Quantum criticality
and
high temperature superconductivity

Syracuse University
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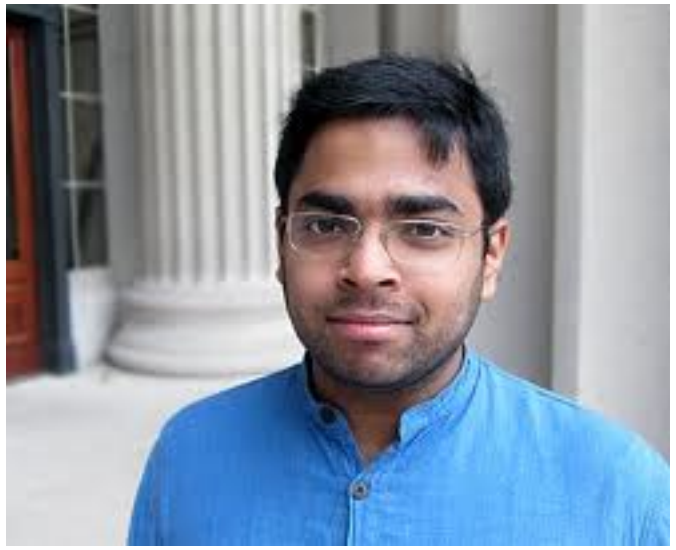


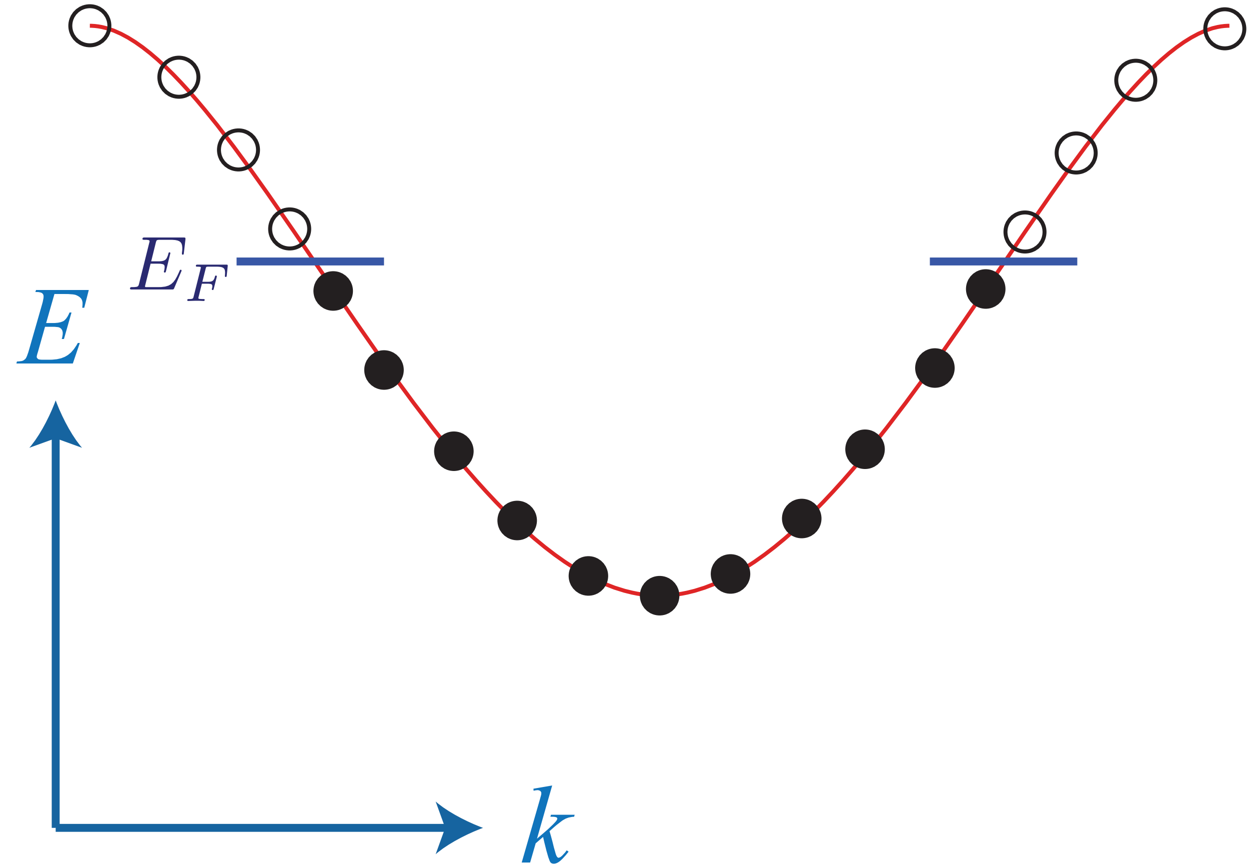
Raghu Mahajan
Stanford

Matthias Punk
Innsbruck

Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

Metals

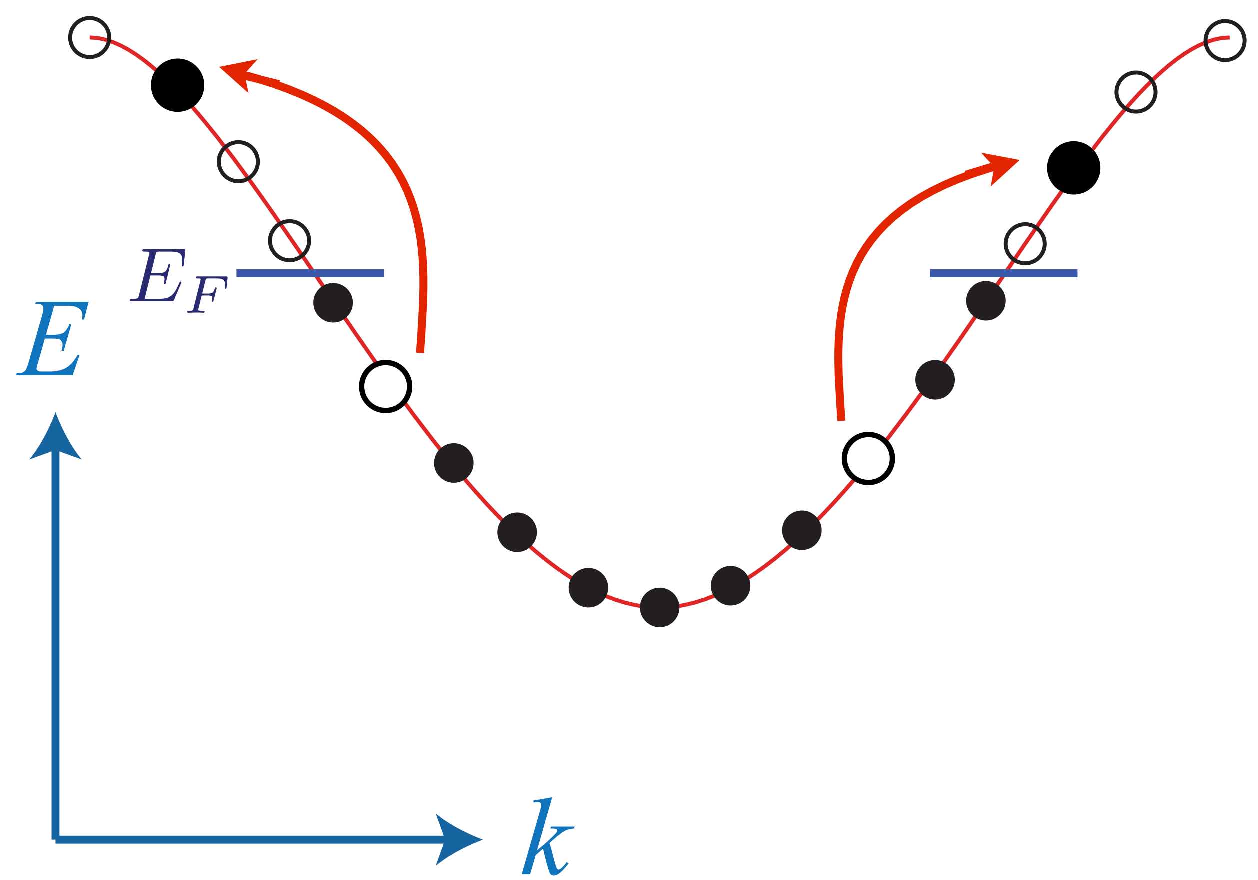


Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Quasiparticle structure of excited states

Metals



Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a *quasiparticle* basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

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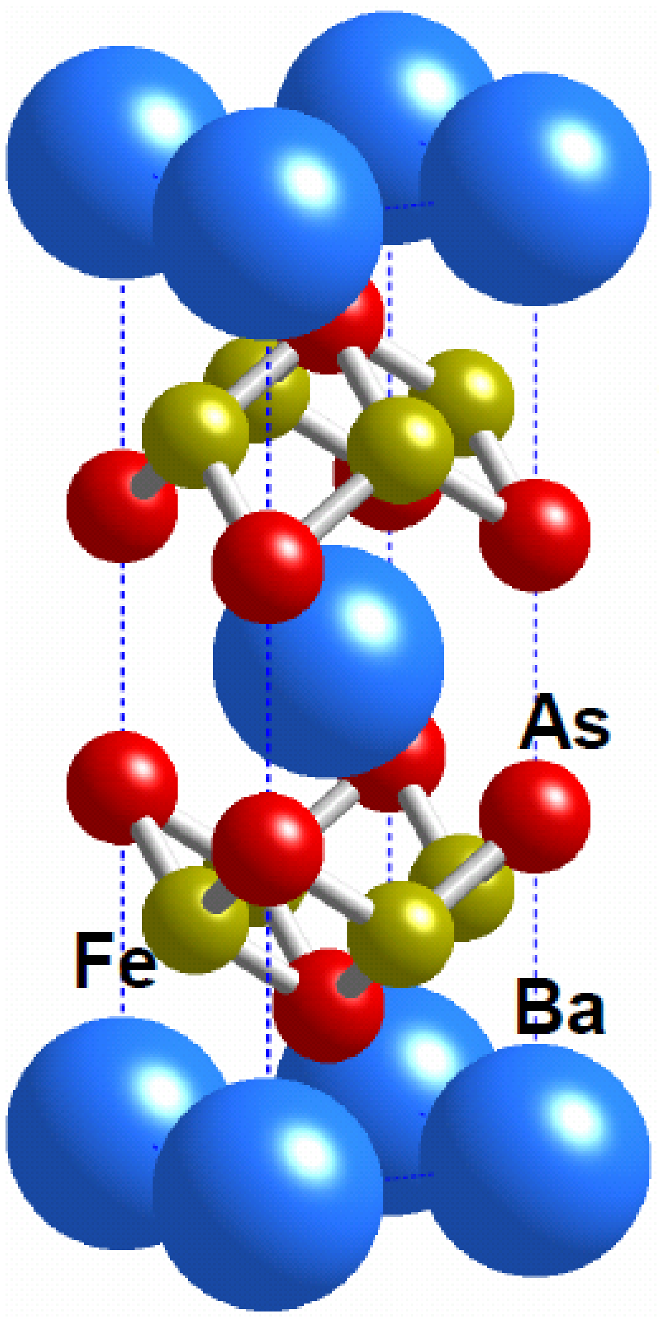
Only 2 examples:

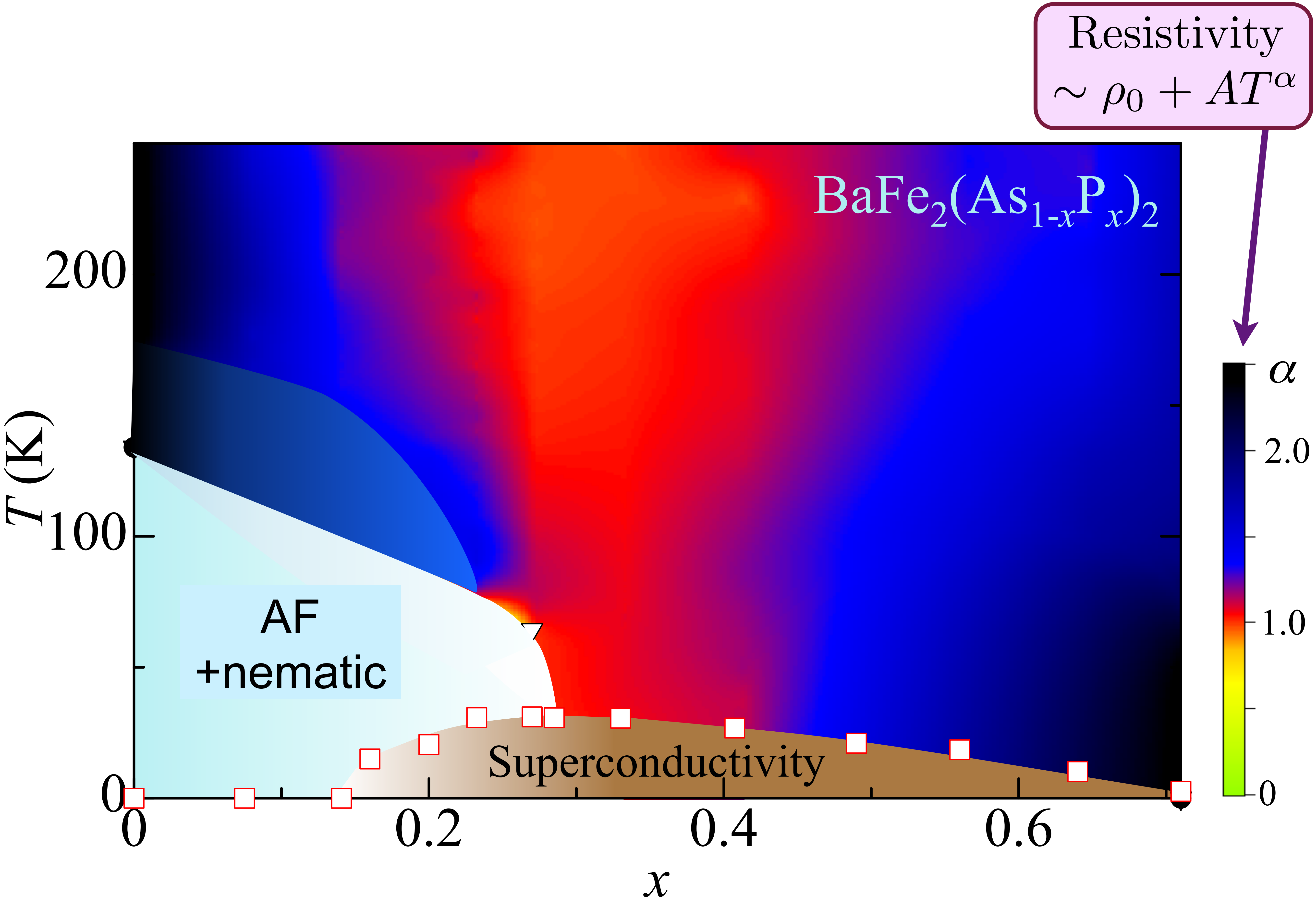
1. Conformal field theories in spatial dimension $d > 1$

2. Quantum critical metals in dimension $d = 2$

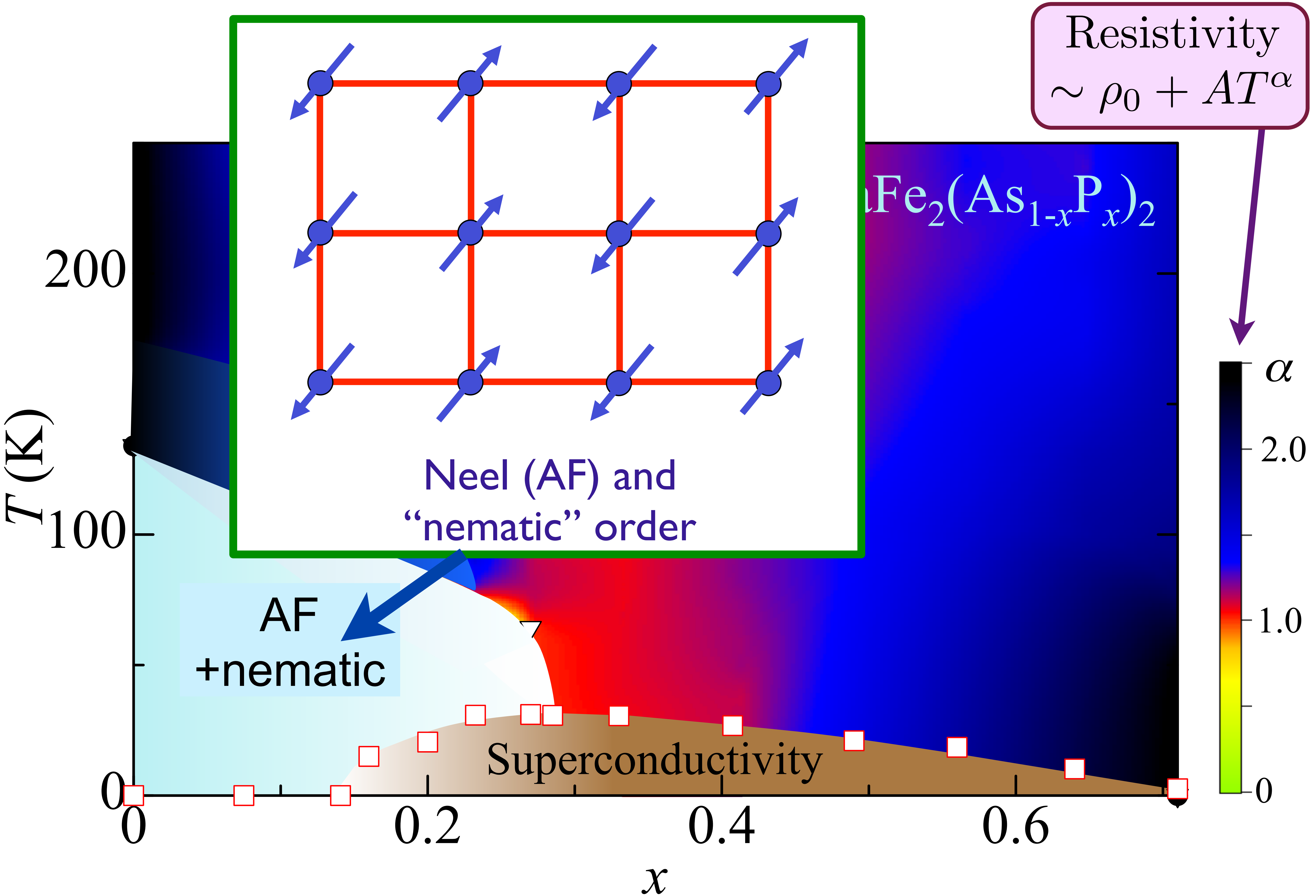
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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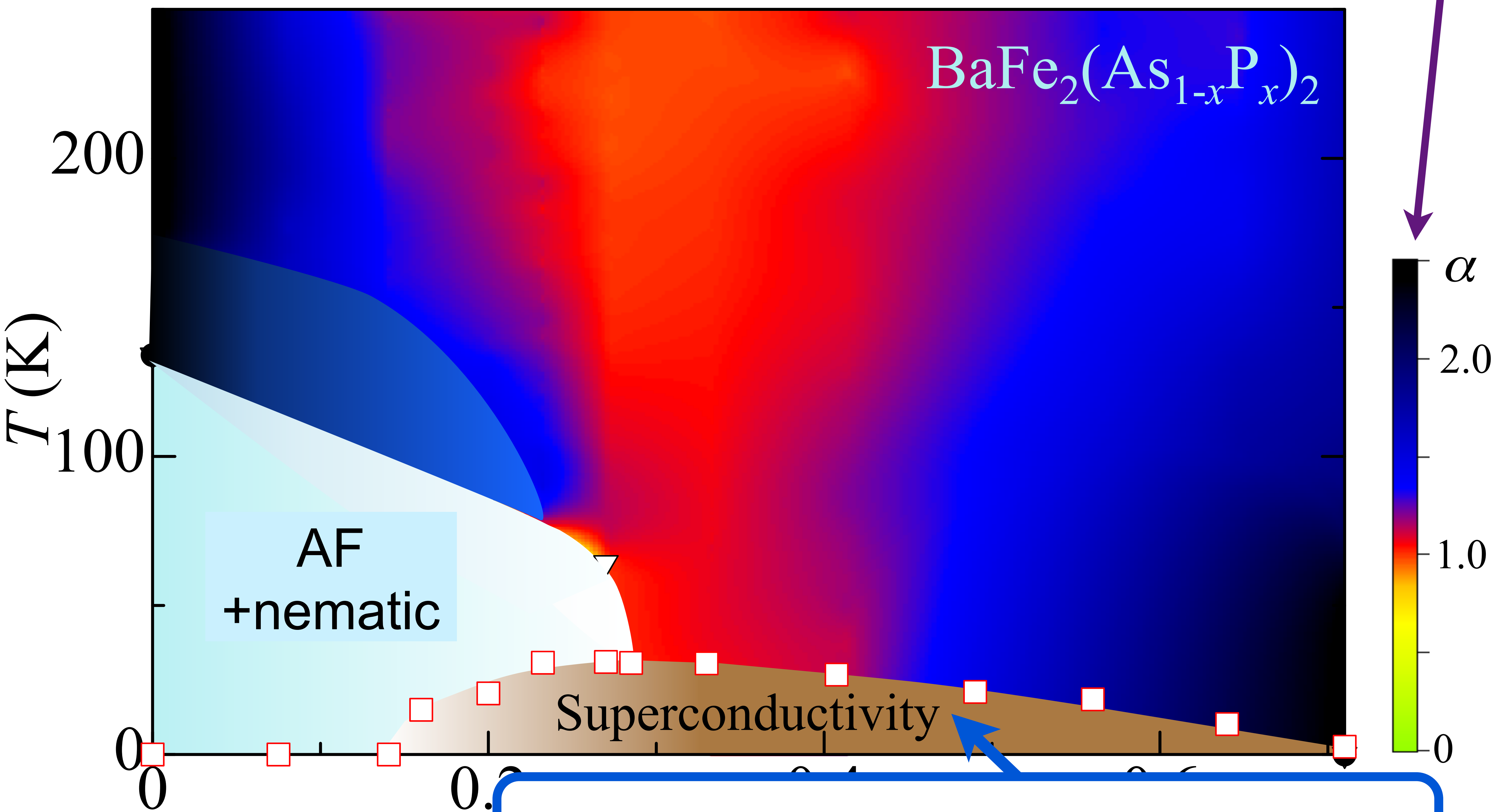
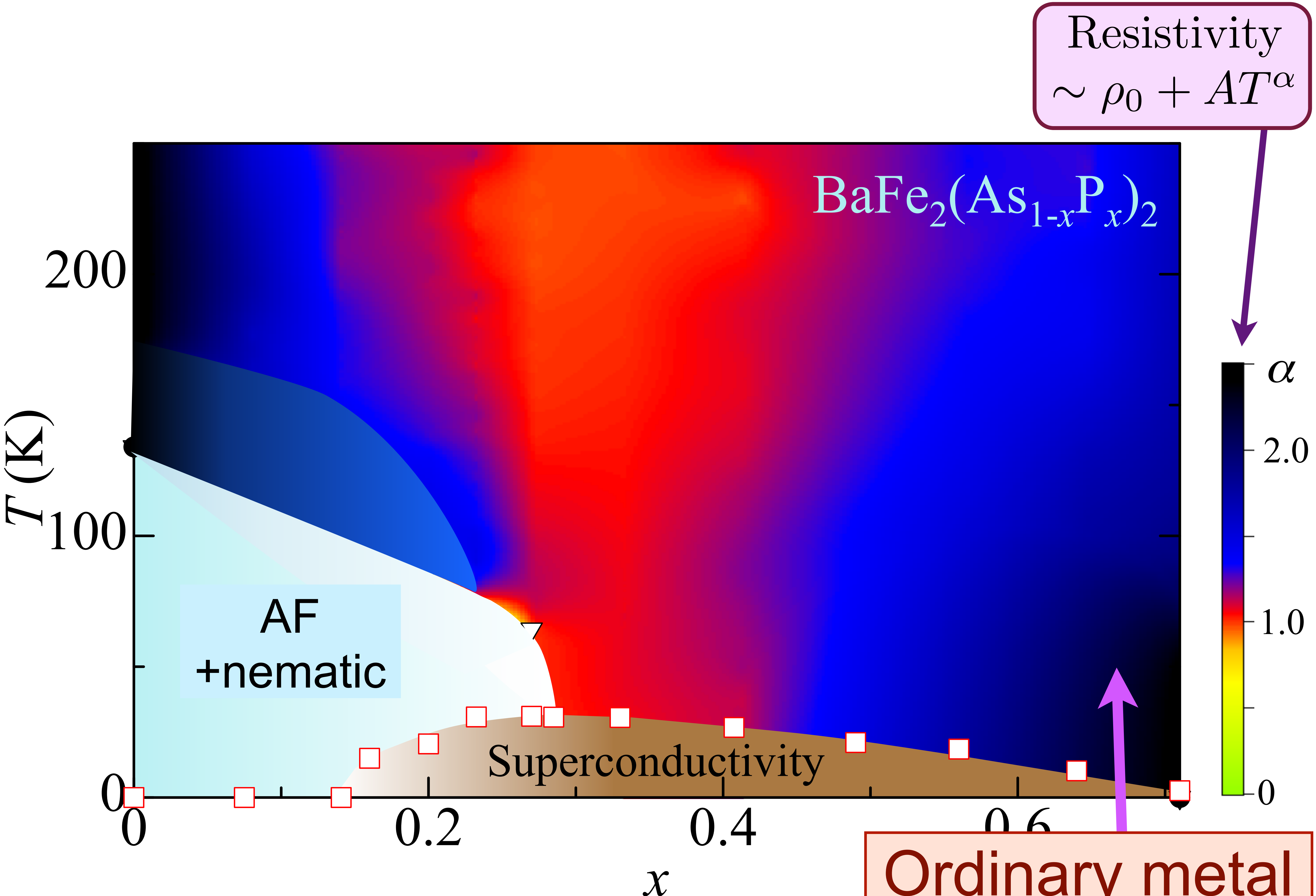
Resistivity
$\sim \rho_0 + AT^\alpha$



Superconductor
Bose condensate of pairs of electrons

S. Kasahara, T. Shiba
H. Ike



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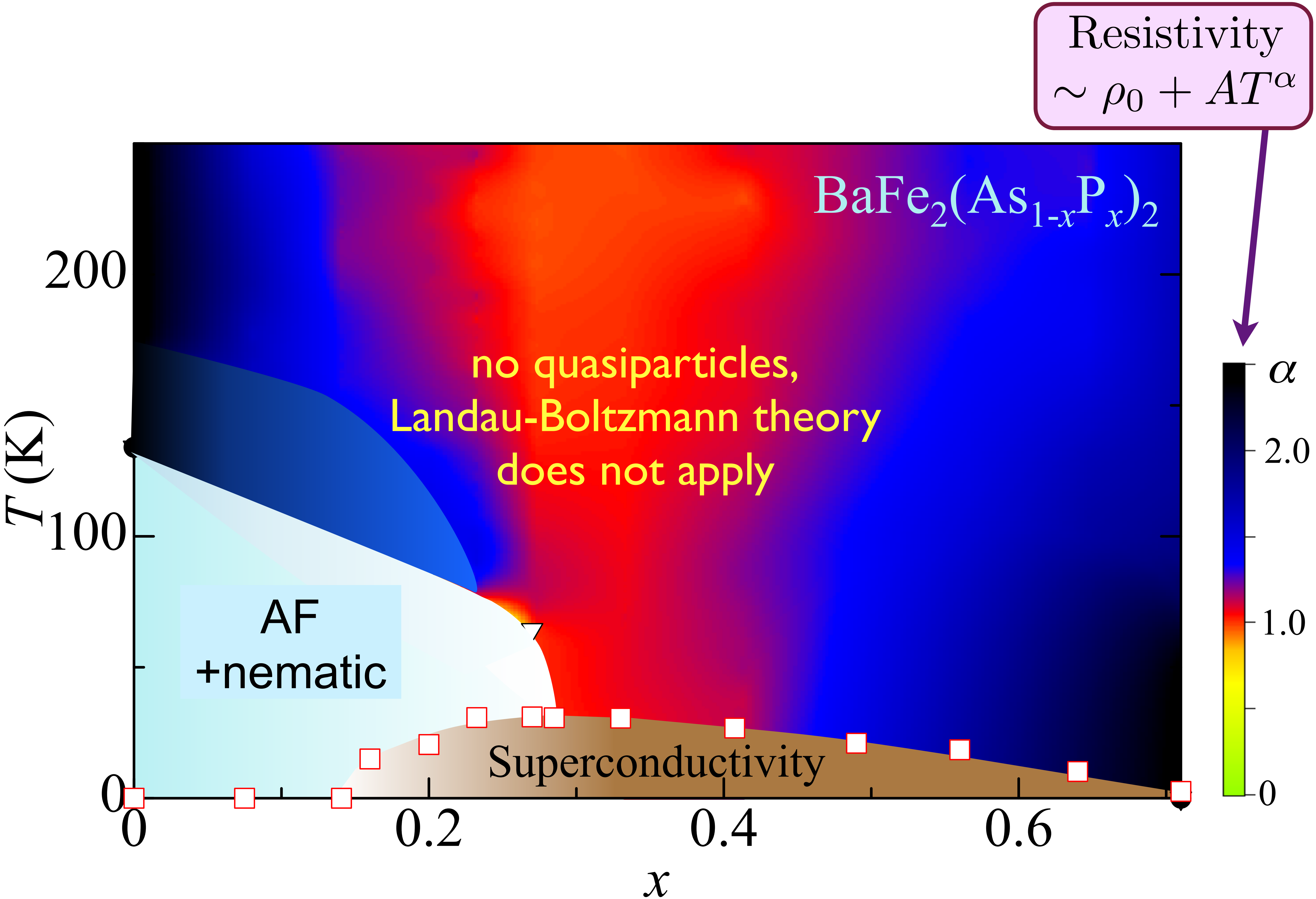

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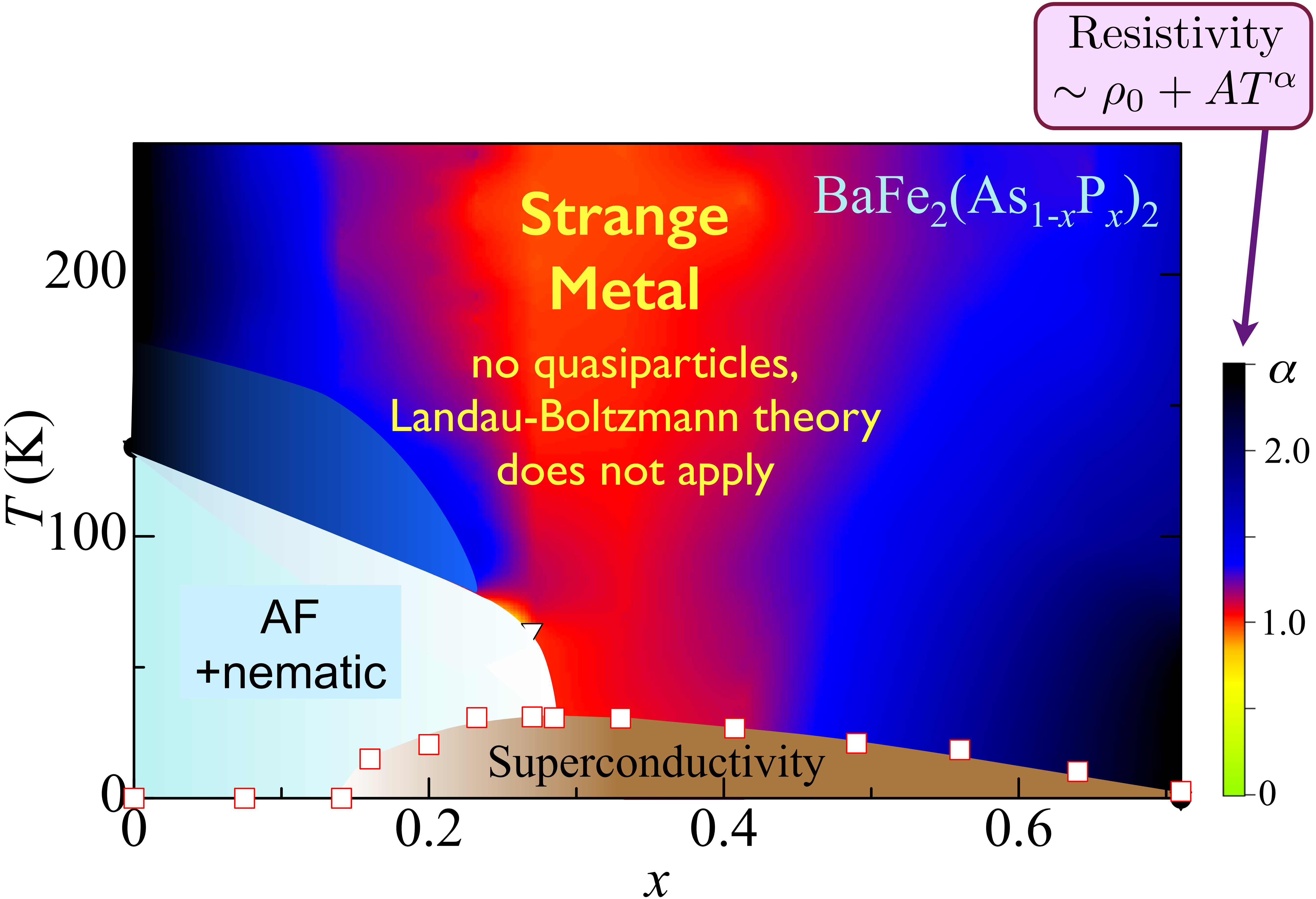

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Outline

1. The simplest model without quasiparticles

   A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice

   B. Conformal field theories in 2+1 dimensions, the AdS/CFT correspondence, and transport without quasiparticles.

2. Strange metals in the high T_c superconductors

   A. The onset of antiferromagnetism in a metal

   B. Non-quasiparticle transport at the Ising-nematic quantum critical point

   C. Entanglement, holography, and strange metals

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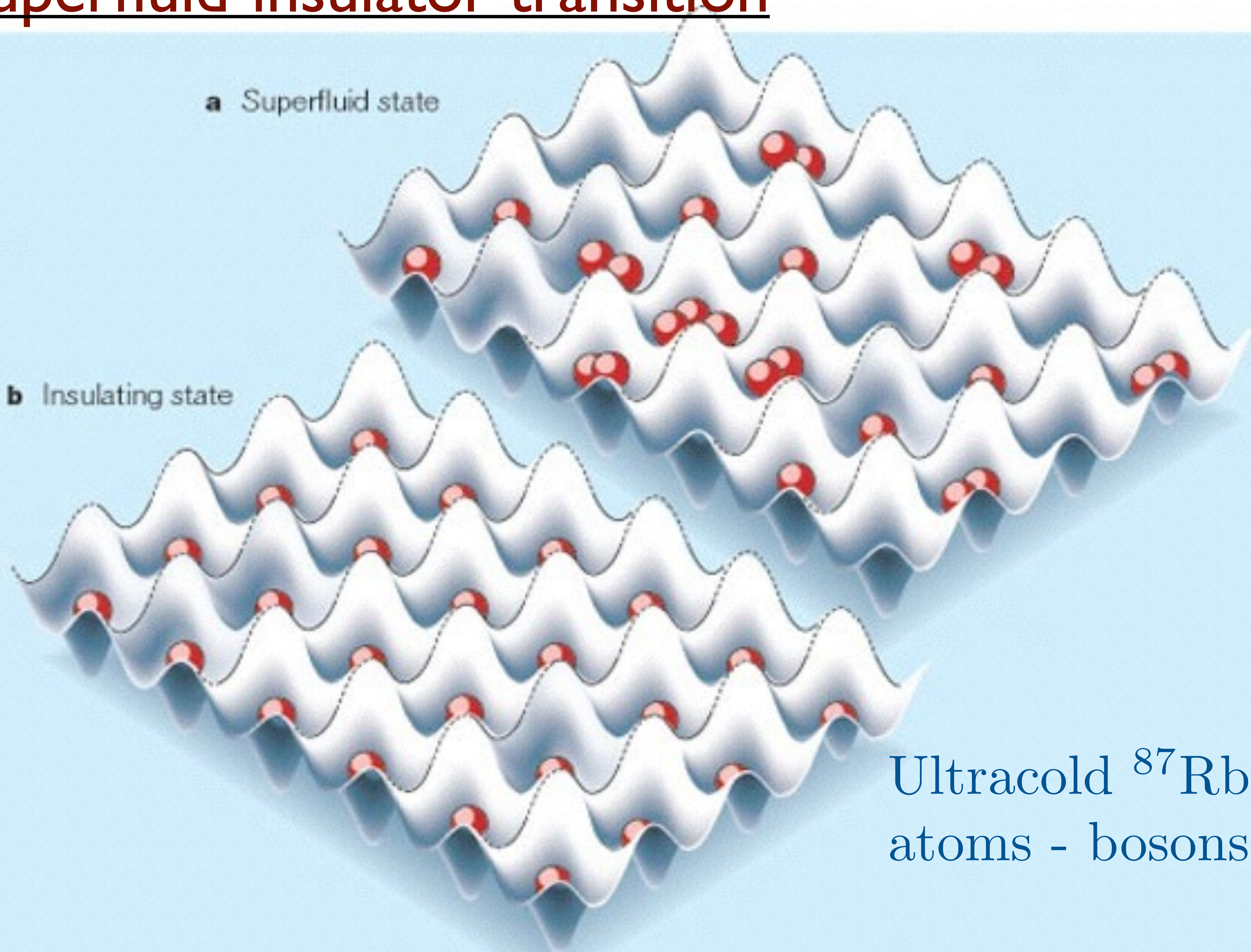
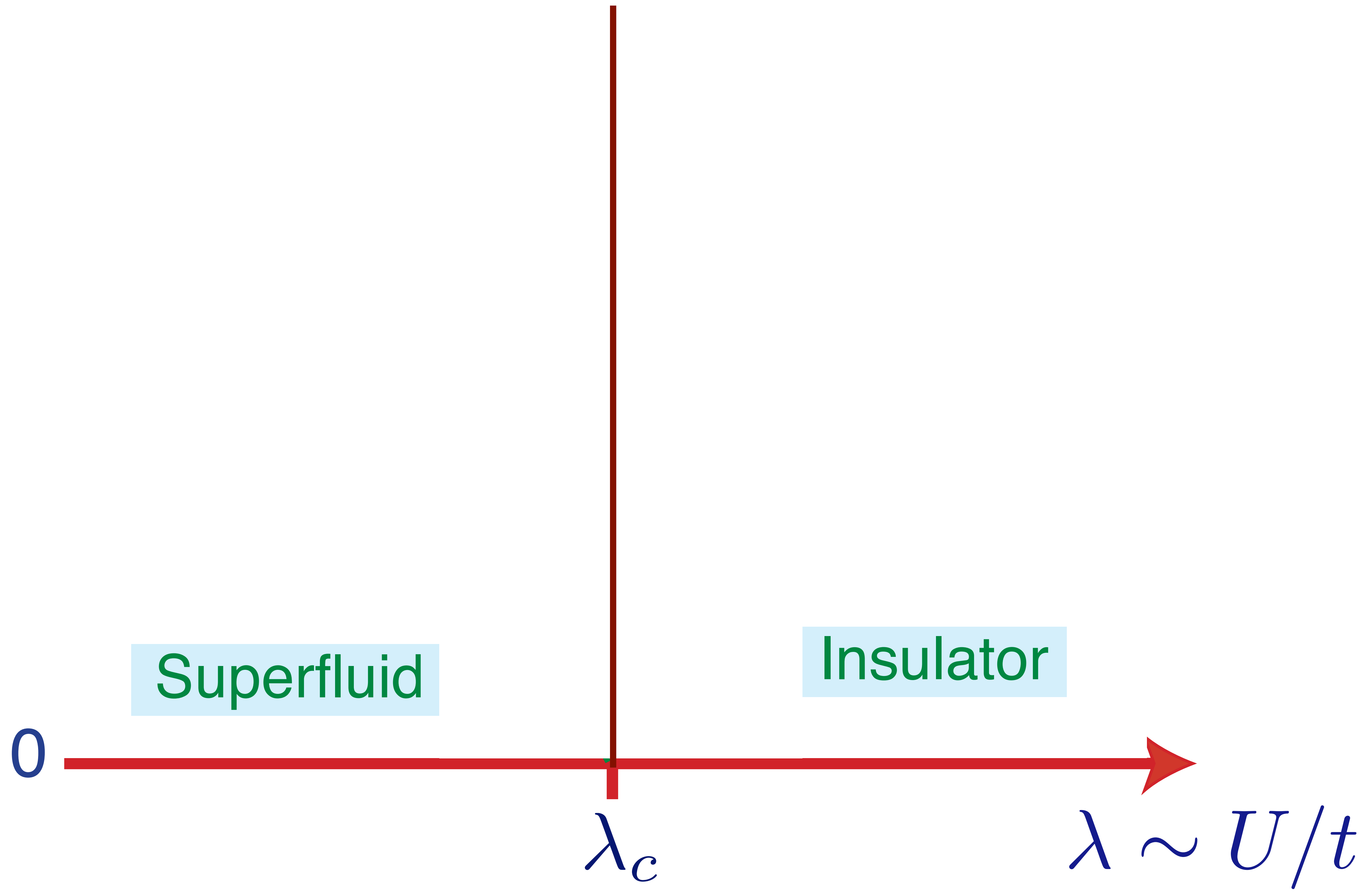
Superfluid-insulator transition



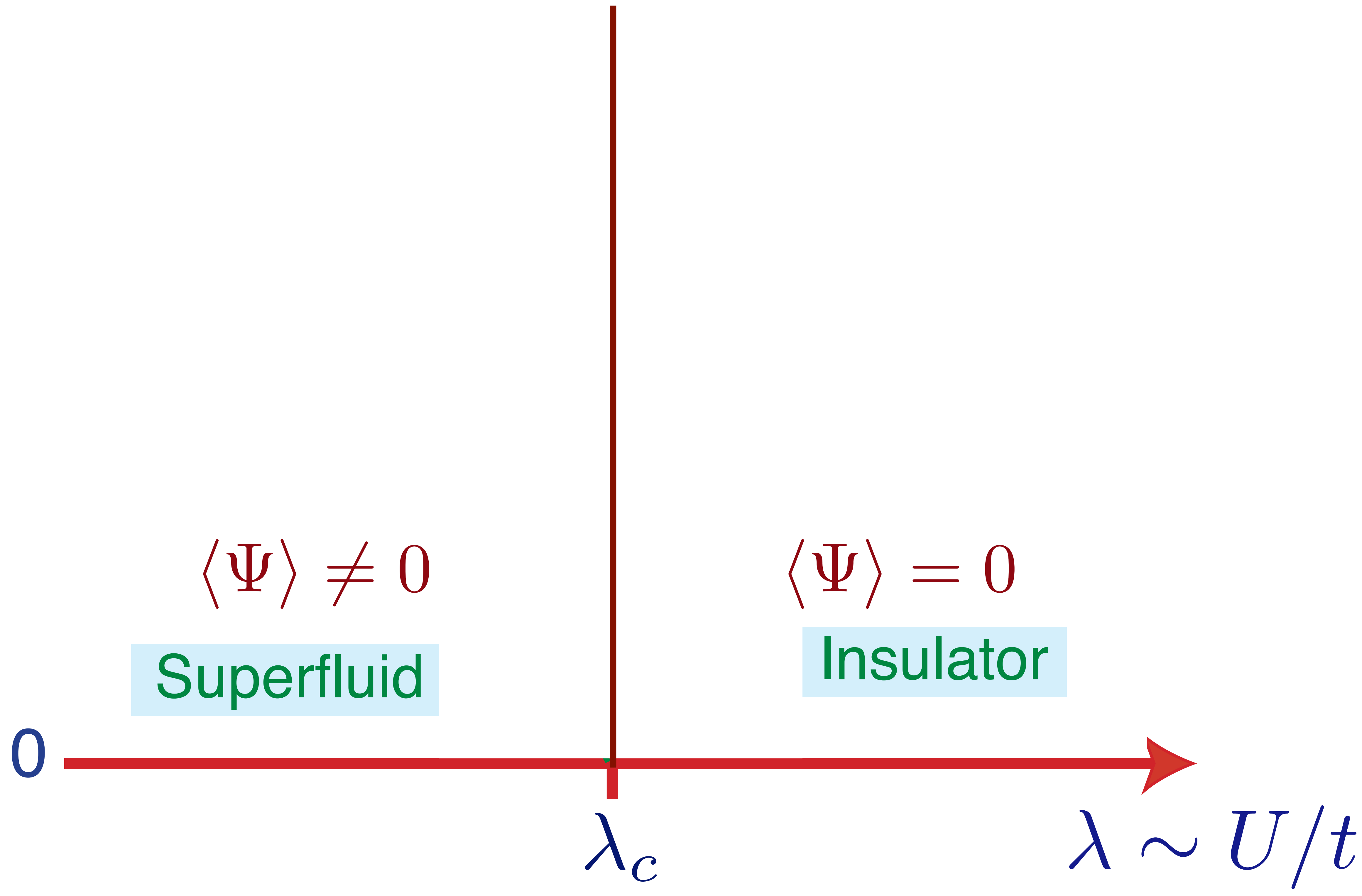
Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



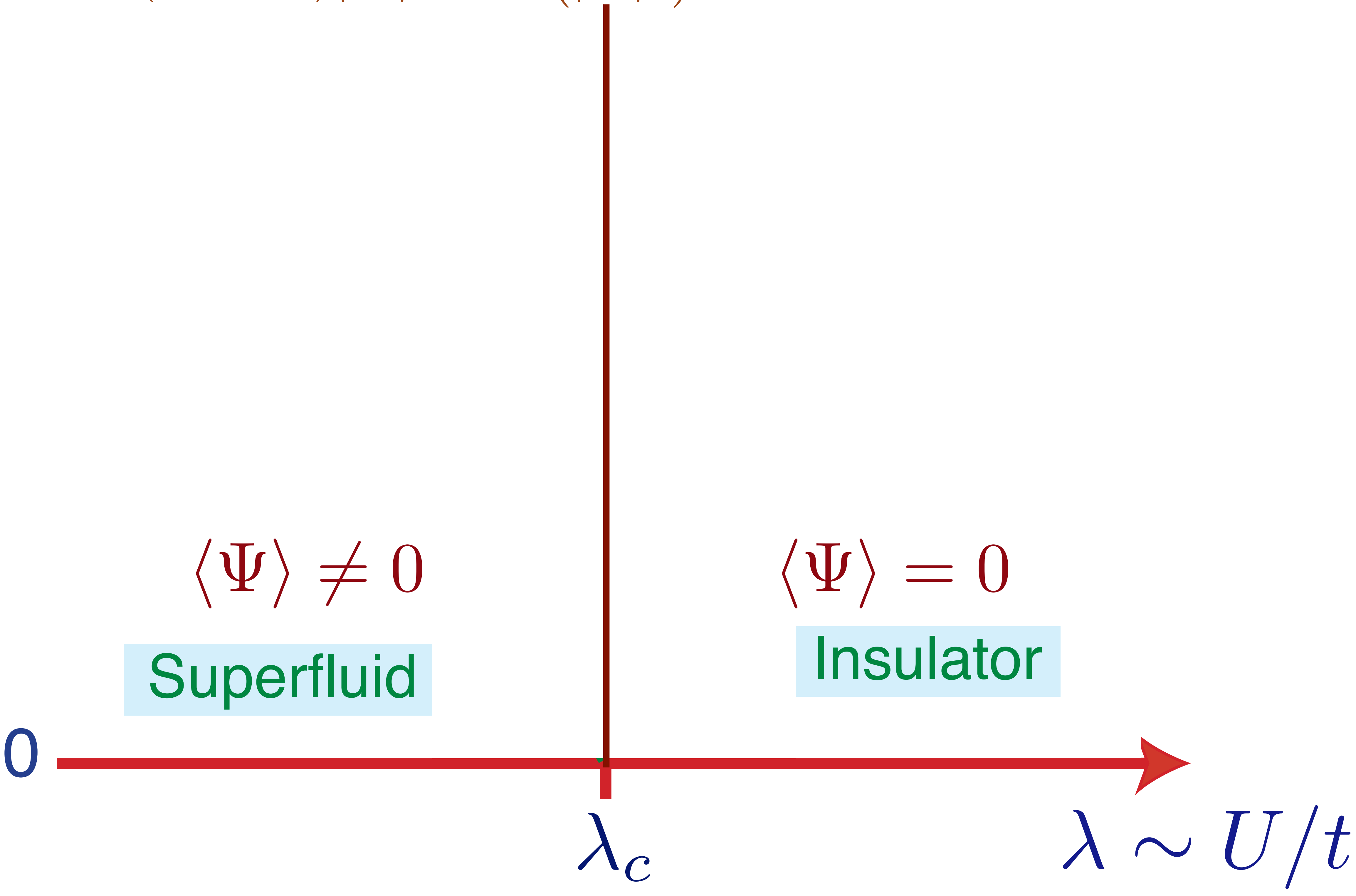
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$

$\Psi \;\rightarrow\;$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



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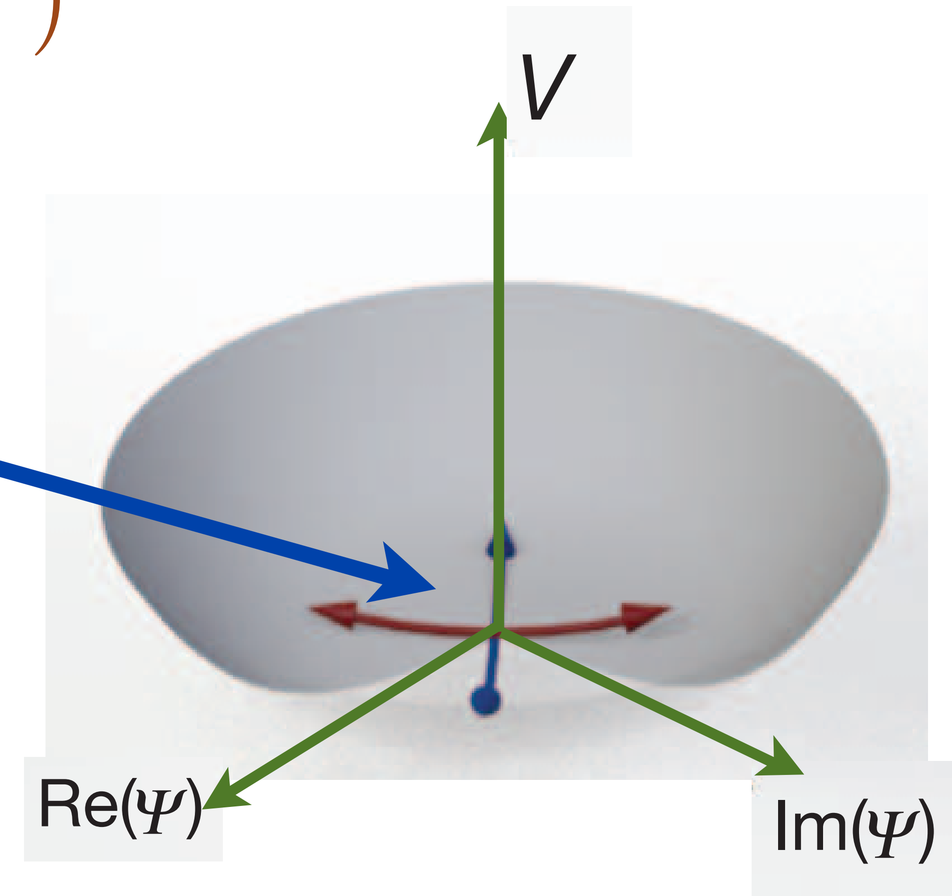
Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$\langle \Psi \rangle \neq 0$

Superfluid

$\langle \Psi \rangle = 0$

Insulator

0

λ_c

$\lambda \sim U/t$

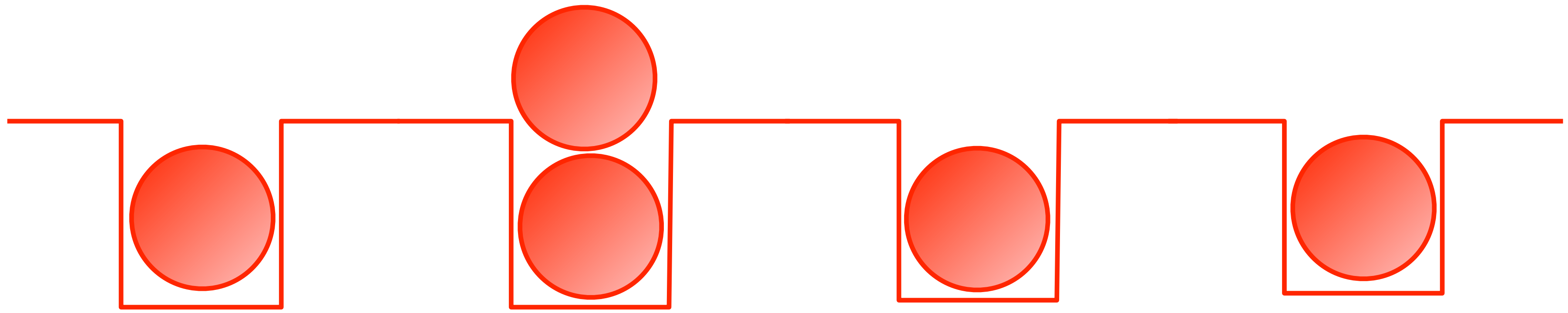
$$U \gg t$$

Insulator (the vacuum)
at large repulsion between bosons

$$|\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle$$

$U \gg t$

Excitations of the insulator:



Particles $\sim \psi^{\dagger}$

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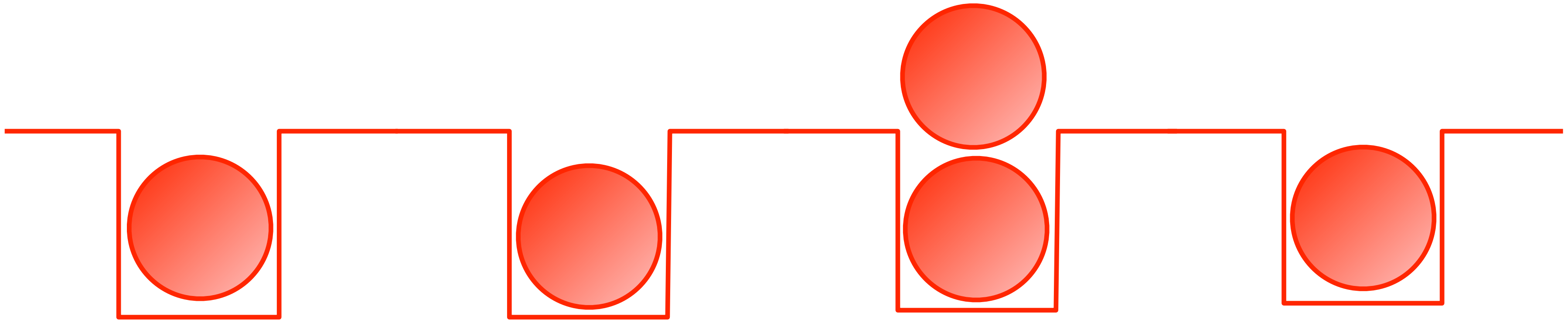

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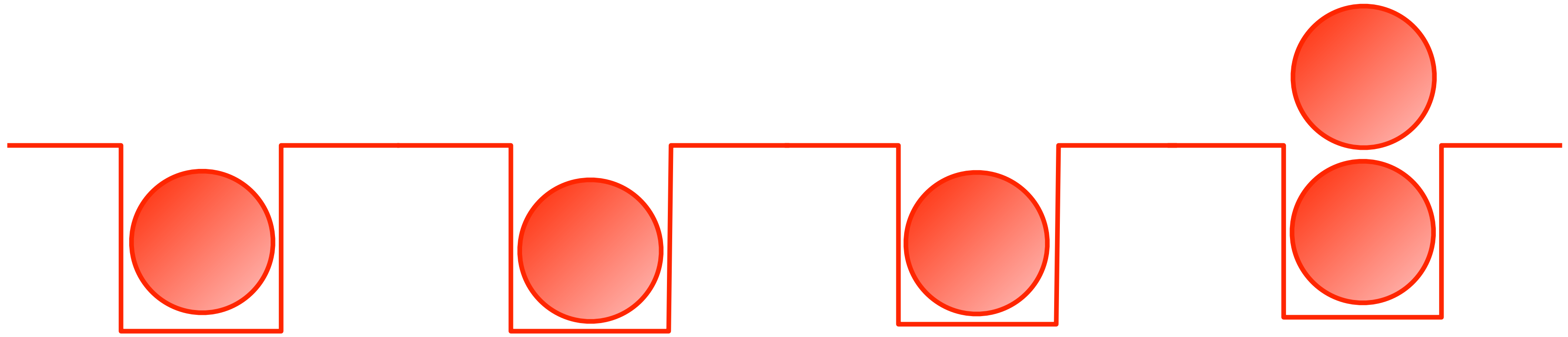
Excitations of the insulator:



Particles $\sim \psi^{\dagger}$

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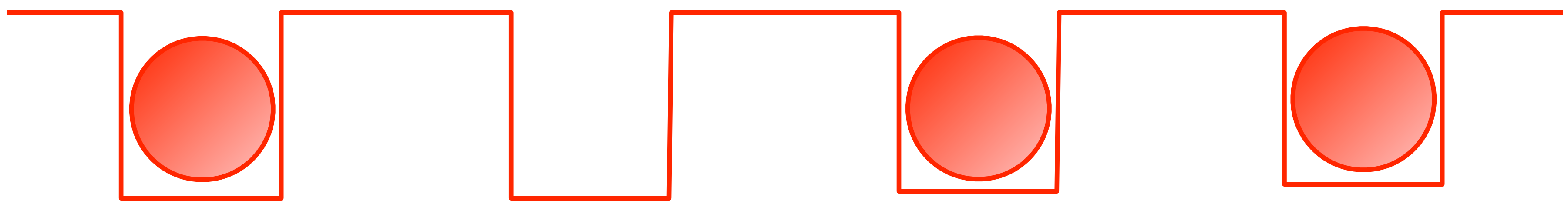
Excitations of the insulator:

Holes $\sim \psi$

$U \gg t$

Excitations of the insulator:



Holes ~ ψ

$U \gg t$

Excitations of the insulator:

Holes $\sim \psi$

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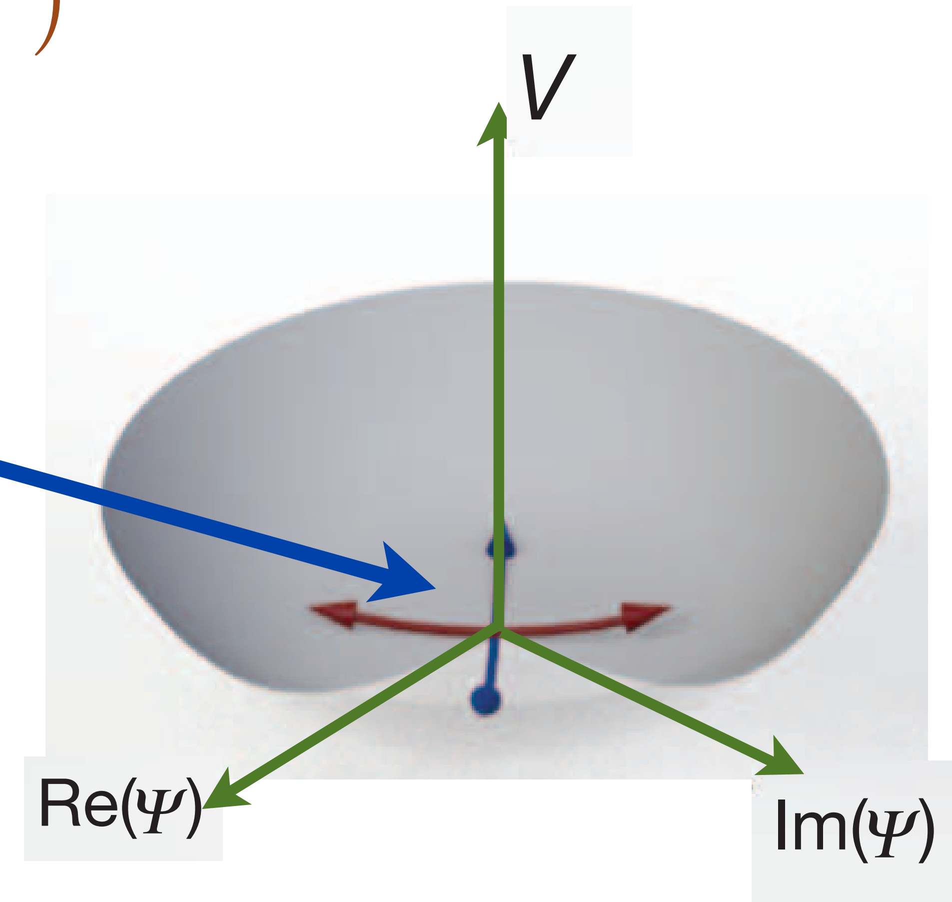

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Superfluid

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Insulator

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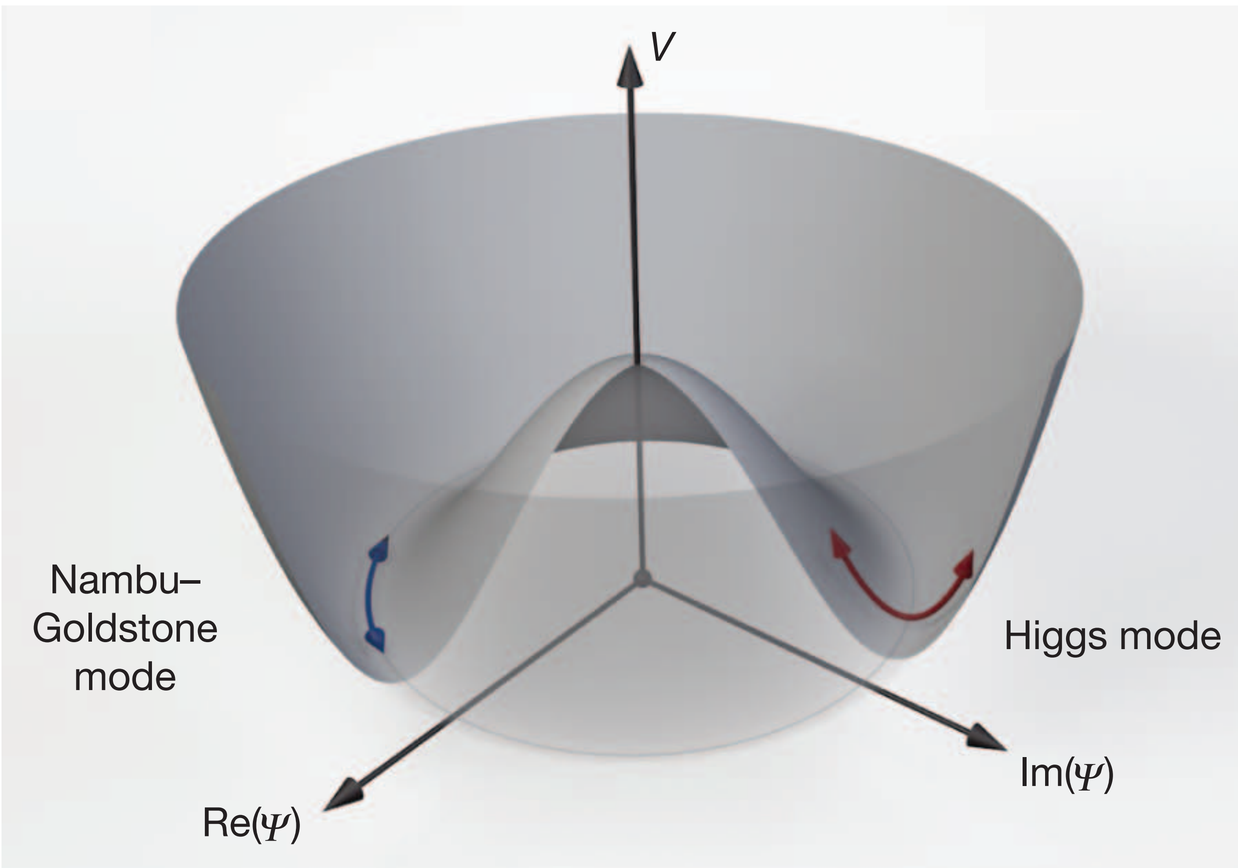

$\langle \Psi \rangle \neq 0$

Superfluid

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Insulator

0

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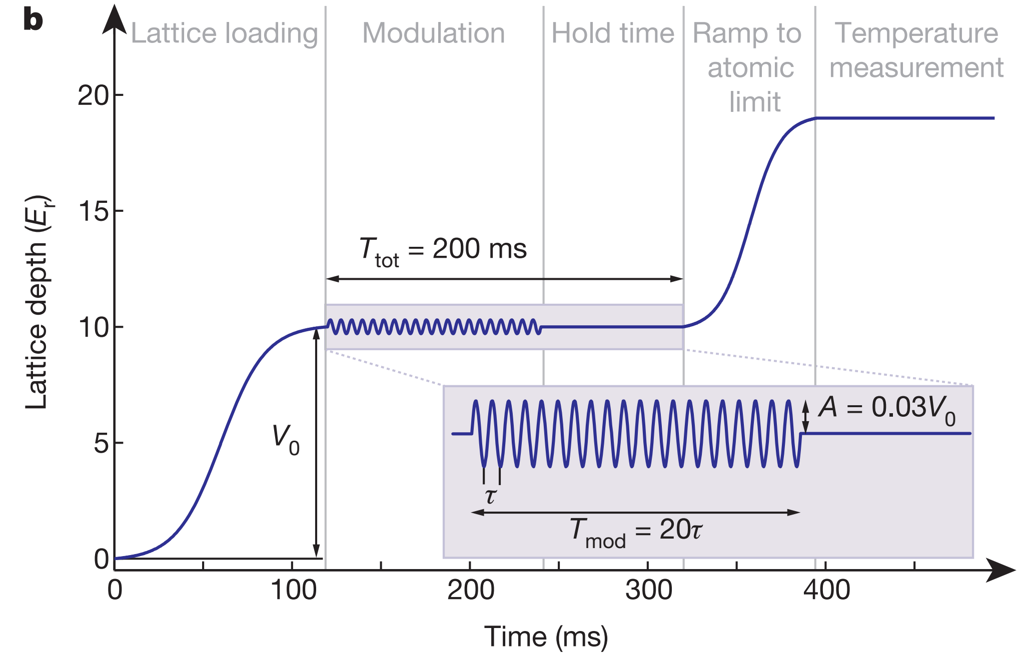
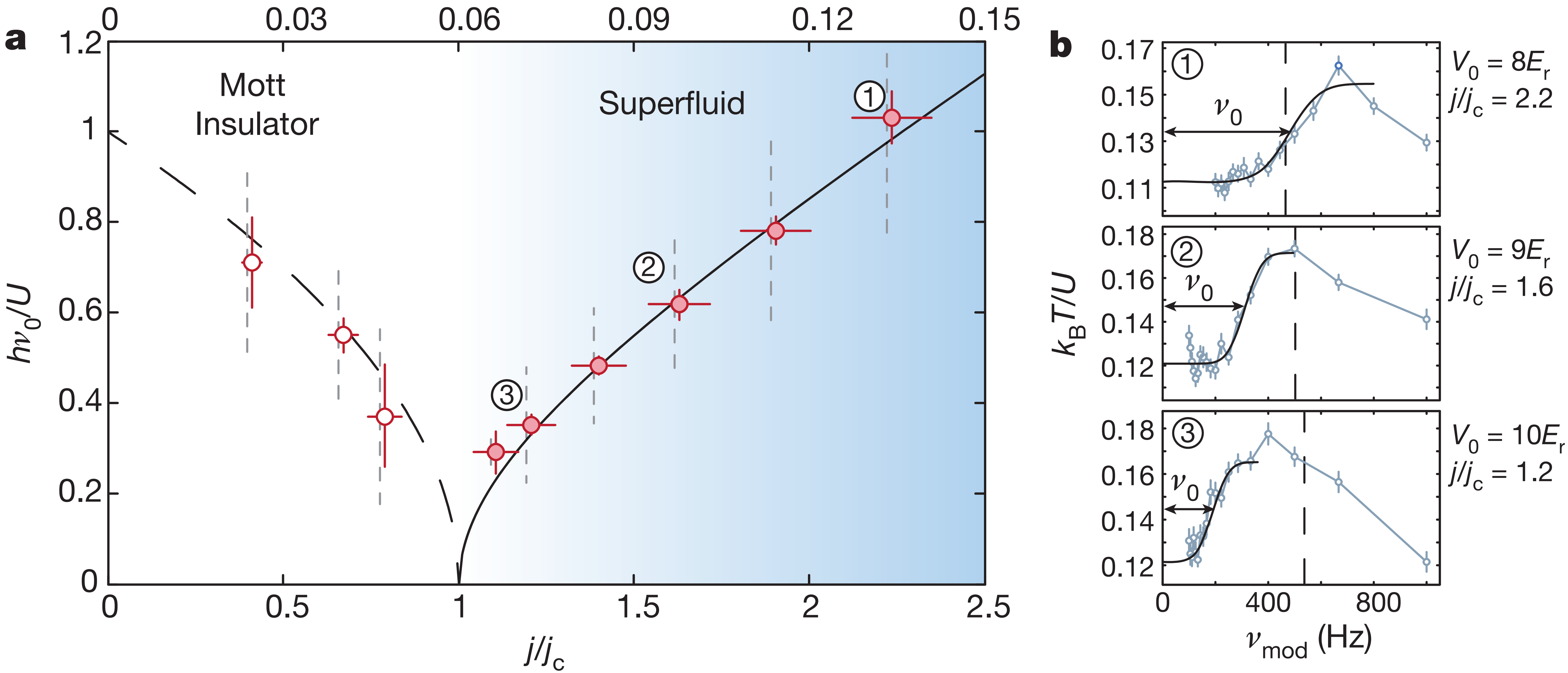
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole





Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

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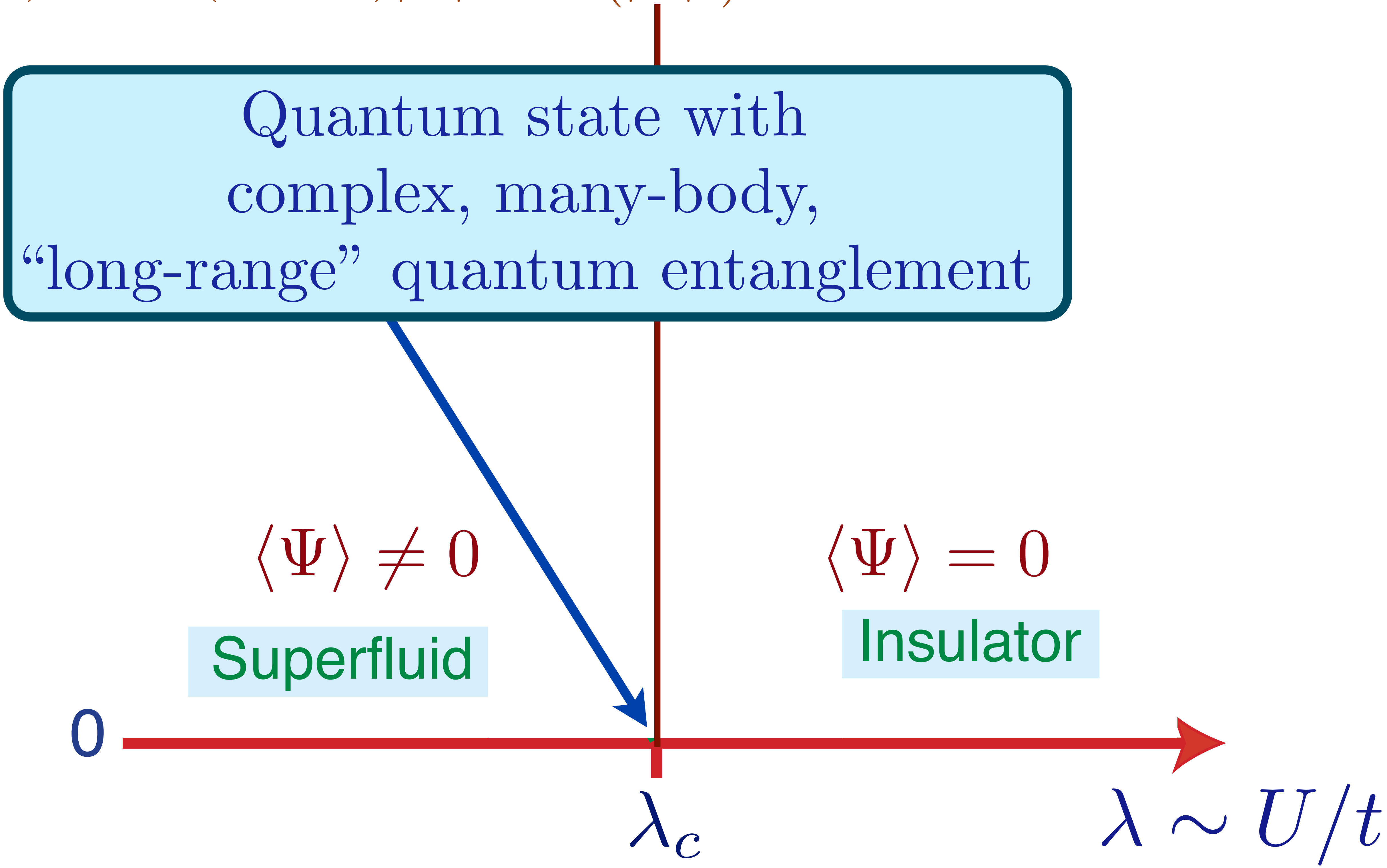

Characteristics of quantum critical point

- Long-range entanglement

- No quasiparticles - no simple description of excitations.

- The low energy excitations are described by a theory which has the same structure as Einstein's theory of special relativity, but with the sound velocity playing the role of the velocity of light.

- The theory of the critical point is strongly-coupled because the quartic-coupling u flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a **CFT3**

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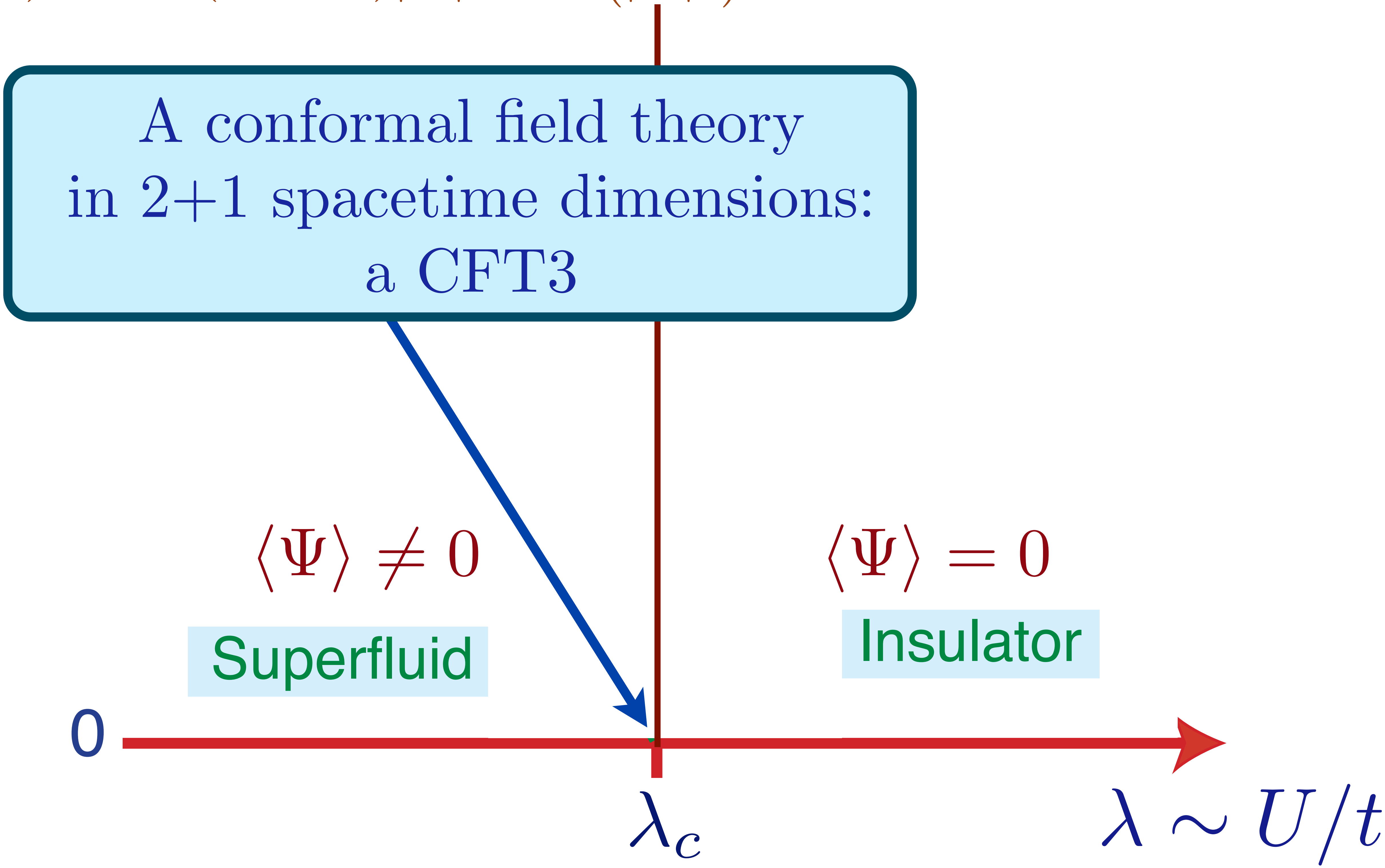
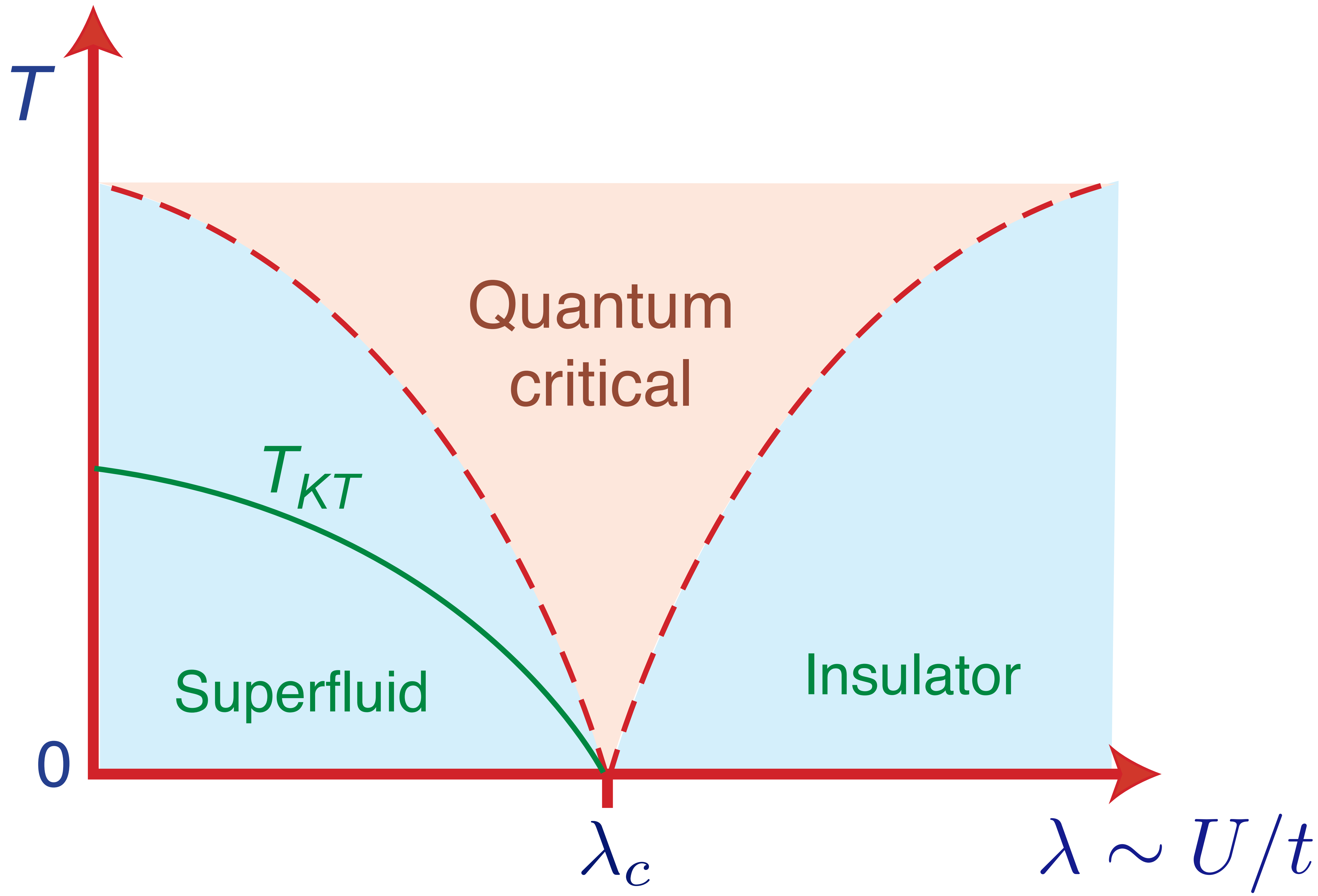
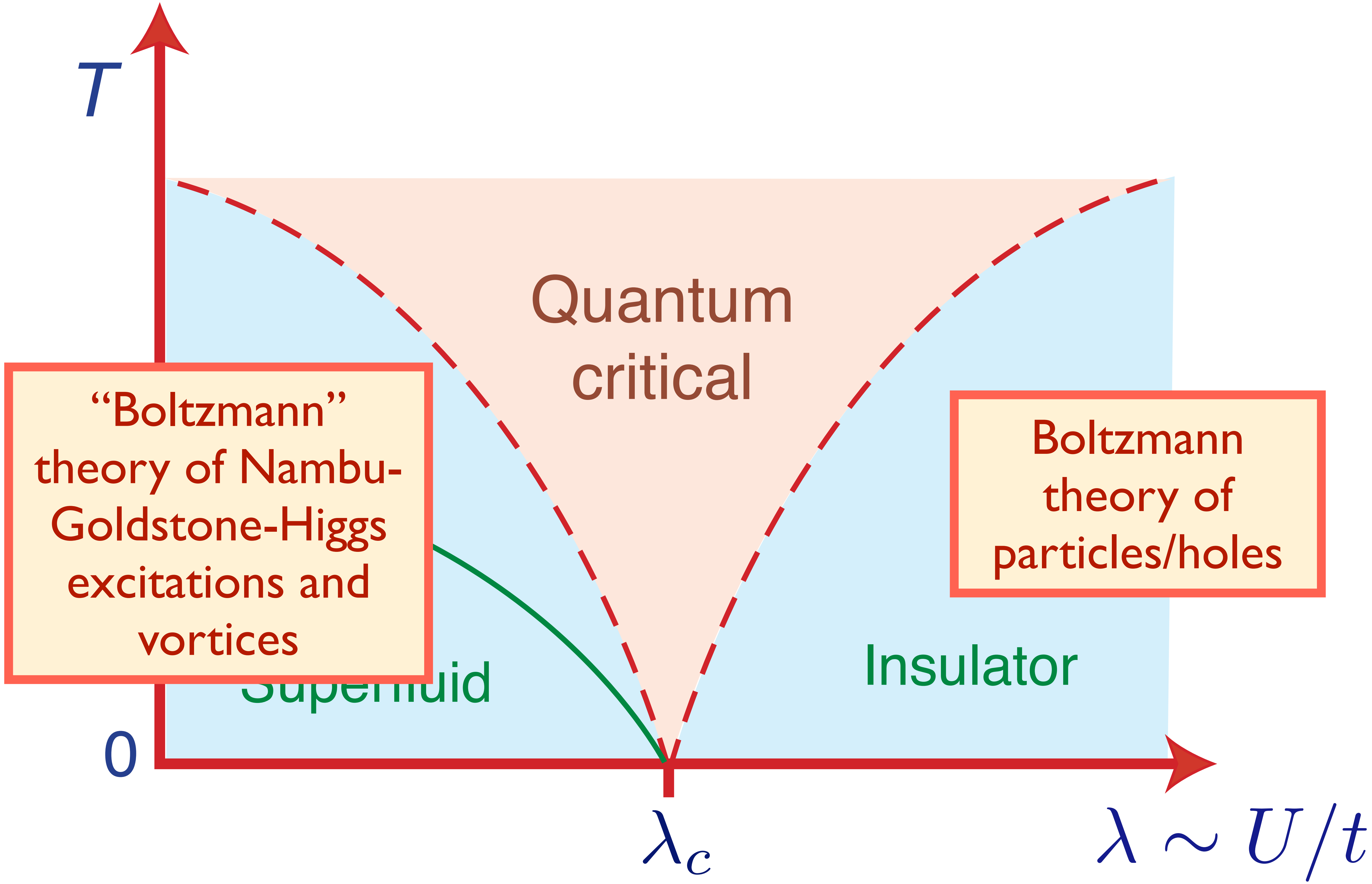




T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
λ_c
$\lambda \sim U/t$


T
0
Quantum
critical
"Boltzmann"
theory of Nambu-
Goldstone-Higgs
excitations and
vortices
Superfluid
Boltzmann
theory of
particles/holes
Insulator
λ_c
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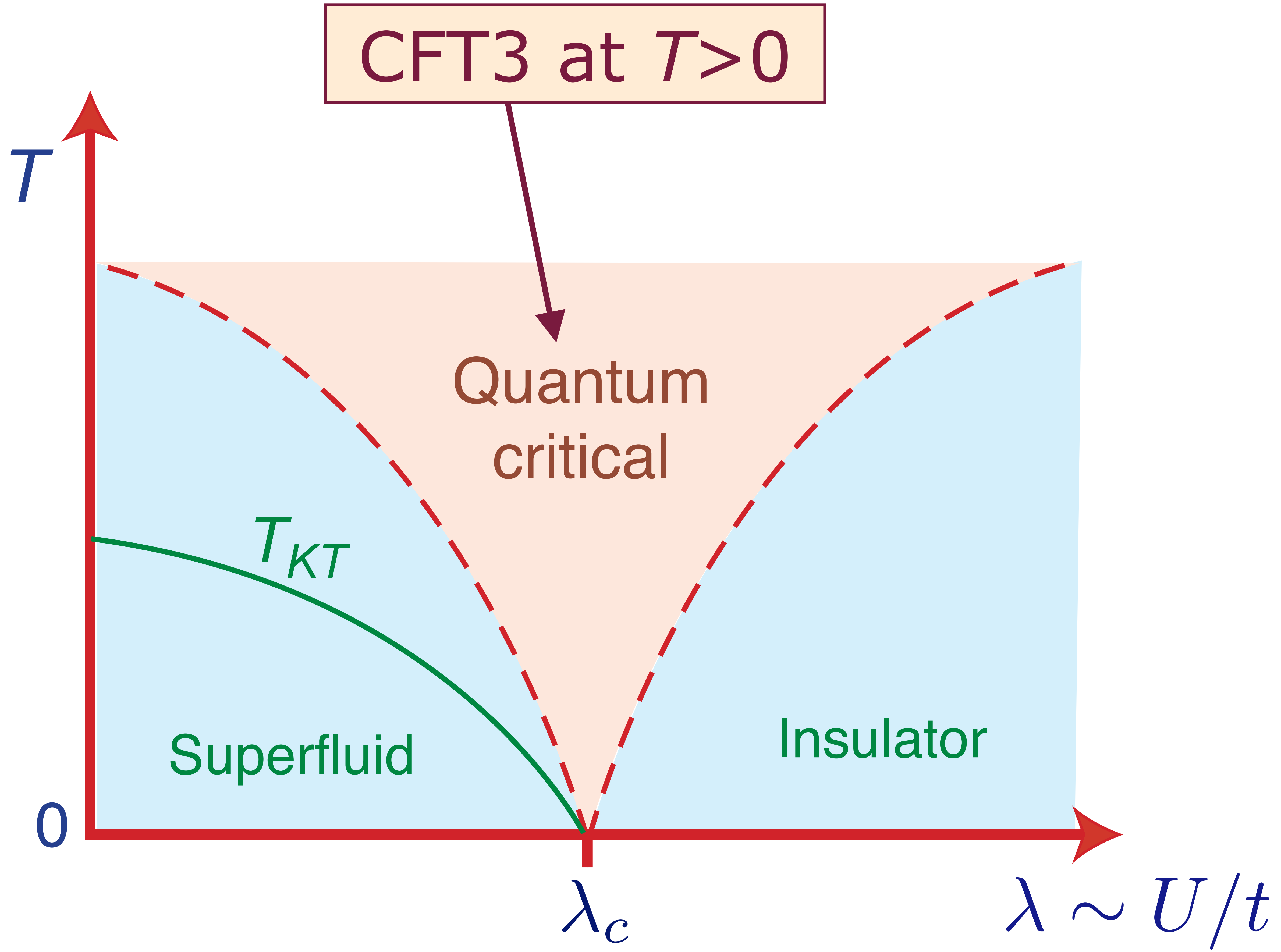
CFT3 at $T>0$
T
Quantum
critical
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Superfluid
Insulator
0
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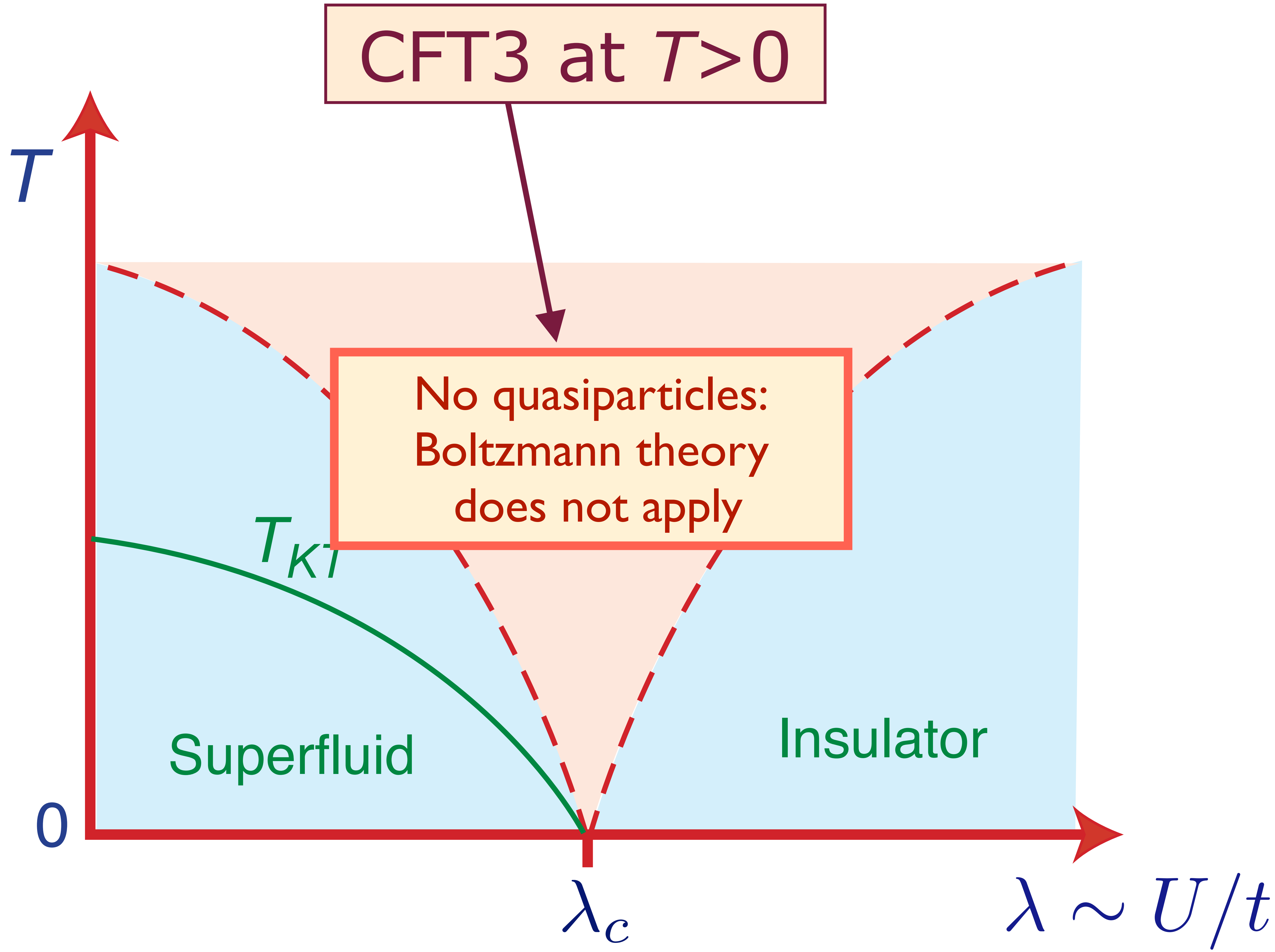
CFT3 at $T>0$
T
No quasiparticles:
Boltzmann theory
does not apply
T_{KT}
Superfluid
Insulator
0
λ_c
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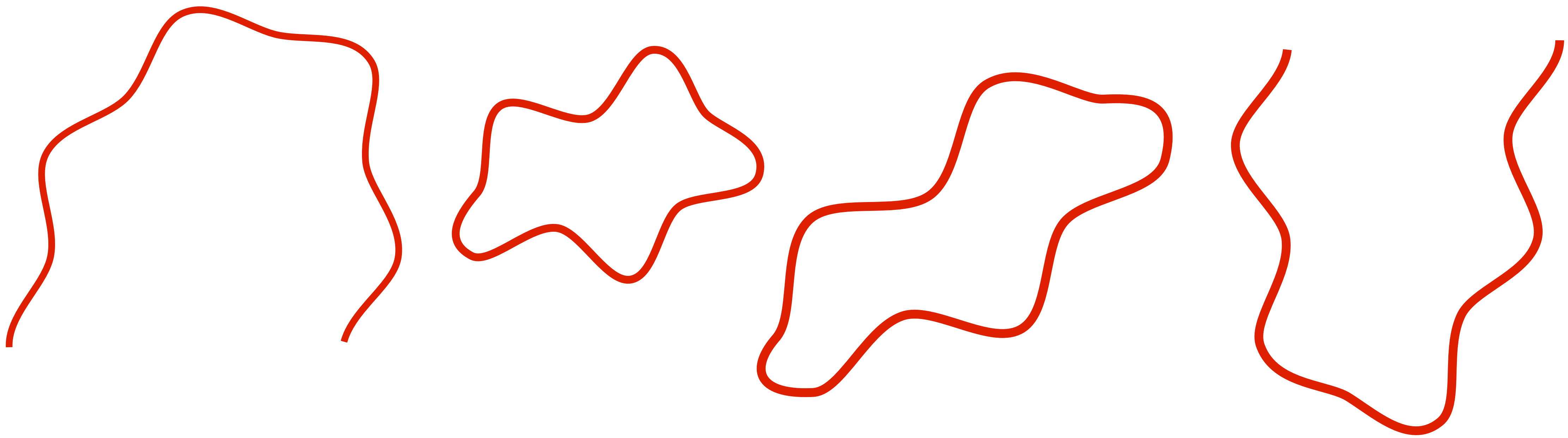
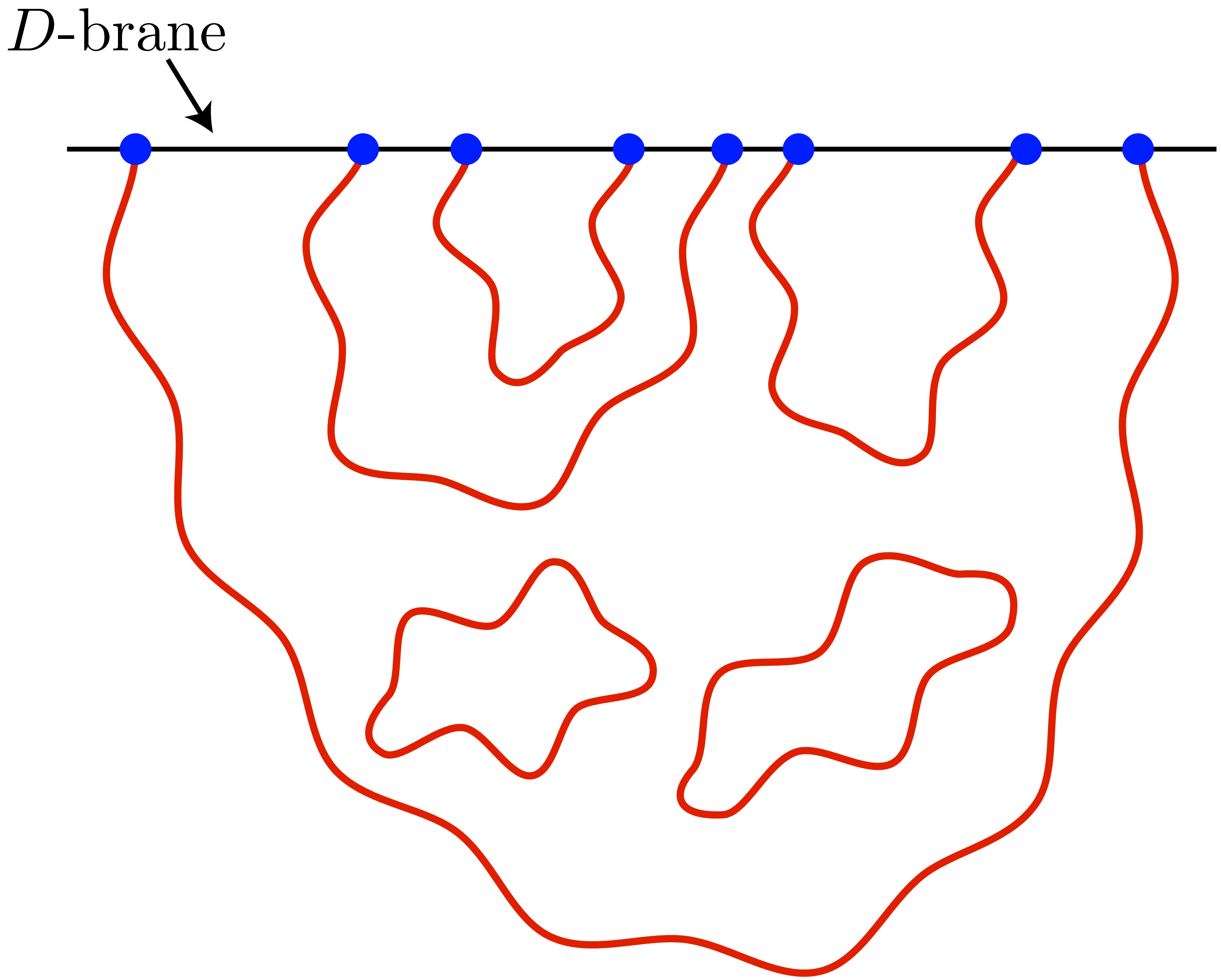
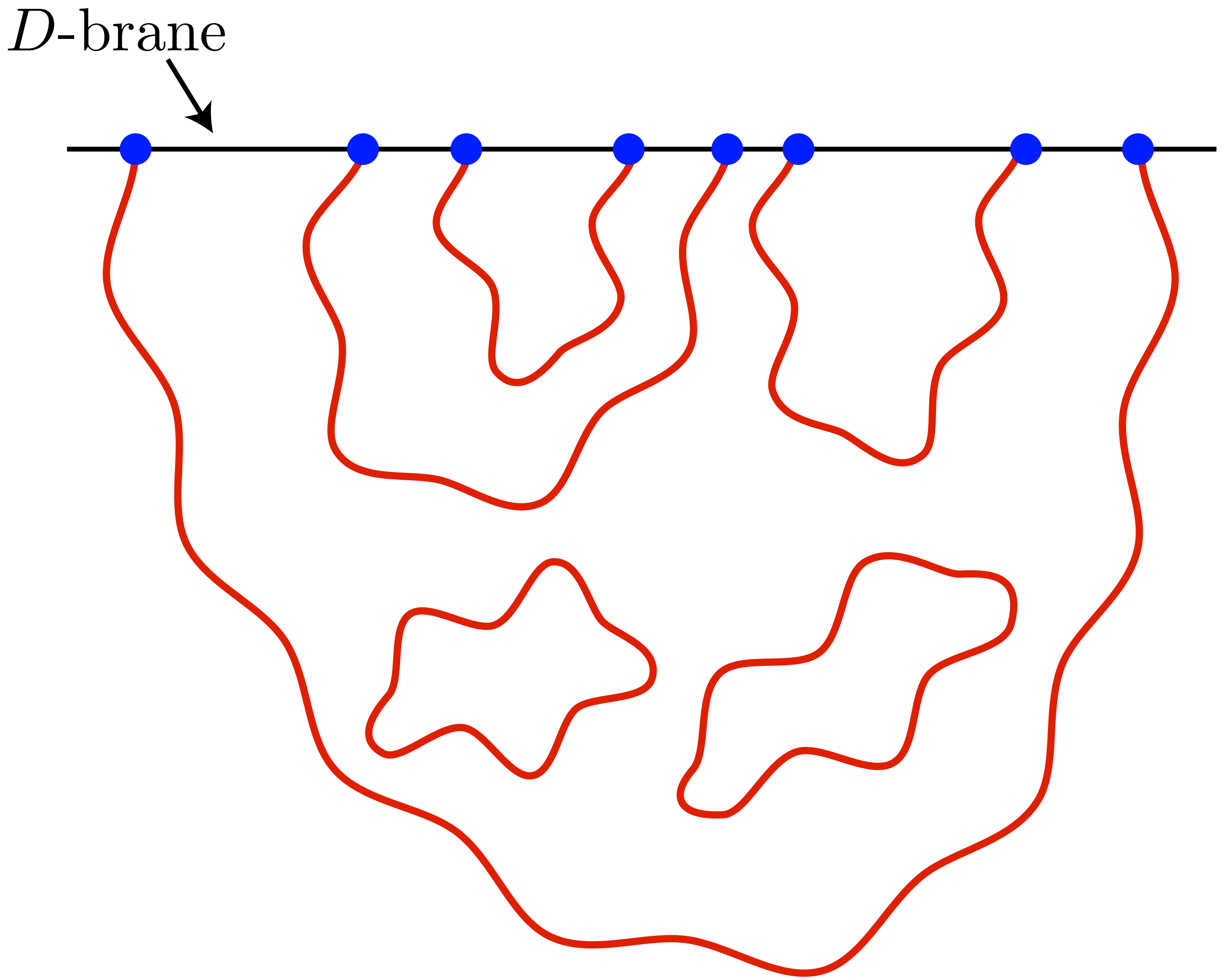
String theory



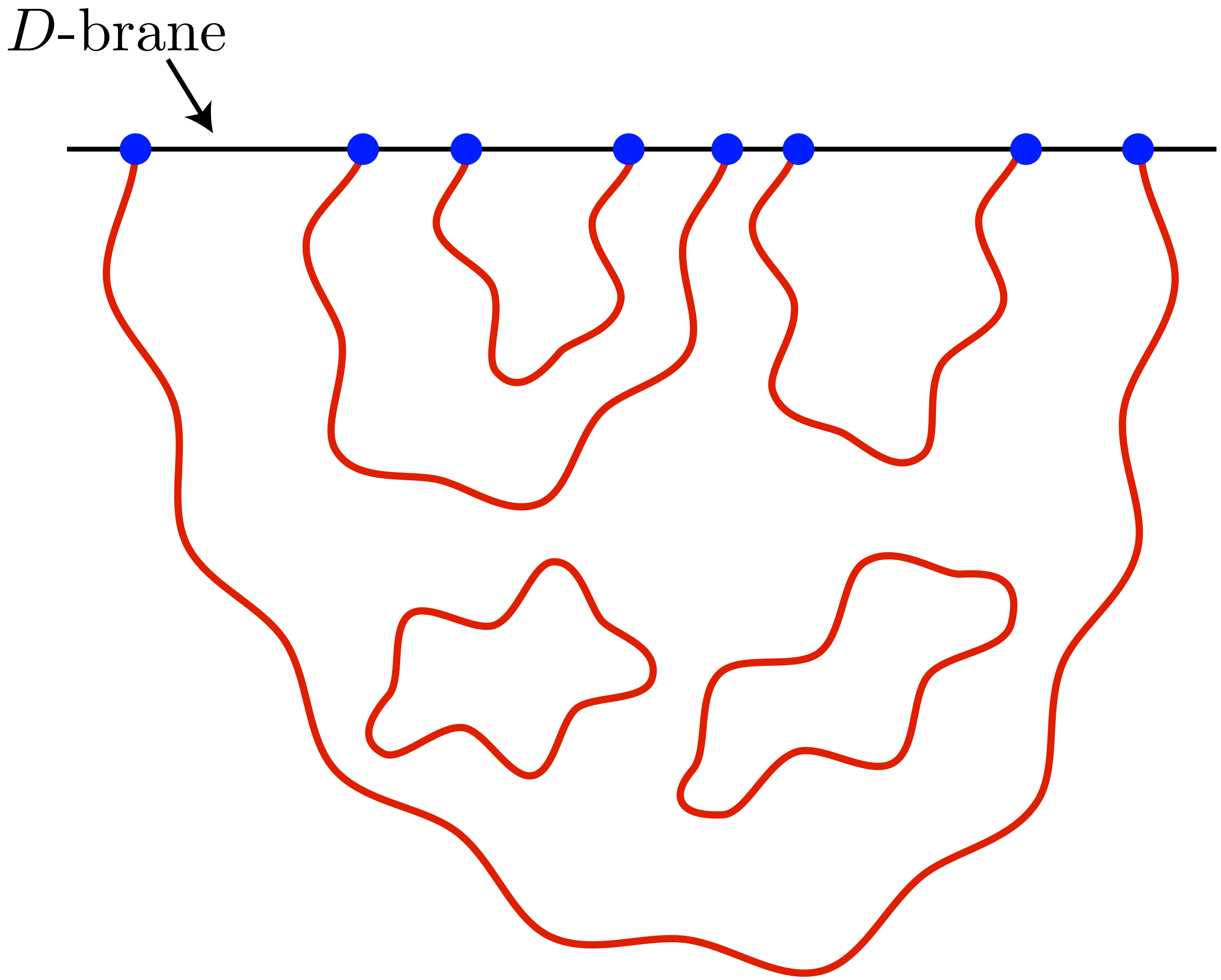
- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .



- A D-brane is a d-dimensional surface on which strings can end.



- A D-brane is a d-dimensional surface on which strings can end.
- The low-energy theory on a D-brane has no gravity, similar to theories of entangled electrons of interest to us: the strings connecting the particles on the D-brane encode the entanglement!



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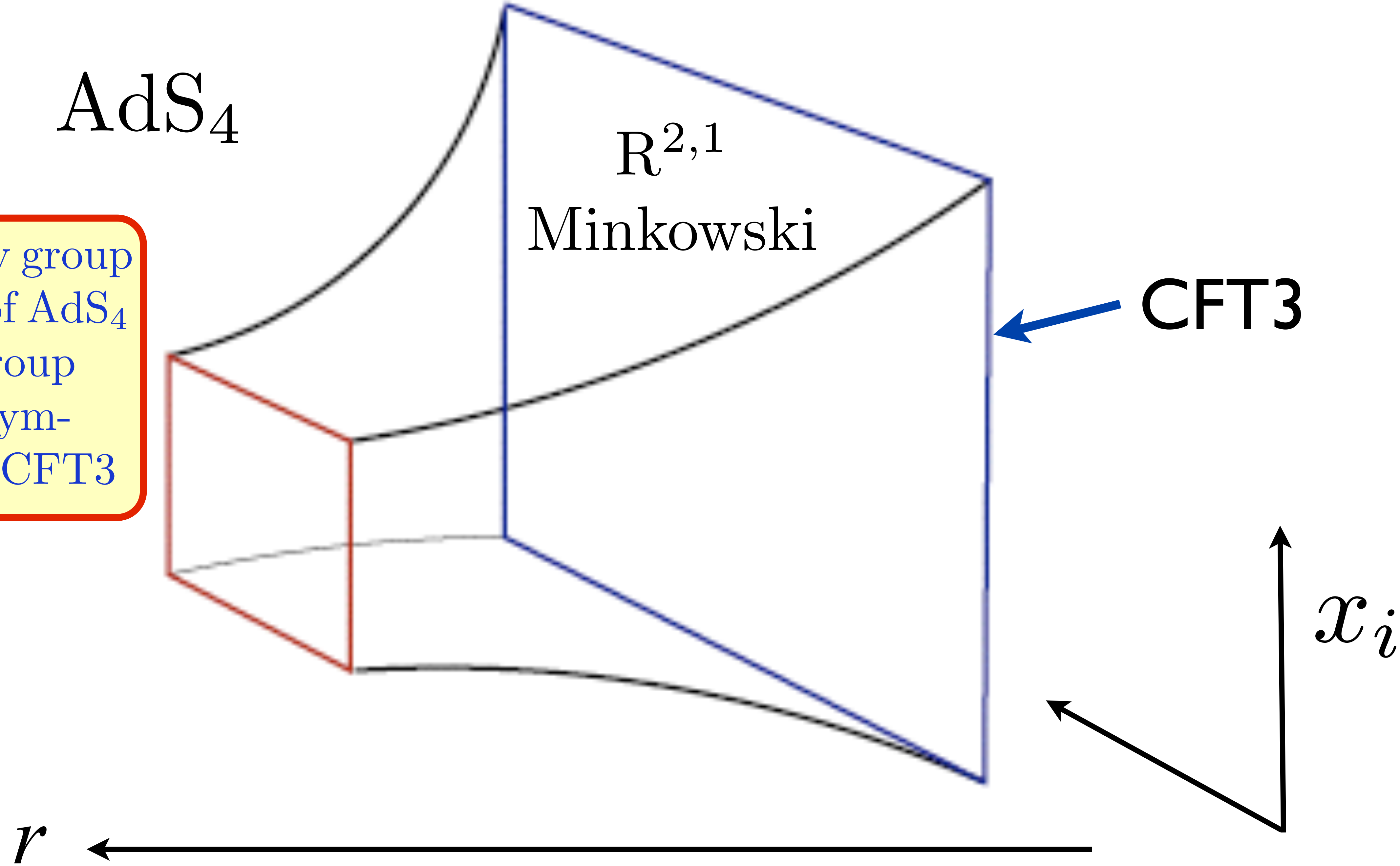
- In $d = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

AdS/CFT correspondence at zero temperature

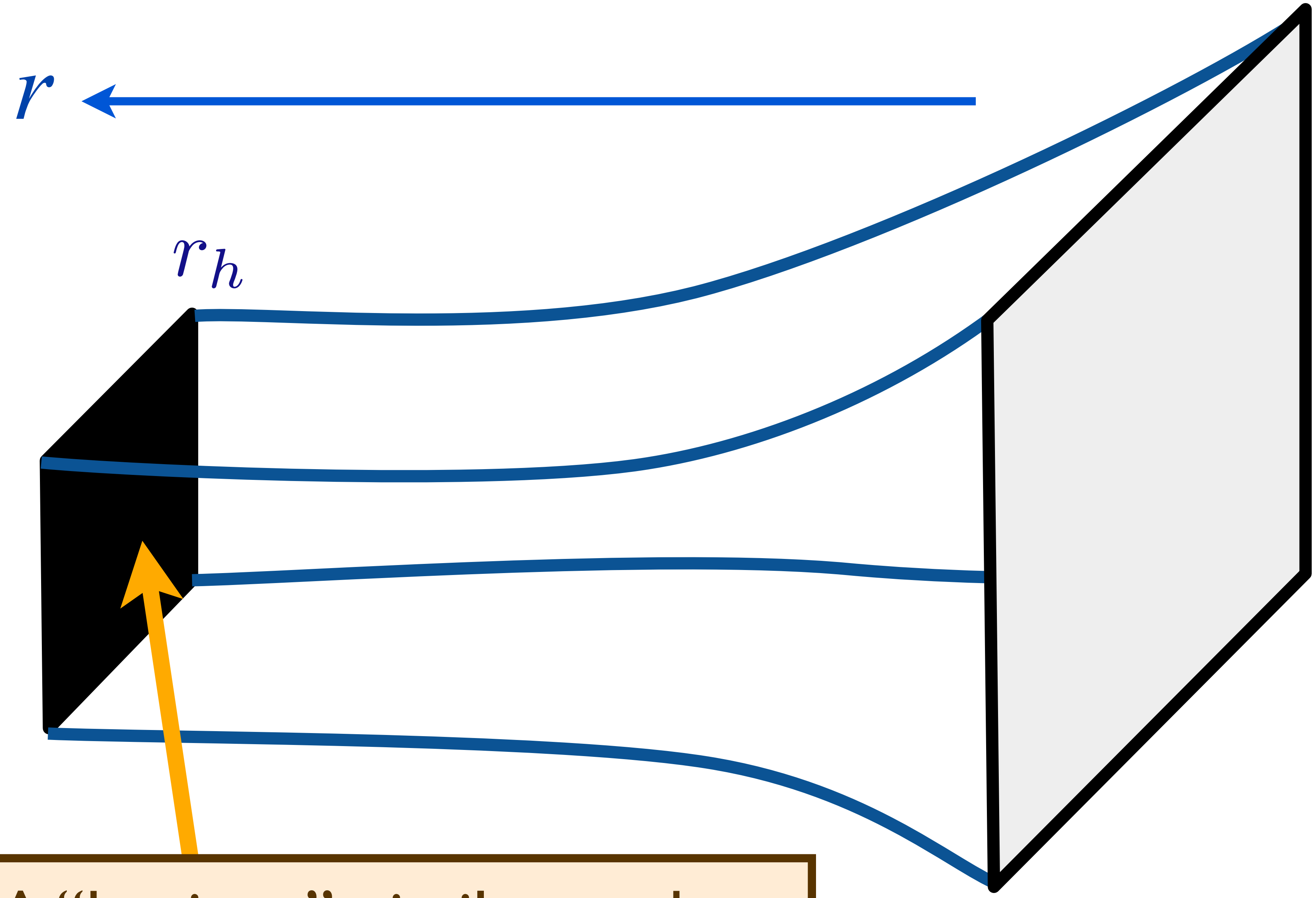
The symmetry group of isometries of AdS_4 maps to the group of conformal symmetries of the CFT3



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Gauge-gravity duality at non-zero temperatures



A "horizon", similar to the surface of a black hole !

There is a family of solutions of Einstein's equations which are AdS_4 as $r \to 0$, but which have horizons at $r = r_h$.

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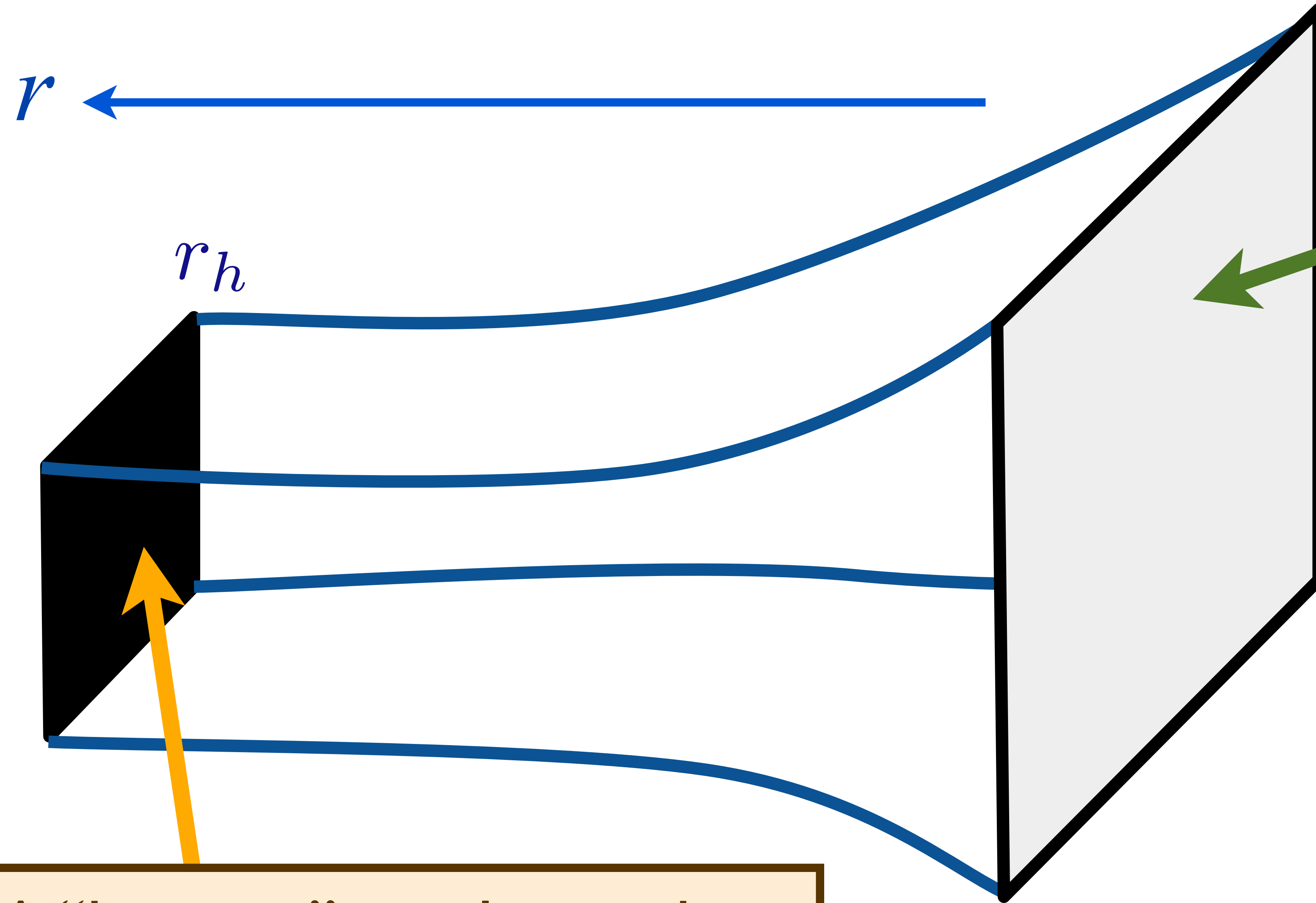

A CFT3 at a temperature $T \sim 1/r_h$ equal to the Hawking temperature of the horizon.

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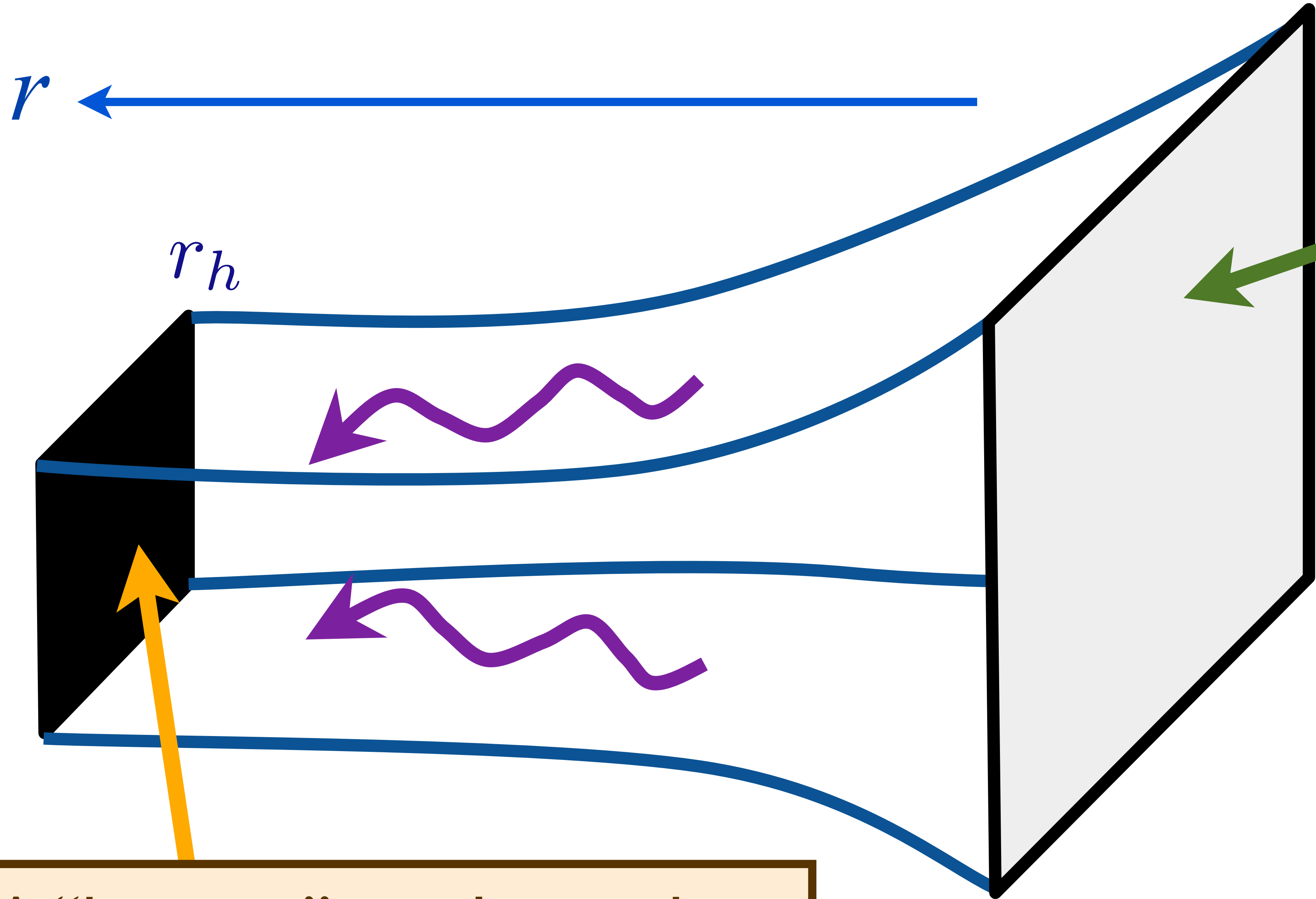

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A "horizon", similar to the surface of a black hole !

Dissipation and friction in the CFT3 = waves falling past the horizon

Traditional CMT

- Identify quasiparticles and their dispersions

- Compute scattering matrix elements of quasiparticles (or of collective modes)

- These parameters are input into a quantum Boltzmann equation

- Deduce dissipative and dynamic properties at non-zero temperatures

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- Start with strongly interacting CFT without particle- or wave-like excitations

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Computable by Wilsonian theory or computer simulations

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- Relate OPE co-efficients to couplings of an effective graviational theory on AdS
- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

Traditional CMT

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Computable by Wilsonian theory or computer simulations

matrix elements of quasiparticles (or of collective modes)

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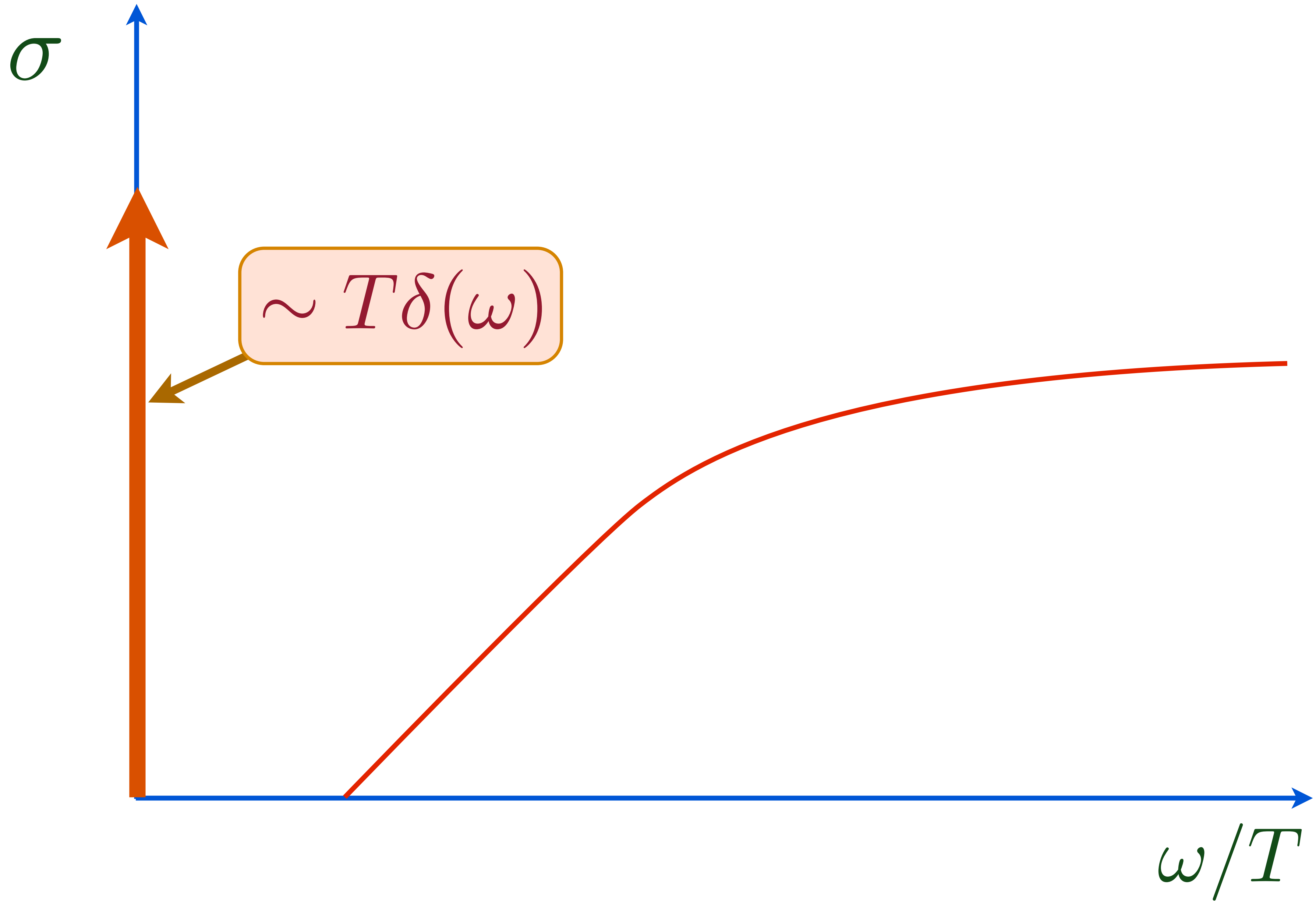
Beyond Wilsonian theory

- Deduce dissipative and dynamic properties at non-zero temperatures

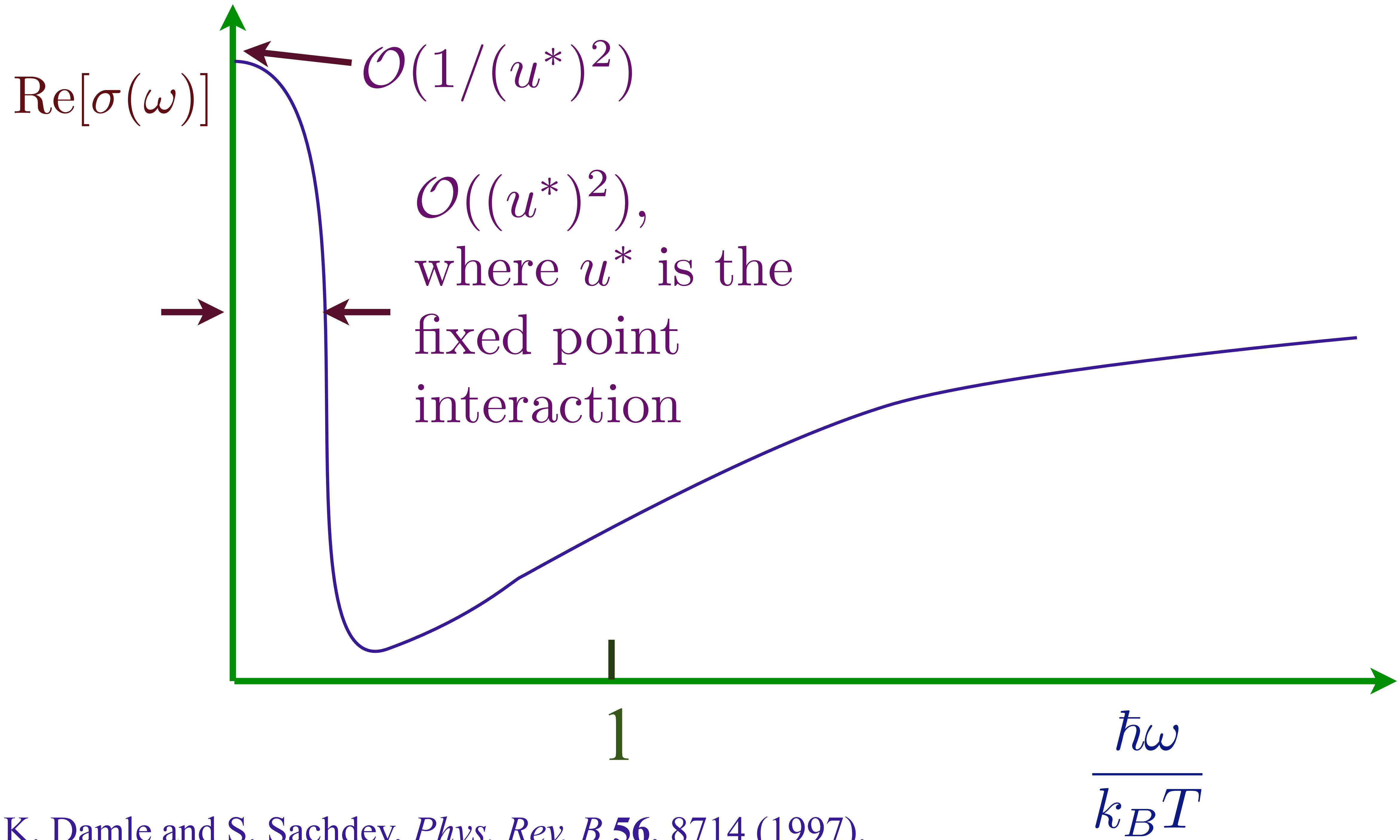
Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT
- Relate OPE co-efficients to couplings of an effective graviational theory on AdS
- Solve Einsten-Maxwell equations. Dynamics of quasi-normal modes of black branes.

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a free CFT3



Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

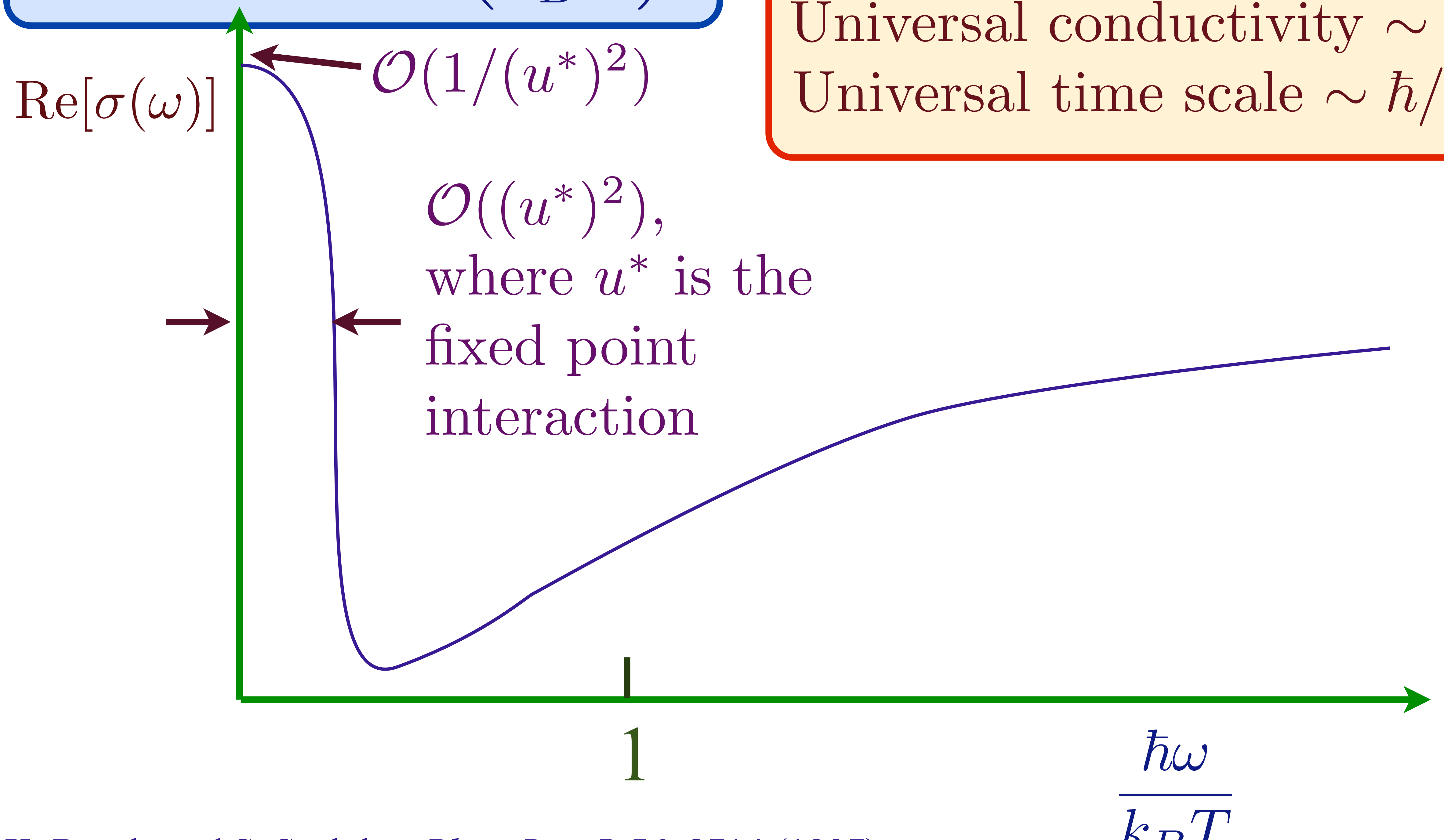
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \;;$$

$\Sigma \rightarrow$ a universal function

Universal conductivity $\sim e^2/h$

Universal time scale $\sim \hbar/k_B T$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

AdS_4 theory of quantum criticality

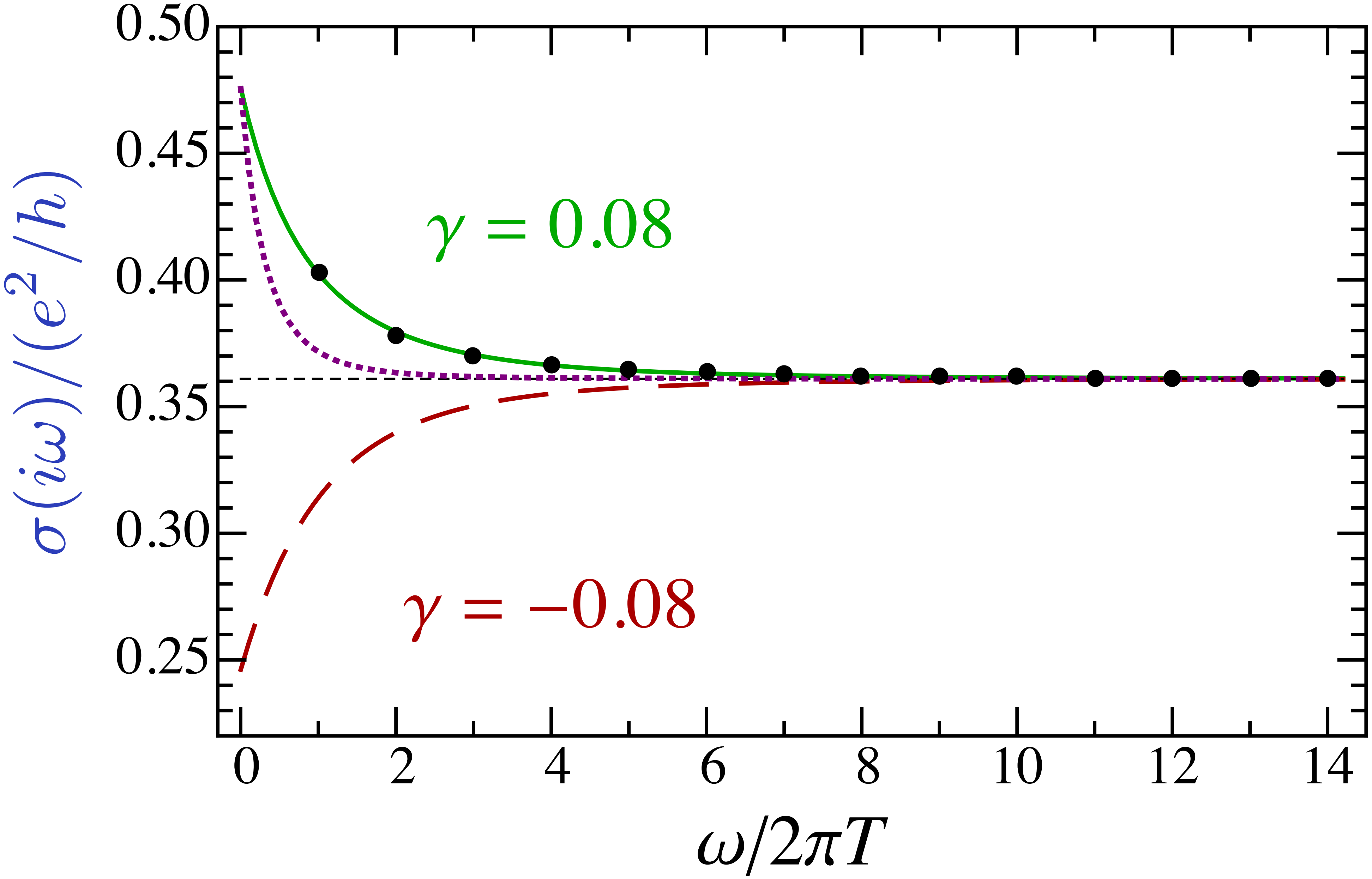
Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833$..

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

Holography+quantum Monte Carlo



The holographic theory provides an excellent fit to imaginary-time quantum Monte Carlo on the boson Hubbard model with $\gamma = 0.08$, and we combine these methods to obtain absolute predictions for the frequency-dependent conductivity.

W. Witczak-Krempa, E. Sorensen, and S. Sachdev, arXiv:1309.2941

See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, arXiv:1309.5635

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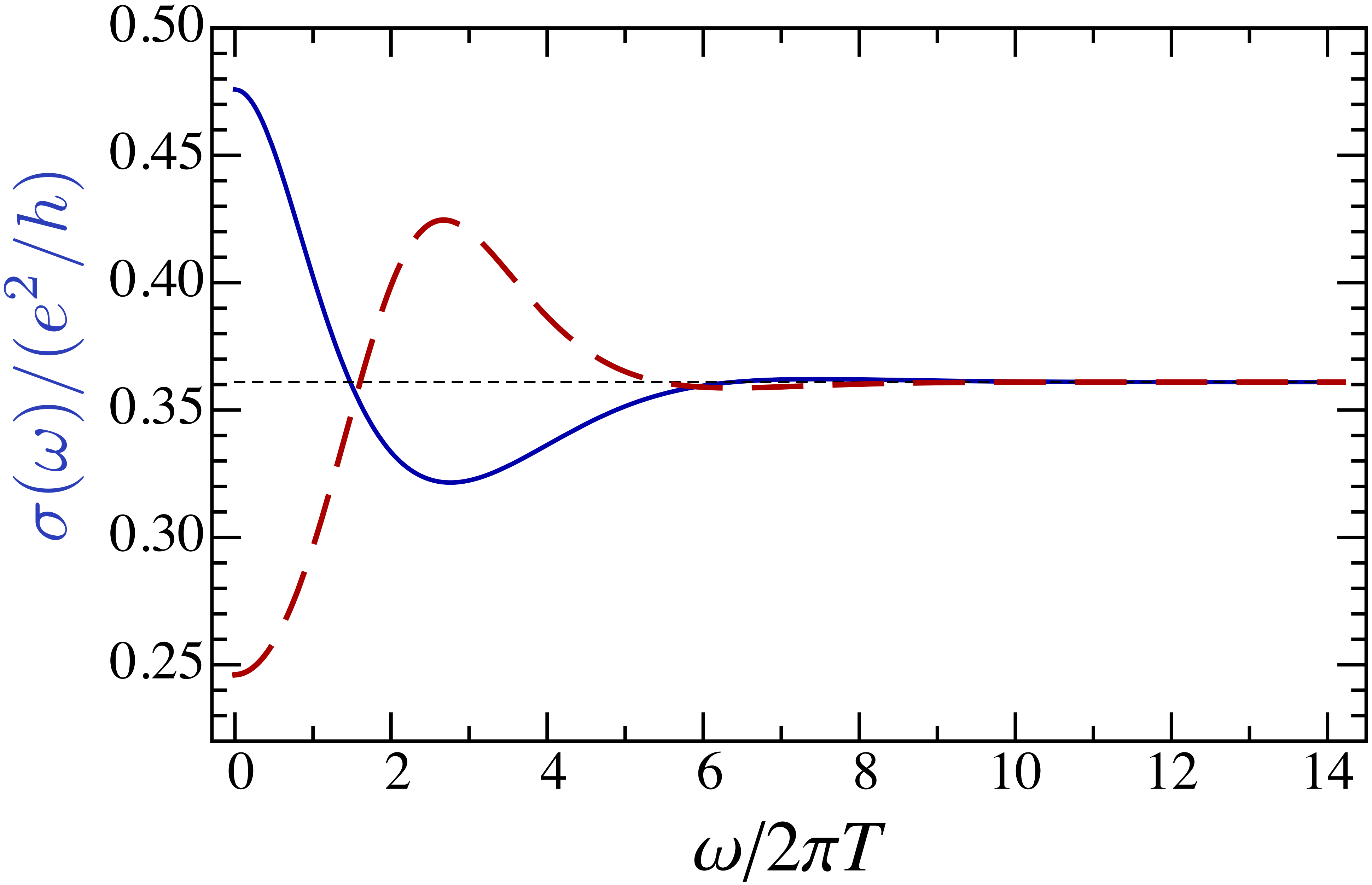

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Outline

1. The simplest model without quasiparticles

A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice

B. Conformal field theories in 2+1 dimensions, the AdS/CFT correspondence, and transport without quasiparticles.

2. Strange metals in the high T_c superconductors

A. The onset of antiferromagnetism in a metal

B. Non-quasiparticle transport at the Ising-nematic quantum critical point

C. Entanglement, holography, and strange metals

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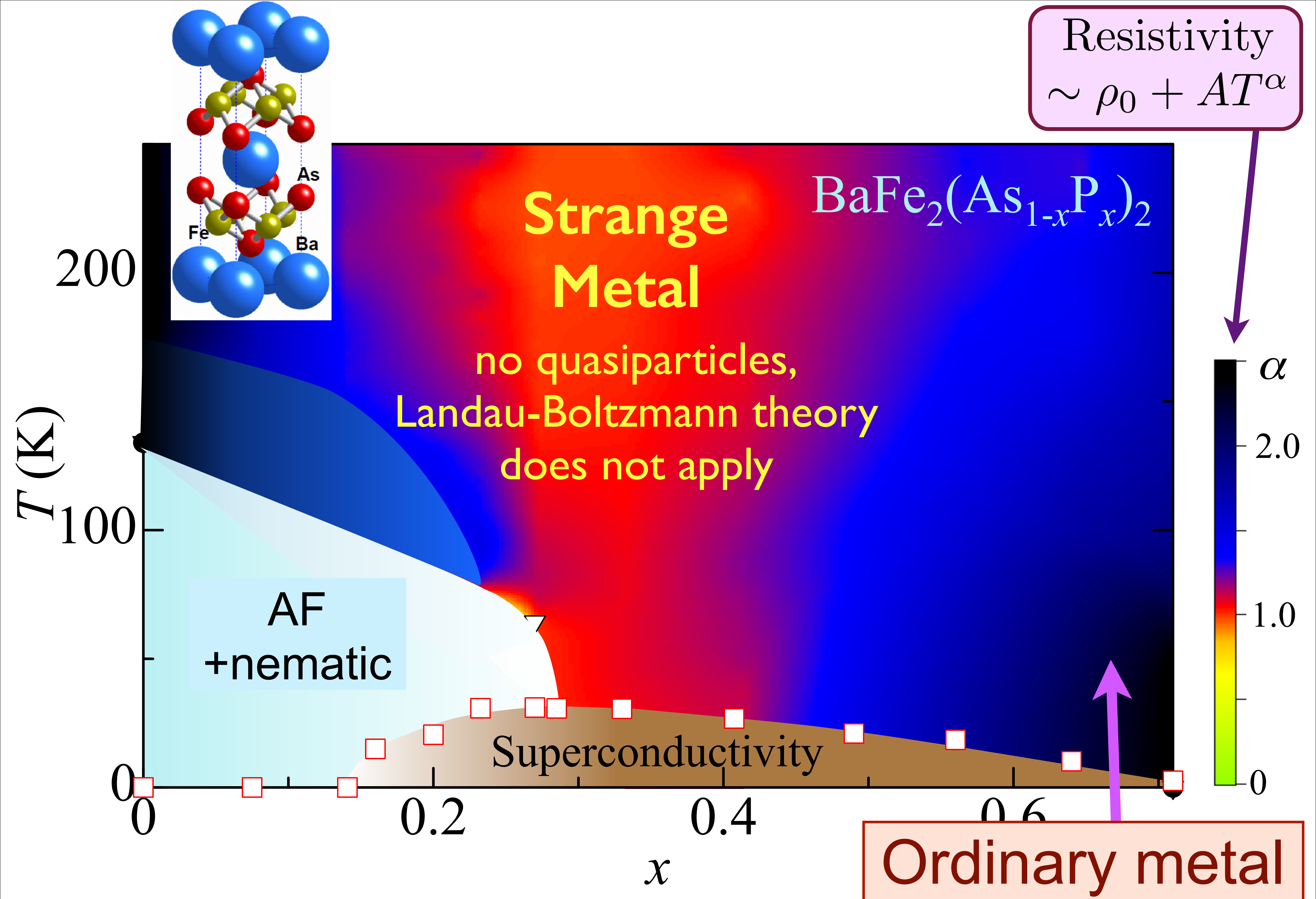
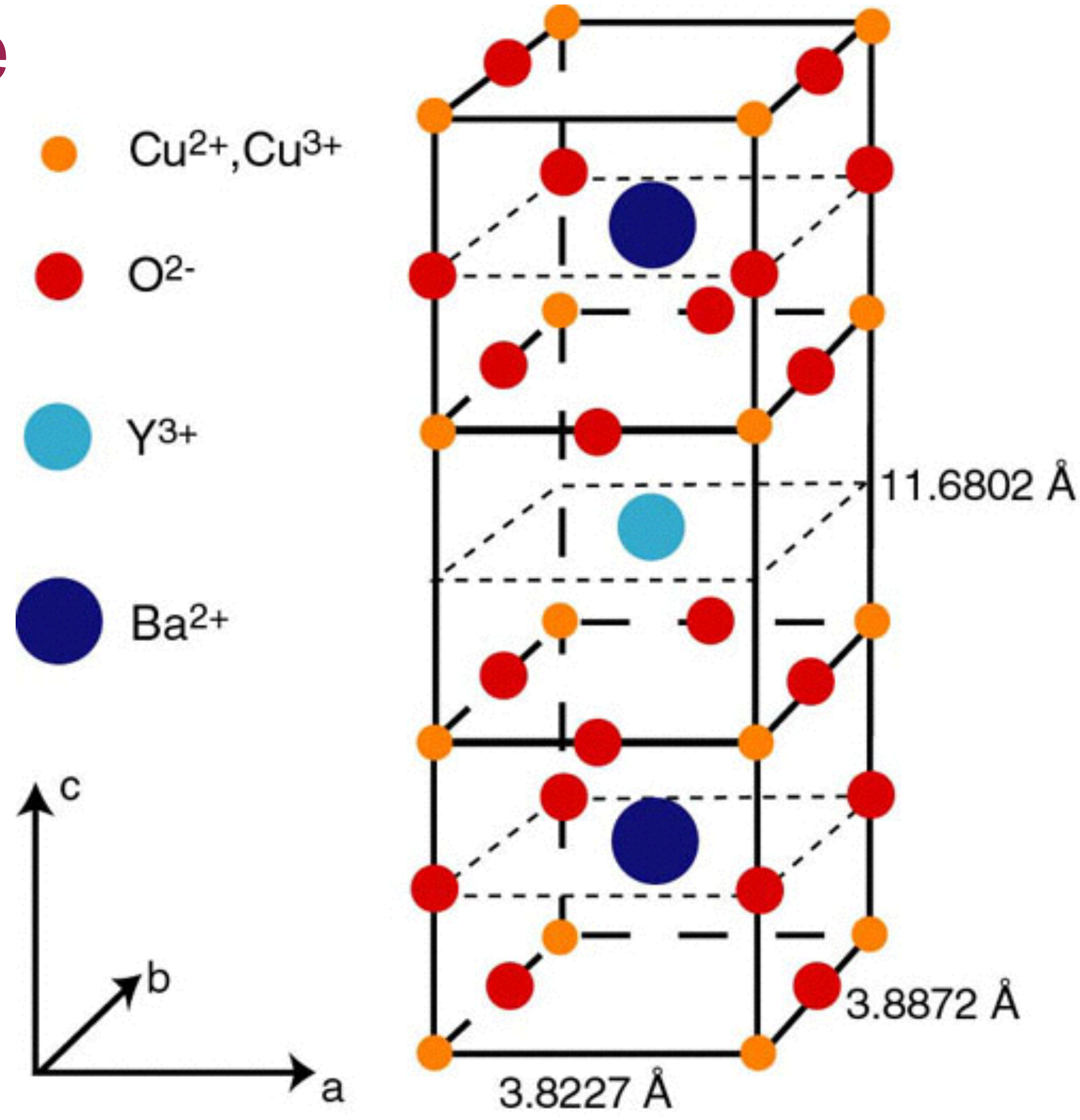
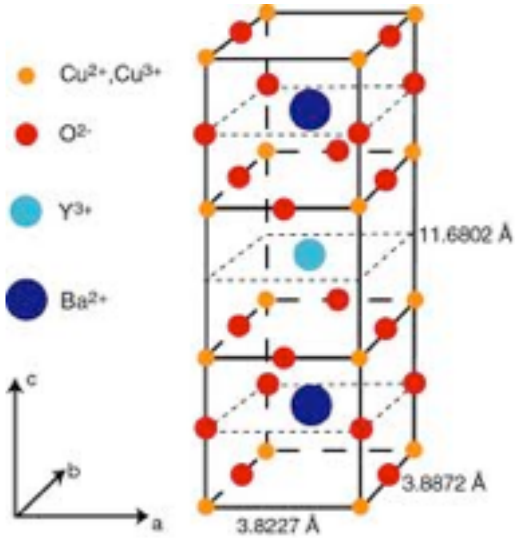


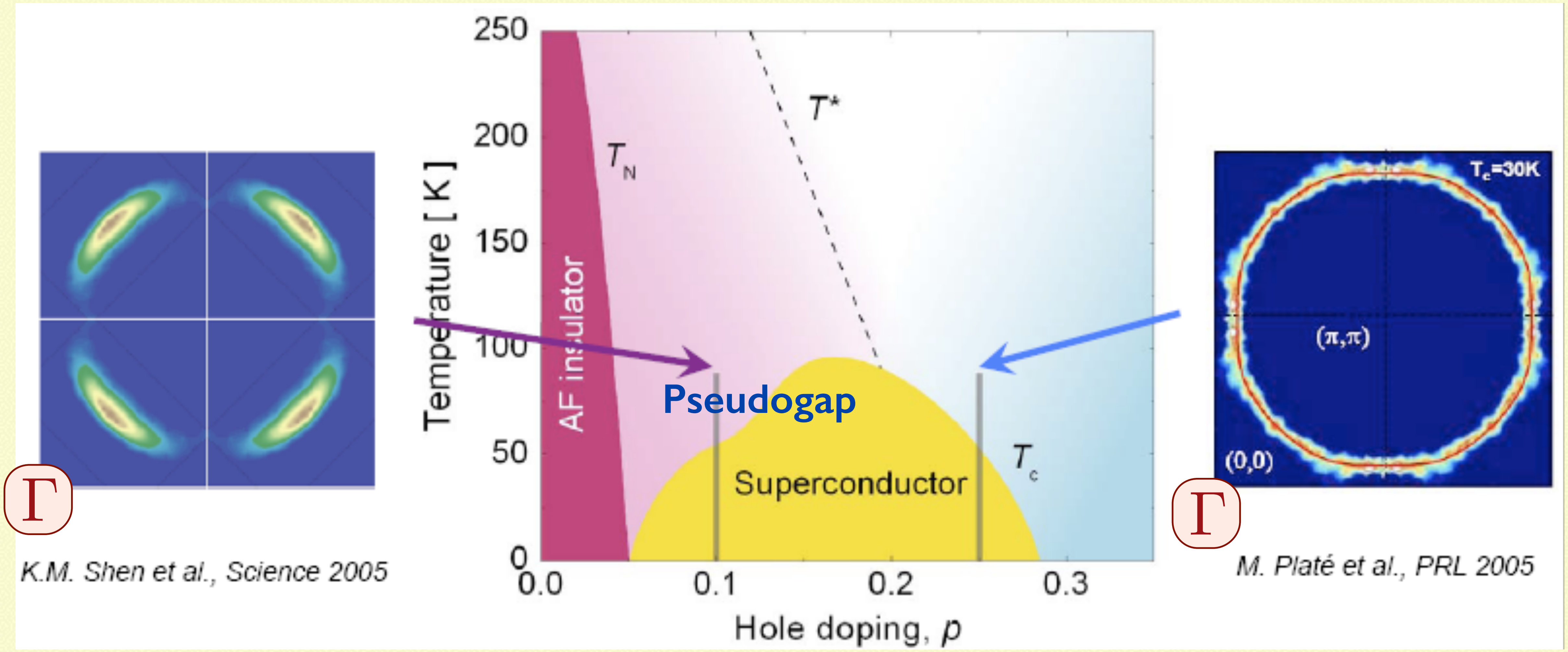
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

High temperature superconductors



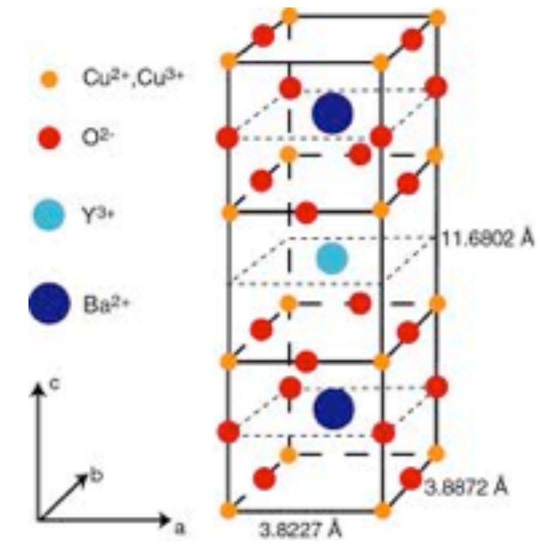
$YBa_2Cu_3O_{6+x}$

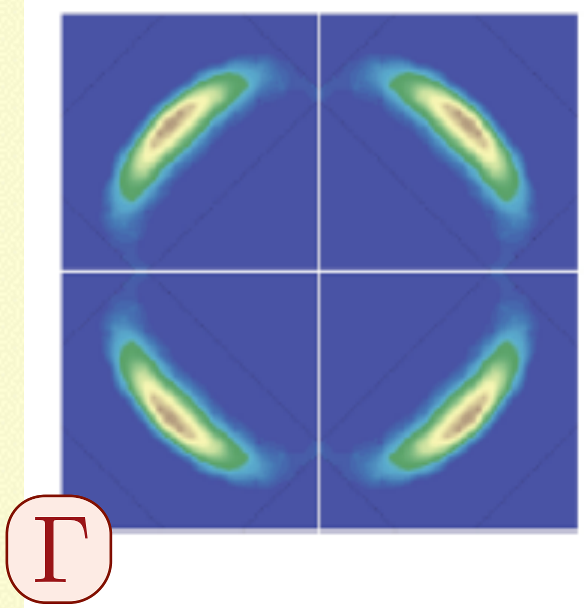




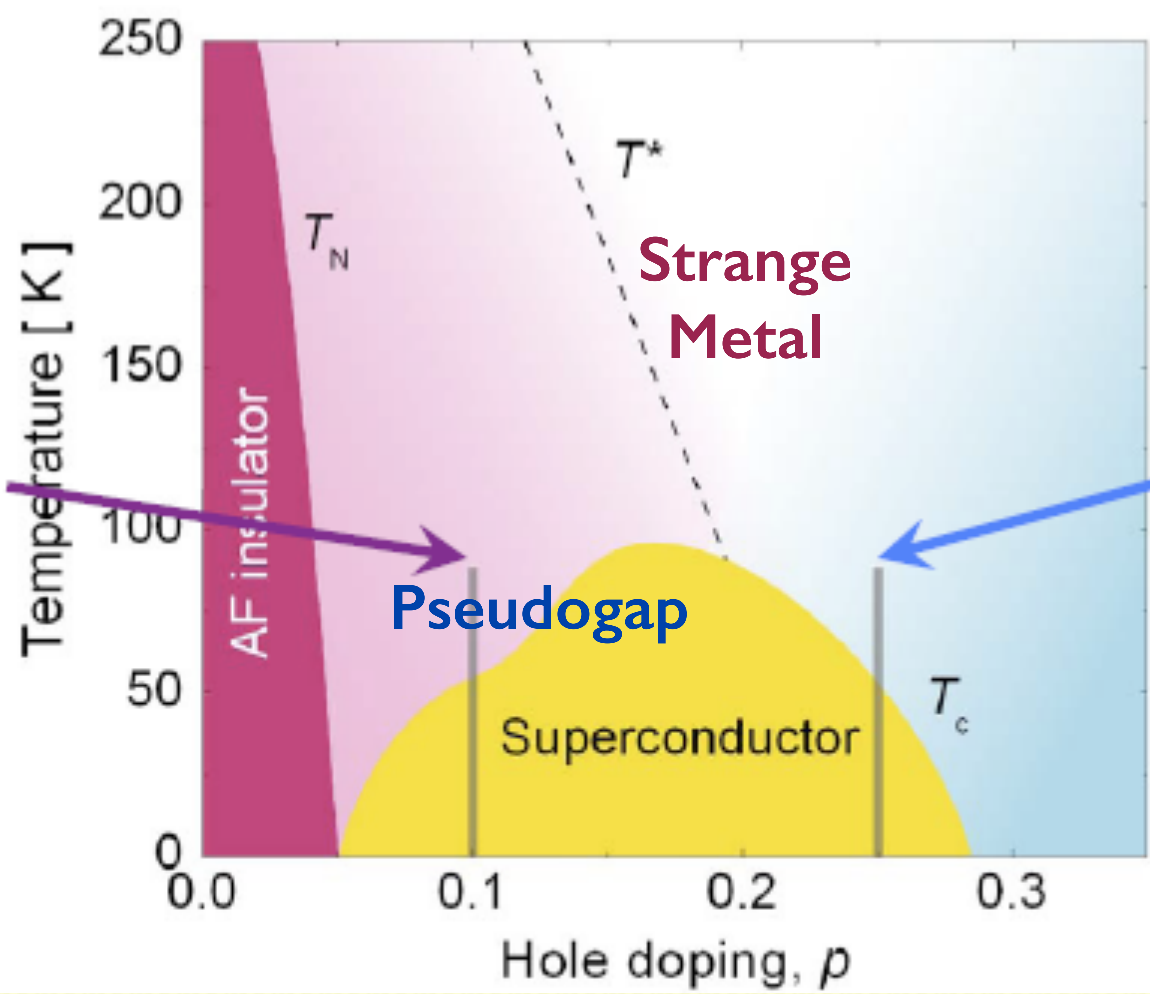
Smaller hole
Fermi-pockets

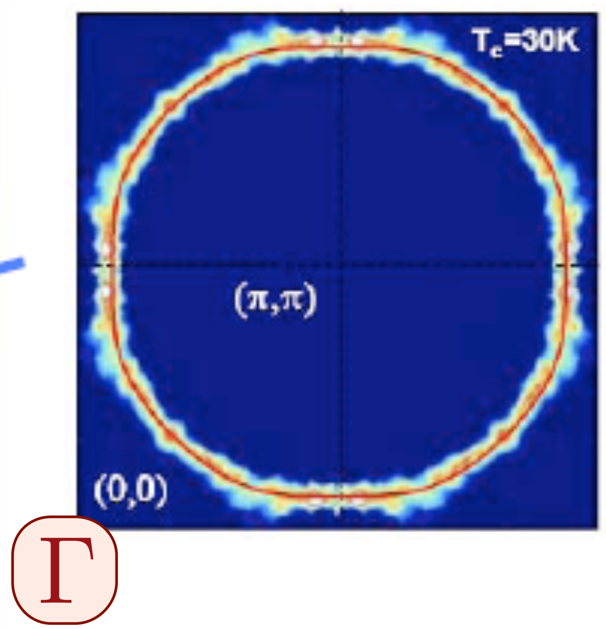
Large hole
Fermi surface





K.M. Shen et al., Science 2005





M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

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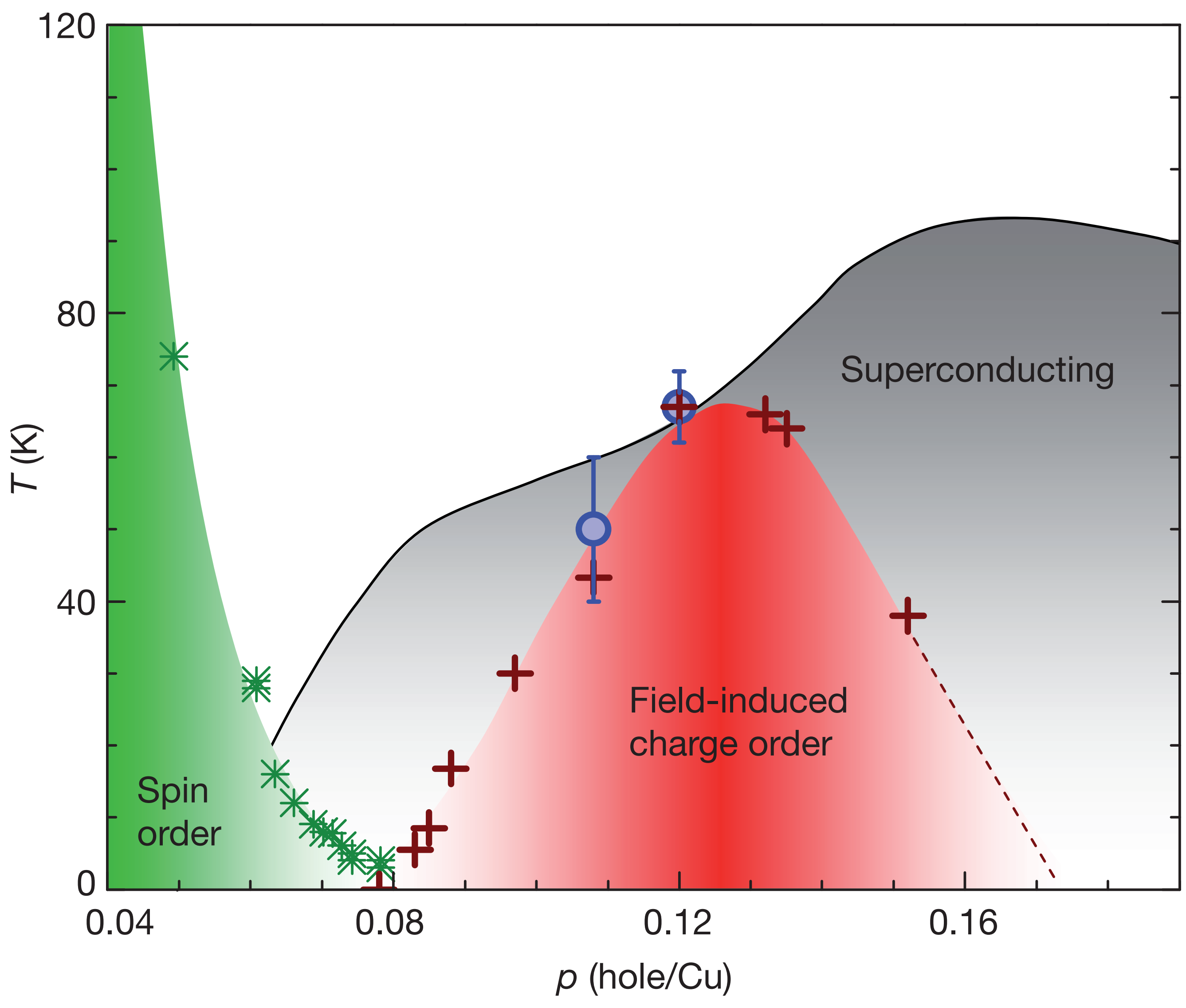
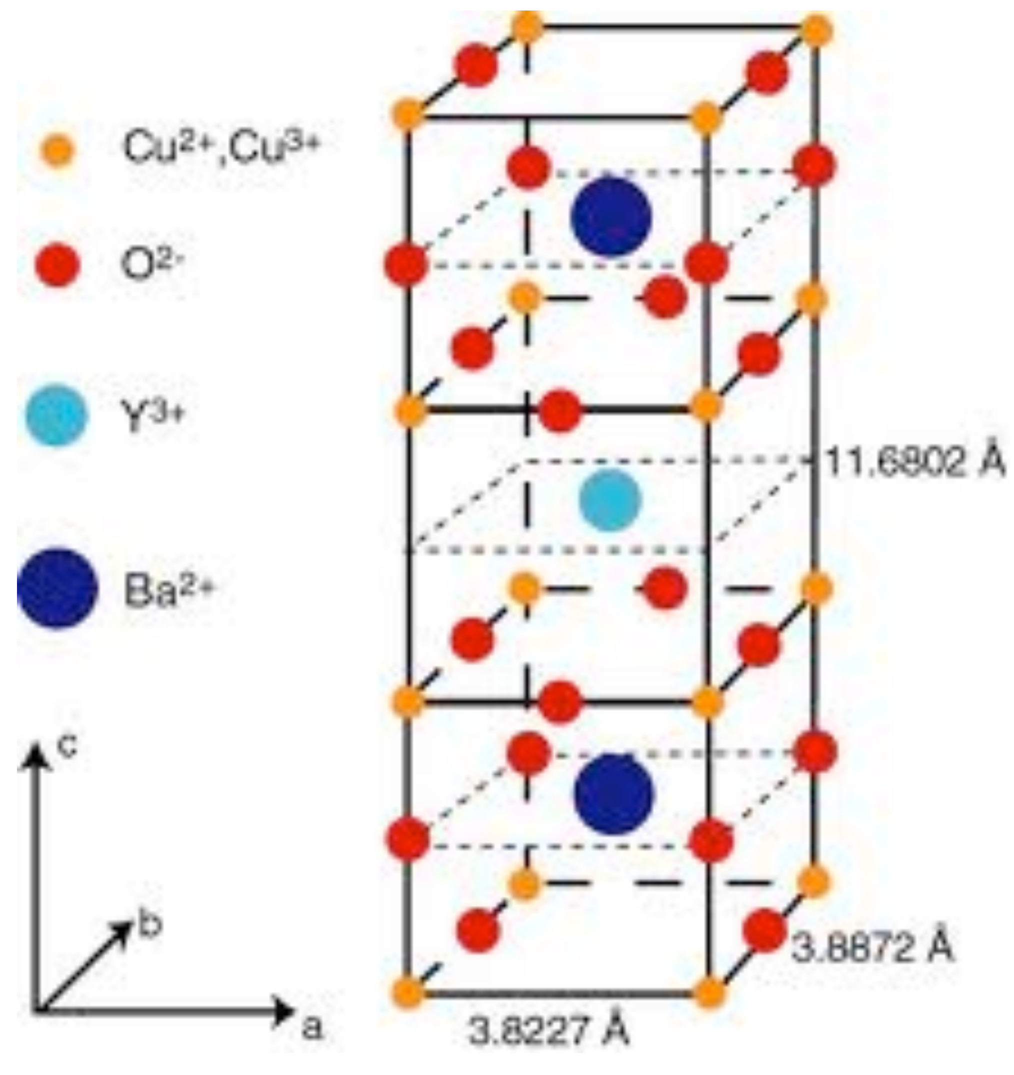
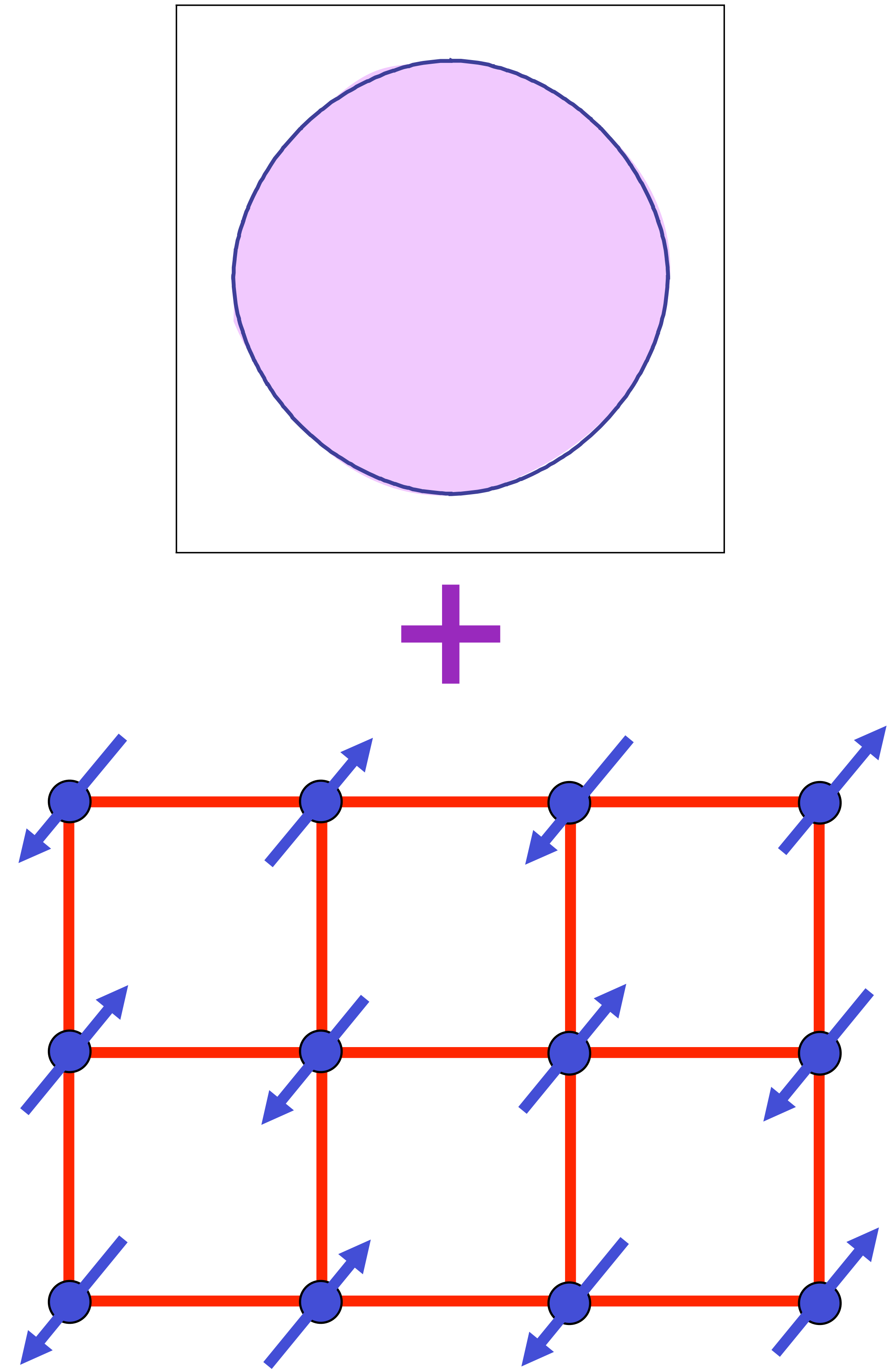




Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface

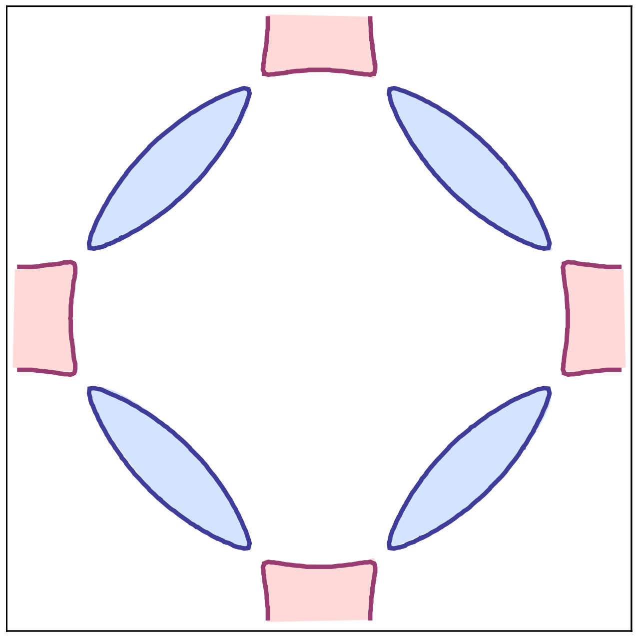


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

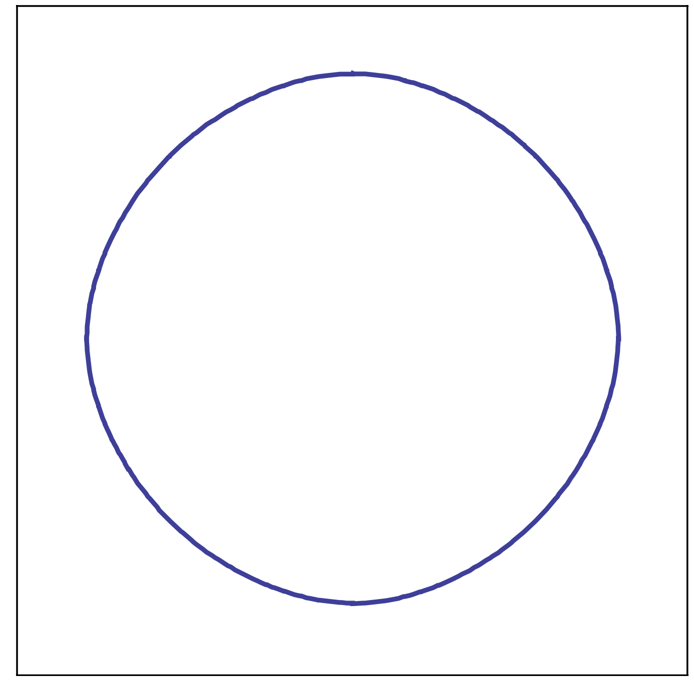
where $\mathbf{K}$ is the ordering wavevector.

Quantum phase transition with onset of antiferromagnetism in a metal



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets



$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

r

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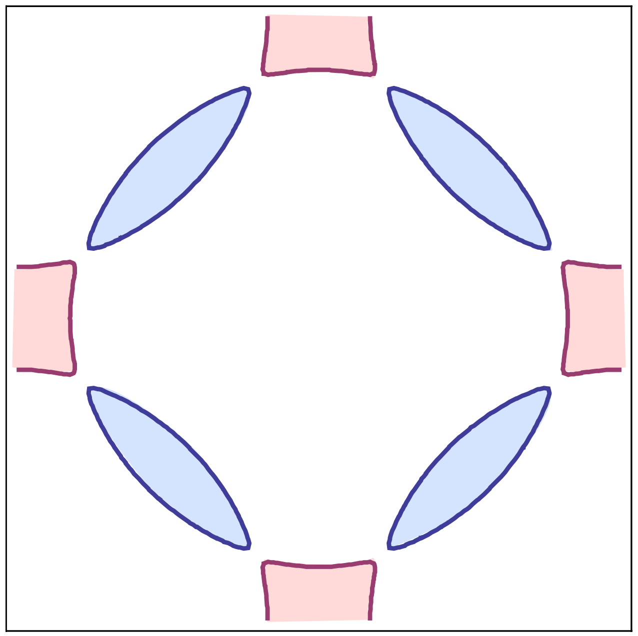

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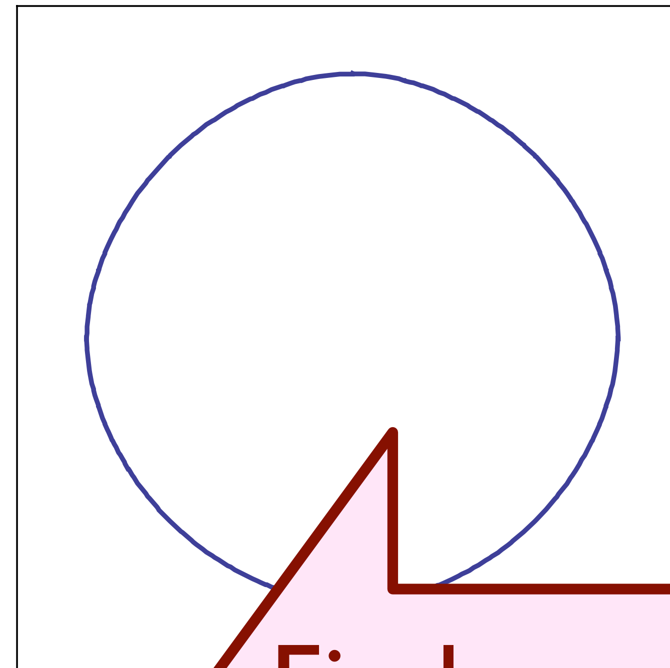


Find new instabilities upon approaching critical point

Metal w th “large” Fermi surface

r

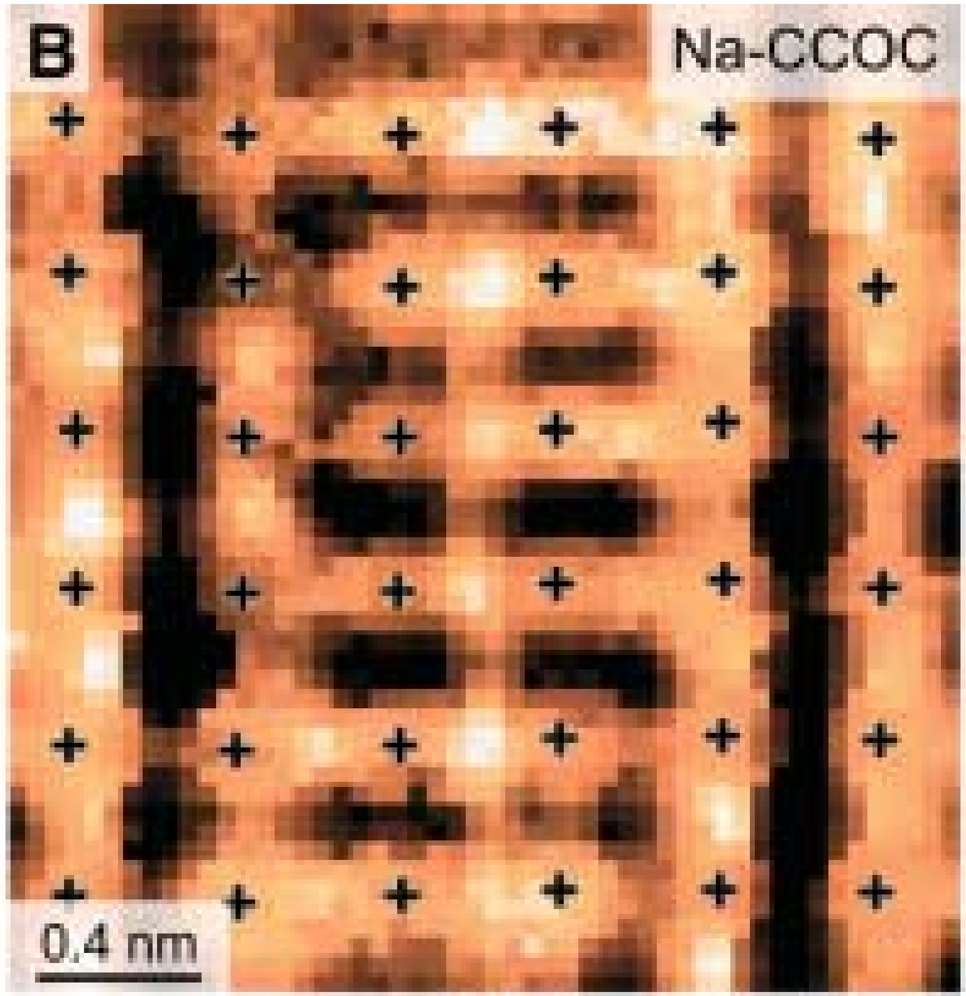
New physics in metals with antiferromagnetic correlation

- Weak-coupling instability to d-wave superconductivity, Ψ.
- Superconductivity survives at strong-coupling with critical antiferromagnetism
- Secondary instability to incommensurate d-wave bond order, Φ_x, Φ_y
- X-ray scattering in underdoped YBCO is described by angular thermal fluctuations of a 6-component order parameter, Ψ, Φ_x, Φ_y.

An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8*]

9 MARCH 2007 VOL 315 SCIENCE



Phys. Rev. Lett. **109**, 167001 (2012).

Distinct Charge Orders in the Planes and Chains of Ortho-III-Ordered $YBa_2Cu_3O_{6+\delta}$ Superconductors Identified by Resonant Elastic X-ray Scattering

A. J. Achkar,[1] R. Sutarto,[2,3] X. Mao,[1] F. He,[3] A. Frano,[4,5] S. Blanco-Canosa,[4] M. Le Tacon,[4] G. Ghiringhelli,[6] L. Braicovich,[6] M. Minola,[6] M. Moretti Sala,[7] C. Mazzoli,[6] Ruixing Liang,[2] D. A. Bonn,[2] W. N. Hardy,[2] B. Keimer,[4] G. A. Sawatzky,[2] and D. G. Hawthorn[1,*]

In such a case, the energy shifts may in fact be a signature of a novel electronic state, such as a valence bond solid

New physics in metals with antiferromagnetic correlation

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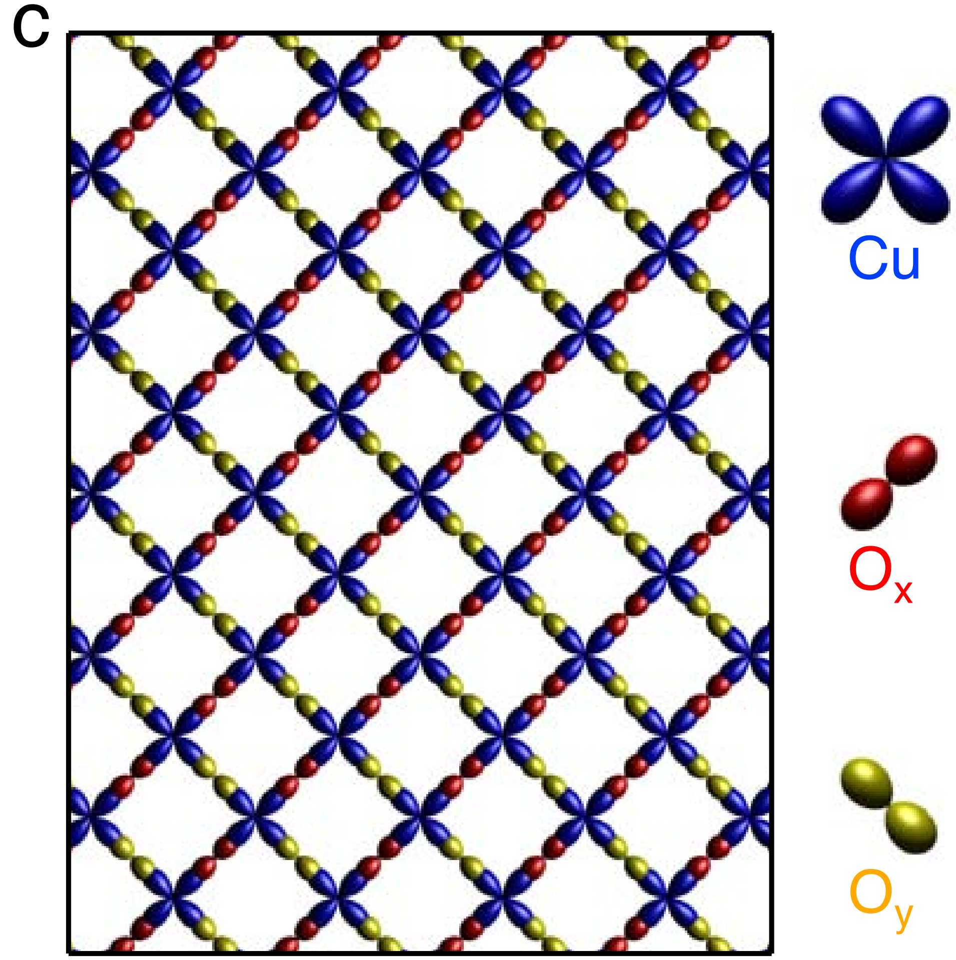
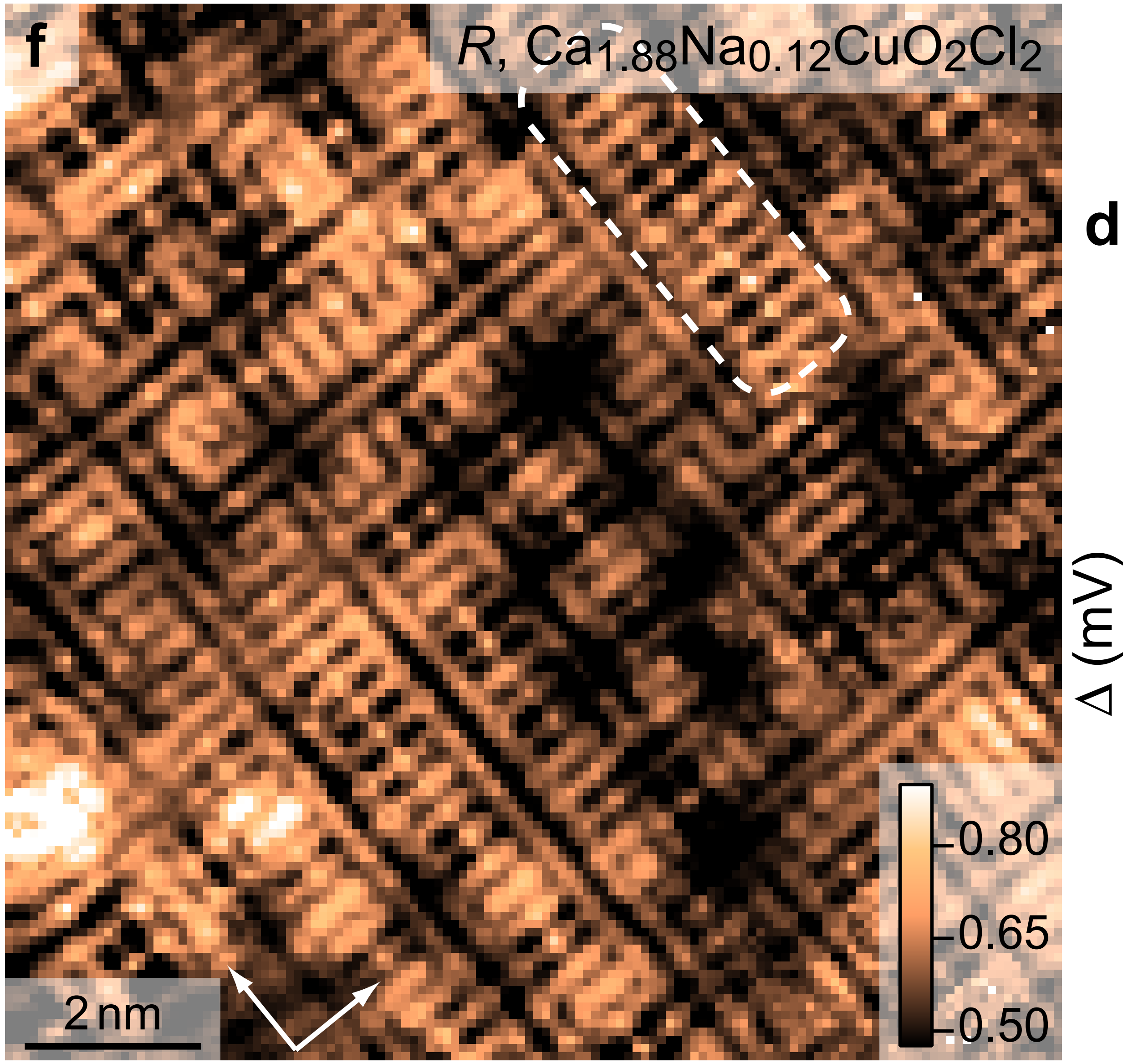
Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi

Nature Physics, **8**, 534 (2012).





Evidence for "nematic" order (*i.e.* breaking of 90° rotation symmetry) in $Ca_{1.88}Na_{0.12}CuO_2Cl_2$.

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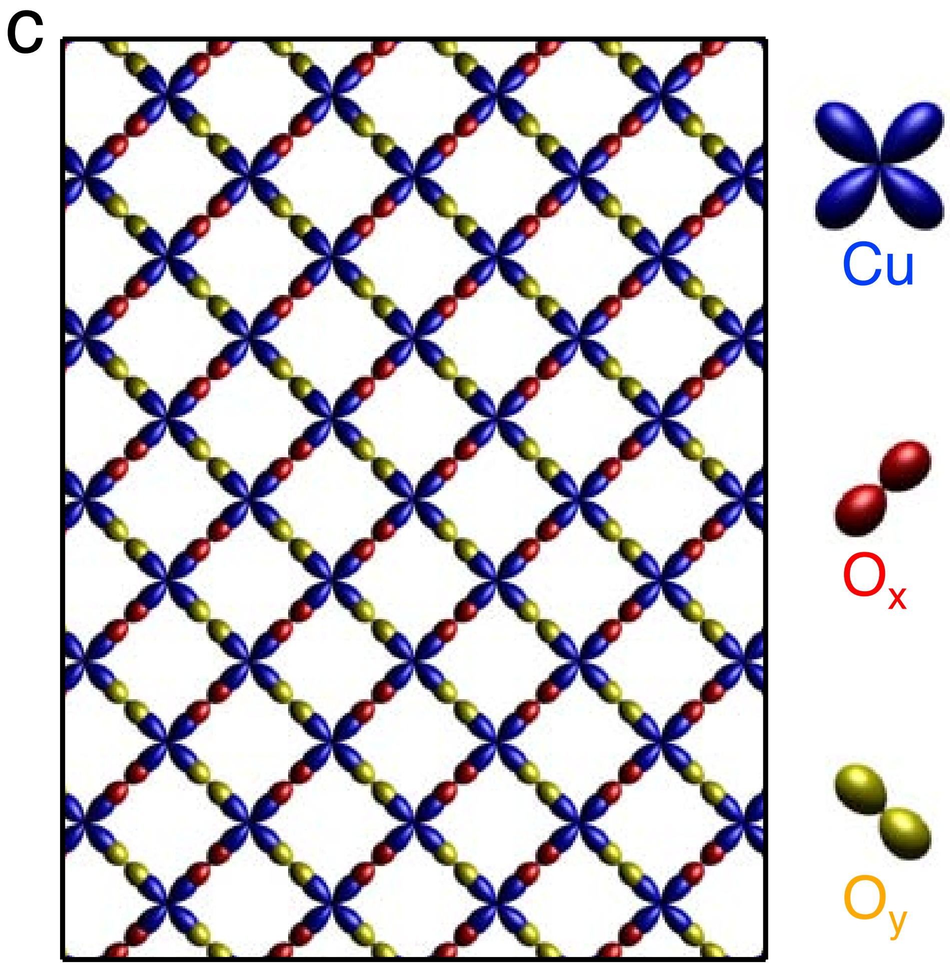

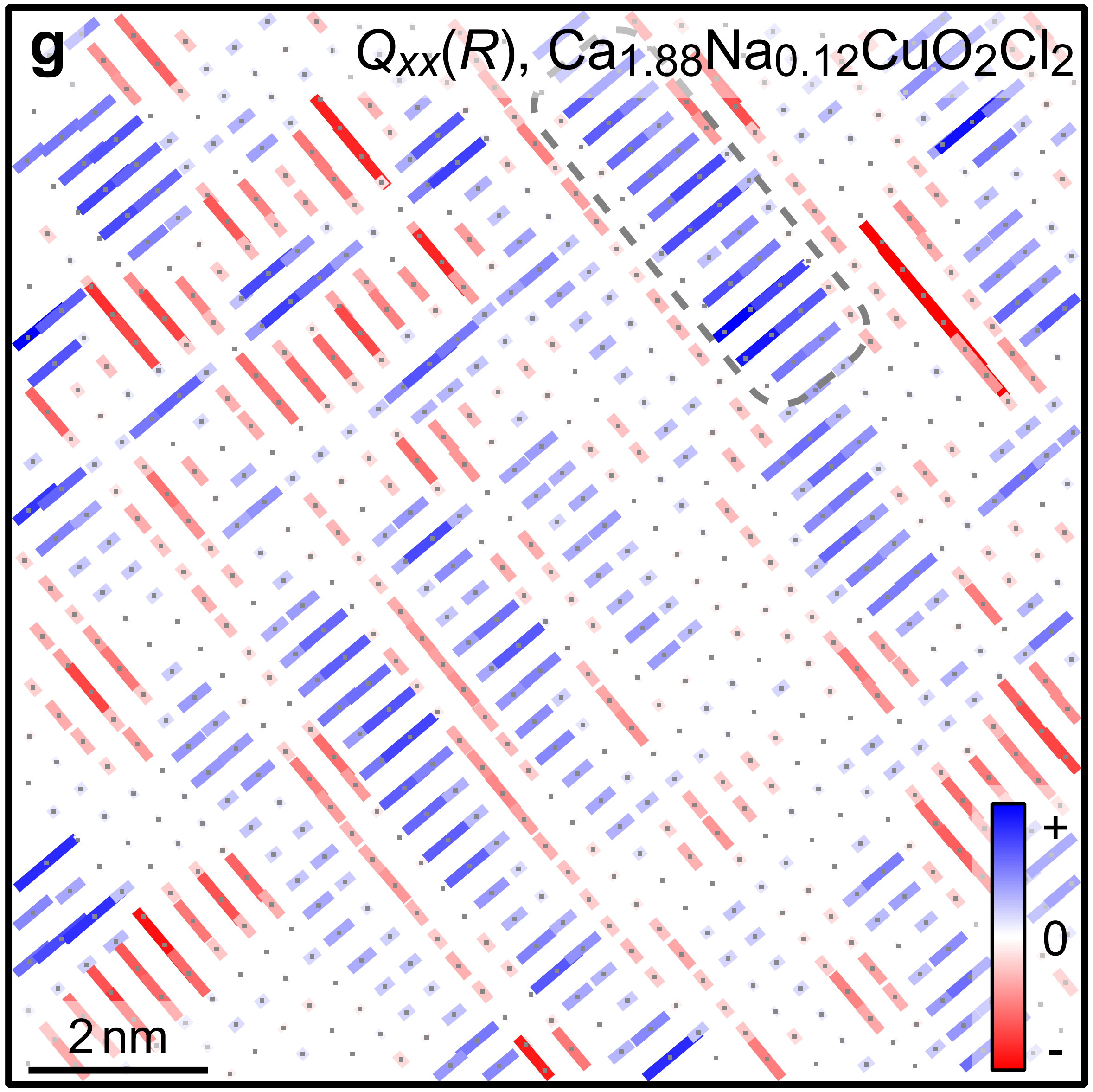

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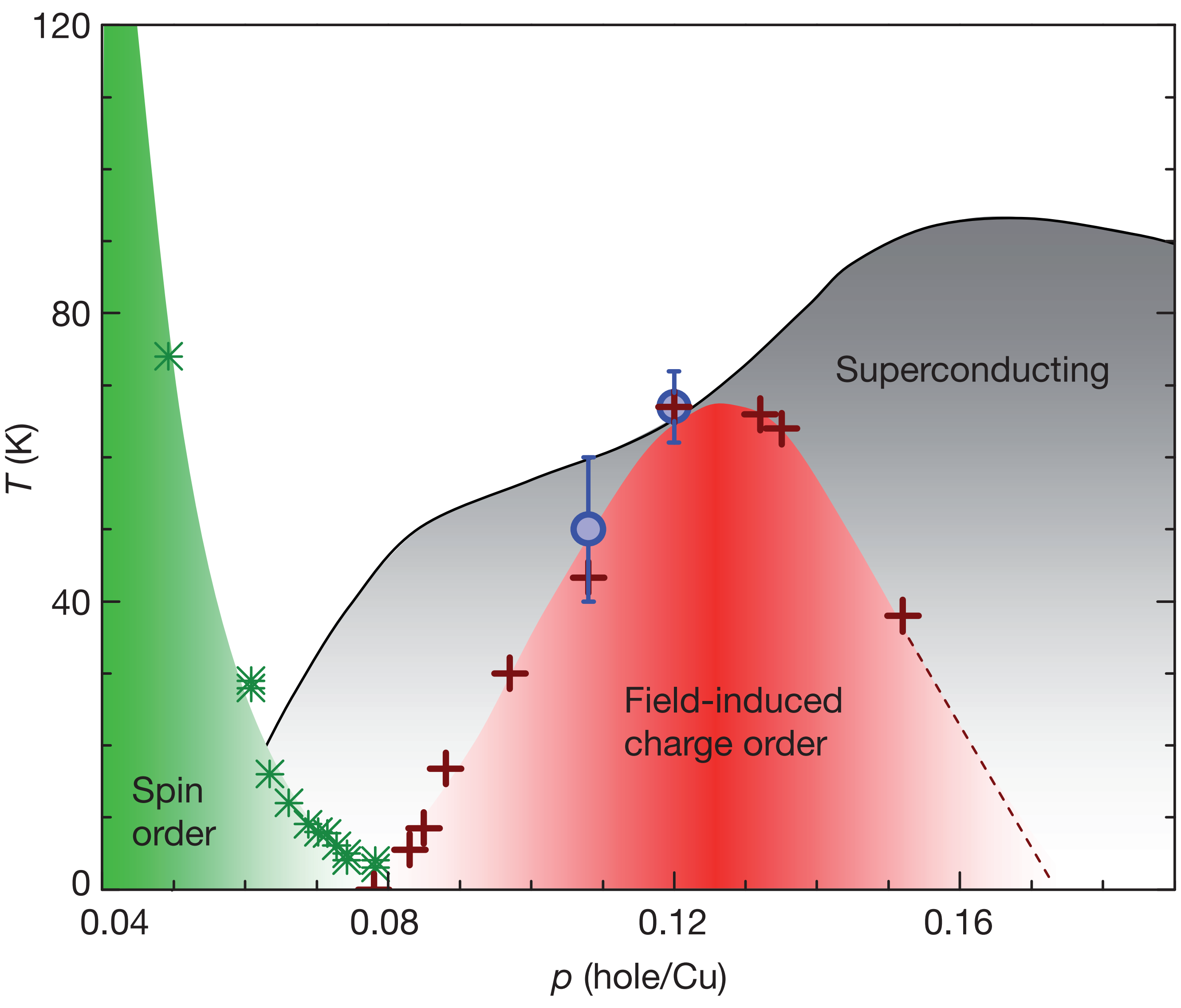
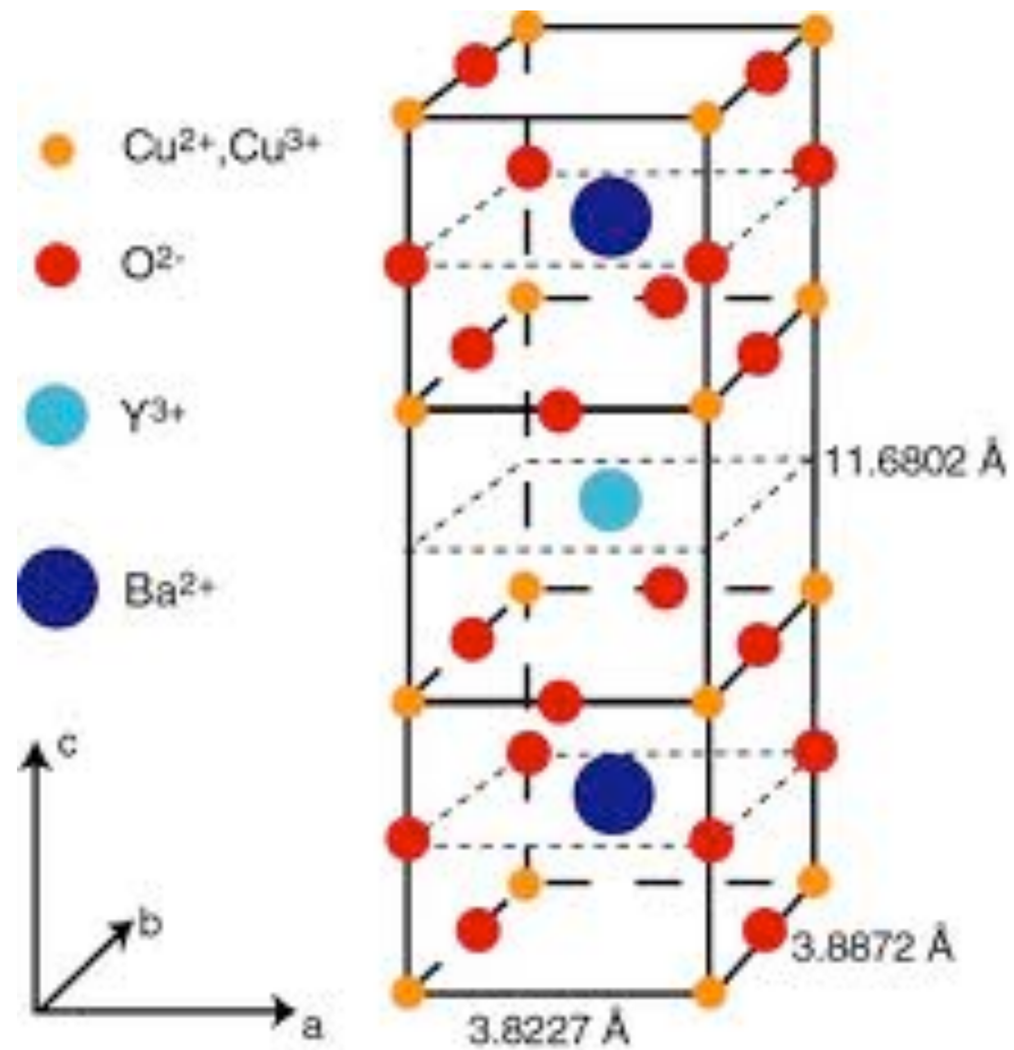
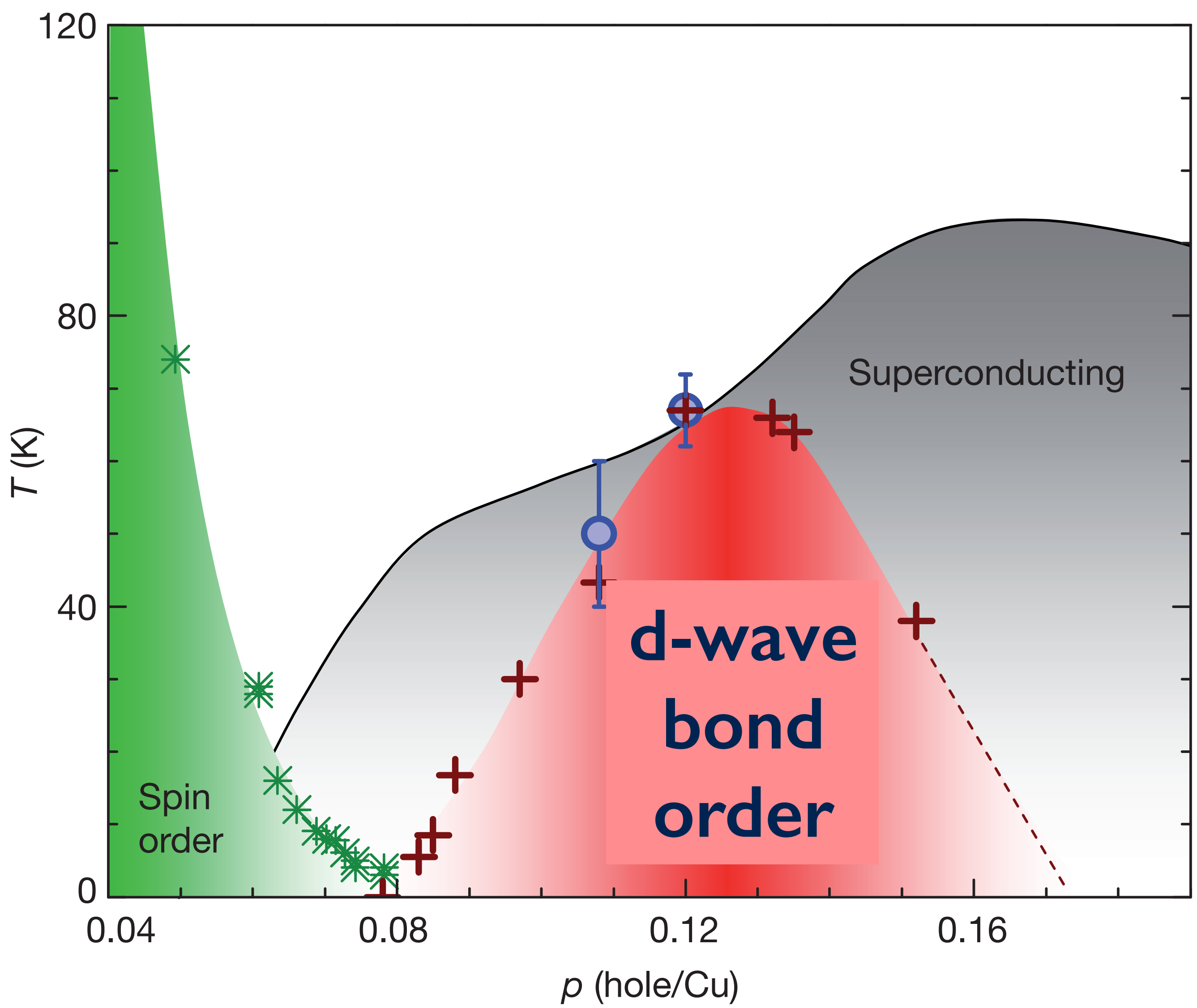
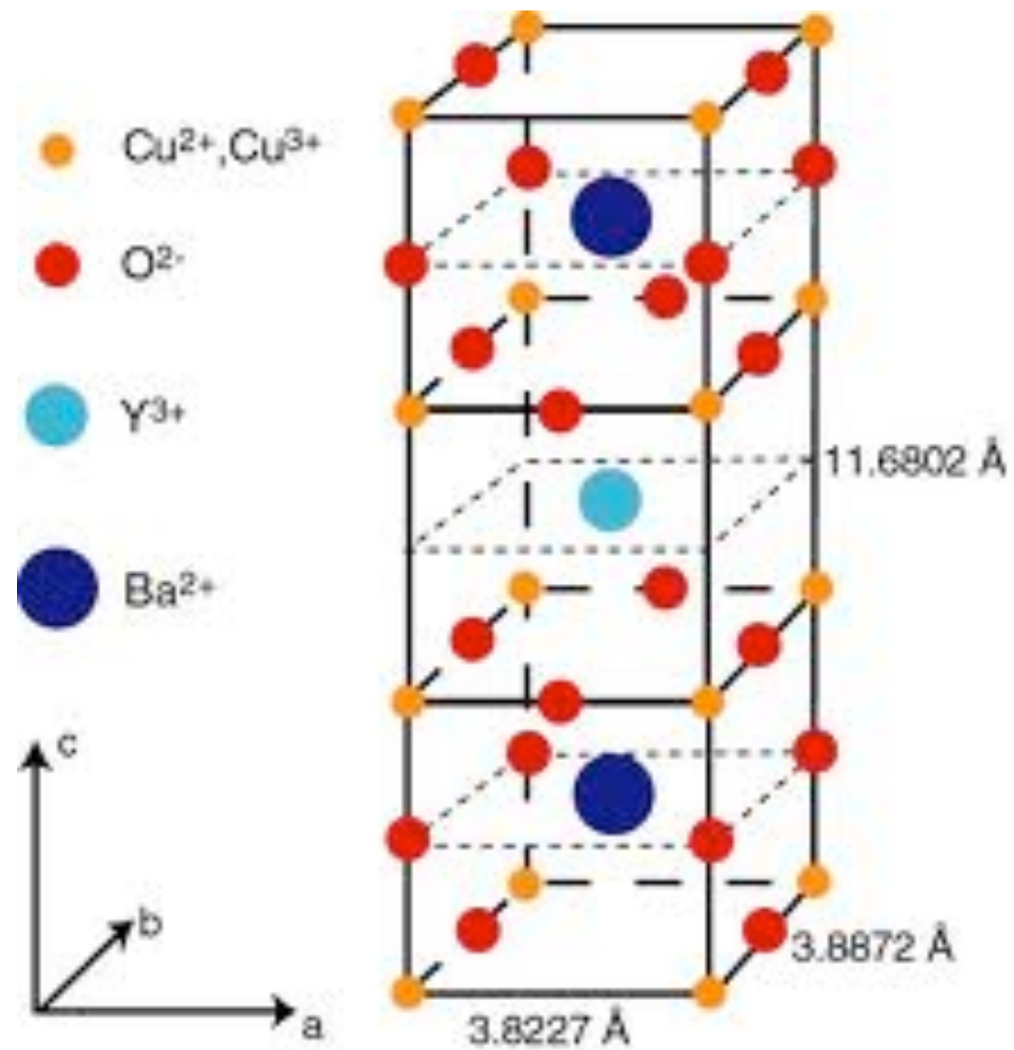
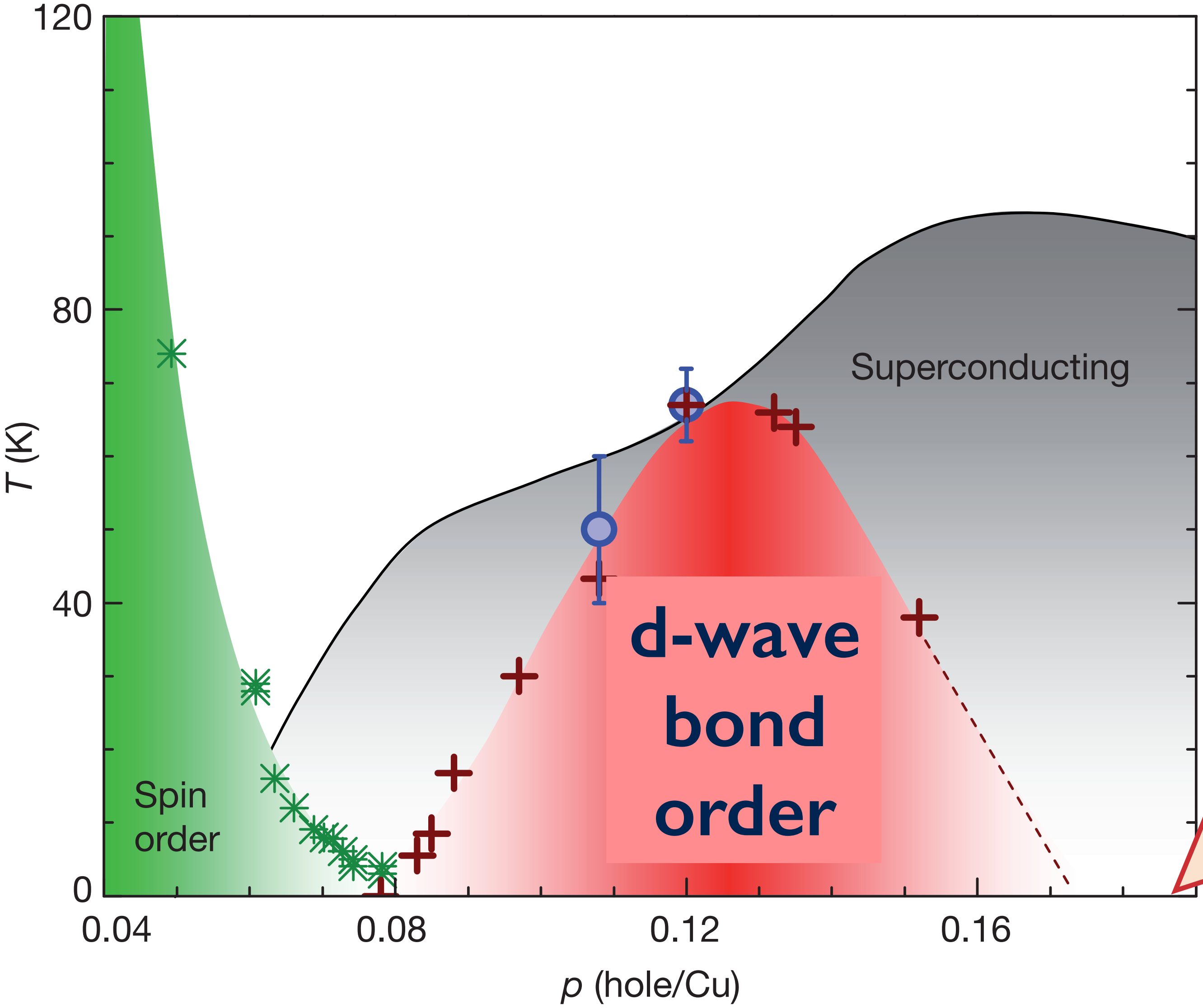


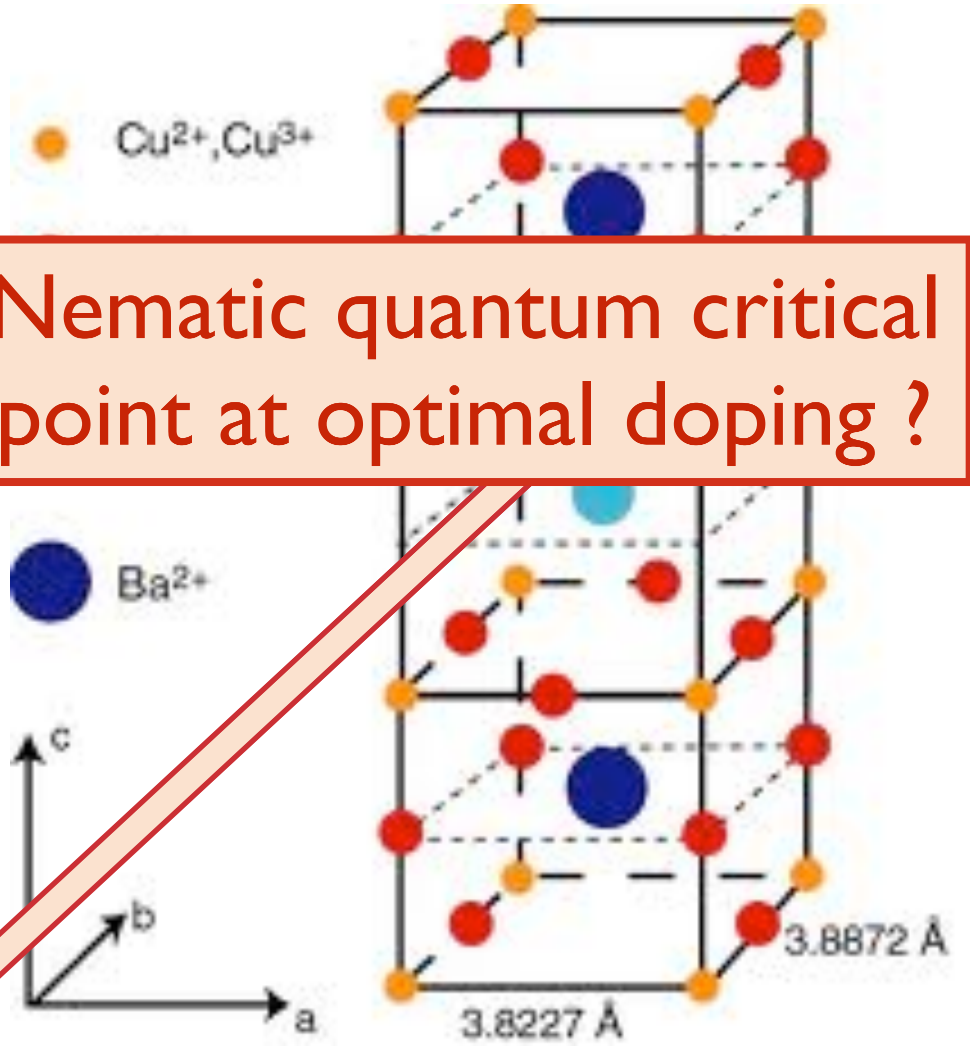


M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
M. Vojta and O. Rosch, Physical Review B **77**, 094504 (2008)
M. Metlitski and S. Sachdev, Physical Review B **82**, 075128 (2010)
S. Sachdev and R. La Placa, Physical Review Letters **111**, 027202 (2013)
A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807



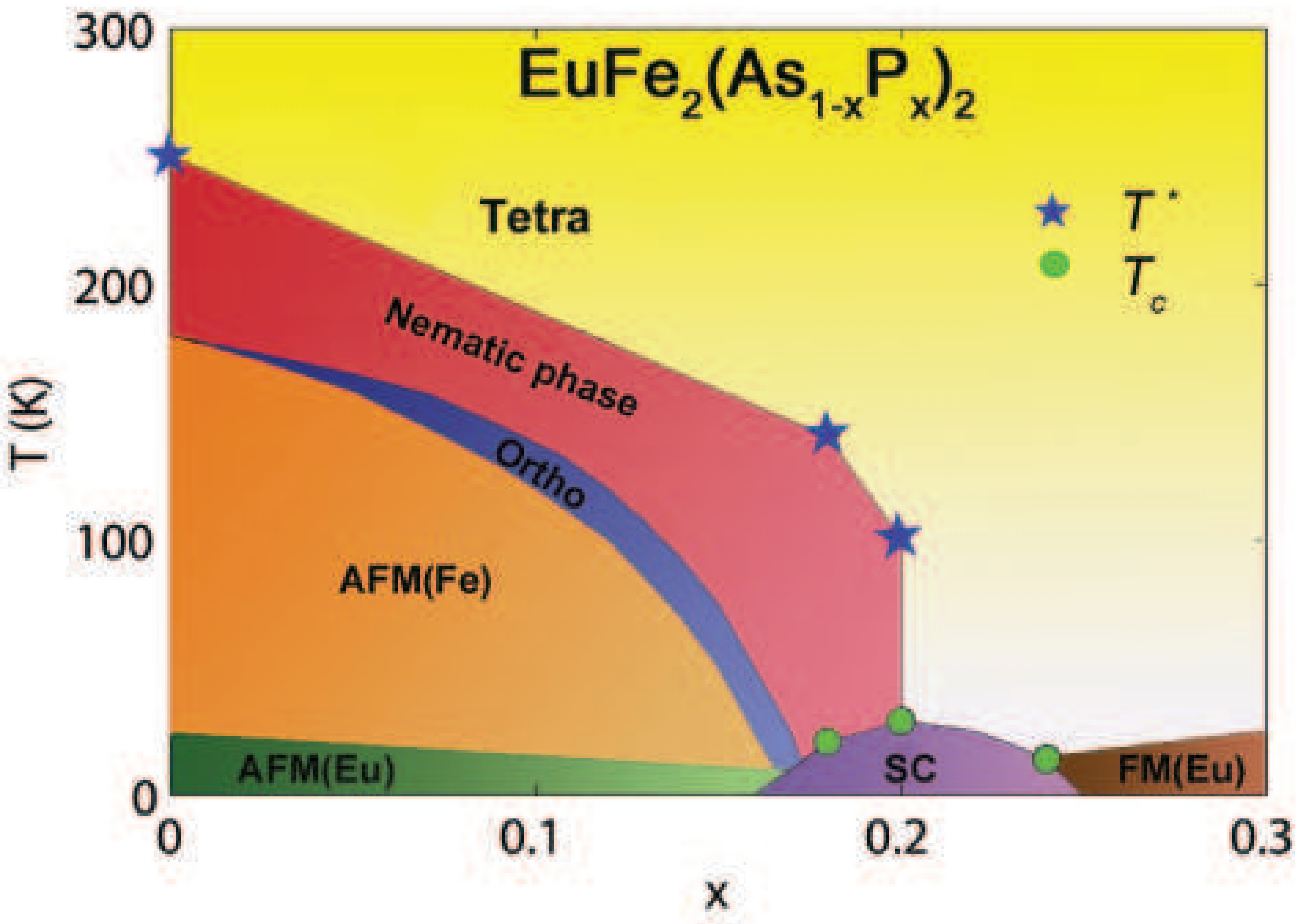


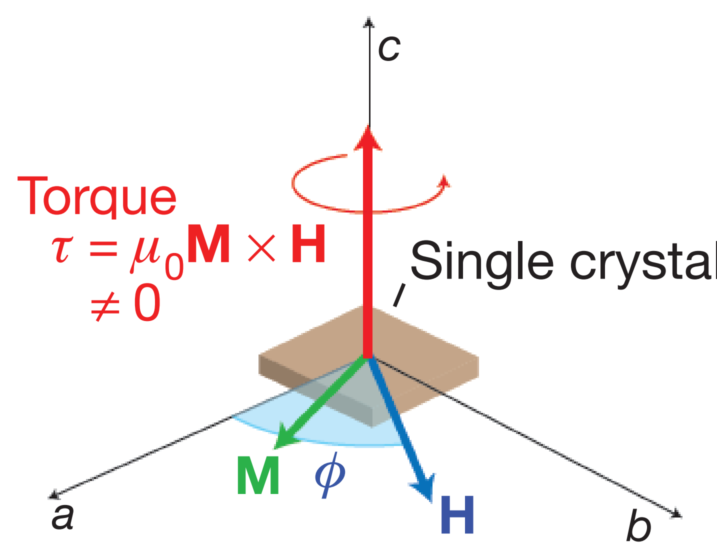




Nematic quantum critical point at optimal doping ?

Xiaofeng Xu, W. H. Jiao, N. Zhou, Y. K. Li, B. Chen, C. Cao, Jianhui Dai, A. F. Bangura, and Guanghan Cao, arXiv:1402.4124





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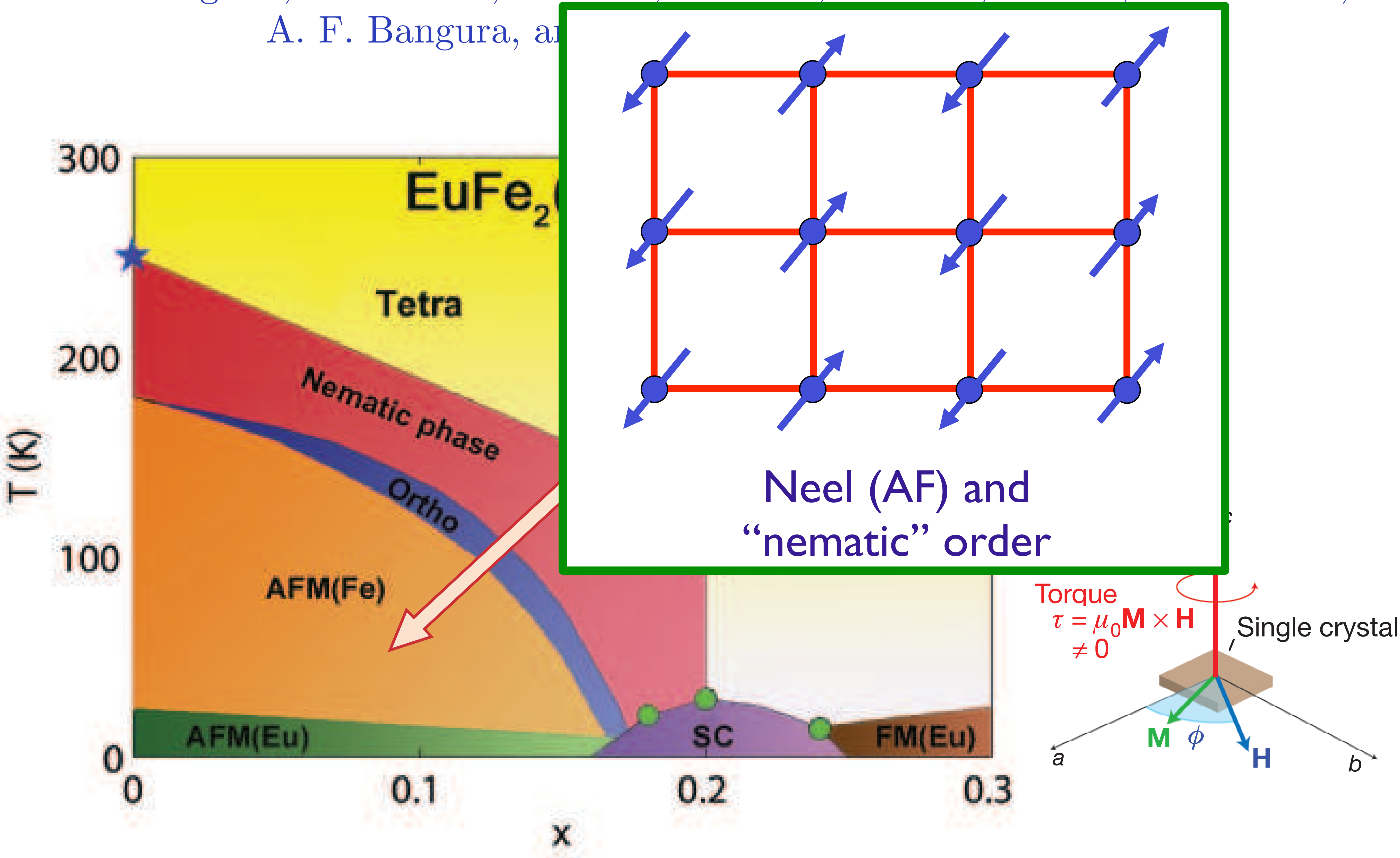

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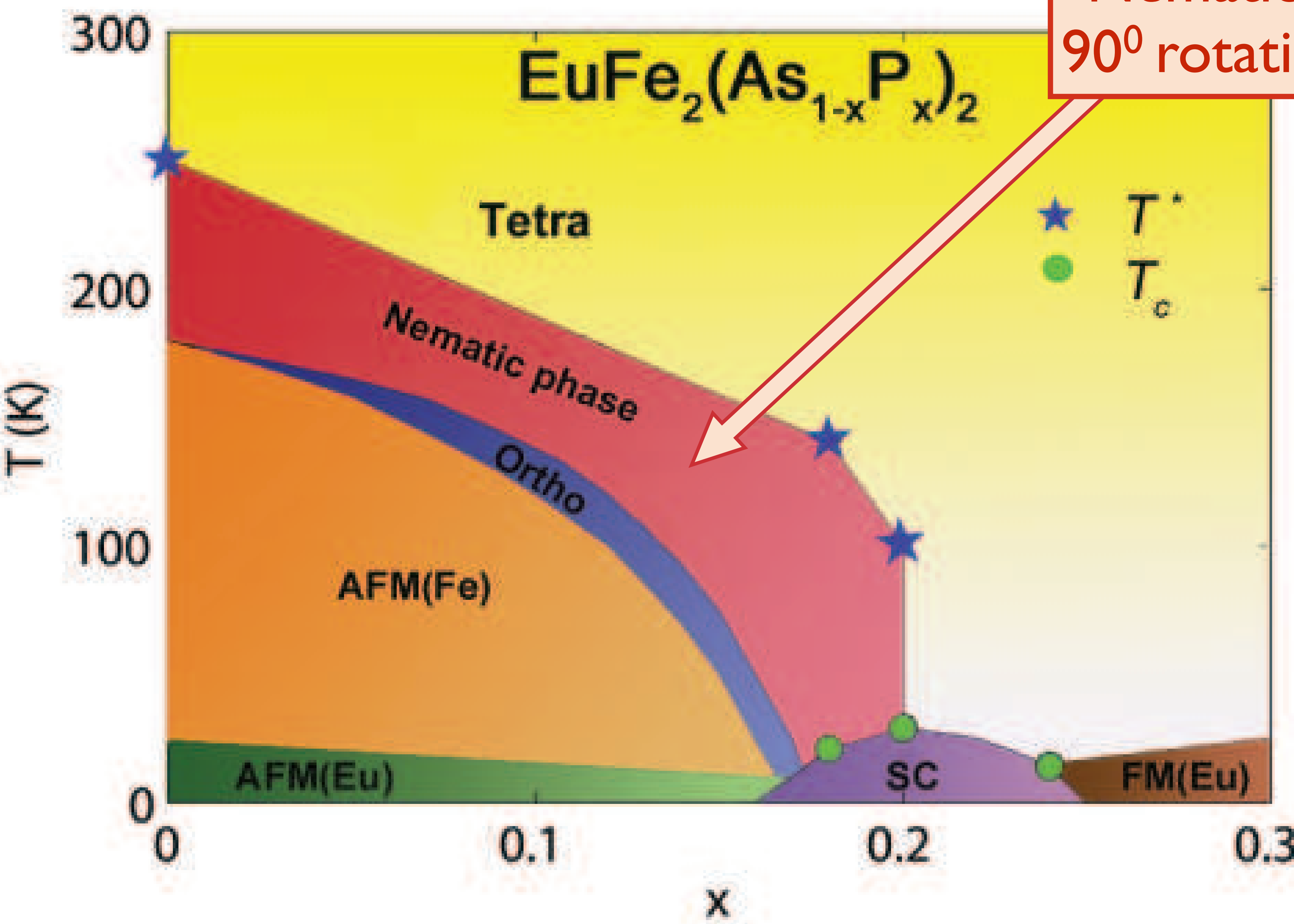
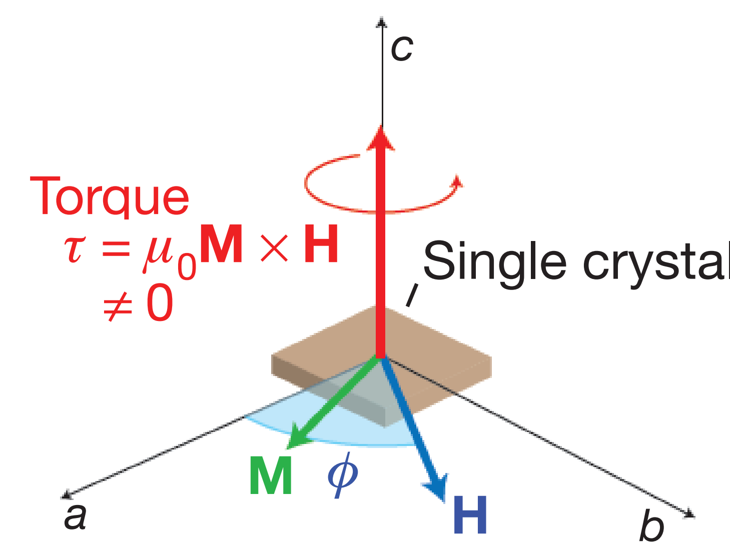
Nematic phase breaks 90^0 rotational symmetry





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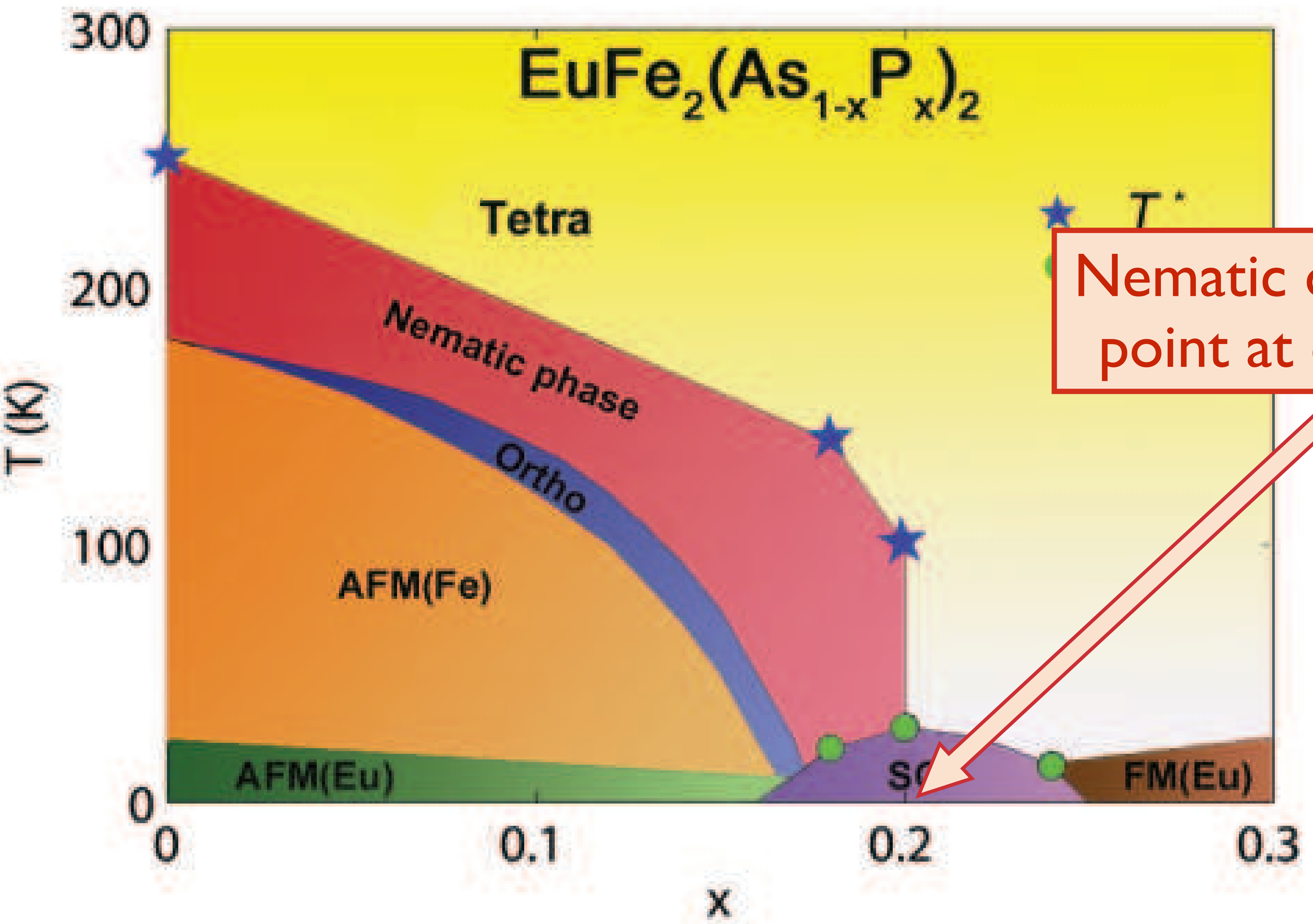

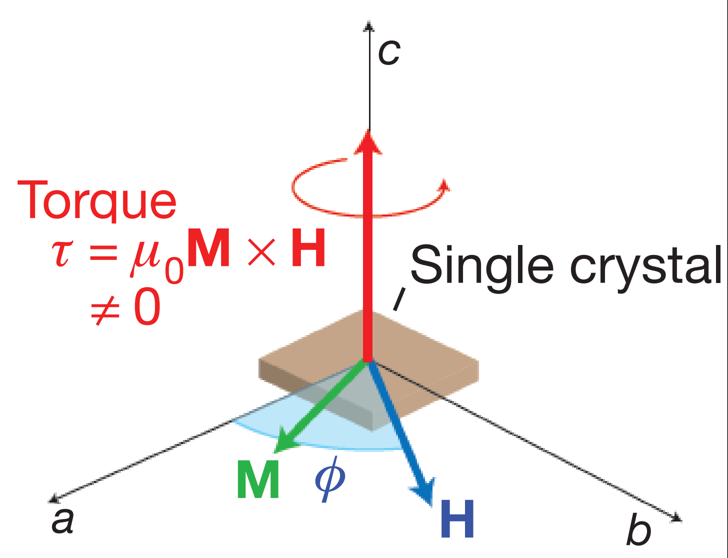

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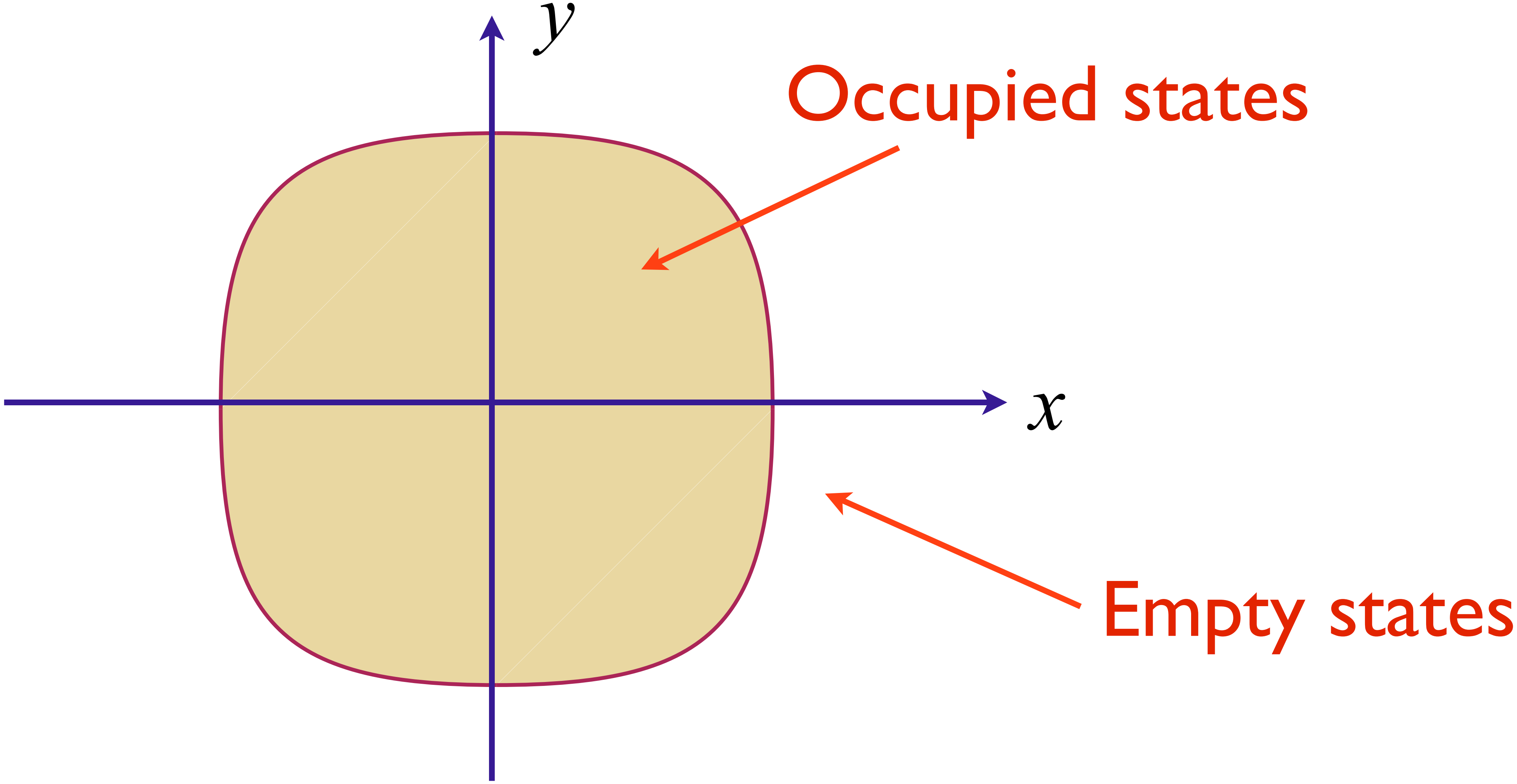
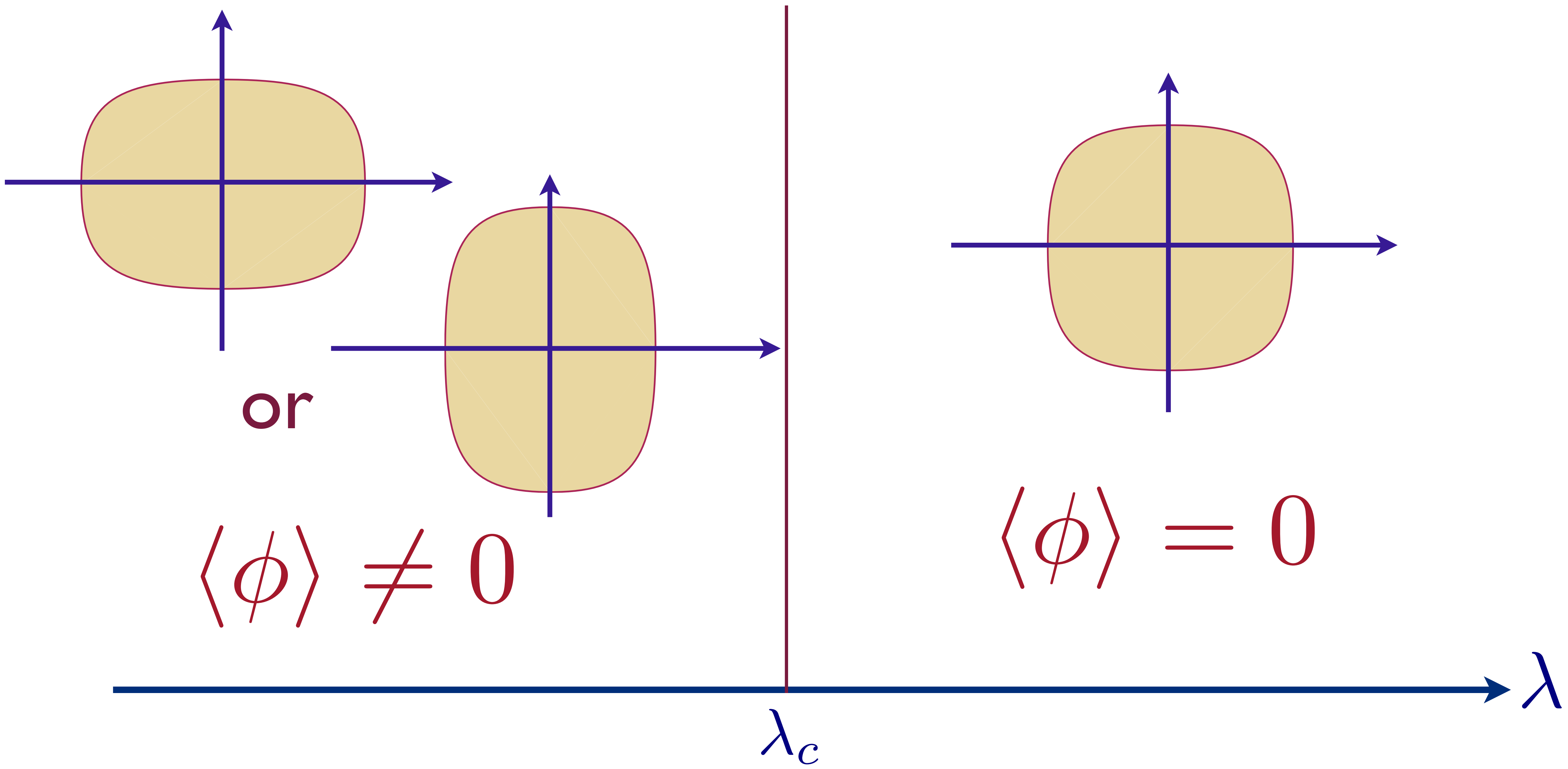
Quantum criticality of Ising-nematic ordering in a metal



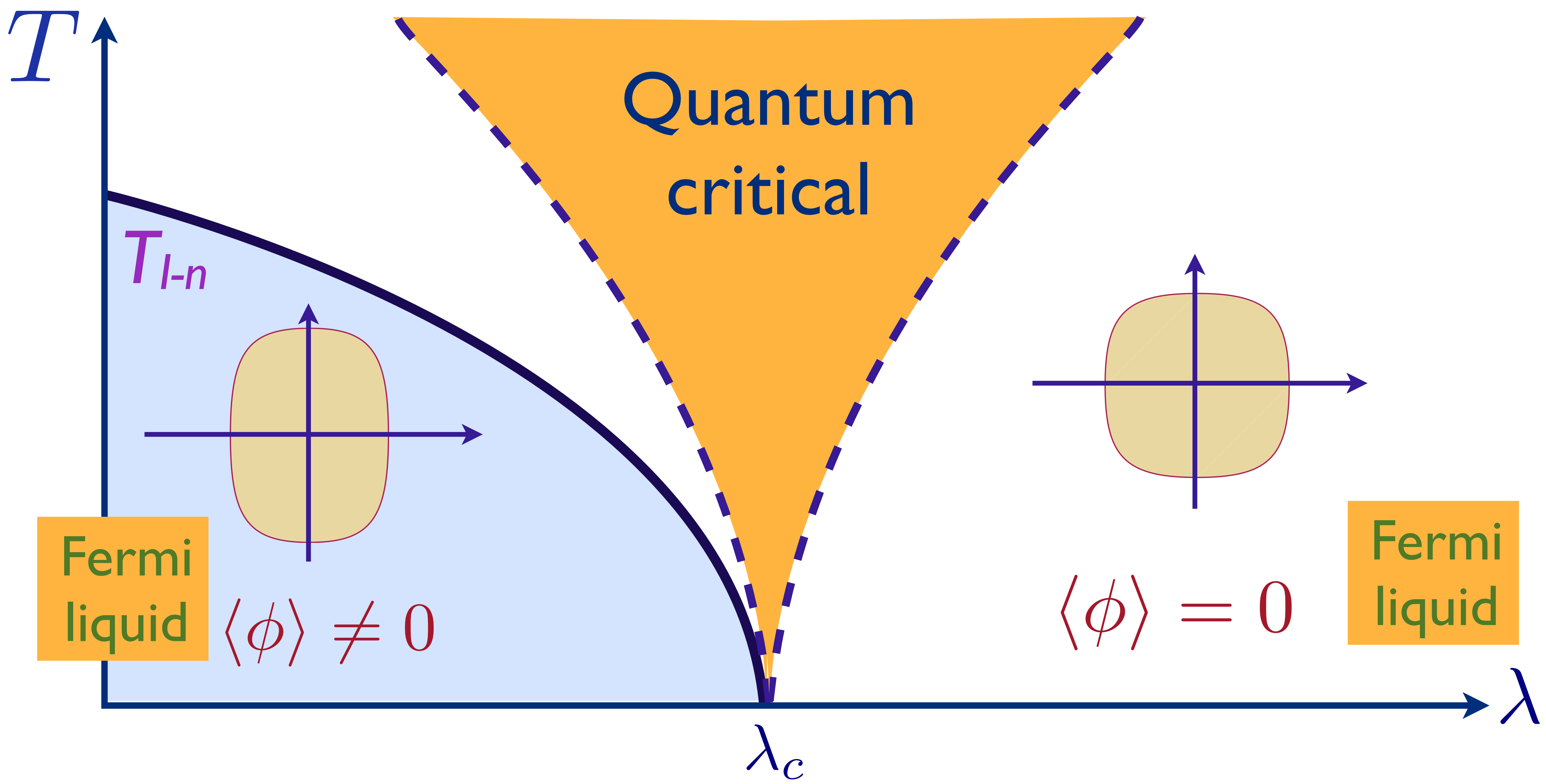
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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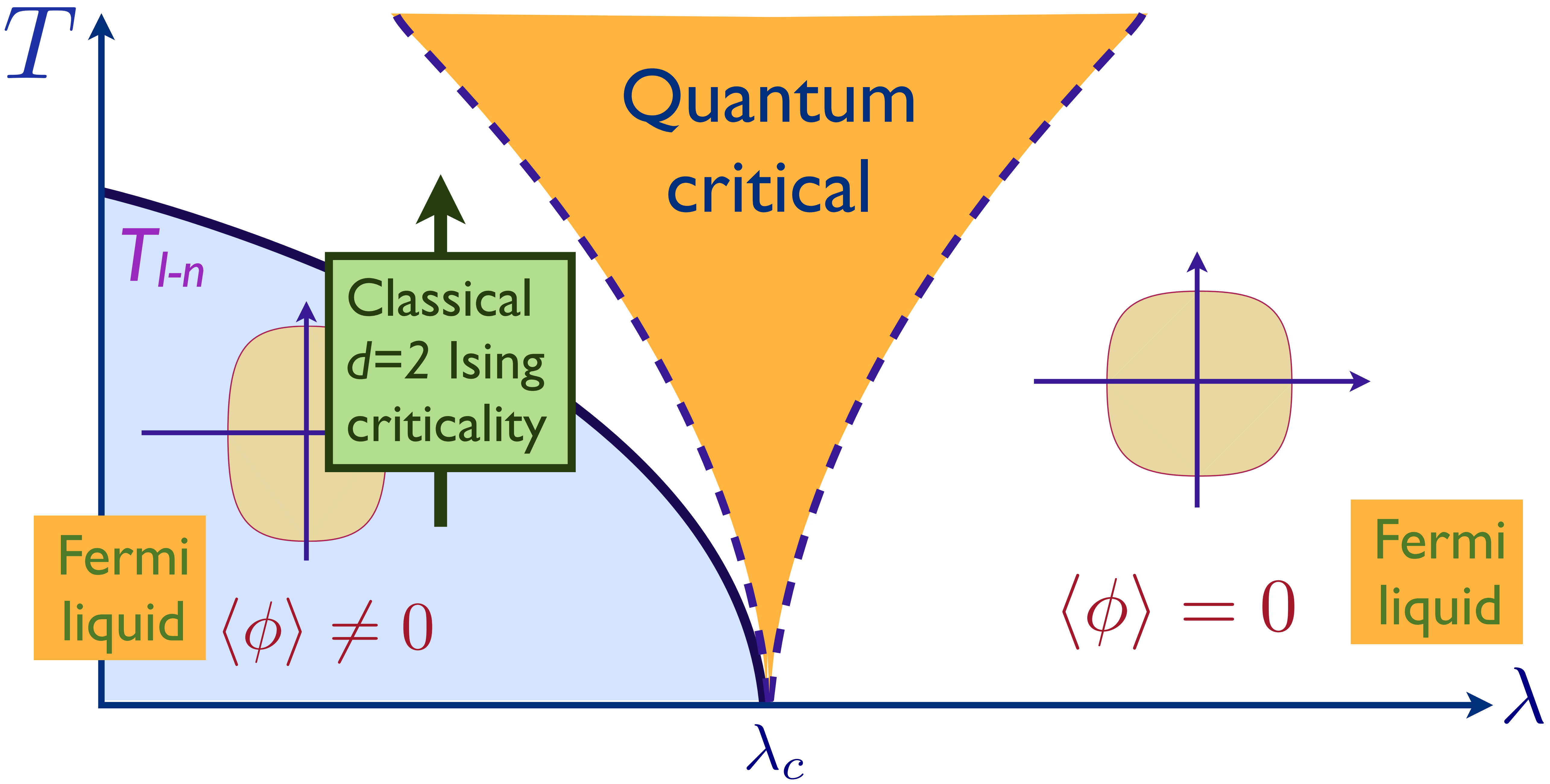

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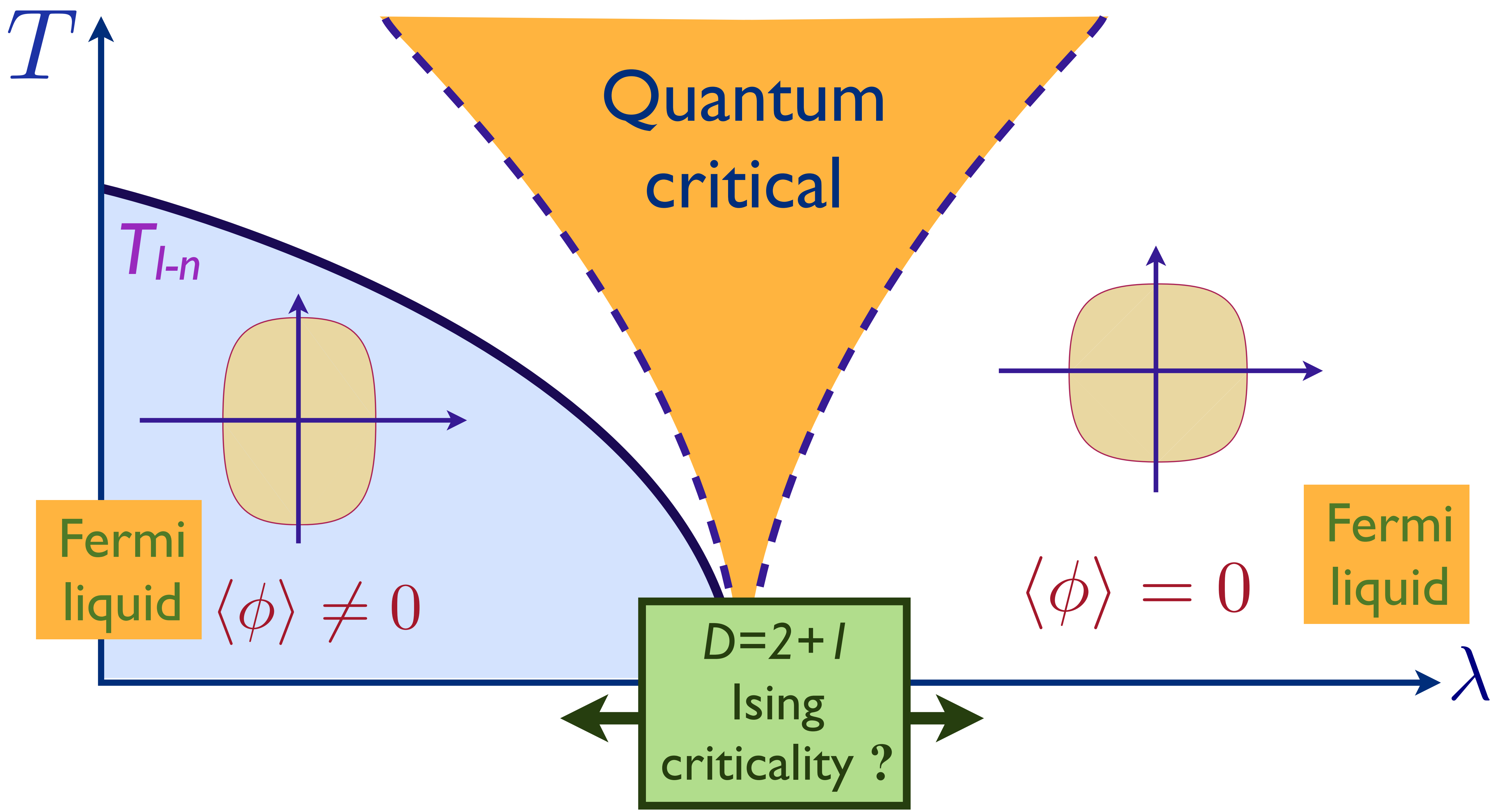

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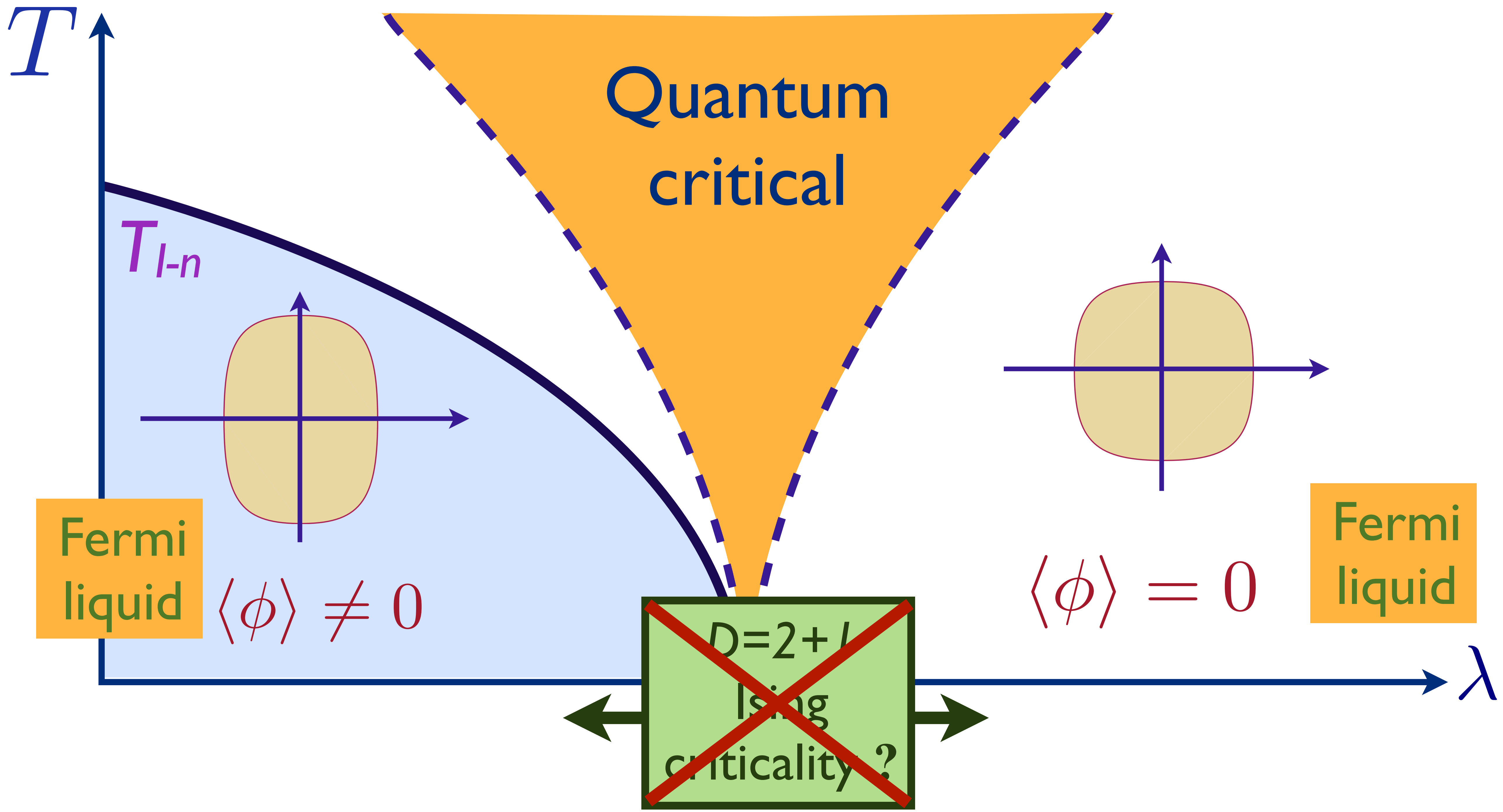

Phase diagram as a function of T and λ

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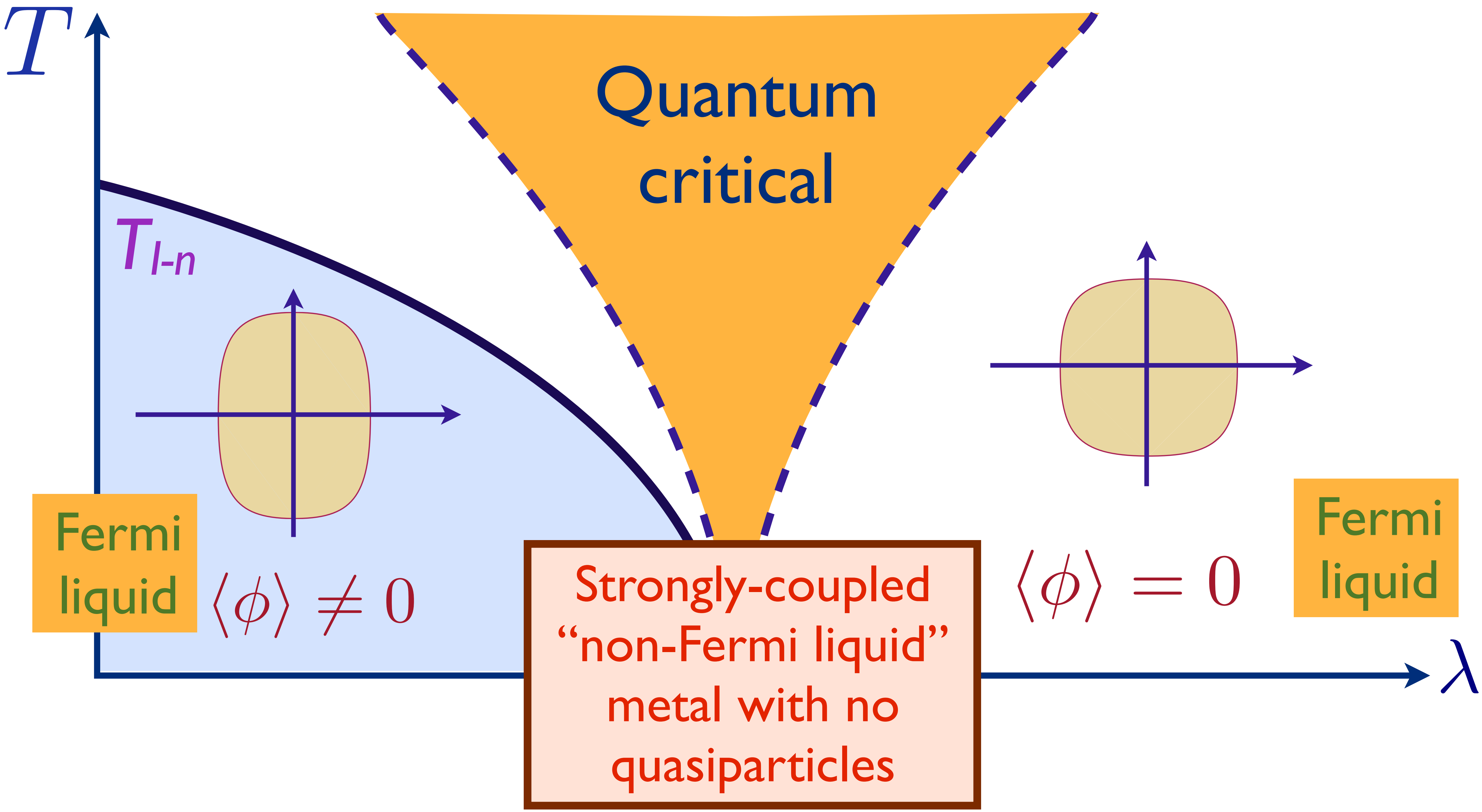

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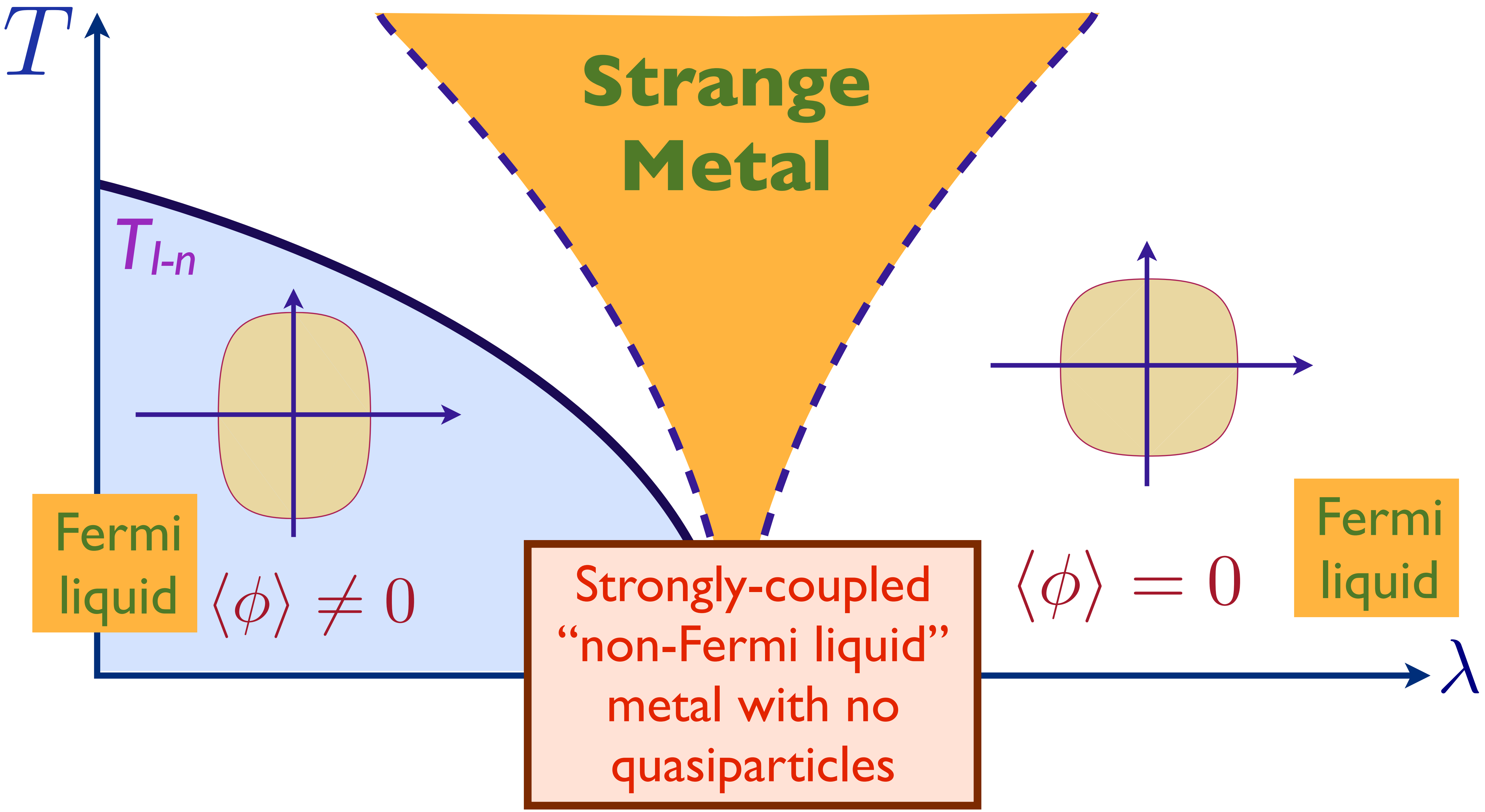

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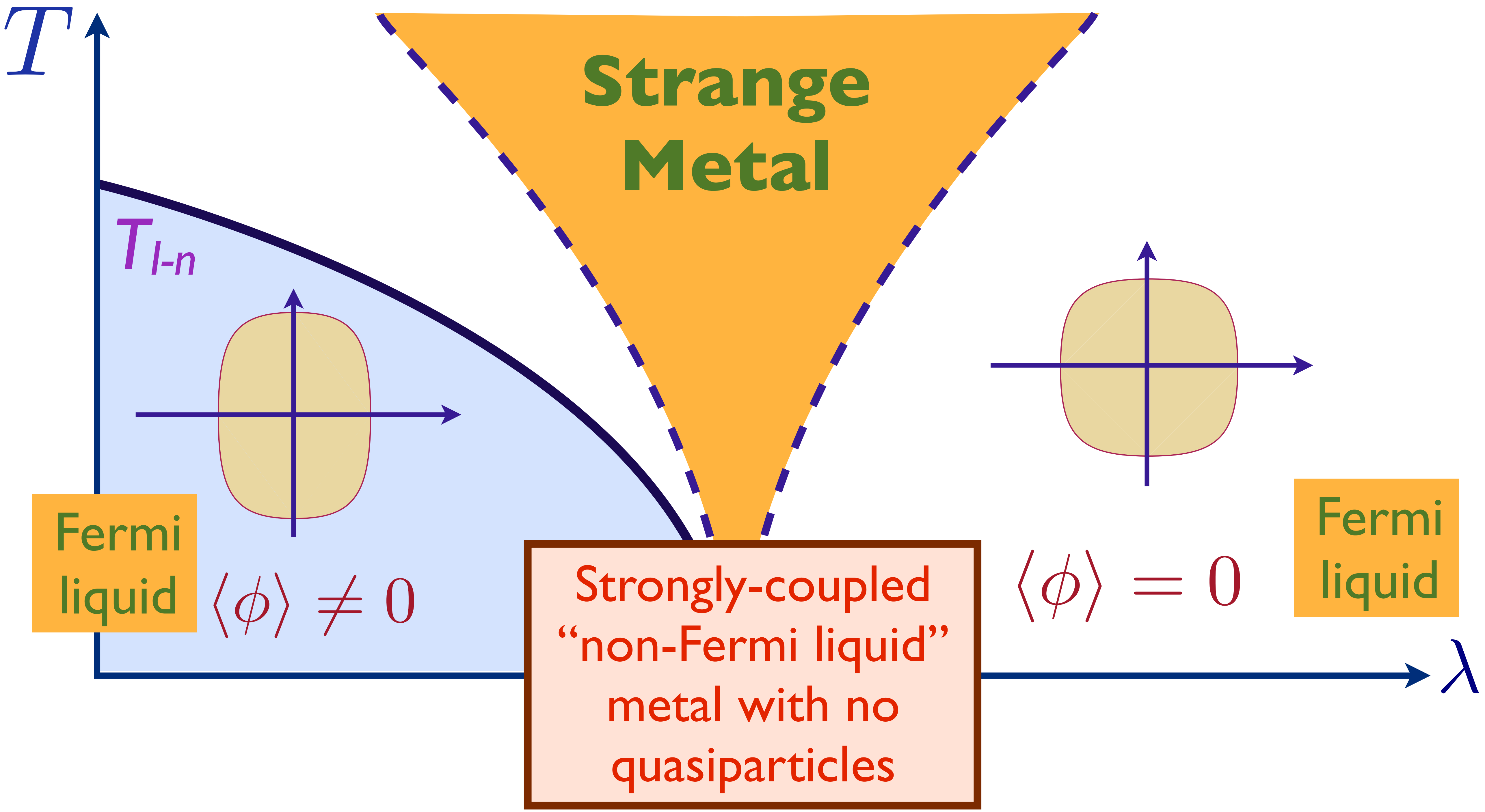

Common theoretical belief from an analysis of scattering of charged electronic quasiparticles off bosonic ϕ fluctuations: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

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Field theory of bosonic order parameter

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Electrons with a Fermi surface: $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - \mu \ldots$

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"Yukawa" coupling between bosons and fermions

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$$\mathcal{S}_{\phi c} = -g \int d^2 r d\tau \sum_{\alpha=1}^{N_f} \phi \left[c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2) c_\alpha^\dagger \right\} c_\alpha \right]$$

This continuum theory has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$, and so the resistivity $\rho(T) = 0$.

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The resistivity of the strange metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} \quad = \quad \int d^2 r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] \,,$$

$$\overline{V(\mathbf{r})} = 0 \qquad ; \qquad \overline{V(\mathbf{r}) V(\mathbf{r}')} = V_0^2\, \delta(\mathbf{r} - \mathbf{r}')\,,$$

$$\overline{h(\mathbf{r})} = 0 \qquad ; \qquad \overline{h(\mathbf{r}) h(\mathbf{r}')} = h_0^2\, \delta(\mathbf{r} - \mathbf{r}')\,,$$

we obtain the resistivity for current along angle ϑ

$$\rho(T) = \frac{1}{\chi^2_{\mathbf{J},\mathbf{P}}} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2}\, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2\, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2\, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right)$$

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`.

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Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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Bosonic term: Dominant contribution:

$$\rho(T) \sim T^{(d-z+\eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim T$ at higher T,
to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.

Outline

1. The simplest model without quasiparticles

   A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice

   B. Conformal field theories in 2+1 dimensions, the AdS/CFT correspondence, and transport without quasiparticles.

2. Strange metals in the high T_c superconductors

   A. The onset of antiferromagnetism in a metal

   B. Non-quasiparticle transport at the Ising-nematic quantum critical point

   C. Entanglement, holography, and strange metals

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2. Fermi liquids and non-Fermi liquids
3. Generalized holography beyond CFTs

1. Entanglement, holography, and CFTs

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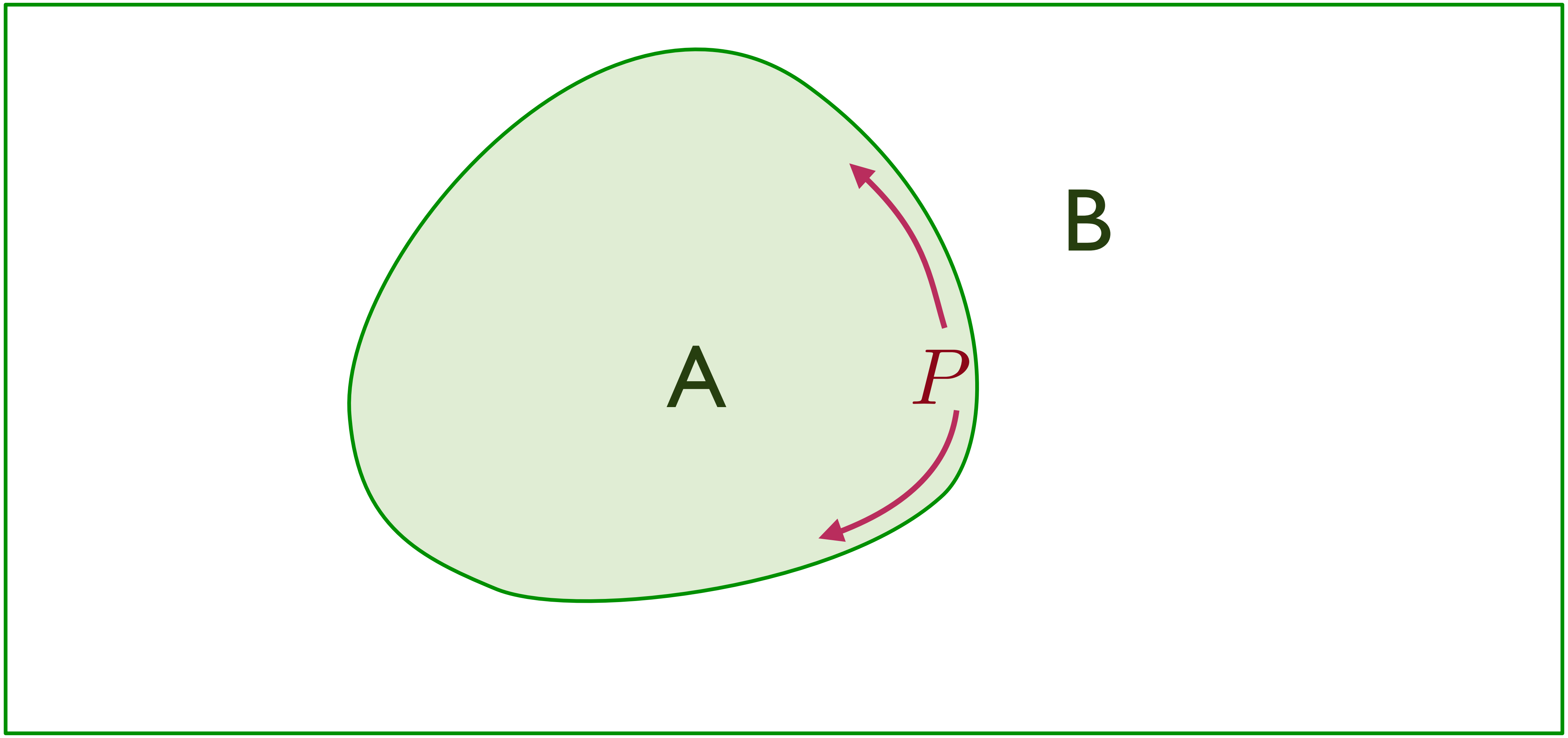
Entanglement entropy



$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy

$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

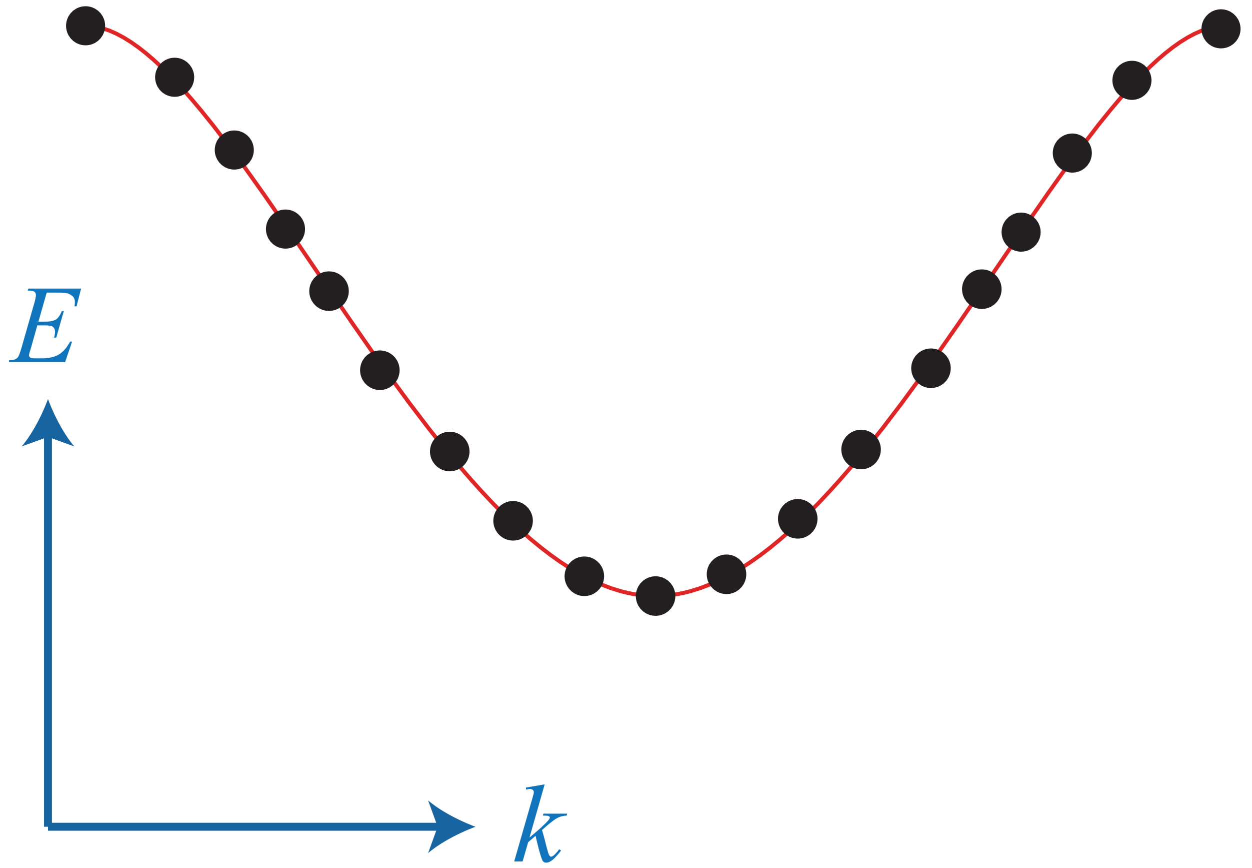
$$\text{Take } |\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B\right)$$

$$\text{Then } \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$
$$= \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A + |\downarrow\rangle_A \langle\downarrow|_A\right)$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$
$$= \ln 2$$

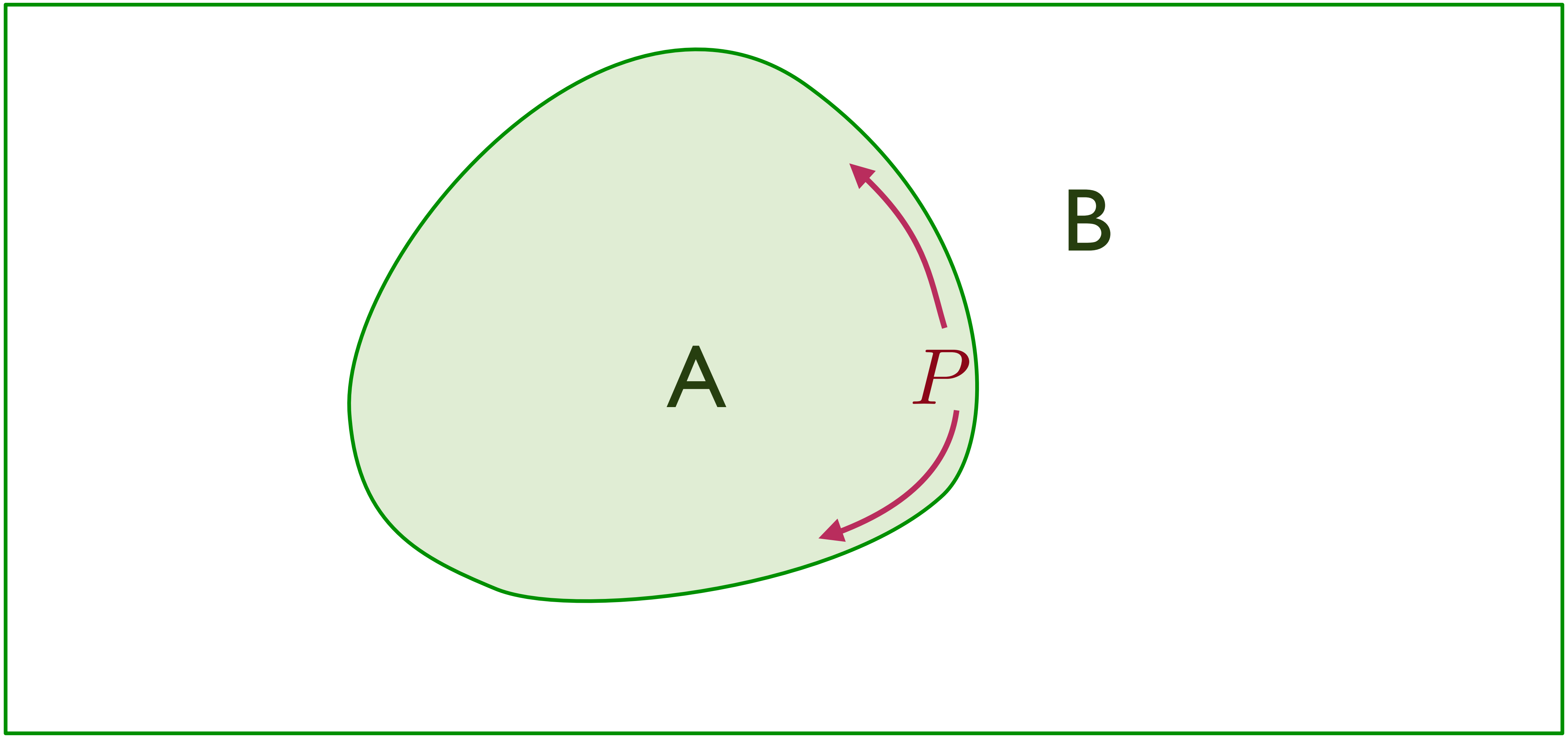
Entanglement entropy of a band insulator

Band insulators



An even number of electrons per unit cell

Entanglement entropy of a band insulator

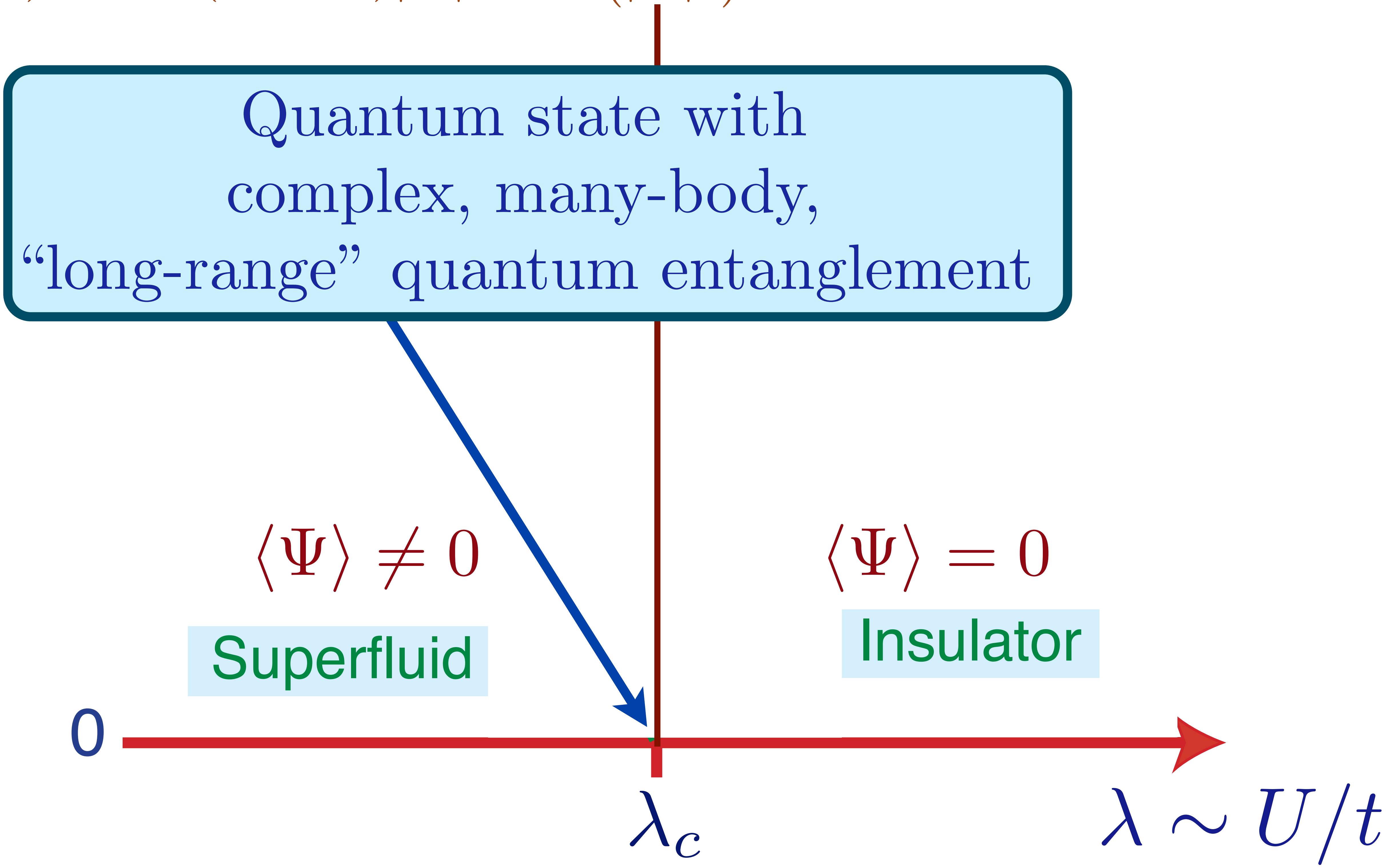


$$S_E = aP - b\exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

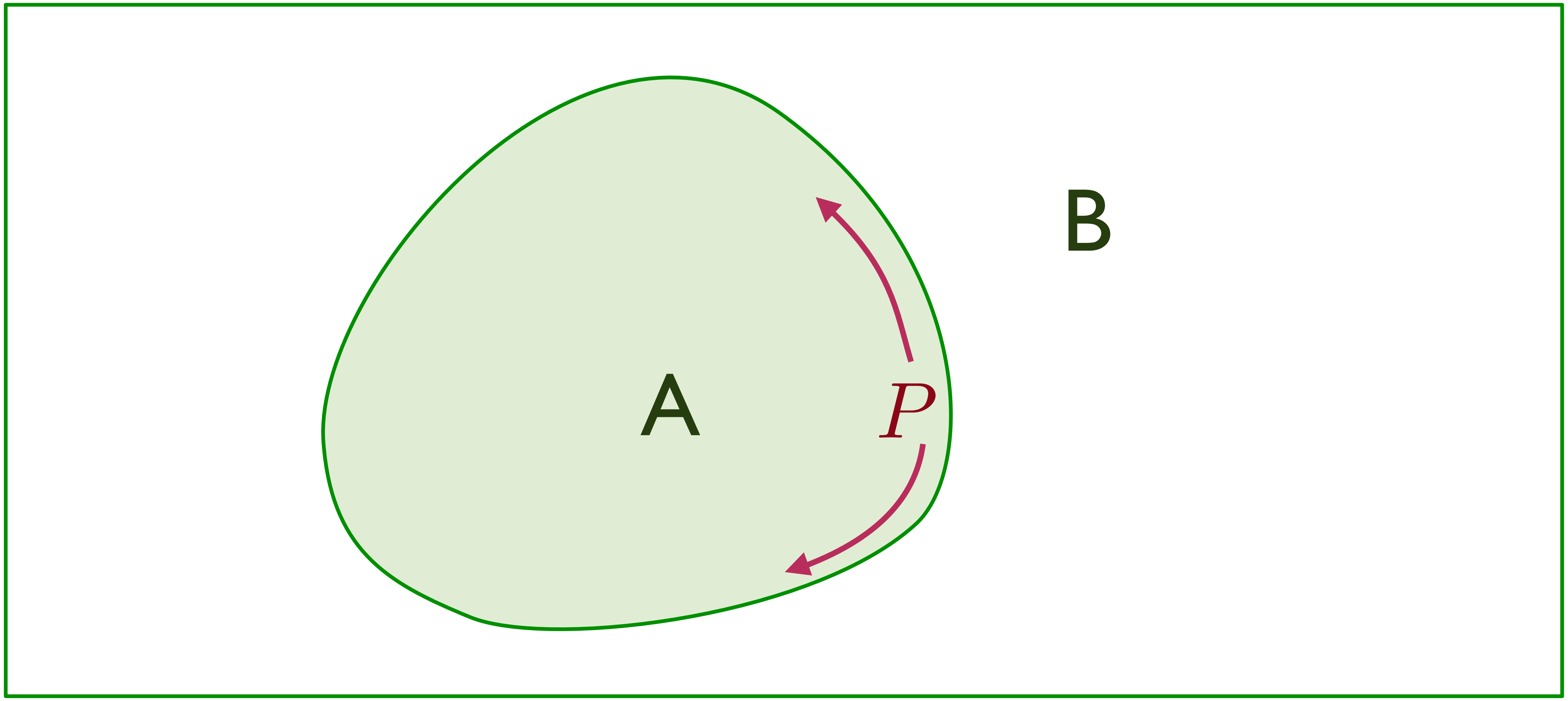
$$\mathcal{S} = \int d^2r dt \left[|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$$

$$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u\left(|\Psi|^2\right)^2$$



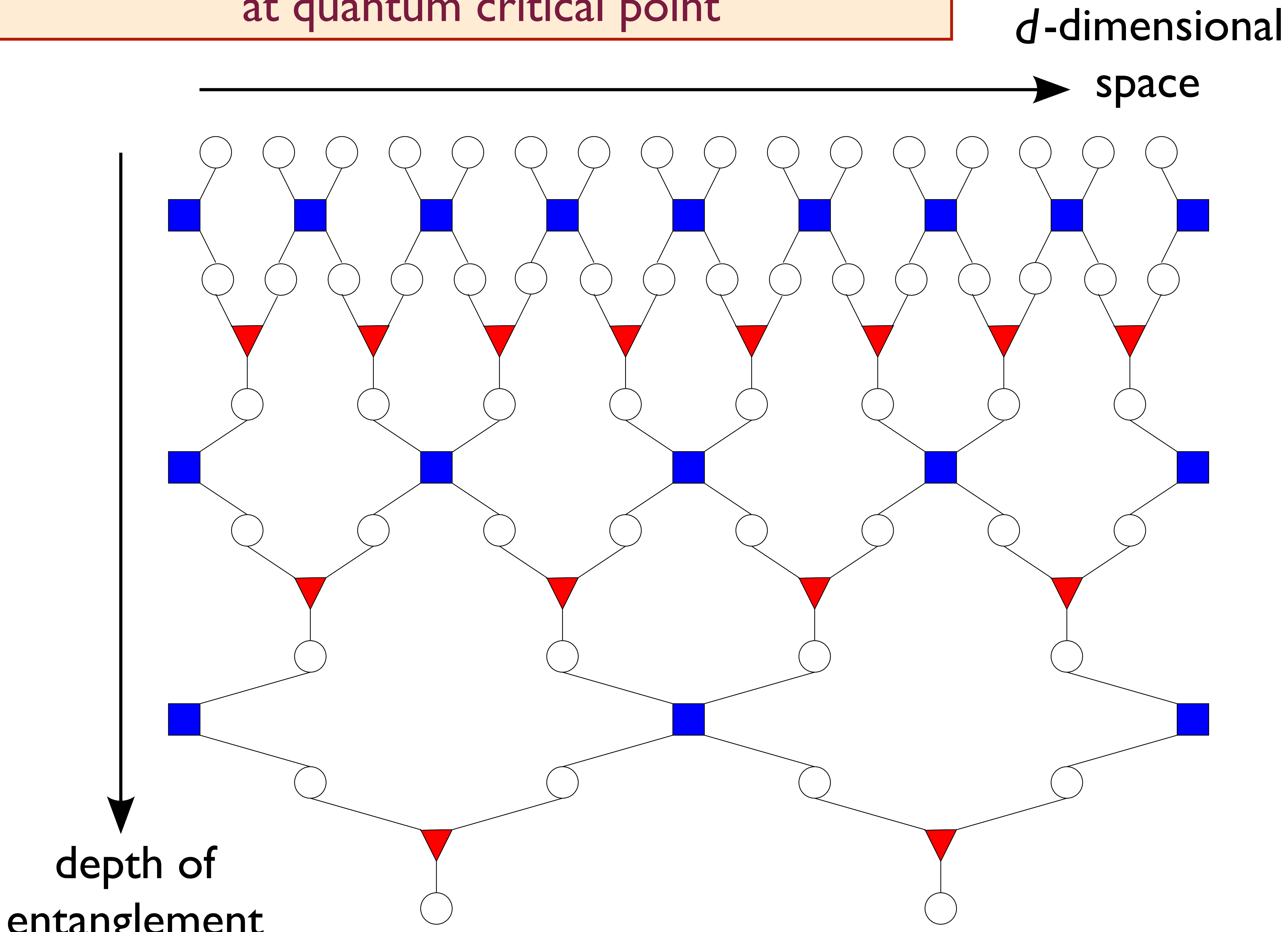
Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.



M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009); H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011); I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Tensor network representation of entanglement at quantum critical point



G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)

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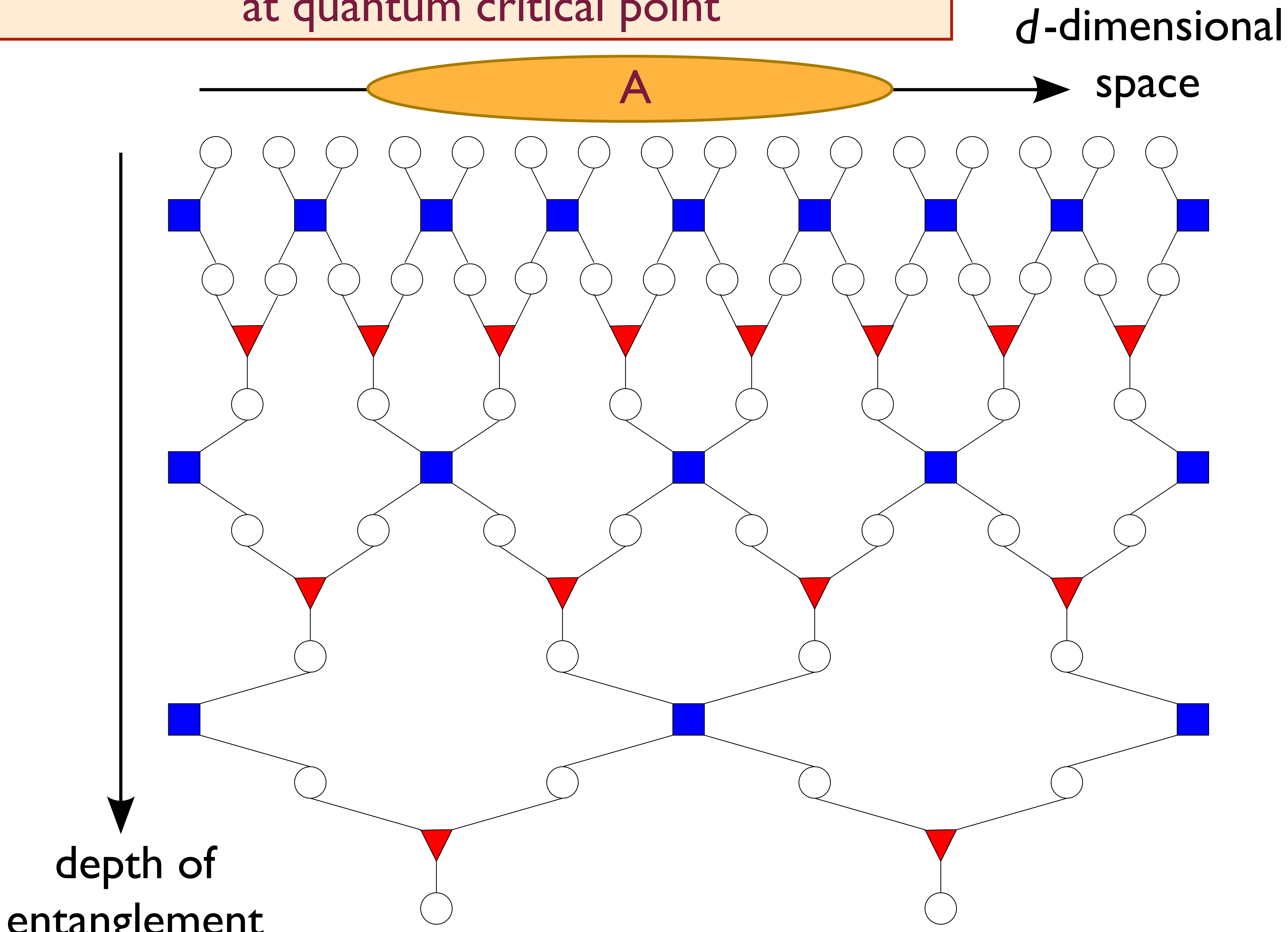

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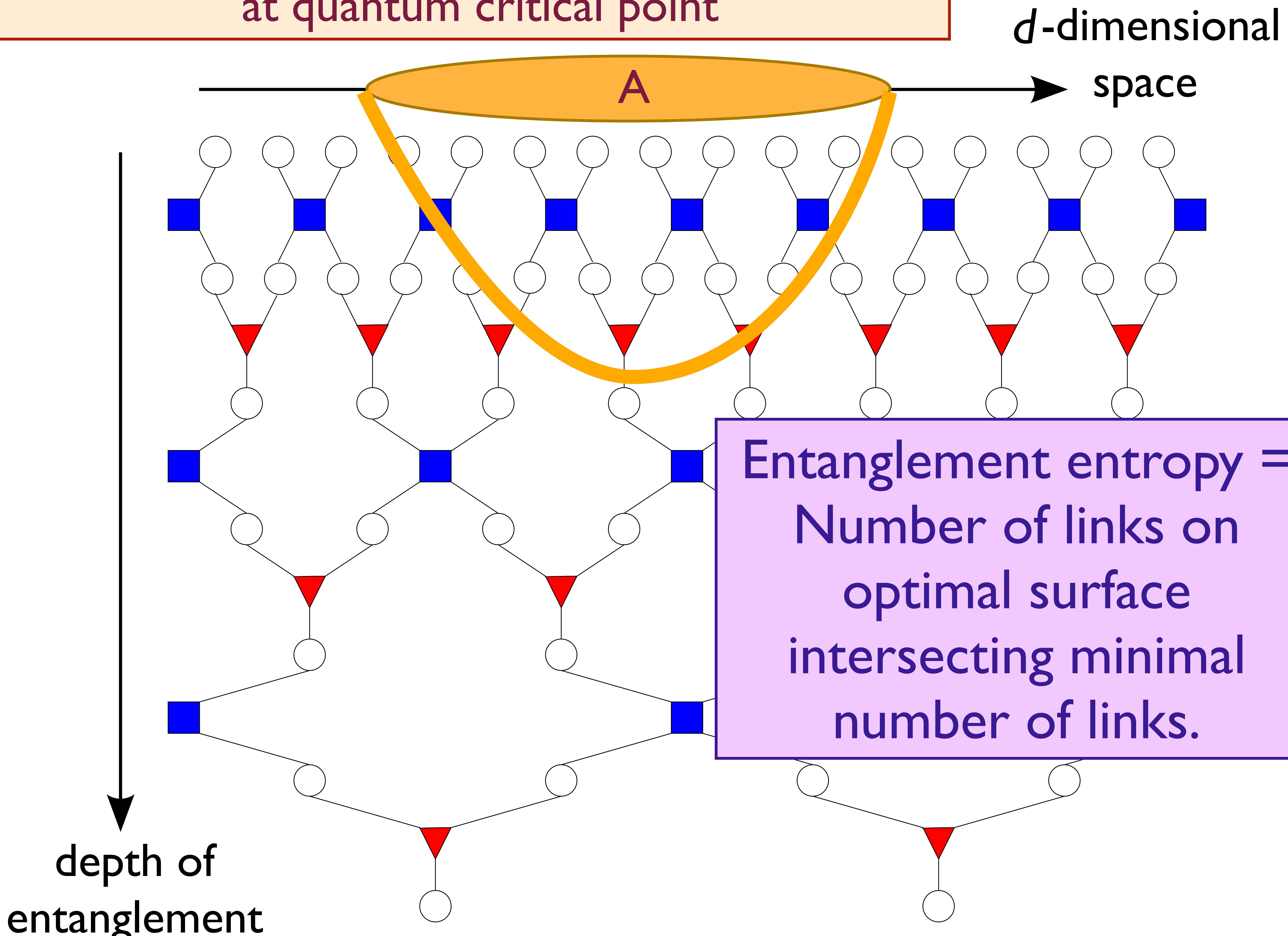

Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Brian Swingle, arXiv:0905.1317

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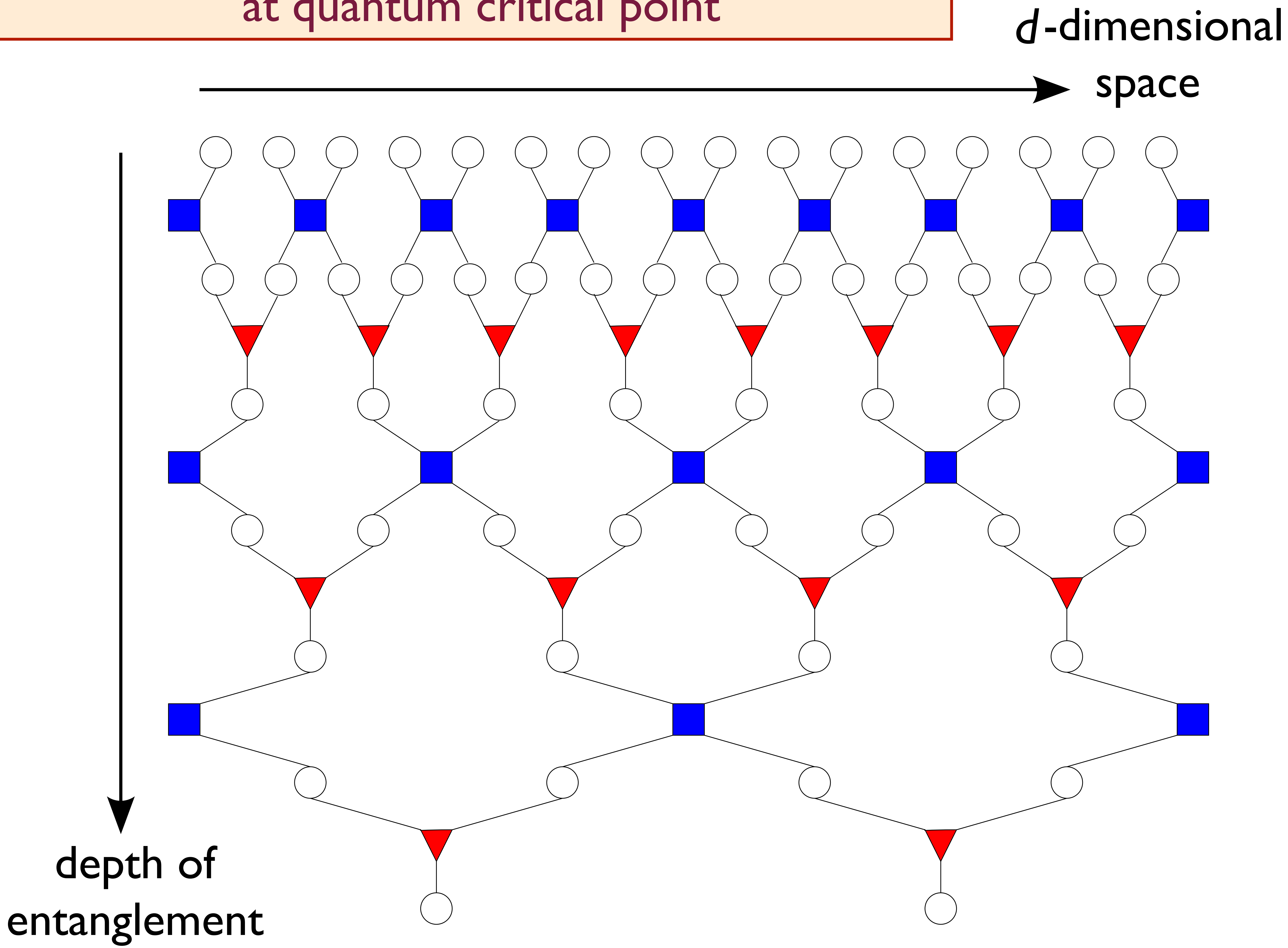

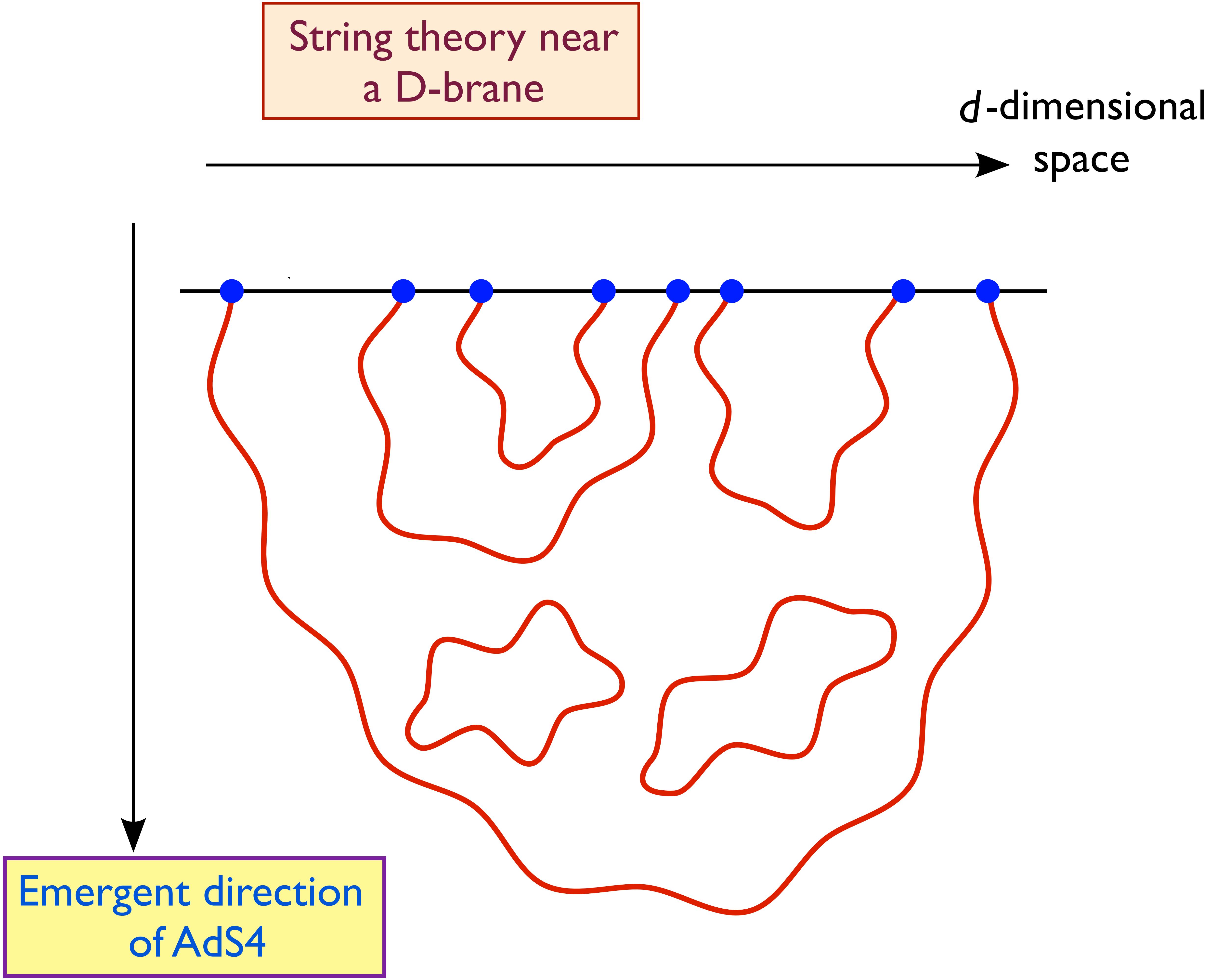
String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

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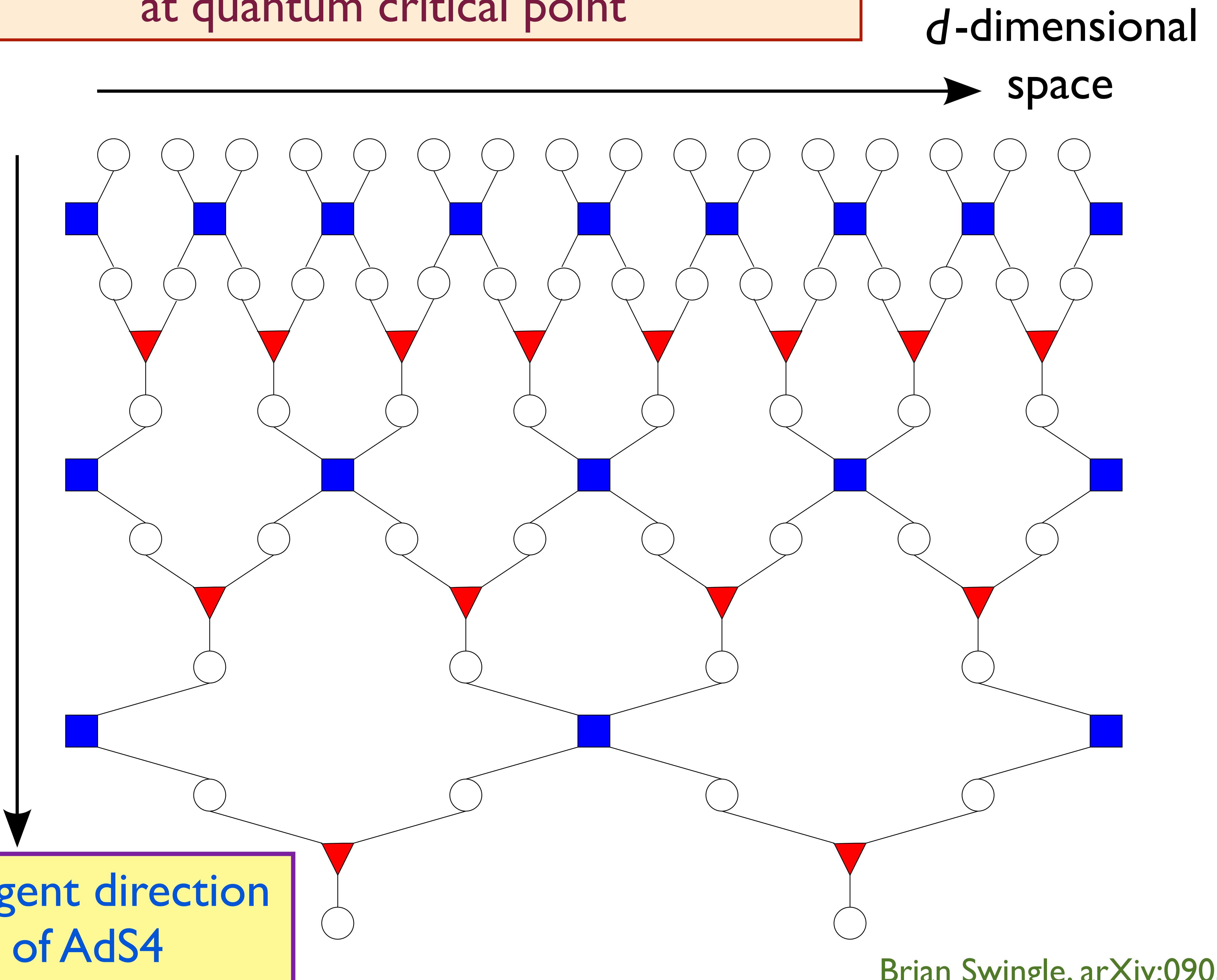

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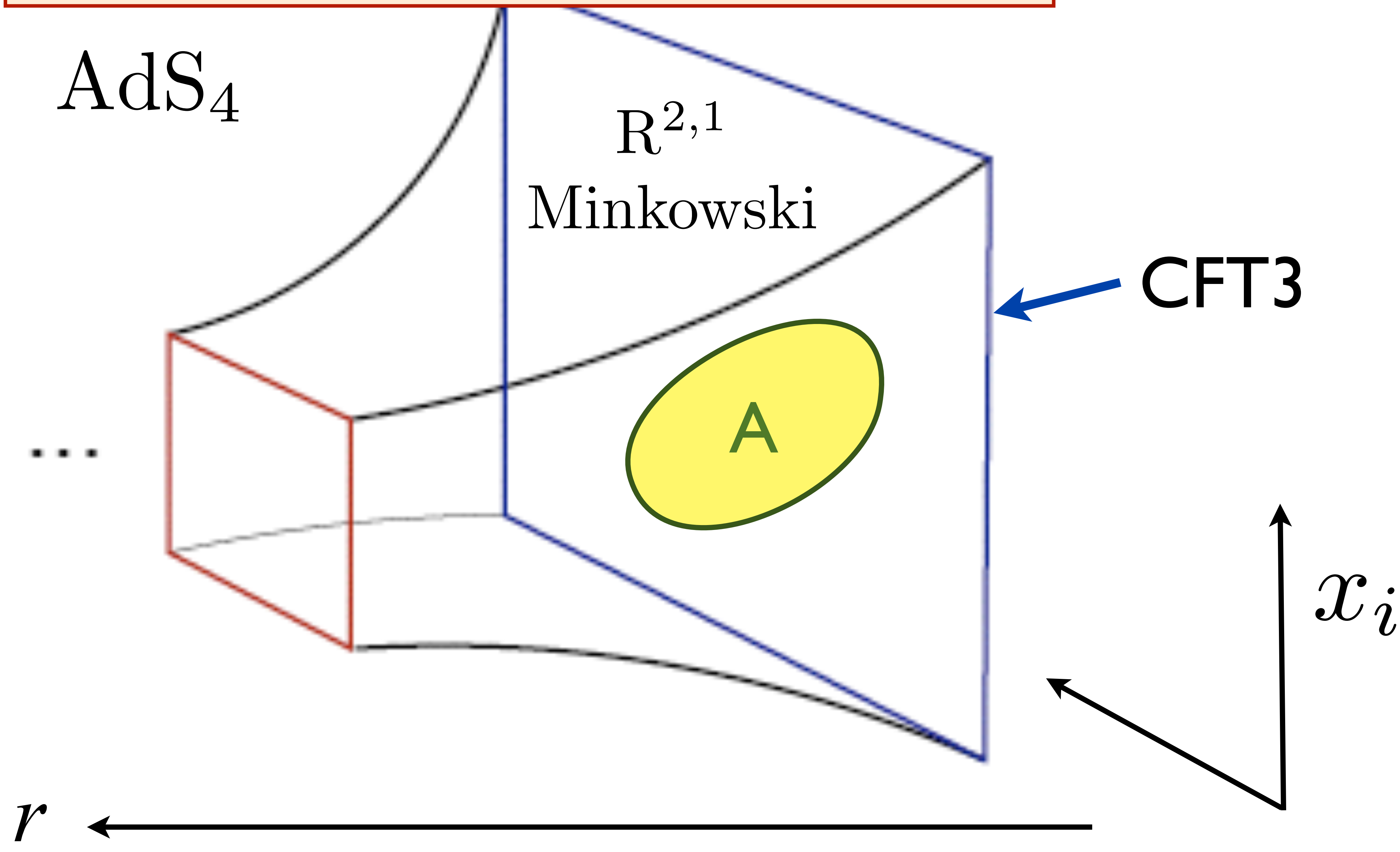
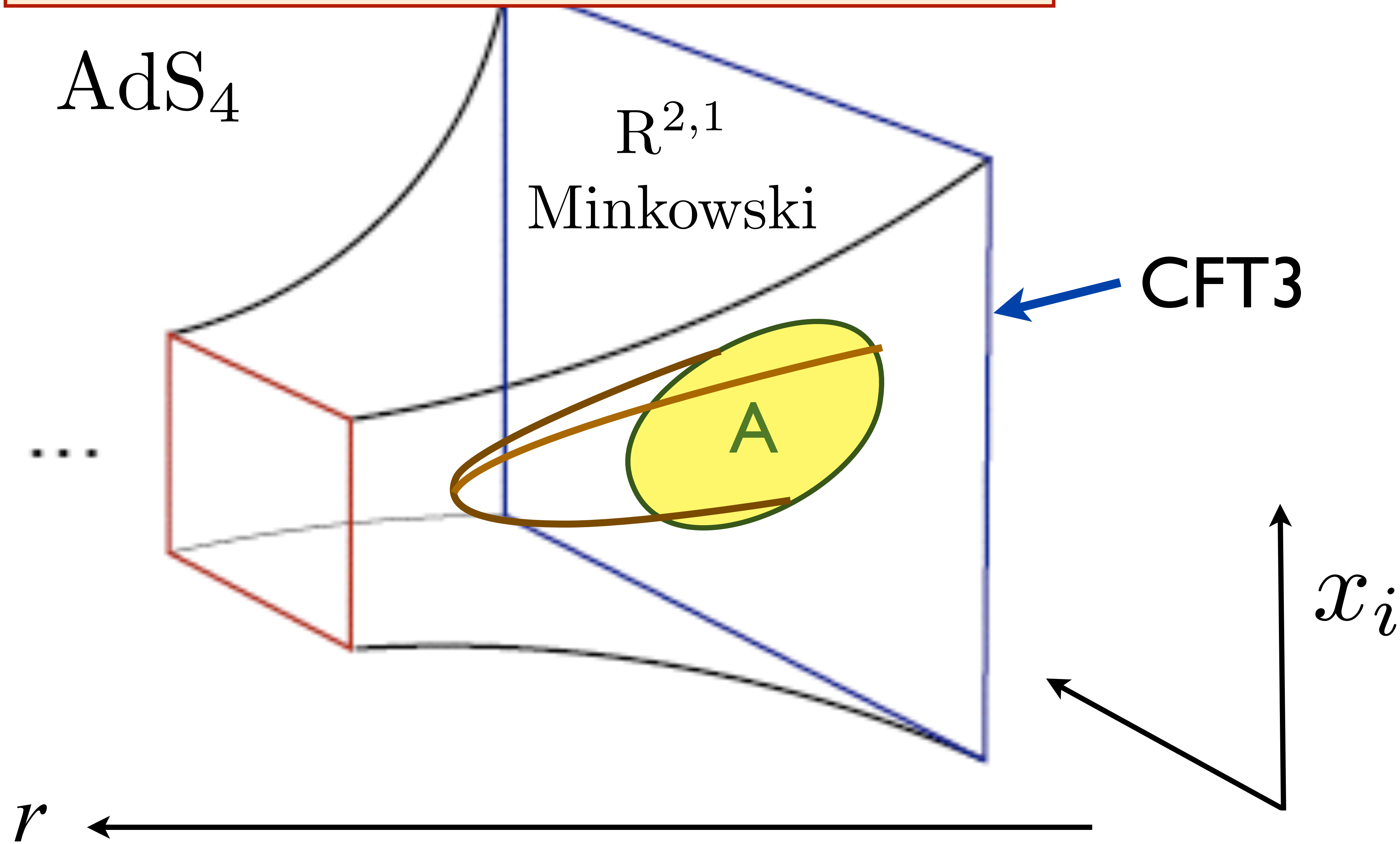
Holography and Entanglement



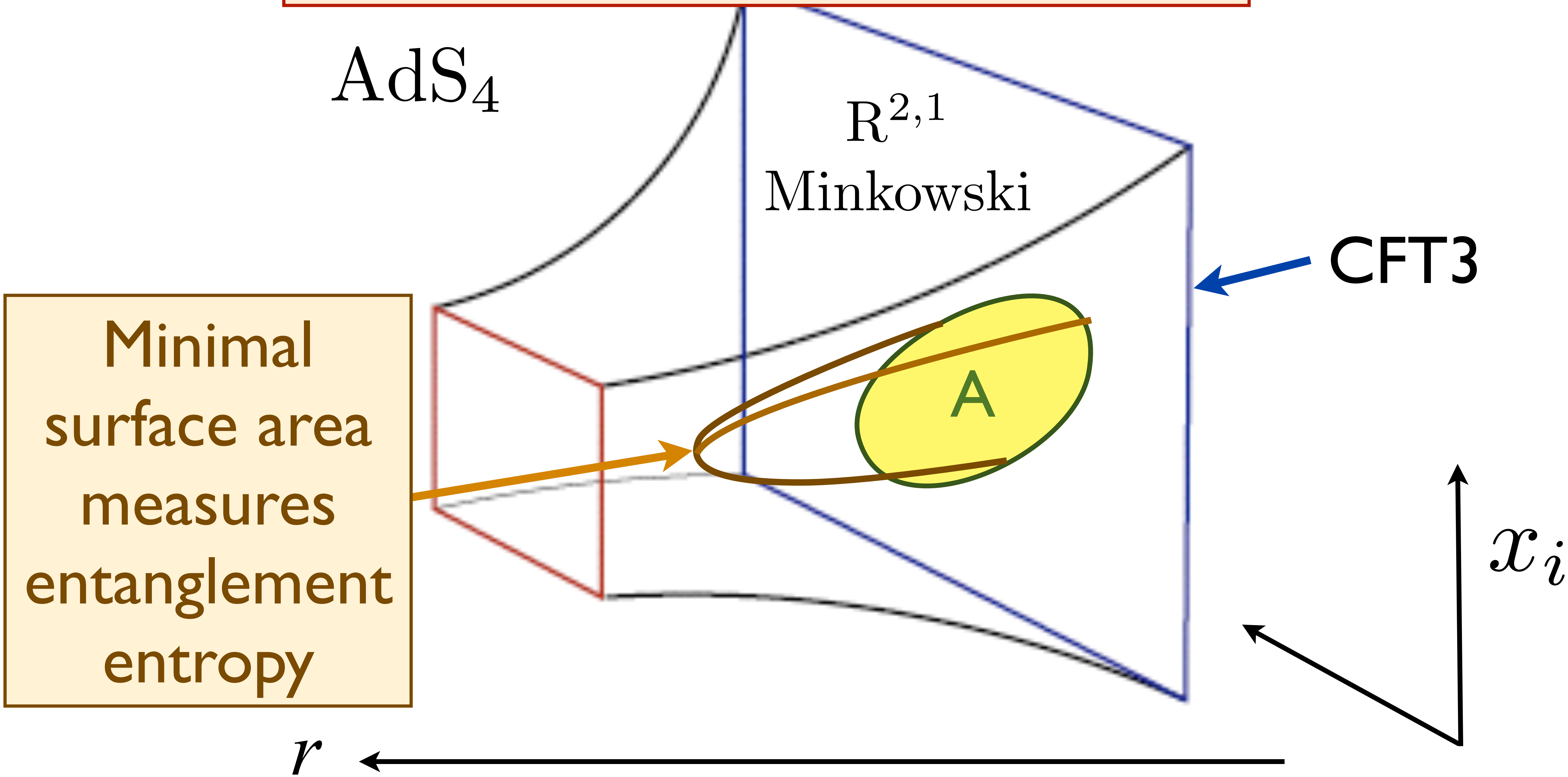
Holography and Entanglement



Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe "outside" : *i.e.* the region is surrounded by an imaginary horizon.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography and Entanglement



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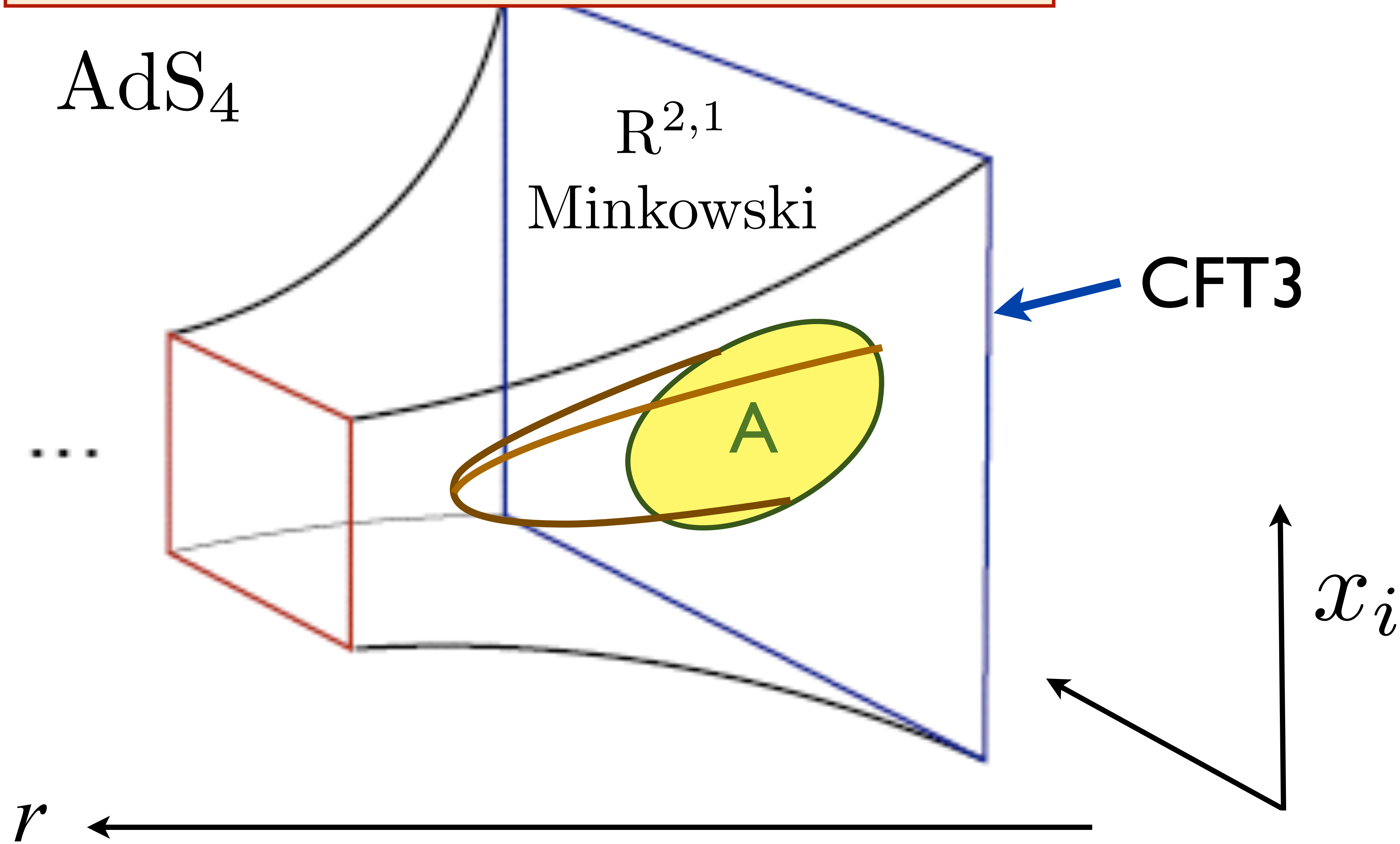

- Computation of minimal surface area yields

$$S_E = aP - \gamma,$$

where γ is a shape-dependent universal number.

M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009); H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011); I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

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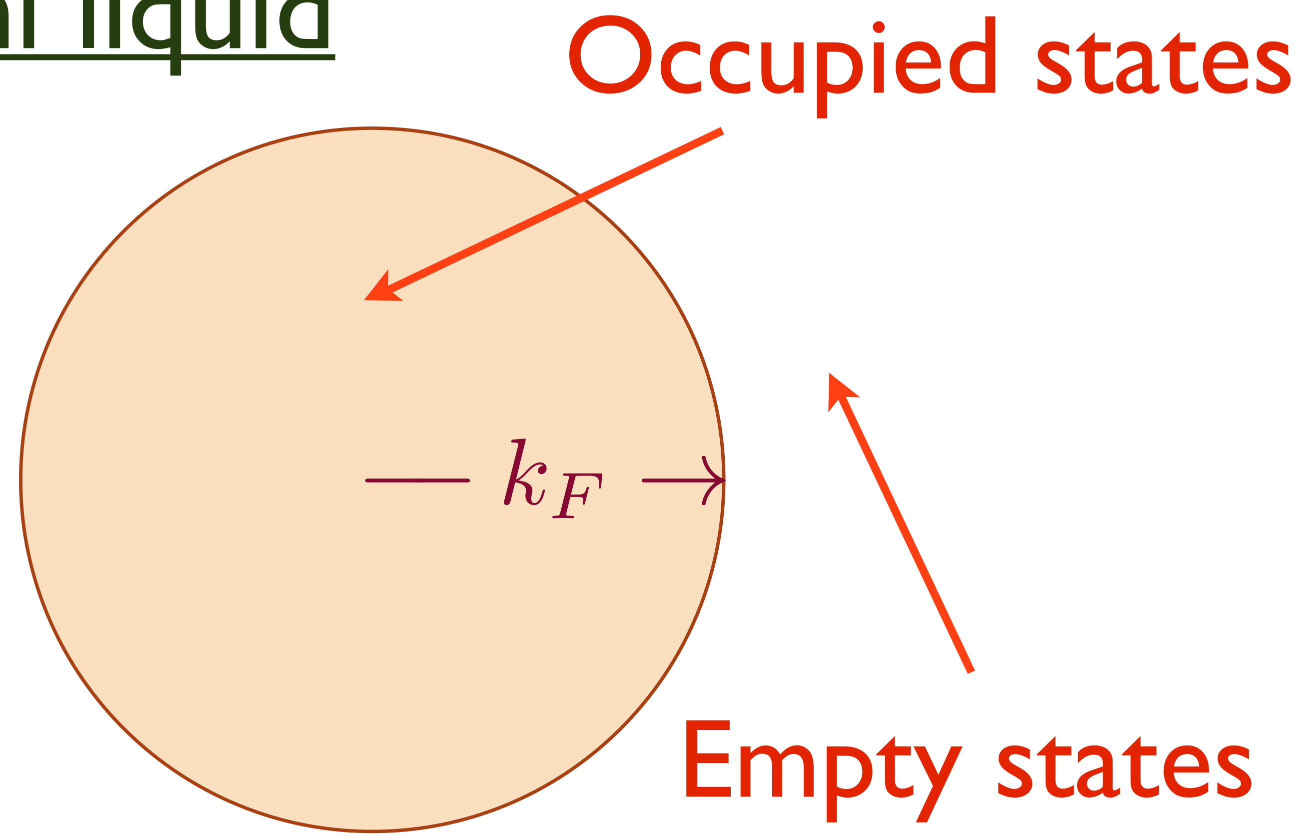
The Fermi liquid

$$\mathcal{L} = f^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f$$

$$+ \, 4 \text{ Fermi terms}$$



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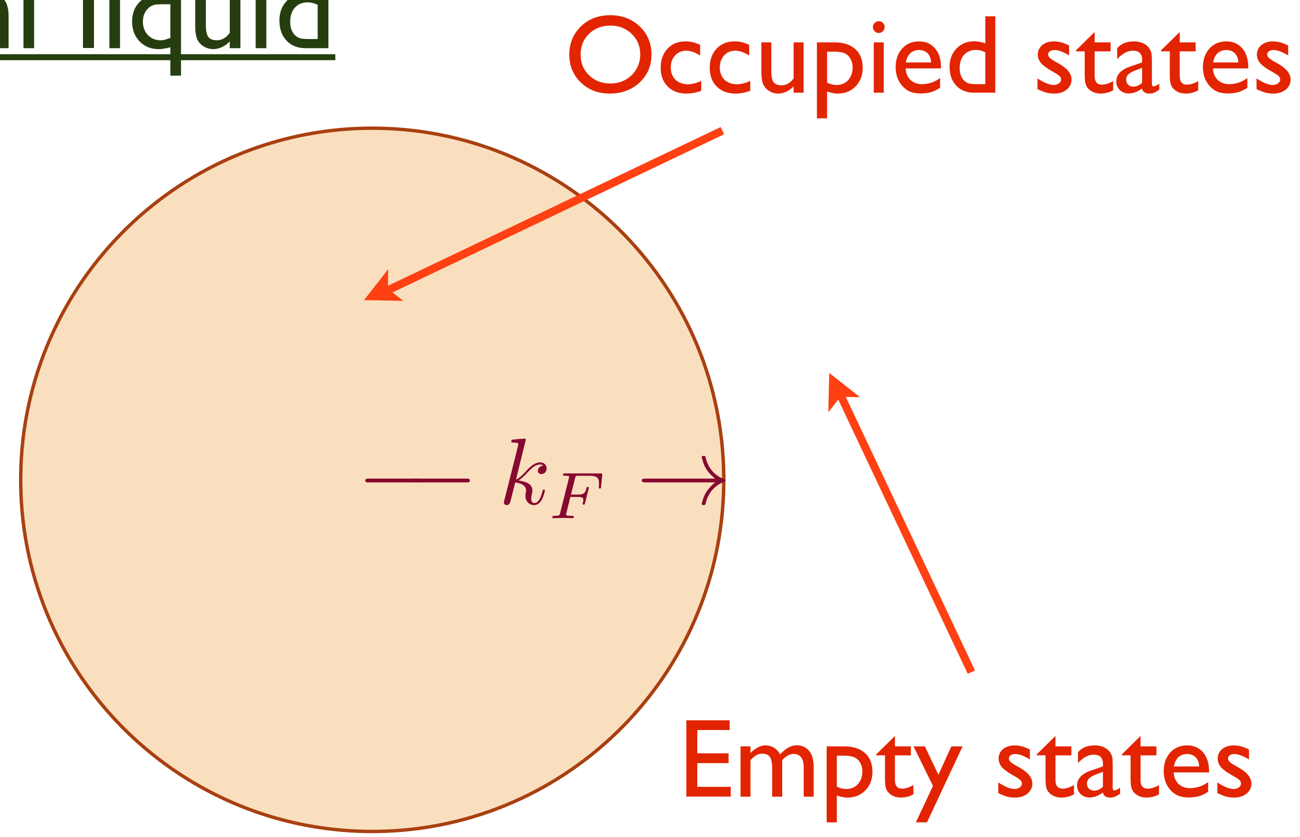
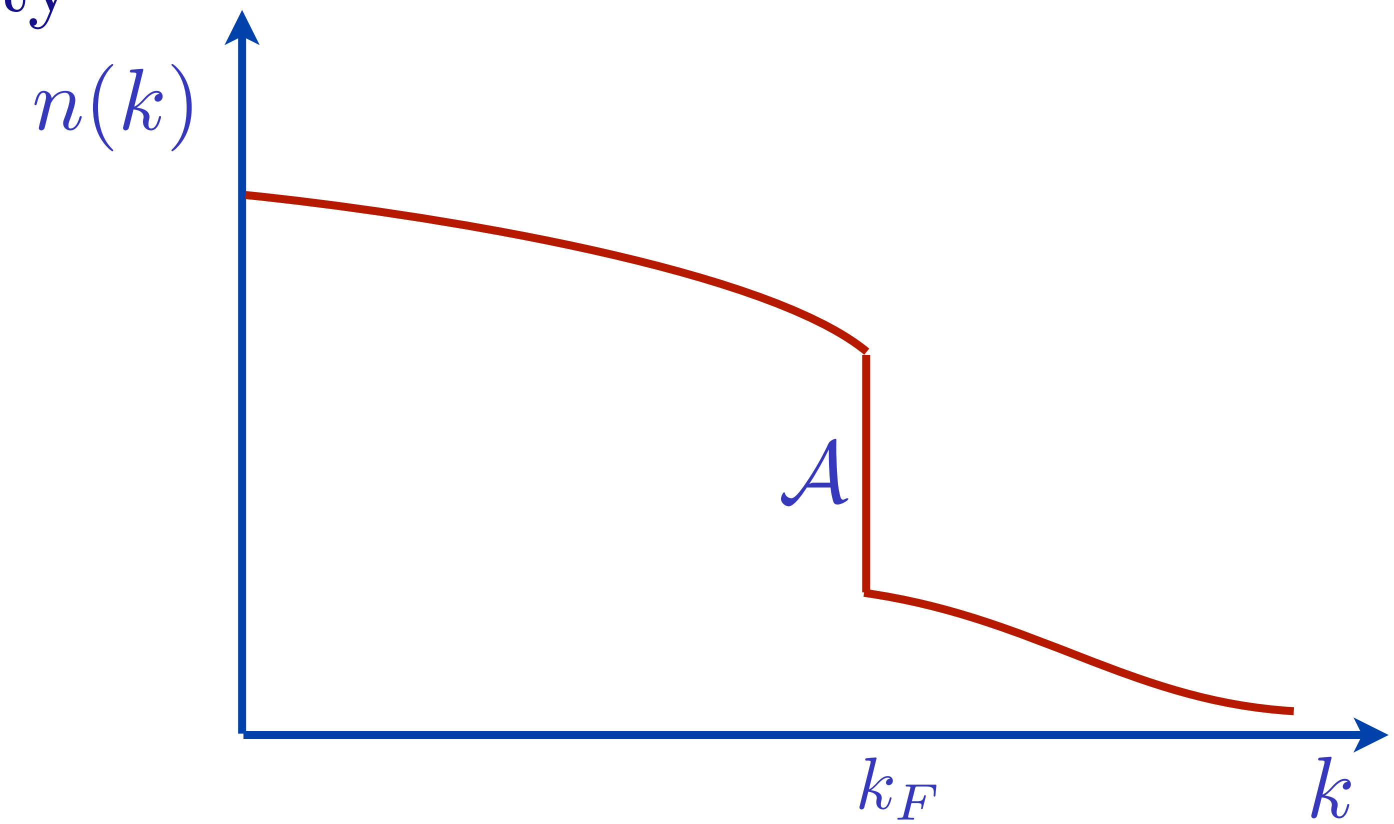
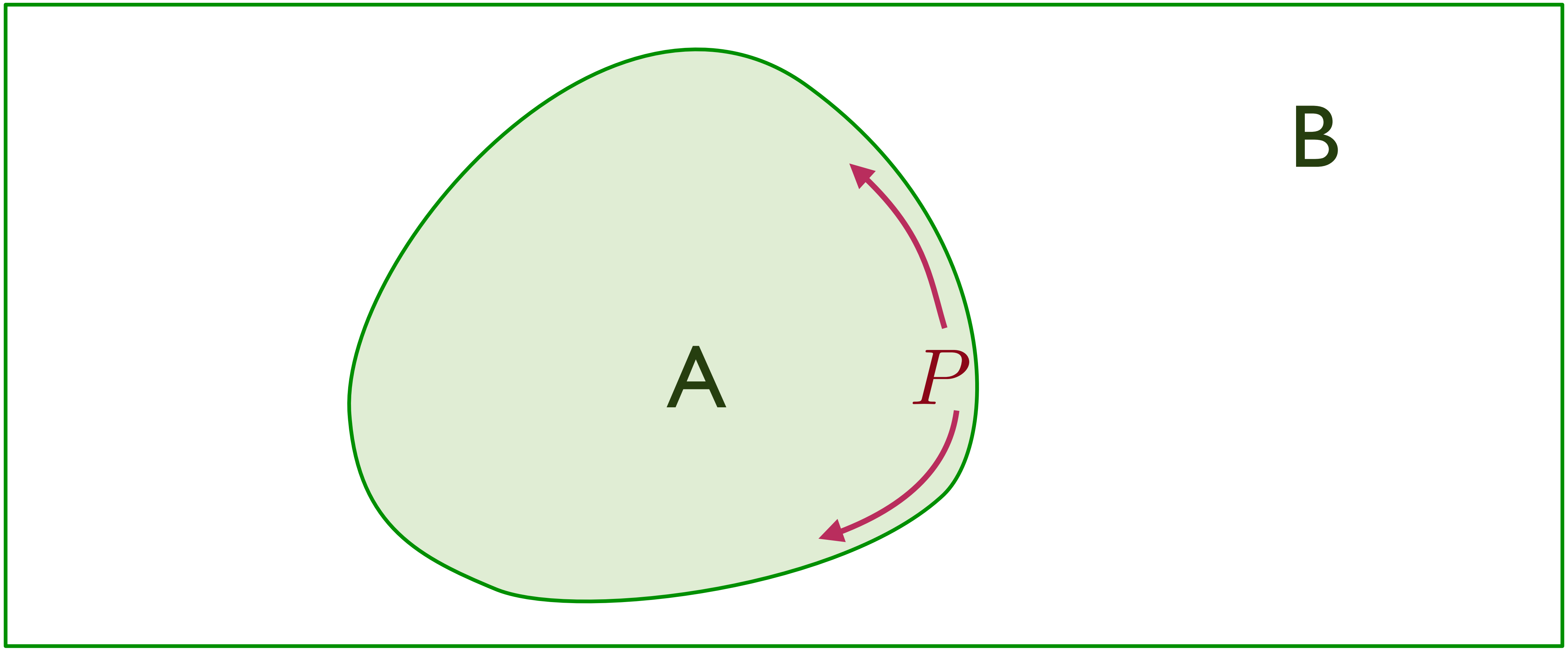


- Fermi wavevector obeys the Luttinger relation $k_F^d \sim \mathcal{Q}$, the fermion density



Entanglement entropy of the Fermi liquid



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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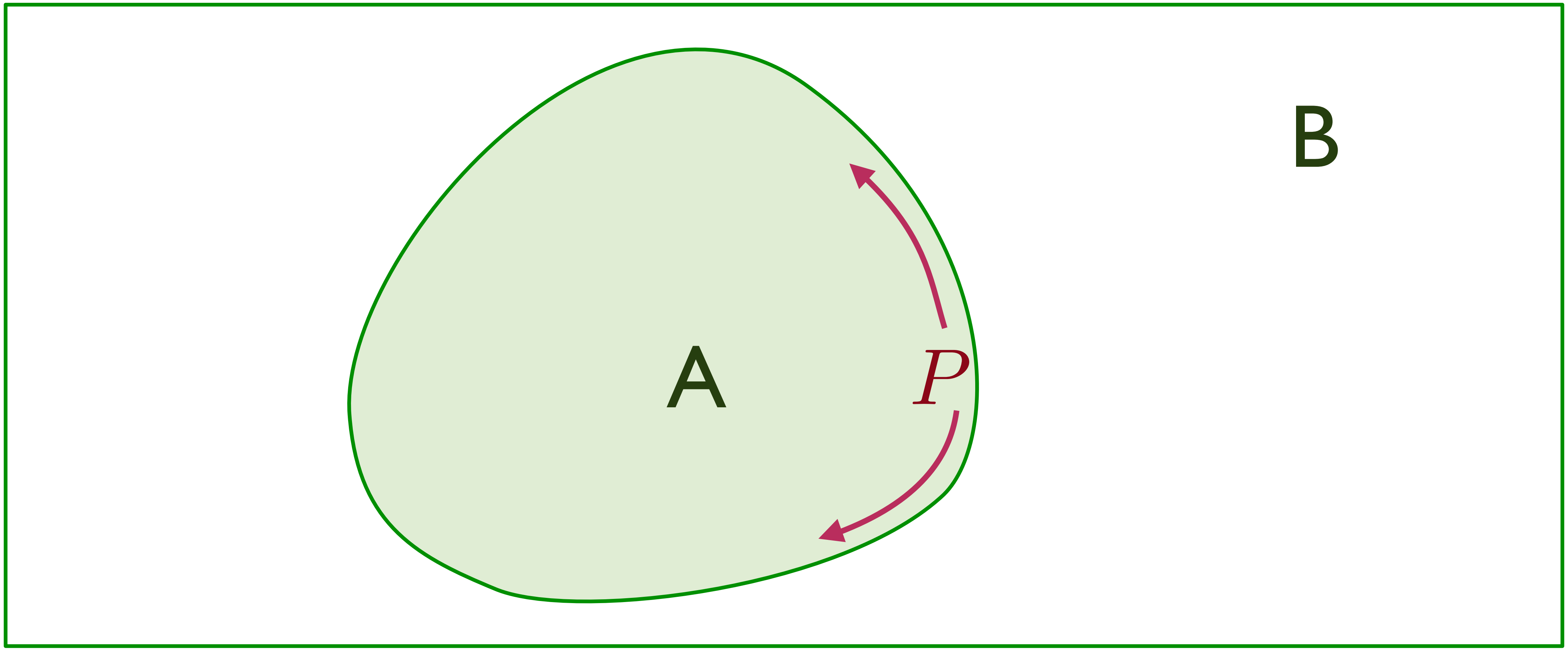

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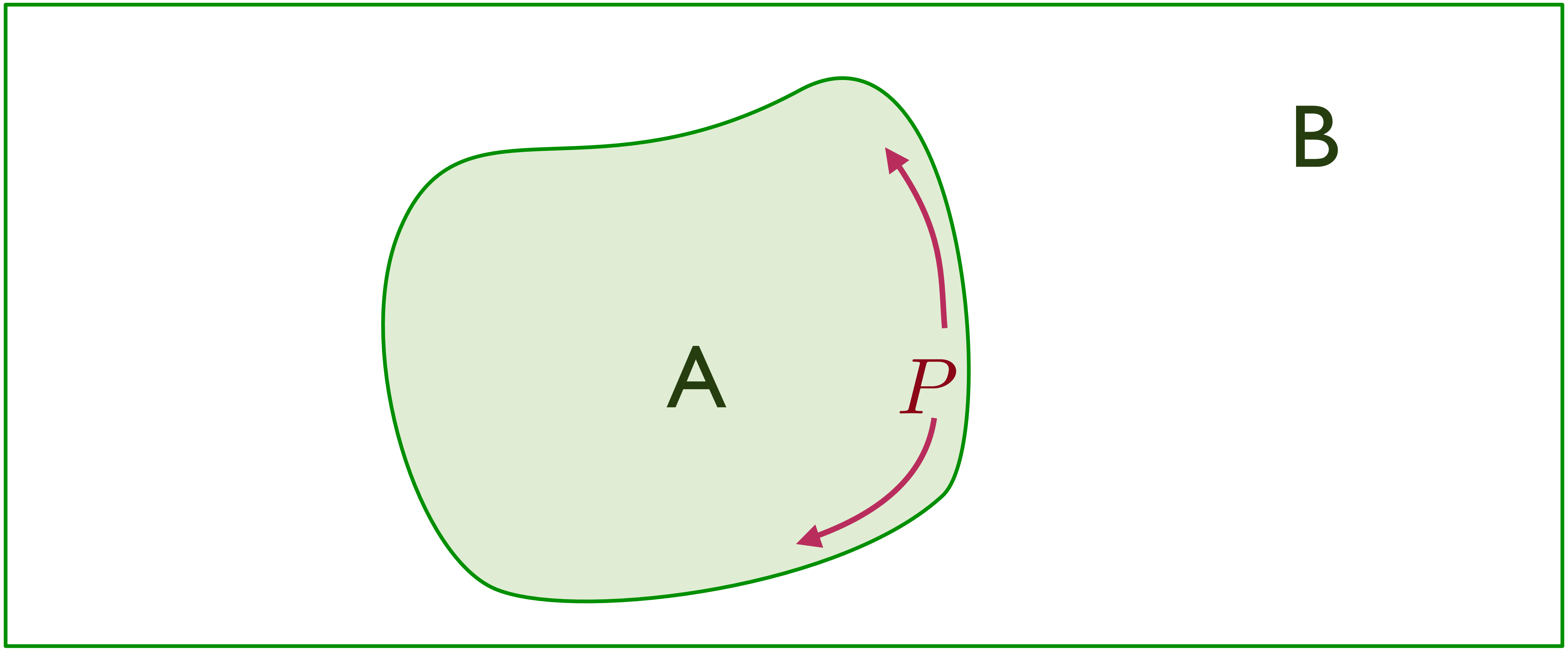

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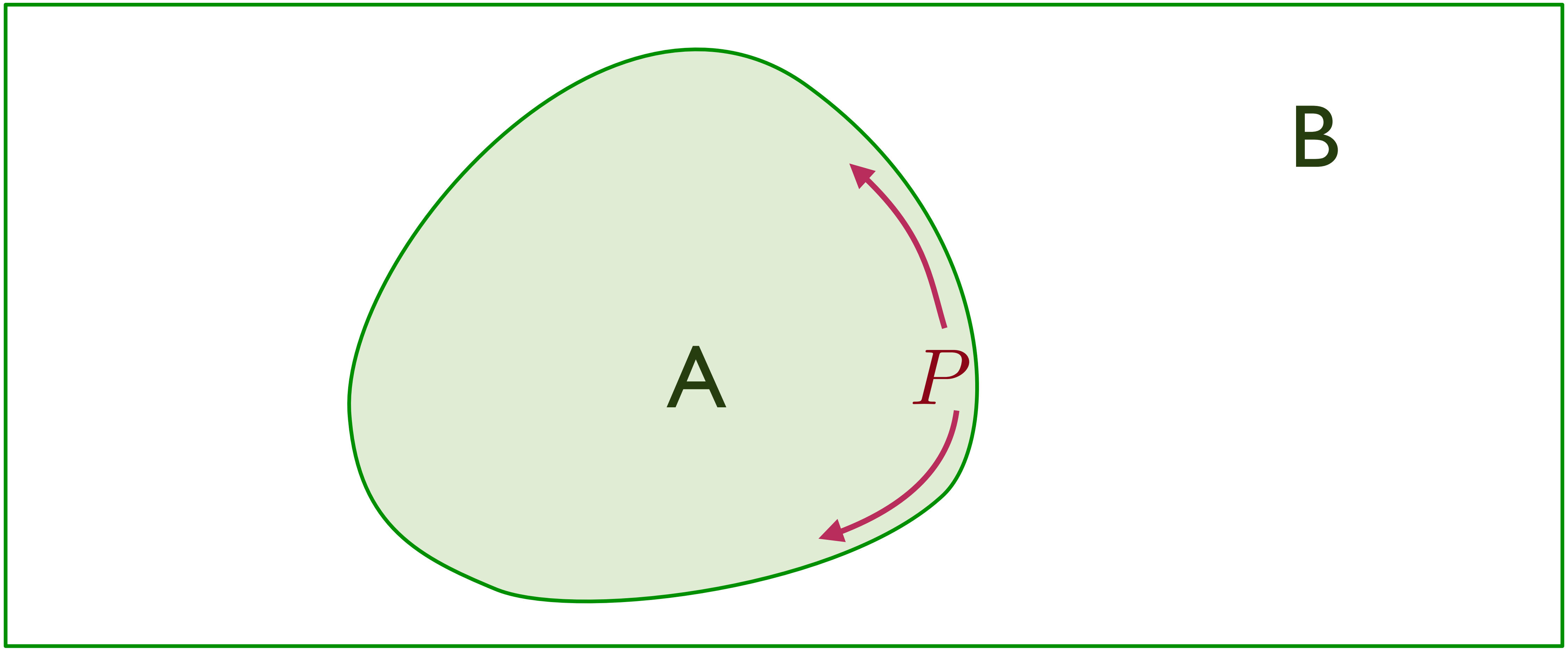

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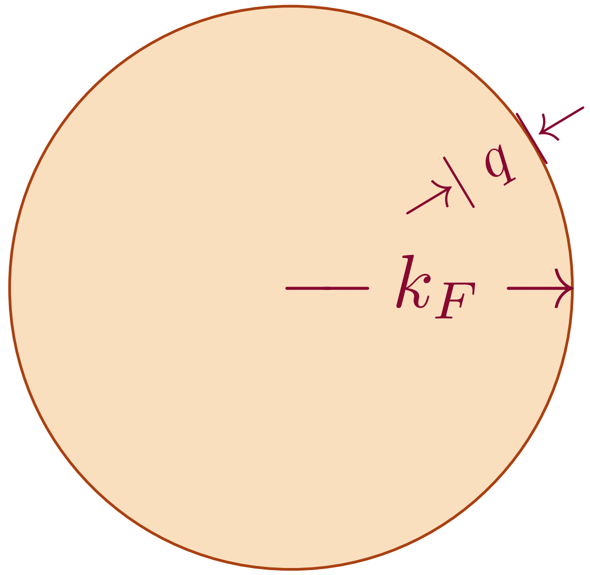
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

FL Fermi liquid



- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

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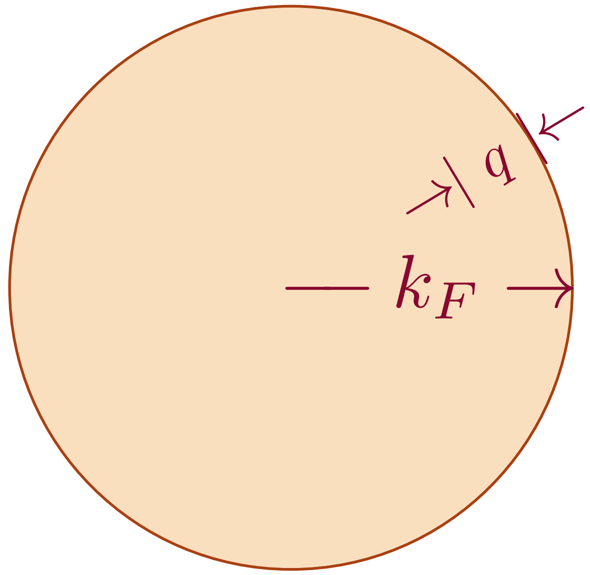

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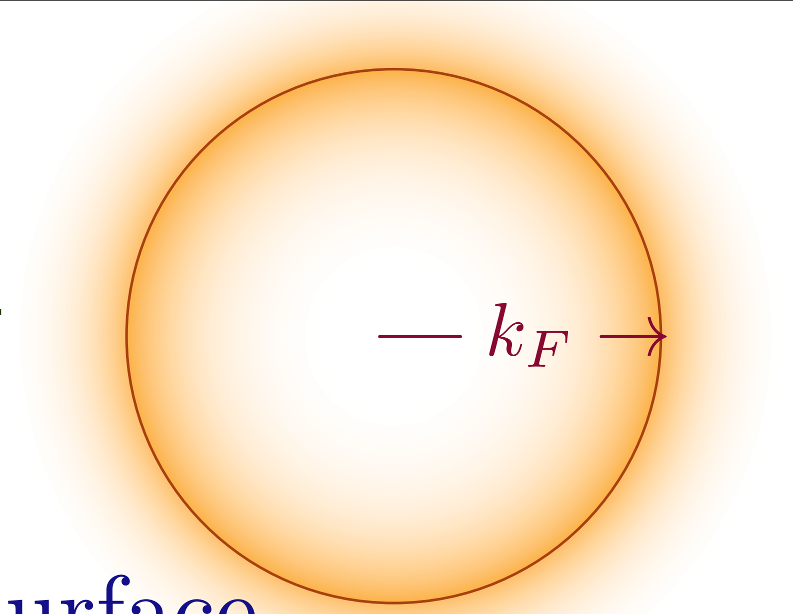
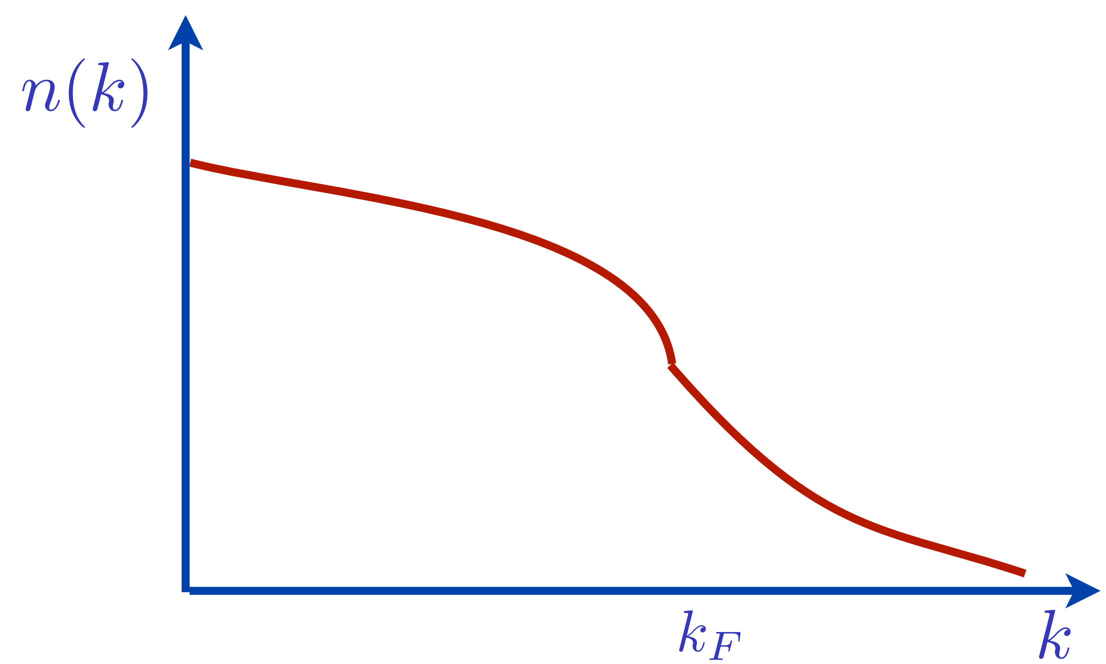
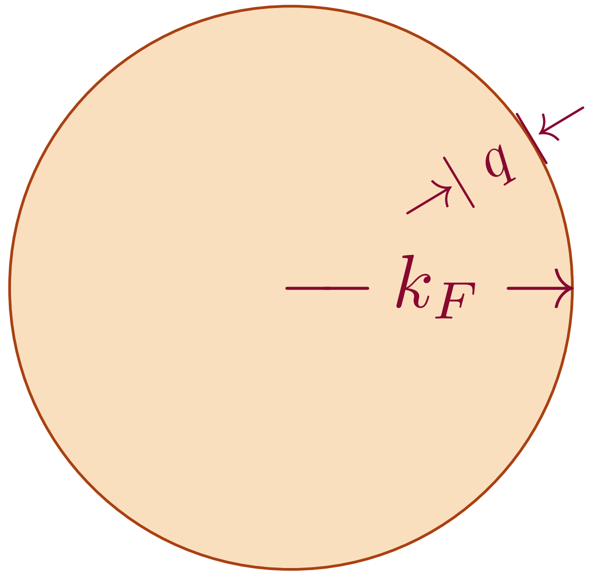
NFL Nematic QCP



- Fermi surface with $k_F^d \sim \mathcal{Q}$.



FL
Fermi liquid



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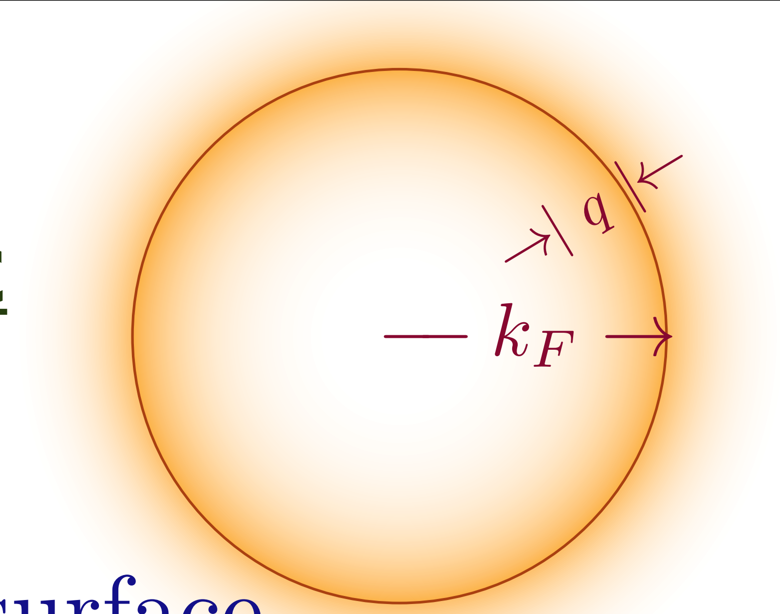

- Fermi surface with $k_F^d \sim \mathcal{Q}$.
- Diffuse fermionic excitations with $z = 3/2$ to three loops.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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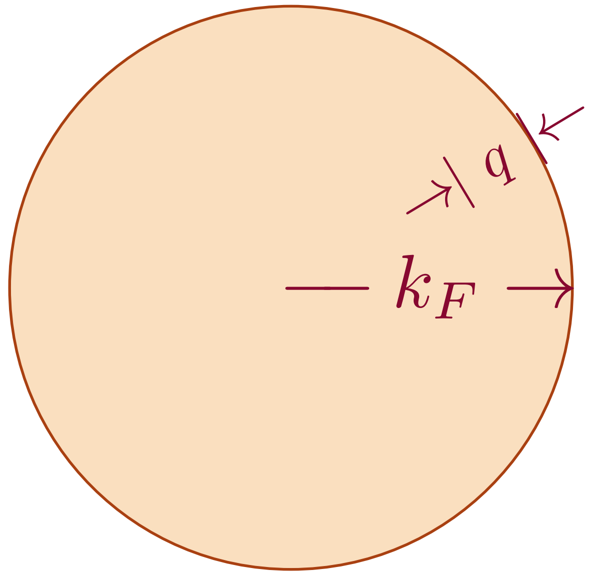	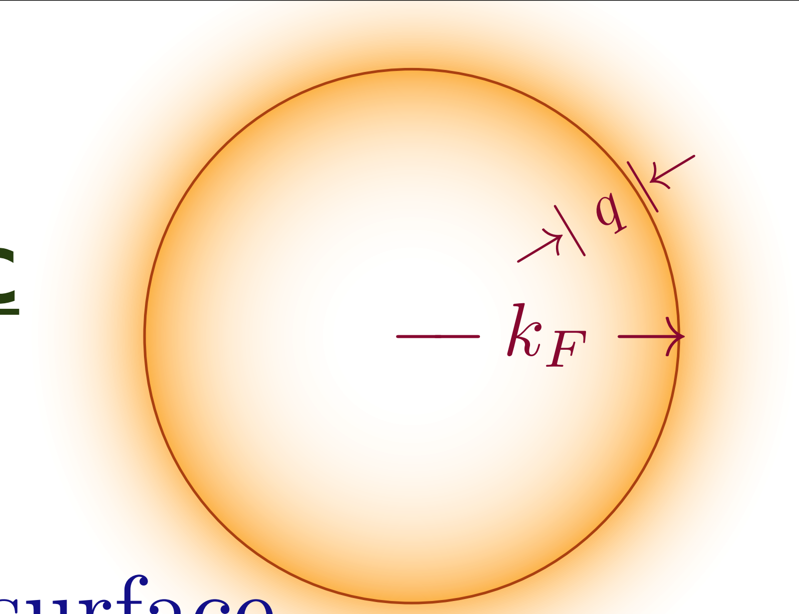
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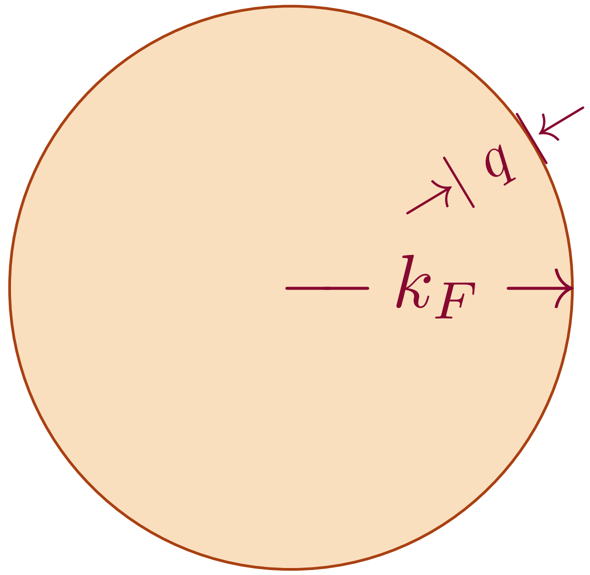	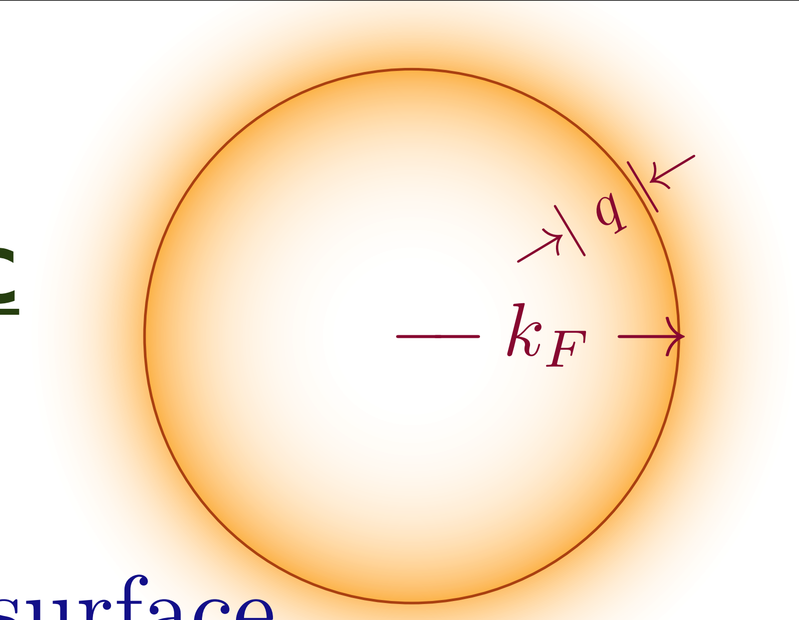
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
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1. Entanglement, holography, and CFTs
2. Fermi liquids and non-Fermi liquids
3. Generalized holography beyond CFTs

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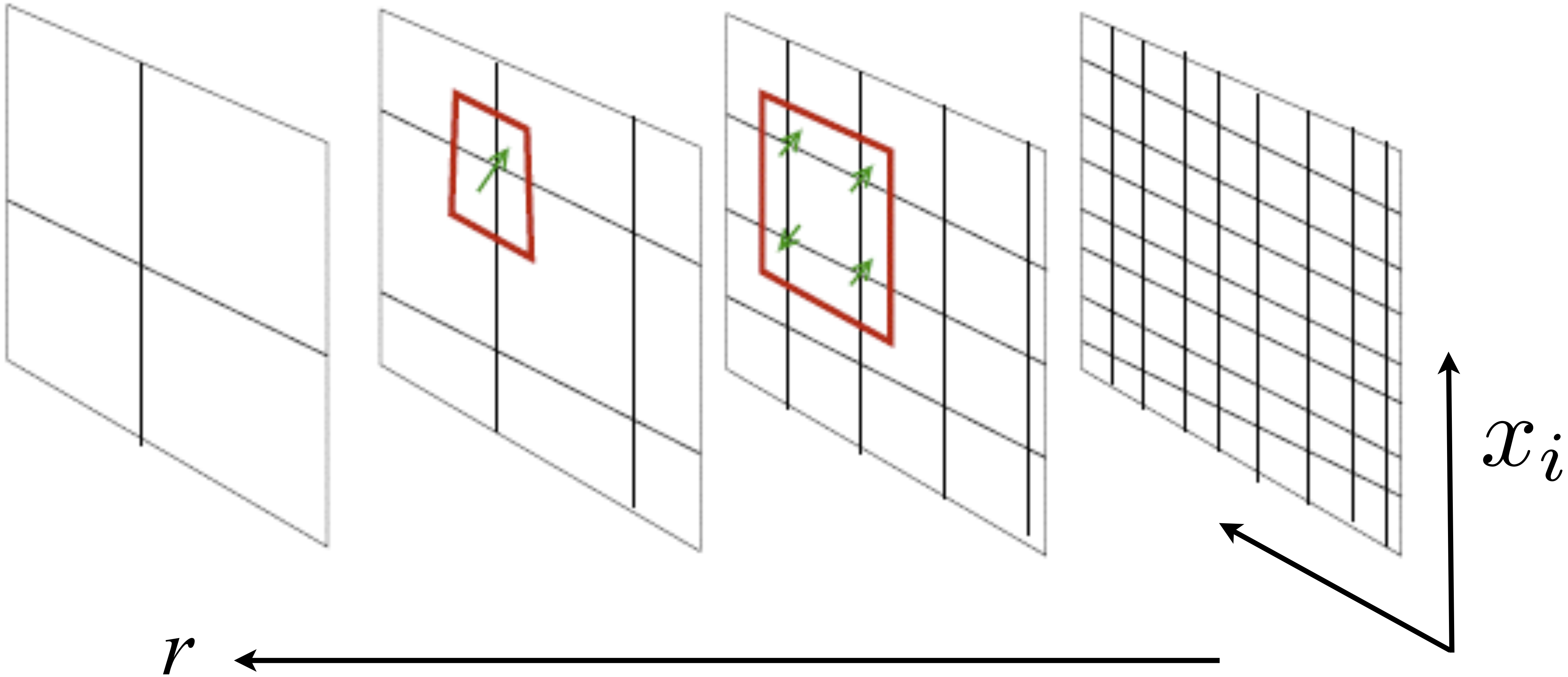
Generalized holography



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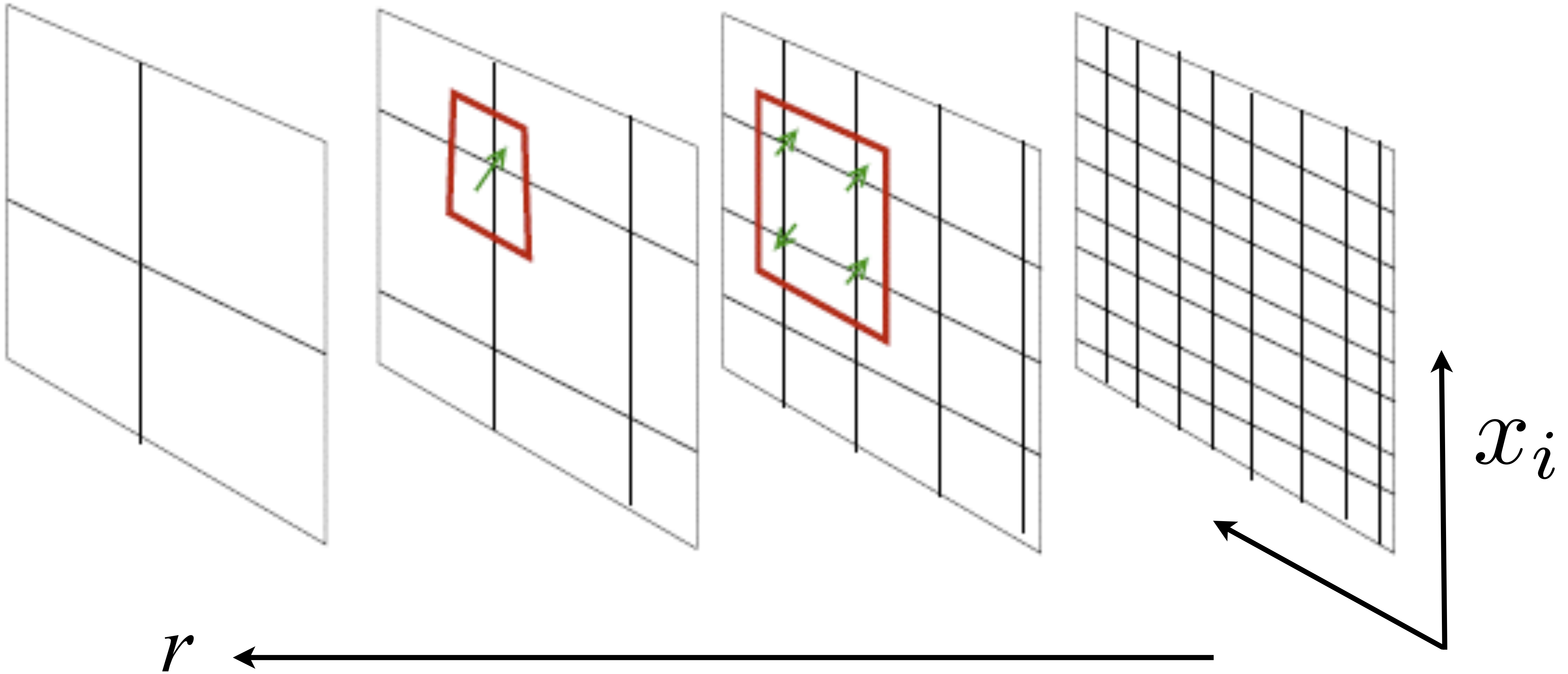
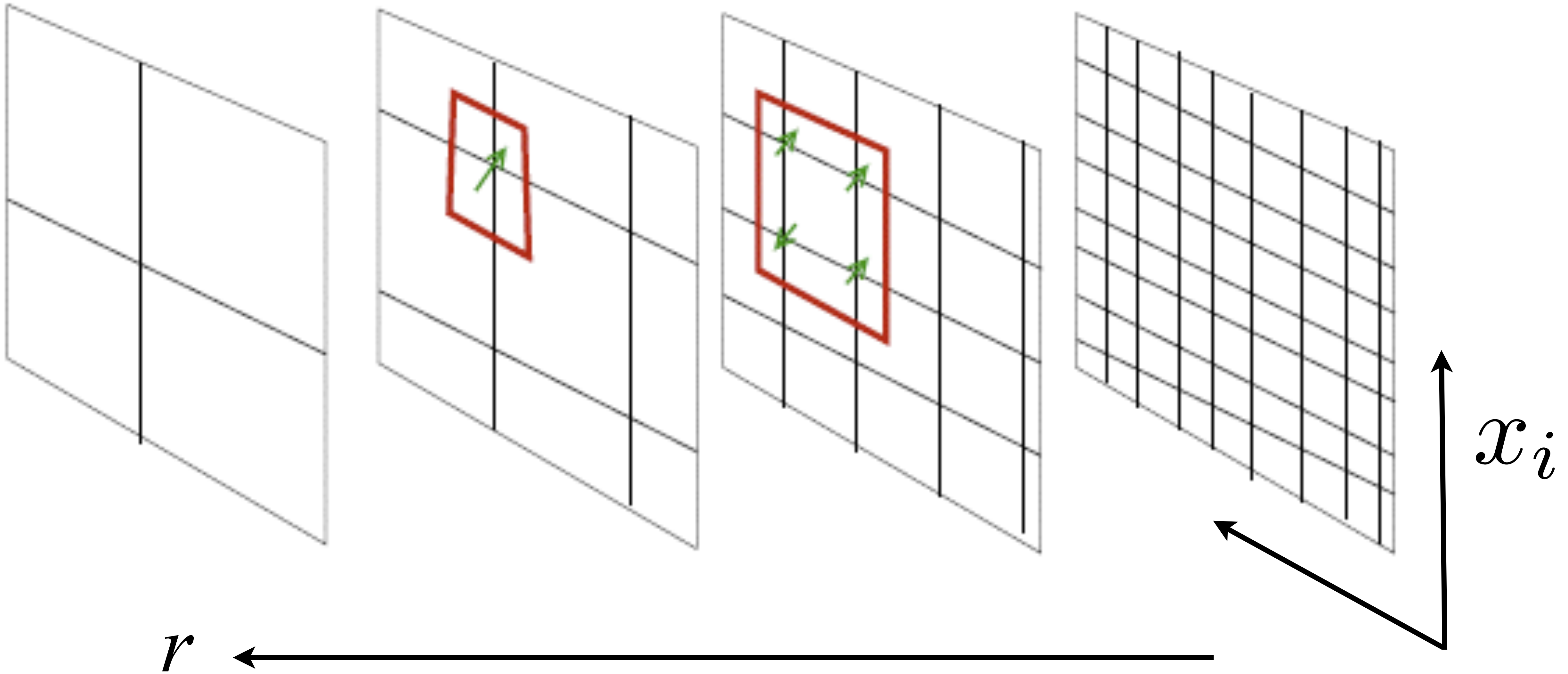


Consider a metric which transforms under rescaling as

$$x_i \to \zeta\, x_i, \quad t \to \zeta^z\, t, \quad ds \to \zeta^{\theta/d}\, ds.$$

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter

Generalized holography



The most general such metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Generalized holography

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This is the most general metric which is invariant under the scale transformation

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points). We will see shortly that θ is the violation of hyperscaling exponent.
We have used reparametrization invariance in r to define it so that it scales as

$$r \rightarrow \zeta^{(d-\theta)/d} r\,.$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Generalized holography

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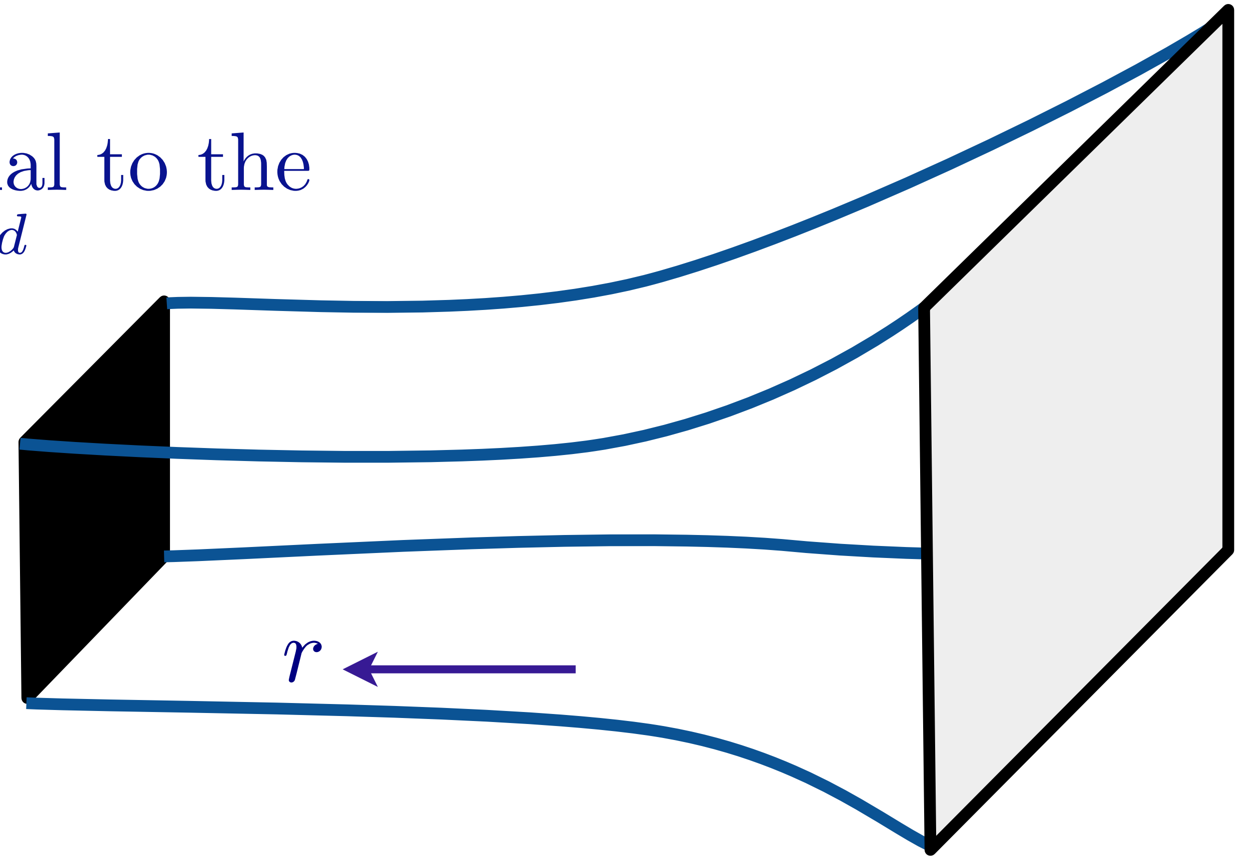
At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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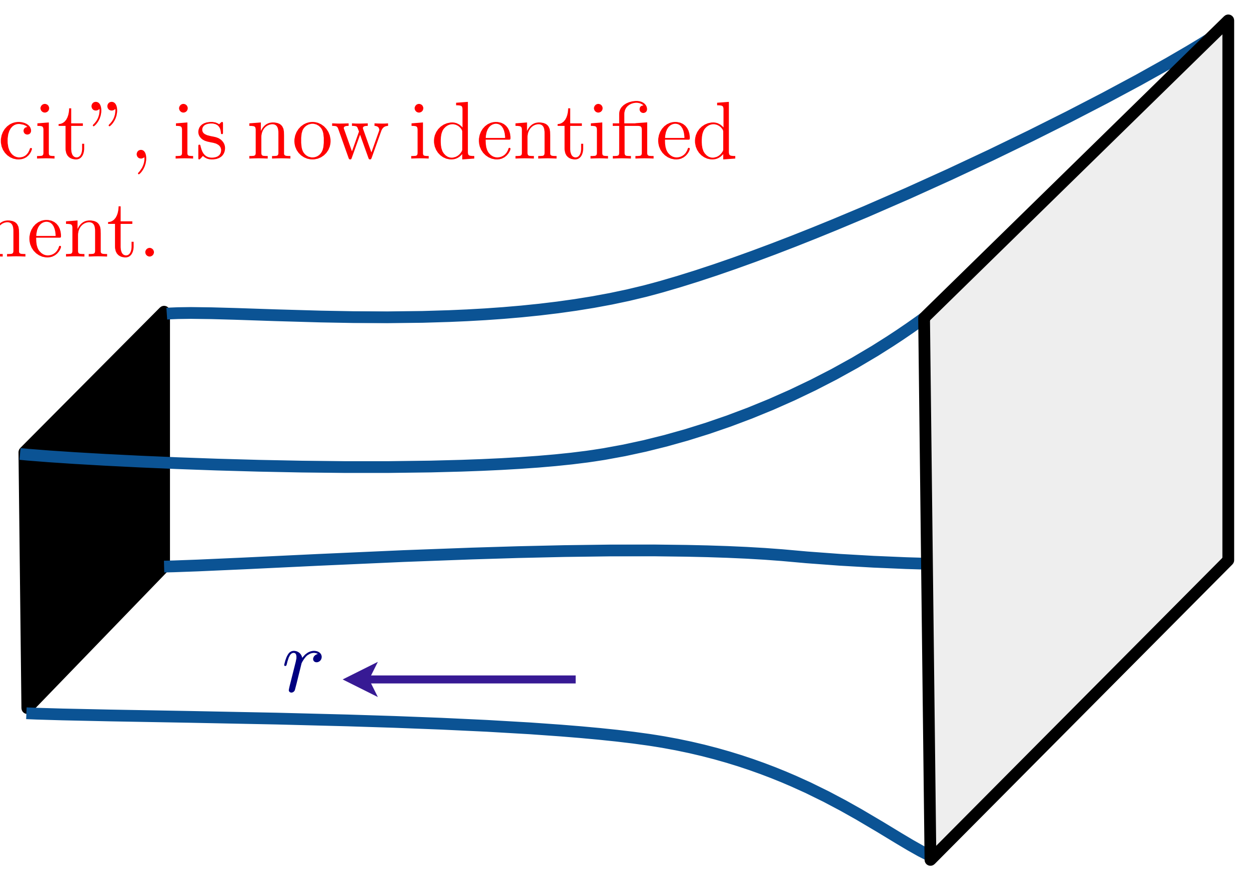
Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$, the "dimension deficit", is now identified as the violation of hyperscaling exponent.



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$$z \geq 1 + \frac{\theta}{d}$$

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The non-Fermi liquid in $d = 2$ has $\theta = d-1$, and this implies $z \geq 3/2$. So the lower bound is precisely the value obtained for the non-Fermi liquid!

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Holography of a non-Fermi liquid

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S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, `arXiv:1401.7012`
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Solution of Einstein-Maxwell equations for small h_0 yields the resistivity

$$\rho(T) \sim h_0^2\, T^{(d-z+\eta)/z}\,,$$

where $\dim[\mathcal{O}] = (d + z - 2 + \eta)/2$. This agrees with the *memory function* computation of the bosonic contribution of the "standard model" field theory. The crossover at higher energies to the Wilson-Fisher CFT (with $z = 1$, $\eta \approx 0$) yields $\rho(T) \sim T$.

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