Competing orders in the underdoped cuprates

Talk online: sachdev.physics.harvard.edu

Competing orders in the underdoped cuprates

Destruction of Neel order in the cuprates by electron doping, R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu, *Physical Review B* **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates, V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Competing orders in the underdoped cuprates, Eun Gook Moon and S. Sachdev, *to appear*

Outline

1. Survey of experiments and theory
   (a) Quantum oscillations
   (b) Competing orders
   (c) Nodal-anti-nodal dichotomy

2. Spin-fluctuation exchange mechanism of d-wave superconductivity
   Successful at large doping, but cannot account for competing orders, the nodal-anti-nodal dichotomy (and other phenomena) at low doping

3. Superconductivity of electron and hole pockets in a background of fluctuating antiferromagnetism
   Pairing by gauge forces the unusual d-wave superconductivity of the underdoped cuprates.

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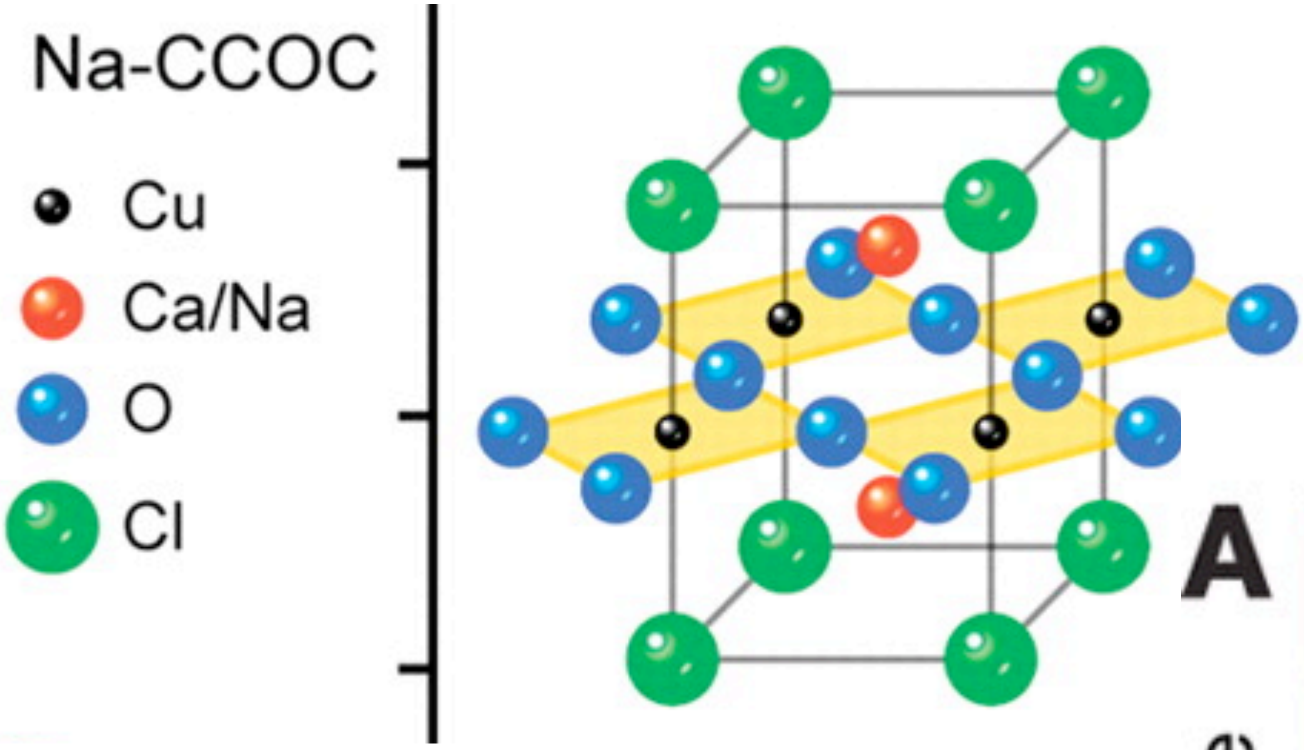
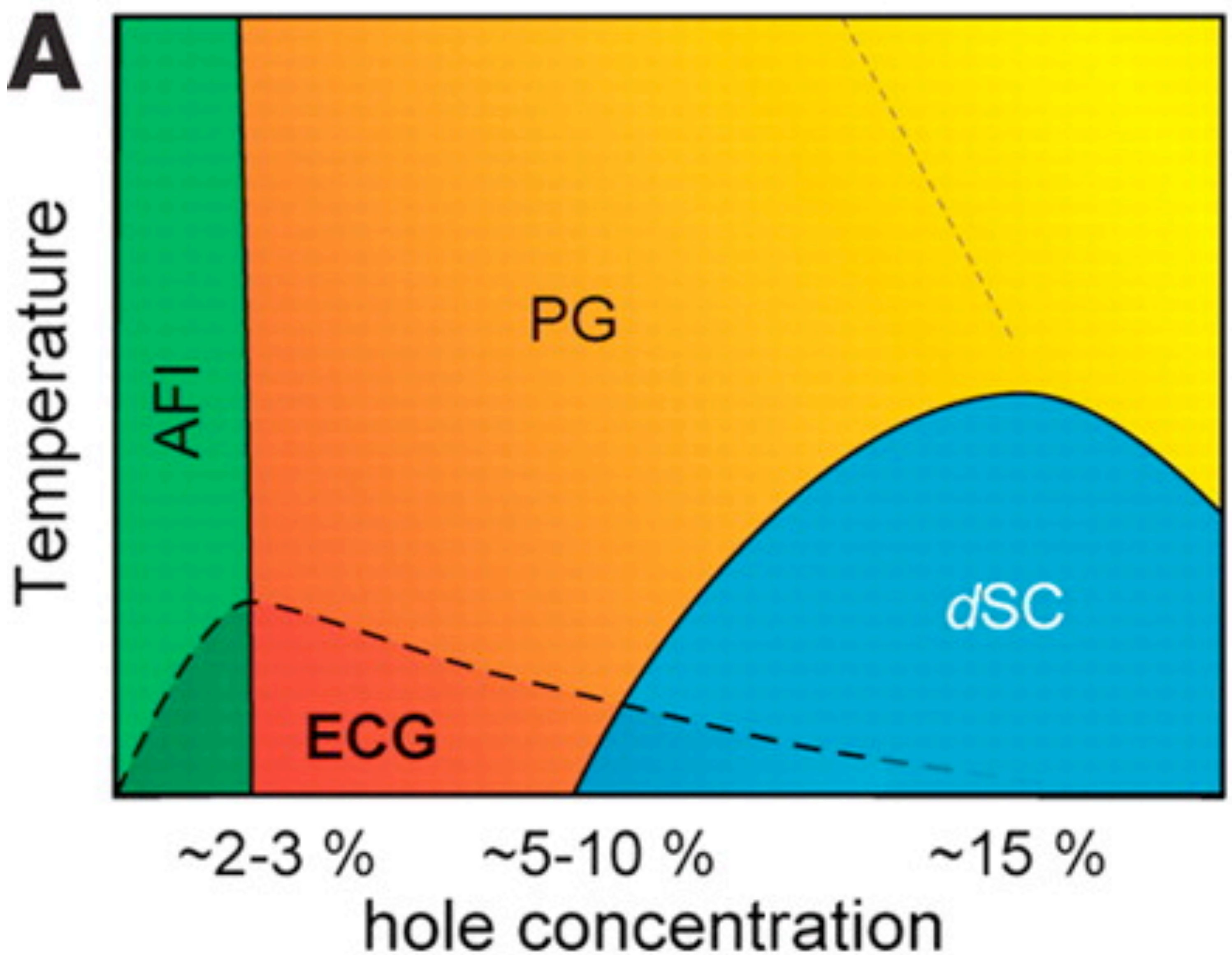
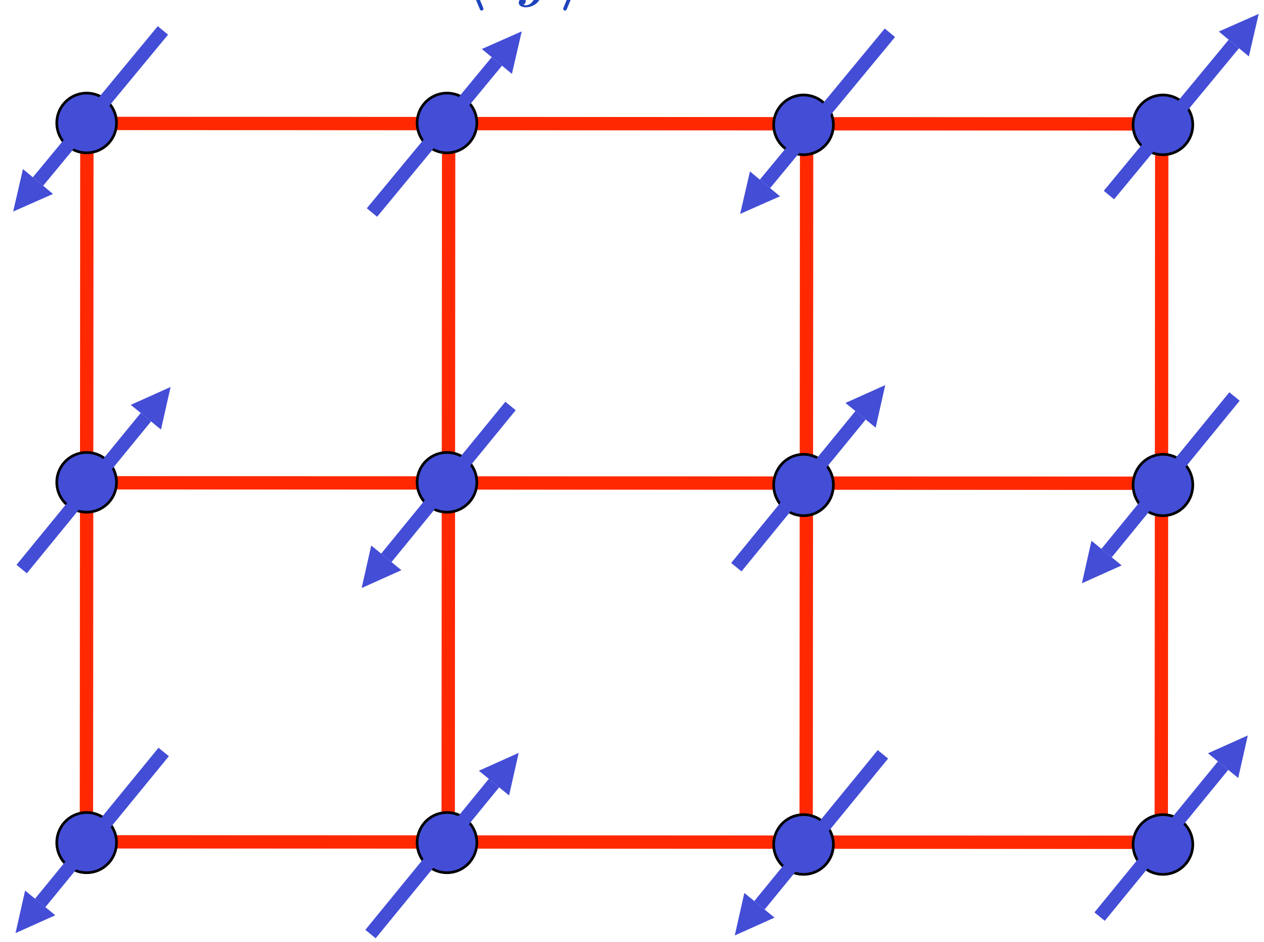
The cuprate superconductors





Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



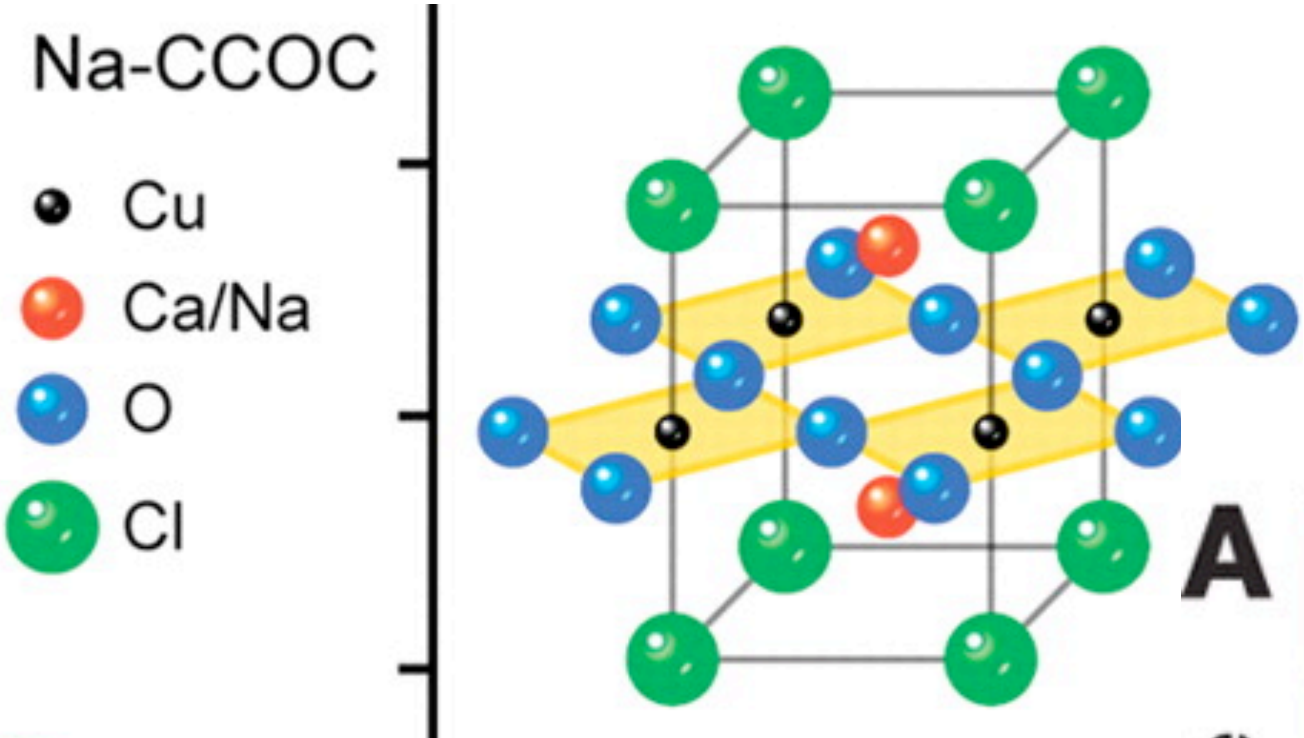
Ground state has long-range spin density wave (SDW) order

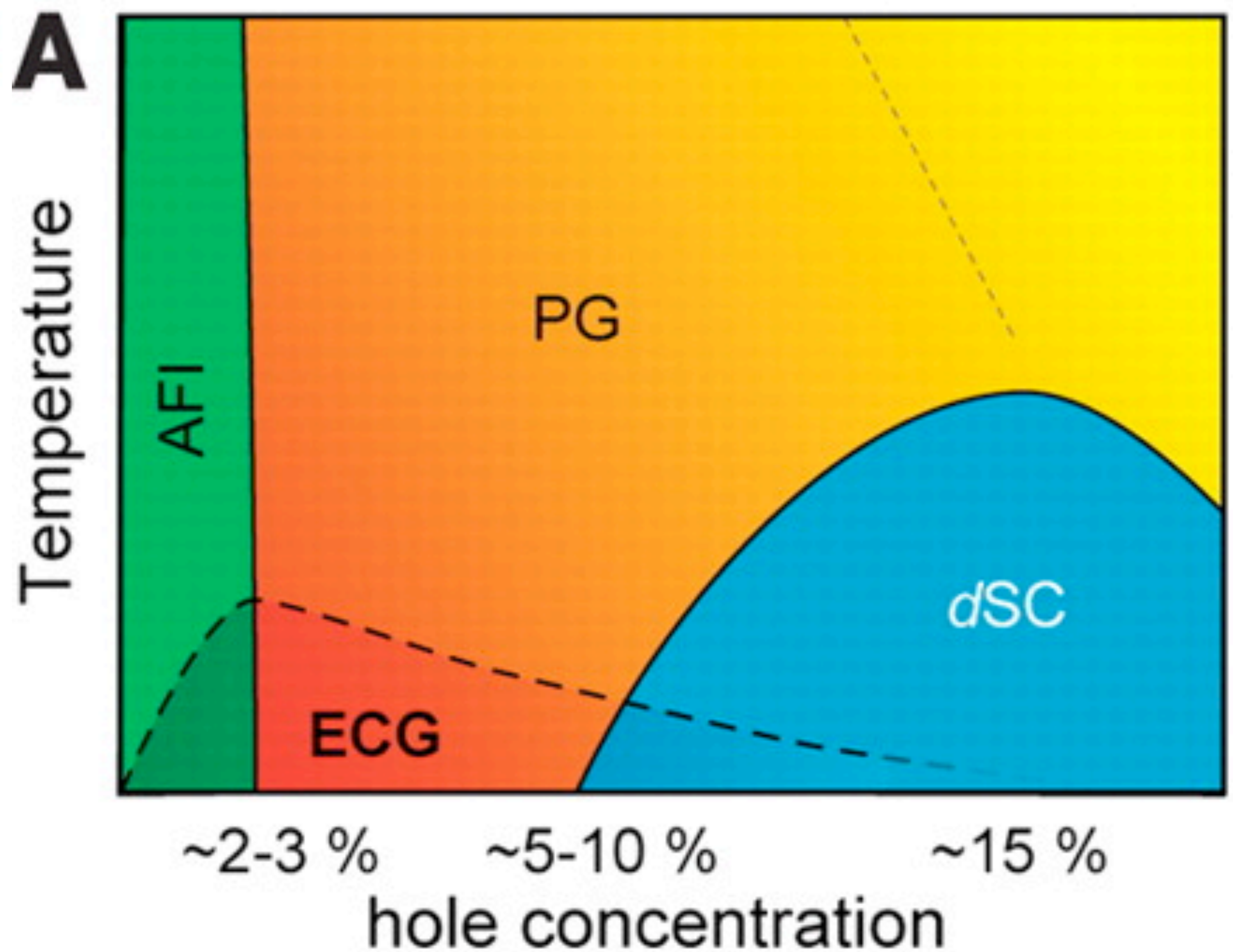
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

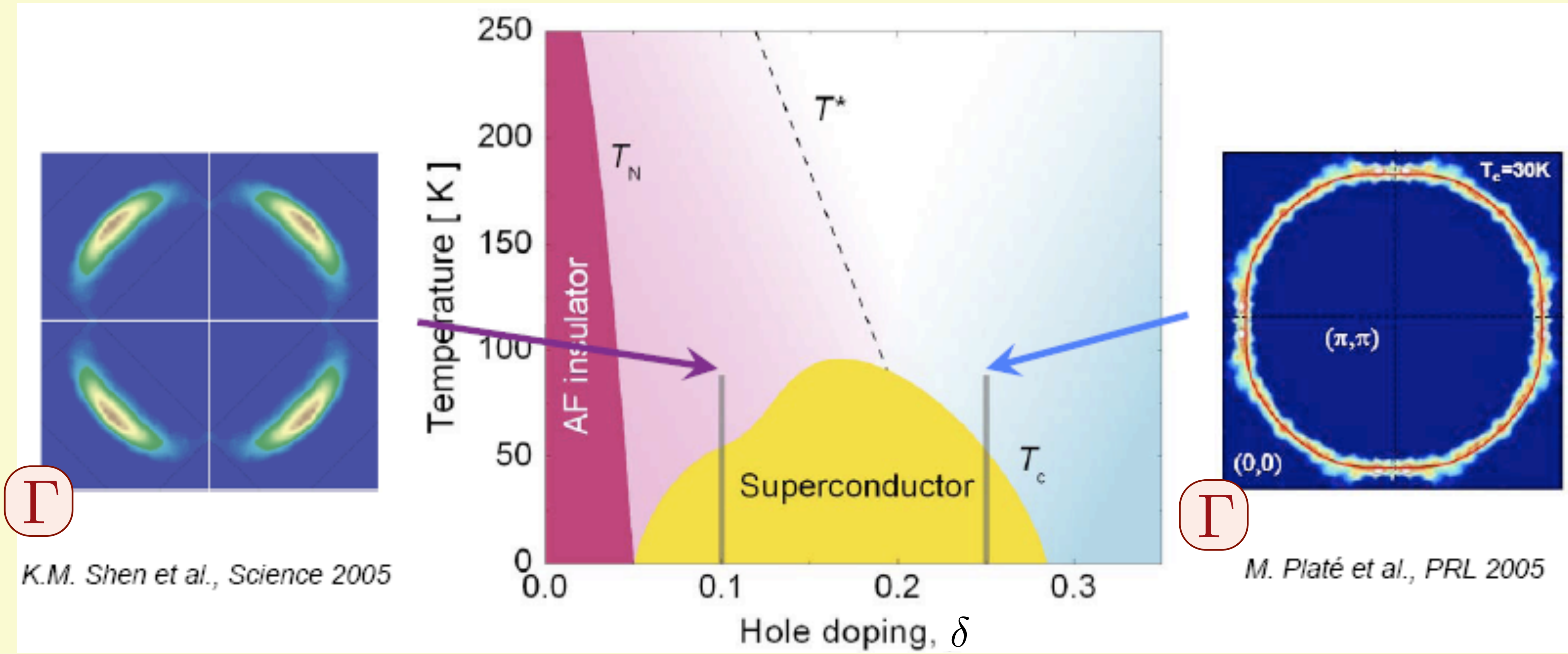
$\langle \vec{\varphi} \rangle \neq 0$ in SDW state.

The cuprate superconductors





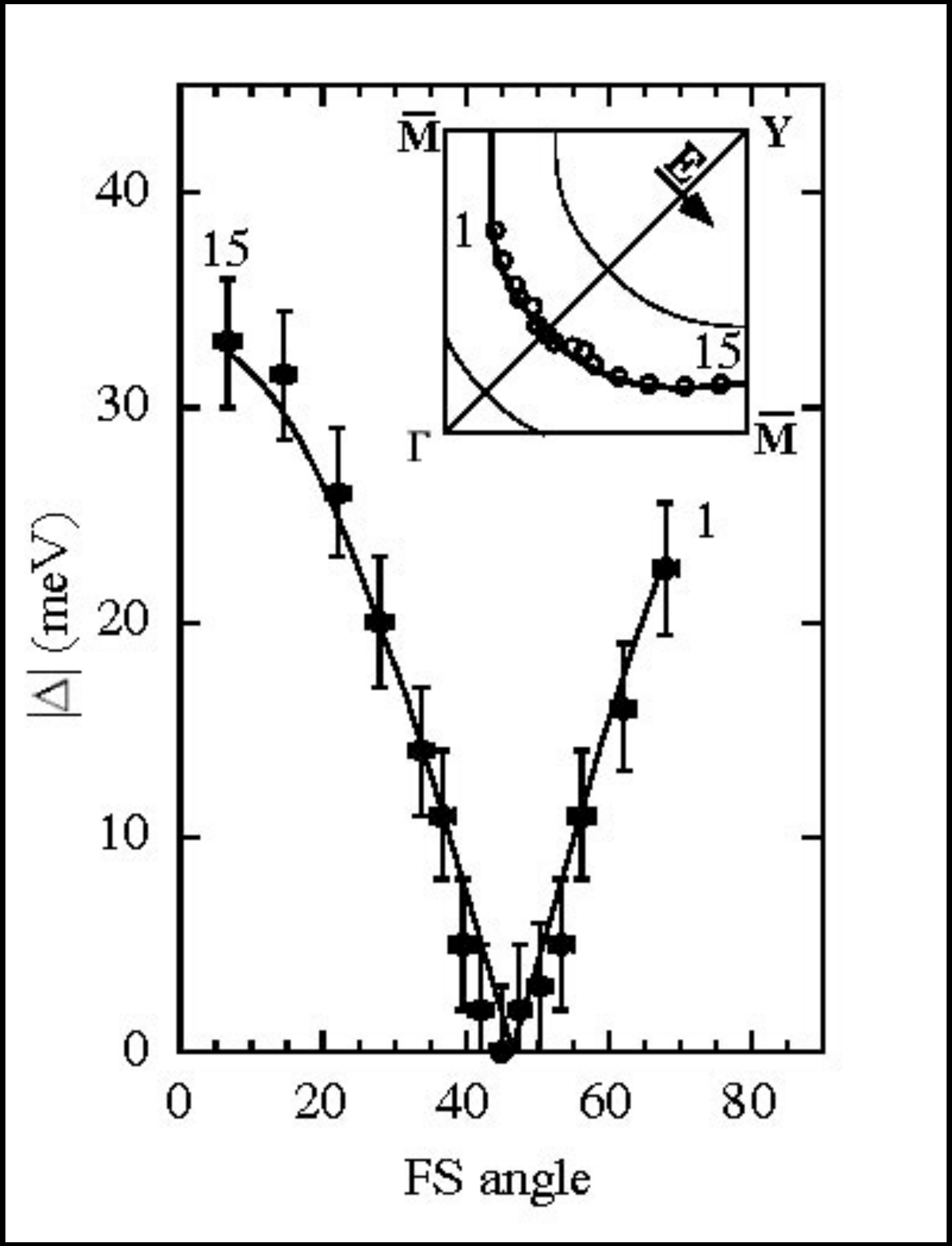
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



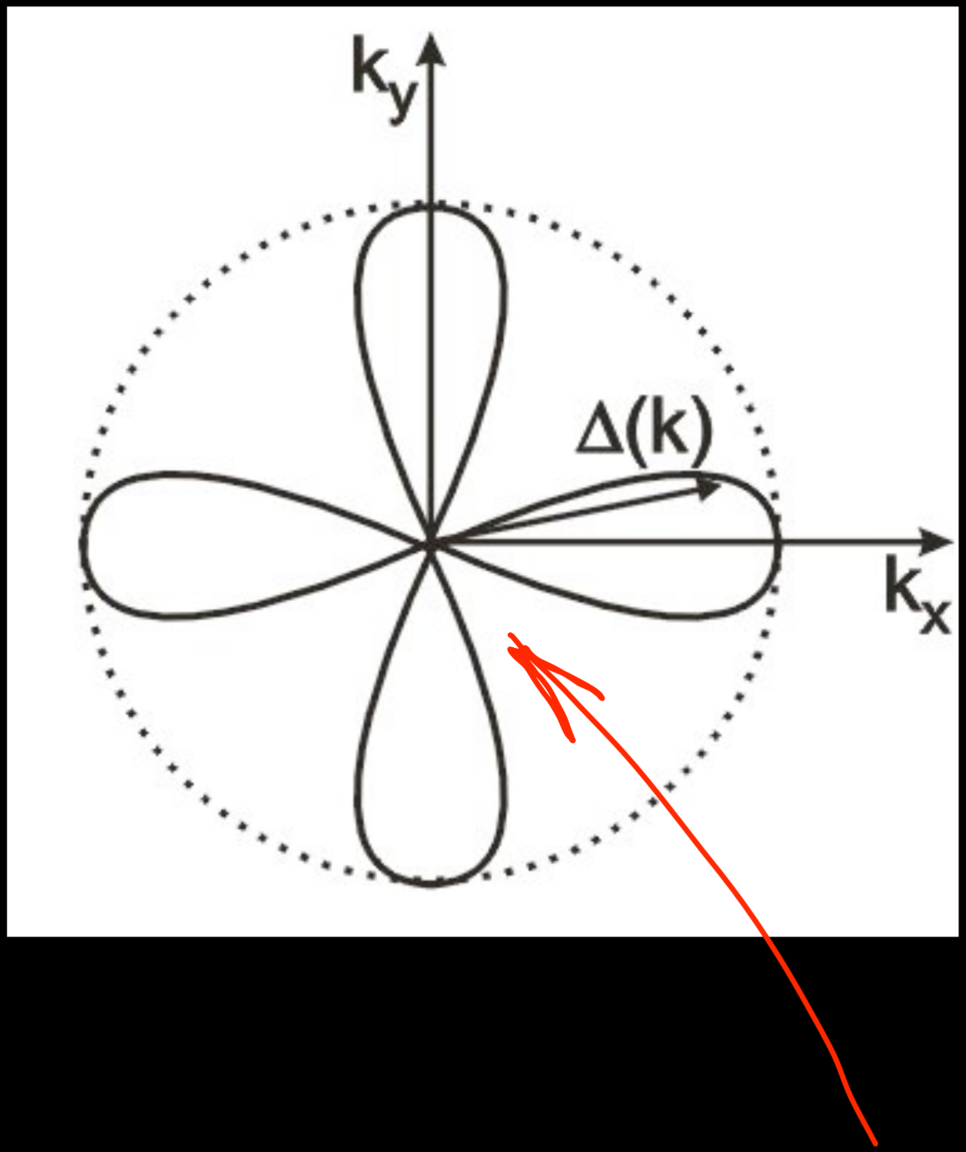
Smaller hole Fermi-pockets

Large hole Fermi surface

Overdoped SC State: Momentum-dependent Pair Energy Gap $\Delta(\vec{k})$

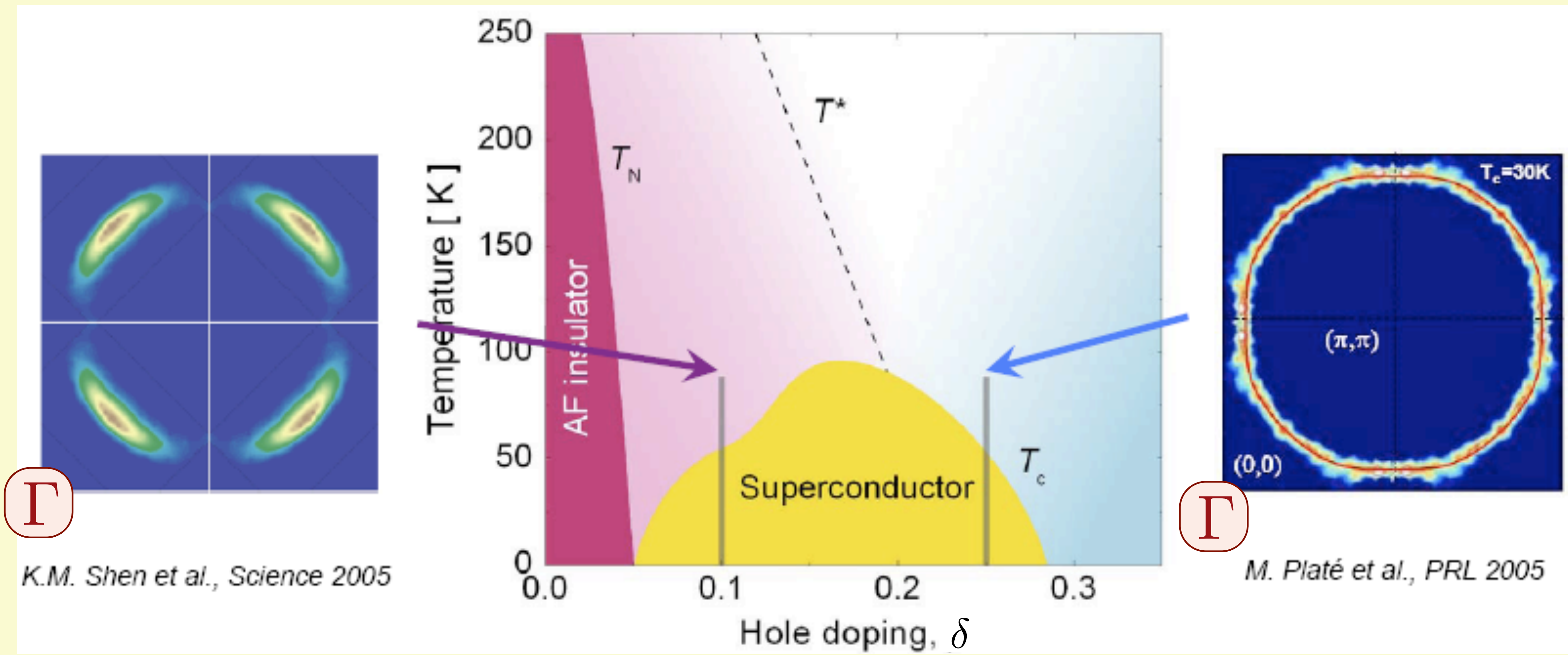


Shen et al PRL **70**, 3999 (1993)
Ding et al PRB **54** 9678 (1996)
Mesot et al PRL **83** 840 (1999)



The SC energy gap $\Delta(\vec{k})$ has four nodes.

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram

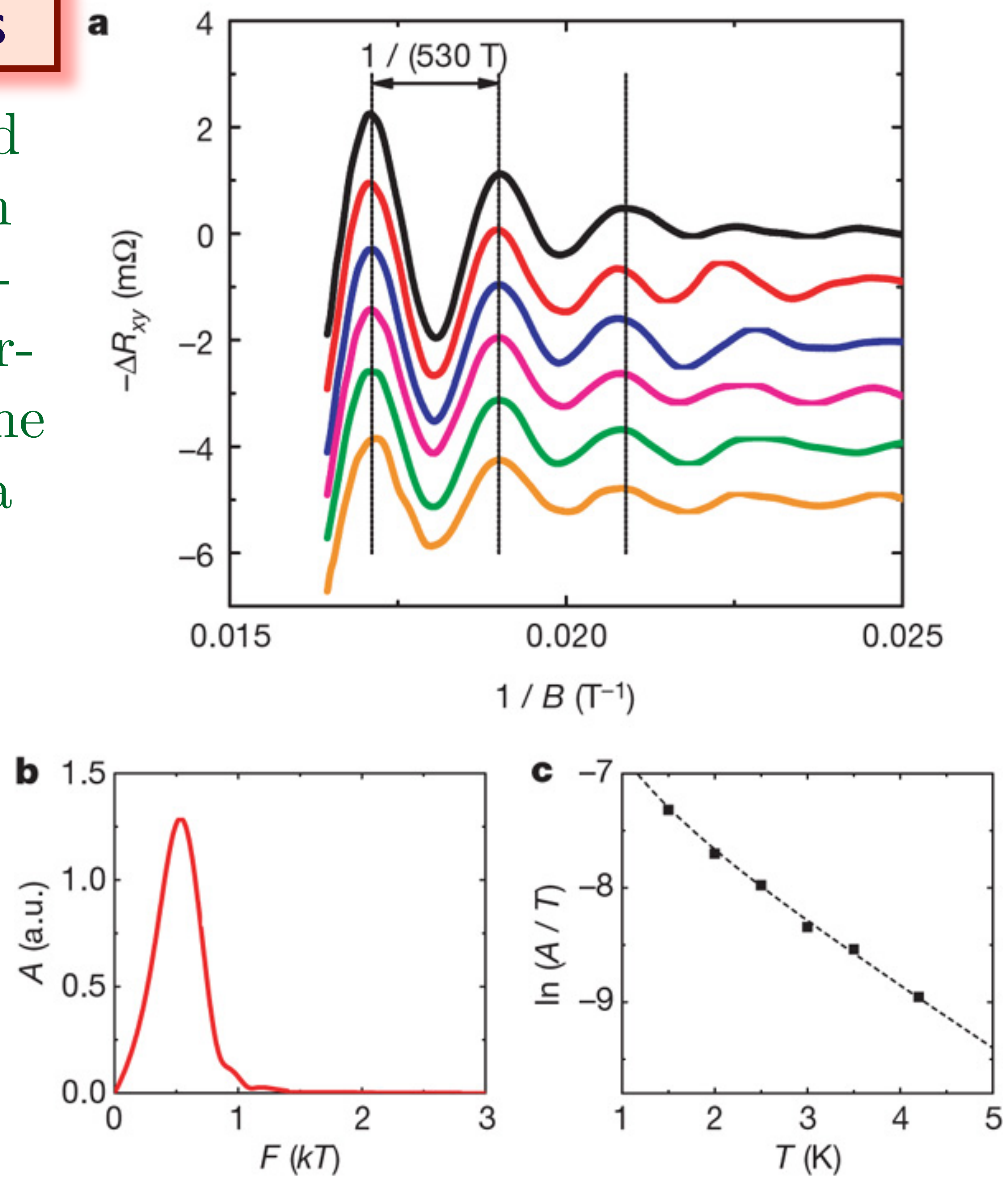


Smaller hole Fermi-pockets

Large hole Fermi surface

(a) Quantum oscillations

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$). The period corresponds to a carrier density ≈ 0.076.



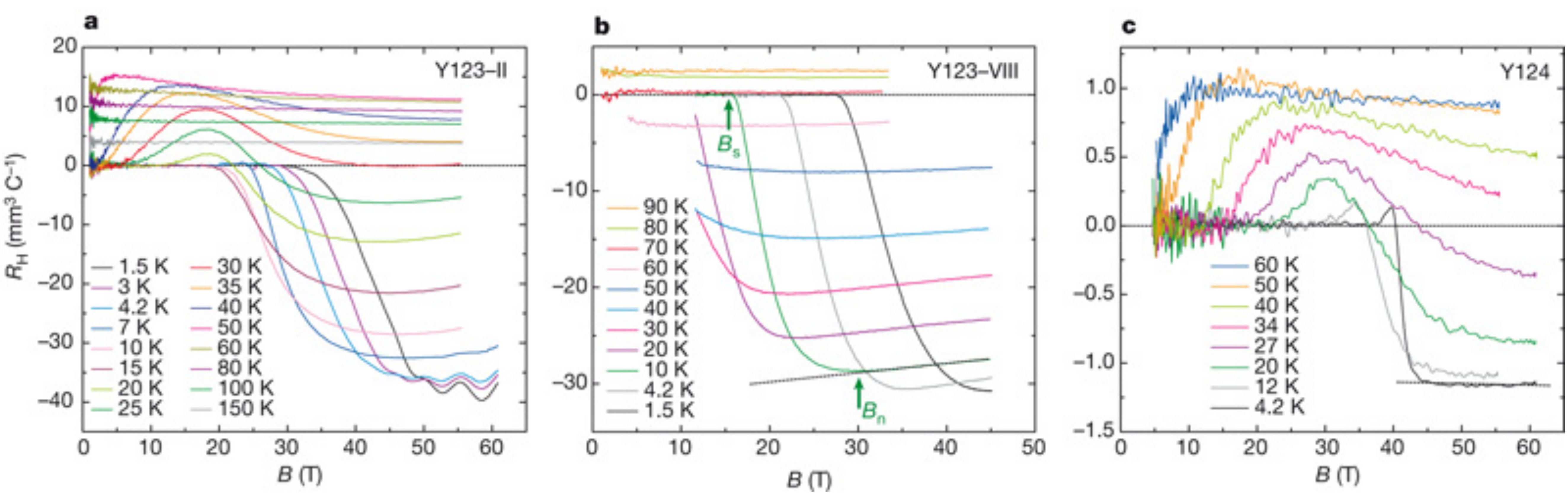
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

(a) Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

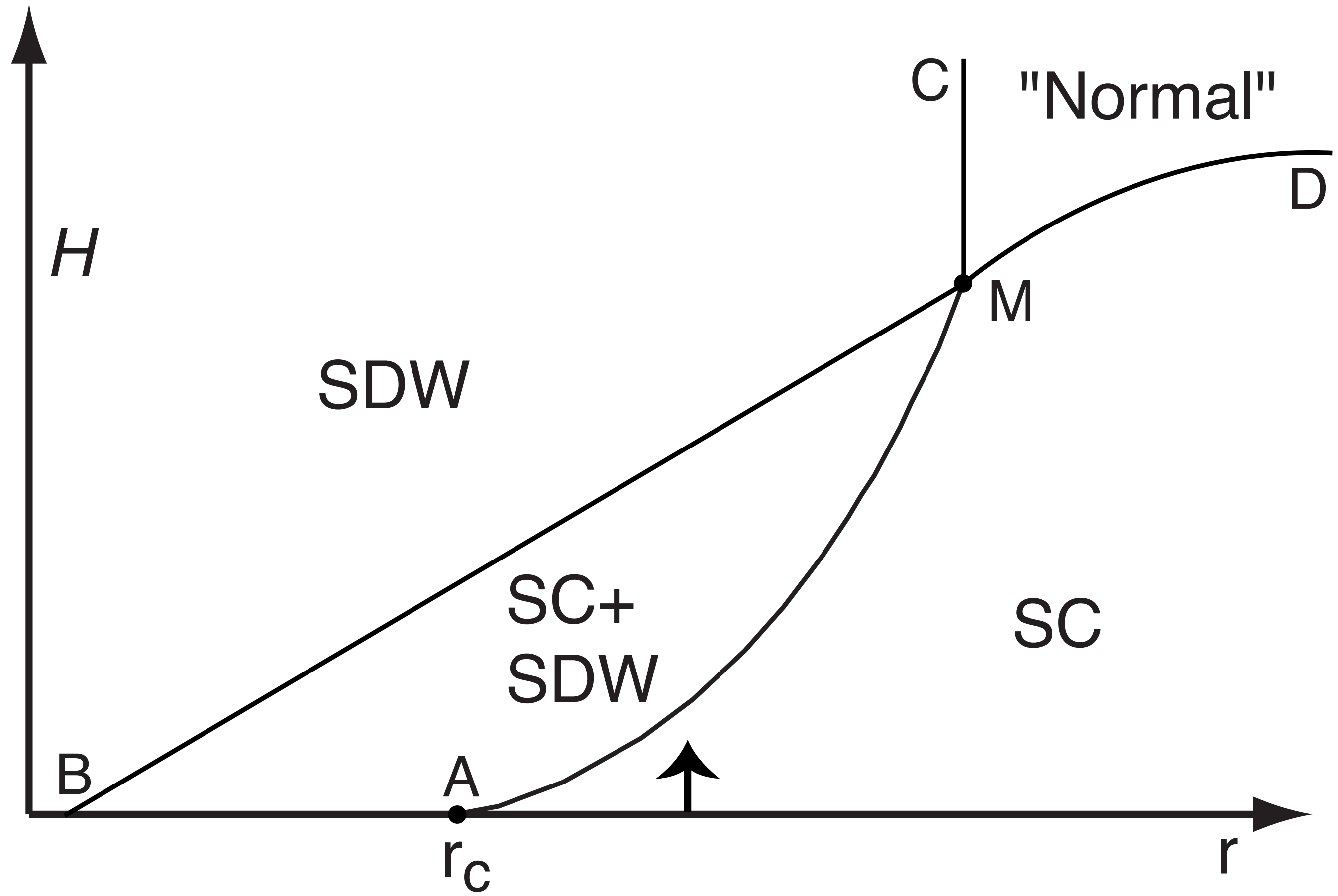
David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



(b) Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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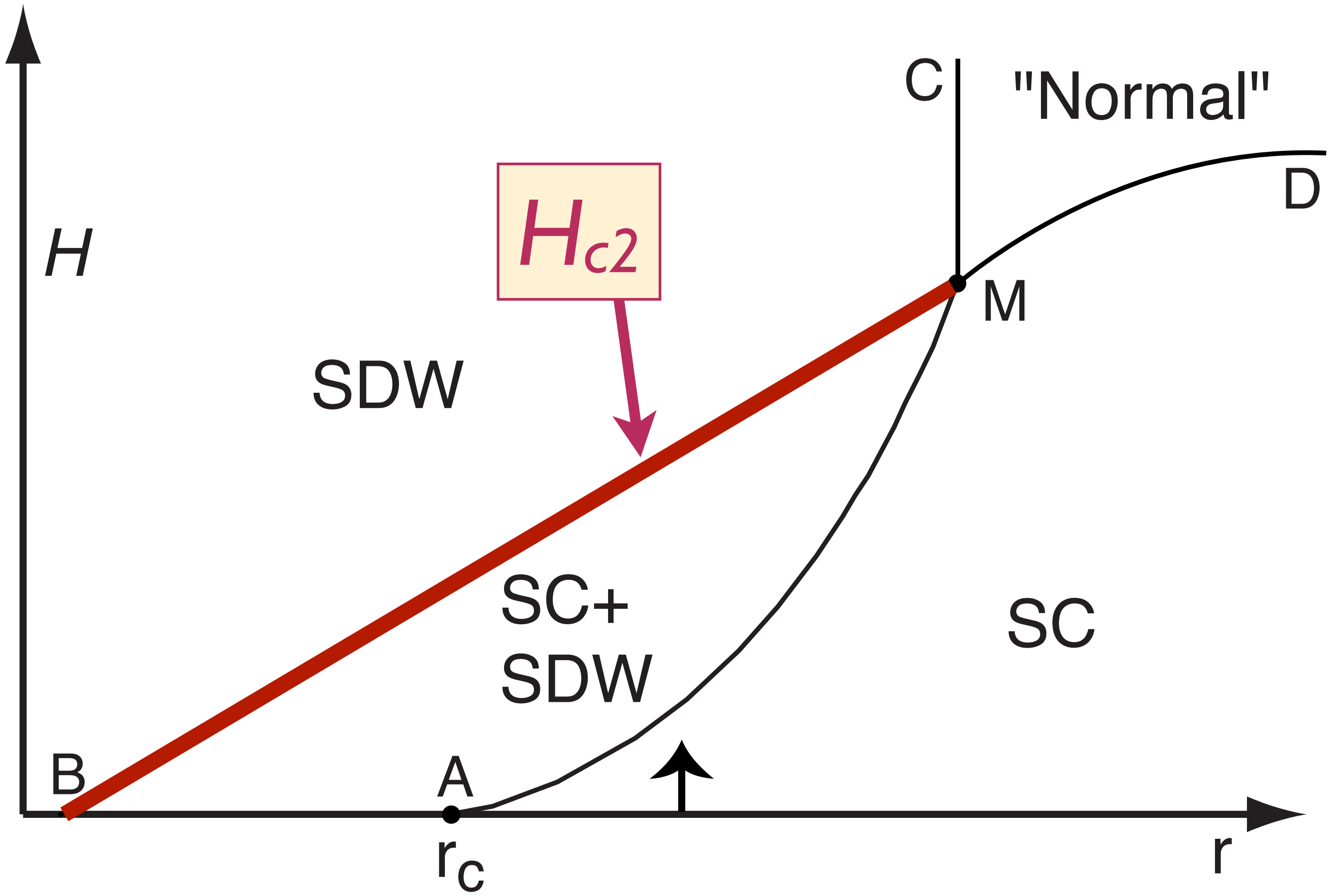

Competing order predictions:

- Upper-critical field, H_{c2}, decreases as SDW is enhanced with decreasing doping (r)

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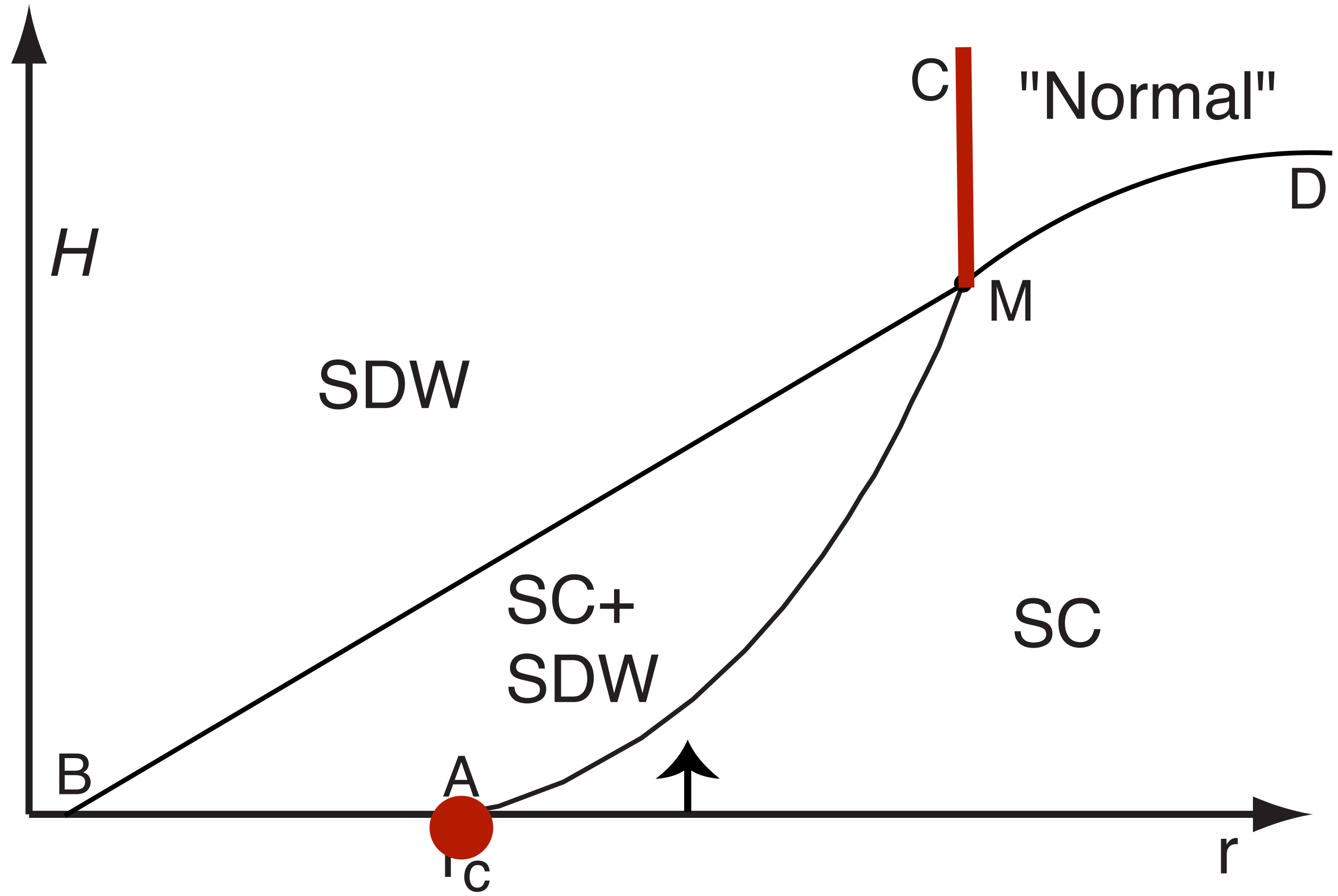

Competing order predictions:

- Upper-critical field, H_{c2}, decreases as SDW is enhanced with decreasing doping (r)
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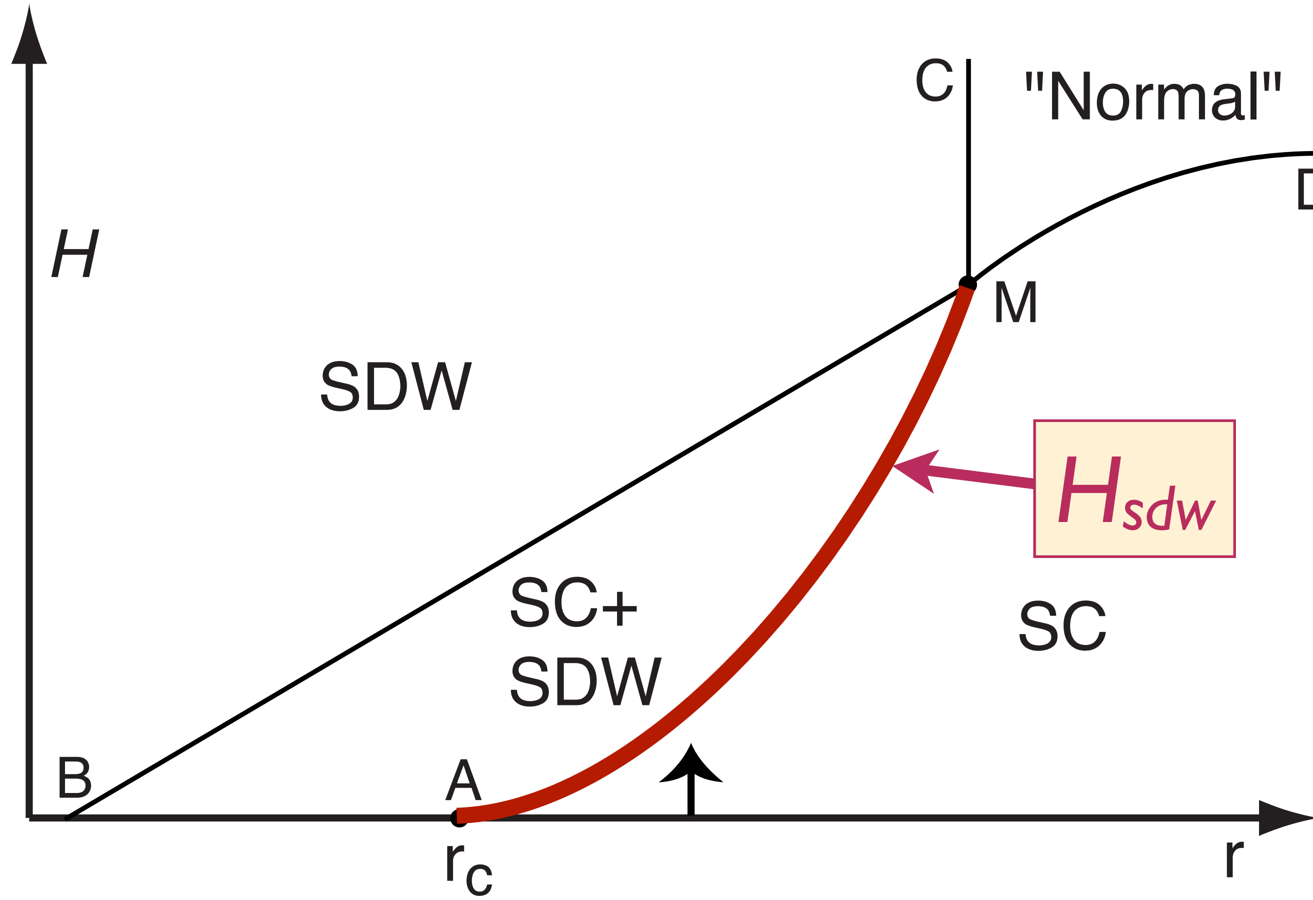


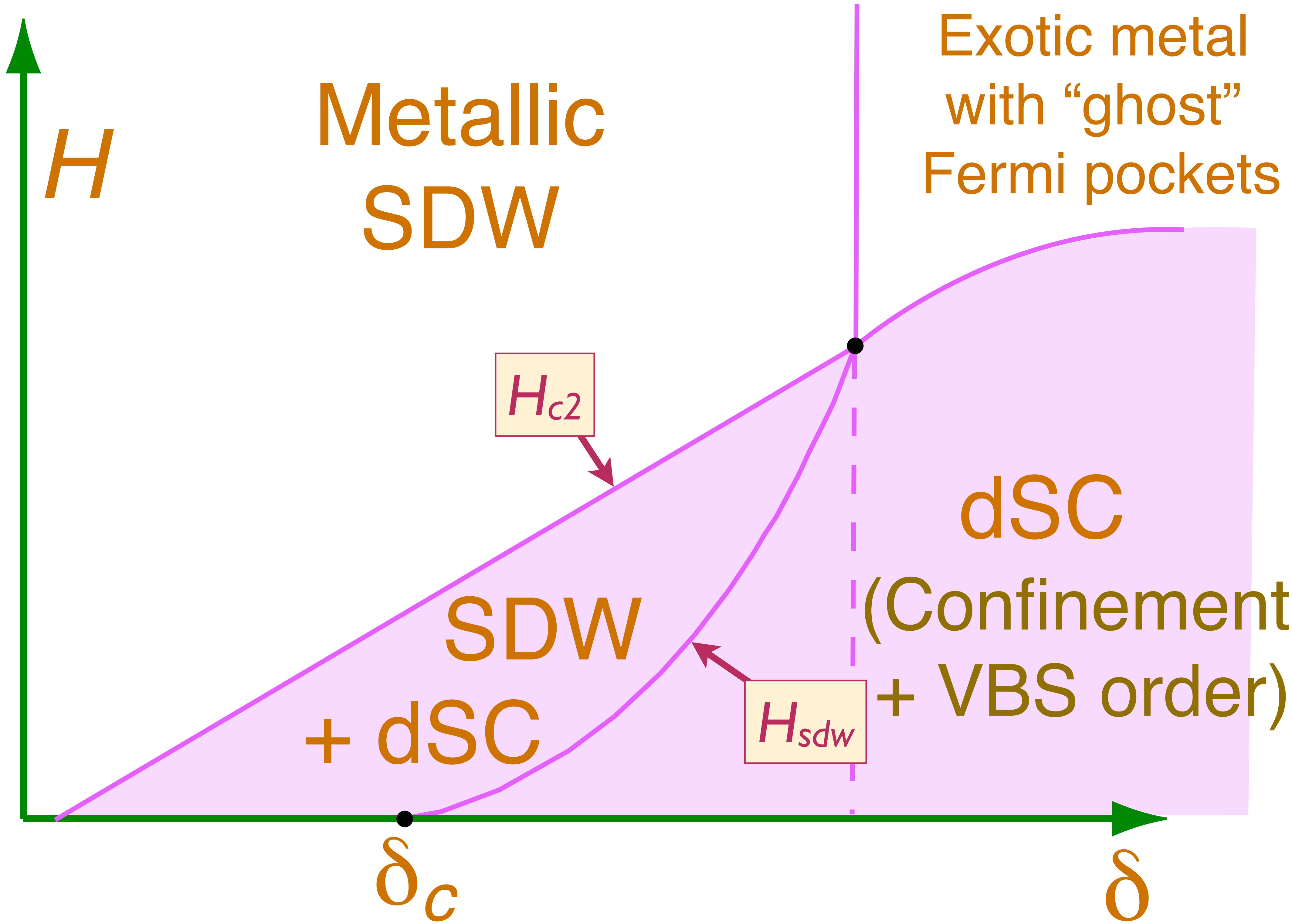
Competing order predictions:

- Upper-critical field, H_{c2}, decreases as SDW is enhanced with decreasing doping (r)
- Onset of SDW order occurs at a larger doping in the normal state (line CM) than in the superconductor (point A).
- For $r > r_c$, there is a field-induced quantum phase transition (line AM) at $H = H_{sdw}$ involving onset of SDW order.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field H.



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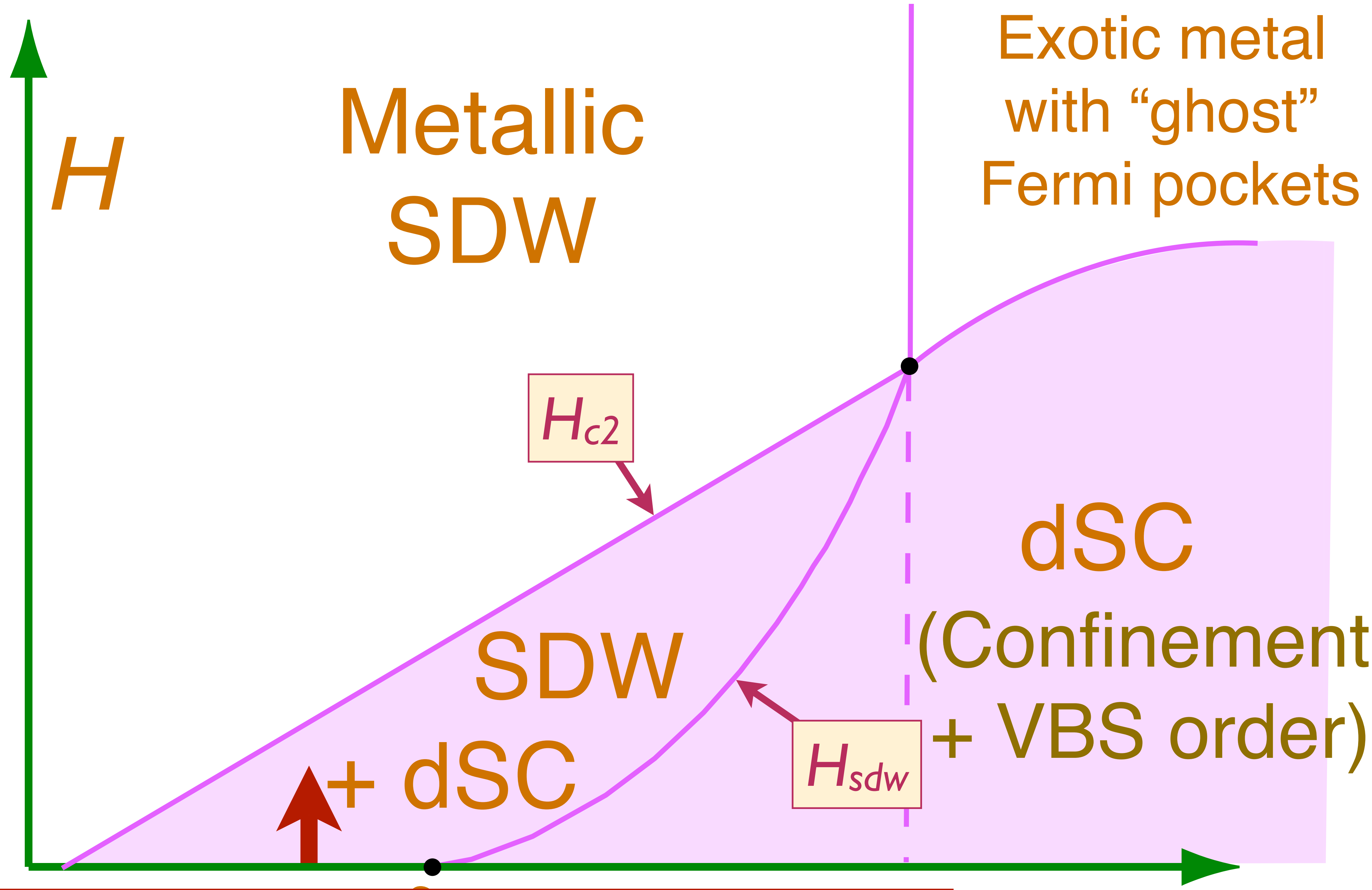

Neutron scattering on $La_{1.9}Sr_{0.1}CuO_4$
B. Lake *et al.*, *Nature* **415**, 299 (2002)

Phase diagram

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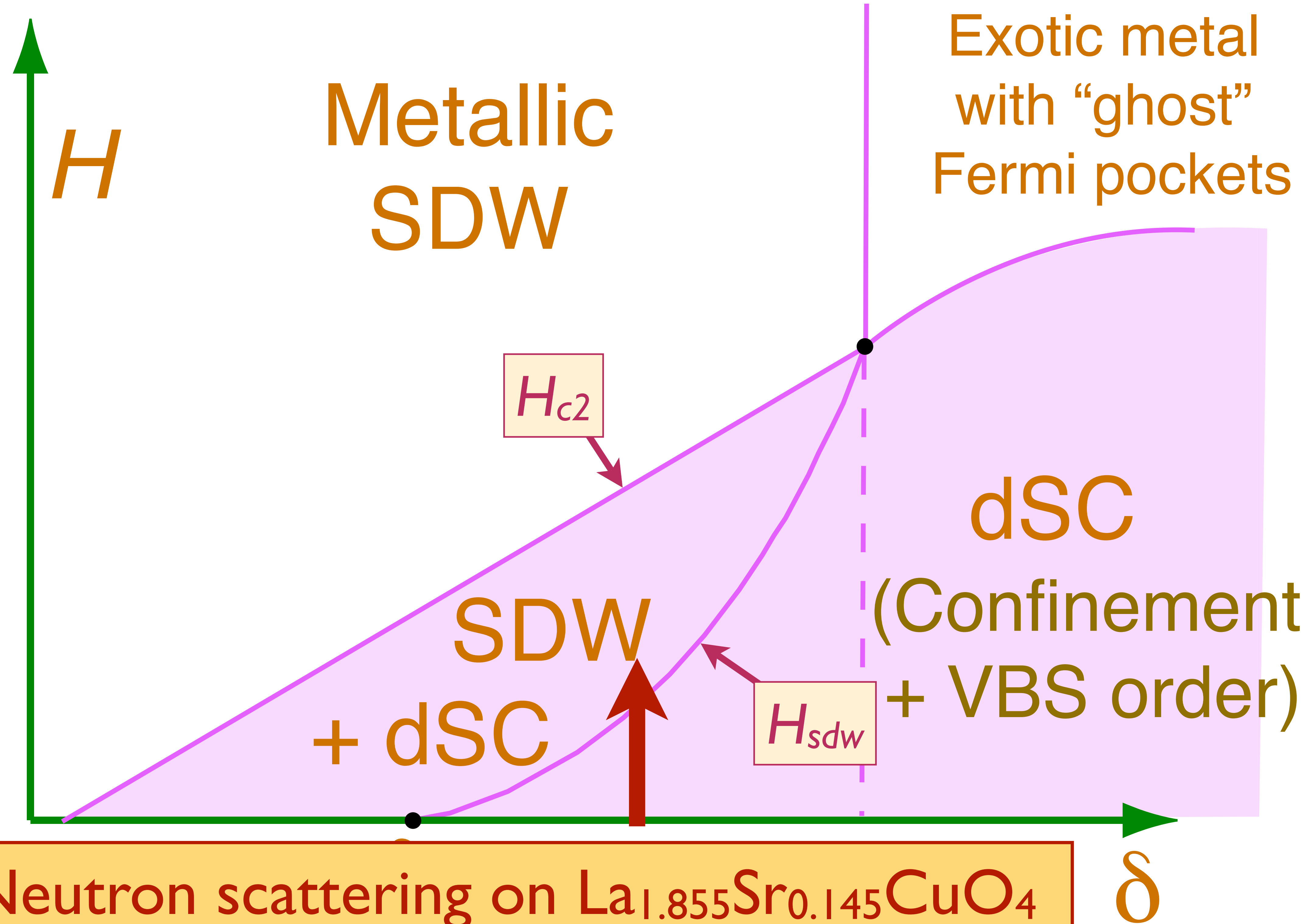
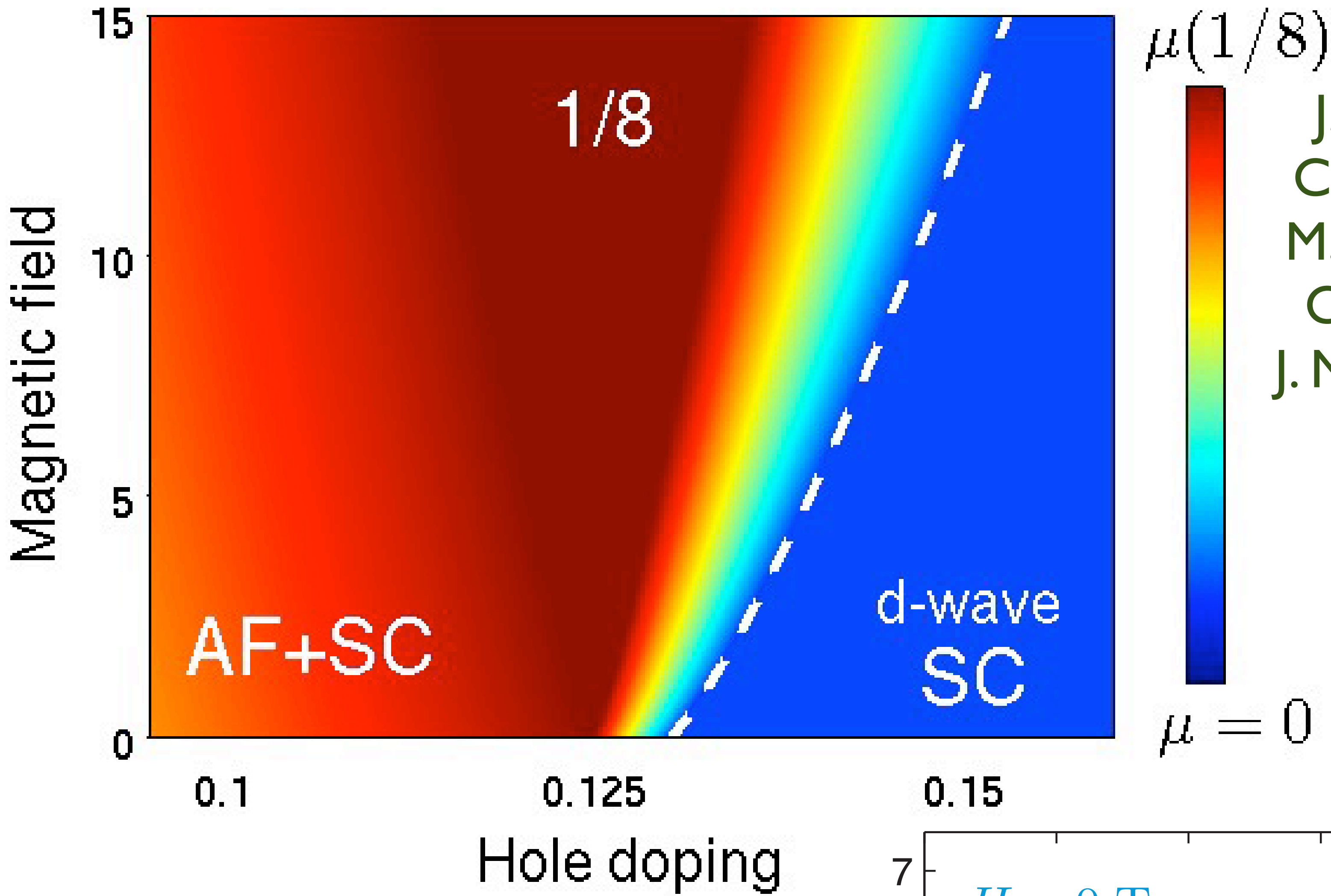
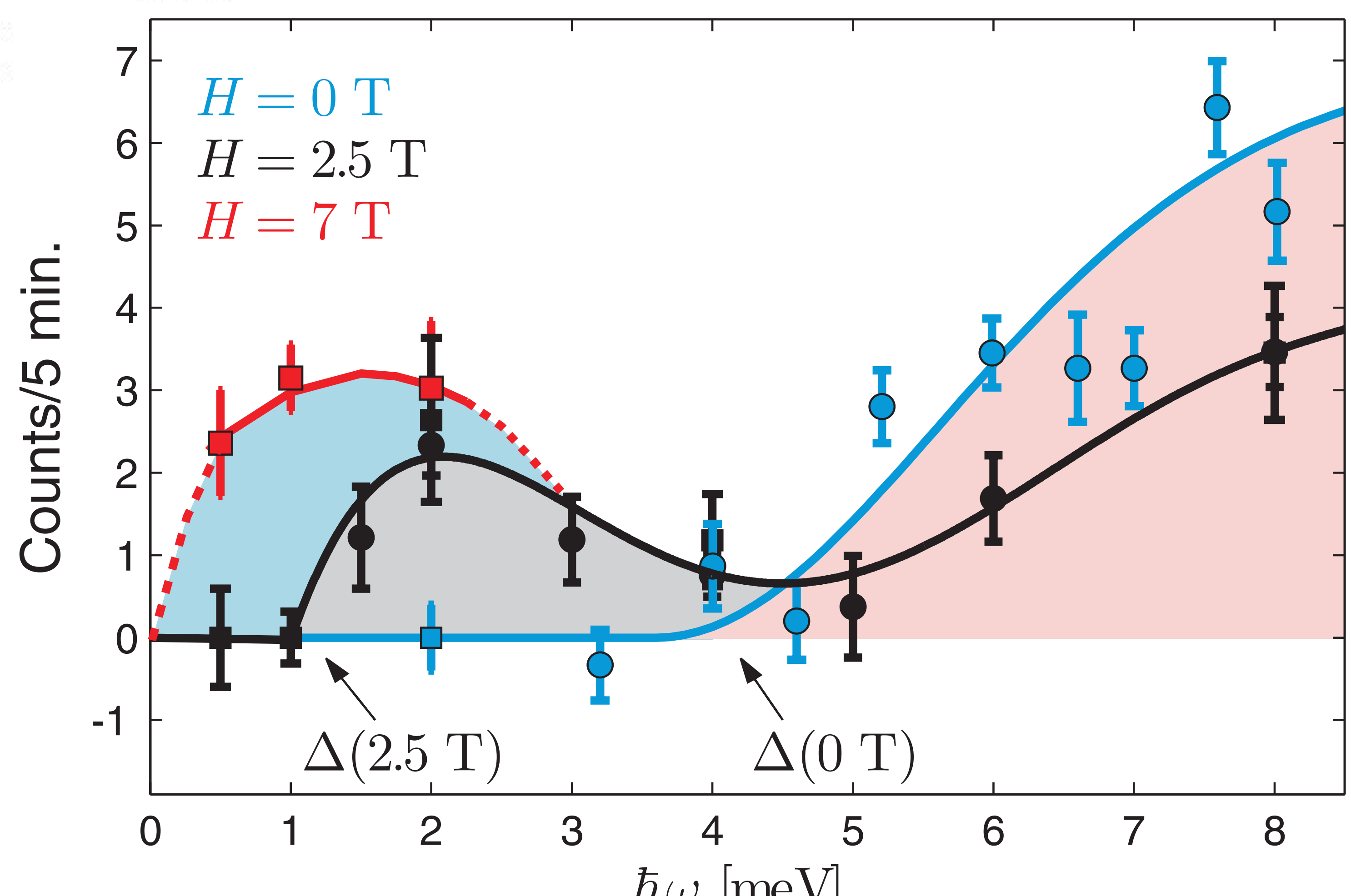


Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, arXiv:0902.1191



J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *arXiv:0902.1191*

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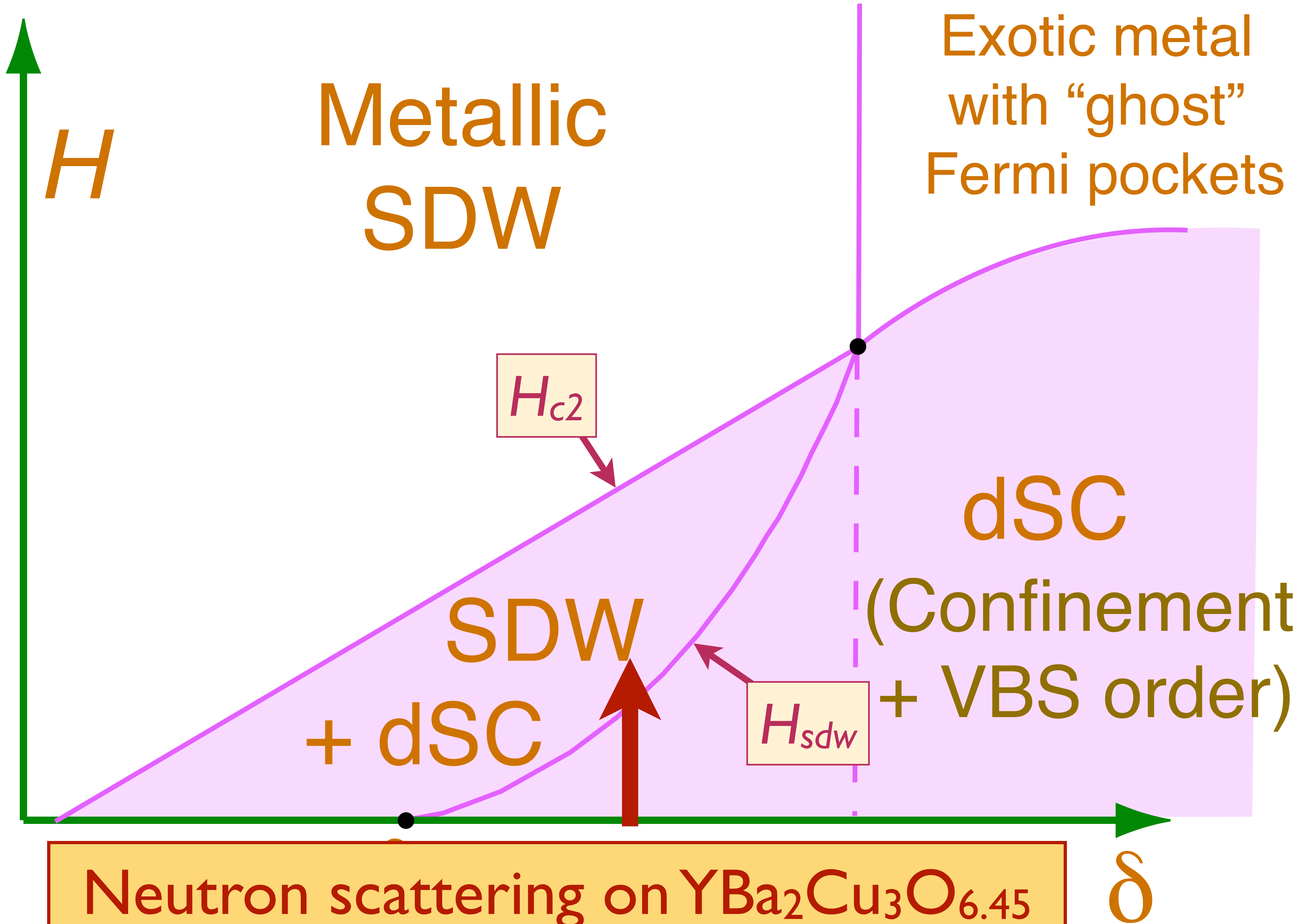
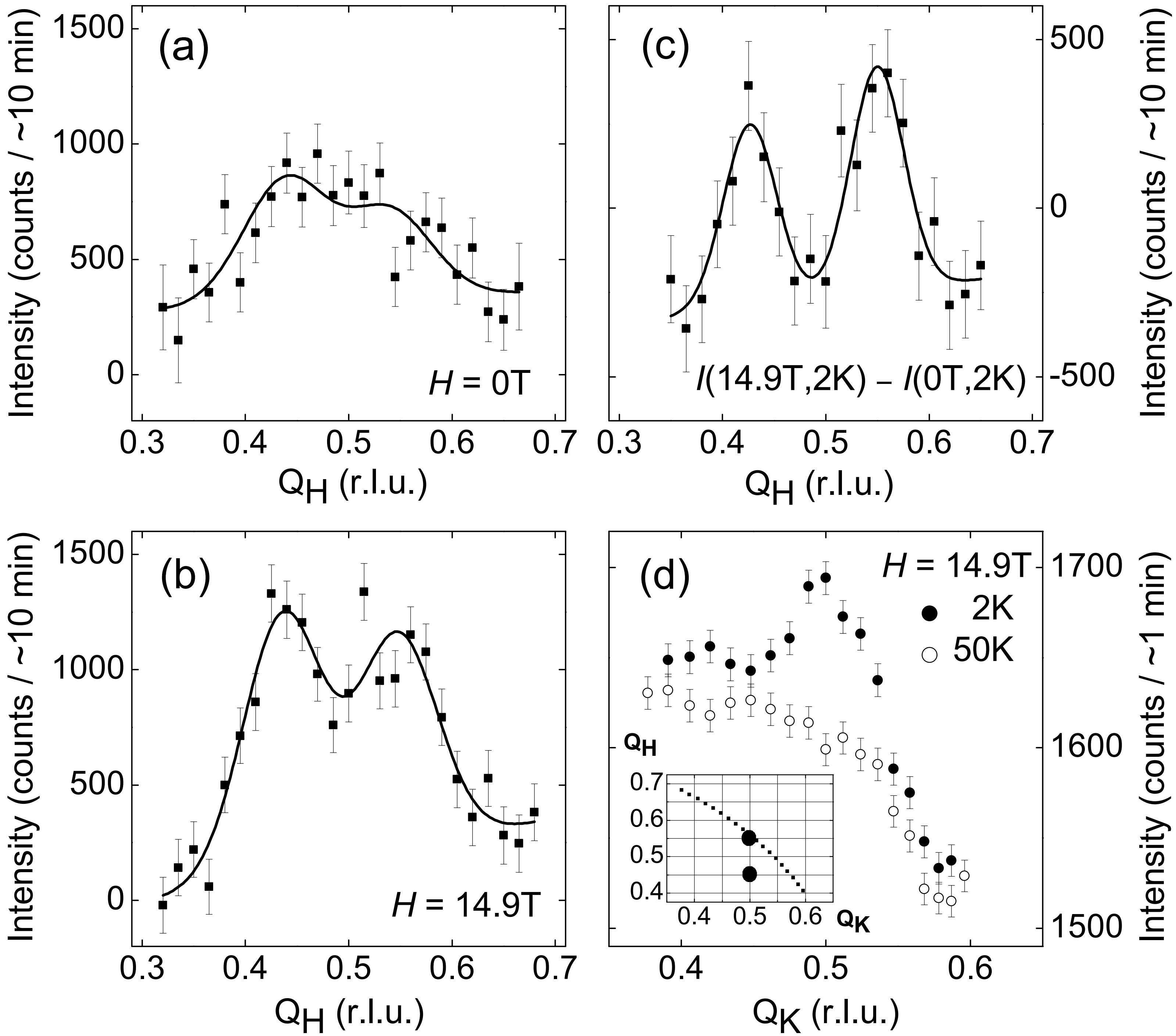


Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, arXiv:0902.3335



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *arXiv:0902.3335*.

Phase diagram

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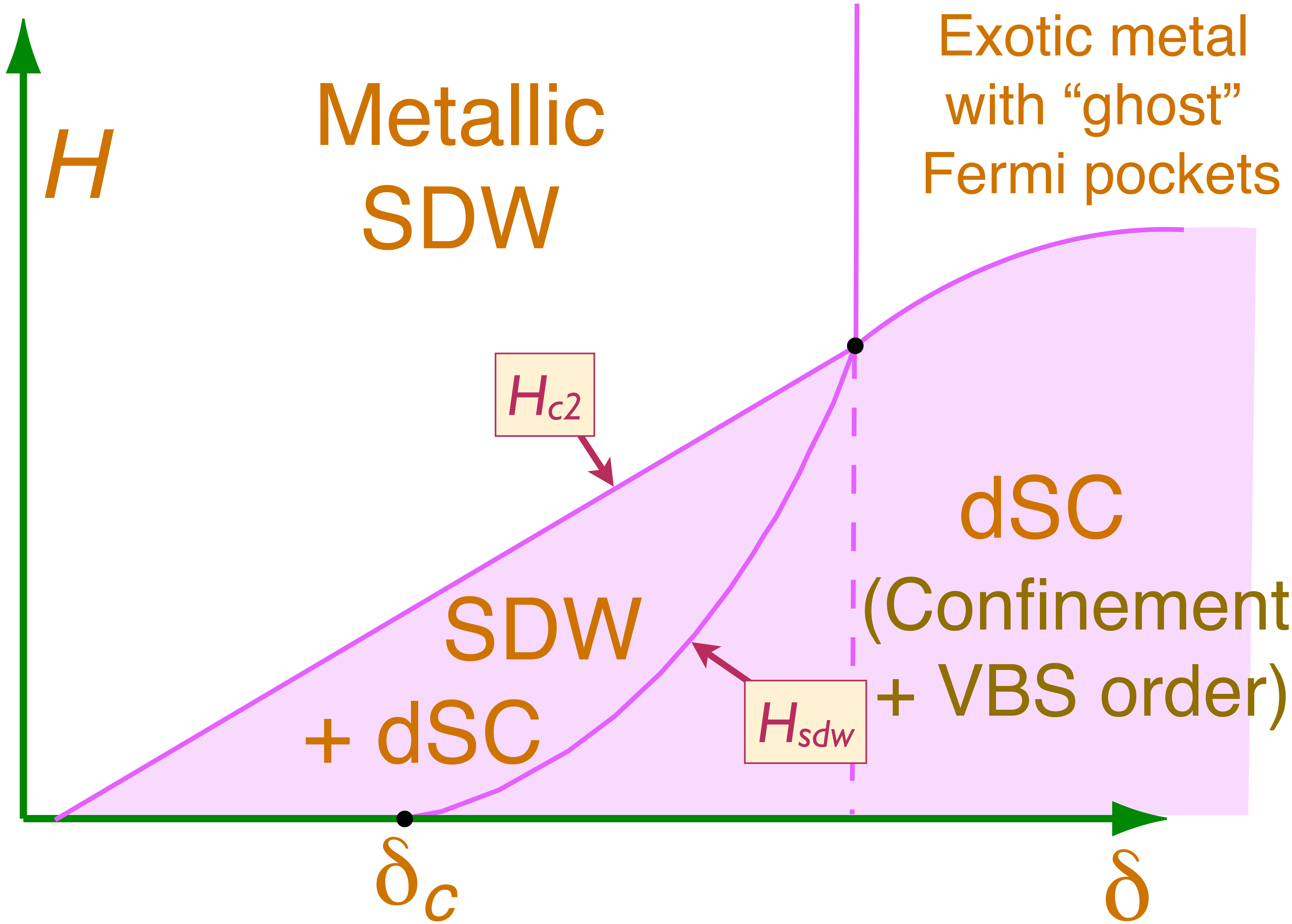

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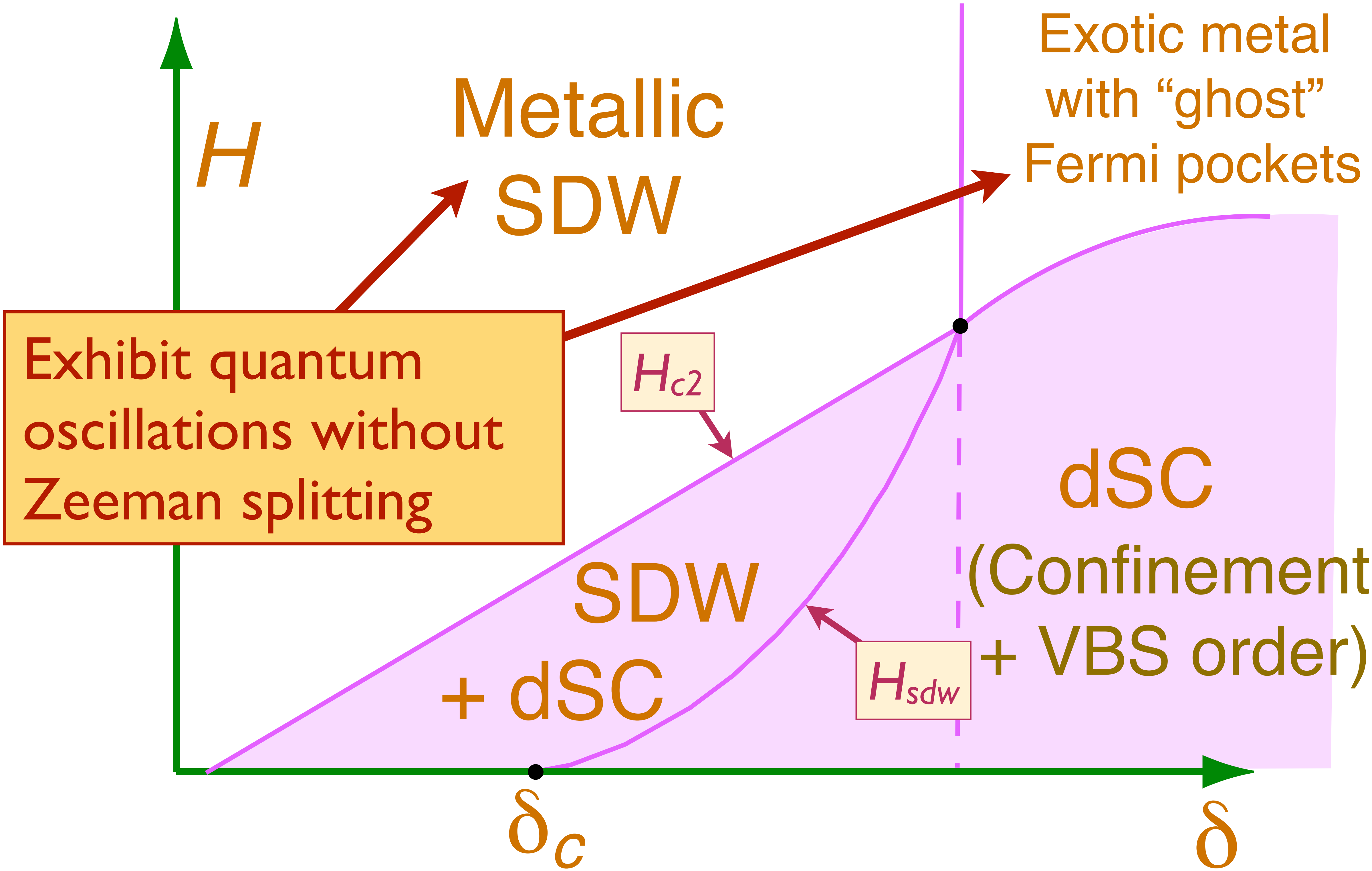

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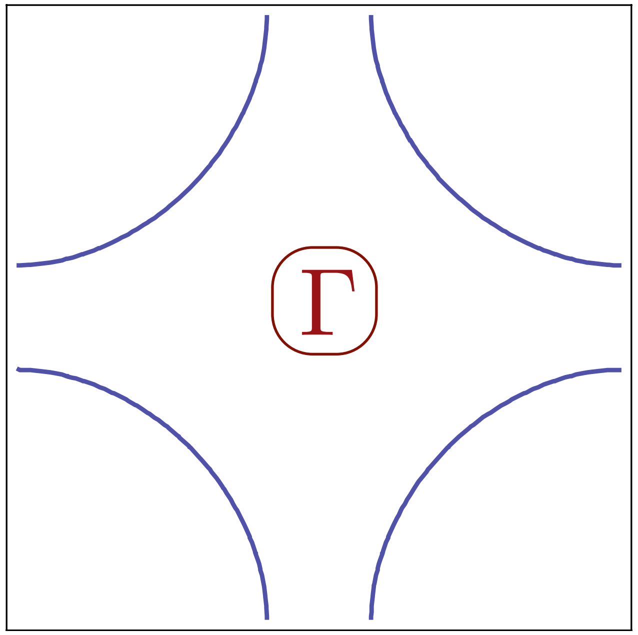
Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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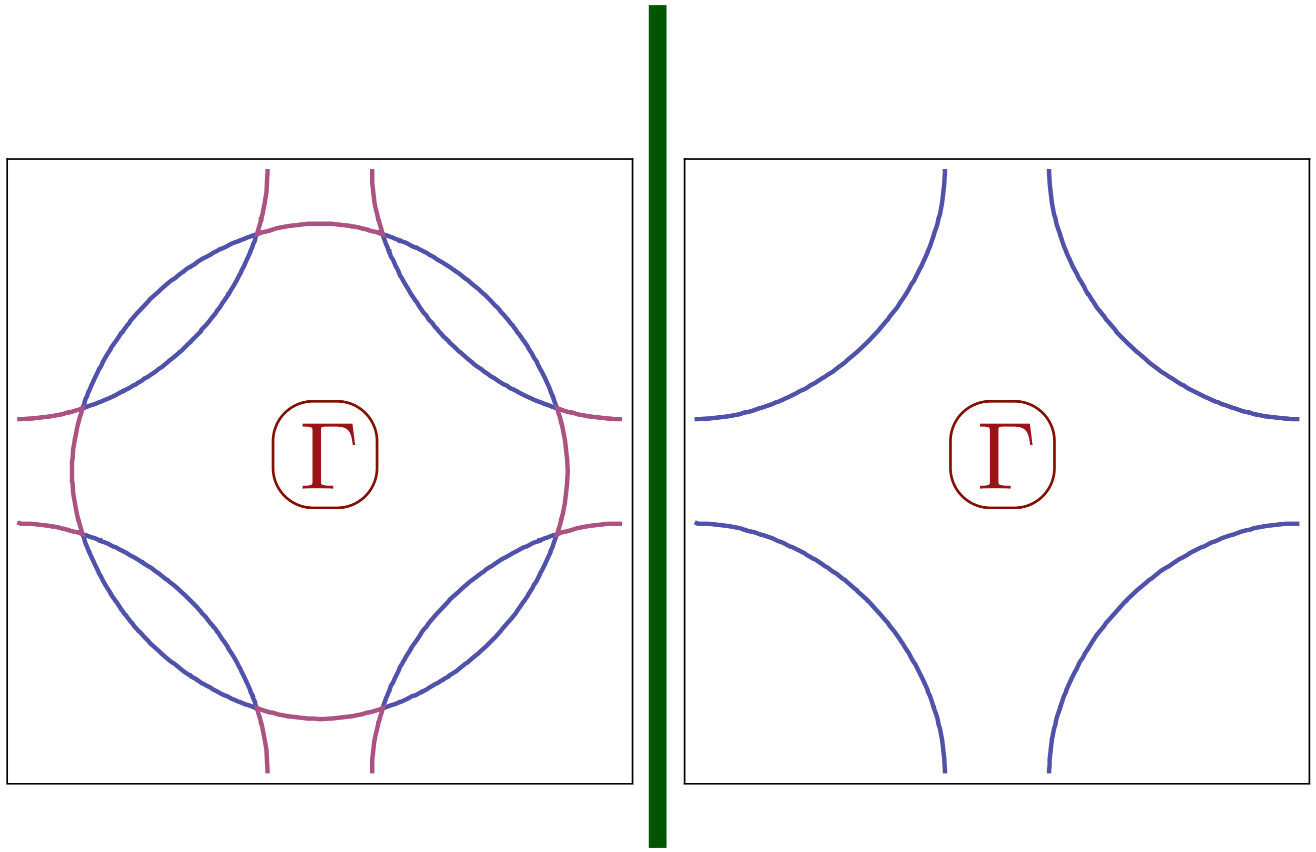

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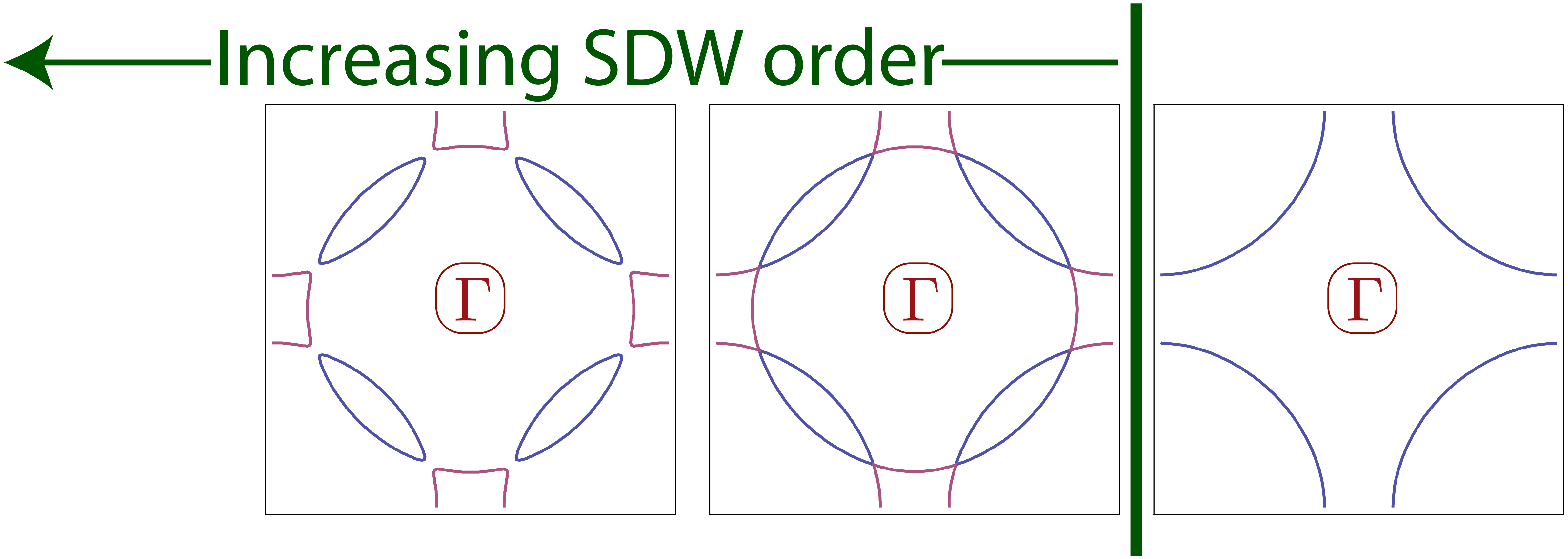

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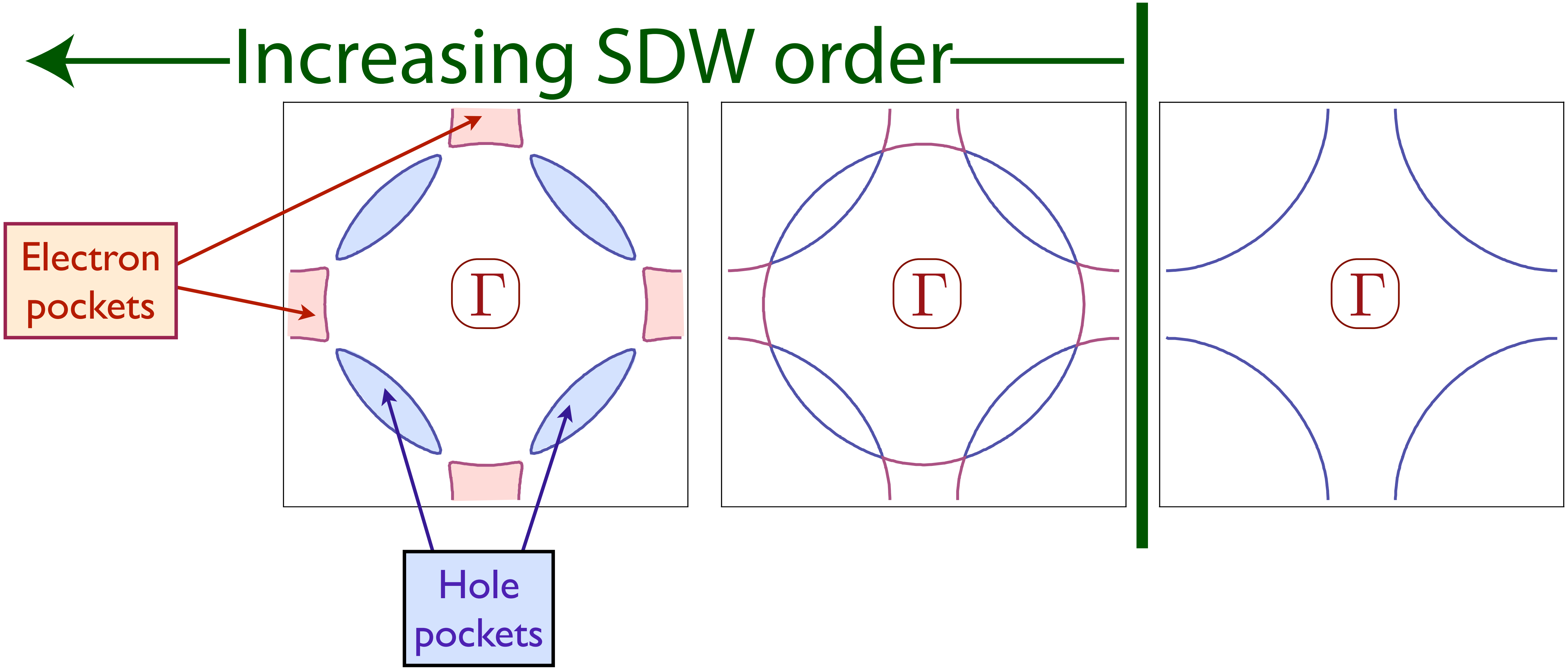

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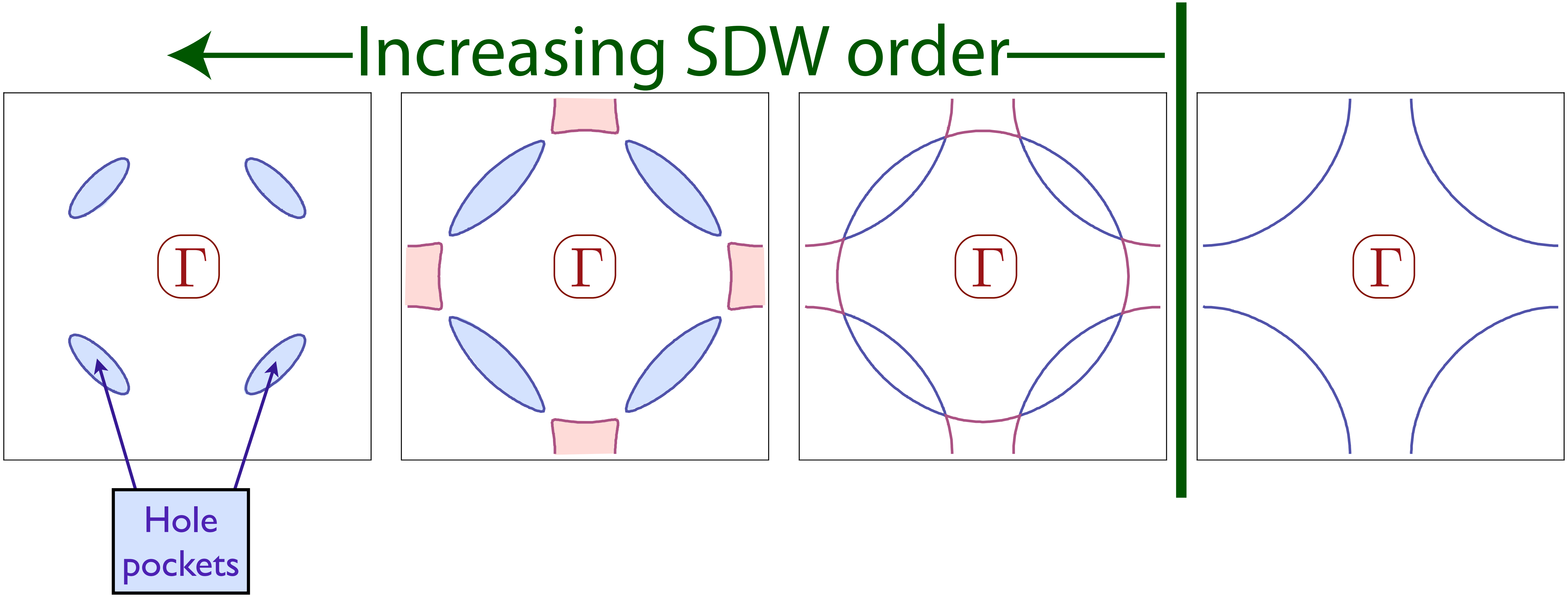

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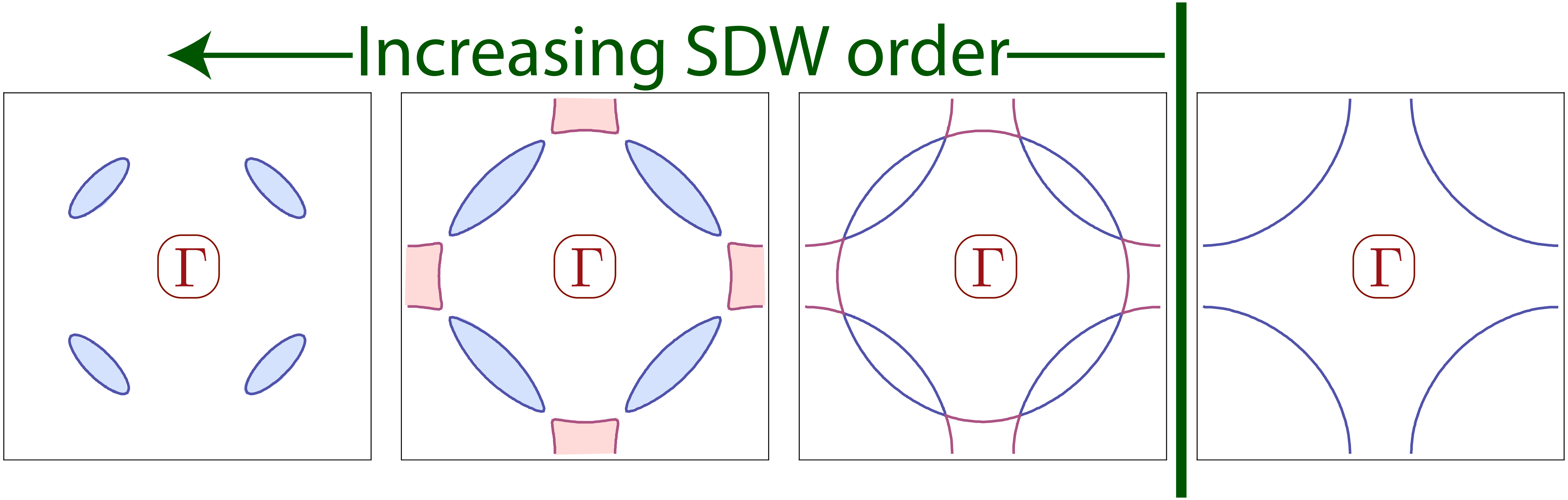

$$\text{SDW order parameter is a vector, } \vec{\varphi}\text{,}$$
$$\text{whose amplitude vanishes at the transition}$$
$$\text{to the Fermi liquid.}$$

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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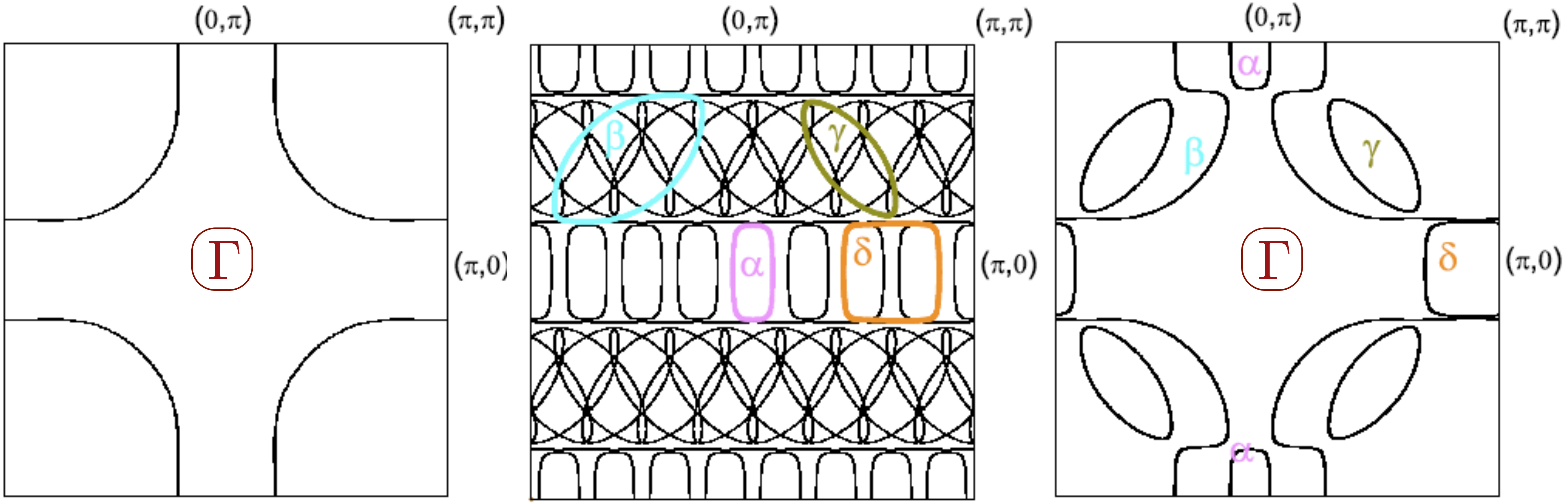
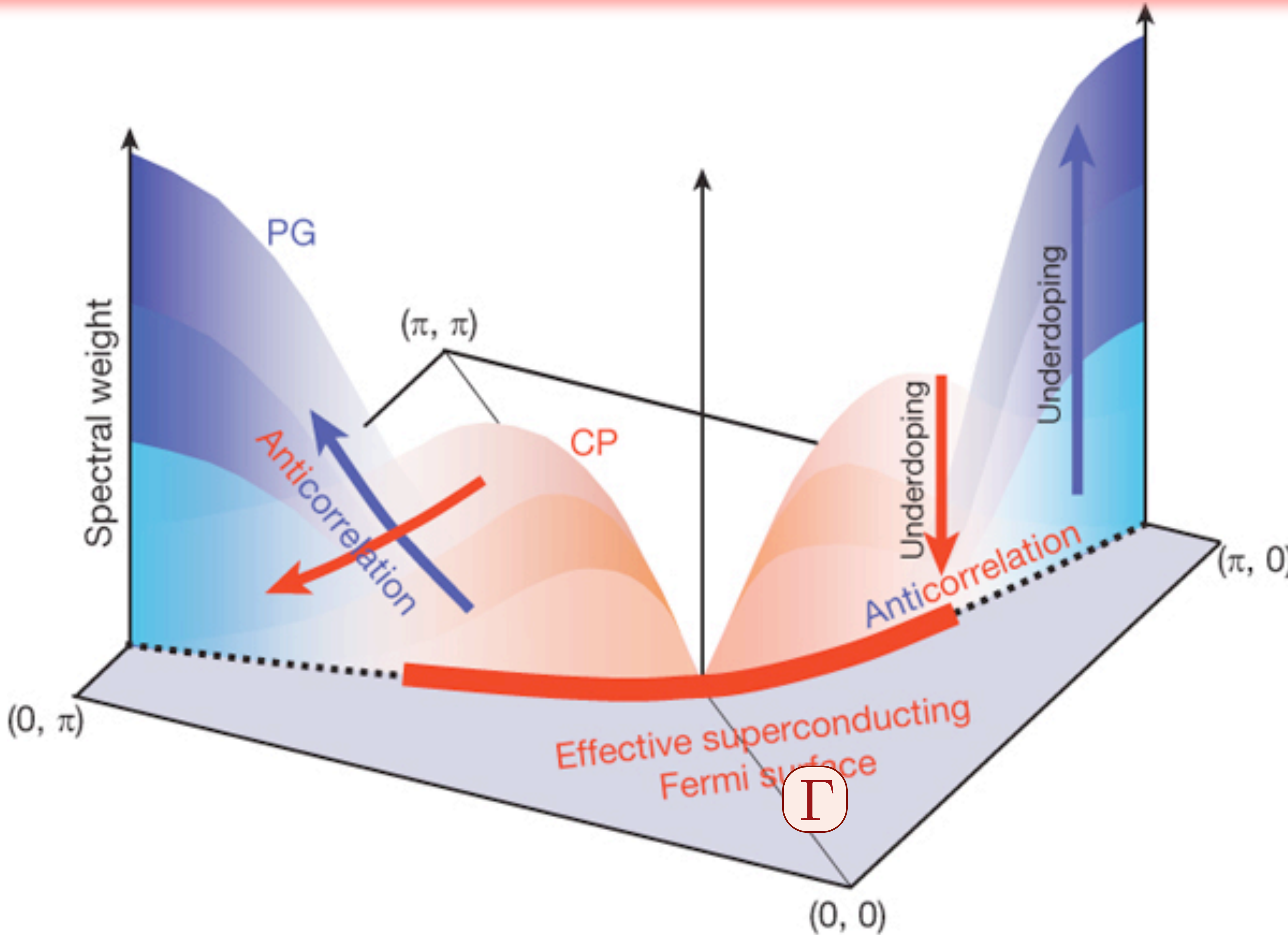


Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, arXiv:0902.2741.

(c) Nodal-anti-nodal dichotomy in the underdoped cuprates



Competition between the pseudogap and superconductivity in the high-Tc copper oxides

T. Kondo, R. Khasanov, T. Takeuchi, J. Schmalian, A. Kaminski, *Nature* **457**, 296 (2009)

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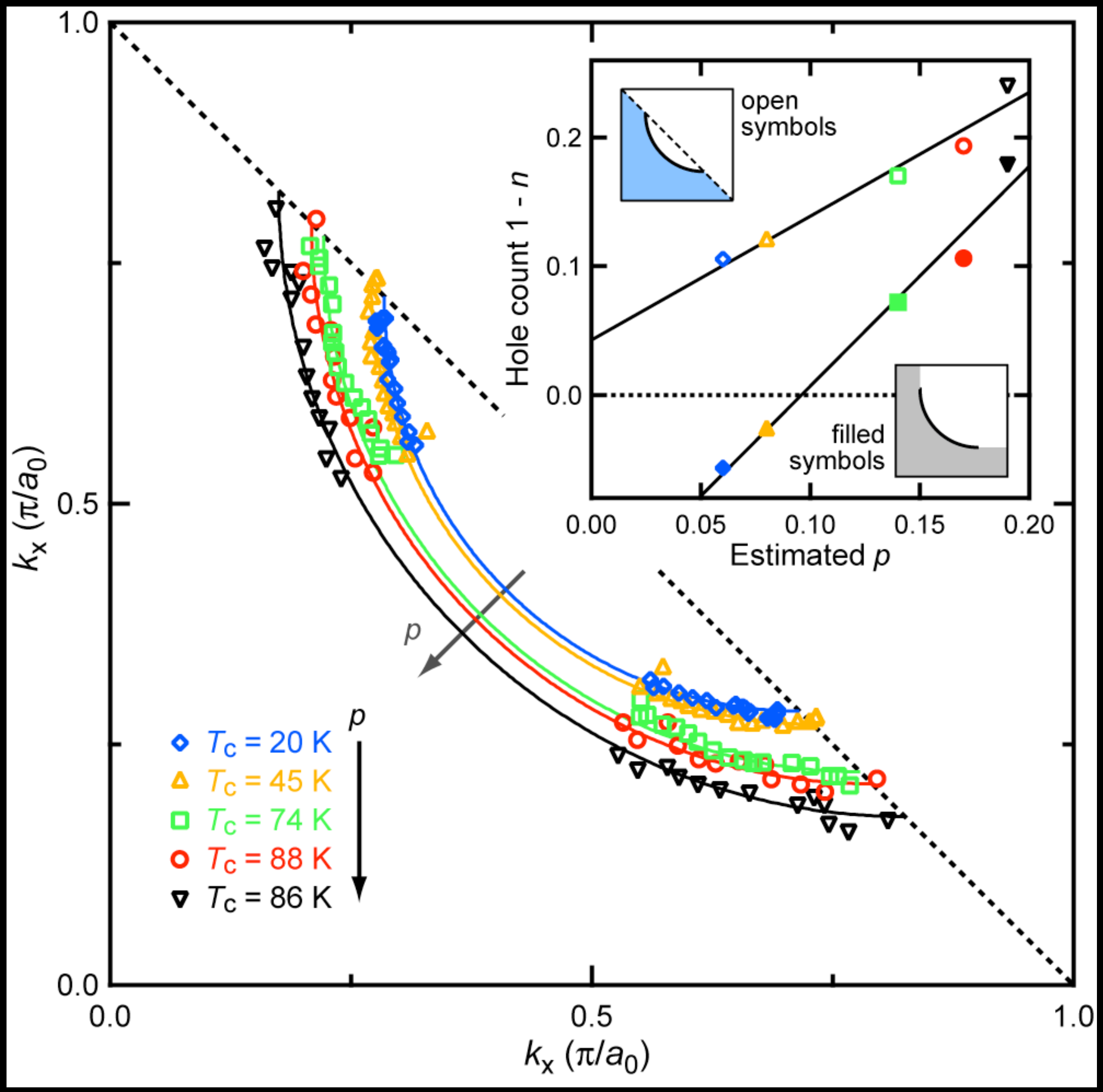

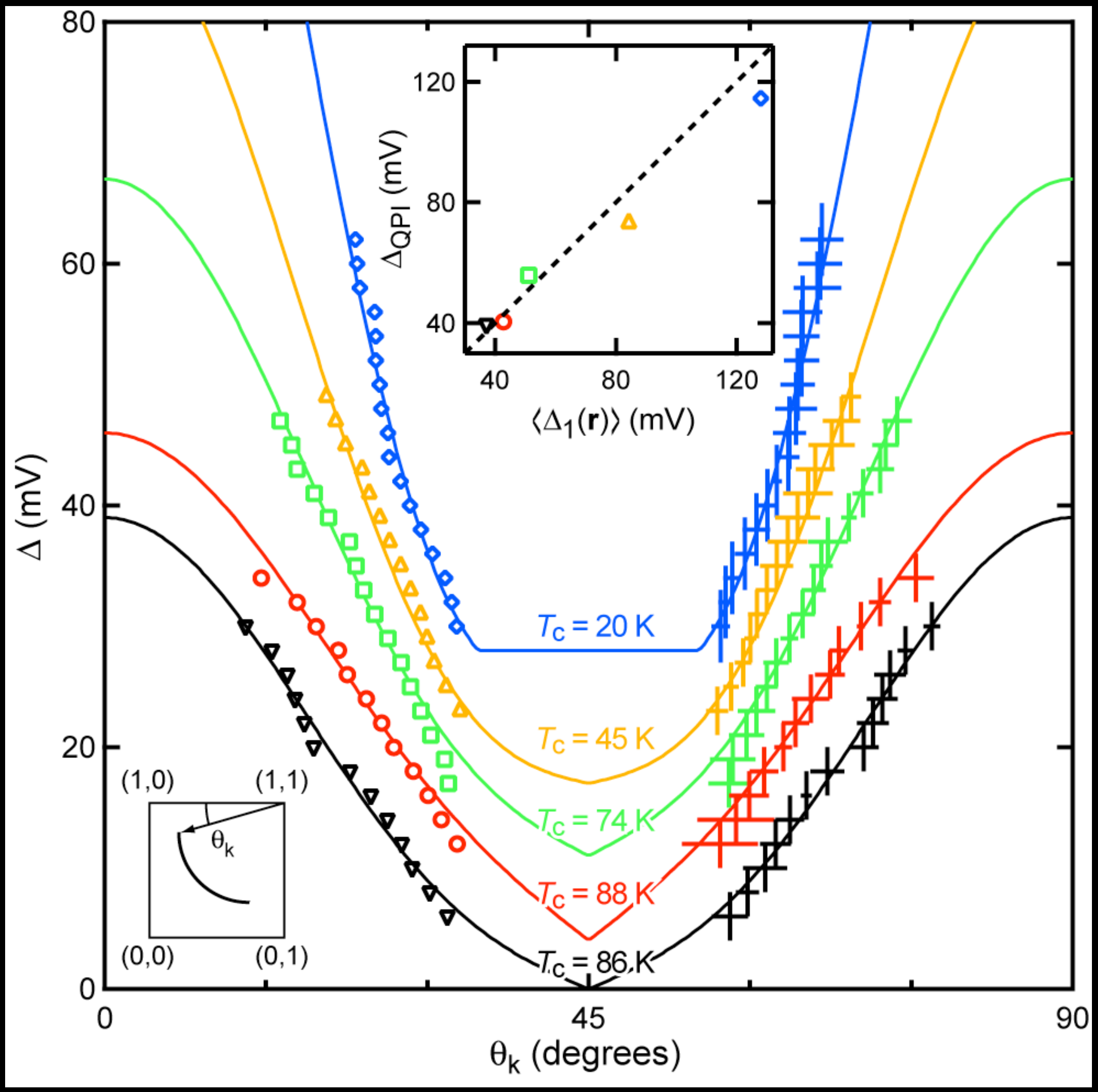

Y. Kohsaka et al., Nature 454, 1072, (2008)

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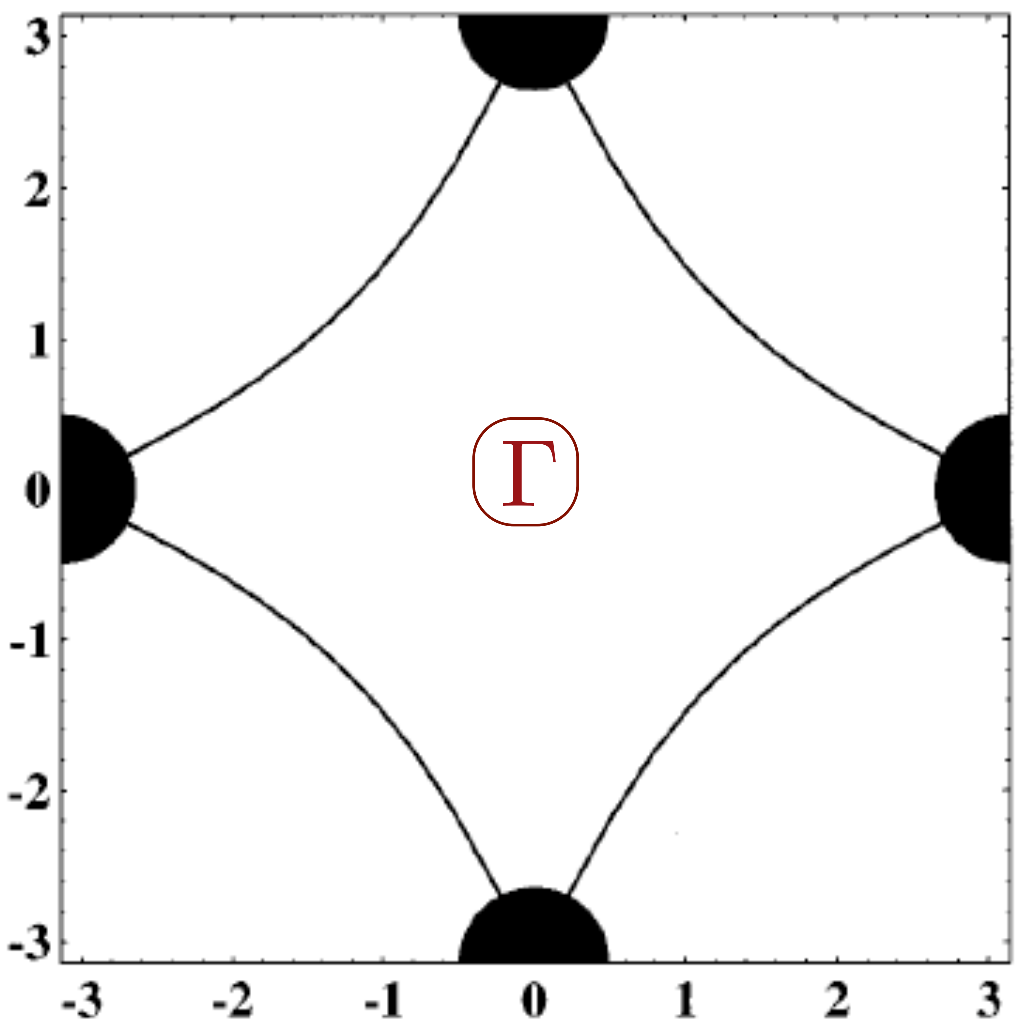

FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}{}' V_{p,q}(b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \mathrm{H.c.}) + \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma};$$

$$V_{p,q} = V a^2 (p_x^2 - p_y^2)$$

-2e bosons at antinodes, +e fermion "arcs" at nodes, and proximity "Josephson" coupling

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

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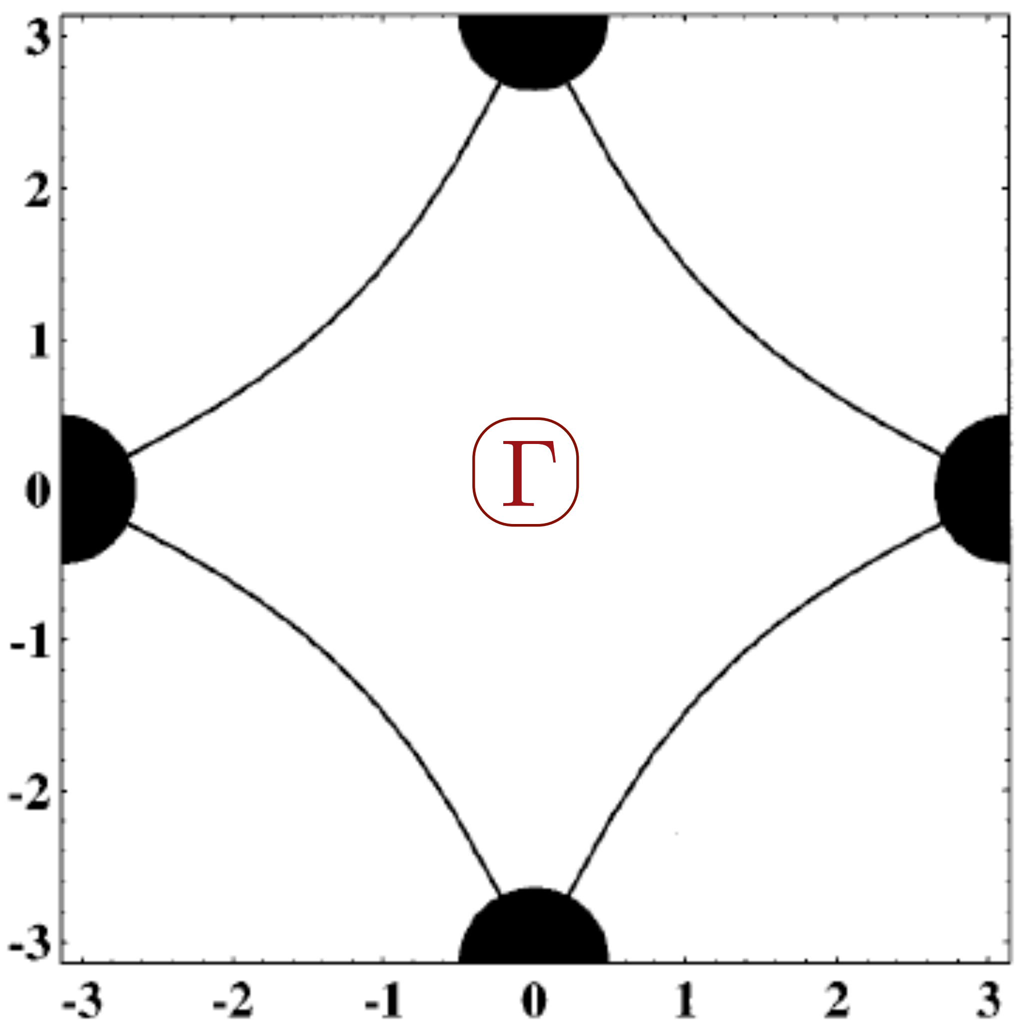

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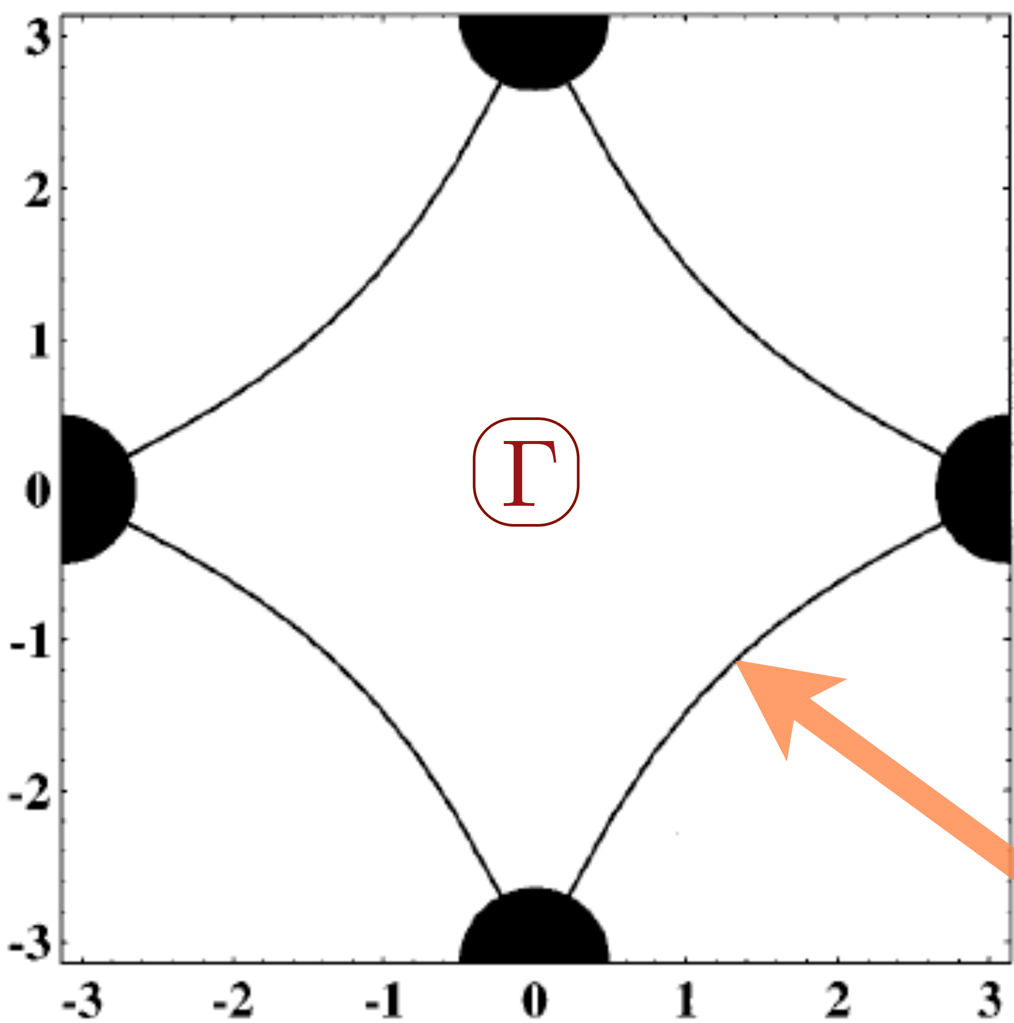

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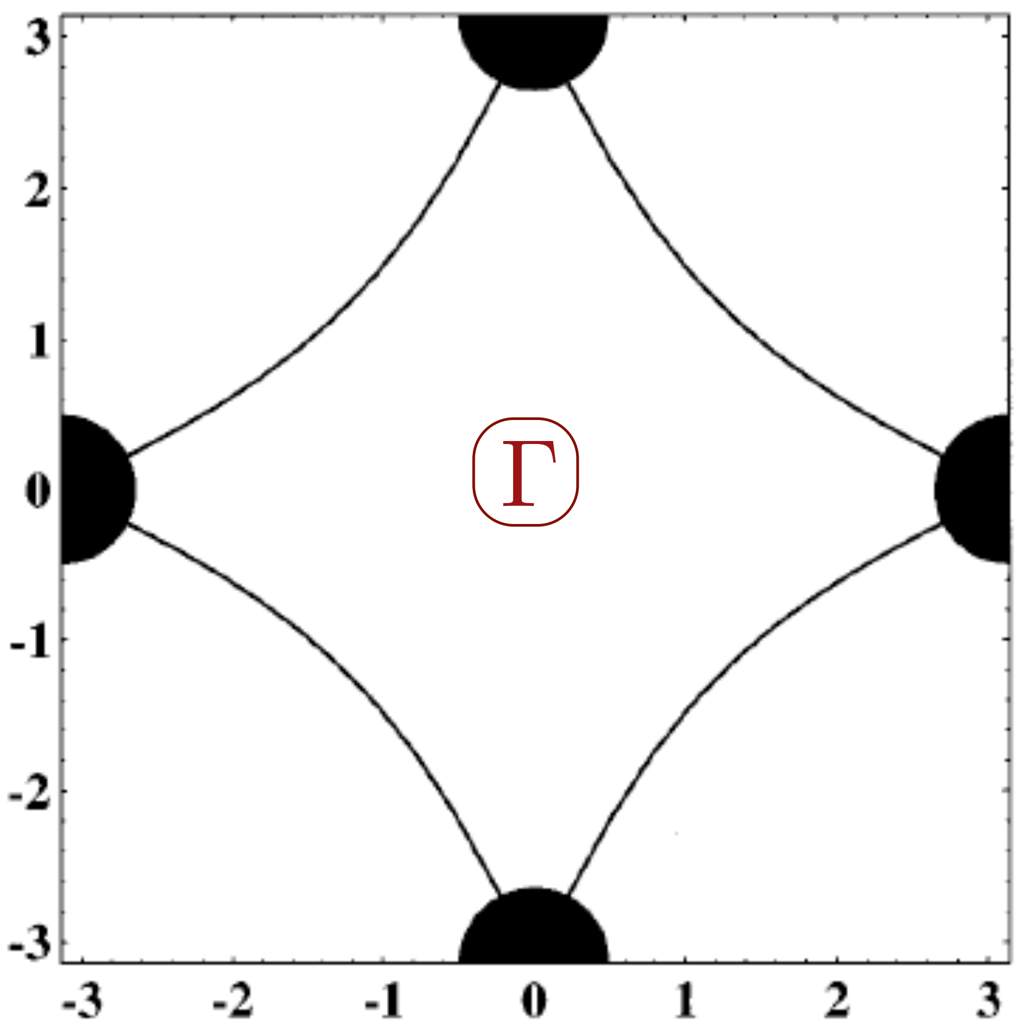

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(c) Nodal-anti-nodal dichotomy in the underdoped cuprates

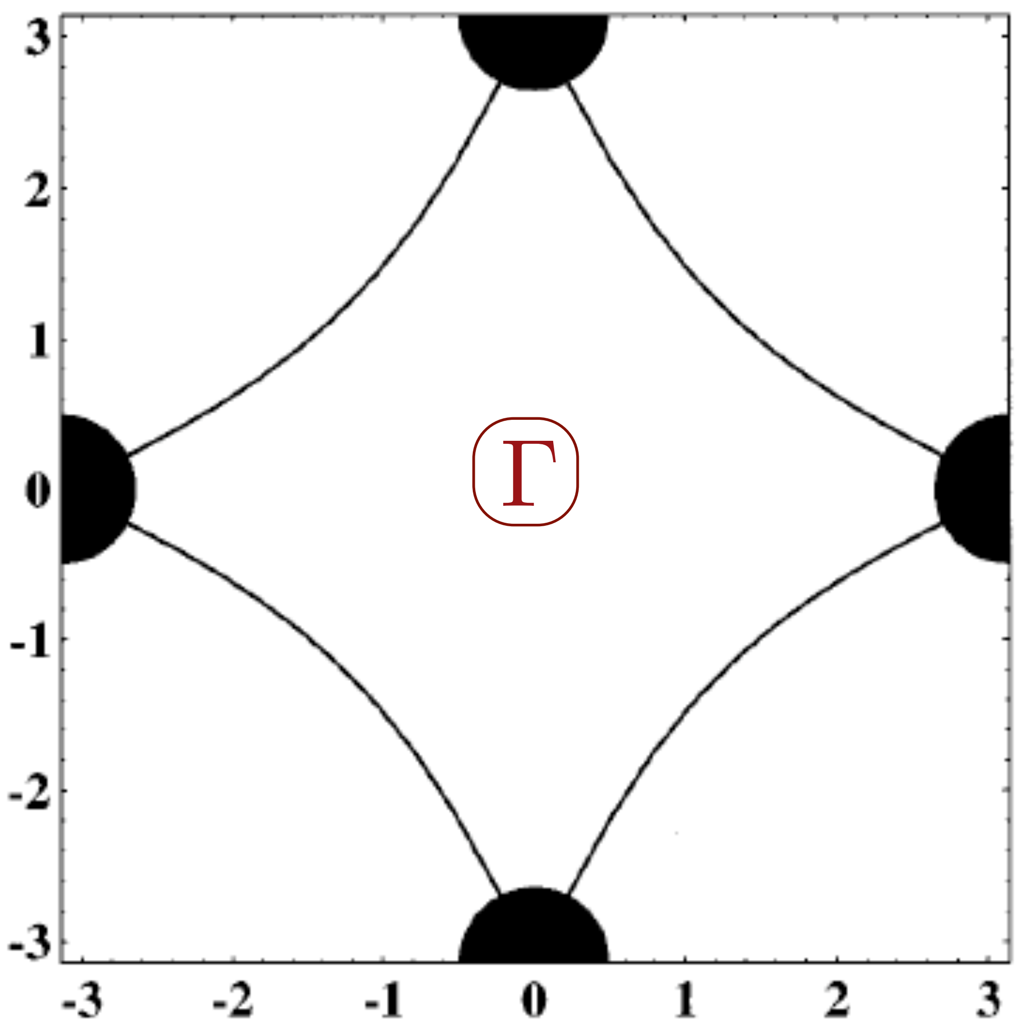

FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}{}' V_{p,q}(b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \text{H.c.})$$

$$+ \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma};$$

$$V_{p,q} = V a^2 (p_x^2 - p_y^2)$$

Attractive phenomenological model, but theoretical and microscopic basis is unclear

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

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   (a) Quantum oscillations
   (b) Competing orders
   (c) Nodal-anti-nodal dichotomy

2. Spin-fluctuation exchange mechanism of d-wave superconductivity

   Successful at large doping, but cannot account for competing orders, the nodal-anti-nodal dichotomy (and other phenomena) at low doping

3. Superconductivity of electron and hole pockets in a background of fluctuating antiferromagnetism

   Pairing by gauge forces the unusual d-wave superconductivity of the underdoped cuprates.

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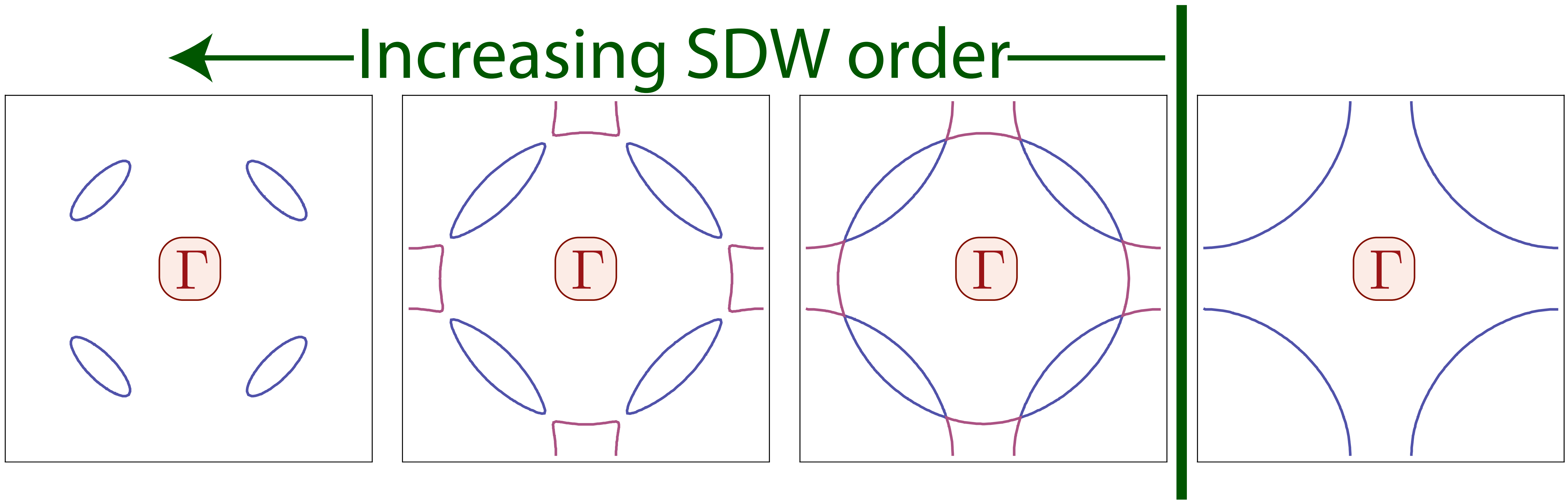
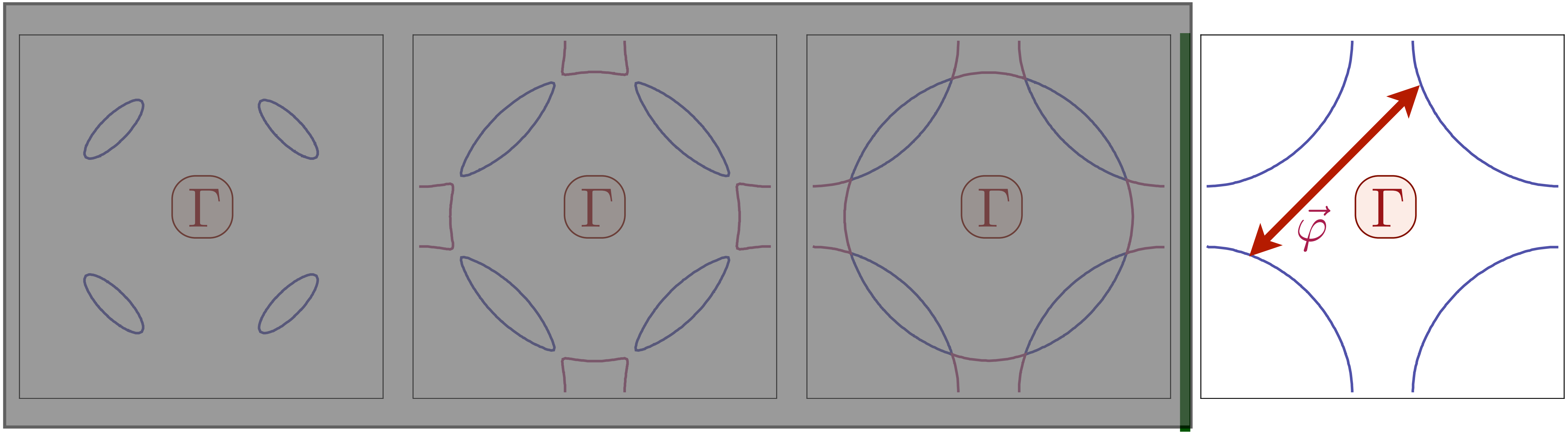
Spin density wave theory in hole-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

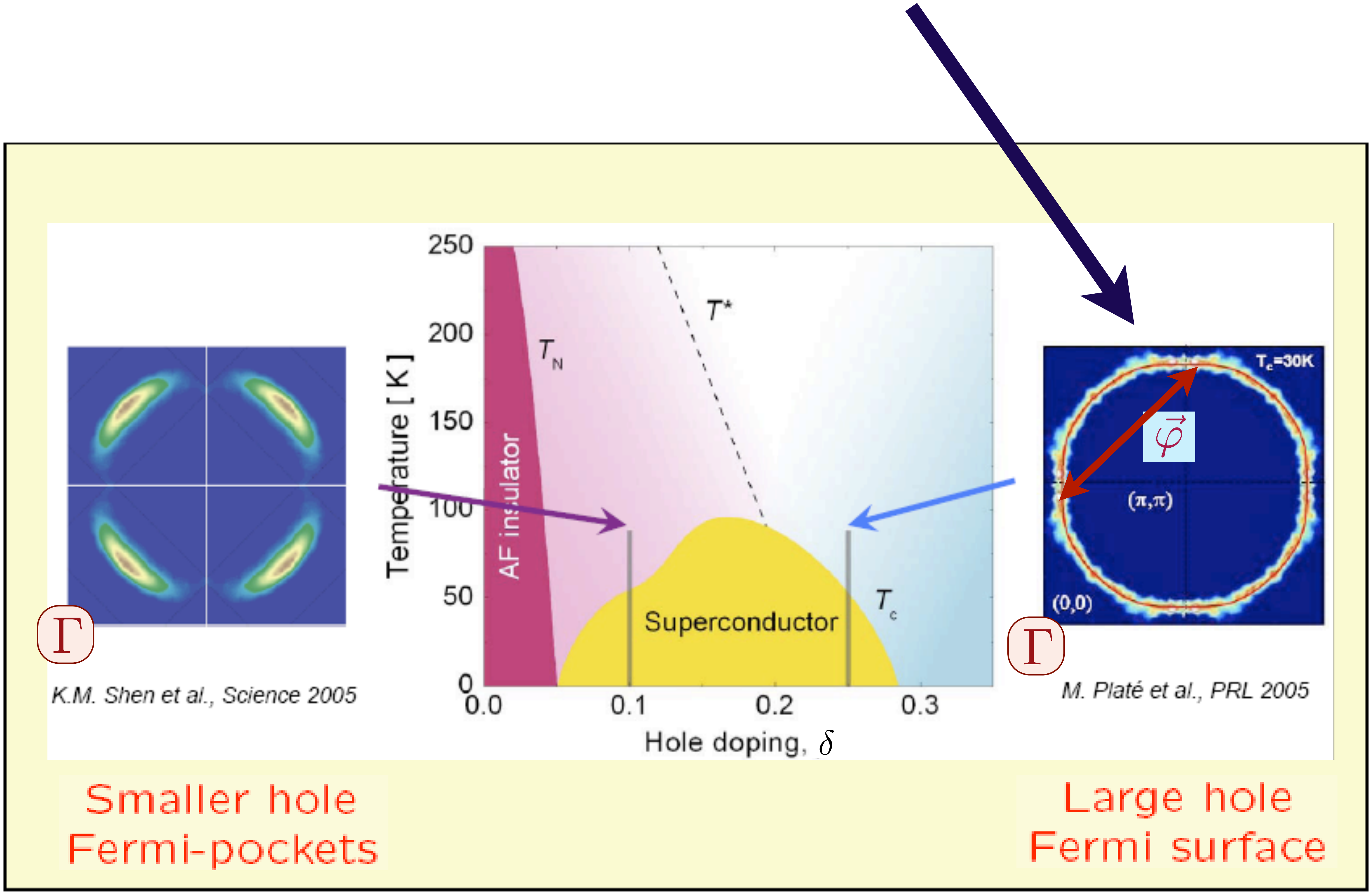
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



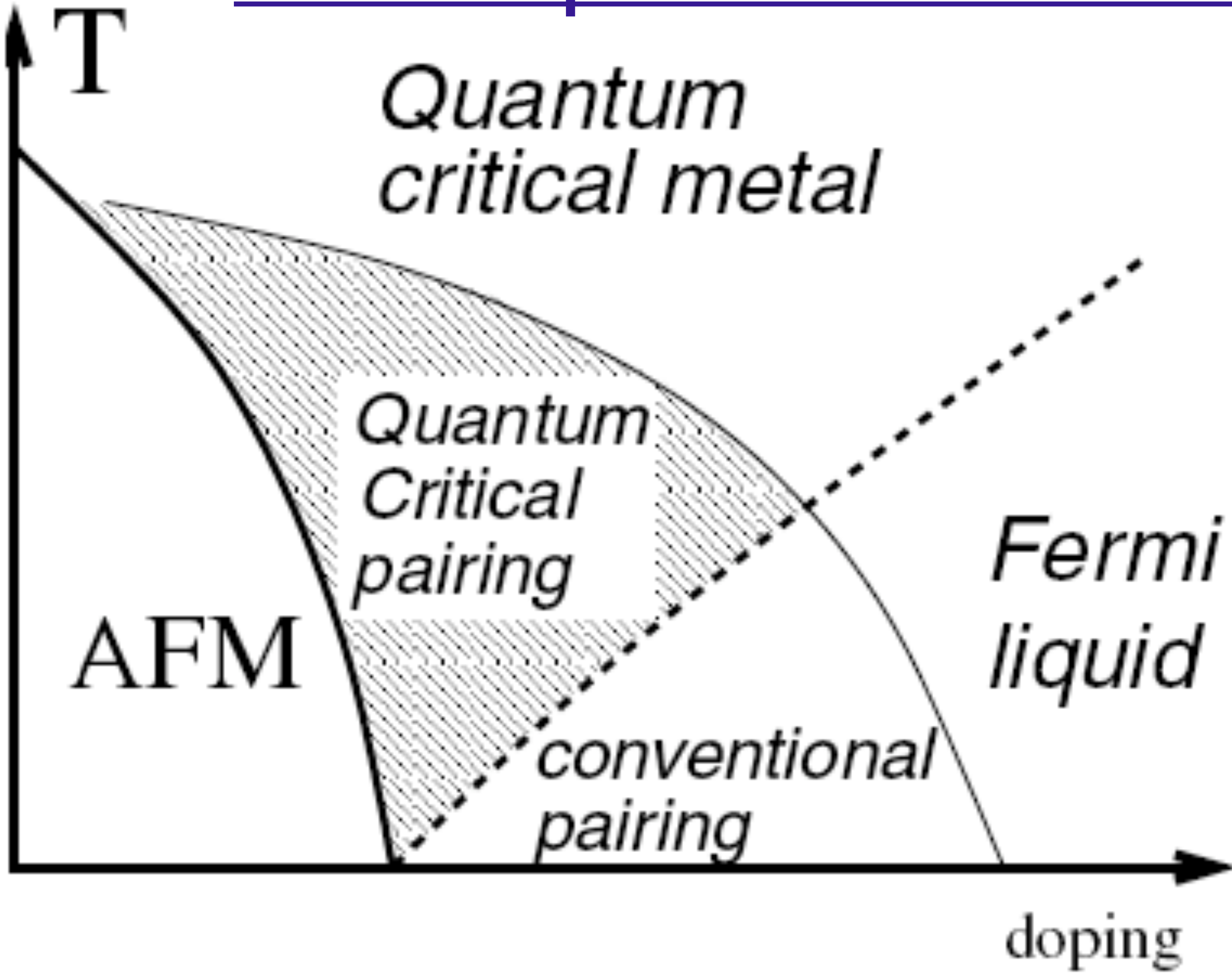
Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

David Pines, Douglas Scalapino

Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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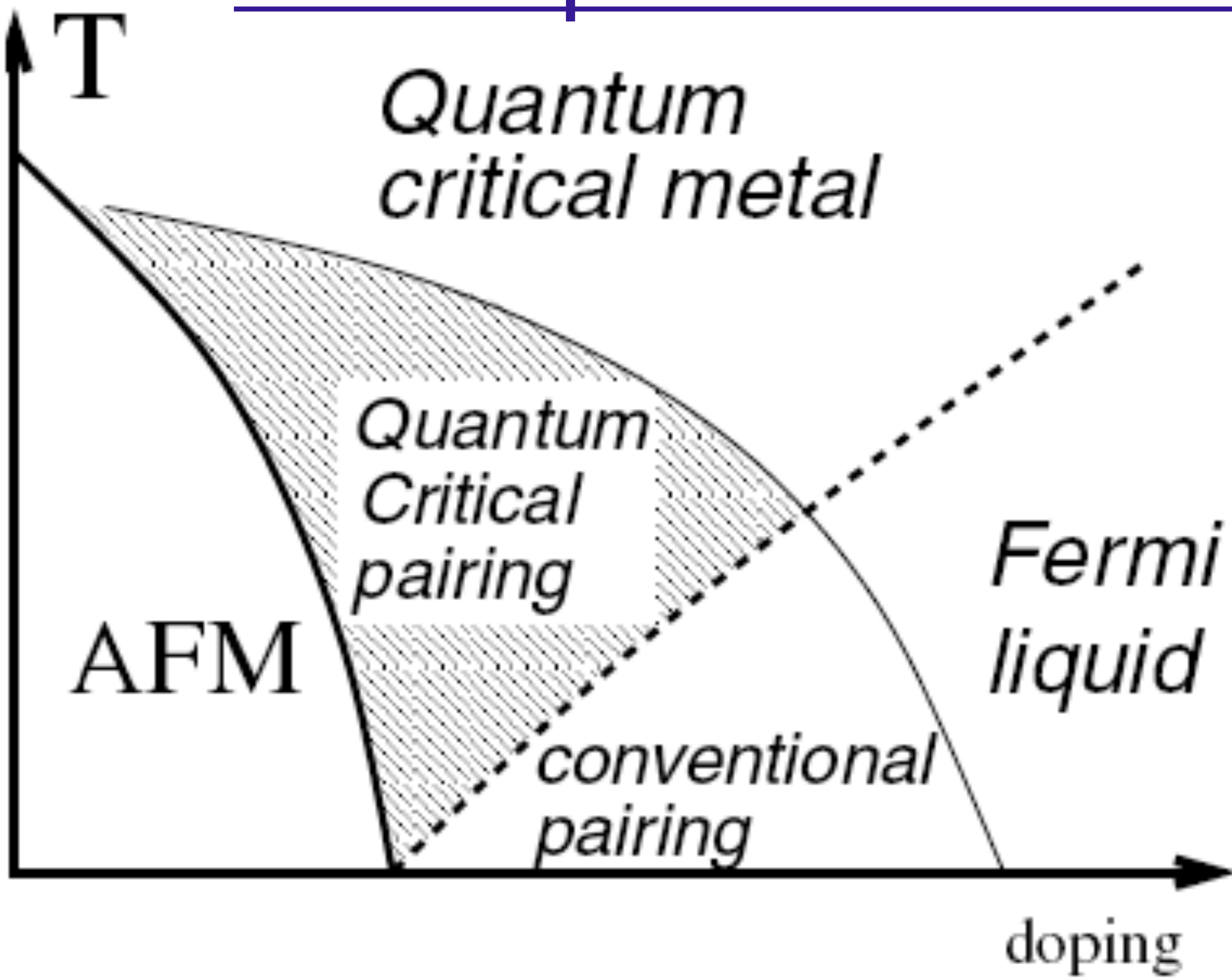

- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.
- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

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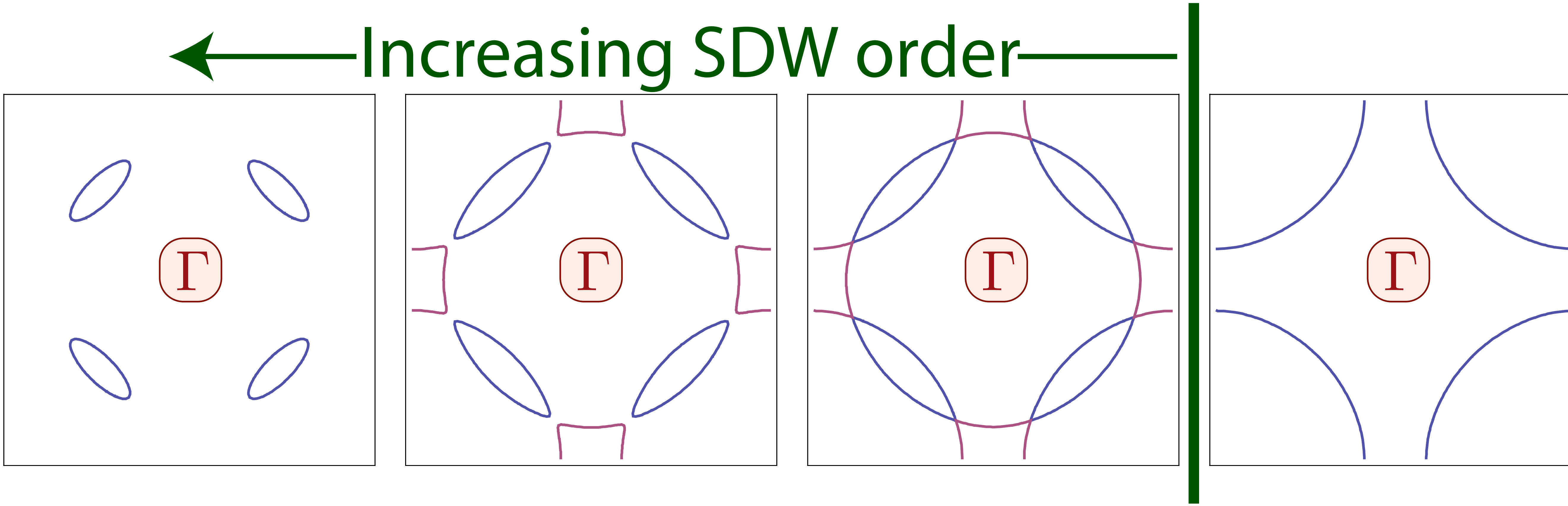
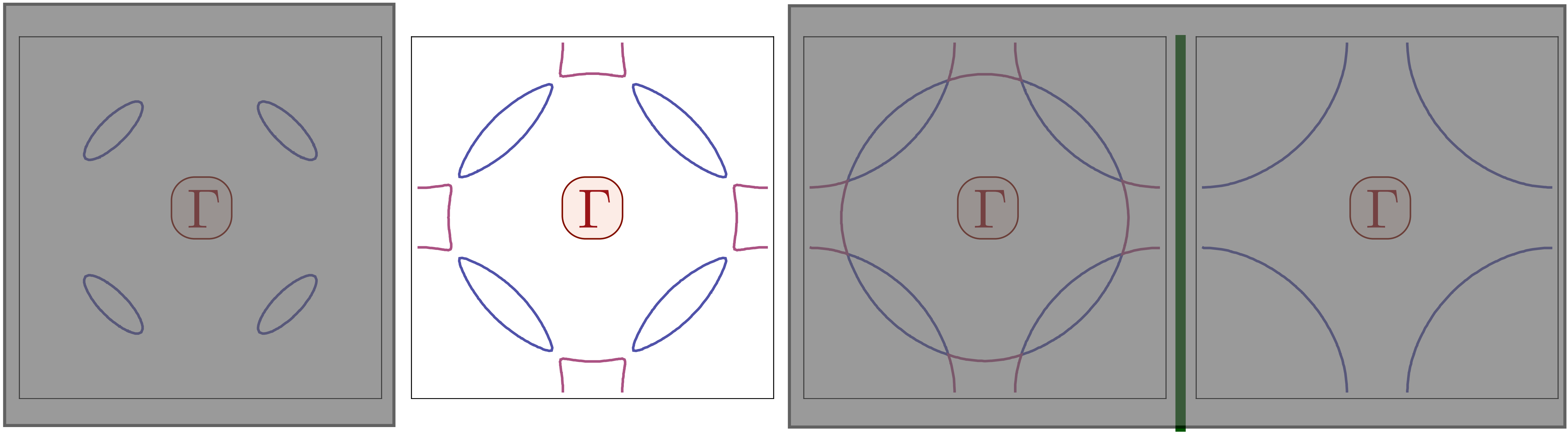
Spin density wave theory in hole-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

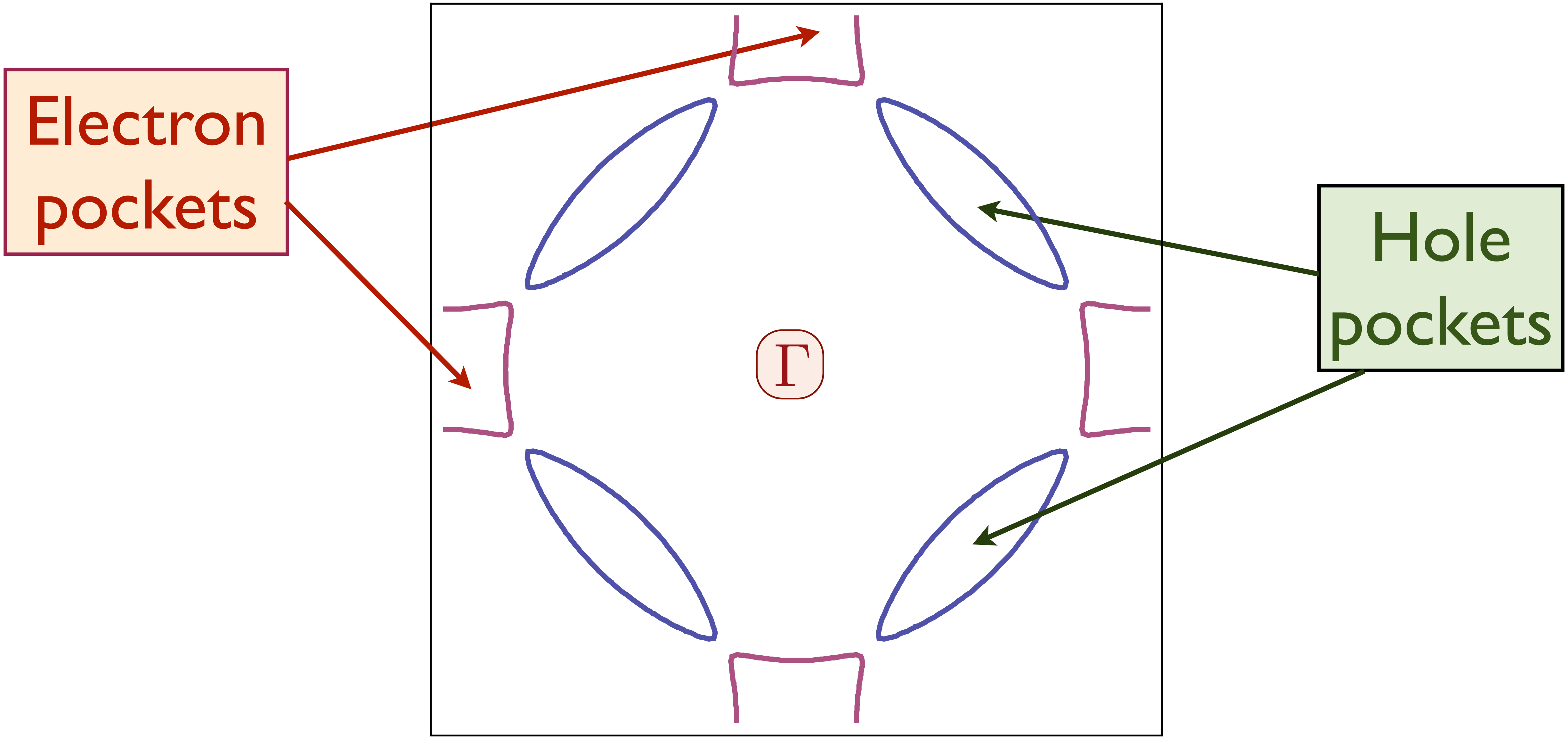
Fermi pockets in hole-doped cuprates

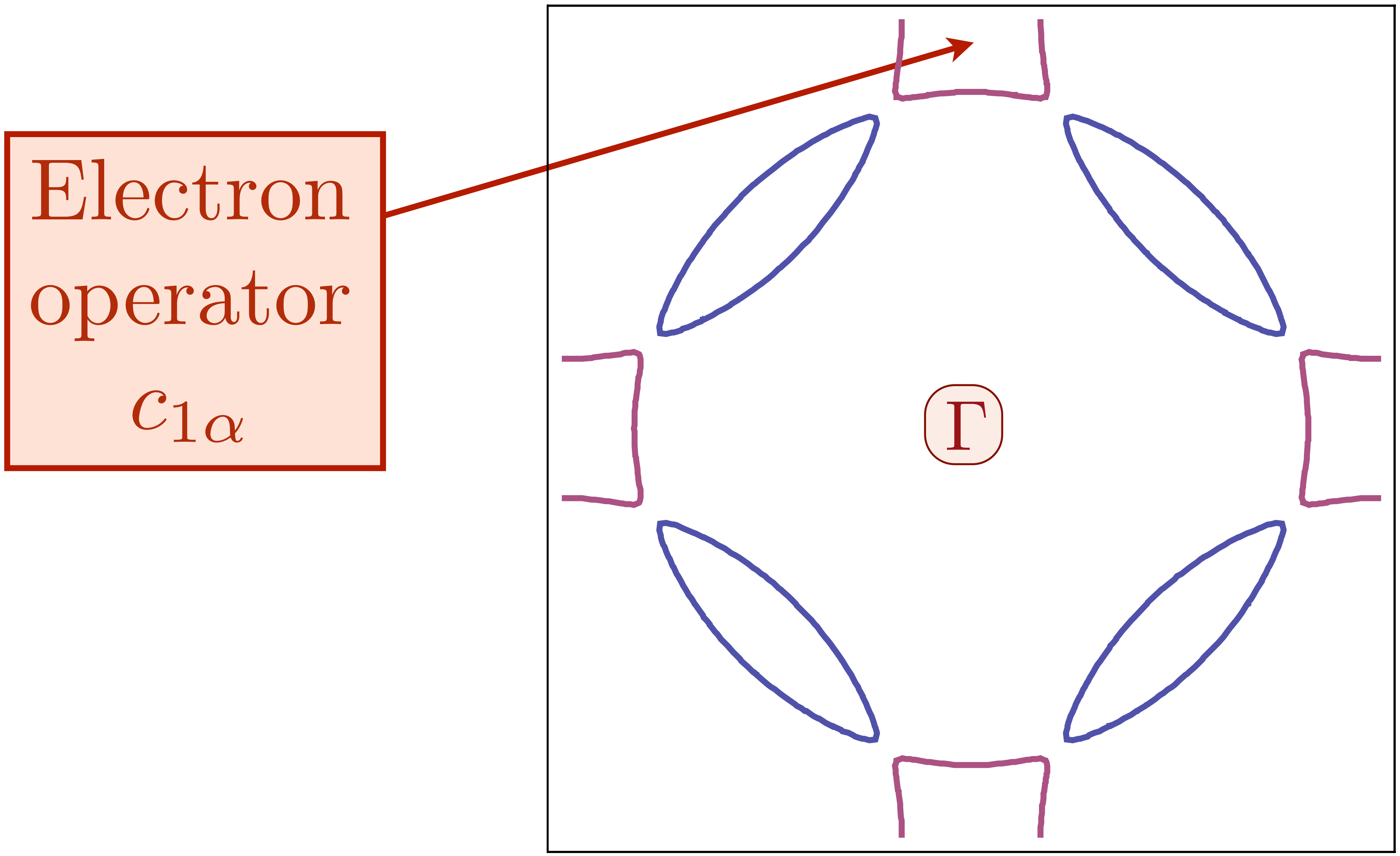


Begin with SDW ordered state, and focus on fluctuations in the *orientation* of $\vec{\varphi}$, by using a unit-length bosonic spinor z_α

$$\vec{\varphi} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$$

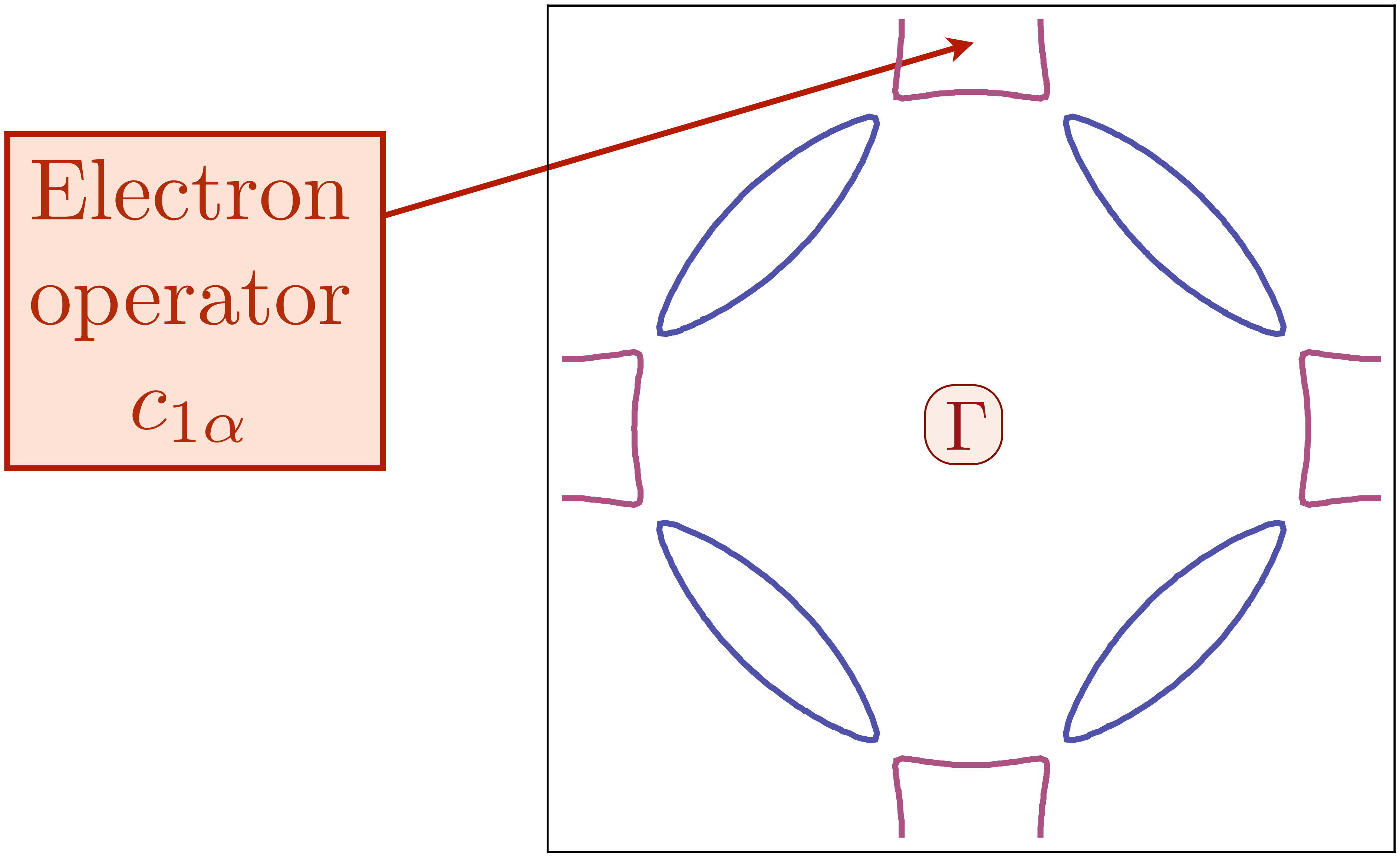
Charge carriers in the lightly-doped cuprates with Neel order





For a uniform SDW order with $\vec{\varphi} \propto (0, 0, 1)$, write

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \begin{pmatrix} g_{+} \\ g_{-} \end{pmatrix}$$

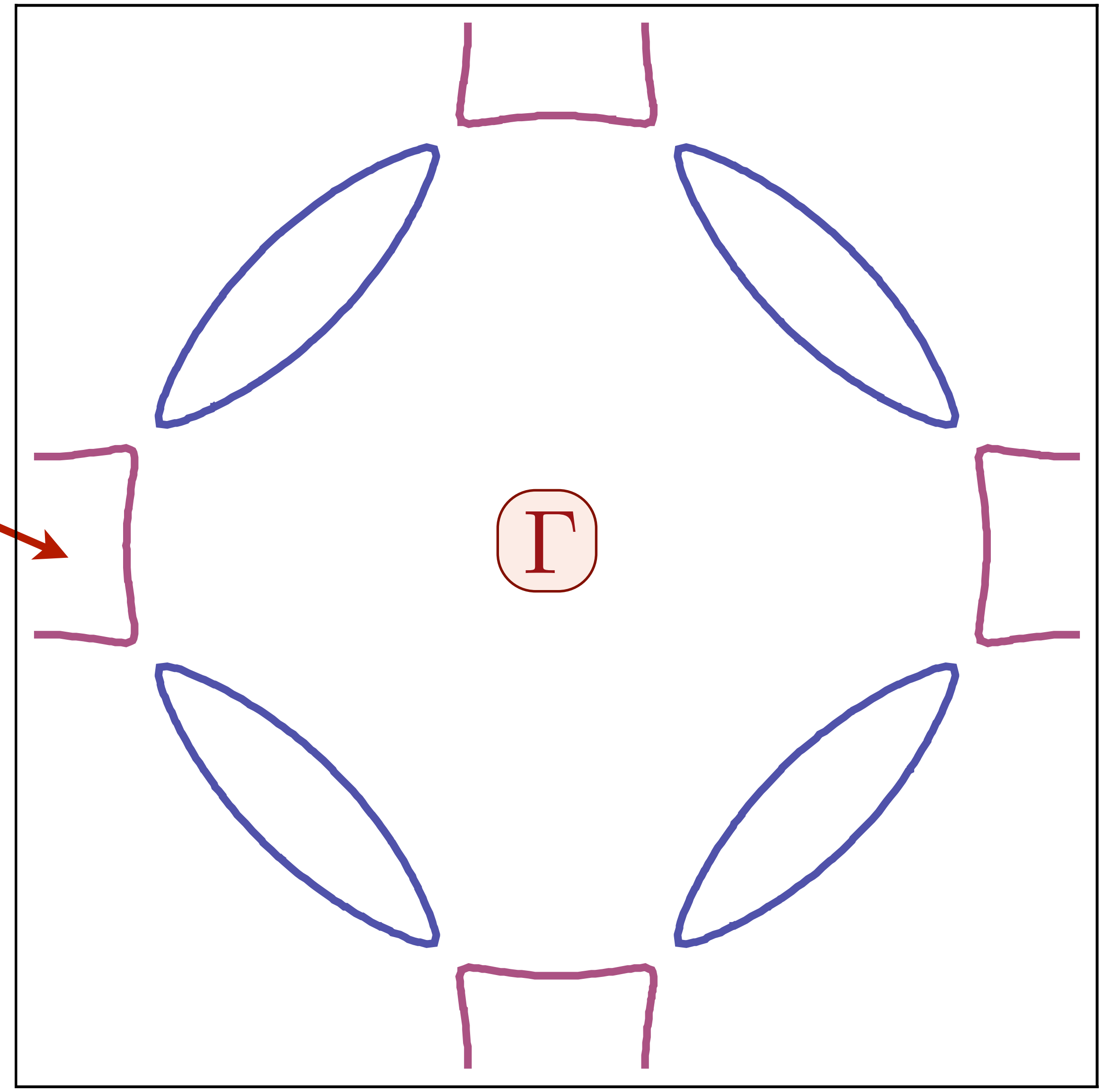


For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\left(\begin{array}{c} c_{1\uparrow} \\ c_{1\downarrow} \end{array} \right) = \mathcal{R}_z \left(\begin{array}{c} g_+ \\ g_- \end{array} \right) \quad ; \quad \mathcal{R}_z \equiv \left(\begin{array}{cc} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{array} \right).$$

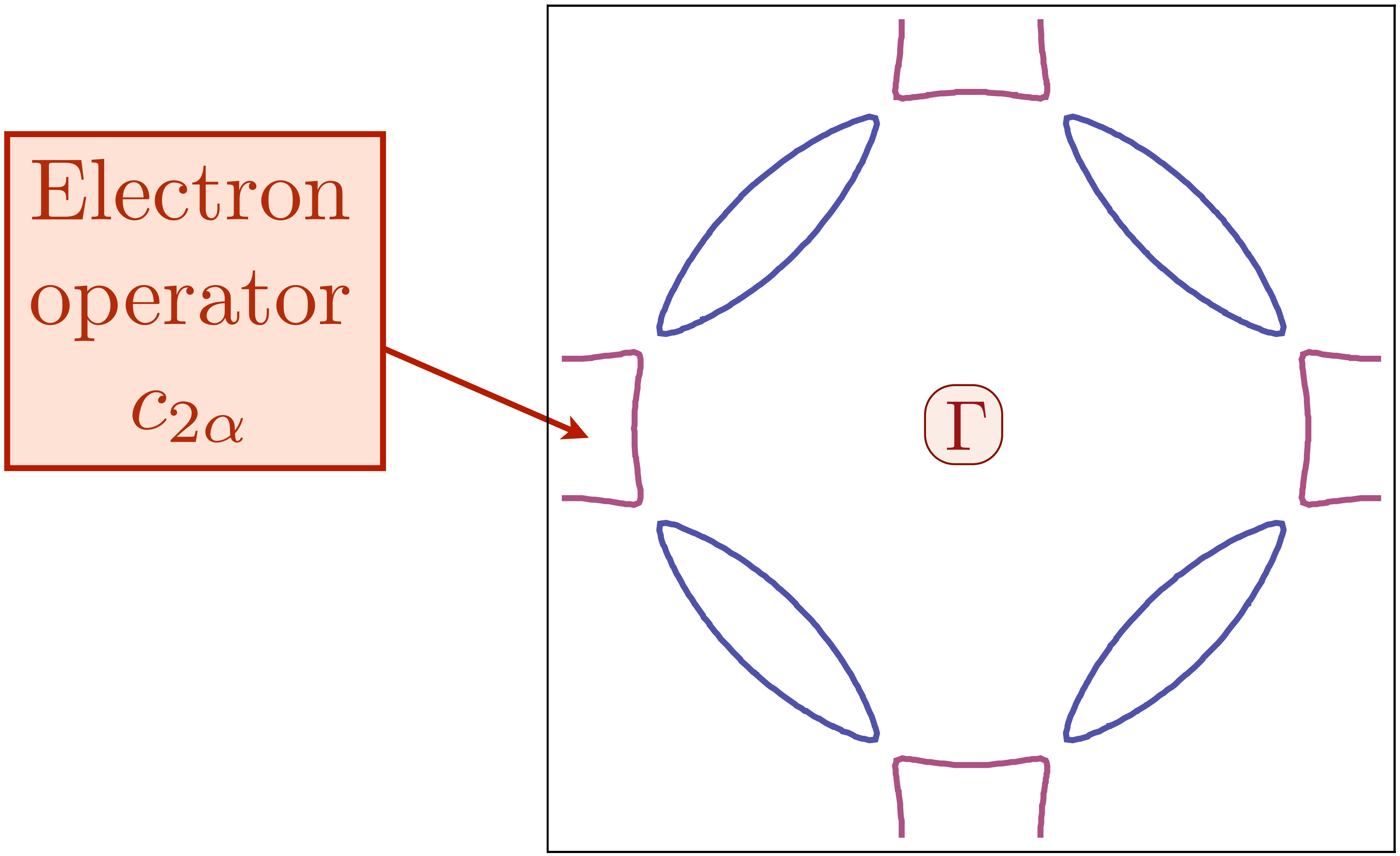
So $g_\pm$ are the “up/down” electron operators
in a rotating reference frame defined by the local SDW order

Electron operator $c_{2\alpha}$



SDW theory also specifies electrons at second pocket for $\vec{\varphi} \propto (0,0,1)$

$$\left(\begin{array}{c} c_{2\uparrow} \\ c_{2\downarrow} \end{array} \right) = \left(\begin{array}{c} g_{+} \\ -g_{-} \end{array} \right) ,$$



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Same SU(2) matrix also rotates electrons in second pocket.

Low energy theory for spinless, charge $-e$ fermions $g_{\pm}$, and spinful, charge 0 bosons z_{α}:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z &= \frac{1}{t}\left[|(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 \right] \\
&+ \text{Berry phases of monopoles in } A_\mu .
\end{aligned}
$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ.
Coupling t tunes the strength of SDW orientation fluctuations.

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CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

$$\begin{aligned}\mathcal{L}_g &= g_+^\dagger\left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu\right]g_+ \\ &\quad + g_-^\dagger\left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu\right]g_-\end{aligned}$$

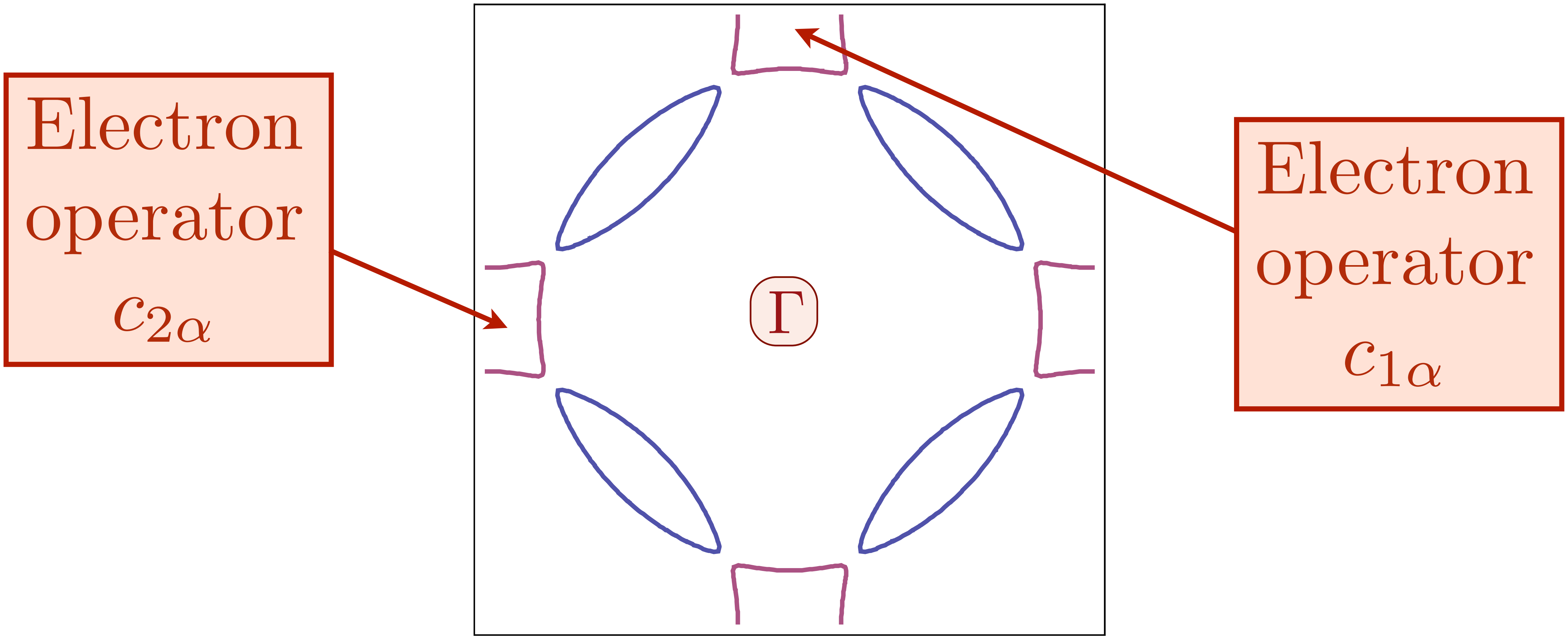
Two Fermi surfaces coupled to the emergent U(1) gauge field A_μ with opposite charges

Strong pairing of the $g_{\pm}$ electron pockets

- Gauge forces lead to a s-wave paired state with a T_c of order the Fermi energy of the pockets. Inelastic scattering from low energy gauge modes lead to very singular $g_{\pm}$ self energy, but is *not* pair-breaking.

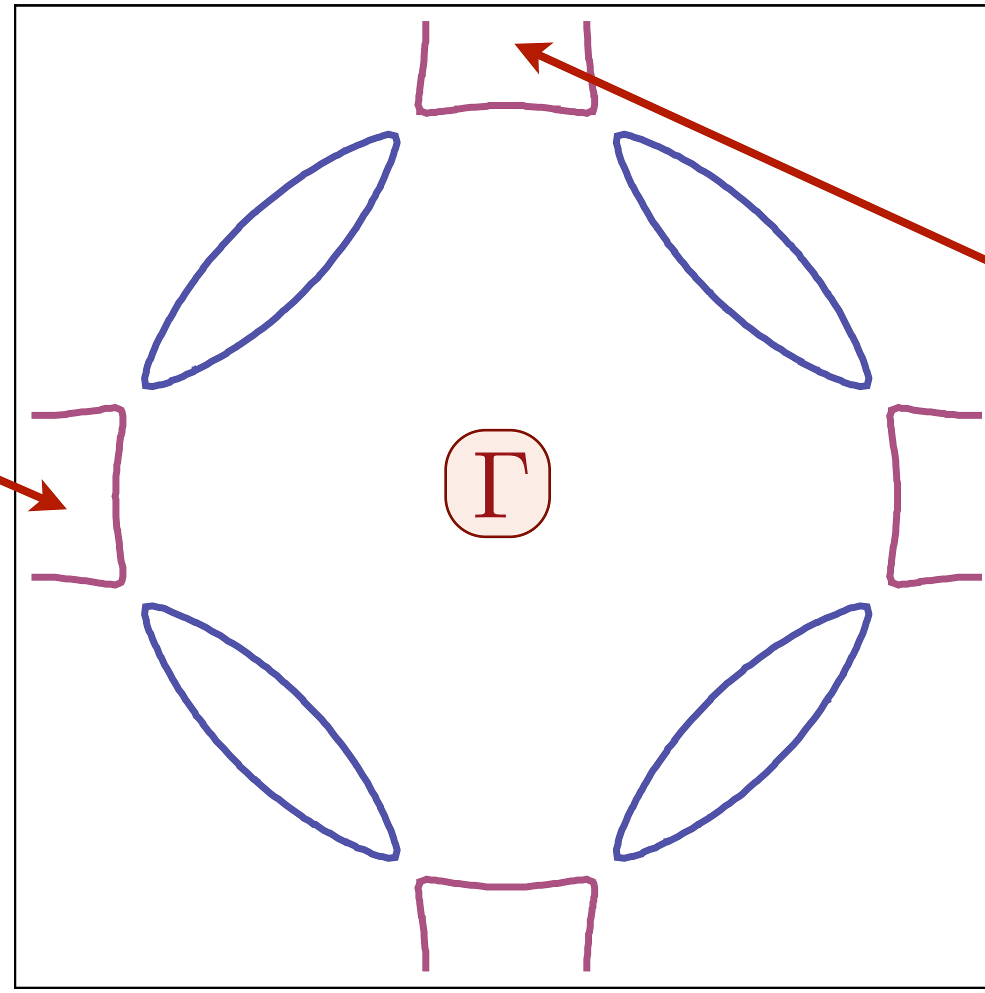
$$\langle g_+ g_- \rangle = \Delta$$

Strong pairing of the $g_\pm$ electron pockets



Strong pairing of the $g_\pm$ electron pockets

Electron operator $c_{2\alpha}$



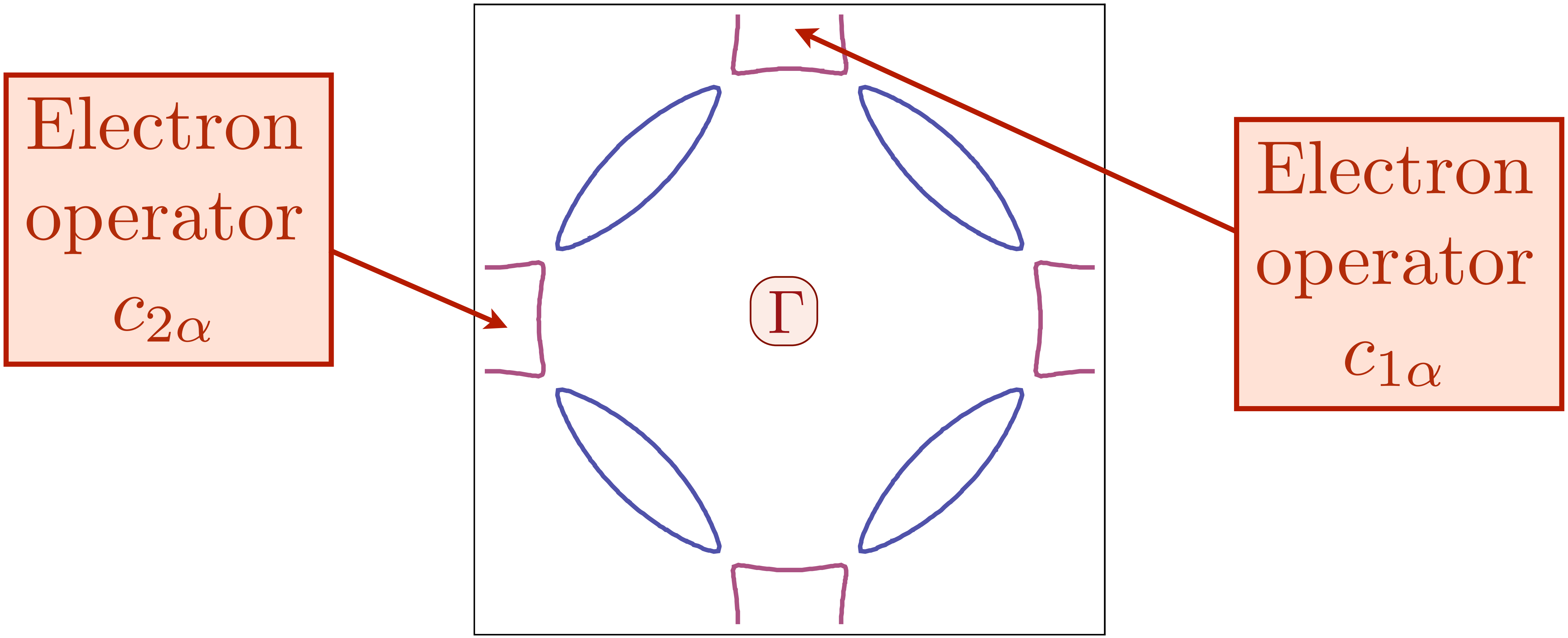
Electron operator $c_{1\alpha}$

- Transforming back to the physical fermions:

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} g_+ \\ -g_- \end{pmatrix},$$

we find: $\langle c_{1\uparrow} c_{1\downarrow} \rangle = - \langle c_{2\uparrow} c_{2\downarrow} \rangle \sim \left\langle |z_\uparrow|^2 + |z_\downarrow|^2 \right\rangle \langle g_+ g_- \rangle \,;$

Strong pairing of the $g_\pm$ electron pockets



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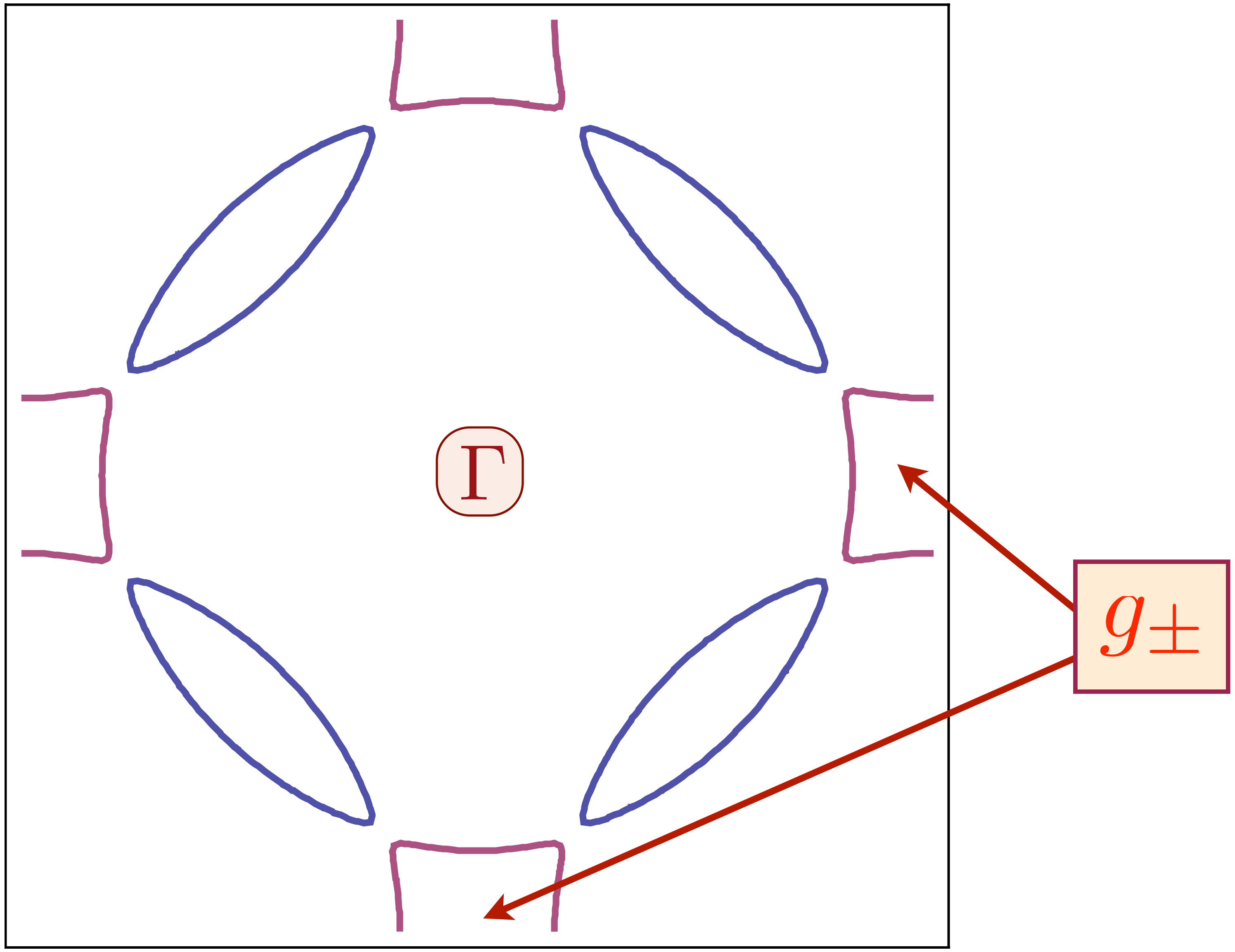
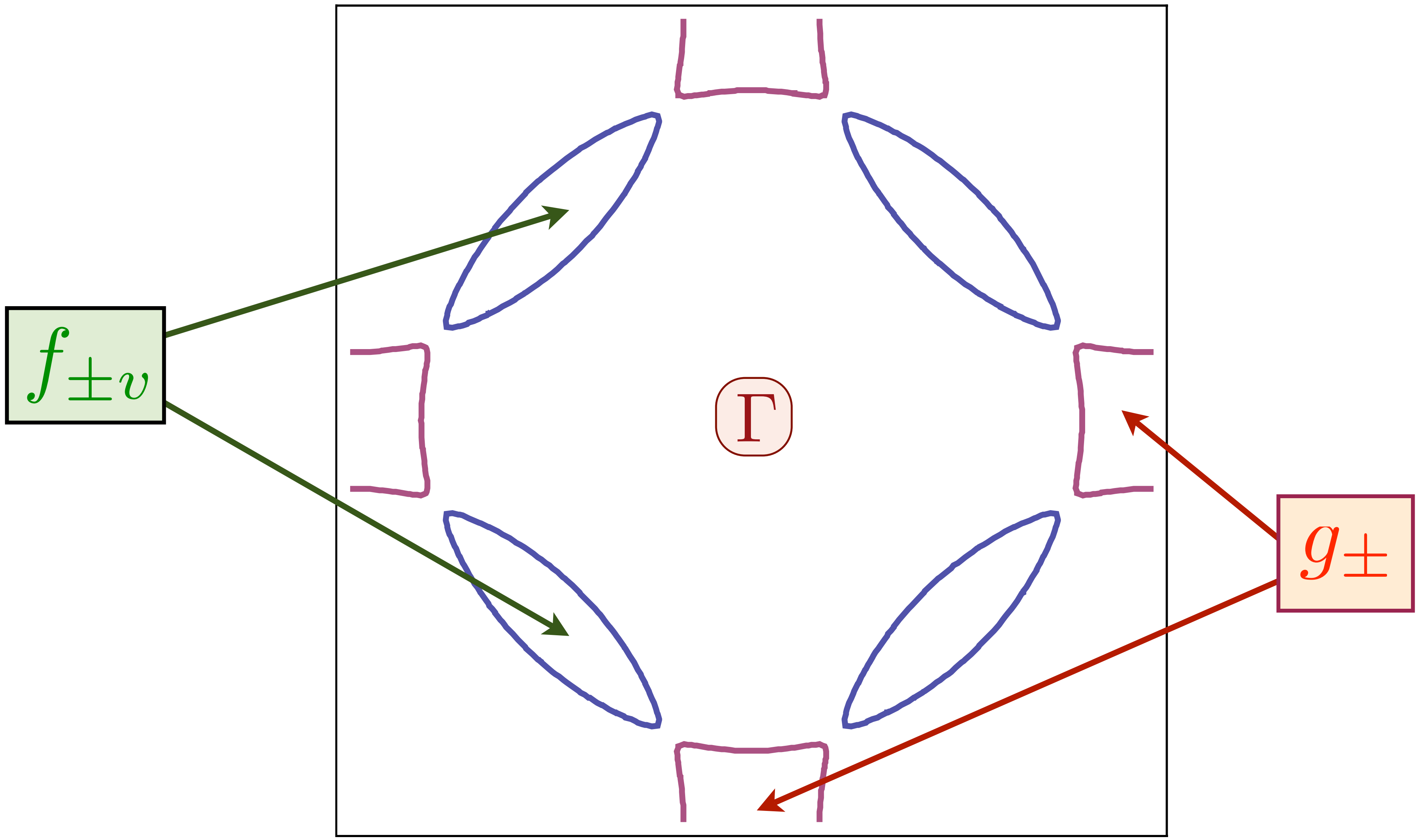
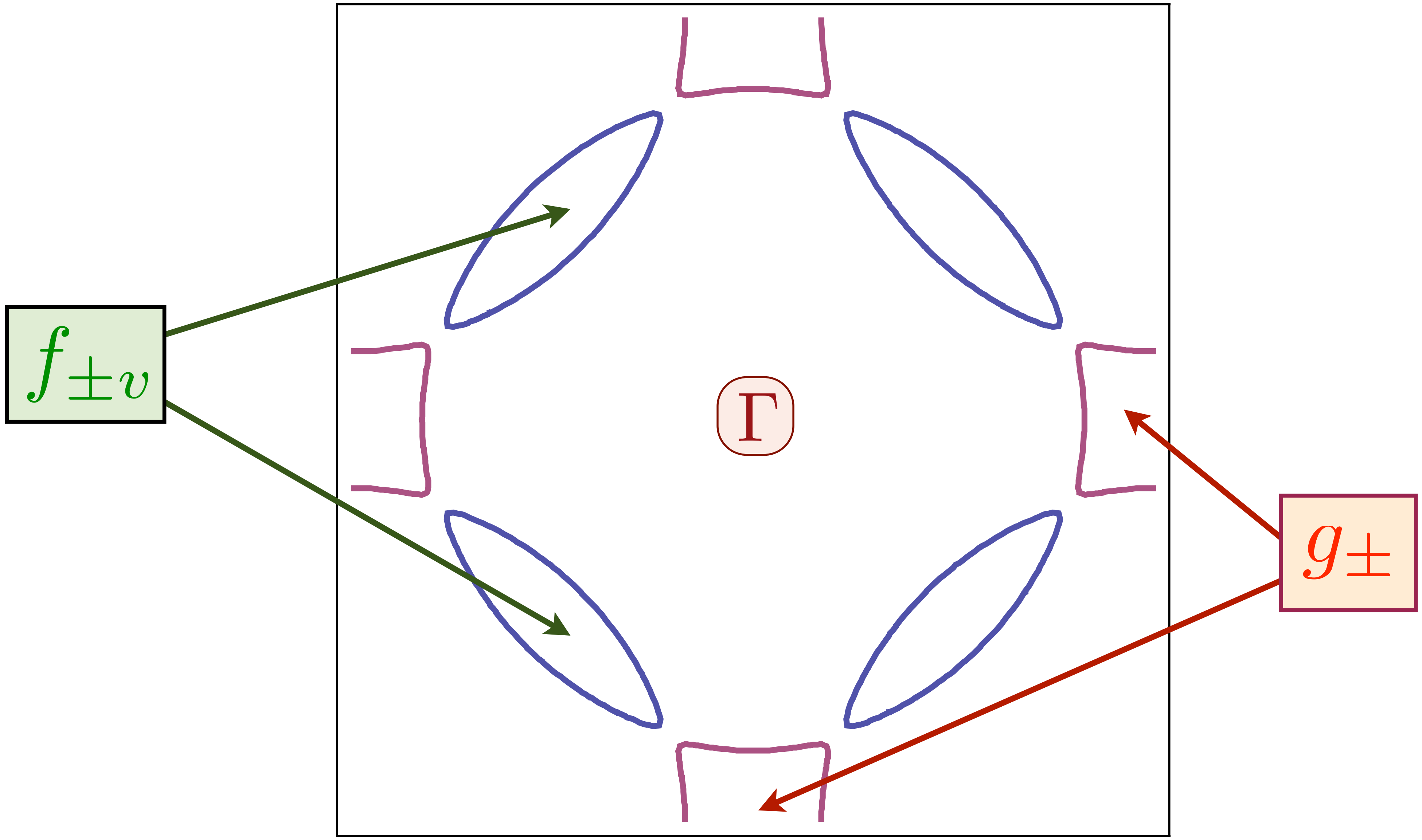
we find: $\langle c_{1\uparrow} c_{1\downarrow} \rangle = - \langle c_{2\uparrow} c_{2\downarrow} \rangle \sim \left\langle |z_\uparrow|^2 + |z_\downarrow|^2 \right\rangle \langle g_+ g_- \rangle$;

i.e. the pairing signature for the electrons is d-wave.


Γ
$g_{\pm}$


$f_{\pm v}$
Γ
$g_{\pm}$



Low energy theory for spinless, charge $+e$ fermions $f_{\pm v}$:

$$\mathcal{L}_f = \sum_{v=1,2} \left\{ f_{+v}^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] f_{+v} + f_{-v}^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] f_{-v} \right\}$$

Weak pairing of the $f_\pm$ hole pockets

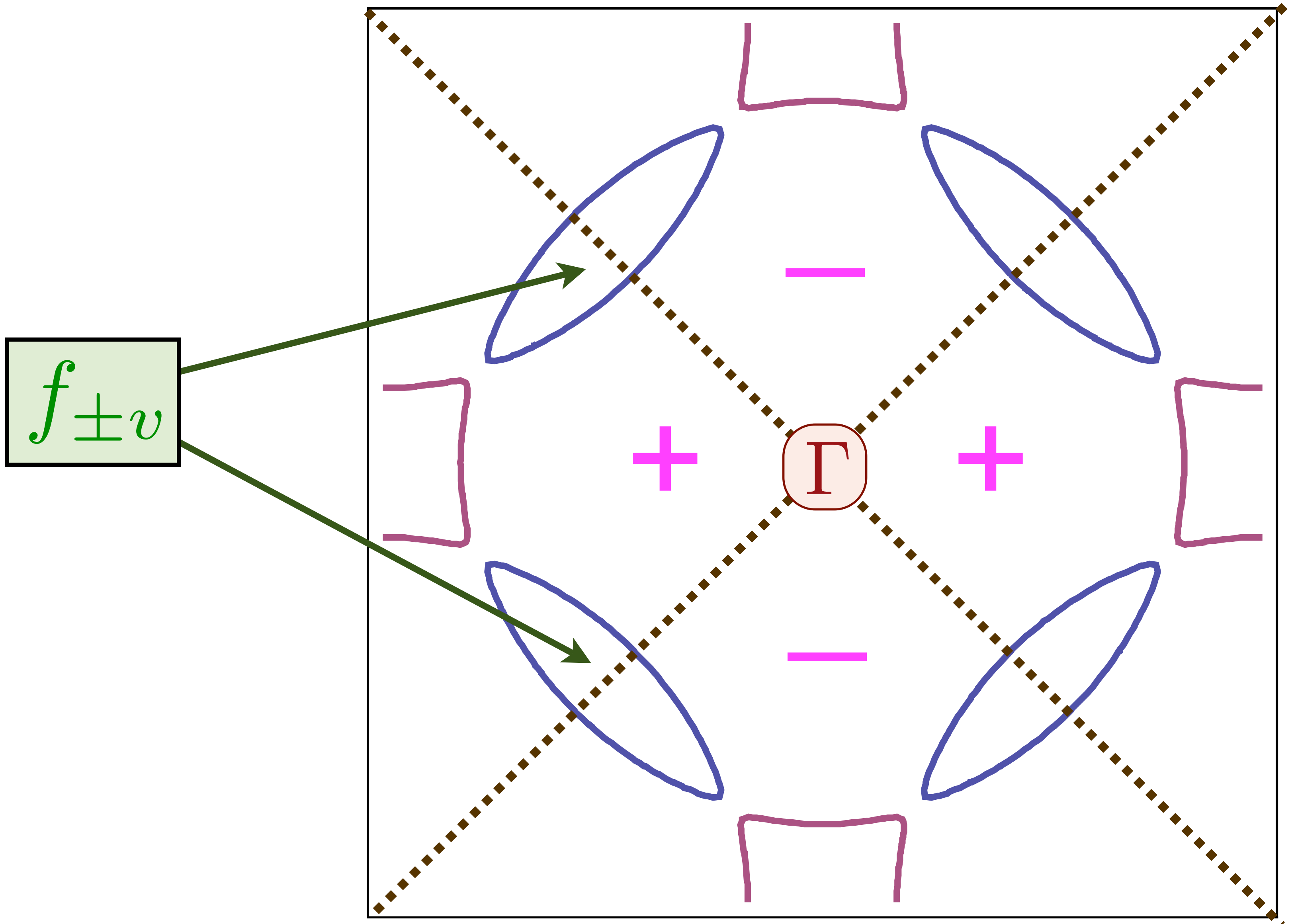
$$\mathcal{L}_{\text{Josephson}} = iJ\left[g_+ g_-\right]\Big[f_{+1}\overleftrightarrow{\partial}_x f_{-1} - f_{+1}\overleftrightarrow{\partial}_y f_{-1} + f_{+2}\overleftrightarrow{\partial}_x f_{-2} + f_{+2}\overleftrightarrow{\partial}_y f_{-2}\Big] + \text{H.c.}$$

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

Proximity Josephson coupling J to $g_\pm$ fermions leads to p-wave pairing of the $f_{\pm v}$ fermions. The A_μ gauge forces are pair-breaking, and so the pairing is weak.

$$\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}$$

Weak pairing of the $f_\pm$ hole pockets



$$\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle &\sim (k_x - k_y) J \langle g_+ g_- \rangle; \\
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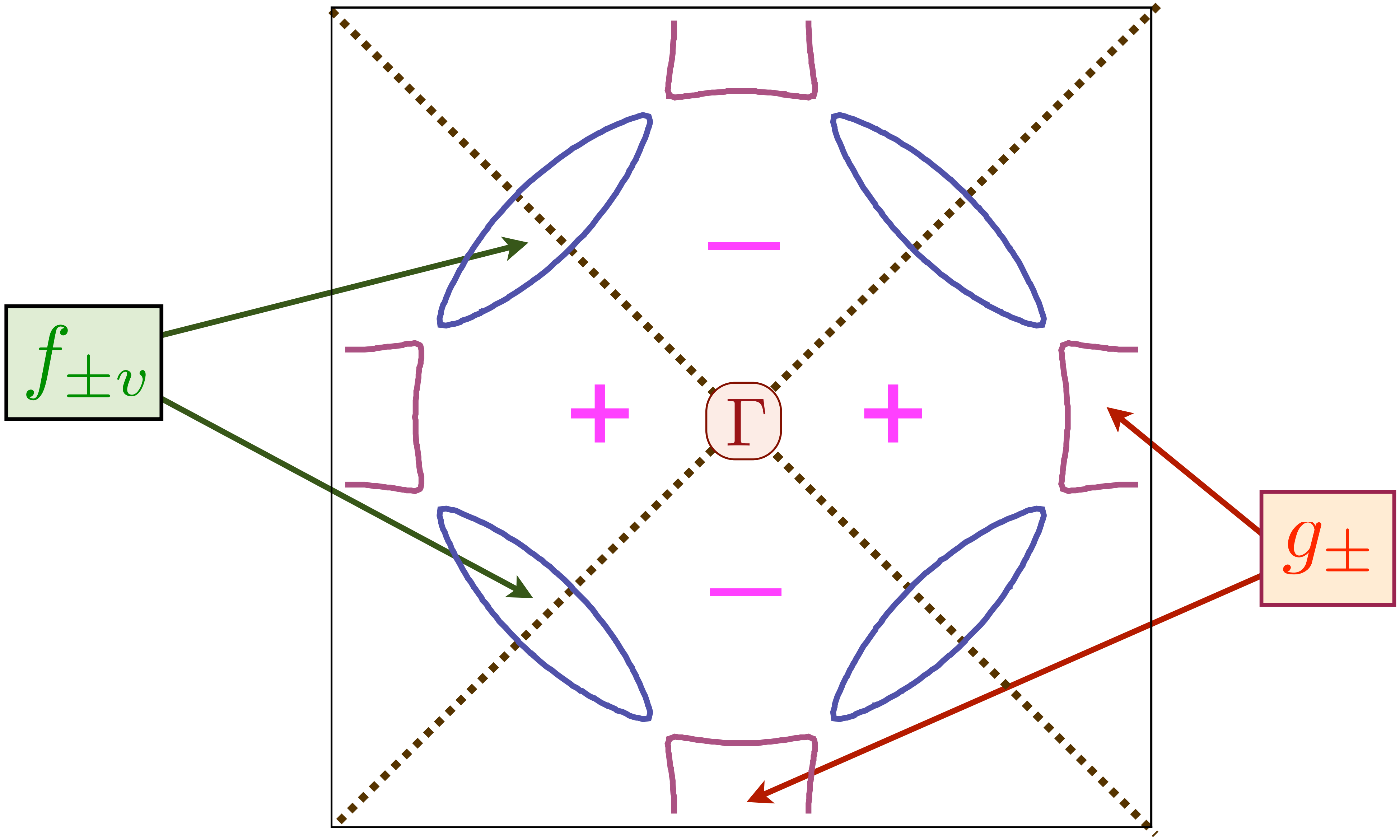

d-wave pairing of the electrons is associated with

- Strong *s*-wave pairing of $g_{\pm}$
- Weak *p*-wave pairing of $f_{\pm v}$.

Emergence of preformed Cooper pairs from the doped Mott insulating state in $Bi_2Sr_2CaCu_2O_{8+\delta}$

H.-B. Yang[1], J. D. Rameau[1], P. D. Johnson[1], T. Valla[1], A. Tsvelik[1] & G. D. Gu[1]

Here we report a photoemission study of the underdoped copper oxide $Bi_2Sr_2CaCu_2O_{8+\delta}$ that shows the opening of a symmetric gap only in the anti-nodal region, contrary to the expectation that pairing would take place in the nodal region. It is therefore evident that the pseudogap does reflect the formation of preformed pairs of electrons and that the pairing occurs only in well-defined directions of the underlying lattice.

Nature **456**, 77 (2008).

Universal theory of superconductivity

$$\mathcal{L} = \frac{1}{t}\left[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2\right]$$
$$+ \quad g_+^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu\right] g_+$$
$$+ g_-^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu\right] g_-$$

- Complex, 'relativistic' bosons z_α, with $|z_\alpha|^2 = 1$
- Fermions $g_\pm$
- Gauge field $(A_\tau, \mathbf{A})$

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- Theory fully characterized by two dimensionless parameters:
  - $(1/t_c - 1/t)/m^*$ measures distance from SDW ordering quantum transition
  - $k_F/(m^* v)$

Universal theory of superconductivity

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  - $k_F/(m^* v)$
- Characteristic length scale: $1/k_F$.
- Characteristic energy scale: $m^* v^2$.

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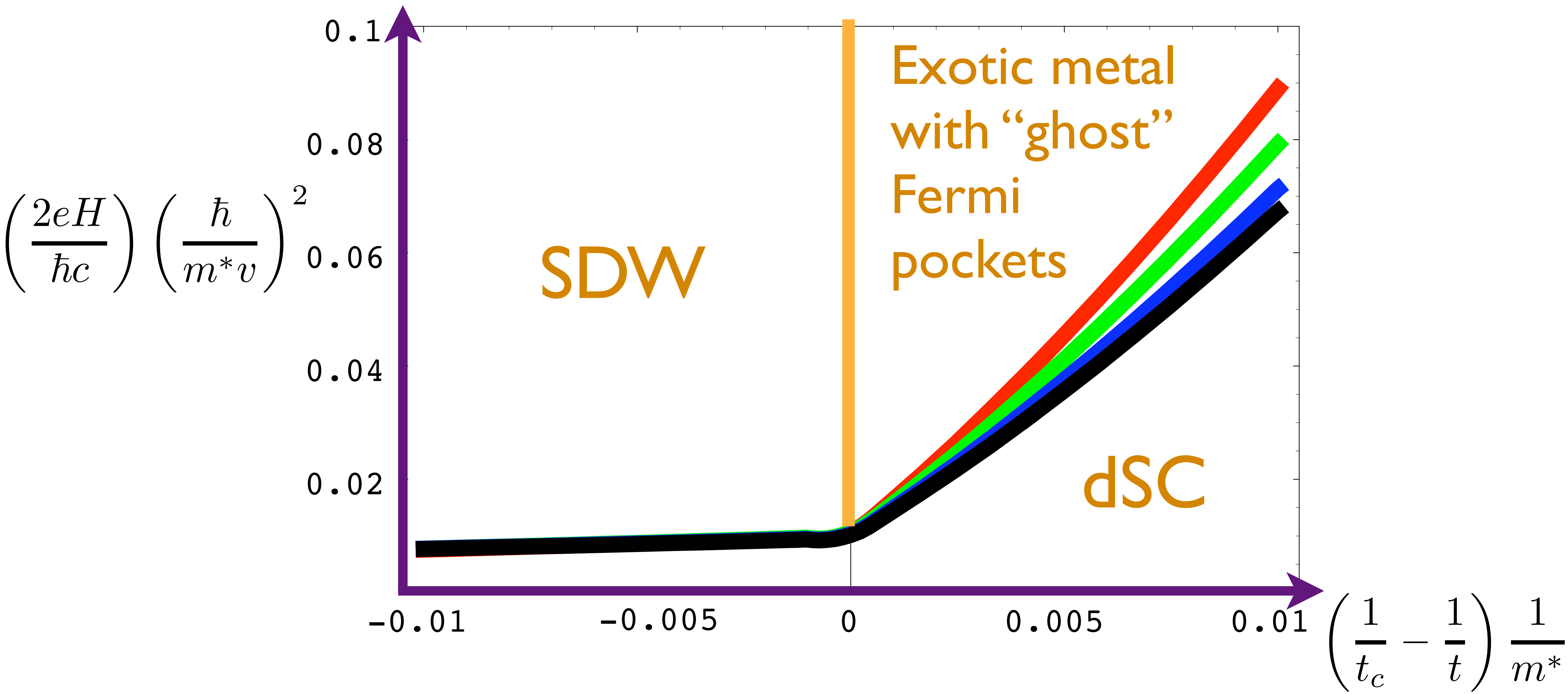
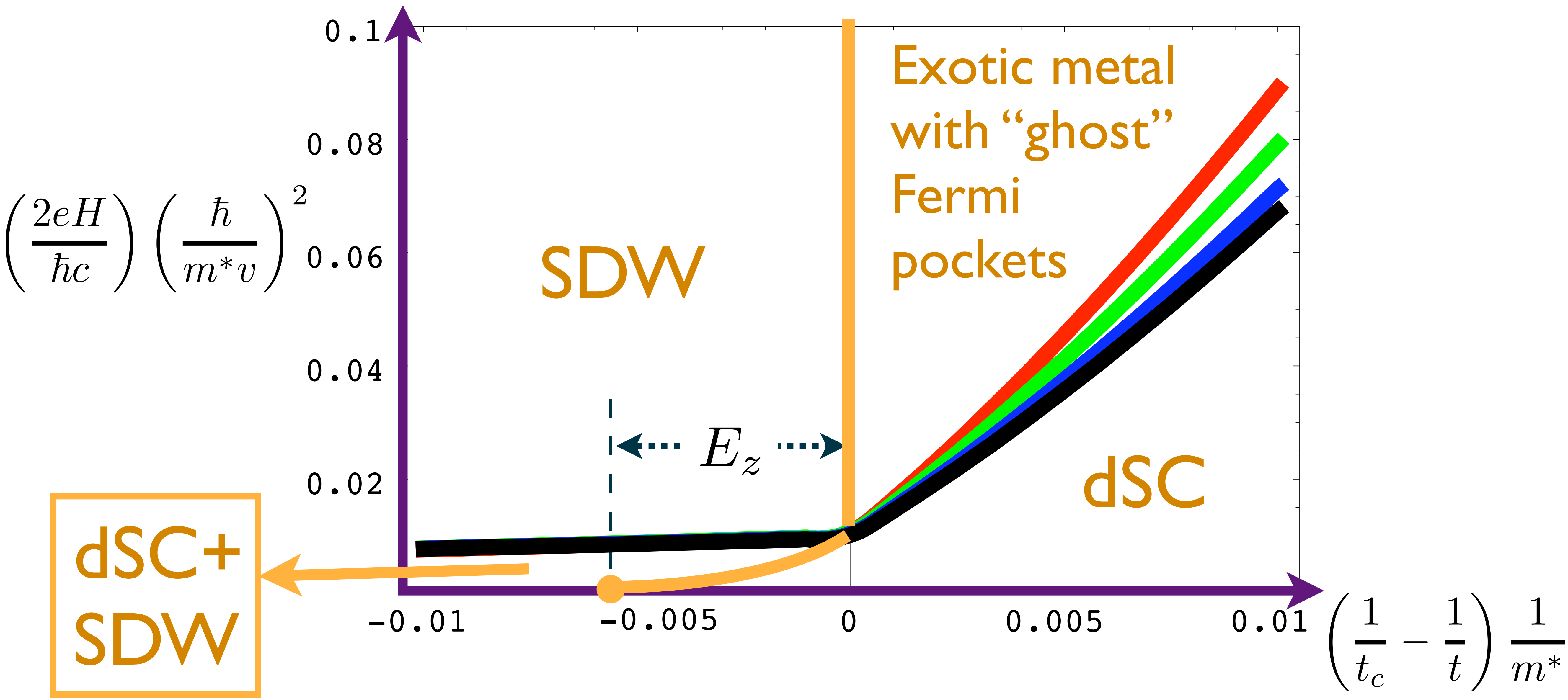


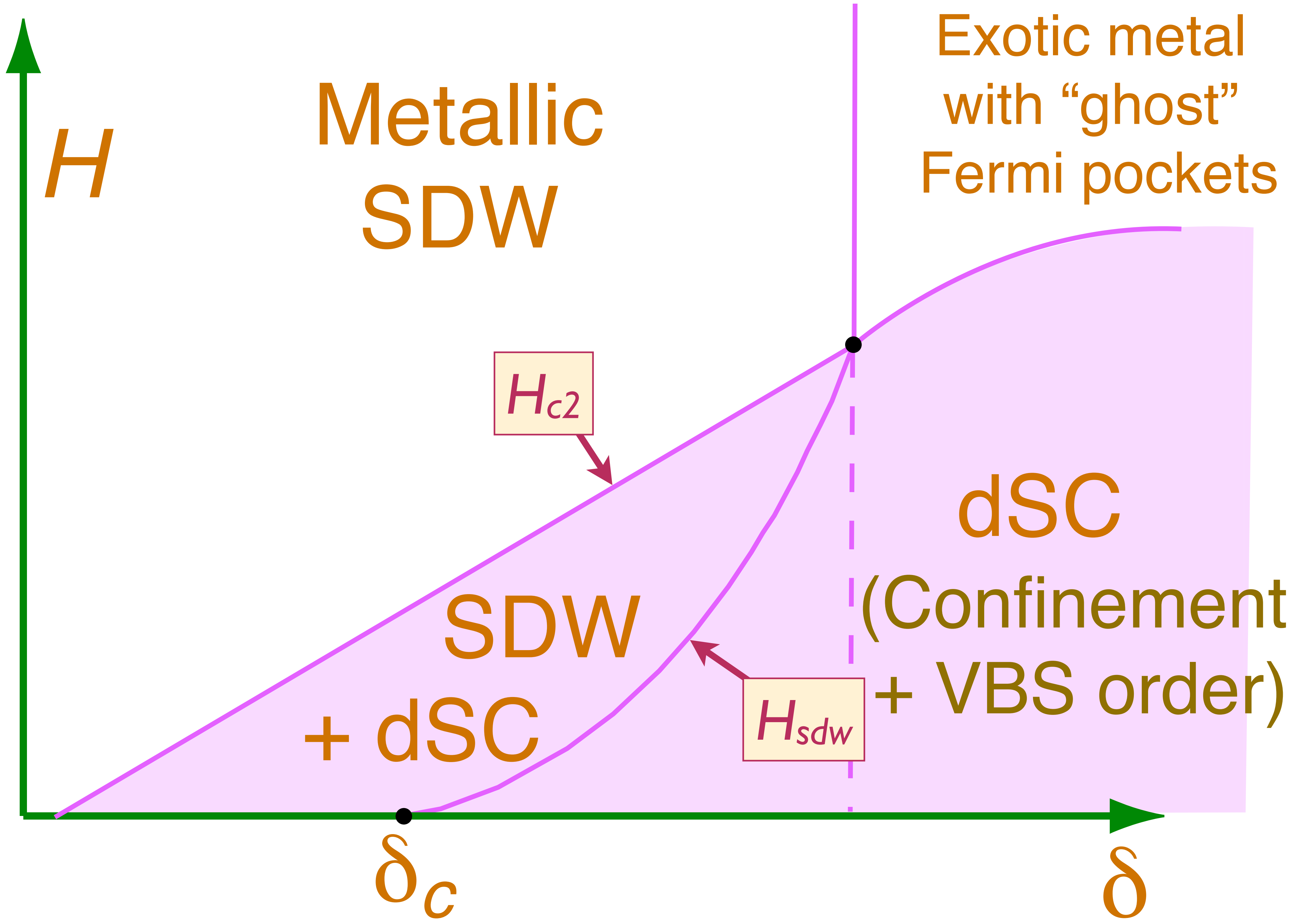
Universal theory of superconductivity

SDW order is suppressed in the superconductor, *i.e.* $E_z > 0$, by enhancement of gauge field fluctuations, which are screened only in the metallic phases.



Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field H.



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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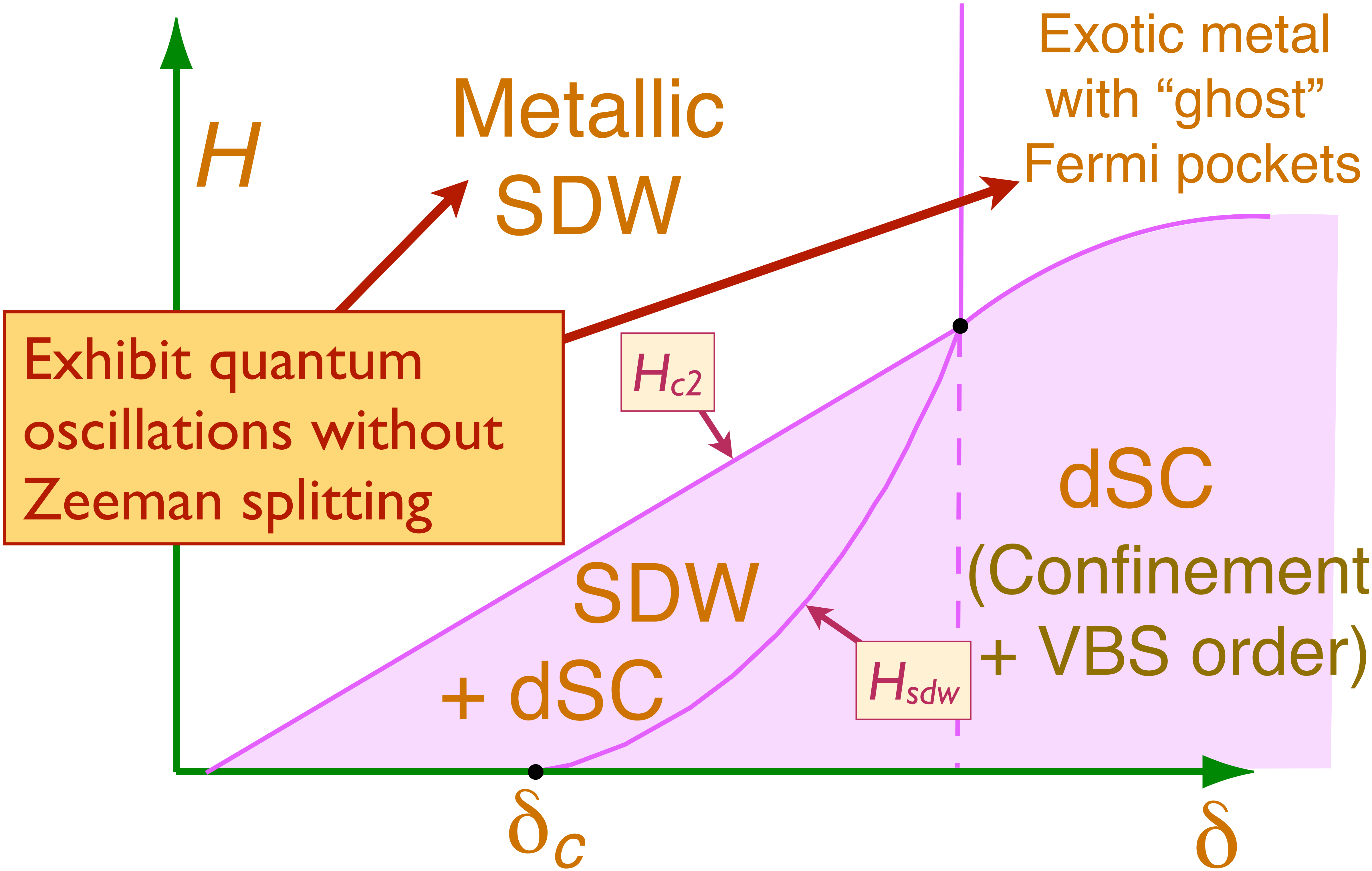

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Conclusions

- ★ Gauge theory for pairing in the underdoped cuprates, describing "angular" fluctuations of spin-density-wave order
- ★ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes

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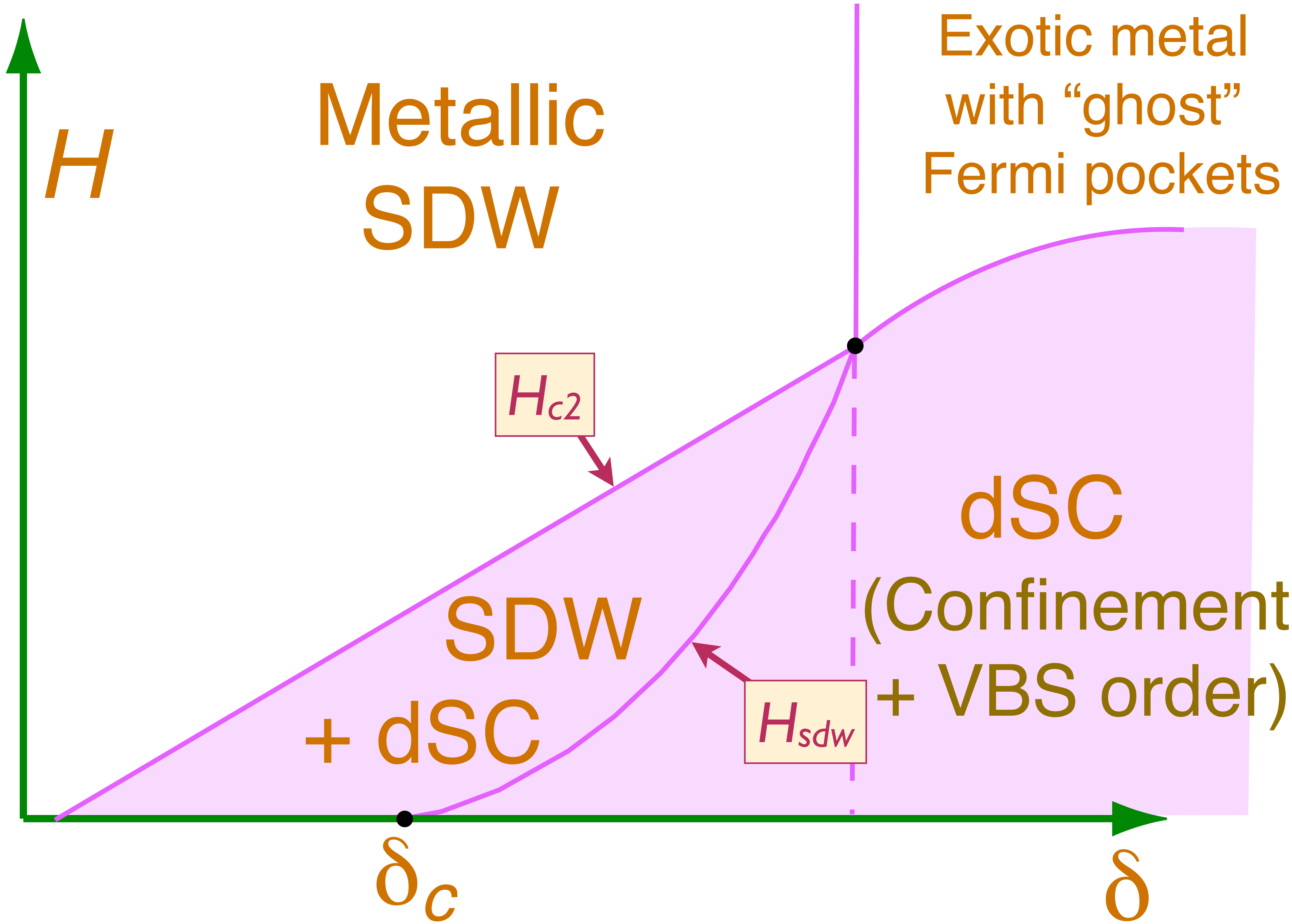

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- ★ New metallic state, an *algebraic charge liquid*, with "ghost" electron and hole pockets, could describe the finite temperature "pseudo-gap" regime.

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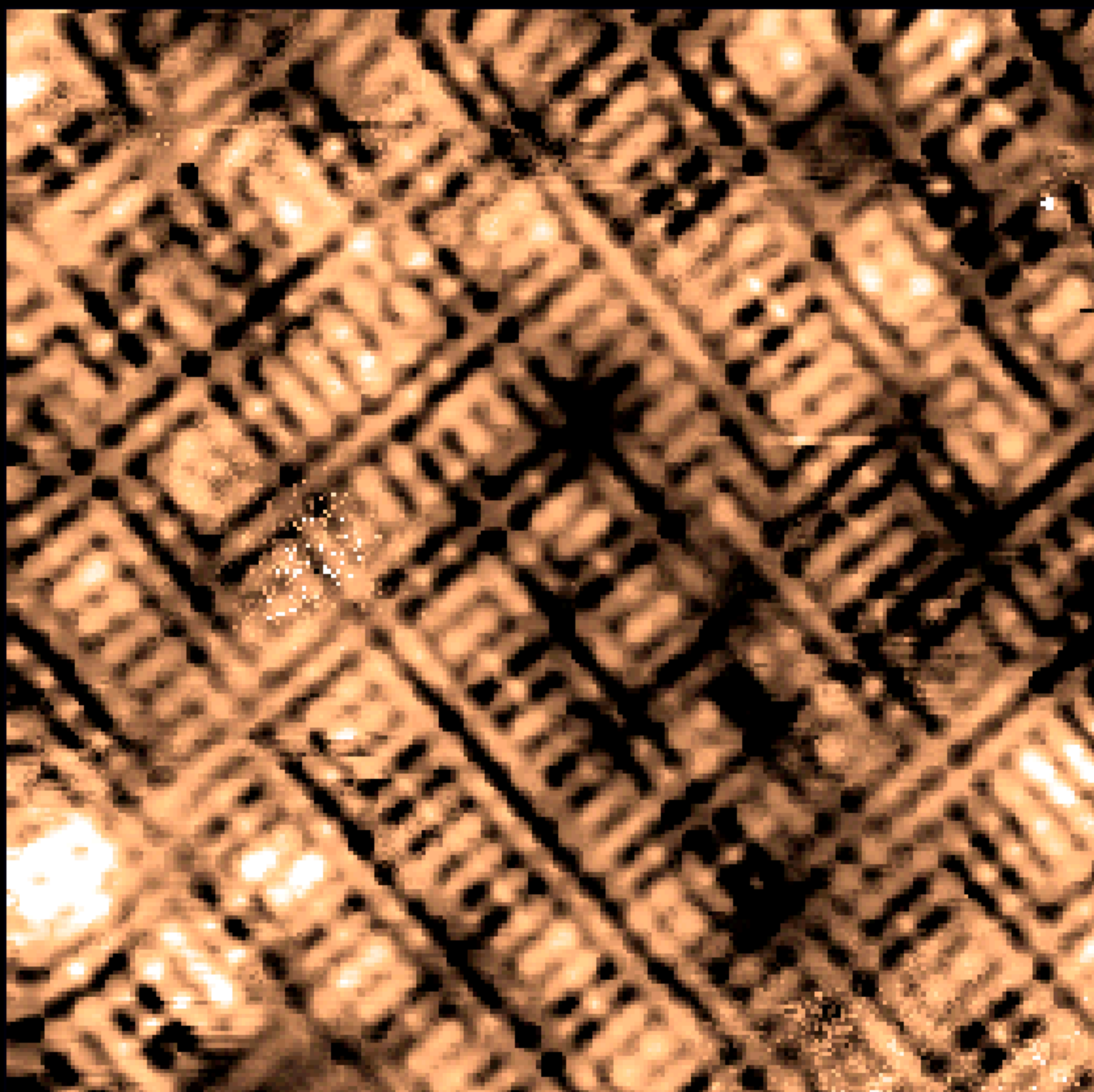
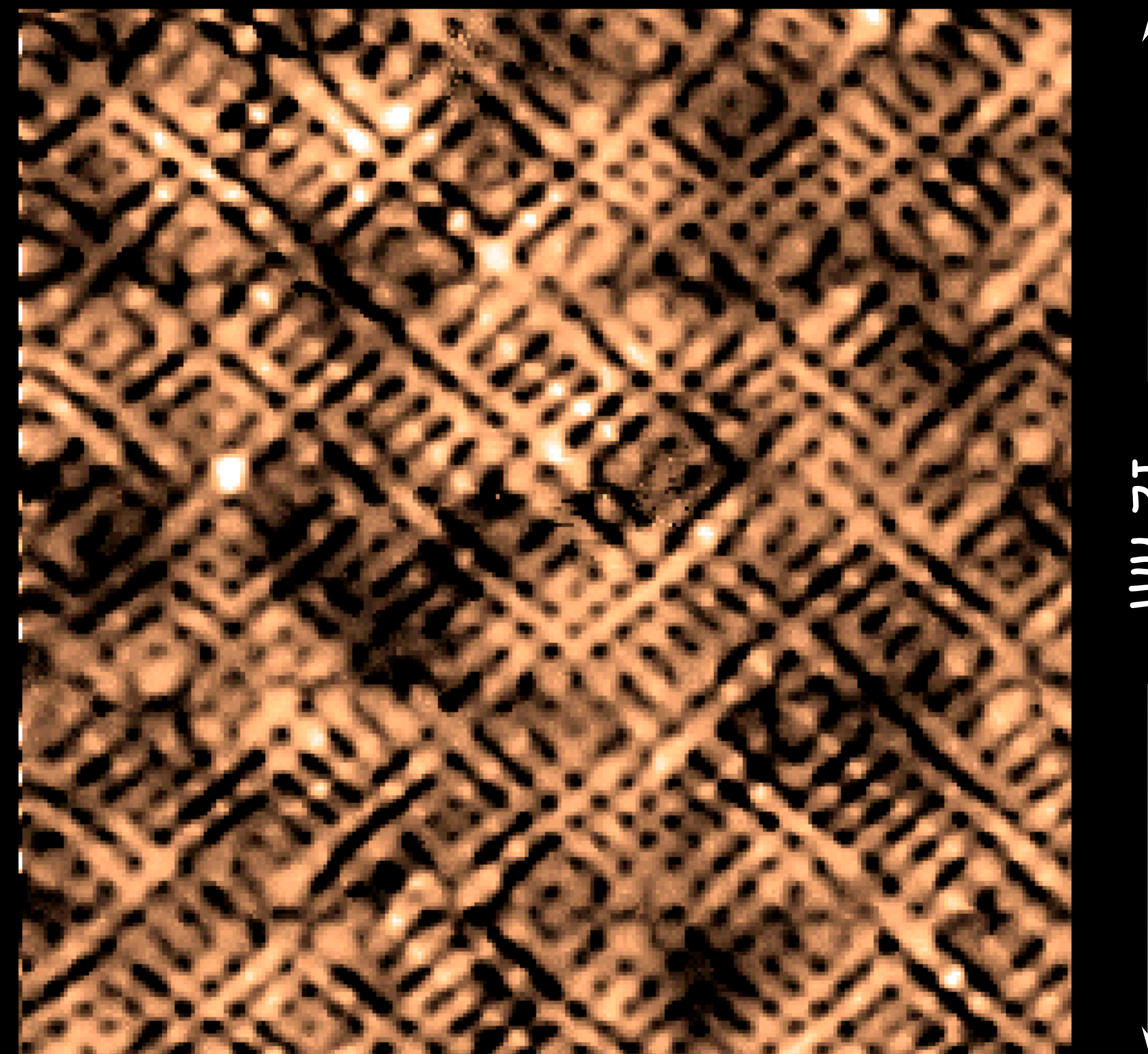
Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

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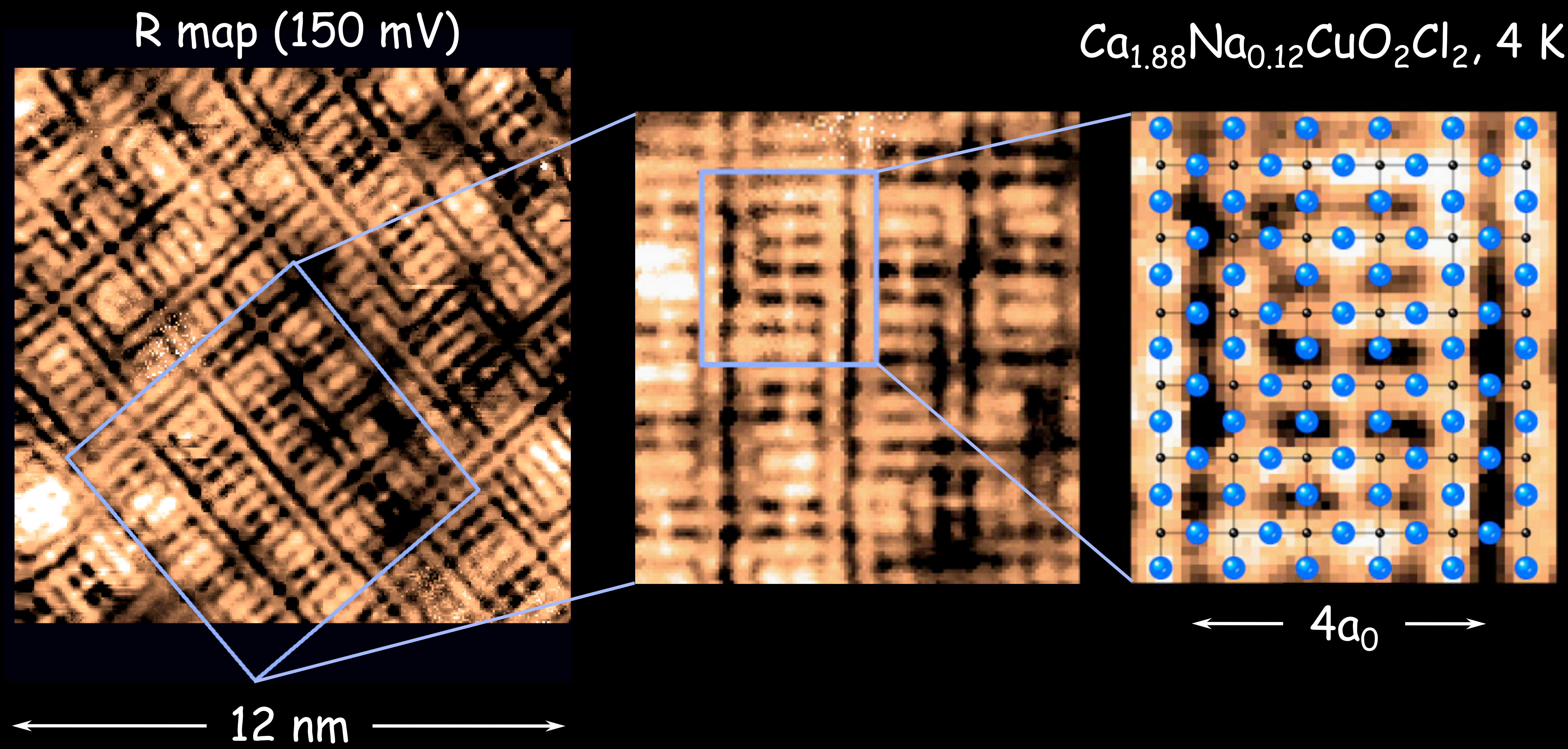
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



Y. Kohsaka et al. **Science 315**, 1380 (2007)

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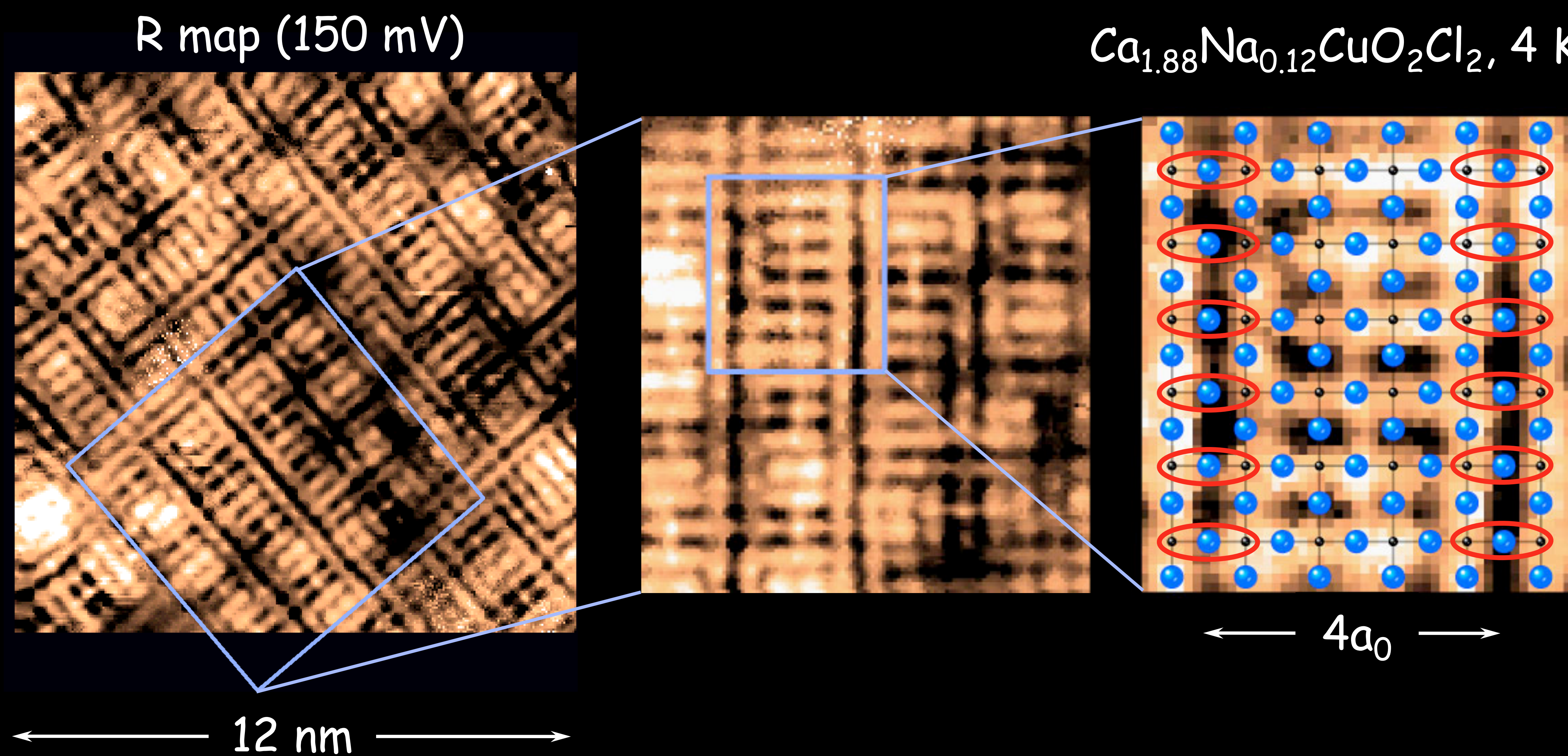

Evidence for a predicted valence bond supersolid

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).

Conclusions

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★ Needed: theory for transition to "large" Fermi surface at higher doping