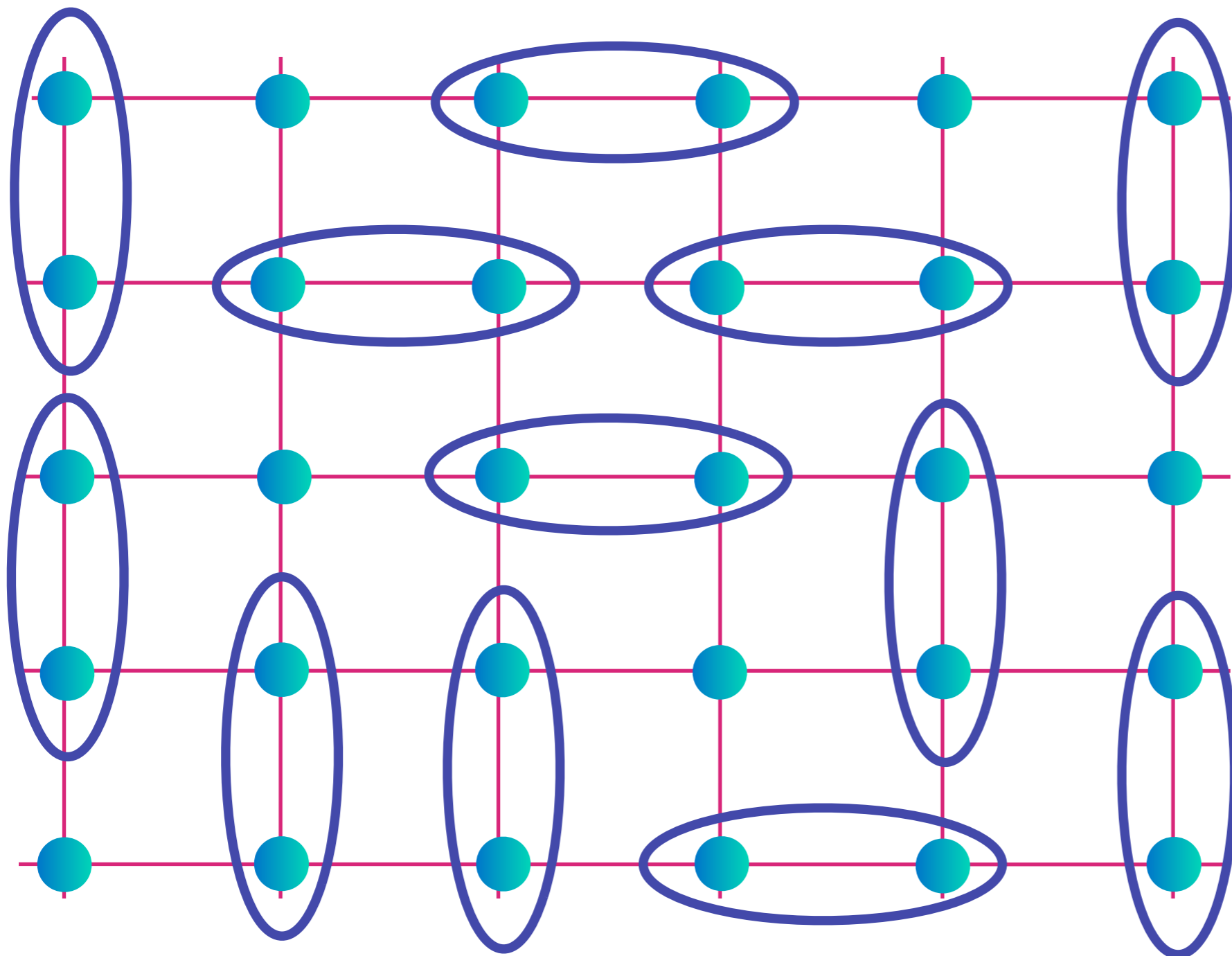


Square lattice of Cu sites



I. Remove
some electrons

Square lattice of Cu sites

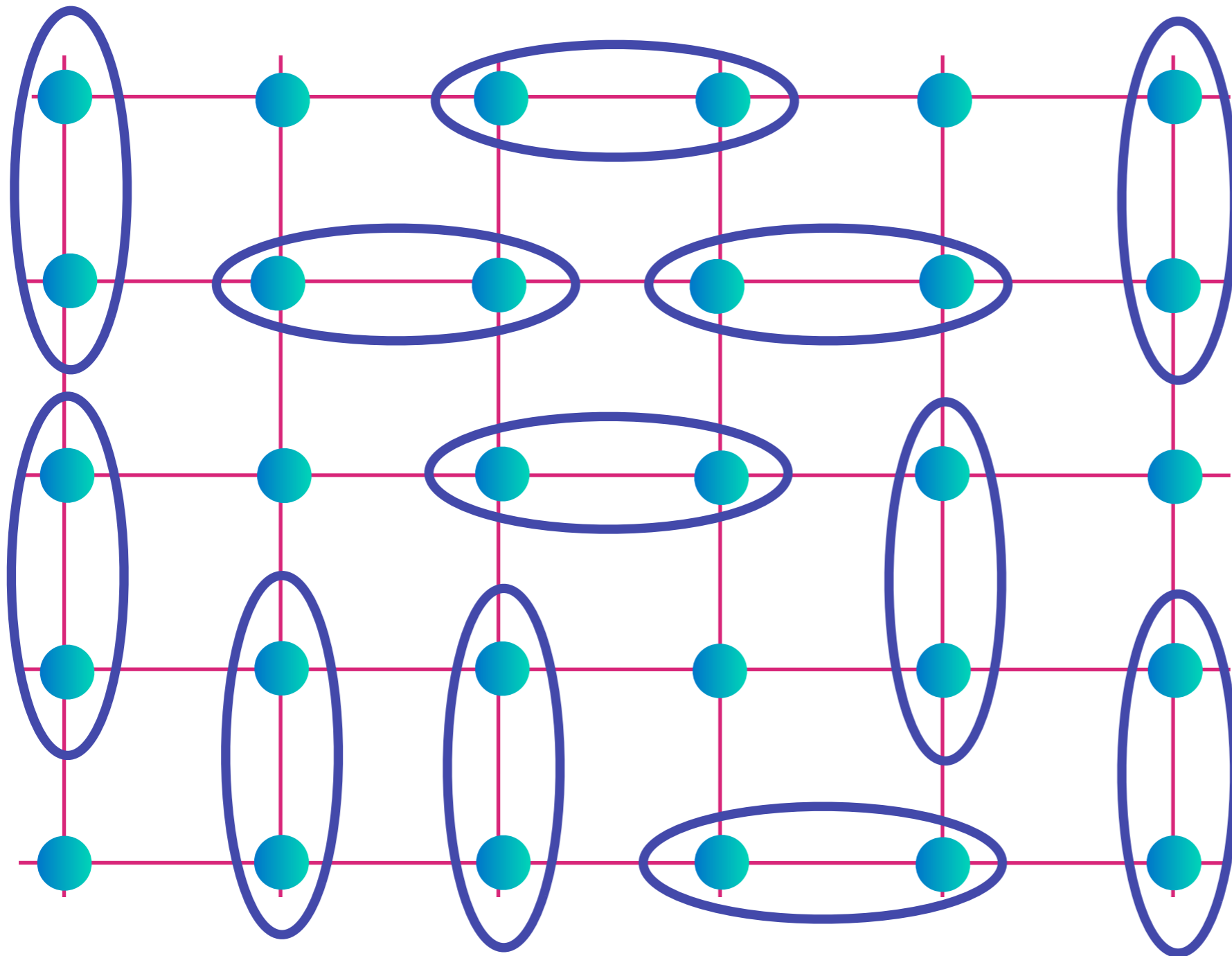


1. Remove
some electrons

2. Electrons
entangle into
chemical bonds

$$\text{[Diagram of two sites in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites



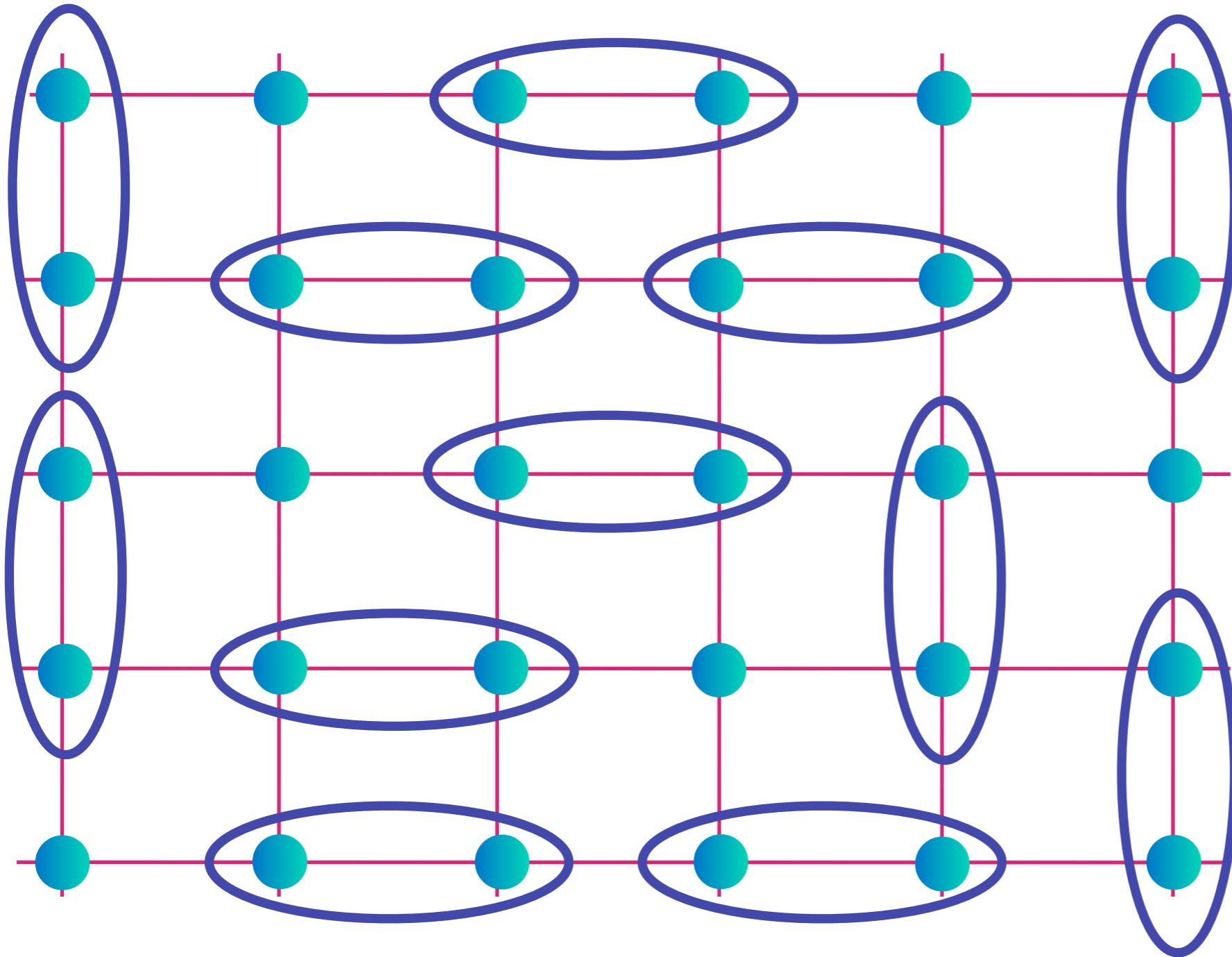
1. Remove some electrons

2. Electrons entangle into chemical bonds

3. Chemical bonds undergo Bose-Einstein condensation

$$\text{Oval with two sites} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Square lattice of Cu sites



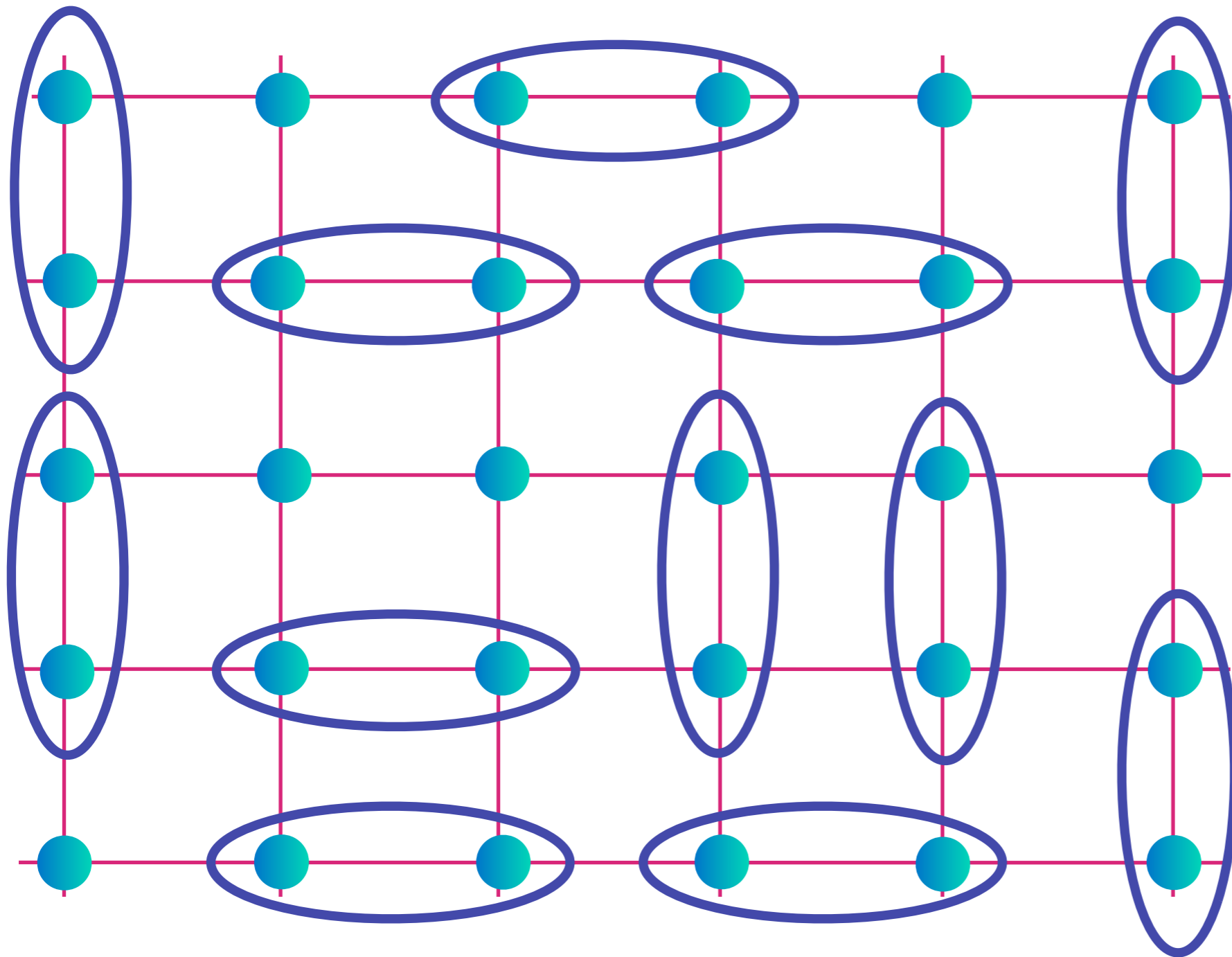
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Square lattice of Cu sites



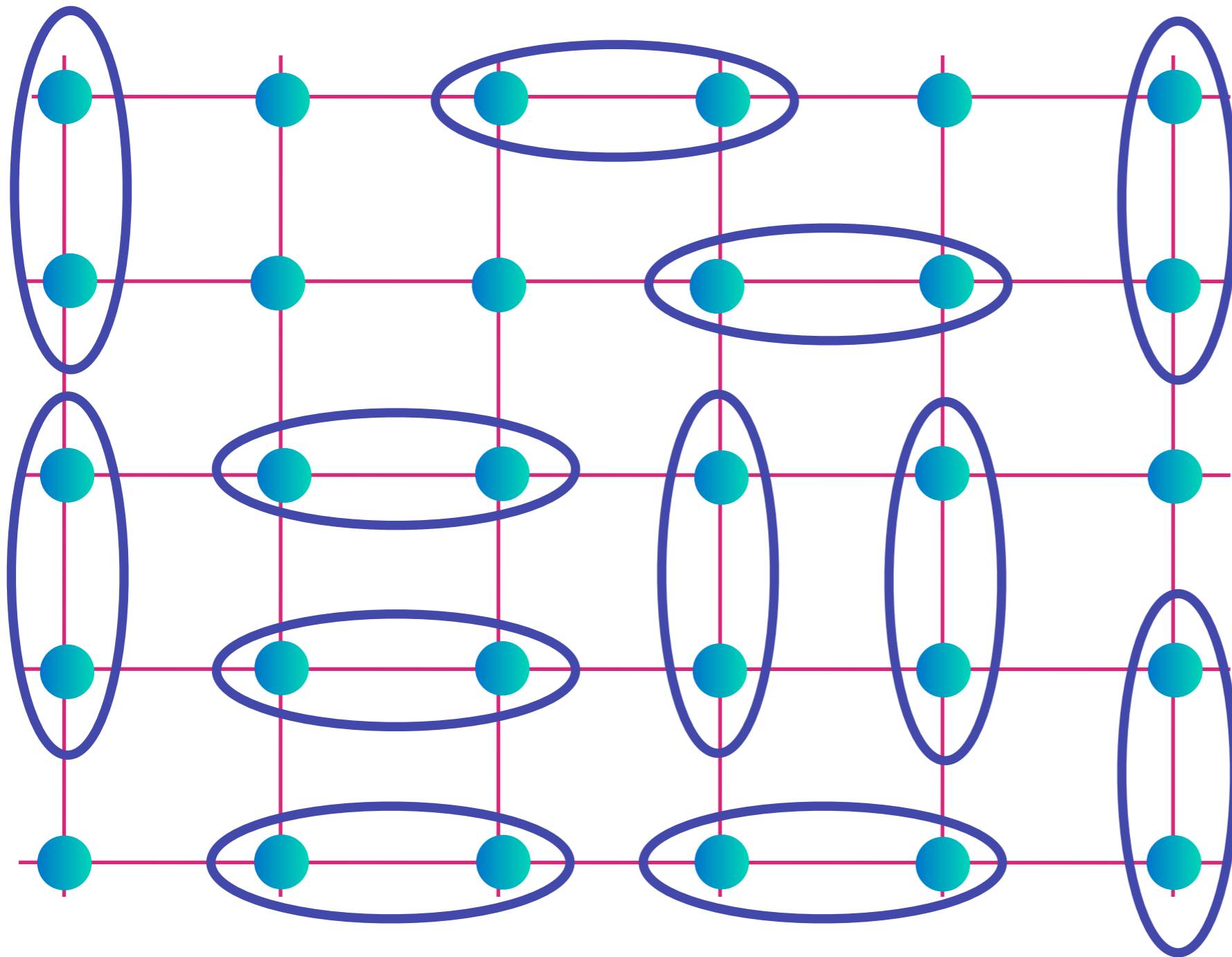
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Square lattice of Cu sites



1. Remove some electrons

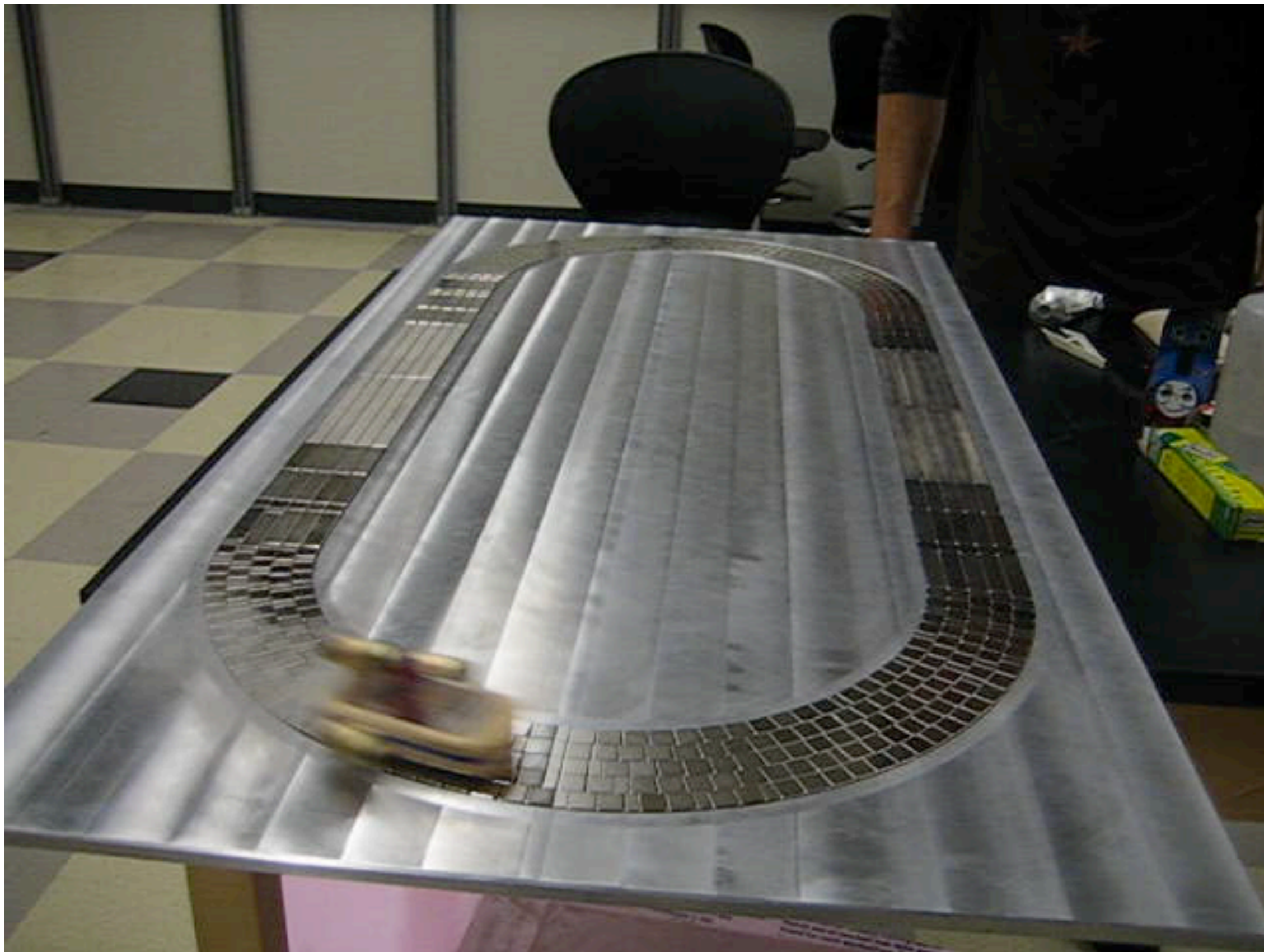
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Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



Nd-Fe-B magnets, YBaCuO superconductor

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**Quantum
superposition and
entanglement**

Superconductivity

**Black Holes and
String Theory**

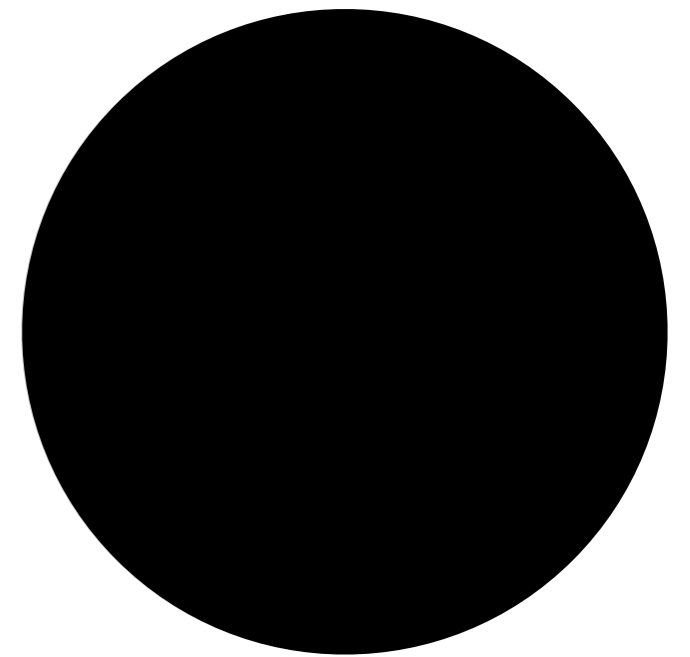
**Quantum
superposition and
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**Black Holes and
String Theory**

Black Holes

Objects so massive that light is gravitationally bound to them.

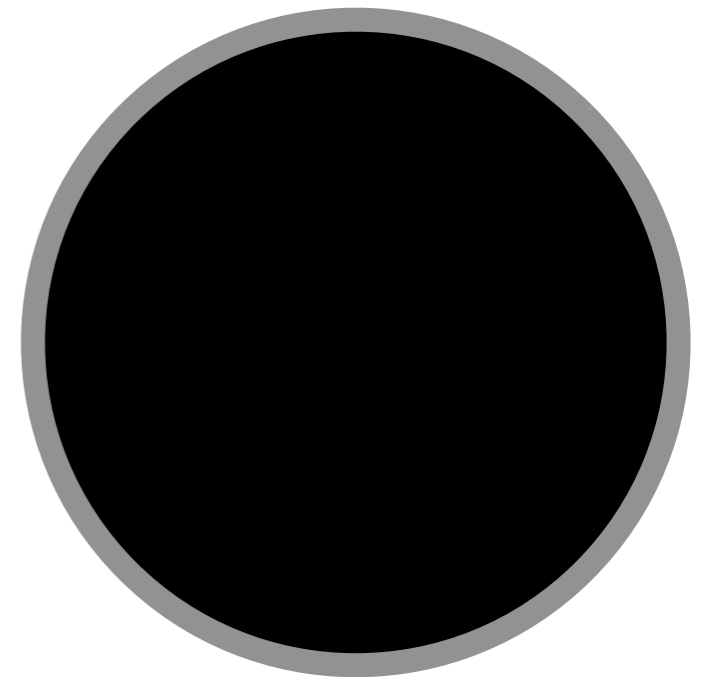


Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

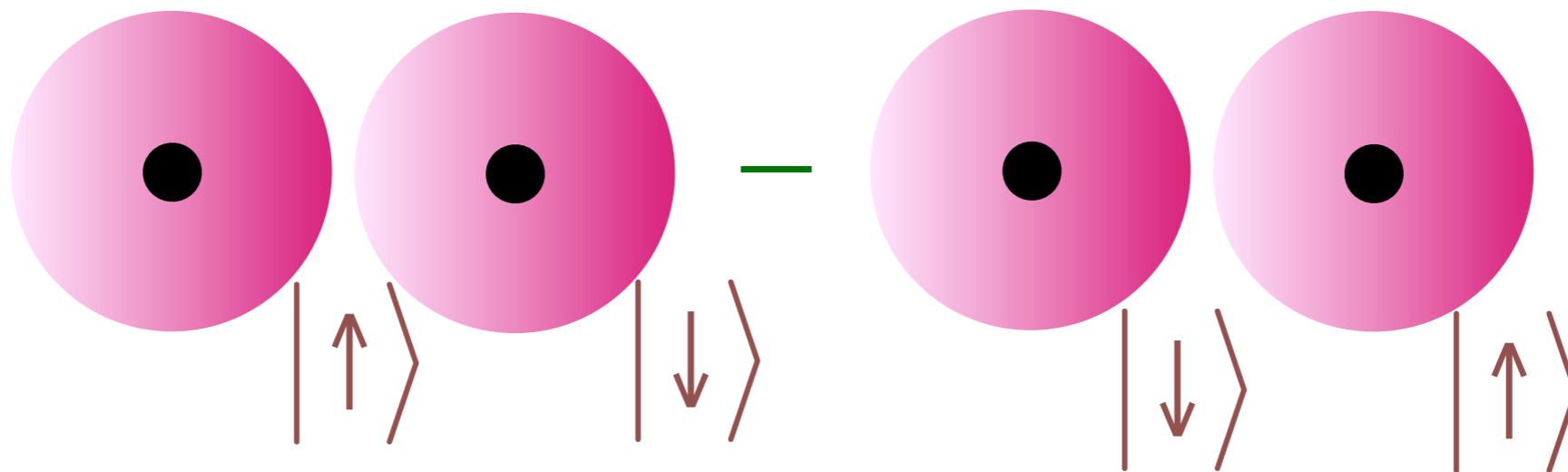
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



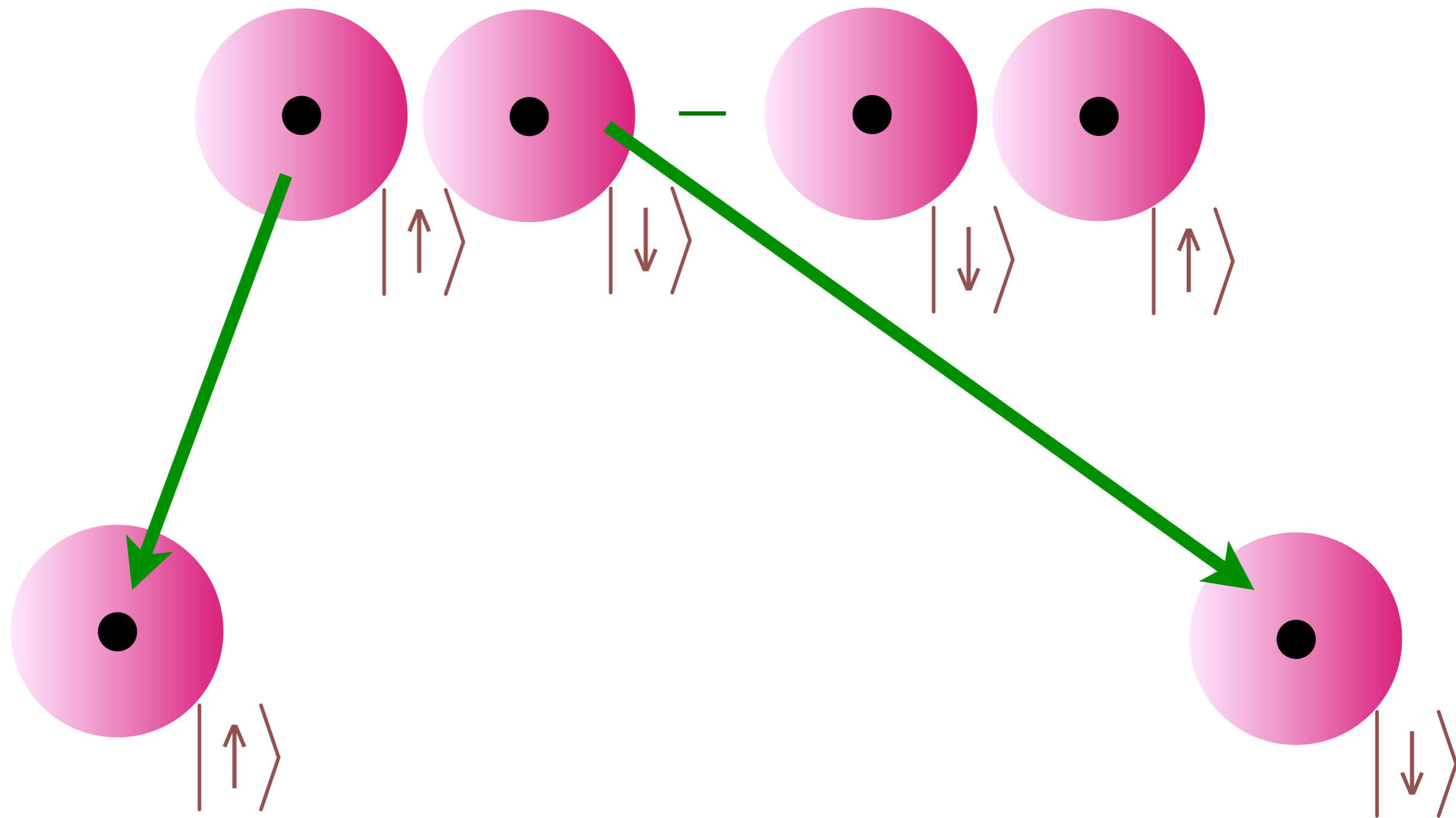
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

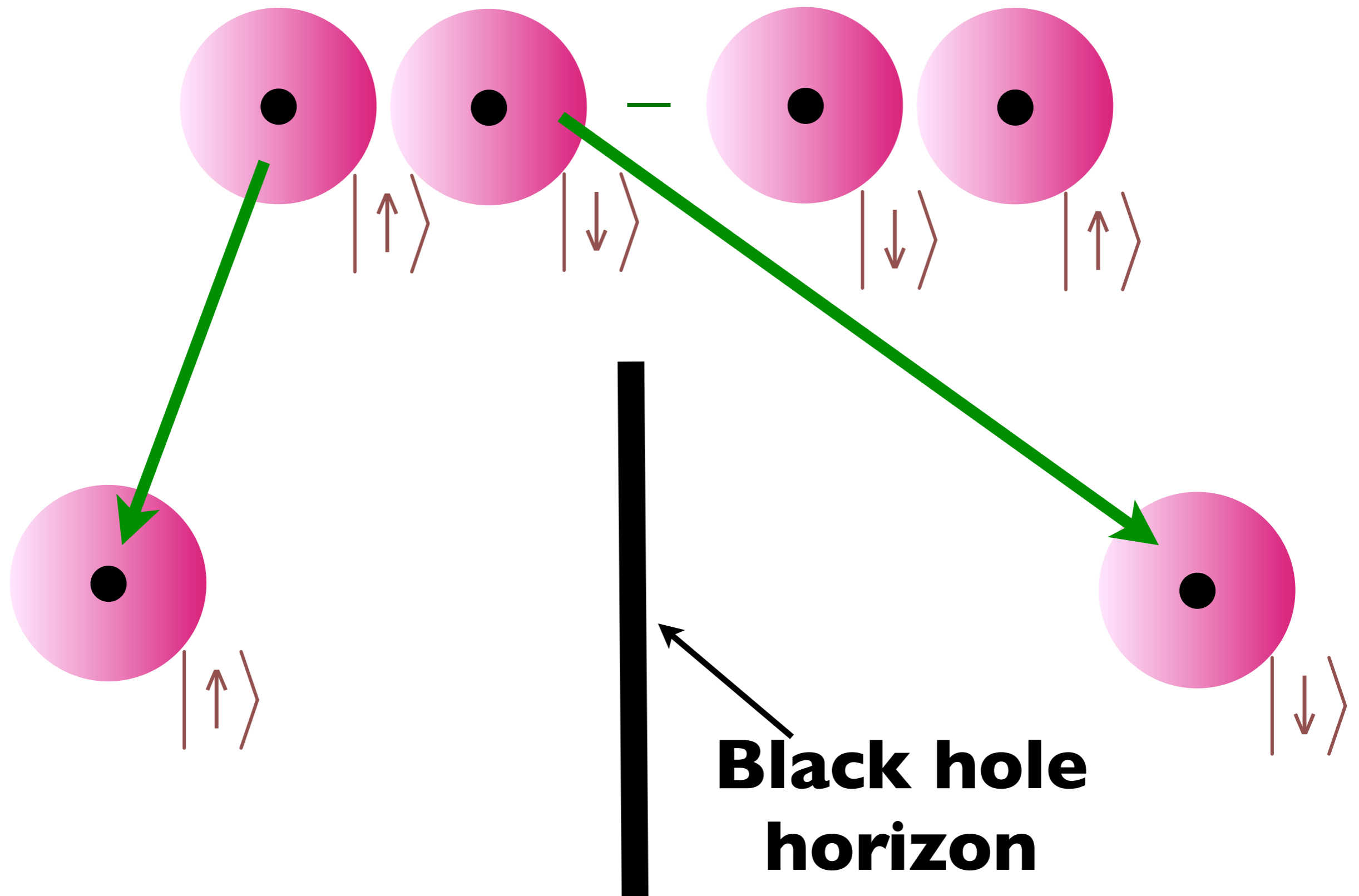
Quantum Entanglement across a black hole horizon



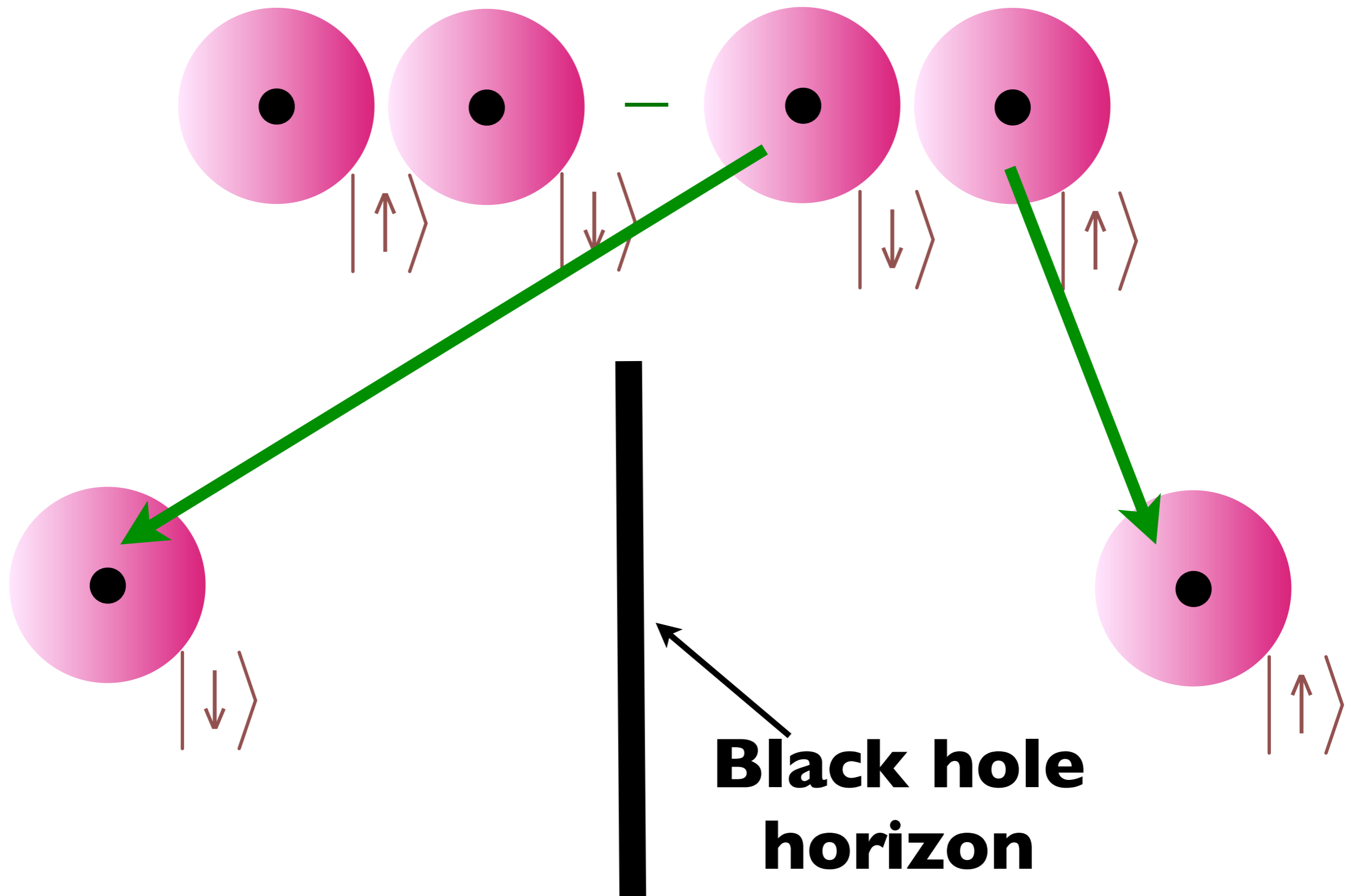
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

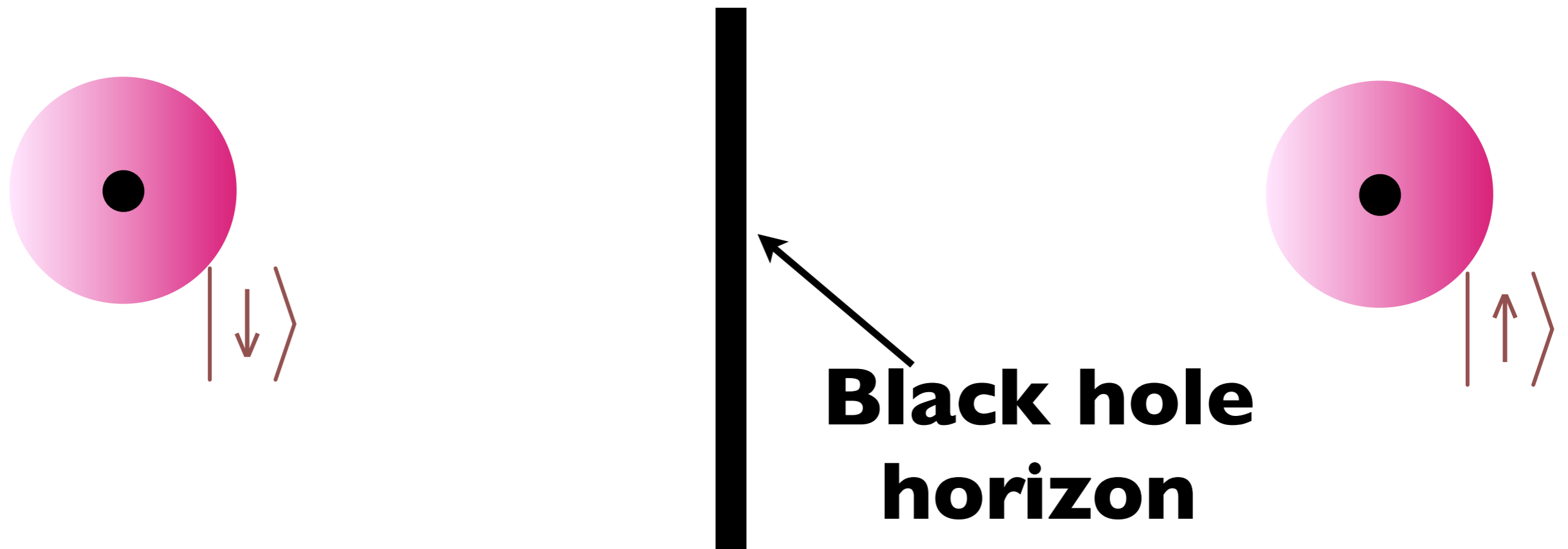


Quantum Entanglement across a black hole horizon



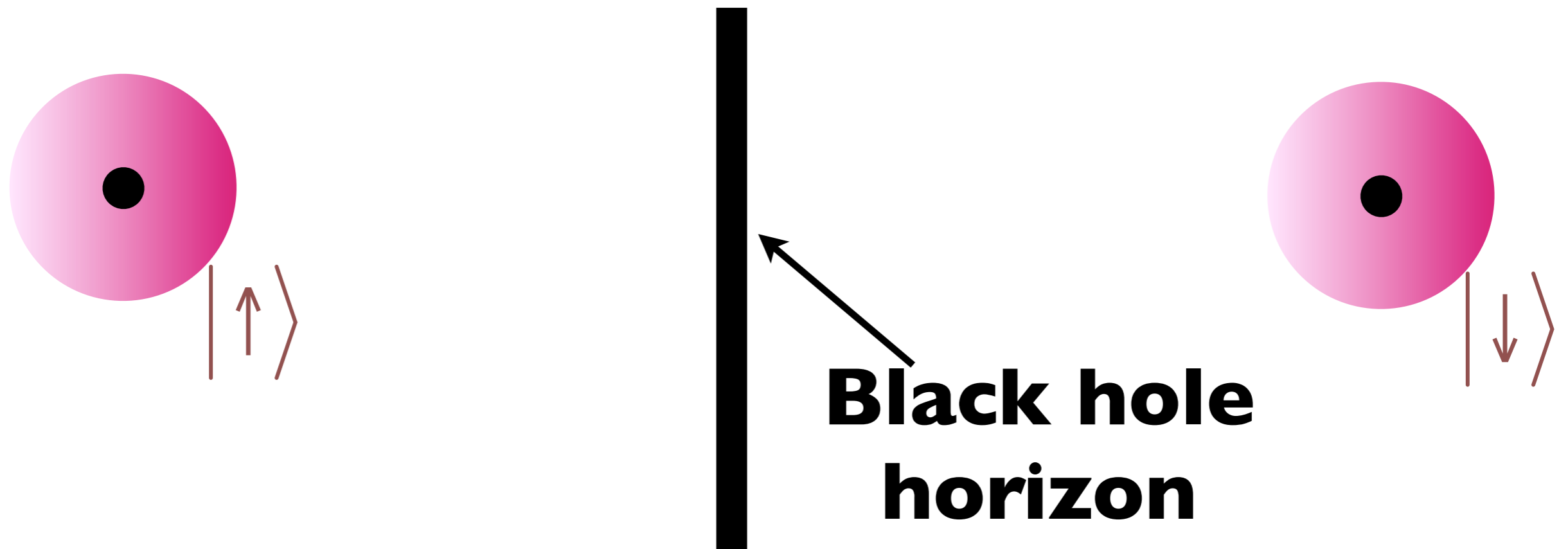
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

**Quantum
superposition and
entanglement**

Superconductivity

**Black Holes and
String Theory**

**Quantum
superposition and
entanglement**

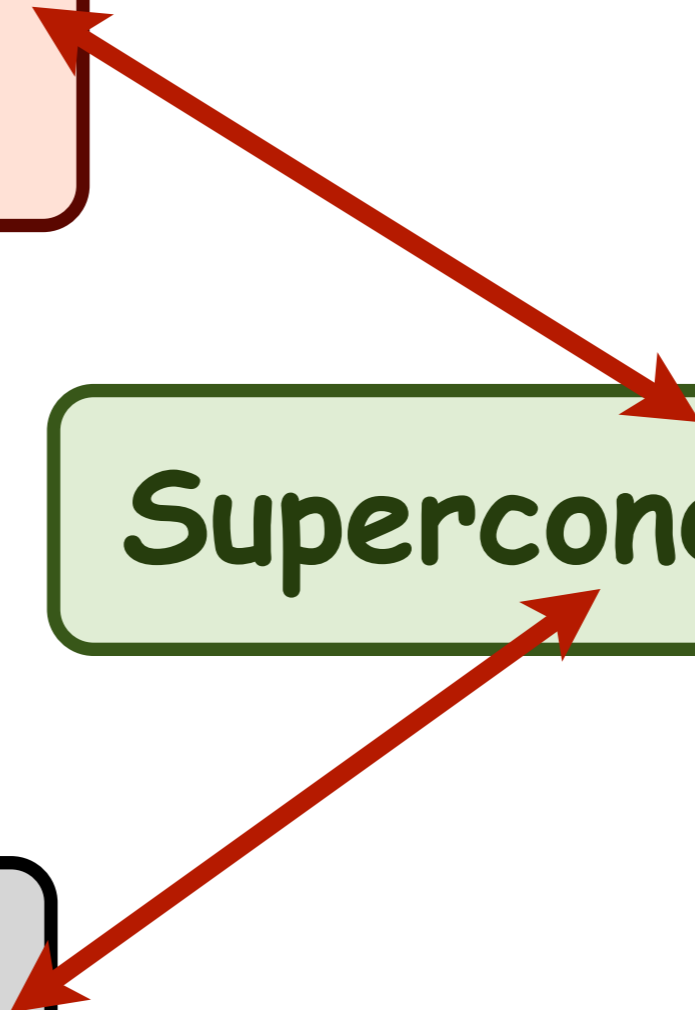
Superconductivity

**Black Holes and
String Theory**

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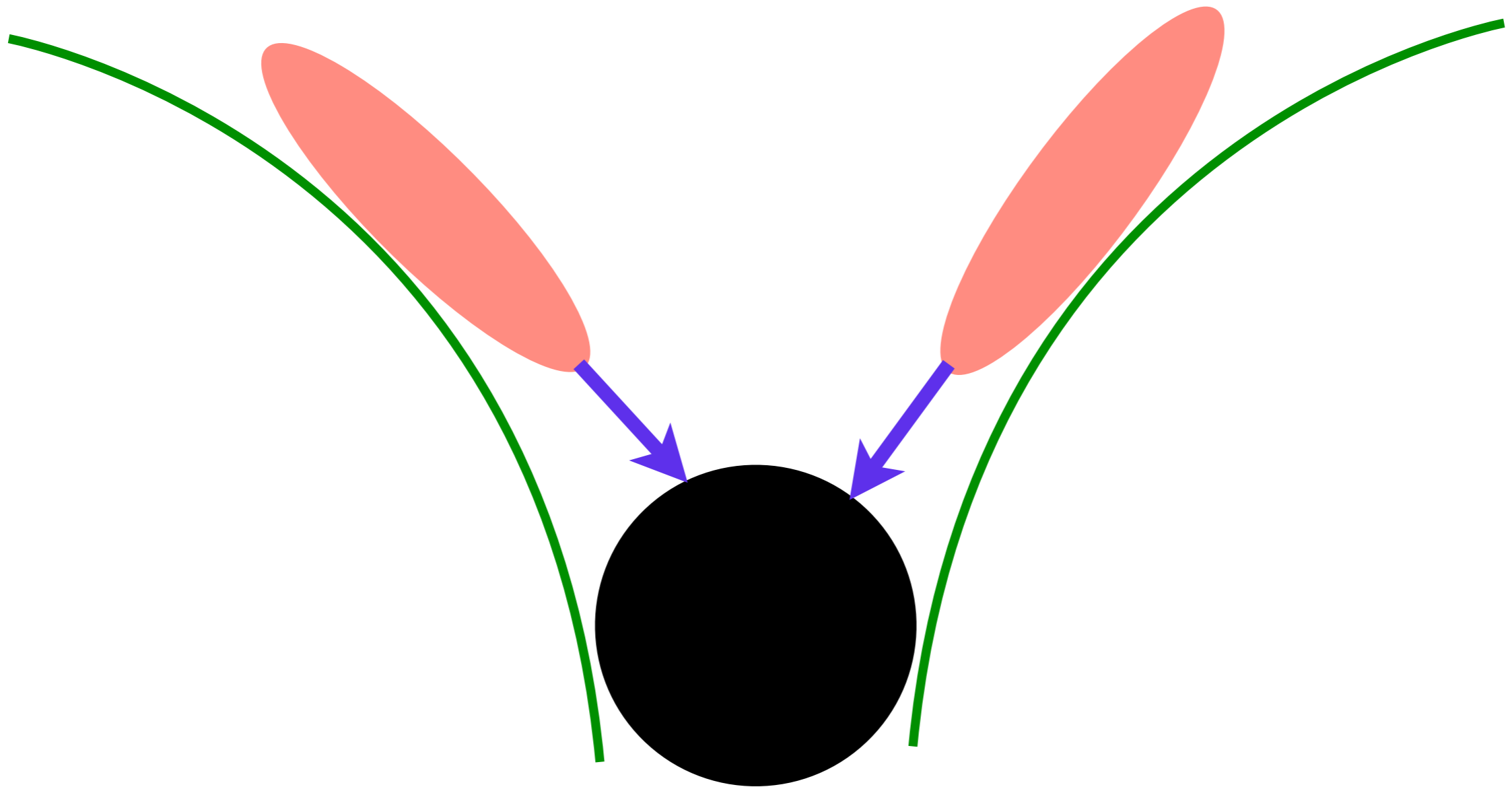
Superconductivity

**Black Holes and
String Theory**



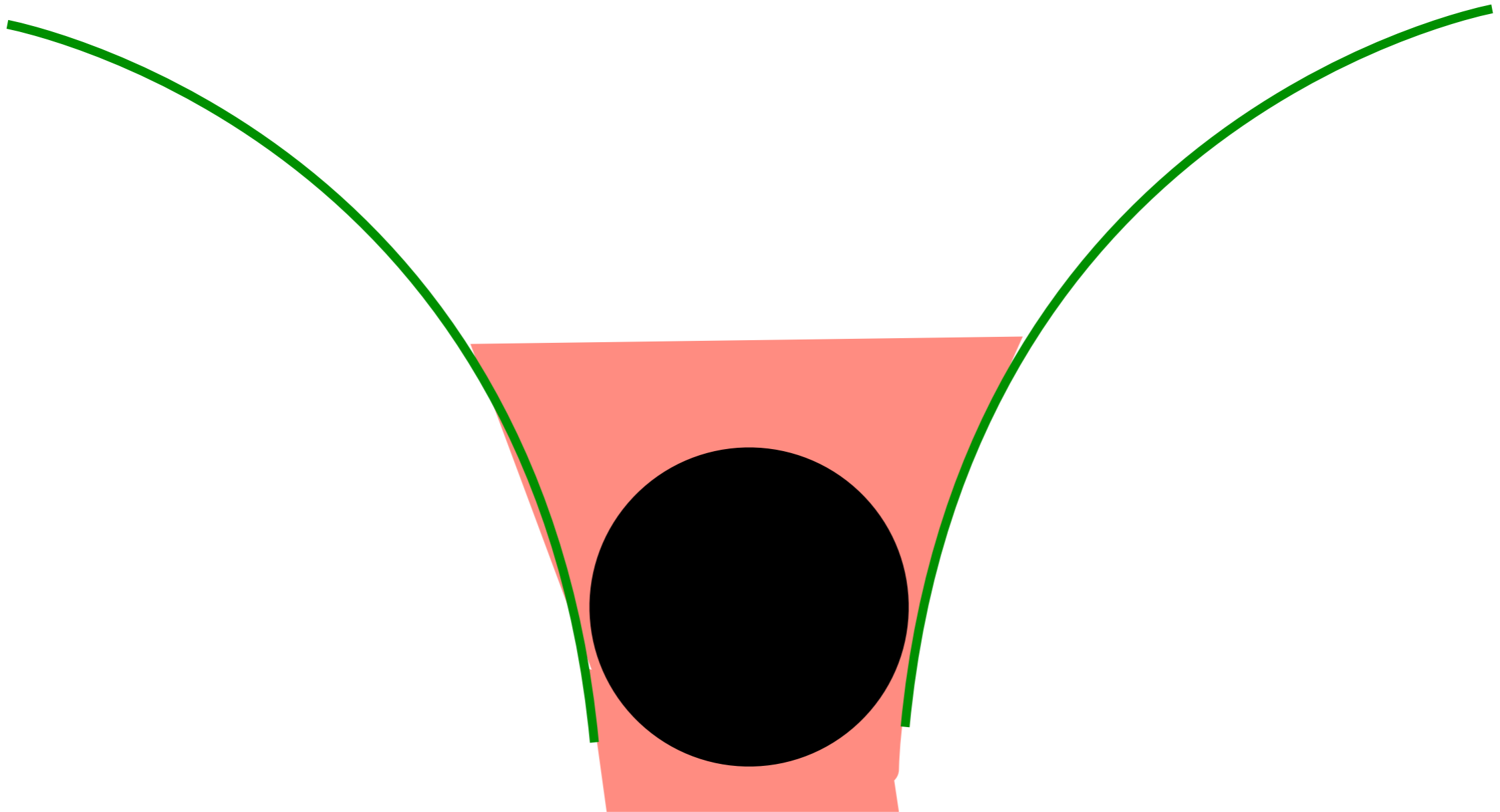
Superconducting Black Holes

Add electrical charge to a black hole in a curved spacetime: initially the charges fall past the horizon into the black hole



Superconducting Black Holes

However, eventually there is a balance between the gravitational forces pulling the charges into the black hole, and the repulsive electrical forces which push them out, and the resulting state is a superconductor !



More generally, string theory shows that there is a deep correspondence between the states of a black hole, and the quantum phases of matter (AdS/CFT correspondence)

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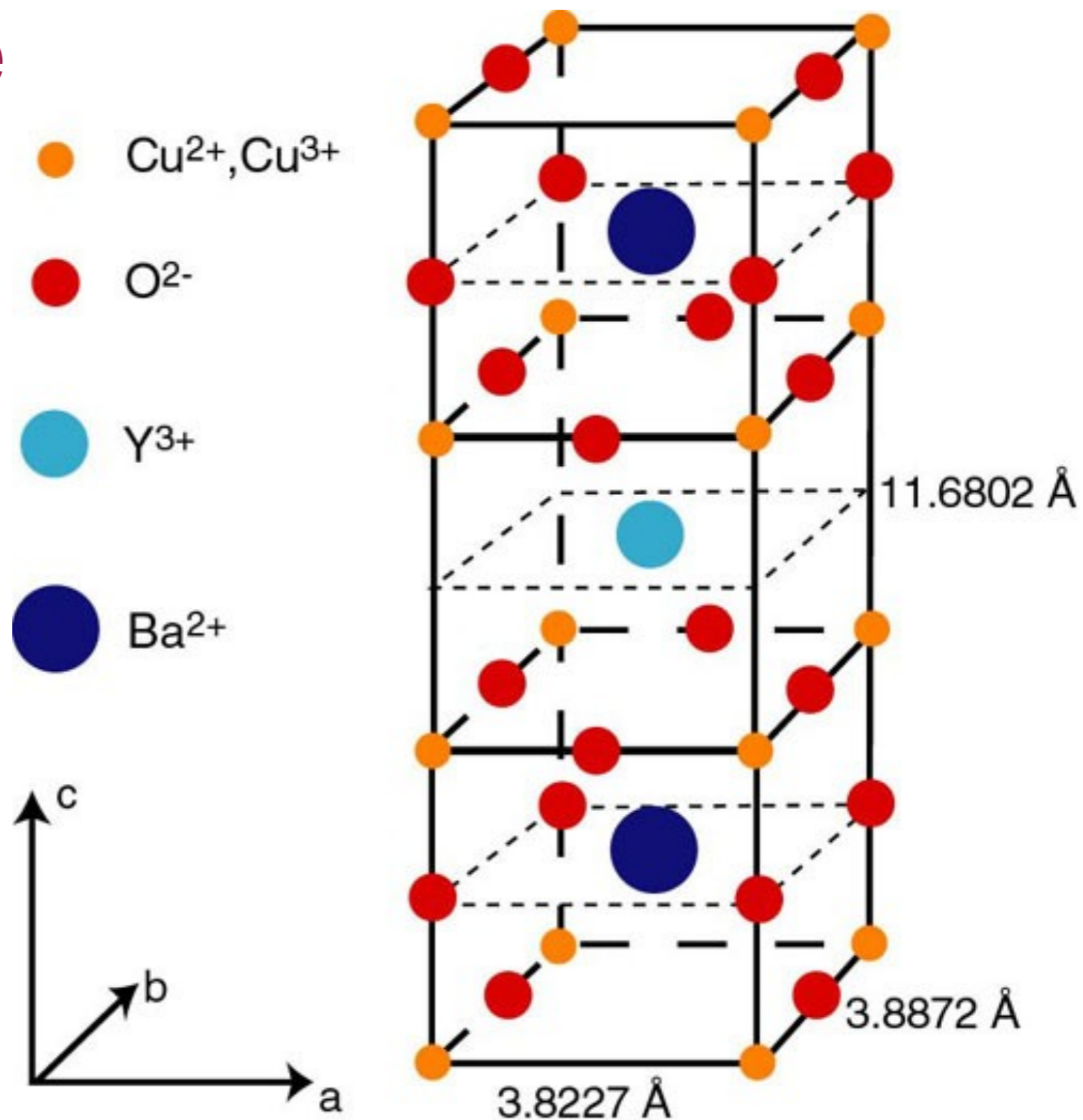
This has helped enrich our understanding of the physics of black holes, and also of the possible quantum phases of electrons in crystals

Quantum phases we do
not understand yet:

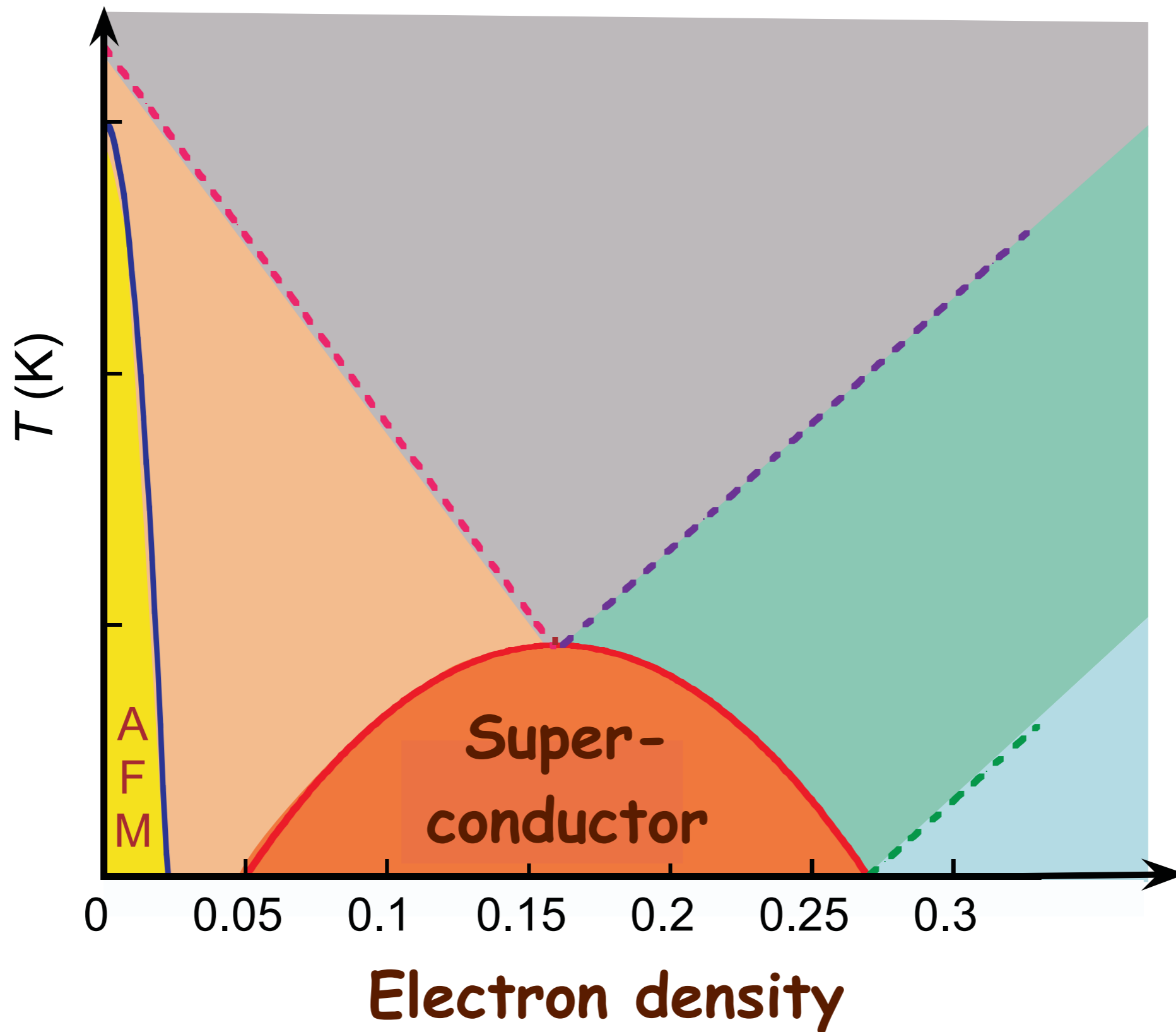
Quantum phases we do
not understand yet:

The phases around the high
temperature superconductor YBCO
as we vary the density of electrons

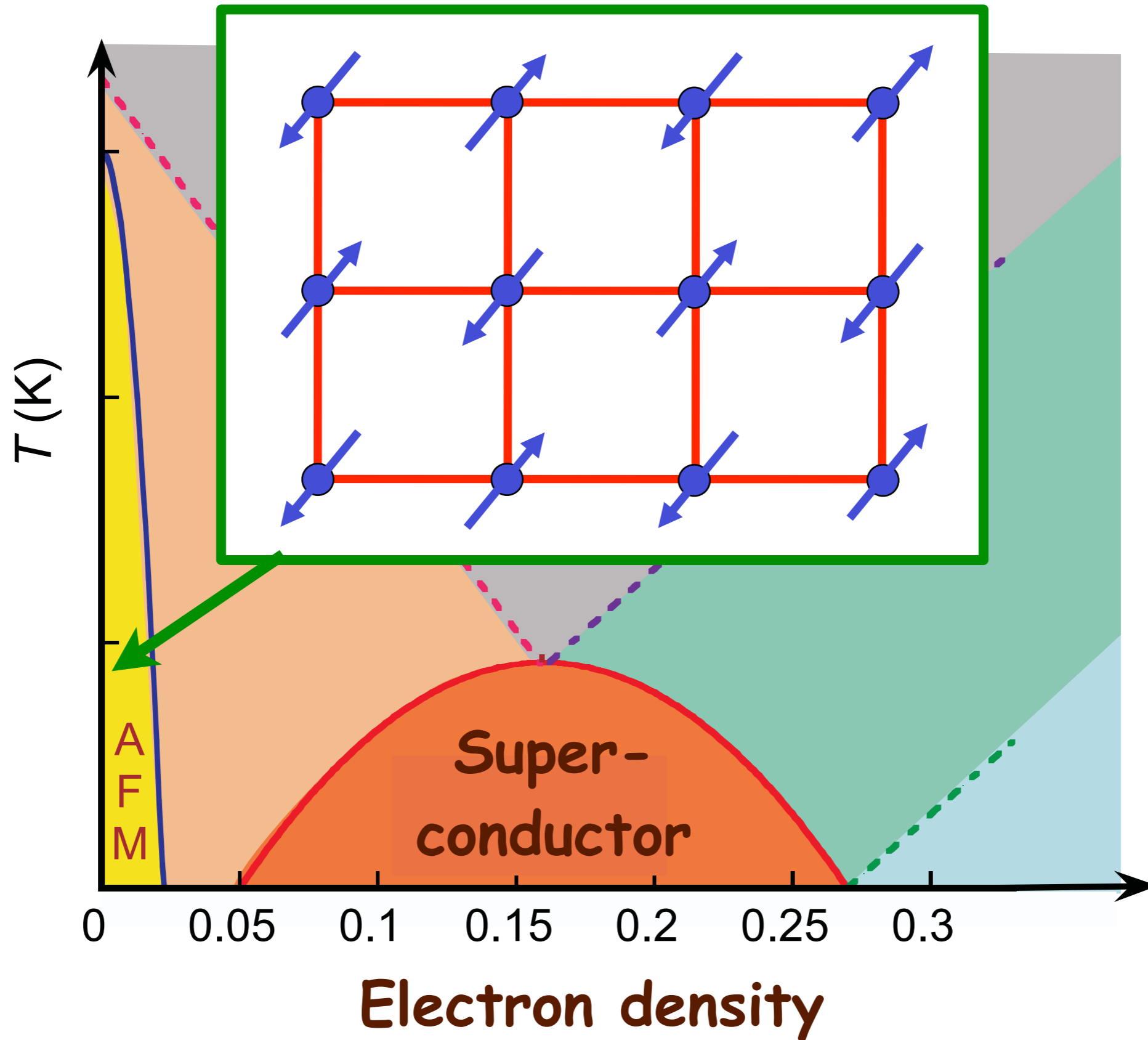
High temperature superconductors



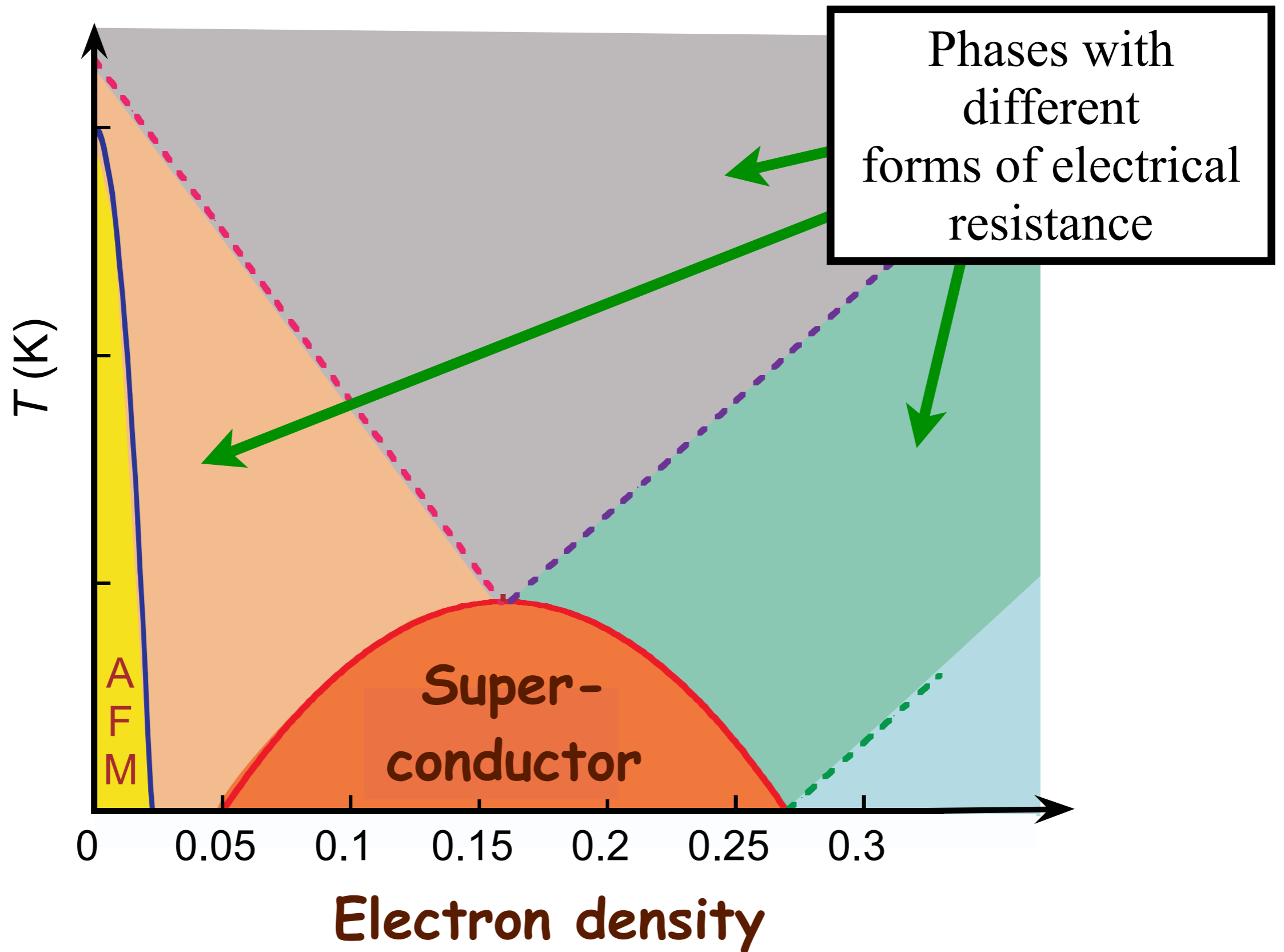
Phase diagram of YBCO



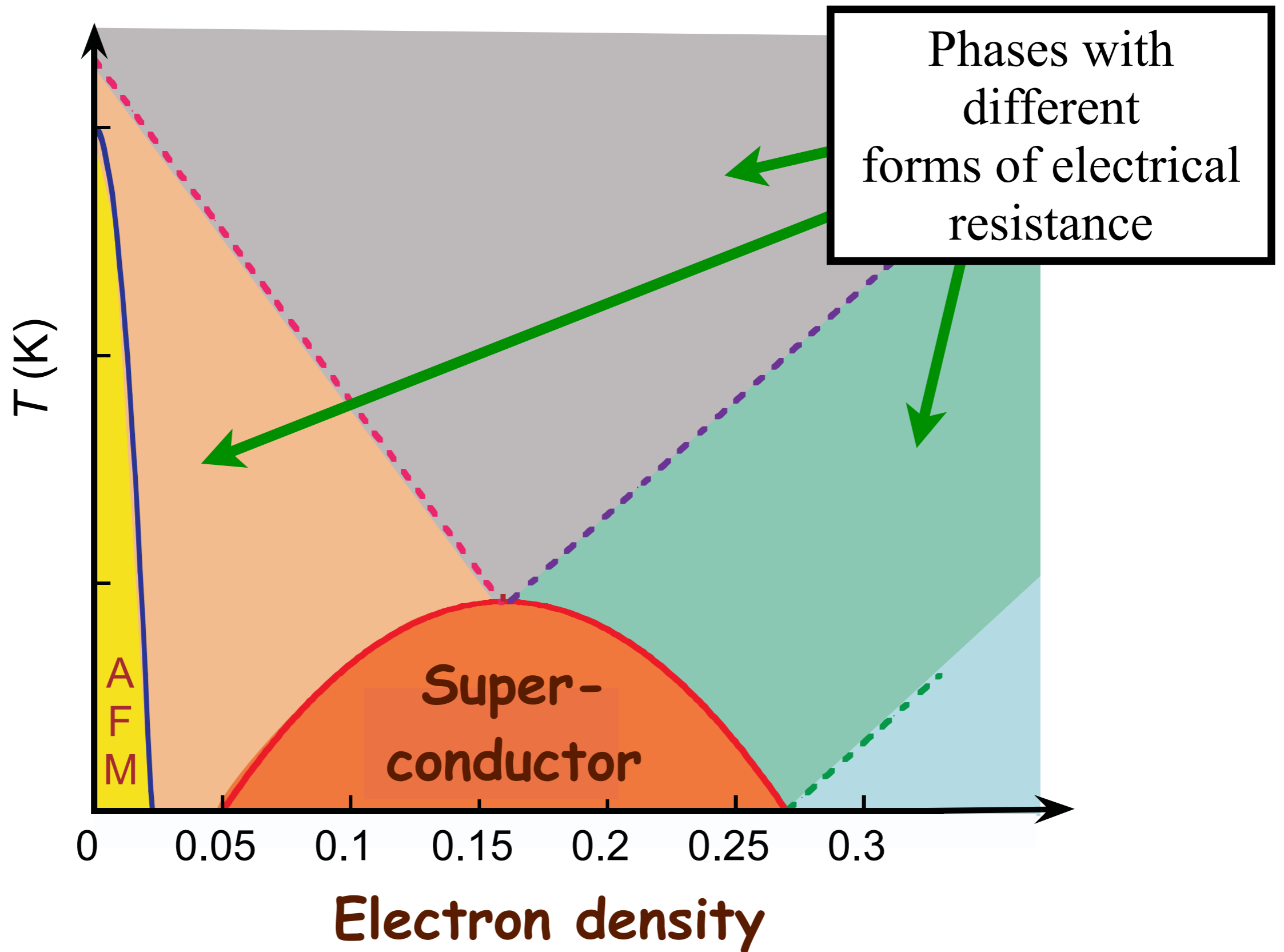
Phase diagram of YBCO



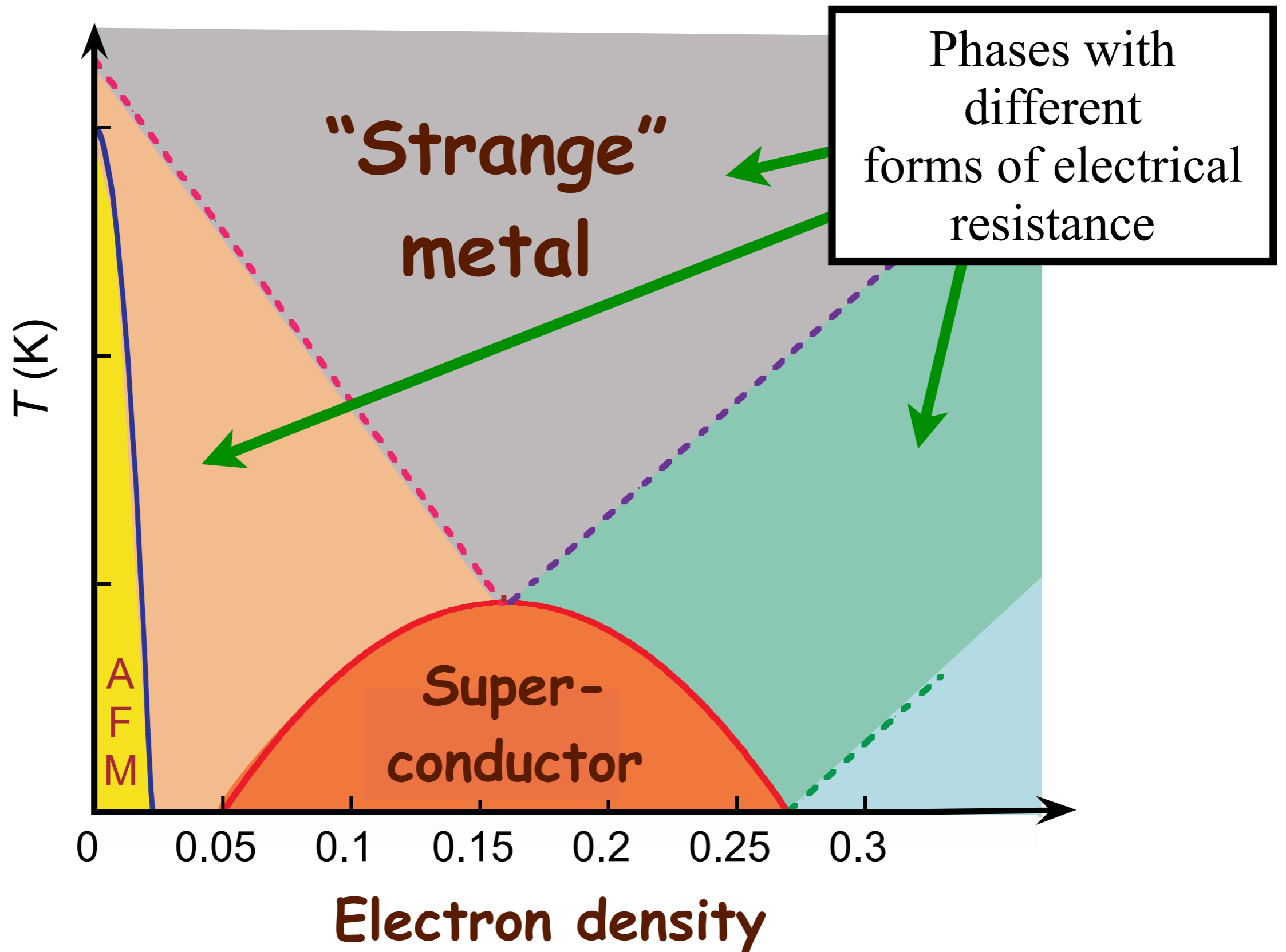
Phase diagram of YBCO



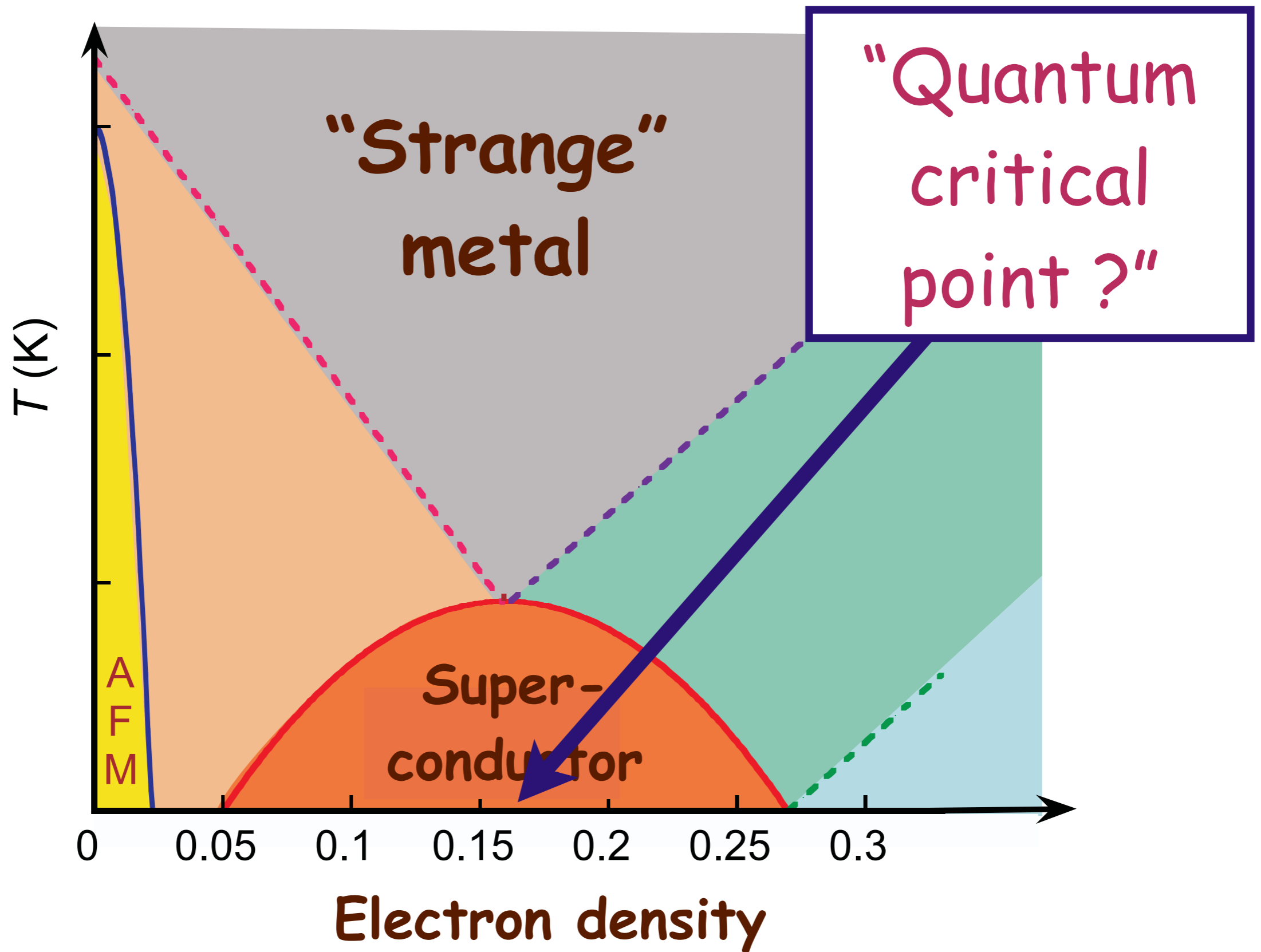
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Phase diagram of YBCO

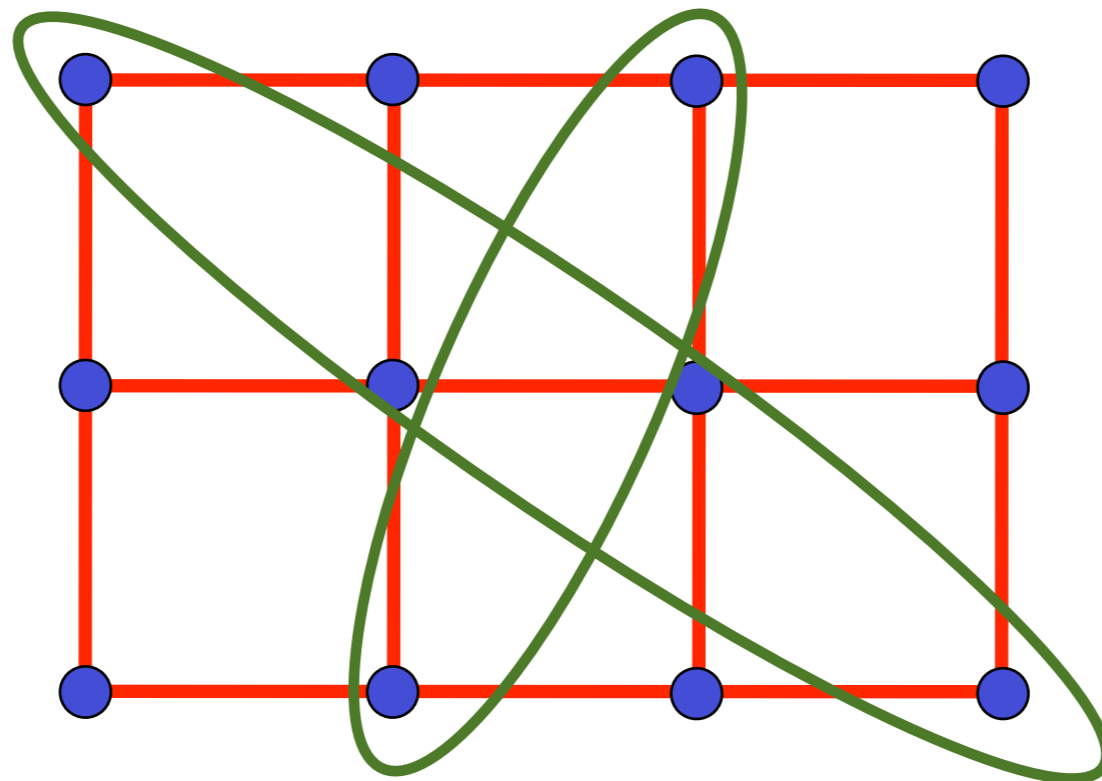


Phase diagram of YBCO



A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range

A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range



A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range

Long-range quantum entanglement is also found in string theories of black holes

A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range

Can string theory improve our understanding of quantum critical points, and of high temperature superconductors like YBCO ?