

# The microstructure of charge order in the cuprates, and implications for the pseudogap

Joint CIFAR - Max Planck Society Quantum Materials Meeting,  
Max Planck Institute for Solid State Research, Stuttgart  
October 14, 2014

Subir Sachdev



PERIMETER INSTITUTE  
FOR THEORETICAL PHYSICS



GORDON AND BETTY  
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FOUNDATION

JOHN TEMPLETON  
FOUNDATION

PHYSICS



HARVARD

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

# Theorists at Harvard



Max Metlitski  
(KITP, UCSB)



Andrea Allais



Matthias Punk  
(Innsbruck)



Debanjan  
Chowdhury



Alexandra  
Thomson

# Cornell



Kazuhiro Fujita  
Cornell/ BNL



Mohammad Hamidian  
Cornell / BNL



Stephen Edkins  
Cornell / St Andrews



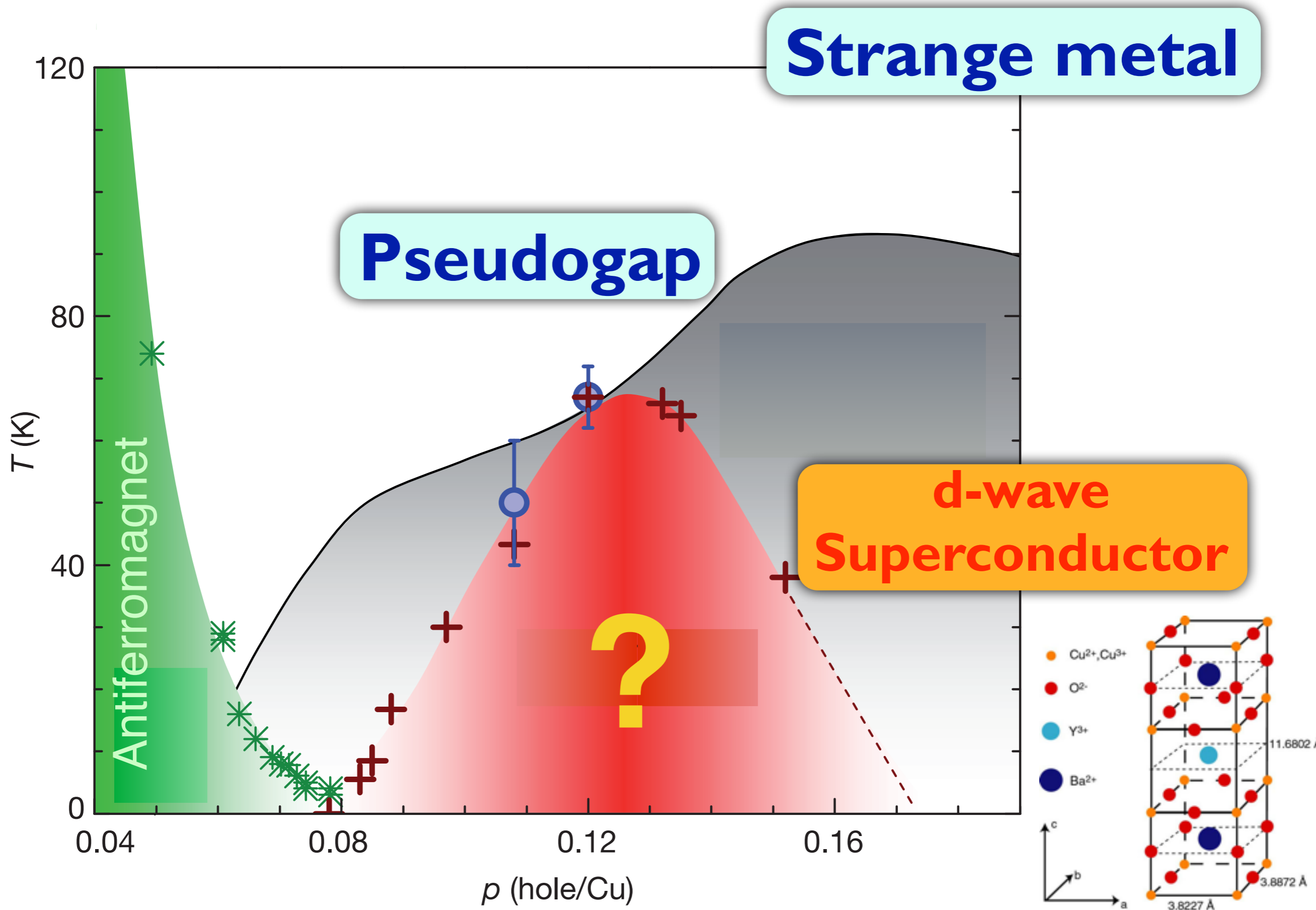
Michael Lawler



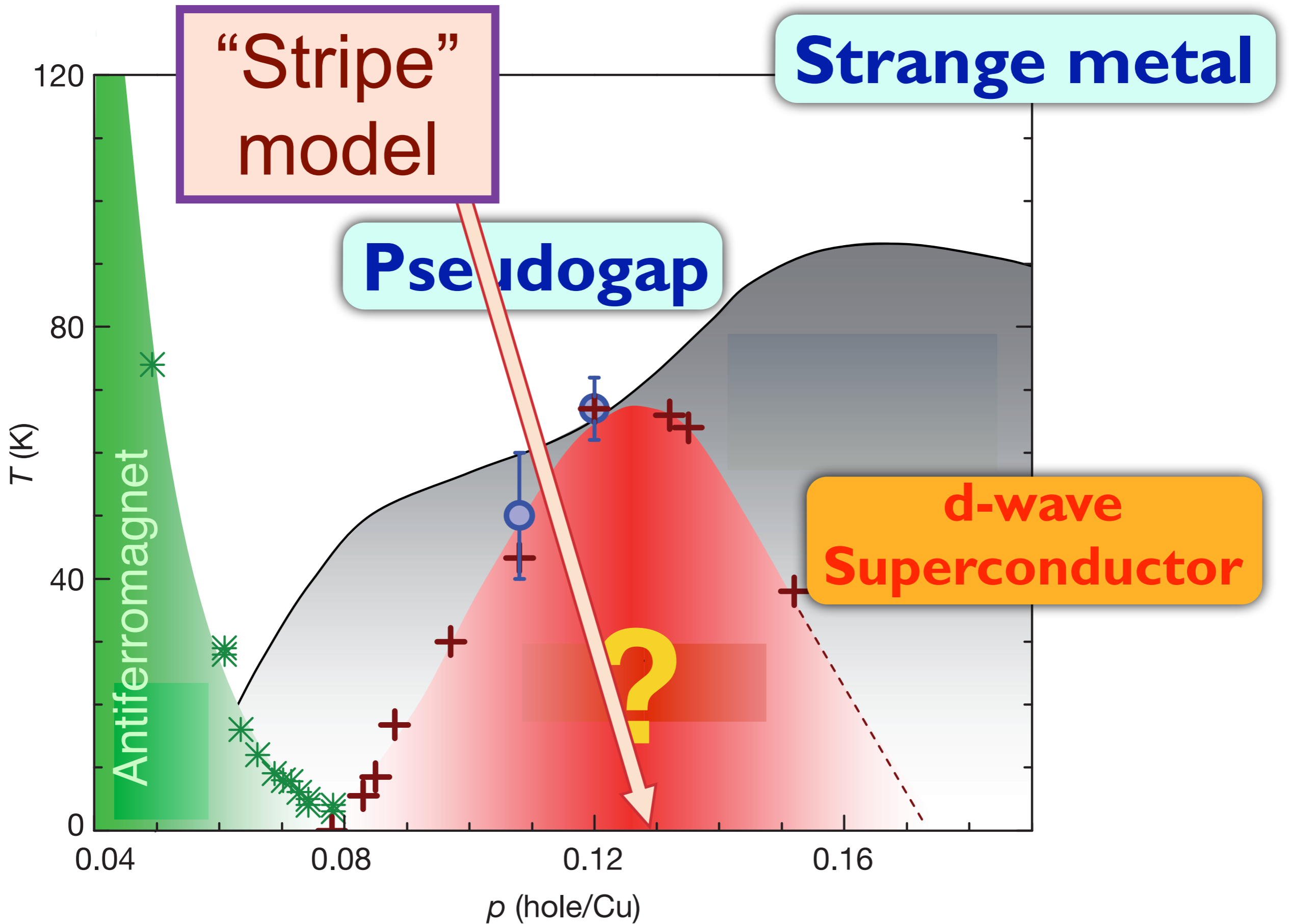
J. C. Seamus Davis



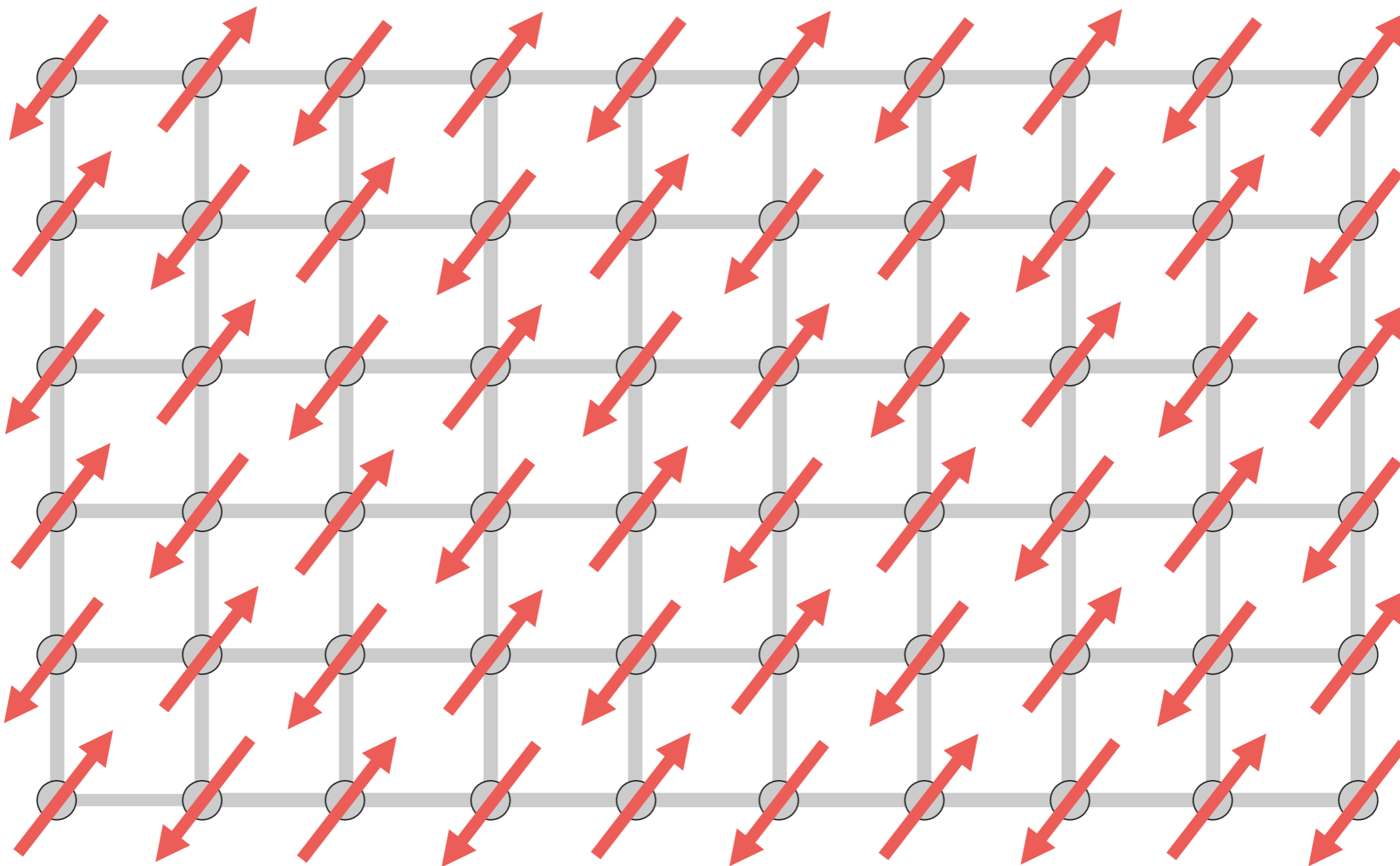
Eun-Ah Kim



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).

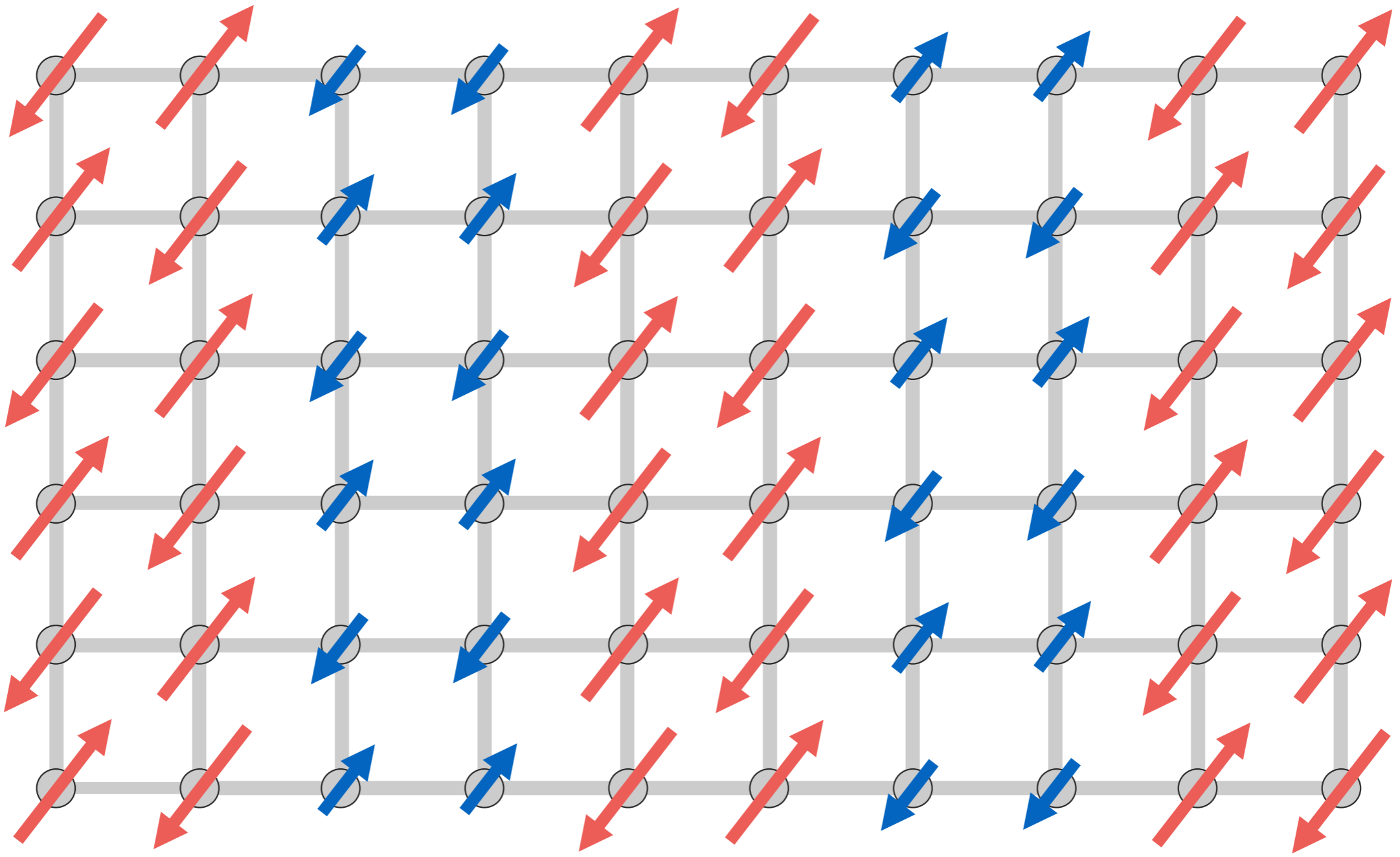


# “Stripe” model



Start with an antiferromagnet

# “Stripe” model

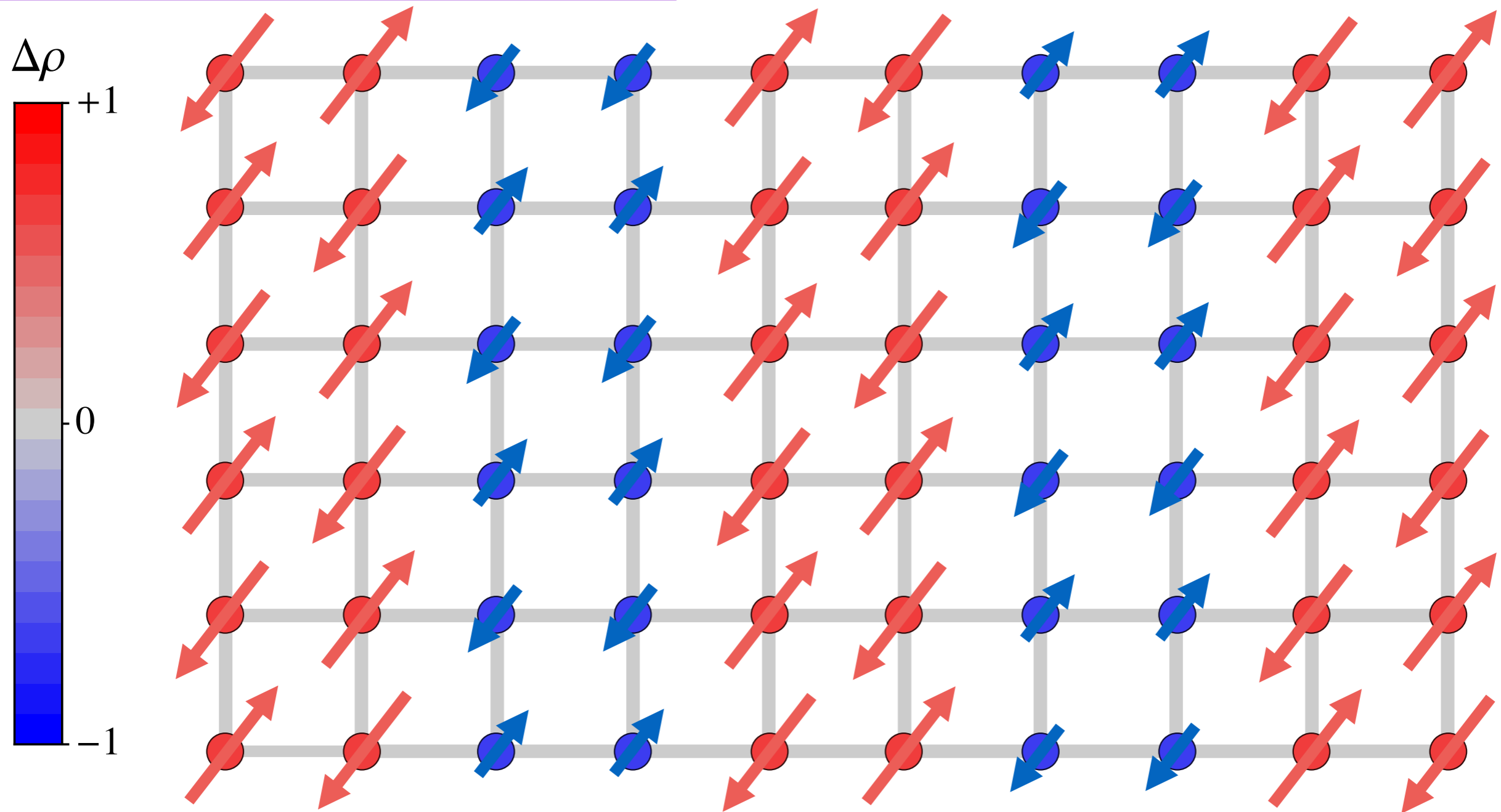


Domain walls 4 lattice spacings apart

# “Stripe” model

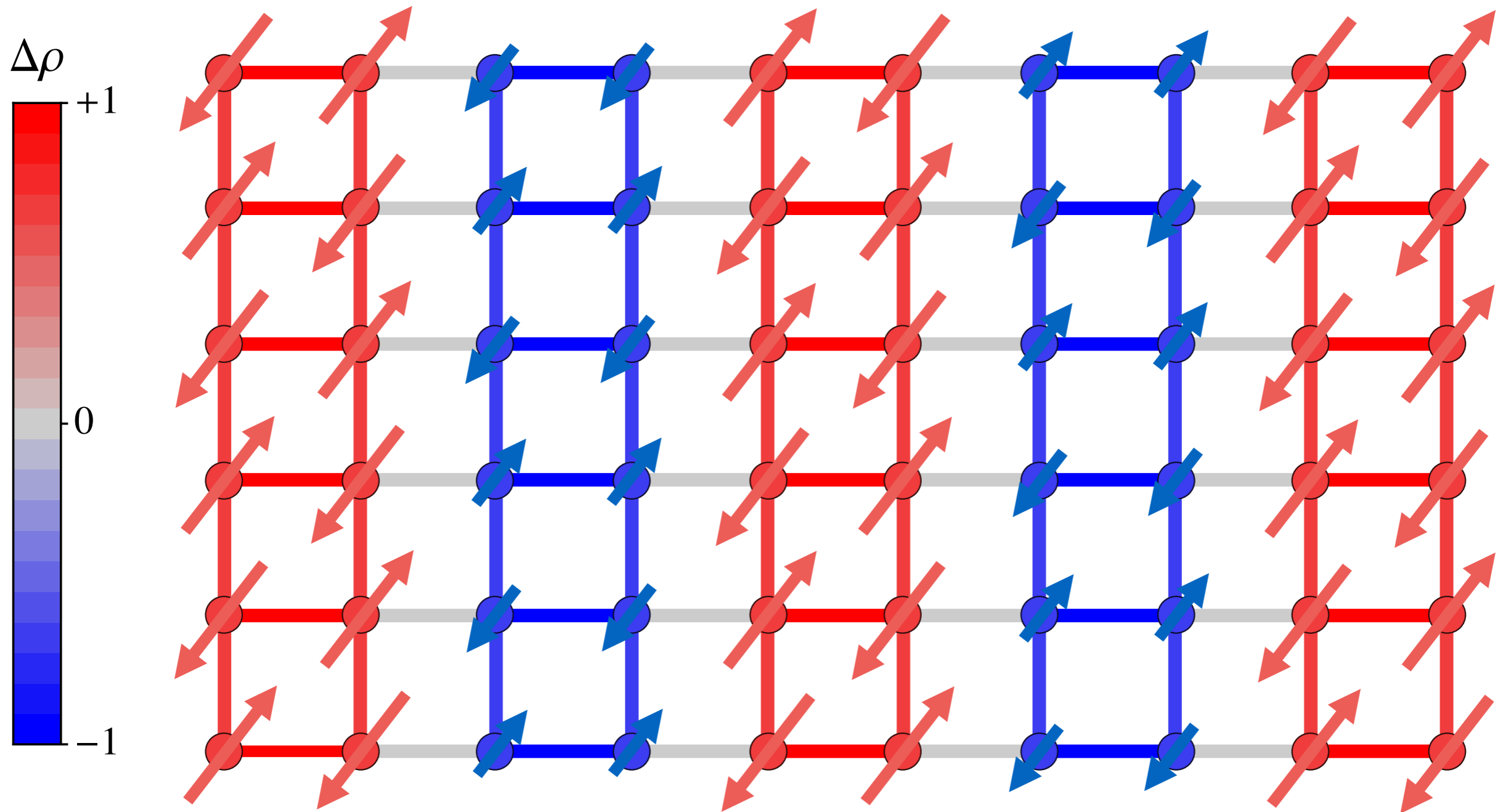
Observed in La-based  
compounds (Tranquada..)

Theory: Zaanen, Kivelson, Fradkin....



Put the holes in the domain walls

# “Stripe” model



Colors on the bonds map the local exchange energy

**Unconventional density wave (DW) :**  
**Bose condensation of particle-hole pairs**

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2}$$

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Crucial “center-of-mass” co-ordinate.  
(Not used in previous work)  
Simplifies action of time-reversal

Unconventional density wave (DW) :  
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$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2}$$

Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires  $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$ .

We expand (using reflection symmetry for  $\mathbf{Q}$  along axes or diagonals)

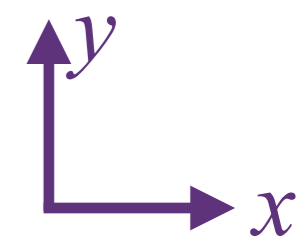
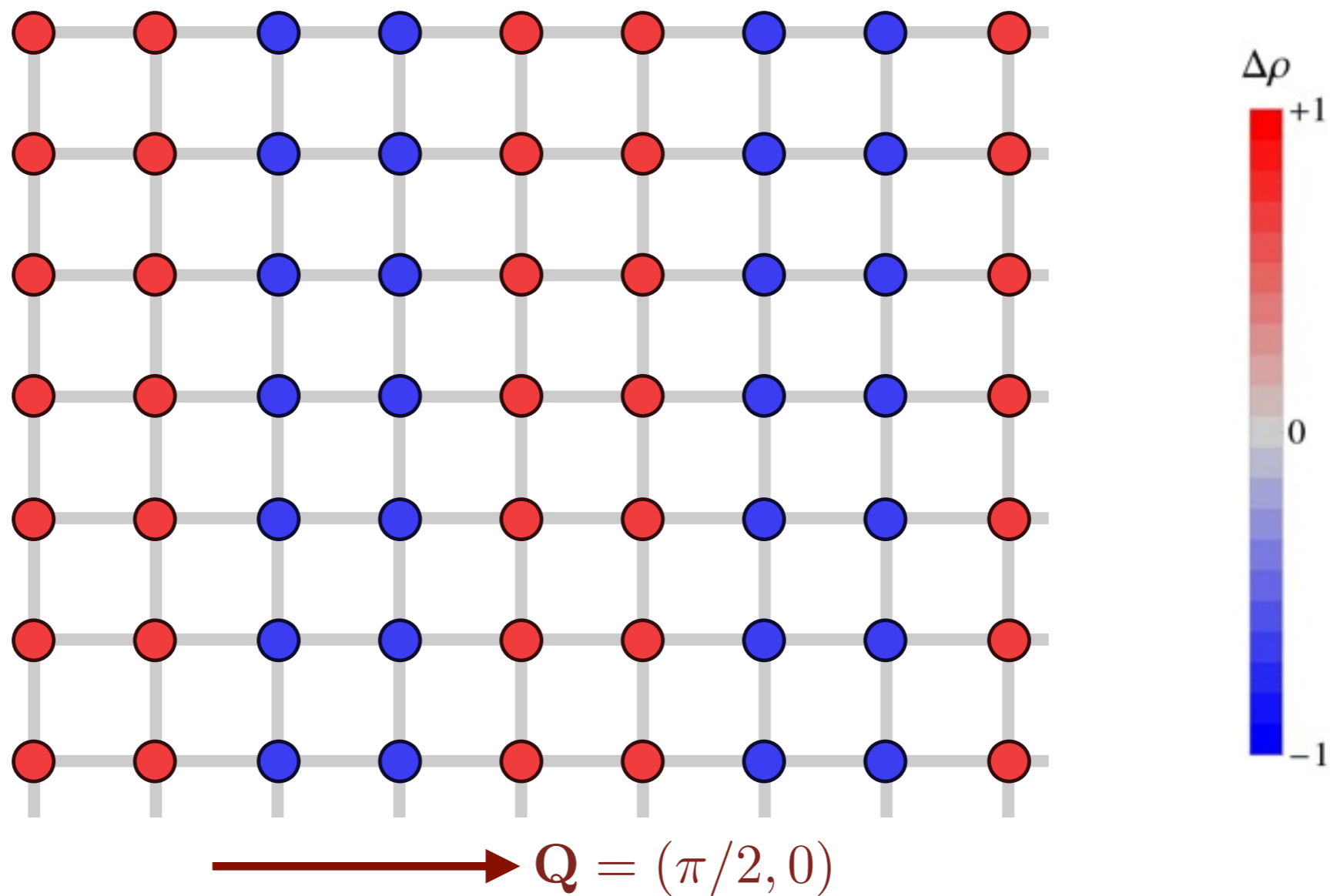
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

# Conventional CDW order: $s$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

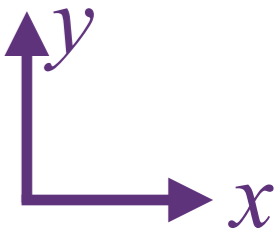
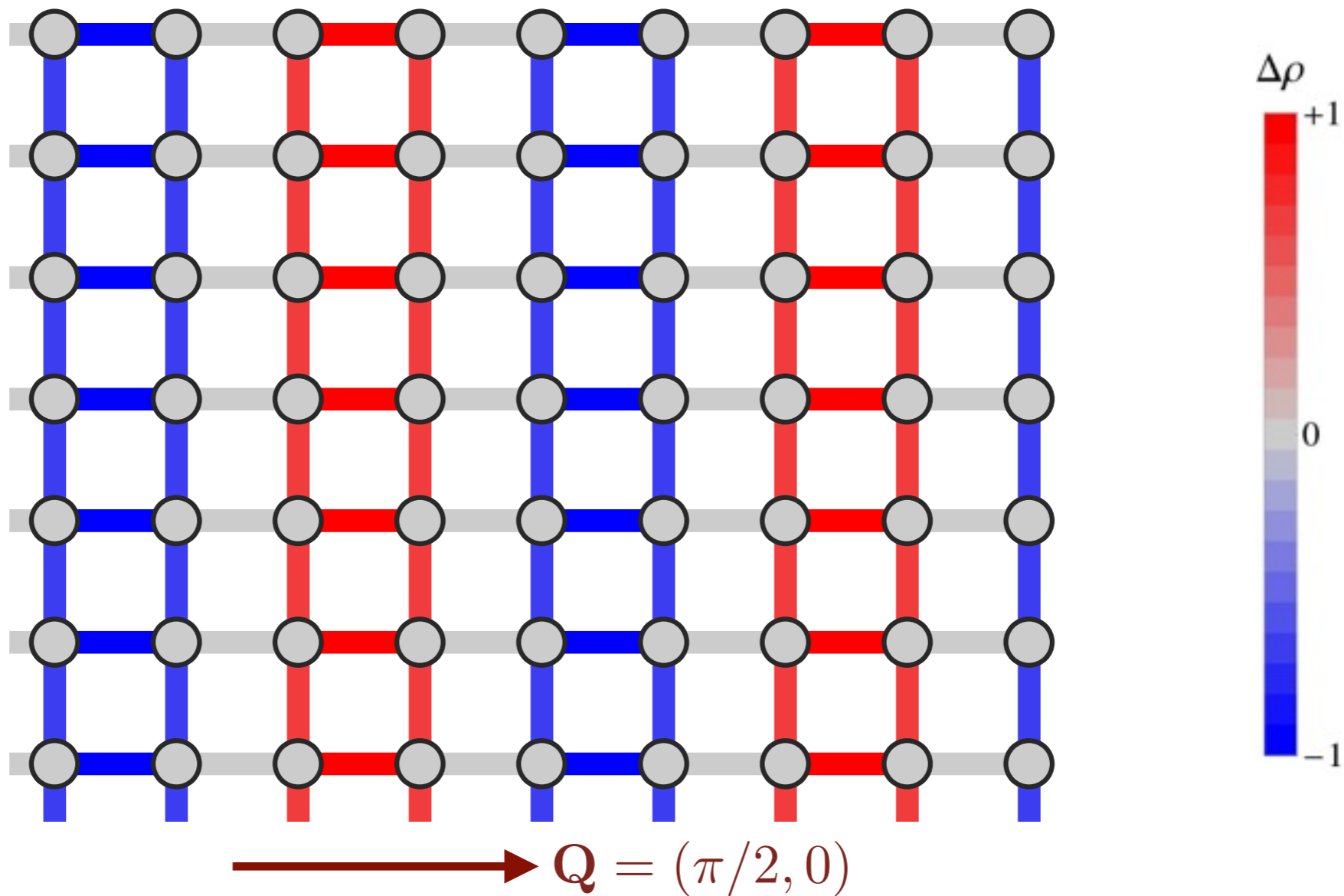


# Unconventional DW order: $s'$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

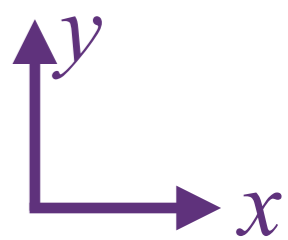


# Unconventional DW order: $s'$ -form factor

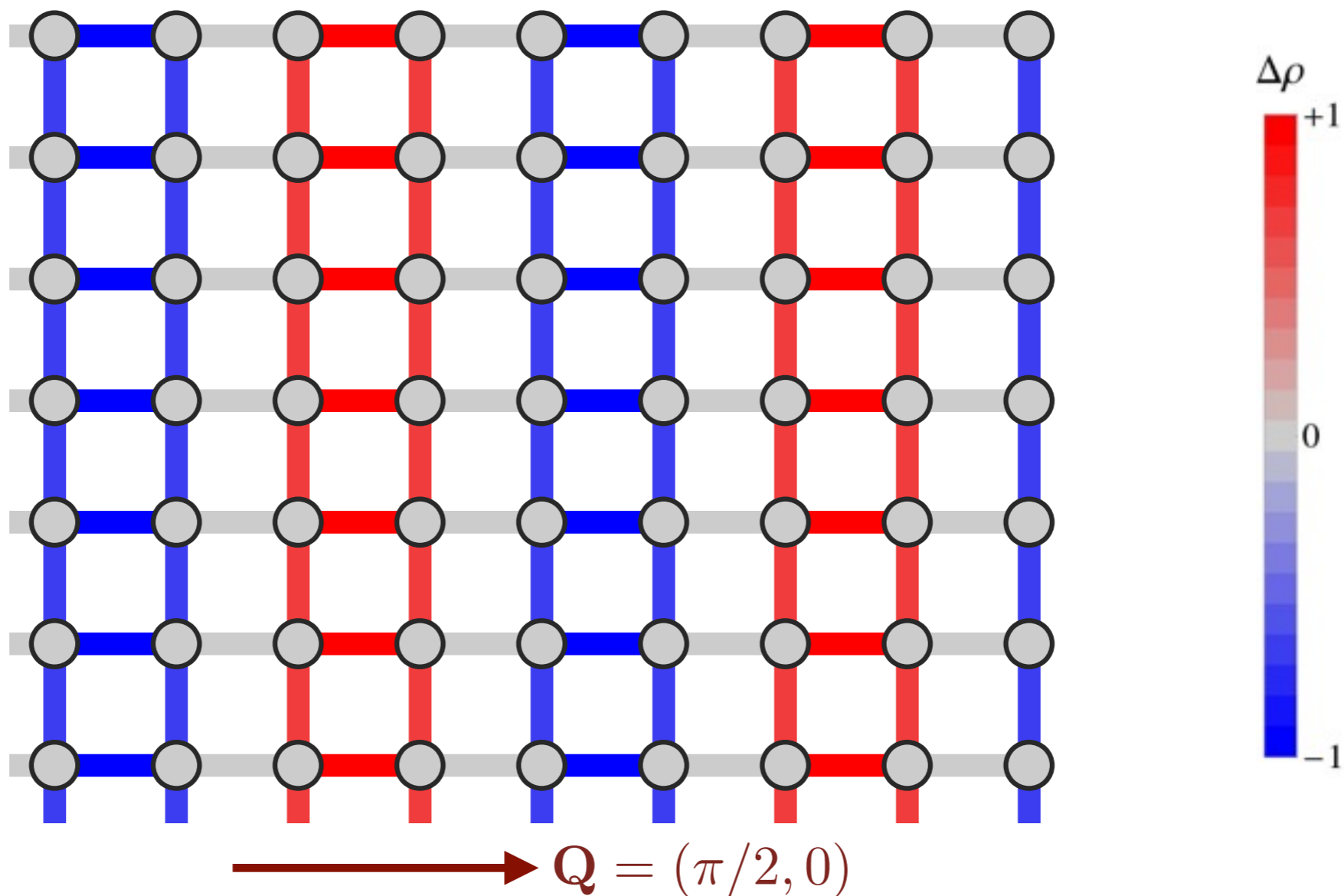
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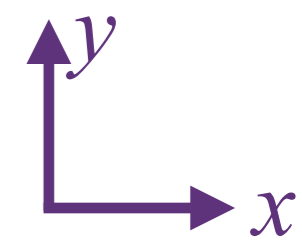


“Stripe”  
model !



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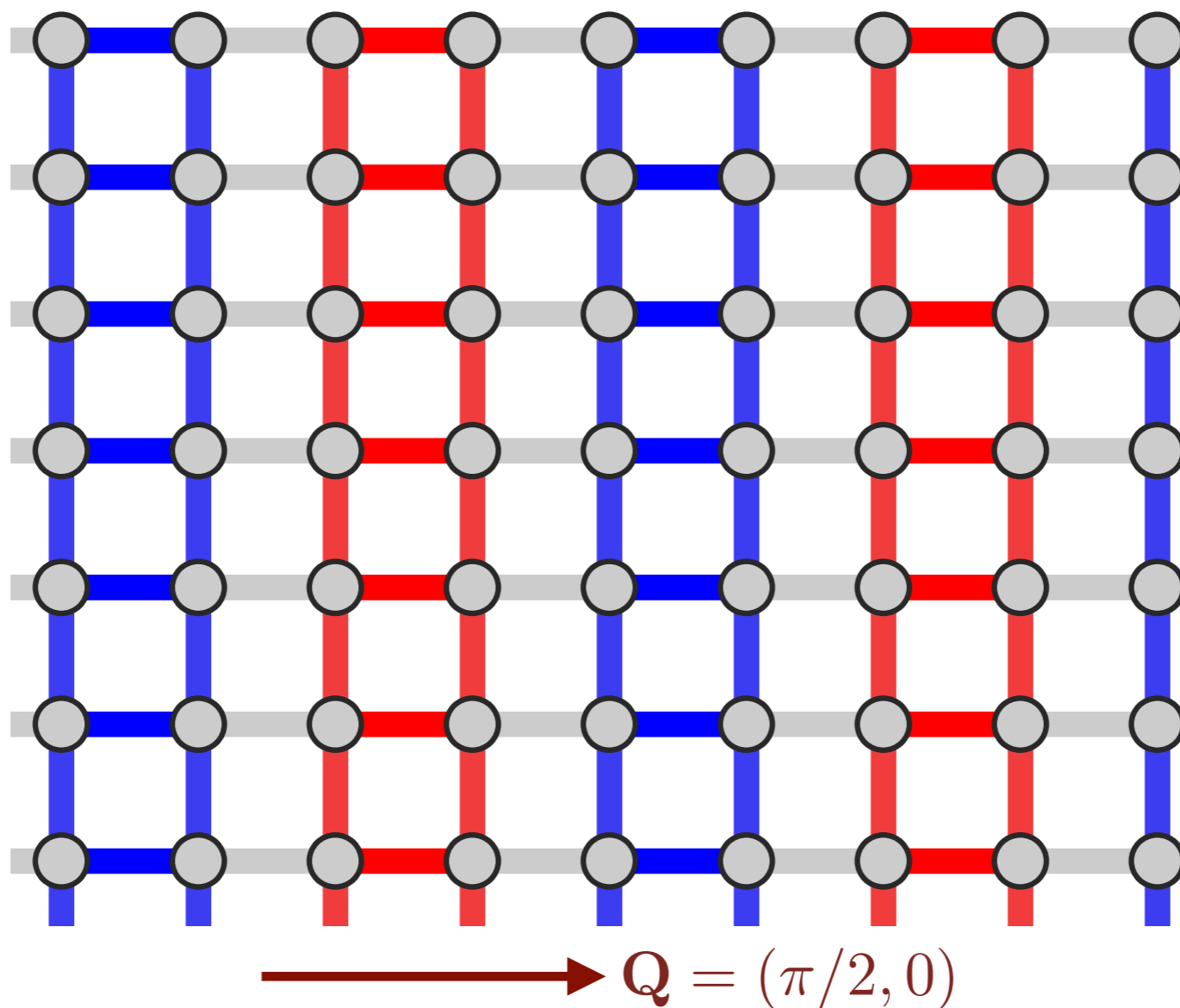


$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

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“Stripe”  
model !

X-ray  
observations  
indicate  
strong  $s'$   
component in  
LBCO



David  
Hawthorn



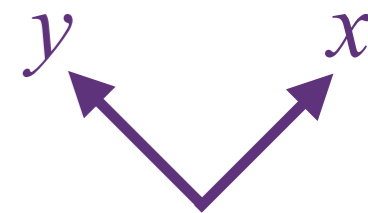
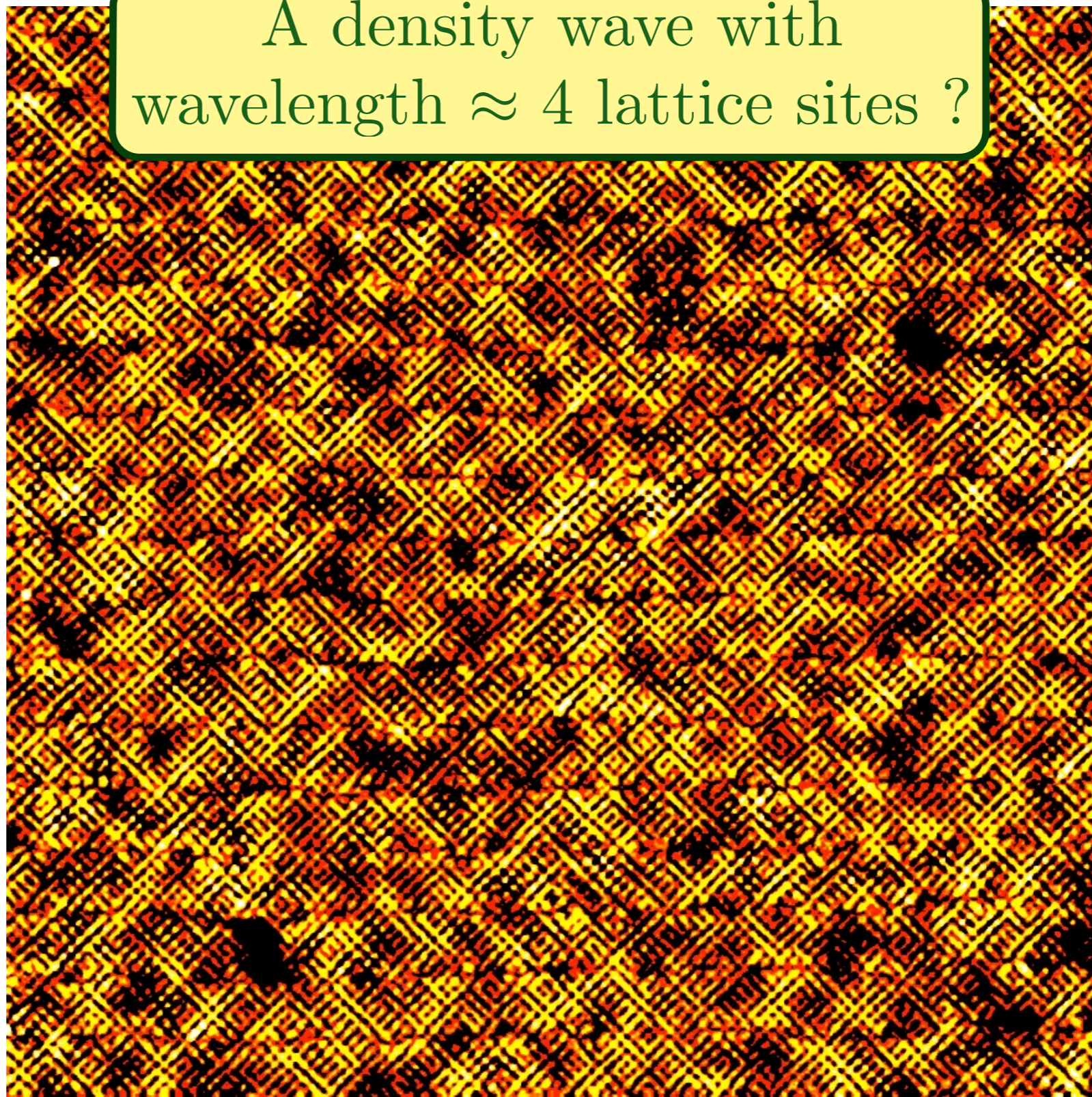
See also

C. Howald, H. Eisaki,  
N. Kaneko, M. Greven,  
and A. Kapitulnik,  
*Phys. Rev. B* **67**,  
014533 (2003);

M. Vershinin, S. Misra,  
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W. D. Wise, M. C. Boyer,  
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A density wave with  
wavelength  $\approx 4$  lattice sites ?



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). **Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.**

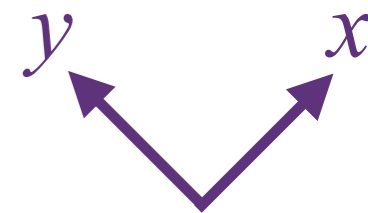
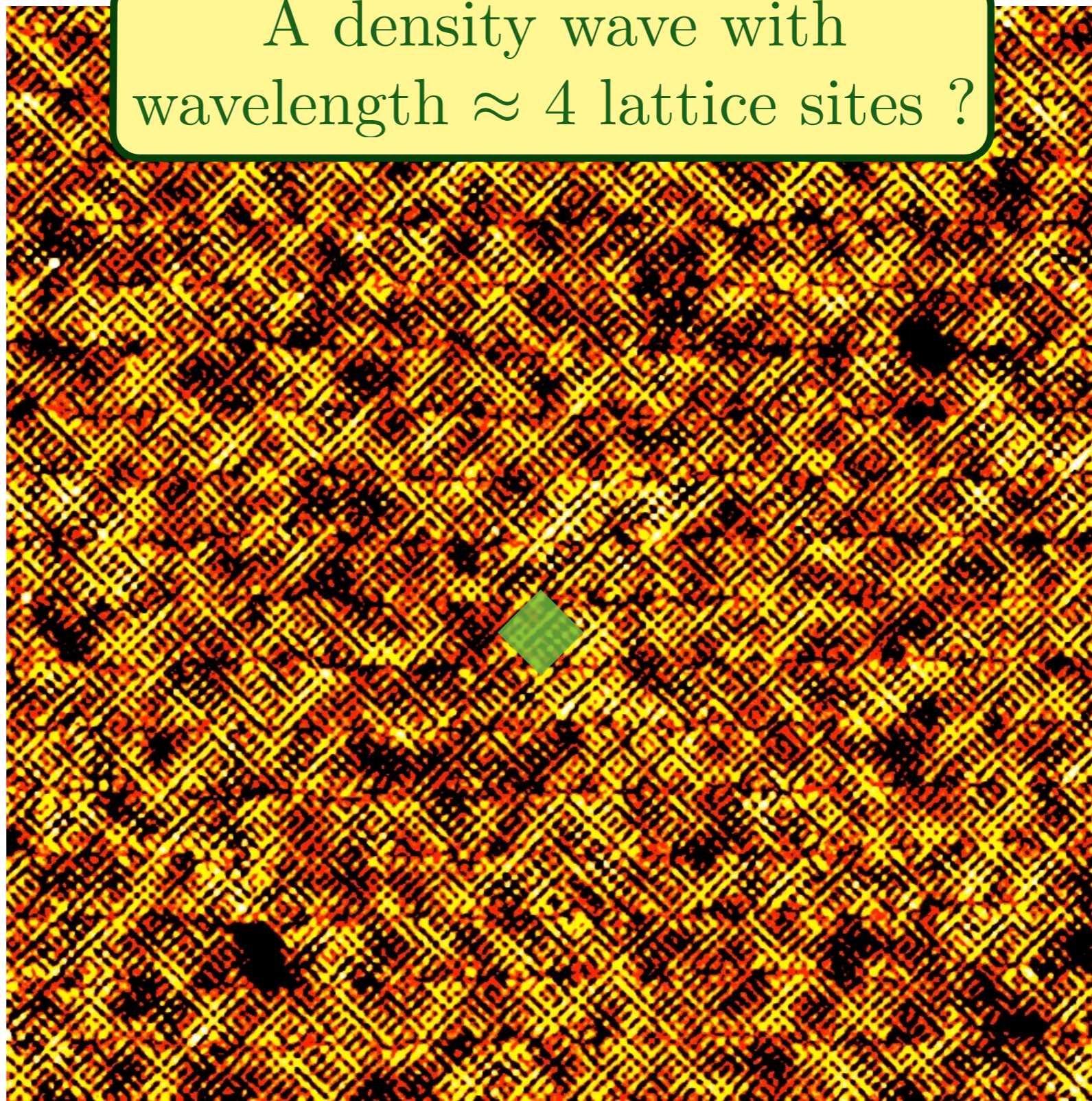
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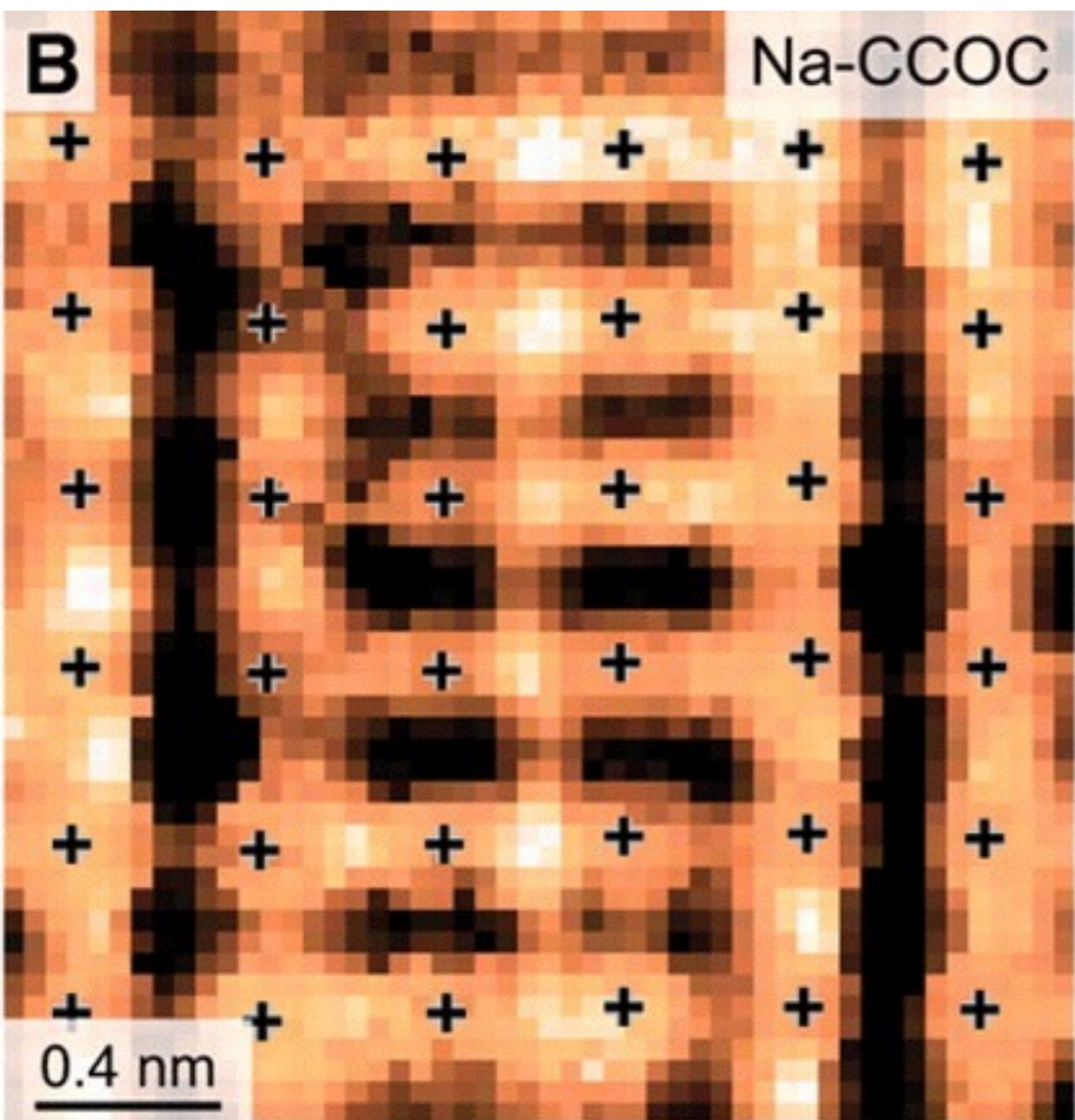
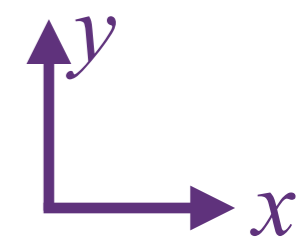
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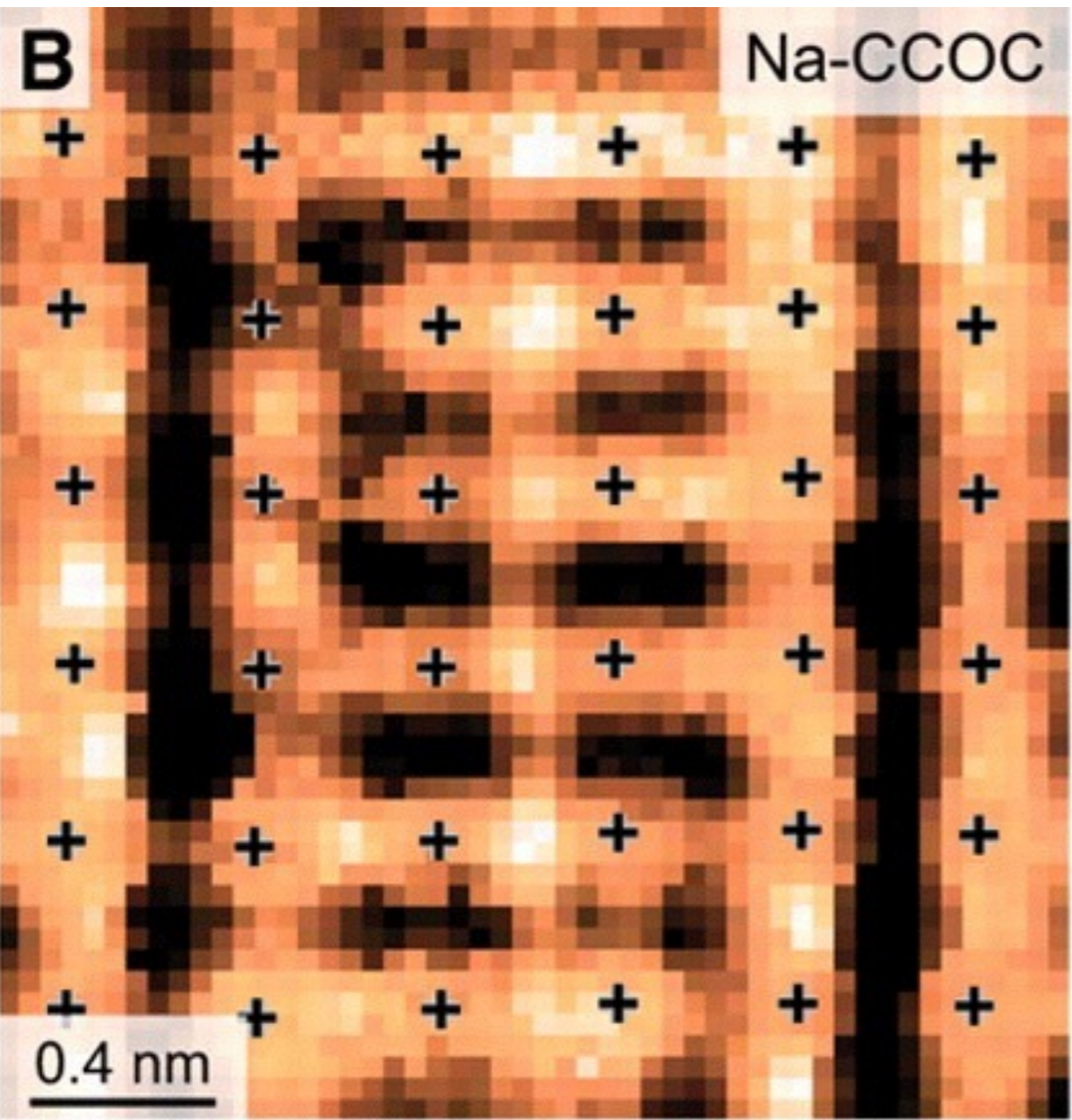
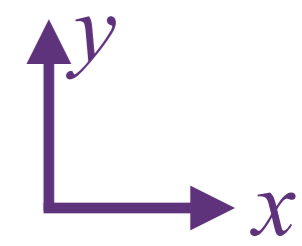
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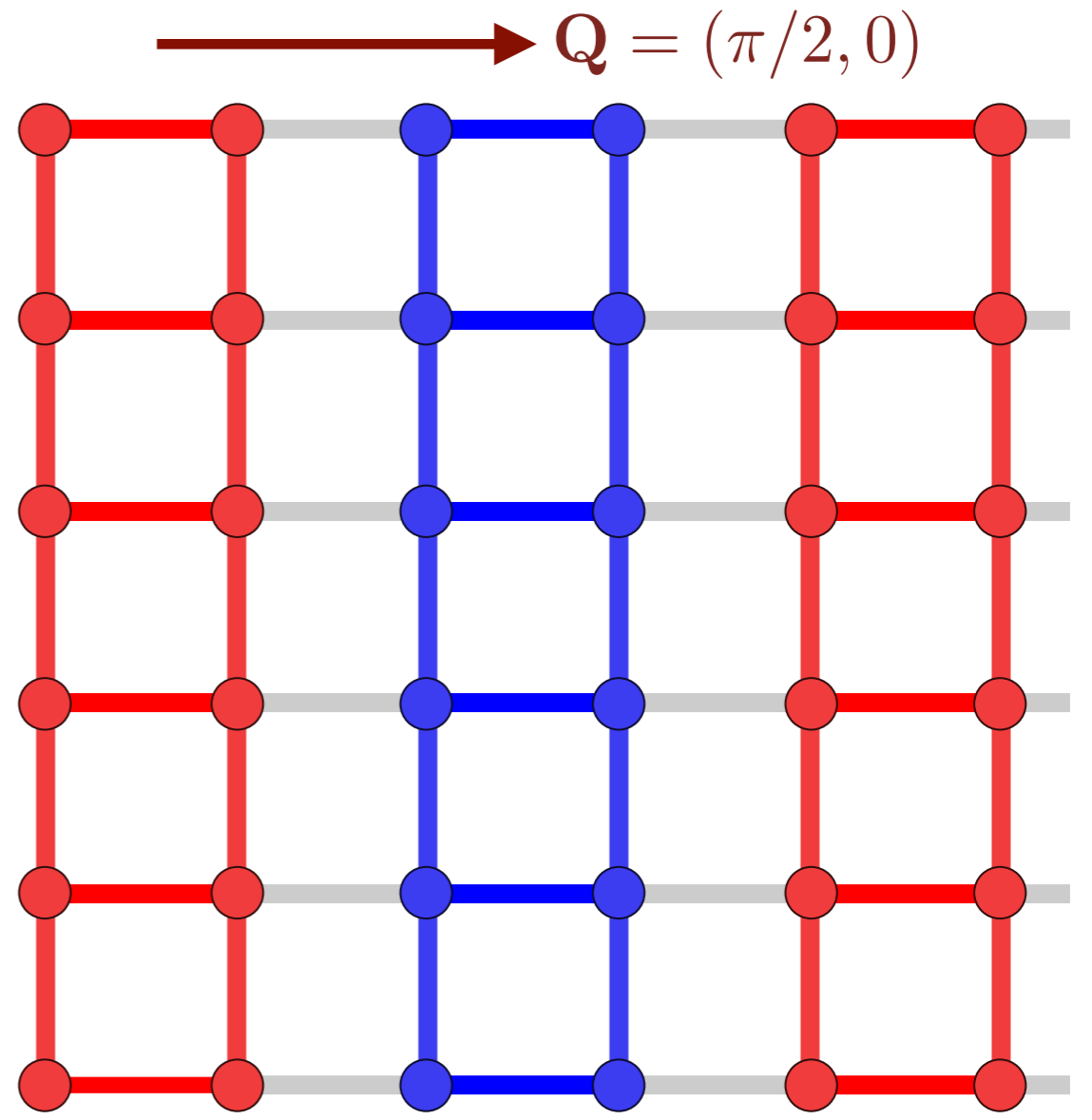
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Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



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$s + s'$ -form factor density wave

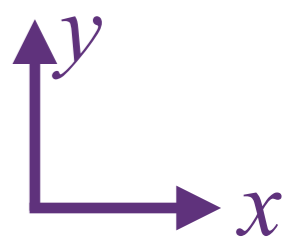
$s + s'$  form factor does not match STM measurements on BSCCO, Na-CCOC.

# Unconventional DW order: $d$ -form factor

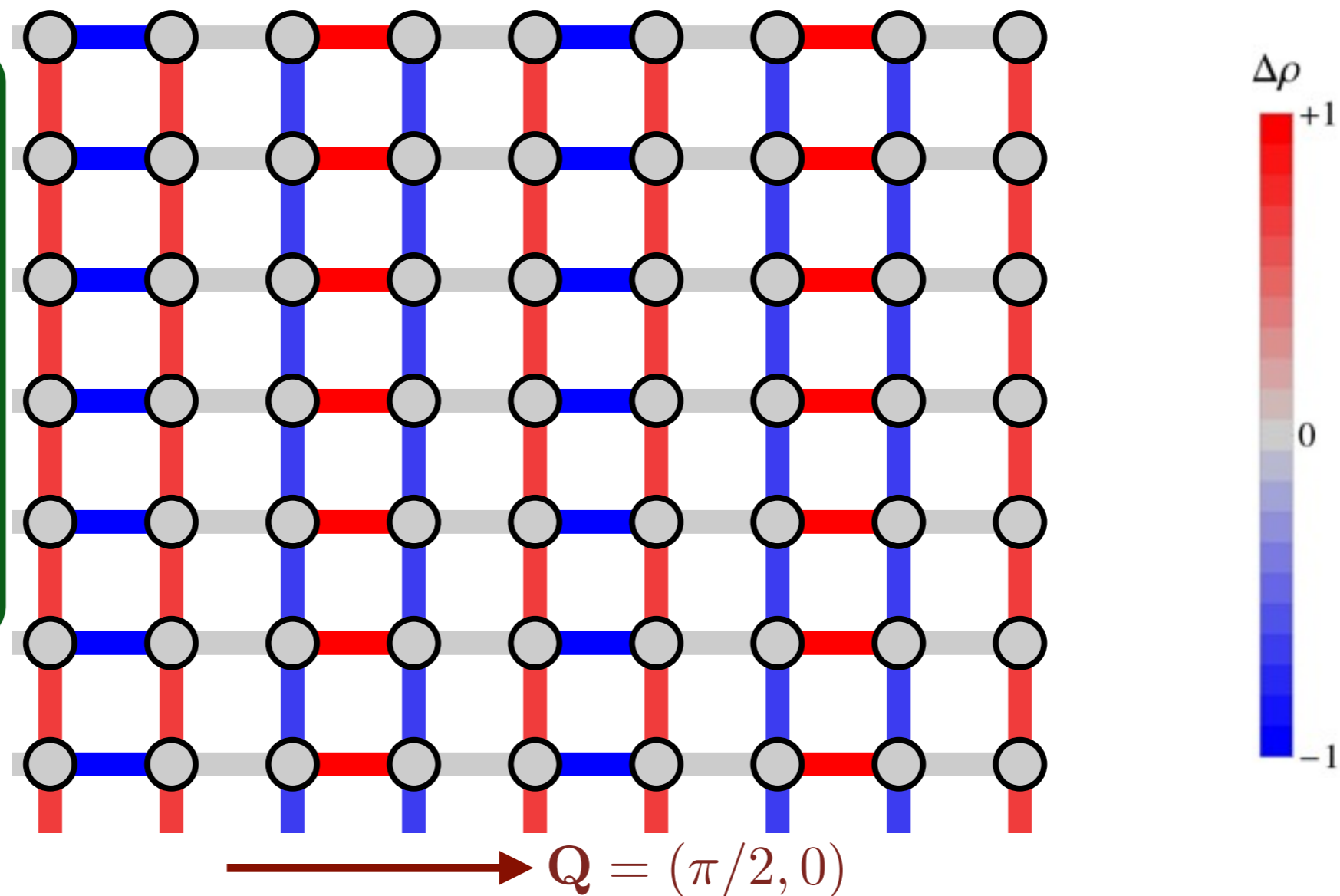
Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

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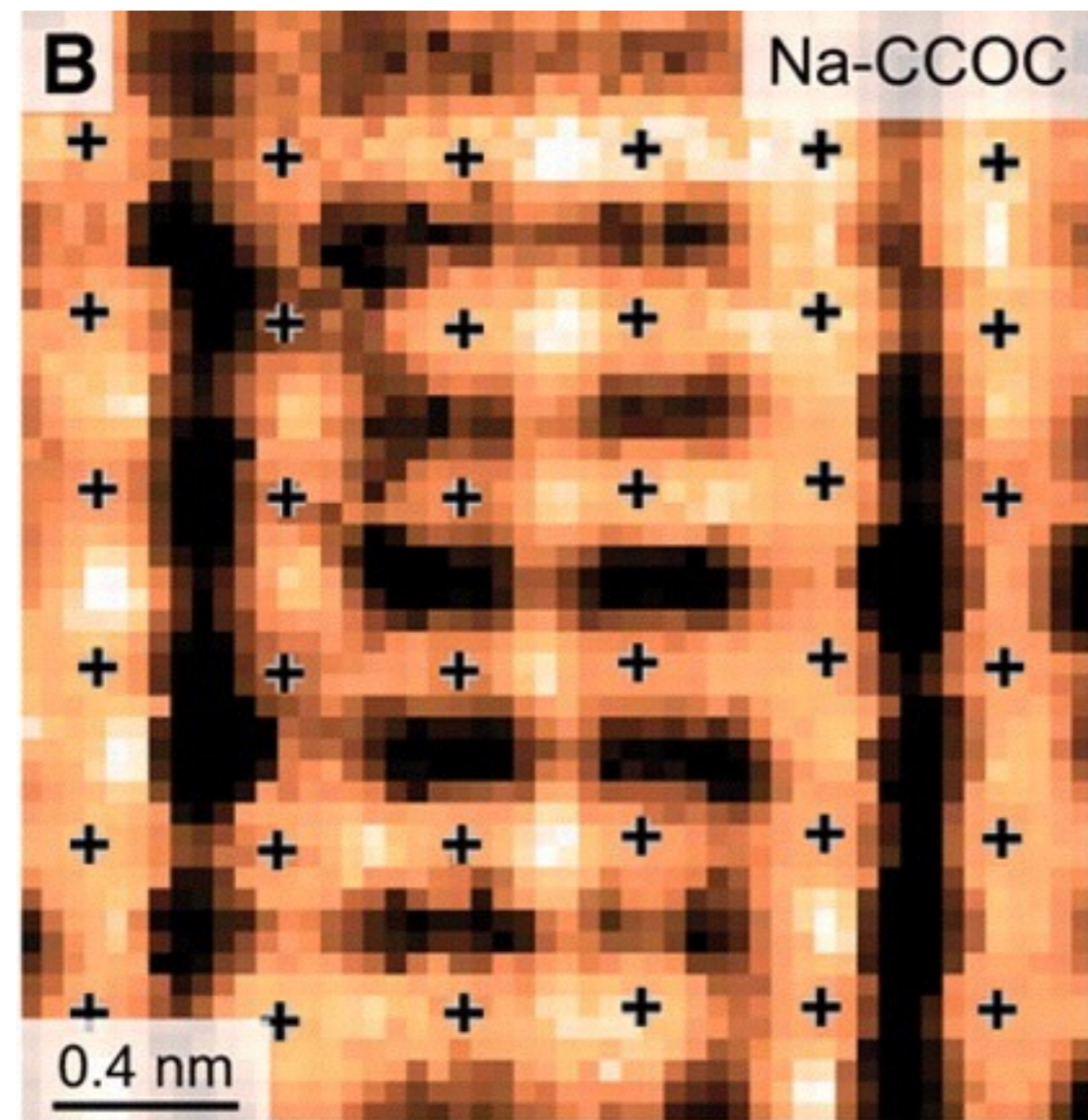
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



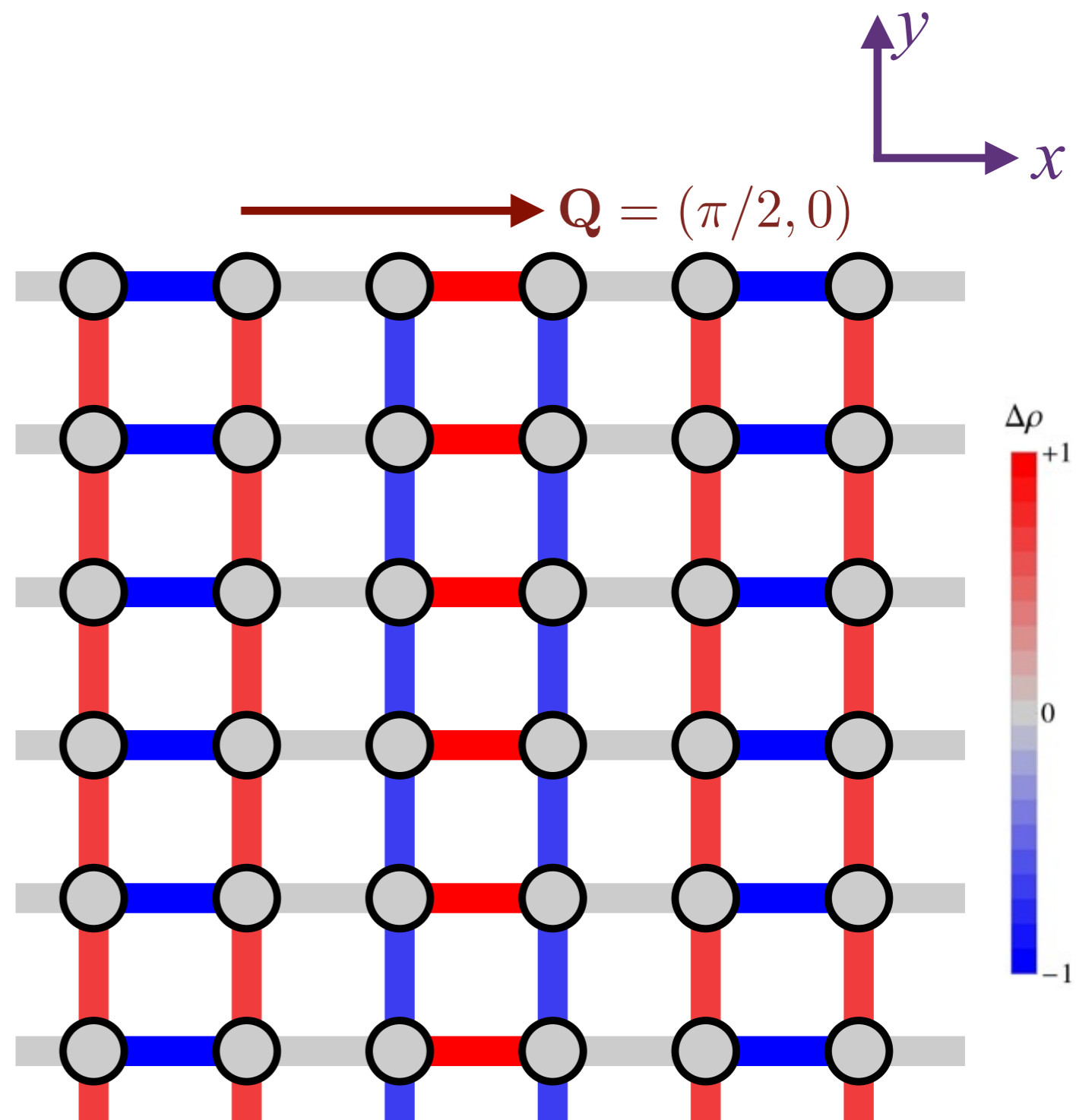
Our prediction:  
Density wave on  
horizontal  
bonds has a  
phase-shift of  $\pi$   
relative to the  
wave on vertical  
bonds



M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).  
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

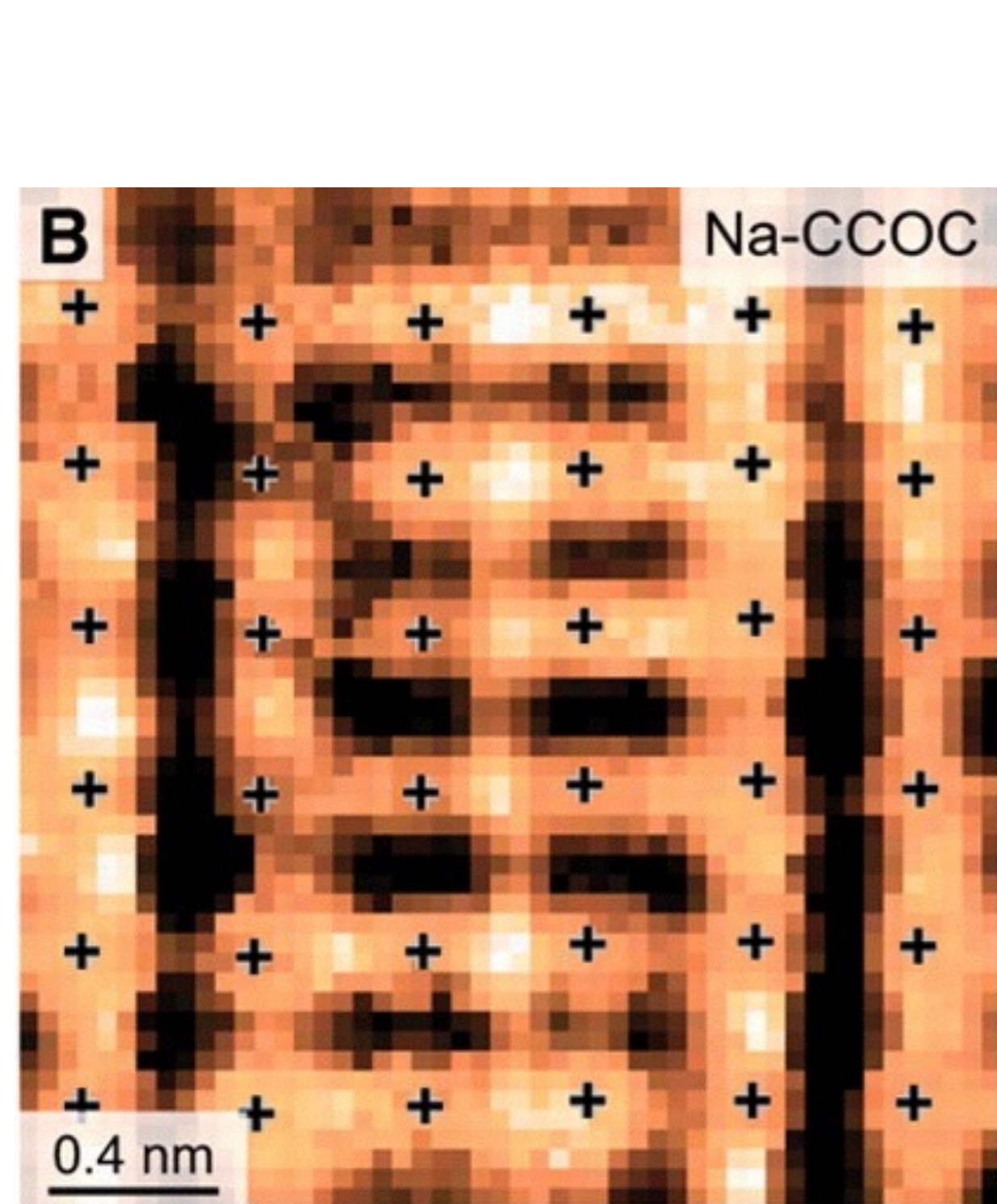


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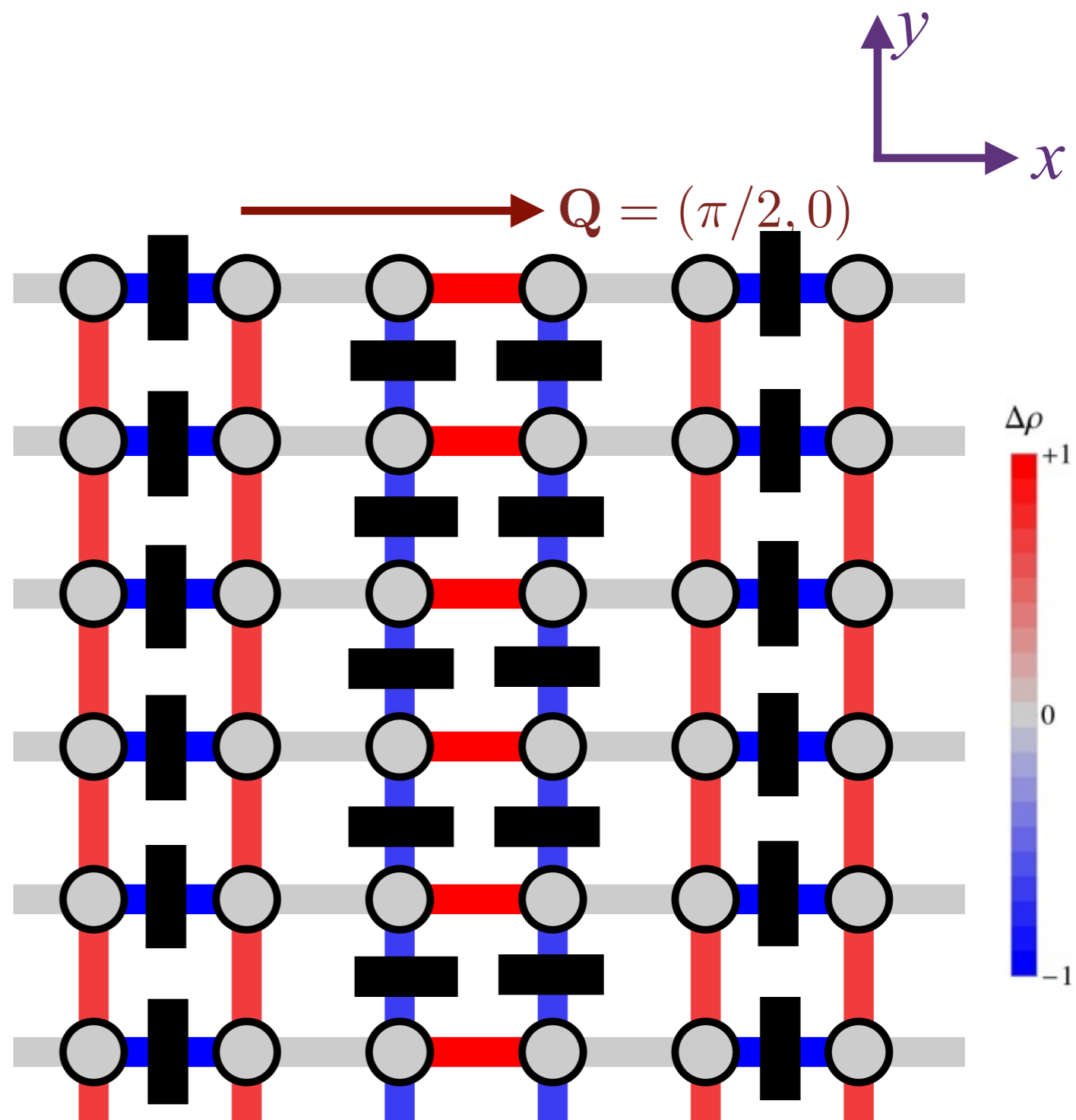


*d*-form factor density wave order

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*d*-form factor density wave order

*d* form factor is compatible with STM measurements  
on BSCCO, Na-CCOC !

# Direct phase-sensitive identification of a $d$ -form factor density wave in underdoped cuprates

Kazuhiro Fujita<sup>a,b,c,1</sup>, Mohammad H. Hamidian<sup>a,b,1</sup>, Stephen D. Edkins<sup>b,d</sup>, Chung Koo Kim<sup>a</sup>, Yuhki Kohsaka<sup>e</sup>, Masaki Azuma<sup>f</sup>, Mikio Takano<sup>g</sup>, Hidenori Takagi<sup>c,h,i</sup>, Hiroshi Eisaki<sup>j</sup>, Shin-ichi Uchida<sup>c</sup>, Andrea Allais<sup>k</sup>, Michael J. Lawler<sup>b,l</sup>, Eun-Ah Kim<sup>b</sup>, Subir Sachdev<sup>k,m</sup>, and J. C. Séamus Davis<sup>a,b,d,2</sup>

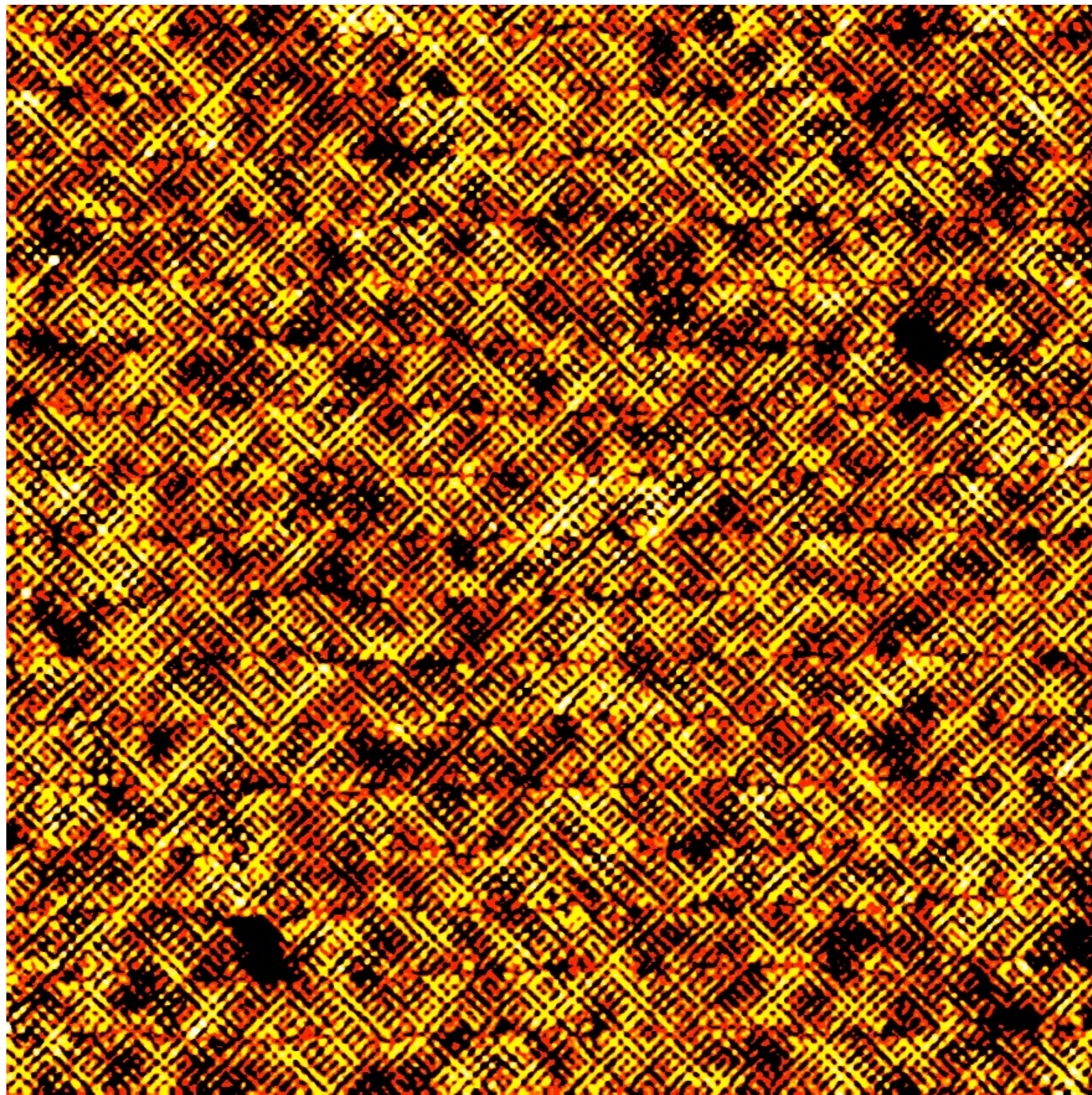
The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each  $\text{CuO}_2$  unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [ $\text{Cu}(r)$ ] and only the  $x/y$  axis O sites [ $\text{O}_x(r)$  and  $\text{O}_y(r)$ ]. Phase-resolved Fourier analysis reveals directly that the modulations in the  $\text{O}_x(r)$  and  $\text{O}_y(r)$  sublattice images consistently exhibit a relative phase of  $\pi$ . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly  $d$ -symmetry form factor.

See also

C. Howald, H. Eisaki,  
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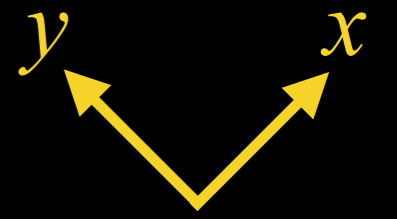
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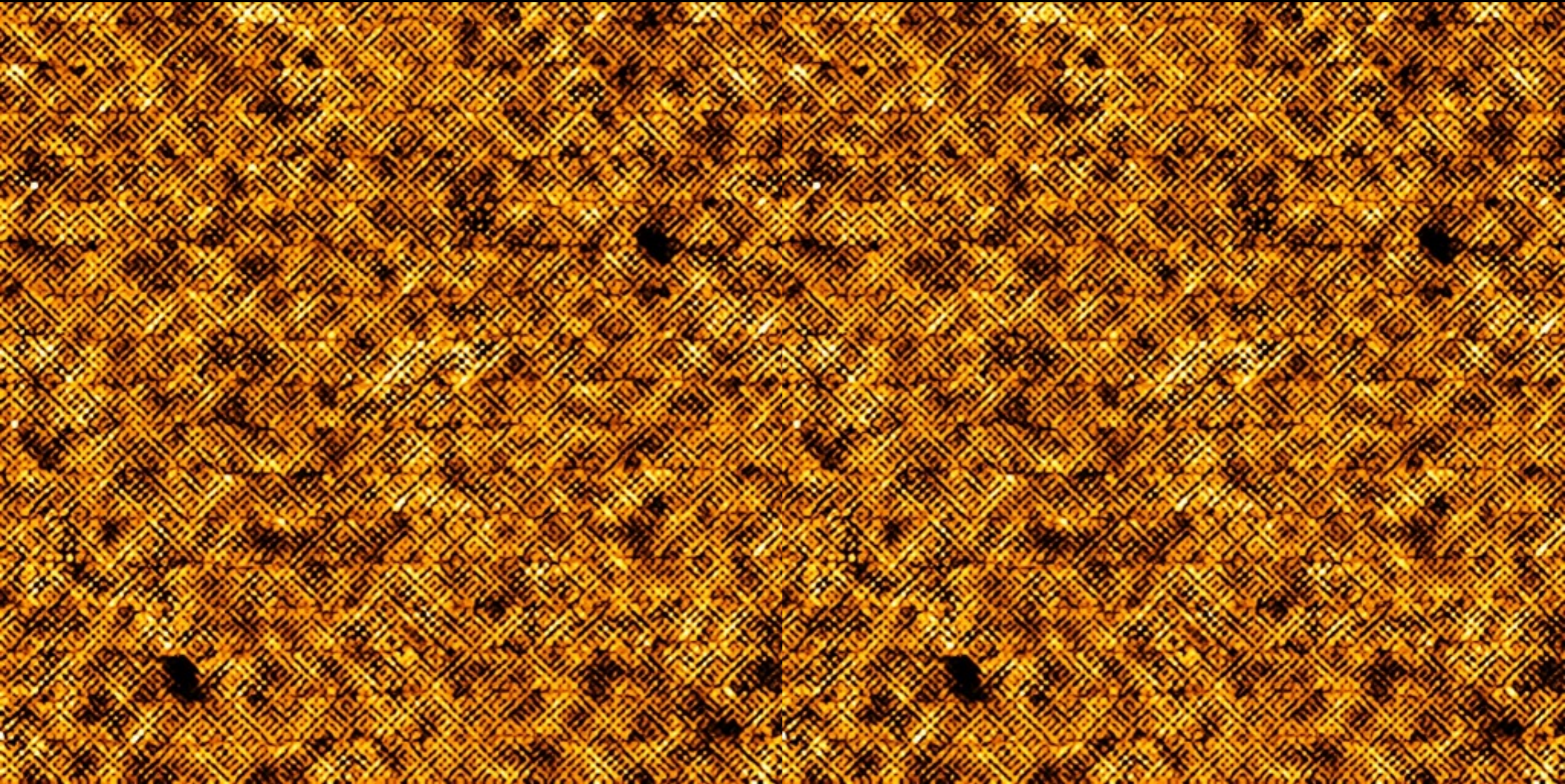
UD45K  
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



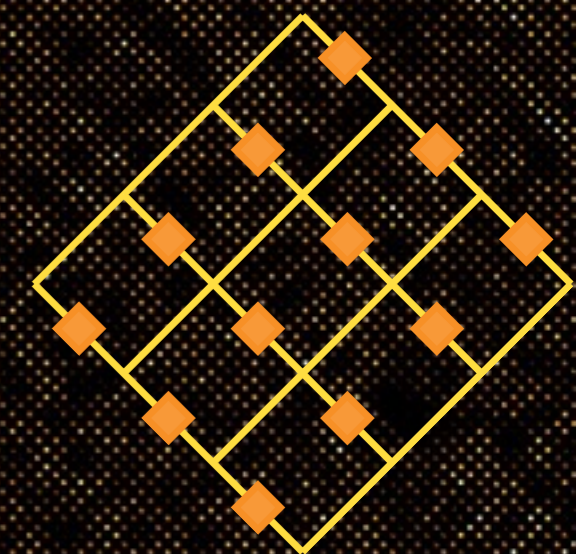
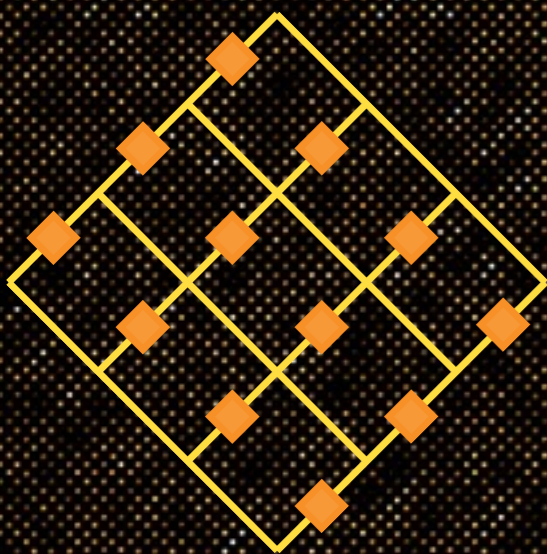
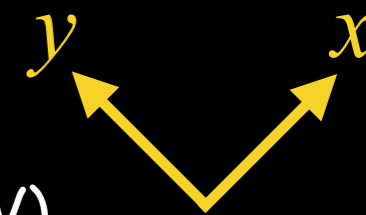
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

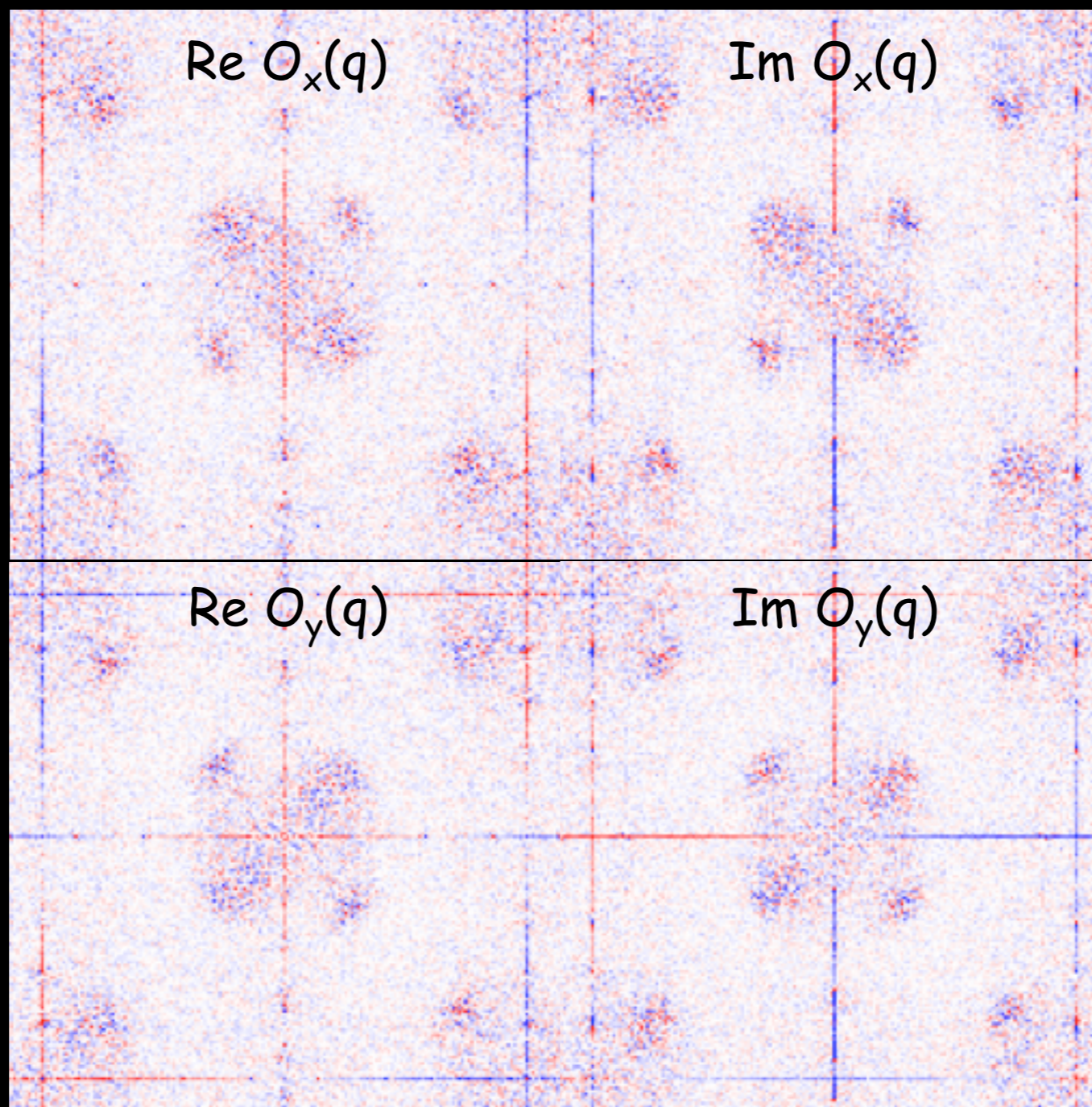
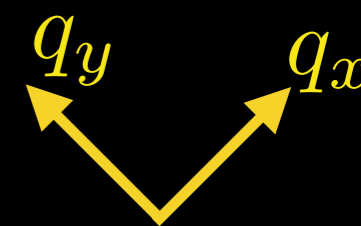
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

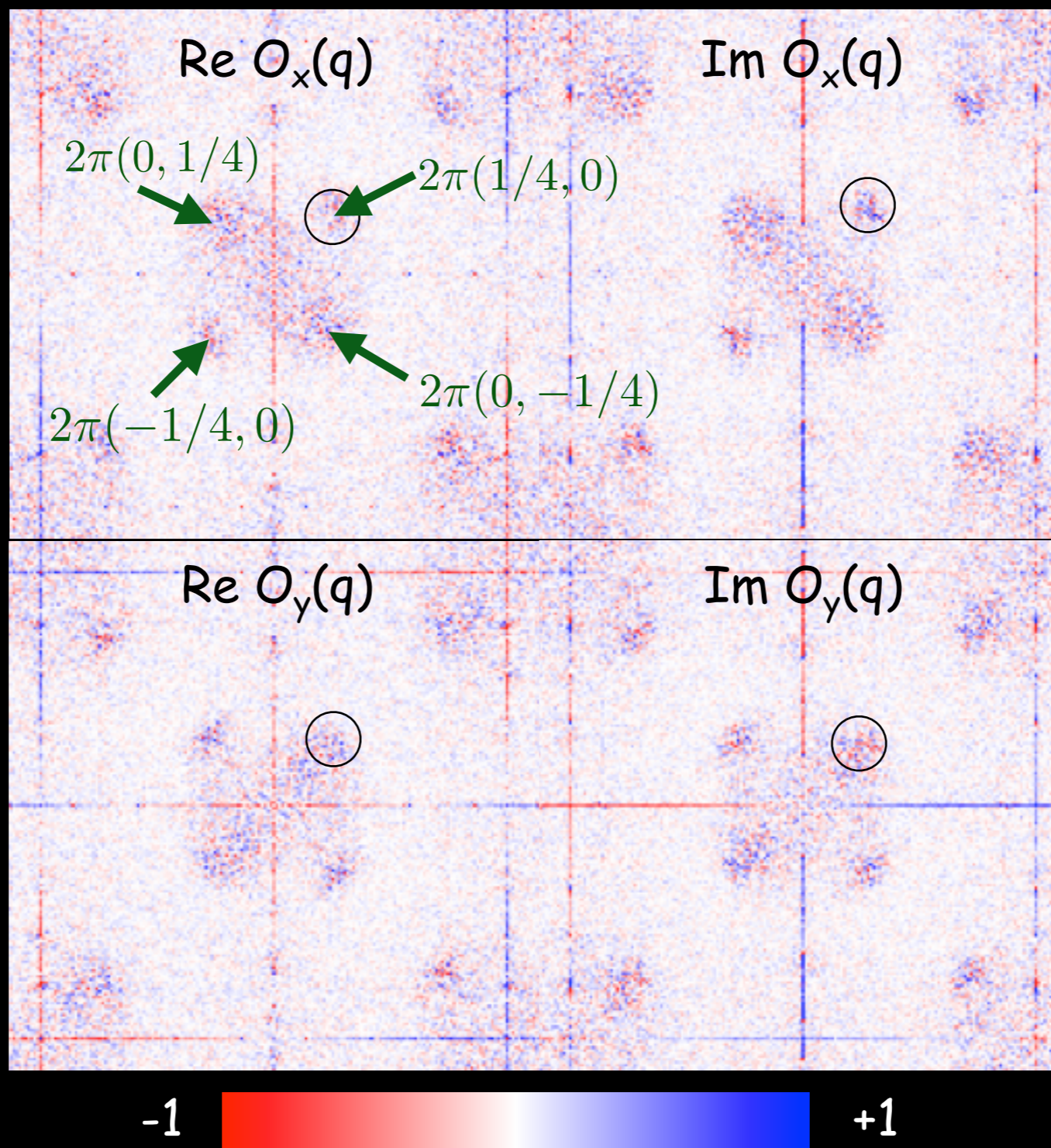
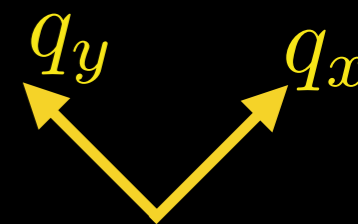


UD45K

# Broad (0,Q) and (Q,0) DW Features

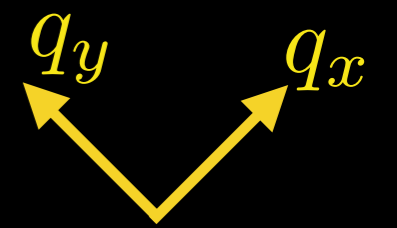


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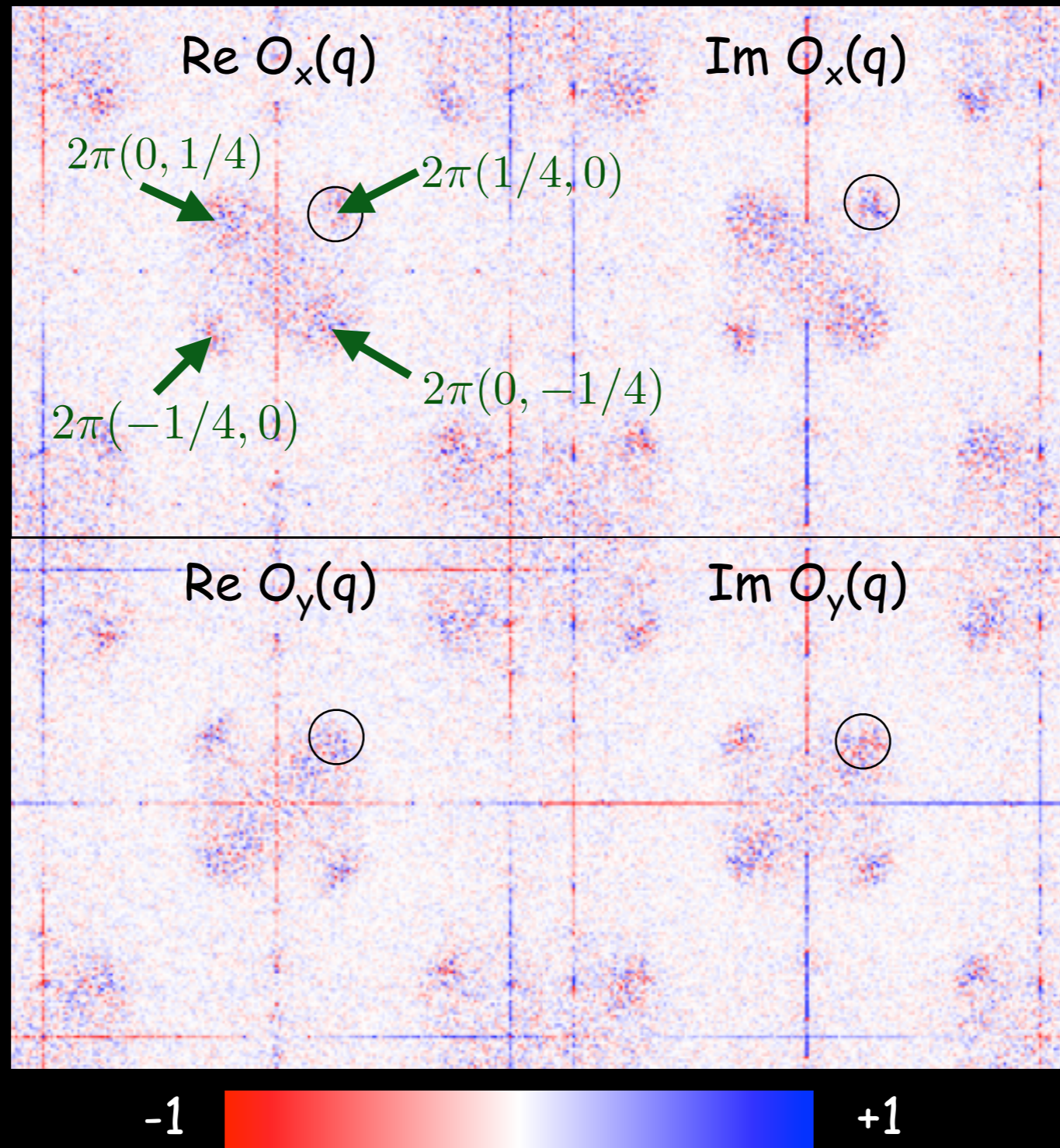


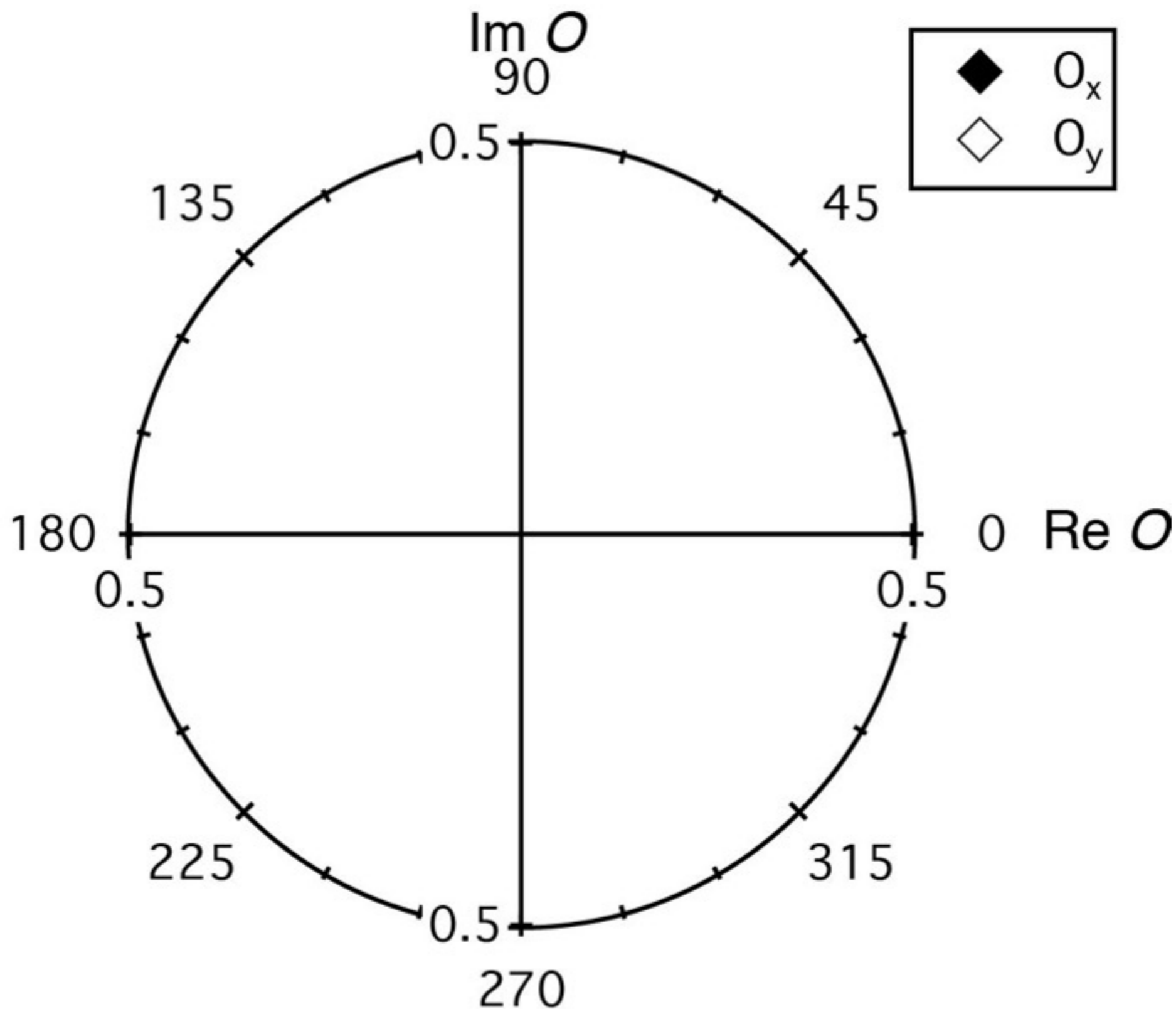
UD45K

## Broad (0,Q) and (Q,0) DW Features

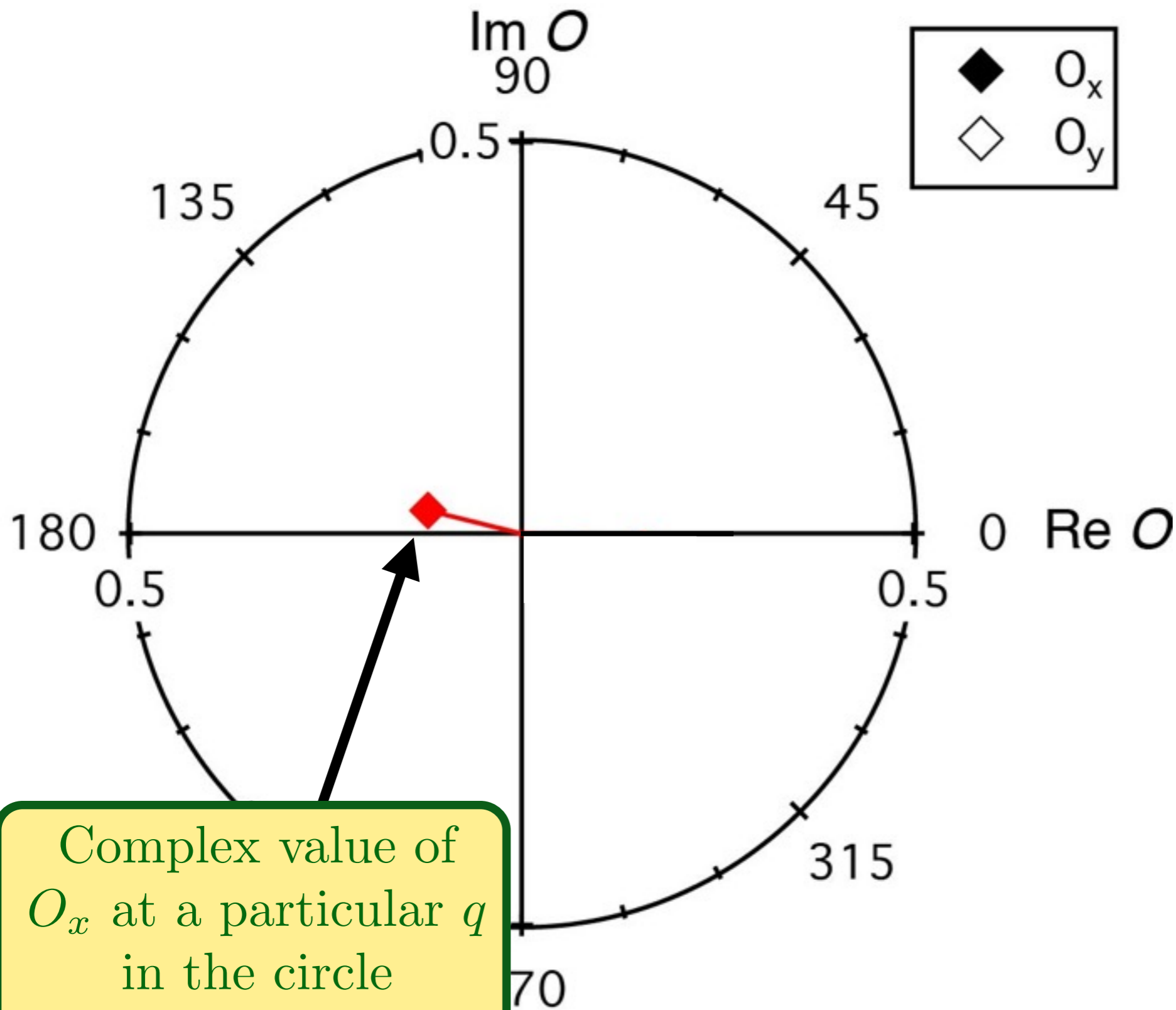


For each pixel in the circles, we obtain 2 complex numbers,  $O_x(q)$  and  $O_y(q)$ .



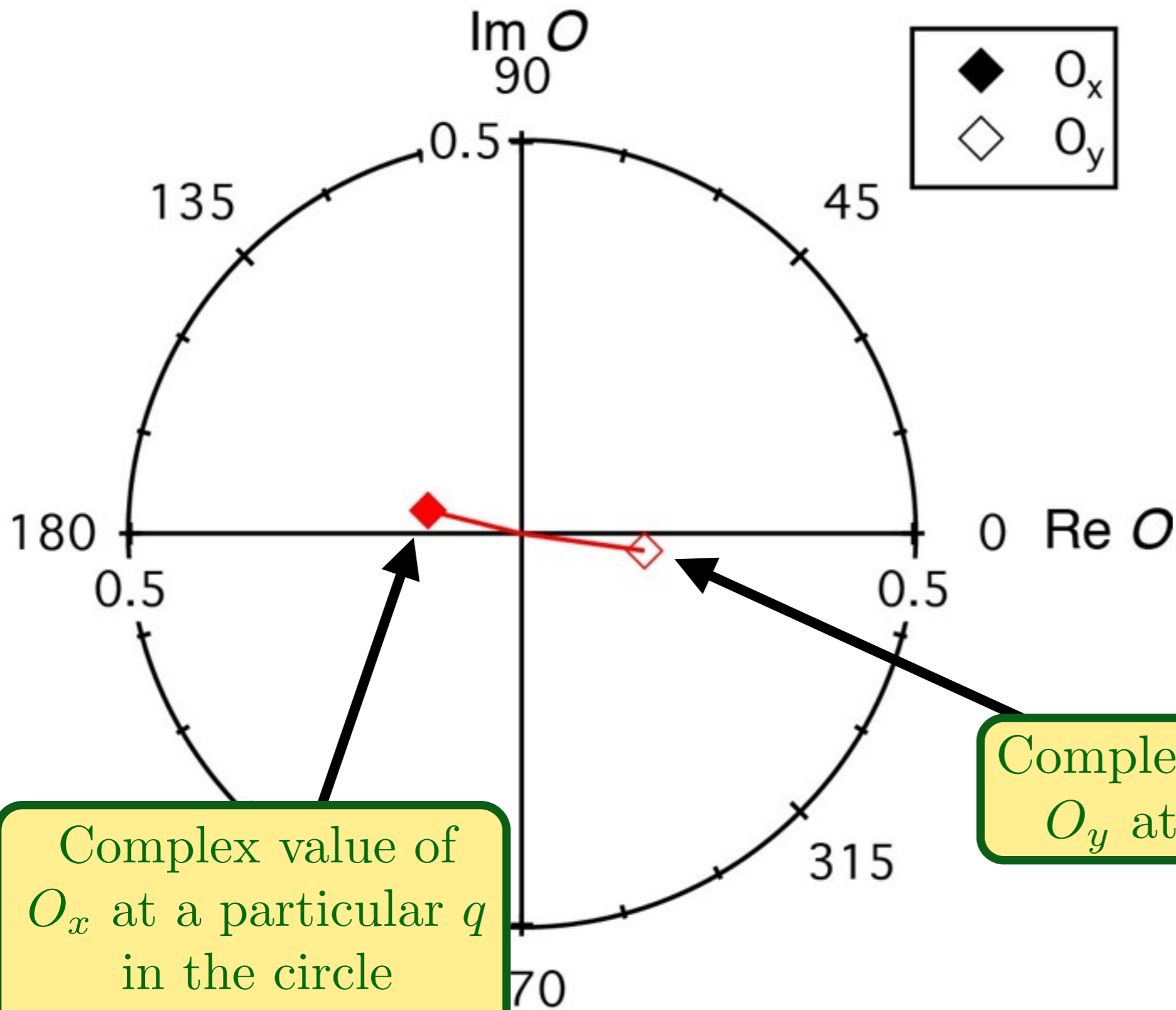


**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



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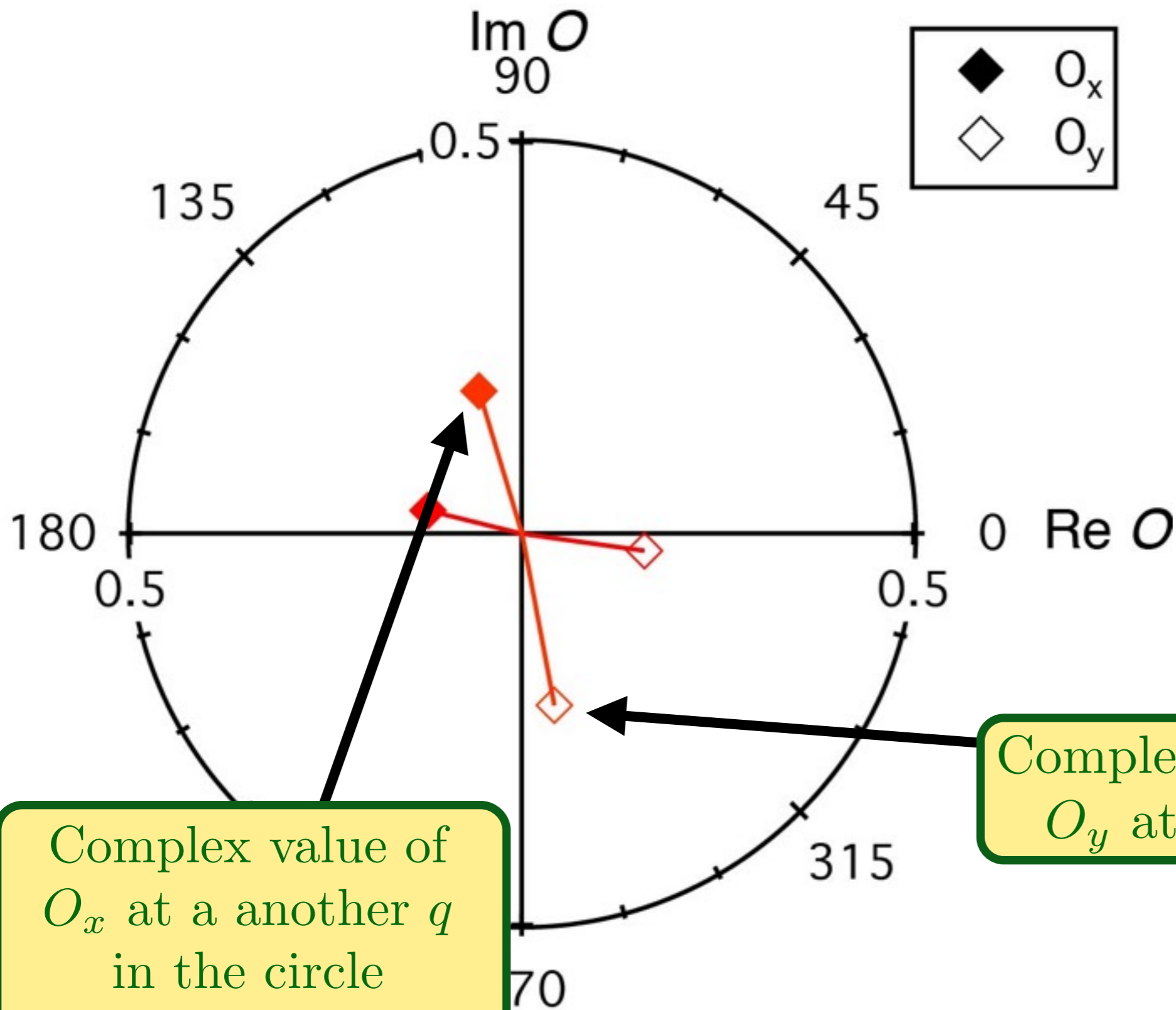
Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .



**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .

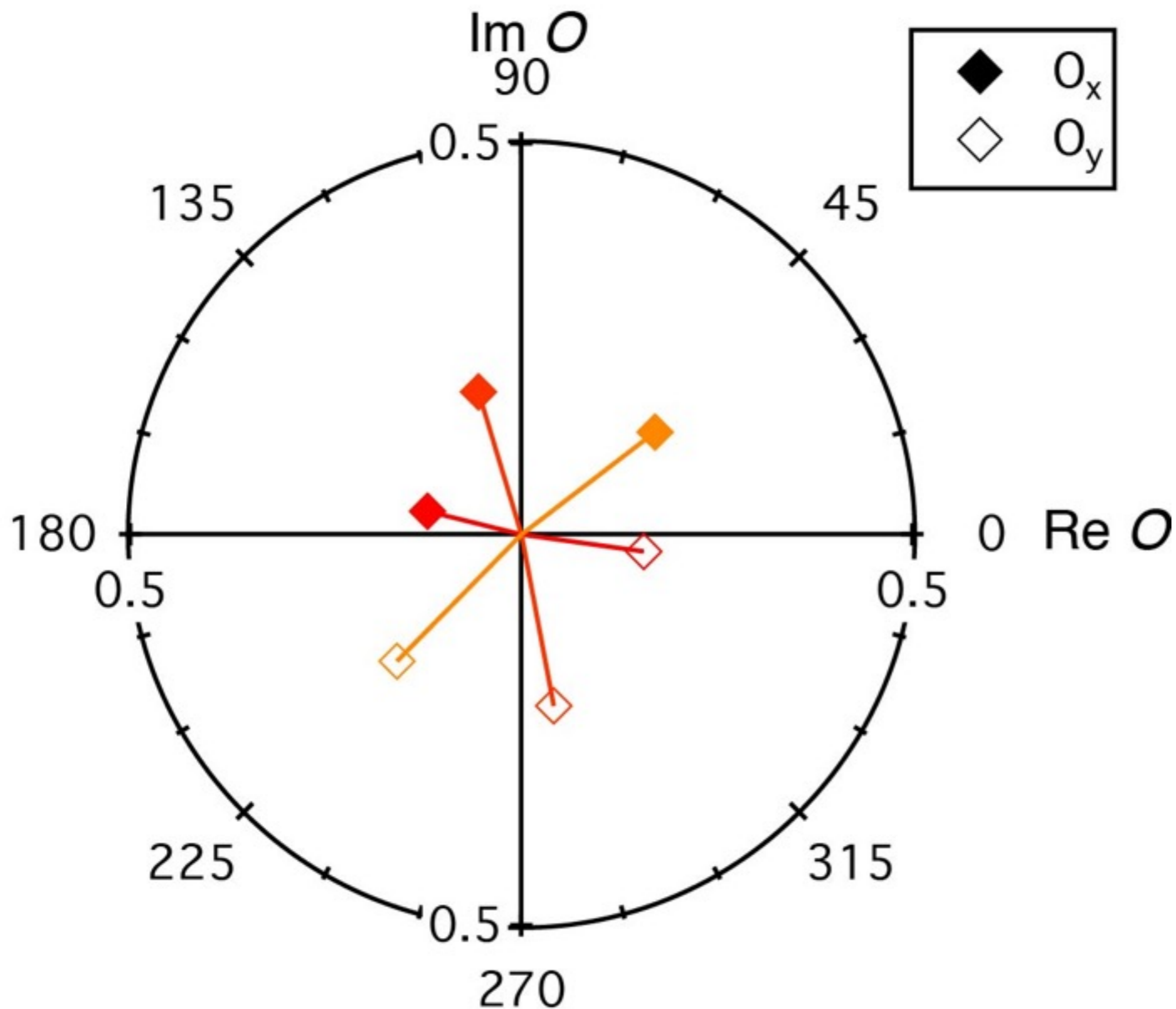
Complex value of  $O_y$  at same  $q$



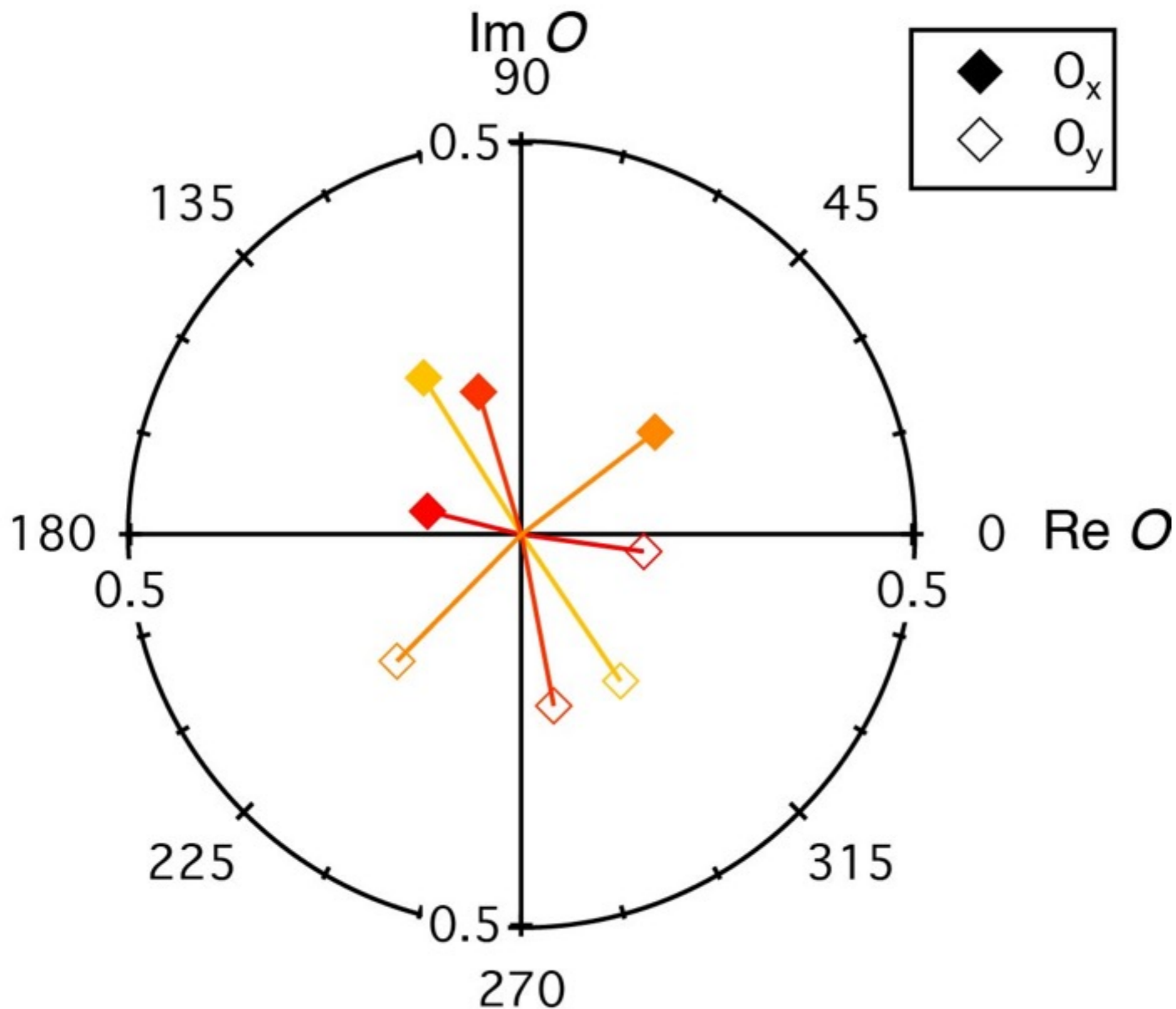
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

Complex value of  $O_x$  at a another  $q$  in the circle around  $2\pi(1/4, 0)$ .

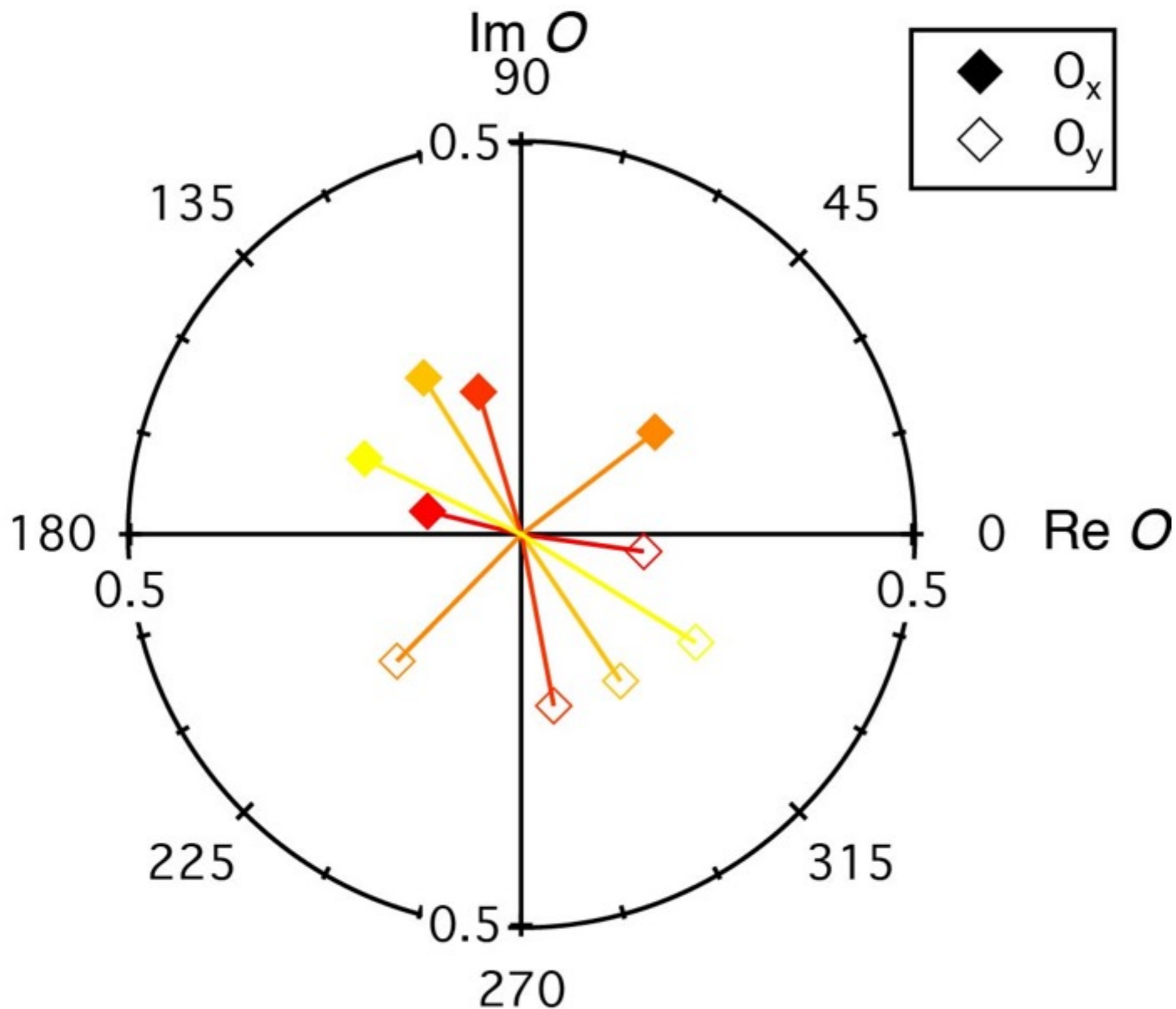
Complex value of  $O_y$  at same  $q$



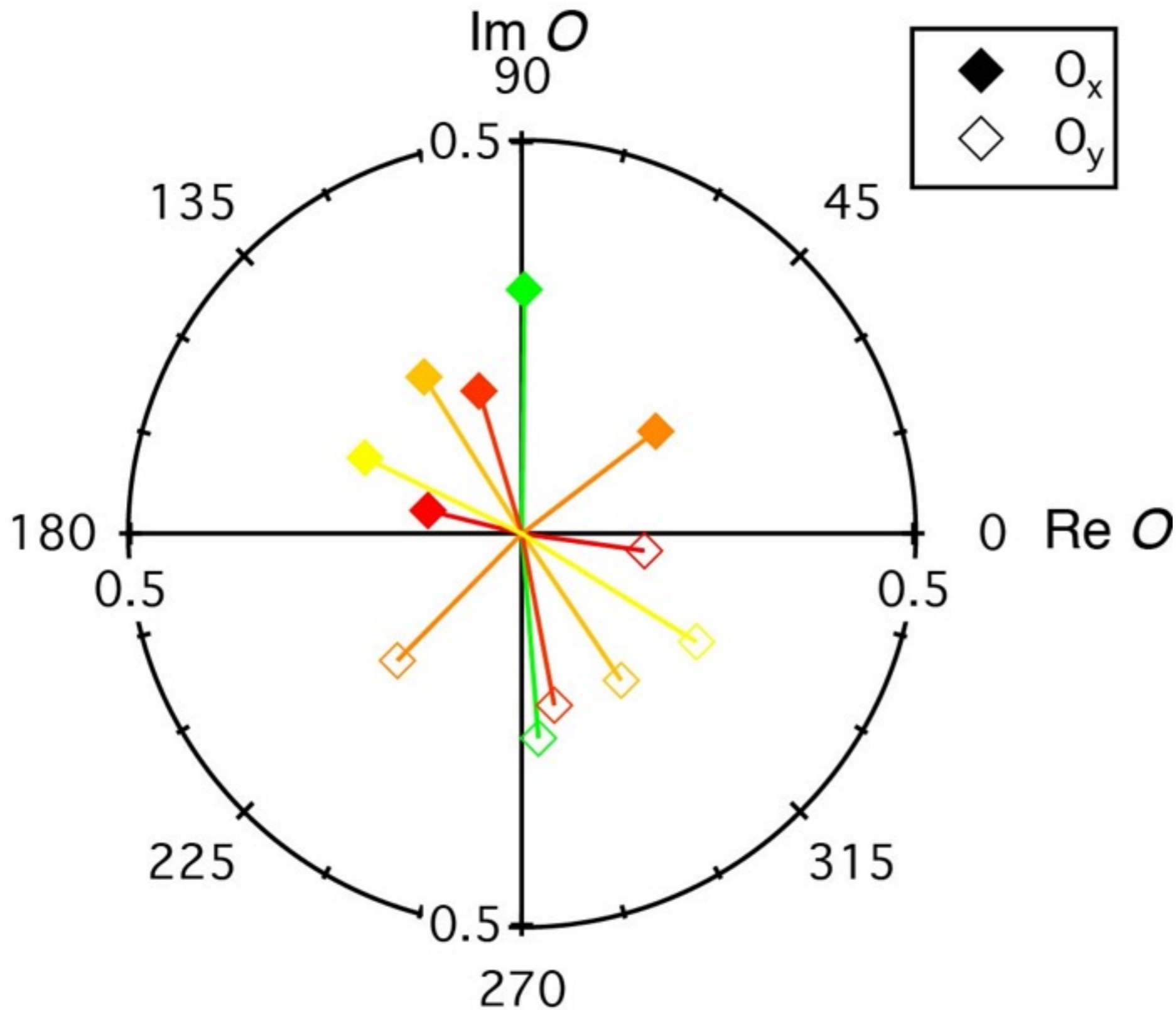
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



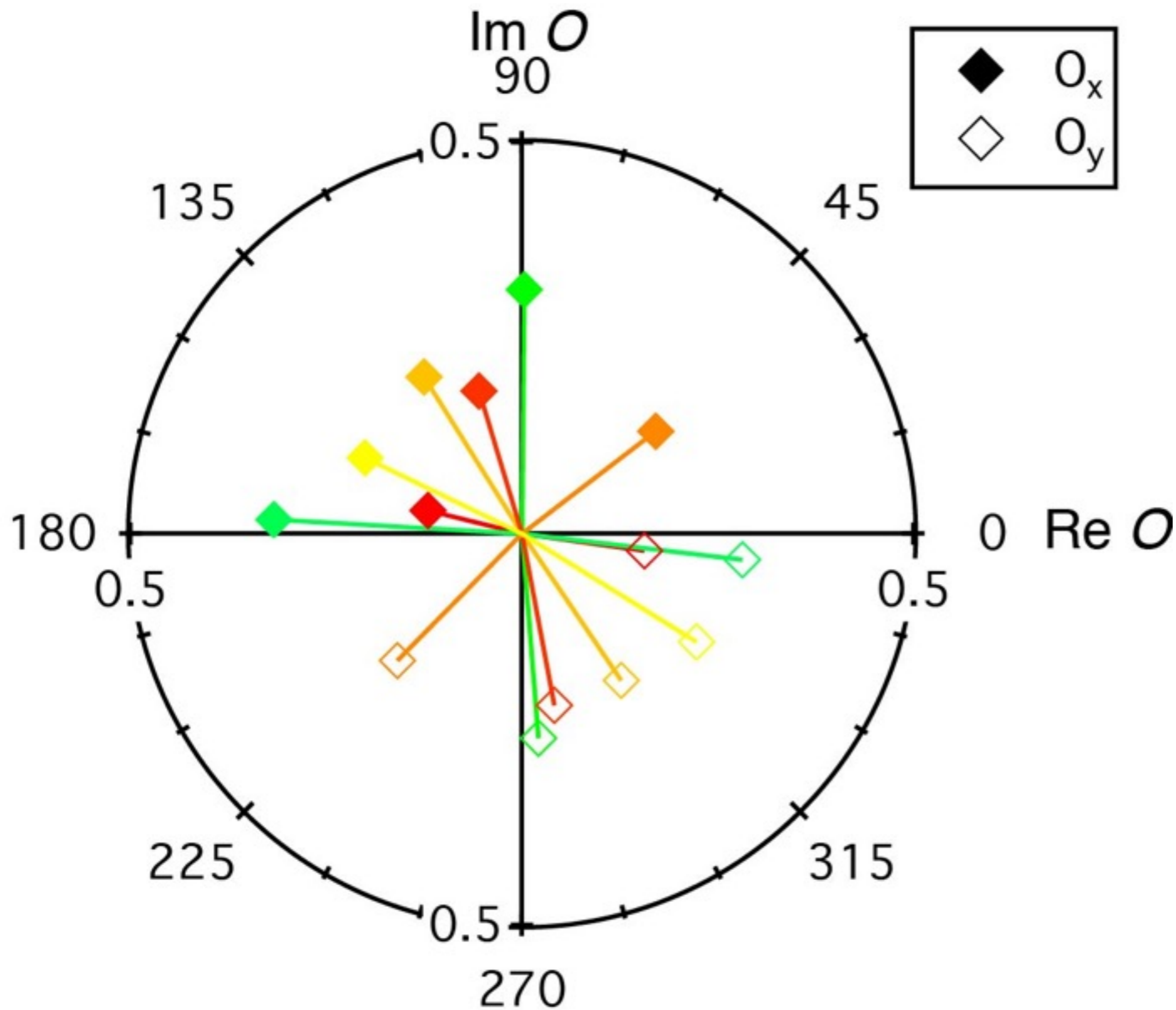
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



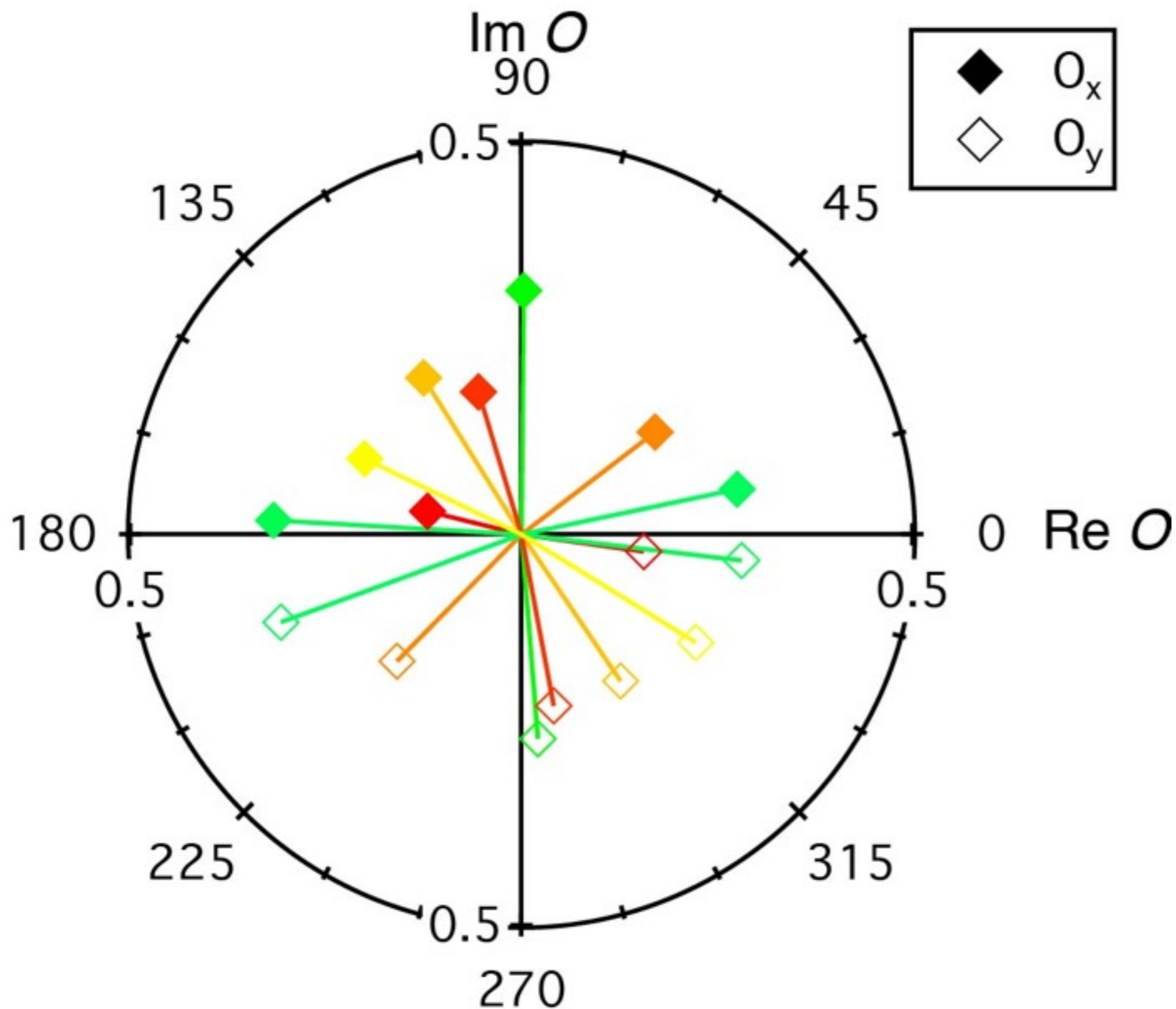
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



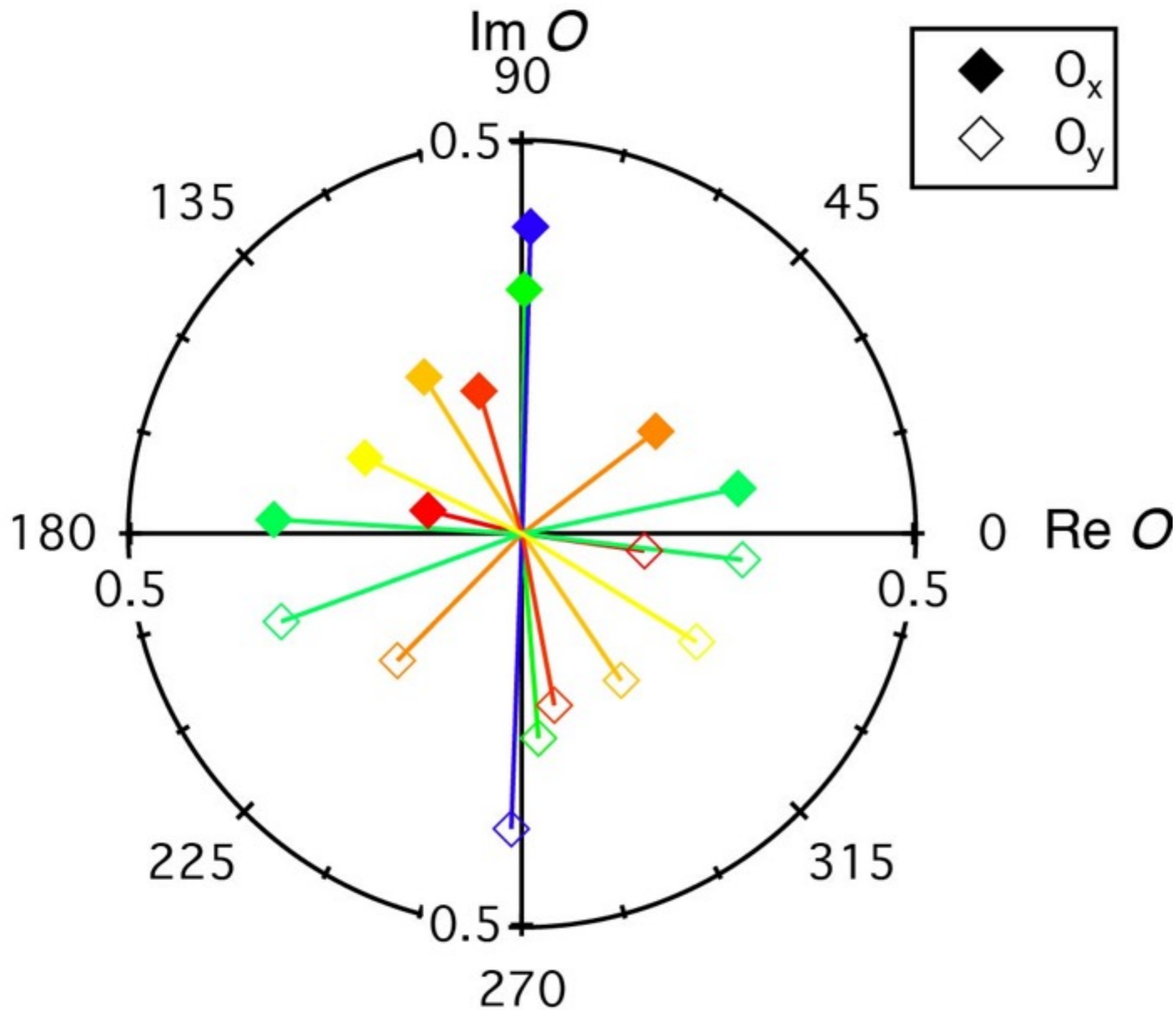
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



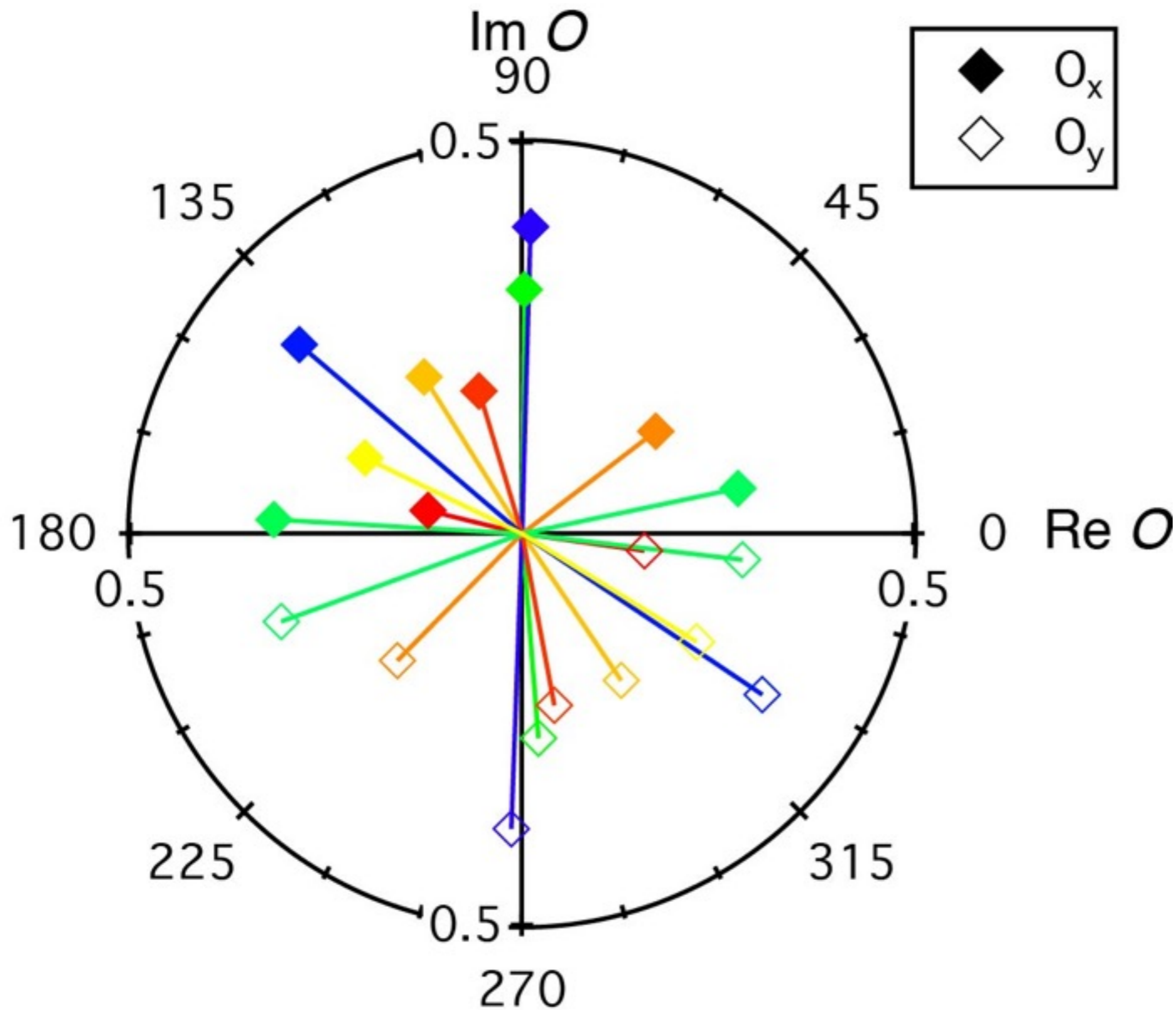
**Phase-sensitive measurement of the *d*-form factor of density wave order**



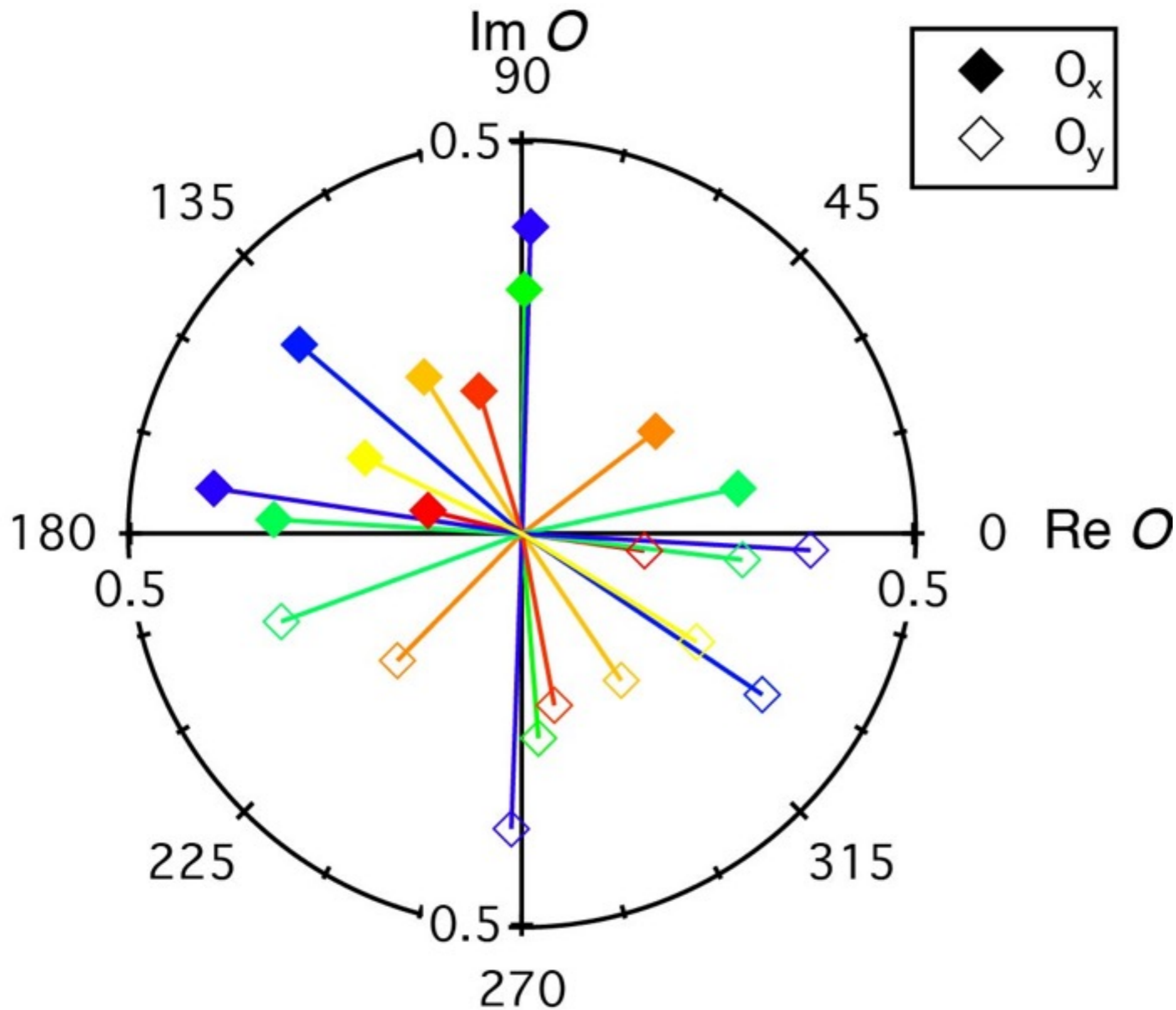
**Phase-sensitive  
measurement of  
the  $d$ -form factor  
of density wave  
order**



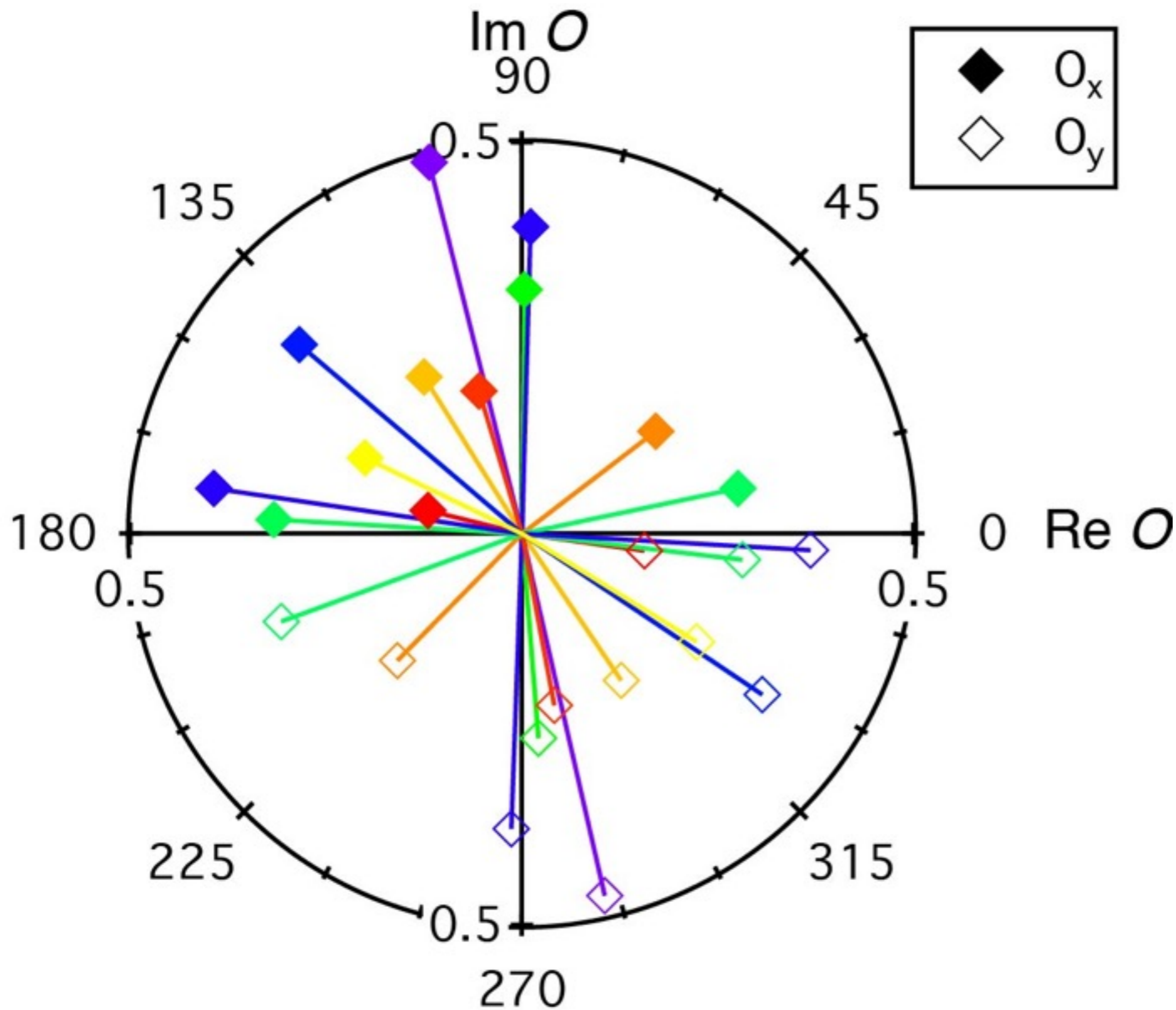
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



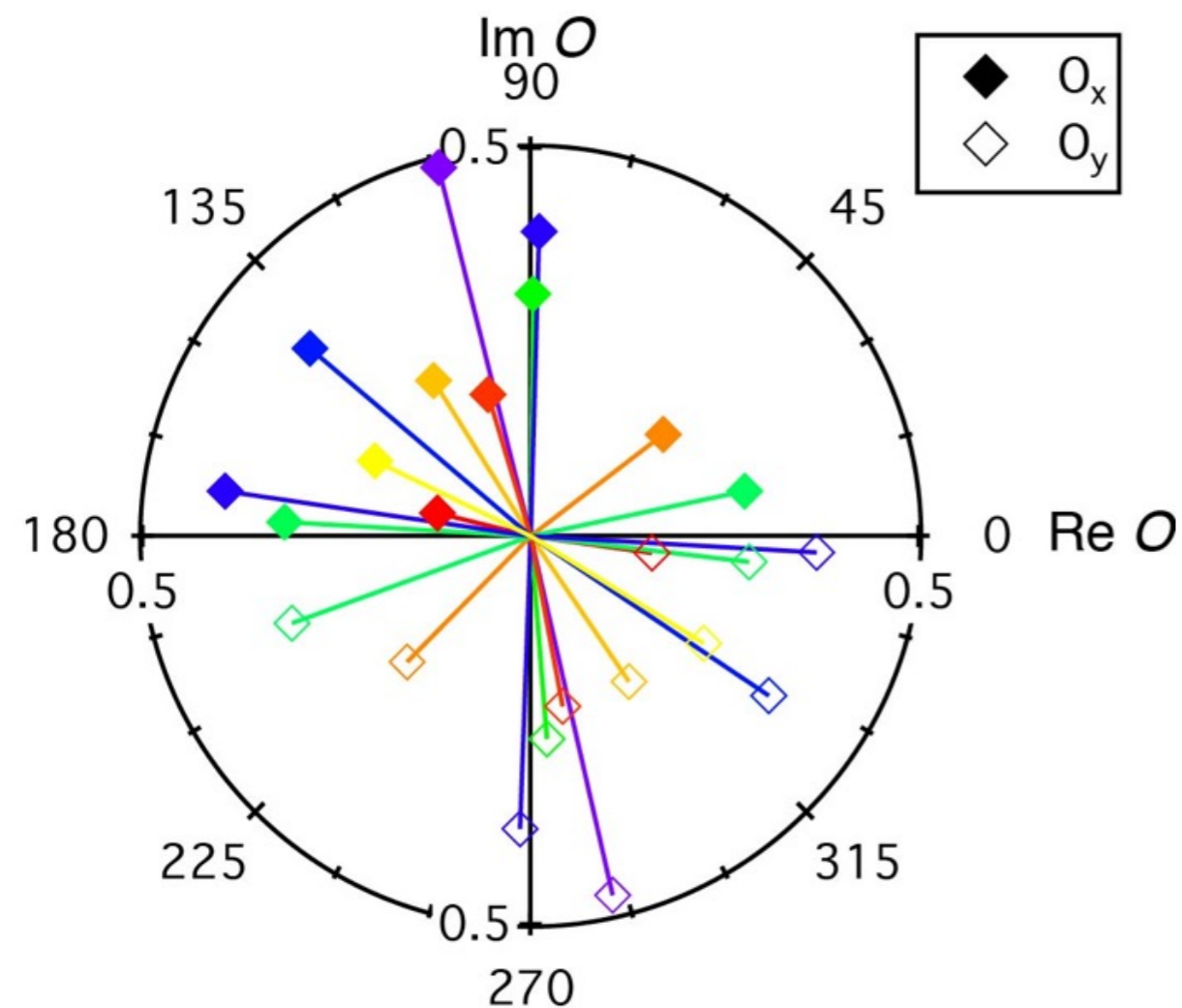
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



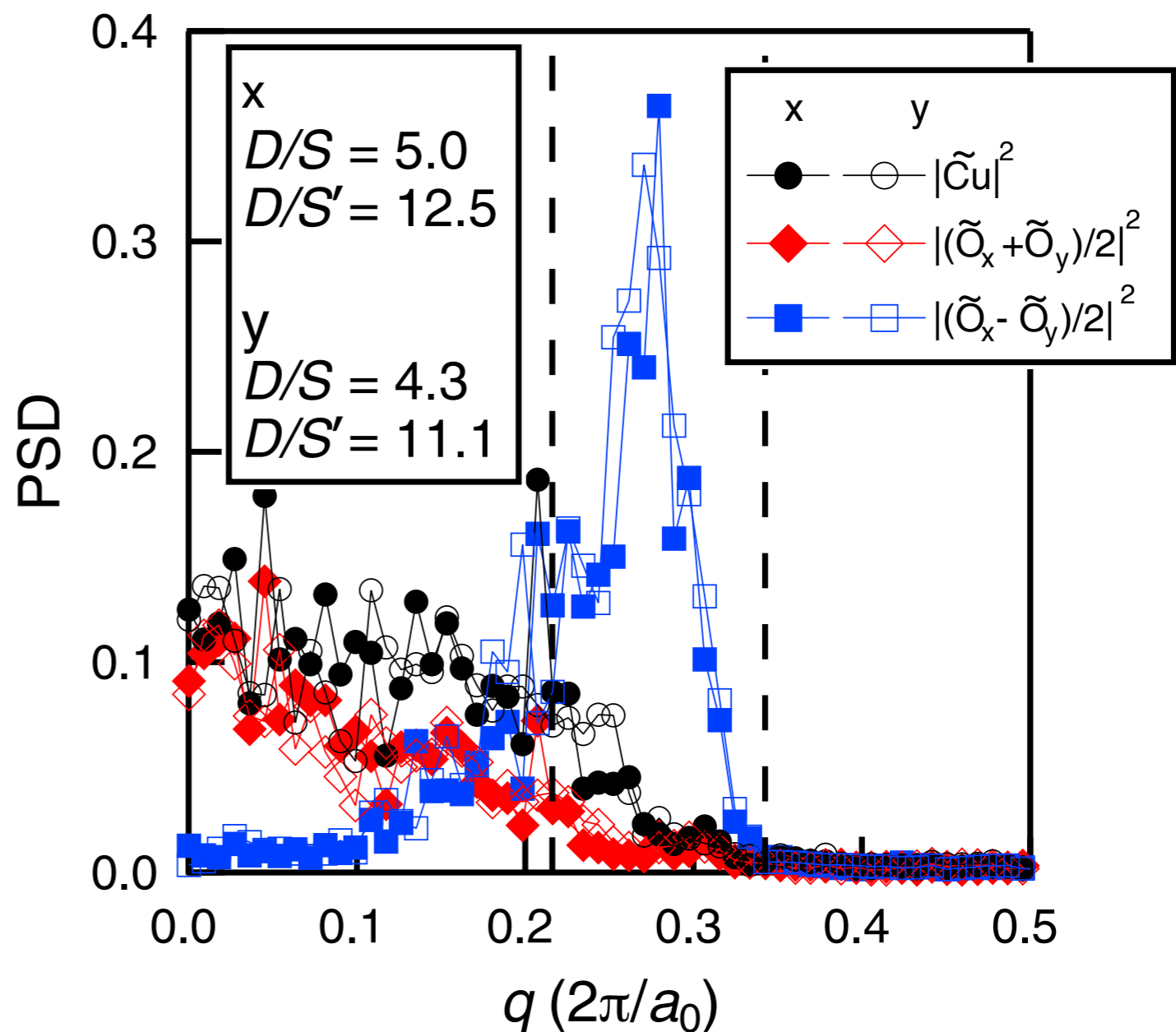
**Phase-sensitive measurement of the *d*-form factor of density wave order**



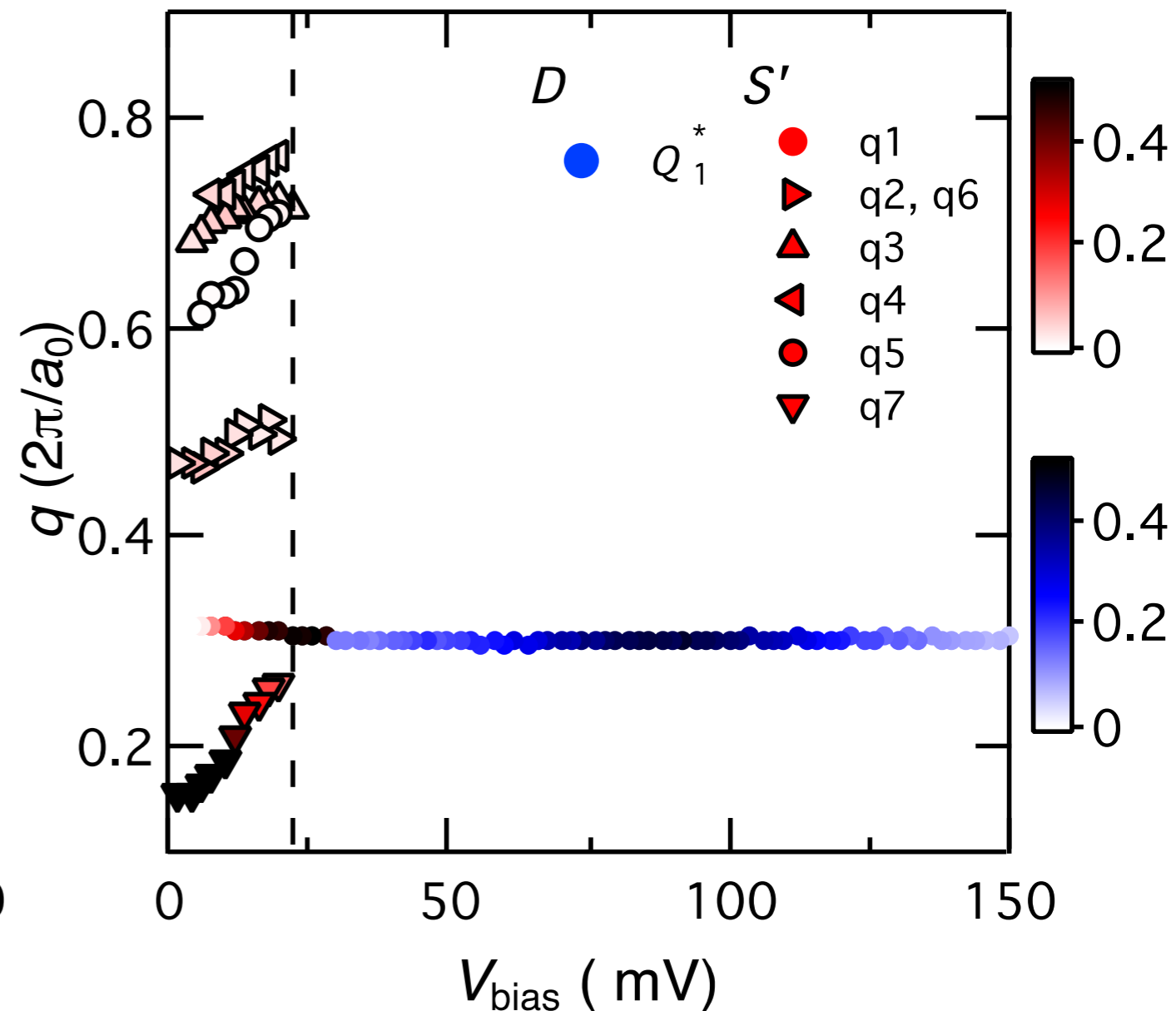
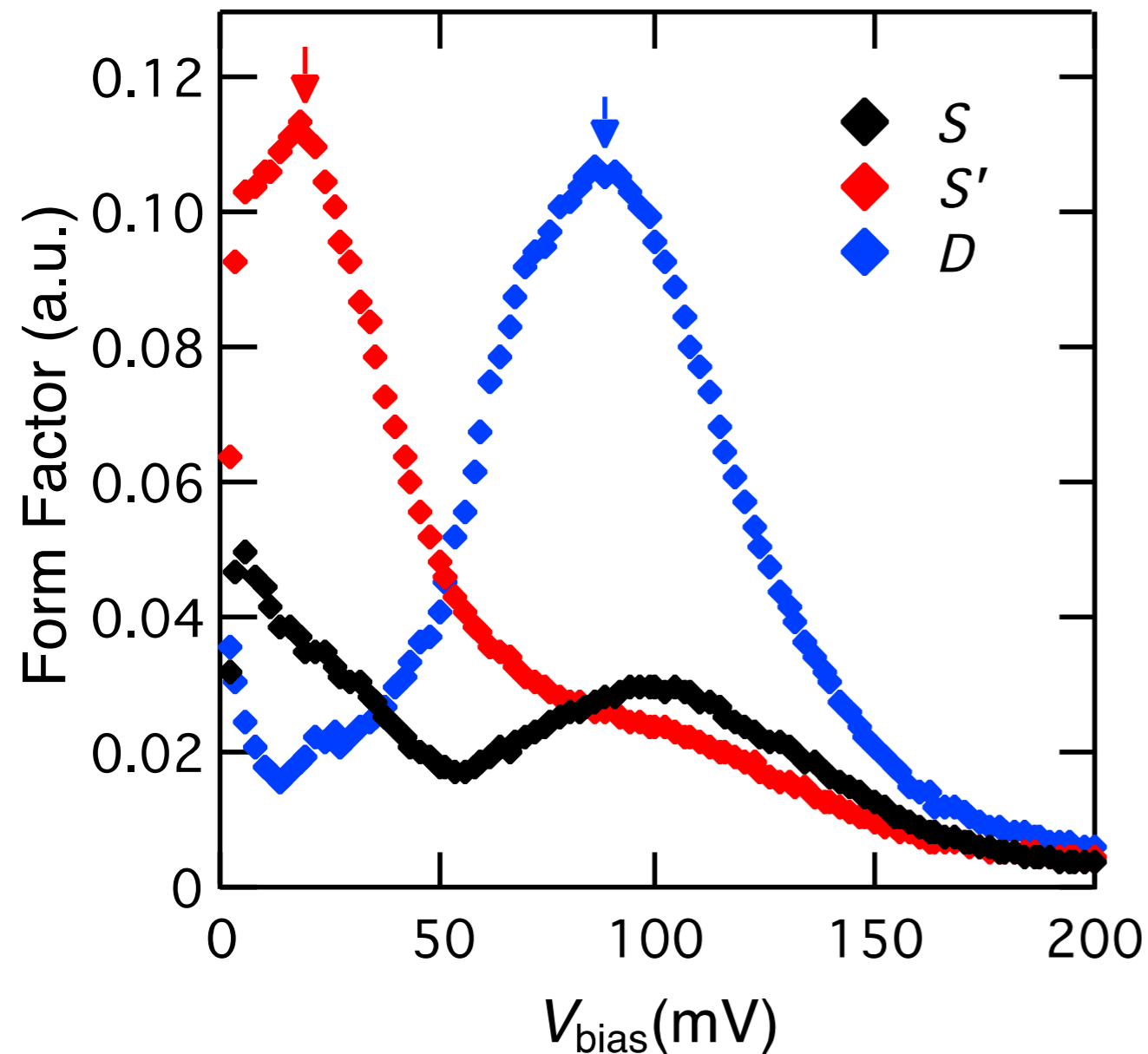
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



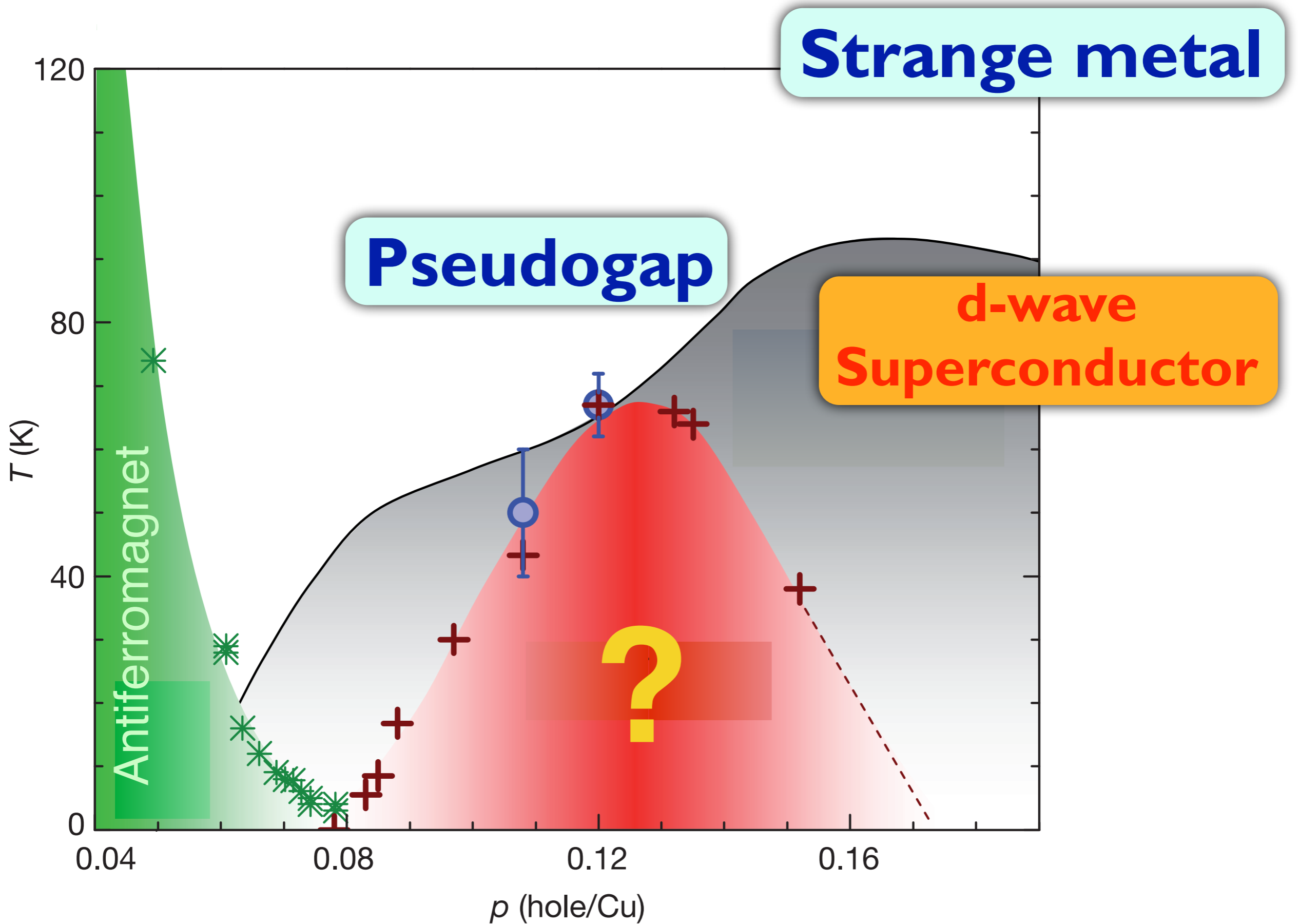
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

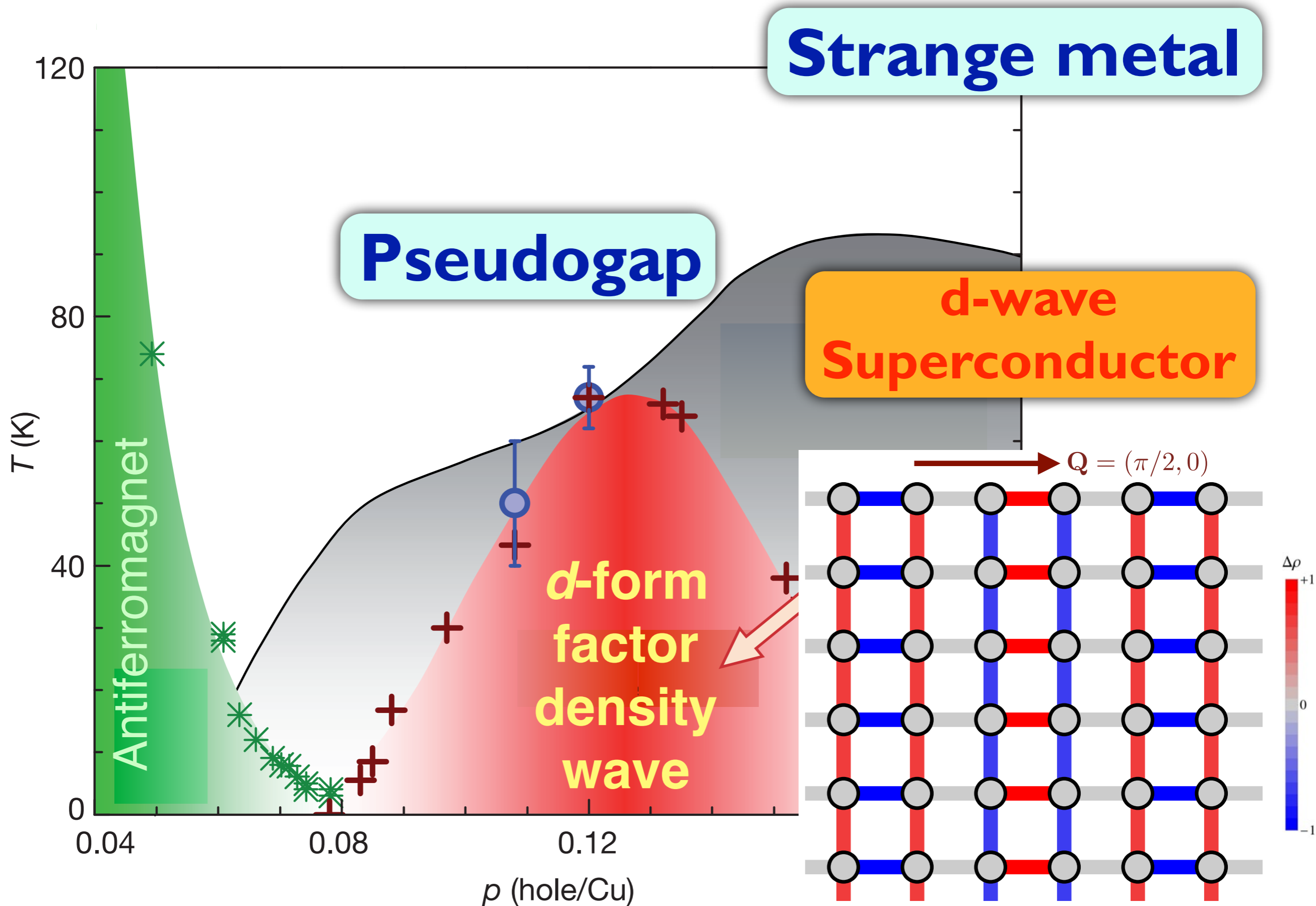


$d$ -form factor is peaked at the pseudogap energy, and does not disperse as a function of wavevector

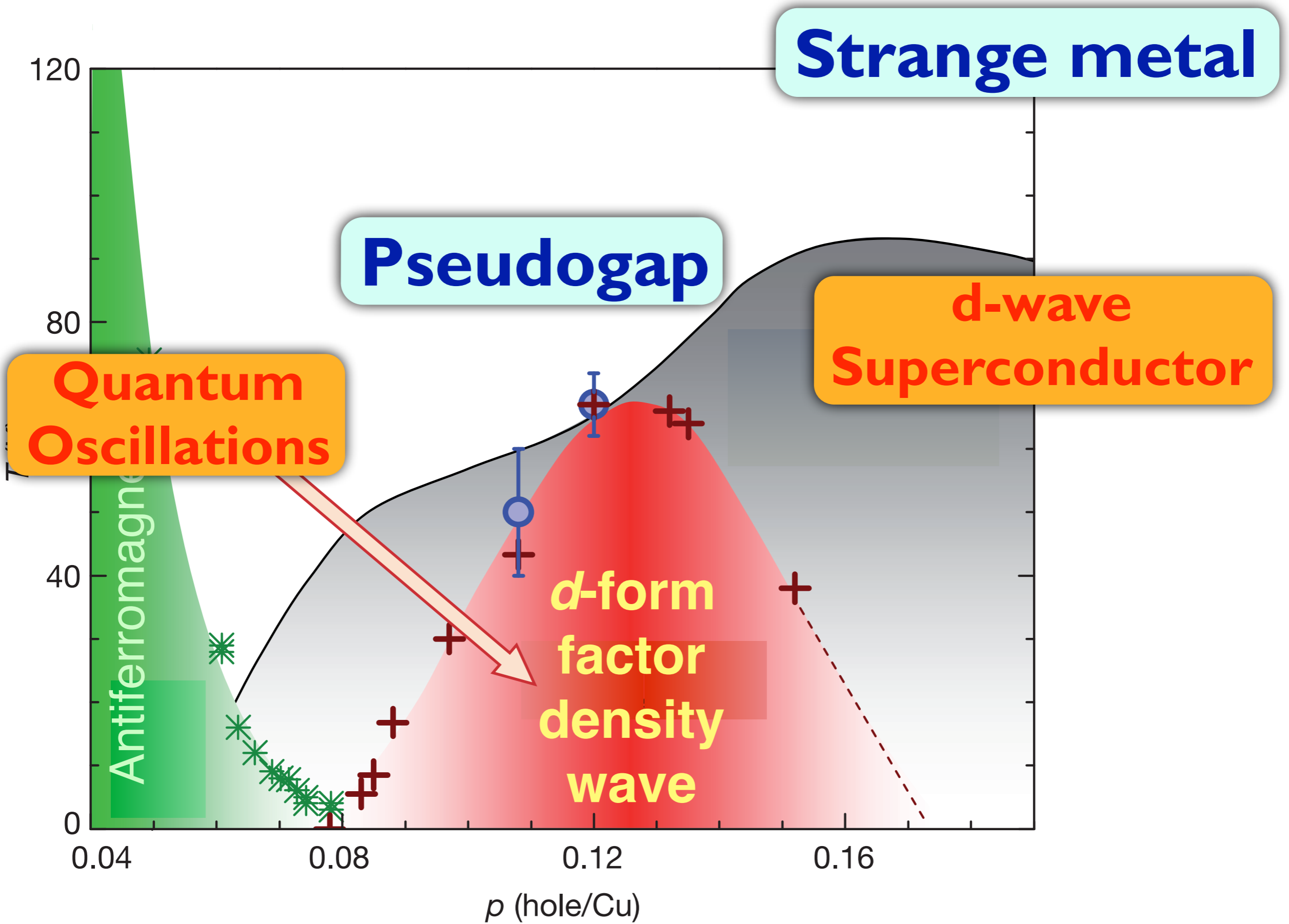


K. Fujita, M. H. Hamidian, S. D. Edkins, Chung Koo Kim, A. P. MacKenzie, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, to appear





K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)



K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

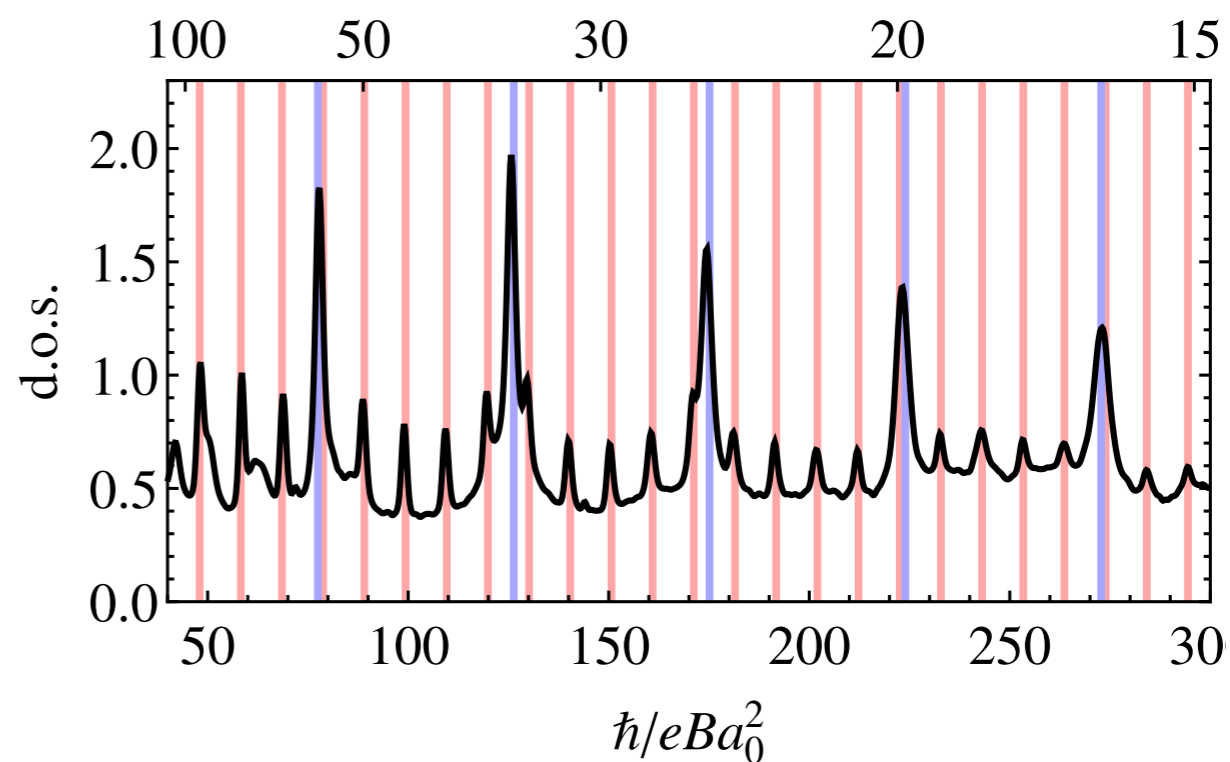
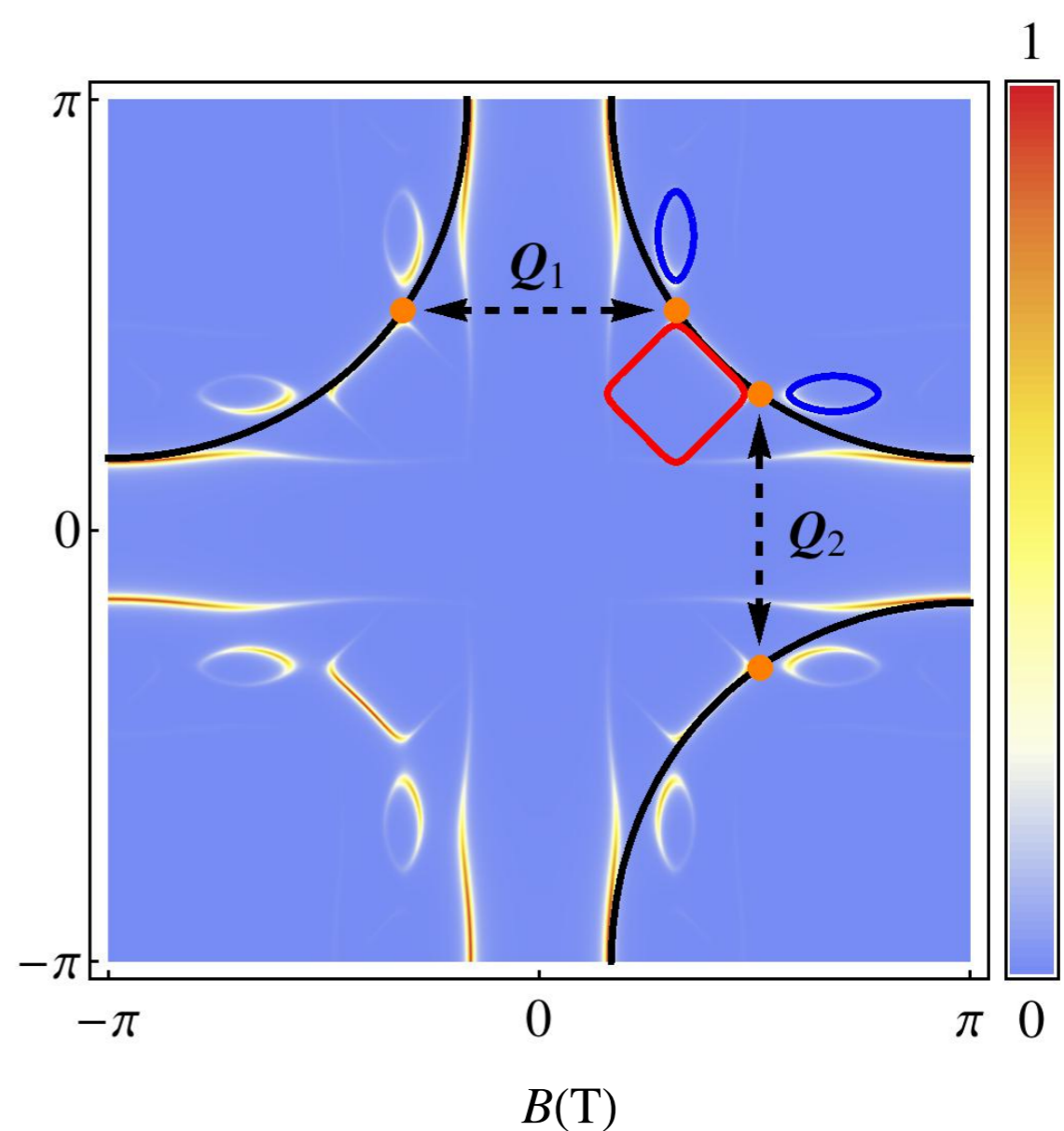


Andrea Allais  
(see poster)

# Electron spectral density and quantum oscillations from bi-directional density wave order

Electron pocket: 432 Tesla  
Hole pocket: 90.9 Tesla

Andrea Allais,  
Debanjan Chowdhury,  
and Subir Sachdev,  
arXiv:1406.0503



# Anisotropic charge order

Andrea Allais, Debanjan Chowdhury,  
and Subir Sachdev, arXiv:1406.0503

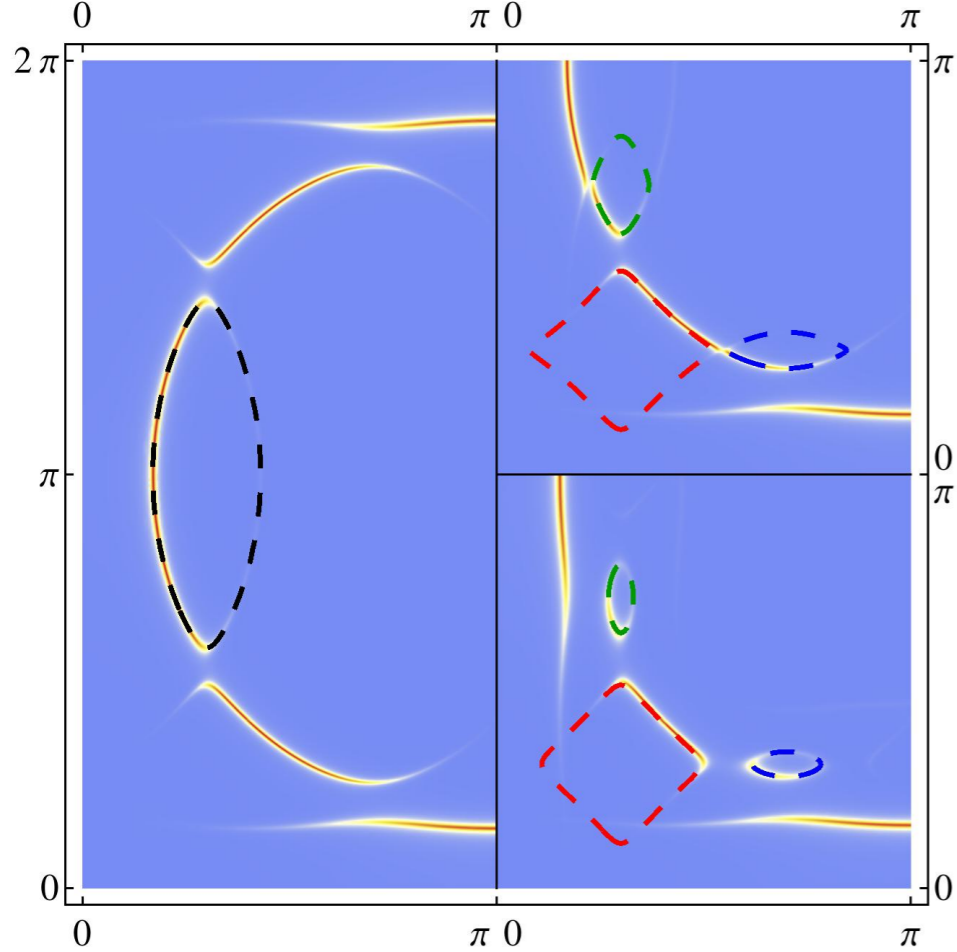
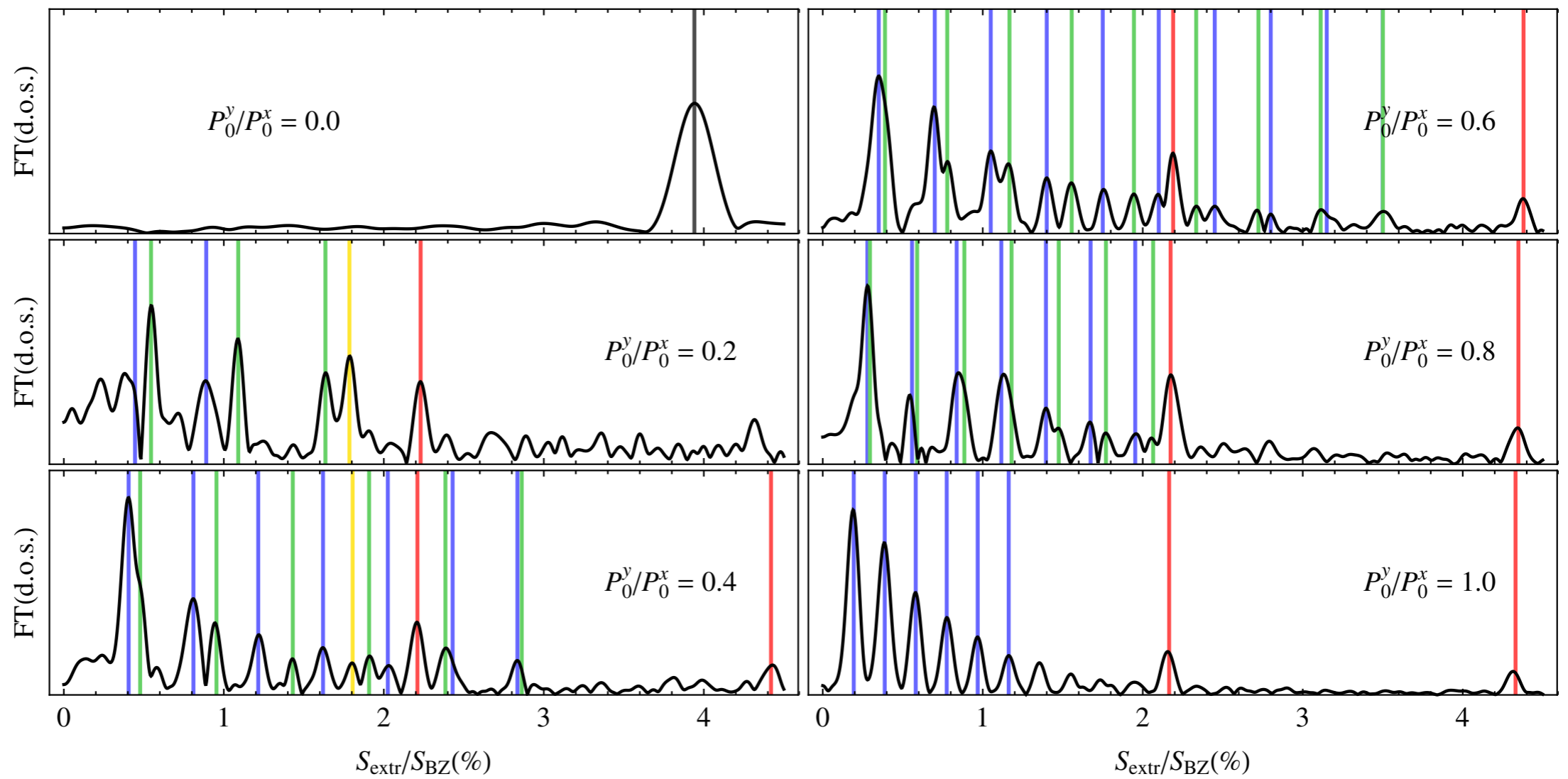
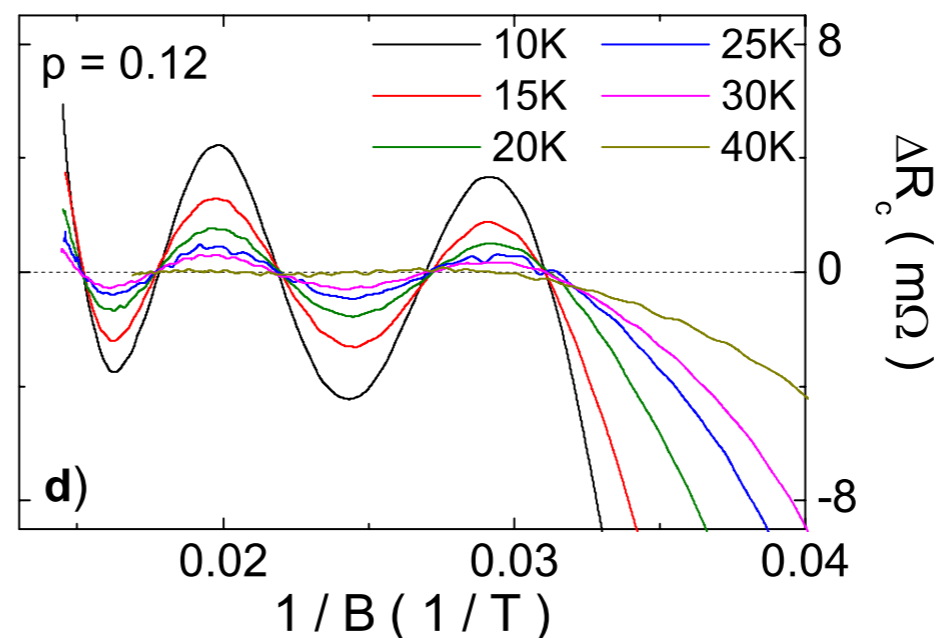
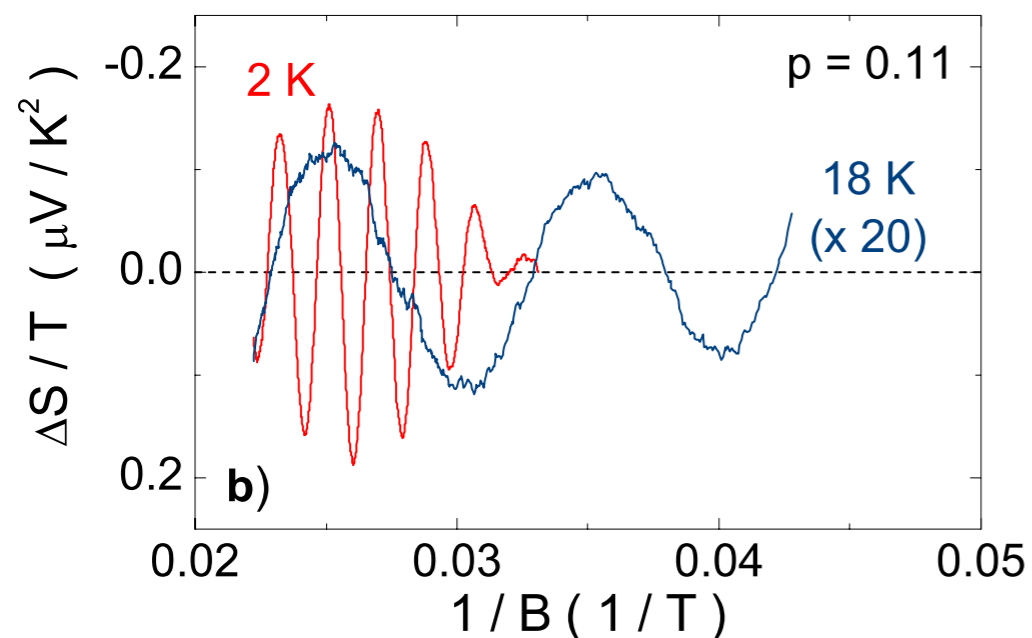
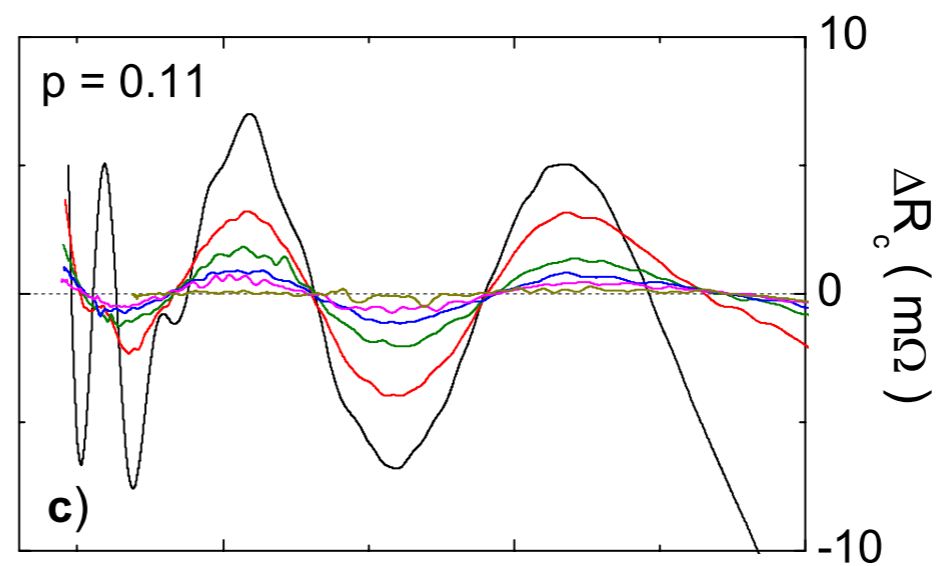
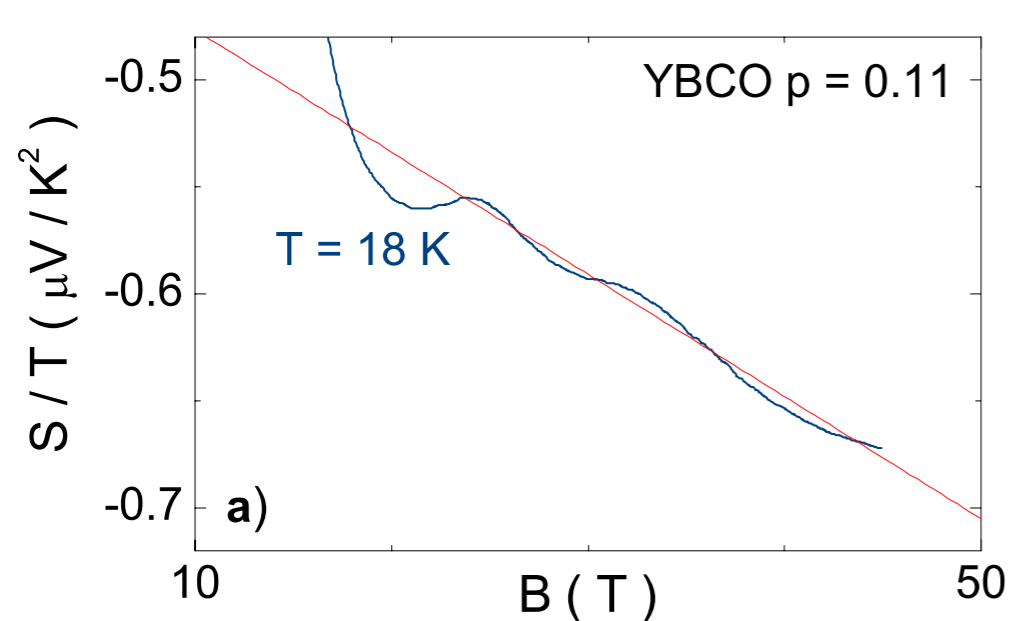


FIG. 3. **Evolution of  $A(\mathbf{k}, \omega = 0)$  with increasingly isotropic order.** Electron spectral function with resonant orbits in evidence, as the order transitions from stripe to checkerboard: stripe order ( $P_0^y/P_0^x = 0$ ) on the left, anisotropic checkerboard ( $P_0^y/P_0^x = 0.2$ ) top right, isotropic checkerboard ( $P_0^y/P_0^x = 1$ ) bottom right.  $p = 11\%$ ,  $P_0^x = 0.15$ ,  $\delta = 0.3$ .



# Evidence for a small hole pocket in the Fermi surface of underdoped $\text{YBa}_2\text{Cu}_3\text{O}_y$

N. Doiron-Leyraud, S. Badoux, S. René de Cotret, S. Lepault, D. LeBoeuf, F. Laliberté, E. Hassinger, B. J. Ramshaw, D. A. Bonn, W. N. Hardy, R. Liang, J.-H. Park, D. Vignolles, B. Vignolle, L. Taillefer, & C. Proust, arXiv:1409.2788



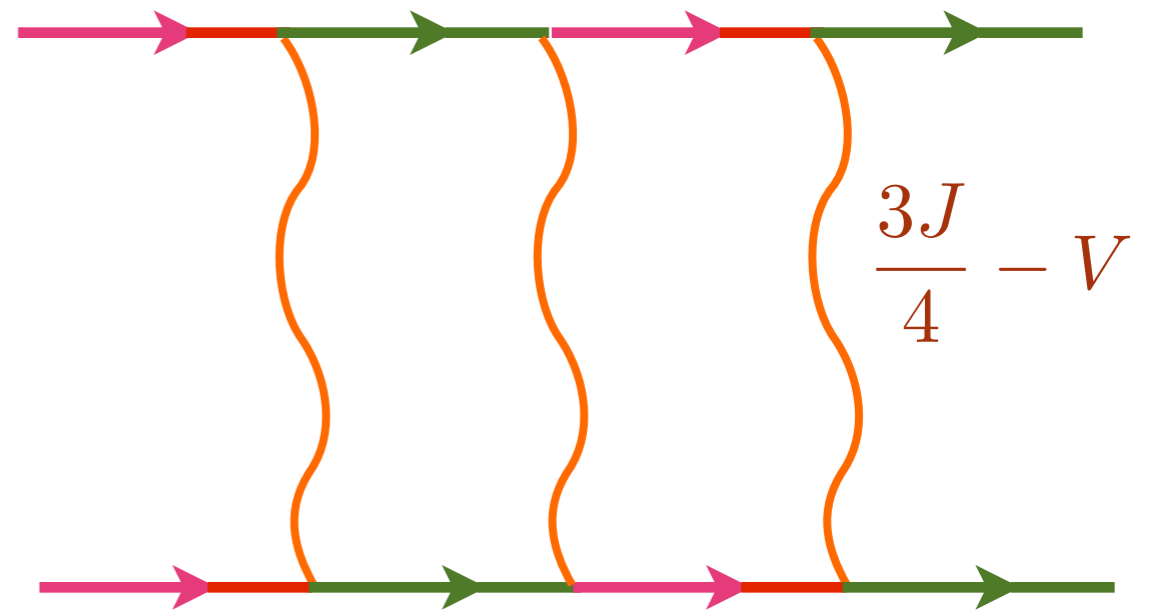
Hole pocket  
 $\approx 100T$



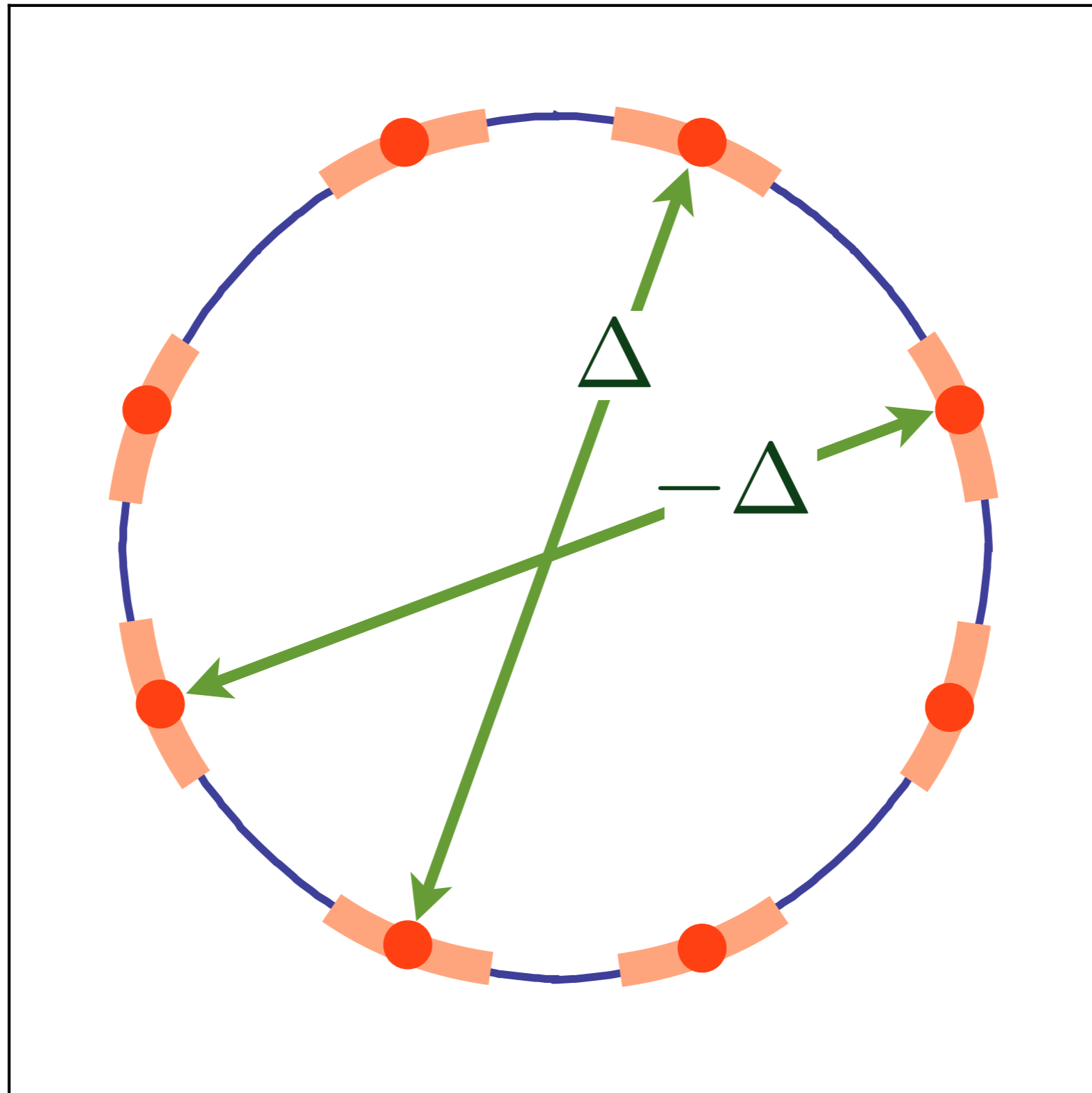
Theory

# Pairing “glue” from antiferromagnetic fluctuations

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$



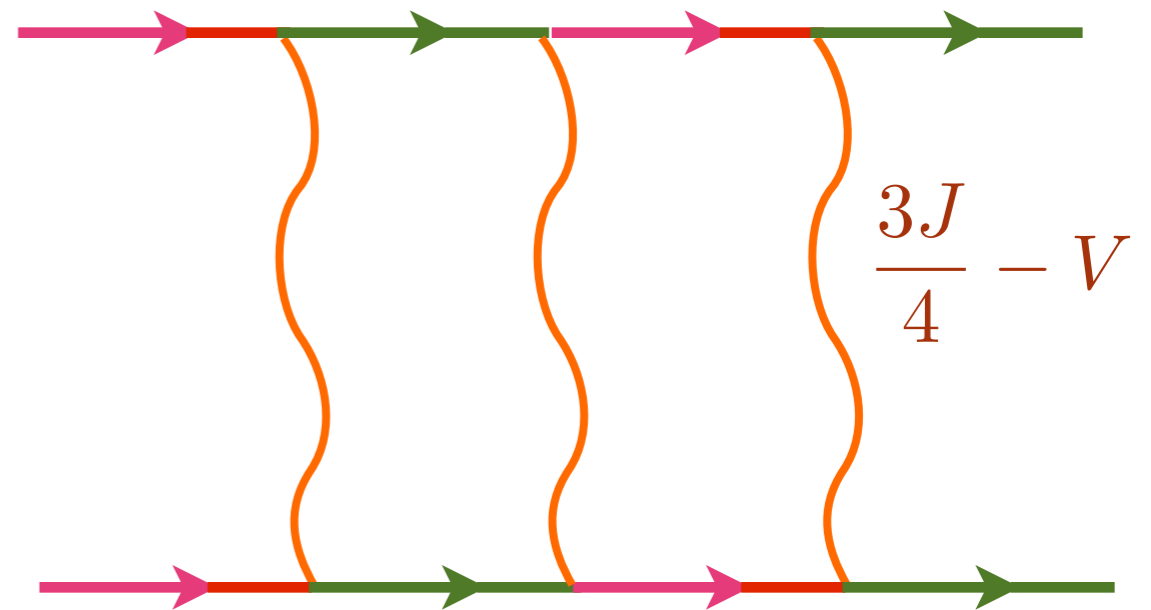
$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



Unconventional pairing at and near hot spots

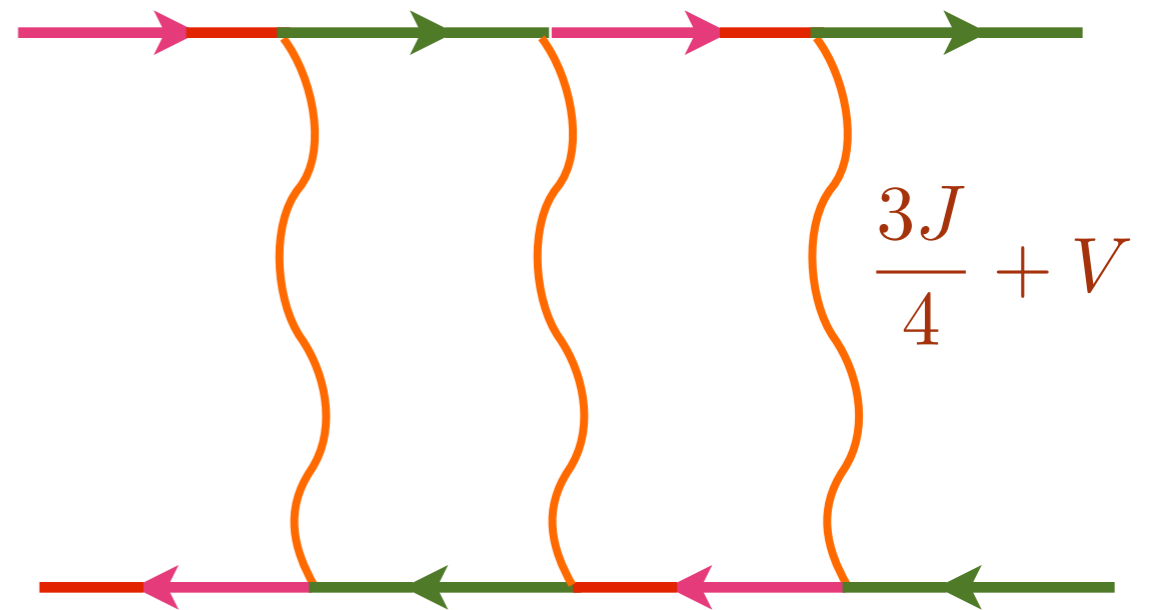
# Pairing “glue” from antiferromagnetic fluctuations

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$



# Same “glue” can lead to particle-hole pairing

$$H = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + V \sum_{\langle ij \rangle} n_i n_j + \dots$$

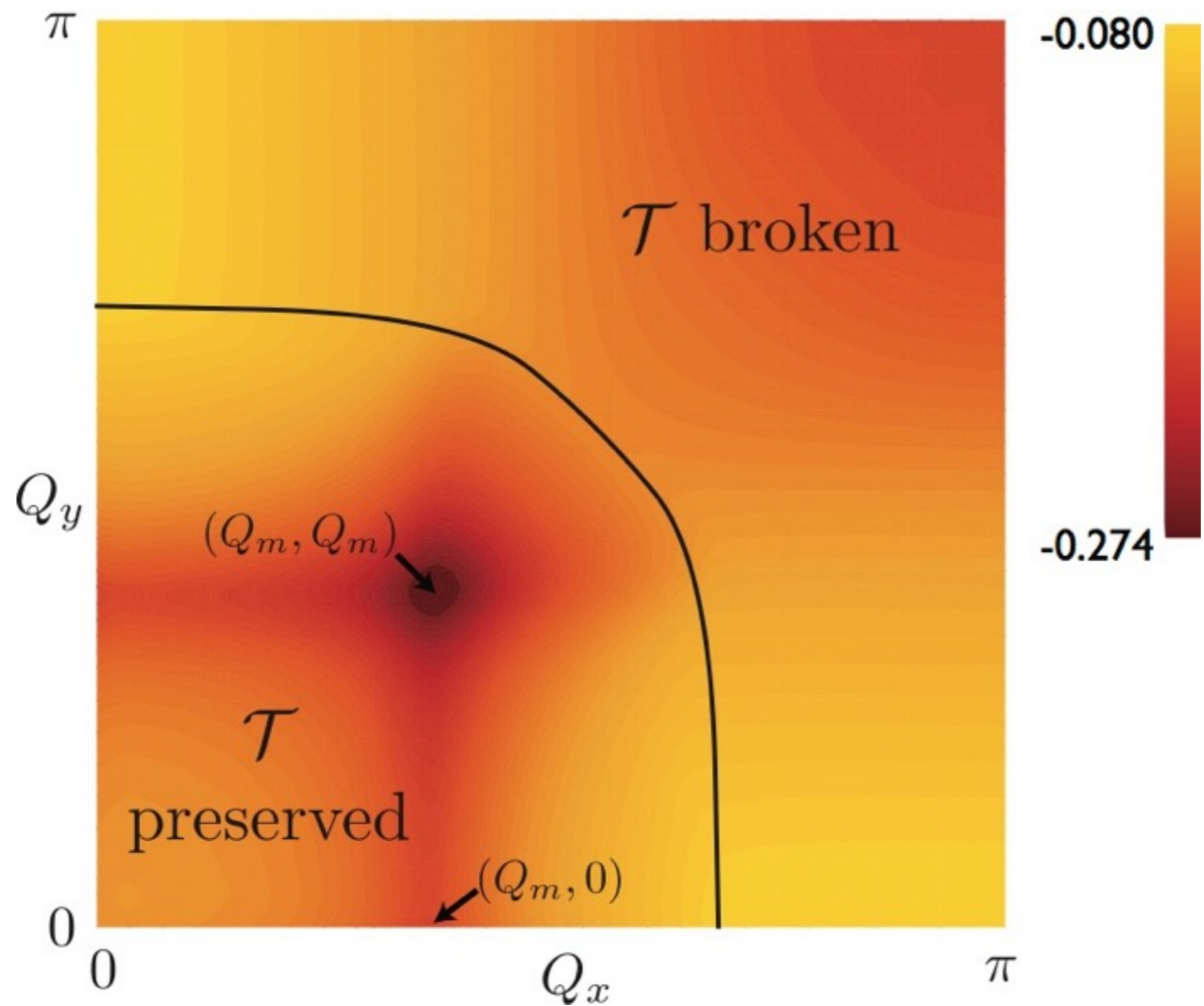


M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

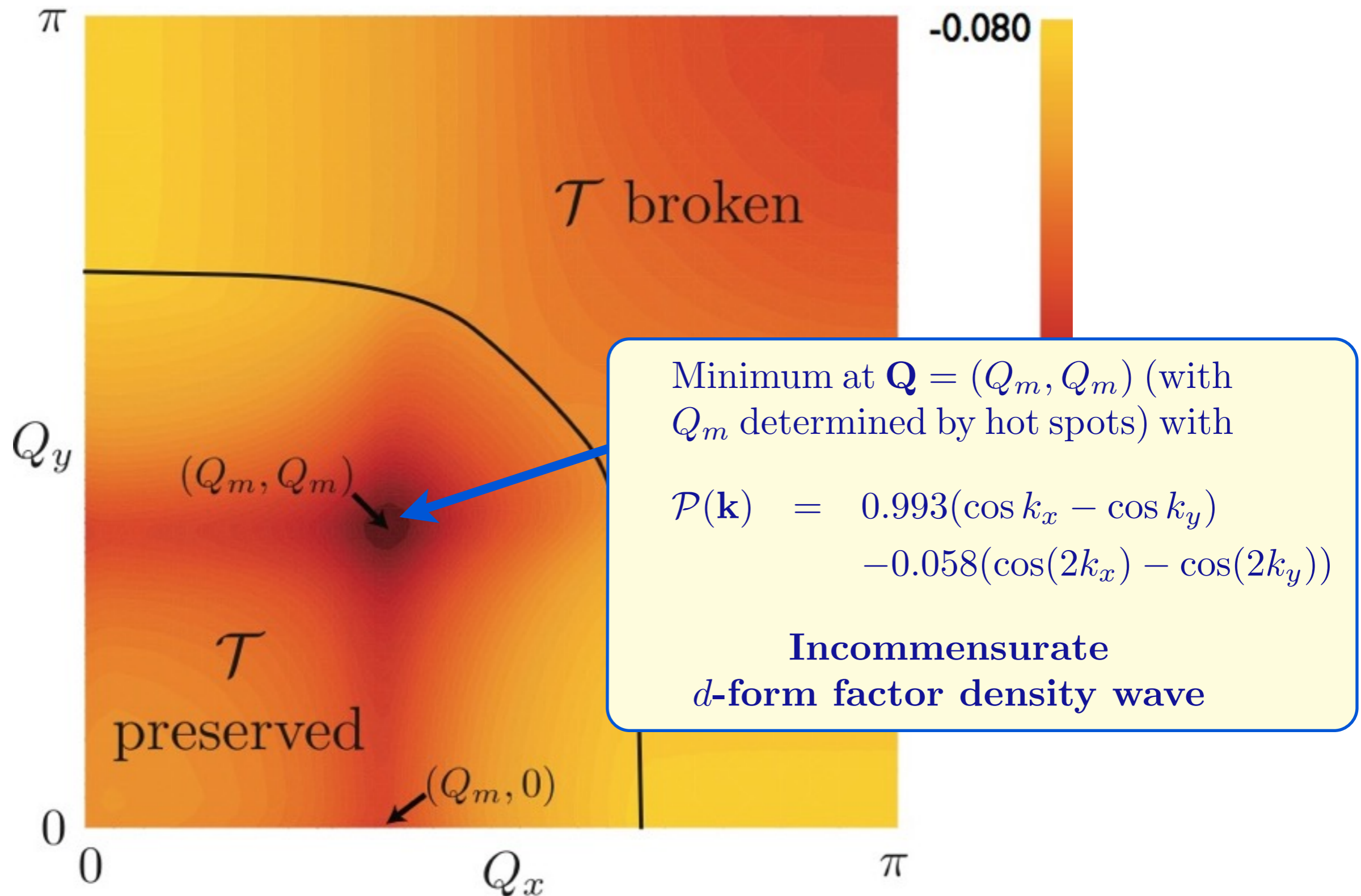
T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)

M. Bejas, A. Greco, and H. Yamase, Phys. Rev. B **86**, 224509 (2012)

S. Sachdev and R. La Placa, Phys. Rev. Lett. **111**, 027202 (2013)

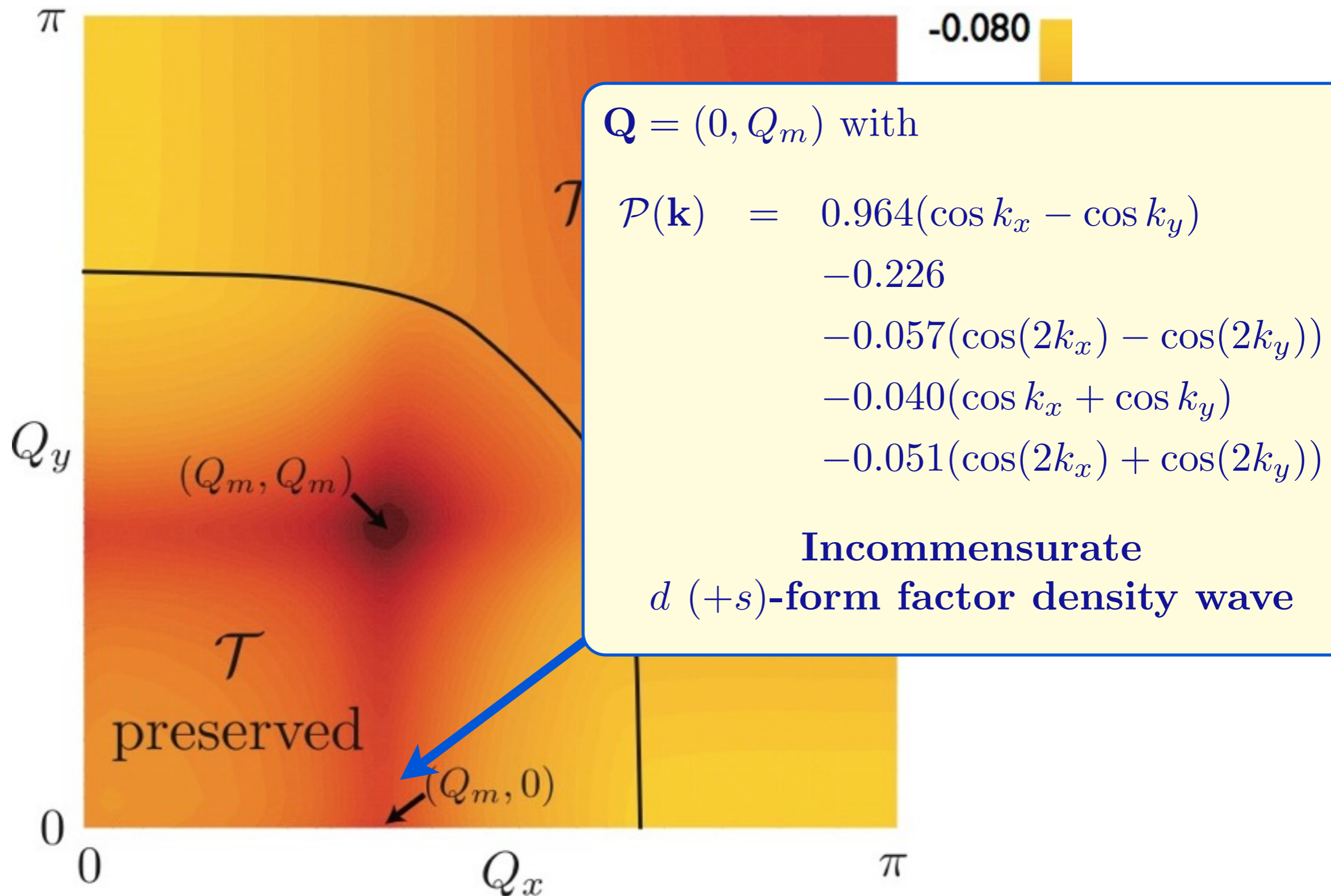


Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order  $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$



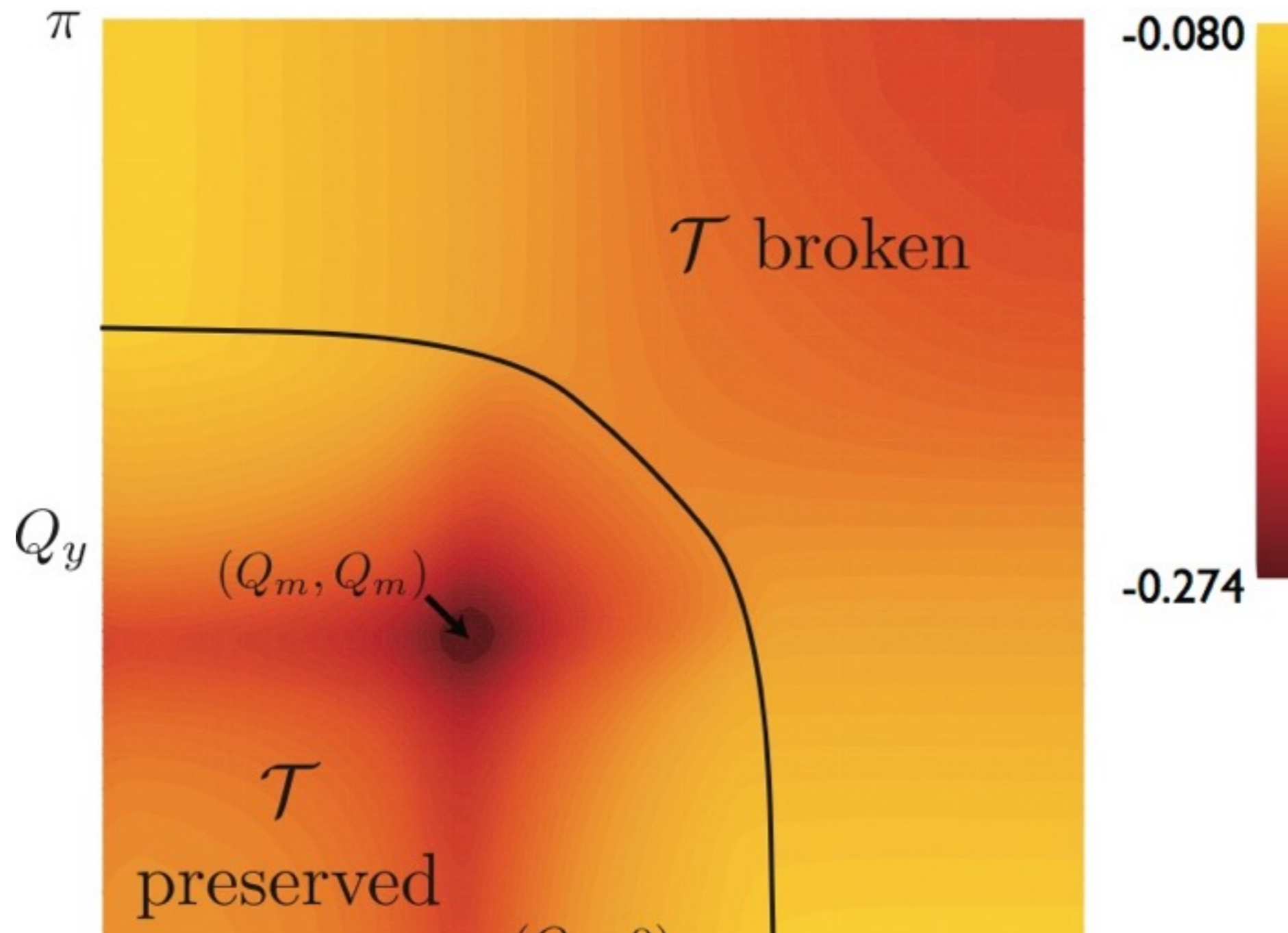
Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

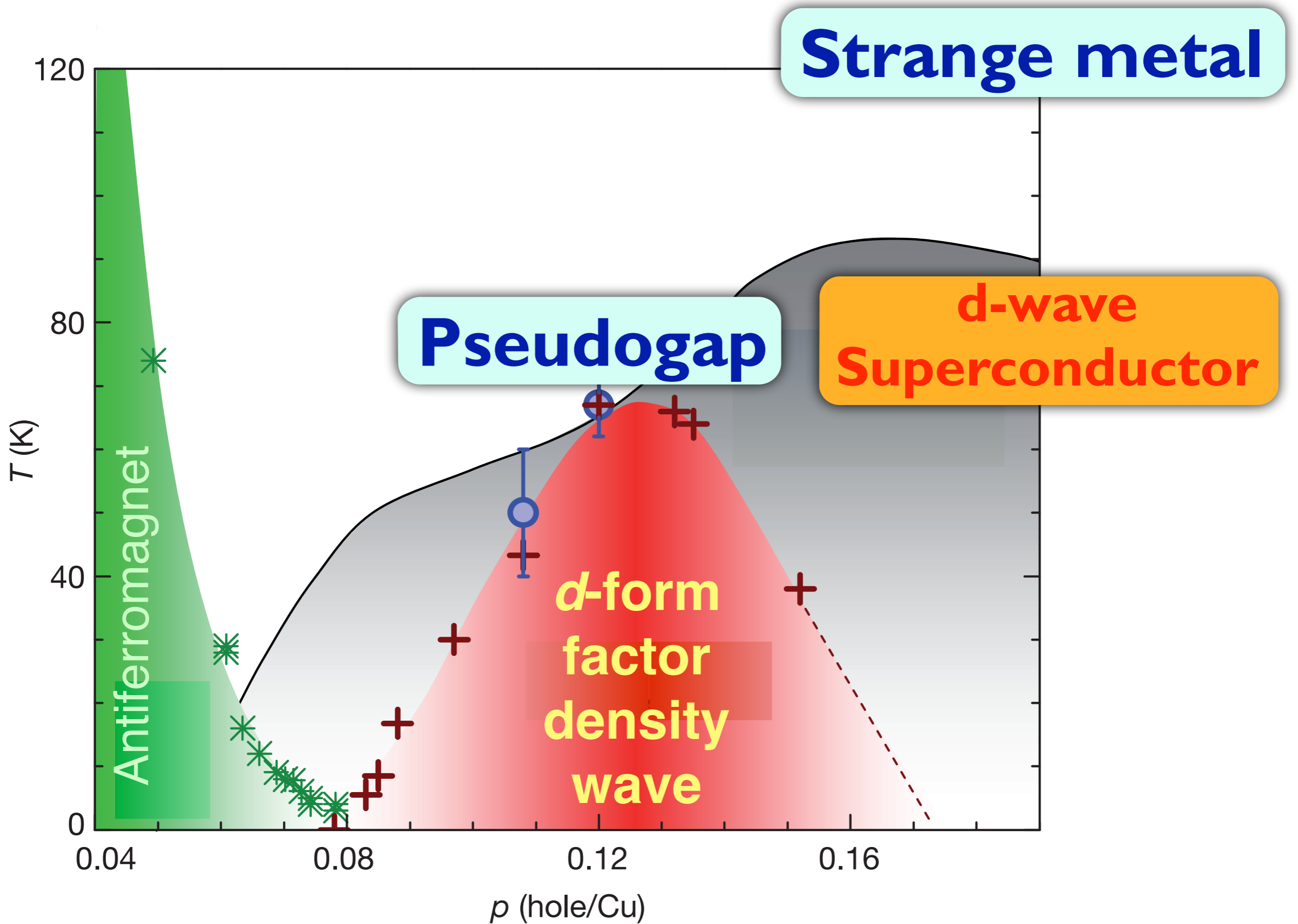


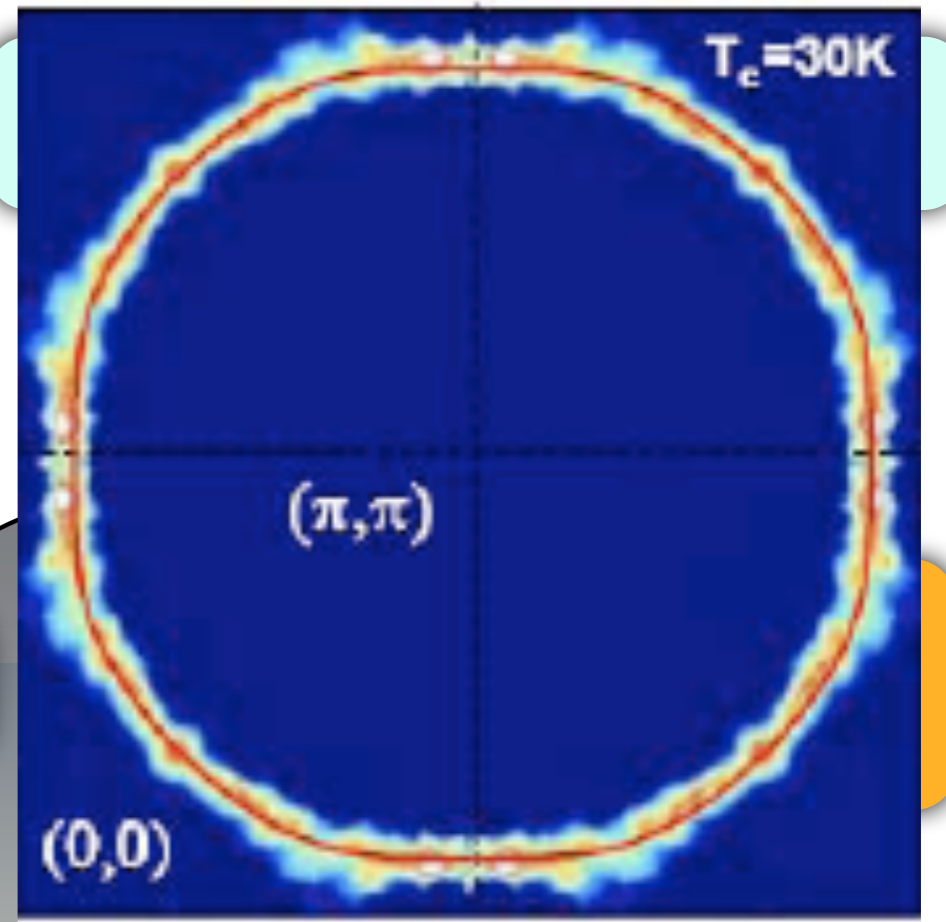
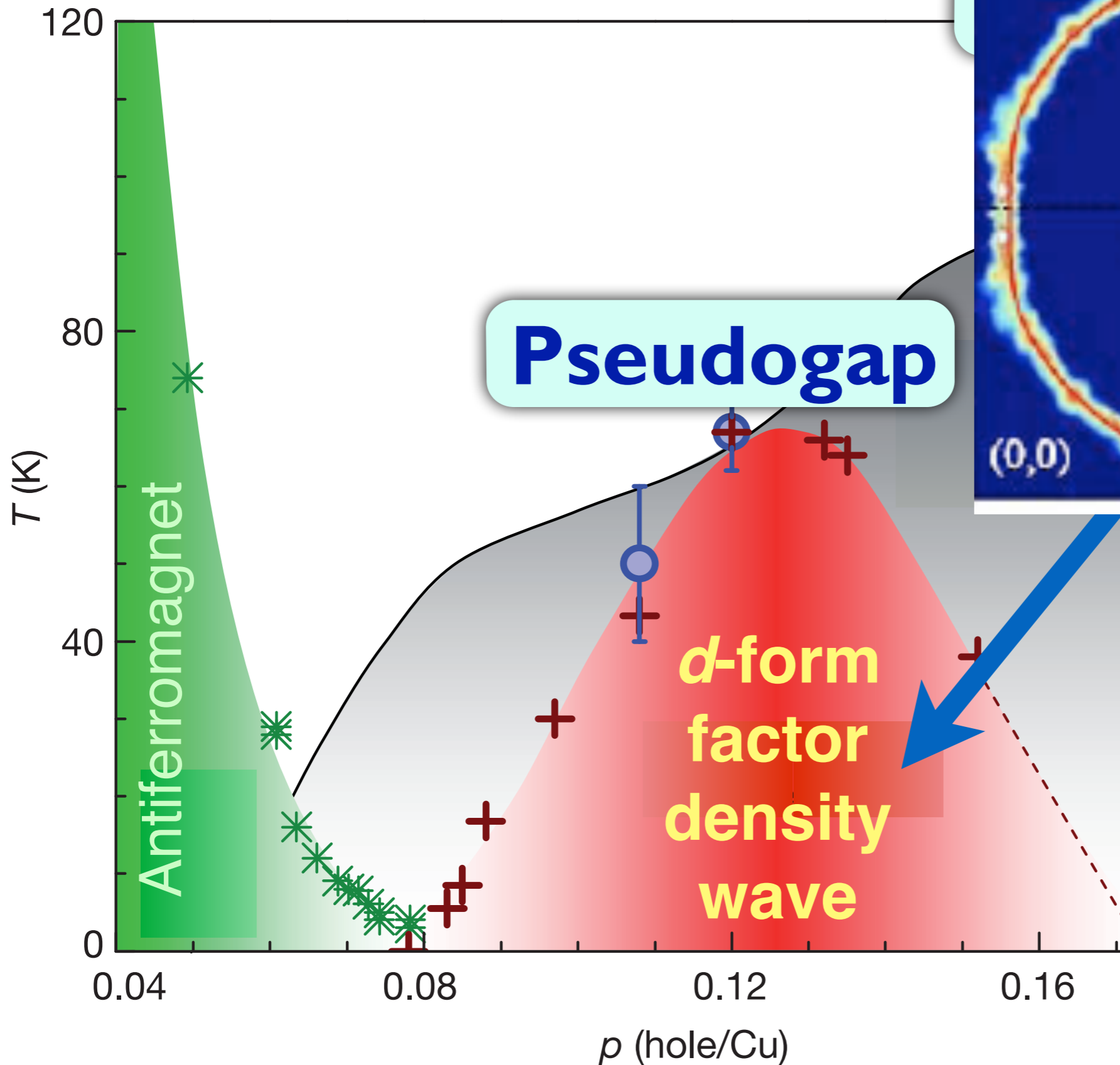
Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

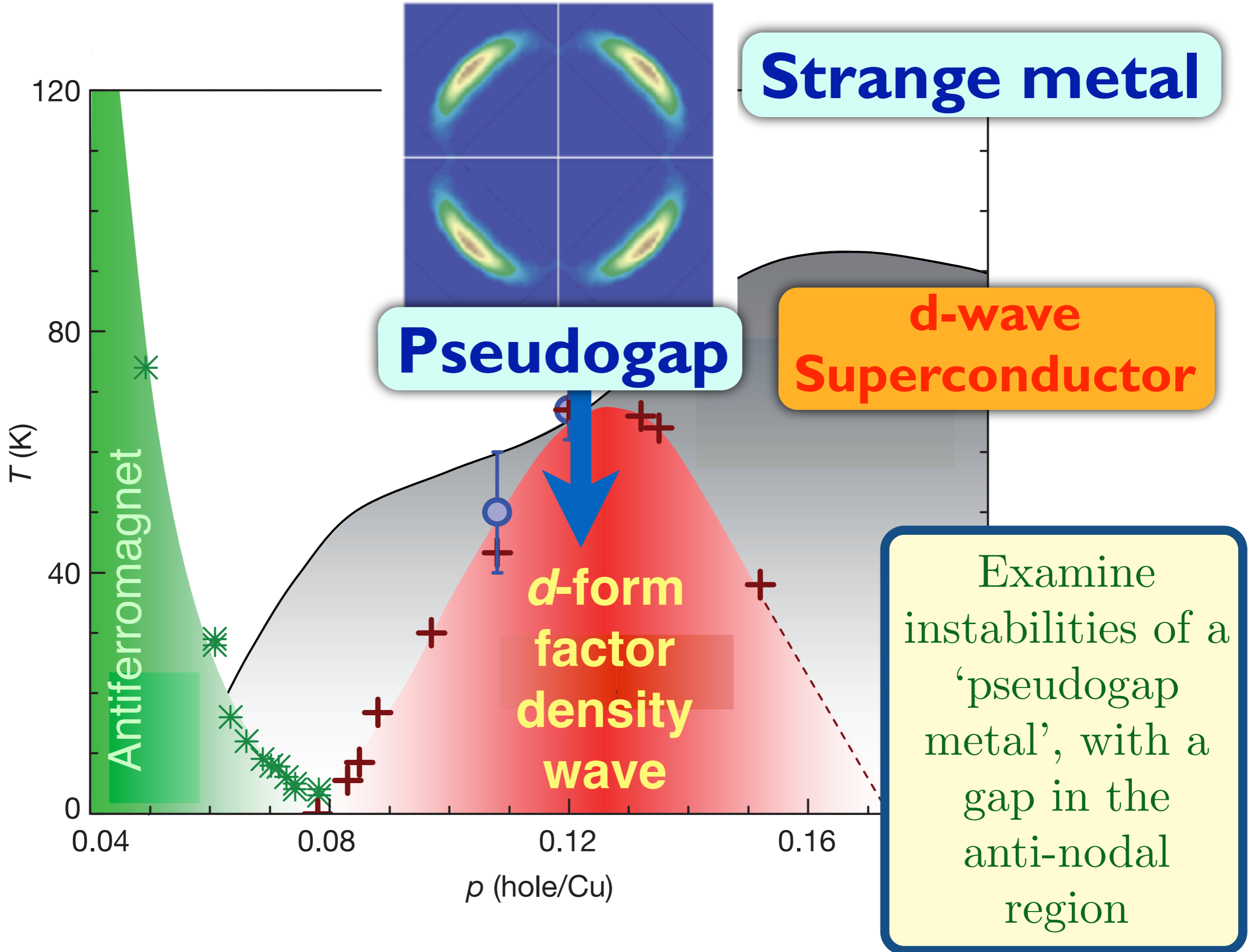


This theory yields the correct form factor, but the incorrect  $\mathbf{Q}$ . We have found the same features in all theories (one- and three-band) with a large Fermi surface

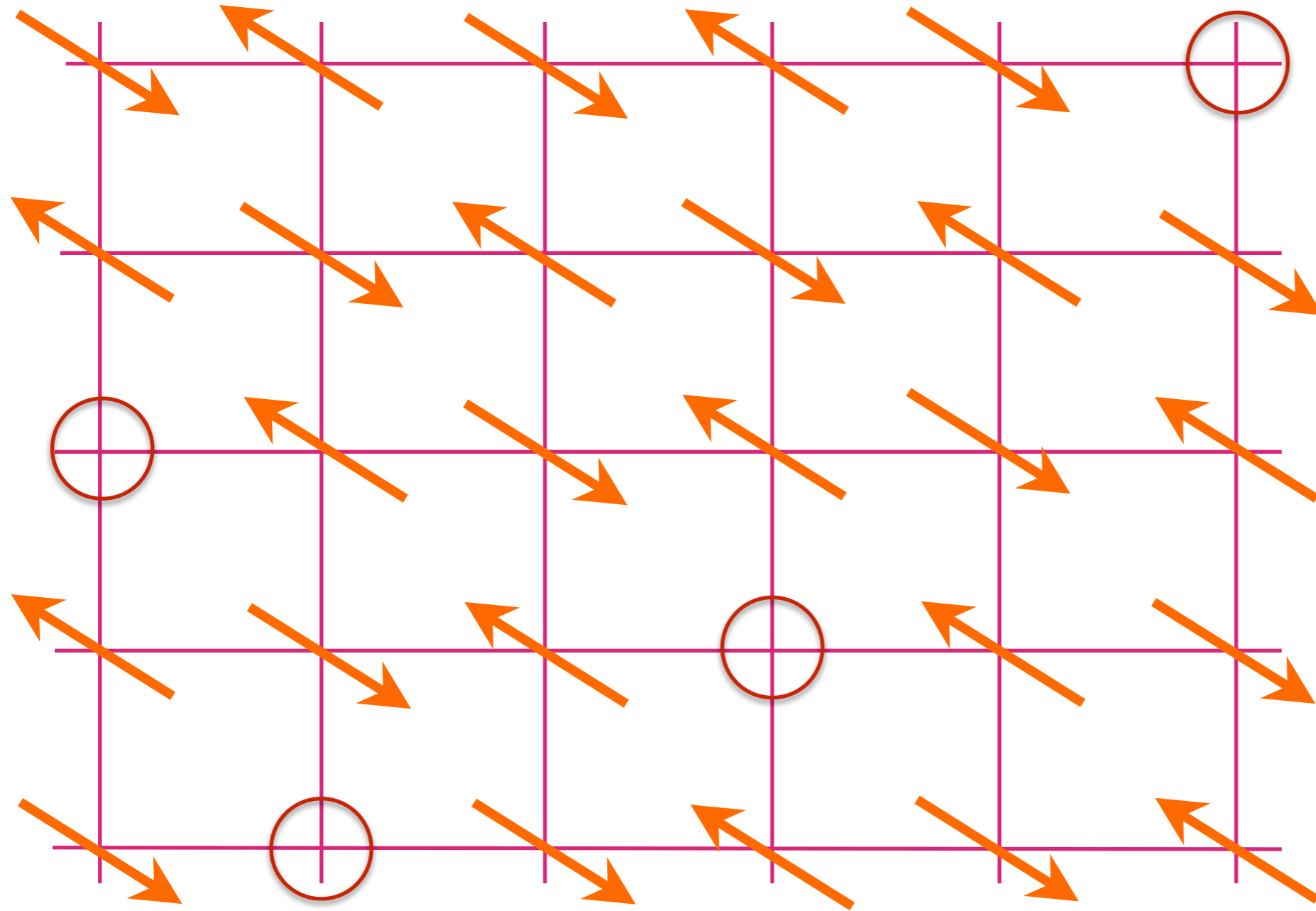




Instabilities of the large Fermi surface do lead to  $d$ -form factors, but at “diagonal” wavevectors.

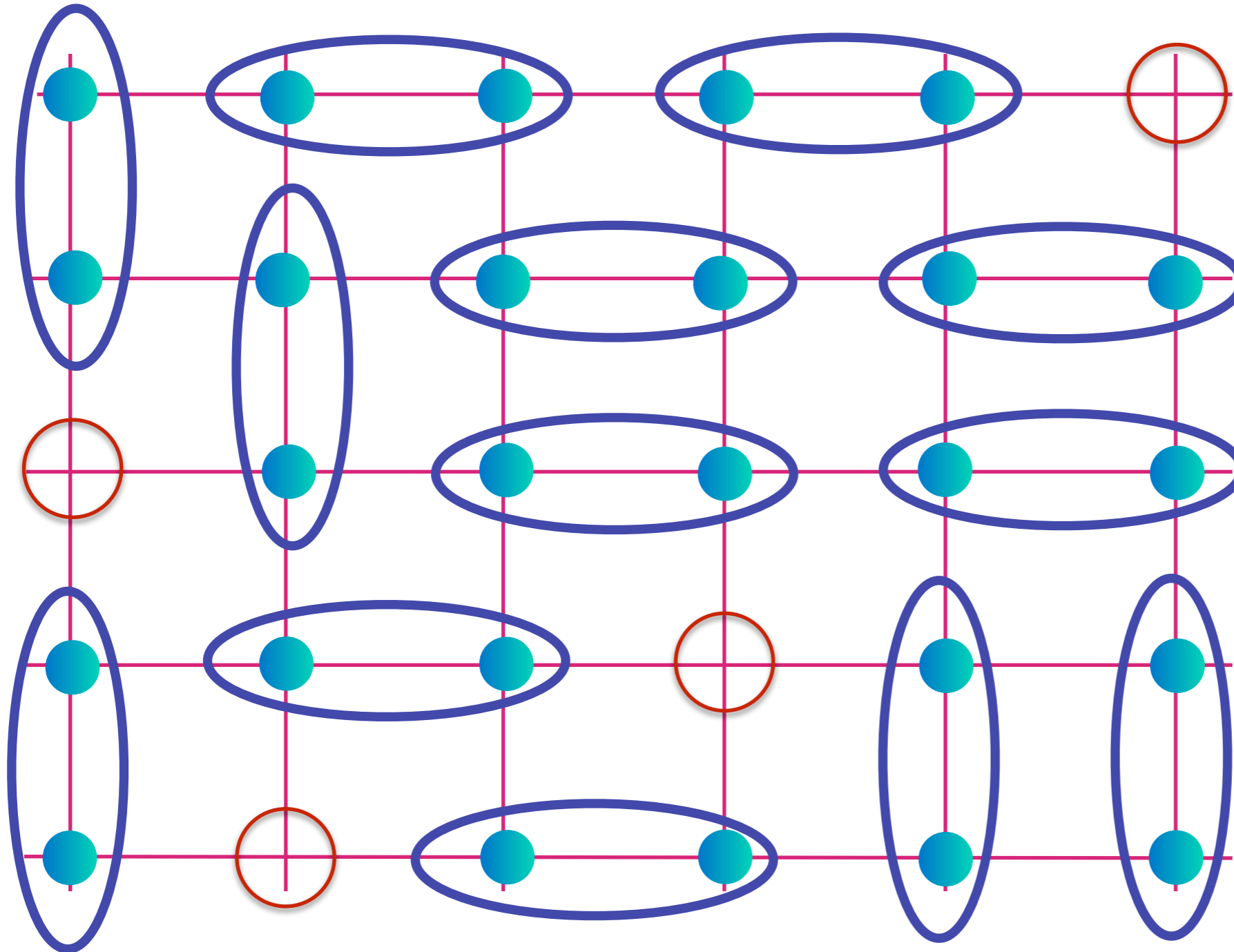


# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



Doped  
anti-  
ferromagnet

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



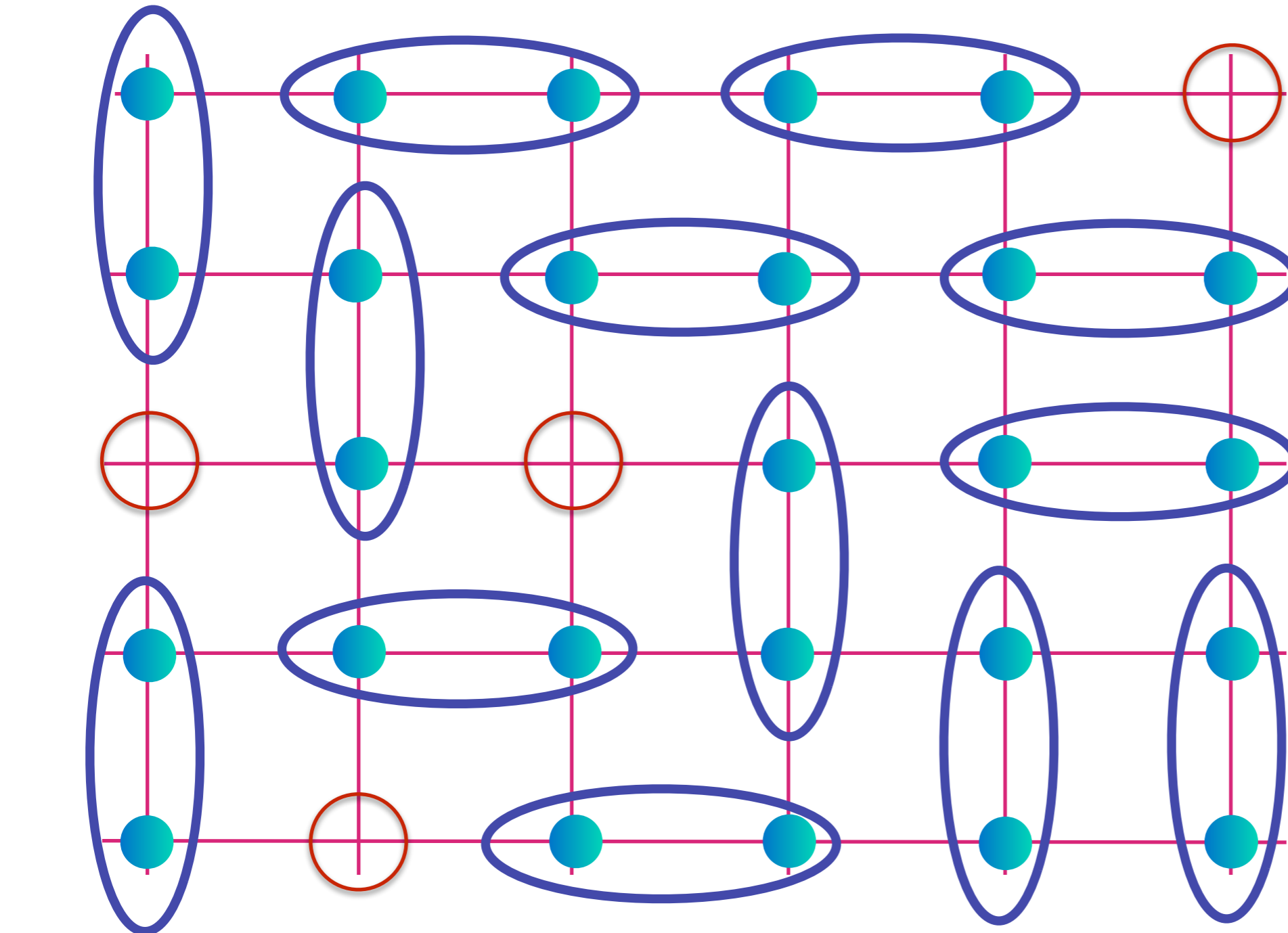
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



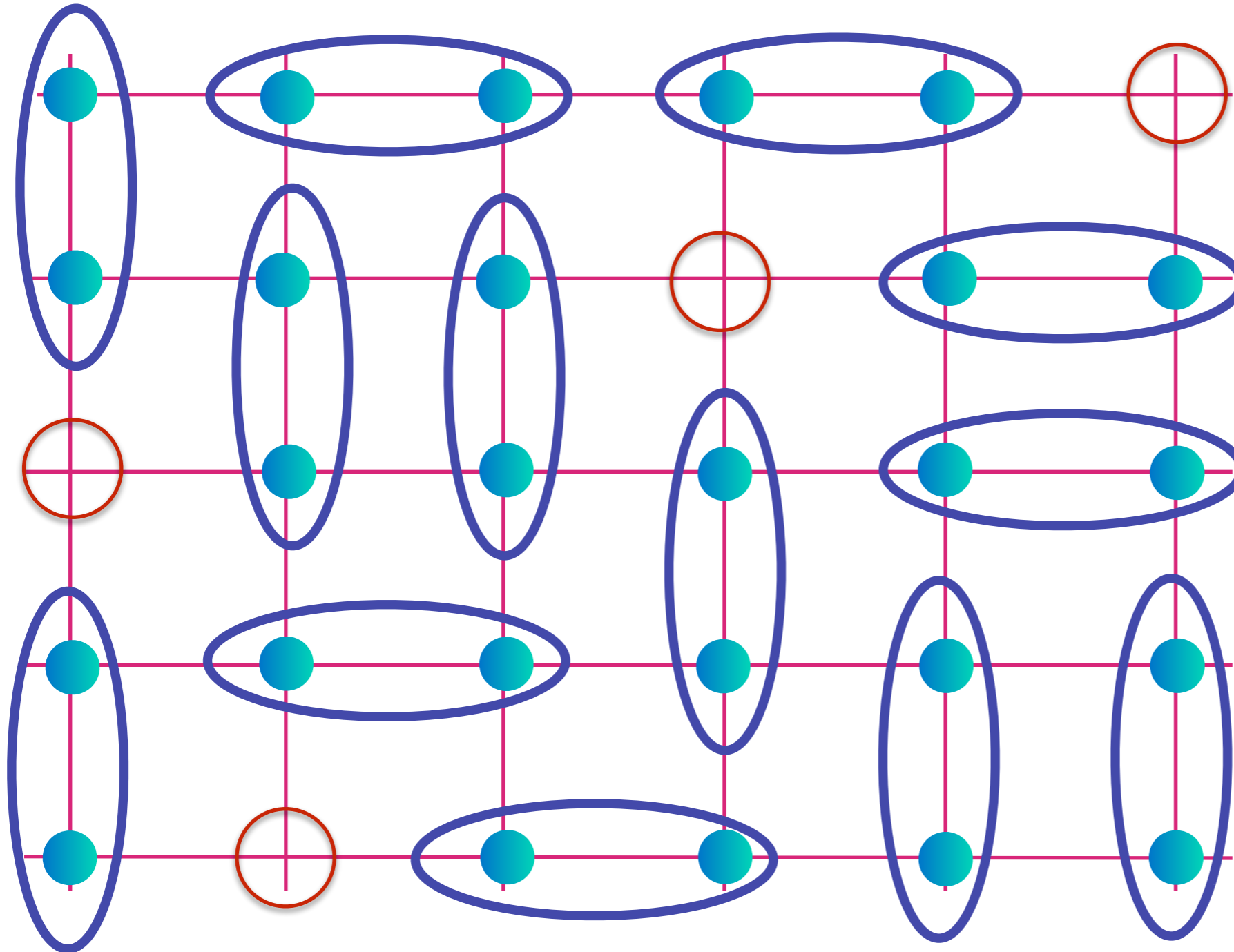
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Blue oval with two teal dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



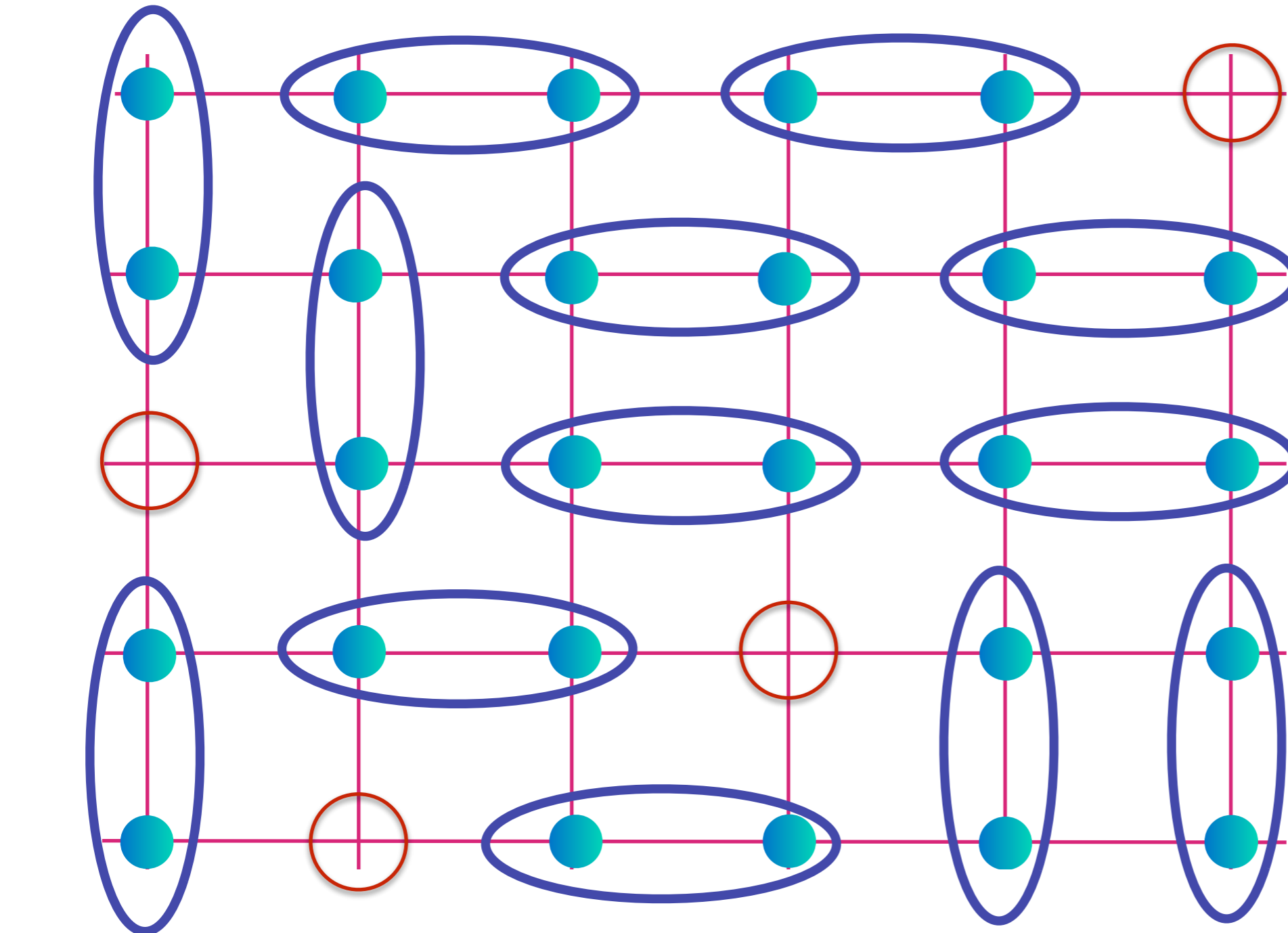
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Pair of dots in oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



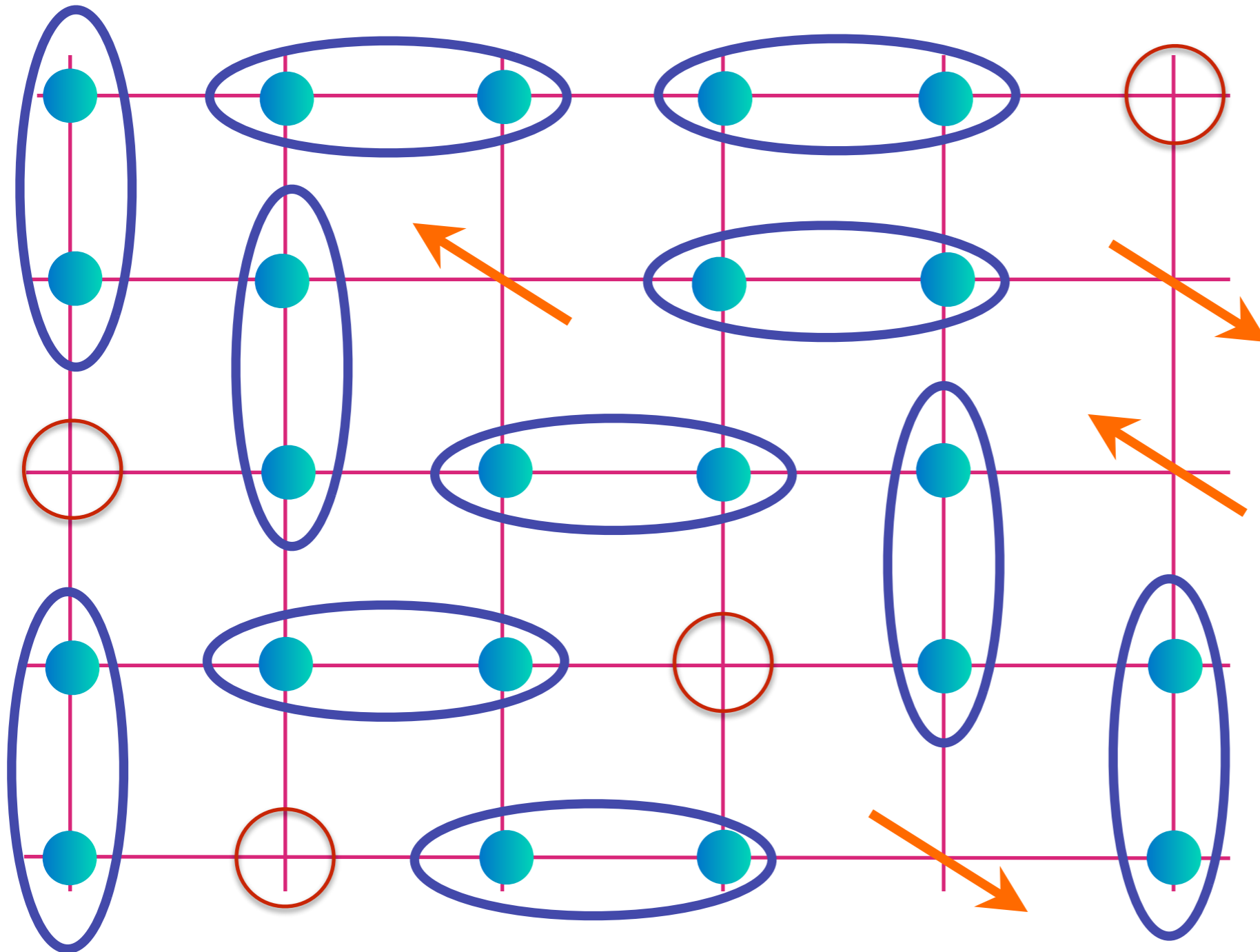
Spin  
liquid

Spinless  
charge  $+e$   
holons

$$\text{[Pair of dots in oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

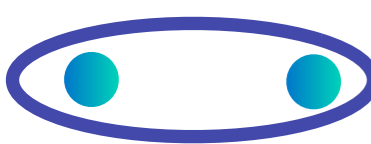
Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

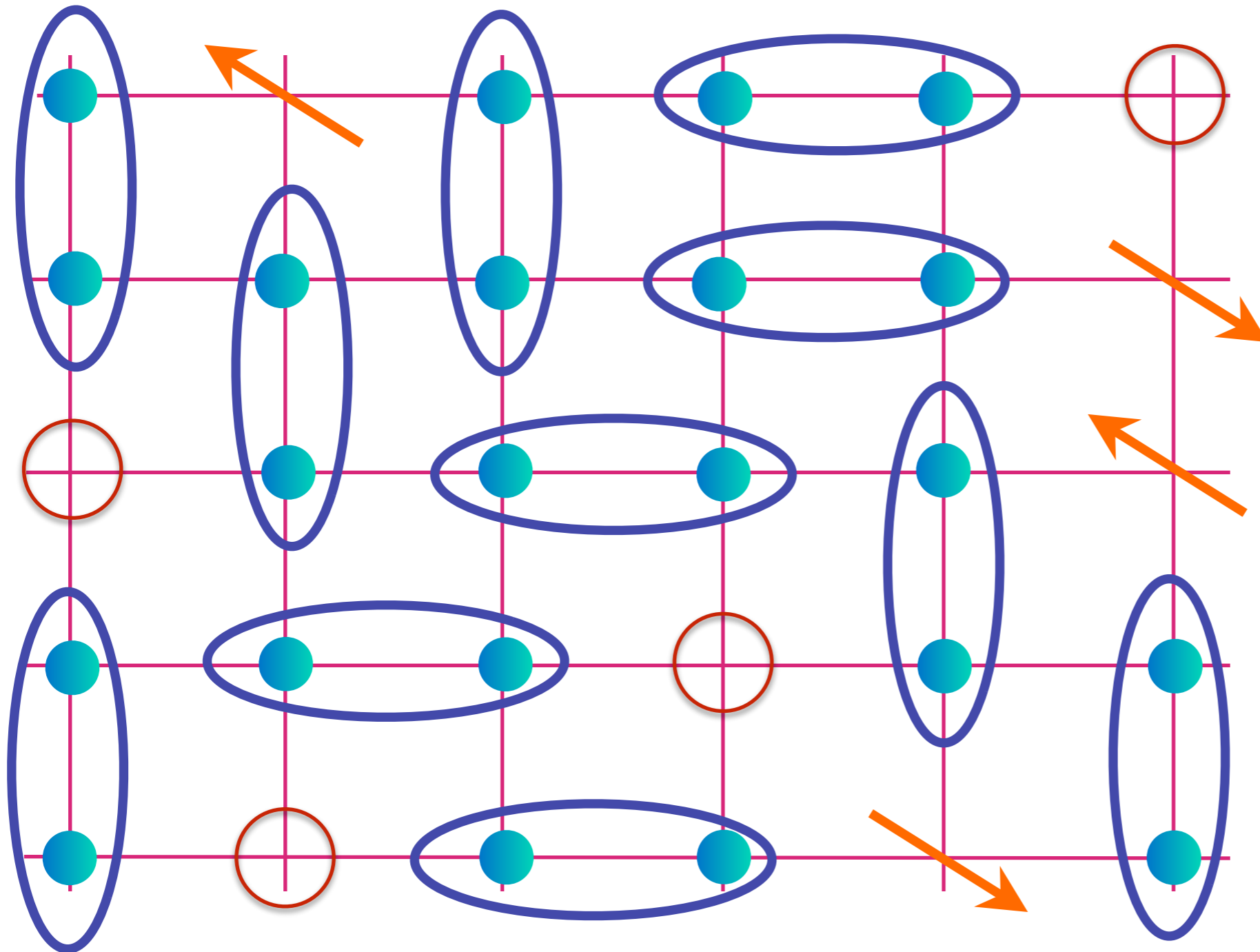


Spin  
liquid

Spinless  
charge  $+e$   
holons  
*and*  
 $S=1/2$   
neutral  
spinons

 =  $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



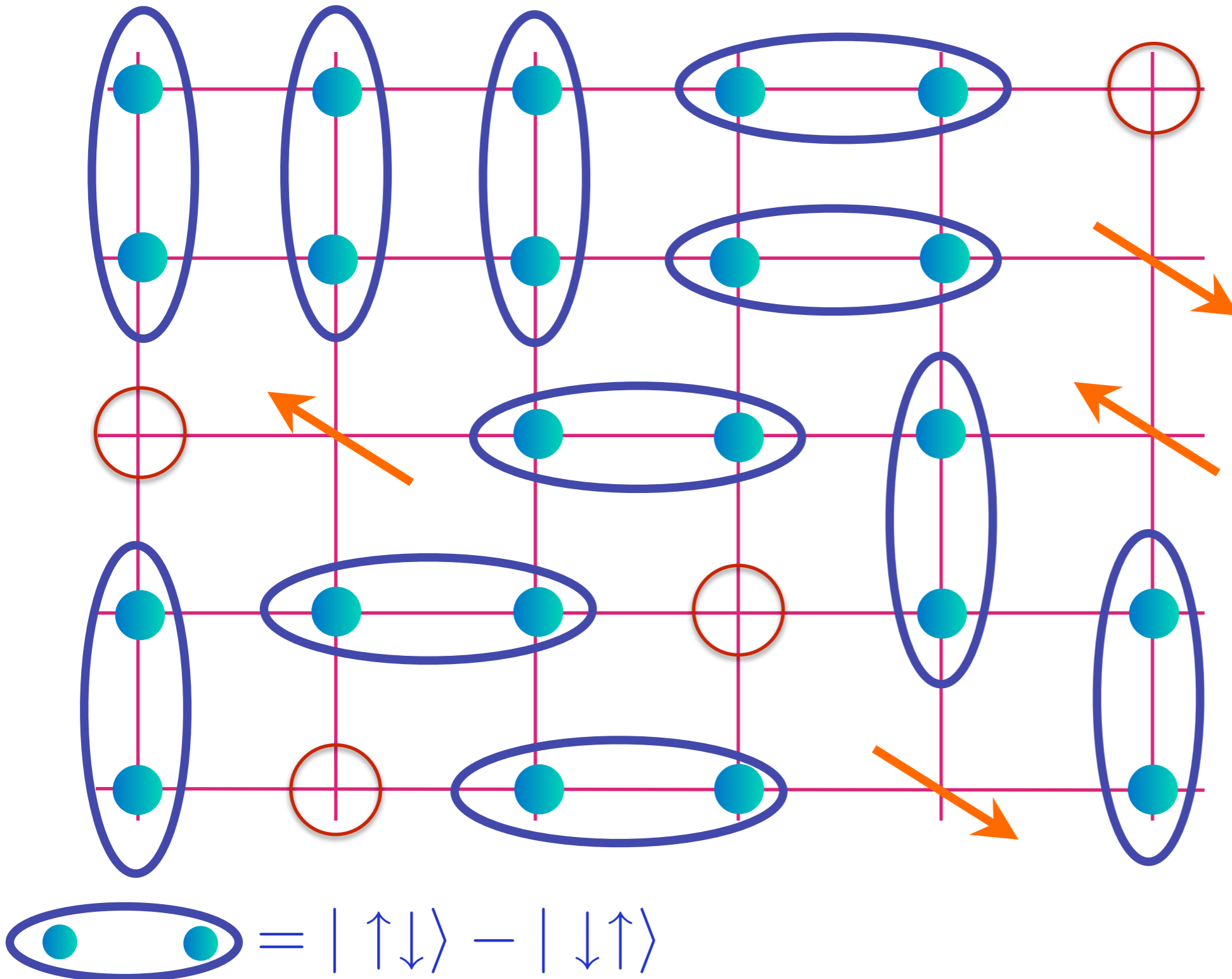
Spin  
liquid

Spinless  
charge  $+e$   
holons  
*and*  
 $S=1/2$   
neutral  
spinons

 =  $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

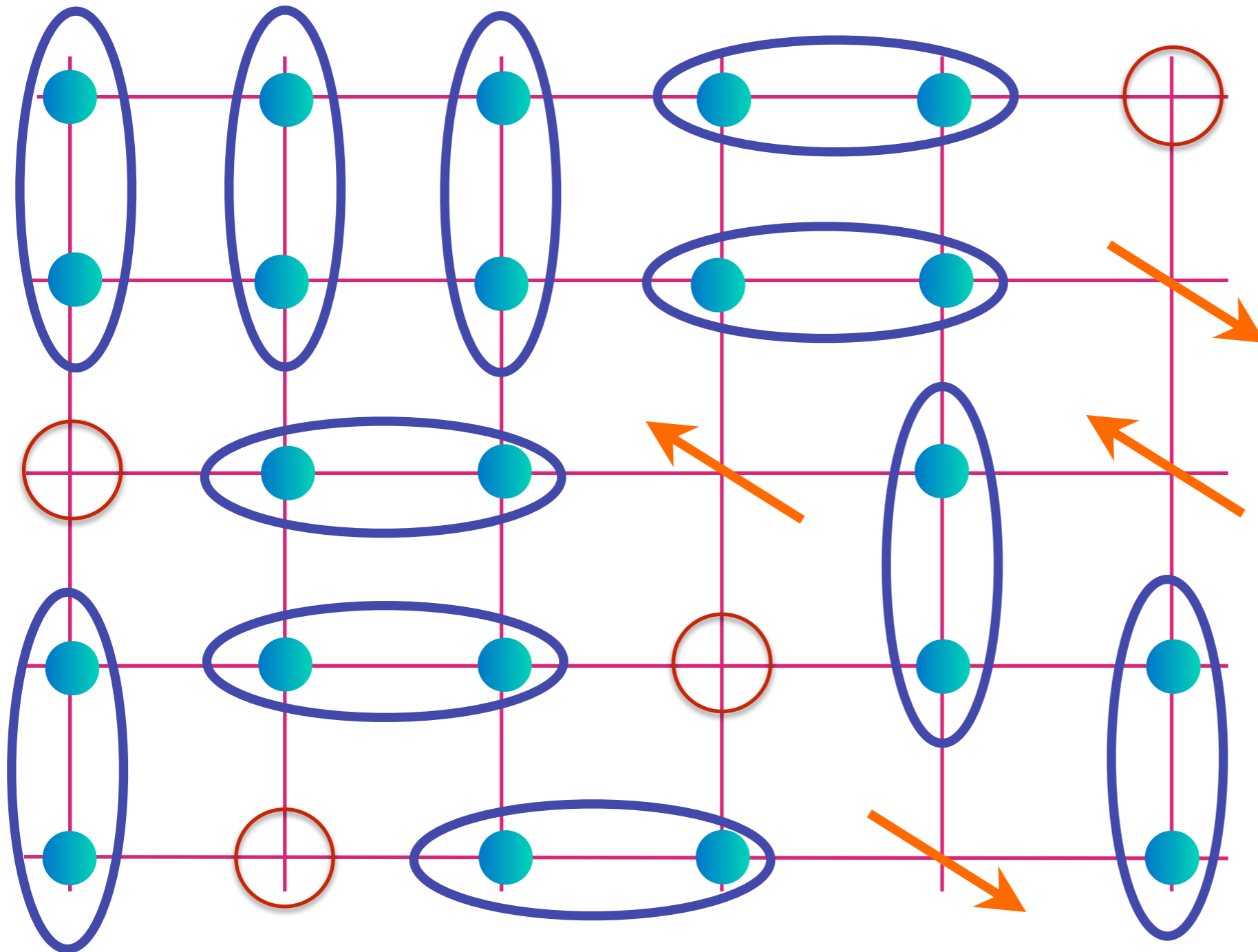


Spin  
liquid

Spinless  
charge  $+e$   
holons  
*and*  
 $S=1/2$   
neutral  
spinons

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



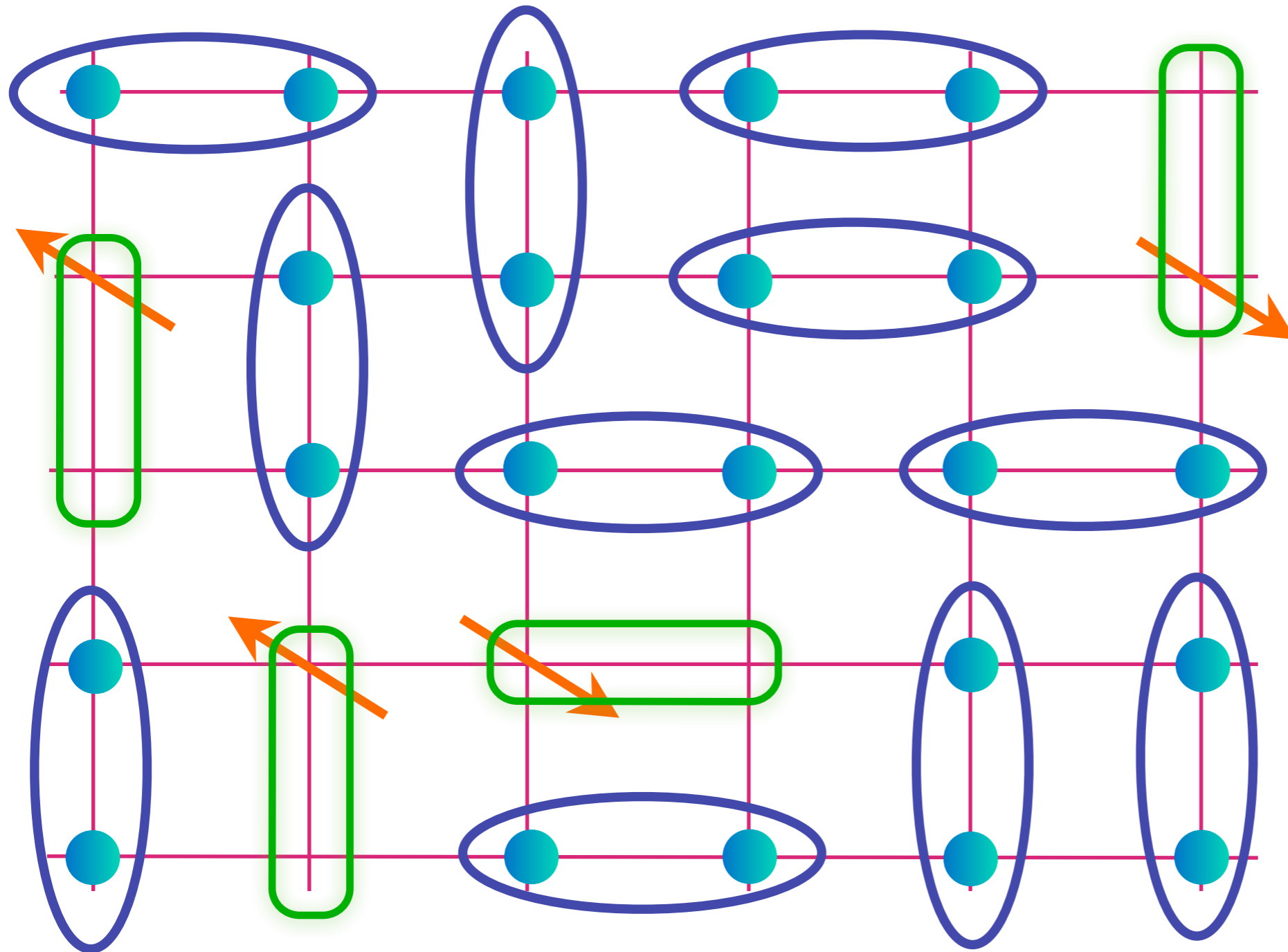
Spin  
liquid

Spinless  
charge  $+e$   
holons  
*and*  
 $S=1/2$   
neutral  
spinons

$$\text{blue oval} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Baskaran, Zou, Anderson, Fradkin, Kivelson...

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

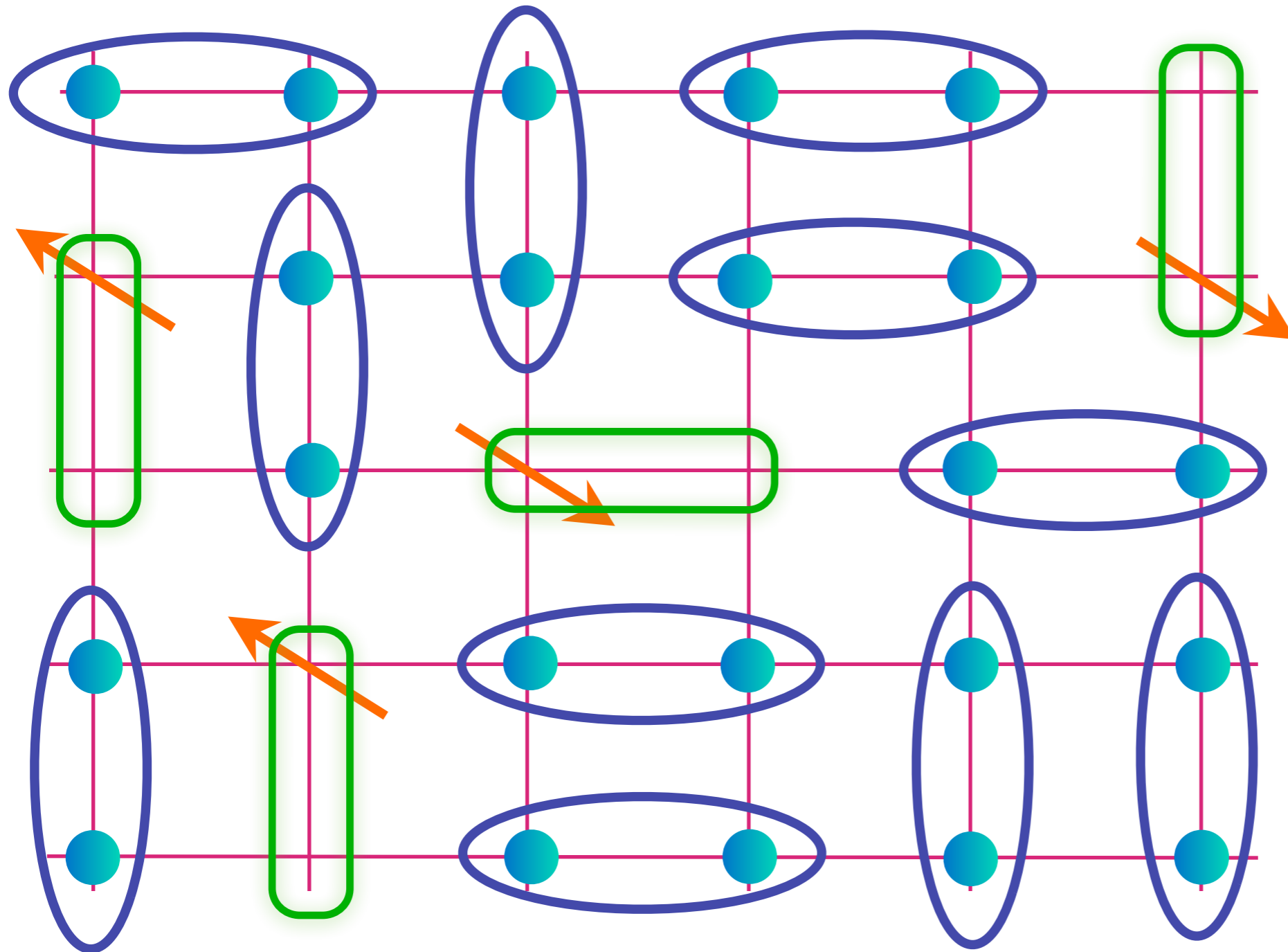


**FL\*** !

Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
holes  
of density  $p$

Charge  $+e$ , spin  $S = 1/2$  holes form Fermi surfaces of total volume  $p$   
(and not  $1 + p$  as in a Fermi liquid).

# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

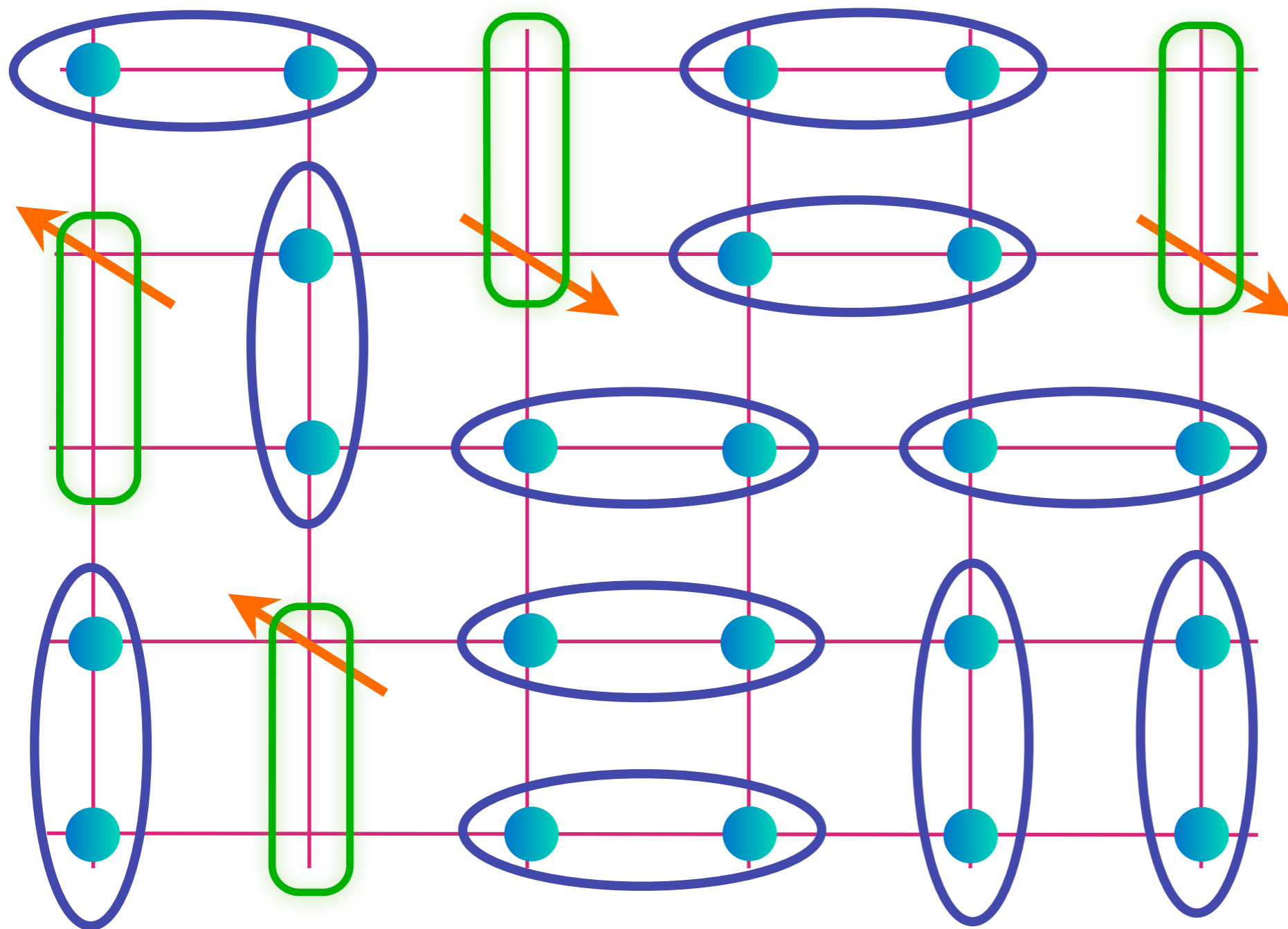


**FL\*** !

Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
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Charge  $+e$ , spin  $S = 1/2$  holes form Fermi surfaces of total volume  $p$   
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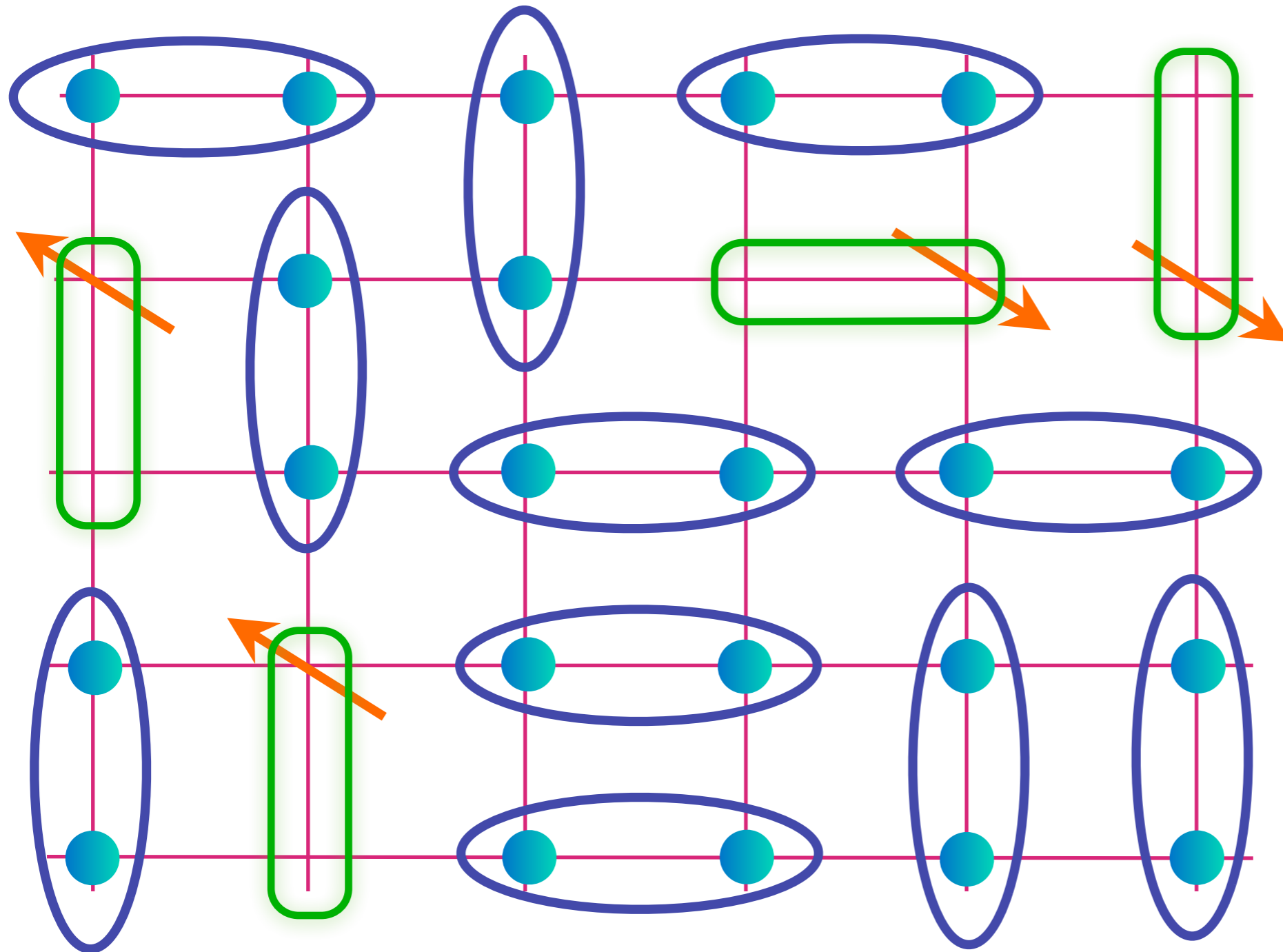
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Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
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of density  $p$

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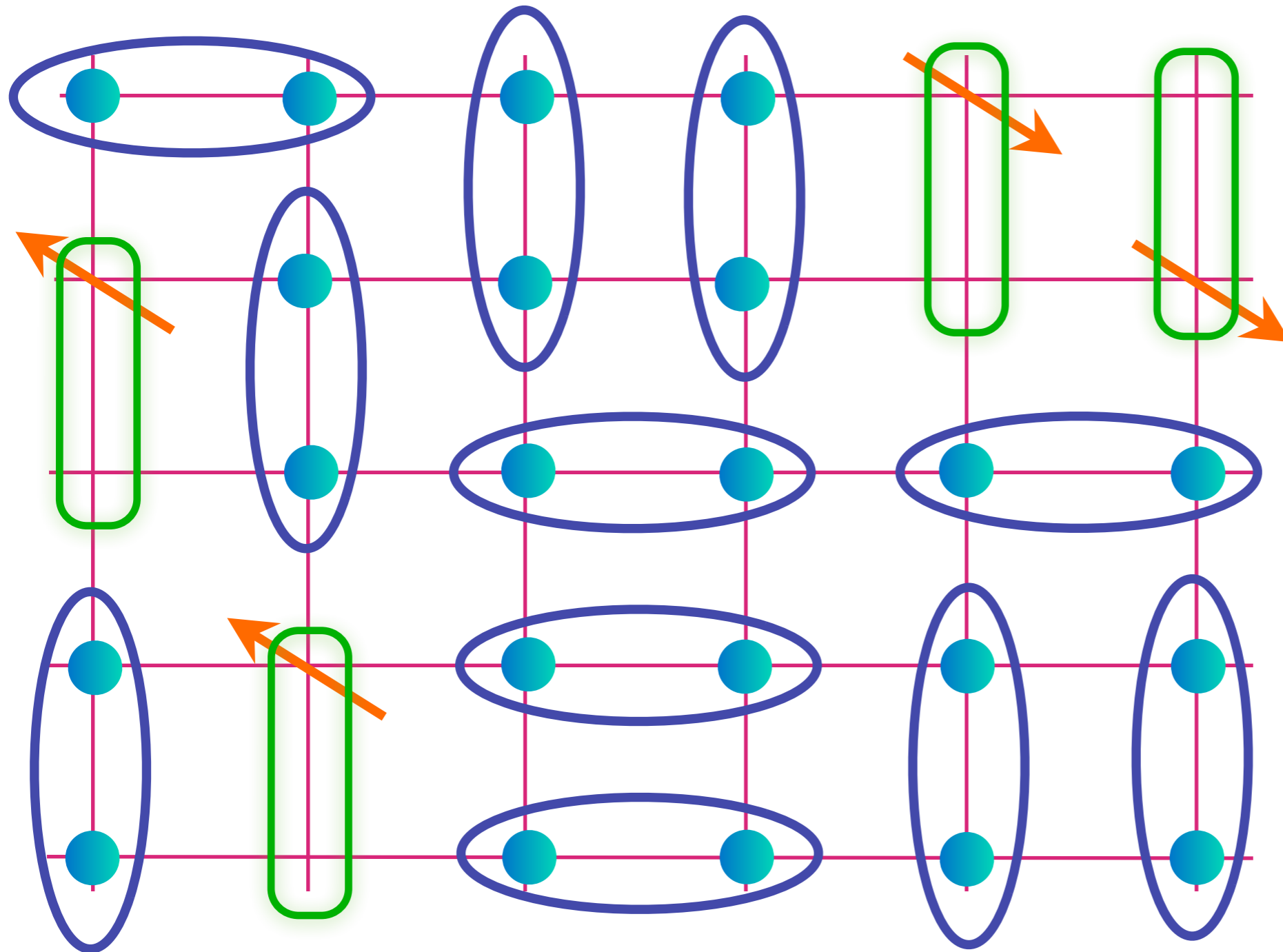


**FL\*** !

Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
holes  
of density  $p$

Charge  $+e$ , spin  $S = 1/2$  holes form Fermi surfaces of total volume  $p$   
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# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)

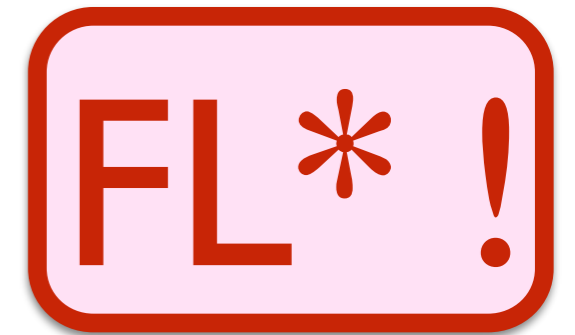
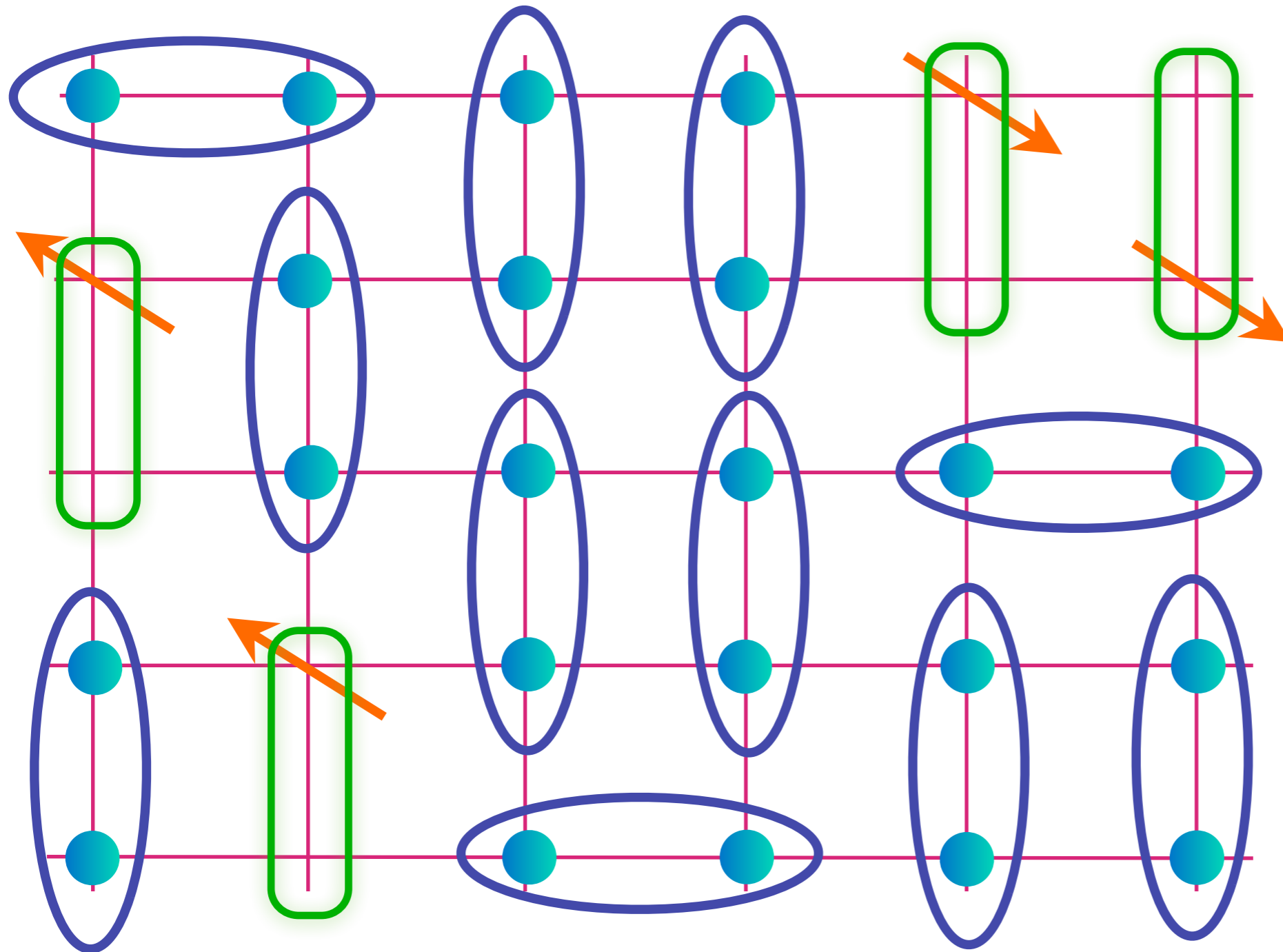


**FL\*** !

Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
holes  
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Charge  $+e$ , spin  $S = 1/2$  holes form Fermi surfaces of total volume  $p$   
(and not  $1 + p$  as in a Fermi liquid).

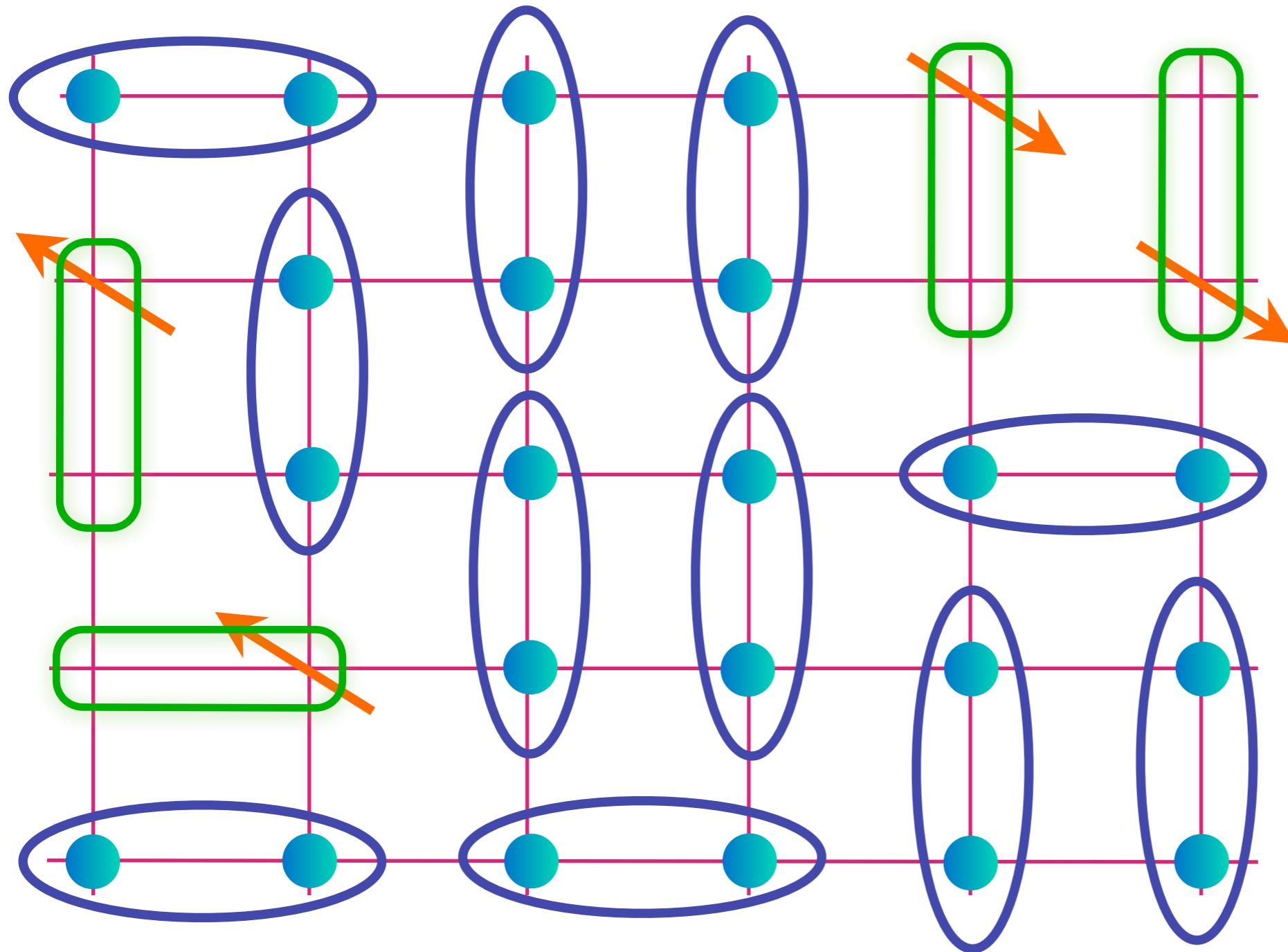
# Theory of the Pseudogap: Fractionalized Fermi liquid (FL\*)



Spin  
singlets  
*and*  
charge  $+e$   
 $S=1/2$   
holes  
of density  $p$

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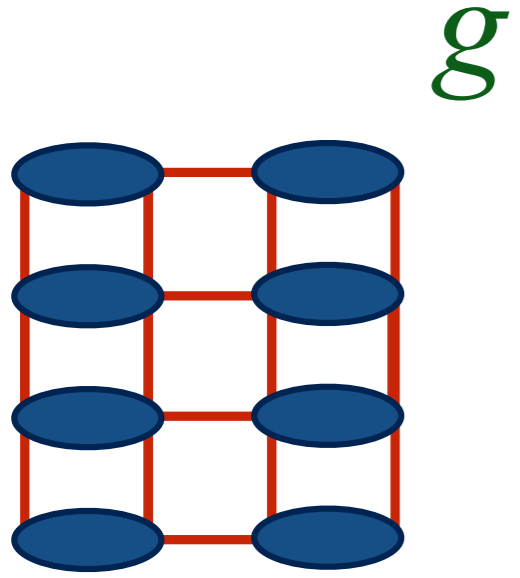
**FL\*** !

Spin  
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## Characteristics of FL\* phase

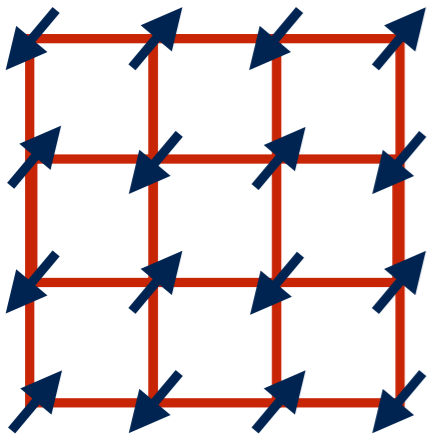
- Fermi surface volume does not count all electrons.
- Such a phase *must* have low energy collective gauge excitations (“topological” order).
- These low energy gauge excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.



$g$

$U(1) \text{ SL} \rightarrow \text{VBS} + \text{confinement}$

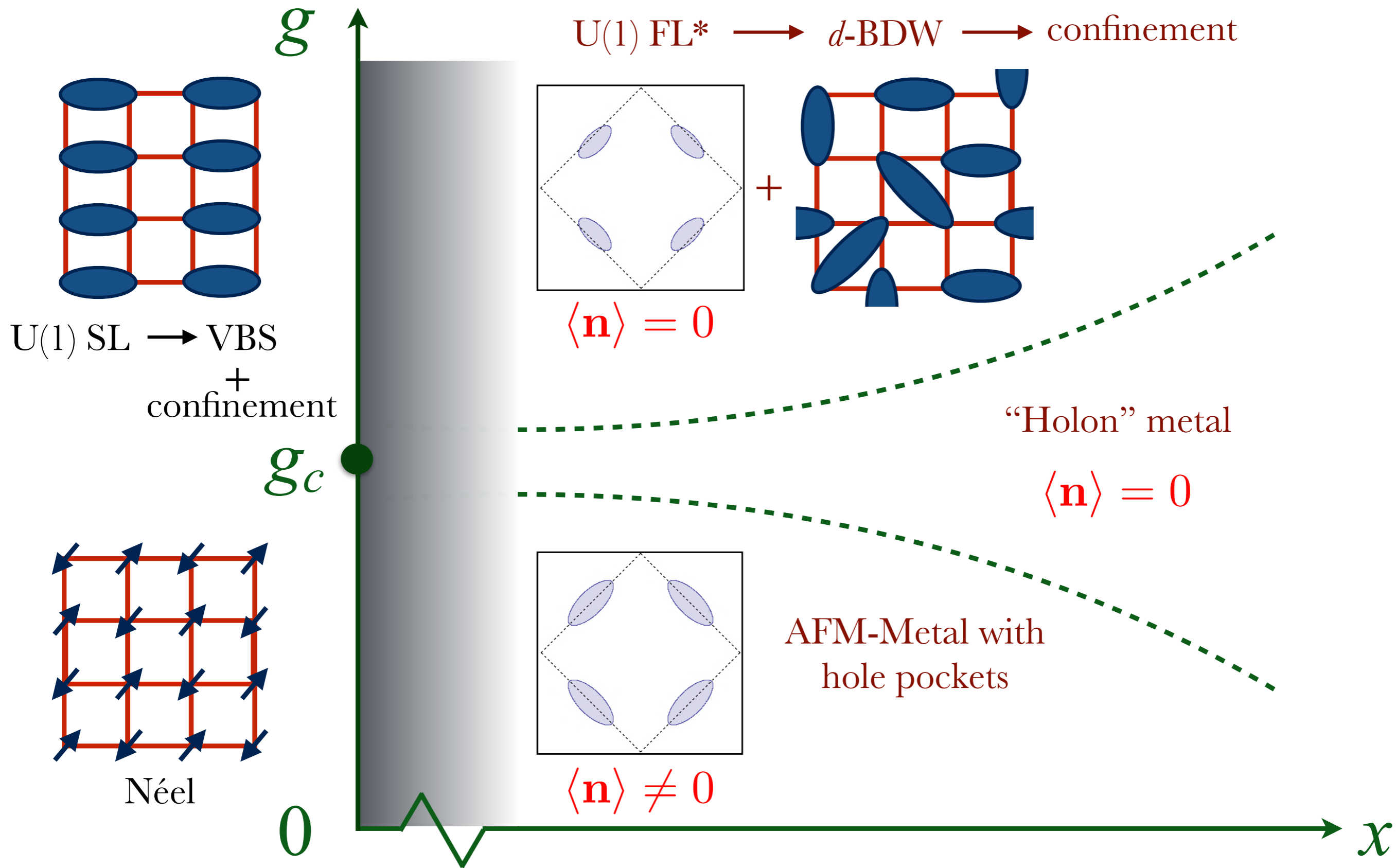
$g_c$



Néel

0

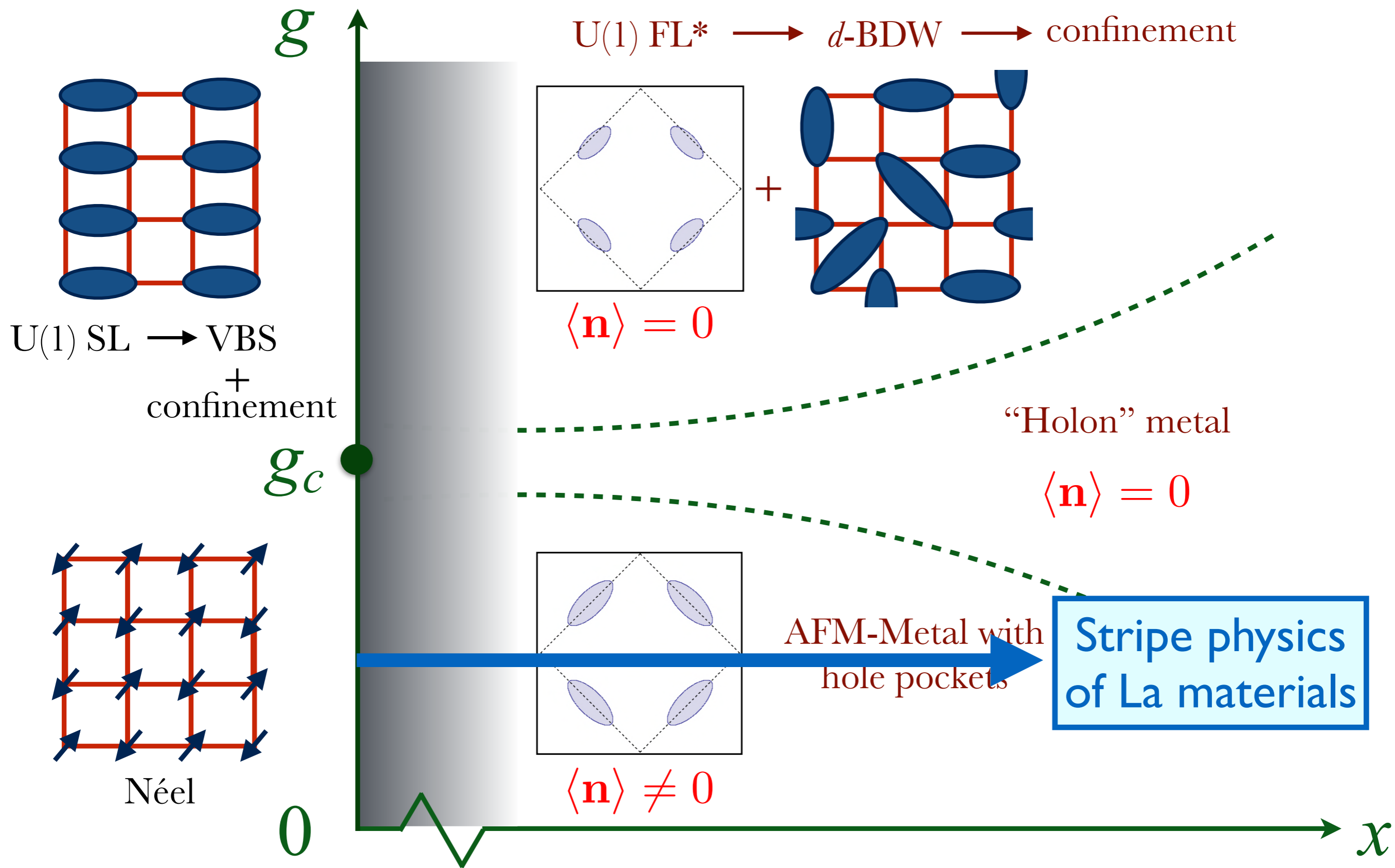


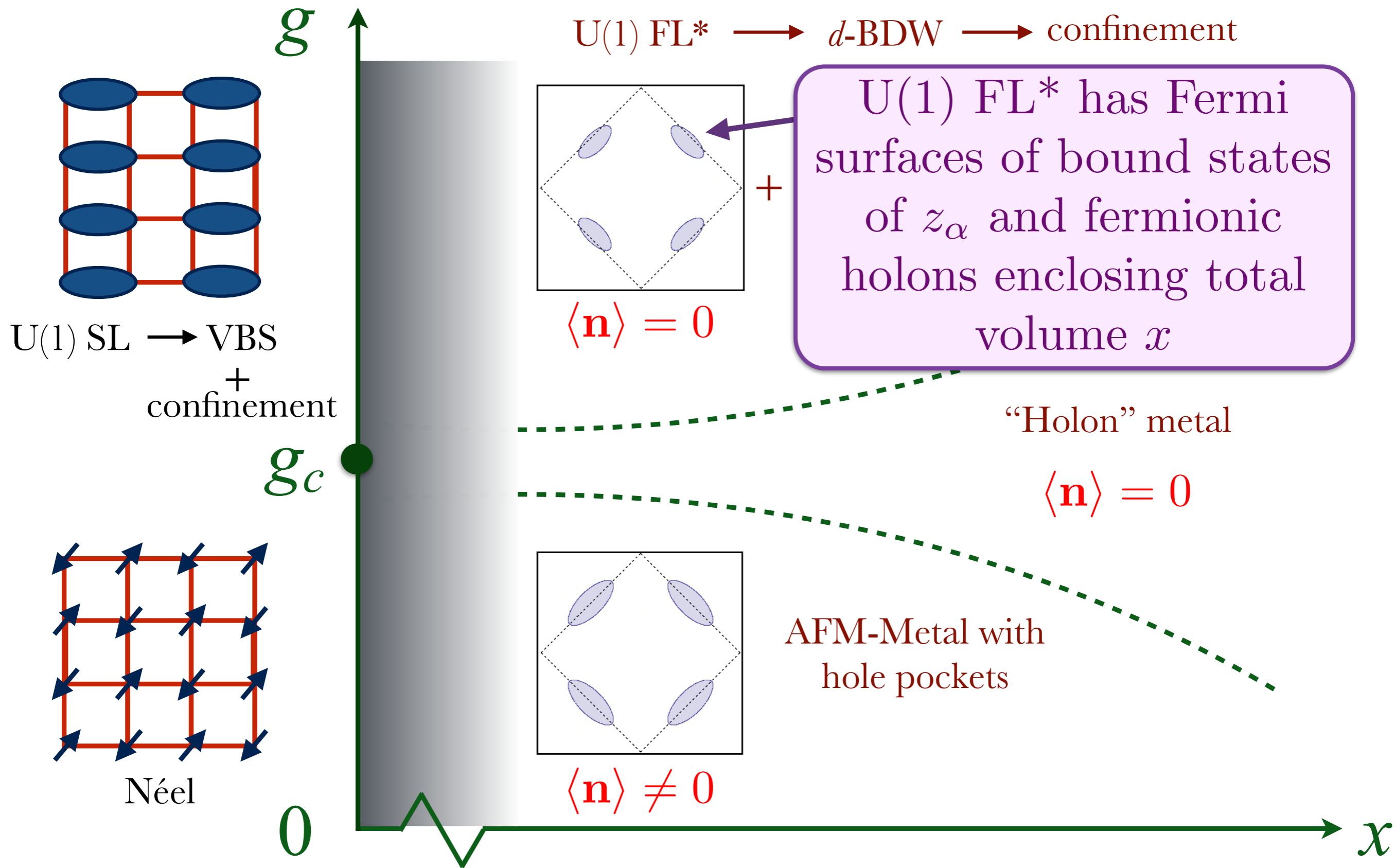


R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, Phys. Rev. B **75**, 235122 (2007)

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)

D. Chowdhury and S. Sachdev, arXiv:1409.5430

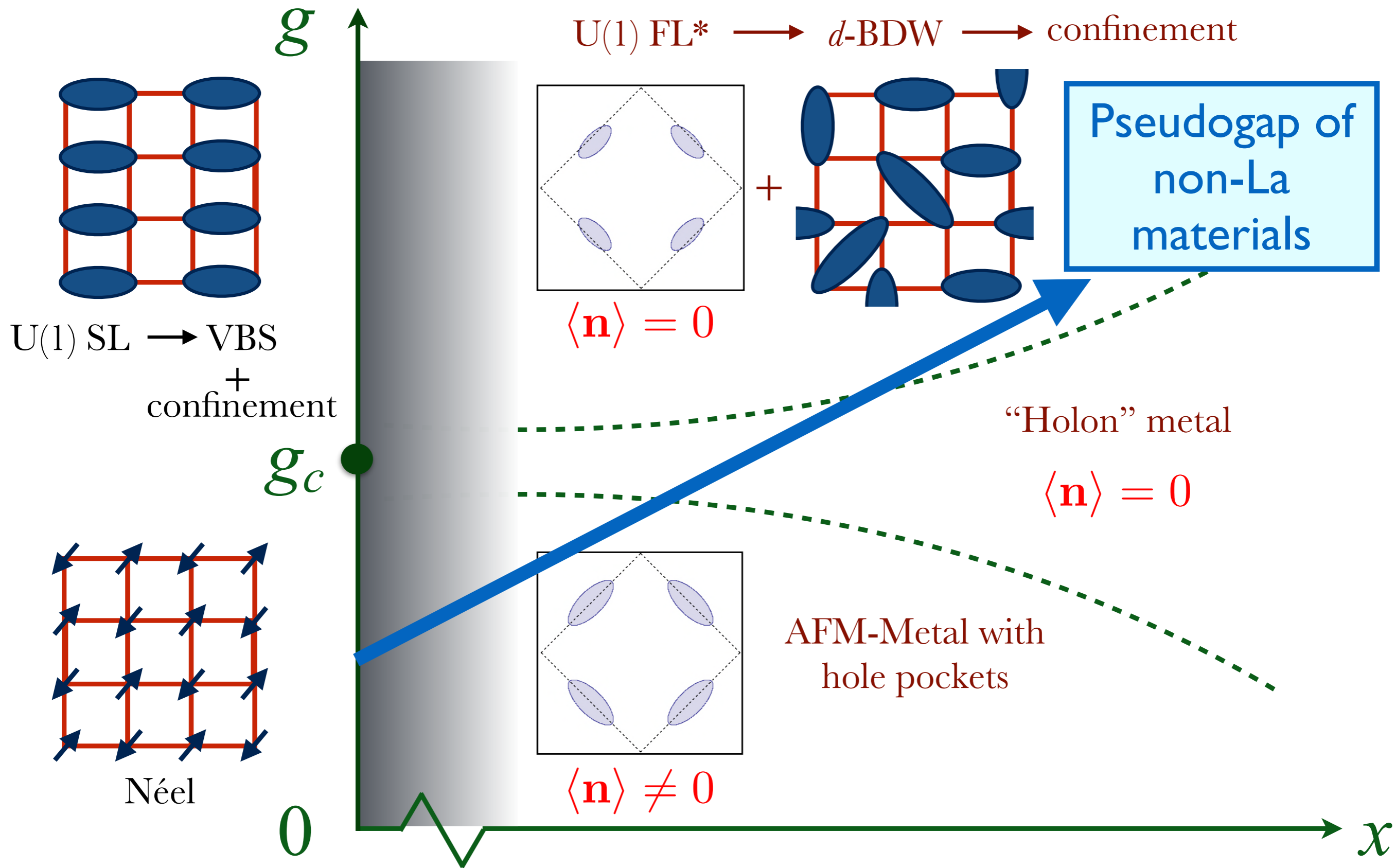


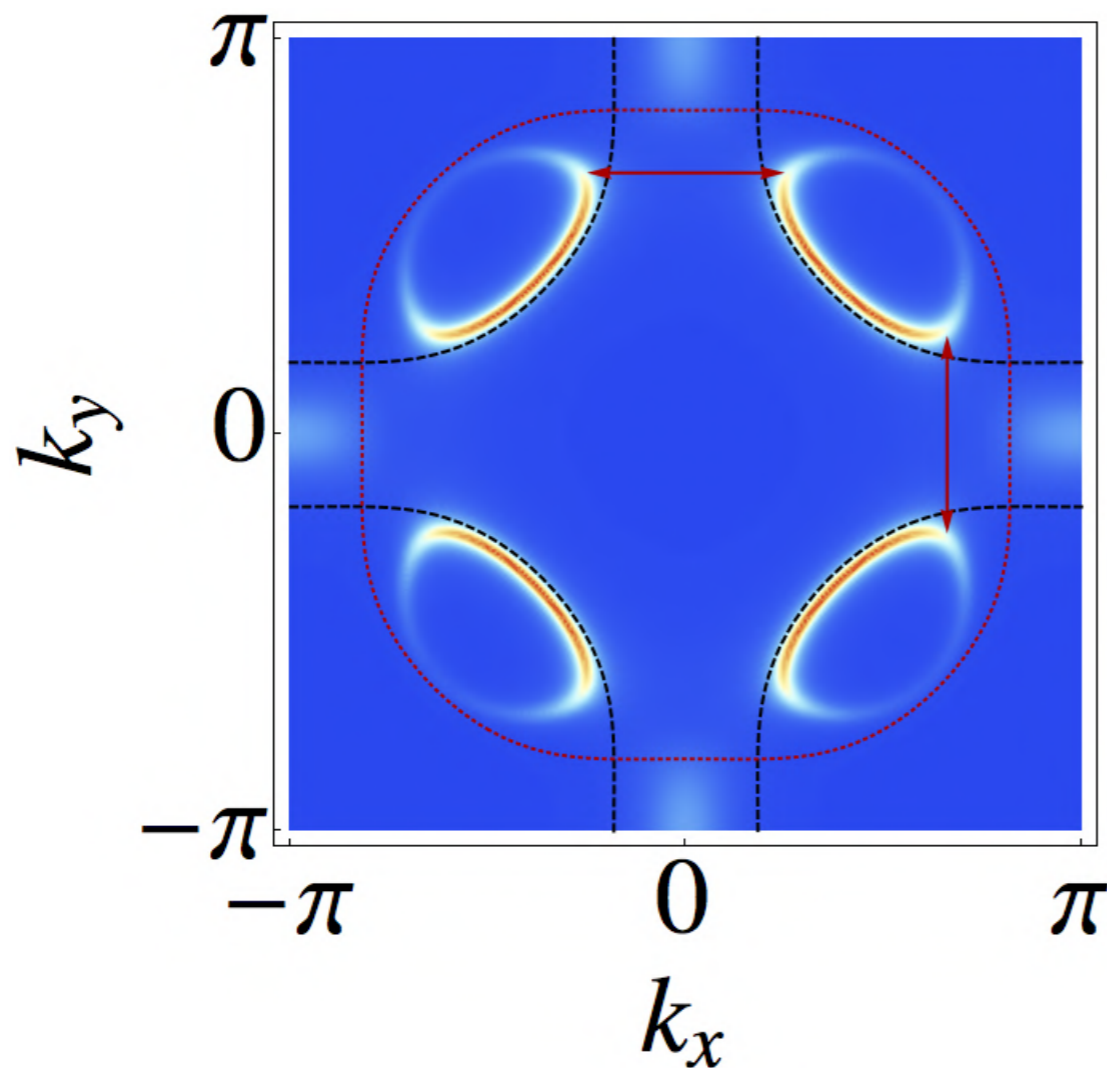


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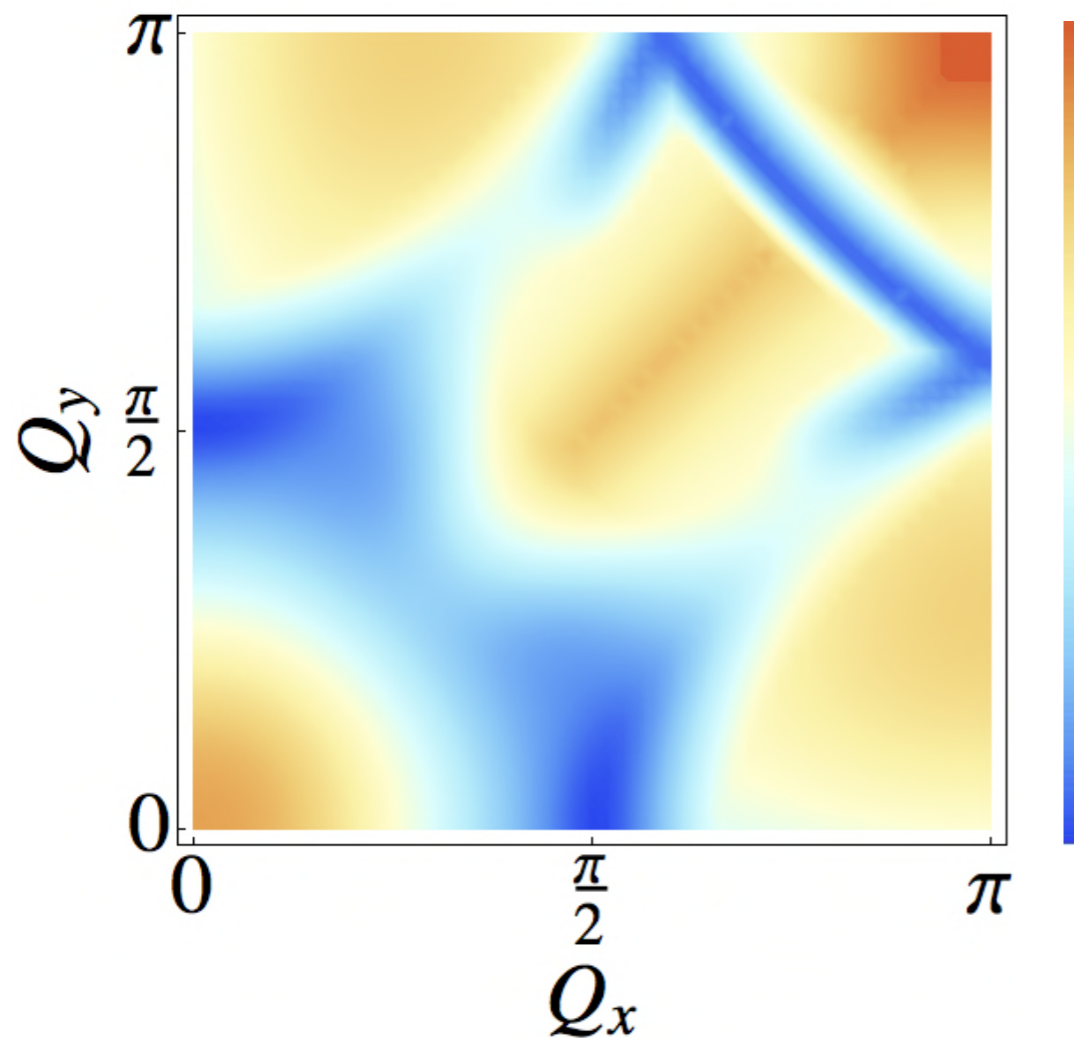




Electron spectral  
function of FL\*

The pseudogap is described by the U(1)-FL\*: a state with hole pockets on a background of a spin-liquid described by a U(1) gauge theory. Its dominant density wave instability is a predominantly  $d$ -form factor density wave with a wavevector  $\mathbf{Q}$  along the  $(1, 0)$  and  $(0, 1)$  square lattice directions, in agreement with observations on the non-La-based cuprates.

- Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)  
M. Punk and S. Sachdev, Phys. Rev. B **85**, 195123 (2012)  
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Eigenvalues of spin-singlet,  
time-reversal-preserving  
particle-hole propagator

The pseudogap is described by the U(1)-FL\*: a state with hole pockets on a background of a spin-liquid described by a U(1) gauge theory. Its dominant density wave instability is a predominantly  $d$ -form factor density wave with a wavevector  $\mathbf{Q}$  along the  $(1, 0)$  and  $(0, 1)$  square lattice directions, in agreement with observations on the non-La-based cuprates.

# Conclusions

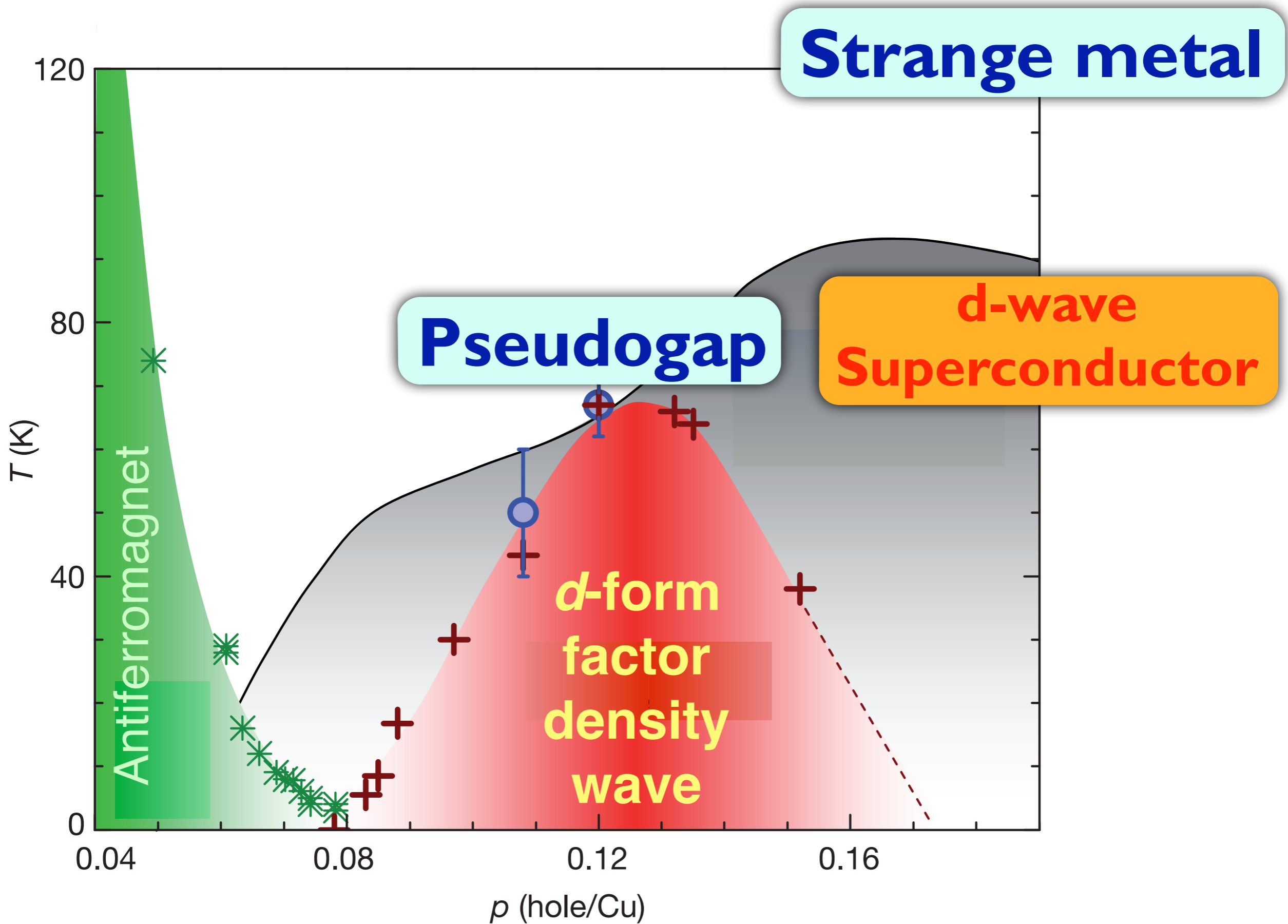
1.  $d$ -form factor density wave order observed in the non-La hole-doped cuprate superconductors.

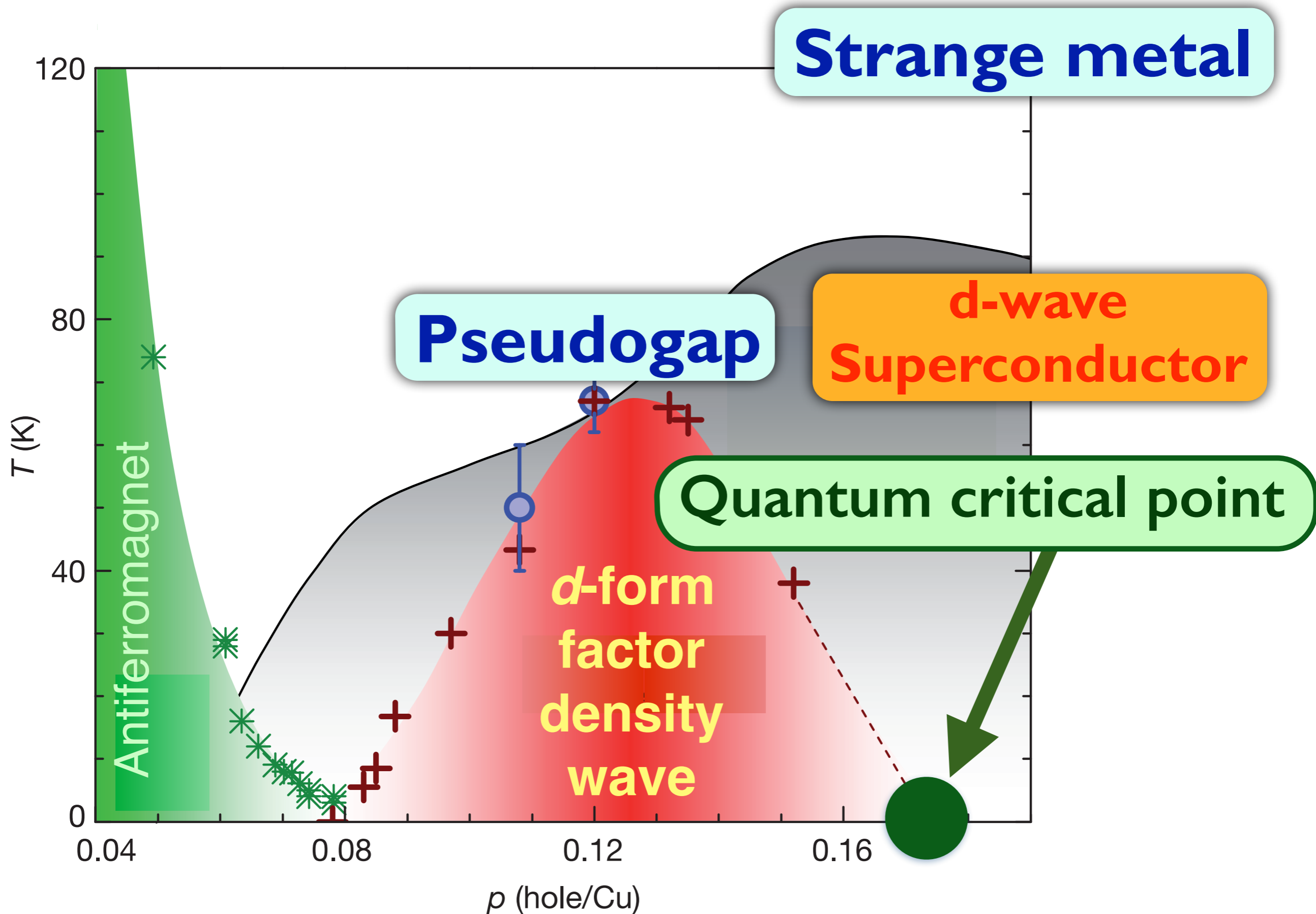
# Conclusions

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2. The “stripe” model corresponds to a  $s'$ -form factor, and this describes the La-based, lower  $T_c$ , hole-doped cuprate superconductors.

# Conclusions

1.  $d$ -form factor density wave order observed in the non-La hole-doped cuprate superconductors.
2. The “stripe” model corresponds to a  $s'$ -form factor, and this describes the La-based, lower  $T_c$ , hole-doped cuprate superconductors.
3. The  $d$ -form factor appears to be an unexpected window into the spin-liquid physics of the pseudogap.





Y. He *et al.*, Science **344**, 608 (2014)  
 K. Fujita *et al.*, Science **344**, 612 (2014)