Metals with long-range entanglement

University of New South Wales
Sydney
August 31, 2015

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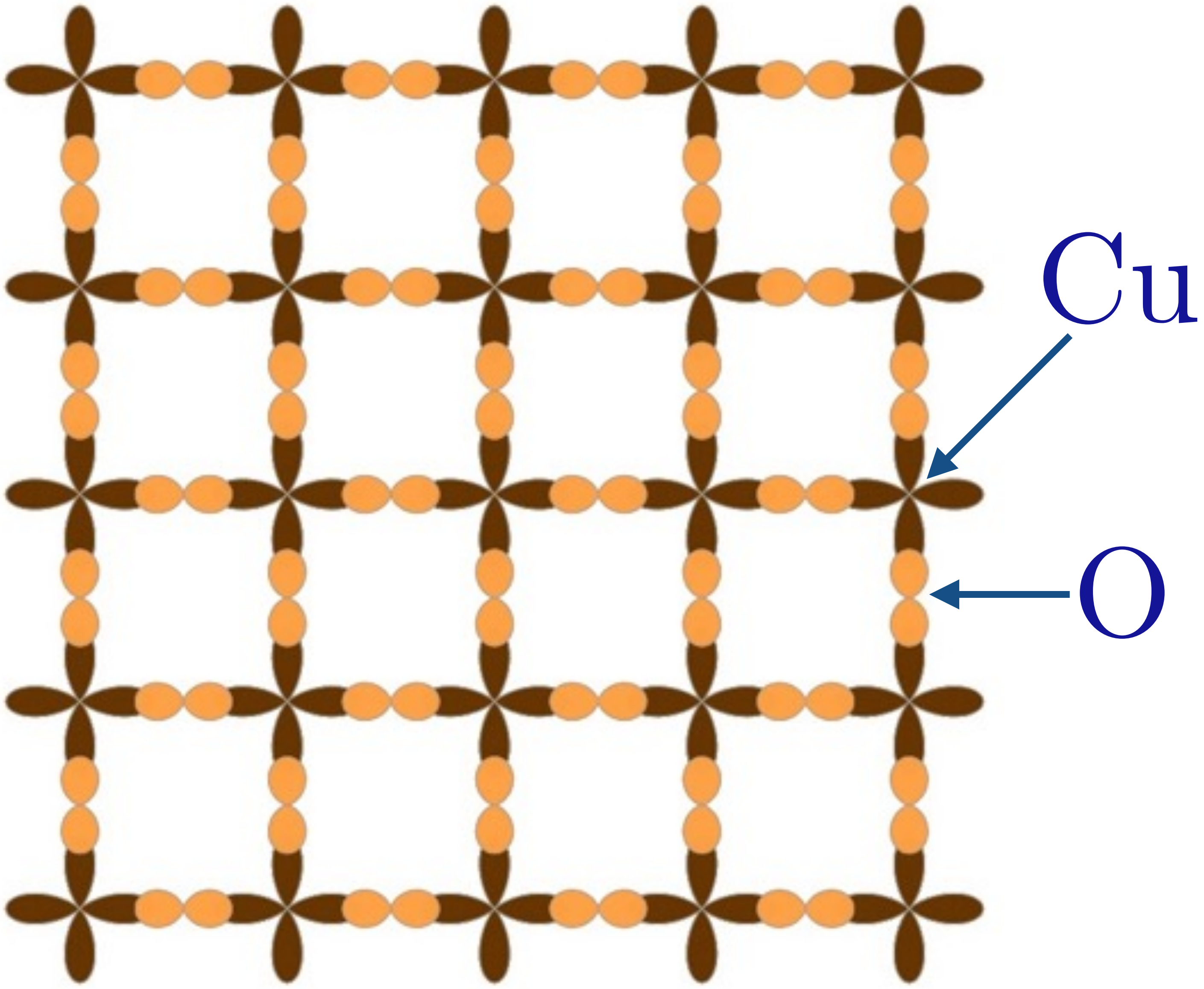
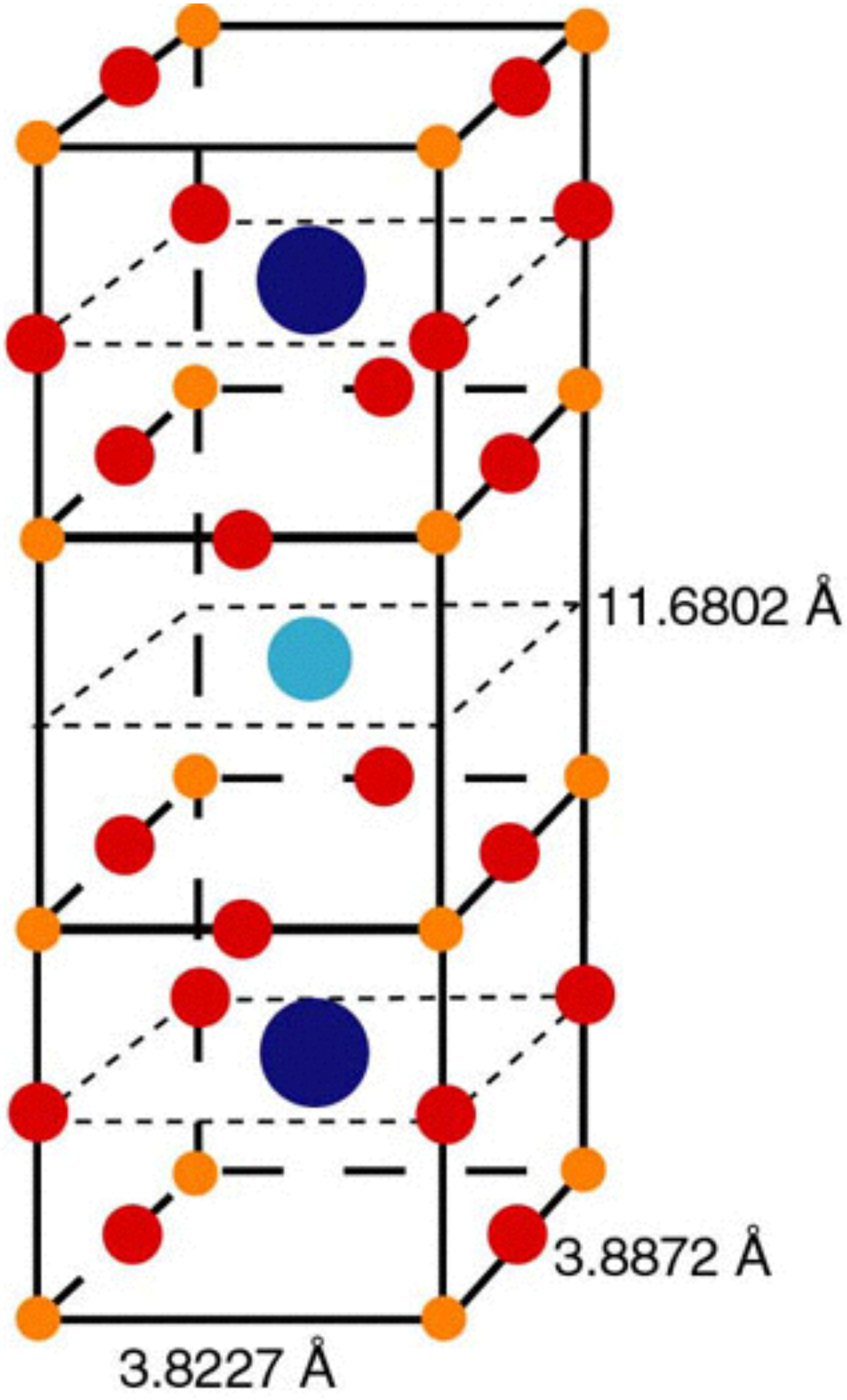
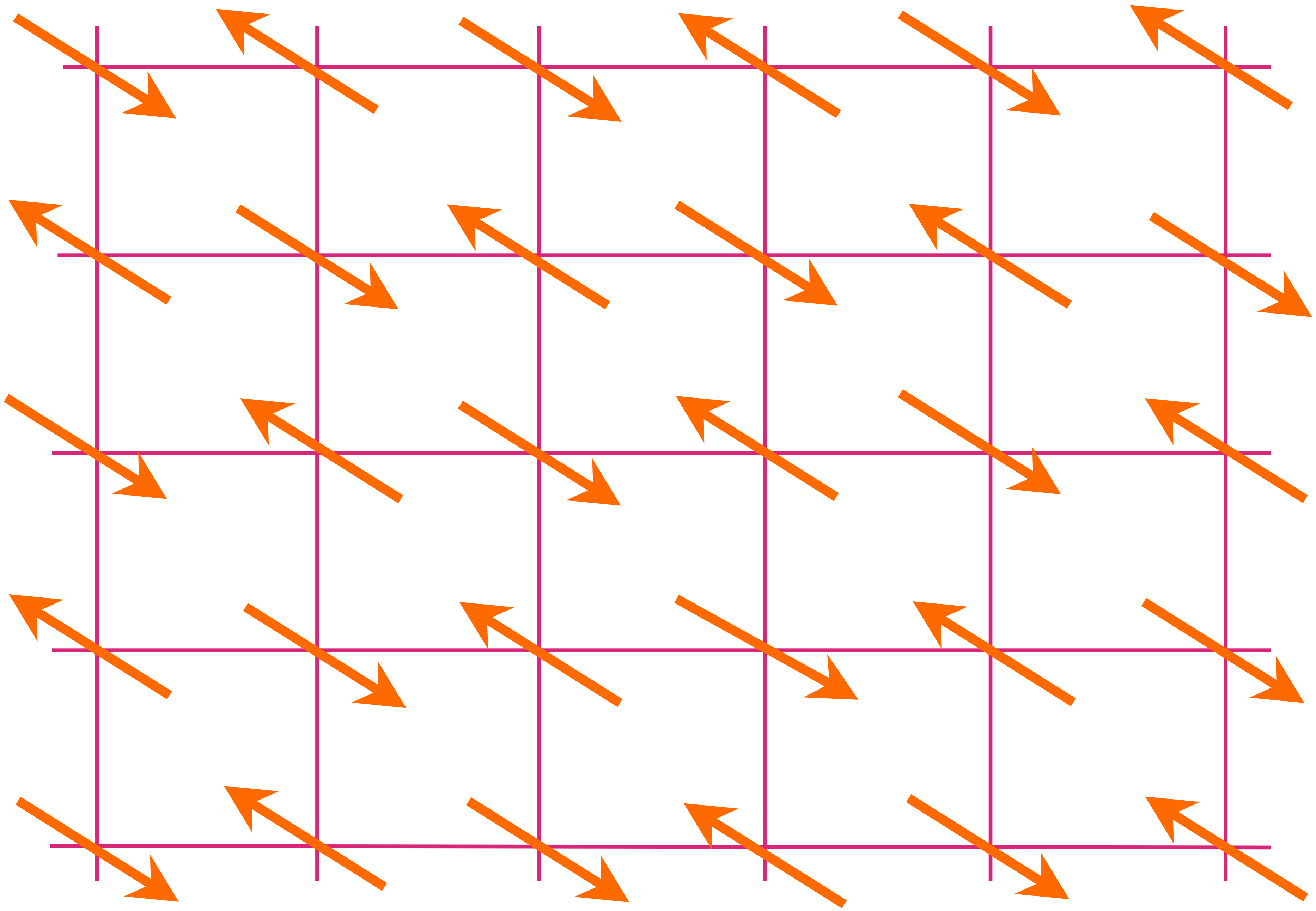
High temperature superconductors



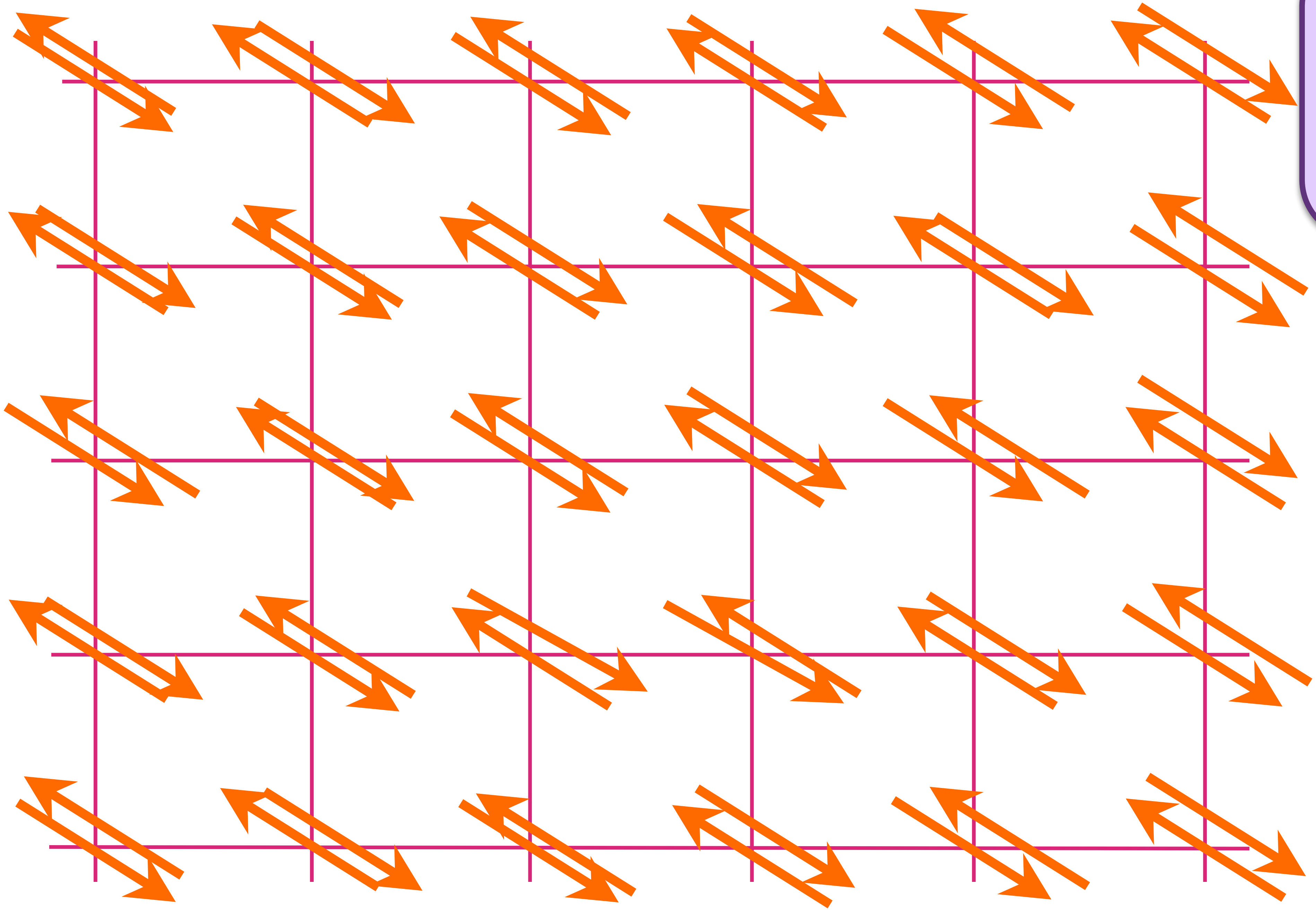
CuO$_2$ plane



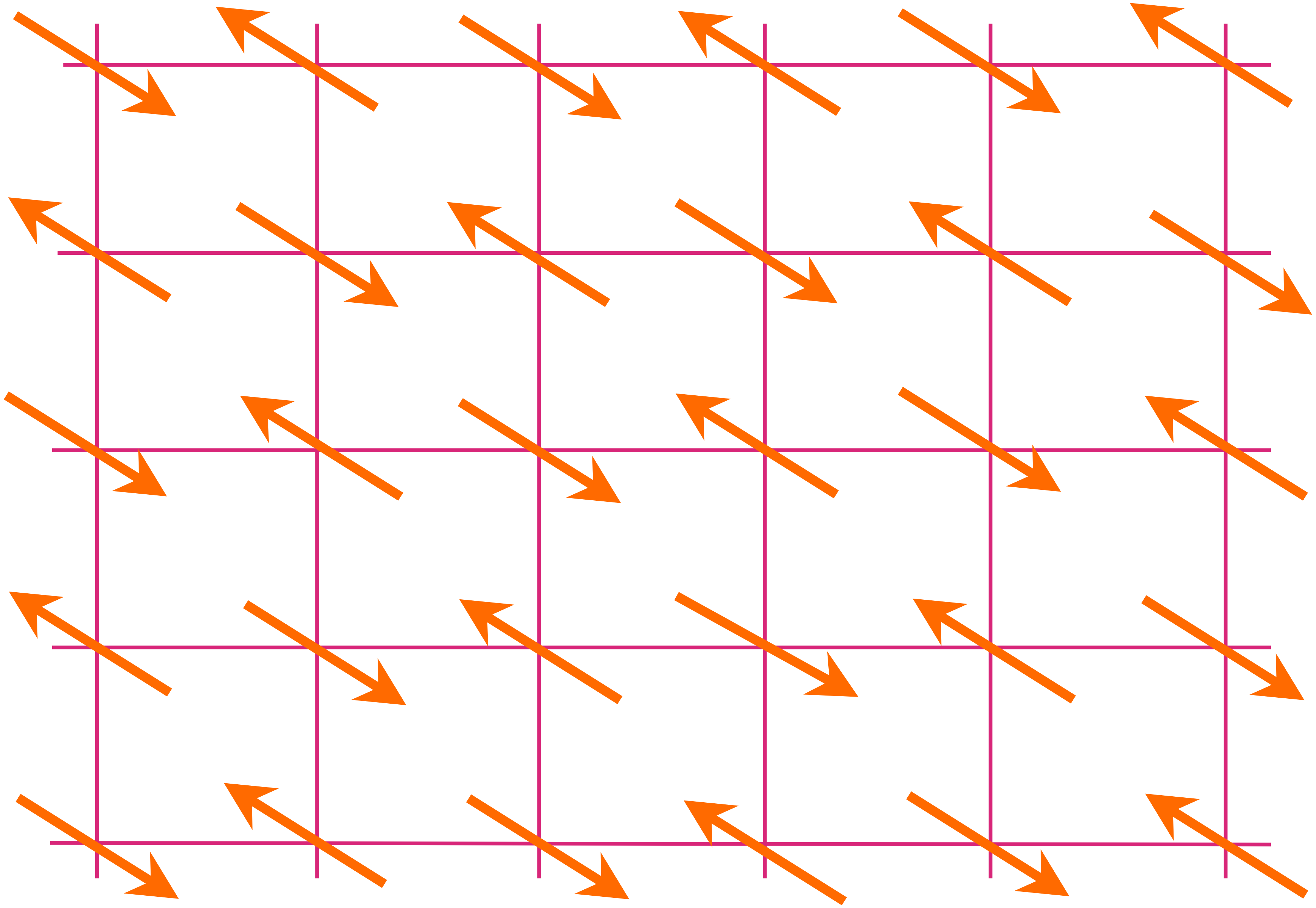
$\mathrm{YBa_2Cu_3O}_{6+x}$



“Undoped”
Anti-
ferromagnet



Filled Band



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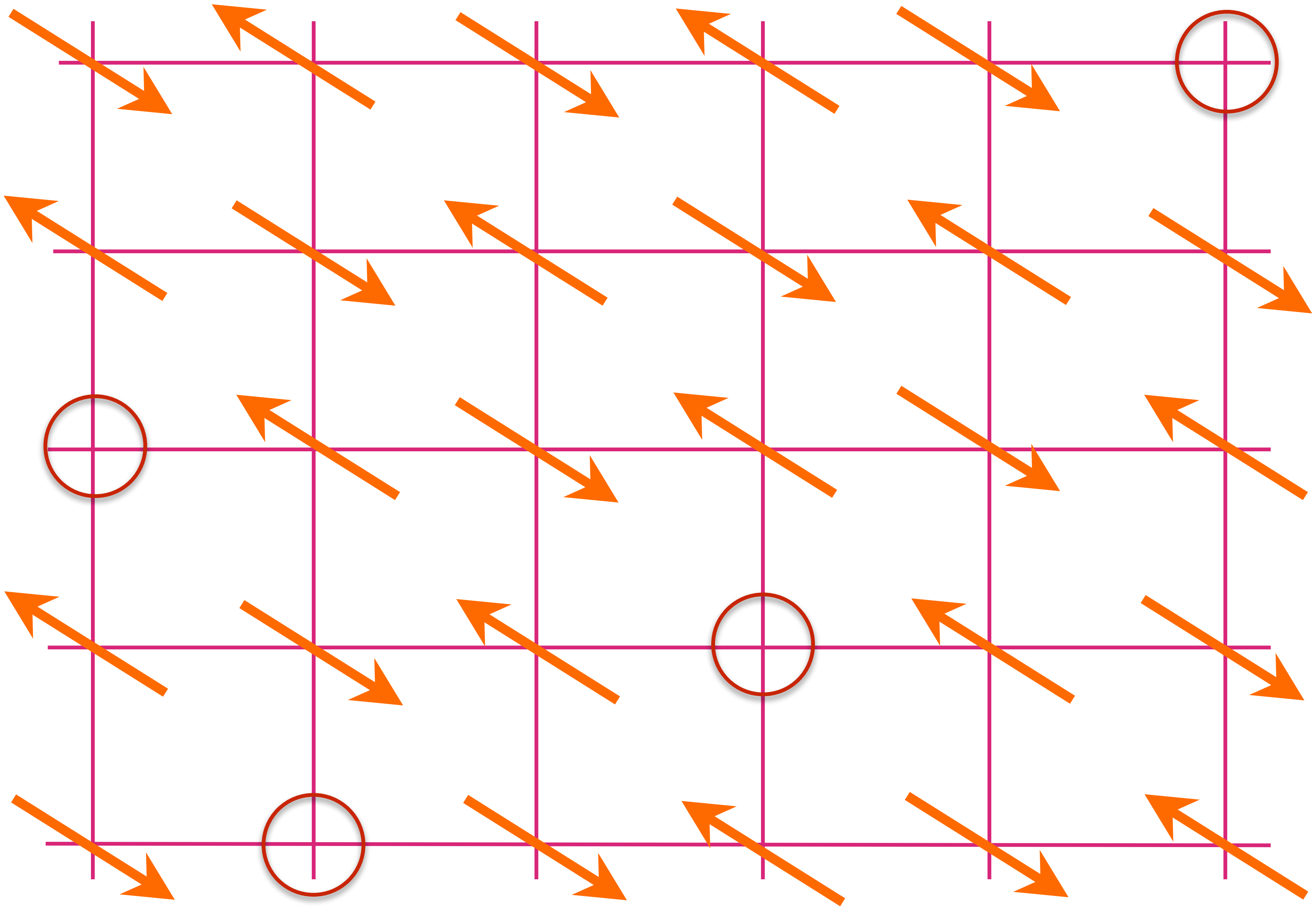
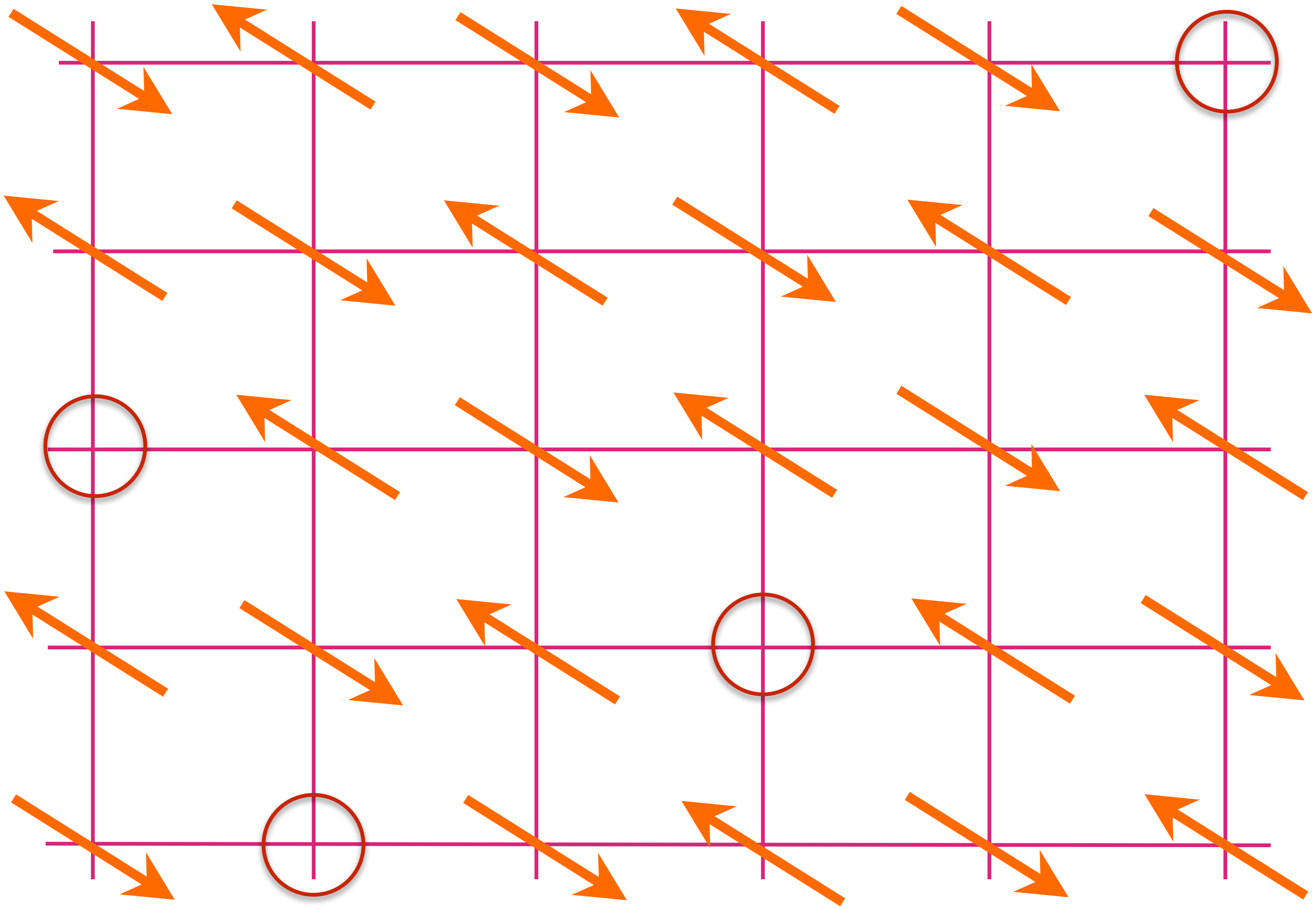
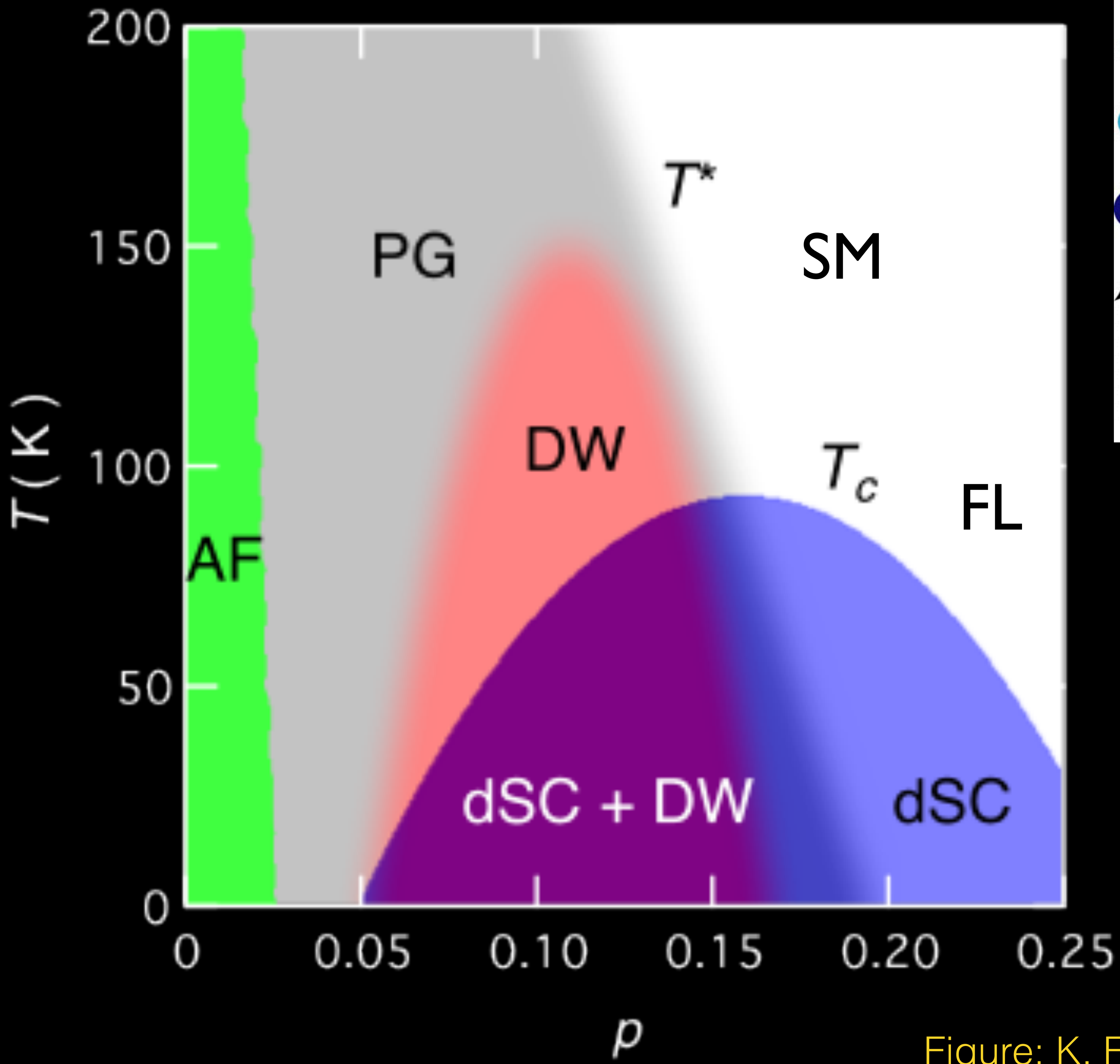


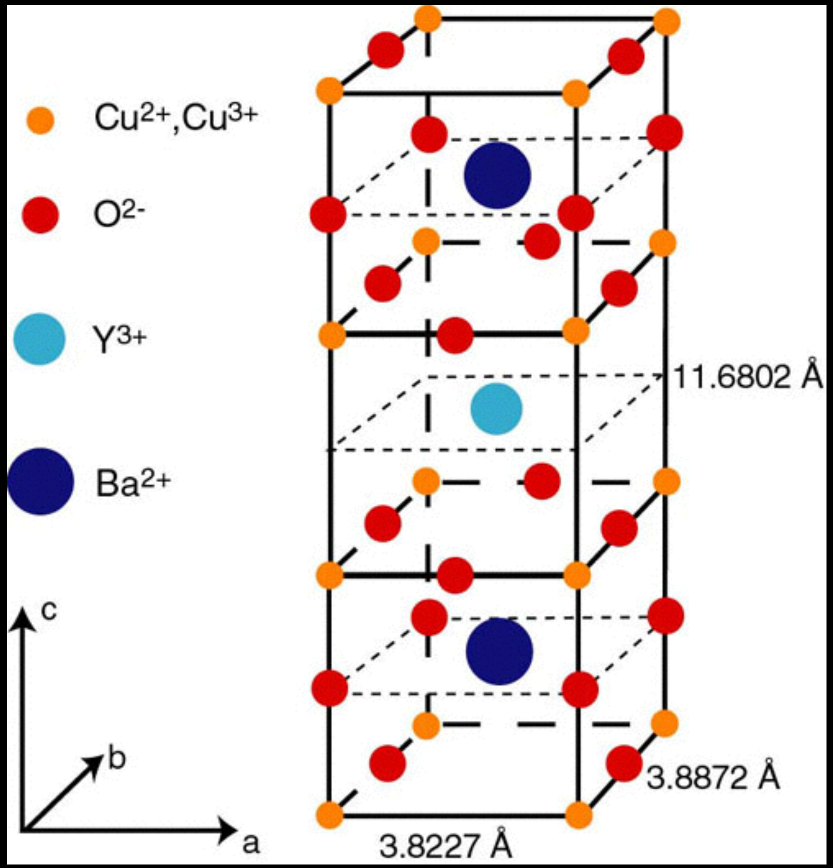
Anti-ferromagnet with *p* holes per square



Anti-ferromagnet with p holes per square

But relative to the band insulator, there are $1+p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1+p$





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

1. Topological order in insulating spin liquids

2. Review of Fermi liquid theory

   Topological argument for the Luttinger theorem

3. Fractionalized Fermi liquid

   A Fermi liquid co-existing with topological order for the pseudogap metal

4. Strange metals without quasiparticles

   (a) A mean-field model of a non-Fermi liquid, and charged black holes

   (b) A (slightly less) strange metal in graphene

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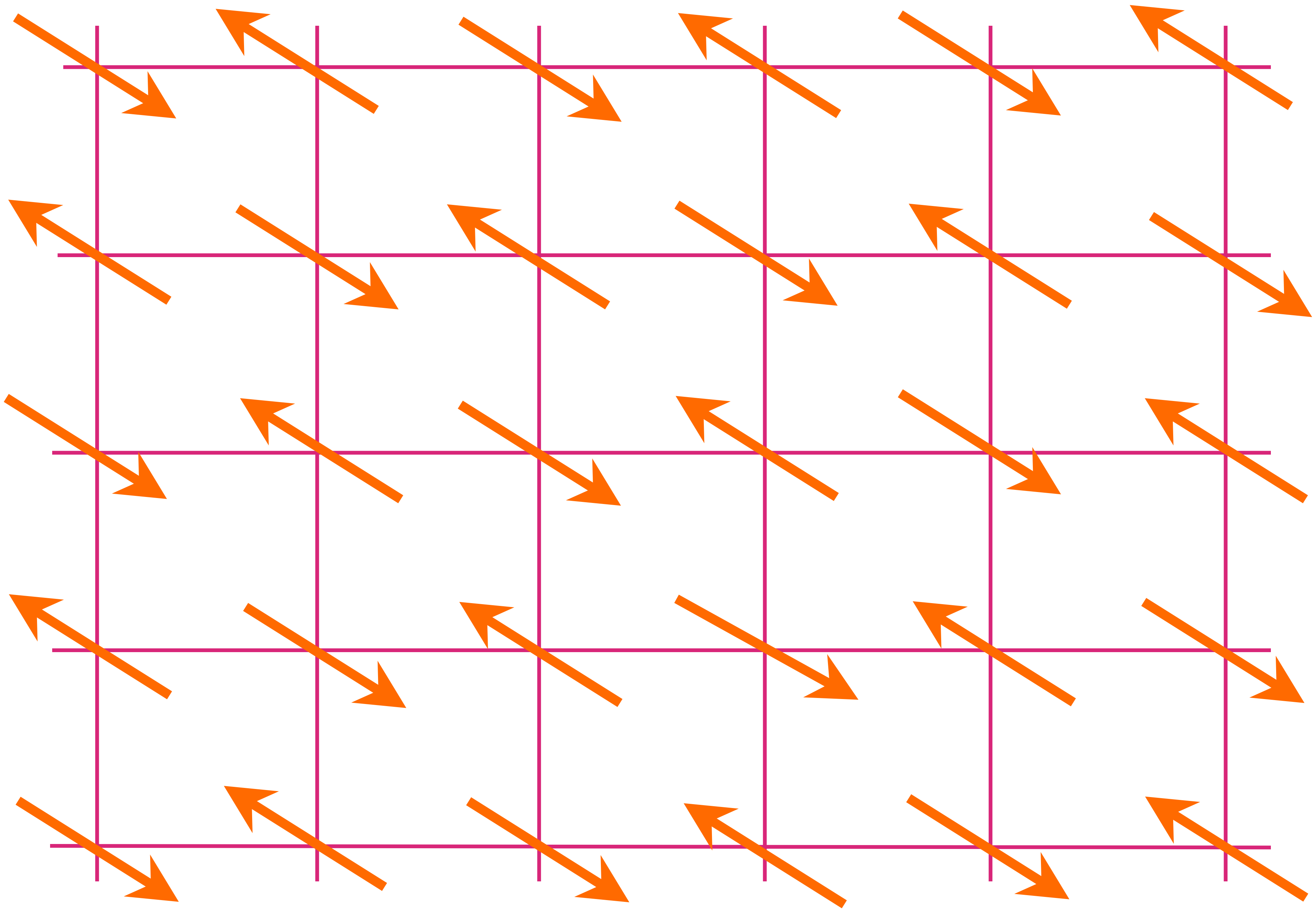
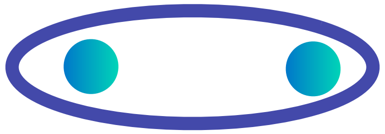


"Undoped" Anti-ferromagnet

Insulating spin liquid



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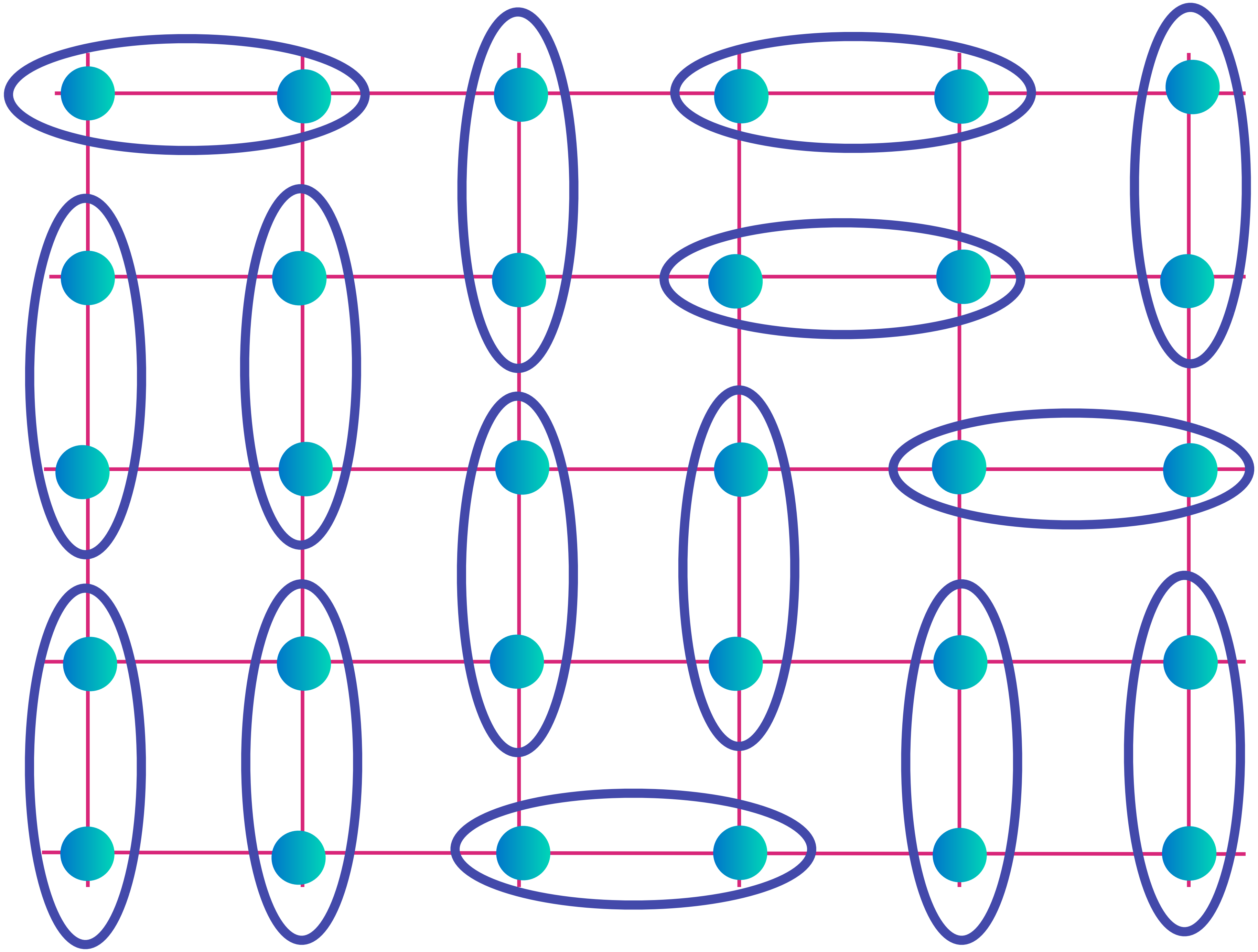

An insulator
at $p=0$
with
"topological
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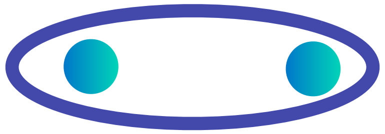
Insulating spin liquid



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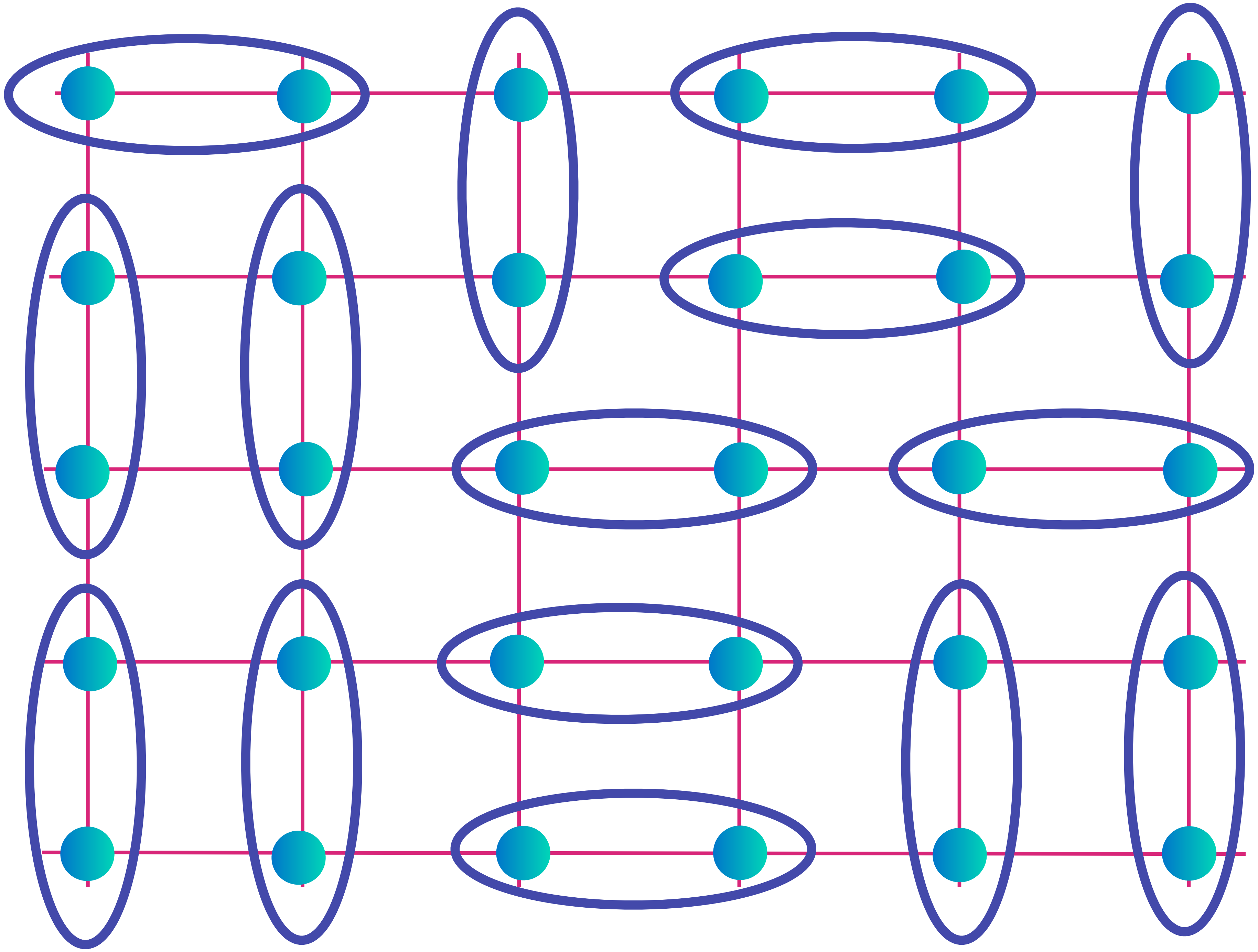

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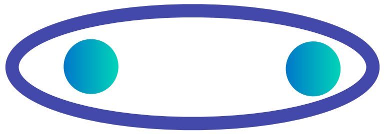

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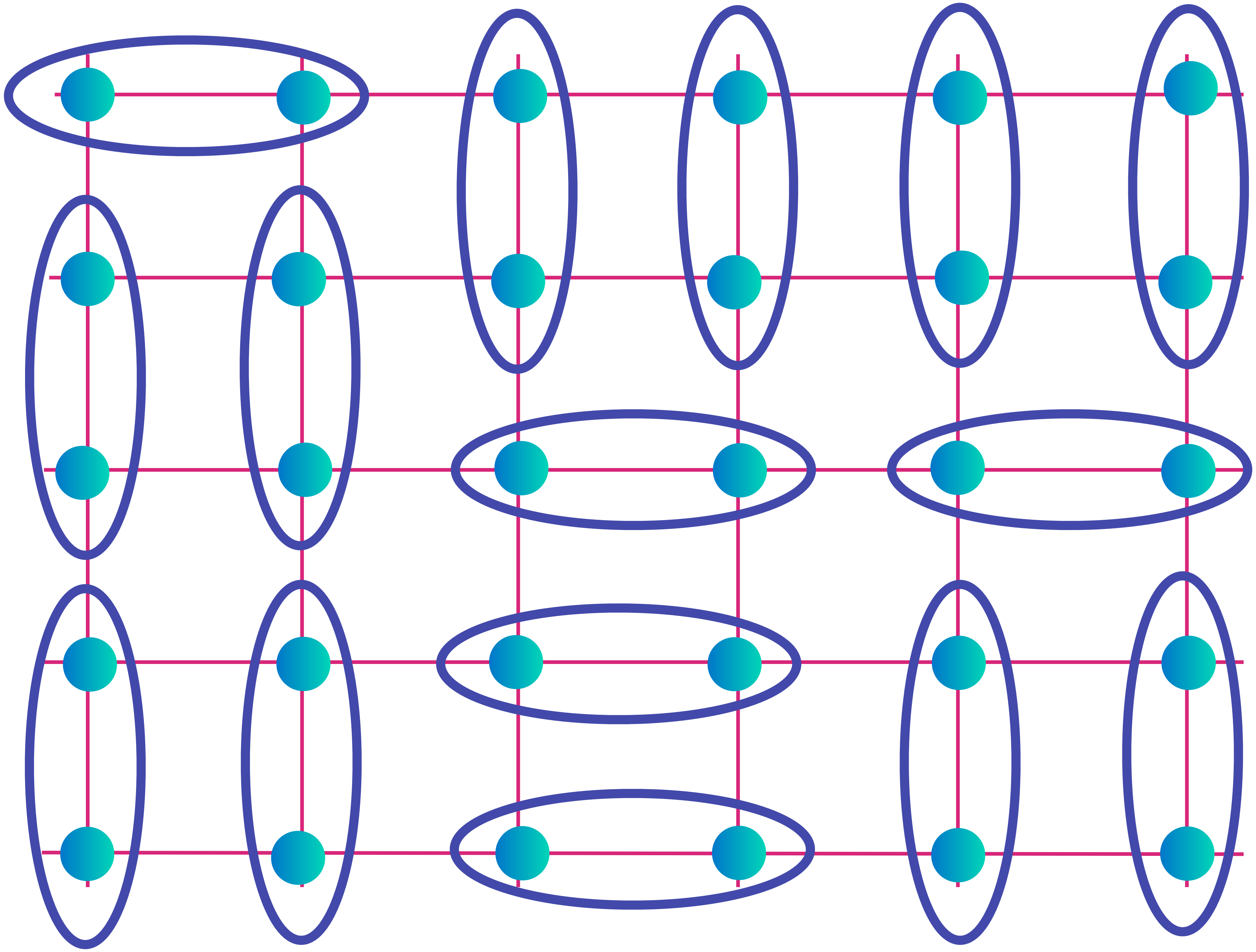

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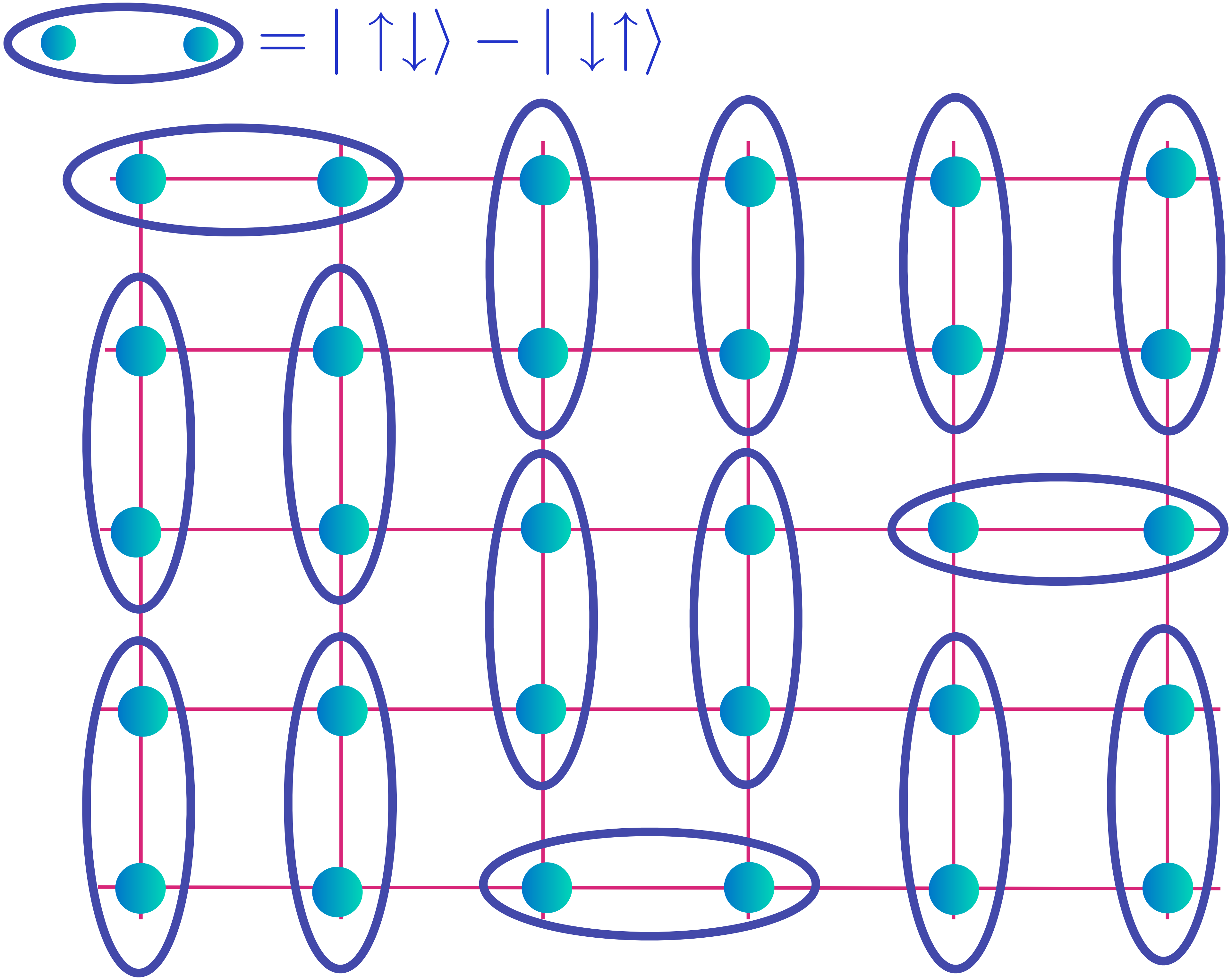

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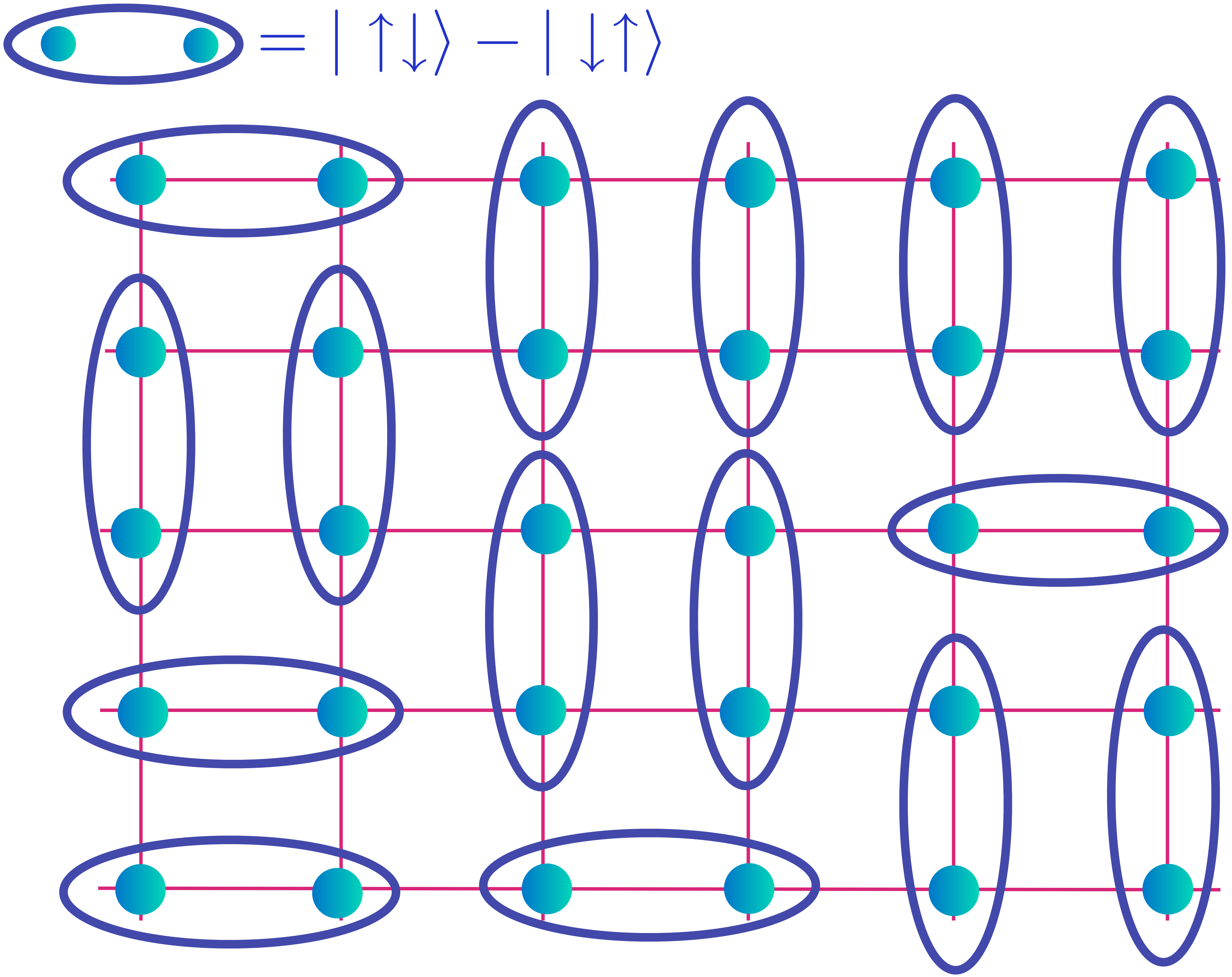

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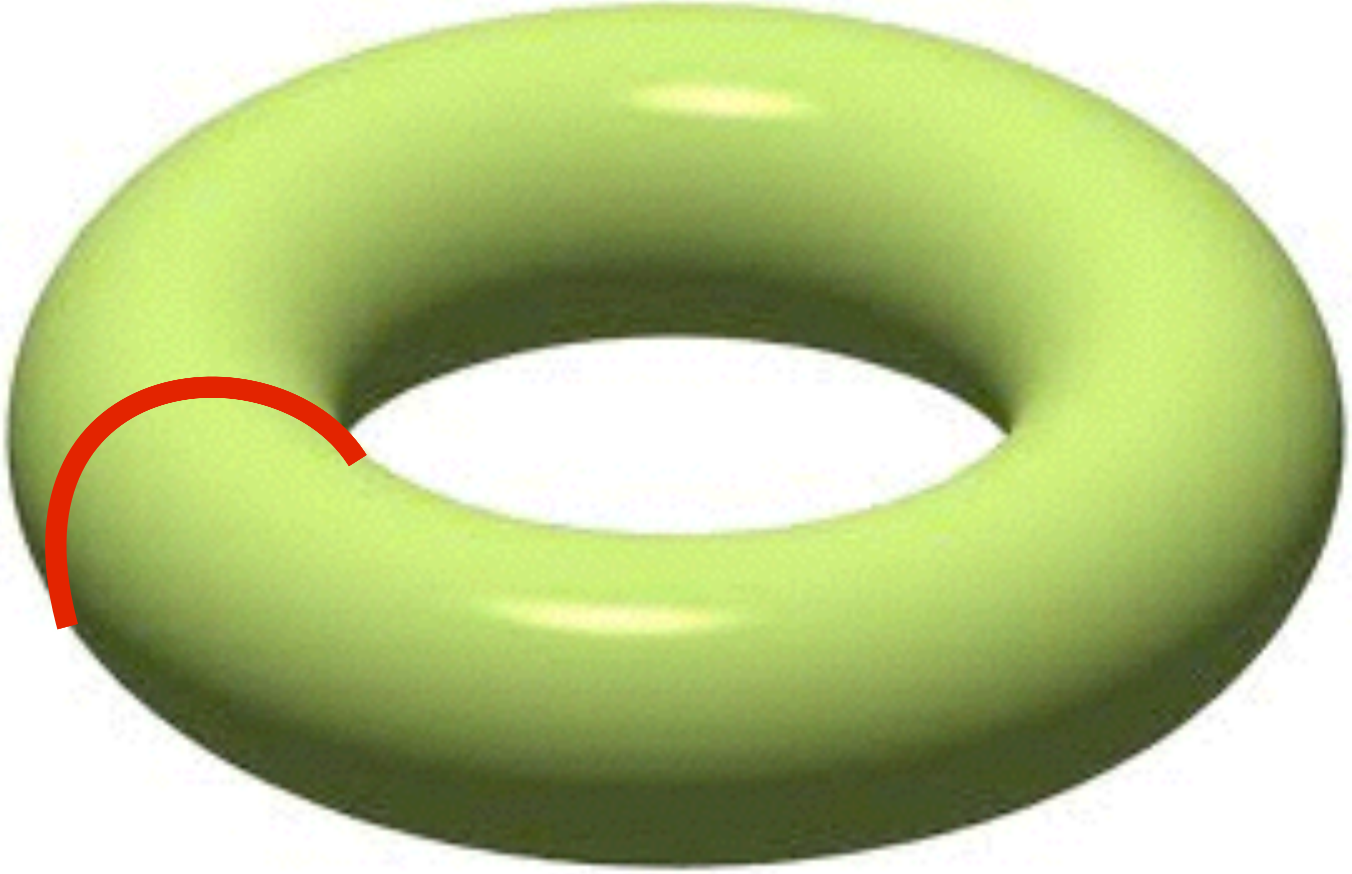
Topological order

Place insulator on a torus;

Topological order

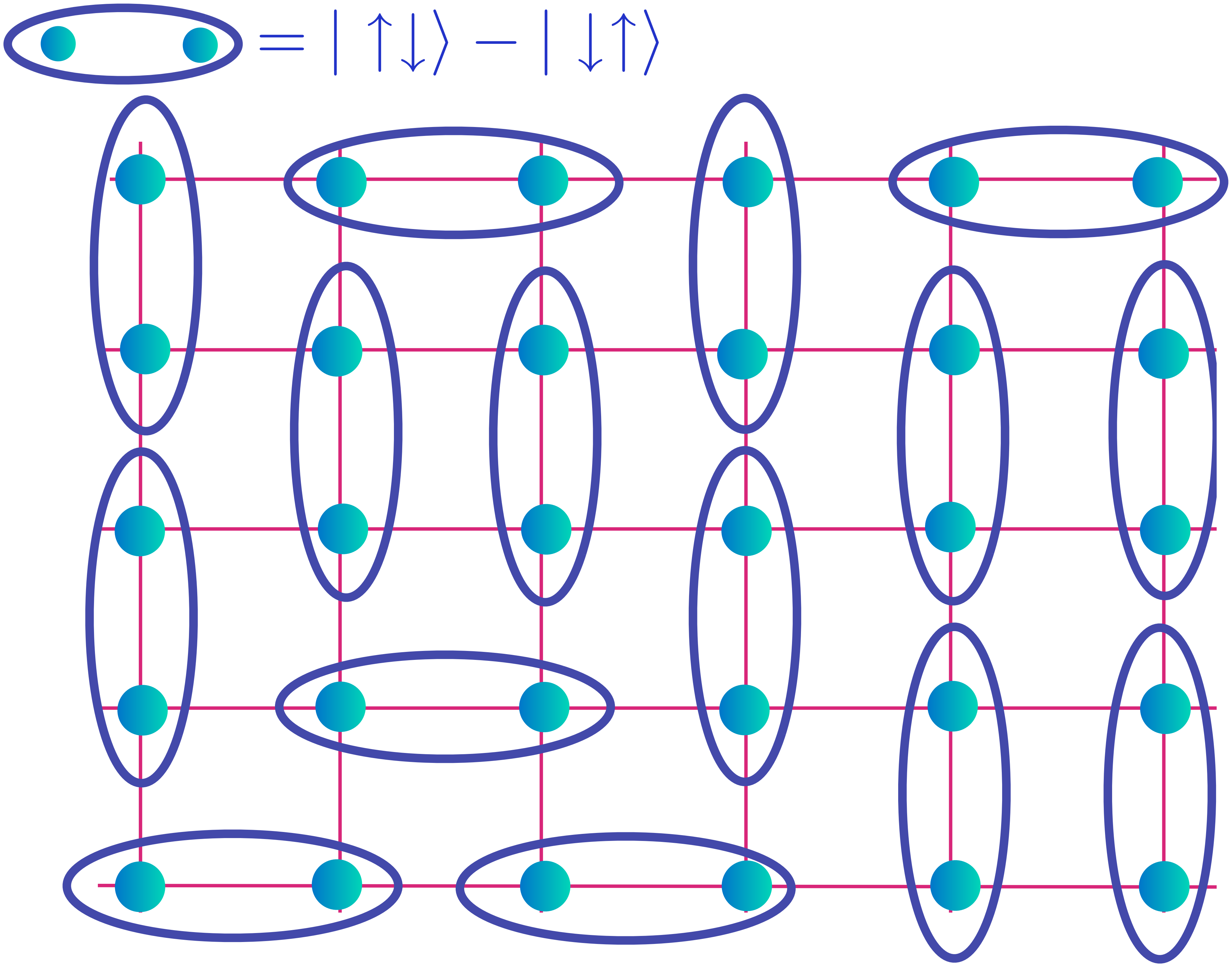


Place insulator on a torus;

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

N. Read and S. Sachdev, PRL **66**, 1773 (1991); X.-G. Wen PRB **44**, 2664 (1991)

Topological order



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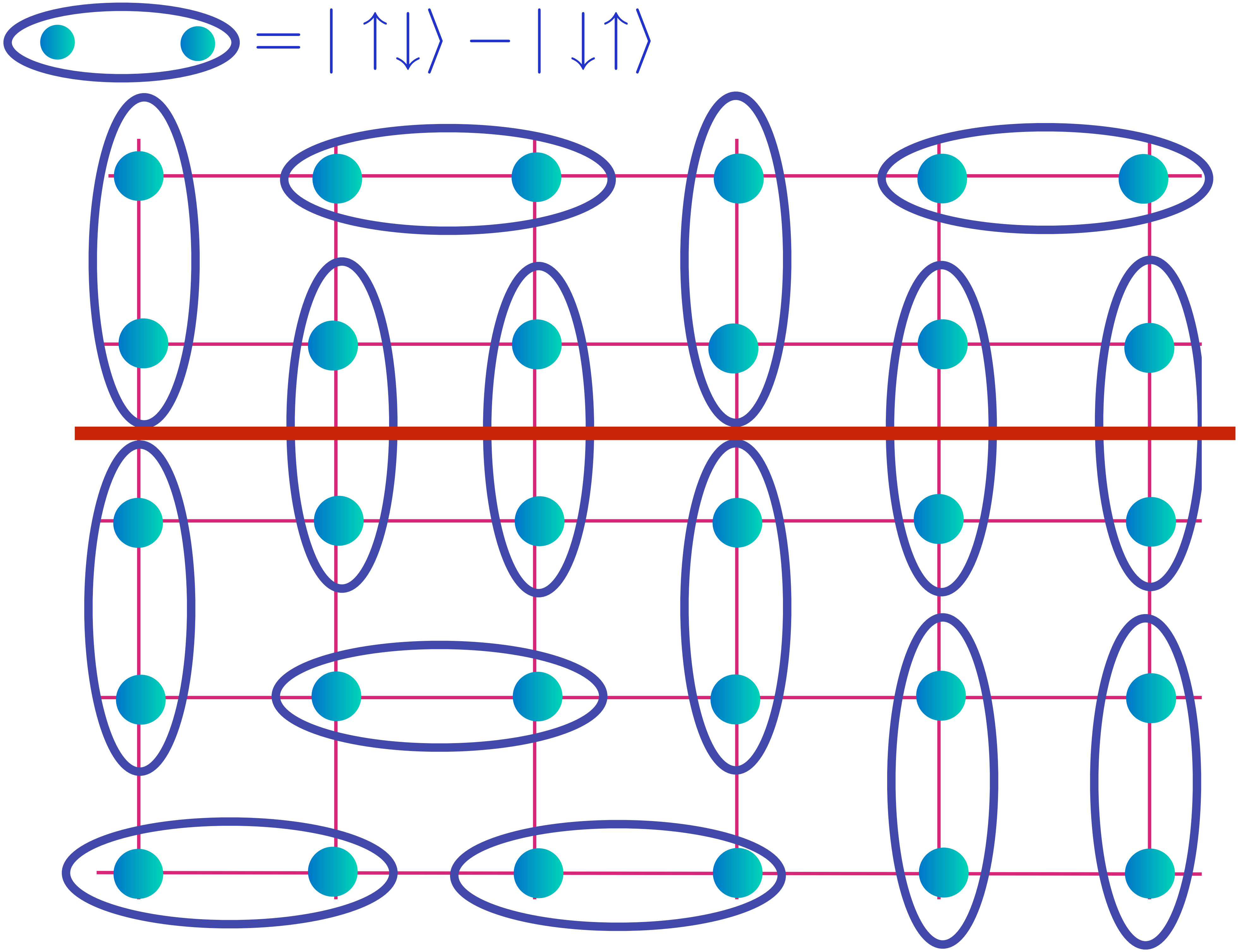

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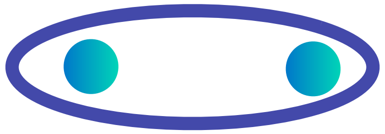

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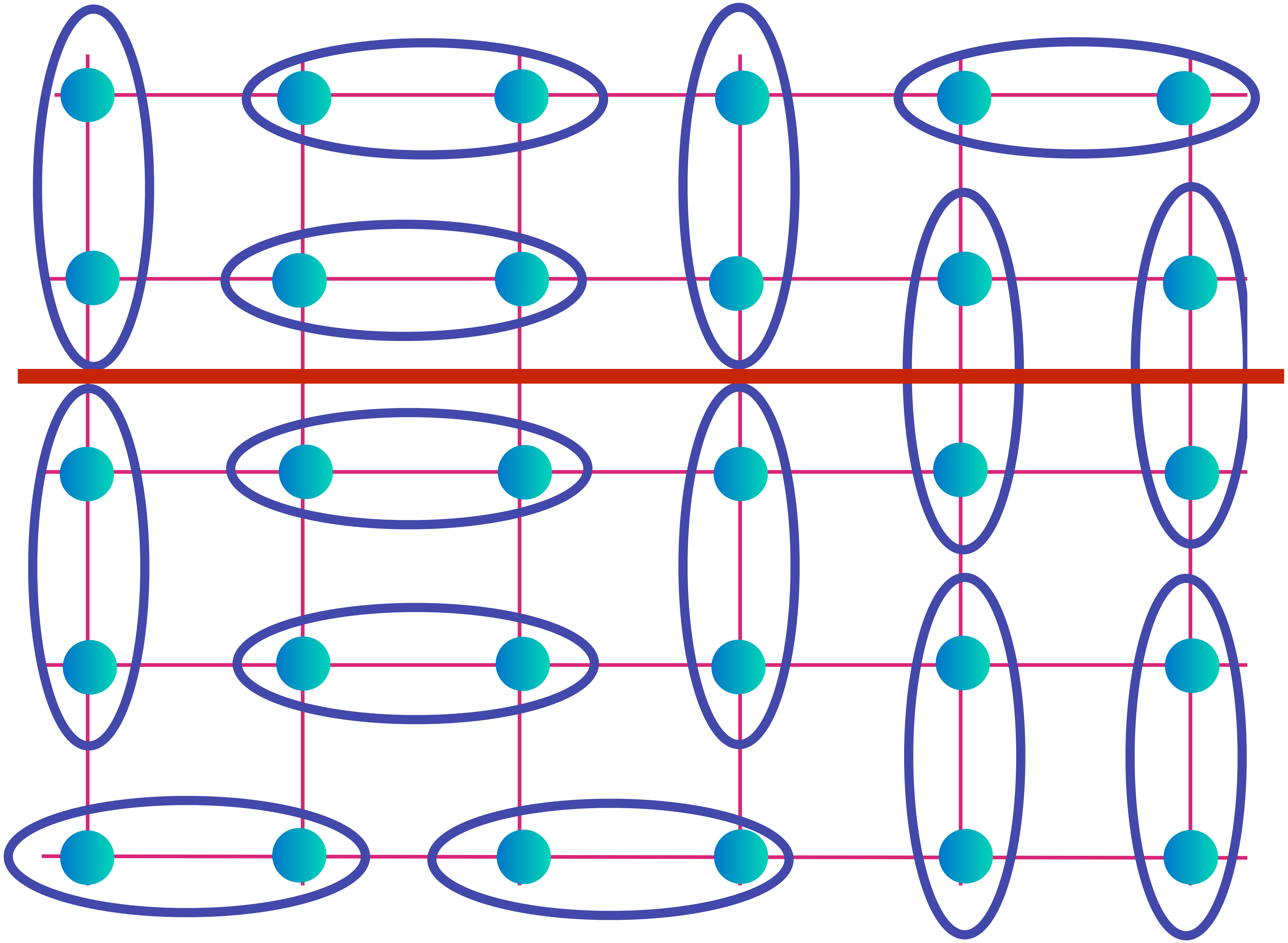

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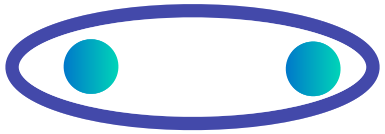
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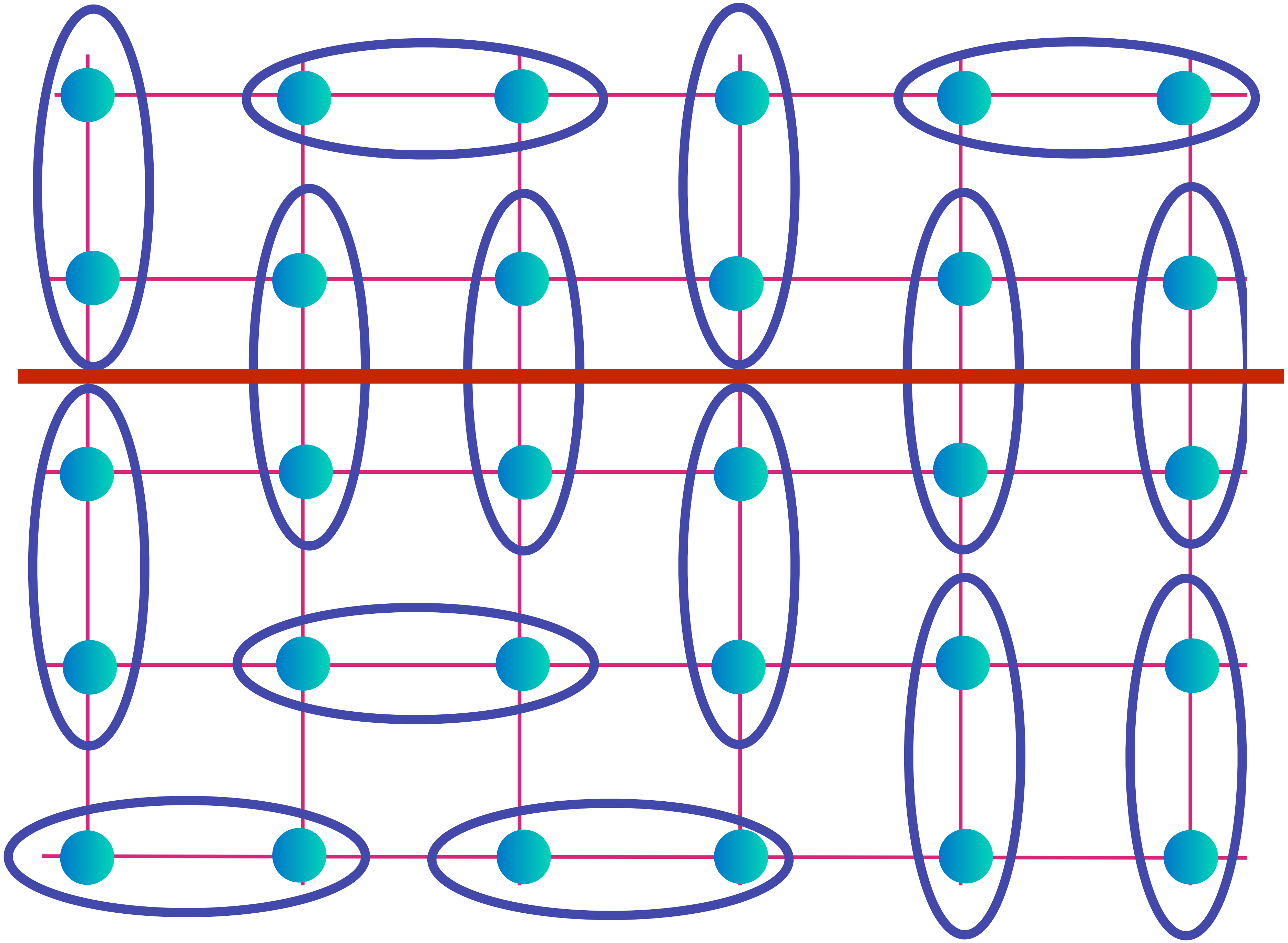

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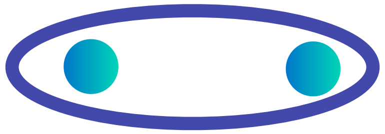

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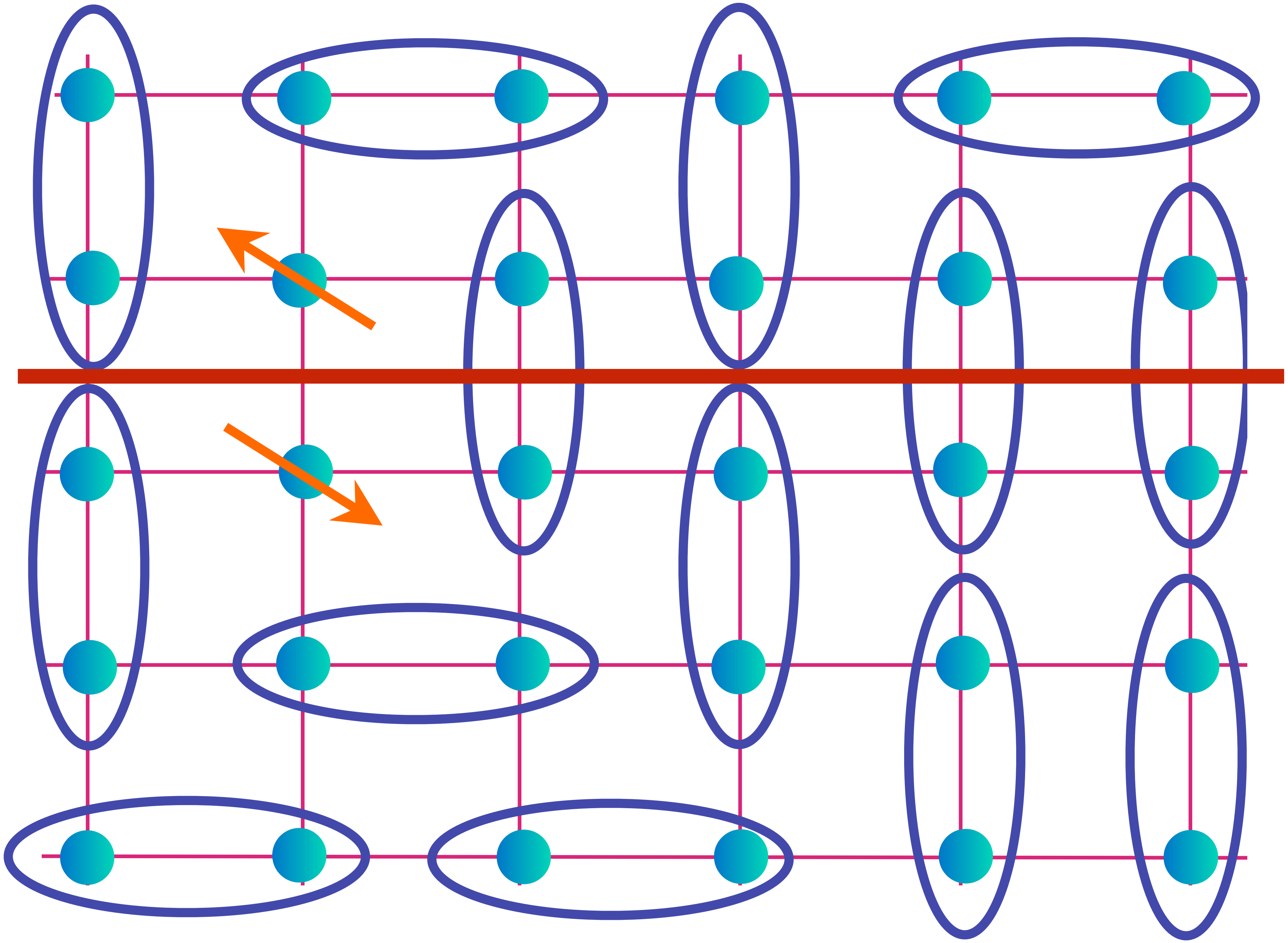

Place insulator on a torus;

to change dimer number parity across red line, it is necessary to create a pair of unpaired spins and move them around the sample.

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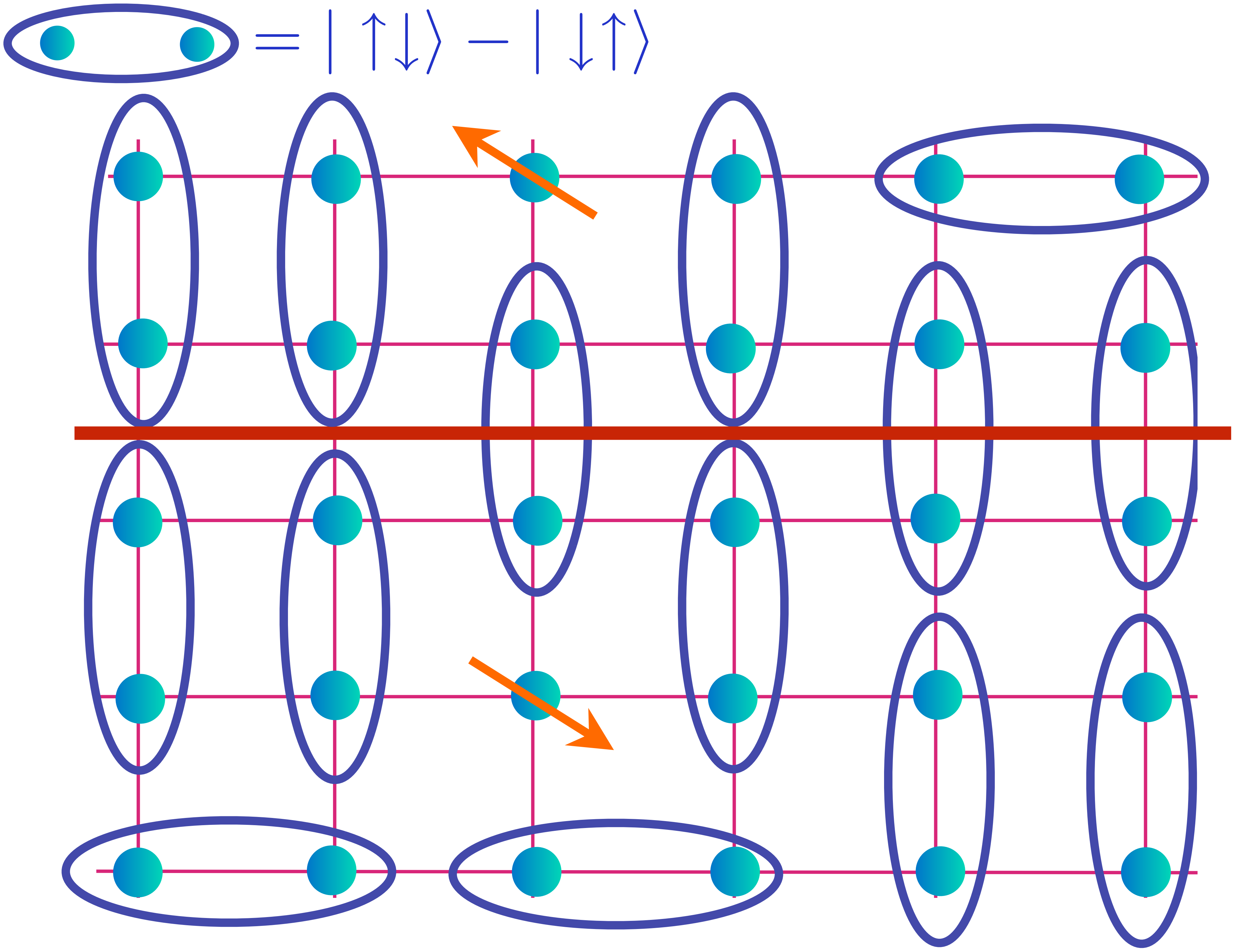

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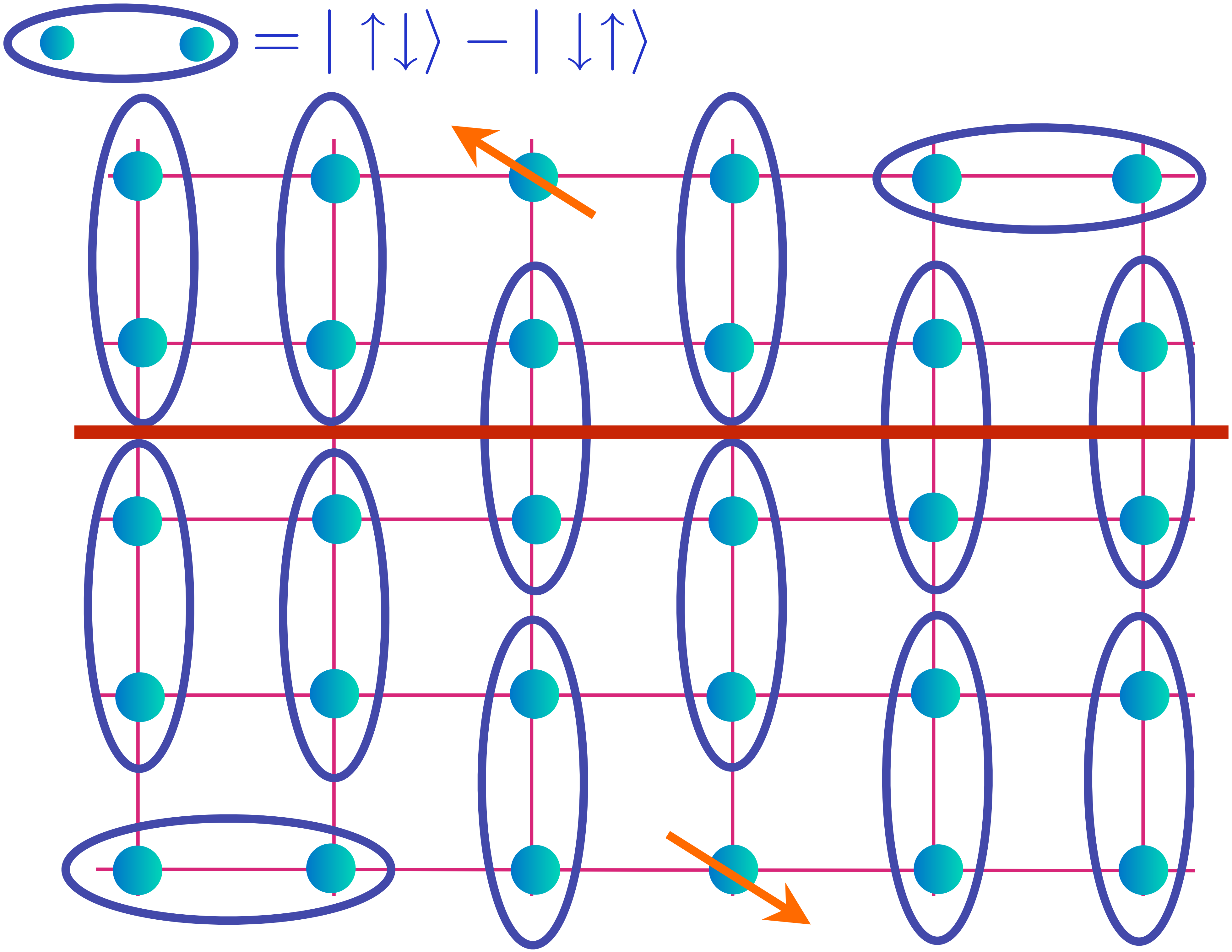

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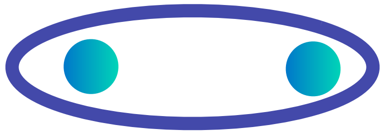

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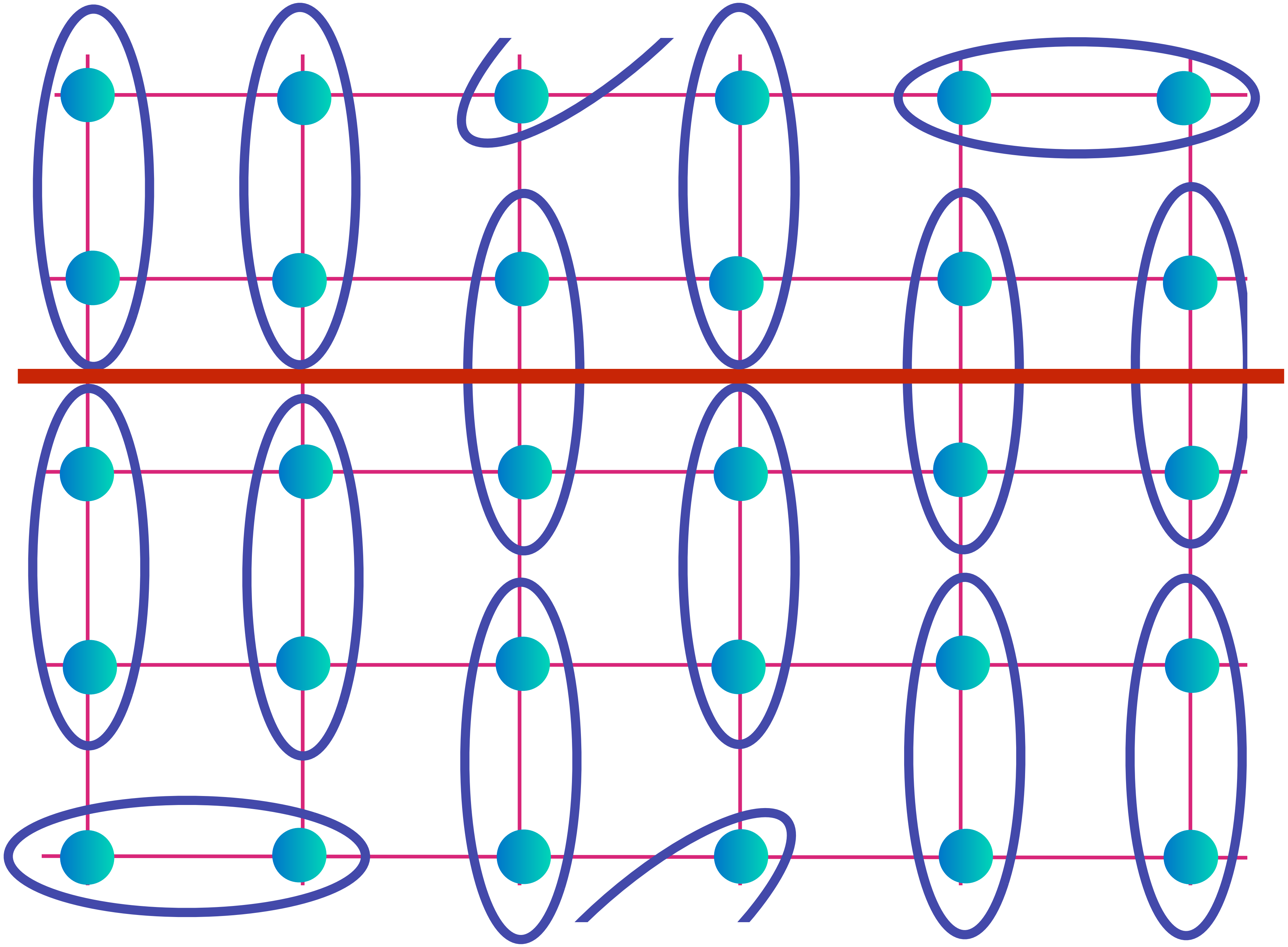

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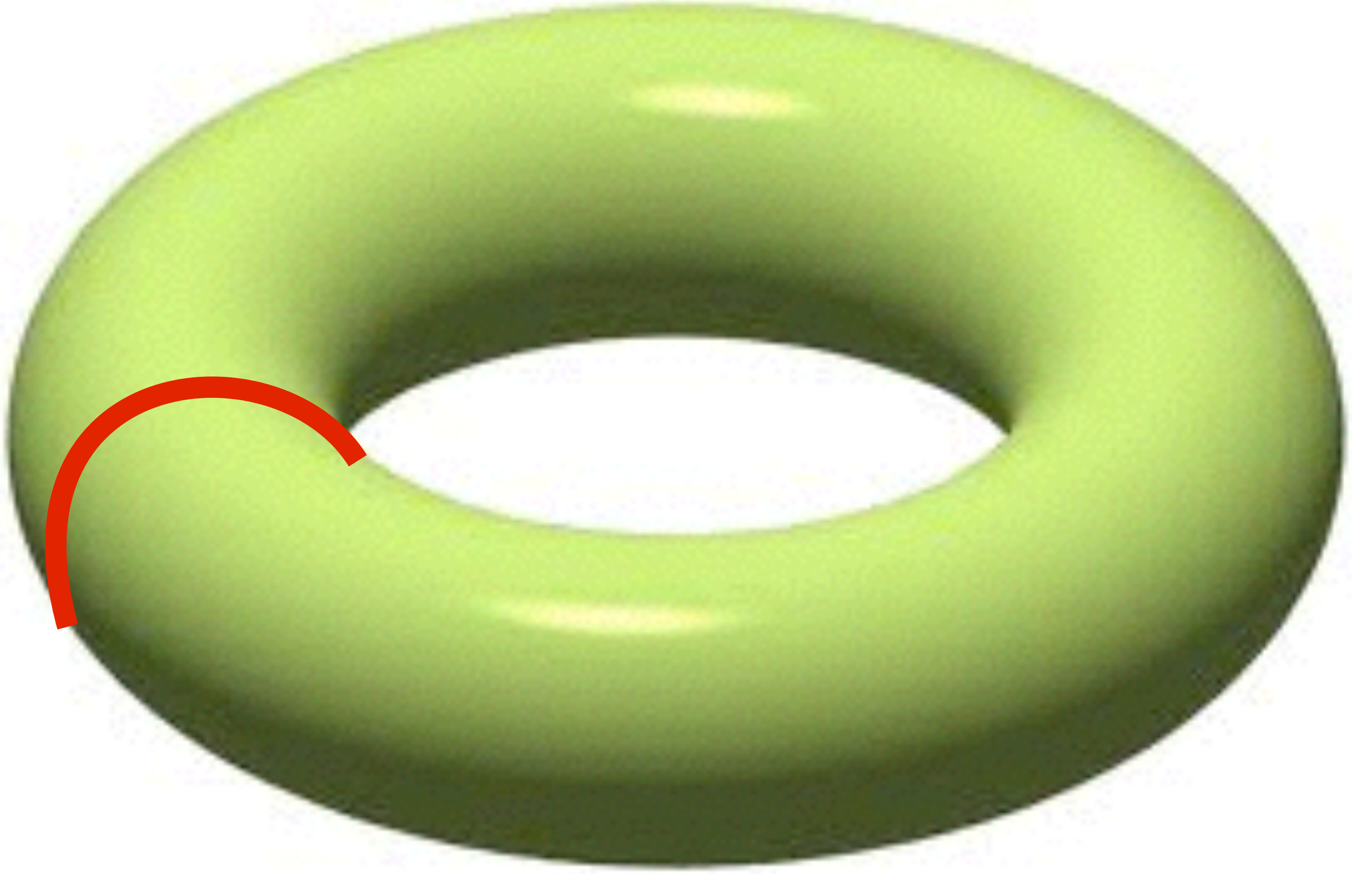

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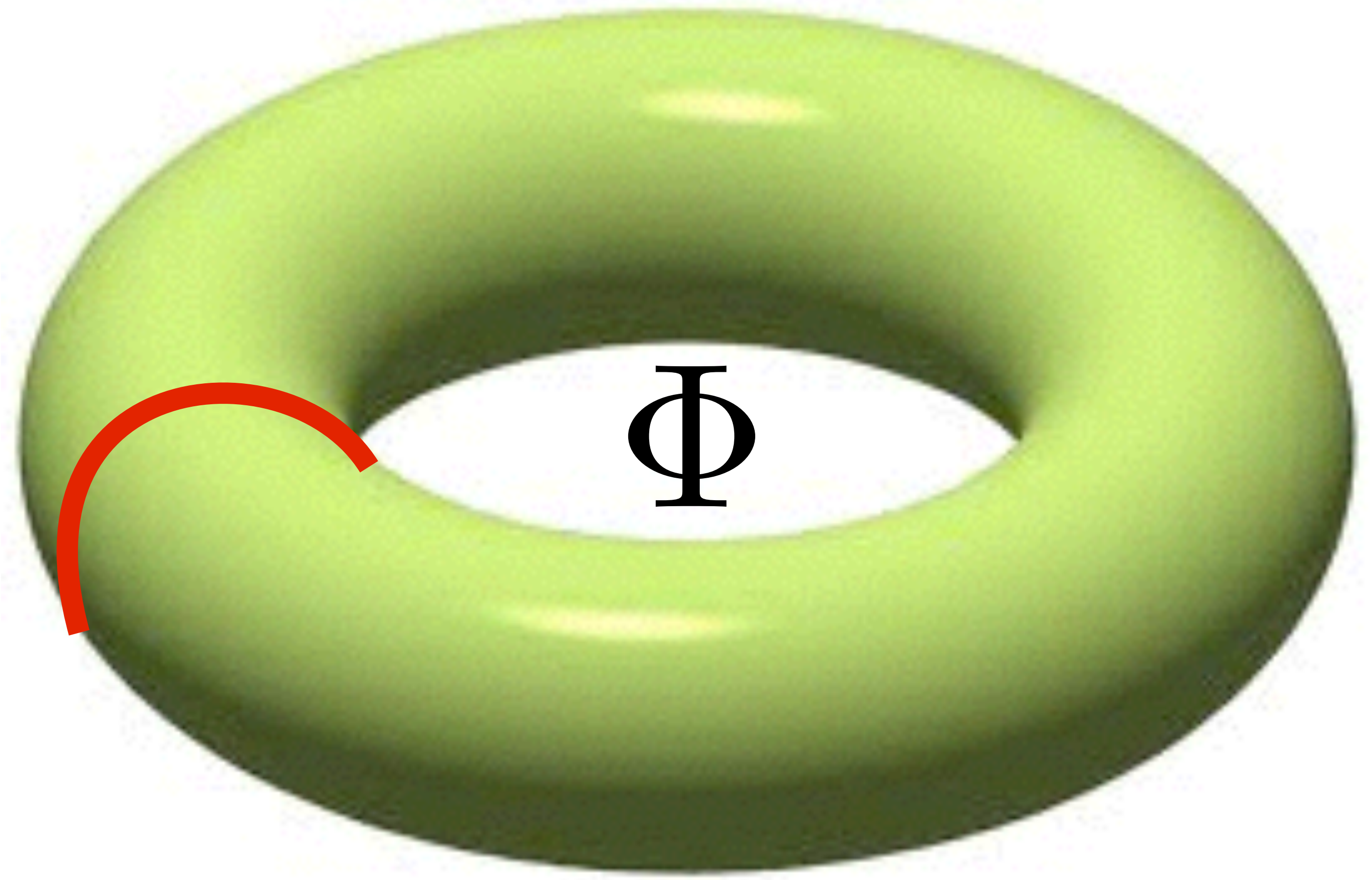
N. Read and S. Sachdev, PRL **66**, 1773 (1991); X.-G. Wen PRB **44**, 2664 (1991)

Topological order: flux insertion



Upon inserting a flux quantum $\Phi = h/e$ coupling only to $\uparrow$ electrons (say), the states with an even (odd) number of dimers crossing the cut pick up a factor of $+1$ (-1). The hole of the torus has a "*vison*" after the flux insertion.

N. E. Bonesteel, PRB **40**, 8954 (1989).

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Ordinary quantum matter: the Fermi liquid (FL)

- Fermi surface separates empty and occupied states in momentum space.

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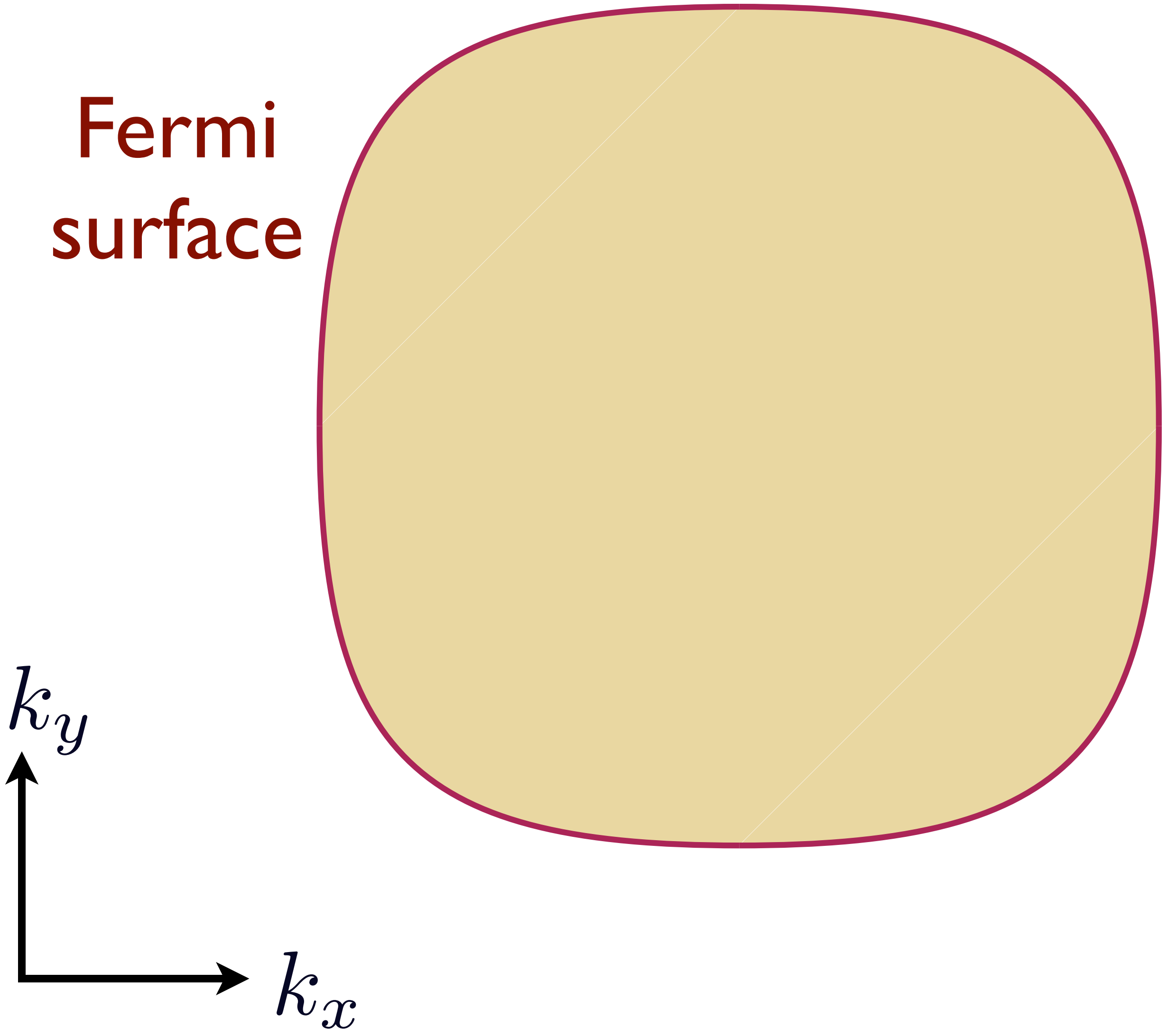

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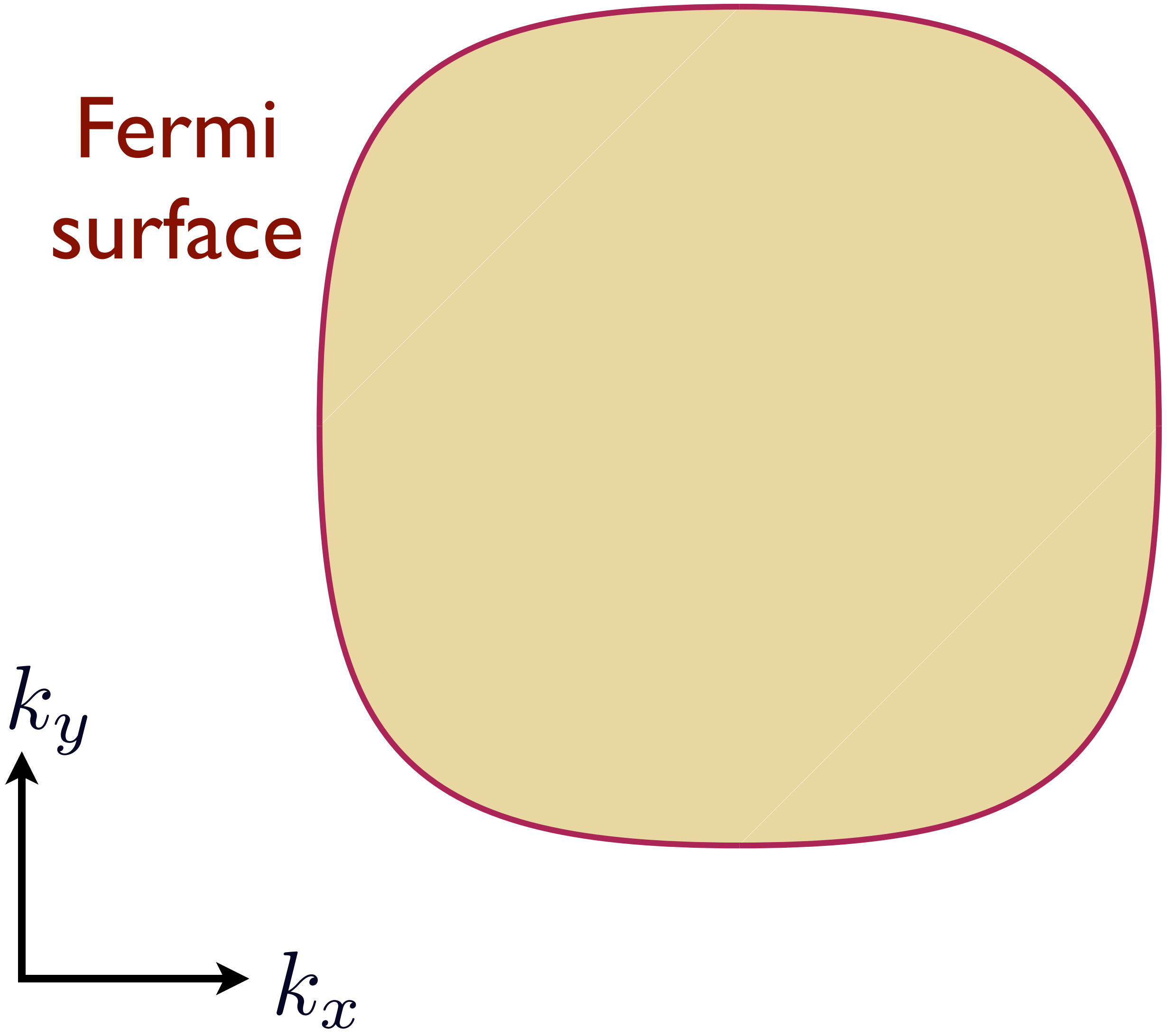
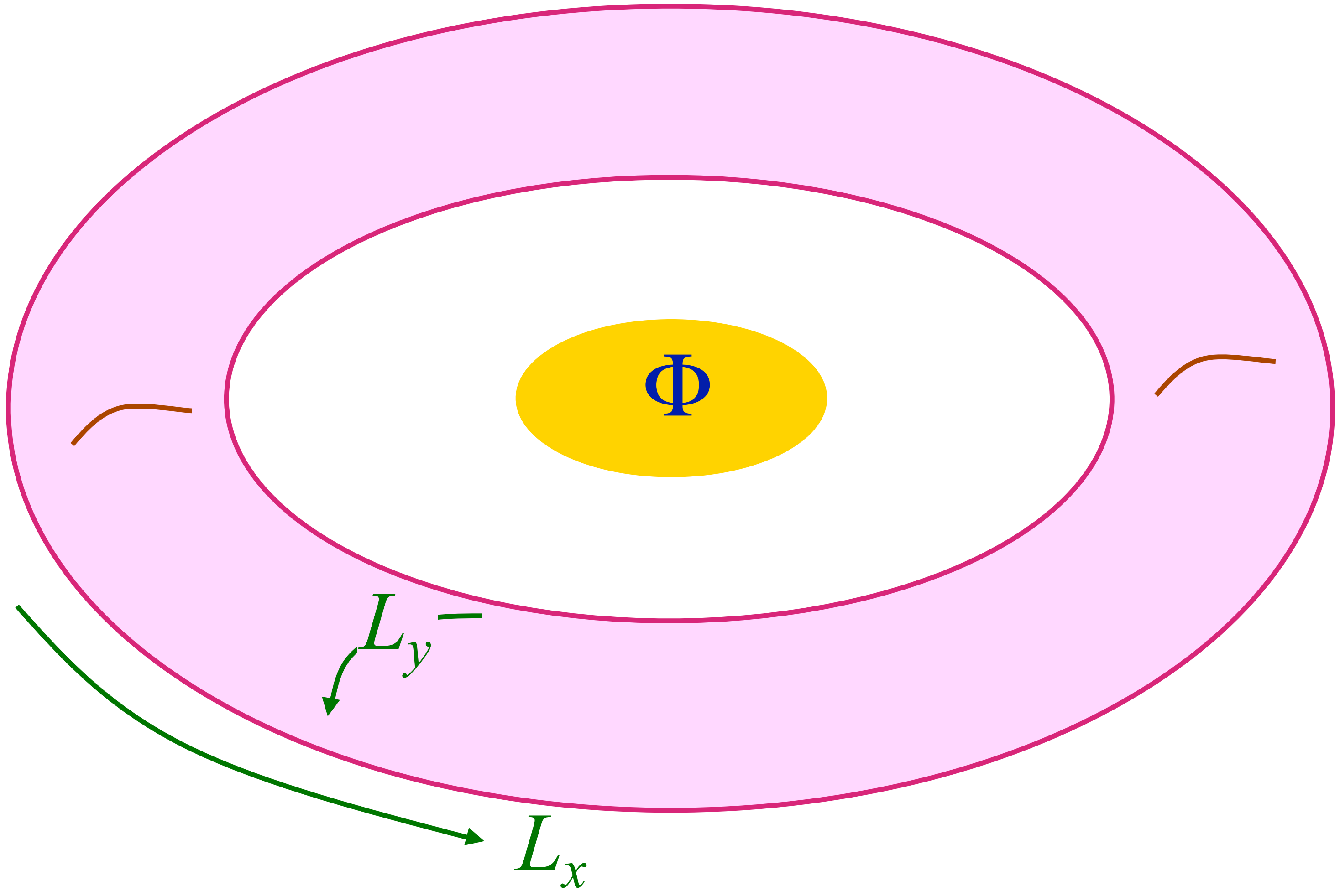


- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2)
- Density of electrons can be continuously varied at zero temperature.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

Oshikawa's non-perturbative proof of the Luttinger theorem

M. Oshikawa, PRL **84**, 3370 (2000)
A. Paramekanti and A. Vishwanath,
PRB **70**, 245118 (2004)



We take N particles, each with charge Q, on a $L_x \times L_y$ lattice on a torus. We pierce flux $\Phi = hc/Q$ through a hole of the torus. An exact computation shows that the change in crystal momentum of the many-body state due to flux piercing is

$$\boxed{P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\text{mod } 2\pi) = 2\pi \nu L_y (\text{mod } 2\pi)}$$

where $\nu = N/(L_x L_y)$ is the density.

Oshikawa’s non-perturbative proof of the Luttinger theorem

Proof of

$$P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\mathrm{mod}\ 2\pi) = 2\pi\nu L_y (\mathrm{mod}\ 2\pi).$$

The initial and final Hamiltonians are related by a gauge transformation

$$\mathcal{U}_G H_f \mathcal{U}_G^{-1} = H_i \quad , \quad \mathcal{U}_G = \exp\left(i \frac{2\pi}{L_x} \sum_i x_i \hat{n}_i \right) .$$

while the wavefunction evolves from $|\Psi_i\rangle$ to $\mathcal{U}_T |\Psi_i\rangle$, where $\mathcal{U}_T$ is the time evolution operator. We want to work in a fixed gauge in which the initial and final Hamiltonians are the same: in this gauge, the final state is $|\Psi_f\rangle = \mathcal{U}_G \mathcal{U}_T |\Psi_i\rangle$. Let $\hat{T}_x$ be the lattice translation operator. Then we can show that

$$\hat{T}_x |\Psi_i\rangle = e^{-iP_{xi}} |\Psi_i\rangle \quad , \quad \hat{T}_x |\Psi_f\rangle = e^{-iP_{xf}} |\Psi_f\rangle ,$$

using the easily established properties

$$\hat{T}_x \mathcal{U}_T = \mathcal{U}_T \hat{T}_x \quad , \quad \hat{T}_x \mathcal{U}_G = \exp\left(-i2\pi \frac{N}{L_x} \right) \mathcal{U}_G \hat{T}_x$$

Oshikawa's non-perturbative proof of the Luttinger theorem

$$\boxed{\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)}$$

Now we compute the momentum balance *assuming* that the only low energy excitations are quasiparticles near the Fermi surface, and these react like free particles to a sufficiently slow flux insertion. Then we can write

$$\Delta P_x = \left(\frac{2\pi}{L_x}\right)\frac{L_x L_y}{4\pi^2} V_{\rm FS} \quad , \quad \Delta P_y = \left(\frac{2\pi}{L_y}\right)\frac{L_x L_y}{4\pi^2} V_{\rm FS}$$

where $V_{\rm FS}$ is the volume of the Fermi surface. Actually, the quasiparticles are only defined near the Fermi surface, but by using Gauss's Law on the momentum acquired by quasiparticles near the Fermi surface, we can convert the answer to an integral over the volume enclosed by the Fermi surface, as shown above.

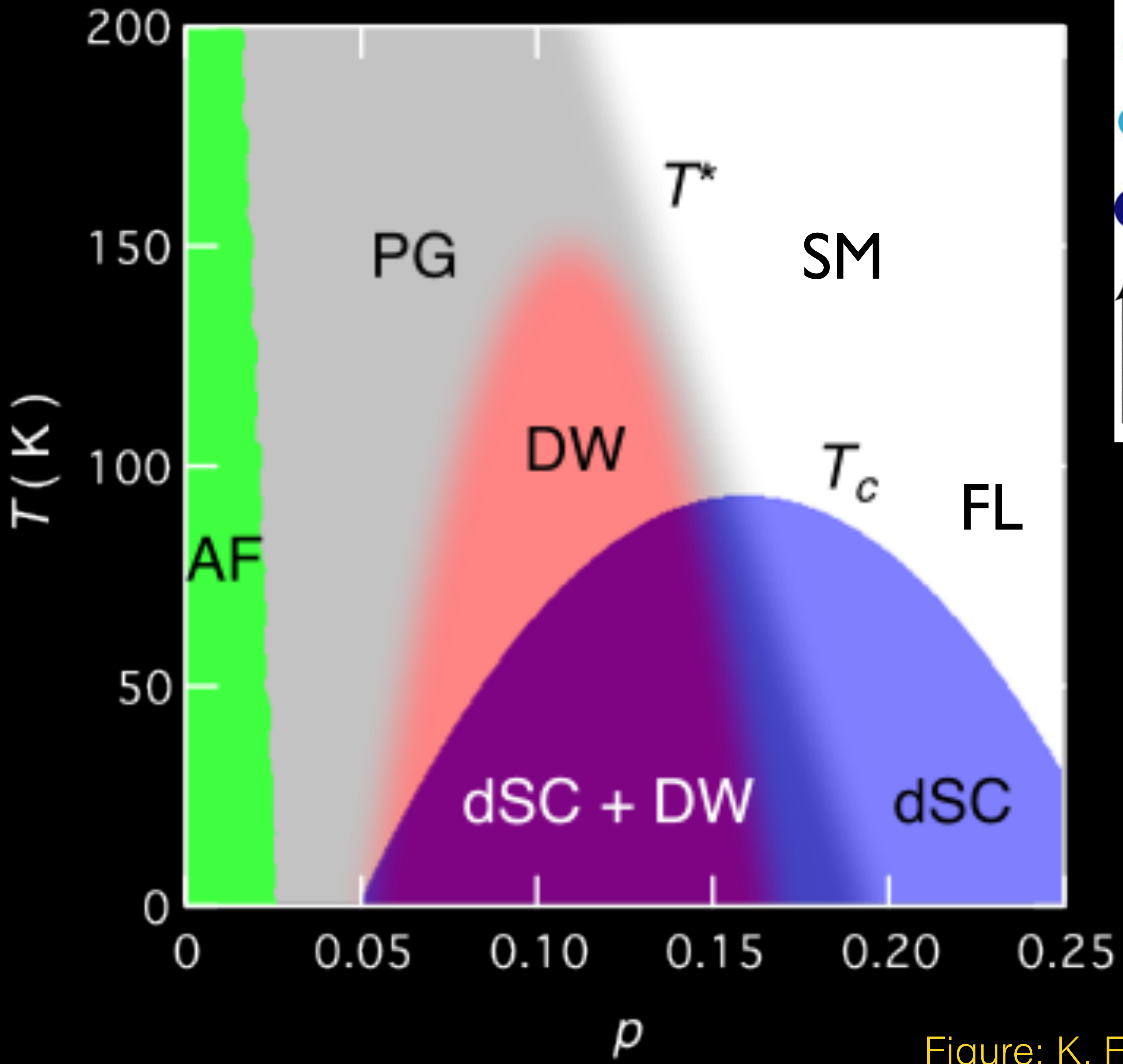
Now we equate these values to those obtained above, and obtain

$$N - L_x L_y \frac{V_{\rm FS}}{4\pi^2} = L_x m_x \quad , \quad N - L_x L_y \frac{V_{\rm FS}}{4\pi^2} = L_y m_y$$

for some integers m_x, m_y. By choosing L_x, L_y mutually prime integers we can now show

$$\nu = \boxed{\frac{N}{L_x L_y} = \frac{V_{\rm FS}}{4\pi^2} + m}$$

for some integer m: this is Luttinger's theorem.



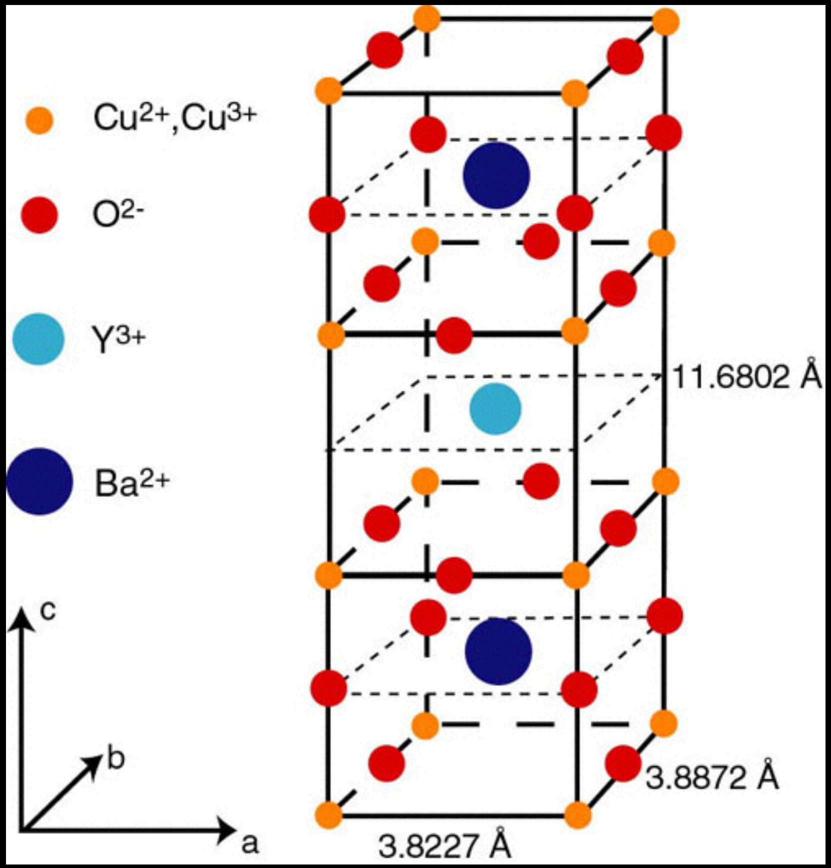


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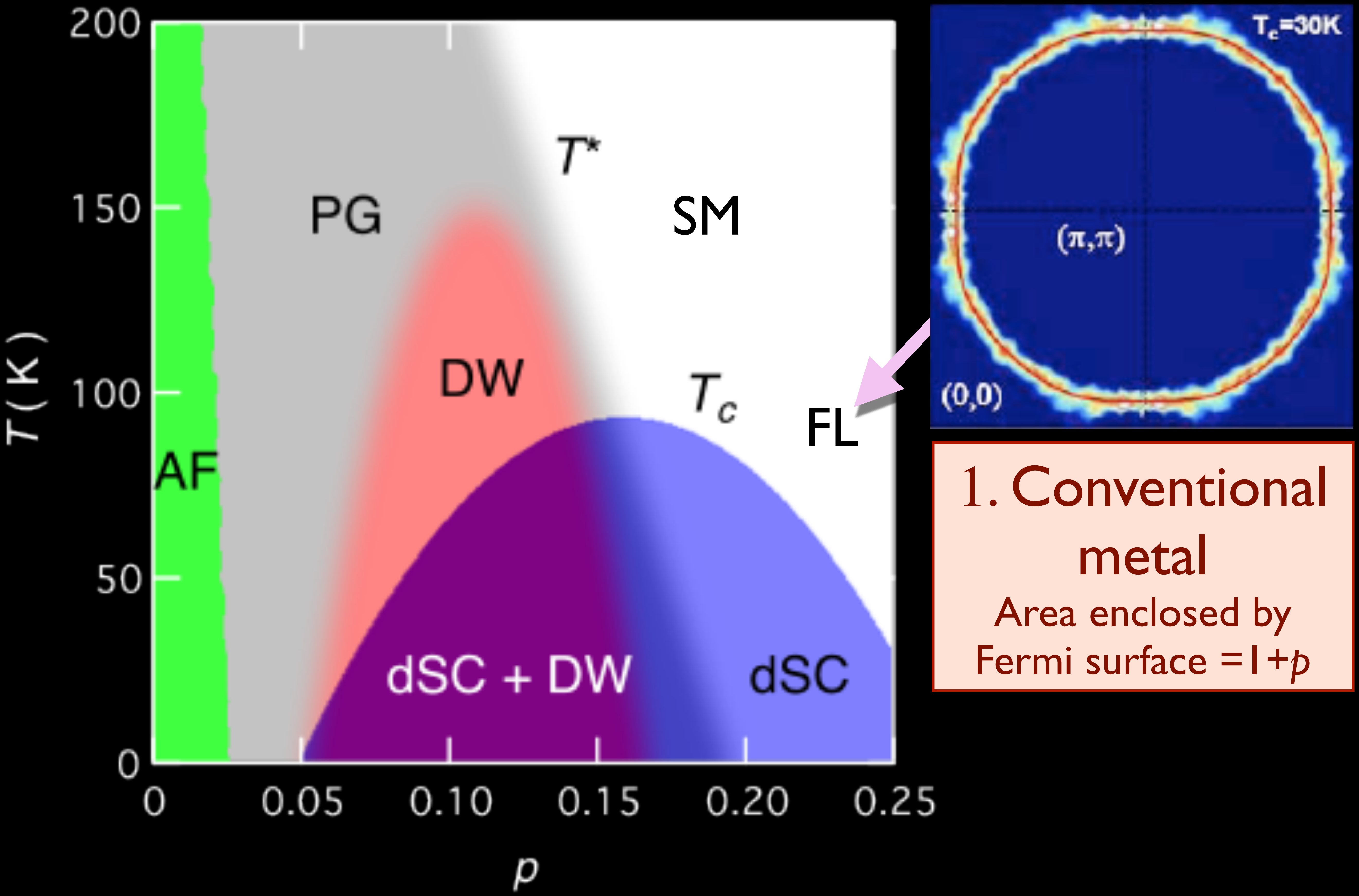
Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)



1. Conventional metal

Area enclosed by Fermi surface =1+p

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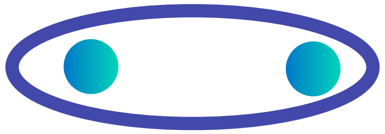
Insulating spin liquid



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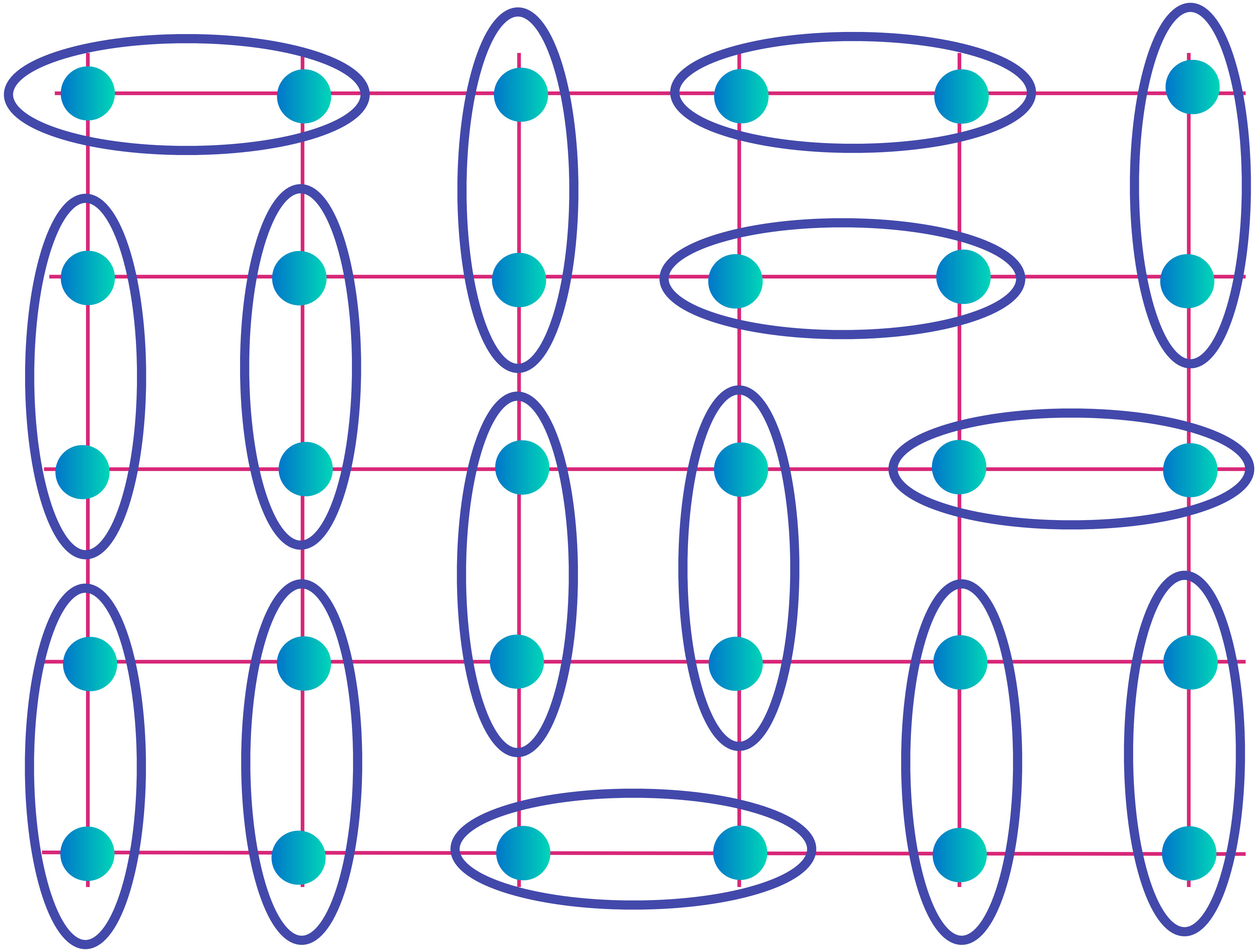

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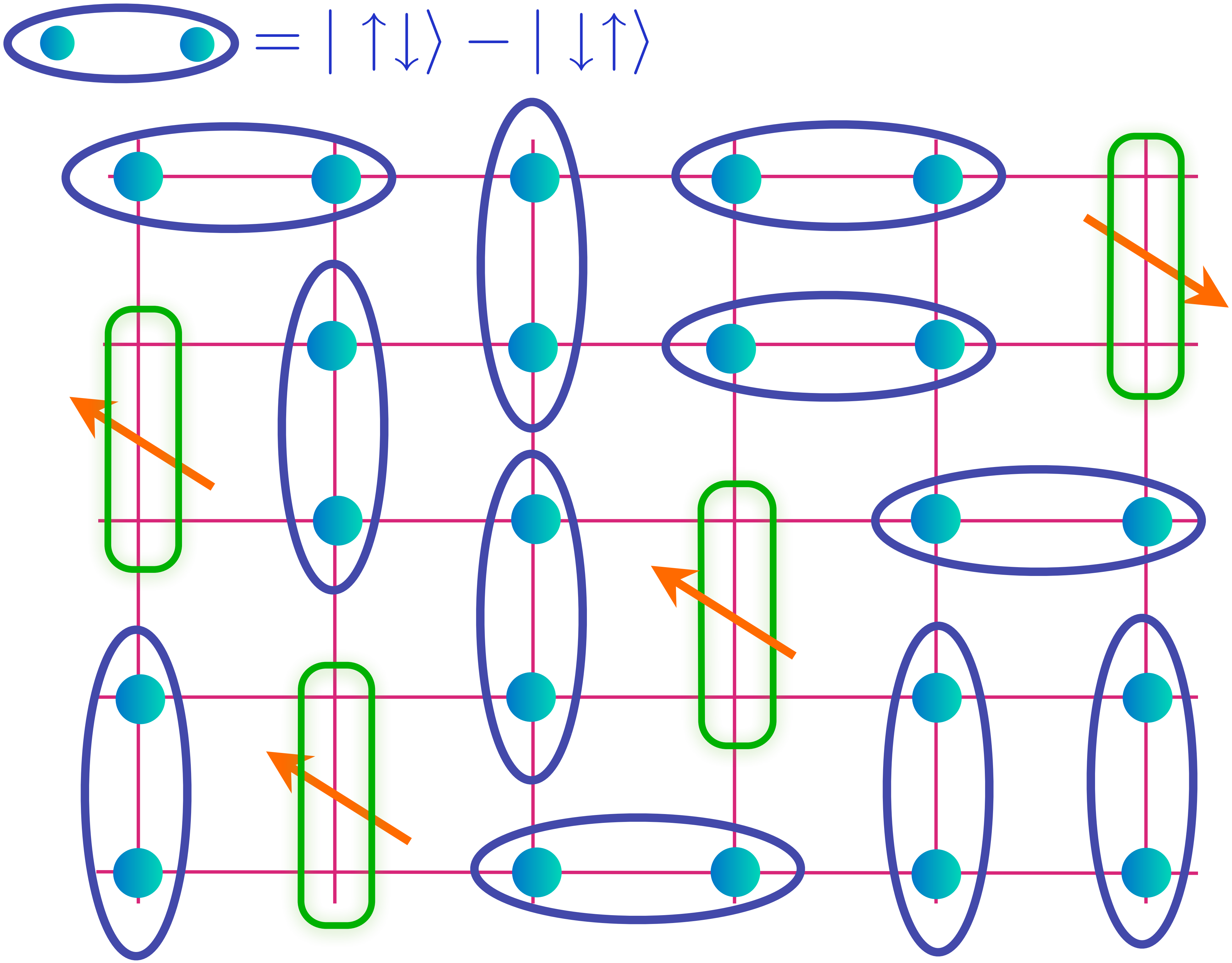
Fractionalized Fermi liquid (FL*)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Realizes a metal with a Fermi surface of area p co-existing with "topological order"

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

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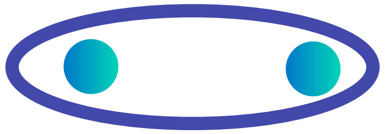

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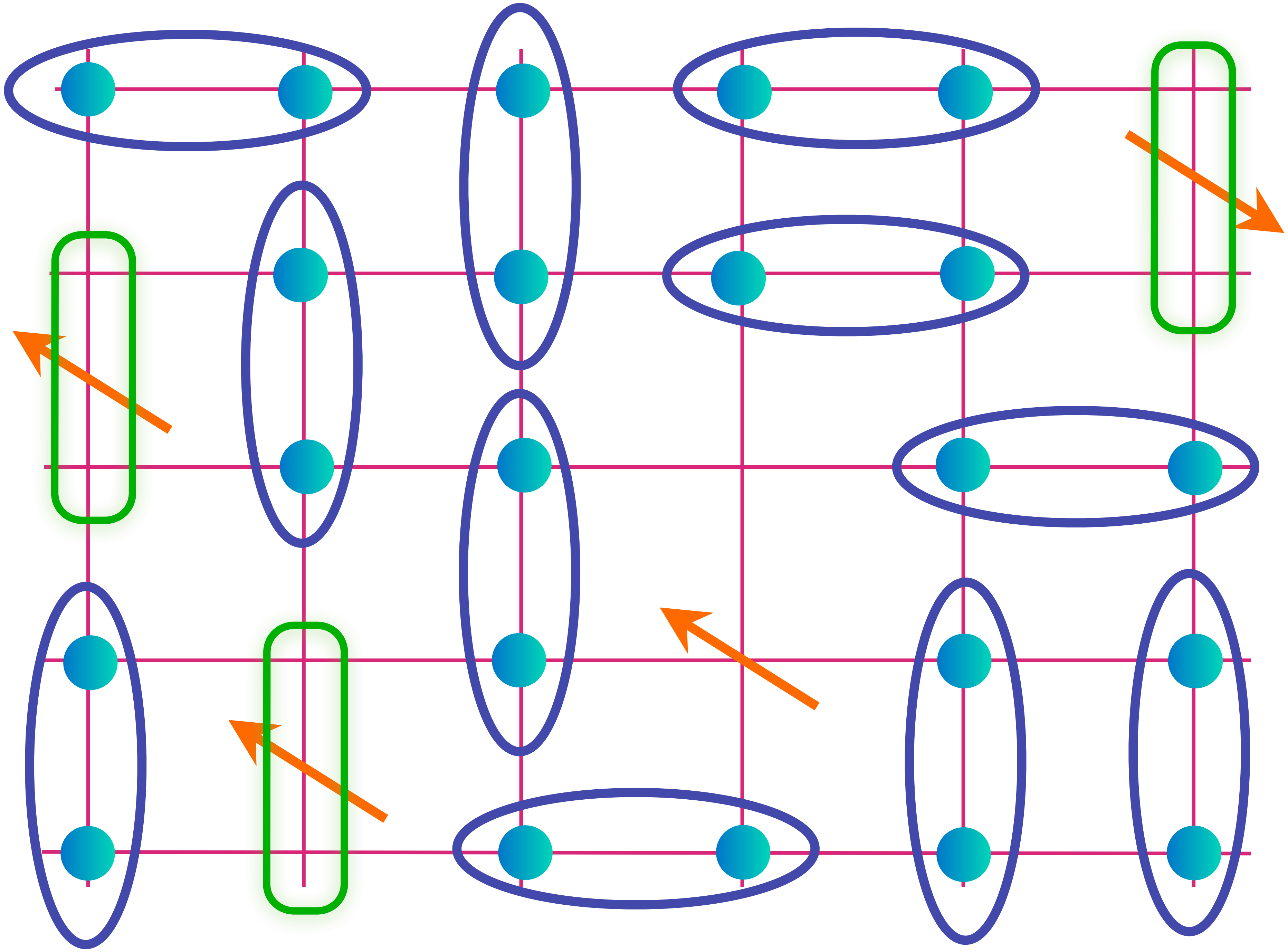

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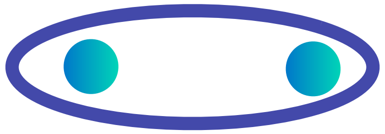

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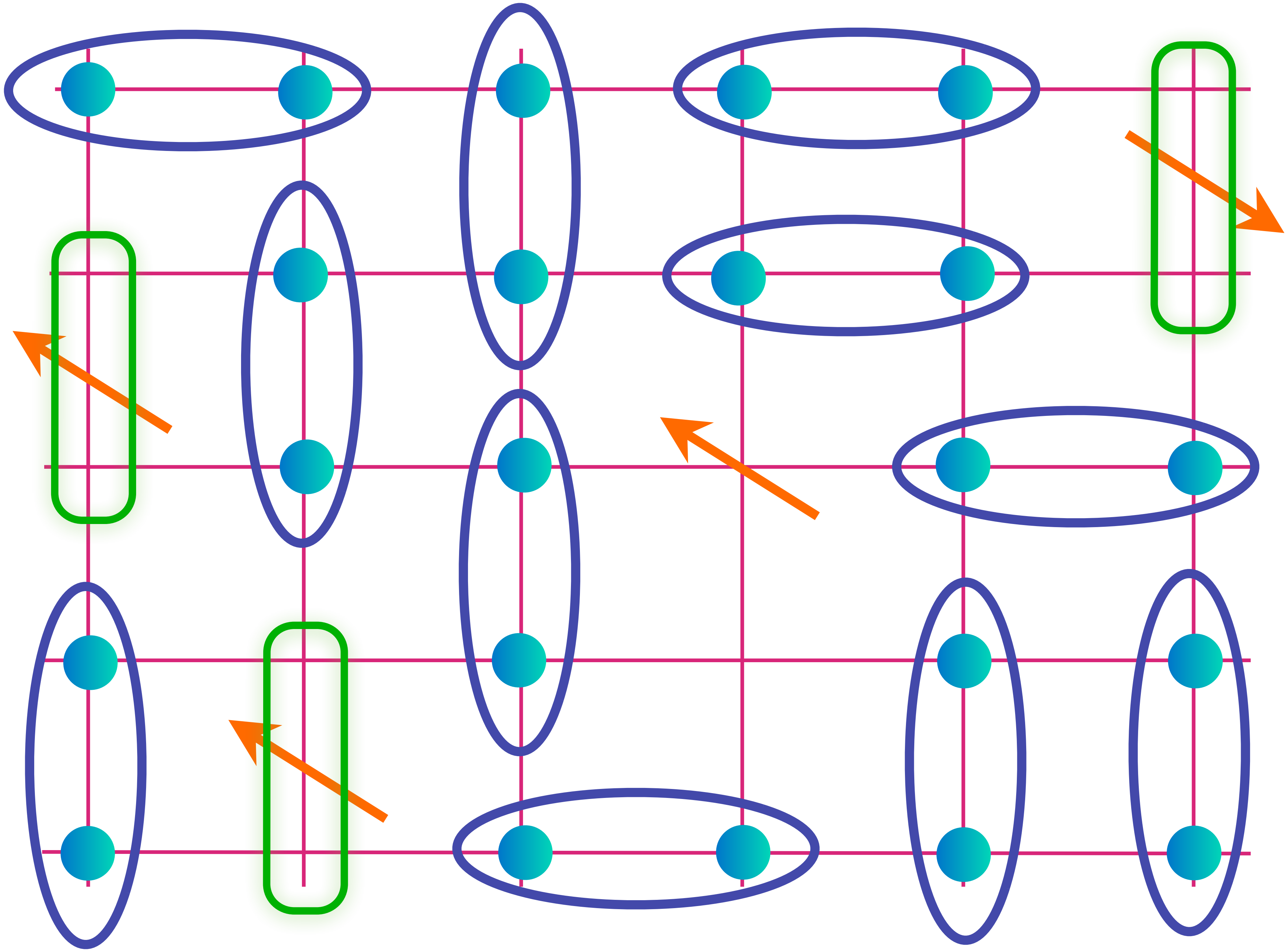
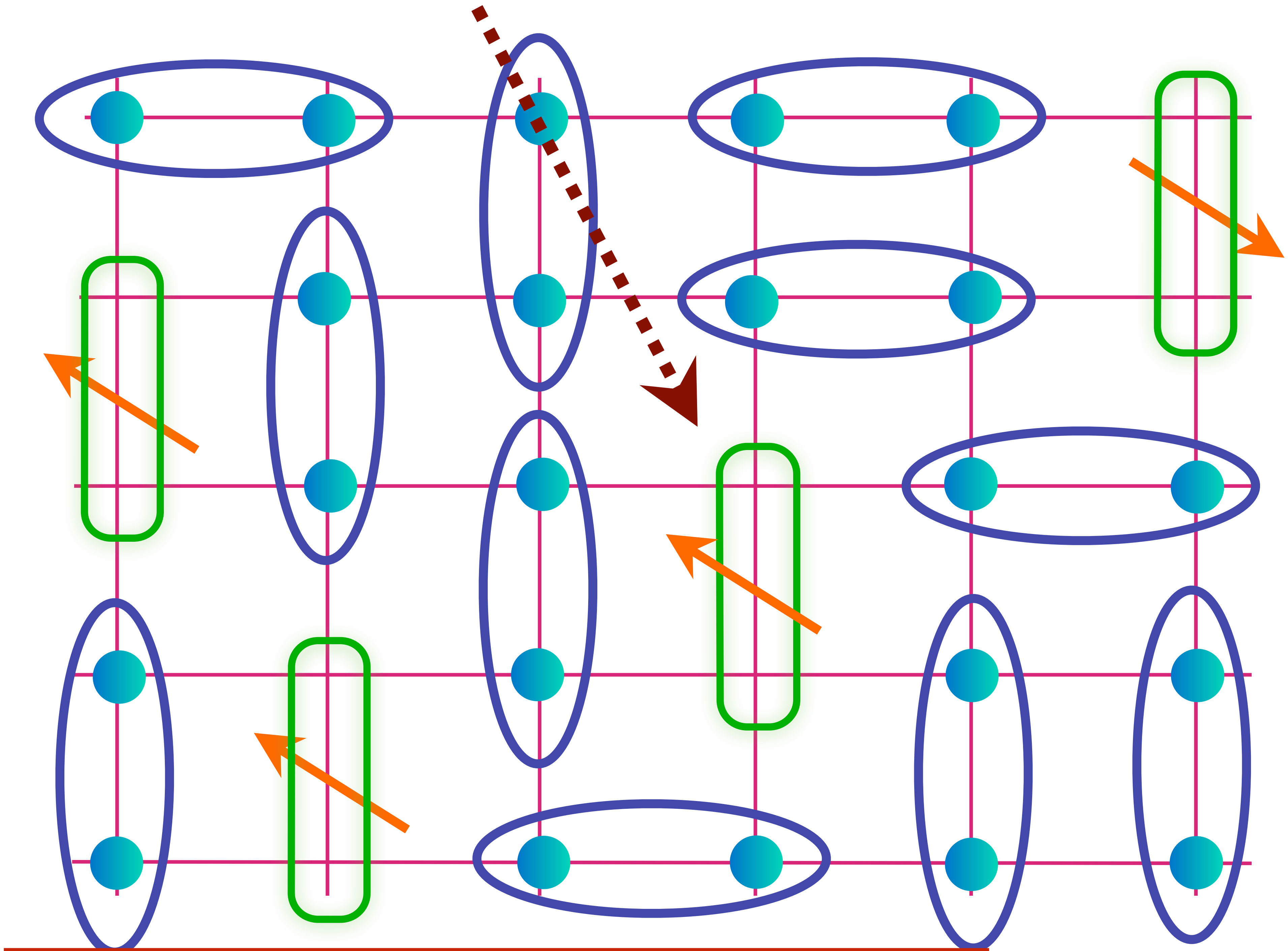


Realizes a metal with a Fermi surface of area p co-existing with "topological order"

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

A fermionic "dimer" describing a "bonding" orbital between two sites

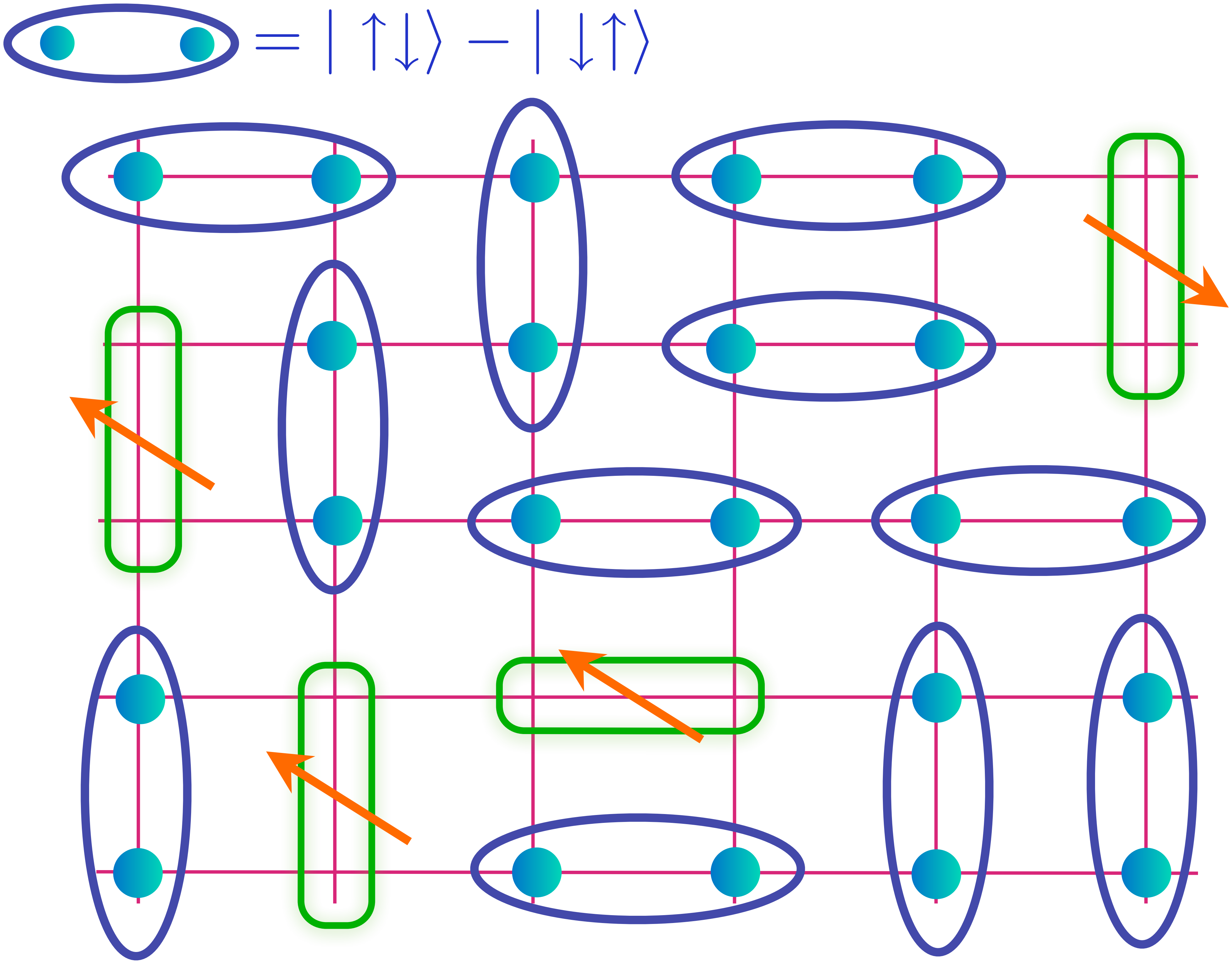


Realizes a metal with a Fermi surface of area p co-existing with "topological order"

Density of fermionic dimers = p;
density of holes relative to filled band = $1 + p$

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Fractionalized Fermi liquid (FL*)



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M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

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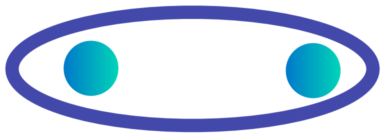

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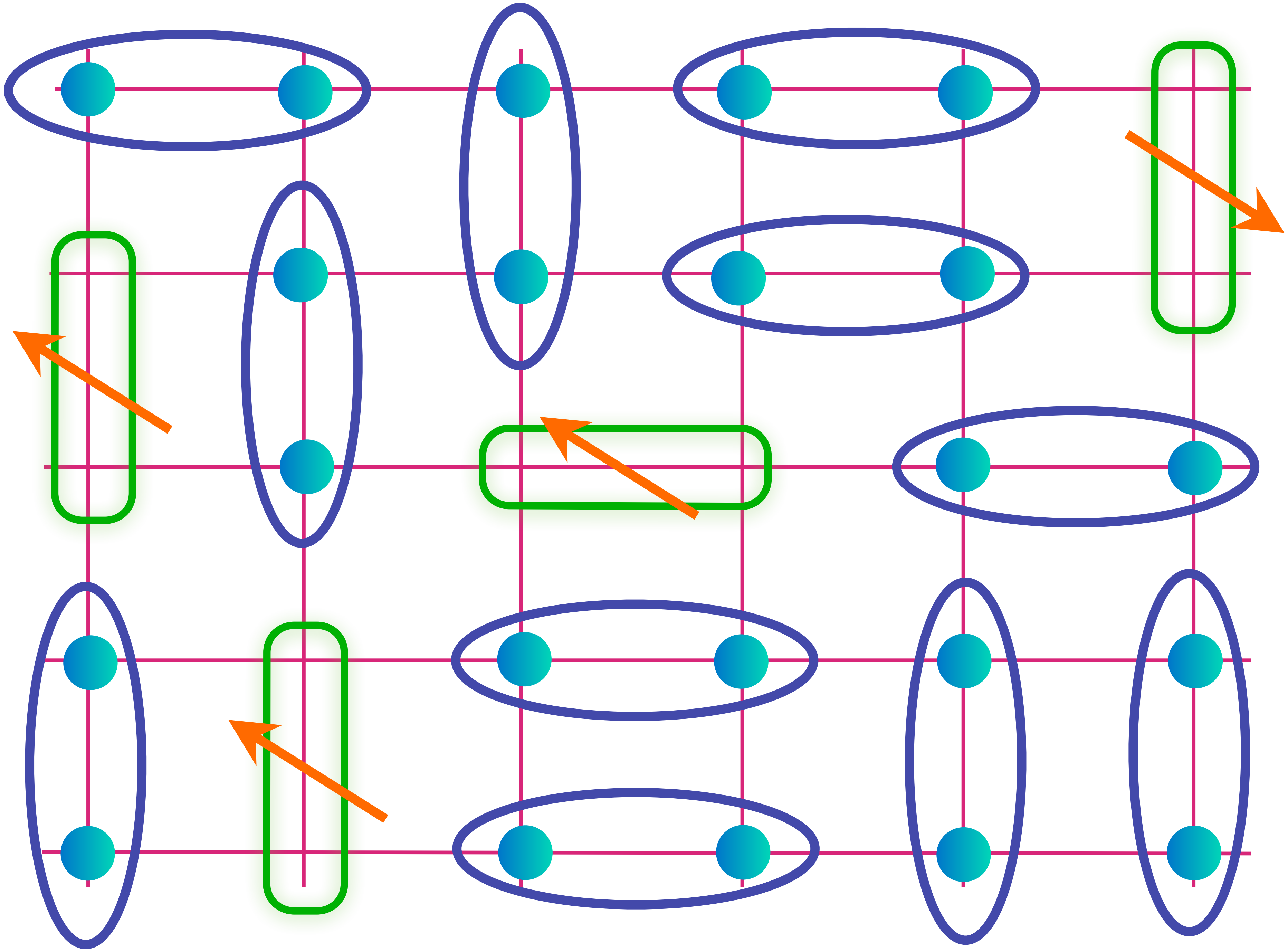

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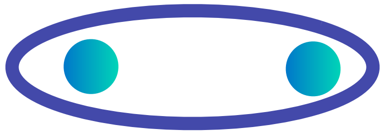

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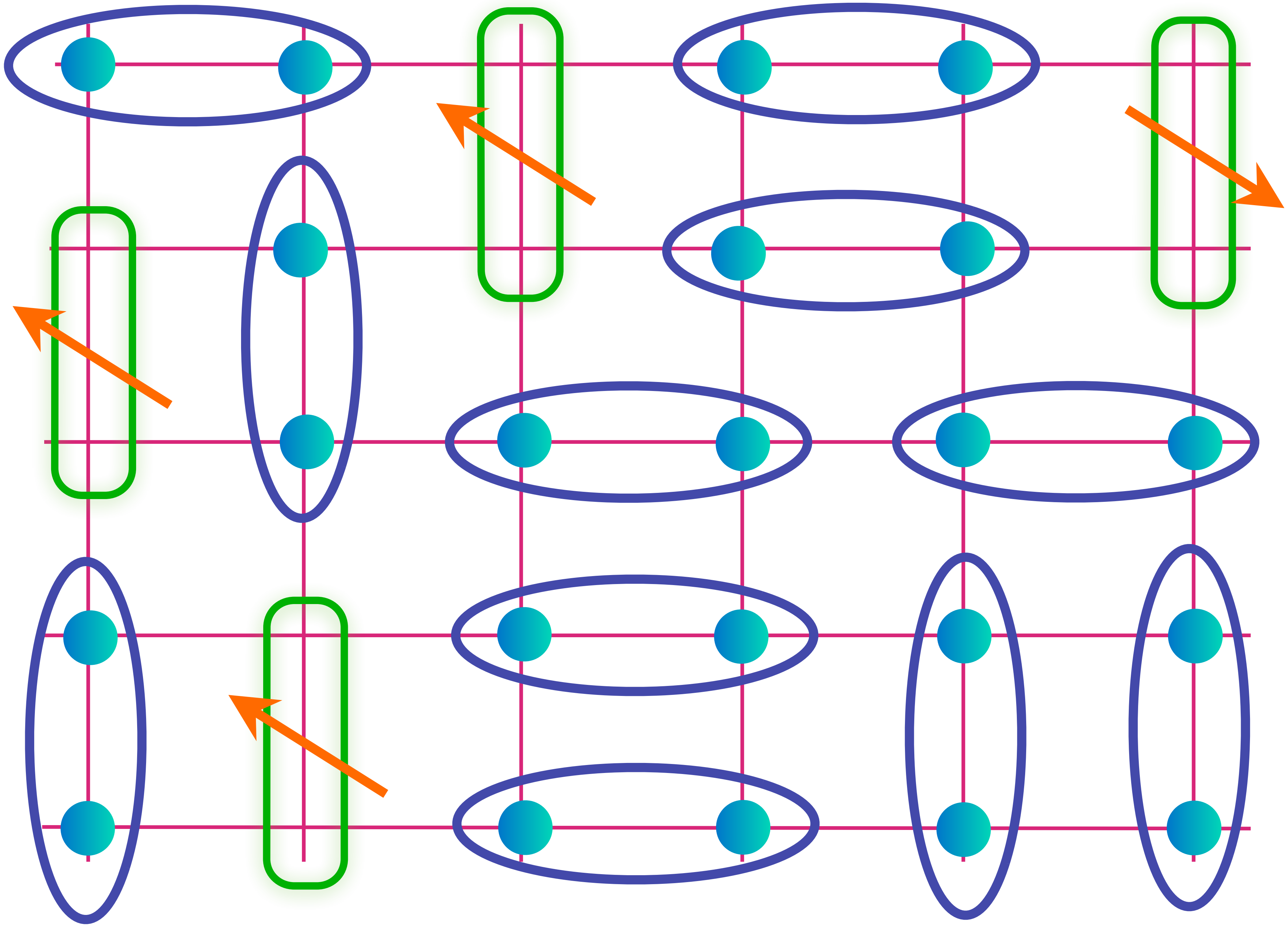

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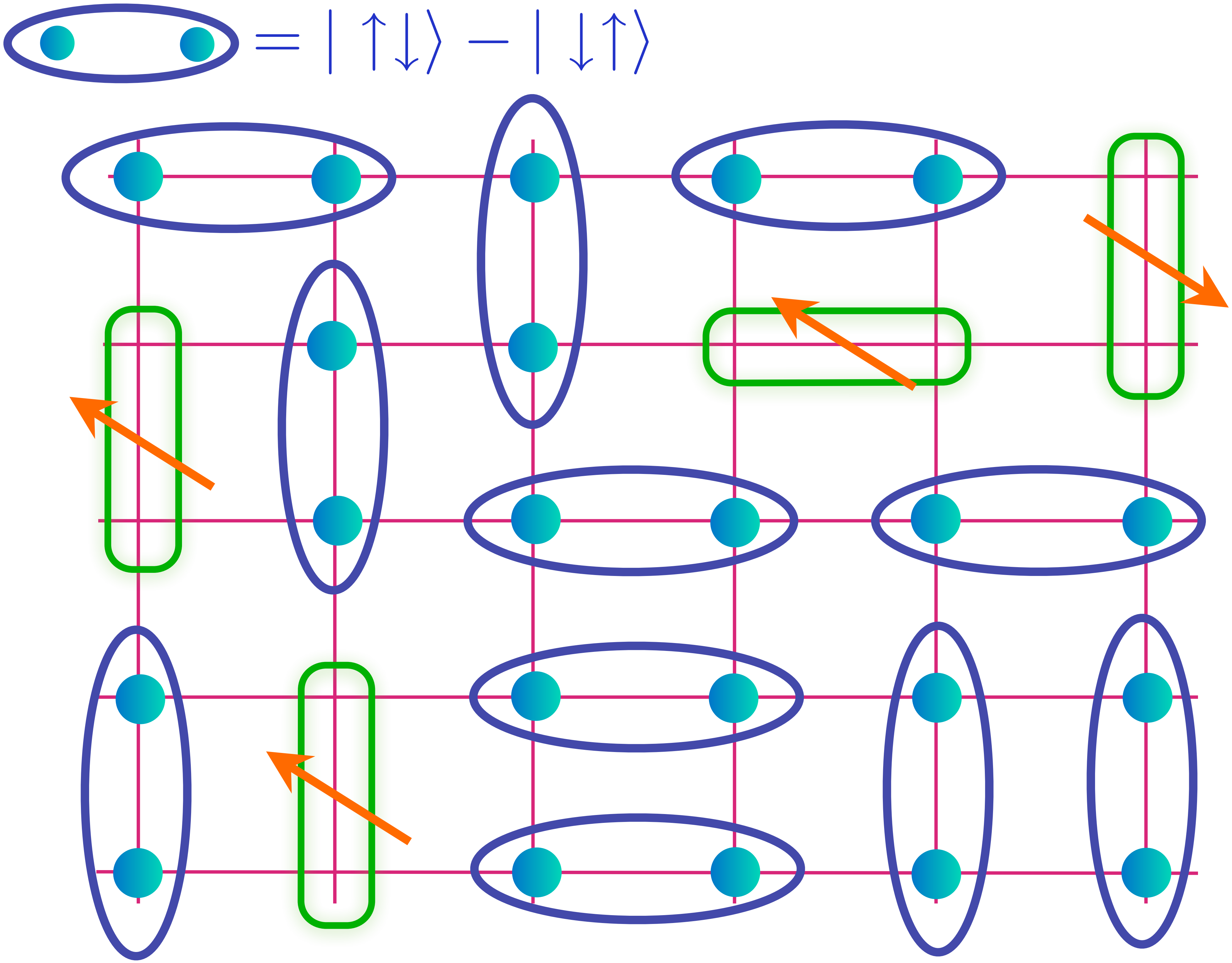

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M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

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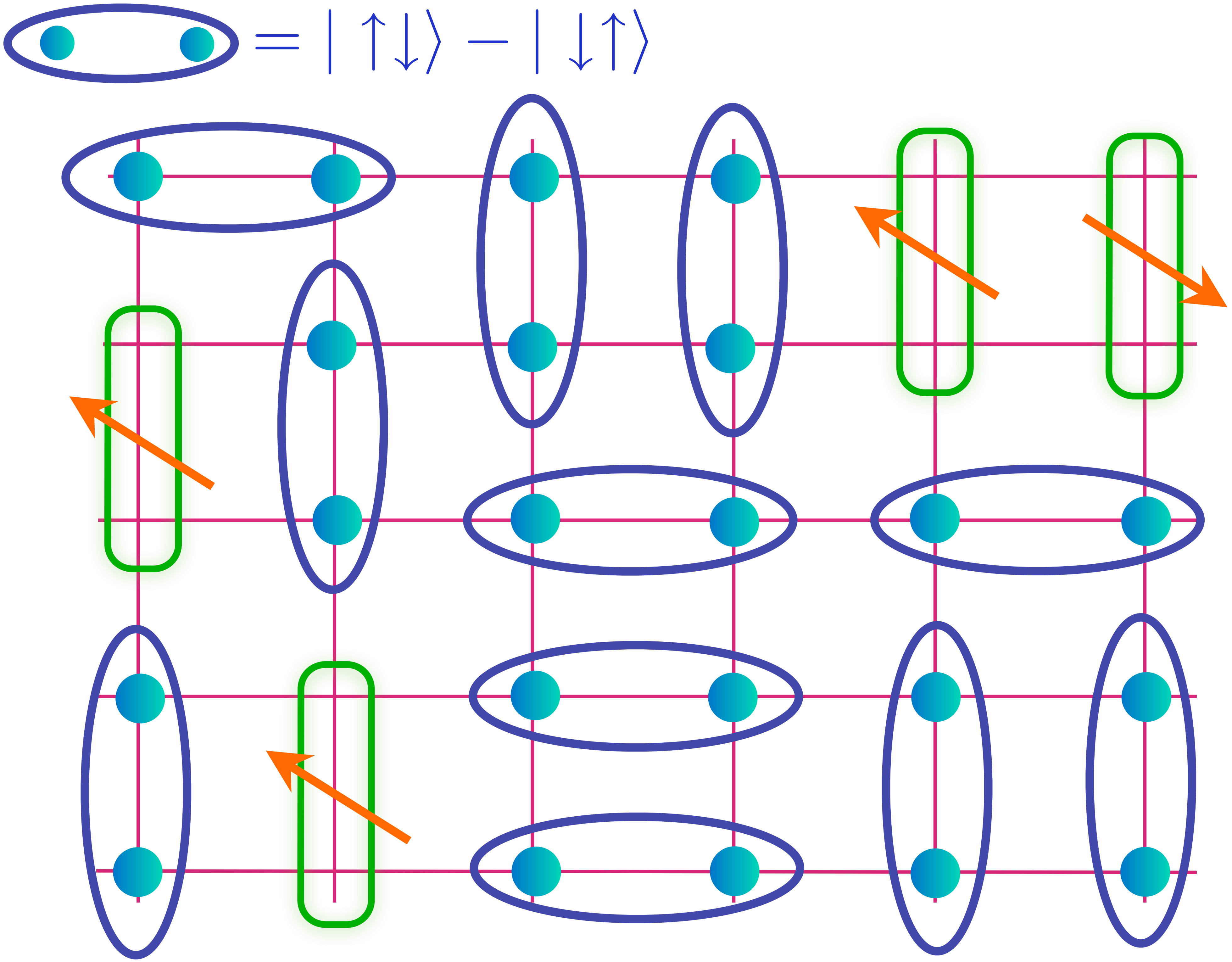

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Realizes a metal with a Fermi surface of area p co-existing with "topological order"

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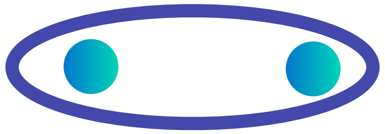
Fractionalized Fermi liquid (FL*)



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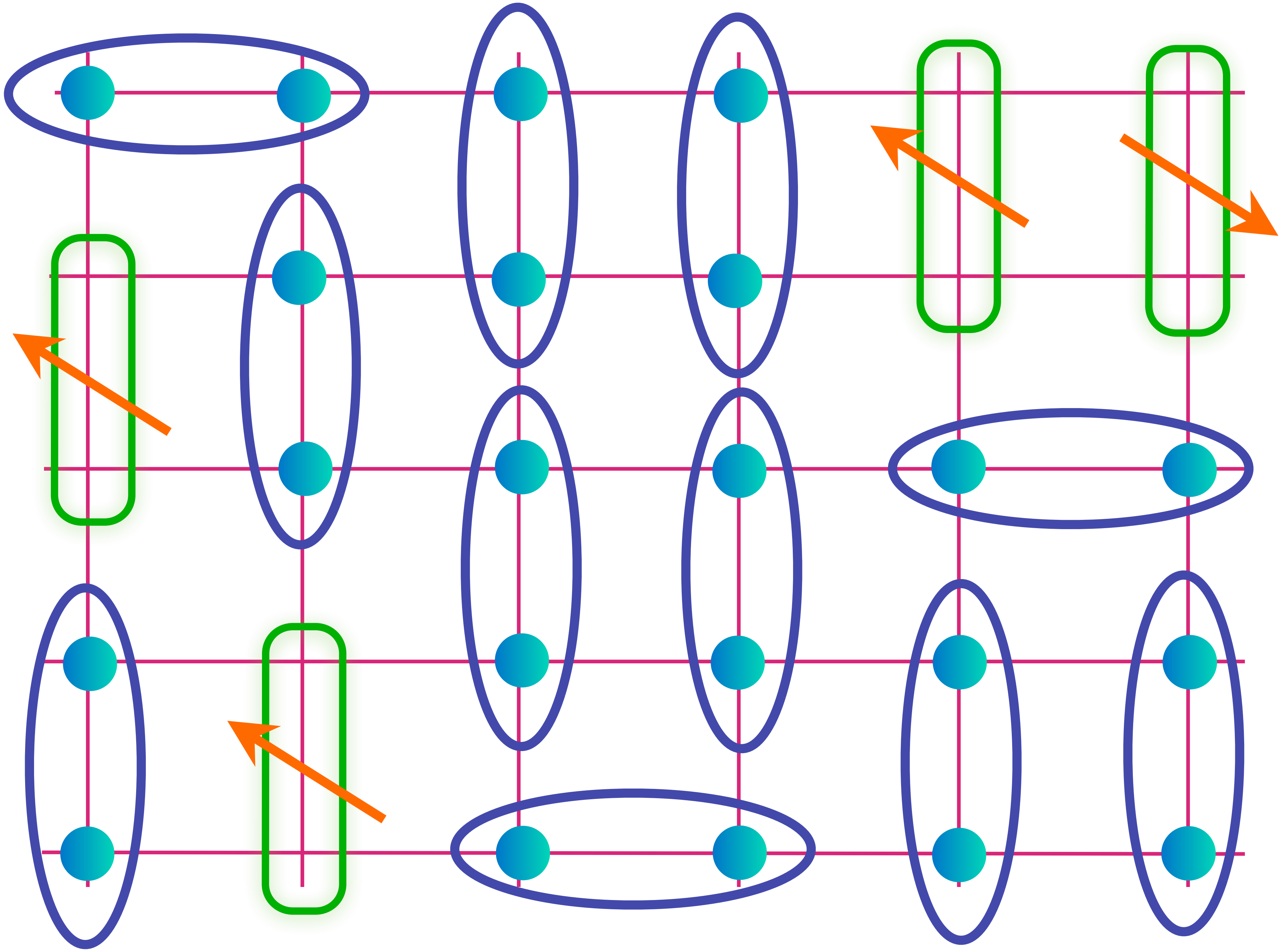

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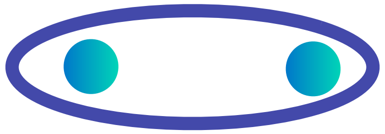

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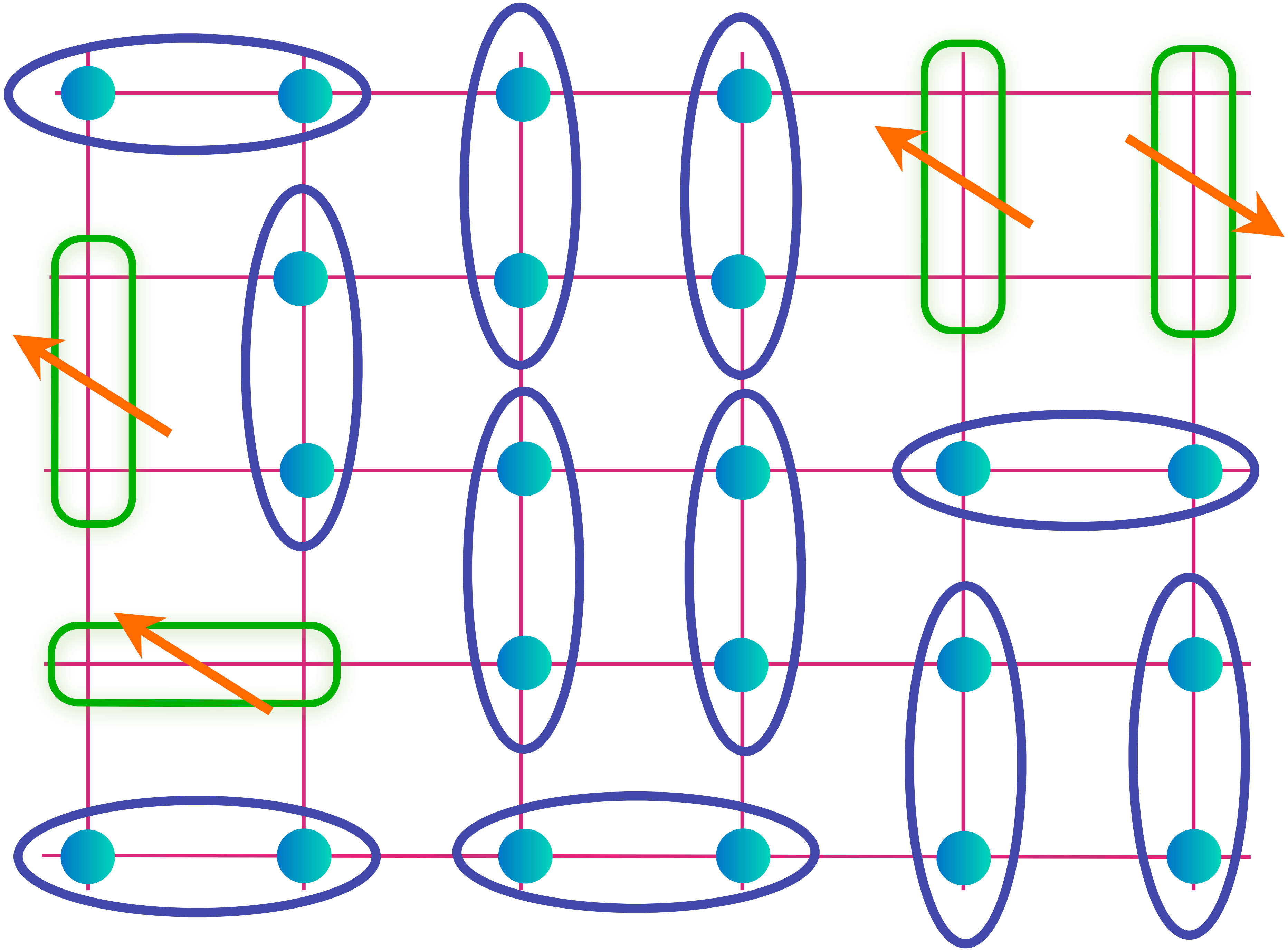


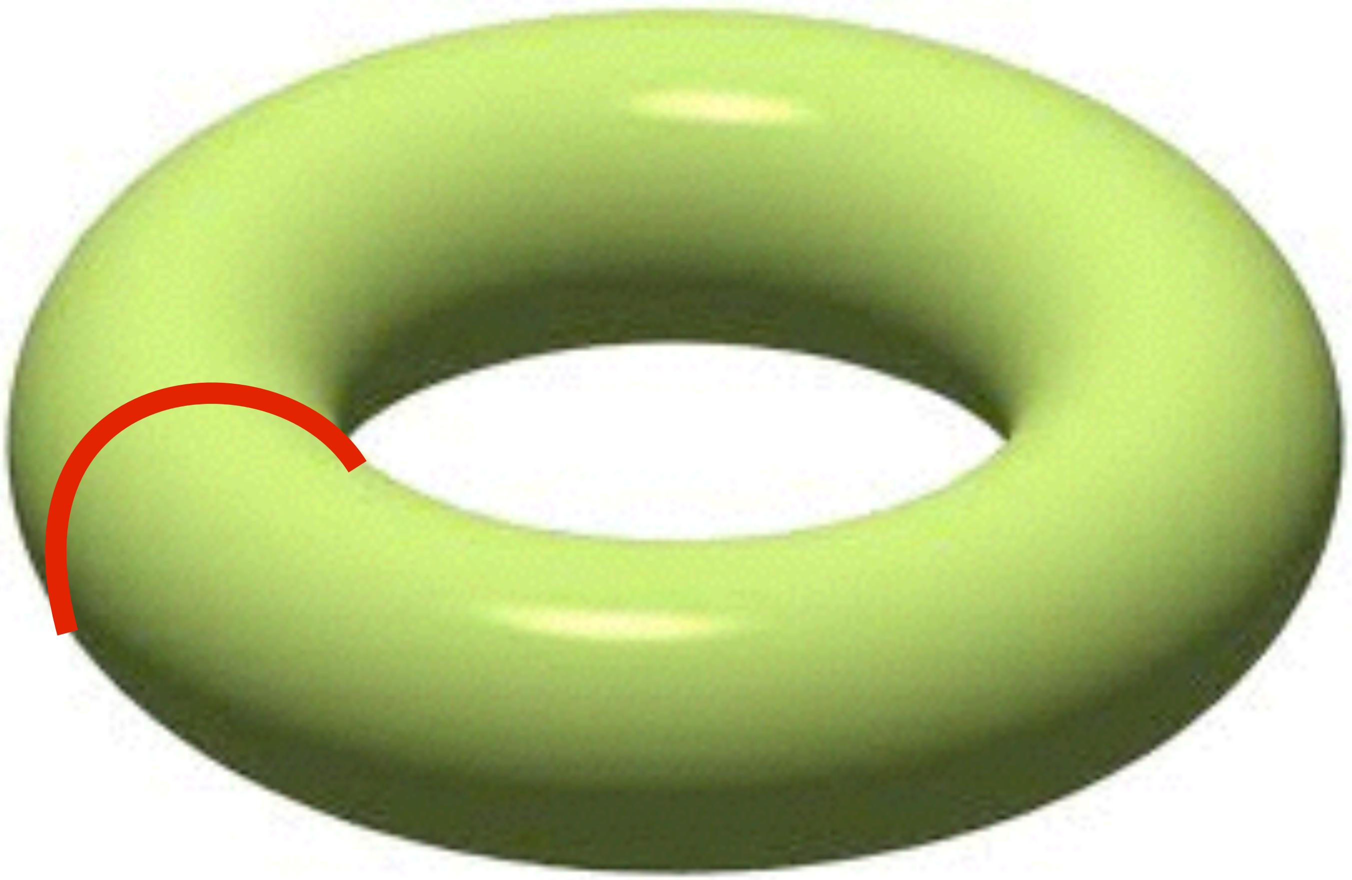
Realizes a metal with a Fermi surface of area p co-existing with "topological order"

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Topological order

Place pseudogap metal on a torus;

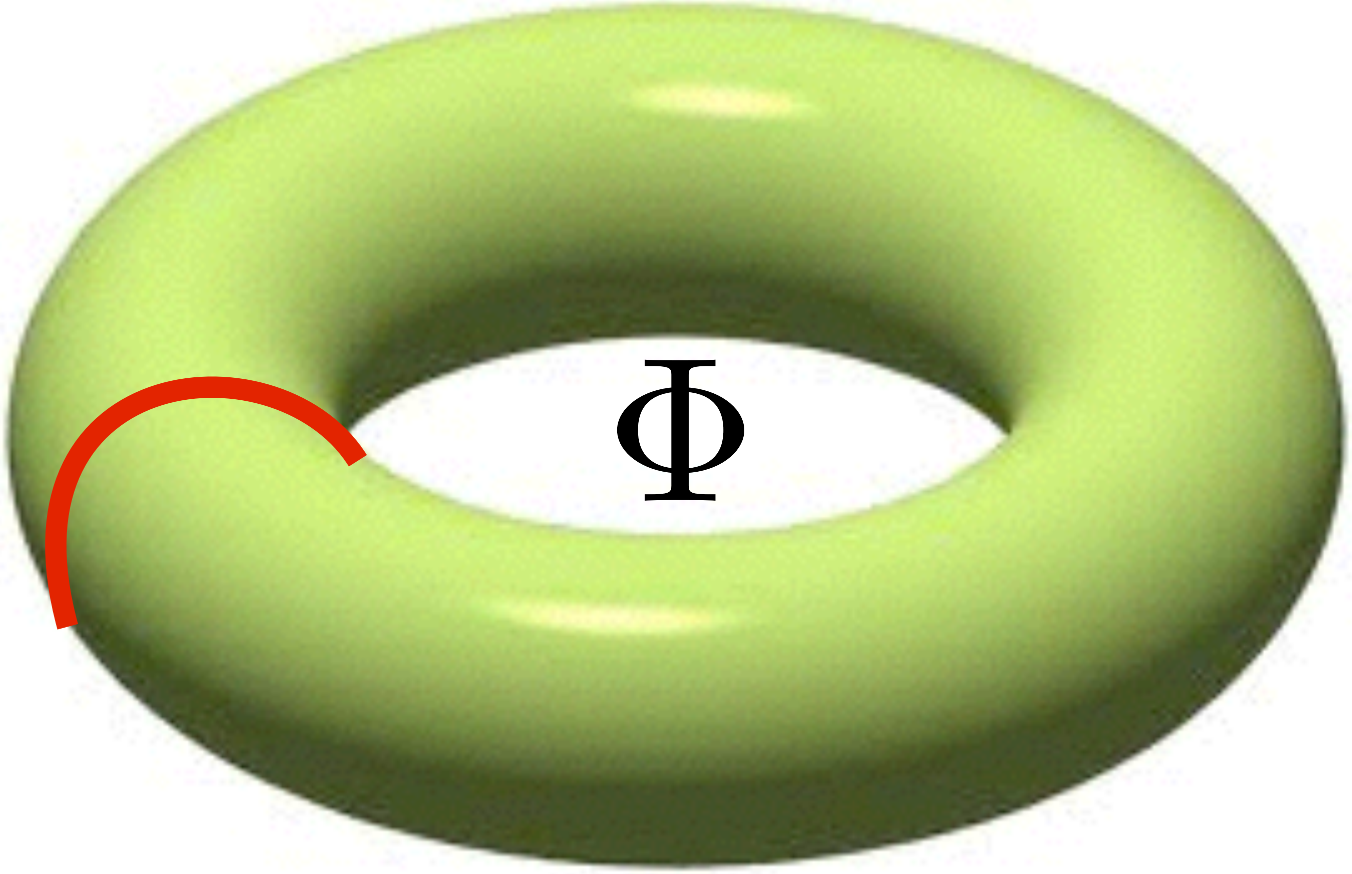
Topological order



Place pseudogap metal on a torus;

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

Topological order: flux insertion



Upon inserting a flux quantum $\Phi = h/e$ coupling only to $\uparrow$ electrons (say), the states with an even (odd) number of dimers crossing the cut pick up a factor of $+1$ (-1). The hole of the torus has a "*vison*" after the flux insertion.

N. E. Bonesteel, PRB **40**, 8954 (1989).

Fractionalized Fermi liquid (FL*)

$$\boxed{\Delta P_x = 2\pi\nu L_y (\text{mod } 2\pi) \quad , \quad \Delta P_y = 2\pi\nu L_x (\text{mod } 2\pi)}$$

Momentum balance for a Z_2 fractionalized Fermi liquid:

Momentum acquired by quasiparticles near the Fermi surface:

$$(\Delta P_x)_{\text{quasiparticles}} = \left(\frac{2\pi}{L_x}\right)\frac{L_x L_y}{4\pi^2} V_{\text{FS}}$$

Momentum acquired by vison:

$$(\Delta P_x)_{\text{vison}} = \pi L_y$$

Using $\Delta P_x = (\Delta P_x)_{\text{quasiparticles}} + (\Delta P_x)_{\text{vison}}$ we obtain the Fermi surface volume:

$$\nu = \frac{N}{L_x L_y} = 2\frac{V_{\text{FS}}}{4\pi^2} + 2m + 1$$ for some integer m. For the cuprates, this implies a Fermi volume of density p and not $1+p$.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

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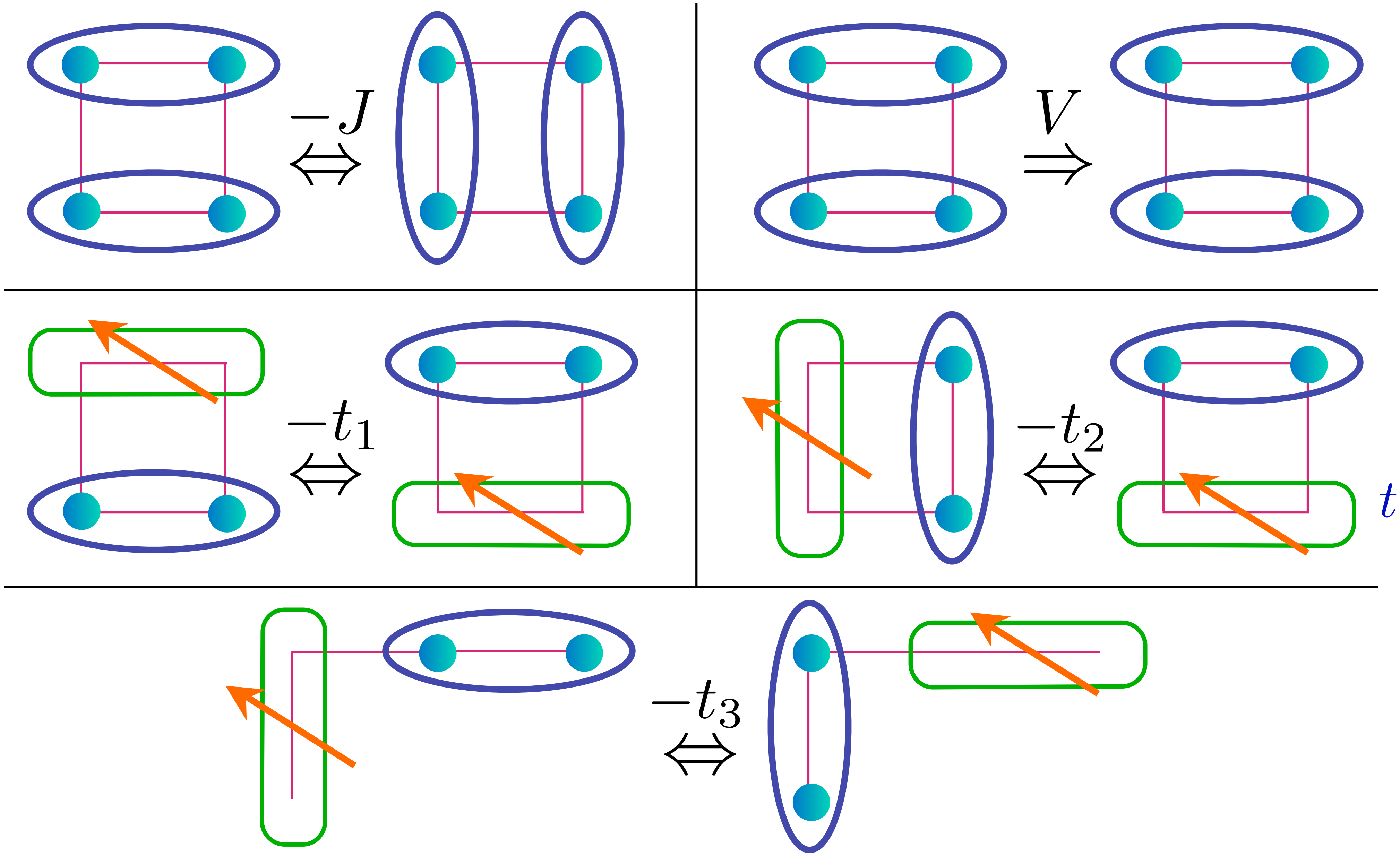
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Quantum dimer model with bosonic and fermionic dimers



Connection to the t-t'-t''-J model:

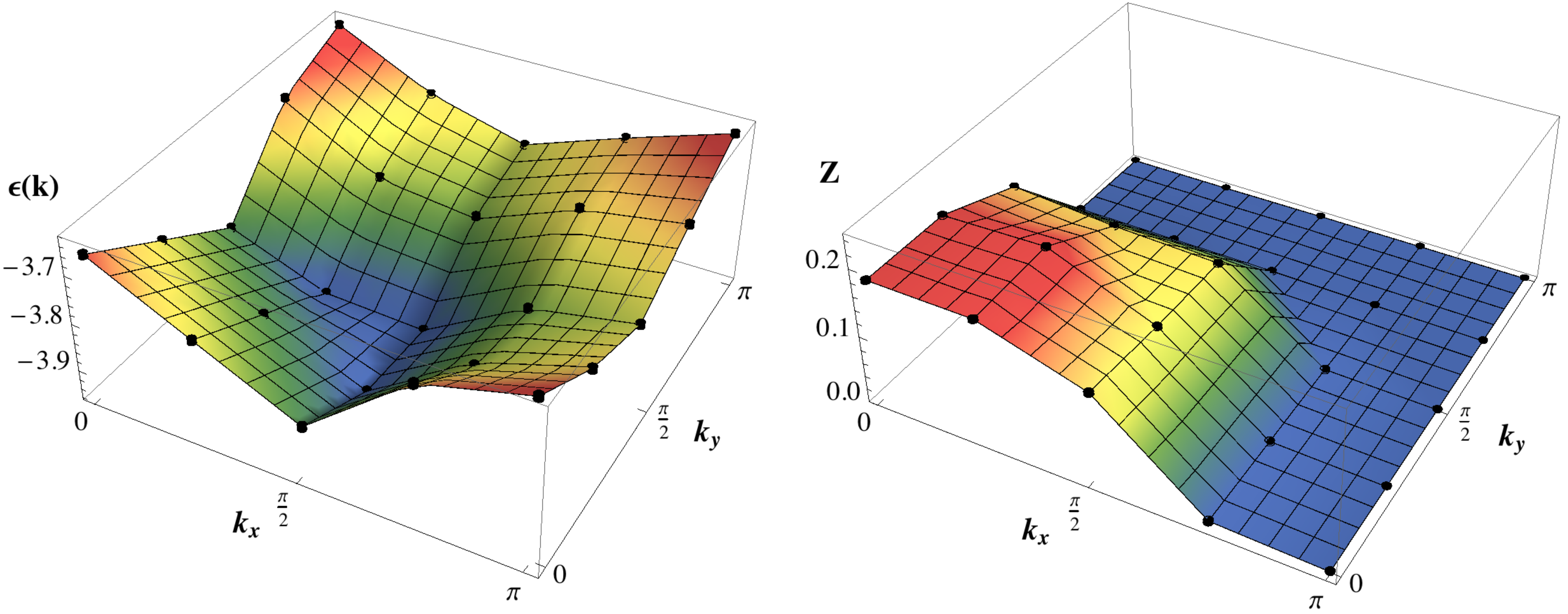
$$t_1 = -(t+t')/2$$

$$t_2 = (t-t')/2$$

$$t_3 = -(t+t'+t'')/4$$

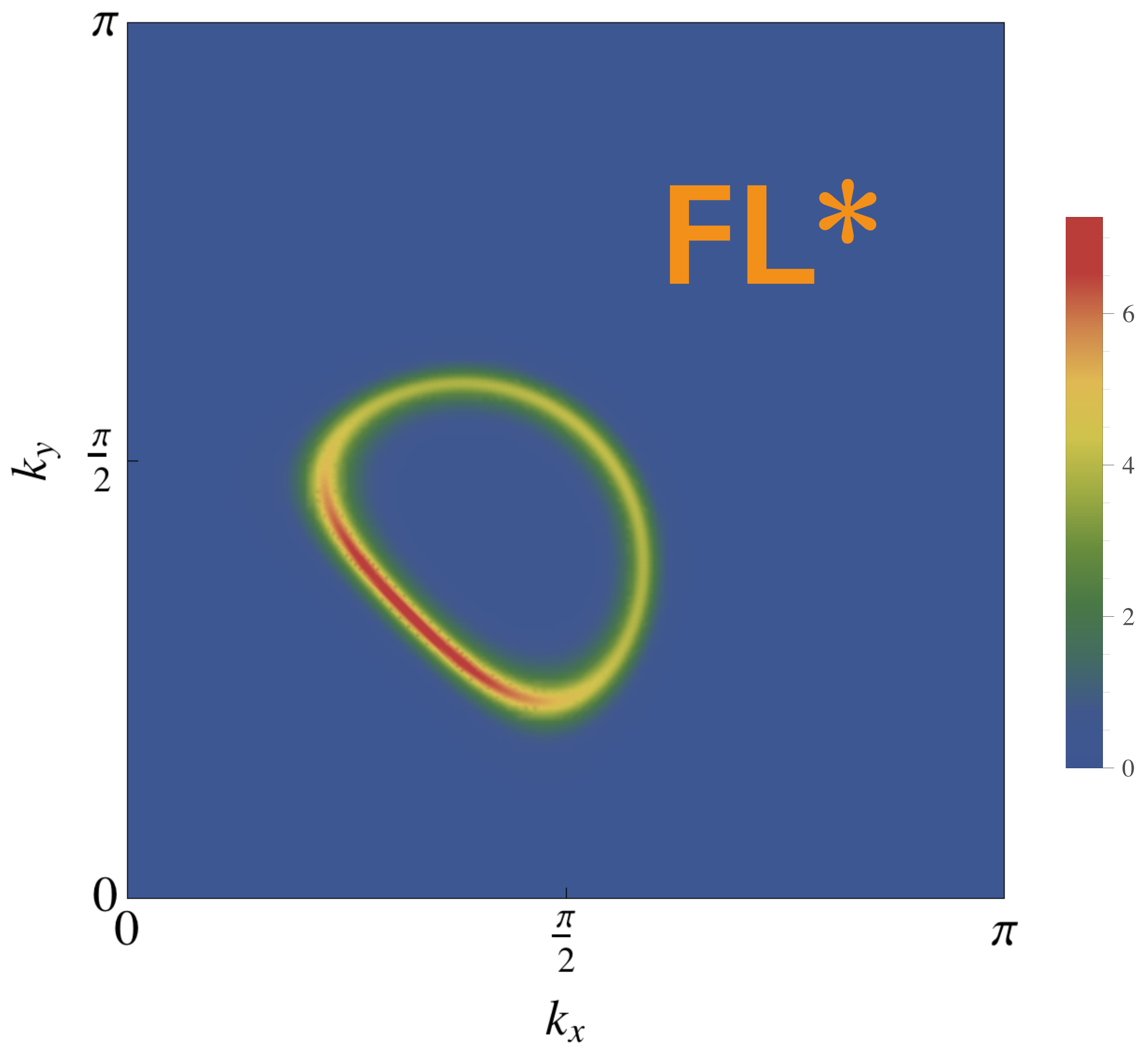
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)

Quantum dimer model with bosonic and fermionic dimers

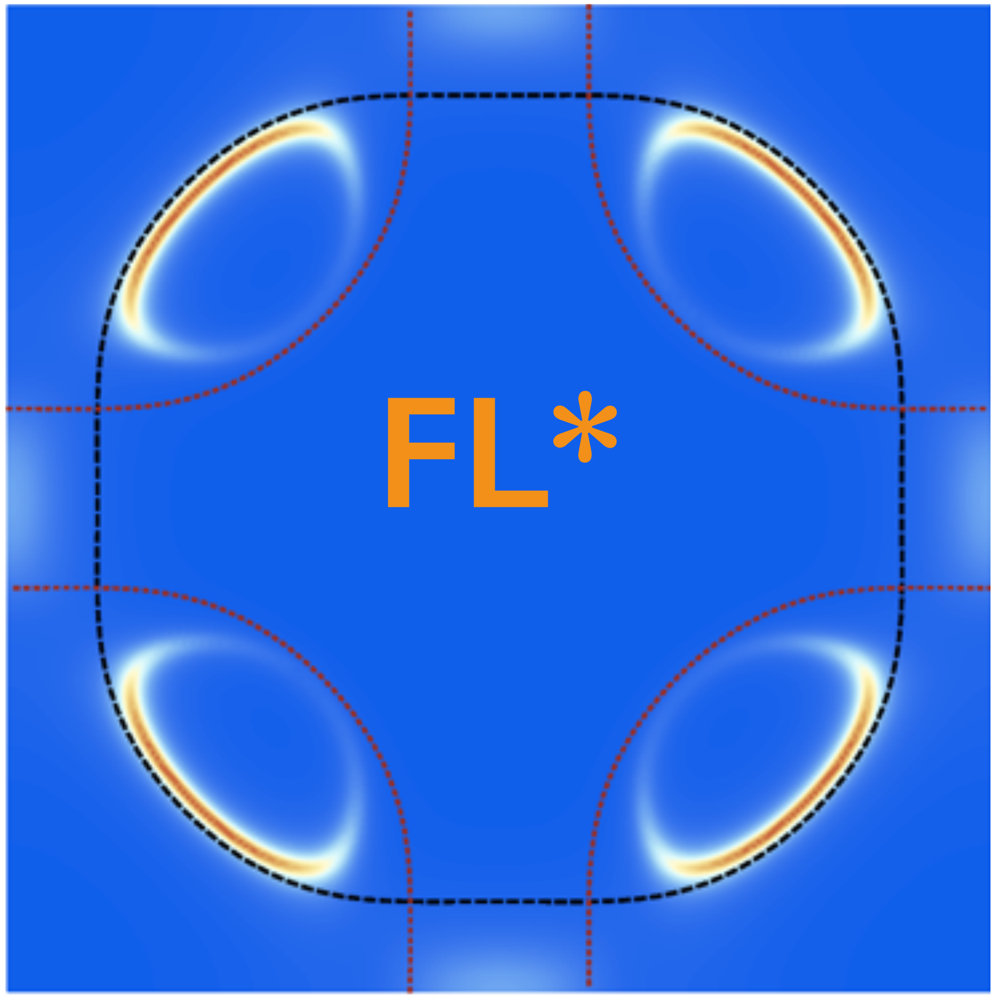


Dispersion and quasiparticle residue of a single fermionic dimer for $J = V = 1$, and hopping parameters obtained from the t-J model for the cuprates, $t_1 = -1.05$, $t_2 = 1.95$ and $t_3 = -0.6$, on a 8×8 lattice.

M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)



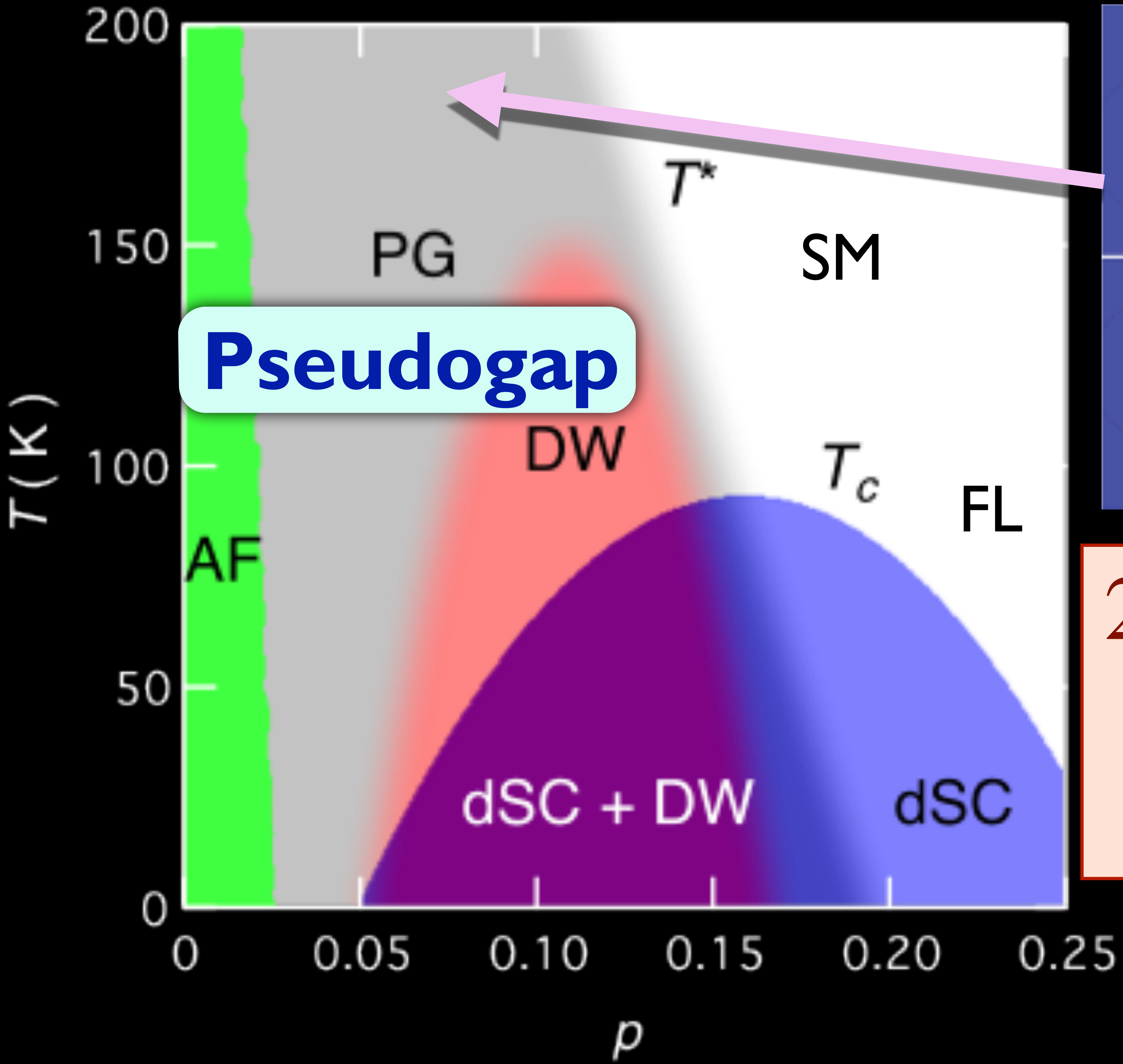
M. Punk, A. Allais, and S. Sachdev,
PNAS **112**, 9552 (2015)

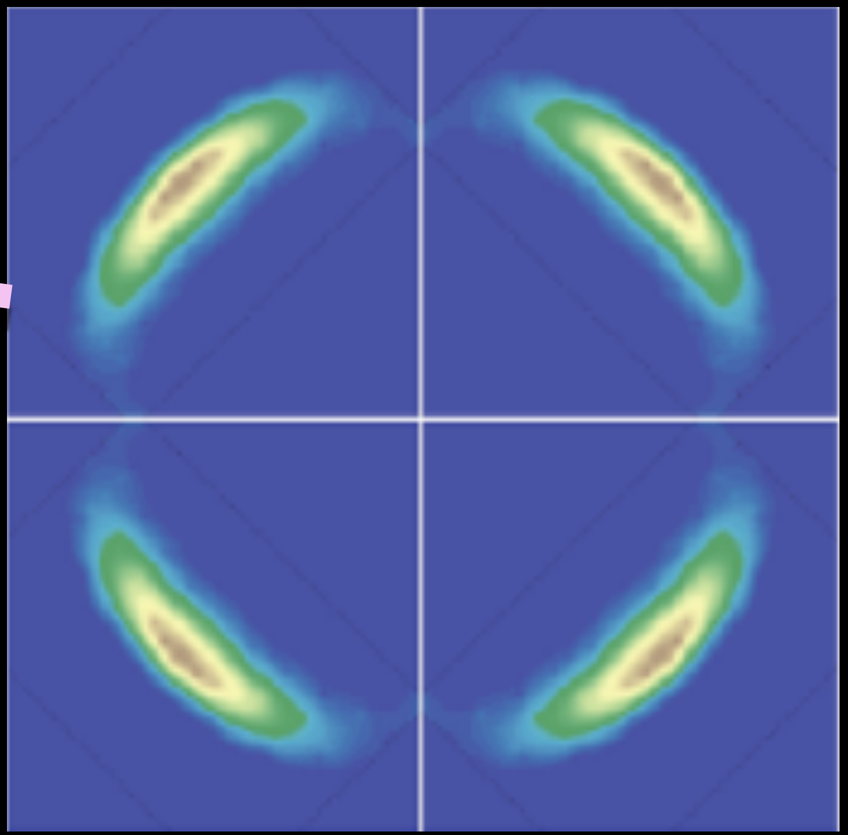


Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the square lattice

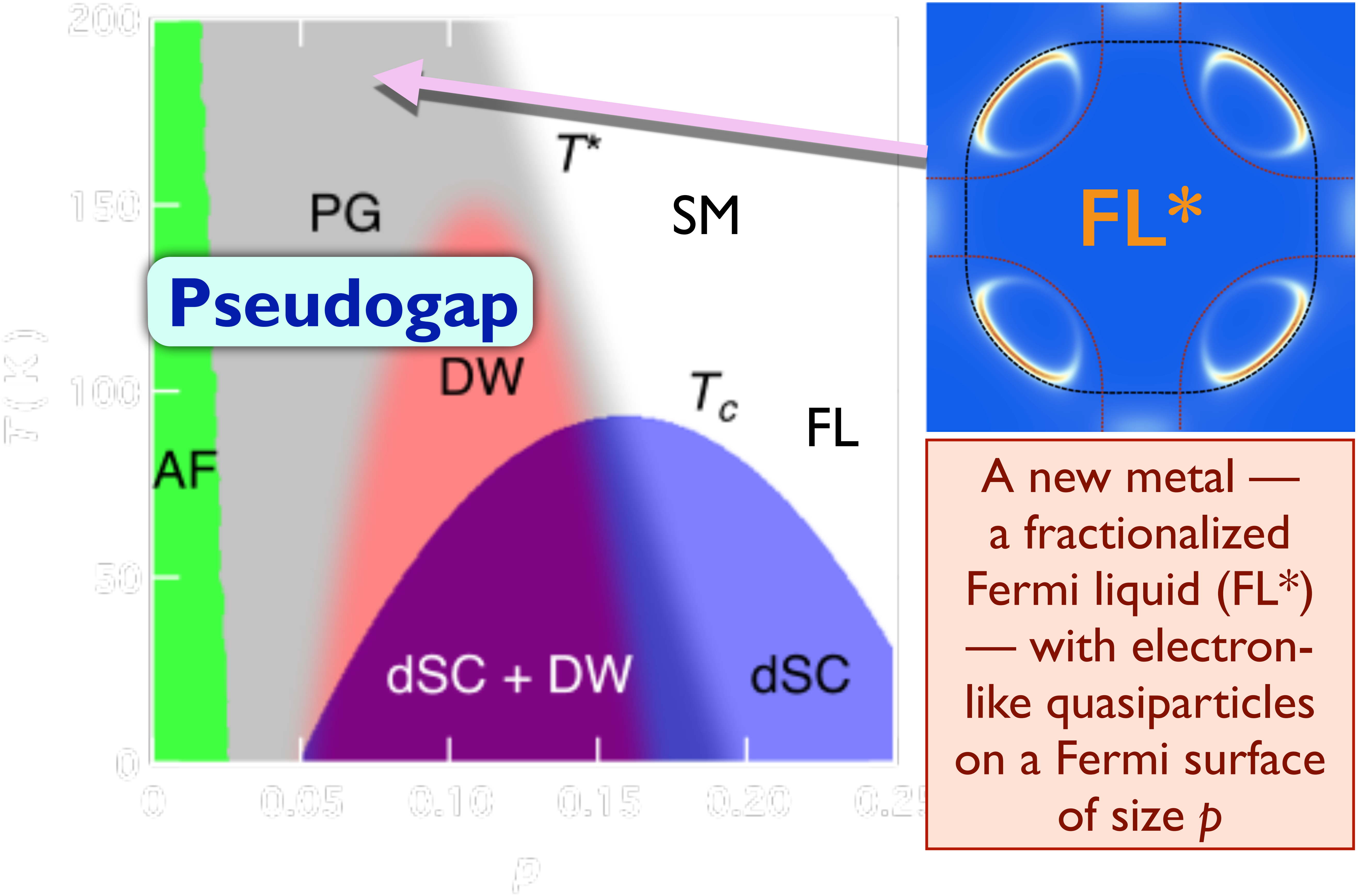
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)





2. Pseudogap metal at low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)



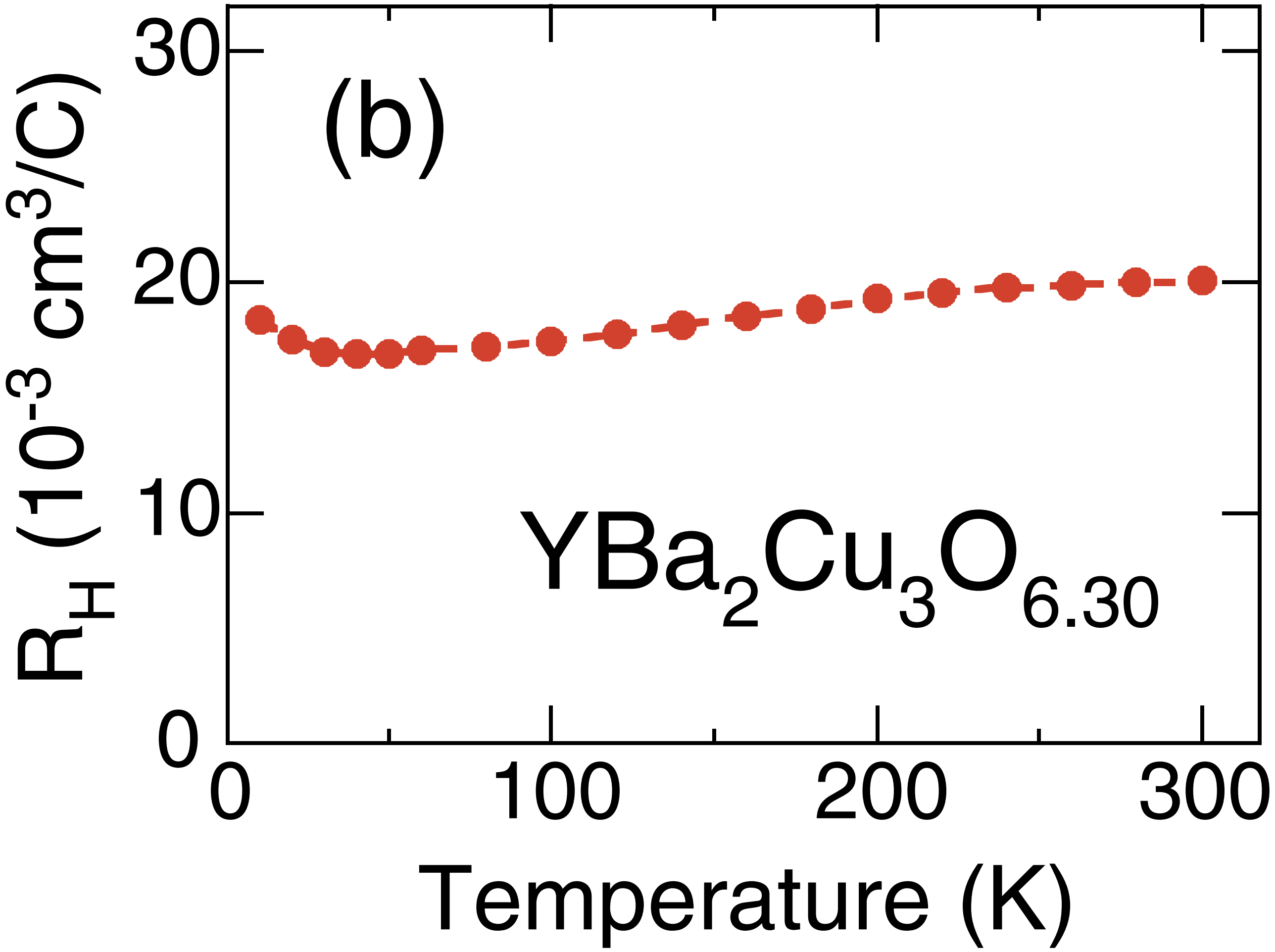
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density *p*

Evolution of the Hall Coefficient and the Peculiar Electronic Structure of the Cuprate Superconductors

Yoichi Ando,* Y. Kurita,† Seiki Komiya, S. Ono, and Kouji Segawa

PRL **92**, 197001 (2004)



T-independent Hall effect in a magnetic field of fermions of charge +e and density *p*

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density p

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei[a], Damien Stricker[a], Jason N. Hancock[a,b], Christophe Berthod[a], Antoine Georges[a,c,d], Erik van Heumen[a,e], Mun K. Chan[f], Xudong Zhao[f,g], Yuan Li[h], Martin Greven[f], Neven Barišić[f,i,j], and Dirk van der Marel[a,1]

PNAS **110**, 5774 (2013)

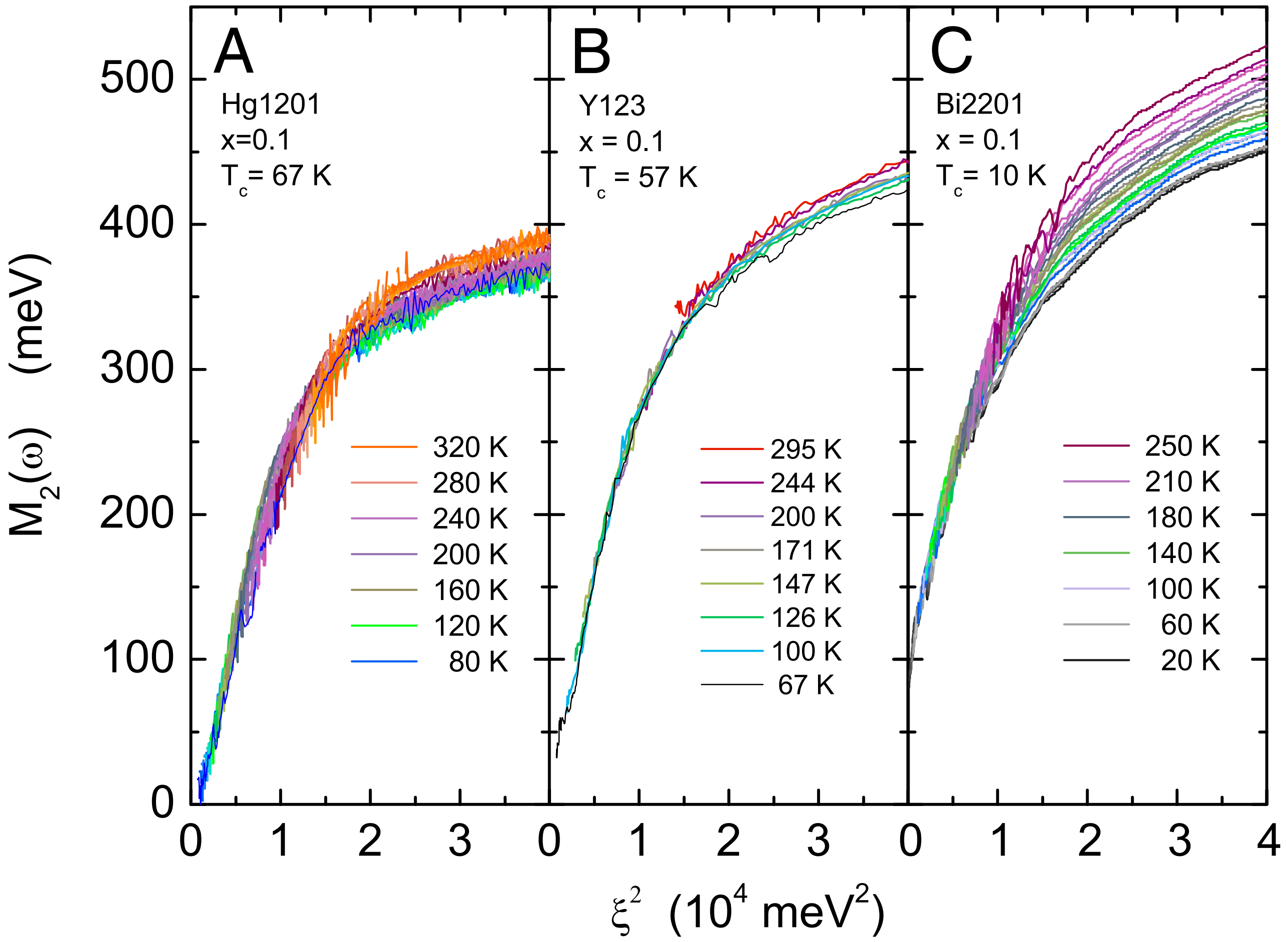
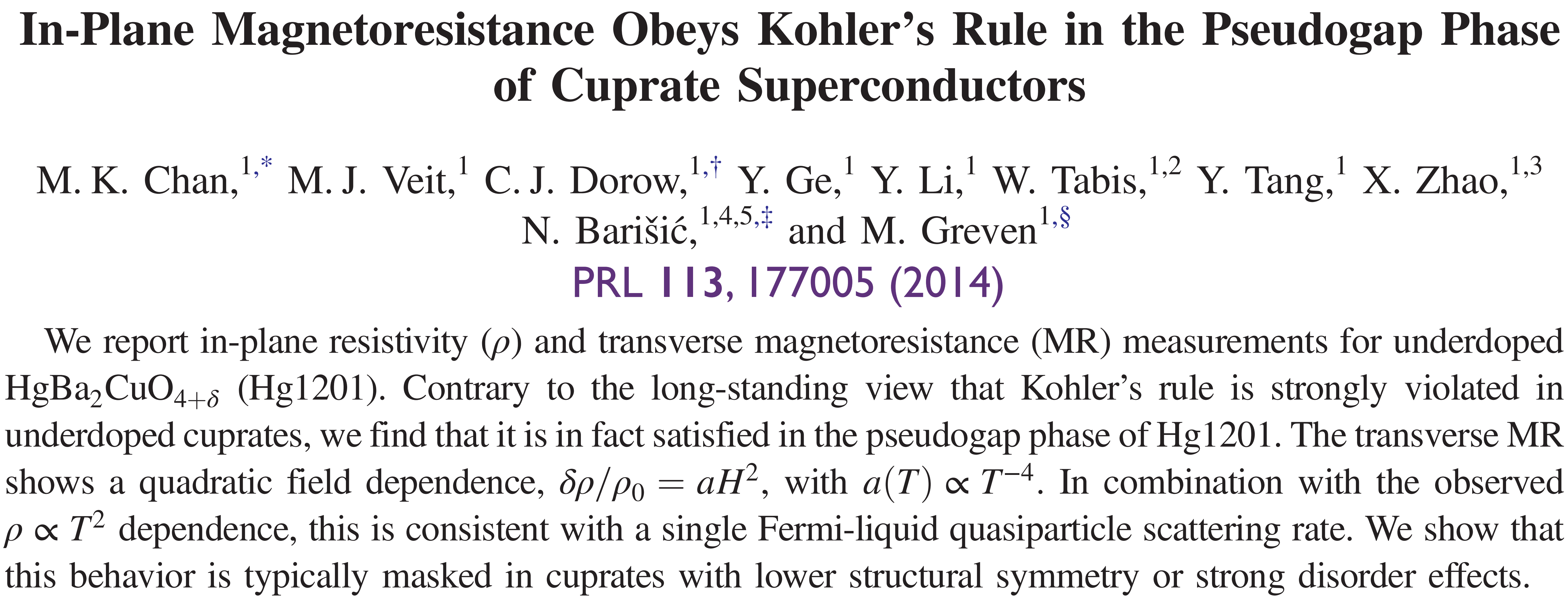


Fig. 6. Collapse of the frequency and temperature dependence of the relaxation rate of underdoped cuprate materials. Normal state $M_2(\omega, T)$ as a function of $\xi^2 \equiv (\hbar\omega)^2 + (p\pi k_B T)^2$

$$\sigma_{xx} \sim \frac{1}{(-i\omega + 1/\tau)}$$

$$\text{with } \frac{1}{\tau} \sim \omega^2 + T^2$$

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density *p*



In-Plane Magnetoresistance Obeys Kohler's Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,[1,*] M. J. Veit,[1] C. J. Dorow,[1,†] Y. Ge,[1] Y. Li,[1] W. Tabis,[1,2] Y. Tang,[1] X. Zhao,[1,3]
N. Barišić,[1,4,5,‡] and M. Greven[1,§]

PRL **113**, 177005 (2014)

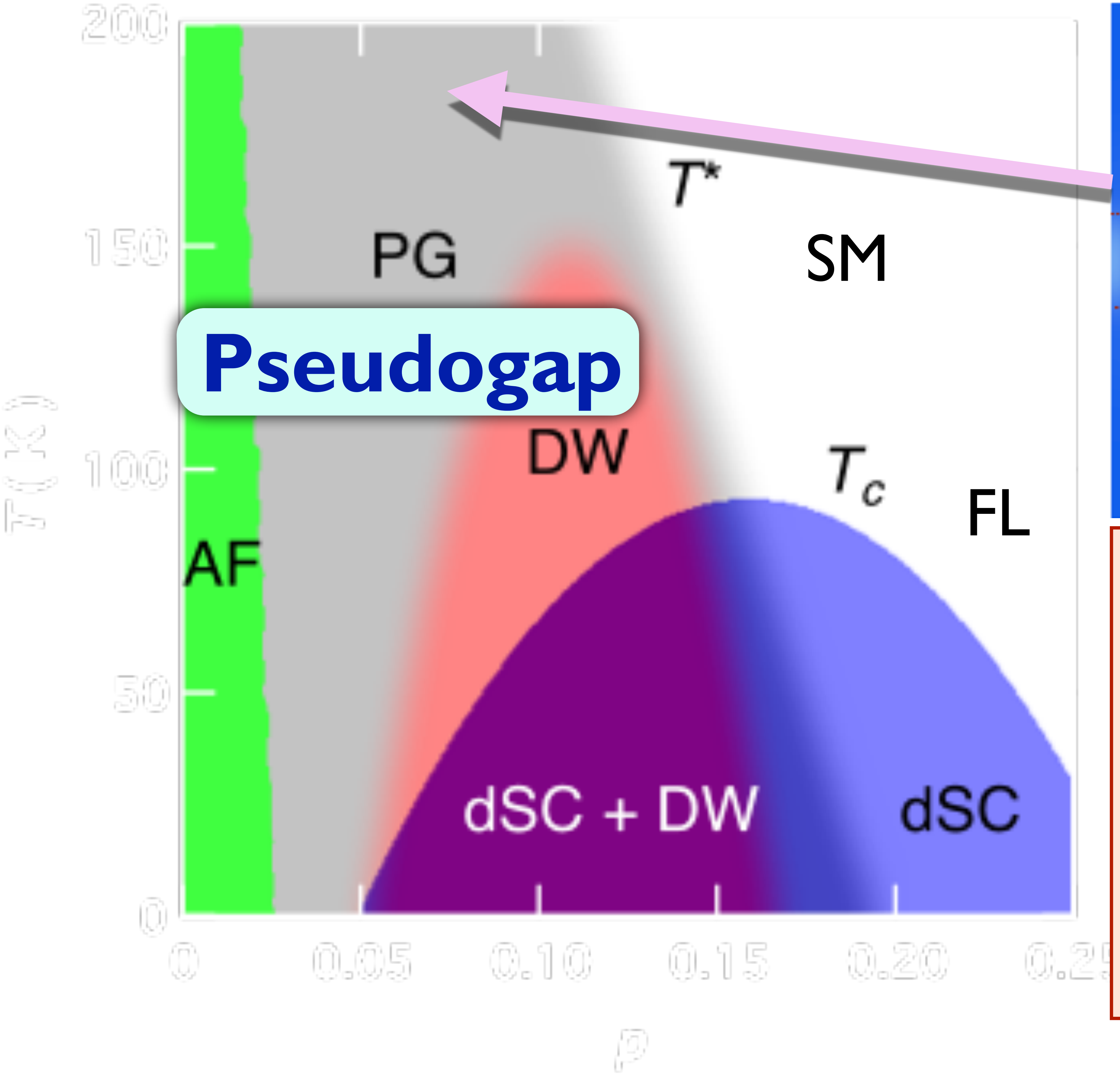
We report in-plane resistivity (ρ) and transverse magnetoresistance (MR) measurements for underdoped $HgBa_2CuO_{4+\delta}$ (Hg1201). Contrary to the long-standing view that Kohler's rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\delta\rho/\rho_0 = aH^2$, with $a(T) \propto T^{-4}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau}(1 + aH^2\tau^2 + \ldots)$$

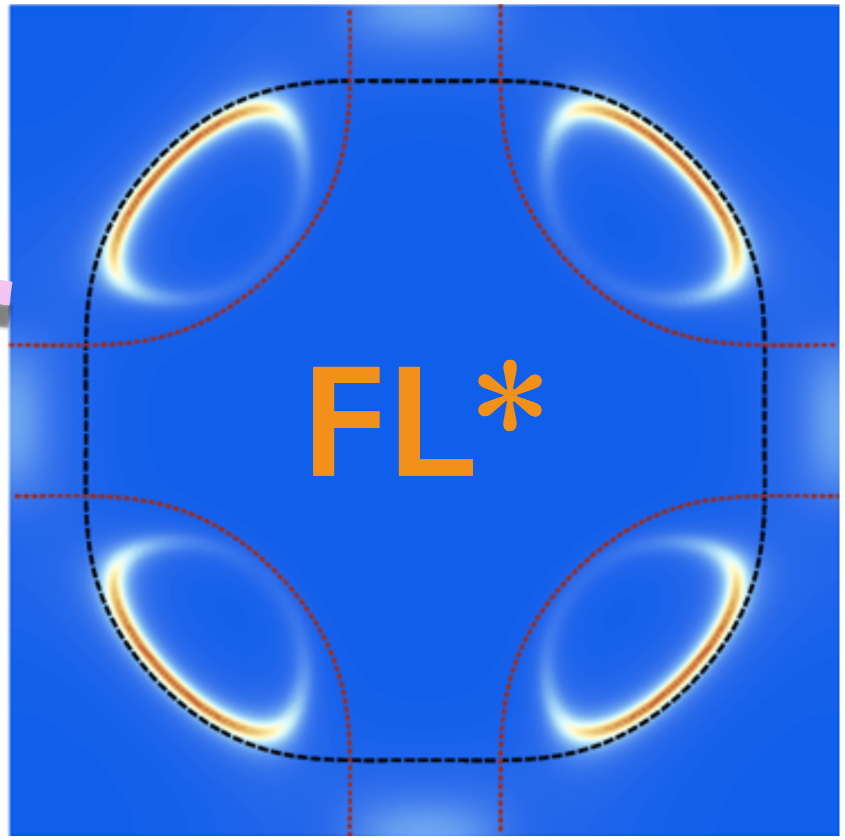
$$\text{with } \frac{1}{\tau} \sim T^2$$

Strong evidence from transport measurements of the existance of a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

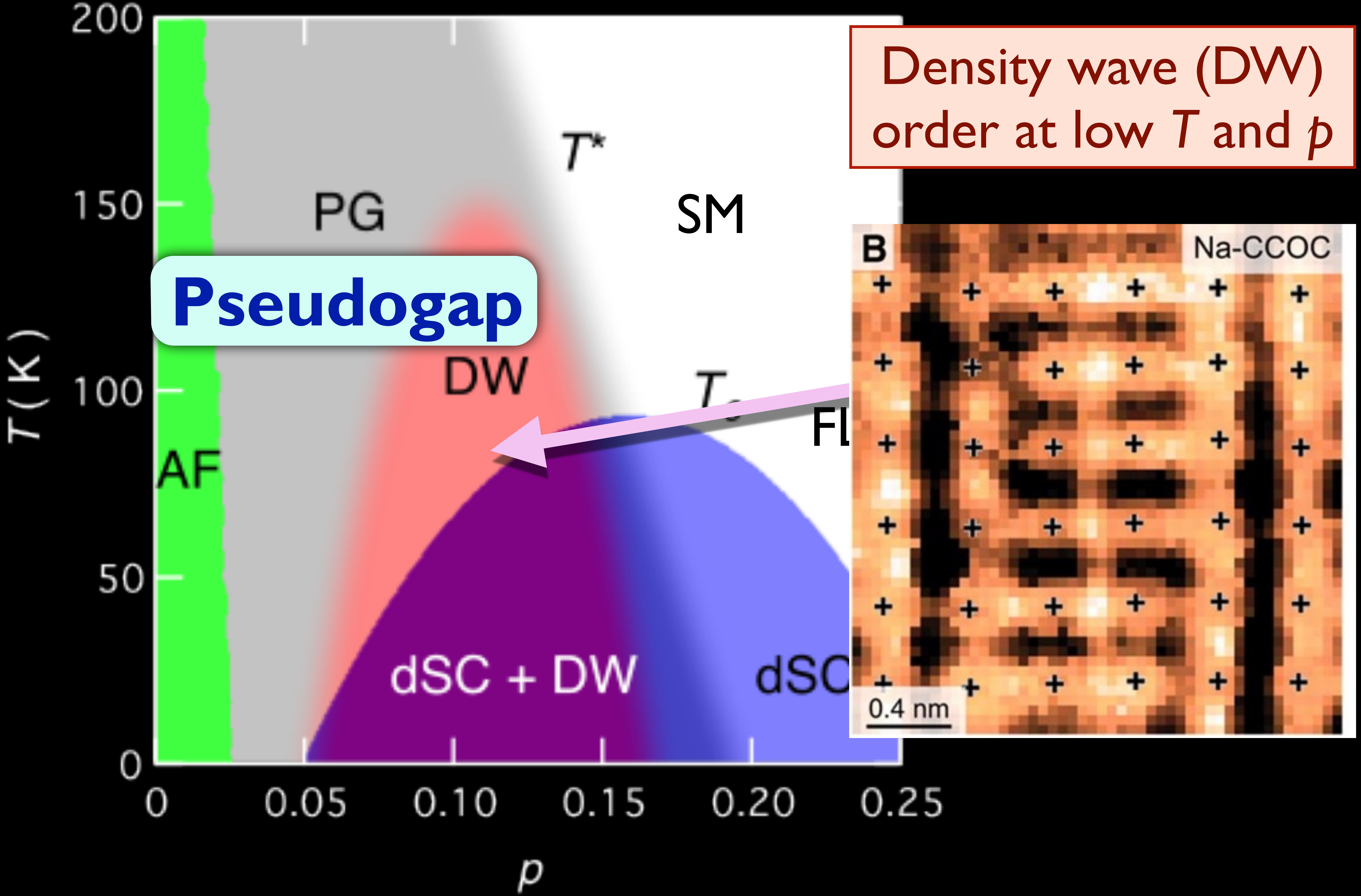


Pseudogap



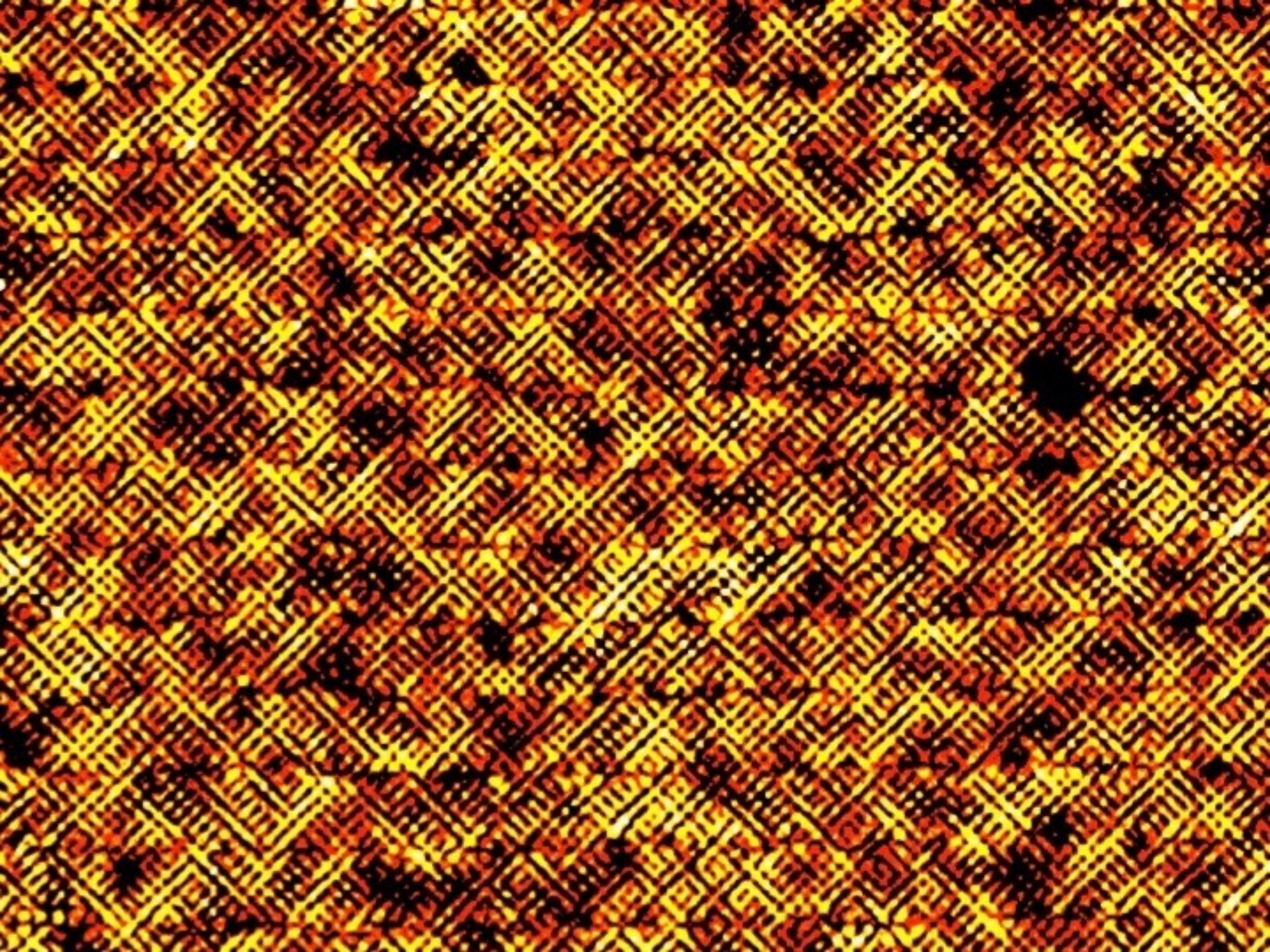
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

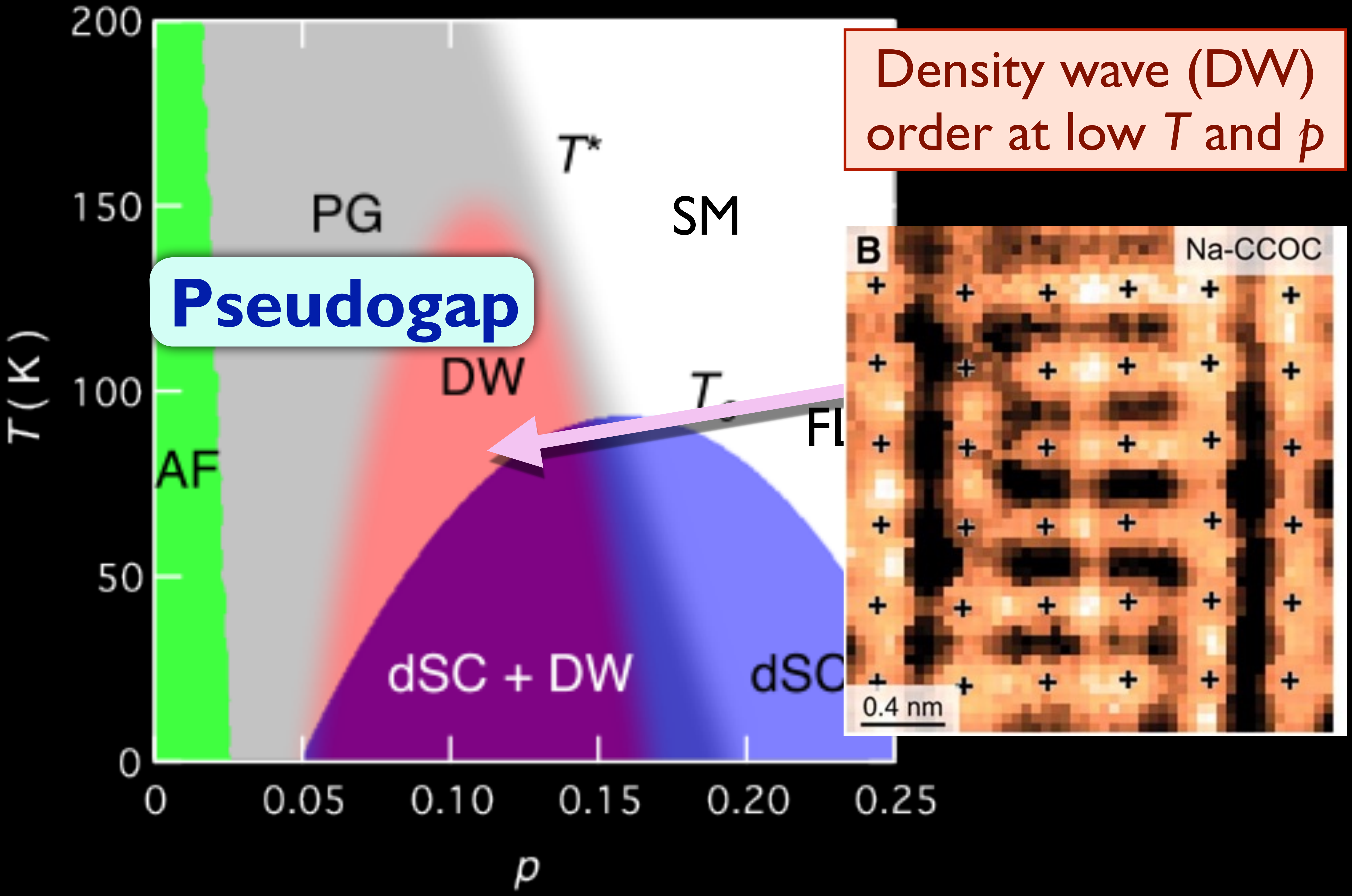


Pseudogap

Density wave (DW) order at low T and p



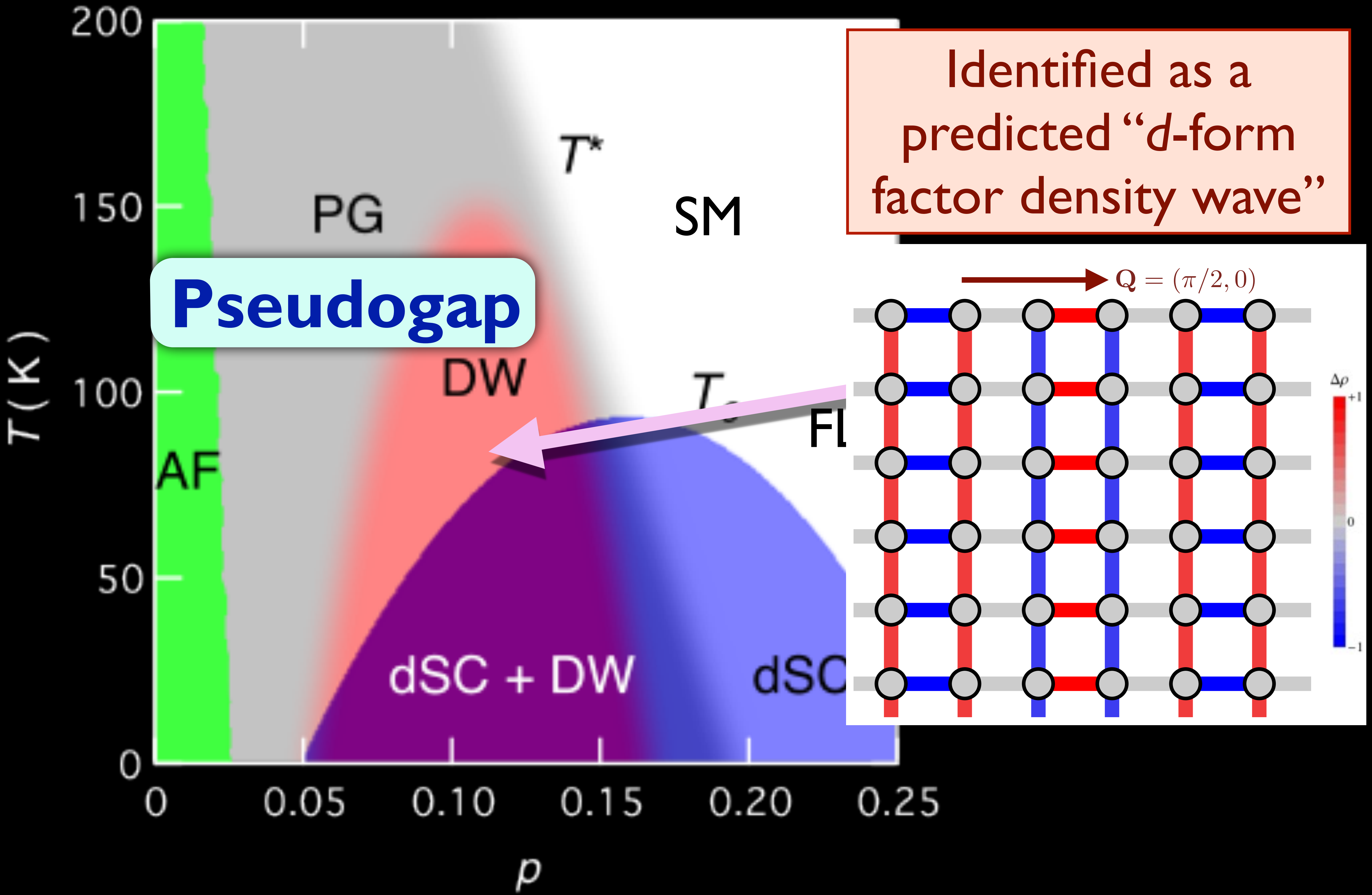
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



Pseudogap

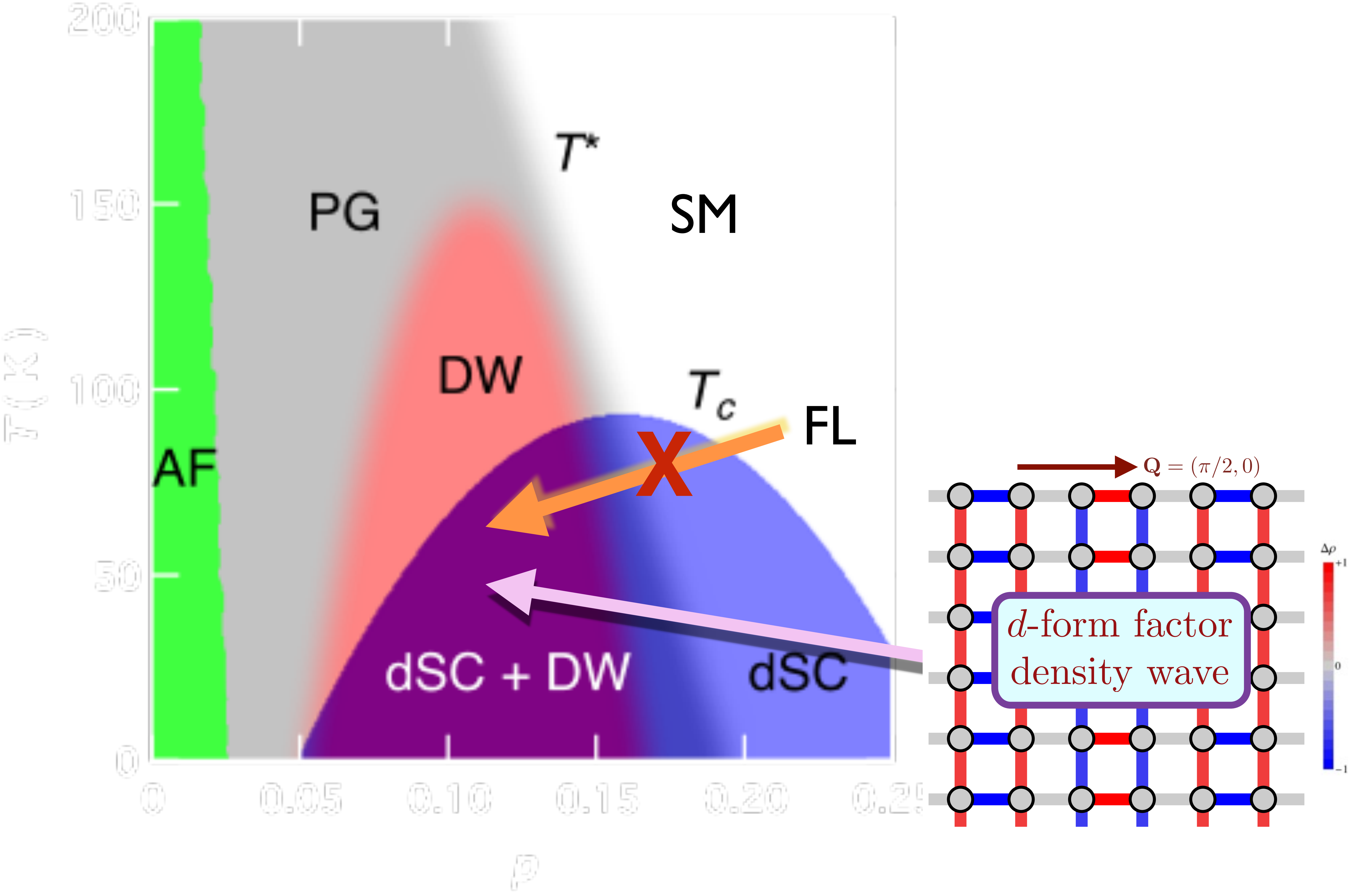
Density wave (DW) order at low T and p

M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

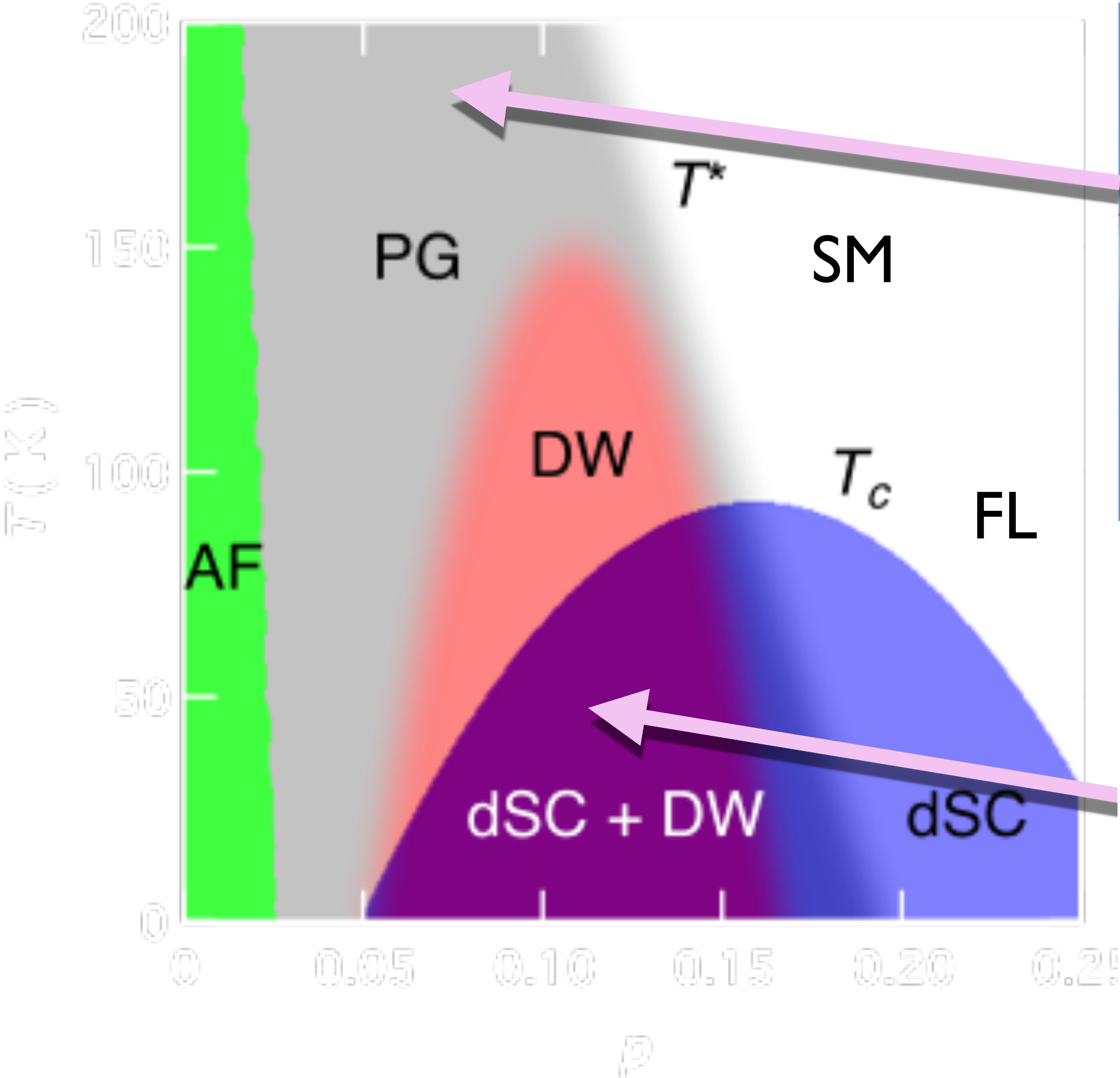


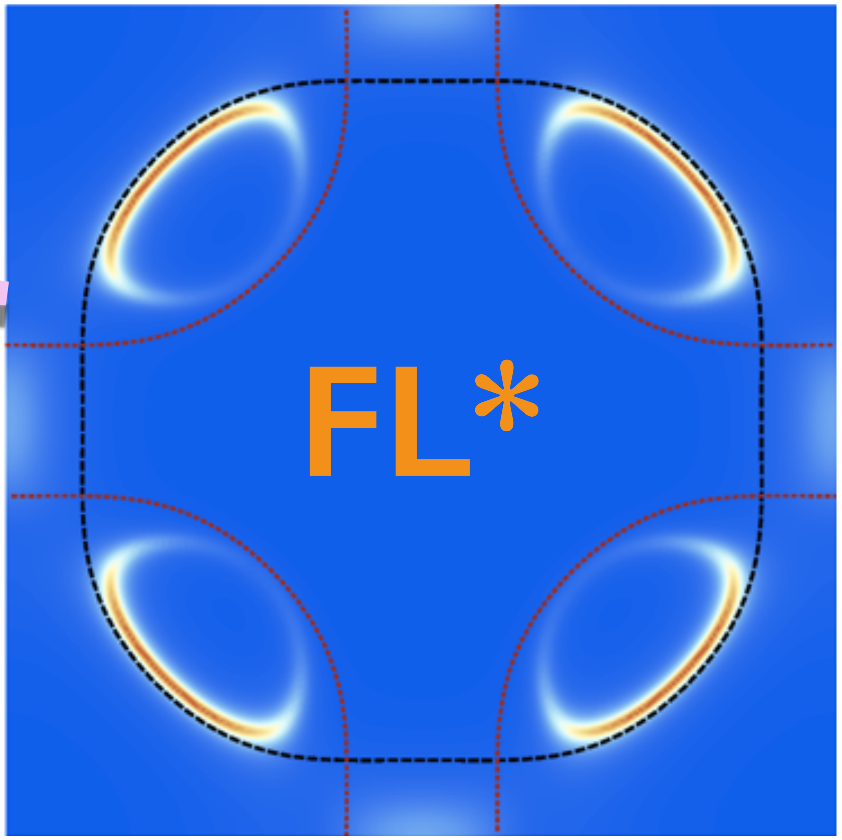
Pseudogap

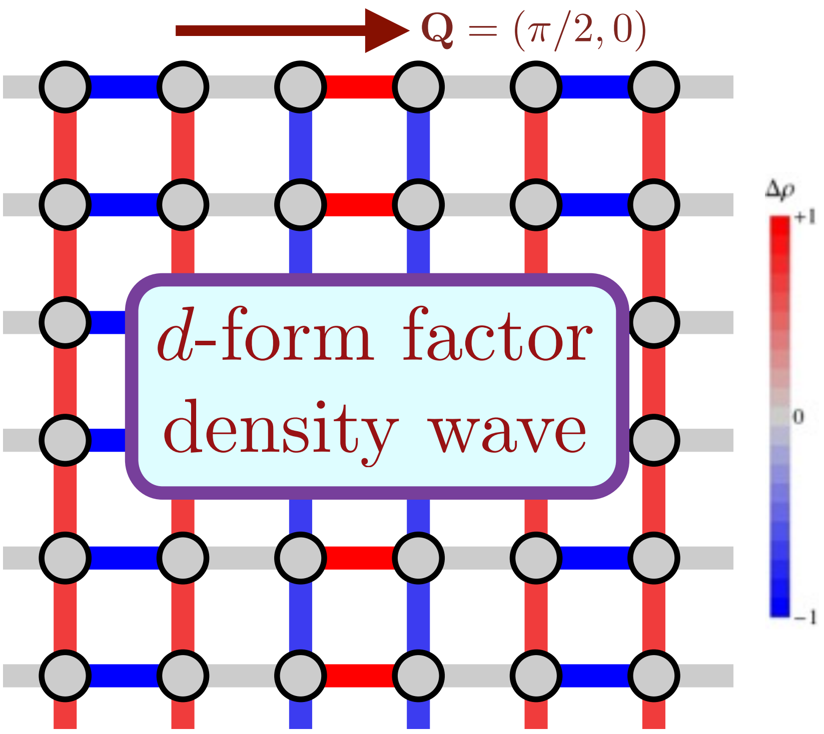
Identified as a predicted "*d*-form factor density wave"



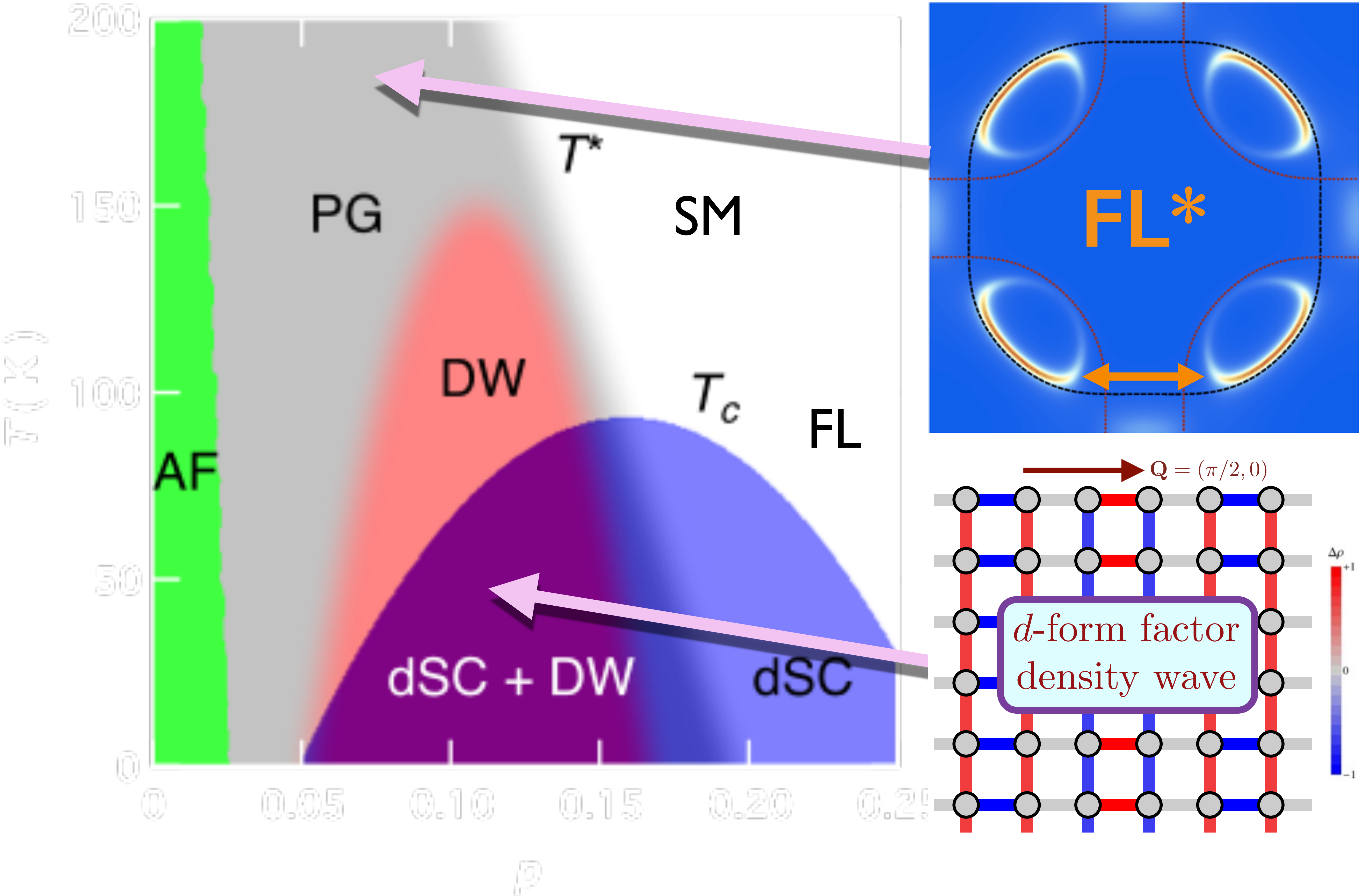
d-form factor density wave





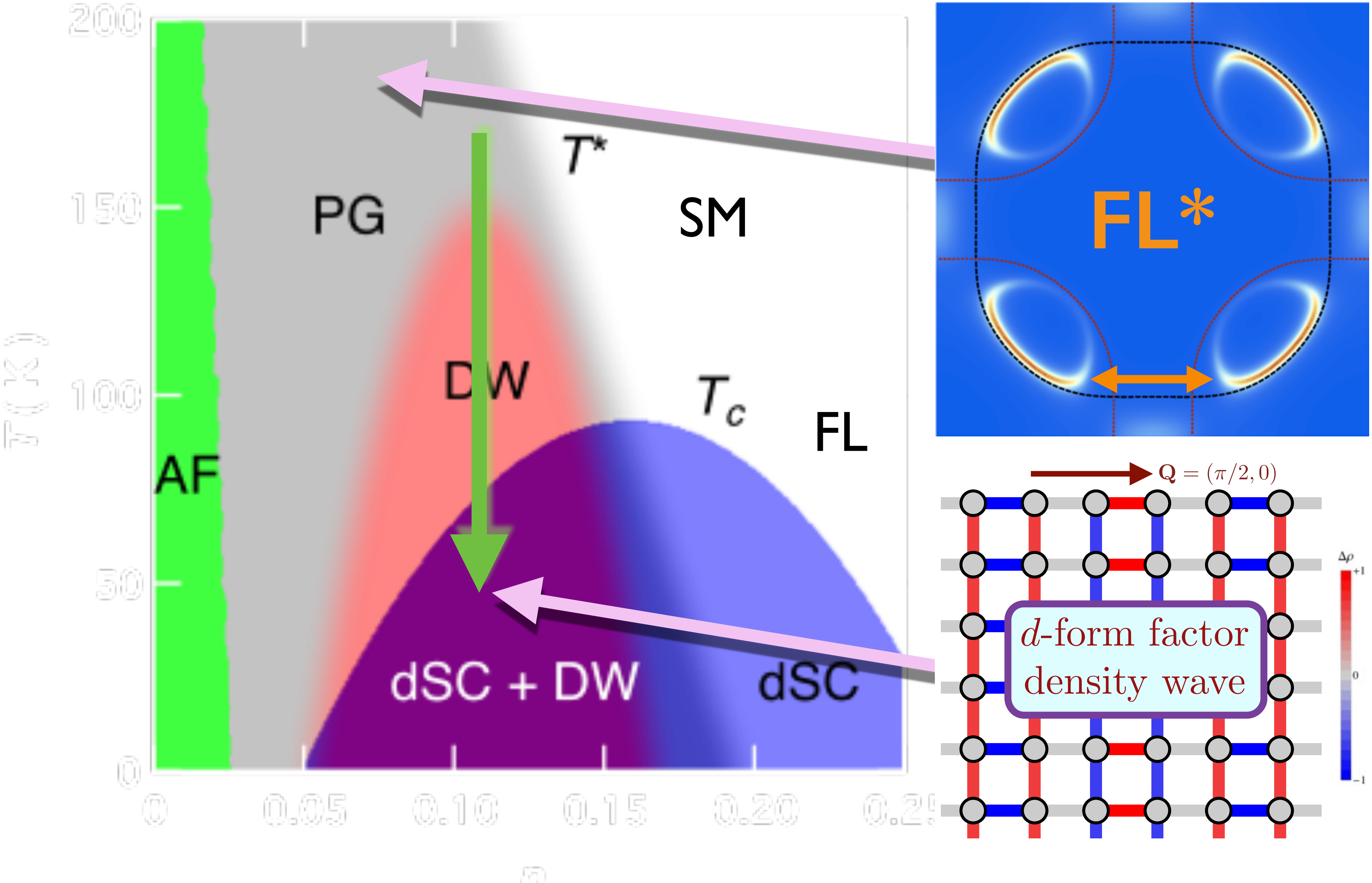


d-form factor density wave



d-form factor density wave

The high T FL* can help explain the "d-form factor density wave" observed at low T



D. Chowdhury and S.S., Phys. Rev. B **90**, 245136 (2014)

1. Topological order in insulating spin liquids

2. Review of Fermi liquid theory

   Topological argument for the Luttinger theorem

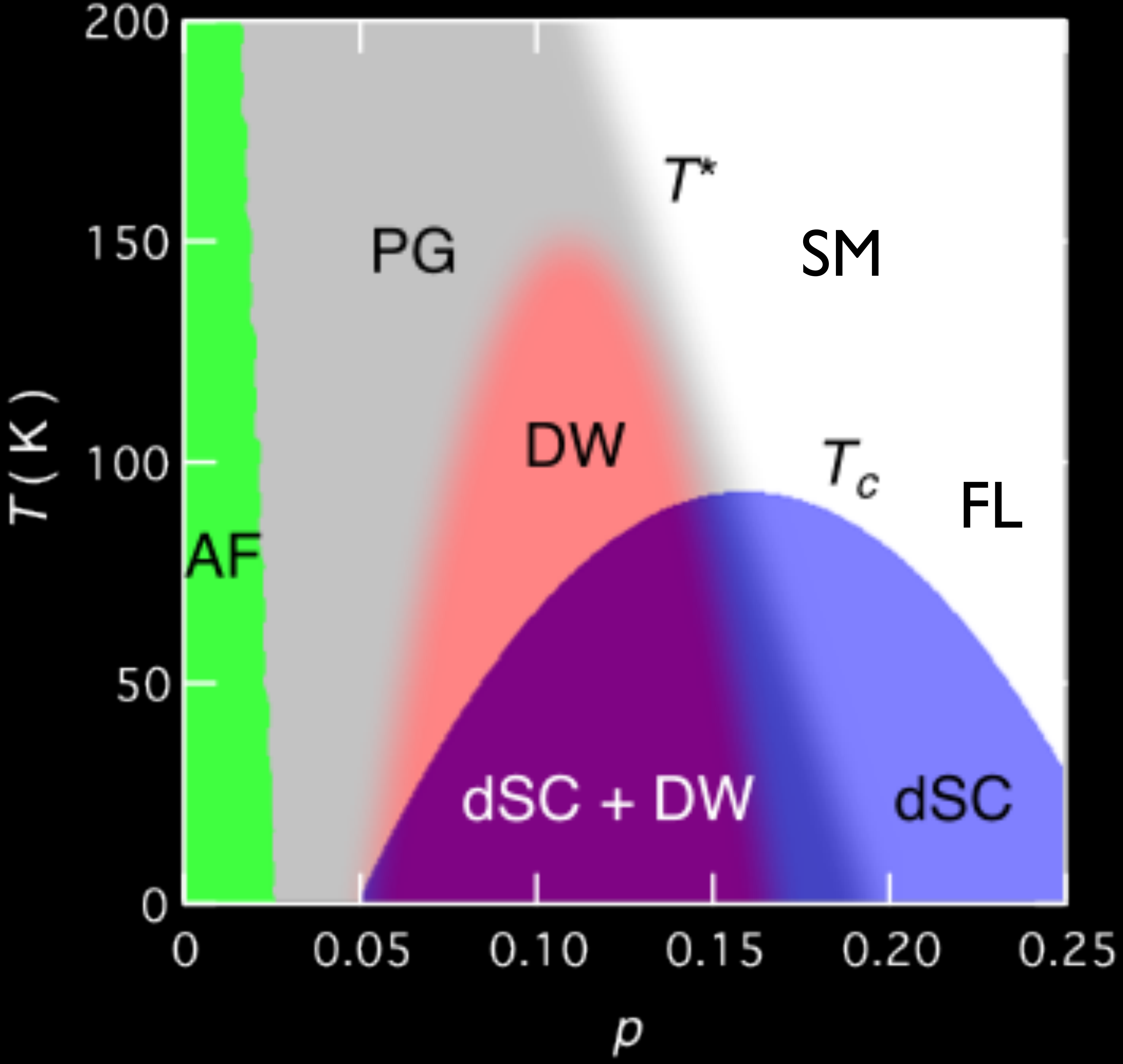
3. Fractionalized Fermi liquid

   A Fermi liquid co-existing with topological order for the pseudogap metal

4. Strange metals without quasiparticles

   (a) A mean-field model of a non-Fermi liquid, and charged black holes

   (b) A (slightly less) strange metal in graphene


200
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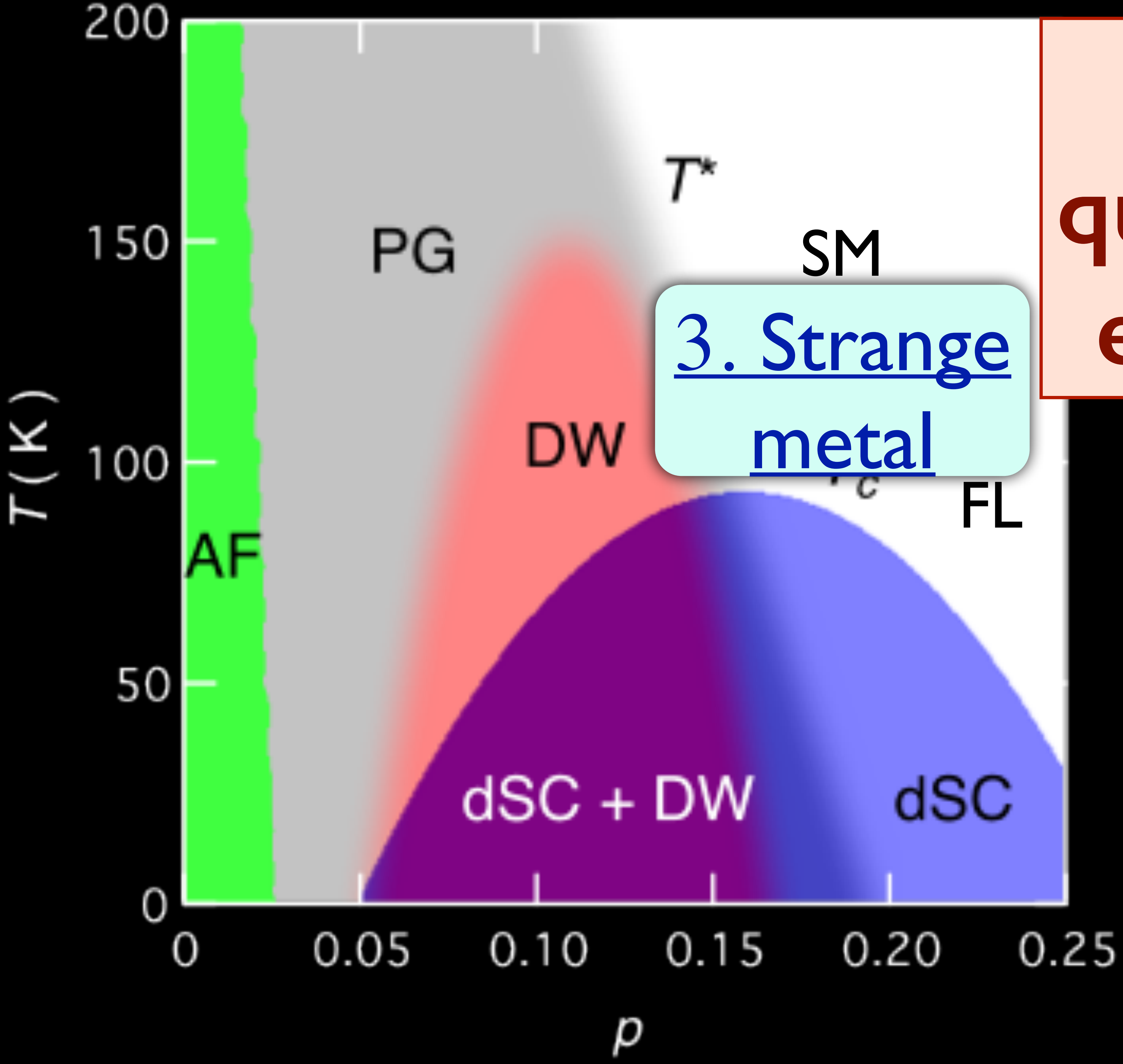
FL
dSC + DW
dSC



3. Strange metal

No quasiparticle excitations

- Consider an infinite quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the “electron density”) in spatial dimension $d > 1$.

- Describe $\underline{\textit{zero temperature}}$ “compressible” phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the “chemical potential”) which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

- Such systems cannot have an energy gap.

- Consider an infinite quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

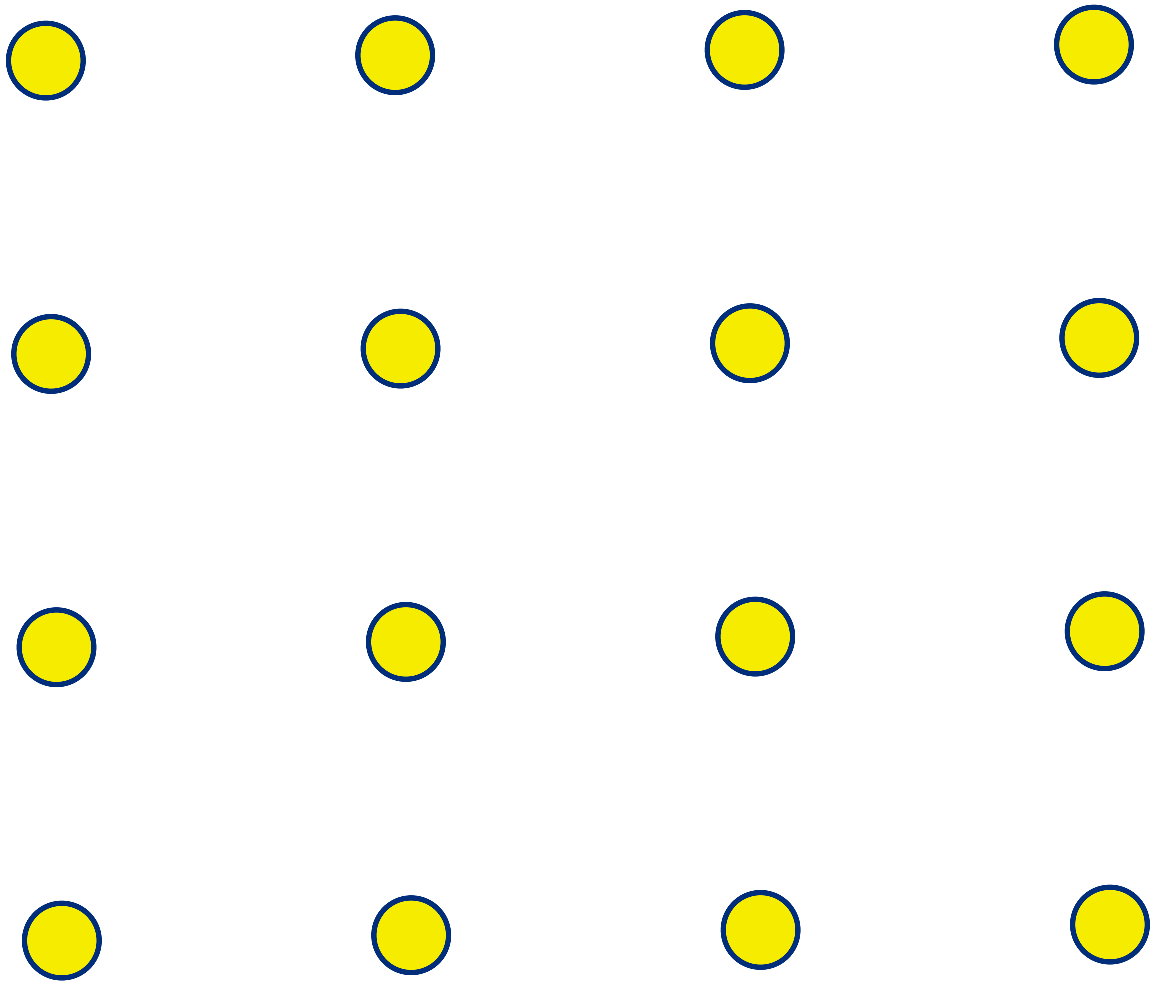
- Describe *zero temperature* "compressible" phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

- Such systems cannot have an energy gap.

- Conformal field theories are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.

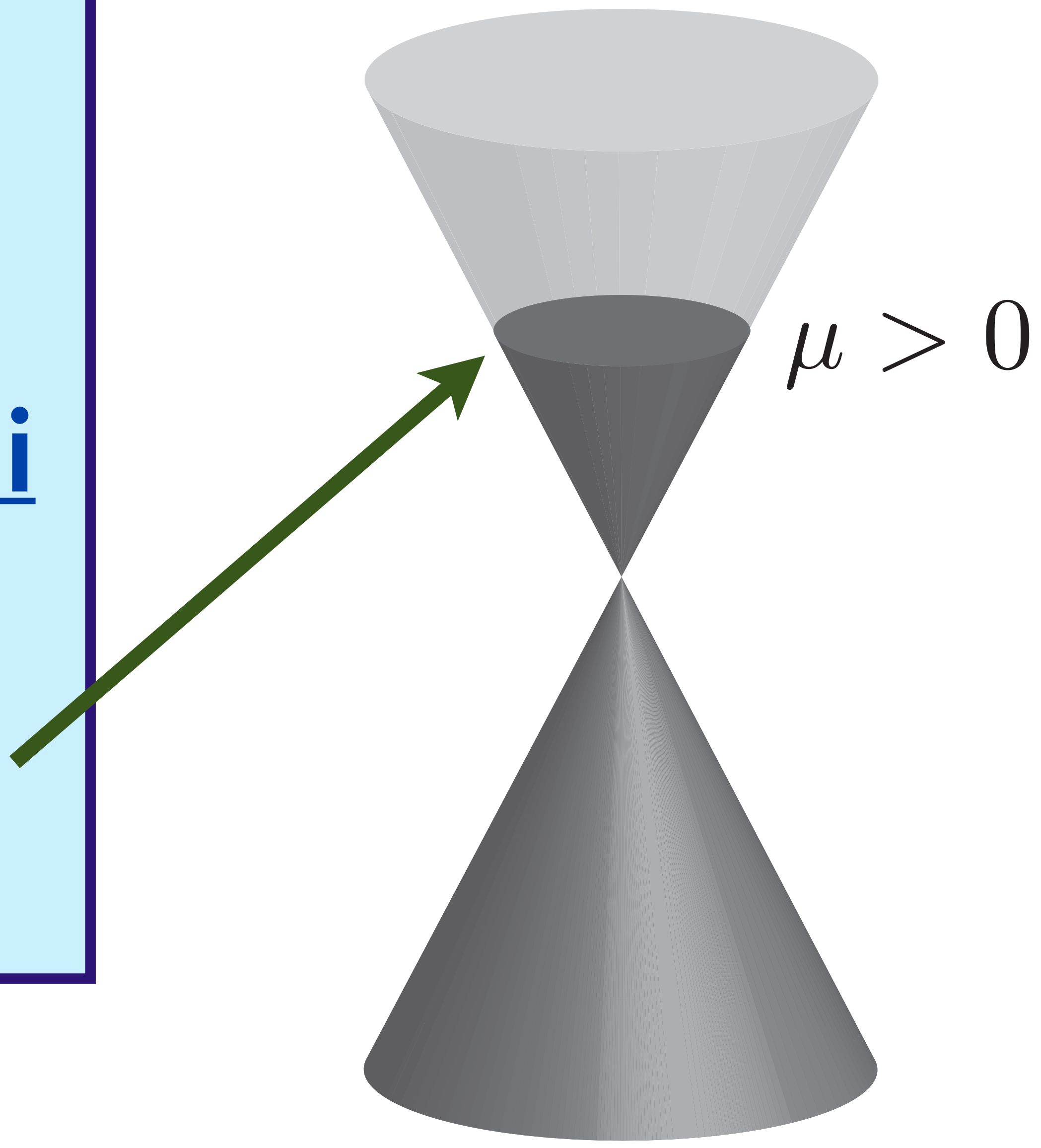


Compressible quantum matter

Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



Graphene

- Consider an infinite quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* "compressible" phases where $d\langle\mathcal{Q}\rangle/d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu\mathcal{Q}$.

- Such systems cannot have an energy gap.

- Conformal field theories are compressible in $d = 1$, but not for $d > 1$.

- I label any compressible/conducting state without quasiparticle excitations a strange metal.

Infinite-range strange metals

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^{\dagger}_{i\alpha} c_{i\beta} c^{\dagger}_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^{\dagger}_{j\beta} + c^{\dagger}_{j\beta} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J^2_{ij}} = J^2$

$N \to \infty$ at $M = 2$ yields spin-glass ground state.

$N \to \infty$ and then $M \to \infty$ yields critical strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^{\dagger} c_{i\beta} c_{j\beta}^{\dagger} c_{j\alpha}$$

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S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

OR

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^{\dagger} c_j^{\dagger} c_k c_\ell - \mu \sum_i c_i^{\dagger} c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^{\dagger} c_i$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields <u>same</u> critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, arXiv:1506.05111

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

At frequencies $\ll J$, the equations for G and Σ can be written as

$$\int d\tau_2 G(\tau_1, \tau_2)\widetilde{\Sigma}(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = -J^2 \left[G(\tau_1, \tau_2)\right]^2 G(\tau_2, \tau_1)$$

These equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

These equations and invariances have similarities to those of the large N limit of quantum spins at the spatial boundary of a CFT2 (multi-channel Kondo problems)

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
PRB **58**, 3794 (1998)

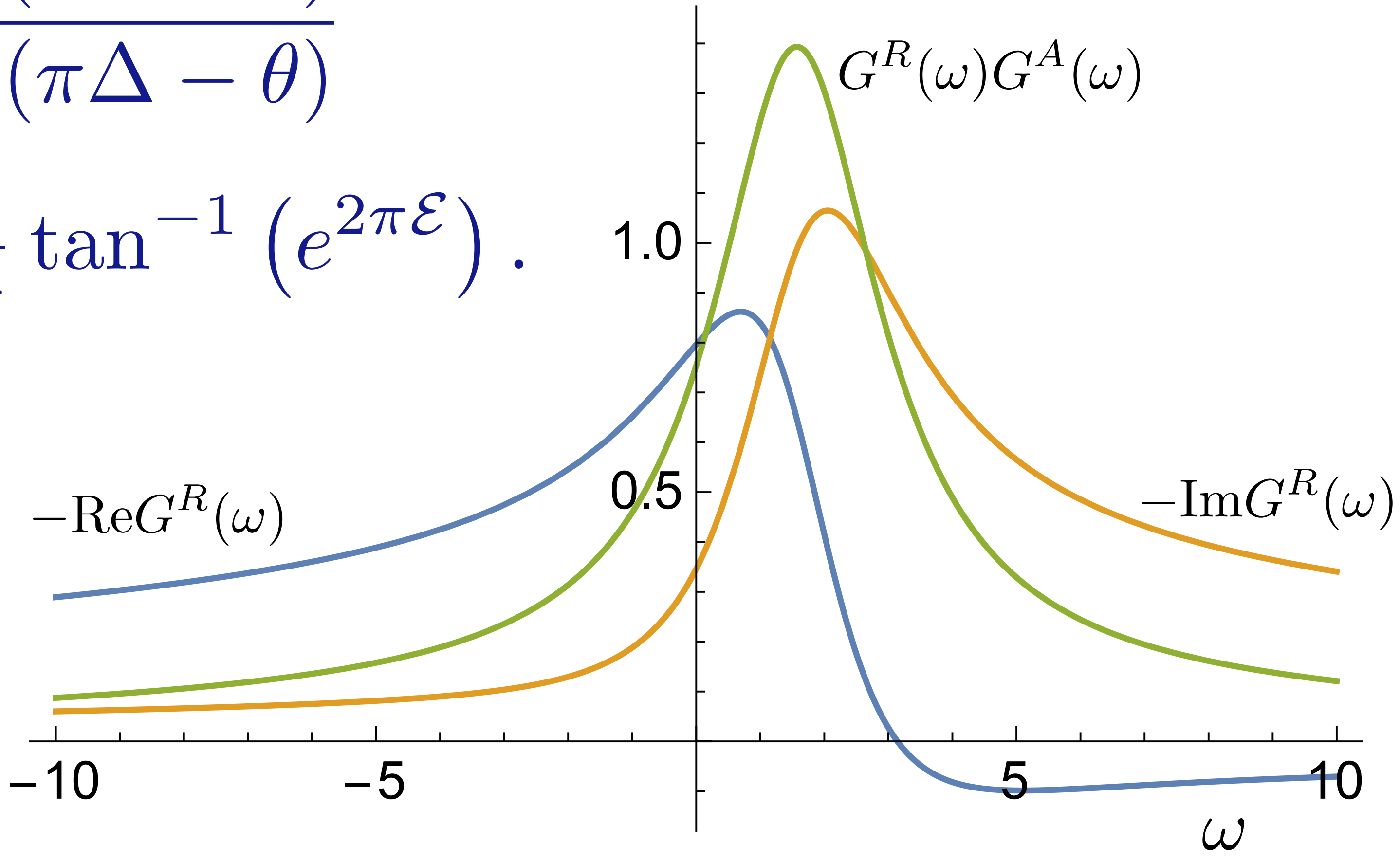
Infinite-range strange metals

From these expressions we obtain the Green's function

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$

and $\mathcal{Q} = \dfrac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \dfrac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$



S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).

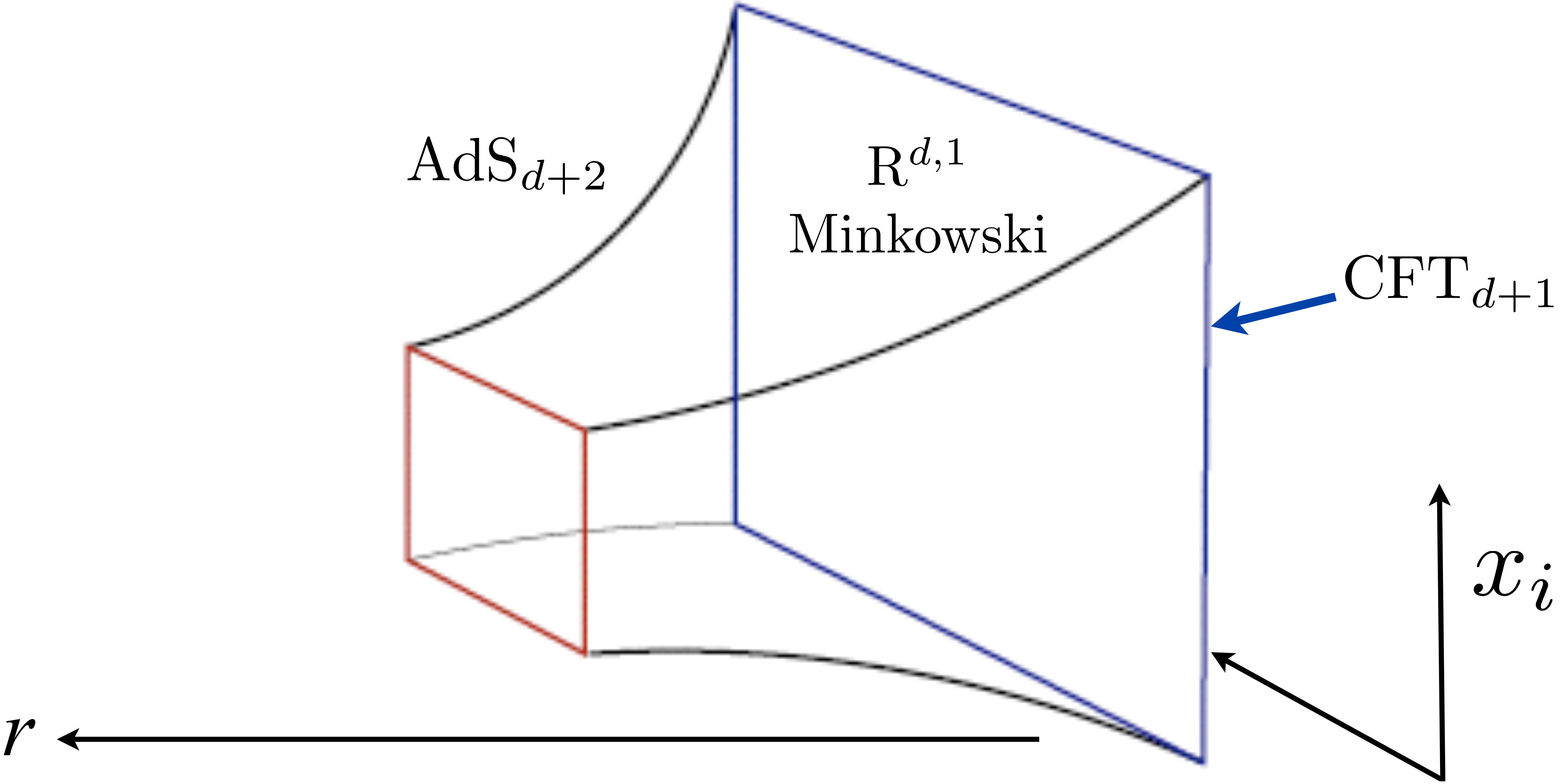
This entropy obeys

$$\boxed{\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

AdS/CFT correspondence at zero temperature

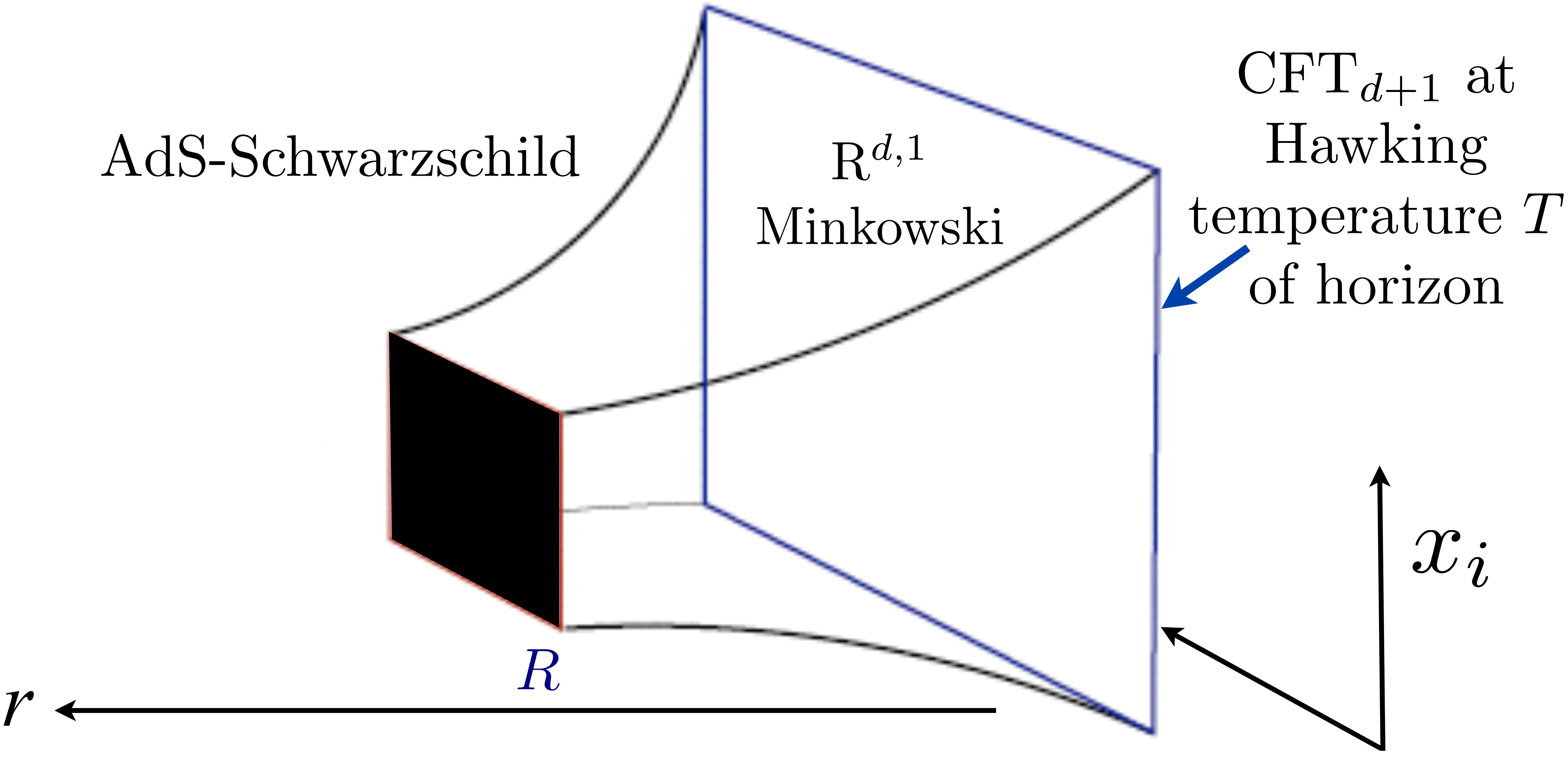
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



AdS-Schwarzschild

$\mathrm{R}^{d,1}$
Minkowski

CFT$_{d+1}$ at
Hawking
temperature T
of horizon

x_i

R

r

$$ds^2 = \left(\frac{L}{r}\right)^2\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

with $f(r) = 1 - (r/R)^{d+1}$ and $T = (d+1)/(4\pi R)$.

AdS/CFT correspondence at non-zero temperature

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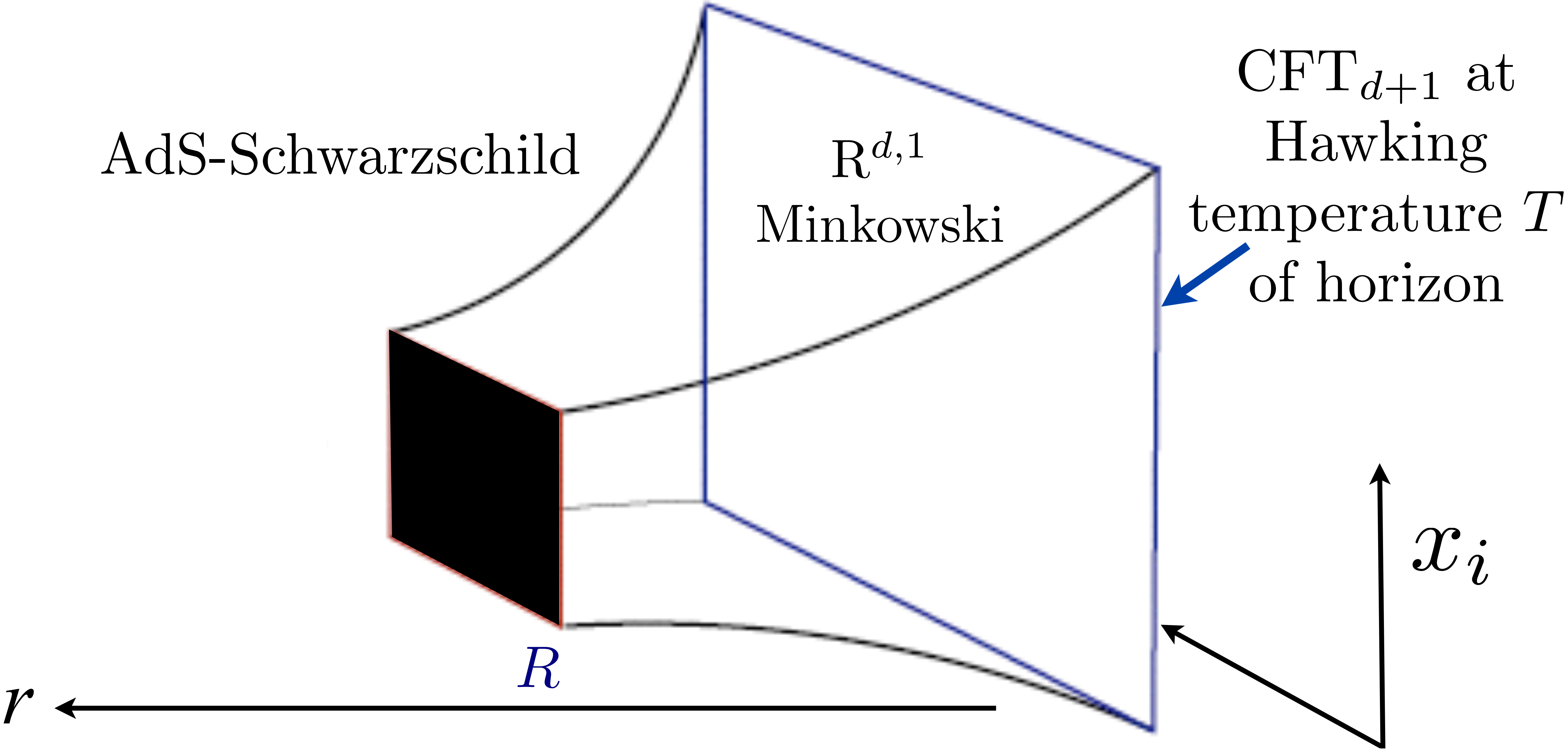


AdS-Schwarzschild

$\mathrm{R}^{d,1}$ Minkowski

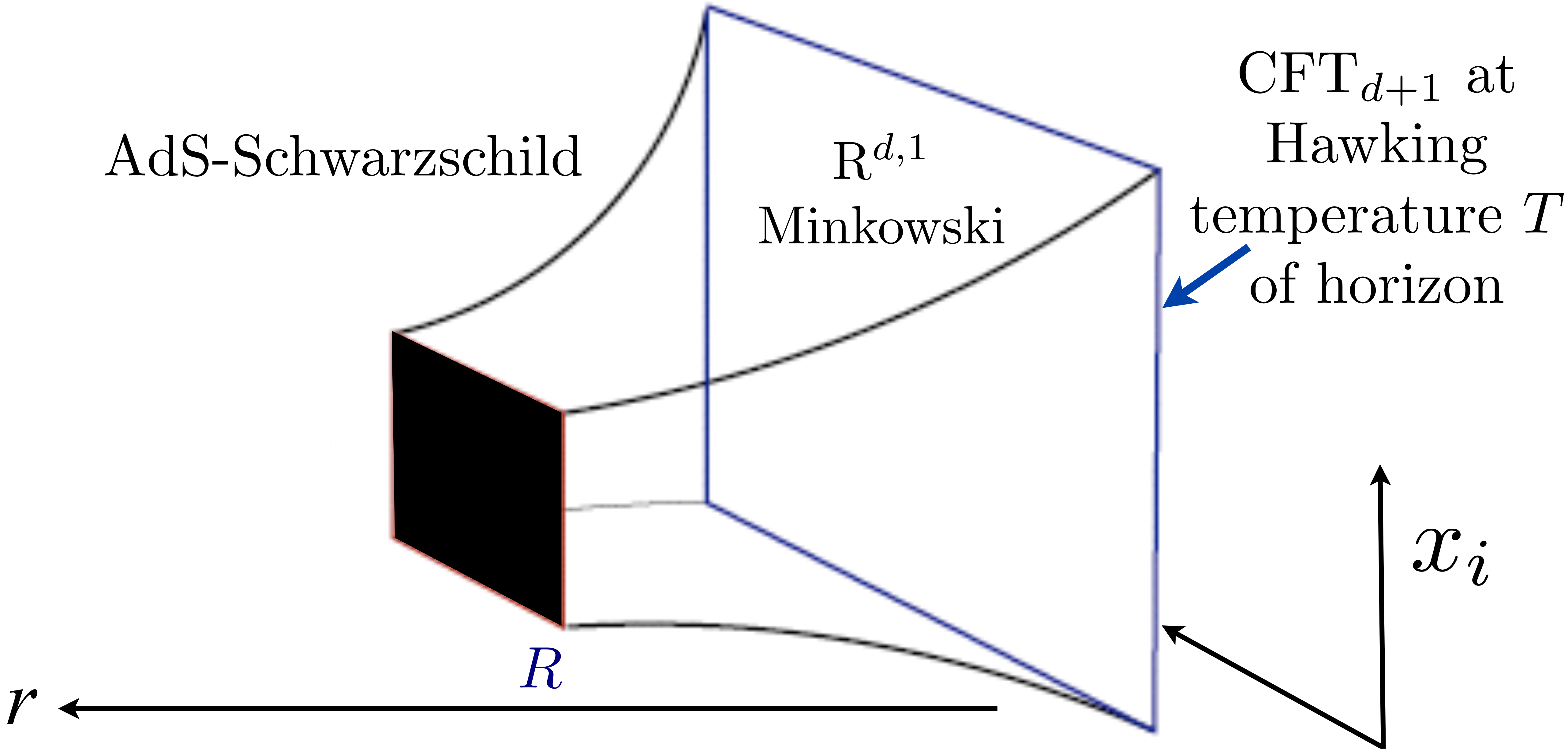
CFT_{d+1} at Hawking temperature T of horizon

Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R}+\frac{d(d+1)}{L^2}\right)\right]$



AdS-Schwarzschild

$\mathrm{R}^{d,1}$
Minkowski

CFT_{d+1} at
Hawking
temperature T
of horizon

R

r

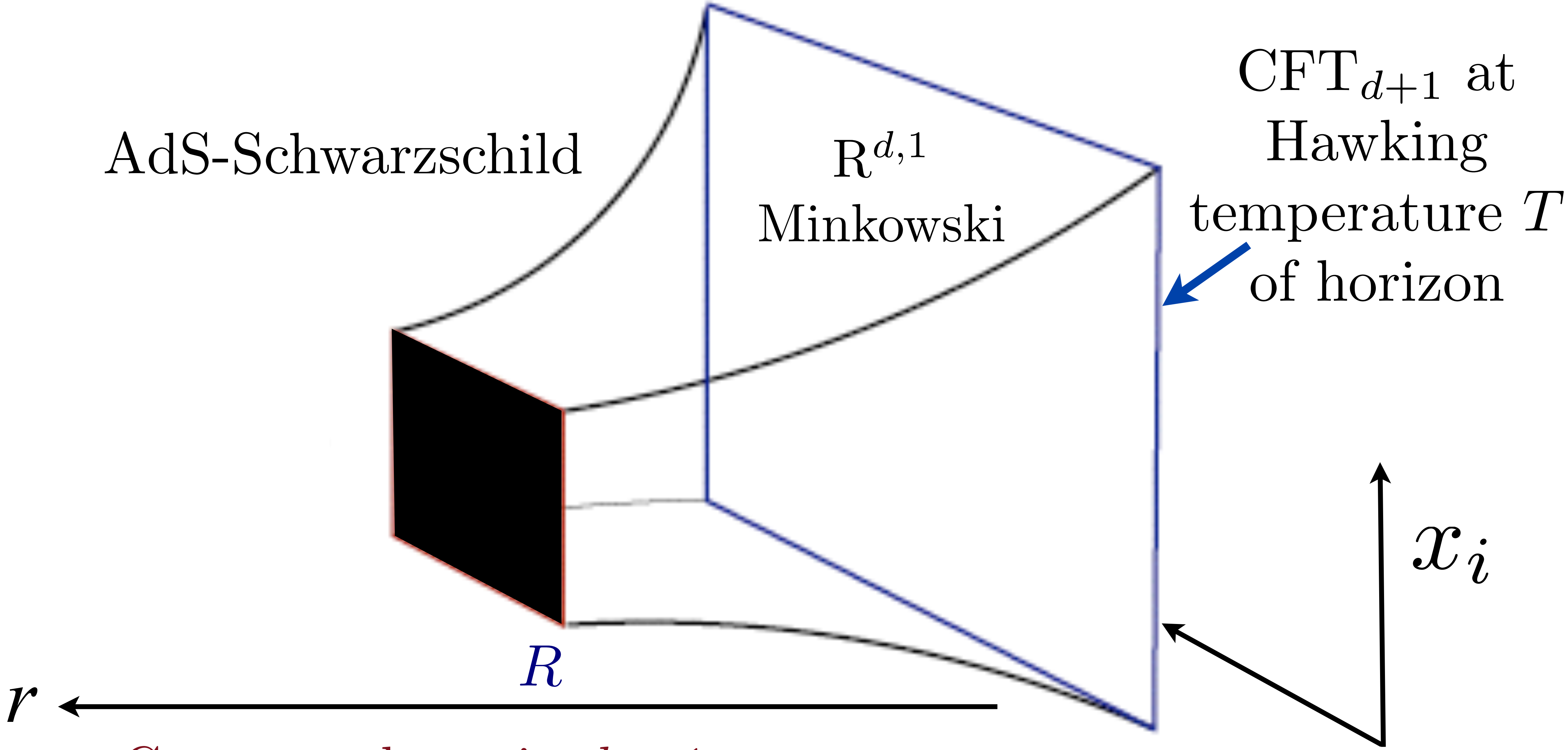
x_i

For SU(N) SYM in $d = 3$, $\mathcal{S}_{\mathrm{BH}} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

Gubser, Klebanov, Peet 96

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$



AdS-Schwarzschild

$\mathrm{R}^{d,1}$
Minkowski

CFT_{d+1} at
Hawking
temperature T
of horizon

x_i

R

r

Correspondence in $d = 1$:

$$\mathcal{S} = \mathcal{S}_{\mathrm{BH}} = \frac{\pi}{3} c T,$$

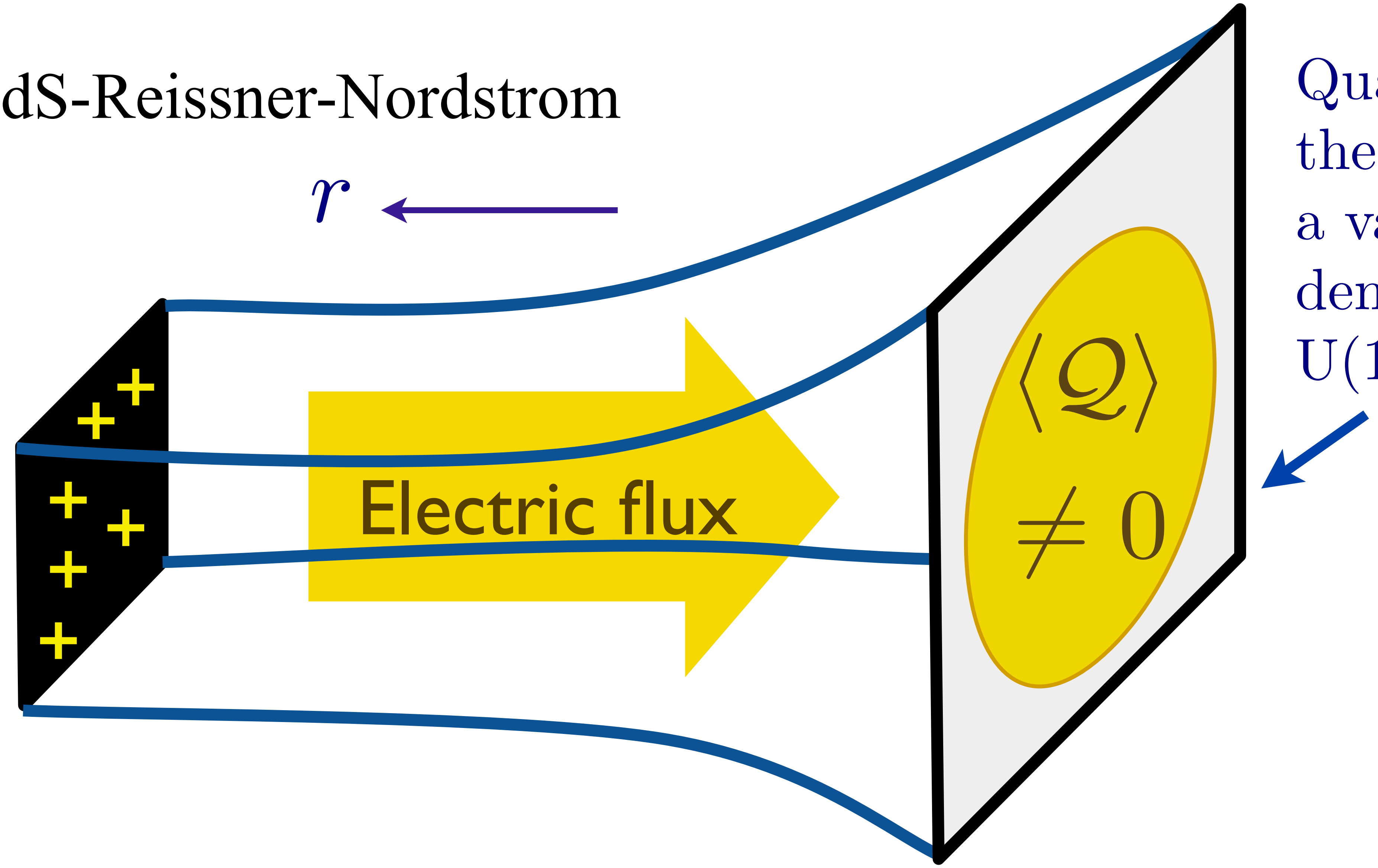
where $c = 12\pi L/\kappa^2$ is the central charge of the CFT_2.

Strominger 97

Charged black branes

Einstein-Maxwell theory $$\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$$

AdS-Reissner-Nordstrom



Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

Charged black branes

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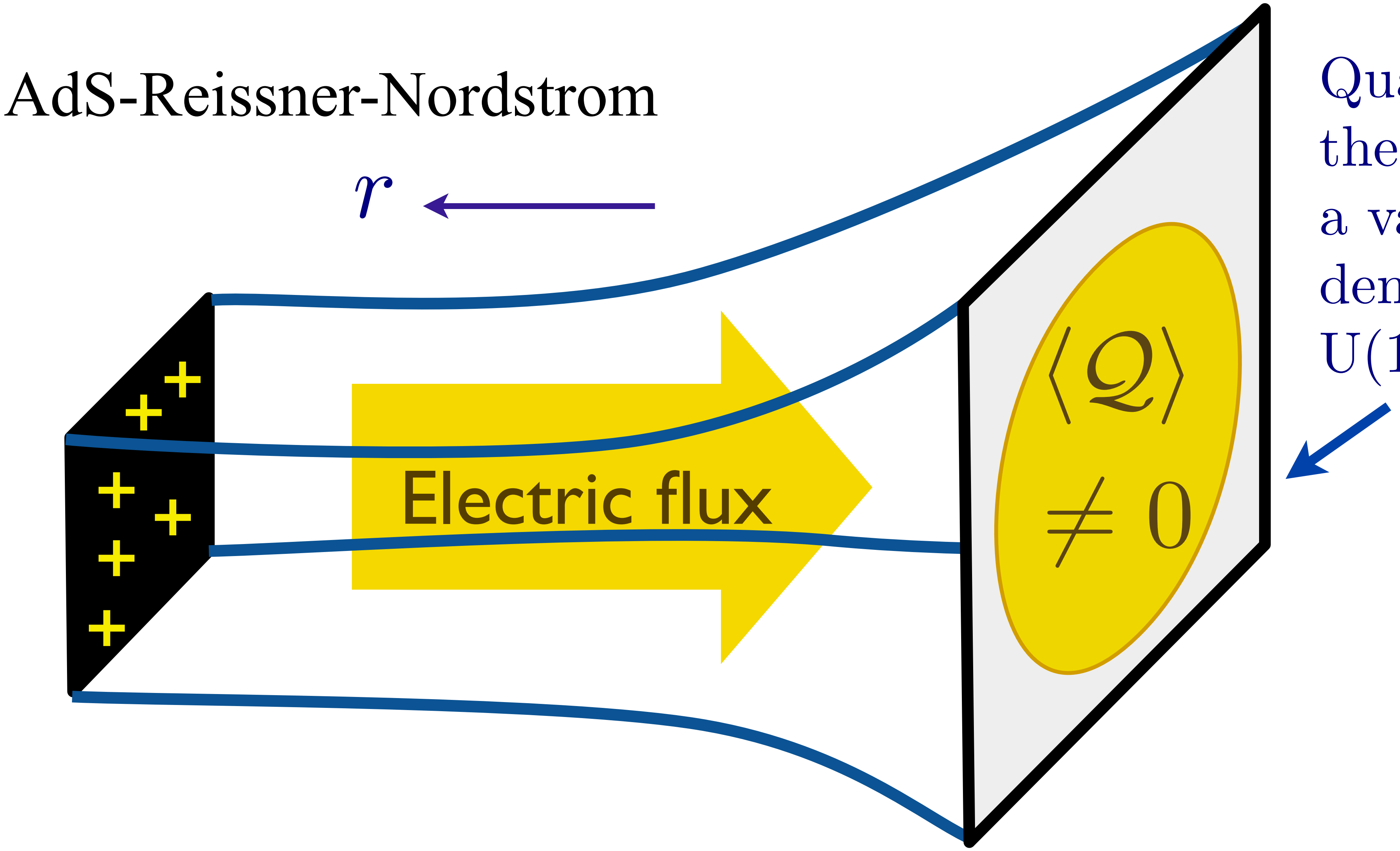
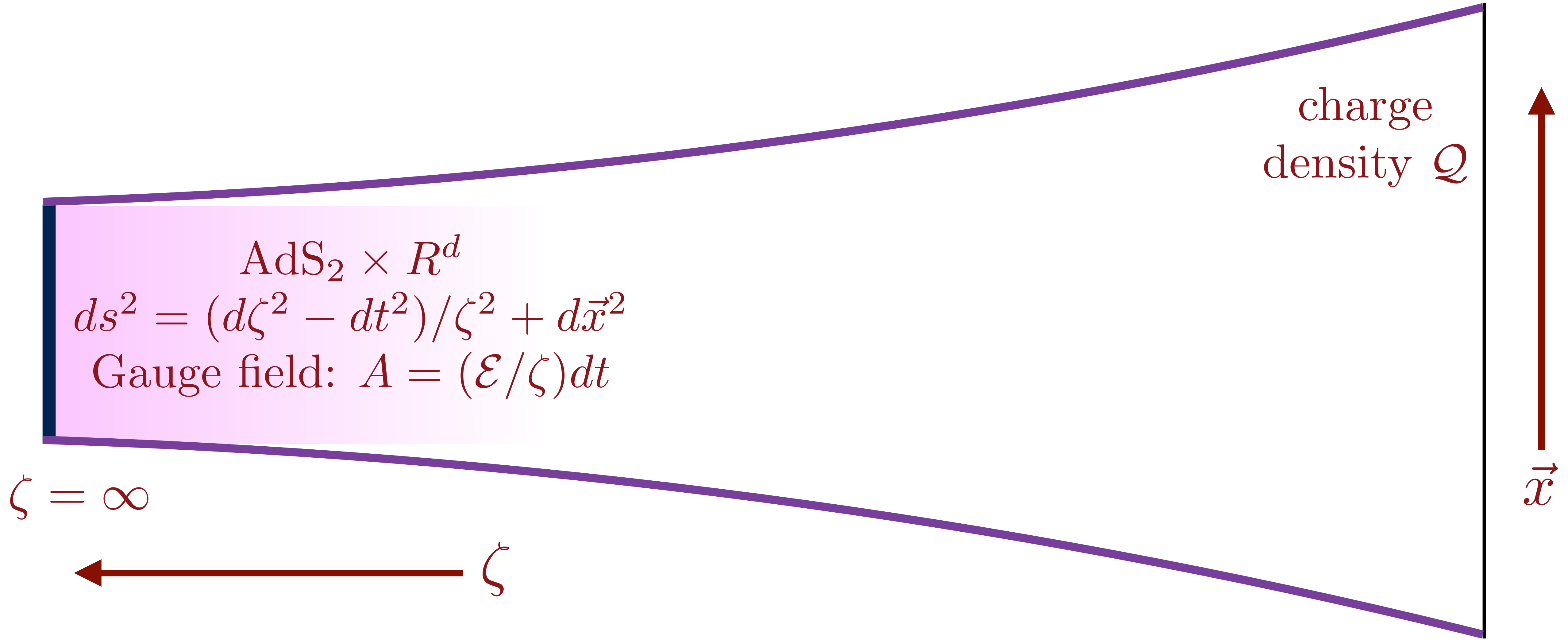


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

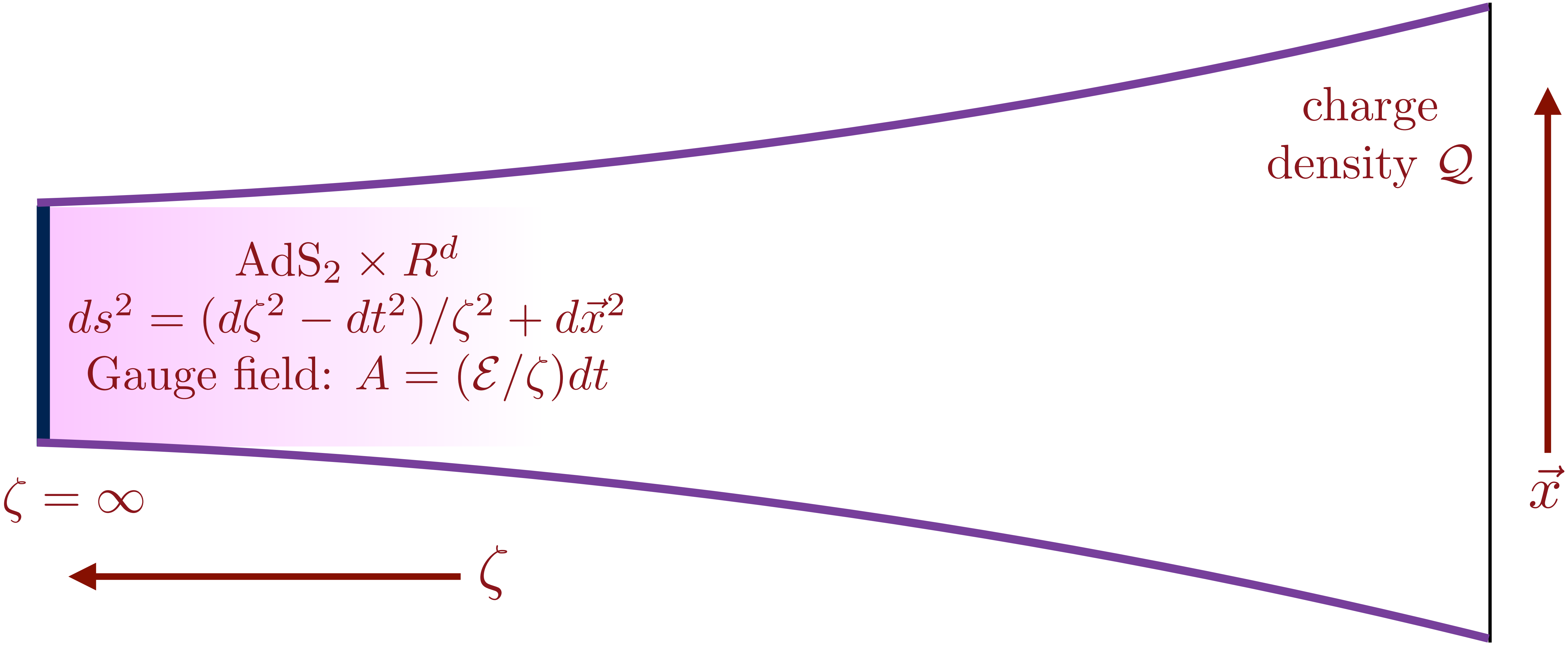
A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

General Relativity of charged black branes



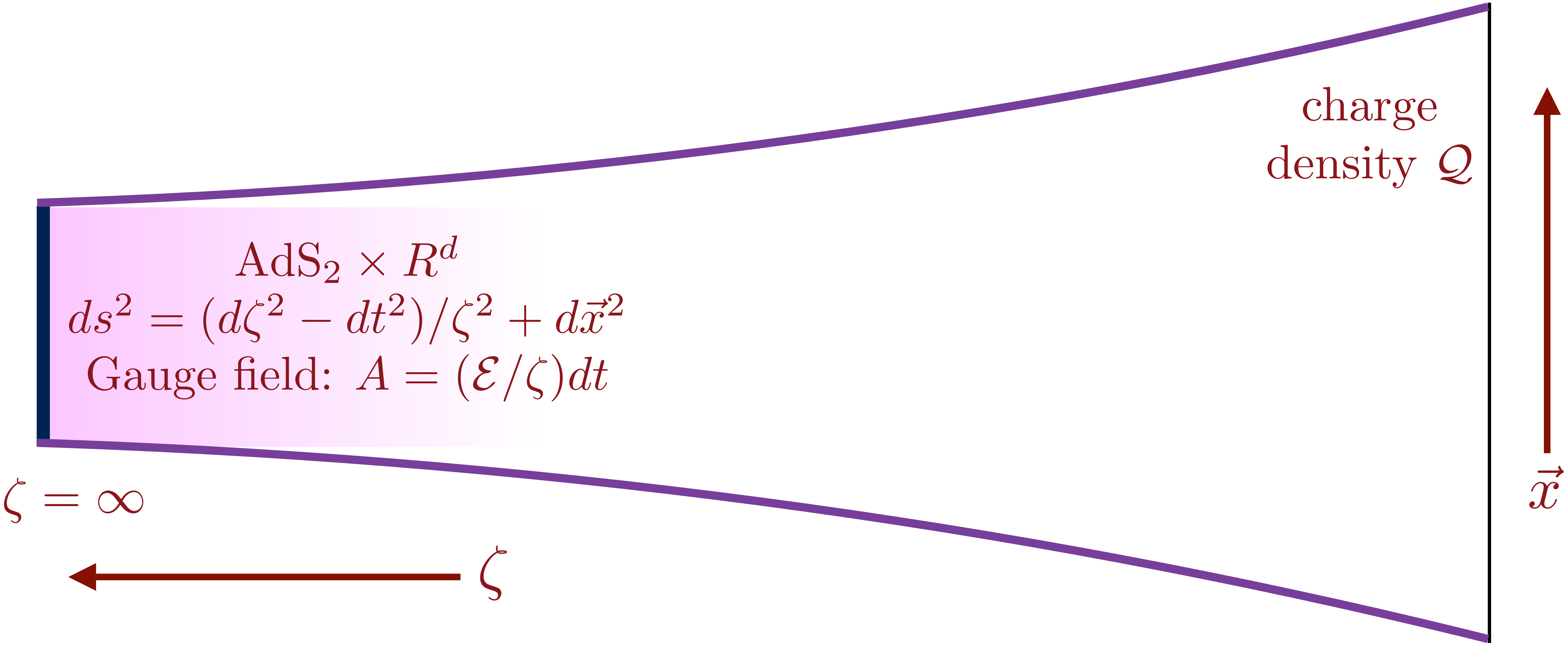
- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.

General Relativity of charged black branes



- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{\mathrm{BH}}$

General Relativity of charged black branes



charge density $\mathcal{Q}$

$\mathrm{AdS}_2 \times R^d$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

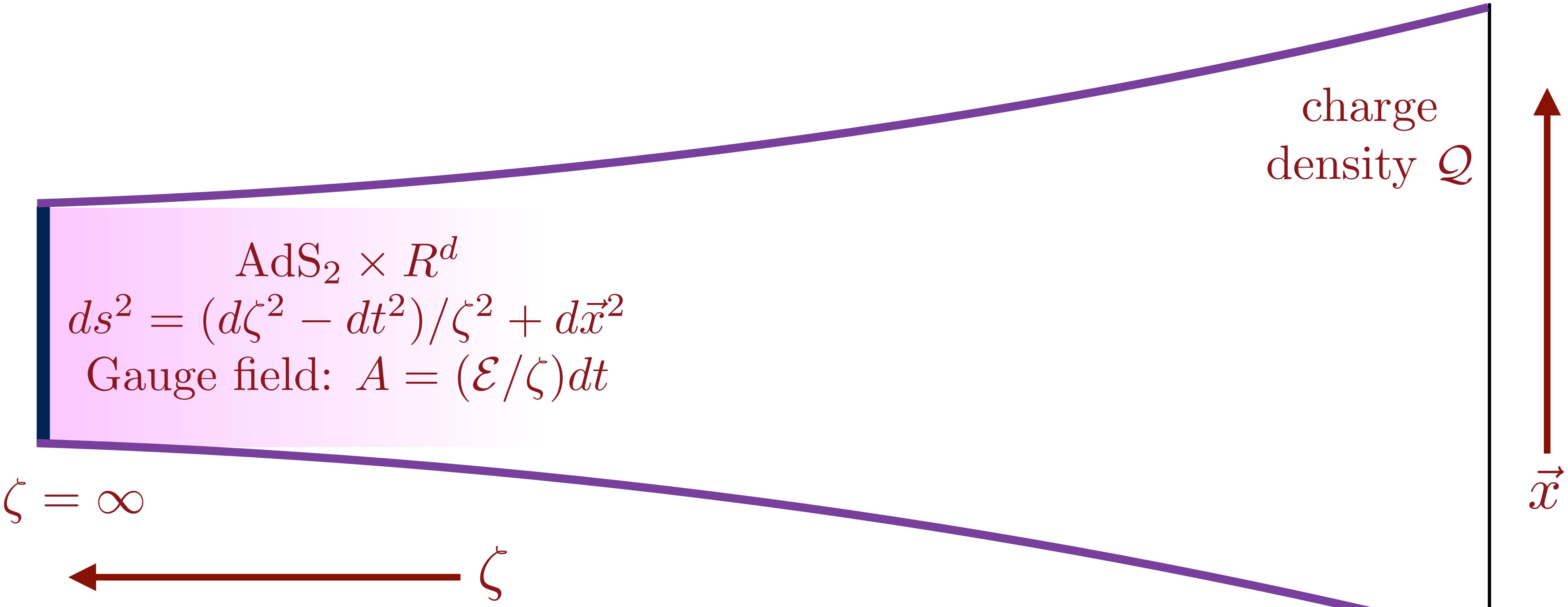
$\zeta = \infty$

ζ

$\vec{x}$

- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.
- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{\mathrm{BH}}$
- Both $\mathcal{E}$ and $\mathcal{S}_{\mathrm{BH}}$ are determined by $\mathcal{Q}$, and both vanish as $\mathcal{Q} \to 0$.

General Relativity of charged black branes



$\mathrm{AdS}_2 \times R^d$

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$\vec{x}$

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ζ

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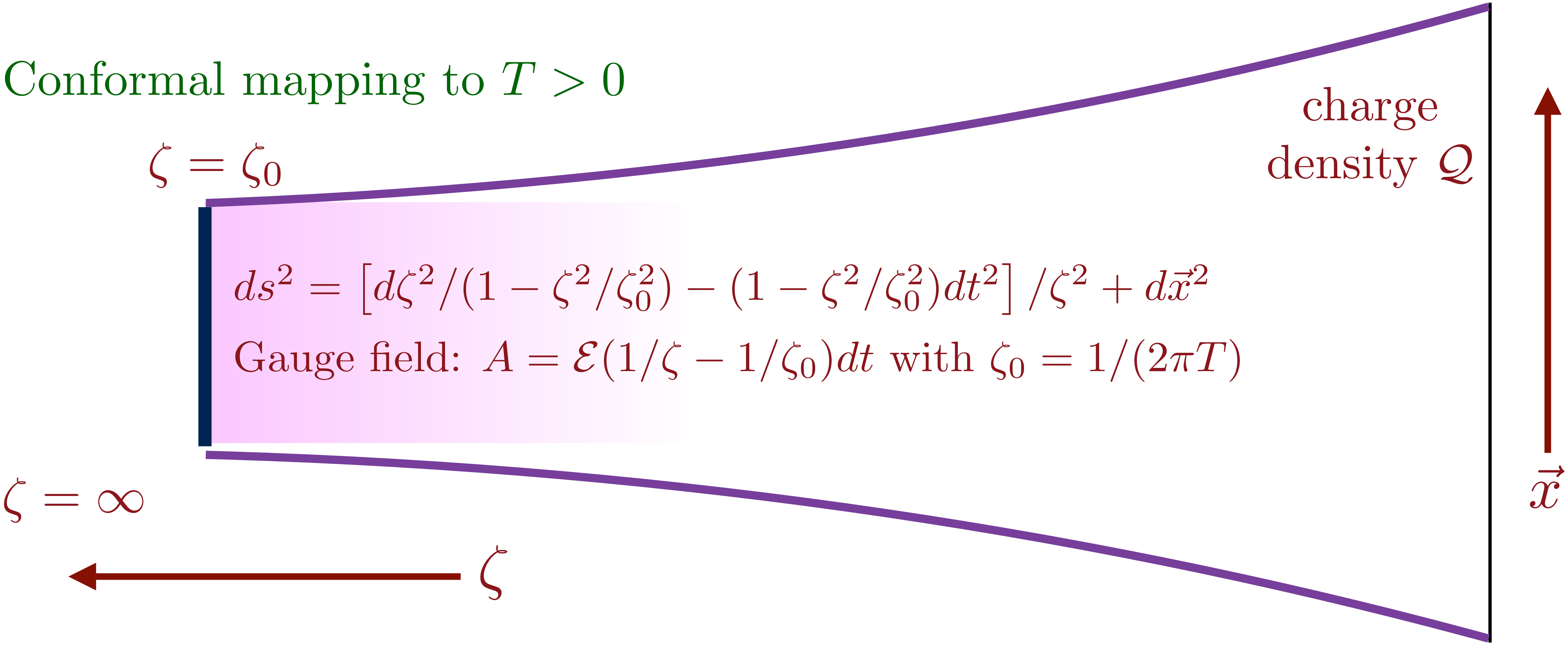
- Both $\mathcal{E}$ and $\mathcal{S}_{\mathrm{BH}}$ are determined by $\mathcal{Q}$, and both vanish as $\mathcal{Q} \to 0$.
- Near the boundary, $A = \mu\, dt$, where μ is the chemical potential

General Relativity of charged black branes

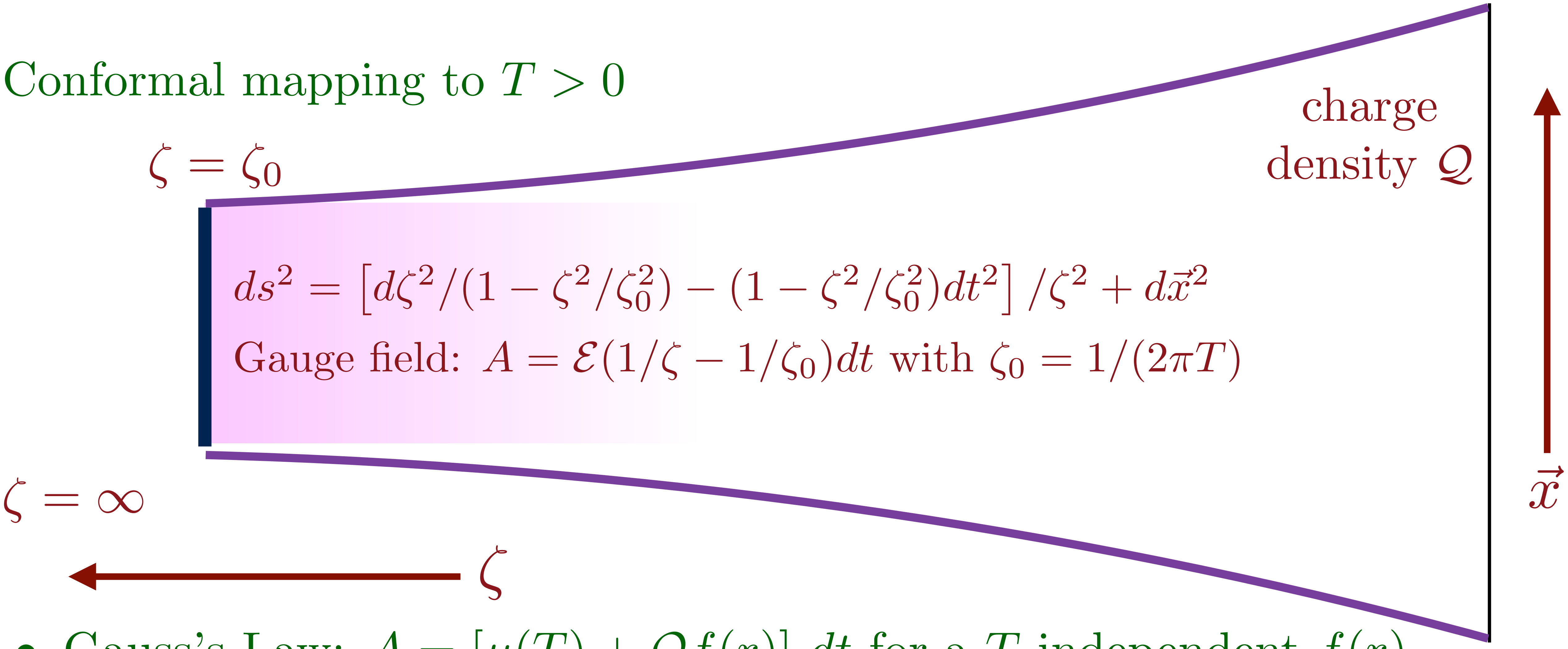
Conformal mapping to $T > 0$



$$ds^2 = \left[d\zeta^2/(1-\zeta^2/\zeta_0^2) - (1-\zeta^2/\zeta_0^2)dt^2\right]/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)dt$ with $\zeta_0 = 1/(2\pi T)$

General Relativity of charged black branes



Conformal mapping to $T > 0$

$\zeta = \zeta_0$

charge density $\mathcal{Q}$

$$ds^2 = \left[d\zeta^2/(1-\zeta^2/\zeta_0^2) - (1-\zeta^2/\zeta_0^2)dt^2\right]/\zeta^2 + d\vec{x}^2$$

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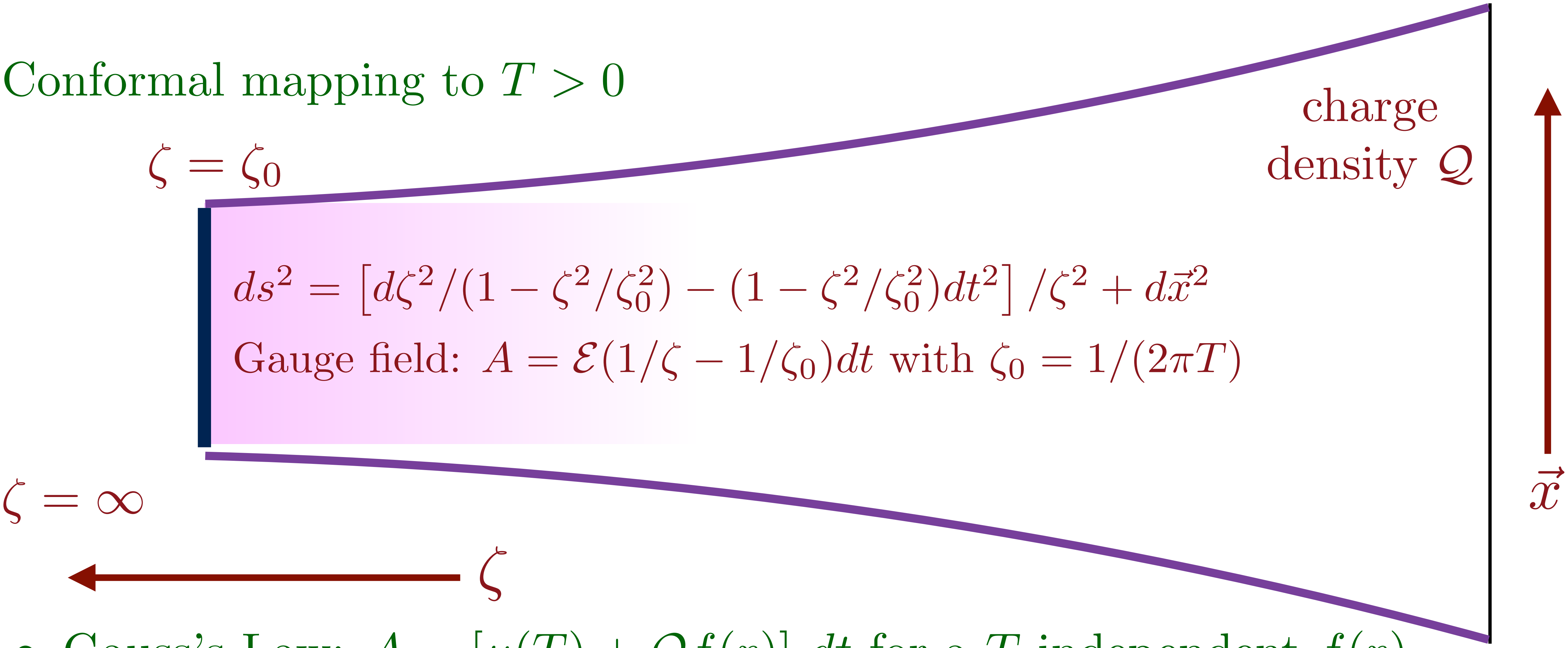
- Gauss's Law: $A = \left[\mu(T) + \mathcal{Q} f(r)\right] dt$ for a T-independent $f(r)$.

General Relativity of charged black branes



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$\zeta = \infty$

$\vec{x}$

ζ

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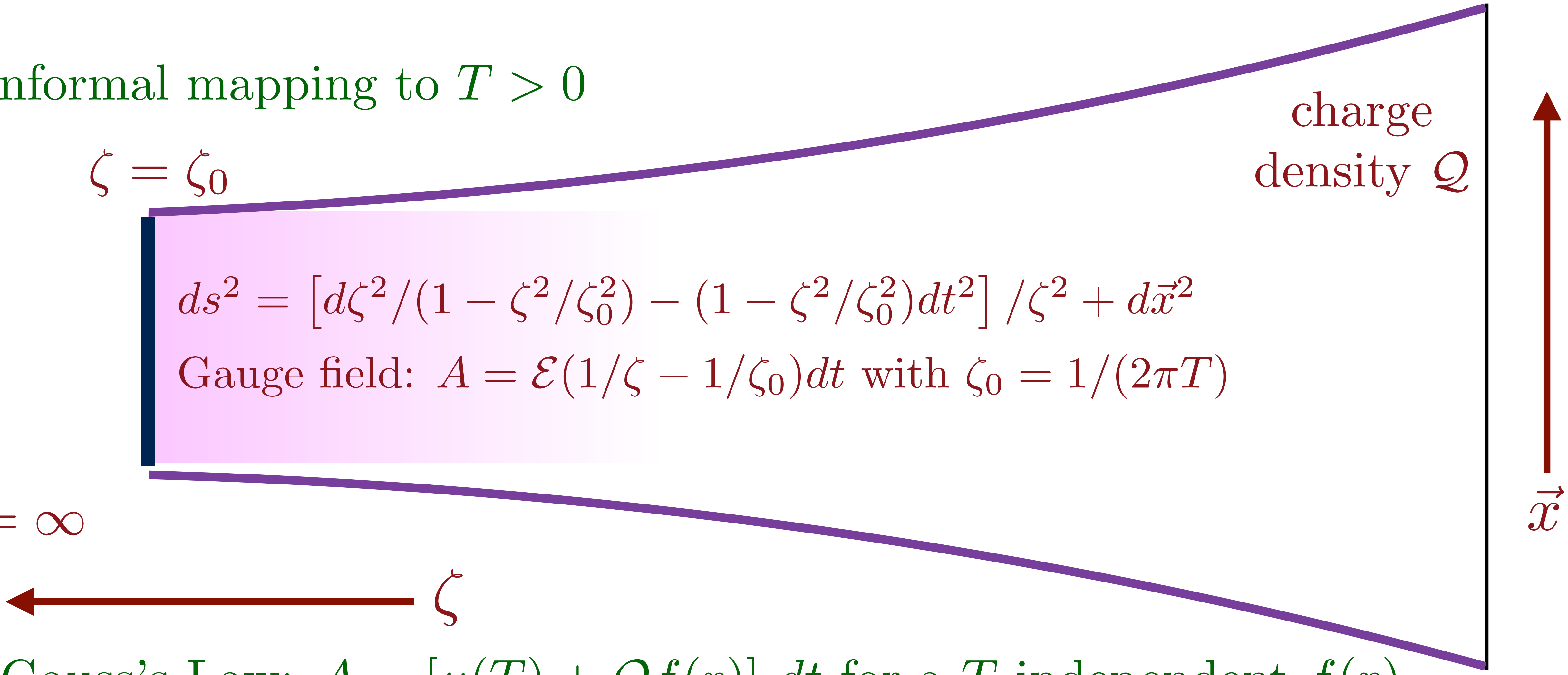
- Comparing with the near-horizon: $\mu(T) = -2\pi\mathcal{E}T +$ constant as $T \to 0$.

General Relativity of charged black branes



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$\zeta = \infty$

$\vec{x}$

ζ

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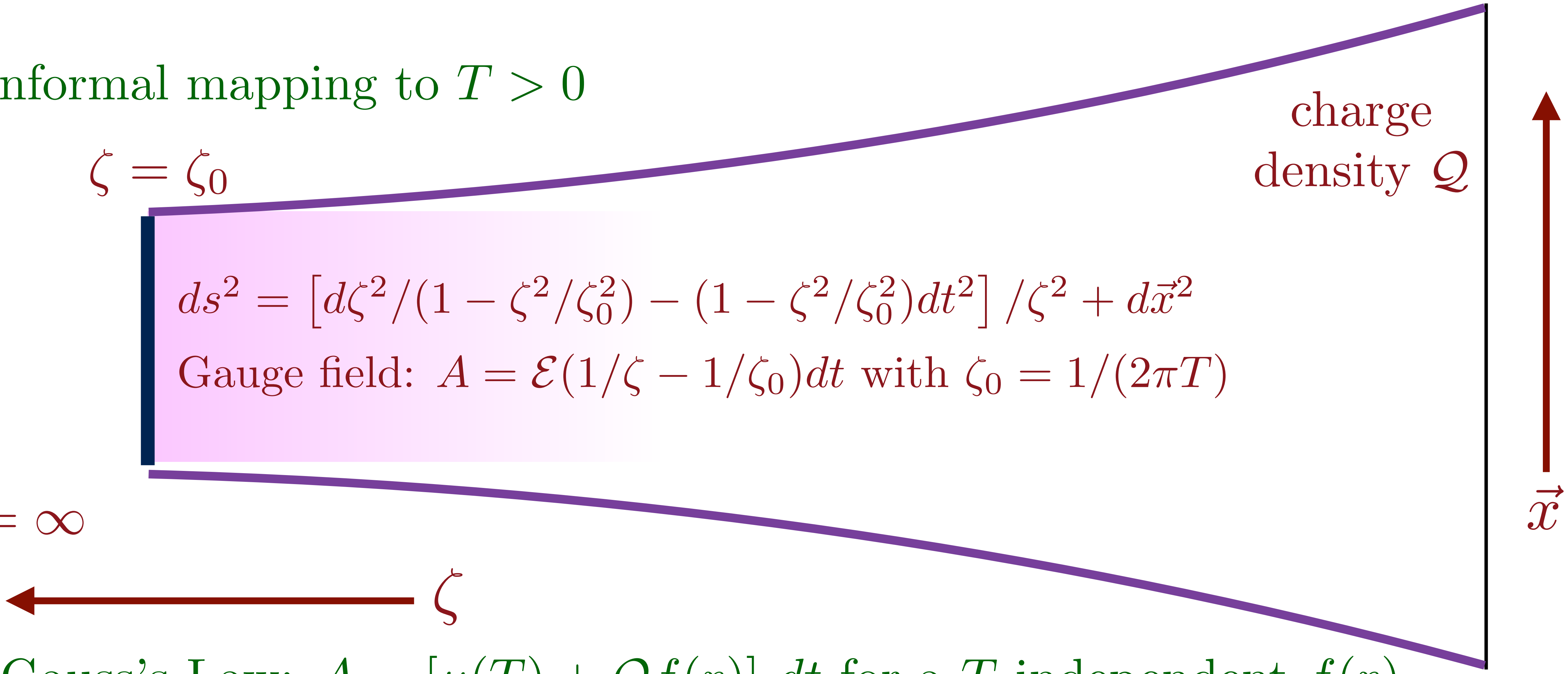
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

$$\boxed{\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

General Relativity of charged black branes

Conformal mapping to $T > 0$



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A. Sen
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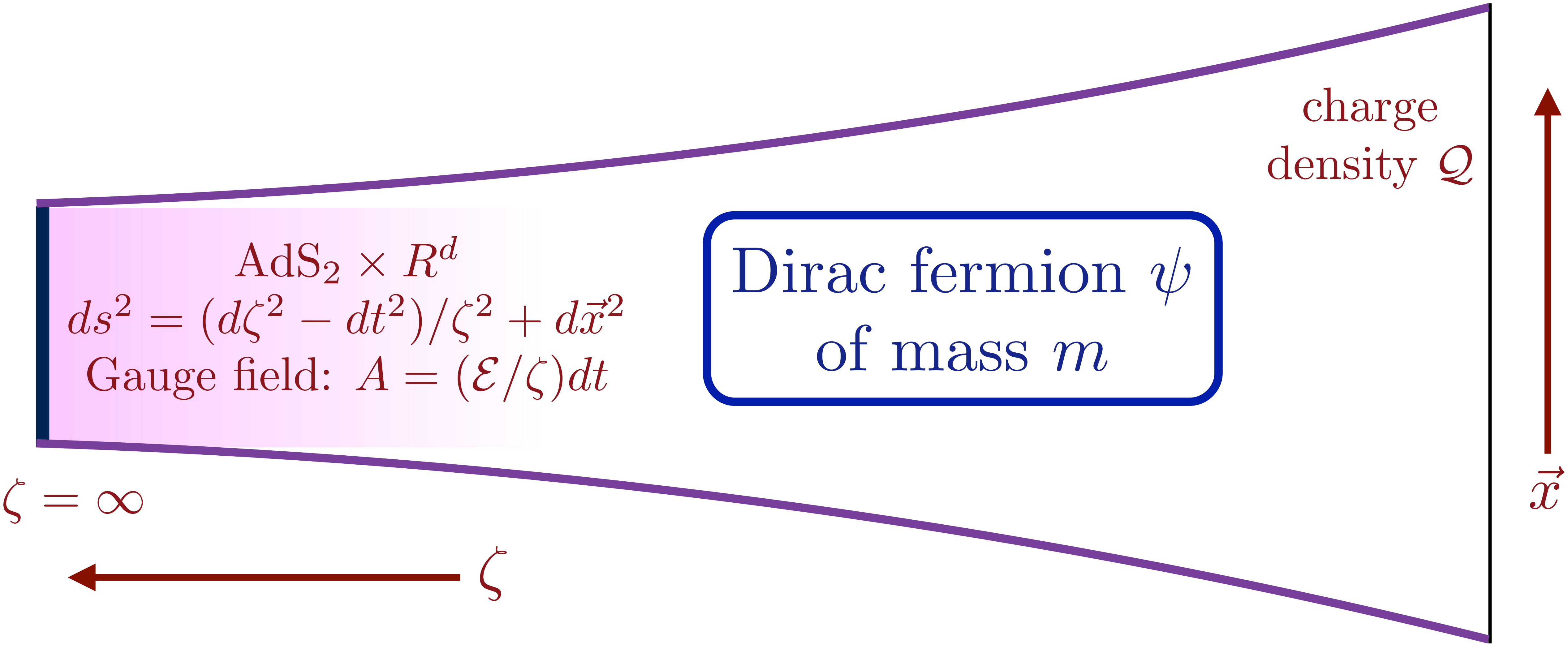
$$\boxed{\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}}$$

- Also obeyed by the Wald entropy in higher derivative gravity.

Quantum fields on charged black branes



$\mathrm{AdS}_2 \times R^d$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

Dirac fermion ψ of mass m

charge density $\mathcal{Q}$

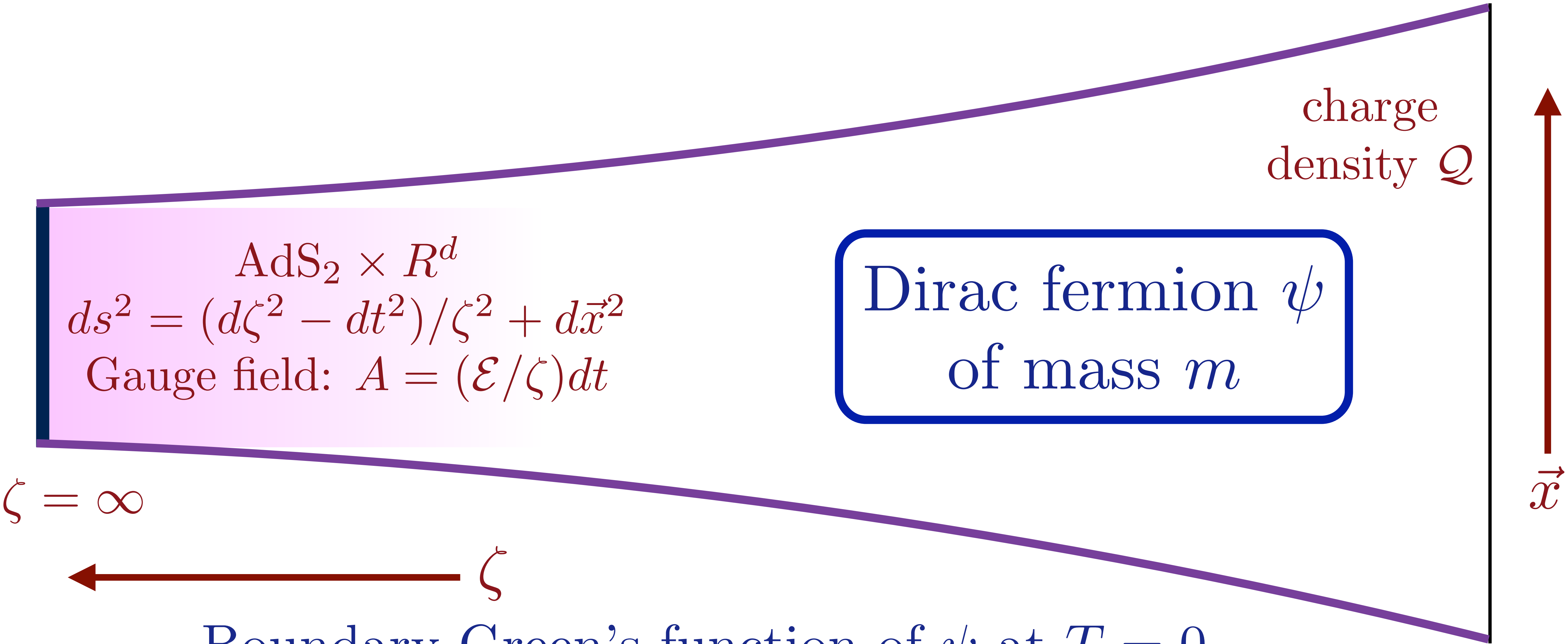
$\zeta = \infty$

ζ

$\vec{x}$

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

Quantum fields on charged black branes



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Gauge field: $A = (\mathcal{E}/\zeta)dt$

Dirac fermion ψ of mass m

$\zeta = \infty$

ζ

$\vec{x}$

Boundary Green's function of ψ at $T = 0$

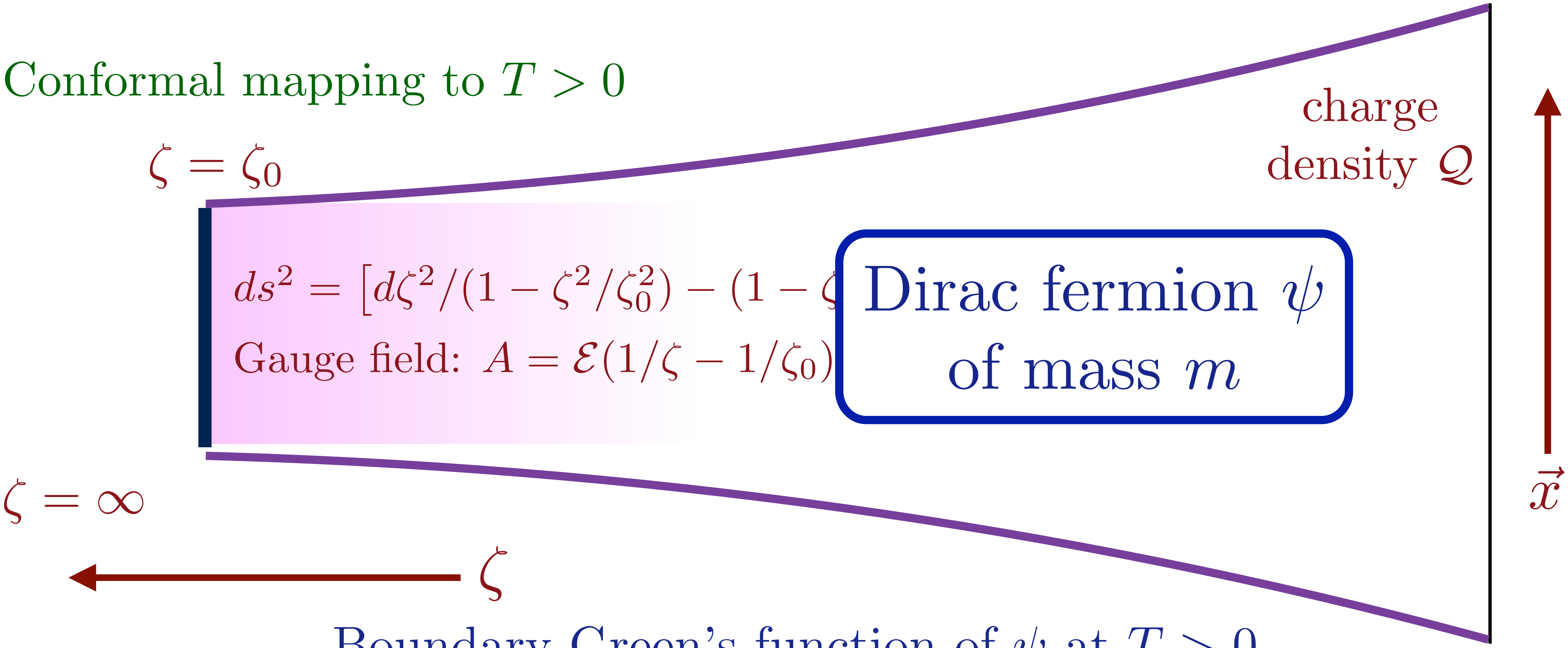
$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \ \omega < 0. \end{cases}$$

where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

Quantum fields on charged black branes



Conformal mapping to $T > 0$

$\zeta = \zeta_0$

$ds^2 = \left[d\zeta^2/(1-\zeta^2/\zeta_0^2) - (1-\zeta\right.$

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)$

Dirac fermion ψ
of mass m

charge
density $\mathcal{Q}$

$\zeta = \infty$

ζ

$\vec{x}$

Boundary Green's function of ψ at $T > 0$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)

What is a possible quantum theory on the boundary ?

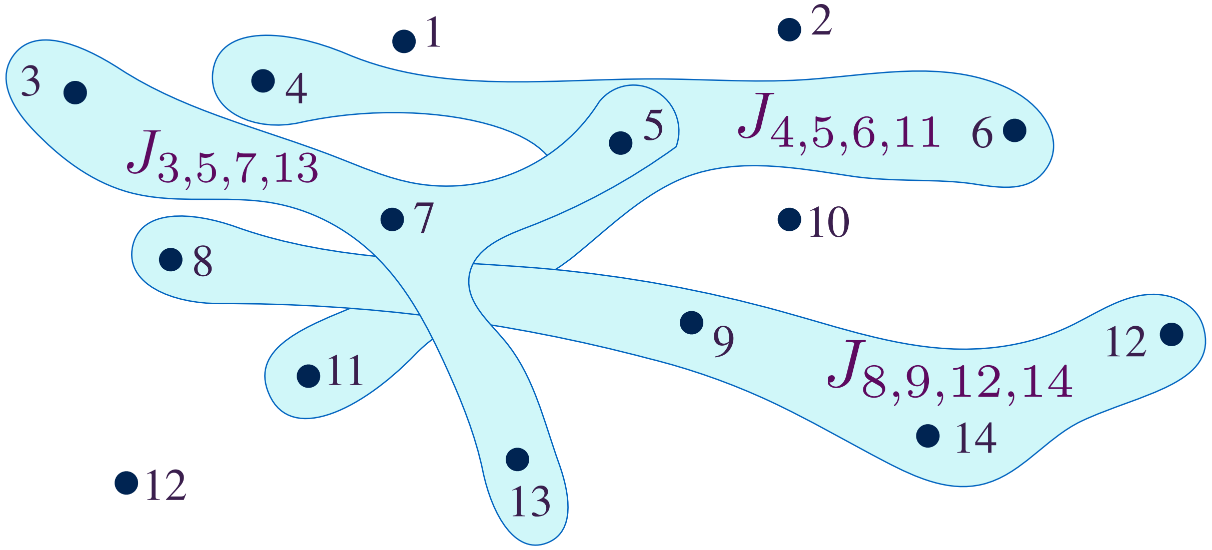
What is a possible quantum theory on the boundary ?

A critical strange metal state with infinite-range interactions obtained in

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N}\sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

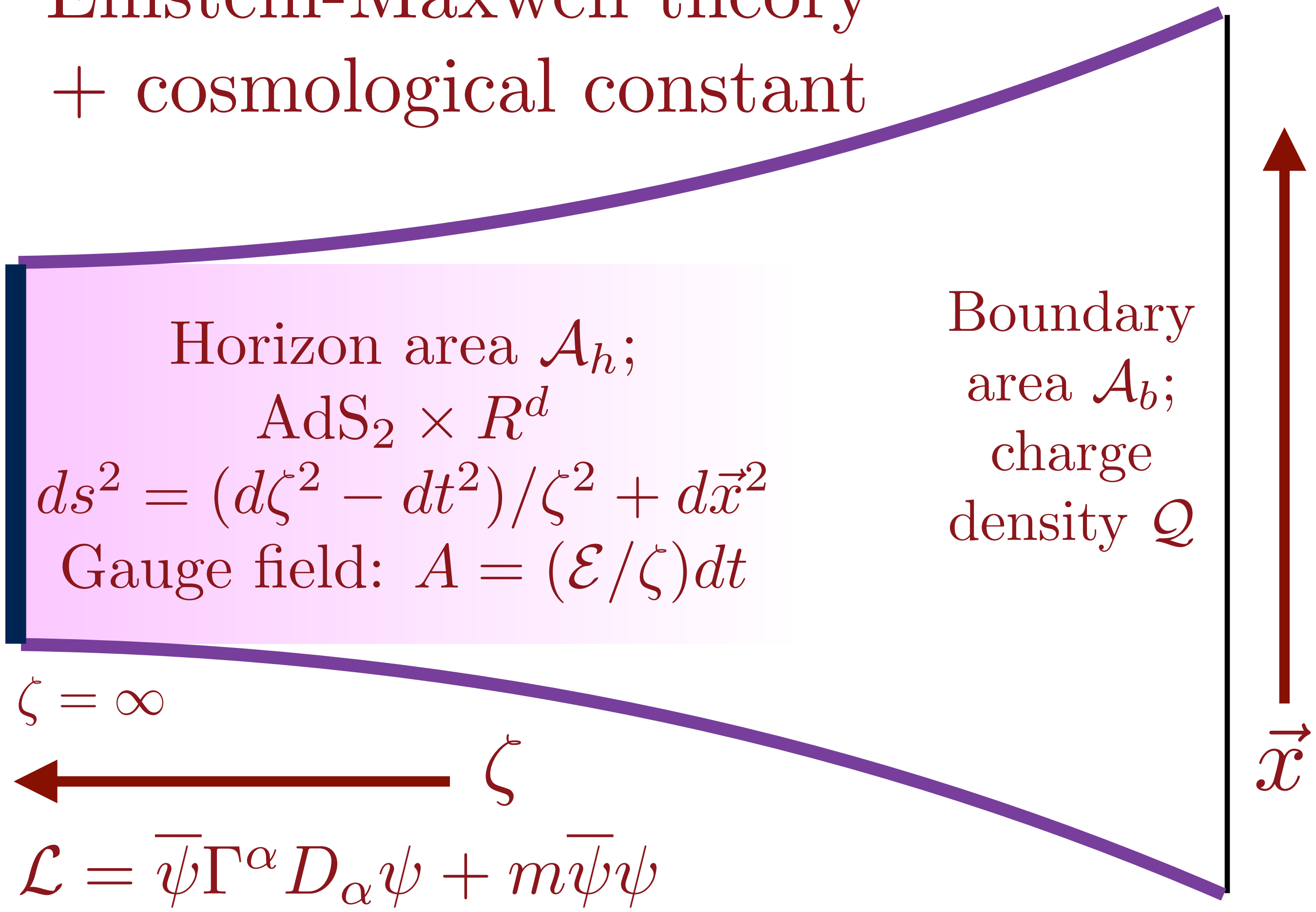
$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known 'equation of state' determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

$$\frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

Einstein-Maxwell theory
+ cosmological constant



Horizon area $\mathcal{A}_h$;
AdS$_2$ × R^d
$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
Gauge field: $A = (\mathcal{E}/\zeta)dt$

Boundary
area $\mathcal{A}_b$;
charge
density $\mathcal{Q}$

$$\mathcal{L} = \overline{\psi}\Gamma^\alpha D_\alpha \psi + m\overline{\psi}\psi$$

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'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS$_2$

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511

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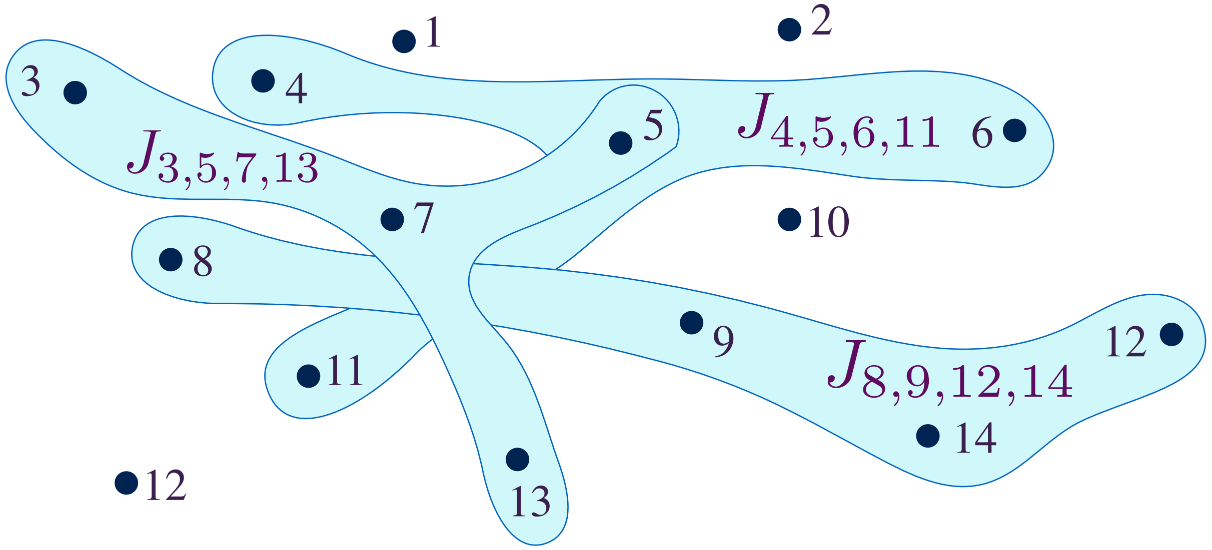

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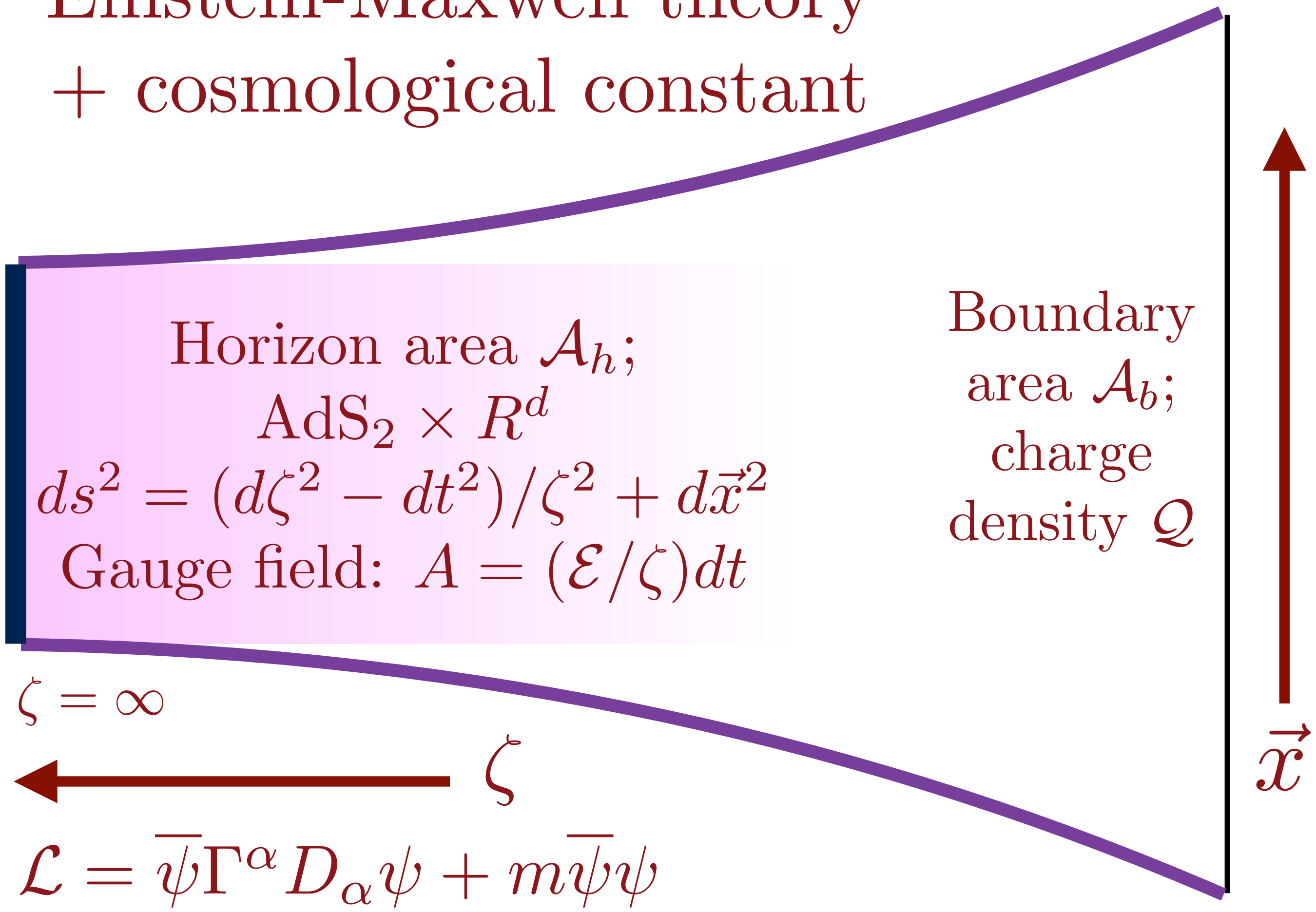

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Evidence for
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dual of H

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S. Sachdev, arXiv:1506.05111

1. Topological order in insulating spin liquids

2. Review of Fermi liquid theory

   Topological argument for the Luttinger theorem

3. Fractionalized Fermi liquid

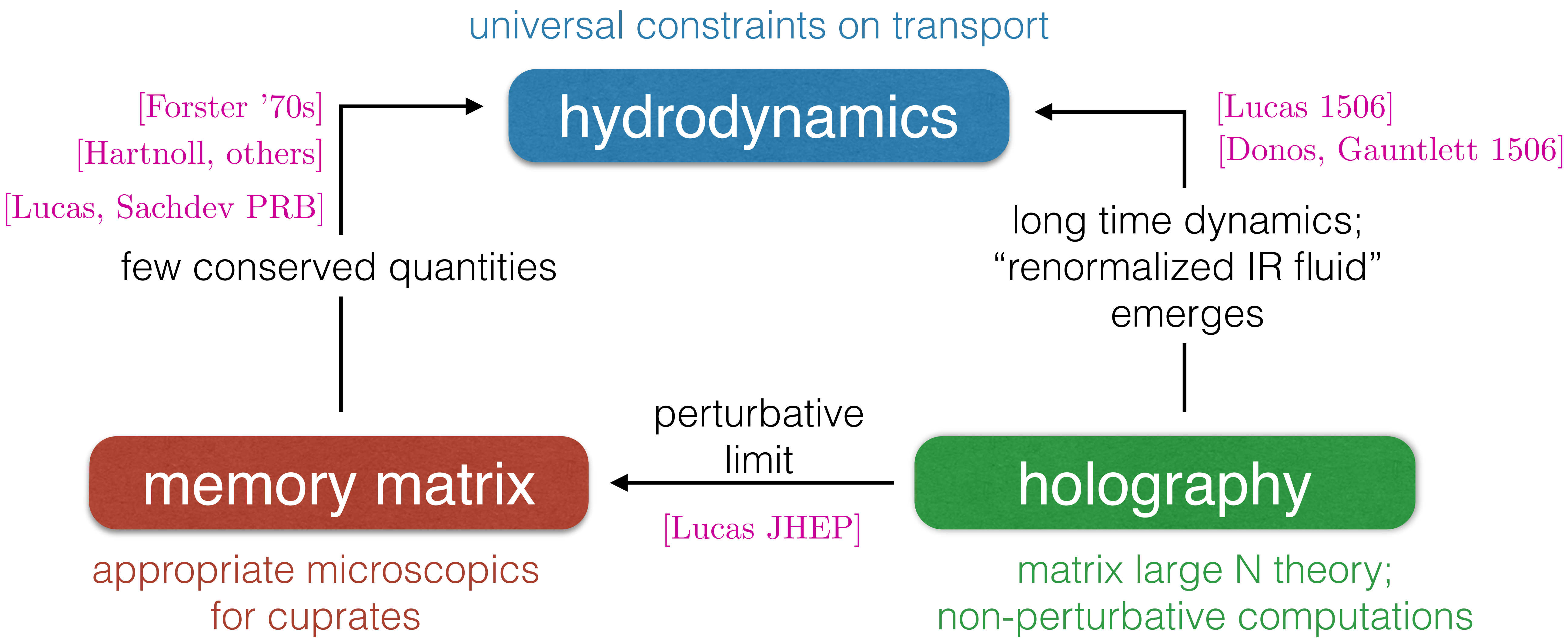
   A Fermi liquid co-existing with topological order for the pseudogap metal

4. Strange metals without quasiparticles

   (a) A mean-field model of a non-Fermi liquid, and charged black holes

   (b) A (slightly less) strange metal in graphene

Transport in Strange Metals



universal constraints on transport

hydrodynamics

[Forster '70s]
[Hartnoll, others]
[Lucas, Sachdev PRB]

few conserved quantities

[Lucas 1506]
[Donos, Gauntlett 1506]

long time dynamics;
"renormalized IR fluid"
emerges

memory matrix

appropriate microscopics
for cuprates

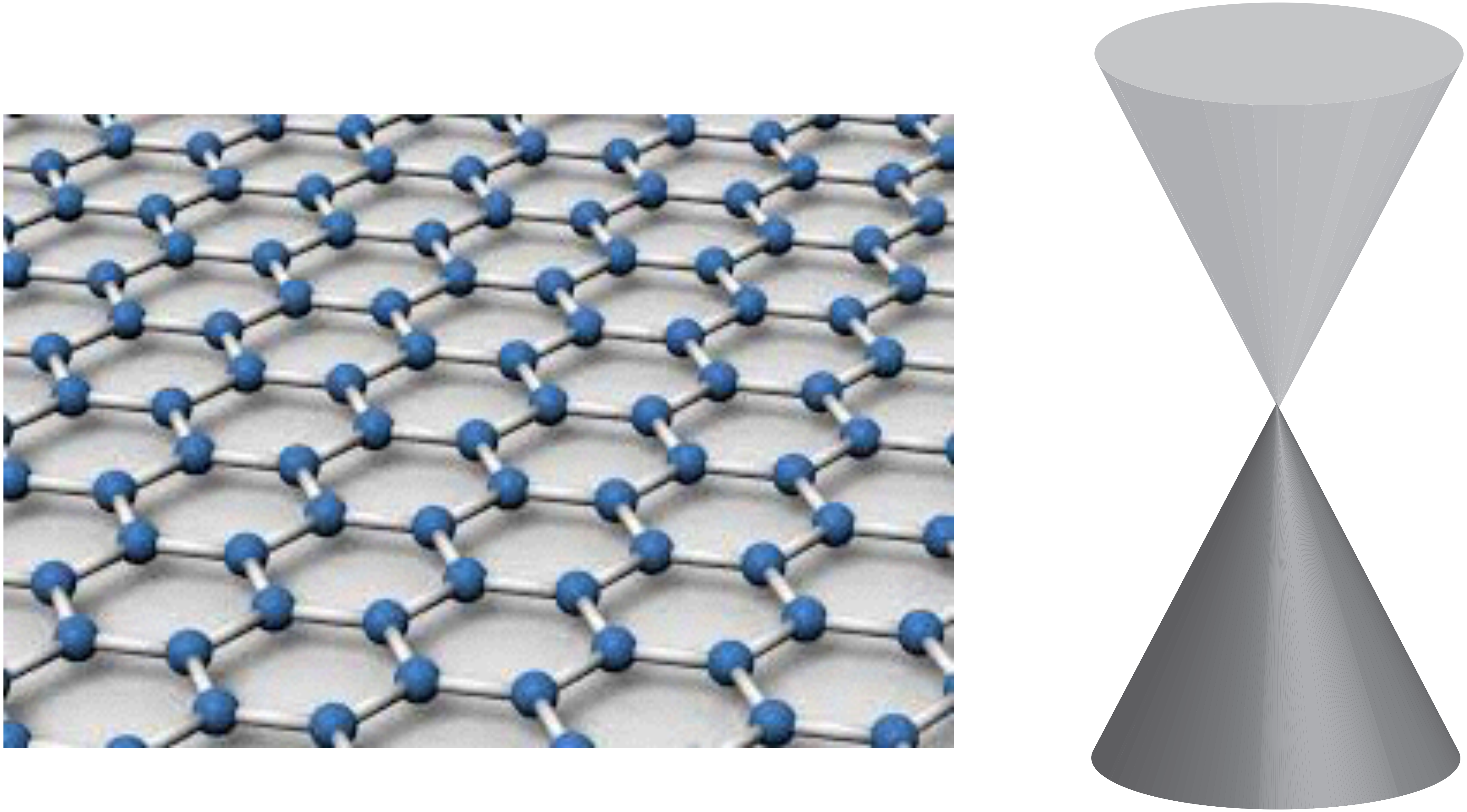
perturbative
limit

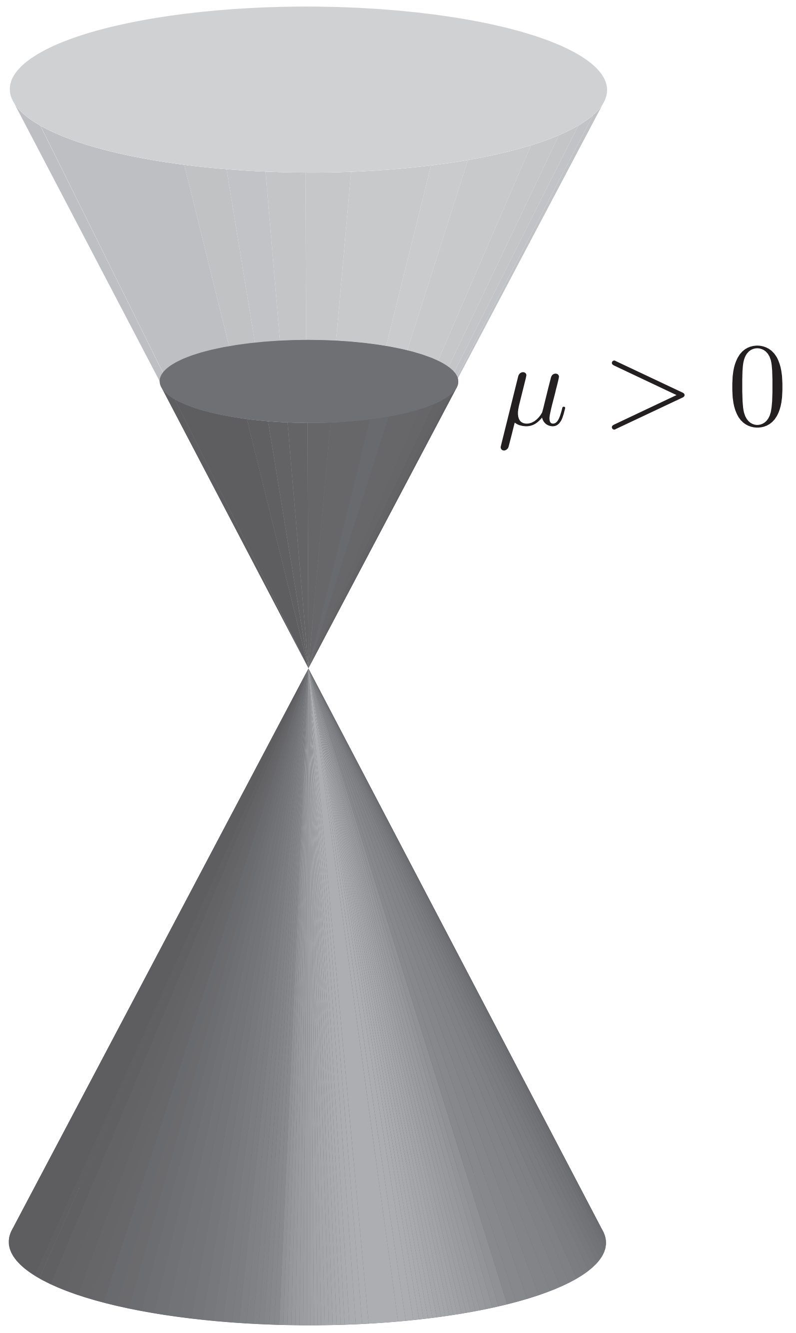
[Lucas JHEP]

holography

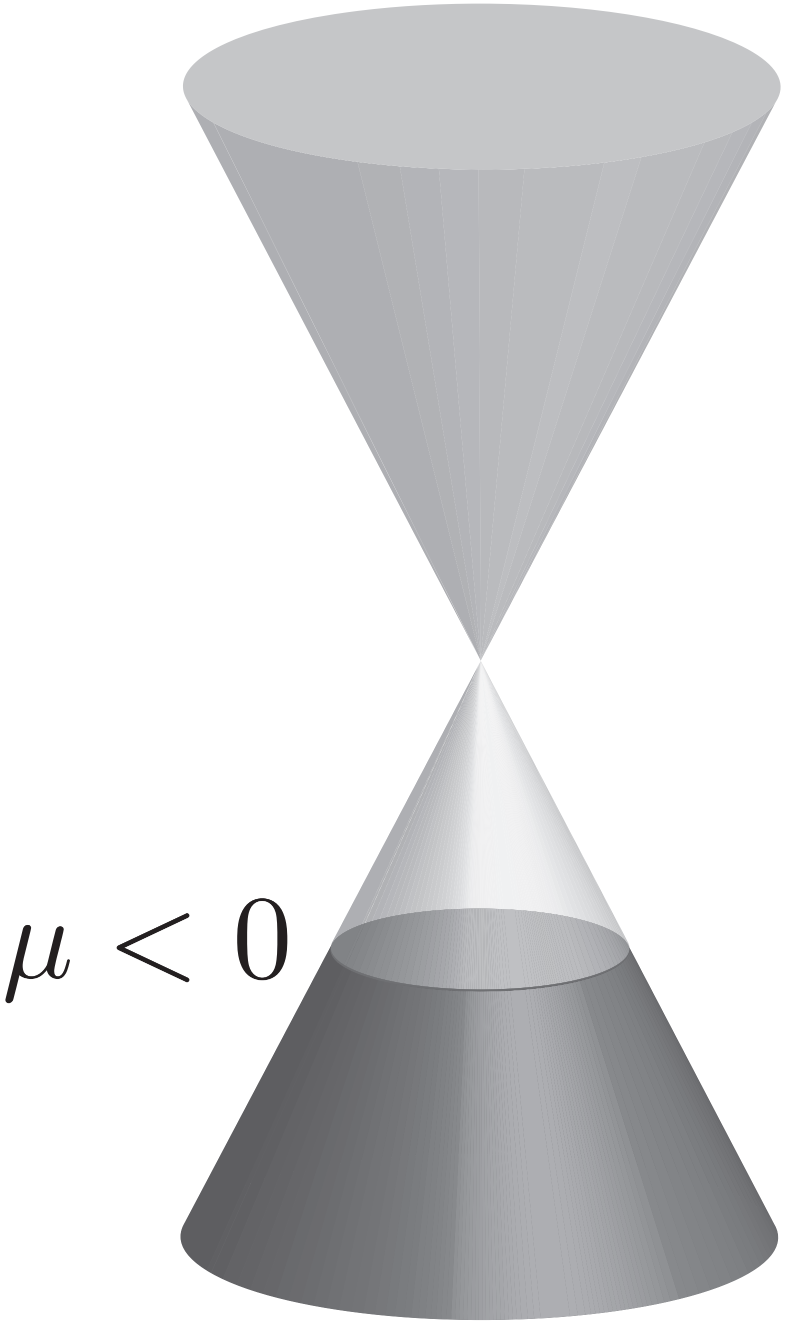
matrix large N theory;
non-perturbative computations

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

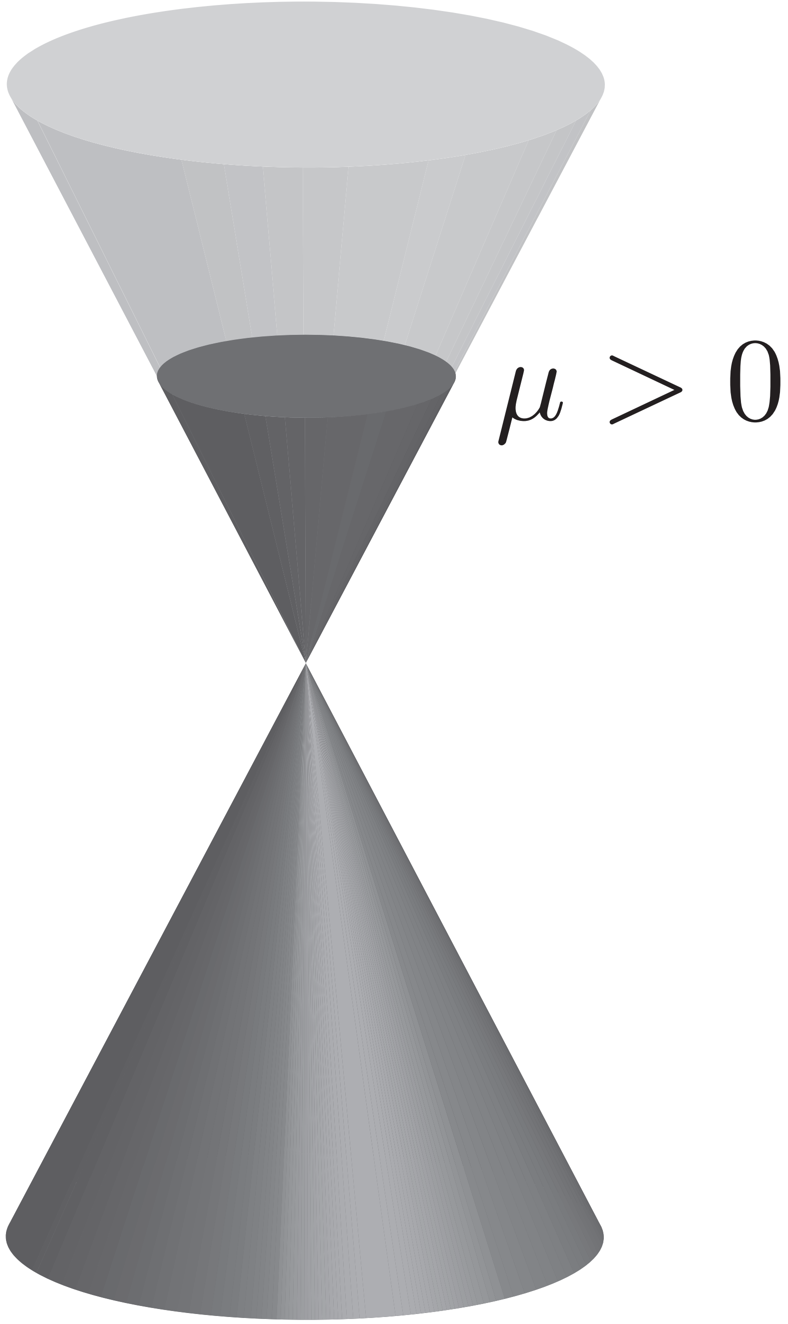




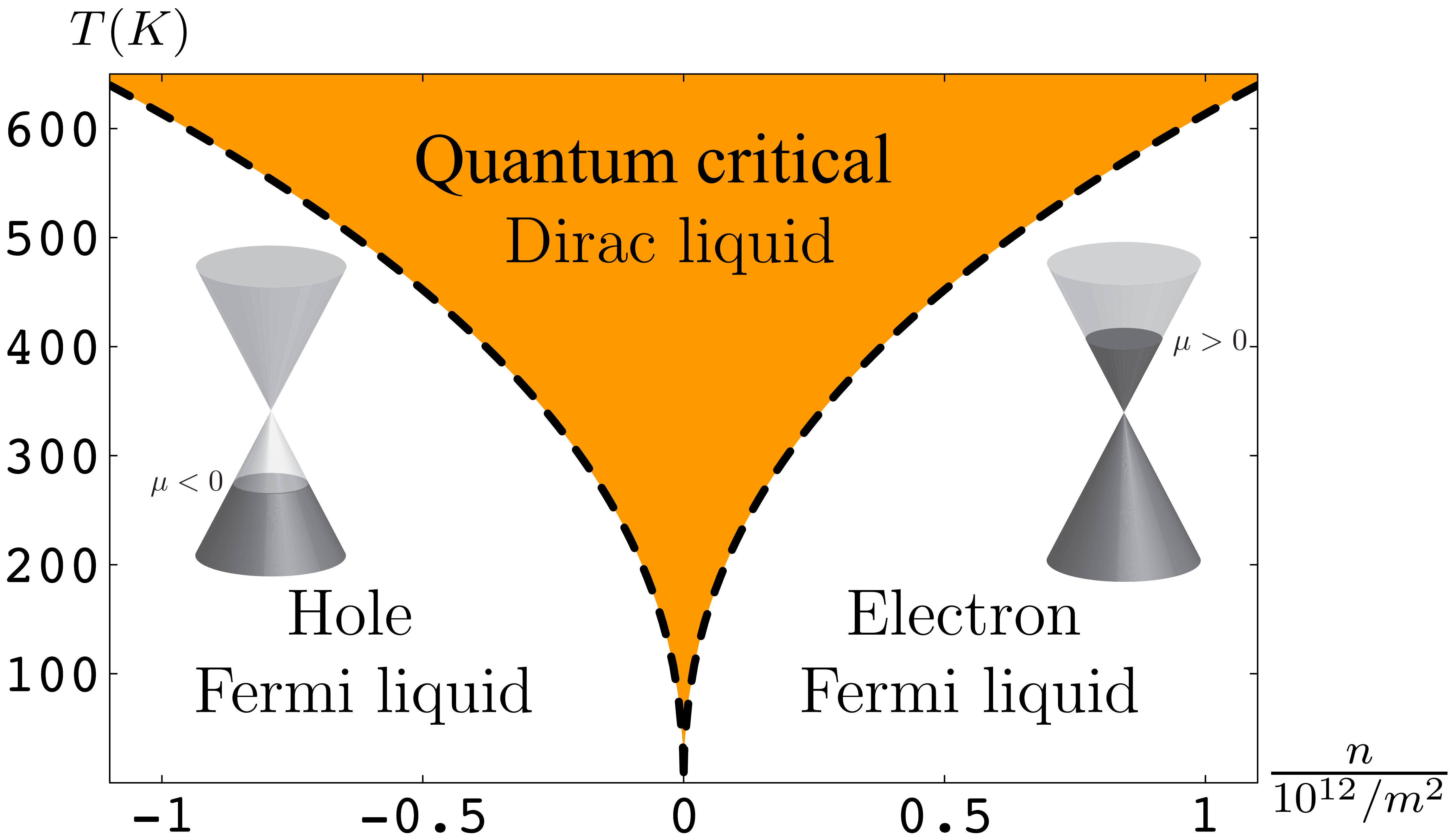
Electron
Fermi surface



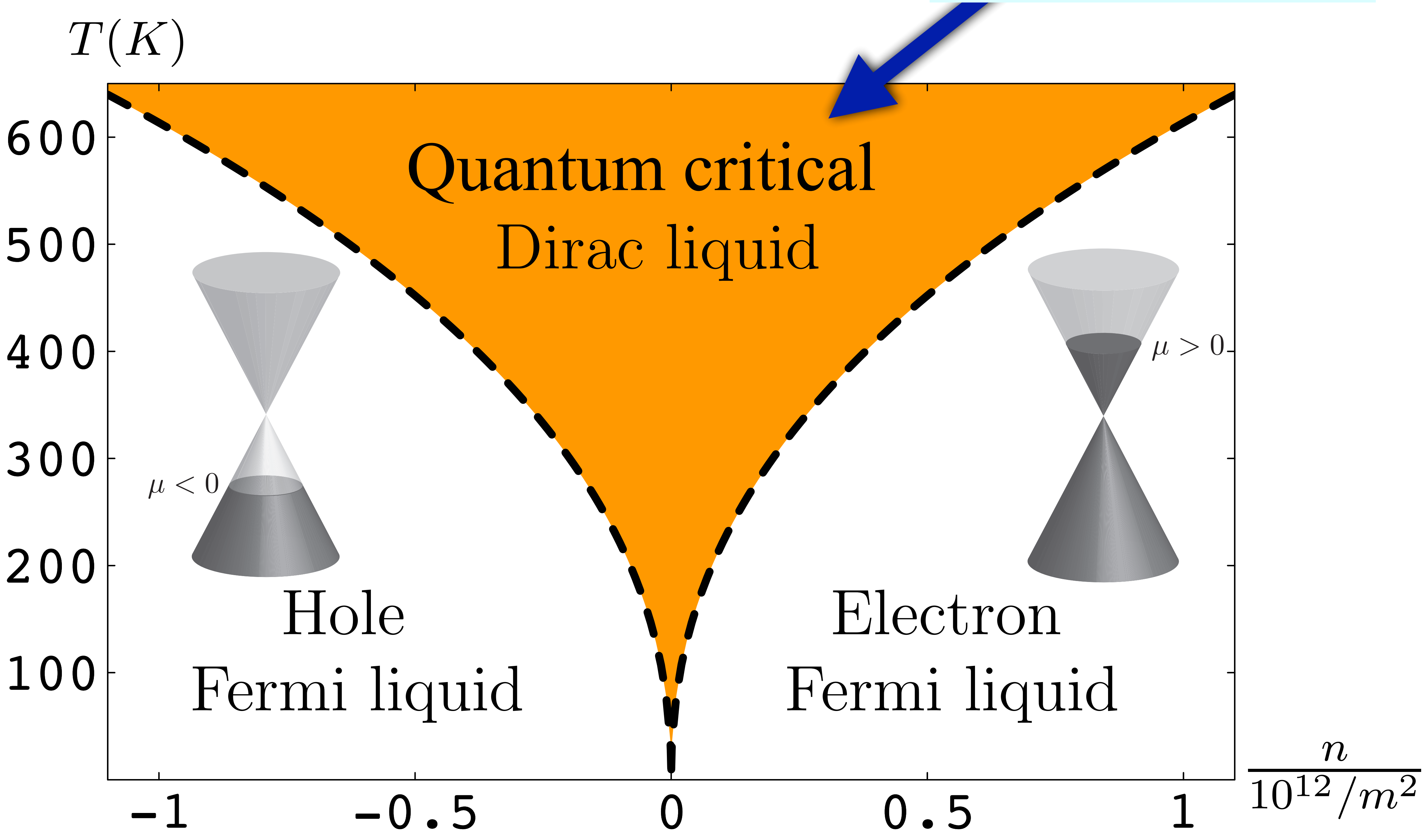
Hole
Fermi surface



Electron
Fermi surface


$T(K)$
600
500
400
300
200
100
Quantum critical
Dirac liquid
$\mu < 0$
$\mu > 0$
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
0
0.5
1
$\frac{n}{10^{12}/m^2}$

(slightly less) strange metal



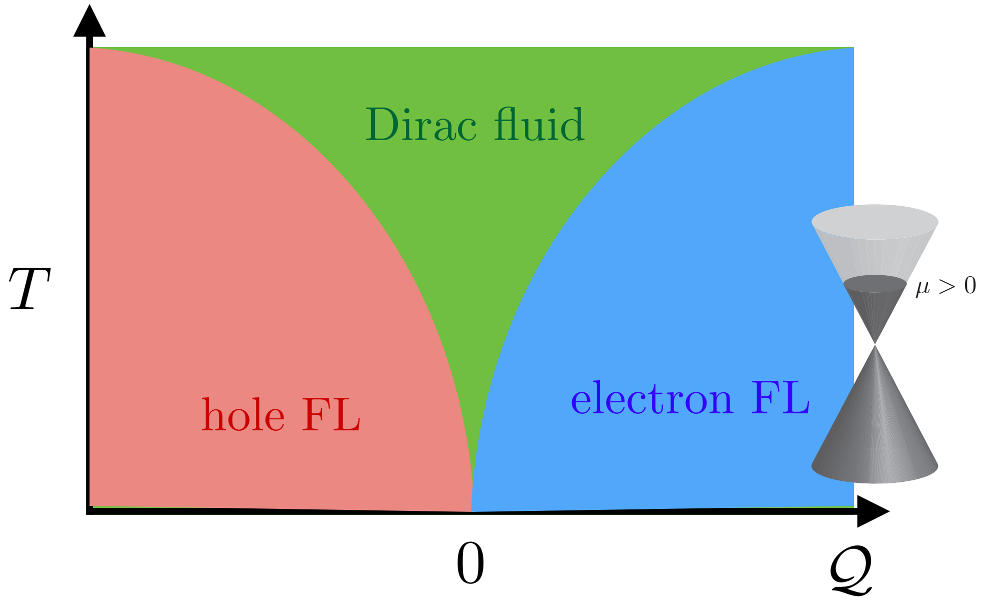
Dirac Fluid in Graphene

The Dirac Fluid

$$\epsilon_{a\sigma} = \hbar v_{\mathrm{F}} k$$

$$+$$

$$V_{\mathrm{int}} = \frac{\alpha_{\mathrm{eff}}}{r}$$



- marginally irrelevant $1/r$ Coulomb interactions:

$$\alpha_{\mathrm{eff}} = \frac{\alpha_0}{1+(\alpha_0/4)\log((10^5\ \mathrm{K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\mathrm{F}}\epsilon_{\mathrm{r}}} \sim 0.5.$$

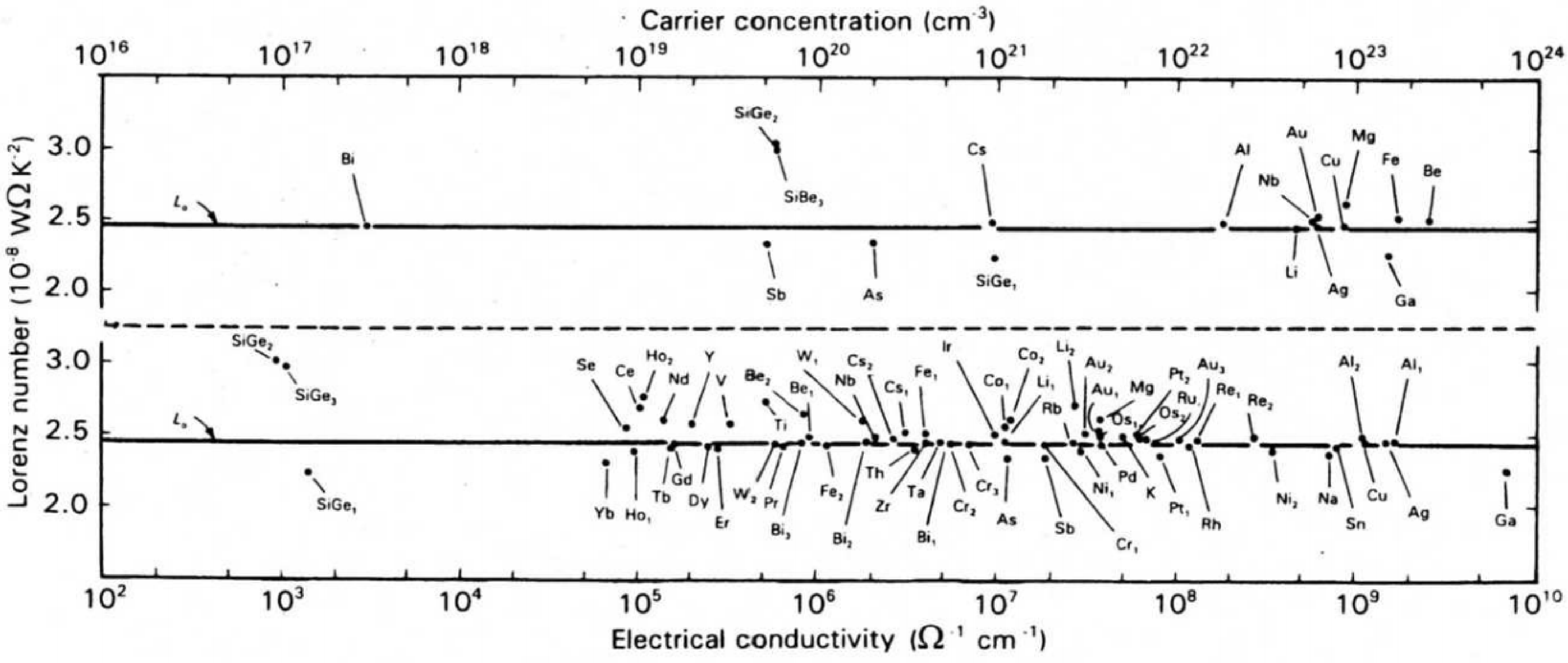
- thermo/hydro nearly that of relativistic theory
- $\alpha_{\mathrm{eff}} \sim 0.3$ at $T = 100$ K

[Müller, Fritz, Sachdev, *Physical Review* **B78** 115406 (2008)]

Wiedemann-Franz Law

- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\ \frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$



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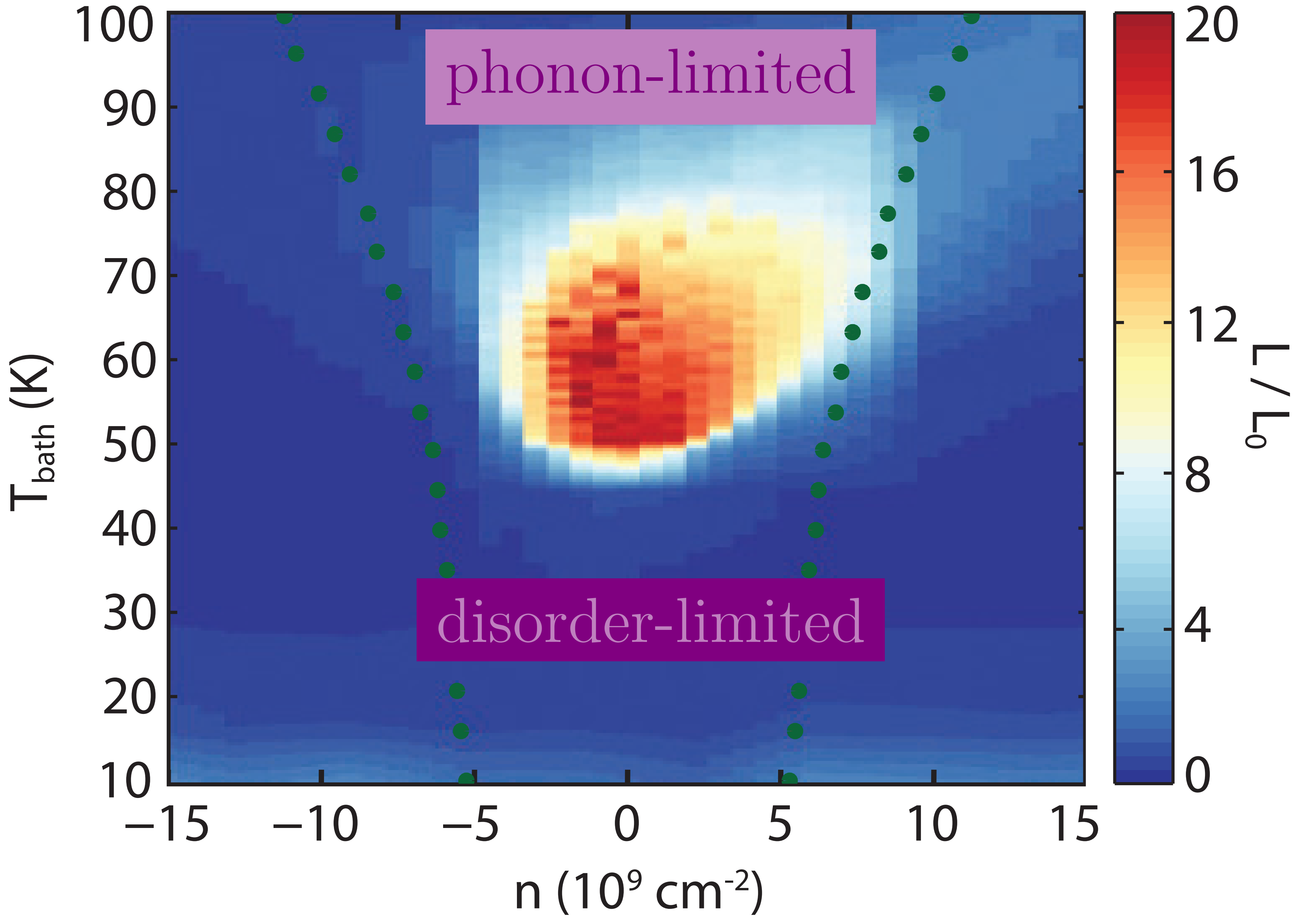
In theory of the (slightly less) strange metal without quasiparticles, we find a large enhancement of the Lorentz ratio $L = \kappa/(T\sigma)$ with

$$L = \frac{\mathcal{H}\tau}{T^2 \sigma_Q} \frac{1}{\left(1 + n^2 \tau/(\mathcal{H}\sigma_Q)\right)^2},$$

where $\mathcal{H}$ is the enthalpy density, τ is the momentum relaxation rate due to scattering from impurities, and σ_Q is an intrinsic "quantum critical" conductivity.

Dirac Fluid in Graphene

Wiedemann-Franz Law Violations in Experiment

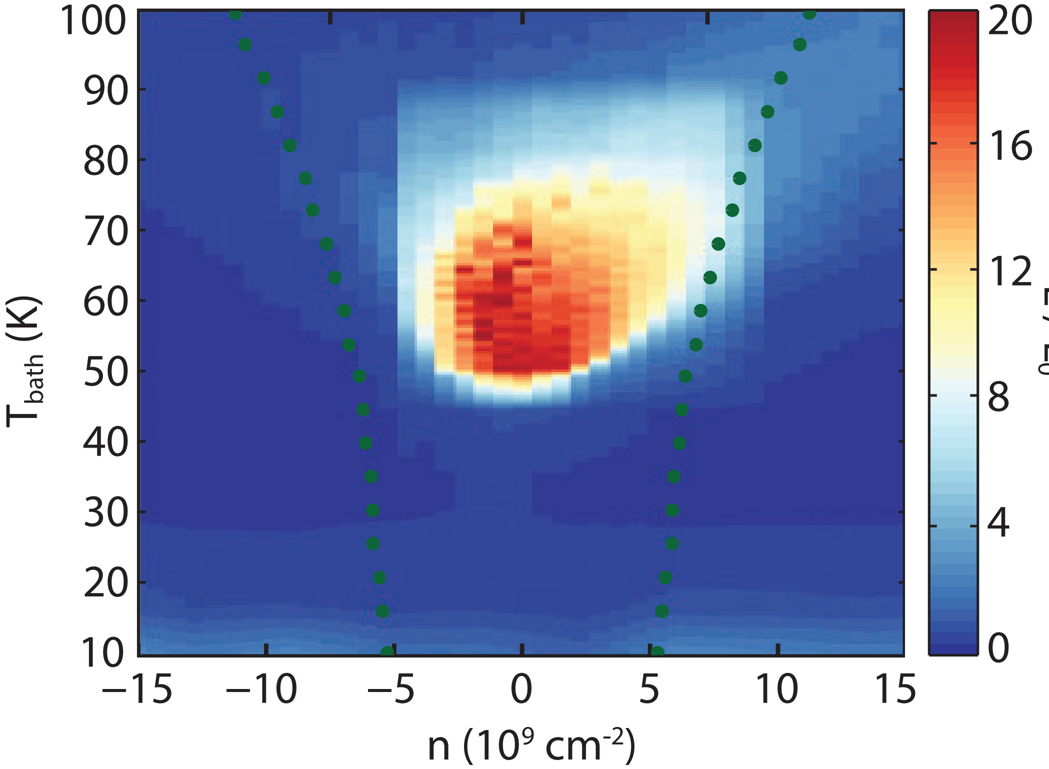


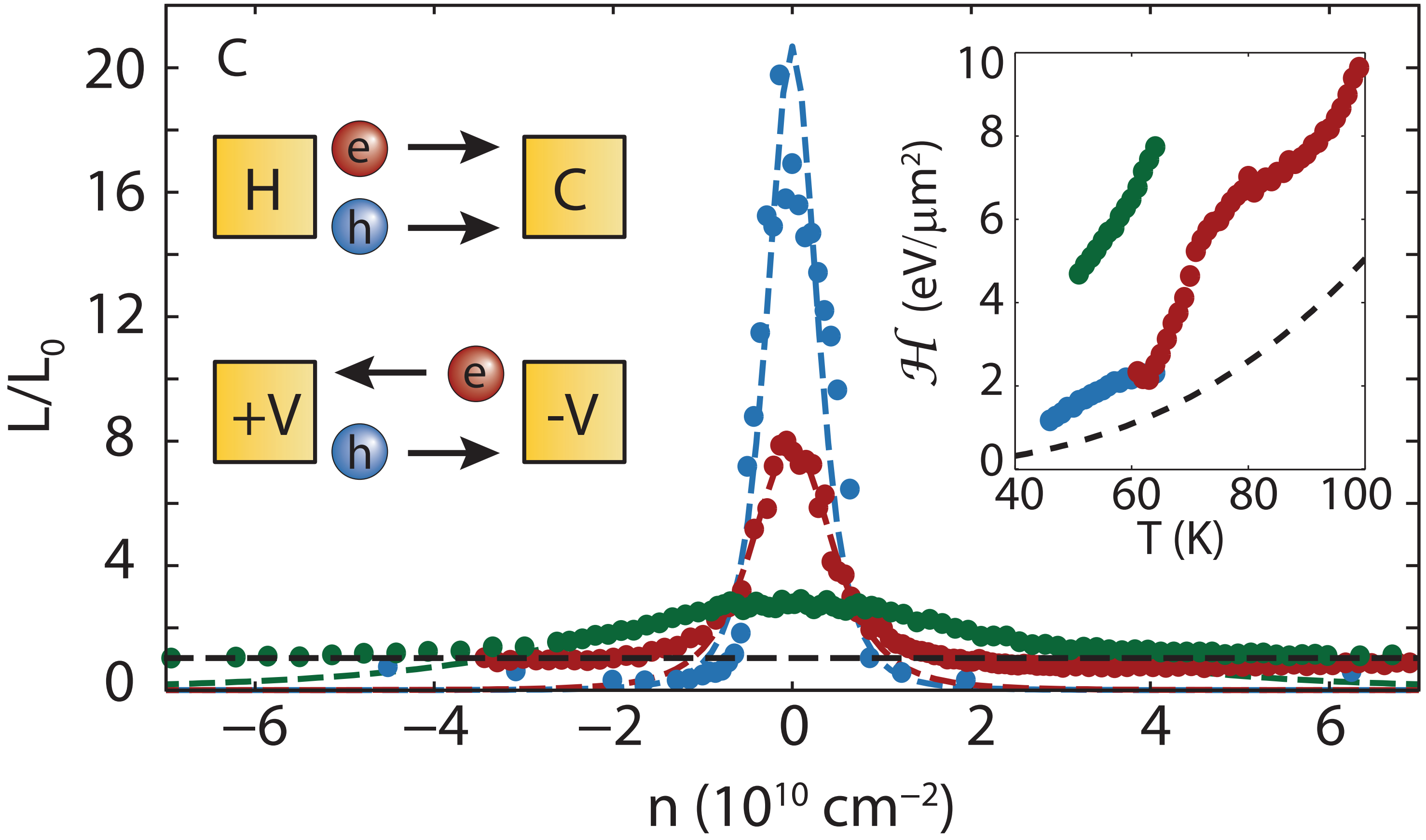
[Crossno *et al, submitted*]

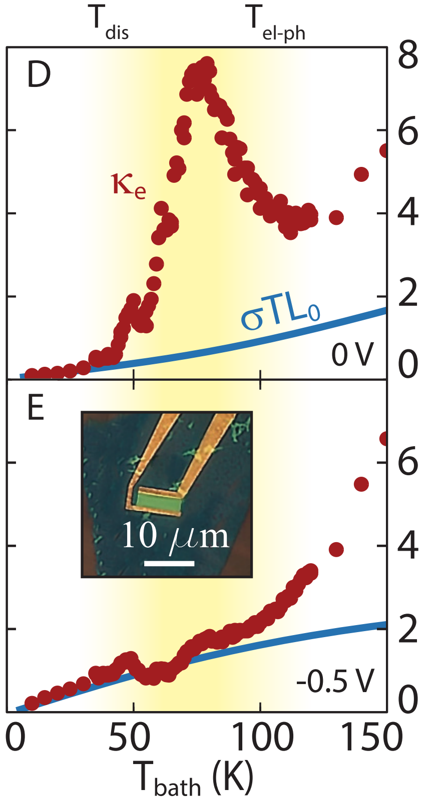
(submitted)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]







$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$

$$= \frac{\mathcal{H}\tau}{T^2\sigma_Q}\,\frac{1}{\left(1+n^2\tau/(\mathcal{H}\sigma_Q)\right)^2}$$

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