

The phase diagrams of the high temperature superconductors

Talk online: sachdev.physics.harvard.edu

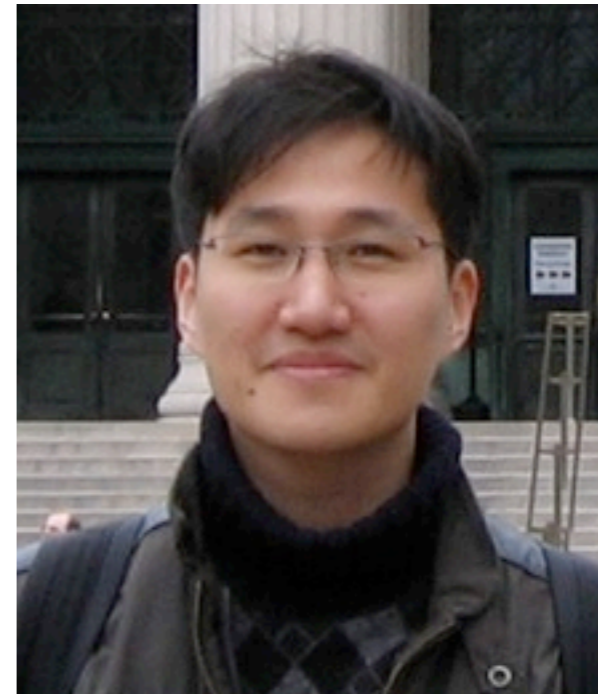
PHYSICS



HARVARD



Max Metlitski, Harvard



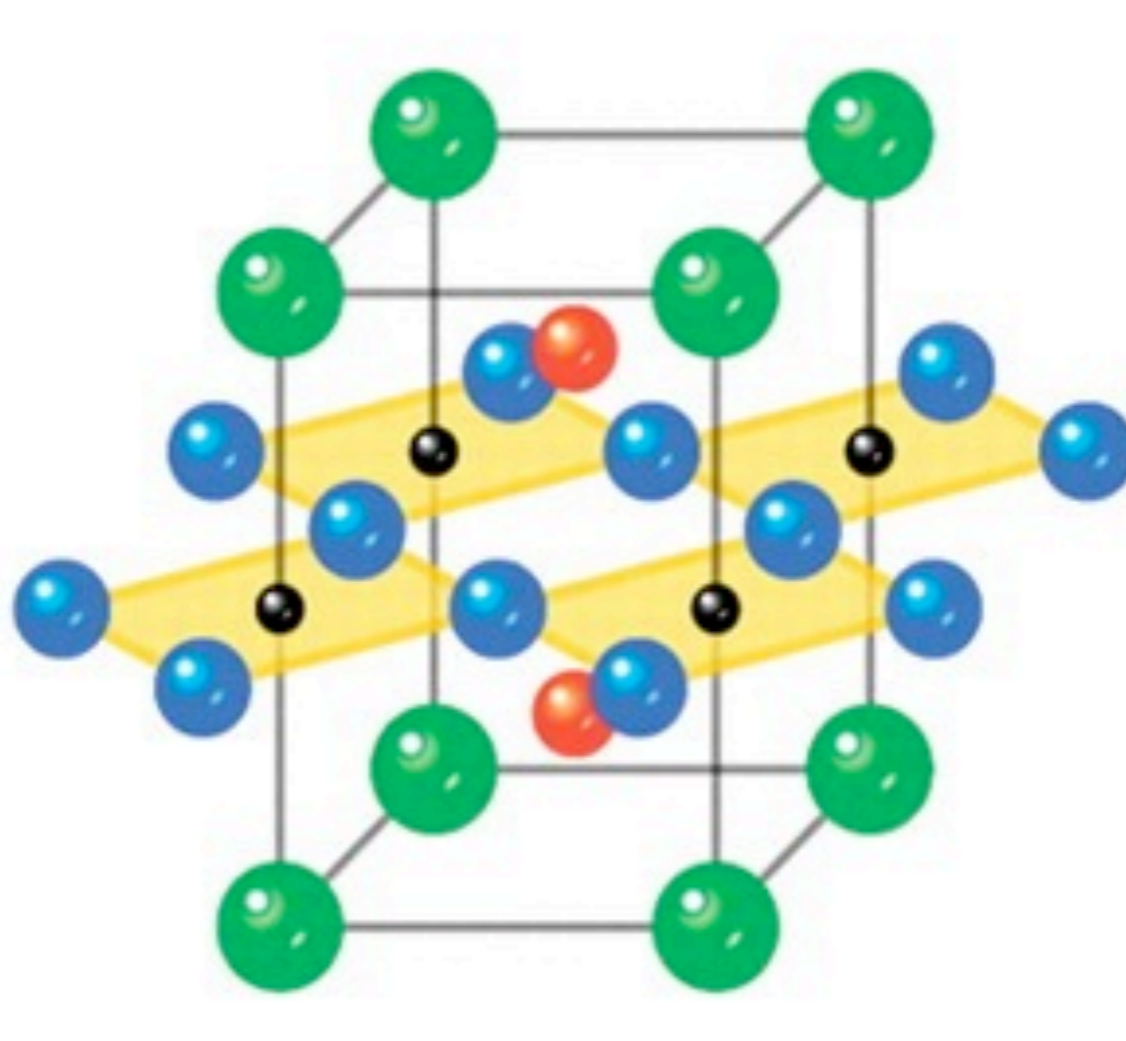
Eun Gook Moon, Harvard



The cuprate superconductors

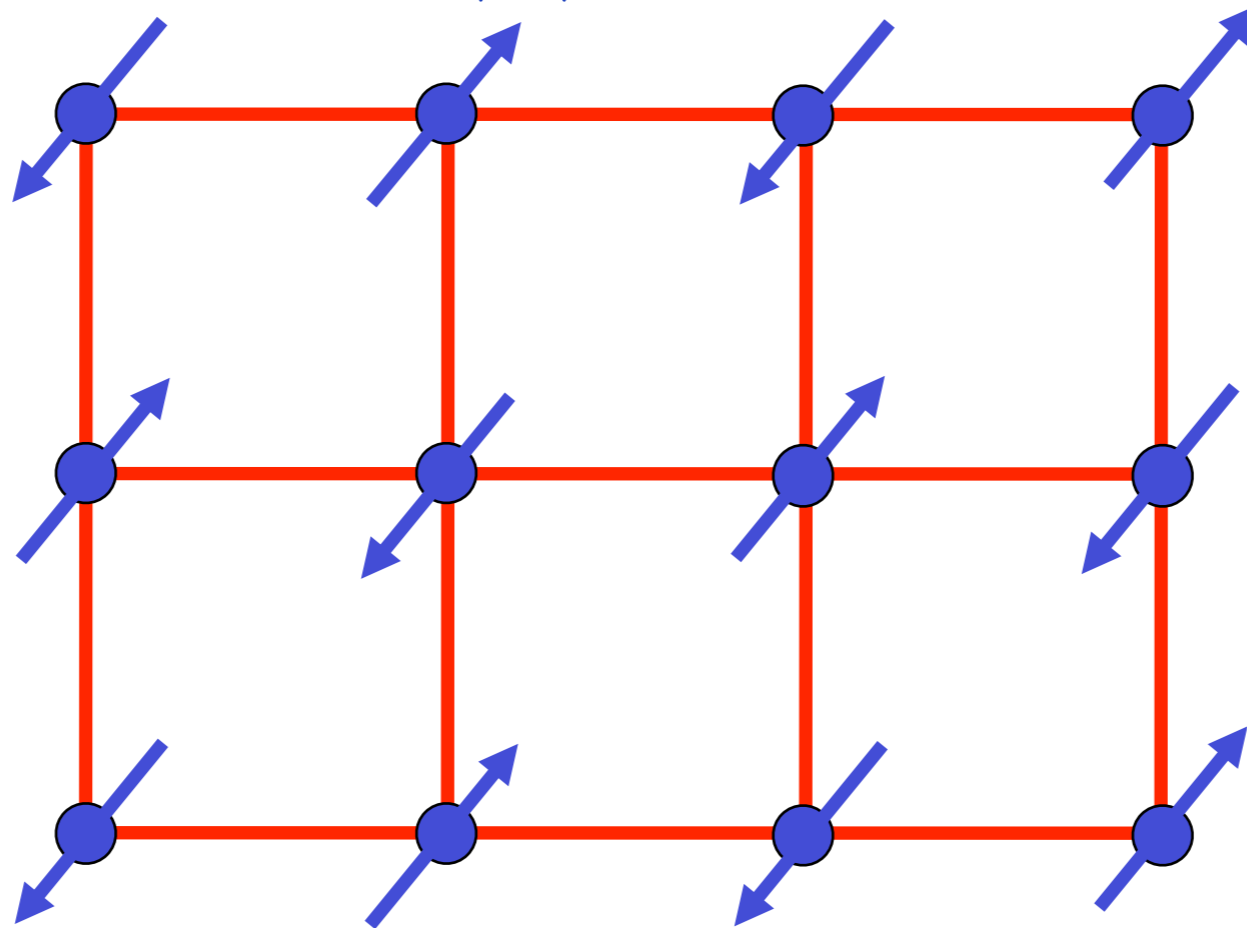
Na-CCOC

- Cu
- Ca/Na
- O
- Cl



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

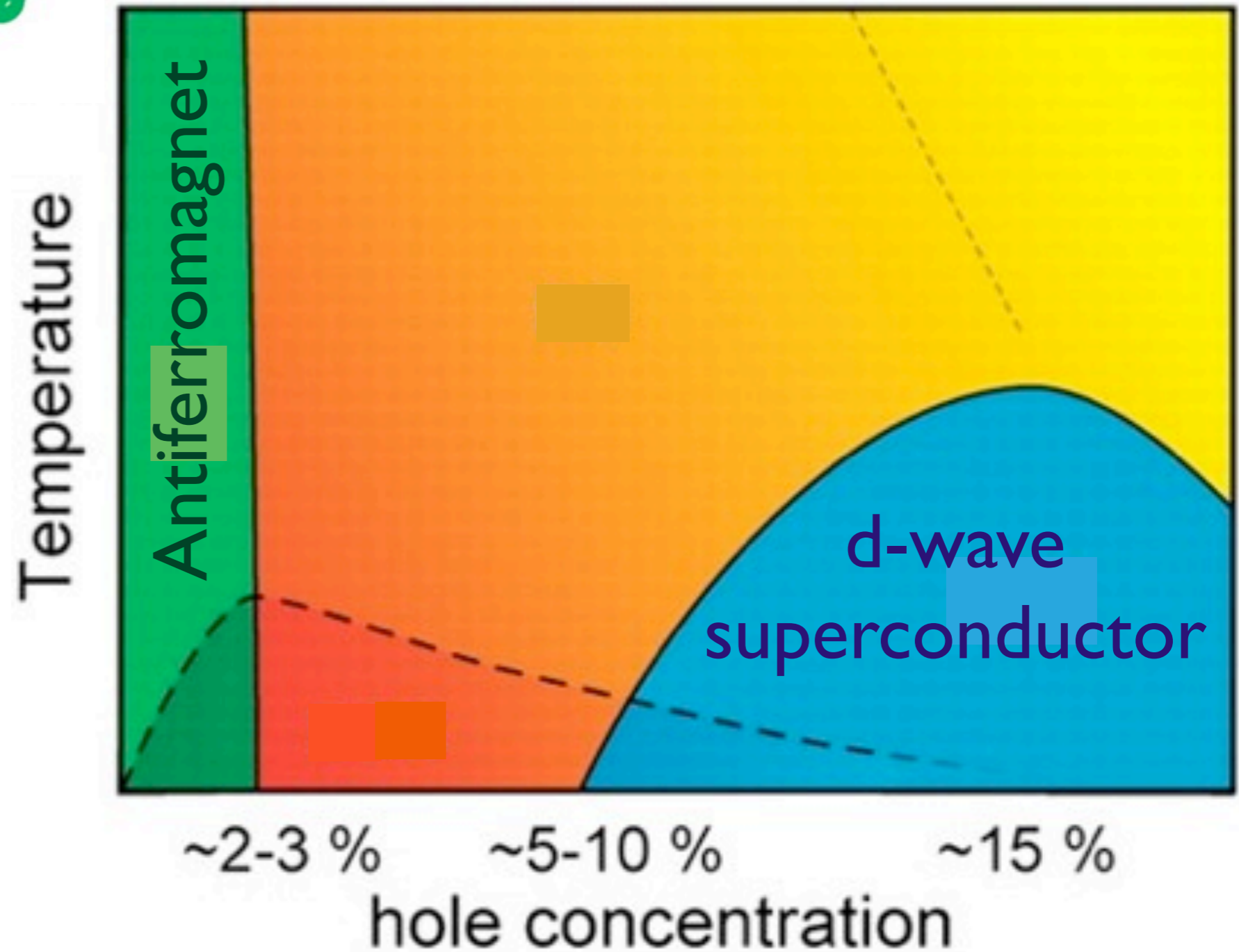
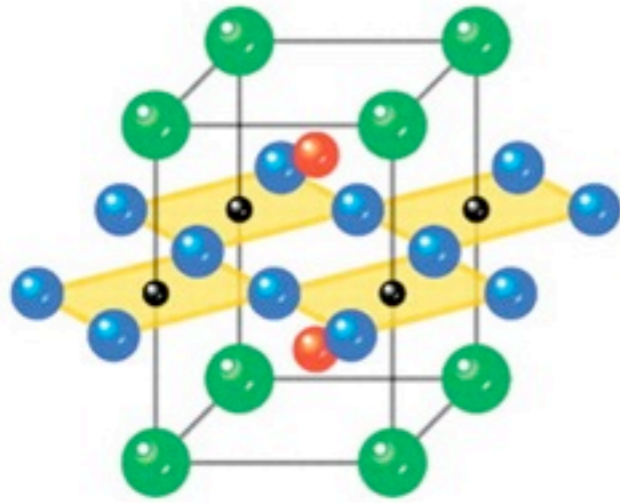
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

The cuprate superconductors

Na-CCOC

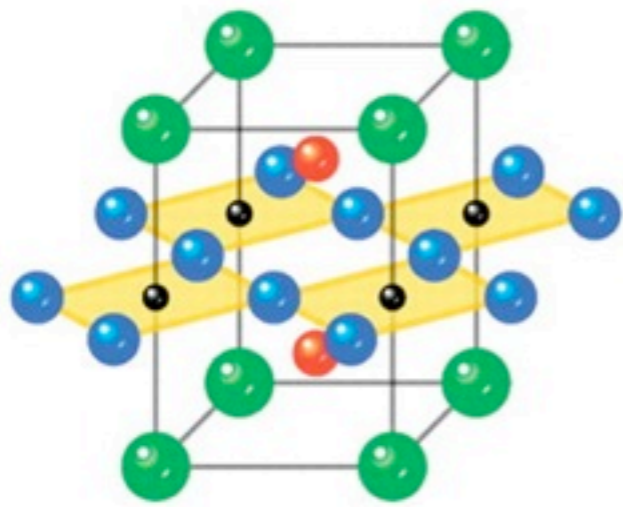
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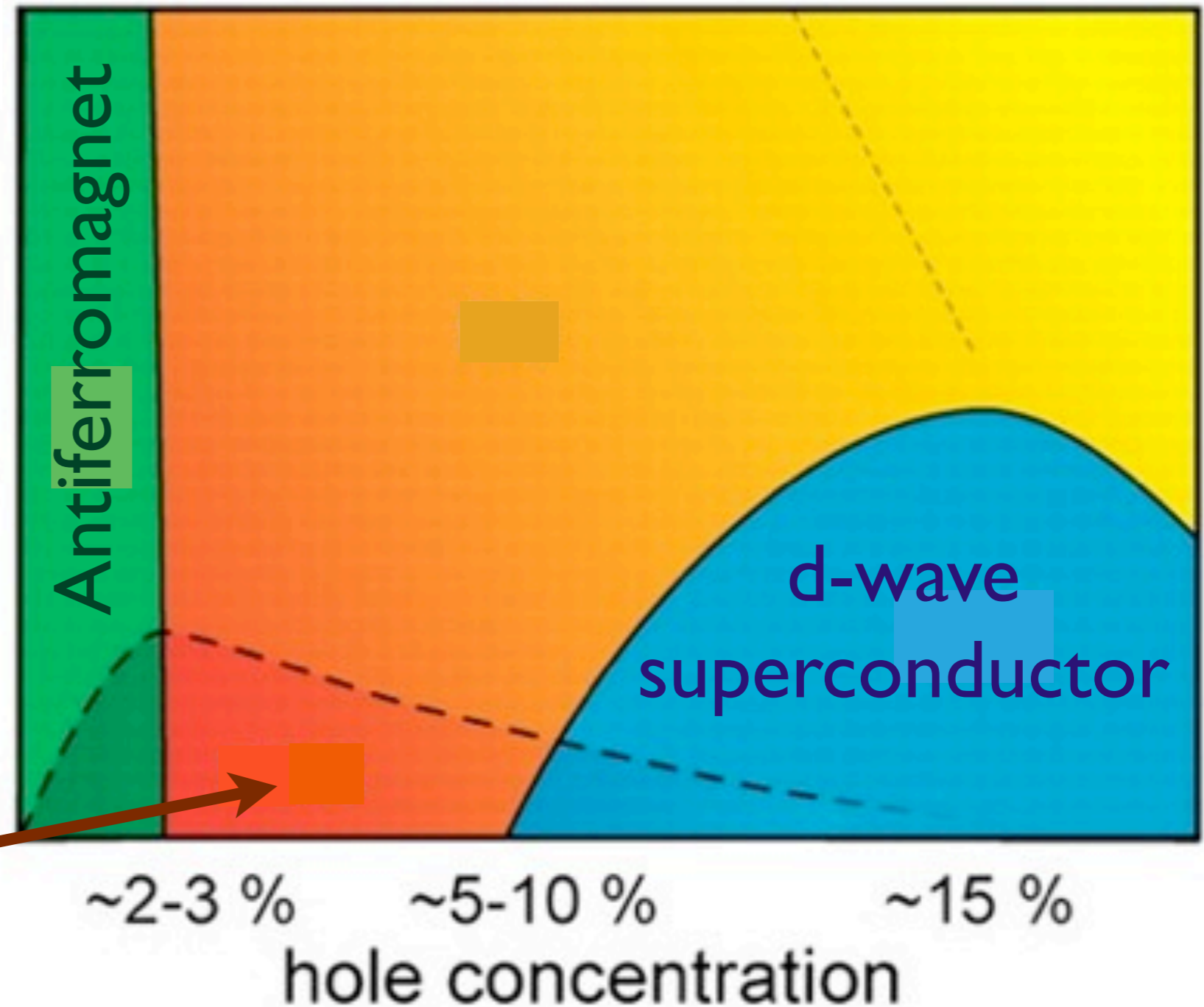
The cuprate superconductors

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Temperature

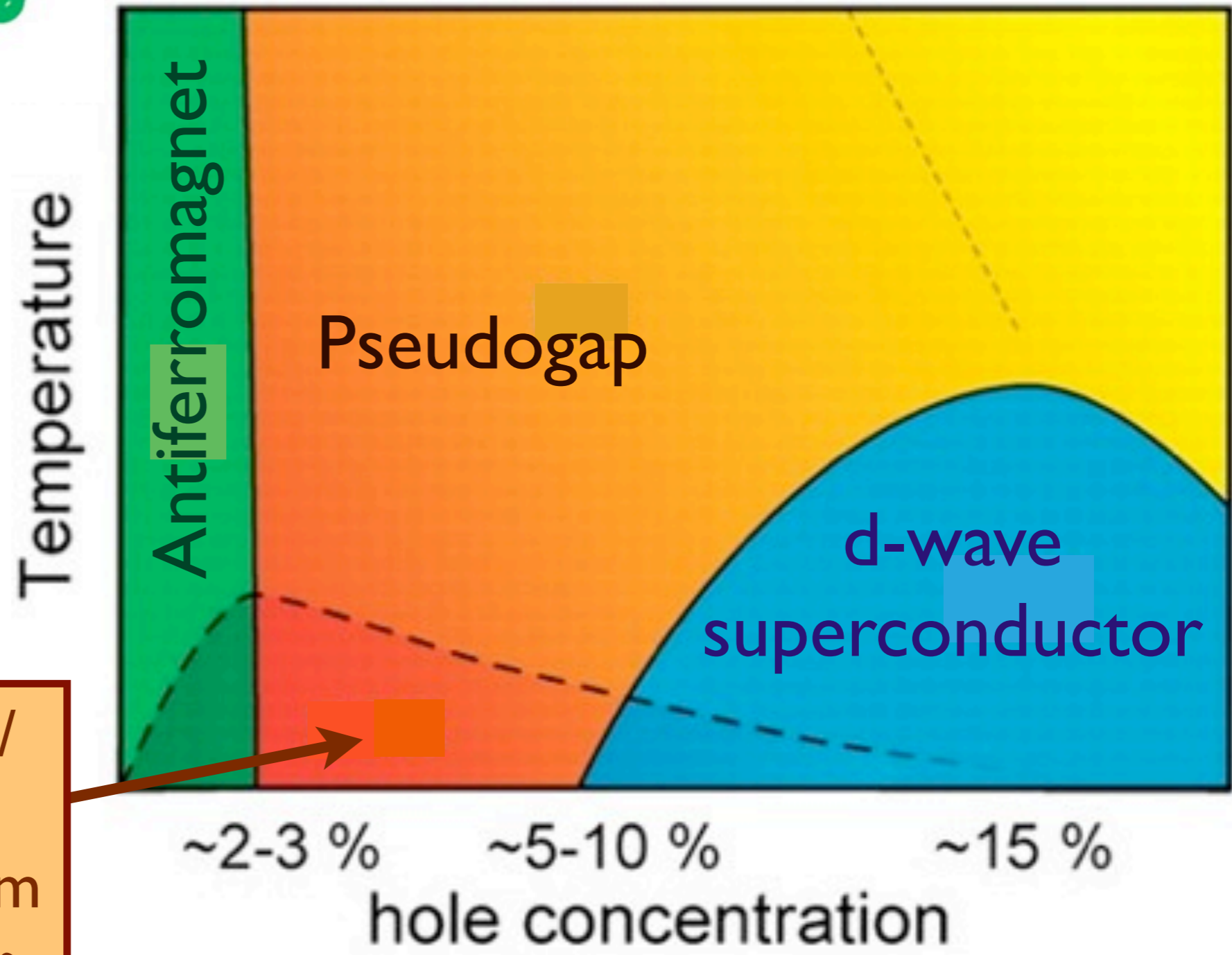
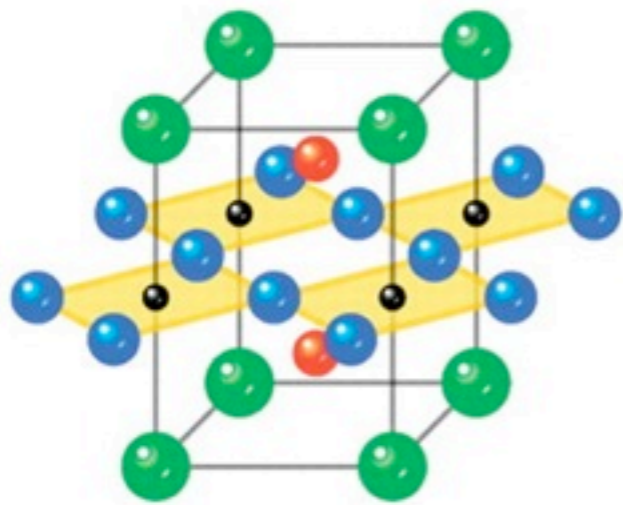


Incommensurate/
disordered
antiferromagnetism
and charge order

The cuprate superconductors

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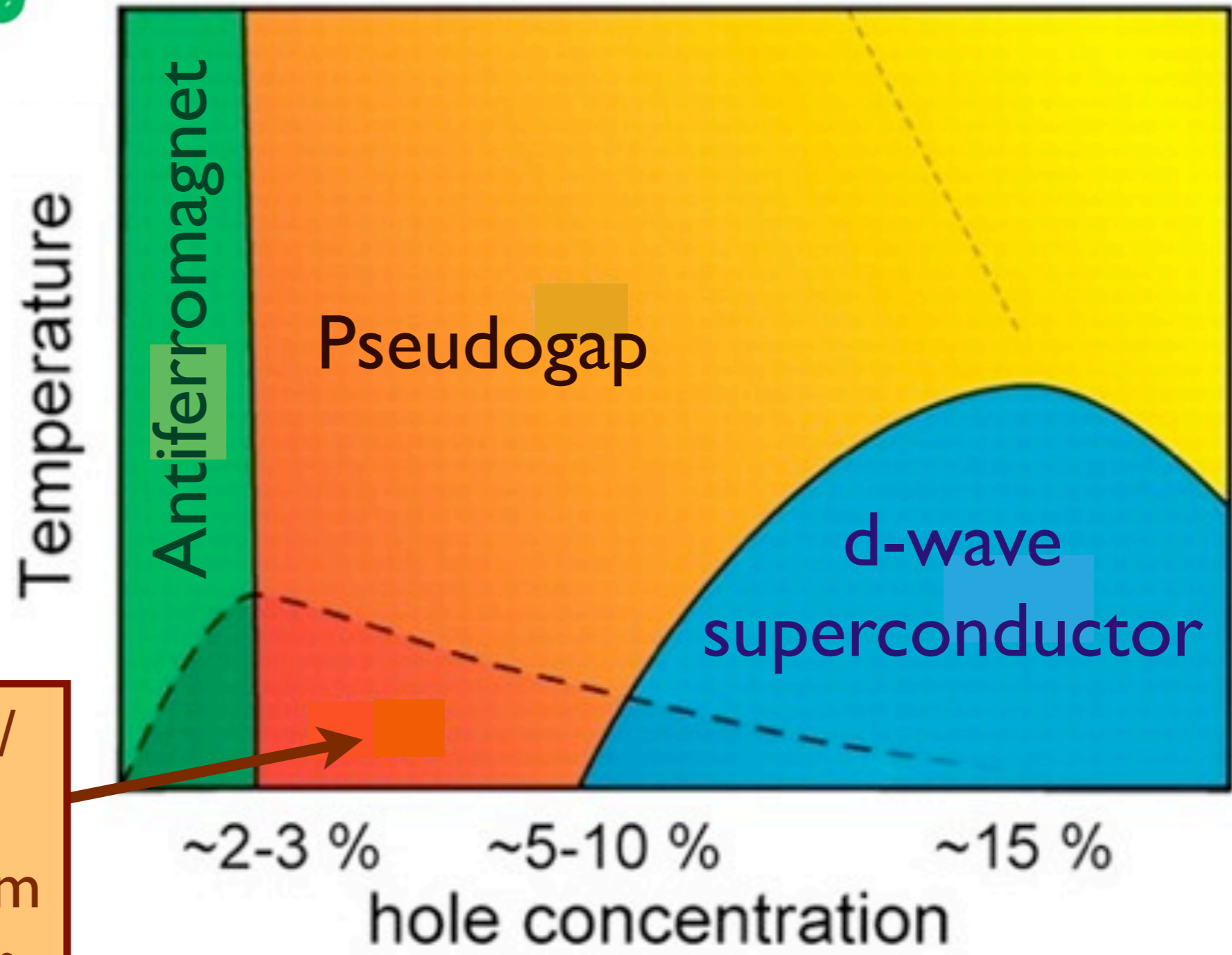
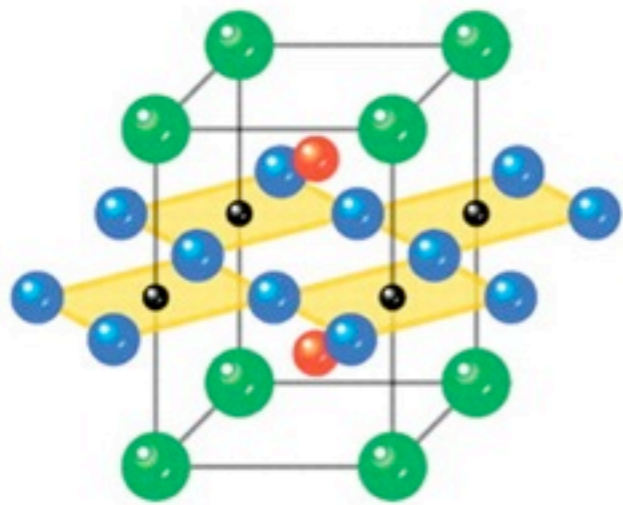


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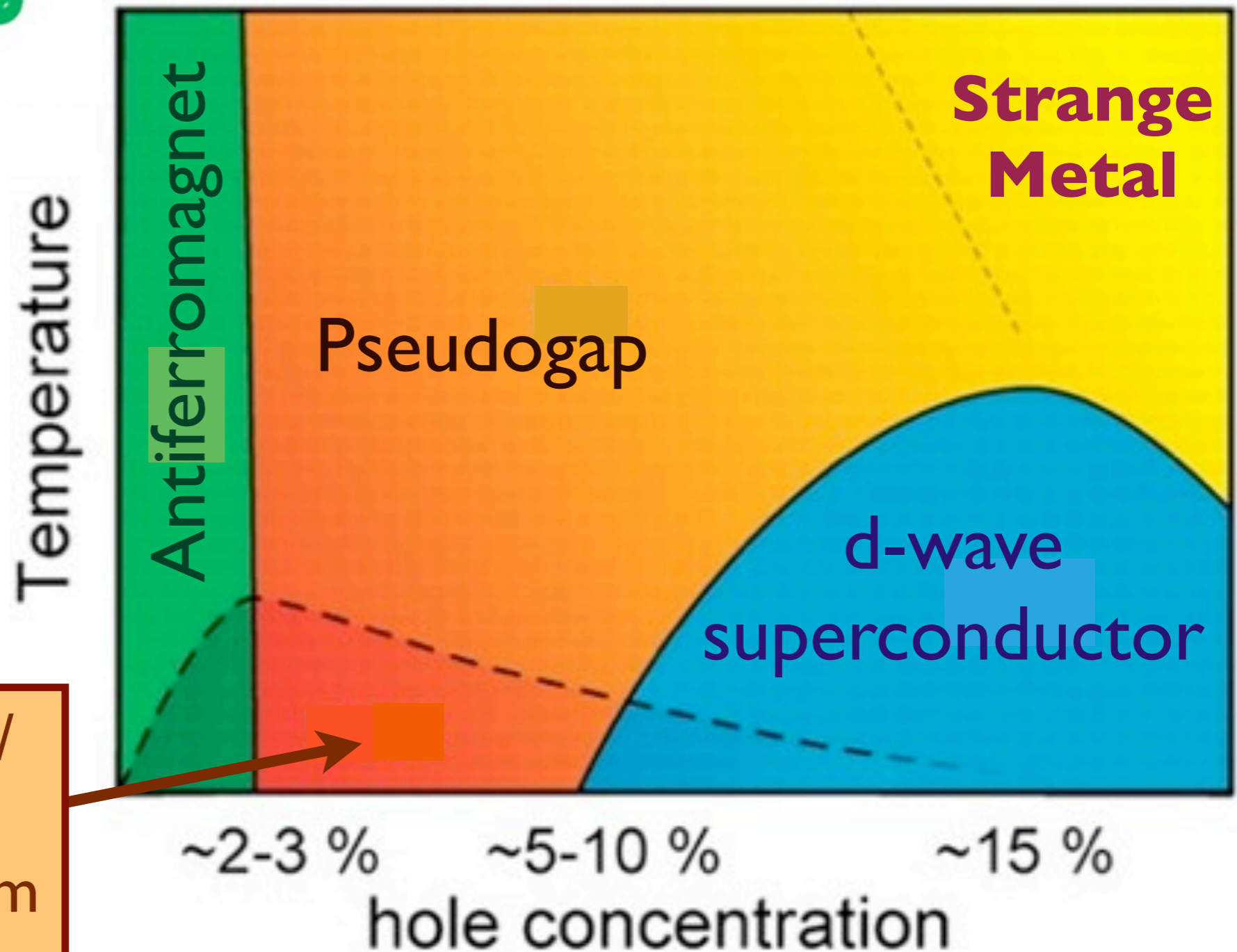
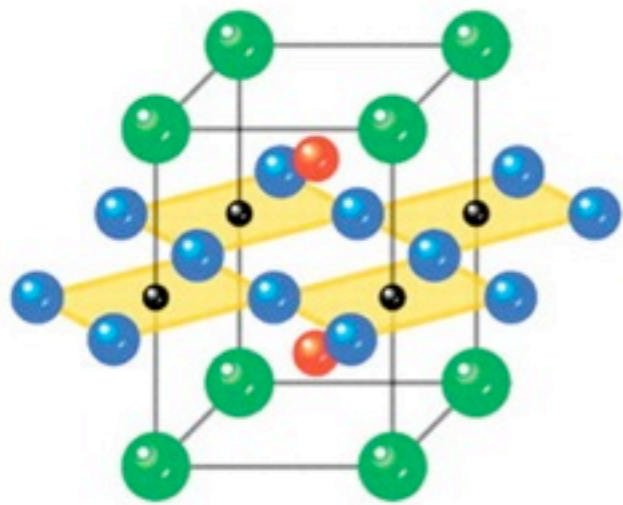


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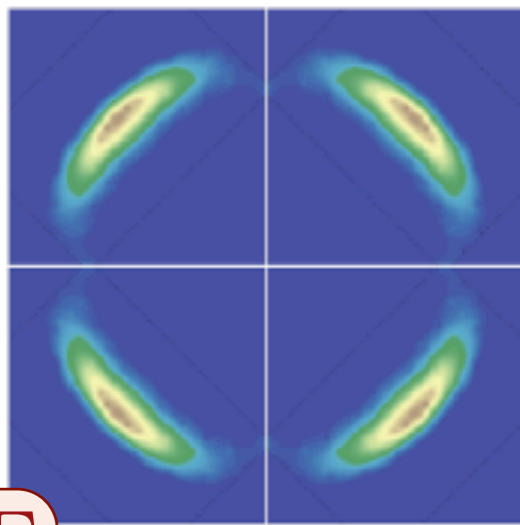
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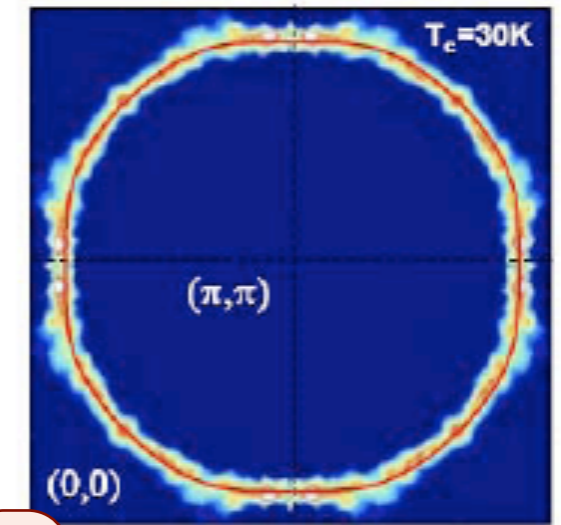
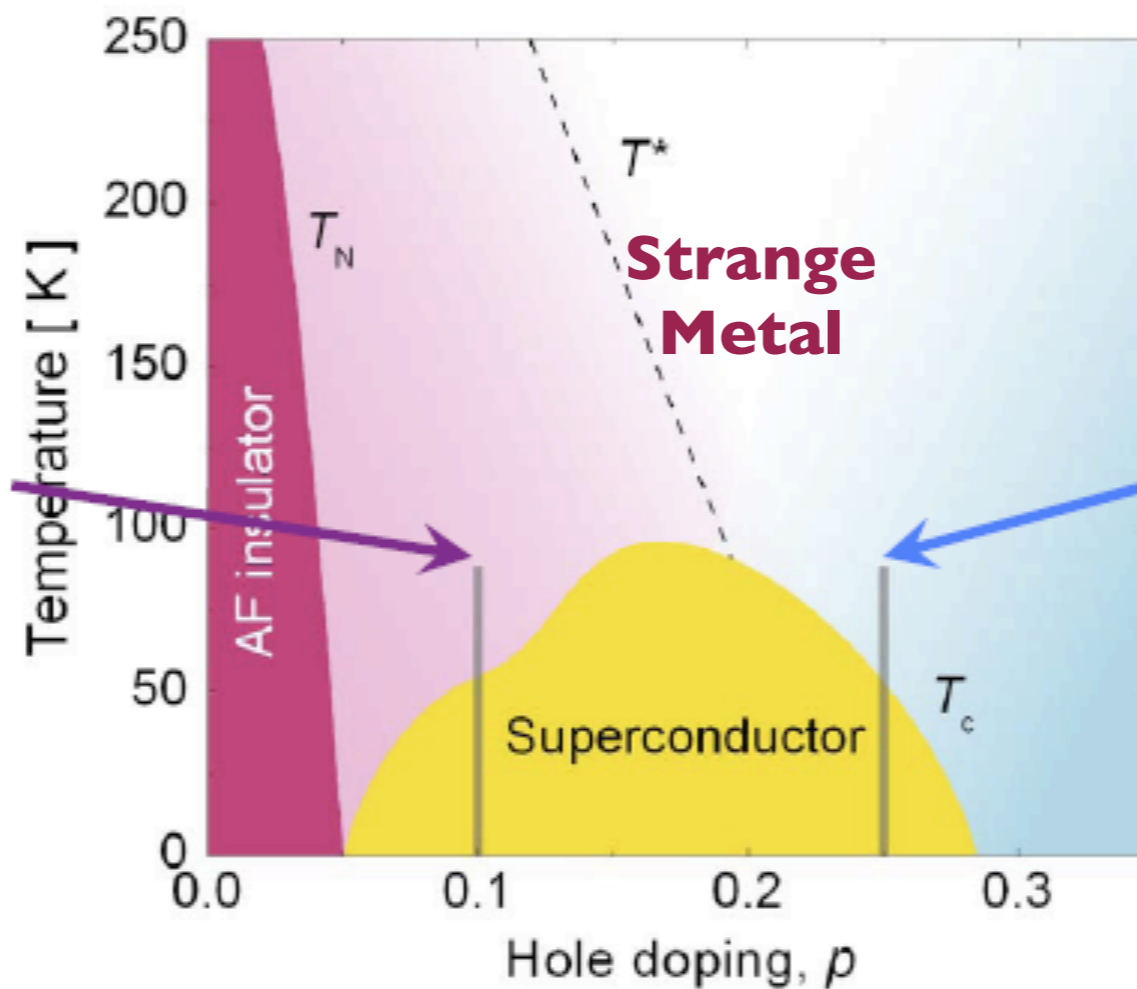


Incommensurate/
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Γ

K.M. Shen et al., Science 2005



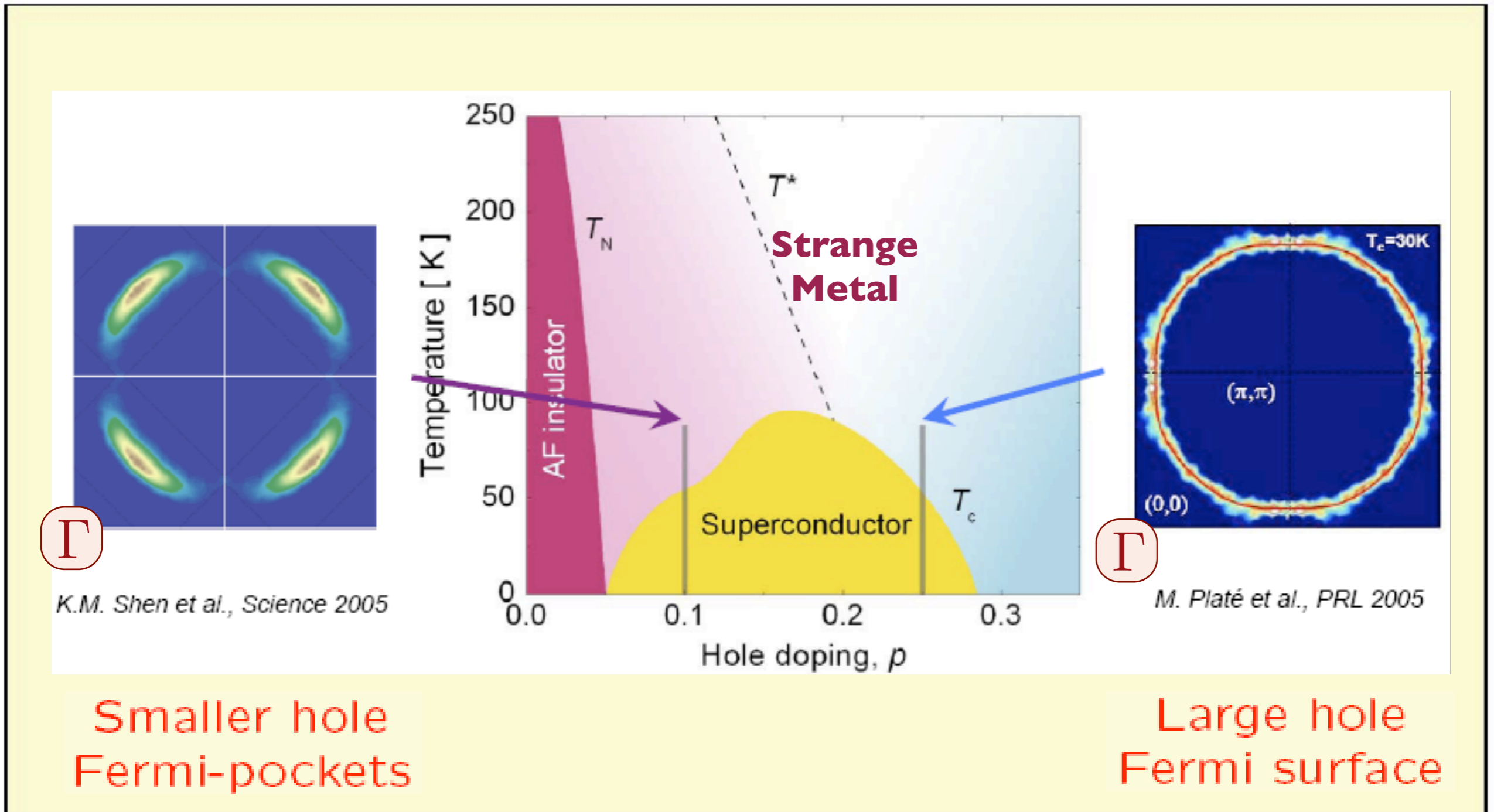
Γ

M. Platé et al., PRL 2005

Smaller hole
Fermi-pockets

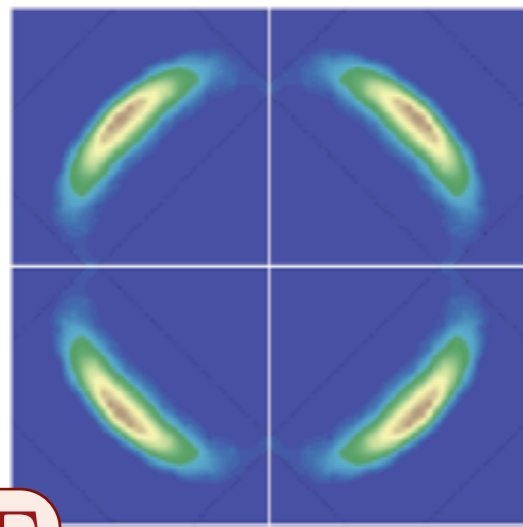
Large hole
Fermi surface

Key Ingredients:



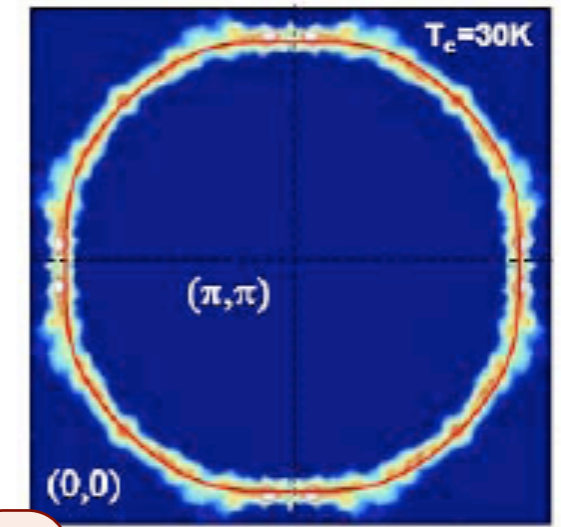
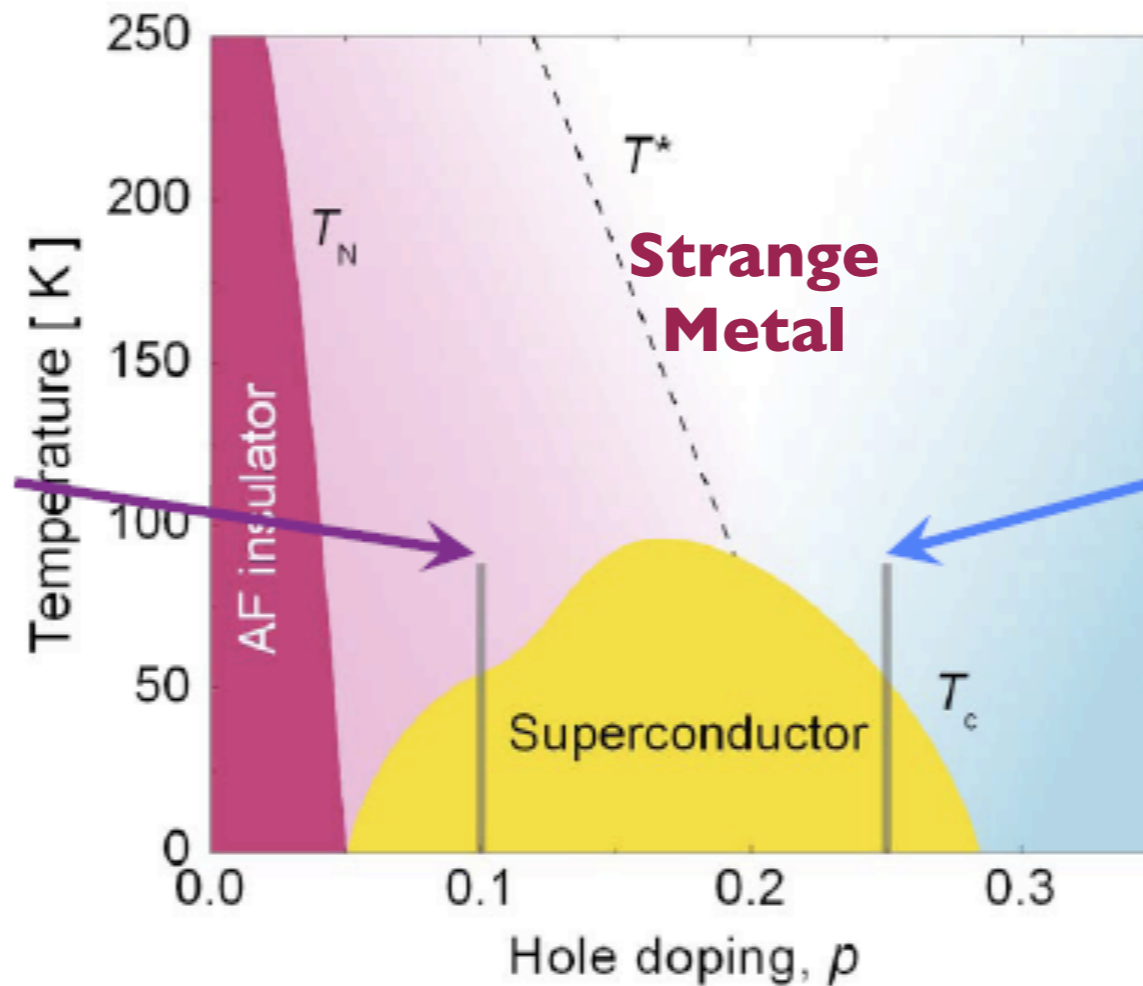
Key Ingredients:

Antiferromagnetism (AF)
Spin density wave (SDW)



Γ

K.M. Shen et al., Science 2005



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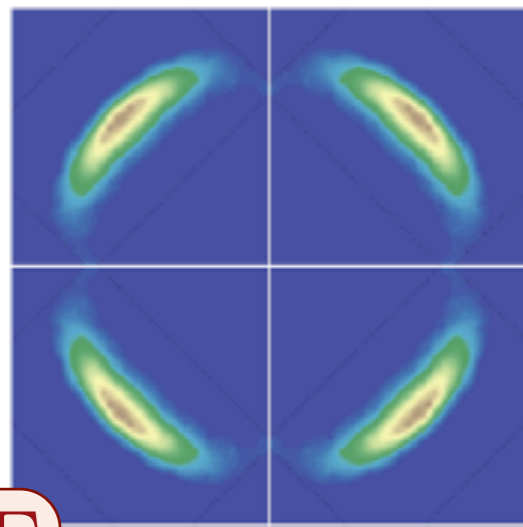
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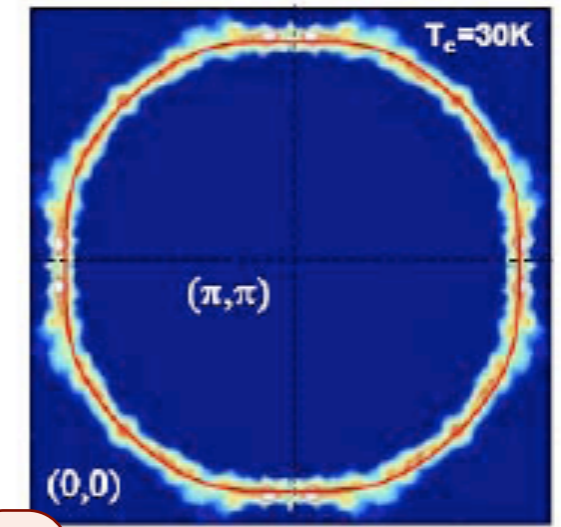
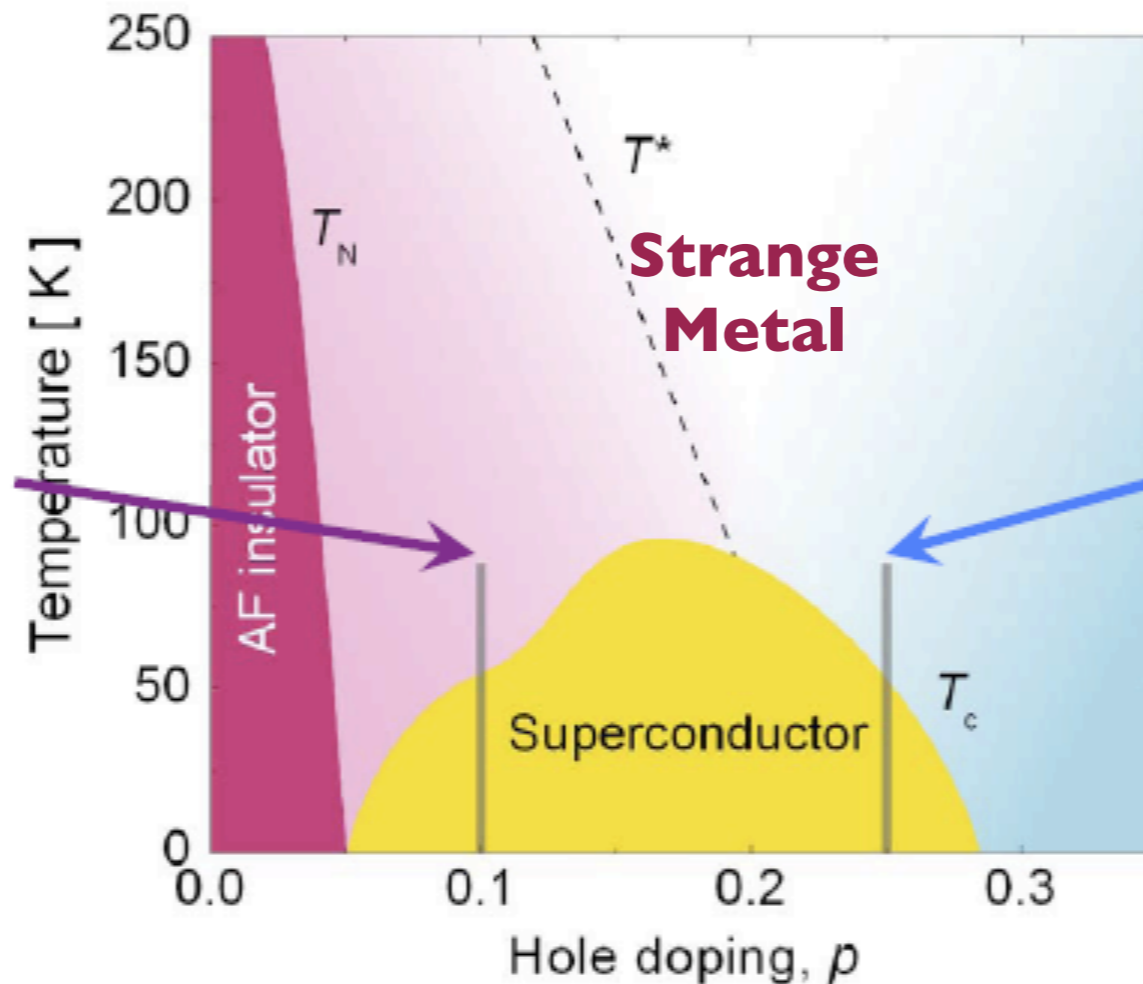
d-wave superconductivity

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K.M. Shen et al., Science 2005



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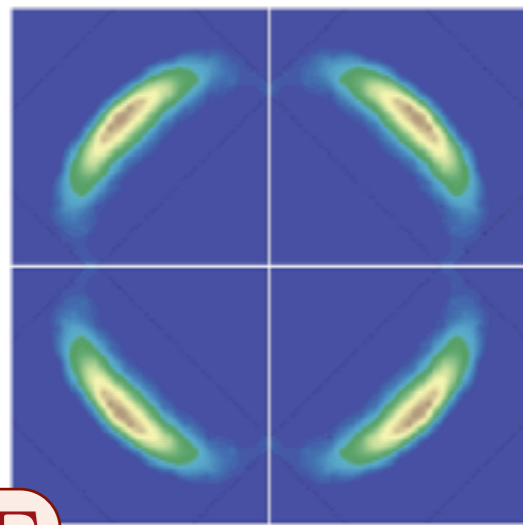
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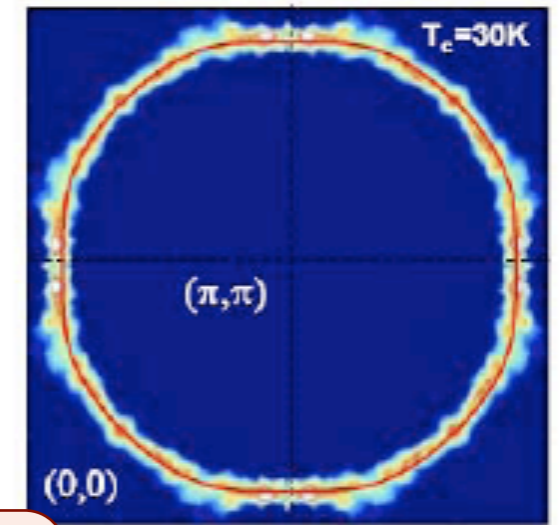
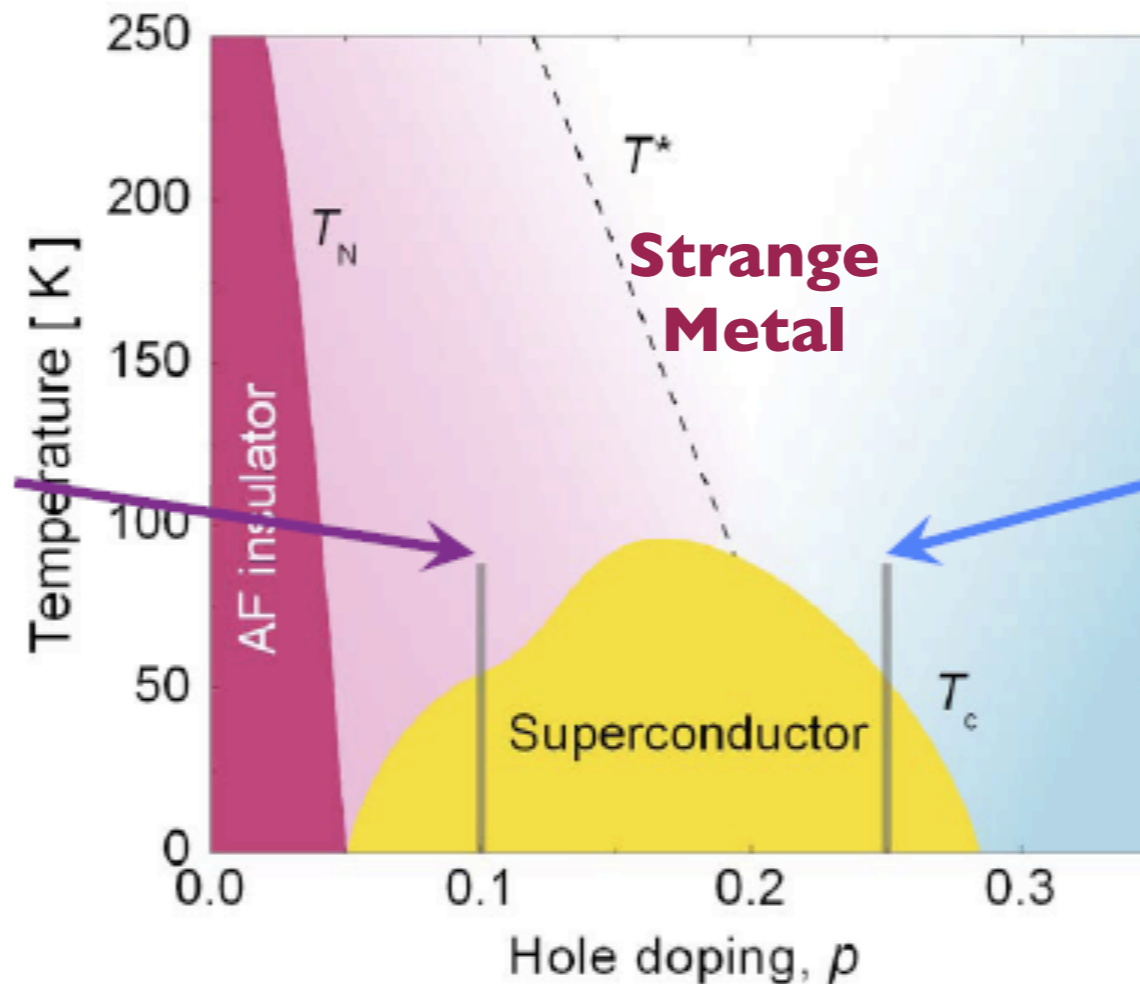
Antiferromagnetism (AF)
Spin density wave (SDW)

Fermi surface reconstruction



Γ

K.M. Shen et al., Science 2005

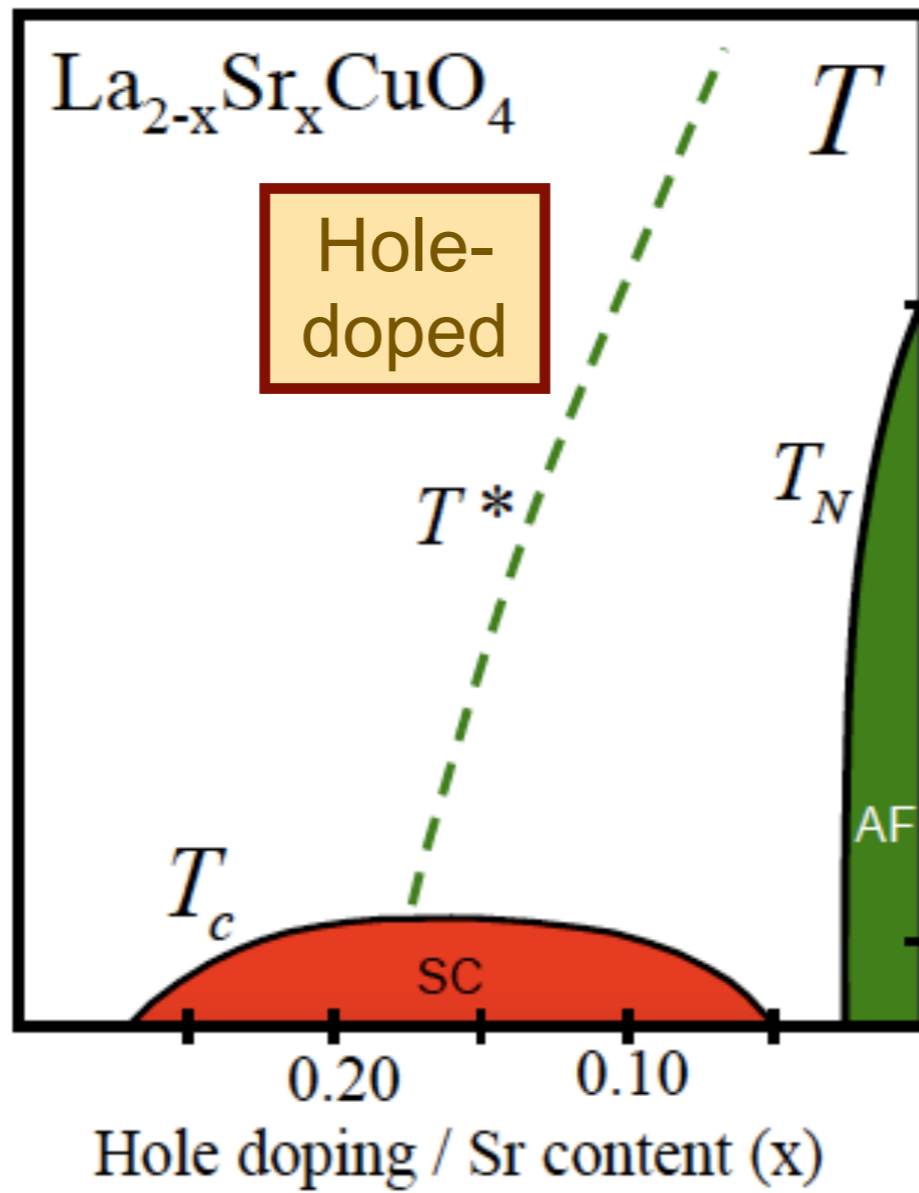


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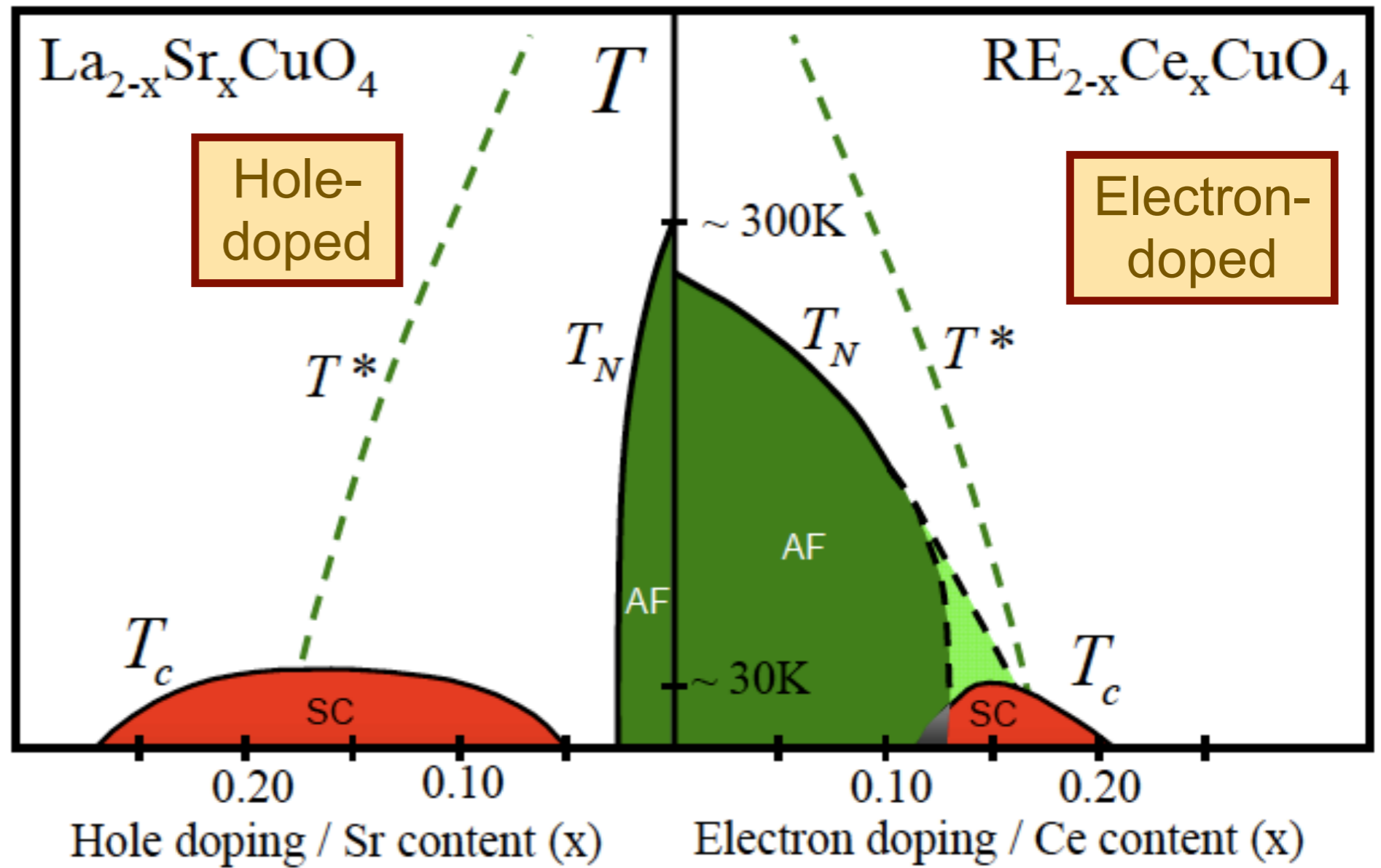
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Electron-doped cuprate superconductors



Electron-doped cuprate superconductors

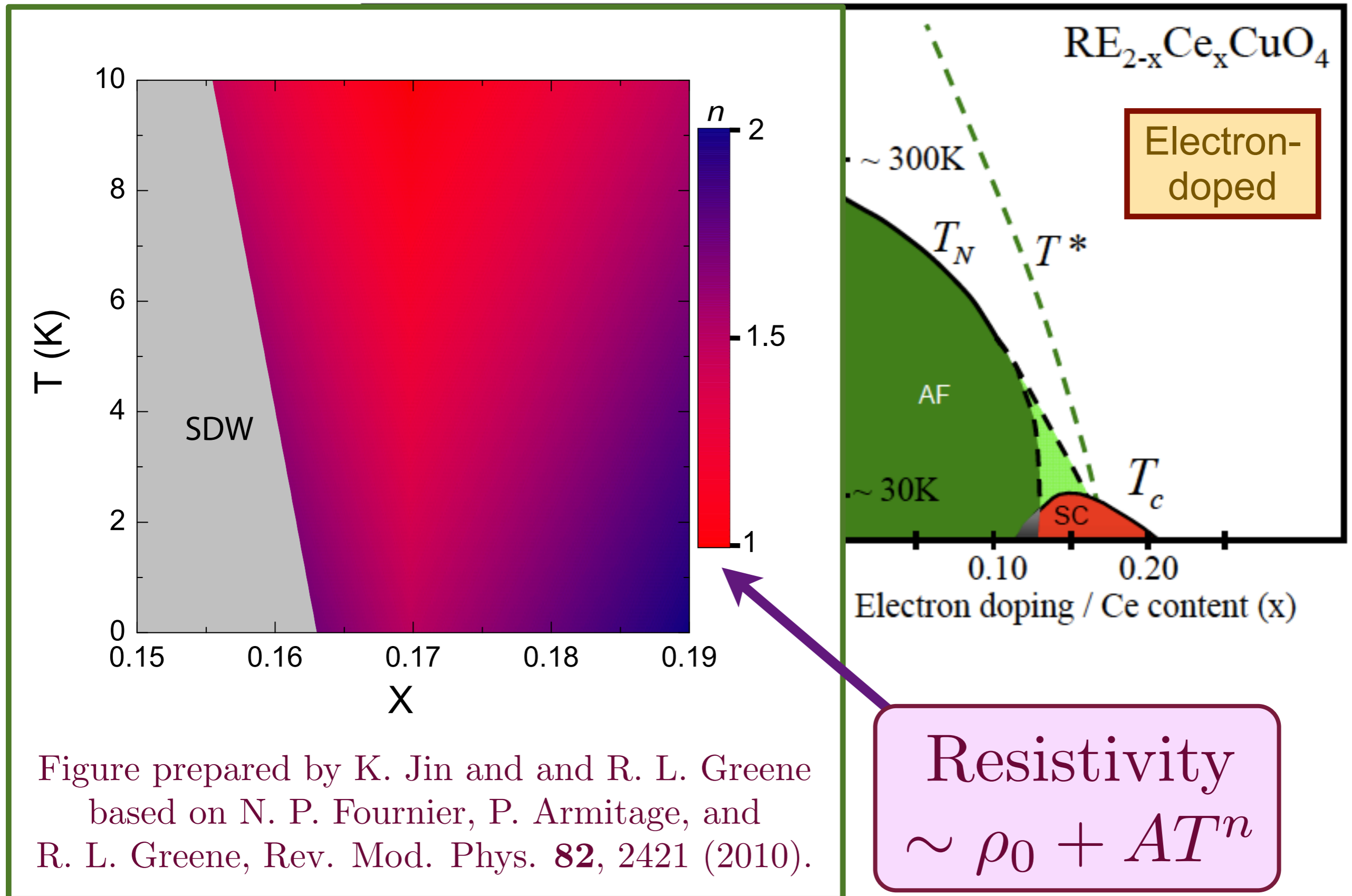


Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Resistivity
 $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors

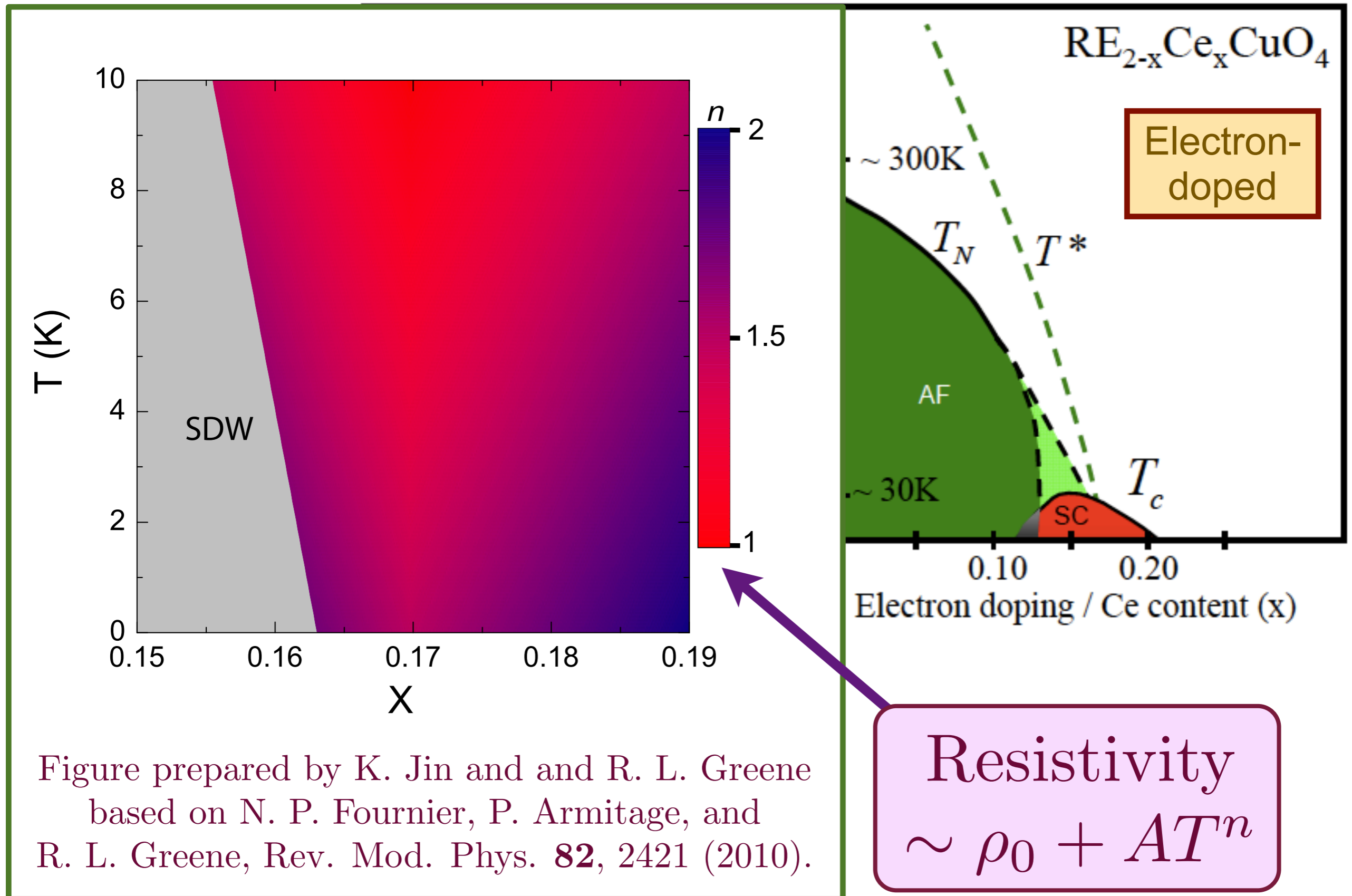


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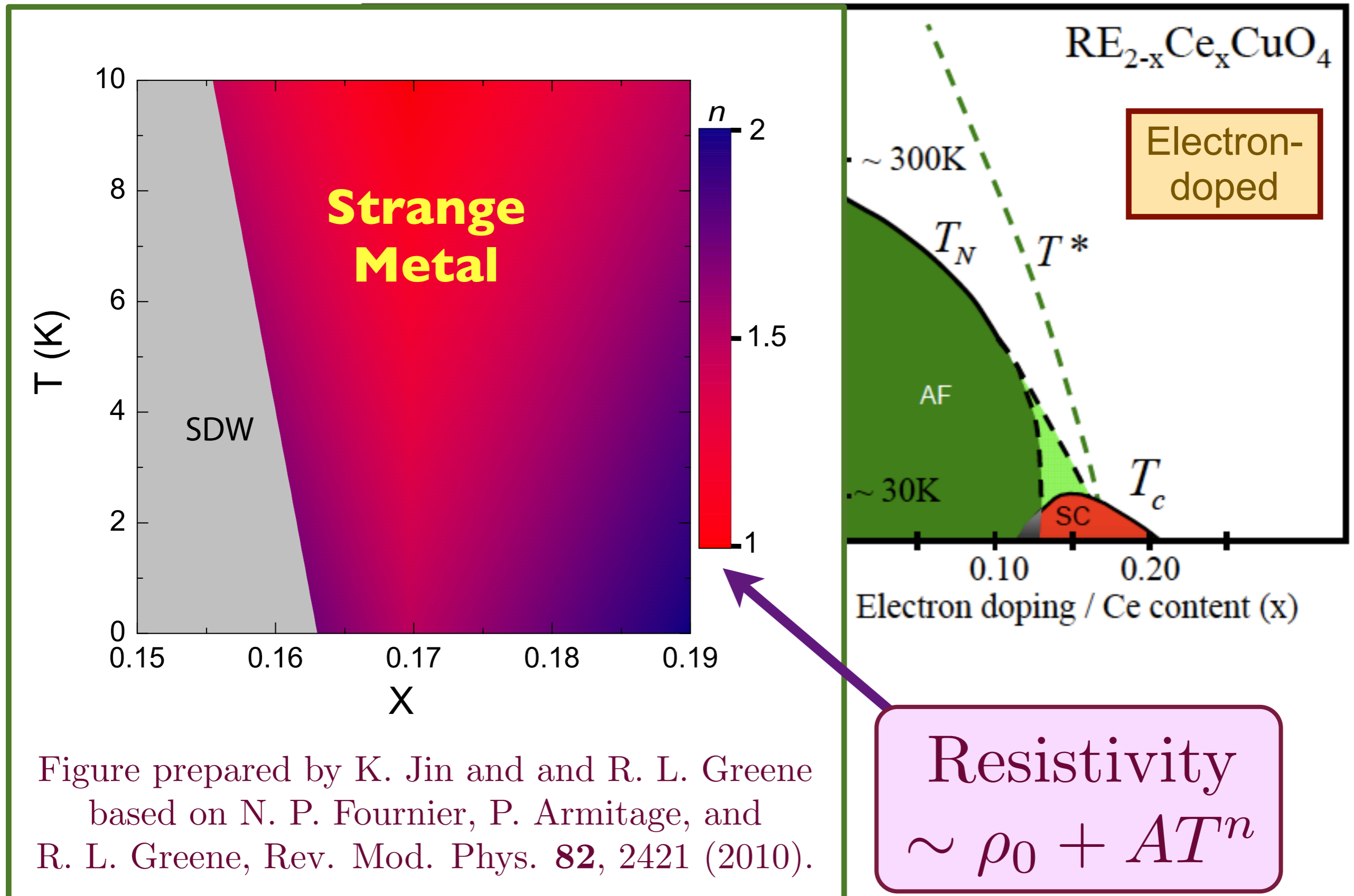
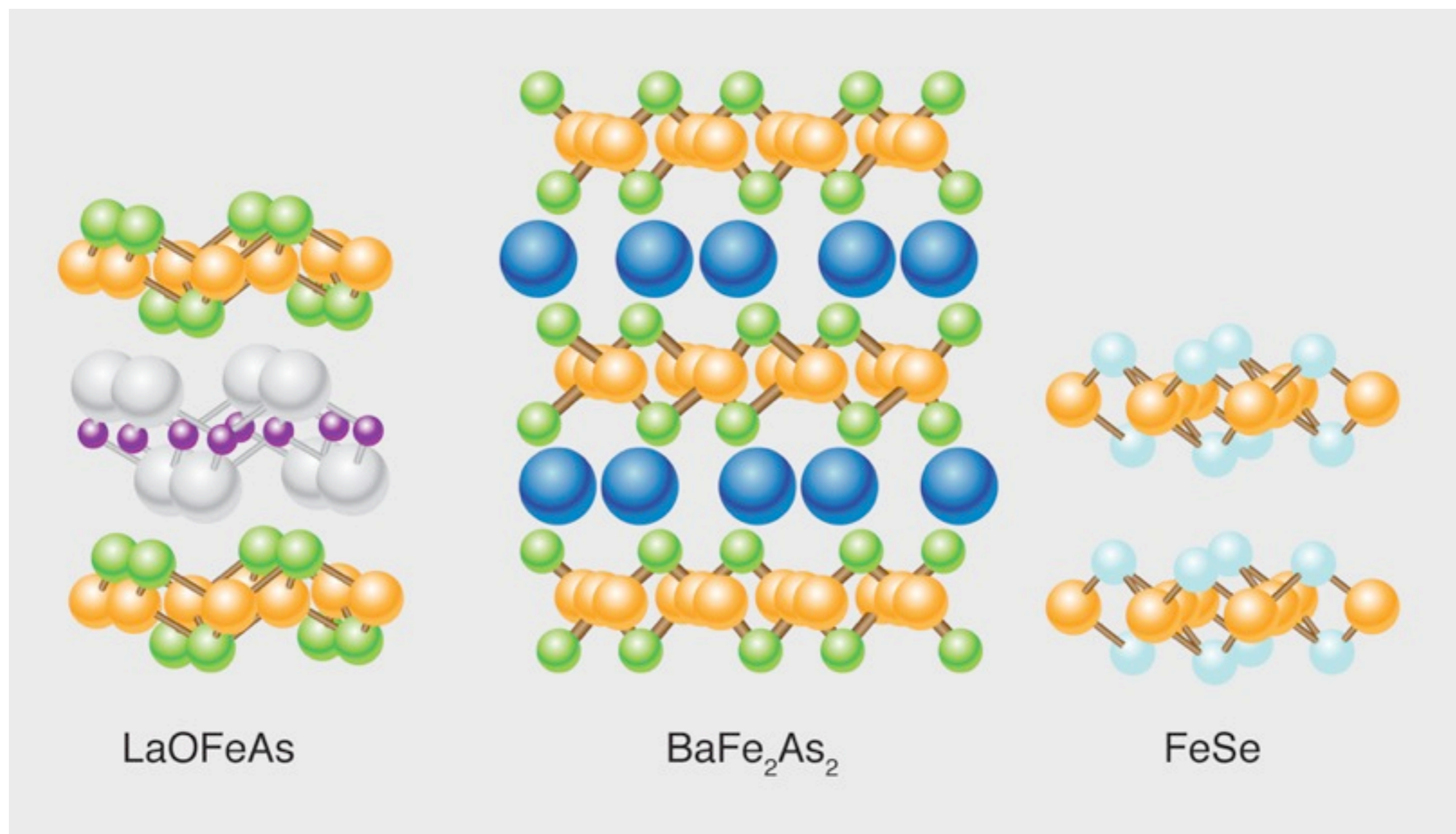


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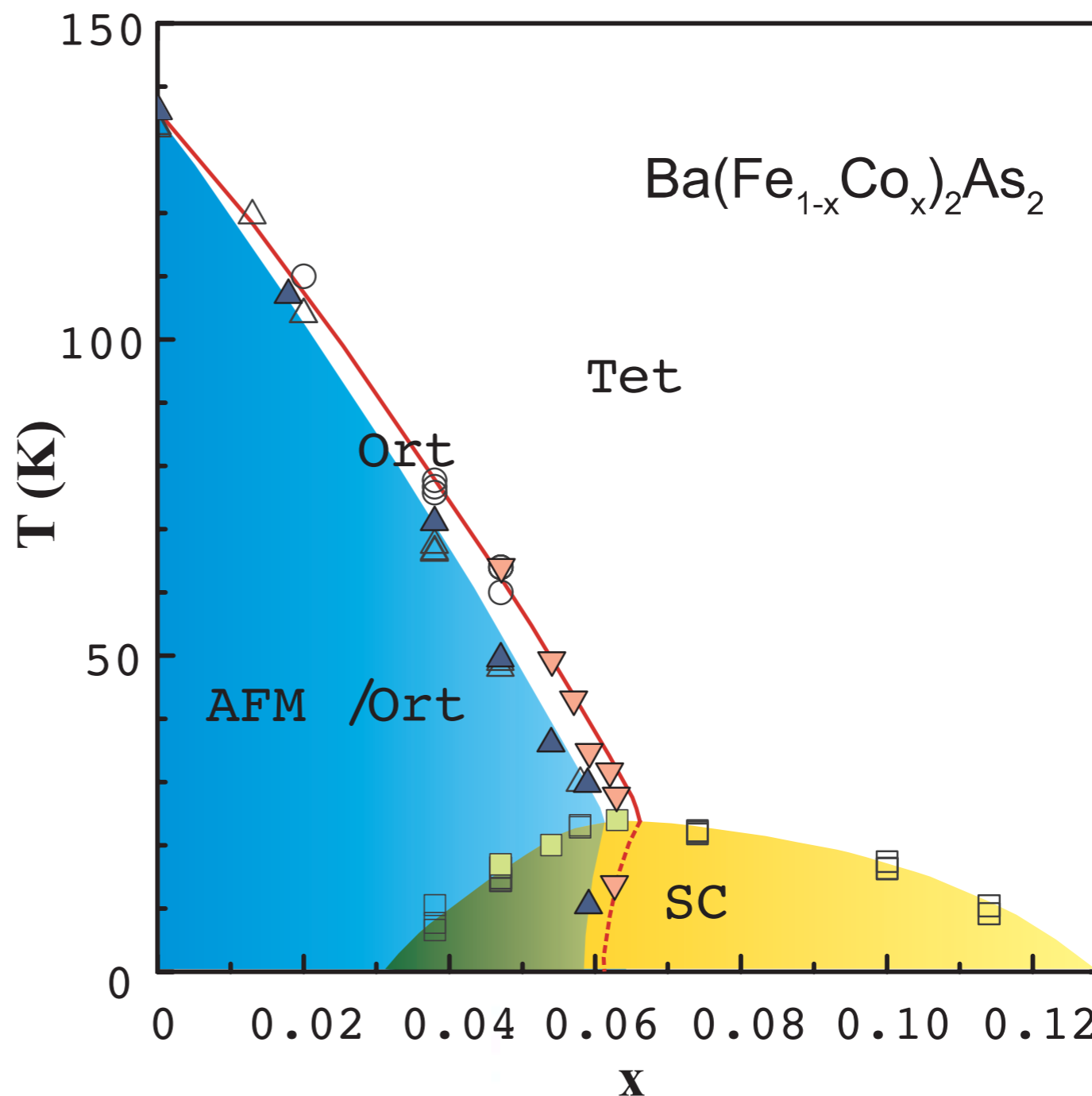
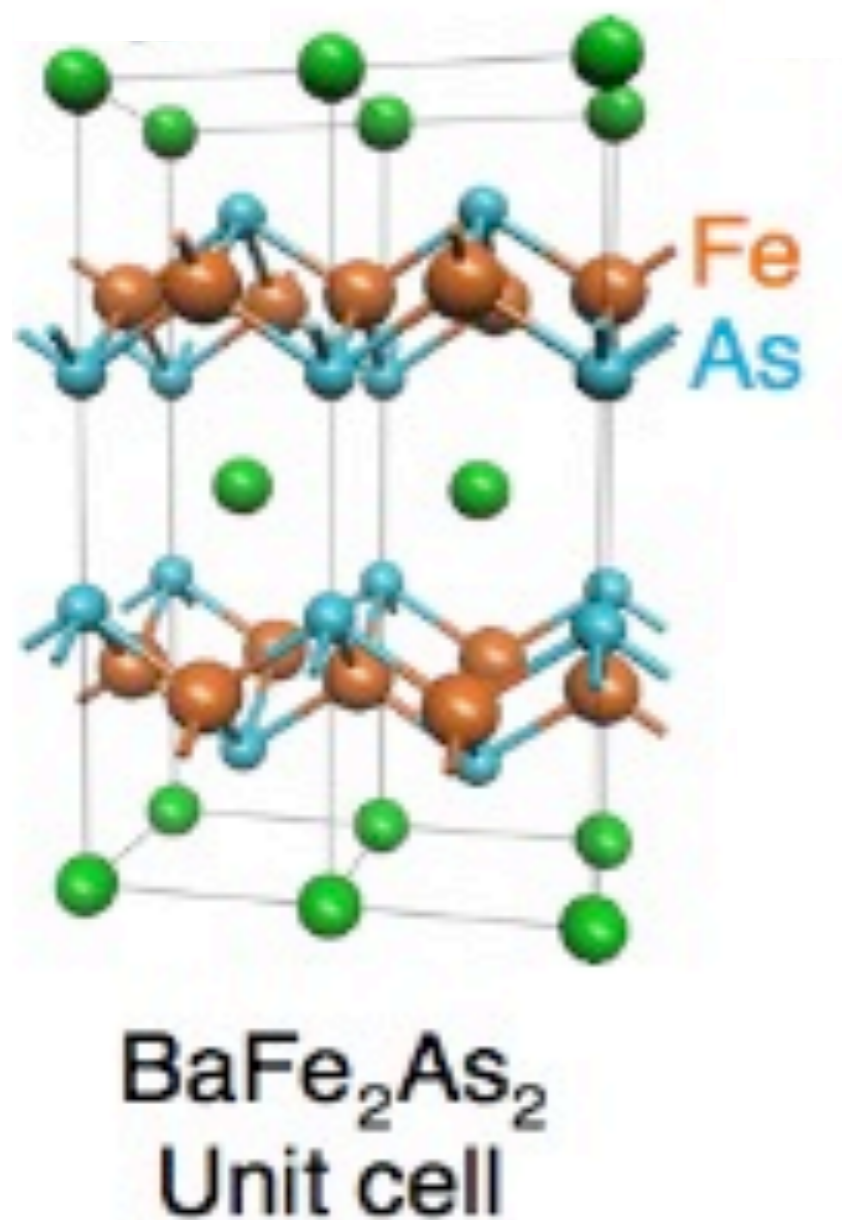
Iron pnictides:

a new class of high temperature superconductors



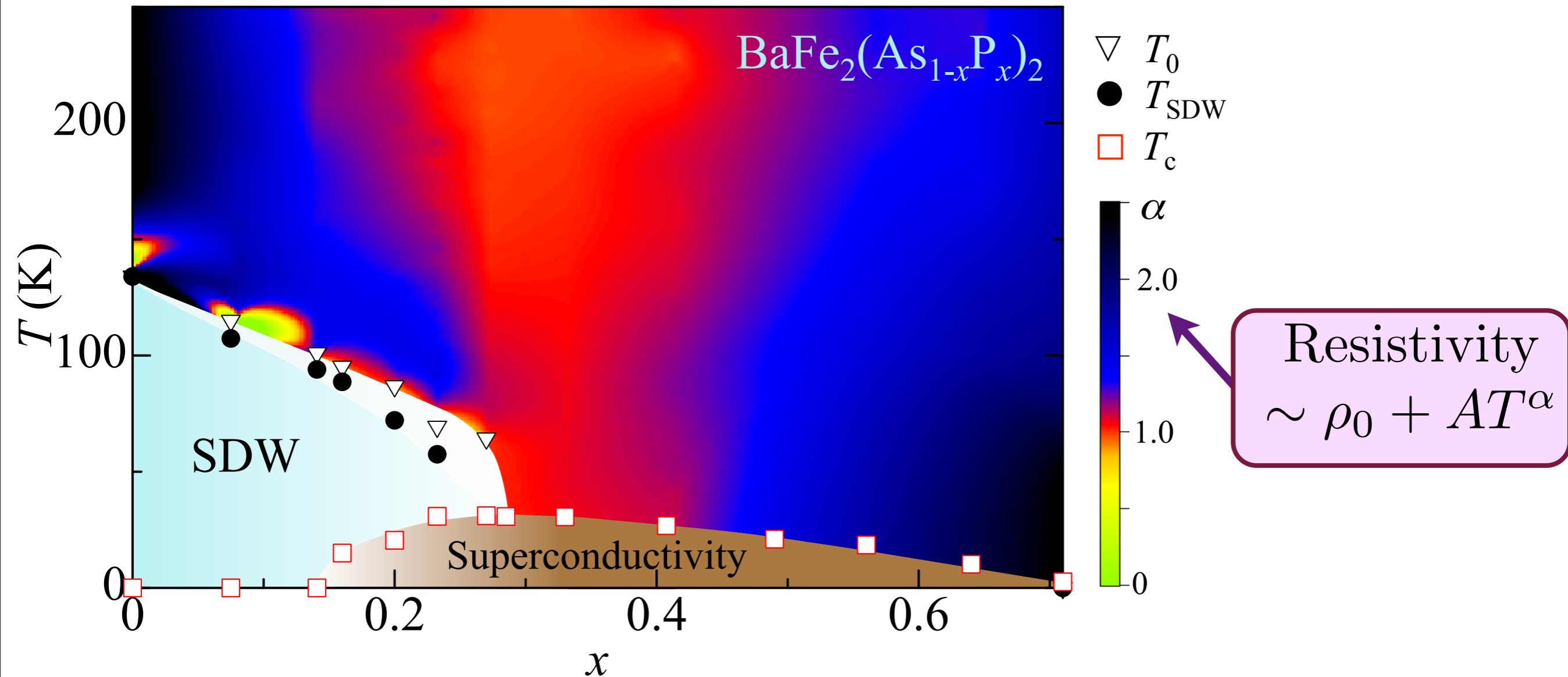
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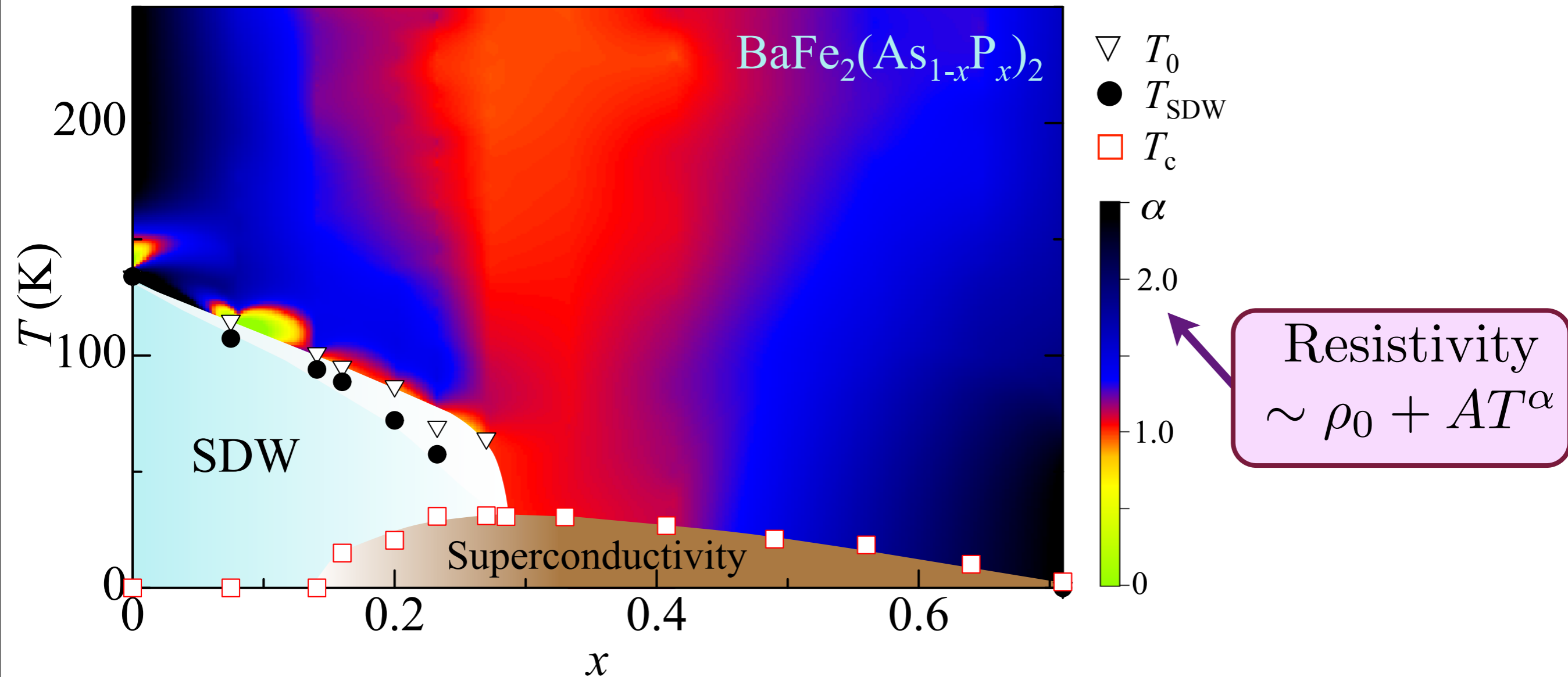
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



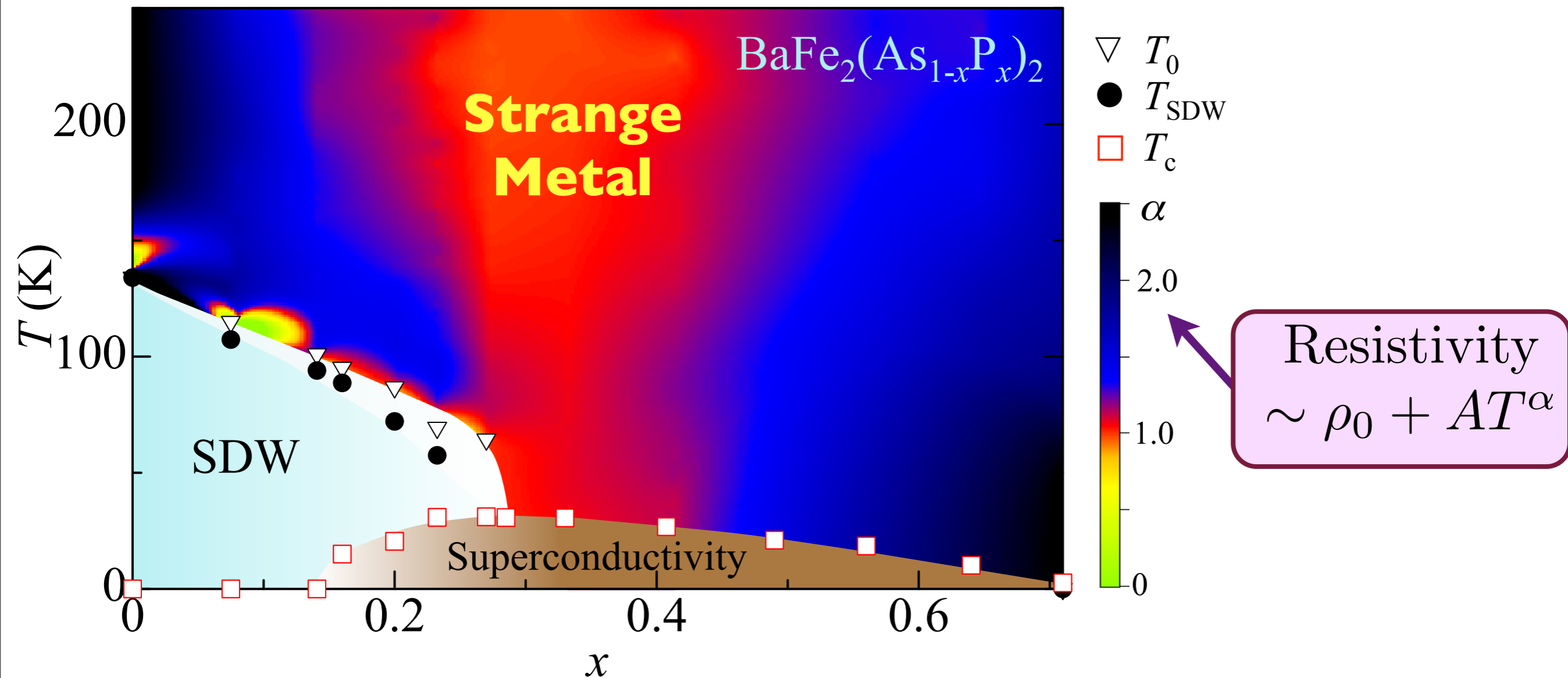
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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Questions

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- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

“Mechanism” of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

Outline

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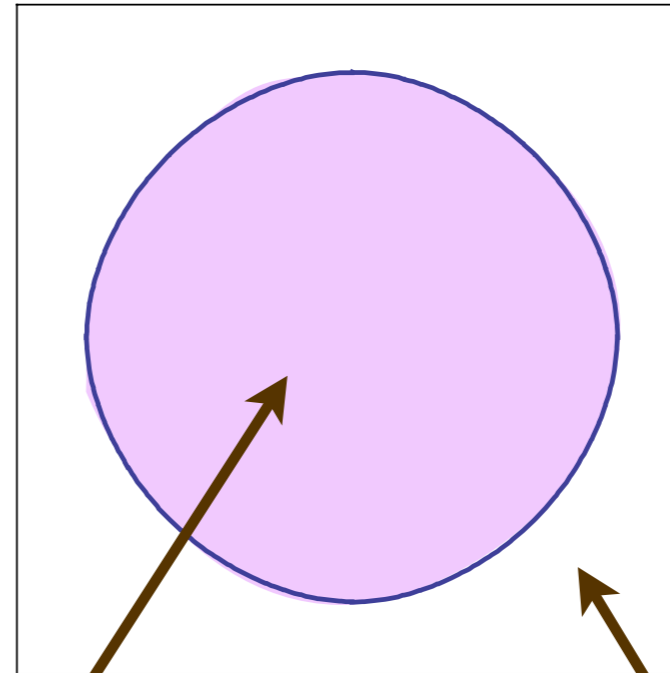
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Fermi surface

Metal with “large”
Fermi surface

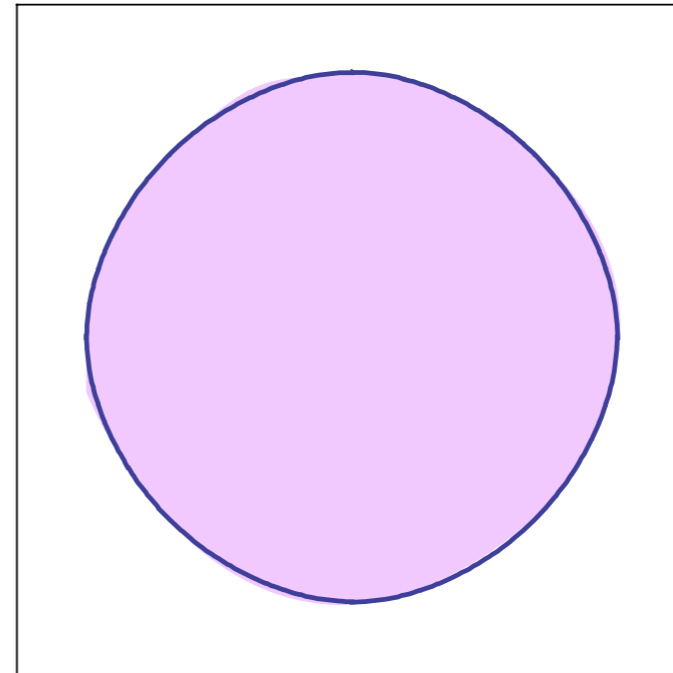


Momenta with
electronic
states empty

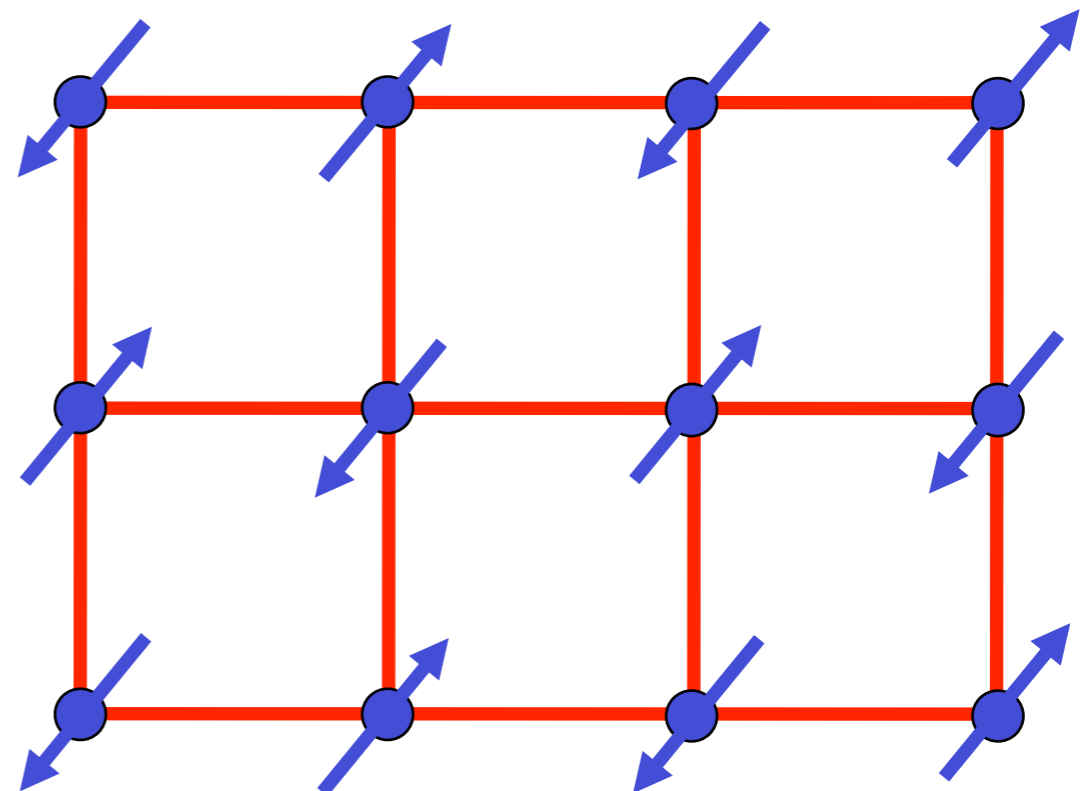
Momenta with
electronic
states
occupied

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



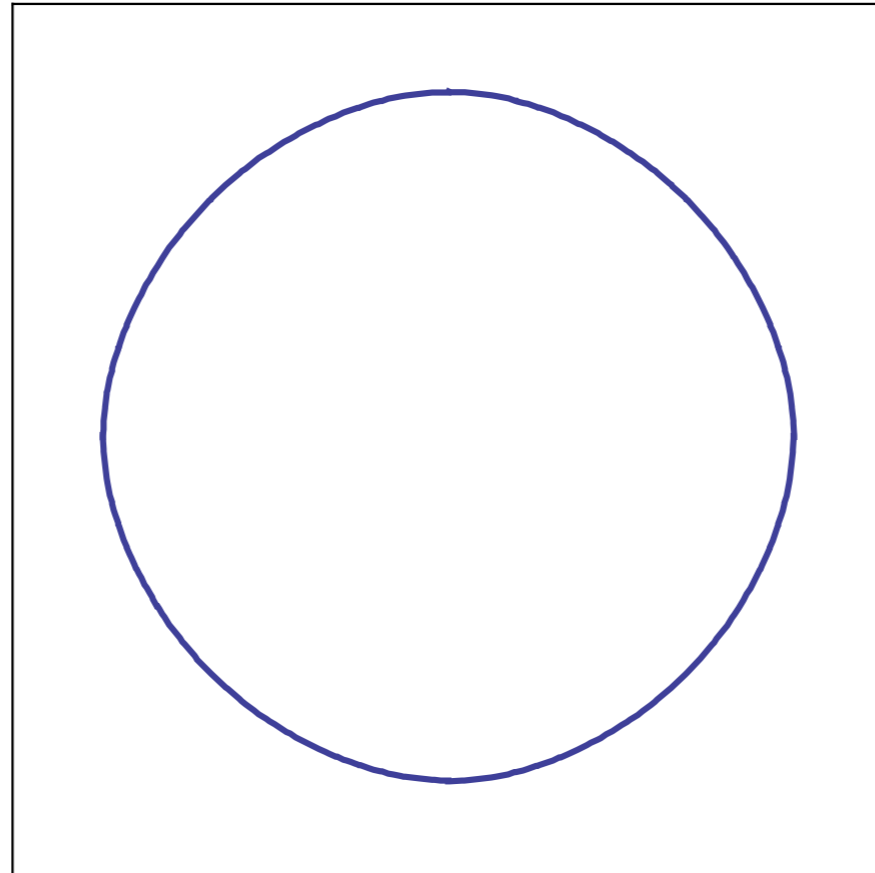
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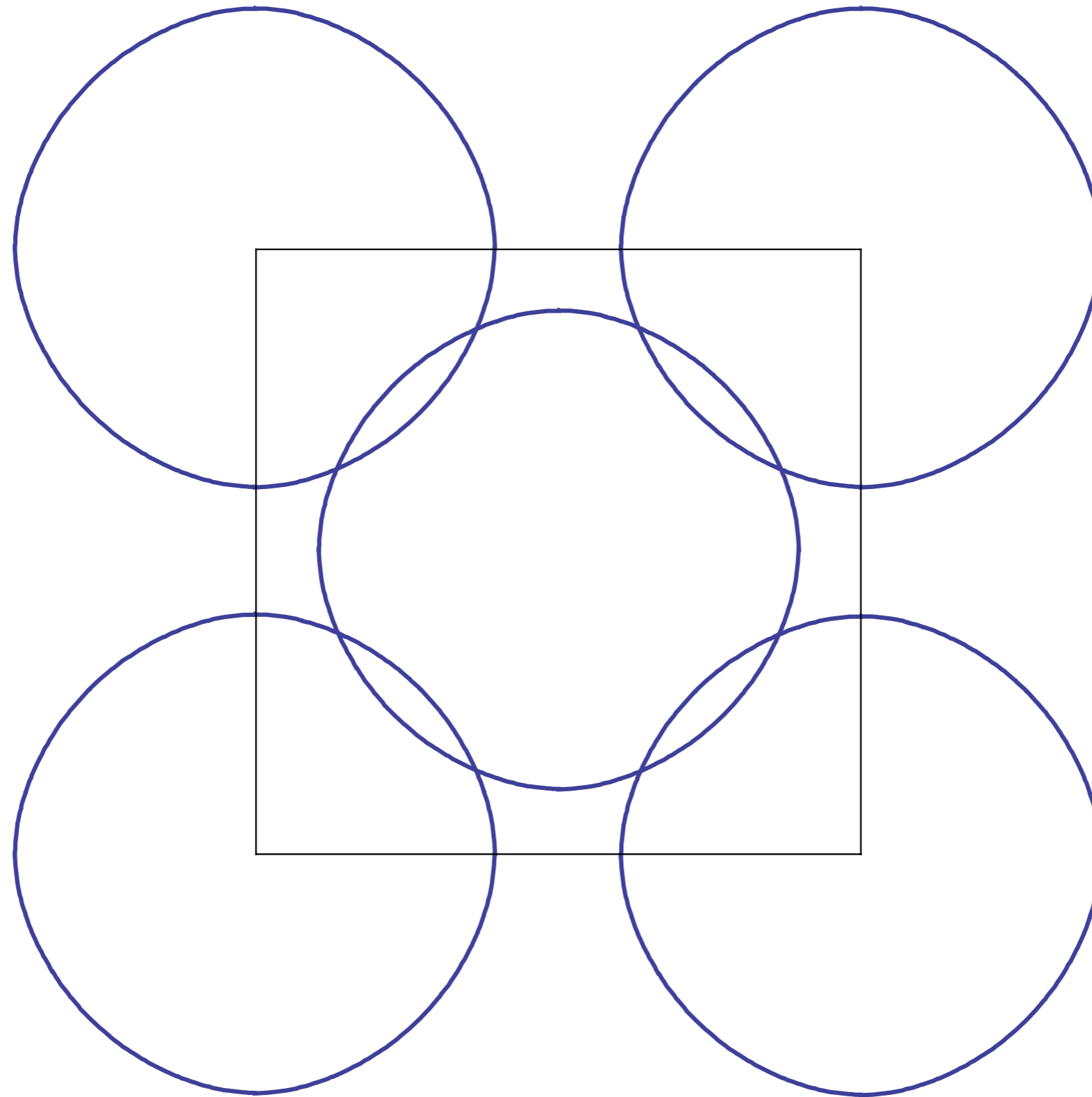
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

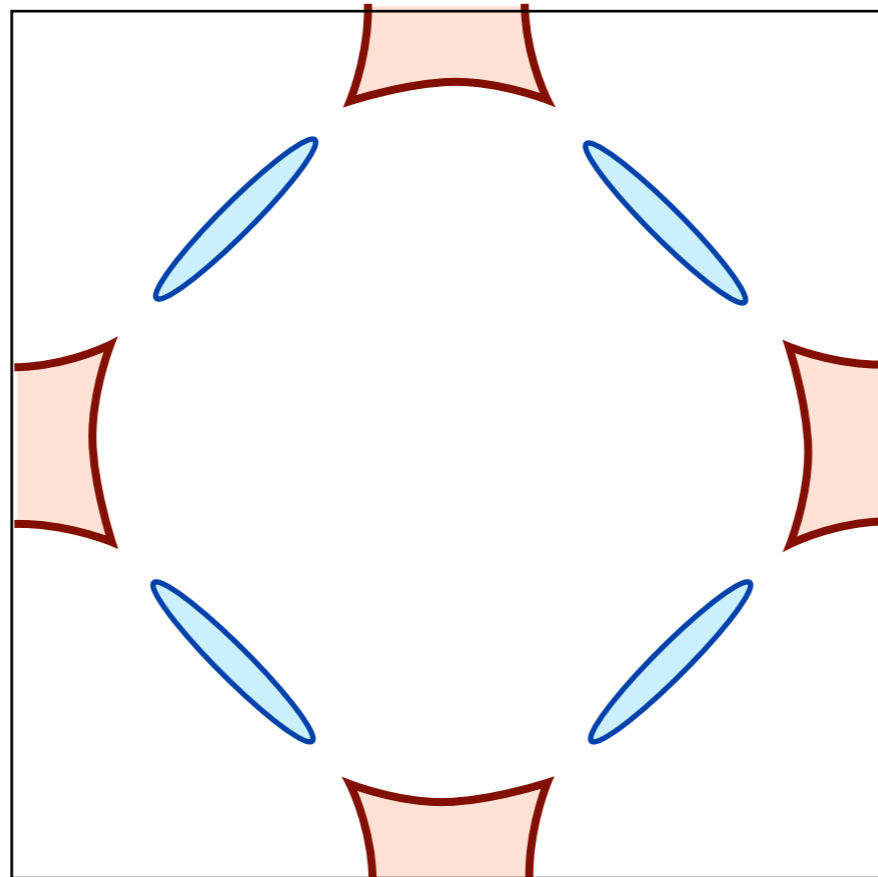
where \mathbf{K} is the ordering wavevector.



Metal with “large” Fermi surface

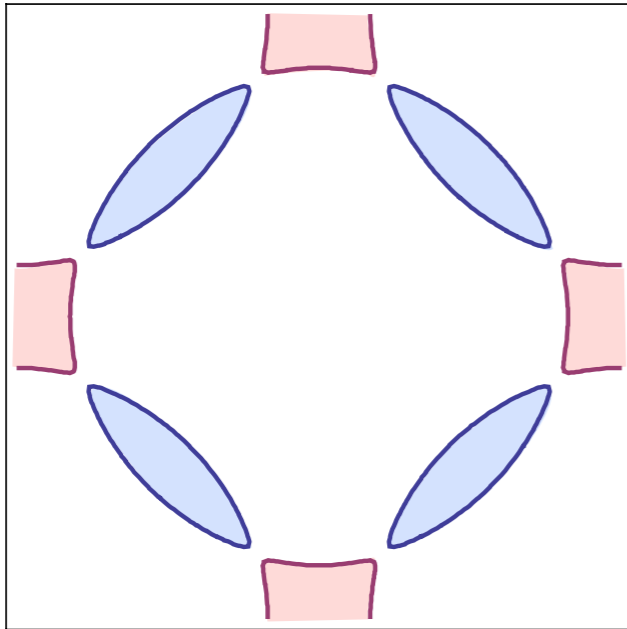


Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



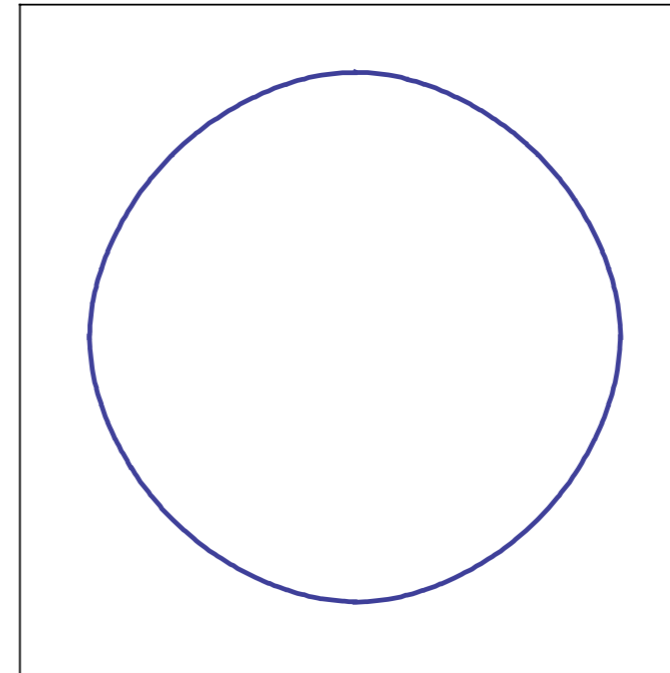
Electron and hole pockets in
antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

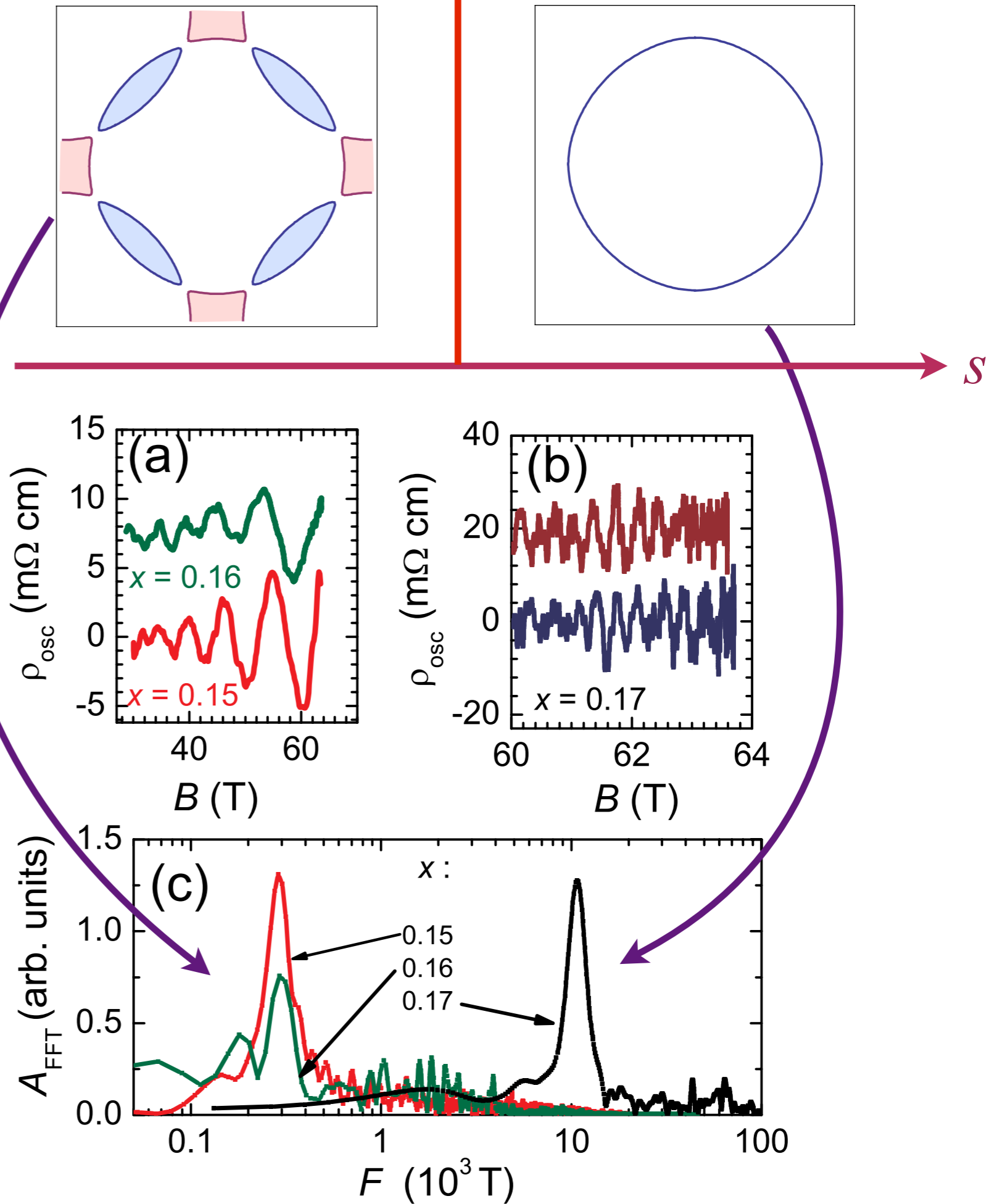
← Increasing interaction

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

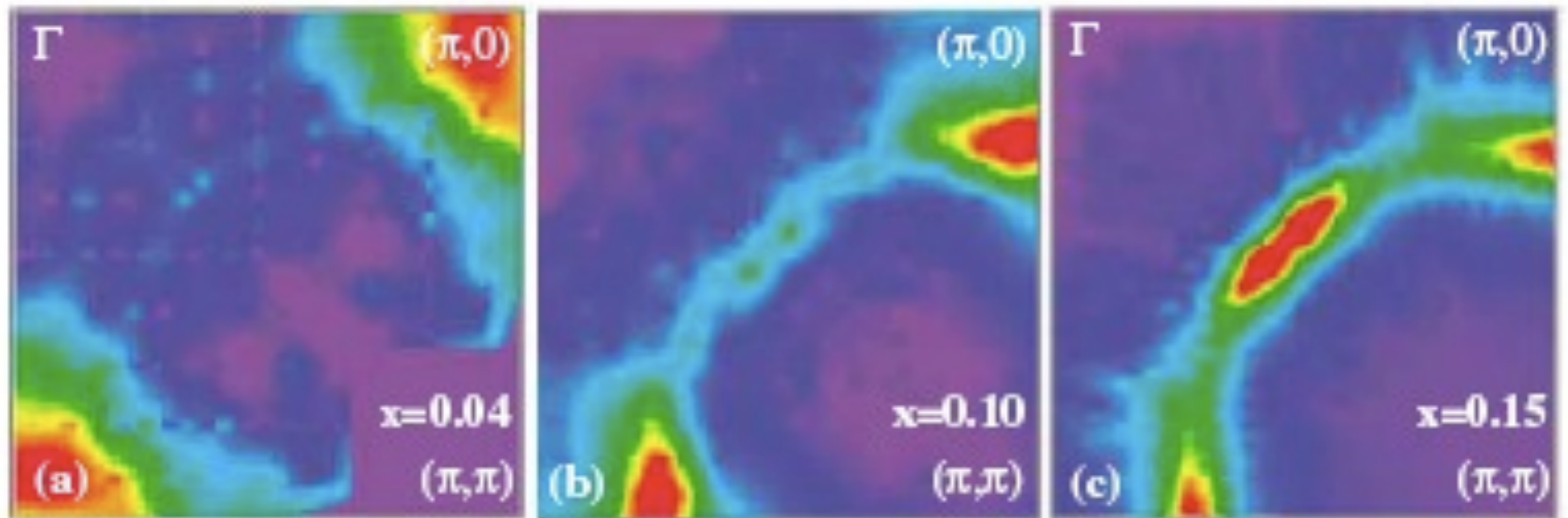
Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

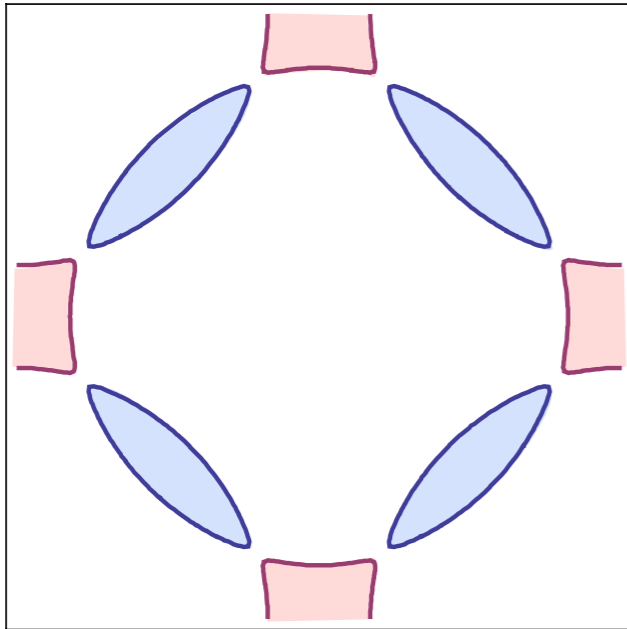


Photoemission in $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$



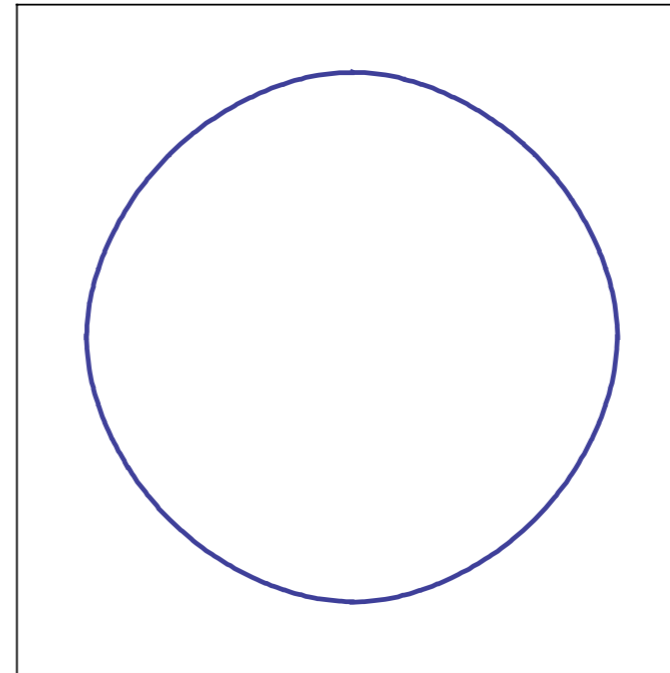
N. P. Armitage *et al.*, Phys. Rev. Lett. **88**, 257001 (2002).

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
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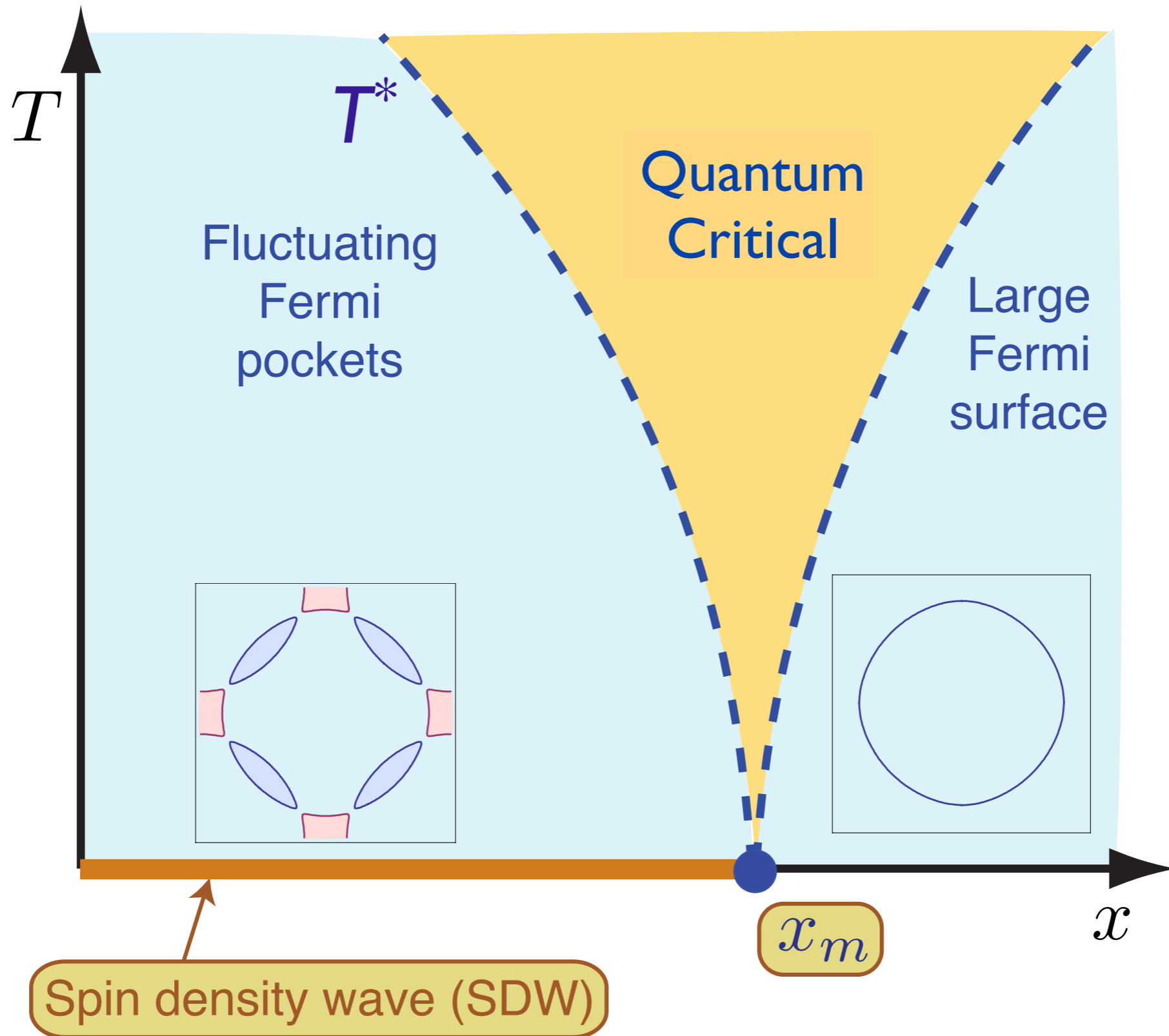
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

S

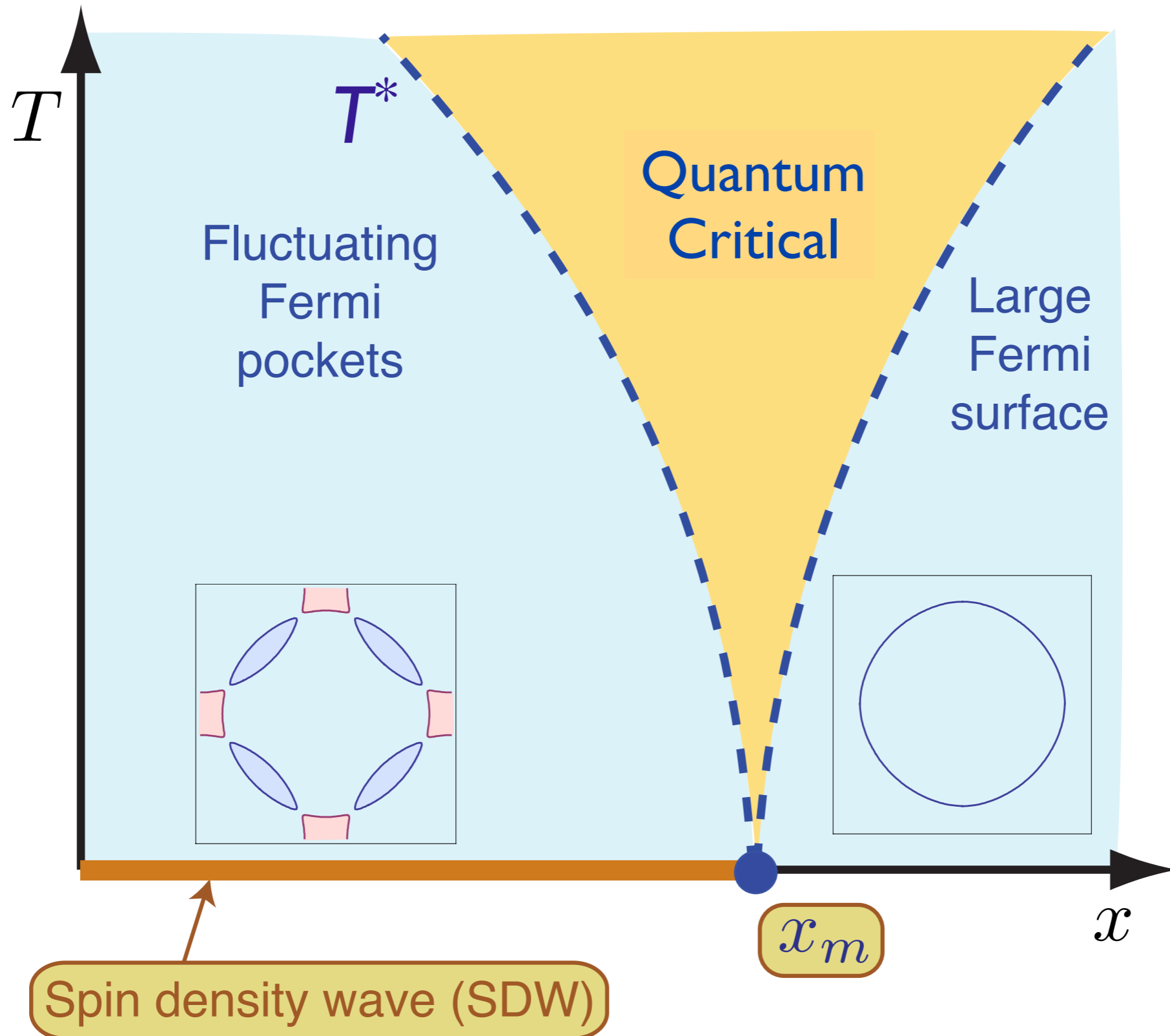
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Theory of quantum criticality in the cuprates



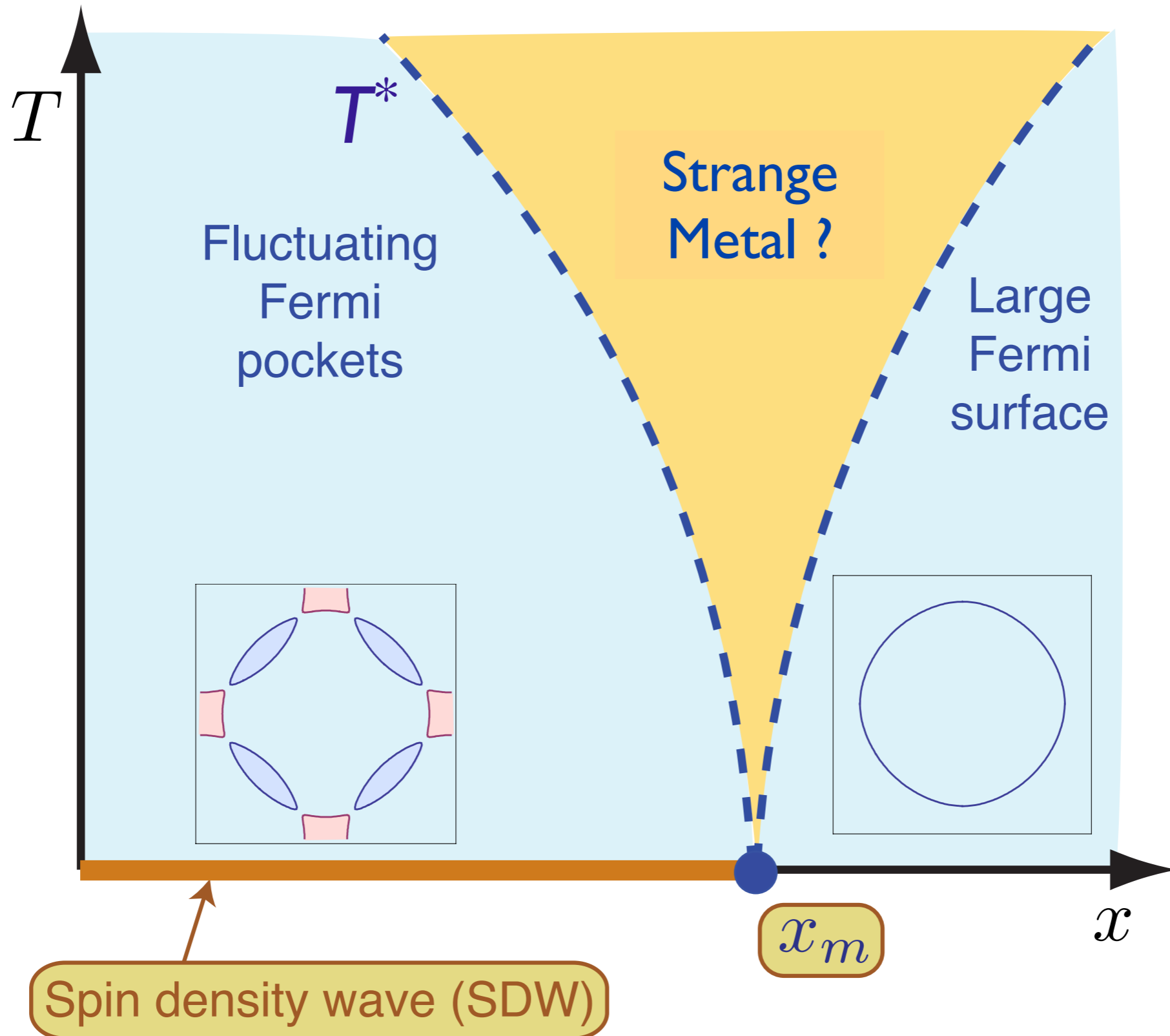
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Theory of quantum criticality in the cuprates



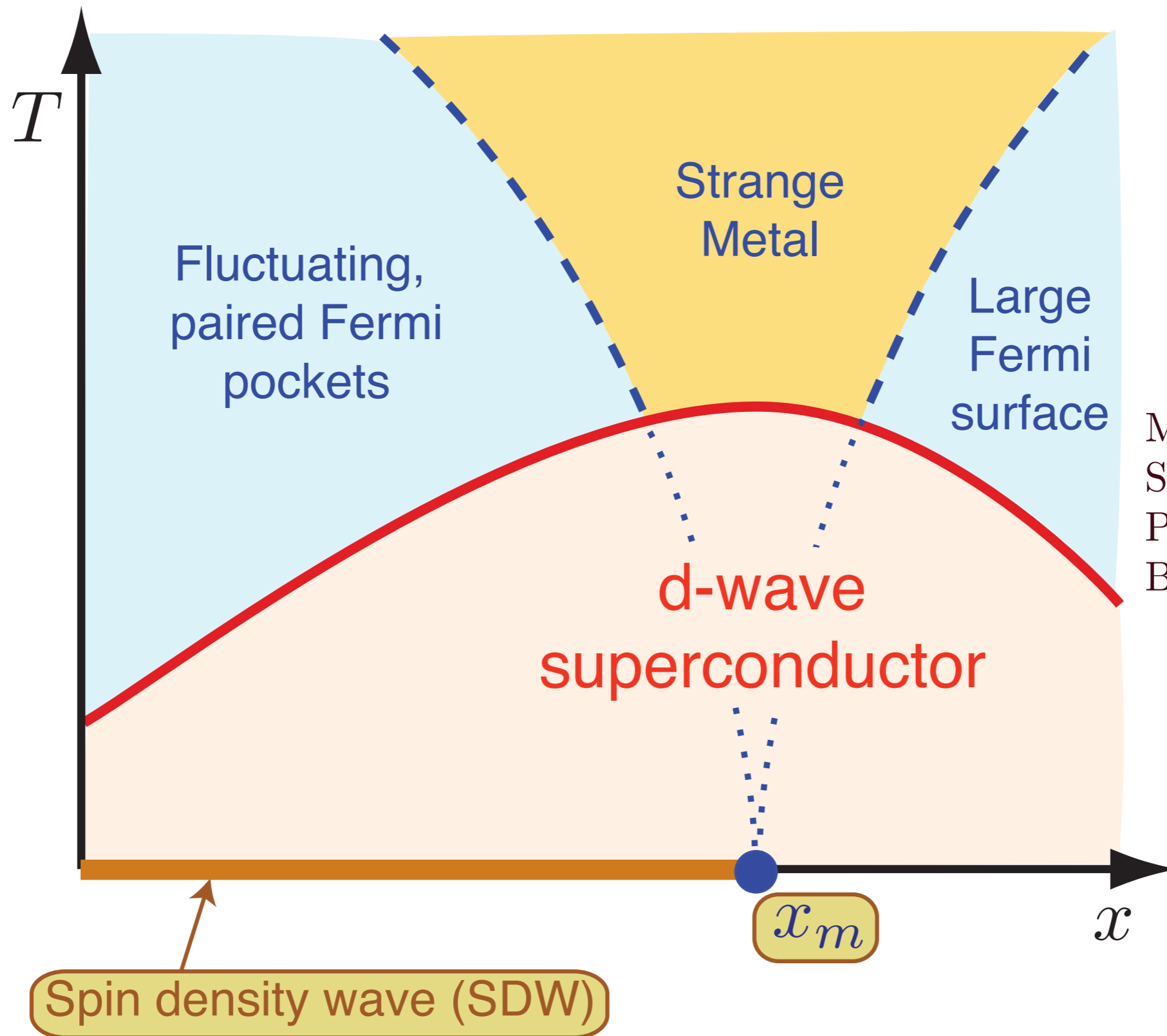
Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

Theory of quantum criticality in the cuprates



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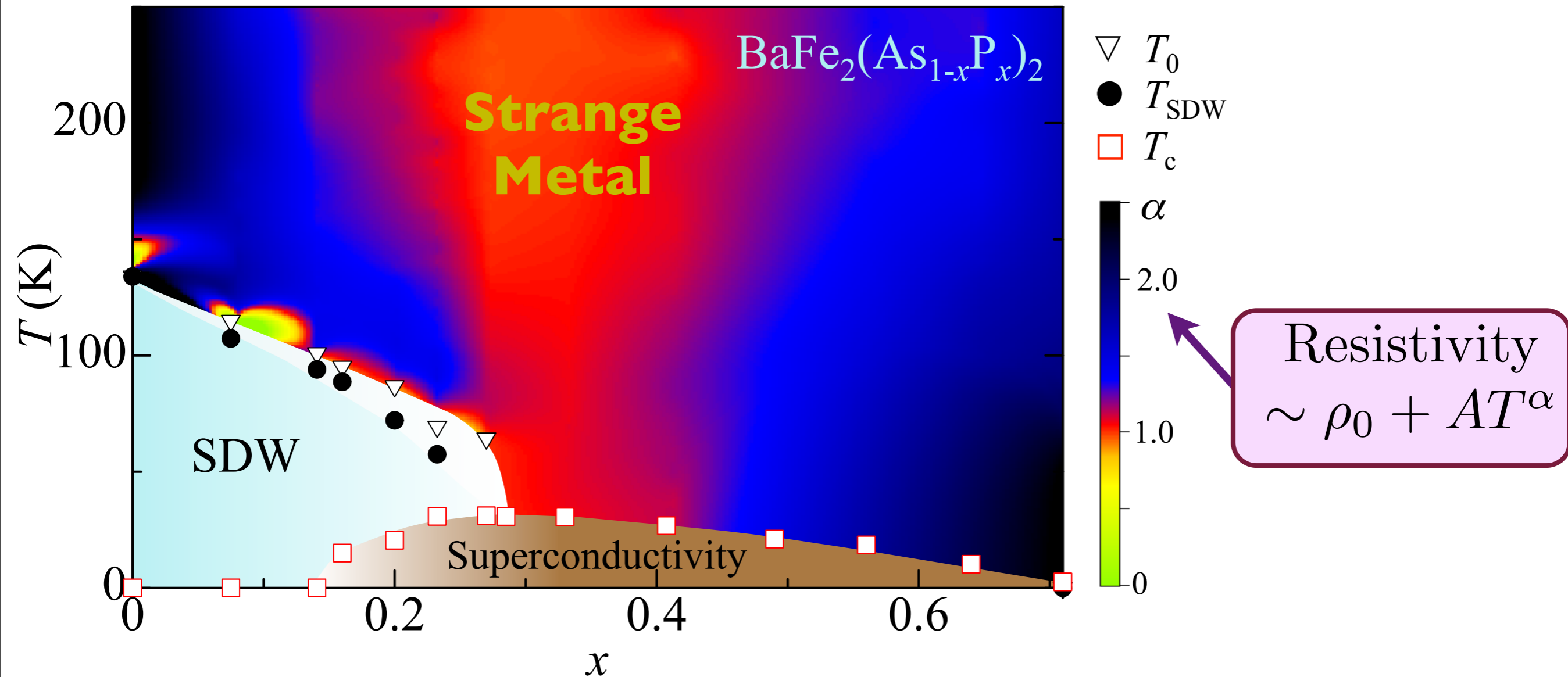
Theory of quantum criticality in the cuprates



M. A. Metlitski and
S. Sachdev,
Physical Review
B **82**, 075128 (2010)

SDW quantum critical point is unstable to d -wave superconductivity
This instability is stronger than that in the BCS theory

Temperature-doping phase diagram of the iron pnictides:

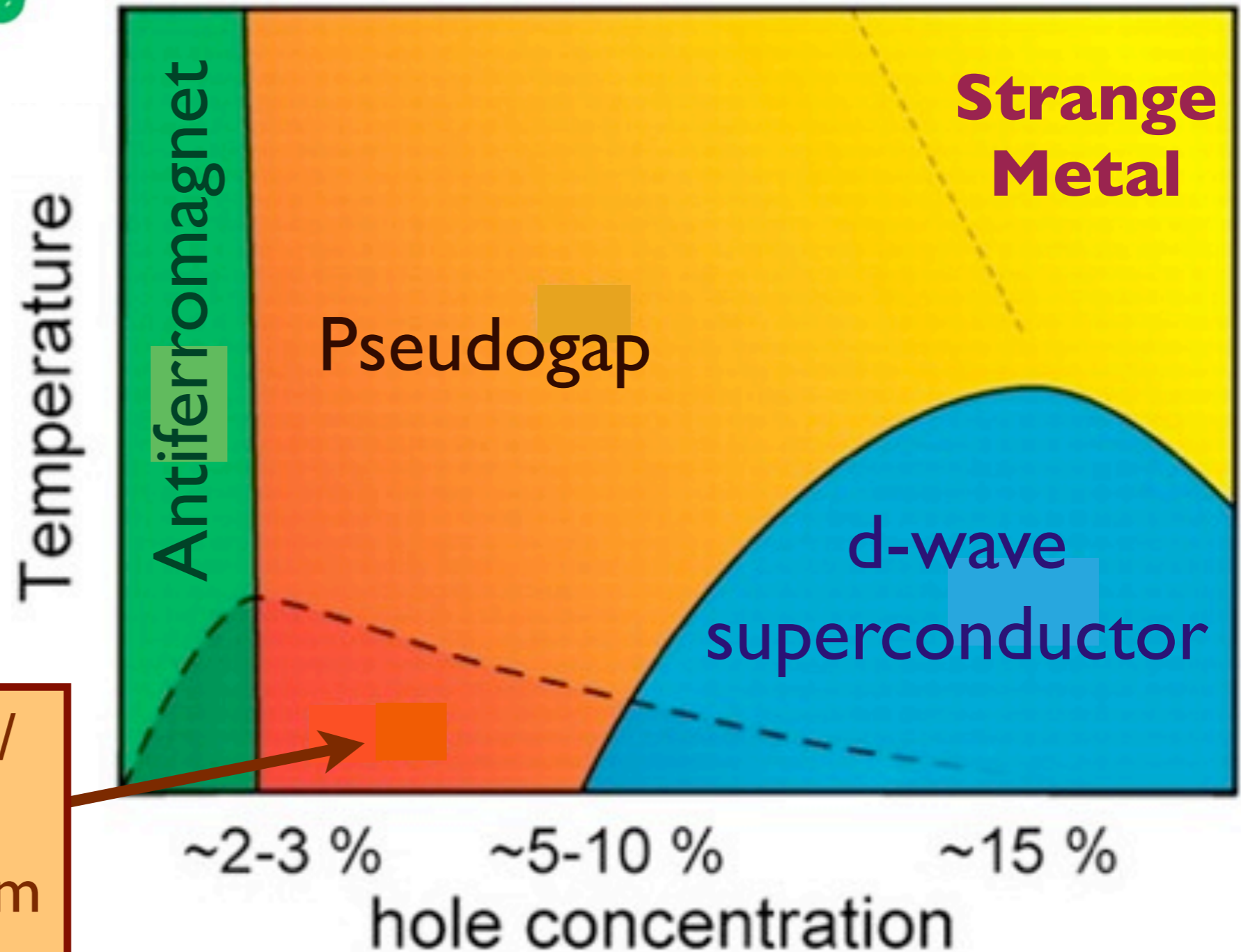
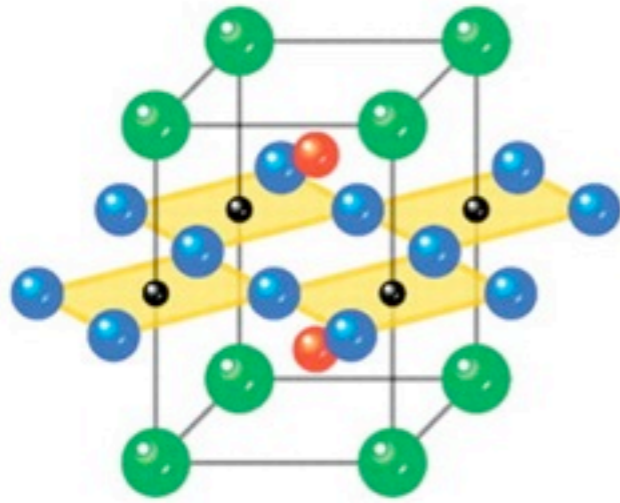


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The cuprate superconductors

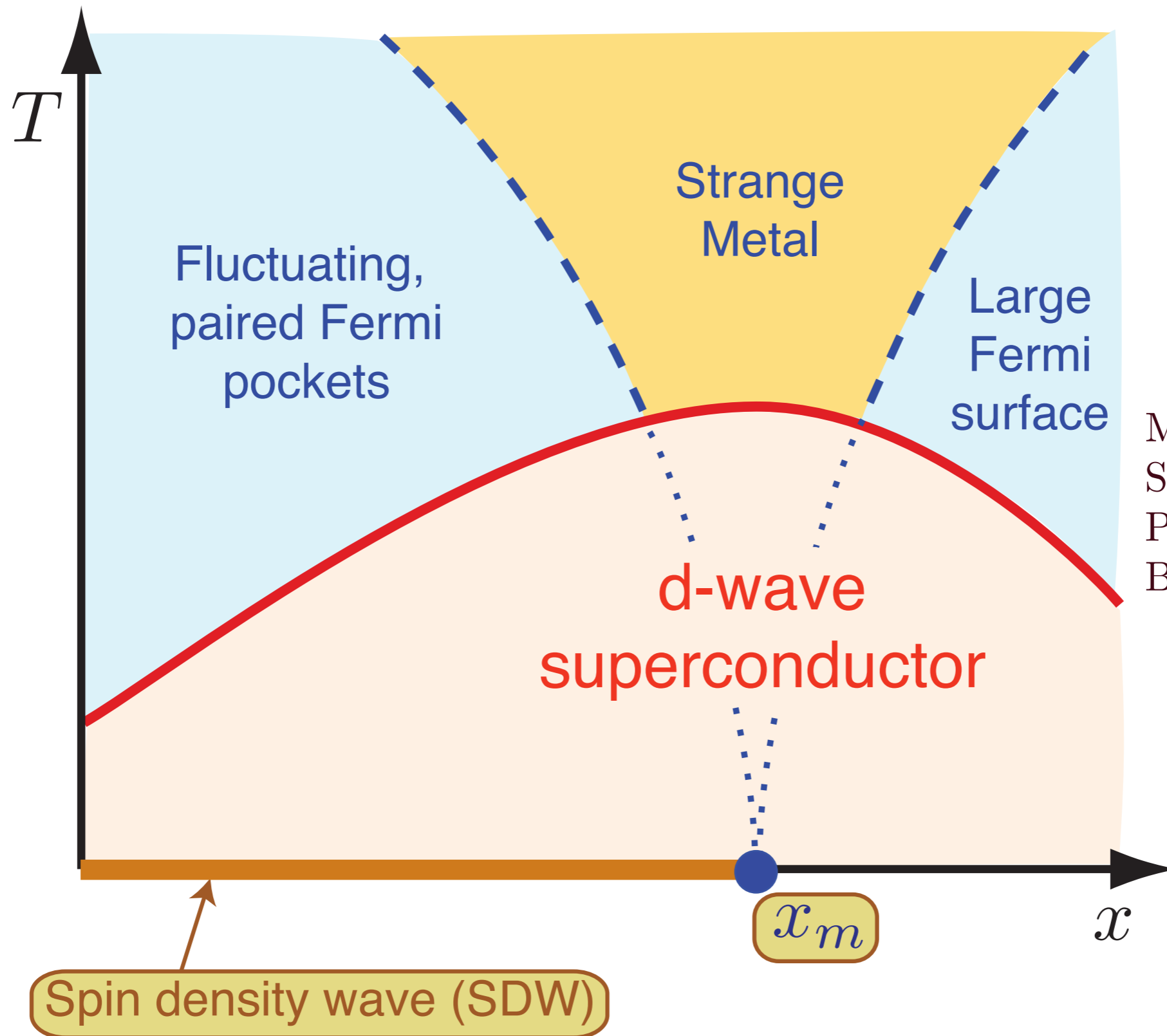
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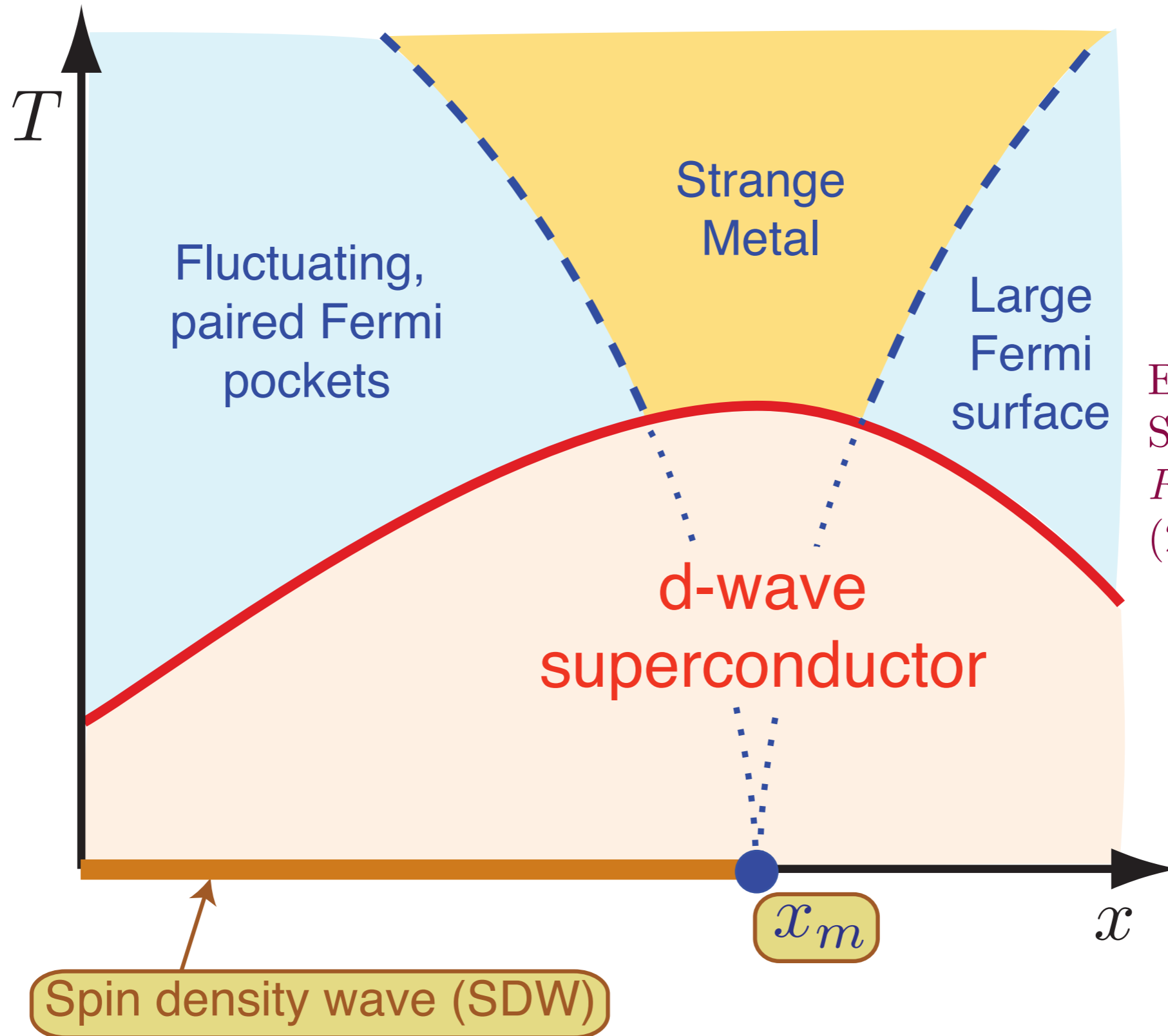
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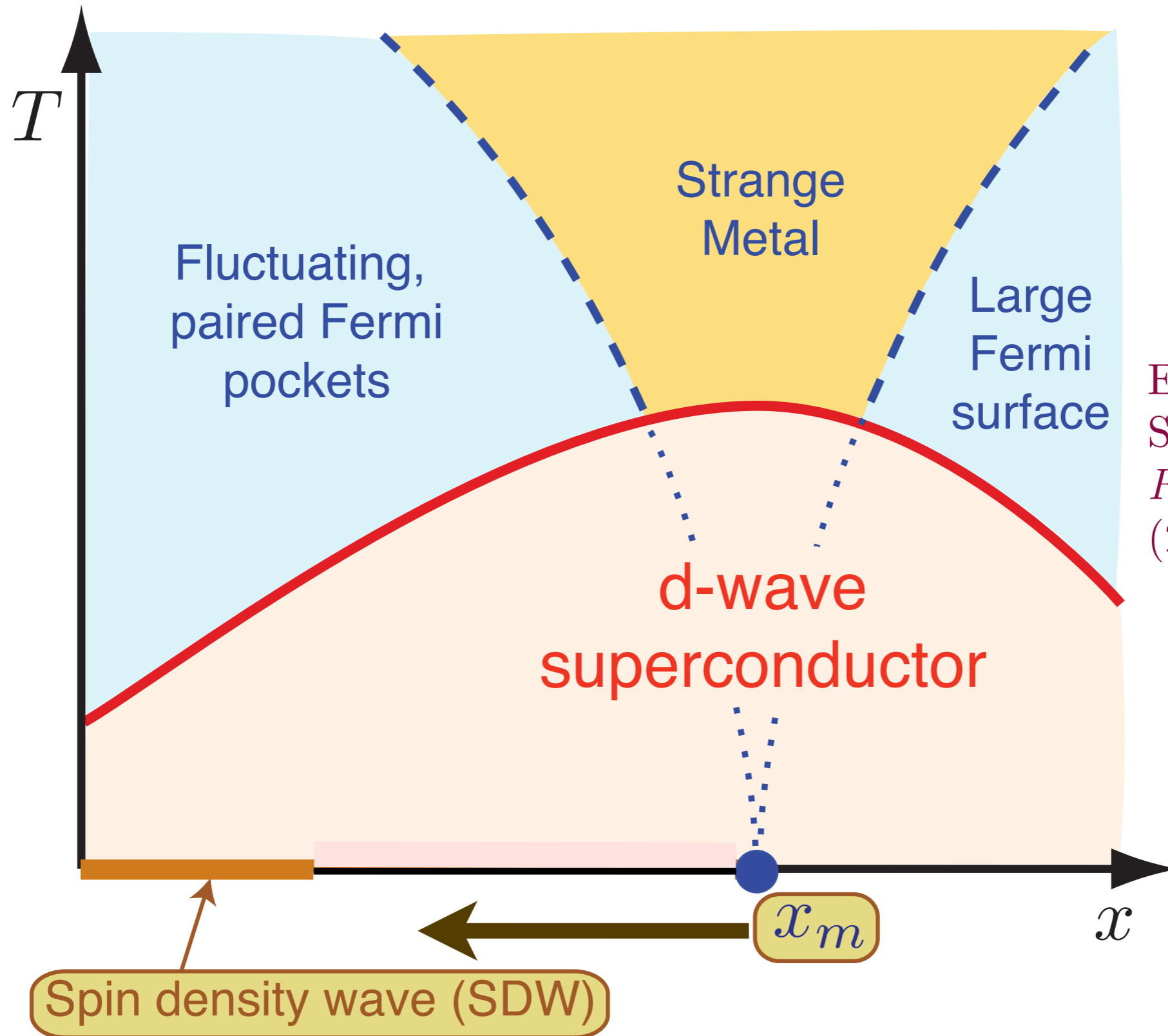
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

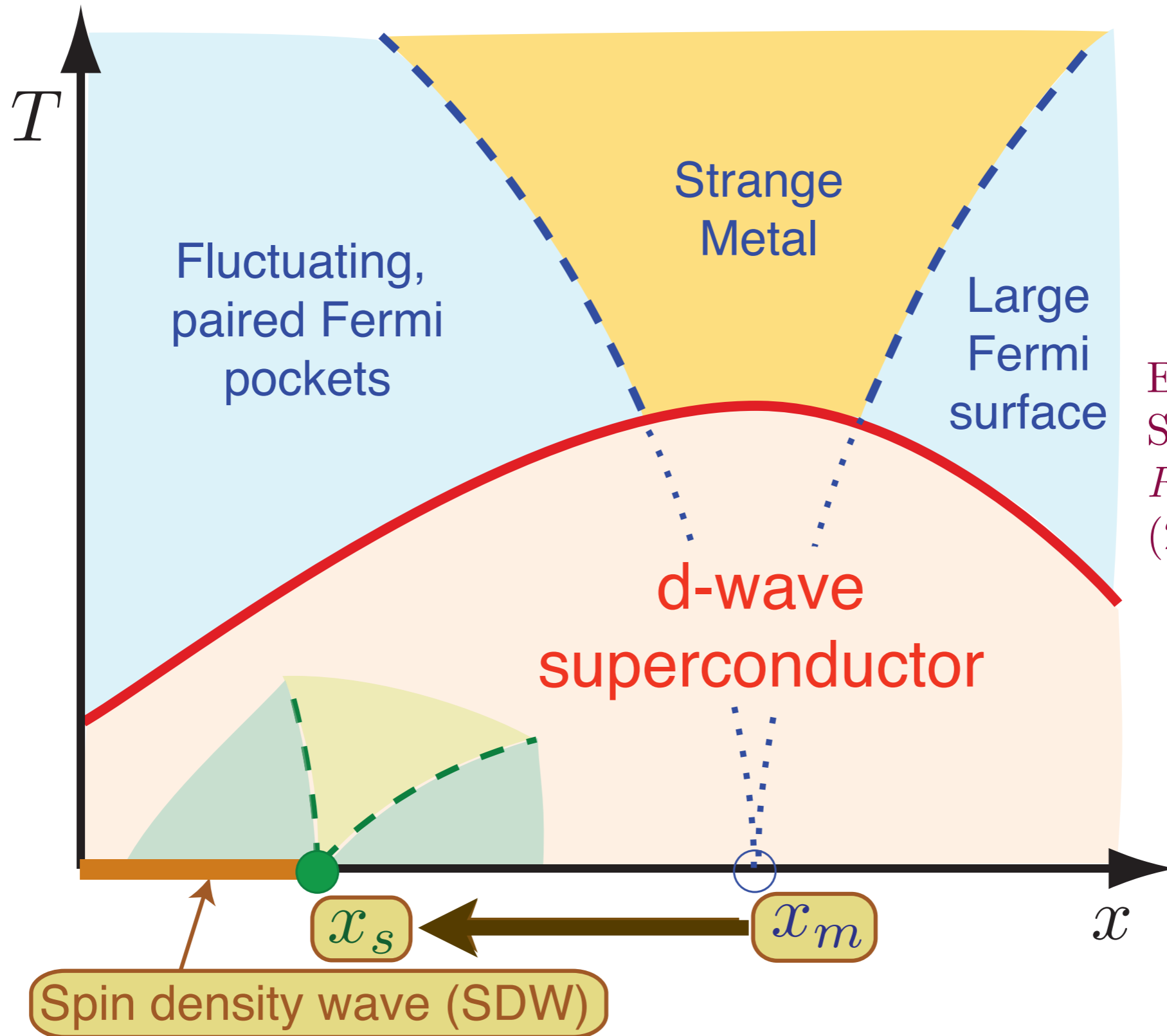
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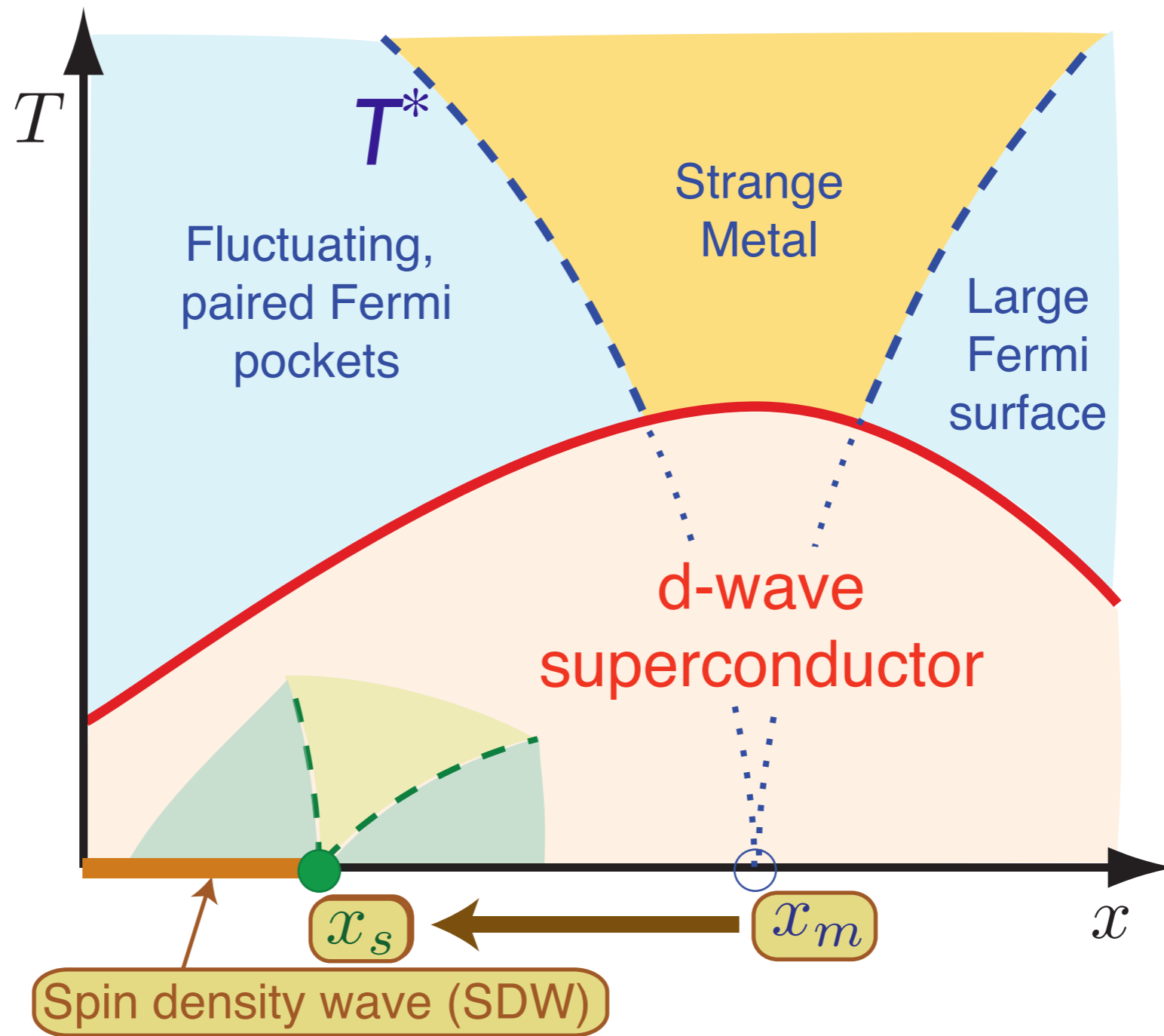
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Theory of quantum criticality in the cuprates

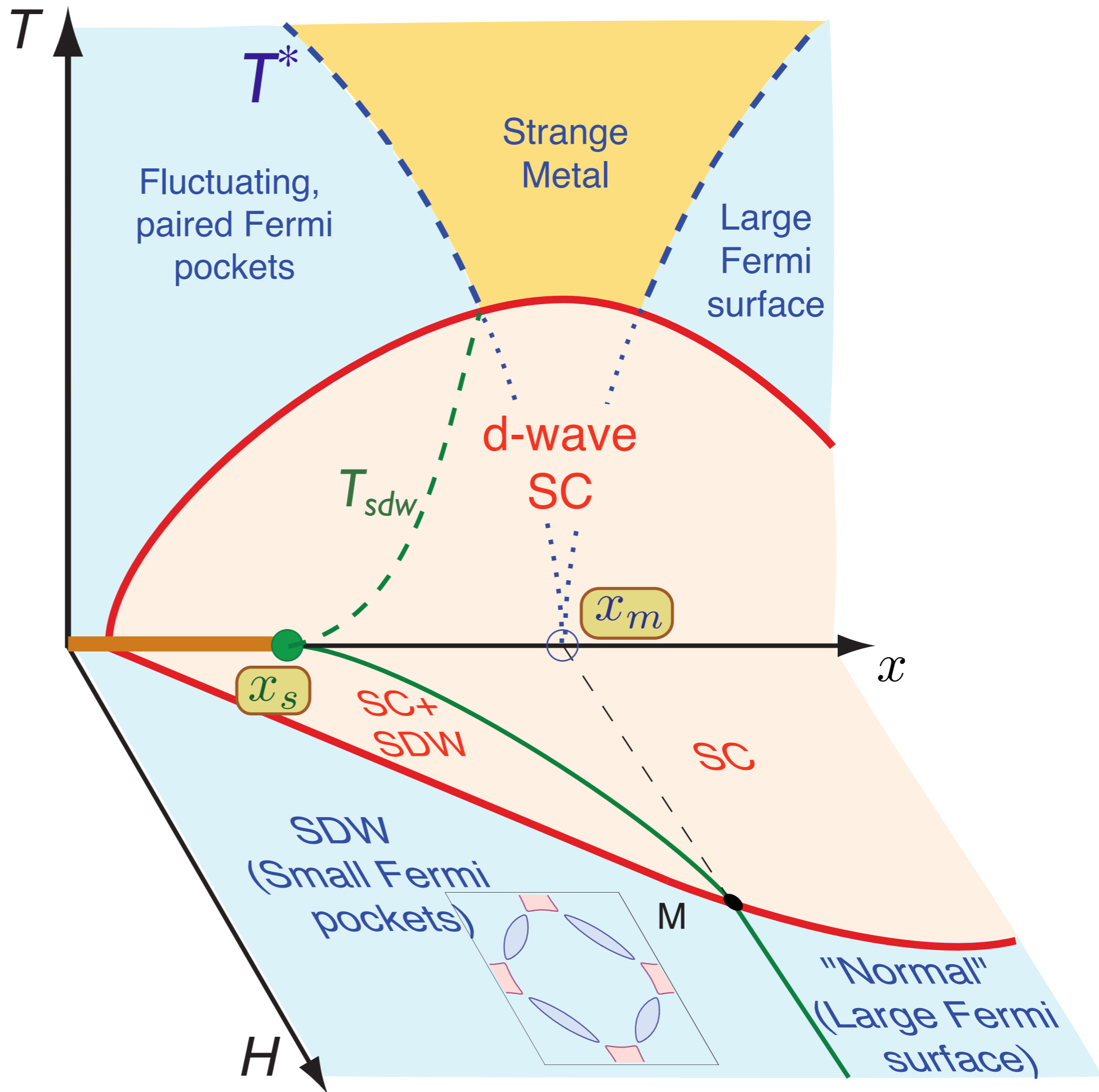


E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

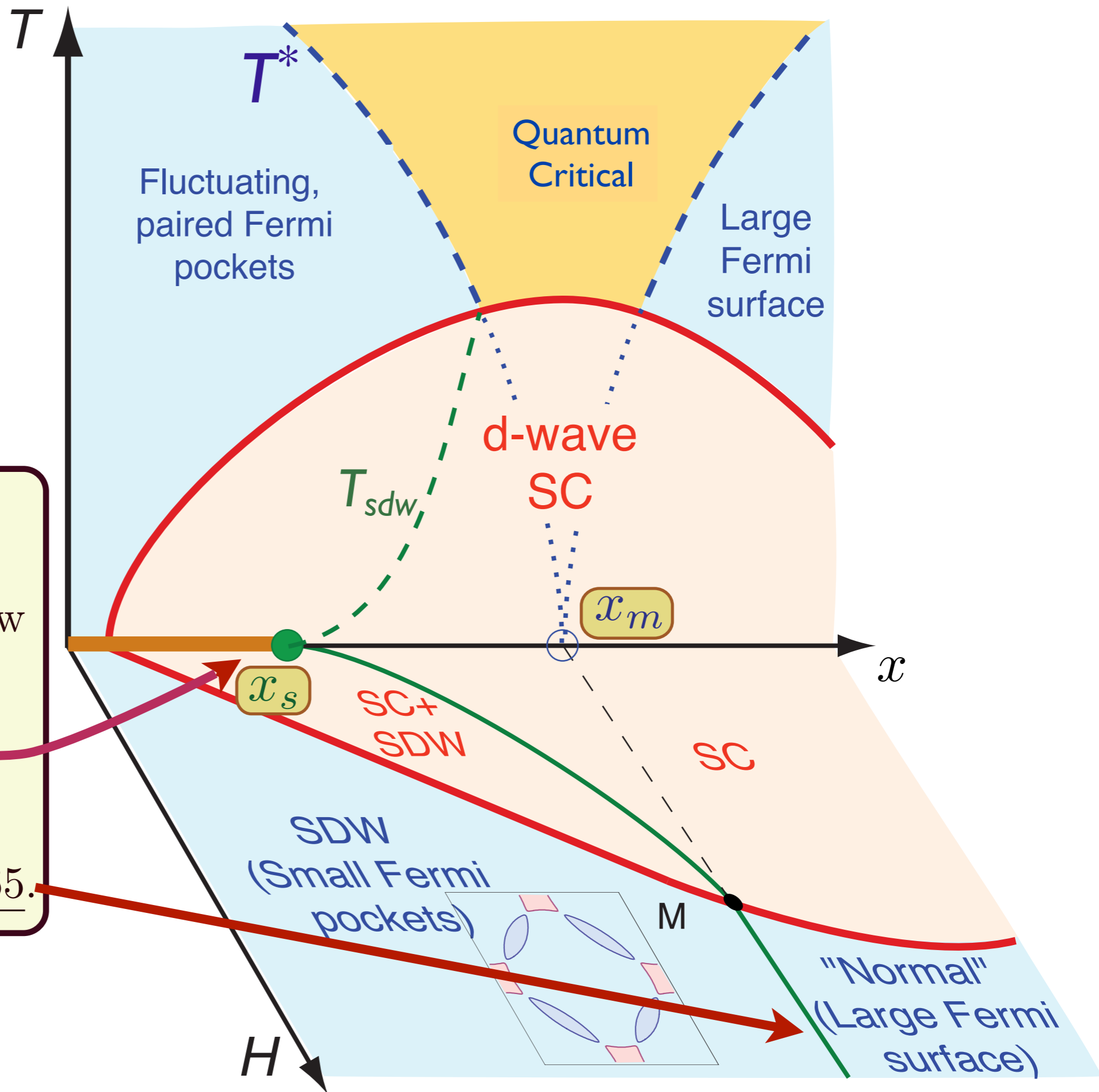


E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).



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and Y. Zhang, *Phys.
Rev. Lett.* **87**,
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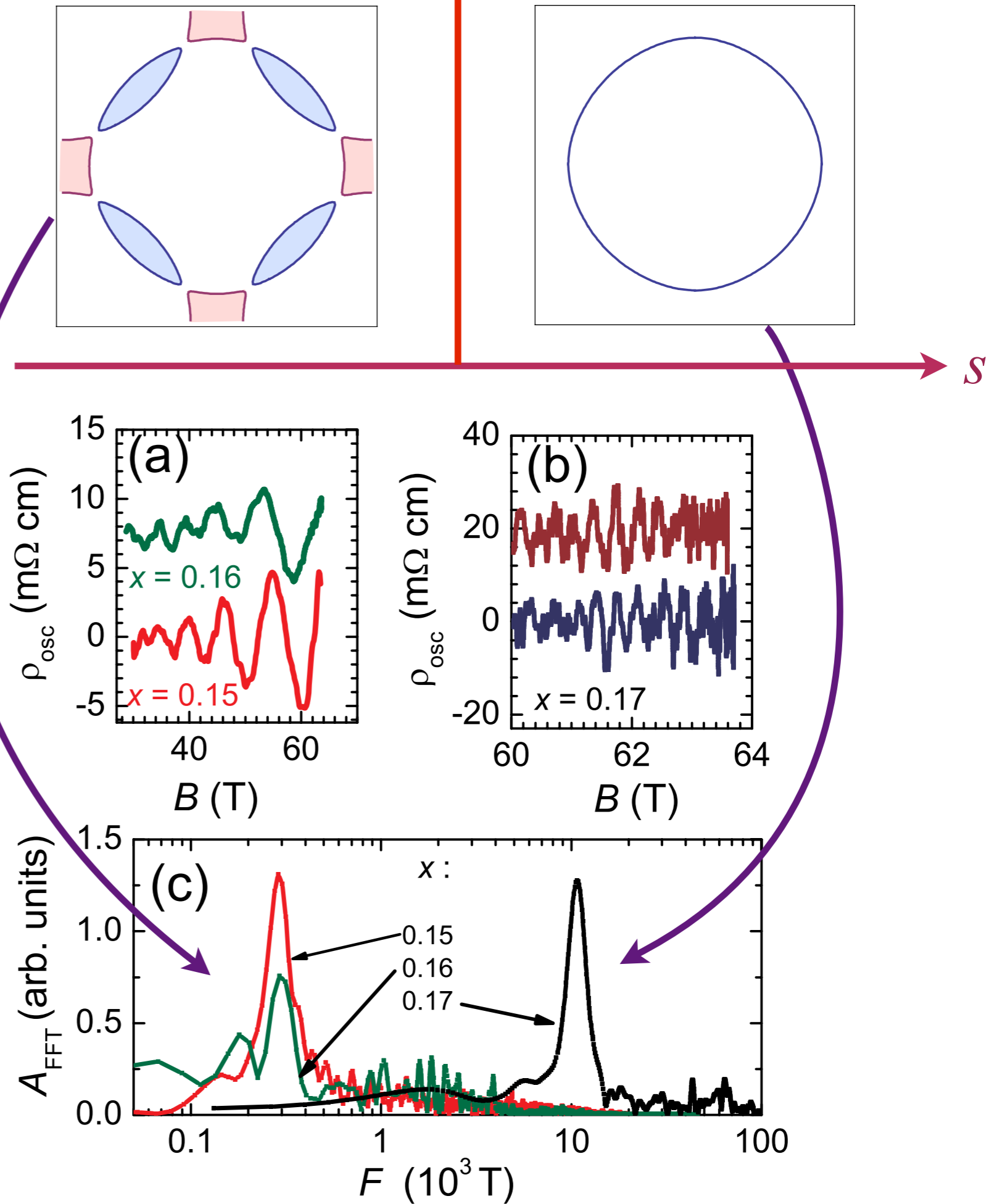
Neutron scatter-
ing experiments on
 $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show
that at low fields
 $x_s = 0.14$, while
quantum oscilla-
tions at high fields
show that $x_m = 0.165$.

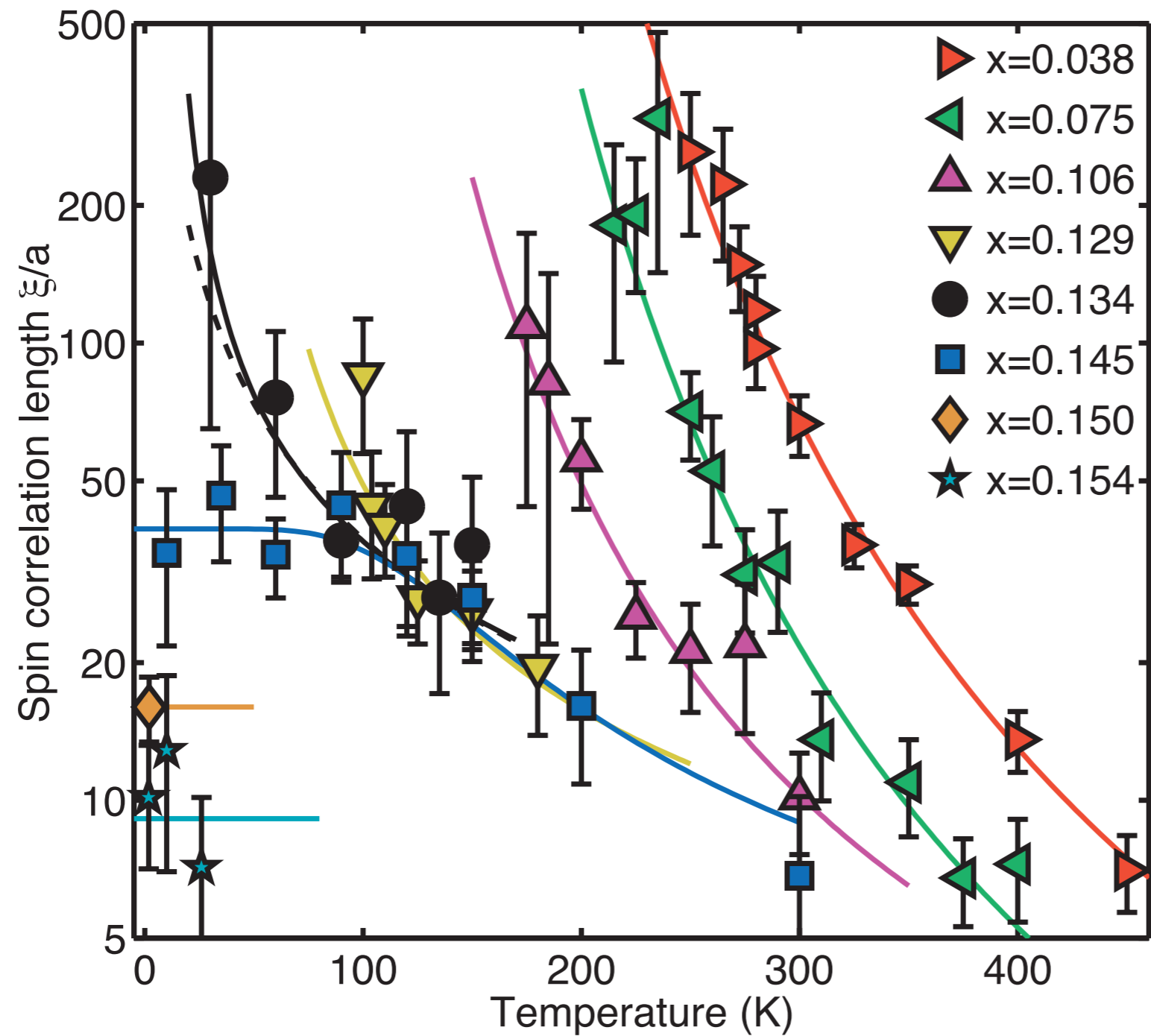


Quantum oscillations



T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

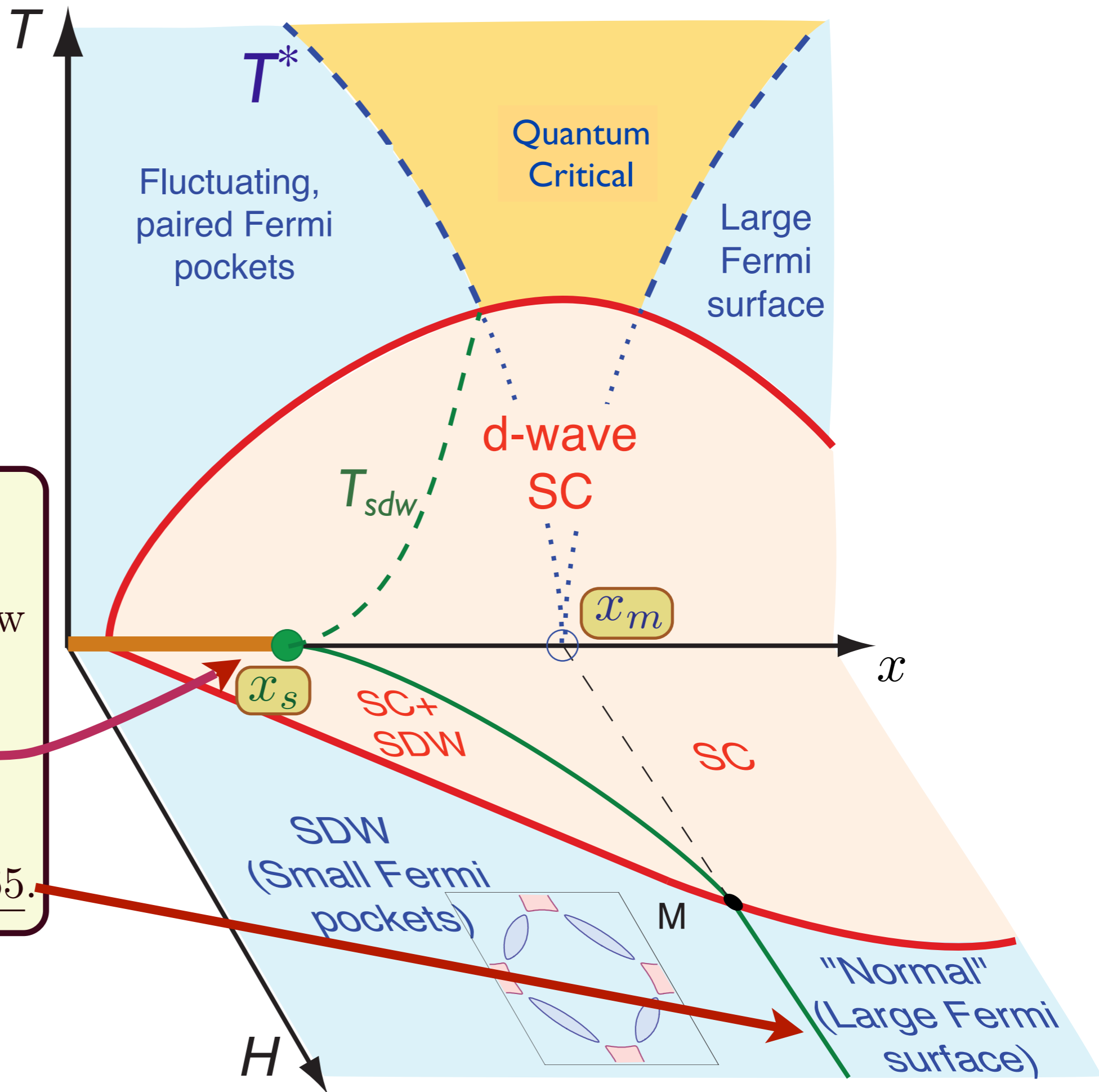




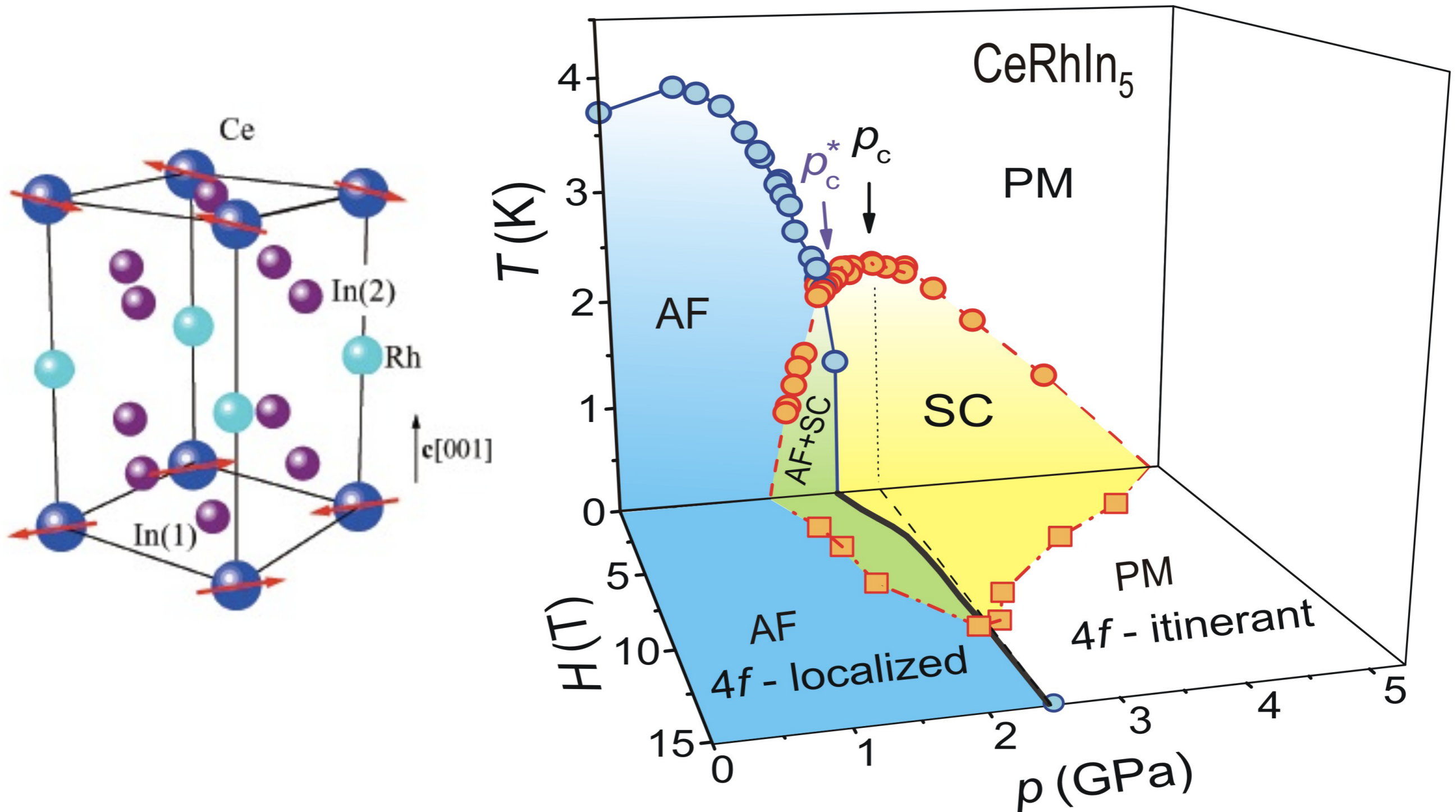
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).

E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).

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 $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ show
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quantum oscilla-
tions at high fields
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Similar phase diagram for CeRhIn₅

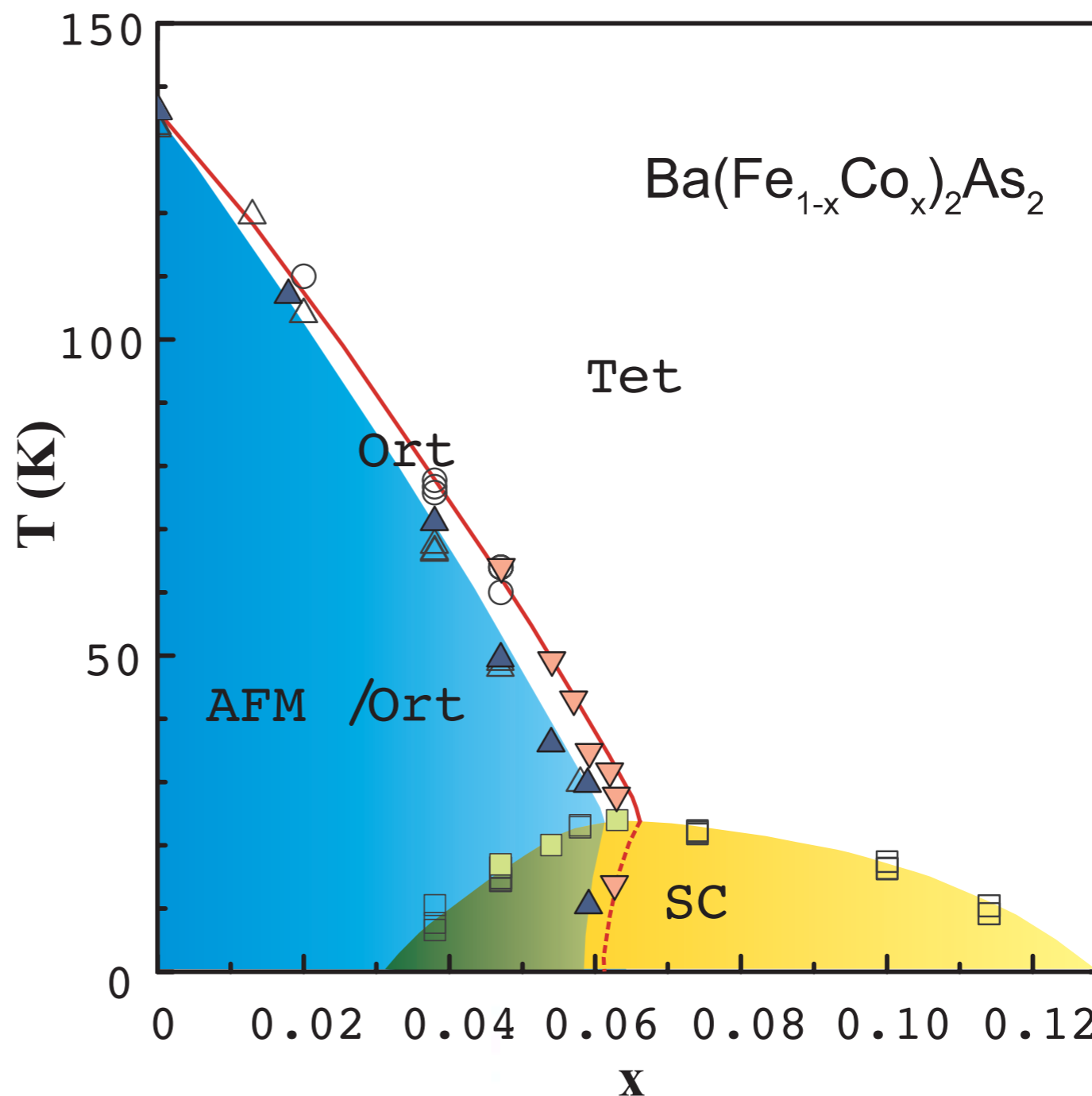
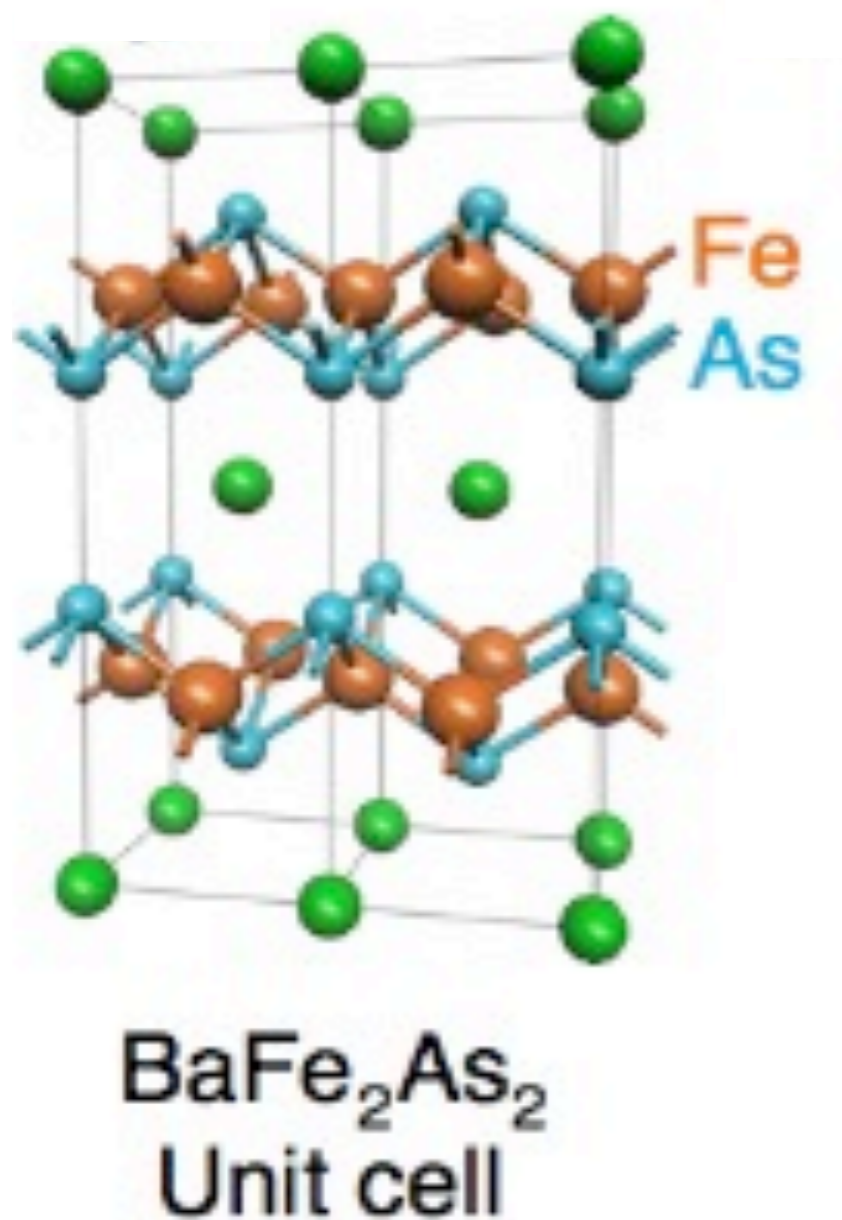


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.

Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

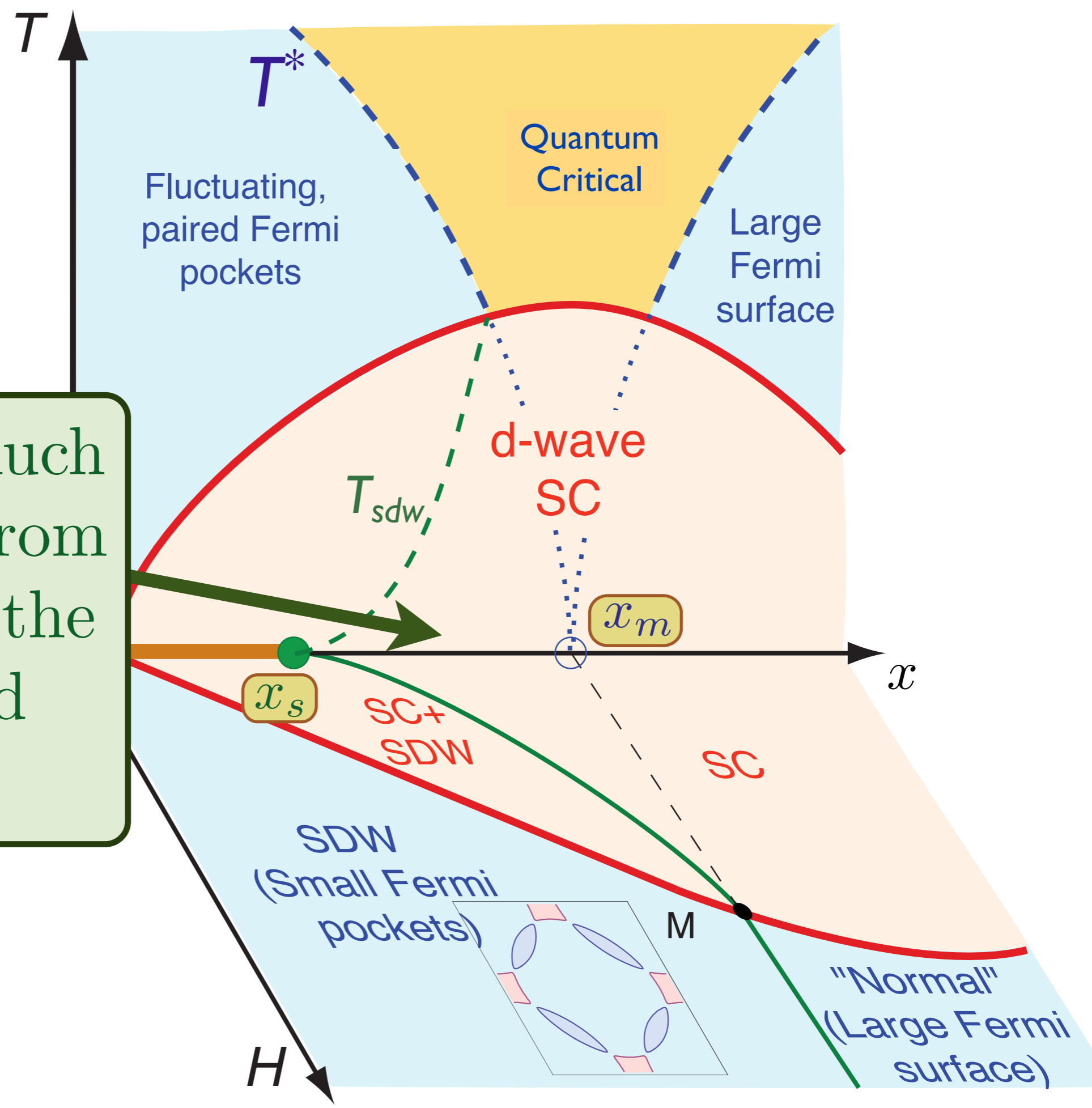
Iron pnictides:

a new class of high temperature superconductors

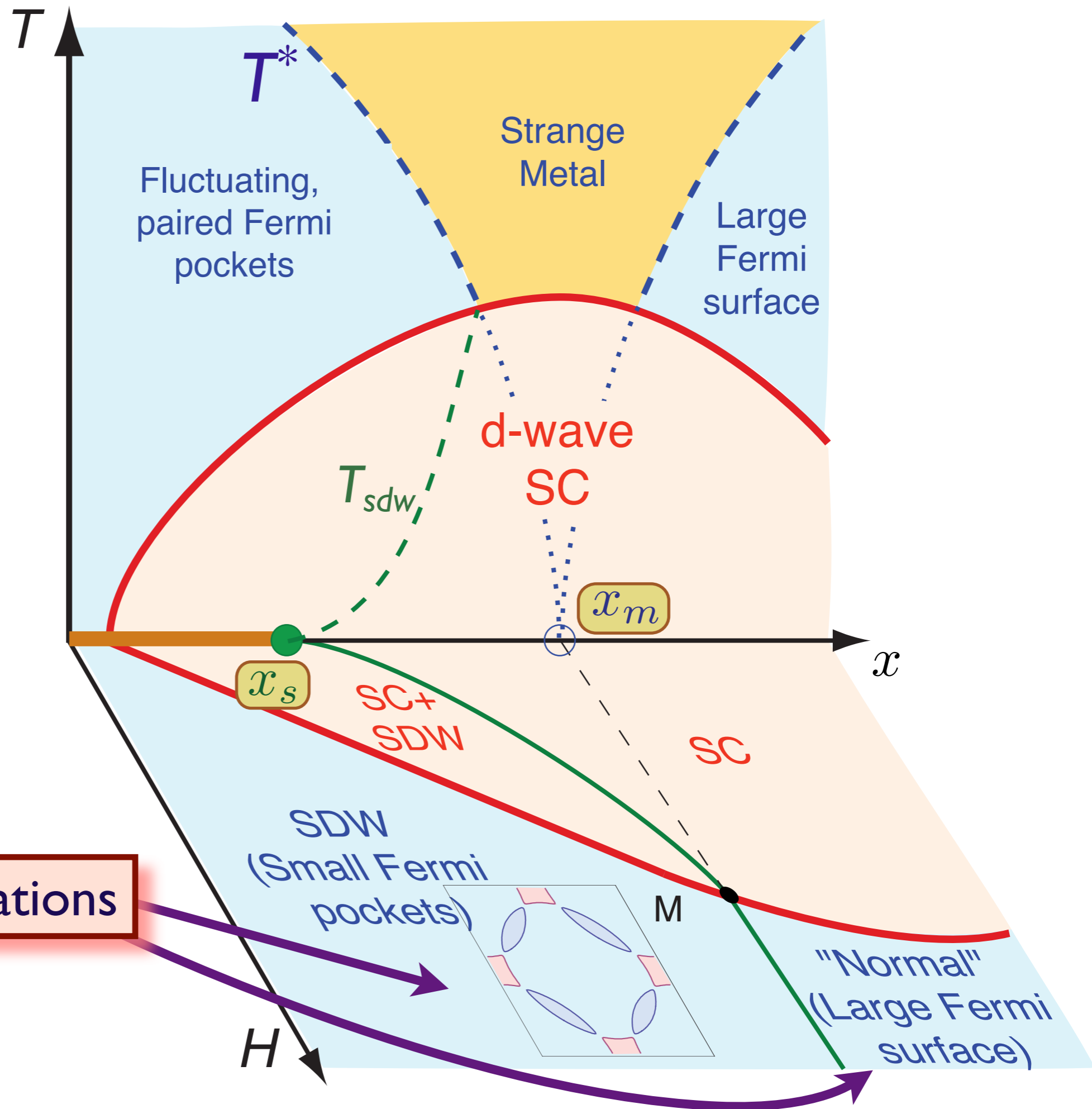


S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni,
S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman,
Physical Review Letters **104**, 057006 (2010).

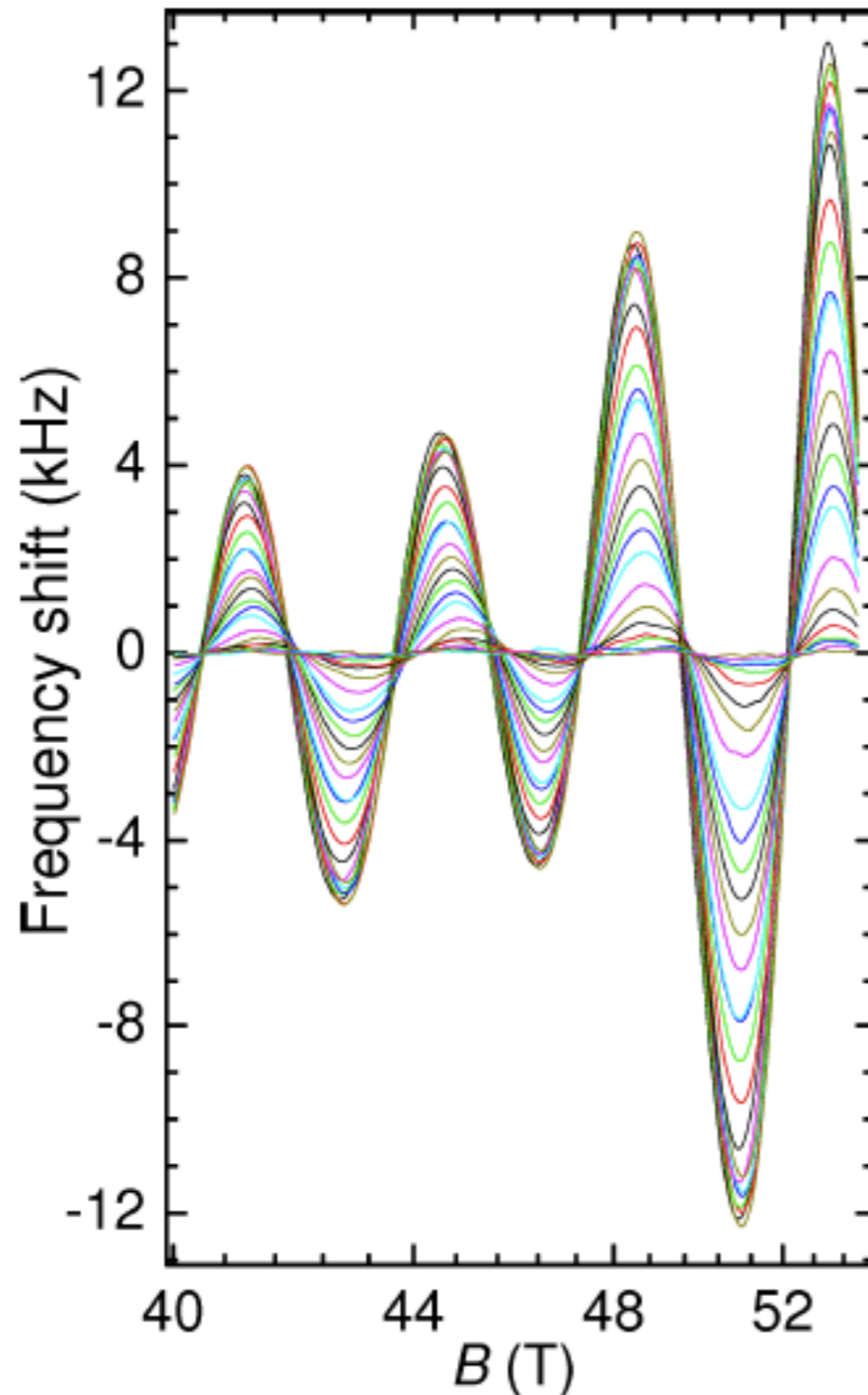
There is a much larger shift from x_m to x_s in the hole-doped cuprates.



E. Demler, S. Sachdev
and Y. Zhang, *Phys.
Rev. Lett.* **87**,
067202 (2001).



Evidence for small Fermi pockets in hole-doped cuprates



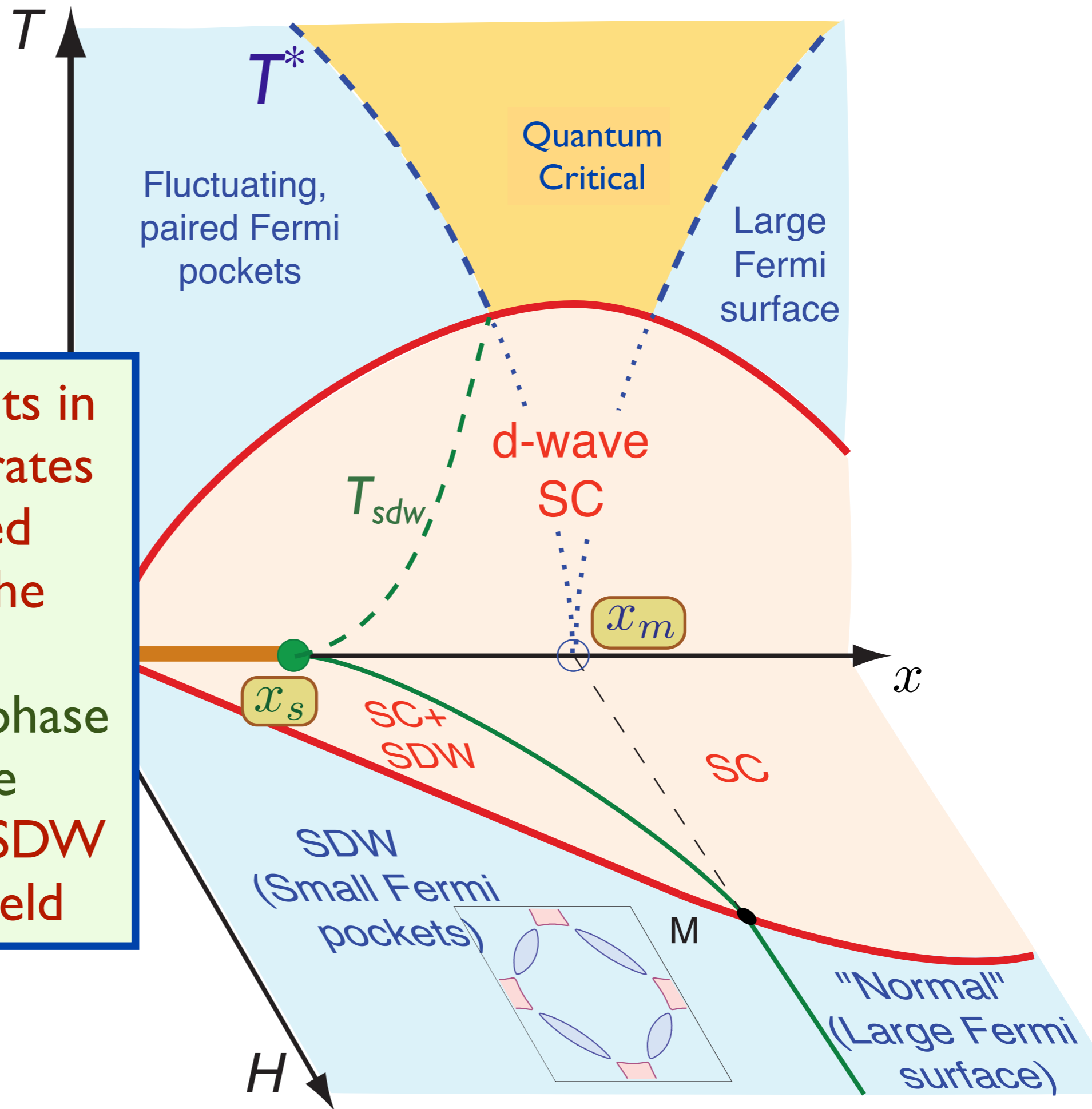
Suchitra E. Sebastian, N. Harrison,
M. M. Altarawneh, Ruixing Liang, D.A. Bonn,
W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

Original observation:
N. Doiron-Leyraud, C. Proust,
D. LeBoeuf, J. Levallois,
J.-B. Bonnemaïson, R. Liang,
D.A. Bonn, W. N. Hardy,
and L. Taillefer,
Nature **447**, 565 (2007)

FIG. 2: Magnetic quantum oscillations measured in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of ~ 50 dB between $T = 1$ and 18 K. The actual T values are provided in Fig. 3.

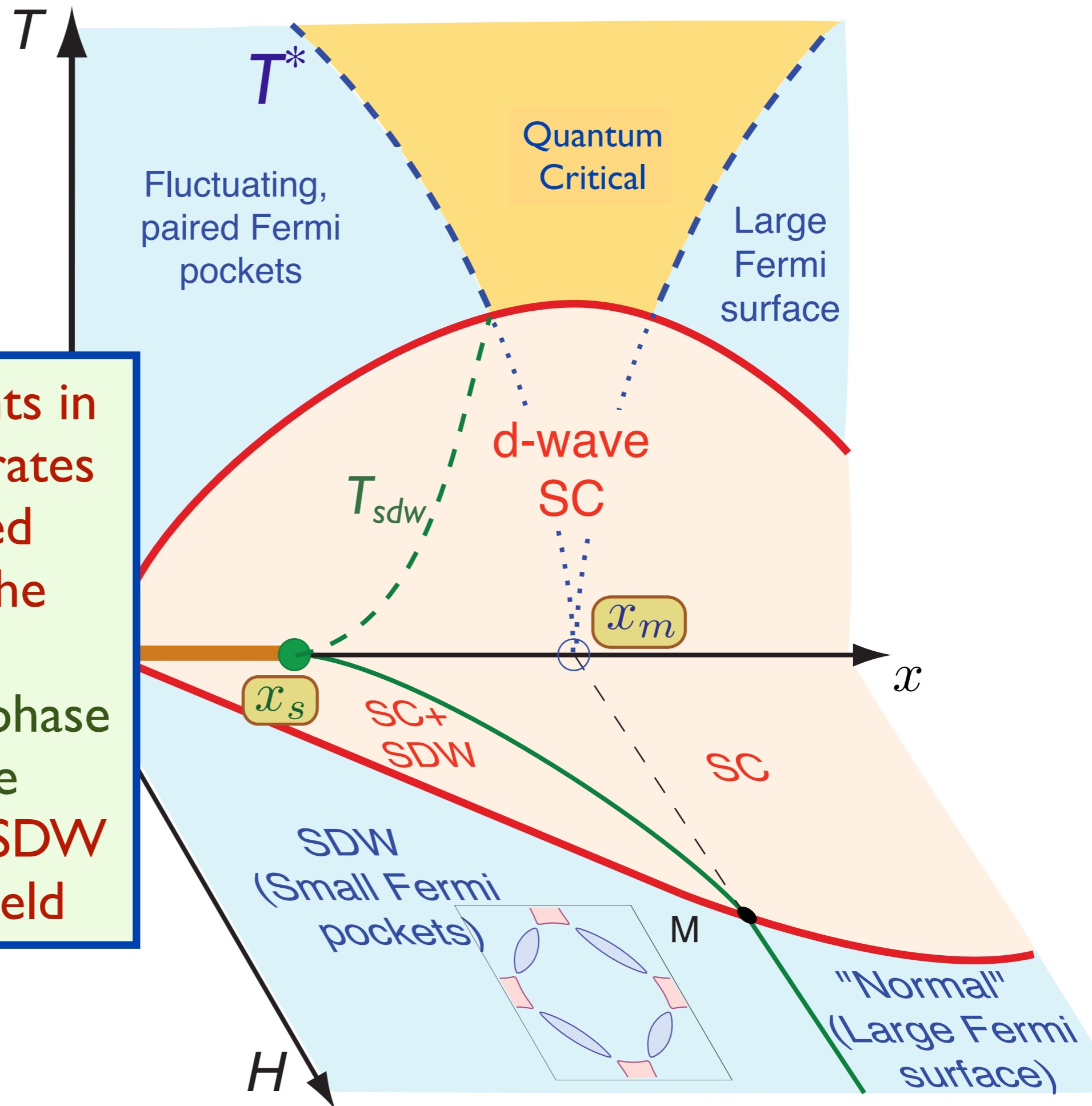
E. Demler, S. Sachdev
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Many experiments in
hole-doped cuprates
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evidence for the
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transition line
from SC to SC+SDW
in a magnetic field



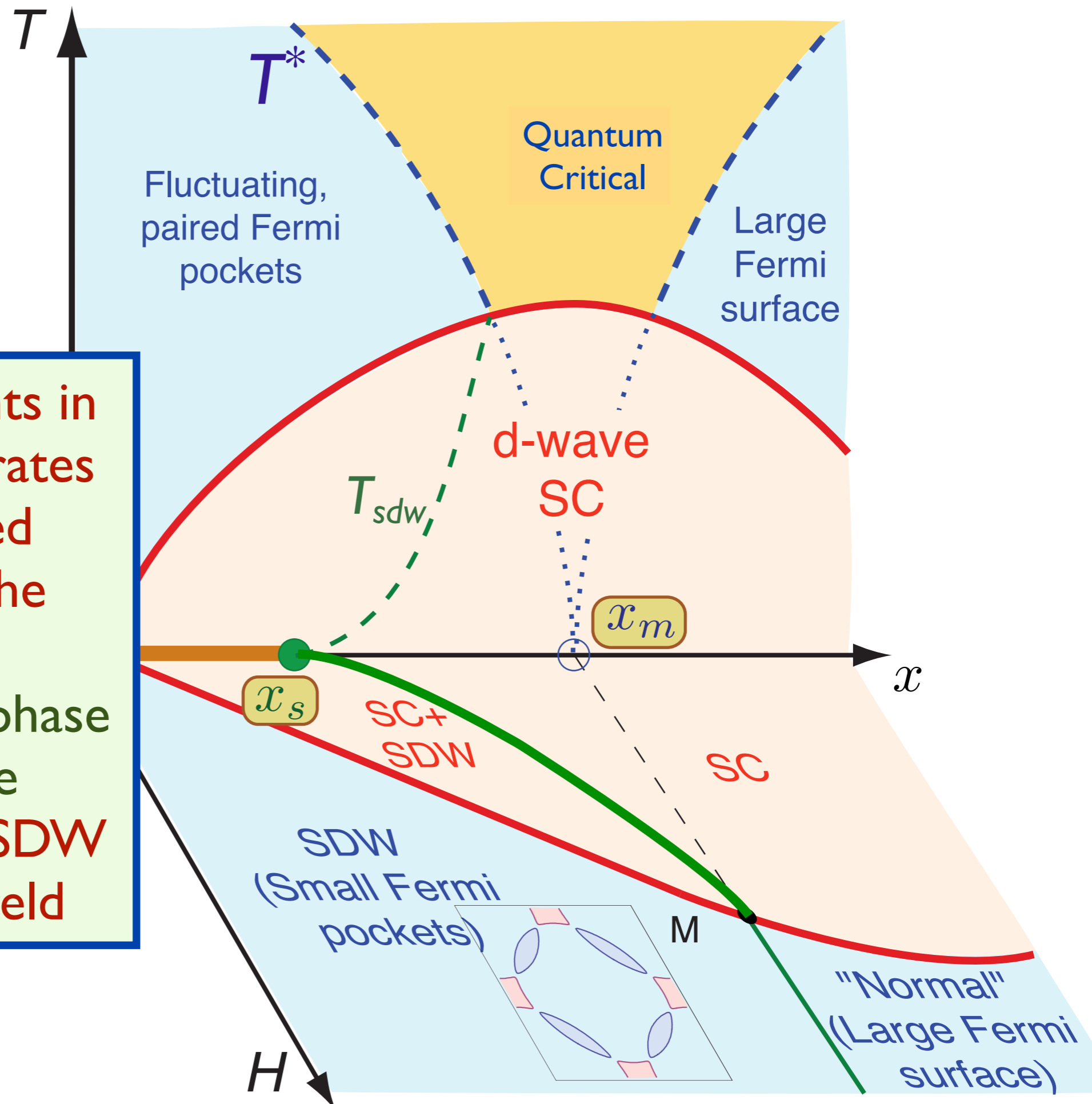
E. Demler, S. Sachdev
and Y. Zhang, *Phys.
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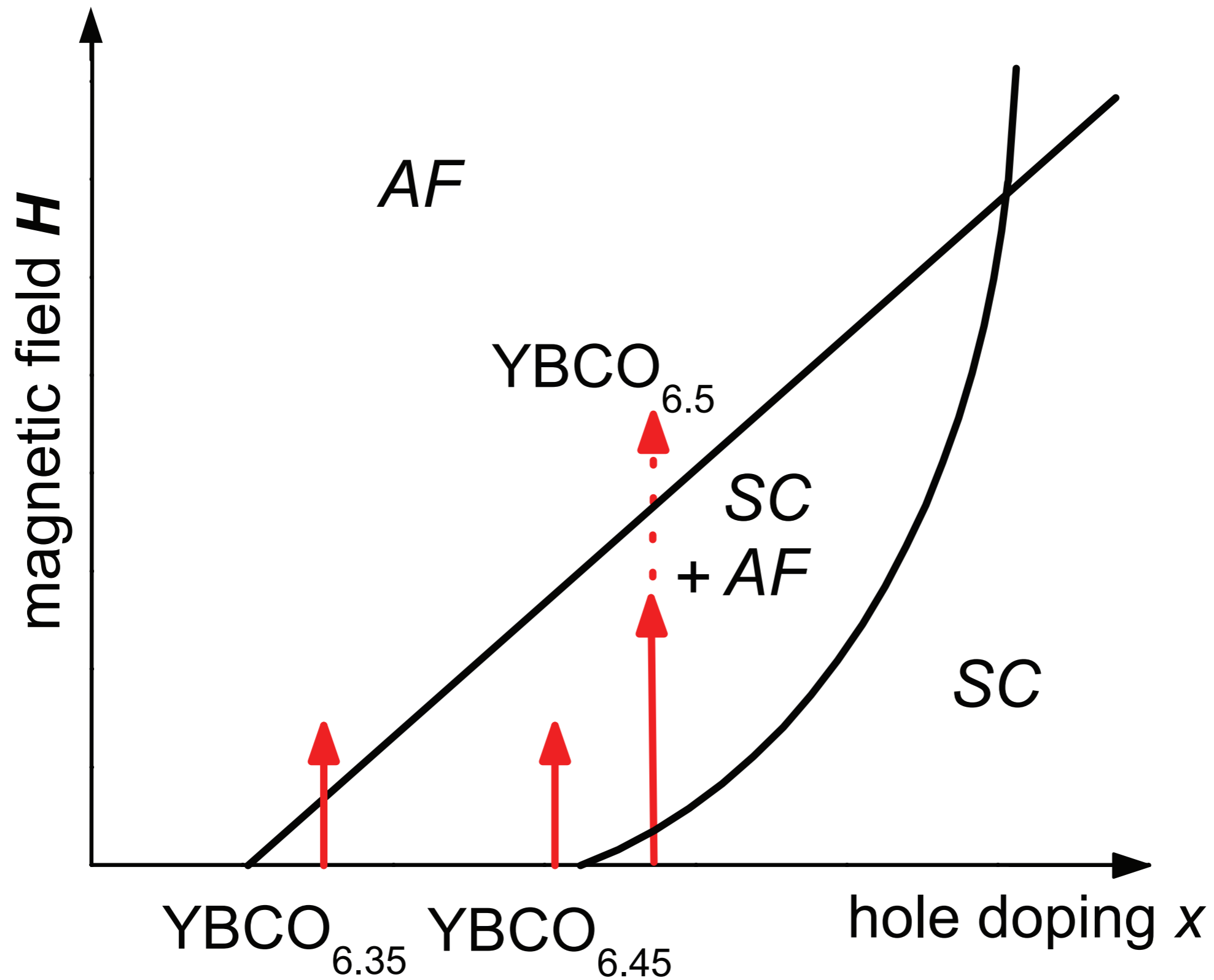
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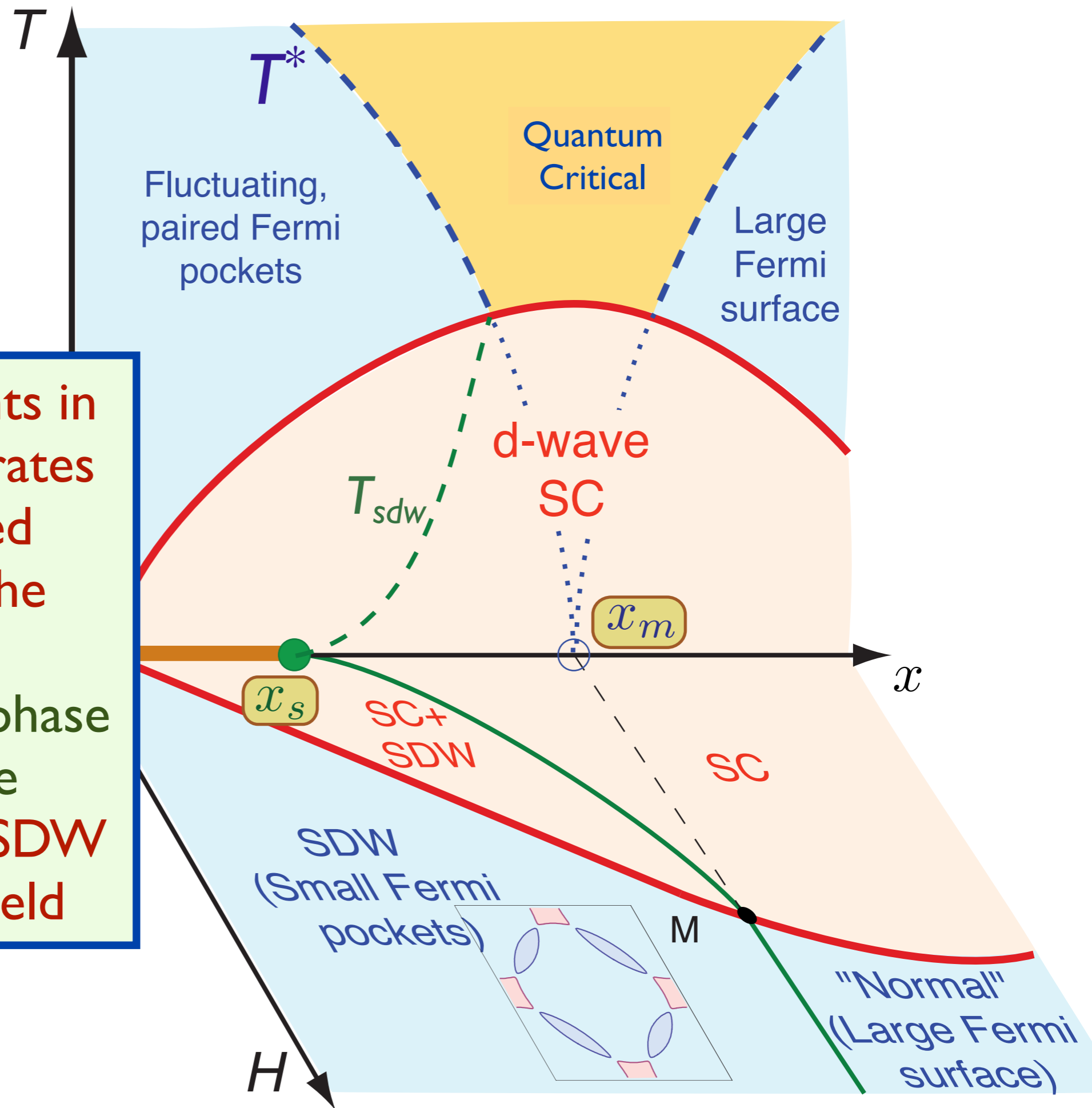




D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, *New J. Phys.* **12**, 105006 (2010)

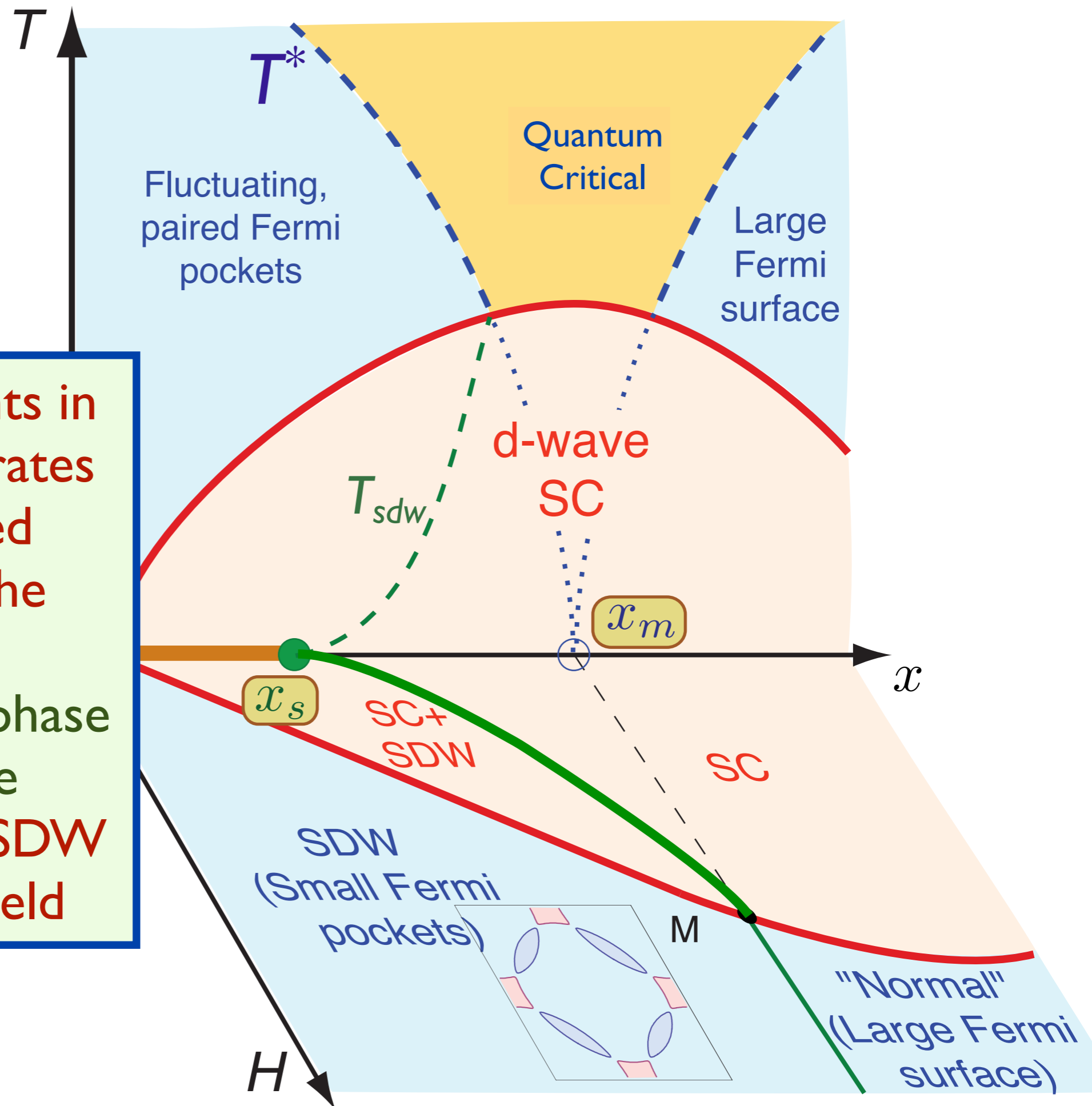
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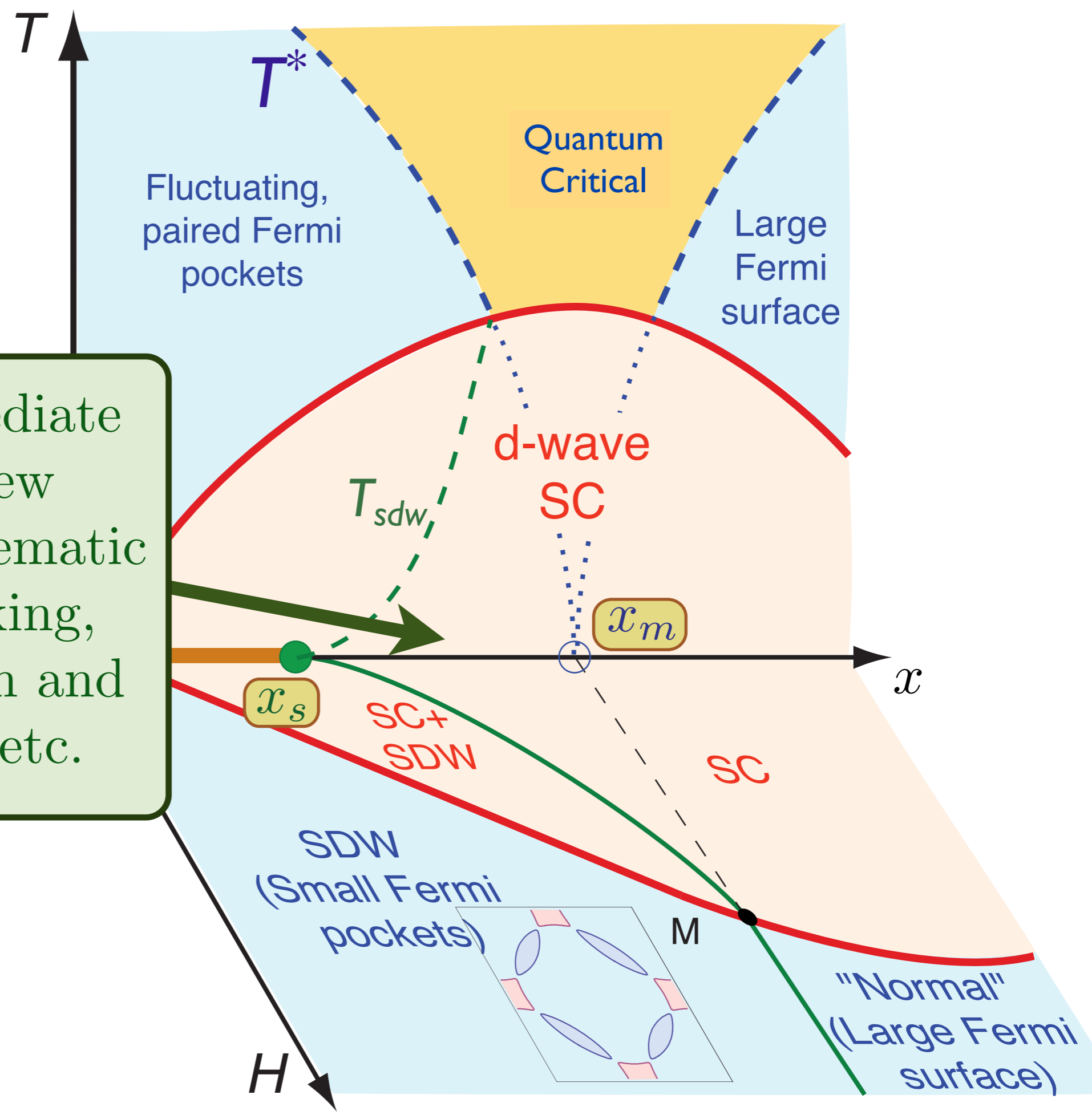


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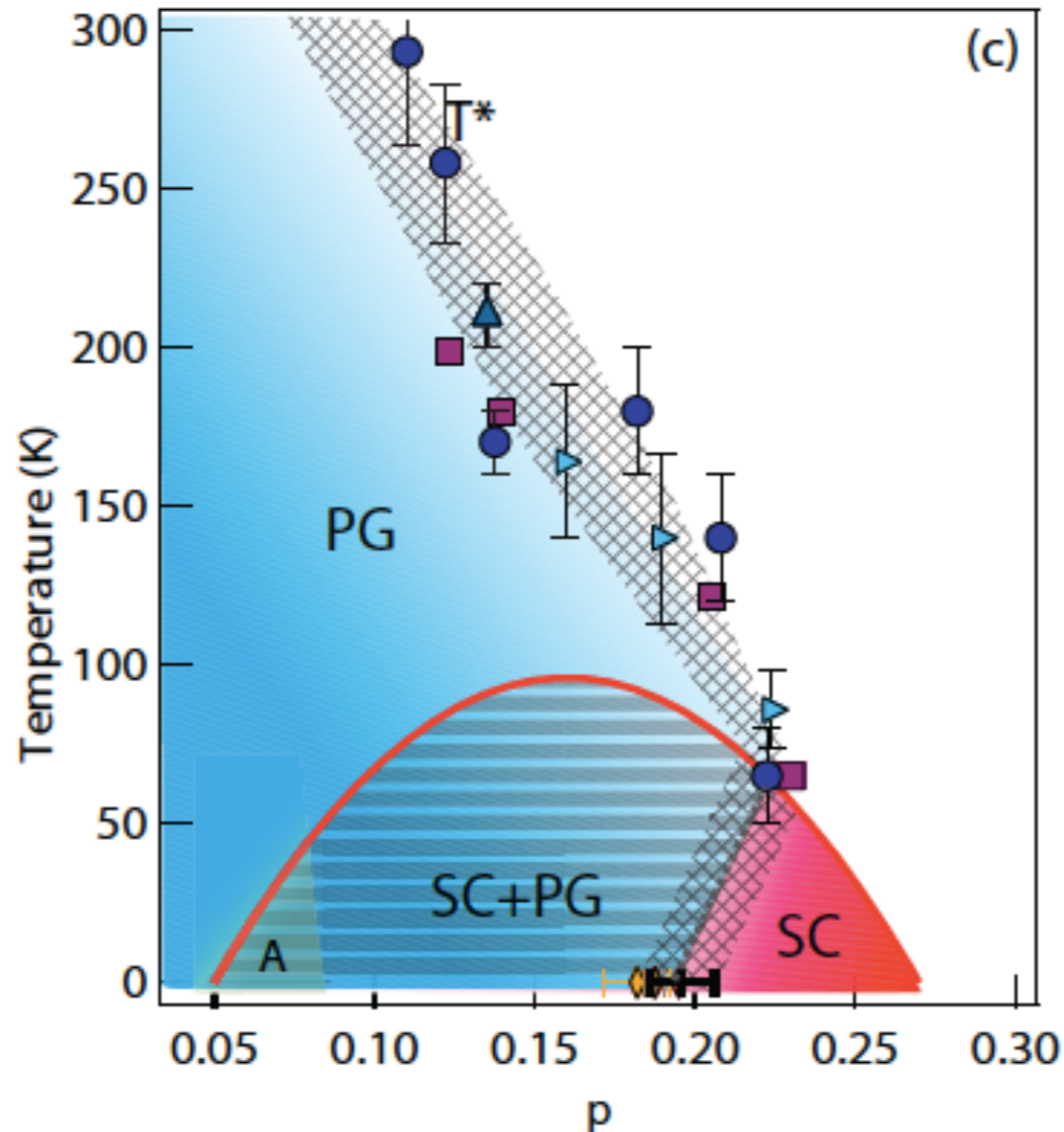


A wide intermediate regime for new physics: bond-nematic order, T -breaking, fractionalization and Mott physics etc.



A revised cuprate phase diagram due to dynamic gap competition

I. M. Vishik,^{1,2} M Hashimoto,³ R.-H. He,⁴ W. S. Lee,^{1,2} F. Schmitt,^{1,2} D. H. Lu,³ R. G. Moore,³ C. Zhang,⁵ W. Meevasana,⁶ T. Sasagawa,⁷ S. Uchida,⁸ K. Fujita,⁹ S. Ishida,⁸ M. Ishikado,¹⁰ Y. Yoshida,¹¹ H. Eisaki,¹¹ Z. Hussain,⁴ T. P. Devereaux,^{1,2} and Z. X. Shen^{1,2}



preprint

Photoemission measurements in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$
Distinguish superconducting gap (SC) and pseudogap (PG)
by distinct dependence on angle, T , and p

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

“Mechanism” of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

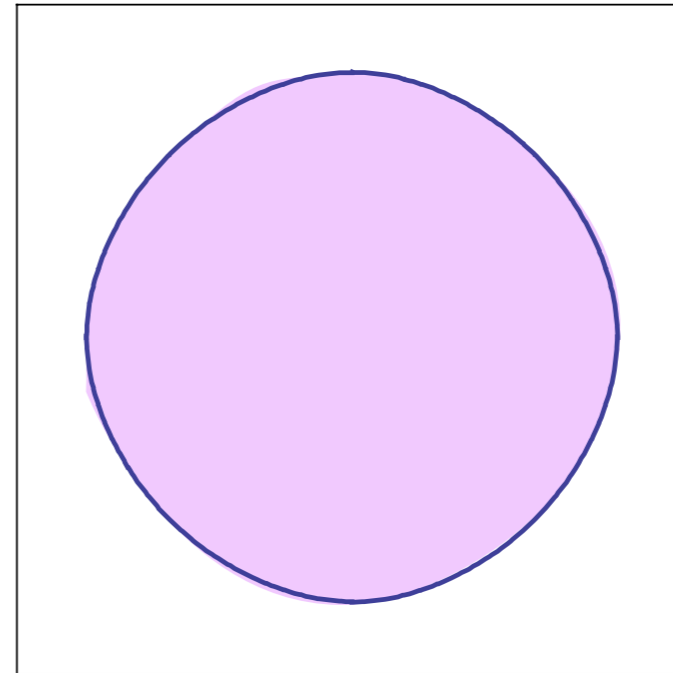
Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals *“Mechanism” of higher temperature superconductivity*

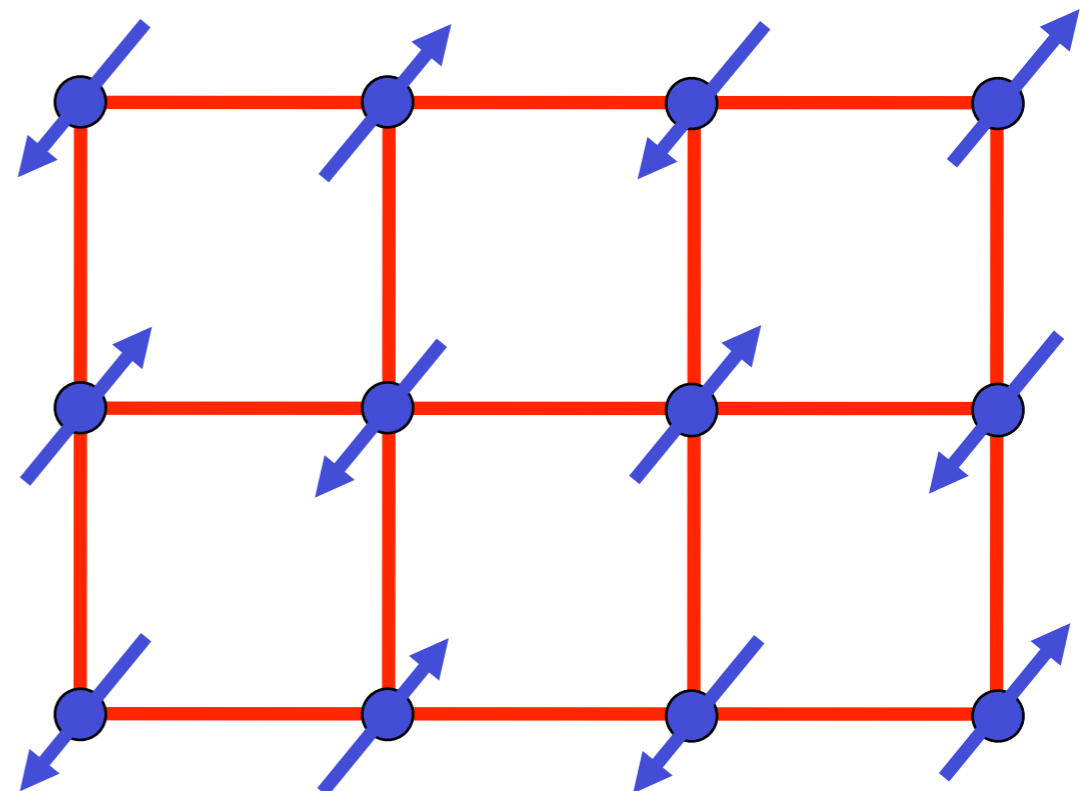
3. Theory of the competition between superconductivity and antiferromagnetism

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



+



The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where \mathbf{K} is the ordering wavevector.

Spin-fermion model: Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_{\alpha} \mathcal{D}\vec{\varphi} \exp(-\mathcal{S})$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2r \left[\frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \dots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{r}_i} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

Coupling between fermions and antiferromagnetic order:
 $\lambda^2 \sim U$, the Hubbard repulsion

A technical aside.....

Hertz-Moriya-Millis theory

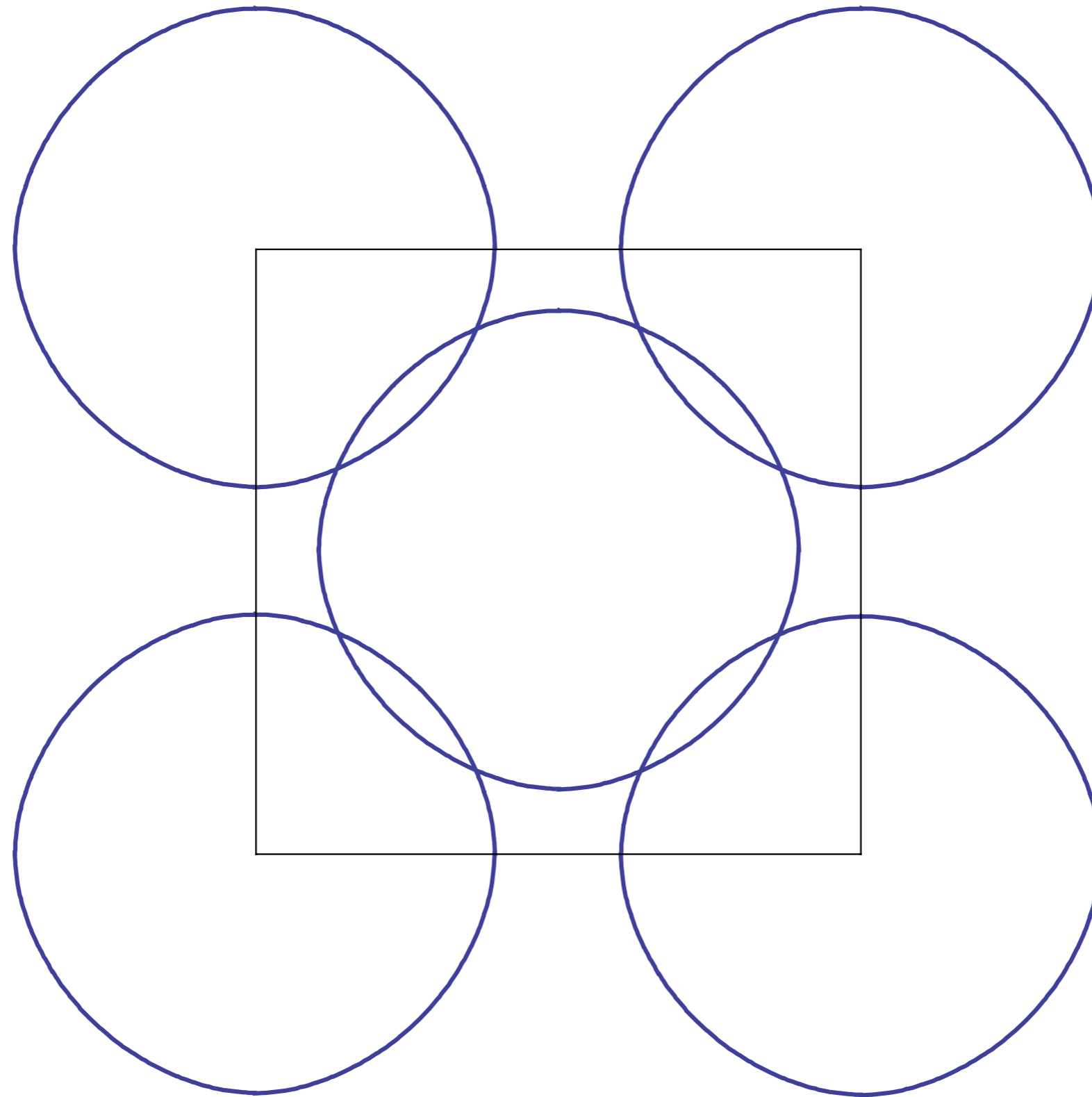
Integrate out fermions and obtain an effective action for the boson field $\vec{\varphi}$ alone. Because the fermions are gapless, this is potentially dangerous, and will lead to non-local terms in the $\vec{\varphi}$ effective action. Hertz focused on only the simplest such non-local term. However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in $d = 2$.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

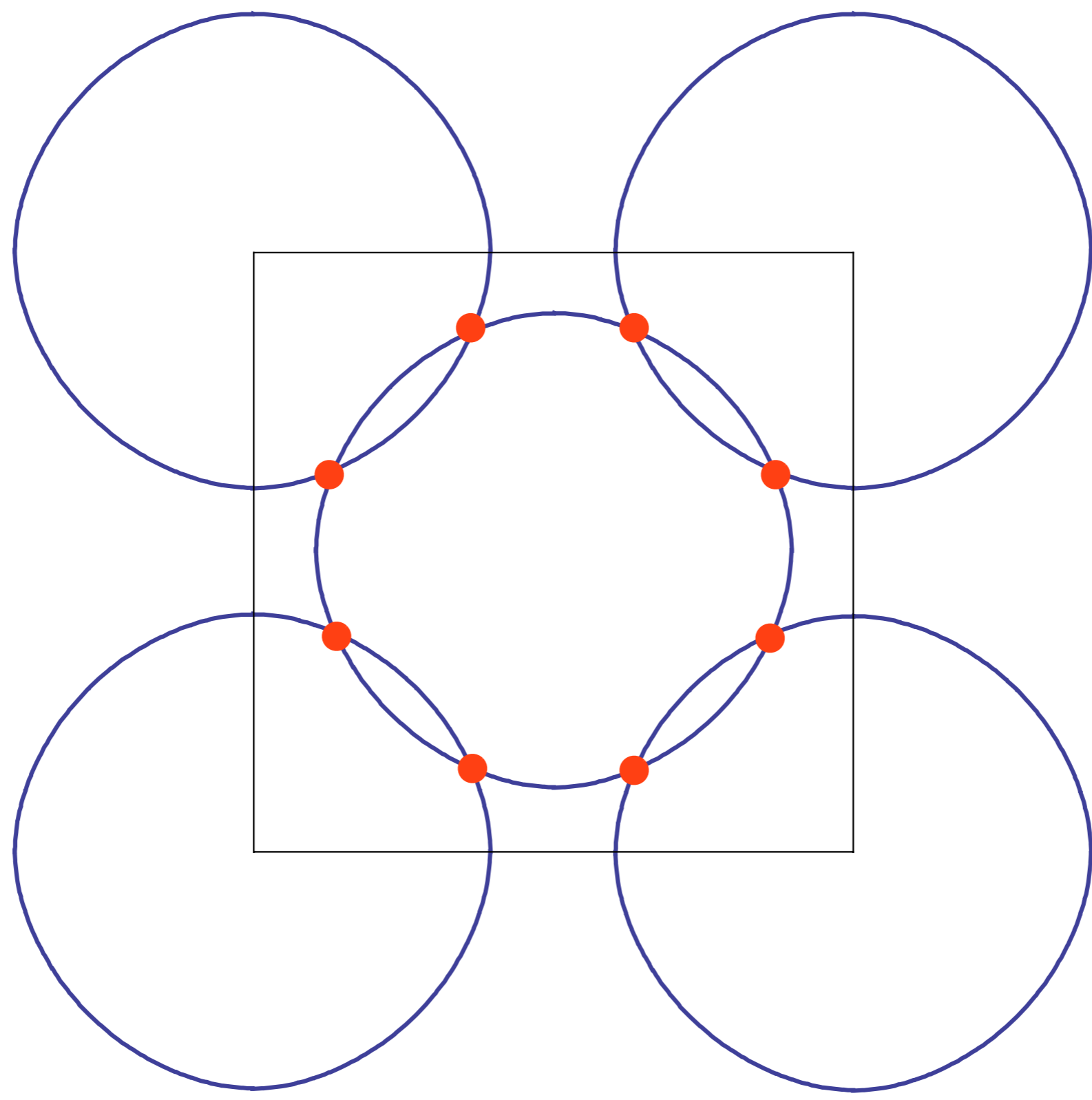
A technical aside.....

We need to perform an RG analysis on a local theory of both the fermions and the $\vec{\varphi}$. It appears that such a theory can be analyzed using a $1/N$ expansion, where N is the number of fermion flavors. At two-loop order, the $1/N$ expansion is well-behaved, and we can determine consistent RG flow equations. However, at higher loops we find corrections to the renormalizations which require summation of all planar graphs even at the leading order in $1/N$, and the $1/N$ expansion appears to be organized as a genus expansion of random surfaces. But even this genus expansion breaks down in the renormalization of a quartic coupling of $\vec{\varphi}$. In the following, I will describe some of the two loop results.

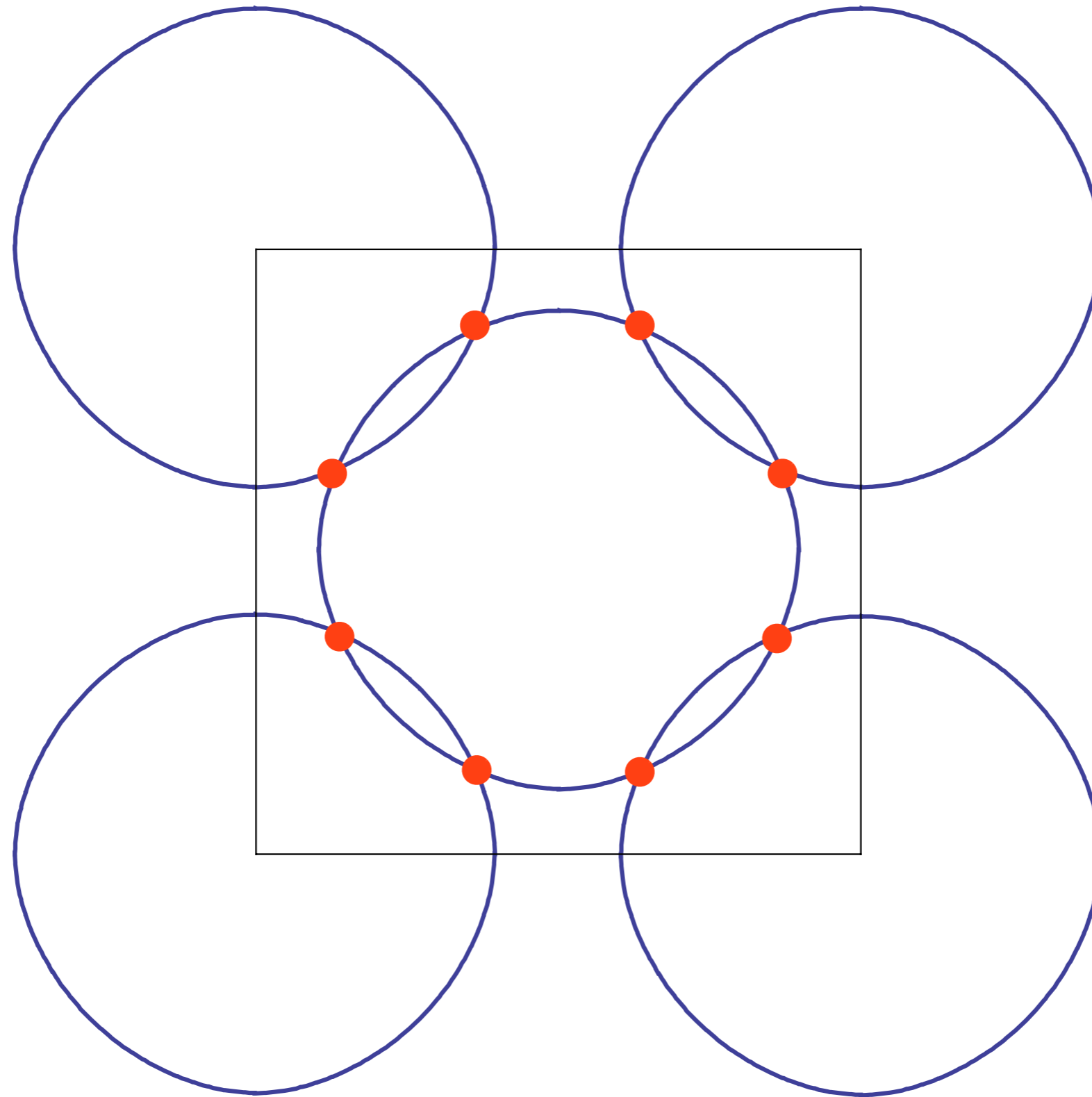
M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



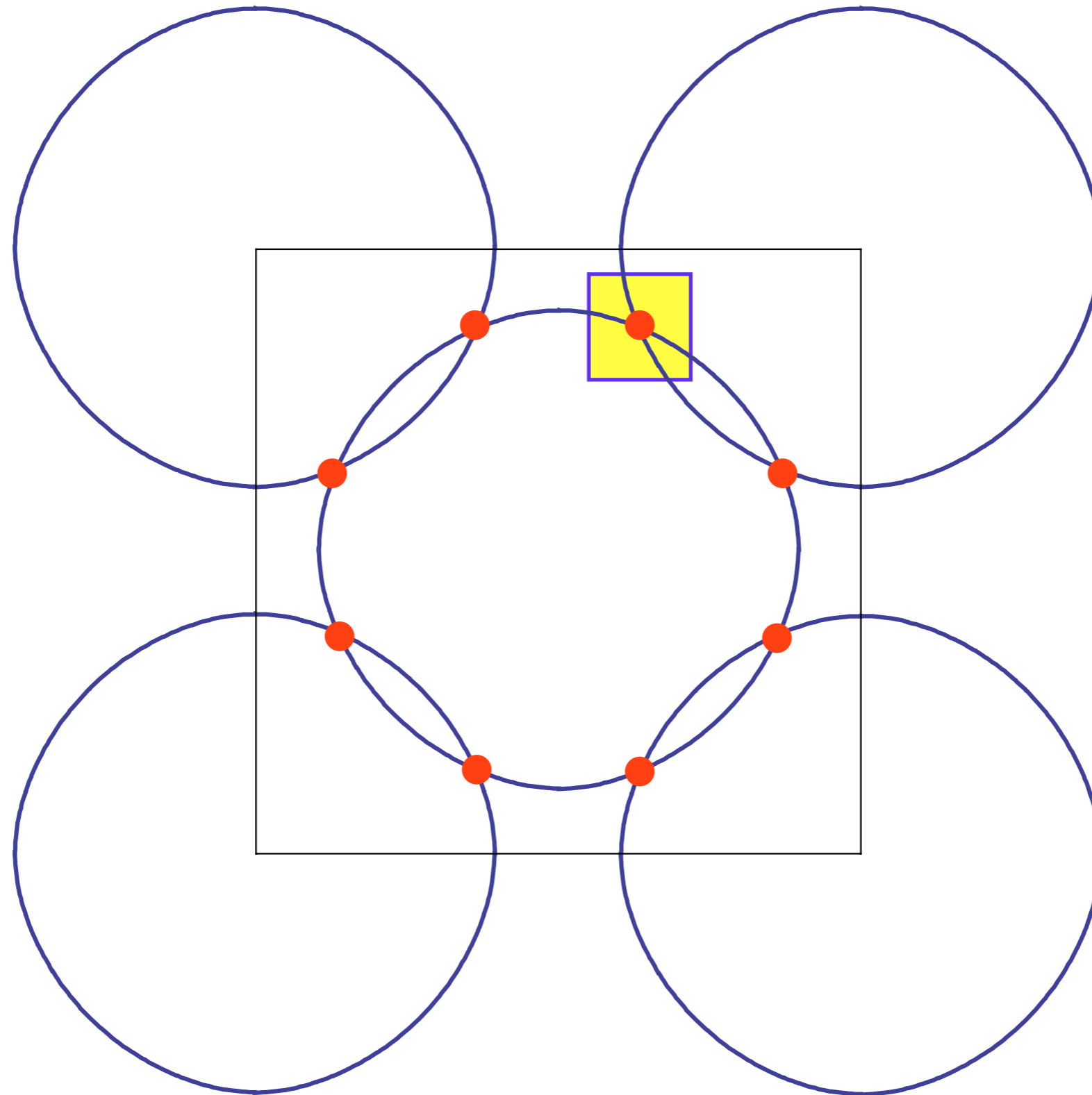
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.



“Hot” spots

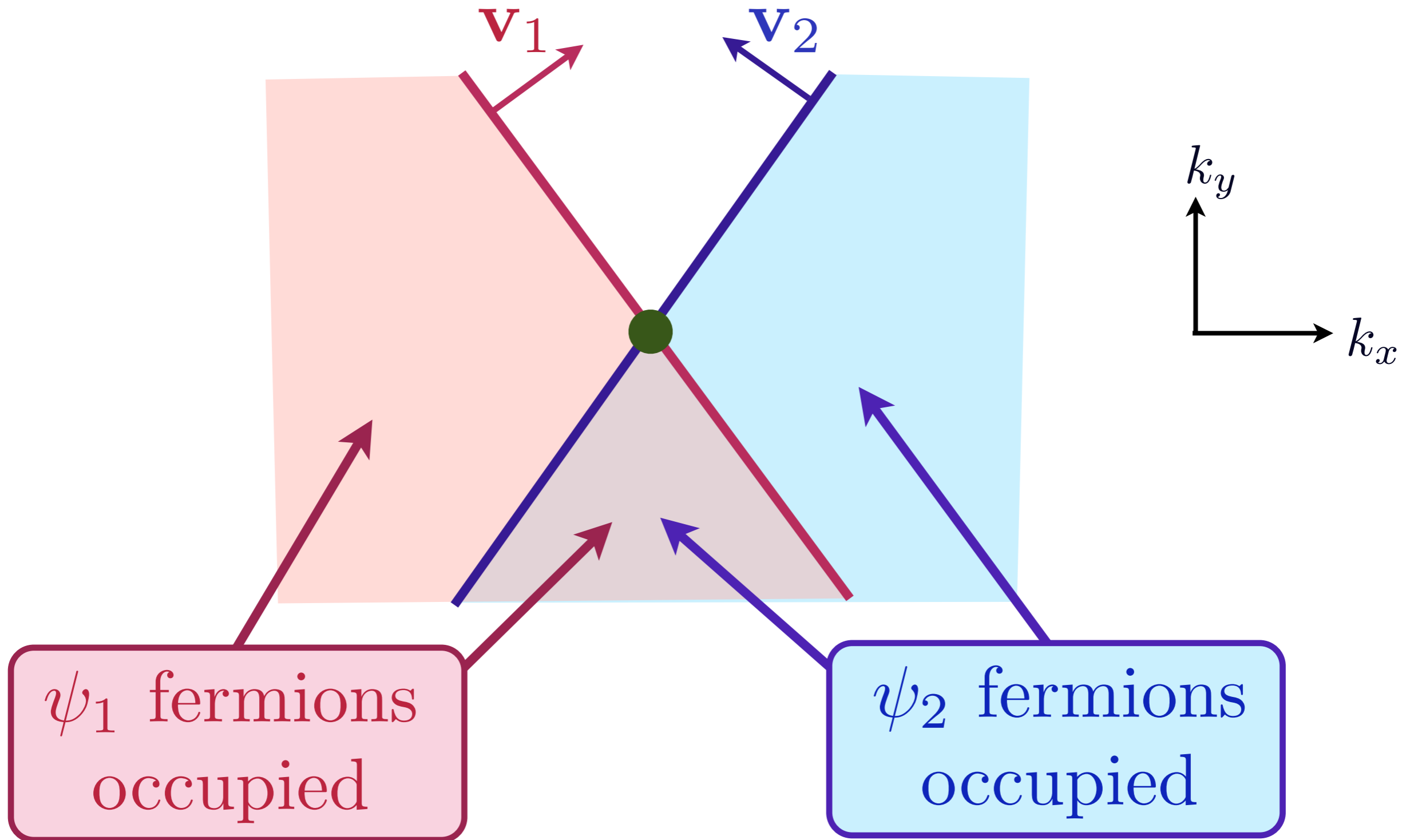


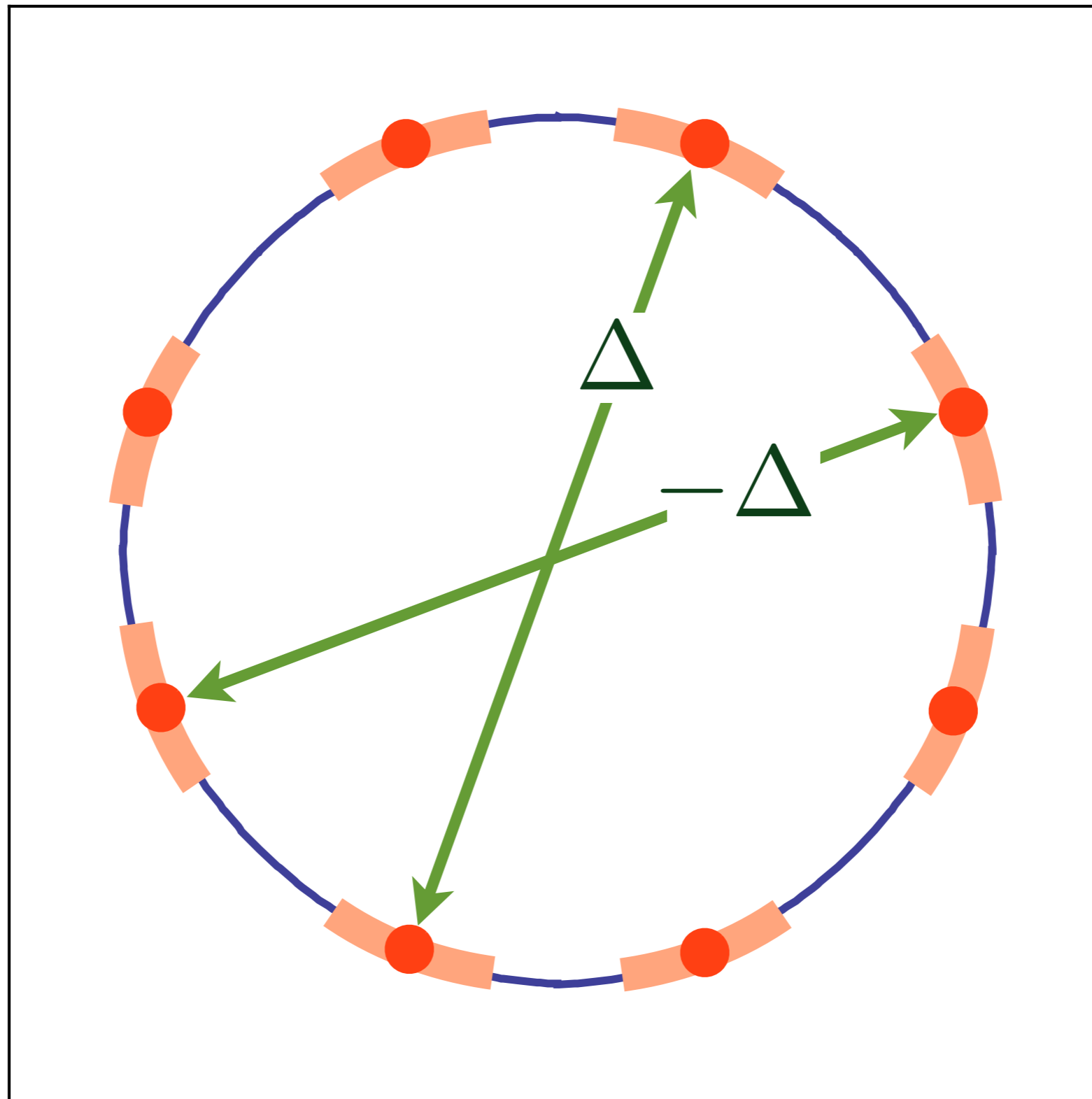
Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

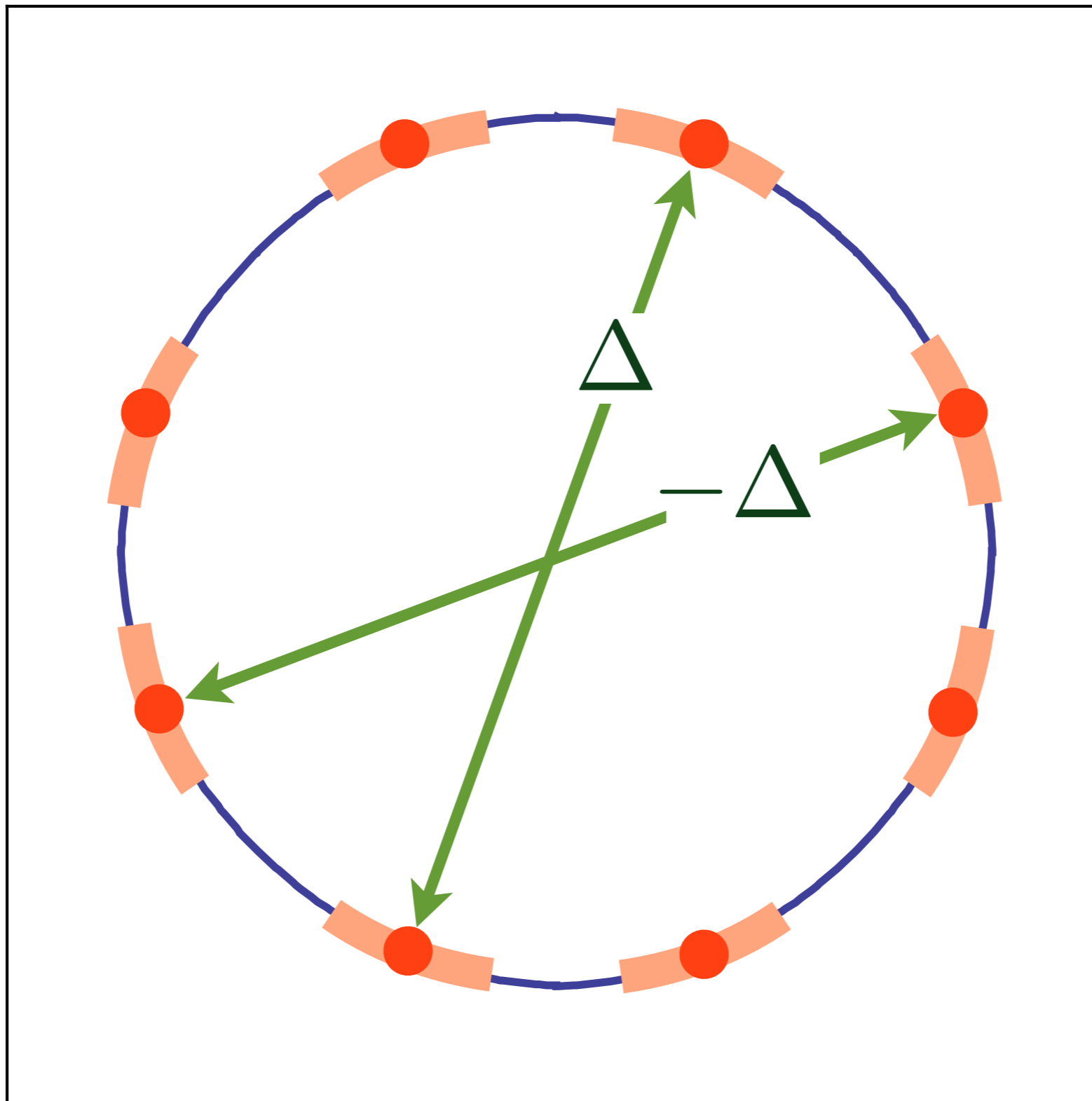
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ





Unconventional pairing at and near hot spots

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$



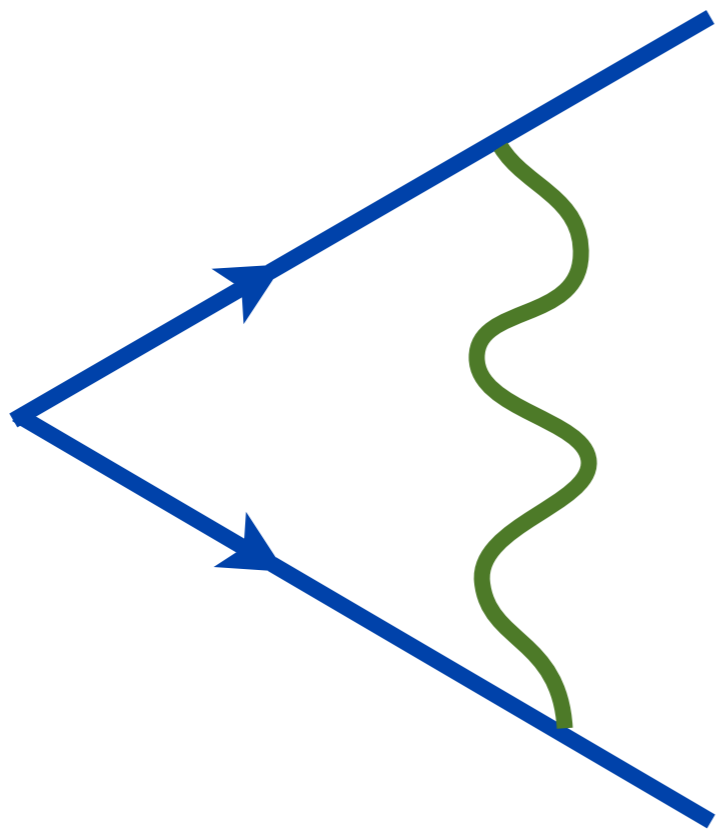
Unconventional pairing at and near hot spots

BCS theory

$$1 + \lambda_{\text{e-ph}} \log \left(\frac{\omega_D}{\omega} \right)$$



Cooper
logarithm



BCS theory

$$1 + \lambda_{\text{e-ph}} \log \left(\frac{\omega_D}{\omega} \right)$$

Electron-phonon
coupling



BCS theory

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Electron-phonon
coupling

Debye
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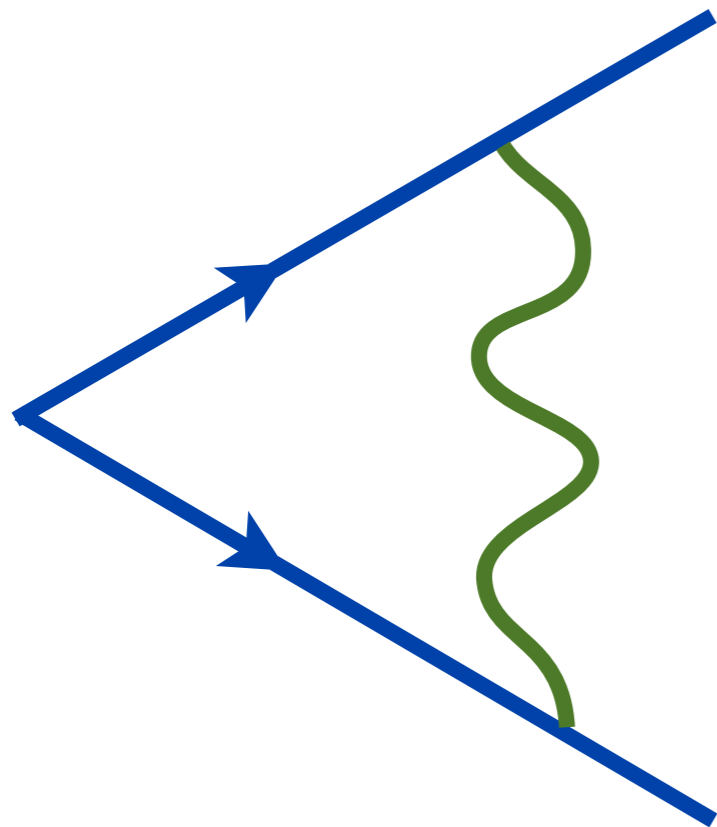
Implies

$$T_c \sim \omega_D \exp(-1/\lambda)$$

Enhancement of pairing susceptibility by interactions

Antiferromagnetic fluctuations: weak-coupling

$$1 + \left(\frac{U}{t}\right)^2 \log\left(\frac{E_F}{\omega}\right)$$



Cooper
logarithm

V.J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

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Implies

$$T_c \sim E_F \exp\left(-\left(t/U\right)^2\right)$$

V.J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

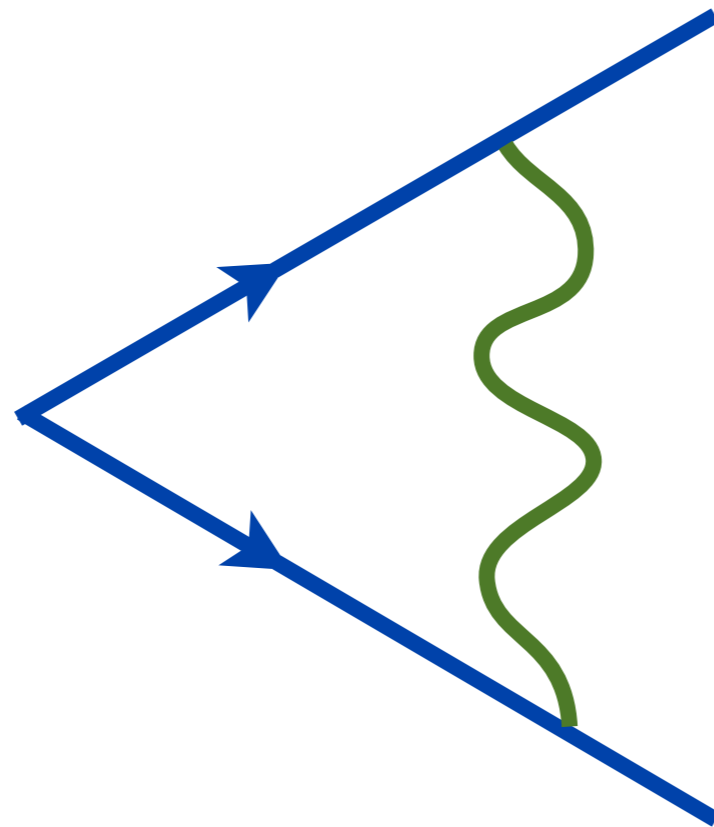
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S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1 + \alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$



M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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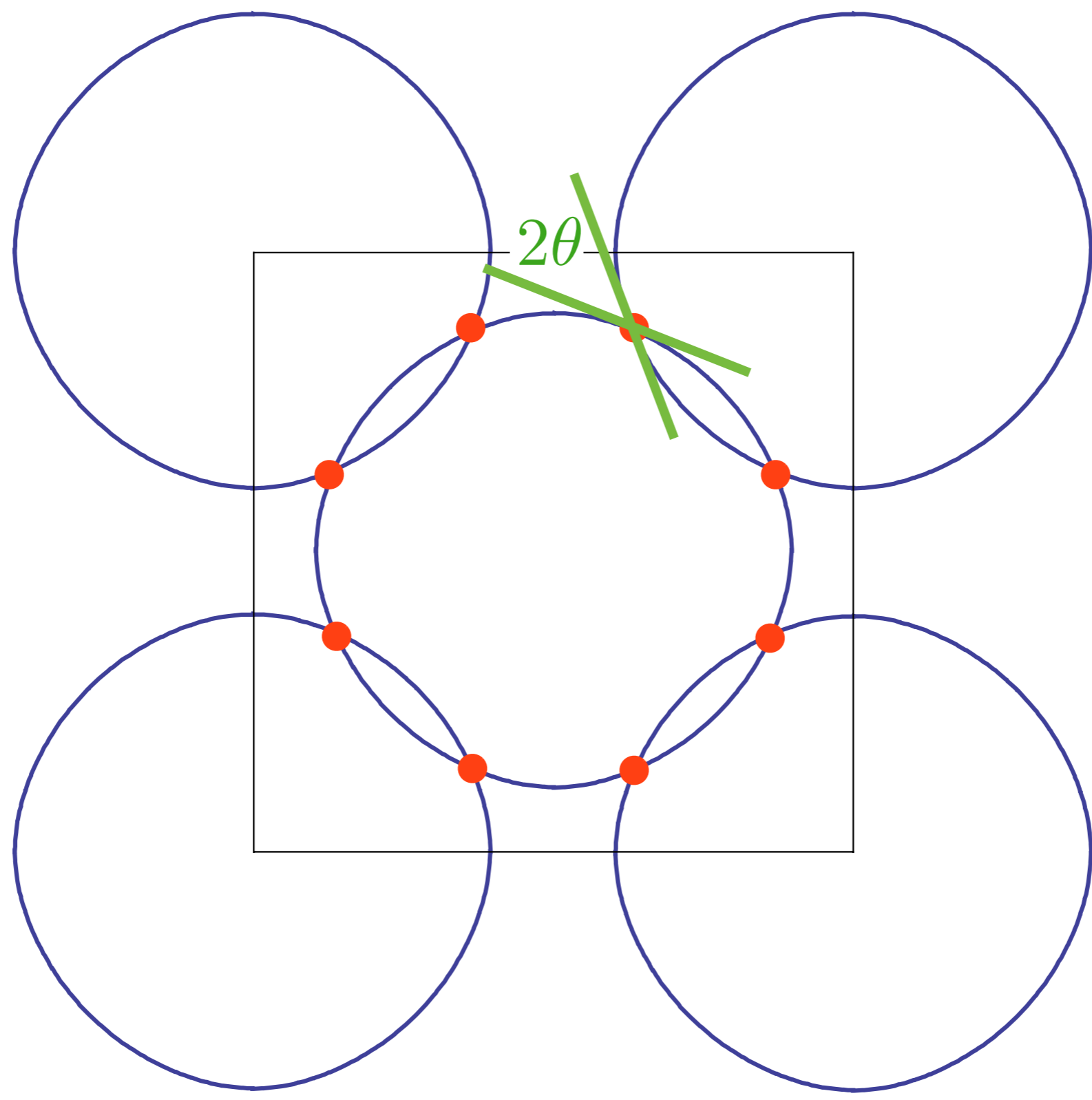
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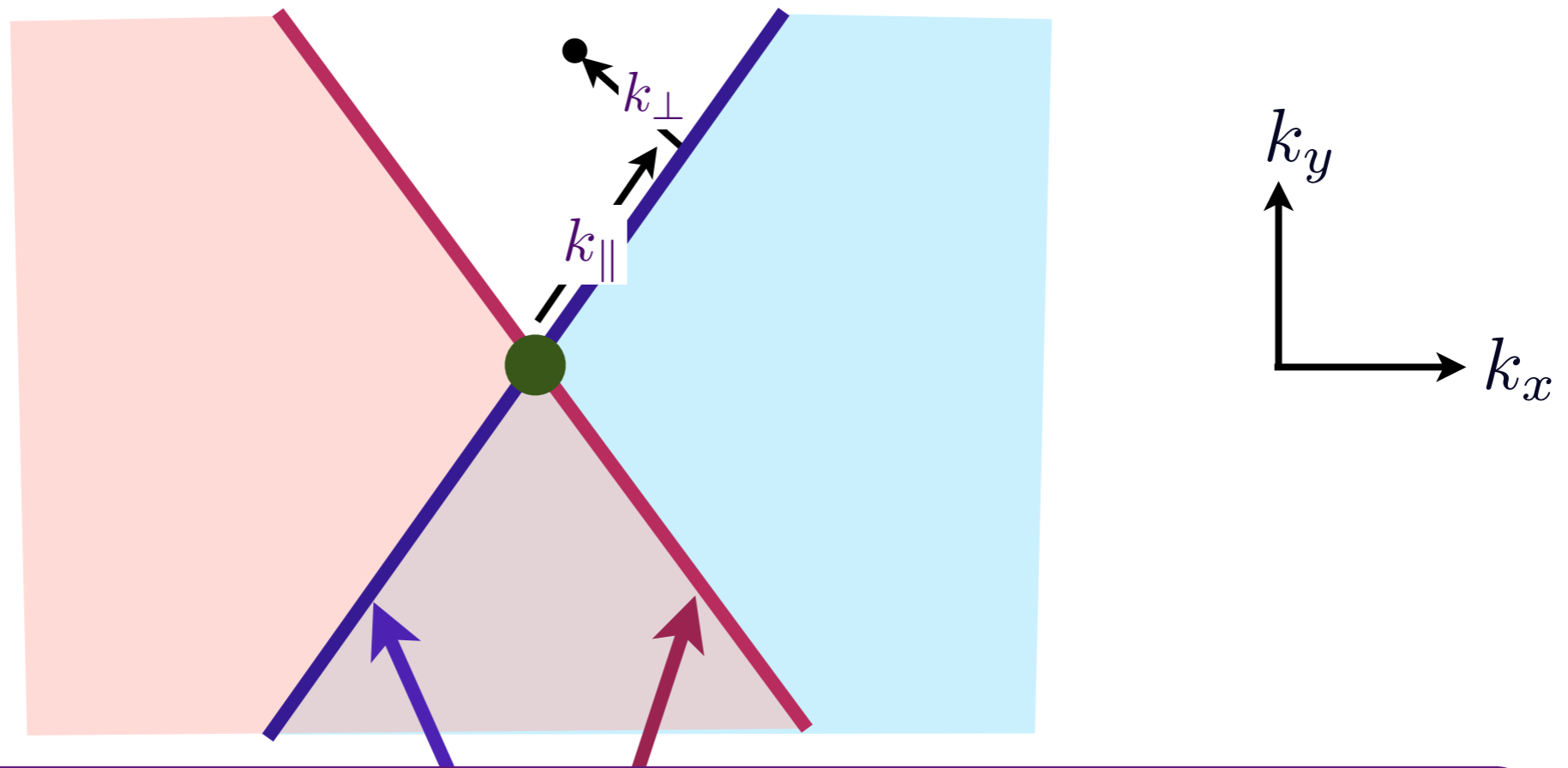
Fermi
energy

$\alpha = \tan \theta$, where 2θ is
the angle between Fermi lines.
Independent of interaction strength
 U in 2 dimensions.

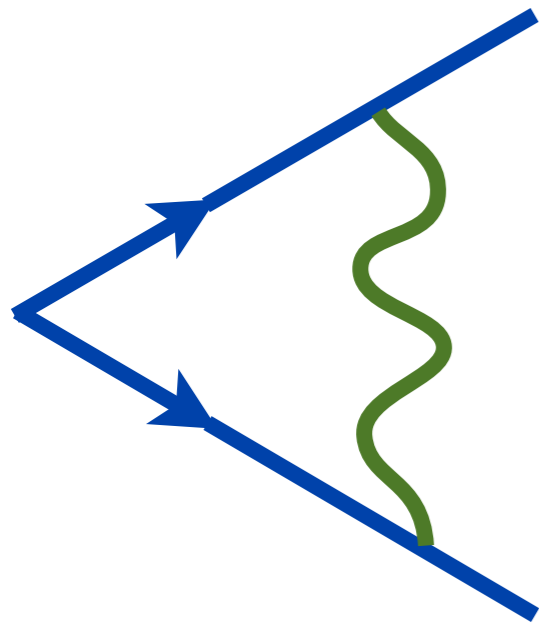
(see also Ar. Abanov, A. V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001))
M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)



M.A. Metlitski
and S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)

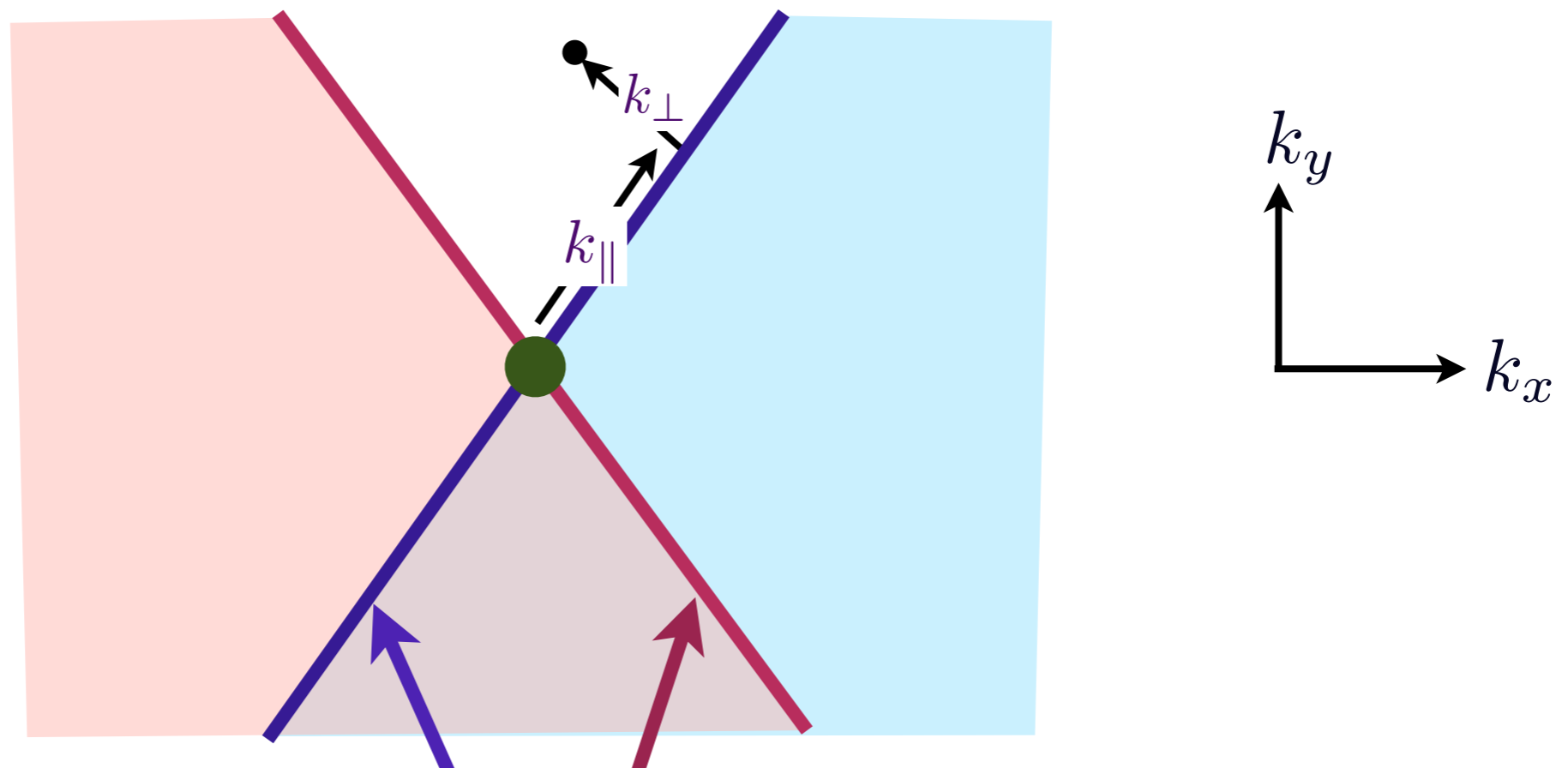


$$G_{\text{fermion}} = \frac{Z(k_{\parallel})}{i\omega - v_F(k_{\parallel})k_{\perp}}, \quad Z(k_{\parallel}) \sim v_F(k_{\parallel}) \sim k_{\parallel}$$

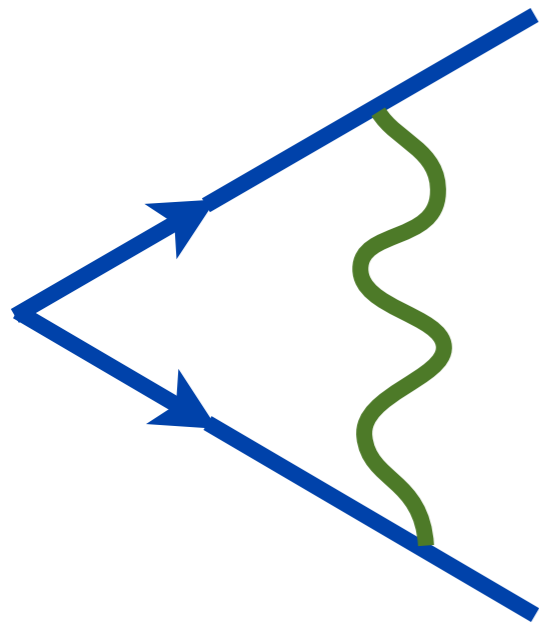


$$\int dk_{\parallel} \frac{1}{k_{\parallel}^2} \left(\frac{Z^2(k_{\parallel})}{v_F(k_{\parallel})} \right) \log \frac{k_{\parallel}^2}{\omega}$$

M.A. Metlitski
and S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)



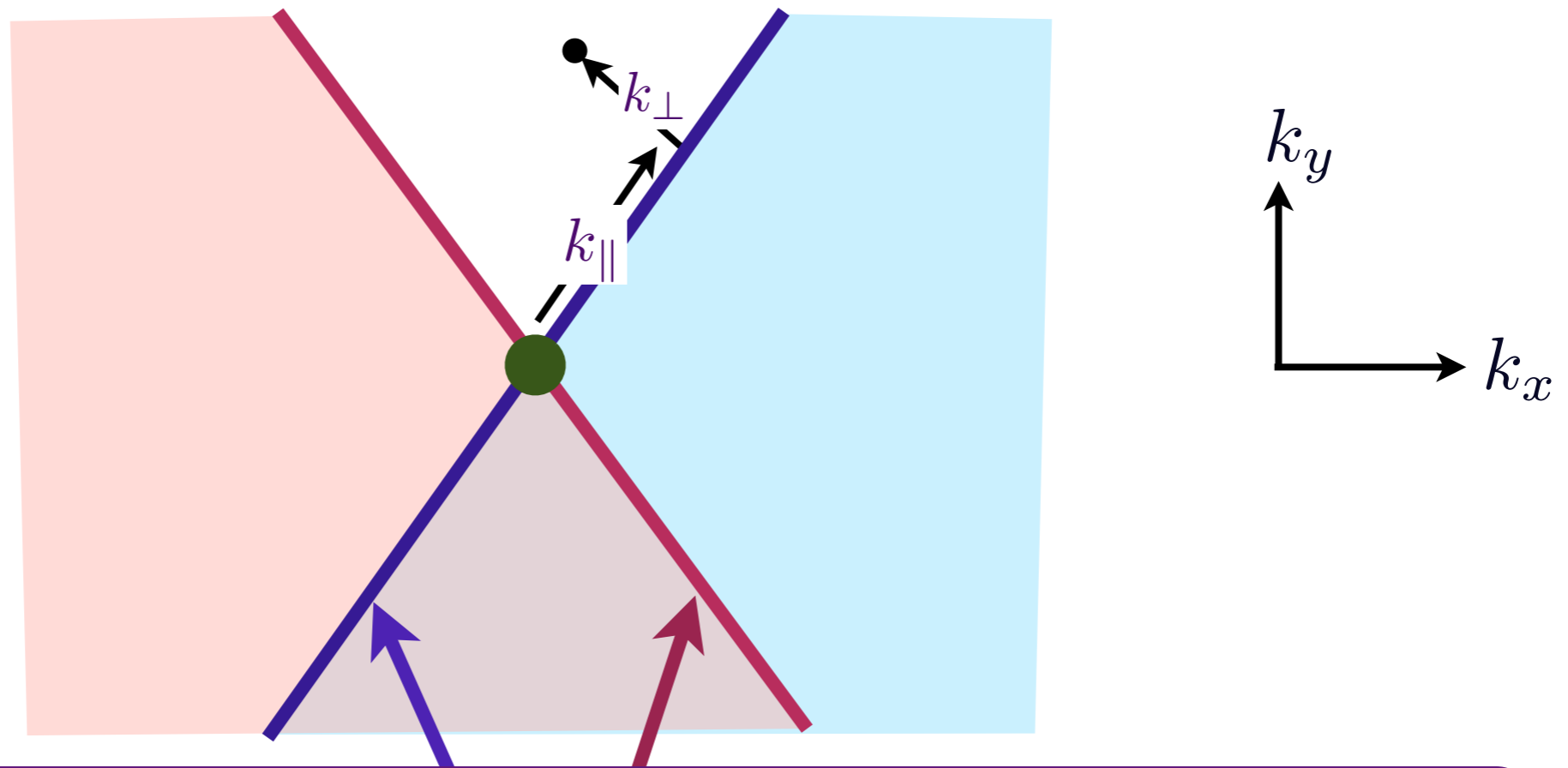
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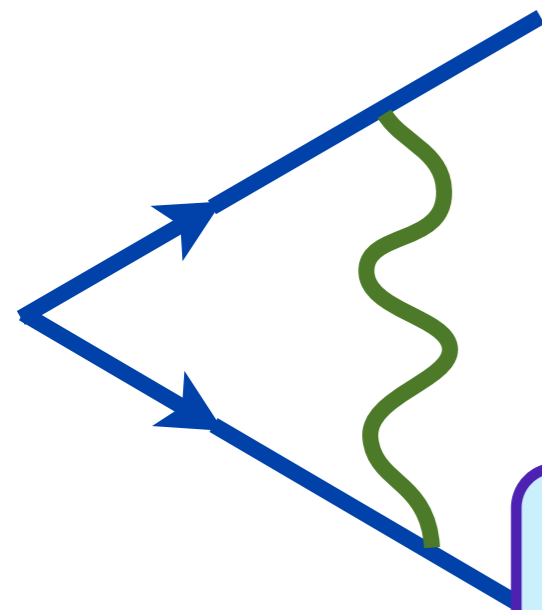
$$\int dk_{\parallel} \frac{1}{k_{\parallel}^2} \underbrace{\left(\frac{Z^2(k_{\parallel})}{v_F(k_{\parallel})} \right)}_{\text{Cooper logarithm}} \log \frac{k_{\parallel}^2}{\omega}$$

Cooper
logarithm

M.A. Metlitski
and S. Sachdev,
Phys. Rev. B **85**,
075127 (2010)



$$G_{\text{fermion}} = \frac{Z(k_{\parallel})}{i\omega - v_F(k_{\parallel})k_{\perp}}, \quad Z(k_{\parallel}) \sim v_F(k_{\parallel}) \sim k_{\parallel}$$



$$\int dk_{\parallel} \frac{1}{k_{\parallel}^2} \underbrace{\left(\frac{Z^2(k_{\parallel})}{v_F(k_{\parallel})} \right)}_{\text{Cooper logarithm}} \log \frac{k_{\parallel}^2}{\omega}$$

Spin fluctuation
propagator

Cooper
logarithm

Enhancement of pairing susceptibility by interactions

Spin density wave quantum critical point

$$1 + \frac{\alpha}{\pi(1 + \alpha^2)} \log^2 \left(\frac{E_F}{\omega} \right)$$

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Enhancement of pairing susceptibility by interactions

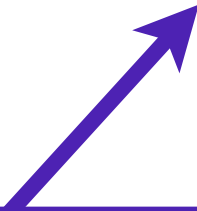
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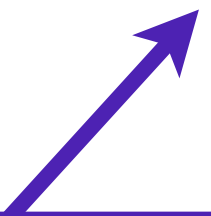
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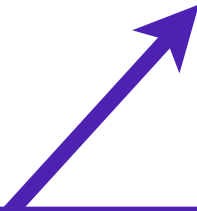
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M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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Ar. Abanov, A. V. Chubukov, and A. M. Finkel'stein, *Europhys. Lett.* **54**, 488 (2001)

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Outline

1. Phenomenology of the onset of antiferromagnetism in a metal

Quantum criticality of Fermi surface reconstruction, and the phase diagram in a magnetic field

2. Strongly-coupled quantum criticality in metals

“Mechanism” of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

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“Mechanism” of higher temperature superconductivity

3. Theory of the competition between superconductivity and antiferromagnetism

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (Δ):

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} (\vec{\varphi}^2)^2 + \kappa \vec{\varphi}^2 |\Delta|^2 \right] + \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta|^2 - |\Delta|^2 + \frac{|\Delta|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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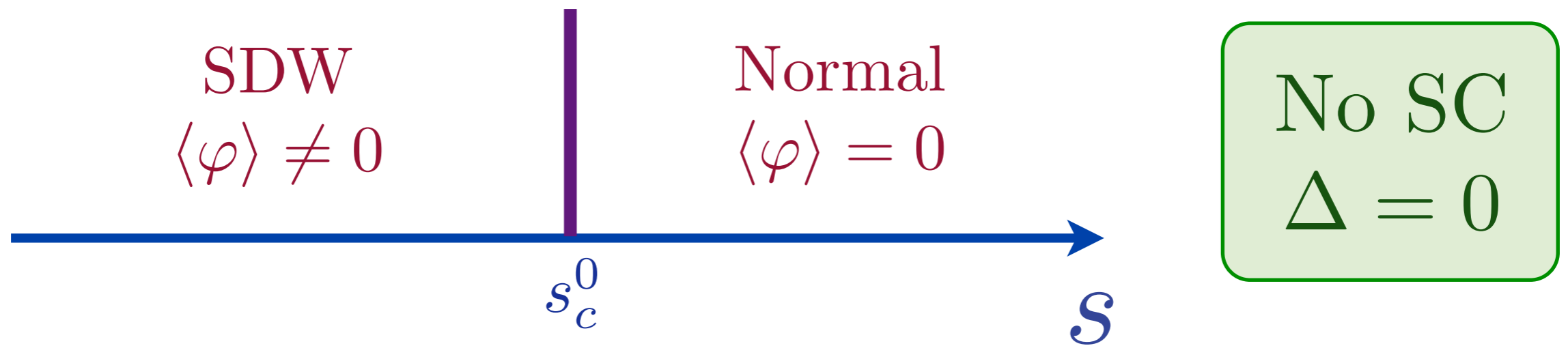
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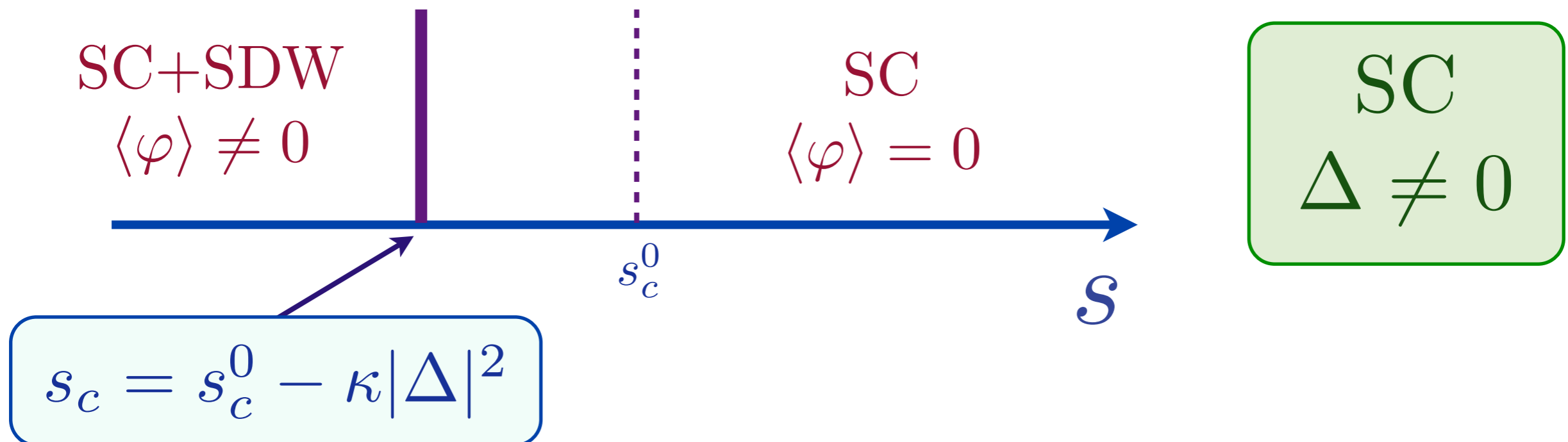
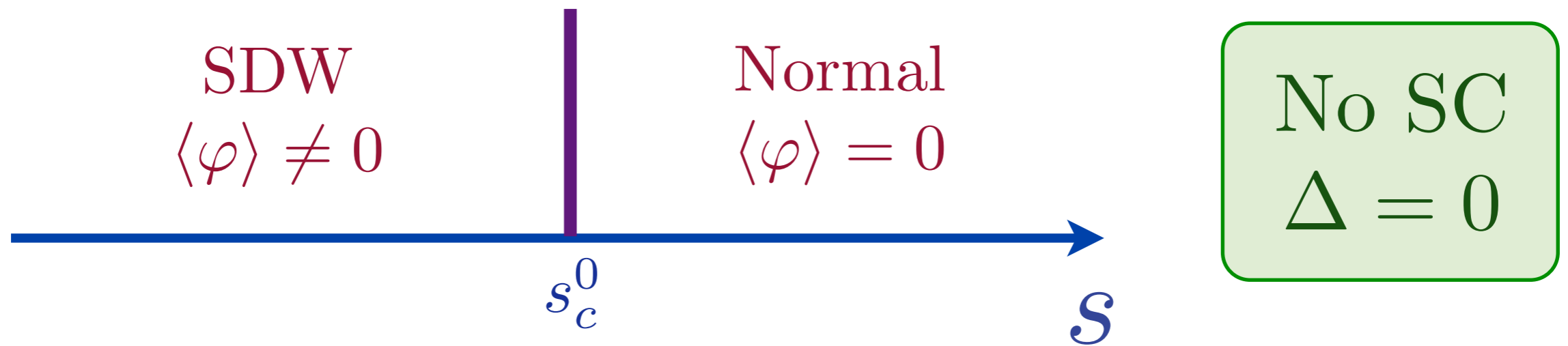
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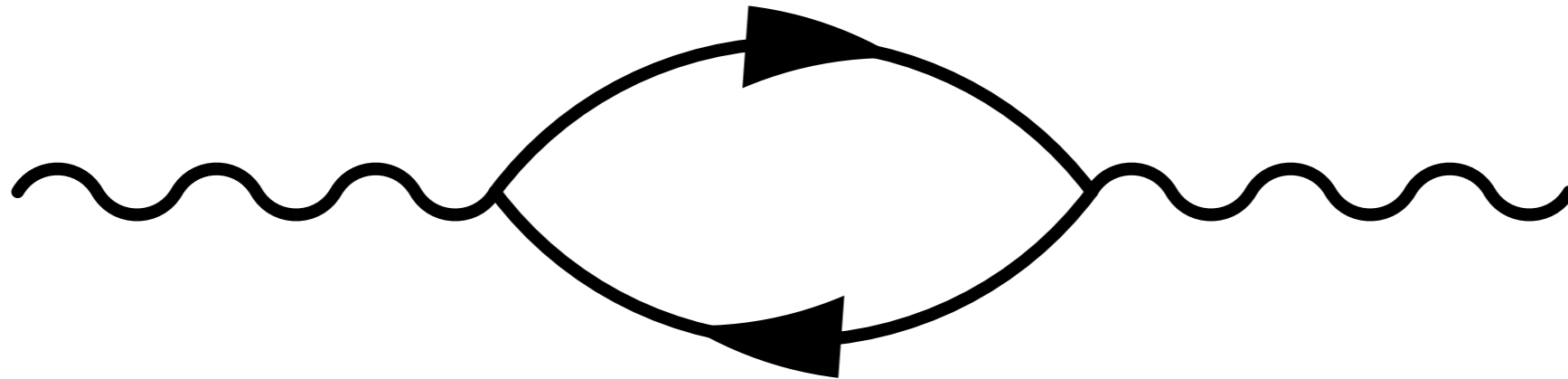
Phenomenological quantum theory of competing orders

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Fermi surface theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



Compute the SDW susceptibility, χ , in the superconducting state. As $\Delta \rightarrow 0$, we find

$$\chi(\Delta) = \chi(0) - C|\Delta|$$

where C is a universal constant dominated by the vicinity of the hot spots.

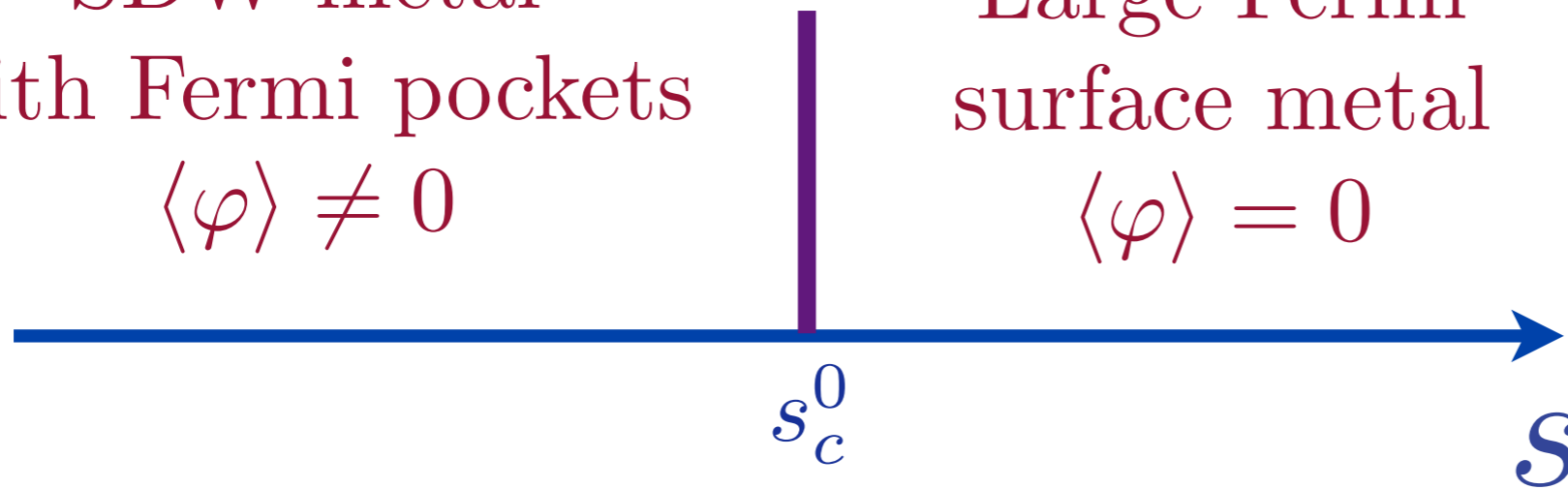
E. G. Moon and S. Sachdev, *Phy. Rev. B* **82**, 104516 (2010)

Fermi surface theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

SDW metal
with Fermi pockets
 $\langle \varphi \rangle \neq 0$

Large Fermi
surface metal
 $\langle \varphi \rangle = 0$



No SC
 $\Delta = 0$

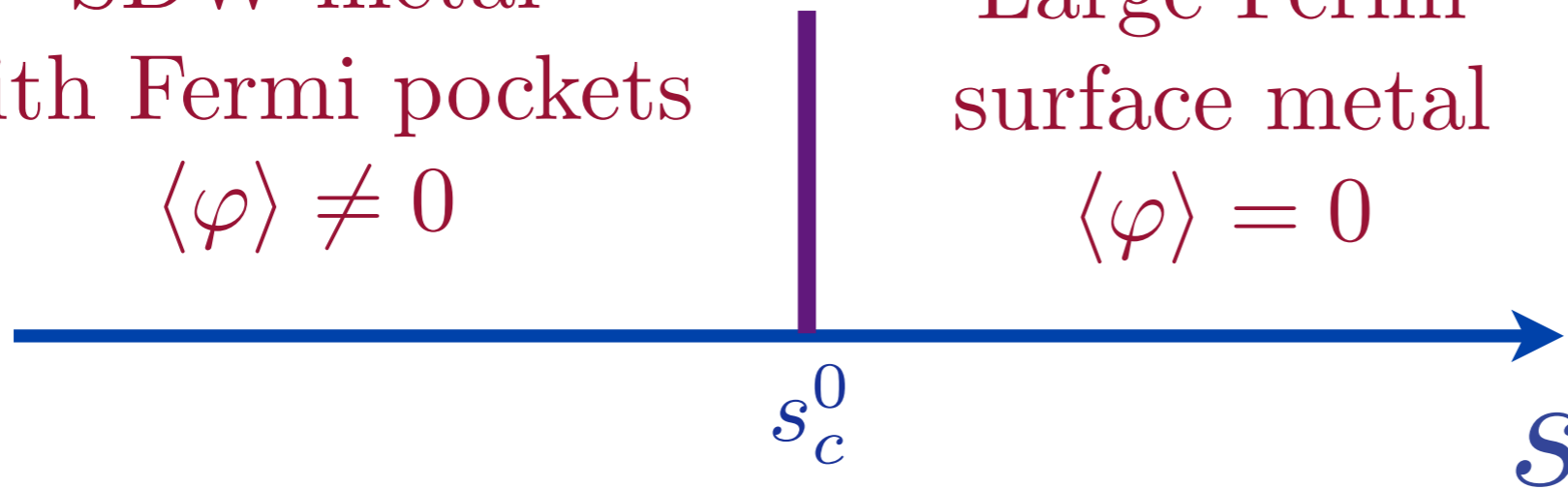
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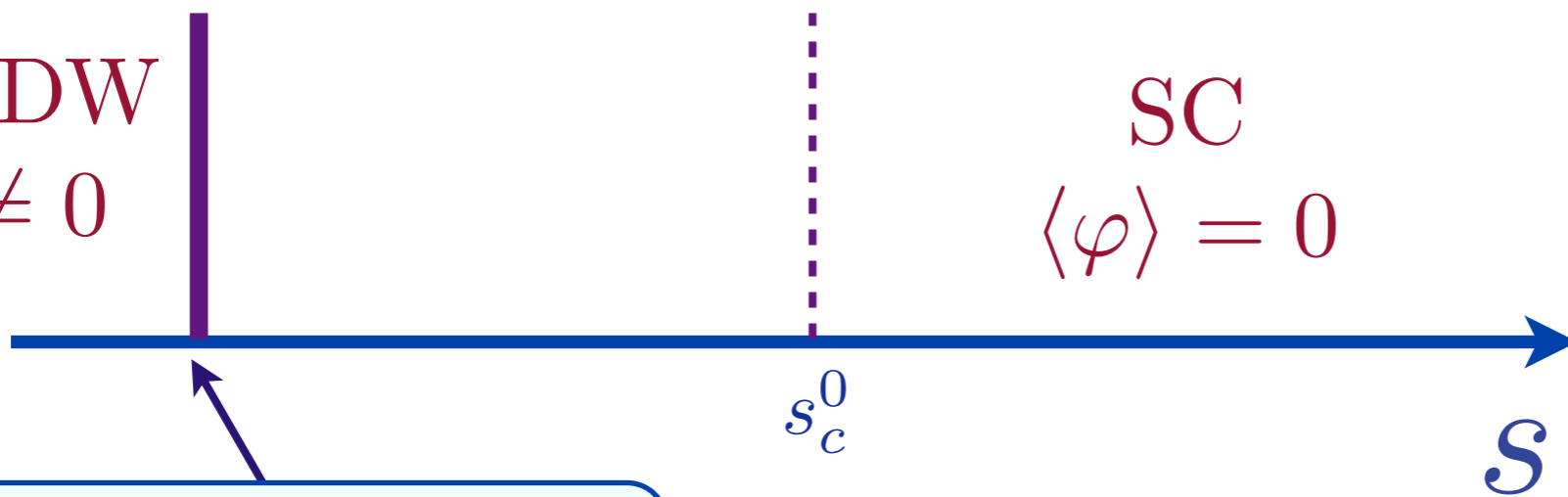
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No SC
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SC+SDW
 $\langle \varphi \rangle \neq 0$

SC
 $\langle \varphi \rangle = 0$

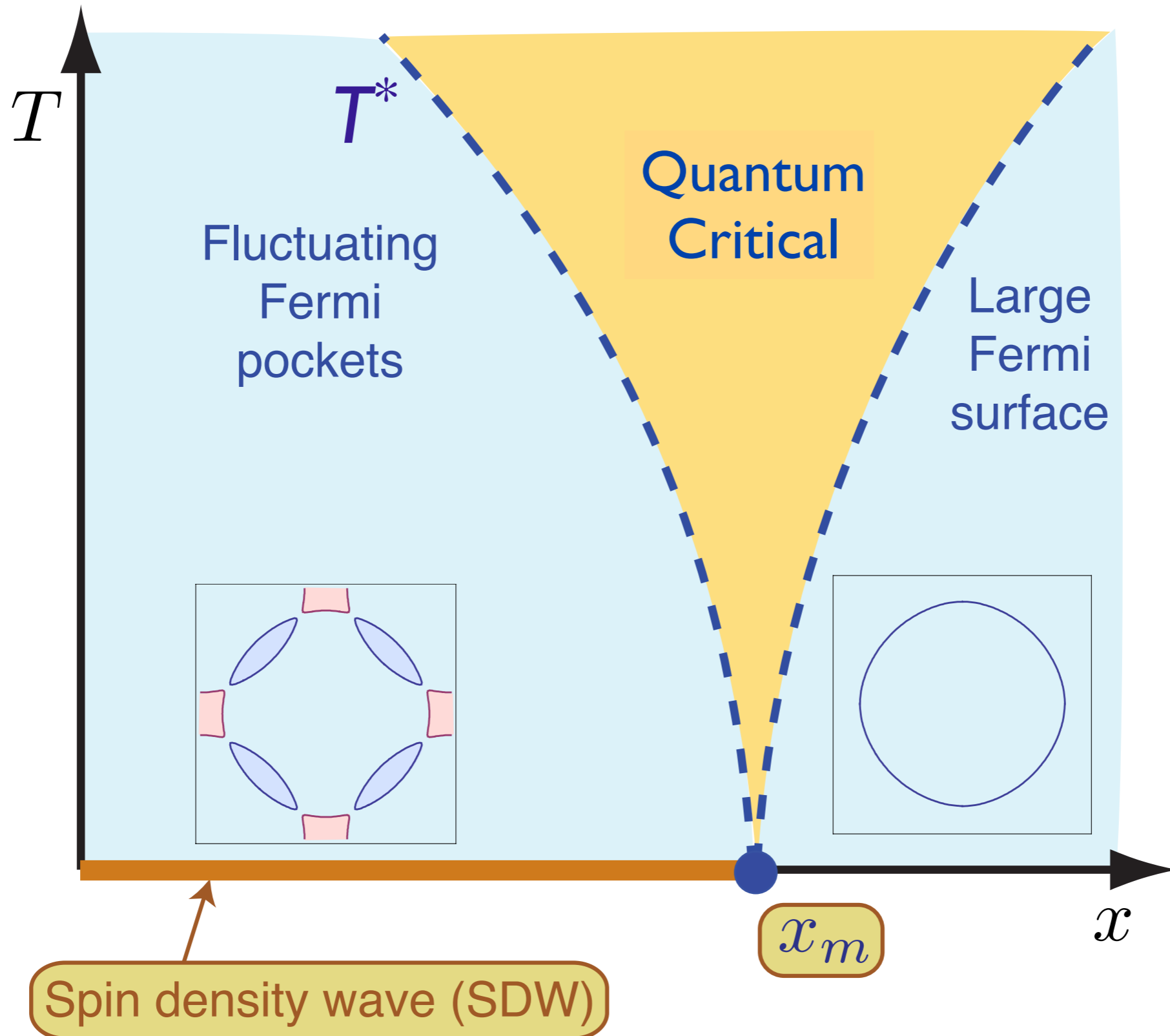


SC
 $\Delta \neq 0$

$$s_c^0 - s_c \sim |\Delta|$$

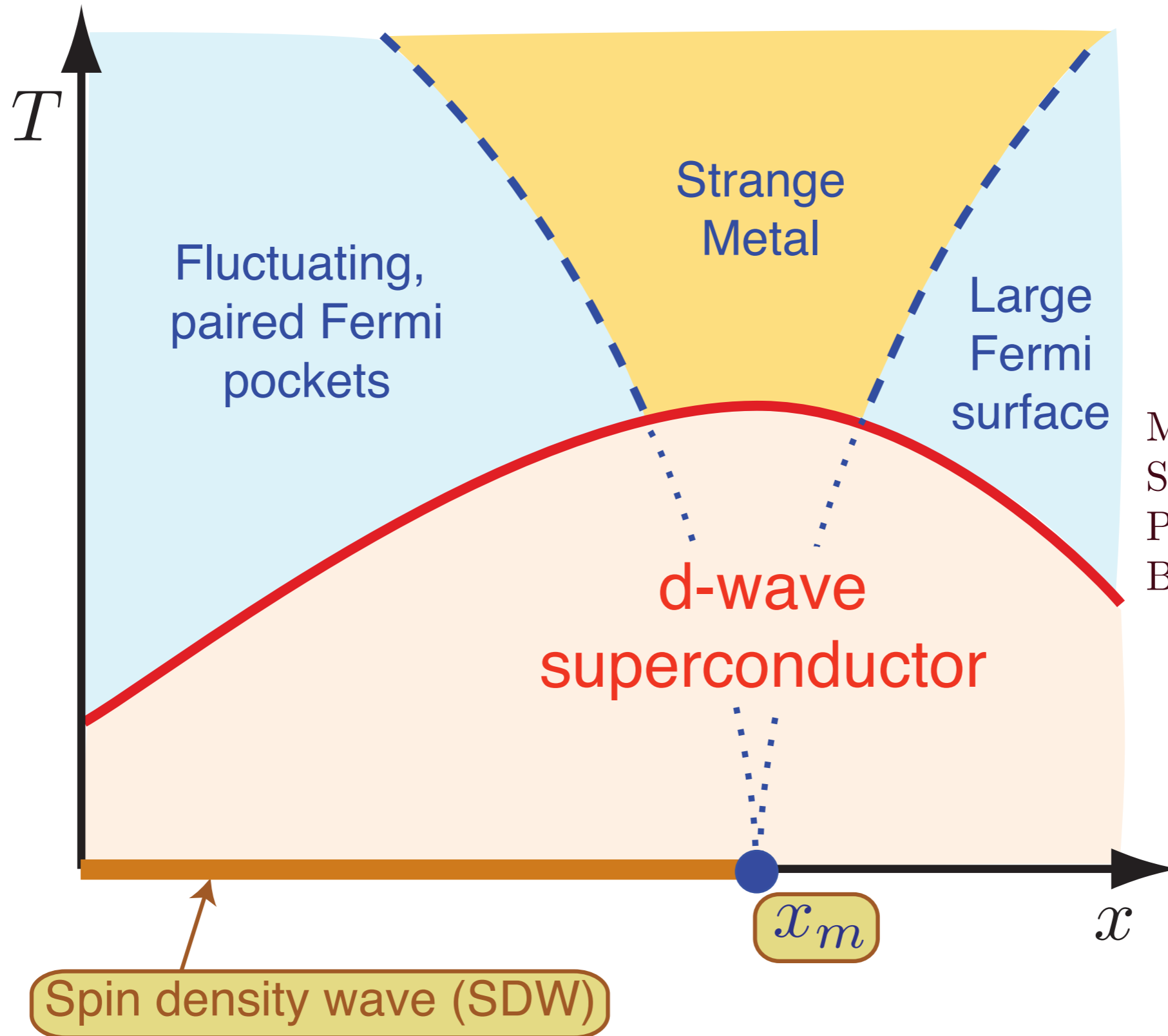
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Theory of quantum criticality in the cuprates



Relaxation and equilibration times $\sim \hbar/k_B T$ are robust properties of strongly-coupled quantum criticality

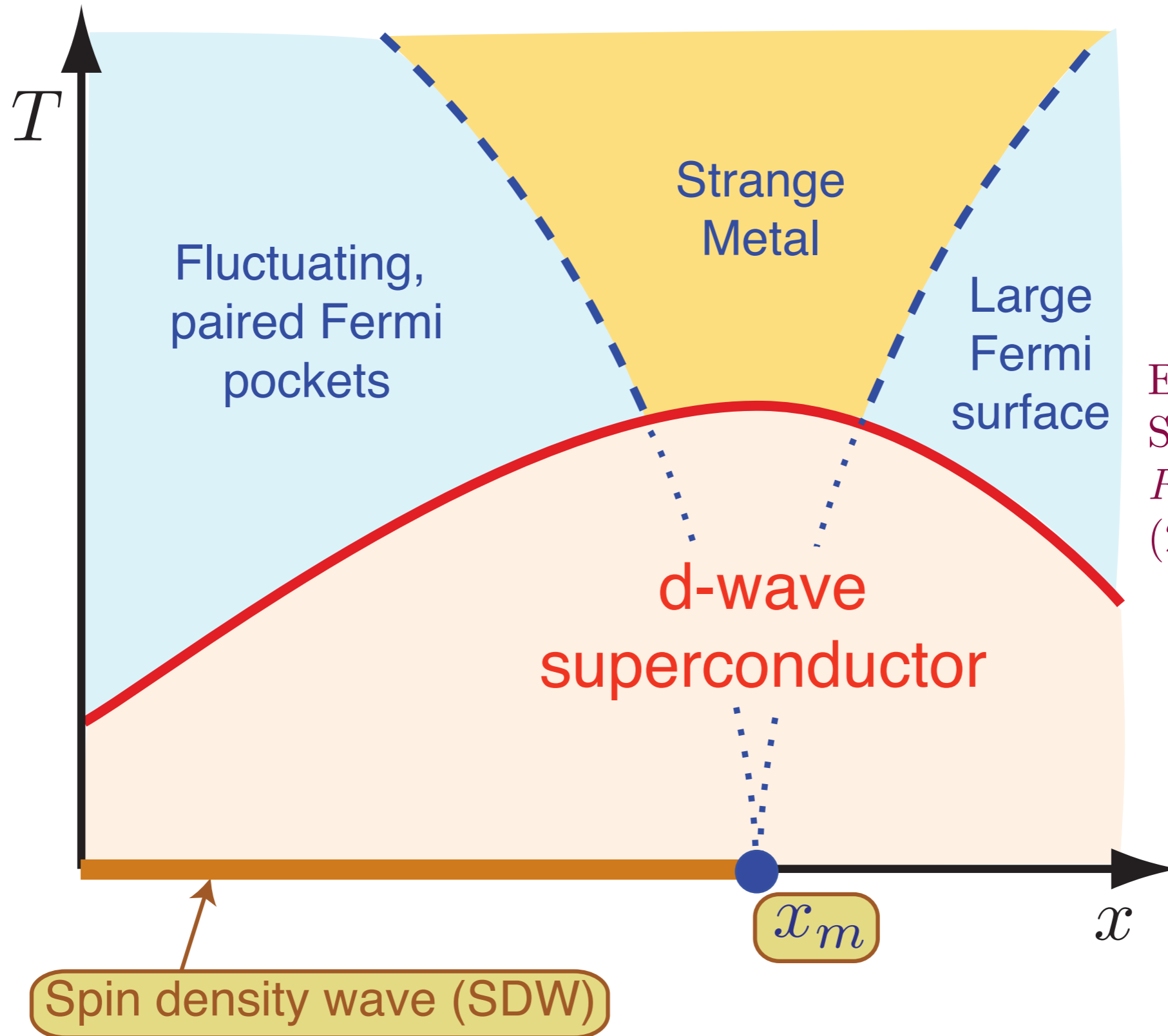
Theory of quantum criticality in the cuprates



M. A. Metlitski and
S. Sachdev,
Physical Review
B **82**, 075128 (2010)

SDW quantum critical point is unstable to d -wave superconductivity
This instability is stronger than that in the BCS theory

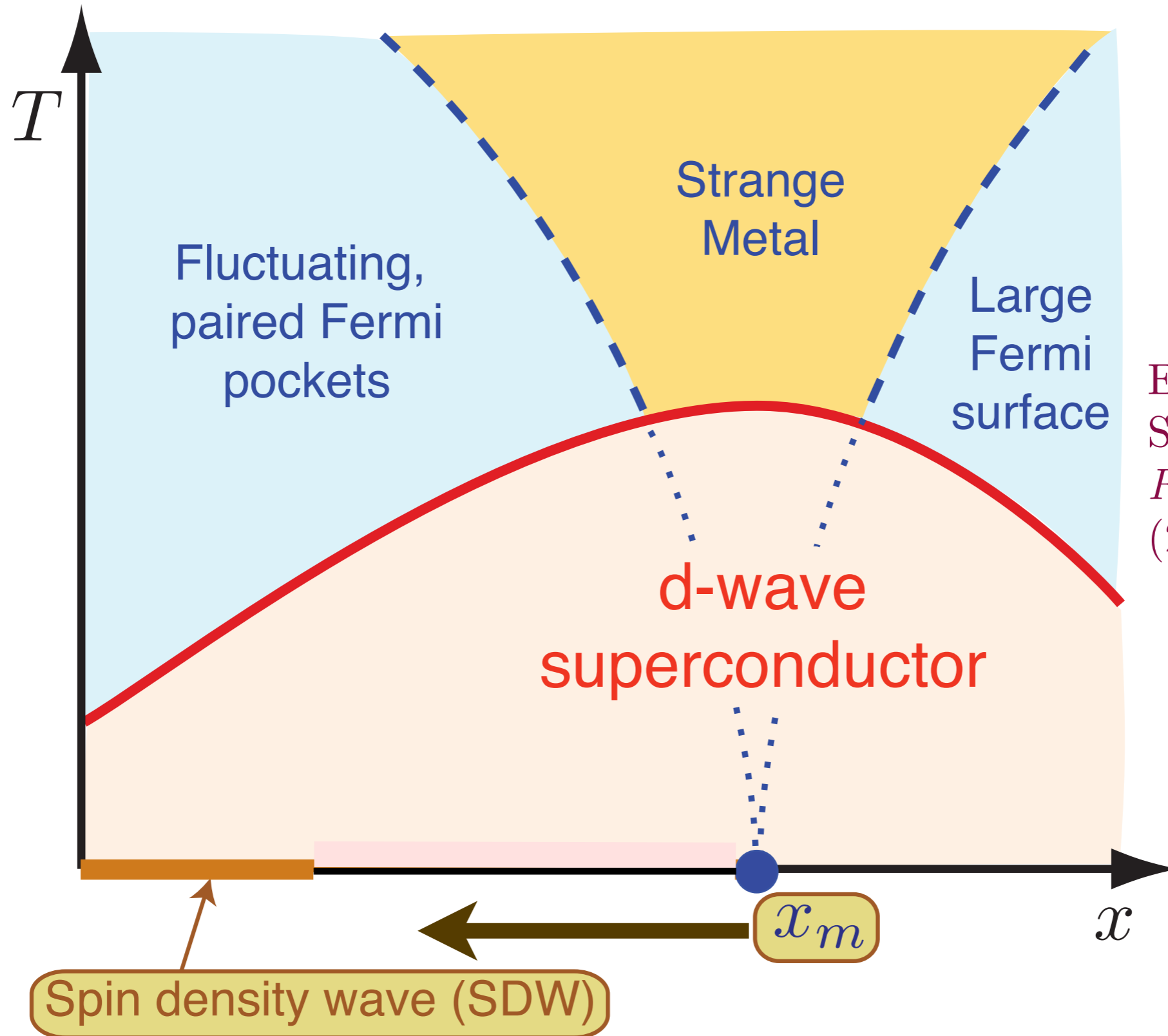
Theory of quantum criticality in the cuprates



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Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

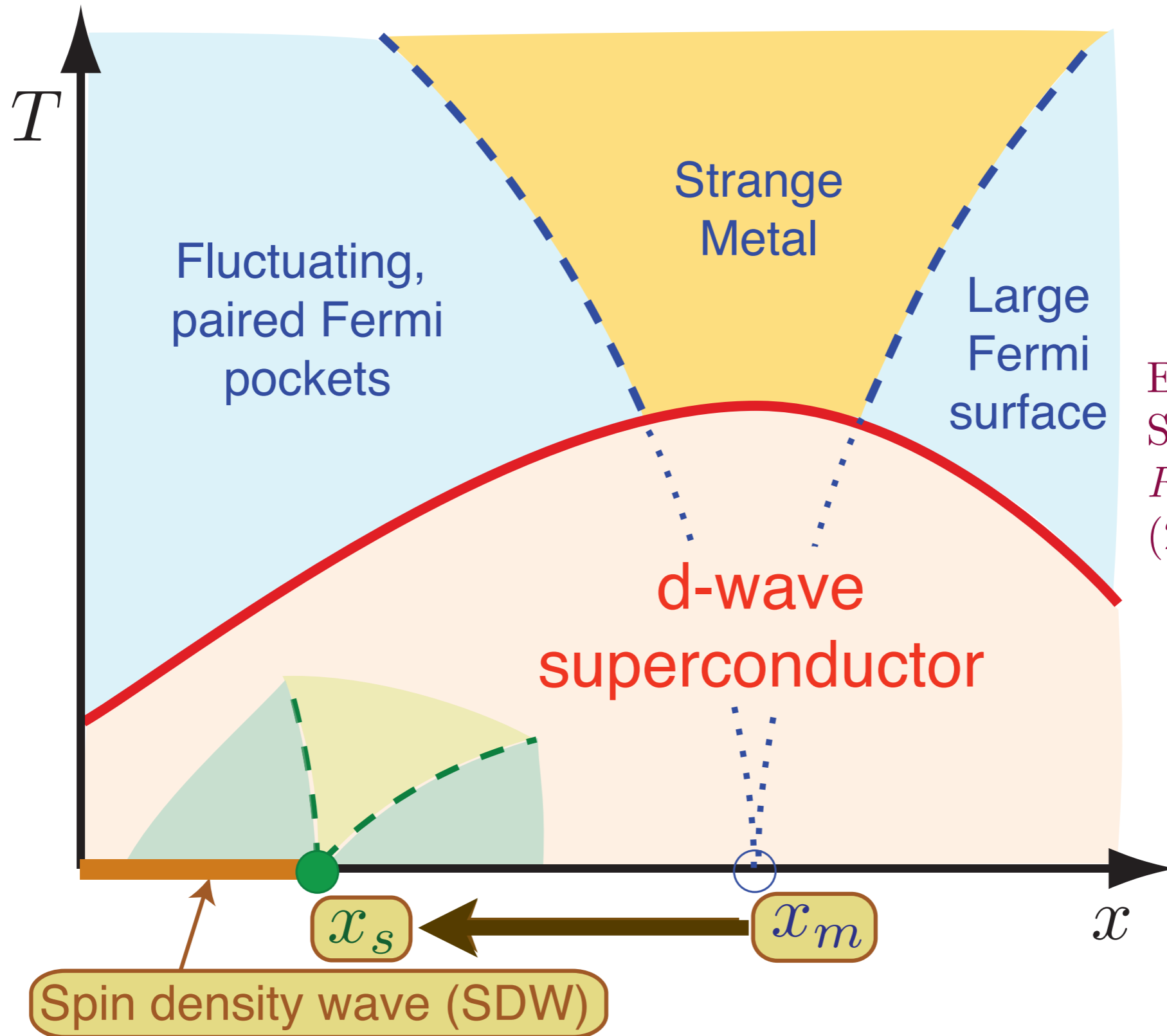
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Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*
- *If so, why is there no antiferromagnetism in the hole-doped cuprates near the point where the superconductivity is strongest ?*
- *What is the physics of the strange metal ?*

Questions and answers

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Yes

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Competition between antiferromagnetism and superconductivity has shifted the antiferromagnetic quantum-critical point (QCP), and shrunk the region of antiferromagnetism. This QCP shift is largest in the cuprates

● *What is the physics of the strange metal ?*

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Proposal: strongly-coupled quantum criticality of Fermi surface change in a metal