Quantum criticality and the cuprate superconductors

Talk online: sachdev.physics.harvard.edu

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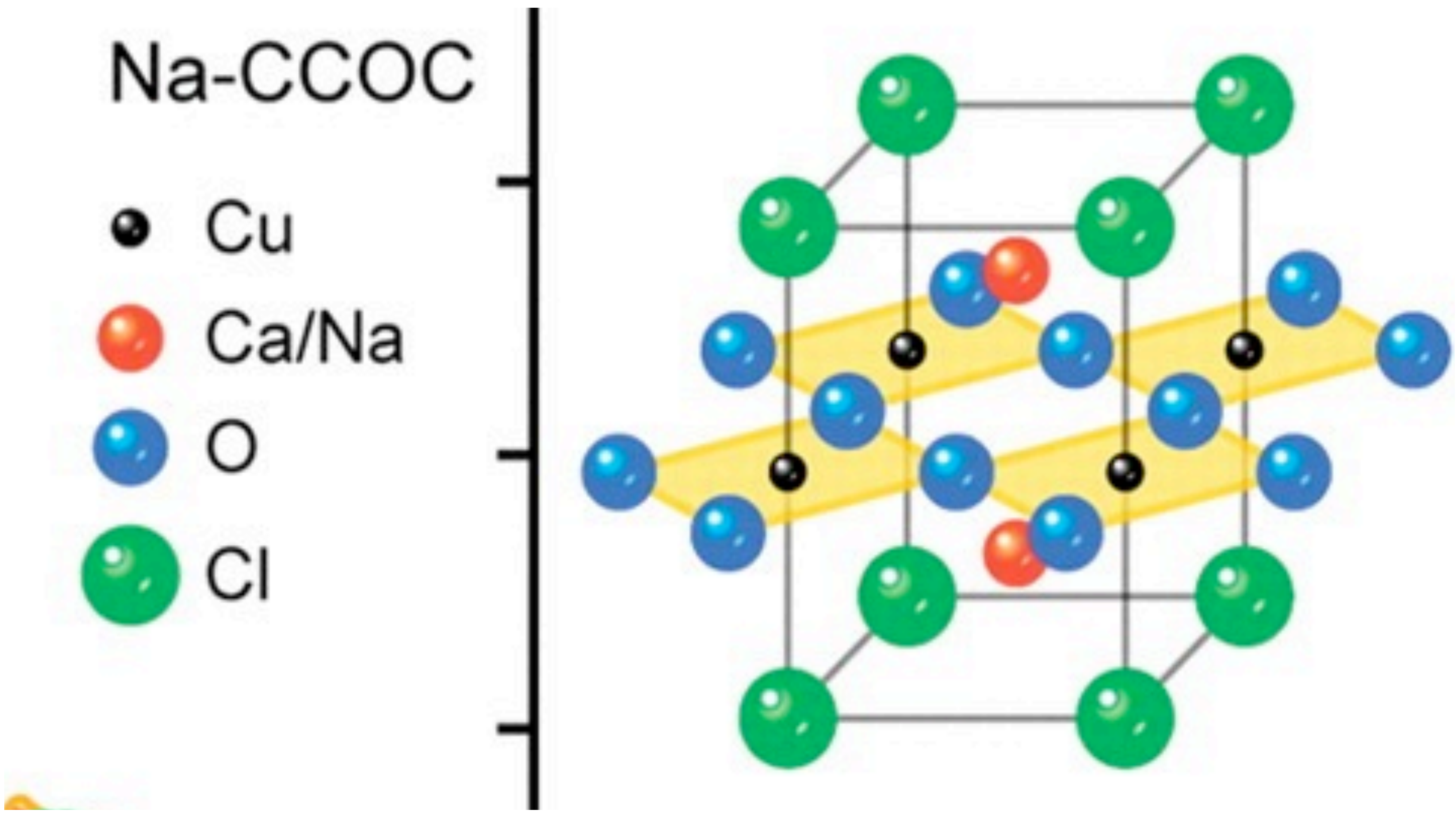
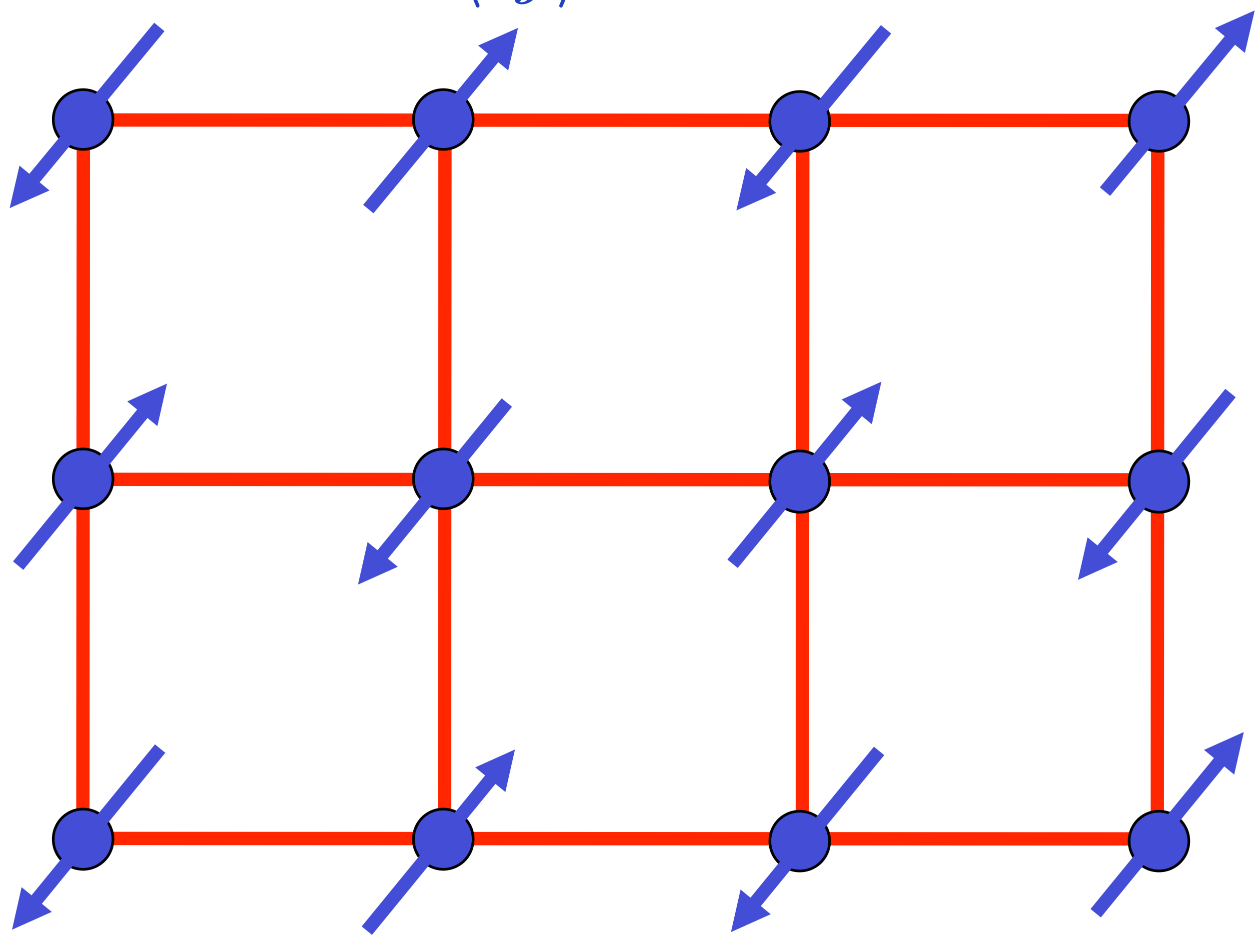
The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



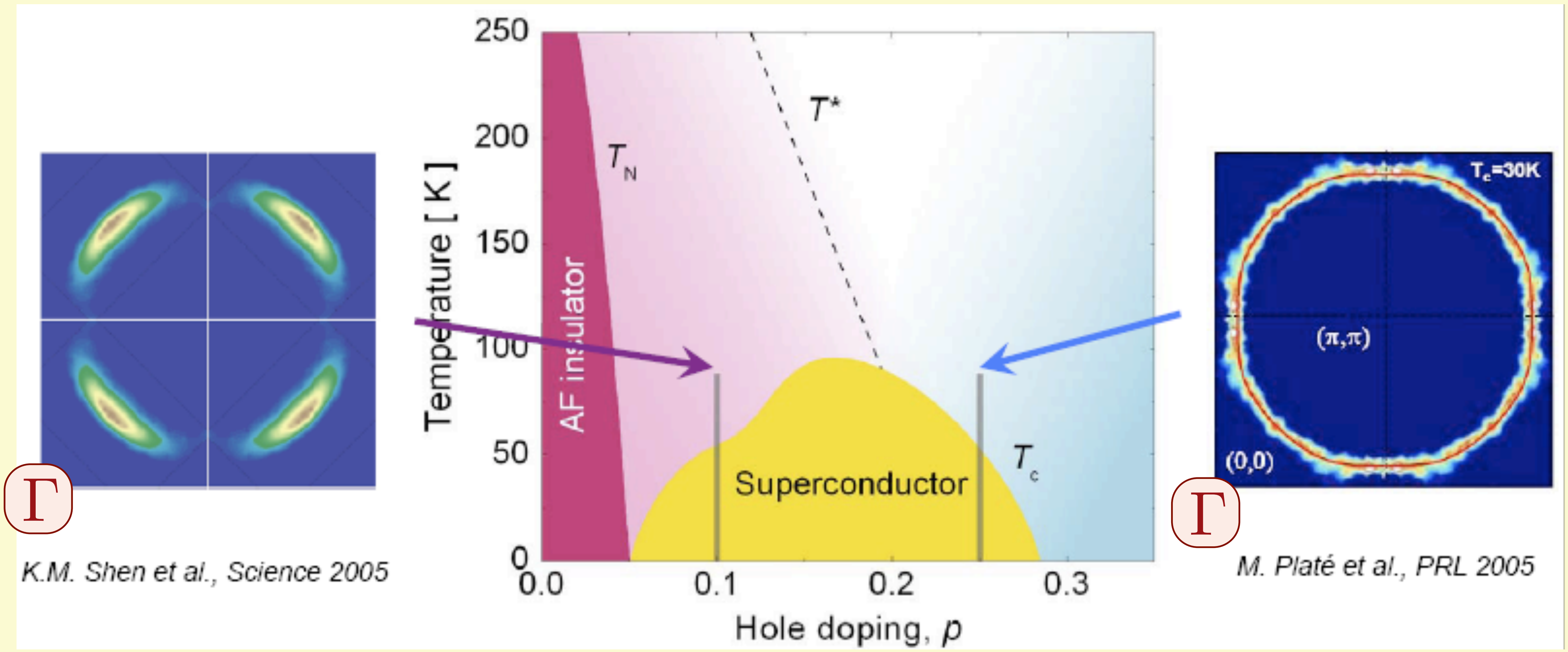
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

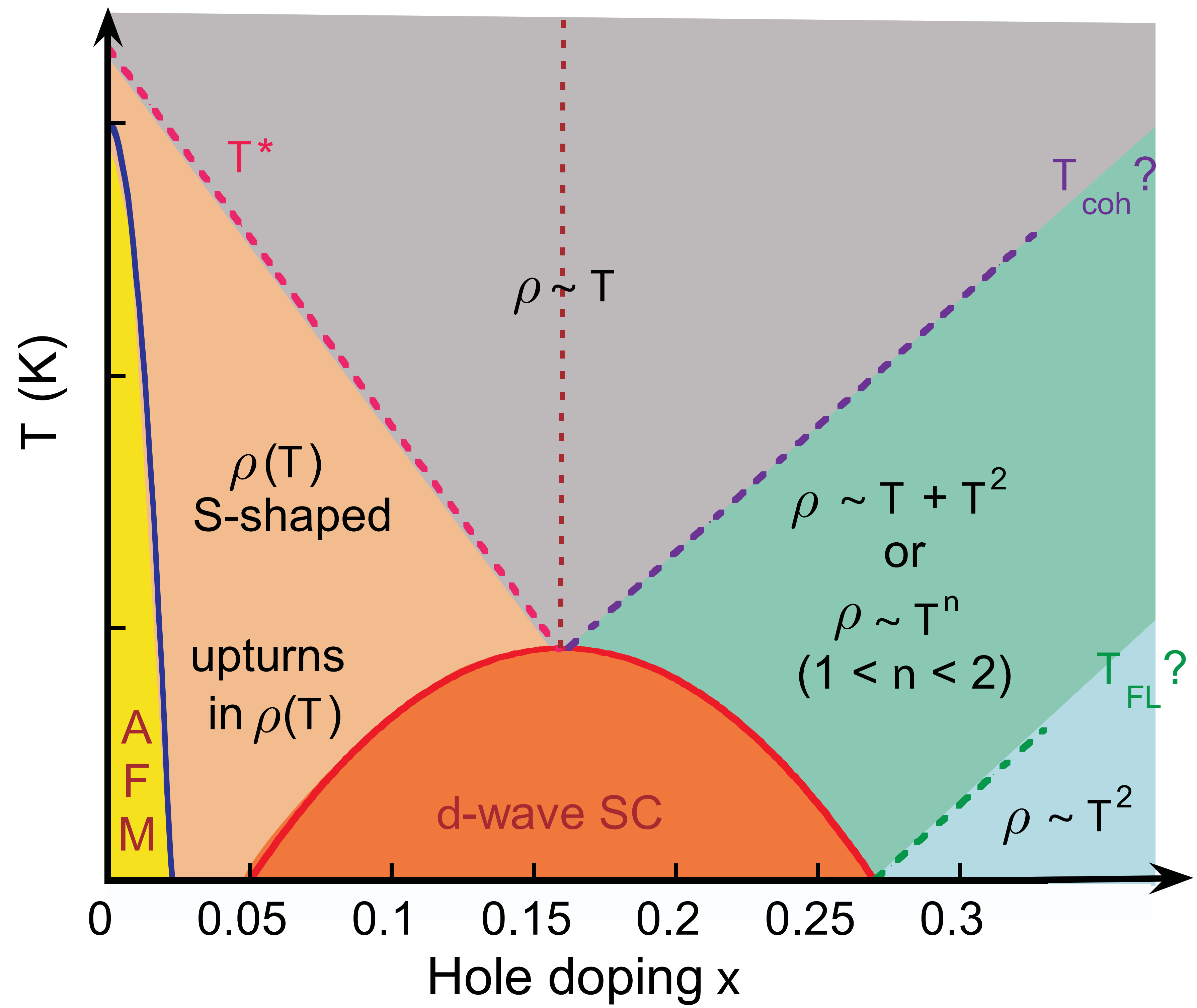
Large hole Fermi surface

Antiferro-
magnetism

d-wave
supercon-
ductivity

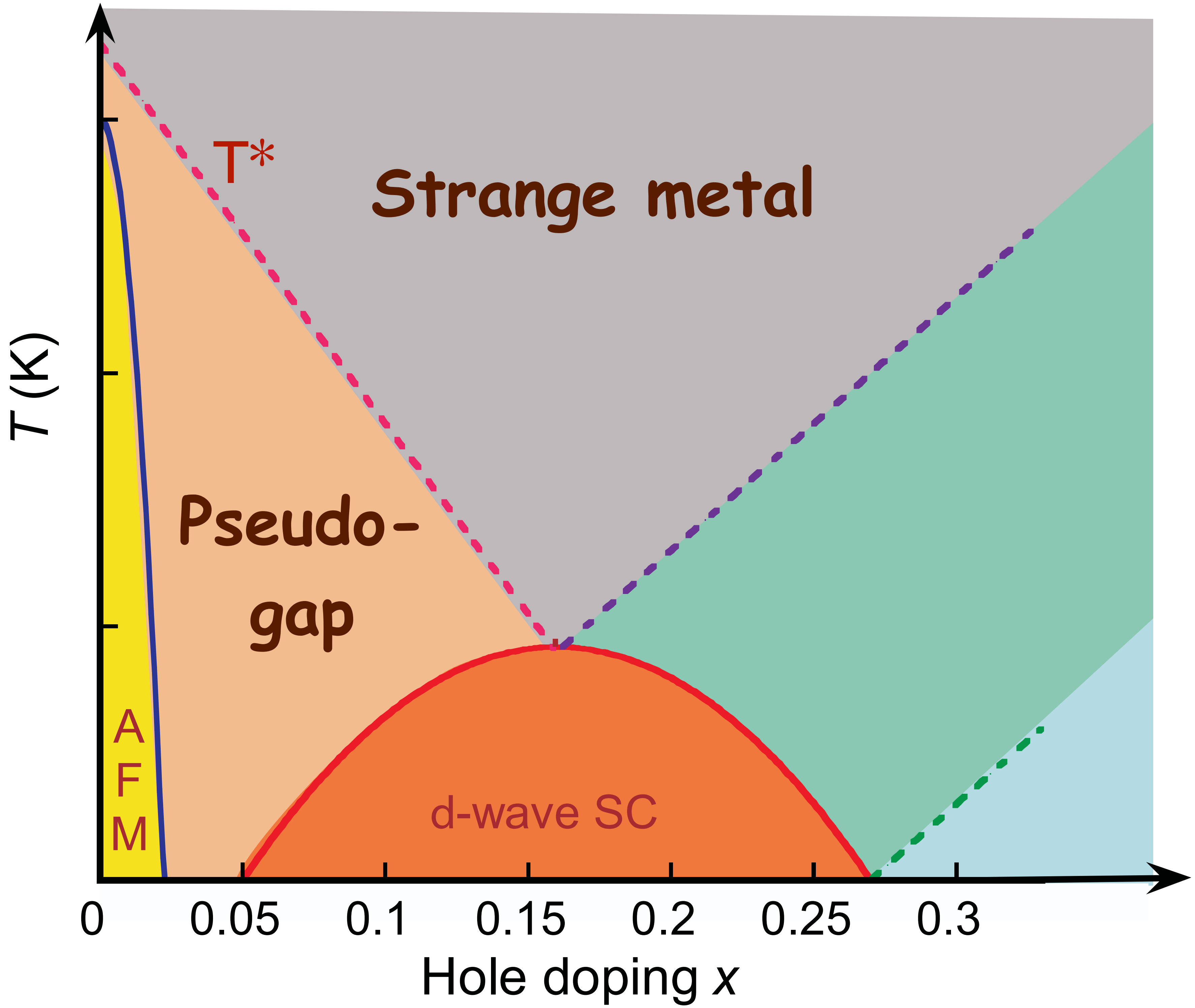
Fermi
surface

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



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Outline

1. Coupled dimer antiferromagnets

   Introduction to quantum criticality

2. Phase diagram of the cuprates

   Quantum criticality of the competition between antiferromagnetism and superconductivity

3. Influence of an applied magnetic field

   Theoretical predictions and experimental tests

4. Theory of Ising-nematic ordering in a metal

   Strong-coupling problems and the AdS/CFT correspondence

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Antiferro-
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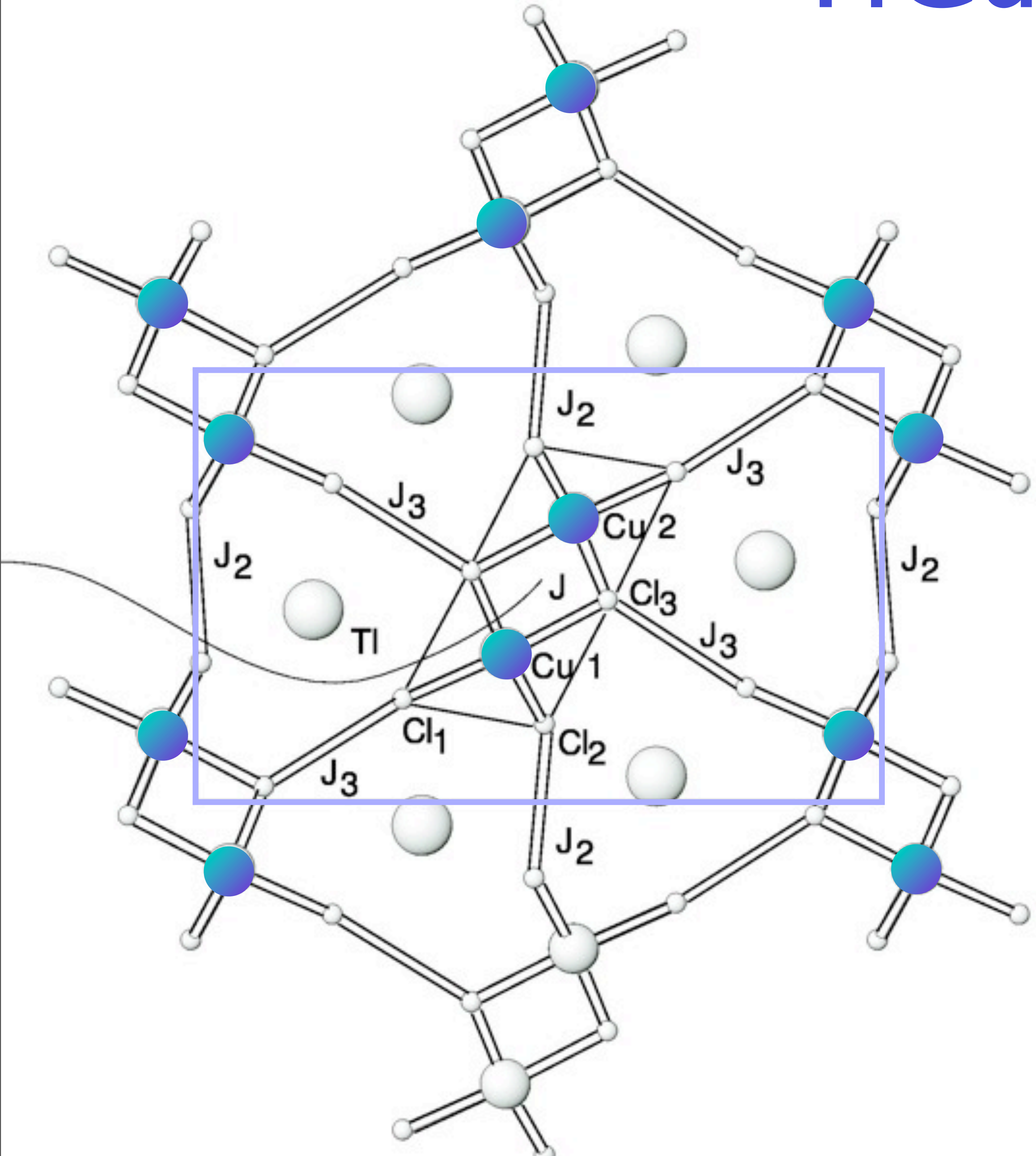
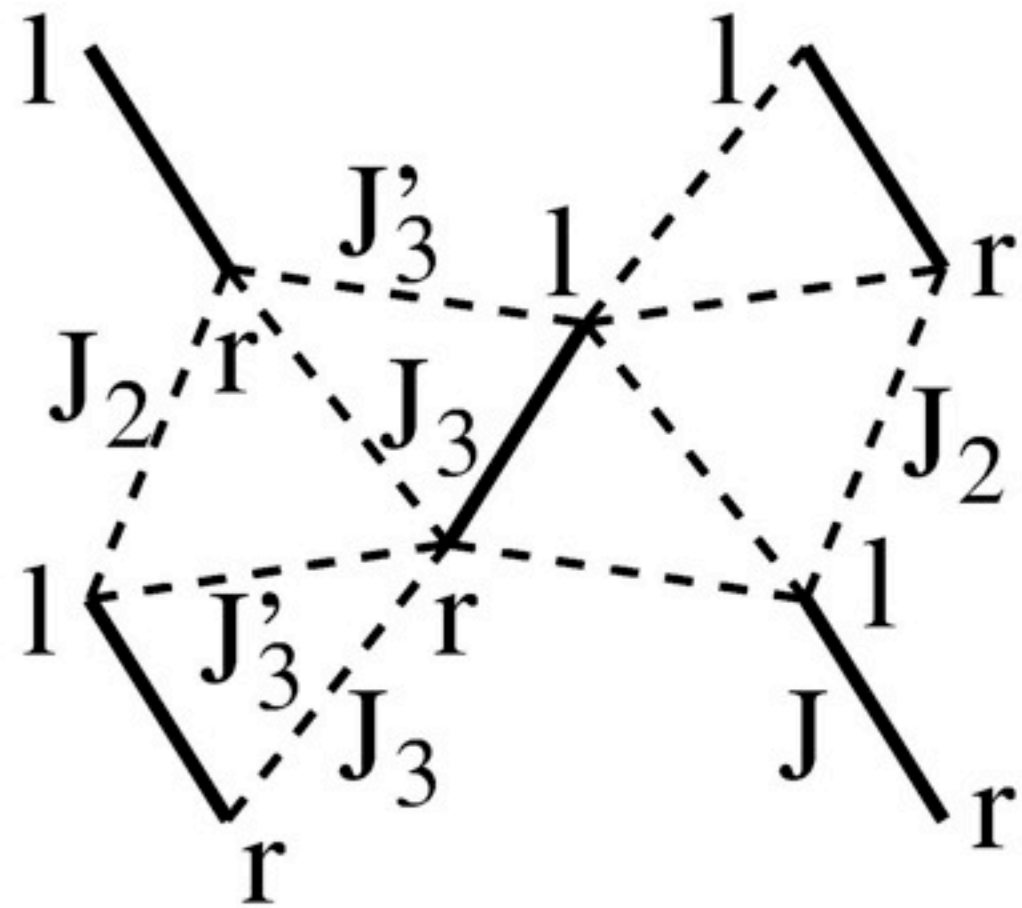
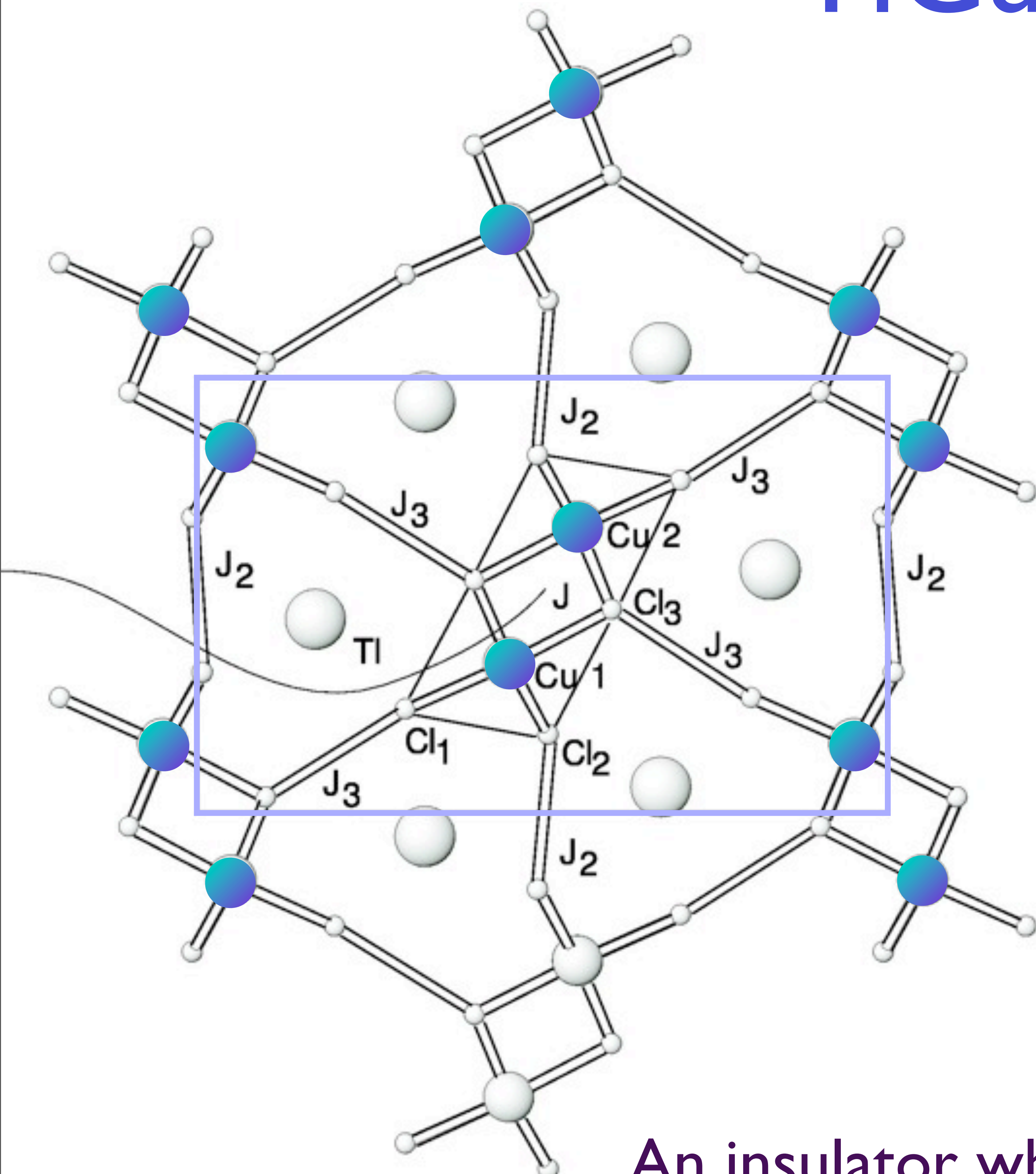
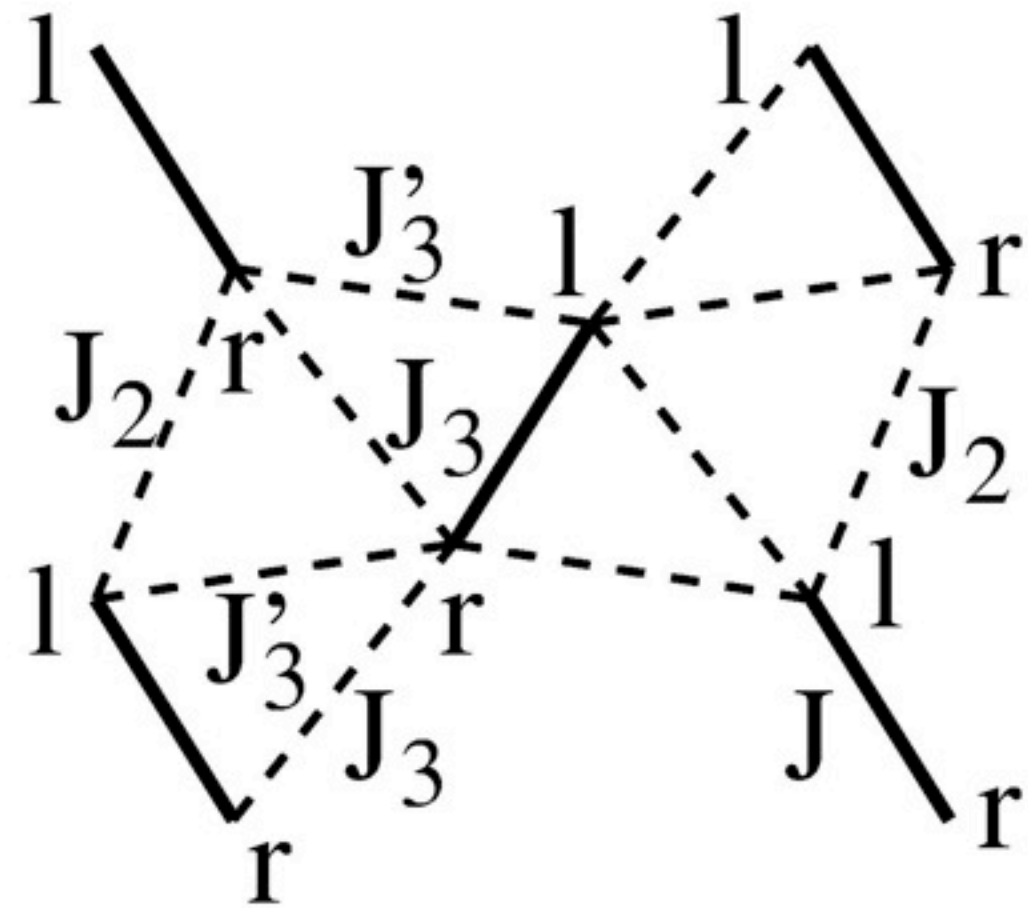
$TlCuCl_3$





TlCuCl$_3$

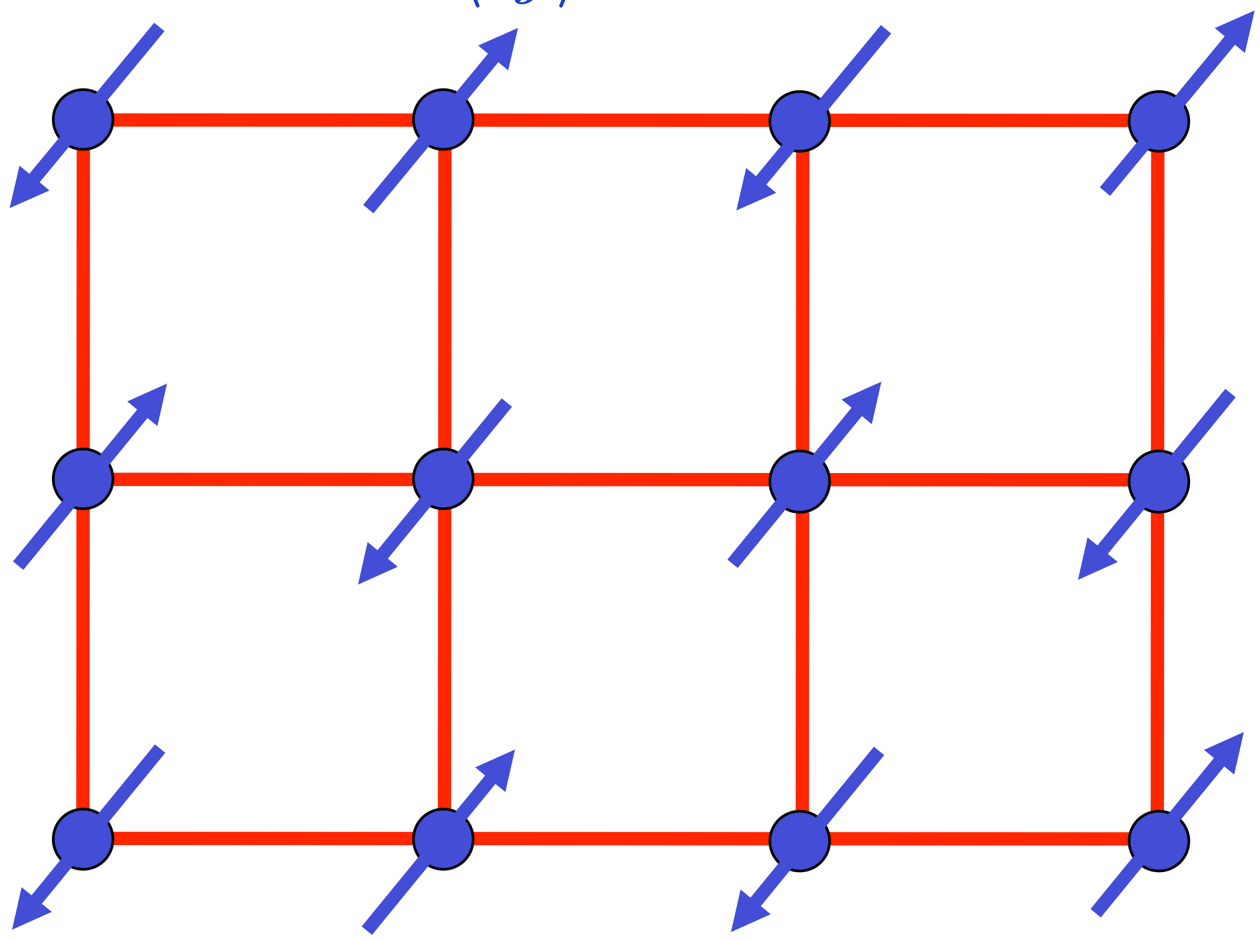




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



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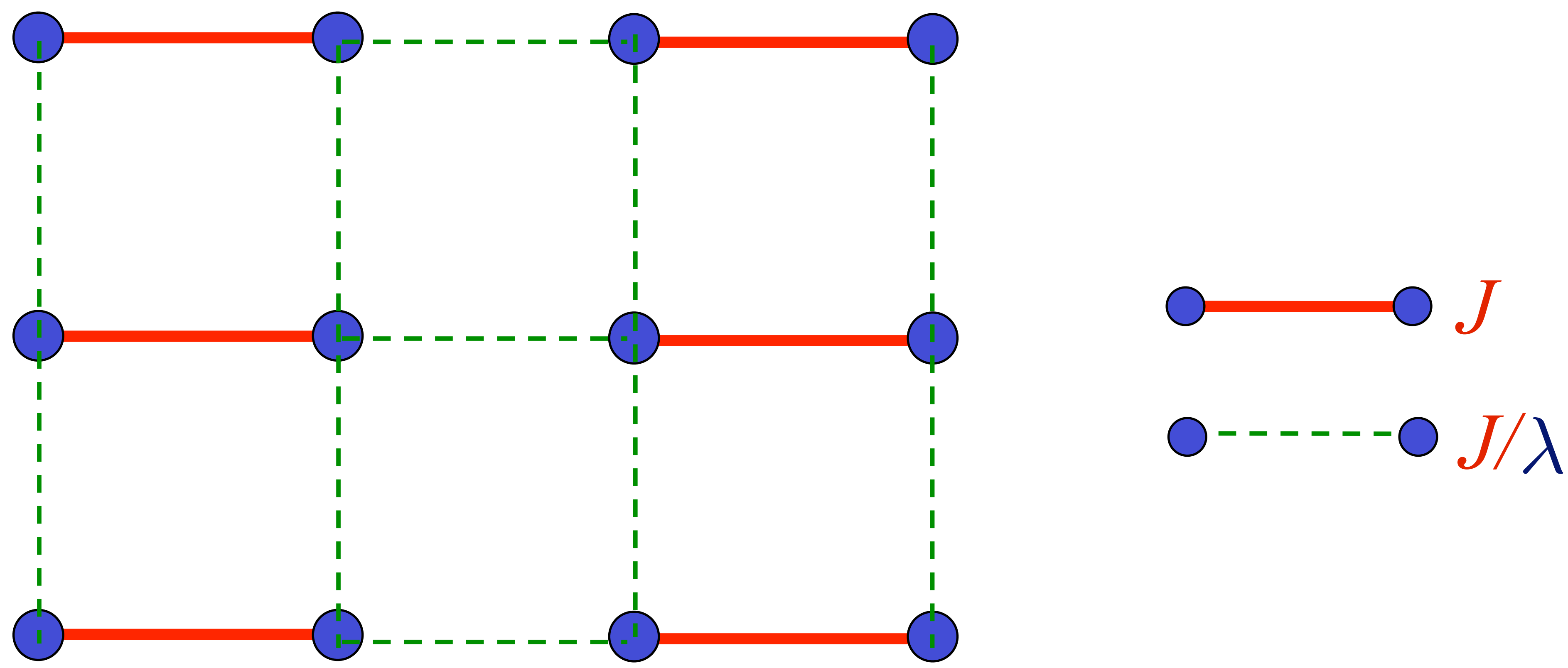

Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

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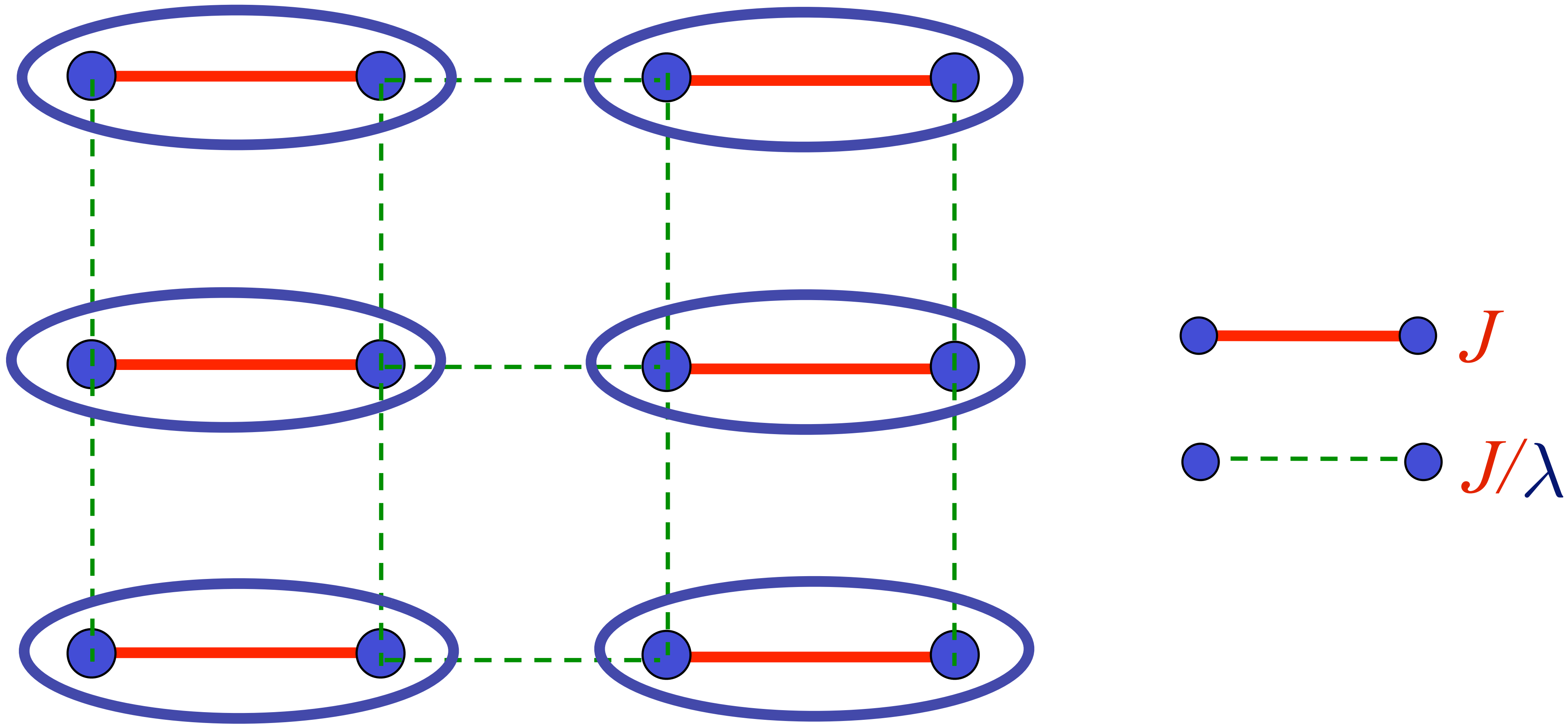
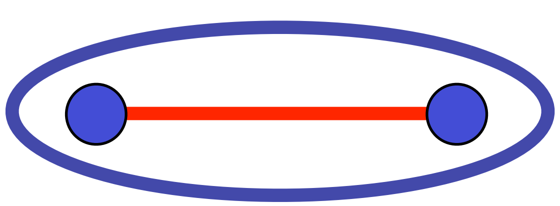
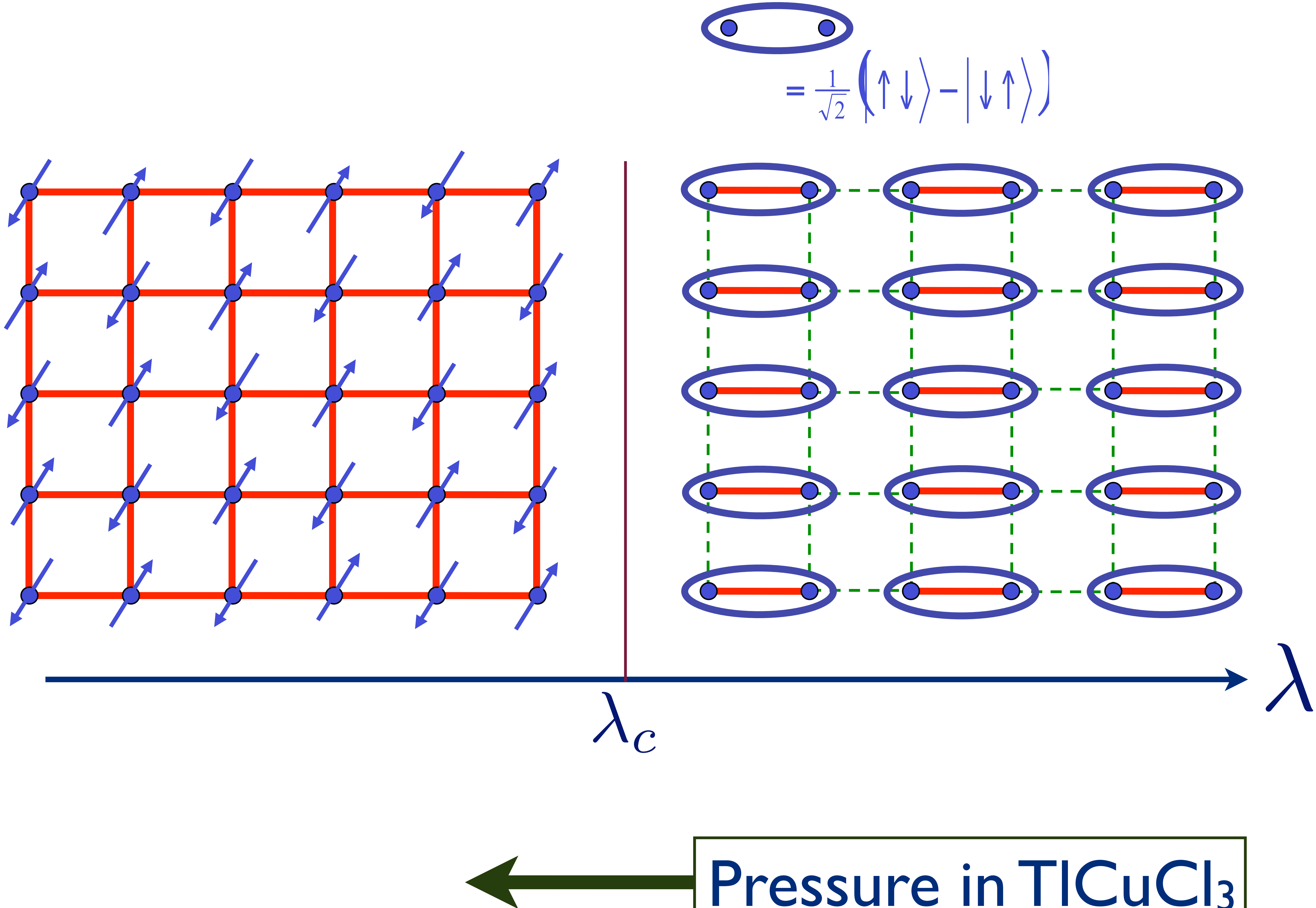


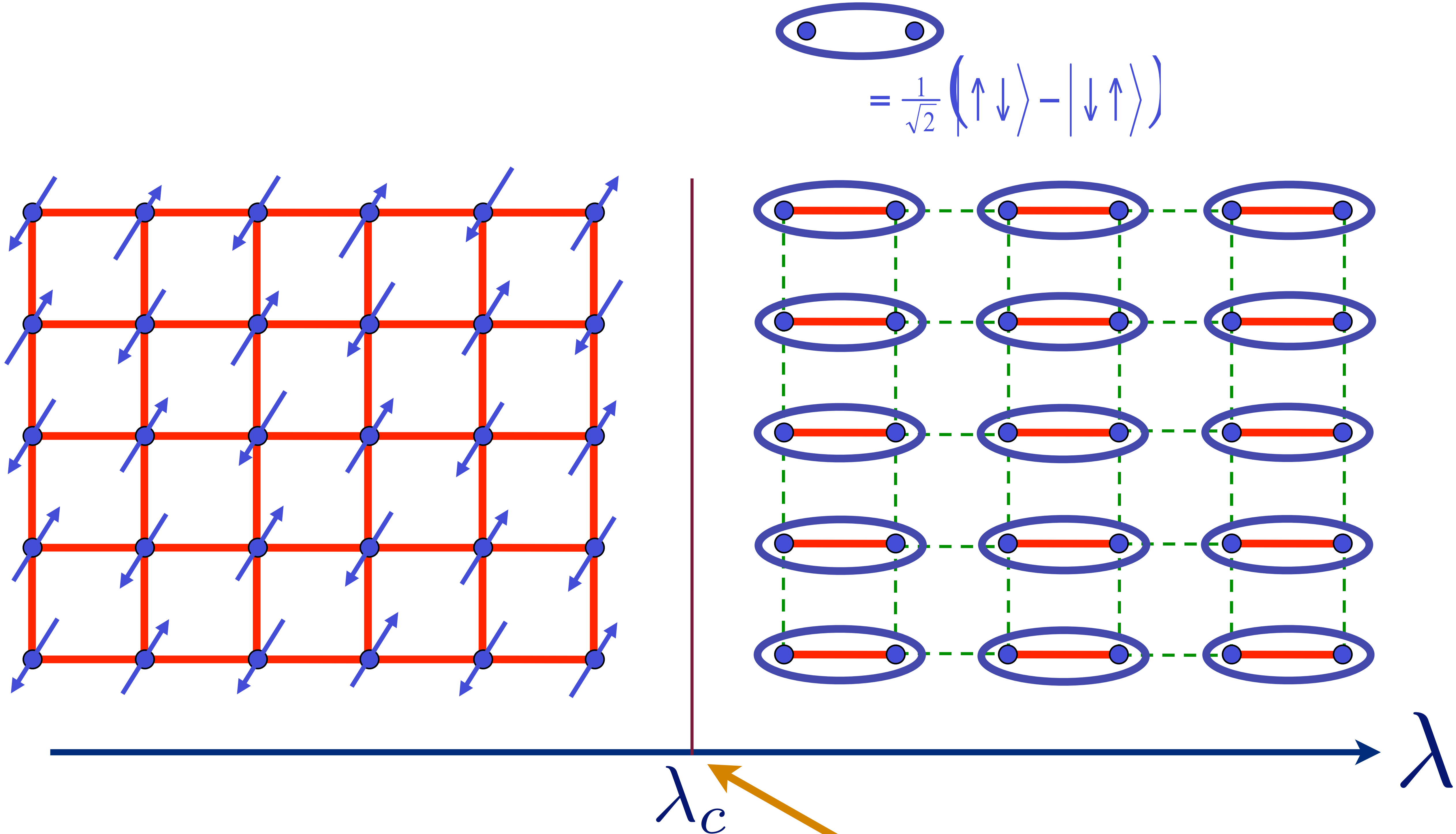
Ground state is a "quantum paramagnet" with spins locked in valence bond singlets



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

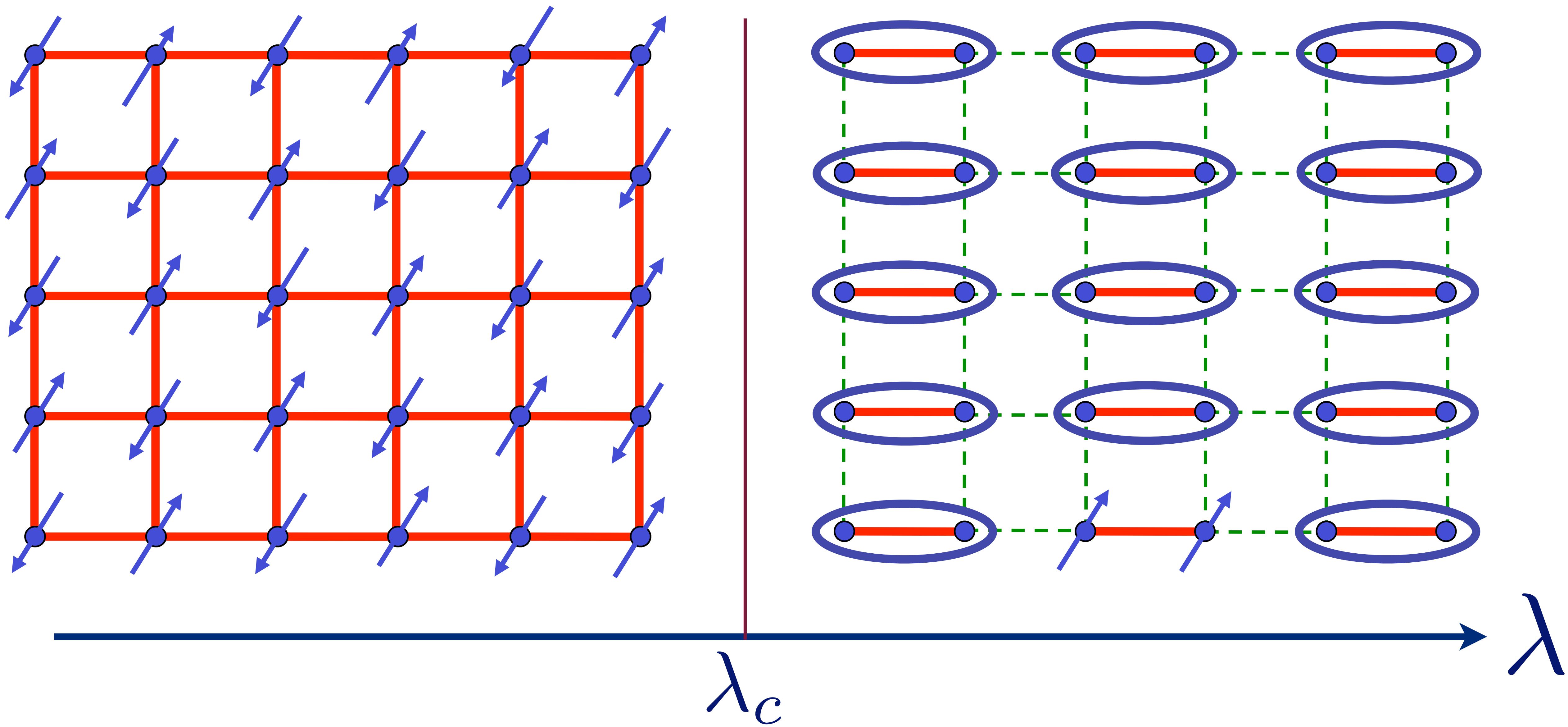


$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$
λ
λ_c
Pressure in $TlCuCl_3$



Quantum critical point with non-local entanglement in spin wavefunction

Excitation spectrum in the paramagnetic phase



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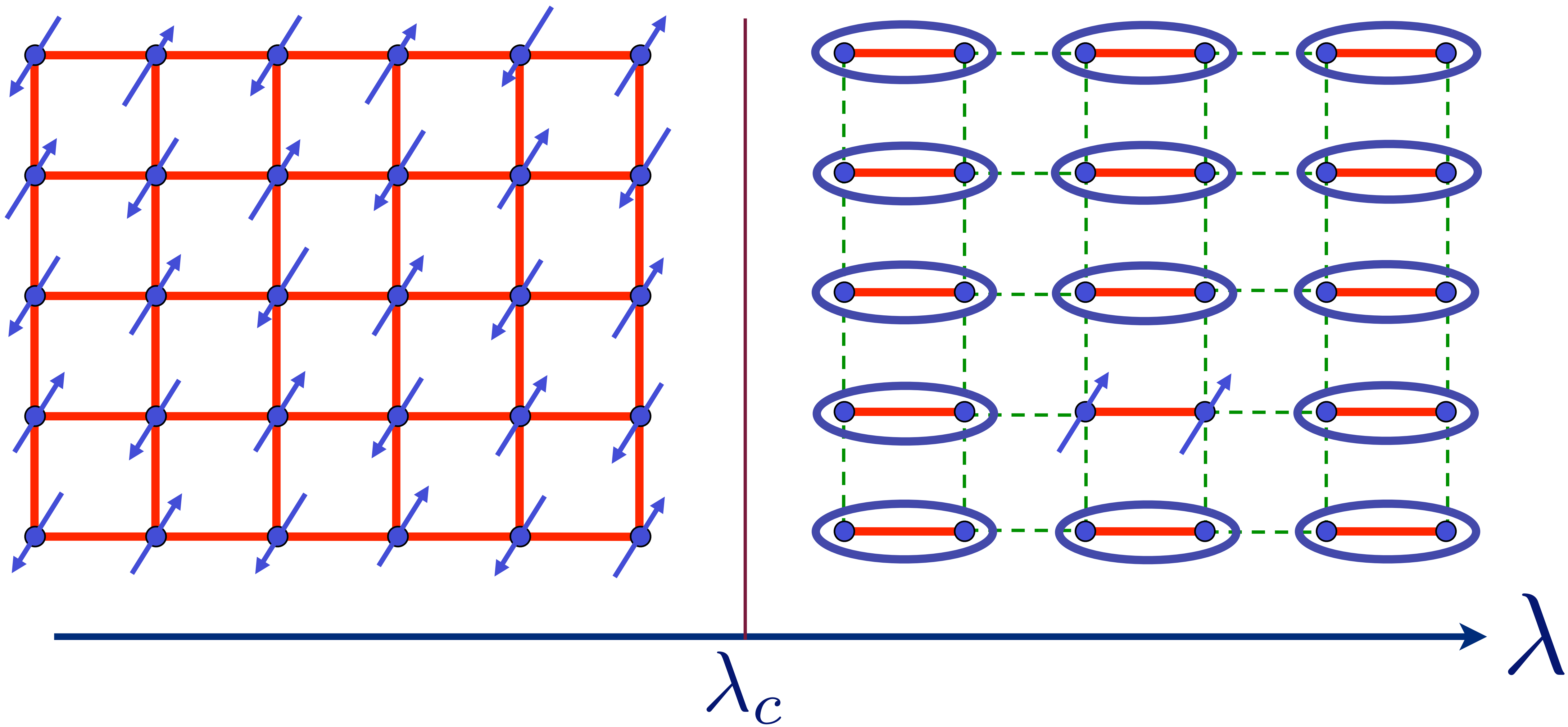

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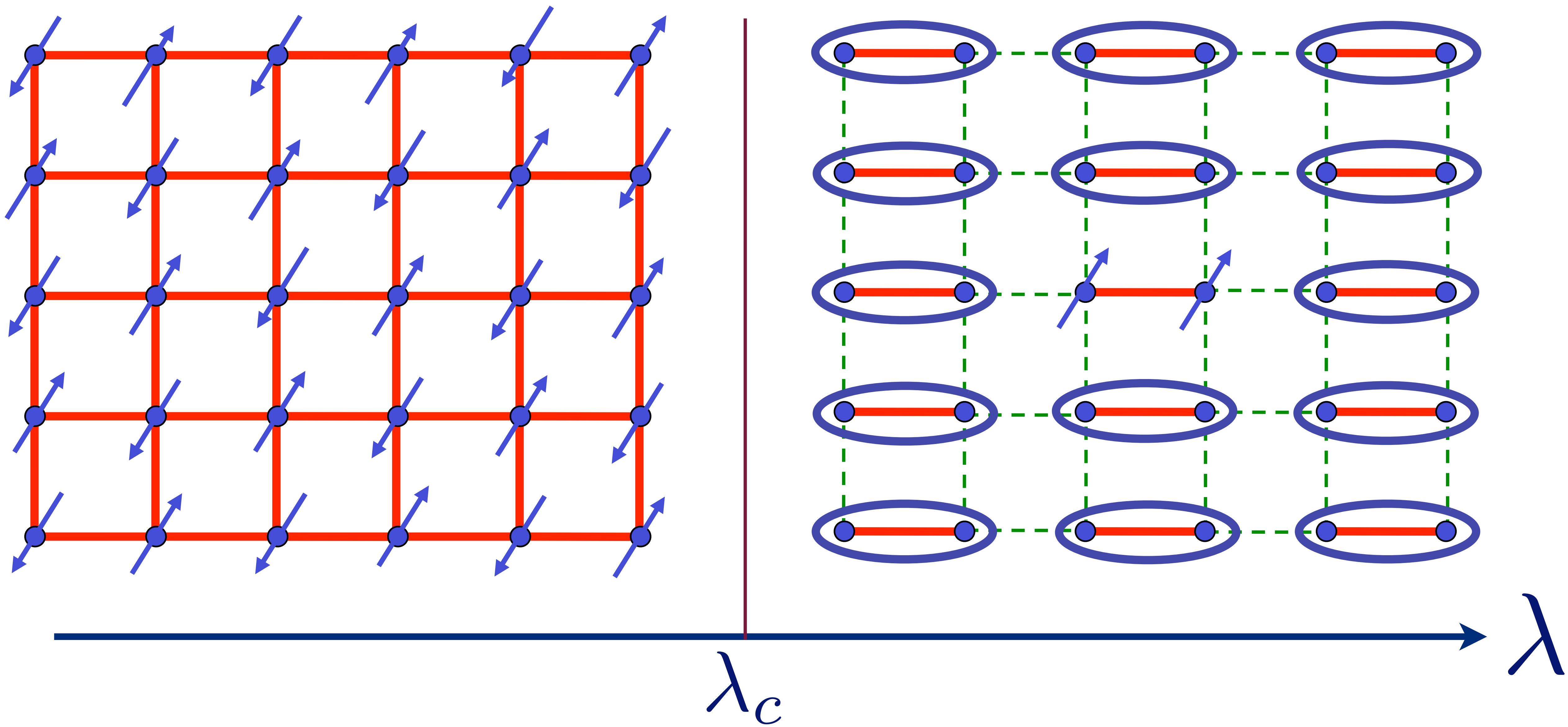

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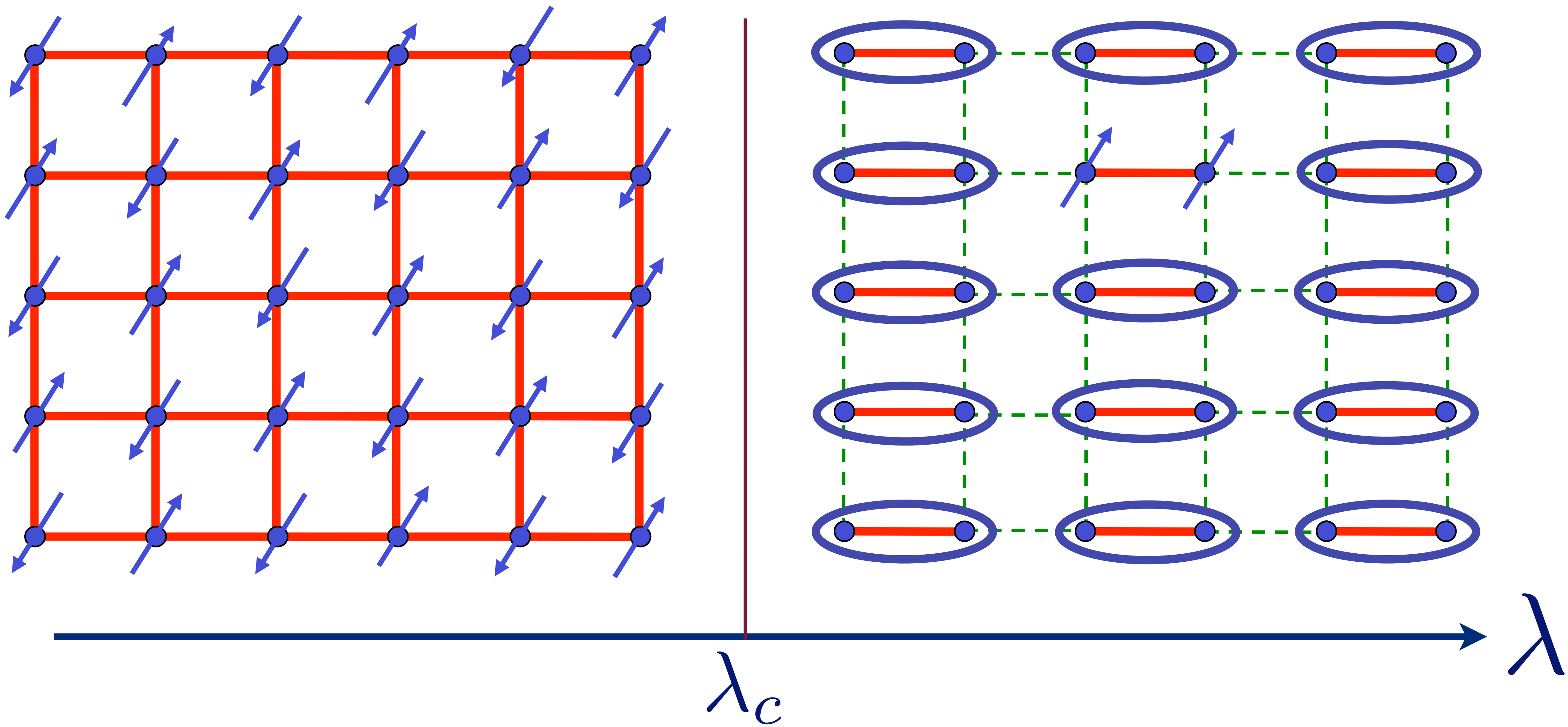

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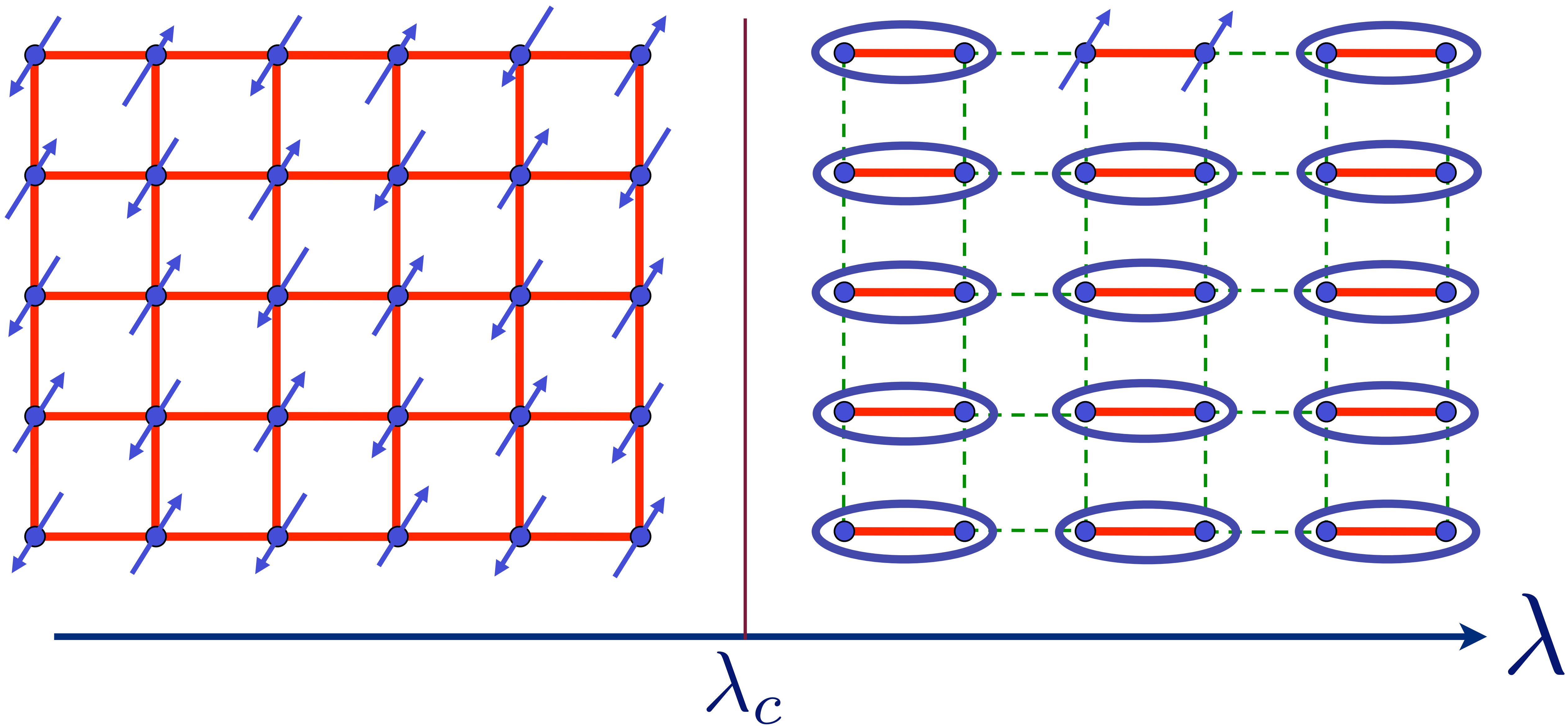
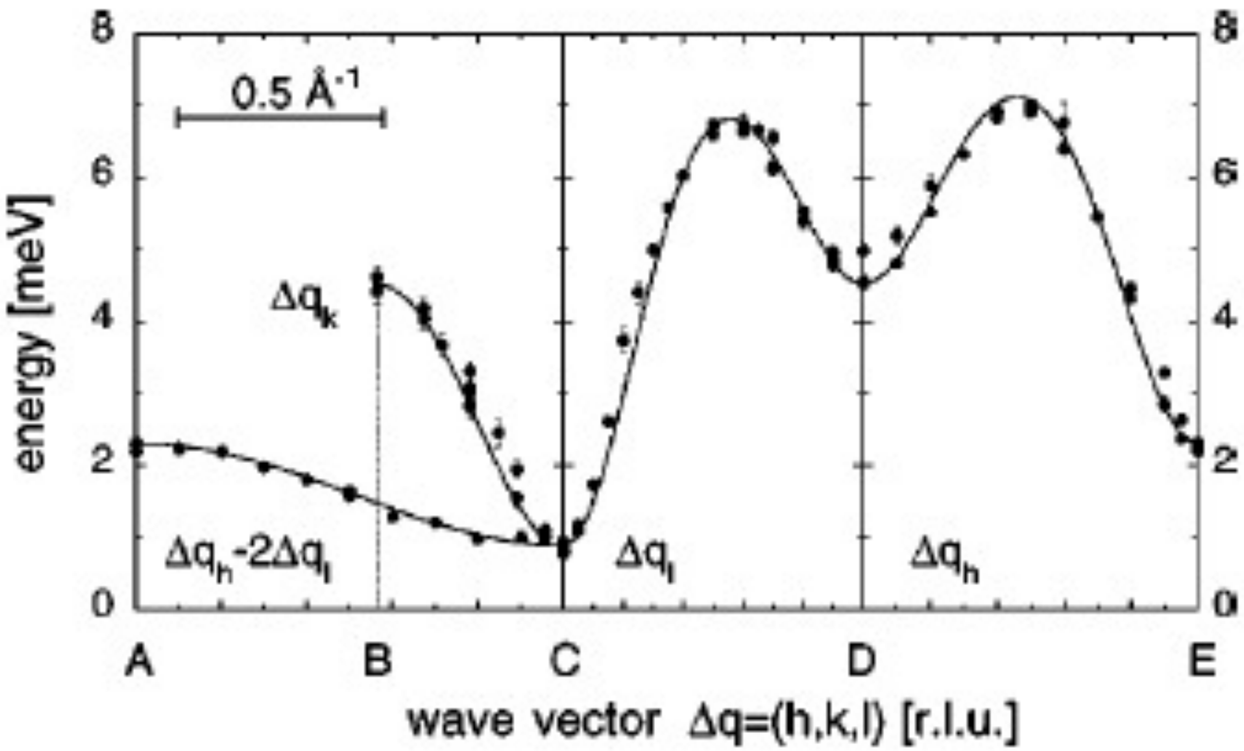


$TlCuCl_3$ at ambient pressure



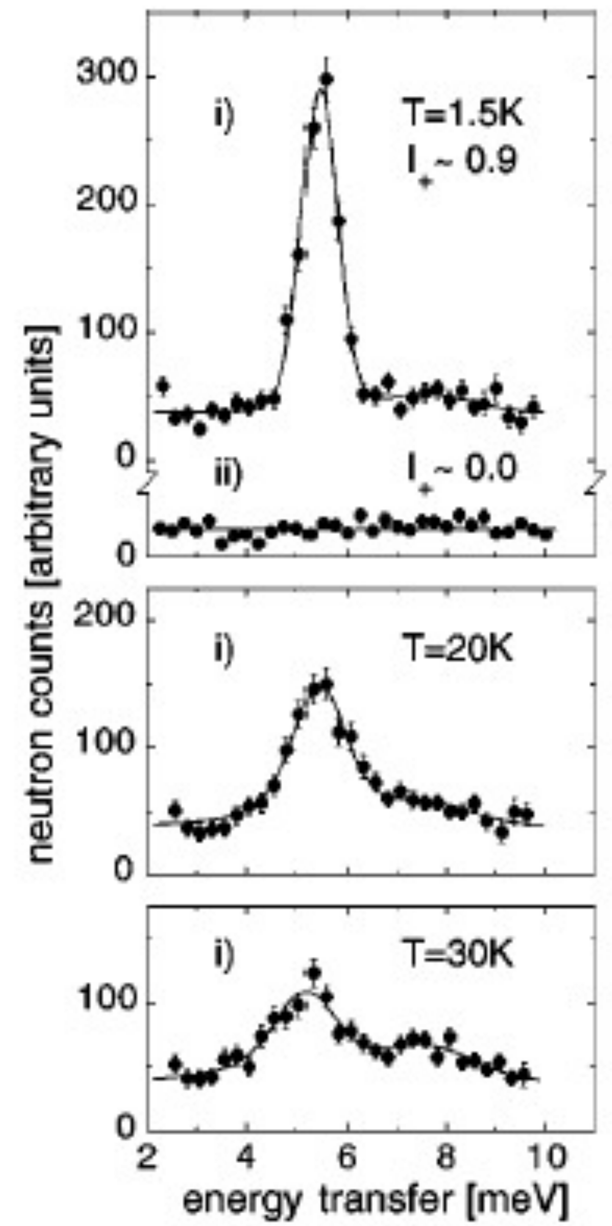


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

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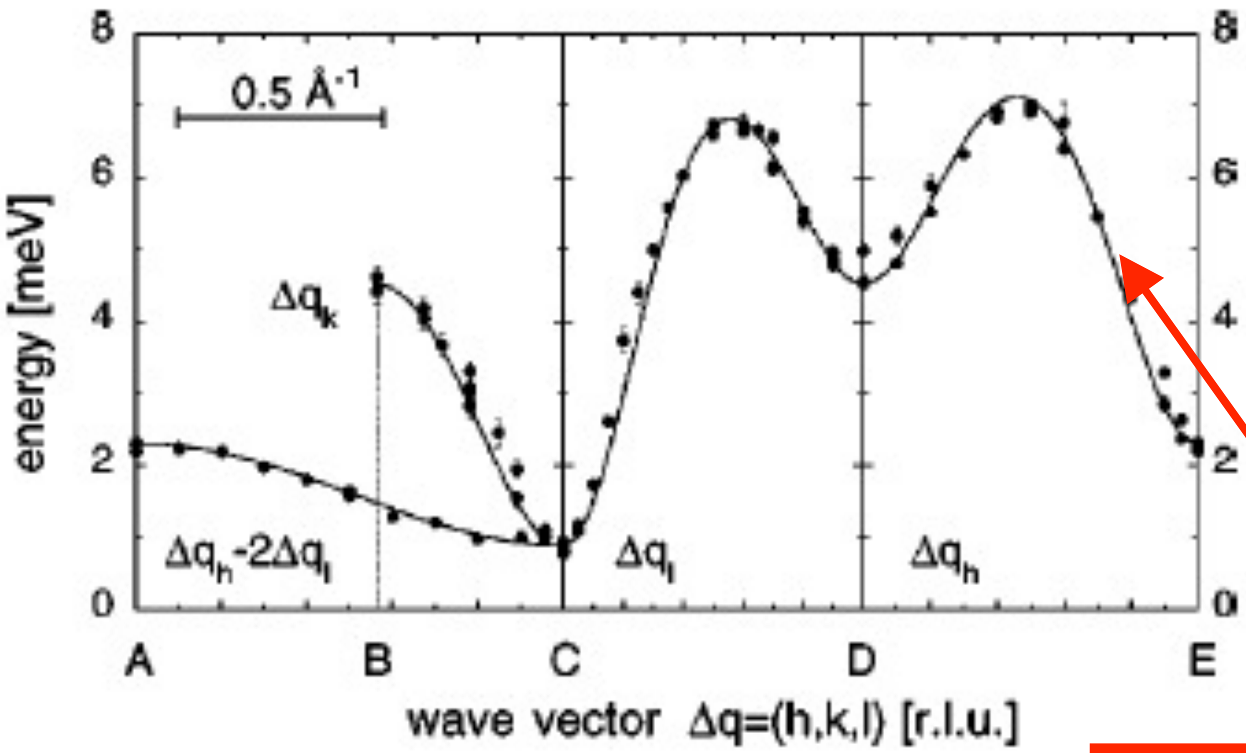

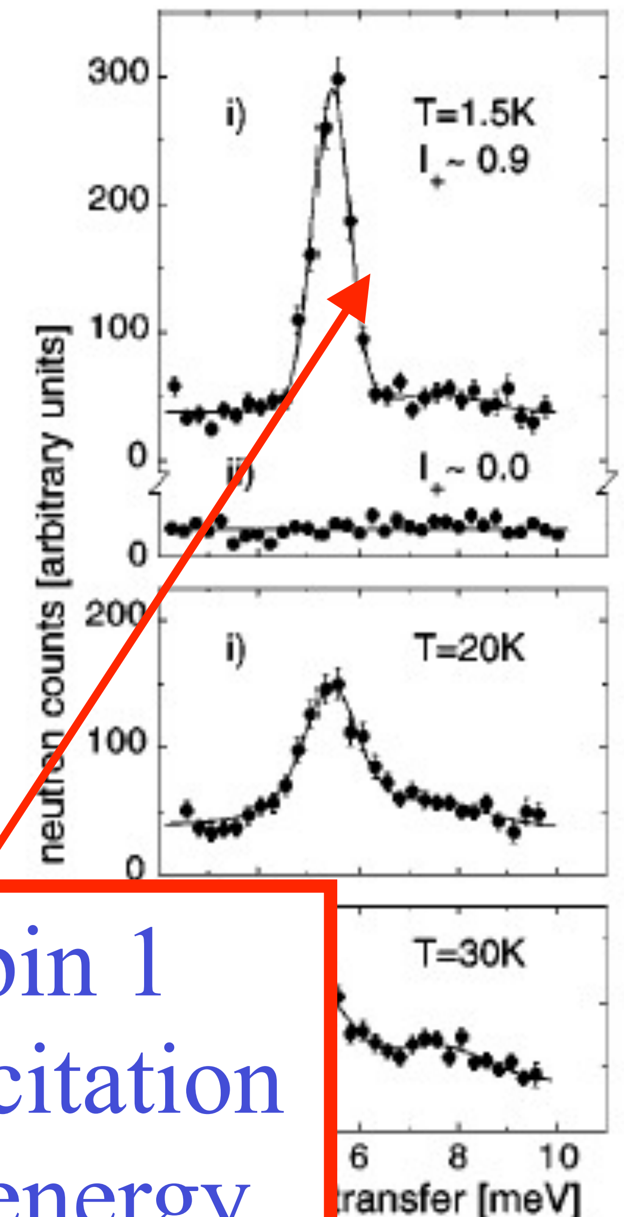

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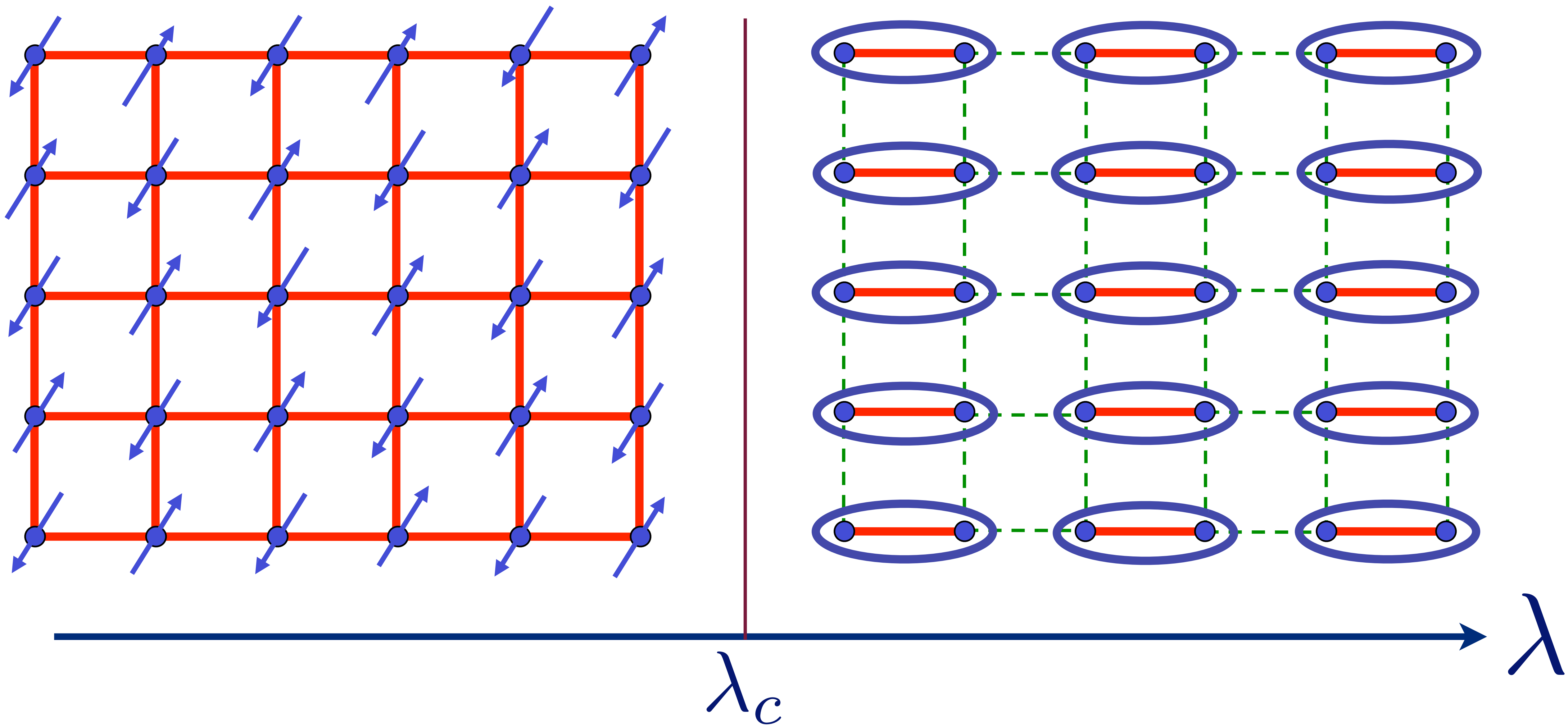
Sharp spin 1 particle excitation above an energy gap (spin gap)

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

Excitation spectrum in the Néel phase



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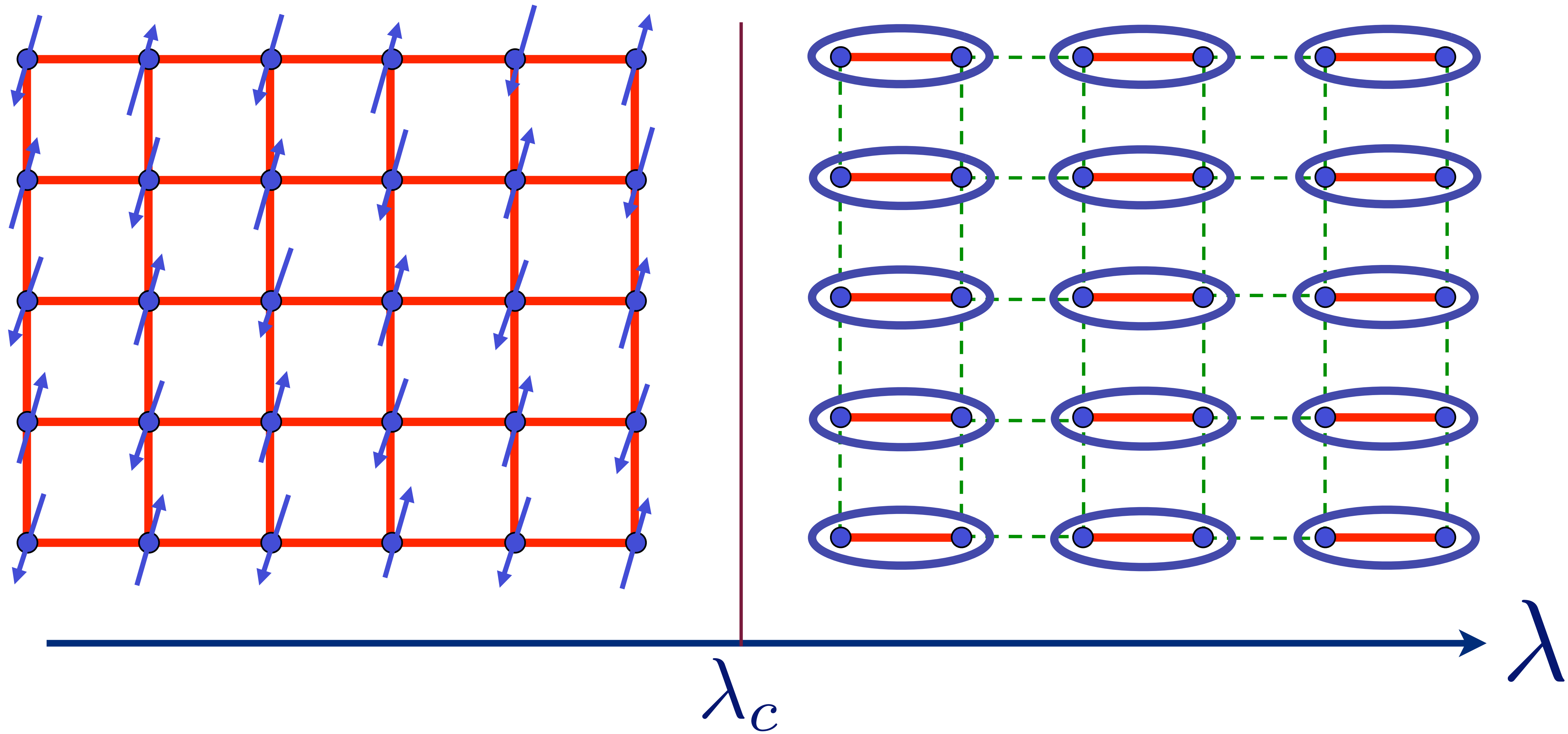

Spin waves

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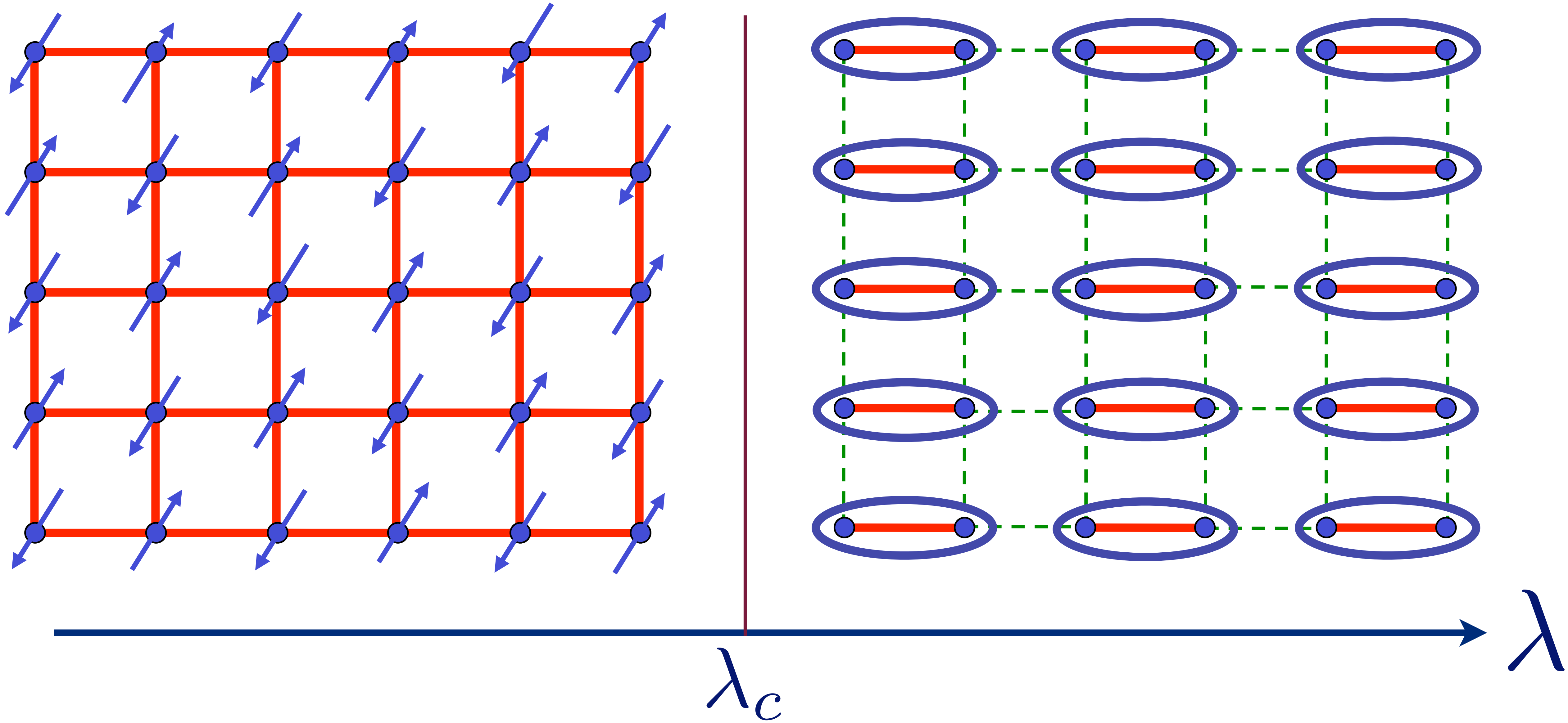
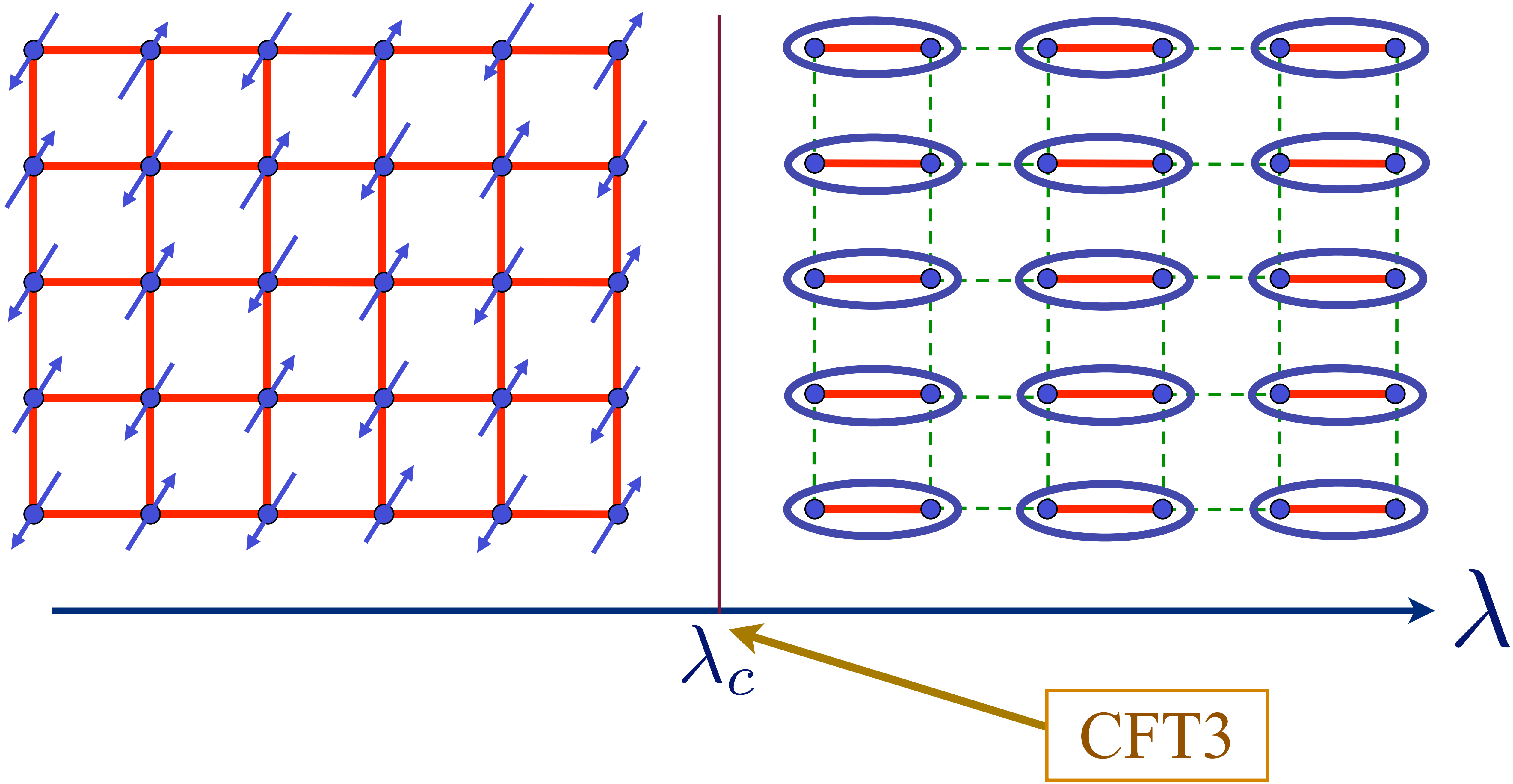


Spin waves

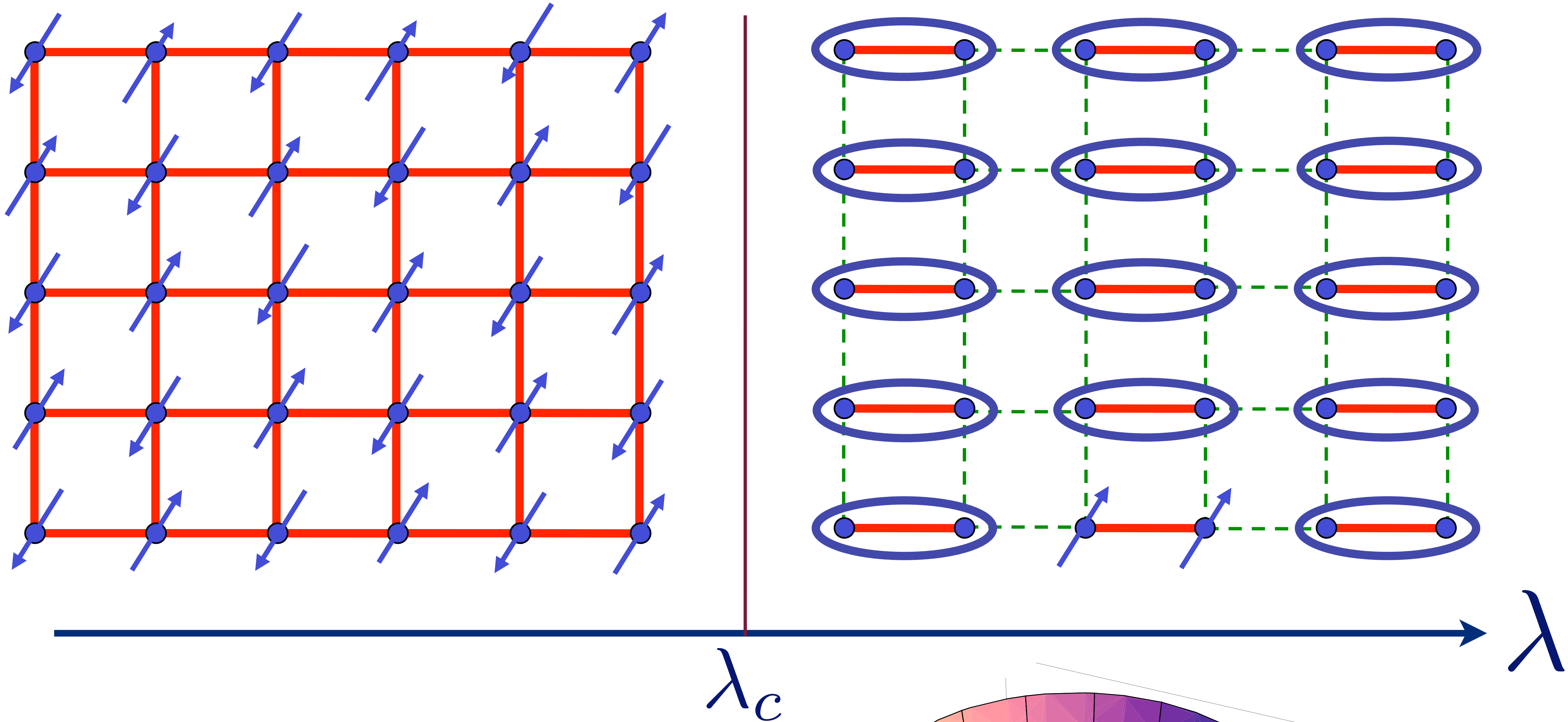
Description using Landau-Ginzburg field theory



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



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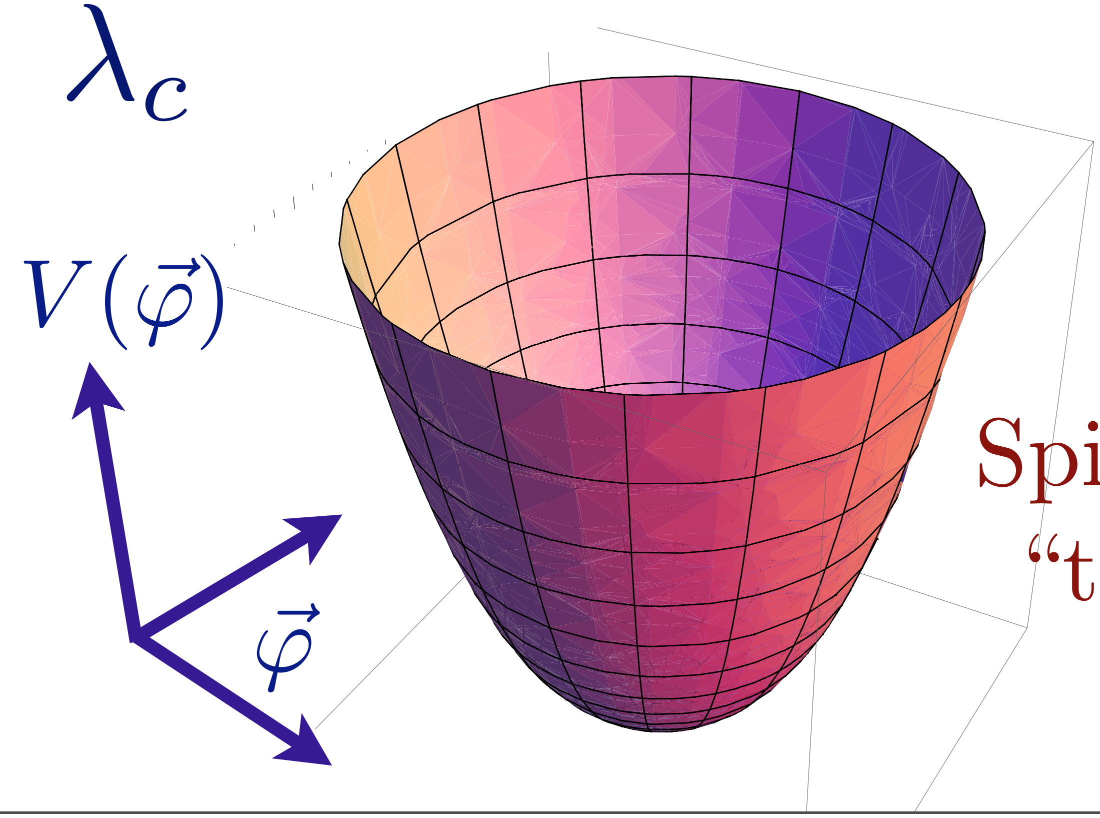
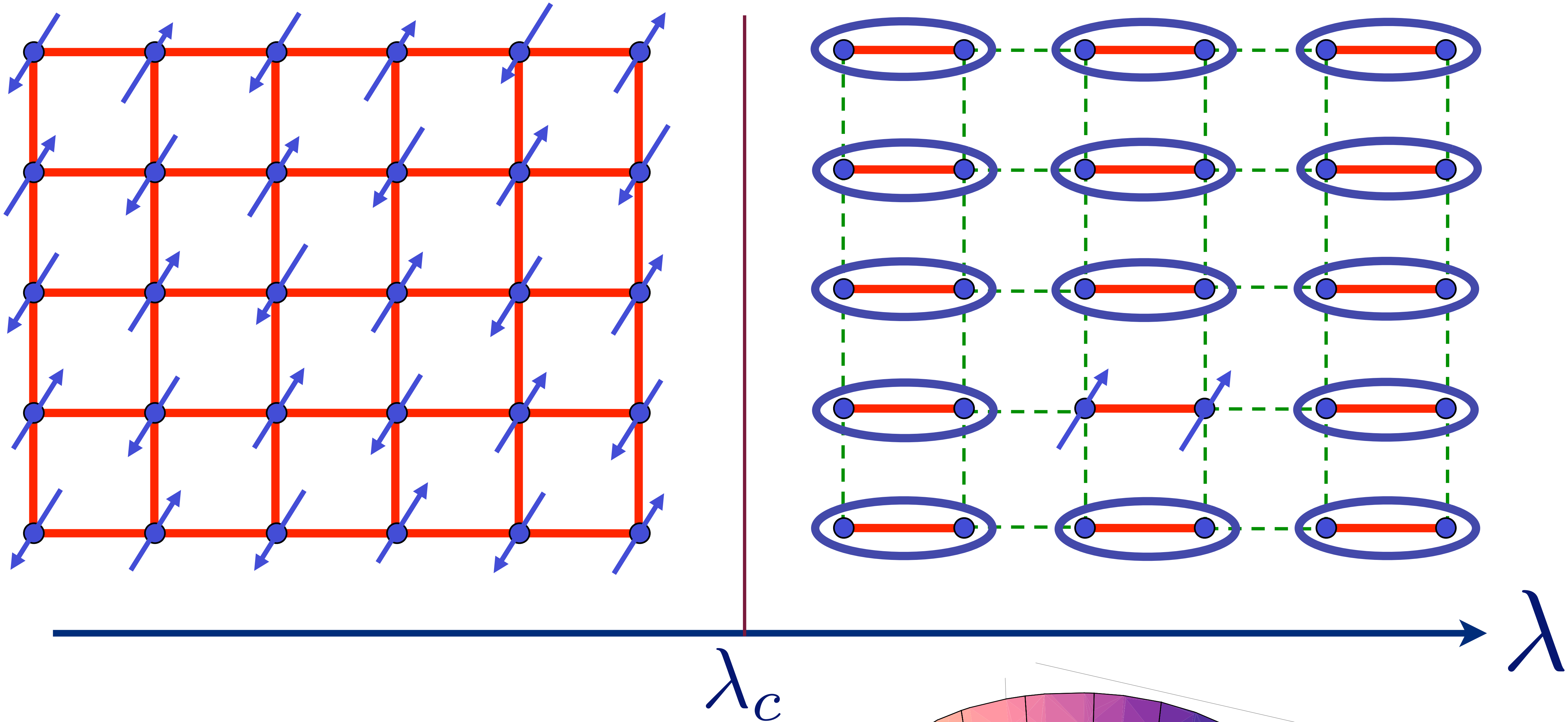


Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



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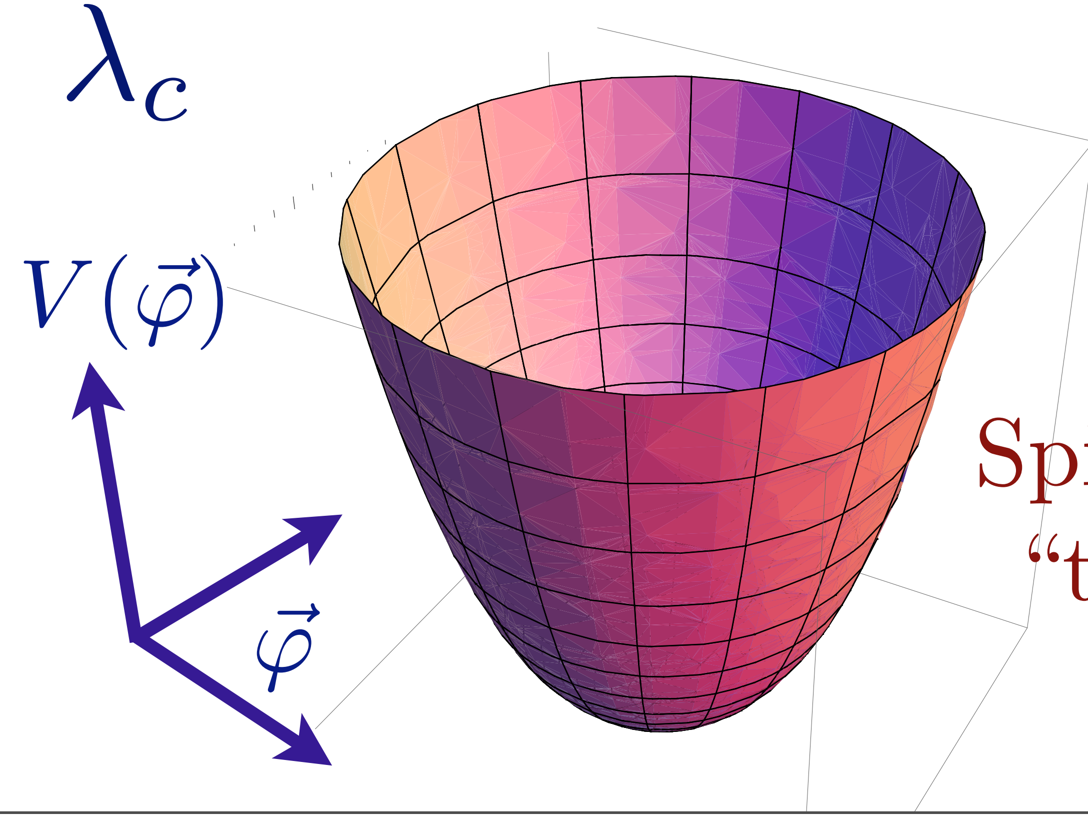

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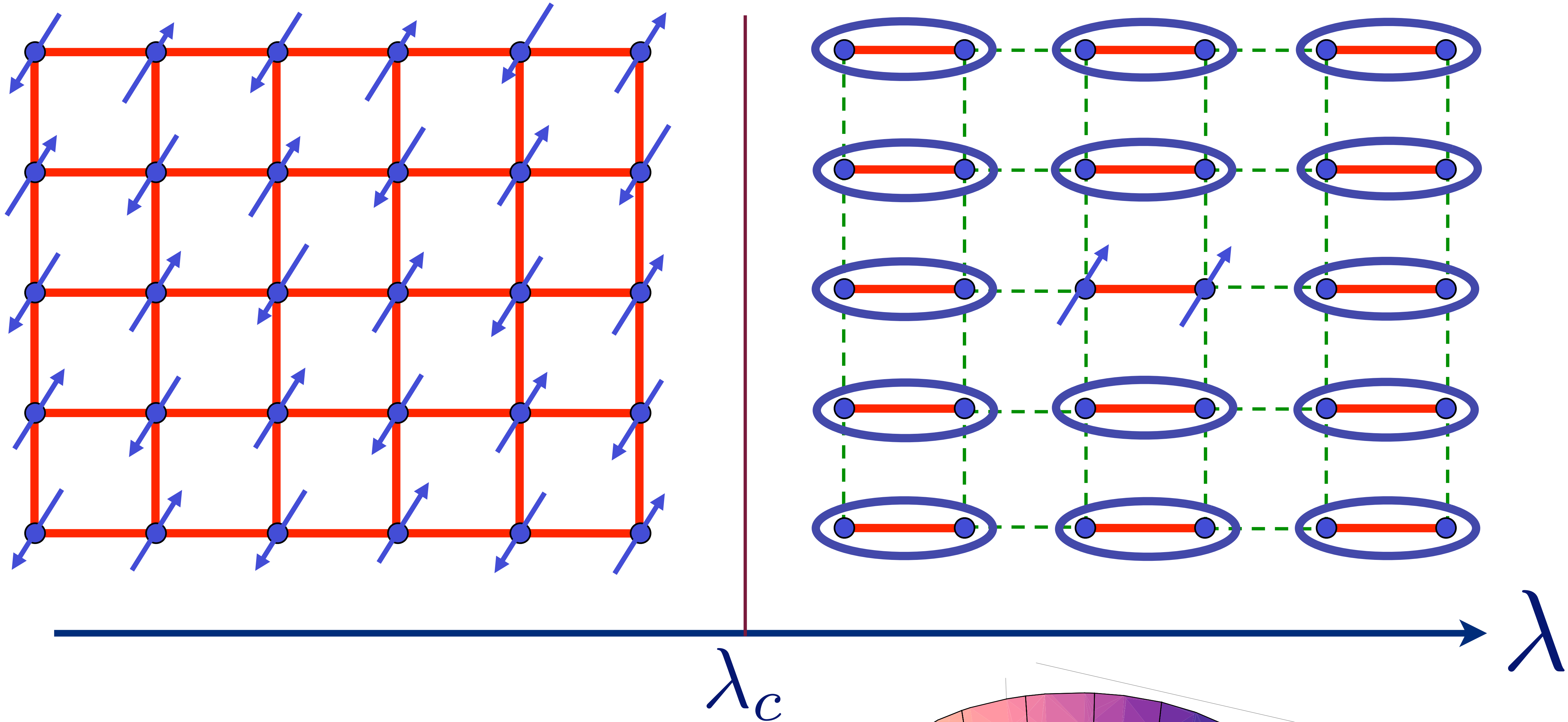

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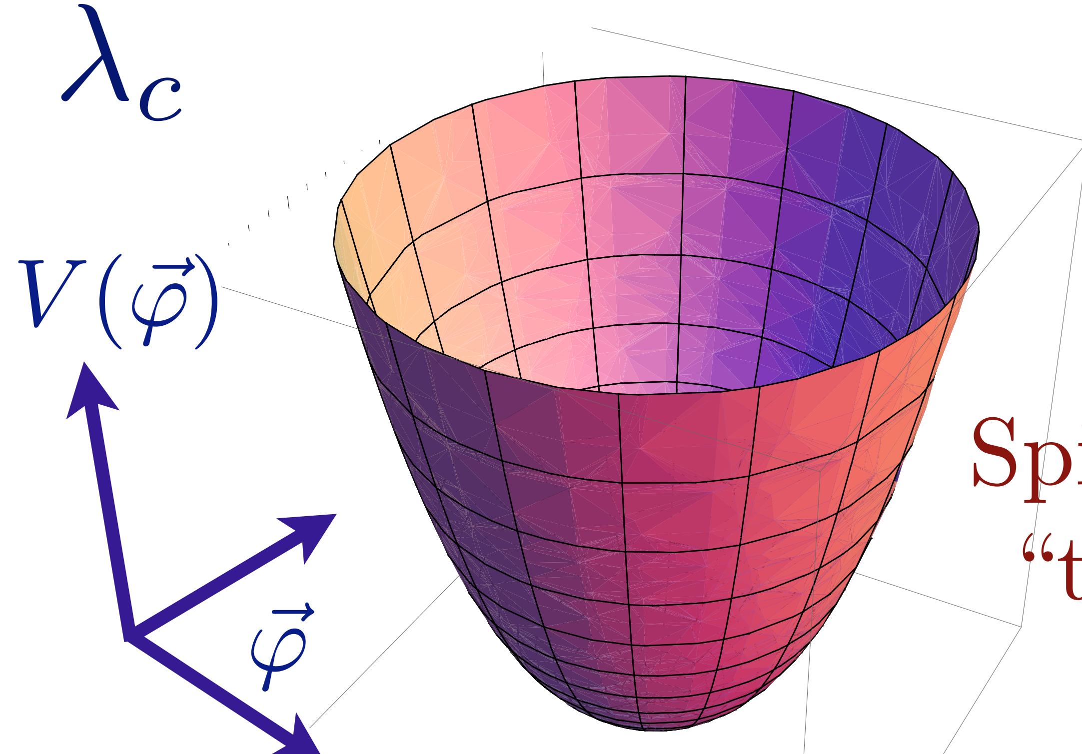

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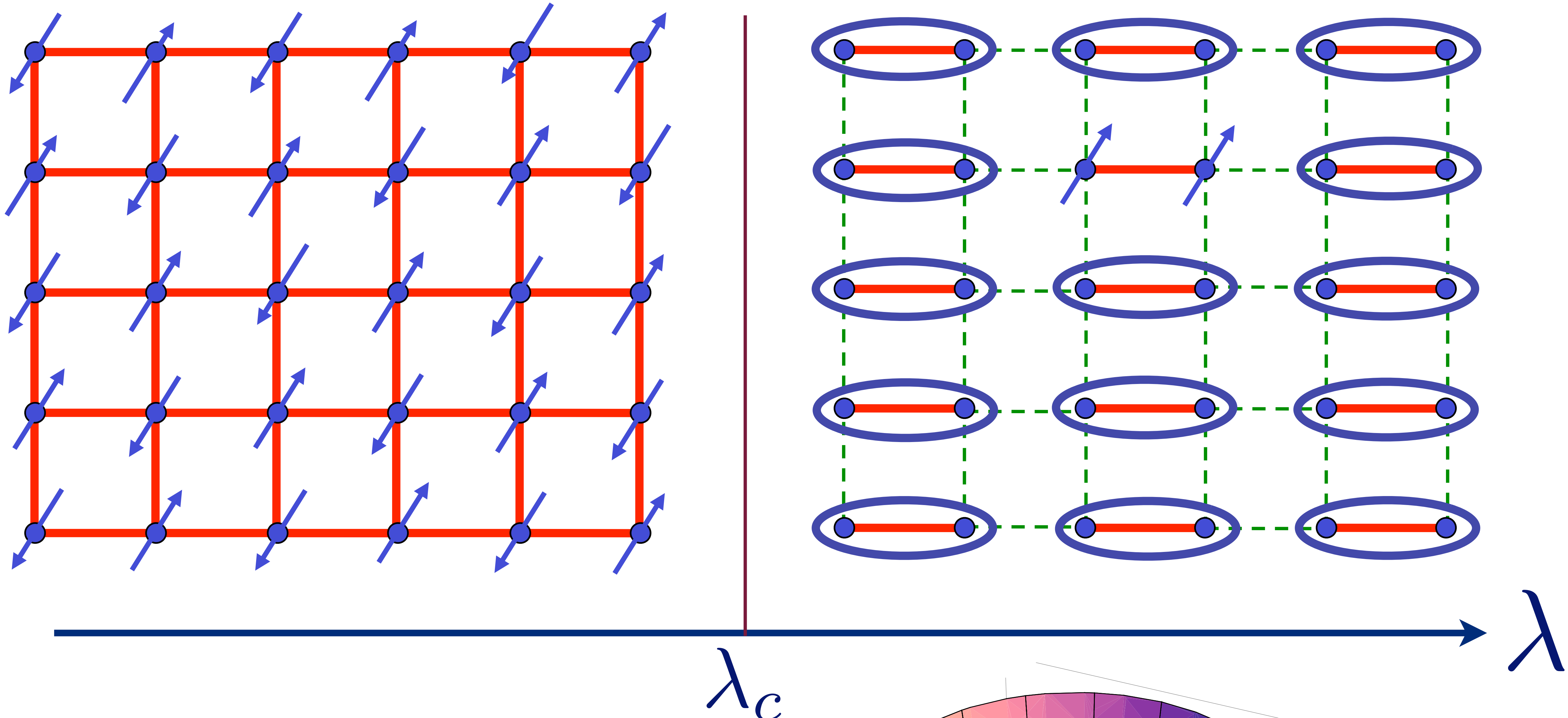

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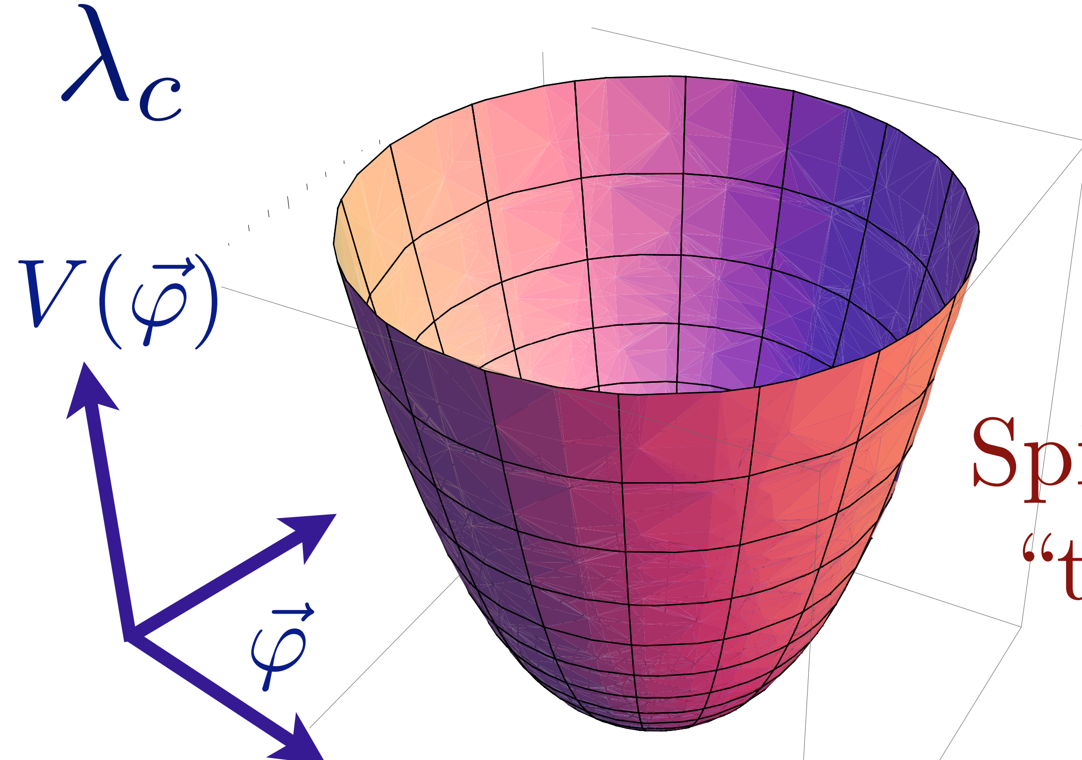

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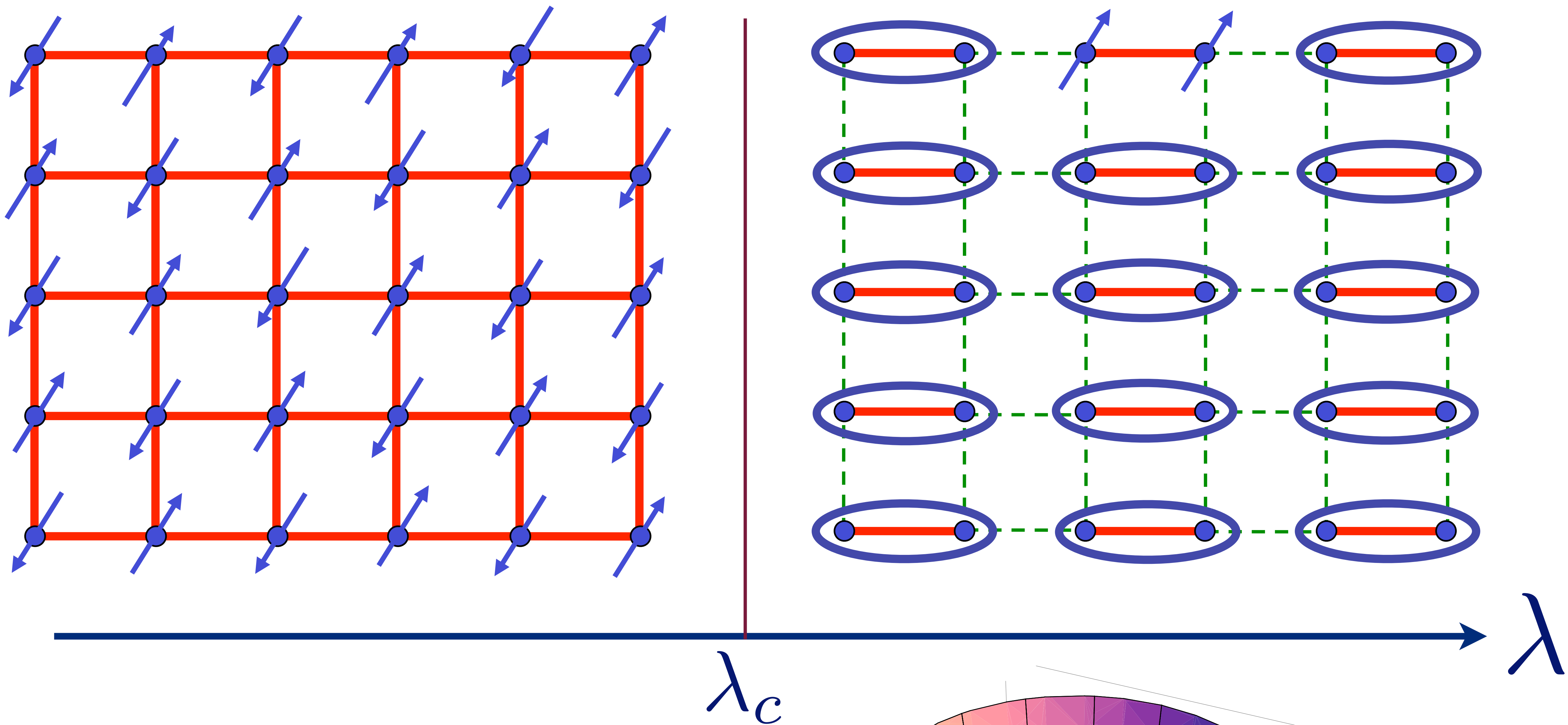

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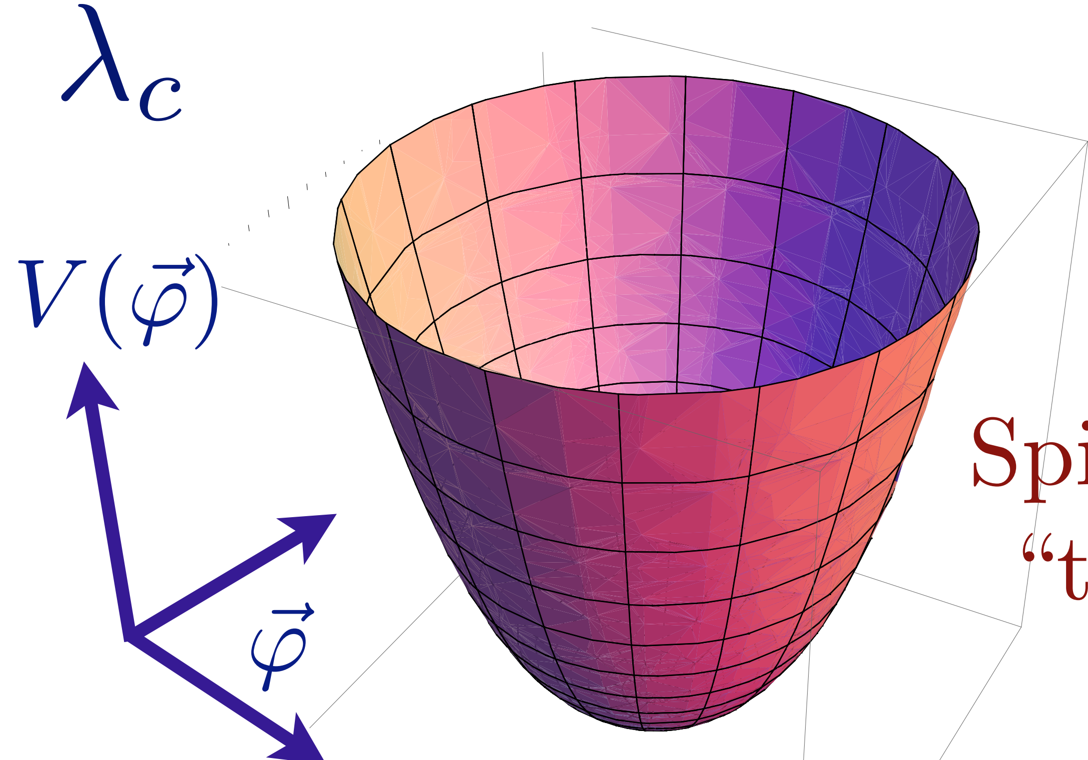

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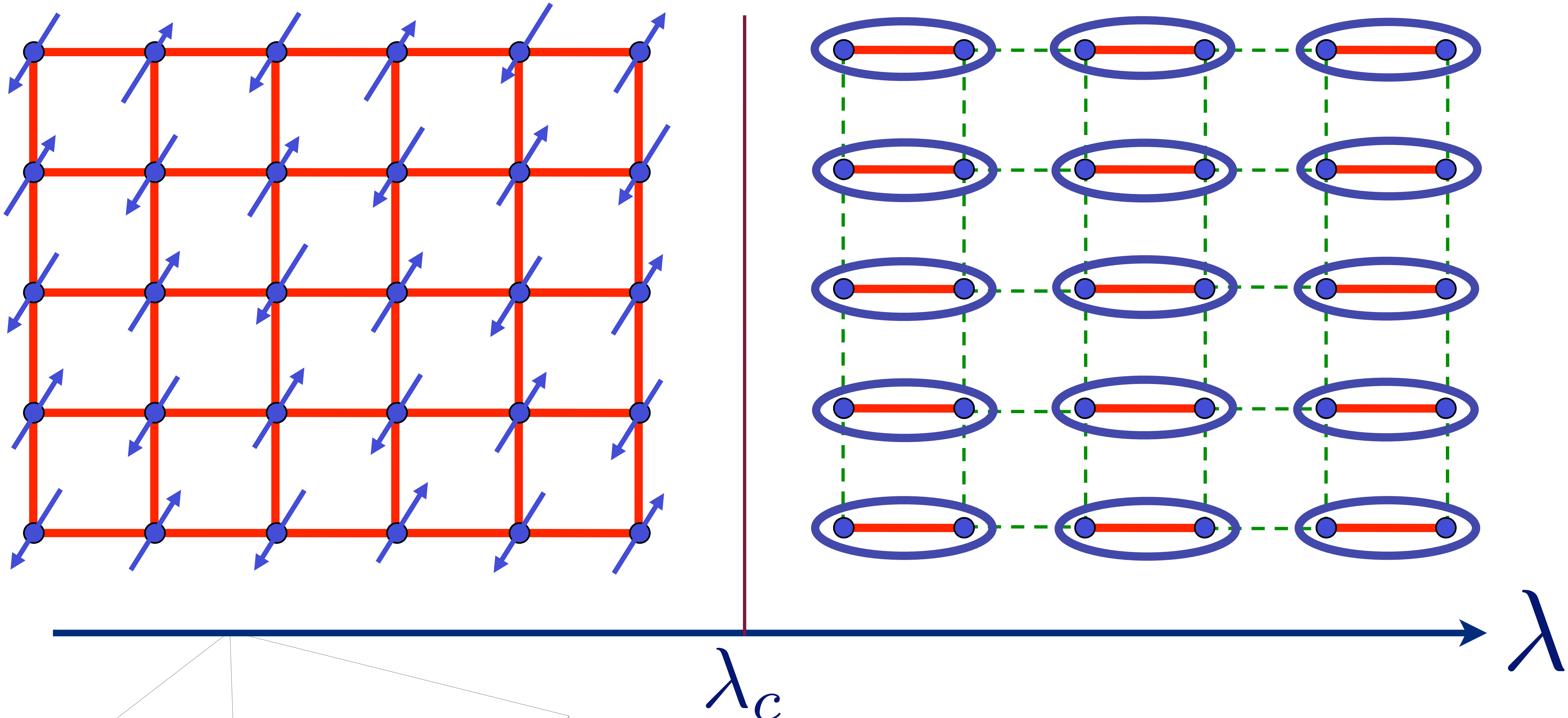

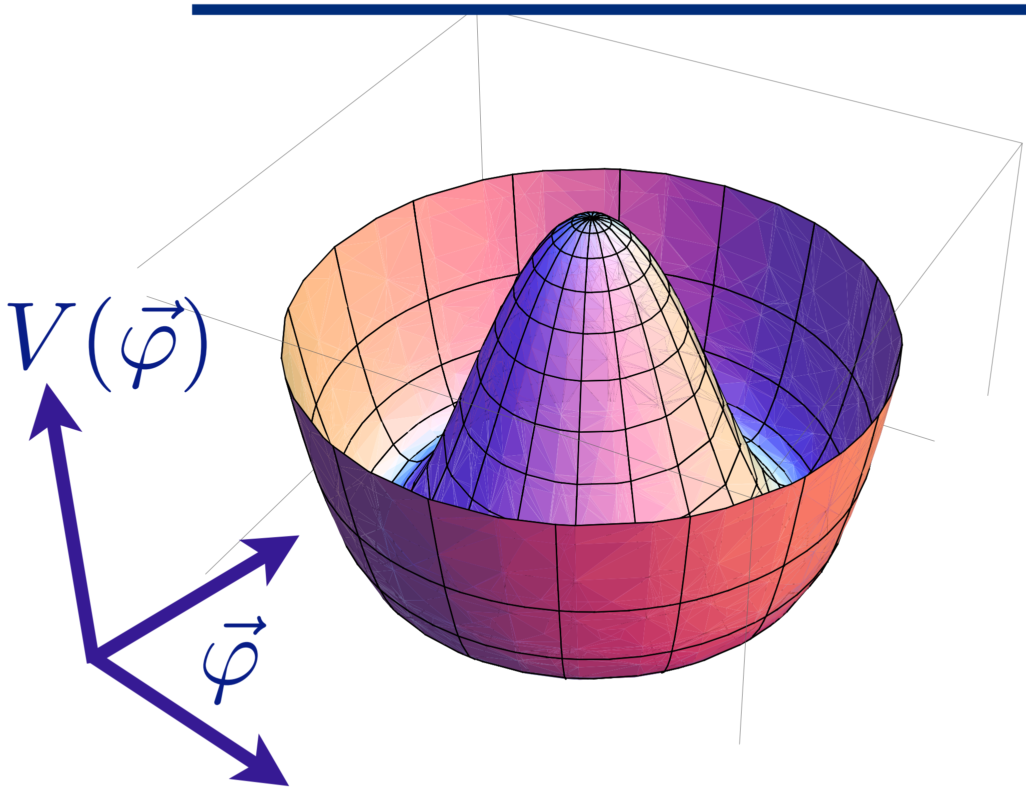

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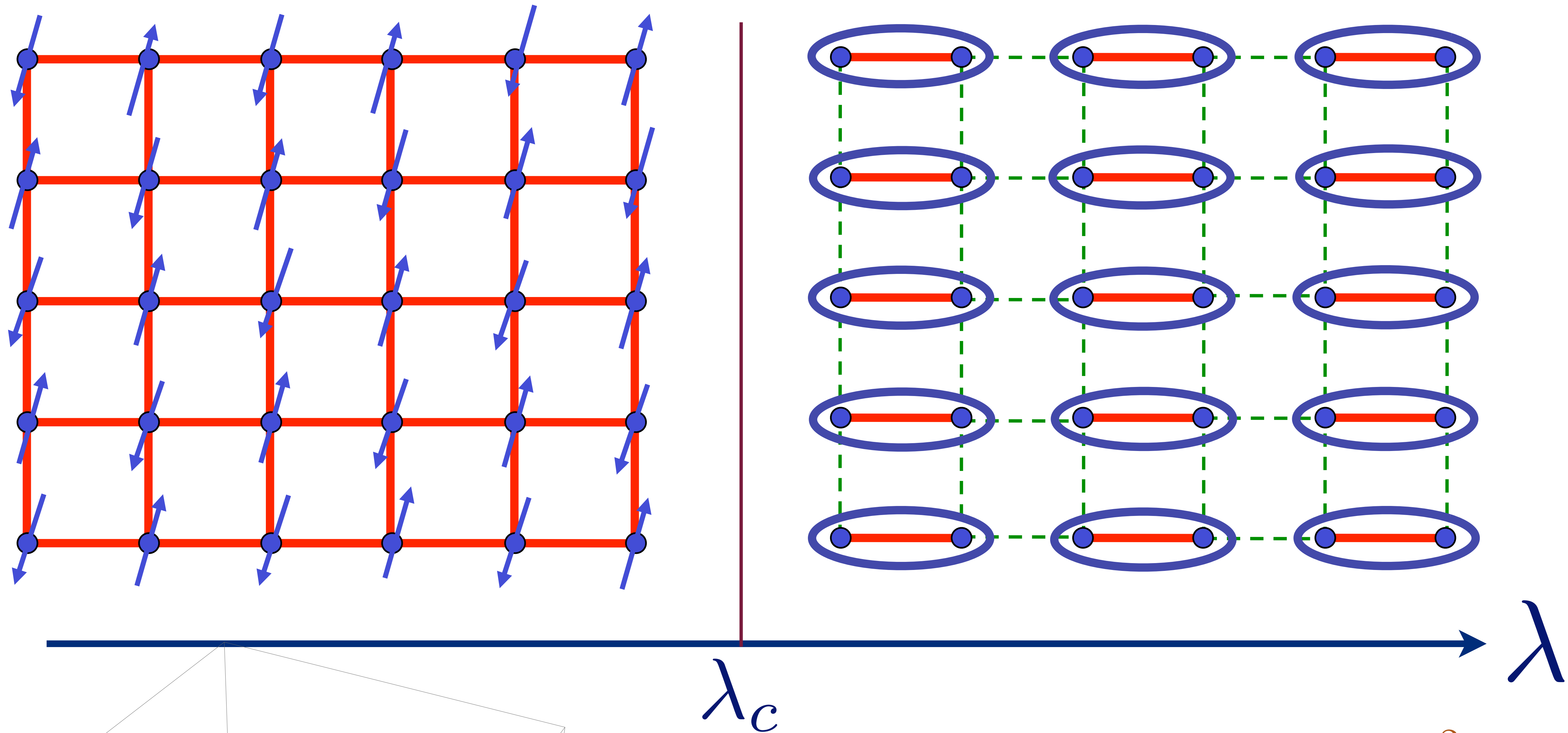

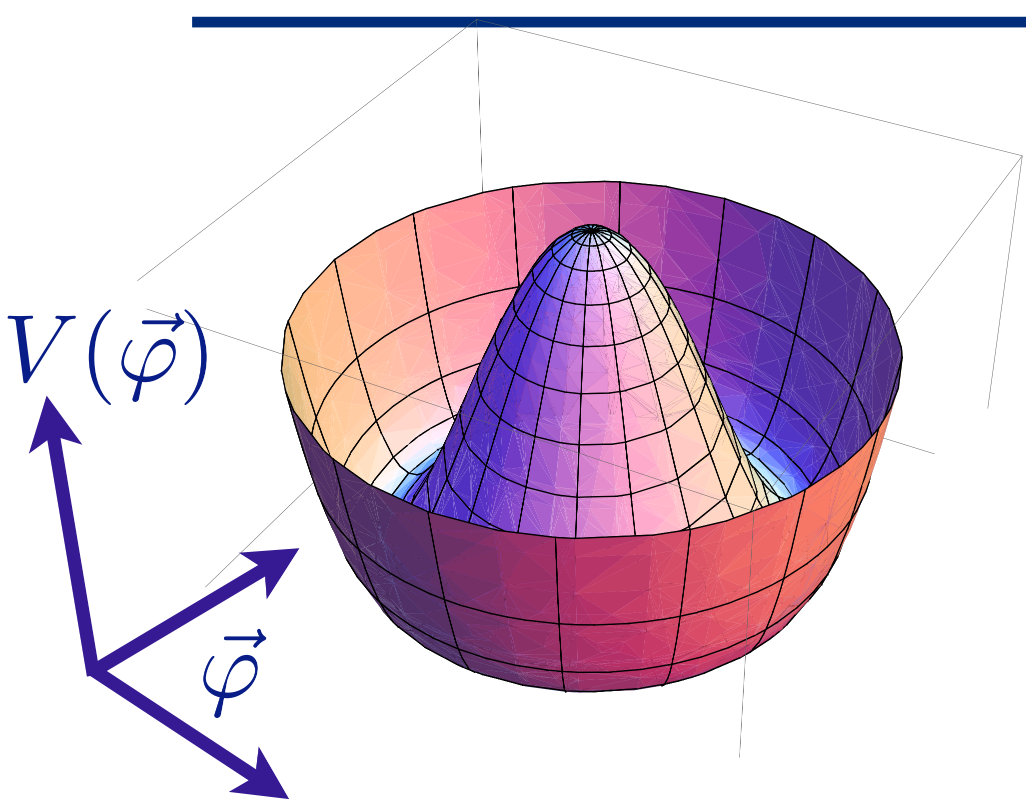

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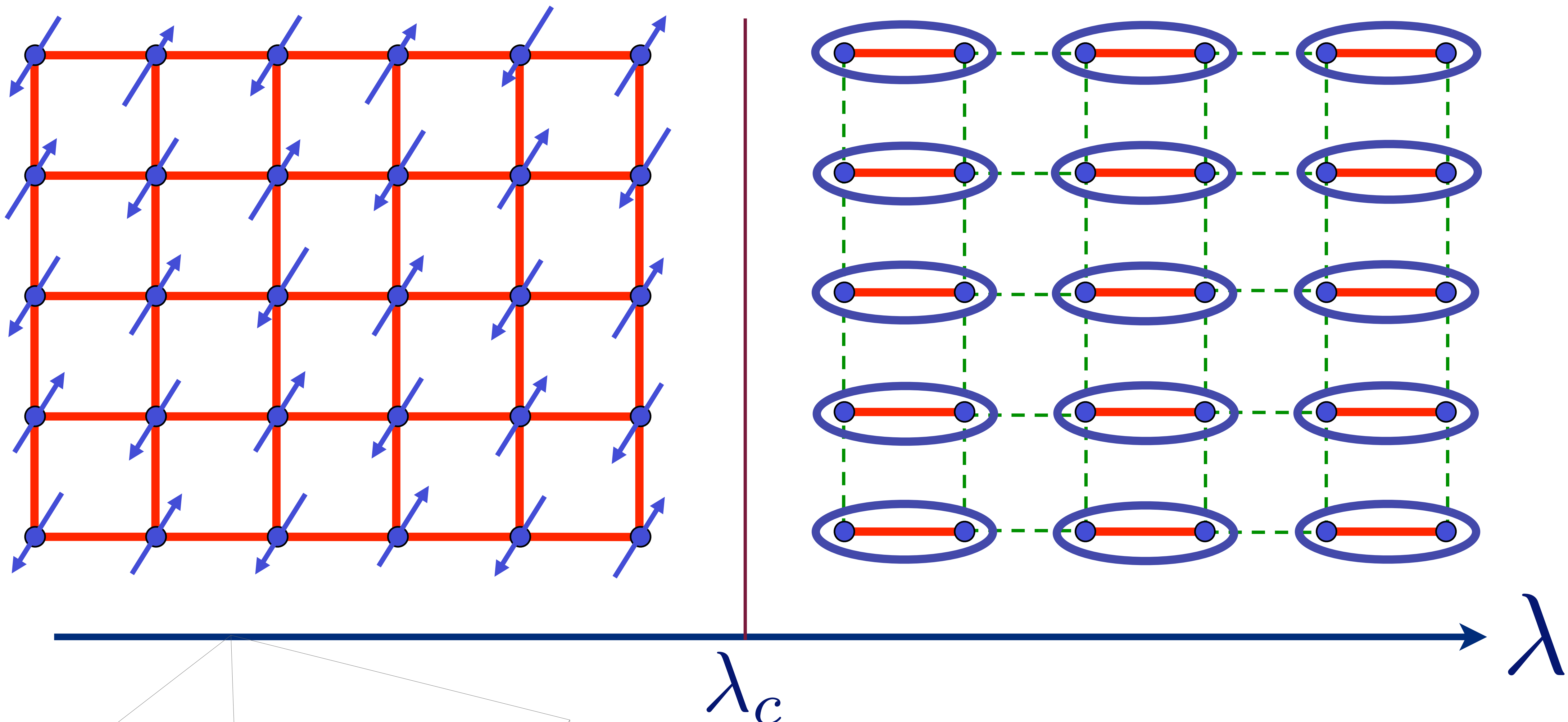

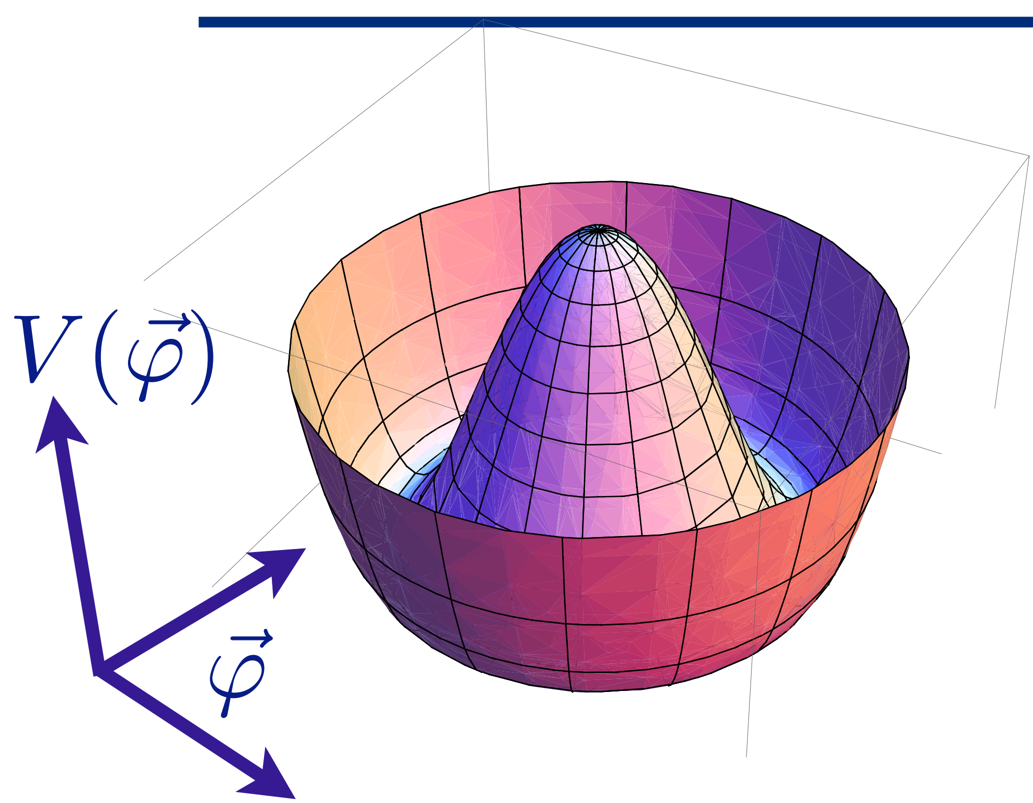

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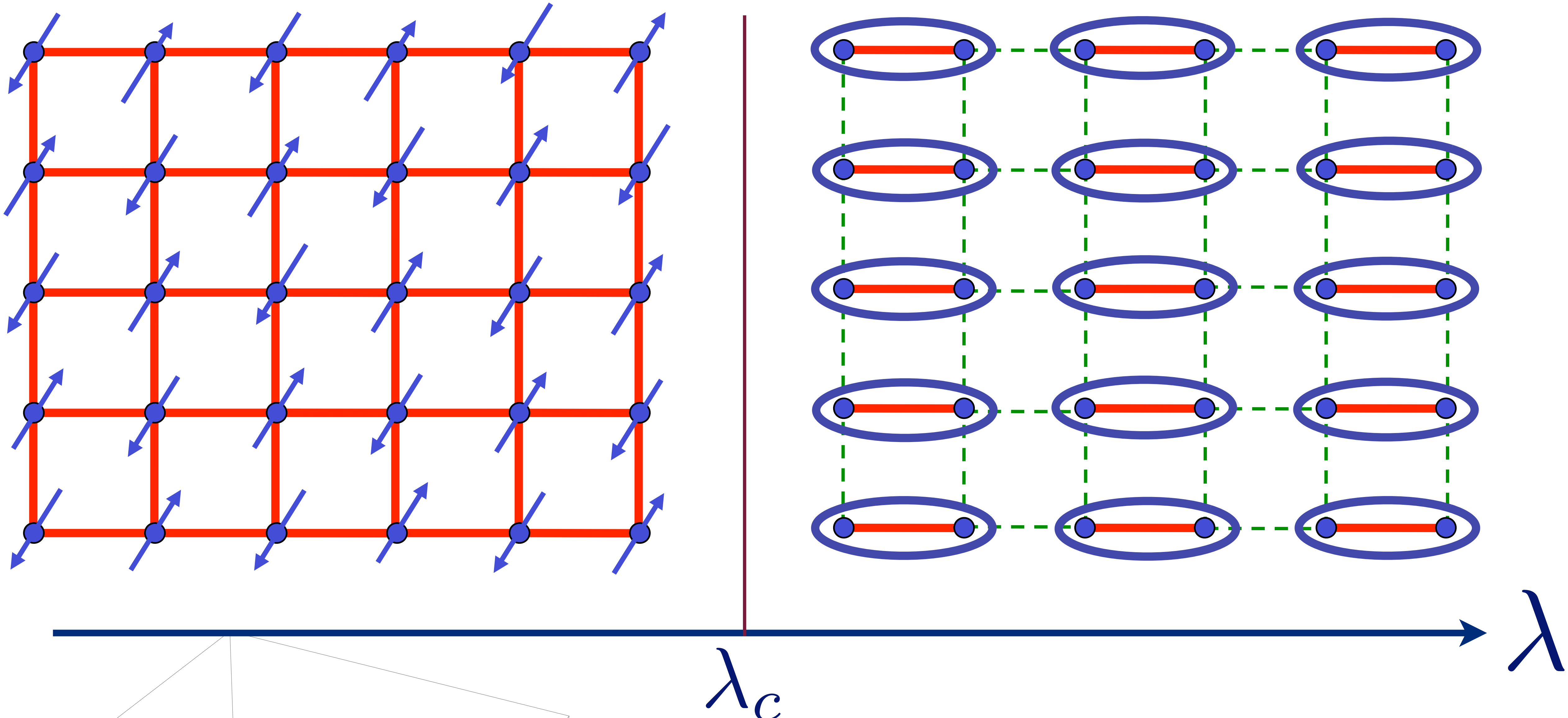

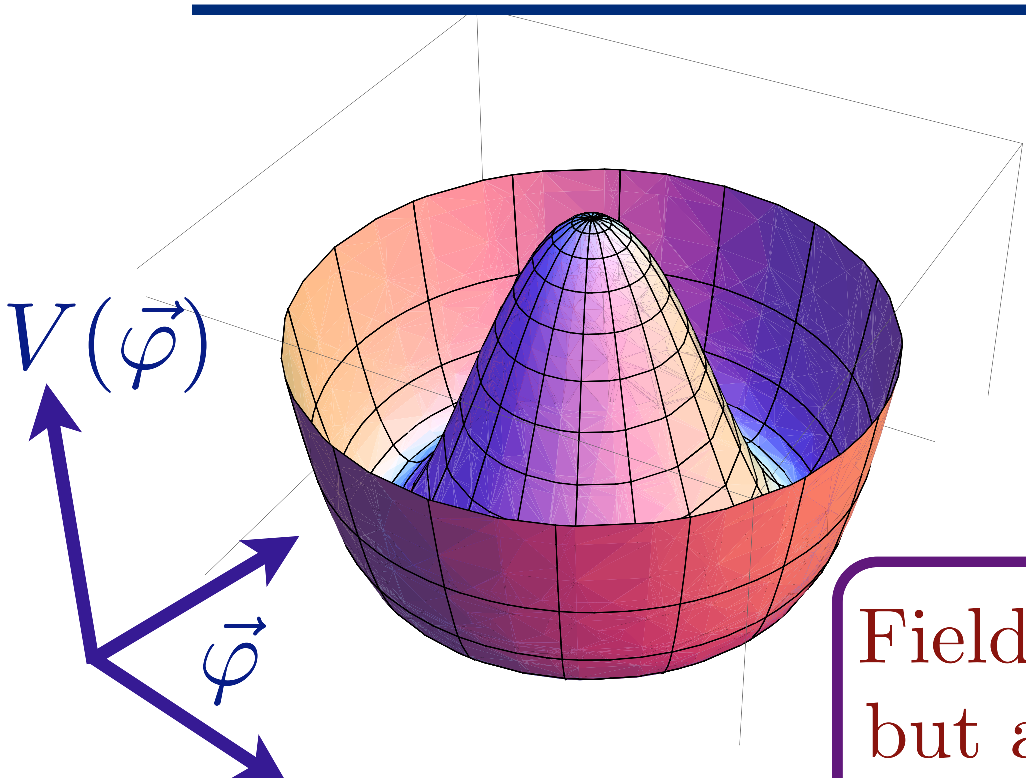

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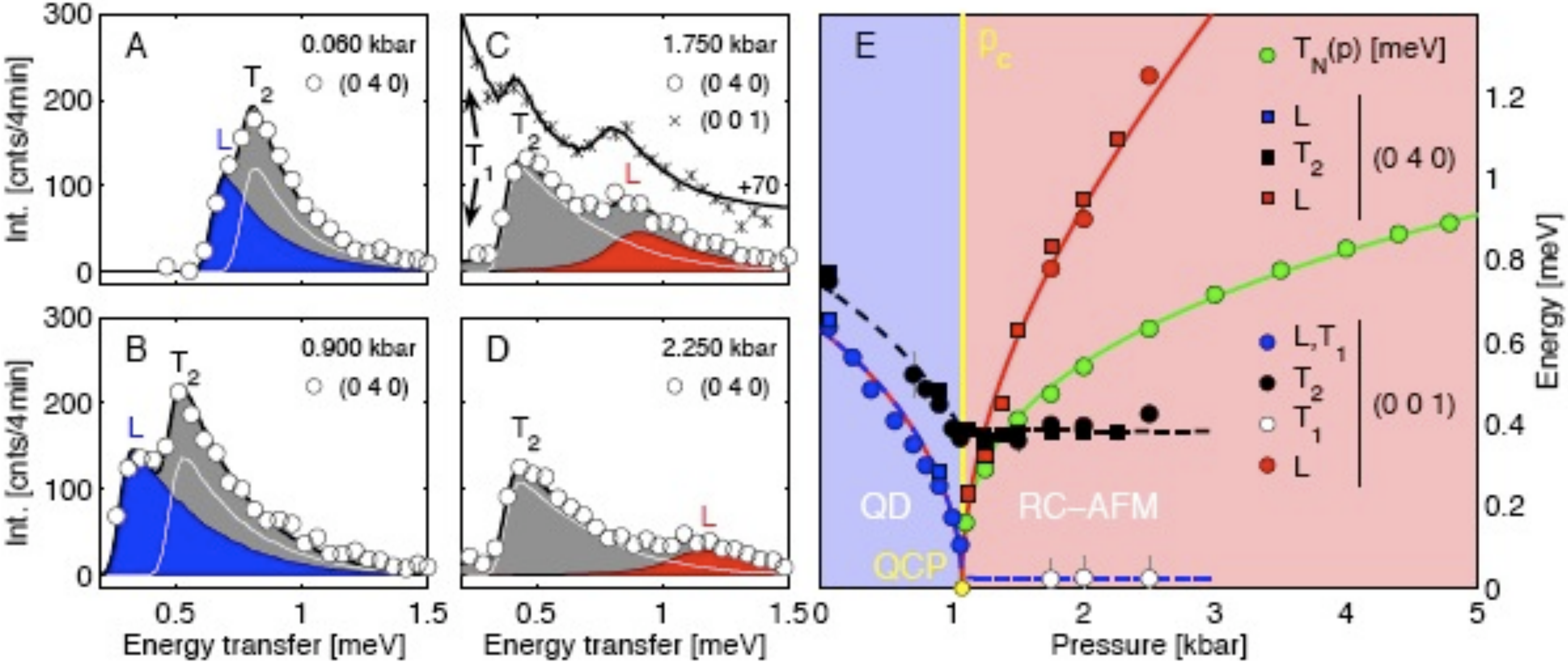
Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

$TlCuCl_3$ with varying pressure



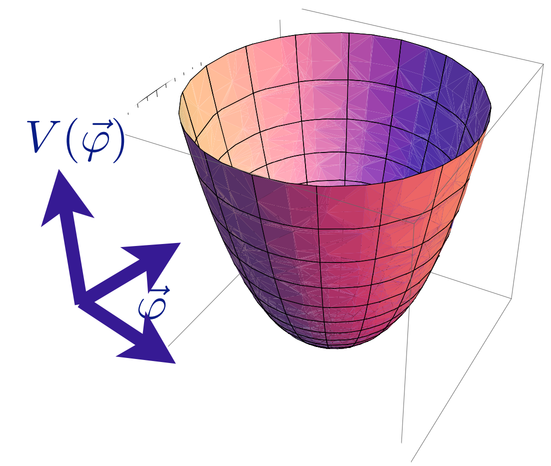
Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$



Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

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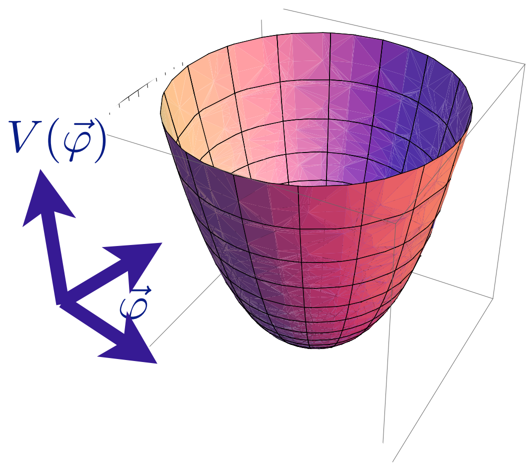

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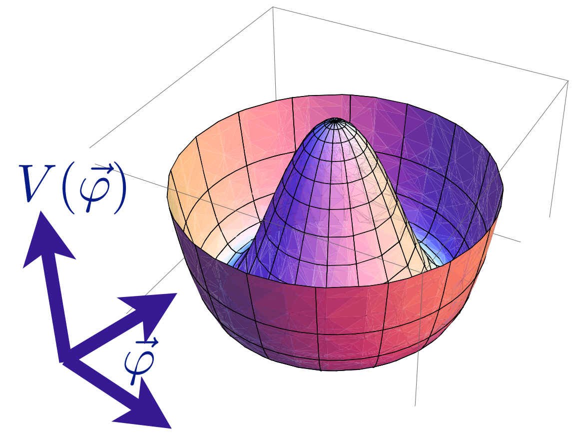


Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

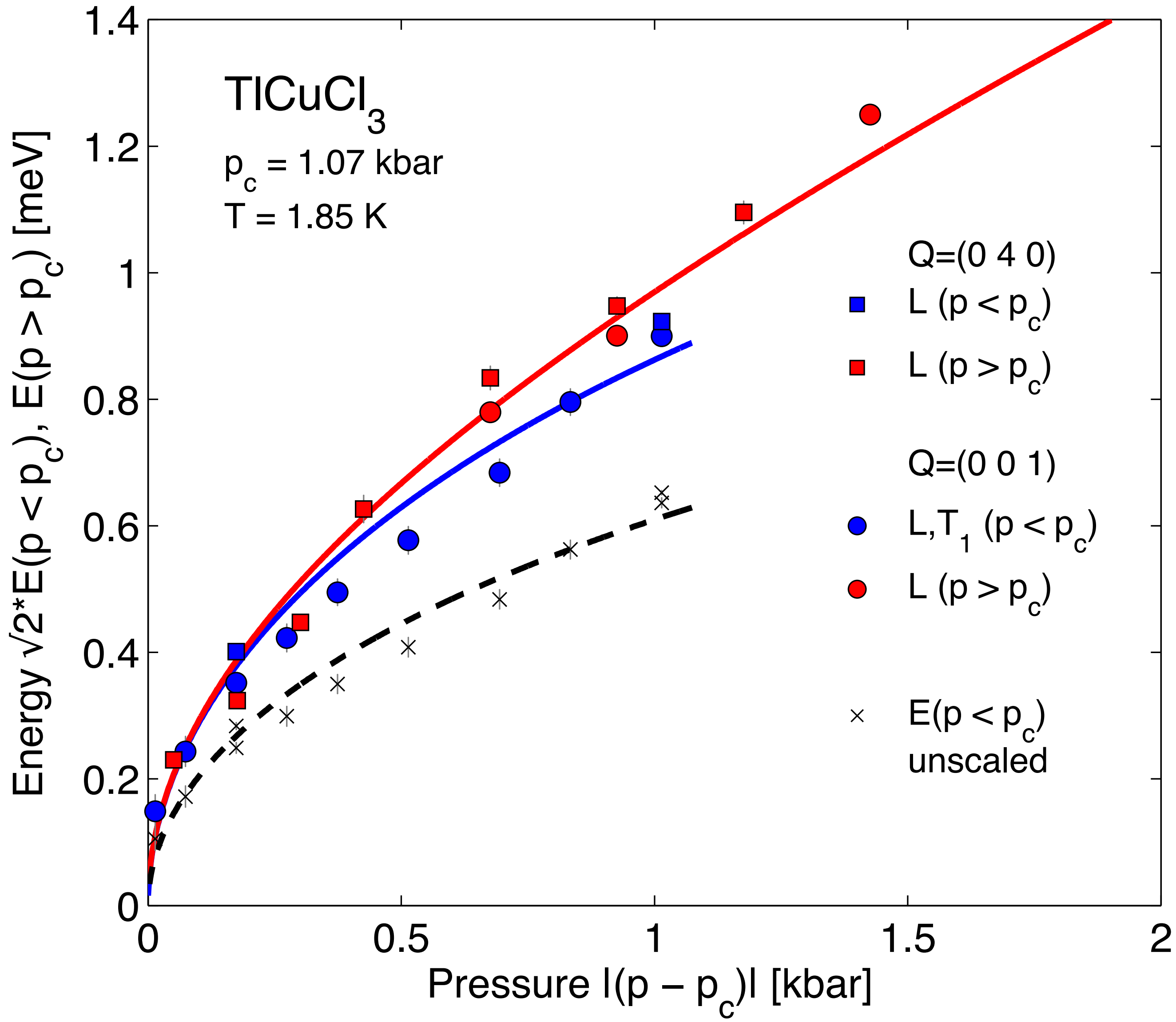
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

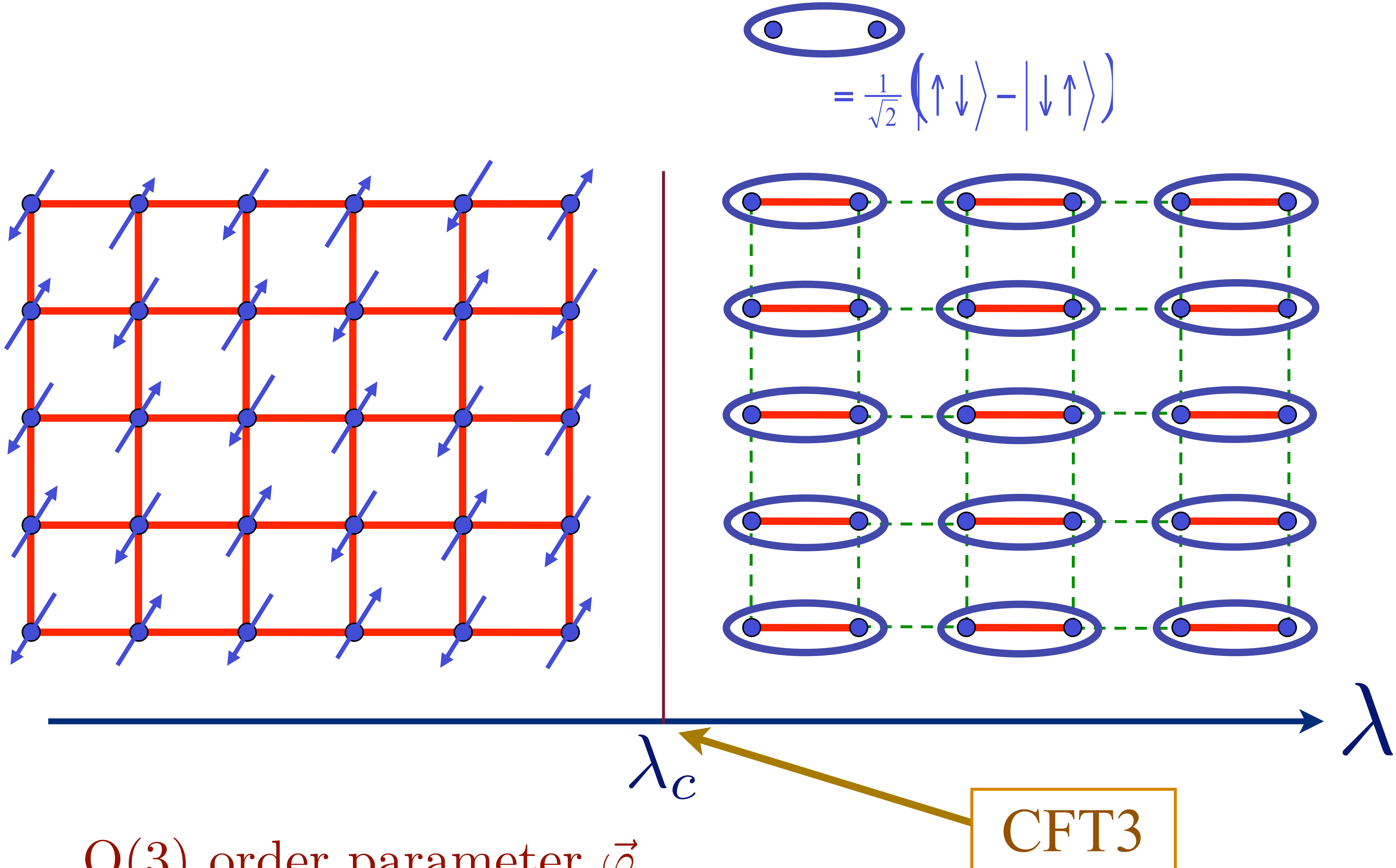
Prediction of quantum field theory

$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

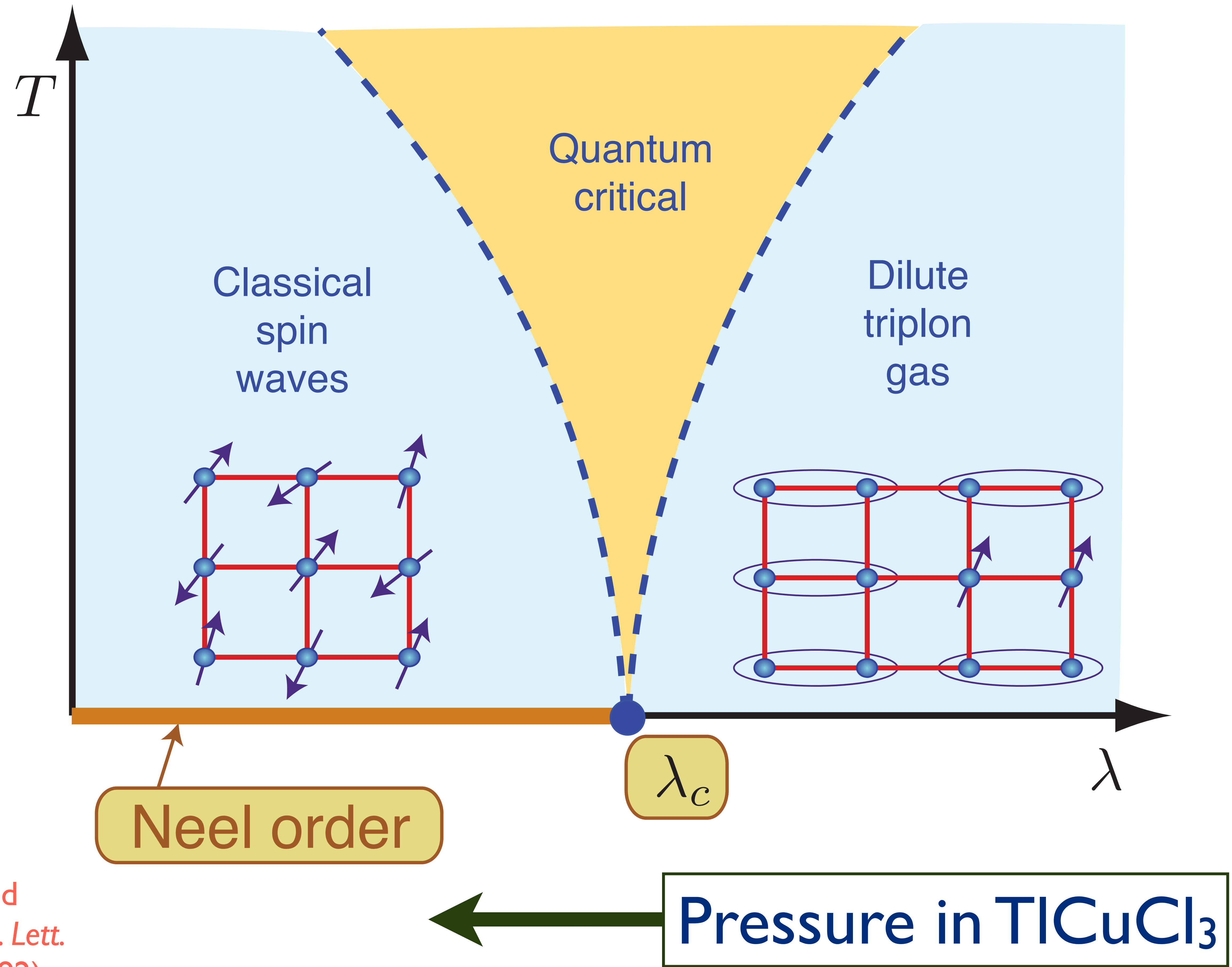


S. Sachdev, arXiv:0901.4103

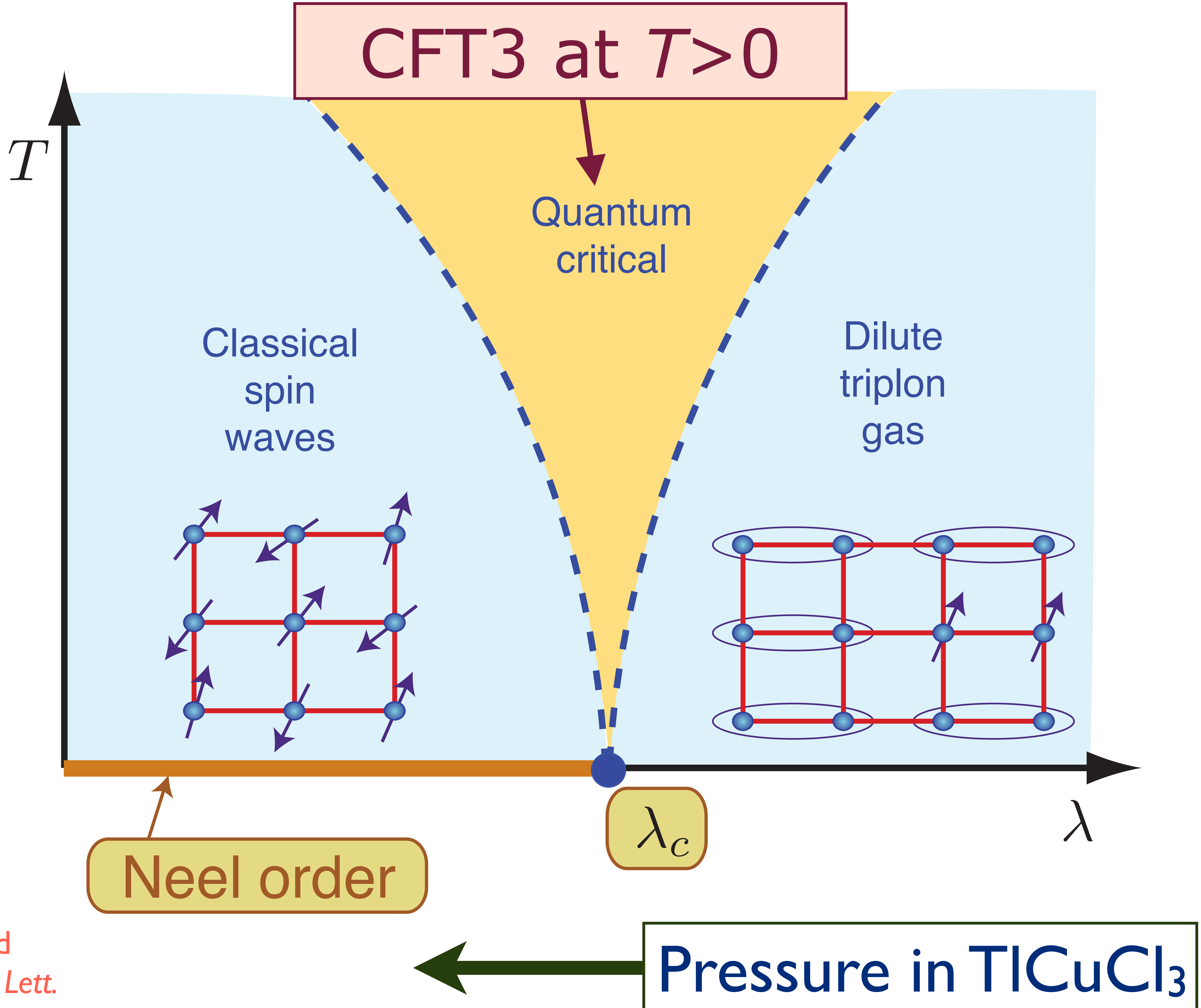


O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s\vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).



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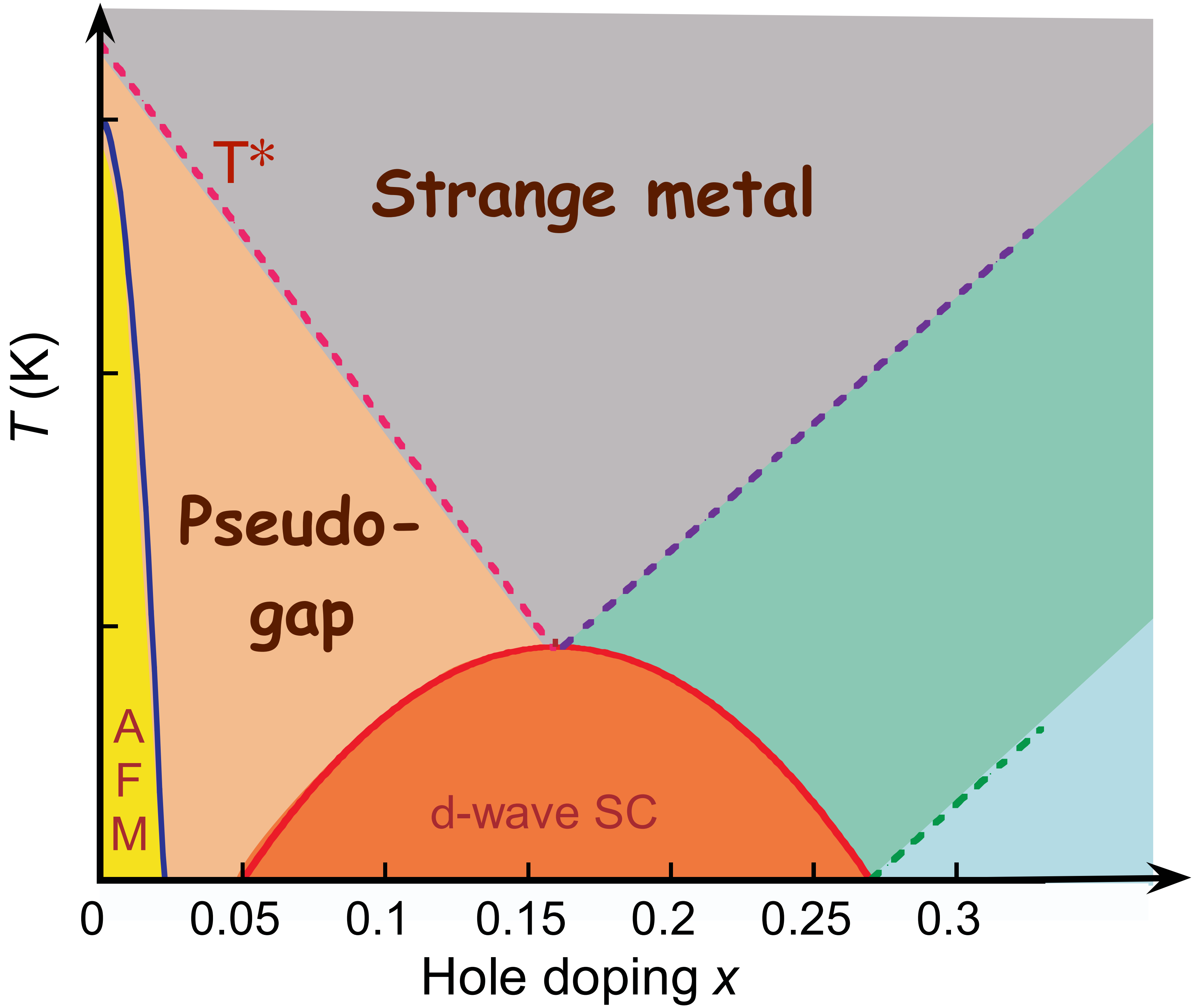
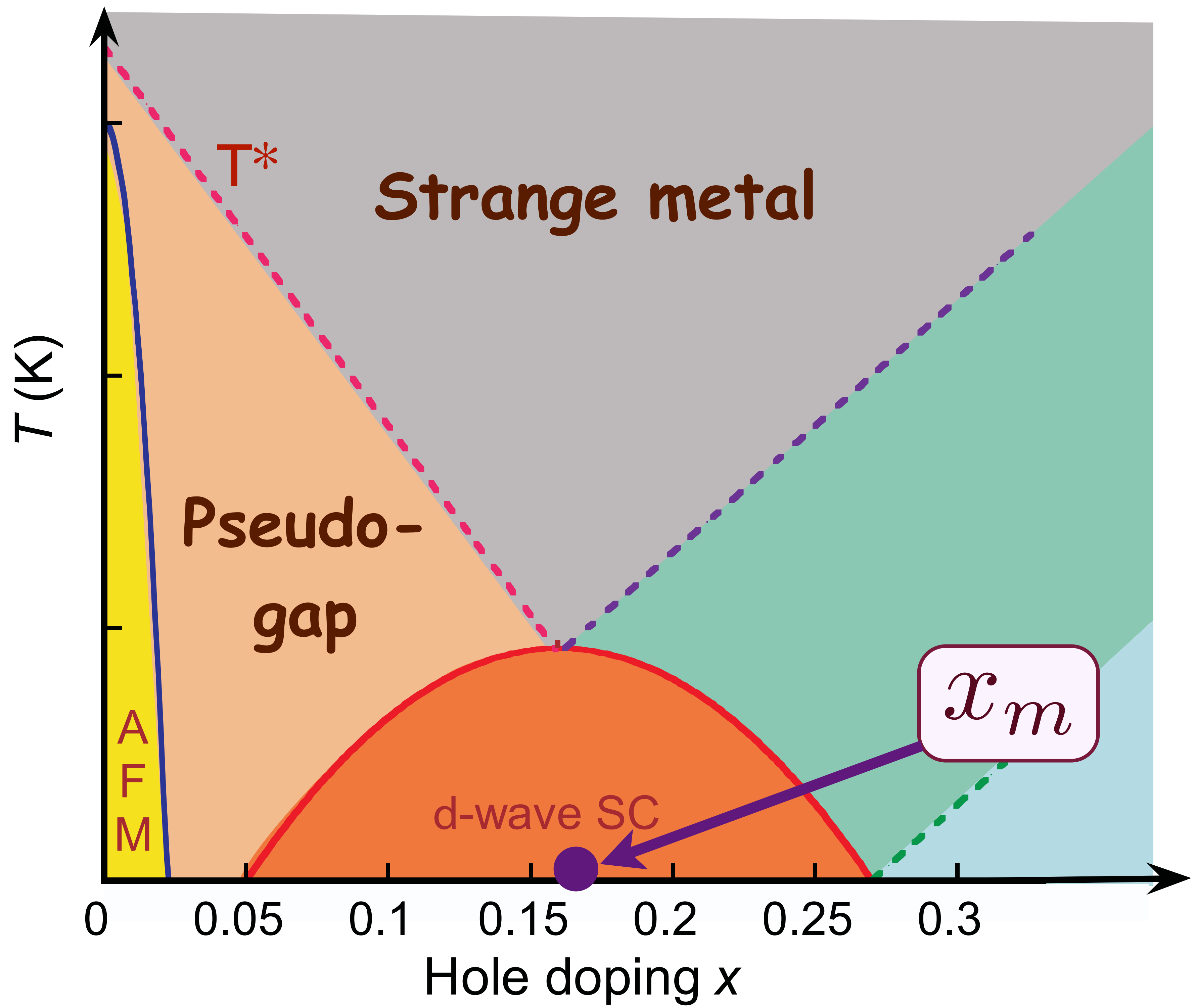
Crossovers in transport properties of hole-doped cuprates



Crossovers in transport properties of hole-doped cuprates



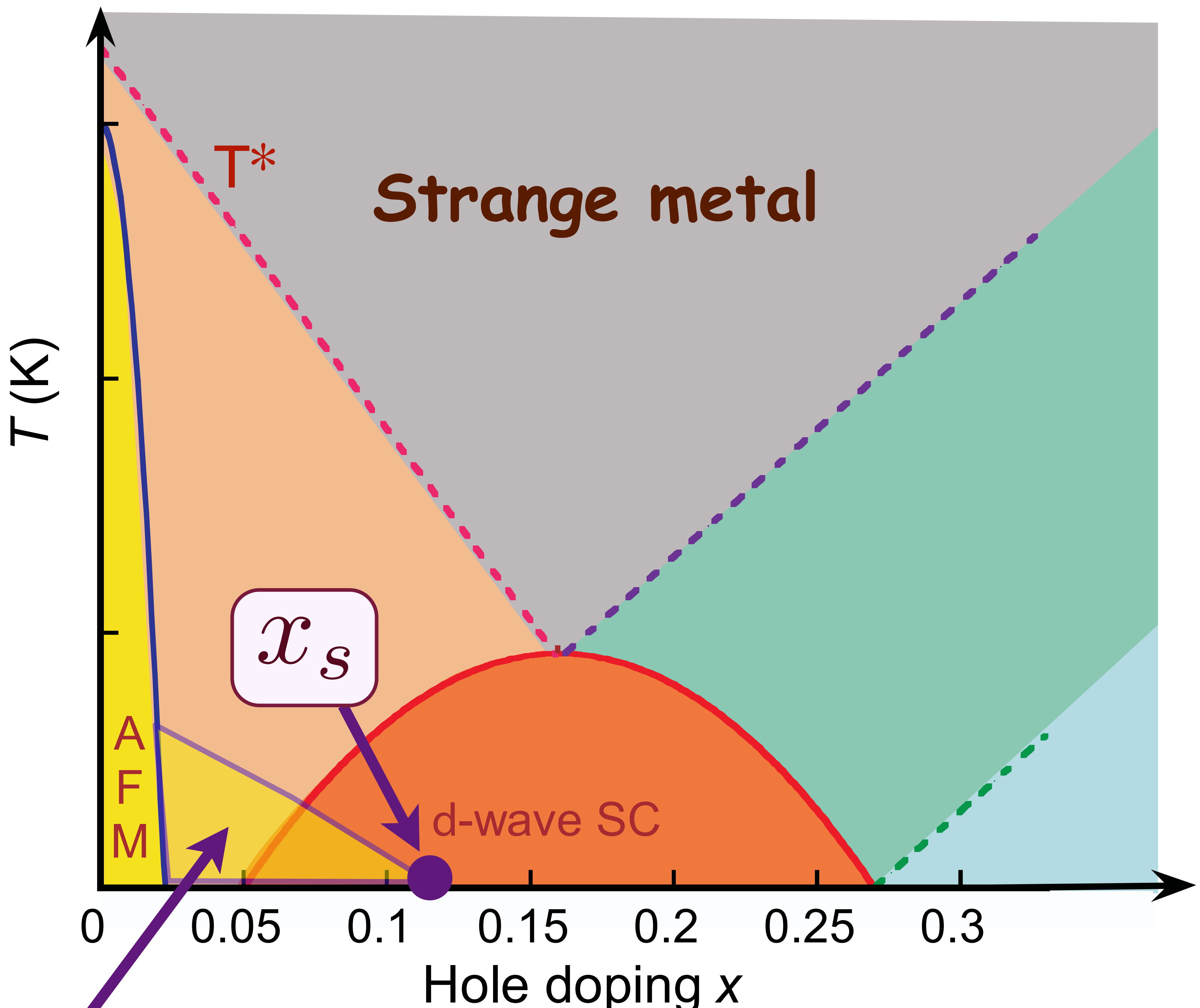
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low *T*



Spin density wave order present
below a quantum critical point at $x = x_s$
with $x_s \approx 0.12$ in the La series of cuprates

Outline

1. Coupled dimer antiferromagnets
   Introduction to quantum criticality

2. Phase diagram of the cuprates
   Quantum criticality of the competition between antiferromagnetism and superconductivity

3. Influence of an applied magnetic field
   Theoretical predictions and experimental tests

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   Strong-coupling problems and the AdS/CFT correspondence

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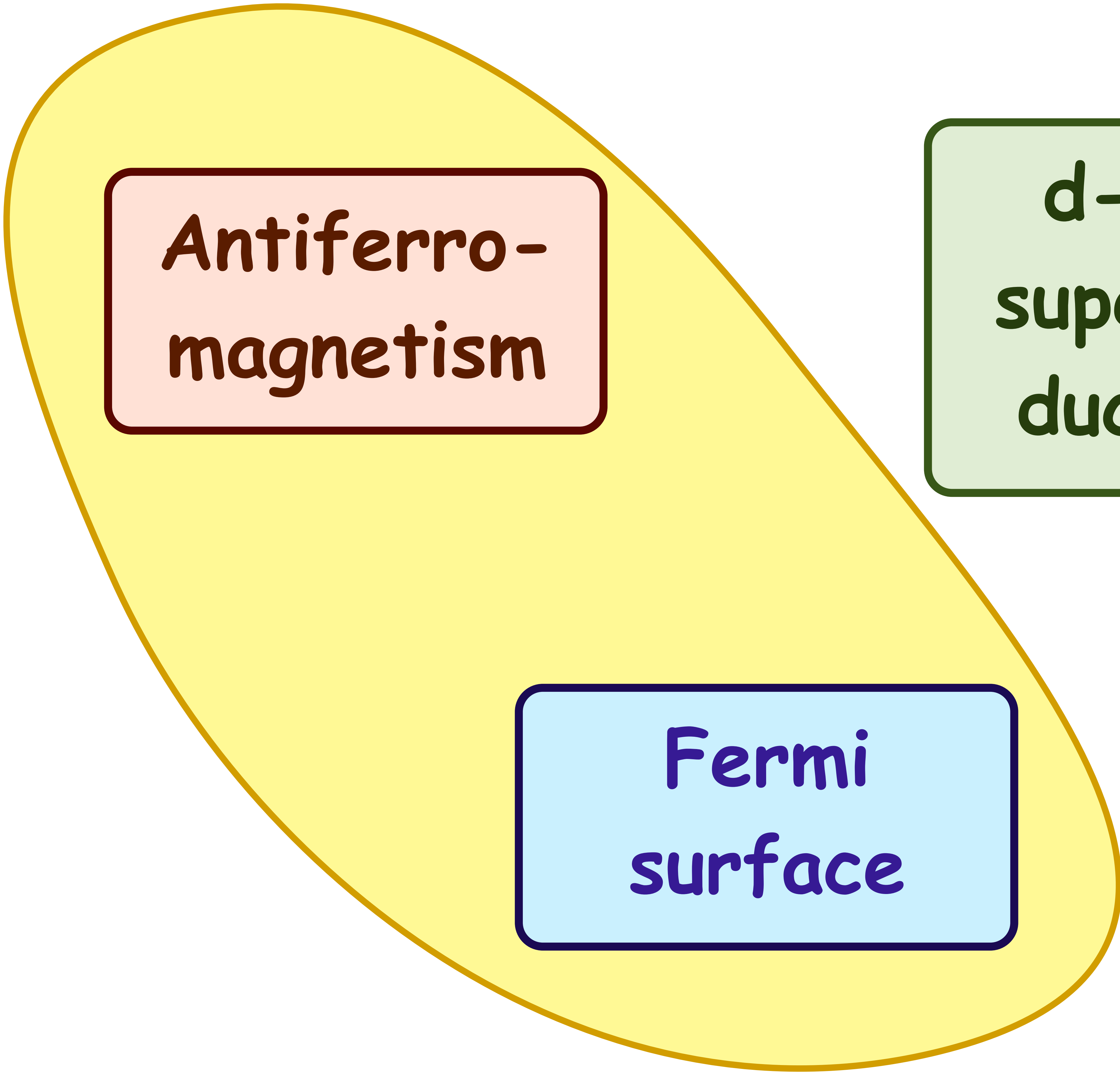
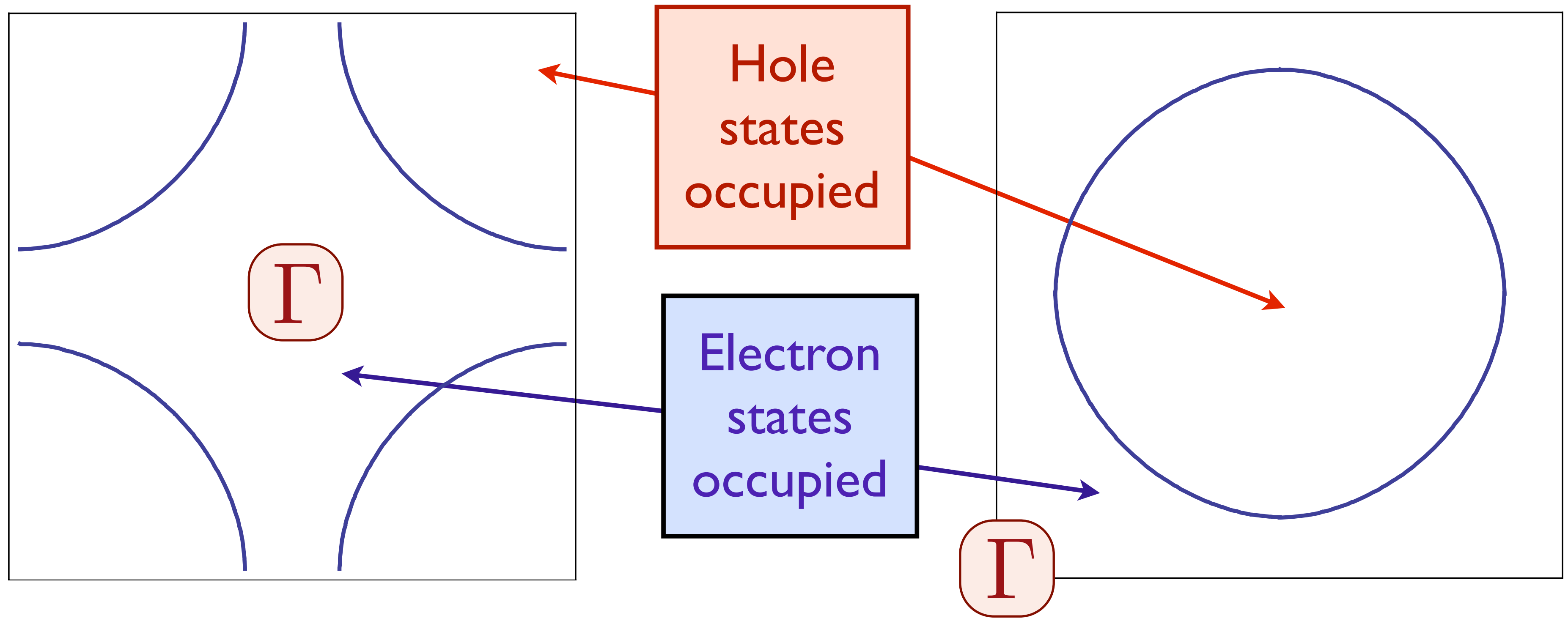
Antiferro-
magnetism

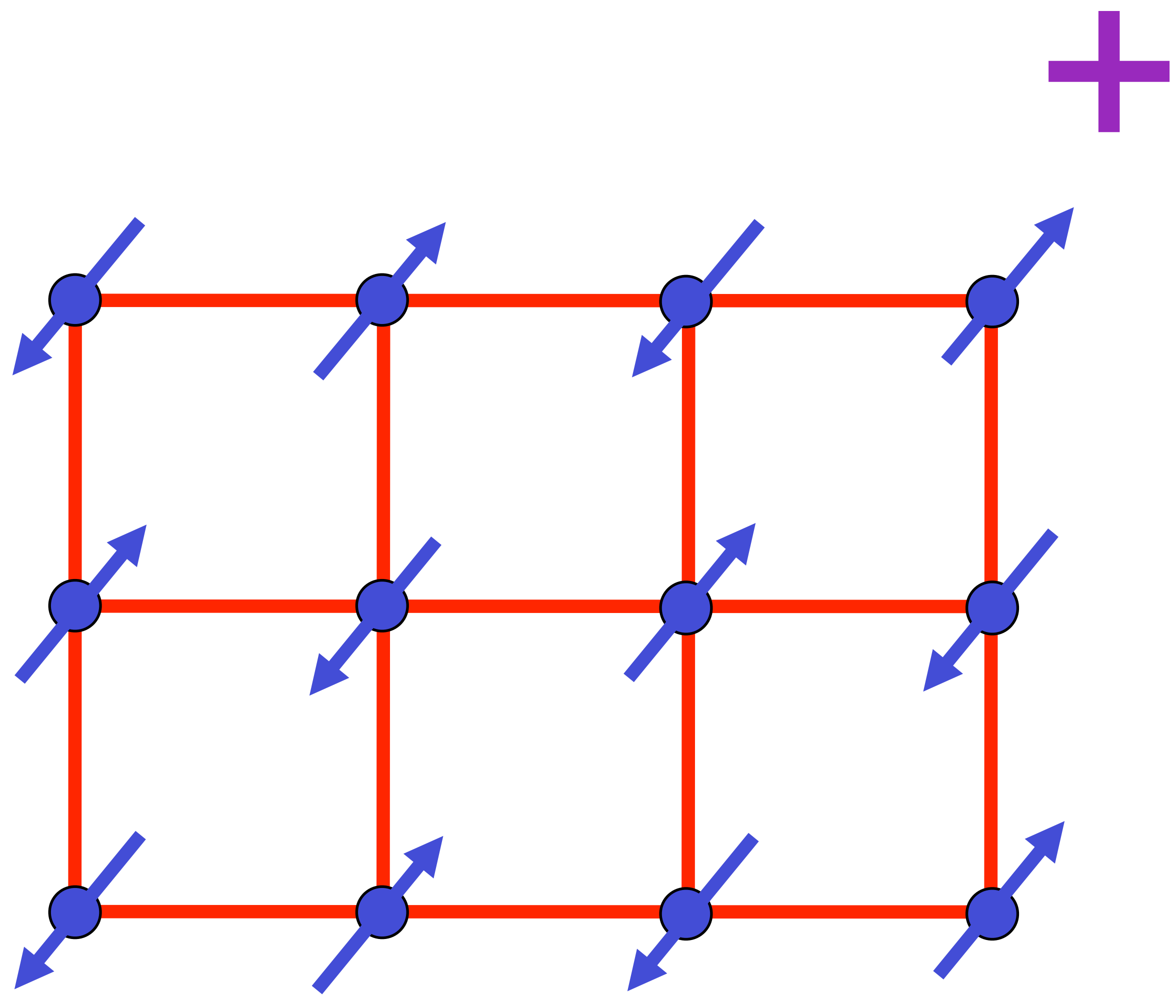
d-wave
supercon-
ductivity

Fermi
surface


Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface

Fermi surface+antiferromagnetism





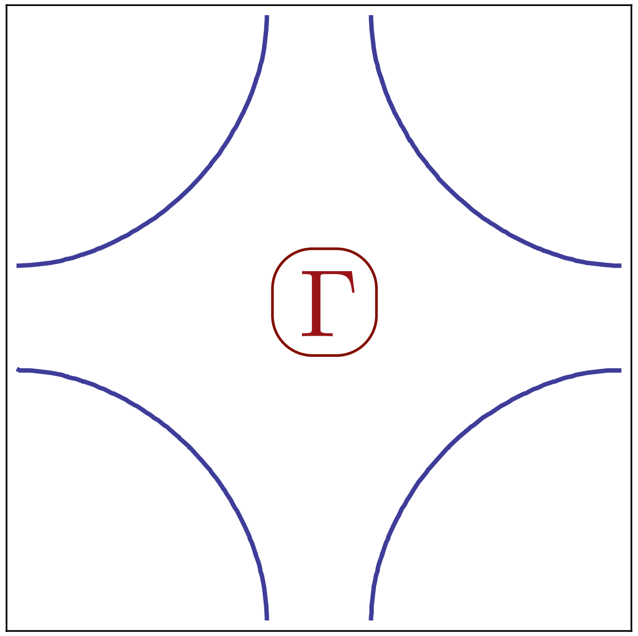
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

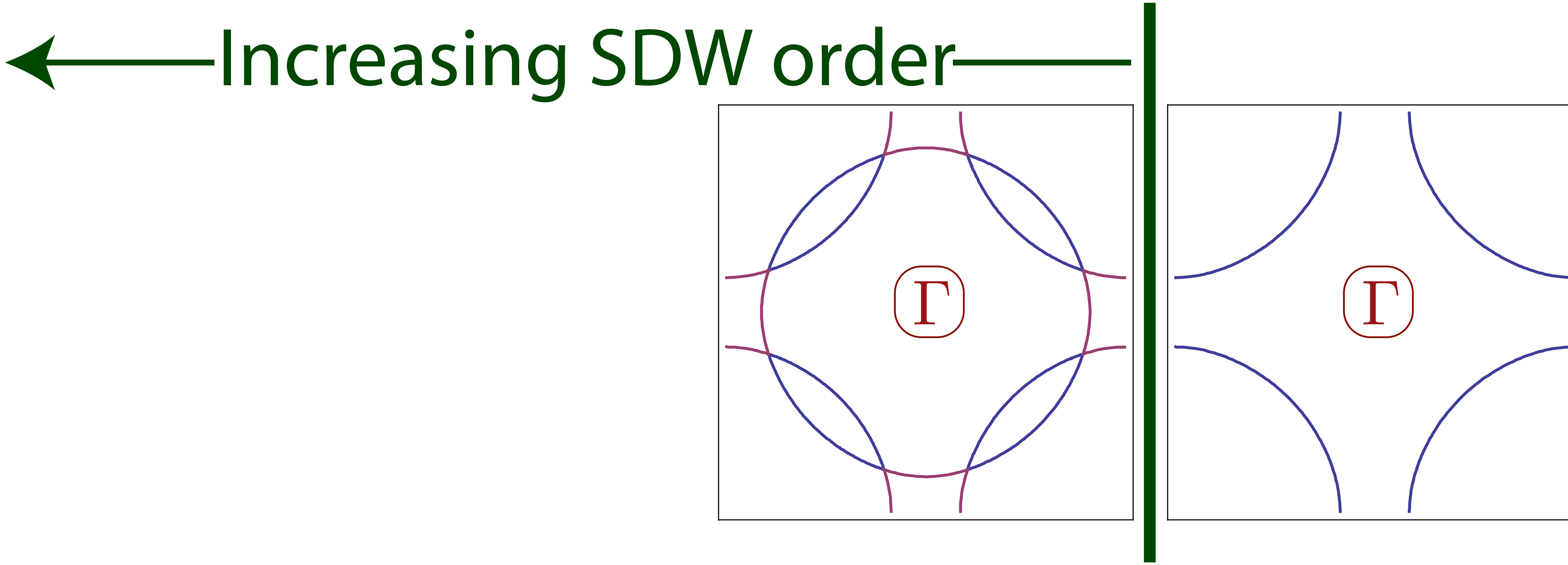
Hole-doped cuprates

Increasing SDW order



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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Hole-doped cuprates

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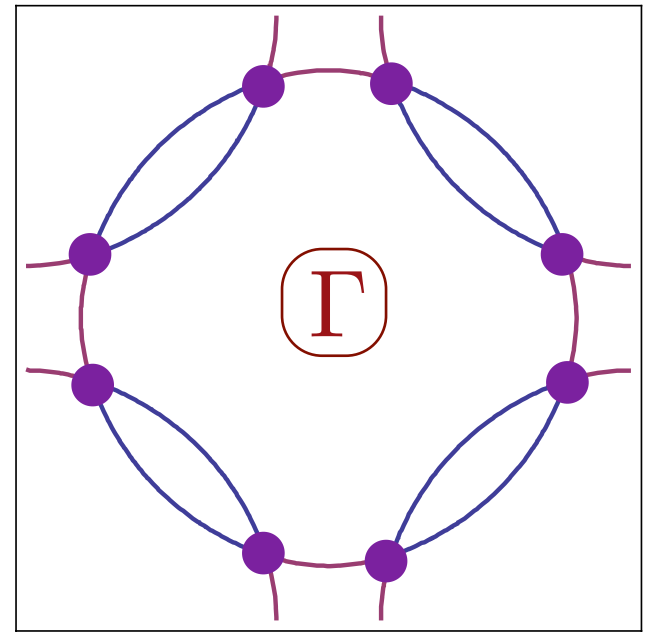

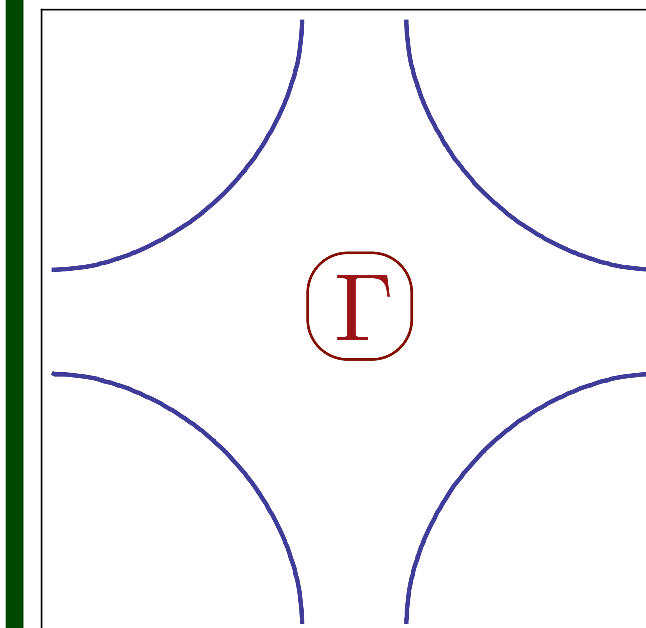

Hot spots

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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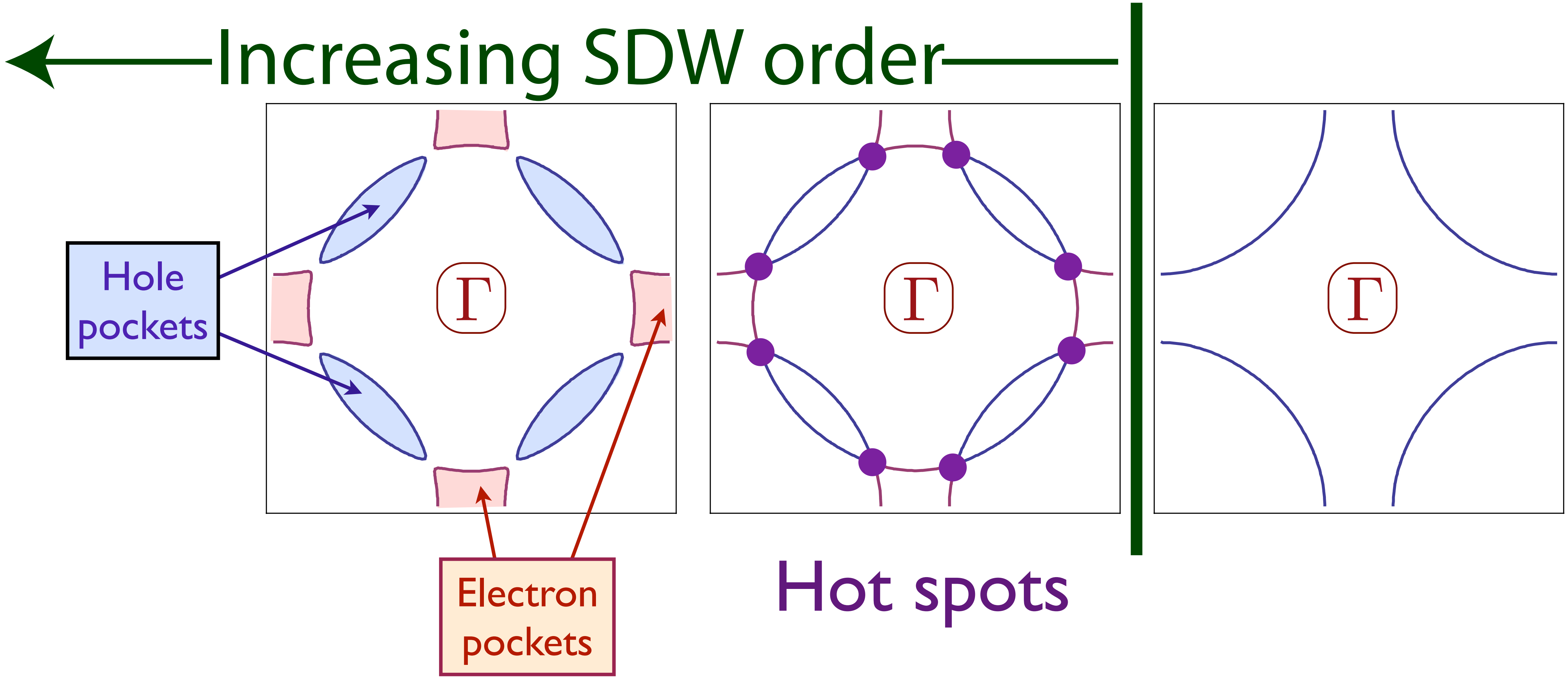
Hole-doped cuprates



Fermi surface breaks up at hot spots into electron and hole "pockets"

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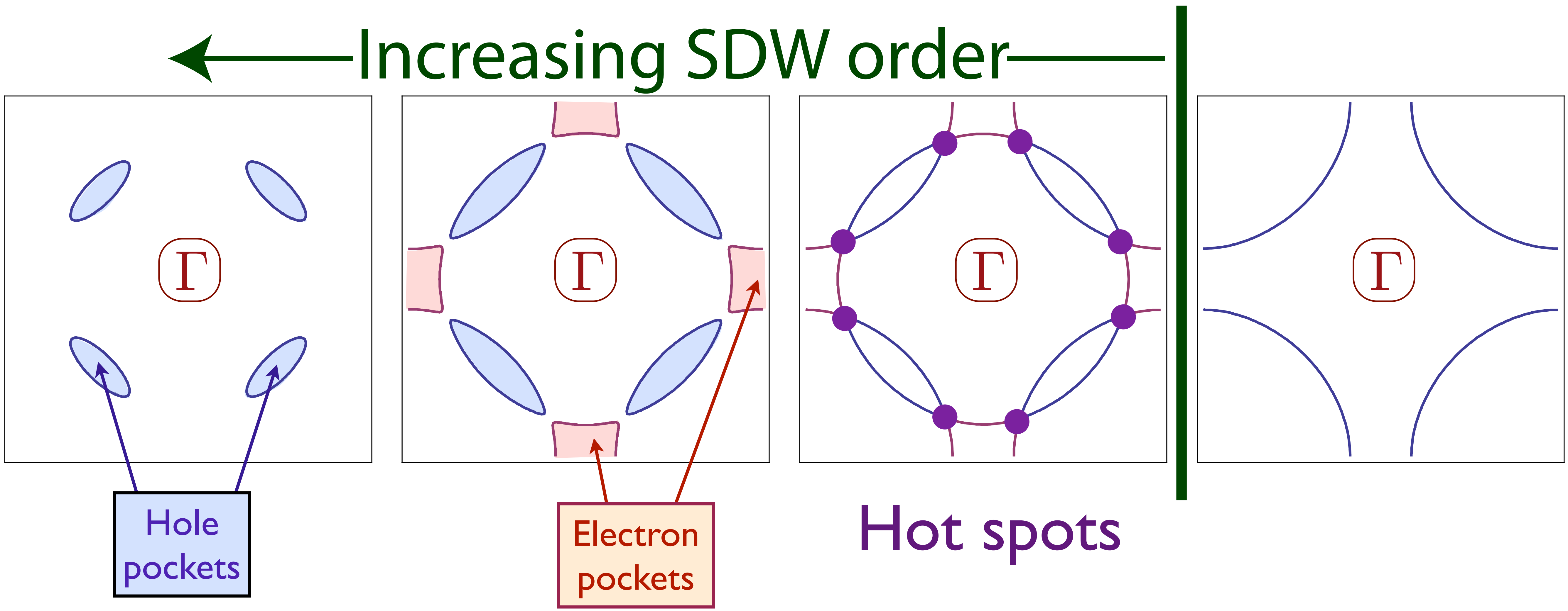

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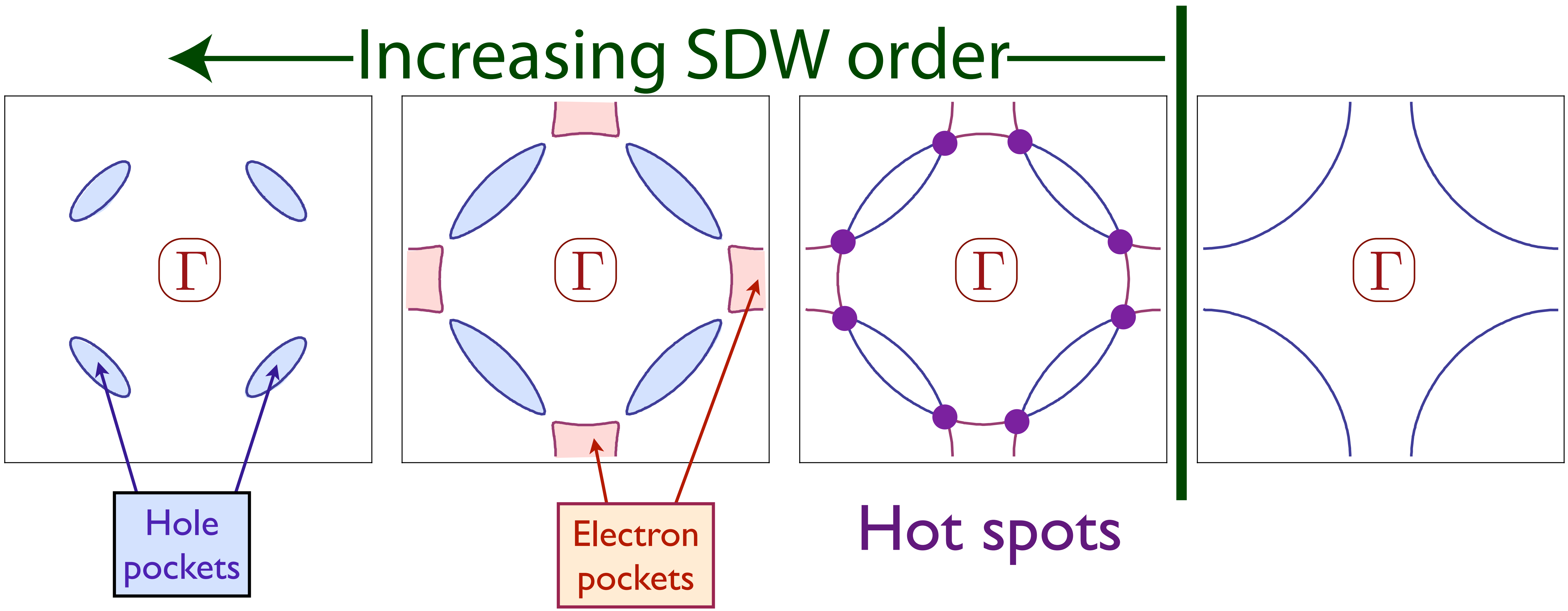

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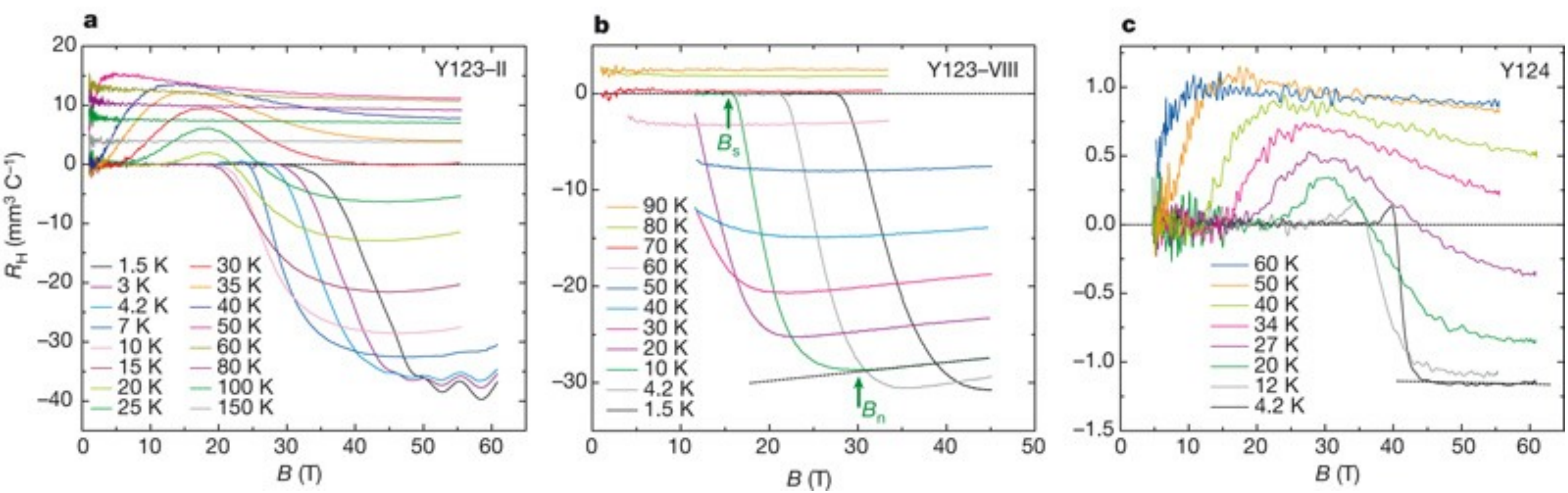
Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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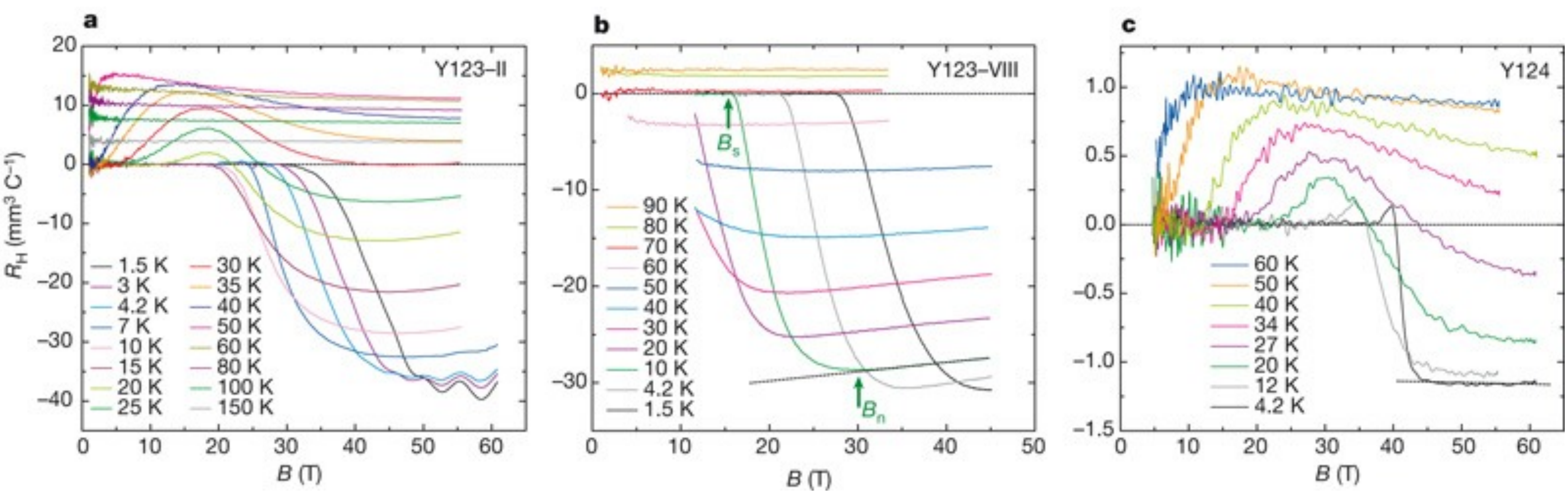


Evidence for small Fermi pockets

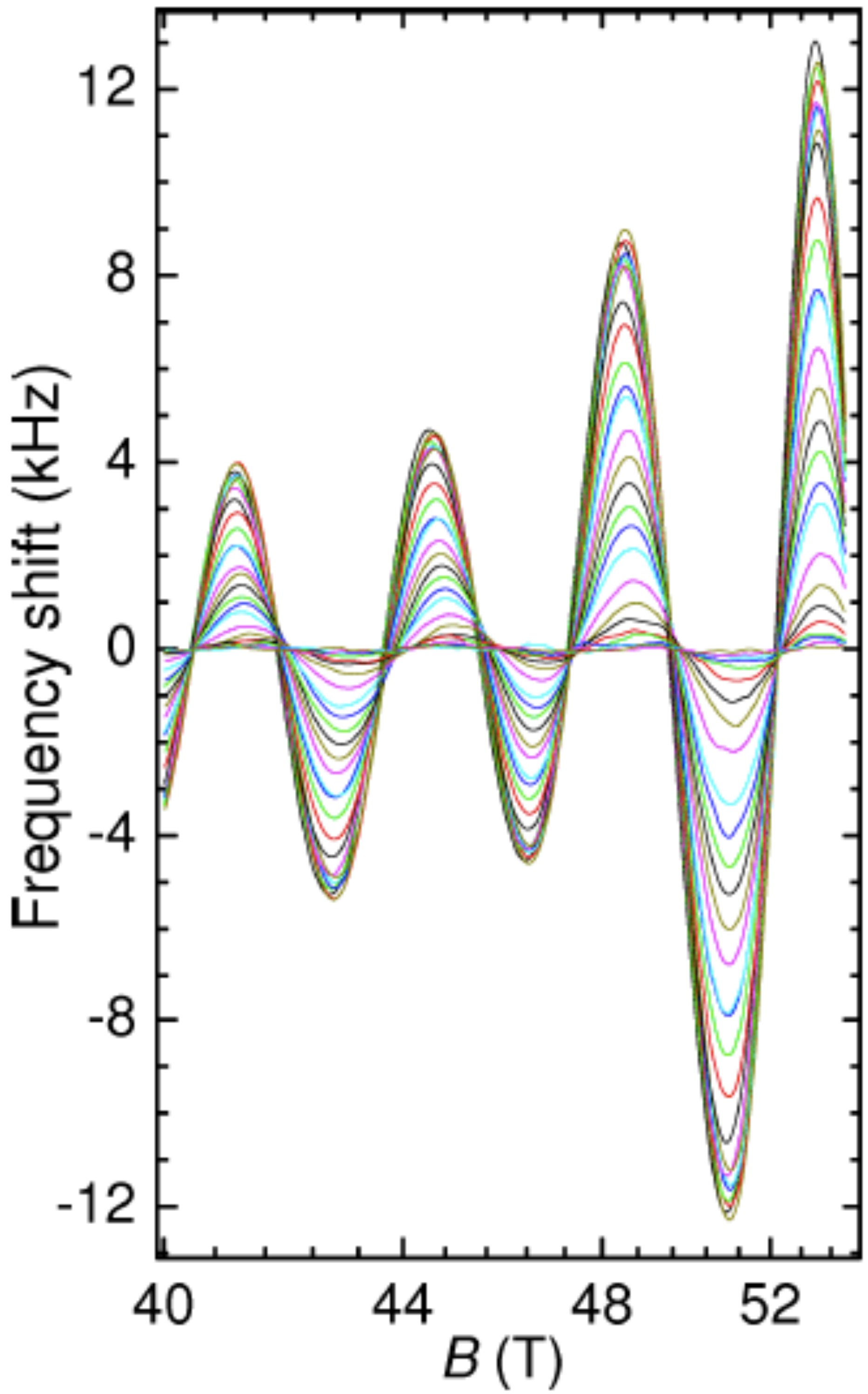


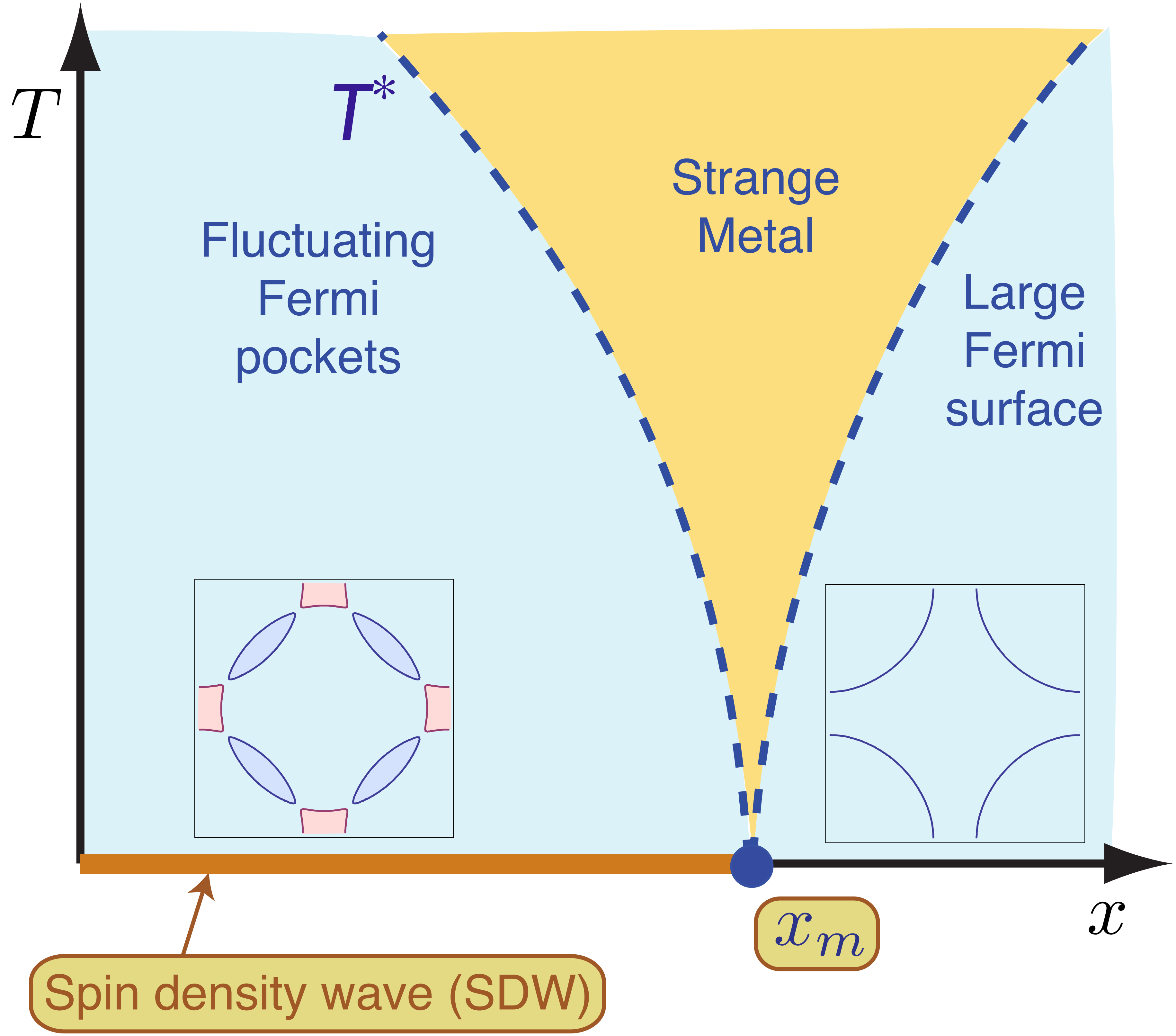
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D. A. Bonn,
W. N. Hardy, and G. G. Lonzarich

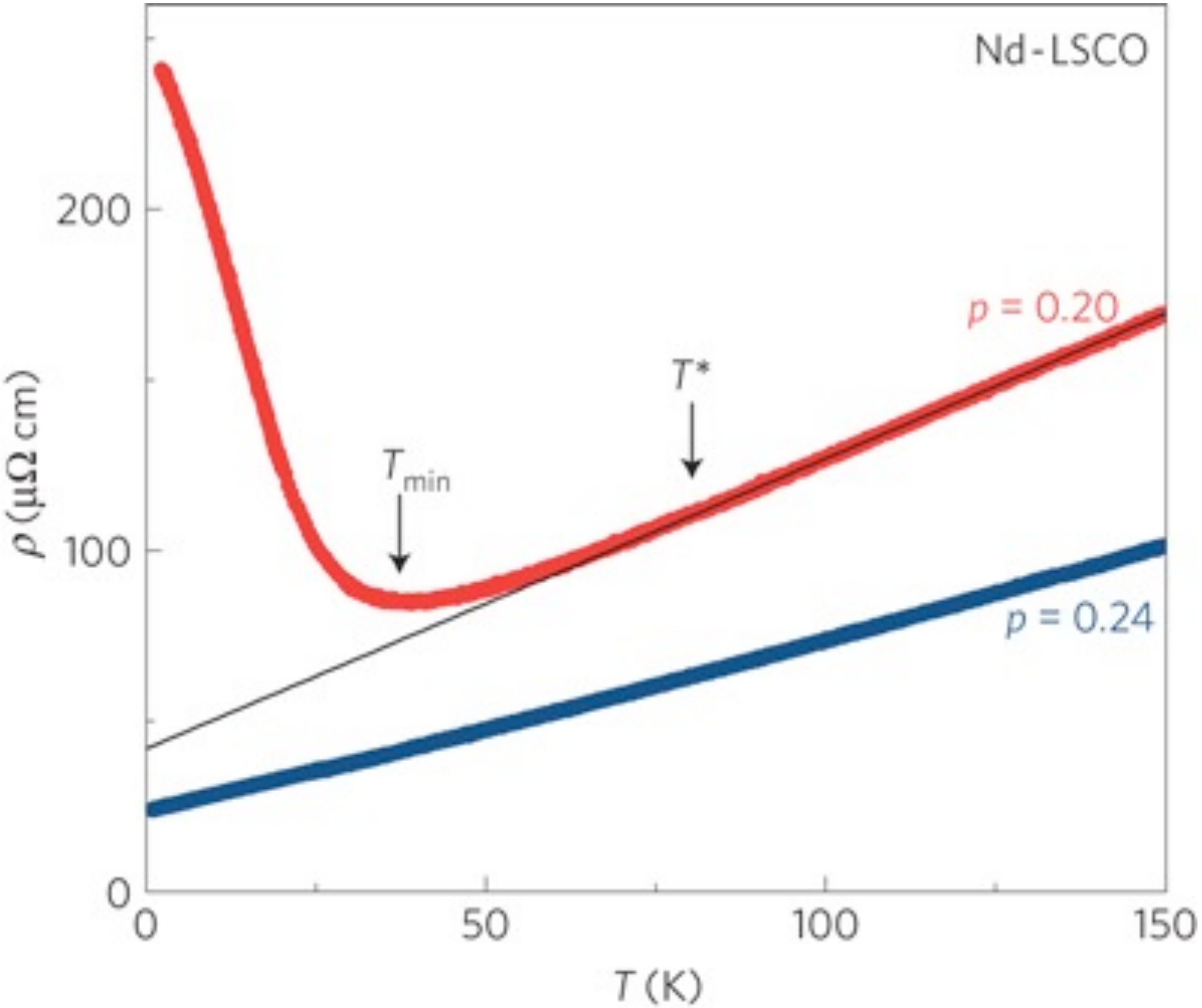
arXiv:0912.3022

Theory of quantum criticality in the cuprates



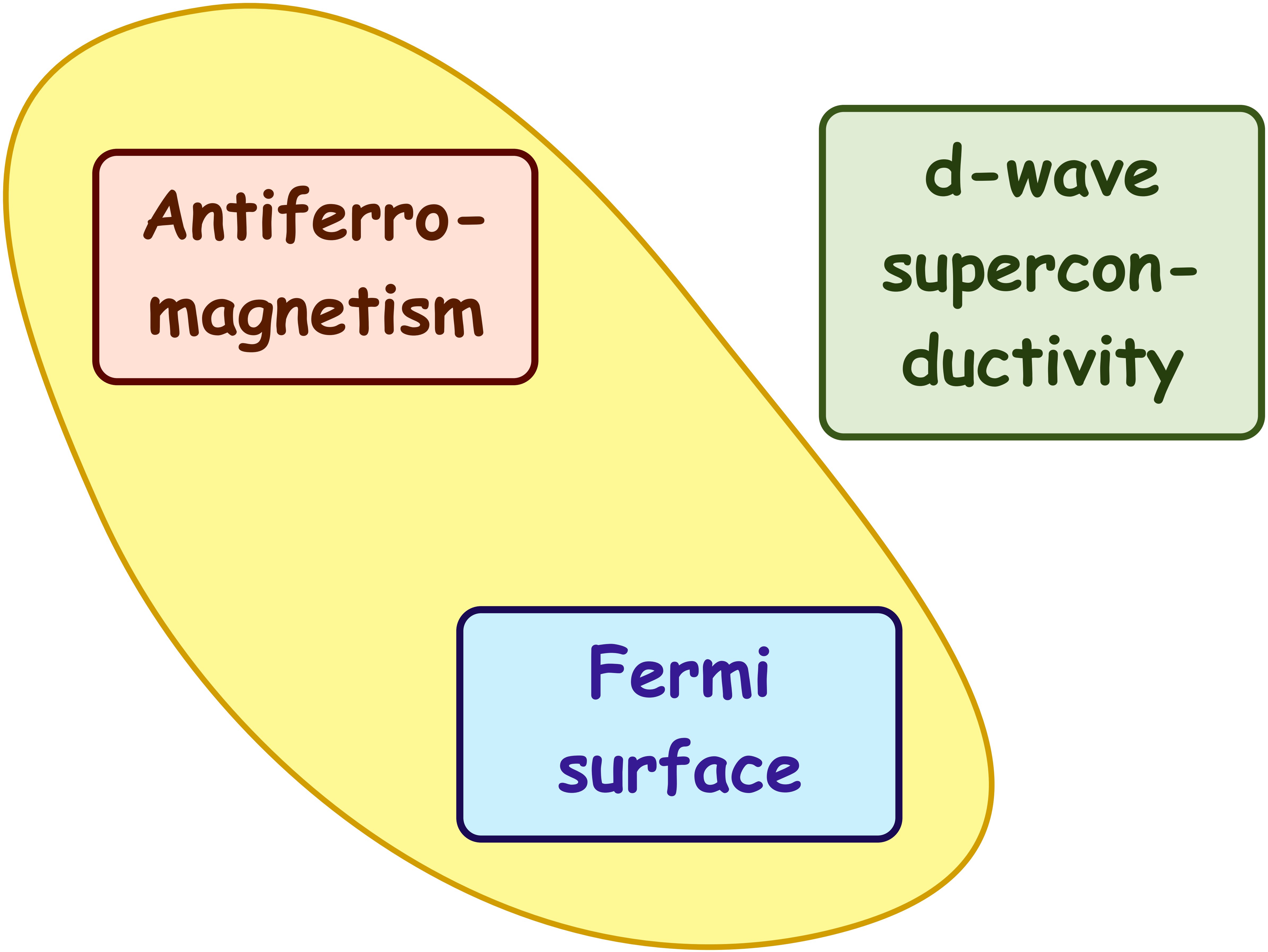
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c



- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor
R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)


Antiferro-
magnetism
d-wave
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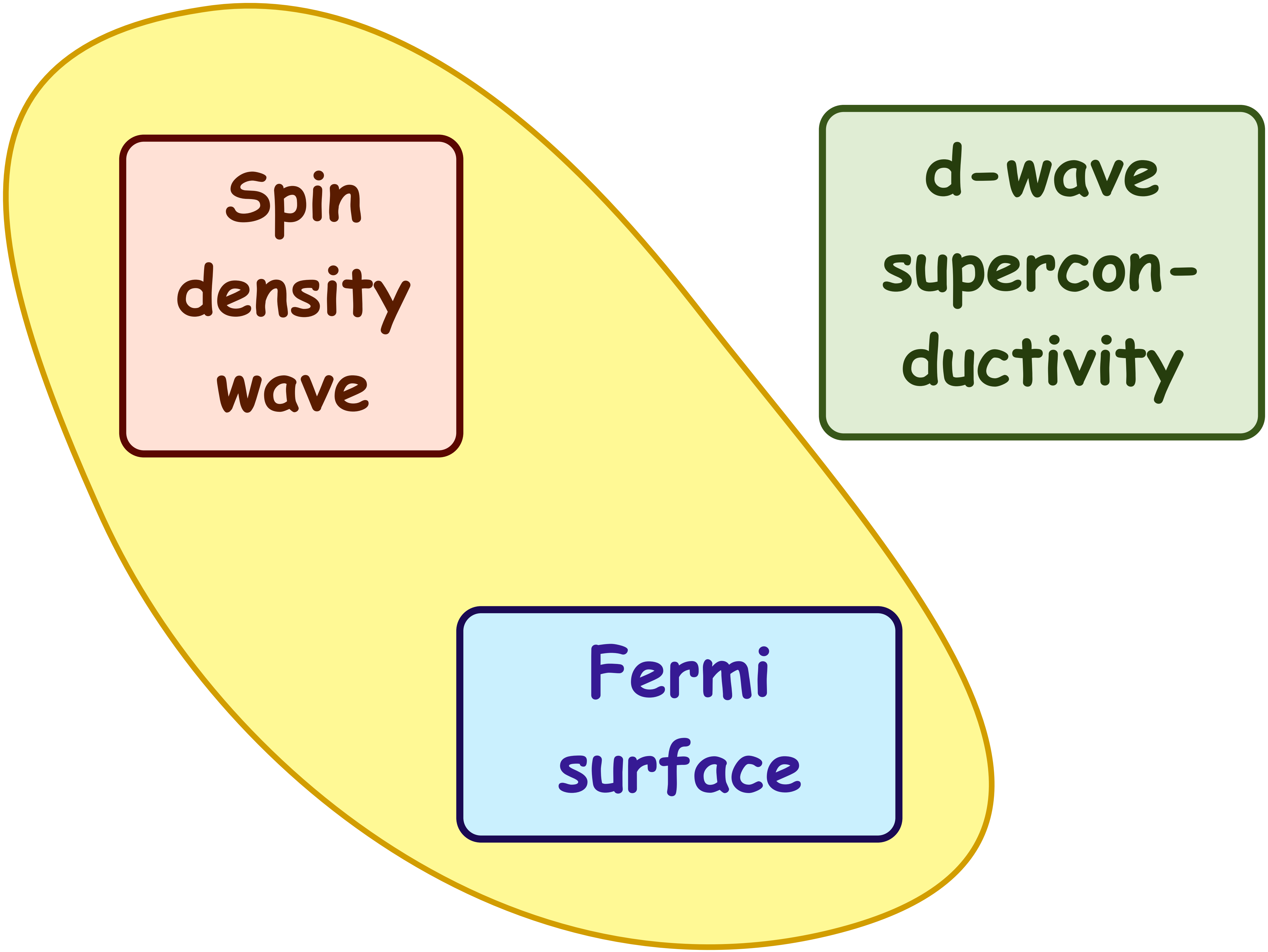
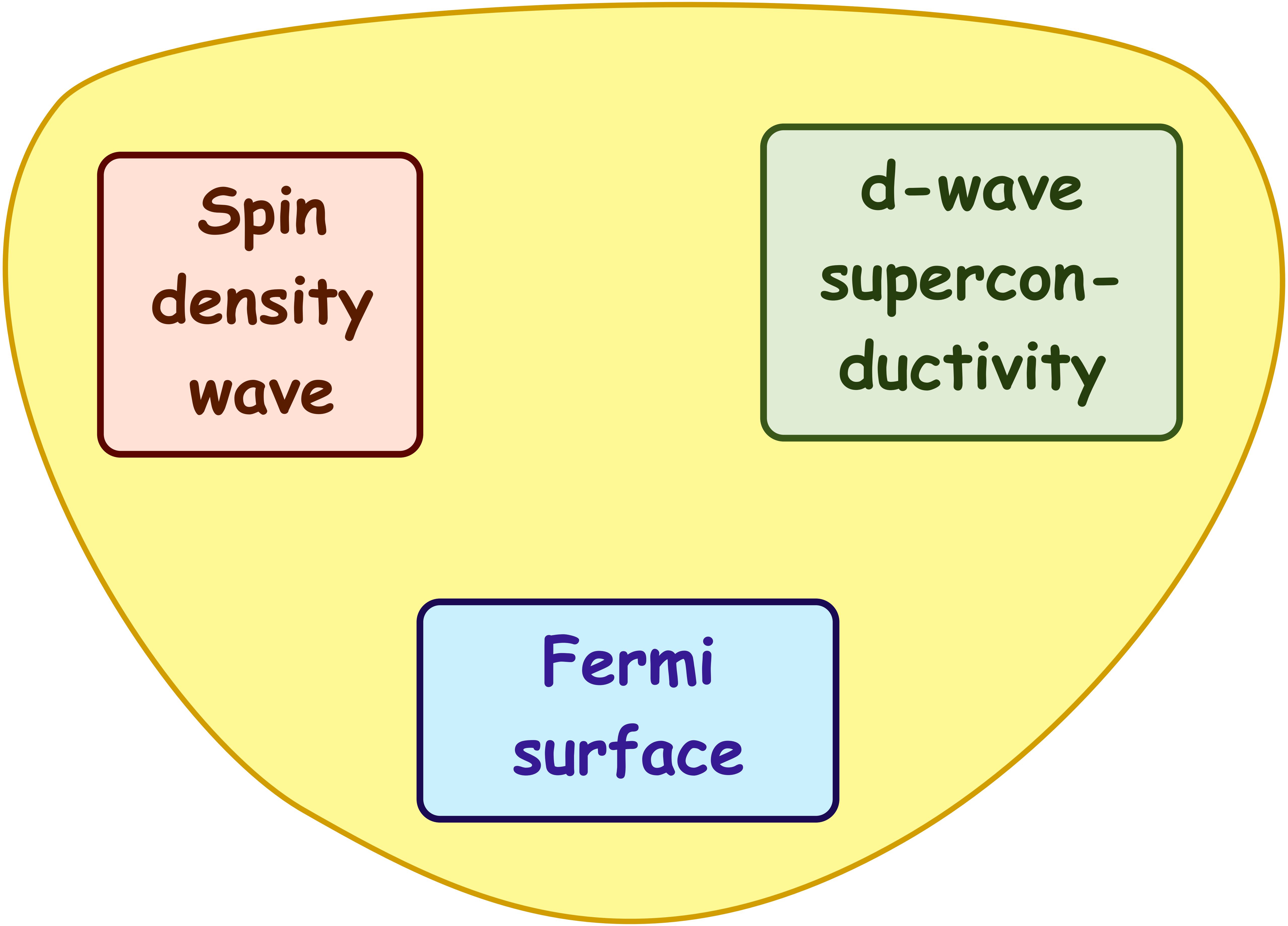
ductivity
Fermi
surface


Spin density wave
d-wave supercon-ductivity
Fermi surface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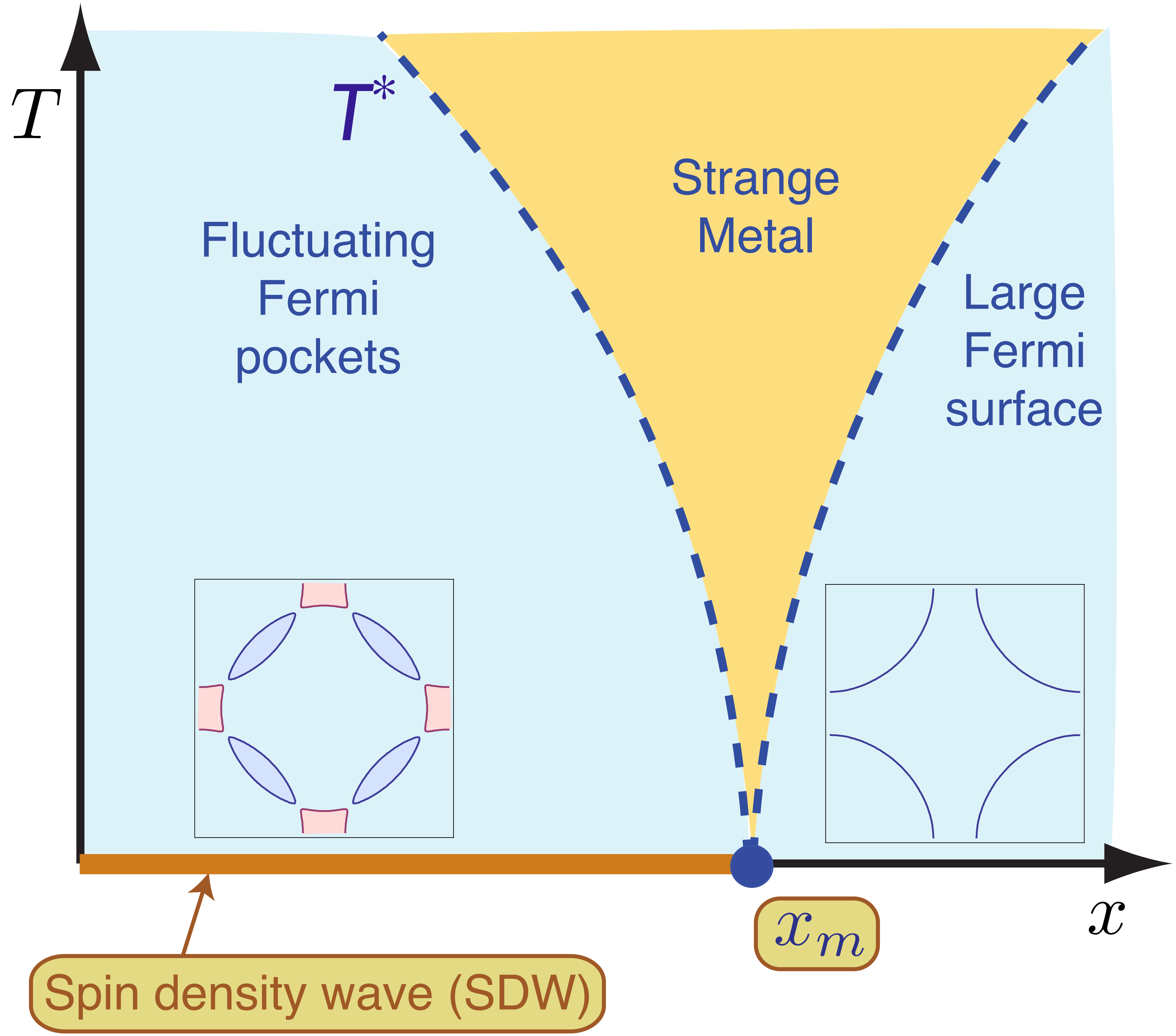

Underlying SDW ordering quantum critical point
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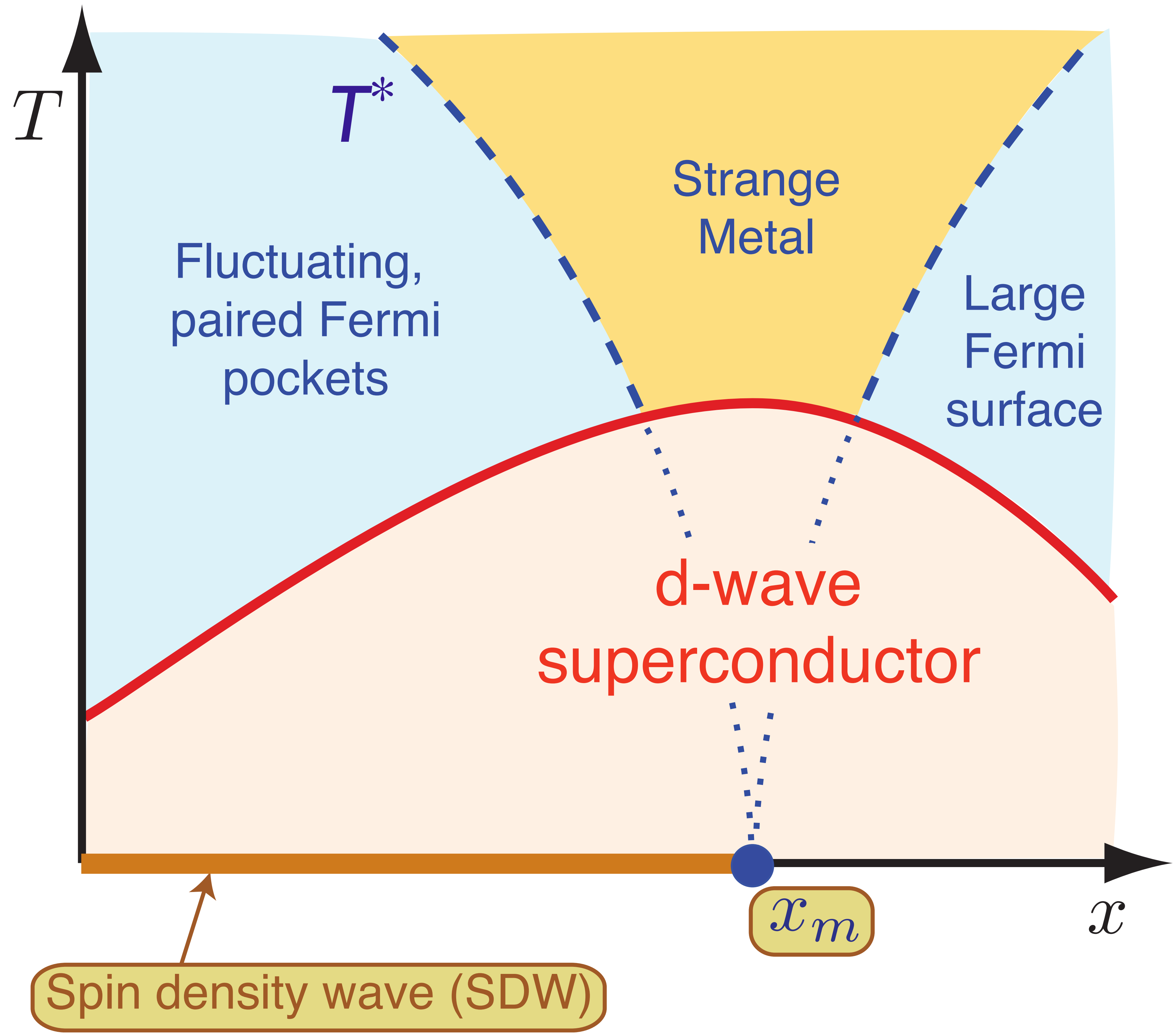
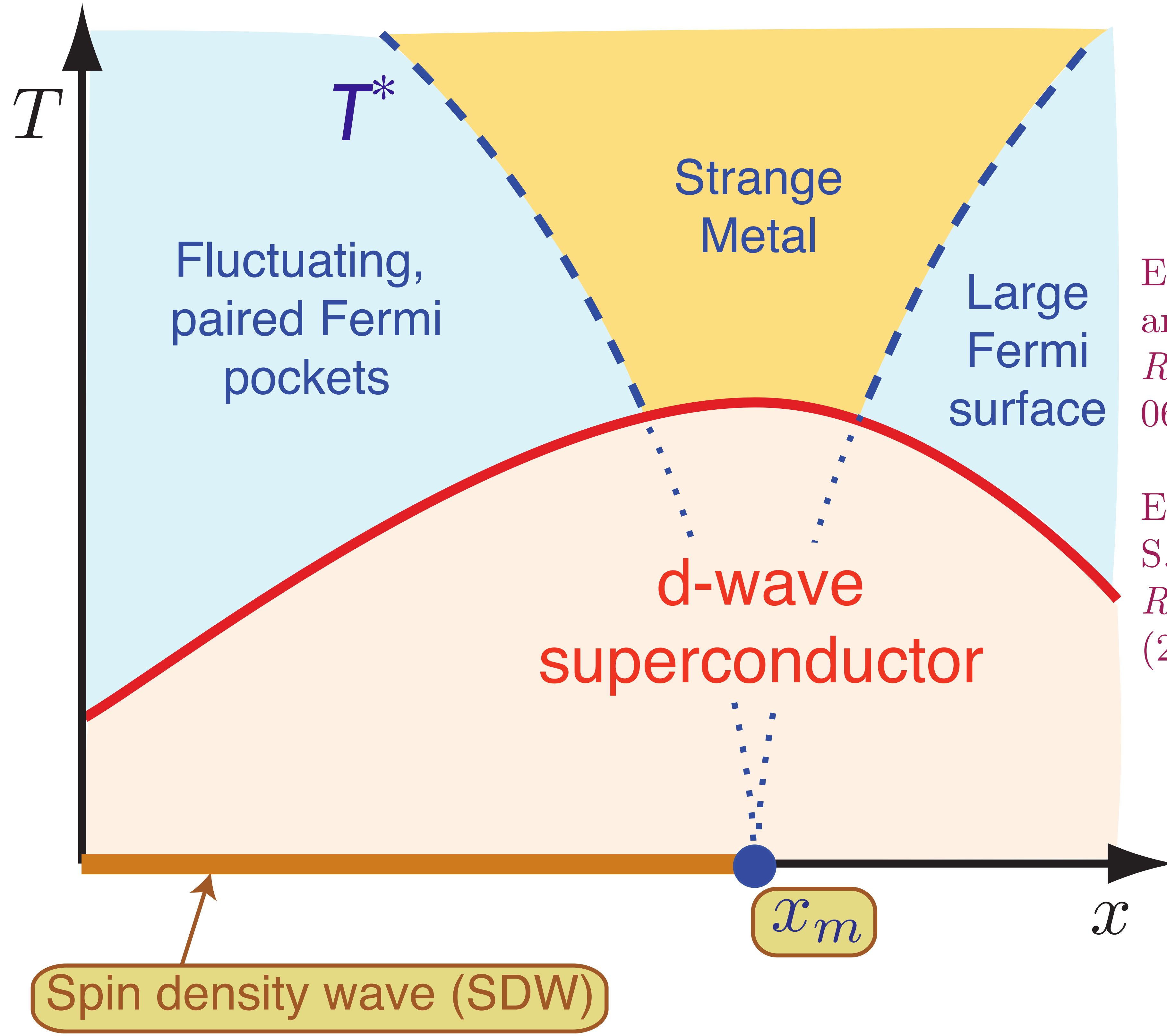
Theory of quantum criticality in the cuprates



Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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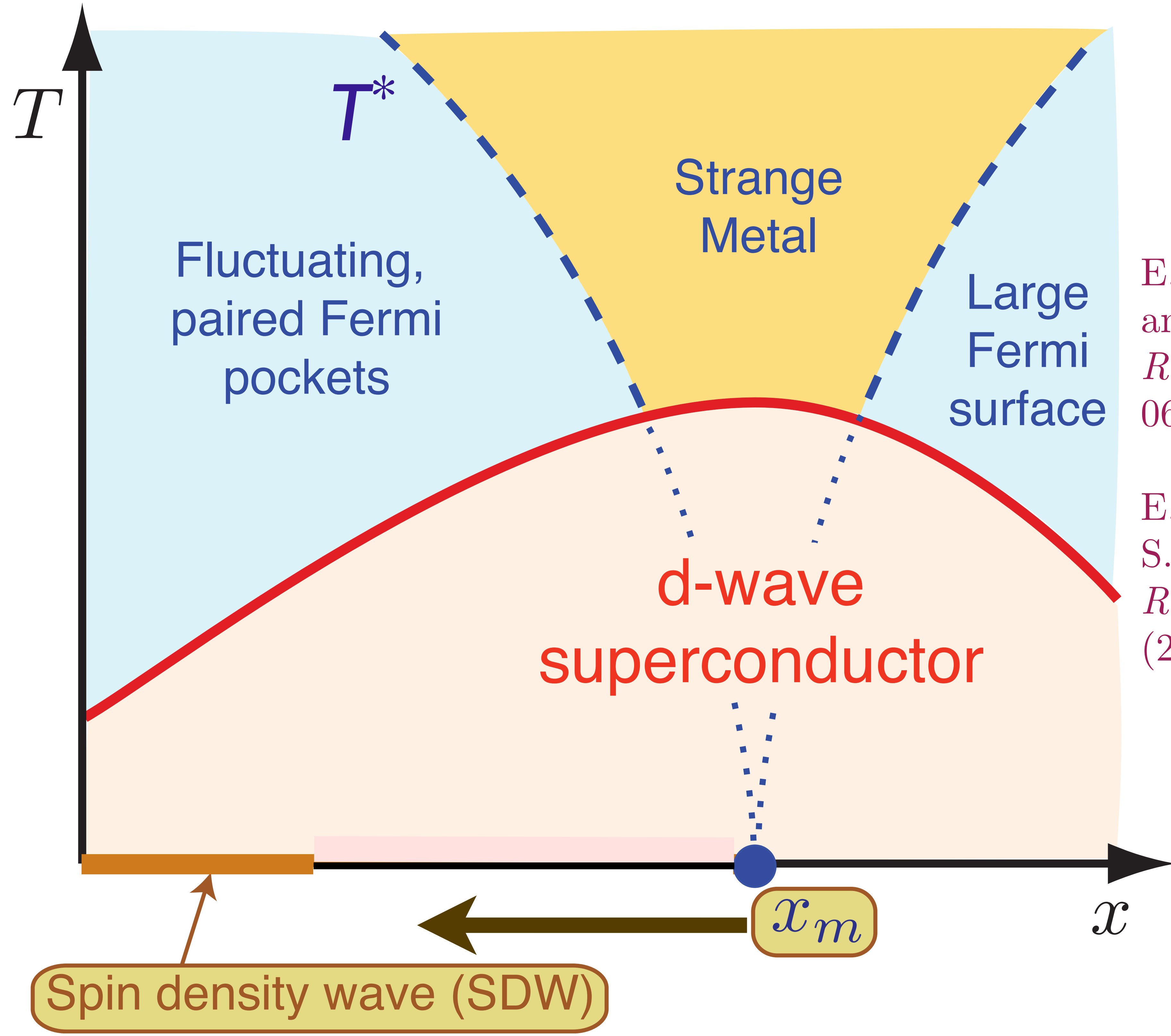

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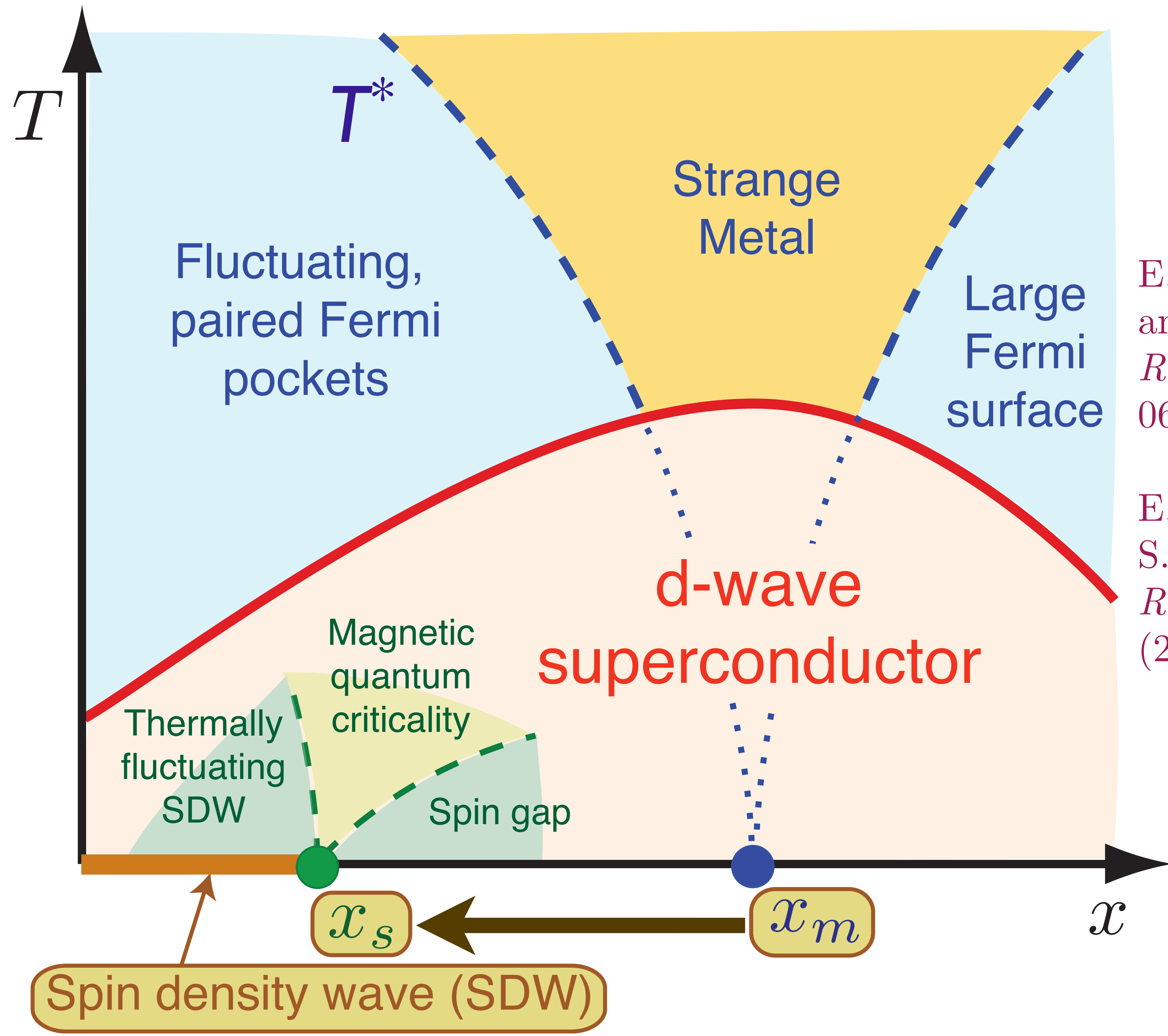

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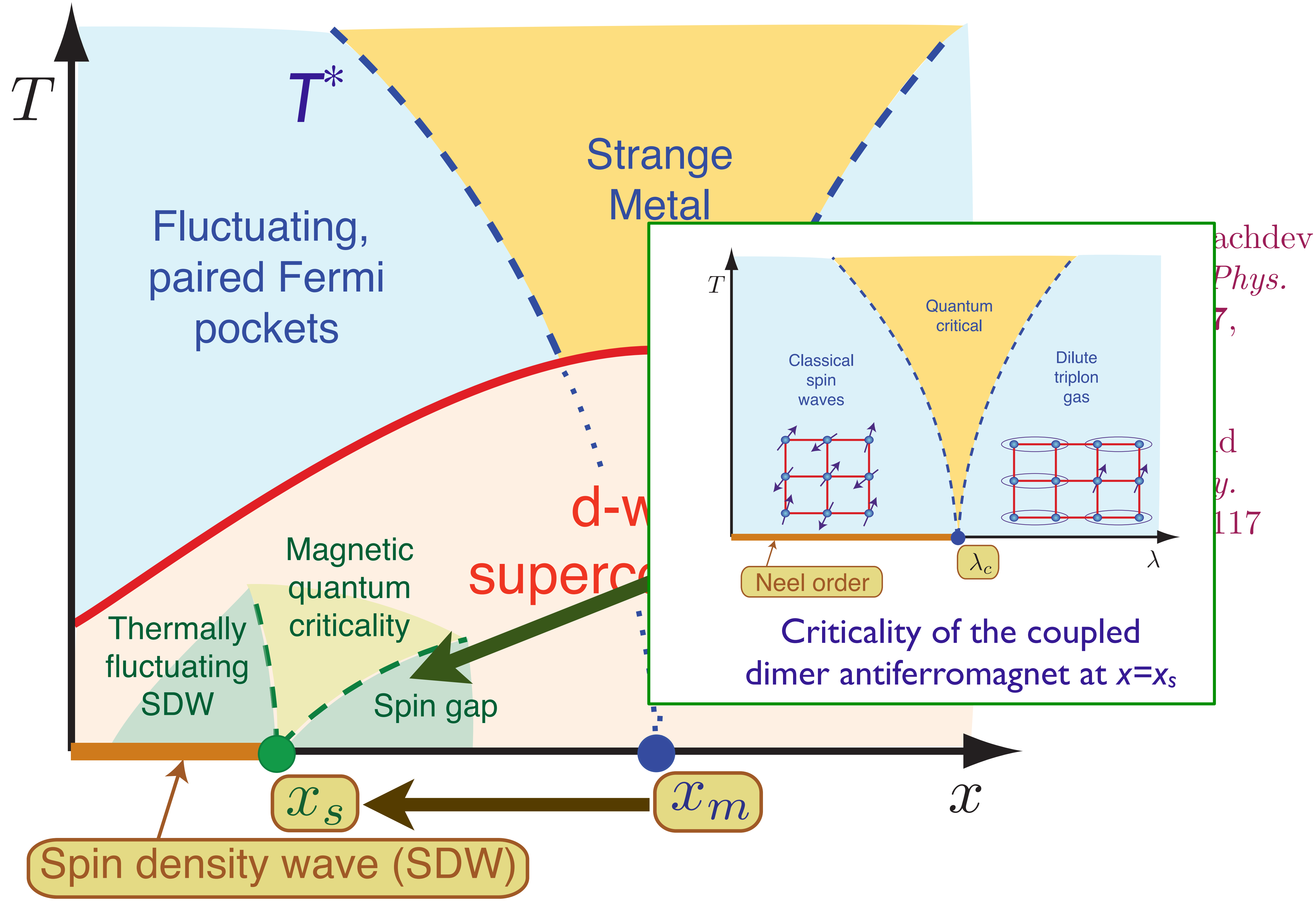

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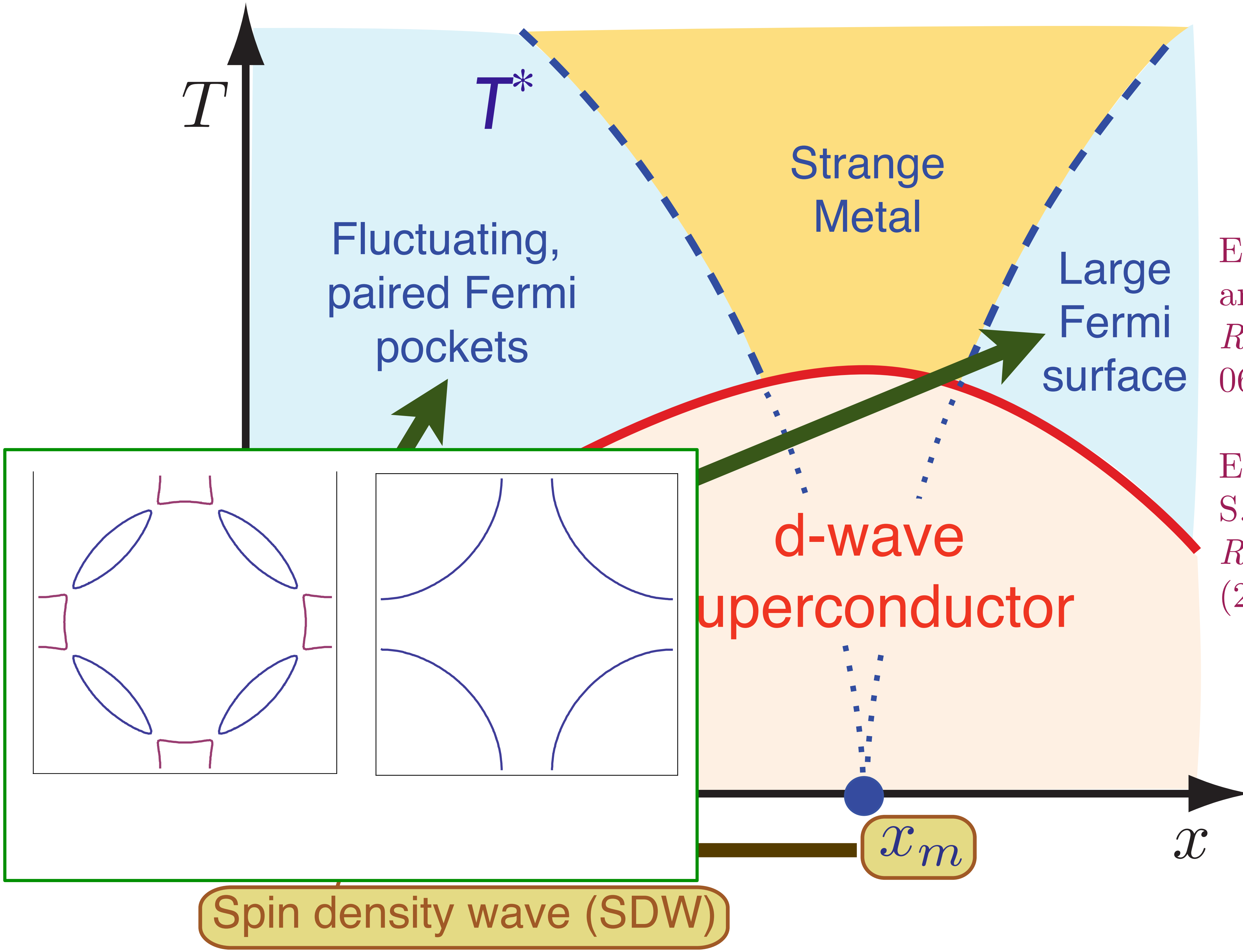

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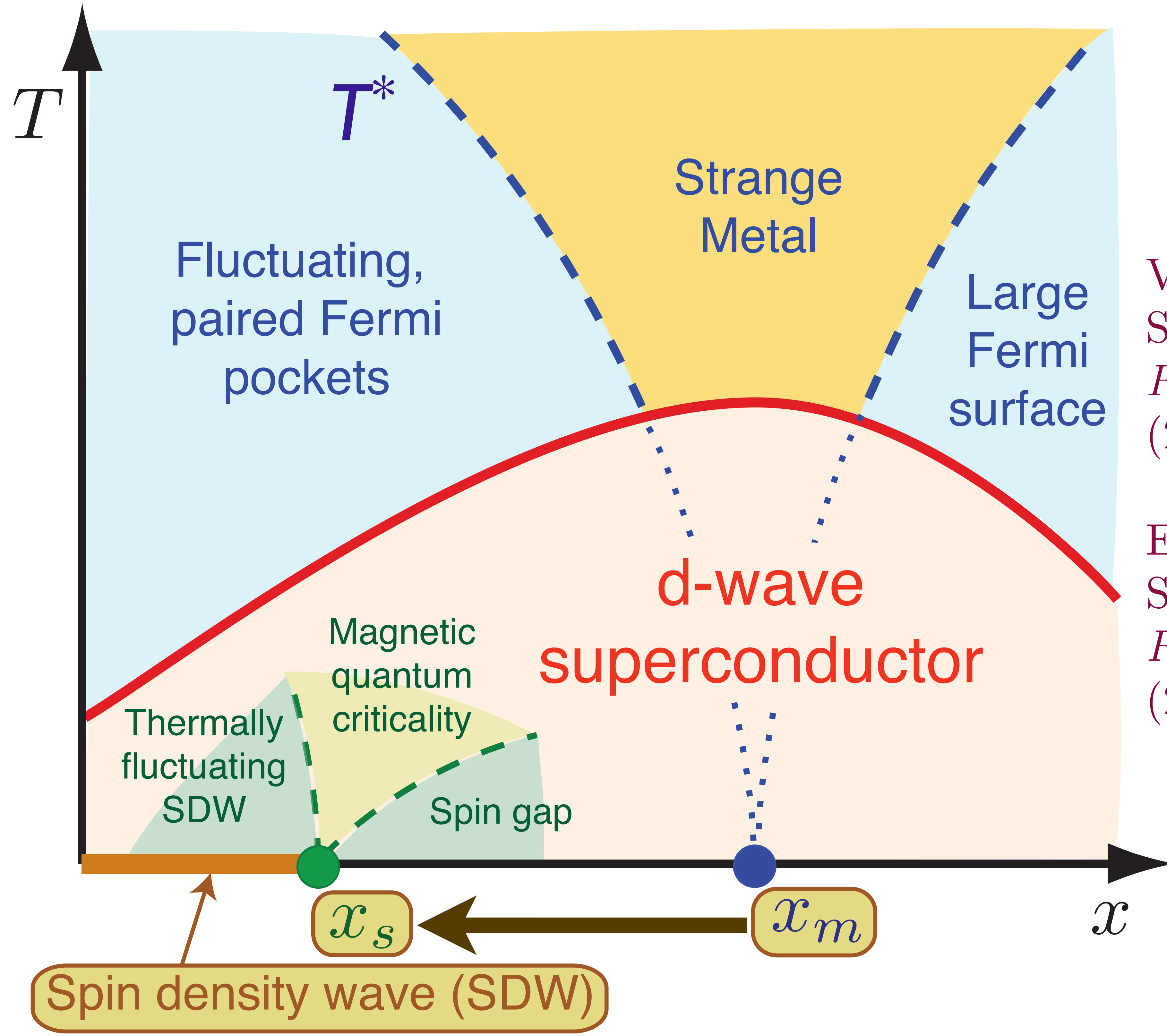

V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

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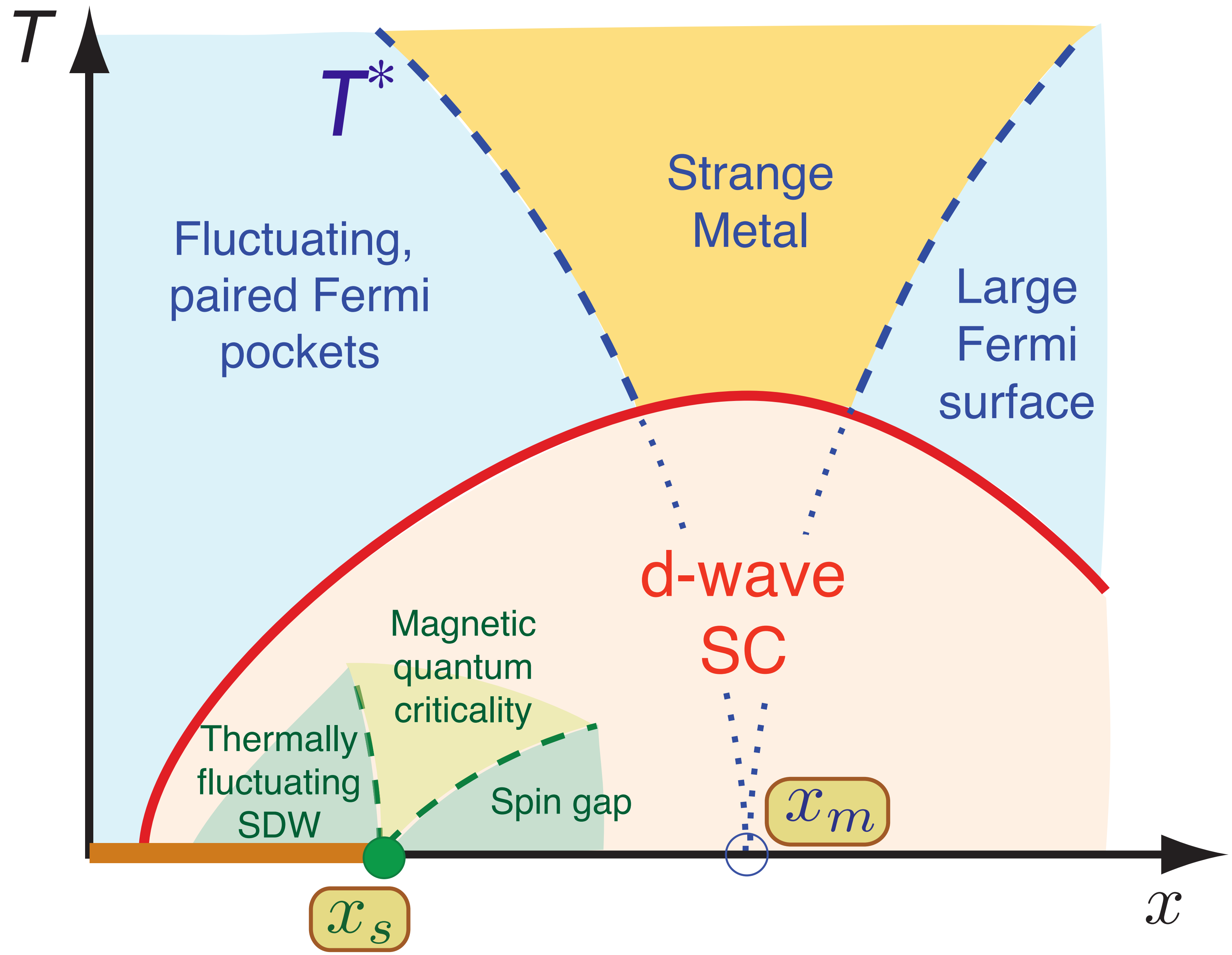
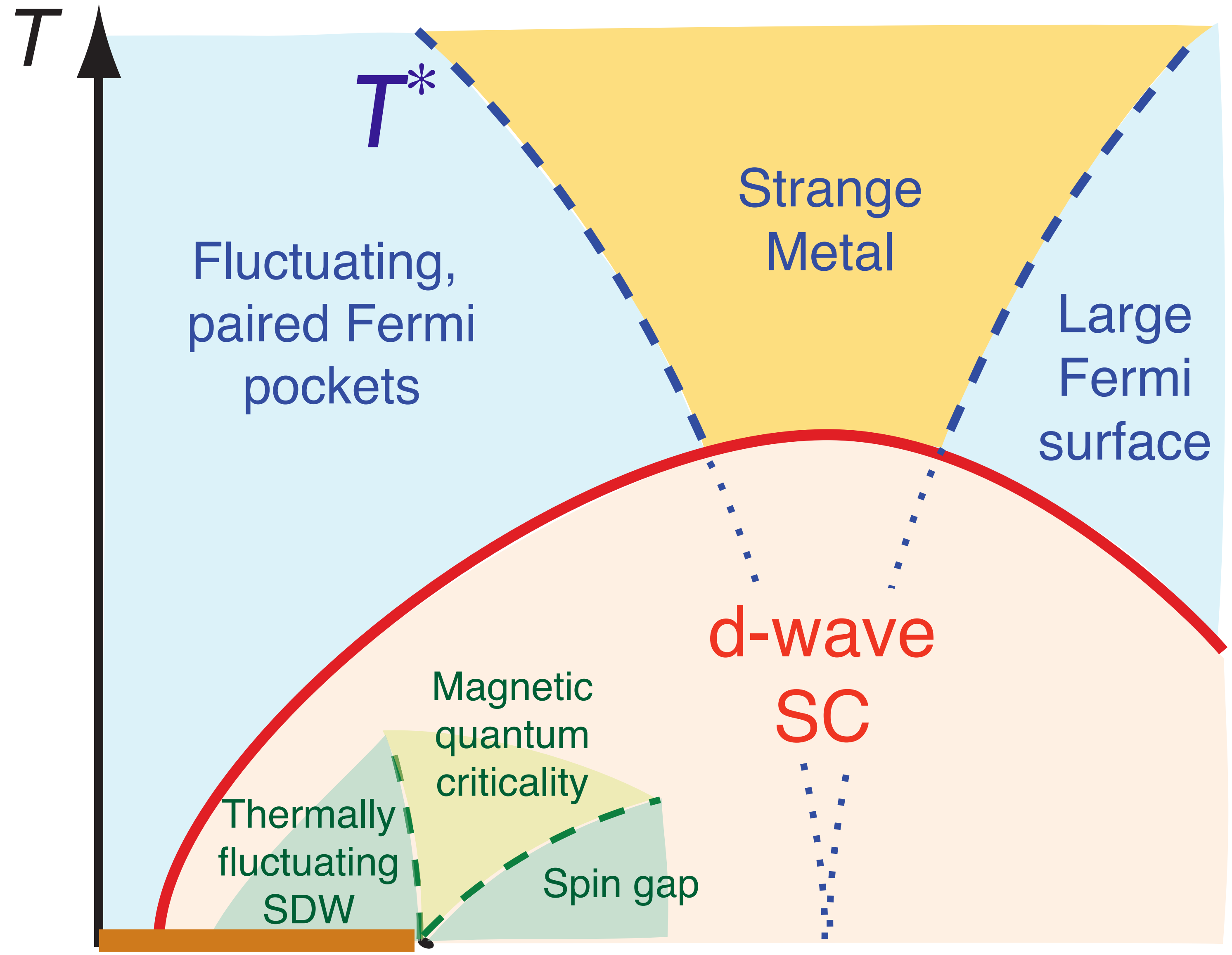

T
T*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x



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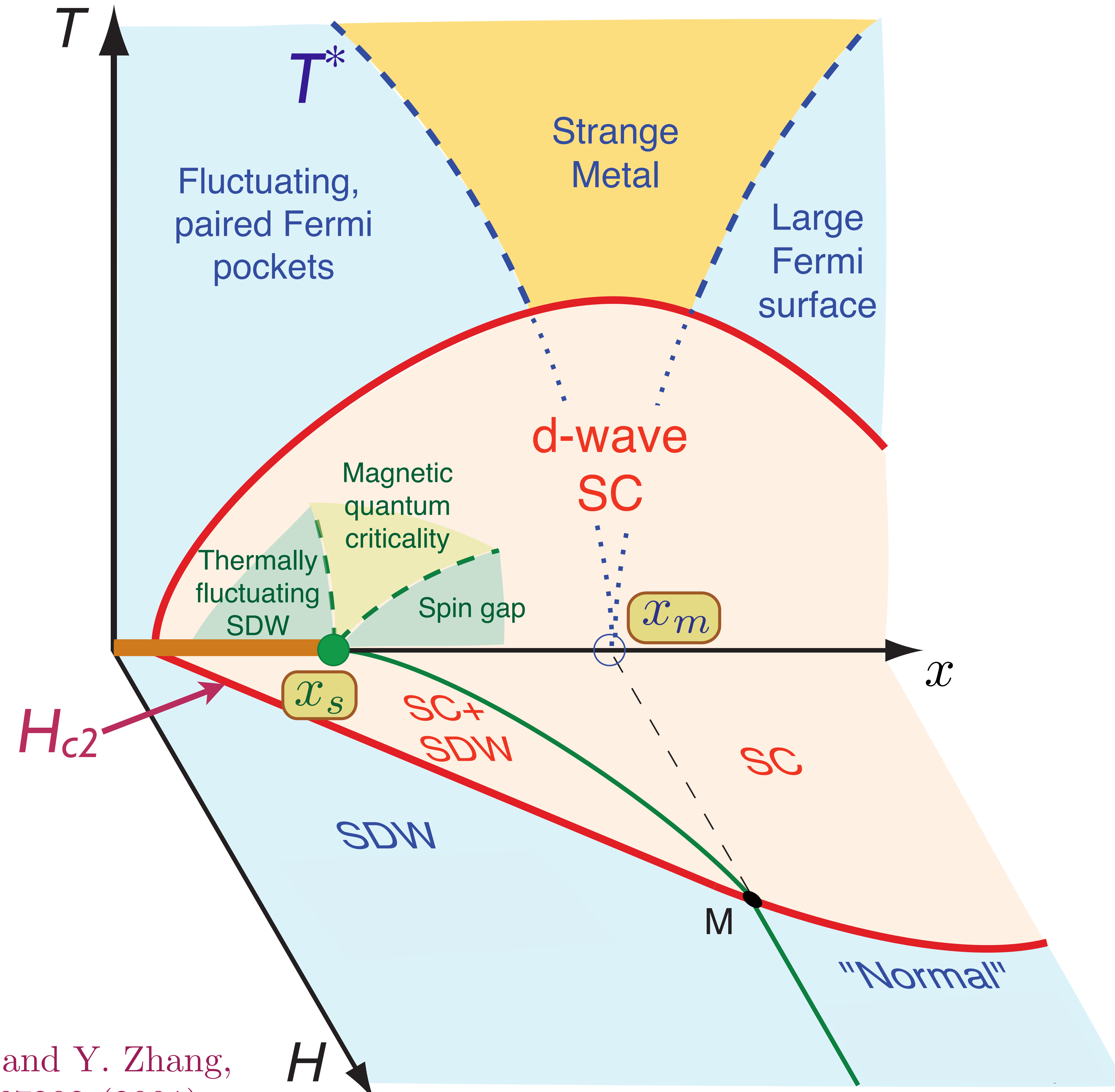

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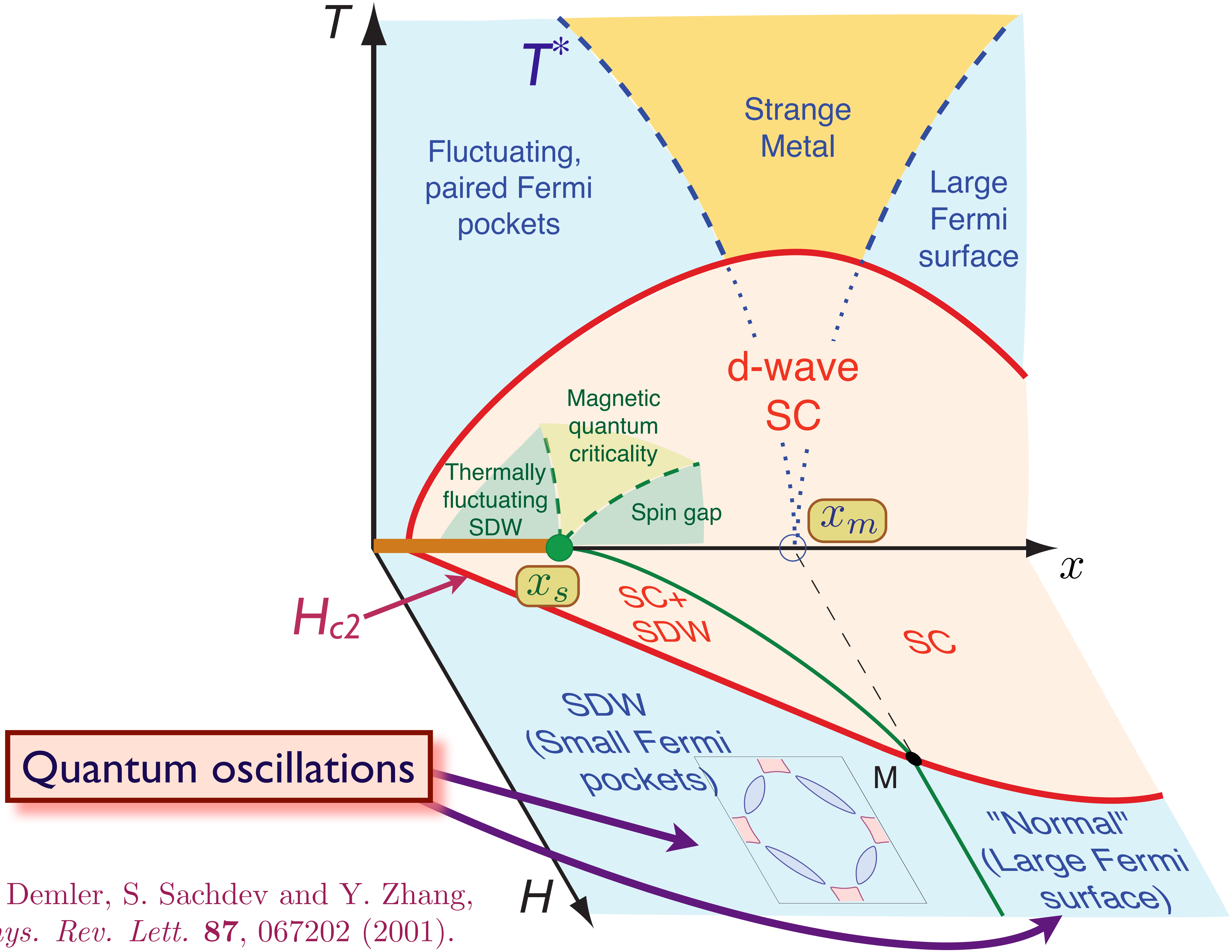

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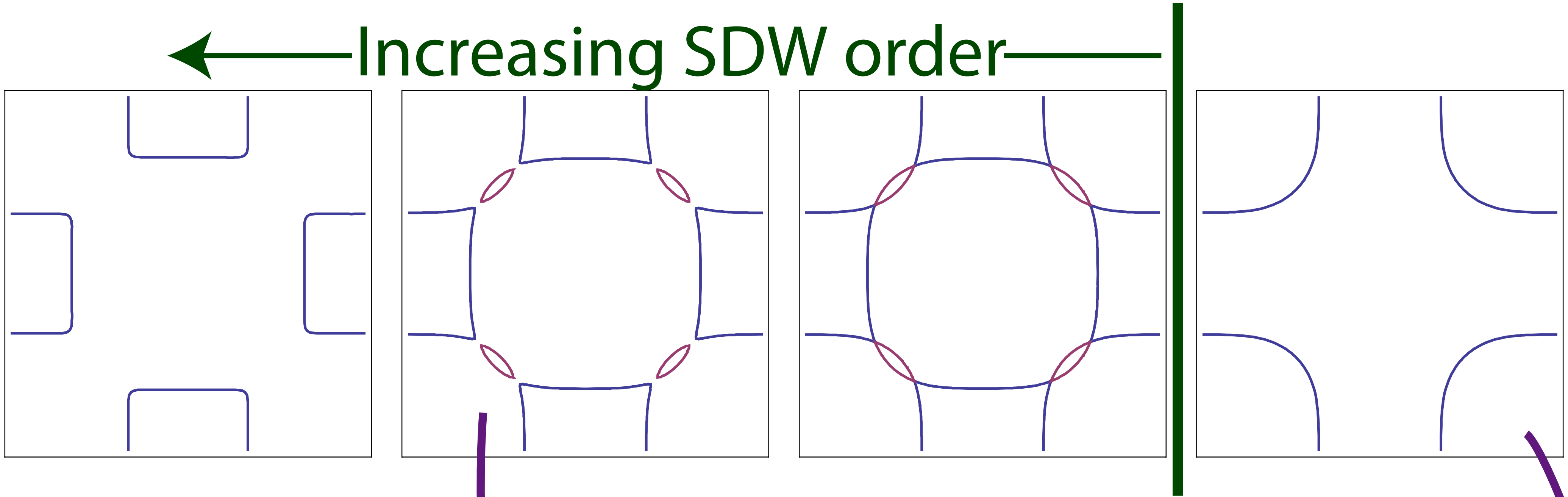
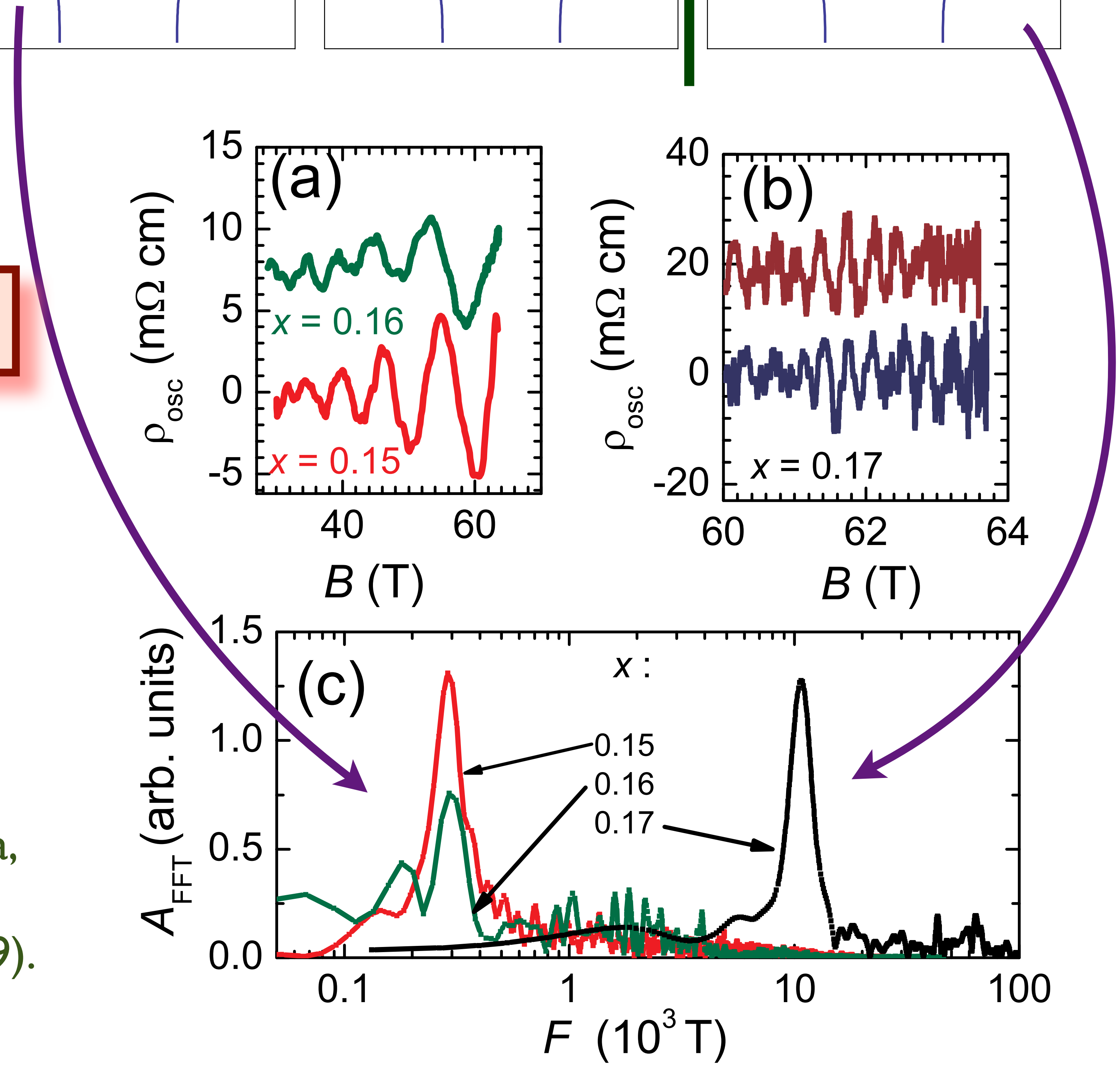
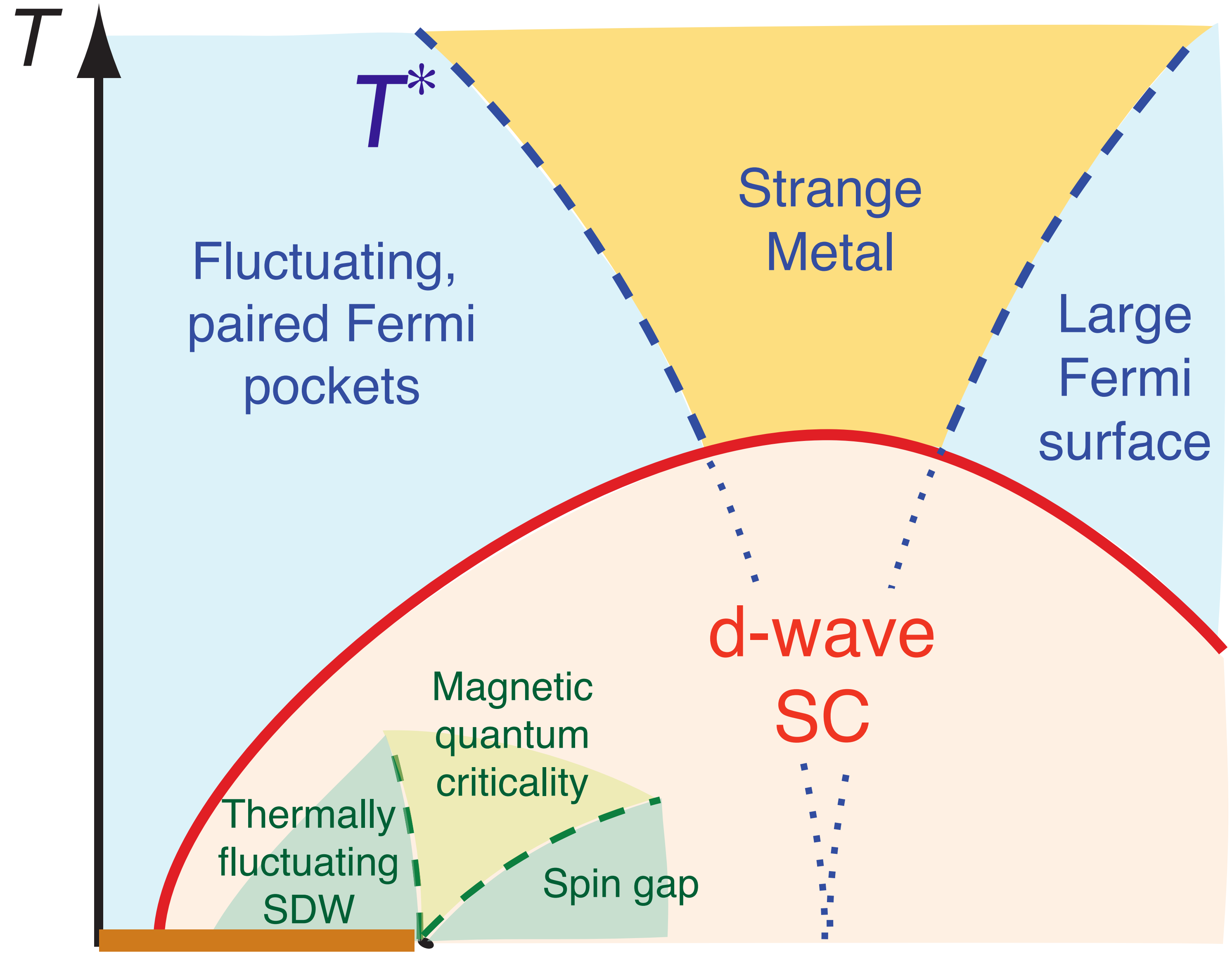


Quantum oscillations



$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).



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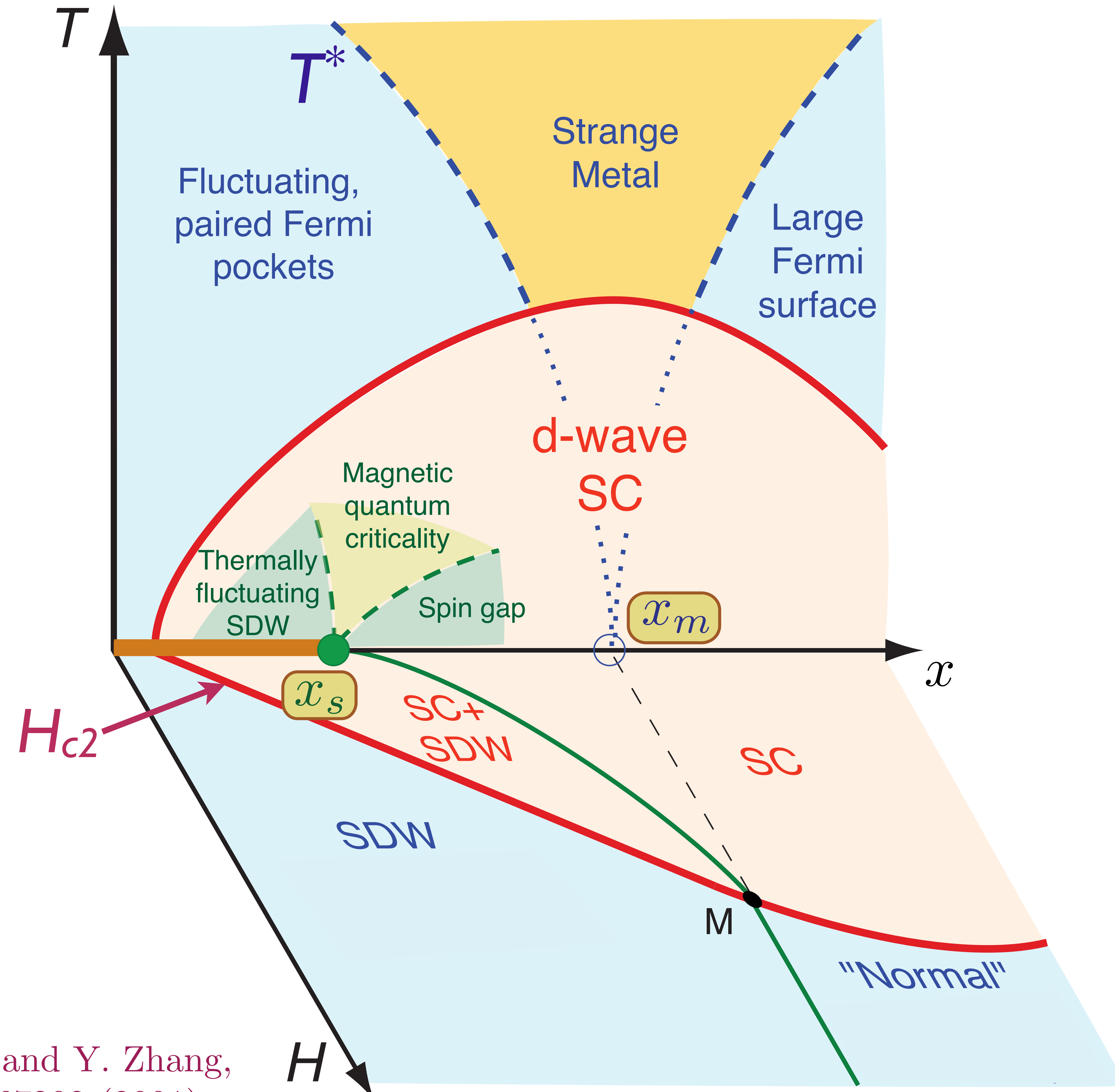

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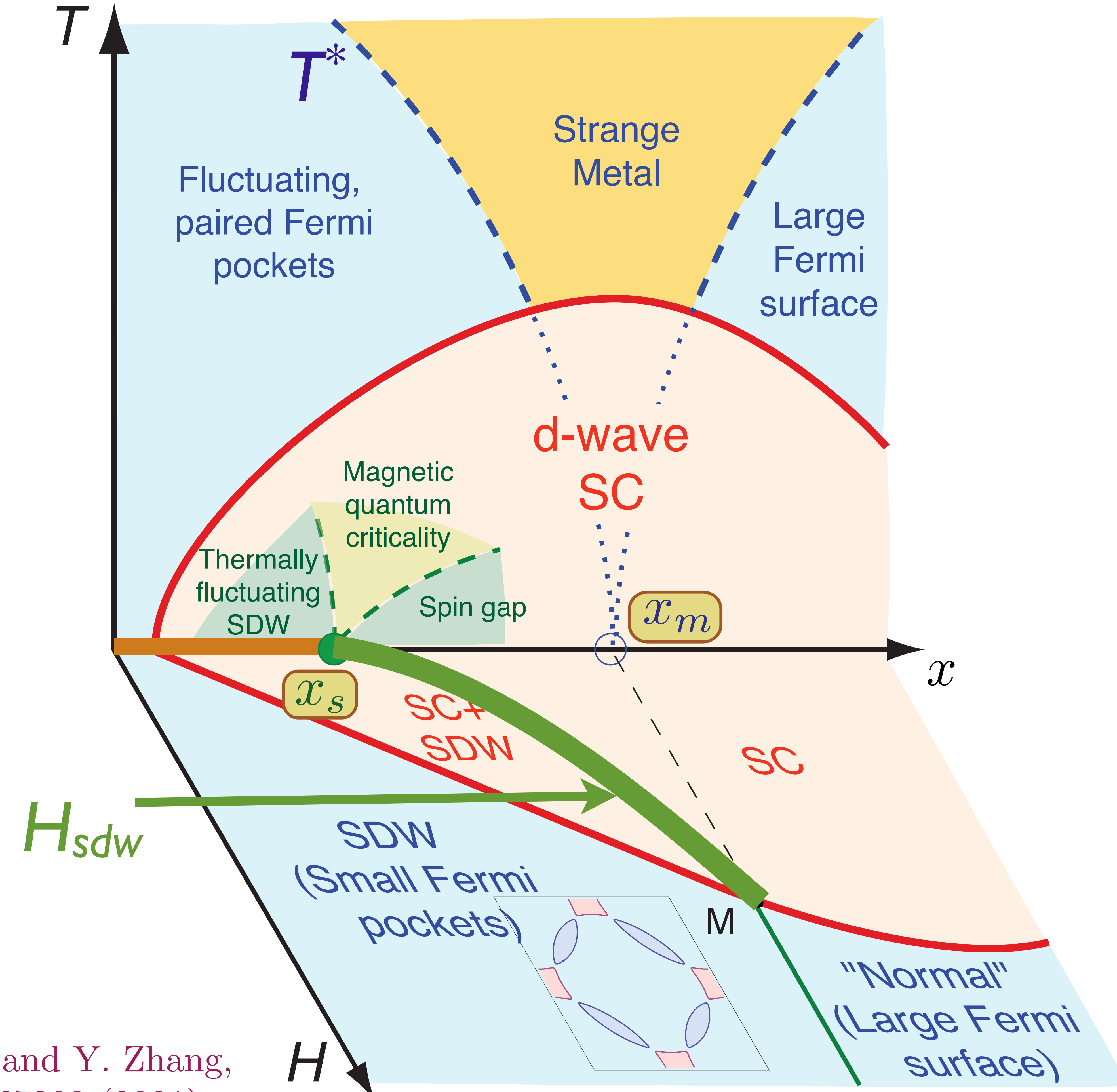

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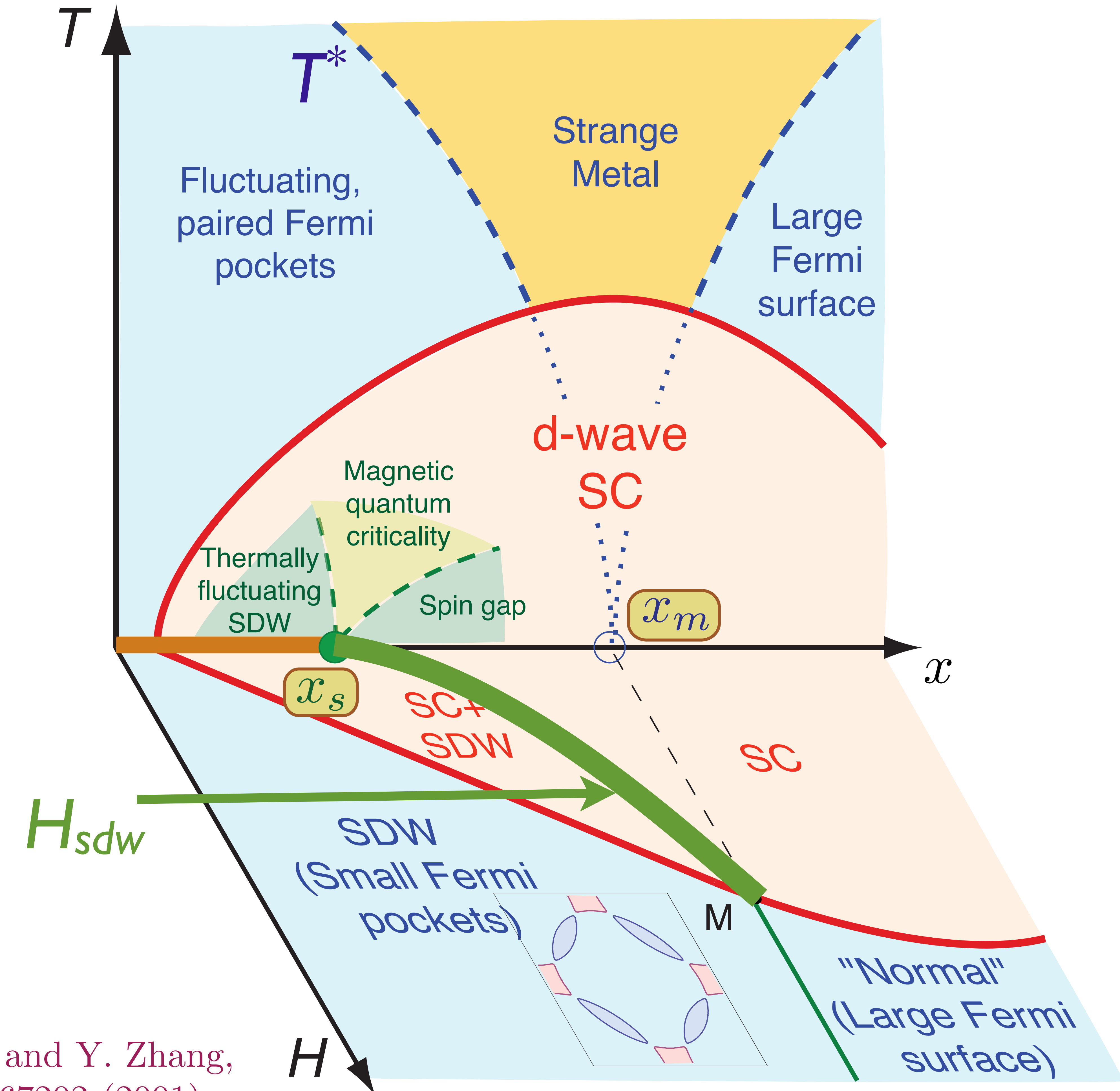

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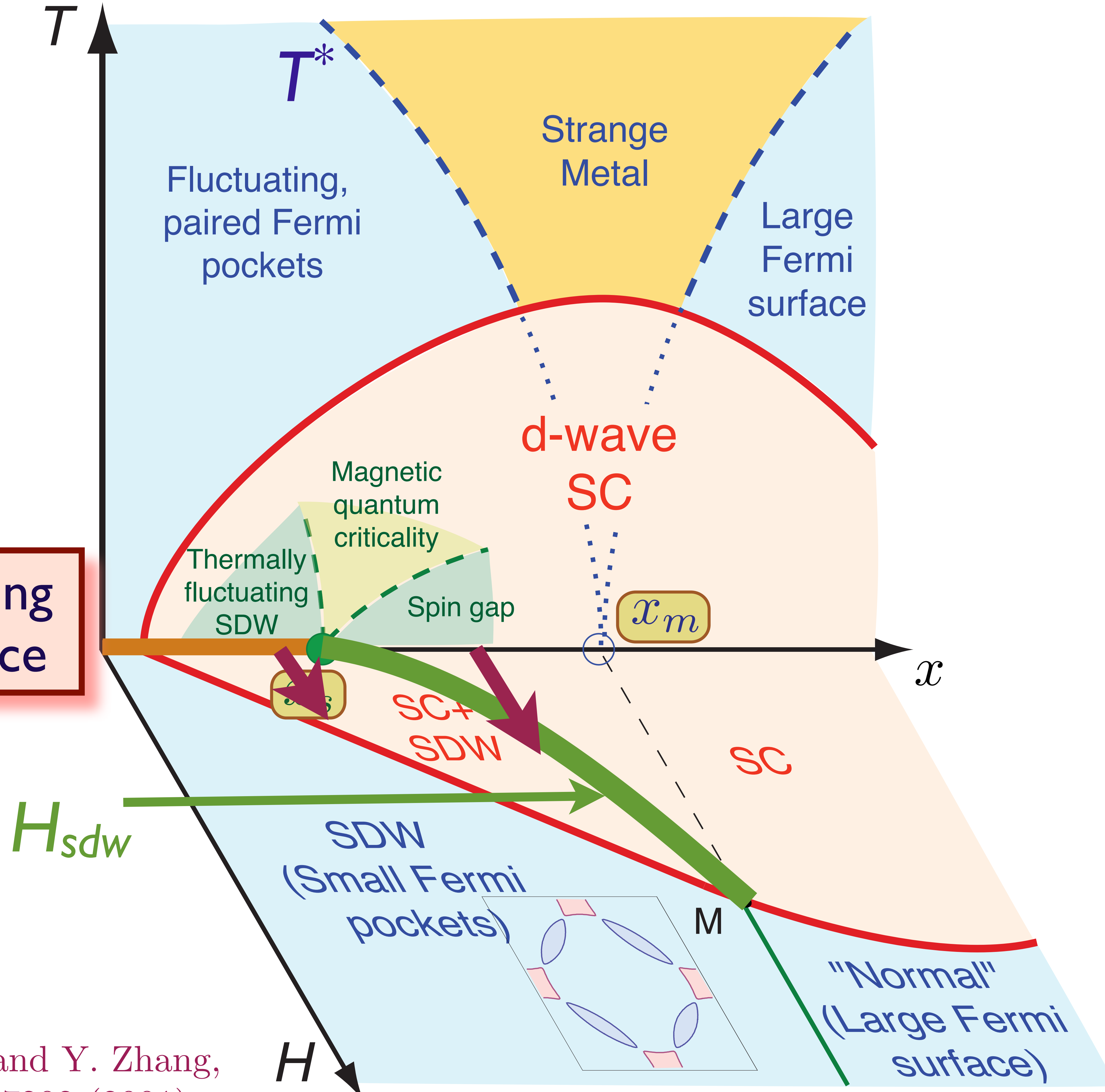
Neutron scattering & muon resonance



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PHYSICAL REVIEW B **71**, 220508(R) (2005)

Field-induced transition between magnetically disordered and ordered phases in underdoped $La_{2-x}Sr_xCuO_4$

B. Khaykovich,[1] S. Wakimoto,[2] R. J. Birgeneau,[3] M. A. Kastner,[1] Y. S. Lee,[1] P. Smeibidl,[4] P. Vorderwisch,[4] and K. Yamada[5]

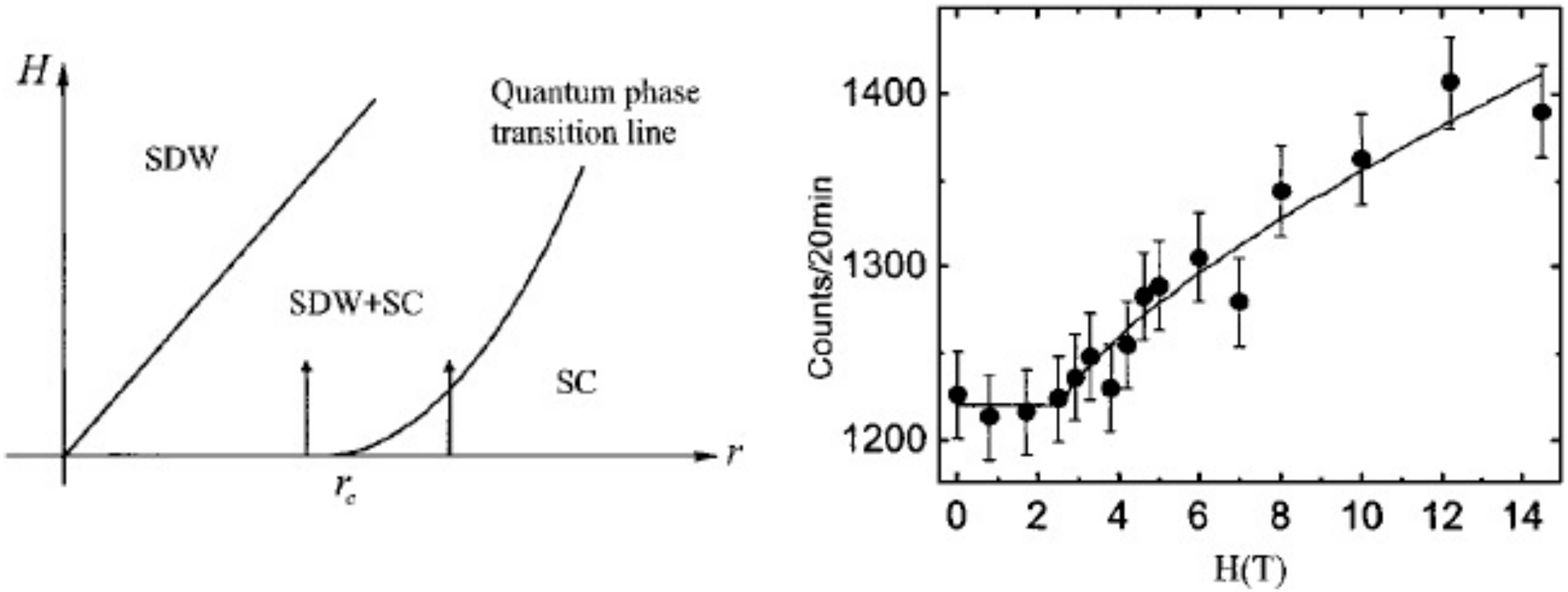
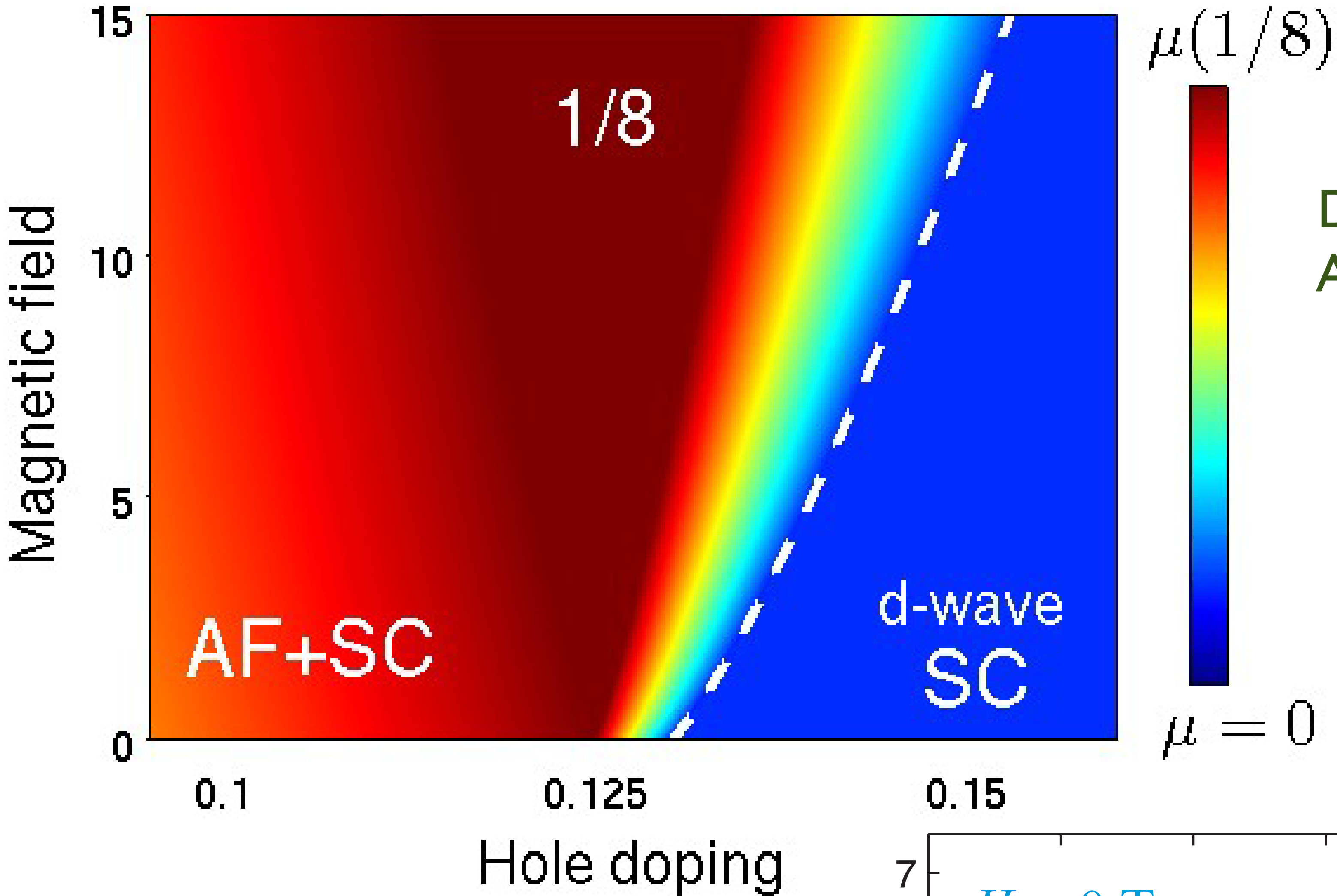
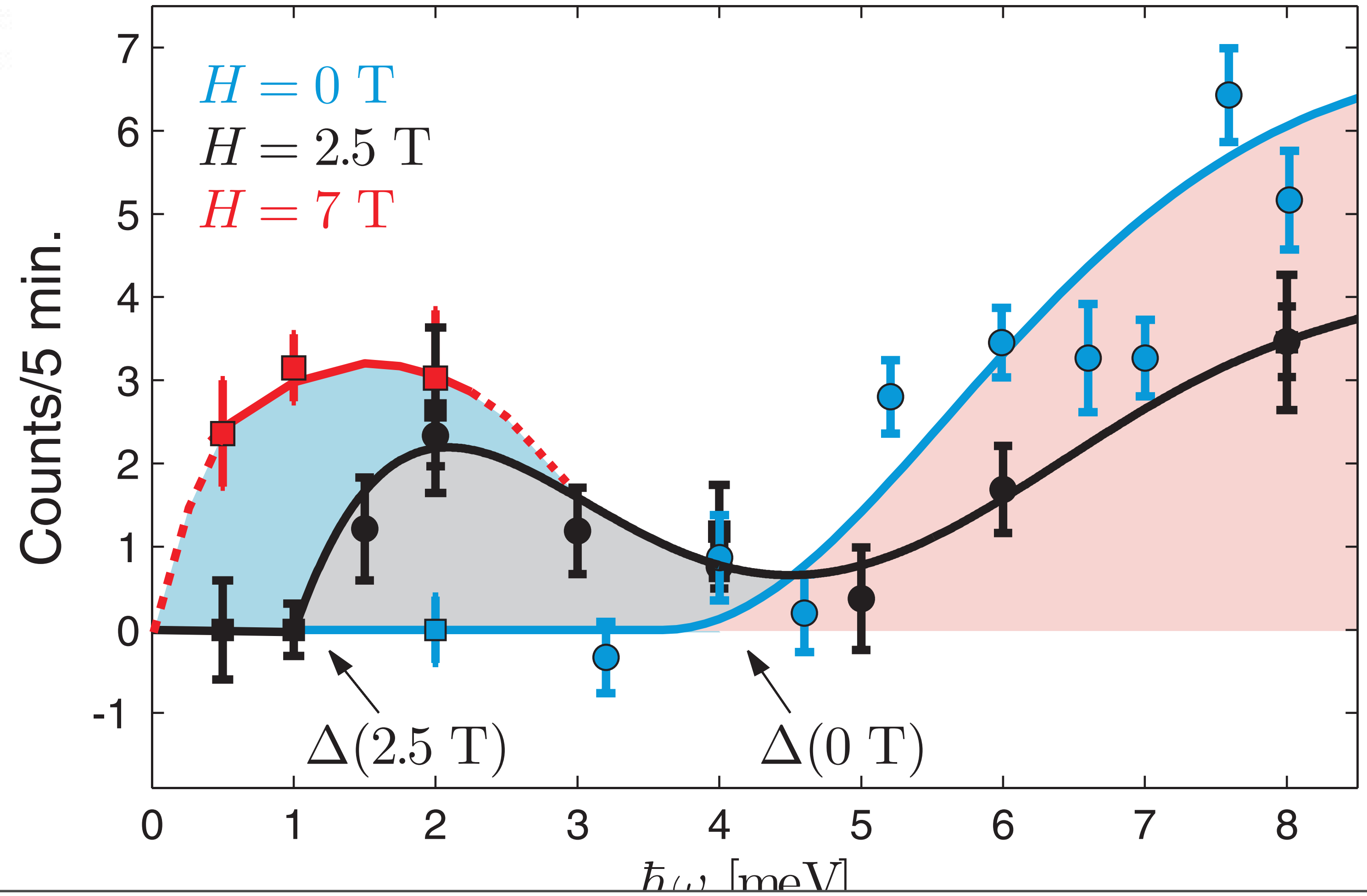


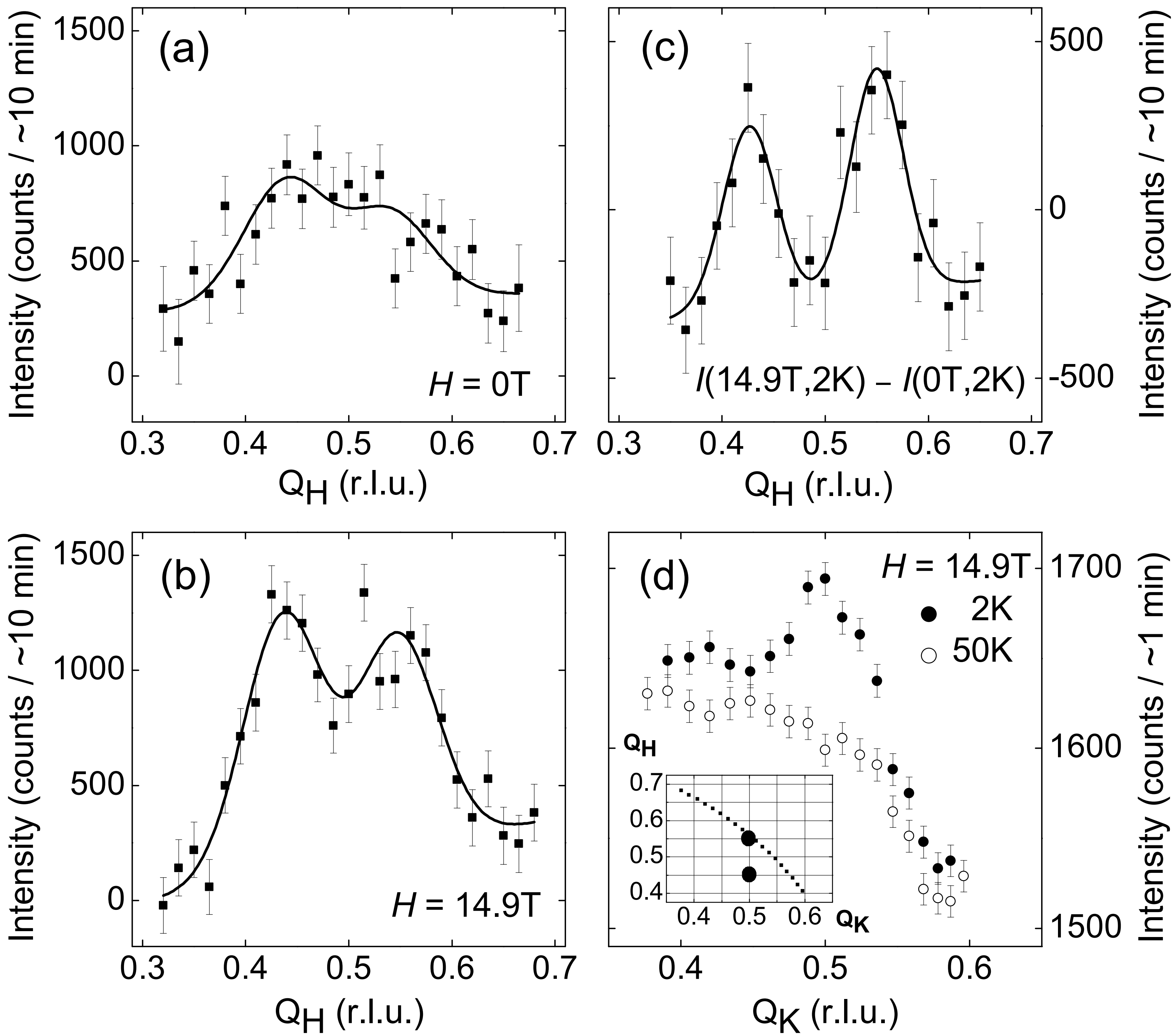
FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125,0.125,0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.

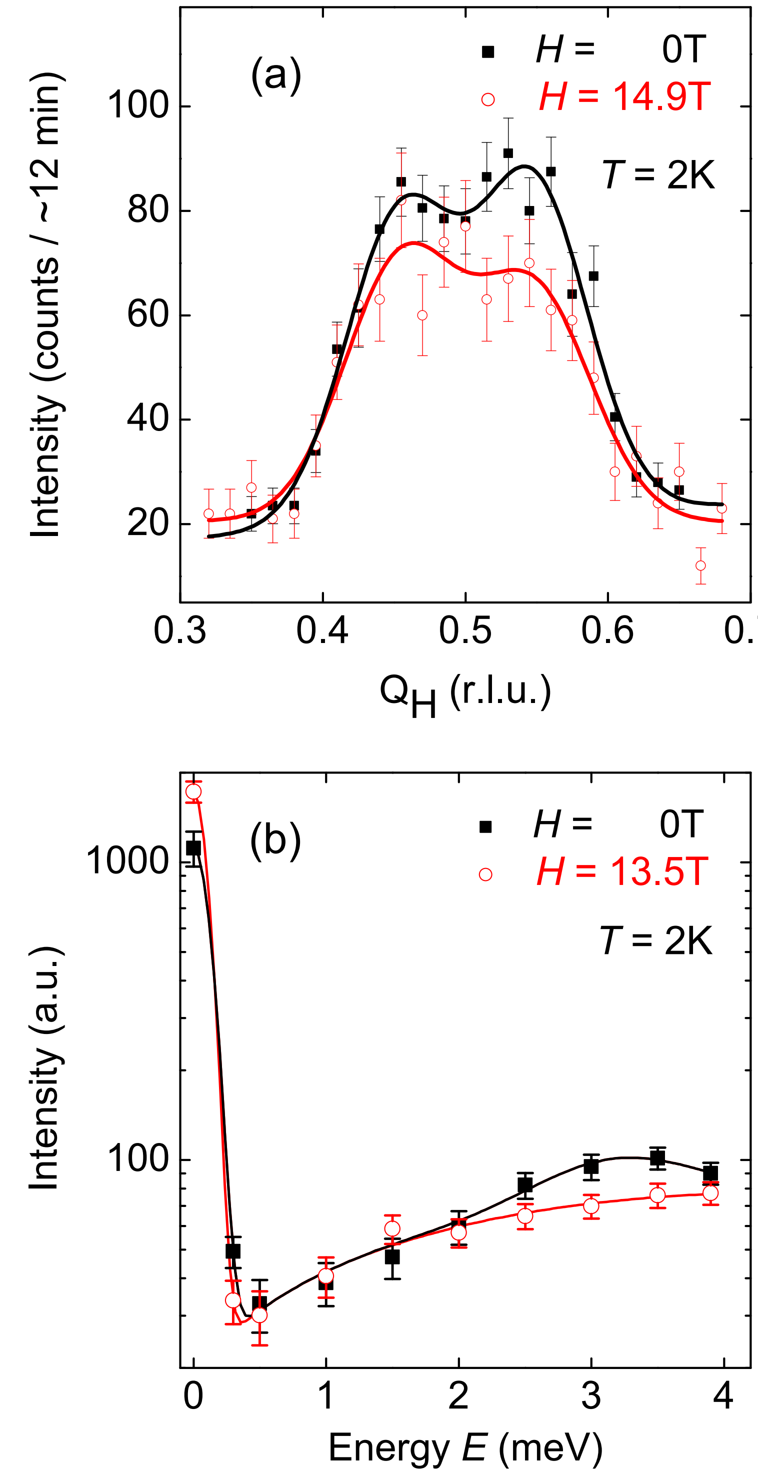


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

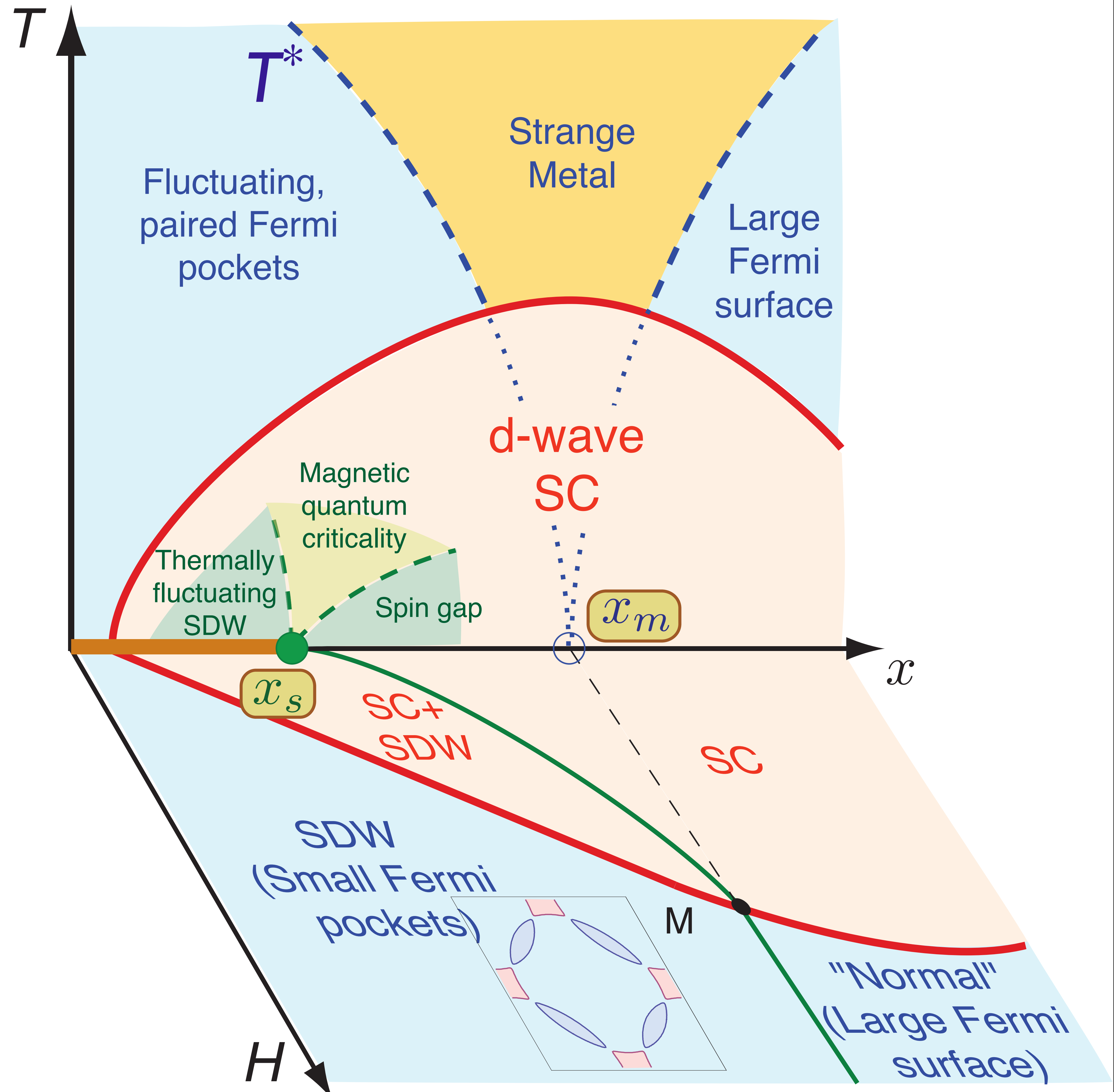


J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

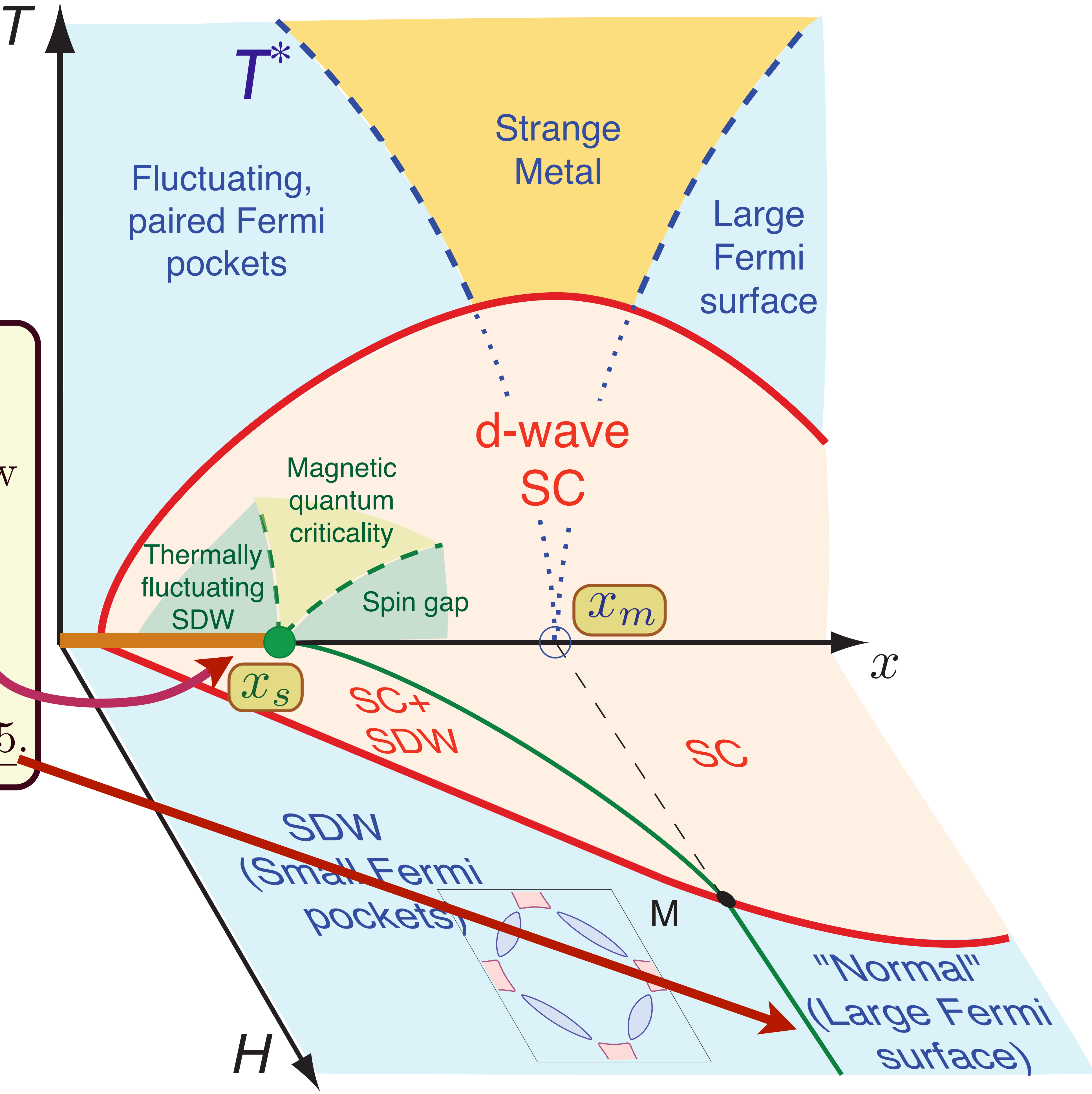




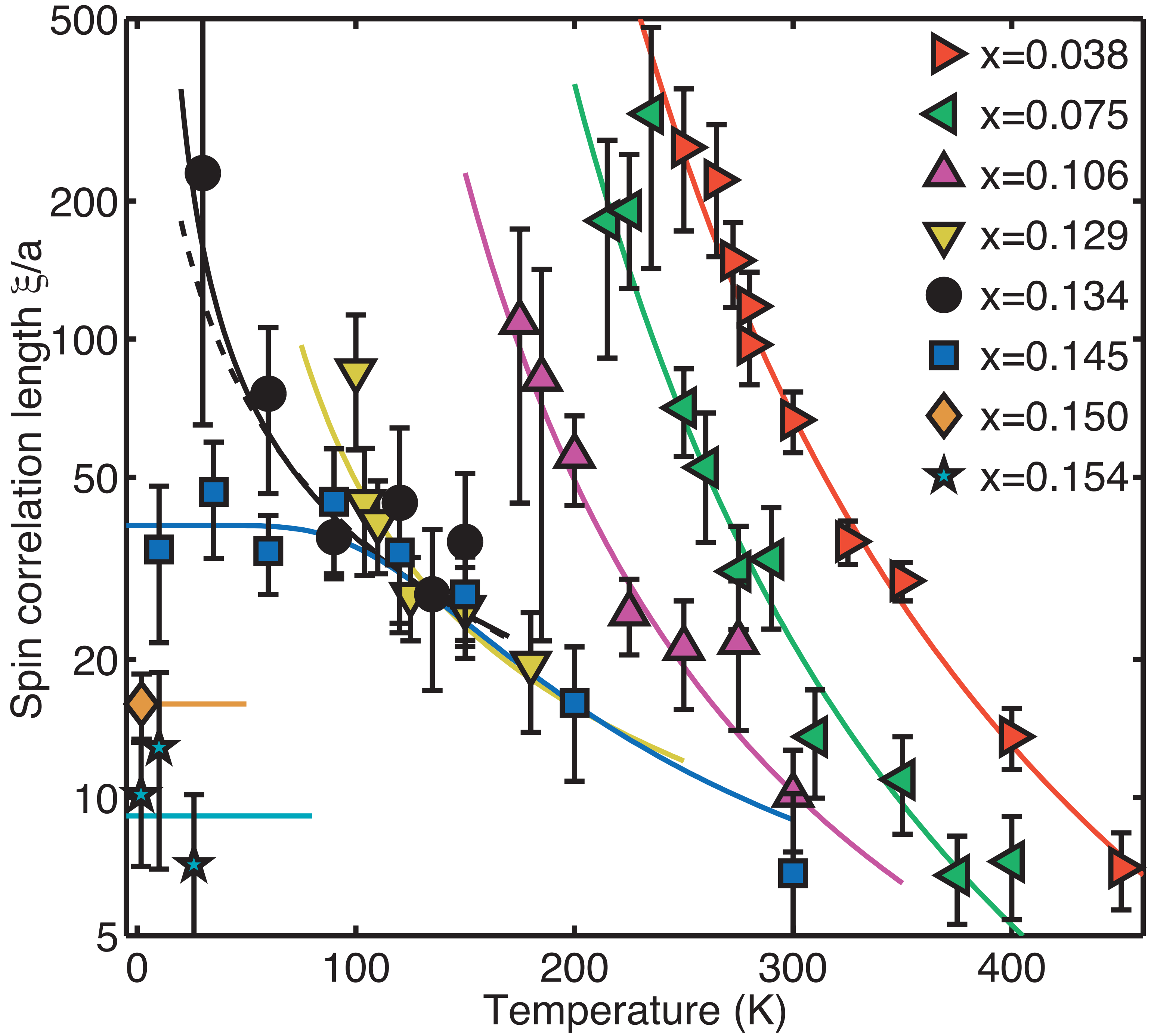
D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)


T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

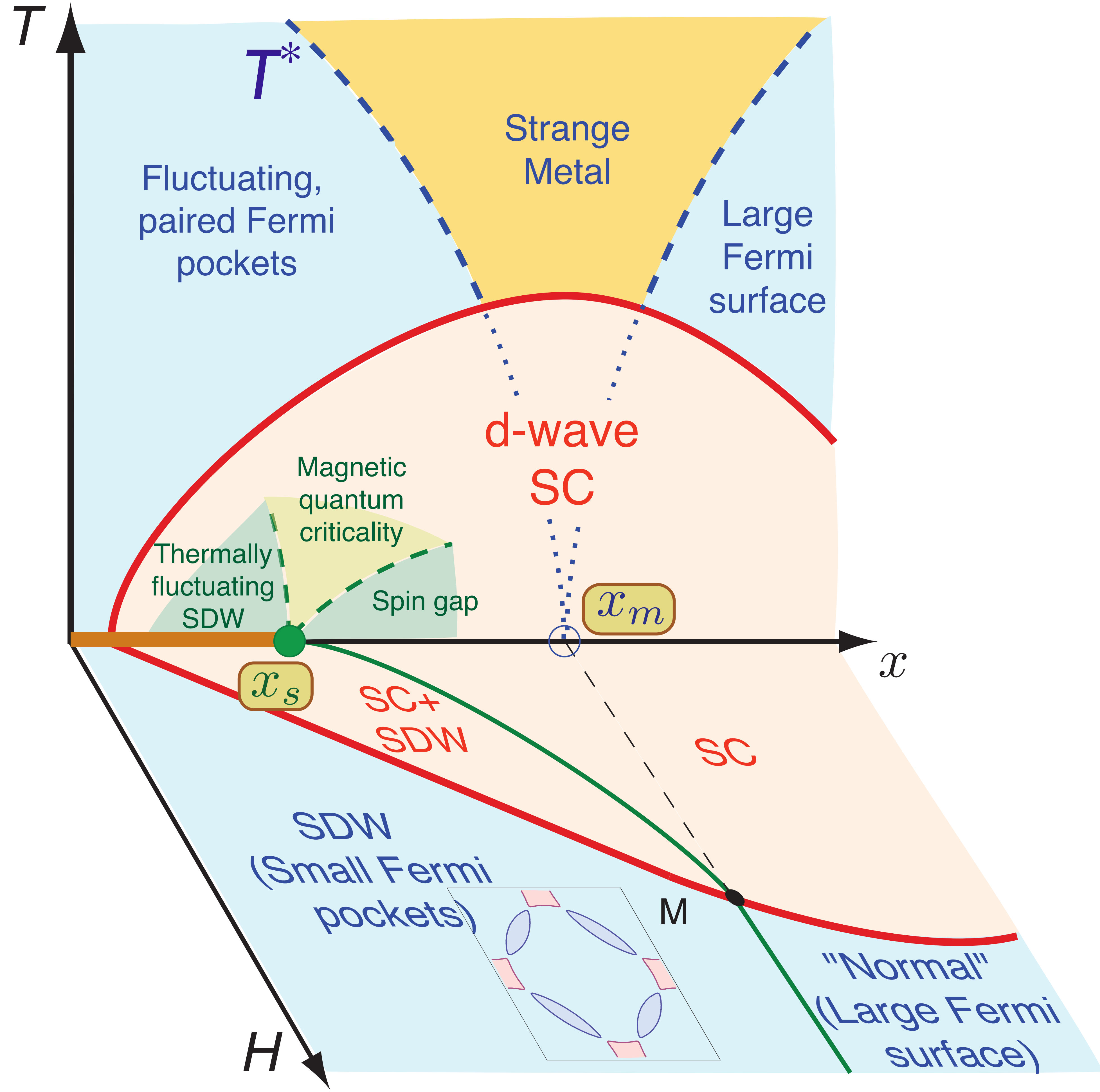
Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.



$Nd_{2-x}Ce_xCuO_4$

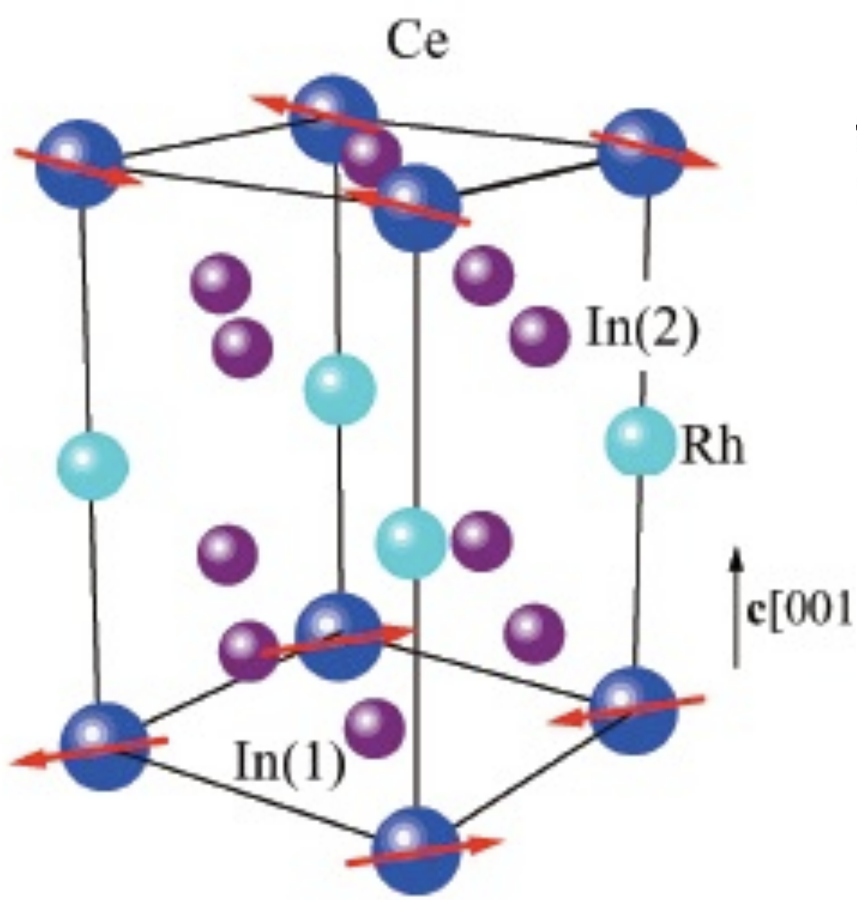


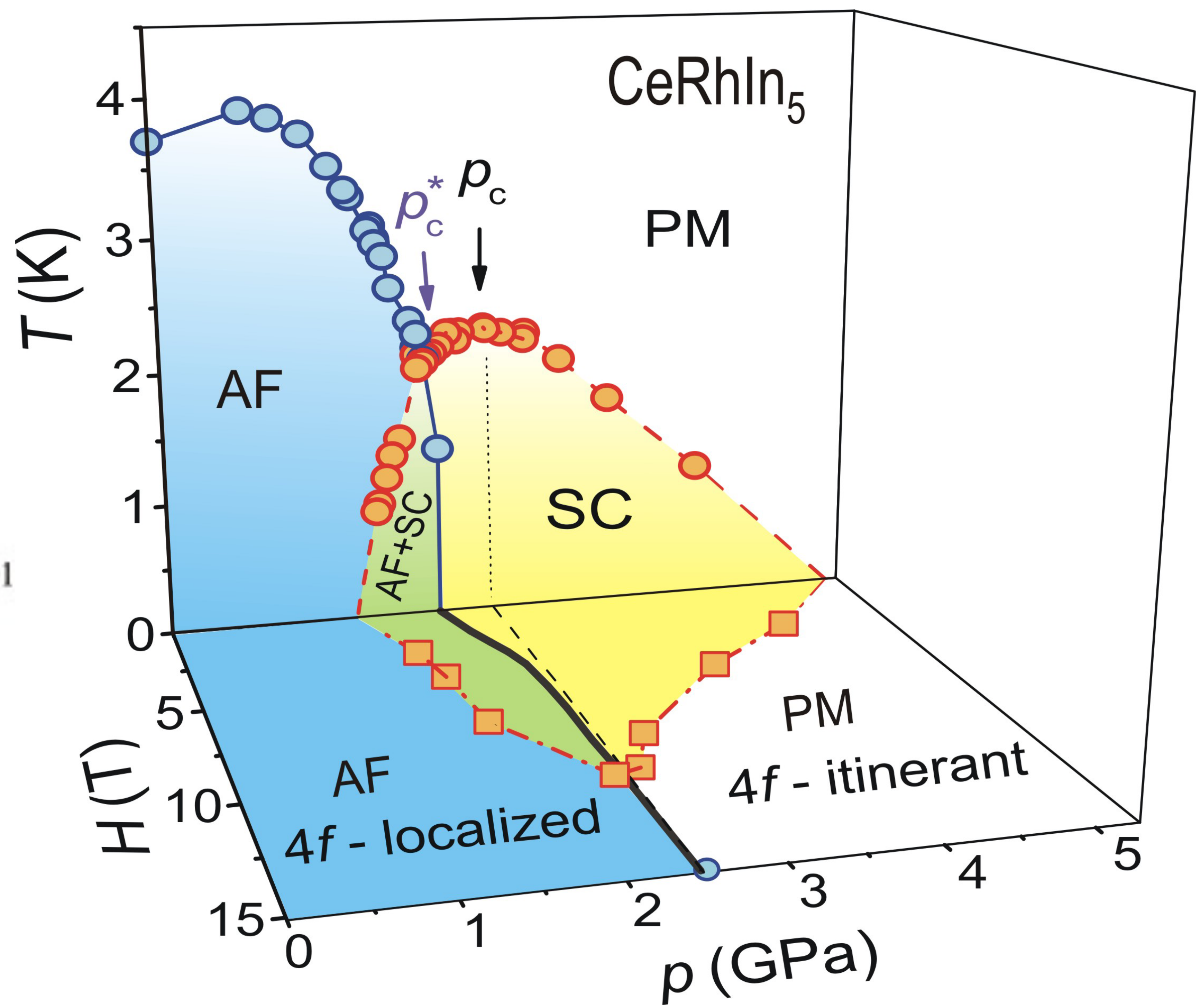
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).



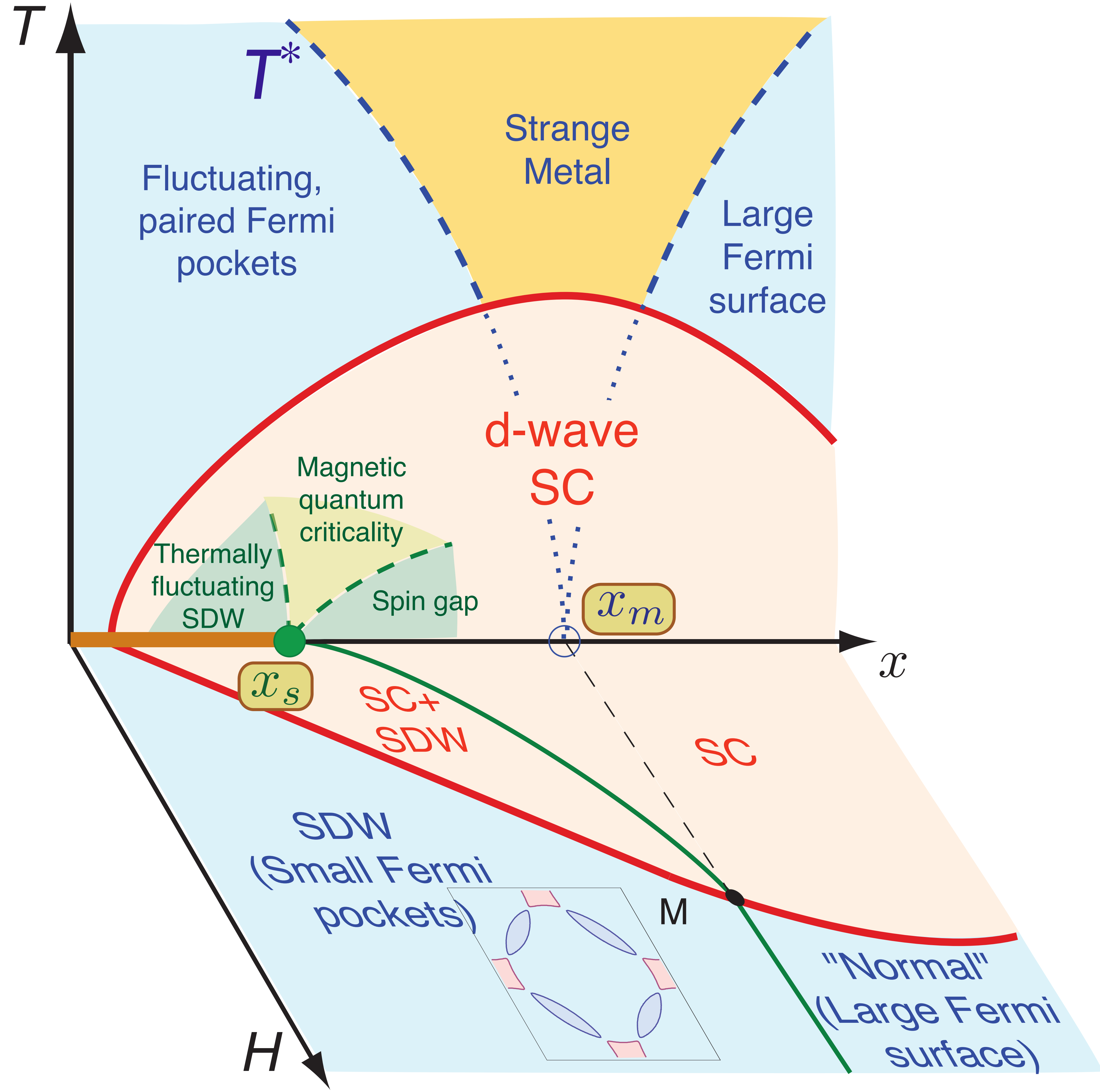
Saturday, January 23, 2010

Similar phase diagram for $CeRhIn_5$

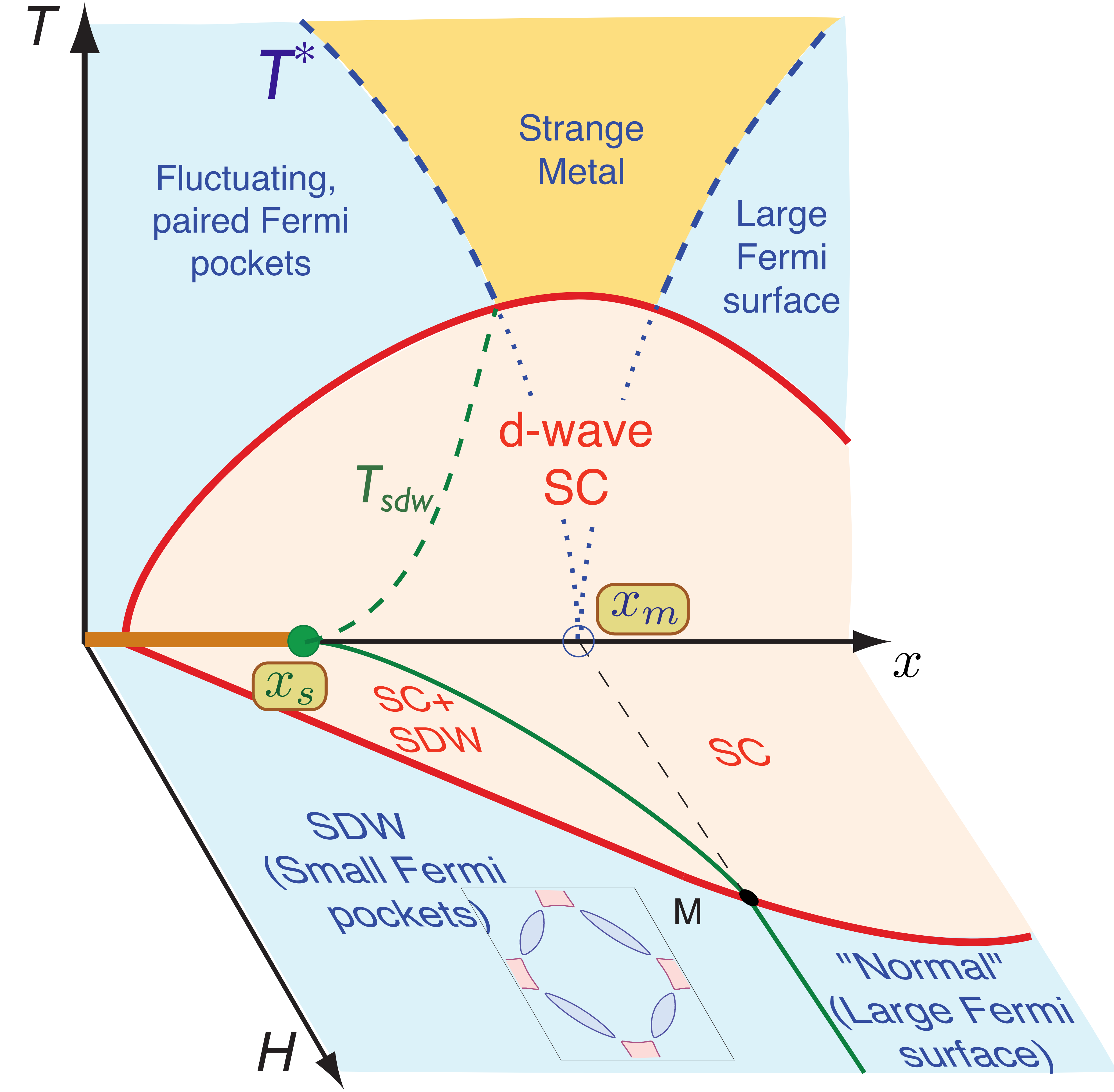




G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223



Saturday, January 23, 2010



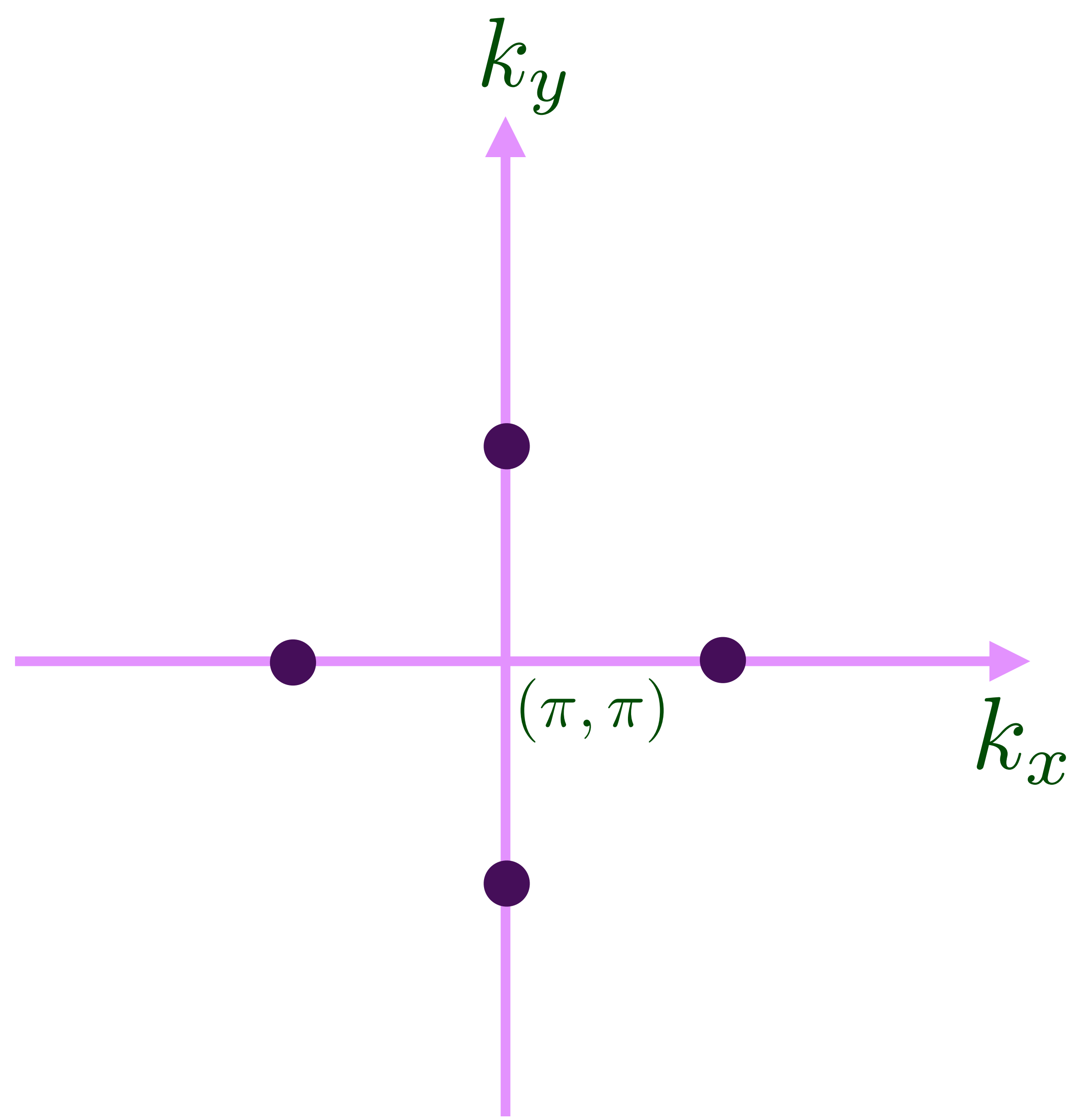
Saturday, January 23, 2010

Remnants of SDW order for $x_s < x < x_m$

For incommensurate ordering, the SDW order parameter consists of 2 complex 3-component vectors $\vec{\Phi}_x$, $\vec{\Phi}_y$:

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}_x(\mathbf{r},\tau) e^{i\mathbf{K}_x \cdot \mathbf{r}} + \vec{\Phi}_y(\mathbf{r},\tau) e^{i\mathbf{K}_y \cdot \mathbf{r}} + \text{c.c.}$$

where $\mathbf{K}_x = (\pi(1-\vartheta), \pi)$ and $\mathbf{K}_y = (\pi, \pi(1-\vartheta))$, with $\vartheta = 1/4$ near $1/8$ doping.

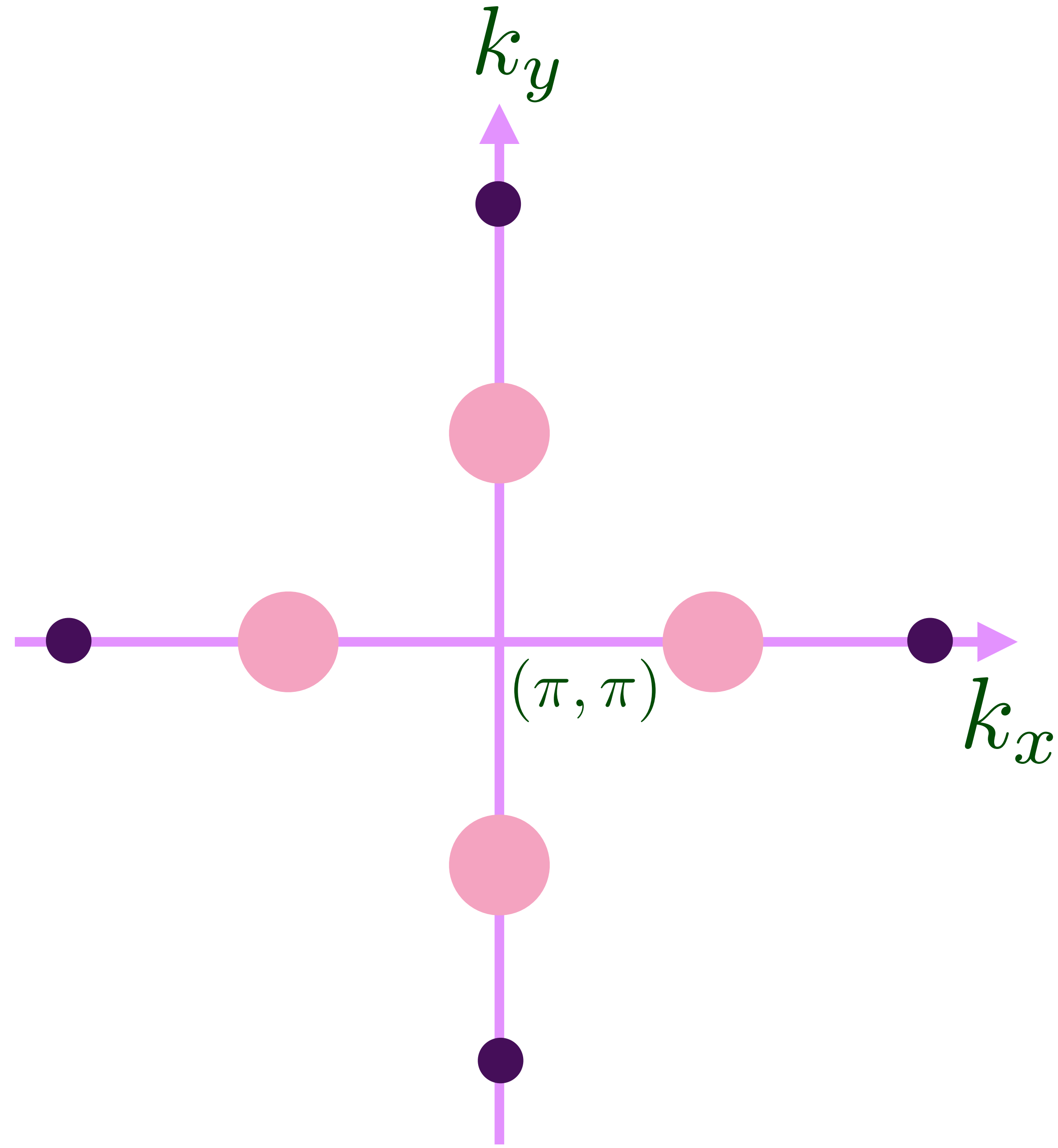


Remnants of SDW order for $x_s < x < x_m$

SDW correlations induce valence bond solid/charge density wave (VBS/CDW) order $\propto \vec{\Phi}_x^2$, $\vec{\Phi}_y^2$

$$\langle \delta\rho(\mathbf{r},\tau)\rangle = \vec{\Phi}_x^2(\mathbf{r},\tau)e^{i2\mathbf{K}_x\cdot\mathbf{r}} + \vec{\Phi}_y^2(\mathbf{r},\tau)e^{i2\mathbf{K}_y\cdot\mathbf{r}} + \text{c.c.}$$

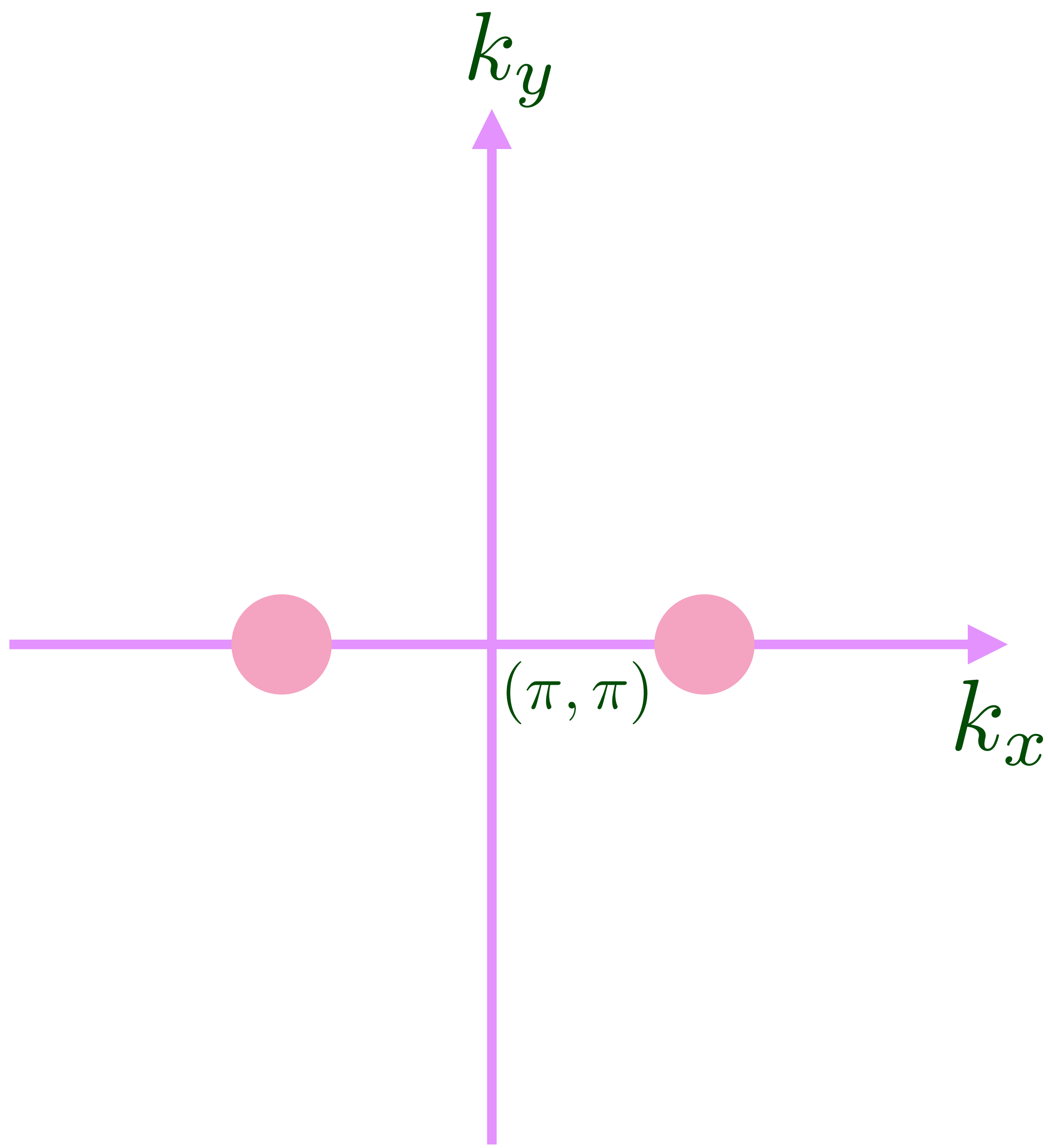
which can be long-ranged, even when SDW correlations are short-ranged.



S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

Remnants of SDW order for $x_s < x < x_m$

SDW correlations also Ising nematic order $\phi \propto |\Phi_x|^2 - |\Phi_y|^2$, which can be long-ranged, with SDW and VBS/CDW order all short ranged. This implies of preferential enhancement of electronic exchange/pairing energies along the x or y directions.



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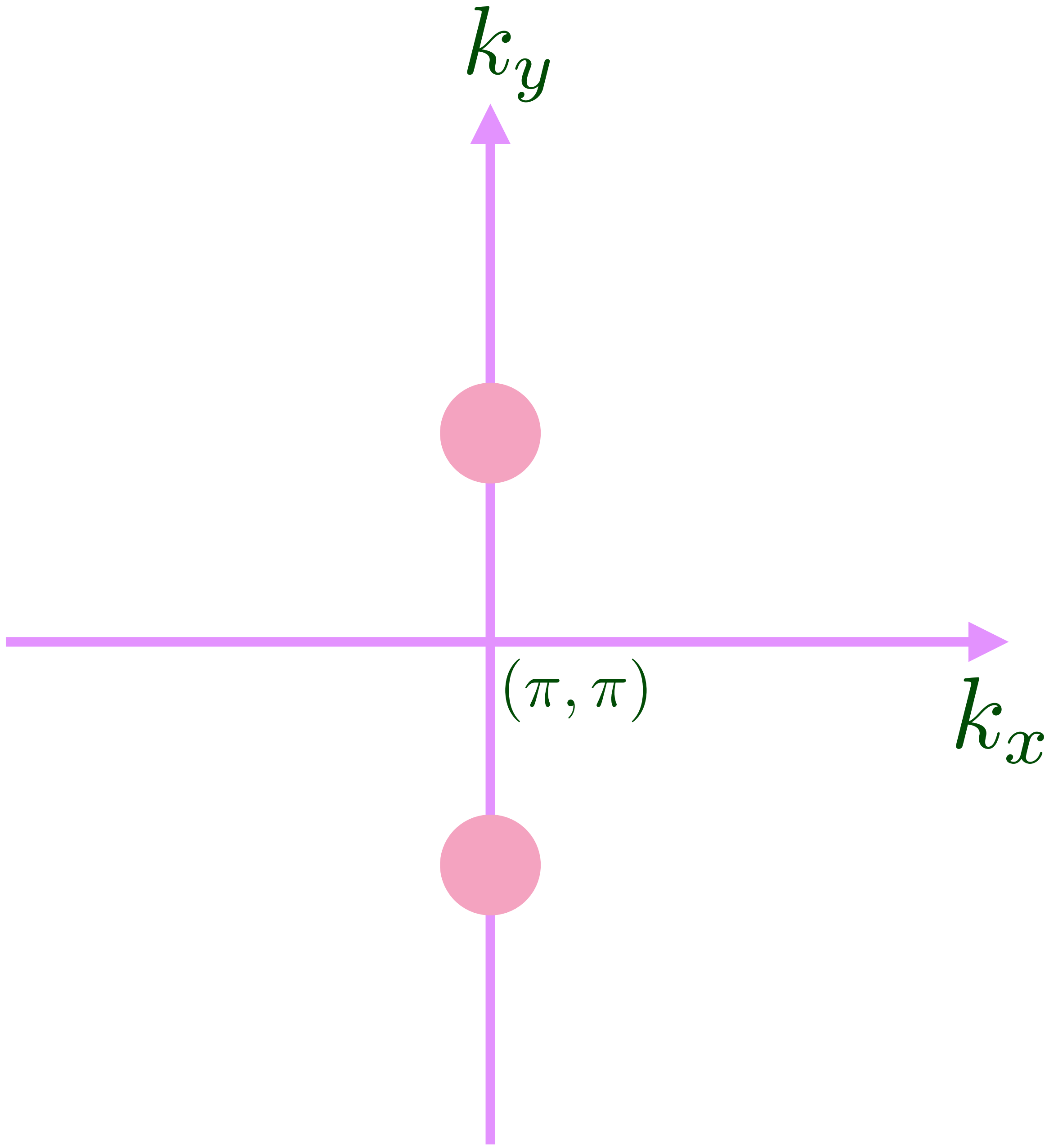

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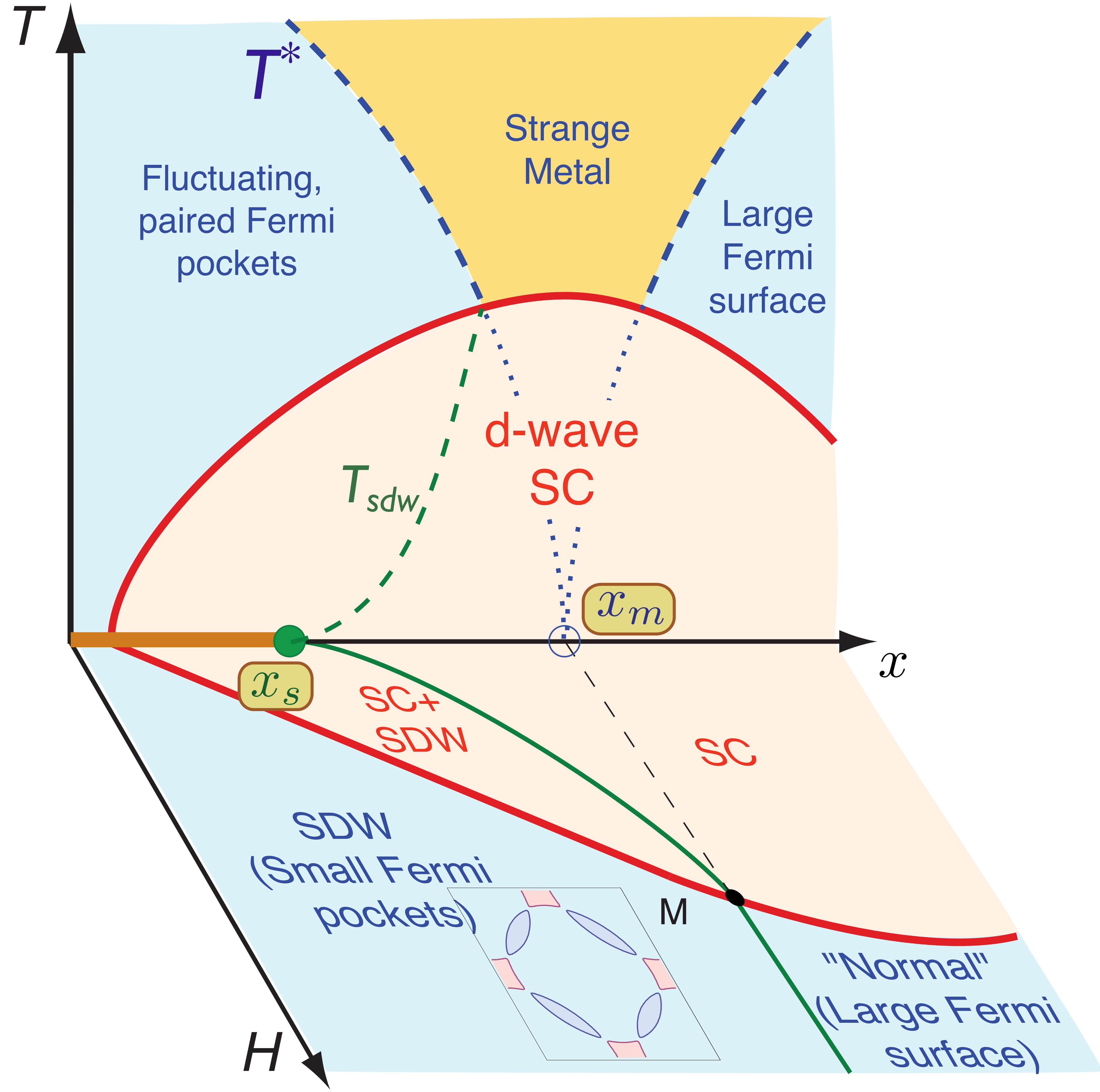
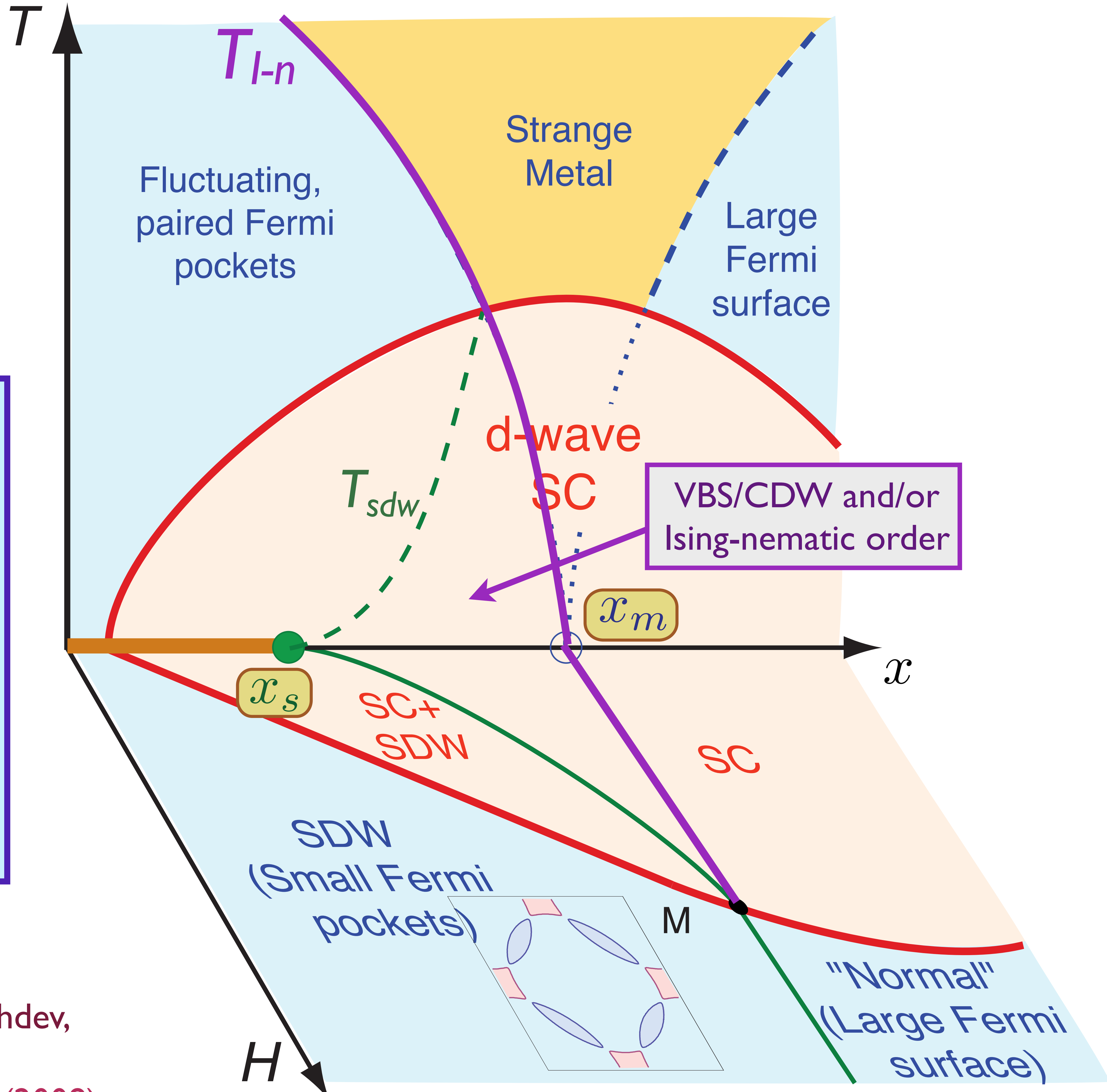


T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
T_{sdw}
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Onset of superconductivity disrupts SDW order, but VBS/CDW/ Ising-nematic ordering can survive



R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

Outline

1. Coupled dimer antiferromagnets

   Introduction to quantum criticality

2. Phase diagram of the cuprates

   Quantum criticality of the competition between antiferromagnetism and superconductivity

3. Influence of an applied magnetic field

   Theoretical predictions and experimental tests

4. Theory of Ising-nematic ordering in a metal

   Strong-coupling problems and the AdS/CFT correspondence

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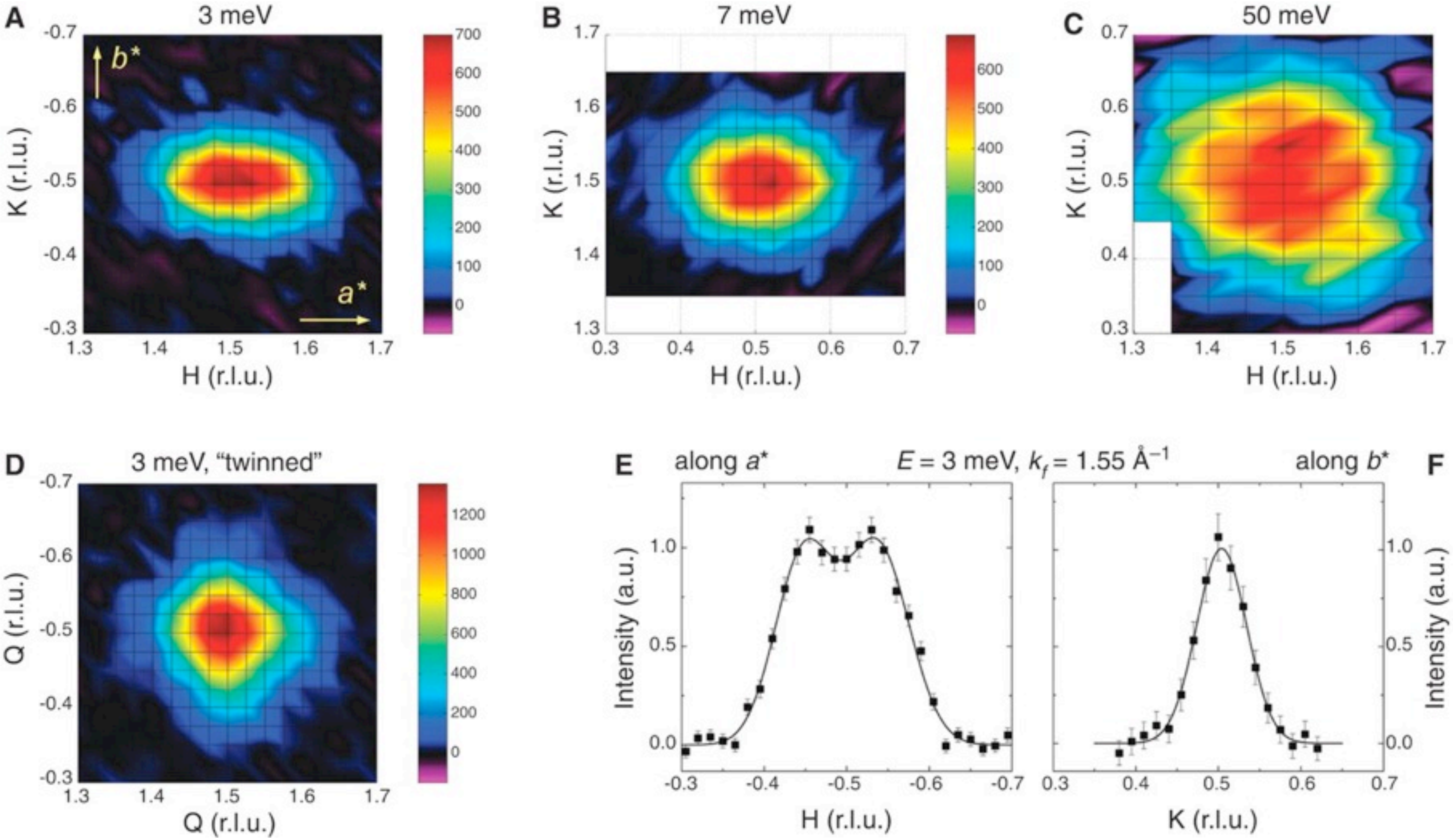
Max Metlitski, Harvard

arXiv:1001.1153

PHYSICS
HARVARD

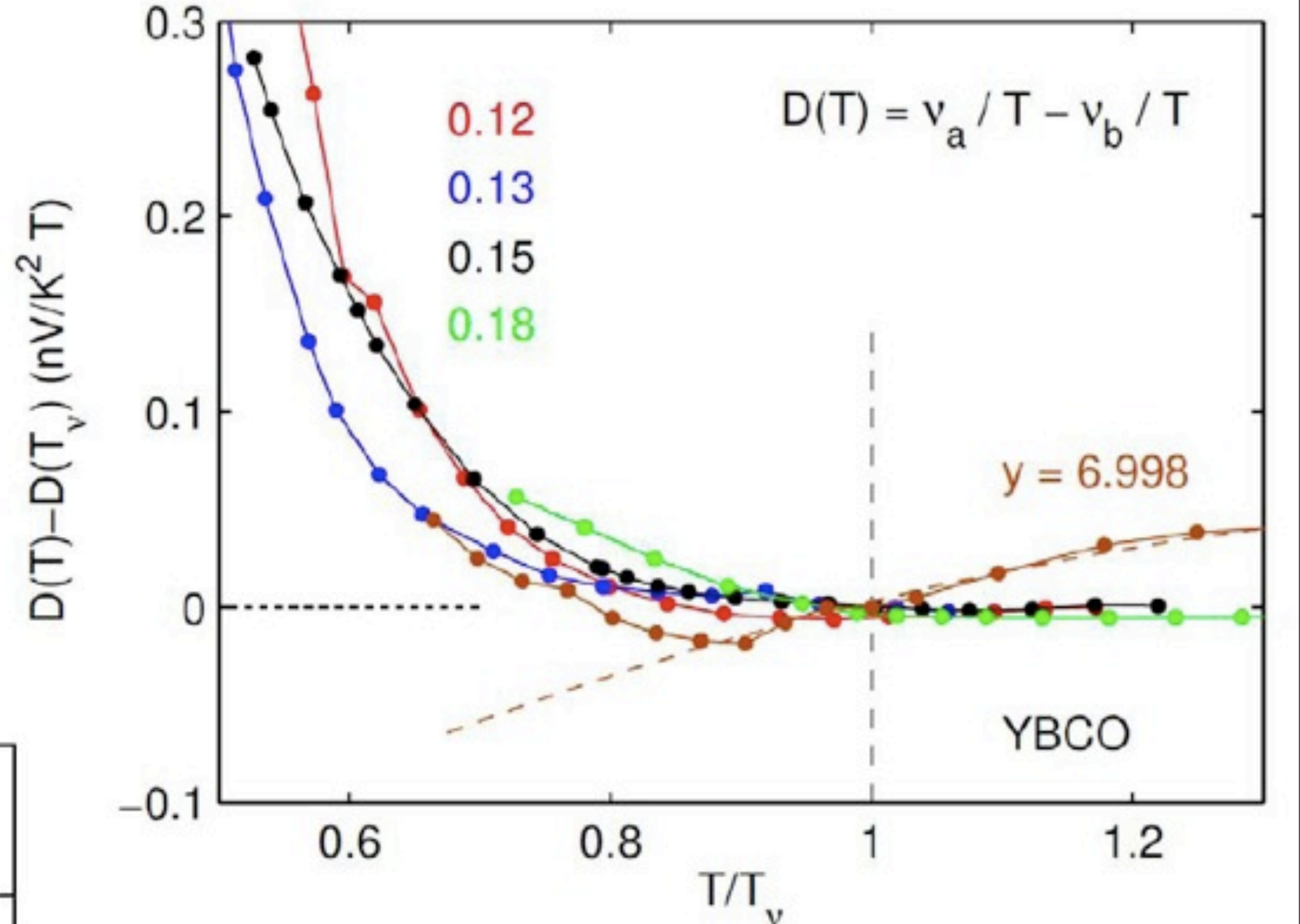


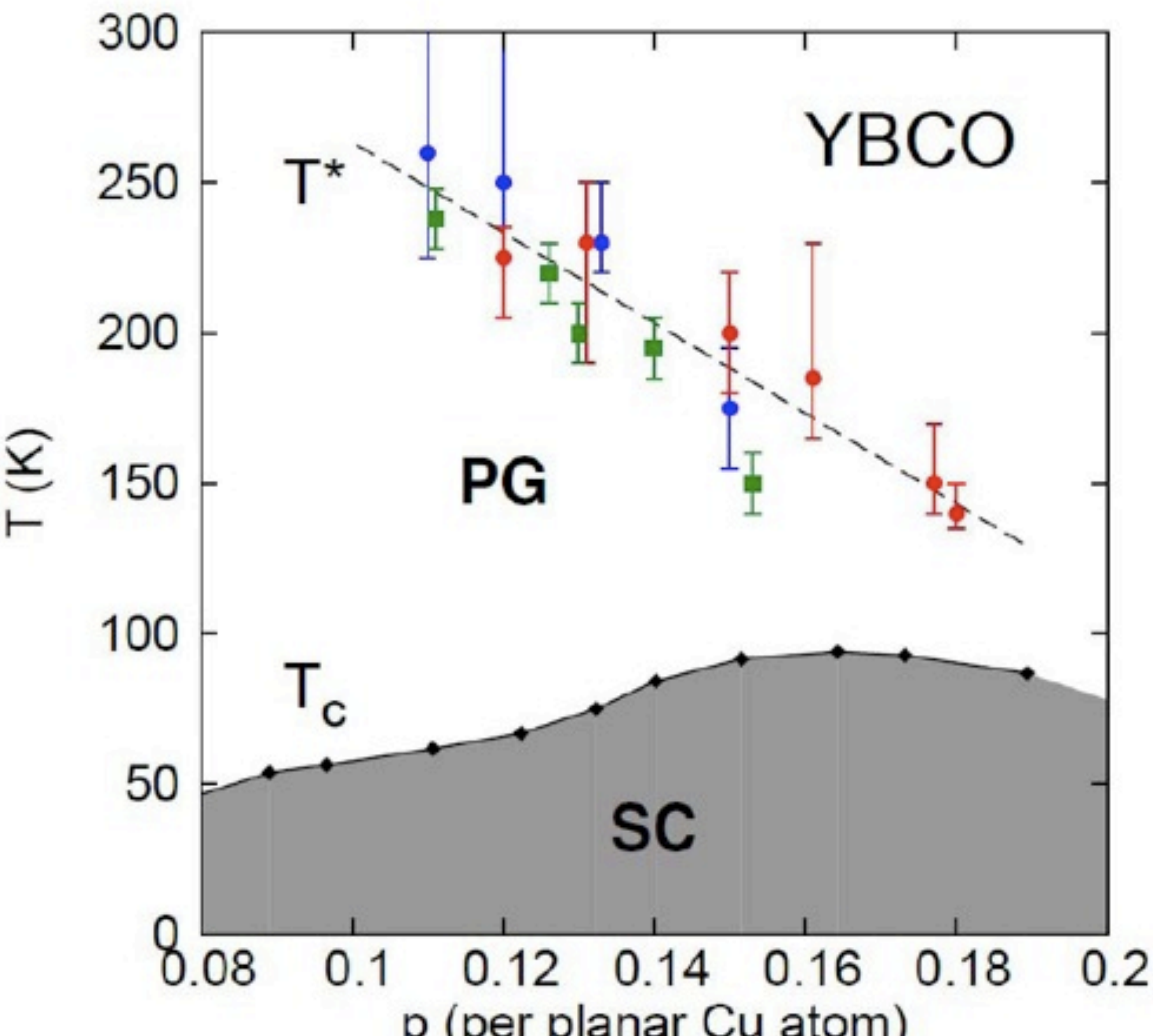
Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

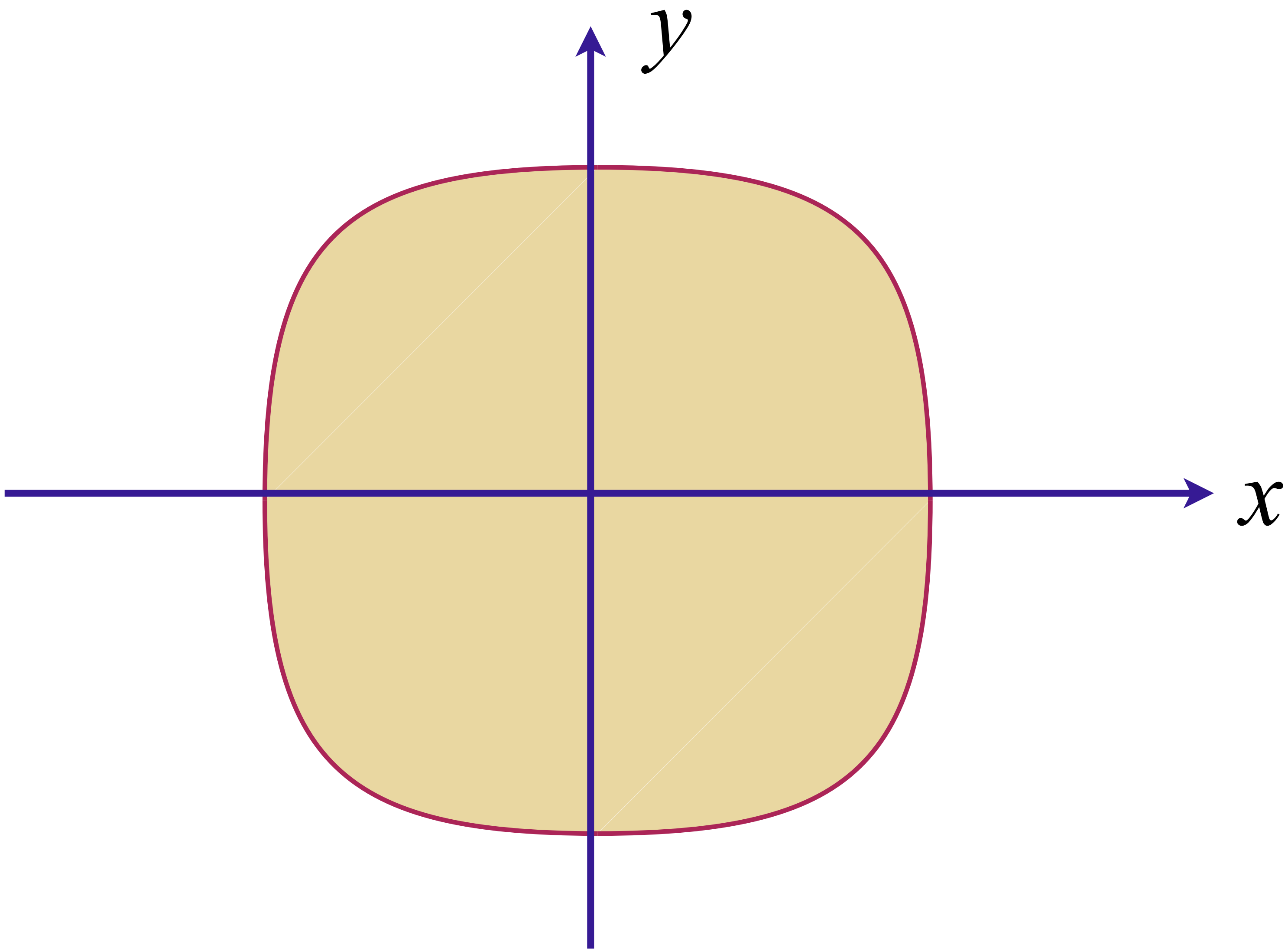
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430, Nature, in press.



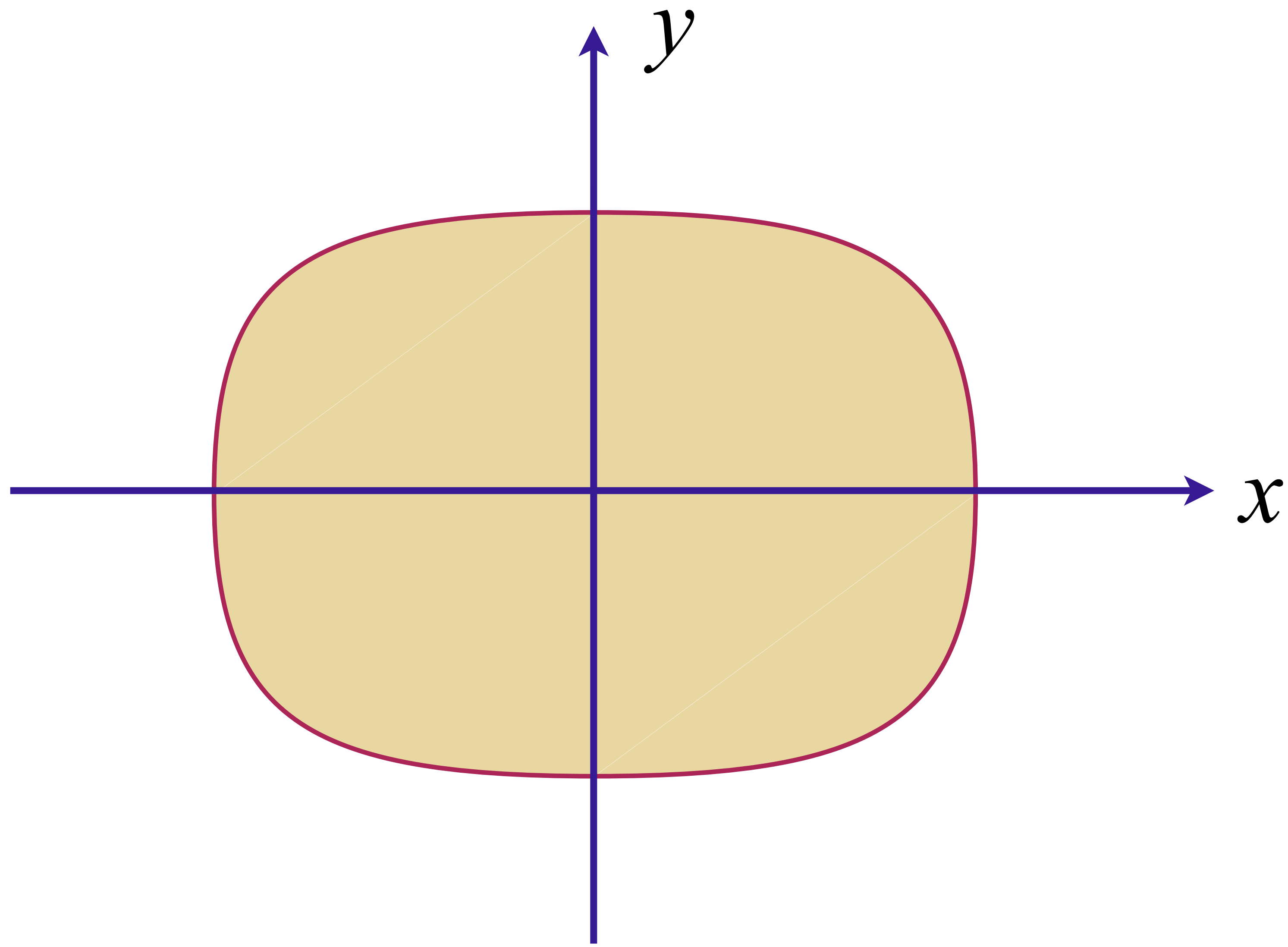


Quantum criticality of Pomeranchuk instability



Fermi surface with full square lattice symmetry

Quantum criticality of Pomeranchuk instability



Spontaneous elongation along x direction:

H. Yamase and H. Kohno, J. Phys. Soc. Jpn. **69**, 2151 (2000).
C. J. Halboth and W. Metzner, Phys. Rev. Lett. **85**, 5162 (2000).

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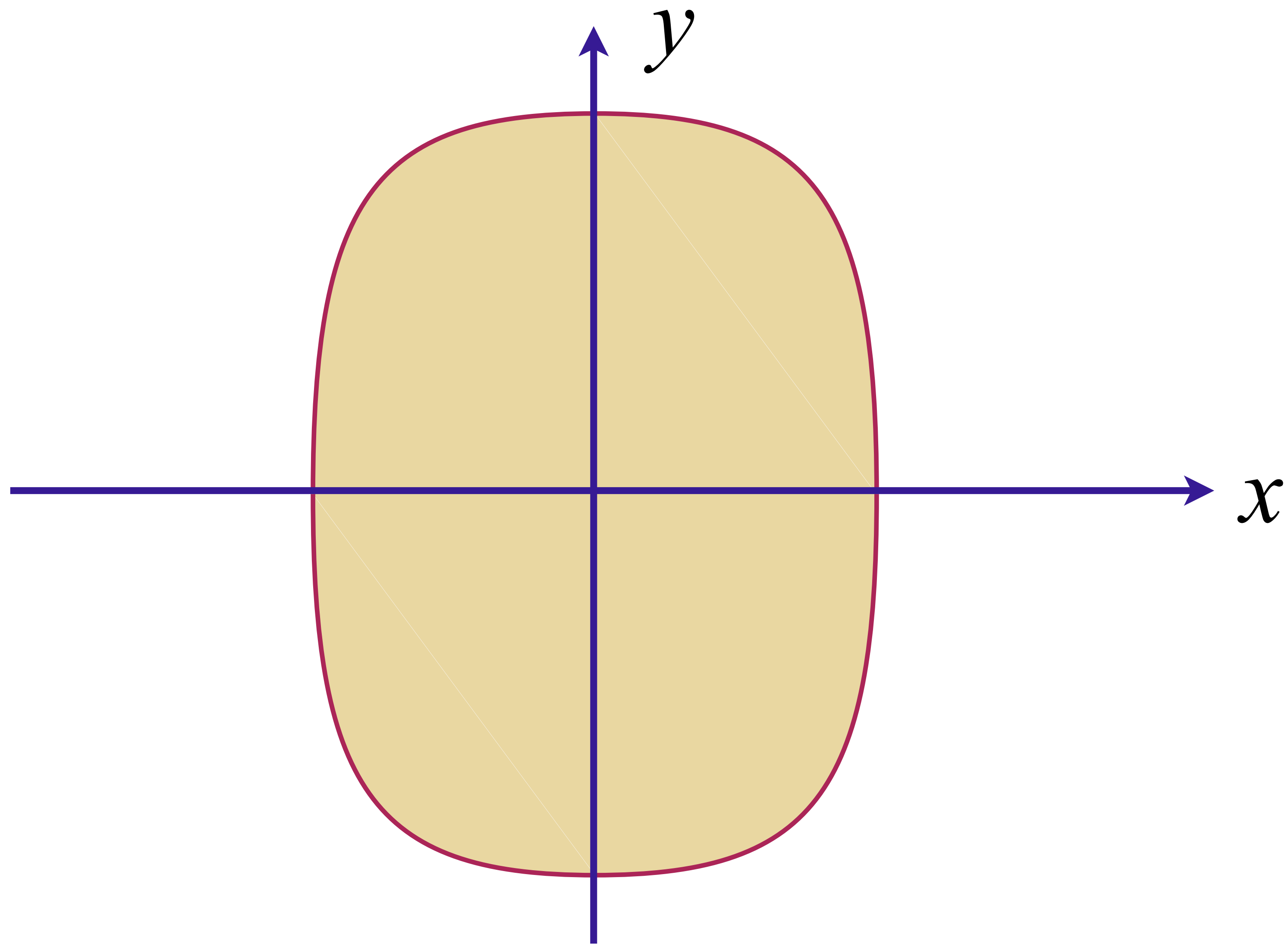

Spontaneous elongation along y direction:

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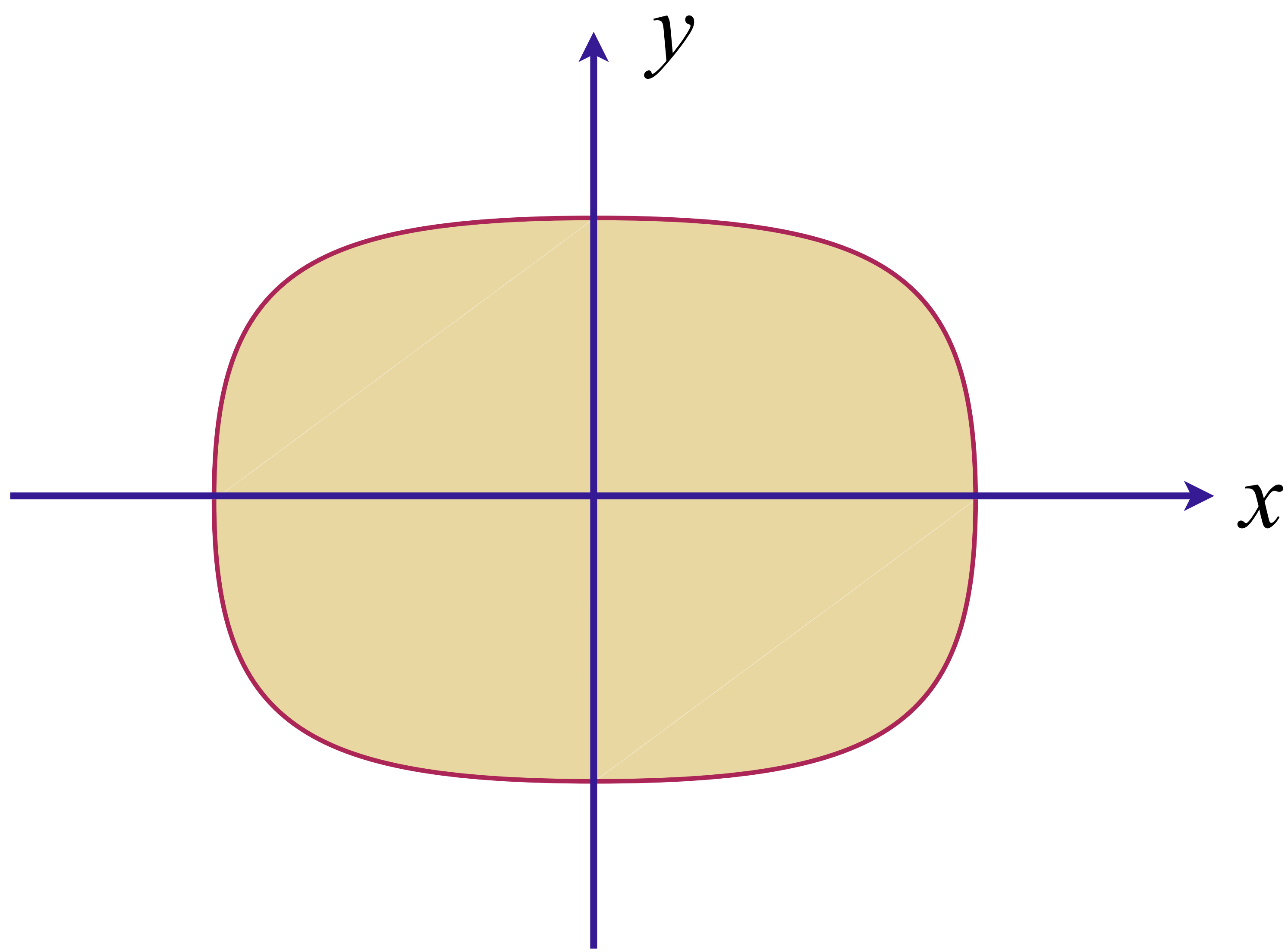
Ising-nematic order parameter

$$\phi \sim \int d^2k \, (\cos k_x - \cos k_y) \, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Pomeranchuk instability



Spontaneous elongation along x direction:
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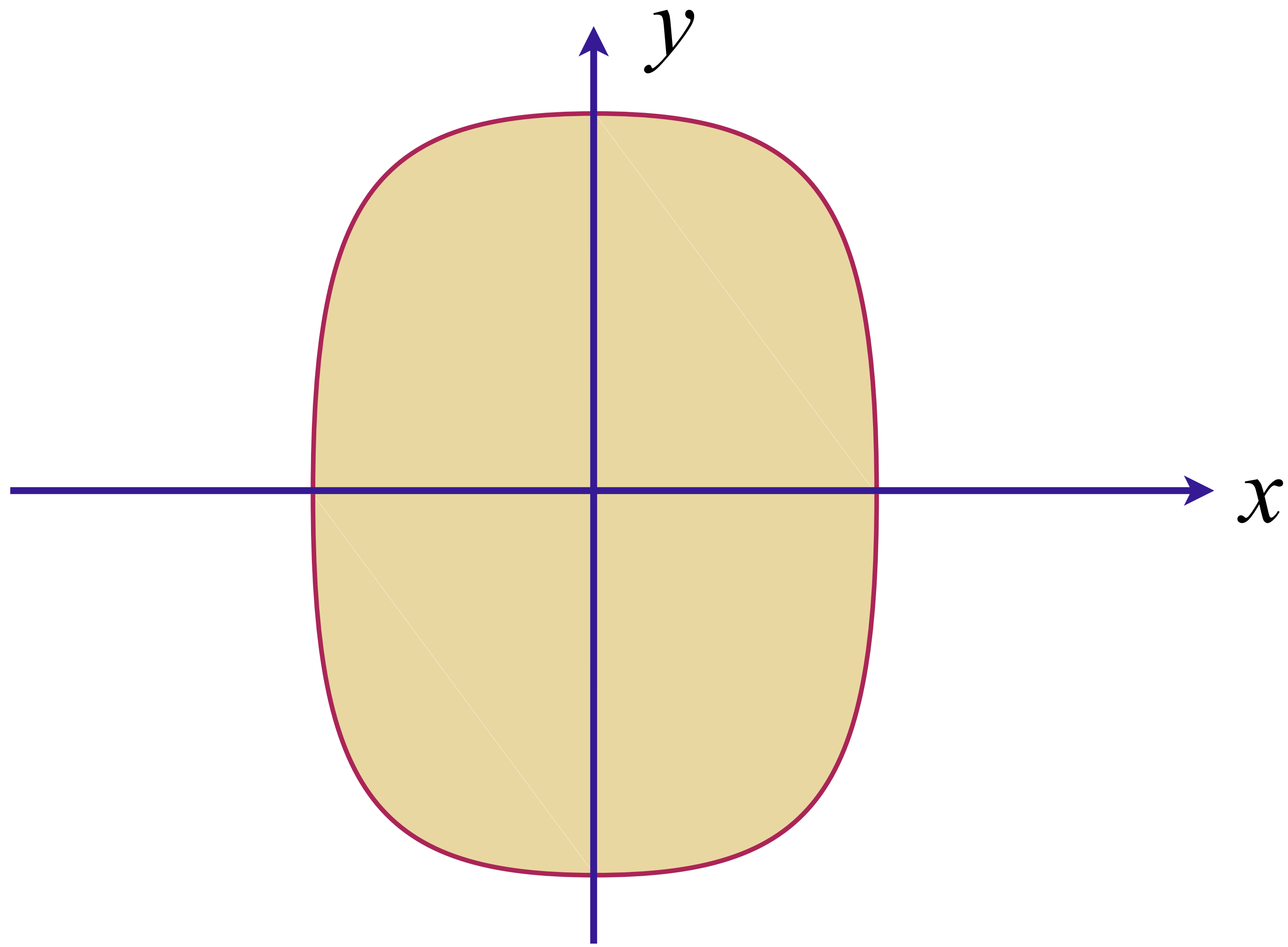

Spontaneous elongation along y direction:
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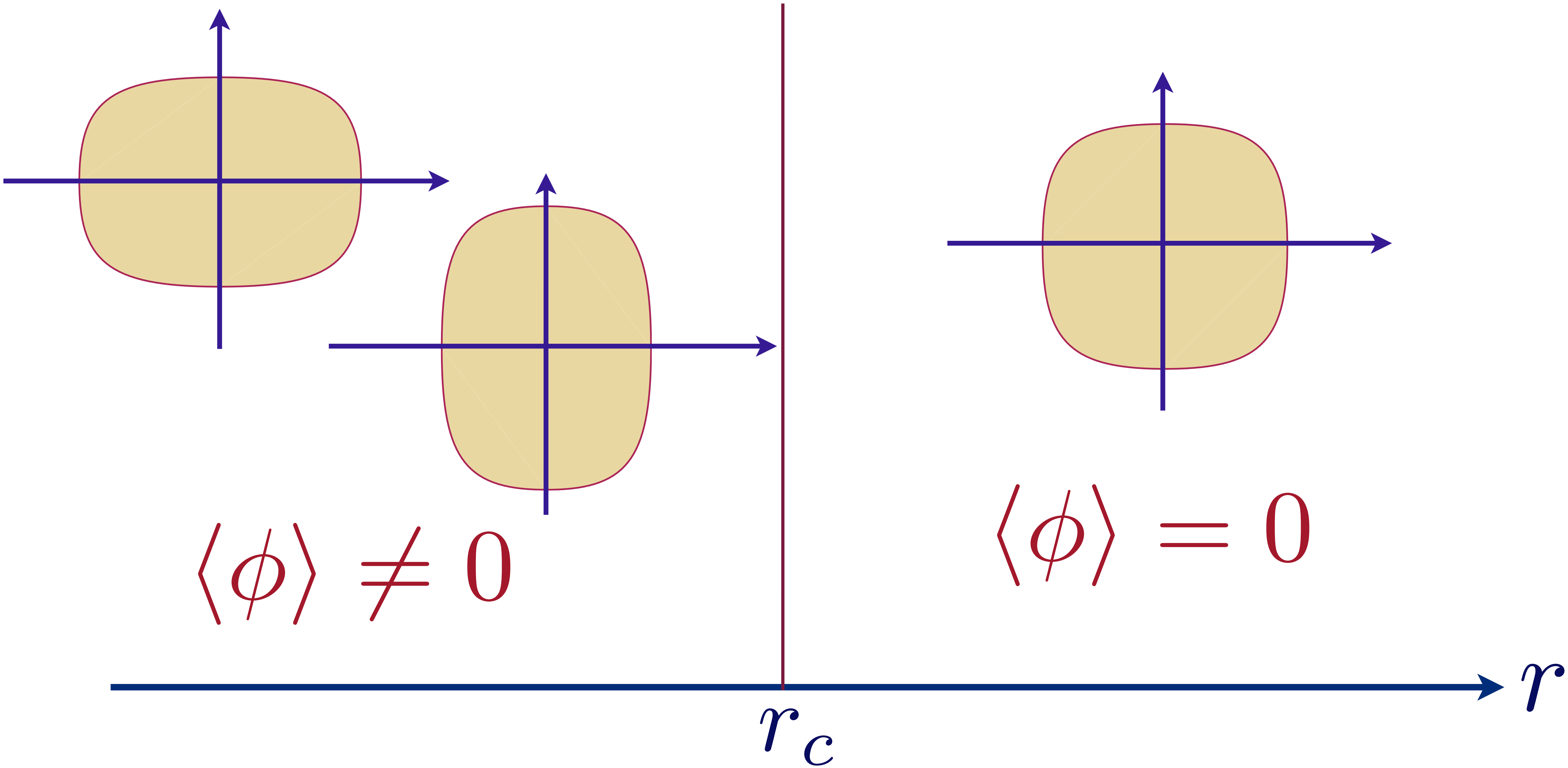

Pomeranchuk instability as a function of coupling r

H. Yamase and H. Kohno, J. Phys. Soc. Jpn. **69**, 2151 (2000).
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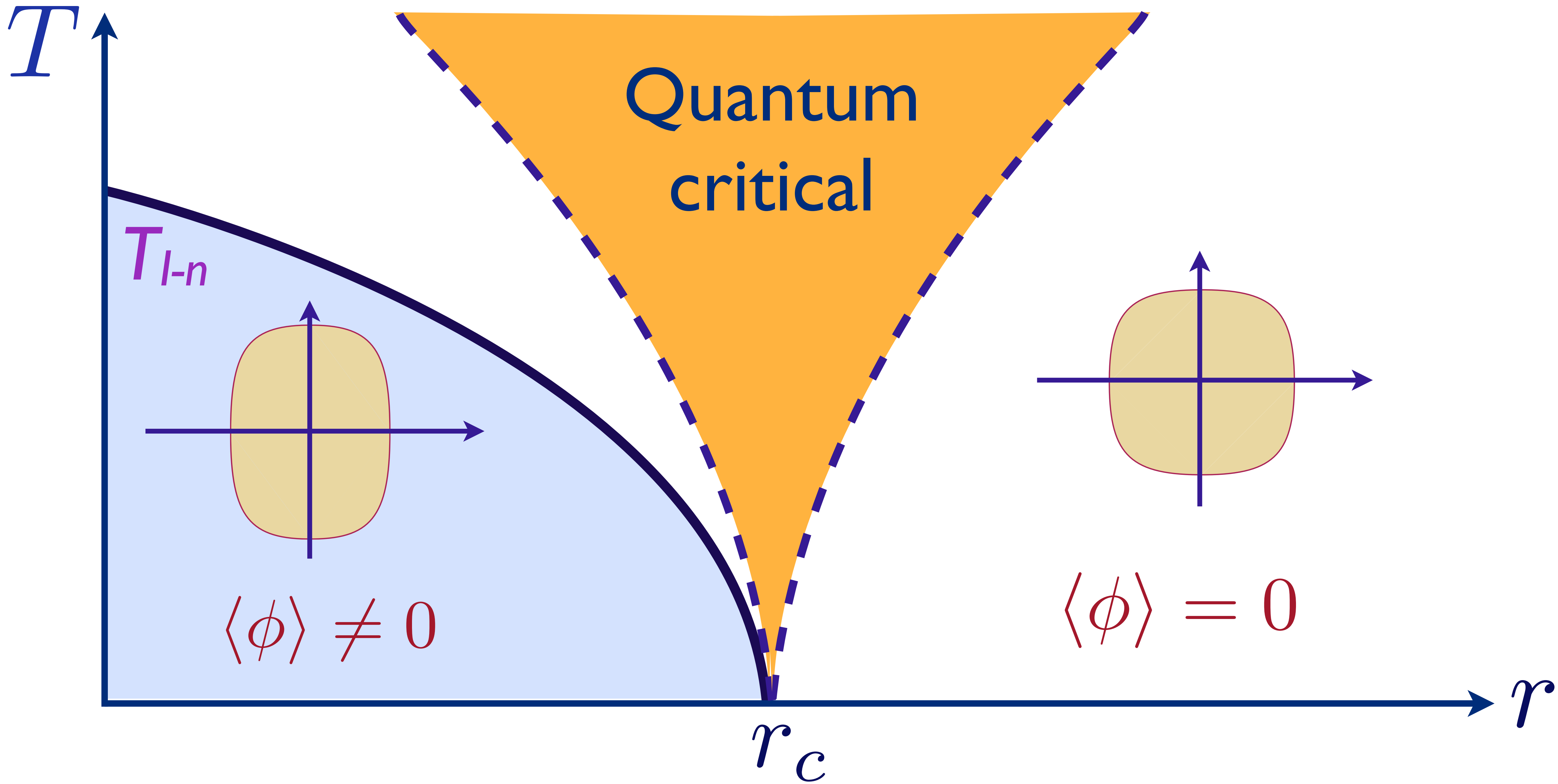
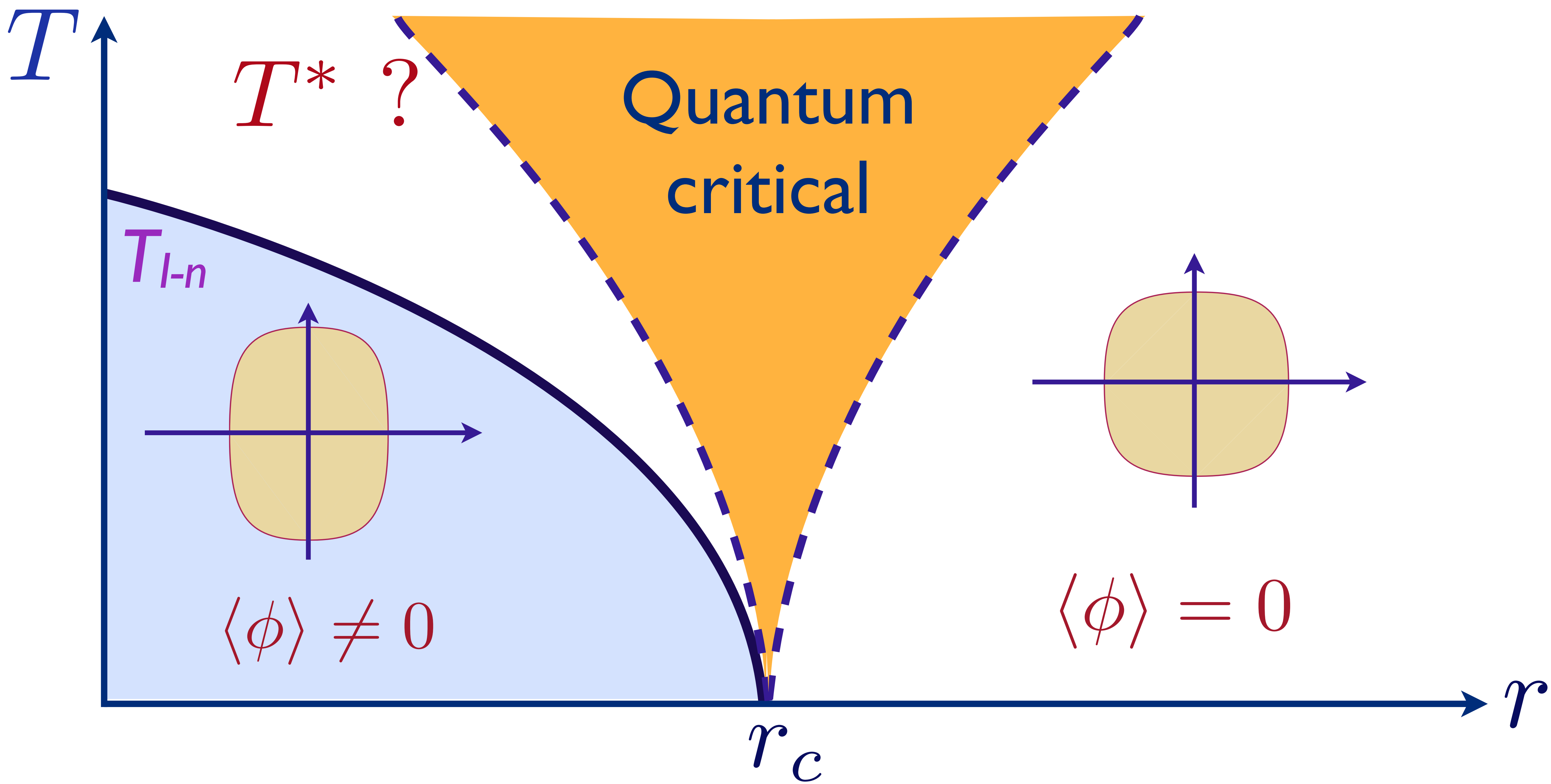
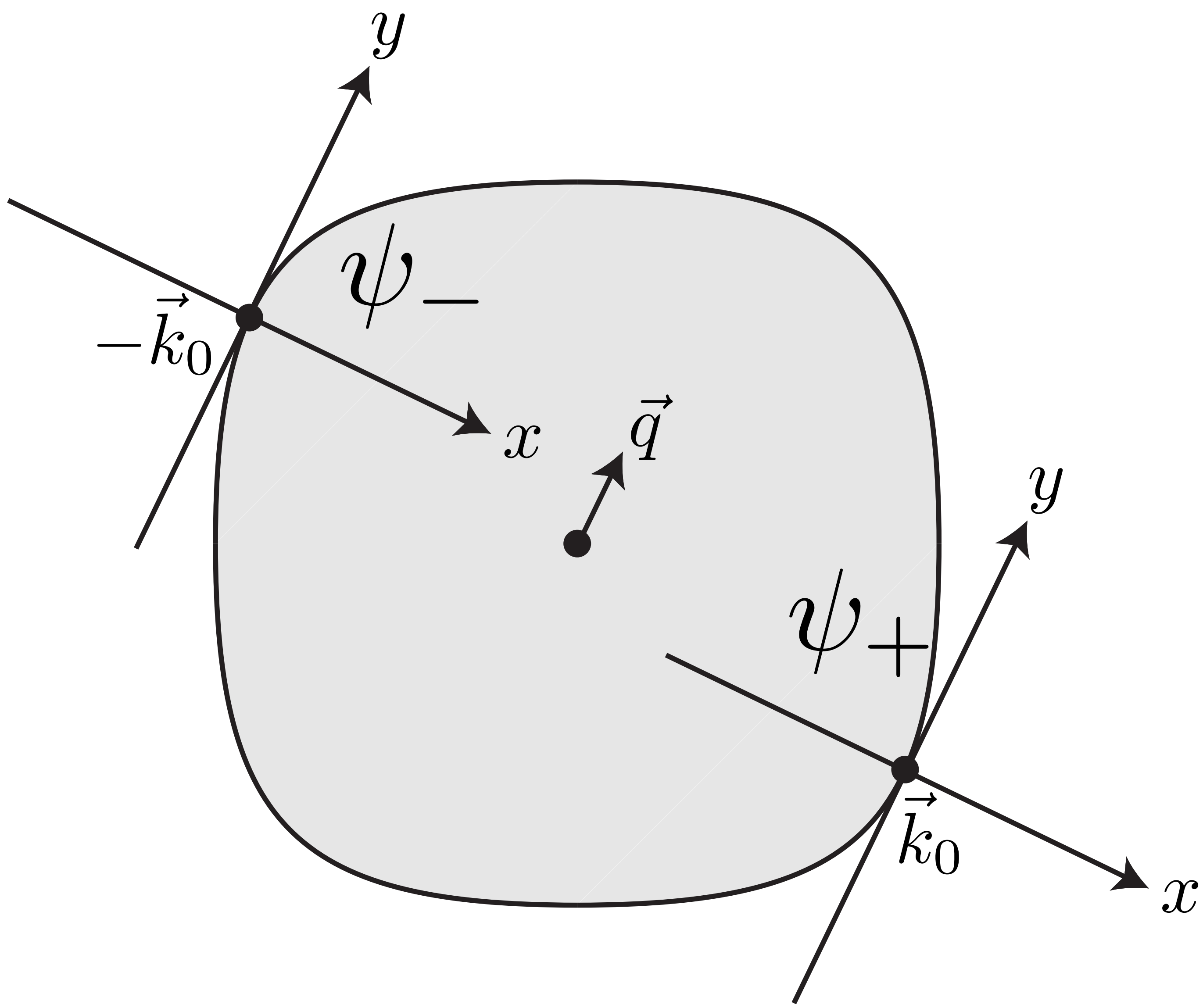


Phase diagram as a function of T and r

Quantum criticality of Pomeranchuk instability



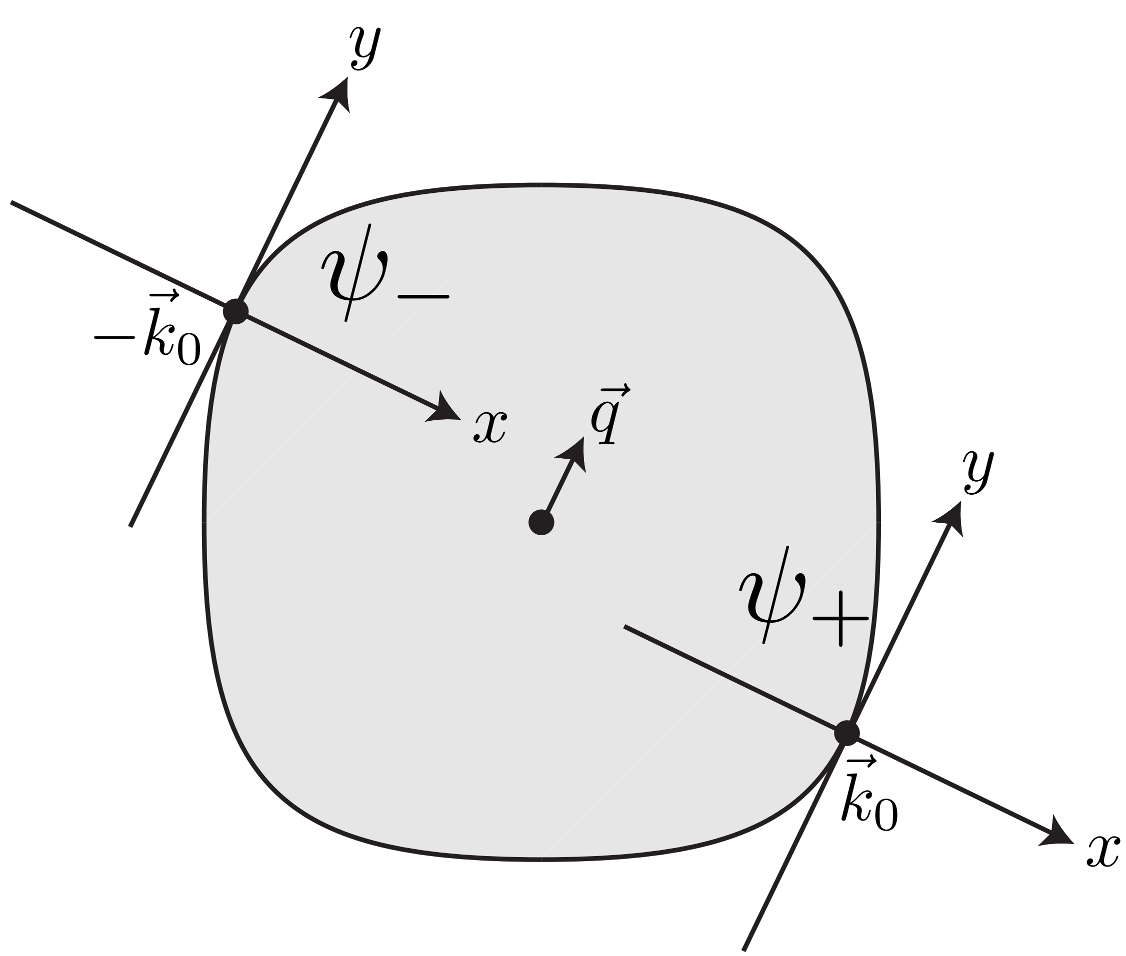
Phase diagram as a function of T and r



A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

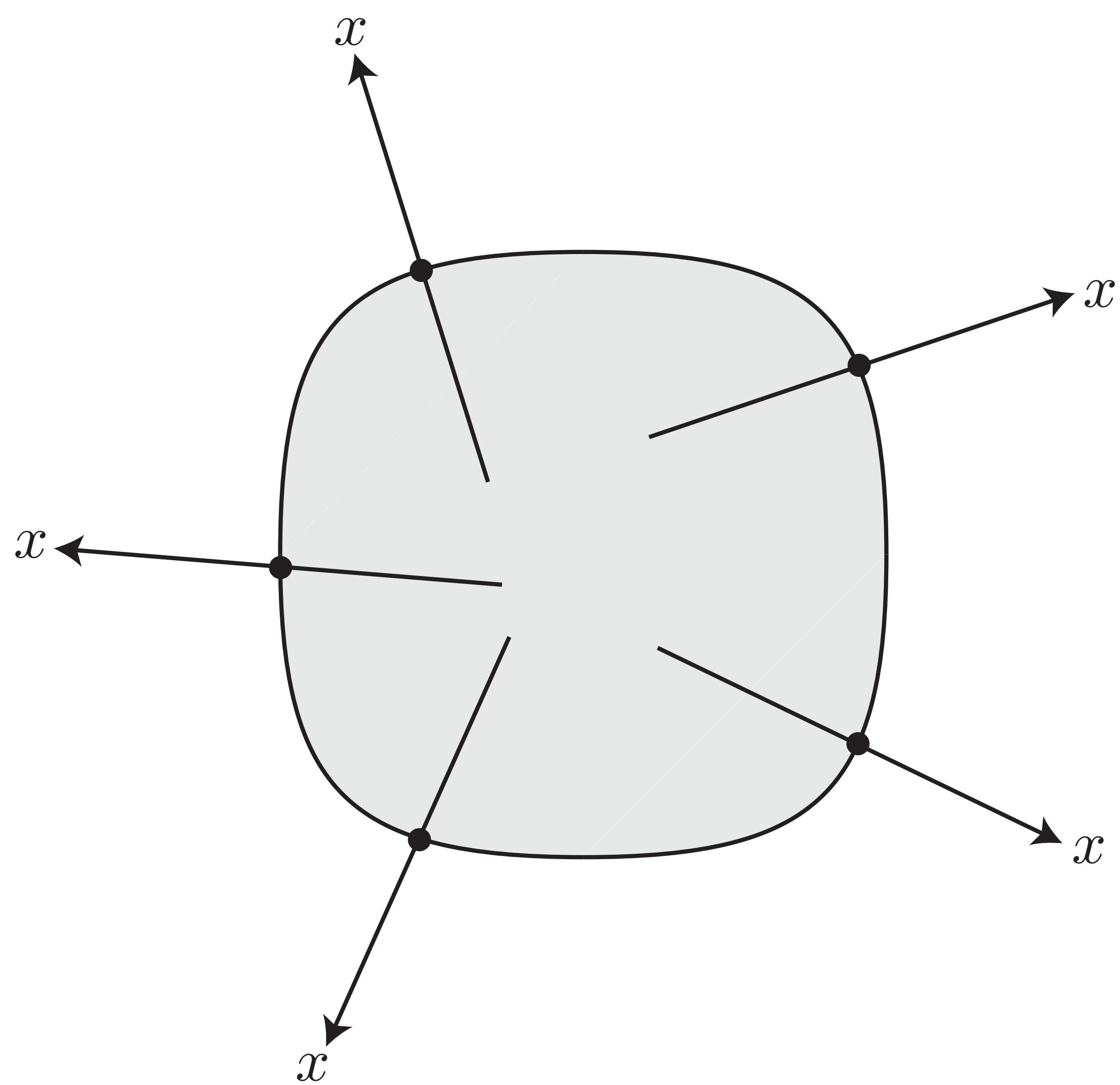
Expand fermion kinetic energy at wavevectors about $\vec{k}_0$

- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.



- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.



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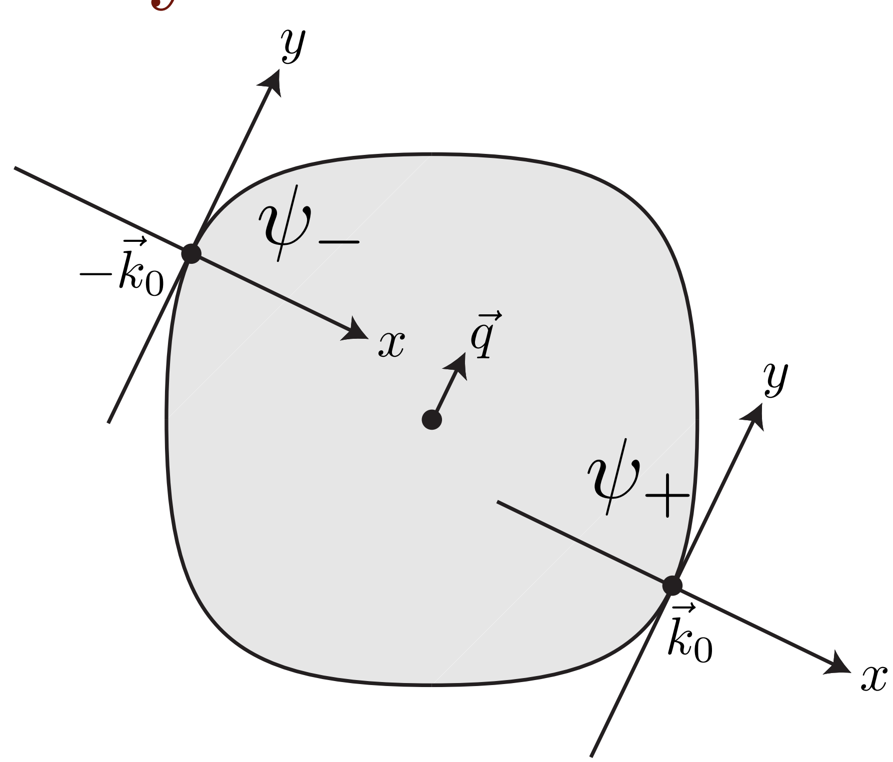
- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.
- Our approach leads to a redundant description of underlying degrees of freedom. A "Galilean symmetry" ensures consistency of redundant description.



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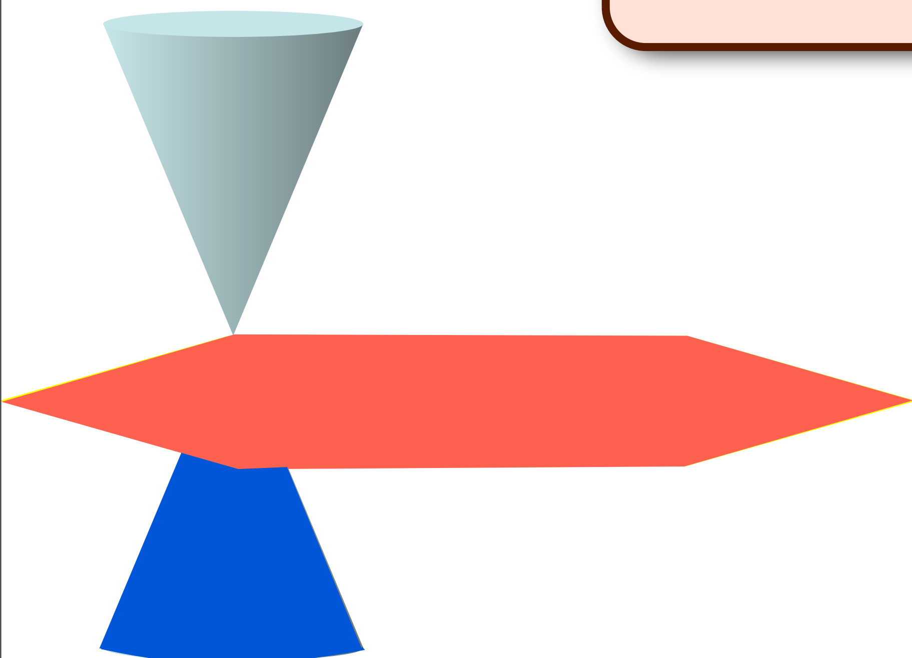
- Our approach leads to a redundant description of underlying degrees of freedom. A "Galilean symmetry" ensures consistency of redundant description.

- <u>Infinite set of 2+1 dimensional field theories</u>: implies an emergent dimension of spacetime, and suggests a string-theoretic description and application of the AdS/CFT correspondence.

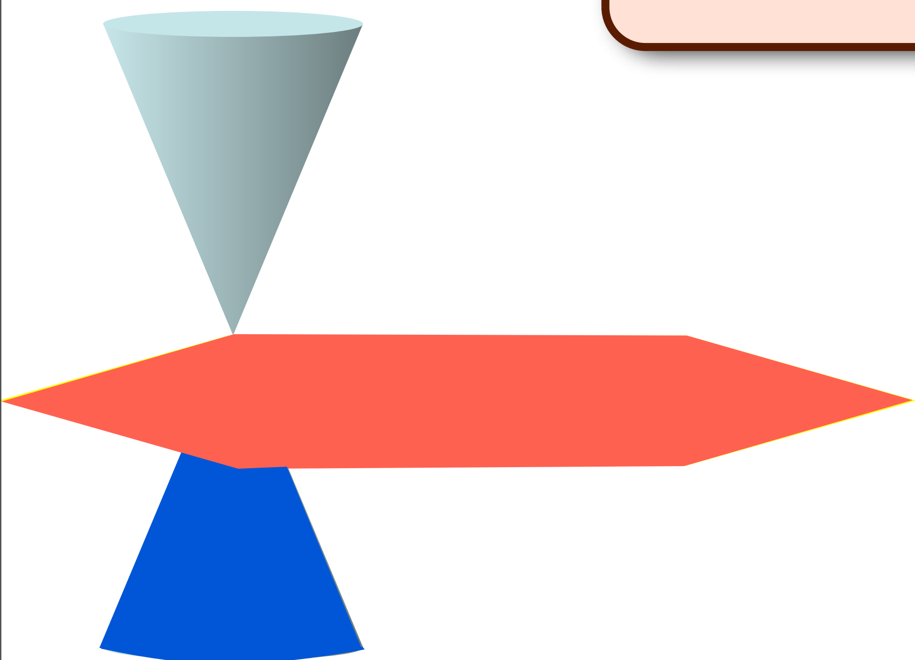
Conformal field theory
in 2+1 dimensions at $T = 0$



$\Updownarrow$

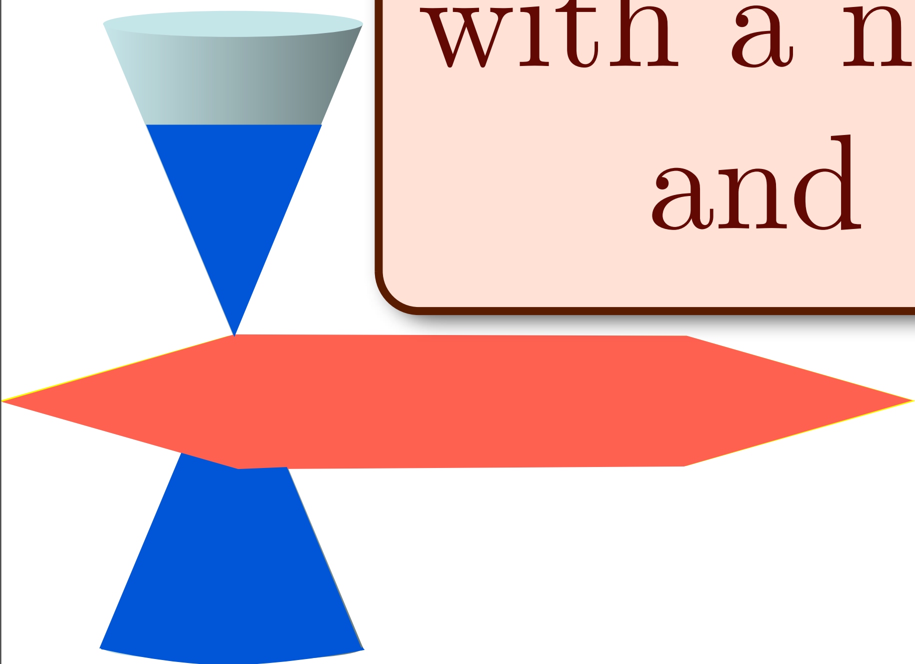
Einstein gravity
on AdS_4

Conformal field theory
in 2+1 dimensions at $T > 0$



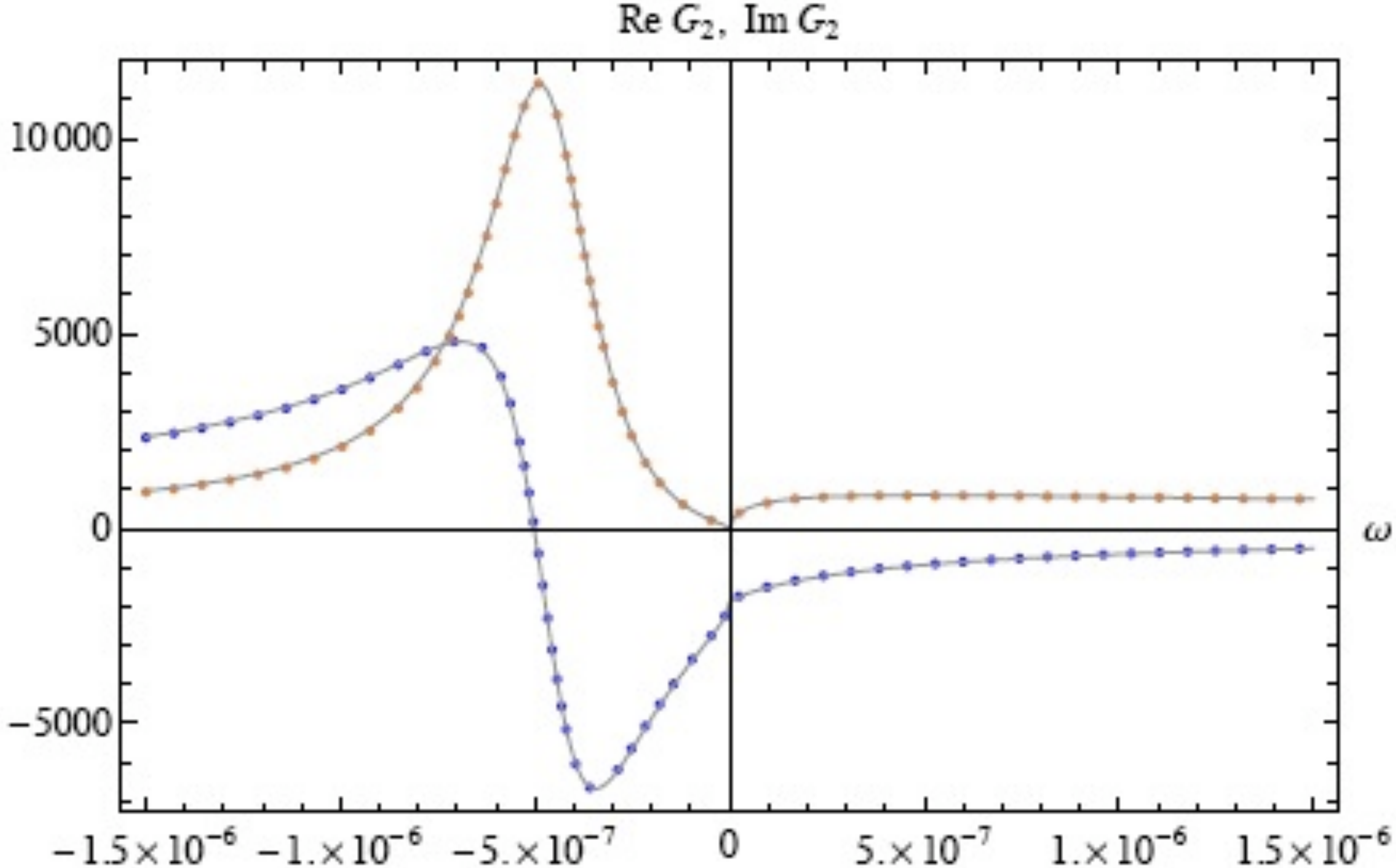
Einstein gravity on AdS_4
with a Schwarzschild
black hole

Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges

Green's function of a fermion

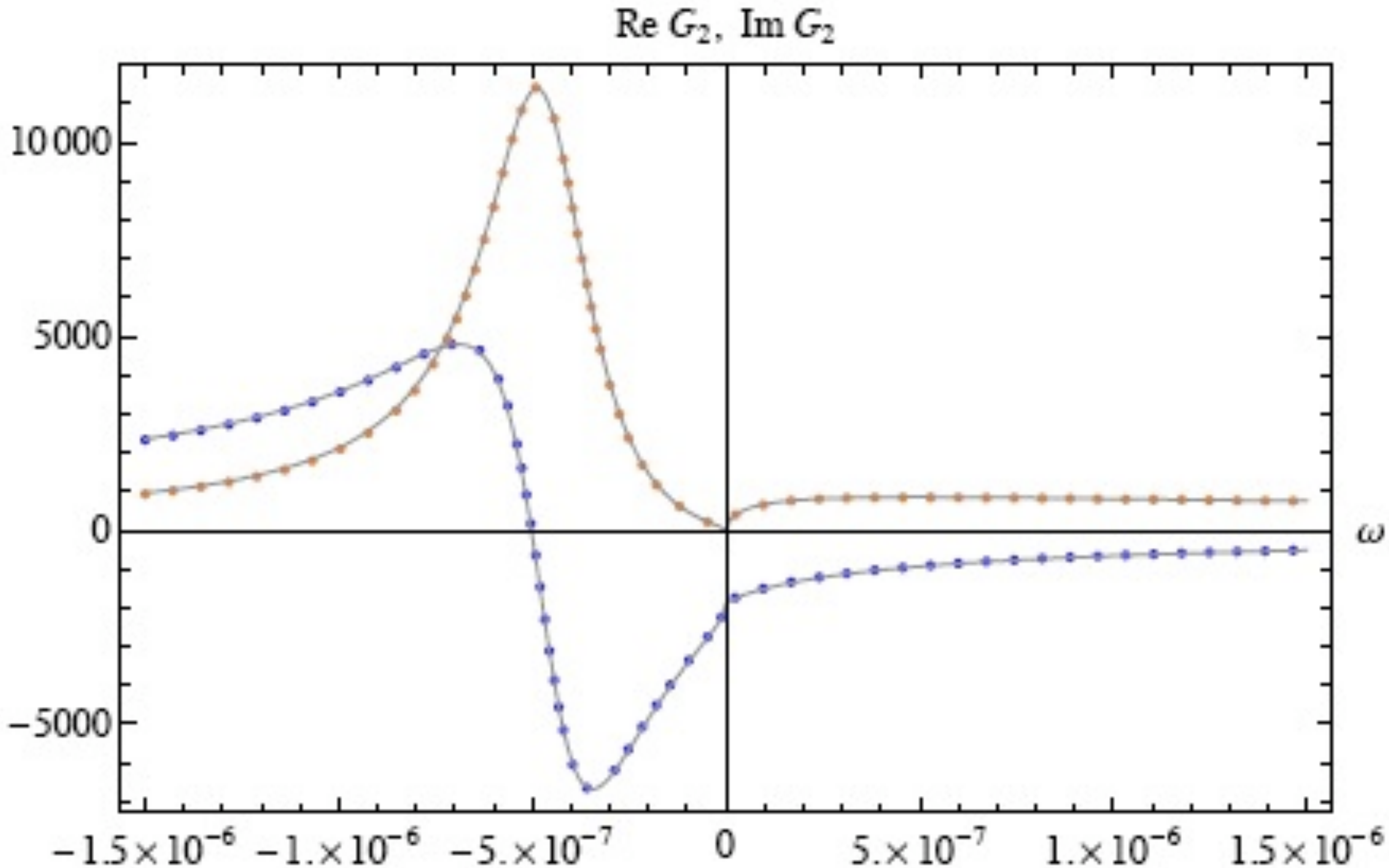


T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
F. Denef, S.A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to our theory of the singular Fermi surface near the Ising-nematic quantum critical point

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theories for the onset of spin density wave and Ising-nematic order in metals are *strongly* coupled in two dimensions